

SUM RATE MAXIMIZATION FOR HYBRID RELAY-RIS-ASSISTED MU-MISO SYSTEMS: MULTIPLE ACCESS TECHNIQUES

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Sum rate maximization for hybrid relay-RIS-assisted MU-MISO
systems: multiple access techniques

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September 2024

We certify that we have read this thesis and that in our opinion it is fully adequate,
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ABSTRACT

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M.S. in Electrical and Electronics Engineering

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Space Division Multiple Access (SDMA) is a pivotal multiple access technique in modern wireless communication systems. Additionally, Reconfigurable Intelligent Surfaces (RIS) is recognized as fundamental technology for next-generation wireless communications. This work investigates a multiuser downlink Multiple-Input Single-Output (MISO) system with an underloaded or critically loaded network. Here, a multiantenna base station (BS) communicates with multiple single-antenna users by leveraging a combination of a half-duplex decode-and-forward (DF) relay and a full-duplex RIS. In this study, the aim is to maximize the sum rate by joint design of active beamforming at the BS and the DF relay in addition to passive beamforming at the RIS under maximum power constraints, minimum SINR constraints at the relay, and unit-modulus constraint for the RIS elements. The complex design problem is addressed using Lagrangian Dual Transformation (LDT), Quadratic Transformation (QT), and Semidefinite Relaxation (SDR). Also, an alternating optimization algorithm is proposed. In the context of performance evaluation, benchmarks such as Non-Orthogonal Multiple Access (NOMA) and Orthogonal Multiple Access (OMA) are considered. Furthermore, Particle Swarm Optimization (PSO) is utilized for RIS phase profile optimization as a benchmark for the model-based proposed algorithm. This comparative analysis provides insights into the effectiveness of the proposed SDMA-enabled hybrid RIS-Relay communication system and the efficiency of the metaheuristic PSO algorithm for large sizes of the RIS.

Keywords: multiple access, reconfigurable intelligent surface (RIS), decode-and-forward relay, alternating optimization, metaheuristic methods.

ÖZET

HİBRİT RÖLE-RİS DESTEKLİ MU-MISO SİSTEMLERİ İÇİN TOPLAM VERİ ORAN MAKSİMİZASYONU: ÇOKLU ERİŞİM TEKNİKLERİ

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Uzay bölmeli çoklu erişim (SDMA), modern kablosuz iletişim sistemlerinde önemli bir çoklu erişim tekniğidir. Ayrıca, yeniden yapılandırılabilir zeki yüzeyler (RIS), gelecek nesil kablosuz iletişim için temel teknoloji olarak tanınmaktadır. Bu çalışma, bir alt yüklü veya kritik yüklü bir ağla çok kullanıcı bir alıcı-yönlendirme çoklu giriş tek çıkış (MISO) sistemi araştırmaktadır. Burada, çok antenli bir baz istasyonu (BS), yarı çift yönlü çöz-ve-ilet (DF) röle ve tam çift yönlü RIS kombinasyonunu kullanarak birden fazla tek antenli kullanıcı ile iletişim kurar. Bu çalışmada, BS’de aktif ışın formu ve DF röle ile birlikte RIS’de pasif ışın formunun en fazla güç kısıtlamaları altında, rölede minimum SINR kısıtlamaları ve RIS elemanları için birim-modül kısıtlaması ile birleşik tasarımıyla toplam hızı maksimize etmek amaçlanmıştır. Karmaşık tasarım problemi lagrange çift dönüşümü (LDT), kare dönüşüm (QT) ve yarı-sürekli gevşeme (SDR) kullanılarak ele alınmıştır. Ayrıca, bir sırayla optimizasyon algoritması önerilmektedir. Performans değerlendirmesi bağlamında, NOMA ve OMA gibi kıyaslama noktaları dikkate alınmaktadır. Ayrıca, RIS faz profili optimizasyonu için model tabanlı önerilen algoritmanın bir referans olarak kullanılması için parçacık sürü optimizasyonu (PSO) kullanılmaktadır. Bu karşılaştırmalı analiz, önerilen SDMA-destekli hibrit RIS-Röle iletişim sistemi ve büyük RIS boyutları için meta-sezgisel PSO algoritmasının etkinliği hakkında içgörüler sağlar.

Anahtar sözcükler: çoklu erişim, yeniden yapılandırılabilir akıllı yüzey, çöz ve ilet rölesi, sırayla optimizasyon, meta-sezgisel yöntemler.

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List of Abbreviations

AF Amplifier and Forward

AO Alternating Optimization

AoD Angle of Departure

AoA Angle of Arrival

B5G Beyond 5G

BS Base Station

CDMA Code Division Multiple Access

CF Compress and Forward

CSI Channel State Information

DF Decode and Forward

FDMA Frequency Division Multiple Access

FP Fractional Programming

IoT Internet of Things

LoS Line of Sight

MIMO Multiple Input Multiple Output

MU-MISO Multi User Multiple Input-Multiple Output

NLoS	Non Line of sight
NOMA	Non-Orthogonal Multiple Access
OFDMA	Orthogonal Frequency Division Multiple Access
OMA	Orthogonal Multiple Access
PSD	Positive Semidefinite
PSO	Particle Swarm Optimization
QT	Quadratic Transform
RIS	Reconfigurable Intelligent Surfaces
SDR	Semidefinite Relaxation
SDMA	Space Division Multiple Access
SIC	Successive Interference Cancellation
SDP	Semidefinite Programming
SINR	Signal to Interference and Noise Ratio
SVD	Singular Value Decomposition
SROCR	Sequential Rank-One Constraint Relaxation
TDMA	Time Division Multiple Access

List of Symbols

ω_k	BS beamforming vector for user k in phase I
$\mathbf{v}_k$	Relay beamforming vector for user k in phase II
Φ_1	RIS phase shift matrix in phase I
Φ_2	RIS phase shift matrix in phase II
M	Number of antennas at the BS
L	Number of antennas at the DF relay
K	Number of single-antenna users
$\mathcal{L}$	Set of antennas at the DF relay ($ \mathcal{L} = L$)
$\mathcal{K}$	Set of users ($ \mathcal{K} = K$)
$\mathbf{H}_{\text{BS,R}}$	Channel matrix between BS and relay
$\mathbf{h}_{\text{BS},k}$	Channel matrix between BS and user k
$\mathbf{h}_{\text{R},k}$	Channel matrix between relay and user k
γ_k^{I}	received SINR/SNR at user k in phase I
γ_k^{R}	received SINR/SNR at relay for user k
γ_k^{II}	received SINR/SNR at user k in phase II
α	Path loss exponent in rayleigh-fading channels
PL	Path loss between transmitter and receiver
PL_0	Reference path loss at $d_0 = 1\text{m}$
σ_{I}^2	Variance of AWGN at user in phase I
σ_{R}^2	Variance of AWGN at the relay
σ_{II}^2	Variance of AWGN at user in phase II
s_k	Data stream of user k

$\mathbb{E}$	Expectation value
$\mathbf{I}_{(K)}$	$K \times K$ identity matrix
$\gamma_{\text{R}}^{\text{th}}$	Minimum SINR threshold at the relay
$P_{\text{BS}}^{\text{max}}$	Maximum transmit power constraint at BS
$P_{\text{R}}^{\text{max}}$	Maximum transmit power constraint at the relay
δ^0	Initial step size in SROCR
θ_{SIC}	SIC constraint parameter
ϵ	Convergence threshold in alternating optimization
ϵ_1	Convergence threshold in SROCR
n_{max}	Maximum number of iterations in alternating optimization
$R_T^{(n)}$	Total achieved rate at iteration n
$\succcurlyeq 0$	Positive semi-definite
ω_{max}	Maximum inertia weight
ω_{min}	Minimum inertia weight in PSO
p_{best}	Personal best in PSO
g_{best}	Global best in PSO
$f(\boldsymbol{\theta})$	Fitness function in PSO
$(\cdot)^*$	Optimal or suboptimal value
$(\cdot)^*$	A complex conjugate of a complex number
$\text{tr}(\cdot)$	Trace of a matrix
$\mathcal{CN}(\cdot, \cdot)$	Complex Gaussian distribution
$(\cdot)^H$	Hermitian transpose
$\text{tr}\{\cdot\}$	Trace of a matrix
$\text{diag}(\cdot)$	Diagonal matrix
$\bar{\mathbf{u}}(\cdot)$	Eigenvector corresponding to the maximum eigenvalue
$\lambda_{\text{max}}(\cdot)$	Maximum eigenvalue
$\otimes$	Kronecker product
$\diamond$	Khatri-Rao product

Chapter 1

Introduction

Over the last three decades, the rapid development of wireless communications has led to an enormous increase in global data transmission rates, enabling the support for the ever-growing number of interconnected devices [1–3]. One of the main challenges in wireless network evolution is the uncontrollable radio environment with reflections and scattering. Recently, reconfigurable intelligent surfaces (RIS) has emerged as a promising technique for enhancing wireless signal propagation [4–6]. Furthermore, by increasing the number of devices and users, various multiple access techniques have been introduced. These techniques allow multiple users to share a communication medium efficiently by allocating physical resources, such as frequency channels, time slots, transmission power, etc. Also, they fulfill the diverse requirements of a massive number of users in terms of spectral efficiency, reliability and quality of service, efficient utilization of radio resources, etc. In recent investigations, there has been a growing interest in enhancing the performance of wireless communication systems through the integration of these multiple access techniques with relay-assisted systems [7–10], and RISs [11–13]. This thesis aims to investigate and compare the efficacy of the space division multiple access (SDMA) method and compares its performance with the non-orthogonal multiple access (NOMA), and orthogonal multiple access (OMA) approaches in maximizing the downlink sum rate of hybrid RIS and decode and

forward (DF) relay assisted multiuser MISO systems with *underloaded* and *critically loaded* network, where the total number of antennas on the receiver side is smaller than or equal to the number of antennas at the transmitter.

1.1 Background

Orthogonal Multiple Access (OMA): OMA is a multiple access method with frequency, time, and code domain sharing among users, thus preventing the interference of transmissions from different users. These techniques are known as Frequency Division Multiple Access (FDMA) for 1G [14], Time Division Multiple Access (TDMA) for 2G [14, 15], and Code Division Multiple Access (CDMA) for 3G [14, 16, 17], respectively. In addition, a more robust technique, known as Orthogonal Frequency Division Multiple Access (OFDMA) for 4G [18, 19], combines time and frequency domains and was incorporated into LTE (Long Term Evolution). These techniques are shown in Fig. 1.1 in their sharing domains [20].

Non-Orthogonal Multiple Access (NOMA): NOMA has been introduced in the 5G wireless networks [21–23]. As shown in Figs. 1.1 and 1.2, in the NOMA technique, multiple users share the same time-frequency resource block, and their signals overlap each other. Users' signals can be distinguished through power domain multiplexing, where signals have different power allocation levels. The receiver uses successive interference cancellation (SIC) or other advanced signal processing techniques to decode individual user signals by eliminating interference from other users, enabling simultaneous decoding of multiple packets that are transmitted to the receiver. In SIC, the receiver first decodes the strongest signal and subtracts it from the combined received signal. Sequentially, the remaining signal, representing weaker signals, is decoded. This process continues until all signals are decoded, as presented in Fig. 1.3.

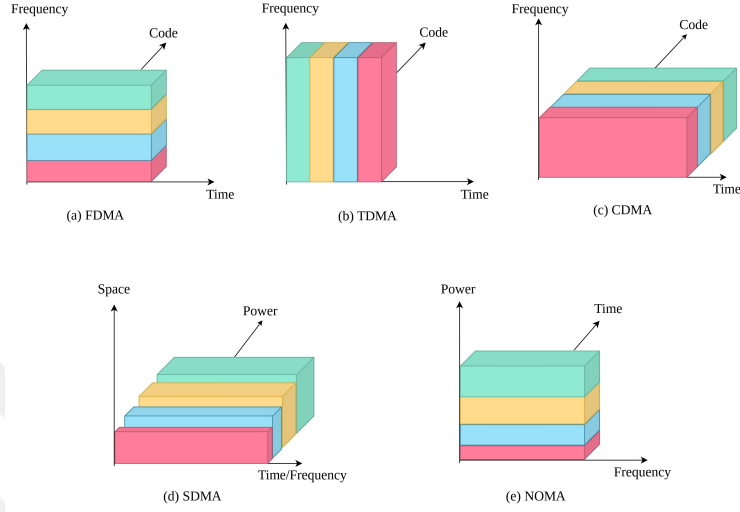


Figure 1.1: Multiple access schemes

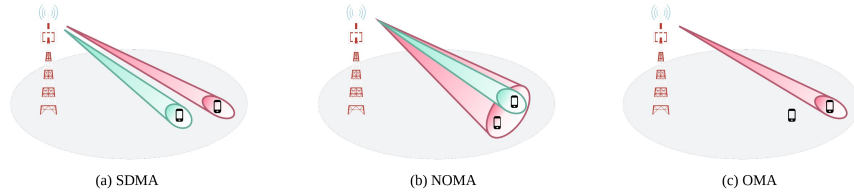


Figure 1.2: Multiple access techniques in downlink transmission

Space Division Multiple Access (SDMA): SDMA is a multiple access technique in wireless communication systems which enables multiple users to send or receive at the same time over the same frequency band by using directed spatial dimension beams, thus enhancing the capacity of the system [24, 25]. SDMA requires the utilization of multiple antennas on either the transmitter, receiver, or both sides to separate users in the spatial domain, as opposed to other traditional multiple access techniques such as FDMA or TDMA, which allocate users to different frequencies or time slots, respectively [25]. Another important aspect of the SDMA technique, in addition to its multiple access capacity, is increasing spectral efficiency by enabling frequency reuse within a particular cell [26, 27]. Since higher frequencies and a large number of antennas at the BS and/or receivers are used in 5G networks, focusing spatial signals for transmission

is easier, which makes the SDMA technique more appropriate for 5G networks [25, 27, 28].

There are some advantages of SDMA over NOMA in underloaded networks:

- In NOMA, increasing the number of users leads to an increase in the receivers' complexity, such as more SIC layers at the receivers, and increased power consumption. For instance, in a network with K users, the user with the strongest channel gain requires $(K - 1)$ SIC layers in order to decode the messages of the weaker users and remove the interference. This inefficient use of SIC results in the sum Degree of Freedom (DoF) loss [29–31]. However, SDMA can achieve better sum-DoF, and since the linear precoding approach is utilized, the complexity of both transmitter and receiver is low. This is because the receivers decode their intended messages directly by fully treating any residual multi-user interference.
- Since decoding the message of a specific user depends on the messages of the users with lower channel gains, SIC error in one of the users can lead to information loss for the remaining undecoded messages. Therefore, the number of users being served by each cell is restricted by this constraint.

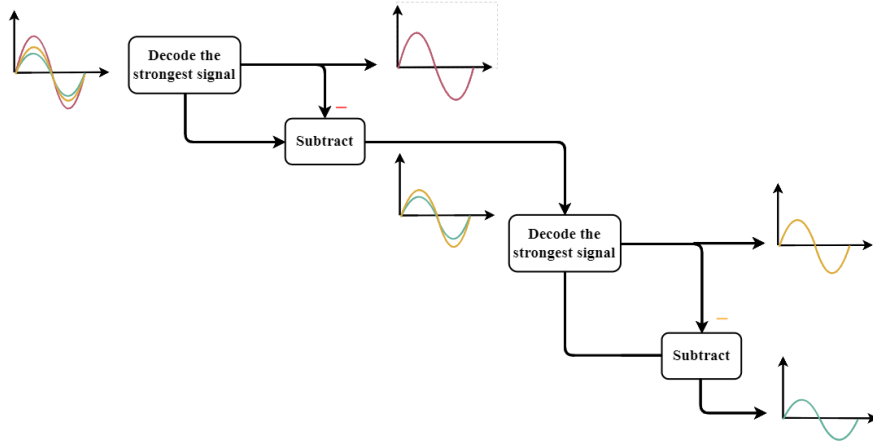


Figure 1.3: Successive Interference Cancellation (SIC)

1.2 Motivation

- This work aims to investigate joint beamforming in hybrid relay–RIS systems. In related work such as [32–38], the general emphasis is on hybrid RIS and relay network, as well as the impact of RIS on the system and performance of hybrid RIS and DF relay. Meanwhile, in this study, we focus on designing the active and passive beamforming utilizing different 4G and 5G multiple access techniques and comparing them under different scenarios.
- In the model-based optimization methods described in the literature, the suboptimal solution is obtained by applying an eigenvalue decomposition or Gaussian randomization for the CVX solution. However, these approaches may lead to a large optimality gap. In this study, by employing the SROCR rank reduction method, we reduce the probable optimality gap between the global optimal and the suboptimal solution of the SDR obtained by the CVX.
- Obtaining a solution for large sizes of RIS is not possible in the related studies that use optimization toolboxes. In this study, we design a constrained metaheuristic PSO method, which is able to obtain a near optimum solution and is able to design the RIS phase profile for large sizes with low computational complexity.

1.3 Contributions

The sum rate maximization in a hybrid RIS and DF relay assisted MU-MISO systems has been investigated in various studies in the literature [32, 33, 36]. In these studies, different reformulation and approximation methods are utilized in order to reformulate the non-convex sum rate maximization problem. The main focus of the aforementioned studies is to maximize the sum rate under certain constraints, and to the best of our knowledge, there exists no work in the literature

which explores different multiple access techniques for signal transmission in a hybrid RIS and DF relay assisted system.

In this thesis, we have investigated active and passive beamforming design in order to maximize the downlink sum rate in hybrid RIS-DF relay-assisted multiuser MISO systems. In addition, three multiple access techniques are considered for the transmission and compared. Moreover, two model-based and metaheuristic algorithms are studied to optimize the RIS phase shift profile.

The main contributions of this thesis are as follows:

- For the first time in the literature, we have investigated different multiple access techniques for the sum rate maximization in a hybrid RIS-relay-aided system with an *underloaded* and *critically loaded* network. We have shown that the SDMA technique outperforms the NOMA and OMA techniques since BS can adjust the spatial beamforming vectors better when the number of antennas at the BS is larger than or equal to the total number of antennas at the users.
- To handle the non-convex optimization problem, we developed an algorithm based on fractional programming (FP), lagrangian dual transformation (LDT), and quadratic transformation (QT). To be specific, the original non-convex sum rate maximization problem is first reformulated as a more solvable one with the introduction of auxiliary variables by exploiting the LDT and QT. Subsequently, the equivalent problem is decomposed into four subproblems by the alternating optimization (AO) framework, where the optimization variables are optimized iteratively until convergence is achieved. In addition, the semidefinite relaxation (SDR) method is used to transform the subproblems into convex formulations by relaxing rank-one constraints. Consequently, a rank reduction method named sequential rank-one constraint relaxation (SROCR) is utilized to obtain the closest rank-one solution and minimize the optimality gap.
- The proposed model-based algorithm obtains the suboptimal solution using the CVX convex optimization toolbox. However, this toolbox is capable

of handling the problems up to a certain number of optimization variables. Therefore, to address this size limitation in the CVX toolbox, a metaheuristic particle swarm optimization (PSO) algorithm is provided to design the RIS phase profiles. Although the metaheuristic algorithm is not always able to find the optimal solution and faces infeasibility when the number of constraints increases, such as in the NOMA case, it is much simpler than the model-based algorithms, and it can be applied to large sizes of RIS. Also, the PSO method demonstrates comparable performance to the model-based algorithm, particularly in the proposed SDMA technique.

- Extensive simulations under various system parameters are conducted to validate that the performance is enhanced with the assistance of RIS, DF relay, and the hybrid RIS-DF relay and that the sum rate achieved by the proposed SDMA method is superior to other baselines.

1.4 Thesis outline

The remainder of the thesis is structured as follows. In Chapter 2, the literature focusing on multiple access techniques, RIS and relays are studied. Then, the optimization methods and related work are discussed. Chapter 3 provides a description of the system model for a DF relay aided MU-MISO system and introduces the problem formulation. Also, the rate expressions and transmission phases are discussed analytically. In addition, the proposed approaches for the active beamforming vectors' optimizations for both phases are provided for the multiple access techniques. In Chapter 4, an RIS is added to the previous system, and the sum rate maximization problem is formulated for a hybrid RIS and relay aided system. Also, the passive beamforming design for the RIS in both transmission phases is discussed. Chapter 5, presents various numerical results for investigating the performance of the proposed approaches in different scenarios. Finally, Chapter 6 provides conclusions and recommendations for future research.

Chapter 2

Literature review

In this chapter, we start with exploring the space division multiple access and its integration with other multiple access techniques. Following this, we discuss the relay-aided system, investigating various types of relays and their functions. Next, we survey studies on reconfigurable intelligent surfaces, their working principles, and potential applications. Lastly, the optimization algorithms and rank reduction methods are presented.

2.1 Space Division Multiple Access

SDMA is a multiple access technique in wireless communication systems that enables serving several users simultaneously over the same frequency band by using the spatial dimension, and the superposition of the beam [24, 39–41]. In the SDMA method, the user decodes the intended message and treats any residual interference as noise. This technique increases the system capacity by accommodating several users within one frequency band. In addition, it enhances spectral efficiency by spatially separating the users, leading to interference reduction. This

technique plays a key role in enabling the next generation of wireless communication systems to support the high-speed and reliable connectivity required in diverse applications and scenarios. According to the previous studies in [29, 30, 42], SDMA outperforms the NOMA and OMA techniques in underloaded regimes. Additionally, the complexity of the transmitter and receiver is lower than that of the NOMA since no SIC is required in SDMA.

In [43], the authors investigate the effect of scheduling of energy harvesting (SEH) on the optimization of minimum length scheduling for SDMA uplink transmission compared to TDMA uplink transmission. In this study, the aim is to minimize the total downlink energy and uplink information transmission time by optimizing the hybrid beamforming vectors and transmission time in the downlink, and transmit power and time for the uplink. In [44], the downlink performance of mmWave Systems under the assumption of a limited scattering environment are examined, and the throughput for each BS beam is approximated.

Also, SDMA for VLC transmission is considered in [45], and the spectral efficiency is studied and has shown that the SDMA outperforms TDMA. In addition, [46] investigates the sum rate maximization for the MU-MIMO VLC systems under a limited dynamic range with two precoding methods, zero-forcing (ZF) and singular value decomposition (SVD). Also, the results show that the achievable sum rate by SDMA performs better than OFDMA for both precodings. Similarly, in [47], the achievable sum rate maximization is studied for VLC-MIMO systems with distributed user grouping, and the superiority of SDMA over conventional OFDMA is demonstrated.

2.1.1 Combination of SDMA with other MA techniques

SDMA can also improve the performance of the system by being combined with other MA techniques such as TDMA, OFDMA, and NOMA. There are a number of studies on the combination of NOMA and SDMA in the literature. These studies have shown that there would be a notable loss of multiplexing gain in

case NOMA is integrated into multi-antenna networks improperly [31].

In [48], the outage characteristic of a downlink SDMA-NOMA single cellular network within a comprehensive capacity-limited feedback scenario is studied. The authors suggest a user dynamic scheduling and grouping strategy, where users are arranged in separate groups. From SDMA, every group uses individual beams, but multicasting, such as with users in the same cluster, is served simultaneously using NOMA. In [49], the authors design the NOMA beamforming within an SDMA legacy system, where primary and secondary users are served by SDMA and NOMA, respectively. In this research, two beamforming strategies, named leveraging existing SDMA beams or forming new beams, are investigated and compared in order to acquire different trade-offs between the system performance and the complexity. Furthermore, as an extension of [49] and by utilizing the same ideas, [50] investigates a massive MIMO downlink network, where multiple prearranged spatial beams are utilized for near-field users, whereas NOMA is applied to a certain number of additional far-field users. In [51] and [52], a hybrid SDMA and NOMA technique is examined in a downlink mmWave communication system, taking multiuser precoding and interference mitigation issues into account. In [53] and [54], the Reconfigurable Intelligent Surface (RIS) aided downlink transmission is studied. In these investigations, the hybrid NOMA and SDMA technique is designed, and the outage performance of the system is analyzed. Also, in [55], a joint SDMA and NOMA method is designed for an uplink MISO system with imperfect channel state information (CSI), and the outage performance of the system is studied.

2.2 Relay-assisted systems

in the modern communication era, traditional direct source-destination communication has transformed into source-relay-destination communication. In this new paradigm, the role of the relay can vary depending on the mode of operation, and this variation can significantly impact the total achievable rate of the system. Furthermore, in half-duplex relay-assisted communication systems, the

resource allocated for each phase of the relay-assisted transmission significantly influences the total achievable rate [56]. Generally, in relay-aided systems, the signal received at a relay is first processed and then forwarded and re-transmitted to the next relay or destination node. Although there are many signal-relaying systems, the most common ones are the amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) relay systems. The selection of a relay strategy depends on different factors, such as channel conditions, system requirements, and implementation constraints [56–58].

2.2.1 Amplifier and Forward relays

In non-regenerative systems, the intended signal is first received by the relay node, and without encoding, it is amplified by the relay and forwarded to the next node, which can be another relay or the receiver [59]. The AF relay is usually considered to be simpler to implement than DF relaying systems since it does not require decoding or encoding capabilities. Hence, it is better suited for wireless applications that require simple relay units. Meanwhile, the AF relay may introduce additional noise and interference due to the amplification process, potentially limiting its performance compared to the DF relay, especially in scenarios with unfavorable channel conditions [60, 61].



Figure 2.1: Non-regenerative Amplifier and Forward (AF) relay

2.2.2 Decode and Forward relays

In regenerative systems, the relay node receives the signal transmitted from the source, decodes it, and then re-encodes and forwards it to the receiver or the next

relay [56, 57, 61]. This relaying strategy is called decode-and-forward (DF) relaying, and in this relay, the performance of the system depends on the successful decoding of the message. In addition, containing decoding capabilities adds more complexity to the system in comparison to AF relays, but as a trade-off, it can improve the reliability and throughput of the system [56].



Figure 2.2: Regenerative Decode and forward (DF) relay

2.2.3 Compress and Forward relays

In this strategy, the relay utilizes the Wyner-Ziv lossy source coding [62] on the estimated symbols of the received signal instead of decoding the data directly. Then, the relay re-transmits the compressed signal to the destination. Depending on the channel gains of the different links, CF may perform better than the DF strategy [56]. This approach is suggested in [63] (Theorem 6). However, due to the additional complexity introduced by this strategy to the system, it will not be taken into consideration in this investigation.

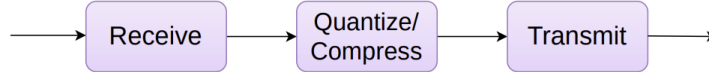


Figure 2.3: Compress and Forward (CF) relay

2.3 Reconfigurable Intelligence Surfaces (RIS)

Unprecedented growth in wireless data traffic and ever-increasing demand for highly secured, and low-latency wireless communication has drawn the attention of academic and industrial communities in order to move towards 6th generation

(6G) technology and the possible new applications and challenging requirements of future 6G and beyond wireless networks, such as enhancing the data rate, latency, and space-air-ground coverage, make The future of mobile communications exciting. With the advent of the modern era of wireless communications, the propagation medium has been considered a randomly acting entity between the transmitter and the receiver, and the quality of the received signal is degraded due to the uncontrollable interactions of the transmitted radio waves with the surrounding artifacts. Further, reconfigurable intelligent surfaces (RIS), also known as intelligent reflecting surfaces (IRS) or large intelligent surfaces (LIS), are flat structures that contain a vast number of passive reflecting elements, usually exposed on surfaces like walls, ceilings, or buildings, and with the potential to realize smart radio environments, it enables network operators in wireless communications to control the scattering, reflection, and refraction characteristics of the radio waves by overcoming the negative effects of environmental wireless propagation and recent results have demonstrated that RIS can effectively control the wavefront, e.g., the phase, amplitude, frequency, and even polarization, of the impinging signals without the need for complex decoding, encoding and radio frequency processing operations.

2.3.1 RIS advantages

The key advantages of RIS are listed as follows:

- **Easy implementation:** RISs can be deployed on several structures, including but not limited to building facades, indoor walls [64, 65], aerial platforms, roadside billboards, highway polls, vehicle windows, as well as pedestrians' clothes due to their low-cost. Furthermore, RIS can easily be designed into a variety of shapes to enhance the existing roof antenna, which can overcome the drawbacks of the traditional active antenna. For instance, RIS can be placed on the roof surface, which is easy to install without affecting the surface shape of the car. In addition, the shape installed to the roof surface is also easy to deploy RIS with a larger antenna aperture to

provide higher antenna gain [66].

- **Spectral and energy Efficiency:** RIS can reconfigure the wireless propagation environment and enhance the Spectral efficiency by overcoming the power loss over long distances and providing extra spatial diversity gain. [67–73]. In addition, since the power consumption of reflecting elements and RIS is low, it can result in the high energy efficiency of the system [74, 75]. Virtual line-of-sight (LoS) links between base stations (BSs) and mobile users can be formed by passively reflecting the incoming radio signals. This passive reflection becomes particularly beneficial when direct LoS links between BSs and users are blocked by obstacles such as high-rise buildings. Due to the intelligent deployment and design of RIS, a software-defined wireless environment may be constructed, potentially improving the received signal-to-interference-plus noise ratio (SINR) [65].
- **Cost efficiency:** Low-cost passive reflecting elements are employed in RIS and RIS does not need ADC/DAC and amplifier equipment as known in conventional relays. Furthermore, since RIS elements do not require active power sources or signal processing units, they have low power consumption. Hence, the cost efficiency of RIS arises from its simplicity and low power consumption.
- **Environmentally friendly:** Unlike traditional relay systems such as AF and DF relays, which rely on power amplifiers [76], RIS shape incoming signals by adjusting the phase shift of the reflecting elements [77, 78]. Thus, deploying RIS provides higher energy efficiency and environmental friendliness compared to conventional AF and DF systems [65, 79].

The aforementioned benefits and properties have made RIS an efficient solution for addressing various issues in commercial and civilian applications in 6G communication systems.

2.3.2 The working principle of RIS

In RIS-assisted systems, the incident radio waves are divided into two groups: transmitted wave and reflected wave. In this transmission, the incident wave is received by the unit cells, and they re-radiate it back into free space with a different amplitude and/or phase. Adjusting the electromagnetic characteristics of the unit cell enables this modulation. For instance, a switch placed at the feed of a patch antenna can change the flow of electric current and modulate the amplitude and phase of the re-radiated field. Employing multiple switches enables multi-bit wavefront modulation, enhancing beam control and efficiency. In addition, a biasing circuit and control unit (e.g., microcontroller) in the RIS systems is required in order to control the state of the adjustable devices by altering the biasing voltage across the device terminals [80, 81]. In the transmissions where the LoS direct link is blocked by obstacles, the receiver can receive the message via the reflected wave. In addition, in some systems, the distance between the transmitter and the receiver is large, and the received signal is weak at the user. In these cases, the performance can be enhanced by using the RIS, and the user will receive the message from both the direct link and the reflected one.

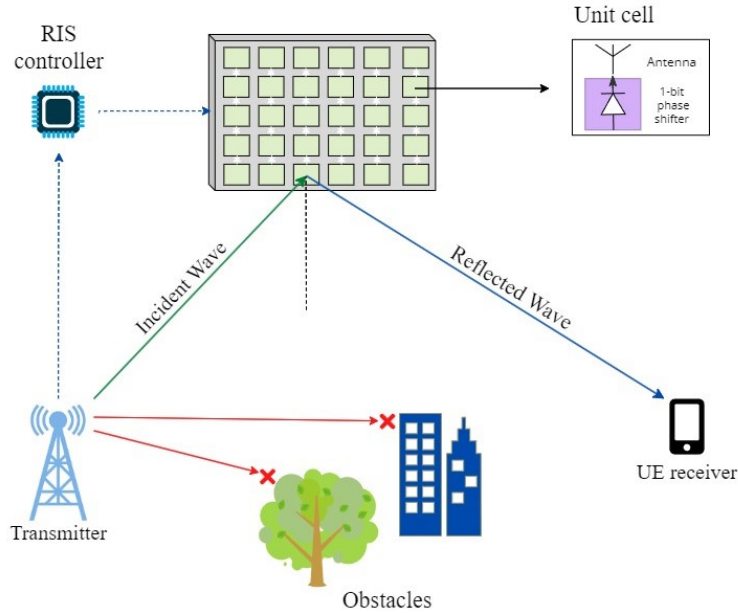


Figure 2.4: Working principle of RIS

2.3.3 Hybrid RIS-relay-assisted systems

One of the key limitations of RIS in wireless communication systems is the channel quality compared to the direct channel between the transmitter (BS) and the receiver (user). This limitation is due to the fact that the reflected channel is the product of the BS-RIS and the RIS-user channels. Thus, the reflected channel is often weaker than the LoS channel. One possible solution to address this issue is to replace the RIS with a relay node. Although the RIS and relay technologies are similar in forwarding signals, relays receive, amplify, and retransmit signals rather than reflecting them. Utilizing a relay node instead of an RIS would allow the system to transmit signals with higher strength and reliability. On the other hand, this technique has some drawbacks as well. Using a relay node increases energy consumption and complexity. Therefore, although the relay node could outperform RIS in certain cases, its trade-offs and consequences must be considered when designing the entire system.

There are numerous studies that compare the RIS and relaying technologies such as DF and AF [82–87]. The authors in [82] compare RIS- and DF-relay-assisted SISO systems and show that the RIS-assisted system with a sufficient number of reflecting elements is able to provide higher energy efficiency and data rate compared to the use of a DF-relaying system. This is due to the fact that an RIS consumes very little power to forward the signal, contrary to a relay. However, in [83], it is shown that an RIS-aided system cannot achieve a higher SNR than that in a MIMO DF-relay-assisted system for any number of reflecting elements and this comes at the cost of higher power consumption for relay-assisted system. This is due to the fact that the equivalent channel of the DF-relay is much better than the reflected channel caused by RIS. In [84], the maximization of the minimum SINR in a downlink MISO system is investigated. In this study, RIS and full-duplex/half-duplex AF relaying are compared. It is shown that RIS can outperform full-duplex and half-duplex AF relaying when the number of the RIS reflecting elements is sufficiently large. In addition, in [85], the energy efficiency of downlink MISO systems utilizing RIS or AF relay is investigated. It shows that RIS-assisted systems are able to provide up to 300% higher energy efficiency in

comparison with the AF relay-assisted ones. This large gain is due to the active power amplification of the AF relay and low power consumption at the RIS. Authors of [86] discuss similarities and differences between RIS and relays and investigate the energy efficiency of downlink RIS-assisted MISO systems. This study shows that RIS-assisted systems can outperform the AF relay-assisted ones in terms of data rate and spectral efficiency while reducing the implementation complexity when the RIS is sufficiently large. This is due to the fact that RIS operates in a full-duplex manner. However, unlike full-duplex relays, RIS do not suffer from loop interference issues. Furthermore, in [87], it has been shown that when the distance between the source and the destination is long, using relays has more advantages than the RIS. This is due to the aforementioned reason, which says the channel between the BS and the user in the RIS-aided system is subject to double fading effects, which are not present in the relay-aided systems.

Based on the aforementioned information, it can be concluded that the RIS-assisted system can outperform the AF relay-assisted system, while it performs worse than a DF relay-assisted system when the relay is equipped with massive antennas. Nevertheless, the RIS can also show better performance than the DF relay in terms of data rates or energy efficiency if the number of reflecting elements considered for the RIS structure is much larger than the number of antennas utilized at the relay [34, 35]. Generally, each of the DF relay and the RIS have their own advantages and disadvantages in terms of energy efficiency, hardware, and software complexity, etc., and we can leverage the benefits of both by combining these technologies in a system [34].

A hybrid relay–RIS system can have the following advantages:

- **Enhanced coverage and range:** The passive nature of RIS and the high path loss introduced by the double fading effects in the RIS reflection channel limits the service range of RIS-only systems [88]. Thus, relay forwarding can reduce the impact of large-scale path loss, and the service range could increase in hybrid relay–RIS systems. Also, using a relay can help to reduce the number of reflecting elements used in the RIS in order to achieve the same performance, and the communication coverage can also be improved

with appropriate deployment [32, 33, 38].

- **Channel estimation robustness:** RIS usually requires a sufficiently large number of reflecting elements to achieve comparable performance as DF relays [34]. However, utilizing a large number of RIS elements can lead to inaccurate and low-pilot-overhead channel estimation due to the high number of channel coefficients for the cascaded channel (BS–RIS–users), which results in high training overhead. Therefore, hybrid relay–RIS systems address this issue and facilitate robust channel estimation with low training overhead by utilizing active relays with more powerful signal processing capabilities instead of RIS with a large number of elements [32].
- **Energy efficiency:** Since the RIS has a passive nature, utilizing hybrid relay–RIS systems can be more energy-efficient since it requires a low transmit power at both the BS and relay, as compared to relay-only systems, and it can improve the achievable rate of the system as well [32, 34, 36]. Furthermore, the error probability is significantly reduced in comparison with either RIS-only or relay-only systems [37, 38].
- **Interoperability and compatibility:** Interoperability and compatibility refers to the ability of different systems or components to work together effectively and efficiently. In this context, hybrid relay–RIS systems offer technology interoperability by integrating the new RIS technology with the classical relaying technology [32].
- **Improved signal quality:** RIS can dynamically adjust signal reflections to optimize signal quality during transmission to the intended receiver, while relays can mitigate signal fading and interference. Combining these technologies can result in improved overall signal quality and reliability [33].

In order to integrate the advantages of RIS and relay, several authors have studied the cooperative communication system combining relays and RIS. Therefore, in this paper, we study a multi-user system employing both a DF relay and an RIS and optimize their parameters (beamforming and reflection phases) to maximize the multi-user system sum rate. Different multiple access techniques

have been considered in this study for the *underloaded and critically loaded* MU-MISO system, and model-based in addition to heuristic algorithms have been used in order to optimize the variables.

2.3.4 Potential applications of RIS in 6G and beyond

The RIS tunes the phase shift of the reflecting elements, and this enhances the power of desired signals compared to the signals from the direct path or mitigates the detrimental effects of multiuser interference. Therefore, utilizing RIS adds degrees of freedom, which improves the system's performance. In the following, future research directions for RIS in some 6G networks are presented.

1) RIS-aided THz and mmWave networks

The THz and mmWave networks have the potential to provide a high data rate due to their high bandwidth. However, one of the significant limitations of these networks is blockages in the direct links, such as cars, trees, etc., which makes receivers unable to communicate with the transmitter. [89–91]. Fortunately, RIS are capable of constructing an auxiliary transmission link even when the direct link is blocked and addresses this limitation [92, 93].

2) RIS-aided UAV networks

These days, the Unmanned Aerial Vehicle (UAV) plays an important role in long-distance communications. In addition, UAVs can also improve the quality of air-ground transmission due to their flexible deployment and high mobility. However, the direct link between the UAV and the ground user can be blocked in a dense urban environment by obstacles such as tall buildings [90, 94]. Hence, deploying RIS in these networks can enable virtual LoS paths by reflecting the desired signal received from the UAV to the ground receivers and improve the performance of the system by enhancing the signal power [95, 96].

3) RIS-aided SWIPT networks

Simultaneous Wireless Information and Power Transfer (SWIPT) provides cost-effective power delivery for energy-constrained low-power networks. In such networks, the BS, with a constant power supply, transmits wireless signals to provide power for the energy users (EUs) and information for the information users (IUs). Explicitly, the EUs usually require more power than that required by the IUs. Therefore, the efficiency of the WPT at EUs is considered the performance factor of the SWIPT networks, and hence, the low efficiency of far-field WPT is one of the key challenges in these networks. In addition, in order to have an efficient energy reception, the EUs are located closer to the BS. This also causes wireless security issues. Given that the IUs may be located farther from the BS, and the information intended for the IUs can be easily wiretapped by the potential malicious EUs, who tend to have access to stronger signals than the IUs [91, 97]. Consequently, integration of RIS in a location closer to the EUs can mitigate these issues. The RIS can improve the strength of the signal [98–100] by dynamically adjusting the reflection coefficients, including amplitudes and phase shifts, and guarantee information security [101, 102].

4) RIS-aided physical layer security networks

The broadcast nature of the wireless transmission makes wireless links vulnerable to security threats like eavesdropping and jamming attacks. In order to tackle these concerns, physical layer security (PLS) techniques have been introduced. These techniques can avoid complex key exchange protocols and, hence, are suitable for latency-sensitive applications. However, there are some limitations to these techniques, such as correlated channels for both the legitimate user and the eavesdropper or a shorter distance between the eavesdropper and the BS than that between the legitimate user and BS [89, 91]. Therefore, RIS offers a potential solution for these issues. The RIS can enhance the PLS of the communication links by simultaneously boosting the signal beam at the legitimate user and suppressing the beam at the eavesdropper [90, 91, 103].

2.4 Optimization algorithms

In this study, we introduced two types of optimization techniques: model-based and metaheuristic approaches. In addition, we have applied a model-based approach for the active beamforming design in the BS and the relay and both model-based and metaheuristic algorithms for the passive beamforming design in RIS. In order to select the most efficient algorithm for specific problems, we have evaluated and compared these approaches, including the main features, advantages, drawbacks, difficulties, and applications for RIS. Additionally, we have demonstrated the performance of each approach and compared them for different sizes of RIS in Chapter 5.

2.4.1 Model-based algorithms

Model-based algorithms usually require complete knowledge of the defined problem for transformation and approximation and finding near-optimal solutions. In addition, these approaches can provide efficient and stable solutions, and the aim is to find the global optimal or at least sub-optimal results once the problem is appropriately reformulated. However, a notable drawback of model-based algorithms lies in their design complexity, which stems from the necessity of multiple transformations and approximations from the requirement of multiple transformations and approximations, such as decoupling the denominator and numerator in terms of Signal-to-Interference-plus-Noise Ratio (SINR) and relaxing integer constraints. Furthermore, environmental uncertainties can have a significant impact on the performance of model-based algorithms as they need complete knowledge of the optimization parameters [104].

2.4.2 Stochastic optimization algorithms

Stochastic optimization algorithms do not promise optimal solutions. However, within a reasonable computational time, they give near-suboptimal solutions. Therefore, they are applied to solve complex and larger problems by their nature [105]. Stochastic algorithms are further divided into heuristic and meta-heuristic algorithms.

A. Heuristic algorithms: Heuristic algorithms are usually problem-dependent methods. Therefore, in solving hard and complex optimization problems, heuristic algorithms seem to be better, especially when the conventional methods fail. Greedy algorithms, local search algorithms, and constructive algorithms are common types of heuristic algorithms.

B. Metaheuristic algorithms: Meta-heuristic algorithms are iterative problem-independent approaches, and these methods follow a high-level strategy of intelligence. Therefore, the metaheuristic algorithms can be applied to solve a wide range of problems [106]. These algorithms aim to find the best solutions in a search space. The main advantage of this group of algorithms lies in their ability to use a multiple descent mechanism, thereby avoiding the trap of local optima. Hence, they can be used to solve the hard and complex problems. Most metaheuristic methods take inspiration from biological or natural phenomena [107]. Metaheuristic algorithms can be categorized in various ways, such as population-based, trajectory-based, nature-inspired, evolutionary-based, and swarm intelligence-based [104, 108]. For instance, genetic algorithm and PSO are well-known evolutionary-based and swarm intelligence-based examples, respectively.

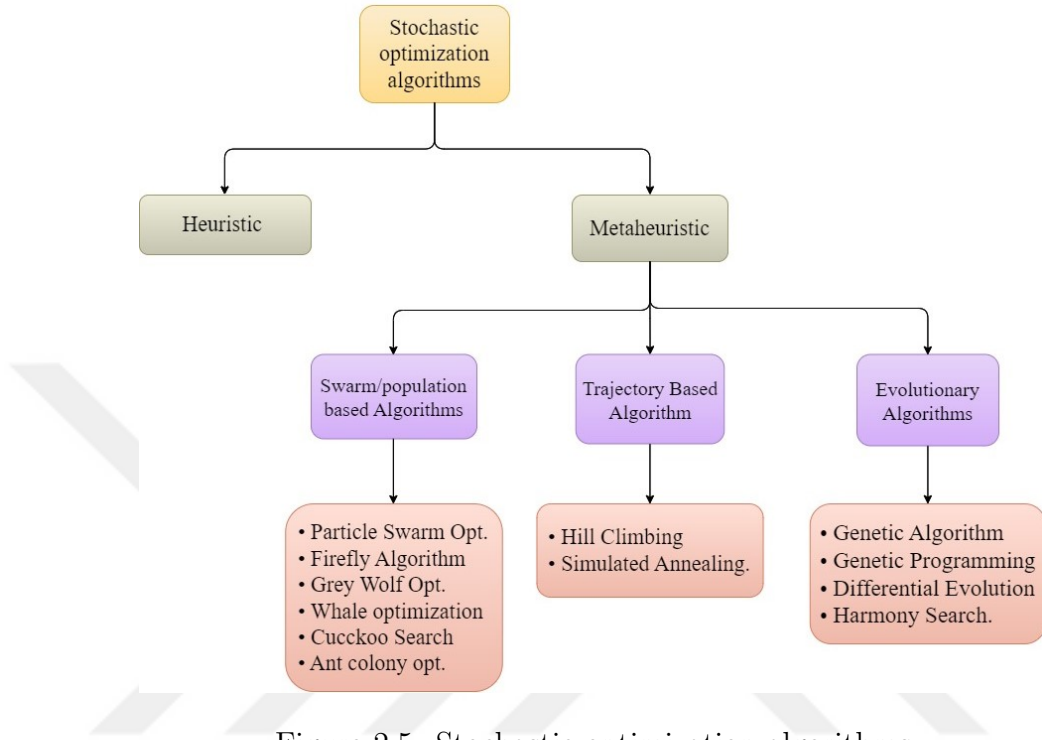


Figure 2.5: Stochastic optimization algorithms

B.1) Particle swarm optimization

Particle Swarm Optimization (PSO) is a population-based stochastic optimization algorithm introduced by Eberhart and Kennedy in 1995 [109,110]. This algorithm is inspired by the intelligent collective behavior of organisms of some animals, such as a school of fish or a flock of birds that moves in a group, and it is based on the swarm theory. PSO belongs to a class of optimization methods that are problem-independent and can be applied to a wide range of optimization problems. Therefore, it is considered a metaheuristic algorithm.

Similar to the other evolutionary computation techniques, PSO is a population-based search algorithm where the population is called a swarm. This method is initialized with a population of random solutions, called particles, each representing a possible solution to the optimization problem. Unlike the evolutionary process techniques, each particle is also associated with a velocity. Consequently, the particles fly in a multidimensional search space with velocities dynamically

adjusted according to their historical behaviors towards the better and better search area over the course of the search process. The behavior of these particles is influenced by two key factors:

- Personal best (p_{best}): Each particle remembers its own best-known position in the search space that it has visited so far.
- Global best (g_{best}): The swarm as a whole remembers the best-known position found by any particle in the entire population.

The general steps of this algorithm are as follows:

1. Initialize population and velocity : Initialize the population and velocities of each particle with random positions in the D-dimensional problem space. In this algorithm, each particle represents a potential solution to the optimization problem, and the velocities determine the direction and distance that the particle will move.

2. Evaluate fitness: Evaluate the desired optimization fitness of each particle in the population. The fitness function quantifies how good each particle's solution is with respect to the optimization problem.

3. Update personal best and global best: For each particle, compare its fitness evaluation with its p_{best} . If the current value is better than p_{best} , then set the corresponding p_{best} equal to the current value and update the location in D-dimensional space. In addition, identify the particle with the best fitness value among all particles in the population and consider its position as the g_{best} position.

4. Update velocities and positions: Update the velocity of each particle based on its current velocity, its distance to its p_{best} and g_{best} positions. This step enables particles to adjust their movement in the search space towards better solutions. Likewise, update the position of each particle based on its velocity. This step determines how particles move within the search space.

5. Convergence check: Check if convergence criteria are met. These conditions include a maximum number of iterations or achieving a sufficiently good fitness. If termination conditions are not met, go back to step 2 and continue the optimization process. Otherwise, terminate the algorithm and return the best solution found.

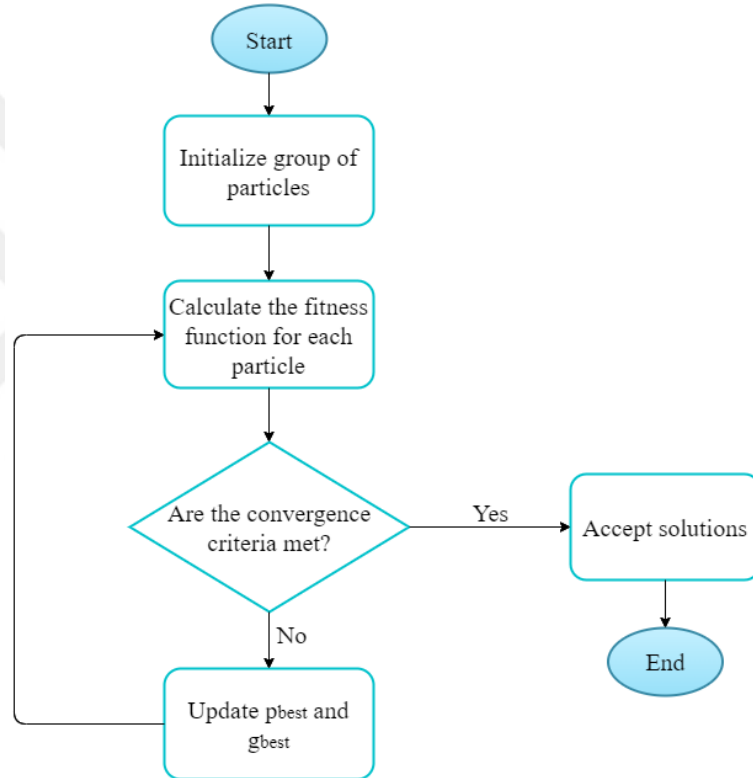


Figure 2.6: Particle Swarm Optimization (PSO)

2.4.3 Model-based vs metaheuristic algorithms

In this subsection, we compare the benefits and drawbacks of model-based and metaheuristic algorithms, including stability, optimality, generalization capability, robustness for environment uncertainty, algorithm design/tuning difficulty, and computational resource requirements.

- **Optimality:** Optimality refers to the quality of solutions, which is one

of the key metrics for optimization problems. Model-based methods can usually achieve the best feasible solution by having complete knowledge of the problem, particularly when closed-form solutions are attainable. Meanwhile, heuristic methods may find local optima instead of achieving global optimal solutions. [104, 111].

- **Generalization capability:** Generalization capability refers to the ability of an optimization model to adapt to new problems. Some metaheuristic methods demonstrate a high generalization capability and can be easily generalized. Examples include GA and PSO algorithms, which employ unified fitness functions to represent optimization objectives. In contrast, model-based methods have limited generalization capability since the optimization model requires a specific case-by-case design. In other words, the optimization problem should be redesigned for new problems [104].
- **Robustness for the environmental uncertainty:** Wireless networks are highly dynamic, and hence, optimization techniques must be robust to environmental uncertainties. Some heuristic algorithms can adapt to environmental changes by applying heuristic rules. Nevertheless, the performance of model-based algorithms may significantly be impacted by such uncertainty since these methods need complete knowledge of the optimization problem. One potential approach is to assume environment changes follow some specific distributions, but the optimization over distributions will further increase the complexity. Alternatively, employing Monte Carlo sampling and repeating the optimization process to obtain average results can address this challenge. However, this approach is time-consuming [104].
- **Algorithm design/tuning difficulty:** This metric denotes the difficulty of designing or tuning algorithms. Metaheuristic algorithms are sensitive to key parameters such as population numbers and inertia weight. Similarly, model-based methods are usually complicated to design due to the stringent requirements for problem forms. Hence, various transformation, relaxation, and approximation techniques must be employed, increasing the algorithm design difficulty [104, 111].

- **Computational resources requirements:** This metric evaluates the number of computing resources, including both time and space, that are consumed when running a particular algorithm. Many model-based methods require iterations to solve the problem, and the matrix operations further add to storage space requirements, while metaheuristic methods have low computational resource requirements due to the simplification of problem-solving complexity achieved through heuristic rules [104].
- **Stability:** Stability is another significant metric to determine whether the optimization algorithm can provide a stable result. Model-based methods can present a very stable result when the full knowledge of the problem is available, particularly when closed-form expressions are obtained. However, most heuristic algorithms have low stability because heuristic rules do not guarantee a stable output [104].

In summary, model-based solutions have high optimality and stability, but their generalization capability is low. These algorithms are challenging to design and vulnerable to environmental uncertainties. On the other hand, heuristic algorithms focus on feasible solutions with low design difficulty and computational resource requirements, but optimality and stability cannot be guaranteed.

2.5 Rank reduction methods

From the mathematics perspective, the RIS phase shift and beamforming optimization problems are usually related to the method of semidefinite relaxation (SDR) due to the unit-modulus constraint for the amplitude of the RIS phase shift elements and non-convex quadratic constraints for the beamforming vectors. After applying the SDR, the rank-one constraint, which is a non-convex constraint, is introduced, making the problem NP-hard. In practice, this type of problem is challenging to solve. Therefore, several heuristic methods exist that can be applied in order to obtain a near-optimal rank-one solution. These methods are generally classified into three groups. (i) Rank-one constraint relaxation: In this

approach, the rank-one constraint is relaxed, and the convex problem is optimized without this constraint. If the solution is rank-one, it is the global optimal solution of the problem. Otherwise, the rank-reduction is applied by approaches such as eigenvalue decomposition [112–115], Cholesky decomposition [116–119], and Gaussian randomization [115, 120–122]. The main drawback of these methods is that the obtained solution of the relaxed problem is usually high-rank, and in addition, the constructed rank-one solution is usually sub-optimal or even infeasible for the rank-one constrained problem [115]. *(ii)* Rank minimization: In this approach, which is denoted as penalty-based norm approximation, the rank function is approximated by some traceable functions, and then it is added to the main objective function as a penalty term. Finally, the problem is solved iteratively until the rank is reduced and the defined convergence criteria are satisfied [123–126]. The challenge with this method is that its performance depends on the quality of the rank-function approximation and the penalty parameter setting, and both the performance and complexity of penalty-based algorithms are affected by the penalty parameters. *(iii)* Quadratic optimization: In this approach, the rank-one positive semidefinite (PSD) matrix defined by the SDR is rewritten as the outer product of a vector variable in quadratic form such that the rank-one constraint can be avoided, and a quadratic optimization problem is solved to obtain the vector variable. The difficulty with this approach is that solving a quadratic problem is even harder. Therefore, semidefinite programming (SDP) is usually applied to reformulate a quadratic optimization problem as a rank-one constrained optimization problem.

The aforementioned heuristic approaches are not always applicable due to their drawbacks, and they are suitable for some particular cases not generic problems. Therefore, a novel method has been introduced as the sequential rank-one constraint relaxation (SROCR) [127]. The SROCR addresses the subsequent non-convex and, in general, NP-hard problems. In this method, an eigenvalue-related constraint is defined instead of dropping the rank-one constraint, and a convex problem is solved iteratively in order to generate a locally optimal solution to a generic rank-one constrained optimization problem if the relaxed optimization problem without a rank-one constraint is a convex optimization

problem [119, 127–129].



Chapter 3

Sum rate maximization in relay-aided MU-MISO systems

In this chapter, the aim is to maximize the sum rate of the relay-aided system by optimizing the beamforming vectors at both BS and the relay. This optimization is subject to constraints, including the minimum SINR at the relay, as well as maximum transmit power constraints for the BS and relay. Also, SIC constraint is added to the problem for both phases in the NOMA technique due to the SIC method used for decoding, which will be mentioned in the NOMA section. The optimization at each transmission phase is discussed individually, for each of which the multiple access (MA) techniques are studied separately. Finally, we introduce the alternating optimization (AO) algorithm and discuss the overall computational complexity of the AO algorithm and the MA techniques.

3.1 System model

In this work, an *underloaded* or *critically loaded* downlink multi-user MISO network has been considered. As shown in Fig. 3.1, the network includes one Base Station (BS) equipped with a set $\mathcal{M}$ of M antennas, a DF relay with a set $\mathcal{L}$ of

L antennas, and a set of $\mathcal{K}$ of K single-antenna users, where $K \leq \min\{M, L\}$ is for ensuring the sufficient degrees of freedom.

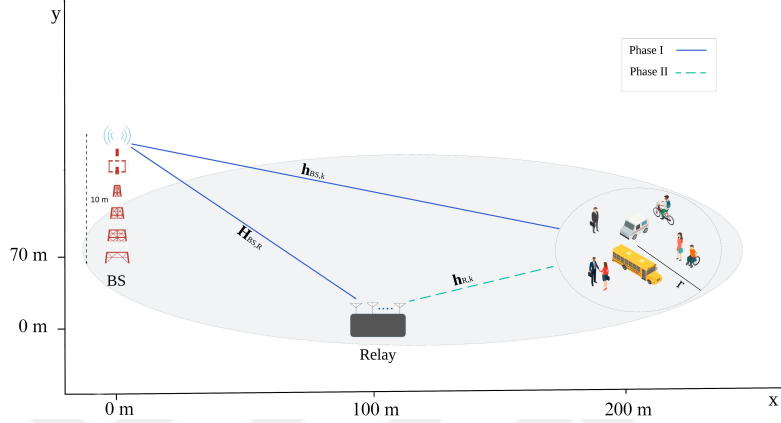


Figure 3.1: A relay-assisted multi-user MISO system

The channels between the BS and relay, BS and user k , and relay and user k are described as $\mathbf{H}_{BS,R} \in \mathbb{C}^{L \times M}$, $\mathbf{h}_{BS,k} \in \mathbb{C}^{1 \times M}$, and $\mathbf{h}_{R,k} \in \mathbb{C}^{1 \times L}$, respectively. In addition, the beamforming vectors for the message of user k in phase I and phase II are denoted as $\boldsymbol{\omega}_k \in \mathbb{C}^{M \times 1}$ and $\mathbf{v}_k \in \mathbb{C}^{L \times 1}$, respectively. All channels are assumed to be quasi-static Rayleigh flat-fading. The path loss between transmitter and receiver is given by $PL(d) = PL_0 - 10\alpha \log_{10}(\frac{d}{d_0})$, where α represents the path loss exponent, d is the distance between the transmitter and the receiver, and $PL_0 = -30\text{dB}$ denotes the reference path loss at $d_0 = 1\text{m}$. The channels are given by,

$$\mathbf{H}_{x,y} = \sqrt{PL(d_{x,y})} \bar{\mathbf{H}}, \quad (x,y) \in \{(BS,R), (R,k), (BS,k)\} \quad (3.1)$$

where $\bar{\mathbf{H}}$ is complex Gaussian with zero mean and unit variance. The channel state information (CSI) of all channels is assumed to be perfectly known for the exposition. Also, the data stream of the user k is named s_k . Thus, we can define $\mathbf{s} = [s_1, \dots, s_K] \in \mathbb{C}^K$, and $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{(K)}$. Without the loss of generality, the channel gains are considered to be $|\mathbf{h}_{BS,1}|^2 \leq |\mathbf{h}_{BS,2}|^2 \leq \dots, |\mathbf{h}_{BS,K}|^2$ for phase I, and $|\mathbf{h}_{R,1}|^2 \leq |\mathbf{h}_{R,2}|^2 \leq \dots, |\mathbf{h}_{R,K}|^2$ for phase II.

3.2 Signals

For the system shown in Fig. 3.1, the transmission is divided into two phases. In phase I, the messages are encoded by the BS using beamforming vectors $\boldsymbol{\omega}_k, \forall k \in \mathcal{K}$, and transmitted to the users and the relay. Subsequently, the DF relay re-encodes the messages received at phase I using new beamforming vectors denoted as $\mathbf{v}_k, \forall k \in \mathcal{K}$ and re-transmit them to the users. For each transmission phase, the mathematical equations of the transmitted and received signals are discussed separately in forthcoming sections.

3.2.1 Phase I transmission

In phase I, the messages are encoded by beamforming vectors $\boldsymbol{\omega}_j, \forall j \in \mathcal{K}$ and transmitted from BS, which is formulated as $\mathbf{x} = \sum_{j=1}^K \boldsymbol{\omega}_j s_j$. On the receiver side, the received signal at user k is represented by

$$y_k^I = \mathbf{h}_{\text{BS},k} \mathbf{x} + z_k = \mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k s_k + \sum_{j=1, j \neq k}^K \mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j s_j + z_k \quad (3.2)$$

where $z_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the Additive White Gaussian Noise (AWGN) at the receiver, user k . Furthermore, in phase I, the signals are also received by the relay and re-transmitted in phase II. Thus, the received signal at the relay can be formulated as,

$$\mathbf{y}_R = \mathbf{H}_{\text{BS},R} \mathbf{x} + \mathbf{z}_R = \mathbf{H}_{\text{BS},R} \boldsymbol{\omega}_k s_k + \sum_{j=1, j \neq k}^K \mathbf{H}_{\text{BS},R} \boldsymbol{\omega}_j s_j + \mathbf{z}_R \quad (3.3)$$

where $\mathbf{z}_R \sim \mathcal{CN}(0, \sigma_R^2 \mathbf{I}_L)$ is AWGN at the relay.

3.2.2 Phase II transmission

After decoding the signals at the relay, they are encoded and re-transmitted to the users in phase II, and the transmitted signal from the relay is given by

$\mathbf{x}_R = \sum_{j=1}^K \mathbf{v}_j s_j$. Hence, the received signal of phase II transmission at user k is presented by,

$$y_k^{\text{II}} = \mathbf{h}_{R,k} \mathbf{x}_R + n_k = \mathbf{h}_{R,k} \mathbf{v}_k s_k + \sum_{j=1, j \neq k}^K \mathbf{h}_{R,k} \mathbf{v}_j s_j + n_k \quad (3.4)$$

where $n_k \sim \mathcal{CN}(0, \sigma_{\text{II}}^2)$ is the AWGN at the user k . Additionally, it is considered that the time is uniformly allocated between two transmission phases, and each uses half of the time, and δ_{time} is set to $\frac{1}{2}$. Therefore, after decoding both signals transmitted from BS and the relay, the total rate of the message at user k is calculated by,

$$R_k = \delta_{\text{time}} \log_2(1 + \gamma_k^{\text{I}}) + (1 - \delta_{\text{time}}) \log_2(1 + \gamma_k^{\text{II}}) \quad (3.5)$$

where γ_k^{I} and γ_k^{II} are the SINR of the user k in phase I and phase II, respectively, which are discussed for each multiple access technique in the next subsections, separately.

According to the introduced objective, constraints and parameters, the main optimization problem is formulated as follows,

$$\begin{aligned} (\text{P0}) \quad & \max_{\boldsymbol{\omega}_k, \mathbf{v}_k} \sum_{k=1}^K R_k \\ & \text{s.t.} \quad \gamma_j^{\text{R}} \geq \gamma_{\text{R}}^{\text{th}}, \quad \forall j \in \mathcal{K} \end{aligned} \quad (3.6a)$$

$$\sum_{k=1}^K \|\boldsymbol{\omega}_k\|^2 \leq P_{\text{BS}}^{\text{max}} \quad (3.6b)$$

$$\sum_{k=1}^K \|\mathbf{v}_k\|^2 \leq P_{\text{R}}^{\text{max}} \quad (3.6c)$$

where (3.6a) is the minimum SINR constraint at the relay for the signal of user k decoded at the relay, and (3.6b) and (3.6c) are maximum power constraints at BS and relay, respectively.

Since the problem (P0) is hard to solve directly, the Alternating Optimization (AO) algorithm outlined in Algorithm 2 is employed. In this algorithm, (P0) is decoupled into two subproblems. Thus, the optimization variables are divided

into two groups, each of which contains the beamforming vectors of one of the transmission phases. In addition, since there is no relation between phase I and phase II objective functions and constraints, and the variables of each phase do not have any effect on the other phase's performance, there is no need to consider them to be fixed in the optimization process of the other phase, and they only affect the objective value, which will be considered in the convergence criteria.

The subproblems and utilized reformulations for each multiple access technique are discussed separately in the next sections.

3.3 Achievable rate in DF relay networks

In the DF relay aided networks, as the system operates as a series of links (BS to relay and relay to users), the overall data rate is constrained by the weakest link in this chain.

In related studies, different expressions are used to define the achievable rate at the receiver for phase II transmission. Consider a transmitter, a DF relay and a single user, with γ_1 , γ_2 , and γ_R representing the SINR in phase I, in phase II, and at the relay, respectively. In [130–132], the achievable rate at the user for phase II is given by $\log_2(1 + \min\{\gamma_2, \gamma_R\})$. This expression ensures that the lower SINR is selected, reflecting the weakest link. Additionally, [133, 134] suggest that the link with the lowest channel gain can be selected for calculating the achievable rate. Furthermore, in other related studies such as [32, 33, 135], the achievable rate for phase II is defined as $\log_2(1 + \gamma_2)$, without taking into account the SINR at the relay.

According to [136], in practice, the amount of harvested energy at the relay is always small, leading to much lower transmit power than the BS. Therefore, it is reasonable to assume that the SINR at the user is lower than at the relay. Based on this knowledge, we assume that the BS has higher transmit power than the relay in our proposed system, ensuring that signals transmitted from the BS to

the relay are stronger than the signals transmitted from the relay to the users. Furthermore, it is also assumed that the height of the BS and relay are different. The BS is located at a higher altitude, which allows a better LoS link, whereas the relay, at a lower altitude, does not have such a favorable LoS path. This assumption indicates that the channel gain of BS-relay link can be higher than that of the relay-user links.

By these assumptions, we approximate $\log_2(1 + \min\{\gamma_2, \gamma_R\}) \approx \log_2(1 + \gamma_2)$, since the SINR at the relay is significantly larger than the SINR in phase II ($\gamma_R \gg \gamma_2$).

3.4 BS beamforming design (ω_k)

The first subproblem represents the optimization of the BS beamforming vectors. For this aim, only ω_k are considered as the optimizing variables. During this process, new auxiliary variables are introduced to reformulate the subproblems as solvable convex problems, and since they are dependent on the optimization variables, they should be considered fixed when the ω_k vectors are optimized. In (P0), the constraints (3.6a), and (3.6b) are related to phase I, and therefore, by considering them, the subproblem 1 is represented by

$$(P1) \quad \max_{\omega_k} \quad \sum_{k=1}^K R_k \quad (3.7a)$$

$$\text{s.t.} \quad \gamma_j^R \geq \gamma_R^{\text{th}}, \quad \forall j \in \mathcal{K} \quad (3.7b)$$

$$\sum_{k=1}^K \|\omega_k\|^2 \leq P_{\text{BS}}^{\text{max}} \quad (3.7c)$$

According to the differences between the multiple access techniques, their mathematical equations for the SINRs and the sum rate are not the same. Therefore, this subproblem needs to be reformulated separately for each multiple access technique.

3.4.1 Problem transformation methods

In this study, in order to transform the original non-convex problem to a traceable convex form, we first use the Lagrangian Dual Transformation (LDT) to remove the SINR term from the logarithm. Following this transformation, we apply the Quadratic Transformation (QT) to reshape the objective function into a more solvable form. Within this approach, we find closed-form optimal solutions for auxiliary variables and optimize corresponding optimization variables using the Alternating Optimization (AO) method. Here, we have explained the transformation methods for the objective function in a general form, and then we have extended the methods to each MA techniques, separately according to the SINR/SNR equations of the MA technique.

Lemma 1: According to the LDT method introduced in [137], the sum-logarithms term in the sum rate of each transmission phase represented by $\sum_{k=1}^K \log_2(1 + \gamma_k)$ for a general form of $\gamma_k = \frac{A_k(\mathbf{x}_k)}{B_k(\mathbf{x}_k)}$, can be transformed into a new function in the form of the sum of ratios by introducing a group of new auxiliary variables, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_K]$, and the new objective function is defined as,

$$f(\mathbf{x}_k, \boldsymbol{\alpha}) = \sum_{k=1}^K (\log_2(1 + \alpha_k) - \alpha_k) + \sum_{k=1}^K (1 + \alpha_k) f_k(\mathbf{x}_k) \quad (3.8)$$

where $\mathbf{x}_k$ is the optimization variables, and $f_k(\mathbf{x}_k) = \frac{A_k(\mathbf{x}_k)}{B_k(\mathbf{x}_k) + A_k(\mathbf{x}_k)}$. This new defined objective function, $f(\mathbf{x}_k, \alpha_k)$, is equivalent to the original sum rate function in (P0), $\sum_{k=1}^K \log_2(1 + \gamma_k)$, if and only if when $\alpha_k = \gamma_k$. (The transformation for phase II is the same since the formulas are in the same mathematical form, and only the parameters are different.)

Proof: The proof is given in Appendix A. ■

Subsequently, after applying the LDT transformation, the transformed objective function is in the form of multiple-ratio fractional programming (MRFP). Therefore, according to the [137] and [138], the quadratic transform (QT) can be applied in order to recast the objective function to a more solvable one, and this

transformation is stated as follows.

Lemma 2: In the QT, new auxiliary variables are introduced as $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_K]$, and the MRFP term of the previous part, $\sum_{k=1}^K f_k(\mathbf{x}_k)$ is transformed and written as

$$g(\mathbf{x}_k, \boldsymbol{\alpha}, \boldsymbol{\beta}) = \sum_{k=1}^K \left(2\sqrt{1 + \alpha_k} \Re \left\{ \beta_k^* \sqrt{A_k(\mathbf{x}_k)} \right\} - |\beta_k|^2 (B_k(\mathbf{x}_k) + A_k(\mathbf{x}_k)) \right) \quad (3.9)$$

The $g(\mathbf{x}_k, \boldsymbol{\alpha}, \boldsymbol{\beta})$ function is equivalent to the $\sum_{k=1}^K f_k(\mathbf{x}_k)$ if and only if when

$$\beta_k = \sqrt{1 + \alpha_k} \frac{\sqrt{A_k(\mathbf{x}_k)}}{B_k(\mathbf{x}_k) + A_k(\mathbf{x}_k)} \quad (3.10)$$

Proof: The proof is given in Appendix B. ■

3.4.2 Space Division Multiple Access (I)

In the SDMA technique, the signals of all other users received at user k are considered interference and are treated as noise. Therefore, according to Eq. (3.2), the SINR of the message decoded by user k at phase I, denoted as γ_k^I , is given by

$$\gamma_k^I = \frac{|\mathbf{h}_{BS,k} \boldsymbol{\omega}_k|^2}{\sum_{j=1, j \neq k}^K |\mathbf{h}_{BS,k} \boldsymbol{\omega}_j|^2 + \sigma_I^2} \quad (3.11)$$

In addition, the messages are received at the relay in phase I. Thus, for the SINR of decoded message of user k at the relay, γ_k^R , we have,

$$\gamma_k^R = \frac{\|\mathbf{H}_{BS,R} \boldsymbol{\omega}_k\|^2}{\sum_{j=1, j \neq k}^K \|\mathbf{H}_{BS,R} \boldsymbol{\omega}_j\|^2 + \sigma_R^2} \quad (3.12)$$

Next, we have the SINRs for phase II of the transmission. According to Eq. (3.4), the SINR of the message decoded by user k in the second phase, γ_k^{II} , is calculated by,

$$\gamma_k^{\text{II}} = \frac{|\mathbf{h}_{R,k} \mathbf{v}_k|^2}{\sum_{j=1, j \neq k}^K |\mathbf{h}_{R,k} \mathbf{v}_j|^2 + \sigma_{\text{II}}^2} \quad (3.13)$$

By considering Eqs. (3.11) and (3.13), the objective function of the main problem is formulated as,

$$\begin{aligned} \sum_{k=1}^K R_k &= \sum_{k=1}^K \frac{1}{2} \log_2 \left(1 + \frac{|\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k|^2}{\sum_{j=1, j \neq k}^K |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2} \right) \\ &\quad + \sum_{k=1}^K \frac{1}{2} \log_2 \left(1 + \frac{|\mathbf{h}_{\text{R},k} \mathbf{v}_k|^2}{\sum_{j=1, j \neq k}^K |\mathbf{h}_{\text{R},k} \mathbf{v}_j|^2 + \sigma_{\text{II}}^2} \right) \end{aligned} \quad (3.14)$$

As can be seen in Eq. (3.14), only the first term is a function of $\boldsymbol{\omega}_j, \forall j \in \mathcal{K}$, and transmission phases do not have any effect on each other. Therefore, the first sum of the logarithms term will be used as the objective function of the subproblem (P1). To relax this term, fractional programming (FP) can be applied. to convert it into a solvable polynomial expression. Specifically, based on the Lagrangian Dual Transformation (LDT) [137] represented in *Lemma 1*, the sum of logarithmic ratios can be reformulated by defining auxiliary variables, α_k 's, as follows,

$$\begin{aligned} &\sum_{k=1}^K \log_2 \left(1 + \frac{|\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k|^2}{\sum_{j=1, j \neq k}^K |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2} \right) \\ &= \sum_{k=1}^K (\log_2 (1 + \alpha_k) - \alpha_k) + \sum_{k=1}^K \frac{(1 + \alpha_k) |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k|^2}{\sum_{j=1}^K |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2} \end{aligned} \quad (3.15)$$

where the optimal value of α_k is given by,

$$\alpha_k^* = \frac{|\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k|^2}{\sum_{j=1, j \neq k}^K |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2} \quad (3.16)$$

Furthermore, since the sum of the multiple-ratios term in Eq. (3.15) still hinders a direct solution, we further apply the quadratic transform [137, 138], represented in *Lemma 2*, to convert it into a convex form by introducing a group of new auxiliary variables given by,

$$\begin{aligned} &\sum_{k=1}^K (1 + \alpha_k) \frac{|\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k|^2}{\sum_{j=1}^K |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2} = \\ &\sum_{k=1}^K \left(2\sqrt{1 + \alpha_k} \Re \{ \beta_k^* \mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k \} - |\beta_k|^2 \left(\sum_{j=1}^K |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2 \right) \right) \end{aligned} \quad (3.17)$$

where the optimal value of β_k is presented by

$$\beta_k^* = \sqrt{1 + \alpha_k} \frac{\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k}{\sum_{j=1}^K |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2} \quad (3.18)$$

By considering Eqs. (3.15) and (3.17), the final objective function can be rewritten as follow,

$$\begin{aligned} & \sum_{k=1}^K 2\Re\{\beta_k^* \mathbf{a}_k \boldsymbol{\omega}_k\} - \sum_{k=1}^K |\beta_k|^2 \sum_{j=1}^K \boldsymbol{\omega}_j^H \mathbf{H}_{\text{BS},k} \boldsymbol{\omega}_j + b_k \\ &= \sum_{k=1}^K 2\Re\{\beta_k^* \mathbf{a}_k \boldsymbol{\omega}_k\} - \sum_{k=1}^K \boldsymbol{\omega}_k^H \left(\sum_{j=1}^K |\beta_j|^2 \mathbf{H}_{\text{BS},j} \right) \boldsymbol{\omega}_k + b_k \\ &= \sum_{k=1}^K 2\Re\{\beta_k^* \mathbf{a}_k \boldsymbol{\omega}_k\} - \sum_{k=1}^K \boldsymbol{\omega}_k^H \mathbf{B}_k \boldsymbol{\omega}_k + b_k \end{aligned} \quad (3.19)$$

where the introduced parameters are $\mathbf{a}_k = \sqrt{1 + \alpha_k} \mathbf{h}_{\text{BS},k}$, $\mathbf{B}_k = \sum_{j=1}^K |\beta_j|^2 \mathbf{H}_{\text{BS},j}$, $\mathbf{H}_{\text{BS},j} = \mathbf{h}_{\text{BS},j}^H \mathbf{h}_{\text{BS},j}$, $\forall j \in \mathcal{K}$ and $b_k = \log_2(1 + \alpha_k) - \alpha_k - \sigma_{\text{I}}^2 |\beta_k|^2$.

Next step is to reformulate the constraints in order to convert the problem to a convex one. The first constraint, (3.6a), is rewritten by considering Eq. (3.12) as follows,

$$\begin{aligned} & \frac{\|\mathbf{H}_{\text{BS},\text{R}} \boldsymbol{\omega}_j\|^2}{\sum_{i=1, i \neq j}^K \|\mathbf{H}_{\text{BS},\text{R}} \boldsymbol{\omega}_i\|^2 + \sigma_{\text{R}}^2} \geq \gamma_{\text{R}}^{th}, \quad \forall j \in \mathcal{K} \\ & \rightarrow -\boldsymbol{\omega}_j^H \mathbf{H}'_{\text{BS},\text{R}} \boldsymbol{\omega}_j + \gamma_{\text{R}}^{th} \sum_{i=1, i \neq j}^K \boldsymbol{\omega}_i^H \mathbf{H}'_{\text{BS},\text{R}} \boldsymbol{\omega}_i + e \leq 0 \\ & \rightarrow \sum_{i=1}^K \boldsymbol{\omega}_i^H \mathbf{E}_{i,j} \boldsymbol{\omega}_i + e \leq 0, \quad \forall j \in \mathcal{K} \end{aligned}$$

where $\mathbf{H}'_{\text{BS},\text{R}} = \mathbf{H}_{\text{BS},\text{R}}^H \mathbf{H}_{\text{BS},\text{R}}$, $e = \gamma_{\text{R}}^{th} \sigma_{\text{R}}^2$ and,

$$\mathbf{E}_{i,j} = \begin{cases} \gamma_{\text{R}}^{th} \mathbf{H}'_{\text{BS},\text{R}} & , i \neq j \\ -\mathbf{H}'_{\text{BS},\text{R}} & , i = j \end{cases}, \quad \forall \{i \in \mathcal{K}, j \in \mathcal{K}\} \quad (3.20)$$

So, the equivalent problem of the first subproblem (P1) is represented by

$$\begin{aligned} \max_{\boldsymbol{\omega}_k} \quad & \sum_{k=1}^K 2\Re\{\mathbf{a}_k \boldsymbol{\omega}_k\} - \sum_{k=1}^K \boldsymbol{\omega}_k^H \mathbf{B}_k \boldsymbol{\omega}_k + b_k \\ \text{s.t.} \quad & \sum_{i=1}^K \boldsymbol{\omega}_i^H \mathbf{E}_{i,j} \boldsymbol{\omega}_i + e \leq 0, \quad j \in \mathcal{K} \end{aligned} \quad (3.21a)$$

$$\sum_{j=1}^K \|\boldsymbol{\omega}_j\|^2 \leq P_{\text{BS}}^{\max} \quad (3.21b)$$

This problem is a nonconvex inhomogeneous QCQP. Therefore, firstly, we define new parameters in order to reformulate and homogenize the objective function; subsequently, by defining $\tilde{\boldsymbol{\omega}}_k = \begin{bmatrix} \boldsymbol{\omega}_k \\ 1 \end{bmatrix}$, and $\mathbf{W}_k = \tilde{\boldsymbol{\omega}}_k \tilde{\boldsymbol{\omega}}_k^H$, the problem of finding $\boldsymbol{\omega}_k$ is transformed into one of obtaining a rank-one positive semidefinite (PSD) matrix $\mathbf{W}_k$ to address the non-convexity of constraints in (3.21a). This transformation eventually leads to a convex semidefinite programming (SDP) problem through the technique of semidefinite relaxation (SDR) [139].

$$\begin{aligned} \text{(P1.1)} \quad \max_{\mathbf{W}_k} \quad & \sum_{k=1}^K \text{tr}\{\mathbf{U}_k \mathbf{W}_k\} \\ \text{s.t.} \quad & \sum_{i=1}^K \text{tr}\{\mathbf{E}'_{i,j} \mathbf{W}_i\} + e \leq 0, \quad \forall j \in \mathcal{K} \end{aligned} \quad (3.22a)$$

$$\sum_{j=1}^K \text{tr}\{\mathbf{W}_j\} \leq P_{\text{BS}}^{\max} \quad (3.22b)$$

$$[\mathbf{W}_j]_{M+1, M+1} = 1, \quad \forall j \in \mathcal{K} \quad (3.22c)$$

$$\mathbf{W}_j \succeq 0, \quad \forall j \in \mathcal{K} \quad (3.22d)$$

$$\text{rank}(\mathbf{W}_j) = 1, \quad \forall j \in \mathcal{K} \quad (3.22e)$$

The newly introduced parameters are given by

$$\begin{cases} \mathbf{U}_k = \begin{bmatrix} -\mathbf{B}_k & \mathbf{a}_k^H \\ \mathbf{a}_k & 0 \end{bmatrix}, & \forall k \in \mathcal{K} \\ \mathbf{E}'_{i,j} = \begin{bmatrix} \mathbf{E}_{i,j} & \mathbf{0}^T \\ \mathbf{0} & 0 \end{bmatrix}, & \forall \{i \in \mathcal{K}, j \in \mathcal{K}\} \end{cases} \quad (3.23)$$

where $\mathbf{0}$ is a row vector of zeros with size M . Still, (3.22e) is a non-convex constraint. Therefore, it is relaxed, and a method named SROCR is used in order to find the best rank one solution for the problem. In the SROCR algorithm, which is represented in Algorithm 1, the rank one constraint is replaced by a new eigenvalue-based constraint, which depends on the optimal solution obtained in the previous iteration. In other words, the problem is solved iteratively until finding the best rank one suboptimal solution. The added constraint for the SROCR is formulated as follows,

$$\bar{\mathbf{u}}(\mathbf{W}_k^{(n)})^H \mathbf{W}_k \bar{\mathbf{u}}(\mathbf{W}_k^{(n)}) \geq m_k^{(n)} \text{tr}(\mathbf{W}_k) \quad (3.24)$$

where $\bar{\mathbf{u}}$ is the eigenvector corresponding to the maximum eigenvalue of the optimal solution $\mathbf{W}_k^{(n)}$, obtained in iteration n , and $m_k^{(n)}$ is,

$$m_k^{(n)} = \min \left(1, \frac{\lambda_{\max}(\mathbf{W}_k^{(n)})}{\text{tr}(\mathbf{W}_k^{(n)})} + \delta_k^{(n)} \right) \quad (3.25)$$

where $\lambda_{\max}(\mathbf{W}_k^{(n)})$ is the maximum eigenvalue of the $\mathbf{W}_k^{(n)}$, and the initial $\delta_k^{(0)}$ is set randomly by $\delta_k^{(0)} \in \left(0, 1 - \frac{\lambda_{\max}(\mathbf{W}_k^{(0)})}{\text{tr}(\mathbf{W}_k^{(0)})} \right)$. Then, at each iteration, if the solution is feasible it does not change but if the problem is infeasible, the value of $\delta^{(n)}$ is decreased by $\delta^{(n-1)}/3$, where $\delta = [\delta_1, \delta_2, \dots, \delta_K]$.

So, the final form of the problem is presented by,

$$\begin{aligned}
 (\text{P1.2}) \quad & \max_{\mathbf{W}_k} \sum_{k=1}^K \text{tr}\{\mathbf{U}_k \mathbf{W}_k\} \\
 \text{s.t.} \quad & \sum_{i=1}^K \text{tr}\{\mathbf{E}'_{i,j} \mathbf{W}_i\} + e \leq 0, \quad \forall j \in \mathcal{K} \quad (3.26a) \\
 & \sum_{j=1}^K \text{tr}\{\mathbf{W}_j\} - K \leq P_{\text{BS}}^{\max} \quad (3.26b) \\
 & [\mathbf{W}_j]_{M+1, M+1} = 1, \quad \forall j \in \mathcal{K} \quad (3.26c) \\
 & \mathbf{W}_j \succcurlyeq 0, \quad \forall j \in \mathcal{K} \quad (3.26d) \\
 & \bar{\mathbf{u}}(\mathbf{W}_j^{(n)})^H \mathbf{W}_j \bar{\mathbf{u}}(\mathbf{W}_j^{(n)}) \geq m_j^{(n)} \text{tr}(\mathbf{W}_j), \quad \forall j \in \mathcal{K} \quad (3.26e)
 \end{aligned}$$

Now, the problem (P1.2) is convex, and the CVX toolbox can solve it.

Algorithm 1 Rank reduction by SROCR method.

```

1: Initialize iteration index  $n = 0$ , feasible  $\boldsymbol{\omega}_k^{(0)}$ , convergence threshold  $\epsilon_1$ 
2: First iteration:
3: solve (P1.1) with relaxed rank-one constraint to obtain  $\mathbf{W}_k^{(0)}$  for  $m_k^{(0)} = 0$ 
4: define initial step size:  $\delta_k^0 \in \left(0, 1 - \frac{\lambda_{\max}(\mathbf{W}_k^{(0)})}{\text{tr}(\mathbf{W}_k^{(0)})}\right)$ 
5: repeat
6:   solve (P1.2) with  $\mathbf{W}_k^{(n)}$  and  $m_k^{(n)}$ 
7:   if Problem (P1.2) is solvable then
8:     obtain optimal  $\mathbf{W}_k^{(n)}$  and  $m_k^{(n)}$  for Eq. (3.26)
9:      $\delta_k^{(n+1)} \leftarrow \delta_k^{(n)}$ 
10:  else
11:     $\mathbf{W}_k^{(n+1)} \leftarrow \mathbf{W}_k^{(n)}$ 
12:     $\delta_k^{(n+1)} \leftarrow \delta_k^{(n)}/3$ 
13:  end if
14:   $m_k^{(n+1)} = \min\left(1, \frac{\lambda_{\max}(\mathbf{W}_k^{(n)})}{\text{tr}(\mathbf{W}_k^{(n)})} + \delta_k^{(n)}\right)$ 
15:   $n = n + 1$ 
16: until  $|1 - m_k^{(n-1)}| \leq \epsilon_1$  or  $n = I_{it}$ 

```

3.4.3 Nonorthogonal Multiple Access (I)

In the NOMA technique, as mentioned in the first chapter, user k decodes the signals of the weaker users and cancels them from the received signal. Additionally, it treats the signals of the stronger users as interference. Moreover, for the received signals at the relay, since they are decoded by one receiver (relay), the SINRs are considered the same as the SDMA case, and the residual signals are considered as interference when the relay decodes the signal of user k . Therefore, the SINR of the signal of user k decoded at the relay will be Eq. (3.12). In addition, without the loss of generality, the channel gains are assumed to be $|\mathbf{h}_{\text{BS},1}|^2 \leq |\mathbf{h}_{\text{BS},2}|^2 \leq \dots \leq |\mathbf{h}_{\text{BS},K}|^2$ for phase I, and $|\mathbf{h}_{\text{R},1}|^2 \leq |\mathbf{h}_{\text{R},2}|^2 \leq \dots \leq |\mathbf{h}_{\text{R},K}|^2$ for phase II. Thus, the equations for SINRs of the received signals at user k in both phases are given by,

$$\gamma_k^{\text{I}} = \frac{|\mathbf{h}_{\text{BS},k}\boldsymbol{\omega}_k|^2}{\sum_{j=k+1}^K |\mathbf{h}_{\text{BS},k}\boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2} \quad (3.27)$$

$$\gamma_k^\Pi = \frac{|\mathbf{h}_{R,k}\mathbf{v}_k|^2}{\sum_{j=k+1}^K |\mathbf{h}_{R,k}\mathbf{v}_j|^2 + \sigma_\Pi^2} \quad (3.28)$$

In addition, as each user employs the SIC method, an SIC constraint is considered for each of them at both phases represented by,

$$|\mathbf{h}_{BS,k+1}\boldsymbol{\omega}_k|^2 - \sum_{j=k+1}^K |\mathbf{h}_{BS,k+1}\boldsymbol{\omega}_j|^2 \geq \theta_{\text{SIC}}, \quad \forall k \in \{1, 2, \dots, K-1\} \quad (3.29)$$

$$|\mathbf{h}_{R,k+1}\mathbf{v}_k|^2 - \sum_{j=k+1}^K |\mathbf{h}_{R,k+1}\mathbf{v}_j|^2 \geq \theta_{\text{SIC}}, \quad \forall k \in \{1, 2, \dots, K-1\} \quad (3.30)$$

In this subproblem, the reformulation methods are the same as in the SDMA case, and the difference is in the interval of the summations of the objective function, which results in,

$$\sum_{k=1}^K 2\Re\{\beta_k^* \mathbf{a}_k \boldsymbol{\omega}_k\} - \sum_{k=1}^K \boldsymbol{\omega}_k^H \mathbf{B}_k \boldsymbol{\omega}_k + b_k \quad (3.31)$$

where the defined parameters are $\mathbf{a}_k = \sqrt{1 + \alpha_k} \mathbf{h}_{BS,k}$, $\mathbf{B}_k = \sum_{j=1}^k |\beta_j|^2 \mathbf{H}_{BS,j}$, $\mathbf{H}_{BS,j} = \mathbf{h}_{BS,j}^H \mathbf{h}_{BS,j}$, $\forall j \in \mathcal{K}$ and $b_k = \log_2(1 + \alpha_k) - \alpha_k - \sigma_I^2 |\beta_k|^2$. Also, the optimal solutions for the auxiliary variables are given by,

$$\alpha_k^* = \frac{|\mathbf{h}_{BS,k}\boldsymbol{\omega}_k|^2}{\sum_{j=1+k}^K |\mathbf{h}_{BS,k}\boldsymbol{\omega}_j|^2 + \sigma_I^2} \quad (3.32)$$

$$\beta_k^* = \sqrt{1 + \alpha_k} \frac{\mathbf{h}_{BS,k}\boldsymbol{\omega}_k}{\sum_{j=k}^K |\mathbf{h}_{BS,k}\boldsymbol{\omega}_j|^2 + \sigma_I^2} \quad (3.33)$$

Furthermore, the added SIC constraint are reformulated as,

$$\begin{aligned} & |\mathbf{h}_{BS,k+1}\boldsymbol{\omega}_k|^2 - \sum_{j=k+1}^K |\mathbf{h}_{BS,k+1}\boldsymbol{\omega}_j|^2 \geq \theta_{\text{SIC}}, \quad \forall k \in \{1, 2, \dots, K-1\} \\ & \text{tr}\{\mathbf{H}_{BS,k+1} \mathbf{W}_k\} - \sum_{j=k+1}^K \text{tr}\{\mathbf{H}_{BS,k+1} \mathbf{W}_j\} \geq \theta_{\text{SIC}}, \quad \forall k \in \{1, 2, \dots, K-1\} \end{aligned} \quad (3.34)$$

Since the minimum SINR and the SIC constraints are non-convex, SDR is applied. Consequently, the final form of the problem is as follows,

$$\begin{aligned} \max_{\mathbf{W}_k} \quad & \sum_{k=1}^K \text{tr}\{\mathbf{U}_k \mathbf{W}_k\} \\ \text{s.t.} \quad & \sum_{i=1}^K \text{tr}\{\mathbf{E}'_{i,j} \mathbf{W}_i\} + e \leq 0, \quad \forall j \in \mathcal{K} \end{aligned} \quad (3.35a)$$

$$\text{tr}\{\mathbf{H}_{\text{BS},j+1} \mathbf{W}_j\} - \sum_{i=j+1}^K \text{tr}\{\mathbf{H}_{\text{BS},j+1} \mathbf{W}_i\} \geq \theta_{\text{SIC}}, \quad \forall j \in \{1, 2, \dots, K-1\} \quad (3.35b)$$

$$\sum_{j=1}^K \text{tr}\{\mathbf{W}_j\} - K \leq P_{\text{BS}}^{\text{max}} \quad (3.35c)$$

$$[\mathbf{W}_j]_{M+1,M+1} = 1, \quad \forall j \in \mathcal{K} \quad (3.35d)$$

$$\mathbf{W}_j \succcurlyeq 0, \quad \forall j \in \mathcal{K} \quad (3.35e)$$

$$\bar{\mathbf{u}}(\mathbf{W}_j^{(n)})^H \mathbf{W}_j \bar{\mathbf{u}}(\mathbf{W}_j^{(n)}) \geq m_j^{(n)} \text{tr}(\mathbf{W}_j), \quad \forall j \in \mathcal{K} \quad (3.35f)$$

Similar to the SDMA problem, the SROCR is utilized, and the rank one constraint is relaxed and replaced by the eigenvalue-related constraint, (3.35f).

3.4.4 Orthogonal Multiple Access (I)

For the OMA technique, it is assumed that the degrees of freedom (time/frequency) are equally divided among the K users [140], referred to as OFDMA, as mentioned in the first chapter. Without loss of generality, the achievable sum rate received by users is considered to be $\frac{1}{K} \sum_{k=1}^K R_k$, where R_k is the sum of rates at phases received by user k with the SNRs at phase I, phase II and relay described respectively by,

$$\gamma_k^{\text{I}} = \frac{|\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k|^2}{\frac{1}{K} \sigma_{\text{I}}^2} \quad (3.36)$$

$$\gamma_k^{\text{II}} = \frac{|\mathbf{h}_{\text{R},k} \mathbf{v}_k|^2}{\frac{1}{K} \sigma_{\text{II}}^2} \quad (3.37)$$

$$\gamma_k^{\text{R}} = \frac{\|\mathbf{H}_{\text{BS,R}} \boldsymbol{\omega}_k\|^2}{\frac{1}{K} \sigma_{\text{R}}^2} \quad (3.38)$$

We avoid repeating the same reformulation and transformation steps, and by replacing the parameters, we formulate the objective function for the OMA technique as follows,

$$\sum_{k=1}^K 2\Re\{\beta_k^* \mathbf{a}_k \boldsymbol{\omega}_k\} - \sum_{k=1}^K \boldsymbol{\omega}_k^H \mathbf{B}_k \boldsymbol{\omega}_k + b_k \quad (3.39)$$

where the defined parameters are $\mathbf{a}_k = \sqrt{1 + \alpha_k} \mathbf{h}_{\text{BS},k}$, $\mathbf{B}_k = |\beta_k|^2 \mathbf{h}_{\text{BS},k}^H \mathbf{h}_{\text{BS},k}$ and $b_k = \log_2(1 + \alpha_k) - \alpha_k - \frac{1}{K} \sigma_1^2 |\beta_k|^2$.

Furthermore, the optimal value of the auxiliary variables is given by,

$$\alpha_k^* = \frac{|\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k|^2}{\frac{1}{K} \sigma_1^2} \quad (3.40)$$

$$\beta_k^* = \sqrt{1 + \alpha_k} \frac{\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k}{|\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k|^2 + \frac{1}{K} \sigma_1^2} \quad (3.41)$$

Moreover, the minimum SNR constraint as the relay is also reformulated by the same techniques as used in (3.20). Similarly, due to the non-convexity of the SNR constraint, the SDR and SROCR are applied. Therefore, the final SROCR-based subproblem 1 for the OMA technique is reformulated as,

$$\max_{\mathbf{W}_k} \quad \frac{1}{K} \sum_{k=1}^K \text{tr}\{\mathbf{U}_k \mathbf{W}_k\} \quad (3.42a)$$

$$\text{s.t.} \quad -\text{tr}\{\mathbf{H}'_{\text{BS},R} \mathbf{W}_j\} + e \leq 0, \quad \forall j \in \mathcal{K} \quad (3.42a)$$

$$\text{tr}\{\mathbf{W}_j\} - 1 \leq P_{\text{BS}}^{\max}, \quad \forall j \in \mathcal{K} \quad (3.42b)$$

$$[\mathbf{W}_j]_{M+1,M+1} = 1, \quad \forall j \in \mathcal{K} \quad (3.42c)$$

$$\mathbf{W}_j \succcurlyeq 0, \quad \forall j \in \mathcal{K} \quad (3.42d)$$

$$\bar{\mathbf{u}}(\mathbf{W}_j^{(n)})^H \mathbf{W}_j \bar{\mathbf{u}}(\mathbf{W}_j^{(n)}) \geq m_j^{(n)} \text{tr}(\mathbf{W}_j), \quad \forall j \in \mathcal{K} \quad (3.42e)$$

3.4.5 Complexity analysis

Regarding the computational complexity of optimizing the beamforming vectors in phase I, we consider the semidefinite programming (SDP) problem in (3.26),

(3.35) and (3.42) for the SDMA, NOMA, and OMA techniques, respectively. In addition, the SROCR method is also employed for each of them by applying the algorithm 1. According to [141], the worst-case computational complexity of solving a generic SDP by the CVX toolbox is given by $\mathcal{O}(\max\{m, n\}^4 n^{1/2} \log(1/\epsilon_1))$, where m is the number of constraints, n is the size of the positive semidefinite matrix in the objective, and $\epsilon_1 > 0$ is a constant reflecting the solution accuracy of the CVX solver. Accordingly, the complexity order of algorithm 1 for SDMA in (3.26) is $\mathcal{O}(I_{it} \max\{4K + 1, K(M + 1)\}^4 (K(M + 1))^{1/2} \log(1/\epsilon_1))$, for the NOMA in (3.35) and the OMA in (3.42) is given by $\mathcal{O}(I_{it} \max\{5K, K(M + 1)\}^4 (K(M + 1))^{1/2} \log(1/\epsilon_1))$, where I_{it} is the maximum iteration number at the Algorithm 1 for the SROCR. As the product of two positive integers tends to grow faster than a linear increase, we simplify and deduce the complexity orders to $\mathcal{O}(I_{it} (K(M + 1))^{4.5} \log(1/\epsilon_1))$ for all MA techniques.

3.5 Relay beamforming design ($\mathbf{v}_k$)

In this section, the beamforming vectors of phase II transmission, $\{\mathbf{v}_k, \forall k \in \mathcal{K}\}$, with the origin of the DF relay are optimized. For this subproblem, the second term of Eq. (3.14) is considered as the objective function, and the same LDT, and QT methods are applied to convert the problem into an equivalent convex form. The optimization for each multiple-access technique is discussed in the forthcoming subsections.

3.5.1 Space Division Multiple Access (II)

For optimizing the relay beamforming vectors $\{\mathbf{v}_k, \forall k \in \mathcal{K}\}$, the newly defined auxiliary variables for the LDT and QT are assumed fixed. By considering the

steps done in phase I, this subproblem also can be written as follows,

$$(P2) \quad \max_{\mathbf{v}_k} \quad \sum_{k=1}^K (2\Re\{\mathbf{a}_k \mathbf{v}_k\} - \mathbf{v}_k^H \mathbf{B}_k \mathbf{v}_k + b_k) \quad (3.43a)$$

$$\text{s.t.} \quad \sum_{i=1}^K \mathbf{v}_i^H \mathbf{v}_i \leq P_R^{\max} \quad (3.43b)$$

where the defined parameters are $\mathbf{a}_k = \sqrt{1 + \alpha_k} \mathbf{h}_{R,k}$, $\mathbf{B}_k = \sum_{j=1}^K |\beta_j|^2 \mathbf{H}_{R,j}$, $\mathbf{H}_{R,j} = \mathbf{h}_{R,j}^H \mathbf{h}_{R,j}$, $\forall j \in \mathcal{K}$. Also, the optimal values of the auxiliary variables, α_k and β_k , are calculated by replacing phase II parameters in Eqs. 3.16 and 3.18. Problem (P2) is a convex problem, hence, the optimal solutions can be obtained by standard optimization tools.

3.5.2 Nonorthogonal Multiple Access (II)

Due to the non-convexity of the SIC constraints, SDR is applied, and the optimization problem for the second phase of the NOMA with the SROCR method is given by,

$$\max_{\mathbf{V}_k} \quad \sum_{k=1}^K \text{tr}\{\mathbf{U}_k \mathbf{V}_k\} \quad (3.44)$$

$$\text{s.t.} \quad \text{tr}\{\mathbf{H}_{R,j+1} \mathbf{V}_j\} - \sum_{i=j+1}^K \text{tr}\{\mathbf{H}_{R,j+1} \mathbf{V}_i\} \geq \theta_{\text{SIC}}, \quad \forall j \in \{1, 2, \dots, K-1\} \quad (3.45)$$

$$\sum_{j=1}^K \text{tr}\{\mathbf{V}_j\} - K \leq P_R^{\max} \quad (3.46)$$

$$[\mathbf{V}_j]_{L+1, L+1} = 1, \quad \forall j \in \mathcal{K} \quad (3.47)$$

$$\mathbf{V}_j \succcurlyeq 0, \quad \forall j \in \mathcal{K} \quad (3.48)$$

$$\bar{\mathbf{u}}(\mathbf{V}_j^{(n)})^H \mathbf{V}_j \bar{\mathbf{u}}(\mathbf{V}_j^{(n)}) \geq m_j^{(n)} \text{tr}(\mathbf{V}_j), \quad \forall j \in \mathcal{K} \quad (3.49)$$

where $\mathbf{H}_{R,j} = \mathbf{h}_{R,j}^H \mathbf{h}_{R,j}$, $\forall j \in \mathcal{K}$, and the reformulation of the objective function is done using the same methods introduced in phase I. $\mathbf{U}_k$ can be defined as described in 3.23, replacing phase II parameters. Also, the optimal values of the

auxiliary variables, α_k and β_k , are calculated by replacing phase II parameters in Eqs. 3.32 and 3.33.

3.5.3 Orthogonal Multiple Access (II)

Phase II beamforming design subproblem for the OMA technique is a convex problem and can be formulated similarly to the SDMA case by using the reformulation methods. The final convex subproblem is given by,

$$\max_{\mathbf{v}_k} \quad \frac{1}{K} \sum_{k=1}^K (2\Re\{\mathbf{a}_k \mathbf{v}_k\} - \mathbf{v}_k^H \mathbf{B}_k \mathbf{v}_k + b_k) \quad (3.50)$$

$$\text{s.t.} \quad \mathbf{v}_j^H \mathbf{v}_j \leq P_R^{\max}, \quad \forall j \in \mathcal{K} \quad (3.51)$$

where the defined parameters are $\mathbf{a}_k = \sqrt{1 + \alpha_k} \mathbf{h}_{R,k}$, $\mathbf{B}_k = |\beta_k|^2 \mathbf{h}_{R,k}^H \mathbf{h}_{R,k}$. Also, the optimal values of the auxiliary variables, α_k and β_k , are calculated by replacing phase II parameters in Eqs. 3.40 and 3.41. Similar to the SDMA subproblem, (3.50) is a convex QCQP, which can be solved by standard convex optimization toolboxes such as the CVX.

3.5.4 Complexity analysis

In the optimization of relay beamforming vectors in phase II, the subproblems for the SDMA and OMA techniques are reformulated as convex QCQP problems without using the SDR or SROCR methods. Therefore, their complexity order is given by $\mathcal{O}(L^3)$ [142, 143]. However, due to the SIC constraint, the NOMA technique subproblem is solved using the SROCR and SDR methods. Therefore, similar to the previous section, the worst-case computational complexity of the NOMA subproblem is calculated as $\mathcal{O}(I_{it} \max\{4K, K(L+1)\}^4 (K(L+1))^{1/2} \log(1/\epsilon_1))$. Considering that the product of two positive integers tends to grow faster than a linear increase, the complexity order is deduced to $\mathcal{O}(I_{it} (K(L+1))^{4.5} \log(1/\epsilon_1))$

3.6 Alternating optimization

The alternating optimization method is employed to solve the problem (P0). In this algorithm, the main problem of this chapter (P0) is decoupled into two subproblems, each of which optimizes the beamforming vectors of one of the transmission phases. These steps are repeated iteratively until the increase in the objective value becomes less than a defined threshold value, signifying convergence.

AO algorithm for the relay-assisted system is described below in Algorithm 2 and Figure 3.2. Since the rate formulas and the beamforming vectors at each transmission phase are independent of the other phases and also they rely on their respective auxiliary variables, this algorithm can be introduced as a two-layer Alternating Optimization method. In the inner layer, the beamforming vectors of each phase are optimized by using the corresponding auxiliary variables separately, and finally, the convergence criteria are applied for the total objective value of the main problem, which consists of both phases.

Algorithm 2 Alternating optimization

- 1: Initialize iteration index $n = 0$, feasible $\omega_k^{(0)}, \mathbf{v}_k^{(0)}$, convergence threshold ϵ
 - 2: **First iteration:**
 - 3: update $\alpha_{\omega,k}^{*(1)}$ and $\beta_{\omega,k}^{*(1)}$ by $\omega_k^{(0)}$
 - 4: $\omega_k^{(1)} \leftarrow$ solution of P1 for fixed $\mathbf{v}_k^{(0)}, \alpha_{\omega,k}^{*(1)}$ and $\beta_{\omega,k}^{*(1)}$
 - 5: update $\alpha_{v,k}^{*(1)}$ and $\beta_{v,k}^{*(1)}$ by $\mathbf{v}_k^{(0)}$
 - 6: $\mathbf{v}_k^{(1)} \leftarrow$ solution of P2 for fixed $\omega_k^{(1)}, \alpha_{v,k}^{*(1)}$ and $\beta_{v,k}^{*(1)}$
 - 7: **repeat**
 - 8: update $\alpha_{\omega,k}^{*(n+1)}$ and $\beta_{\omega,k}^{*(n+1)}$ by $\omega_k^{(n)}$
 - 9: $\omega_k^{(n+1)} \leftarrow$ solution of P1 for fixed $\mathbf{v}_k^{(n)}, \alpha_{\omega,k}^{*(n+1)}$ and $\beta_{\omega,k}^{*(n+1)}$
 - 10: update $\alpha_{v,k}^{*(n+1)}$ and $\beta_{v,k}^{*(n+1)}$ by $\mathbf{v}_k^{(n)}$
 - 11: $\mathbf{v}_k^{(n+1)} \leftarrow$ solution of P2 for fixed $\omega_k^{(n+1)}, \alpha_{v,k}^{*(n+1)}$ and $\beta_{v,k}^{*(n+1)}$
 - 12: $n = n + 1$
 - 13: **until** $R_T^{(n-1)} - R_T^{(n-2)} \leq \epsilon$ or $n = n_{\max}$
 - 14: $\mathbf{v}_k^* = \mathbf{v}_k^{(n)}, \omega_k^* = \omega_k^{(n)}$
-

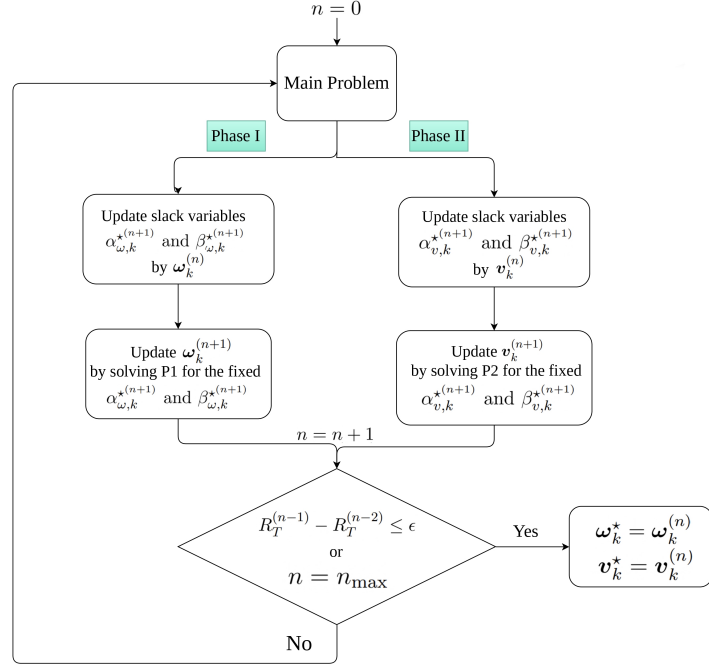


Figure 3.2: AO algorithm for relay-assisted system

3.6.1 Complexity analysis

Combining the complexities of the subproblems discussed separately, it is concluded that the worst-case complexity of the entire proposed algorithm for the SDMA and OMA techniques is given by $\mathcal{O}(n_{\max}[I_{it}(K(M+1))^{4.5} \log(1/\epsilon_1) + L^3])$. Also, the complexity of the AO in the NOMA is $\mathcal{O}((n_{\max}[I_{it}K^{4.5}[(M+1)^{4.5} + (L+1)^{4.5}] \log(1/\epsilon_1)])$.

Chapter 4

Sum rate maximization in hybrid RIS and relay aided system

In this chapter, RIS is added to the previous system presented in Fig. 3.1 in order to enhance the performance. Our objective is to maximize the sum rate of the hybrid RIS-relay-assisted system under the minimum SINR constraints at the relay, maximum transmits power constraints for the BS and relay, in addition to the unit-modulus constraint for the RIS elements. The optimization of the beam-forming vectors has been discussed in detail in the previous chapter. The addition of RIS-related channels to the existing links does not alter the formulation significantly, and only the channel parameters are rewritten as the combination of the direct link and the auxiliary RIS-related link. Furthermore, the RIS phase profile optimization based on AO algorithm is investigated separately for each MA technique in the next sections. Finally, the metaheuristic constrained PSO algorithm is introduced for the RIS phase profile optimization, and the computational complexity is discussed and compared for both model-based and metaheuristic algorithms, separately, for the MA techniques and AO algorithm.

4.1 System model

In this investigation, the RIS phase profile is designed separately for each time slot, with the assumption that the RIS phase shift matrices are independent of each other in both transmission phases. These subproblems are added to the corresponding phase in the inner layer of the AO algorithm of the hybrid RIS-relay-aided systems.

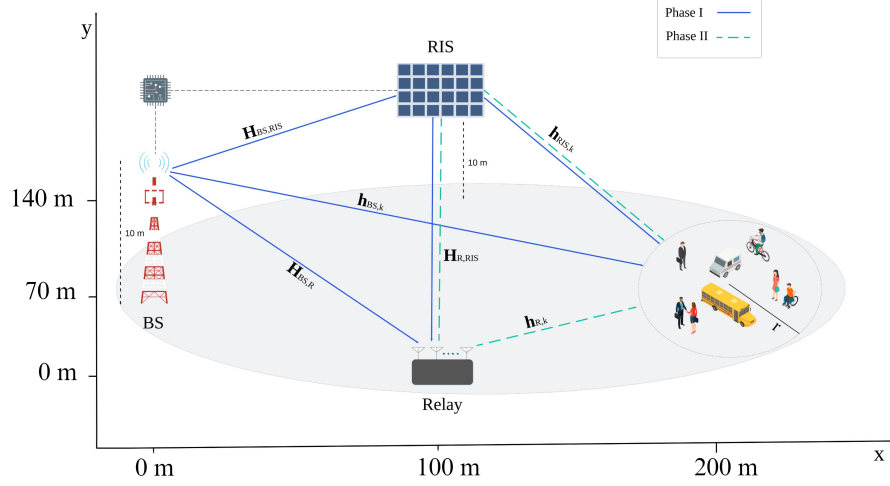


Figure 4.1: A RIS-relay-assisted multi-user MISO system

Adding RIS to the system introduces RIS-related channels and new optimization variables for the RIS phase shift matrices, as detailed in Table 4.1. It is assumed that Line-of-Sight (LoS) communication links exist between the BS and RIS, relay and RIS, and the links between RIS and the users. Therefore, they experience small-scale fading that follows a rician fading. Now, we can express the channel coefficients as follows,

$$\mathbf{H}_{i,\text{RIS}} = \sqrt{\text{PL}(d_{i,\text{RIS}})} \left(\sqrt{\frac{K_1}{K_1 + 1}} \mathbf{H}_{\text{LoS},i} + \sqrt{\frac{1}{K_1 + 1}} \mathbf{H}_{\text{NLoS},i} \right), \quad i \in \{\text{BS}, \text{R}, k\} \quad (4.1)$$

where K_1 is the Rician factor and is set to 10, and $\mathbf{H}_{\text{LoS},i}$ and $\mathbf{H}_{\text{NLoS},i}$ are line-of-sight (LoS) and non-line-of-sight (NLoS) components given by

$$\mathbf{H}_{\text{LoS},i} = a(\omega)a^H(\zeta), \quad i \in \{\text{BS}, \text{R}, k\} \quad (4.2)$$

$$\mathbf{H}_{\text{NLoS},i} = \bar{\mathbf{H}}, \quad i \in \{\text{BS}, \text{R}, k\} \quad (4.3)$$

where $\mathbf{H}_{\text{LoS},i}$ represents the Line-of-Sight (LoS) channel, and $a(\omega) = \left[1, e^{j2\pi \frac{d}{\lambda} \sin(\omega)}, \dots, e^{j2\pi \frac{d}{\lambda} (N-1) \sin(\omega)}\right]^T \in \mathbb{C}^{N \times 1}$ is the array response vector, where d is the RIS inter-element spacing, λ is the wavelength, and ω is the angle of arrival (AoA); Also, $a(\zeta) \in \mathbb{C}^{M \times 1}$ with ζ being the angle of departure (AoD) is defined in a similar manner. We set $\frac{d}{\lambda} = \frac{1}{2}$ and the values of AoA and AoD are randomly generated between 0 and 2π . In addition, $\mathbf{H}_{\text{NLoS},i}$ represents the Non-LoS (NLoS) channel, and $\bar{\mathbf{H}}$ follows a Rayleigh distribution with zero mean and unit variance.

$\mathbf{H}_{\text{BS,RIS}} \in \mathbb{C}^{N \times M}$	channel between BS and RIS
$\mathbf{H}_{\text{RIS,R}} \in \mathbb{C}^{L \times N}$	channel between RIS and Relay
$\mathbf{h}_{\text{RIS},k} \in \mathbb{C}^{1 \times N}$	channel between RIS and user k
$\boldsymbol{\Theta}_1 \in \mathbb{C}^{N \times N}$	RIS phase shift matrix in phase I
$\boldsymbol{\Theta}_2 \in \mathbb{C}^{N \times N}$	RIS phase shift matrix in phase II

Table 4.1: List of RIS-related notations

By adding the RIS to the network, the unit-modulus constraint is added to the main problem. Therefore, the sum rate maximization problem of the hybrid system is given by,

$$\begin{aligned} \max_{\boldsymbol{\omega}_k, \mathbf{v}_k, \boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2} \quad & \sum_{k=1}^K R_k \\ \text{s.t.} \quad & \gamma_j^{\text{R}} \geq \gamma_{\text{R}}^{\text{th}}, \quad \forall j \in \mathcal{K} \end{aligned} \quad (4.4a)$$

$$\sum_{k=1}^K \|\boldsymbol{\omega}_k\|^2 \leq P_{\text{BS}}^{\text{max}} \quad (4.4b)$$

$$\sum_{k=1}^K \|\mathbf{v}_k\|^2 \leq P_{\text{R}}^{\text{max}} \quad (4.4c)$$

$$|[\boldsymbol{\Theta}_1]_{i,i}| = 1, \quad \forall i \in \mathcal{N} \quad (4.4d)$$

$$|[\boldsymbol{\Theta}_2]_{i,i}| = 1, \quad \forall i \in \mathcal{N} \quad (4.4e)$$

Similar to chapter 3, each optimization subproblem is discussed for each multiple access separately in the next subsections. Since adding RIS to the system does

not change the general equations of the received signals at users and the relay, and SINR/SNRs w.r.t to the beamforming vectors at each phase, and it only affects the channel parameters numerically, we do not repeat the beamforming vectors optimizations subproblems. Therefore, only the RIS phase profile design for both transmission phases is discussed in this chapter as follows.

4.2 RIS phase shift (Θ_1) design

Since the RIS phase shift matrices of the transmission phases are assumed to be independent of each other, the optimization problem of these variables is also divided into two subproblems and they are added to the general AO algorithm. The first subproblem is optimizing the RIS phase shift matrix in phase I. In order to reformulate this subproblem and rewrite it as a convex one, some transformation and approximation approaches, in addition to the reformulations in the signals equations, are required. Hence, the channel equation $\mathbf{h}'_{\text{BS},k} = \mathbf{h}_{\text{BS},k} + \mathbf{h}_{\text{RIS},k}\Theta_1\mathbf{H}_{\text{BS,RIS}}$ can be reformulated as $\mathbf{h}_{\text{BS},k} + \theta_1\text{diag}(\mathbf{h}_{\text{RIS},k})\mathbf{H}_{\text{BS,RIS}} = \mathbf{h}_{\text{BS},k} + \theta_1\mathbf{A}_k$ where $\theta_1 = \text{diag}(\Theta_1)$ is considered as a row vector of $\theta_1 = [\theta_{1,1}, \theta_{1,2}, \dots, \theta_{1,N}]$. Therefore, the received signal in the first phase can be rewritten as,

$$\begin{aligned}
y_k^{\text{I}} &= (\mathbf{h}_{\text{BS},k} + \theta_1\mathbf{A}_k)\mathbf{x} + z_k = \mathbf{h}_{\text{BS},k}\mathbf{x} + \theta_1\mathbf{A}_k\mathbf{x} + z_k \\
&= \mathbf{h}_{\text{BS},k}\boldsymbol{\omega}_k s_k + \sum_{j=1, j \neq k}^K \mathbf{h}_{\text{BS},k}\boldsymbol{\omega}_j s_j + \theta_1\mathbf{A}_k\boldsymbol{\omega}_k s_k + \sum_{j=1, j \neq k}^K \theta_1\mathbf{A}_k\boldsymbol{\omega}_j s_j + z_k \\
&= (q_{k,k} + \theta_1\mathbf{p}_{k,k}) + \sum_{j=1, j \neq k}^K (q_{k,j} + \theta_1\mathbf{p}_{k,j}) + z_k
\end{aligned} \tag{4.5}$$

4.2.1 Space Division Multiple Access (I)

According to Eq. (3.11) and Eq. (4.5), and by applying the aforementioned reformulations, the SINR of the received signal at user k in phase I for the SDMA

technique is given by

$$\gamma_k^I = \frac{|q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}|^2}{\sum_{j=1, j \neq k}^K |q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}|^2 + \sigma_I^2} \quad (4.6)$$

With the same LDT and QT transformations introduced previously, the objective function of the SDMA subproblem is reformulated and represented by

$$\sum_{k=1}^K 2\Re \left\{ \sqrt{1 + \eta_k} \lambda_k^* (q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}) \right\} - |\lambda_k|^2 \left(\sum_{j=1}^K |q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}|^2 + \sigma_I^2 \right) \quad (4.7)$$

where the optimal values of the auxiliary variables, η_k and λ_k , are given by

$$\eta_k^* = \frac{|q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}|^2}{\sum_{j=1}^K |q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}|^2 + \sigma_I^2} \quad \lambda_k^* = \sqrt{1 + \eta_k} \frac{q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}}{\sum_{j=1}^K |q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}|^2 + \sigma_I^2} \quad (4.8)$$

By simplifying Eq. (4.7), we can rewrite it as

$$\begin{aligned} & \sum_{k=1}^K 2\Re \{ \sqrt{1 + \eta_k} \lambda_k^* q_{k,k} \} + \sum_{k=1}^K 2\Re \{ \sqrt{1 + \eta_k} \lambda_k^* \boldsymbol{\theta}_1 \mathbf{p}_{k,k} \} \\ & - |\lambda_k|^2 \left(\sum_{j=1}^K (q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j})^H (q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}) + \sigma_I^2 \right) \\ & = 2\Re \{ \boldsymbol{\theta}_1 \sum_{k=1}^K (\sqrt{1 + \eta_k} \lambda_k^* \mathbf{p}_{k,k}) \} - 2\Re \{ \boldsymbol{\theta}_1 \sum_{k=1}^K \sum_{j=1}^K (|\lambda_k|^2 q_{k,j}^* \mathbf{p}_{k,j}) \} \\ & + \boldsymbol{\theta}_1 \left(\sum_{k=1}^K \sum_{j=1}^K |\lambda_k|^2 \mathbf{p}_{k,j} * \mathbf{p}_{k,j}^H \right) \boldsymbol{\theta}_1^H \\ & = 2\Re \{ \mathbf{g} \boldsymbol{\theta}_1^H \} - \boldsymbol{\theta}_1 \mathbf{G} \boldsymbol{\theta}_1^H + g \end{aligned} \quad (4.9)$$

Furthermore, by considering $\boldsymbol{\phi}_1 = \begin{bmatrix} \boldsymbol{\theta}_1^H \\ 1 \end{bmatrix}$, we can write the final objective function as,

$$\boldsymbol{\phi}_1^H \mathbf{U} \boldsymbol{\phi}_1 + g, \quad \mathbf{U} = \begin{bmatrix} -\mathbf{G} & \mathbf{g}^H \\ \mathbf{g} & 0 \end{bmatrix}$$

Then, the constraints should be reformulated and written as convex ones. According to the reformulated channel term, $\mathbf{h}_{\text{BS},k} + \boldsymbol{\theta}_1 \text{diag}(\mathbf{h}_{\text{RIS},k}) \mathbf{H}_{\text{BS,RIS}} =$

$\mathbf{h}_{\text{BS},k} + \boldsymbol{\theta}_1 \mathbf{A}_k$, and by considering $\phi_1 = \begin{bmatrix} \boldsymbol{\theta}_1^H \\ 1 \end{bmatrix}$, we have $\phi_1^H \begin{pmatrix} \mathbf{A}_k \\ \mathbf{h}_{\text{BS},k} \end{pmatrix} = \phi_1^H \mathbf{A}'_k$. For the minimum SINR constraint at the relay, the channel term $\mathbf{H}'_{\text{BS},R} = \mathbf{H}_{\text{BS},R} + \mathbf{H}_{\text{RIS},R} \boldsymbol{\Theta}_1 \mathbf{H}_{\text{BS},\text{RIS}}$ can be reformulated as follows,

$$\begin{aligned} \mathbf{y}_R &= \mathbf{H}'_{\text{BS},R} \mathbf{x} + \mathbf{z}_R = \sum_{j=1}^K (\mathbf{H}_{\text{BS},R} + \mathbf{H}_{\text{RIS},R} \boldsymbol{\Theta}_1 \mathbf{H}_{\text{BS},\text{RIS}}) \boldsymbol{\omega}_j s_j + \mathbf{z}_R \\ &\rightarrow (\mathbf{H}_{\text{BS},R} + \mathbf{H}_{\text{RIS},R} \boldsymbol{\Theta}_1 \mathbf{H}_{\text{BS},\text{RIS}}) \boldsymbol{\omega}_j s_j \\ &= \mathbf{H}_{\text{BS},R} \boldsymbol{\omega}_j s_j + \mathbf{H}_{\text{RIS},R} \boldsymbol{\Theta}_1 \mathbf{H}_{\text{BS},\text{RIS}} \boldsymbol{\omega}_j s_j = \mathbf{h}_{j,R} s_j + \mathbf{H}_{j,R} \boldsymbol{\theta}_1^T s_j \end{aligned} \quad (4.10)$$

where $\mathbf{H}_{j,R} = (\boldsymbol{\omega}_j^T \otimes \mathbf{I}_M)(\mathbf{H}_{\text{BS},\text{RIS}}^T \diamond \mathbf{H}_{\text{RIS},R})$ [144, 145], and $\boldsymbol{\theta}_1 = [\theta_{1,1}, \theta_{1,2}, \dots, \theta_{1,N}]$ is a row vector. Now, the SINRs in the relay can be rewritten as,

$$\gamma_k^R = \frac{\|\mathbf{h}_{k,R} + \mathbf{H}_{k,R} \boldsymbol{\theta}_1^T\|^2}{\sum_{j=1, j \neq k}^K \|\mathbf{h}_{j,R} + \mathbf{H}_{j,R} \boldsymbol{\theta}_1^T\|^2 + \sigma_R^2} \quad (4.11)$$

Therefore, by replacing (4.11) in (3.12), the minimum SINR constraint in the RIS phase shift optimization subproblem is given by,

$$\begin{aligned} \frac{\|\mathbf{h}_{j,R} + \mathbf{H}_{j,R} \boldsymbol{\theta}_1^T\|^2}{\sum_{i=1, i \neq j}^K \|\mathbf{h}_{i,R} + \mathbf{H}_{i,R} \boldsymbol{\theta}_1^T\|^2 + \sigma_R^2} &\geq \gamma_R^{th}, \quad \forall j \in \mathcal{K} \\ \rightarrow \|\mathbf{h} + \mathbf{H} \boldsymbol{\theta}_1^T\|^2 &= (\mathbf{h} + \mathbf{H} \boldsymbol{\theta}_1^T)^H (\mathbf{h} + \mathbf{H} \boldsymbol{\theta}_1^T) = \mathbf{h}^H \mathbf{h} + 2\Re\{\mathbf{h}^H \mathbf{H} \boldsymbol{\theta}_1^T\} + \boldsymbol{\theta}_1^* \mathbf{H}^H \mathbf{H} \boldsymbol{\theta}_1^T \end{aligned} \quad (4.12)$$

Theorem 1 If $\mathbf{A}$ is a $n \times n$ Hermitian matrix, then $\mathbf{x}^H \mathbf{A} \mathbf{x}$ is real for all $x \in \mathcal{C}^n$.

Proof: $(\mathbf{x}^H \mathbf{A} \mathbf{x})^H = \mathbf{x}^H \mathbf{A}^H \mathbf{x}^{HH} = \mathbf{x}^H \mathbf{A} \mathbf{x}$. That is, the number $\mathbf{x}^H \mathbf{A} \mathbf{x}$ is equal to its conjugate. Thus, it is real.

Theorem 2 If $\mathbf{A}$ is a $n \times n$ Hermitian matrix, then all its eigenvalues are real. In particular, all real symmetric matrices have real eigenvalues.

Proof: Let λ be an eigenvalue of $\mathbf{A}$ and $\mathbf{x}$ a corresponding eigenvector. Then, $\mathbf{A} \mathbf{x} = \lambda \mathbf{x}$ and $\mathbf{x}^H \mathbf{A} \mathbf{x} = \mathbf{x}^H \lambda \mathbf{x} = \lambda \|\mathbf{x}\|^2$. Since the left side is real, and so is $\|\mathbf{x}\|^2$, it follows that λ is real too.

By using the fact of $\Re\{x\} = \Re\{x^*\}$, where x is a complex number, for the second term and considering **Theorem 1** and **Theorem 2** for the quadratic term, the (4.12) can be simplified as,

$$h' + 2\Re\{\mathbf{h}'\boldsymbol{\theta}_1^H\} + \boldsymbol{\theta}_1\mathbf{H}'\boldsymbol{\theta}_1^H \quad (4.13)$$

where $\mathbf{h}' = (\mathbf{h}^H\mathbf{H})^* = \mathbf{h}^T\mathbf{H}^*$ and $\mathbf{H}' = \mathbf{H}^T\mathbf{H}^*$, and it should be mentioned that the conjugate of a hermitian matrix is also a hermitian matrix.

Next, the linearization is applied to address the nonconvexity of the quadratic constraint as follows,

$$h' + 2\Re\{\mathbf{h}'\boldsymbol{\theta}_1^H\} + \boldsymbol{\theta}_1\mathbf{H}'\boldsymbol{\theta}_1^H = h' + \boldsymbol{\phi}_1^H\mathbf{E}\boldsymbol{\phi}_1 = h' + \text{tr}\{\mathbf{E}\boldsymbol{\Phi}_1\}$$

where $\boldsymbol{\phi}_1 = \begin{bmatrix} \boldsymbol{\theta}_1^H \\ 1 \end{bmatrix}$, $\boldsymbol{\Phi}_1 = \boldsymbol{\phi}_1\boldsymbol{\phi}_1^H$ and $\mathbf{E}$ is given by,

$$\mathbf{E} = \begin{bmatrix} \mathbf{H}' & \mathbf{h}'^H \\ \mathbf{h}' & 0 \end{bmatrix} \quad (4.14)$$

Therefore, by using these reformulations and applying the SDR method, the final form of minimum SINR constraint for the RIS phase profile design in phase I is given by,

$$\begin{aligned} & \rightarrow \text{tr}\{\mathbf{E}_j\boldsymbol{\Phi}_1\} - \gamma_R^{th} \sum_{i=1, i \neq j}^K \text{tr}\{\mathbf{E}_i\boldsymbol{\Phi}_1\} + h'_j - \sum_{i=1, i \neq j}^K \gamma_R^{th} h'_i + \gamma_R^{th} \sigma_R^2 \leq 0 \\ & \rightarrow \text{tr}\{(\mathbf{E}_j - \gamma_R^{th} \sum_{i=1, i \neq j}^K \mathbf{E}_i)\boldsymbol{\Phi}_1\} + h'_j - \sum_{i=1, i \neq j}^K \gamma_R^{th} h'_i + \gamma_R^{th} \sigma_R^2 \leq 0 \\ & \rightarrow \text{tr}\{\mathbf{L}_j\boldsymbol{\Phi}_1\} + l_j \leq 0 \end{aligned}$$

Therefore, the optimization problem w.r.t $\boldsymbol{\Phi}_1$ is presented by,

$$\begin{aligned} & \max_{\boldsymbol{\Phi}_1} \quad \text{tr}\{\mathbf{A}\boldsymbol{\Phi}_1\} \\ & \text{s.t.} \quad \text{tr}\{\mathbf{L}_j\boldsymbol{\Phi}_1\} + l_j \leq 0, \quad \forall j \in \mathcal{K} \end{aligned} \quad (4.15a)$$

$$\text{diag}(\boldsymbol{\Phi}_1) = \mathbf{1} \quad (4.15b)$$

$$\boldsymbol{\Phi}_1 \succcurlyeq 0 \quad (4.15c)$$

$$\text{rank}(\boldsymbol{\Phi}_1) = 1 \quad (4.15d)$$

In problem (4.15), the rank-one constraint (4.15d) is non-convex and it is relaxed. Therefore, the SROCR method is utilized instead. After adding the eigenvalue-related constraint, the final convex form of the RIS phase optimization problem using SROCR is given as follows,

$$\begin{aligned} \max_{\Phi_1} \quad & \text{tr}\{\mathbf{A}\Phi_1\} \\ \text{s.t.} \quad & \text{tr}\{\mathbf{L}_j\Phi_1\} + l_j \leq 0, \quad \forall j \in \mathcal{K} \end{aligned} \quad (4.16a)$$

$$\bar{\mathbf{u}}(\Phi_1^{(n)})^H \Phi_1 \bar{\mathbf{u}}(\Phi_1^{(n)}) \geq m^{(n)} \text{tr}(\Phi_1), \quad (4.16b)$$

$$\text{diag}(\Phi_1) = \mathbf{1} \quad (4.16c)$$

$$\Phi_1 \succcurlyeq 0 \quad (4.16d)$$

4.2.2 Nonorthogonal Multiple Access (I)

In the subproblem of the RIS matrix optimization in phase I for the NOMA technique, the SINR of the user k can be rewritten by using the previous reformulations. Hence, the SINRs of the first phase are rewritten as follows,

$$\gamma_k^I = \frac{|q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}|^2}{\sum_{j=1+k}^K |q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}|^2 + \sigma_I^2} \quad (4.17)$$

By employing the LDT, and QT to this subproblem, the objective function is reformulated as,

$$\sum_{k=1}^K (2\Re\{\sqrt{1+\eta_k} \lambda_k^* (q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k})\} - |\lambda_k|^2 (\sum_{j=1}^k |q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}|^2 + \sigma_I^2)) \quad (4.18)$$

where the optimal value of the auxiliary variables, η_k and λ_k , are given by,

$$\eta_k^* = \frac{|q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}|^2}{\sum_{j=k+1}^K |q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}|^2 + \sigma_I^2} \quad \lambda_k^* = \sqrt{1+\eta_k} \frac{q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}}{\sum_{j=k}^K |q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}|^2 + \sigma_I^2} \quad (4.19)$$

By simplifying (4.18), we can rewrite it as follows,

$$\begin{aligned}
& \sum_{k=1}^K (2\Re\{\sqrt{1+\eta_k}\lambda_k^* q_{k,k}\} + \sum_{k=1}^K 2\Re\{\sqrt{1+\eta_k}\lambda_k^* \boldsymbol{\theta}_1 \mathbf{p}_{k,k}\}) \\
& - |\lambda_k|^2 \left(\sum_{j=1}^K (q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j})^H (q_{k,j} + \boldsymbol{\theta}_1 \mathbf{p}_{k,j}) \right) + \sigma_1^2) \\
& = 2\Re\{\boldsymbol{\theta}_1 \sum_{k=1}^K (\sqrt{1+\eta_k}\lambda_k^* \mathbf{p}_{k,k})\} - 2\Re\{\boldsymbol{\theta}_1 \sum_{k=1}^K \sum_{j=1}^K (|\lambda_k|^2 q_{k,j}^* \mathbf{p}_{k,j})\} \\
& + \boldsymbol{\theta}_1 \left(\sum_{k=1}^K \sum_{j=1}^K |\lambda_k|^2 \mathbf{p}_{k,j} * \mathbf{p}_{k,j}^H \right) \boldsymbol{\theta}_1^H \\
& = 2\Re\{\mathbf{g} \boldsymbol{\theta}_1^H\} - \boldsymbol{\theta}_1 \mathbf{G} \boldsymbol{\theta}_1^H + g
\end{aligned} \tag{4.20}$$

In addition, by considering $\phi_1 = \begin{bmatrix} \boldsymbol{\theta}_1^H \\ 1 \end{bmatrix}$, we can write the objective function in a convex quadratic form, given as,

$$\phi_1^H \mathbf{U} \phi_1 + g, \quad \mathbf{U} = \begin{bmatrix} -\mathbf{G} & \mathbf{g}^H \\ \mathbf{g} & 0 \end{bmatrix}$$

Then, since the SINR formulas of the NOMA case at the relay are the same as the SDMA technique, for the minimum SINR constraint at the relay for the NOMA technique subproblem, the same reformulations applied to (4.15) are valid and the same results can be used for this technique too.

Moreover, the SIC constraint should be reformulated in the NOMA case. According to (3.29) and the transformation of $\mathbf{h}'_{\text{BS},k} = \mathbf{h}_{\text{BS},k} + \boldsymbol{\theta}_1 \text{diag}(\mathbf{h}_{\text{RIS},k}) \mathbf{H}_{\text{BS,RIS}} = \phi_1^H \mathbf{A}'_k$, we can reformulate the SIC constraint as,

$$\begin{aligned}
|\mathbf{h}'_{\text{BS},k+1}\boldsymbol{\omega}_k|^2 - \sum_{j=k+1}^K |\mathbf{h}'_{\text{BS},k+1}\boldsymbol{\omega}_j|^2 &\geq \theta_{\text{SIC}}, \quad \forall k \in \{1, 2, \dots, K-1\} \\
|\phi_1^H \mathbf{A}'_{k+1}\boldsymbol{\omega}_k|^2 - \sum_{j=k+1}^K |\phi_1^H \mathbf{A}'_{k+1}\boldsymbol{\omega}_j|^2 &\geq \theta_{\text{SIC}}, \quad \forall k \in \{1, 2, \dots, K-1\}
\end{aligned} \tag{4.21}$$

$$\text{tr}\{\mathbf{X}_{k+1}^{(k)} \boldsymbol{\Phi}_1\} - \sum_{j=k+1}^K \text{tr}\{\mathbf{X}_{k+1}^{(j)} \boldsymbol{\Phi}_1\} \geq \theta_{\text{SIC}}, \quad \forall k \in \{1, 2, \dots, K-1\}$$

$$\text{tr}\{\mathbf{X}_{k+1} \boldsymbol{\Phi}_1\} \geq \theta_{\text{SIC}}, \quad \forall k \in \{1, 2, \dots, K-1\}$$

where $\mathbf{X}_i^{(j)} = \mathbf{A}'_i \boldsymbol{\omega}_j$, and $\mathbf{X}_i = \mathbf{X}_i^{(i-1)} - \sum_{j=i}^K \mathbf{X}_i^{(j)}$.

Finally, by using SDR and the SROCR method, the final convex form of the NOMA problem for RIS phase profile design in phase I is as follows,

$$\begin{aligned}
\max_{\boldsymbol{\Phi}_1} \quad & \text{tr}\{\mathbf{A}\boldsymbol{\Phi}_1\} \\
\text{s.t.} \quad & \text{tr}\{\mathbf{L}_j \boldsymbol{\Phi}_1\} + l_j \leq 0, \quad \forall j \in \mathcal{K}
\end{aligned} \tag{4.22a}$$

$$\text{tr}\{\mathbf{X}_{j+1} \boldsymbol{\Phi}_1\} \geq \theta_{\text{SIC}}, \quad \forall j \in \{1, 2, \dots, K-1\} \tag{4.22b}$$

$$\bar{\mathbf{u}}(\boldsymbol{\Phi}_1^{(n)})^H \boldsymbol{\Phi}_1 \bar{\mathbf{u}}(\boldsymbol{\Phi}_1^{(n)}) \geq m^{(n)} \text{tr}(\boldsymbol{\Phi}_1), \tag{4.22c}$$

$$\text{diag}(\boldsymbol{\Phi}_1) = \mathbf{1} \tag{4.22d}$$

$$\boldsymbol{\Phi}_1 \succcurlyeq 0 \tag{4.22e}$$

4.2.3 Orthogonal Multiple Access (I)

Using the same reformulation techniques applied previously for other cases and techniques, the SNR of the signal of user k at the relay for the OMA-based system is written as,

$$\gamma_k^{\text{I}} = \frac{|q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}|^2}{\frac{1}{K} \sigma_1^2} \tag{4.23}$$

Next, applying the LDT and QT reformulation methods to this subproblem results in the objective function in the form of

$$\sum_{k=1}^K 2\Re\{\sqrt{1 + \eta_k} \lambda_k^* (q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k})\} - |\lambda_k|^2 (|q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}|^2 + \frac{1}{K} \sigma_1^2)$$

where the optimal values for the auxiliary variables, η_k and λ_k , are obtained by the following equations,

$$\eta_k^* = \frac{|q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}|^2}{\frac{1}{K} \sigma_1^2} \quad \lambda_k^* = \sqrt{1 + \eta_k} \frac{q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}}{|q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}|^2 + \frac{1}{K} \sigma_1^2} \quad (4.24)$$

Afterwards, further simplification is applied for the objective function and converts it into a convex quadratic form, which is represented by,

$$\begin{aligned} & \sum_{k=1}^K 2\Re\{\sqrt{1 + \eta_k} \lambda_k^* q_{k,k}\} + \sum_{k=1}^K 2\Re\{\sqrt{1 + \eta_k} \lambda_k^* \boldsymbol{\theta}_1 \mathbf{p}_{k,k}\} \\ & - |\lambda_k|^2 ((q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k})^H (q_{k,k} + \boldsymbol{\theta}_1 \mathbf{p}_{k,k}) + \sigma_1^2) \\ & = 2\Re\{\boldsymbol{\theta}_1 \sum_{k=1}^K (\sqrt{1 + \eta_k} \lambda_k^* \mathbf{p}_{k,k})\} - 2\Re\{\boldsymbol{\theta}_1 \sum_{k=1}^K (|\lambda_k|^2 q_{k,k}^* \mathbf{p}_{k,k})\} \\ & + \boldsymbol{\theta}_1 (\sum_{k=1}^K (|\lambda_k|^2 \mathbf{p}_{k,k} * \mathbf{p}_{k,k}^H)) \boldsymbol{\theta}_1^H \\ & = 2\Re\{\mathbf{g} \boldsymbol{\theta}_1^H\} - \boldsymbol{\theta}_1 \mathbf{G} \boldsymbol{\theta}_1^H + g \end{aligned} \quad (4.25)$$

Then, the SNRs of the messages decoded in the relay are given by,

$$\gamma_k^R = \frac{\|\mathbf{h}_{k,R} + \mathbf{H}_{k,R} \boldsymbol{\theta}_1\|^2}{\frac{1}{K} \sigma_R^2} \quad (4.26)$$

Therefore, by simplifying (4.26), the minimum SNR constraint at the relay is reformulated and finalized as,

$$\begin{aligned} & \frac{\|\mathbf{h}_{j,R} + \mathbf{H}_{j,R} \boldsymbol{\theta}_1\|^2}{\frac{1}{K} \sigma_R^2} \geq \gamma_R^{th}, \quad \forall j \in \mathcal{K} \\ & \rightarrow \text{tr}\{\mathbf{E}_j \boldsymbol{\Phi}_1\} + h'_j + \frac{1}{K} \gamma_R^{th} \sigma_R^2 \leq 0 \\ & \rightarrow \text{tr}\{\mathbf{E}_j \boldsymbol{\Phi}_1\} + l_j \leq 0 \end{aligned} \quad (4.27)$$

where $\mathbf{E}_j$ is defined by the reformulation methods introduced in 4.14, and $l_j = h'_j + \frac{1}{K} \gamma_R^{th} \sigma_R^2$.

Finally, the SDR and SROCR methods are employed, which forms the final convex subproblem for the OMA technique as follows,

$$\begin{aligned} \max_{\Phi_1} \quad & \text{tr}\{\mathbf{A}\Phi_1\} \\ \text{s.t.} \quad & \text{tr}\{\mathbf{E}_j\Phi_1\} + l_j \leq 0, \quad \forall j \in \mathcal{K} \end{aligned} \quad (4.28a)$$

$$\bar{\mathbf{u}}(\Phi_1^{(n)})^H \Phi_1 \bar{\mathbf{u}}(\Phi_1^{(n)}) \geq m^{(n)} \text{tr}(\Phi_1), \quad (4.28b)$$

$$\text{diag}(\Phi_1) = \mathbf{1} \quad (4.28c)$$

$$\Phi_1 \succcurlyeq 0 \quad (4.28d)$$

4.2.4 Complexity analysis

Based on the complexity of the SDP outlined in previous sections, the worst-case computational complexity for the RIS phase profile optimization in phase I is given by $\mathcal{O}(\max\{K+3, N+1\}^4(N+1)^{1/2} \log(1/\epsilon_1))$ for the SDMA and OMA techniques. Given that the size of RIS is typically large and $N+1$ is usually greater than $K+3$, we simplify the complexity to $\mathcal{O}((N+1)^{4.5} \log(1/\epsilon_1))$. Moreover, the NOMA subproblem contains $K-1$ additional constraints for the SIC constraint. Thus, the complexity of the NOMA technique in this subproblem is calculated as $\mathcal{O}(\max\{2(K+1), N+1\}^4(N+1)^{1/2} \log(1/\epsilon_1))$.

4.3 RIS phase shift (Θ_2) design

In this section, the RIS phase profile is designed for phase II transmission. The channel equation between the relay and user k is written as $\mathbf{h}'_{\text{R},k} = \mathbf{h}_{\text{R},k} + \mathbf{h}_{\text{RIS},k} \Theta_2 \mathbf{H}_{\text{R},\text{RIS}}$, which can be reformulated as $\mathbf{h}_{\text{R},k} + \boldsymbol{\theta}_2 \text{diag}(\mathbf{h}_{\text{RIS},k}) \mathbf{H}_{\text{R},\text{RIS}} = \mathbf{h}_{\text{R},k} + \boldsymbol{\theta}_2 \mathbf{A}_k$ where $\boldsymbol{\theta}_2 = \text{diag}(\Theta_2)$ is considered as a row vector of $\boldsymbol{\theta}_2 =$

$[\theta_{2,1}, \theta_{2,2}, \dots, \theta_{2,N}]$. Therefore, the received signal in the first phase can be rewritten as,

$$\begin{aligned}
y_k^{\text{II}} &= (\mathbf{h}_{\text{R},k} + \boldsymbol{\theta}_2 \mathbf{A}_k) \mathbf{x}_{\text{R}} + n_k = \mathbf{h}_{\text{R},k} \mathbf{x}_{\text{R}} + \boldsymbol{\theta}_2 \mathbf{A}_k \mathbf{x}_{\text{R}} + n_k \\
&= \mathbf{h}_{\text{R},k} \mathbf{v}_k s_k + \sum_{j=1, j \neq k}^K \mathbf{h}_{\text{R},k} \mathbf{v}_j s_j + \boldsymbol{\theta}_2 \mathbf{A}_k \mathbf{v}_k s_k + \sum_{j=1, j \neq k}^K \boldsymbol{\theta}_2 \mathbf{A}_k \mathbf{v}_j s_j + n_k \\
&= (q_{k,k} + \boldsymbol{\theta}_2 \mathbf{p}_{k,k}) + \sum_{j=1, j \neq k}^K (q_{k,j} + \boldsymbol{\theta}_2 \mathbf{p}_{k,j}) + n_k \tag{4.29}
\end{aligned}$$

Similar techniques as before are applied for the problem reformulation, and since the objective functions of both subproblems follow the same form, and only parameters are different, we avoid repeating the same equations, and the final form of the subproblem is written for each MA technique.

4.3.1 Space Division Multiple Access (II)

Employing the same techniques outlined in phase I reformulation, the subproblem for optimizing the RIS phase shift matrix in phase II for the SDMA is expressed as follows,

$$\begin{aligned}
\max_{\boldsymbol{\Phi}_2} \quad & \text{tr}\{\mathbf{A}\boldsymbol{\Phi}_2\} \\
\text{s.t.} \quad & \bar{\mathbf{u}}(\boldsymbol{\Phi}_2^{(n)})^H \boldsymbol{\Phi}_2 \bar{\mathbf{u}}(\boldsymbol{\Phi}_2^{(n)}) \geq m^{(n)} \text{tr}(\boldsymbol{\Phi}_2), \tag{4.30a}
\end{aligned}$$

$$\text{diag}(\boldsymbol{\Phi}_2) = \mathbf{1} \tag{4.30b}$$

$$\boldsymbol{\Phi}_2 \succcurlyeq 0 \tag{4.30c}$$

The only constraint that affects this subproblem is the unit-modulus constraint for the RIS elements, and due to the high non-convexity of this equality, the SDR technique is considered for (4.30). Also, similar to the previous subproblems, the SROCR method is applied in order to find the nearest rank-one solution.

4.3.2 Nonorthogonal Multiple Access (II)

The SIC constraint in phase II is added to the subproblem for the NOMA case. By utilizing the same techniques introduced in the previous sections, the reformulation is similar to (4.21). Therefore, the convex optimization problem for the RIS phase shift matrix in phase II for the NOMA is given by,

$$\begin{aligned} \max_{\Phi_2} \quad & \text{tr}\{\mathbf{A}\Phi_2\} \\ \text{s.t.} \quad & \text{tr}\{\mathbf{X}_{j+1}\Phi_2\} \geq \theta_{\text{SIC}}, \quad \forall j \in \{1, 2, \dots, K-1\} \end{aligned} \quad (4.31a)$$

$$\bar{\mathbf{u}}(\Phi_2^{(n)})^H \Phi_2 \bar{\mathbf{u}}(\Phi_2^{(n)}) \geq m^{(n)} \text{tr}(\Phi_2), \quad (4.31b)$$

$$\text{diag}(\Phi_2) = \mathbf{1} \quad (4.31c)$$

$$\Phi_2 \succcurlyeq 0 \quad (4.31d)$$

where $\mathbf{X}_i = \mathbf{X}_i^{(i-1)} - \sum_{j=i}^K \mathbf{X}_i^{(j)}$ and $\mathbf{X}_i^{(j)} = \mathbf{A}'_i \mathbf{v}_j$ represent the reformulations done similar to phase I, and $\mathbf{A}'_k$ is defined as $\begin{bmatrix} \text{diag}(\mathbf{h}_{\text{RIS},k}) \mathbf{H}_{\text{R},\text{RIS}} \\ \mathbf{h}_{\text{R},k} \end{bmatrix}$ for the second phase.

4.3.3 Orthogonal Multiple Access (II)

The same reformulation steps are repeated for the OMA case, and the subproblem for optimizing the RIS phase shift profile in phase II is represented by

$$\begin{aligned} \max_{\Phi_2} \quad & \text{tr}\{\mathbf{A}\Phi_2\} \\ \text{s.t.} \quad & \bar{\mathbf{u}}(\Phi_2^{(n)})^H \Phi_2 \bar{\mathbf{u}}(\Phi_2^{(n)}) \geq m^{(n)} \text{tr}(\Phi_2), \end{aligned} \quad (4.32a)$$

$$\text{diag}(\Phi_2) = \mathbf{1} \quad (4.32b)$$

$$\Phi_2 \succcurlyeq 0 \quad (4.32c)$$

All of these convex subproblems can be handled by the CVX toolbox, and they are added to the AO algorithm. Further explanation is discussed in the next section.

4.3.4 Complexity analysis

According to the complexity of the SDP, the worst-case computational complexity for the RIS phase profile optimization in phase II is given by $\mathcal{O}(\max\{3, N + 1\}^4(N + 1)^{1/2} \log(1/\epsilon_1))$ for the SDMA and OMA techniques. Given that the size of RIS is usually large and $N + 1$ is greater than 3, we deduce $\max\{3, N + 1\} = N + 1$, leading to a simplified complexity of $\mathcal{O}((N + 1)^{4.5} \log(1/\epsilon_1))$. Moreover, the NOMA subproblem includes $K - 1$ additional constraints for the SIC constraint. Therefore, the complexity of the NOMA technique is calculated as $\mathcal{O}(\max\{K + 2, N + 1\}^4(N + 1)^{1/2} \log(1/\epsilon_1))$. Similarly, as the size of RIS typically is considered to be a large number, we can simplify the complexity and write it as $\mathcal{O}((N + 1)^{4.5} \log(1/\epsilon_1))$.

4.4 Alternating optimization

The integration of the relay-aided system with the RIS adds two subproblems to the main problem in order to optimize the RIS phase shift matrix at each transmission phase. Similar to the beamforming vectors, the RIS phase shift matrix at each transmission phase is independent of the other phase. Therefore, these subproblems can be added to the inner layer of the AO algorithm, each to the corresponding transmission phase.

The new auxiliary variables have been introduced for optimizing the Θ_1 and Θ_2 , at each phase, which are dependent on the beamforming vectors of the corresponding transmission phase, as well as the RIS phase profile. Hence, firstly beamforming vectors are optimized in the AO. Subsequently, the auxiliary variables are updated according to the suboptimal beamforming vectors and the RIS phase values of the previous iteration. Finally, the RIS phase profiles are designed according to the updated auxiliary variables and beamforming vectors. Moreover, similar to the relay-only case, these steps are repeated iteratively until the total objective value converges. The resulting AO algorithm for the RIS-Relay assisted

system is given in Algorithm 3 and Figure 4.2.

Algorithm 3 Alternating optimization

- 1: Initialize iteration index $n = 0$, feasible $\omega_k^{(0)}, \mathbf{v}_k^{(0)}$, convergence threshold ϵ
 - 2: **First iteration:**
 - 3: update $\alpha_{\omega,k}^{*(1)}$ and $\beta_{\omega,k}^{*(1)}$ by $\omega_k^{(0)}$
 - 4: $\omega_k^{(1)} \leftarrow$ solution of P1 for fixed $\Theta_1^{(0)}, \alpha_{\omega,k}^{*(1)}$ and $\beta_{\omega,k}^{*(1)}$
 - 5: update $\alpha_{v,k}^{*(1)}$ and $\beta_{v,k}^{*(1)}$ by $\mathbf{v}_k^{(0)}$
 - 6: $\mathbf{v}_k^{(1)} \leftarrow$ solution of P2 for fixed $\Theta_2^{(1)}, \alpha_{v,k}^{*(1)}$ and $\beta_{v,k}^{*(1)}$
 - 7: update $\eta_{1,k}^{*(1)}$ and $\lambda_{1,k}^{*(1)}$ by $\Theta_1^{(0)}$ and $\omega_k^{(1)}$
 - 8: $\Theta_1^{(1)} \leftarrow$ solution of P3 for fixed $\omega_k^{(1)}, \eta_{1,k}^{*(1)}$ and $\lambda_{1,k}^{*(1)}$
 - 9: update $\eta_{2,k}^{*(1)}$ and $\lambda_{2,k}^{*(1)}$ by $\Theta_2^{(0)}$ and $\mathbf{v}_k^{(1)}$
 - 10: $\Theta_2^{(1)} \leftarrow$ solution of P4 for fixed $\mathbf{v}_k^{(1)}, \eta_{2,k}^{*(1)}$ and $\lambda_{2,k}^{*(1)}$
 - 11: **repeat**
 - 12: update $\alpha_{\omega,k}^{*(n+1)}$ and $\beta_{\omega,k}^{*(n+1)}$ by $\omega_k^{(n)}$
 - 13: $\omega_k^{(n+1)} \leftarrow$ solution of P1 for fixed $\Theta_1^{(n)}, \alpha_{\omega,k}^{*(n+1)}$ and $\beta_{\omega,k}^{*(n+1)}$
 - 14: update $\alpha_{v,k}^{*(n+1)}$ and $\beta_{v,k}^{*(n+1)}$ by $\mathbf{v}_k^{(n)}$
 - 15: $\mathbf{v}_k^{(n+1)} \leftarrow$ solution of P2 for fixed $\Theta_2^{(n+1)}, \alpha_{v,k}^{*(n+1)}$ and $\beta_{v,k}^{*(n+1)}$
 - 16: update $\eta_{1,k}^{*(n+1)}$ and $\lambda_{1,k}^{*(n+1)}$ by $\Theta_1^{(n)}$ and $\omega_k^{(n+1)}$
 - 17: $\Theta_1^{(n+1)} \leftarrow$ solution of P3 for fixed $\omega_k^{(n+1)}, \eta_{1,k}^{*(n+1)}$ and $\lambda_{1,k}^{*(n+1)}$
 - 18: update $\eta_{2,k}^{*(n+1)}$ and $\lambda_{2,k}^{*(n+1)}$ by $\Theta_2^{(n)}$ and $\mathbf{v}_k^{(n+1)}$
 - 19: $\Theta_2^{(n+1)} \leftarrow$ solution of P4 for fixed $\mathbf{v}_k^{(n+1)}, \eta_{2,k}^{*(n+1)}$ and $\lambda_{2,k}^{*(n+1)}$
 - 20: $n = n + 1$
 - 21: **until** $R_T^{(n-1)} - R_T^{(n-2)} \leq \epsilon$ or $n = n_{\max}$
 - 22: $\mathbf{v}_k^* = \mathbf{v}_k^{(n)}, \omega_k^* = \omega_k^{(n)}, \Theta_1^* = \Theta_1^{(n)}, \Theta_2^* = \Theta_2^{(n)}$
-

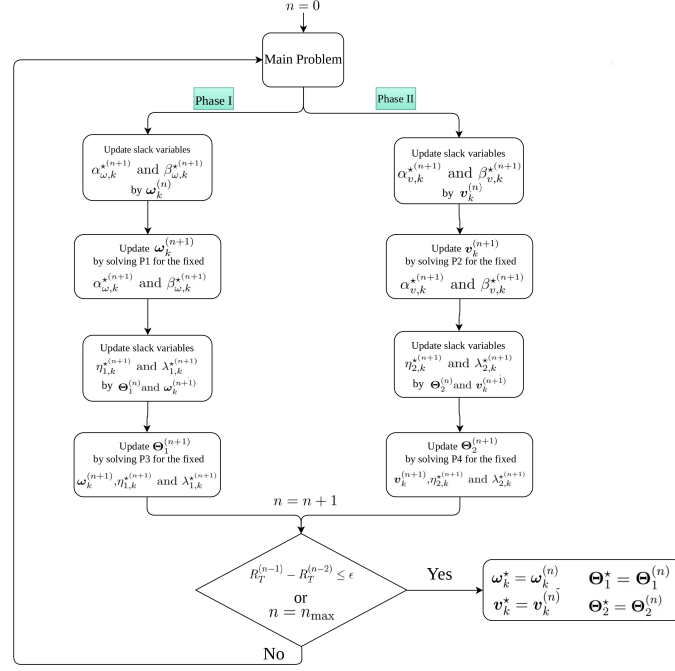


Figure 4.2: AO algorithm for RIS-relay-assisted system

4.4.1 Complexity analysis

Summing the complexities of the four subproblems mentioned in each section results in the overall complexity of the AO for each MA technique separately. For the SDMA and OMA techniques, the worst-case complexity of the AO is given by $\mathcal{O}(n_{\max}(I_{it}[K^{4.5}(M+1)^{4.5} + 2(N+1)^{4.5}]\log(1/\epsilon_1) + L^3))$, where $n_{\max}$ is the maximum number of iterations in AO. In addition, for the NOMA technique we can calculate the AO complexity by $\mathcal{O}(n_{\max}(I_{it}[K^{4.5}(M+1)^{4.5} + K^{4.5}(L+1)^{4.5} + 2(N+1)^{4.5}]\log(1/\epsilon_1)))$.

4.5 PSO algorithm

The PSO algorithm, outlined in Algorithm 4, randomly generates the position and velocity of particles at the beginning and then efficiently explores the solution

space to get the optimal or near-to-optimal solution by maintaining a population of particles with associated phase shift vectors. In each iteration, the particle updates its velocity and position by tracking two extremes until the upper limit of the number of iterations is reached.

Remark 1 In this study, the PSO algorithm is employed to solve two subproblems of an AO algorithm. Therefore, at each iteration of the AO, new random particle positions and velocities are generated. In addition, according to step 6 of Algorithm 4, the suboptimal solution of the previous AO iteration is set as one initial position of the PSO algorithm. This approach aims to maintain the increasing convergence behaviour of the AO and avoid fluctuation. In other words, if the PSO algorithm obtains a superior solution compared to the previous AO iteration, the suboptimal solution of the subproblem will be updated accordingly.

In this study, the optimization objective aims to maximize the sum rate of the system. Thus, the total rate calculation function is set to the fitness function, denoted as R_T in this manuscript. However, due to the constraints of the optimization problem, the fitness function needs to be adjusted such that the problem be reformulated as an unconstrained one. For the transformation from a constrained to an unconstrained problem, the penalty term is usually added to the objective function.

In this investigation, three constraints have been considered in different subproblems and MA techniques: The minimum SINR and unit-modulus constraints for all RIS phase shift optimization problems and the SIC constraint for the NOMA technique. The unit-modulus constraint implies that the amplitude of the phase shift elements in RIS must be one. This condition is automatically satisfied in the PSO algorithm, as the phase of these elements is optimized only while the amplitudes are fixed and set to 1. Moreover, the minimum SINR constraint at the relay is added as a penalty term to the fitness function as follows:

$$\text{SINR}_k(\boldsymbol{\theta}) = \text{sgn}(\gamma_k^R(\boldsymbol{\theta}) - \gamma_R^{th}) \quad (4.33)$$

where $\text{sgn}(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ is defined such that it is set to one of the constraint holds and to zero if the constraint is not satisfied.

Also, for the SIC technique in the NOMA technique, we have,

$$\text{SIC}_k(\boldsymbol{\theta}) = \text{sgn}(p_k(\boldsymbol{\theta}) - \theta_{\text{SIC}}) \quad (4.34)$$

where $p_k(\boldsymbol{\theta}) = |\mathbf{h}'_{\mathbf{x},k+1}\boldsymbol{\omega}_k|^2 - \sum_{j=k+1}^K |\mathbf{h}'_{\mathbf{x},k+1}\boldsymbol{\omega}_j|^2$ and for the channel between the transmitter and receiver we have $\mathbf{h}'_{\mathbf{x},k+1} = \mathbf{h}_{\mathbf{x},k+1} + \boldsymbol{\theta} \text{diag}(\mathbf{h}_{\text{RIS},k+1})\mathbf{H}_{\mathbf{x},\text{RIS}}$ where $\mathbf{x}$ is either the BS or R and are defined for phase I and phase II, respectively.

Therefore, the final penalty-based fitness function for the PSO algorithm in SDMA and OMA techniques is given by,

$$f(\boldsymbol{\theta}) = -R_T - G \left(\sum_{k=1}^K (\text{SINR}_k(\boldsymbol{\theta})) - K \right) \quad (4.35)$$

where G is the punishment factor and is a sufficiently large number, which is set to 10^5 in this study.

Also, in the NOMA technique, by adding the SIC, the fitness function is reformulated as,

$$f(\boldsymbol{\theta}) = -R_T - G \left(\sum_{k=1}^K (\text{SINR}_k(\boldsymbol{\theta}) + \text{SIC}_k(\boldsymbol{\theta})) - (2K - 1) \right) \quad (4.36)$$

4.5.1 Update formula

Each particle indexed $i = 1, 2, \dots, Z$ at time t is associated with a phase shift vector $\boldsymbol{\theta}_i^{(t)} = (\theta_{i1}, \dots, \theta_{iN})$ with size $1 \times N$, where each component is limited within the range $[0, 2\pi]$. The fitness value of each particle is evaluated by the fitness function denoted as $f(\boldsymbol{\theta})$. To improve the search ability, the particle velocity should have inertia. Generally, a larger inertia weight facilitates the global search, while a smaller weight is more conducive to the local search. Here,

the weight factor is set to

$$\omega = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \cdot (t/T)^2 \quad (4.37)$$

where $\omega_{\min}$ and $\omega_{\max}$ represent the minimum and maximum inertia weight, respectively [146,147] and T denotes the total number of iterations, and ω diminishes as the number of iterations increases. With this adjustment, the particle search has a strong global search ability in the early stage and a stronger local search ability in the later stage of iteration. Each velocity and particle is updated according to the following equations in t th iteration:

$$\begin{aligned} \mathbf{u}_i^{(t+1)} &= \omega \mathbf{u}_i^{(t)} + c_1 r_1 (\mathbf{p}_i^{(t)} - \boldsymbol{\theta}_i^{(t)}) + c_2 r_2 (\mathbf{g}^{(t)} - \boldsymbol{\theta}_i^{(t)}), \\ \boldsymbol{\theta}_i^{(t+1)} &= \boldsymbol{\theta}_i^{(t)} + \mathbf{u}_i^{(t+1)} \end{aligned} \quad (4.38)$$

where $\mathbf{p}_i^{(t)}$ represents the individual historical optimal solution (personal best) of any particle i in the t th iteration, $\mathbf{g}^{(t)}$ denotes the global optimal solution (global best) in the t iteration, c_1 and c_2 denote the learning factor, and r_1 and r_2 are the random numbers between 0 and 1. The main parameters for the proposed PSO algorithm are set as follows: $Z = \min\{200, 10N\}$, $T = 10N$, $\omega = 0.9$, $c_1 = c_2 = 1.49$, $\mathbf{u}_i^{(t)} = (u_{i1}, \dots, u_{iN})$, each of which is limited within $[-u_{\max}, u_{\max}]$, with $u_{\max} = \frac{\pi}{8}$.

4.5.2 Complexity analysis

According to [146], the complexity of the PSO algorithm is given by $\mathcal{O}(MT)$, where T is the number of iterations, and M is the number of particles. Hence, the complexity of the PSO algorithm defined in Algorithm 4 is $\mathcal{O}(ZT)$. Typically, the RIS size is considered large. Therefore, we can approximate $Z = \min\{200, 10N\}$ to 200. Also, we have the $T = 10N$. So, the resulting complexity can be rewritten as $\mathcal{O}(N)$.

Furthermore, the AO complexity when the PSO is used for the RIS phase profile design for both phases instead of the CVX toolbox can be reduced to $\mathcal{O}(n_{\max}(I_{it}K^{4.5}(M+1)^{4.5}\log(1/\epsilon_1) + L^3 + N))$ for the SDMA

Algorithm 4 Particle swarm optimization algorithm (PSO)

```

1: Particle swarm parameters initialization:
2: Initialize  $Z, T, c_1, c_2, r_1, r_2, \omega_{\max}, \omega_{\min}$ 
3: for  $i = 1, 2, \dots, Z$  do
4:   Initialize  $\theta_i^{(0)}, \mathbf{u}_i^{(0)}, \mathbf{p}_i^{(0)} = \theta_i^{(0)}$ 
5: end for
6: Set the suboptimal point of the previous iteration of the AO algorithm as one of
   the initial particles:  $\mathbf{p}_1^{(0)} = \theta_1^{(0)} = \theta^{*(n-1)}$ 
7: Find  $f(\mathbf{p}^{*(0)}) = \min\{f(\mathbf{p}_1^{(0)}), f(\mathbf{p}_2^{(0)}), \dots, f(\mathbf{p}_Z^{(0)})\}$  and set  $\mathbf{g}^{(0)} = \mathbf{p}^{*(0)}$ 
8:  $\omega = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \cdot (1/T)^2$ 
9: while  $t \leq T$  do
10:  for  $i = 1, 2, \dots, Z$  do
11:    Update the velocity and position of particles:
12:     $\mathbf{u}_i^{(t+1)} = \omega \mathbf{u}_i^{(t)} + c_1 r_1 (\mathbf{p}_i^{(t)} - \theta_i^{(t)}) + c_2 r_2 (\mathbf{g}^{(t)} - \theta_i^{(t)})$ 
13:     $\theta_i^{(t+1)} = \theta_i^{(t)} + \mathbf{u}_i^{(t+1)}$ 
14:    Evaluate fitness value;
15:    Calculate the historical optimal position of particle  $i$ :
16:     $\mathbf{p}_i^{(t+1)} = \begin{cases} \mathbf{p}_i^{(t)}, & f(\mathbf{p}_i^{(t)}) \leq f(\theta_i^{(t+1)}) \\ \theta_i^{(t+1)}, & f(\mathbf{p}_i^{(t)}) > f(\theta_i^{(t+1)}) \end{cases}$ 
17:    Find  $f(\mathbf{p}^{*(t)}) = \min\{f(\mathbf{p}_1^{(t+1)}), \dots, f(\mathbf{p}_Z^{(t+1)})\}$ 
18:  end for
19:  Calculate the historical optimal position of the population:
20:   $\mathbf{g}^{(t+1)} = \begin{cases} \mathbf{g}^{(t)}, & f(\mathbf{g}^{(t)}) \leq f(\mathbf{p}^{*(t+1)}) \\ \mathbf{p}^{*(t+1)}, & f(\mathbf{g}^{(t)}) > f(\mathbf{p}^{*(t+1)}) \end{cases}$ 
21:  Find  $f(\mathbf{p}^{*(t)}) = \min\{f(\mathbf{p}_1^{(t+1)}), \dots, f(\mathbf{p}_Z^{(t+1)})\}$ 
22:  set  $t \leftarrow t + 1$ 
23:   $\omega = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \cdot (t/T)^2$ 
24: end while

```

and OMA cases. Additionally, the complexity order for NOMA is given by $\mathcal{O}(n_{\max}(I_{it}[K^{4.5}(M+1)^{4.5} + K^{4.5}(L+1)^{4.5}]\log(1/\epsilon_1) + N))$. Therefore, the complexity of the AO algorithm significantly decreases by using the PSO instead of the CVX toolbox, as can be concluded from Fig. 5.7-(a).



Chapter 5

Numerical results

In this chapter, we explain the scenarios, performance metrics, and comparisons based on the results of a series of simulations. In the forthcoming sections, the results for the relay-aided systems from Chapter 3 and hybrid RIS-relay systems from Chapter 4 are discussed separately. In general, we compare three multiple access techniques named "Space Division Multiple Access (SDMA)," "Non-Orthogonal Multiple Access (NOMA)," and "Orthogonal Multiple Access (OMA)" for each case. In addition, the performance of different systems is represented as the proposed approach and benchmarks for each section according to the investigated network in chapters 3 and 4, and they are discussed more explicitly in the following sections.

5.1 Relay-aided system results

In this section, we investigate the theoretical results and the proposed approach based on various numerical examples for the relay-aided systems described in Chapter 3. The simulation setting includes one BS, one Relay, and K users, and according to Fig. 3.1, their locations are given in a three-dimensional coordinate system as (0m, 70m, 10m), and (100m, 0m, 0m), respectively. Also, K users are

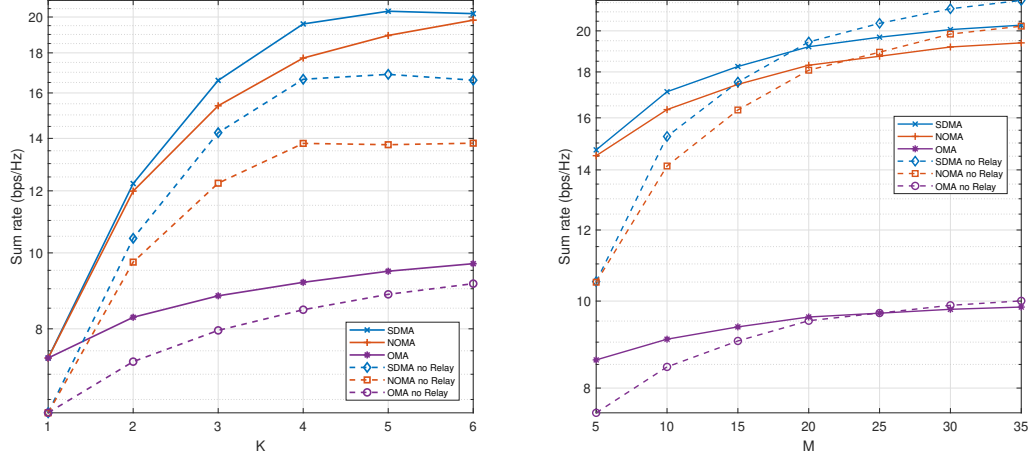
Table 5.1: Channel characteristics for relay aided system

Channel parameter	Value
$\sigma_I^2, \sigma_{II}^2$	-90 dBm
PL_0	-30 dB
α	3.5
θ_{SIC} (SIC detection threshold)	10^{-11}

randomly distributed within a circle centered at (200m, 70m, 0m) with a radius of $r = 20$ m. As mentioned in Chapter 3, the channels between BS and Relay, Relay and the users, and BS and the users are considered to be Rayleigh fading. The path loss exponent, α , for BS-Relay, BS-user, and Relay-user channels is considered to be 3.5, and noise variances for both transmission phases, $\sigma_I^2, \sigma_{II}^2$ are set to be -90 dBm.

Two systems are considered for the numerical results of this section: with-relay system as the proposed approach and without considering relay (no Relay) serving as the benchmark; in each system, all three multiple access techniques have been investigated. In addition, the maximum transmit power at BS ($P_{\max, BS}$) and the maximum transmit power at relay (P_R) are set to 33 dBm and 30 dBm, respectively for all cases except the ones that have defined their values specifically. Also, the number of Antennas at the BS (M), at the relay (L), and the number of users (K) are considered as $(K, M, L) = (3, 6, 6)$ for all cases except the ones that have specific values. Also, in relay-aided systems, δ_{time} is set to $\frac{1}{2}$, and in no relay systems, it is set to 1. Furthermore, the NOMA technique needs an SIC threshold (θ_{SIC}) for the SIC constraint. According to [148], the SIC threshold is set to be 10dB larger than the noise variance. Therefore, it is set to $\theta_{SIC} = 10^{-11}$. The simulations are performed on a laptop computer with an AMD Ryzen 7, 3.20 GHz CPU, and 16GB RAM, and MATLAB R2022b is used for the proposed algorithm.

In Fig. 5.1-(a), sum rate versus different K values are demonstrated. It can be seen that for a single user, all techniques are reduced to equal form since there is no interference, and the sum rate achieved by the user is the same for all MA



(a) Sum rate vs K

(b) Sum rate vs M

Figure 5.1: Sum rate comparison for different K and M values in relay-aided system

techniques. Moreover, for the K larger than one, the SDMA technique can outperform the NOMA and OMA techniques, which confirms the better performance of the SDMA technique in *underloaded* and *critically loaded* networks. It is also clear that the sum rate of all schemes rises with K and reaches a peak at a certain point. At this point, the lower transmit power allocated for each user (given fixed BS and relay transmit power constraints), coupled with the heightened multiuser interference, negatively impacts the performance [32, 149]. In addition, adding a DF-relay with the same number of antennas as BS to the system can increase the achieved sum rate. This is due to the channel gains between the relay and users, which are stronger, and half of the rate achieved by the user is attributed to the signals transmitted by the relay.

Next, Fig. 5.1-(b) compares the sum rate vs the number of BS antenna (M) performance. It is seen that for lower numbers of antennas, which are near the number of antennas at the relay, the relay-assisted system can outperform the system without a relay. However, as the number of antennas M increases, the base station (BS) can adjust the beamforming vectors with more elements. This enhances the gain of the transmitted signal received by the user. Since half of the rate is attributed to the transmitted signal from the relay with a low number

of antennas, the no-relay system with a large M can exhibit a higher sum rate. However, adding a DF-relay introduces a trade-off, as increasing the number of antennas at the BS may not always be cost-efficient. Also, it is clear that SDMA performs better than NOMA and OMA techniques, and SDMA in a no-relay system needs fewer additional antennas than NOMA and OMA to outperform the relay-aided system.

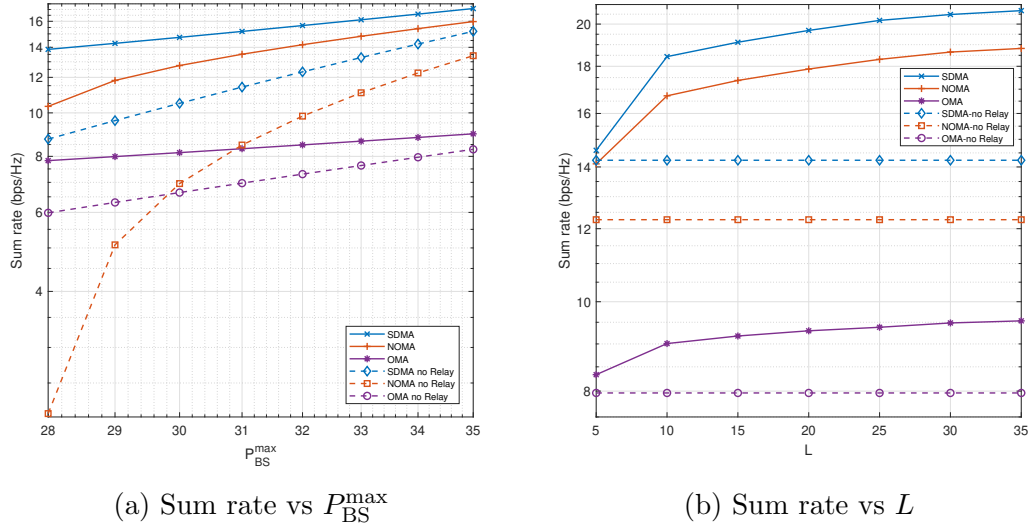


Figure 5.2: Sum rate comparison for different $P_{max,BS}$ and L values in relay-aided system

Fig. 5.2-(a) represents the sum rate for different values of maximum transmit power at the BS. It is shown that the sum rate of all schemes increases with the increase of the total transmit power since the gains of signals increase. Notably, the proposed SDMA scheme is superior to other benchmark schemes. Furthermore, the NOMA technique exhibits inferior performance than OMA in lower powers. This is due to the SIC constraint, whereby the lower power limits the intended signal gain received by the user, thus constraining performance. In addition, it can be seen that the gap between the SDMA and NOMA techniques is narrower in the relay-aided system compared to the direct transmission without a relay. This is because the sum rate of the system is dominated by phase II transmission in a relay-aided system, and the sum rate achieved in phase II of the transmission is larger than in phase I. Additionally, due to the larger channel gain between the relay and the users compared to that between the BS and users,

the impact of the SIC constraint on the suboptimal optimization solution is less in Phase II than in Phase I. Consequently, this leads to a higher sum rate during phase II transmission, achieving a sum rate for the NOMA closer to that of the SDMA technique.

Additionally, Fig. 5.2-(b) illustrates the sum-rate vs number of relay antennas (L) performance. The results indicate that the sum rate of the relay-aided system for all MA techniques increases by increasing L since the relay has more degrees of freedom to adjust the beamforming vectors better. Conversely, the value of L does not affect the systems without the relay. Also, it can be seen that when L increases from 5 to 35, the gain of the relay-aided system gradually decreases for all MA techniques.

Furthermore, in Fig. 5.3, the AO algorithm is compared to two benchmarks, denoted as “BS beamforming only” and “Relay beamforming only”. At the benchmarks, the beamforming vectors of the BS and relay are optimized individually, by considering the other’s beamforming vectors to be random and fixed, respectively. It is seen that the AO outperforms the benchmarks, as all of the beamforming vectors are optimized, and a suboptimal objective value is computed jointly.

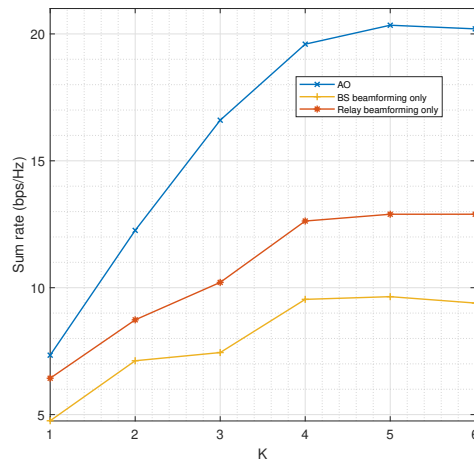


Figure 5.3: Comparison of AO with each phase in relay-aided system

5.2 RIS-relay-aided system results

In this section, we investigate the theoretical results and the proposed approach based on various numerical examples for the systems outlined in Chapter 4. According to Fig. 4.1, an RIS is located at coordinates (100m, 140m, 10m) of the previous system depicted in Fig. 3.1. For the RIS-aided communication links, both small-scale and large-scale fading are considered. It is assumed that line-of-sight (LoS) communication links exist between the BS and RIS, Relay and RIS, and the links between RIS and the users. Therefore, the RIS-related channels experience small-scale fading that follows a Rician fading. In addition, other channels follow the Rayleigh distribution with zero mean and unit variance, as mentioned in the previous section. For the RIS-aided channels, PL_0 is set to 10^{-3} , and the path loss exponent for the LoS components, α_{LoS} , and the NLoS components, α_{NLoS} , are set to 2 and 2.5, respectively, for all rician fading RIS-related channels. Also, for the time allocation between the transmission phases, for the hybrid RIS and relay system, and relay-aided systems δ_{time} is set to $\frac{1}{2}$; otherwise, it is considered as 1.

Channel parameter	Value
$\sigma_I^2, \sigma_{II}^2$	-90 dBm
α for rayleigh fading channels	3.5
α_{LoS} for rician fading channels	2
α_{NLoS} for rician fading channels	2.5

Table 5.2: Channel characteristics for hybrid RIS-relay-aided system

In the results, the proposed approach for this section is labeled as “hybrid,” referring to the performance of the hybrid RIS and relay aided system. Additionally, “relay-only,” “RIS-only,” and “random RIS” denote the relay-aided system, RIS-aided system, and RIS-aided system with a random RIS phase profile, respectively. Also, the legends “CVX” and “PSO” refer to the scenarios where the RIS phase profiles are designed by the model-based algorithm using the CVX toolbox and the metaheuristic PSO algorithm, respectively. Also, the number of Antennas at the BS (M), at the relay (L), the number of users (K), and the number of RIS elements (N) are set as $(K, M, L, N) = (3, 6, 6, 50)$ for all scenarios,

except the cases that have specific values.

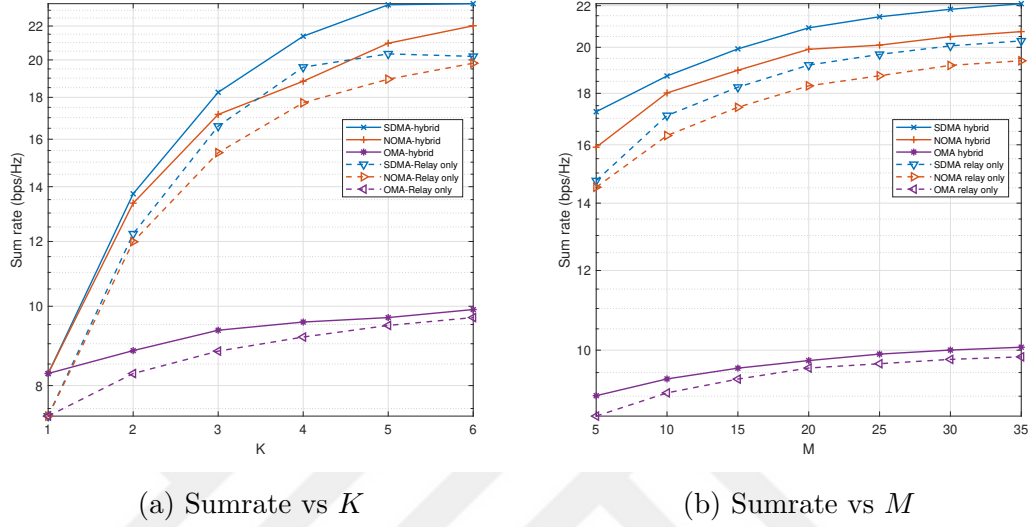


Figure 5.4: Sum rate comparison for different K and M values in hybrid RIS-relay-aided and relay-only systems.

In Fig. 5.4-(a), the sum rate of the network is illustrated for the hybrid and relay-only systems for all MA techniques. It can be seen that adding RIS can improve the sum rate of the system for all MA techniques. Consistent with previous results, SDMA outperforms NOMA and OMA techniques. Furthermore, by increasing the number of users, the gap between the sum rate in the hybrid and relay-only systems for SDMA and NOMA techniques increases, while in the OMA technique, it diminishes. This is because the RIS adjusts the phase of propagation, thereby reducing the interference between the users in SDMA and NOMA. In other words, the SDMA and NOMA leverage both the higher combined channel gain and interference control by the added auxiliary link gain and DoF in RIS, respectively. However, since there is no interference in the OMA technique, only the increase in the channel gain causes a rise in the sum rate of this MA technique.

Fig. 5.4-(b) represents the sum rate of the hybrid RIS-relay-aided system versus the different numbers of BS antennas. It can be observed that the performance of the hybrid systems can also be enhanced by increasing the number of antennas at the BS, similar to the other schemes in this figure and Fig. 5.1-(b). Additionally, a

hybrid system with joint optimization of active and passive beamforming vectors can achieve a better sum rate than the relay-aided system in all multiple access techniques. Moreover, increasing M generally provides diminishing returns for all schemes.

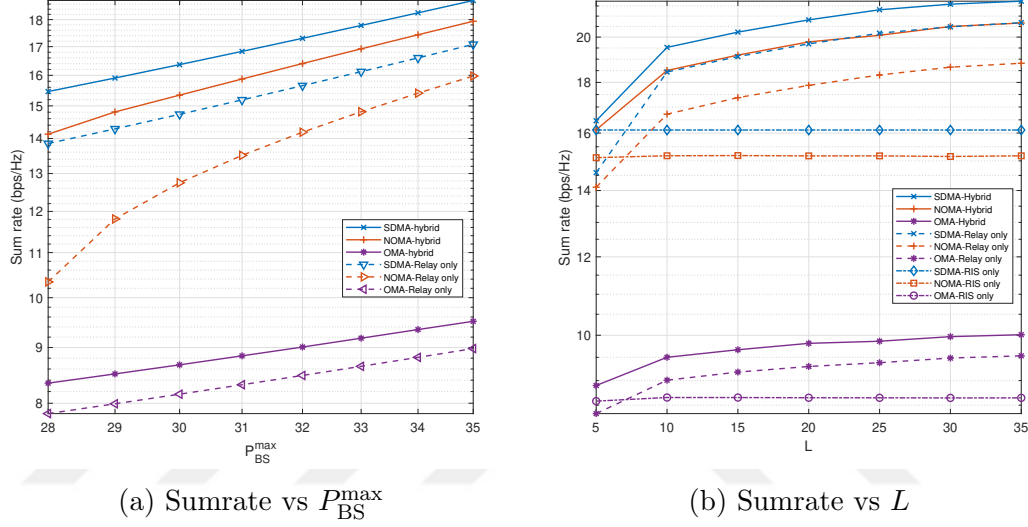


Figure 5.5: Sum rate comparison for different L and $P_{BS}^{\max}$ values in hybrid RIS-relay-aided and relay-only systems.

In Fig. 5.5-(a), the sum rate of the hybrid system is compared to that of the relay-aided system for different values of maximum transmit power at the BS. From this figure, it is observed that as the maximum transmit power increases, the sum rate of all schemes rises. In addition, the sum rate of the hybrid system outperforms that of the relay-only system. This improvement stems from leveraging the RIS to adjust the propagation direction of the signals by designing passive beamforming vectors, which adds more degrees of freedom for transmission. Moreover, RIS adds an auxiliary transmission link to the direct link, which helps to increase the channel gain and leads to a higher sum rate for the users. Furthermore, although the SDMA outperforms other techniques, the gap between the SDMA and NOMA in the hybrid system is smaller than in the relay-only system. This is due to the SIC constraint. In other words, RIS increases the channel gain between the transmitter and the user, which makes the SIC constraint hold easier in the hybrid system. Conversely, this constraint has more impact on the optimization in the relay-only system, leading to a lower sum rate for the NOMA

technique.

Furthermore, Fig. 5.5-(b) plots the sum rate of the hybrid, relay-only, and RIS-only systems and compares the sum rate achieved by MA techniques for different numbers of antennas at the relay, ranging from 5 to 35. It can be seen that by increasing L , the degree of freedom at phase II increases, which helps the relay adjust the beamforming vectors better and achieve a higher sum rate for the users for all MA techniques. Additionally, it can be concluded that adding RIS also improves the performance of the system by adding degrees of freedom to the performance optimization and adjusting the propagation direction of the signals, resulting in a higher signal gain. Finally, according to the fact that the RIS performs better than a DF relay if the number of RIS phase shift elements is significantly larger than the relay antennas. Hence, it is observed that when the number of antennas at the relay, L , is less than 8, the number of RIS elements N , which is set to 50, far exceeds the L . Consequently, the RIS-only system can outperform the DF relay-only system for all MA techniques in this range.

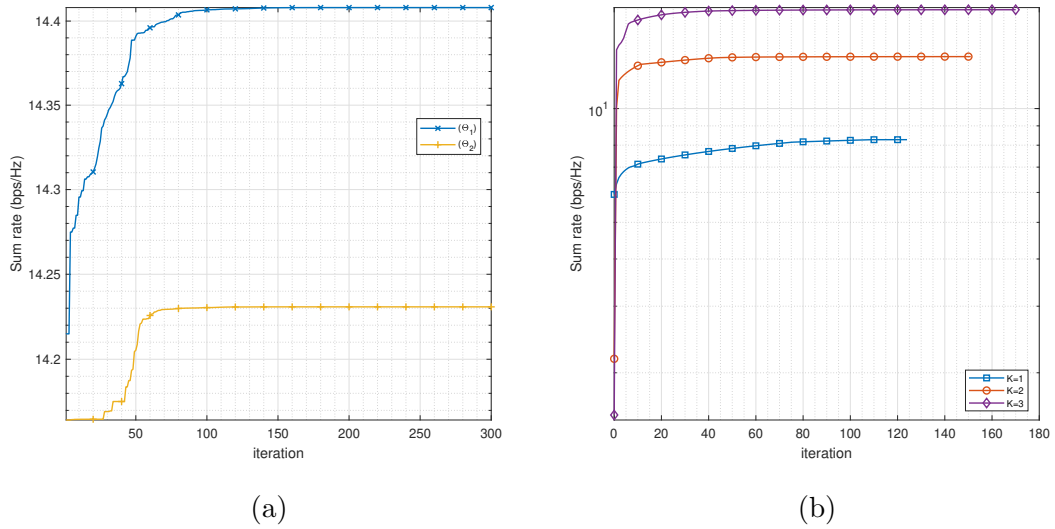


Figure 5.6: (a) Convergence behavior of PSO algorithm in the second iteration of AO algorithm for the SDMA technique for both phases. (b) Convergence behaviour for different numbers of users for SDMA technique.

Fig. 5.6-(a) presents the convergence behaviour of the PSO technique in the second iteration of AO, for $N = 50$, for the SDMA technique. In this figure, it

can be seen that the algorithm converges until a local optimal solution is found. In addition, Fig. 5.6-(b) examines the iterative improvement of the AO for the hybrid RIS and relay system for different K values. It is clear that by increasing the number of users, the number of optimization variables rises, hence requiring more iterations for holding the convergence criteria and obtaining the suboptimal solution.

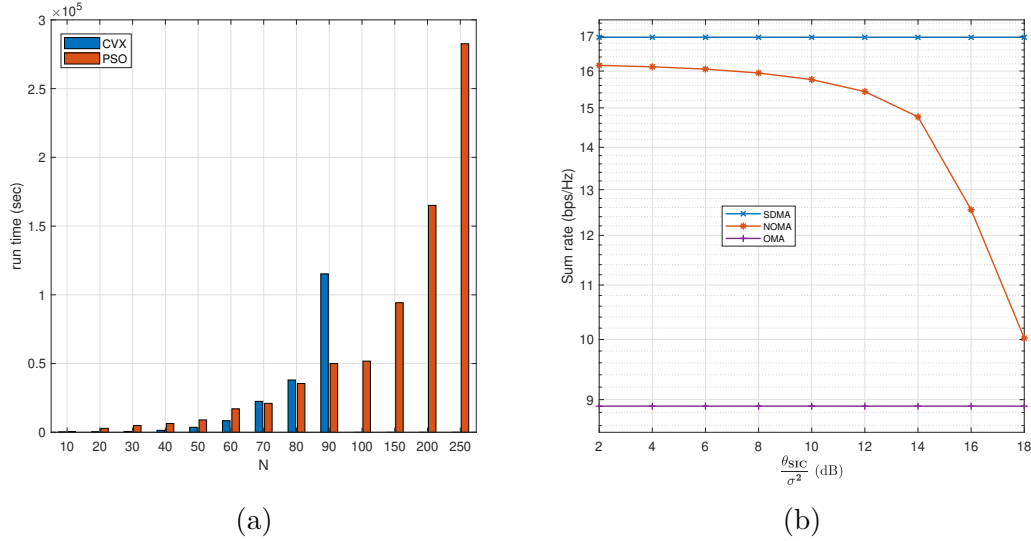


Figure 5.7: (a) run-time comparison between the CVX and PSO (b) Sum rate comparison for different ratios of the SIC threshold (θ_{SIC}) and noise variance (σ^2)

In Fig. 5.7-(a), the required run time for the PSO and model-based method using the CVX toolbox is compared. It can be seen that CVX can solve the problem faster than the PSO algorithm for smaller numbers of variables. However, as the number of optimization variables increases, the complexity of the model-based method rises, and solving by the CVX toolbox becomes less time-efficient. Also, after a certain number of variables, the CVX is not able to solve the problem, which depends on the hardware system. In contrast, the PSO algorithm is capable of solving problems with larger sizes much more rapidly. Due to the fact that the number of required iterations for the PSO algorithm, T , increases by 10 times N , thus, in the larger sizes of RIS, T becomes a very large amount. Therefore, the run time for the PSO algorithm in larger RIS sizes also increases significantly.

Fig. 5.7-(b) demonstrates the impact of the SIC constraint and the value of the

SIC threshold on the performance of the NOMA technique for $(K, L, M, N) = (3, 6, 6, 10)$. It is observed that when θ_{SIC} is small and close to the noise variance, the NOMA achieves a performance comparable to the SDMA technique since the SIC constraint is easier to satisfy. However, as θ_{SIC} increases, the performance of the NOMA significantly drops, approaching the sum rate achieved by the OMA technique.

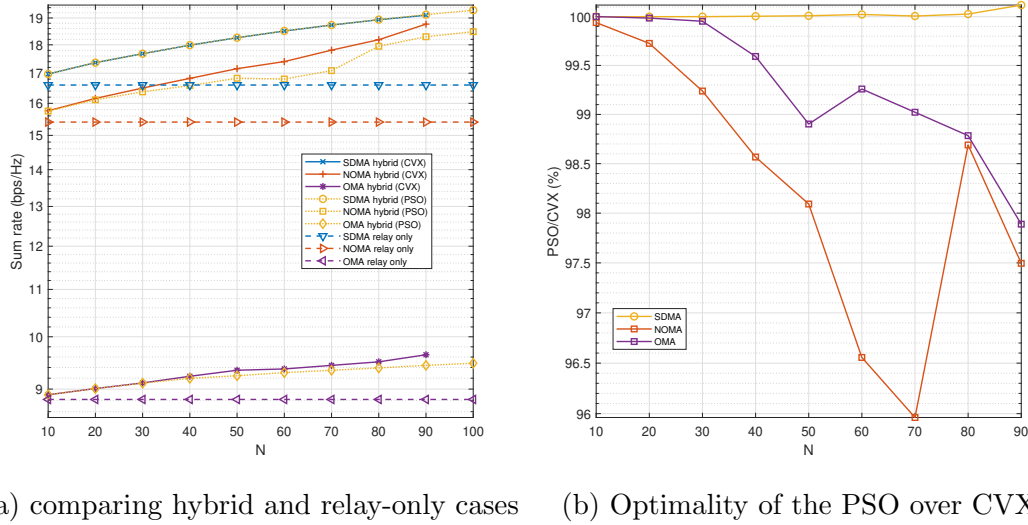


Figure 5.8: sum rate vs N for different scenarios comparing hybrid and relay-only systems and comparing the optimality gap of the PSO over CVX solutions for MA techniques in the hybrid system shown in (a).

In Fig. 5.8-(a), the sum rate of the hybrid RIS and relay system is illustrated for various numbers of RIS phase shift elements, N , and compared to the relay-only system for all MA techniques. It can be concluded that using RIS improves the sum rate of the system for all optimized schemes in the hybrid system. Moreover, it can be seen that in the SDMA and OMA techniques, the PSO is capable of obtaining the near solution to the CVX, while in NOMA, the additional SIC constraint increases the complexity of the PSO technique and limits the subspace for the particles, resulting in an optimality gap between the CVX and PSO solutions. In addition, it is demonstrated that the PSO is capable of solving the problem for larger sizes, whereas the CVX is limited in handling a certain size and complexity.

Furthermore, in Fig. 5.8-(b), we calculate the optimality gap between the solutions derived from the PSO approach and those obtained via the CVX toolbox for the schemes presented in Fig. 5.8-(a), considering SDMA, NOMA, and OMA. Notably, it is observed that PSO performs remarkably for the proposed MA technique, SDMA, yielding solutions in close proximity to those of the CVX toolbox. Even by increasing the number of RIS elements, PSO is able to obtain a better solution than the CVX toolbox. However, in the NOMA and OMA systems, by increasing the RIS size, the optimality gap generally increases, and it becomes hard for the PSO to obtain a closer solution to the CVX toolbox. Particularly in the NOMA scenario that has additional SIC constraints, which limit the solution space, the optimality gap between the PSO and CVX can reach up to 4%, while the maximum optimality gap for the OMA method is approximately 2%.

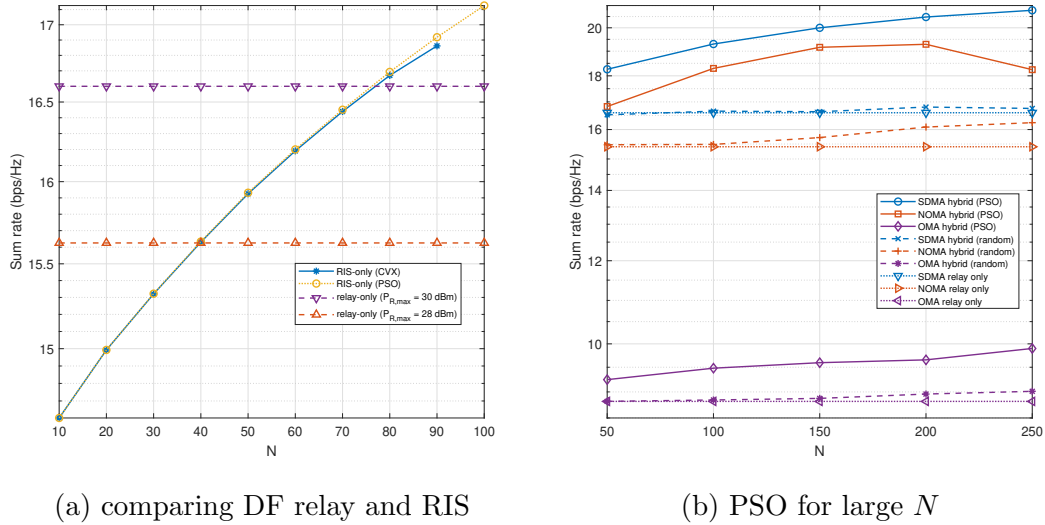


Figure 5.9: (a) Sum rate vs N for comparing the RIS-only and relay-only systems. (b) Comparison of different scenarios in hybrid system with optimized RIS profiles by the PSO, with the random RIS phase profile, and relay-only systems for large N .

Fig. 5.9-(a) illustrates the sum rate of the relay-only and RIS-only systems. As discussed in the previous sections, the RIS-only system can outperform the DF relay-assisted system when the RIS size significantly exceeds the number of antennas at the relay. This threshold size of the RIS also varies depending on the maximum transmit power at the relay. In other words, by increasing

the maximum transmit power at the relay, the required number of phase shift elements at the RIS in order to outperform DF-relay increases.

Finally, Fig. 5.9-(b) illustrates the sum rate for larger RIS sizes, where the CVX toolbox reaches its computational limits and is not able to solve. This figure compares the sum rate in the hybrid system with optimized RIS profiles, random RIS phase profiles, and the relay-only system. In particular, with an increase in the RIS size, the sum rate of the system rises for the MA techniques due to the higher DoF of the RIS and the combined channel gain, as previously highlighted. Furthermore, we can see that in the NOMA technique, the PSO optimizes the RIS profile, resulting in the increase in the sum rate of the system until a certain amount, and beyond a specific RIS size, the PSO is unable to obtain a better solution and increase the sum rate for larger RIS sizes. According to the optimality gap and the performance of the PSO technique with large N for the NOMA technique, we can conclude that PSO may not be well-suited for this MA technique. Conversely, it demonstrates good performance for the proposed SDMA method and OMA, thereby enhancing performance in these scenarios. The fluctuation in the optimality percentage of all schemes is due to the fact that at each AO iteration and for each size of RIS, the particles, and velocities of the PSO algorithm are generated randomly, and initial positions are not the same. Therefore, in some sizes, PSO obtains closer and better solutions, and in some other N values, the optimality gap increase to as large as 4%.

Chapter 6

Conclusions and future work

6.1 Conclusions

In this thesis, we investigate the active beamforming design for three multiple access techniques, SDMA, NOMA, and OMA, for maximizing the downlink sum rate in a DF relay-aided MU-MISO system. Then, we extend the network by adding an RIS and investigate the active and passive beamforming design for the hybrid RIS-relay system. Due to the non-convexity of the sum rate maximization problem, it is decoupled into four subproblems. By reformulating the non-convex subproblems into equivalent convex ones using LDT, QT, and SDR methods, the suboptimal solutions are obtained by the model-based alternative optimization algorithm with the CVX optimization toolbox followed by the SROCR rank reduction method in order to minimize the optimality gap. We show that the SDMA technique outperforms the NOMA and OMA methods in the *underloaded* and *critically loaded* multiuser MISO systems. In addition, the results illustrate that adding a DF relay enhances performance. Also, the hybrid RIS-relay systems outperform the relay-only and RIS-only systems. Furthermore, it is shown that the CVX toolbox is capable of obtaining the suboptimal solutions until a certain number of sizes are obtained for the RIS. Therefore, a constrained meta-heuristic PSO algorithm is designed for the RIS phase profile design, and it is

shown that for the SDMA and OMA techniques, the PSO can obtain a solution near the values obtained by the model-based algorithm. However, in the NOMA technique, the additional SIC constraints that restrict the feasible subspace may select a wrong searching direction and miss the near global optimality, leading to an optimality gap between the model-based and metaheuristic approaches.

6.2 Future work

- **Rate Splitting Multiple Access (RSMA):** In this thesis, the system was limited to *underloaded* and *critically loaded* systems, and we showed that SDMA performs better than NOMA in underloaded systems with weak interference levels. Meanwhile, the NOMA technique can perform well in overloaded systems with a strong interference level. However, none of these multiple access techniques are uniformly suitable for all load regimes. Therefore, as a future work, a general scenario for the network that contains all load regimes can be studied. According to the previous studies in [29, 30, 42, 150], it is illustrated that the newly introduced multiple access technique called Rate Splitting Multiple Access (RSMA) can enhance the performance in all types of systems and interference levels, and this technique can lead to a higher achieved sum rate at the receivers. Furthermore, both SDMA and NOMA techniques are vulnerable to channel state information at the transmitter (CSIT) estimation errors. In other words, both of these techniques are suitable for cases where CSIT is perfect, and when CSIT is inaccurate, the performance drops significantly. However, in the aforementioned studies, it is claimed that the RSMA technique is more robust against imperfect CSIT.
- **AI-enabled 6G networks:** In recent years, AI-enabled communication networks have drawn research interest, and machine learning (ML) and deep learning (DL) algorithms have been conceived as breakthrough technologies for addressing the challenges in 5G and beyond wireless networks. Therefore, the problem formulated in this study can be solved using these

algorithms. Although the ML and DL algorithms have a higher computational complexity due to the iterative model training compared to the model-based and metaheuristic algorithms, these approaches are capable of being generalized to other cases, while model-based algorithms should be redesigned for new problems. Furthermore, since wireless networks are highly dynamic, the optimization approaches have to be robust to environmental uncertainties. Therefore, the model-based algorithms may be affected by these uncertainties since they require full knowledge of the optimization problem in order to obtain the solution. However, the ML techniques can adapt well to dynamic environments.

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Appendix A

Proof of Lemma 1

According to the transformed objective function in Eq. (3.8), it is observed that $f(\boldsymbol{\omega}_k, \boldsymbol{\alpha})$ is a concave differentiable function over $\boldsymbol{\alpha}$ when $\boldsymbol{\omega}_k, \forall k \in \mathcal{K}$ are held fixed. Therefore, the closed form optimal solution for the auxiliary variables of $\boldsymbol{\alpha}$ can be obtained by setting $\nabla_{\alpha_k} f(\boldsymbol{\alpha}) = 0, \forall k \in \mathcal{K}$, and they are found as $\alpha_k^* = \gamma_k^I$. Substituting $\alpha_k^* = \gamma_k^I$ in the new objective function introduced in (3.8) as $f(\boldsymbol{\omega}_k, \boldsymbol{\alpha})$ recovers the original sum rate function in form of sum of logarithms, $\sum_{k=1}^K \log(1 + \gamma_k^I)$. Hence, the equivalence of $f(\boldsymbol{\omega}_k, \boldsymbol{\alpha})$ and $\sum_{k=1}^K \log(1 + \gamma_k^I)$ is exactly established.

Thus, this completes the proof.

Appendix B

Proof of Lemma 2

The reformulated function introduced as $g(\boldsymbol{\omega}_k, \boldsymbol{\alpha}, \boldsymbol{\beta})$ in Eq. (3.9) is a concave differentiable function w.r.t the auxiliary variables, $\boldsymbol{\beta}$, for the fixed $\boldsymbol{\omega}_k, \forall k \in \mathcal{K}$ and $\boldsymbol{\alpha}$. Therefore, the closed-form optimal value of the $\beta_k, \forall k \in \mathcal{K}$ is obtained by solving $\nabla_{\beta_k} g(\boldsymbol{\beta}) = 0, \forall k \in \mathcal{K}$ and the final solution is given by,

$$\beta_k^* = \sqrt{1 + \alpha_k} \frac{\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_k}{\sum_{j=1}^K |\mathbf{h}_{\text{BS},k} \boldsymbol{\omega}_j|^2 + \sigma_{\text{I}}^2} \quad (\text{B.1})$$

Next, by substituting the optimal β_k^* in the transformed function, $g(\boldsymbol{\omega}_k, \boldsymbol{\alpha}, \boldsymbol{\beta})$, the MRFP term defined in (3.8) as $\sum_{k=1}^K (1 + \alpha_k) f_k(\boldsymbol{\omega}_k)$ will be accurately recovered, thereby establishing the equivalence.

Hence, *Lemma 2* is proved.