

A STOCHASTIC GEOMETRY APPROACH FOR CELLULAR WIRELESS NETWORK ANALYSIS

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A STOCHASTIC GEOMETRY APPROACH FOR CELLULAR WIRELESS NETWORK ANALYSIS

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To my beloved wife Seil

ABSTRACT

Cellular wireless communications systems transformed the way people communicated by combining communication and mobility. Since the early 1970s, the mobile industry has been driving the development and evolution of cellular technology. The first generation only offered voice communication, whereas the second generation offered both voice and data services. Furthermore, as cellular wireless technology advanced, 3G enabled video conferencing and other services. With rising demand, 4G emerged, ensuring us with ultra-broadband internet access. 5G will be able to provide us with services that we have never had before.

Cellular wireless networks are typically modeled by placing base stations on a regular hexagonal grid and mobile users either randomly or deterministically distributed. These models have been widely used, but they have the disadvantage of being both highly idealized and not very tractable, so complex system-level simulations are used to assess coverage probability and rate. Models that are more tractable have long been desired.

In this dissertation, we develop novel methods based on stochastic geometry to analyze the coverage and average spectral efficiency of downlink cellular wireless networks. In the first part of the dissertation, we consider to analyze the performance of SISO (Single Input Single Output) transmission scheme. Regarding this analysis, both the positions of base stations (BS) and mobile users are modeled as points of an independent Poisson Point Process' (PPP). We consider to use the max C/I (Carrier to Interference Ratio) scheduling method, to select the user with the best SIR (Signal to Interference Ratio) value among the users that are connected to the same base station. Since the base stations are randomly placed and users are linked to their closest

base stations, the C/I ratios of multiple users become conditionally dependent. For such a realistic deployment scenario, we utilize stochastic geometry tools and copula functions to take into account this correlation and analyze the coverage probability and spectral efficiency. Our simulation results demonstrate that the commonly used independence assumption of the user C/I values overestimates both actual coverage probability and spectral efficiency of the system.

In the second part, we analyze the coverage performance of multi-user multiple-input multiple-output (MU-MIMO) downlink cellular networks. We model the locations of the base stations and mobile users using Poisson Point Processes and employ maximum ratio transmission precoding at the base station. Our analysis builds upon Prony's method to approximately calculate the coverage probability using the sum of exponentials which facilitates the computation of Laplace transform of both intra-cell and inter-cell interference. We demonstrate the accuracy of derived approximate coverage probability expressions through numerical simulations for different MU-MIMO configurations.

In the third part, we analyze the downlink coverage performance of general user, arbitrarily located in the 2-D region, edge user and worst user sit on the boundary between two neighbor cells and vertex where three neighbor cells meet respectively, considering the Coordinated Multipoint (CoMP) transmission scheme by using stochastic geometry approach. In addition, we analyze the performance of two users, in terms of average spectral efficiency in the CoMP transmission scheme where the users are connected to the closest and second closest BS's, by using stochastic geometry approach. The correlation of SIR values of two users are again modeled by using copula functions. The max C/I scheduling method is employed to select the user in this scheme.

ÖZETÇE

Hücresel kablosuz iletişim sistemleri, iletişim ve mobilitayı birleştirerek insanların iletişim kurma şeklini değiştirmiştir. 1970'lerin başından beri, mobil endüstri, hücresel teknolojinin gelişimini ve evrimini yönlendirmektedir. Birinci nesil (1G) kablosuz haberleşme sistemleri sadece sesli iletişim sunarken, ikinci nesil (2G) hem ses hem de veri hizmetleri sunuyordu. Ayrıca, hücresel kablosuz teknoloji geliştikçe, 3G haberleşme sistemleri video konferans ve benzeri diğer hizmetleri etkinleştirmiştir. Artan taleple birlikte 4G kablosuz haberleşme sistemleri geliştirilmiş ve bu sayede ultra geniş bant internet erişimi sağlanmıştır. Halihazırda geliştirilmekte olan 5G ise bize daha önce hiç sahip olmadığımız hizmetleri sunabilecektir.

Hücresel kablosuz ağlar, genel olarak, baz istasyonlarının düzenli bir altıgen ızgaraya yerleştirilmesi ve mobil kullanıcıların rastgele veya deterministik olarak dağıtılmasıyla modellenir. Bu modeller yaygın olarak kullanılmaktadır, ancak hem idealize edilmiş olmaları dezavantajına hem de matematiksel olarak çözümlenebilir olmama dezavantajına sahiptirler. Bu nedenle kapsama olasılığını ve kanal kapasitesini hesaplamak için karmaşık, sistem düzeyinde simulasyonlar yapılmaktadır. Sonuç olarak, matematiksel olarak çözümlenebilir modeller uzun zamandır arzu edilmektedir.

Bu tezde, indirme yönündeki hücresel kablosuz ağların kapsama alanını ve ortalama spektral verimliliğini analiz etmek için stokastik geometriye dayalı yeni yöntemler geliştirilmiştir. Tezin ilk bölümünde, tek girişli tek çıkışlı (SISO-Single Input Single Output) iletim şemasının performansı analiz edilmiştir. Bu analiz kapsamında, hem baz istasyonlarının (BS) hem de mobil kullanıcıların konumları, bağımsız bir Poisson Noktası Süreci'nin (PPP-Poisson Point Process) noktaları olarak modellenmiştir.

Aynı baz istasyonuna bağı kullanıcılar arasında en iyi SIR (Sinyal-Girişim Oranı) değerine sahip kullanıcıyı seçmek için maksimum C/I (Carrier to Interference Ratio-Taşıyıcı Girişim Oranı) zamanlama yöntemi kullanılmıştır. Baz istasyonları rastgele yerleştirildiğinden ve kullanıcılar en yakın baz istasyonlarına bağı olduğundan, birden fazla kullanıcının C/I oranları koşullu olarak bağımlı hale gelir. Böyle gerçekçi bir yerleştirme senaryosu için, bu korelasyonu hesaba katmak ve kapsama olasılığı ile spektral verimliliği analiz etmek için stokastik geometri araçları ve kopula fonksiyonları kullanılmıştır. Simülasyon sonuçlarımız, kullanıcı C/I değerleri için yaygın olarak kullanılan bağımsızlık varsayımının, sistemin hem gerçek kapsama olasılığının hem de spektral verimliliğinin olduğundan fazla hesaplanmasına neden olduğunu göstermektedir.

İkinci bölümde, çok kullanıcılı çok girişli çok çıkışlı (MU-MIMO-Multiuser Multiple Input Multiple Output) aşağı yönlü hücresele ağların kapsama performansı analiz edilmiştir. Baz istasyonlarının ve mobil kullanıcıların konumları PPP kullanarak modellenmiş ve baz istasyonunda maksimum oranlı iletim (MRT-Maximum Ratio Transmission) ön kodlaması kullanılmıştır. Analizimiz, hem hücre içi hem de hücreler arası girişimin Laplace dönüşümünün hesaplanmasını kolaylaştıran üstellerin toplamını kullanarak kapsama olasılığını yaklaşık olarak hesaplamak için Prony'nin yöntemini temel almaktadır. Farklı MU-MIMO konfigürasyonları için nümerik simülasyonlar yoluyla, analitik olarak türetilmiş yaklaşık kapsama olasılığı ifadelerinin doğruluğu gösterilmiştir.

Üçüncü bölümde, 2 boyutlu bölgeye rastgele yerleştirilen genel kullanıcı, iki komşu hücre arasındaki sınırda yer alan uç kullanıcı ve üç komşu hücrenin birleştiği köşe noktasındaki en kötü kullanıcı için, Koordineli Çoklu Nokta (CoMP-Coordinated Multipoint) yöntemiyle iletim yapıldığı göz önünde bulundurularak indirme yönündeki kapsama performansı, stokastik geometri yaklaşımıyla analiz edilmiştir. Ayrıca, kullanıcıların en yakın ve ikinci en yakın BS'lere bağı olduğu, CoMP yöntemiyle servis

verilen iki kullanıcının performansı stokastik geometri yaklaşımı kullanılarak analiz edilmiştir. İki kullanıcının SIR değerlerinin korelasyonu yine kopula fonksiyonları kullanılarak modellenmiştir. Bu CoMP iletim şemasında, kullanıcıyı seçmek için maksimum C/I zamanlama yöntemi kullanılmıştır.



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CHAPTER I

INTRODUCTION

1.1 Background, Motivation and Objectives

The population of people using the wireless cellular infrastructure for Internet connectivity, as well as the traffic demand per user, is rapidly increasing due to the rapid growth of smart phones and tablets with significant processing capability. According to Cisco, the number of connected devices is estimated to reach 500 billion by 2030, which is roughly 59 times bigger than the expected world population of 8.5 billion at the time [1].

The first generation (1G) of public phone service, which had a speed of up to 2.4kbps, was based on an analog system, whereas the second generation was based on a digital system [2]. As a result of the second generation's support for text messaging, its popularity and demand grew. Therefore, data connectivity has improved. In 1998, 3G networks were launched [3]. Information can be transferred at a rate of at least 200 kbit/s using 3G telecommunication services [3]. Later editions of 3G include 3.5G and 3.75G [2]. The advancement of packet-switched 3G technologies gave birth to the 4G network. Voice communication is used in 4G, which is an IP-based technology.

The introduction of 4G signaled the beginning of the smartphone and handheld mobile device era in the wireless industry. However when the ITU-R (International Telecommunications Union Radiocommunication Sector) established the required minimum speeds for 4G (12.5 Mbps), they were unattainable. The ITU-R decided that LTE (Long Term Evolution) could be labeled as 4G if it provided a significant improvement over 3G, in response to the amount of money that technology manufacturers were putting into achieving this goal. 4G was the first generation to

use LTE technology to deliver theoretical download speeds ranging from 10 Mbps to 1 Gbps, providing end users with lower latency, better voice quality, instant messaging services, and social media [4].

Recognizing that 4G LTE networks would eventually reach capacity, the ITU-R defined the 5G networking requirements specification in 2015. Whereas 3G and 4G networking focused on voice and data services and were primarily associated with smart phones, 5G networking promises to connect billions of devices of almost any type. The 5G roll-out has only recently begun. Verizon and AT&T launched the first 5G services in the United States. 5G networks have already been deployed in 17 countries, including South Korea, the United Kingdom and Germany [4].

In order to analyze the performance of wireless cellular networks, one traditional method which is utilized by the communication researchers is the Wyner model [5]-[7]. This model is one-dimensional and assumes a unit gain from each base station to the tagged user and an equal gain that is less than one to the two users in the two neighboring cells. Wyner model is a tractable but unnecessarily basic downlink model. Unless there is a considerable quantity of interference averaging over space, such as in the uplink of heavily-loaded CDMA (Code Division Multiple Access) systems [8], this is also a severely erroneous model. The Wyner model and related mean-value approaches are especially inaccurate for cellular systems that use orthogonal multiple access techniques, such as 4G and 5G because SINR (signal-to-interference-plus-noise-ratio) values vary dramatically across a cell.

However, researchers in need of more realistic models typically model a 2-D network of base stations on a regular hexagonal lattice. Tractable analysis can be obtained for a fixed user with a small number of interfering base stations, for example, by considering the "worst-case" user location which is the cell corner and calculating the SINR [9], [10]. In the case of shadowing and/or fading, the resulting SINR remains a random variable from which performance metrics such as (worst-case) average rate

and (worst-case) coverage probability relative to some target rate can be calculated. Naturally, such a technique produces pessimistic results that provide no insight to the performance of the system's majority of users. Tractable SINR expressions for a random user location in the cell are generally unavailable, so more general results that provide guidance into typical SINR or the probability of coverage over the entire cell must be obtained through complex time-consuming simulations. In addition to being difficult to build and run, such private simulations have issues with repeatability and transparency, and they rarely inspire "optimal" or innovative algorithms or designs. It is also important to recognize that, while widely accepted, grid-based models are highly idealized and they are becoming increasingly inaccurate for the heterogeneous and ad hoc deployments. These types of deployments are more common in urban and suburban areas, where cell radii vary significantly due to differences in transmission power, tower height, and user density. For instance, picocells are frequently placed into an existing cellular network near high-traffic locations, while short-range femto-cells may be dispersed randomly throughout a centrally organized cellular network.

In order to address the problems mentioned above, an approach based on stochastic geometry was introduced for the analysis of wireless cellular networks. This method adds another source of randomness which is the positions of base stations. Adversely to the regular hexagonal grid assumption, where the base stations are placed deterministically, using this approach, the locations of the base stations are modeled as a Poisson Point Process (PPP). Therefore, the spatial randomness inherent in wireless cellular networks can be captured in the analysis. In addition, stochastic geometry models are naturally extensible to account for additional sources of uncertainty such as fading, shadowing, and power control. In some cases, stochastic geometry analysis can result in closed-form expressions governing the system behavior. These expressions facilitate network operation and provide insightful design guidelines, which are frequently difficult to obtain from computationally intensive simulations.

Attempts to raise stochastic geometry to model cellular networks can be backtracked to the late 90s [43], [89], motivated by its tractability. Success, however, did not come until a decade later [13]- [15]. According to the theoretical and statistical analyses provided in [13]- [15], wireless cellular networks diverge from the idealized hexagonal grid layout and follow an irregular topology that varies randomly from one geographic area to the next. Andrews et al. [13] show that in a simulation with actual base station (BS) locations, the signal-to-interference-plus-noise-ratio (SINR) experienced by users is upper bounded by the SINR of users in an idealized grid network, and lower bounded by the SINR of users in a random network. Surprisingly, the random network yields a lower bound that is as tight as the idealized grid network's upper bound. It can be found a taxonomy of the state-of-the-art stochastic geometry models for cellular networks in [17]. Stochastic geometry based modeling for wireless and cellular networks is widely recognized by both industry and academia as a result of the findings in [13]- [22], [28]- [45] and [81]-[116].

A cellular wireless network should guarantee a certain quality of service (QoS) level for its connected users. There are different performance metrics specified to evaluate the QoS level of users such as throughput, end to end delay etc. Coverage probability is one of these QoS performance metrics, which is defined as the probability that the signal-to-interference ratio (SIR) of a user is greater than a reliability threshold. Coverage probability is considered as a key QoS performance metric regarding the network planning and deployment of emerging communication systems [28]. Another performance metric is the average achievable rate. It is calculated considering adaptive modulation/coding is utilized so that each user can tune their rate to achieve Shannon bound for their instantaneous SINR i.e $\ln(1 + SINR)$.

Many researchers have been drawn to the planning and optimization of wireless networks and radio resources due to the enormous popularity of cellular wireless networks [11]. The process of designing a network structure and defining network

elements while meeting various design requirements is referred to as RF(Radio Frequency) network planning. RF network planning is linked to network dimensioning and detailed planning, which are two critical network life phases because an adopted plan imposes additional hard limits on network performance.

Regarding the commercial RF network planning software tools such as Cloud-RF, Wireless InSite and Atoll, they rely on complex and time-consuming system level simulations to characterize the performance of cellular wireless networks. So, these tools do not provide tractable SINR expressions that are used to evaluate the performance. However, as we have already stated, it is hard to get insights from these simulations to optimize the system performance.

Traditionally, RF network planning has been thought of as a one-dimensional process. However, in heterogeneous radio environments, the concept of dynamic network reconfigurability, which incorporates the software defined radio concept [12], has lately gained traction. As a result, dynamic network planning, in which the network infrastructure and network procedures are defined dynamically, has become a popular research topic. In order to implement the dynamical network planning approach, complexity is an important feature to be considered.

Last but not least, system improvements necessitate an appropriate cellular wireless network architecture in order to provide users with a good mobile experience. As more wireless networks transition from 4G to 5G, adopting a 5G radio planning tool or analytics software to assist with the transition offers significant benefits. Therefore, considering the aforementioned RF network planning issues with this transition from 4G to 5G, there is a motivation to develop a tractable and accurate framework with low-complexity to model and analyze the performance of cellular wireless networks. This framework can be utilized for effective RF network planning.

In this dissertation, our main objective is to develop a tractable, accurate and low-complex framework to analyze the coverage and rate performance of downlink wireless cellular networks using an approach based on stochastic geometry. We analyze the performance considering the transmission schemes namely SISO, MU-MIMO and CoMP. Regarding the SISO case we aim at analyzing the performance of a multiuser multi-cell system with multiuser diversity. Regarding the MU-MIMO, our objective is to analyze the performance of a typical user considering random scheduling and for the CoMP case we analyze the performance of general user, cell-edge user, worst-case user and a 2-user multi-cell system with multi-user diversity. In the following, we explain our motivation and contributions for the analysis of SISO, MU-MIMO and CoMP systems respectively.

1.2 Performance Analysis of SISO Systems

Regarding the analysis of SISO cellular networks, while the majority of stochastic geometry studies analyze system performance metrics for a typical user at the origin (e.g., [13, 16] and references therein), a few of them also consider multiple users [18, 19, 20, 21, 22]. The studies [18, 19] assume that one randomly selected user is served at a given time, out of many that experience independent channel conditions, and consider a scenario where multiuser diversity is absent.

Multiuser diversity is a technique used with aim of optimizing the multi-user wireless system capacity in a frequency flat fading channel by scheduling the user with the best instantaneous channel condition and allocating all system resources to it [23]. This technique is omnipresent in modern telecommunication systems and the impact of it increases with the number of active users, which in turn increases the system capacity.

In [20, 21, 22], the multiuser diversity performance is analyzed in PPP distributed cellular networks. These studies consider that the user with the maximum normalized

signal to noise ratio (SNR) value is scheduled in each time slot. This normalized SNR corresponds to an instantaneous SNR normalized by the short-time average SNR. Effectively, in this scheduling method, the scheduler selects the user with the largest fading gain which is equal to the gain of the channel between the selected user and its associated BS. So, the effect of interference on the channel quality is ignored while scheduling the user. However, in modern cellular systems including 4G and 5G networks, in order to exploit the multiuser diversity gain, the mobile users are scheduled according to their channel quality indicator (CQI) values that are determined based on the observed carrier-to-interference (C/I) ratios [24, 25].

In the first part of this dissertation, the performance of the downlink of a SISO (Single Input Single Output) cellular network under a multiuser diversity scenario is analyzed, where both users and base stations are distributed according to independent PPPs and maximum C/I scheduling is performed at each cell to select the user to be served. When base stations are randomly distributed, contrary to the common assumption in the literature, given that a set of users are connected to the closest base station, their C/I values become conditionally dependent. Utilizing the stochastic geometry approach, the multiuser diversity is analyzed by considering the correlation between C/I values of the users connected to the closest base station. In order to incorporate this correlation into the analysis, we use copula functions [26] to compute the joint distribution of C/I values. These functions couple a multivariate distribution function to its marginals in such a way that the correlation structure and the marginals are modeled separately. Coverage probability and average system rate metrics estimated with this approach give more accurate results compared to using the simplistic assumption of independent channel gains, which overestimates the aforementioned wireless network performance metrics.

1.3 Performance Analysis of MIMO Systems

The downlink coverage performances of SU-MIMO and MU-MIMO transmission schemes have been widely studied in the literature [29], [31], [32], [33], [34], [35], [53], [30] utilizing stochastic geometry tools. Locations of the BSs are generally modeled using Poisson Point Process (PPP) and users are assumed to be connected to the nearest BS [29]. Under the assumption of Rayleigh fading, the received signal gain is Gamma distributed [29]. Therefore, the exact coverage probability of MIMO requires the calculation of the n -th derivative of Laplace transform of aggregate interference [31]. In an effort to calculate the n -th derivative of Laplace transform of interference, various approaches have been investigated such as Taylor expansion [32], Faà di Bruno's formula [33, 34, 30] and Toeplitz matrix representation [31], [35].

The work in [32] focuses on multi-antenna ad-hoc networks and presents an outage probability expression containing integral and nested sum expressions. The derived coverage probability of downlink MIMO cellular networks in [34] is also not in closed form and contains an integral operation. Although the methods proposed in [31], [33], [35] & [30] lead to closed-form solutions, the derived expressions are complicated and include special functions. The computational complexity of these expressions grow with the number of transmit antennas and number of simultaneously scheduled users on the same frequency resource.

In the second part of this dissertation, in order to facilitate the evaluation of the coverage performance of downlink cellular MU-MIMO networks, we propose to approximate the complementary cumulative distribution function (CCDF) of received signal gain by sum of exponential functions. This kind of approximation is first used in [36] to analyze downlink coverage of single antenna heterogeneous networks. Our proposed approach lets us derive the coverage probability without taking the n -th derivative of Laplace transform of intra-cell and inter-cell interference.

In the second part, we derive a novel expression for downlink coverage probability of MU-MIMO networks. The derived expression is approximate, yet accurate and it has closed-form. This expression is a one-fold summation and has also lower complexity compared to aforementioned methods. We analyze the downlink coverage performance of MU-MIMO cellular networks using the derived expression and show that the coverage probability decreases with the number of simultaneously scheduled users on the same frequency resource without employing any smart interference cancellation scheme or interference-aware scheduling at the transmitter. We further verify the accuracy of the derived expression through Monte Carlo simulations for different MU-MIMO configurations.

1.4 Performance Analysis of CoMP Systems

In the information-theoretic literature, the problem of base station cooperation has been widely studied. Several studies including [61], [62], [63], [64] analyze the advantages of cooperation within the framework of the Wyner model [65]. This model is extensively used to analyze the capacity of cellular systems for downlink communication. A drawback of this model is that, its trading off simplicity and analytic tractability at the expense of accuracy [66]. [67] gives an overview on the information theoretic techniques to study multi-cell MIMO cooperation in wireless networks. In [68], a fundamental information theoretic limit on the achievable spectral efficiency due to cooperation is derived. A new approach which is based on stochastic geometry is to assume that base stations are randomly located, so tools from stochastic geometry can be used to characterize the SINR and as a result, coverage probability [16] and [44]. [69] models a heterogeneous networks as the union of independently distributed tiers according to a PPP. This approach is validated by [70] comparing the coverage probability derived analytically with this model and the one measured in a real network deployment. The average rate in the downlink of a heterogeneous

network is analyzed in [71].

The impact of backhaul delays in wireless networks where zero forcing beamforming is implemented at cooperating base stations is studied in [72]. In [73] the effect of a non-ideal backhaul network is examined and the performance of a specific two-base-station cooperative scheme based on rate-splitting, which is similar to the one proposed in [74] for the multiple access channel with conferencing encoders is analyzed. In [75], a cooperative scheme where base stations are randomly clustered by nulling the inter-cell interference. The problem of non-coherent joint transmission is also considered in [54]. In [55], the benefits of joint transmission in heterogeneous networks for the general and worst case users are analyzed.

In the third part of the dissertation, by using stochastic geometry approach, we first analyze the downlink coverage performance of three types of mobile users namely general user, cell edge user and worst case user. General user is located at an arbitrary location in inner Voronoi cell region. Cell edge user sits on the boundary between two Voronoi cells and worst case user is located at vertex, where three Voronoi cells meet. Second, we analyze the spectral efficiencies of two users, whose locations are also arbitrary, in the stochastic geometry framework, under CoMP transmission structure by using copula function. We again use the max C/I scheduling method, like non-CoMP case, where we select the user with the highest C/I. We consider six different scenarios to evaluate the average spectral efficiencies of two users. To the best of our knowledge our method is the first method which considers analyzing the average spectral efficiency of two users, by using stochastic geometry approach under CoMP transmission scheme.

1.5 Outline

After the introduction, which contains a brief summary of some useful terms that will be used in this dissertation, a detailed overview of stochastic geometry phenomenon is

provided in Chapter 2. In Chapter 3, multiuser diversity performance is characterized in the stochastic geometry framework. The first part of Chapter 3 concerns with the analytical solution while in the second part the numerical results are given. In Chapter 4, the downlink coverage performance of MU-MIMO system is analyzed. In Chapter 5, in the first part a detailed overview on the CoMP transmission method is given with the analysis of general, cell edge and worst case users by using two different CoMP transmission strategies namely joint transmission and dynamic cell selection with stochastic geometry approach. In the second part, the numerical analysis of average spectral efficiencies of two users in six different scenarios is undertaken. The last chapter is entirely dedicated to the discussion and interpretation of the presented results.

Notation: $\mathcal{C}^{N \times 1}$ denotes N -dimensional complex space, $(\cdot)^T$ denotes transpose operation, $(\cdot)^H$ denotes Hermitian (complex conjugate transpose) operation, $B'(a, b, z) = \int_z^1 t^{a-1}(1-t)^{b-1}dt$ denotes complementary incomplete Beta function, $\gamma(a, b) = \int_0^b t^{a-1}e^{-t}dt$ denotes lower incomplete Gamma function and $\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt$ denotes Gamma function.

CHAPTER II

AN OVERVIEW ON STOCHASTIC GEOMETRY ANALYSIS OF CELLULAR NETWORKS

In this chapter, we provide some mathematical preliminaries on the stochastic geometry modeling that helps to understand the discussions presented later in this dissertation. Stochastic geometry is also a basis for our proposed model, that is used to accurately compute the distribution of the spectral efficiency of the downlink of a cellular system utilizing multiuser diversity.

Stochastic geometry has been used as a mathematical and statistical tool for the modeling, analysis and design of wireless networks with random topologies. It is usage of modeling and characterizing the interference in wireless networks goes back to days of 1978 [37]. It was further enhanced by Sousa and Silvester in the early 1990's [38], [39], [40]. In the late 1990's the impact of channel effects, such as fading, shadowing, path loss and the combinations of these effects on the aggregate interference was characterized by Iltis and Hatzinakos [41]. Baccelli et al. developed tools based on Poisson Voronoi tessellations and Delaunay triangulations for the optimization of hierarchical networks and mobility management in cellular networks [42], [43]. A detailed survey of these models can be found in [44].

2.1 Stochastic Geometry Analysis

Stochastic geometry is a mathematical tool which is concerned with random geometric structures, ranging from simple points or line segments to arbitrary closed sets. This tool provides spatial averages, which is taken over large number of nodes at different locations or over many network realizations, for the quantities of interest

like aggregate interference, outage probability and achieved data rate. It is inherently related to the theory of point processes. At the beginning, its development was excited by applications in biology, astronomy and material sciences. Nowadays, it is also used in the context of wireless communication networks and image processing. In this section, after presenting the necessary preliminaries, we will expatiate on how stochastic geometry captures the topological randomness while accounting for the system characteristics in cellular networks [29].

2.1.1 Point Processes

The origins of theory of point processes trace back to ancient days. In the past, man had been counting stars in regions of the sky, recording natural events such as floodings, earthquakes, and appearances of comets. In recent times, point processes, which depends heavily on measure theory, have been used in counting problems, particle physics, life tables, communication engineering and population processes.

A point process is a type of stochastic or random process for which any realization of this process is composed of isolated points either in time or geographical space, or even in more general spaces. In some cases, like the occurrence of lightning strikes, if each point is recorded according to its location in time and space, this point process can be classified as a process both in time and space. Since a point process can take on only one of two possible values indicating whether or not an event occurs at that time and/or space, they are opposed to continuous valued processes, which can take on any countless value at each point in time and/or space. For our proposed model, we deal with point processes that are composed of points in 2-D space. Therefore, in the following, the planar case is regarded only, although most of the results presented in this dissertation can be generalized easily to $\mathbb{R}^d$ for $d \geq 2$.

A spatial point process Φ is a random, finite or countably infinite collection of points in the space $\mathbb{R}^2$, without accumulation points. Any given realization ϕ of a

point process can be considered as a discrete subset $\phi = \{X_i\} \subset \mathbb{R}^2$ of the 2-D space.

A single point $X_i \in \mathbb{R}^2$ in a discrete subset ϕ can be defined by the Dirac measures δ_{X_i} as [45]

$$\delta_{X_i}(A) = \mathbb{1}_A(X_i) \quad (1)$$

where $\mathbb{1}_A$ denotes the indicator function of the set A i.e. if $X_i \in A$ $\delta_{X_i}(A) = 1$ and if $X_i \notin A$ $\delta_{X_i}(A) = 0$.

A point process can be defined in a formal manner as, a measurable mapping Φ from some probability space to the space of point measures on some space A . A point measure is locally finite and that takes only integer values. This measure can be identified as a discrete countable sum of Dirac measures on A . As a result,

$$\phi = \sum_i \delta_{X_i}(A) \quad (2)$$

where δ_{X_i} is the Dirac measure at X_i . As a result, $\phi(A)$ gives the number of “points” of ϕ in A . The random variables X_i take their values in A are the points of Φ .

2.1.1.1 Some properties of point processes

Let Φ be any point process in $\mathbb{R}^2$. If we define the probability measure P_X defined on N by $P_\Phi(A) = \mathbb{P}(A) \in A$ for $A \in N$ is called the distribution of Φ . A point process Φ is called

- stationary, if its distribution is translation invariant, i.e. if $P_X = P_{t_X}$ for any $x \in \mathbb{R}^2$,
- isotropic, if its distribution is rotation invariant, i.e. if $P_X = P_{r_X}$ for all rotations r around the origin and,
- motion-invariant, Φ is both stationary and isotropic.

- simple, if the multiplicity of a point is at most one (no two points are at the same location).

If Φ is a stationary point process then, it is always assumed that $\mathbb{P}(\Phi(\mathbb{R}^2) = \infty) = 1$ and $\mathbb{P}(\Phi(\mathbb{R}^2) = 0) = 0$. The intensity measure $\Lambda : \mathbb{R}^2 \rightarrow [0, \infty]$ of a point process Φ is defined by

$$\Lambda(A) = \mathbb{E}\Phi(A), \quad A \in \mathbb{R}^2 \quad (3)$$

We can interpret from the formula Equation 3 that $\Lambda(A)$ is equal to the expected number of points Φ in the Borel set A . In the following it is assumed that Λ is locally finite and not equal to the zero measure, i.e. $\Lambda(B) < \infty$ for all $A \in \mathbb{R}^2$ and $\Lambda(\mathbb{R}^2) > 0$. If Φ is stationary with intensity measure Λ , then

$$\Lambda(A) = \mathbb{E}\Phi(A) = \mathbb{E}\Phi(A + x) = \Lambda(A + x) \quad (4)$$

for any $x \in \mathbb{R}^2$ and any Borel set $A \in \mathbb{R}^2$.

Since every translation-invariant and locally finite measure on $\mathbb{R}^2$ is proportional to the Lebesgue measure $\Lambda(A) = \lambda\nu(A)$ for some constant $\lambda > 0$. λ is the intensity of the stationary point process Φ . It can be interpreted as the mean number of points per unit area since $\lambda = \mathbb{E}\Phi(A)$ for each $A \in \mathbb{R}^2$ with $\nu(A) = 1$.

In the following we state the Campbell's theorem, which is an important result in the theory of point processes, [45]

Theorem 1. *Let $\Phi = X_n$ be a point process and let $f : \mathbb{R}^2 \rightarrow [0, \infty)$ be a measurable function. Then $\sum_{x_n \in \Phi} f(X_n) \rightarrow [0, \infty)$ is measurable and*

$$\mathbb{E} \sum_{x_n \in \Phi} f(X_n) = \mathbb{E} \int_{\mathbb{R}^2} f(x) \Phi(dx) = \mathbb{E} \int_{\mathbb{R}^2} f(x) \Lambda(dx)$$

If Φ is stationary with intensity λ , then

$$\mathbb{E} \sum_{x_n \in \Phi} f(X_n) = \lambda \int_{\mathbb{R}^2} f(x) \nu(dx)$$

The proof of Theorem 1 is provided in [45].

2.1.2 Poisson Point Processes

In this subsection, the concept of Poisson Point Process (PPP) is introduced. PPP is the most frequently studied point process model. The reasons for that can be summarized as follows:

- PPP is analytically tractable and many theoretical results can be derived for PPP's.
- PPP can be used to model complete randomness since there is no interaction between the points.
- PPP is used to construct the other types of point processes such as Poisson cluster process, specific hardcore process and Cox process.

Let Λ be a locally finite measure on some metric space E . A point process Φ with intensity measure Λ is Poisson if

- The random variables $\Phi(A_i)$ are independent for all disjoint subsets $A_1, A_2, \dots, A_n$ of E .
- For all sets of A of E , the random variables $\Phi(A)$ are Poisson random variables.

$$\mathbb{P}(\Phi(A) = k) = e^{-(\Lambda(A))} \frac{\Lambda(A)^k}{k!} \quad A \in \mathbb{R}^2, k \in N_0 \quad (5)$$

Since the points of a PPP $\Phi(A)$ are independently (and uniformly for homogeneous PPP) located in A , we can explicitly represent the Laplace functional of PPP Φ . If $E = \mathbb{R}^2$, this Laplace functional is defined for general point processes as:

$$\mathcal{L}_\Phi(f) = \mathbb{E} \left[e^{-\int_{\mathbb{R}^2} f(x) \Phi(dx)} \right] = \mathbb{E} \left[e^{-\sum_{x \in \Phi} f(x)} \right], \quad (6)$$

where f is a nonnegative function on $\mathbb{R}^2$. In the PPP case,

$$\mathcal{L}_\Phi(f) = \exp \left(- \int_{\mathbb{R}^2} (1 - e^{-f(x)}) \Lambda(dx) \right) \quad (7)$$

This formula constitutes a basis for evaluating a large number of formulas like Laplace transform of the interference. In the following we give some important features of PPP's, which are their invariance to a large number of key operations. Specifically,

- The superposition of two or more independent PPP's (which is defined as the sum of the associated point measures) is again a PPP.
- The independent thinning of a PPP is again a PPP; this can be extended to the case of location-dependent thinning, where a point is retained or not depending on its location.

2.2 Baseline Stochastic Geometry Models and Performance Metrics

In order to understand and establish the analytical paradigm to be used in more practical and complicated cases, baseline models are used. In this section, we will review the expressions for the probability density function (pdf) and the cumulative distribution function (CDF) of performance metrics, such as coverage probability and the average data rate, based on the baseline stochastic geometry model for single tier cellular networks.

In the baseline model, spatial distribution of the positions of base stations follow PPP Φ_b with the density λ_b . It is assumed that each base station transmits with the same transmit power and each mobile user associates with the closest base station based on the received signal strength (RSS). This means, a line bisecting the distance between each two neighboring base stations, separates their coverage regions. The planar graph which is constructed by perpendicular lines that separates the points of a PP is called a Voronoi tessellation. As a special case, the Voronoi tessellation captures the hexagonal grid. Figure 1 shows the Voronoi tessellation for PPP. In the figure, blue triangles correspond to the center of the cell where the base station is located and red dots correspond to the mobile users. It is assumed that the radio

channel attenuation is dependent on path loss and Rayleigh fading.

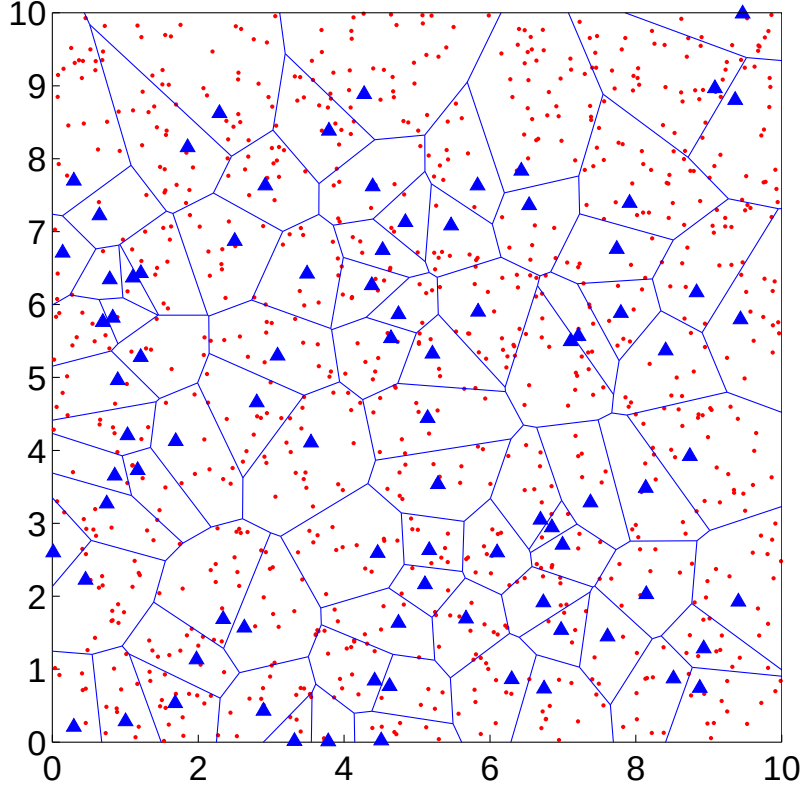


Figure 1: An example placement of the BSs (triangles) and users (dots) according to PPPs with densities λ_b and λ_u , respectively. The blue solid boundaries denote the Voronoi cells associated with each BS.

2.3 Performance Metrics of Downlink Cellular Networks

In this section, we review the expressions of the pdf and CDF's of performance metrics, such as coverage probability and average data rate, when the location of BS's are distributed according to homogenous PPP Φ with density λ_b .

The carrier-to-interference-ratio (C/I) distribution is the most important and general metric in cellular networks. Once the C/I distribution is known, the probability of coverage and average achievable data rate can be easily computed.

At a generic time instant, the C/I experienced by typical user depends on its location, the locations of the interference sources as well as the instantaneous channel gains. As a result, given the effect of network geometry on interference, the C/I is a random variable that strongly depends on the network geometry and significantly varies from one receiver to another and from one time instant to another. According to Slivnyak's theorem [45], the statistics seen from a PPP is independent from the test location. So, without loss of generality, the statistics seen by the user which is located at the origin can be studied.

In the baseline model, we use the standard power loss propagation model with path loss coefficient $\alpha > 2$. We assume the channel gain between the user and its associated base station i is random and modeled via Rayleigh fading with mean 1 and base station transmits at a constant power of $1/\mu$. The received power at a user which has a distance R from its associated base station is $H_i R^{-\alpha}$ where the random variable H_i is exponentially distributed with mean $1/\mu$.

The interference power at the receiver, I_r is the sum of the received powers from all other base stations other than the associated base station. Due to orthogonal multiple access within a cell there is no inter-cell interference.

We can express C/I of the typical mobile user at a random distance R_i from its home base station i as;

$$\Psi = \frac{H_i R_i^{-\alpha}}{I_r + \sigma^2} \quad (8)$$

where

$$I_r = \sum_{j \neq i} H_j R_j^{-\alpha}$$

is the aggregate interference from all the other base stations except the home base station, which have distance R_i from that user and have a fading value H_i and σ^2 is the power of the additive white Gaussian noise (AWGN).

Following [13], we neglect the noise term in (8) for the remainder of the analysis as it is usually dominated by the interference term.

The aggregate interference I_r is a random process which depends on the location of typical user and time. Stochastic geometry analysis, gives the statistics of the interference, experienced by the nodes existing in the network. This interference is averaged with respect to the spatial domain. As a result, interference can be completely characterized by its pdf or equivalently, its CDF. Generally, the exact expression of pdf of the aggregate interference in cellular networks, is not known. Hence, the aggregate interference is usually characterized by using the Laplace transform of the pdf or equivalently the moment generating function or characteristic function of the pdf.

The Laplace transform of the aggregate interference is given by

$$\mathcal{L}_{I_r} = \mathbb{E}[e^{-sI_r}] \quad (9)$$

Since the aggregate interference is a strictly positive random variable, its Laplace transform always exists. The Laplace transform of the aggregate interference is derived by using stochastic geometry techniques. In the following, the derivation of Laplace transform of the aggregate interference is used to express the pdf/CDF's of performance metrics.

2.3.1 Coverage Analysis of a Typical User in Downlink Cellular Systems

The coverage probability of typical mobile user is defined as

$$P_{cov} \triangleq \mathbb{P}(\Psi > \gamma) \quad (10)$$

We can think coverage probability equivalently as

- the probability that a randomly chosen user can achieve a target C/I ψ
- the average fraction of users who at any time achieve C/I ψ
- the average fraction of the network area that is in "coverage" at any time

Since the CDF is equal to $P(\Psi \leq \psi)$, the probability of coverage is directly equal to the complementary cumulative distribution function (CCDF) of C/I of the entire network.

2.3.1.1 Distance to the Closest Base Station

Before expressing the coverage probability, first, we give the expressions of the pdf of the distance R_i of typical mobile user to its home base station. No other base station can be closer than R_i to typical mobile user, since each user communicates with its closest base station. In other words, all interfering base stations must be further than R_i to typical mobile user. Therefore, the pdf of the distance R_i of typical mobile user to its home base station can be specified as:

$$f_{\mathcal{R}_i}(R_i) = 2\pi\lambda_b R_i e^{-\lambda_b \pi R_i^2} \quad (11)$$

The proof of pdf of the distance R_i can be found in [13]. At the same time, the interference created by a Poisson point process of intensity λ outside a disc of radius R_i , for which some useful results are known and applied in [13] to evaluate the coverage probability P_{cov} .

Theorem 2. *The probability of coverage, which is equal to the complementary cumulative distribution function (CCDF) of the C/I of the typical mobile user, when the path loss coefficient $\alpha = 4$:*

$$P(\Psi > \psi) = \frac{1}{1 + \sqrt{\psi} [\pi/2 - \arctan(1/\sqrt{\psi})]} \quad (12)$$

The proof of Theorem 2 is provided in [13].

From the result of this theorem, we can observe that the probability of coverage does not depend on the base station density. Hence, when thermal noise is negligible, for both very dense and very sparse networks, the probability of coverage has a positive value. This means that increasing the number of base stations does not affect the probability of coverage, because the increase in signal power is equal to the increase in interference power. In other words, without employing any interference management technique such as frequency reuse, one can not increase the probability of coverage in interference-limited networks. Note that, this result can be achieved under the assumption that there is at least one mobile user per cell.

By using the CCDF of C/I of the typical mobile user, we can calculate CDF of C/I of the typical mobile user as

$$F_{\Psi}(\psi) = P(\Psi \leq \psi) = 1 - P(\Psi > \psi) \quad (13)$$

$$= 1 - \frac{1}{1 + \sqrt{\psi} [\pi/2 - \arctan(1/\sqrt{\psi})]} \quad (14)$$

$$= \frac{\sqrt{\psi} [\pi/2 - \arctan(1/\sqrt{\psi})]}{1 + \sqrt{\psi} [\pi/2 - \arctan(1/\sqrt{\psi})]} \quad (15)$$

In order to get the pdf of C/I of the typical mobile user, we should take derivative of Eq. 34 with respect to ψ . Then we can express the pdf of C/I of the typical mobile user as

$$f_{\Psi}(\psi) = \frac{dF_{\Psi}(\psi)}{d\psi} \quad (16)$$

$$= \frac{\frac{1}{2+2\psi} + \frac{\pi/2 - \arctan(1/\sqrt{\psi})}{2\sqrt{\psi}}}{(\sqrt{\psi}(\pi/2 - \arctan(1/\sqrt{\psi})) + 1)^2} \quad (17)$$

2.3.2 Average Spectral Efficiency Analysis of a Typical User in Downlink Cellular Systems

In this section, average spectral efficiency of a typical user in downlink wireless cellular systems is expressed. Specifically, average spectral efficiency is computed in units of nats (1 bit = $\log(2) = 0.693$ nats) for a typical user. According to Shannon bound, the maximum data rate per frequency, is given as $\log(1 + \Psi)$. In this setup, interference is treated as noise. Before expressing the average spectral efficiency, we first specify the pdf of spectral efficiency of a typical user. Following the work provided in [13], the pdf of $\Omega = \log(1 + \Psi)$ can be expressed as:

$$f_{\Omega}(\omega) = \frac{e^{\omega} \left(\frac{1}{2+2(e^{\omega}-1)} + \frac{\pi/2 - \arctan(1/\sqrt{e^{\omega}-1})}{2\sqrt{e^{\omega}-1}} \right)}{\left(\sqrt{e^{\omega}-1}(\pi/2 - \arctan(1/\sqrt{e^{\omega}-1})) + 1 \right)^2} \quad (18)$$

In order to calculate the average spectral efficiency, the mean of random variable Ω should be taken. So, we get the average spectral efficiency as:

$$\mathbb{E}[\Omega] = \int_0^{\infty} \omega f_{\omega}(\Omega) d\omega \quad (19)$$

Hence, the average spectral efficiency is expressed to a single numerical integration that yields a precise scalar

$$\mathbb{E}[\Omega] = \int_0^\infty \omega \left(\frac{e^\omega \left(\frac{1}{2+2(e^\omega-1)} + \frac{\pi/2 - \arctan(1/\sqrt{e^\omega-1})}{2\sqrt{e^\omega-1}} \right)}{\left(\sqrt{e^\omega-1}(\pi/2 - \arctan(1/\sqrt{e^\omega-1})) + 1 \right)^2} \right) d\omega \quad (20)$$

$$\approx 1.49 \text{nats/sec/Hz}. \quad (21)$$

As in the case of coverage, increasing the base station density does not increase the average spectral efficiency per user in the downlink because the distance from the mobile user to the nearest base station and the average distance to the nearest interferer scale equally. But since it is assumed that there is at least one mobile user per cell, the area spectral efficiency of the network do increase linearly with the number of base stations.

CHAPTER III

PERFORMANCE ANALYSIS OF DOWNLINK SISO CELLULAR NETWORKS

In this chapter, we model a cellular wireless network and investigate the effect of correlation among the carrier-to-interference (C/I) ratios of multiple users on the network downlink performance with multiuser diversity. Base stations are randomly placed and users are linked to their closest base stations. The C/I ratios of multiple users become conditionally dependent. For such a realistic deployment scenario, we utilize stochastic geometry tools and copula functions to take into account this correlation and analyze the coverage probability and spectral efficiency.

The rest of the chapter is organized as follows. The system model is given in Section 3.1. In Section 3.2, the coverage probability and cumulative distribution function (CDF) of achieved spectral efficiency is derived when the users and base stations are distributed according to independent PPPs and maximum C/I scheduling is performed. Here, the dependency of the C/I values of different users and the use of copula functions to incorporate this dependency into the analysis is explained. In Section 3.3, results for the analysis performed in Section 3.2 are given.

3.1 System Model

We consider a cellular network plan where BSs and users are placed randomly and independently according to two PPPs with densities λ_b and λ_u , respectively. Each user is attached to its nearest BS. Thus, the geography is partitioned to Voronoi cells [46] each associated with a BS, and the users within each Voronoi cell is connected to its associated BS.

We assume a time division multiple access (TDMA) scheme, where a single user is scheduled in each time slot so that there is no intra-cell interference. We consider a frequency reuse factor of 1 for the downlink, thus, inter-cell interference is caused by all other base stations. Assume that all base stations transmit at a constant power level of unity. The downlink signal is assumed to experience path loss with an exponent of α , as such the average power of the signal is decreased by d^α where d is the distance between the base station and user. Since the signal amplitude further experiences Rayleigh fading, the received power of the user i that is at a distance R_{ij} from the j^{th} base station signal is equal to $H_{ij}R_{ij}^{-\alpha}$ where H_{ij} follows an exponential distribution with mean 1. Then, the C/I experienced by that user can be written as,

$$\Psi_i = \frac{H_{ij}R_{ij}^{-\alpha}}{\sigma^2 + I_i}, \quad (22)$$

where σ^2 is the variance of the additive white Gaussian noise (AWGN) term. I_i is the total interference experienced by the user, described as,

$$I_i = \sum_{k \neq j} H_{ik}R_{ik}^{-\alpha}. \quad (23)$$

Following [13], we neglect the noise term in (22) for the remainder of the analysis as it is usually dominated by the interference term.

Assume there are N_j users connected to BS j , and that these users are numbered $i = 1, 2, \dots, N_j$. The connection of user i to BS j is denoted by $j = \text{BS}(i)$ throughout the paper. We assume that at a given time slot, each BS schedules its user according to the maximum C/I scheduler. That is, BS j schedules the user ℓ_j , if

$$\ell_j = \arg \max_i (\Psi_i), \quad j = \text{BS}(i), \quad \forall i = 1, 2, \dots, N_j. \quad (24)$$

Then, corresponding coverage probability and spectral efficiency (bits/sec/Hz) are expressed respectively as:

$$P_C = \Pr(\Psi_{\ell_j} > \psi) = \Pr(\max_i (\Psi_i) > \psi), \quad i = 1, 2, \dots, N_j \quad (25)$$

$$\Omega_j = \log_2(1 + \Psi_{\ell_j}) = \log_2(1 + \max_i (\Psi_i)), \quad i = 1, 2, \dots, N_j. \quad (26)$$

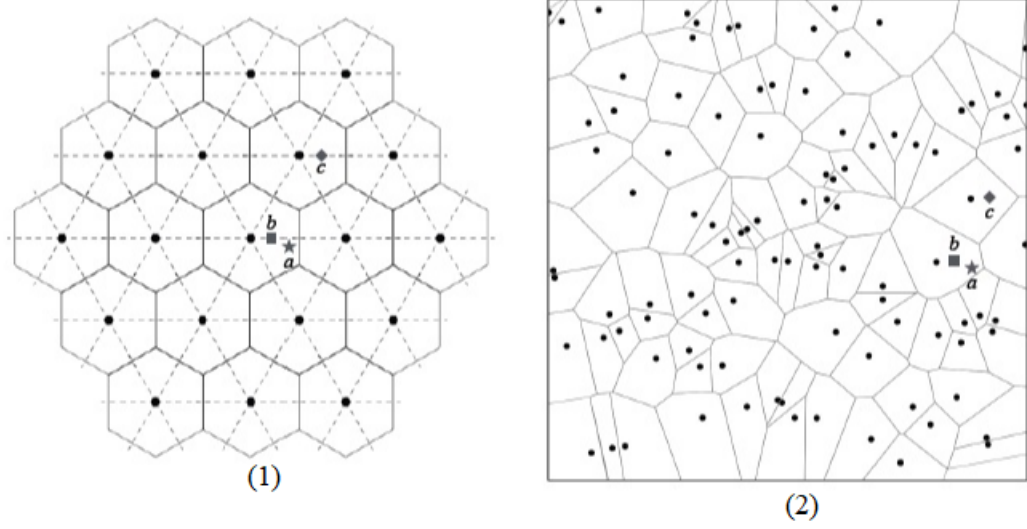


Figure 2: The symmetry in hexagonal cell layout (1) enables independence whereas the randomness in the Voronoi cell layout (2) introduces correlation

3.2 Performance Analysis

When many users are connected to a base station, the common approach in the literature is to conduct the analysis for a typical user, and assume that all users' C/I distributions are i.i.d. Under this assumption, the joint cumulative distribution function (CDF) or probability density function (pdf) of the C/I values are the products of the marginals. However, when base stations are randomly distributed, conditioned on the information that the individual users are assigned to their closest base stations, the C/I values become conditionally dependent.

As illustrated in Figure 2, we consider two cases: 1) traditional hexagonal cell structure, and 2) the case when the base stations and the users are placed randomly according to two PPPs. For both cases, assume the users a and b are in cell A and the user c is in cell B. For the hexagonal structure, the conditional joint distribution $F_{\Psi_a, \Psi_b}(\psi_a, \psi_b | \{\text{BS}(a) = A\}, \{\text{BS}(b) = A\})$ is identical to $F_{\Psi_a, \Psi_c}(\psi_a, \psi_c | \{\text{BS}(a) = A\}, \{\text{BS}(c) = B\})$. This is due to the symmetry in the hexagonal structure, assuming that the geography is infinitely large in all directions. However this is not the case

for the randomly deployed structure. In a practical deployment scenario, the Voronoi cells are almost surely unique in shape and dimensions. This means that the received signal and interference characteristics at different cells are also statistically different, and hence, conditional joint distributions $F_{\Psi_a, \Psi_b}(\psi_a, \psi_b | \{\text{BS}(a) = A\}, \{\text{BS}(b) = A\})$ and $F_{\Psi_a, \Psi_c}(\psi_a, \psi_c | \{\text{BS}(a) = A\}, \{\text{BS}(c) = B\})$ are different.

The example above illustrates how the unique shapes and dimensions of the resulting cells in a random deployment may change the joint distribution $F_{\Psi_1, \dots, \Psi_n}(\psi_1, \dots, \psi_n)$, when it is conditioned upon the fact that all n users are connected to the same cell. Thus, the C/I values are conditionally dependent given they are known to be connected to the same cell.

For a randomly selected base station, we now derive the coverage probability P_C and CDF of the spectral efficiency, Ω . Let $\Psi_{\max} = \max_i(\Psi_i)$, $i = 1, 2, \dots$. Then, P_C and the CDF of Ω are expressed respectively as,

$$P_C = \Pr(\Psi_{\max} > \psi) = 1 - F_{\Psi_{\max}}(\psi), \quad (27)$$

$$F_{\Omega}(\omega) = \Pr(\log_2(1 + \Psi_{\max}) \leq \omega) = F_{\Psi_{\max}}(2^{\omega} - 1). \quad (28)$$

Using total probability theorem we can write,

$$F_{\Psi_{\max}}(\psi) \approx \sum_{n=0}^{N_{\max}} F_{\Psi_{\max}|N}(\psi|n) \Pr(N = n). \quad (29)$$

where N is the number of users in the Voronoi cell. For the approximation to be exact, we need $N_{\max} \rightarrow \infty$. However in practice we cannot calculate infinitely many conditional CDFs. Therefore in our results, for each λ_u and λ_b combination, we choose the constant $N_{\max}$ such that

$$\sum_{n=0}^{N_{\max}} \Pr(N = n) \geq 1 - \epsilon. \quad (30)$$

where ϵ is a number which should be chosen very small. In this paper, regarding the performance analysis, we consider to take this number as 10^{-4} .

Before computing the probability mass function (pmf) of number of mobile users connected to the same base station, we give the pdf of size of a Voronoi cell. The pdf of size of a typical Voronoi cell which is derived by the Monte Carlo fit is given as [46]:

$$f_X(x) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp(-bx) \quad (31)$$

where b and a are constant values and $\Gamma(\cdot)$ is the Gamma function. In this equation the size of the Voronoi cell X is scaled by the value $1/\lambda_b$.

Following [18], the pmf of number of users in a Voronoi cell can be derived using (31) as:

$$\Pr[N = n] = \frac{b^a \Gamma(n + a) (\lambda_u / \lambda_b)^n}{\Gamma(a) n! (\lambda_u / \lambda_b + b)^{n+a}} \quad (32)$$

Next, we compute the conditional CDF $F_{\Psi_{\max}|N}(\psi|n)$ in (29) for all N . For $N = 1$, following the steps of [18], we have

$$\begin{aligned} F_{\Psi_i}(\psi) &= 1 - \pi \lambda_b \int_0^\infty e^{-\pi \lambda_b (1 + (1 - \Pr[N=0])k)x} dx \\ &= \frac{\hat{\lambda}_b \psi^{2/\alpha} \int_{\psi^{-2/\alpha}}^\infty 1/(1 + u^{\alpha/2}) du}{\lambda_b + \hat{\lambda}_b \psi^{2/\alpha} \int_{\psi^{-2/\alpha}}^\infty 1/(1 + u^{\alpha/2}) du}. \end{aligned} \quad (33)$$

where $k = \psi^{2/\alpha} \int_{\psi^{-2/\alpha}}^\infty 1/(1 + u^{\alpha/2}) du$ and $\hat{\lambda}_b = \lambda_b (1 - \Pr[N = 0])$. If we take $\alpha = 4$, corresponding to the case of suburban cellular radio, an analytical solution exists for the equation $\psi^{2/\alpha} \int_{\psi^{-2/\alpha}}^\infty 1/(1 + u^{\alpha/2}) du$. Therefore, we would have a closed form expression for CDF of the C/I of a single user by taking $\alpha = 4$, resulting in:

$$F_{\Psi_i}(\psi) = \frac{\hat{\lambda}_b \sqrt{\psi} [\pi/2 - \arctan(1/\sqrt{\psi})]}{\lambda_b + \hat{\lambda}_b \sqrt{\psi} [\pi/2 - \arctan(1/\sqrt{\psi})]}. \quad (34)$$

For all other values of α , $F_{\Psi_i}(\psi)$ needs to be computed numerically. So there is no closed form solution for these values.

While computing $F_{\Psi_{\max}|N_j}(\psi|n)$, $n \geq 2$ the conditional dependency of the C/I values in the Voronoi consellation need to be taken into account. The dependency may be defined using copula functions [26]. A copula function $C(\cdot)$ is the joint CDF

of multiple random variables $U_1, U_2, \dots, U_N$, whose marginals are uniform over $[0, 1]$,

$$C(u_1, \dots, u_N) = \Pr(U_1 \leq u_1, \dots, U_N \leq u_N). \quad (35)$$

Sklar's Theorem [47] states that for any multivariate CDF, $F(x_1, \dots, x_N)$ with marginal CDFs $F_1(x_1), \dots, F_N(x_N)$, there exists a copula function $C(\cdot)$ such that

$$F(x_1, \dots, x_N) = C(F_1(x_1), \dots, F_N(x_N)). \quad (36)$$

The copula framework enables the description of the joint distribution of a number of correlated random variables in terms of their marginal distributions and the copula. Therefore it allows a separate description of the marginal distributions and their associations. Several families of copulas have been described in the literature. They are classified as Archimedean copulas and elliptical copulas [26]. Archimedean copulas are explicit copulas which depend on a handful of parameters, whereas elliptical copulas are implicitly derived from well-known multivariate distributions such as Gaussian copula and t-copula. In this paper, we consider to use both the Archimedean copulas and the elliptical copulas to model the joint distribution of C/I values. Regarding the Archimedean copulas we focus our attention on Clayton, Rotated Clayton, Plackett, Frank, Gumbel, Rotated Gumbel, and Symmetrised Joe-Clayton copulas and for the elliptical copulas we consider to use Gaussian copula and t-copula which are widely used. For each $n \geq 2$, we select the copula which results in the best fit for the joint C/I distribution in the maximum likelihood sense.

Utilizing the copula framework, the conditional CDF $F_{\Psi_{\max}|N}(\psi|2)$ can be computed as:

$$\begin{aligned} F_{\Psi_{\max}|N}(\psi|2) &= \Pr(\Psi_1 \leq \psi, \Psi_2 \leq \psi) \\ &= C_{\rho_1}(F_{\Psi_1}(\psi), F_{\Psi_2}(\psi)) \end{aligned} \quad (37)$$

where $F_{\Psi_1}(\psi) = F_{\Psi_2}(\psi) = F_{\Psi_i}(\psi)$.

In (37), following [48], the optimal copula function is selected and the correlation parameter ρ_1 is computed in a semiparametric manner, as the value that maximizes

the following pseudo log-likelihood equation:

$$[C_{\rho_1}, \rho_1] = \arg \max_{\rho \in (0, \infty)} \left(\sum_{k=1}^K \log \left(\frac{\partial^2}{\partial u \partial v} C_{\rho}(u^{(k)}, v^{(k)}) \right) \right). \quad (38)$$

Here, $u^{(k)} = F_{\Psi_i}(\psi_{1k})$ and $v^{(k)} = F_{\Psi_i}(\psi_{2k})$ and ψ_{1k} and ψ_{2k} , are the k 'th C/I values experienced by users residing in a two user Voronoi cell, obtained experimentally for K samples, with K sufficiently large.

Next, the conditional density, $F_{\Psi_{\max}|N}(\psi|3)$ can be computed as,

$$\begin{aligned} F_{\Psi_{\max}|N}(\psi|3) &= \Pr(\max(\Psi_1, \Psi_2) \leq \psi, \Psi_3 \leq \psi) \\ &= \Pr(Z \leq \psi, \Psi_3 \leq \psi) = C_{\rho_2}(F_Z(\psi), F_{\Psi_3}(\psi)) \end{aligned} \quad (39)$$

where Ψ_1, Ψ_2 and Ψ_3 are the C/I values experienced by users residing in a three user Voronoi cell and $F_Z(\psi) = F_{\Psi_{\max}|N}(\psi|2)$. The optimal copula function C_{ρ_2} and the corresponding correlation parameter ρ_2 may be computed using the semiparametric method given in (38) by letting $u^{(k)} = F_Z(\max(\psi_{1k}, \psi_{2k}))$ and $v^{(k)} = F_{\Psi_i}(\psi_{3k})$ with $\psi_{1k}, \psi_{2k}, \psi_{3k}$ being the k 'th C/I values experienced by users residing in a three user Voronoi cell, obtained experimentally for K samples, with K sufficiently large.

In general, for any $n \geq 3$, the conditional cdf may be computed recursively using

$$F_{\Psi_{\max}|N}(\psi|n) = C_{\rho_{n-1}}(F_{\Psi_{\max}|N}(\psi|n-1), F_{\Psi_n}(\psi)). \quad (40)$$

It is clear that (40) requires the estimation of $n-1$ copula functions with correlation parameters, C_{ρ_i} , $i = 1, 2, \dots, N_j - 1$. Once all conditional CDFs are computed, one can calculate (29) to obtain (44) and (28) respectively.

3.3 Results

The stochastic-geometry-based analysis presented in the previous section takes into account the correlation between the C/I values experienced by users when base stations are placed randomly. In this section, we present results for this analysis. We

compare these results with those computed without taking this correlation into account. We also present results of a Monte Carlo simulation to show the accuracy of the analysis presented herein.

We consider a rectangular geography of size 200×200 units. We assume a user density of $\lambda_u = 4$. We let $\lambda_b = 1$ and investigate cases where path loss exponent of α takes on values 2.7, 3 and 4 corresponding to the urban, shadowed urban and suburban scenarios respectively. To conduct the analysis, we first numerically compute $N_{\max}$ using (30) as well as the parameters a and b in (32) for the considered geometry. As a result of this computation, the values of $N_{\max}$, a and b are calculated as 24, 2.88 and 2.80 respectively.

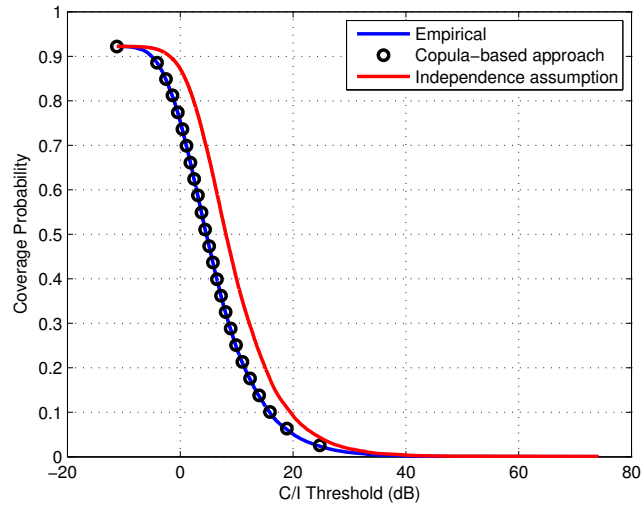
Next, we compute coverage probability and the CDF of the spectral efficiency given in (25) and (26) using (44) and (28) respectively. We also repeat the analysis without taking the correlation into account. Assuming independence,

$$F_{\Psi_{\max}|N}(\psi|n) = [F_{\Psi_i}(\psi)]^n, \quad \forall n \geq 2. \quad (41)$$

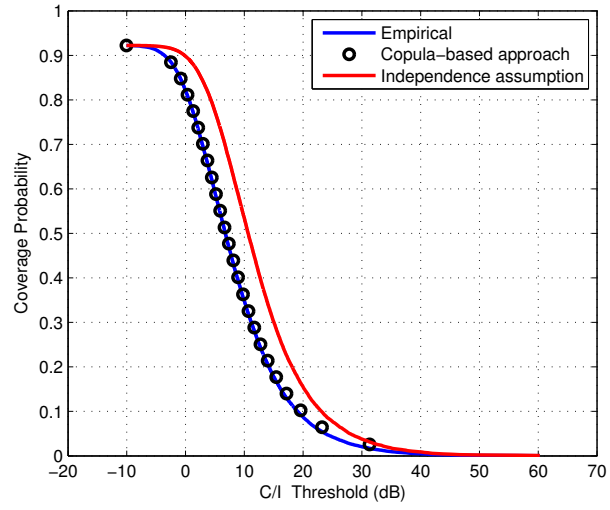
may be used to compute (29).

To assess the accuracy of the copula-based analysis, we conduct Monte Carlo simulations. Within the 200×200 unit geometry, we drop base stations and users according to their PPPs with densities λ_u and λ_b , respectively, independent of each other. The users are associated with the closest base station, and their C/I values are computed according to (22). One such drop gives C/I data for N_b cell realizations, where N_b is a Poisson random variable with mean $200 \times 200 \times \lambda_b$. We conduct a total of 3×10^5 drops, yielding C/I data for many cell realizations with varying number of users. For each cell, we determine the maximum C/I value and compute an empirical CDF from these values.

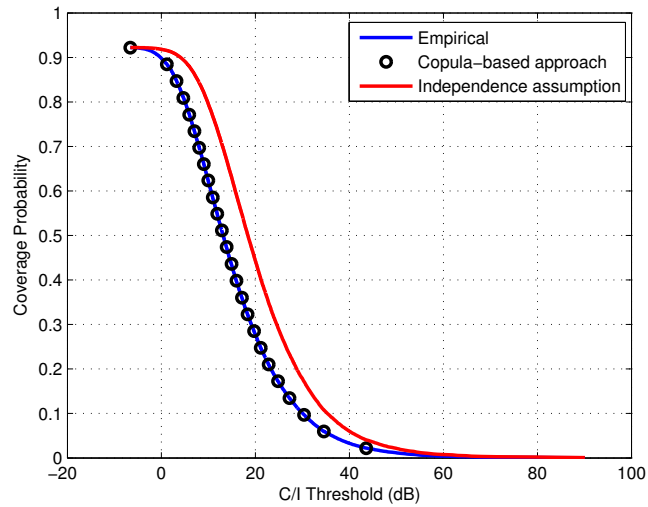
We report the coverage probabilities and CDF of spectral efficiencies computed using the Copula-based analysis and the independence assumption in Figure 7(a)–(c) and Figure 4(a)–(c), respectively, for all cases along with the empirical plots resulting



(a)

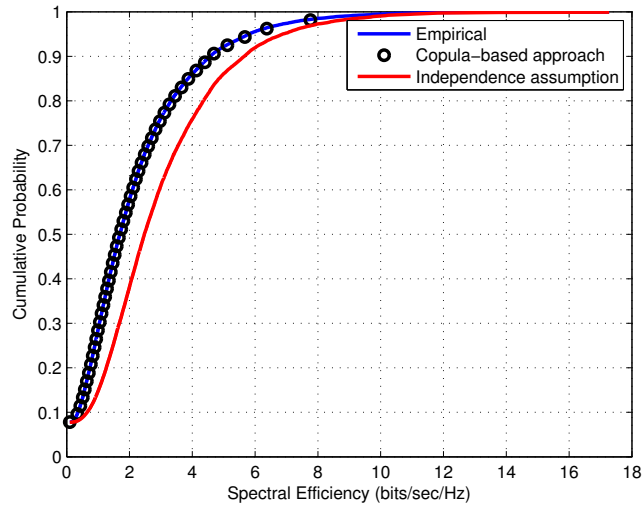


(b)

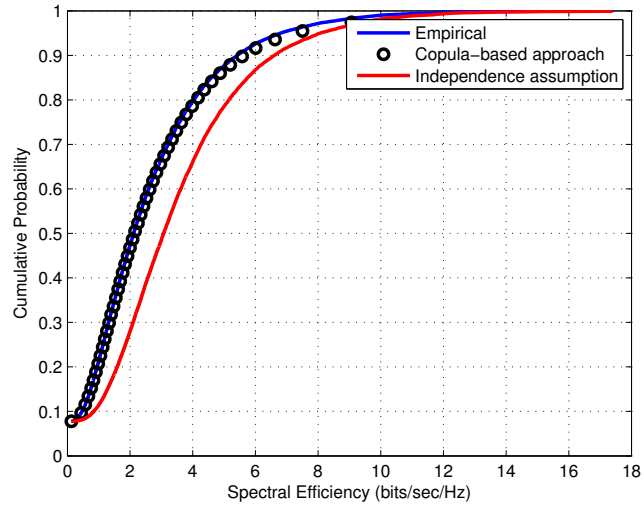


(c)

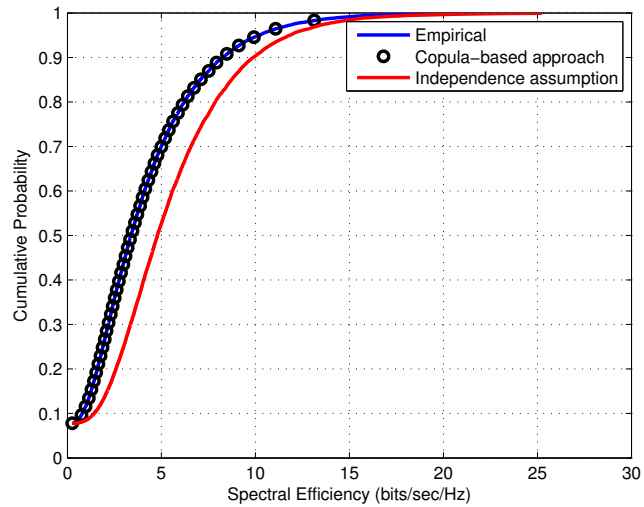
Figure 3: Coverage probability when $\lambda_b \stackrel{32}{=} 1$, and $\lambda_u = 4$ and (a) $\alpha = 2.7$, (b) $\alpha = 3$, (c) $\alpha = 4$.



(a)



(b)



(c)

Figure 4: Spectral efficiency CDF when $\lambda_b = 1$, and $\lambda_u = 4$ and (a) $\alpha = 2.7$, (b) $\alpha = 3$, (c) $\alpha = 4$.

from Monte Carlo simulations. From the figures it can be seen that the independence assumption, which is commonly used in the literature, is not very accurate when base stations are placed randomly since both the coverage probabilities and spectral efficiency CDFs obtained with this assumption do not agree with the empirical results obtained from Monte Carlo simulations. On the other hand, both the coverage probabilities and the spectral efficiency CDFs obtained using the copula-based analysis agree well with the empirical results, validating the accuracy of our approach.

We can state in general, that the independence assumption overestimates both the coverage probability and the average spectral efficiency. According to Figure 7(a)–(c), regarding 90% coverage probability, the amount of overestimation is equal to 3 dB, 3.8 dB and 4.3 dB for the cases where path loss exponent of α equals to 2.7, 3 and 4 respectively. The situation is similar for the average spectral efficiency such that according to Figure 4(a)–(c) regarding the 90 percentile average spectral efficiency, the amount of overestimation is equal to 0.95 bits/sec/Hz, 1.03 bits/sec/Hz and 1.44 bits/sec/Hz for the cases where α equals to 2.7, 3 and 4 respectively. So, according to the both coverage probability and the average spectral efficiency figures we can also state that the amount of overestimation increases as path loss exponent α increases. This is because as path loss exponent increases, the signal from the serving base station becomes more prominent compared to the interference from neighbor base stations, which makes the C/I values in the same cell more dependent since the signal source is the same for all.

CHAPTER IV

COVERAGE ANALYSIS OF DOWNLINK MU-MIMO CELLULAR NETWORKS

In the previous chapter, we analyzed the coverage and average spectral efficiency performance of SISO cellular networks considering multiuser diversity is employed. We showed that the C/I values of users connected to the same base stations are conditionally dependent and we modeled this dependency using the copula functions.

Multiple-input multiple-output (MIMO) transmission improves the data rates and link reliability through spatial diversity and multiplexing gains with respect to SISO systems. In single-user MIMO (SU-MIMO), the BS transmits one or multiple data streams, called layers, from one antenna array to a single user. On the other hand, in multi-user MIMO (MU-MIMO) systems, antenna array at the BS simultaneously transmits different layers in separate beams to different users using the same frequency resource based on spatial division multiplexing [27].

In this chapter, we analyze the coverage performance of multi-user multiple-input multiple-output (MU-MIMO) downlink cellular networks. Similar to the SISO case, we model the locations of the base stations and mobile users using Poisson Point Processes. We employ maximum ratio transmission precoding at the base station. Our analysis builds upon Prony's method to approximately calculate the coverage probability using the sum of exponentials which facilitates the computation of Laplace transform of both intra-cell and inter-cell interference. We demonstrate the accuracy of derived approximate coverage probability expressions through numerical simulations for different MU-MIMO configurations.

The rest of the chapter is organized as follows. In Section 4.1, we present the MU-MIMO system model under consideration. In Section 4.2, we present the derivation of the coverage probability based on our proposed approach. In Section 4.3, we present numerical results to corroborate the analytical derivations.

4.1 *System Model*

As assumed in the previous chapter, a cellular network is assumed where BSs and users are distributed randomly and independently according to two PPPs with densities λ_b and λ_u , respectively. Each user is associated with its closest BS¹. As a result, the geography is divided into Voronoi cells [29] each connected to a BS, and the users within each Voronoi cell are associated with its connected BS.

In the previous chapter, we assumed a transmission scheme where both the BSs and users have single antenna. In this chapter, as illustrated in Figure 5, we assume a MU-MIMO transmission scheme where each BS has N transmit antennas and each user has a single antenna. In the previous chapter, a TDMA transmission scheme was assumed where a single user with the maximum C/I value was scheduled in each time slot. Regarding this chapter, in each time slot, randomly selected M users are assumed to be scheduled for service by a BS. This random user selection scheme is assumed for the ease of the analysis. Similar to the previous chapter, all BSs transmit on the same carrier frequency, so inter-cell interference is generated by all other BSs other than the serving BS. But different than the previous chapter, since multiple users are scheduled on the same frequency resource in each cell, there also exists intra-cell interference. For the sake of simplicity of the analysis, we assume that all BSs transmit at a constant power level of unity and have full Channel State Information (CSI) of all connected users which was also assumed in the previous chapter. Based on CSI, maximum ratio

¹It is worth noting that the analysis proposed in this dissertation can be extended considering a dynamical association rule on the basis of channel gain. But it is beyond the scope of this dissertation.

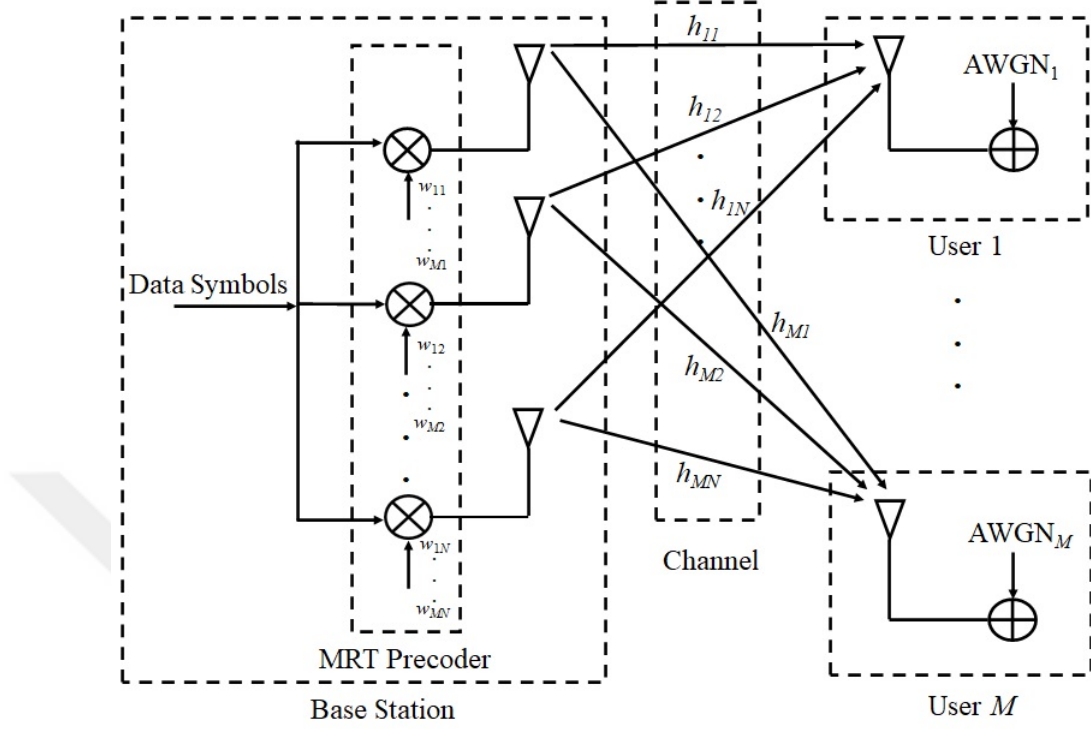


Figure 5: The downlink MU-MIMO transmission scheme

transmission (MRT) precoding [49]² is employed at the BS. In MRT, the data symbol of user i is multiplied by the precoding vector $\mathbf{w}_i = [w_{i1}, w_{i2}, \dots, w_{iN}]^T \in \mathcal{C}^{N \times 1}$. So, the received signal gain at the user i is maximized.

The downlink signals transmitted by multiple antennas are attenuated by path loss with a coefficient of α , as such the average power of the signals are decreased by r^α where r is the distance between the BS and the user. They further experience Rayleigh fading. Therefore, the received signal power of a user i at a distance r_{ij} from the j^{th} BS is given by $|\mathbf{h}_{ij}\mathbf{w}_i r_{ij}^{-\alpha/2}|^2$ where $\mathbf{h}_{ij} = [h_{ij1}, h_{ij2}, \dots, h_{ijN}]$, $\mathbf{h}_{ij} \sim \mathcal{CN}(0, \mathbf{I}_N)$ is a complex Gaussian random vector with N dimensions. It should be also noted that the precoding vector will then take the form of $\mathbf{w}_i = \mathbf{h}_{ij}^H$.

²It is also important to note that the analysis proposed in this dissertation can be extended considering the optimization of the precoder. Since our aim is to propose an approach that can be used as a basis for the analysis of downlink MU-MIMO cellular networks, we choose MRT precoder which is simple and practical. So the extension of the analysis is left as a future work.

The signal-to-interference ratio (SIR) of the mobile user i connected to BS j at a random distance r_{ij} can be therefore expressed as

$$\Psi_{ij} = \frac{|\mathbf{h}_{ij}\mathbf{w}_i r_{ij}^{-\alpha/2}|^2}{\sigma^2 + I_i} \quad (42)$$

where σ^2 is the variance of the additive white Gaussian noise (AWGN) term. I_i is the total interference incurred by the user and expressed as:

$$I_i = I_{i1} + I_{i2} = \sum_{k \neq i} |\mathbf{h}_{ik}\mathbf{w}_k r_{ik}^{-\alpha/2}|^2 + \sum_{m \neq i, n \neq j} |\mathbf{h}_{in}\mathbf{w}_m r_{in}^{-\alpha/2}|^2 \quad (43)$$

which consists of the intra-cell interference (I_{i1}) and inter-cell interference (I_{i2}). For the remainder of the analysis, we ignore the noise term in (42) as it is usually dominated by the total interference term I_i [13].

4.2 Coverage Analysis

Let us assume that the randomly selected user i is connected to the BS j . Furthermore, let us define $\Omega_{ij} = |\mathbf{h}_{ij}\mathbf{w}_i|^2$. The conditional coverage probability P'_C conditioned on r_{ij} can be then expressed as:

$$P'_C = \Pr(\Psi_{ij} > \theta | r_{ij}) = \Pr(\Omega_{ij} > \theta r_{ij}^\alpha I_i | r_{ij}) \quad (44)$$

where θ denotes the threshold value. Then under the assumption that correlation between the SIR values of users connected to the same cell is ignored, the average coverage probability, which we call “coverage probability” for the sake of brevity, is obtained by averaging P'_C over the plane and expressed as³:

$$\begin{aligned} P_C &\approx \mathbb{E}_{r_{ij}}[P'_C] = \mathbb{E}_{r_{ij}} [\Pr(\Omega_{ij} > \theta r_{ij}^\alpha (I_{i1} + I_{i2}) | r_{ij})] \\ &= \int_0^\infty \Pr(\Omega_{ij} > \theta r_{ij}^\alpha (I_{i1} + I_{i2}) | r_{ij}) f(r_{ij}) dr_{ij} \end{aligned} \quad (45)$$

where $f(r_{ij})$ is the probability density function (pdf) of the distance. This pdf can be derived easily considering the fact that the null probability of a 2-D PPP with density

³We show in [50] that the SIR values of users connected to same cell are dependent.

λ_b in an area A is $\exp(-\lambda_b A)$ [13]. Since all interfering BSs must be farther than r_{ij} , the probability that there is no BS which is closer than r_{ij} is equal to $\exp(-\lambda_b \pi r_{ij}^2)$. Then following [13], the pdf of the distance can be derived as $f(r_{ij}) = e^{-\lambda_b \pi r_{ij}^2} 2\pi \lambda_b r_{ij}$.

In order to calculate the coverage probability, we also need to average it over the intra-cell and inter-cell interferences respectively. Mathematically speaking, we have:

$$\Pr(\Omega_{ij} > \theta r_{ij}^\alpha (I_{i1} + I_{i2}) | r_{ij}) = \mathbb{E}_{I_{i1}, I_{i2}} [\Pr(\Omega_{ij} > \theta r_{ij}^\alpha (I_{i1} + I_{i2}) | r_{ij}, I_{i1}, I_{i2})] \quad (46)$$

Since Ω_{ij} is Gamma distributed [29], we can calculate the coverage probability which is conditioned on distance, intra-cell and inter-cell interferences as:

$$\Pr(\Omega_{ij} > \theta r_{ij}^\alpha (I_{i1} + I_{i2}) | r_{ij}, I_{i1}, I_{i2}) = 1 - \frac{\gamma(N, \theta r_{ij}^\alpha (I_{i1} + I_{i2}))}{\Gamma(N)} \quad (47)$$

Prony's method was proposed by Gaspard de Prony in 1795 to explain the expansion of gases as a linear sum of damped exponentials [51]. This proposed method interpolates uniformly sampled data points that are obtained through the measurements by fitting an exponential model to them. Nowadays, due to advances in digital computing systems, Prony's method has been successfully applied in various engineering fields such as materials science, antennae and biomedical systems[52].

To facilitate the calculation of expectation over the intra-cell and inter-cell interferences, we can approximate $\Pr(\Omega_{ij} > \theta r_{ij}^\alpha (I_{i1} + I_{i2}) | r_{ij}, I_{i1}, I_{i2})$ using Prony's method [36, 51] as:

$$1 - \frac{\gamma(N, \theta r_{ij}^\alpha (I_{i1} + I_{i2}))}{\Gamma(N)} \approx \sum_{n=1}^{N_p} a_n \exp(b_n \theta r_{ij}^\alpha (I_{i1} + I_{i2}) N) \quad (48)$$

where N_p denotes the number of exponential functions used in the approximation and a_n and b_n are the variables that control the accuracy of the approximation. Therefore, we can express the average of coverage probability conditioned on distance, I_{i1} and

I_{i2} as:

$$\begin{aligned}
& \mathbb{E}_{I_{i1}, I_{i2}} [\Pr (\Omega_{ij} > \theta r_{ij}^\alpha (I_{i1} + I_{i2}) \mid r_{ij}, I_{i1}, I_{i2})] \\
& \stackrel{(a)}{\approx} \sum_{n=1}^{N_p} a_n \mathbb{E}_{I_{i1}} [\exp(-b_n r_{ij}^\alpha \theta N I_{i1})] \mathbb{E}_{I_{i2}} [\exp(-b_n r_{ij}^\alpha \theta N I_{i2})] \\
& \stackrel{(b)}{=} \sum_{n=1}^{N_p} a_n \mathcal{L}_{I_{i1}} [b_n r_{ij}^\alpha \theta N] \mathcal{L}_{I_{i2}} [b_n r_{ij}^\alpha \theta N]
\end{aligned} \tag{49}$$

where $\mathcal{L}_{I_{i1}}[s]$ and $\mathcal{L}_{I_{i2}}[s]$ denote the Laplace transform of I_{i1} and I_{i2} respectively. Here, (a) follows since I_{i1} and I_{i2} are independent and (b) follows from the definition of Laplace transform.

Following [53] and noting $I_{i1} \sim \text{Gamma}(M-1, r_{ij}^{-\alpha})$, we can express the Laplace transform of intra-cell interference I_{i1} as

$$\mathcal{L}_{I_{i1}}(s) = \frac{1}{(1 + s r_{ij}^{-\alpha})^{M-1}} \tag{50}$$

The Laplace transform of I_{i2} can be calculated as

$$\begin{aligned}
\mathcal{L}_{I_{i2}}(s) &= \mathbb{E}_{\Phi, Z_n} \left[e^{-s \sum_{n \in \Phi \setminus \{j\}} Z_n r_{in}^{-\alpha}} \right] \\
&\stackrel{(a)}{=} \mathbb{E}_{\Phi} \left[\prod_{n \in \Phi \setminus \{j\}} \mathbb{E}_{Z_n} \left[e^{-s Z_n r_{in}^{-\alpha}} \right] \right] \\
&\stackrel{(b)}{=} \mathbb{E}_{\Phi} \left[\prod_{n \in \Phi \setminus \{j\}} \frac{1}{(1 + s r_{in}^{-\alpha})^M} \right] \\
&\stackrel{(c)}{=} \exp \left(-2\pi \lambda_b \int_{r_{ij}}^{\infty} \left(1 - \frac{1}{(1 + s x^{-\alpha})^M} \right) x dx \right) \\
&\stackrel{(d)}{=} \exp \left(-2\pi \lambda_b \sum_{k=1}^M \binom{M}{k} \frac{s^{2/\alpha}}{\alpha} \int_z^1 t^{a-1} (1-t)^{b-1} dt \right) \\
&= \exp(-\lambda_b s^{2/\alpha} \Delta)
\end{aligned} \tag{51}$$

where r_{in} denotes the distance between user i and interfering BS n , $Z_n \triangleq \sum_{m \neq i} |\mathbf{h}_{in} \mathbf{w}_m|^2$ and $Z_n \sim \text{Gamma}(M, 1)$, Φ denotes the PPP with density λ_b (used to model the locations of the BSs), $\Delta = \frac{2\pi}{\alpha} \sum_{k=1}^M \binom{M}{k} B'(a, b, z)$, $a = \frac{2}{\alpha} - k + M$, $b = \frac{-2}{\alpha} + k$,

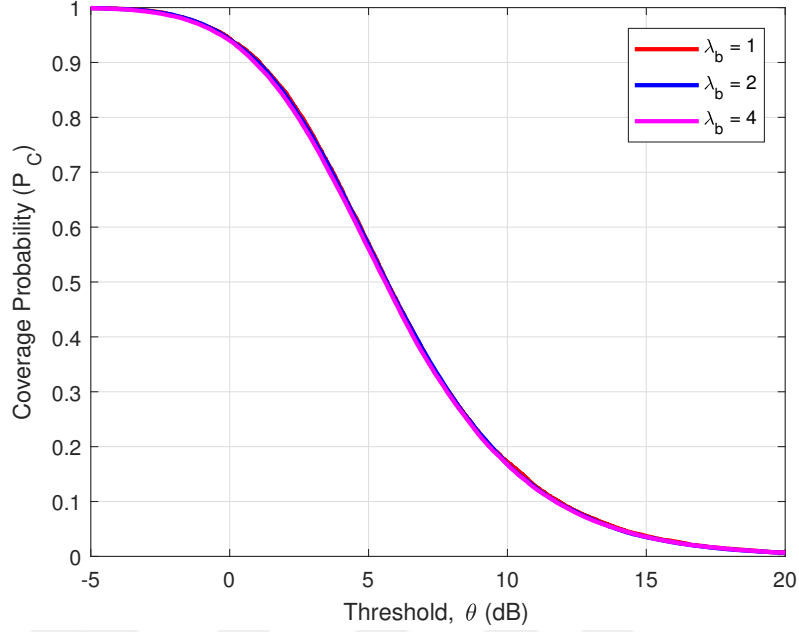


Figure 6: Coverage probability when $N = 8, M = 2$ and $\lambda_b = 1, 2, 4$

$z = \frac{1}{1+sr_{ij}^{-\alpha}}$. In the above, (a) follows from the i.i.d (independent and identically distributed) property of Z_n and its further independence from the PPP Φ , (b) follows from the Laplace transform of Z_n , (c) follows from the definition of probability generating functional (PGFL) [13] of Φ , which is expressed for some function $f(x)$ as $\mathbb{E}_\Phi [\prod_{x \in \Phi} f(x)] = \exp(-\lambda_b \int_{\mathbb{R}^2} (1 - f(x)) dx)$ and (d) is obtained by some mathematical manipulations and change of variables $1/(1 - sx^{-\alpha}) \rightarrow t$.

Replacing $s = b_n r_{ij}^\alpha \theta N$ and substituting Eq. (49) into Eq. (45), we obtain the final form of coverage probability as

$$P_C \approx \sum_{n=1}^{N_p} \frac{a_n \pi}{(1 + b_n \theta N)^{M-1} (\pi + (b_n \theta N)^{2/\alpha} \Delta)} \quad (52)$$

It is obvious that the downlink coverage probability of a typical user in MU-MIMO cellular networks is independent of BS density λ_b according to Eq. (52). Therefore, increasing the number of BS, does not affect the downlink coverage probability, as it is also observed for the single antenna case according to [13]. This result is verified

according to the simulation results presented in Fig.6. This is because, increasing the signal power is counter-weighted by the increase in interference power. As a result, increasing the number of base stations does not affect the downlink coverage probability.

4.3 Numerical Results

We consider a rectangular geometry which has an area of 200×200 unit square. We assume the number of simultaneously scheduled users M on the same frequency resource takes values of 1, 2 and 4. We also assume a user density of $\lambda_u = 8$, $\lambda_u = 16$, $\lambda_u = 32$ and $\lambda_u = 64$ for the cases where number of transmit antennas N takes on values 8, 16, 32 and 64 respectively. Additionally, we assume a BS density of $\lambda_b = 1$, and path loss exponent of $\alpha = 4$.

To assess the accuracy of our proposed approach, we run Monte Carlo simulations. Within the 200×200 unit square rectangular area, we distribute BSs and users with respect to their PPPs with densities λ_b and λ_u , respectively, independent of each other. The users are connected to the nearest BS, and their SIR values are calculated according to Eq. (42). We consider to simulate SIR data for N_b cell realizations, where N_b is a Poisson random variable with mean $200 \times 200 \times \lambda_b$.

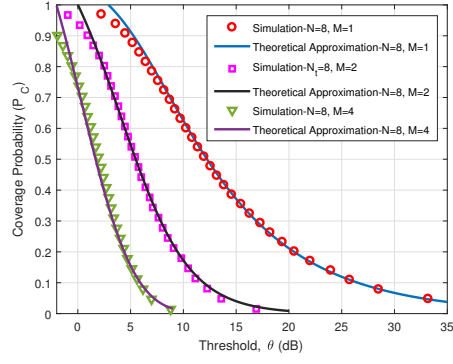
The parameters a_n and b_n for $n = 1, 2, \dots, N_p$ required in Eq. (52) are obtained by minimizing the approximation error defined by:

$$\left| 1 - \frac{\gamma(N, \theta r_{ij}^\alpha (I_{i1} + I_{i2}))}{\Gamma(N)} - \sum_{n=1}^{N_p} a_n \exp(b_n \theta r_{ij}^\alpha (I_{i1} + I_{i2}) N) \right|^2 \quad (53)$$

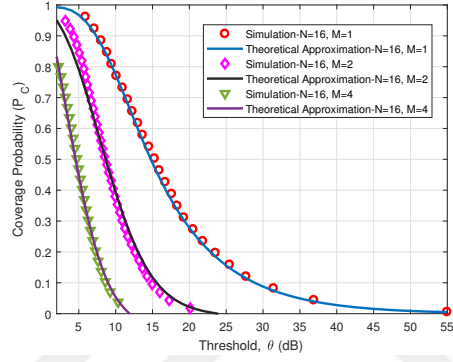
Therefore we solve the following optimization problem numerically using the MATLAB's built-in function `fminsearch`.⁴:

$$\arg \min_{a_n, b_n} \left| 1 - \frac{\gamma(N, u)}{\Gamma(N)} - \sum_{n=1}^{N_p} a_n \exp(b_n u N) \right|^2 \quad (54)$$

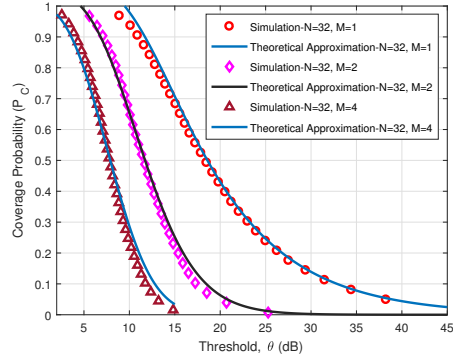
⁴MATLAB's `fminsearch` function of optimization toolbox is known to be successful for optimization problems consisting of small numbers of variables. Regarding our optimization problem, since we have two variables namely, a_n and b_n to optimize, we select "fminsearch" to calculate them.



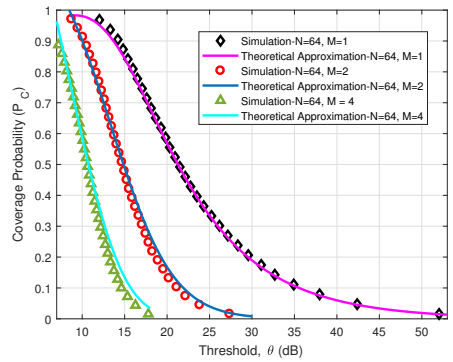
(a)



(b)



(c)



(d)

Figure 7: Coverage probability when $\lambda_b = 1, \alpha = 4$ (a) $\lambda_u = 8, N = 8$, (b) $\lambda_u = 16, N = 16$, (c) $\lambda_u = 32, N = 32$, (d) $\lambda_u = 64, N = 64$.

where $u = \theta r_{ij}^\alpha I_i$. `fminsearch` function uses Nelder-Mead simplex direct search algorithm for optimization. In order to solve the problem given by (12) using MATLAB's "fminsearch", approximately 10,000 iterations and 13,000 function counts are required in most cases.

In our calculations, the number of exponentials N_p in Eq. (48) is taken as 10, which is a sufficiently large number for the approximation error to converge to a small value on the order of magnitude of 10^{-6} .⁵ Additionally, regarding the N_p which is greater than 10, the accuracy of our downlink coverage probability approximation does not improve. Therefore we select N_p as 10.

In Figure 7(a)–(d), we plot the derived coverage probabilities of Eq. (52) along with the Monte Carlo simulations. It is observed from Fig. 7(a)–(d) that the derived expression agrees well with the simulation results. According to these simulation results, there are only small deviations observed for the low threshold values regarding the cases where $N = 8-M = 1$, $N = 8-M = 2$, $N = 8-M = 4$, $N = 16-M = 2$, $N = 32-M = 1$ and for the high threshold values regarding the cases where $N = 32-M = 4$ and $N = 64-M = 4$.⁶

It is observed from these figures that as the number of simultaneously scheduled users M on the same frequency resource increases, the coverage probability decreases for all MU-MIMO configurations. For instance, according to the results plotted in Figure 7(a), for SIR threshold $\theta = 7$ dB, the coverage probability is equal to 0.79, 0.37 and 0.08 for the cases where $M = 1, 2, 4$ respectively. In other words, the performance decrease in coverage is equal to 53% and 78% for the cases where number of simultaneously scheduled users is equal to 2 and 4 respectively, with respect to the case where single user is scheduled at a time. The trend is similar for the results

⁵We run Monte Carlo simulations, trying 100,000 number of different initial points that are selected randomly for all considered MU-MIMO configurations. The ratios of number of simulations, where the approximation error stayed under 10^{-6} take on values between 84% and 98%.

⁶These deviations are because of the correlation between the SIR values of users connected to same cell. In our analysis, we ignore this correlation as we have already stated.

plotted in Figure 7(b)–(d). Additionally, regarding the case when $M = 1$, target SIR value θ in dB that a typical user can achieve is 3.5 dB and 5 dB more than the cases when $M = 2$ and $M = 4$ respectively with 70% probability. The main reason behind these results is that the total received interference power of typical user increases with number of simultaneously scheduled users M . This observation also points out the necessity of employing interference-aware scheduling algorithm or interference cancellation scheme. Furthermore, it is observed that as the number of transmit antennas increases, the coverage probability increases as expected. According to the results plotted in Figure 7(a)–d, regarding SIR threshold $\theta = 10$ dB and $M = 2$, the coverage probability is equal to 0.17, 0.39, 0.65, 0.90 for the cases where $N = 8, 16, 32, 64$ respectively. The trend is also similar for $M = 1$ and $M = 4$. This is because of the diversity gain increases with the number of transmit antennas.

CHAPTER V

PERFORMANCE ANALYSIS OF DOWNLINK COORDINATED MULTIPOINT SYSTEMS

In the previous chapter, we analyzed the coverage performance of downlink MU-MIMO cellular networks. Regarding this analysis, the performance was limited by the interference. In order to tackle this interference, Coordinated Multipoint (CoMP) transmission and reception techniques are proposed. These techniques actually refer to a wide range of techniques that enable dynamic coordination of transmission and reception with multiple geographically separated BS's. Its aim is to enhance the overall system performance by means of utilizing the resources more effectively and improve end user service quality. CoMP requires close coordination between a number of geographically separated base stations. They dynamically coordinate to provide joint scheduling and transmissions as well as providing joint processing of the received signals. In this way a mobile user at the edge of a cell is able to be served by two or more BS's to improve signal reception / transmission and increase throughput particularly under cell edge conditions.

In this chapter, our main purpose is to analyze the performance of CoMP transmission scheme by using stochastic geometry techniques. Before this analysis, we give some brief information about the schemes which are used to employ the CoMP transmission/reception. Also, we review the performance analyses of CoMP transmission scheme, where the positions of Base Stations (BSs) follow a Poisson point process distribution and where Voronoi cells define the planar areas associated with them. This analysis considers only single user which can be located at an arbitrary location [54], [55]. Our analysis considers both single user and 2-user cases. Regarding the

2-user case, we use copula functions to model the dependency of C/I values of mobile users.

5.1 Overview of CoMP Techniques

Principally, cooperative scheduling / beamforming and joint processing are the two techniques which are employed in the downlink. In the first scheme, like the noncooperative case, there is only one BS transmitting data to the mobile user; however, the mobile user receives data from two or more BS's in the second scheme. It is expected that the CoMP transmission technique is beneficial only when the (C/I) of the cell edge users is low. However, not only cell-edge user spectral efficiency, but also the average cell spectral efficiency is also increased, by employing CoMP techniques according to the recent simulation results and analytical results [56].

5.1.1 Coordinated Scheduling/Beamforming

In order to control and coordinate the interference at the mobile terminal, the data is transmitted from one of the base stations, which are coordinated according to their channel state information (CSI) in coordinated scheduling/beamforming scheme. This coordination can take place in a centralized manner, where all the CSI information is collected at the central unit or in a distributed manner where CSI information is exchanged between the coordinated BS's. In order to mitigate the intercell interference, coordinated scheduling is attained by muting the base stations with critical interference towards the victim mobile user. This scheme only allows transmission from serving BS. Particularly, as can be seen in the figure, mobile terminal MS_1 get served by BS_1 . However, other users are selected by BS_2 in such a way that, it causes little interference to the MS_1 .

As described in [57] coordinated scheduling/beamforming can be represented in the scenario which is depicted in 8. As can be seen from the figure, in this scenario, there are two BS's which are denoted by BS_1 and BS_2 and two mobile terminals,

which are denoted as MS_1 and MS_2 . This two mobile terminals are served by BS_1 and BS_2 respectively. Y_1 and Y_2 , which are the received signals by MS_1 and MS_2 can be written as:

$$Y_1 = H_{11}W_1X_1 + H_{21}W_2X_2 + Z_1 \quad (55)$$

$$Y_2 = H_{12}W_1X_1 + H_{22}W_2X_2 + Z_2 \quad (56)$$

where H_{ij} is the channel vector between BS_i and MS_j , W_i is the precoding matrix at BS_i , X_i is the transmitted signal and Z_i is additive white gaussian noise (AWGN) at mobile terminal i .

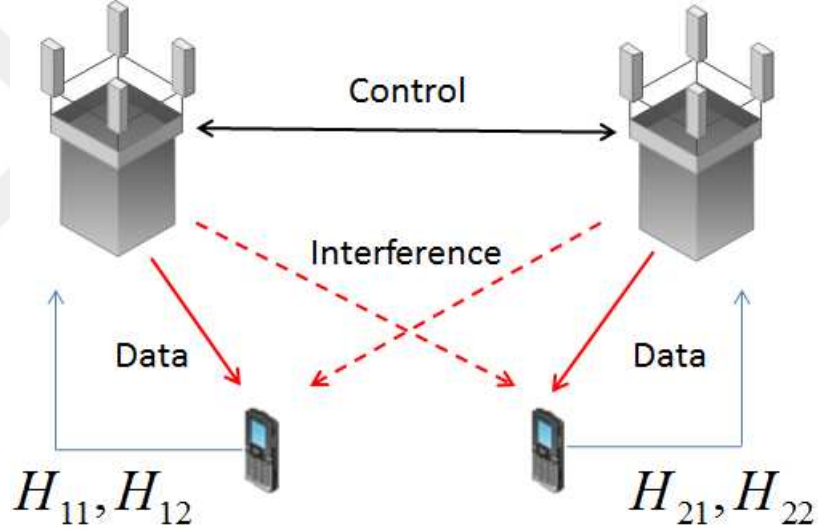


Figure 8: Coordinated scheduling/beamforming approach

5.1.2 Joint Processing

In order to improve the received signal quality and cancel the inter-cell interference, purposed data for a particular mobile terminal are jointly transmitted from multiple BS's in the category of joint processing (JP). The exploited paradigm of information theory is the following: when the antennas are uncorrelated, the number of independent communication channels is equal to the product of number of transmitting and receiving antennas. Even though, inherently, there is low correlation between different

cell site locations, this approximation gives an upper bound for the system capacity. Hence, there is high potential for achieving high capacity gain. Joint transmission and dynamic cell selection are the two different methods, which are being studied for JP. In joint transmission scheme data are transmitted from several BS, while in dynamic cell selection, a fast cell selection approach is used and only one of the cells transmits data at a time. Figure 9 and Figure 10 show a simplified scheme of joint transmission dynamic cell selection respectively. These two techniques increase the cell edge throughput and they are preceded to be the dominant application of CoMP.

In order to share the needed data among base stations, it is required a very fast link interconnecting the two base stations. In the joint transmission scheme, a complex signal processing operation is needed and it is considered that the transmission points correspond to different BS's. It is required that a cluster of base stations must jointly decide on the transmission scheme of a signal to the mobile terminal. Precoding must be applied among the coordinating cells in this context. In order to take the advantage of this technique several design considerations are needed to be studied. Serving set determination and the coherent versus non-coherent transmission approach are the two relevant techniques.

5.1.2.1 Serving Set Determination

There are different ways to determine the serving base stations (also called BS cluster). The two relevant approaches are summarized in the following. In the first way, the network can determine the base stations forming the CoMP cluster or the mobile user [58] or adaptively user and network can determine the coordinating base stations together [59]. In the second way, the cluster of base stations could be formed by a fixed or flexible set of serving base stations.

5.1.3 Coherent and Non-Coherent Transmission

Joint transmission can be classified as coherent and non-coherent transmission techniques, in terms of ways to transmit and combine the signals. In coherent transmission, channel state information from all the coordinating base stations must be obtained. In order to adjust the phase of the transmitted signal to the CSI, the receiver should be able to combine them at symbol level coherently. Combining also affects signaling support design for the downlink of CoMP. Besides the single-cell and multi-cell precoding gains, this also allows for an additional multi-cell array gain. There are two possible implementations of coherent transmission. One of the implementation approach could be a different phase correction applied to each cell. The other implementation approach could be a global precoding matrix based on all the CSI between coordinating base stations and the intended mobile user where each BS uses part of the global matrix as precoder.

For non-coherent transmission, the network does not know the the relationship of the channels among coordinating base stations. The received signals arriving at the mobile terminal can not be coherently combined, since cell-edge users calculate channel quality indicators and report them to their serving base stations, without providing channel phase information. Although the gains are not as big as coherent approach, this kind of transmission has been widely used for single-user MIMO to simplify the system overhead and backhaul capacity [57]. SFBC, Cyclic Delay Diversity (CDD) [59], or multi-user eigenmode transmission (MET) scheduling [60] could be given as an examples of implementations of this technique. Analogously to hybrid automatic repeat request (HARQ) combining, non-coherent combining can be performed at soft-bit level after demodulation.

For the dynamic cell selection approach, the transmission to the intended mobile user only takes place from one BS at a time. This BS must be selected from the set of cooperating base stations, which serve the same mobile user. The switching

operation can be performed at most on a subframe-by-subframe basis. This allows a dynamic change in the transmission point, which is transparent to the mobile user. The related radio resource management and packet scheduling are always performed by the serving BS. Since there is only one BS which serves the mobile user at a time, there is no need for the BS's to have a tight phase synchronization. As a result, it is an appropriate technique for small cell networks, since dynamic cell selection can be implemented with relaxed RF performance requirements.

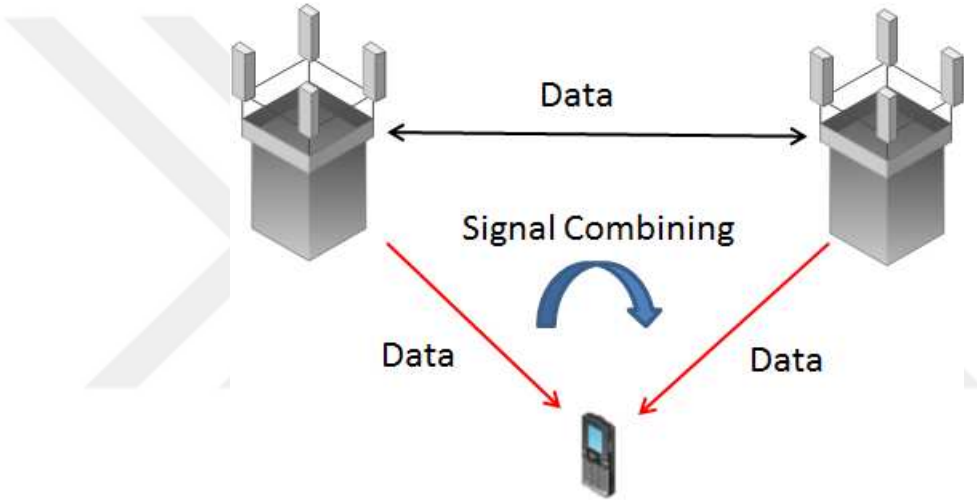


Figure 9: Joint transmission approach

5.2 Coverage Analysis of General, Cell Edge and Worst Case Users

In this section, by using our approach which is based on stochastic geometry, we derive the downlink coverage probability of cell edge user, worst case user and general user respectively considering joint processing is implemented at the base station. These three types of users are depicted in Fig. 11. According to this figure, general user corresponds to the general user which is located at an arbitrary location in the $\mathbb{R}^2$ plane independent of the BS distribution process. Cell edge user is located on the boundary between two neighbor cells whereas the worst case user is located on the

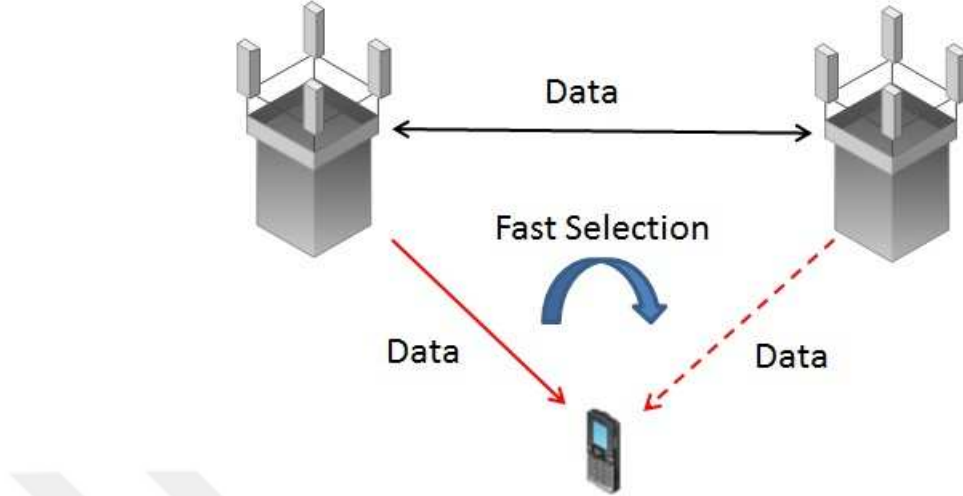


Figure 10: Dynamic cell selection approach

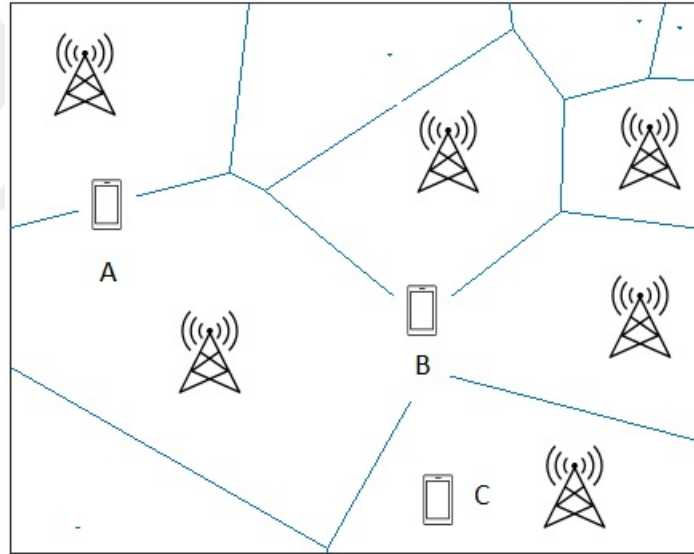


Figure 11: Three types of mobile users where A denotes cell edge user, B denotes worst case user and C denotes general user

vertex, where three cells meet. We consider to analyze the performance of joint processing as a CoMP transmission technique since it promises high potential gain [76]. Regarding the joint processing, we consider two different transmission strategies such as non-coherent joint transmission and dynamic cell selection.

5.2.1 Coverage Analysis of Cell Edge User

Let us first consider the non-coherent joint transmission scheme. Regarding this scheme, we assume that the cell edge user i is jointly served by two nearest base stations. The distances between these base stations and the cell edge user are given as r_{i1} and r_{i2} respectively. These two distances are equal to each other such as $r_{i1} = r_{i2} = r$.

We assume that all BSs transmit at a constant power level of unity. The downlink signals transmitted by the two coordinating base stations are attenuated by path loss with a coefficient of α , as such the average power of the signals are decreased by r , where r is the distance between the BS and the user. They further experience Rayleigh fading. Therefore, the received power of the cell edge user i is equal to $|h_{i1}r_{i1}^{-\alpha/2} + h_{i2}r_{i2}^{-\alpha/2}|^2$. Since $r_{i1} = r_{i2} = r$, this expression is equal to $|h_{i1} + h_{i2}|^2 r^{-\alpha}$, where both h_{i1} and h_{i2} follow standard complex Gaussian distribution.

Under these conditions, the SIR of cell edge user i is defined as:

$$\Psi_i = \frac{|h_{i1} + h_{i2}|^2 r^{-\alpha}}{I_i} \quad (57)$$

where I_i is the total interference experienced by the user, described as,

$$I_i = \sum_{k \neq 1,2} |h_{ik}|^2 r_{ik}^{-\alpha}. \quad (58)$$

Let us define $\Omega_i = |h_{i1} + h_{i2}|^2$. The conditional coverage probability P'_C conditioned on r can be then expressed as:

$$P'_C = \Pr(\Psi_i > \theta | r_{ij}) = \Pr(\Omega_i > \theta r^\alpha I_i | r) \quad (59)$$

where θ denotes the SIR threshold. Then coverage probability of cell edge user is obtained by averaging P'_C over the plane and expressed as:

$$\begin{aligned} P_C &= \mathbb{E}_r[P'_C] = \mathbb{E}_r[\Pr(\Omega_i > \theta r^\alpha I_i | r)] \\ &= \int_0^\infty \Pr(\Omega_i > \theta r^\alpha I_i | r) f(r) dr \end{aligned} \quad (60)$$

where $f(r)$ is the probability density function (pdf) of the distance. As shown in [78], this pdf can be expressed as:

$$f(r) = 4\pi\lambda_b^{3/2}r^2 \exp(-\pi\lambda_b r^2) \quad (61)$$

where λ_b is the density of PPP used to model the locations of base stations. After averaging P'_C over inter-cell interference, we have:

$$\Pr(\Omega_i > \theta r^\alpha I_i | r) = \mathbb{E}_{I_i} [\Pr(\Omega_i > \theta r^\alpha I_i | r, I_i)] \quad (62)$$

Since Ω_i is exponentially distributed with mean $1/2$, we can calculate the coverage probability which is conditioned on distance, and inter-cell interference as:

$$\Pr(\Omega_i > \theta r^\alpha I_i | r, I_i) = \exp\left(-\frac{1}{2}\theta r^\alpha I_i\right) \quad (63)$$

As a result, we can express the average of coverage probability conditioned on distance and I_i as:

$$\begin{aligned} \mathbb{E}_{I_i} [\Pr(\Omega_i > \theta r^\alpha I_i | r, I_i)] \\ = \mathbb{E}_{I_i} \left[\exp\left(-\frac{1}{2}\theta r^\alpha I_i\right) \right] \\ = \mathcal{L}_{I_i} \left[\frac{1}{2}\theta r^\alpha \right] \end{aligned} \quad (64)$$

where $\mathcal{L}_{I_i}[s]$ corresponds to the Laplace transform of I_i .

Following [13], the Laplace transform of I_i can be calculated as:

$$\mathcal{L}_{I_i} \left[\frac{1}{2}\theta r^\alpha \right] = \exp(-\lambda_b \pi r^2 \rho(\theta, \alpha)) \quad (65)$$

where $\rho(\theta, \alpha) = \theta^{\frac{2}{\alpha}} \int_{\theta^{-\frac{2}{\alpha}}}^{\infty} \frac{1}{2u^{\frac{\alpha}{2}+1}} du$. Substituting Eq.(65) into Eq.(60), we get the coverage probability:

$$\begin{aligned} P_C &= \int_0^\infty 4\pi\lambda_b^{\frac{3}{2}}r^2 \exp(-\pi r^2 \lambda_b \rho(\theta, \alpha)) \exp(-\pi r^2 \lambda_b) dr \\ &= \int_0^\infty 4\pi\lambda_b^{\frac{3}{2}}r^2 \exp(-\pi r^2 \lambda_b (\rho(\theta, \alpha) + 1)) dr \end{aligned} \quad (66)$$

Secondly, we consider to analyze the dynamic sell selection transmission scheme. Then Ω_i is expressed as $\Omega_i = \max(h_{i1}, h_{i2})$. Therefore, the cdf of Ω_i is given as:

$$F(\Omega_i) = (1 - \exp(-\Omega_i))^2 \quad (67)$$

$$= \sum_{k=0}^{k=2} \binom{2}{k} (-1)^k \exp(-k\Omega_i) \quad (68)$$

$$= 1 - 2 \exp(-\Omega_i) + \exp(-2\Omega_i) \quad (69)$$

In this case, the coverage probability conditioned on distance and I_i can be calculated as:

$$\Pr(\Omega_i > \theta r^\alpha I_i | r, I_i) = 1 - F(\theta r^\alpha I_i) = 2 \exp(-\theta r^\alpha I_i) - \exp(-2\theta r^\alpha I_i) \quad (70)$$

Therefore, the average of coverage probability can be expressed as:

$$\begin{aligned} & \mathbb{E}_{I_i} [\Pr(\Omega_i > \theta r^\alpha I_i | r, I_i)] \\ &= \mathbb{E}_{I_i} [2 \exp(-\theta r^\alpha I_i) - \exp(-2\theta r^\alpha I_i)] \\ &= 2\mathcal{L}_{I_i} [\theta r^\alpha] - \mathcal{L}_{I_i} [2\theta r^\alpha] \end{aligned} \quad (71)$$

After taking Laplace transforms of $\mathcal{L}_{I_i} [\theta r^\alpha]$ and $\mathcal{L}_{I_i} [2\theta r^\alpha]$ we have:

$$\mathcal{L}_{I_i} [\theta r^\alpha] = \exp(-\lambda_b \pi r^2 \rho_2(\theta, \alpha)) \quad (72)$$

$$\mathcal{L}_{I_i} [2\theta r^\alpha] = \exp(-\lambda_b \pi r^2 \rho_3(\theta, \alpha)) \quad (73)$$

where $\rho_2(\theta, \alpha) = \theta^{\frac{2}{\alpha}} \int_{\theta^{\frac{2}{\alpha}}}^{\infty} \frac{1}{u^{\frac{\alpha}{2}+1}} du$ and $\theta^{\frac{2}{\alpha}} \int_{(\frac{1}{2})\theta^{\frac{2}{\alpha}}}^{\infty} \frac{1}{u^{\frac{\alpha}{2}+1}} du$. Substituting Eq.(71) into Eq.(45) we obtain the average coverage probability of typical cell edge user, when dynamic cell selection is implemented at the base station:

$$P_C = \int_0^\infty 4\pi \lambda_b^{\frac{3}{2}} r^2 (2 \exp(-\pi r^2 \lambda_b (\rho_2(\theta, \alpha) + 1)) - \exp(-\pi r^2 \lambda_b (\rho_3(\theta, \alpha) + 1))) dr \quad (74)$$

Following the work presented in [79], the coverage probability of cell edge user, when there is no coordination between the neighboring base stations is implemented can be expressed as:

$$P_C = \int_0^\infty 4\pi \lambda_b^{\frac{3}{2}} r^2 \exp(-\pi r^2 \lambda_b (\rho_2(\theta, \alpha) + 1)) dr \quad (75)$$

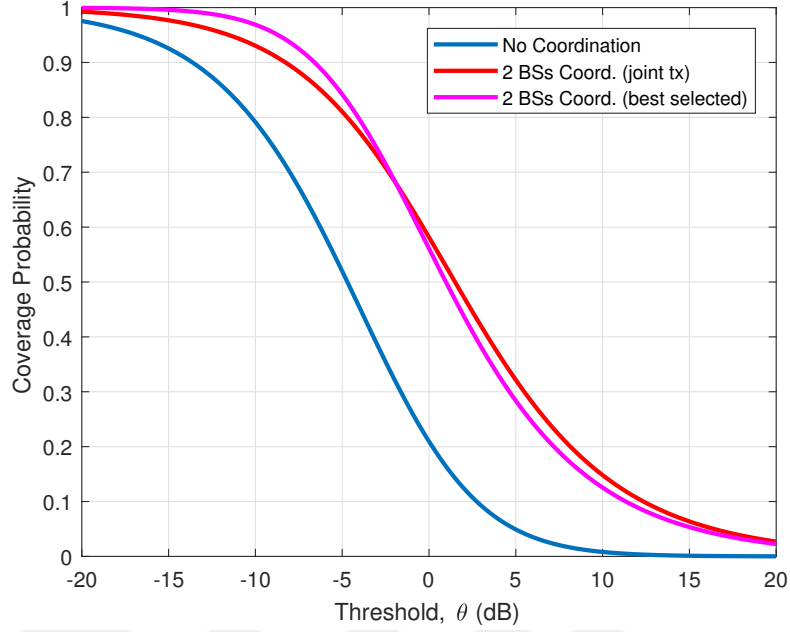


Figure 12: Downlink Coverage Probability of Cell Edge User

In order to evaluate the downlink coverage performance, we take the path loss coefficient $\alpha = 4$ and numerically evaluate the integrals given in Eq. (66) and Eq.(74) respectively. Regarding the case where there is no coordination implemented, we use the equation given in Eq.(75). In Figure 12, we plot the derived coverage probabilities of Eq. (75),Eq. (66) and Eq. (74) respectively. From the figures we observe that both of the methods, namely joint transmission and dynamic cell selection improve the downlink coverage performance of cell edge user with respect to the case where there is no coordination implemented. It can also be observable that, in terms of downlink coverage performance, there is a cross-over point between the dynamic cell selection and joint transmission methods at $\theta = -2.5$ dB. Below this cross-over threshold value, the dynamic cell selection provides better performance, whereas beyond this value joint transmission is the better method. Dynamic cell selection provides 7 dB power gain for 90% coverage and joint transmission provides 6.2 dB power gain for 50% coverage with respect to the case where there is no coordination between the

base stations.

5.2.2 Coverage Analysis of Worst Case User

Regarding the non coherent joint transmission scheme, since the worst case user is located on the intersection of three neighboring cells, we consider that this user is jointly served by these three nearest cells. If we make the same assumptions of typical cell edge user except the number of coordinating cells for the worst case user j , then the SIR of this user can be expressed as:

$$\Psi_j = \frac{|h_{j1} + h_{j2} + h_{j3}|^2 r^{-\alpha}}{I_j} \quad (76)$$

where I_j is the total interference experienced by the user, described as,

$$I_j = \sum_{k \neq 1,2,3} |h_{jk}|^2 r_{jk}^{-\alpha}. \quad (77)$$

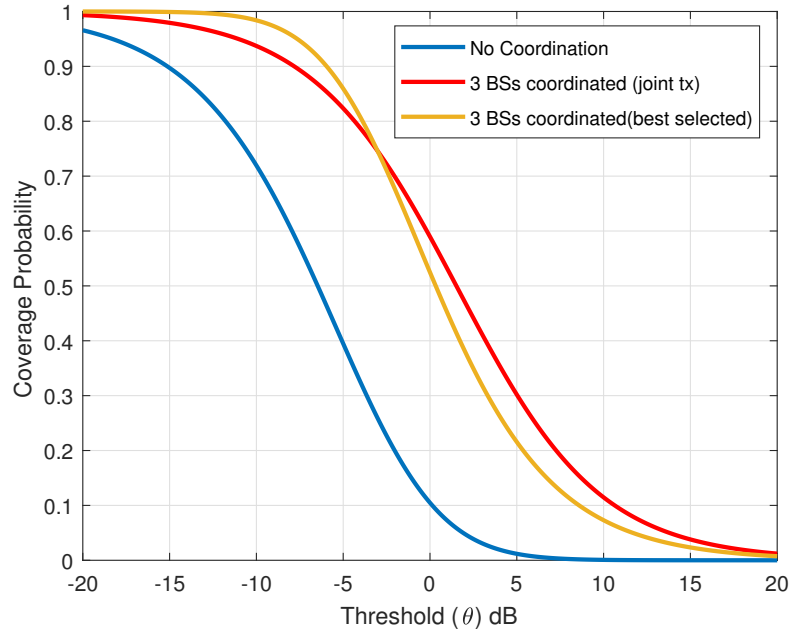


Figure 13: Downlink Coverage Probability of Worst Case User

Let us define $\Gamma_j = |h_{j1} + h_{j2} + h_{j3}|^2$. The average coverage probability P_C of worst case user can then be found by averaging the conditional coverage probability over

the plane:

$$\begin{aligned} P_C &= \mathbb{E}_r [\Pr(\Gamma_j > \theta r^\alpha I_j | r)] \\ &= \int_0^\infty \Pr(\Gamma_j > \theta r^\alpha I_j | r) f(r) dr \end{aligned} \quad (78)$$

where $f(r)$ is the pdf of the distance. Based on the Palm distribution theory, as shown in [77], this pdf can be expressed as:

$$f(r) = 2(\pi\lambda_b)^2 r^3 \exp(-\pi\lambda_b r^2) \quad (79)$$

Based on the fact that Γ_i is exponentially distributed with mean $1/3$, the average of coverage probability conditioned on distance and inter-cell interference is given as:

$$\begin{aligned} &\mathbb{E}_{I_j} [\Pr(\Gamma_j > \theta r^\alpha I_j | r, I_j)] \\ &= \mathbb{E}_{I_j} \left[\exp \left(-\frac{1}{3} \theta r^\alpha I_j \right) \right] \\ &= \mathcal{L}_{I_j} \left[\frac{1}{3} \theta r^\alpha \right] \end{aligned} \quad (80)$$

where $\mathcal{L}_{I_j}[s]$ corresponds to the Laplace transform of I_j . The Laplace transform of I_j can be calculated as:

$$\mathcal{L}_{I_j} \left[\frac{1}{3} \theta r^\alpha \right] = \exp(-\lambda_b \pi r^2 \phi_1(\theta, \alpha)) \quad (81)$$

where $\phi_1(\theta, \alpha) = \theta^{\frac{2}{\alpha}} \int_{\theta^{\frac{2}{\alpha}}}^\infty \frac{1}{3u^{\frac{\alpha}{2}+1}} du$. The coverage probability can be calculated by substituting Eq.(81) into Eq.(78):

$$P_C = \int_0^\infty 2(\pi\lambda_b)^2 r^3 \exp(-\pi r^2 \lambda_b (\phi_1(\theta, \alpha) + 1)) dr \quad (82)$$

Secondly, we analyze the coverage performance of dynamic sell selection transmission scheme for worst case user. In this case, Γ_j is expressed as $\Gamma_j = \max(h_{j1}, h_{j2}, h_{j3})$.

Therefore, the cdf of Γ_j is given as:

$$F(\Gamma_j) = (1 - \exp(-\Gamma_j))^3 \quad (83)$$

$$= \sum_{k=0}^{k=3} \binom{3}{k} (-1)^k \exp(-k\Omega_j) \quad (84)$$

$$= 1 - 3 \exp(-\Omega_j) + 3 \exp(-2\Omega_j) - \exp(-3\Omega_j) \quad (85)$$

Then, the conditional coverage probability can be expressed as:

$$\Pr(\Omega_i > \theta r^\alpha I_i | r, I_i) = 1 - F(\theta r^\alpha I_i) = 3 \exp(-\Omega_j) - 3 \exp(-2\Omega_j) + \exp(-3\Omega_j) \quad (86)$$

Therefore, the average of coverage probability can be expressed as:

$$\begin{aligned} & \mathbb{E}_{I_i} [\Pr(\Omega_i > \theta r^\alpha(I_i) | r, I_i)] \\ &= \mathbb{E}_{I_i} [3 \exp(-\Omega_j) - 3 \exp(-2\Omega_j) + \exp(-3\Omega_j)] \\ &= 3\mathcal{L}_{I_i}[\theta r^\alpha] - 3\mathcal{L}_{I_i}[2\theta r^\alpha] + \mathcal{L}_{I_i}[3\theta r^\alpha] \end{aligned} \quad (87)$$

As a result, the coverage probability of worst case user, when dynamic cell selection is implemented at the base station can be expressed as:

$$P_C = \int_0^\infty (2\pi\lambda_b)^2 r^3 (3 \exp(-\pi r^2 \lambda_b(\phi_2(\theta, \alpha) + 1)) - 3 \exp(-\pi r^2 \lambda_b(\phi_3(\theta, \alpha) + 1)) + \exp(-\pi r^2 \lambda_b(\phi_4(\theta, \alpha) + 1))) dr \quad (88)$$

where $\phi_2(\theta, \alpha) = \theta^{\frac{2}{\alpha}} \int_{\theta^{\frac{-2}{\alpha}}}^\infty \frac{1}{u^{\frac{\alpha}{2}+1}}$, $\phi_3(\theta, \alpha) = \theta^{\frac{2}{\alpha}} \int_{\theta^{\frac{-2}{\alpha}}}^\infty \frac{1}{(\frac{1}{2})u^{\frac{\alpha}{2}+1}}$ and $\phi_4(\theta, \alpha) = \theta^{\frac{2}{\alpha}} \int_{\theta^{\frac{-2}{\alpha}}}^\infty \frac{1}{(\frac{1}{3})u^{\frac{\alpha}{2}+1}}$.

Following the work presented in [76], the coverage probability of worst case user, when there is no coordination between the neighboring base stations is implemented can be expressed as:

$$P_C = \int_0^\infty 2(\pi\lambda_b)^2 r^3 \exp(-\pi r^2 \lambda_b(\phi_2(\theta, \alpha) + 1)) dr \quad (89)$$

In order to evaluate the downlink coverage performance, we take the path loss coefficient $\alpha = 4$ and numerically evaluate the integrals given in Eq. (82) and Eq.(88) respectively. Regarding the case where there is no coordination implemented, we use the equation given in Eq.(89). In Figure 13, we plot the derived coverage probabilities of Eq.(89),Eq. (82) and Eq. (88) respectively. Similar to cell edge user, from the figures we observe that both of the methods, namely joint transmission and dynamic cell selection improve the downlink coverage performance of worst case user with respect to the case where there is no coordination implemented. It can be also deduced that, in terms of downlink coverage performance, there is a cross-over point between

the dynamic cell selection and joint transmission methods at $\theta = -3.1$ dB. Below this cross-over threshold value, the dynamic cell selection provides better performance, whereas beyond this value joint transmission is the better method. Dynamic cell selection provides 9 dB power gain for 90% coverage and joint transmission provides 8 dB power gain for 50% coverage with respect to the case where there is no coordination between the base stations. These power gains are approximately 2 dB greater than the gains achieved for cell edge user.

5.2.3 Coverage Analysis of General User

Let us consider the general user k , which is located at an arbitrary location. First, we consider the non coherent joint transmission scheme, in which this user is assumed to be served by n nearest neighboring base stations. In this case, the SIR of this user k can be expressed as:

$$\Psi_k = \frac{|h_{k1}r_{k1}^{-\alpha/2} + h_{k2}r_{k2}^{-\alpha/2} + \dots + h_{kn}r_{kn}^{-\alpha/2}|^2}{I_k} \quad (90)$$

where I_k is the total interference experienced by the user, described as,

$$I_k = \sum_{l \neq 1, 2, \dots, n} |h_{kl}|^2 r_{kl}^{-\alpha}. \quad (91)$$

Let us define $X_k = |h_{k1}r_{k1}^{-\alpha/2} + h_{k2}r_{k2}^{-\alpha/2} + \dots + h_{kn}r_{kn}^{-\alpha/2}|^2$. The average coverage probability P_C of general user can then be found by averaging the conditional coverage probability over the plane:

$$\begin{aligned} P_C &= \mathbb{E}_{\mathbf{r}_k} [\Pr(X_k > \theta I_k | \mathbf{r}_k)] \\ &= \int \dots \int_0^\infty \Pr(X_k > \theta I_k | \mathbf{r}_k) f(\mathbf{r}_k) d\mathbf{r}_k \end{aligned} \quad (92)$$

where $\mathbf{r}_k = [r_{k1}, r_{k2}, \dots, r_{kn}]$ is the random distance vector and $f(\mathbf{r}_k)$ is the joint pdf of the distance vector. Following [80], this pdf can be expressed as:

$$f(\mathbf{r}_k) = e^{-\lambda_b \pi r_{kn}^2} (2\lambda_b \pi)^n r_{k1} r_{k2} \dots r_{kn} \quad (93)$$

Since X_k is exponentially distributed with mean $\mu = \frac{1}{r_{k1}^{-\alpha} + r_{k2}^{-\alpha} + \dots + r_{kn}^{-\alpha}}$, the average of coverage probability conditioned on distance and inter-cell interference is given as:

$$\begin{aligned} \mathbb{E}_{I_k} [\Pr(X_k > \theta I_k | \mathbf{r}_k, I_k)] \\ = \mathbb{E}_{I_k} [\exp(-\mu \theta I_k)] \\ = \mathcal{L}_{I_k} [\mu \theta] \end{aligned} \quad (94)$$

where $\mathcal{L}_{I_j}[s]$ corresponds to the Laplace transform of I_j . The Laplace transform of I_j can be calculated as:

$$\mathcal{L}_{I_k} [\mu \theta] = \exp(-2\pi \lambda_b \tau_1(\theta, \alpha)) \quad (95)$$

where $\tau_1(\theta, \alpha) = \int_{r_{kn}}^{\infty} \frac{\theta}{\frac{v^\alpha}{\mu} + \theta} v dv$. The coverage probability can be calculated by substituting Eq.(95) into Eq.(92):

$$P_C = \int \dots \int_0^{\infty} (2\pi \lambda_b)^n \exp(-\pi \lambda_b (2\tau_1(\theta, \alpha) + r_{kn}^2)) dr_{k1} dr_{k2} \dots dr_{kn} \quad (96)$$

Secondly, we analyze the coverage performance of dynamic sell selection transmission scheme for general user where n nearest base stations to the general user are coordinated. In this case, X_k is expressed as:

$X_k = \max(|h_{k1} r_{k1}^{-\alpha/2}|^2, |h_{k2} r_{k2}^{-\alpha/2}|^2, \dots, |h_{kn} r_{kn}^{-\alpha/2}|^2)$. Therefore, the cdf of X_k is given as:

$$F(X_k) = \prod_{i=1}^n (1 - e^{-r_{ki}^\alpha x_k}) \quad (97)$$

Then, the conditional coverage probability can be expressed as:

$$\Pr(X_k > \theta I_k | \mathbf{r}_k, I_k) = 1 - F(\theta I_k) = 1 - \prod_{i=1}^n (1 - e^{-r_{ki}^\alpha \theta I_k}) \quad (98)$$

For instance, if we take the number of coordinating base stations n as 2, then this probability can be expressed as:

$$\Pr(X_k > \theta I_k | \mathbf{r}_k, I_k) = 1 - ((1 - e^{-r_{k1}^\alpha \theta I_k})(1 - e^{-r_{k2}^\alpha \theta I_k})) \quad (99)$$

$$= e^{-r_{k2}^\alpha \theta I_k} - e^{-(r_{k1}^\alpha + r_{k2}^\alpha) \theta I_k} + e^{-r_{k1}^\alpha \theta I_k} \quad (100)$$

After taking the average of this coverage probability conditioned on distance vector and inter-cell interference, we have:

$$\begin{aligned}
& \mathbb{E}_{I_k} [\Pr (X_k > \theta I_k | \mathbf{r}_k, I_k)] \\
&= \mathbb{E}_{I_k} [e^{-r_{k2}^\alpha \theta I_k}] - \mathbb{E}_{I_k} [e^{-(r_{k1}^\alpha + r_{k2}^\alpha) \theta I_k}] + \mathbb{E}_{I_k} [e^{-r_{k1}^\alpha \theta I_k}] \\
&= \mathcal{L}_{I_k} [r_{k2}^\alpha \theta] - \mathcal{L}_{I_k} [(r_{k1}^\alpha + r_{k2}^\alpha) \theta] + \mathcal{L}_{I_k} [r_{k1}^\alpha \theta]
\end{aligned} \tag{101}$$

In this case, the coverage probability of general user can be expressed as:

$$\begin{aligned}
P_C &= \int_0^\infty \int_0^\infty (2\pi\lambda_b)^2 \exp(-\pi\lambda_b(2v_1(\theta, \alpha) + r_{k2}^2)) dr_{k1} dr_{k2} \\
&\quad - \int_0^\infty \int_0^\infty (2\pi\lambda_b)^2 \exp(-\pi\lambda_b(2v_2(\theta, \alpha) + r_{k2}^2)) dr_{k1} dr_{k2} + \\
&\quad \int_0^\infty \int_0^\infty (2\pi\lambda_b)^2 \exp(-\pi\lambda_b(2v_3(\theta, \alpha) + r_{k2}^2)) dr_{k1} dr_{k2}
\end{aligned} \tag{102}$$

where $v_i(\theta, \alpha) = \int_{r_{k2}}^\infty \frac{\theta}{\frac{v^\alpha}{\mu_i} + \theta} v dv$ $i = 1, 2, 3$, $\mu_1 = r_{k2}^\alpha$, $\mu_2 = r_{k1}^\alpha + r_{k2}^\alpha$ and $\mu_3 = r_{k1}^\alpha$.

Similar to the case where $n = 2$, the downlink coverage probability of general user for the cases where $n = 3$ and $n = 4$ can be found respectively as follows:

$$P_C = \sum_{j=4}^{10} \int_0^\infty \int_0^\infty \int_0^\infty (2\pi\lambda_b)^2 \exp(-\pi\lambda_b(2v_j(\theta, \alpha) + r_{k2}^2)) dr_{k1} dr_{k2} dr_{k3} \tag{103}$$

where $v_i(\theta, \alpha) = \int_{r_{k2}}^\infty \frac{\theta}{\frac{v^\alpha}{\mu_i} + \theta} v dv$ $i = 4, \dots, 10$, $\mu_4 = r_{k3}^\alpha$, $\mu_5 = r_{k1}^\alpha$, $\mu_6 = r_{k1}^\alpha + r_{k3}^\alpha$, $\mu_7 = r_{k2}^\alpha$, $\mu_8 = r_{k2}^\alpha + r_{k3}^\alpha$, $\mu_9 = r_{k1}^\alpha + r_{k2}^\alpha$ and $\mu_{10} = r_{k1}^\alpha + r_{k2}^\alpha + r_{k3}^\alpha$

$$P_C = \sum_{j=11}^{25} \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty (2\pi\lambda_b)^2 \exp(-\pi\lambda_b(2v_j(\theta, \alpha) + r_{k4}^2)) dr_{k1} dr_{k2} dr_{k3} dr_{k4} \tag{104}$$

where $v_i(\theta, \alpha) = \int_{r_{k4}}^\infty \frac{\theta}{\frac{v^\alpha}{\mu_i} + \theta} v dv$ $i = 11, \dots, 25$, $\mu_{11} = r_{k3}^\alpha$, $\mu_{12} = r_{k1}^\alpha$, $\mu_{13} = r_{k1}^\alpha + r_{k3}^\alpha$, $\mu_{14} = r_{k2}^\alpha$, $\mu_{15} = r_{k2}^\alpha + r_{k3}^\alpha$, $\mu_{16} = r_{k1}^\alpha + r_{k2}^\alpha$, $\mu_{17} = r_{k1}^\alpha + r_{k2}^\alpha + r_{k3}^\alpha$, $\mu_{18} = r_{k4}^\alpha$, $\mu_{19} = r_{k3}^\alpha + r_{k4}^\alpha$, $\mu_{20} = r_{k1}^\alpha + r_{k4}^\alpha$, $\mu_{21} = r_{k1}^\alpha + r_{k3}^\alpha + r_{k4}^\alpha$, $\mu_{22} = r_{k2}^\alpha + r_{k4}^\alpha$, $\mu_{23} = r_{k2}^\alpha + r_{k3}^\alpha + r_{k4}^\alpha$, $\mu_{24} = r_{k1}^\alpha + r_{k2}^\alpha + r_{k4}^\alpha$, $\mu_{25} = r_{k1}^\alpha + r_{k2}^\alpha + r_{k3}^\alpha + r_{k4}^\alpha$.

In order to evaluate the downlink coverage performance, we take the path loss coefficient $\alpha = 4$, and consider the number of coordinating base stations as $n = 2, 3, 4$. We numerically evaluate the integral given in Eq. (96) by taking $n = 2, 3$ and 4

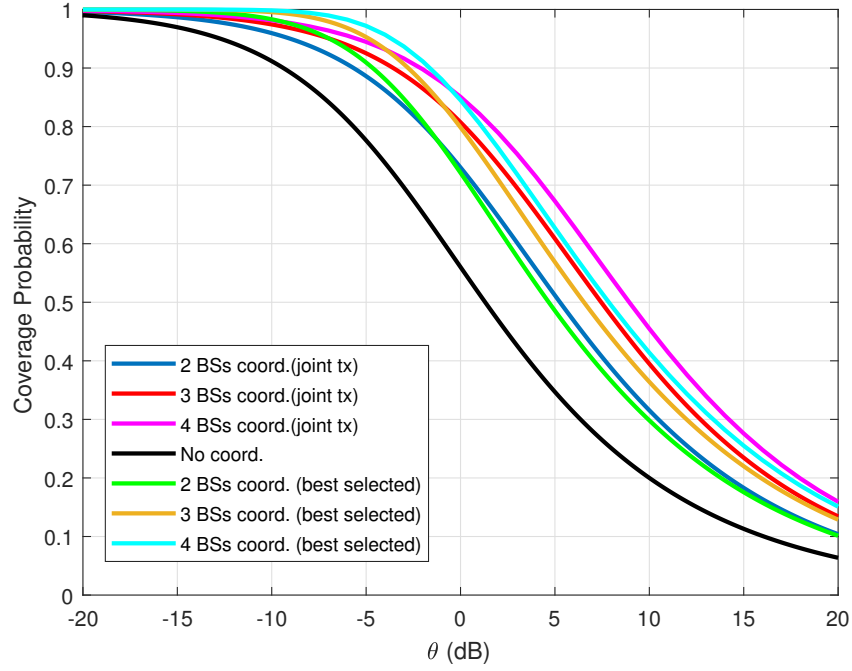


Figure 14: Downlink Coverage Probability of General User

respectively to obtain the downlink coverage performance of joint transmission. We also numerically evaluate Eq.(102), Eq.(103) and Eq.(104) to obtain the downlink coverage performance of the dynamic cell selection where $n = 2, 3$ and 4 respectively. Regarding the case where there is no coordination implemented, we use the equation given in Eq.(??). Similar to cell edge user and worst case user, from the Fig.(14), we observe that both of the methods, namely joint transmission and dynamic cell selection improve the downlink coverage performance of worst case user with respect to the case where there is no coordination implemented. It can be also observed that, increasing the number of coordinating base stations, improves the downlink coverage performance. In terms of downlink coverage performance, there are cross-over points between the dynamic cell selection and joint transmission methods at $\theta = -1.4$ dB, $\theta = -0.8$ dB and $\theta = -0.5$ dB. Below these cross-over threshold values, the dynamic cell selection provides better performance, whereas beyond these values

joint transmission is the better method. Dynamic cell selection provides 4.7 dB, 5.8 dB and 7.9 dB power gains for $n = 2, 3$ and 4 respectively regarding 90% coverage and joint transmission provides 3.9 dB, 6.1 dB and 7.7 dB power gains for $n = 2, 3$ and 4 respectively regarding 50% coverage with respect to the case where there is no coordination between the base stations.

5.3 Performance Analysis of Downlink 2-User CoMP System

In this section, we evaluate average spectral density of two users, under different scenarios, by using our CoMP transmission approach which is explained in detail in the previous section and the copula function which is introduced in Chapter 3 to model the (C/I) dependencies of two users. For multiuser CoMP transmission scenarios, when the number of users are increased, the correlation is also increased, and it is very hard to model the dependency of (C/I) of users, since all users are connected to more than one base station. So, in this section, without loss of generality we just consider two users, which are served by the two closest base stations.

In the first scenario which is depicted in Figure 15, two users are connected to the same base stations. In other words, closest and the second closest base stations of two users are the same. In the second scenario, which is depicted in Figure 16, the closest base stations of two users are the same but the second closest base stations are different. In the third scenario which is depicted in Figure 17, the closest base stations of user 1 and second closest base station of user 2 are the same and, the closest base station of user 2 and second closest base station of user 1 are the same. In the fourth scenario which is depicted in Figure 18, the closest base station of user 1 and the second closest base station of user 2 are the same. In the fifth scenario which is depicted in Figure 19, second closest base station of user 1 and second closest base station of user 2 are the same. In the last scenario, which is depicted in Figure 20 both the first and second closest base stations of user 1 and user 2 are different. For all

scenarios, we assume that the two base stations are connected to a central unit over a perfect unlimited backhaul network. In all figures, the dashed line with number 1 corresponds to the closest link of user 1 and user 2, and the dashed line with number 2 corresponds to the second closest link of user 1 and user 2.

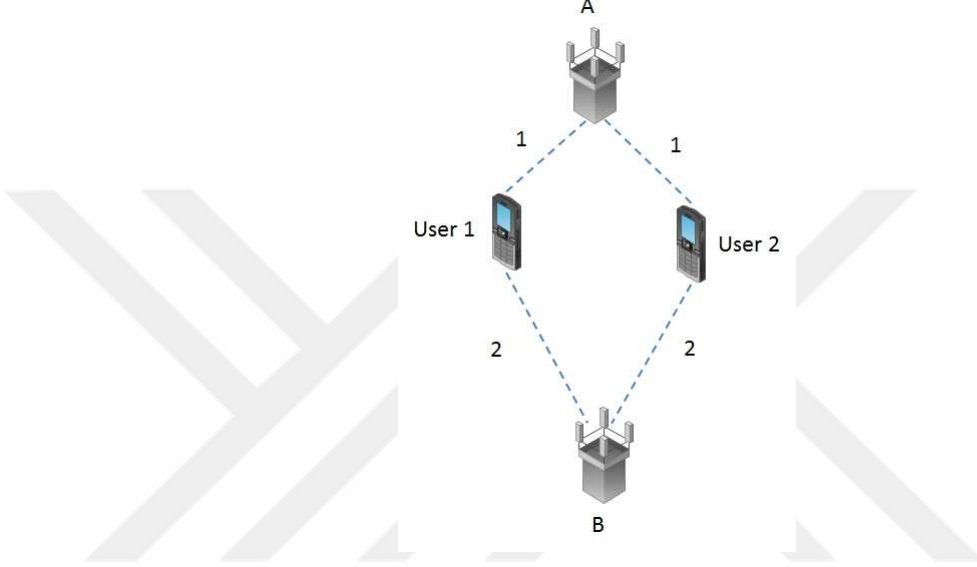


Figure 15: Scenario 1

In order to evaluate the average spectral density of 2 users, we should first, calculate the probabilities of the four different situations. Here we denote these situations with s_i , $i = 1, 2, 3, 4$. We also denote $P((C/I)_{1B}) = X$ and $P((C/I)_{2B}) = Y$ for Scenario 1, $P((C/I)_{1B}) = X$ and $P((C/I)_{2C}) = Y$ for Scenario 2, $P((C/I)_{1B}) = X$ and $P((C/I)_{2A}) = Y$ for Scenario 3, $P((C/I)_{1B}) = X$ and $P((C/I)_{2A}) = Y$ for Scenario 4, $P((C/I)_{1A}) = X$ and $P((C/I)_{2A}) = Y$ for Scenario 5, $P((C/I)_{1B}) = X$ and $P((C/I)_{2D}) = Y$ for Scenario 6. The probability of each situation can be calculated

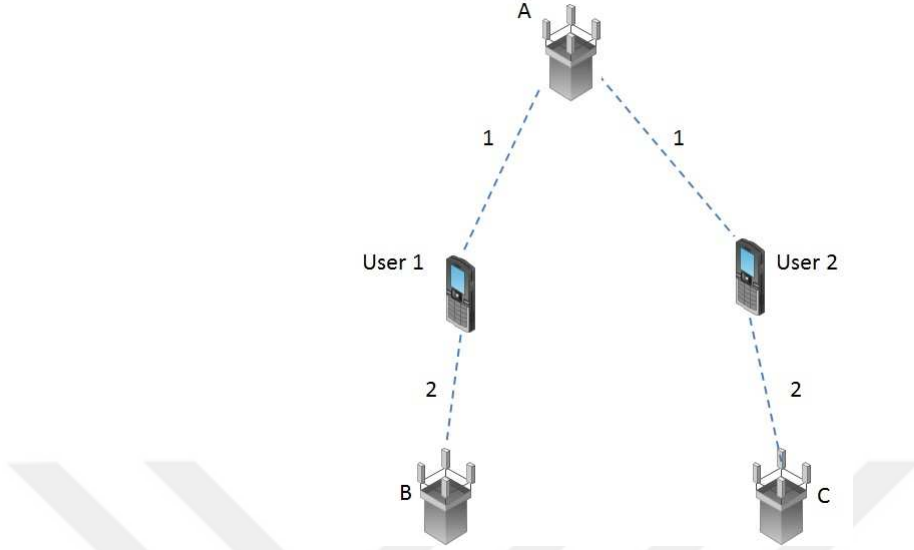


Figure 16: Scenario 2

as:

$$P(s_1) = P(X \geq \psi, Y \geq \psi) \quad (105)$$

$$= F_{XY}(\infty, \infty) - F_{XY}(F_X(\psi), \infty) - F_{XY}(\infty, F_Y(\psi)) + F_{XY}(F_X(\psi), F_Y(\psi)) \quad (106)$$

$$= 1 - F_X(\psi) - F_Y(\psi) + F_{XY}(F_X(\psi), F_Y(\psi)) \quad (107)$$

$$P(s_2) = P(X \geq \psi, Y < \psi) \quad (108)$$

$$= F_{XY}(\infty, F_Y(\psi)) - F_{XY}(F_X(\psi), F_Y(\psi)) - F_{XY}(\infty, 0) + F_{XY}(F_X(\psi), 0) \quad (109)$$

$$= F_Y(\psi) - F_{XY}(F_X(\psi), F_Y(\psi)) - F_Y(0) + F_{XY}(F_X(\psi), 0) \quad (110)$$

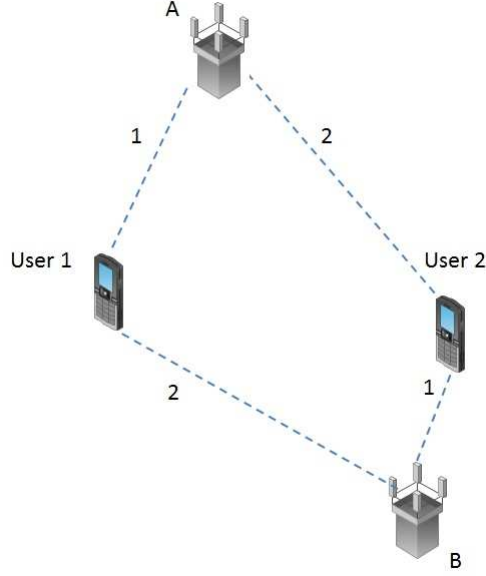


Figure 17: Scenario 3

$$P(s_3) = P(X < \psi, Y \geq \psi) \quad (111)$$

$$= F_{XY}(F_X(\psi), \infty) - F_{XY}(0, \infty) - F_{XY}(F_X(\psi), F_Y(\psi)) + F_{XY}(0, F_Y(\psi)) \quad (112)$$

$$= F_X(\psi) - F_X(0) - F_{XY}(F_X(\psi), F_Y(\psi)) + F_{XY}(0, F_Y(\psi)) \quad (113)$$

$$P(s_4) = P(X < \psi, Y < \psi) \quad (114)$$

$$= F_{XY}(F_X(\psi), F_Y(\psi)) - F_{XY}(0, F_Y(\psi)) - F_{XY}(F_X(\psi), 0) + F_{XY}(0, 0) \quad (115)$$

where, X and Y are identically distributed, $F_X(\cdot)$ and $F_Y(\cdot)$ are the marginal CDF's of random variables X and Y respectively, and $F_{XY}(\cdot, \cdot)$, is the joint CDF of X and Y . We use copula function method to evaluate the joint CDF $F_{XY}(\cdot, \cdot)$. By using copula function method which is introduced in Chapter 3, we can express $F_{XY}(x, y)$ as

$$F_{XY}(x, y) = C_\rho^C(F_X(x), F_Y(y)) \quad (116)$$

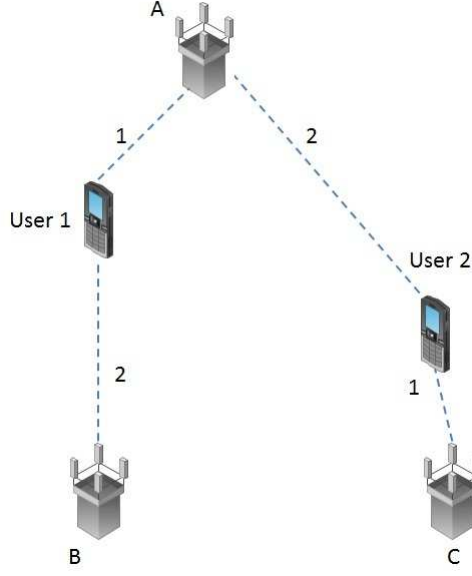


Figure 18: Scenario 4

where ρ is the correlation parameter, which can be calculated as the maximum likelihood approach, which is explained in detail, in Chapter 3.

5.3.1 Scenario 1

In the first scenario, the decision algorithm is given below.

- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2B} \geq \psi) \implies$ select the user with maximum $(C/I)_{CoMP}$
- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2B} < \psi) \implies (C/I)_{CoMP_1} \geq (C/I)_{2A}$ select user 1,
else $(C/I)_{CoMP_1} < (C/I)_{2A}$ select user 2
- $((C/I)_{1B} < \psi) \& ((C/I)_{2B} \geq \psi) \implies (C/I)_{CoMP_2} \geq (C/I)_{1A}$ select user 2,
else $(C/I)_{CoMP_2} < (C/I)_{1A}$ select user 1
- $((C/I)_{1B} < \psi) \& ((C/I)_{2B} < \psi) \implies (C/I)_{1A} \geq (C/I)_{2A}$ select user 1,
else $(C/I)_{1A} < (C/I)_{2A}$ select user 2

In this algorithm, $(C/I)_{CoMP_1} = \frac{|h_{1A}r_{1A}^{-\alpha/2} + h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 1, $(C/I)_{CoMP_2} = \frac{|h_{2A}r_{2A}^{-\alpha/2} + h_{2B}r_{2B}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 2, $(C/I)_{1A} = \frac{|h_{1A}r_{1A}^{-\alpha/2}|^2}{I_r}$,

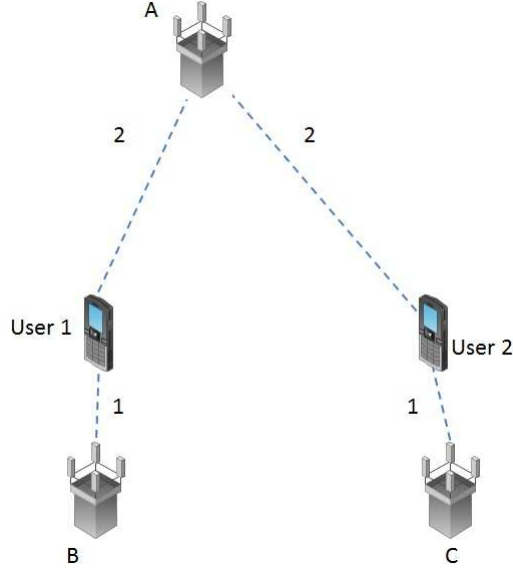


Figure 19: Scenario 5

is the (C/I) of user 1 from base station A, $(C/I)_{1B} = \frac{|h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 1 from base station B, $(C/I)_{2A} = \frac{|h_{2A}r_{2A}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station A and $(C/I)_{2B} = \frac{|h_{2B}r_{2B}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station B respectively.

The average spectral densities of each situations can be evaluated as:

$$\Omega_{s_1}|s_1 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{CoMP_2}))] \quad (117)$$

$$\Omega_{s_2}|s_2 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{2A}))] \quad (118)$$

$$\Omega_{s_3}|s_3 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_2}, (C/I)_{1A}))] \quad (119)$$

$$\Omega_{s_4}|s_4 = \mathbb{E}[\ln(1 + \max((C/I)_{1A}, (C/I)_{2A}))] \quad (120)$$

In order to evaluate these average spectral densities, we should first find the CDF of $\ln(1 + \max(s_{i1}, s_{i2}))$ for each of the average spectral densities $\Omega_{s_i}|s_i$ corresponding to each situation. If we denote the random variable $\max(s_{i1}, s_{i2})$ as Θ_{s_i} . The CDF of $\ln(1 + \Theta_{s_i}) \triangleq \Psi_i$ can be expressed as:

$$F_{\Psi_i} = F_{\Theta_{s_i}}(e^{\Psi_i} - 1) \quad (121)$$

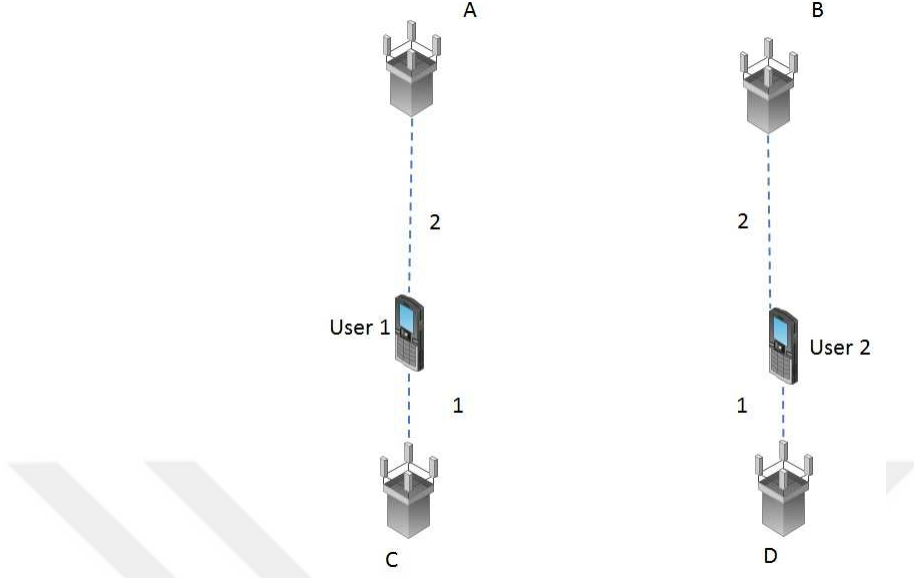


Figure 20: Scenario 6

For each situation, the CDF of Θ_{s_i} can be evaluated as:

$$F_{\Theta_{s_1}}(\theta_{s_1}) = P(\max((C/I)_{CoMP_1}, (C/I)_{CoMP_2}) < \theta_{s_1}) = C_\rho^C(F_{(C/I)_{CoMP_1}}(\theta_{s_1}), F_{(C/I)_{CoMP_2}}(\theta_{s_1}))$$

$$F_{\Theta_{s_2}}(\theta_{s_2}) = P(\max((C/I)_{CoMP_1}, (C/I)_{2A}) < \theta_{s_2}) = C_\rho^C(F_{(C/I)_{CoMP_1}}(\theta_{s_2}), F_{(C/I)_{2A}}(\theta_{s_2}))$$

$$F_{\Theta_{s_3}}(\theta_{s_3}) = P(\max((C/I)_{CoMP_2}, (C/I)_{1A}) < \theta_{s_3}) = C_\rho^C(F_{(C/I)_{CoMP_2}}(\theta_{s_3}), F_{(C/I)_{1A}}(\theta_{s_3}))$$

$$F_{\Theta_{s_4}}(\theta_{s_4}) = P(\max((C/I)_{1A}, (C/I)_{2A}) < \theta_{s_4}) = C_\rho^C(F_{(C/I)_{1A}}(\theta_{s_4}), F_{(C/I)_{2A}}(\theta_{s_4}))$$

The CDF's of $(C/I)_{1A}$ and $(C/I)_{2A}$ are identical and we derived these CDF in Chapter 2 as 34. $(C/I)_{CoMP_1}$ and $(C/I)_{CoMP_2}$ are also identically distributed. We first find the CCDF of $(C/I)_{CoMP_i}$, and then we can easily find the CDF which is equal to 1-CCDF of $(C/I)_{CoMP_i}$. The CCDF of (C/I) of a user which is connected to two nearest base stations when the path loss coefficient $\alpha = 4$ can be expressed as:

$$P(U_i > u_i) = \int_0^\infty (2\lambda\pi)^2 r_1 \int_{r_1}^\infty r_2 e^{-\lambda\pi r_2^2} \mathcal{L}_{I_r}(\mu u_i) dr_1 dr_2 \quad (122)$$

where,

$$\mathcal{L}_{I_r}(\mu u_i) = \exp\left(-\pi\lambda\left(\pi/2 - \arctan\left(\frac{r_2^2}{\sqrt{(u_i)\mu}}\right)\right)\sqrt{u_i\mu}\right)$$

and

$$\mu = \frac{1}{r_1^{-4} + r_2^{-4}}$$

Once we get the CDF of U_i , by using copula functions we can obtain the CDF of Θ_{s_i} .

After obtaining the CDF of Θ_{s_i} , we calculate the CDF of Ψ_i . In order to obtain the pdf of Ψ_i we should take derivative of the CDF of Ψ_i .

Thus the pdf of Ψ_i $f_{\Psi_i}(\psi_i)$ is

$$f_{\Psi_i}(\psi_i) = \frac{\partial}{\partial \psi_i} F_{\Psi_i}(\psi_i) = e^{\psi_i} f_{\Theta_{s_i}}(e^{\psi_i} - 1) \quad (123)$$

In order to calculate the average spectral efficiency, the mean of random variable Ψ_i should be taken. So, we get the average spectral efficiency as

$$\Omega_{s_i}|s_i = \mathbb{E}[\ln(1 + \max(s_{i1}, s_{i2}))] = \mathbb{E}[\Psi_i] = \int_0^\infty \psi_i f_{\Psi_i}(\Psi_i) d\psi_i \quad (124)$$

As a result, we express the overall average spectral density of 2 users, whose closest and second closest base stations are common as,

$$\Omega_{av} = \sum_{i=1}^4 (\Omega_{s_i}|s_i) P(s_i) \quad (125)$$

5.3.2 Scenario 2

In the second scenario, the decision algorithm is given below.

- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2C} \geq \psi) \implies$ select the user with maximum $(C/I)_{CoMP}$
- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2C} < \psi) \implies (C/I)_{CoMP_1} \geq (C/I)_{2A}$ select user 1,
else $(C/I)_{CoMP_1} < (C/I)_{2A}$ select user 2
- $((C/I)_{1B} < \psi) \& ((C/I)_{2C} \geq \psi) \implies (C/I)_{CoMP_2} \geq (C/I)_{1A}$ select user 2,
else $(C/I)_{CoMP_2} < (C/I)_{1A}$ select user 1
- $((C/I)_{1B} < \psi) \& ((C/I)_{2C} < \psi) \implies (C/I)_{1A} \geq (C/I)_{2A}$ select user 1,
else $(C/I)_{1A} < (C/I)_{2A}$ select user 2

In this algorithm, $(C/I)_{CoMP_1} = \frac{|h_{1A}r_{1A}^{-\alpha/2} + h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 1, $(C/I)_{CoMP_2} = \frac{|h_{2A}r_{2A}^{-\alpha/2} + h_{2C}r_{2C}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 2, $(C/I)_{1A} = \frac{|h_{1A}r_{1A}^{-\alpha/2}|^2}{I_r}$, is the (C/I) of user 1 from base station A, $(C/I)_{1B} = \frac{|h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 1 from base station B, $(C/I)_{2A} = \frac{|h_{2A}r_{2A}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station A and $(C/I)_{2C} = \frac{|h_{2C}r_{2C}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station C.

The overall average spectral density of 2 users can be expressed for the second scenario as,

$$\Omega_{av} = \sum_{i=1}^4 (\Omega_{s_i} | s_i) P(s_i) \quad (126)$$

where

$$\Omega_{s_1} | s_1 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{CoMP_2}))] \quad (127)$$

$$\Omega_{s_2} | s_2 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{2A}))] \quad (128)$$

$$\Omega_{s_3} | s_3 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_2}, (C/I)_{1A}))] \quad (129)$$

$$\Omega_{s_4} | s_4 = \mathbb{E}[\ln(1 + \max((C/I)_{1A}, (C/I)_{2A}))]. \quad (130)$$

5.3.3 Scenario 3

In the third scenario, the decision algorithm is given below.

- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2A} \geq \psi) \implies$ select the user with maximum $(C/I)_{CoMP}$
- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2A} < \psi) \implies (C/I)_{CoMP_1} \geq (C/I)_{2B}$ select user 1,
else $(C/I)_{CoMP_1} < (C/I)_{2A}$ select user 2
- $((C/I)_{1B} < \psi) \& ((C/I)_{2A} \geq \psi) \implies (C/I)_{CoMP_2} \geq (C/I)_{1A}$ select user 2,
else $(C/I)_{CoMP_2} < (C/I)_{1A}$ select user 1
- $((C/I)_{1B} < \psi) \& ((C/I)_{2A} < \psi) \implies (C/I)_{1A} \geq (C/I)_{2B}$ select user 1,
else $(C/I)_{1A} < (C/I)_{2B}$ select user 2

In this algorithm, $(C/I)_{CoMP_1} = \frac{|h_{1A}r_{1A}^{-\alpha/2} + h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 1, $(C/I)_{CoMP_2} = \frac{|h_{2B}r_{2B}^{-\alpha/2} + h_{2A}r_{2A}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 2, $(C/I)_{1A} = \frac{|h_{1A}r_{1A}^{-\alpha/2}|^2}{I_r}$,

is the (C/I) of user 1 from base station A, $(C/I)_{1B} = \frac{|h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 1 from base station B, $(C/I)_{2B} = \frac{|h_{2B}r_{2B}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station B and $(C/I)_{2A} = \frac{|h_{2A}r_{2A}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station A.

The overall average spectral density of 2 users can be expressed for the third scenario as,

$$\Omega_{av} = \sum_{i=1}^4 (\Omega_{s_i} | s_i) P(s_i) \quad (131)$$

where

$$\Omega_{s_1} | s_1 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{CoMP_2}))] \quad (132)$$

$$\Omega_{s_2} | s_2 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{2B}))] \quad (133)$$

$$\Omega_{s_3} | s_3 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_2}, (C/I)_{1A}))] \quad (134)$$

$$\Omega_{s_4} | s_4 = \mathbb{E}[\ln(1 + \max((C/I)_{1A}, (C/I)_{2B}))]. \quad (135)$$

5.3.4 Scenario 4

In the fourth scenario, the decision algorithm is given below.

- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2A} \geq \psi) \implies$ select the user with maximum $(C/I)_{CoMP}$
- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2A} < \psi) \implies (C/I)_{CoMP_1} \geq (C/I)_{2C}$ select user 1,
else $(C/I)_{CoMP_1} < (C/I)_{2A}$ select user 2
- $((C/I)_{1B} < \psi) \& ((C/I)_{2A} \geq \psi) \implies (C/I)_{CoMP_2} \geq (C/I)_{1A}$ select user 2,
else $(C/I)_{CoMP_2} < (C/I)_{1A}$ select user 1
- $((C/I)_{1B} < \psi) \& ((C/I)_{2A} < \psi) \implies (C/I)_{1A} \geq (C/I)_{2C}$ select user 1,
else $(C/I)_{1A} < (C/I)_{2C}$ select user 2

In this algorithm, $(C/I)_{CoMP_1} = \frac{|h_{1A}r_{1A}^{-\alpha/2} + h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 1, $(C/I)_{CoMP_2} = \frac{|h_{2C}r_{2C}^{-\alpha/2} + h_{2A}r_{2A}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 2, $(C/I)_{1A} = \frac{|h_{1A}r_{1A}^{-\alpha/2}|^2}{I_r}$, is the (C/I) of user 1 from base station A, $(C/I)_{1B} = \frac{|h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user

1 from base station B, $(C/I)_{2C} = \frac{|h_{2C}r_{2C}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station C and $(C/I)_{2A} = \frac{|h_{2A}r_{2A}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station B.

The overall average spectral density of 2 users can be expressed for the fourth scenario as,

$$\Omega_{av} = \sum_{i=1}^4 (\Omega_{s_i} | s_i) P(s_i) \quad (136)$$

where

$$\Omega_{s_1} | s_1 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{CoMP_2}))] \quad (137)$$

$$\Omega_{s_2} | s_2 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{2C}))] \quad (138)$$

$$\Omega_{s_3} | s_3 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_2}, (C/I)_{1A}))] \quad (139)$$

$$\Omega_{s_4} | s_4 = \mathbb{E}[\ln(1 + \max((C/I)_{1A}, (C/I)_{2C}))]. \quad (140)$$

5.3.5 Scenario 5

In the fifth scenario, the decision algorithm is given below.

- $((C/I)_{1A} \geq \psi) \& ((C/I)_{2A} \geq \psi) \implies$ select the user with maximum $(C/I)_{CoMP}$
- $((C/I)_{1A} \geq \psi) \& ((C/I)_{2A} < \psi) \implies (C/I)_{CoMP_1} \geq (C/I)_{2C}$ select user 1,
else $(C/I)_{CoMP_1} < (C/I)_{2C}$ select user 2
- $((C/I)_{1A} < \psi) \& ((C/I)_{2A} \geq \psi) \implies (C/I)_{CoMP_2} \geq (C/I)_{1B}$ select user 2,
else $(C/I)_{CoMP_2} < (C/I)_{1B}$ select user 1
- $((C/I)_{1A} < \psi) \& ((C/I)_{2A} < \psi) \implies (C/I)_{1B} \geq (C/I)_{2C}$ select user 1,
else $(C/I)_{1B} < (C/I)_{2C}$ select user 2

In this algorithm, $(C/I)_{CoMP_1} = \frac{|h_{1B}r_{1B}^{-\alpha/2} + h_{1A}r_{1A}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 1, $(C/I)_{CoMP_2} = \frac{|h_{2C}r_{2C}^{-\alpha/2} + h_{2A}r_{2A}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 2, $(C/I)_{1B} = \frac{|h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$, is the (C/I) of user 1 from base station B, $(C/I)_{1A} = \frac{|h_{1A}r_{1A}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 1 from base station A, $(C/I)_{2C} = \frac{|h_{2C}r_{2C}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station C and $(C/I)_{2A} = \frac{|h_{2A}r_{2A}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station A.

The overall average spectral density of 2 users can be expressed for the fifth scenario as,

$$\Omega_{av} = \sum_{i=1}^4 (\Omega_{s_i} | s_i) P(s_i) \quad (141)$$

where

$$\Omega_{s_1} | s_1 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{CoMP_2}))] \quad (142)$$

$$\Omega_{s_2} | s_2 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{2C}))] \quad (143)$$

$$\Omega_{s_3} | s_3 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_2}, (C/I)_{1B}))] \quad (144)$$

$$\Omega_{s_4} | s_4 = \mathbb{E}[\ln(1 + \max((C/I)_{1B}, (C/I)_{2C}))]. \quad (145)$$

5.3.6 Scenario 6

In the sixth scenario, the decision algorithm is given below.

- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2D} \geq \psi) \implies$ select the user with maximum $(C/I)_{CoMP}$
- $((C/I)_{1B} \geq \psi) \& ((C/I)_{2D} < \psi) \implies (C/I)_{CoMP_1} \geq (C/I)_{2C}$ select user 1,
else $(C/I)_{CoMP_1} < (C/I)_{2C}$ select user 2
- $((C/I)_{1B} < \psi) \& ((C/I)_{2D} \geq \psi) \implies (C/I)_{CoMP_2} \geq (C/I)_{1A}$ select user 2,
else $(C/I)_{CoMP_2} < (C/I)_{1A}$ select user 1
- $((C/I)_{1B} < \psi) \& ((C/I)_{2D} < \psi) \implies (C/I)_{1A} \geq (C/I)_{2C}$ select user 1,
else $(C/I)_{1A} < (C/I)_{2C}$ select user 2

In this algorithm, $(C/I)_{CoMP_1} = \frac{|h_{1A}r_{1A}^{-\alpha/2} + h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 1, $(C/I)_{CoMP_2} = \frac{|h_{2C}r_{2C}^{-\alpha/2} + h_{2D}r_{2D}^{-\alpha/2}|^2}{I_r}$ is the total (C/I) of user 2, $(C/I)_{1A} = \frac{|h_{1A}r_{1A}^{-\alpha/2}|^2}{I_r}$, is the (C/I) of user 1 from base station A, $(C/I)_{1B} = \frac{|h_{1B}r_{1B}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 1 from base station B, $(C/I)_{2C} = \frac{|h_{2C}r_{2C}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station C and $(C/I)_{2D} = \frac{|h_{2D}r_{2D}^{-\alpha/2}|^2}{I_r}$ is the (C/I) of user 2 from base station D.

The overall average spectral density of 2 users can be expressed for the sixth scenario as,

$$\Omega_{av} = \sum_{i=1}^4 (\Omega_{s_i}|s_i)P(s_i) \quad (146)$$

where

$$\Omega_{s_1}|s_1 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{CoMP_2}))] \quad (147)$$

$$\Omega_{s_2}|s_2 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_1}, (C/I)_{2C}))] \quad (148)$$

$$\Omega_{s_3}|s_3 = \mathbb{E}[\ln(1 + \max((C/I)_{CoMP_2}, (C/I)_{1A}))] \quad (149)$$

$$\Omega_{s_4}|s_4 = \mathbb{E}[\ln(1 + \max((C/I)_{1A}, (C/I)_{2C}))]. \quad (150)$$

5.4 Numerical Results and Discussion

In this section, the analytical and simulated results of average spectral efficiencies of two user are given for six different scenarios which are explained in the previous section. In order to verify the derivation of the average spectral efficiency, we conduct Monte Carlo simulations with 10^5 independent samples of the locations of base stations and for each of six scenarios. For each trial, the locations of two mobile users are fixed. We use MATLAB environment to both numerically evaluate the mean of average spectral efficiency and to conduct the simulations. In 21, 22, 23, 24, 25 and 26 the average spectral efficiencies of 2 users in scenario 1,2,3,4,5 and 6 are plotted respectively. For each of these scenario, in order to evaluate the analytical spectral efficiency of 2 user the copula functions are used to model the dependency. In order to model the dependency, we select the copula function with respect to the maximum log likelihood value among other Archimedean copulae. The Archimedean copulae that we tried were Clayton, Rotated Clayton, Plackett, Frank, Gumbel, Rotated Gumbel, and Symmetrised Joe-Clayton copulae [26]. According to our maximum likelihood estimation results, Plackett or Clayton copulas are used in six scenarios to model the

dependency. Plackett copula and Clayton copula are expressed respectively as:

$$C_{\rho}^P(u, v) = \frac{1 + (\rho - 1)(u + v) - \sqrt{(1 + (\rho - 1)(u + v))^2 - 4\rho(\rho - 1)uv}}{2(\rho - 1)} \quad (151)$$

$$C_{\rho}^C(u, v) = \max([u^{-\alpha} + v^{-\alpha} - 1]^{-1/\alpha}, 0) \quad (152)$$

In all figures, blue line corresponds to the analytical spectral efficiency, whereas the red dots correspond to the simulated spectral efficiency.

The correlation parameters and the copula types that are used to model the dependency of (C/I) values are given for scenarios 1,2,3,4,5 and 6 in Tables 1, 2, 3, 4, 5 and 6 respectively. From the tables, we can inspect that, the copula type which is used to model both the correlation between the $(C/I)_{CoMP_1}$ and $(C/I)_{CoMP_2}$ values, the correlation between the $(C/I)_1$ and $(C/I)_{CoMP_2}$ values and the correlation between the $(C/I)_2$ and $(C/I)_{CoMP_1}$ values is Plackett for the scenarios 1,2,4,5 and 6. For scenarios 2,4,5 and 6, the copula type which is used to model the correlation between the (C/I) values of the second closest base stations is Clayton whereas for the scenario 1, the copula type is Plackett. For scenario 3, the situation is just the opposite of scenarios 2,4,5 and 6. This time, the copula type which is used to model both the correlation between the $(C/I)_{CoMP_1}$ and $(C/I)_{CoMP_2}$ values, the correlation between the $(C/I)_1$ and $(C/I)_{CoMP_2}$ values and the correlation between the $(C/I)_2$ and $(C/I)_{CoMP_1}$ values is Clayton and the copula type which is used to model the correlation between the (C/I) values of the second closest base stations is Plackett. We can also inspect that, among the correlation parameters, which are modeled by Plackett copula, scenario 5, has the biggest values. In scenario 1, situation is just the opposite.

If we look at the average spectral efficiency plots, we can inspect that, the analytical model which is used to estimate the dependency works well for all scenarios. We can also inspect that the efficiency is highest in scenario 1 and lowest in scenario 5. This is reasonable, because it is consistent with the correlation parameters. Because

Table 1: Correlation Parameters of (C/I) values of 2 Users in Scenario 1

	Copula Type	ρ
$(C/I)_{CoMP_1}, (C/I)_{CoMP_2}$	Plackett	0.6307
$(C/I)_{CoMP_2}, (C/I)_{1A}$	Plackett	0.6307
$(C/I)_{CoMP_1}, (C/I)_{2A}$	Plackett	0.6279
$(C/I)_{1A}, (C/I)_{2A}$	Plackett	0.6261
$(C/I)_{1B}, (C/I)_{2B}$	Clayton	0.0131

Table 2: Correlation Parameters of (C/I) values of 2 Users in Scenario 2

	Copula Type	ρ
$(C/I)_{CoMP_1}, (C/I)_{CoMP_2}$	Plackett	0.6688
$(C/I)_{CoMP_2}, (C/I)_{1A}$	Plackett	0.6581
$(C/I)_{CoMP_1}, (C/I)_{2B}$	Plackett	0.6586
$(C/I)_{1A}, (C/I)_{2B}$	Plackett	0.6472
$(C/I)_{1B}, (C/I)_{2C}$	Plackett	1.1549

Table 3: Correlation Parameters of (C/I) values of 2 Users in Scenario 3

	Copula Type	ρ
$(C/I)_{CoMP_1}, (C/I)_{CoMP_2}$	Clayton	0.0885
$(C/I)_{CoMP_2}, (C/I)_{1A}$	Clayton	0.0892
$(C/I)_{CoMP_1}, (C/I)_{2B}$	Clayton	0.0854
$(C/I)_{1A}, (C/I)_{2B}$	Clayton	0.0883
$(C/I)_{1B}, (C/I)_{2A}$	Plackett	1.1668

Table 4: Correlation Parameters of (C/I) values of 2 Users in Scenario 4

	Copula Type	ρ
$(C/I)_{CoMP_1}, (C/I)_{CoMP_2}$	Plackett	1.4216
$(C/I)_{CoMP_2}, (C/I)_{1A}$	Plackett	1.4221
$(C/I)_{CoMP_1}, (C/I)_{2C}$	Plackett	1.3895
$(C/I)_{1A}, (C/I)_{2C}$	Plackett	1.4180
$(C/I)_{1B}, (C/I)_{2A}$	Clayton	1.1720

Table 5: Correlation Parameters of (C/I) values of 2 Users in Scenario 5

	Copula Type	ρ
$(C/I)_{CoMP_1}, (C/I)_{CoMP_2}$	Plackett	1.5965
$(C/I)_{CoMP_2}, (C/I)_{1B}$	Plackett	1.5649
$(C/I)_{CoMP_1}, (C/I)_{2C}$	Plackett	1.0194
$(C/I)_{1B}, (C/I)_{2C}$	Plackett	1.6272
$(C/I)_{1A}, (C/I)_{2A}$	Plackett	1.2038

we have the highest efficiency in scenario 1 and lowest efficiency in scenario 5.

Table 6: Correlation Parameters of (C/I) values of 2 Users in Scenario 6

	Copula Type	ρ
$(C/I)_{CoMP_1}, (C/I)_{CoMP_2}$	Plackett	1.5077
$(C/I)_{CoMP_2}, (C/I)_{1A}$	Plackett	1.4908
$(C/I)_{CoMP_1}, (C/I)_{2A}$	Plackett	1.4963
$(C/I)_{1A}, (C/I)_{2A}$	Plackett	1.4842
$(C/I)_{1B}, (C/I)_{2B}$	Plackett	1.2796

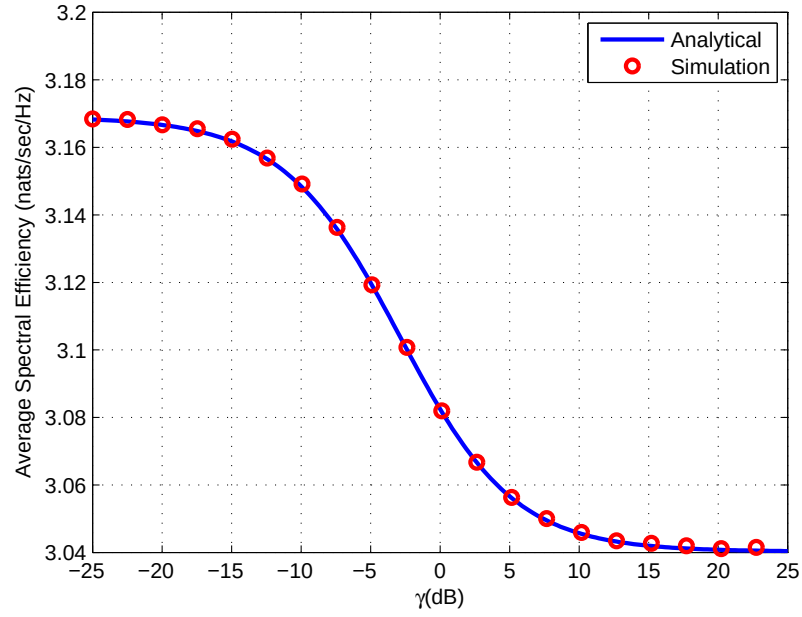


Figure 21: Average Capacity of 2 Users in Scenario 1

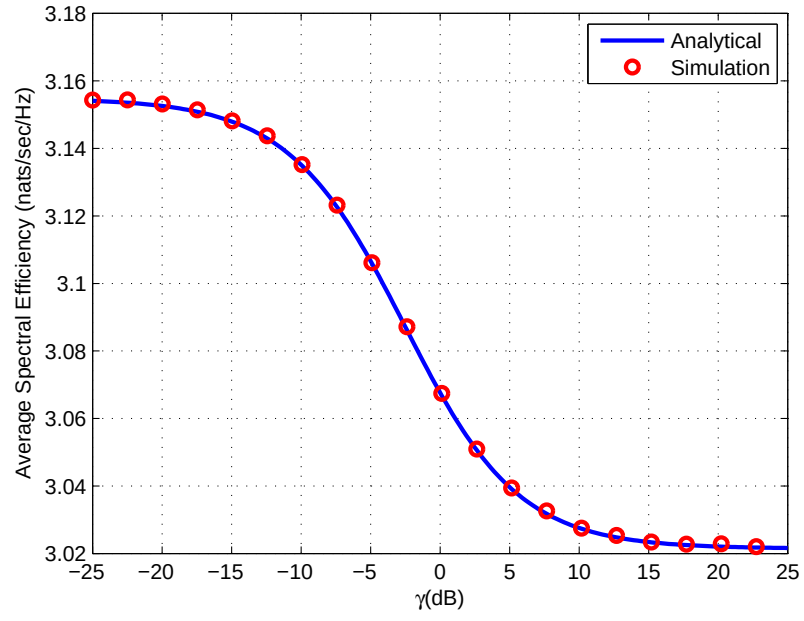


Figure 22: Average Capacity of 2 Users in Scenario 2

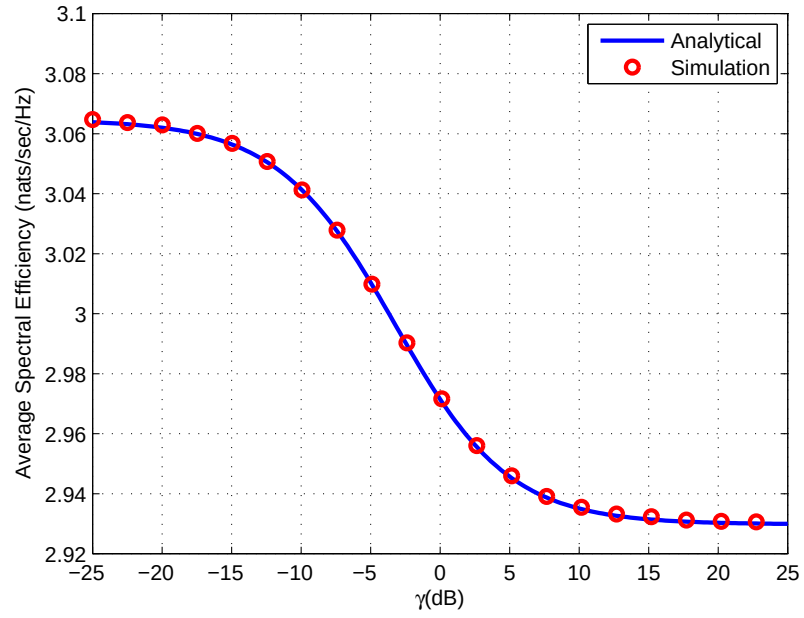


Figure 23: Average Capacity of 2 Users in Scenario 3

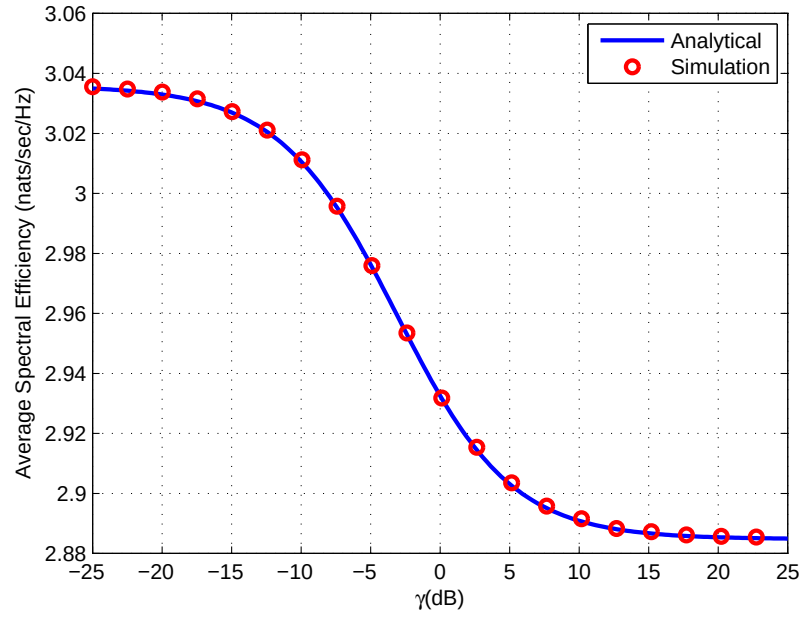


Figure 24: Average Capacity of 2 Users in Scenario 4

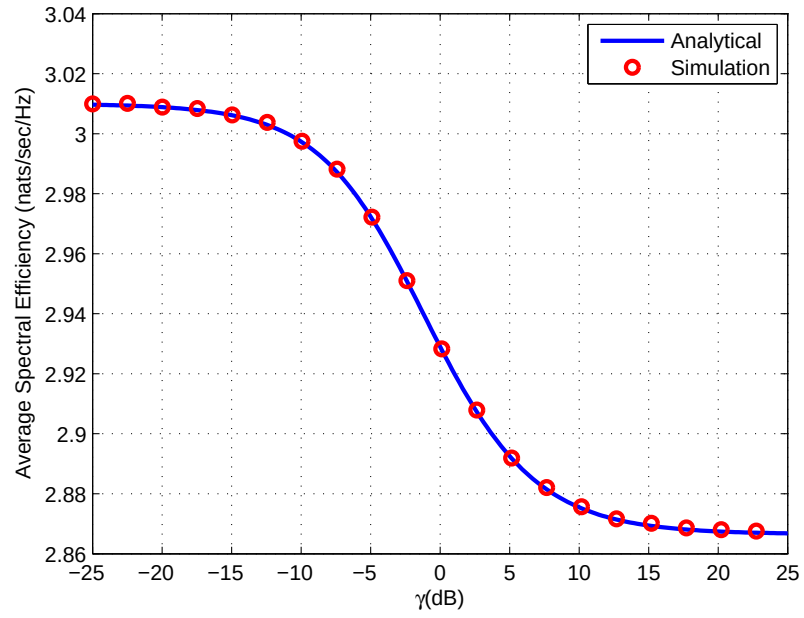


Figure 25: Average Capacity of 2 Users in Scenario 5

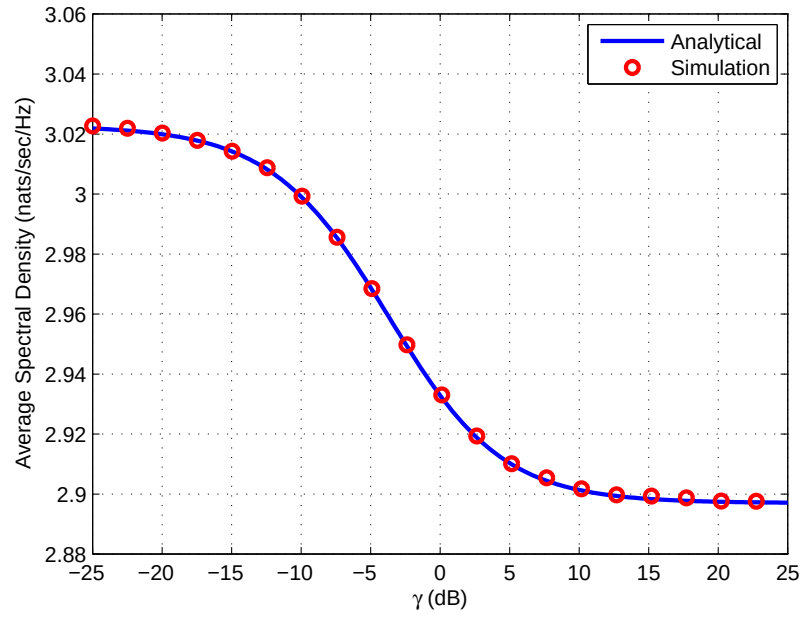


Figure 26: Average Capacity of 2 Users in Scenario 6

CHAPTER VI

CONCLUSION

In this dissertation, in the first part, a copula-based analysis is introduced to analyze the performance of the downlink of a SISO cellular network where the base stations as well as users are randomly distributed. The analysis takes into account the conditional dependence between the C/I values observed by the users connected to the same cell due to the randomness of the base station locations. The copula-based analysis is assessed by the numerical results obtained via Monte Carlo simulations. The numerical results also illustrate that the commonly used independence assumption of the user C/I values is increasingly more inaccurate as path loss exponent increases and this assumption overestimates both the actual coverage probability and the actual spectral efficiency of the system.

In the second part, we analyzed the coverage performance of downlink MU-MIMO cellular networks where the BSs as well as users are randomly distributed. We propose an approach based on Prony's method to approximate the CCDF of received signal gain of a typical user by sum of exponential functions which facilitates the analysis of downlink coverage probability. Through comparisons with Monte Carlo simulations, we demonstrated the accuracy of our derived expression. Our numerical results further highlight that the downlink coverage performance decreases with the number of simultaneously scheduled users on the same frequency resource in a MU-MIMO system if any intelligent scheduling/interference cancellation algorithm is not employed.

In the third part, we first analyze the performance of three types of mobile users namely cell edge, worst case and general user that are served by coordinated base

stations in the stochastic geometry framework. In this analysis, we consider joint transmission and dynamic cell selection transmission techniques. For both types of users, there is a cross-over threshold. Below this threshold, dynamic cell selection provides better downlink coverage performance whereas beyond this threshold joint transmission is the better technique. Both of the CoMP techniques provide the most power gain to worst case user. For low number of coordinating base stations (e.g $n = 2$), these techniques provide more power gain to cell edge user with respect to the general user. Regarding the general user, increasing the number of coordinating base stations improves the downlink coverage performance.

In the third part, we also analyze the performance of two users by using stochastic geometry approach in the CoMP transmission scheme. In order model to the dependency of (C/I) values we use copula functions like in the first part. We implement six different scenarios, and among these scenarios the correlation of (C/I) values is highest in the scenario where the second closest base stations of two users are the same and the closest base stations of two users are different. In consistent with this result the worst spectral efficiency is observed in this scenario. In the scenario, where the two users are connected to the same base stations, the correlation between the (C/I) values are lowest and the average spectral efficiency performance is the highest among the other scenarios.

In this dissertation, we worked on the analysis of downlink cellular networks including the SISO, MU-MIMO and downlink CoMP transmission schemes. Regarding the SISO case, we considered to analyze the performance of multiuser diversity where the user with maximum C/I value was scheduled. However, for MU-MIMO we analyzed the coverage performance of a randomly selected user for the sake of simplicity of the analysis. As a result, we ignored the dependency of SIR values of users connected to same cell for this transmission scheme. Furthermore, we ignored to utilize any intelligent scheduling/interference cancellation algorithm for the MU-MIMO case.

We also assumed to employ MRT precoder which is simple and practical. But we did not consider to optimize the precoder. In order to analyze the downlink performance of CoMP, we considered the general user, cell edge user and worst case user with the 2-user case. We did not consider a multiuser case where there is more than 2 users are scheduled for the ease of the analysis. Regarding the analysis of all transmission schemes we assumed a simple association rule, which dictated the connection of user to its nearest base station.

For future work, we would like to extend our analysis considering a dynamic association rule for all of the transmission schemes. We also consider to extend the analysis for MU-MIMO both considering the utilization of an intelligent scheduling/interference cancellation scheme and optimization of the precoder. Furthermore, we would like to extend the analysis by utilizing mmWave frequency bands, which are planning to be used in 5G and also considered to be used in next generation wireless cellular standard namely 6G. Regarding the downlink CoMP scheme, we consider to analyze the performance of multiuser diversity for the cases where there are more than 2 users connected to the base station.

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