

SELECTIVE ROUTING PROBLEMS IN HUMANITARIAN OPERATIONS

A DISSERTATION SUBMITTED TO
THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN
INDUSTRIAL ENGINEERING

By
Çağla Fatma Dursunoğlu
December 2025

SELECTIVE ROUTING PROBLEMS IN HUMANITARIAN OPERATIONS

By Çağla Fatma Dursunoğlu

December 2025

We certify that we have read this dissertation and that in our opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

Bahar Yetiş Kara (Advisor)

Okan Arslan (Co-Advisor)

Özlem Karsu

Eda Yücel

Oya Ekin Kardeşan

Çağrı Koç

Approved for the Graduate School of Engineering and Science:

Orhan Arikan
Director of the Graduate School



ABSTRACT

SELECTIVE ROUTING PROBLEMS IN HUMANITARIAN OPERATIONS

Çağla Fatma Dursunoğlu
Ph.D. in Industrial Engineering
Advisor: Bahar Yetiş Kara
Co-Advisor: Okan Arslan
December 2025

In this thesis, we investigate the critical role of demand frequency in disaster response. We categorize demand into three types: one-time, continuous, and periodic. We define three distinct problems for each demand type. For one-time demand, we define a location-allocation problem with service duration decisions. For continuous demand, we define a maximal covering location problem. For periodic demand, we define a scheduling problem with duration decisions. For each problem, we develop tailored mathematical formulations that integrate realistic demand functions.

Firstly, we consider one-time demand, which is characterized by immediate and non-recurring needs in the aftermath of a disaster. We develop three integer programming models for location-allocation problem with service duration decisions which incorporate a stepwise demand function that captures the gradual decline in demand over time. We compare the mathematical models on real-world datasets in terms of solution time and optimality gap for large instances. Through extensive computational experiments, we observe that the best-performing model achieves optimal or near-optimal solutions significantly faster with smaller optimality gaps, especially when solving large-scale instances. The results show that adopting temporal coverage of demand at different locations satisfies more demand over time. We observe that dynamically changing disaster environment requires adopting more agile deployment strategies for mobile service units.

Secondly, we consider continuous demand to represent critical and ongoing service

requirements. We take into account different types of mobile service units (high-capacity, medium-capacity, and low-capacity). We provide a mathematical formulation maximal covering location problem that determines the optimal locations of mobile service units and allocation of demand points to mobile service units. We also integrate a demand function that accounts for diminishing demand due to coverage. We consider two concepts for how demand is satisfied from covered areas: binary and gradual coverage. In binary coverage, demand is fully satisfied if the demand location is within the coverage radius of a mobile service unit. On the other hand, in gradual coverage, the total demand satisfied drops significantly because it follows a decay function to reflect a decrease with respect to distance. According to our results, we observe that the model prioritizes locating mobile service units closer to demand areas to decrease the decay effect.

Finally, we consider periodic demand to represent recurring service needs over specific time intervals. We introduce four mathematical formulations for a scheduling problem with duration decisions: compact 5-index, 4-index models and two 3-index non-compact models. The latter two are derived through a Benders-type projection method and solved using a branch-and-cut algorithm strengthened with valid inequalities. Although decomposition-friendly formulations are attractive, they face significant computational overhead. We also conduct a sensitivity analysis on the parameters and identify the most influential setting. Additionally, we analyze demand functions, including linear, exponential, sigmoid, quadratic, logarithmic, and step functions. We evaluate those functions by post-processing their solutions with a neutral demand function for fair comparison. We finally provide a case study using data from the 2023 Kahramanmaraş earthquakes to validate the model's practical applicability. According to the solutions, the high-density districts receive extended service durations, while the low-density districts receive the minimum visit requirements. It is noted that to enhance fairness, the parameters regarding the minimum requirement can be updated for low-density areas.

Based on these comprehensive computational studies across all three demand types, we provide strategic insights for deploying mobile service units in complex disaster settings.

Keywords: Humanitarian Logistics, Disaster Response, Facility Location, Mobile

Service Units, One-time Demand, Continuous Demand, Periodic Demand, Integer Programming, Benders Decomposition, Coverage Functions, Demand Functions.



ÖZET

İNSANİ YARDIM OPERASYONLARINDA SEÇİCİ ROTALAMA PROBLEMLERİ

Çağla Fatma Dursunoğlu

Endüstri Mühendisliği, Doktora

Tez Danışmanı: Bahar Yetiş Kara

İkinci Tez Danışmanı: Okan Arslan

Aralık 2025

Bu tezde, afet müdahalesinde talep sıklığının kritik rolünü inceliyoruz. Talebi tek seferlik, sürekli ve periyodik olacak şekilde üç türe ayırıyoruz. Her talep türü için matematiksel formülasyonlar öneriyoruz ve gerçek hayat senaryolarına yakın talep fonksiyonlarını bu modellere entegre ediyoruz.

İlk olarak, bir afet sonrası acil ve tekrarlanmayan ihtiyaçlarla karakterize edilen tek seferlik talebi ele alıyoruz. Bu talep türü için, talebin zamanla kademeli olarak azalmasını modellediğimiz bir talep fonksiyonunu içeren üç adet tamsayılı programlama modeli geliştiriyoruz. Modelleri, gerçek hayat veri setleri üzerinde büyük ölçekli problemler için çözüm süresi ve optimalite açığı kriterleriyle karşılaştırıyoruz. Kapsamlı deneyler sonucunda, en iyi performans gösteren modelin, özellikle büyük ölçekli örneklerde daha düşük optimalite açıklarıyla optimal veya optimale yakın çözümlere daha hızlı ulaştığını gözlemliyoruz. Sonuçlara göre talep zamanla azaldıkça, mobil hizmet birimlerinin daha fazla talebi etkili bir şekilde karşılamak için farklı konumlarda talebin zamansal kapsamını benimsemesi gerektiğini ortaya koyuyor. Dinamik olarak değişen bir afet ortamında, mobil hizmet birimleri için daha çevik konuşlandırma stratejileri benimsememiz gerektiğini gözlemliyoruz.

İkinci olarak, kritik ve devam eden hizmet gereksinimlerini temsil etmek için sürekli talebi ele alıyoruz. Farklı türde mobil hizmet birimleri (yüksek kapasiteli, orta kapasiteli ve düşük kapasiteli) dikkate alıyoruz. Mobil hizmet birimlerinin optimal yerleşimini ve talep noktalarının bu birimlere tahsisini belirleyen bir matematiksel formülasyon sunuyoruz. Ayrıca, kapsama nedeniyle azalan talebi hesaba katan bir talep fonksiyonunu entegre ediyoruz. Talebin karşılanma biçimini tanımlamak

için ikili kapsama ve kademeli kapsama olmak üzere iki kavramı inceliyoruz. İkili kapsamada, bir talep noktası bir mobil hizmet biriminin erişim yarıçapı içindeyse, talep tamamen karşılanır. Öte yandan, kademeli kapsamada ise, mesafeye göre azal-
lan talep varsaydığı için karşılanan toplam talep önemli ölçüde düşer. Elde ettiğimiz sonuçlara göre, modelimiz uzaklığa bağlı azalan talebin etkisini azaltmak için mobil hizmet birimlerini talep yoğunluğu yüksek bölgelere daha yakın konumlandırmayı önceliklendirdiğini gözlemliyoruz.

Son olarak, belirli zaman aralıklarında tekrarlayan hizmet ihtiyaçlarını temsil et-
mek için periyodik talebi ele alıyoruz. Bu talep türü için dört matematiksel for-
mülasyon sunuyoruz: kompakt 5-indeksli, 4-indeksli modeller ve iki 3-indeksli kom-
pakt olmayan model. Son iki modeli, Benders-tipi projeksiyon yöntemiyle türetiyor
ve geçerli eşitsizliklerle güçlendirilmiş bir dal-kesme algoritması kullanılarak çözü-
yoruz. Ayırıştırma dostu formülasyonlar teorik olarak avantajlı olsa da, önemli
hesaplama yükü getiriyorlar. Parametreler üzerinde duyarlılık analizi yapıyor ve
en etkili parametreyi belirliyoruz. Ayrıca, doğrusal, üstel, sigmoid, kuadratik, log-
aritmik ve basamak fonksiyonları dahil olmak üzere talep fonksiyonlarını da analiz
ediyoruz. Bu fonksiyonları, adil bir karşılaştırma yapabilmek için, elde ettiğimiz
çözümleri nötr bir talep fonksiyonu kullanarak sonradan işleme tabi tutarak değer-
lendiriyoruz. Nihayet, modelimizin pratik uygulanabilirliğini doğrulamak amacıyla
2023 Kahramanmaraş depremlerine ait verilerle bir vaka çalışması gerçekleştiriy-
yoruz. Çözümlere göre, yüksek yoğunluklu bölgeler uzatılmış hizmet süreleri alırken,
düşük yoğunluklu bölgeler yalnızca asgari ziyaret gereksinimlerini karşılayacak kadar
hizmete erişiyor. Daha adil çözümler sunabilmek maksadıyla, düşük yoğunluklu alan-
lar için minimum gereksinime ilişkin parametrelerin güncellenmesi gerektiği sonucuna
varıyoruz.

Her üç talep türü üzerindeki bu kapsamlı hesaplama çalışmaları temelinde, kar-
maşık afet ortamlarında mobil hizmet birimlerinin konuşlandırılması için stratejik
çıkarımlar sunuyoruz.

Anahtar sözcükler: İnsani Yardım Lojistiği, Afet Müdahalesi, Tesis Yerleşimi, Mo-
bil Hizmet Birimleri, Tek Seferlik Talep, Sürekli Talep, Periyodik Talep, Tamsayı
Programlama, Benders Ayırıştırması, Kapsama Fonksiyonları, Talep Fonksiyonları.

Acknowledgement

I would like to express my sincere gratitude to all those who have contributed to completing this thesis. First, I sincerely appreciate my principal advisor, Prof. Bahar Yetiş Kara, and my co-advisor, Assoc. Prof. Okan Arslan, for their invaluable guidance, unwavering support, and insightful mentorship throughout my PhD journey. Their profound knowledge, constructive suggestions, and continuous encouragement were instrumental in shaping this research and significantly contributed to my academic and personal growth.

I also want to thank the members of my thesis committee, Assoc. Prof. Özlem Karsu and Assoc. Prof. Eda Yücel, for their time, valuable insights, and thoughtful critiques. Their rigorous review and insightful questions pushed me to refine my arguments and improve the clarity of this thesis. I also want to thank Prof. Oya Ekin Karaşan and Prof. Çağrı Koç for agreeing to be a member of my examination committee and for all their valuable suggestions.

Beyond my academic mentors, my dear friends' unwavering companionship and trustworthy support were essential. My profound gratitude goes to Parinaz Toufani and Milad Malekipirbazari, who were not just friends but like older siblings throughout my PhD journey, consistently believing in me and standing by my side. Equally vital was the boundless love, patience, understanding, and unwavering emotional fortitude offered by Aşlı Semerci, Safa Öztürk, Ozanalp Eryılmaz, and Gözde Yazıcı. Their collective belief in my capabilities, especially during moments of doubt, was an irreplaceable source of strength and inspiration.

Finally, and most importantly, I dedicate this work to my beloved family. Their unwavering love, patience, and boundless support have been the cornerstone of my journey. To my mother, Aşlıhan, for her endless sacrifices, encouragement, and for instilling in me the values of perseverance and dedication. Your belief in me never wavered, and your love was a constant source of strength. To my siblings, Günalp and Batuhan, along with my sister-in-law Duha, and Birsen, Günalp's future wife,

for their constant cheer, understanding, and for reminding me to take breaks and find joy amidst the everyday demands of life. To my precious niece, Güneş Gülce, whose vibrant presence and infectious love for dance were my brightest motivation, providing boundless joy and a wonderful sense of perspective throughout this challenging academic journey. This thesis is as much a reflection of your collective support as it is of my own efforts. Thank you for everything.



Contents

1	Introduction	1
1.1	Motivation	6
2	Literature Review	10
2.1	One-time Demand	10
2.2	Continuous Demand	13
2.3	Periodic Demand	15
2.4	Selective Routing Problems	19
3	Datasets	30
3.1	Seoul Dataset	31
3.2	Defne Dataset	32
3.3	Kartal Dataset	32
3.4	Istanbul Dataset	33
3.5	Kilis Dataset	33
3.6	Synthetic Dataset	35
4	One-time Demand in the Post-Disaster Operations	36
4.1	Formal Problem Definition	38
4.1.1	Problem Setup and Notation	38
4.1.2	Demand Function	40
4.1.3	Problem Definition	43

4.2	Mathematical Models	44
4.2.1	Model 1	47
4.2.2	Model 2	52
4.2.3	Model 3	54
4.3	Computational Study	56
4.3.1	Datasets and Parameter Settings	56
4.3.2	Performance of Models	57
4.3.3	Analysis of Optimal Solutions	66
4.4	Conclusion of Chapter 4	72
5	Continuous Demand in the Post-Disaster Operations	75
5.1	Formal Problem Definition	76
5.1.1	Problem Setup and Notation	77
5.1.2	Coverage Function	80
5.1.3	Problem Definition	82
5.2	Mathematical Model	83
5.3	Computational Study	86
5.3.1	Datasets and Parameter Settings	87
5.3.2	Analysis of Optimal Solutions	90
5.4	Conclusion of Chapter 5	104
6	Periodic Demand in the Post-Disaster Operations	107
6.1	Formal Problem Definition	108
6.1.1	Problem Setup and Notation	109
6.1.2	The Demand Function	111
6.1.3	Problem Definition	114
6.2	Mathematical Modeling	118
6.2.1	5-index Model	119
6.2.2	4-index Model	123
6.2.3	3-index Model	126

6.2.4	Extended 3-index Model	134
6.3	Computational Study	135
6.3.1	Datasets and Parameter Settings	135
6.3.2	Performance of Models	138
6.3.3	Results and Discussion	145
6.3.4	Sensitivity Analysis	147
6.3.5	Demand Function Analysis	152
6.3.6	Case Study on Hatay Dataset	156
6.4	Conclusion of Chapter 6	163
7	Conclusion	166
A	Proof of Proposition 1	188
B	Proof of Lemma 1	190
C	Proof of Lemma 2	192
D	Proof of Proposition 2	194
E	Demand Functions	196
F	Proof of Proposition 3	198
G	Proof of Lemma 3	200
H	Proof of Lemma 4	202
I	Proof of Lemma 5	204
J	Proof of Lemma 6	205
K	Proof of Proposition 4	206

L	Proof of Proposition 5	212
M	Proof of Proposition 6	215
N	Proof of Proposition 7	217



List of Figures

1.1	Collection of mobile service units during the Kahramanmaraş earthquake 2023	3
1.2	Mobile service units emerging during the Kahramanmaraş earthquake 2023	4
1.3	Disaster management timeline showing phase transitions and demand	6
2.1	Relationships between the sets V , $\bar{V}$, W and $\bar{W}$	24
2.2	Classification diagram for selective routing problems	25
3.1	Potential locations in Seoul, South Korea	31
3.2	Potential locations in Defne, Hatay	32
3.3	Potential locations in Kartal, İstanbul	33
3.4	Potential locations in Istanbul, Türkiye	34
3.5	Potential locations in Kilis, Türkiye	34
3.6	Potential locations in Synthetic dataset	35
4.1	Schematic illustration of the tours of MSUs for one-time demand . . .	39
4.2	Step-wise Demand Function	42
4.3	Uniform Demand Function	42
4.4	Results of the solutions for the uniform demand function in the Seoul dataset	67

4.5	Results of the solutions for the uniform demand function in the Kartal dataset	68
4.6	Results of the solutions for decreasing demand function in Seoul dataset	70
4.7	Results of the solutions for decreasing demand function in Kartal dataset	71
5.1	Schematic illustration of the location and allocation of HCap MSU, MCap MSU and LCap MSU for continuous demand type of services .	79
5.2	Different Coverage Functions	81
5.3	Solutions of Kartal dataset with demand rates (a_{ij}^u) defined as binary coverage	96
5.4	Solutions for Kartal dataset with demand rates (a_{ij}^u) defined as gradual decay functions with uniform decay	102
5.5	Solutions for Istanbul dataset with different demand rates (a_{ij}^u) . . .	105
6.1	Service schedule showing MSU assignments. Each 1×1 grid square represents one period. Each colored block represents service periods $[t, t + d - 1]$ with unit numbers (MSU 1/MSU 2), where d denotes the total service period	111
6.2	Operational timeline: MSU deployment across districts with service durations	111
6.3	Relationships between mathematical formulations	118
6.4	Service schedule optimization results for the base case scenario using Seoul dataset	140
6.5	Service schedule optimization results for the base case scenario using Kartal dataset	141
6.6	Service schedule optimization results for Defne case study	160
6.7	Service schedule optimization results for Defne case study with change in h_{it}	161
E.1	Comparison of different growth functions and their cumulative versions with $\hat{p}_{si} = 1$ for simplicity	196

E.1 Comparison of different growth functions and their cumulative ver-
sions with $\hat{p}_{si} = 1$ for simplicity (continued) 197



List of Tables

1.1	Classification of demand types	6
2.1	Selective routing problems considered in this framework	20
3.1	Dataset characteristics	30
4.1	Sets and parameters	48
4.2	Comparison of mathematical models for maximizing total demand satisfied under decreasing demand	58
4.3	Comparison of mathematical models for maximizing total demand satisfied under uniform demand	60
4.4	Comparison of LP relaxations of models	62
4.5	Comparison models based on relaxation of decision variables regarding time (LP_{time})	63
4.6	Number of decision variables and constraints of models	64
5.1	Sets and parameters	85
5.2	Results for Kartal dataset with demand rates (a_{ij}^u) defined as binary coverage	94
5.3	Results for Kartal dataset with demand rates (a_{ij}^u) defined as gradual decay functions with uniform decay	99
5.4	Results for Istanbul dataset with different demand rates (a_{ij}^u)	103

6.1	Variants of demand function	113
6.3	Sets and parameters	119
6.4	Dataset characteristics	136
6.5	Service-type parameters	136
6.6	Service-type vehicle number for different datasets	136
6.7	Visit frequencies ℓ_{si} by demand weight and service type	137
6.8	Model comparison results for base scenario	139
6.9	Comprehensive sensitivity analysis results	148
6.10	Different demand function results	153
6.11	Comparison of different demand function results under PP-OFV-NDF	155
6.12	Service-type vehicle number for Defne datasets	157

Chapter 1

Introduction

Türkiye is located in a seismically active region and has experienced devastating earthquakes in the past, such as the 1999 İzmit, 1999 Düzce, 2011 Van, 2020 Elazığ, 2020 İzmir and 2022 Düzce earthquakes. In the past, disasters such as the 1999 İzmit and Düzce earthquakes made evident systemic vulnerabilities in coordination, communication, and logistics in disaster preparedness and response. More recently, the earthquakes in 2023 around Kahramanmaraş re-emphasized these vulnerabilities. This highlighted a critical and constant need to improve disaster response mechanisms.

On February 6, 2023, two major earthquakes with moment magnitude (M_w) of 7.7 and 7.6 struck Kahramanmaraş, with epicentres in the districts of Pazarcık and Elbistan. Those major earthquakes occurred over a vast area, leading to the official designation of 11 provinces as disaster zones: Kahramanmaraş, Gaziantep, Hatay, Adana, Adıyaman, Elazığ, Diyarbakır, Kilis, Malatya, Osmaniye and Şanlıurfa. Hatay, Kahramanmaraş, and Adıyaman suffered the most severe impacts. The scale of the disaster was immense, resulting in over 50,000 fatalities, damage

to more than half a million buildings, and severe disruption to communication and energy infrastructure, alongside significant financial losses [1].

Afet ve Acil Durum Yönetimi Başkanlığı (AFAD), numerous non-governmental organizations (NGOs) and municipalities have been actively handling the affected population's needs. According to insights obtained from victims and volunteers, coordination and logistics have been challenged, especially in the acute period of the response phase. Consequently, those organizations struggled to fulfil critical daily needs such as food, shelter, and sanitation. Initially, some permanent facilities were established to provide essential services and resources. However, those static locations often proved inaccessible to specific segments of the dispersed population, leaving many victims without crucial support.

Various Mobile Service Units (MSUs) have emerged as a vital and flexible supplement in response to these challenges. They are instrumental for quickly reaching areas with widespread damage and crippled infrastructure, so these MSUs represented a significant evolution in response operations. As illustrated in Figures 1.1 and 1.2, NGOs, metropolitan municipalities, and AFAD deployed various novel mobile services during the Kahramanmaraş earthquake response [2]–[13]. These units included mobile health clinics operated from field hospitals (e.g., X-ray, dentistry, ultrasonography, and pharmacy units), mobile internet and satellite communication tools to support damaged infrastructure, and mobile generators to handle power demand. Furthermore, mobile travel repair units maintained vehicles and machinery, end-of-life buses were converted into mobile facilities for hygiene (restrooms, showers) and personal care (barbers, laundries), and mobile cash units were established to accommodate financial needs due to inoperable ATMs. Library and child-friendly areas were also formed to provide psychological assistance. These mobile services were utilized by victims in disaster areas, rural regions, and temporary housing, as well as by personnel operating in the field.



(a) Mobile dental clinic
Konya Metropolitan Municipality [2]



(b) Mobile kitchen gendarmerie command [3]



(c) Mobile pharmacy
Turkish Pharmacists Association [4]



(d) Solar mobile charging station
My City Project [5]



(e) Mobile toilet [6]



(f) Mobile shower
Ankara [7]



(g) Mobile hairdresser [8]



(h) Mobile shower [9]



(i) Mobile PTT branch
post office services [10]



(j) Mobile communication
Haberleşme A.Ş.[11]



(k) Mobile barber [12]



(l) Mobile laundry [13]

Figure 1.1: Collection of mobile service units during the Kahramanmaraş earthquake 2023

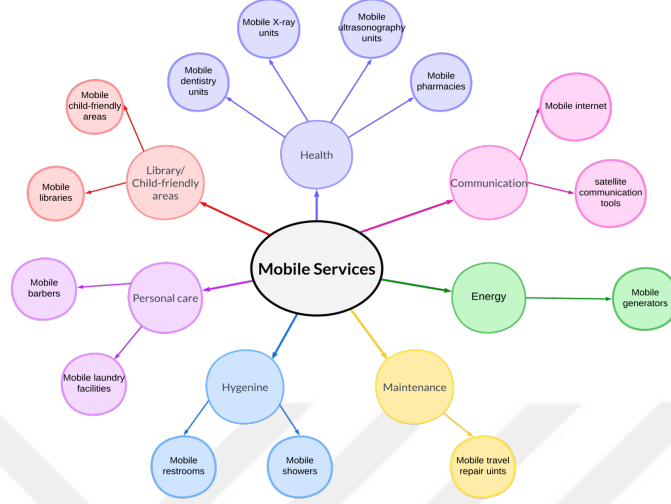


Figure 1.2: Mobile service units emerging during the Kahramanmaraş earthquake 2023

During a disaster, the primary purpose is to ensure the victim’s survival by effectively delivering and distributing various goods and services. Essential commodities required during disaster response include items such as covers (blankets, sleeping bags, boots, and clothing), personal medical care products (soap, sanitary pads, diaper, toilet paper, towels, disinfectants, and wet tissues), food (water, baby food, instant food, bars, canned food, pasta, rice, kitchen tools, and bowls), and medical equipment (first aid kits, vitamin pills, painkillers, and bandages) alongside machinery and personnel. The distribution of these commodities has been examined in the disaster logistics literature [14]–[15]. However, their distribution depends significantly on the demand, which determines the items to be delivered, the frequency of delivery, and the quantity to be delivered. The demand evolves across the four main phases of the disaster management cycle: mitigation, preparedness, response, and recovery [16]–[17]. This thesis focuses on the response phase, which occurs in the immediate aftermath of a disaster and is crucial for satisfying victims’ immediate and evolving needs. The response phase is subdivided into an acute period focused on life-saving operations under high uncertainty, followed by a stabilization period

aimed at establishing reliable provision for medium-term sustenance [18].

A critical aspect of managing the response phase is understanding the demand for aid, which can be classified into three categories based on frequency:

- One-time demand: Classified as urgent, single-fulfilment needs that must be satisfied once to maximize immediate survival.
- Continuous demand: Classified as needs that must be satisfied constantly and without interruption to ensure daily survival and well-being.
- Periodic demand: Classified as needs that arise at regular intervals over a longer term to sustain community health and life standards.

The relative prevalence and intensity of these demand types evolve significantly through the transition from the acute to the stabilization period and into recovery. This relationship between the disaster cycle and the demand is visualized in Figure 1.3. Table 1.1 provides a detailed classification of these demand types, listing concrete examples for each category.

Although the humanitarian logistics literature addresses various demand types, they are frequently not categorized explicitly, and their distinct characteristics are often overlooked in modeling. This gap is critically important, as evidenced by recent large-scale disasters such as the 2023 Kahramanmaraş earthquakes in Türkiye. The unprecedented scale of destruction led to the emergence of novel MSUs and new demand types (e.g., the widespread need for hemodialysis due to crush syndrome [19]–[20] and the need for hygiene facilities and regular mental health support), highlighting the necessity for advanced models. To address this, we review the literature through the lens of demand types and modeling approaches.

	Pre-disaster Era		Disaster	Post-disaster Era	
Phases	Mitigation	Preparedness		Response	Recovery
Sub-phases				Acute	Stabilization
Demand types				One-Time or Continuous	Continuous or Periodic

Figure 1.3: Disaster management timeline showing phase transitions and demand

Table 1.1: Classification of demand types

	One-time Demand	Continuous Demand	Periodic Demand
Examples of Demand	- Shelter needs (tent, blanket, pillow, etc.) - Clothes, shoes - Initial rescue supplies (tools, flashlights, etc.) - Basic medical kits	- Food - Water bottles - Emergency healthcare - Hygiene (WC) - Pharmacy (ongoing medication) - Power generation (portable generators) - Communication systems (emergency broadcasting)	- Personal Care (Long-term Medical Treatment) - Maintenance (facilities upkeep) - Library/Child-friendly areas - Hygiene (Showers, laundry) - Mental health counseling - Health (routine medical, dental check-ups)

1.1 Motivation

In this dissertation, we formally define different demand types based on their frequency in the response phase. We also develop mathematical models to capture the distinct characteristics among those demand types. For one-time demand, the models address urgent and non-recurrent needs. We enforce uninterrupted service delivery under resource and capacity limits for continuous demand. We finally introduce a long-horizon scheduling for periodic demand to ensure regular service visits with parametric control on visit frequency and duration.

The proposed approach is relevant to multiple stakeholders. Disaster response planners, especially AFAD and large NGOs, can effectively deploy MSUs in the response phase. They can allocate the scarce resources across different demands

while keeping service disruptions to a minimum. Municipalities can coordinate MSU operations when access is unavailable due to infrastructure damage. The academic community gains a modeling framework which contributes to real-life practices. Also, we provide new research areas to be explored and strengthen the practical relevance. Finally, policy makers can use the resulting insights to design and evaluate response protocols that account for the evolution of demand in disaster response.

The contribution is practical as well as methodological. In large-scale events such as the Kahramanmaraş earthquakes, the models support the routing of mobile clinics, the scheduling of generator operations to widen power coverage, the rotation of hygiene services, and the prioritization of mental-health support in high-need areas. Using quantitative allocation methods rather than ad-hoc decisions helps to avoid service disturbances. The provided formulations are valuable primarily for a large-scale and complementary service system.

The dissertation introduces a unified perspective that explicitly models transition between demand types along the response phases. As urbanization increases the frequency and severity of disasters, the need for such tools becomes critical. The broad-scale aim is to use optimization tools to support robust, evidence-based decisions under severe constraints.

We provide different problem formulations for different demand types; however, it should be noted that they share some underlying principles. For consistency, those principles, such as coverage and demand functions, are kept notationally consistent throughout the thesis. Even though we use the same parameter notations for the coverage, the superscripts denote the service units' varying characteristics and their respective sets. Due to the problem definitions, the set notations for each demand type change, but the meaning of coverage remains the same. We adopt a similar approach for the demand functions changing over time. Even though the concept of demand change over time is used for different demand types, we adapt parameters to

capture the unique characteristics of each demand type due to different formulations. Due to the problem dynamics of each demand type, the mathematical formulations change along with the decision variables, objective function, and constraints. Therefore, the decision variables in this thesis are defined chapter-specifically. The same notation in decision variables may represent different entities across chapters. However, we tried to maintain consistency in the notations of parameters regarding the proposed problems.

The remainder of this thesis is organized as follows. Chapter 2 provides a comprehensive review of humanitarian logistics and selective routing problems. Based on our classification, we review and segregate the papers in disaster logistics literature based on demand-based services and methodological approaches. Due to scarce resources, those problems are generally modeled under the Selective Routing Problems in the literature. Therefore, we further provide another classification of the selective routing problems in the literature. Chapter 3 details all datasets utilized throughout the thesis. Chapter 4 provides a formal definition of the problem and a discussion of the complexity of one-time demand. We later propose three mathematical formulations and introduce the parameter settings and datasets to be used for the computational study. We compare models under different demand functions with various datasets and derive insights from the computational studies. Also, we provide a summary and conclusion of the findings of Chapter 4. Chapter 5 provides a formal definition of the problem along with a discussion of the complexity of continuous demand. We introduce the parameter settings and datasets. We also provide a mathematical formulation for continuous demand. We analyze the problem with different coverage perspectives (binary and gradual). We derive insights from the computational studies conducted and present a conclusion for Chapter 5. Chapter 6 provides a formal definition and discusses the complexity of periodic demand. We also present notation and propose mathematical models. We prove that they are reformulations of each other. We also provide details on Benders reformulation, since some mathematical formulations were generated from Benders reformulation. We introduce the datasets

and functions to be used in the experimental setup. We further provide a detailed comparison of our mathematical models, followed by a sensitivity analysis on parameters and an analysis of demand functions with the best-performing model. We also provide a case study on the Hatay/Defne dataset and present a conclusion for Chapter 6. Finally, we summarize the thesis contributions and practical implications and present the conclusions derived from our study in Chapter 7, where we discuss the directions for future research.



Chapter 2

Literature Review

This chapter provides a broad literature review in humanitarian and disaster logistics by categorizing based on the proposed demand types. We classify the literature to provide a comprehensive understanding of how demand types have been addressed and mathematically modeled. We present the literature on one-time, continuous and periodic demand in Sections 2.1, 2.2 and 2.3, respectively. Furthermore, due to the scarcity of resources, we extend the literature review by analyzing the Selective Routing Problems and providing a unifying framework for these problems in Section 2.4. Finally, we integrate the proposed demand types within the selective routing literature.

2.1 One-time Demand

In the immediate aftermath of a disaster, during the *acute period*, the primary focus is on addressing urgent, single-fulfilment needs to maximize the victims' survival. We refer to them as one-time demand. They include shelter and clothing to protect

victims from environmental conditions, as well as initial supplies such as tools and flashlights, which are required by search and rescue teams. Such demand has a short-term nature and is designed to address urgent, non-recurring, one-time demands. From a modeling perspective, one-time demand can be modeled as a time-dependent variant of the orienteering problem, where profit is defined as a function of time [21]–[22]. This approach directly captures the urgency of these demands, which is a defining characteristic of the acute period.

Santini and Archetti(2023) [23] propose the Hazardous Orienteering Problem as a stochastic variant of the Orienteering Problem in the context of routing hazardous material. In the Hazardous Orienteering Problem, visiting “hazardous” or “time-bomb” consumers risks losing the entire earned profit of the vehicle. Thus, there is a chance of profit loss, which increases with the number of hazardous clients visited and the time spent in transit following each visit to a hazardous customer. During debris removal, hazardous materials, such as asbestos, can be classified as a one-time demand due to their non-recurring nature. Erdoğan and Laporte(2013) [21] study the Orienteering Problem with Variable Profits, with an application area in humanitarian logistics where a helicopter is looking to collect information and find survivors. In this application, multiple passes or longer stays at target locations increase the expected number of survivors that can be found. Moreover, Afsar and Labadie(2013) [24] introduce the Team Orienteering Problem with Decreasing Profits in the application of search and rescue operations in the aftermath of an earthquake. Similarly, Yu et al.(2022) [25] introduce the Team Orienteering with Time-Varying Profit via its application in humanitarian search and rescue operations. The rescue teams help trapped people in multiple locations, and the survival rate of people depends on the time that rescue teams spend at each location. In this context, they present a variant of the team orienteering problem with varying service and arrival time varying times, resulting in profits.

Search and rescue operations are considered a core emergency healthcare need and

align with continuous demand. However, the problems in [24, 25] are mathematically modeled as one-time demand to maximize the number of survivors, with the probability of success decreasing over time. These demands are highly time-critical; delays exponentially reduce their utility. The probability of rescuing trapped victims decreases rapidly during the post-disaster phase. Yu et al.(2022) [25] model exponential decay in survival probabilities, while Afsar and Labadie(2013) [24] assume linearly decreasing profits per visit. On the other hand, Erdoğan and Laporte(2013) [21] use both convex and concave profit collection over time.

Additionally, Dewilde et al.(2013) [26] propose the Traveling Repairman Problem with Profits as a routing problem in the aftermath of disasters. After the earthquake, they considered routing locations that were in urgent need of medicine to increase the number of people who survived. Regarding demand, during the *acute period* the need for medicine can be categorized as a one-time demand for a basic medical kit. Basic medical kits fall under one-time demand because they are used exclusively for initial treatment. Ekici and Retharekar (2013) [22] study the Multiple Agents Maximum Collection Problem with Time-Dependent Rewards with the application on the last mile distribution of the relief chain. They aim to distribute supplies from local distributors to people in affected areas after a disaster to increase the number of saved lives. They consider the materials distributed as an urgent need in the acute period; therefore, mathematically, they present a one-time demand.

Overall, the existing literature on one-time demand often models problems as time-dependent variants of the Orienteering Problem, focusing on maximizing collected profit or satisfied demand where utility diminishes over time. However, the literature does not explicitly integrate the *inventory constraints and the necessity of reloading mobile units* into time-dependent routing and service duration decisions.

2.2 Continuous Demand

Similar to one-time demand, continuous demand also emerges in the immediate aftermath of a disaster. However, unlike one-time demand, continuous demand must be satisfied continuously without interruption. Such demand includes food and water distribution, emergency healthcare, and personal hygiene needs, such as access to restrooms or bathrooms. Satisfying such demand is essential during both the *acute* and *stabilization period*, since they ensure consistent fulfilment of the daily needs of victims. From a modeling perspective, continuous demand can be modeled as variants of the Maximal Covering Location Problem (MCLP) and the Covering Tour Problem (CTP), aiming to serve as many people as possible with scarce resources but highlighting continuity of those demands [27]–[28].

Balcik and Beamon(2008) [29] study pre-disaster and post-disaster relief funding for quick-onset disasters. They propose a variant of MCLP to integrate location and inventory decisions of multiple items with different criticalities. They also include response time requirements and capture budgetary and capacity constraints. Their problem is categorized under continuous demand due to its modeling perspective and continuity of the supply system. Ozdamar et al.(2004) [30] examine macro-level logistics planning in emergencies while proposing a hybrid problem that integrates the multi-commodity network flow problem and the vehicle routing problem. Their problem aligns with continuous demand due to recurring delivery of resources, even though they tackle the problem as one-time demand over each period. Berman et al.(2009) [31] consider a defensive MCLP, assuming that one of the network links will become unusable due to a natural disaster. In this case, they locate new facilities in order to cover more people in the disaster area. Although they do not explicitly specify the type of demand provided by the facilities, the problem is categorized as continuous demand from a modeling perspective.

Hodgson et al.(1998) [27], Hachicha et al.(2000) [32], and Doerner et al.(2006) [28] study variants of CTP in the context of routing mobile healthcare delivery teams. In the studies, the healthcare teams can be considered a continuous demand since the proposed demands are expected to operate uninterrupted over a long period. Nolz et al.(2010) [33] discuss the distribution of providing medicine, food, shelter, and drinking water for an area affected by a disaster. Naji-Azimi et al.(2012) [34] study the distribution of survival items to affected people by locating temporary satellite relief centres and routing relief vehicles. Afsar et al.(2014) [35] propose Generalized Vehicle Routing Problems in the application area of disaster relief operations. They aim to deliver resources such as medical staff or equipment to the disaster zone by dropping them off by plane. Allahyari et al.(2015) [36] also propose Multi-Depot CTP to distribute items to the people in an area affected by a disaster. They use the coverage concept to allocate unvisited customers to the nearest visited customer due to scarce resources. Alinaghian et al.(2019) [37] consider a dynamic tour covering model to locate temporary relief centers and route air rescue vehicles to distribute relief supplies. Mete and Zabinsky(2010) [38] study the stochastic optimization approach to select the storage areas for medical supplies and required inventory levels for each type of medical supply. Their objective function is to minimize the transportation duration and the unsatisfied demand by limiting the penalties incurred from unsatisfied demand.

Across these studies, it is evident that most problems related to continuous demand mathematically align with variants of the MCLP and CTP. Furthermore, existing studies on MSUs treat demand as binary (fully covered or not), overlooking the diminishing utility of coverage with distance. The studies like [29], [30], [27]–[33], [35] assume binary coverage (full/no coverage). Even though the binary coverage concept is useful, it cannot fully capture the demand change in real life. Thus, the gradual coverage is introduced by [39] to better reflect real-life demand decay. It has application areas such as school attendance, healthcare participation, and emergency response. There are numerous studies focusing on the gradual decay

or gradual coverage functions, and several forms have been utilized throughout the literature. Researchers have proposed various decay functions—including stepwise linear [40], uniform decay [41], and Fermi-type coverage functions leading to nonlinear forms [42]. These functions have been applied across diverse contexts, including preventive healthcare [43], where the willingness to participate declines with increasing travel distance; ambulance location [44], where survival rates decrease with longer response times; and supply chain and recycling logistics [45], where service adoption depends on the distance.

In summary, the literature on continuous demand primarily utilizes MCLP and CTP variants, with a growing emphasis on gradual coverage functions to reflect diminishing demand with respect to distance. In this thesis, we address the continuous demand problem by incorporating heterogeneous MSUs. We also consider the coverage concept through the a_{ij}^u parameter to increase accessibility to services and to offer a realistic representation of uninterrupted service delivery for such needs in the response phase. Further, there exist some problems in the literature integrating the *inventory constraints* into MCLP to be closer to real-life practices.

2.3 Periodic Demand

The final category is periodic demand, which plays crucial role during the *stabilization* and particularly the *recovery period* of the disaster cycle. Such demand includes long-term treatments, maintenance, providing child-friendly areas, shower and laundry needs, and health check-ups. They improve the long-term life standards of the affected community, particularly in terms of health, hygiene, and education. Unlike continuous demand, they arise at regular periods. Those types of problems have been extensively studied within the healthcare context. [46] –[47] are some notable studies, including the scheduling of long-term medical treatments for patients.

Steeg and Schröder(2008) [48] propose a multi-period home healthcare problem to maximize the nurse–patient loyalty. González et al.(2023) [49] present a multi-period location-routing problem to select the locations, schedules, and routes of the mobile clinics. They use a multi-period representation to capture the notion of continuity of care. Nickel et al.(2012) [50] propose a home healthcare problem that seeks weekly plans for nurses and present a two-stage approach to solve it. Savaşer and Kara(2022) [51] propose a periodic location routing problem to determine monthly visit schedules for doctors via mobile healthcare units. They determine the location of the base healthcare facilities, which will be used as depots for mobile service units, while assigning fixed periodic schedules to them and generating routes over a time horizon. They minimize the total travel distance while satisfying visit frequencies and visiting rules. Li and Teo(2019) [52] develop a two-tier optimization model for post-disaster recovery, integrating road repair scheduling (upper level) with relief logistics coordination (lower level). They provide an approach that dynamically schedules repair crews to restore damaged nodes. Tarhan et al.(2023) [53] focus on the Dynamic Disaster Personnel Routing and Scheduling problem, which involves the dynamic routing of mobile personnel. Multi-skilled teams (e.g., medics, search-and-rescue) are dynamically routed across demand points to minimize unmet demand, completion times, and risk. They also categorize demand, where it can be discrete or continuous. Moreover, Ozmemiş et al.(2025) [54] develop a robust optimization framework for post-disaster hemodialysis care, integrating allocation of hemodialysis centers to treatment schedules, assignment of patients to sessions, and transportation routes under uncertain travel times. In order to incorporate travel time uncertainty, they propose an integer programming model based on the shortest path problem and develop an efficient decomposition-based constructive heuristic and matheuristic to solve the resulting problem.

Health- and medical-related needs can be considered one-time demands if they are non-recurrent. This includes distribution of basic medical kits, initial rescue supplies, and search and rescue operations in small-scale disasters. Search and rescue

operations are conducted in the acute period and typically occur once per affected location to save as many people as possible. Once the site has been cleared and all survivors have been accounted for, the demand will cease. However, it should be noted that if a regular resupply is required due to ongoing medical needs, it is considered emergency healthcare. As seen in the categorization, emergency healthcare is considered a continuous demand due to the sustained need for treatment. Additionally, in large-scale disasters, search and rescue operations often require continuous efforts to address different locations sequentially. For instance, after an earthquake with aftershocks, search and rescue operations may need to be repeated, resulting in continuous demand. On the other hand, although emergency healthcare is often urgent and continuous throughout the stabilization period and recovery phases, it can shift to a periodic demand. Emergency healthcare is not required instantaneously throughout the recovery phase, as the chaotic environment has eased. Thus, healthcare needs can be addressed through planned routine services. This evolution highlights the change in demand during the disaster response phase.

The demand evolution can be observed also in the domain of communication, which serves as the central nervous system of any disaster response. Initially, in the acute phase, the primary demand is often for a rapid, one-time intervention to restore a minimum level of connectivity. Such demand could involve the deployment of drones equipped with communication relays to provide temporary coverage over a critical area like a collapsed hospital or an isolated community, or a single satellite phone delivery to a command post. The goal here is singular and time-bound: establish an initial communication lifeline.

Following the acute phase, as response efforts stabilize and transition into stabilization phase, the demand shifts to a continuous one. The temporary solutions must be replaced or supplemented by reliable systems. This entails the restoration of mobile networks, the establishment of continuous satellite internet feeds for field hospitals and coordination centers, and the maintenance of radio networks for aid

workers. This continuous communication demand is indispensable for coordinating long-term logistics, disseminating public safety information, enabling financial services, and supporting the socio-economic recovery of the affected population. Thus, much like healthcare, communication needs are not static; they transform from one-time demand into continuous demand, mirroring the broader progression of disaster management phases.

Regarding periodic demand and the implicit routing structure of our problem, we also include the literature on the Periodic Vehicle Routing Problem (PVRP) and Periodic Location-Routing Problem (PLRP). While our problem regarding periodic demand does not optimize explicit travel routes or routing costs, it shares structural similarities with the PVRP and PLRP through its scheduling of repeated visits under frequency and spacing constraints. We reference these works (e.g., [55], [56], [57], [58]) only to position our problem within the broader class of periodic service-scheduling problems. Unlike traditional PVRP formulations, our problem abstracts away explicit routing, focusing instead on the temporal deployment and duration decisions of mobile service units.

The PVRP, rooted in the work of [55], and PLRP, which is rooted in [56]. The literature on the PVRP is extensive, and the surveys by [57] and [58] are referred to the interested reader. The PVRP literature has recently integrated new variants such as PVRP with service choice [59] and the Flexible PVRP [60]. More recently, for the Flexible PVRP, [61] propose a memetic algorithm implemented through a population-based approach that includes genetic and local searches to find a suitable sequence of visits. They achieve significant improvements in solution quality and execution time.

The PVRP with service choice is motivated by delivery operations for interlibrary loan items, and the service is characterized by the schedule of visits and is associated with visit frequency. In this problem, they determine the visit frequency on a

predetermined schedule, the routing configuration, and the amount delivered to a node while putting a lower bound on the number of visits per period and allowing nodes to be visited by multiple vehicles. On the other hand, the Flexible PVRP is motivated by the delivery operations, and it establishes a distribution plan to serve customers over a given planning horizon. Unlike PVRP with service choice, the Flexible PVRP is much more flexible regarding its dynamics since there is no fixed frequency, schedule, or quantity to be distributed while allowing multiple visits to each customer over the planning horizon.

In the literature, periodic demand is discussed under different contexts. With Flexible PVRP and the PVRP with service choice, new perspectives on the periodic demand concept have been introduced. We build upon these established problems by applying them to the humanitarian context. Therefore, we focus on the *frequency and duration* of services to ensure that *all* affected communities receive their minimum requirements over the long-term recovery phase.

2.4 Selective Routing Problems¹

Along with classifying the literature based on demand types, we also provide a unifying framework for Selective Routing Problems (SRP), which aims to determine optimal vehicle routes to serve a subset of vertices while covering another subset. An extended version of this framework is published in [62]. In the classical routing problems, routing through every vertex may be restrictive and is not always applicable in every context. In many real-world contexts, like disaster response and mobile healthcare, service can be provided by visiting a location directly or by “covering” it from another visited location. For example, the Covering Tour Problem [63] finds an optimal tour that covers a set of locations within a radius. At the same time, the

¹An extended version of this subsection is published in EJOR [62]

Orienteering Problem [64] selects a subset of locations to visit to maximize collected profit under a budget. We classify SRPs according to their *selectiveness degree*.

Table 2.1: Selective routing problems considered in this framework

Problem number	Problem Names	Proposed by
0-i	TSP	[65]
0-ii	VRP	[66]
0-iii	LRP	[67]
1	Generalized TSP	[68]–[69]
2	Generalized VRP	[70]
3	School Bus Routing Problem	[71]–[72]
4	Traveling Purchaser Problem	[73]
5	Traveling Purchaser VRP	[74]
6	Covering Tour Problem/Covering TSP	[63] [75]
7	Multi-vehicle Covering Tour Problem	[32]
8	Orienteering Problem	[64], [76]
	Multi-objective Vending Problem	[77], [78]
	Maximum Collection Problem	[79]
	Selective TSP	[80]
	Bank Robber Problem	[81]
9	Multiple Tour Maximum Collection Problem	[82]
	Team Orienteering Problem	[83]
10	Prize Collecting TSP	[84]
11	Prize Collecting VRP	[85]
12	Single VRP with Selective Backhauls	[86]
	Single VRP with Deliveries and Selective Pickups	[87]

continues on next page

13	Single VRP with Fixed Delivery and Optional Collections	[88]
	One-commodity TSP with Selective Pickup and Delivery	[89]
	VRP with Deliveries and Selective Pickups	[90]–[91]
	Selective Pickup and Delivery Problem	[92]
14	Covering Salesman Problem	[93]
15	Generalized Orienteering Problem	[94]
16	Generalized K-Peripatetic Salesman Problem	[95]
17	Profitable Tour Problem	[96], [97]
18	Multi-vehicle Profitable Tour Problem	[98]
19	Traveling Circus Problem	[99]
	Median Tour Problem	[100]
20	Maximal Covering Tour Problem	[100]
21	Geometric Covering Salesman Problem/TSP with Neighbourhood	[101]
22	Single Vehicle Routing Allocation Problem	[102]
23	Orienteering Problem with Variable Profits	
	Orienteering Problem with Arrival Time Dependent Profits	[103]
	Traveling Repairman Problem with Profits	[104]
	Orienteering Problem with Service Time Dependent Profits	[105]
24	Team Orienteering Problem with Variable Profits	

continues on next page

	Team Orienteering Problem with Arrival Time Dependent Profits	[106]
	Team Orienteering Problem with Decreasing Profits	[107]
	Team Orienteering Problem with Service Time Dependent Profits	[108]
25	Quota TSP	[81]
26	Median Cycle Problem	[109]
27	Time Dependent Orienteering Problem	[110]
28	Time Dependent Team Orienteering Problem	[111]
29	Ring Star Problem	[112]
30	m-Ring Star Problem	[113]
31	Close-Enough TSP	[114]
32	Close-Enough VRP	[115]
33	Attractive TSP	[116]
34	Location and Selective Routing Problem	[117]
35	Min-Max Selective VRP	[118]
36	Generalized LRP with Profits	[119], [120]
37	Generalized Covering Salesman Problem	[121]
38	Share-a-Ride Problem	[122]
39	Clustered Orienteering Problem	[123]
40	Clustered Team Orienteering Problem	[124]
41	Family TSP	[125]
42	Selective VRP with Integrated Tours	[126]

continues on next page

43	Selective Assessment Routing Problem	[127], [128]
44	The Set Orienteering Problem	[129]
45	Generalized Median Tour Problem	[130]
46	Location-or-Routing Problem	[131]
47	Generalized Close-Enough TSP	[132]

We distinguish two concepts:

- **Coverage (Service):** The act of providing service to a vertex. A vertex is considered *served* if it is covered.
- **Visiting:** The tool used to provide coverage. A vertex can be covered *by being visited* or *from another visited vertex*.

This leads to the core objective of an SRP: to determine optimal vehicle routes to serve a *subset of vertices to be covered* (set W) by either visiting them directly (from set V) or by covering them from a visited vertex. Therefore, we establish how a visited vertex can cover another.

- **Coverage by Visit (V):** A vertex must be visited to be served. ($S_i = \{i\}$)
- **Coverage by Radius (R):** A vertex is served if it is within a coverage radius r of a visited vertex. ($S_i = \{j \in V \mid d_{ij} \leq r\}$)
- **Coverage by Clusters (C):** A vertex is considered to be served if a certain number of vertices in its cluster are visited. ($S_i = \{j \in V \mid j \text{ is in the same cluster of } i\}$)
- **Coverage by Allocation (A):** A vertex is served by being allocated to one of the visited vertices (e.g., assigned to a depot or a mobile unit). ($S_i = V$)

Further, we define the presence of mandatory vertices:

- $\bar{V}$: Set of vertices that *must be visited* (e.g., central depots).
- $\bar{W}$: Set of vertices that *must be covered* (e.g., high-priority disaster zones).

Problems can be classified based on their *selectiveness degree* concerning these sets. This ranges from *full selectiveness* ($\bar{V} = \emptyset, \bar{W} = \emptyset$), where all visit/cover decisions are optimized, to *zero selectiveness* ($\bar{V} = V, \bar{W} = W$), which corresponds to classical routing problems where all must be visited.

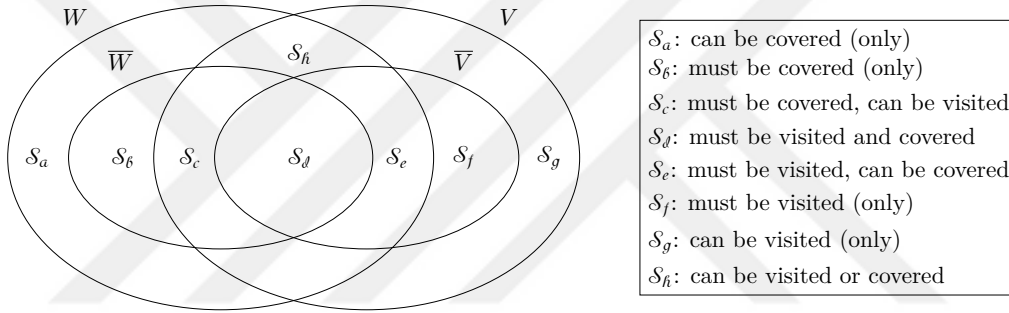


Figure 2.1: Relationships between the sets V , $\bar{V}$, W and $\bar{W}$

We organize routing problems through a three-level hierarchical classification. At the first level, we use **coverage selectiveness** to establish the degree of freedom in serving vertices, ranging from full selectiveness ($\bar{W} = \emptyset$), through partial selectiveness ($\bar{W} \subset W$), to zero selectiveness ($\bar{W} = W$). They present that there are no mandatory vertices to cover, some vertices require coverage, and all vertices must be covered. Further, the second level defines **visiting selectiveness**, which governs the flexibility in choosing vertices to visit, spanning from full selectiveness ($\bar{V} = \emptyset$) to partial selectiveness ($\bar{V} \subset V$), to zero selectiveness ($\bar{V} = V$). Finally, the third level establishes the **structural relationship between vertex sets**, distinguishing between problems with disjoint sets ($W \cap V = \emptyset$), subset relationships ($W \subset V$), and identical sets ($W = V$). We investigate SRPs according to these nine classes.

For every class, we identify the most general problem. These *main* problems have less restrictive properties and broader definitions.

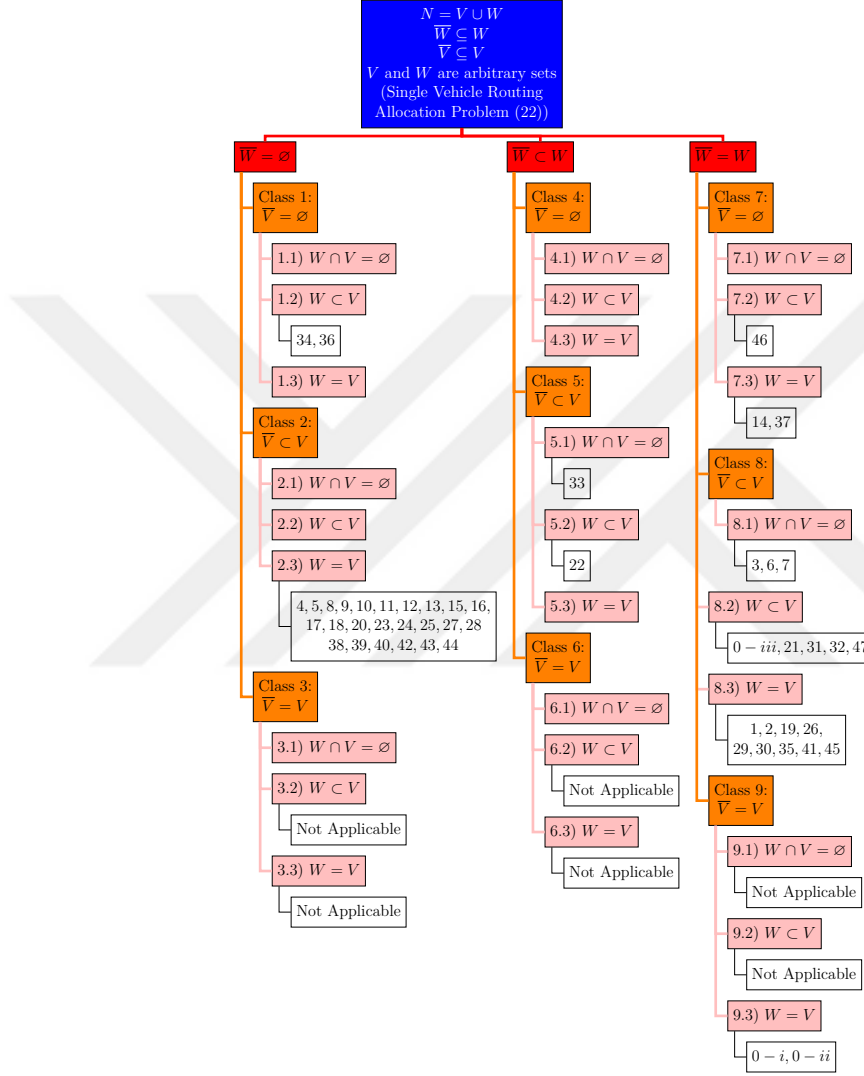


Figure 2.2: Classification diagram for selective routing problems

This research helps identify potential future research areas. Our classification in Figure 2.2 shows that valid SRPs have yet to be studied. In particular, according to our classification, Classes 1, 3, 4, 5 and 6 have not yet been explored and offer potential research areas. While we have not identified any publications in these

classes, each class represents a valid research area.

In Class 1 (subclasses 1.1 with $W \cap V = \emptyset$ and 1.3 with $W = V$), consider an example with the route of a new bus line being designed with a set of candidate stations, where each station can cover certain regions. However, not all candidate stations may be on the route. Also, there may not be any initial or terminal vertices, as an arbitrary station on the resulting route can be selected as the initial point. If the problem is defined with school buses and houses where children are picked up from the nearest visited house, we have $W = V$. When this problem is defined with houses and separate bus stations, it yields the definition of $W \cap V = \emptyset$. If there is also an option to visit arbitrary stops or intersections of coverage radii of houses, as in the discretization of continuous-space problems, then $W \subset V$. Therefore, these three relationships between sets W and V do not affect the full selectiveness of the problem and can be presented as subclasses of Class 1. As another application in Class 1, consider, for example, mobile healthcare facilities maximizing their reach to the public in a humanitarian context. Such facilities may visit potential locations with medical staff and equipment to service the impacted population. Similar to the school bus routing example, while visiting a location, the same neighbourhood may receive service by the same mobile unit, in which case we have $W = V$. If the stops of the healthcare facility and the households are distinct sets, we then have $W \cap V = \emptyset$. We also have the $W \subset V$ case when mobile facilities visit the intersections of coverage radii of households.

According to our classification, Class 4, regarding partial selectiveness in terms of vertices to cover ($\overline{W} \subset W$) and full selectiveness in vertices to visit ($\overline{V} = \emptyset$), has not been studied, even though it corresponds to a valid application area. For example, determining a bus route including the stations near the contracted malls yields the classification of $\overline{W} \subset W$ and $\overline{V} = \emptyset$. Similarly, there are problems in Class 5 that have not been investigated. For example, $W \subset V$ can be defined for a bus routing problem with mandatory stops in certain schools ($\overline{W}$) and certain stations

$(\overline{V})$, which represents an interesting application and a future research topic in this class. Moreover, Classes 3.1 and 6.1 regarding zero selectiveness in terms of vertices to visit ($\overline{V} = V$) and partial and full selectiveness in terms of vertices to cover ($\overline{W} \subset W$ and $\overline{W} = \emptyset$) have not been studied. When W and V are not identical sets, vertices in V have capacities and vertices in W are covered by allocation; visiting every vertex in V does not imply covering all vertices in W . Therefore, there would still be the question of which vertices to serve.

It is worth highlighting that, concerning both current and prospective research areas, there does not exist a pattern or a common relationship among sets from $\mathcal{S}_a$ through $\mathcal{S}_h$. This lack of pattern suggests that each subclass represents a distinct and unique research area. Moreover, the most studied subclasses are 2.3 and 8.3, which are two of the subclasses with partial selectiveness in terms of vertices to visit and $W = V$. In real-life practices, the set of vertices that can be visited and the set of vertices that can be covered are generally identical due to their direct applicability.

As shown in Figure 2.2, the subclasses 3.2, 3.3, 6.2, 6.3, 9.1 and 9.2 are defined as impractical. We have noted that they share a combination of non-empty sets $\mathcal{S}_d$, $\mathcal{S}_e$, and $\mathcal{S}_f$, alongside other sets being empty. This characteristic results in a lack of selectiveness in the vertices to cover and the vertices to visit, which limits their practical relevance.

Understanding the connections between different problem classes and their practical implications provides insights into which formulations are best for real-life challenges and which are more theoretical. Concurrently, it allows for the identification of problem classes that may be more abstract or theoretical in nature, providing clarity on their potential utility within practical contexts.

In this thesis, we provide problem definitions for different demands based on frequency during disaster response and we establish their connection with the SRP

framework presented in [62]. As previously mentioned, **one-time demand** can be modeled as a *service time-dependent orienteering problem* where the value of service decays with delay. Also, **continuous demand** can be formulated as a *gradual decay maximal cover problem* where coverage quality diminishes gradually over distance. According to the classification, those problems can be considered under the class of $\overline{W} = \emptyset$ and $\overline{V} \subset V$, which presents full selectiveness regarding vertices to cover, meaning that there is no requirement to cover any vertex. All problems in this class include decisions on the vertices to cover and visit with $W = V$. Further, the **periodic demand** can be characterized under two different classes regarding the choice of its parameters. It can be characterized under the class of $\overline{W} = \emptyset$ and $\overline{V} = \emptyset$ if the minimum requirements on visit frequency and duration for each district and service types are considered as zero. Since there are no restrictions on starting and ending points, there are no mandatory visits or coverage requirements, providing full selectiveness in both visiting and coverage decisions. However, with those requirements, the problem falls under the class of $\overline{W} = W$ and $\overline{V} = V$ since every node should be visited for a given minimum requirements on visit frequency, elapsed time and visit duration.

To summarize the connection within our framework:

- One-time demand problems typically fall into the subclass 2.3 with full coverage selectiveness ($\overline{W} = \emptyset$) and partial visiting selectiveness ($\overline{V} \subset V$), as the goal is to select which points to visit to maximize time-dependent reward, without mandatory coverage points.
- Continuous demand problems also align with subclass 2.3 due to its full coverage selectiveness ($\overline{W} = \emptyset$) and aim to select points to visit to maximize coverage, often with $W = V$.
- Periodic demand problems are more complex. With minimum service requirements, they exhibit zero selectiveness in both coverage and visiting, aligning with subclass 9.3 ($\overline{W} = W$, $\overline{V} = V$), as all points must be served. Without

strict requirements, they allow for full selectiveness, aligning with subclass 1.3 ($\overline{W} = \emptyset, \overline{V} = \emptyset$), focusing on selecting an optimal service schedule.



Chapter 3

Datasets

In this thesis, we utilize six datasets, which are used to validate and test our proposed mathematical models. The datasets are diverse in terms of geographic scope and demographic characteristics. The real-world datasets allow us to test our models against real-life scenarios and challenges. We also have a synthetic dataset used to test models' scalability. Table 3.1 summarizes the key characteristics of datasets. We also provide a detailed description of each dataset in the sections 3.1–3.6.

Table 3.1: Dataset characteristics

Dataset	Nodes	Average population	Geographic area (km ²)
Seoul	25	379,606	605.21
Defne	37	3,810	155
Kartal	45	21,619	38.54
Istanbul	99	3,536	2,576.85
Kilis	187	454	1,642
Synthetic	1,000	554	10,000

Note: Population values were averaged across all nodes within each dataset and rounded to the nearest integer for presentation purposes.

3.1 Seoul Dataset

Seoul is the capital and the largest city in South Korea. It has an area of 605.21 km^2 and has more than 10 million inhabitants. The population data are obtained from the Turkish Statistical Institute, and the maximal demand parameters are created proportional to the nodes' populations. The data include 25 districts of Seoul, Korea. The distances between the nodes are also taken from the Seoul dataset obtained from Google Maps. According to [133], the data were calculated using the coordinates of the locations.

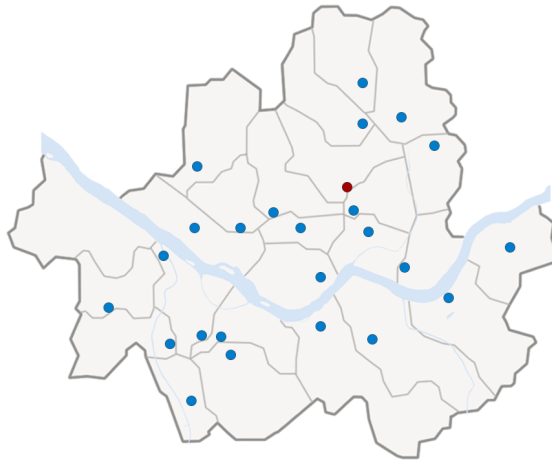


Figure 3.1: Potential locations in Seoul, South Korea

3.2 Defne Dataset

Defne is one of the 15 districts in Hatay. It has an area of 155 km² inhabitants. It is a post-disaster dataset from the 2023 earthquakes in Türkiye, detailing population and infrastructure across 37 locations, emphasizing Defne. The population data are taken from [134], and the dataset will be used for the case study.

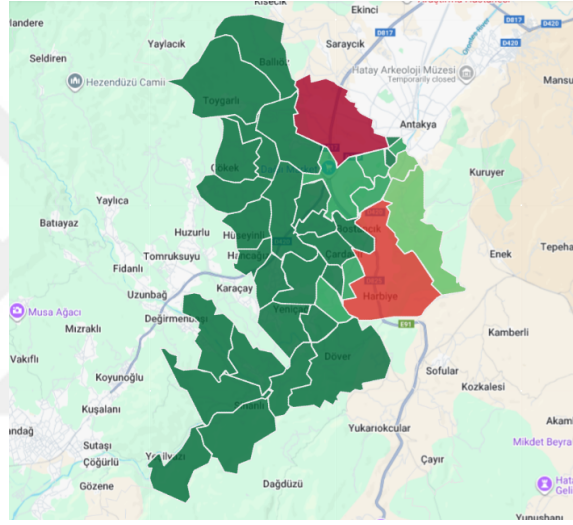


Figure 3.2: Potential locations in Defne, Hatay

3.3 Kartal Dataset

Kartal is one of the 39 districts in Istanbul. It has an area of 38.54 km² inhabitants. The dataset has 25 potential shelter locations and 20 demand points. In total, there are 45 potential locations. According to [135], the district data are obtained from Google Maps, and the population data are obtained from the Turkish Statistical Institute. The maximal demand parameters are created proportional to the districts' populations.

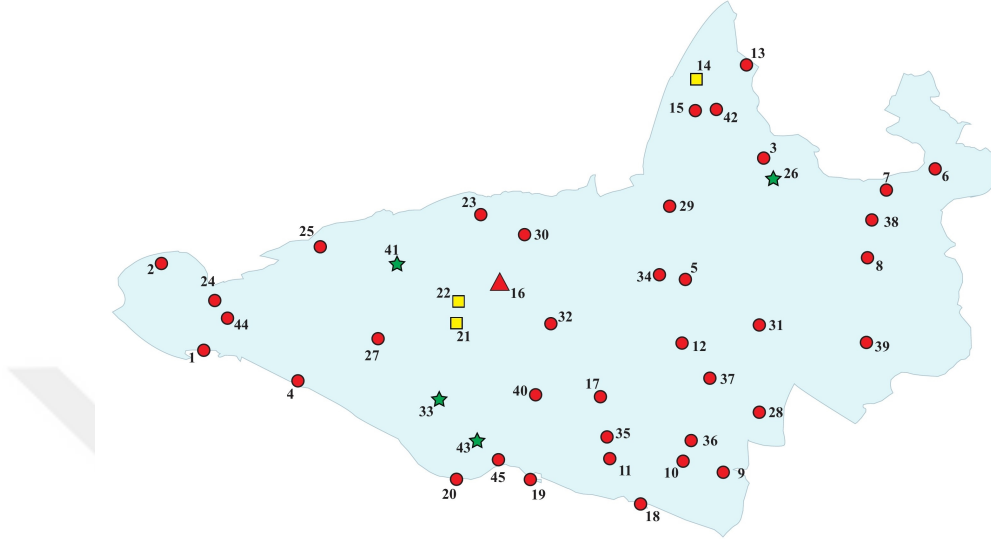


Figure 3.3: Potential locations in Kartal, İstanbul

3.4 Istanbul Dataset

The Istanbul dataset is a challenging dataset [133] that is constructed by 99 points from distinct neighbourhoods of 22 districts of Istanbul, Türkiye. The maximal demand parameters are assumed to be equal to the neighbourhood population of the point and are obtained from the population data of districts in Istanbul, taken from the Turkish Statistical Institute.

3.5 Kilis Dataset

The Kilis dataset includes the GIS coordination and population information of a municipality building, 2 hospitals, 1 Red Crescent office, and 15 schools, together with



Figure 3.4: Potential locations in Istanbul, Türkiye

109 villages and 78 neighbourhoods in the Kilis province in southern Türkiye. The population of each identified demand location was obtained from the local government in 2018. The dataset is created by [136] for a humanitarian operations project. The maximal demand parameters are created based on the districts' populations.

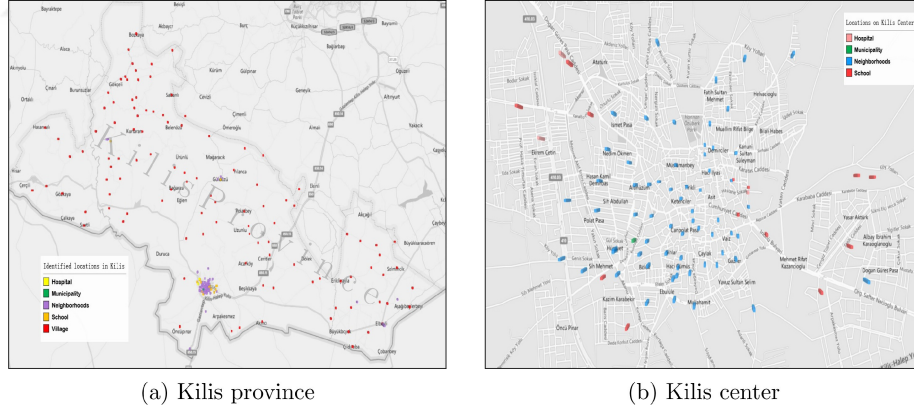


Figure 3.5: Potential locations in Kilis, Türkiye

3.6 Synthetic Dataset

The synthetic dataset is an artificial dataset with 1,000 uniformly distributed locations across a 100×100 km² area. Population and distances are generated randomly for scalability tests.

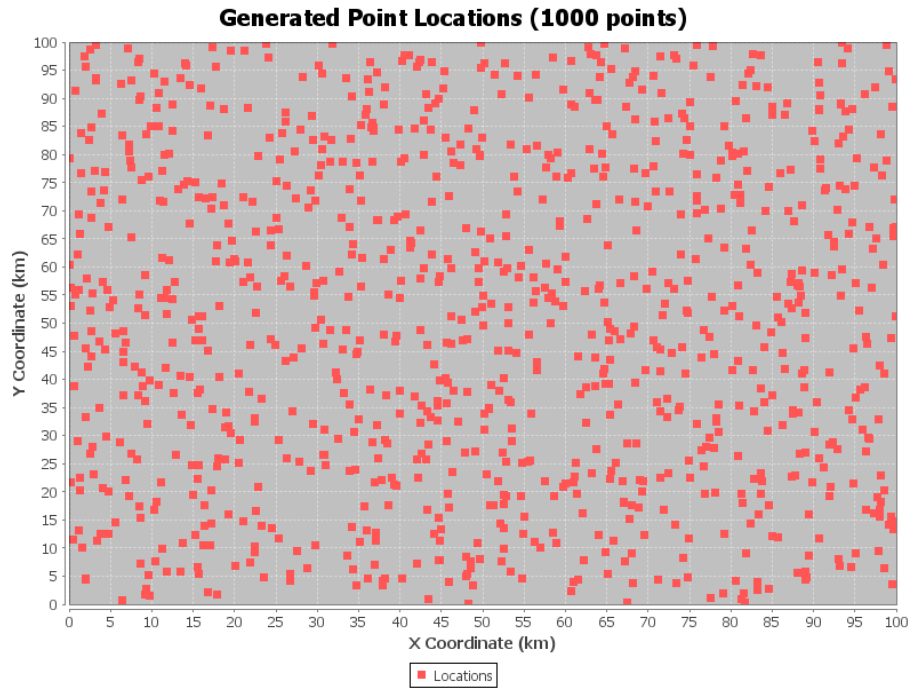


Figure 3.6: Potential locations in Synthetic dataset

Chapter 4

One-time Demand in the Post-Disaster Operations

One-time demand involves immediate and non-recurring survival needs such as shelter, clothing, hygiene kits, and first-aid supplies. These essentials provide instant support and contribute to a stable environment for victims. Crucially, they are not expected to recur once these needs are met. This section focuses on one-time demand, which is fundamental for addressing immediate physical and psychological needs.

Disasters inflict significant physical and economic damage upon affected regions. Beyond material losses, the literature consistently reports profound psychological consequences for victims, including mental health disorders such as post-traumatic stress disorder, depression, fear, and anxiety [137, 138, 139]. In the aftermath of such devastating events, victims often exhibit behaviours driven by a desire to avoid further loss, profound trust issues, and an urgent need to regain control over their lives. These emotional and behavioural responses, compounded by resource scarcity,

can lead victims to cling to available resources to ensure safety, security, and the fulfilment of basic needs. Organizations like AFAD and various NGOs play crucial roles in addressing these needs, aiming to help disaster-affected individuals restore a sense of normalcy.

Kürkçü (2023) [140] reported an audio recording alleging aid diversion, explicitly claiming that an aid truck was not fully unloaded in the Afşin district. After the allegations, the news report emphasized that AFAD coordinates aid delivery and such diversions violate established official protocols. However, the allegations provoked significant public criticism. In practical scenarios, aid trucks are often unable to retain inventory and disaster victims who are driven by fear and uncertainty often prevent them from leaving a location. Consequently, traditional multi-stop routing is not a common practice and we consider direct shipment by MSUs to reflect real-world aid distribution challenges and constraints accurately.

The remainder of this chapter is structured as follows. We formally define the One-time Demand Problem (OTDP) and analyze its computational complexity in Section 4.1. We develop three novel mathematical models in Section 4.2: Model 1 utilizing integer time durations, Model 2 which uses a binary variable for discrete time periods, and Model 3 which provides another binary variable for total duration of stays. These models incorporate a non-linear, stepwise demand function to capture time-dependent demand decrement accurately. An extensive computational study is conducted in Section 4.3 to compare these models under both uniform and decreasing demand functions, providing an analysis of their relaxations and structural components. We provide strategic insights into selected locations for CDs and MSUs along with duration and routing strategies of MSUs. Finally, we provide a conclusion drawn from the study in Section 4.4.

4.1 Formal Problem Definition

In OTDP, we determine the optimal locations for central depots (CDs), allocation of available MSUs, and duration of service provided by each MSU. We aim to maximize the total demand satisfied across locations within a given planning horizon T . Such services are highly dynamic and often decrease as needs are met. The demand is presented using a decay function to reflect the diminishing effect over time. We also implement the concept of coverage, which means that demand can be satisfied either by a direct visit from a MSU or by being within the coverage radius of a CD or a MSU. Furthermore, our models incorporate integer time durations for MSU stays in order to simplify planning and coordination in real-life disaster response scenarios.

4.1.1 Problem Setup and Notation

Consider a network $G = (V, A)$, where V represents the set of districts and A is the set of arcs connecting these districts. Let M be the set of available MSUs and m_f be the maximum number of CDs that can be established. Each CD and MSU has an associated coverage threshold, λ^f for CDs and λ^m for MSUs. They define the allowable distance within which demand nodes can receive service. The maximum potential demand per period for a location $i \in V$ is denoted by b_{fi} if a CD provides service, and b_{mi} if a MSU provides service.

In Figure 4.1, a system consisting of one CD and two MSUs is presented as an illustration of the proposed system. As illustrated in Figure 4.1, the CD is located in District 1, and two MSUs depart from this CD. It should be noted that the people in Districts 7 and 8 are within the coverage radius of District 1. Therefore, people in Districts 7 and 8 can travel to the CD in District 1. Moreover, MSUs depart from this CD and visit some districts to provide service. The first MSU visits District 6

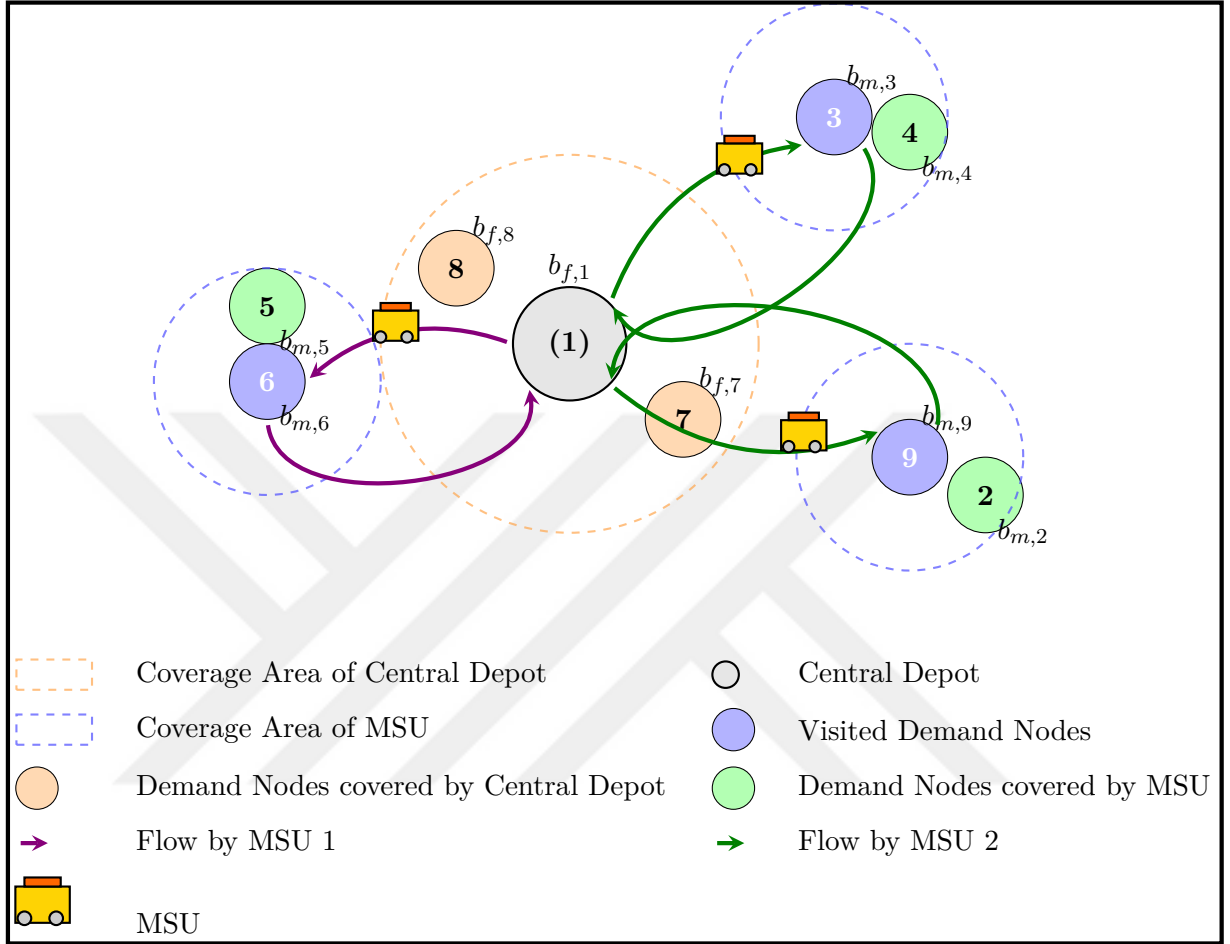


Figure 4.1: Schematic illustration of the tours of MSUs for one-time demand

and stays there as long as possible. After providing its service, it returns to the CD. The second MSU visits District 3 and stays there for some time periods. Also, people in District 4 can travel to the MSU in District 3 to get service since it is located within the coverage radius of District 3. Also, excess inventory is not allowed, then the MSU distributes all its products at District 3 before returning to the CD. The MSU must be reloaded at the CD in order to visit other locations.. After reloading, it moves to District 9 and stays there for the time left. Similar to the other cases, since District 2 is within the coverage radius of District 9, people of District 2 can

travel to District 9 for service.

The proposed system aims to enhance the availability of the service provided while also increasing the total demand satisfied. Thus, the main decisions are taken via the system as follows:

- the locations for CDs,
- the locations visited by MSUs among candidate visiting points,
- the length of stay of the MSUs

In the proposed problem, the performance measure is considered as the maximization of total demand satisfied during the disaster response.

4.1.2 Demand Function

When a MSU arrives in a district, we assume that the demand will be high. This assumption is vital for considering the immediate and critical needs of the population affected by a disaster. One of the aims of MSUs is to increase accessibility to these aids. However, it is important to note that demand will decrease over time as it is met, and the needs of people change over time. Therefore, MSUs must explore other points while adapting to changes in demand. We employ a non-linear demand function to capture the time-dependent decrement of demand. We adopt integer time durations to reflect real-world dynamics and simplify planning. This discretization allows us to linearize functions that exhibit non-linear behaviour. We specifically utilize a 2-step-function denoting the demand decrement over time. This function considers the duration of stays of the vehicles and divides them into two parts as presented in Figure 4.2. The first part represents the initial phase when the demand

is high, and the second part represents the subsequent phase with lower demand. There is no decrement in the first part; there is a lower demand in the second part. To present such a change in demand for MSUs, we use the following parameters, δ_n , ρ_n , θ_n , and α^c .

- δ_n : The time horizon is separated into smaller time segments which are presented by the breakpoints. δ_n parameter denotes the upper limit of the segment up to the breakpoint n . The breakpoints allow a discretized representation of diminishing demand. It should be noted that the upper limit for the final breakpoint should coincide with the total length of the planning horizon. Therefore, the following assumption holds for consistency, $\delta_N = |T|$. Note that $|T|$ is the total count of the time slots and N is the index of the final breakpoint.
- ρ_n : The percentage of original demand remaining in each segment (δ_{n-1} and δ_n) is presented by the ρ_n parameter. It typically takes values in the range $[0, 1]$ and it is monotonically decreasing ($\rho_1 \geq \rho_2 \geq \dots \geq \rho_{|N|}$). At the initial segment, it is assumed that $\rho_1 = 1$.
- θ_n : θ_n parameter is closely related to ρ_n since θ_n represents the marginal utility rate satisfied from the service provided by an MSU during the time segment δ_{n-1} and δ_n . In other words, it depicts the slope of demand during the time segment δ_{n-1} and δ_n . The relation between those two parameters can be denoted as follows: $\theta_n = \frac{\rho_n \times \delta_n - \rho_{n-1} \times \delta_{n-1}}{\delta_n - \delta_{n-1}}$.
- α^c : We incorporate the demand decay with respect to coverage. Therefore, we utilize a parameter α^c denoting the decrease in demand if a node is within the coverage area of a CD (α^f) or a MSU (α^m). It reflects how much the demand from indirectly served locations (those within coverage but not directly visited) is affected. Typically, α^c is a constant value in the range $[0, 1]$.

In the step function presented in Figure 4.2, the demand in the first part is high.

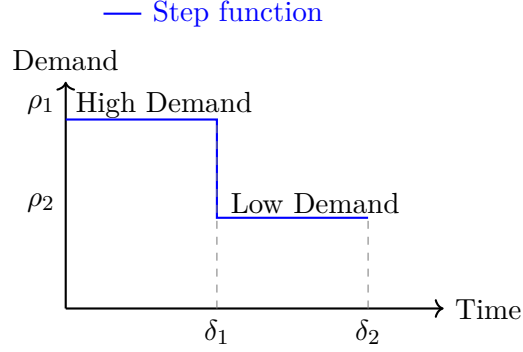


Figure 4.2: Step-wise Demand Function

Then, it may be beneficial to allocate more aid materials in order to increase the total demand satisfied. Moreover, the demand in the second part is lower due to a decrease over time. In this case, it may be more reasonable to allocate fewer resources. It can be noted that if we set $\rho_n = 1$ and $\theta_n = 1$ for all $n \in \mathcal{N}$, then we assume uniform demand over all time segments during the planning horizon. The uniform demand function is presented in Figure 4.3. We similarly assume a decreasing demand for CDs, and due to their higher functional capability and non-mobility, the demand change over time during each phase is represented as a step function of parameter b_{fj} .

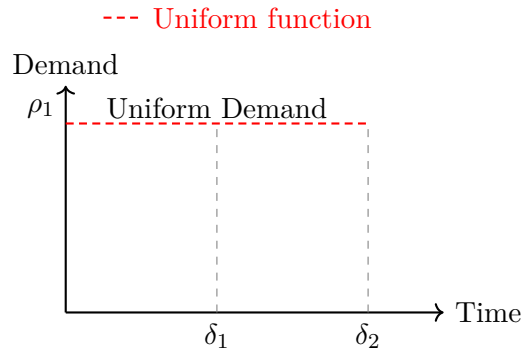


Figure 4.3: Uniform Demand Function

Another aspect of this system is that the people in the neighbourhood of a visited

district can walk to the service; this will be called 'coverage'. Therefore, providing service to non-visited points in districts within a specific distance by coverage is also possible. Due to the chaotic environment in the post-disaster period, people may be late learning about services arriving at certain distances. Therefore, we assume that there will be a change in demand in the coverage area. This assumption is to comply with real-life practices. In the proposed problem, these assumptions are given by the real-life applications of the MSUs in the disaster management cycle.

4.1.3 Problem Definition

The formal definition and computational complexity analysis of the OTDP constitute a key contribution of this chapter. We integrate location, allocation and duration decisions that captures time-dependent demand decay. We also extend the classical facility location and coverage literature by integrating its application to humanitarian context. The NP-hardness proof (Proposition 1) establishes the computational complexity of the OTDP, justifying the need for the efficient mathematical models developed in the subsequent sections.

Definition 1. *OTDP is defined as a mobile service system across a network $G = (V, A)$ over a given time horizon T . We determine the nodes to visit, allocation of nodes to MSUs and the duration of MSUs during each visit. Formally, the decision version of the problem is as follows:*

INSTANCE: $\langle G, T, |M|, m_f, \lambda^f, \lambda^m, b_{fi}, b_{mi}, \bar{c}, \underline{c} \rangle$ where: $G = (V, A)$ is a graph with V denoting the nodes and A denoting the arcs with edge travel times $c_{ij} \geq 0$ for each $(i, j) \in A$, T is the set of time periods, $|M|$ is the total number of MSUs available, $\bar{c}, \underline{c}$ are allowable duration limits for MSUs, m_f is the number of CDs to be planned, λ^f, λ^m are the coverage thresholds for each CDs and MSUs respectively, b_{fi} and b_{mi} are the maximum potential demand per period for location $i \in V$ if CDs or MSUs provide service to that location and a positive number Q .

QUESTION: Is there a service coverage that locates m_f CDs and allocates available MSUs from the set M to nodes in network G within a given time horizon T such that each node $i \in V$ is either visited or covered at most once while determining the duration of mobile units between $\bar{c}$ and $\underline{c}$ with a total coverage more than or equal to Q ?

The solution of the OTDP Problem is a triplet $\langle y, x, z \rangle$ where y is a vector of size V representing locations of CDs, x is the allocation vector of MSUs to nodes, and z is a vector denoting the service duration of MSUs at each node within the time horizon.

Consider the MCLP, which is known to be NP-Hard [141].

INSTANCE: Consider a graph $G' = (V', A')$, $p \leq |V'|$, β represent the coverage distance, weights w_i for each vertex $i \in V'$, distance $d_{ij} \quad \forall i, j \in V'$ and a positive number P .

QUESTION: Is there a set of vertices $H' \subseteq V'$ and $|H'| = p$ such that $\sum_{\substack{i: \exists j \in H' \\ d_{ij} \leq \beta}} w_i \geq P$?

Proposition 1. *The OTDP is NP-hard.*

Proof. See Appendix A. □

4.2 Mathematical Models

In this section, we present three mathematical formulations for the OTDP. The proposed models utilize different approaches to discretize time. We first provide a direct and intuitive formulation which models the duration of MSU at a location

using integer variables. Model 1 naturally handles the step-function decay. Later, we present Model 2, which introduces binary variables to indicate whether an MSU is providing service at each discrete time period t . In Model 2, we discretize time more finely for potentially greater control and modeling flexibility. Even though this allows the model to explicitly track the schedule over time, it significantly increases the number of decision variables. Finally, we propose Model 3 to achieve a tighter formulation than Model 2 while still using time-discretized binary variables. It introduces a new binary variable that indicates whether service is provided for a total duration of exactly t time periods, rather than at each period t . This change in perspective leads to a mathematically stronger formulation. It typically allows optimization solvers to find proven optimal solutions faster, even though the number of variables remains similar to Model 2.

Note that each model is provided through the vehicle indices k since it is essential to accurately represent the physical reality of MSU operations, even when vehicles are homogeneous. While the MSUs may be identical in terms of equipment and capacity, each remains a distinct physical entity with its own operational timeline and spatial constraints. The vehicle index k enables the model to track individual vehicle schedules, prevent spatiotemporal conflicts, and enforce per-vehicle limits on total working time, travel, and service duration. Also, this modelling approach control individual resource usage and routing continuity while preventing impractical or infeasible schedules.

In this section, we developed three distinct mathematical formulations to explore different modeling approaches for representing time and duration decisions in the OTDP. Each model offers unique advantages in terms of formulation strength and computational performance. Note that the advanced decomposition techniques such as Benders decomposition could potentially improve scalability for very large-scale instances. However our main focus here is to establish effective formulations that provide clear insights into the problem structure and can be solved efficiently using

standard solvers. The comparative analysis presented in this chapter demonstrates that Model 3 achieves best computational performance. Therefore, Model 3 is suitable for practical application without requiring additional algorithmic enhancements for the tested instances.

The models assume that each MSU carries sufficient supplies to meet the demand at its visited location and its coverage area during its stay. This is reflected in the objective function, which sums the satisfied demand without explicit per-unit capacity constraints. In practice, MSUs have finite carrying capacities. However, our approach is motivated by the urgent nature of post-disaster response: the main goal is to deliver as much aid as possible to meet immediate survival needs. Therefore, we model the system from a demand-driven perspective. It includes the implicit assumption that supply is continuously available at the central depot to reload MSUs. This simplifies the problem and allows us to focus on maximizing total demand satisfied and service periods.

On the other hand, the capacity is managed implicitly through two key operational decisions: (1) limiting the maximum service duration per visit ($\bar{c}$), and (2) respecting the total planning horizon $|T|$. The maximum service duration indirectly determines the volume and amount of aid distributed at a single stop and the time horizon bounds the total operational time which determines that the total aid delivered per vehicle. In other words, the duration is treated as a proxy for commodity quantity and how much to distribute is also determined alongside location and routing decisions. This simplification aligns with real-world practices where aid trucks are dispatched fully loaded with standardized kits and are expected to distribute their entire inventory before returning. However, future work could explicitly incorporate detailed vehicle capacity constraints to model scarce resource allocation more precisely.

Moreover, the proposed demand function models the aggregate decay of one-time

demand for a single demand type. In reality, there are multiple commodities with distinct demand patterns. The mixed commodity demand scenario is an important extension of our work. Our current model could serve as a foundation for such an extension by introducing commodity-specific parameters and extending the indices of decision variables with a commodity index p . Models can utilize separate decay rates ρ_n^p and coverage factors α_c^p for each commodity (p), and potentially incorporate multi-commodity capacity constraints for MSUs and CDs. This would allow for more realistic planning of aid delivery. However, we focus on the core decisions of location, allocation, and duration under a cohesive demand structure and leave the multi-commodity extension for future research.

4.2.1 Model 1

Let y_i be a binary decision variable which determines whether a CD is established at location $i \in V$ and let x_{ijk} be another binary decision variable denoting whether MSU $k \in M$, departing from a CD located at $i \in V$, is assigned to provide service at location $j \in V$. We also define z_{ijk} and u_{nijk} as integer decision variables representing the duration that MSU $k \in M$, departing from $i \in V$, spends providing service at district $j \in V$. z_{ijk} denotes the total duration while u_{nijk} denotes duration within the specific time segment δ_{n-1} and δ_n (for $n \in \mathcal{N} \setminus \{0\}$). Table 4.1 lists the sets and parameters for the problem. Accordingly, the mathematical formulation Model 1 is as follows:

Table 4.1: Sets and parameters

Sets and Parameters	
V	Set of potential service locations
C	Set of types of service units- service by CD (f) or service by MSU (m)
M	Set of MSUs
T	Set of time $\{1, 2, \dots, T \}$
$\mathcal{n}$	Set of breakpoint indices $n \in \{0, 1, 2, \dots, N\}$
m_f	Total number of CDs to be planned
λ^c	Coverage threshold for service type $c \in C$
α^c	Demand change due to coverage $\in [0, 1]$ for service type $c \in C$
c_{ij}	Travel time between $i \in V$ and $j \in V$
a_{ij}^c	$\begin{cases} 1 & \text{if } 0 = c_{ij} \\ \alpha^c & \text{if } 0 < c_{ij} \leq \lambda^c \text{ for each service type } c \in C \\ 0 & \text{otherwise} \end{cases}$
F_i^c	$\{l \in V : c_{il} \leq \lambda^c\}$: Set of districts in the allowable walking distance to district $i \in V$ which is served by service type $c \in C$
b_{ci}	Maximum potential demand to be served at location $i \in V$ by a service unit type $c \in C$
θ_n	Slope of time segment between breakpoints n and $n - 1$ ($n \in \mathcal{n} \setminus \{0\}$) to represent the change in utility taken from service by an MSU
ρ_n	Demand change due to time $\in [0, 1]$ between breakpoints n and $n - 1$ ($n \in \mathcal{n} \setminus \{0\}$)
δ_n	Upper bound of time segment up to breakpoint $n \in \mathcal{n}$ where $\delta_N = T $
T_n	Set of time periods after breakpoint $n - 1$ ($n \in \mathcal{n} \setminus \{0\}$) until end of planning horizon
$\bar{c}$	Maximum number of periods that a MSU can serve in total
$\underline{c}$	Minimum number of periods that a MSU can serve in total (with $\underline{c} \leq \bar{c} \leq T $)

Decision Variables:

$$y_i = \begin{cases} 1 & \text{if a CD is located at } i \in V \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ijk} = \begin{cases} 1 & \text{if MSU } k \in M \text{ departing from CD } i \in V \text{ is allocated to} \\ & \text{location } j \in V \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ijk} = \text{total duration of MSU } k \in M \text{ departing from } i \in V \text{ at} \\ \text{district } j \in V$$

$$u_{nijs} = \text{duration of MSU } k \in M \text{ departing from } i \in V \text{ between} \\ \text{consecutive breakpoints } \delta_{n-1} \text{ and } \delta_n \text{ where } n \in \mathcal{N} \setminus \{0\} \\ \text{at district } j \in V$$

$$\begin{aligned} (\text{Model 1}) \text{ maximize } & \sum_{i \in V} \sum_{n \in \mathcal{N} \setminus \{0\}} y_i b_{fi} (\delta_n - \delta_{n-1}) \rho_n + \sum_{i \in V} \sum_{\substack{l \in F_i^f \\ l \neq i}} \sum_{n \in \mathcal{N} \setminus \{0\}} y_i b_{fl} a_{il}^f (\delta_n - \delta_{n-1}) \rho_n + \\ & \sum_{i \in V} \sum_{j \in V} \sum_{k \in M} \left[\theta_1 z_{ijk} b_{mj} - \sum_{n \in \{1, \dots, N-1\}} (\theta_n - \theta_{n+1}) \sum_{s \in \{(n+1), \dots, N\}} u_{sijk} b_{mj} \right] + \\ & \sum_{i \in V} \sum_{j \in V} \sum_{\substack{l \in F_j^m \\ l \neq j}} \sum_{k \in M} \left[\theta_1 z_{ijk} b_{ml} - \sum_{n \in \{1, \dots, N-1\}} (\theta_n - \theta_{n+1}) \sum_{s \in \{(n+1), \dots, N\}} u_{sijk} b_{ml} a_{jl}^m \right] \end{aligned} \quad (4.1)$$

subject to

$$\sum_{i \in V} y_i \leq m_f \quad (4.2)$$

$$x_{ijk} \leq y_i \quad i \in V, j \in V, k \in M \quad (4.3)$$

$$z_{ijk} \geq \underline{c} x_{ijk} \quad i \in V, j \in V, k \in M \quad (4.4)$$

$$z_{ijk} \leq \bar{c} x_{ijk} \quad i \in V, j \in V, k \in M \quad (4.5)$$

$$y_j + \sum_{\substack{i \in F_j^f \\ i \neq j}} y_i + \sum_{i \in V} \sum_{k \in M} x_{ijk} + \sum_{i \in V} \sum_{\substack{l \in F_j^m \\ l \neq j}} \sum_{k \in M} x_{ilk} \leq 1 \quad j \in V \quad (4.6)$$

$$\sum_{j \in V} c_{ij} x_{ijk} + \sum_{j \in V} z_{ijk} \leq |T| \quad i \in V, k \in M \quad (4.7)$$

$$z_{ijk} - \sum_{n=l}^N u_{nijs} \leq \delta_{l-1} \quad i \in V, j \in V, k \in M, l \in \mathcal{N} \setminus \{0\} \quad (4.8)$$

$$u_{nijs} \leq \delta_n - \delta_{n-1} \quad i \in V, j \in V, k \in M, n \in \mathcal{N} \setminus \{0\} \quad (4.9)$$

$$x_{ijk} \in \{0, 1\} \quad i \in V, j \in V, k \in M \quad (4.10)$$

$$y_i \in \{0, 1\} \quad i \in V \quad (4.11)$$

$$z_{ijk} \geq 0 \text{ and integer} \quad i \in V, j \in V, k \in M \quad (4.12)$$

$$u_{nijs} \geq 0 \text{ and integer} \quad i \in V, j \in V, k \in M, n \in \mathcal{N} \setminus \{0\}. \quad (4.13)$$

The performance measure is represented by equation (4.1) and denotes the total demand satisfied. Note that the objective function can be written by utilizing ρ_n as follows:

$$\begin{aligned} & \max \sum_{i \in V} \sum_{n \in \mathcal{N} \setminus \{0\}} y_i b_{fi} (\delta_n - \delta_{n-1}) \rho_n + \sum_{i \in V} \sum_{\substack{l \in F_i^f \\ l \neq i}} \sum_{n \in \mathcal{N} \setminus \{0\}} y_i b_{fl} a_{il}^f (\delta_n - \delta_{n-1}) \rho_n \\ & + \sum_{i \in V} \sum_{j \in V} \sum_{k \in M} \sum_{n \in \mathcal{N} \setminus \{0\}} \rho_n u_{nijs} b_{mj} + \sum_{i \in V} \sum_{j \in V} \sum_{\substack{l \in F_j^m \\ l \neq j}} \sum_{k \in M} \sum_{n \in \mathcal{N} \setminus \{0\}} \rho_n u_{nijs} b_{ml} a_{jl}^m. \end{aligned}$$

In the mathematical model, the initial form of the objective function is utilized, but the latter form is presented as an alternative. The first part of the equation represents the demand of a node i where a CD is located. The second term considers the total demand satisfied only from locations l covered when the CD is located at location i . Those parts reflect the diminishing demand for coverage and time for CD. The third part of the constraint represents the demand of a visited node j . The fourth term considers the people receiving service only from locations l covered when the MSU is visiting location j . It reflects the diminishing demand over coverage through a_{ij}^c .

parameters. The diminishing demand over time is represented by the aforementioned step-function and is adopted in the formulation.

Constraint set (4.2) is used to provide an upper bound for the total number of CDs to be opened. Also, constraint set (4.3) prevents the service by a MSU departing from a location if there is no CD at that location. Constraint set (4.4) and (4.5) are to ensure the stay of the MSUs is limited by lower and upper bounds, respectively. Constraint set (4.6) is to ensure that each location can be served at most once. If a location j is served, there can be 4 cases: i) a CD is located at j , ii) the location j is within the coverage threshold of another CD, iii) the location j is served by a MSU departing from a CD at i , and finally, iv) the location j is within the coverage threshold of a MSU visiting the location i which is departed from a CD at l . The aim is to increase the availability of the service by reaching different areas. Constraint set (4.7) ensures that the total travel and service times do not exceed the given planning horizon $|T|$. Note that constraint set (4.7) bounds the total operational time including service and travel time for each MSU over the entire planning horizon. On the other hand, constraint set (4.5) limits the duration of any single service assignment. These constraints address different operational considerations: $\bar{c}$ may represent maximum continuous service time per location, while $|T|$ represents the total available operational period. In the special case where $\bar{c} = |T|$, Constraint (4.5) is mathematically redundant but is retained for model clarity and potential computational benefits.

Constraint set (4.8) represents the relation between total service time and the service times after the breakpoint. For each breakpoint index l , the constraint ensures that the cumulative time assigned to all segments up to $l - 1$ cannot exceed the upper bound of segment $l - 1$ (which is δ_{l-1}). Thus, it ensures that the cumulative filling respects breakpoints. Constraint set (4.9) represents the upper bound for the duration time after the breakpoint of the functions. It ensures that the service time assigned to segment n cannot exceed the length of that segment. In other words,

it acts as a capacity constraint for each duration spent at each time segment. For instance If $\delta_0 = 0$ and $\delta_1 = 4$, then segment 1 has length 4 hours. This constraint ensures that $u_{1ijk} \leq 4$. You cannot assign 5 hours of service to a 4-hour segment. Therefore, it ensures that no single segment is overfilled. Constraints (4.10)–(4.13) are domain constraints.

4.2.2 Model 2

We define q_{ijkt} as a binary decision variable which determines whether a MSU $k \in M$ departing from a CD $i \in V$ stays at district $j \in V$ at the timer period $t \in T$. The decision variable simultaneously captures location, allocation and period decisions. By incorporating $t \in T$ as an index, a new set definition T_n is introduced. T_n denotes the set of time after δ_{n-1} where $n \in \mathcal{N} \setminus \{0\}$. For instance, T_1 denotes the time segments after δ_0 until the end of the planning horizon. Similarly, T_2 denotes the time segments after δ_1 until the end of the planning horizon. When we generalize the notation, T_n denotes the time segments after δ_{n-1} until the end of the planning horizon. Accordingly, the mathematical formulation Model 2 is provided as follows:
Decision Variables:

$$q_{ijkt} = \begin{cases} 1 & \text{if a MSU } k \in M \text{ departing from CD } i \in V \text{ stays the district } j \in V \text{ at} \\ & \text{the time period } t \in T \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} (\text{Model 2}) \text{ maximize } & \sum_{i \in V} \sum_{n \in \mathcal{N} \setminus \{0\}} y_i b_{fi} (\delta_n - \delta_{n-1}) \rho_n + \sum_{i \in V} \sum_{\substack{l \in F_i^f \\ l \neq i}} \sum_{n \in \mathcal{N} \setminus \{0\}} y_i b_{fl} a_{il}^f (\delta_n - \delta_{n-1}) \rho_n + \\ & \sum_{i \in V} \sum_{j \in V} \sum_{k \in M} \left[\theta_1 \sum_{t \in T} q_{ijkt} b_{mj} - \sum_{n \in \{1, \dots, N-1\}} (\theta_n - \theta_{n+1}) \sum_{t \in T_{(n+1)}} q_{ijkt} b_{mj} \right] + \end{aligned}$$

$$\sum_{i \in V} \sum_{j \in V} \sum_{\substack{l \in F_j^m \\ l \neq j}} \sum_{k \in M} \left[\theta_1 \sum_{t \in T} q_{ijkt} b_{ml} - \sum_{n \in \{1, \dots, N-1\}} (\theta_n - \theta_{n+1}) \sum_{t \in T_{(n+1)}} q_{ijkt} b_{ml} a_{jl}^m \right] \quad (4.14)$$

subject to

$$\sum_{i \in V} y_i \leq m_f \quad (4.15)$$

$$x_{ijk} \leq y_i \quad i \in V, j \in V, k \in M \quad (4.16)$$

$$\sum_{t \in T} q_{ijkt} \geq \underline{c} x_{ijk} \quad i \in V, j \in V, k \in M \quad (4.17)$$

$$\sum_{t \in T} q_{ijkt} \leq \bar{c} x_{ijk} \quad i \in V, j \in V, k \in M \quad (4.18)$$

$$y_j + \sum_{\substack{i \in F_j^f \\ i \neq j}} y_i + \sum_{i \in V} \sum_{k \in M} x_{ijk} + \sum_{i \in V} \sum_{\substack{l \in F_j^m \\ l \neq j}} \sum_{k \in M} x_{ilk} \leq 1 \quad j \in V \quad (4.19)$$

$$\sum_{j \in V} c_{ij} x_{ijk} + \sum_{j \in V} \sum_{t \in T} q_{ijkt} \leq |T| \quad i \in V, k \in M \quad (4.20)$$

$$x_{ijk} \in \{0, 1\} \quad i \in V, j \in V, k \in M \quad (4.21)$$

$$y_i \in \{0, 1\} \quad i \in V \quad (4.22)$$

$$q_{ijkt} \in \{0, 1\} \quad i \in V, j \in V, k \in M, t \in T. \quad (4.23)$$

The performance measure is represented by equation (4.14) and it is the total demand satisfied. Constraint sets (4.15)–(4.23) are the representation of the constraints (4.2)–(4.12) in Model 1 by utilizing the new decision variables q_{ijkt} . Also, by utilizing ρ_n , the objective function can be re-written as follows:

$$\begin{aligned} & \sum_{i \in V} \sum_{j \in V} \sum_{k \in M} \sum_{n \in \{2, \dots, N\}} \rho_{n-1} \left(\sum_{t \in T_{n-1}} q_{ijkt} - \sum_{t \in T_n} q_{ijkt} \right) b_{mj} \\ & + \sum_{i \in V} \sum_{j \in V} \sum_{\substack{l \in F_j^m \\ l \neq j}} \sum_{k \in M} \sum_{n \in \{2, \dots, N\}} \rho_{n-1} \left(\sum_{t \in T_{n-1}} q_{ijkt} - \sum_{t \in T_n} q_{ijkt} \right) b_{ml} a_{jl}^m. \end{aligned}$$

Lemma 1. *The following equality holds for any feasible z_{ijk} and u_{nik} variables of Model 1 and feasible q_{ijkt} variables of Model 2:*

$$z_{ijk} = \sum_{t \in T} q_{ijkt} \quad i \in V, j \in V, k \in M$$

$$u_{nik} = \sum_{t \in T_n} q_{ijkt} - \sum_{t \in T_{n+1}} q_{ijkt} \quad i \in V, j \in V, k \in M.$$

Proof. See Appendix B. □

4.2.3 Model 3

We define r_{ijkt} as a binary decision variable which determines whether a MSU $k \in M$ departing from a CD $i \in V$ stays at district $j \in V$ for a total duration of exactly $t \in T$ time periods. This section provides the Model 3 formulation by the new decision variables.

Decision Variables:

$$r_{ijkt} = \begin{cases} 1 & \text{if MSU } k \text{ departing from CD } i \text{ is allocated to provide service at district } \\ & j \text{ for a total duration of exactly } t \text{ time periods.} \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} (\text{Model 3}) \text{ maximize } & \sum_{i \in V} \sum_{n \in N \setminus \{0\}} y_i b_{fi} (\delta_n - \delta_{n-1}) \rho_n + \sum_{i \in V} \sum_{\substack{l \in F_i^f \\ l \neq i}} \sum_{n \in N \setminus \{0\}} y_i b_{fl} a_{il}^f (\delta_n - \delta_{n-1}) \rho_n + \\ & \sum_{i \in V} \sum_{j \in V} \sum_{k \in M} \left[\theta_1 \sum_{t \in T} t r_{ijkt} b_{mj} - \sum_{n \in \{1, \dots, N-1\}} (\theta_n - \theta_{n+1}) \sum_{t \in T_{(n+1)}} (t - \delta_n) r_{ijkt} b_{mj} \right] + \\ & \sum_{i \in V} \sum_{j \in V} \sum_{\substack{l \in F_j^m \\ l \neq j}} \sum_{k \in M} \left[\theta_1 \sum_{t \in T} t r_{ijkt} b_{ml} - \sum_{n \in \{1, \dots, N-1\}} (\theta_n - \theta_{n+1}) \sum_{t \in T_{(n+1)}} (t - \delta_n) r_{ijkt} b_{ml} a_{jl}^m \right] \end{aligned} \quad (4.24)$$

subject to

$$\sum_{i \in V} y_i \leq m_f \quad (4.25)$$

$$x_{ijk} \leq y_i \quad i \in V, j \in V, k \in M \quad (4.26)$$

$$\sum_{t \in T} r_{ijkt} \leq 1 \quad i \in V, j \in V, k \in M \quad (4.27)$$

$$\sum_{t \in T} tr_{ijkt} \geq \underline{c}x_{ijk} \quad i \in V, j \in V, k \in M \quad (4.28)$$

$$\sum_{t \in T} tr_{ijkt} \leq \bar{c}x_{ijk} \quad i \in V, j \in V, k \in M \quad (4.29)$$

$$y_j + \sum_{\substack{i \in F_j^f \\ i \neq j}} y_i + \sum_{i \in V} \sum_{k \in M} x_{ijk} + \sum_{i \in V} \sum_{\substack{l \in F_j^m \\ l \neq j}} \sum_{k \in M} x_{ilk} \leq 1 \quad j \in V \quad (4.30)$$

$$\sum_{j \in V} c_{ij} x_{ijk} + \sum_{j \in V} \sum_{t \in T} tr_{ijkt} \leq |T| \quad i \in V, k \in M \quad (4.31)$$

$$x_{ijk} \in \{0, 1\} \quad i \in V, j \in V, k \in M \quad (4.32)$$

$$y_i \in \{0, 1\} \quad i \in V \quad (4.33)$$

$$r_{ijkt} \in \{0, 1\} \quad i \in V, j \in V, k \in M, t \in T. \quad (4.34)$$

Constraint set (4.27) ensures that at most one time period is chosen, allowing the service to continue for a total duration of exactly t time periods. The objective function (4.24) includes the demand satisfied by CD and MSUs, similar to the previous models. The coverage decay and time decay factors influence the demand. Constraints (4.25), (4.26) and (4.28)–(4.34) are the representation of the constraints (4.15)–(4.23) in Model 2 by utilizing the new decision variables, r_{ijkt} .

Lemma 2. *The following equality holds for any feasible q_{ijkt} variables of Model 2 and feasible r_{ijkt} variables of Model 3:*

$$\sum_{t \in T} tr_{ijkt} = \sum_{t \in T} q_{ijkt} \quad i \in V, j \in V, k \in M.$$

Proof. See Appendix C. □

4.3 Computational Study

We introduce the dataset and parameter settings. Then we provide an analysis of the performance of the proposed models and an analysis of optimal solutions. We compare the mathematical models under uniform and decreasing demand functions. In order to present the effect of decreasing demand on models and optimal solutions, we utilize a uniform demand function. With uniform demand functions, we adopt models so that there is no decrement in demand over time, but it only includes demand change due to coverage, and we take $\rho_n = 1$ and $\theta_n = 1$ for all $n \in \mathcal{N}$. With a decreasing demand function, we adopt models such that there is a decrease over time, in the form of a step function.

4.3.1 Datasets and Parameter Settings

The proposed model is applied to Seoul, Istanbul, and Kartal datasets, which are used by [133] and by [135], respectively. This selection serves two purposes: (1) enabling direct benchmarking against established literature, and (2) providing representative urban contexts at international, metropolitan, and district-level scales for robust preliminary analysis. This focused approach to initial model validation ensures sufficient insight without imposing unnecessary computational overhead.

Both MSUs and CDs have an availability parameter a_{ij}^c and in all datasets, α^c is chosen as 0.5 to reflect the reduction in demand due to coverage. For both MSUs and CD, the predefined coverage threshold for availability (λ^m, λ^f) is 0.04 hours (2 kilometers). We consider the upper bound on length of stay $\bar{c}$ to be 8 time periods, and the lower bound on length of stay $\underline{c}$ to be 1 time period. Further, we utilize 2-step function by letting set $\mathcal{N} = \{0, 1, 2\}$ and setting $\delta_0 = 0, \delta_1 = 4$ and $\delta_2 = 8$. Therefore, the first and second halves of the stay are to be 4 time periods for the

base case.

We perform a comparative analysis of the models across these different scenarios, specifically focusing on the total demand satisfied as our key performance metric. We investigate how this metric changes with varying numbers of MSUs and CDs. All experiments were conducted on an Intel Xeon Silver 4210 with 14 GB RAM. We solved the models using IBM ILOG CPLEX 12.10.0, with each experiment limited to a 2-hour computational time.

4.3.2 Performance of Models

Tables 4.2 and 4.3 present a comparison of the models under decreasing and uniform demand functions over time. The solution times, best integer solution found, best bound and gap are used as comparison criteria for models.

According to both tables, the objective function values increase as the number of MSUs and CDs increases. This is expected, since a larger number of available resources enables the system to reach more affected individuals, thereby increasing the total satisfied demand. It should be noted that increasing the number of MSUs assigned to each CD also increases the total number of mobile service units (TMSU) available in the system. Specifically, in Tables 4.2 and 4.3, # MSU represents the number of MSUs allowed to depart from each CD, while # CD denotes the number of central depots. Therefore, the TMSU is calculated as the product of the number of MSUs per CD and the number of CDs. As TMSU increases, the system gains greater operational capacity, which leads to higher coverage levels and improved objective function values.

According to Table 4.2, Model 3 consistently surpasses Model 1 and Model 2. It can be noted that Model 3 achieves a 0% optimality gap across all instances

# Nodes*	# MSU*	# CD*	# TMSU*	Decreasing Demand											
				Model 1				Model 2				Model 3			
				CPU (sec.)	Best Integer	Best Bound	Gap (%)	CPU (sec.)	Best Integer	Best Bound	Gap (%)	CPU (sec.)	Best Integer	Best Bound	Gap (%)
25	2	1	2	1.12	217821	217821	0	1.58	217821	217821	0	0.41	217821	217821	0
		2	4	9.70	376884	376884	0	110.67	376884	376884	0	1.57	376884	376884	0
		3	6	392.86	512670	512670	0	7436.56	512670	522460.49	1.91	3.46	512670	512670	0
	3	1	3	3.59	268254	268254	0	3.20	268254	268254	0	2.24	268254	268254	0
		2	6	2088.46	459528	459528	0	7337.30	459528	471742.94	2.66	4.80	459528	459528	0
		3	9	562.84	619079	619079	0	7254.63	619079	631939.08	2.08	8.02	619079	619079	0
45	2	1	2	11.67	15946	15946	0	6.55	15946	15946	0	5.11	15946	15946	0
		2	4	559.25	26012	26012	0	1235.73	26012	26012	0	47.27	26012	26012	0
		3	6	7203.05	35489	35577.99	0.25	7200.20	35489	35543.197	0.15	469.85	35489	35489	0
	3	1	3	86.64	19198	19198	0	17.18	19198	19198	0	19.54	19198	19198	0
		2	6	7204.95	31820	32075.93	0.80	7243.71	31820	32872.17	3.30	1203.78	31820	31820	0
		3	9	7200.21	43646	44002.71	0.82	7200.21	43646	45519.89	4.29	700.51	43646	43646	0
99	2	1	2	223.54	45708	45708	0	97.40	45708	45708	0	19.28	45708	45708	0
		2	4	7214.36	74369	75151.93	1.05	7200.24	74369	202668.35	172.52	41.80	74369	74369	0
		3	6	7200.96	98924	187279.70	89.31	7200.21	98924	206030.93	108.27	41.58	98924	98924	0
	3	1	3	7003.59	54554	54554	0	1978.72	54554	54554	0	74.57	54554	54554	0
		2	6	7207.02	888743	188059.30	111.91	7200.37	88802	208227.56	134.49	85.80	88931	88931	0
		3	9	7206.36	116793	192616	64.92	7200.36	116609	210572.40	80.58	76.78	116874	116874	0

* # Nodes denotes the number of nodes in the used dataset. # MSU denotes the number of MSUs and # CD denotes the number of CDs. # TMSU denotes the total number of MSUs in the proposed system.

Table 4.2: Comparison of mathematical models for maximizing total demand satisfied under decreasing demand

and achieves optimality within the given time limit. On the other hand, Model 1 and Model 2 face computational challenges. They achieve 0% gaps for smaller instances; however, their performance significantly gets worse with larger instances. Additionally, they fail to converge to an optimal or near-optimal solution within the given time limit.

We can also observe that the solution times get higher as the number of nodes, MSUs, and CDs increases. The rate of increase and solution times differ dramatically. Model 3 consistently achieves lower solution times. For instance, Model 3 solves the largest instance in 76.78 seconds, and it shows the model's effectiveness in handling complex problem instances. On the other hand, Model 1 and Model 2 experience computational challenges with medium and large-sized instances. Neither of those models can solve the largest instance optimally within the 2-hour time limit.

Increasing the number of MSUs and CDs generally increases the complexity of the problem. While Model 3 handles these challenges with only a small increase in solution time, Model 1 and Model 2 exhibit a significantly steeper increase in computational effort and a wider gap. Increasing the number of MSUs and CDs poses a challenge for the formulations of Model 1 and Model 2.

Table 4.3 presents an analysis of three mathematical models under a uniform demand function. For most small and moderately sized instances, all three models can find and prove the optimal solution. Model 3 consistently finds the optimal solutions within the given 2-hour limits. Generally, lower solution times are observed; however, in the medium dataset, higher solution times are noted. In general, Model 3 demonstrates significantly superior performance in achieving optimality. Model 3 is observed to be the fastest model across almost all instances and the solution times of Model 3 are significantly lower than those of the other two models. On the other hand, Model 1 and Model 2 generally fail to converge to the optimal solutions within the 2-hour limit. Among all models, Model 1 is observed to be the slowest.

# Nodes*	# MSU*	# CD*	# TMSU*	Uniform Demand											
				Model 1				Model 2				Model 3			
				CPU (sec.)	Best Integer	Best Bound	Gap (%)	CPU (sec.)	Best Integer	Best Bound	Gap (%)	CPU (sec.)	Best Integer	Best Bound	Gap (%)
25	2	1	2	0.88	237477	237477	0	0.79	237477	237477	0	0.47	237477	237477	0
		2	4	2.21	410508	410508	0	2.00	410508	410508	0	1.37	410508	410508	0
		3	6	40.33	559227	559227	0	30.28	559227	559227	0	1.63	559227	559227	0
	3	1	3	1.65	292497	292497	0	1.74	292497	292497	0	0.67	292497	292497	0
		2	6	89.44	499793	499793	0	205.81	499793	499793	0	4.55	499793	499793	0
		3	9	4933.54	677051	677051	0	375.71	677051	677051	0	5.46	677051	677051	0
45	2	1	2	6.34	17273	17273	0	4.20	17273	17273	0	5.03	17273	17273	0
		2	4	45.64	28430	28430	0	882.36	28430	28430	0	25.51	28430	28430	0
		3	6	135.71	38310	38310	0	3994.31	38310	38310	0	565.531	38310	38310	0
	3	1	3	12.74	20927	20927	0	12.18	20927	20927	0	13.12	20927	20927	0
		2	6	7201.49	34338	34338	0.19	1826.99	34338	34406.19	0	1871.44	34338	34338	0
		3	9	2116.11	47130	47130	0	7203.11	47130	47749.71	1.31	2822.93	47130	47130	0
99	2	1	2	39.16	50217	50217	0	20.25	50217	50217	0	290.74	50217	50217	0
		2	4	2001.04	81465	81465	0	1997.05	81465	81465	0	126.29	81465	81465	0
		3	6	7200.15	108286	190682.30	76.09	7200.18	108286	196336.57	81.31	122.11	108286	108286	0
	3	1	3	64.38	59779	59779	0	35.49	59779	59779	0	538.23	59779	59779	0
		2	6	7202.44	97269	19081.21	96.34	7200.21	97269	179984.45	85.03	275.86	97269	97269	0
		3	9	7202.47	128747	206568.60	60.44	7200.22	128747	206204.77	60.16	255.21	128747	128747	0

* # Nodes denotes the number of nodes in the used dataset. # MSU denotes the number of MSUs, and # CD denotes the number of CDs. # TMSU denotes the total number of MSUs in the proposed system.

Table 4.3: Comparison of mathematical models for maximizing total demand satisfied under uniform demand

Model 1 generally hits the time limit before solving the problem optimally. Although Model 2 outperforms Model 1 in most cases, it still struggles significantly with larger instances.

We further compare the models based on their Linear Programming (LP) relaxations. We use the LP relaxation as an indicator for the strength and efficiency of models. The tighter the relaxation, the stronger the formulation. Stronger formulations lead to smaller optimality gaps and faster convergence to the optimal solution. In Table 4.4, we present a detailed comparison of the LP relaxations for our proposed models across different instances.

According to Table 4.4, LP relaxations of each model yielded the same objective function values across different instances and under different demand functions. It shows that all formulations provide an equally strong upper bound for the total demand satisfied. Due to their similarities, it can be noted that the LP relaxation is not enough to differentiate the strength of models.

We analyze a partial linear programming relaxation, referred to as LP_{time} , for a more detailed analysis. In general, the models differ in how time-related decision variables are defined. We relax the integrality constraints exclusively for the duration variables. It should be noted that the location and allocation decisions remain integer to preserve the model's structure. The results are presented in Table 4.5. According to Table 4.5, we observe that for smaller instances, all models typically achieve 0% gaps in their LP_{time} relaxations within reasonable solution times. However, it should be noted that Models 2 and 3 struggle to solve the problem instances as the instances get larger. Model 1 also fails to converge optimally within 2 hours in some large instances. However, it can be observed that the behaviour of models reversed in LP_{time} analysis. Continuous duration variables challenge Model 3 computationally. Even though we have gained some insights about the model performances, we further analyze the decision variables and constraints. We aim to understand the reversed

# Nodes	# MSU	# CD	Optimal objective function of IP	Optimal objective function of LP relaxation of Model 1	Optimal objective function of LP relaxation of Model 2	Optimal objective function of LP relaxation of Model 3
25	2	1	217821	966814.1	966814.1	966814.1
		2	376884	966820	966820	966820
		3	512670	966824	966824	966824
	3	1	268254	966814.9	966814.9	966814.9
		2	459528	966820	966820	966820
		3	619079	966824	966824	966824
45	2	1	15946	94842.32	94842.32	94842.32
		2	26012	94846.33	94846.33	94846.33
		3	35489	94848	94848	94848
	3	1	19198	94843	94843	94843
		2	31820	94847	94847	94847
		3	43646	94848	94848	94848
99	2	1	45708	237218.5	237218.5	237218.5
		2	74369	237222.6	237222.6	237222.6
		3	98924	237226.7	237226.7	237226.7
	3	1	54554	237219.4	237219.4	237219.4
		2	88931	237223.4	237223.4	237223.4
		3	116874	237227.5	237227.5	237227.5

Table 4.4: Comparison of LP relaxations of models

relation between IP and LP_{time} versions. We note the structure of their decision variables and constraints. Table 4.6 provides a detailed breakdown of the decision variables and constraints.

Model 2 and Model 3 have a significantly higher number of decision variables compared to Model 1 due to the explicit time index t in their q_{ijkt} and r_{ijkt} variables. Even though they have more decision variables, Model 2 and 3 have a lower number of constraints than Model 1. More specifically, Model 3 generally has more constraints than Model 2, but both have fewer constraints than Model 1.

# Nodes	# MSU	# CD	Obj	Relaxation of decision variables regarding time (LP_{time}) in Model 1					Relaxation of decision variables regarding time (LP_{time}) in Model 2					Relaxation of decision variables regarding time (LP_{time}) in Model 3				
				CPU (sec.)	Best ger	Inte- ger	Best Bound	Gap (%)	CPU (sec.)	Best ger	Inte- ger	Best Bound	Gap (%)	CPU (sec.)	Best ger	Inte- ger	Best Bound	Gap (%)
25	2	1	217821	0.69	222445.5	222445.5	0	0.94	222445.5	222445.5	222445.5	0	1.15	222445.5	222445.5	222445.5	0	
		2	376884	1.10	389547.9	389547.9	0	4.37	389547.9	389547.9	389547.9	0	5.81	389547.9	389547.9	389547.9	0	
		3	512670	1.16	533548.6	533548.6	0	27.72	533548.6	533548.6	533548.6	0	7.96	533548.6	533548.6	533548.6	0	
	3	1	268254	1.28	276967.6	276967.6	0	0.80	276967.6	276967.6	276967.6	0	2.08	276967.6	276967.6	276967.6	0	
		2	459528	2.47	479833.9	479833.9	0	4.79	479833.9	479833.9	479833.9	0	16.57	479833.9	479833.9	479833.9	0	
		3	619079	3.21	649687.7	649687.7	0	11.86	649687.7	649687.7	649687.7	0	524.13	649687.7	649687.7	649687.7	0	
45	2	1	15946	4.03	16162.56	16162.56	0	2.70	16162.56	16162.56	16162.56	0	11.76	16162.56	16162.56	16162.56	0	
		2	26012	6.18	27009.68	27009.68	0	59.46	27009.68	27009.68	27009.68	0	496.34	27009.68	27009.68	27009.68	0	
		3	35489	12.27	36874.04	36874.04	0	59.47	36874.04	36874.04	36874.04	0	11776.48	36874.04	36874.04	36874.04	0	
	3	1	19198	11.15	19633.68	19633.68	0	5.40	19633.68	19633.68	19633.68	0	19.85	19633.68	19633.68	19633.68	0	
		2	31820	24.70	33041.60	33041.60	0	273.46	33041.60	33041.60	33041.60	0	924.47	33041.60	33041.60	33041.60	0	
		3	43646	668.50	45553.56	45553.56	0	1691.47	45553.56	45553.56	45553.56	0	7200.30	45545.40	91550.04	101.00	101.00	
99	2	1	45708	92.11	47614.49	47614.49	0	155.14	47614.49	47614.49	47614.49	0	124.43	47614.49	47614.49	47614.49	0	
		2	74369	3172.58	76688.5	76688.5	0	7200.24	76667.72	2009470.94	173.22	2445.74	76688.5	76688.5	76688.5	0		
		3	98924	7200.39	101879.5	100496.78	96.80	7200.47	101710.8	211754.52	108.19	7200.26	101339.6	203387.9	100.70	100.70		
	3	1	54554	174.23	56899.63	56899.63	0	155.14	56899.63	56899.63	56899.63	0	253.79	56899.63	56899.63	56899.63	0	
		2	88931	6007.52	92353.5	92353.5	0	7201.74	92353.5	212444.75	130.03	7200.78	92353.5	237208.36	156.85	156.85		
		3	116874	7205.69	121334.00	204293.10	68.37	7202.90	121538.9	214826.09	76.75	7200.45	121538.9	237093.34	95.07	95.07		

Table 4.5: Comparison models based on relaxation of decision variables regarding time (LP_{time})

# Nodes	# MSU	# CD	Model 1		Model 2		Model 3	
			Decision Variables	Constraints	Decision Variables	Constraints	Decision Variables	Constraints
25	2	1	3,775	6,326	11,275 B:11,275 I:0	3,826	11,275 B:11,275 I:0	5,076
		2	B:1,275					
		3	I:2,500					
	3	1	5,650	9,476	16,900 B:16,900 I:0	5,726	16,900 B:16,900 I:0	7,601
		2	B:1,900					
		3	I:3,750					
45	2	1	12,195	20,386	36,495 B:36,495 I:0	12,286	36,495 B:36,495 I:0	16,336
		2	B:4,095					
		3	I:8,100					
	3	1	18,270	30,556	54,720 B:54,720 I:0	18,406	54,720 B:54,720 I:0	24,481
		2	B:6,120					
		3	I:12,150					
99	2	1	58,905	98,308	176,517 B:176,517 I:0	59,104	176,517 B:176,517 I:0	78,706
		2	B:19,701					
		3	I:39,204					
	3	1	88,308	147,412	264,726 B:264,726 I:0	88,606	264,726 B:264,726 I:0	118,726
		2	B:29,502					
		3	I:58,806					

Table 4.6: Number of decision variables and constraints of models

Despite having a larger number of decision variables and constraints compared to Model 1 and Model 2, Model 3 outperforms them in Table 4.2. It shows its superior formulation. Model 3 appears to be a tighter IP formulation due to its definition of binary decision variables combined with more effective constraints. This multifaceted analysis of LP relaxations, partial relaxations, and structural components provides a comprehensive understanding of each model’s strengths and weaknesses, guiding the selection of the most appropriate formulation for a given problem context.

The scalability of the proposed formulations is systematically assessed by increasing problem size along three key dimensions: number of demand nodes, number of central depots (CDs), and total number of mobile service units (MSUs). As shown in Tables 4.2 and 4.3, scaling any of these dimensions increases computational complexity and solution time across all models.

The results demonstrate that Model 3 exhibits significantly better scalability than Models 1 and 2. While Models 1 and 2 experience severe performance degradation, Model 3 consistently solves all tested instances to optimality within short computational times. Notably, for the largest instances, Model 3 achieves optimality in under 300 seconds, whereas the other formulations struggle or fail to converge. The results indicate that formulation strength is the profound factor for computational performance. Model 3 has tighter formulation due to its binary time-indexed variable definitions and constraint design. Thus, it leads to faster branch-and-bound convergence and improved computational robustness.

Overall, the computational study confirms that Model 3 scales effectively to realistically sized urban instances and can efficiently handle increases in network complexity and resource availability. This makes it suitable for practical, large-scale humanitarian logistics and disaster-response planning applications where computational efficiency is critical.

4.3.3 Analysis of Optimal Solutions

This section analyzes the optimal solutions of different instances over different demand functions. We first analyze the solutions with the uniform demand function.

- Uniform demand function:

In Figures 4.4 and 4.5, we analyze the solutions with a uniform demand function across both the Seoul and Kartal datasets. We observe a consistent pattern regarding selecting CD locations, particularly notable in districts 12, 13, and 19 for Seoul, and 15, 22, and 39 for Kartal. These are high-density demand locations; therefore, they are chosen to contribute to the total demand satisfied. District 19 in Seoul and District 15 in Kartal are prime locations with exceptionally high demand. Districts 1 and 18 are within the coverage area of District 12, while Districts 6 and 21 are within the coverage area of District 13. Even though Districts 12 and 13 in Seoul are not individually high-density demand areas like Districts 19 and 15, other districts within their coverage area affect the total demand satisfied. In the Kartal dataset, one of the CDs is located at District 22. District 21 being within the coverage area of District 22 significantly increases the total demand satisfied by CD at District 22. When the number of CDs is reduced to 2, CDs are strategically located in Districts 12 and 13 in Seoul, whereas in Kartal, they are positioned in Districts 15 and 39. Also, it is observed that one of the MSUs departing from the CD in District 12 is routed through District 19. Similarly, in Kartal, a MSU departing from District 39 visits District 22. In scenarios where only one CD is designated, Districts 13 in Seoul and District 15 in Kartal are consistently chosen as the optimal locations. MSUs dispatched from these CDs visit Districts 12 and 19 in Seoul, and Districts 22 and 39 in Kartal. There are clear patterns in the results which indicate that the CD locations are chosen as high-density demand areas, including the demand within their coverage areas. If a CD is not located

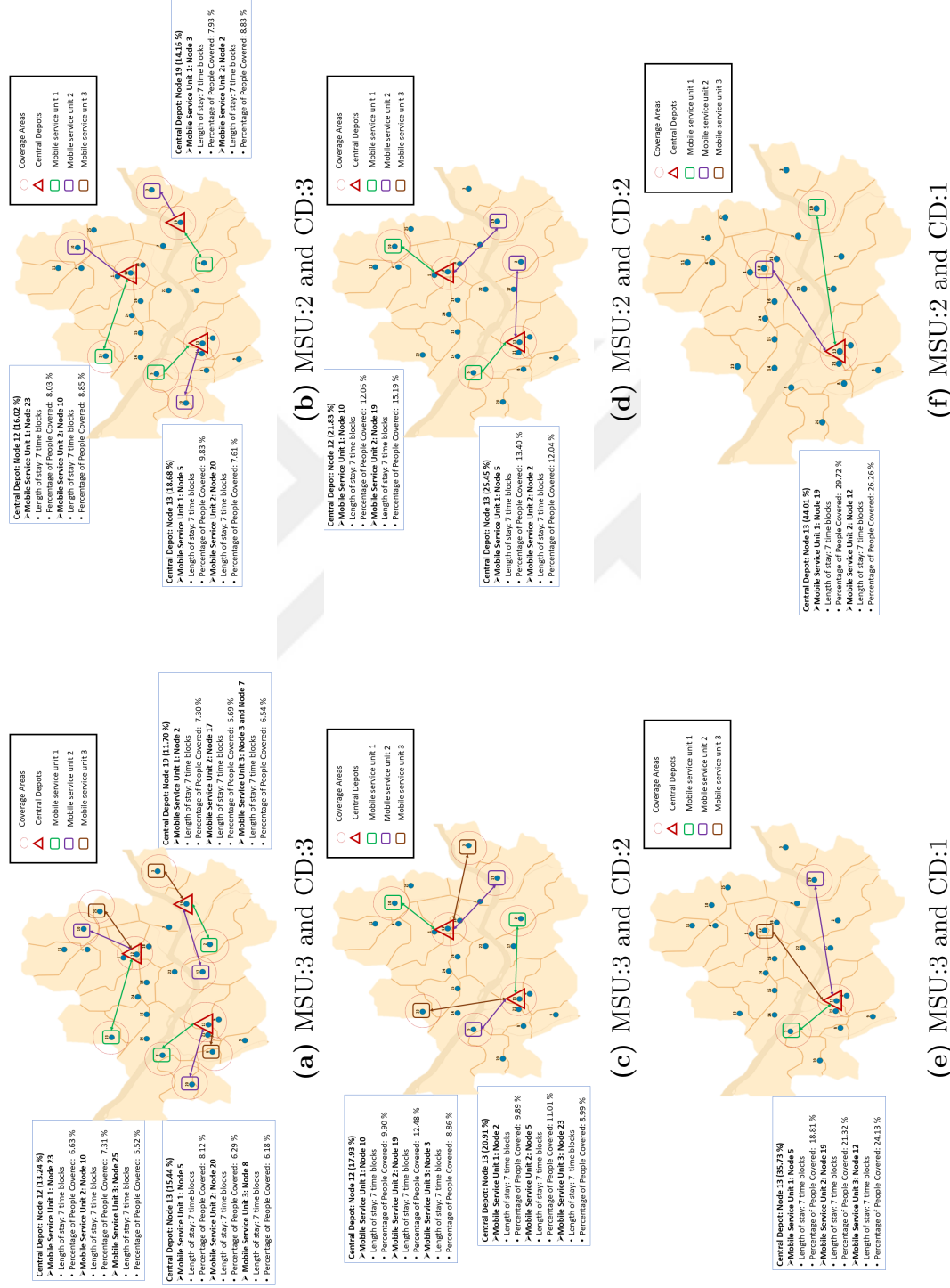


Figure 4.4: Results of the solutions for the uniform demand function in the Seoul dataset

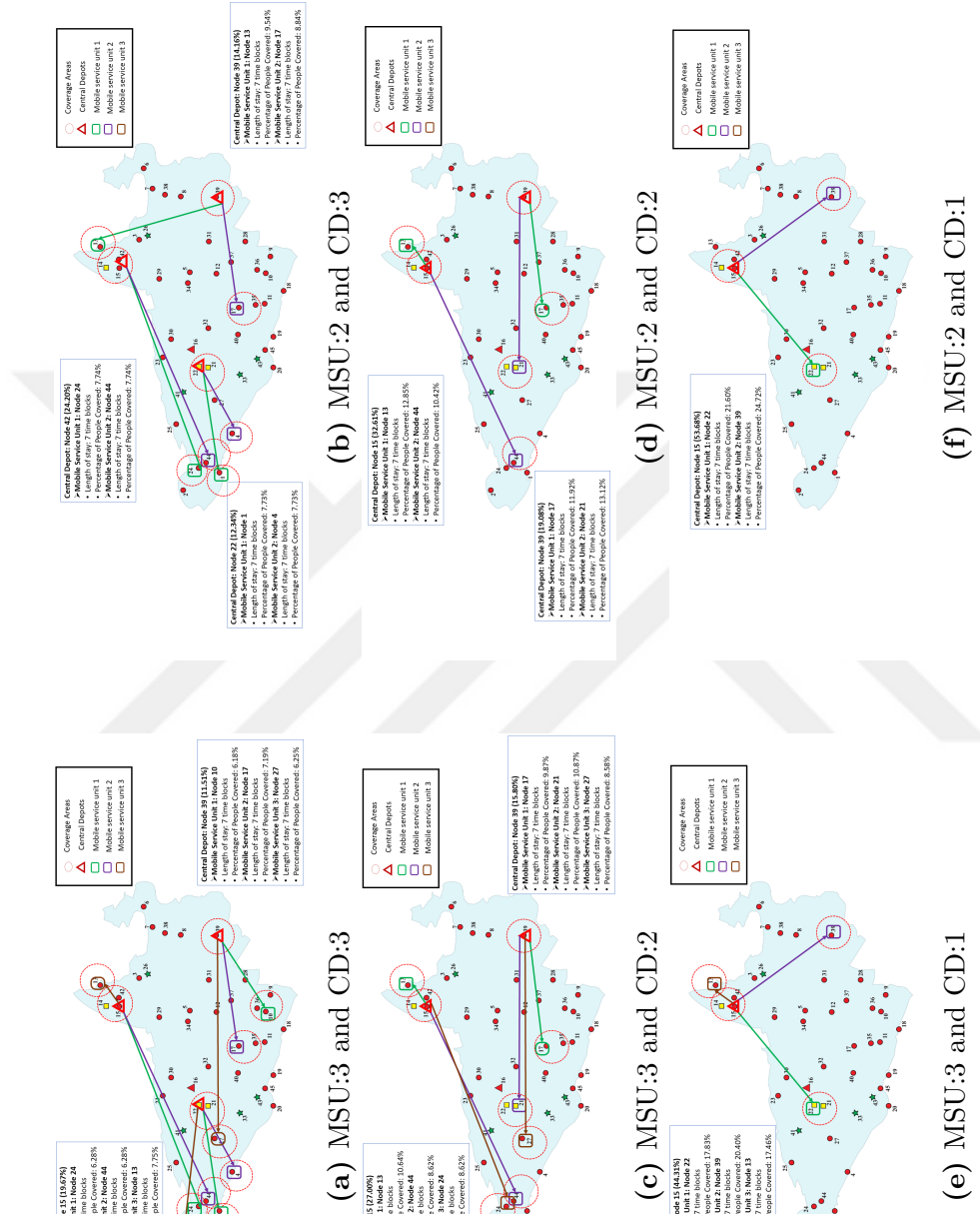


Figure 4.5: Results of the solutions for the uniform demand function in the Kartal dataset

at those high-density demand locations, then those locations are visited by a MSU.

It can be observed that MSUs do not visit multiple locations within a given planning horizon. The uniform demand function ensures that MSUs stay at the high-density demand locations as long as time allows to optimize their impact.

- Decreasing demand function:

In Figures 4.6 and 4.7, we extend our analysis to explore the solutions within the decreasing demand function. Likewise, with the decreasing demand function, the models select Districts 12, 13, and 19 in Seoul as CD locations. When the number of CDs is 2, Districts 12 and 13 are chosen as CD locations, while a MSU serves District 19. Similarly, when only one CD is located, District 13 is chosen as the optimal choice while MSUs are visiting Districts 12 and 19.

However, in the Kartal dataset, the CD locations are different from the results with a uniform demand function. Even though District 39 has a higher demand than District 17, District 17 is chosen over District 39. When we closely analyzed the case, it was observed that District 39 is located far from other high-density demand locations. Therefore, a MSU departing from District 39 would not satisfy the total time duration requirements and would have short stays at critical locations. This could result in a decrease in total demand satisfied. On the other hand, District 17 is located in a geographically more accessible position with high demand. Thus, even though District 39 has higher demand, due to time restrictions, District 17 is selected as the optimal CD location in the Kartal dataset.

With a decreasing demand function, MSUs adopt a flexible strategy to prioritize high-density demand areas but visit multiple locations. In the Seoul dataset, with 2 MSUs and 1 CD, one MSU strategically only visits District 12, which has high demand. We observe a similar trend in the same instance of the

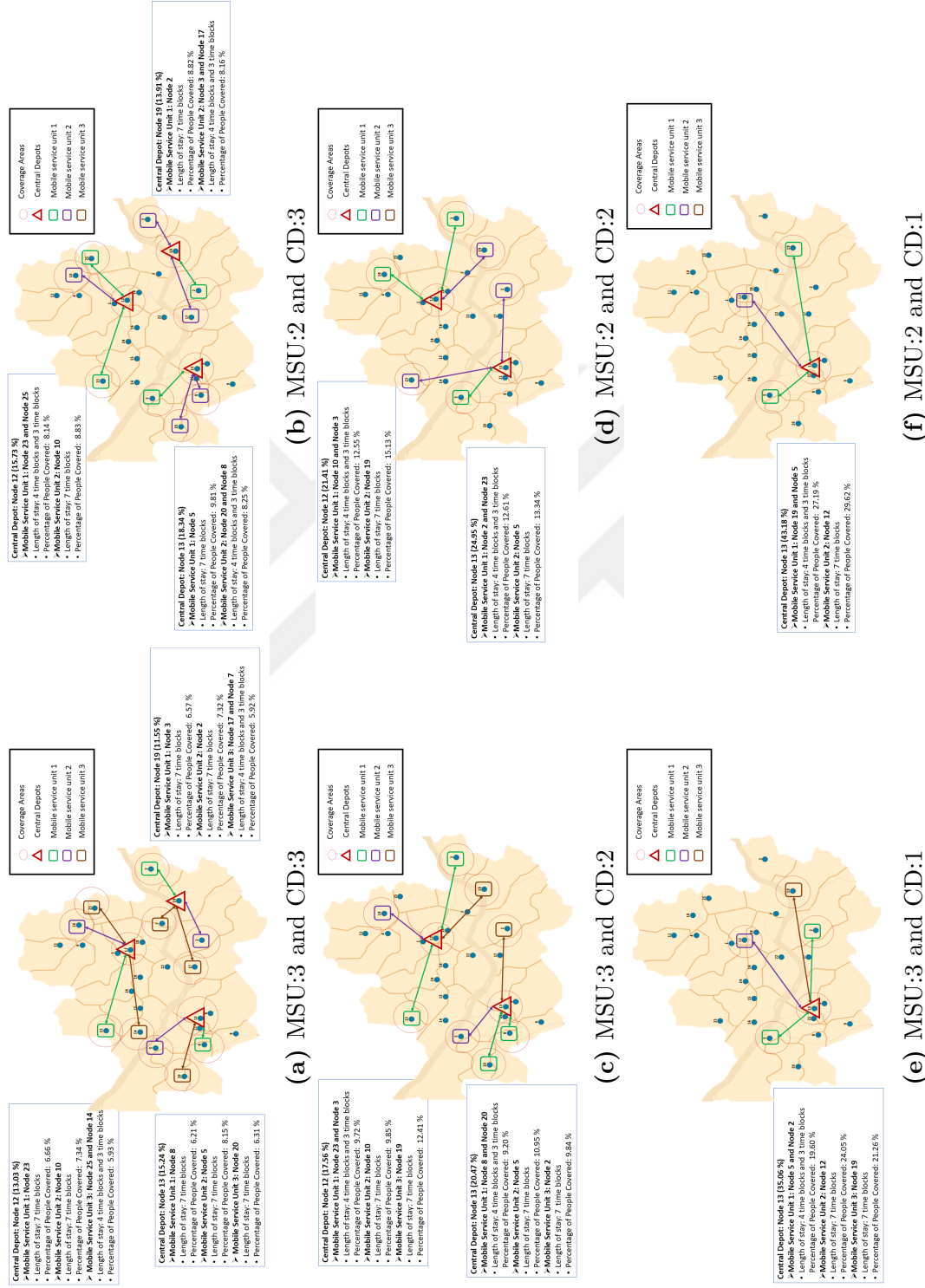


Figure 4.6: Results of the solutions for decreasing demand function in Seoul dataset

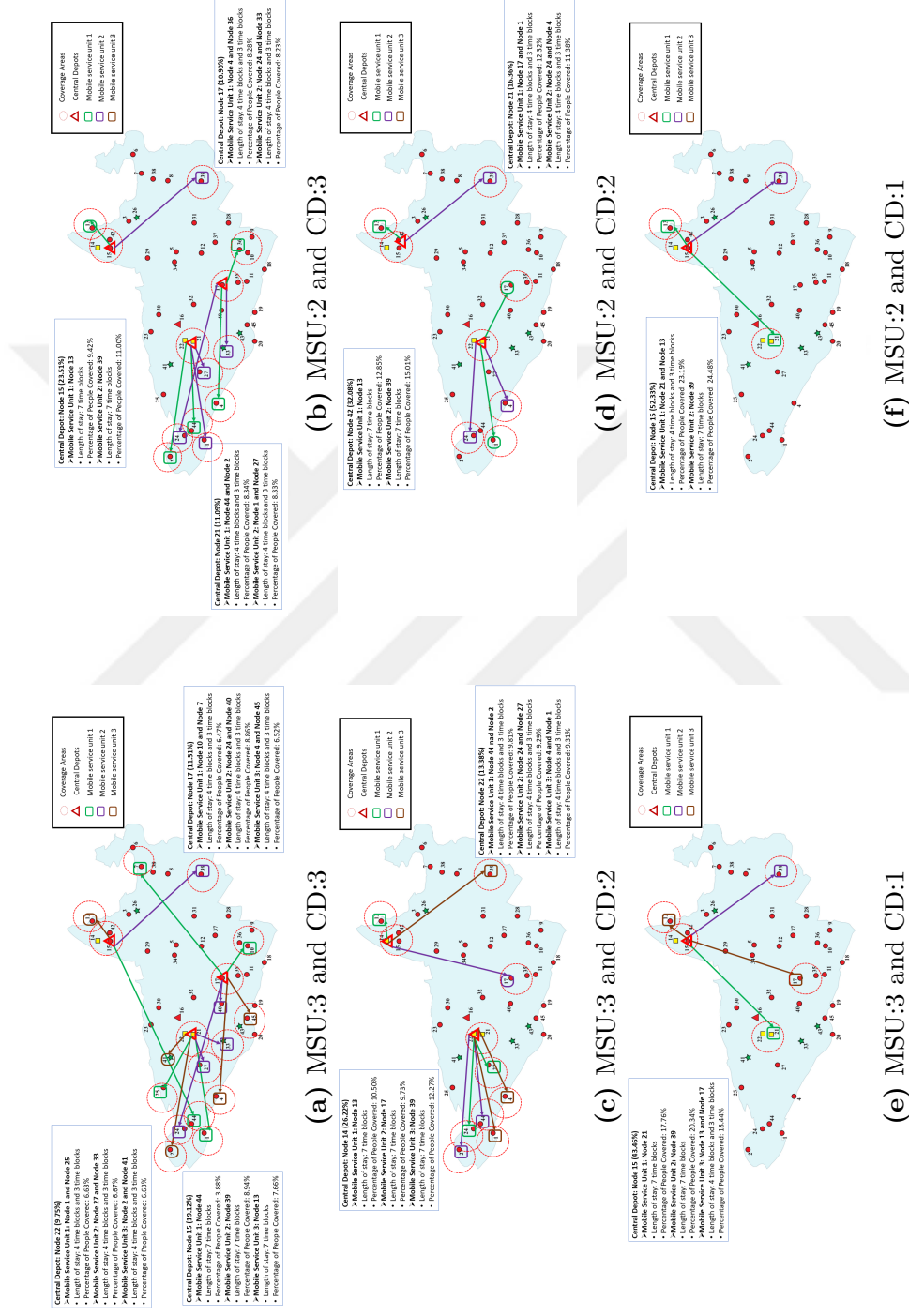


Figure 4.7: Results of the solutions for decreasing demand function in Kartal dataset

Kartal dataset. One of the MSUs only visits District 39 during the whole planning horizon. It shows a strategic movement in service high-density demand locations to maximize total demand served.

Conversely, the other MSU in both datasets follows a more dynamic trajectory. In Seoul, it initially targets District 19 before proceeding to District 5, whereas in Kartal, it begins its journey in District 21 before moving on to District 13. Despite the diminishing demand in District 12 (Seoul) and District 39 (Kartal) over time, the MSU unit remains stationed there due to the intrinsic value of these districts in terms of demand. However, as the demand begins to decline, the other MSU departs to explore other districts where a higher demand is anticipated, thereby ensuring efficient allocation of resources and sustained service effectiveness.

The adaptable approach remains consistent even when 2 or 3 CDs exist in both the Seoul and Kartal datasets. In Seoul, each MSU departing from the CDs visits multiple locations to increase the total demand served. Similarly, when there are 2 CDs in Kartal, the MSUs departing from one of the CDs visit multiple locations. When there are 3 CDs, in both the Kartal Seoul datasets, we observe that one of the MSUs visits two locations. This strategy shows an adaptation to change in demand over time and accordingly adapts by allocating resources to serve various areas efficiently.

4.4 Conclusion of Chapter 4

This chapter focuses on mobile service systems in the context of one-time demand during disaster response. We determine the locations of CDs and allocations and service durations of MSUs to maximize total demand satisfied for immediate, non-recurring survival needs.

We first formally define the OTDP. We utilized a non-linear, stepwise demand function to account for demand decrement over time. We mathematically proposed three novel models for the OTDP. Model 1 is an integer programming model which utilizes integer time durations, while Model 2 handles duration with discrete time periods using binary variables. The final model, Model 3, explicitly models stay durations with binary variables and represents a refined approach to the problem.

We then provided a detailed computational study on varying-sized datasets, under both uniform and decreasing demand functions. We further provided an analysis of LP relaxations and model sizes. According to our comparison, Model 3 is the best-performing model. Even though it includes a higher number of constraints and decision variables, Model 3 consistently achieves optimal solutions for larger problem instances within substantially shorter computational times. In contrast, Models 1 and 2 fail to converge optimality within the given 2-hour time limit, particularly for larger instances. We first analyze their LP relaxations by relaxing all integer variables. All models yielded identical objective function values, implying the same upper bound. Later, we have analyzed their LP relaxations by relaxing only integer duration variables, LP_{time} . Unexpectedly, Model 3 sometimes performed worse than other models and its integer counterpart. This result indicates that computational complexities arise when time-related variables are relaxed.

We derived strategic insights from the analysis of the optimal solutions. It is observed that high-density demand locations with high-density demand coverage areas are consistently preferred as CD locations. With a uniform demand function, MSUs prefer long and single-location visits, while with a decreasing demand function, MSUs adapt to a more flexible strategy by visiting multiple locations.

This study can be extended for future research through exploring advanced decomposition techniques or column generation approaches to enhance further the computational efficiency of models, particularly for larger problem instances. Integrating

a multi-objective approach could explicitly quantify equity-performance trade-offs, ensuring that aid distribution considers not only total demand satisfaction but also equitable access across all affected populations.



Chapter 5

Continuous Demand in the Post-Disaster Operations

Continuous demand services are essential for sustaining life and must be provided without interruption, especially to disaster victims. These services require an uninterrupted supply of necessary materials and an uninterrupted service process. The most well-known services that fall under this category are hot meals, bottled water, and WCs. When individuals are faced with a disaster, their immediate needs may sometimes be unknown or uncertain. However, it is crucial that these needs are addressed continuously to ensure their well-being and survival. Therefore, providing services that require continuity is of utmost importance in such situations.

Typical examples for continuous demand services include food distribution via food trucks, providing emergency health services, and finally, personal hygiene needs such as WCs. For instance, regarding emergency healthcare services, emergency response teams provide immediate medical care to injured people. They treat injuries ranging from minor cuts to life-threatening conditions, starting with the search and

rescue operations. Moreover, disaster victims struggle to access basic necessities such as food and water. Therefore, providing hot meals and bottled water becomes crucial to sustain their lives. By ensuring a continuous supply of these resources, the immediate needs of disaster victims can be met, providing them with the sustenance required to survive in the aftermath of a disaster. This requires adequate planning, coordination, and a consistent supply of materials. By addressing the evolving needs of disaster victims without interruption, these services restore normalcy and ensure the well-being of those affected by a disaster.

The remainder of this chapter is structured as follows. Section 5.1 formally defines the Continuous Demand Services Problem (CDSP), analyzing its computational complexity. In Section 5.2, we develop a mathematical model that optimizes the location and allocation of these heterogeneous MSUs to maximize the total demand served, integrating both binary and gradual coverage functions to represent demand satisfaction. A detailed computational study is performed on various datasets in Section 5.3. We demonstrate how different MSU types fulfil distinct and complementary roles in maximizing coverage. We also provide an analysis highlighting the impact of gradual coverage and emphasize the importance of proximity to MSUs for maximizing service participation and overall demand satisfaction. Finally, in Section 5.4, we provide insights and a conclusion driven by the study.

5.1 Formal Problem Definition

In continuous demand, we disregard the mobility of services in the sense that they can move to other districts to serve. For instance, in distributing soup or hot meals, the MSUs can only be located at one district, and they should always serve at that district. Victims in that district require food during the whole day and every day.

Thus, MSUs are not allowed to visit another district to provide service. We consider that there are three types of MSUs: high-capacity (HCap), medium-capacity (MCap), and low-capacity (LCap) MSUs. Those services are also utilized in the continuous demand to improve service accessibility. However, none utilize their mobility option since they have to offer non-stop service wherever they are stationed. Their distinction is built on their ability to serve. HCap MSUs are huge centers that can serve a significant population. While MCap MSUs can serve a significant population, not as many as a HCap MSU, LCap MSUs can only serve a small population compared to other units.

5.1.1 Problem Setup and Notation

Similar to OTDP, we present the continuous time demand problem over a network $G = (V, A)$ where V lists all the districts and A is the set of arcs connecting these districts. The populations are denoted as b_{mi} representing the population that can be served if a MSU is located at $i \in V$. The aim is to locate m_h HCap MSUs, m_m MCap MSUs and m_l LCap MSUs to maximize the people served while allocating the districts to get service from those MSUs. All the travel times in the network are represented as c_{ij} where $i, j \in V$.

The coverage concept is also considered here, by allowing people in the neighbourhood of a visited district to walk for the service. Therefore, providing service to non-visited points in districts within a specific distance by coverage is also possible. The demand points located within a certain distance threshold to a MSU can be assigned to that MSU via the coverage radius. We define coverage radius's for HCap MSU (λ^h), MCap MSUs (λ^m) and LCap MSUs (λ^l). The allocations within the coverage radius of MSUs are considered. CDSP decides the locations of HCap MSUs (v_i^h), MCap MSUs (v_i^m), and LCap MSUs (v_i^l) from the set of districts in the network (G). The model determines the districts that are assigned to HCap MSUs

(n_{ij}^h) , MCap MSUs (n_{ij}^m) and LCap MSUs (n_{ij}^l) to get service. Due to the chaotic environment in the post-disaster period, people may be late learning about services arriving at certain distances. Therefore, we assume that there will be a change in maximum potential demand in the coverage area, which is defined as demand rates for HCap MSU (a_{ij}^h), for MCap MSU (a_{ij}^m) and LCap MSU (a_{ij}^l). This assumption is to comply with real-life practices.

Figure 5.1 visually depicts the location and allocation of HCap MSU, MCap MSUs and LCap MSU within this proposed system across geographically dispersed districts. In the proposed system, the coverage areas may intersect, and the allocation of districts is also decided. The locations which are within a predetermined distance of the visited locations can be served, but when they are within the intersection of two MSUs, their allocation is decided.

In Figure 5.1, a system consisting of one HCap MSU, one MCap MSU and three LCap MSUs is presented as an illustration of the proposed system. As illustrated in Figure 5.1, a HCap MSU is located in district 1. It should be noted that the people in districts 7 and 8 are within the coverage radius of district 1. Thus, people in districts 7 and 8 can travel to the HCap MSU in district 1. Moreover, LCap MSUs visit some districts to provide service. The first LCap MSU is located at district 3 to provide service. Moreover, people in district 4 can travel to the LCap MSU in district 3 to get service since it is located within the coverage radius of district 3. The second LCap MSU is located at district 2, and people in districts 9 and 7 are within the coverage radius of district 3. However, only district 9 is allocated to LCap MSU at district 2 to get service. On the other hand, district 7 is allocated to HCap MSU at district 1 since it is also within the coverage radius of HCap MSU at 1. The third LCap MSU is located at district 5, while the MCap MSU is located at district 10. district 6 is within the coverage radius of both MCap MSU and the third LCap MSU. However, it is allocated to MCap MSU rather than the LCap MSU.

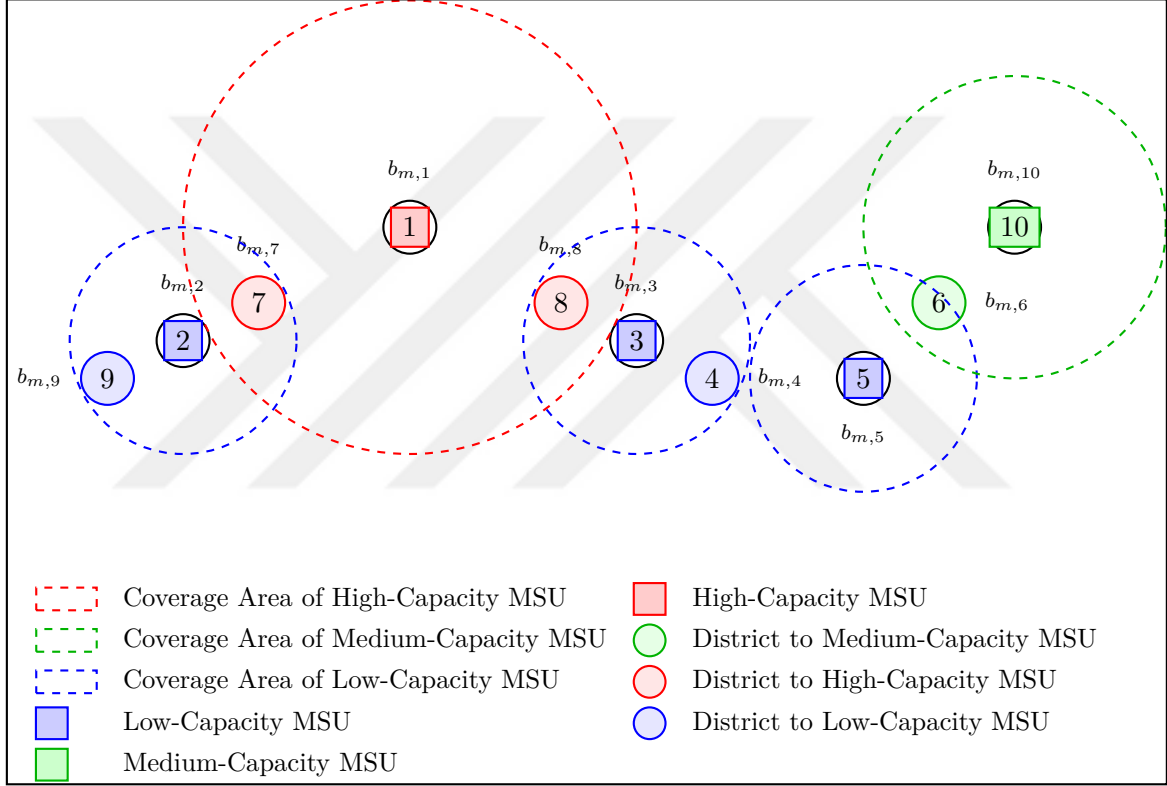


Figure 5.1: Schematic illustration of the location and allocation of HCap MSU, MCap MSU and LCap MSU for continuous demand type of services

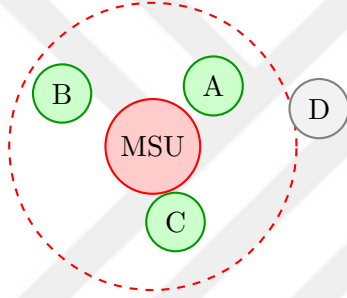
5.1.2 Coverage Function

In the proposed problem, along with location and allocation decisions, we also consider the demand from the covered and allocated districts. In the analysis part, we will be considering two cases: binary coverage and gradual coverage. Figure 5.2 presents the visualization of those coverage functions. In the binary coverage case, we assume that if a district is within the predetermined distance of a center, then the demand at the district will be satisfied fully. In other words, all people in covered districts will receive service from the located MSUs.

In the literature, the location coverage models generally assume that the service level is equally distributed for the districts within the coverage radius of a facility. Even though binary coverage is not the most suitable approach to analyze the victims' participation to the MSUs, we utilize binary coverage to comprehend the demand within the coverage areas of the located MSU which may utilize its services.

Even though the binary coverage assumption is valid in some cases, we extend this assumption by defining a gradual decay function. In the second case, we consider that the people in covered districts may receive service partially. This assumption is realistic since once the locations of the MSUs are decided, people in the covered districts travel to utilize those services. Moreover, due to the chaotic environment in the post-disaster period, people may be late learning about services arriving at certain distances. Therefore, there will likely be a change in maximum potential demand in the coverage area. In this case, even if a district may be within the coverage radius of a MSU, the participation from a distant district is lower than that of a nearby district. We will be complying this case by defining a gradual decay function for demand rates. In binary coverage, we implemented the difference between heterogeneous MSU by variation in their coverage areas. Therefore, allocating the district within the coverage radii of two service units is not highlighted in binary

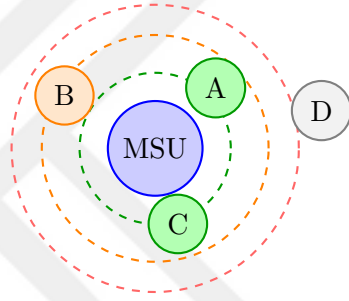
Binary Coverage



$$a_{ij}^u = \begin{cases} 1 & \text{if } c_{ij} \leq \lambda^u \\ 0 & \text{if } c_{ij} > \lambda^u \end{cases}$$

(a) Binary Coverage

Distance-Based Coverage



$$a_{ij}^u = \begin{cases} 1 & \text{if } c_{ij} \leq 1 \\ 1 - \frac{c_{ij}-1}{\lambda^u-1} & \text{if } 1 < c_{ij} \leq \lambda^u \\ 0 & \text{if } c_{ij} > \lambda^u \end{cases}$$

(b) Distance-Based Coverage

Figure 5.2: Different Coverage Functions

coverage. However, by providing a gradual decay function, we define demand rates as a decay function depending on the distance between the visited district and the covered district. In this case, we highlight the allocation of districts to service units by integrating their distance to the service units.

5.1.3 Problem Definition

The formal definition and computational complexity analysis of the CDSP constitute a key contribution of this chapter. We integrate location and allocation decisions that captures coverage-dependent demand decay. We also extend the maximal coverage location literature by introducing a new application area to humanitarian context. The NP-hardness proof (Proposition 2) establishes the computational complexity of the CDSP, justifying the need for the efficient mathematical models developed in the subsequent sections.

Definition 2. *The Continuous Demand Services Problem (CDSP) is defined is defined on a network $G = (V, A)$, where the goal is to determine locations to be visited by a set of heterogeneous MSUs (high-, medium-, and low-capacity) types. Formally, the decision version of the problem is as follows:*

INSTANCE: $\langle G, S, M_U, m_u, \lambda^u, b_{mi} \rangle$ where: $G = (V, A)$ is a graph with V denoting the nodes and A denoting the arcs with edge travel times $c_{ij} \geq 0$ for each $(i, j) \in A$, M_U is the set of MSU types in the system, m_u is the number of MSUs of type $u \in M_U$ to be planned, λ^u is the coverage thresholds for each MSU type $u \in M_U$, b_{mi} is the demand of location $i \in V$ and a positive number Q denoting the required minimum total demand satisfied.

QUESTION: Is there a assignment of m_u MSUs for each service unit type $u \in M_U$ in network G such that each node $i \in V$ is either visited or covered at most once with a total coverage of more than or equal to Q ?

The solution of the CDSP Problem is a 2-tuple $\langle v^u, n^u \rangle$ where v^u is a vector of size $V \times M_U$ representing locations of high-, medium- and low- capacity MSUs, and n is the allocation vector of those service units to nodes.

Consider the MCLP, which is known to be NP-Hard [141].

INSTANCE: Consider a graph $G' = (V', A')$, $p \leq |V'|$, β represent the coverage distance, weights w_i for each vertex $i \in V'$, distance $d_{ij} \quad \forall i, j \in V'$ and a positive number P .

QUESTION: Is there a set of vertices $H' \subseteq V'$ and $|H'| = p$ such that $\sum_{\substack{i: \exists j \in H' \\ d_{ij} \leq \beta}} w_i \geq P$?

Proposition 2. *The CDSP is NP-hard.*

Proof. See Appendix D. □

5.2 Mathematical Model

We present a single integer programming formulation for the CDSP, as our analysis focuses on establishing a foundational model for heterogeneous facility location with coverage. Unlike the one-time demand chapter where we explored alternative time representations, in this chapter, we have strategic decisions without routing or scheduling complexities and a single formulation is appropriate. Regarding replenishment, continuous demand services indeed require sustained supply chains. Our model implicitly assumes that MSUs are continuously resupplied from the central logistics network and ensure that service is provided uninterruptedly. We focus on service continuity perspective rather than a detailed inventory management. Note

that the operational-level decisions such as replenishment schedules and vehicle routing for supply can be modeled explicitly, but they are beyond the strategic focus of this chapter. They can be studied for a future integration with distribution logistics.

The model maximizes demand satisfaction under the assumption that each MSU has sufficient service capacity to meet the demand arising from its assigned nodes. It does not imply infinite physical supply, but rather reflects a planning priority in real-life. In the acute phase of a disaster, the main aim is to deploy resources to satisfied as much demand as possible while assuming that the supply chain will strive to provide the necessary throughput.

On the other hand, the capacity is implicitly handled through two aspects: (1) the heterogeneity of MSU types, and (2) the coverage decay function. The different types of MSUs (HCap, MCap, LCap) represent different scales of service capability and the coverage decay function naturally limits the demand allocated to a facility by decaying based on distance. Note that the model can explicitly utilize capacity constraints or service capacity parameters and then it would become a capacitated facility location problem. This approach would lead to more dispersed locations to balance load. Our current model focuses on strategic first-response planning while capacity refinement represents an operational dimension that can be incorporated in future work to enable more detailed planning.

We define v_j^u as a binary decision variable which determines the locations of HCap, MCap and LCap MSUs and define n_{ij}^u as a binary decision variable deciding allocations of districts to receive service from those MSUs. Accordingly, we formulate the CDP as follows:

Table 5.1: Sets and parameters

Sets and Parameters	
V	Set of potential service locations
M_U	Set of different service unit types of MSUs (h: High-Capacity, m: Medium-Capacity, l: Low-Capacity)
m_u	Total number of different types of MSUs to be planned where $u \in M_U$
λ^u	Coverage threshold for different types of MSUs to be planned where $u \in M_U$
c_{ij}	Travel distance between $i \in V$ and $j \in V$
F_i^u	$\{l \in V : c_{il} \leq \lambda^u\}$: Set of nodes in the allowable walking distance to node $i \in V$ where a different types of MSUs where $u \in M_U$ is located
b_{mi}	Maximum potential demand to be served at location $i \in V$ by a MSU
a_{ij}^u	$\begin{cases} 1 & \text{if } c_{ij} \leq 1 \\ 1 - \frac{c_{ij}-1}{\lambda^u-1} & \text{if } 1 < c_{ij} \leq \lambda^u \\ 0 & \text{if } c_{ij} > \lambda^u \end{cases}$

Decision Variables:

$$v_j^u = \begin{cases} 1 & \text{if a type } u \in M_U \text{ MSU is located at } j \in V \\ 0 & \text{otherwise} \end{cases}$$

$$n_{ij}^u = \begin{cases} 1 & \text{if people in location } i \in V \text{ is assigned to type } u \in M_U \text{ MSU at } j \in F_i^u \\ 0 & \text{otherwise} \end{cases}$$

$$\text{maximize } \sum_{u \in M_U} \sum_{j \in V} b_{mj} v_j^u + \sum_{u \in M_U} \sum_{i \in V} \sum_{\substack{j \in F_i^u \\ i \neq j}} a_{ij}^u b_{mi} n_{ij}^u \quad (5.1)$$

subject to

$$\sum_{i \in V} v_i^u \leq m_u \quad u \in M_U \quad (5.2)$$

$$n_{lj}^u \leq v_j^u \quad l \in V, j \in F_l^u, u \in M_U \quad (5.3)$$

$$\sum_{u \in M_U} v_j^u + \sum_{\substack{j \in F_i^u \\ i \neq j}} n_{ij}^u \leq 1 \quad i \in V \quad (5.4)$$

$$v_i^u \in \{0, 1\} \quad i \in V, u \in M_U \quad (5.5)$$

$$n_{ij}^u \in \{0, 1\} \quad i \in V, j \in F_i^u, u \in M_U. \quad (5.6)$$

The objective (5.1) is to maximize the people served by each service unit type and demand served allocated to those center within their coverage radius. Constraint (5.2) restricts the number of MSUs located to be less than or equal to m_u for different types $u \in M_U$. Constraint set (5.3) ensures that if a type of MSU is located at a node, then the nodes within their coverage radius can be assigned to those MSUs to get service. Constraint (5.4) ensures that each demand location is covered at most once by all different MSUs. Constraint sets (5.5)–(5.6) corresponds to domain constraints.

5.3 Computational Study

In the analysis section, we provide an analysis based on the coverage. We first provide a base case scenario analysis by setting a_{ij}^u to binary coverage. This is a simple case in which we assume that all people in the covered areas can get service from the nodes where MSUs are located. If a node is not within the coverage area of a MSU, then it simply assumes that it cannot be served. In this case, we do not include the difference in serving by directly visiting the node or by covering the node. The differences in their coverage areas highlight the differences between the heterogeneous MSUs.

We later provide a_{ij}^u as a gradual decay function and define it as a uniform decay function depending on the distance between the visited node and the covered node. Therefore, the demand rate from the covered nodes is at the top level within a minimum critical distance. However, beyond this minimum critical distance, the rate decreases with distance until the maximum critical distance. Finally, there will be no participation from the nodes that are beyond the maximum critical distance.

This idea is proposed by [142] and we use it in our results. We present an analysis focusing on the performance metric of the total demand served with a changing number of LCap MSUs, MCap MSUs and HCap MSUs.

All experiments are performed on an Intel Xeon Silver 4210 with 14 GB RAM. The models are solved using IBM ILOG CPLEX 12.10.0. All runs in this section are in seconds.

5.3.1 Datasets and Parameter Settings

The proposed model is applied to the Kartal and Istanbul datasets. Kartal dataset is used by [135] while Istanbul dataset is used by [133]. The Kartal dataset provides a well-established and benchmarked network structure. Istanbul dataset represent a large-scale metropolitan with high population density and spatial heterogeneity. Continuous demand services are particularly critical in densely populated urban environments. Istanbul dataset therefore allows us to assess the scalability and practical relevance of the proposed model under realistic urban conditions. On the other hand, other datasets are not included since the objective of this chapter is not to provide an exhaustive empirical comparison across multiple regions. Our aim is to establish and demonstrate a generalizable modeling framework for continuous demand services with heterogeneous MSUs and gradual coverage. Thus, by utilizing those aforementioned datasets, we provide a balance between methodological validation and practical applicability.

In both datasets, predefined coverage thresholds are used to represent service accessibility based on walking distance for disaster victims. Specifically, the predefined coverage threshold for availability is 5 kilometers for HCap MSU (λ^h), 3 kilometers for MCap MSU (λ^m) and 2 kilometers for LCap MSUs (λ^l). These values reflect

both the operational nature of different unit types and behavioral realities in post-disaster environments. Large, high-capacity units (such as field hospitals or major distribution centers) are typically well-publicized and attract people from wider areas; hence, a 5 km radius is appropriate. Medium-capacity units serve more localized needs with a 3 km range. Low-capacity mobile units (e.g., small health teams or local food points) are often less visible and provide services that require frequent access; therefore, a shorter 2 km radius ensures realistic accessibility. This tiered coverage structure aligns with humanitarian practice and allows our model to capture the complementary roles of different MSU types in a layered response network. While the specific distances can be adjusted based on context, our model's structure allows for easy parameter variation. A sensitivity analysis on coverage radius is indeed a relevant extension; it would reveal how service accessibility and optimal location strategies change with different mobility assumptions or infrastructural conditions. However, such an analysis falls outside the scope of this chapter. Our main objective is to establish and demonstrate the core modeling framework for heterogeneous MSU location under continuous demand. Further to present demand structure in continuous demand, we utilize binary and gradual coverage paradigms. Conducting a full parameter sensitivity study would significantly expand the computational analysis, potentially diluting the focus on the key managerial insights regarding facility heterogeneity and coverage decay. We therefore leave the systematic exploration of sensitivity analysis for future research.

For the base case scenario, we use binary coverage for a_{ij}^u to reflect the total demand that can be served. The binary coverage is defined as follows for demand rates:

$$a_{ij}^h = \begin{cases} 1 & \text{if } c_{ij} \leq \lambda^h \\ 0 & \text{if } c_{ij} > \lambda^h \end{cases}$$

$$a_{ij}^m = \begin{cases} 1 & \text{if } c_{ij} \leq \lambda^m \\ 0 & \text{if } c_{ij} > \lambda^m \end{cases}$$

$$a_{ij}^l = \begin{cases} 1 & \text{if } c_{ij} \leq \lambda^l \\ 0 & \text{if } c_{ij} > \lambda^l. \end{cases}$$

Later, for analysis on gradual coverage, a_{ij}^u is chosen as a function depending on distance to reflect the reduction in population served due to coverage. Accordingly, we define the gradual coverage as follows for demand rates:

$$a_{ij}^h = \begin{cases} 1 & \text{if } c_{ij} \leq 1 \\ 1 - \frac{c_{ij}-1}{\lambda^h-1} & \text{if } 1 < c_{ij} \leq \lambda^h \\ 0 & \text{if } c_{ij} > \lambda^h \end{cases}$$

$$a_{ij}^m = \begin{cases} 1 & \text{if } c_{ij} \leq 1 \\ 1 - \frac{c_{ij}-1}{\lambda^m-1} & \text{if } 1 < c_{ij} \leq \lambda^m \\ 0 & \text{if } c_{ij} > \lambda^m \end{cases}$$

$$a_{ij}^l = \begin{cases} 1 & \text{if } c_{ij} \leq 1 \\ 1 - \frac{c_{ij}-1}{\lambda^l-1} & \text{if } 1 < c_{ij} \leq \lambda^l \\ 0 & \text{if } c_{ij} > \lambda^l. \end{cases}$$

The shift from binary to gradual coverage fundamentally changes the interpretation of coverage distance. Under binary coverage, distance acts as a fixed threshold. Service is assumed to be fully available within the threshold and entirely unavailable beyond it. In contrast, gradual coverage treats distance as an upper bound on willingness to travel. Also, demand satisfaction or service utilization is not uniform but progressively declines as distance from the facility increases. In practice, although individuals may be physically capable of traveling longer distances (e.g., 5 km), the probability of doing so for routine needs such as daily meals declines with distance, terrain, and individual circumstances. Consequently, change in coverage distance in the gradual model represents more than a geographic expansion; it also reflects decreasing participation rates and levels of demand satisfaction. This formulation

enables the model to prioritize proximity and supports the design of a more resilient network by ensuring populations have accessible service options nearby.

5.3.2 Analysis of Optimal Solutions

This section analyzes the optimal solutions of different instances over demand changing due to different coverage functions. We first analyze the solutions with the binary coverage.

5.3.2.1 Binary Coverage

In Table 5.2, we present the population served for different configurations of HCap MSUs, MCap MSUs and LCap MSUs located. The numbers of HCap, MCap and LCap range from 1 to 5. The number of nodes served by HCap ranges from 8 to 35, while the number of nodes served by MCap ranges from 2 to 18. Finally the number of nodes served by LCap ranges from 1 to 14. The HCap MSU generally provides service to a greater number of districts than MCap MSUs and LCap MSUs. This is due to the variation in their coverage areas, with HCap MSUs associated with the highest coverage radius. HCap MSUs are associated with a greater coverage radius due to their strategic utilization and their greater capacities and resources compared to MCap MSUs and LCap MSUs. Also, the locations of each MSU type are strategically determined by the larger populations that can be served by those centres within their coverage radii.

When there is only one HCap MSU, the model generally chooses to locate the HCap MSU at District 17, since within its coverage radius, there are more districts and densely populated areas to cover. There are only two exceptional cases: locating 1 MCap MSU with 3 and 5 LCap MSUs. In those cases, the model chooses to locate

the HCap MSU at District 21 rather than District 17. When a HCap MSU is located at District 21, compared to District 17, there is a smaller number of districts to cover; however, they are more dense areas. In those cases, since the number of LCap MSUs is increased, the districts with high-density populations within the coverage radius of District 17 are now visited by LCap MSUs. Therefore, the locating a HCap MSU on District 17 is no longer effective. When a HCap MSU is located at District 21, only the low-density populations within the coverage radius of District 17 are not served.

According to Table 5.2, as the number of HCap, MCap and LCap MSUs increases, the total number of districts served increases; therefore, the total demand served by those MSUs increases. When there are only 1 HCap MSU, 1 MCap MSU and 1 LCap MSU, only 44.02% of the whole population can be served in the system. As the number of HCap MSU, MCap MSU and LCap MSU increases, the percentage of demand served increases. In the cases of 5-3-5, 5-5-3 and 5-5-5 combinations of number of HCap, MCap and LCap MSUs, respectively, the whole demand of Kartal can be served by reaching every 45 districts by MSUs. In the binary coverage case, the allocations of districts within the coverage radii of two MSUs may change, and the optimal objective function value stays the same. This is because the districts within the coverage radii are served fully. Since we allow single allocation, there can be alternative allocations for the districts that are within the coverage radii of different service units. However, due to single allocation, the coverage radii of MSUs are not preferred to be intersected to maximize the population served. For instance, in Figure 5.3d, a LCap MSU is located at District 20, and District 45 is within its coverage radius. Also, District 45 is within the coverage radius of HCap MSU located at District 35. Allocating District 45 to HCap MSU at District 35 or LCap MSU at District 20 will provide the same objective function values. In this analysis, we do not include a distance-based coverage model but integrate binary coverage. In this case, only being within the coverage radius is important. However, the allocation is not important if it is within the coverage radius of a MSU. We will provide a gradual

decay analysis later to highlight the allocation decisions.

Also, when we analyze the percentages of total demand served, we observe that there are combinations of HCap, MCap and LCap MSUs which provide the same objective function value and the same percentage of total demand served. With the cases of 3-3-3 and 3-5-1 combinations of HCap, MCap and LCap MSUs, respectively, 89.26% of total demand can be served. When we chose 3-3-5 and 3-5-3 for the HCap, MCap and LCap MSUs, respectively, we observe that 94.55% of the total demand is to be served. Finally with the cases of 5-1-3 and 5-5-3, 96.33% of total demand and with the cases of 5-1-5, 5-3-3 and 5-5-1, 98.94% of total demand can be served. When we analyze the cases of 3-3-3 and 3-5-1, we observe that HCap MSU locations are exactly the same in each combination. In the case of 3-5-1, Districts 5, 6, 14, 23 and 39 are chosen for the locations of MCap MSUs, while District 25 is chosen as the location for LCap MSU. On the other hand, in the case of 3-3-3, Districts 5, 14 and 23 are chosen for the locations of MCap MSUs while Districts 25, 38 and 39 are chosen as locations for LCap MSUs. Even though the coverage radii of MCap MSUs and LCap MSUs are different, the districts within the coverage areas of Districts 38 and 39 are no further than 2 km. Therefore, locating a LCap MSU in those districts provides the same coverage area.

In all cases, the locations of MSUs are strategically determined based on the population density of visited districts and covered districts. HCap MSUs have a greater coverage radius; therefore, they are located in areas with high population density. This allows HCap MSUs to serve a higher number of districts. On the other hand, MCap MSUs are generally located in areas with a larger population, although not as densely populated as HCap MSUs due to their lower coverage areas. Finally, generally, LCap MSUs are located in districts such that within their coverage radius, there is at most one district other than themselves. They serve as supplementary vehicles for high-population districts, but are located far away from most of the districts. It should be noted that even though a district visited by LCap MSUs

might have a high population, the population density within the coverage area of LCap MSUs is generally low. This is because there are not many districts within its coverage area.

In Figure 5.3, we provide some of the solutions of the results presented in Table 5.2. The solutions are provided for some combinations of HCap, MCap and LCap MSUs. When there is only one HCap MSU, it is generally located at District 17. Meanwhile, MCap MSUs are located at the districts whose coverage radii include districts with higher populations. In case of HCap:1, MCap:3 and LCap:3, the MCap MSUs are located at Districts 14, 24 and 27. Even though their coverage areas are smaller than HCap MSUs, they are strategically located to cover other high-population districts in those areas. Also, the LCap MSUs are located at 4, 7 and 39. Their coverage radii are much smaller than MCap MSUs. Only one of the LCap MSUs covers districts other than itself.

When we analyze the case of HCap:3, MCap:1 and LCap:3, there is HCap MSU at District 35, which is very close to District 17 and covers almost all of the districts within District 17 except District 32. However, different from District 17, HCap MSU at District 35 covers District 18, which was not served by District 17. Also, there are other HCap MSUs at Districts 22 and 44. HCap MSU at District 22 covers District 32 and other highly populated districts such as 21 and 27. In the case of 1 HCap MSU, Districts 27 and 24 were chosen as the location for MCap MSUs due to dense population. In the case of HCap:3, MCap:1, and LCap:3, Districts 22 and 44 are chosen as HCap MSU locations since they cover Districts 24 and 27 and their coverage areas. Moreover, 44 and 22 are now chosen as HCap MSU, implying they have larger coverage radii, reach more districts, and increase the number of people served. In this case, District 4 is already within the coverage radius of District 44; therefore, no LCap MSU is located at District 4. In Figure 5.3b, there is only one MCap MSU located, which aligns with the MCap MSU in Figure 5.3a. Finally, the LCap MSUs locations also align with the location of LCap MSUs in Figure 5.3a.

# HCap*	# MCap*	# LCap*	Total demand served	Number of nodes served by HCap	Number of nodes served by MCap	Number of nodes served by LCap	Total number of nodes served	% of demand served (%)
1	1	1	428,278	13	4	3	20	44.02
1	1	3	517,740	8	3	9	20	53.22
1	1	5	604,213	8	3	14	25	62.11
1	3	1	599,296	13	11	1	25	61.60
1	3	3	670,597	13	11	5	29	68.93
1	3	5	729,476	13	11	8	32	74.98
1	5	1	685,210	13	16	1	30	70.43
1	5	3	755,597	11	17	5	33	77.67
1	5	5	814,476	10	18	8	36	83.72
3	1	1	725,091	25	4	4	33	74.53
3	1	3	794,977	25	4	5	34	81.72
3	1	5	852,052	25	4	9	38	87.58
3	3	1	811,005	25	9	1	35	83.36
3	3	3	868,328	25	8	5	38	89.26
3	3	5	919,790	25	7	9	41	94.55
3	5	1	868,328	25	12	1	38	89.26
3	5	3	919,790	25	13	3	41	94.55
3	5	5	950,898	20	15	8	43	97.74
5	1	1	881,153	35	2	1	38	90.57
5	1	3	937,180	35	2	5	42	96.33
5	1	5	962,547	33	2	9	44	98.94
5	3	1	937,180	34	7	1	42	96.33
5	3	3	962,547	29	10	5	44	98.94
5	3	5	972,849	32	7	6	45	100.00
5	5	1	962,547	31	11	2	44	98.94
5	5	3	972,849	27	13	5	45	100.00
5	5	5	972,849	25	12	8	45	100.00

* # HCap, # MCap and # LCap denote the number of high-capacity, medium-capacity and low-capacity MSUs, respectively.

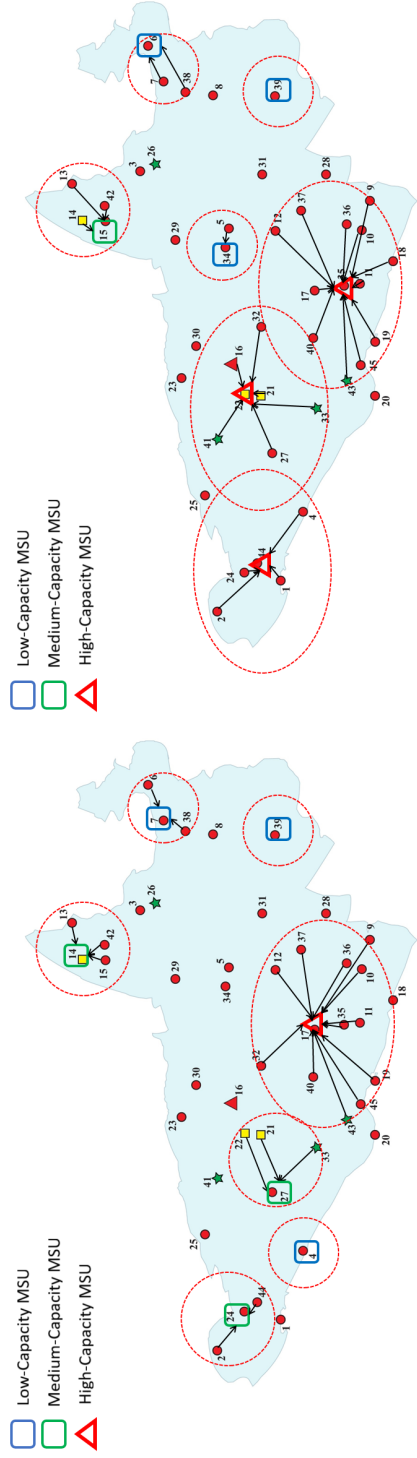
Table 5.2: Results for Kartal dataset with demand rates (a_{ij}^u) defined as binary coverage

However, in this case, since District 4 is already covered by HCap MSU at 44, the third LCap MSU is located at District 34, which covers District 5.

When we analyze the case with HCap:3, MCap:3 and LCap:1, the HCap MSU locations are the same as in the Figure 5.3b. One of the MCap MSUs is located at District 15, as in Figure 5.3b. However, other MCap MSUs are located at 23 and 38. District 38 is chosen as the location for LCap MSU in Figure 5.3b. In Figure 5.3c, District 38 is chosen as MCap MSU location even though the districts within its coverage do not change. Finally, a LCap MSU is located at District 39. Note that District 39 can be chosen as MCap MSU location while District 38 can be chosen as LCap MSU location as an alternative solution. Even if we locate a MCap MSU or LCap MSU at District 39, within its 3 km coverage radius, there are no districts. Similarly, for District 38, Districts 6 and 7 are within its 2 km coverage radius, so locating a MCap MSU and LCap MSU at District 38 will lead to the same number of people served. As the number of HCap MSUs increases, more densely populated areas are covered due to the larger coverage radius of HCap MSUs. However, as the number of MCap MSUs and LCap MSUs increases, they act as supplementary units.

5.3.2.2 Gradual Coverage

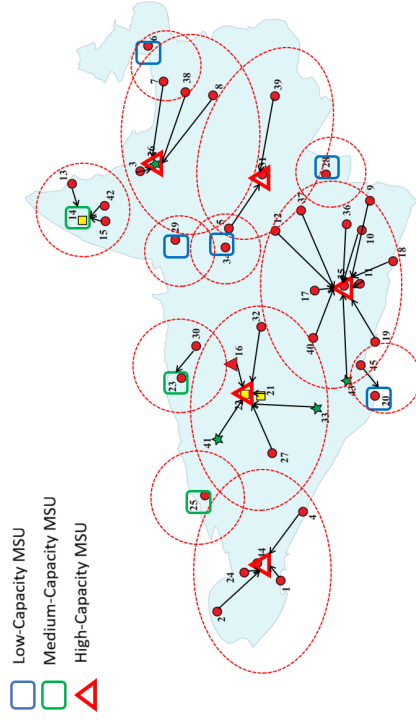
In Table 5.3, we present the population served for different numbers of HCap, MCap and LCap MSU by integrating the gradual decay function. The number of HCap, MCap and LCap MSUs ranges from 1 to 5. The number of districts served by HCap MSUs ranges from 8 to 27, while the number of districts served by MCap MSUs ranges from 3 to 14. Finally, the number of districts served by LCap MSUs ranges from 1 to 6. Unlike the binary coverage, the number of districts allocated to the service units changes drastically. The primary reason is the change in demand resulting from the decay function. In most cases, HCap MSUs generally provide service to a greater number of districts compared to MCap and LCap MSUs. However, there



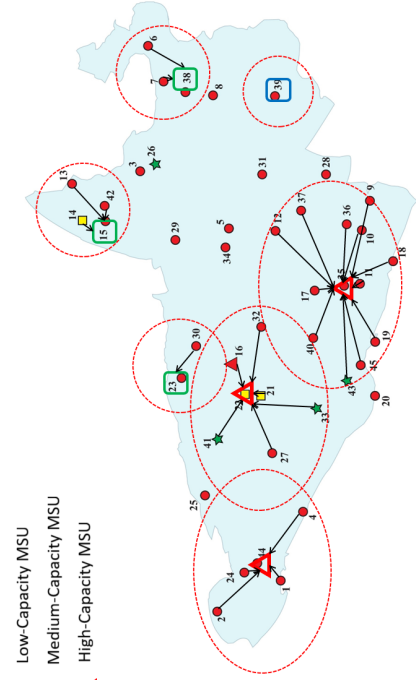
(a) HCcap:1, MCcap:3, LCap:3



(b) HCcap:3, MCcap:1, LCap:3



(c) HCcap:3, MCcap:3, LCap:1



(d) HCcap:5, MCcap:3, LCap:5

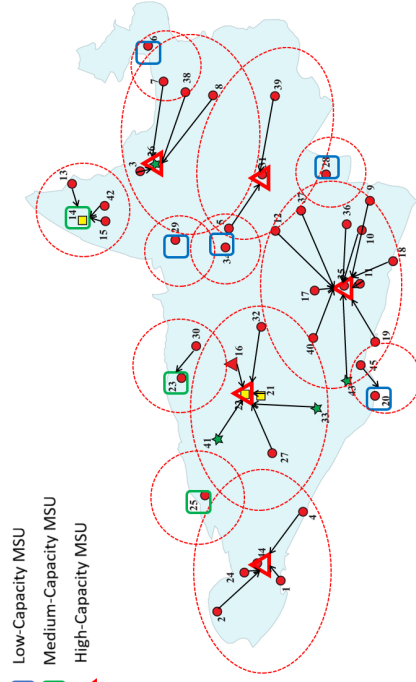


Figure 5.3: Solutions of Kartal dataset with demand rates (a_{ij}^u) defined as binary coverage

are some exceptional cases such as 1-3-1, 1-5-1, 1-5-3 and 1-5-5. In those cases, even if some districts are within the coverage radius of HCap MSUs, they are very far away from the HCap MSUs. Then, due to gradual decrement, those districts are not served by HCap MSUs. On the other hand, the districts within the coverage radius of MCap MSUs are mostly not that far away from MCap MSUs, so they are served by MCap MSUs even though there is a gradual decrement.

Moreover, as the number of MCap MSUs and LCap MSUs increases, the districts served by HCap MSUs decrease. There are some districts which are within the coverage radius of a HCap MSU; however, due to a gradual decrement, they could not be served, or a small portion of their population could be served. Some of those districts may have high population. In this case, another MSU is located to provide service to those locations or provide service to a higher portion of the population since newly located MSUs are considerably closer to those areas.

According to Table 5.3, as the number of HCap, MCap and LCap MSUs increases, the total number of districts served increases; therefore, the total demand served by those units increases. When there are only 1 HCap, 1 MCap and 1 LCap MSUs, only 22.88% of the whole population can be served in the system. This percentage is almost half of 44.02%, which is the percentage covered without a gradual decrement. Due to a gradual decrease, the percentage of the population served decreases drastically. Moreover, as the number of HCap, MCap and LCap MSUs increases, the percentage of demand served increases. In the binary coverage case, it is possible to cover all districts and serve everyone with some combinations of HCap, LCap and LCap MSUs. Due to the gradual decrement, it is not possible to serve the whole population unless there are 41 one type of MSUs. Since there is a gradual decrement due to distance, even though all districts are served by being within the coverage radius of a MSU, it is not possible to reach the total demand. Reaching the whole population is only possible with 41 MSUs in this dataset. For instance, if there are only 41 LCap MSUs, 41 districts will be directly served by those units, and the other

4 districts will be within their 1 km distance. Unlike the binary coverage case, in the gradual decrement case, the allocations of districts within the coverage radii of two MSUs depend on their distance to the service units. Therefore, allocating a district to a closer MSU will improve the objective function value. This is because the districts within the coverage radii are served partially. Moreover, in this case, again, single allocation is allowed. Therefore, a district can be served only by a closer MSU, even though the district can be within the coverage radius of two service units. For instance, in Figure 5.4d, a HCap MSU is located at District 22, and District 33 is within its coverage radius. It can be noted that District 33 is also within the coverage radius of HCap MSU located at District 43. Since District 33 is exactly 5 km away from District 22, it cannot be served, or 0% of its population can get service from HCap MSU at 22. However, District 33 has a high population and locating another HCap MSU at District 43 and allocating District 33 to this HCap MSU improves the objective function value. In the gradual decrement case, apart from being within the coverage radius, being closer to the MSU is more important, affecting the number of people served.

In this case, the allocations of MSUs are strategically determined based on the population density of districts, as well as the distance between districts and MSUs. HCap MSUs have a greater coverage radius; therefore, they are located in areas with high population density to maximize accessibility. However, the population receiving service decreases due to their distance from HCap MSUs. Unlike the binary coverage case, MSUs can be located at districts which are within the coverage radius of another MSU. Thus, the districts, within the coverage radius of one MSU but not served, can be served by another MSU.

In Figure 5.4, for some combinations of HCap, MCap and LCap MSUs, the solutions of the results are presented in Table 5.3. In case of HCap:1, HC:3 and MC:3, HCap MSU is located at District 40 while MCap MSUs are located at Districts 7, 36 and 42. Moreover, HCap MSU provides service to all of the districts within its

# HCap*	# MCap*	# LCap*	Total demand served	Number of nodes served by HCap	Number of nodes served by MCap	Number of nodes served by LCap	Total number of nodes served	% of demand served (%)
1	1	1	222,561	6	3	1	10	22.88
1	1	3	293,627	9	3	4	16	30.18
1	1	5	353,921	9	3	6	18	36.38
1	3	1	296,772	6	7	1	14	30.51
1	3	3	364,990	9	9	4	22	37.52
1	3	5	425,284	9	9	6	24	43.72
1	5	1	365,507	6	12	1	19	37.57
1	5	3	425,801	6	12	3	21	43.77
1	5	5	486,095	6	12	5	23	49.97
3	1	1	354,073	17	3	1	21	36.40
3	1	3	419,505	16	3	3	22	43.12
3	1	5	478,643	15	3	5	23	49.20
3	3	1	423,653	16	7	1	24	43.55
3	3	3	484,728	16	9	3	28	49.83
3	3	5	543,466	16	9	5	30	55.86
3	5	1	484,728	16	11	1	28	49.83
3	5	3	543,466	16	11	3	30	55.86
3	5	5	597,538	14	14	5	33	61.42
5	1	1	461,199	27	3	1	31	47.41
5	1	3	521,494	27	3	3	33	53.60
5	1	5	575,345	27	3	5	35	59.14
5	3	1	521,493	27	5	1	33	53.60
5	3	3	575,345	27	5	3	35	59.14
5	3	5	623,063	23	7	5	35	64.05
5	5	1	575,345	27	7	1	35	59.14
5	5	3	623,063	23	9	3	35	64.05
5	5	5	670,060	20	12	5	37	68.88

* # LCap, # MCap and # HCap denote the number of high-capacity, medium-capacity and low-capacity MSUs, respectively.

Table 5.3: Results for Kartal dataset with demand rates (a_{ij}^u) defined as gradual decay functions with uniform decay

coverage radius. The coverage areas of MCap MSUs are smaller than HCap MSUs and they are strategically located to cover closer districts so that they can improve the number of people served. For instance, the population in District 24 is higher than in District 36. However, rather than locating a MCap MSU at District 24, which covers Districts 2 and 44, one MCap MSU is located at District 36, which covers Districts 9 and 10. Even though District 24 covers Districts 2 and 44, it cannot provide service to those locations since they are exactly 3 km away from District 24. Due to a gradual decrease, those locations are not served. However, MCap MSU at District 36 provides service to Districts 9 and 10. Moreover, it provides service to the whole population of those districts since they are 1 km away from MCap MSU at District 36. It is better to locate a MCap MSU at 36 to improve the objective function value. Furthermore, LCap MSUs are located at 13, 21 and 39. Their coverage radii are much smaller than MCap MSUs. Only one of the LCap MSUs covers districts other than itself. One of the LCap MSUs is located at District 13, which is close to a MCap MSU at District 42 but does not lie within its coverage radius. Since District 13 has high population density, a LCap MSU is located.

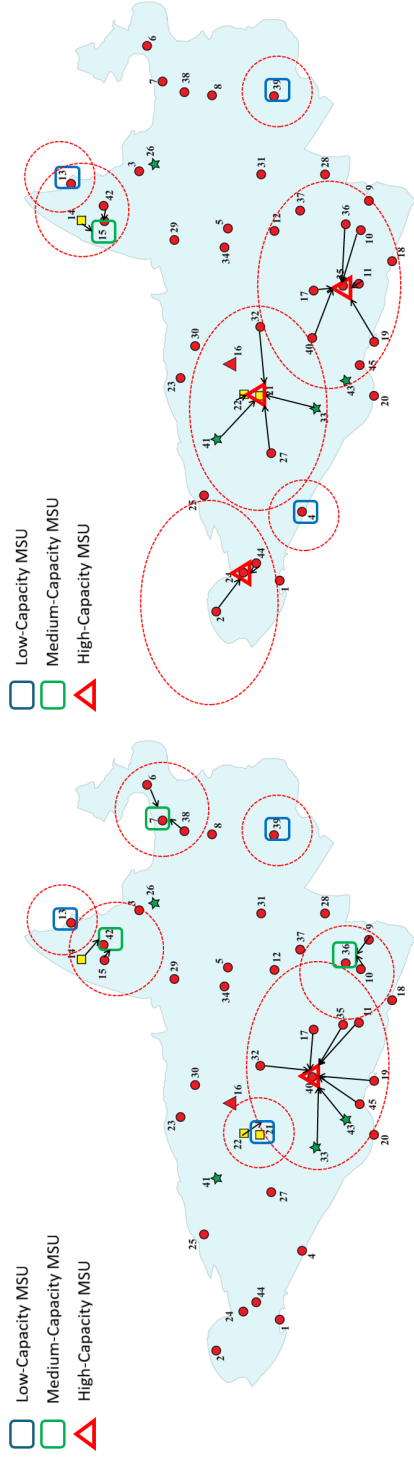
When HCap:3, MCap:1 and LCap:3, HCap MSUs are located at 21, 24 and 35. Districts 21 and 35 cover all the districts covered by District 40 and cover even more districts. The other HCap MSU is located at District 24, covering Districts 2 and 44, which have high populations. Since HCap MSUs have large coverage radius, Districts 2 and 44 are served. In Figure 5.4b, there is only one MCap located, which aligns with the MCap in Figure 5.4a. Finally, the LCap MSUs locations also align with the location of LCap MSUs in Figure 5.4a. However, in Figure 5.4b, District 21 is now chosen as HCap MSU; therefore, there is no LCap MSU located at District 21. Also, District 4 is not within the coverage radius of District 24, therefore, a LCap MSU is located at District 4.

When we move to the case with HCap:3, MCap:3 and LCap:1, the HCap MSU locations are the same as in the Figure 5.4b. One of the MCap MSUs is located at

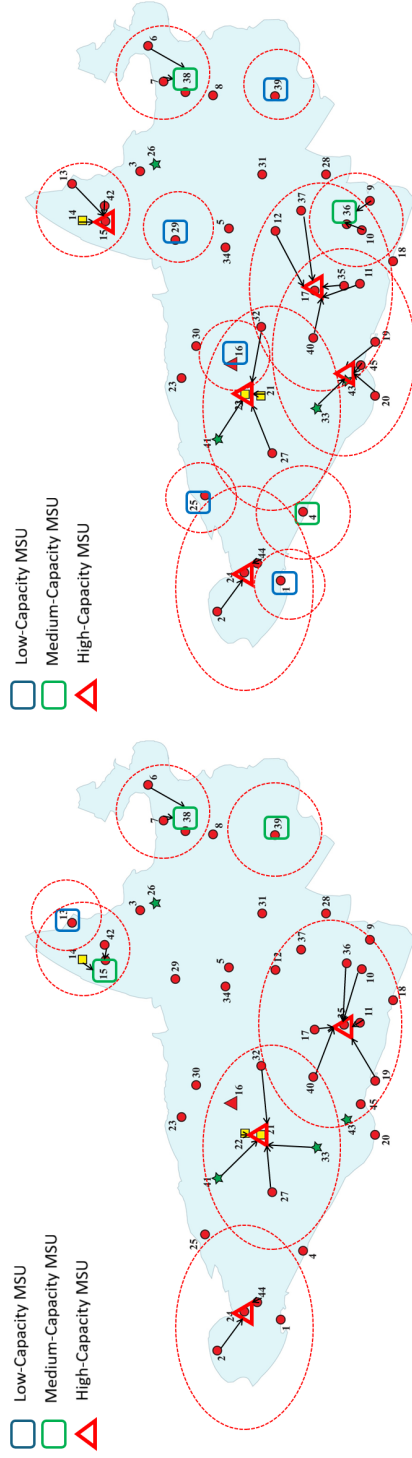
District 15, as in Figure 5.4b. However, other MCap MSUs are located at 38 and 39. District 39 was chosen as the location for LCap MSU in Figure 5.4b. In Figure 5.4c, it is chosen as MCap MSU location even though there are no other districts within its coverage. Finally a LCap MSU is located at District 13, similar to Figure 5.4b. However, District 39 can be chosen as the MCap MSU location, while District 38 can be chosen as the LCap MSU location as an alternative solution. As the number of HCap MSUs increases, more densely populated areas are covered due to the larger coverage radius of HCap MSUs. Also, HCap MSUs are more closely located to cover locations. If one of HCap MSUs cannot cover a location, then another HCap MSU can easily cover this location. Moreover, as the number of MCap MSUs and LCap MSUs increases, they again act as supplementary units to HCap MSUs. They are located in areas closer to HCap MSU but are not served by HCap MSU. Also, they are located in areas which have high population density and are far from other high-density areas.

After analyzing the Kartal dataset, we have solved the mathematical model for both binary coverage and gradual coverage cases in the Istanbul dataset. For binary coverage, we have provided the results and solutions for a combination of the number of HCap, MCap and LCap MSUs in which the whole population is served. For the gradual coverage case, we have provided the results and solutions for the same combination of number of HCap, MCap and LCap MSUs as in the binary coverage case for comparison. The results are provided in Table 5.4 and the solutions are provided in Figure 5.5.

According to the Table 5.4, in binary coverage, 44 districts are served by HCap MSUs while 45 districts are served by MCap MSUs. HCap and MCap MSUs are located in crowded areas. On the other hand, the LCap MSUs are located in distinct areas with high population. When we analyze the solutions of each result presented in Table 5.4, we observe that in the binary coverage case, MCap MSUs are able to serve more districts and more people. The reason is that they are located in crowded



(a) HCcap:1, MCcap:3, LCap:3



(b) HCcap:3, MCcap:1, LCap:3

(c) HCcap:3, MCcap:3, LCap:1

(d) HCcap:5, MCcap:3, LCap:5

Figure 5.4: Solutions for Kartal dataset with demand rates (a_{ij}^u) defined as gradual decay functions with uniform decay

Coverage Type	# HCap*	# MCap*	# LCap*	Total demand served	Number of nodes served by HCap	Number of nodes served by MCap	Number of nodes served by LCap	Total number of nodes served	% of demand served
Binary coverage	7	7	7	350,032.4	44	45	10	99	100.00%
Gradual coverage	7	7	7	290,381.7	44	29	10	83	82.95%

*# LCap, # MCap and # HCap denote the number of high-capacity, medium-capacity and low-capacity MSUs, respectively.

Table 5.4: Results for Istanbul dataset with different demand rates (a_{ij}^u)

areas and serve the crowded areas. The HCap MSUs are located in crowded areas; however, due to single allocation, HCap MSUs are not located close to each other. If they are located close to each other, one district can be within the coverage radius of two HCap MSUs, but it can only get service from one of those units. Therefore, the service availability of a HCap MSU decreases. To increase the availability, HCap MSU locations are chosen at a distance from each other. Also, since their coverage radius is larger than that of MCap MSUs, there are some areas in which MCap MSUs may not be able to cover the neighbouring districts within a 5 km distance. However, in the crowded areas with more districts, MCap MSUs can be located without intersecting with another MSU and covering other highly populated areas. In this case, rather than 1 HCap MSU, 2 MCap MSUs may be needed.

In the gradual coverage case, the number of districts served by MCap MSUs decreases. HCap MSUs are located in crowded areas again in this case. However, due to a gradual decrease, some locations may be within the coverage radius, but only a very small portion of their population can be served. MCap MSUs are located at such locations to cover these highly populated areas by being closer to increase the attendance rates. LCap MSUs also play similar roles in the gradual decrement

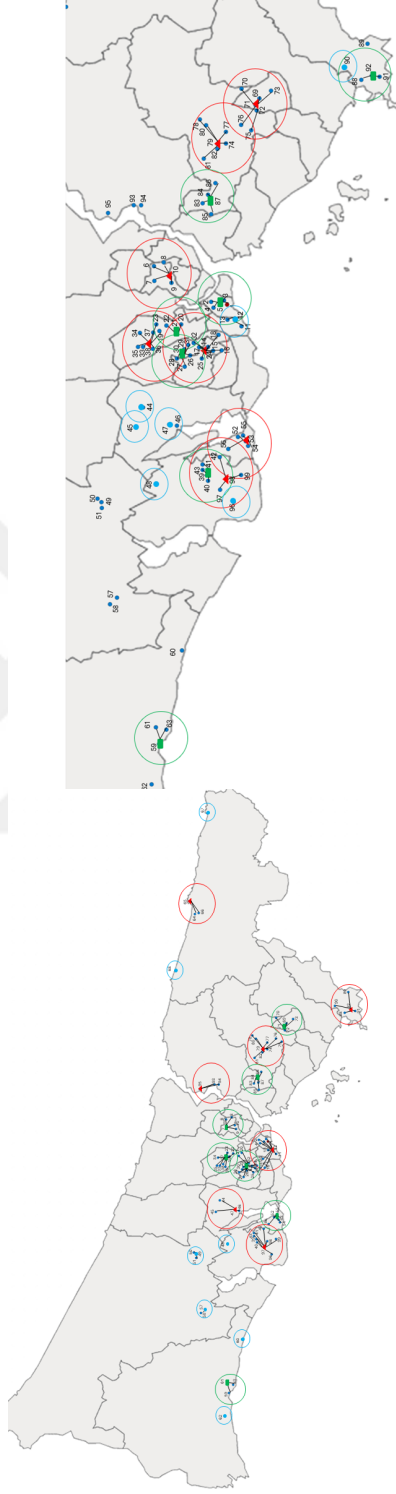
case. They reach out to areas that are located very distant within the coverage radius. However, in the gradual decrement case, the total percentage of people served decreases to 82.95% and more MSUs are required to reach more people.

5.4 Conclusion of Chapter 5

In this chapter, we focus on continuous demand during disaster response. The continuous demand includes services such as hot meals and medical care, which are essential for survival after a disaster. They should be provided uninterrupted; therefore, MSUs should be stationed at their assigned locations to provide constant availability. We utilize different types of MSUs -HCap, MCap, and LCap- and allocate demand districts to them to maximize the total demand served. We also consider the coverage concept, which assumes that people can travel from nearby areas within a coverage radius to receive aid.

We consider two concepts for how demand is satisfied from covered areas. In binary coverage, we assume that all individuals within a service unit's coverage radius are fully served. However, in gradual coverage, we assume that the demand rates are defined as a decay function to reflect a decrease with respect to distance. We mathematically formulated the problem and analyzed the results on the Kartal and Istanbul datasets.

Under binary coverage, we observe that high-capacity MSUs are placed in high-density areas, and it is possible to achieve 100% population coverage with sufficient units. However, under the more realistic assumption of gradual coverage, the total demand satisfied drops significantly because distant individuals are less likely to access the services. Therefore, the model prioritizes locating MSUs closer to demand areas to decrease the decay effect.



(a) Binary coverage with HCap:7, MCap:7, LCap:7 (b) Gradual coverage with HCap:7, MCap:7, LCap:7

Figure 5.5: Solutions for Istanbul dataset with different demand rates (a_{ij}^u)

The analysis demonstrates that MSU types play distinct and complementary roles. HCap units serve major population areas, while MCap and LCap units act as supplementary forces. This heterogeneous fleet approach allows for a more efficient and responsive deployment of limited resources.



Chapter 6

Periodic Demand in the Post-Disaster Operations

In the disaster cycle, each demand comes with its own characteristics and challenges. Among these, periodic demand stands out due to the need to account for both the frequency and duration across multiple locations. Addressing this complexity, the purpose of this chapter is to propose different mathematical models for the scheduling the periodic demand in disaster response. The aim is to determine a schedule for a set of distinct MSUs across a network over a given planning horizon while selecting the nodes to serve and determining the duration of service at each visit.

In this section we focus on periodic demand which determines a schedule similar to the Flexible PVRP and PVRP with service choice; however, our application area is on periodic demand under humanitarian logistics. In our problem, each demand location has some demand requirements that determine how frequently it must be visited. The problem requires making two key decisions: first, determining where and when to deploy each MSU, and second, scheduling the start period and duration for

each service visit. Each visit must last between a minimum and maximum number of consecutive periods, with mandatory gaps between consecutive visits to the same location to ensure service regularity. It should be noted that our problem does not optimize explicit routing costs; however it shares structural similarities with those problems through its scheduling constraints.

The scheduling of periodic demand requires consideration of various factors. In Section 6.1, we introduce the Periodic Demand Services Problem (PDSP) and formally analyze its computational complexity. Later, in Section 6.2, we develop mathematical models for the PDSP, including a linearization strategy for the non-linear demand function that captures demand changes in time. We develop four mathematical models with increasing levels of complexity: 5-index and 4-index compact models and two 3-index non-compact models, which are obtained through a Benders-type projection method. The first of these 3-index models requires both feasibility and optimality cuts, while the latter only has optimality cuts. We solve the models by a branch-and-cut algorithm strengthened through valid inequalities. In Section 6.3, an extensive computational study is conducted to compare the proposed models, performing sensitivity analysis, and evaluating the robustness of solutions against a range of demand functions. We test our approach using a dataset based on the Hatay and Defne region, which was severely affected by the 2023 Kahramanmaraş earthquakes, providing insights for emerging demands and MSUs in post-disaster logistics. Finally, we provide insights and conclusion driven from study in Section 6.4.

6.1 Formal Problem Definition

The PDSP entails scheduling a fleet of MSUs to provide essential services across a disaster-affected region over a finite planning horizon. The objective is to maximize total demand served, where the demand is a nonlinear function of service duration

and repeats periodically. The problem incorporates constraints on minimum visit frequency, allowable minimum and maximum duration, and mandatory elapsed period between consecutive visits to the same node.

6.1.1 Problem Setup and Notation

Similar to the previous demand types, we also represent the periodic demand over a network $G = (V, A)$ where V and A are the set of nodes and arcs, respectively. Let S be the set of services provided to the nodes and T be the set of periods when services are provided. In this demand type, rather than distinguishing between specific types of service units (like CD versus MSU in one-time demand, or different capacity MSUs in continuous demand), S denotes distinct categories of services themselves (e.g., medical care, food distribution, sanitation). At each node $i \in V$ and on period $t \in T$, the maximum number of service types offered is denoted by h_{it} . For each service $s \in S$, a fleet of MSUs, with size m_s , is available to provide services to the nodes. A visit is defined as a single, unbroken deployment of a MSU to a node.

The parameter $\hat{p}_i$ denotes the raw population of node $i \in V$. To calculate the total demand served, we introduce two population-related parameters. A demand weight, p_{si} , for service $s \in S$ at node $i \in V$, representing the relative priority and urgency of providing service s at location i . This weight is scaled between a minimum and a maximum value, where higher values indicate greater need for the specific service. Also, the maximum daily potential demand, b_{si} , for service $s \in S$ at node $i \in V$, represents the maximum daily demand that could be served per period.

The demand weight p_{si} directly determines the required minimum service frequency ℓ_{si} , representing the minimum number of distinct visits node i must receive from service $s \in S$ during the planning horizon. The parameter $\ell_{si}^{\max}$ denotes the maximum number of distinct visits a node $i \in V$ can receive from service $s \in S$. A natural

upper bound for this value is the length of the planning horizon, i.e., $\ell_{si}^{\max} \leq |T|$. Each visit by a MSU of service $s \in S$ must have a service duration d between a minimum $\underline{c}_s$ and maximum $\overline{c}_s$ number of consecutive periods (i.e., $d \in \{\underline{c}_s, \dots, \overline{c}_s\}$). A minimum number of e_s periods is required between the end of one visit by service $s \in S$ to a node and the start of the next visit by the same service to that node. These constraints ensure service regularity and prevent resource hoarding.

The schedule implicitly accounts for travel between nodes under the following assumptions. Let us assume two consecutive deployments of a single MSU. We have a visit to district i starting on period t with duration d . The service start at time t and ends at time $t + d - 1$. After the service at district i ends, the MSU must be located to a new district j . Note that the previous service at district i and travel between districts i and j must be completed before the new service begins at district j . We assume that travel between nodes occurs instantaneously relative to service duration; thus, travel times are negligible and are not explicitly modeled.

The main decisions in PDSP are:

- determining which node(s) each MSU will serve and on which periods, and
- scheduling the *start periods* and *allocating the duration* for each service visit.

A sample solution illustrating a schedule and the resulting routes for two MSUs is provided in Figures 6.1 and 6.2, respectively.

The objective function is to maximize the total demand served during the planning horizon. The service duration directly influences the daily demand served for services $s \in S$ at a node $i \in V$ during a visit duration, which is presented by f_{sid} where $d \in \{1, \dots, \overline{c}_s\}$. It also influences total demand served during a visit duration d , which is presented by g_{sid} . In other words, $g_{sid} = \sum_{\theta=1}^d f_{si\theta}$ capturing the aggregated

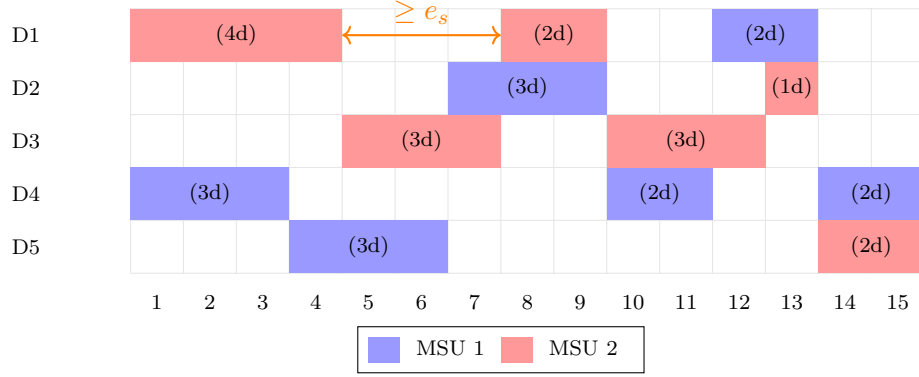
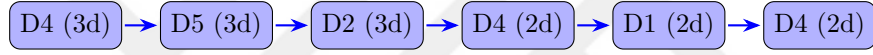


Figure 6.1: Service schedule showing MSU assignments. Each 1×1 grid square represents one period. Each colored block represents service periods $[t, t + d - 1]$ with unit numbers (MSU 1/MSU 2), where d denotes the total service period

MSU 1



MSU 2

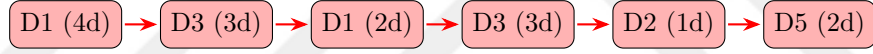


Figure 6.2: Operational timeline: MSU deployment across districts with service durations

effect of service duration on demand served. The demand function $\phi(\cdot)$ is important to the problem and is discussed in detail in the following subsection.

6.1.2 The Demand Function

In disaster response, the demand may evolve non-linearly over time due to information diffusion, infrastructure damage, and changing population needs. This evolution was observed in the 2023 Kahramanmaraş earthquakes [143], where demand varied significantly by service, location, and response phase. The objective function must capture this non-linearity to model the total demand served accurately. Therefore,

we use the function $\phi(\cdot)$, the demand function, with support between 0 and 1, representing the proportion of the maximum daily demand.

The non-linear function, $\phi(\cdot)$ can take various forms to model different real-world dynamics. We present several forms in Table 6.1 and visualize them in Appendix A. Growth functions represent scenarios where longer stays have increasing returns. They capture situations where communities initially hesitant about new services become more receptive over time, medical teams become more effective as they learn local conditions and patient needs, and MSUs establish better connections and logistics over extended periods. Conversely, decay functions present scenarios where longer stays have diminishing return. They capture situations where initial high demand for emergency supplies decreases as stockpiles are established, most urgent cases are addressed early leaving less critical needs for later periods, and affected populations develop alternative solutions, reducing dependence on mobile services.

Although $\phi(d)$ is a nonlinear function of duration d , the fact that the duration is expressed in periods allows for a linearization by precomputation. Therefore, we provide an effective linearization for our model. Let $d \in \{1, \dots, \bar{c}_s\}$ index be the remaining periods within a visit and keep track of each period within a visit. The function $f_{sid} = b_{si}\phi(d)$ denotes the demand served for a node $i \in V$ when service $s \in S$ is provided on d^{th} period during a visit. The demand served per period must be calculated separately for each period. Similarly, we use the function $g_{sid} = b_{si} \sum_{\theta=1}^d \phi(\theta)$ where the summation $\sum_{\theta=1}^d \phi(\theta)$ represents the cumulative demand served by providing service $s \in S$ for a visit of total d periods. As previously discussed, $g_{sid} = \sum_{\theta=1}^d f_{si\theta}$ is capturing the aggregated effect of service duration on demand served. The values $f_{si\theta}$ and g_{sid} are pre-computed for every possible integer service duration.

Table 6.1: Variants of demand function

Case	Growth Functions ($0 \leq \phi(d) \leq 1$)	Decay Functions ($0 \leq \phi(d) \leq 1$)
(i) Neutral	$\phi(d) = 1$	$\phi(d) = 1$
(ii) Linear	$\min(1, d/\gamma \bar{c}_s)$ • Constant marginal returns • $\gamma \geq 1$ for bounding	$\max(0, 1 - d/\gamma \bar{c}_s)$ • Constant demand loss • $\gamma \geq 1$ for bounding
(iii) Exponential	$\min(1, \lambda e^{\gamma(d-\bar{c}_s)})$ • Saturates at $\lambda \leq 1$ • γ controls growth rate	$\max(0, \lambda e^{-\gamma d})$ • Initial value at $\lambda \leq 1$ • γ controls decay rate
(iv) Sigmoid	$(1 + \lambda e^{-\gamma(d-c_0)})^{-1}$ • Inflection at $d = c_0$ where $c_0 = (\bar{c}_s + c_s - 1)/2$. • Naturally bounded	$(1 + \lambda e^{\gamma(d-c_0)})^{-1}$ • Inflection at $d = c_0$ where $c_0 = (\bar{c}_s + c_s - 1)/2$. • Naturally bounded
(v) Quadratic	$\min(1, (d/\gamma \bar{c}_s)^2)$ • Accelerating returns • Convex growth	$\max(0, 1 - (d/\gamma \bar{c}_s)^2)$ • Accelerating demand loss • Convex decay
(vi) Logarithmic	$\min(1, \frac{\log(1+\gamma d)}{\log(1+\gamma \bar{c}_s)})$ • Diminishing returns • γ controls curvature	$\max(0, 1 - \frac{\log(1+\gamma d)}{\log(1+\gamma \bar{c}_s)})$ • Diminishing demand loss • γ controls curvature
(vii) Step	$\begin{cases} \rho_{max}, & \text{if } \bar{c}_s - 1 < d \leq \bar{c}_s \\ \rho_\delta, & \text{if } \delta - 1 < d \leq \delta, \\ & \delta \in \{\underline{c}_s + 1, \dots, \bar{c}_s - 1\} \\ \rho_{min}, & \text{if } 0 \leq d \leq \underline{c}_s \\ 0, & \text{if } d < 0 \end{cases}$ • Abrupt increase at threshold • High selectivity	$\begin{cases} \rho_{max}, & \text{if } 0 \leq d \leq \underline{c}_s \\ \rho_\delta, & \text{if } \delta - 1 < d \leq \delta, \\ & \delta \in \{\underline{c}_s + 1, \dots, \bar{c}_s - 1\} \\ \rho_{min}, & \text{if } \bar{c}_s - 1 < d \leq \bar{c}_s \\ 0, & \text{if } \bar{c}_s < d \end{cases}$ • Abrupt decrease at threshold • High selectivity

6.1.3 Problem Definition

This section formally defines the PDSP, examines the structural characteristics of the PDSP and identifies special cases that admit polynomial-time solutions.

Definition 3. A PDSP instance is 11-tuple $\langle G, T, S, m_s, \ell_{si}, \ell_{si}^{max}, \overline{c_s}, \underline{c_s}, e_s, h_{it}, g_{sid} \rangle$ where $G = (V, A)$ is the transportation network with V denoting the set of nodes and A denoting the set of arcs, T is the set of periods, S is the set of services, m_s is the number of service units available for each service $s \in S$, ℓ_{si} and ℓ_{si}^{max} are the minimum and maximum number of times service $s \in S$ must visit node $i \in V$, $\underline{c_s}$ and $\overline{c_s}$ are the minimum and maximum number of consecutive periods service vehicle can serve a node, e_s is the minimum number of periods required between consecutive visits to the same node by service $s \in S$, h_{it} is the maximum number of service types that can be provided simultaneously at node $i \in V$ during period $t \in T$, g_{sid} is the demand served at node $i \in V$ if service $s \in S$ is provided for $d \in \{\underline{c_s}, \dots, \overline{c_s}\}$ consecutive periods.

Definition 4. Given a PDSP instance $\langle G, T, S, m_s, \ell_{si}, \ell_{si}^{max}, \overline{c_s}, \underline{c_s}, e_s, h_{it}, g_{sid} \rangle$, the PDSP is defined as determining a schedule for a set of distinct services S with m_s number of vehicles in graph G over a given planning horizon T , such that the total demand served g_{sid} is maximized while ensuring that each node i is visited at least ℓ_{si} times and at most ℓ_{si}^{max} times by service $s \in S$, allowing at most h_{it} number of service types to be operated simultaneously at each node $i \in V$ on period $t \in T$, maintaining a service gap e_s between consecutive visits to the same node, and keeping the duration of each visit within predefined limits $\overline{c_s}$ and $\underline{c_s}$. Formally, the decision version of the problem is as follows:

QUESTION: Is there a service schedule that assigns available service units m_s to nodes in network G within a given planning horizon T such that each node $i \in V$ is visited by each service type $s \in S$ at least ℓ_{si} times and at most ℓ_{si}^{max} times with duration of d days for each visit, allowing no more than h_{it} services to be active

concurrently at each node $i \in V$ on period $t \in T$, and respecting the minimum gap e_s between consecutive visits with a population coverage greater than or equal to Q ?

6.1.3.1 Structural Properties and Polynomially Solvable Special Cases

The NP-hardness of the PDSP, as will be formally established in Proposition 3, arises from the simultaneous interaction of its fundamental scheduling components. These components comprise timing decisions regarding the start of each visit, duration decisions determining the length of each visit, and frequency decisions governing how often visits occur, as enforced by minimum visit requirements and minimum elapsed time constraints. In addition, these decisions are coupled through the shared capacity constraints that limit concurrent services at a node (h_{it}) and per service fleet (m_s). In classical routing and scheduling problems, typically at most one of these components is flexible. In contrast, in the PDSP all of these components are jointly decision-dependent.

Intuitively, the PDSP does not require selecting isolated service visits, nor does it impose a strictly periodic schedule. Instead, it requires designing a *temporal service pattern* for each node, which specifies when visits start, how long they last, and how they are spaced over the planning horizon. This pattern is shaped by minimum visit requirements, minimum gap constraints between consecutive visits, and flexible service durations. As a result, different nodes may exhibit markedly different visit frequencies within the same solution, ranging from relatively frequent service to sparse and irregular visit patterns. The resulting temporal service pattern therefore emerges from the interaction of these constraints.

This explains why the PDSP remains NP-hard even under simple demand structures. Note that the computational difficulty of the problem does not primarily stem

from the non-linearity of the demand function $\phi(\cdot)$. Even when the demand is simplified to a trivial form (e.g., $\phi(d) = 1$), the combinatorial challenge remains. As these decisions remain tightly interdependent, the problem retains its NP-hard nature.

Although PDSP is strongly NP-hard in its general form, a number of structurally restricted cases admit polynomial-time solutions. Identifying such cases is useful both for building intuition and for clarifying which modeling components are responsible for the computational difficulty. In this subsection, we characterize polynomially solvable subclasses of the PDSP. Each case is defined as a self-contained set of structural assumptions under which the problem can be solved in polynomial time. Cases that require further restrictions beyond those stated are not included.

6.1.3.1.1 Case 1: Single node ($|V| = 1$). Suppose the network consists of a single node. All other parameters remain arbitrary but polynomially bounded. In this case, all spatial decisions disappear and the PDSP reduces to scheduling visits for a single location over a finite planning horizon. The problem becomes a one-dimensional temporal scheduling problem with visit frequency requirements, service durations, and minimum separation constraints.

Since no competition across nodes exists, feasibility and optimality can be determined using dynamic programming over time. The state space is polynomial in the planning horizon $|T|$ and the number of services $|S|$. Therefore, the PDSP is polynomially solvable when $|V| = 1$, regardless of service durations or frequency requirements.

6.1.3.1.2 Case 2: Unit service durations ($\underline{c}_s = \overline{c}_s = 1$ for all $s \in S$). Assume that all service visits have unit duration, that is, $\underline{c}_s = \overline{c}_s = 1$ for all service types. Under this assumption, each visit occupies exactly one time period and no duration decisions are required.

The PDSP then reduces to assigning unit-length visits to time periods while satisfying visit requirements ℓ_{si} , minimum separation constraints e_s , service fleet capacities m_s , and node-time capacities h_{it} . Minimum separation constraints can be enforced by restricting feasible assignments between time periods in the network construction.

This problem can be modeled on a time-expanded network whose nodes represent (i, t) pairs and whose arcs encode feasible visit assignments. Feasible schedules correspond to feasible flows or matchings in this network. Since minimum-cost flow and bipartite matching problems are solvable in polynomial time, the PDSP with unit service durations is polynomially solvable.

Apart from degenerate cases such as a single node or unit service durations, the PDSP remains NP-hard even under severe structural simplifications, including fixed visit frequencies, zero separation times, single service types, and unit demand.

The solution of the PDSP is a vector $\langle x \rangle$ where x denotes the assignment of service units to nodes, specifying the start time and service duration of each service units at each node within the planning horizon.

Consider the Bin Packing Problem which is known to be strongly NP-Hard [144].
 INSTANCE: Finite set I of items, a size $s(i) \in \mathbb{Z}^+$ for each $i \in I$, a positive integer bin capacity B , and a positive integer K . QUESTION: Is there a partition of I into disjoint sets $I_1, I_2, \dots, I_K$ such that the sum of the sizes of the items in each I_j is B or less?

Proposition 3. *The PDSP is NP-hard.*

Proof. See Appendix F. □

6.2 Mathematical Modeling

We now present four different mathematical models for the PDSP. We first describe a more intuitive explicit 5-index compact model (M-5i). The model employs indices to explicitly track the remaining duration and the number of visits within its decision variables, resulting in a complete but large-scale representation. In Section 6.2.2, we present a 4-index model (M-4i). By reinterpreting the temporal index from a remaining duration to a cumulative duration, we achieve the same goal using 4 indices. This transformation reduces the model's dimension significantly while preserving problem requirements. We then advance to a 3-index non-compact formulation using a Benders type decomposition in Section 6.2.3, which we refer to as M-3i. This model iteratively adds feasibility and optimality cuts to enforce constraints. Finally, in Section 6.2.4, we introduce an enhanced variant of M-3i which is also a 3-indexed non-compact formulation, but only generates optimality cuts. In Figure 6.3, the relationship between the models is explained.

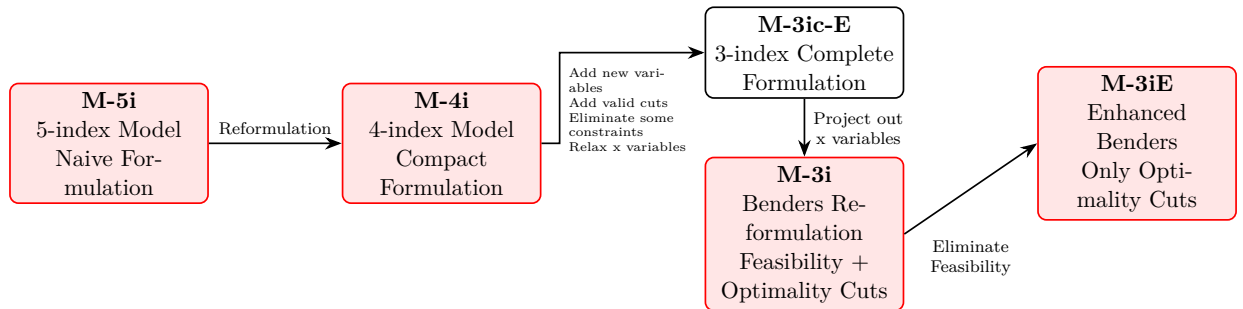


Figure 6.3: Relationships between mathematical formulations

Accordingly, the mathematical formulation M-5i is as follows:

6.2.1 5-index Model

We define q_{sikt} to be a binary decision variable indicating whether the k^{th} visit to district $i \in V$ for service $s \in S$ is conducted on period $t \in T$. Furthermore, let $r_{sikt d}$ be a binary decision variable representing whether the remaining duration is d periods given the visit occurs on period $t \in T$, or not. Table 6.3 lists the set and parameters for the problem.

Table 6.3: Sets and parameters

Sets and parameters	
V	Set of potential service locations
T	Set of periods within the planning horizon
S	Set of service types
$\overline{c}_s$	Maximum number of periods that a service vehicle can serve in total
$\underline{c}_s$	Minimum number of periods that a service vehicle has to serve a node (with $\underline{c}_s \leq \overline{c}_s \leq T $)
ℓ_{si}	Minimum number of times a node i has to be visited by service $s \in S$
$\ell_{si}^{\max}$	Maximum number of times a node i can be visited by service $s \in S$
D	Set of all durations: $\{1, \dots, \overline{c}_s\}$
$\hat{D}$	Set of minimum and maximum durations: $\{\underline{c}_s, \dots, \overline{c}_s\}$
K	Set of visit orders: $\{1, \dots, \ell_{si}^{\max}\}$
h_{it}	Maximum number of service types offered at node i simultaneously on period t
m_s	Maximum number of MSUs available per service $s \in S$
e_s	Minimum gap in terms of number of periods between two consecutive visits at the same node by service $s \in S$
f_{sid}	Daily population covered at node $i \in V$ if service $s \in S$ is provided on period $d \in D$ during a visit
g_{sid}	Total population covered at node $i \in V$ if service $s \in S$ is provided $d \in \hat{D}$ consecutive periods
$\hat{p}_i$	Total demand at node i
p_{si}	Scaled demand weight proportional to the population at node $i \in V$ for service $s \in S$
b_{si}	Maximum daily potential demand to be served proportional to the population at node $i \in V$ for service $s \in S$
$\phi(d)$	Demand function where $d \in D$ encodes the period within duration of the visit

Decision variables:

$$\begin{aligned}
 q_{sikt} &= \begin{cases} 1 & \text{if node } i \in V \text{ is visited } k^{th} \text{ time } k \in K \\ & \text{by a MSU for service } s \in S \text{ on period} \\ & t \in T \\ 0 & \text{otherwise} \end{cases} \\
 r_{siktd} &= \begin{cases} 1 & \text{if node } i \in V \text{ is visited } k^{th} \text{ time } k \in K \\ & \text{by a MSU for service } s \in S \text{ on period} \\ & t \in T \text{ with remaining duration of } d \in D \\ & \text{periods} \\ 0 & \text{otherwise} \end{cases} \\
 v_{sik} &= \begin{cases} 1 & \text{if node } i \in V \text{ is visited } k^{th} \text{ time } k \in K \\ & \text{by a MSU for service } s \in S \\ 0 & \text{otherwise} \end{cases}
 \end{aligned}$$

$$(M-5i) \text{ maximize } \sum_{d \in D} \sum_{s \in S} \sum_{i \in V} \sum_{t \in T} \sum_{k \in K} f_{sid} r_{siktd} \quad (6.1)$$

subject to

$$\sum_{i \in V} \sum_{k \in K} q_{sikt} \leq m_s \quad s \in S, t \in T \quad (6.2)$$

$$\sum_{k \in K} q_{sikt} \leq 1 \quad i \in V, s \in S, t \in T \quad (6.3)$$

$$\sum_{t \in T} r_{siktd} \leq 1 \quad i \in V, s \in S, k \in K, d \in D \quad (6.4)$$

$$\sum_{d \in D} r_{siktd} = q_{sikt} \quad i \in V, s \in S, k \in K, t \in T \quad (6.5)$$

$$dr_{siktd} - q_{sikt} \leq (d-1)r_{sik(t+1)(d-1)} \quad i \in V, s \in S, k \in K, t \in T \setminus \{|T|\}, d \in D \quad (6.6)$$

$$\begin{aligned}
& t' q_{si(k+1)t'} - (t+1)q_{sikt} \\
& + 2|T|(2 - q_{si(k+1)t'} - q_{sikt}) \geq e_s \quad i \in V, s \in S, k \in K \setminus \{\ell_{si}^{\max}\}, t, t' \in T : t' > t
\end{aligned} \tag{6.7}$$

$$\sum_{d \in D} \sum_{t \in T} r_{siktd} \leq \bar{c}_s v_{sik} \quad i \in V, s \in S, k \in K \tag{6.8}$$

$$\sum_{d \in D} \sum_{t \in T} r_{siktd} \geq \underline{c}_s v_{sik} \quad i \in V, s \in S, k \in K \tag{6.9}$$

$$\sum_{k \in K} v_{sik} \geq \ell_{si} \quad i \in V, s \in S \tag{6.10}$$

$$\sum_{k \in K} v_{sik} \leq \ell_{si}^{\max} \quad i \in V, s \in S \tag{6.11}$$

$$\sum_{t \in T} q_{sikt} \geq v_{sik} \quad i \in V, s \in S, k \in K \tag{6.12}$$

$$\sum_{t' \in T : t' < t} q_{si(k-1)t'} \geq q_{sikt} \quad i \in V, s \in S, k \in K \setminus \{1\}, t \in T \tag{6.13}$$

$$\sum_{t' \in T : t' \geq t} q_{si(k-1)t'} \leq |T|(1 - q_{sikt}) \quad i \in V, s \in S, k \in K \setminus \{1\}, t \in T \tag{6.14}$$

$$\sum_{s \in S} \sum_{k \in K} q_{sikt} \leq h_{it} \quad i \in V, t \in T \tag{6.15}$$

$$q_{sikt} \in \{0, 1\} \quad i \in V, s \in S, k \in K, t \in T \tag{6.16}$$

$$v_{sik} \in \{0, 1\} \quad i \in V, s \in S, k \in K \tag{6.17}$$

$$r_{siktd} \in \{0, 1\} \quad i \in V, s \in S, k \in K, t \in T, d \in D : t + d - 1 \leq |T|. \tag{6.18}$$

The objective function (6.1) seeks to maximize the total demand served by MSUs. The objective function coefficients for the r_{siktd} variables are computed directly as $f_{sid} = b_{si}\phi(d)$, where d is the remaining service duration by Lemma 3. This approach assigns a value to each period of service based on its position relative to the start and end of the visit, allowing implementation of decay and growth functions properly.

Constraints (6.2) ensure that at any given period $t \in T$, the total number of MSUs for each service type $s \in S$ cannot exceed the available units m_s . Constraints (6.3) guarantee that, for k^{th} visit, at most one MSU can visit node $i \in V$ for service type $s \in S$ on period t . Constraints (6.4) ensure that for any given service $s \in S$, node $i \in V$ and visit $k \in K$, each possible remaining duration $d \in D$ is assigned to at most one period $t \in T$. Therefore, it prevents any illegal visits. Constraints (6.5) ensure that r_{siktd} can only be positive if the district is scheduled to be visited on the given period. Constraints (6.6) ensure that dr_{siktd} decreases consistently across periods during a visit. For instance, if k^{th} visit a district i starts on period 4, the service duration is 3 periods. This constraints then ensure that $r_{sik43} = 1, r_{sik52} = 1, r_{sik61} = 1$, representing the periods remaining before leaving district i . Constraints (6.7) ensure that there is a minimum gap between consecutive visits to each district for each service type $s \in S$. Constraints (6.8) impose an upper limit on the duration of stays during each visit, while Constraints (6.9) ensure that the service duration meets a minimum limit on the duration of stays. Constraints (6.10) and (6.11) ensure that there occurs at least ℓ_{si} visits and at most ℓ_{si}^{max} visits. Constraints (6.12) link the q variables with the v variables to ensure consistency. Constraints (6.13) enforce that a k^{th} visit to a district can only occur after all previous visits have been completed, maintaining the correct sequence of visits. Constraints (6.14) enforce that $k - 1^{st}$ visit must finish before the k^{th} visit starts. Constraints (6.15) limit the the maximum services offered at node $i \in V$ simultaneously on period $t \in T$. These constraints basically ensure that nodes do not receive more than h_{it} different service types at the same time and simultaneously ensure that service types don't overcrowd a single location. Constraints (6.16) and (6.18) are the domain restrictions of decision variables.

Lemma 3. *Let $D = \{1, 2, \dots, \bar{c}_s\}$ denote the set of all possible duration indices. Due to constraint (6.6), the remaining durations of an activity decrease by 1 in each consecutive period. In a feasible solution for a given $s \in S, i \in N, k \in K$, we define*

the set of active durations:

$$D^{\text{act}} = \{d \in D : r_{sikt d} = 1 \text{ for some consecutive } t \in T\} = \{d_{\max}, d_{\max} - 1, \dots, 1\}$$

where $d_{\max}$ is the initial remaining duration with $1 \leq d_{\max} \leq \bar{c}_s$. For a given $s \in S, i \in N, k \in K$, we define the following function:

$$\pi : D^{\text{act}} \longrightarrow \{1, 2, \dots, d_{\max}\}$$

With set D^{act} and function π defined, the objective function term for a given $s \in S, i \in N, k \in K$ satisfies

$$\sum_{t \in T} \sum_{d \in D^{\text{act}}} f_{sid} r_{sikt d} = \sum_{t \in T} \sum_{d \in D^{\text{act}}} f_{si \pi(d)} r_{sikt d}.$$

Proof. See Appendix G. □

Model M-5i provides a naive formulation for the periodic demand problem; however, it includes scheduling, visit, and duration variables separately. This separation actually increases the complexity of the problem. Once the visit schedule is finalized, the route of the service units is decided implicitly. The main focus of the proposed problem is on the duration decision during each visit, which is tackled through the binary decision variable $r_{sikt d}$. It is noted that with an increase in instance sizes, the problem is getting difficult to handle via the proposed naive formulation. Although M-5i captures the essentials of the problem, the formulation becomes computationally intractable for larger instances. In the following section, we propose another formulation based on the naive formulation; however, it includes binary decision variables that consider scheduling, visit, and duration decisions simultaneously.

6.2.2 4-index Model

We define $x_{sit d}$ as a binary decision variable which determines whether service $s \in S$ starts serving node $i \in V$ on period $t \in T$ for a total duration of $d \in \hat{D}$ periods

where $\hat{D} = \{\underline{c}_s, \dots, \overline{c}_s\}$. The decision variable simultaneously captures the scheduling, visit, and service duration decisions. By incorporating duration index as cumulative duration rather than the remaining duration within the decision variable, new parameters, such as $g_{sid} = \sum_{\theta=1}^d f_{si\theta}$, are introduced to align with this new formulation. Accordingly, the mathematical formulation (M-4i) is provided as follows:

Decision variables:

$$x_{sitd} = \begin{cases} 1 & \text{if service } s \in S \text{ starts serving node } i \in \\ & V \text{ on period } t \in T \text{ for a total duration} \\ & \text{of } d \in \hat{D} \text{ periods (} t + d - 1 \leq |T| \text{)} \\ 0 & \text{otherwise} \end{cases}$$

$$(M-4i) \text{ maximize } \sum_{d=\underline{c}_s}^{\overline{c}_s} \sum_{t \in T} \sum_{s \in S} \sum_{i \in V} g_{sid} x_{sitd} \quad (6.19)$$

subject to

$$\sum_{i \in V} \sum_{d=\underline{c}_s}^{\overline{c}_s} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} \leq m_s \quad s \in S, t \in T \quad (6.20)$$

$$\sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c}_s\}}^{\min\{|T|-t+1, \overline{c}_s\}} x_{sitd} \geq \ell_{si} \quad i \in V, s \in S \quad (6.21)$$

$$\sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c}_s\}}^{\min\{|T|-t+1, \overline{c}_s\}} x_{sitd} \leq \ell_{si}^{max} \quad i \in V, s \in S \quad (6.22)$$

$$\sum_{d=\underline{c}_s}^{\overline{c}_s} \sum_{\hat{t}=\max\{1, t-\underline{c}_s-d+1\}}^t x_{sitd} \leq 1 \quad i \in V, s \in S, t \in T \quad (6.23)$$

$$\sum_{s \in S} \sum_{d=\underline{c}_s}^{\overline{c}_s} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} \leq h_{it} \quad i \in V, t \in T \quad (6.24)$$

$$x_{sitd} \in \{0, 1\} \quad i \in V, s \in S, t \in T, d \in \hat{D} : t + d - 1 \leq |T|. \quad (6.25)$$

Objective function (6.19) maximizes the total population coverage, similar to Model M-5i. To maintain consistency with our decision variables, we introduce g_{sid} to represent the *total demand served* instead of f_{sid} . Constraints (6.20) ensure that the number of MSUs available to provide service at any given period $t \in T$ does not exceed the total available number, m_s , for each service $s \in S$. Constraints (6.21) ensure that service $s \in S$ is offered to node $i \in V$ at least ℓ_{si} distinct visits to the same node while constraints (6.22) provides an upper bound for the number of distinct visits. Constraints (6.23) ensure that the minimum gap requirement between the two service periods is respected. Constraints (6.24) enforce that node $i \in V$ cannot receive more than h_{it} simultaneous service types at any period $t \in T$. Constraints (6.25) are the domain restrictions for the decision variables.

Lemma 4. *The following equality holds for any feasible r_{siktd} and q_{sikt} variables of M-5i and feasible x_{sitd} variables of M-4i:*

$$x_{sitd} = \begin{cases} \sum_{k \in K} r_{siktd}(1 - q_{sik(t-1)}) & \text{if } t \in \{2, \dots, |T|\}, \\ \sum_{k \in K} r_{siktd} & \text{if } t = 1. \end{cases}$$

$$i \in V, s \in S, d \in \hat{D}.$$

Proof. See Appendix H. □

Lemma 5. *The following equality holds for any feasible q_{sikt} variables of M-5i and feasible x_{sitd} variables of M-4i:*

$$\sum_{k \in K} q_{sikt} = \sum_{d=\underline{c}_s}^{\overline{c}_s} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} \quad i \in V, s \in S, t \in T.$$

Proof. See Appendix I. □

Lemma 6. *The following equality holds for any feasible v_{sik} variables of M-5i and feasible x_{sitd} variables of M-4i:*

$$\sum_{k \in K} v_{sik} = \sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c}_s\}}^{\min\{|T|-t+1, \overline{c}_s\}} x_{sitd} \quad i \in V \quad s \in S.$$

Proof. See Appendix J. □

Proposition 4. *M-4i is a valid reformulation of M-5i.*

Proof. See Appendix K. □

6.2.3 3-index Model

In this section, we provide a decomposition-friendly formulation by introducing new decision variables.

Decision variables:

$$y_{sit} = \begin{cases} 1 & \text{if node } i \in V \text{ receives service } s \in S \text{ on} \\ & \text{period } t \in T \\ 0 & \text{otherwise} \end{cases}$$

$$z_{sit} = \begin{cases} 1 & \text{if node } i \in V \text{ starts receiving service} \\ & s \in S \text{ on period } t \in T \\ 0 & \text{otherwise} \end{cases}$$

$$(M-3ic) \text{ maximize } \sum_{d=\underline{c}_s}^{\overline{c}_s} \sum_{t \in T} \sum_{s \in S} \sum_{i \in V} g_{sid} x_{sitd} \tag{6.19}$$

subject to

$$\sum_{i \in V} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{si\hat{t}d} \leq m_s \quad s \in S, t \in T \quad (6.20)$$

$$\sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c_s}\}}^{\min\{|T|-t+1, \overline{c_s}\}} x_{sitd} \geq \ell_{si} \quad i \in V, s \in S \quad (6.21)$$

$$\sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c_s}\}}^{\min\{|T|-t+1, \overline{c_s}\}} x_{sitd} \leq \ell_{si}^{max} \quad i \in V, s \in S \quad (6.22)$$

$$\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-e_s-d+1\}}^t x_{si\hat{t}d} \leq 1 \quad i \in V, s \in S, t \in T \quad (6.23)$$

$$\sum_{s \in S} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{si\hat{t}d} \leq h_{it} \quad i \in V, t \in T \quad (6.24)$$

$$\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{si\hat{t}d} = y_{sit} \quad i \in V, s \in S, t \in T \quad (6.26)$$

$$\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} = z_{sit} \quad i \in V, s \in S, t \in T \quad (6.27)$$

$$\sum_{i \in V} y_{sit} \leq m_s \quad s \in S, t \in T \quad (6.28)$$

$$y_{sit} + \sum_{\hat{t}=t+1}^{\min\{|T|, t+e_s\}} z_{si\hat{t}} \leq 1 \quad i \in V, s \in S, t \in T \quad (6.29)$$

$$\sum_{t \in T} z_{sit} \geq \ell_{si} \quad i \in V, s \in S \quad (6.30)$$

$$\sum_{t \in T} z_{sit} \leq \ell_{si}^{max} \quad i \in V, s \in S \quad (6.31)$$

$$\sum_{s \in S} y_{sit} \leq h_{it} \quad i \in V, t \in T \quad (6.32)$$

$$y_{si(t+\overline{c_s}-1)} \leq \sum_{\hat{t}=t}^{\min\{|T|, t+\overline{c_s}-1\}} z_{si\hat{t}} \quad i \in V, s \in S, t \in T : t + \overline{c_s} - 1 \leq |T| \quad (6.33)$$

$$\sum_{\hat{t}=\max\{1, t-\underline{c}_s+1\}}^t z_{sit} \leq y_{sit} \quad i \in V, s \in S, t \in T \quad (6.34)$$

$$z_{si(|T|-d+1)} \leq 0 \quad i \in V, s \in S, d \in \{1, \dots, \underline{c}_s - 1\} \quad (6.35)$$

$$y_{sit}, z_{sit} \in \{0, 1\} \quad i \in V, s \in S, t \in T \quad (6.36)$$

$$x_{sitd} \in \{0, 1\} \quad i \in V, s \in S, t \in T, d \in \hat{D} : t + d - 1 \leq |T|. \quad (6.25)$$

The objective function (6.19) and constraints (6.20)–(6.25) are taken from M4-i. Constraints (6.26) and (6.27) link x variables with y and z variables, respectively. Constraints (6.28) enforce the fleet size limit, m_s , for each service $s \in S$ at every period $t \in T$. Constraints (6.29) impose a minimum gap, preventing a new service from starting until this mandatory gap has been fulfilled. Constraints (6.30) guarantee that each node $i \in V$ receives service $s \in S$ for at least the required service frequency, ℓ_{si} . Constraints (6.31) provide an upper bound the service frequency. Furthermore, constraints (6.32) ensure that no node $i \in V$ receives more than its allowable number of simultaneous service types, h_{it} , at any period $t \in T$. Constraints (6.33) and (6.34) enforce the lower and upper bounds on service duration, respectively. Constraints (6.35) ensure that no service is scheduled to start so late that its duration would extend beyond the planning horizon. Finally, constraints (6.36) define the domains for decision variables.

Proposition 5. *M-3ic is a valid reformulation of M-4i.*

Proof. See Appendix L. □

Proposition 6. *There exists at least one extreme point of the polyhedron, formed by relaxing the integrality requirement of x_{sitd} variables, where the x_{sitd} variables assume binary values provided that the y_{sit} and z_{sit} are binary.*

Proof. See Appendix M. □

Note that the valid inequalities (6.28)–(6.35) implies constraints (6.20)–(6.24). The constraints (6.20)–(6.24) are eliminated from M-3ic and we keep the x variables only to calculate the objective function value. Also, by Proposition 6, we can relax the x variables and then, we have the following model:

$$(M-3ic-E) \text{ maximize } \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t \in T} \sum_{s \in S} \sum_{i \in V} g_{sid} x_{sitd} \quad (6.19)$$

subject to

$$\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} = y_{sit} \quad i \in V, s \in S, t \in T \quad (6.26)$$

$$\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} = z_{sit} \quad i \in V, s \in S, t \in T \quad (6.27)$$

$$\sum_{i \in V} y_{sit} \leq m_s \quad s \in S, t \in T \quad (6.28)$$

$$y_{sit} + \sum_{\hat{t}=t+1}^{\min\{|T|, t+\underline{c_s}\}} z_{sit\hat{t}} \leq 1 \quad i \in V, s \in S, t \in T \quad (6.29)$$

$$\sum_{t \in T} z_{sit} \geq \ell_{si} \quad i \in V, s \in S \quad (6.30)$$

$$\sum_{t \in T} z_{sit} \leq \ell_{si}^{\max} \quad i \in V, s \in S \quad (6.31)$$

$$\sum_{s \in S} y_{sit} \leq h_{it} \quad i \in V, t \in T \quad (6.32)$$

$$y_{si(t+\overline{c_s}-1)} \leq \sum_{\hat{t}=t}^{\min\{|T|, t+\overline{c_s}-1\}} z_{sit\hat{t}} \quad i \in V, s \in S, t \in T : t + \overline{c_s} - 1 \leq |T| \quad (6.33)$$

$$\sum_{\hat{t}=\max\{1, t-\underline{c_s}+1\}}^t z_{sit\hat{t}} \leq y_{sit} \quad i \in V, s \in S, t \in T \quad (6.34)$$

$$z_{si(|T|-d+1)} \leq 0 \quad i \in V, s \in S, d \in \{1, \dots, \underline{c_s} - 1\} \quad (6.35)$$

$$y_{sit}, z_{sit} \in \{0, 1\} \quad i \in V, s \in S, t \in T \quad (6.36)$$

$$x_{sitd} \geq 0$$

$$i \in V, s \in S, t \in T, d \in \hat{D} : t + d - 1 \leq |T|.$$

Next, we implement a Benders reformulation to project out the continuous x variables in Model M-3ic-E. For given $\bar{y}, \bar{z} \in \{0, 1\}$, the subproblem of the M-3i, referred to as $SP_{M-3i}(x|\bar{y}, \bar{z})$ is presented below:

$$\text{maximize } \sum_{d=\underline{c}_s}^{\bar{c}_s} \sum_{t \in T} \sum_{s \in S} \sum_{i \in V} g_{sid} x_{sitd} \quad (6.19)$$

subject to

$$\sum_{d=\underline{c}_s}^{\bar{c}_s} \sum_{t=\max\{1, t-d+1\}}^t x_{sitd} = \bar{y}_{sit} \quad i \in V, s \in S, t \in T \quad (6.37)$$

$$\sum_{d=\underline{c}_s}^{\bar{c}_s} x_{sitd} = \bar{z}_{sit} \quad i \in V, s \in S, t \in T \quad (6.38)$$

$$x_{sitd} \geq 0 \quad i \in V, s \in S, t \in T, d \in \hat{D}, t + d - 1 \leq |T|. \quad (6.19)$$

Let μ_{sit} and β_{sit} be the dual variables associated with constraints (6.37) and (6.38), respectively. Then, the dual subproblem, referred as $DSP_{M-3i}(\mu, \beta|\bar{y}, \bar{z})$, is:

$$\text{minimize } \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \mu_{sit} \bar{y}_{sit} + \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \beta_{sit} \bar{z}_{sit} \quad (6.39)$$

subject to

$$\sum_{t=\min\{1, t+d-1\}}^{\min\{|T|, t+d-1\}} \mu_{sit} + \beta_{sit} \geq g_{sid} \quad i \in V, s \in S, t \in T, d \in \hat{D}, t + d - 1 \leq |T| \quad (6.40)$$

$$\mu_{sit}, \beta_{sit} \in \mathbb{R} \quad i \in V, s \in S, t \in T. \quad (6.41)$$

Let $D_E(DSP_{M-3i})$ denote the set of extreme points of $DSP_{M-3i}(\mu, \beta|\bar{y}, \bar{z})$. Let $D_R(DSP_{M-3i})$ denote the set of extreme rays of $DSP_{M-3i}(\nu, \alpha|\bar{y}, \bar{z})$. The new formulation which we refer to as M-3i is:

$$(M-3i) \text{ maximize } \eta \quad (6.42)$$

subject to (6.28)–(6.36)

$$\sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \nu_{sit} y_{sit} + \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \alpha_{sit} z_{sit} \geq 0 \quad (\nu, \alpha) \in D_R(DSP_{M-3i}) \quad (6.43)$$

$$\eta \leq \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \mu_{sit} y_{sit} + \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \beta_{sit} z_{sit} \quad (\mu, \beta) \in D_E(DSP_{M-3i}) \quad (6.44)$$

The objective function (6.42) reflects that of Model M-5i, aiming to maximize total demand served. It is important to note that in the decomposed 3-index Model (M-3i), this master problem objective is subject to additional constraints: feasibility cuts (6.43) and optimality cuts (6.44) generated from the subproblems.

Since the subproblem may be infeasible, M-3i includes both feasibility and optimality cuts (constraints (6.43) and (6.44), respectively). Details regarding the implementation of Benders Decomposition to M-3i is given in the following subsections 6.2.3.1–6.2.3.3. Below, we discuss aforementioned Benders cut selection schemes.

6.2.3.1 Benders cut Selection Scheme 1: single cut

For a given $\hat{y}, \hat{z}$, the dual values are computed, then a cut is added to the master problem at each iteration in the form of inequality of type (6.44). Those are called as optimality cuts. If the sub-problem is feasible and bounded, then the dual sub-problem is also feasible and bounded by duality theory. In this case only the optimality cuts are generated and added to the master problem. Moreover, for a given $\hat{y}, \hat{z}$, the dual subproblem can be either unbounded or feasible if the feasible region is not empty. In the case of unboundedness, given the set of extreme rays, there is a direction of unboundedness. This direction infeasibility of subproblem and therefore it must be avoided. Therefore, the extreme rays are computed, then feasibility cuts are added to the master problem in the form (6.43).

6.2.3.2 Benders cut Selection Scheme 2: multicut

To implement the multi-cut version we need to modify our master problem. For this purpose, let η_{si} be new surrogate variables associated with $DSP_{M-3i}^{si}(\mu, \beta | \bar{y}, \bar{z})$ and Let $D_E^{si}(DSP_{M-3i})$ and $D_R^{si}(DSP_{M-3i})$ denote the set of extreme points and the set of extreme rays of respective problems. Then the modified master problem can be expressed as follows:

$$(M-3i)^{si} \text{ maximize } \sum_{s \in S} \sum_{i \in V} \eta_{si} \quad (6.45)$$

subject to (6.28)–(6.36)

$$\sum_{t \in T} \nu_{sit} y_{sit} + \sum_{t \in T} \alpha_{sit} z_{sit} \geq 0 \quad (\nu, \alpha) \in D_R^{si}(DSP_{M-3i}) \quad (6.46)$$

$$\eta_{si} \leq \sum_{t \in T} \mu_{sit} y_{sit} + \sum_{t \in T} \beta_{sit} z_{sit} \quad (\mu, \beta) \in D_E^{si}(DSP_{M-3i}) \quad (6.47)$$

For a given $\hat{y}, \hat{z}$, each subproblem have extreme points. By modifying the master problem, we can use the given information for each subproblem and add a cut corresponding to each extreme point. At each iteration, a cut pool is generated in this fashion. In our case, we add the most violated cuts of those disaggregated cuts since adding $|V \times S|$ cuts at each iteration can enlarge the size of LP and decrease the efficiency.

6.2.3.3 Benders cut selection Scheme 3: Pareto-optimal cut

The standard application of Benders Decomposition algorithm may result in excessive computing time and memory. Decreasing the number of iterations may increase the effectiveness of the implementation. Single cut generation scheme produces one

cut at each iteration and adds it to the master problem. On the other hand, the multicut implementation produces several cuts at once by solving each partition of subproblem. In our implementation rather than adding all those generated cuts at once, we pick the most violated cut to be added to the master problem. Finally, we present Pareto-optimal cut generation scheme which is used to select a cut that might reduce the number of iterations in the algorithm. We implement the Pareto-optimal cut generation scheme based on the study by [145]. Let $\mathcal{K}$ be the feasible set of the master problem in the iteration where all feasibility cuts added, and $opt(\mathcal{P})$ be the optimal objective function value of the problem $\mathcal{P}$. Also the function $C(y, z, \mu, \beta)$ is defined as the follows:

$$C(y, z, \mu, \beta) = \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \mu_{sit} y_{sit} + \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \beta_{sit} z_{sit} \quad (6.48)$$

[145] introduce the concept of cut domination, which involves selecting a dual solution that dominates other alternative cuts in terms of Pareto-optimality. More precisely, the cut generated by the dual solution $(\hat{\mu}, \hat{\beta})$ dominates the cut generated by the dual solution $(\tilde{\mu}, \tilde{\beta})$ if and only if $C(y, z, \hat{\mu}, \hat{\beta}) \leq C(y, z, \tilde{\mu}, \tilde{\beta})$ for all $y, z \in \mathcal{K}$ with a strict inequality for at least one point. If there are no other cuts dominating over a cut, then this cut is called Pareto-optimal cut. In order to obtain a Pareto-optimal cut induced by $\hat{y}, \hat{z}$, the following linear problem should be solved:

$$\min C(\bar{y}, \bar{z}, \mu, \beta) \quad (6.49)$$

subject to (6.40)–(6.41)

$$C(\hat{y}, \hat{z}, \mu, \beta) = opt(DSP_{M-3i}(\mu, \beta | \hat{y}, \hat{z})) \quad (6.50)$$

where $\bar{y}, \bar{z} \in ri(\mathcal{K}^c)$. $\bar{y}, \bar{z}$ is a point in the relative interior of the convex hull of the set $\mathcal{K}$ and it is called *core point*. This LP will be referred as $MW(\mu, \beta | \hat{y}, \bar{y}, \hat{z}, \bar{z})$. Our initial core point is the first feasible solution found by the master problem. Moreover the average of the current core point and the active solution of the MP are used to determine the new core point value in the following iterations.

In our reformulation, we add both feasibility and optimality cuts. It should be noted that the number of iterations is closely related to the strength of the cuts generated. In order to accelerate our solution method by optimality cuts, we have provided three cut selection schemes in the previous parts. Another way to strengthen our formulation is to improve the master problem by decreasing the number of feasibility cuts. It should be noted that feasibility cuts are undesirable since they do not improve the upper bound. In order to avoid the unboundedness of the dual subproblem, we can add a set of valid inequalities that exclude the infeasible solutions. We next investigate the extreme rays of the dual subproblem $D_R(DSP_{M-3i})$ to find those valid cuts and those cuts are added to the master problem. There are different rays to be considered and explored.

Proposition 7. *Equations (6.51)–(6.55) eliminate all the feasibility cuts (6.43) generated by the Benders Reformulation applied for 3-index Model. Therefore, Extended 3-index Model (M-3iE) is a valid reformulation of 3-index Model (M-3i).*

$$z_{si1} = y_{si1} \quad i \in V, s \in S \quad (6.51)$$

$$z_{sit} \leq y_{si(t-1)} + y_{sit} \quad i \in V, s \in S, t \in T \quad (6.52)$$

$$y_{sit} \leq y_{si(t-1)} + z_{sit} \quad i \in V, s \in S, t \in T \quad (6.53)$$

$$z_{sit} + y_{si(t-1)} \leq y_{sit} + 1 \quad i \in V, s \in S, t \in T \quad (6.54)$$

$$z_{sit} + y_{si(t-1)} + y_{sit} \leq 2 \quad i \in V, s \in S, t \in T. \quad (6.55)$$

Proof. See Appendix N. □

6.2.4 Extended 3-index Model

In this section, we built on the previous formulation M-3i by introducing new constraints. The formulation M-3iE is:

$$(M-3iE) \text{ maximize } \eta \quad (6.42)$$

subject to (6.28)–(6.36),(6.44),(6.51)–(6.55)

Objective function (6.42) and constraints (6.28)–(6.36),(6.44) are already present in 3-index Model (M-3i) and have been retained in Enhanced 3-index Model (M-3iE) with the same numbering. Constraints (6.51)–(6.55) are added to eliminate the feasibility cuts generated by constraints (6.43). The constraints (6.51) ensure that at the initial period, the decision to start a service is equivalent to actually providing the service. These constraints guarantee consistency in the initial period. Constraints (6.52) and (6.53) ensure that if a service starts on period t or if there is ongoing service from the previous period, then the node must be serviced on period t . Constraints (6.54) and (6.55) prevent the beginning of a service on period t if the service is already provided in the previous consecutive period.

6.3 Computational Study

We now introduce the datasets, demand functions, and parameter settings. We then provide an analysis of the performance of the proposed models, a sensitivity analysis of parameters, and an analysis of demand functions. Finally, we provide a real-world case study.

6.3.1 Datasets and Parameter Settings

We evaluate the proposed models on five datasets. Table 6.4 summarizes their key characteristics. The parameter settings to be used in models are summarized in Table 6.5.

The numbers of vehicles for different service types in the base case scenario are presented in Table 6.6. Table 6.7 outlines the required visit frequencies for different

Table 6.4: Dataset characteristics

Dataset	Nodes	Planning horizon (periods)
Seoul	25	40
Defne	37	60
Kartal	45	60
Kilis	187	90
Synthetic	1,000	150

Table 6.5: Service-type parameters

Service	s	Description	e_s	$(\underline{c}_s, \overline{c}_s)$
HY	S1	Hygiene	3	(3,5)
PC	S2	Personal Care	3	(2,6)
CF	S3	Child-Friendly Area	2	(2,4)
MH	S4	Mental Health	7	(1,4)
HC	S5	Health Checkups	7	(1,2)

service types. It should be noted that the parameter $l_{si}^{\max}$ is set to $|T|$ for all services, indicating no upper bound on the service frequency within the planning horizon. For the base case scenario, services types of hygiene and personal care will be used. For a comprehensive evaluation of the proposed models, the experimental study is conducted in three parts.

Table 6.6: Service-type vehicle number for different datasets

Service	s	m_s			
		Seoul	Kartal	Kilis	Synthetic
HY	S1	7	10	25	100
PC	S2	8	11	30	150

Initially, we compare the four mathematical models, M-5i, M-4i, M-3i, M-3iE, to identify the most promising formulation in terms of computational efficiency and solution quality. All five datasets are used with the base case scenario for this comparison. The models are compared using the *sigmoid decay demand function*

Table 6.7: Visit frequencies ℓ_{si} by demand weight and service type

Scaled Demand Weights	HY	PC	CF	MH	HC
$0 \leq p_{si} < 2$	3	4	3	1	2
$2 \leq p_{si} < 4$	3	4	4	2	2
$4 \leq p_{si} < 6$	4	5	4	3	3
$6 \leq p_{si} < 8$	4	5	5	4	3
$p_{si} \geq 8$	5	5	5	5	4

(see Table 6.8) based on metrics include total population covered, solution time and optimality gap. Moreover, corresponding to each of the two Benders Decomposition implementations, there are seven and six columns for M-3i and M-3iE, respectively. *Sub* presents the number of subproblems solved. *Feas. Cuts* and *Opt. Cuts* denote the total number of feasibility and optimality cuts added to the master problem in M-3i. On the other hand, *Cuts* shows the total number of cuts added to the master problems for M-3iE. To measure the solution time, we integrate 3 columns *Prep*, *Master* and *Subp* showing the preprocessing time, master problem solution time and the subproblem solution time, respectively.

Upon selecting the best-performing model, we analyze its behavior under different parameter settings and demand function specifications. Four scenarios for each parameter $(\ell_{si}, e_s, m_s, \overline{c_s}, \underline{c_s})$ along with different T values will be used for sensitivity analysis. For further sensitivity analysis, 6 functional forms are applied for the demand functions, including linear, exponential, sigmoid, quadratic, logarithmic and step through all 5 datasets. Finally, we evaluate model applicability in a real-world post-disaster scenario on Hatay dataset by including different service types.

All experiments were performed on a Windows 11 OS with an 11th Gen Intel® Core™ i7-1165G7 (2.80 GHz, 4 cores and 8 threads) processor and 16.0 GB RAM. The models and algorithms were implemented in Java and solved using Gurobi.

6.3.2 Performance of Models

We evaluate the performance of four mathematical formulations, M-5i, M-4i, M-3i, and M-3iE, on service scheduling problems with base case services hygiene and personal care. All computational experiments were conducted with a one-hour time limit to ensure consistent benchmarking. The results are summarized in Table 6.8 and the solutions for Seoul (N25_T40) and Kartal (N45_T60) datasets are provided in Figures 6.4 and 6.5, respectively.

The mathematical formulation, M-5i, proves computationally intractable even for small problem instances. For the case of N25_T40, the model contains 2,472,130 constraints and 522,000 binary variables before presolve. While presolve reduces the problem size by approximately 80%, yielding 174,036 constraints and 63,475 binary variables, the formulation remains unsolvable within the time limit. The root node relaxation, obtained after 335.79 seconds, produces an objective function value of 327,534, but the model fails to progress beyond this stage due to severe degeneracy and computational complexity. The reason for M-5i being computationally expensive is due to the visit order constraints and additional binary decision variables v denoting visit orders. They expand the mathematical model such that it cannot perform computationally efficiently.

In contrast to M-5i, M-4i displays significantly enhanced computational performance. For the same instance, the M-4i formulation begins with 3,130 constraints and 7,400 binary variables, which is reduced to 938 constraints and 4,375 variables after presolve. Also, the root node relaxation value of 319,230 was obtained in merely 0.20 seconds. This formulation consistently achieves optimal solutions in Table 6.8. Also, the formulation typically requires less than one minute of computation time. Only for the largest synthetic dataset does the model reach the time limit, yet still maintains an exceptional 0.01% optimality gap. In this large instance, it takes around

Table 6.8: Model comparison results for base scenario

Instance	Metric	Model and Method							
		M5-i		M-4i		M-3i		M-3Ei	
		Single Cut	Multi-Cut	Single Cut	Multi-Cut + MW	Single Cut	Multi-Cut	Single Cut	Multi-Cut + MW
N25_T40	Objective	319,230	319,230	319,230	319,230	319,230	319,230	319,230	319,230
	Time (s)	168.13	37.43	TL	TL	28.55	3.89	5.23	
	Gap (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	Subproblems	277	1,900	3,559	31	900	1,389		
	Feas. Cuts	39	246	242	-	-	-		
	Opt./Total Cuts	238	594	402	30*	483*	482*		
	Master Time (s)	163.93	36.92	3,597.99	28.10	3.52	4.58		
	Subp. Time (s)	4.14	0.42	1.99	0.35	0.27	0.60		
N45_T60	Av. Subp. (s)	0.0149	0.0002	0.0006	0.0113	0.0003	0.0004		
	Objective	621,034	621,034	619,637	621,034	621,034	621,034		
	Time (s)	2.34	288.56	TL	2,813.91	265.78	66.61		
	Gap (%)	0.00	0.00	0.22	0.00	0.00	0.00		
	Subproblems	550	9,630	11,468	142	6,570	3,338		
	Feas. Cuts	450	1,585	1,135	-	-	-		
	Opt. Cuts	92	500	453	113*	1,731*	1,286*		
	Master Time (s)	3,579.82	286.57	3,595.50	2,805.47	262.62	64.40		
N187_T90	Subp. Time (s)	20.03	1.85	4.43	8.21	2.87	2.04		
	Av. Subp. (s)	0.0364	0.0002	0.0004	0.0727	0.0004	0.0006		
	Objective	-	-	TL	807.485	819.684	799.824		
	Time (s)	63.60	TL	TL	TL	TL	TL		
	Gap (%)	0.00	-	-	4.16	2.71	5.07		
	Subproblems	545	5,984	3,740	24	34,408	29,534		
	Feas. Cuts	541	578	2,533	-	-	-		
	Opt. Cuts	4	5,058	664	21*	10,766*	11,036*		
N1000_T150	Master Time (s)	3,339.52	3,585.06	3,578.74	3,592.45	3,587.65	3,563.92		
	Subp. Time (s)	259.50	12.42	19.95	6.35	11.01	35.05		
	Av. Subp. (s)	0.4761	0.0021	0.0005	0.2645	0.0003	0.001		
	Objective	-	-	-	-	-	-		
	Time (s)	23,247,048	TL	TL	TL	TL	TL		
	Gap (%)	0.01	-	-	-	-	-		
	Subproblems	355	8,000	2,000	1	4,000	2,000		
	Feas. Cuts	354	1,031	292	-	-	-		
N1000_T150	Opt. Cuts	1	6,969	1,708	1*	4,000*	2,000*		
	Master Time (s)	2,343.86	3,484.87	3,549.22	3,536.64	3,573.1	3,577.74		
	Subp. Time (s)	1,234.93	73.58	37.78	15.99	14.08	12.05		
	Av. Subp. (s)	3.4786	0.0091	0.018	15.99	0.0004	0.006		

Notations: TL: Time Limit (3,600s); '-': No feasible solution found within TL; MW: Magnanti-Wong.

Note: M-3Ei represents the enhanced version with Magnanti-Wong Pareto-optimal cuts. Gaps are relative to M-4i baseline.

* For M-3Ei methods, this represents the total cuts generated.

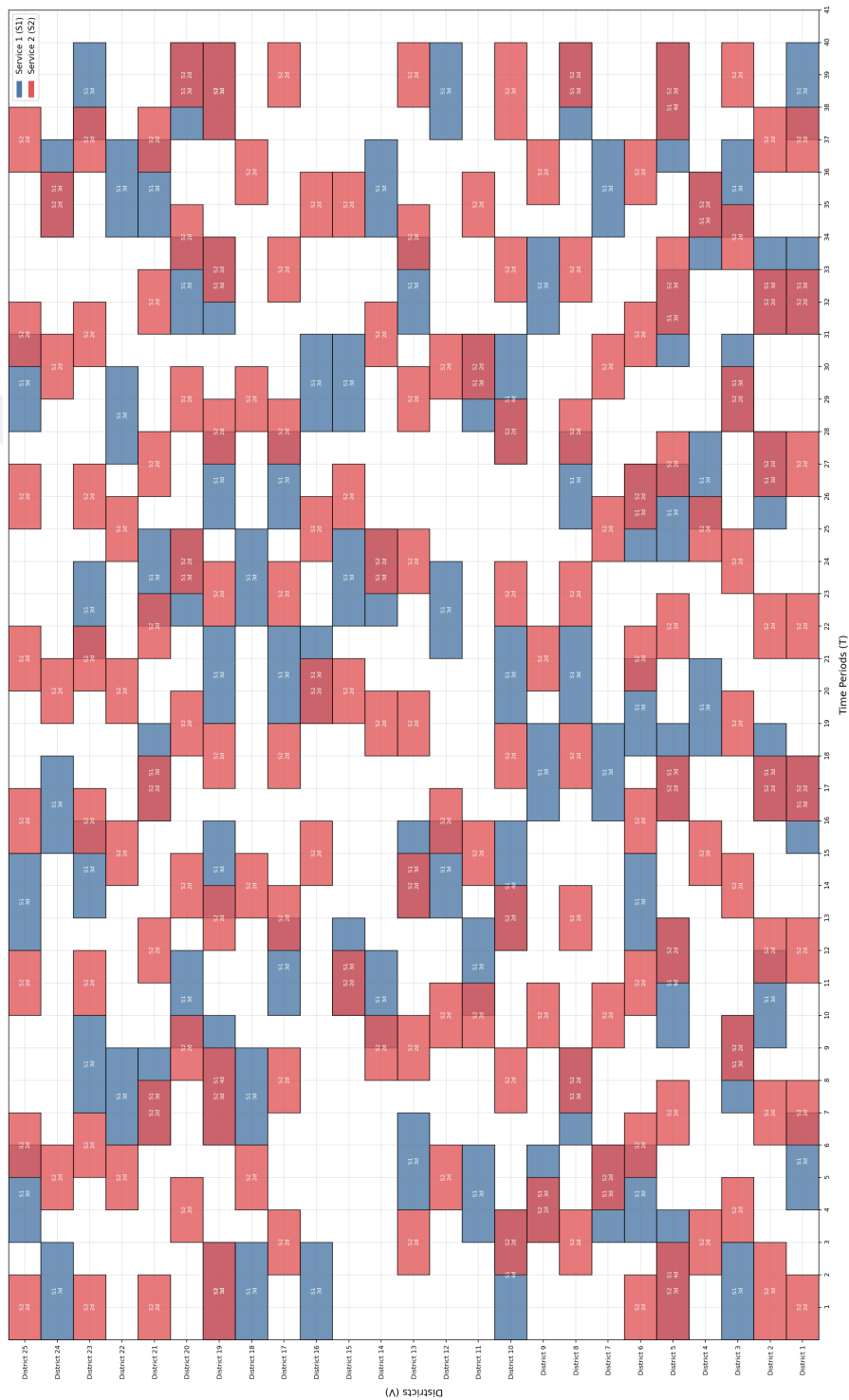


Figure 6.4: Service schedule optimization results for the base case scenario using Seoul dataset

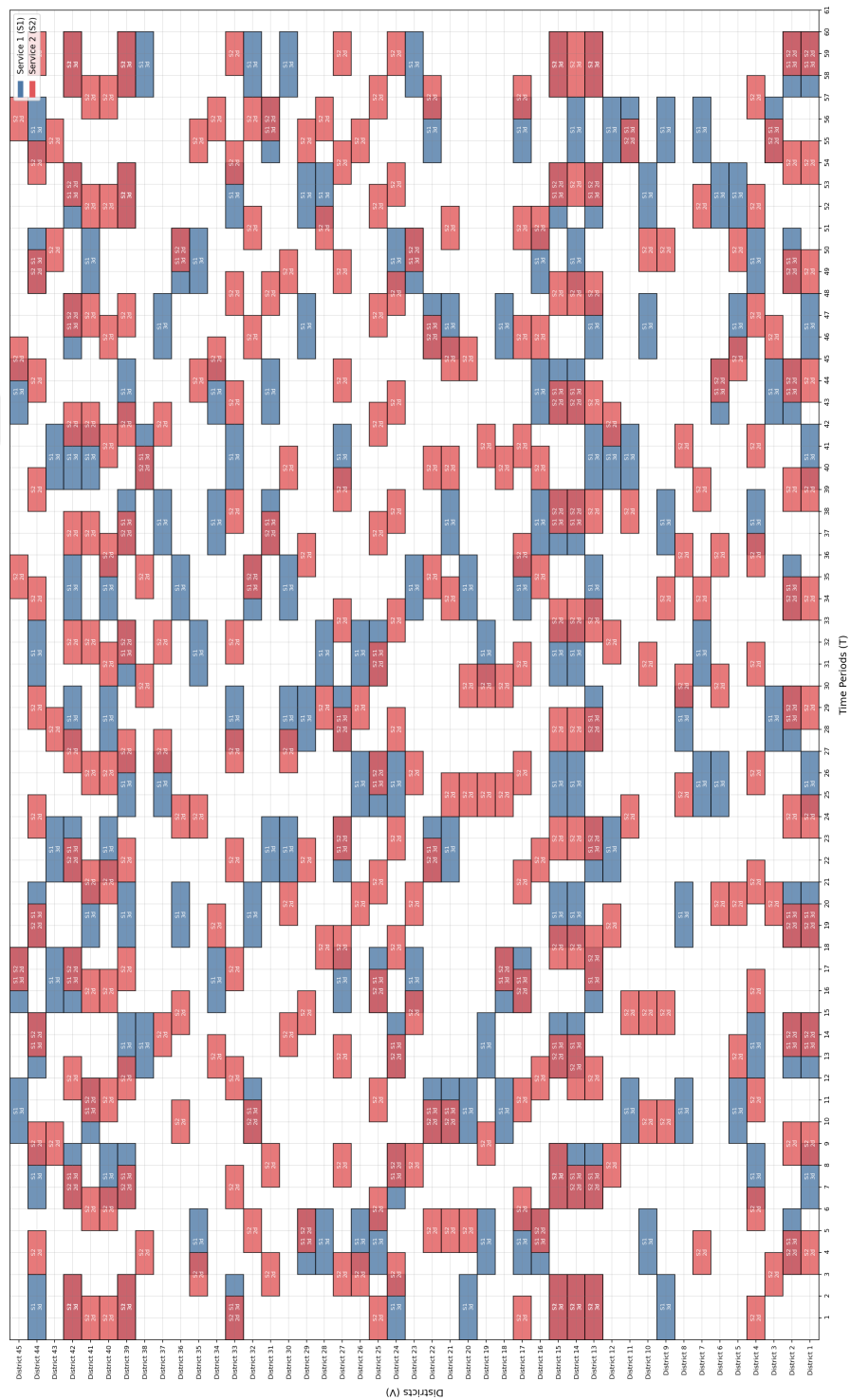


Figure 6.5: Service schedule optimization results for the base case scenario using Kartal dataset

2,000 seconds to find root-node relaxation before the branch-cut procedure. Based on the comparative analysis, M-4i’s superior performance stems from its tight mathematical formulation. M-4i can solve problems that are shown to be intractable for other approaches. M-4i represents each service visit as an atomic decision variable x_{sitd} that simultaneously encodes the visit start time, total service duration, and visit occurrence. This integrated representation preserves the natural structure of the PDSP and yields a substantially tighter linear relaxation.

M-3i represents a Benders reformulation of M-4i, incorporating both optimality and feasibility cuts. With the reformulation, we project out the x variables, and new decision variables y and z are added. The constraints are reformulated by those new decision variables. Although theoretically attractive, this approach introduces significant computational overhead and cannot exceed M-4i. For the N25_T40 case, the model contains 6,805 constraints and 4,000 binary variables, and 50 continuous variables. The model’s presolve reduces to 5,680 constraints and 3,925 binary variables. The model finds its first feasible solution at node 632 in 18 seconds with the objective function value of 318,032. Eventually, it finds the optimal solution at node 64,193 in 2,286 seconds with the objective function value of 319,230.

However, the introduction of different cut generation strategies reveals critical insights. Single Cut implementation requires 168.13 seconds for N25_T40, while Multi-Cut reduces this to 37.43 seconds. Surprisingly, adding Magnanti-Wong acceleration to Multi-Cut degrades performance to time limit (TL). It shows that theoretical improvements do not always translate to empirical gains when feasibility cuts are present.

For larger instances such as N187_T90, M-3i fails to identify any feasible solution within the time limit. It initially contains 115,933 constraints and 67,320 binary variables, and after presolve, it is reduced to 98,168 constraints, 374 continuous, and 66,759 binary decision variables. It could not find a feasible solution despite

exploring 3,009 nodes in 1 hour. Table 6.8 reveals that this failure persists across all cut generation strategies (Single, Multi, and Multi+MW), confirming that the fundamental limitation stems from two factors: (1) the addition of feasibility cuts and (2) the computational burden of the master problem, which becomes particularly severe with tight Pareto-optimal cuts.

M-3iE addresses the first limitation by eliminating feasibility cuts and it only generates optimality cuts. This modification yields substantial improvements for small and medium problem instances. In the N25_T40 case, the model solves optimally in just 5.23 seconds, compared to M-3i's timeout. The formulation begins with 14,655 constraints and 4,000 binary variables, which presolve reduces to 7,604 constraints and 3,900 binary variables.

The comparison across cut strategies within M-3Ei reveals important patterns. Multi-Cut consistently outperforms Single Cut across all instances (3.89s vs 28.55s for N25_T40; 265.78s vs 2,813.91s for N45_T60). Magnanti-Wong acceleration provides mixed benefits: It improves performance for N45_T60 (66.61s vs 265.78s) but shows minimal benefit for N25_T40 (5.23s vs 3.89s).

For the more challenging N187_T90 instance, where M-3i fails completely, M-3Ei with Multi-Cut finds a feasible solution with a 2.71% optimality gap, while Multi-Cut+MW achieves 5.07%. This represents a critical finding: while MW acceleration can improve solution times for medium instances, it may compromise solution quality for larger problems due to increased master problem complexity from dense cuts. However, like its predecessor, M-3iE cannot solve the largest N1000_T150 instance within the time limit. This demonstrates that while eliminating feasibility cuts resolves one bottleneck, it does not overcome the fundamental issue of master problem for very large-scale problems.

The comparative analysis clearly shows that neither M-3i nor M-3iE can surpass

the performance of the compact M-4i formulation. While M-3i offers theoretical advantages for certain problem structures, its practical implementation faces significant computational challenges. Moreover, M-3iE demonstrates that elimination of the feasibility cuts can yield meaningful improvements, though not sufficient to overcome all computation time issues regarding the master problem. The results show that M-3i and M-3iE cannot surpass M-4i. Note that the effectiveness of Benders decomposition fundamentally relies on a clear separation between master and subproblem decisions. In the PDSP, such a separation is created artificially. In M-3i and M-3iE, constraints governing visit patterns $(\ell_{si}, m_s, h_{it}, e_s)$ are enforced in the master problem, making it a complex scheduling problem in itself. The subproblem then only optimizes durations for a fixed pattern. The master problem lacks information on the marginal benefit of duration choices. The information in dual subproblem dual yields cuts that cannot effectively discriminate between high-quality and low-quality schedules. This coupling prevents effective master-subproblem separation and leads to weak convergence in Benders-based formulations.

The performance differences among cut generation strategies can be explained by the trade-off between cut strength and master problem complexity. Multi-Cut schemes generate multiple sparse cuts per iteration. They provide detailed feedback and keep the master problem simple per iteration. Thus it often reduces the total iteration count. On the other hand, Magnanti-Wong method produces stronger, non-dominated cuts that are theoretically optimal for tightening the bound. However those cuts are denser cuts which implies that they contain non-zero coefficients for many (y, z) variables. This density drastically increases the solution time of each master problem iteration.

Consequently, for medium-sized instances (N45_T60), the master problem is manageable. The reduction in iteration count achieved by stronger cuts compensates the increased time per iteration. Meanwhile, for large-scale instances (N1000_T150), the master problem is already a bottleneck. Adding dense cuts only increases the time

per iteration significantly and we cannot observe the benefit of the fewer iterations.

In conclusion, our computational study demonstrates that the compact M-4i formulation proves to be the best-performing approach for the PDSP. Although Benders reformulations are elegant, they are fundamentally limited by the weak decomposition between the master problem and the subproblem.

6.3.3 Results and Discussion

In this section, we present a comprehensive analysis of the solutions for the base case scenario across distinct instances, including Seoul (N25_T40), Kartal (N45_T60), and Kilis (N187_T90). We observe a consistent differentiation between S1 (HY) and S2 (PC). Across all instances, service type S1 (HY) is characterized by a 3–4 period duration per visit and less frequent visits, while service type S2 (PC) involves a 2-period duration per visit and more frequent visits. This structural distinction suggests that S1 is designed for intensive, in-depth tasks, while S2 is optimized for regular, high-frequency support. In the Seoul instance, 91 visits and 274 periods of service occurred for S1. Similarly, for S2, 156 visits and 320 periods of service were incurred. This results in 3.64 visits per district for S1 and 6.24 visits per district for S2, indicating that S2 is 71% more active than S1. In the Kartal instance, a total of 200 visits and 600 periods of service occur for S1, while for S2, a total of 323 visits and 660 periods of service occur. On average, 4.44 visits per district for S1 and 7.18 visits per district for S2 occur. It can be noted that S2 is 62% more active than S1. In the Kilis instance, 748 total visits with 2,250 total periods of service occur for S1, and 1,261 total visits with 2,575 total periods of service occur for S2. On average, 4.00 visits per district occur for S1 and 6.74 visits per district for S2. It can be noted that S2 is 69% more active than S1. Across the instances, the ratio of S2 to S1 shows that the optimal solution inherently requires 60–70% more operational capacity for S2.

Furthermore, we observe prioritization based on the daily potential demand of districts. Notably, Districts 5 and 19 in the Seoul dataset are observed to receive the most intensive service, with totals of 40 and 41 periods of service, respectively. District 5 receives 6 visits of S1 with a total duration of 21 periods and 8 visits of S2 with 19 periods. District 19 receives 7 visits of S1 with a total duration of 22 periods and 8 visits of S2 with 19 periods. Districts 13 and 25 receive 3 visits of S1 with a total duration of 9 periods and 8 visits for S2 with a total duration of 16 periods. Ten districts receive 4 visits for S1 with 12 periods and 8 for S2 with 16–17 periods. Eleven districts are low-priority districts and are visited at the bare minimum levels, which are 3 visits with a total of 9 periods for S1 and 4 visits with a total of 8 periods for S2. We observe a similar priority based on the daily potential demands in the Kartal dataset. Seven districts are the high-priority demand zones. Four out of those seven districts receive 10 visits from S1 for 30 periods, while three of them receive 7–8 visits from S1 with 21–24 periods. There are also fifteen medium-priority districts receiving 4 visits of S1 with 12 periods. District 24 is also considered medium-priority but receives slightly more visits, with 5 visits and a total duration of 15 periods. Also, there are twenty-one districts receiving baseline requirements since they are low-priority districts; they receive 3 visits of S1 with 9 periods. For S2, we observe more intense service visits. Notably, four districts receive 12 visits of S2 with 27 total service periods. Eleven districts also receive 12 visits with a slightly reduced duration of 24 periods. There are three medium-priority districts for S2 receiving 7 visits with 14 periods. Further, the other twenty-six districts are considered low-priority districts. District 32 receives 6 visits of S2 with 12 periods, and Districts 23, 29, 30, and 31 receive marginally enhanced service for S2 with 5 visits and 10 periods. Finally, the last twenty-one districts receive the baseline service for S2. For the Kilis instance, we observe a similar distinction of high-, medium-, and low-priority districts. 129 districts receive exactly 3 visits of S1 (9 periods) and 4 visits of S2 (8 periods), representing the baseline service. For S1, 47 districts receive a modest increase to 4 visits (12 periods), with a subset of 13 districts (164–176)

receiving 5 visits (15 periods). It is noted that District 177 receives 10 visits with 30 total service periods. Eight districts (178–185) receive 15 visits for 45 periods. Two districts are the most visited, receiving 15 visits for 48 periods. For S2, twenty-one districts (130–150) receive enhanced service with 5 visits and 10 periods, and they are low-priority districts. Thirteen districts (151–164) receive 18 visits with 36 periods, while thirteen districts (165–177) also receive 18 visits with longer total durations of 38–39 periods. Finally, there are high-priority districts (178–187). Among them, eight districts receive 16 visits with a total duration of 45 periods, and two districts receive service for 48 periods.

In summary, the analysis demonstrates that the proposed optimization model consistently allocates greater resources to S2 and successfully generates demand-driven prioritization schemes.

6.3.4 Sensitivity Analysis

This subsection presents a comprehensive sensitivity analysis conducted on the Seoul dataset (N25_T40) to evaluate how variations in key parameters impact the objective function value (Obj. Value), computational time (CPU Time), and solution feasibility (Status) relative to the base case scenario. The analysis examines both individual parameter variations and simultaneous changes to multiple parameters, with all results systematically presented in Table 6.9.

We initially focus on singular changes in problem parameters, and the analysis reveals several important patterns regarding parameter sensitivity. Increasing parameter m_1 improves the objective function, and the improvements are +7.56% for a +1 unit increment, +14.26% for a +2 unit increment on m_1 . On the other hand, decreasing its value leads to infeasibility since there are not enough MSUs providing service while maintaining minimum visit, duration and elapsed period requirements

Table 6.9: Comprehensive sensitivity analysis results

Scenario	Obj. Value	CPU Time (s)	Status	ΔObj (%)
Base Case	319,230	0.68	Opt	-
m1_-2	-	0.46	Inf	-
m1_-1	-	0.53	Inf	-
m1_+1	343,358	0.55	Opt	7.56
m1_+2	364,742	0.68	Opt	14.26
m2_-2	272,021	0.71	Opt	-14.79
m2_-1	297,490	1.14	Opt	-6.81
m2_+1	337,750	1.64	Opt	5.80
m2_+2	352,987	3.34	Opt	10.57
l1_-2	335,715	0.51	Opt	5.16
l1_-1	329,607	0.67	Opt	3.25
l1_+1	-	0.50	Inf	-
l1_+2	-	0.42	Inf	-
l2_-2	323,412	1.57	Opt	1.31
l2_-1	321,664	0.83	Opt	0.76
l2_+1	315,834	0.53	Opt	-1.06
l2_+2	-	0.51	Inf	-
e1_-2	319,821	0.49	Opt	0.19
e1_-1	319,597	0.50	Opt	0.11
e1_+1	318,564	0.53	Opt	-0.21
e1_+2	318,291	0.56	Opt	-0.29
e2_-2	328,518	0.71	Opt	2.91
e2_-1	324,571	0.42	Opt	1.67
e2_+1	313,370	4.21	Opt	-1.84
e2_+2	305,376	510.32	Opt	-4.34
cl1_-2	321,496	93.35	Opt	0.71
cl1_-1	329,423	1.06	Opt	3.19
cl1_+1	-	0.31	Inf	-
cl1_+2	-	0.18	Inf	-
cl2_-2	311,557	3.21	Opt	-2.40
cl2_-1	311,557	3.21	Opt	-2.40
cl2_+1	-	0.38	Inf	-
cl2_+2	-	0.31	Inf	-
cu1_-2	285,249	0.36	Opt	-10.64
cu1_-1	303,939	0.44	Opt	-4.79
cu1_+1	331,583	0.52	Opt	3.87
cu1_+2	340,904	0.52	Opt	6.79
cu2_-2	287,123	0.72	Opt	-10.06
cu2_-1	305,272	0.65	Opt	-4.37
cu2_+1	329,350	2.39	Opt	3.17
cu2_+2	336,918	0.91	Opt	5.54
T_-2	-	0.64	Inf	-
T_-1	312,575	0.67	Opt	-2.08
T_+1	325,697	0.74	Opt	2.03
T_+2	339,708	2.51	Opt	6.41

Scenario	Obj. Value	CPU Time (s)	Status	ΔObj (%)
cl1_cu1_-2	282,588	844.25	Opt	-11.48
cl1_cu1_-1	313,086	1.33	Opt	-1.92
cl1_cu1_+1	-	0.42	Inf	-
cl1_cu1_+2	-	0.40	Inf	-
cl2_cu2_-2	273,466	9.84	Opt	-14.34
cl2_cu2_-1	294,318	172.10	Opt	-7.80
cl2_cu2_+1	-	0.34	Inf	-
cl2_cu2_+2	-	0.37	Inf	-
m1_-2_m2_-2	-	0.39	Inf	-
m1_-2_m2_-1	-	0.37	Inf	-
m1_-2_m2_+1	-	0.31	Inf	-
m1_-2_m2_+2	-	0.35	Inf	-
m1_-1_m2_-2	-	0.37	Inf	-
m1_-1_m2_-1	-	0.52	Inf	-
m1_-1_m2_+1	-	0.43	Inf	-
m1_-1_m2_+2	-	0.37	Inf	-
m1_+1_m2_-2	296,150	0.61	Opt	-7.23
m1_+1_m2_-1	321,618	0.94	Opt	0.75
m1_+1_m2_+1	361,878	1.46	Opt	13.36
m1_+1_m2_+2	377,115	3.05	Opt	18.13
m1_+2_m2_-2	317,534	0.79	Opt	-0.53
m1_+2_m2_-1	343,003	1.03	Opt	7.45
m1_+2_m2_+1	383,263	1.46	Opt	20.06
m1_+2_m2_+2	398,500	3.24	Opt	24.83
cl1_cu1_-2_cl2_cu2_-2	236,824	2,683.87	Opt	-25.81
cl1_cu1_-2_cl2_cu2_-1	257,676	671.61	Opt	-19.28
cl1_cu1_-2_cl2_cu2_+1	-	10.19	Inf	-
cl1_cu1_-2_cl2_cu2_+2	-	10.67	Inf	-
cl1_cu1_-1_cl2_cu2_-2	267,322	38.83	Opt	-16.26
cl1_cu1_-1_cl2_cu2_-1	288,175	373.19	Opt	-9.73
cl1_cu1_-1_cl2_cu2_+1	-	4.42	Inf	-
cl1_cu1_-1_cl2_cu2_+2	-	4.53	Inf	-
cl1_cu1_+1_cl2_cu2_-2	-	3.84	Inf	-
cl1_cu1_+1_cl2_cu2_-1	-	0.46	Inf	-
cl1_cu1_+1_cl2_cu2_+1	-	0.41	Inf	-
cl1_cu1_+1_cl2_cu2_+2	-	0.46	Inf	-
cl1_cu1_+2_cl2_cu2_-2	-	3.78	Inf	-
cl1_cu1_+2_cl2_cu2_-1	-	0.41	Inf	-
cl1_cu1_+2_cl2_cu2_+1	-	0.46	Inf	-
cl1_cu1_+2_cl2_cu2_+2	-	0.45	Inf	-
l1_-2_l2_-2	339,897	1.31	Opt	6.47
l1_-2_l2_-1	338,150	0.82	Opt	5.93
l1_-2_l2_+1	332,320	0.50	Opt	4.10
l1_-2_l2_+2	-	0.48	Inf	-
l1_-1_l2_-2	333,790	1.33	Opt	4.56
l1_-1_l2_-1	332,042	0.82	Opt	4.01
l1_-1_l2_+1	326,212	0.52	Opt	2.19
l1_-1_l2_+2	-	0.49	Inf	-
l1_+1_l2_-2	-	0.42	Inf	-
l1_+1_l2_-1	-	0.45	Inf	-
l1_+1_l2_+1	-	0.43	Inf	-
l1_+1_l2_+2	-	0.42	Inf	-
l1_+2_l2_-2	-	0.37	Inf	-
l1_+2_l2_-1	-	0.37	Inf	-
l1_+2_l2_+1	-	0.35	Inf	-
l1_+2_l2_+2	-	0.37	Inf	-

Obj. Value: Objective Function Value, ΔObj : Percentage change in objective function value from base scenario
Positive values indicate an increase from the base scenario; negative values indicate a decrease.

for S1. We can observe a similar trend with parameter m_2 . Decreasing m_2 by two units reduces the objective function value by -14.79% , while increasing m_2 by two units improves the objective function value by 10.57% . Thus, it can be easily stated that parameter m_s has a positive correlation with the objective function value. More vehicles for any service type allow broader coverage and more service, especially for high-demand nodes. It should be noted that there is a threshold level to maintain feasibility, since there will be insufficient service capacity under this level.

Furthermore, the minimum visit frequency parameters l_{1i} and l_{2i} show an inverse relation. Unlike parameter m_s , the decrement in l_{si} values improves the objective function value. For l_{1i} the decrements $(-1, -2)$ improve the objective function value by 3.25% and 5.16% , respectively. For l_{2i} , the same decrements improve the objective function value by 0.76% and 1.31% . The analysis show that lowering minimum visit frequencies reduces operational burden, but increasing them beyond capacity leads to infeasibility.

Similar to l_{si} , parameter e_s negatively correlates with the objective function value. As the e_s value increases, the objective function value decreases. Even though the results do not show infeasibility in those cases, it will be infeasible if we increase e_s values until the model cannot satisfy both minimum visit and duration requirements within a given planning horizon.

While performing sensitivity analysis on duration parameters, we allow the range to be strictly greater than 0. If the parameter value goes below 1, we allow the minimal value of 1, but not zero or negative. The most important parameters of our problem are the upper and lower duration limit since they have a much bigger impact than m_s, e_s, l_{si} parameters. Especially the upper bound $\bar{c}_s$ is the most influential parameter in the model, with adjustments generating the largest objective function value swings ($\pm 10\%$). We must focus on the sigmoid decay demand function in the objective function to understand the significance of duration limits. The inflection

point c_0 is defined as the midpoint between $\overline{c_s}$ and $\underline{c_s} - 1$. Therefore, any change to $\overline{c_s}$ and $\underline{c_s}$ directly alters the decay dynamics. When $\overline{c_1}$ increases from 5 to 7 periods, the inflection point shifts rightward (e.g., from 3.5 to 4.5). It delays the decay, implying that visits lasting 4-5 periods now contribute more substantially to the objective function value. Conversely, decreasing $\overline{c_1}$ from 5 to 3 shifts the inflection point leftward (from 3.5 to 2.5). It ensures that even 3-period visits are strongly penalized by decay. Thus, a decrement in $\overline{c_s}$ leads to dramatic objective function value drops (-10%).

Similarly, $\underline{c_s}$ affects the shape of the sigmoid decay function. When $\underline{c_s}$ increases by one unit, the inflection point shifts rightward by 0.5 periods. It delays the decay for more extended visits. Also, increasing $\underline{c_s}$ values may cause infeasibility since it may not be possible to satisfy the minimum visit requirements and the minimum elapse gap requirement for a given planning horizon. Decreasing $\underline{c_s}$ by one unit causes a leftward shift for inflection point and introduces earlier and more aggressive decay. For instance, for the case of a decrease in the $\underline{c_2}$ from 2 periods to 1 period, we observe a decrease in the objective function value (-2.40%). In this case, the left-shifted inflection point (from 3.5 to 3) penalizes more extended visits (4–6 periods). It also means that 3-period visits are penalized more, and generally 1–2 period visits are preferred. We observe a decrease in the objective function value for this service type due to a decrease in the inflection point.

On the other hand, as the minimum required duration decreases, the model can easily provide service to low-density districts for a minimum number of visits in a shorter time. Then, high-density districts also receive visits with a minimum duration but with more frequent visits. For instance, the base case’s minimum 3-period visit requirement for $\underline{c_1}$ was reduced to 2 and 1. The leftward shift in the inflection point (from 3.5 to 3 and 2.5) implies that 3-period visits are penalized more than the base case. In other words, more extended visits face more substantial penalties. Due to a change in the lower bound on duration, the model now schedules shorter visits than

3 periods. It also implies that the minimum visit frequency with a lower duration limit is satisfied faster for low-density locations. As a result, more locations with higher benefits can be served more frequently. In the base case, S1 made four visits, each lasting at least 3 periods, resulting in 12 periods in service but only covering 4 locations. With the decrement in duration, it makes six visits with the lower duration of 2 periods, resulting again in 12 periods in service, but now it covers 6 locations. Visiting more locations means better coverage and a higher objective function value. This trade-off creates an unexpected correlation between $\underline{c}_s$ and the objective function value.

The final parameter on the sensitivity analysis is the planning horizon (T). Reducing T by just one unit decreases the objective function by 2.08%, while one unit increment yields a 2.03% improvement. However, a two unit decrement makes the problem infeasible, which shows that time horizon operates near a critical minimum threshold required to satisfy all mandatory service constraints.

We observe similar trends in the multi-parameter sensitivity analysis as in the singular parameter analysis. For instance, decreasing the values of $\overline{c}_s$ and $\underline{c}_s$ simultaneously for specific services, we observe that there is a moderate drop in the objective function values (-1.92% to -14.34%). Inflection point decreases from 3.5 to 1.5 by a decrement of 2 units in both upper and lower duration bounds of S1. It implies that shorter durations are allowed and more extended visits are penalized. Moreover, we can observe the effect of duration bounds for two services simultaneously. The objective function value decreases (up to -25.81%) as the bounds decrease since shorter visits reduce per-visit efficiency, and the sigmoid decay function penalizes deviations from optimal durations more harshly.

We also observe similar trends in the change of m_s values simultaneously. Since m_1 is already at lower limits, decreasing it even more causes infeasibility, even though the values of m_2 increase. An increase in m_1 by 1 or 2 units generally leads to an increase

in the objective function value (ranging from +0.75% to +24.83%). This positive correlation is reversed when 2 units decrement in m_2 is applied. Further, the analysis reveals that there are infeasible cases also for the simultaneous changes in parameters l_1 and l_2 . Unlike parameter m_s , a decrement in l_{si} values causes an increment in the objective function value. Even though their effect is much more significant than the singular changes, the effect of parameters m_s is much more significant than the l_{si} parameters. The multi-parameter analysis confirms and extends single-parameter findings.

6.3.5 Demand Function Analysis

In this section, we run our model with different demand functions and compare the behaviour of the model. The neutral demand function serves as our baseline, assuming equal benefits for visiting the same node on different periods. This approach provides a reference point for comparing the effectiveness of different demand functions while maintaining computational simplicity. The neutral function solutions demonstrate distinct service allocation patterns across service types. S1 is allocated in 3-period blocks, while S2 visits are allocated in 2-period or 5–6-period blocks. Also, S2 is provided more frequently than S1.

The solutions reveal a consistent 7–10 period spacing between service visits across all districts and service types. The spacing ensures sustainable service delivery while maintaining regular visits with districts.

The neutral function solutions demonstrate clear differentiation based on population density and minimum visit requirements. Due to higher population densities and corresponding minimum visit requirements, certain districts are visited more frequently. High-density districts, such as districts 5 and 19 in the N25_T40 instance, receive more frequent and longer-duration services, with up to 5 visits lasting 5–6

periods each for both service types. Conversely, low-density districts like District 24 receive services at minimum required levels in terms of both duration and visit frequency.

To evaluate the effectiveness of different demand functions, we implemented twelve distinct variants representing both growth and decay patterns. Table 6.10 presents the raw objective function values and computational performance for each demand function across three instance sizes (N25_T40, N45_T60, N187_T90).

Table 6.10: Different demand function results

Demand Function	N25_T40		N45_T60		N187_T90	
	Obj. Value	Time (s)	Obj. Value	Time (s)	Obj. Value	Time (s)
Neutral	399,297	0.7	775,829	2.0	1,121,420	47.6
Linear Growth	176,025	8.3	381,023	3.1	591,013	164.2
Exponential Growth	295,221	0.7	588,061	3.8	869,455	1,223.3
Sigmoid Growth	120,647	557.0	278,452	3.8	449,482	59.4
Quadratic Growth	102,863	6.9	251,236	3.8	403,949	194.3
Logarithmic Growth	246,238	1.5	503,736	3.5	763,922	TL
Step Growth	94,603	2.4	27,713	2.6	386,677	84.2
Linear Decay	260,218	0.5	506,699	2.7	687,282	51.9
Exponential Decay	326,749	1.8	632,489	2.4	881,100	98.1
Sigmoid Decay	319,230	0.7	621,033	2.3	842,555	63.6
Quadratic Decay	336,661	1.4	654,892	6.8	897,437	81.8
Logarithmic Decay	182,472	1.1	355,926	2.5	480,357	87.2
Step Decay	382,052	0.6	744,825	5.5	997,092	74.7

Notes: TL = Time Limit (3,600s); '-' = No solution

The results reveal significant variation in computational efficiency across demand functions. Linear Decay (0.5s, 2.7s, 51.9s) and Sigmoid Decay (0.7s, 2.3s, 63.6s) demonstrate the fastest solution times. In contrast, Sigmoid Growth shows highly inconsistent performance. The time spent on solving the sigmoid growth function is 557.0s on small instances, but only 3.8s on medium instances. Moreover, Exponential Growth exhibits poor scalability due to a change in its solution time from 0.7s in a small instance to 1223.3s in a large instance. Logarithmic Growth fails to converge within the time limit on the largest instance, highlighting computational challenges for certain function types.

In Table 6.10, we record the objective function values for different demand functions. However, it should be noted that we cannot properly compare those values. Therefore, in order to provide a valid comparison, we use solutions obtained from the demand functions and post-process those solutions with the neutral demand function. Those results are shown in Table 6.11 which presents the post-processed objective function value with the neutral demand function (PP-OFV-NDF), objective change percentage, and demand satisfaction rates.

According to Table 6.11, the demand functions exhibit distinct behaviors compared to the neutral demand function. Growth functions consistently outperform decay functions in the objective function value when post-processed with the neutral demand function. The decay functions assume that the highest benefit is obtained in the initial periods, and as duration increases, the benefit decreases. Therefore, shorter durations gain more benefits and provide higher coverage. Among the growth functions, exponential growth achieves the highest PP-OFV-NDF values (398,855, 775,503, 1,120,119), while logarithmic growth (397,914, 763,864, 1,115,694) and linear growth follow closely (397,606, 755,368, 1,106,731). Quadratic growth performs worst among growth functions with the lowest PP-OFV-NDF values (397,705, 747,721, 1,095,081). In the decay functions, exponential decay performs best with PP-OFV-NDF values of 392,201, 758,889, and 1,096,635, while logarithmic decay performs worst with values of 387,742, 748,391, and 1,016,753. In the decay functions, the service durations remain around the lower bound (S1: 3–4 periods or S2: 2–3 periods), and more frequent visits are performed. On the other hand, the growth functions assume that more extended visits gain more benefit as the planning horizon and minimum requirements are satisfied. The durations in the growth functions are more flexible and variable. Service durations range between 2 and 6 periods for S2 and 3–5 periods for S1. Also, districts like 5, 10, and 19 in the N25_T40 case have higher population densities and are visited longer. In the decay functions, the visits were shorter but more frequent; in growth functions, visits satisfy the minimum requirements, and durations are more extended, reaching upper bounds.

Table 6.11: Comparison of different demand function results under PP-OFV-NDF

Demand Function	N25_T40			N45_T60			N187_T90		
	PP-OFV-NDF	Obj.Cha.%	Dem.Sat. (%)	PP-OFV-NDF	Obj.Cha.%	Dem.Sat. (%)	PP-OFV-NDF	Obj.Cha.%	Dem.Sat. (%)
Neutral	399,297	-	100.00	775,829	-	100.00	1,121,420	-	100.00
Linear Growth	397,606	0.42	99.58	755,368	2.64	97.36	1,106,731	1.31	98.69
Exponential Growth	398,855	0.11	99.89	775,503	0.04	99.96	1,120,119	0.12	99.88
Sigmoid Growth	397,606	0.42	99.58	751,809	3.10	96.90	1,098,853	2.01	97.99
Quadratic Growth	396,705	0.65	99.35	747,721	3.62	96.38	1,095,081	2.35	97.65
Logarithmic Growth	397,914	0.35	99.65	763,864	1.54	98.46	1,115,694	0.51	99.49
Step Growth	397,812	0.37	99.63	751,809	3.10	96.90	1,095,496	2.31	97.69
Linear Decay	388,591	2.68	97.32	749,083	3.45	96.55	1,033,970	7.80	92.20
Exponential Decay	392,201	1.78	98.22	758,889	2.18	97.82	1,096,635	2.21	97.79
Sigmoid Decay	388,591	2.68	97.32	750,177	3.31	96.69	1,040,928	7.18	92.82
Quadratic Decay	391,053	2.06	97.94	751,706	3.11	96.89	1,053,443	6.06	93.94
Logarithmic Decay	387,742	2.89	97.11	748,391	3.54	96.46	1,016,753	9.33	90.67
Step Decay	387,742	2.89	97.11	748,391	3.54	96.46	1,025,341	8.57	91.43

PP-OFV-NDF: post-processed objective function value with the neutral demand function.

Obj.Cha.(%): Change in the value of the objective function of the solution post-processed with the neutral demand function.

Dem.Sat. (%): Total percentage of demand satisfied.

Even though sigmoid decay does not achieve the highest PP-OFV-NDF values, our primary choice was the sigmoid decay function due to several factors. The function has diminishing returns, which aligns with real-life scenarios for humanitarian operations where early intensive intervention is more valuable than extended stays. The smooth efficiency transitions, rather than abrupt changes, provide operational flexibility and realistic service planning capabilities. It can be noted that sigmoid decay achieves lower PP-OFV-NDF values (388,591, 750,177, 1,040,928) while it maintains performance levels at 97.3%, 96.7%, and 92.8% of the neutral baseline, respectively. Further, the solution times of the sigmoid decay are consistently fast (0.7s, 2.3s, 63.6s) across instances.

6.3.6 Case Study on Hatay Dataset

The February 2023 earthquakes caused catastrophic destruction in Hatay, leading to widespread building collapses, significant loss of life, and mass displacement [146]. The disaster severely disrupted daily life, forcing many people to leave the city. Even in 2025, reconstruction efforts continue, including road repairs and building rehabilitation, while many residents remain in temporary container housing or tents provided by AFAD [147].

The 2023 earthquakes present a critical case study on Hatay for analysing periodic demand in post-disaster scenarios. Hatay could not be reached for 72 hours, and this isolation period created a unique demand trajectory. The periodic demand became crucial for long-term recovery after meeting initial emergency needs. The extreme damage caused aid to come in waves. The order is as follows: immediate rescue first, daily survival needs next and long-term recovery last. Further, the slow recovery proved that services like counselling and childcare need careful long-term planning. This case validates using sigmoid decay demand functions due to demand patterns like starting extremely urgent, then slowly levelling off over time.

This section presents a case study on Defne with all five service types. Here in Table 6.12, we present the parameter values for the Defne case study for five service types.

Table 6.12: Service-type vehicle number for Defne datasets

Service	s	m_s
HY	S1	10
PC	S2	11
CF	S3	12
MH	S4	7
HC	S5	8

The solution for the case study on Defne is presented in Figure 6.6. S1 mainly provides service for 3-period deployments but extends to 4 periods in high-density districts. High-density districts (10, 19, and 24) receive frequent visits, while low-demand districts (15 and 33) receive service only to meet minimum visit requirements (3 instances in 60 periods). For instance, District 33 has a 20-period gap in S1.

S2 shows flexibility in duration (2–3 periods). It mostly visits districts 8, 10 and 19, and in District 19, it follows a pattern of 7-period recurrence, proposing a pattern like weekly visits. S3 displays a high-frequency, short-duration pattern especially in frequently served districts such as districts 3, 8, 10 and 22. For example, District 10 is visited by S3 every 4–6 periods. S4 generally provides a short service duration, like 1–2 periods. However, it increases the duration to 3 in highly populated districts such as districts 24 and 35. S5 provides 2-period visits in general, and the services are spaced around 9 periods in districts 5, 6, and 7. Moreover, the results show that districts 10, 24, 27 and 35 get most of their services packed into a short period—right at the end of the planning horizon. The frequency of S4 and S5 is low due to high e_s values.

The solution demonstrates uneven service distribution, with some districts receiving near-continuous coverage while others experience sparse visits. There are highly

prioritized 12 districts. They receive continuous coverage, especially by S3, since it is scheduled chiefly in 2-period visits with minimal elapse gaps. For instance, District 24 receives S3 every four periods (periods 3–4, 7–8, 11–12). Similarly, S1 in the same district is provided in three-period intervals (periods 1–3, 7–9, and 13–15). This reflects a real-life situation where urban centres or high-density zones need constant support.

On the other hand, eight districts are visited less frequently. They experience sparse, clustered service windows. For instance, District 15 has been provided S1 three times. This indicates that lower population density areas are visited only for the minimum visit requirement levels. Also, seventeen districts with medium-density areas are treated almost as frequently as high-density areas; however, the durations are generally at the lower bound for some service types. For instance, District 21 is visited by S3 fourteen times during the planning horizon, but each visit lasts 2 periods. However, it should be noted that to ensure fairness and equity, minimum visit, minimum/maximum duration and minimum elapsed period requirements are considered. Without those, only high-density areas would be served, and the low-density regions would not be visited to maximize benefit.

The parameter l_{si} , denoting the minimum number of visits, guarantees base coverage for all districts. For example, S1 is provided to District 15 exactly thrice. Across the planning horizon, the visit occurs on periods 4–6, 19–21, and 35–37, and even though it is a low-density area, it is not neglected. Moreover, the parameters $\underline{c}_s, \overline{c}_s$ ensure meaningful engagement. For instance, S1 has a minimum 3-period visit duration, eliminating inefficient short-duration visits. Moreover, the minimum elapsed period e_s captures gaps between services to avoid continuous unnecessary services and regulates service frequency. These parameters also naturally bound the maximum elapsed time, which avoids prolonged deprivation. For instance, S5 services in Districts 5–7 provide service to a 9-period maximum interval (e.g., periods 5–6, 14–15, 23–24). This natural bound also ensures regular visits in the schedule. If

these parameters were not adopted, high-density districts would be served primarily, and if their schedule is full, then low-density areas would be served. For instance, S3 would run daily in District 10, while S2 would have visits with 60-period gaps in District 32. Therefore, the demand and fairness are balanced with high-density districts receiving frequent services while lower-demand areas are ensured a minimum requirements. By adjusting parameters on these service requirements, low-density areas may even get more frequent or higher-duration services.

In this case study, we further consider the change in h_{it} parameter by allowing only three services to be provided simultaneously pre- $T/2$ and allowing all five services post- $T/2$. The solution is presented in Figure 6.7. The objective function value in the base case is 342,492, obtained in 544.46 seconds. On the other hand, the objective function of the new instance is 341,962 and was obtained within a 1-hour time limit. As the constraint gets tight, it becomes harder to solve. We observe that service distribution and duration patterns follow similar patterns to the base case. Identical to the base case, some districts receive more frequent visits while others have sparse schedules. For instance, district 35 receives S1 ten times, while S1 is provided to district 15 three times during the planning horizon. Moreover, similar to the base case, most services are provided at lower durations due to the decay function property and the minimum limit requirement. For instance, S1 is provided for mostly three periods, S2 and S3 are provided for mostly two periods, and S4 and S5 are provided for 1 period and 2 periods.

We similarly observe uneven service distribution among the districts, but the tight constraint creates a schedule in which active services are carefully selected pre- $T/2$. We can evaluate the difference based on the population densities. High service density is observed in highly populated areas, and service sparsity is observed in less populated regions. More consistent service patterns are provided to high-density districts. For instance, District 10 receives ten visits for S1 and S2 at every five- to six-period interval throughout the planning horizon. Likewise, District 19 shows identical

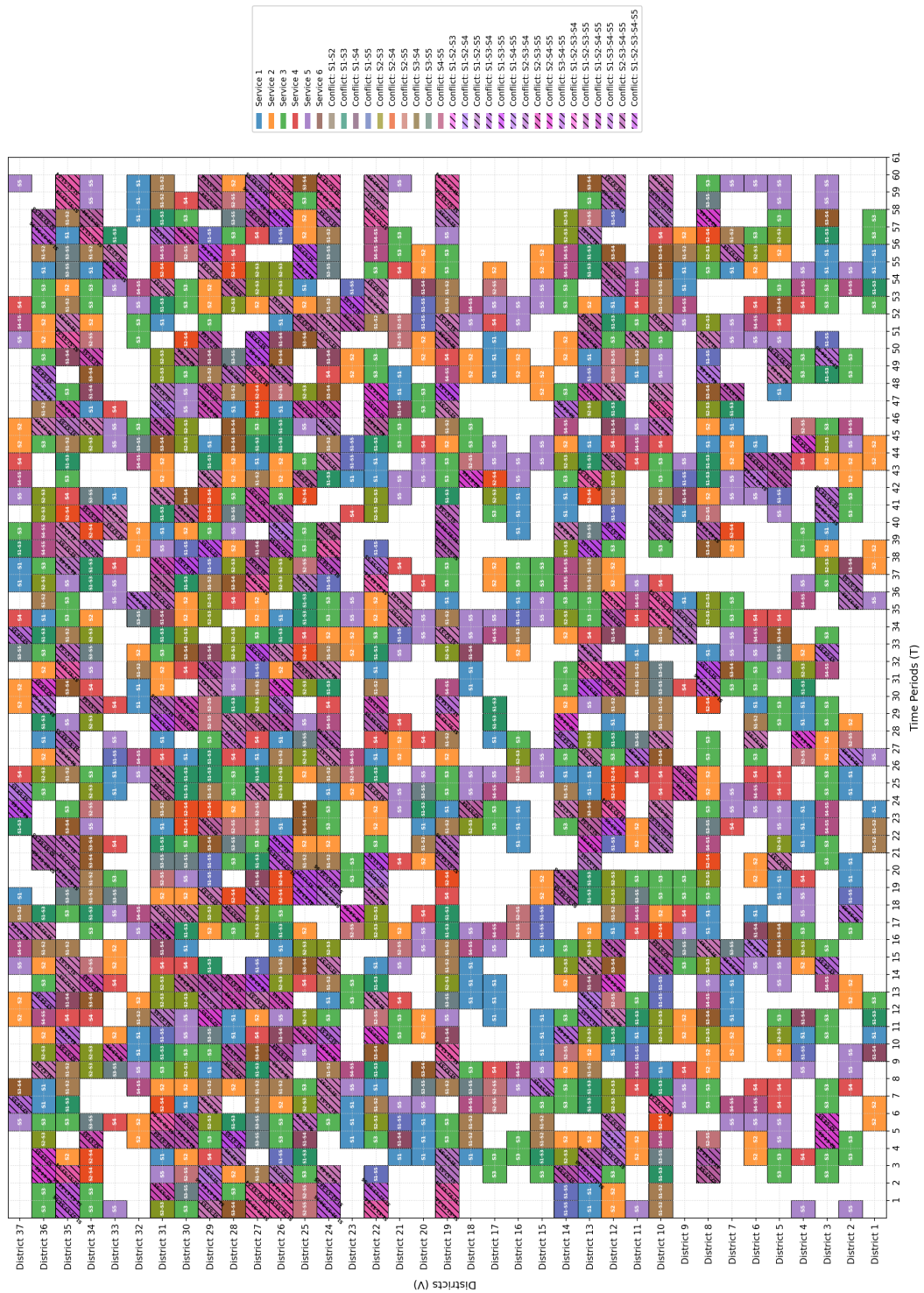


Figure 6.6: Service schedule optimization results for Defne case study

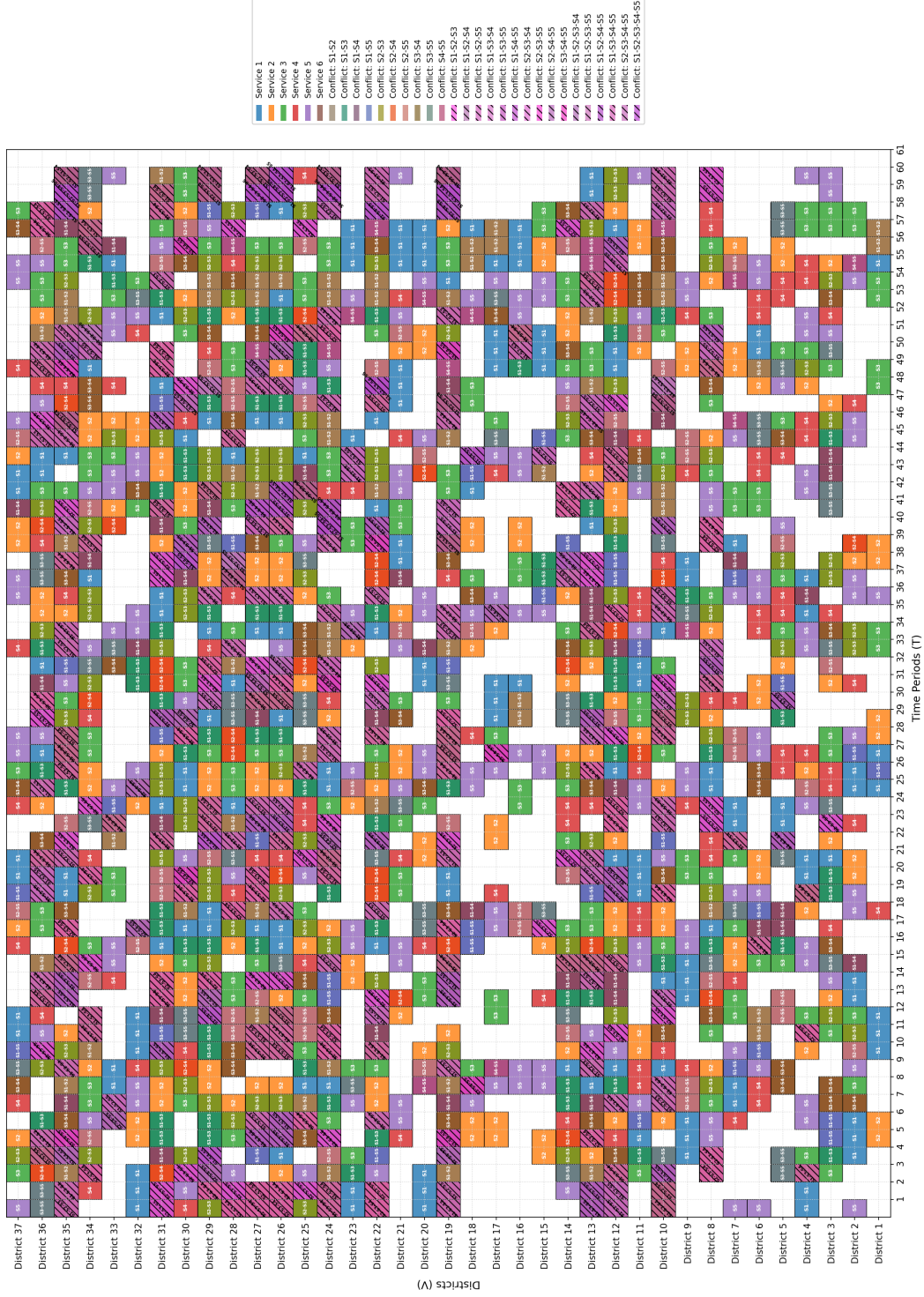


Figure 6.7: Service schedule optimization results for Defne case study with change in h_{it}

patterns for S1 with ten visits spaced every six-seven periods and S2 with nine visits every six periods. Generally, these services are provided for three periods, but sometimes extended to four periods. The results show that high-density areas receive near-uninterrupted service. Also, high-priority districts use different combinations of services to include S4 and S5 pre- $T/2$ by satisfying $h_{it} = 3$. Even though S4 and S5 have lower frequencies than other service types, they have longer elapsed durations. Thus, they are squeezed into a schedule with different combinations of 3 services.

In contrast, low-density districts receive a minimum visit requirement, and the services are significantly sparser and more irregularly distributed. For instance, District 15 receives S1 only three times on periods 36, 43, and 49, while S3 is provided for only three visits on periods 17, 37, and 57. In the base case, S1 is provided on periods 4, 10 and 16 while S3 is provided on periods 2, 7, and 37. As another example, in the base case, District 1 is visited twice by S1 and S2 and once by S3, S4, S5 pre- $T/2$. However, when the constraint tightens, S3 does not visit District 1 pre- $T/2$, and S3 only provides service to District 1 post- $T/2$. For low-priority districts, some S3–S5 activities are delayed post- $T/2$.

Finally, medium-density areas exhibit a distinct pattern of short-duration but high-frequency visits, with District 21 as a notable example. The service allocation adapts strategically to the tightened constraint while preserving some characteristics of the base case. For S4, District 21 maintains nearly identical high-frequency, single-period visits (periods 5, 13, 21, 29, 37, 45, 53) under both scenarios. Further, the schedule for S5 remains unchanged. However, the tight constraint significantly reshapes other services. In the base case, S1 is provided on periods 4, 34, and 47, but is delayed post- $T/2$ to periods 37, 47, and 55. Conversely, the activities of S3 are shifted earlier, on periods 19, 23, 29, 40, compared to base-case periods 11, 35, 45, 56. These changes balance both early slots for high-density areas and balance the delayed service types, or exploit underutilized pre- $T/2$ capacity.

In summary, we observe three main strategies: 1) High-density districts are prioritized and dominate early period slots. 2) Medium-density areas retain high-frequency but low-duration services. 3) Low-density regions face the most pronounced service postponements.

6.4 Conclusion of Chapter 6

We formally defined the Periodic Demand Services Problem (PDSP). In order to incorporate the change in demand over time, we developed a linearization scheme. We then formulated four novel mathematical formulations for the PDSP. We developed a 5-indexed compact model (M-5i), a more efficient 4-indexed model (M-4i), and two 3-indexed non-compact models (M-3i and M-3iE) based on Benders decomposition.

We then provided a detailed computational study on varying-sized datasets, using the sigmoid decay function for our base case, sensitivity analysis, and case study. According to our comparison of four models, M-5i is computationally intractable even for small instances. On the other hand, M-4i is the best-performing model since it mostly achieves optimal solutions across instances within acceptable solution times. Although M-3i and M-3iE are decomposition-friendly formulations with theoretical advantages, they face significant computational overhead.

Our sensitivity analysis shows that the upper bound $\bar{c}_s$ is the most influential parameter in the model, with changes to $\bar{c}_s$ causing the most significant objective function value swings of $\pm 10\%$. We also analyze demand functions, including linear, exponential, sigmoid, quadratic, logarithmic, and step functions. We observe that different demand patterns significantly impact solution quality and computational performance. We also evaluate different demand functions by post-processing their solutions with a neutral demand function for fair comparison. Growth functions

consistently outperformed decay functions. Exponential functions achieve the highest post-processed objective function values among the growth and decay functions.

Finally, we provide a case study on Defne to validate the model’s practical applicability. According to the solutions, the high-density districts receive extended service durations. Also, minimum visit, minimum/maximum duration, and minimum elapsed period requirements are crucial to ensure fairness and equity, especially for low-density districts. Without those requirements, the low-density regions would not be visited. Therefore, to enhance fairness, the parameters can be updated for low-density areas, and it is important to note that these minimum requirements ensure that demand in those areas is also satisfied.

This study can be extended for future research through computational enhancements for M-3i and M-3iE to find a better formulation utilizing the Benders reformulation. It can be easy to quantify equity-performance trade-offs explicitly by integrating a multi-objective approach. Finally, all demand types can be integrated to capture a more realistic real-life case study. Dynamic demand patterns can be a future research area in which to extend the models. More sophisticated demand functions can be developed to better capture the complex relationships between duration and elapsed period parameters. A promising direction for future work lies in the relaxation of the simultaneous service constraint, 6.15, 6.24, 6.32 in the proposed models. This relaxation would immediately decompose the model into $|S|$ fully independent single-service subproblems. Therefore, it would enable perfect parallelization and reductions in computational complexity. More importantly, the relaxed problem provides a strong and valid upper bound that can be effectively used within a branch-and-bound framework or to evaluate the quality of heuristic solutions. Moreover, an incremental or iterative approach can be designed: initially solve the fully decomposed problem to obtain a bound and an initial allocation; then identify the most critical or violated h_{it} constraints and reintroduce them sequentially. This progressive tightening allows the algorithm to focus computational effort

on the binding constraints that truly shape the solution. Such a methodology would not only yield excellent bounds early in the solution process but also maintain scalability for very-large-scale instances that are currently intractable. This approach directly aligns with the Benders decomposition philosophy embedded in M-3i and M-3Ei, offering a natural and computationally efficient pathway to extend the current modelling framework.



Chapter 7

Conclusion

This study addresses a gap in humanitarian logistics literature by introducing a classification of demand types in disaster response. The frequency and the intensity of demand change during different phases of the disaster response operations. While existing humanitarian logistics literature focuses on different demand types, they do not explicitly consider their frequency patterns. We categorize them based on their frequency patterns as one-time, continuous, and periodic demands.

In Chapter 4, we explored one-time demand for urgent, non-repeating needs of disaster victims, like shelter and first-aid. We first defined the problem OTDP and proved its complexity. Later, we mathematically formulated the problem and used a stepwise demand function to show how demand naturally changes over time. We proposed three integer programming models (Model 1, Model 2, and Model 3). We have tested our models on Seoul, Kartal and Istanbul datasets and observed that Model 3 is the fastest and best, finding optimal solutions even for large instances. This chapter highlighted that MSUs need to be flexible and placed wisely to get the most aid out as immediate needs lessen.

In Chapter 5, we formally defined CTD for steady, ongoing needs crucial for survival after a disaster, such as food, water, and healthcare. We introduced a system with different types of MSUs: high-capacity (HCap), medium-capacity (MCap), and low-capacity (LCap) MSUs, each with their own service ability. Our mathematical model determined the optimal locations and allocation of MSUs. We utilized both binary and gradual coverage. In binary coverage, the results show that everyone can be covered while with gradual coverage it is shown that that being close and assigning MSUs precisely is key to helping the most people, calling for clever strategies for ongoing aid.

Furthermore, the complex scheduling requirements of periodic demand, such as medical treatments, mental health support, and maintenance services, remain under-explored. In Chapter 6, we are interested in scheduling of MSUs to respond to these varying demand categories that evolve throughout the disaster cycle. We introduce the PDSP for scheduling MSUs to meet recurring service requirements across disaster-affected regions. We develop four mathematical formulations with increasing complexity, incorporating non-linear demand functions that capture the change in demand over time. While two of our formulations are compact, they involve 5- and 4-indexed variables. On the other hand, we also introduce two models with 3-indexed variables with exponential number of constraints. We solve the latter two by a branch-and-cut algorithm, strengthened through valid inequalities. We study efficiency of our formulations on datasets ranging from 25 to 1,000 nodes. We also carry out a case-study using real-world data from the 2023 Kahramanmaraş earthquakes in Türkiye. Our results demonstrate the effectiveness of the proposed methodology in handling complex periodic demand scenarios.

Our main contributions are as follows:

- We categorize the emerging demands in the response phase as one-time, continuous, and periodic demand, and synthesize and re-frame the existing literature.

- We formally define problems regarding demand types and analyze their computational complexity.
- We develop mathematical model for each demand types, including a linearization strategy for the non-linear demand function that captures demand changes in time.
- For one-time demand services, we propose three novel mathematical models (Model 1, Model 2, and Model 3). We also incorporate a stepwise demand function to capture time-dependent demand decrement to be close real-life applications. We conduct an extensive computational study, comparing these models under both uniform and decreasing demand functions. To further derive insight on the strengths of models, we analyzed their LP relaxations, number of constraints and decision variables. We finally derive strategic insights from optimal solutions, identifying patterns in CD locations and MSU deployment strategies that adapt to different demand scenarios.
- For continuous demand services, we develop a location and allocation model heterogeneous MSUs to maximize the total demand satisfied. Due to constant service requirement, we do not include a demand function changing over time but we consider demand change due to coverage. We consider both binary and gradual coverage functions and conduct a detailed analysis of optimal solutions. We highlight the significant impact of gradual coverage, showing a more realistic representation of demand satisfaction where proximity to service centers is crucial for maximizing participation.
- For periodic demand services, we develop four mathematical models with increasing levels of complexity: 5-index and 4-index compact models and two 3-index non-compact models, which are obtained through a Benders-type projection method. The first of these 3-index models requires both feasibility and optimality cuts, while the latter only has optimality cuts. We solve the models by a branch-and-cut algorithm strengthened through valid inequalities. We conduct an extensive computational study comparing the proposed

models, performing sensitivity analysis, and evaluating the robustness of solutions against different demand functions. Finally, we test our approach using a dataset based on the Hatay and Defne region, which was severely affected by the 2023 Kahramanmaraş earthquakes, providing insights for emerging demands and MSUs in post-disaster logistics.

This thesis has several areas to develop for future work. For one-time demand models, advanced decomposition methods can be adopted to improve computational efficiency, especially for larger instances. Also, in order to quantify equity-performance trade-off, a multi-objective approach can be integrated. Further, to incorporate uncertainty regarding demand and supply, stochastic modeling approaches can be considered.

For continuous demand, different types of decay functions can be explored to integrate real-time demand rates. Also, the inventory replenishment, inventory management and transshipment constraint can be added to models to ensure that supply is continuous for uninterrupted service delivery.

For periodic demand, the study can be extended for future research through computational enhancements for M-3i and M-3iE to find a better formulation utilizing the Benders reformulation. It can be easy to quantify equity-performance trade-off explicitly by integrating a multi-objective approach. Finally, all demand types can be integrated to capture a more realistic real-life case study. Dynamic demand patterns can be a future research area in which to extend the models. More sophisticated demand functions can be developed to better capture the complex relationships between duration and elapsed period parameters.

Bibliography

- [1] Türkiye Cumhuriyeti İçişleri Bakanlığı, “Türkiye’nin birlik ve dayanışma gücü depremle sınandı: Asrın felaketi asrın dayanışmasına dönüştü,” 2023. <https://www.icisleri.gov.tr/turkiyenin-birlik-ve-dayanisma-gucu-depremlerle-sinandi-asrin-felaketi-asrin-dayanismasina-donustu8> Access date: September 15, 2025.
- [2] T.C. Cumhurbaşkanlığı İletişim Başkanlığı, “Konya Büyükşehir Belediyesi’nin hayata geçirdiği mobil dış aracı Hatay’da hizmet vermeye başladı,” 2023. https://www.iletisim.gov.tr/turkce/yerel_basin/detay/konya-buyuksehir-belediyesinin-hayata-gecirdigi-mobil-dis-araci-hatayda-hizmet-vermeye-basladi Access date: September 15, 2025.
- [3] Alkın Group, “Mobil mutfak tırı teslim edildi,” 2023. <https://www.alkangroup.com.tr/mobil-mutfak-tiri-teslim-edildi/> Access date: September 15, 2025.
- [4] Anadolu Ajansı, “Bozkurt’ta mobil eczane sayesinde ilaç ihtiyacı kesintisiz karşılandı,” 2022. <https://www.aa.com.tr/tr/yasam/bozkurtta-mobil-eczane-sayesinde-ilac-ihciyaci-kesintisiz-karsilandi/2376816> Access date: September 15, 2025.
- [5] Anadolu Ajansı, “Konya Büyükşehir Belediyesi depremzedeler için başlattığı çalışmalarını sürdürüyor,” 2023. <https://www.aa.com.tr/tr/asrin-felak>

eti/konya-buyuksehir-belediyesi-depremzedeler-icin-baslattigi-calismalarini-surduruyor-/2833384 Access date: September 15, 2025.

- [6] Merzifon Belediyesi, “Mobil tuvalet aracımızı belediyemize kazandırdık,” 2023. <https://merzifon.bel.tr/mobil-tuvalet-aracimizi-belediyemize-kazandirdik> Access date: September 15, 2025.
- [7] Haberin Nabzı, “Ankara Büyükşehir Kahramanmaraş’a duş alanları ile mobil çamaşırhane ve berber gönderdi,” 2023. <https://www.haberinnabzi.com/ankara-buyuksehir-kahramanmaras-a-dus-alanlari-ile-mobil-camasirhane-ve-berber-gonder-34961h> Access date: September 15, 2025.
- [8] Belediye Haber, “Mobil kuaför depremzedeler için Alanya’da,” 2023. <https://belediyehaber.net/2023/02/16/mobil-kuafor-depremzedeler-icin-alanyada/> Access date: September 15, 2025.
- [9] Haberin Nabzı, “Ankara Büyükşehir Belediyesi Kahramanmaraş’a bu kez tırlardan dönüştürülen sabit duş,” 2023. <https://www.haberinnabzi.com/ankara-buyuksehir-belediyesi-kahramanmaras-a-bu-kez-tirlardan-donus-turulen-sabit-dus--35035h> Access date: September 15, 2025.
- [10] Taşıyanlar, “PTT’nin 40 mobil aracı bankacılık hizmeti veriyor,” 2023. <https://tasiyanlar.com/en-yeniler/pttnin-40-mobil-araci-bankacilik-hizmeti-veriyor.html> Access date: September 15, 2025.
- [11] En Son Haber, “Deprem bölgelerine mobil baz istasyonları kuruldu,” 2023. <https://www.ensonhaber.com/ekonomi/deprem-bolgelerine-mobil-baz-istasyonlari-kuruldu> Access date: September 15, 2025.
- [12] EGO Genel Müdürlüğü, “Ankara Büyükşehir’in atıl otobüslerden dönüştürdüğü mobil çamaşırhane ve berber deprem bölgesine doğru yola çıktı,” 2023. <https://www.ego.gov.tr/tr/haber/6021/ankara-buyuksehi>

rin-atil-otobuslerden-donusturdugu-mobil-camasirhane-ve-berber-deprem-bolgesine-dogru-yola-cikti Access date: September 15, 2025.

- [13] Hürriyet, “Atıl otobüslerden mobil duş,” 2023. <https://www.hurriyet.com.tr/gundem/atil-otobuslerden-mobil-dus-42224062> Access date: September 15, 2025.
- [14] S.-C. Oh and A. Haghani, “Testing and evaluation of a multi-commodity multi-modal network flow model for disaster relief management,” *Journal of Advanced Transportation*, vol. 31, no. 3, pp. 249–282, 1997.
- [15] W. Yi and A. Kumar, “Ant colony optimization for disaster relief operations,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 43, no. 6, pp. 660–672, 2007.
- [16] W. Green III, “Four phases of emergency management,” in *Electronic Encyclopedia of Civil Defense and Emergency Management*, 2002.
- [17] N. Altay and W. G. Green III, “OR/MS research in disaster operations management,” *European Journal of Operational Research*, vol. 175, no. 1, pp. 475–493, 2006.
- [18] R. Gensch, A. Jennings, S. Renggli, and P. Reymond, “Ecompendium of sanitation technologies in emergencies, Emergency WASH Knowledge Portal,” 2020. <https://www.emersan-compendium.org/en/faq/> Access date: September 15, 2025.
- [19] E. H. Erdem, B. Aydinli, S. Dogru, H. Ozmen, M. Atilla, and I. Yildirim, “Postoperative outcomes in earthquake victims with orthopedic trauma: a focus on mortality and dialysis needs,” *BMC Anesthesiol*, vol. 25, no. 141, 2025.
- [20] E. Erek, M. S. Sever, K. Serdengeçti, R. Vanholder, E. Akoğlu, M. Yavuz, H. Ergin, M. Tekçe, N. Duman, and N. Lameire, “An overview of morbidity

and mortality in patients with acute renal failure due to crush syndrome: the marmara earthquake experience,” *Nephrology Dialysis Transplantation*, vol. 17, no. 1, pp. 33–40, 2002.

- [21] G. Erdoğan and G. Laporte, “The orienteering problem with variable profits,” *Networks*, vol. 61, pp. 104–116, 2013.
- [22] A. Ekici and A. Retharekar, “Multiple agents maximum collection problem with time dependent rewards,” *Computers & Industrial Engineering*, vol. 64, no. 4, pp. 1009–1018, 2013.
- [23] A. Santini and C. Archetti, “The hazardous orienteering problem,” *Networks*, vol. 81, no. 2, pp. 235–252, 2023.
- [24] H. M. Afsar and N. Labadie, “Team orienteering problem with decreasing profits,” *Electronic Notes in Discrete Mathematics*, vol. 41, pp. 285–293, 2013.
- [25] Q. Yu, Y. Adulyasak, L.-M. Rousseau, N. Zhu, and S. Ma, “Team orienteering with time-varying profit,” *INFORMS Journal on Computing*, vol. 34, no. 1, pp. 262–280, 2022.
- [26] T. Dewilde, D. Cattrysse, S. Coene, F. C. Spieksma, and P. Vansteenwegen, “Heuristics for the traveling repairman problem with profits,” *Computers & Operations Research*, vol. 40, no. 7, pp. 1700–1707, 2013.
- [27] M. J. Hodgson, G. Laporte, and F. Semet, “A covering tour model for planning mobile health care facilities in Suhum district, Ghana,” *Journal of Regional Science*, vol. 38, pp. 621–638, 1998.
- [28] K. Doerner, A. Focke, and W. Gutjahr, “Multicriteria tour planning for mobile healthcare facilities in a developing country,” *European Journal of Operational Research*, vol. 179, pp. 1078–1096, 2007.

- [29] B. Balcik and B. M. Beamon, "Facility location in humanitarian relief," *International Journal of Logistics Research and Applications*, vol. 11, no. 2, pp. 101–121, 2008.
- [30] L. Özdamar, E. Ekinçi, and B. Küçükyazıcı, "Emergency logistics planning in natural disasters," *Annals of Operations Research*, vol. 129, pp. 217–245, 2004.
- [31] O. Berman, T. Drezner, Z. Drezner, and G. O. Wesolowsky, "A defensive maximal covering problem on a network," *International Transactions in Operational Research*, vol. 16, no. 1, pp. 69–86, 2009.
- [32] M. Hachicha, M. Hodgson, G. Laporte, and F. Semet, "Heuristics for the multi-vehicle covering tour problem," *Computers & Operations Research*, vol. 27, pp. 29–42, 2000.
- [33] P. C. Nolz, K. F. Doerner, W. J. Gutjahr, and R. F. Hartl, "A bi-objective metaheuristic for disaster relief operation planning," *Advances in Multi-Objective Nature Inspired Computing*, vol. 272, pp. 167–187, 2010.
- [34] Z. Naji-Azimi, J. Renaud, A. Ruiz, and M. Salari, "A covering tour approach to the location of satellite distribution centers to supply humanitarian aid," *European Journal of Operational Research*, vol. 222, pp. 596–605, 2012.
- [35] H. Afsar, C. Prins, and A. Santos, "Exact and heuristic algorithms for solving the generalized vehicle routing problem with flexible fleet size," *International Transactions in Operational Research*, vol. 21, no. 1, pp. 153–175, 2014.
- [36] S. Allahyari, M. Salari, and D. Vigo, "A hybrid metaheuristic algorithm for the multi-depot covering tour vehicle routing problem," *European Journal of Operational Research*, vol. 242, pp. 756–768, 2015.
- [37] M. Alinaghian, M. Aghaei, and M. Sabbagh, "A mathematical model for location of temporary relief centers and dynamic routing of aerial rescue vehicles," *Computers & Industrial Engineering*, vol. 131, pp. 227–241, 2019.

- [38] H. O. Mete and Z. B. Zabinsky, “Stochastic optimization of medical supply location and distribution in disaster management,” *International Journal of Production Economics*, vol. 126, pp. 76–84, July 2010.
- [39] R. L. Church and K. L. Roberts, “Generalized coverage models and public-facility location,” *Papers of the Regional Science Association*, vol. 53, no. 1, pp. 117–135, 1983.
- [40] O. Berman and D. Krass, “The generalized maximal covering location problem,” *Computers & Operations Research*, vol. 29, no. 6, pp. 563–581, 2002.
- [41] Z. Drezner, G. O. Wesolowsky, and T. Drezner, “The gradual covering problem,” *Naval Research Logistics*, vol. 51, no. 6, pp. 841–855, 2004.
- [42] M. Karatas, “A multi-objective facility location problem in the presence of variable gradual coverage performance and cooperative cover,” *European Journal of Operational Research*, vol. 262, no. 1, pp. 1040–1051, 2017.
- [43] V. Verter and S. Lapierre, “Location of preventive health care facilities,” *Annals of Operations Research*, vol. 110, no. 1, pp. 123–132, 2002.
- [44] E. Erkut, A. Ingolfsson, and G. Erdoğan, “Ambulance location for maximum survival,” *Naval Research Logistics*, vol. 55, no. 1, pp. 42–58, 2008.
- [45] F. Rahim and C. Sepil, “A location-routing problem in glass recycling,” *Annals of Operations Research*, vol. 223, pp. 329–353, 2014.
- [46] Z. Liu, Z. Liu, Z. Zhu, Y. Shen, and J. Dong, “Simulated annealing for a multi-level nurse rostering problem in hemodialysis service,” *Applied Soft Computing*, vol. 64, pp. 148–160, 2018.
- [47] M. Reihaneh, S. Ansari, and F. Farhadi, “Discrete optimization patient appointment scheduling at hemodialysis centers: An exact Branch and Price

- approach,” *European Journal of Operational Research*, vol. 309, pp. 35—52, 2023.
- [48] J. Steeg and M. Schröder, “A hybrid approach to solve the periodic home health care problem,” in *Operations Research Proceedings 2007* (J. Kalcsics and S. Nickel, eds.), pp. 297–302, Springer Berlin Heidelberg, 2008.
- [49] R. S. González, M. Cherklesly, T. G. Crainic, and M. Ève Rancourt, “Multi-period location routing: An application to the planning of mobile clinic operations in iraq,” *Computers & Operations Research*, vol. 159, p. 106288, 2023.
- [50] S. Nickel, M. Schröder, and J. Steeg, “Mid-term and short-term planning support for home health care services,” *European Journal of Operational Research*, vol. 219, no. 3, pp. 574–587, 2012.
- [51] S. K. Savaşer and B. Y. Kara, “Mobile healthcare services in rural areas: an application with periodic location routing problem,” *OR Spectrum*, vol. 44, no. 33, pp. 875–910, 2022.
- [52] S. Li and K. L. Teo, “Post-disaster multi-period road network repair: Work scheduling and relief logistics optimization,” *Annals of Operations Research*, vol. 283, pp. 1345–1385, 2019.
- [53] I. Tarhan, K. G. Zografos, J. Sutanto, A. Kheiri, and H. Suhartanto, “A multi-objective rolling horizon personnel routing and scheduling approach for natural disasters,” *Transportation Research Part C: Emerging Technologies*, vol. 149, p. 104029, 2023.
- [54] C. Ozmemis, B. Balcik, I. Yanıkoğlu, and B. Akca, “Integrated treatment scheduling and logistics planning for a hemodialysis center after a disaster with robust travel times,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 198, p. 104110, 2025.

- [55] E. J. Beltrami and D. B. Lawrence, “Networks and vehicle routing for municipal waste collection,” *Networks*, vol. 4, no. 1, pp. 65–94, 1974.
- [56] C. Prodhon, “A metaheuristic for the periodic location-routing problem,” in *Operations Research Proceedings 2007* (J. Kalcsics and S. Nickel, eds.), pp. 159–164, Springer Berlin Heidelberg, 2007.
- [57] A. M. Campbell and J. H. Wilson, “Forty years of periodic vehicle routing,” *Networks*, vol. 63, no. 1, pp. 2–15, 2014.
- [58] P. M. Francis, K. R. Smilowitz, and M. Tzur, *The Period Vehicle Routing Problem and its Extensions*, pp. 73–102. Boston, MA: Springer US, 2008.
- [59] P. Francis, K. Smilowitz, and M. Tzur, “The period vehicle routing problem with service choice,” *Transportation Science*, vol. 40, no. 4, pp. 439–454, 2006.
- [60] C. Archetti, E. Fernández, and D. L. Huerta-Muñoz, “The flexible periodic vehicle routing problem,” *Computers & Operations Research*, vol. 85, pp. 58–70, 2017.
- [61] B. Amiri, K. Ziarati, and S. Sohrabi, “A memetic algorithm for the flexible periodic vehicle routing problem,” *Journal of Heuristics*, vol. 31, no. 7, pp. 1–25, 2025.
- [62] C. F. Dursunoglu, O. Arslan, S. M. Demir, B. Y. Kara, and G. Laporte, “A unifying framework for selective routing problems,” *European Journal of Operational Research*, vol. 320, no. 1, pp. 1–19, 2025.
- [63] J. R. Current, “Multiobjective design of transportation networks,” ph.D. Thesis, The Johns Hopkins University, 1981.
- [64] T. Tsiligirides, “Heuristic methods applied to orienteering,” *Journal of the Operational Research Society*, vol. 35, no. 9, pp. 797–809, 1984.

- [65] G. B. Dantzig, D. R. Fulkerson, and S. M. Johnson, "Solution of a large-scale traveling-salesman problem," *Journal of the Operations Research Society of America*, vol. 2, no. 4, pp. 393–410, 1954.
- [66] G. B. Dantzig and J. Ramser, "The truck dispatching problem," *Management Science*, vol. 6, no. 1, pp. 80–91, 1959.
- [67] G. Laporte, Y. Nobert, and D. Arpin, "An exact algorithm for solving a capacitated location-routing problem," *Annals of Operations Research*, vol. 6, pp. 291–310, 1986.
- [68] A. L. Henry-Labordère, "The record balancing problem: A dynamic programming solution of a generalized traveling salesman problem," *Revue française d'automatique d'informatique et de recherche opérationnelle*, vol. 3, no. 2, pp. 43–49, 1969.
- [69] G. Laporte and Y. Nobert, "Generalized travelling salesman problem through n sets of nodes: An integer programming approach," *INFOR: Information Systems and Operational Research*, vol. 21, no. 1, pp. 61–75, 1983.
- [70] G. Ghiani and G. Improta, "Efficient transformation of the generalized vehicle routing problem," *European Journal of Operational Research*, vol. 122, no. 1, pp. 11–17, 2000.
- [71] R. M. Newton and W. H. Thomas, "Design of school bus routes by computer," *Socio-Economic Planning Sciences*, vol. 3, no. 1, pp. 75–85, 1969.
- [72] A. Fügenschuh, "Solving a school bus scheduling problem with integer programming," *European Journal of Operational Research*, vol. 193, no. 3, pp. 867–884, 2009.
- [73] T. Ramesh, "Traveling purchaser problem," *Opsearch*, vol. 18, no. 1-3, pp. 78–91, 1981.

- [74] M. Choi and S. Lee, “The multiple traveling purchaser problem for maximizing system’s reliability with budget constraints,” *Expert Systems with Applications*, vol. 38, no. 8, pp. 9848–9853, 2011.
- [75] M. Gendreau, G. Laporte, and F. Semet, “The covering tour problem,” *Operations Research*, vol. 45, no. 4, pp. 568–576, 1997.
- [76] B. L. Golden, L. Levy, and R. Vohra, “The orienteering problem,” *Naval Research Logistics*, vol. 34, pp. 304–318, 1987.
- [77] C. P. Keller, “Muitiobjective routing through space and time: The MVP and TDVP problems,” PhD Thesis, University of Western Ontario, 1985.
- [78] C. Keller and M. F. Goodchild, “Multi-objective vending problem: A generalization of the travelling salesman problem,” *Environment Planning B: Planning and Design*, vol. 15, pp. 447–460, 1988.
- [79] S. Kataoka and S. Morito, “An algorithm for single constraint maximum collection problem,” *Journal of the Operations Research Society of Japan*, vol. 31, pp. 515–530, 1988.
- [80] G. Laporte and S. Martello, “The selective travelling salesman problem,” *Discrete Applied Mathematics*, vol. 26, no. 2–3, pp. 193–207, 1990.
- [81] B. Awerbuch, Y. Azar, A. Blum, and S. Vempala, “New approximation guarantees for minimum-weight k-trees and prize-collecting salesmen,” *SIAM Journal on Computing*, vol. 28, no. 1, pp. 254–262, 1998.
- [82] S. E. Butt and T. M. Cavalier, “A heuristic for the multiple tour maximum collection problem,” *Computers & Operations Research*, vol. 21, no. 1, pp. 101–111, 1994.
- [83] I-M. Chao, B. L. Golden, and E. A. Wasil, “The team orienteering problem,” *European Journal of Operational Research*, vol. 88, no. 3, pp. 464–474, 1996.

- [84] E. Balas, “The prize collecting traveling salesman problem,” *Networks*, vol. 19, pp. 621–636, 1989.
- [85] L. Tang and X. Wang, “Iterated local search algorithm based on very large-scale neighborhood for prize-collecting vehicle routing problem,” *The International Journal of Advanced Manufacturing Technology*, vol. 29, no. 11-12, pp. 1246–1258, 2006.
- [86] H. Süral and J. H. Bookbinder, “The single-vehicle routing problem with unrestricted backhauls,” *Networks*, vol. 41, no. 3, pp. 127–136, 2003.
- [87] I. Gribkovskaia, G. Laporte, and A. Shyshou, “The single vehicle routing problem with deliveries and selective pickups,” *Computers & Operations Research*, vol. 35, no. 9, pp. 2908–2924, 2008.
- [88] G. Gutiérrez-Jarpa, V. Marianov, and C. Obreque, “A single vehicle routing problem with fixed distribution and optional collections,” *IIE Transactions*, vol. 41, pp. 1067–1079, 2009.
- [89] R. Falcon, X. Li, A. Nayak, and I. Stojmenovic, “The one-commodity traveling salesman problem with selective pickup and delivery: an ant colony approach,” in *Proceedings of the 2010 IEEE Congress on Evolutionary Computation (CEC2010)*, pp. 1–8, 2010.
- [90] C. A. Yano, T. J. Chan, L. K. Richter, T. Cutler, K. G. Murty, and D. McGettigan, “Vehicle routing at Quality Stores,” *Interfaces*, vol. 17, no. 2, pp. 52–63, 1987.
- [91] G. Gutiérrez-Jarpa, G. Desaulniers, G. Laporte, and V. Marianov, “A branch-and-price algorithm for the vehicle routing problem with deliveries, selective pickups and time windows,” *European Journal of Operational Research*, vol. 206, no. 2, pp. 341–349, 2010.

- [92] C. Ting and X. Liao, “The selective pickup and delivery problem: Formulation and a memetic algorithm,” *International Journal of Production Economics*, vol. 141, no. 1, pp. 199–211, 2013.
- [93] J. R. Current and D. A. Schilling, “The covering salesman problem,” *Transportation Science*, vol. 23, no. 3, pp. 208–213, 1989.
- [94] R. Ramesh and K. M. Brown, “An efficient four-phase heuristic for the generalized orienteering problem,” *Computers & Operations Research*, vol. 18, pp. 151–165, 1991.
- [95] J. D. Kort and A. Volgenant, “On the generalized 2-peripatetic salesman problem,” *European Journal of Operational Research*, vol. 73, no. 1, pp. 175–180, 1994.
- [96] D. Bienstock, M. Goemans, and D. Simchi-Levi, “A note on the prize collecting traveling salesman problem,” *Mathematical Programming*, vol. 59, pp. 413–420, 1993.
- [97] M. Dell’Amico, F. Maffioli, and P. Värbrand, “On prize-collecting tours and the asymmetric travelling salesman problem,” *International Transactions in Operational Research*, vol. 2, pp. 297–308, 1995.
- [98] C. Archetti, D. Feillet, A. Hertz, and M. Speranza, “The capacitated team orienteering and profitable tour problems,” *JORS*, vol. 60, pp. 831–842, 2009.
- [99] C. S. ReVelle and G. Laporte, “New directions in plant location,” *Studies in Locational Analysis*, vol. 5, pp. 31–58, 1993.
- [100] J. R. Current and D. A. Schilling, “The median tour and maximal covering tour problems: Formulations and heuristics,” *European Journal of Operational Research*, vol. 73, no. 1, pp. 114–126, 1994.

- [101] E. M. Arkin and R. Hassin, “Algorithm approximations for the geometric covering salesman problem,” *Discrete Applied Mathematics*, vol. 55, pp. 197–218, 1994.
- [102] J. Beasley and E. Nascimento, “The vehicle routing-allocation problem: A unifying framework,” *TOP*, vol. 4, pp. 65–86, 1996.
- [103] E. Erkut and J. Zhang, “The maximum collection problem with time-dependent rewards,” *Naval Research Logistics*, vol. 43, pp. 749–763, 1996.
- [104] S. Coene and F. C. R. Spieksma, “Profit-based latency problems on the line,” *Operations Research Letters*, vol. 43, no. 3, pp. 333–337, 2008.
- [105] G. Erdoğan and G. Laporte, “The orienteering problem with variable profits,” *Networks*, vol. 61, pp. 104–116, 2013.
- [106] H. Tang, E. Miller-Hooks, and R. Tomastik, “Scheduling technicians for planned maintenance of geographically distributed equipment,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 43, pp. 591–609, 2007.
- [107] N. Afsar, H. M. and Labadie, “Team orienteering problem with decreasing profits,” *Electronic Notes in Discrete Mathematics*, vol. 42, pp. 285–293, 2013.
- [108] A. Gunawan, K. Ng, G. Kendall, and J. Lai, “An iterated local search algorithm for the team orienteering problem with variable profits,” *Engineering Optimization*, vol. 50, no. 7, pp. 1148–1163, 2017.
- [109] M. Labbé, G. Laporte, I. R. Martín, and J. J. Salazar-González, “Locating median cycles in networks,” *European Journal of Operational Research*, vol. 160, no. 2, pp. 457–470, 2005.
- [110] F. V. Fomin and A. Lingas, “Approximation algorithms for time-dependent orienteering,” *Information Processing Letters*, vol. 83, no. 2, pp. 57–62, 2002.

- [111] J. Li, “Model and algorithm for time-dependent team orienteering problem,” in *Advanced Research on Computer Education, Simulation and Modeling* (S. Lin and X. Huang, eds.), vol. 175, pp. 1–7, Springer, 2011.
- [112] M. Labbé, G. Laporte, I. Rodríguez-Martín, and J. J. Salazar-González, “The ring star problem: Polyhedral analysis and exact algorithm,” *Networks: An International Journal*, vol. 43, no. 3, pp. 177–189, 2004.
- [113] R. Baldacci, M. Dell’Amico, and J. J. Salazar-González, “The capacitated m-ring-star problem,” *Operations Research*, vol. 55, no. 6, pp. 1147–1162, 2007.
- [114] D. Gulczynski, J. Heath, and C. Price, *The Close Enough Traveling Salesman Problem: A Discussion of Several Heuristics*, vol. 36, pp. 271–283. 2006.
- [115] W. Mennell, “Heuristics for solving three routing problems: Close-enough traveling salesman problem, close-enough vehicle routing problem, sequence-dependent team orienteering problem,” PhD Thesis, University of Maryland, 2009.
- [116] G. Erdogan, J.-F. Cordeau, and G. Laporte, “The attractive traveling salesman problem,” *European Journal of Operational Research*, vol. 203, pp. 59–69, 2010.
- [117] N. Aras, D. Aksen, and T. Tekin, “Location and selective routing problem with pricing for the collection of used products,” in *The 40th International Conference on Computers & Industrial Engineering*, pp. 1–6, IEEE, 2010.
- [118] C. A. Valle, L. C. Martinez, A. S. da Cunha, and G. R. Mateus, “Heuristic and exact algorithms for a min–max selective vehicle routing problem,” *Computers & Operations Research*, vol. 38, no. 7, pp. 1054–1065, 2011.
- [119] J. Ahn, O. de Weck, and J. Hoffman, “An optimization framework for global planetary surface exploration campaigns,” *Journal of the British Interplanetary Society*, vol. 61, pp. 487–498, 2008.

- [120] J. Ahn, O. de Weck, Y. Geng, and D. Klabjan, "Column generation based heuristics for a generalized location routing problem with profits arising in space exploration," *European Journal of Operational Research*, vol. 223, no. 1, pp. 47–59, 2012.
- [121] B. Golden, Z. Naji-Azimi, S. Raghavan, M. Salari, and P. Toth, "The generalized covering salesman problem," *INFORMS Journal on Computing*, vol. 24, pp. 534–553, 2012.
- [122] B. Li, D. Krushinsky, H. Reijers, and T. Van Woensel, "The share-a-ride problem: People and parcels sharing taxis," *European Journal of Operational Research*, vol. 238, no. 1, pp. 31–40, 2014.
- [123] E. Angelelli, C. Archetti, and M. Vindigni, "The clustered orienteering problem," *European Journal of Operational Research*, vol. 238, pp. 404–414, 2014.
- [124] A.-E. Yahiaoui, A. Moukrim, and M. Serairi, "The clustered team orienteering problem," *Computers & Operations Research*, vol. 111, pp. 386–399, 2019.
- [125] L. Morán-Mirabal, J. L. Gonzalez-Velarde, and M. C. Resende, "Randomized heuristic for family travelling salesman problem," *International Transactions in Operational Research*, vol. 21, pp. 41–57, 2014.
- [126] F. G. Şahinyazan, B. Y. Kara, and M. R. Taner, "Selective vehicle routing for a mobile blood donation system," *European Journal of Operational Research*, vol. 245, no. 1, pp. 22–34, 2015.
- [127] B. Balcik, "Selective routing for post-disaster needs assessments," in *Dynamics of Disasters—Key Concepts, Models, Algorithms, and Insights* (I. S. Kotsireas, A. Nagurney, and P. M. Pardalos, eds.), (Cham), pp. 15–36, Springer International Publishing, 2016.

- [128] B. Balcik, “Site selection and vehicle routing for post-disaster rapid needs assessment,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 101, pp. 30–58, 2017.
- [129] C. Archetti, F. Carrabs, and R. Cerulli, “The set orienteering problem,” *European Journal of Operational Research*, vol. 267, no. 1, pp. 264–272, 2018.
- [130] C. Obreque, G. Paredes-Belmar, P. Miranda, G. Campuzano, and G. Gutiérrez-Jarpa, “The generalized median tour problem: Modeling, solving and an application,” *IEEE Access*, vol. 8, pp. 178097–178107, 2020.
- [131] O. Arslan, “The location-or-routing problem,” *Transportation Research Part B: Methodological*, vol. 147, pp. 1–21, 2021.
- [132] A. Di Placido, C. Archetti, C. Cerrone, and B. Golden, “The generalized close enough traveling salesman problem,” *European Journal of Operational Research*, vol. 310, no. 3, pp. 974–991, 2023.
- [133] I. Ozdemir, C. F. Dursunoglu, B. Y. Kara, and M. Dora, “Logistics of temporary testing centers for coronavirus disease,” *Transportation Research Part C: Emerging Technologies*, vol. 145, p. 103954, 2022.
- [134] Endeksa, “Defne İlçesi demografik analiz raporu,” 2025. <https://www.endeksa.com/tr/analiz/turkiye/hatay/defne/demografi> Access date: September 15, 2025.
- [135] F. Kılıcı, B. Y. Kara, and B. Bozkaya, “Locating temporary shelter areas after an earthquake: A case for Turkey,” *European Journal of Operational Research*, vol. 243, no. 1, pp. 323–332, 2015.
- [136] R. Kian, M. H. Demir, G. Erdogan, B. Kara, S. Salman, S. de Leeuw, and E. Sabet, “Distribution network design for syrian refugees in kilis optimising cost and reach: parameters and solutions.” Mendeley Data, V3, 2021.

- [137] S. Bland, E. Farinaro, F. Jossa, and M. Trevisan, “Long-term psychological effects of natural disasters,” *Psychosomatic*, vol. 58, no. 1, pp. 18–24, 1996.
- [138] J. Cénat and D. Derivois, “Long-term outcomes among child and adolescent survivors of the 2010 haitian earthquake,” *Depress Anxiety*, vol. 32, no. 1, pp. 57–63, 2015.
- [139] J. Cénat, S. McIntee, and C. Blais-Rochette, “Symptoms of posttraumatic stress disorder, depression, anxiety and other mental health problems following the 2010 earthquake in haiti: A systematic review and meta-analysis,” *Journal of Affective Disorders*, vol. 273, pp. 55–85, 2020.
- [140] F. M. Kürkcü, “Kahramanmaraş’a giden yardım tırlarının boşaltılmadığı iddiasıyla ilgili açıklama,” 2023. <https://www.aa.com.tr/tr/gundem/kahraman-marasa-giden-yardim-tirlarinin-bosaltilmadigi-iddiasıyla-ilgili-aciklama/2815818> Access date: September 15, 2025.
- [141] N. Megiddo, E. Zemel, and S. L. Hakimi, “The maximum coverage location problem,” *SIAM Journal on Algebraic Discrete Methods*, vol. 4, no. 2, pp. 253–261, 1983.
- [142] O. Karasakal and E. K. Karasakal, “A maximal covering location model in the presence of partial coverage,” *Computers & Operations Research*, vol. 31, no. 9, pp. 1515–1526, 2004.
- [143] Presidency of Strategy & Budget of the Republic of Türkiye, “Kahramanmaraş and Hatay Earthquakes Report,” 2023. Access date: September 15, 2025.
- [144] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*. A Series of Books in the Mathematical Sciences, San Francisco, Calif.: W. H. Freeman and Co., 1979.

- [145] T. L. Magnanti and R. T. Wong, “Accelerating Benders decomposition: Algorithmic enhancement and model selection criteria,” *Operations Research*, vol. 29, no. 3, pp. 464–484, 1981.
- [146] Hatay Büyükşehir Belediyesi, “Hatay Afet sonrası Planlama Çalışmaları: Analitik Etüt Raporu,” 2023. Access date: September 15, 2025.
- [147] AFAD, “06 Şubat 2023 Kahramanmaraş (Pazarcık ve Elbistan) Depremleri Saha Çalışmaları Ön Değerlendirme Raporu,” 2023. https://deprem.afad.gov.tr/assets/pdf/Arazi_Onrapor_28022023_surum1_revize.pdf Access date: September 15, 2025.

Appendix A

Proof of Proposition 1

Proof. We will construct an instance of the OTDP from an instance of the MCLP by the following polynomial-time transformations. For MCLP, consider an arbitrary instance of the graph G' where vertices denote the set of customers and potential sites for the facilities. Let w_i denote the demand of customer $i \in V'$, and if $d_{ij} \leq \beta$ for $i \in V', j \in H'$, then w_i is covered. At most p facilities can be located at the candidate locations, and MCLP is known to be NP-Hard [141]. Now consider an instance of OTDP $\langle G = (V, A), T = \{1\}, |M| = \emptyset, b_{mi} = 0, b_{fi} = w_i, \lambda^m = 0, \lambda^f = \beta, m_f = p, \bar{c} = 0, \underline{c} = 0 \rangle$. Let OTDP graph be exactly MCLP graph $G = (V, A) := (V', A')$. We set $T = \{1\}$ to eliminates mobility over time, $|M| = 0, b_{mi} = 0, \lambda^m = 0$ and $\bar{c} = \underline{c} = 0$ to ensure that MSUs play no role. Then, also set $b_{fi} = w_i, \lambda^f = \beta, m_f = p, c_{ij} = d_{ij} \forall i, j \in V$ and $Q = P$. This construction clearly requires time polynomial in $|V'|$.

If MCLP has a solution, then OTDP must also have a solution. Assume there exists a set $H' \subseteq V'$ with $|H'| = p$ such that $\sum_{i: \exists j \in H' \atop d_{ij} \leq \beta} w_i \geq P$. We construct an OTDP solution by locating CDs exactly at the nodes in H' . Also note that since $|M| = 0$, the MSU allocation and durations are no longer relevant to the problem.

Then we have the following: A node i is covered in OTDP if and only if there exists a CD $j \in H'$ with $d_{ij} \leq \lambda^f = \beta$. The total demand satisfied in OTDP equals $\sum_{i: \exists j \in H' \atop c_{ij} \leq \lambda^f} b_{fi} = \sum_{i: \exists j \in H' \atop d_{ij} \leq \beta} w_i \geq P = Q$. Note that all other constraints are trivially satisfied. Thus, OTDP has a solution.

Next, we show the reverse direction. Assume the constructed OTDP instance has a feasible solution with total demand satisfied at least Q . Note that we have $|M| = 0$, which eliminates all MSUs and allows coverage to be satisfied through CDs. Let $H' = \{j \in V : y_j = 1\}$ and observe that $|H'| = m_f = p$. Thus, H' is a valid MCLP facility set. In the OTDP instance, a node i is covered if and only if there exists $j \in H'$ such that $c_{ij} \leq \lambda^f = \beta$. Thus, the total demand satisfied can be written as follows:

$$\sum_{i: \exists j \in H' \atop d_{ij} \leq \beta} w_i = \sum_{i: \exists j \in H' \atop c_{ij} \leq \lambda^f} b_{fi} \geq Q = P.$$

H' is a feasible solution for the MCLP instance.

Therefore, two problems are equivalent. Since the reduction is polynomial and correctness holds in both directions, OTDP is NP-hard. $\square$

Appendix B

Proof of Lemma 1

Proof. The equivalences between the decision variables from Model 1 (z_{ijk} and u_{nijk}) and Model 2 (q_{ijkt}) can be demonstrated by considering variable definitions and how time periods are segmented. Note that z_{ijk} represents the total time duration of MSU k at district j departing from CD i . Also, q_{ijkt} denotes whether a MSU k stays at district j at time period t . The summation of q_{ijkt} over all possible time periods $t \in T$, gives the definition of variable z_{ijk} and the first equation provides it: $z_{ijk} = \sum_{t \in T} q_{ijkt} \quad \forall i, j \in V, k \in M$.

Furthermore, u_{nijk} denotes the duration of MSU k 's stay at district j within the specific time segment between δ_{n-1} and δ_n . We have defined the set T_n as the set of time periods after time segment δ_{n-1} until the end of the planning horizon. It implies $T_n = \{\delta_{n-1} + 1, \dots, |T|\}$ and similarly we have $T_{n+1} = \{\delta_n + 1, \dots, |T|\}$. Next, let us subtract the cumulative sum of q_{ijkt} values from $\delta_n + 1$ onwards (which is $\sum_{t \in T_{n+1}} q_{ijkt}$) from the cumulative sum of q_{ijkt} values from $\delta_{n-1} + 1$ onwards (which is $\sum_{t \in T_n} q_{ijkt}$). We will derive the duration of stay exclusively within the segment

from $\delta_{n-1} + 1$ to δ_n , and we have the following:

$$\left(\sum_{t \in T_n} q_{ijkt} \right) - \left(\sum_{t \in T_{n+1}} q_{ijkt} \right) = \left(\sum_{t=\delta_{n-1}+1}^{|T|} q_{ijkt} \right) - \left(\sum_{t=\delta_n+1}^{|T|} q_{ijkt} \right).$$

It simplifies to summing q_{ijkt} only for the time periods $\{\delta_{n-1} + 1, \dots, \delta_n\}$ and it corresponds to the definition of u_{nijk} . Therefore, the second equality also holds:

$$u_{nijk} = \sum_{t \in T_n} q_{ijkt} - \sum_{t \in T_{n+1}} q_{ijkt} \quad \forall i, j \in V, k \in M, n \in \mathcal{N} \setminus \{0\}.$$

□

Appendix C

Proof of Lemma 2

Proof. Based on the definition of q_{ijkt} , we can calculate the total duration and denote it as D_{ijk} representing total number of periods that a MSU k spends at district j :

$$D_{ijk} = \sum_{t \in T} q_{ijkt} \quad \forall i, j \in V, k \in M.$$

By the definition of r_{ijkt} , we can observe that r_{ijkt} will be one if and only if the total duration D_{ijk} is exactly equal to t . Then we can mathematically rewrite it as follows:

$$r_{ijkt} = \begin{cases} 1 & \text{if } D_{ijk} = t \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j \in V, k \in M, t \in T.$$

Further, we have an additional restriction which enforces $r_{ijkt} = 1$ for at most one t period. Also, it can be noted that in the following summation $\sum_{t \in T} tr_{ijkt}$, only the term t contributes to the summation if its corresponding multiplier r_{ijkt} is equal to one. Further, it can be noted that $t = D_{ijk}$. Therefore:

$$\sum_{t \in T} tr_{ijkt} = D_{ijk} r_{ijk, D_{ijk}}.$$

Since $r_{ijk,D_{ijk}}$ is 1 by definition (as D_{ijk} is indeed equal to D_{ijk}), we get:

$$\sum_{t \in T} tr_{ijk t} = D_{ijk} 1 = D_{ijk}.$$

Finally, substituting our initial definition of D_{ijk} back into equation, we get the proposed equality:

$$\sum_{t \in T} tr_{ijk t} = \sum_{t \in T} q_{ijk t} \quad \forall i, j \in V, k \in M.$$

□



Appendix D

Proof of Proposition 2

Proof. We will construct an instance of the CDSP from an instance of the MCLP using the following polynomial-time transformations. For MCLP, consider an arbitrary instance of the graph G' where vertices denote the set of customers and potential sites for the facilities. Let w_i denote the demand of customer $i \in V'$, and if $d_{ij} \leq \beta$ for $i \in V', j \in H'$, then w_i is covered. At most p facilities can be located at the candidate locations, and MCLP is known to be NP-Hard [141]. Now consider an instance of CDSP $\langle G = (V, A), b_{mi} = w_i, \lambda^h = \beta, \lambda^m = 0, \lambda^l = 0, m_h = p, m_m = 0, m_l = 0 \rangle$. Let CDSP graph be exactly MCLP graph $G = (V, A) := (V', A')$. We set $\lambda^m = \lambda^l = 0$ and $m_m = m_l = 0$ to reduce the type of MSU to a single type. We also set $b_{mi} = w_i, \lambda^h = \beta, m_h = p, c_{ij} = d_{ij} \forall i, j \in V$ and $Q = P$. This construction clearly requires time polynomial in $|V'|$.

If MCLP has a solution, then CDSP must also have a solution. Assume there exists a set $H' \subseteq V'$ with $|H'| = p$ such that $\sum_{i: \exists j \in H' \atop d_{ij} \leq \beta} w_i \geq P$. We construct a CDSP solution by locating high-capacity MSUs exactly at the nodes in H' . Also, since $\lambda^m = \lambda^l = 0$ and $m_m = m_l = 0$, medium- and low-capacity MSUs are no longer relevant to the problem. Then we have the following: A node i is covered in CDSP if and only if

there exists a high-capacity MSU $j \in H'$ with $d_{ij} \leq \lambda^h = \beta$. The total demand satisfied in CDSP equals $\sum_{\substack{i: \exists j \in H' \\ c_{ij} \leq \lambda^h}} b_{mi} = \sum_{\substack{i: \exists j \in H' \\ d_{ij} \leq \beta}} w_i \geq P = Q$. Note that all other constraints are trivially satisfied. Thus, CDSP has a solution.

Next, we show the reverse direction. Assume the constructed CDSP instance has a feasible solution with total demand satisfied at least Q . Note that we have $\lambda^m = \lambda^l = 0$ and $m_m = m_l = 0$, which eliminates medium- and low-capacity MSUs, allowing coverage to be satisfied through only high-capacity MSUs. Let $H' = \{j \in V : v_j^h = 1\}$ and observe that $|H'| = m_h = p$ since exactly p high-capacity MSUs exist. Thus, H' is a valid MCLP facility set. In the CDSP instance, a node i is covered if and only if there exists $j \in H'$ such that $c_{ij} \leq \lambda^h = \beta$. Thus, the total demand satisfied can be written as follows:

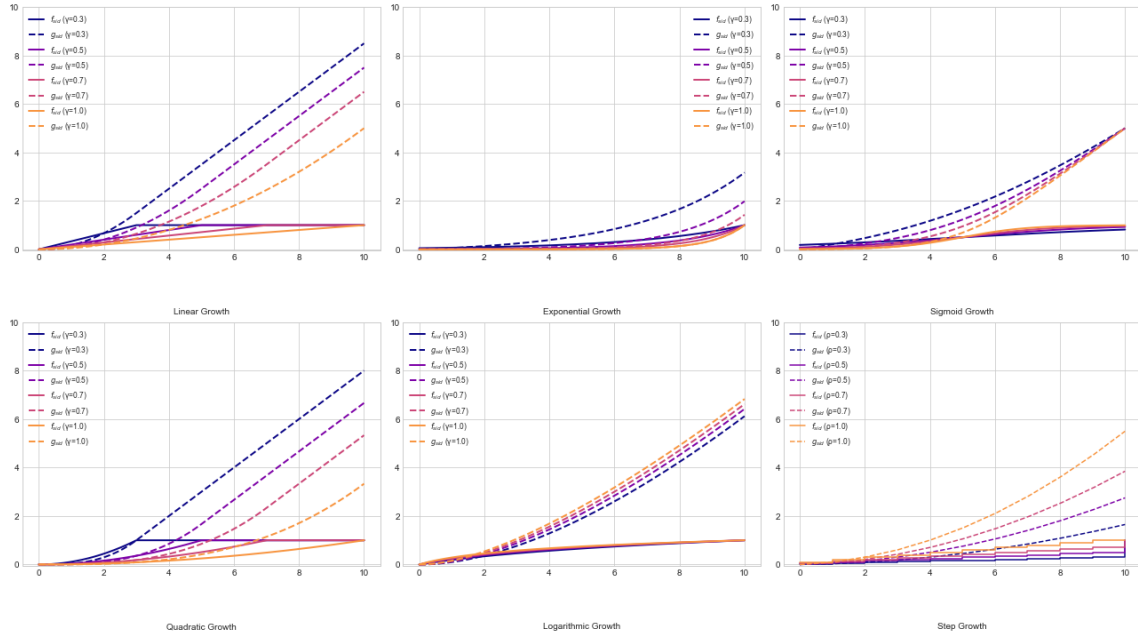
$$\sum_{\substack{i: \exists j \in H' \\ d_{ij} \leq \beta}} w_i = \sum_{\substack{i: \exists j \in H' \\ c_{ij} \leq \lambda^h}} b_{mi} \geq Q = P.$$

H' is a feasible solution for the MCLP instance.

Therefore, two problems are equivalent. Since the reduction is polynomial and correctness holds in both directions, CDSP is NP-hard. $\square$

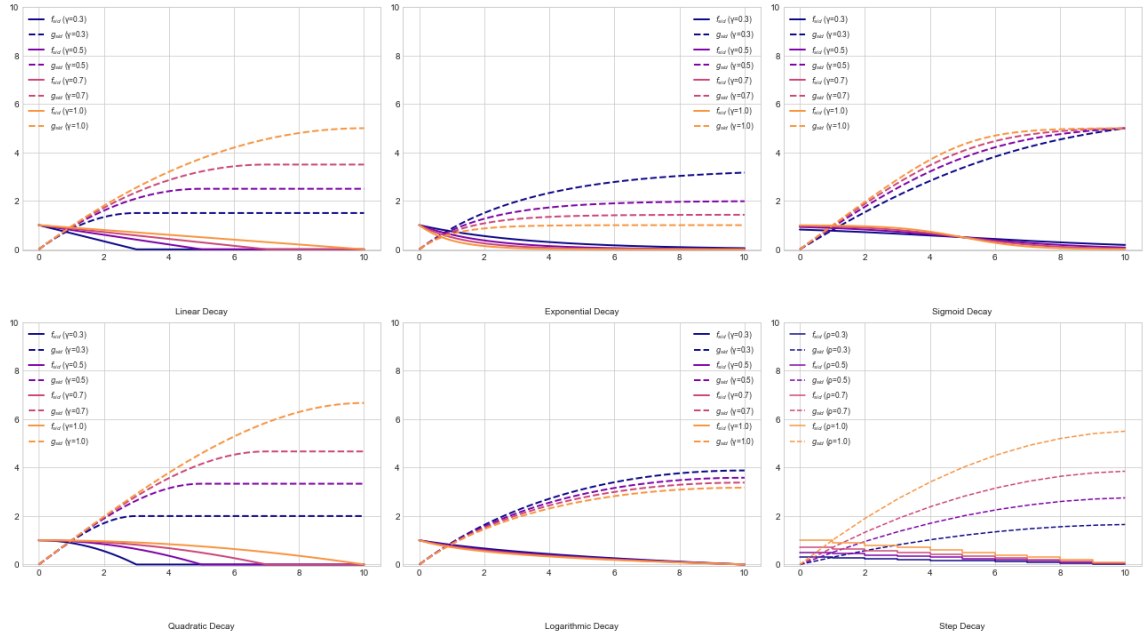
Appendix E

Demand Functions



(a) Growth functions f_{sid} and g_{sid}

Figure E.1: Comparison of different growth functions and their cumulative versions with $\hat{p}_{si} = 1$ for simplicity



(b) Decay functions f_{sid} and g_{sid}

Figure E.1: Comparison of different growth functions and their cumulative versions with $\hat{p}_{si} = 1$ for simplicity (continued)

Appendix F

Proof of Proposition 3

Proof. In order to show that PDSP is NP-Hard, we will construct an instance of the PDSP from an instance of the Bin Packing Problem by the following polynomial time transformations. Additionally, we assume, without loss of generality, that the data is polynomially bounded. Even under this assumption, the Bin Packing Problem is known to be strongly NP-hard. We will provide a transformation under the polynomially bounded data. For the Bin Packing Problem, consider an arbitrary instance of a finite set I of items, a size $s(i) \in \mathbb{Z}^+$ for each $i \in I$, a positive integer bin capacity B , and a positive integer K . I is partitioned into at most K subsets $I_1, \dots, I_K$ such that $\sum_{i \in I_j} s(i) \leq B$ for each $j = 1, \dots, K$. Now consider an instance of PDSP $\langle G = (I, A), T = \{1, \dots, B\}, S = \{s\}, m_s = K, \ell_{si} = 1, \ell_{si}^{\max} = 1, \underline{c}_s = 1, \overline{c}_s = B, e_s = 0, h_{it} = 1, g_{sid} = s(i) \text{ if } d = s(i) \rangle$. Let the set of nodes V in PDSP be exactly the set of items I in the Bin Packing Problem, and the arc set A can be arbitrary. Note that the transportation network will not play any role in the reduction. Also, set $T = \{1, \dots, B\}$, where the time horizon contains exactly B periods, which corresponds to the bin capacity. We also set a single service type $S = \{s\}$ with $h_{it} = 1$ and $m_s = K$. Since there is only one service type, each node

can receive at most one service type simultaneously through h_{it} . There are also m_s MSUs available, which correspond to K bins in the Bin Packing instance. Also, we set $\ell_{si} = 1, \ell_{sv_i}^{\max} = 1 \forall i \in I$ to ensure that each item can be scheduled exactly once with visit durations $\underline{c}_s = 1, \overline{c}_s = B$. We do not require any elapsed time between consecutive visits, so it can be zero by setting $e_s = 0$. Let each visit contribute as follows: Let $g_{sid} = \begin{cases} s(i) & \text{if } d = s(i) \\ 0 & \text{otherwise} \end{cases}$. If the Bin Packing Problem has a solution, then PDSP has a solution. Suppose the items are partitioned into at most K bins $I_1, \dots, I_K$ such that each bin I_j satisfies $\sum_{i \in I_j} s(i) \leq B$. For each bin j , we can assign one MSU of service type $s \in S$. Then, there will be a schedule of visits to the corresponding nodes $i \in I$ for consecutive periods $s(i)$. Since $\overline{c}_s = B$ and total demand satisfied will be $\sum_{i \in I} \sum_{s \in S} \sum_{d \in \hat{D}} g_{sid} = \sum_{i \in I} s(i)$, the PSDP schedule is feasible. If PDSP has a solution, then the Bin Packing Problem has a solution. A feasible PDSP solution must visit each node exactly once ($\ell_{si} = \ell_{sv_i}^{\max} = 1$). Additionally, the durations of each visit are added to the summation if they have a length of $s(i)$ through one of the m_s MSUs. Let I_j denote the set of items whose nodes are visited by service unit $j \in \{1, \dots, K\}$. The planning horizon is given B periods, which implies that the total duration of MSUs cannot exceed B , $\sum_{i \in I_j} s(i) \leq B$. Then, the partition $I_1, \dots, I_K$ is a feasible solution to the Bin Packing Problem. Therefore, two problems are equivalent. Since the reduction is polynomial and correctness holds in both directions, PDSP is NP-hard. $\square$

Appendix G

Proof of Lemma 3

Proof. For a given $s \in S, i \in N, k \in K$, the set D^{act} and the function π , we need to prove that the objective function term satisfies $\sum_{t \in T} \sum_{d \in D^{\text{act}}} f_{sid} r_{siktd} = \sum_{t \in T} \sum_{d \in D^{\text{act}}} f_{si\pi(d)} r_{siktd}$. In other words, we need to prove that reordering the objective function coefficients by reordering of the index d keep the value of objective function the same. For a given k^{th} visit to node i by service s starts at period t_0 with duration $d_{\max}$, assume that we have the following feasible solution for binary r variables:

$$r_{sikt_0 d_{\max}} = 1, \quad r_{sik(t_0+1)(d_{\max}-1)} = 1, \quad \dots, \quad r_{sik(t_0+d_{\max}-1)1} = 1$$

and all other $r_{siktd} = 0$. Then, we analyze the following summations:

$$\sum_{d \in D^{\text{act}}} \sum_{t \in T} f_{sid} r_{siktd} = f_{sid_{\max}} + f_{si(d_{\max}-1)} + \dots + f_{si1}$$

and

$$\sum_{d \in D^{\text{act}}} \sum_{t \in T} f_{si\pi(d)} r_{siktd} = f_{si\pi(d_{\max})} + f_{si\pi(d_{\max}-1)} + \dots + f_{si\pi(1)}$$

Note that each active remaining duration index $d \in D^{\text{act}}$ is assigned exactly for a unique $t \in T$. Then, each term $f_{sid} r_{siktd}$ appears exactly once on the first summation

$(\sum_{d \in D^{\text{act}}} \sum_{t \in T} f_{sid} r_{sikt d})$. Further, π is a bijective function defined for reversing the order of d indices. Then, the second summation also contains the same set of terms but with the indices relabeled. Thus, the following holds:

$$\pi(d_{\max}) = 1, \quad \pi(d_{\max} - 1) = 2, \quad \dots, \quad \pi(1) = d_{\max}$$

Consequently, we observe that the summations becomes: $f_{sid_{\max}} + f_{si(d_{\max}-1)} + \dots + f_{si1} = f_{si1} + f_{si2} + \dots + f_{sid_{\max}}$. Note that the equality holds for each $s \in S, i \in V, k \in K$, and the objective function is a linear function, the equality holds for all $s \in S, i \in V, k \in K$ with feasible $r_{sikt d}$ variables.

□

Appendix H

Proof of Lemma 4

Proof. We prove the provided relation in Lemma 4 by considering two cases:

- Case 1: $t = 1$

$t = 1$ is the starting period and there is no previous period to consider, so the service period starting at beginning period cannot overlap with any previous service. Also this aligns with the case $|T| = 1$.

$$x_{si1d} = \sum_{k \in K} r_{sik1d} \quad i \in V, s \in S, d \in \hat{D}.$$

- If there exists a k such that $r_{sik1d} = 1$, then $x_{si1d} = 1$. It implies that indicating that service s starts at node i on period $t = 1$ with duration d .
 - If $r_{sik1d} = 0$ for all k , then $x_{si1d} = 0$ which implies that there is no service starting at $t = 1$.
- Case 2: $t \in \{2, \dots, |T|\}$

On period $t > 1$, the service period starting at t should not overlap with a

service period continuing at $t - 1$.

$$x_{sitd} = \sum_{k \in K} r_{siktd} (1 - q_{sik(t-1)}) \text{ if } t \in \{2, \dots, |T|\} \quad i \in V, s \in S, d \in \hat{D}.$$

- If $r_{siktd} = 1$ and $q_{sik(t-1)} = 0$ ($(1 - q_{sik(t-1)}) = 1$), then $x_{sitd} = 1$. There is a service s at node i on period t with duration d , and there is no overlapping service from the previous period. Therefore, period t is the starting period.
- If $r_{siktd} = 1$ and $q_{sik(t-1)} = 1$ ($(1 - q_{sik(t-1)}) = 0$), then $x_{sitd} = 0$. The service at t overlaps with a service from the previous period, so it cannot be the beginning period for the service.
- If $r_{siktd} = 0$ and $q_{sik(t-1)} = 1$ ($(1 - q_{sik(t-1)}) = 0$), then $x_{sitd} = 0$. There is no service at t , but there is a service from the previous period. It means that the service has ended in the previous period therefore period t cannot be the beginning period.
- If $r_{siktd} = 0$ and $q_{sik(t-1)} = 0$ ($(1 - q_{sik(t-1)}) = 1$), then $x_{sitd} = 0$. There is no service at t , and there is no service from the previous period. It means there is no beginning, ending or continuing service on period t .

□

Appendix I

Proof of Lemma 5

Proof. We prove the provided relation in Lemma 5 as follows:

- By constraints (6.3), there can be at most one k where $q_{sikt} = 1$ and $\sum_{k=1}^{\ell_{si}^{\max}} q_{sikt} = 1$. Let $q_{sikt} = 1$ for some k^{th} occurrence of service s , then node i is visited on period t . Note that the visit must be part of an ongoing service that started at some $\hat{t} \leq t$ with duration $d \geq t - \hat{t} + 1$. Thus, by definition of x variable, there exist $x_{si\hat{t}d} = 1$ for some $\hat{t} \leq t$ and $d \geq t - \hat{t} + 1$.
- For a given t , $\sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{si\hat{t}d}$ ensures that only start times $\hat{t}$ where $t \in [\hat{t}, \hat{t} + d - 1]$ are considered. Also, $\sum_{d=c_s}^{\bar{c}_s}$ considers all possible service durations. Therefore, if $x_{si\hat{t}d} = 1$, then service s starts at node i on period $\hat{t}$ with duration d . This implies visits on periods $\hat{t}, \hat{t} + 1, \dots, \hat{t} + d - 1$. By Lemma 4 and constraints (6.6), for each such period $t \in [\hat{t}, \hat{t} + d - 1]$, there exists some k such that $r_{siktd} = 1$ and $q_{sikt} = 1$.

□

Appendix J

Proof of Lemma 6

Proof. We prove the provided relation in Lemma 6 as follows:

- In model M-5i, $v_{sik} = 1$ indicates that the k^{th} visit to node i by service s occurs. Constraints (6.10) counts the total number of distinct visits and there can be at least l_{si} values where $v_{sik} = 1$.
- In model M-4i, each variable $x_{sitd} = 1$ represents a distinct visit starting at period t with duration d . For each visit k with $v_{sik} = 1$ in M-5i, there exists exactly one starting period t and duration d . This corresponds to exactly one $x_{sitd} = 1$ in M-4i. Conversely, each $x_{sitd} = 1$ in M-4i represents a unique visit that would be counted by exactly one $v_{sik} = 1$ in M-5i.

□

Appendix K

Proof of Proposition 4

Proof. In the first part, we assume that (r, q, v) is a feasible integer solution to M-5i. So, (r, q, v) satisfies all constraints of M-5i and we will show that the corresponding solution x , defined by the previous lemmas, satisfies all constraints of M-4i. In the second part, we assume that x is an feasible integer solution for M-4i, then we will show that the corresponding solution (r, q, v) satisfies all constraints of M-5i. We start with the first part as follows:

- If $\sum_{i \in V} \sum_{k \in K} q_{sikt} \leq m_s \quad s \in S, t \in T$ then $\sum_{i \in V} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{si\hat{t}d} \leq m_s \quad s \in S, t \in T$. Lemma 4 is used to show the following:

$$\begin{aligned}
 m_s &\geq \sum_{i \in V} \sum_{k \in K} q_{sikt} \geq \sum_{i \in V} \sum_{k \in K} \left(\sum_{\hat{t}=1}^t q_{sik\hat{t}} (1 - q_{sik(\hat{t}-1)}) \right) \geq \\
 &\sum_{i \in V} \sum_{k \in K} \left(\sum_{d=\max\{\underline{c_s}, t-1+1\}}^{\overline{c_s}} r_{sik1d} + \sum_{\hat{t}=2}^t \sum_{d=\max\{\underline{c_s}, t-\hat{t}+1\}}^{\overline{c_s}} r_{sik\hat{t}d} (1 - q_{sik(\hat{t}-1)}) \right) = \\
 &\sum_{i \in V} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{si\hat{t}d} \quad s \in S, t \in T
 \end{aligned}$$

- If $\sum_{k \in K} v_{sik} \geq \ell_{si}$ $i \in V, s \in S$ then $\sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c}_s\}}^{\min\{|T|-t+1, \overline{c}_s\}} x_{situd} \geq \ell_{si}$ $i \in V, s \in S$.

Lemma 6 is used to show following:

$$\ell_{si} \leq \sum_{k \in K} v_{sik} = \sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c}_s\}}^{\min\{|T|-t+1, \overline{c}_s\}} x_{situd} \quad i \in V \quad s \in S.$$

- If $\sum_{k \in K} v_{sik} \leq \ell_{si}^{max}$ $i \in V, s \in S$ then $\sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c}_s\}}^{\min\{|T|-t+1, \overline{c}_s\}} x_{situd} \leq \ell_{si}^{max}$ $i \in V, s \in S$.

Lemma 6 is used to show following:

$$\ell_{si}^{max} \geq \sum_{k \in K} v_{sik} = \sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c}_s\}}^{\min\{|T|-t+1, \overline{c}_s\}} x_{situd} \quad i \in V \quad s \in S.$$

- We prove that if the inequality

$$t' q_{si(k+1)t'} - (t+1) q_{sikt} + 2|T|(2 - q_{si(k+1)t'} - q_{sikt}) \geq e_s \quad (\text{K.1})$$

holds for $i \in V, s \in S, k \in K \setminus \{\ell_{si}^{max}\}$, and $t, t' \in T$ with $t' > t$, then

$$\sum_{d=\underline{c}_s}^{\overline{c}_s} \sum_{\hat{t}=\max\{1, t-e_s-d+1\}}^t x_{si\hat{t}d} \leq 1 \quad i \in V, s \in S, t \in T. \quad (\text{K.2})$$

Assume to contrary that there exists $i \in V, s \in S, t_0 \in T$ such that

$$\sum_{d=\underline{c}_s}^{\overline{c}_s} \sum_{\hat{t}=\max\{1, t_0-e_s-d+1\}}^{t_0} x_{si\hat{t}d} \geq 2, \quad (\text{K.3})$$

In other words, there exists some i, s, t_0, d such that there are at least two distinct services being active in periods $[\max\{1, t_0 - e_s - d + 1\}, t_0]$. We denote each such x values as distinct visits with start times t_1, t_2 with given duration d_1, d_2 . Without loss of generality, we can assume $t_1 \leq t_2$. For those given non-zero x variables, we have $x_{sit_1d_1} = 1$ and $x_{sit_2d_2} = 1$ such that $t_1 \in [\max\{1, t_0 -$

$e_s - d_1 + 1\}$, $t_0]$ and $t_2 \in [\max\{1, t_0 - e_s - d_2 + 1\}, t_0]$. Then, we have $\max\{1, t_0 - e_s - d_1 + 1\} \leq t_1$ and $t_2 \leq t_0$. From active periods, we derive $t_2 - t_1 \leq \min\{e_s + d_1 - 1, t_0 - 1\}$. Further, we consider last service day for service starting at t_1 with duration d_1 to find service gap relation between those services and derive the following $t_2 - t_1 - d_1 \leq \min\{e_s - 1, t_0 - d_1 - 1\}$. However, it can be easily noted that $t_2 - t_1 - d_1 < e_s$. In M-5i, we consider the consecutive visits. Note that if $t_2 \leq t_1 + d_1 - 1$, then the visits overlap, which violates constraint (6.14) in M-5i. If $t_2 > t_1 + d_1 - 1$, let k^{th} visit start at t_1 and let $k + 1^{th}$ visit start at t_2 . From constraints (6.6)–(6.9), the visit represented by q is adjacent. If $q_{sikt_1} = 1$ then the visit occurs in consecutive periods $t_1, \dots, t_1 + d_1 - 1$. Similarly, if $q_{si(k+1)t_2} = 1$ then the visit occurs in consecutive periods $t_2, \dots, t_2 + d_2 - 1$. For $t = t_1 + d_1 - 1$ (last period of k^{th} visit) and $t' = t_2$ (first period of $k + 1^{th}$ visit), we have $q_{sikt} = 1$ and $q_{si(k+1)t'} = 1$. The gap between the end of k^{th} visit and the start of $(k + 1)^{th}$ visit is $t_2 - (t_1 + d_1)$ and the constraints (K.1) enforces $t_2 - (t_1 + d_1) \geq e_s$. However, it contradicts with the previous result $t_2 - t_1 - d_1 < \min\{e_s, t_0 - d_1\}$ from M-4i. This contradiction proves that our assumption (K.3) is false, thus (K.2) is correct.

- If $\sum_{s \in S} \sum_{k \in K} q_{sikt} \leq h_{it} \quad i \in V, t \in T$ then $\sum_{s \in S} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{si\hat{t}d} \leq h_{it} \quad i \in V, t \in T$. Lemma 4 is used to show the following:

$$\begin{aligned}
h_{it} &\geq \sum_{s \in S} \sum_{k \in K} q_{sikt} \geq \sum_{s \in S} \sum_{k \in K} \left(\sum_{\hat{t}=1}^t q_{sik\hat{t}} (1 - q_{sik(\hat{t}-1)}) \right) \geq \\
&\sum_{s \in S} \sum_{k \in K} \left(\sum_{d=\max\{\underline{c_s}, t-1+1\}}^{\overline{c_s}} r_{sik1d} + \sum_{\hat{t}=2}^t \sum_{d=\max\{\underline{c_s}, t-\hat{t}+1\}}^{\overline{c_s}} r_{sik\hat{t}d} (1 - q_{sik(\hat{t}-1)}) \right) = \\
&\sum_{s \in S} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{si\hat{t}d} \quad i \in V, t \in T.
\end{aligned}$$

- The remaining constraints in M-5i ensure that x variables are defined properly via feasible solution (r, q, v) :

Constraints (6.3)–(6.6) collectively ensure that r and q variables are properly

defined to form a valid decreasing duration pattern with unique duration assignments. They prevent multiple start periods for a given k^{th} visit and prevent multiple duration assignments for the same visit at a given period t . They also ensure that r variables are defined properly with a decreasing duration index d while increasing period index t . Based on those constraints, the feasible r and q variables are used to define x variables through Lemma 4 while ensuring x variables to represent contiguous service blocks with unique start times and durations. Constraints (6.8) and (6.9) provide lower and upper service duration limits for each visit. Through Lemma 4, feasible x variables are ensured to provide service for a feasible range of d indices ($d \in \hat{D}$). Therefore, it maintains the feasibility in M-4i. Constraints (6.12) maintains consistency between visit indicators and service periods by linking q and v variables. They ensure that the transformations in Lemmas 4-6 preserve the correct occurrence of and counting of visits. Constraints (6.13) and (6.14) guarantee strict temporal ordering of visits without overlap. Then, the service periods by x variables don't temporally overlap. Finally, the domain restrictions (6.16)-(6.18) ensure that x variables defined through Lemma 4 are binary and satisfy $t + d - 1 \leq |T|$. The remaining duration indices in M-5i also cannot extend beyond the planning horizon.

- By Lemma 3, we have observed that the M-5i objective function equals $\sum_{d \in D^{act}} \sum_{t \in T} f_{sid} r_{siktd} = \sum_{d \in D^{act}} \sum_{t \in T} f_{si \pi(d)} r_{siktd}$ which aggregates the same marginal contributions across the visit duration. Thus the M-4i objective function $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t \in T} g_{sid} x_{sitd}$ equals the M-5i objective for the mapped solution.

Therefore, all M-4i constraints are satisfied by the x variables constructed from feasible r, q and v variables in M-5i.

Next, we assume that x is an feasible integer solution for M-4i, then we will show that the corresponding solution (r, q, v) satisfies all constraints of M-5i. Therefore, the second part is as follows:

- If $\sum_{i \in V} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t=\max\{1, t-d+1\}}^t x_{s\hat{i}d} \leq m_s \quad s \in S, t \in T$, then $\sum_{i \in V} \sum_{k \in K} q_{sikt} \leq m_s \quad s \in S, t \in T$.

$$m_s \geq \sum_{i \in V} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t=\max\{1, t-d+1\}}^t x_{s\hat{i}d} = \sum_{i \in V} \sum_{k \in K} q_{sikt} \quad s \in S, t \in T.$$

- If $\sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c_s}\}}^{\min\{|T|-t+1, \overline{c_s}\}} x_{s\hat{i}d} \geq \ell_{si} \quad i \in V, s \in S$ then $\sum_{k \in K} v_{sik} \geq \ell_{si} \quad i \in V, s \in S$. Lemma 6 is used to show following:

$$\ell_{si} \leq \sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c_s}\}}^{\min\{|T|-t+1, \overline{c_s}\}} x_{s\hat{i}d} = \sum_{k \in K} v_{sik} \quad i \in V \quad s \in S.$$

- If $\sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c_s}\}}^{\min\{|T|-t+1, \overline{c_s}\}} x_{s\hat{i}d} \leq \ell_{si}^{max} \quad i \in V, s \in S$ then $\sum_{k \in K} v_{sik} \leq \ell_{si}^{max} \quad i \in V, s \in S$. Lemma 6 is used to show following:

$$\ell_{si}^{max} \geq \sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c_s}\}}^{\min\{|T|-t+1, \overline{c_s}\}} x_{s\hat{i}d} = \sum_{k \in K} v_{sik} \quad i \in V \quad s \in S.$$

- If $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t=\max\{1, t-e_s-d+1\}}^t x_{s\hat{i}d} \leq 1 \quad i \in V, s \in S, t \in T$ then $t' q_{si(k+1)t'} - (t+1)q_{sikt} + 2|T|(2 - q_{si(k+1)t'} - q_{sikt}) \geq e_s \quad i \in V, s \in S, k \in \{1, \dots, \ell_{si}^{max} - 1\}$. Assume to the contrary that there exist $t, t' \in T$ (with $t' > t$), $k \in K \setminus \{\ell_{si}^{max}\}$ such that

$$t' q_{si(k+1)t'} - (t+1)q_{sikt} + 2|T|(2 - q_{si(k+1)t'} - q_{sikt}) < e_s. \quad (\text{K.4})$$

The constraint set (K.4) becomes $t' - (t+1) < e_s$ if both $q_{sikt} = 1$ and $q_{si(k+1)t'} = 1$. Let k^{th} visit start at time t_1 with duration d_1 , and $k+1^{th}$ visit start at time t_2 with duration d_2 . Without loss of generality, we can assume $t_1 \leq t_2$. Note that for a feasible solution, we should have $q_{sikt} = 1$ for all $t \in \{t_1, \dots, t_1 + d_1 - 1\}$ and $q_{si(k+1)t'} = 1$ for all $t' \in \{t_2, \dots, t_2 + d_2 - 1\}$. We have the following relations, $t \leq t_1 + d_1 - 1$ and $t' \geq t_2$. We can derive following relation between t and t' , $t_2 - (t_1 + d_1 - 1 + 1) \leq t' - (t+1) < e_s$. Simply, we

have $t_2 - t_1 - d_1 < e_s$. Consider t_1, d_1 in M-4i with constraint set (K.2) for some $t^* \in T$, we have $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t^*-e_s-d+1\}}^{t^*} x_{s\hat{t}d} \leq 1$ for k^{th} visit starting at time t_1 , so $x_{sit_1d_1} = 1$ is counted in this sum. For those given non-zero x variables, we have $t_1 \in [\max\{1, t^* - e_s - d_1 + 1\}, t^*]$ for some $t^* \in T$. From our assumption, we have $t_2 - t_1 - d_1 < e_s$ and $t_1 + d_1 + e_s > t_2$. We have $t_1 \leq t_2 \leq t_1 + d_1 + e_s - 1 \leq t^* + e_s + d_1 - 1$. It can be observed that $k + 1^{th}$ visit starts between the periods t_1 and $t_1 + d_1 + e_s - 1$. It simply implies that $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t^*-e_s-d+1\}}^{t^*} x_{s\hat{t}d} \geq 2$ and it contradicts our assumption that the constraint set (K.2) in M4-i holds.

- If $\sum_{s \in S} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{s\hat{t}d} \leq h_{it} \quad i \in V, t \in T$, then $\sum_{s \in S} \sum_{k \in K} q_{sikt} \leq h_{it} \quad i \in V, t \in T$.

$$h_{it} \geq \sum_{s \in S} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{s\hat{t}d} = \sum_{s \in S} \sum_{k \in K} q_{sikt} \quad i \in V, t \in T.$$

- The rest of the constraints ((6.3), (6.4), (6.5), (6.6), (6.8), (6.9), (6.10), (6.12), (6.13), (6.14)) are provided to ensure that the model prevent illegal visits and visit orders while ensuring link and consistency between r, q and v variables. They are satisfied by construction of the (r, q, v) solution from x .
- Given the construction of r and the equivalence of aggregated demand function we have $\sum_{d \in D} \sum_{t \in T} f_{sid} r_{sikt} = \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t \in T} g_{sid} x_{sikt}$. Therefore objective function value of the M-5i solution equals that of the M-4i solution.

Therefore, we have shown that any feasible solution to M-4i maps to a feasible solution to M-5i.

In summary, we have shown that every feasible solution in M-5i maps to a feasible solution in M-4i with identical objective value. Later, we have shown that every feasible solution in M-4i maps to a feasible solution in M-5i with identical objective value. Thus, M-4i is a valid reformulation of M-5i. $\square$

Appendix L

Proof of Proposition 5

Proof. Models M-3ic and M-4i share the same objective function (6.19), same variables x_{sitd} , and same constraints (6.20)–(6.25). M-3ic introduces additional variables y_{sit} and z_{sit} , along with the constraint (6.26)–(6.36). We need to show that (6.28)–(6.35) are valid inequalities by using the relation defined by constraints (6.26) and (6.27).

- By relation $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} = y_{sit} \quad i \in V, s \in S, t \in T$, we can easily show that if $\sum_{i \in V} y_{sit} \leq m_s \quad s \in S, t \in T$ then $\sum_{i \in V} \sum_d \sum_t x_{sitd} \leq m_s \quad s \in S, t \in T$. Hence (6.28) is a valid inequality.
- To prove that constraint set (6.29) is a valid inequality, we use both relations $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} = y_{sit} \quad i \in V, s \in S, t \in T$ and $\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} = z_{sit} \quad i \in V, s \in S, t \in T$. The constraint set (6.29) is written for all $t \in T$. Therefore, we can modify the constraint as follows:

$$y_{si(\max\{1, t-e_s\})} + \sum_{\hat{t}=\max\{1, t-e_s+1\}}^{\min\{|T|, t\}} z_{sit} \leq 1 \quad i \in V, s \in S, t \in T.$$

and by relations we have:

$$\begin{aligned} \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-e_s-d+1\}}^{\max\{1, t-e_s\}} x_{sitd} + \sum_{\hat{t}=\max\{1, t-e_s+1\}}^{\min\{|T|, t\}} \sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} \leq 1 \quad i \in V, s \in S, t \in T. \\ \rightarrow \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-e_s-d+1\}}^t x_{sitd} \leq 1 \quad i \in V, s \in S, t \in T. \end{aligned}$$

Therefore, constraints (6.29) is a valid inequality.

- By relation $\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} = z_{sit} \quad i \in V, s \in S, t \in T$, we can easily show that if $\ell_{si}^{max} \geq \sum_{t \in T} z_{sit} \geq \ell_{si} \quad i \in V, s \in S$ then $\ell_{si}^{max} \geq \sum_{t \in T} \sum_{d=\min\{|T|-t+1, \underline{c_s}\}}^{\min\{|T|-t+1, \overline{c_s}\}} x_{sitd} \geq \ell_{si} \quad i \in V, s \in S$. Hence (6.30) and (6.31) are valid inequalities.
- By relation $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} = y_{sit} \quad i \in V, s \in S, t \in T$, we can easily show that if $\sum_{s \in S} y_{sit} \leq h_{it} \quad i \in V, t \in T$ then $\sum_{s \in S} \sum_d \sum_{\hat{t}} x_{sitd} \leq h_{it} \quad i \in V, t \in T$. Hence (6.32) is a valid inequality.
- Similarly, we use both relations to show that constraint set (6.33) is valid inequality. We re-write it as follows:

$$\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t+\overline{c_s}-1-d+1\}}^t x_{sitd} \leq \sum_{\hat{t}=t}^{\min\{|T|, t+\overline{c_s}-1\}} \sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} \quad i \in V, s \in S, t \in T : t+\overline{c_s}-1 \leq |T|.$$

Note that the constraint set hold since under the condition $t + \overline{c_s} - 1 \leq |T|$, the first summation contains at most one x variable which is $x_{sit\overline{c_s}}$ that is also counted in the second summation. Also the second summation includes additional nonnegative terms which makes the inequality to hold.

- To show that constraint set (6.34) is valid inequality, we similarly use those relations. We re-write it as follows:

$$\sum_{\hat{t}=\max\{1, t-\underline{c_s}+1\}}^t \sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} \leq \sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} \quad i \in V, s \in S, t \in T.$$

Note that for a given d , due to condition $\hat{t} \geq t - d + 1$ on the second summation and $d \geq \underline{c}_s$, there are more start times than the first summation. Therefore, the second summation includes additional nonnegative terms which makes the inequality to hold.

- We re-write the constraint set (6.35) as follows:

$$\sum_{d=\underline{c}_s}^{\overline{c}_s} x_{si(|T|-d'+1)d} \leq 0 \quad i \in V, s \in S, d' \in \{1, \dots, \underline{c}_s - 1\}.$$

This is valid since for $d' \in \{1, \dots, \underline{c}_s - 1\}$, the given time $(|T| - d' + 1)$ is an illegal start time for any service with a duration of at least $\underline{c}_s$ periods. The illegal visits are prevented by definition of x variables with condition $t + d - 1 \leq |T|$ and constraint set (6.35) prevent such visit by taking summation equal to zero.

As a conclusion, we have shown that constraints (6.28)–(6.35) are valid inequalities. Therefore, M-3ic is a valid reformulation of M-4i. $\square$

Appendix M

Proof of Proposition 6

Proof. Let P be the polyhedron obtained by relaxing the integrality constraints on the variables x_{sitd} while keeping y_{sit} and z_{sit} binary. We have the following projections:

$$\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sit\hat{t}d} = y_{sit} \quad i \in V, s \in S, t \in T$$

$$\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sit\hat{t}d} = z_{sit}.$$

The constraints define the relation among x , y and z variables. Since $y_{sit} \in \{0, 1\}$ and $z_{sit} \in \{0, 1\}$, there are four cases. Let us consider each case separately:

- **Case 1:** $y_{sit} = 0, z_{sit} = 0$.

Having $y_{sit} = 0$ implies that $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sit\hat{t}d} = 0$. It means that $x_{sit\hat{t}d} = 0 \quad d \in \hat{D}, \hat{t} \in \{\max\{1, t-d+1\}, \dots, t\}$, implying no fractional values of x variables due to domain constraints.

Having $z_{sit} = 0$ implies that $\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sit\hat{t}d} = 0$. It means that $x_{sit\hat{t}d} = 0 \quad d \in \hat{D}$, implying no fractional values of x variables due to domain constraints.

- **Case 2:** $y_{sit} = 0, z_{sit} = 1$.

Having $y_{sit} = 0$ implies that $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t=\max\{1, t-d+1\}}^t x_{sitd} = 0$.

Having $z_{sit} = 1$ implies that $\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} = 1$. It should be noted that this is infeasible since $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t=\max\{1, t-d+1\}}^t x_{sitd} = 0$ but $\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} = 1$.

- **Case 3:** $y_{sit} = 1, z_{sit} = 0$.

Having $y_{sit} = 1$ and $z_{sit} = 0$ imply that $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t=\max\{1, t-d+1\}}^{t-1} x_{sitd} = 1$.

Define a feasible solution $\bar{x}$ by selecting exactly one index $(\hat{t}, d)$ in the above summation and setting $\bar{x}_{sitd} = 1$, while setting all remaining associated x -variables equal to zero. This assignment is binary and satisfies all constraints.

We claim that $(\bar{x}, y, z)$ is an extreme point of P . Suppose, to the contrary, that $\bar{x} = \lambda x^1 + (1 - \lambda)x^2$ for some $\lambda \in (0, 1)$ with two distinct feasible points (x^1, y, z) and (x^2, y, z) in P . Since $0 \leq x_{sitd} \leq 1$ for all variables, the equality $\bar{x}_{sitd} = 1$ implies $x_{sitd}^1 = x_{sitd}^2 = 1$, while $\bar{x}_{sit\tilde{d}} = 0$ implies $x_{sit\tilde{d}}^1 = x_{sit\tilde{d}}^2 = 0$ for all other indices. Thus, $x^1 = x^2 = \bar{x}$, contradicting their distinctness. Hence, $(\bar{x}, y, z)$ is an extreme point of P .

- **Case 4:** $y_{sit} = 1, z_{sit} = 1$.

Having $y_{sit} = 1$ and $z_{sit} = 1$ imply that $\sum_{d=\underline{c_s}}^{\overline{c_s}} \sum_{t=\max\{1, t-d+1\}}^{t-1} x_{sitd} = 0$ and $\sum_{d=\underline{c_s}}^{\overline{c_s}} x_{sitd} = 1$.

Define $\bar{x}$ by choosing exactly one d such that $\bar{x}_{sitd} = 1$ and setting all remaining associated variables to zero. Using the same argument as in Case 3, $(\bar{x}, y, z)$ cannot be expressed as a strict convex combination of two distinct feasible points of P , and is therefore an extreme point.

□

Appendix N

Proof of Proposition 7

Proof. For any given $\bar{y}_{sit}, \bar{z}_{sit}$ the primal subproblem $SP_{M-3i}(x|\bar{y}, \bar{z})$ is feasible if the following constraint are satisfied:

$$\sum_{d=\underline{c_s}}^{\bar{c_s}} \sum_{\hat{t}=\max\{1, t-d+1\}}^t x_{sitd} = \bar{y}_{sit} \quad i \in V, s \in S, t \in T \quad (6.37)$$

$$\sum_{d=\underline{c_s}}^{\bar{c_s}} x_{sitd} = \bar{z}_{sit} \quad i \in V, s \in S, t \in T \quad (6.38)$$

$$x_{sitd} \geq 0 \quad i \in V, s \in S, t \in T, d \in \hat{D}, t + d - 1 \leq |T|. \quad (6.19)$$

Therefore, we need to show that y_{sit} , and z_{sit} from the master problem are always feasible, implying that no feasibility cuts are needed in the master problem. Therefore, we only need to prove that (6.51)–(6.55) eliminate all the feasibility cuts (6.43) generated by Benders reformulation applied to 3-index Model.

The primal subproblem $SP_{M-3i}(x|\bar{y}, \bar{z})$ is feasible if and only if its dual $DSP_{M-3i}(\mu, \beta|\bar{y}, \bar{z})$ has a finite optimal solution. If $SP_{M-3i}(x|\bar{y}, \bar{z})$ is infeasible, then

$DSP_{M-3i}(\mu, \beta | \bar{y}, \bar{z})$ is unbounded. By Farkas' Lemma, the primal subproblem is infeasible if and only if there exists a ray $r_q = (\nu^q, \alpha^q) \in \mathcal{C}$ such that:

$$\sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \nu_{sit}^q y_{sit} + \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \alpha_{sit}^q z_{sit} < 0.$$

The dual subproblem's feasible cone is defined as follows:

$$\mathcal{C} = \{(\nu, \alpha) \mid \sum_{\hat{t}=t}^{\min\{|T|, t+d-1\}} \nu_{sit} + \alpha_{sit} \geq 0 \quad i \in V, s \in S, t \in T, d \in \{\underline{c}_s, \dots, \overline{c}_s\}, t+d-1 \leq |T|\}.$$

To avoid infeasibility, we add a feasibility cut to the master problem for each extreme ray $r_q = (\nu^q, \alpha^q)$ that satisfies the above condition. The feasibility cut is given by:

$$\sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \nu_{sit}^q y_{sit} + \sum_{s \in S} \sum_{t \in T} \sum_{i \in V} \alpha_{sit}^q z_{sit} \geq 0.$$

This cut ensures that the master solution $(\bar{y}, \bar{z})$ does not lead to an unbounded dual problem. Each extreme ray $(\nu, \alpha) \in \mathcal{C}$ corresponds to an infeasible (y, z) configuration and we can derive them systematically:

- Infeasible Case 1: $y_{si1} = 0, z_{si1} = 1$ and $y_{si1} = 1, z_{si1} = 0$.
 - a. $y_1 = 0, z_1 = 1$ indicates that a service starts at period 1 but is not active at period 1. However, it is contradictory since starting a service implies it must be active in that period. We derive the regarding feasibility cut through a valid ray of $\mathcal{C}$.

$$\begin{aligned} \nu_{si1} &= 1, \alpha_{si1} = -1 \quad (\text{All other components } 0) \\ \Rightarrow \text{Cut: } y_{si1} - z_{si1} &\geq 0 \end{aligned}$$

- b. $y_1 = 1, z_1 = 0$ indicates that a service is active at period 1 but did not start at period 1. Since the period $t = 1$ is the initial period, this is also contradictory. We similarly derive the regarding feasibility cut through a valid ray of $\mathcal{C}$.

$$\begin{aligned} \nu_{si1} &= -1, \alpha_{si1} = 1 \quad (\text{All other components } 0) \\ \Rightarrow \text{Cut: } z_{si1} - y_{si1} &\geq 0 \end{aligned}$$

Combining those two cases, we have $y_1 = z_1$ which is exactly Constraint (6.51).

- Infeasible Case 2: $y_{si(t-1)} = 0, y_{sit} = 0, z_{sit} = 1$ for some $t \in T, t > 1$. This case indicates that a service starts at period t , but is neither active at period $t - 1$ nor at period t . We derive the regarding feasibility cut through a valid ray of $\mathcal{C}$.

$$\begin{aligned} \nu_{si(t-1)} &= 1, \nu_{sit} = 1, \alpha_{sit} = -1 \quad (\text{All other components } 0) \\ \Rightarrow \text{Cut: } y_{si(t-1)} + y_{sit} - z_{sit} &\geq 0 \quad i \in V, s \in S, t \in T \\ \equiv z_{sit} &\leq y_{si(t-1)} + y_{sit} \quad i \in V, s \in S, t \in T \quad (\text{Constraint (6.52)}). \end{aligned}$$

- Infeasible Case 3: $y_{si(t-1)} = 0, y_{sit} = 1, z_{sit} = 0$ for some $t \in T, t > 1$. This case indicates that a service is active at period t , but it is not active at period $t - 1$ or it does not start at period t . We derive the regarding feasibility cut through a valid ray of $\mathcal{C}$.

$$\begin{aligned} \nu_{si(t-1)} &= 1, \nu_{sit} = -1, \alpha_{sit} = 1 \quad (\text{All other components } 0) \\ \Rightarrow \text{Cut: } y_{si(t-1)} - y_{sit} + z_{sit} &\geq 0 \quad i \in V, s \in S, t \in T \\ \equiv y_{sit} &\leq y_{si(t-1)} + z_{sit} \quad i \in V, s \in S, t \in T \quad (\text{Constraint (6.53)}). \end{aligned}$$

- Infeasible Case 4: $y_{si(t-1)} = 1, y_{sit} = 0, z_{sit} = 1$ for some $t \in T, t > 1$. This case indicates that a service ends at $t - 1$ but a new one starts at t , which is an inconsistent case.

$$\begin{aligned} \nu_{sit} &= 1, \alpha_{sit} = -1 \quad (\text{All other components } 0) \\ \Rightarrow \text{Cut: } y_{sit} - z_{sit} &\geq 0 \quad i \in V, s \in S, t \in T \end{aligned}$$

This is a valid ray, and its corresponding feasibility cut is $y_t \geq z_t$. This cut directly addresses the core contradiction of $y_t = 0, z_t = 1$. However, it is not the cut we have derived. Unlike the previous cases, this case is a valid inequality that strengthens the master problem and guarantees subproblem feasibility. Constraint (6.54) is a valid inequality that eliminates the specific infeasible configuration $y_{t-1} = 1, y_t = 0, z_t = 1$.

- Infeasible Case 5: $y_{si(t-1)} = 1, y_{sit} = 1, z_{sit} = 1$ for some $t \in T, t > 1$. This case indicates that a service is active at $t - 1$ and t , and a new service also starts at t . This is an infeasible scenario since a new service cannot begin if there is an ongoing service in the current period. Constraint (6.55) is a valid inequality that explicitly prevents the corresponding infeasible subproblem.

Constraints (6.51), (6.52), and (6.53) are directly derived from extreme rays of the dual feasible cone, while constraints (6.54) and (6.55) are valid inequalities that prevent specific infeasible combinations of y and z variables. This proves that Extended 3-index Model (M-3iE) is a valid reformulation of 3-index Model (M-3i). $\square$