HIGGS COSMOLOGY

by

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ABSTRACT

HIGGS COSMOLOGY

In this thesis, a cosmological model is developed that unifies the scalar sector of gravity with spontaneous symmetry breaking by extending the Jordan–Brans–Dicke (JBD) framework through the introduction of a specific potential, inspired by the hypothesis that the Higgs field and the cosmic scale factor are fundamentally linked. The scalar mode of gravity is treated as a conformal factor of the metric, leading to a reduced action in Minkowski space that mirrors the Higgs mechanism by simultaneously describing a massive scalar excitation and an expanding spacetime background. Within this framework, a coupled Lagrangian is derived involving the JBD scalar field and the Higgs field, the latter reinterpreted to reflect the dynamics of the cosmic scale factor. Through a field-space coordinate transformation applied to the reduced JBD action—whose kinetic sector takes the form of a nonlinear sigma model—the theory is reformulated such that the two scalar fields decouple, thereby allowing a Higgs-like mechanism to manifest. This reformulation leads naturally to spontaneous symmetry breaking, resulting in a time-dependent vacuum expectation value (VEV) of the Higgs field and associated Higgs bosons as quantized fluctuations. By embedding the Standard Model into the Einstein–JBD Lagrangian, the time-dependence of the Higgs field is shown to affect the effective gravitational constant G_{eff}, giving rise to a dual Big Bang scenario: the first characterized by a vanishing spatial volume and $G_{\text{eff}} = 0$, evolving through a phase of negative, repulsive gravity dominated by JBD dynamics, and a second Big Bang where G_{eff} transitions from negative infinity to positive values. To explore internal symmetries, an SO(3) symmetric triplet of Higgs fields is introduced, and it is shown that the conserved quantity associated with this symmetry induces spontaneous symmetry breaking and Higgs mass generation.

ÖZET

HIGGS KOZMOLOJİSİ

Bu çalışmada, Higgs alanı ile kozmik ölçek faktörü arasında temel bir bağ bulunduğu hipotezinden yola çıkılarak, Jordan–Brans–Dicke (JBD) kuramı özel bir potansiyel terimi ile genişletilmekte ve yerçekiminin skaler modu ile kendiliğinden simetri kırınımı birleştirilerek özgün bir kozmolojik model geliştirilmektedir. Yerçekiminin skaler modu metrik tensörün konformal bir çarpanı olarak ele alınmakta ve bu yaklaşım, Minkowski uzayında Higgs mekanizmasını andıran indirgenmiş bir eylem elde edilmesine olanak tanımaktadır; bu eylem hem kütleli bir skaler uyarımı hem de genişleyen bir uzay-zaman arka planını birlikte tanımlar. Bu çerçevede, JBD skaler alanı ile kozmik ölçek faktörünün dinamiklerini yansıtacak biçimde yeniden yorumlanan Higgs alanını içeren bağlantılı bir Lagrange yoğunluğu türetilmiştir. İndirgenmiş JBD eylemine, kinetik kısmı doğrusal olmayan sigma modeli biçiminde olan bir alan uzayı koordinat dönüşümü uygulandığında, iki skaler alan birbirinden ayrışmakta ve bu sayede Higgs-benzeri bir mekanizma kendiliğinden ortaya çıkmaktadır. Bu yeniden formülasyon, kendiliğinden simetri kırınımını doğal olarak üretmekte; zamanla değişen bir vakum beklenti değeri (VEV) ve bu değerin çevresindeki kuantize salınımlar olarak yorumlanan Higgs bozonlarının ortaya çıkmasına neden olmaktadır. Standart Model'in Einstein–JBD Lagrangyenine gömülmesiyle, Higgs alanının zamana bağlılığı, etkili kütleçekim sabiti G_{eff}'i etkilemekte ve çift aşamalı bir Büyük Patlama senaryosuna yol açmaktadır: İlk aşama, uzaysal hacmin sıfıra ve $G_{\text{eff}} = 0$'a indirgenmesiyle tanımlanır; burada dinamikler JBD kuramındaki itici, negatif kütleçekim fazı tarafından yönlendirilir. İkinci aşamada ise G_{eff} negatif sonsuzluktan pozitif değerlere geçiş yapar. İç simetrileri incelemek amacıyla, $SO(3)$ simetrisine sahip üçlü bir Higgs alanı tanıtılmakta ve bu simetriyle ilişkili korunmuş niceliğin hem kendiliğinden simetri kırınımına hem de Higgs kütlesinin oluşumuna neden olduğu gösterilmektedir.

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LIST OF SYMBOLS

A	Annihilation operator
$A^\dagger$	Creation operator
a	Scale factor of the universe
$\dot{a}$	Time rate of the scale factor of the universe
a_0	Current value of the scale factor of the universe
G_N	Newtonian constant of gravitation
G_{eff}	Effective Newtonian constant of gravitation
$G_{\mu\nu}$	Einstein tensor
$\mathcal{H}$	Hamiltonian density
H	Hubble parameter
$\dot{H}$	Time rate of Hubble parameter
H'	Scale size rate of Hubble parameter
ds	Line element
d^4x	Four dimensional volume element
p	Pressure
R	Ricci scalar
$R_{\mu\nu}$	Ricci tensor
c	Speed of light
h	Planck constant
t	Cosmological time
$\hbar$	Reduced Planck constant
g	Determinant of the metric tensor
$g_{\mu\nu}$	Metric tensor
L	Lagrangian
$\mathcal{L}$	Lagrangian density
M_p	Planck mass
S	Action
S_{EH}	Einstein Hilbert action

S_{JBD}	Jordan Brans Dicke action
S_{matter}	Matter action
SO	Special orthogonal group
$SO(3)$	Three dimensional rotation group
$SO(4)$	Four dimensional rotation group
$SO(5)$	Five dimensional rotation group
$SU(2)$	special unitary group of degree 2
t	cosmological time
$T_{\mu\nu}$	Stress energy tensor
V	Potential
V_eff	Effective potential
W	W boson
Z	Z boson
ρ	energy density
ω	Brans Dicke parameter
ξ	Inverse Brans Dicke parameter
Λ	Cosmological constant
λ	Coupling constant
δ	Dirac delta function
$\eta_{\mu\nu}$	Minkowski metric
π	Canonical momentum density
$\Gamma^{\lambda}_{\mu\nu}$	Christoffel symbols
τ	Conformal time
$\Box$	D'Alembertian operator

LIST OF ACRONYMS/ABBREVIATIONS

FLRW	Two Dimensional
JBD	Jordan Brans Dicke
VEV	Vacuum expectation value
WEC	Weak energy condition

1. INTRODUCTION

Since ancient times, humans have known that the scale of the universe is enormous. The size of the observable universe is approximately 93 billion light-years at present [1]. Cosmology, therefore, permits us to study the vast universe as a whole [2]. While the Standard Model of particle physics does not directly describe the large-scale structure of the universe [3], it can nevertheless be unified with cosmology to study the behaviour of particles during its formation [4, 5].

Consequently, the Higgs field plays crucial roles in connecting the Standard Model with cosmology [6–8]. Most notably, the electroweak phase transition in the early universe [9,10] is intimately related to the Higgs field, as it is responsible for electroweak symmetry breaking and for generating the mass of all fundamental particles [11–13]. As the universe evolved from a hot, dense state to a cold, sparse one, the dynamics of the Higgs field during the electroweak phase transition becomes relevant in understanding the universe's thermal history [14]. Historically, the Higgs field, which gives mass to elementary particles, has also been proposed as a candidate for the inflaton [15], responsible for driving the exponential expansion of the universe shortly after the Big Bang [16].

Moreover, Einstein's theory of gravity is among the most remarkable achievements of modern physics. General Relativity [17–19] not only explained the observed perihelion advance of Mercury and predicted the correct bending of light near the Sun, but also anticipated the expansion of the universe and gravitational waves. From a modern perspective, Einstein's equations in vacuum can be derived by considering Pauli–Fierz field theory [20] for a massless spin-2 graviton and coupling the canonical energy–momentum tensor as a source. This derivation, iterated to all orders [21, 22], is more involved than Einstein's original argument based on covariance and conservation. While gravitational waves have now been observed [23], confirming key aspects of spin-2 gravity, the possibility of spin-0 gravitational waves remains open.

Nevertheless, although Einstein's vacuum equations describe gravity as mediated by a massless spin-2 particle, the FLRW cosmological model [24–29] includes only a scalar degree of freedom — the time-dependent scale factor — due to the assumptions of isotropy and homogeneity. This thesis, therefore, focuses on this scalar mode to examine how far cosmological phenomena can be explained without invoking the full tensor structure of General Relativity. Such scalar theories were first considered by Einstein and Nordström [30–32], but were largely set aside following the empirical successes of General Relativity. More recently, scalar fields have regained importance due to the triumph of the Standard Model [33–35], which describes particle masses as arising from dimensionless couplings to the vacuum expectation value of the Higgs field.

Historically speaking, since the Renaissance, two dominant ideas about the nature of space have shaped scientific thought. The first, derived from Descartes' vortex theory [36] and Newton's absolute space, envisions space-time as an entity with its own physical structure. This classical view continued through aether theories that proposed a medium for the transmission of electromagnetic or gravitational interactions. In contrast, the relativistic viewpoint holds that only relative motion is physically meaningful. Following Maxwell's equations and the Michelson–Morley experiment [37], Poincaré proposed that all fundamental interactions, including gravity, should obey relativity [38]. In the same year, Einstein introduced the foundational postulates of special relativity [39], and a decade later formulated General Relativity [40], in which gravity emerges from spacetime curvature induced by matter and energy. This framework elegantly explains gravitational lensing, black holes, gravitational waves, and the universe's origin and expansion [2].

Accordingly, General Relativity requires both classical and quantum corrections in strong and weak gravity regimes to explain certain phenomena such as binary pulsar orbits, black hole mergers, and exotic sources [41, 42]. These considerations motivate modified gravity theories, categorized as higher-derivative extensions, extra-field theories, non-minimal couplings, or higher-dimensional models. Among the scalar–tensor theories, the Jordan–Brans–Dicke (JBD) theory remains one of the most influential.

The JBD theory incorporates both scalar and tensor fields to represent gravitational interactions. Despite issues such as naturalness [43], indeterminacy [44], experimental constraints [41], and interpretational challenges [45], it continues to attract interest. Besides predicting classical tests of gravity [46], JBD theory has been used to study inflation [47], cosmic homogeneity [48], dark energy [49], and stellar stability [50]. In this thesis, we therefore consider a hybrid model in which the early universe is governed by JBD dynamics, transitioning into an Einsteinian framework with a constant Newtonian gravitational constant.

This thesis synthesizes the results of five individual research papers, each presented and analyzed in a dedicated chapter. The structure of this thesis is as follows. Chapter 2 [51] investigates the scalar mode of gravity, highlighting the importance of the scalar sector of the Standard Model in cosmology. Chapter 3 [52] examines the interplay between the Higgs field and Jordan–Brans–Dicke cosmology. Chapter 4 [53] develops the Higgs-mode perspective within modified cosmology, focusing on the consequences of a time-dependent Higgs vacuum expectation value. Chapter 5 [54] explores spontaneous symmetry breaking induced by internal SO(3) symmetry, relating angular momentum in field space to the scale of the universe. Chapter 3 provides a more realistic and detailed treatment compared to Chapter 2. The motivations behind Chapters 1 and 2 are closely aligned, as both investigate scalar degrees of freedom in cosmological settings. Additionally, natural units will be employed throughout this thesis by setting $\hbar = c = 1$, which simplifies the equations by eliminating explicit factors of Planck's constant and the speed of light.

1.1. Motivations for Extending Einstein Gravity to Jordan–Brans–Dicke Theory

Einstein's theory of General Relativity represents a landmark in our understanding of gravitation, unifying the geometry of spacetime with the distribution of matter and energy. Its predictions have been repeatedly confirmed, and its formulation—based on the Einstein–Hilbert action,

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R + S_{\text{matter}}[g_{\mu\nu}, \Psi], \tag{1.1}$$

has proven robust across a wide range of physical regimes. Nevertheless, certain conceptual gaps and cosmological observations have long motivated the search for a more flexible framework. One such extension is offered by the Jordan–Brans–Dicke (JBD) theory, a scalar–tensor model in which the gravitational coupling is no longer constant, but instead evolves as a field determined by the dynamics of the universe.

A key motivation for this generalization arises from the philosophical legacy of *Mach's principle*, which suggests that local inertial properties should somehow reflect the influence of distant matter. Although General Relativity was partly inspired by this idea, it ultimately does not satisfy it: Newton's constant G_N remains fixed and unresponsive to the distribution of cosmic matter. The JBD theory, introduced in 1961 by Brans and Dicke [56], directly addresses this issue by replacing G_N with a dynamical scalar field $\Phi(x)$, such that the effective strength of gravity becomes

$$G_{\text{eff}}(x) \sim \frac{1}{\Phi(x)}. \tag{1.2}$$

In this setting, gravitational interactions are mediated by both the spacetime metric and a scalar degree of freedom, whose evolution is governed by the following action:

$$S_{\text{JBD}} = \int d^4x \sqrt{-g} \left[\frac{1}{2}\Phi R - \frac{\omega}{2\Phi} g^{\mu\nu} \partial_\mu \Phi \, \partial_\nu \Phi \right] + S_{\text{matter}}[g_{\mu\nu}, \Psi], \tag{1.3}$$

where ω is a dimensionless coupling constant characterizing the kinetic term of the scalar field.

Beyond its conceptual elegance, the JBD framework offers room to accommodate physical scenarios where the gravitational constant may evolve slowly over cosmological timescales. Such possibilities are not excluded by observational data and may even help to resolve puzzles in early-universe cosmology, such as the dynamics of inflation or the onset of cosmic acceleration [57, 58]. In these contexts, a dynamical scalar field can serve as a source of effective dark energy or play the role of a cosmological inflaton.

Equally important is the connection between scalar–tensor models and the broader landscape of high-energy physics. Theories that seek to unify gravity with quantum field theory—such as string theory, supergravity, and higher-dimensional models—generically predict scalar fields that couple non-minimally to curvature. Notably, the dilaton field in string theory plays a role closely analogous to the Brans–Dicke scalar [59, 60]. Thus, the JBD action may be interpreted as a low-energy effective description of more fundamental dynamics arising from a quantum theory of gravity.

An attractive feature of the theory is its compatibility with observational constraints. In the formal limit $\omega \to \infty$, the scalar field becomes effectively frozen, recovering Einstein's theory as a special case:

$$\lim_{\omega \to \infty} S_{\mathrm{JBD}} \to S_{\mathrm{EH}}. \tag{1.4}$$

This ensures that the model remains consistent with current solar system tests, while still allowing for non-trivial departures from General Relativity in cosmological settings where deviations could be more pronounced [61].

Moreover, the replacement of a fixed gravitational scale with a field-dependent one enables the theory to accommodate *scale invariance*, a symmetry of growing interest in gravitational and quantum field models. Under a local conformal rescaling of the form

$$g_{\mu\nu} \to \lambda^2(x) g_{\mu\nu}, \quad \Phi(x) \to \lambda^{-2}(x)\Phi(x), \tag{1.5}$$

the JBD action remains invariant (in the absence of matter), suggesting a deeper geometric structure that may play a role in ultraviolet completions of gravity [62, 63].

In light of these motivations, the Jordan–Brans–Dicke framework stands out as a natural and well-grounded extension of Einstein gravity. It retains the geometric essence of General Relativity while enriching it with a scalar dynamical structure, allowing gravity itself to evolve in concert with the universe it shapes.

1.2. The Higgs Mechanism and the Origin of Mass

One of the most profound insights in modern physics is that mass is not necessarily a fundamental property. Rather, it can emerge dynamically from the quantum structure of a field through a process known as the Higgs mechanism [64], which bridges the abstract symmetries of the Standard Model with the tangible properties of matter [65].

At the core of this mechanism is the Higgs field, denoted Φ, a scalar field that fills all of spacetime. Unlike many physical fields, the Higgs field has a non-zero vacuum expectation value (vev) even in its lowest energy state. This vev leads to a spontaneous breaking of electroweak symmetry and reshapes the vacuum of quantum field theory in a nontrivial way.

The potential energy associated with the Higgs field is written as:

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \tag{1.6}$$

where $\mu^2 < 0$ and $\lambda > 0$. This so-called "Mexican hat potential" has a minimum not at the origin but at a circular valley of minima, leading the system to spontaneously select a non-zero field configuration:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \text{with} \quad v = \sqrt{\frac{-\mu^2}{\lambda}}. \tag{1.7}$$

This spontaneous symmetry breaking causes gauge bosons of the weak interaction—$W^\pm$ and Z^0—to acquire mass through their interaction with the Higgs field. Starting from the gauge-invariant kinetic term:

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi), \tag{1.8}$$

where D_μ is the covariant derivative incorporating the $SU(2)_L \times U(1)_Y$ gauge fields, one obtains the masses:

$$m_W = \frac{1}{2} g v, \qquad m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v. \tag{1.9}$$

Here, g and g' are the coupling constants associated with $SU(2)$ and $U(1)_Y$ symmetries, respectively. The photon remains massless, as required by unbroken $U(1)_{em}$.

Fermions also acquire mass via their Yukawa couplings to the Higgs field:

$$\mathcal{L}_{Yukawa} = -y_f \bar{\psi}_L \Phi \psi_R + \text{h.c.}, \tag{1.10}$$

which, after symmetry breaking, becomes:

$$m_f = \frac{y_f v}{\sqrt{2}}. \tag{1.11}$$

In this framework, mass becomes a measure of how strongly a particle couples to the Higgs field. The Higgs mechanism thus offers a unified, symmetry-based explanation for why some particles are massive while others remain massless. The discovery of the *Higgs boson* in 2012 by the ATLAS [67] and CMS [68] collaborations at the Large Hadron Collider (LHC) confirmed this elegant theoretical structure and completed the Standard Model's particle content.

The Higgs mechanism is not only vital for electroweak unification but also influences modern cosmology, vacuum stability, and inflationary models—making it a profound concept with implications far beyond particle physics alone [70].

2. THE SCALAR MODE OF GRAVITY

A cosmological model can be investigated by introducing a conformal factor in the metric, in a framework motivated by the Jordan–Brans–Dicke action. Although gravitational waves have been observed in experiments that are consistent with the spin-2 nature of the graviton, these observations do not yet exclude the possibility of spin-0 gravitational waves.

Although vacuum Einstein equations unquestionably describe a spin-2 massless particle as the carrier of the gravitational force, the FLRW model of cosmology [24–29] contains only the scale factor of the universe, whose time dependence arises from the idealized assumptions of isotropy and homogeneity. This paper concentrates on this scalar mode and investigates how much can be explained by neglecting the tensor mode of general relativity. Historically, first Einstein and then Nordstrom [30–32] initially considered such theories, but they were later abandoned due to the successes of Einstein's General Relativity. Scalar fields in physics have gained renewed importance due to the triumph of the Standard Model [33–35] of particle physics which says that the masses of all the massive elementary particles (the leptons, the quarks, and the weak bosons) are given by multiplication of a dimensionless coupling and the vacuum expectation value of the Higgs field. The trajectory of a particle in cosmic time scales is given by the geodesic action

$$S = -m \int ds, \tag{2.1}$$

where m is the mass of the particle and ds is the line element.

Since a particle gains mass by its coupling to the Higgs field, the mass of the particle should depend on time if the vacuum expectation value of the Higgs field is time dependent. Therefore, the action (2.1) can be rewritten as

$$S = -m_0 \int \frac{< \phi(\tau) >}{\Phi_0} \sqrt{g_{\mu\nu} dx^\mu dx^\nu} \tag{2.2}$$

where, $< \phi(\tau) >$ is the vacuum expectation value of the Higgs field and Φ_0 denotes its present value. The line element of the conformal form of the FLRW model for a flat universe ($k = 0$) can be expressed as

$$ds^2 = a^2(t)\eta_{\mu\nu} dx^\mu dx^\nu, \tag{2.3}$$

where $a(t)$ is the conformal factor of the universe which is treated as dimensionless in this chapter. Therefore, the time-dependent cosmological expectation value of the Higgs field $\Phi(t)$ can be embedded into the metric, and the line element comes to be

$$ds^2 = (\Phi(\tau)/\Phi_0)^2 \eta_{\mu\nu} dx^\mu dx^\nu. \tag{2.4}$$

Due to Eq.(2.2), there may be a connection between the vacuum expectation value of the Higgs field and the scale factor of the universe as

$$a(t) = \Phi(\tau)/\Phi_0. \tag{2.5}$$

The Jordan-Brans-Dicke action in the Jordan form is

$$S = \int d^4x \sqrt{-\det g} \left(\frac{\xi^2}{2} \chi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right), \tag{2.6}$$

where, χ is the Jordan-Brans-Dicke field, ξ^2 is proportional to inverse Brans-Dicke parameter, and $V(\chi)$ is pure quartic potential

$$V(\chi) = \frac{1}{4}\lambda\chi^4, \tag{2.7}$$

which provides scale invariant action and λ is a coupling constant.

More generally, the field Φ may also depend on the space variables, so the conformal metric can be written as

$$g_{\mu\nu} = \frac{\Phi^2(\tau, x, y, z)}{\Phi_0^2}\eta_{\mu\nu}, \tag{2.8}$$

where the signature is (+, -, -, -) in this chapter. There is non-minimal coupling term in action (2.6) involving Ricci scalar R, which is defined as

$$R = g^{\mu\nu}R_{\mu\nu}, \tag{2.9}$$

where $g^{\mu\nu}$ is the inverse metric, and for the metric given in Eq. (2.8), it takes the form

$$g^{\mu\nu} = \left(\frac{\Phi^2}{\Phi_0^2}\right)^{-1}\eta^{\mu\nu}, \tag{2.10}$$

and, $R_{\mu\nu}$ is Ricci tensor with its definition of

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} \tag{2.11}$$

where, the Riemann tensor in terms of Christoffel symbols is

$$R^{\lambda}{}_{\mu\rho\nu} = \partial_\rho\Gamma^{\lambda}_{\nu\mu} - \partial_\nu\Gamma^{\lambda}_{\rho\mu} + \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\nu\mu} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu}, \tag{2.12}$$

and, its contracted form is

$$g^{\lambda\rho}R^{\lambda}{}_{\mu\rho\nu} = R_{\mu\nu} = \partial_\lambda \Gamma^{\lambda}_{\mu\nu} - \partial_\nu \Gamma^{\lambda}_{\mu\lambda} + \Gamma^{\lambda}_{\lambda\sigma}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\mu\lambda}. \tag{2.13}$$

The Christoffel symbols are defined by

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu} \right). \tag{2.14}$$

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The Ricci scalar can be written in terms of metric,

$$\begin{aligned} R = g^{\mu\nu}\Big[&\partial_\alpha g^{\alpha\beta}\partial_\beta g_{\mu\nu} - \partial_\alpha g^{\alpha\beta}\partial_\mu g_{\beta\nu} - \partial_\beta g_{\mu\nu}\partial_\alpha g^{\alpha\beta} - g_{\rho\nu}\partial_\lambda\partial_\sigma g_{\mu\rho} \\ &- g^{\alpha\beta}\partial_\mu\partial_\alpha g_{\nu\beta} - \frac{3}{4}\partial_\mu g_{\alpha\beta}\partial_\nu g^{\alpha\beta} - \frac{1}{4} g^{\alpha\beta} g^{\rho\sigma}\partial_\mu g_{\alpha\beta}\partial_\nu g_{\rho\sigma} + \frac{1}{2} g^{\alpha\beta}\partial_\mu g_{\alpha\beta}\partial_\nu g_{\rho\sigma}\Big] \\ = g^{\mu\nu}\Big[&-\partial_\alpha g^{\alpha\beta}\partial_\beta g_{\mu\nu} - g^{\alpha\beta}\partial_\mu\partial_\alpha g_{\nu\beta} - \frac{3}{4}\partial_\mu g_{\alpha\beta}\partial_\nu g^{\alpha\beta} - \frac{1}{4} g^{\alpha\beta} g^{\rho\sigma}\partial_\mu g_{\alpha\beta}\partial_\nu g_{\rho\sigma} \\ &- \frac{1}{2} g^{\alpha\beta}\partial_\mu g_{\alpha\beta}\partial_\nu g_{\rho\sigma} - \partial_\nu\partial_\mu g^{\mu\nu}\Big]. \end{aligned} \tag{2.15}$$

For the metric (2.10), it can be expressed as

$$R = -6\,\Phi^{-3}\Phi_0^2\partial_\mu\partial^\mu\Phi, \tag{2.16}$$

or its equivalent description is

$$R = -6\,\Phi^{-3}\Phi_0^2\Box\Phi, \tag{2.17}$$

where, $\Box$ is D'Alembertian Operator,

$$\Box\Phi = \eta^{\mu\nu}\partial_\mu\partial_\nu\Phi = \frac{\partial^2\Phi}{\partial t^2} - \frac{\partial^2\Phi}{\partial x^2} - \frac{\partial^2\Phi}{\partial y^2} - \frac{\partial^2\Phi}{\partial z^2}. \tag{2.18}$$

The action can be rewritten as

$$S = \int d^4x \left(-\frac{\xi^2}{2}\chi^2\Phi\partial_\mu\partial^\mu\Phi + \frac{1}{2}\Phi^2\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{4}\chi^4\Phi^4 \right), \tag{2.19}$$

where, the factor in front of (2.16), 6, has been embedded in inverse Brans-Dicke parameter, ξ^2, and Φ_0 has also been absorbed into χ. Now, the indices are lowered and raised by the Minkowski metric. Note that although (2.6) defines a field theory in curved spacetime with metric $g_{\mu\nu}$, (2.19) defines a field theory in Minkowski space. Here, this convention is chosen because the metric is conformally flat, and this choice leads to the simplest expression for the action (2.19). The first term in this action can be separated, then

$$S = \int d^4x \left(-\frac{\xi^2}{2}\chi^2(\partial_\mu(\Phi\partial^\mu\Phi) - \partial_\mu\Phi\partial^\mu\Phi) + \frac{1}{2}\Phi^2\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{4}\chi^4\Phi^4 \right). \tag{2.20}$$

Here, the first term in Eq.(2.20) is total divergence and can therefore be omitted

$$S = \int d^4x \left(\frac{\xi^2}{2}\chi^2\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\Phi^2\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{4}\chi^4\Phi^4 \right). \tag{2.21}$$

Note that the dimensions of the scalar fields are taken as the square root of mass, $(mass)^{\frac{1}{2}}$, so that the parameters ξ and λ are dimensionless.

The Lagrangian density, motivated by the conformally flat metric given in Eq. (2.8), is expressed as

$$\mathcal{L} = \frac{1}{2}\zeta^2\chi^2\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\Phi^2\partial_\mu\chi\partial^\mu\chi - \frac{\lambda}{4}\chi^4\Phi^4 \tag{2.22}$$

which has SO(1,1) symmetry acting on Φ and χ as $\Phi \to a\Phi$, $\chi \to \frac{1}{a}\chi$, where a is constant. Due to this invariance, the Lagrangian simplifies when new field variables α and γ are defined as

$$\Phi = \alpha^{\frac{1}{2}} e^{-\gamma}, \tag{2.23}$$

$$\chi = \alpha^{\frac{1}{2}} e^{\gamma}, \tag{2.24}$$

so that α has the dimension of mass and γ is dimensionless.

One can show that this Lagrangian density leads to a spontaneous symmetry breaking, even though it does not contain any dimensional parameters. The Minkowski spacetime is isotropic and homogeneous, so the fields may depend only on time,

$$\alpha = \alpha(\tau), \tag{2.25}$$

$$\gamma = \gamma(\tau). \tag{2.26}$$

The Lagrangian density (2.22) in terms of α and γ becomes

$$\mathcal{L} = \frac{1}{8}(1+\zeta^2)\dot{\alpha}^2 + \frac{1}{2}(1+\zeta^2)\alpha^2\dot{\gamma}^2 + \frac{1}{2}(1-\zeta^2)\alpha\dot{\alpha}\dot{\gamma} - \frac{\lambda}{4}\alpha^4. \tag{2.27}$$

The Euler-Lagrange equation with respect to γ is

$$\frac{\partial \mathcal{L}}{\partial \gamma} - \partial_0 \left(\frac{\partial \mathcal{L}}{\partial(\partial_0 \gamma)} \right) = 0. \tag{2.28}$$

Since $\mathcal{L}$ does not contain the field γ, but only its derivatives, it leads to the conserved momentum, π_γ,

$$\frac{\partial \mathcal{L}}{\partial \dot{\gamma}} = \frac{1}{2}(1-\zeta^2)\alpha\dot{\alpha} + (1+\zeta^2)\alpha^2\dot{\gamma} = \mu^3 \tag{2.29}$$

where μ is a constant, and its dimension is mass.

Since the observed space-time is expanding on the cosmological scale, we assume that the cosmological expectation values of α and γ may also depend on the time variable t.

Hamiltonian density is defined by

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial \dot{\gamma}}\dot{\gamma} + \frac{\partial \mathcal{L}}{\partial \dot{\alpha}}\dot{\alpha} - \mathcal{L} \tag{2.30}$$

where the momentum conjugate to α is

$$\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} = \frac{1}{4}(1+\zeta^2)\dot{\alpha} + \frac{1}{2}(1-\zeta^2)\alpha\dot{\gamma}. \tag{2.31}$$

By using Eqs. (2.29),(2.30) and, (2.31), the Hamiltonian density becomes

$$\mathcal{H} = \frac{1}{2}\frac{\zeta^2}{(1+\zeta^2)}\dot{\alpha}^2 + \frac{\mu^6}{2(1+\zeta^2)}\frac{1}{\alpha^2} + \frac{\lambda}{4}\alpha^4, \tag{2.32}$$

where the effective potential for this dynamical system is given by

$$V_{eff} = \frac{\mu^6}{2(1+\zeta^2)}\frac{1}{\alpha^2} + \frac{\lambda}{4}\alpha^4. \tag{2.33}$$

The minimum of the effective potential can be obtained. To find this, the effective potential may be differentiated with respect to α^2, and set to zero,

$$\frac{dV_{\text{eff}}}{d\alpha^2} = -\frac{\mu^6}{(1+\zeta^2)}\frac{1}{\alpha^4} + \frac{\lambda}{2}\alpha^2 = 0. \tag{2.34}$$

Then, the minimum of effective potential, α, is given by

$$\alpha_0 = \frac{\mu}{\left(\lambda(1+\zeta^2)\right)^{\frac{1}{6}}}, \tag{2.35}$$

and, for $\alpha = \alpha_0$, γ is given from (2.29)

$$\gamma_0(\tau) = C + D\tau \tag{2.36}$$

where C is an integration constant and

$$D = \frac{\mu^3}{(1+\zeta^2)\alpha_0^2}. \tag{2.37}$$

Now, α and γ are expanded about α_0 and $\gamma_0(\tau)$ by

$$\alpha = \alpha_0(1 + \epsilon A e^{im\tau} + \epsilon A^\dagger e^{-im\tau}) + \mathcal{O}(\epsilon^2), \tag{2.38}$$

$$\gamma = \gamma_0(\tau) + \epsilon E e^{im\tau} + \epsilon E^\dagger e^{-im\tau} + \mathcal{O}(\epsilon^2) \tag{2.39}$$

where ϵ is a small dimensionless parameter.

By performing a perturbative calculation in (2.32) and neglecting higher powers of ϵ, the mass is extracted from the second derivative of the potential at the minimum.

$$m^2 = \left.\frac{d^2V_{\text{eff}}}{d\alpha^2}\right|_{\alpha=\alpha_0} = \frac{3\mu^6}{(1+\zeta^2)}\frac{1}{\alpha_0^4} + 3\lambda\alpha_0^2. \tag{2.40}$$

After substituting α_0 (2.41), the mass m and frequency in Eqs. (2.38) and (2.39) should be given by

$$m = \sqrt{6}\frac{(1+\zeta^2)^{\frac{1}{3}}}{\zeta}\lambda^{\frac{1}{3}}\mu. \tag{2.41}$$

Furthermore, we find that A and E are not independent (2.29). They are related by

$$E = \left(i\frac{2}{\sqrt{6}}\frac{\zeta}{(1+\zeta^2)} - \frac{1}{2}\frac{(1-\zeta^2)}{(1+\zeta^2)} + \mathcal{O}(\epsilon)\right)A \tag{2.42}$$

in lowest order.

The part $C + D\tau$, in Eq. (2.36) which in quantum field theory would describe a massless particle, in classical physics corresponds to an expanding universe.

Moreover, the system can be quantized by imposing the commutation relation

$$\left[A, A^{\dagger}\right] = 1 \tag{2.43}$$

so that A and $A^{\dagger}$ act as the annihilation and creation operators of the α-particle. On the other hand, E and $E^{\dagger}$ which are not independent of A and $A^{\dagger}$ can be interpreted as quantum corrections to $\gamma_0(\tau)$ which governs the expansion of the universe. This expansion is obtained by taking the average value of $\gamma(\tau)$

$$\langle \gamma \rangle = C + D\tau \tag{2.44}$$

which gives

$$\langle \Phi \rangle = \alpha_0^{\frac{1}{2}} e^{-C-D\tau} = \Phi_0 e^{-D\tau} \tag{2.45}$$

$$\langle \chi \rangle = \alpha_0^{\frac{1}{2}} e^{C+D\tau} = \chi_0 e^{D\tau}. \tag{2.46}$$

Equation (2.45) together with the metric (2.4) defines for $D < 0$ a spacetime that expands exponentially in the conformal time variable t. In terms of cosmological time t, the line element is given by

$$ds^2 = +dt^2 - a(t)^2 d\vec{r}^2 \tag{2.47}$$

where the differential of the cosmological time can be written as

$$dt = a(\tau)\, d\tau. \tag{2.48}$$

When conformal time is transformed into cosmological time, this gives a linearly expanding space.

$$a(t) = -Dt \tag{2.49}$$

Thus, a massive scalar field with mass m, along with an effectively linearly expanding spacetime as expressed in Eq. (2.44), has been derived through the application of the Lagrangian density in Eq. (2.22) formulated within the framework of Minkowski spacetime. This chapter shows that the expanding universe and the Higgs mechanism can be considered as two faces of the same coin presented. The next chapter explores a more realistic framework unifying cosmology with the Standard Model of particle physics.

3. JORDAN BRANS DICKE THEORY AND HIGGS FIELD

The motivation, as introduced in Chapter (2), stems from an intriguing relationship between the Higgs field and the universe's scale factor. This connection allows for two distinct yet algebraically equivalent interpretations of cosmic evolution. In one perspective, the masses of elementary particles vary with time, implying a universe that is otherwise static. In the other, it is the fabric of space that expands while particle masses remain constant. Both interpretations are mathematically consistent and reflect different ways of framing the same underlying physics. This duality serves as the foundation for the current chapter's exploration of how the Higgs field interacts with cosmological dynamics.

Given that a particle acquires mass through its interaction with the Higgs field, any temporal variation in the vacuum expectation value (VEV) of this field naturally implies a corresponding time dependence in the particle's mass. In light of this, the action presented in Equation (2.1) can be recast in the form

$$S = -m_0 \int \frac{\langle \phi(\tau) \rangle}{\Phi_0} \sqrt{g_{\mu\nu} dx^\mu dx^\nu}, \tag{3.1}$$

where $\langle \phi(\tau) \rangle$ represents the cosmological vacuum expectation value of the Higgs field at a given time t, and Φ_0 denotes its value at the present epoch. This expression reflects how the time evolution of the Higgs field directly influences the effective mass of particles propagating through a dynamical spacetime background.

By the way, the line element of an expanding FLRW universe with $k = 0$ is given in conformal coordinates by

$$ds^2 = a^2(\tau)(-d\tau^2 + dx^2 + dy^2 + dz^2) \tag{3.2}$$

where τ is conformal time. If the mass is governed by the time-dependent cosmological expectation value of the Higgs field, then in a macroscopic theory, one can incorporate the factor, $\frac{\langle\phi(\tau)\rangle}{\Phi_0}$ into the metric. In simple terms, the scale factor, $a(\tau)$, can be thought of as related to the time-dependent cosmological vacuum expectation value of the Higgs field.

3.1. Cosmological Solution

Jordan-Brans-Dicke theory has a scalar–tensor structure which uses both a scalar field and a tensor field in order to represent gravitational interactions. The Jordan-Brans-Dicke action extended with a quartic potential term is

$$S = \int d^4x\sqrt{-g}\left(-\frac{\bar{\xi}^2}{2}\bar{\chi}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\chi}\partial_\nu\bar{\chi} - \frac{\bar{\lambda}}{4}\bar{\chi}^4\right), \tag{3.3}$$

where $\bar{\chi}$ is the JBD scalar field and its dimension is mass, $[\bar{\chi}] = [mass]$. Also, $\bar{\xi}^2$ is the inverse JBD parameter. The relation between Brans-Dicke parameter and its inverse is

$$\bar{\xi}^2 = -1/4\omega. \tag{3.4}$$

To present the scalar-tensor theory in a simpler and more tractable form where the Lagrangian involves two scalar fields in Minkowski spacetime, we employ the following two relations:

$$\sqrt{-g} = a^4(t)\sqrt{-\eta} = a^4(\tau), \tag{3.5}$$

$$R = 6\frac{\partial_0^2 a}{a^3}, \tag{3.6}$$

where (2.15) has been used for the metric (3.2). Moreover, $\bar{\chi}$ can be made dimensionless through dimensional transmutation by defining

$$\chi = \frac{\bar{\chi}}{\mu}, \tag{3.7}$$

where μ is constant and its dimension is mass. After dimensional transmutation, χ is dimensionless and dimensionless constants, $\bar{\xi}$ and λ turn to be

$$\xi = \sqrt{6}\mu\bar{\xi}, \tag{3.8}$$

$$\lambda = \mu^4\bar{\lambda}. \tag{3.9}$$

By the way, χ is also only time-dependent due to the isotropic and homogeneous nature of the universe. In terms of a and χ which are dimensionless, the Lagrangian density can be written as

$$\mathcal{L} = -\frac{\xi^2}{2}\chi^2 a\partial_0^2 a + \frac{1}{2}a^2(\partial_0\chi)^2 - \frac{\lambda}{4}a^4\chi^4. \tag{3.10}$$

By applying integration by parts to the first term in Eq. (3.104), the Lagrangian density transforms into the following form

$$\mathcal{L} = -\frac{\xi^2}{2}[\partial_0(\chi^2 a\partial_0 a) - 2a\chi\partial_0 a\partial_0\chi - \chi^2(\partial_0 a)^2] + \frac{1}{2}a^2(\partial_0\chi)^2 - \frac{\lambda}{4}a^4\chi^4. \tag{3.11}$$

Here, the term, $\partial_0(\chi^2 a\partial_0 a)$, is a total divergence, and thus it can be neglected in the action by assuming it vanishes at infinity. Thus, having excluded this term, the Lagrangian density can be rearranged to

$$\mathcal{L} = \frac{\xi^2}{2}a^2\chi^2\left[\left(\frac{\partial_0\chi}{\chi}\right)^2 + 2\frac{\partial_0 a}{a}\frac{\partial_0\chi}{\chi} + \left(\frac{\partial_0 a}{a}\right)^2\right] + \frac{\mu^2-\xi^2}{2}a^2(\partial_0\chi)^2 - \frac{\lambda}{4}a^4\chi^4. \tag{3.12}$$

The positive sign in front of the square bracket containing the kinetic energy terms, suggests that ξ^2 is positive, whereas due to (3.4), the Brans–Dicke parameter ω must be negative

$$\omega < 0.$$

To make it clearer, it is helpful to perform a field coordinate transformation. The following identities, along with the field-space definitions of α and γ, are useful for simplifying the terms in the Lagrangian density.

$$[\partial_0(\ln\chi + \ln a)]^2 = [\partial_0(\ln\chi a)]^2 = [\partial_0\ln\alpha]^2, \tag{3.13}$$

$$\alpha = \chi a, \tag{3.14}$$

$$\gamma = \ln\chi. \tag{3.15}$$

which are dimensionless.

In terms of α and γ, the Lagrangian density is written as

$$\mathcal{L} = \frac{\xi^2}{2}(\partial_0\alpha)^2 + \frac{\mu^2 - \xi^2}{2}\alpha^2(\partial_0\gamma)^2 - \frac{\lambda}{4}\alpha^4. \tag{3.16}$$

The Euler-Lagrange equations for transformed fields are

$$\frac{\partial\mathcal{L}}{\partial\gamma} - \partial_0\left(\frac{\partial\mathcal{L}}{\partial(\partial_0\gamma)}\right) = 0, \tag{3.17}$$

$$\frac{\partial\mathcal{L}}{\partial\alpha} - \partial_0\left(\frac{\partial\mathcal{L}}{\partial(\partial_0\alpha)}\right) = 0. \tag{3.18}$$

Since $\mathcal{L}$ does not contain the field γ

$$\frac{\partial\mathcal{L}}{\partial\gamma} = 0, \tag{3.19}$$

but only its derivatives, it leads to the conserved momentum,

$$\pi_\gamma = \frac{\partial\mathcal{L}}{\partial(\partial_0\gamma)} = (\mu^2 - \xi^2)\alpha^2\partial_0\gamma, \tag{3.20}$$

where the conserved momentum can be equated to constant, C, and its dimension is mass,

$$[C] = mass.$$

Hence,

$$\partial_0\gamma = \frac{C}{(\mu^2 - \xi^2)\alpha^2}. \tag{3.21}$$

Hamiltonian density can be expressed as

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial(\partial_0 \alpha)}\,\partial_0\alpha + \frac{\partial \mathcal{L}}{\partial(\partial_0 \gamma)}\,\partial_0\gamma - \mathcal{L} \tag{3.22}$$

where the momentum conjugate to α is

$$\frac{\partial \mathcal{L}}{\partial(\partial_0 \alpha)} = \xi^2 \partial_0 \alpha \tag{3.23}$$

With some algebra, the Hamiltonian density becomes

$$\mathcal{H} = \frac{\xi^2}{2}(\partial_0\alpha)^2 + \frac{\mu^2 - \xi^2}{2}\alpha^2(\partial_0\gamma)^2 + \frac{\lambda}{4}\alpha^4, \tag{3.24}$$

Applying Eqs. (3.23) and (3.24) leads to

$$\mathcal{H} = \frac{\xi^2}{2}(\partial_0\alpha)^2 + \frac{C^2}{2(\mu^2 - \xi^2)}\frac{1}{\alpha^2} + \frac{\lambda}{4}\alpha^4. \tag{3.25}$$

In this expression, the last two terms in the Hamiltonian density act as an effective potential, which can be expressed as

$$\mathcal{H} = \frac{\xi^2}{2}(\partial_0\alpha)^2 + V_{eff}(\alpha), \tag{3.26}$$

where the effective potential for this dynamical system is given by

$$V_{eff}(\alpha) = \frac{C^2}{2(1 - \xi^2)}\frac{1}{\alpha^2} + \frac{\lambda}{4}\alpha^4. \tag{3.27}$$

One may write the Hamiltonian density in terms of the field and its conjugate momentum as

$$\mathcal{H} = \frac{1}{2\xi^2}\pi_\alpha^2 + \frac{C^2}{2(\mu^2 - \xi^2)}\frac{1}{\alpha^2} + \frac{\lambda}{4}\alpha^4, \tag{3.28}$$

where π_α is the canonical momentum, given by

$$\pi_\alpha = \xi^2 \partial_0 \alpha. \tag{3.29}$$

Moreover, the Hamiltonian equation is

$$-\frac{\partial \mathcal{H}}{\partial \alpha} = \partial_0 \pi_\alpha, \tag{3.30}$$

whose left-hand side is

$$-\frac{\partial \mathcal{H}}{\partial \alpha} = \frac{C^2}{(\mu^2 - \xi^2)}\frac{1}{\alpha^3} - \lambda\alpha^3. \tag{3.31}$$

By using (3.29), (3.30), and (3.31), one can get

$$\partial_0^2 \alpha - \frac{C^2}{\xi^2(\mu^2 - \xi^2)}\frac{1}{\alpha^3} + \frac{\lambda}{\xi^2}\alpha^3 = 0. \tag{3.32}$$

In equation (3.25), the last two terms in the Hamiltonian density are an effective potential which can be expressed as

$$\mathcal{H} = \frac{\xi^2}{2}(\partial_0 \alpha)^2 + V_{eff}(\alpha), \tag{3.33}$$

where the effective potential for this dynamical system is given by

$$V_{eff}(\alpha) = \frac{C^2}{2(\mu^2 - \xi^2)}\frac{1}{\alpha^2} + \frac{\lambda}{4}\alpha^4. \tag{3.34}$$

The minimum of the effective potential can be obtained. In order to find this, the effective potential is found by differentiating with respect to α^2,

$$\frac{dV_{\text{eff}}}{d\alpha^2} = -\frac{C^2}{(\mu^2 - \zeta^2)}\frac{1}{\alpha^4} + \lambda\alpha^2 \tag{3.35}$$

and then set to zero,

$$\frac{dV_{\text{eff}}}{d\alpha^2} = 0. \tag{3.36}$$

The minimum of effective potential, α, is therefore given by

$$\alpha_0 = \left(\frac{C^2}{\lambda(\mu^2 - \xi^2)}\right)^{1/6}, \tag{3.37}$$

where α_0 represents the vacuum expectation value of α that means the field, α, fluctuates about the minimum. By using Eq. (3.21) , one can solve γ at the vacuum,

$$\partial_0\gamma_0 = \frac{C}{(\mu^2 - \xi^2)\alpha_0^2} \tag{3.38}$$

After substituting Eq. (3.37), one gets

$$\partial_0\gamma_0 = \left(\frac{\lambda C}{(\mu^2 - \xi^2)^2}\right)^{1/3} = D \tag{3.39}$$

where C, λ, μ, ξ are constants; consequently, D is also a constant whose dimension is mass. Then γ_0 is obtained.

$$\gamma_0 = D\tau + E, \tag{3.40}$$

where E is an integration constant. Applying the field coordinate transformation (3.15), the Jordan Brans Dicke field χ at the vacuum is obtained as

$$\chi_0 = e^{D\tau+E}. \tag{3.41}$$

The evolution of the universe is determined by the scale factor, $a(\tau)$, which, in this model, also determines the time dependence of the Higgs field. Using the field transformation (3.14), one can determine

$$a = \frac{\alpha_0}{e^{D\tau+E}} = \exp[-D(\tau - \tau_0)], \tag{3.42}$$

where

$$\frac{\alpha_0}{e^E} = e^{D\tau_0}. \tag{3.43}$$

Here, τ_0 is the age of the universe and $a(\tau_0)$=1. Equation (3.42) indicates an exponential expansion in conformal time. In terms of cosmological time, t, the line element is given by

$$ds^2 = -dt^2 + a(\tau)^2 d\vec{r}^2, \tag{3.44}$$

where the differential of the cosmological time can be written as

$$dt = a(\tau)\, d\tau. \tag{3.45}$$

When conformal time is transformed to cosmological time, this gives a linearly expanding space

$$a(t) = \frac{t}{t_0}. \tag{3.46}$$

where t is the age of universe in cosmological time.

In order to determine the evolution of the fields at the vacuum expectation value of α, a small perturbation can be added to α_0

$$\alpha = \alpha_0 + \epsilon(\tau)\alpha_0. \tag{3.47}$$

This expression in Eq. (3.47) can be substituted into Eq. (3.32) to get

$$\partial_0^2\epsilon - \frac{C^2}{\xi^2(\mu^2 - \xi^2)}\alpha_0^{-4}(1+\epsilon)^{-3} + \frac{\lambda}{\xi^2}\alpha_0^2(1+\epsilon)^3 = 0, \tag{3.48}$$

where

$$\alpha_0 \gg \epsilon.$$

So, the second and third terms in equation (3.48) can be expanded by retaining the first two terms. This leads to

$$\partial_0^2\epsilon - \frac{C^2}{\xi^2(\mu^2 - \xi^2)}\alpha_0^{-4}(1-3\epsilon) + \frac{\lambda}{\xi^2}\alpha_0^2(1+3\epsilon) = 0. \tag{3.49}$$

Since, the zeroth-order terms yield

$$-\frac{C^2}{\xi^2(\mu^2 - \xi^2)}\alpha_0^{-3} + \frac{\lambda}{\xi^2}\alpha_0^3 = 0, \tag{3.50}$$

the remaining equation takes the form:

$$\partial_0^2\epsilon + \left(\frac{3C^2}{\xi^2(\mu^2 - \xi^2)}\alpha_0^{-4} + \frac{3\lambda}{\xi^2}\alpha_0^2\right)\epsilon = 0. \tag{3.51}$$

Equation (3.51) can be rewritten in the form of the Klein–Gordon equation as follows

$$\partial_0^2\epsilon + m^2\epsilon = 0, \tag{3.52}$$

where

$$m = \left(\frac{3C^2}{\xi^2(\mu^2-\xi^2)}\alpha_0^{-4} + \frac{3\lambda}{\xi^2}\alpha_0^2\right)^{\frac{1}{2}}. \tag{3.53}$$

By using Eq. (3.37), mass can be written as

$$m = \left(\frac{6(C\lambda)^{\frac{2}{3}}}{\xi^2(\mu^2-\xi^2)^{\frac{1}{3}}}\right)^{\frac{1}{2}}. \tag{3.54}$$

The perturbation term can be written as

$$\epsilon = \epsilon_0(e^{im\tau} + e^{-im\tau}), \tag{3.55}$$

and insert this into

$$\alpha = \alpha_0(1 + \epsilon_0(e^{im\tau} + e^{-im\tau})). \tag{3.56}$$

One of the main goals is to find solutions for the fields χ and a. As before, the procedure begins by solving for γ, which then allows us to determine χ and a. To proceed, equation (3.56) is substituted back into Eq. (3.21)

$$\partial_0\gamma = \frac{C}{(\mu^2-\xi^2)\alpha_0^2(1+\epsilon_0(e^{im\tau}+e^{-im\tau}))^2}, \tag{3.57}$$

by considering only the leading-order contributions in ϵ and omitting terms of higher order

$$\partial_0\gamma = \frac{C}{(\mu^2-\xi^2)}\alpha_0^{-2}\left(1-2\epsilon\right), \tag{3.58}$$

next, using the definition of D (3.39)

$$\partial_0\gamma = D\left(1-2\epsilon\right), \tag{3.59}$$

and after integrating, one obtains γ

$$\gamma = D\tau + \frac{i2D}{m}(e^{im\tau}-e^{-im\tau}) + F. \tag{3.60}$$

where F is an integration constant. After using Eqs. (3.14) and (3.15), one obtains

$$\chi = \exp\left(D\tau + \frac{i2D}{m}(e^{im\tau}-e^{-im\tau}) + F\right), \tag{3.61}$$

$$a(\tau) = \alpha_0(1+\epsilon(\tau))\exp\left(D\tau + \frac{i2D}{m}(e^{im\tau}-e^{-im\tau}) + F\right). \tag{3.62}$$

As before, higher-order terms are neglected since $\epsilon \ll 1$. Additionally, the integration constant F is set to zero to satisfy the condition $a(\tau_0) = 1$. With these simplifications, the evolution of the universe in conformal time becomes

$$a(\tau) = \exp\left(-D(\tau-\tau_0) + \epsilon_0\left(\left(1-\frac{i2D}{m}e^{im\tau}\right)+\left(1+\frac{i2D}{m}e^{-im\tau}\right)\right)\right) \tag{3.63}$$

and in terms of cosmological time, it becomes

$$a(t) = \left(\frac{t}{t_0} + \epsilon_0\left(\left(1 - \frac{i2D}{m}e^{imt}\right) + \left(1 + \frac{i2D}{m}e^{-imt}\right)\right)\right). \tag{3.64}$$

Here, it is important to recognize that the oscillation modes of ϵ can be quantized by expressing them in terms of creation and annihilation operators, $A^\dagger$ and A as follows,

$$\epsilon(\tau) = \epsilon_0\left(Ae^{im\tau} + A^\dagger e^{-im\tau}\right). \tag{3.65}$$

3.2. Non-linear Sigma Model and Field Coordinate Transformation

In the context of a dynamic universe, described by the metric $g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}$, the action in Eq. (3.3) can be expressed more explicitly as

$$S = \int d\tau\left[\frac{1}{2}\left(\xi^2\chi^2(\partial_0 a)^2 + 2\xi^2 a\chi\partial_0 a\partial_0\chi + \mu^2 a^2(\partial_0\chi)^2\right) - \frac{\lambda}{4}a^4\chi^4\right]. \tag{3.66}$$

This structure reveals that the action can be interpreted as a non-linear sigma model, which takes the form

$$S = \frac{1}{2}\int d\tau\left(G_{ab}(\psi)\partial_0\psi^a\partial_0\psi^b - V(\psi)\right) \tag{3.67}$$

where a,b = 1,2, and $\psi^1 = a$, $\psi^2 = \chi$ are two scalar fields that depend only on conformal time τ. Moreover, G_{ab} is the metric in this scalar field space and

$$G_{ab} = \begin{pmatrix} \xi^2\chi^2 & \xi^2 a\chi \\ \xi^2 a\chi & \mu^2 a^2 \end{pmatrix}, \tag{3.68}$$

with $\xi^2 = \frac{1}{4\omega}$, where ω is the Brans-Dicke parameter, which has to be negative for this model.

The Ricci tensor is defined by

$$R_{ab} = \partial_c \Gamma^c_{\ ab} - \partial_b \Gamma^c_{\ ac} + \Gamma^c_{\ ab}\Gamma^d_{\ cd} - \Gamma^c_{\ ad}\Gamma^d_{\ bc}, \tag{3.69}$$

where a, b, c, d are indices, and the Christoffel symbols are defined as

$$\Gamma^a_{bc} = \frac{1}{2} G^{ad} \left(\partial_b G_{cd} + \partial_c G_{bd} - \partial_d G_{bc} \right). \tag{3.70}$$

By contracting the Ricci tensor with the inverse metric, the scalar curvature is obtained as

$$R = G^{ab} R_{ab} \tag{3.71}$$

where

$$G^{ab} = \begin{pmatrix} \frac{1}{\xi^2 \chi^2} & -\frac{1}{\chi a} \\ -\frac{1}{\chi a} & \frac{1}{\mu^2 a^2} \end{pmatrix}. \tag{3.72}$$

After the computation is performed, the scalar curvature for this metric is found to be

$$R = 0, \tag{3.73}$$

which means that the two dimensional field space defined by G_{ab} is flat.

Due to the flatness of the metric, the kinetic term can be mapped to the Klein–Gordon form through an appropriate transformation of the field-space coordinates. In this manner, the action in (3.67) can be explored from a point of view provided by the Higgs mechanism.

To cast the theory as a non-linear sigma model in the Jordan–Brans–Dicke framework, the metric G_{ab}, is transformed to $\hat{G}_{ab}$:

$$G_{ab} = \begin{bmatrix} \xi^2\chi^2 & \xi^2 a\chi \\ \xi^2 a\chi & \mu^2 a^2 \end{bmatrix} \longleftrightarrow \hat{G}_{ab} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.74}$$

In order to shift the Lagrangian density from the Jordan-Brans-Dicke formulation to the Higgs perspective, the first step is to write down the line element of the target space

$$ds^2 = \frac{1}{2}[\xi^2\chi^2 da^2 + 2\xi^2 a\chi da d\chi + \mu^2 a^2 d\chi^2]. \tag{3.75}$$

To simplify the line element(3.75), one can first add and subtract the term $\xi^2 a^2\chi^2$, and then factor out of $a^2\chi^2$, leading to

$$ds^2 = \frac{1}{2}\xi^2 a^2\chi^2 \left[\left(\frac{d\chi}{\chi} + \frac{da}{a}\right)^2 + \frac{\mu^2 - \xi^2}{\xi^2}\left(\frac{d\chi}{\chi}\right)^2\right]. \tag{3.76}$$

Using Eqs (3.14) and (3.15), one can obtain the relations:

$$a(\alpha, \gamma) = \alpha e^{-\gamma}, \tag{3.77}$$

$$\frac{da}{a} = \frac{d\alpha}{\alpha} - d\gamma, \tag{3.78}$$

$$\chi(\gamma) = e^{\gamma}, \tag{3.79}$$

$$\frac{d\chi}{\chi} = d\gamma, \tag{3.80}$$

then, in terms of α and γ, the line element becomes

$$ds^2 = \frac{1}{2}\xi^2 \left(d\alpha^2 + \frac{\mu^2 - \xi^2}{\xi^2}\alpha^2 d\gamma^2 \right). \tag{3.81}$$

Here, we perform one more field coordinate transformation along with its associated relations,

$$\alpha(\rho) = \xi \ln \left(\frac{\rho}{\xi} \right), \tag{3.82}$$

$$d\alpha = \frac{\xi}{\rho} d\rho, \tag{3.83}$$

$$\gamma(\theta) = \xi\theta, \tag{3.84}$$

$$d\gamma = \xi d\theta. \tag{3.85}$$

The result of applying equations (3.83) and (3.85) to the line element (3.81) is

$$ds^2 = \frac{1}{2}d\rho^2 + \frac{1}{2}\rho^2 d\theta^2, \tag{3.86}$$

where, ρ and θ correspond to spherical coordinates. In order to obtain $\hat{G}_{\mu\nu}$ in (3.74), one may introduce

$$\rho(\phi_3, \phi_5) = \sqrt{\phi_3^2 + \phi_5^2}, \tag{3.87}$$

$$\theta(\phi_3, \phi_5) = \arctan\frac{\phi_5}{\phi_3}. \tag{3.88}$$

After using this Euclidean field coordinate system, the line element can be written as

$$ds^2 = \frac{1}{2}(d\phi_3^2 + d\phi_5^2). \tag{3.89}$$

The expressions for a and χ in terms of ϕ_3 and ϕ_5 can be obtained by sequentially applying the full transformations from the initial to the final stage. To begin with, one substitutes the expressions for Eqs (3.82) and (3.84) into Eqs (3.77) and (3.79), respectively, yielding

$$a(\rho, \theta) = \frac{\rho}{\xi}\exp\left(-\frac{\xi\theta}{\sqrt{\mu^2 - \xi^2}}\right), \tag{3.90}$$

$$\chi(\theta) = \exp\left(\frac{\xi\theta}{\sqrt{\mu^2 - \xi^2}}\right). \tag{3.91}$$

Performing the coordinate transformation from spherical to Cartesian form yields

$$a(\phi_3, \phi_5) = \frac{\sqrt{\phi_3^2 + \phi_5^2}}{\xi}\exp\left(-\frac{\xi}{\sqrt{\mu^2 - \xi^2}}\arctan\left(\frac{\phi_5}{\phi_3}\right)\right), \tag{3.92}$$

$$\chi(\phi_3, \phi_5) = \exp\left(\frac{\xi}{\sqrt{\mu^2 - \xi^2}}\arctan\left(\frac{\phi_5}{\phi_3}\right)\right). \tag{3.93}$$

It is now possible to represent ϕ_3 and ϕ_5 as functions of a and χ by retracing the transformation steps in reverse order. Since the coordinates were most recently expressed in spherical form, the corresponding values for ϕ_3 and ϕ_5 are

$$\phi_3(\rho,\theta) = \rho\cos\theta, \tag{3.94}$$

$$\phi_5(\rho,\theta) = \rho\sin\theta. \tag{3.95}$$

By using Eqs. (3.82) and (3.84), ρ and θ can be obtained as

$$\rho(\alpha) = \xi\alpha, \tag{3.96}$$

$$\theta(\gamma) = \frac{\sqrt{\mu^2-\xi^2}}{\xi}\gamma. \tag{3.97}$$

Next, substituting Equations (3.96) and (3.97) into Equations (3.94) and (3.95) yields

$$\phi_3(\alpha,\gamma) = \xi\alpha\cos\left(\frac{\sqrt{\mu^2-\xi^2}}{\xi}\gamma\right), \tag{3.98}$$

$$\phi_5(\alpha,\gamma) = \xi\alpha\sin\left(\frac{\sqrt{\mu^2-\xi^2}}{\xi}\gamma\right). \tag{3.99}$$

Based on the definitions provided in Eqs. (3.77) and (3.79), α and γ can be expressed as

$$\alpha(a,\chi) = a\chi, \tag{3.100}$$

$$\gamma(\chi) = \ln\chi. \tag{3.101}$$

When Eqs. (3.100) and (3.101) are substituted into Eqs. (3.98) and (3.99), the scalar fields ϕ_3 and ϕ_5, expressed in the Higgs framework, are obtained in terms of the fields a and χ as

$$\phi_3(\alpha,\gamma) = \xi a\chi \cos\left(\frac{\sqrt{\mu^2-\xi^2}}{\xi}\ln\chi\right), \tag{3.102}$$

$$\phi_5(\alpha,\gamma) = \xi a\chi \sin\left(\frac{\sqrt{\mu^2-\xi^2}}{\xi}\ln\chi\right). \tag{3.103}$$

Hence, the Lagrangian density in Eq. (3.66) can be rewritten as

$$\mathcal{L} = \frac{1}{2}(\partial_0\phi_3)^2 + \frac{1}{2}(\partial_0\phi_5)^2 - \frac{\kappa}{4}(\phi_3^2+\phi_5^2)^2, \tag{3.104}$$

where κ is a coupling constant,

$$\kappa = \lambda\xi^{-4}. \tag{3.105}$$

3.3. Higgs Formalism

The Lagrangian density of the Higgs field in doublet form is defined as

$$\mathcal{L} = \partial_\mu\Theta^\dagger\partial^\mu\Theta - V(\Theta) \tag{3.106}$$

where Θ is a Higgs doublet,

$$\Theta = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{3.107}$$

in which, ϕ_a are denoted as scalar fields and $a = 1, 2, 3, 4$. Moreover, the potential is taken to be

$$V(\Theta) = -m_s^2 \Theta^\dagger \Theta + \kappa(\Theta^\dagger \Theta)^2 \tag{3.108}$$

where, κ is positive dimensionless constant and, dimension of ϕ_a is mass,

$$[\phi_a] = \text{mass}.$$

Furthermore, the minus sign in front of the tachyonic term leads to spontaneous symmetry breaking. Using ϕ_a as variables, the Lagrangian density becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_a \partial^\mu \phi^a + \frac{1}{2} m_s^2 \phi_a \phi^a - \frac{\kappa}{4}(\phi_a \phi^a)^2 \tag{3.109}$$

where

$$m_s = 0,$$

so that the potential in Eq. (3.109) is purely quartic.

The symmetry group, $SO(5)$, is larger than $SU(2) \times U(1)$,

$$SU(2) \times U(1) \subset SO(5).$$

Therefore, an extension of the Lagrangian density can be achieved by adding an extra scalar field, ϕ_5, thus extending

$$a = 1, 2, 3, 4, 5.$$

Breaking this rotational symmetry annihilates three of four components of Θ Eq. (3.107) such that

$$\phi_1 = \phi_2 = \phi_4 = 0. \tag{3.110}$$

Thus, the same Lagrangian density is recovered as in Eq. (3.104). Now, by applying field coordinate transformations given in (3.102) and (3.103) to the vacuum expectation values and their quantum fluctuations of Jordan-Brans-Dicke fields, one can obtain their correspondence in Higgs picture as

$$\phi_3 = \xi\chi_0 \cos(H(\tau)) \left(1 + \epsilon_0 \left(Ae^{im\tau} + A^\dagger e^{-im\tau} \right) - i\sqrt{\frac{2}{3}} \tan(H(\tau))\, \epsilon_0 \left(Ae^{im\tau} - A^\dagger e^{-im\tau} \right) \right) \tag{3.111}$$

$$\phi_5 = \xi\chi_0 \sin(H(\tau)) \left(1 + \epsilon_0 \left(Ae^{im\tau} + A^\dagger e^{-im\tau} \right) \right) + i\sqrt{\frac{2}{3}} \cot(H(\tau))\, \epsilon_0 \left(Ae^{im\tau} - A^\dagger e^{-im\tau} \right), \tag{3.112}$$

in which

$$H(\tau) = \sqrt{\frac{\mu^2 - \xi^2}{\xi}}\, (D\tau + E)\,, \tag{3.113}$$

and

$$\chi_0 = e^{D\tau_0 + E} \tag{3.114}$$

which is the present value of the Jordan-Brans-Dicke field.

It is important to note that the model is quantized by imposing the commutation relation

$$[A, A^\dagger] = 1, \tag{3.115}$$

where A and $A^\dagger$ denote annihilation and creation operators, respectively.

By the way, the vacuum expectation values of ϕ_3 and ϕ_5 are represented by

$$\langle\phi_3\rangle = \xi\chi_0 \cos\left(\sqrt{\frac{\mu^2 - \xi^2}{\xi}} D\tau\right), \tag{3.116}$$

$$\langle\phi_5\rangle = \xi\chi_0 \sin\left(\sqrt{\frac{\mu^2 - \xi^2}{\xi}} D\tau\right) \tag{3.117}$$

In this model, the time evolution of the Higgs field's vacuum expectation value is governed by the parameter D, which appears in the argument of the cosine in Eq. (3.116). As can be verified by comparing the scale factors expressed in conformal and cosmological time, given in Eq. (3.42) and Eq. (3.46), one finds that

$$D = -\frac{1}{t_0}, \tag{3.118}$$

where t_0 denotes the age of the universe in cosmological time. Consequently, the evolution is extremely slow, ensuring that particle masses remain effectively constant.

4. HIGGS MODE OF MODIFIED COSMOLOGY

A model where the Standard Model is added to the Einstein Lagrangian together with a Jordan-Brans-Dicke(JBD) coupling is considered. The time-dependent Higgs field plays an important role in interpreting the effective gravitational constant, G_{eff}. This may lead to two Big Bangs, the first Big Bang corresponds to the universe having zero size. At this Big Bang, the value of the effective gravitational constant is zero and begins to decrease over time, becoming negative. During this era, the JBD term is significant. In the second Big Bang, the effective gravitational constant passes through infinity and becomes positive. The negative gravitational constant is interpreted as repulsive gravity. The Lagrangian density provides effective potentials that lead to spontaneous symmetry breaking which gives cosmological expectation value of the Higgs field and the Higgs mass both of which depend on curvature and the Brans Dicke parameter.

According to the Standard Model, all fundamental particles obtain their masses from the vacuum expectation value of the Higgs field, which permeates all of space and time. Moreover, the Higgs field of the Standard Model could have significantly impacted cosmology, contributing to the universe's homogeneity, isotropy, and flatness [6]. A model in which the Higgs field affects the growth of curvature perturbations and the structure of the universe as a spectator discussed by Shaposnikov and others [6,15,71–76] motivates this chapter where this model that hybridizes Einstein's relativity and JBD theory with the Higgs field.

4.1. Hybridization of General Relativity and JBD theory with Higgs field

In modern physics, the most extraordinary achievement which describes geometric properties of gravitation and stages of the universe is the General Theory of Relativity. On the other hand, the JBD theory is also consistent in explaining the early stages of the universe. In addition to those theories, the Higgs field is the only known candidate scalar field to have an important role in cosmology [51,52]. In this section, the process of hybridizing general relativity with the JBD theory will be discussed, and the Higgs field will be assigned as a fundamental field in the formation and evolution of the universe. The Planck mass in Einstein's theory is defined by

$$M_p^2 = \frac{1}{8\pi G_N}. \tag{4.1}$$

On the other hand, Jordan Brans Dicke theory extends General Relativity by introducing a dynamical scalar field φ that couples to the Ricci scalar R, thereby promoting the gravitational "constant" to a spacetime-dependent quantity. The JBD action reads

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\varphi R - \frac{\omega}{\varphi} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi \right], \tag{4.2}$$

where the Brans-Dicke parameter, ω, governs the kinetic coupling of the scalar field. Although this form is concise, its kinetic term is non-canonical, and the gravitational coupling appears linearly in φ. To bring the theory closer in form to standard scalar field theories, one may perform a field redefinition that reshapes the action into one with a canonical kinetic term and a non-minimal coupling to curvature. This is achieved by introducing a new scalar field ϕ via the relation $\varphi = \frac{1}{8\omega}\phi^2$, a transformation. Under this change of variables, the scalar field kinetic term transforms into the canonical form $\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi$, while the coupling to gravity becomes non-minimal and takes the form $-\frac{1}{8\omega}\phi^2 R$. The action is thus recast as

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - \frac{1}{8\omega} \phi^2 R \right]. \tag{4.3}$$

This formulation is dynamically equivalent to the original JBD action but offers several advantages. The canonical structure of the kinetic term facilitates quantization and the study of perturbations, while the non-minimal coupling term $\phi^2 R$ appears naturally in various contexts, including inflationary cosmology and scalar-tensor extensions of gravity [51,77]. Moreover, this term enables a more transparent unification of Einstein's

General Relativity with scalar-tensor modifications such as Brans–Dicke theory.

A modified theory of gravitation is considered in which Einstein's theory is hybridized with Jordan-Brans-Dicke theory. In order to describe the effective Newtonian constant of gravitation, one can propose the following relation:

$$\frac{1}{8\pi G_N} + \frac{1}{4\omega}\phi^2 = \frac{1}{8\pi G_{eff}} \tag{4.4}$$

where the effective Newtonian gravitational constant, G_{eff}, depends on cosmological time [78]. It is important to note that a theory with a negative Brans-Dicke parameter, $\omega < 0$, which will be discussed in Section (4.2) suggests that there may be two instants of time in the early universe that can be interpreted as two Big Bangs. In the first Big Bang, the universe starts from zero scale size, $a = 0$, and evolves with a negative gravitational constant, which increases in magnitude and passes to positive values from infinity. One should remark that for positive ω, as ϕ^2 increases, G_{eff} decreases , whereas for negative ω, as ϕ^2 decreases, G_{eff} increases.

In the present era, the value of the Higgs field is much smaller than the Plank mass. This implies that the second term in the left hand side of (4.4) is negligible. This equation may also be interpreted as

$$M_{peff}^2 = \frac{1}{8\pi G_{eff}} \tag{4.5}$$

where M_{peff} is effective Planck mass, which is time-dependent.

Based on these motivations, the hybridized action is given by

$$\mathcal{S}[\Theta, g^{\mu\nu}] = \int \sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\Theta^\dagger\partial_\nu\Theta - \left(\frac{1}{8w}\,\Theta^\dagger\Theta + \frac{M_p^2}{2}\right)R - \Lambda - V(\Theta)\right)d^4x \tag{4.6}$$

where $g^{\mu\nu}$ is the metric, ω is the dimensionless Brans-Dicke parameter, M_p is the Planck mass, Λ is the cosmological constant, R is the Ricci scalar, $V(\Theta)$ is the quartic potential

$$V(\Theta) = -\frac{1}{2}m^2\Theta^\dagger\Theta + \frac{\lambda}{4}(\Theta^\dagger\Theta)^2.$$

and, Θ is the weak isospin Higgs doublet with four real components

$$\Theta = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix} \tag{4.7}$$

where ϕ_a corresponds to scalar fields with $a = 1, 2, 3, 4$.

One sees that Einstein's theory is obtained in the limit $\Theta \to 0$, whereas standard JBD theory is obtained when Planck mass is zero. Regarding the Higgs multiplet, Θ, as the fundamental field, one notices that the action (4.6) takes the form

$$\mathcal{S}[\Theta, g^{\mu\nu}] = \int \sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\Theta^\dagger\partial_\nu\Theta - V_{eff}(\Theta, R)\right)d^4x \tag{4.8}$$

where effective potential, V_{eff}, is curvature dependent

$$V_{eff}(\Theta, R) = \left(\frac{1}{8w}\,\Theta^\dagger\Theta + \frac{M_p^2}{2}\right)R + \Lambda - \frac{1}{2}m^2\Theta^\dagger\Theta + \frac{\lambda}{4}(\Theta^\dagger\Theta)^2. \tag{4.9}$$

One can choose the direction of the cosmological expectation value of the Higgs field, so that the three components of the Higgs multiplet, (4.7), are

$$\phi_1 = \phi_2 = \phi_4 = 0 \tag{4.10}$$

as in the Standard Model. Thus, we identify the scalar field in JBD theory with one chosen real component of the Higgs doublet.

In Minkowski space-time which is static, the Higgs field, ϕ, has to be time independent. However, in the Friedmann–Lemaître–Robertson–Walker universe which is dynamic, it is natural to expect that ϕ can depend on cosmological time

$$\phi_3 = \phi(t). \tag{4.11}$$

which has dimension of mass.

The Robertson-Walker metric that emphasizes ϕ is necessarily spatially homogeneous, given by

$$ds^2 = dt^2 - a^2\left(t\right) \frac{d\vec{x}^2}{\left[1 + \frac{k}{4}\vec{x}^2\right]^2} \tag{4.12}$$

where k is the curvature parameter, with $k = -1,\ 0,\ 1$ corresponding to hyperbolic, flat, and spherical universes, respectively, and $a(t)$ is the scale factor of the universe.

For the FLRW metric, three equations are obtained from the action (4.6). Two of these, namely (4.15) and (4.16), are derived by variation with respect to the metric, where the stress-energy tensor $T_{\mu\nu}$ is assumed to take the perfect fluid form, defined in an orthonormal basis as

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}. \tag{4.13}$$

On the other hand, the last equation can be found by variation with respect to ϕ

$$\frac{\delta \mathcal{S}}{\delta \phi} = 0. \tag{4.14}$$

After substituting the metric, the equations [79] can be written as

$$3\left(\frac{1}{4\omega}\phi^2 + M_p^2\right)\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 - \Lambda + \frac{3}{2\omega}\frac{\dot{a}}{a}\dot{\phi}\phi = \rho, \tag{4.15}$$

$$-\left(\frac{1}{4\omega}\phi^2 + M_p^2\right)\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \frac{1}{\omega}\frac{\dot{a}}{a}\dot{\phi}\phi - \frac{1}{2\omega}\ddot{\phi}\phi - \left(\frac{1}{2} + \frac{1}{2\omega}\right)\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \Lambda = p. \tag{4.16}$$

The equation derived by varying the Higgs field is

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \left[m^2 + \frac{3}{2\omega}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)\right]\phi + \lambda\phi^3 = 0. \tag{4.17}$$

where, ρ is density, P is pressure, and the dot denotes $\frac{d}{dt}$. Scale factor a depends on time so it can be used as an independent variable. For the sake of clearly examining the fate of the scale size of the universe, one may change the independent variable from time t to scale factor a. The relations are given

$$\dot{\phi} = Ha\phi', \tag{4.18}$$

$$\ddot{\phi} = H^2a^2\phi'' + (H'Ha + H^2a)\phi', \tag{4.19}$$

$$\dot{a} = Ha, \tag{4.20}$$

$$\ddot{a} = H'Ha^2 + H^2a, \tag{4.21}$$

H is the Hubble parameter, which depends on the scale factor a and the prime denotes $\frac{d}{da}$. After substituting (4.18), (4.19), (4.20) and (4.21) into (4.15), (4.16), and (4.17), they become

$$3\left(\frac{1}{4\omega}\phi^2 + M_p^2\right)\left(H^2 + \frac{k}{a^2}\right) - \frac{1}{2}\,H^2a^2\phi'^2 + \frac{1}{2}\,m^2\,\phi^2 - \frac{1}{4}\,\lambda\,\phi^4 - \Lambda + \frac{3}{2\omega}\,H^2\,a\phi'\phi = \rho, \tag{4.22}$$

$$-\left(\frac{1}{4\omega}\phi^2 + M_p^2\right)\left(H'Ha + 3H^2 + \frac{k}{a^2}\right) - \frac{1}{2\omega}\,\left(H'Ha^2 + 3H^2\,a\right)\phi'\phi - \frac{1}{2\omega}H^2a^2\phi''\phi$$

$$-\left(\frac{1}{2} + \frac{1}{2\omega}\right)H^2a^2\phi'^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\,\lambda\,\phi^4 + \Lambda = p, \tag{4.23}$$

$$H^2a^2\phi'' + \left[H'Ha^2 + 4H^2a\right]\phi' - \left[m^2 + \frac{3}{2\omega}\left(H'Ha + 2H^2 + \frac{k}{a^2}\right)\right]\phi + \lambda\phi^3 = 0, \tag{4.24}$$

respectively.

4.2. Dynamics of the Universe

The energy density of the universe, ρ, is the key parameter for understanding the dominance of various components of the universe. It may be expanded as

$$\rho = \frac{R_\rho}{a^4} + \frac{M_\rho}{a^3} + \frac{S_\rho}{a^2} + \frac{W_\rho}{a} + \Lambda_\rho \tag{4.25}$$

where R_ρ, M_ρ, S_ρ, W_ρ, Λ_ρ are dimensional constants for radiation, matter, cosmic string, cosmic wall and cosmological constant dominated cases, respectively. Note that each term in the expansion corresponds to a potential source of energy density that can influence cosmic evolution. While some of these (e.g., strings and walls) are speculative or constrained observationally, including them allows for a more general and flexible approach to modeling the universe's dynamics—especially in extended or modified gravity frameworks.

4.2.1. Cosmological Solution

Equations (4.22), (4.23) and (4.24) contain the Hubble parameter which has crucial properties of the universe including its expansion, evolution, and structure, as well as linking cosmological observations to theoretical models. Its dimension is $\frac{1}{length}$, then a simple ansatz that one can formulate which is compatible with both the early universe and the late universe is

$$H^2 = \frac{h_1}{a^2} + \frac{h_2}{a} + h_3 \tag{4.26}$$

where h_1, h_2, and h_3 are constants. $\sqrt{h_1}$ can be interpreted as the velocity of the expanding early universe.

In the Standard Model, the solution of the potential for a constant scalar field is compatible only with Minkowski spacetime. However, if one adds the Standard Model to General Relativity, it would be natural to include a Brans-Dicke term to the action due to the scalar field. Upon examining the solution, one obtains a constant cosmological density parameter, (4.22). One sees that it cannot be possible for the constant scalar field, because radiation, a^{-4}, and matter, a^{-3}, terms are then zero. In addition to this, if the scalar field goes as a^1, its density parameter, (4.22), has zero matter, a^{-3}, term. Therefore, one can propose

$$\phi = \frac{f_1}{a} + f_2 \tag{4.27}$$

where f_1, and f_2 are constants. This is the simplest behaviour of the scalar field, ϕ, which could be consistent with this model.

Anzatzes (4.26) and (4.27) can be substituted into the field equation(4.24), which can be expanded in the inverse power of a

$$0 = \frac{A_L}{a^3} + \frac{B_L}{a^2} + \frac{C_L}{a} + D_L, \tag{4.28}$$

where

$$A_L = f_1\big[-h_1\big(\frac{3}{2\omega}+1\big) - \frac{3k}{2\omega} + \lambda f_1^2\big], \tag{4.29}$$

$$B_L = -f_1\big[\frac{3}{2}h_2\big(\frac{3}{2\omega}+1\big)\big] - f_2\big[\frac{3}{2\omega}(h_1+k) - 3\lambda f_1^2\big], \tag{4.30}$$

$$C_L = -f_1\big[2(\frac{3}{2\omega}+1)h_3 - 3\lambda f_2^2 + m^2\big] - f_2\frac{9}{4\omega}h_2, \tag{4.31}$$

$$D_L = -f_2\big[\frac{3}{\omega}h_3 - \lambda f_2^2 + m^2\big]. \tag{4.32}$$

In a similar manner, coefficients of the energy density (4.25) can also be expressed as

$$R_\rho = f_1^2\big[-h_1\big(\frac{3}{4\omega}+\frac{1}{2}\big)+\frac{3k}{4\omega}-\frac{\lambda}{4}f_1^2\big], \tag{4.33}$$

$$M_\rho = -f_1^2 h_2\big[\frac{3}{4\omega}+\frac{1}{2}\big]+f_1 f_2\big[\frac{3k}{2\omega}-\lambda f_1^2\big], \tag{4.34}$$

$$W_\rho = f_1^2\big[-h_3\big(\frac{3}{4\omega}+\frac{1}{2}\big)+\frac{m^2}{2}\big]+f_2^2\big[\frac{3}{4\omega}(h_1+k)-\frac{3}{2}\lambda f_1^2\big]+3M_p^2(h_1+k), \tag{4.35}$$

$$S_\rho = f_1 f_2 m^2 + f_2^2\big[\frac{3}{4\omega}h_2 - \lambda f_1 f_2\big]+3M_p^2 h_2, \tag{4.36}$$

$$\Lambda_\rho = f_2^2\big[\frac{3}{4\omega}h_3+\frac{m^2}{2}-\frac{\lambda}{4}f_2^2\big]+3M_p^2 h_2+\Lambda. \tag{4.37}$$

The weak energy condition (WEC), which is a crucial concept in general relativity, implies that the energy density has to be non-negative, $\rho \geq 0$ in any frame. It can be assumed that the partial energy densities [80] are also non-negative. Therefore, the coefficients in (4.25) have to be non-negative as well.

In the early universe, the Robertson–Walker scale factor, $a(t)$, is so small that higher order terms in the field equation(4.28) and energy density(4.25) become negligible. Under this condition, they turn into

$$0 \approx \frac{f_1\big[-h_1\big(\frac{3}{2\omega}+1\big)-\frac{3k}{2\omega}+\lambda f_1^2\big]}{a^3}+\frac{-f_1\big[\frac{3}{2}h_2\big(\frac{3}{2\omega}+1\big)\big]-f_2\big[\frac{3}{2\omega}(h_1+k)-3\lambda f_1^2\big]}{a^2}, \tag{4.38}$$

$$\rho \approx \frac{f_1^2\big[-h_1\big(\frac{3}{4\omega}+\frac{1}{2}\big)+\frac{3k}{4\omega}-\frac{\lambda}{4}f_1^2\big]}{a^4}+\frac{-f_1^2h_2\big[\frac{3}{4\omega}+\frac{1}{2}\big]+f_1f_2\big[\frac{3k}{2\omega}-\lambda f_1^2\big]}{a^3}. \tag{4.39}$$

The Brans-Dicke parameter can be found to be positive for the closed FLRW universe. In contrast, a negative Brans-Dicke parameter has been obtained for the open FLRW universe, with $\omega = -3/2$ indicating the presence of a singularity. However, no solution that satisfies WEC exists for the flat universe within this framework. Note that when the Higgs field takes the value

$$\phi^2 \to -\frac{\omega M_p^2}{4}, \tag{4.40}$$

the effective Newtonian constant of gravitation , G_{eff}, goes to infinity. This situation may be referred to as the second Big Bang.

In the Standard Model of particle physics, which is formulated in Minkowski spacetime, the value of the quartic coupling constant, λ, has been measured to be approximately 0.13 [81]. Because of this value, f_2 in (4.27) becomes imaginary, which is incompatible in this model when one tries to solve equation (4.28) as a whole such that energy density (4.25) is partially non-negative. The condition for the field ϕ to be real can only be satisfied if λ is large, which indicates an unstable universe [82]. On the other hand, an exact solution can be obtained for negative partial energy densities [83–87] or for a scalar-field dependent Brans-Dicke parameter [88, 89]. Moreover, this solution indicates nonzero string and cosmic wall contributions to the expanding universe. For the late universe, standard cosmological results are recovered as expected.

4.2.2. Higgs mass

The curvature dependent effective potential can be written in terms of the Higgs field

$$V_{eff}(\phi, R) = \left(\frac{1}{8w}\phi^2 + \frac{M_p^2}{2}\right)R + \Lambda - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4. \tag{4.41}$$

The minimum of the effective potential is given by

$$\upsilon = \frac{1}{\sqrt{\lambda}}\big(m^2 - \frac{R}{4\omega}\big)^{\frac{1}{2}}. \tag{4.42}$$

After performing a perturbative calculation, the Higgs mass is given by

$$m_{Higgs} = \big(4m^2 - \frac{R}{\omega}\big)^{\frac{1}{2}}. \tag{4.43}$$

Note that Higgs mass depends on the Ricci scalar and the Brans-Dicke parameter. At present time, it's value should be compatible with the Standard Model. However, this idea has not yet been successfully incorporated into the present chapter in a consistent manner.

4.3. SO(4) invariance

The weak isospin doublet in Equation (4.7) constitutes an SO(4) symmetric scalar field space. Moreover, the closed FLRW universe possesses SO(4) symmetry. In the

Standard Model, ϕ_3 and ϕ_4 are neutral, whereas ϕ_1 and ϕ_2 correspond to charged particles that absorbed by the $W_\pm$ bosons during symmetry breaking.

Instead of annihilating three of the four scalar fields in the Higgs doublet as in the Standard Model, it may be considered to take the norm of all field components which is defined as

$$|\phi| = \sqrt{\phi_1^2(t) + \phi_2^2(t) + \phi_3^2(t) + \phi_4^2(t)}, \tag{4.44}$$

where

$$\Theta^\dagger \Theta = |\phi|^2. \tag{4.45}$$

On the other hand, if only ϕ_1 is different than zero, the action can be written as

$$\mathcal{S}[\phi, g^{\mu\nu}] = \int \sqrt{g}\left(\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \left(\frac{1}{8w}\phi^2 + \frac{M_p^2}{2}\right)R - \Lambda - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4\right)dt. \tag{4.46}$$

By using closed FLRW metric in Euclidean coordinates [90], field equations similar to Equations (4.15), (4.16), and (4.17) were obtained. However, this idea does not satisfy the continuity equation,

$$\dot{\rho} = -\frac{1}{3}\frac{\dot{a}}{a}(\rho + p). \tag{4.47}$$

This incompatibility may be caused because the field equations are satisfied when they are written in terms of ϕ_1, ϕ_2, ϕ_3, ϕ_4, but not written in terms of the norm of the complex Higgs doublet.

5. ANGULAR MOMENTUM IN THE INTERNAL $SO(3)$ SYMMETRY SPACE RELATING THE SIZE OF THE UNIVERSE AND THE HIGGS MASS

In the Standard Model, the Higgs field, by adding a tachyonic negative mass squared term to the Lagrangian density, induces spontaneous symmetry breaking. On the other hand, it will be demonstrated in this chapter that symmetry breaking can occur even in the absence of a negative mass squared term [51,52]. Note that SU(2) and SO(3) are locally isomorphic [91]. We consider the simplest non-Abelian Higgs model with the SO(3) symmetry. Thus, in our model, the new quantum number j associated with the internal SO(3) invariance of the field space is conserved and may lead to spontaneous symmetry breaking which results in a Higgs mass inversely proportional to the size of the universe. Such a mass can be useful in cosmological models and in explaining the neutrino mass. The Minkowski metric is defined as

$$\eta_{\mu\nu} = (+1, -1, -1, -1)$$

.

The Lagrangian density of the Higgs triplet

$$\mathcal{L} = \partial_\mu \Theta^\dagger \partial^\mu \Theta - V(\Theta), \tag{5.1}$$

with $\Theta = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$, where ϕ_a corresponds to scalar fields and $a = 1, 2, 3$. Furthermore, the potential term is usually defined as

$$V(\Theta) = -\frac{1}{2}\bar{m}^2\Theta^\dagger\Theta + \frac{\lambda}{4}(\Theta^\dagger\Theta)^2, \tag{5.2}$$

where dimensionless constant, λ, is bigger than zero and the scalar fields have the dimension of mass,

$$[\phi_a] = mass.$$

In terms of the fields ϕ_a, the Lagrangian density can be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_a(t,\vec{r})\partial^\mu\phi_a(t,\vec{r}) - \frac{\lambda}{4}(\phi_a(t,\vec{r})\phi_a(t,\vec{r}))^2, \tag{5.3}$$

where $\bar{m} = 0$.

The system can be quantized by imposing the commutation relation

$$\Big[\phi_a(t,\vec{r}), \pi_b(t,\vec{r'})\Big] = i\delta^3(\vec{r} - \vec{r'})\delta_{ab}, \tag{5.4}$$

where $\pi_b(t,\vec{r})$ is the canonical momentum conjugate of the Lagrangian density (5.3). By the way, in homogeneous and isotropic cosmology, ϕ_a may depend on only time

$$\phi_a(t,\vec{r}) = \phi_a(t). \tag{5.5}$$

The Universe is assumed to be a torus that is flat, but finite, then the spatial volume of the universe is a^3. After integrating the commutation relation overall space, the spatial volume of the universe appears

$$\int [\phi_a(t), \pi_b(t)]\, d^3r = i\delta_{ab}\int \delta^3(\vec{r} - \vec{r'})d^3r, \tag{5.6}$$

$$[\phi_a(t), \pi_b(t)]\, a^3 = i\delta_{ab}, \tag{5.7}$$

where a is the size of the universe. Actually, it is changing with time, but a is assumed to be constant in this chapter.

The analogue of angular momentum density in the field space can be written as

$$\mathcal{J}_a = \epsilon_{abc}\phi_b\dot{\phi}_c \tag{5.8}$$

and, its dimension is $mass^3$.

a^3 can be plugged into the commutation(5.7) and distribute evenly

$$\left[\phi_a(t)a^{\frac{3}{2}}, \pi_b(t)a^{\frac{3}{2}}\right] = i\delta_{ab} \tag{5.9}$$

and one may define $\phi_a(t)a^{\frac{3}{2}}$ and $\pi_b(t)a^{\frac{3}{2}}$ as

$$\Phi_a(t) = a^{\frac{3}{2}}\phi_a(t) \tag{5.10}$$

$$\Pi_b(t) = a^{\frac{3}{2}}\pi_b(t) \tag{5.11}$$

whose dimensions are $mass^{-\frac{1}{2}}$ and $mass^{\frac{1}{2}}$,

$$[\Phi_a] = mass^{\frac{1}{2}},$$

$$[\Pi_a] = mass^{-\frac{1}{2}}.$$

In terms of new fields, the quantization of the system can be imposed by the commutation relation,

$$[\Phi_a(t), \Pi_b(t)] = i\delta_{ab}. \tag{5.12}$$

By definition, the integral of the Lagrangian density over whole space gives the Lagrangian of the system,

$$L = \int \mathcal{L} d^3x. \tag{5.13}$$

The Lagrangian density is independent of position variables because of isotropy and homogeneity of the universe, and the Lagrangian can be expressed as

$$L = \mathcal{L} a^3. \tag{5.14}$$

Therefore, we can investigate the system in the perspective of a SO(3) invariant internal space,

$$L = \frac{1}{2}\dot{\Phi}_a(t)\dot{\Phi}_a(t) - \frac{\Lambda}{4}(\Phi_a(t)\Phi_a(t))^2 \tag{5.15}$$

where $\dot{\Phi}_a(t)$ is the time derivative of $\Phi_a(t)$, Λ is defined as

$$\Lambda = a^{-3}\lambda, \tag{5.16}$$

and, its dimension is $mass^3$. Moreover, in the picture of the SO(3) symmetry, the analogue angular momentum in internal field space may be written as

$$J_a = \epsilon_{abc}\Phi_b\dot{\Phi}_c, \tag{5.17}$$

and, from (5.12), it satisfies

$$[J_a, J_b] = -i\epsilon_{abc}J_c, \tag{5.18}$$

which is the commutation relation of analogue angular momentum components.

After some algebraic calculations, the Hamiltonian becomes

$$H = \frac{1}{2}\Pi_a(t)\Pi_a(t) + \frac{\Lambda}{4}(\Phi_a(t)\Phi_a(t))^2. \tag{5.19}$$

One can write this Hamiltonian in terms of $\Phi = \sqrt{\Phi_1^2 + \Phi_2^2 + \Phi_3^2}$ and $\Pi = \sqrt{\Pi_1^2 + \Pi_2^2 + \Pi_3^2}$ in the space that has the SO(3) symmetry

$$H = \frac{1}{2}\Pi^2(t) + \frac{J^2}{2\Phi^2(t)} + \frac{\Lambda}{4}\Phi^4(t), \tag{5.20}$$

where J is the analogue angular momentum operator of the internal field space in the aspect of the conserved quantity associated with the SO(3) symmetry. Here, the effective potential for our dynamical system is given by

$$V_{eff} = \frac{J^2}{2\Phi^2(t)} + \frac{\Lambda}{4}\Phi^4. \tag{5.21}$$

where the eigenvalue of J^2 is $\mathsf{j}(\mathsf{j}+1)$. Then, its minimum is given by

$$\Phi_0 = \left(\frac{\mathsf{j}(\mathsf{j}+1)}{\Lambda}\right)^{\frac{1}{6}}. \tag{5.22}$$

Furthermore, the field, Φ, may be expanded pertubatively as

$$\Phi = \Phi_0 + \epsilon\left(Ae^{imt} + A^* e^{-imt}\right) + \mathcal{O}(\epsilon^2). \tag{5.23}$$

Here, A and A^* are annihilation and creation operators for the time dependent background expectation value. They satify the cannonical commutation relation. After performing a perturbative calculation neglecting higher powers of ϵ, we find that for consistency the mass m and frequency in (5.23) should be given by

$$m = \sqrt{12}\lambda^{\frac{1}{3}}\frac{(\mathsf{j}(\mathsf{j}+1))^{\frac{1}{3}}}{a}. \tag{5.24}$$

Note that $\Lambda = a^{-3}\lambda$ and j has to be big enough to find non-trivial Higgs mass. In other words, there must be an internal analogue angular momentum quantum number of unprecedented magnitude to obtain the measured Higgs mass.

6. CONCLUSION

This thesis synthesizes results from four companion papers, each illuminating a complementary facet of a scalar-field approach to cosmology: (i) the idea that the time-dependent Higgs vev imprints a conformal contribution on the metric and thus on expansion; (ii) an embedding of the Higgs framework into JBD scalar–tensor dynamics; (iii) a GR–JBD–Higgs hybrid model spanning the early and late eras including a two Big Bang scenario for negative ω; and (iv) an internal-symmetry reformulation of scalar mass generation based on SO(3). The concluding narrative below weaves these strands into a single, coherent picture.

Firstly, this work has presented a cosmological framework in which the expansion of the universe is governed by the time evolution of the vacuum expectation value (vev) of the Higgs field. Inspired by a reinterpretation of time-dependent inertial mass, it has been proposed that the temporal dynamics of the Higgs field can be absorbed into the fabric of spacetime itself, effectively contributing to the metric as a conformal factor. In this approach, the Higgs field becomes dynamically linked to the FLRW scale factor, offering a unified view where mass generation and cosmological expansion are interwoven.

To place this idea within a more Machian gravitational setting, attention has been focused on the scalar mode of Jordan–Brans–Dicke (JBD) theory. By assuming that both the scale factor $a(t)$ and the scalar field depend solely on time, a bridge has been constructed between JBD cosmology and the Higgs mechanism via a transformation in field space. While solar system tests constrain the Brans–Dicke parameter ω to be large and positive, a wide range of cosmological models remain viable for negative ω, particularly in effective string-theoretic contexts [52]. In this framework, small oscillations around the vacuum state have been studied and shown to be quantizable, lending additional physical support to the theory.

Moreover, a hybrid model combining General Relativity, Brans–Dicke theory, and the Higgs sector of the Standard Model has also been proposed. The resulting field equations, though too complex for exact solutions, have been explored through simple ansätze that capture key features of the radiation- and matter-dominated eras. Both positive and negative values of the Brans–Dicke parameter are found to be compatible with the model. Interestingly, in the early universe, negative ω values lead to a scenario with two Big Bangs, while the model smoothly reduces to standard cosmology at late times. By analyzing the effective potential, expressions for the Higgs mass have been obtained in terms of the Ricci scalar and the Brans–Dicke parameter.

In extensions with non-minimal curvature couplings and/or scalar–tensor dynamics, the Higgs field acquires small, cosmology-induced shifts to its effective mass through the FLRW curvature and background scalars. These effects can be significant in the early universe, but are strongly constrained at late times, where they must be tiny and consistent with precision bounds.

Complementing these cosmological considerations, it is worth recalling that, without spontaneous symmetry breaking, the Standard Model assigns scalar mass via a dimensionful ghost term—an arbitrary constant with no deeper origin. In contrast, the present framework replaces this with a dimensionless quantum number associated with internal symmetry and a contribution from the inverse size of the universe. Spontaneous symmetry breaking has been reexamined from the perspective of an internal SO(3) symmetric field space, leading to the emergence of a massive scalar field. The starting point is a Lagrangian density that corresponds to an SO(3) reduction of the scalar sector of the Standard Model, which in full admits SO(4) symmetry.

Taken together, these results reflect an ongoing effort to explore deeper connections between particle physics and gravity from a scalar field perspective. Future work will continue in this direction, driven by the belief that the Higgs field holds untapped cosmological significance.

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