DOKUZ EYLÜL UNIVERSITY

GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

HODGES – LEHMANN ESTIMATOR AND USAGE IN QUALITY CONTROL CHART

by

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September, 2016

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HODGES – LEHMANN ESTIMATOR AND USAGE IN QUALITY CONTROL CHART

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HODGES – LEHMANN ESTIMATOR AND USAGE IN QUALITY CONTROL CHART

ABSTRACT

Widely used Shewhart's control chart for monitoring process location parameter is based on mean of the process and constructed under the Normal distribution. Performance of these charts is affected negatively when the process distribution is different from Normal. Nonparametric or distribution-free control charts are useful in such a case because these charts do not require any assumption about process distribution. One of such nonparametric control charts is Hodges – Lehmann control chart which is based on Hodges – Lehmann estimator.

In this study, Hodges – Lehmann control chart is introduced and evaluated the performance of chart for in control and out of control states with average run length (ARL) values by using observations randomly generated from some distributions. Also, Hodges – Lehmann control chart and Shewhart type control chart for mean are compared for Normal distribution with a simulation study. Finally, results are interpreted.

Keywords: Statistical process control, nonparametric quality control charts, Hodges-Lehmann estimator, average run length

HODGES – LEHMANN KESTİRİCİSİ VE KALİTE KONTROL KARTINDA KULLANIMI

ÖZ

Yaygın olarak kullanılan, sürecin konum parametresinin izlendiği Shewhart kontrol kartı sürecin ortalamasına dayanır ve Normal dağılım varsayımı altında geliştirilmiştir. Süreç dağılımı Normal dağılımdan farklı olduğunda kartın performansı bu durumdan negatif etkilenir. Böyle bir durumda, parametrik olmayan ya da diğer adıyla dağılımdan bağımsız kontrol kartları kullanmak yararlı olur. Çünkü parametrik olmayan kontrol kartları dağılım hakkında herhangi bir varsayım gerektirmez. Bu kartlardan biri de Hodges- Lehmann kestiricisine dayalı Hodges – Lehmann kontrol kartıdır.

Bu çalışmada, Hodges – Lehmann kontrol kartı tanıtılmış ve bazı dağılımlardan rasgele üretilen gözlemler kullanılarak hesaplanan ortalama koşum uzunluğu (ARL) değerleri ile kartın performansı değerlendirilmiştir. Aynı zamanda Normal dağılım altında Hodges – Lehmann ve Shewhart kontrol kartları karşılaştırılmıştır. Son olarak, sonuçlar yorumlanmıştır.

Anahtar kelimeler: İstatistiksel süreç kontrolü, parametrik olmayan kalite kontrol kartları, Hodges – Lehmann kestiricisi, ortalama koşum uzunluğu

CONTENTS

Pages

LIST OF FIGURES

LIST OF TABLES

Pages

CHAPTER ONE
INTRODUCTION

Throughout history, innovations in technology have created the foundation of all developments and these rapid changes in technology have affected the economic and social life. In this context, people’s lifestyles have begun to be influenced by changes and their needs have swelled and diversified. Hence, people have looked for ways to satisfy their needs. In years that turned from production society to consumer society, people’s growing demand for high quality and cheap goods also increased worldwide competition. The concept “quality” that has importance not only for production sector but also for health and finance sectors has become an important strength for companies to rise at the international area.

Producing the least defective products and minimizing waste have been purposed with the quality being the main elements of competition. In accordance with this purpose, process control has become more important so statistical methods have begun to be used. Business’ understanding these methods and applying them well will help quality level increase rapidly.

One of the most widely used statistical methods in process control is control charting technique. The main aim of quality control charts is to detect as soon as possible a change in process.

The most popular and widely used control chart is Shewhart type control charts that have been developed under Normal distribution assumption. However, in most of the applications, the researcher does not have enough information about the process distribution or process distribution is not Normal. Widely used Shewhart control charts can give misleading results when the process is not normally distributed. To overcome this, new control charts have been developed and these charts are called “nonparametric” or “distribution-free” control charts.

The most important advantage of nonparametric control charts is that these do not necessitate any information about the underlying distribution of the variable and also these charts are quite robust to outliers. Some of these charts are more complicated than the Shewhart charts but their use have been easier thanks to developing computer technology.

There are various studies about nonparametric control charts in the literature. Janacek & Meikle (1997) proposed a modification of the control charts based on median for overcoming non-normality problems. Chakraborti, Van Der Laan & Bakir (2001) highlighted various advantages of nonparametric control charts and considered Shewhart-type chart for the median. Altukife & Schaeffer (2001) presented a new nonparametric charting scheme based on order statistics and proposed control chart is compared to the traditional Shewhart type chart. Das (2009) compared three nonparametric control charts to detect shifts in location parameters. Park (2013) studied about the relationship between reliability of the control chart and hypothesis testing and proposed a new median control chart with the sign test statistic. Pongpullponsak & Jayathavaj (2014b) presented a nonparametric control chart based on Hodges-Lehmann estimator for standard Normal distribution and Weibull distribution. Also, in another study Pongpullponsak & Jayathavaj (2014c) presented a simulation study on the performance of the sign test, Mann-Whitney test, Hodges-Lehmann estimator and control charts for Normal and Weibull populations.

The purpose of this thesis is to introduce one of such nonparametric control charts (Hodges – Lehmann (HL) control chart) which is based on Hodges – Lehmann estimator and evaluate the performance of this quality control chart with average run length (ARL) values under some distributions. Performances of Hodges – Lehmann and Shewhart control charts are also compared for Normal distribution.

This thesis consist of seven chapters. In chapter two, general information about quality is given. In chapter three, control charting technique which is one of the statistical process control is examined. In chapter four, how to measure performance of control charts is evaluated. Hodges – Lehmann control chart which is nonparametric one is introduced in chapter five. To evaluate the performance of Hodges – Lehmann control chart and compare with Shewhart chart for Normal distribution, simulation study is performed and the results of simulation study is given in chapter six. Finally, conclusions are presented in chapter seven.

CHAPTER TWO
BASIC CONCEPTS OF QUALITY

2.1 Definition of Quality

Quality has been defined in many ways by different scientists and establishments. These can be listed as:

- Philip B. Crosby defined quality as "Conformance to requirements."
- "W. Edwards Deming defined quality as follows: Good quality means a predictable degree of uniformity and dependability with a quality standard suited to the customer" (Chandrupatla, 2009).
- "Definition adopted by the American Society for Quality (ASQ): Quality denotes an excellence in goods and services, especially to the degree they conform to requirements and satisfy customers" (Chandrupatla, 2009).
- "Another frequently used definition comes from Juran: Quality is fitness for use" (Chandrupatla, 2009).

As it can be understood from the descriptions, quality does not mean best of everything or the most expensive of everything. Summarizes the quality is suitability to desired properties.

2.1.1 Dimensions of Quality

The quality of a product can be described and evaluated in several ways…. Garvin (1987) provides an excellent discussion of eight components or dimensions of quality (Montgomery, 2009):

- **Performance:** Suitability for the main use of product.
- **Reliability:** The scale for the frequency of product breaking down.
- **Durability:** The length of the product use lifetime.
- **Serviceability:** It defines whether the product can be repaired easily.
- **Aesthetics:** Physical appearance of the product.

- **Features:** It defines whether the product has other feature except for the main purpose of use.
- **Perceived Quality:** It is the impact of the product or the company on the users.
- **Conformance to Standards:** It shows whether the product is suitable for desired design.

2.1.2 Chance and Assignable Causes of Quality Variation

There are two reasons of the quality variability:

- **Chance Causes:** There is always some natural variability in any production process and even if a process operate with only chance causes of variation, we can say that it is in statistical control. In other words, chance causes are part of the Normal process. Some cases which can give rise to chance causes; measurement errors, ambient conditions, inappropriate procedures, poor maintenance of machines.

- **Assignable Causes:** These causes cannot be estimated and variability due to assignable causes are unacceptable. If process operate with assignable causes, we can say that it is out of control. Some examples of this; machine malfunction, untrained operator, poor adjustment of equipment.

These two causes are illustrated in Figure 2.1. Until time t_1 there is only chance causes of variation and the process is between lower Specification Limit (LSL) and Upper Specification Limit (USL) values so we can say that the process is in control. Also, mean of the process (μ_0) at intended value until time t_1. At time t_1 an assignable cause occurred and because of this reason process mean (μ_0) shifted to the new value. At time t_2 another assignable cause occurred and this time process mean at intended value but process standard deviation shifted to larger value as shown in Figure 2.1. At time t_3 there is another assignable cause present, resulting in both the process mean and standard deviation taking on out-of-control values (Montgomery, 2009; Kumpas, 2006). As a result, process is out of control from t_1 to t_3 of time due to some of the production was not fall between LSL and USL in Figure 2.1.

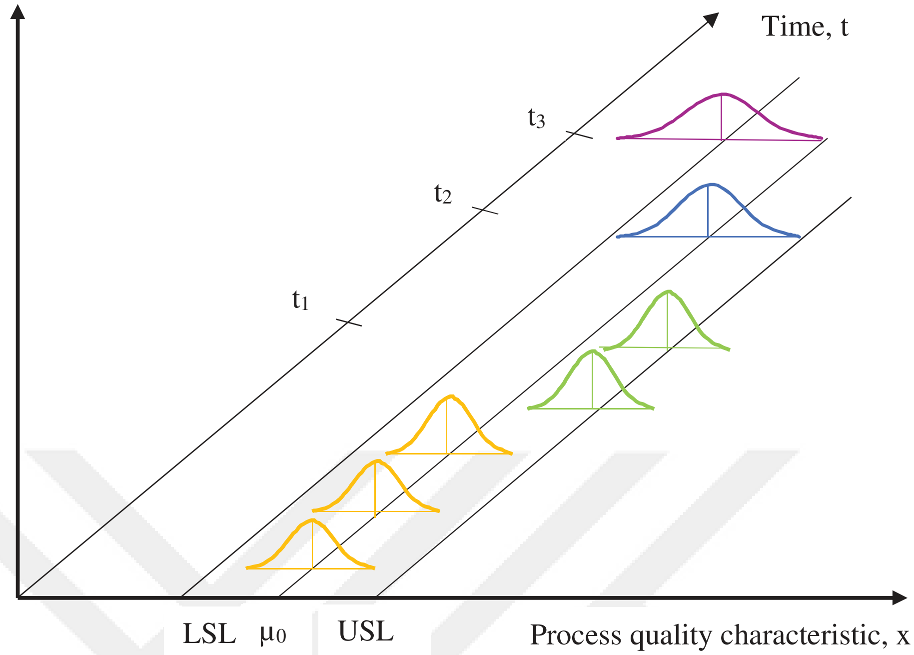

Figure 2.1 Chance and assignable causes of variation (Montgomery, 2009).

2.2 Definition of Quality Control

Quality control may be defined as follows; Quality control involves establishment of quality standards and installation of systems to ensure that these standards are maintained and practiced. In addition to that, J.A. Shubin defines quality control as; “Quality control means the recognition and removal of identifiable causes and defects, and variables from the set standards” (Subramani, 2015).

2.2.1 Benefits of Quality Control

- It provides the development of quality and production.
- It provides solve the manufacturing problems timely.
- Economic losses reduce thanks to quality control.
- It improves the relationship with customers.
- It increases the earning and productivity.

CHAPTER THREE
STATISTICAL PROCESS CONTROL

3.1 Control Charts

The main purpose of Statistical Process Control (SPC) is to identify and eliminate assignable causes that changing the process and to take corrective actions as quickly as possible. Control charts are the most commonly used techniques to fulfill this purpose. By definition, control charts are the statistical tools which are used detect changes that may occur during the manufacturing process (Kılıç, 2006).

There are at least five reasons for their popularity (Montgomery, 2009):

- Control charts are proven technique for improving productivity.
- Control charts are effective in defect prevention.
- Control charts prevent unnecessary process adjustment.
- Control charts provide diagnostic information.
- Control charts provide information about process capability.

3.1.1 Basic Principles

Control charts consist of three base line as in Figure 3.1:

- **Center Line (CL):** The mean of the quality characteristic while process is in control.
- **Lower Control Limit (LCL):** Minimum value can be taken by quality characteristic while process is in control.
- **Upper Control Limit (UCL):** Maximum value can be taken by quality characteristic while process is in control.

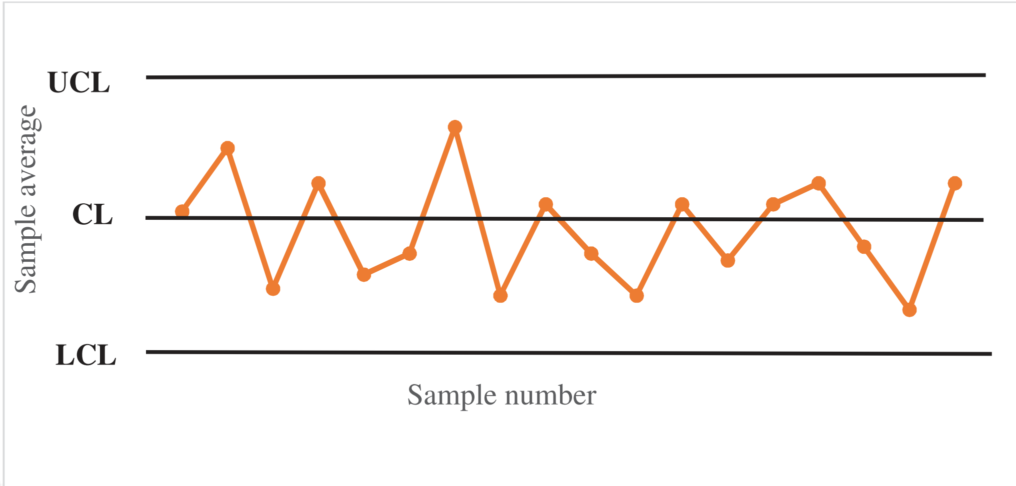

Figure 3.1 A typical control chart

Quality characteristics of interest are placed on chart as in Figure 3.1 and even if a point outside of these limits, we must say that process is out of control.

3.2 Shewhart Control Charts

The first control chart was invented by Walter A. Shewhart while working for "Bell Telephone Laboratories" in 1924. The traditional Shewhart type control charts are parametric with the normality assumption in hypothesis testing to verify that the process is in-control or out-of-control (Pongpullponsak & Jayathavaj, 2014a). Dudding and Jennet (1944) developed a new narrowed confidence interval called as "warning limits (Upper Warning Limit (UWL) and Lower Warning Limit (LWL))" in addition to control limits which is developed by Walter A. Shewhart and so the structure of today's Shewhart control charts formed.

The regions of this chart are shown in Figure 3.2:

- **Region I:** The regions which the process was not operate in accordance with the desired specifications. If an observation point falls here, decided that process is out of control.

- **Region II:** The regions which the process moved away from the desired specifications unsafely. If observations point falls here consecutively, can be decided that process is out of control.
- **Region III:** The regions which the process was operate controlledly in accordance with desired specifications.

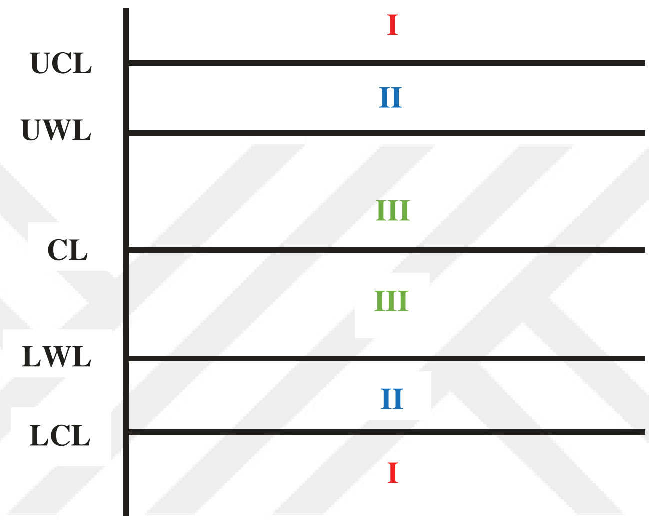

Figure 3.2 Regions of Shewhart control charts

Note that, external limits (UCL and LCL) are usually 3-sigma limits and internal limits (UWL-LWL) are usually 2-sigma limits on Shewhart charts.

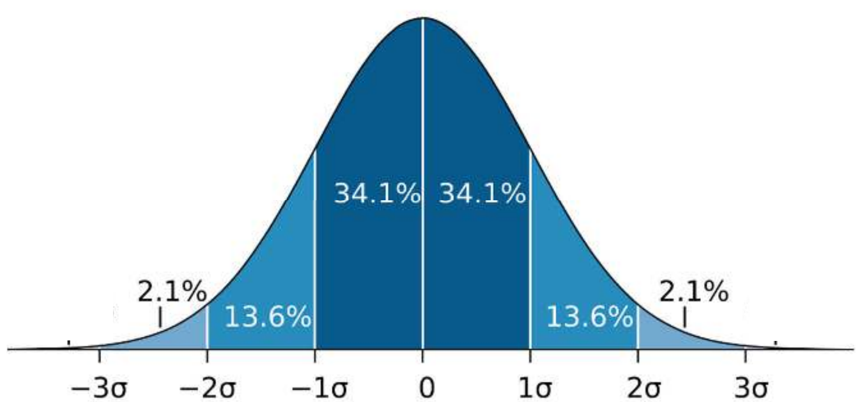

Figure 3.3 Sigma levels

As the sigma level increases, the performance of the process also increases as illustrated in Figure 3.3.

3.2.1 $\overline{X}$ – R *and* $\overline{X}$ – S *Charts*

In process control; need to check both the mean and standard deviation of the variables. For this reason, R or S charts must be used with $\bar{x}$ chart.

If sample size is less than 10 ($2 \leq n \leq 10$), we must use R chart with $\overline{X}$ chart. Otherwise ($n > 10$), we must use S chart with $\overline{X}$ chart.

Suppose that a quality characteristic is normally distributed with mean μ and standard deviation σ. If the standard values (μ and σ) can be specified, we may use these standards for $\overline{X}$ and R charts without analysis of past data.

Table 3.1 Control Limits for Shewhart control charts when standards given

Charts based on standard values (μ and σ are known)		
Chart	**Center Line**	**Control Limits**
$\overline{X}$ chart	μ	$LCL = \mu - 3\frac{\sigma}{\sqrt{n}} = \mu - A\,\sigma$ $UCL = \mu + 3\frac{\sigma}{\sqrt{n}} = \mu + A\,\sigma$
R chart	$d_2\,\sigma$	$LCL = D_1\,\sigma$ $UCL = D_2\,\sigma$
S chart	$c_4\,\sigma$	$LCL = B_5\,\sigma$ $UCL = B_6\,\sigma$

Values of A, D_1, D_2, B_5, B_6, d_2 and c_4 are given in appendix A.

Suppose that we have m samples, each containing n observations. If $x_1, x_2, x_3, \ldots, x_n$ is a sample of size n, then

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} \tag{3.1}$$

$$R = x_{max} - x_{min} \tag{3.2}$$

Let $\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_m$ be the average of each sample, then

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \cdots + \bar{x}_m}{m} \tag{3.3}$$

Let $R_1, R_2, R_3, \ldots, R_m$ be the ranges of the m samples, then

$$\bar{R} = \frac{R_1 + R_2 + R_3 + \cdots + R_m}{m} \tag{3.4}$$

Estimator of σ;

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \tag{3.5}$$

Note that, d_2 value is given in appendix A.

Table 3.2 Control limits for Shewhart control charts when standards not given

Charts based on estimated values (μ and σ are unknown)		
Chart	**Center Line**	**Control Limits**
$\bar{X}$ chart (using R)	$\bar{\bar{x}}$	LCL = $\bar{\bar{x}}-3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}}-A_2 \bar{R}$ UCL = $\bar{\bar{x}}+3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}}+A_2 \bar{R}$
$\bar{X}$ chart (using S)	$\bar{\bar{x}}$	LCL = $\bar{\bar{x}}-A_3 \bar{S}$ UCL = $\bar{\bar{x}}+A_3 \bar{S}$
R chart	$\bar{R}$	LCL = $D_3 \bar{R}$ UCL = $D_4 \bar{R}$
S chart	$\bar{S}$	LCL = $B_3 \bar{S}$ UCL = $B_4 \bar{S}$

Values of A_2, A_3, D_3, D_4, B_3 and B_4 are given in appendix A.

CHAPTER FOUR
PERFORMANCE OF CONTROL CHARTS

4.1 Type I and Type II Errors in Control Charts

Type I and Type II errors are two well-known concepts in quality engineering, which are related to hypothesis testing…. They have many different names (Reliability Engineering Resources, 2016):

- Type I errors also called:
  - Producer's risk
  - False alarm rate
  - α error
- Type II errors also called:
  - Consumer's risk
  - Misdetection
  - β error

Table 4.1 Type I and Type II errors

	H_0 is true	**H_0 is not true**
Reject H_0	Type I error (α)	Correct Decision (1-β) = Power of test
Do not reject H_0	Correct Decision (1-α) = The confidence level of the test	Type II error (β)

Type I error (α) can be defined as concluding the process is out of control when it is actually in control. This is also called “false alarm rate”. It is shown in Figure 4.1.

$$\alpha = P(x < LCL) + P(x > UCL) \tag{4.1}$$

$$\alpha = \alpha_1 + \alpha_2 \tag{4.2}$$

Type II error (β) can be defined as concluding the process is in control when it is actually out of control.

$$\beta = P(LCL < x < UCL) \tag{4.3}$$

The power of the test which here is giving a correct decision is shown in Figure 4.1;

$$1 - \beta = \gamma_1 + \gamma_2 \tag{4.4}$$

Keeping narrow the control limits increase possibility of making a Type I error. On the other side, wide control limits increase possibility of a making Type II error. So, keeping an optimum level of Type I and Type II errors is quite important.

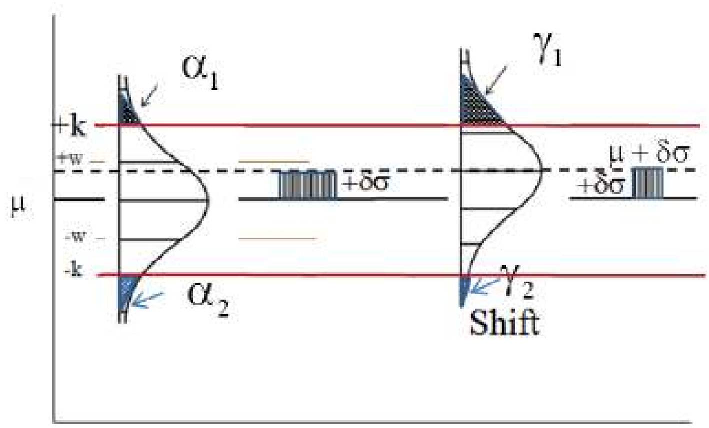

Figure 4.1 Process shift (Pongpullponsak & Jayathavaj, 2014b)

Note that, UCL and LCL are denoted as +k and –k respectively in Figure 4.1. UWL and LWL are also denoted as +w and –w.

4.2 Average Run Length

Run length (RL) is the number of samples or subgroups that need to be collected before the first out of control signal is given by a chart and hence average run length (ARL) is expected value of the run length (Pongpullponsak & Jayathavaj, 2014b). ARL which is a popular measure of chart performance had been considered by Page in 1954 and there has been wide applications in industrial sector. In general ARL is;

$$\text{ARL} = \frac{1}{P(one\ point\ plots\ out\ of\ contol)} \tag{4.5}$$

ARL can be calculated based on Type I error (α) and Type II error (β) values. There are two cases for ARL in any Shewhart chart:

- **In control ARL (ARL_0);** Pongpullponsak & Jayathavaj (2014b) define ARL_0 as "the in control average run length (ARL_0) is the average number of samples before the control chart signals an out of control condition". Process mean is on the target value here. In short $\mu = \mu_0$.

$$\text{ARL}_0 = \frac{1}{P\left(\begin{matrix} one\ point\ plots \\ out\ of\ control \end{matrix} \middle| \begin{matrix} process\ is \\ in\ control \end{matrix}\right)} = \frac{1}{\alpha} \tag{4.6}$$

Where α is the probability of the Type I error just as Equation 4.2 and Figure 4.1. Another important point is that ARL_0 values are desired to have large value.

- **Out of control ARL (ARL_1 or ARL_1^{δ});** Pongpullponsak & Jayathavaj (2014b) also define ARL_1 as "the out of control average run length (ARL_1) is the number of samples to detect the process shift, on the average". The only difference from ARL_0 is that process mean shifted from μ to $\mu+\delta\sigma$ as in Figure 4.1. In short $\mu \neq \mu_0$.

$$\text{ARL}_1 = \frac{1}{P\left(\begin{matrix} one\ point\ plots \\ out\ of\ control \end{matrix} \middle| \begin{matrix} process\ is \\ out\ of\ control \end{matrix}\right)} = \frac{1}{(1-\beta)} \tag{4.7}$$

Where (1-β) is shown in Equation 4.4 and Figure 4.1. Furthermore, ARL_1 values are desired to have small values so as to detect the shift quickly.

Giving a simple example on Shewhart $\overline{X}$ chart will help to better understand the concept of ARL; first suppose that process is in control when the standards given as μ=0 and σ =1 (assuming normality) also sample size of each group n=5. Then, LCL and UCL are:

$$\text{LCL} = \mu - 3\frac{\sigma}{\sqrt{n}} = 0 - 3\frac{1}{\sqrt{5}}$$

$$\text{UCL} = \mu + 3\frac{\sigma}{\sqrt{n}} = 0 + 3\frac{1}{\sqrt{5}}$$

P (one point plots out of control) = $\alpha = P\,[\bar{x} < \text{LCL or } \bar{x} > \text{UCL}]$

$$= 1 - P\,[\text{LCL} < \bar{x} < \text{UCL}]$$

$$= 1 - P\left[\frac{LCL-\mu}{\sigma/\sqrt{n}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{UCL-\mu}{\sigma/\sqrt{n}}\right]$$

$$= 1 - P\left[\frac{{}^{-3}/_{\sqrt{5}}-0}{{}^{1}/_{\sqrt{5}}} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{{}^{3}/_{\sqrt{5}}-0}{{}^{1}/_{\sqrt{5}}}\right]$$

$\alpha = 1 - P\,[-3 < Z < 3] = 0.0027$ (using the standard Normal distribution table)

$$\text{ARL}_0 = \frac{1}{\alpha} = \frac{1}{0.0027} = 370$$

Interpretation: Even if the process remains in control, an out of control signal will be generated every 370 samples, on the average (Montgomery, 2009).

In the second place, suppose that process is out of control and process mean is shifted from its standard value by 1σ (μ+δσ). Sample size of each group n=5. Also, μ and σ values are still standard values like example of ARL_0.

$\beta = P\ [LCL < \bar{x} < UCL]$

$1 - \beta = 1 - P\ [LCL < \bar{x} < UCL]$

$$= 1 - P\ [\frac{LCL-(\mu+\sigma)}{\sigma/\sqrt{n}} < \frac{\bar{x}-(\mu+\sigma)}{\sigma/\sqrt{n}} < \frac{UCL-(\mu+\sigma)}{\sigma/\sqrt{n}}]$$

$$= 1 - P\ [-5.24 < Z < 0.76] = 0.2236$$

$$ARL_1 = \frac{1}{(1-\beta)} = \frac{1}{0.2236} = 4.5$$

Interpretation: $\overline{X}$ chart detect the shift in process mean on the average 4.5th sample when the process mean shifted from its standard value by 1σ.

CHAPTER FIVE

NONPARAMETRIC CONTROL CHART BASED ON HODGES – LEHMANN ESTIMATOR

5.1 Overview to Nonparametric Control Charts

The parametric control charts make strong probabilistic assumptions on the variable that they control. These charts generally assume that the variable under control follows a predefined distribution, e.g., a Normal distribution. The nonparametric control charts, on the other hand, make weaker assumptions. For instance, they may only assume that the distribution of the quality characteristic is symmetric (Chakraborti et al., 2001; Alvarez et al., 2016).

Parametric control charts usually offer a better control of the process when the distribution of the variable is the one that was assumed by the chart. However, when the variable follows a different distribution, the performance of the parametric chart may deteriorate dramatically. Because of their weaker assumptions, nonparametric charts are more robust than their parametric counterparts to changes in the distribution (Chakraborti et al., 2001; Alvarez et al., 2016).

Besides, Khilare and Shirke (2010) stated in their study that “in nonparametric control charts variance of the process need not to be known or estimated in order to apply the control chart. In fact, the nonparametric control charts for controlling median are not affected by changes in the variance as long as location parameter (μ) is constant”.

Finally, it is true that nonparametric methods are not well-known among many control chart practitioners because these methods are not emphasized enough (Chakraborti et al., 2001).

5.2 Hodges – Lehmann Estimator

Hodges-Lehmann estimator (HLE) was proposed by Joseph Hodges and Erich Lehmann in 1963. This estimator was obtained from Wilcoxon signed-rank statistic and based on the median of Walsh averages (mean of the pair of each item and itself) so the Hodges-Lehmann estimator is a nonparametric estimator of a population's location parameter. Main properties of the Hodges – Lehmann estimator can be listed as:

- Ch'ng, Quah & Low (2004) stated in their paper that "the asymptotic relative efficiency of the HLE relative to sample mean is 0.955 if the underlying distribution is Normal. However, the asymptotic relative efficiency of the HLE is often greater than unity if the underlying distribution is non-normal" taking advantage of Lehmann's (1983) study.
- HLE is an unbiased estimator.
- HLE has a breakdown point of 29%. Breakdown point is the most popular measure of robustness. Median has a 0.5 breakdown point while sample mean has a zero breakdown point. So, we can say that HLE is quite robust to outliers.

The computation steps of HLE; Let $Y_1, Y_2, \ldots, Y_n$ be a random sample obtained from some distribution which is continuous and symmetrical about θ. Then,

- Compute,

$$M = \frac{n(n+1)}{2} \tag{5.1}$$

Where M is the number of Walsh averages.

- Compute Walsh averages,

$$W_r = \frac{Y_i + Y_j}{2} \tag{5.2}$$

Where $r = 1, 2, \ldots,$ M and $i \leq j = 1, 2, \ldots, n$.

- Sort the Walsh averages in ascending form, that is,

$$W_{(1)} \leq W_{(2)} \leq \ldots \leq W_{(M)} \tag{5.3}$$

- The HLE for the point of symmetry θ of a continuous and symmetric distribution is defined as:

$$\text{HLE} = \begin{cases} W_{(k+1)} & , \ M \text{ is odd} \\ \left(W_{(k)} + W_{(k+1)}\right)/2 & , \ M \text{ is even} \end{cases} \tag{5.4}$$

$$k = \begin{cases} \frac{M-1}{2} & , \ M \text{ is odd} \\ \frac{M}{2} & , \ M \text{ is even} \end{cases} \tag{5.5}$$

Ch'ng, Quah & Low (2004) give an example to better understand the computations of Hodges – Lehmann estimator. Suppose that n=4 and let $Y_1=7$, $Y_2=10.5$, $Y_3=9.3$ and $Y_4=10.3$.

$M = \frac{n(n+1)}{2} = \frac{4(5)}{2} = 10 \longrightarrow$ number of Walsh averages are 10.

Find Walsh averages values by the help of $W_r = \frac{Y_i+Y_j}{2}$;

$W_1 = \frac{Y_1+Y_1}{2} = \frac{7+7}{2} = 7$ $\qquad$ $W_6 = \frac{Y_2+Y_3}{2} = \frac{10.5+9.3}{2} = 9.90$

$W_2 = \frac{Y_1+Y_2}{2} = \frac{7+10.5}{2} = 8.75$ $\qquad$ $W_7 = \frac{Y_2+Y_4}{2} = \frac{10.5+10.3}{2} = 10.40$

$W_3 = \frac{Y_1+Y_3}{2} = \frac{7+9.3}{2} = 8.15$ $\qquad$ $W_8 = \frac{Y_3+Y_3}{2} = \frac{9.3+9.3}{2} = 9.30$

$W_4 = \frac{Y_1+Y_4}{2} = \frac{7+10.3}{2} = 8.65$ $\qquad$ $W_9 = \frac{Y_3+Y_4}{2} = \frac{9.3+10.3}{2} = 9.80$

$W_5 = \frac{Y_2+Y_2}{2} = \frac{10.5+10.5}{2} = 10.5$ $\qquad$ $W_{10} = \frac{Y_4+Y_4}{2} = \frac{10.3+10.3}{2} = 10.30$

Reorder the Walsh averages;

$W_1 < W_3 < W_4 < W_2 < W_8 < W_9 < W_6 < W_{10} < W_7 < W_5$

Since M=10 is even, $k = \frac{10}{2} = 5$ and k+1 = 6, then with classical median computation;

$$\text{HLE} = \frac{W_8+W_9}{2} = \frac{9.30+9.80}{2} = 9.55$$

5.3 Hodges – Lehmann Control Chart

Das (2009) stated in her paper that "Alloway and Raghavachari (1991) considered a Shewhart-type chart for the point of symmetry θ of a continuous, symmetric population based on a distribution free confidence interval for θ, calculated using the Hodges – Lehmann estimator".

The chart based on Hodges – Lehmann estimator is related to Wilcoxon signed-rank confidence interval. The approach differs from a traditional Shewhart control chart. Control limits are based on the order statistics instead of traditional measure of mean and dispersion in Hodges – Lehmann control chart (Das, 2009; Laha et al., 2005).

Pongpullponsak and Jayathavaj (2014b) stated that if WSR^+ is the Wilcoxon signed rank statistic for testing the null hypothesis that the underlying distribution of the individual observation is symmetric about θ then WSR^+ is the number of Walsh averages greater than θ.

Chakraborti et al. (2001) stated that 100(1-α) % distribution free confidence interval for θ is given by two ordered Wash averages;

$$\mathrm{P}\left(W_{(a_i)} \leq \theta \leq W_{(M-a_i+1)}\right) \geq 1-\alpha \tag{5.6}$$

Where $\mathrm{a_i}$ is the WSR^+ $(^{\alpha}/_{2}, n)$.

The given confidence interval connection with Wilcoxon signed rank statistic and its table where is in appendix C constructed by Wilcoxon, Katti & Wilcox (Chakraborti et al., 2001; Gibbons & Chakraborti, 2003).

Steps to construct the HL control chart can be listed as:

- Suppose that we have m subgroups each of size n. Let x_1, x_2, …, x_n be the observations in each subgroup. Minimum size of n should be 10 for HL control chart. Because, it is require to achieve a significance level closes to ±3 control limits in traditional used (Pongpullponsak & Jayathavaj, 2014a).

- Compute Walsh averages by Equation 5.2 and Hodges – Lehmann estimator by Equation 5.4 for each subgroup after this we have m Hodges – Lehmann estimator.

- The 100(1-α) % distribution free confidence interval is computed for each *m* subgroups like in Equation 5.6 and after this we have *m* confidence intervals.

- Finally, we can find control limits of HL chart taking advantage of Figure 5.1.

	1 **2** . . . ***n***
1	Median of Walsh averages in first subgroup = HL_1 $(W_{(a_{i1})}, W_{(M-a_{i1}+1)})$ is the confidence interval of first subgroup
2	Median of Walsh averages in second subgroup = HL_2 $(W_{(a_{i2})}, W_{(M-a_{i2}+1)})$ is the confidence interval of second subgroup
⋮	⋮ ⋮
m	

Figure 5.1 Simplified representation of HL chart's control limits

LCL = Median of the lower confidence limits.

= Median ($W_{a1}, W_{a2}, ..., W_{am}$)

CL = Mean of the Hodges – Lehmann estimators.

= ((HLE$_1$+HLE$_2$+…+HLE$_m$) / m)

UCL = Median of the upper confidence limits.

= Median ($W_{M-ai1+1}, W_{M-ai2+1}, ..., W_{M-aim+1}$)

Note that, out of control condition of HL chart;

$$\text{HLE} < \text{LCL or HLE} > \text{UCL}$$

5.4 Performance of Hodges – Lehman Control Chart

In previous chapters it is specified that the how to we evaluate the performance of control charts with concept of ARL. In Hodges – Lehmann control chart it is same working as traditional Shewhart-type control charts and ARL values are found with the help of simulation study as follows;

- **Concept of ARL$_0$ for Hodges – Lehmann control chart;** First, Hodges – Lehmann estimators are computed for each subgroup by following the steps in the section 5.2. The condition of being out of control of the HL chart is HLE is lesser than LCL or greater than UCL.

Pongpullponsak & Jayathavaj (2014b) stated that concept of ARL$_0$ for Hodges – Lehmann control chart is "Let RL_i for i=1,2,…,10000 be the 10000 run lengths of Hodges-Lehmann estimators of the sample size n from the given distribution. If the first out-of-control Hodges-Lehmann estimator is found, then the RL equals to the number of the previous Hodges-Lehmann estimators before out-of-control signal". Thus,

$$\text{ARL}_0 = \frac{\sum_{i=1}^{10000} RL_i}{10000} \tag{5.7}$$

- **Concept of ARL_1 for Hodges – Lehmann control chart;** Hodges –Lehmann estimators are computed for each subgroup by following the steps in the section 5.2.

Pongpullponsak & Jayathavaj (2014b) also stated that concept of ARL_1 for Hodges – Lehmann control chart is "Let RL_i^{δ} for i=1,2,…,10000 be the 10000 run lengths of Hodges-Lehmann estimators of the sample size n using the given process distribution with shift in δ times standard deviation (σ). Hodges-Lehmann estimator is computed for each sample. When the first out-of-control Hodges-Lehmann estimator is found, then i^{th} RL equals to the number of this first out-of-control sample". Thus,

$$ARL_1 = \frac{\sum_{i=1}^{10000} RL_i^{\delta}}{10000} \tag{5.8}$$

CHAPTER SIX
SIMULATION STUDY

6.1 Design of Simulation

In this chapter of the thesis, control limits and ARL values of the Hodges – Lehmann control chart are computed for standard Normal, Uniform and Weibull distributions. In addition to that, Shewhart chart and Hodges – Lehmann chart are compared under distribution of standard Normal with a simulation study. Distributions are;

- Normal distribution with μ=0 and σ =1.
- Uniform distribution with a=0 and b=1.
- Weibull distribution with shape parameter (β) =3 and scale parameter (η) =3.

Histograms of the distributions are illustrated in Figures 6.1, 6.2 and 6.3.

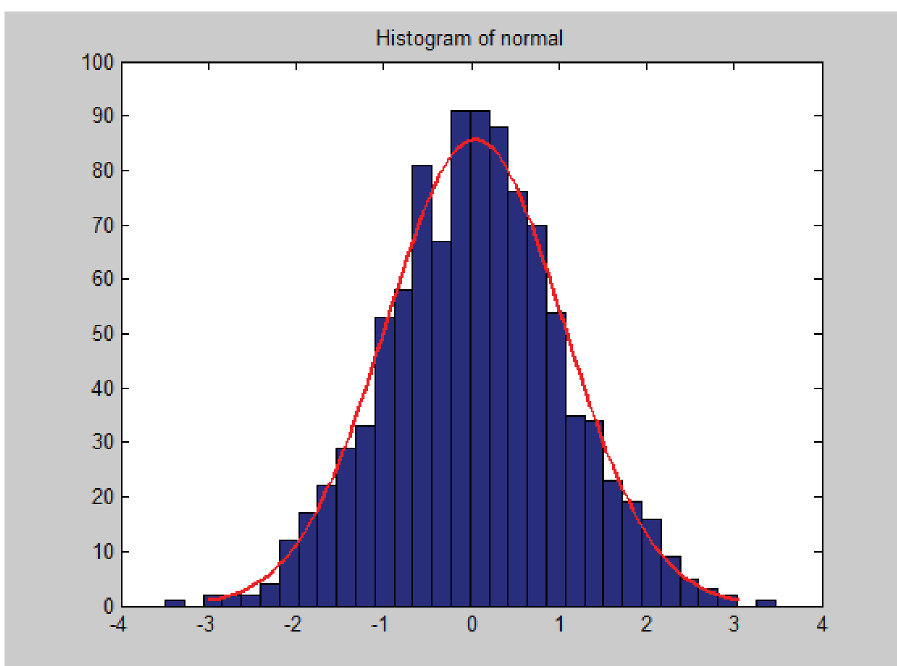

Figure 6.1 Histogram of Normal distribution

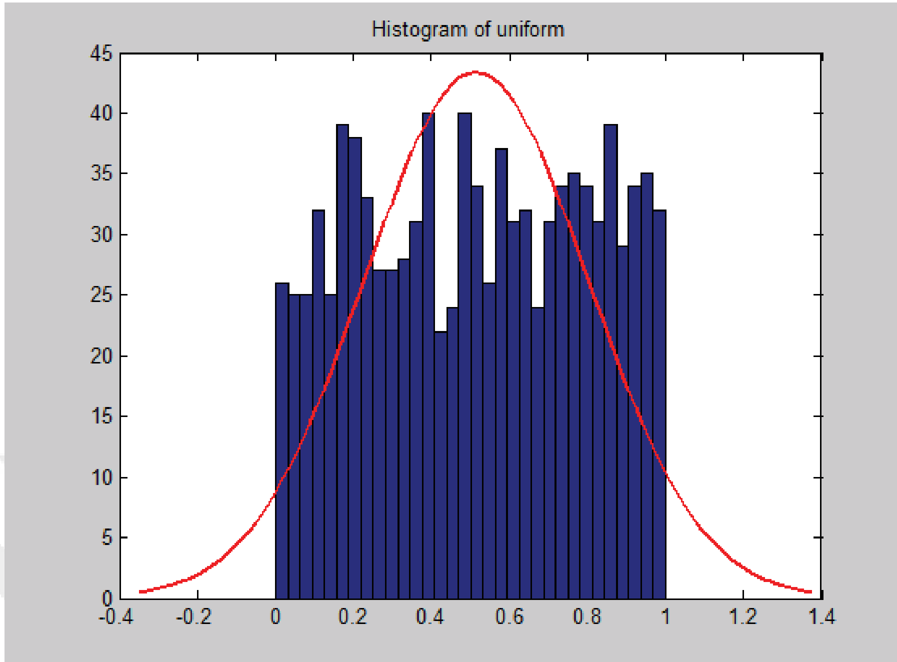

Figure 6.2 Histogram of Uniform distribution

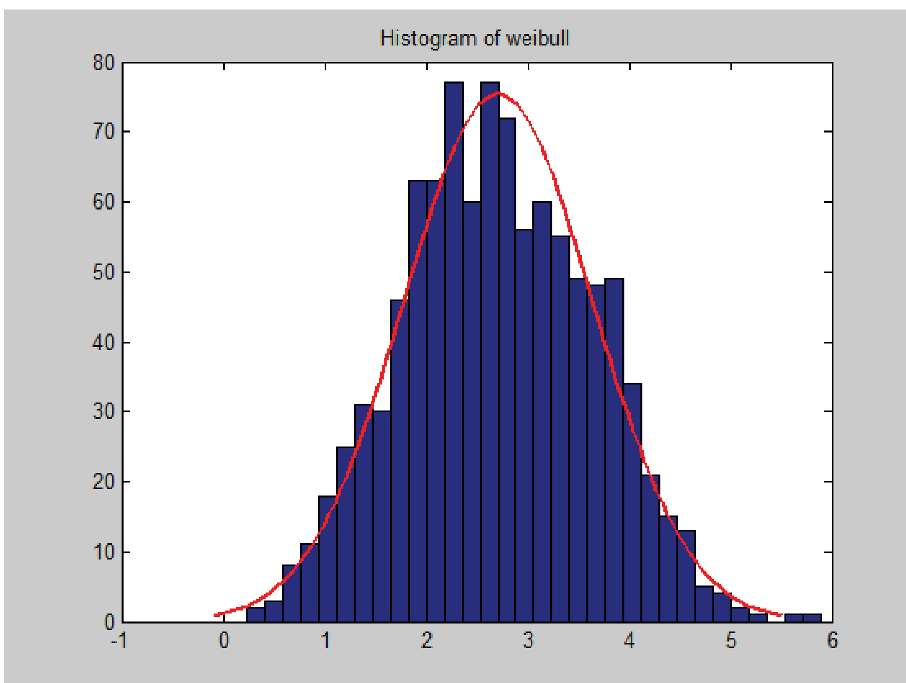

Figure 6.3 Histogram of Weibull distribution

Simulation study is performed by using Matlab and Rstudio. Histograms and Shewhart control chart's ARL values are found by using Matlab and computing of Hodges – Lehmann control chart is done by using Rstudio.

Simulation process is explained step by step as follows;

1. Generate random numbers x_i for $i=1, \ldots, n$ from the intended distribution with the parameter $\mu+\delta\sigma$, then Hodges – Lehmann estimators and control limits (LCL, CL and UCL) are derived from 10000 runs simulation.
2. For computing of run length, again generate random numbers x_i for $i=1, \ldots, n$ from the same distribution.
3. Compute Hodges – Lehmann estimator.
4. Compare the HLE with control limits;
   If LCL ≤ HLE ≤ UCL, go back to second step.
   If HLE < LCL or HLE > UCL, keep the number of sample that appear out of control and repeat this process 1000 times.
5. Finally, compute the ARL (mean of run lengths).

6.2 Results

In this section, simulation results are shown and interpreted. Hodges – Lehmann control chart is constructed for different sample sizes n =10, 11, 12 and 15. To compute ARL, process shifts in δ times of standard deviation from 0, 0.25, 0.50, 1, 2 and 3 are selected.

First, Hodges – Lehmann control chart is constructed for specified distributions while sample size (n) is 10 as in Table 6.1 and Table 6.2. It should be noted that number of Walsh averages is 55 when n =10. So, confidence interval of two ordered Walsh averages are selected as 4^{th} and 52^{th} by using appendix B and appendix C. Correspondingly, acceptable α is selected about 0.004 by using Gibbons' (2003) table in appendix C.

Table 6.1 Control limits of HL chart when n =10

Shift	Distribution		
	Normal	**Uniform**	**Weibull**
0.00	**LCL :** -1.0307 **CL :** 0.0100 **UCL :** 1.0681	**LCL :** 0.1801 **CL :** 0.5000 **UCL :** 0.8205	**LCL :** 1.6682 **CL :** 2.6656 **UCL :** 3.7094
0.25	**LCL :** -0.7937 **CL :** 0.2439 **UCL :** 1.2862	**LCL :** 0.4297 **CL :** 0.7494 **UCL :** 1.0683	**LCL :** 1.9089 **CL :** 2.9191 **UCL :** 3.9606
0.50	**LCL :** -0.5190 **CL :** 0.5091 **UCL :** 1.5347	**LCL :** 0.6824 **CL :** 0.9993 **UCL :** 1.3176	**LCL :** 2.1612 **CL :** 3.1678 **UCL :** 4.2039
1.00	**LCL :** -0.0328 **CL :** 1.0002 **UCL :** 2.0383	**LCL :** 1.1806 **CL :** 1.4989 **UCL :** 1.8182	**LCL :** 2.6696 **CL :** 3.6701 **UCL :** 4.7112
2.00	**LCL :** 0.9700 **CL :** 2.0000 **UCL :** 3.0251	**LCL :** 2.1818 **CL :** 2.5033 **UCL :** 2.8189	**LCL :** 3.6554 **CL :** 4.6633 **UCL :** 5.7099
3.00	**LCL :** 1.9720 **CL :** 3.0024 **UCL :** 4.0349	**LCL :** 3.1788 **CL :** 3.4966 **UCL :** 3.8166	**LCL :** 4.6610 **CL :** 5.6668 **UCL :** 6.7051

Table 6.2 ARL values when n =10

Shift	Distribution		
	Normal	**Uniform**	**Weibull**
0.00	691.926	546.523	643.516
0.25	134.981	4.25	106.324
0.50	17.519	1.032	17.63
1.00	2.189	1.00	1.98
2.00	1.00	1.00	1.001
3.00	1.00	1.00	1.00

In results of tables, shift in 0.00 represents the ARL_0 and it means that there is no any shift in the process ($\mu=\mu_0$). Also, shift in 0.25, 0.50, 1.00, 2.00 and 3.00 gives ARL_1 values. In Table 6.2, the best performance of HL chart is under Normal distribution with ARL_0 =691.926 when n =10. Because high values are intended for ARL_0. Looking at the ARL_1 values, HL chart's performance is quite good under Uniform distribution while δ=0.25, 0.50 and 1.00. Because low values are intended for ARL_1. So we can say that HL chart catches these shifts sooner when process distribution is Uniform according to Table 6.2.

Table 6.3 Control limits of HL chart when n =11

Shift	Distribution		
	Normal	**Uniform**	**Weibull**
0.00	**LCL :** -0.9578 **CL :** -0.0054 **UCL :** 0.9488	**LCL :** 0.2016 **CL :** 0.5007 **UCL :** 0.7986	**LCL :** 1.7407 **CL :** 2.6722 **UCL :** 3.6333
0.25	**LCL :** -0.7090 **CL :** 0.2484 **UCL :** 1.2071	**LCL :** 0.4492 **CL :** 0.7493 **UCL :** 1.0479	**LCL :** 1.9904 **CL :** 2.9146 **UCL :** 3.8789
0.50	**LCL :** -0.4678 **CL :** 0.4950 **UCL :** 1.4543	**LCL :** 0.6999 **CL :** 0.9992 **UCL :** 1.2974	**LCL :** 2.2351 **CL :** 3.1677 **UCL :** 4.1223
1.00	**LCL :** 0.0428 **CL :** 1.0026 **UCL :** 1.9560	**LCL :** 1.2007 **CL :** 1.4984 **UCL :** 1.7978	**LCL :** 2.7397 **CL :** 3.6705 **UCL :** 4.6293
2.00	**LCL :** 1.0457 **CL :** 2.0038 **UCL :** 2.9642	**LCL :** 2.2042 **CL :** 2.5023 **UCL :** 2.7993	**LCL :** 3.7444 **CL :** 4.6701 **UCL :** 5.6301
3.00	**LCL :** 2.0424 **CL :** 2.9998 **UCL :** 3.9487	**LCL :** 3.1988 **CL :** 3.4988 **UCL :** 3.7972	**LCL :** 4.7337 **CL :** 5.6649 **UCL :** 6.6258

Table 6.4 ARL values when n =11

Shift	Distribution		
	Normal	**Uniform**	**Weibull**
0.00	470.102	407.217	477.255
0.25	87.077	3.417	75.251
0.50	14.671	1.019	11.577
1.00	1.845	1.00	1.688
2.00	1.00	1.00	1.001
3.00	1.00	1.00	1.00

In Table 6.3, control limits are much closed to the results of Table 6.1. In Table 6.4, HL chart's performance is better for Weibull distribution with ARL_0=477.255 when there is no any shift in the process. For ARL_1, again Uniform distribution is better than the others.

Table 6.5 Control limits of HL chart when n =12

Shift	Normal	Uniform	Weibull
	Distribution		
0.00	**LCL :** -0.8964 **CL :** -0.0016 **UCL :** 0.8997	**LCL :** 0.2175 **CL :** 0.4993 **UCL :** 0.7817	**LCL :** 1.7884 **CL :** 2.6624 **UCL :** 3.5702
0.25	**LCL :** -0.6533 **CL :** 0.2518 **UCL :** 1.1523	**LCL :** 0.4693 **CL :** 0.7514 **UCL :** 1.0333	**LCL :** 2.0418 **CL :** 2.9213 **UCL :** 3.8286
0.50	**LCL :** -0.4002 **CL :** 0.5039 **UCL :** 1.4113	**LCL :** 0.7197 **CL :** 1.0007 **UCL :** 1.2800	**LCL :** 2.2862 **CL :** 3.1635 **UCL :** 4.0691
1.00	**LCL :** 0.0911 **CL :** 0.9993 **UCL :** 1.9161	**LCL :** 1.2185 **CL :** 1.5012 **UCL :** 1.7829	**LCL :** 2.7945 **CL :** 3.6679 **UCL :** 4.5785
2.00	**LCL :** 1.0985 **CL :** 2.0000 **UCL :** 2.9060	**LCL :** 2.2173 **CL :** 2.4993 **UCL :** 2.7809	**LCL :** 3.7891 **CL :** 4.6645 **UCL :** 5.5708
3.00	**LCL :** 2.0933 **CL :** 2.9993 **UCL :** 3.9006	**LCL :** 3.2179 **CL :** 3.4995 **UCL :** 3.7800	**LCL :** 4.7906 **CL :** 5.6673 **UCL :** 6.5696

Table 6.6 ARL values when n =12

Shift	Normal	Uniform	Weibull
	Distribution		
0.00	361.722	381.388	393.917
0.25	66.386	2.716	62.992
0.50	11.504	1.008	10.438
1.00	1.609	1.00	1.641
2.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00

In Table 6.6, ARL values are lower than results of Tables 6.2 and 6.4. Weibull distribution is better for ARL_0 and Uniform distribution is better for ARL_1. In typical Shewhart charts, desired ARL_0 is 370. So, HL chart's performance is not as good as Shewhart's chart when n =12.

Table 6.7 Control limits of HL chart when n =15

Shift	Distribution		
	Normal	**Uniform**	**Weibull**
0.00	**LCL :** -0.7858 **CL :** 0.0004 **UCL :** 0.7875	**LCL :** 0.2540 **CL :** 0.4991 **UCL :** 0.7446	**LCL :** 1.9027 **CL :** 2.6646 **UCL :** 3.4594
0.25	**LCL :** -0.5394 **CL :** 0.2502 **UCL :** 1.0376	**LCL :** 0.5019 **CL :** 0.7486 **UCL :** 0.9948	**LCL :** 2.1557 **CL :** 2.9181 **UCL :** 3.7108
0.50	**LCL :** -0.2906 **CL :** 0.4972 **UCL :** 1.2905	**LCL :** 0.7504 **CL :** 0.9988 **UCL :** 1.2445	**LCL :** 2.4007 **CL :** 3.1663 **UCL :** 3.9612
1.00	**LCL :** 0.2068 **CL :** 1.0017 **UCL :** 1.7914	**LCL :** 1.2543 **CL :** 1.4995 **UCL :** 1.7462	**LCL :** 2.9011 **CL :** 3.6678 **UCL :** 4.4636
2.00	**LCL :** 1.2105 **CL :** 2.0000 **UCL :** 2.7907	**LCL :** 2.2522 **CL :** 2.4993 **UCL :** 2.7462	**LCL :** 3.9064 **CL :** 4.6722 **UCL :** 5.4606
3.00	**LCL :** 2.2147 **CL :** 3.0005 **UCL :** 3.7967	**LCL :** 3.2537 **CL :** 3.4996 **UCL :** 3.7464	**LCL :** 4.8990 **CL :** 5.6657 **UCL :** 6.4618

Table 6.8 ARL values when n =15

Shift	Distribution		
	Normal	**Uniform**	**Weibull**
0.00	320.17	262.733	323.258
0.25	47.613	1.987	37.729
0.50	7.774	1.002	6.075
1.00	1.274	1.00	1.241
2.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00

In Table 6.8, Weibull and Uniform distributions give better results than Normal distribution that expected. But in general, performance of HL chart quite good even if distribution is Normal.

Table 6.9 ARL values of the $\overline{\mathrm{X}}$ and HL charts for Normal distribution

Shift	$\overline{X}$ chart	HL chart
0.00	370.239	3222.345
0.25	130.43	835.94
0.50	33.71	95.683
1.00	4.55	4.696
2.00	1.001	1.007
3.00	1.00	1.00

In Table 6.9, $\overline{\mathrm{X}}$ and HL charts are compared for Normal distribution. $\overline{\mathrm{X}}$ chart's sample size is 5 and HL chart's sample size is 10. Also, confidence interval of Walsh averages for HL chart is selected as 2th and 53th Walsh averages by using appendix C of Gibbon's study (2003) since achieving the significance level (α) closes to 0.0027 in traditional used. As is seen from Table 6.9, performance of HL chart is better than $\overline{\mathrm{X}}$ chart when there is no shift in the process. However, if there is shift in the process, $\overline{\mathrm{X}}$ chart catch this shift sooner than HL chart.

CHAPTER SEVEN
CONCLUSIONS

In this thesis, the proposed control chart which is based on Hodges – Lehmann estimator is evaluated and compared with $\bar{x}$ chart by computer simulation and below are the conclusions;

- According to results of simulation, control limits are gradually narrowed while sample size increases.
- Probability of one point falling beyond the control limits is increases as long as control limits narrow. Therefore, ARL_0 and ARL_1 values decrease when the sample size increases.
- Looking at the ARL_1 for the all results of tables, HL chart catch the shift sooner as long as sample size increases.
- HL chart also catch the shift sooner while the amount of deviation from the mean increases for all specified distribution.
- The best result for ARL_0=691.926 under Normal distribution and ARL_1 values closer to each other for Normal and Weibull distributions while sample size is 10.
- While sample size is 11, the best ARL_0=477.255 under Weibull distribution. Besides, Normal distribution's ARL_0=470.102 and apparently HL chart is still better than $\bar{X}$ chart. Because, desired ARL_0 for Normal distribution in $\bar{X}$ chart is 370.
- While sample size is 12, the best ARL_0=393.917 under Weibull distribution.
- While sample size is 15, Weibull distribution has still better ARL_0=323.258 than the others.
- In all results of tables, Uniform distribution has the best ARL_1 for in every shift value. It means that HL chart catch the shift sooner for Uniform distribution.
- Confidence interval of Walsh averages are selected from appendix B and appendix C. Value of α is specified as about 0.004 for Tables 6.2, 6.4, 6.6 and 6.8. When the confidence interval of Walsh averages are narrowed, α value increases and if α value increases, ARL values decrease.

- Depending on α value, ARL_0 of HL chart is 3222.345 and it is very high performance when compared to the $\bar{x}$ chart. However, when there is a shift in the process, performance of $\overline{X}$ chart is better than HL chart but as the shift in the process increased, ARL_1 values closed each other.
- All in all, HL chart gives quite good results for Normal distribution although it is one of the nonparametric chart.

As a result, Hodges-Lehmann control chart is an alternative control chart that gives good results for different distributions and it is robust to the violation of normality assumption and outliers since it is based on median. It is worth noting that Type I error probability (α) is selected low enough for making the ARL_0 high enough and ARL_1 low enough.

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APPENDICES

Appendix A: Constant values of Shewhart control charts

	For Estimating Sigma		For $\overline{X}$ Chart		For $\overline{X}$ Chart (Standard Given)	For R Chart		For R Chart (Standard Given)		For s chart (Standard Given)			
n	c_4^*	d_2	A_2	A_3^*	A	D_3	D_4	D_1	D_2	B_3	B_4	B_5^*	B_6^*
2	0.7979	1.128	1.880	2.659	2.121	0	3.267	0	3.686	0	3.267	0	2.606
3	0.8862	1.693	1.023	1.954	1.732	0	2.575	0	4.358	0	2.568	0	2.276
4	0.9213	2.059	0.729	1.628	1.500	0	2.282	0	4.698	0	2.266	0	2.088
5	0.9400	2.326	0.577	1.427	1.342	0	2.115	0	4.918	0	2.089	0	1.964
6	0.9515	2.534	0.483	1.287	1.225	0	2.004	0	5.078	0.030	1.970	0.029	1.874
7	0.9594	2.704	0.419	1.182	1.134	0.076	1.924	0.205	5.203	0.118	1.882	0.113	1.806
8	0.9650	2.847	0.373	1.099	1.061	0.136	1.864	0.387	5.307	0.185	1.815	0.179	1.751
9	0.9693	2.970	0.337	1.032	1.000	0.184	1.816	0.546	5.394	0.239	1.761	0.232	1.707
10	0.9727	3.078	0.308	0.975	0.949	0.223	1.777	0.687	5.469	0.284	1.716	0.276	1.669
15	0.9823	3.472	0.223	0.789	0.775	0.348	1.652	1.207	5.737	0.428	1.572	0.421	1.544
20	0.9869	3.735	0.180	0.680	0.671	0.414	1.586	1.548	5.922	0.510	1.490	0.504	1.470
25	0.9896	3.931	0.153	0.606	0.600	0.459	1.541	1.804	6.058	0.565	1.435	0.559	1.420

Appendix B: Quantiles of the Wilcoxon signed ranks test statistic.

	$w_{0.005}$	$w_{0.01}$	$w_{0.025}$	$w_{0.05}$	$w_{0.10}$	$w_{0.20}$	$w_{0.30}$	$w_{0.40}$	$w_{0.50}$	$\frac{n(n+1)}{2}$
$n = 4$	0	0	0	0	1	3	3	4	5	10
5	0	0	0	1	3	4	5	6	7.5	15
6	0	0	1	3	4	6	8	9	10.5	21
7	0	1	3	4	6	9	11	12	14	28
8	1	2	4	6	9	12	14	16	18	36
9	2	4	6	9	11	15	18	20	22.5	45
10	4	6	9	11	15	19	22	25	27.5	55
11	6	8	11	14	18	23	27	30	33	66
12	8	10	14	18	22	28	32	36	39	78
13	10	13	18	22	27	33	38	42	45.5	91
14	13	16	22	26	32	39	44	48	52.5	105
15	16	20	26	31	37	45	51	55	60	120
16	20	24	30	36	43	51	58	63	68	136
17	24	28	35	42	49	58	65	71	76.5	153
18	28	33	41	48	56	66	73	80	85.5	171
19	33	38	47	54	63	74	82	89	95	190
20	38	44	53	61	70	83	91	98	105	210
21	44	50	59	68	78	91	100	108	115.5	131
22	49	56	67	76	87	100	110	119	126.5	153
23	55	63	74	84	95	110	120	130	138	176
24	62	70	82	92	105	120	131	141	150	300
25	69	77	90	101	114	131	143	153	162.5	325
26	76	85	99	111	125	142	155	165	175.5	351
27	84	94	108	120	135	154	167	178	189	378
28	92	102	117	131	146	166	180	192	203	406
29	101	111	127	141	158	178	193	206	217.5	435
30	110	121	138	152	170	191	207	220	232.5	465
31	119	131	148	164	182	205	221	235	248	496
32	129	141	160	176	195	219	236	250	264	528
33	139	152	171	188	208	233	251	266	280.5	561
34	149	163	183	201	222	248	266	282	297.5	595
35	160	175	196	214	236	263	283	299	315	630
36	172	187	209	228	251	279	299	317	333	666
37	184	199	222	242	266	295	316	335	351.5	703
38	196	212	236	257	282	312	334	353	370.5	741
39	208	225	250	272	298	329	352	372	390	780
40	221	239	265	287	314	347	371	391	410	820
41	235	253	280	303	331	365	390	411	430.5	861
42	248	267	295	320	349	384	409	431	451.5	903
43	263	282	311	337	366	403	429	452	473	946
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128

Appendix C: Probabilities for the Wilcoxon signed-rank statistic.

N	*Left tail*	*P*	*Right tail*	*N*	*Left tail*	*P*	*Right tail*	*N*	*Left tail*	*P*	*Right tail*
2	0	.250	3	7	0	.008	28	9	0	.002	45
	1	.500	2		1	.016	27		1	.004	44
3	0	.125	6		2	.023	26		2	.006	43
	1	.250	5		3	.039	25		3	.010	42
	2	.375	4		4	.055	24		4	.014	41
	3	.625	3		5	.078	23		5	.020	40
4	0	.062	10		6	.109	22		6	.027	39
	1	.125	9		7	.148	21		7	.037	38
	2	.188	8		8	.188	20		8	.049	37
	3	.312	7		9	.234	19		9	.064	36
	4	.438	6		10	.289	18		10	.082	35
	5	.562	5		11	.344	17		11	.102	34
5	0	.031	15		12	.406	16		12	.125	33
	1	.062	14		13	.469	15		13	.150	32
	2	.094	13		14	.531	14		14	.180	31
	3	.156	12	8	0	.004	36		15	.213	30
	4	.219	11		1	.008	35		16	.248	29
	5	.312	10		2	.012	34		17	.285	28
	6	.406	9		3	.020	33		18	.326	27
	7	.500	8		4	.027	32		19	.367	26
6	0	.016	21		5	.039	31		20	.410	25
	1	.031	20		6	.055	30		21	.455	24
	2	.047	19		7	.074	29		22	.500	23
	3	.078	18		8	.098	28	10	0	.001	55
	4	.109	17		9	.125	27		1	.002	54
	5	.156	16		10	.156	26		2	.003	53
	6	.219	15		11	.191	25		3	.005	52
	7	.281	14		12	.230	24		4	.007	51
	8	.344	13		13	.273	23		5	.010	50
	9	.422	12		14	.320	22		6	.014	49
	10	.500	11		15	.371	21		7	.019	48
					16	.422	20		8	.024	47
					17	.473	19		9	.032	46
					18	.527	18		10	.042	45

N	*Left tail*	*P*	*Right tail*	*N*	*Left tail*	*P*	*Right tail*	*N*	*Left tail*	*P*	*Right tail*
10	11	.053	44	11	28	.350	38	13	0	.000	91
	12	.065	43		29	.382	37		1	.000	90
	13	.080	42		30	.416	36		2	.000	89
	14	.097	41		31	.449	35		3	.001	88
	15	.116	40		32	.483	34		4	.001	87
	16	.138	39		33	.517	33		5	.001	86
	17	.161	38	12	0	.000	78		6	.002	85
	18	.188	37		1	.000	77		7	.002	84
	19	.216	36		2	.001	76		8	.003	83
	20	.246	35		3	.001	75		9	.004	82
	21	.278	34		4	.002	74		10	.005	81
	22	.312	33		5	.002	73		11	.007	80
	23	.348	32		6	.003	72		12	.009	79
	24	.385	31		7	.005	71		13	.011	78
	25	.423	30		8	.006	70		14	.013	77
	26	.461	29		9	.008	69		15	.016	76
	27	.500	28		10	.010	68		16	.020	75
11	0	.000	66		11	.013	67		17	.024	74
	1	.001	65		12	.017	66		18	.029	73
	2	.001	64		13	.021	65		19	.034	72
	3	.002	63		14	.026	64		20	.040	71
	4	.003	62		15	.032	63		21	.047	70
	5	.005	61		16	.039	62		22	.055	69
	6	.007	60		17	.0456	61		23	.064	68
	7	.009	59		18	.055	60		24	.073	67
	8	.0125	58		19	.065	59		25	.084	66
	9	.016	57		20	.076	58		26	.095	65
	10	.021	56		21	.088	57		27	.108	64
	11	.027	55		22	.102	56		28	.122	63
	12	.034	54		23	.117	55		29	.137	62
	13	.042	53		24	.133	54		30	.153	61
	14	.051	52		25	.151	53		31	.170	60
	15	.062	51		26	.170	52		32	.188	59
	16	.074	50		27	.190	51		33	.207	58
	17	.087	49		28	.212	50		34	.227	57
	18	.103	48		29	.235	49		35	.249	56
	19	.120	47		30	.259	48		36	.271	55
	20	.139	46		31	.285	47		37	.294	54
	21	.160	45		32	.311	46		38	.318	53
	22	.183	44		33	.339	45		39	.342	52
	23	.207	43		34	.367	44		40	.368	51
	24	.232	42		35	.396	43		41	.393	50
	25	.260	41		36	.425	42		42	.420	49
	26	.289	40		37	.455	41		43	.446	48
	27	.319	39		38	.485	40		44	.473	47
					39	.515	39		45	.500	46

N	*Left tail*	*P*	*Right tail*	*N*	*Left tail*	*P*	*Right tail*	*N*	*Left tail*	*P*	*Right tail*
14	0	.000	105	14	46	.357	59	15	39	.126	81
	1	.000	104		47	.380	58		40	.138	80
	2	.000	103		48	.404	57		41	.151	79
	3	.000	102		49	.428	56		42	.165	78
	4	.000	101		50	.452	55		43	.180	77
	5	.001	100		51	.476	54		44	.195	76
	6	.001	99		52	.500	53		45	.211	75
	7	.001	98	15	0	.000	120		46	.227	74
	8	.002	97		1	.000	119		47	.244	73
	9	.002	96		2	.000	118		48	.262	72
	10	.003	95		3	.000	117		49	.281	71
	11	.003	94		4	.000	116		50	.300	70
	12	.004	93		5	.000	115		51	.319	69
	13	.005	92		6	.000	114		52	.339	68
	14	.007	91		7	.001	113		53	.360	67
	15	.008	90		8	.001	112		54	.381	66
	16	.010	89		9	.001	111		55	.402	65
	17	.012	88		10	.001	110		56	.423	64
	18	.0158	87		11	.002	109		57	.445	63
	19	.018	86		12	.002	108		58	.467	62
	20	.021	85		13	.003	107		59	.489	61
	21	.025	84		14	.003	106		60	.511	60
	22	.029	83		15	.004	105				
	23	.034	82		16	.005	104				
	24	.039	81		17	.006	103				
	25	.045	80		18	.008	102				
	26	.052	79		19	.009	101				
	27	.059	78		20	.011	100				
	28	.068	77		21	.013	99				
	29	.077	76		22	.015	98				
	30	.086	75		23	.018	97				
	31	.097	74		24	.021	96				
	32	.108	73		25	.024	95				
	33	.121	72		26	.028	94				
	34	.134	71		27	.032	93				
	35	.148	70		28	.036	92				
	36	.163	69		29	.042	91				
	37	.179	68		30	.047	90				
	38	.196	67		31	.053	89				
	39	.213	66		32	.060	88				
	40	.232	65		33	.068	87				
	41	.251	64		34	.076	86				
	42	.271	63		35	.084	85				
	43	.292	62		36	.094	84				
	44	.313	61		37	.104	83				
	45	.335	60		38	.115	82				