50312.

INVESTIGATION OF CHAOTIC INSTABILITY IN A DISSIPATIVE CIRCUIT

VEDAT ÖZKANER

Ç.Ü
FEN BİLİMLERİ ENSTİTÜSÜ
ELEKTRİK-ELEKTRONİK ANABİLİM DALI

YÜKSEK LİSANS TEZİ

ADANA
MART - 1996

Çukurova Üniversitesi Fen Bilimleri Enstitüsü Müdürlüğüne

Yrd. Doç. Dr. Ali Okşaşoğlu'nun danışmanlığı altında yürütülen bu çalışma jürimiz tarafından Elektrik - Elektronik Mühendisliği Anabilim Dalında YÜKSEK LİSANS TEZİ olarak kabul edilmiştir.

Başkan: Prof. Dr. A. Hamit SERBEST

Üye: Yrd. Doç. Dr. Arslan YAZICI

Üye: Yrd. Doç. Dr. Z. Gökalp ALTUN

Kod no: 1088

Yukarıdaki imzaların, adı geçen öğretim üyelerine ait olduğunu onaylarım.

T.C.

Prof. Dr. Ural DİNÇ

Enstitü Müdürü

CONTENTS

LIST OF FIGURES

ABSTRACT

In this thesis, a second-order nonautonomous dissipative nonlinear electrical circuit will be studied with respect to its chaotic instability. The chaotic mechanism and the chaotic oscillation properties of the system and the effects of parameter on the chaotic behavior of the system will be investigated numerically. From an engineering point of view, this will give the designer an idea of which set of system parameters should be avoided or exploited. A corresponding computer method will be developed. Also, different ways of controlling and synchronizing chaos will be discussed.

ÖZ

Bu tezde ikinci dereceden, bazı doğrusal olmayan sönümlü ve nonotonom elektrik devrelerinin kaotik kararsızlığı incelenecektir. Sistemin kaotik mekanizması ve kaotik salınımın özellikleri ve parametrelerin sistemin kaotik özellikleri (davranışları) üzerindeki etkileri sayısal olarak araştırılacaktır. Mühendislik açısından bakıldığında, bu tasarımcıya hangi sistem parametrelerinden kaçınılması veya çıkarılması gerektiği hakkında bir fikir verecektir. Aynı zamanda, kaosun değişik yollarla kontrolü ve eşzamanlaması (senkronizasyonu) tartışılacaktır.

CHAPTER 1

INTRODUCTION

Nonlinear system have always played an important role in the study of natural phenomena, but the last decades has seen a heightened interest and renewed vigor in nonlinear systems research.The main reason for this growth is the recent availability of inexpensive computing power. Unlike linear systems where knowledge of the eigenvalues and eigenvectors allows one to write a closed-form solution, few nonlinear systems posses closed-form solutions and, therefore, numerical simulations play an important role in the process of finding and analyzing nonlinear phenomena. Before the advent of low-cost computers, the ability to perform nonlinear simulations was restricted to researchers with access to a large computing facility: now, anyone with a personal computer may simulate a nonlinear system.

The increased interest in nonlinear systems is also due to the recent (within the last sixteen years) discovery of chaos. One of the basic tenets of science is that deterministic systems are predictable: given the initial condition and the equations describing a system, the behavior of the system can be predicted for all time. The discovery of chaotic systems has eliminated this viewpoint. The term chaos is used for deterministic systems which do not possess any random or unpredictable inputs or parameters. Chaos, also called strange behavior, is currently one of the most exciting topics in nonlinear system research.

In order for a dynamical system to be chaotic, it must, first, be nonlinear. Obviously, no linear system can behave chaotically. Second, the system must possess a third degree of freedom. This degree of freedom basically refers to the order of the system. A continuous dynamical system must be of third-order in the autonomous case and of second-order in the nonautonomous case. Because of the difficulty (most often impossibility) of obtaining an analytical solution for nonlinear systems, the investigation of such systems is based on numerical simulations. Even then, it is not an easy task to determine if the system under study is chaotic. Lyapunow exponents, Poincare maps, Fractal dimensions, Fourier transform are some of the available criteria used for such an investigation.

What are the properties of a chaotic system? A chaotic spectrum is not composed solely of discrete frequencies, but has a continuous, broad-band nature. This noise like spectrum is the characteristic of chaotic systems. The limit set for chaotic behavior is not a simple geometrical object like a circle or a torus, but is related to fractals and Cantor sets. Another property of the chaotic systems is sensitive dependence on initial conditions; when the; two different initial conditions are arbitrarily close to one another. The trajectories emanating from these points diverge at a rate characteristic of the system until, for all practical purposes, they are uncorrelated.

CHAPTER 2

THE SIMPLEST DISSIPATIVE NONAUTONOMOUS CHAOTIC CIRCUIT

In this chapter, we numericaly analyze the behavior of a very simple, second-order dissipative nonlinear circuit under single-frequency input and we will investigate the chaotic instability of the circuit which depends on the parameters. This nonlinear circuit is made of three linear elements and a Chua's diode. Chua's diode is a nonlinear element that is modeled by a three-segment piecewise *v-i* charecteristic. Chua's diode can easily be realized by using two op-amps and six linear resistors or as an IC chip.

2.1 CIRCUIT REALIZATION AND CHAOS

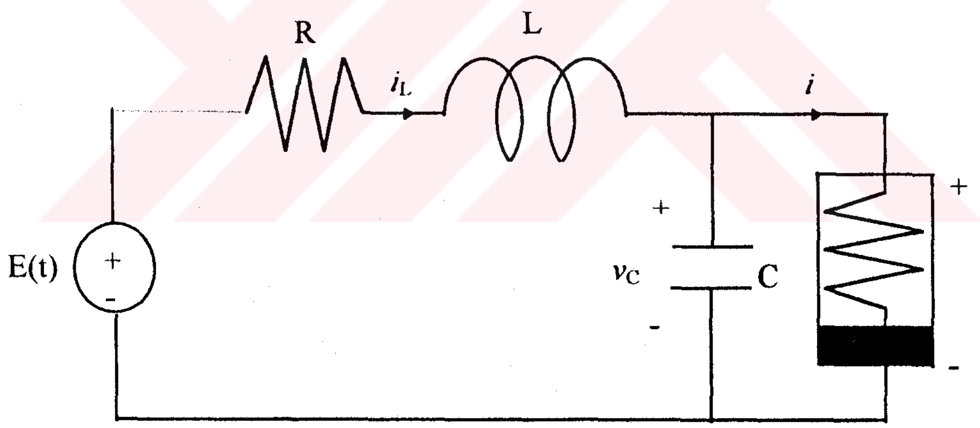

Fig. 2.1 Circuit realization of the simple nonaoutonomous circuit.

The circuit realization of the simple nonatonomous circuit is shown in Fig. 1.1. It contains a linear capacitor, a linear inductor, a linear resistor, an external forcing and a nonlinear element (Chua's diode). By applying Kirchoff's laws to this circuit, the governing equation for the voltage v across the capacitor C and the current i_L through the inductor L are represented by the following differential equations:

$$C\frac{dv_C}{dt} = i_L - g(v_C) \qquad 2.1$$

$$L\frac{di_L}{dt} = -Ri_L - v_C + E(t) \qquad 2.2$$

where $E(t)$ is harmonic signal and we use E(t) as

$$E(t) = E_m Cos\omega_2 t$$

E_m is amplitude and ω_2 is the angular frequency of the external periodic signal. The term

$$g(v_C) = G_1 v_C + 0.5(G_0 - G_1)\left[\left|v_C + B_p\right| - \left|v_C - B_p\right|\right]$$

is the mathematical representation of the characteristic curve of the Chua's diode and G_0, G_1 and B_p are, respectively, the slopes and the break point. At this point, Let us use new variables as $v_C = x_0 x$, $i_L = y_0 y$, $\tau = w_0 t$. where x_0 , y_0 , w_0 are the arbitary voltage, current, and frequency scaling factors, respectively, and x and y are new dimensionless state variables, and τ is the dimensionless time variable. In this case, we change the variables in the first differential equation:

$$L\frac{di_L}{dt} = -Ri_L - v_C + E_m Cos(\omega_2 t)$$

$$C\frac{dv_C}{dt} = i_L - \left(G_1 v_C + 0.5(G_0 - G_1)\left[\left|v_C + B_p\right| - \left|v_C - B_p\right|\right]\right)$$

$$L\frac{d(y_0 y)}{d(\frac{\tau}{\omega_0})} = -Ry_0 y - x_0 x + E_m Cos(\omega_2 / \omega_0)\tau$$

$$C\frac{d(x_0 x)}{d(\frac{\tau}{\omega_0})} = y_0 y - \left(G_1 x_0 x + 0.5(G_0 - G_1)\left[\left|x_0 x + B_p\right| - \left|x_0 x - B_p\right|\right]\right)$$

$$Ly_0\omega_0\frac{dy}{d\tau} = -Ry_0y - x_0x + E_mCos(\omega_2/\omega_0)\tau$$

$$Cx_0\omega_0\frac{dx}{d\tau} = y_0y - \left(G_1x_0x + 0.5(G_0 - G_1)\left[\left|x_0x + B_p\right| - \left|x_0x - B_p\right|\right]\right)$$

If we divide two equations by $Ly_0\omega_o$ and $Cx_0\omega_0$ respectively, then we obtain

$$\frac{dy}{d\tau} = -\frac{Ry_0}{Ly_0\omega_0}y - \frac{x_0}{Ly_0\omega_0}x + \frac{E_m}{Ly_0\omega_0}Cos(\omega_2/\omega_0)\tau$$

$$\frac{dx}{d\tau} = \frac{y_0}{Cx_0\omega_0}y - \frac{G_1x_0}{Cx_0\omega_0}x - 0.5\frac{G_0 - G_1}{Cx_0\omega_0}\left(\left|x_0x + B_p\right| - \left|x_0x - B_p\right|\right)$$

If we divide and multiply x variable by R and y_0 for the first and second differential equations respectively, then, we find

$$\frac{dy}{d\tau} = -\frac{Ry_0}{Ly_0\omega_0}y - \frac{Rx_0}{RLy_0\omega_0}x + \frac{E_m}{Ly_0\omega_0}Cos(\omega_2/\omega_0)\tau$$

$$\frac{dx}{d\tau} = \frac{y_0}{Cx_0\omega_0}y - \frac{G_1x_0y_0}{Cx_0\omega_0y_0}x - 0.5\frac{(G_0 - G_1)y_0x_0}{Cx_0\omega_0y_0}\left(\left|x + \frac{B_p}{x_0}\right| - \left|x - \frac{B_p}{x_0}\right|\right)$$

or

$$\frac{dy}{d\tau} = -\frac{R}{L\omega_0}\left(y + \frac{x_0}{y_0R}x\right) + \frac{E_m}{Ly_0\omega_0}Cos(\omega_2/\omega_0)\tau$$

$$\frac{dx}{d\tau} = \frac{y_0}{Cx_0\omega_0}\left(y - \frac{G_1x_0}{y_0}x - 0.5\left(\frac{G_0x_0}{y_0} - \frac{G_1x_0}{y_0}\right)\left[\left|x + \frac{B_p}{x_0}\right| - \left|x - \frac{B_p}{x_0}\right|\right]\right)$$

Now, let us define some new variables on:

$$\alpha = \frac{y_0}{x_0C\omega_0}, \qquad M_0 = \frac{G_0x_0}{y_0}, \qquad \Omega_2 = \frac{\omega_2}{\omega_0},$$

$$\beta = \frac{R}{L\omega_0}, \qquad M_1 = \frac{G_1x_0}{y_0}, \qquad f = \frac{E_m}{Ly_0\omega_0},$$

$$k = \frac{x_0}{y_0R}, \qquad \varepsilon = \frac{B_p}{x_0},$$

Hence, the system of (2.1) and (2.2) are transformed into new dimensionless system.

$$\frac{dy}{d\tau} = -\beta(y + kx) + fCos(\Omega_2 \tau) \qquad 2.3$$

$$\frac{dx}{d\tau} = \alpha(y - h(x)) \qquad 2.4$$

where, $h(x) = M_1 x + 0.5(M_0 - M_0)[|x + \varepsilon| - |x - \varepsilon|]$

The differential equations (2.3) and (2.4) now depend on the parameters β, α, k, M_0, M_1, Ω_2, and f. For the purpose of numerical simulations we choose $\varepsilon = 1$, $\Omega_2 = 0.6$, $\alpha = 1$, $\beta = 1$, $k = 1$, and M_0, M_1, and f as our system parameters. Before differential equations (2.3) and (2.4) are solved, we have to do equilibrium anlaysis.

2.1.1 Equilibrium Point Analysis

We find the equilibrium points for the differential equations (2.3) and (2.4). For this case, we accept that the input external signal is zero and fix $\alpha = 1$, $\beta = 1$, $k = 1$ and the obtained equations will be as follows:

$$\frac{dy}{d\tau} = -(y + x) \qquad 2.5$$

$$\frac{dx}{d\tau} = (y - h(x)) \qquad 2.6$$

The equilibrium points of (2.5) and (2.6) are identified by setting $\frac{dy}{d\tau} = \frac{dx}{d\tau} = 0$ which yields the conditions:

$$-y - x = 0$$

$$y - h(x) = 0$$

If we add both sides of two equations above, then we obtain;

$$-x - h(x) = 0 \qquad 2.7$$

where, $h(x) = M_1 x + 0.5(M_0 - M_1)[|x + 1| - |x - 1|]$. If we evaluate *h(x)*, then

$$h(x)=\begin{cases} M_1x+(M_0-M_1) & x\ge 1 \\ M_0x & -1<x<1 \\ M_1x-(M_0-M_1) & x\le -1 \end{cases}$$

As shown above, there are three cases of *h(x)* and we find the equilibrium points for each cases of *h(x)*. First, if $x\ge 1$ then $h(x)=M_1x+(M_0-M_1)$. If we substitue *h(x)* in (2.7) then,

$$-x-M_1x-(M_0-M_1)=0$$

If we solve this equation, then we obtain first equilibrium points as:

$$x_{eq1}=-\frac{(M_0-M_1)}{1+M_1}, \qquad y_{eq1}=\frac{(M_0-M_1)}{1+M_1} \qquad 2.8$$

Second, if $-1<x<1$ then $h(x)=M_0x$. If we substitue *h(x)* in (2.7) then,

$$-x-M_0x=0$$

The equilibrium points are;

$$x_{eq2}=0, \qquad y_{eq2}=0, \qquad 2.9$$

This means that if the value of x is between -1 and 1, then the equilibrium points are equal to zero.

Third, if $x\le -1$ then the function $h(x)$ will be; $h(x)=M_1x-(M_0-M_1)$. If we substitue *h(x)* in (2.7) then,

$$-x-M_1x-(M_0-M_1)=0$$

If this equation is solved, we obtain the equilibrium points.

$$x_{eq3}=\frac{(M_0-M_1)}{1+M_1}, \qquad y_{eq3}=-\frac{(M_0-M_1)}{1+M_1} \qquad 2.10$$

If we look at the equations (2.8) and (2.9), we can say that for existing x_{eq1} and x_{eq3} , M_0 must be less or equal to -1.

We find the eigenvalues of the system. For this case, we calculate det(λI-J). Where J is Jacobian matrix and I is identity matrix. Jacobian matrix is defined as the derivative of

(2.5) and (2.6) with respect to x and y at the equilibrium point and we can write it as follows:

$$J = \begin{bmatrix} \frac{df_1(x,y)}{dx} & \frac{df_1(x,y)}{dy} \\ \frac{df_2(x,y)}{dx} & \frac{df_2(x,y)}{dy} \end{bmatrix}$$

where $f_1(x,y) = -y - x$, $f_2(x,y) = y - h(x)$

If we substitue $f_1(x,y)$ and $f_2(x,y)$ in Jacobian matrix, then we obtain

$$j = \begin{bmatrix} -1 & -1 \\ -\frac{dh(x)}{dx} & 1 \end{bmatrix}$$

Now, we calculate det(λI-J)

$$\det(\lambda I - J) = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & -1 \\ -\frac{dh(x)}{dx} & 1 \end{bmatrix} = \begin{bmatrix} \lambda + 1 & -1 \\ -\frac{dh(x)}{dx} & \lambda - 1 \end{bmatrix} = (\lambda+1)(\lambda-1) - \frac{dh(x)}{dx}$$

$$\det(\lambda I - J) = \lambda^2 - 1 + \frac{dh(x)}{dx} \qquad 2.11$$

If we make the right side of equation (2.11) zero, then we can find the roots of (2.11).

$$\lambda^2 - 1 + \frac{dh(x)}{dx} = 0 \qquad 2.12$$

Note that, $\frac{dh(x)}{dx}$ is the derivative of $h(x)$ with respect to x. $\frac{dh(x)}{dx}$ is equal to

$$\frac{dh(x)}{dx} = \begin{cases} M_1 & x \geq 1 \\ M_0 & -1 < x < 1 \\ M_1 & x \leq -1 \end{cases}$$

As shown above, there are two cases of the derivative of *h(x)*. First, $x \geq 1$ and $x \leq -1$ then derivative of *h(x)* is equal to M_1 . If we substitue M_1 in (2.12), then we obtain;

$$\lambda^2 - 1 - M_1 = 0$$

The roots of this equation is equal to

$$\lambda_{1,2} = \pm\sqrt{1 + M_1} \qquad 2.13$$

Second, if x is between -1 and 1 ($-1<x<1$), then the derivative of *h(x)* is equal to M_0 and we substitue M_0 in (2.12), in this case, we can obtain the roots of the equation which is given as

$$\lambda_{3,4} = \pm\sqrt{1 + M_0} \qquad 2.14$$

If we look at the equation (2.14), we can say that we have two complex conjugate eigenvalues, because we have found that M_0 was less or equal to -1. The system which must have two real eigenvalues at least for showing chaotic behavior and one of these eigenvalues must be positive and the another eigenvalue must be negative. In this case, λ_1 must be positive and λ_2 must be negative. For this purpose, M_1 must be bigger then -1. We have found that between which values M_0 and M_1 must be for the system to be chaotic at the equilibrium points analysis.

2.1.2 Numerical Analysis

The differential equations of the system was given by (2.3) and (2.4). If we rewrite these differential equations;

$$\frac{dy}{d\tau} = -(y+x) + f\,Cos(0.6\tau)$$

$$\frac{dx}{d\tau} = (y - h(x))$$

where $h(x) = M_1 x + 0.5(M_0 - M_1)\left[|x+1| - |x-1|\right]$ and we have fixed $\Omega_2 = 0.6$, $\alpha = 1$, $\beta = 1$, $k = 1$. If we choose $M_0 = -1.27$ and $M_1 = -0.68$ and we increase the amplitude of the external signal f from 0.1 to 0.5, then we can see when the system will be chaotic. For this case, the one-parameter bifurcation diagram in the $(x\text{-}f)$ plane is shown in Fig. 2.2.

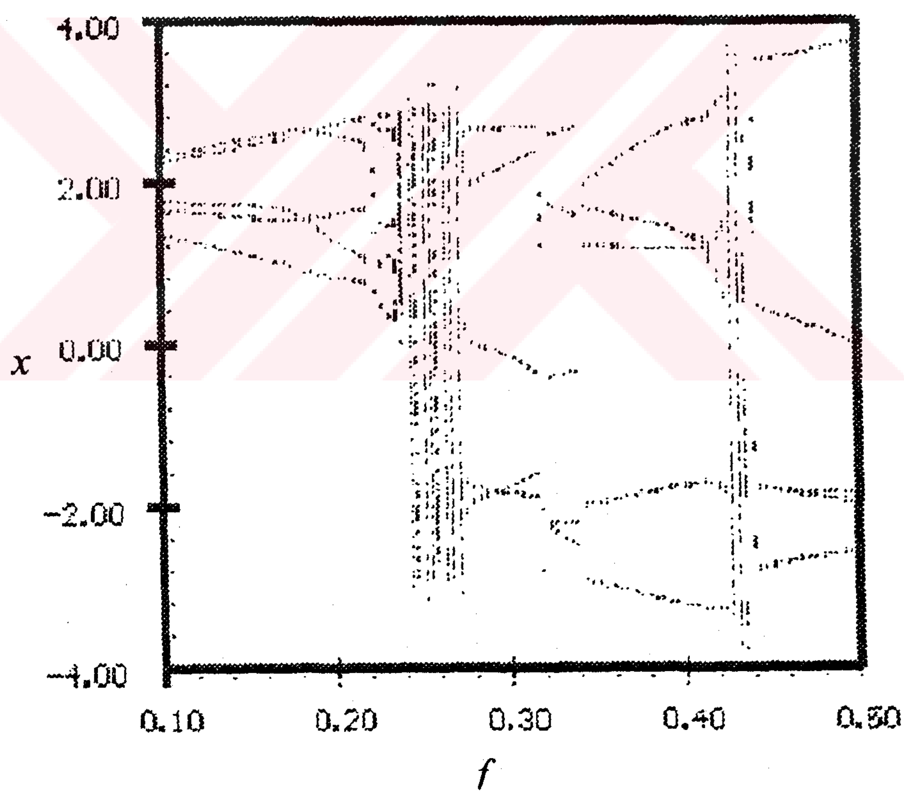

Fig. 2.2 One parameter bifurcation diagram in the $(f - x)$ plane

This bifurcation diagram shows that the system is chaotic in two regions. First region is between 0.24 and 0.28, and second region is between 0.42 and 0.44. We can say that the system shows chaotic behaviour through lese narrow regions.

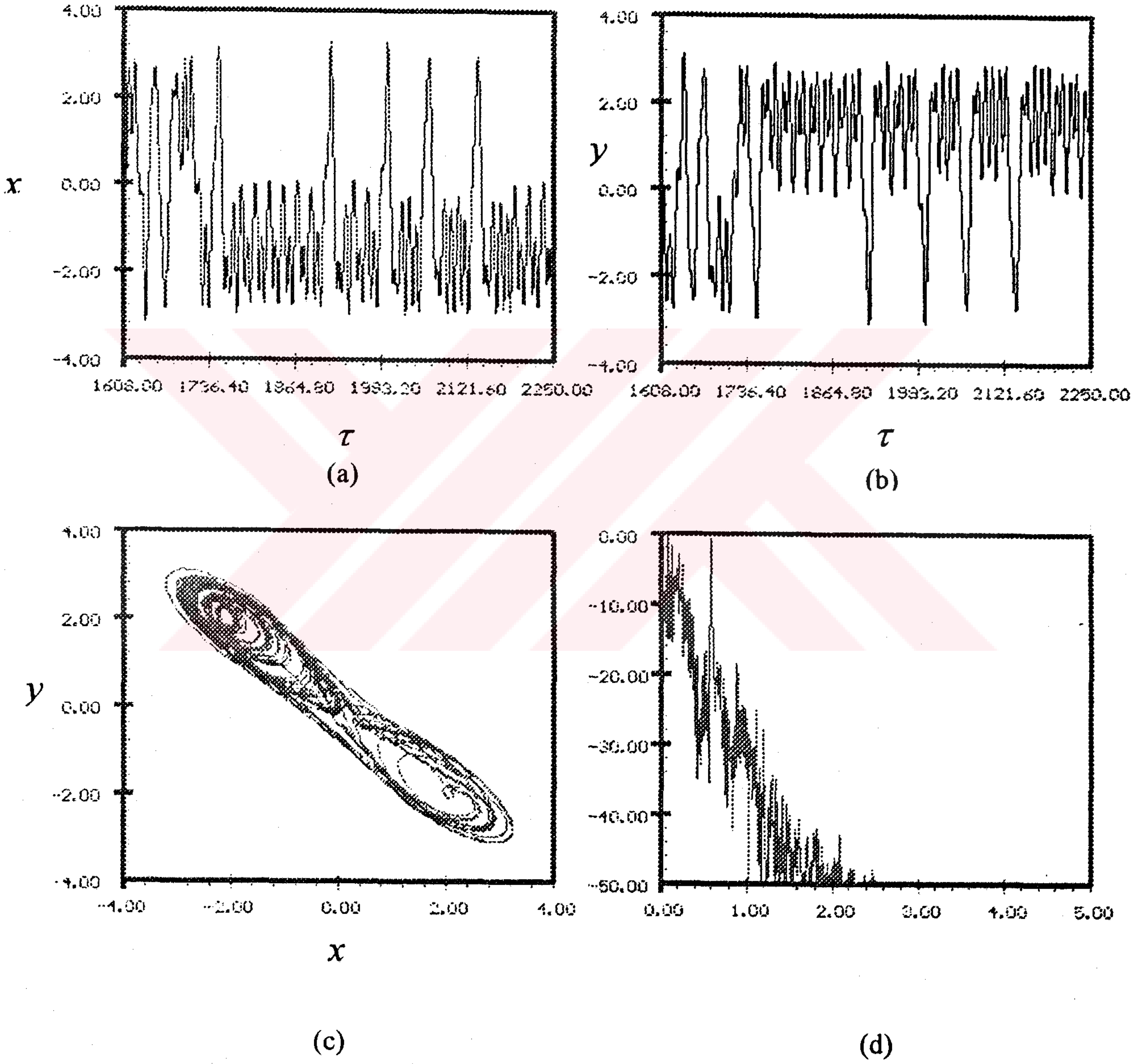

Fig. 2.3 (a) The time waveform of x (b) the time waveform of y (c) Chaotic attractor in the $(x\text{-}y)$ plane (d) The Fourier transform of x for $f = 0.25$.

If we choose $f = 0.25$ and fix M_0 and M_1, then we obtain time waveform of x, y and chaotic attractor and the Fourier transform of x as given in Fig. 2.3.

We now investigate the effects of M_0 and M_1 on the chaotic behavior of the circuit. For this purpose, we fix f and other parameters and we change M_0 and M_1 parameters. The chaotic region in the M_0 and M_1 plane is shown in Fig. 2.4. If we look at this bifurcation diagram, we can say that the system is chaotic in four narrow regions.

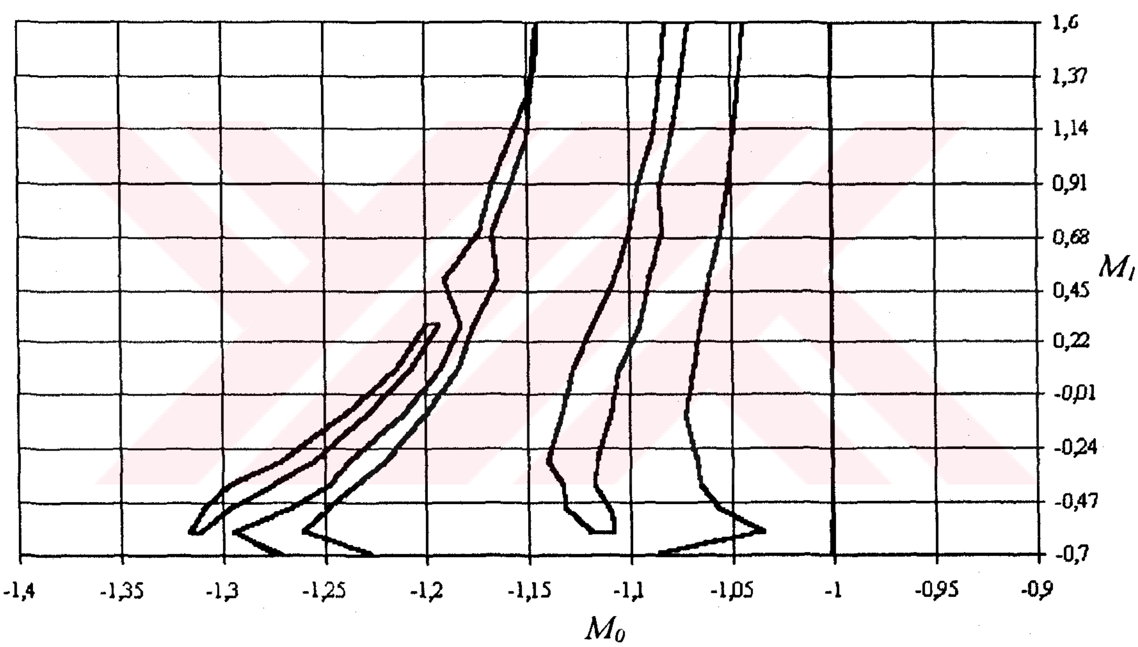

Fig 2.4 Two parameter bifurcation diagram in the $M_0 - M_1$ plane.

As shown Fig. 2.4, the system is chaotic for the case of $M_0 \leq -1$ and $M_1 \geq -1$ and we have explained this case in equilibrium points analysis.

CHAPTER 3

THE SIMPLEST DISSIPATIVE NONAUTONOMOUS CHAOTIC CIRCUIT DRIVEN by MULTI-FREQUENCY SIGNAL

The behavior of nonlinear system under multi-frequency input is, in general, different from that of the single-frequency case. System that can not behave chaotically under harmonic excitation for a given set of parameters can behave chaoticly under multi-frequency input for the same set of parameters. In this chapter, we numerically analyze the behavior of a very simple, second-order dissipative nonlinear circuit under multi-frequency input and we show the effects of the multi-frequency excitation on the behavior of the circuit. We have analyzed this circuit under single-frequency case in Chapter 2.

3.1 ANALYSIS

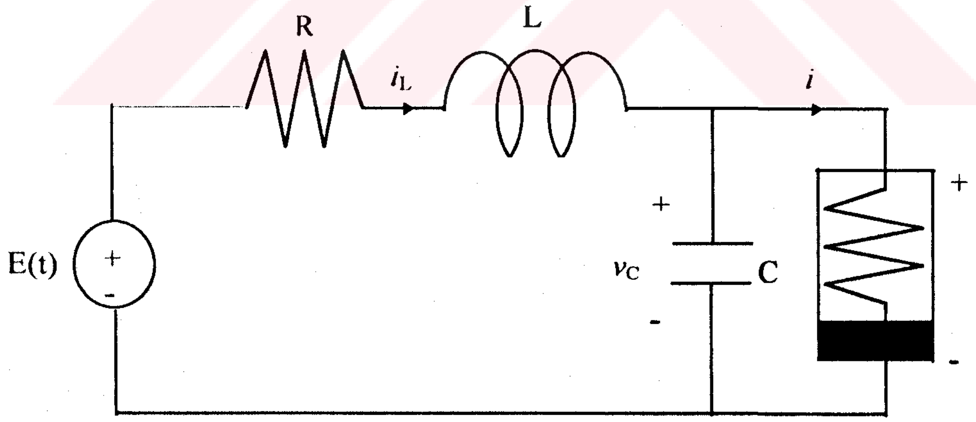

Fig. 3.1 Circuit realization of the simple nonaoutonomous circuit.

The circuit realization of the simple nonatonomous circuit is shown in Fig. 3.1. This circuit is same as Fig. 2.1 but the external signal will be different from in Fig. 2.1. By applying Kirchoff's laws to this circuit, the governing equation for the voltage v across the capacitor C and the current i_L through the inductor L are represented by the following differential equations:

$$C\frac{dv_C}{dt} = i_L - g(v_C) \tag{3.1}$$

$$L\frac{di_L}{dt} = -Ri_L - v_C + E(t) \tag{3.2}$$

The input $E(t)$ is an amplitude modulated signal given as $E(t)=E_m(1+mCos\omega_1)Cos\omega_2$, m is the coefficient of modulation, ω_1 and ω_2 are the angular frequencies. Note that when m=0, $E(t)$ becomes a harmonic signal. The term

$$g(v_C) = G_1 v_C + 0.5(G_0 - G_1)\left[\left|v_C + B_p\right| - \left|v_C - B_p\right|\right]$$

is the mathematical representation of the characteristic curve of the Chua's diode and G_0, $G_1$and B_p are, respectively, the slopes and the break point. At this point, let us use new variables as $v_C = x_0 x$, $i_L = y_0 y$, $\tau = w_0 t$ where x_0 , y_0 , w_0 are the arbitrary voltage, current, and frequency scaling factors, respectively, and x and y are new dimensionless state variables, and τ is the dimensionless time variable. In this case, we change the variables in the first differential equation:

$$L\frac{d(y_0 y)}{d(\frac{\tau}{\omega_0})} = -Ry_0 y - x_0 x + E_m(1+mCos(\omega_1/\omega_0)\tau)Cos(\omega_2/\omega_0)\tau$$

$$C\frac{d(x_0 x)}{d(\frac{\tau}{\omega_o})} = y_0 y - \left(G_1 x_0 x + 0.5(G_0 - G_1)\left[\left|x_0 x + B_p\right| - \left|x_0 x - B_p\right|\right]\right)$$

$$Ly_0\omega_0\frac{dy}{d\tau} = -Ry_0 y - x_0 x + E_m(1+mCos(\omega_1/\omega_0)\tau)Cos(\omega_2/\omega_0)\tau$$

$$Cx_0\omega_0\frac{dx}{d\tau} = y_0 y - \left(G_1 x_0 x + 0.5(G_0 - G_1)\left[\left|x_0 x + B_p\right| - \left|x_0 x - B_p\right|\right]\right)$$

If we divide two equations by $Ly_0\omega_o$ and $Cx_0\omega_0$ respectively, then

$$\frac{dy}{d\tau} = -\frac{Ry_0}{Ly_0\omega_0}y - \frac{x_0}{Ly_0\omega_0}x + \frac{E_m}{Ly_0\omega_0}(1 + mCos(\omega_1 / \omega_0)\tau)Cos(\omega_2 / \omega_0)\tau$$

$$\frac{dx}{d\tau} = \frac{y_0}{Cx_0\omega_0}y - \frac{G_1x_0}{Cx_0\omega_0}x - 0.5\frac{G_0 - G_1}{Cx_0\omega_0}\left(\left|x_0x + B_p\right| - \left|x_0x - B_p\right|\right)$$

If we divide and multiply x variable by R and y_0 for the first and second differential equation respectively, then we obtain;

$$\frac{dy}{d\tau} = -\frac{R}{L\omega_0}\left(y + \frac{x_0}{y_0R}x\right) + \frac{E_m}{Ly_0\omega_0}(1 + mCos(\omega_1 / \omega_0)\tau)Cos(\omega_2 / \omega_0)\tau$$

$$\frac{dx}{d\tau} = \frac{y_0}{Cx_0\omega_0}\left(y - \frac{G_1x_0}{y_0}x - 0.5\left(\frac{G_0x_0}{y_0} - \frac{G_1x_0}{y_0}\right)\left[\left|x + \frac{B_p}{x_0}\right| - \left|x - \frac{B_p}{x_0}\right|\right]\right)$$

Now, let we define some new variables on:

$$\alpha = \frac{y_0}{x_0C\omega_0}, \qquad M_0 = \frac{G_0x_0}{y_0}, \qquad \Omega_2 = \frac{\omega_2}{\omega_0},$$

$$\beta = \frac{R}{L\omega_0}, \qquad M_1 = \frac{G_1x_0}{y_0}, \qquad f = \frac{E_m}{Ly_0\omega_0},$$

$$k = \frac{x_0}{y_0R}, \qquad \Omega_1 = \frac{\omega_1}{\omega_0}, \qquad \varepsilon = \frac{B_p}{x_0},$$

Hence, the system of (3.1) and (3.2) are transformed into new dimensionless system,

$$\frac{dy}{d\tau} = -\beta(y + kx) + f(1 + mCos(\Omega_1\tau))Cos(\Omega_2\tau) \qquad 3.3$$

$$\frac{dx}{d\tau} = \alpha(y - h(x)) \qquad 3.4$$

where, $h(x) = M_1x + 0.5(M_0 - M_0)[|x + \varepsilon| - |x - \varepsilon|]$

The differential equations (3.3) and (3.4) now depend on the parameters β, α, k, M_0, M_1, Ω_1, Ω_2, m and f. For the purpose of numerical simulations we choose $\varepsilon = 1$, $\Omega_1 = 0.11$, $\Omega_2 = 0.6$, $\alpha = 1$, $\beta = 1$, $k = 1$, and M_0, M_1, m and f as our system parameters. If we choose m=0, then the external signal will be periodic signal and differential equations (3.3) and (3.4) will be same as (2.3) and (2.4). For this case, the differential equations have been solved in Chapter 2. Now, we will solve differential equations (3.3) and (3.4) for case of $m \neq 0$. In this case, the external signal will be amplitude modulated signal.

If we choose $m = 0.5$ and fix other parameters except f, and increase f from 0.1 to 0.5, then the one parameter bifurcation diagram in the (x-f) plane is shown in Fig. 3.2. As shown in Fig. 3.2, the system shows chaotic behavior between 0.15 and 0.5. If we compare two bifurcation diagrams (Fig. 2.2 and Fig 3.2), we can see that the chaotic behavior of system is increased. The reason of this is the external signal which is applied to the circuit. If we choose f = 0.25, we obtain time waveform of x, y and chaotic attractor and the Fourier transform of x as given in Fig. 3.3.

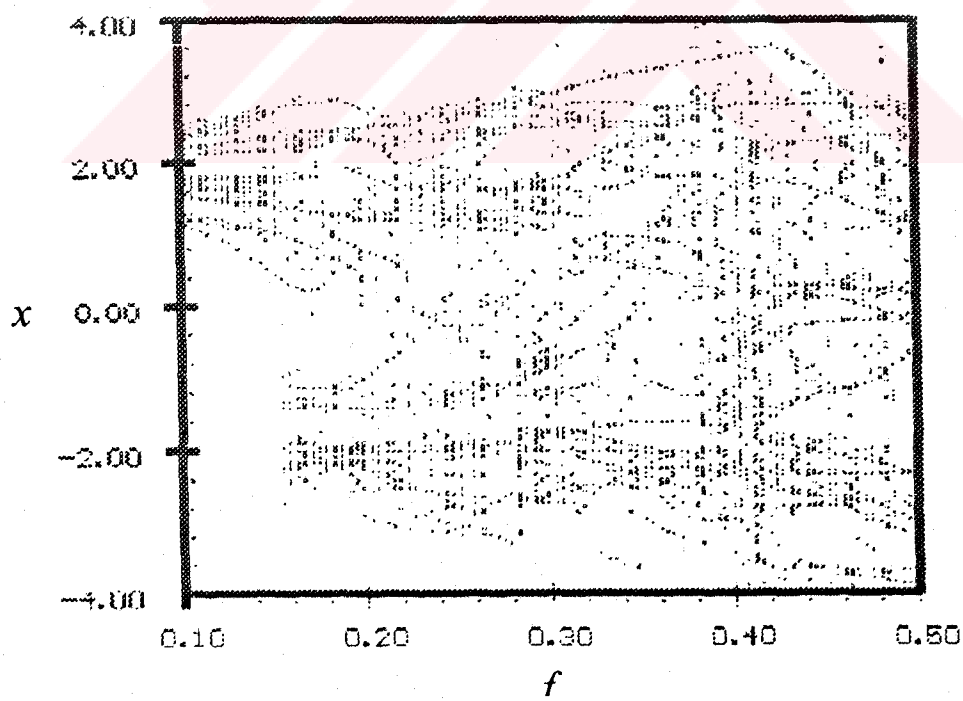

Fig. 3.2 One parameter bifurcation diagram in the (f-x) plane

We now investigate the effects of M_0 and M_1 on the chaotic behavior of the circuit. For this purpose, we fix f and other parameters and we change M_0 and M_1 parameters. The chaotic region in the M_0 and M_1 plane is shown in Fig. 3.4.

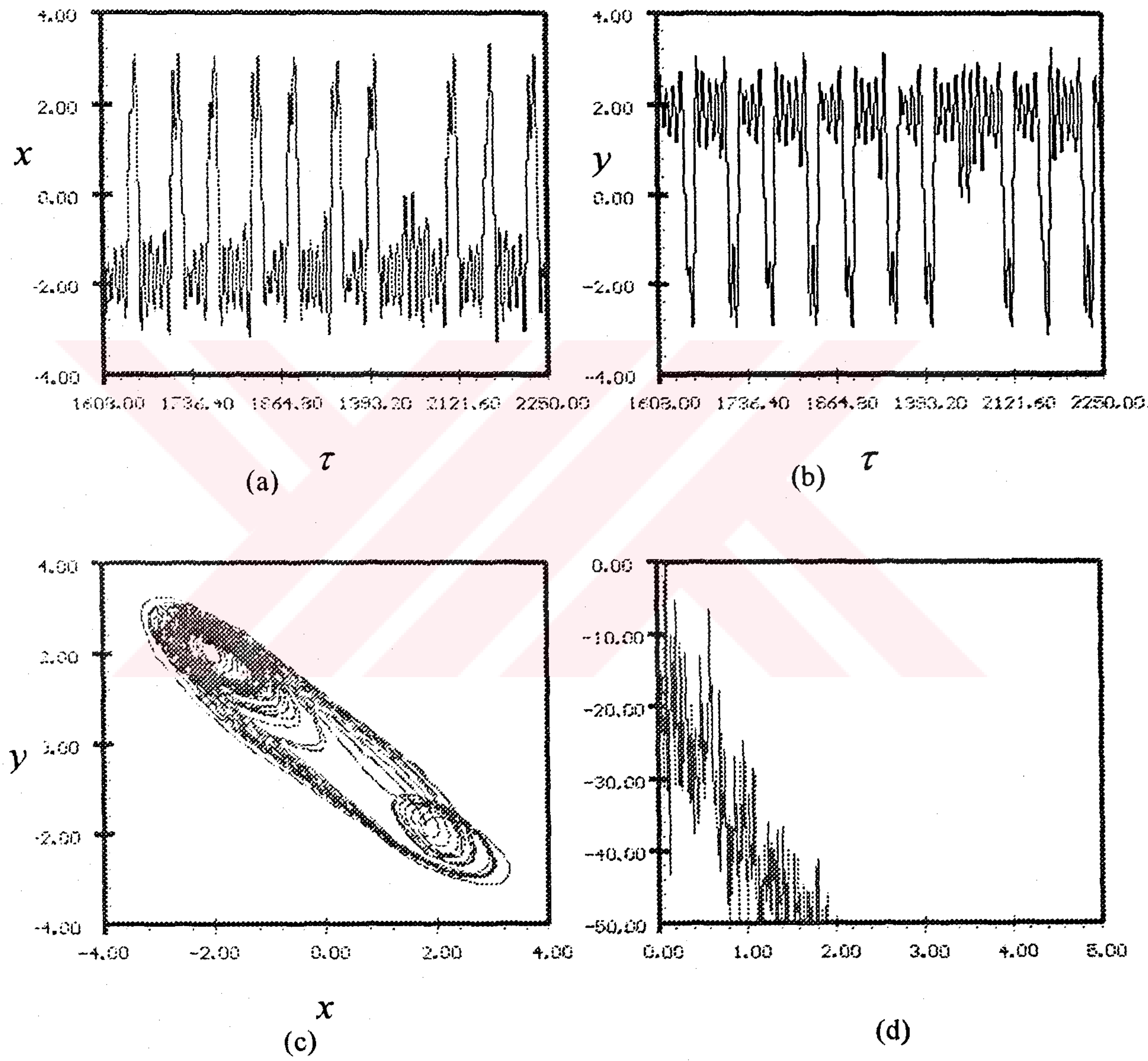

Fig 3.3 (a) The time waveform of x (b) The time waveform of y (c) Chaotic attractor in the (x-y) plane, (d) The Fourier transform of x for m=0.5 and f=0.25.

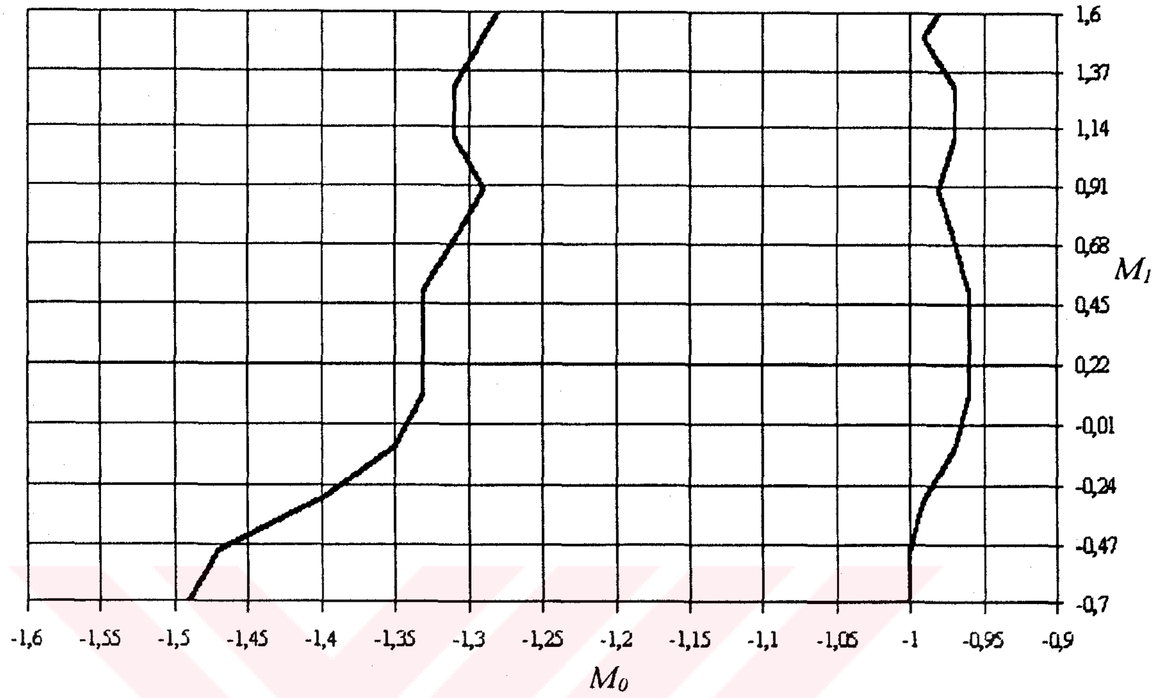

Fig. 3.4 Two-parameter bifurcation diagram in the (M_0 -M_1) plane

If we compare two bifurcation diagrams (Fig. 2.4 and Fig. 3.4), we can say that the chaotic region of the system under multi-frequency excitation is very large and the system shows chaotic behavior in one region. Unlike the system under single-frequency excitation, the system shows chaotic behavior in three regions which are very narrow. We can say that these regions connect and increase under multi-frequency excitation.

The numerical results obtained show that the chaotical dynamics demonstrated by even a very simple system in the case of multi-frequency forcing can differ from harmonic input case. Under multi-frequency excitation, the range of system parameters for chaotic behavior is larger than the case of single-frequency input.

CHAPTER 4

INTERACTION OF CAPACITIVE AND RESISTIVE NONLINEARITIES IN THE CIRCUIT

The chaotic nature and the behavior of the circuit whose only nonlinear element is a three segment piecewise linear resistor has been studied in Chapter 2. This nonlinear resistor, also called the Chua's diode, can easily be realized by using two op-amps and six linear resistors. In this chapter, we numerically investigate the interaction of capacitive and resistive nonlinearities in the circuit, and discuss the effects of capacitive nonlinearities on the chaotic behavior of the circuit. For this purpose, the original simple nonautonomous circuit (shown in Fig. 2.1) is modified by replacing the linear capacitor in the circuit with a nonlinear one.

4.1 ANALYSIS

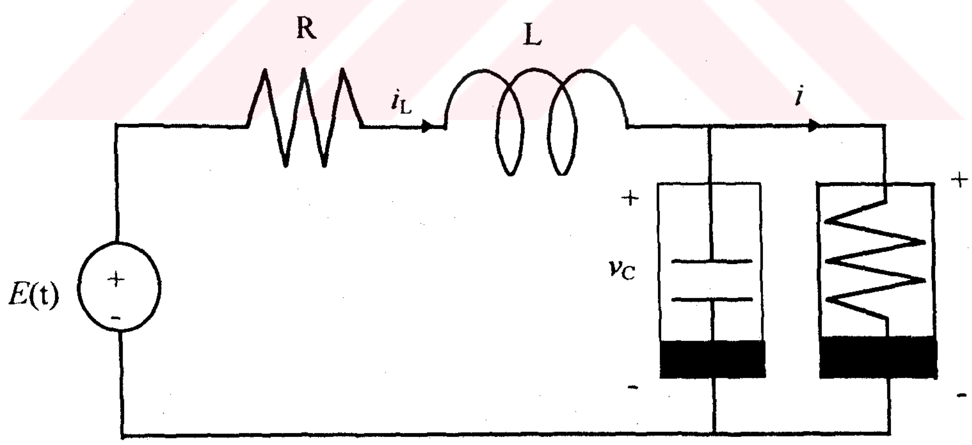

Fig. 4.1 Modified circuit realization

The circuit realization is shown in Fig. 4.1. It contains a linear inductor, a linear resistor, a nonlinear resistor (Chua's diode), an external forcing, and a nonlinear capacitor. This nonlinear capacitor is charge controlled and its *v-q* characteristics is given by $v_C(t)=f(q)=a_1q+a_3q^3$. By applying Kirchoff's laws to this circuit, the governing equations for the voltage v across the nonlinear capacitor C and the current i_L through the inductor L are given by the following:

$$E(t)=Ri_L+L\frac{di_L}{dt}+v_C(q) \tag{4.1}$$

$$i_L=\frac{dq}{dt}+g(v_C(q)) \tag{4.2}$$

where $\frac{dq}{dt}$ is the current through the nonlinear capacitor and q is charge. The term,

$$g(v_C(q))=G_1v_C(q)+0.5(G_0-G_1)\left[\left|v_C(q)+B_p\right|-\left|v_C(q)-B_p\right|\right]$$

is the mathematical representation of the characteristic curve of the Chua's diode and G_0 , G_1 and B_p are, respectively, the slopes and breakpoint, and the v_C is;

$$v_C(q)=a_1q+a_3q^3$$

$E(t)$ is external forcing and we will conduct our analysis for two different cases of $E(t)$. For first case, $E(t)$ will be a harmonic signal and second case, $E(t)$ will be a multi-frequency. $E(t)$ is given as $E(t)=E_m(1+mCos\omega_1t)Cos\omega_2$, where m is the coefficient of modulation. ω_1 and ω_2 are the angular frequency, when m=0, $E(t)$ becomes a harmonic signal. If we again write above equations;

$$\frac{di_L}{dt}=\frac{1}{L}\left[E(t)-R.i_L-v_C(q)\right]$$

$$\frac{dq}{dt}=i_L-g(v_C(q))$$

$$\frac{di_L}{dt}=\frac{1}{L}\left[-Ri_L-(a_1q+a_3q^3)+E_m(1+mCos\omega_1t)Cos(\omega_2t)\right] \tag{4.3}$$

$$\frac{dq}{dt}=i_L-\left(G_1v_C(q)+0.5(G_0-G_1)\left[\left|v_C(q)+B_p\right|-\left|v_C(q)-B_p\right|\right]\right) \tag{4.4}$$

At this point, let us use new variables as $q = x_0 x$,$i_L = y_0 y, \tau = \omega_0 t$. Where ω_0 , x_0 , y_0 are arbitrary frequencies, charge and current scaling factors, respectively, and x, y are the new respective nondimensional charge and current variables. If we substitute these variables in (4.3) ,(4.4), then,

$$\frac{d(y_0 y)}{d\left(\frac{\tau}{\omega_0}\right)} = \frac{1}{L}\left[-Ry_0 y - (a_1 x_0 x + a_3 x_0^3 x^3) + E_m(1 + mCos(\omega_1/\omega_0))Cos(\omega_2/\omega_1)\right]$$

$$\frac{d(x_0 x)}{d\left(\frac{\tau}{\omega_0}\right)} = y_0 y - \left[G_1(a_1 x_0 x + a_3 x_0^3 x^3) + 0.5(G_0 - G_1)\left[\left|a_1 x_0 x + a_3 x_0^3 x^3 + B_p\right|\right.\right.$$

$$\left.\left. - \left|a_1 x_0 x + a_3 x_0^3 x^3 - B_p\right|\right]\ \right]$$

$$y_0 \omega_0 \frac{dy}{d\tau} = \frac{1}{L}\left[-Ry_0 y - (a_1 x_0 x + a_3 x_0^3 x^3) + E_m(1 + mCos(\omega_1/\omega_0))Cos(\omega_2/\omega_1)\right]$$

$$x_0 \omega_0 \frac{dx}{d\tau} = y_0 y - \left[G_1(a_1 x_0 x + a_3 x_0^3 x^3) + 0.5(G_0 - G_1)\left[\left|a_1 x_0 x + a_3 x_0^3 x^3 + B_p\right|\right.\right.$$

$$\left.\left. - \left|a_1 x_0 x + a_3 x_0^3 x^3 - B_p\right|\right]\ \right]$$

If we divide two equations by $y_0\omega_0$ and $x_0\omega_0$ respectively, then

$$\frac{dy}{d\tau} = -\frac{Ry_0}{Ly_0\omega_0}y - \frac{a_1 x_0}{Ly_0\omega_0}x - \frac{a_3 x_0^3}{Ly_0\omega_0}x^3 + \frac{E_m}{Ly_0\omega_0}\left(1 + mCos(\omega_1/\omega_0)\tau Cos(\omega_2/\omega_0)\tau\right)$$

$$\frac{dx}{d\tau} = \frac{y_0}{x_0\omega_0}y - \frac{1}{x_0\omega_0}\left[G_1(a_1 x_0 x + a_3 x_0^3 x^3) + 0.5(G_0 - G_1)\left[\left|a_1 x_0 x + a_3 x_0^3 x^3 + B_p\right|\right.\right.$$

$$\left.\left. - \left|a_1 x_0 x + a_3 x_0^3 x^3 - B_p\right|\ \right]\ \right]$$

If we divide and multiply x variable by R and Ry_0 for the first and second differential equations, respectively, then;

$$\frac{dy}{d\tau} = -\frac{R}{L\omega_0}y - \frac{Ra_1x_0}{RLy_0\omega_0}x - \frac{Ra_3x_0^3}{RLy_0\omega_0}x^3 + \frac{E_m}{Ly_0\omega_0}\left(1 + mCos(\omega_1/\omega_0)\tau Cos(\omega_2/\omega_0)\tau\right)$$

$$\frac{dx}{d\tau} = \frac{y_0}{x_0\omega_0}y - \frac{Ry_0}{Ry_0x_0\omega_0}\left[G_1(a_1x_0x + a_3x_0^3x^3) + 0.5(G_0 - G_1)\left[\left|a_1x_0x + a_3x_0^3x^3 + B_p\right|\right.\right.$$

$$\left.\left. - \left|a_1x_0x + a_3x_0^3x^3 - B_p\right|\right]\ \right]$$

$$\frac{dy}{d\tau} = \frac{R}{L\omega_0}\left[-y - \frac{a_1x_0}{Ry_0}x - \frac{a_3x_0^3}{Ry_0}x^3\right] + \frac{E_m}{Lx_0\omega_0}\left(1 + mCos(\omega_1/\omega_0)\tau\right)Cos(\omega_2/\omega_0)\tau$$

$$\frac{dx}{d\tau} = \frac{y_0}{x_0\omega_0}y - \frac{y_0}{x_0\omega_0}\left[RG_1\left[\frac{a_1x_0}{Ry_0}x + \frac{a_0x_0^3}{Ry_0}x^3\right]\right] + \frac{y_0}{x_0\omega_0}0.5R(G_0 - G_1)\left[\left|\frac{a_1x_0}{Ry_0}x + \frac{a_3x_0^3}{Ry_0}x^3\right.\right.$$

$$\left.\left. + \frac{B_p}{R\ y_0}\ \right|\ \right] - \frac{y_0}{x_0\omega_0}0.5R(G_0 - G_1)\left[\left|\frac{a_1x_0}{Ry_0}x + \frac{a_3x_0^3}{Ry_0}x^3 - \frac{B_p}{R\ y_0}\right|\right]$$

Now, we again change the variables as;

$$\beta = \frac{R}{L\omega_0}, \qquad b = \frac{a_3x_0^3}{Ry_0}, \qquad M_0 = RG_0 \qquad \varepsilon = \frac{B_p}{Ry_0}$$

$$\alpha = \frac{y_0}{x_0\omega_0}, \qquad f = \frac{E_m}{Ry_0}, \qquad \Omega_1 = \frac{\omega_1}{\omega_0}$$

$$a = \frac{a_1x_0}{Ry_0}, \qquad M_1 = RG_1, \qquad \Omega_2 = \frac{\omega_2}{\omega_0}$$

Hence, the system of (4.3) and (4.4) are transformed into a new dimensionless system:

$$\frac{dy}{d\tau} = \beta\left[-y - g(x)\right] + f(1 + mCos\Omega_1\tau)Cos\Omega_2\tau \qquad 4.5$$

$$\frac{dx}{d\tau} = \alpha\left[y - f(g(x))\right] \qquad 4.6$$

where, $g(x) = ax + bx^3$,and $f(g(x)) = M_1 g(x) + 0.5(M_0 - M_1)\left[|g(x)+\varepsilon| - |g(x)-\varepsilon|\right]$

The differential equations (4.5) and (4.6) now depend on the parameters, β, α, M_0, M_1, Ω_1, Ω_2, m, f, ε, a, and b. Here, b is the coefficient of cubic nonlinearity for the nonlinear capacitor. If we choose $b = 0$, the system of (4.5) and (4.6) will be same as (2.3) and (2.4) thus, nonlinear capacitor will be omitted from the circuit given in Fig. 4.1 and the circuit will be same as given in Fig. 2.1.

4.1.1 EQUILIBRIUM POINT ANALYSIS

We will find the equilibrium points of the differential equations (4.5) and (4.6). For this case, we accept that the input external signal is zero and fix β=1, α=1, ε=1, a=1 and the obtained equations will be as follows;

$$\frac{dy}{d\tau} = -y - g(x) \qquad 4.7$$

$$\frac{dx}{d\tau} = y - f(g(x)) \qquad 4.8$$

The equilibrium points of (4.7) and (4.8) are identified by setting $\frac{dy}{d\tau} = \frac{dx}{d\tau} = 0$ which yields the conditions;

$$-y - g(x) = 0$$

$$y - f(g(x)) = 0$$

If we add both sides of two equations above, then we obtain;

$$-g(x)-f(g(x))=0 \qquad 4.9$$

where, $g(x)=bx^3+x$, $f(g(x))=M_1g(x)+0.5(M_0-M_1)\big[|g(x)+1|-|g(x)-1|\big]$

If we rewrite $f(g(x))$ clearly, then

$$f(g(x))=\begin{cases} M_1g(x)+(M_0-M_1) & g(x)\geq 1 \\ M_0g(x) & -1\langle g(x)\langle 1 \\ M_1g(x)-(M_0-M_1) & g(x)\leq -1 \end{cases}$$

As shown above, there are three cases of $f(g(x))$ and we will find the equilibrium points for each cases of $f(g(x))$. First, $f(g(x))\geq 1$ then $f(g(x))=M_1g(x)+(M_0-M_1)$; if we substitute $f(g(x))$ in (4.9), then we obtain ;

$$-g(x)-M_1g(x)-(M_0-M_1)=0$$

or

$$g(x)+\frac{M_0-M_1}{1+M_1}=0$$

If we write the function of g(x), then;

$$bx^3+x+\frac{M_0-M_1}{1+M_1}=0 \qquad 4.10$$

If we look at the equation (4.10), we can see that there are three equilibrium points dependent on b, M_0 , M_1. These equilibrium points can be found by solving the equation (4.10), y_{eq} is equal to $\dfrac{M_0-M_1}{1+M_1}$

The second case is $-1<g(x)<1$, then $f(g(x))=M_0g(x)$ and if we substitute $f(g(x))$ in (4.9), then we obtain;

$$-g(x)-M_0g(x)=0$$

$$g(x)=0$$

$$bx^3+x=0 \qquad 4.11$$

If we look at the equation (4.11), we can say that there are three equilibrium points, these are;

$$x_{eq1}=0, \qquad x_{eq2}=\sqrt{-\frac{1}{b}}, \qquad x_{eq3}=-\sqrt{-\frac{1}{b}}, \qquad y_{eq}=0$$

The third case is $g(x)\le -1$, then $f(g(x))=M_1g(x)-(M_0-M_1)$ and if we substitute $f(g(x))$ in (4.9), then ;

$$-g(x)-M_1g(x)+(M_0-M_1)=0$$

or

$$g(x)-\frac{M_0-M_1}{1+M_1}=0$$

If we write *g(x)*, then

$$bx^3+x-\frac{M_0-M_1}{1+M_1}=0 \qquad 4.12$$

If we look at the equation (4.12), we can see that there are three equilibrium points dependent on b, M_0 , M_1.. These equilibrium points can be found by solving the equation (4.12), y_{eq} is equal to $-\frac{M_0-M_1}{1+M_1}$

We find the eigenvalues of the system. For this case, we calculate det(λI-J), where J is Jacobean matrix and I is identity matrix. Jacobean matrix is defined as derivative of (4.7) and (4.8) with respect to x and y at the equilibrium point. We can write it as follows.

$$J=\begin{bmatrix} \frac{df_1(x,y)}{dx} & \frac{df_1(x,y)}{dy} \\ \frac{df_2(x,y)}{dx} & \frac{df_2(x,y)}{dy} \end{bmatrix}$$

where, $f_1(x,y) = -y - g(x)$, $f_2(x,y) = y - f(g(x))$

If we substitute $f_1(x,y)$, $f_2(x,y)$ in Jacobean matrix, then we obtain;

$$J = \begin{bmatrix} -g'(x) & -1 \\ -f'(g(x)) & 1 \end{bmatrix}$$

Now, we calculate det(λI-J).

$$\det(\lambda I - J) = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -g'(x) & -1 \\ -f'(g(x)) & 1 \end{bmatrix} = \begin{bmatrix} \lambda + g'(x) & 1 \\ f'(g(x)) & \lambda - 1 \end{bmatrix}$$

$$\det(\lambda I - J) = \lambda^2 + \lambda(g'(x) - 1) - [g'(x) + f'(g(x))]$$

If we make the right side of this equation zero, we can find the roots of it.

$$\lambda^2 + \lambda(g'(x) - 1) - [g'(x) + f'(g(x))] = 0 \qquad 4.13$$

where, $g'(x) = 3bx^2 + 1$ and $f'(g(x))$ is;

$$f'(g(x)) = \begin{cases} M_1 g'(x) & g(x) \geq 1 \\ M_0 g'(x) & -1 < g(x) < 1 \\ M_1 g'(x) & g(x) \leq -1 \end{cases}$$

There are three cases of $f'(g(x))$ and first case is equal to third case, in this case $f'(g(x)) = M_1 g'(x)$ ($g(x) \geq 1$ or $g(x) \leq -1$). If we substitute $f'(g(x))$ in (4.13), then we obtain

$$\lambda^2 + \lambda(g'(x) - 1) - [g'(x) + M_1 g'(x)] = 0$$

or

$$\lambda^2 + \lambda 3bx^2 - (3bx^2 + 1)(1 + M_1) = 0 \qquad 4.14$$

The roots of this equation are;

$$\lambda_{1,2} = \frac{-3bx^2 \pm \sqrt{(3bx^2)^2 + 4(1 + M_1)(1 + 3bx^2)}}{2} \qquad 4.15$$

If b is negative and M_l >-1 then, since the system has saddle points, the equilibrium points must be equal to;

$$-\sqrt{-\frac{1}{3b}} < x_{eq} < \sqrt{-\frac{1}{3b}}$$

and for other cases, the system is unstable.

If b is positive and M_l >-1; the system always has saddle point when the equilibrium points are real. The system will have one equilibrium point at least. For the other cases, the system will be unstable.

Second case is $f'(g(x)) = M_0 g'(x)$ (if $-1 < g(x) < 1$). If we substitute $f'(g(x))$ in (4.13), then we obtain;

$$\lambda^2 + \lambda(g'(x) - 1) - [g'(x) + M_0 g'(x)] = 0$$

or

$$\lambda^2 + \lambda 3bx^2 - (3bx^2 + 1)(1 + M_0) = 0 \qquad 4.16$$

The roots of this equation are ;

$$\lambda_{1,2} = \frac{-3bx^2 \pm \sqrt{(3bx^2)^2 + 4(1 + M_0)(1 + 3bx^2)}}{2} \qquad 4.17$$

We have found that one of the equilibrium points is zero ($x_{eq1} = 0$). If we substitute this equilibrium point in (4.17), then we find;

$$\lambda_{1,2} = \pm\sqrt{1 + M_0} \qquad 4.18$$

If we look at this equation, we can say that the system has one positive real eigenvalue and one negative real eigenvalue when $M_0 > -1$. When $M_0 < -1$, the system has two complex conjugate eigenvalues.

The other equilibrium points are equal to $x_{eq2,3} = \pm\sqrt{-\frac{1}{b}}$. If we substitute these equilibrium points in (4.17), then we obtain;

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9 - 8(1 + M_0)}}{2} \qquad 4.19$$

Although the system has one positive and one negative equilibrium points, the system has same eigenvalues. If we look at the equation (4.19), we can see that the system will have two complex eigenvalues when $M_0 > \frac{1}{8}$. When $-1 < M_0 < \frac{1}{8}$, the system will have two positive real eigenvalues. When $M_0 < -1$, the system will have one positive eigenvalue and one negative eigenvalue.

4.1.2 NUMERICAL ANALYSIS

The system equations have been given in (4.5) and (4.6). For the purpose of numerical simulations we choose $\varepsilon = 1$, $\Omega_1 = 0.11$, $\Omega_2 = 0.6$, $\alpha = 1$, $\beta = 1$, $a = 1$, $f = 0.25$ and M_0, M_1, m, b as our system parameters. The values of system parameters of ε, β, α, Ω_1, Ω_2, f, a given above are the same with the values given in Chapter 2. So by changing M_0, M_1, m and b, we investigate the effects of these parameters on the circuit. First, we choose $m = 0$ then the input signal is a cosine signal. If we choose $b = 0$ then the nonlinear capacitor will be a linear one and two parameter bifurcation diagram in $(M_0 - M_1)$ plane was given in Fig. 2.4. The bifurcation diagram in the $(M_0 - M_1)$ plane for values of b as, 0.01, 0.02, 0.1, -0.01, -0.02, -0.1 are given in Figs 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, respectively. The regions between the darker lines in each diagram show the chaotic region of circuit.

If we compare the Fig. 4.2 with the Fig. 2.4, in this case b=0.01, we can see that the system shows chaotic behavior in four regions and these regions are a little large with respect to Fig.2.4. When M_1 and M_0 is decreased, the chaotic region is increasing. When M_1 is increased (M_1>1), the chaotic region is not changing. If we increase b (b=0.02), for this case, the bifurcation diagram is shown in Fig. 4.3. If we compare two bifurcation diagrams (Fig. 4.2 and Fig. 4.3), we can say that the chaotic region is increased but the chaotic region is pulled towards for increasing values of M_1. When $M_1 \leq -0.47$, there are two chaotic regions but for b=0.01, there are three chaotic regions which are large. If we again increase b (b=0.1), for this case, the bifurcation diagram is shown in Fig. 4.4. As shown in Fig.4.4, the chaotic regions of system are enlarged. There are three chaotic regions at the beginning of chaotic behavior. These regions are increasing, especially the first region when M_1 is increased. When b is increased, another problem occurs except the chaotic behavior. This problem is the operating range of the system which is affected. For instance; for b =0.1 and M_1 is less than -0.5, the circuit does not operate until M_0 = -0.8, and up to M_1 =0.1, there is no chaotic region of system and we can say for the increasing values of b operating region of the system is pulled towards the positive values of M_1.

If we choose b=-0.01, for this case, the bifurcation diagram is shown in Fig. 4.5. If we compare this bifurcation diagram with Fig. 2.4, we can see that there are three decreasing chaotic regions. If we decrease b (b=-0.02), the bifurcation diagram is shown in Fig. 4.6. As shown in Fig. 4.6, There are three chaotic regions which is beginning when $M_1 \geq -0.4$ and $M_0 \leq -1$ and the chaotic region is decreased with respect to b=-0.01. If we decrease b (b=-0.1), the bifurcation diagram is shown in Fig. 4.7. There is only one chaotic region which is small according to another values of b and system shows chaotic behavior when $M_1 \geq 0.0$. From these figures, we can see that the region of chaos is the largest when b=0.0, when no capacitive nonlinearity exist, as b decrease, the chaotic motion is confined to a smaller region. We can say that another thing that M_1 is to be more effective on the chaotic behavior of system.

Second, if we choose m = 0.5 then the input signal is amplitude modulated. If we choose b =0.0 than the two parameter bifurcation diagram in (M_0-M_1) plane will be the same as in Fig. 3.4

If we choose b=0.01, the bifurcation diagram is shown in Fig. 4.8. If we compare this bifurcation diagram with Fig. 3.4, we can see that the chaotic region increase and there is only one chaotic region which is large but when m=0, there are three or more chaotic regions which are narrow. For this case, we can say that these small chaotic regions are connected and enlarged when the input signal becomes multi-frequency. If we increase b (b=0.02 and b=0.1), the bifurcation diagrams are shown in Fig. 4.9 and 4.10. As shown in these figures, the chaotic region increases and this region is pulled towards the positive values of M_1

If we choose b=-0.01, the bifurcation diagram is shown in Fig. 4.11. If we compare this bifurcation diagram with Fig 3.4, we can see that the chaotic region is decreased. If we continue decreasing value of b (b=-0.02, -0.1), for this case the bifurcation diagrams are shown in Fig. 4.12. and Fig. 4.13. As shown these figures, the chaotic region is decreased and, especially for b=-0.1, the chaotic region is very small according to another bifurcation diagrams.

As a result of, we can say that the chaotic region of the system enlarge to be independent of other parameters when $m\neq0$. When b is increased positively, the chaotic region increase and operating range of the system is pulled towards the positive values of M_1. When b is decreased negatively, the chaotic region of the system decrease thus, addition of the capacitive nonlinearity (for negative values of b) plays limiting role on chaos.

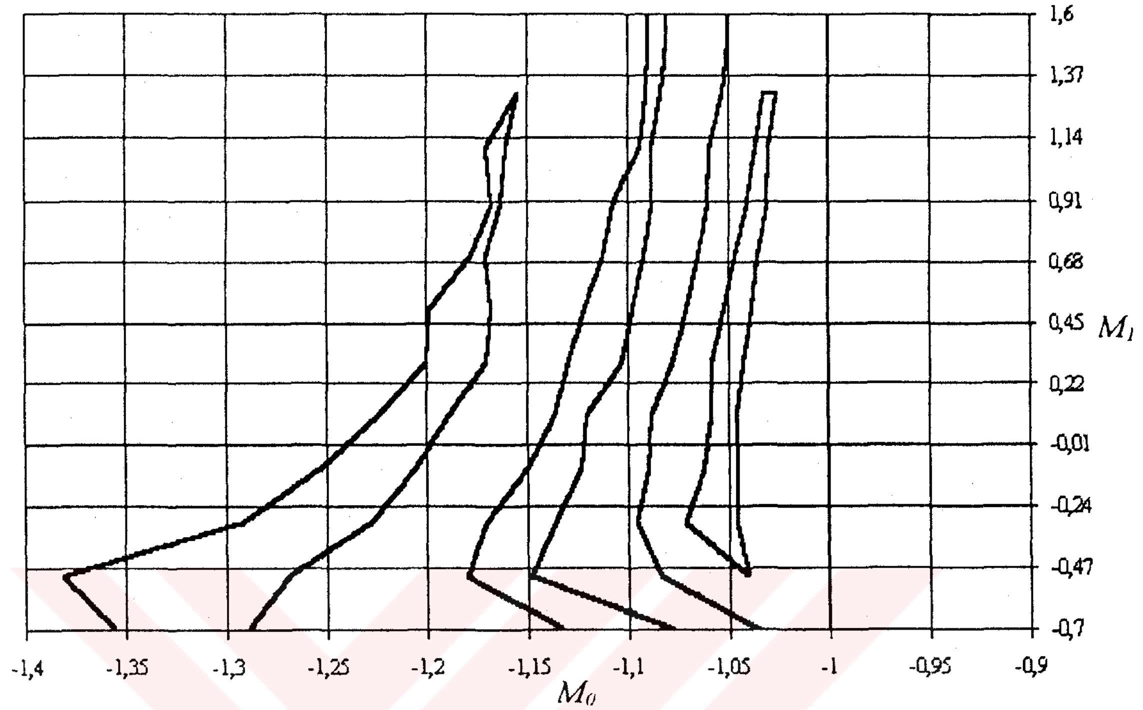

Fig. 4.2 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = 0.01

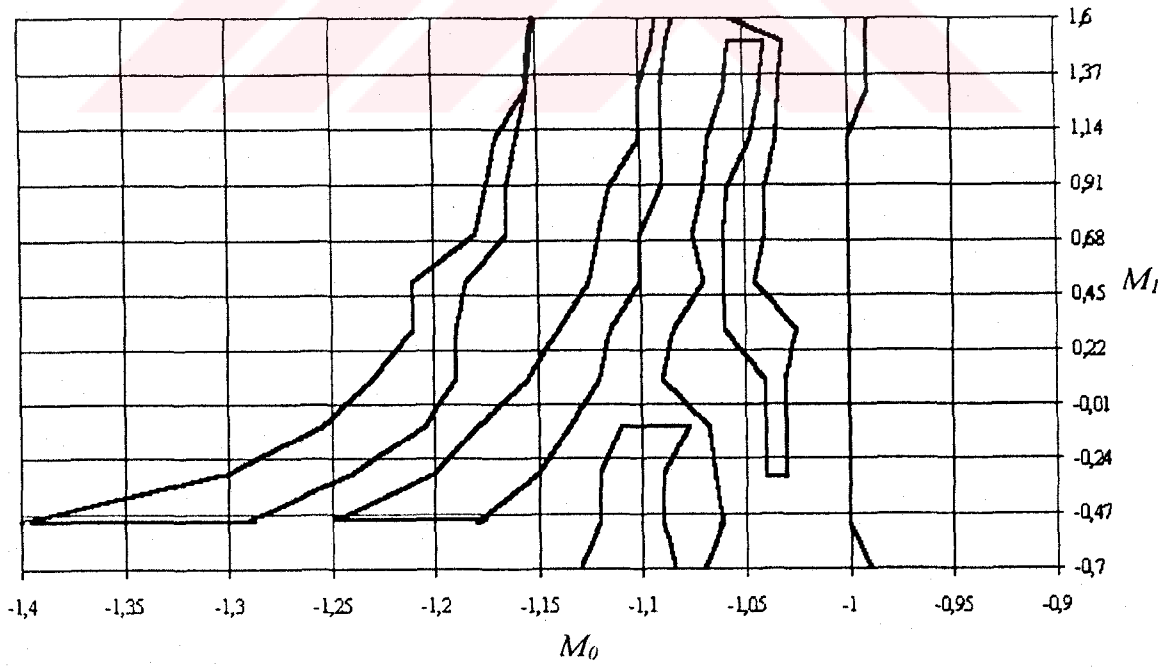

Fig. 4.3 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = 0.02

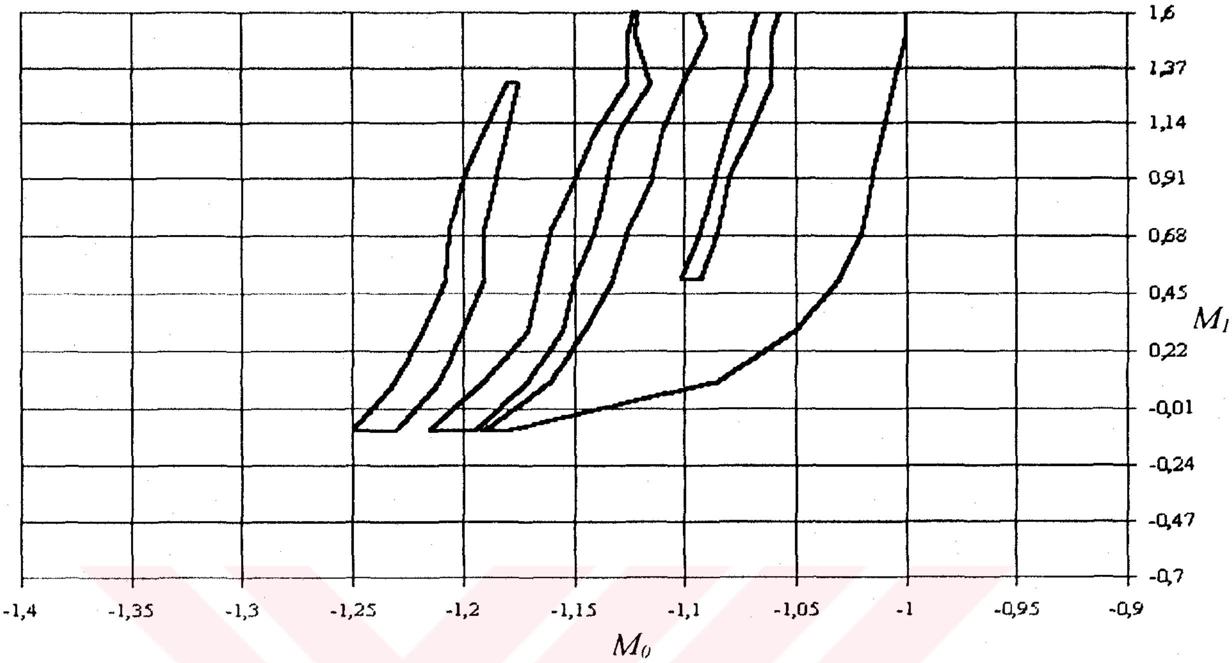

Fig. 4.4 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = 0.1

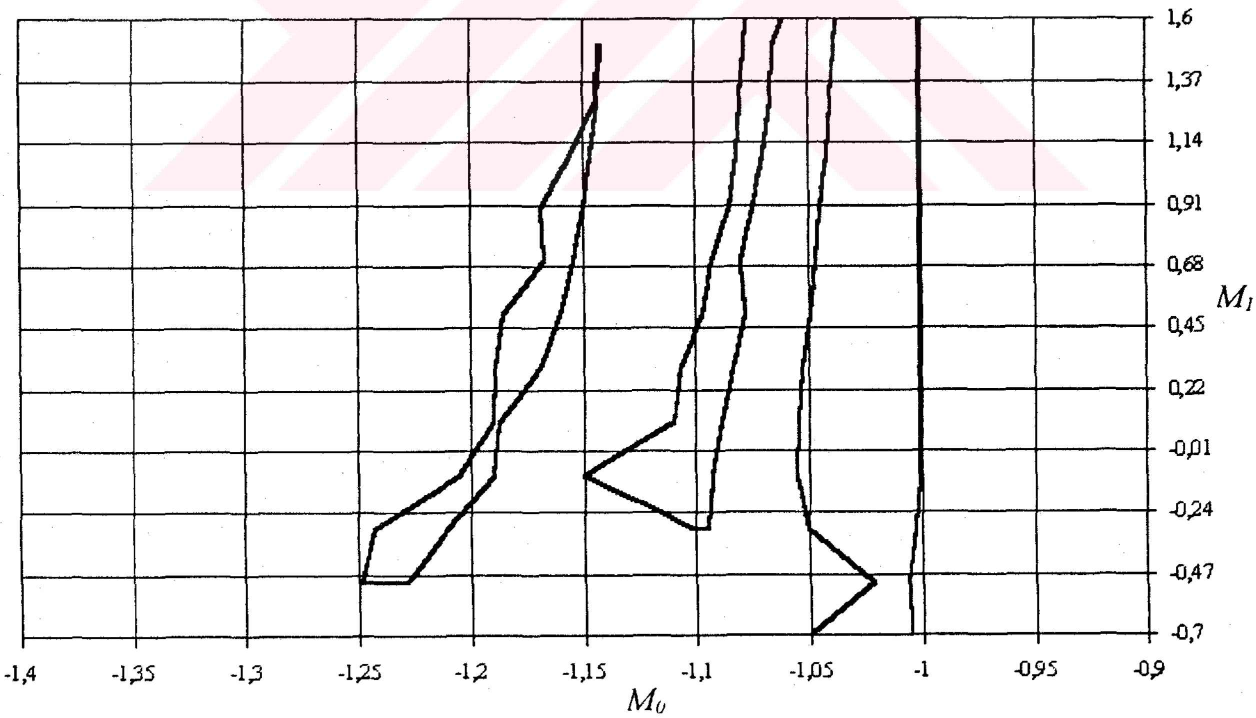

Fig. 4.5 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = - 0.01

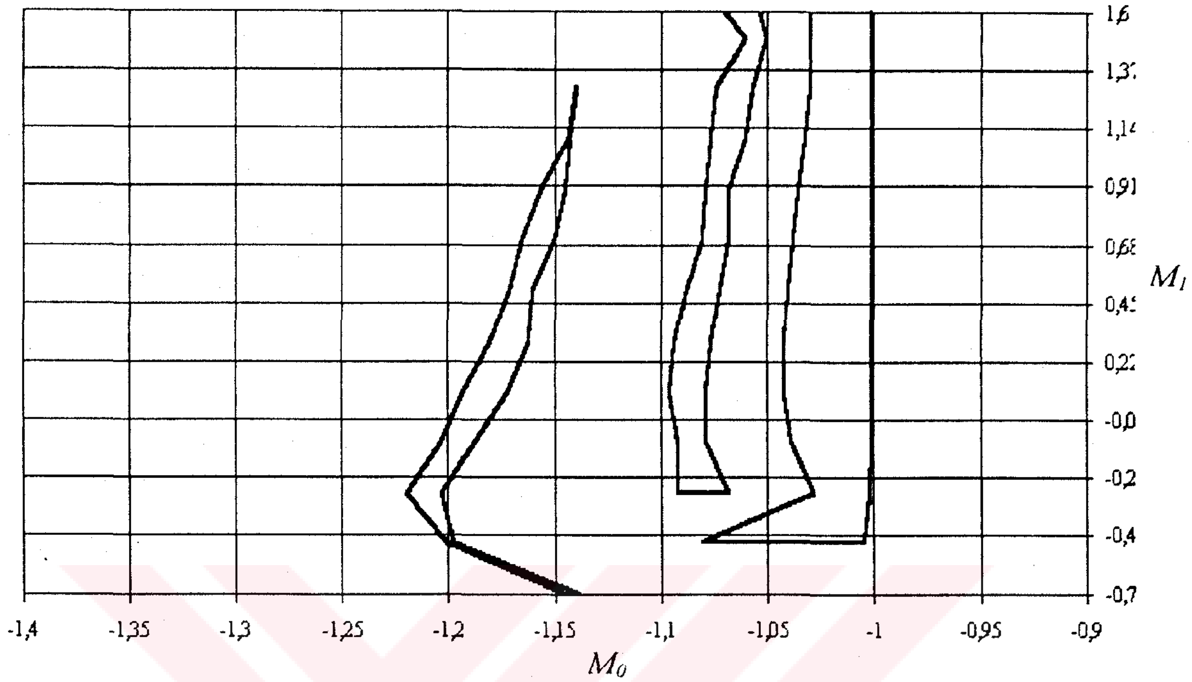

Fig. 4.6 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = -0.02

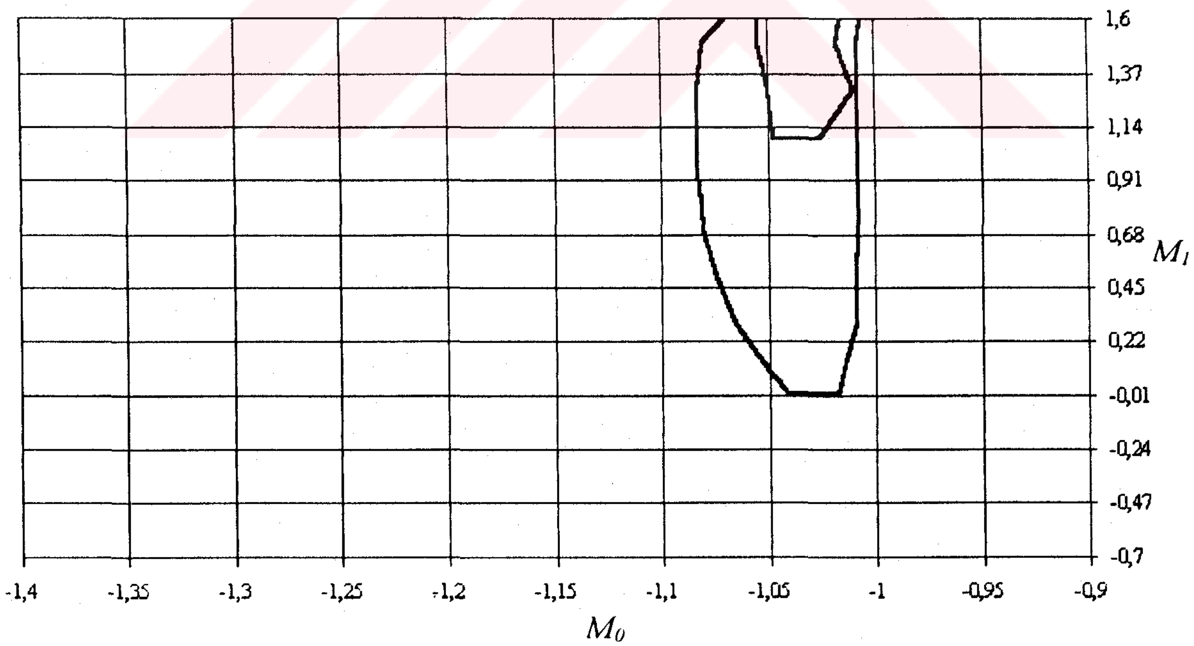

Fig. 4.7 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = -0.1

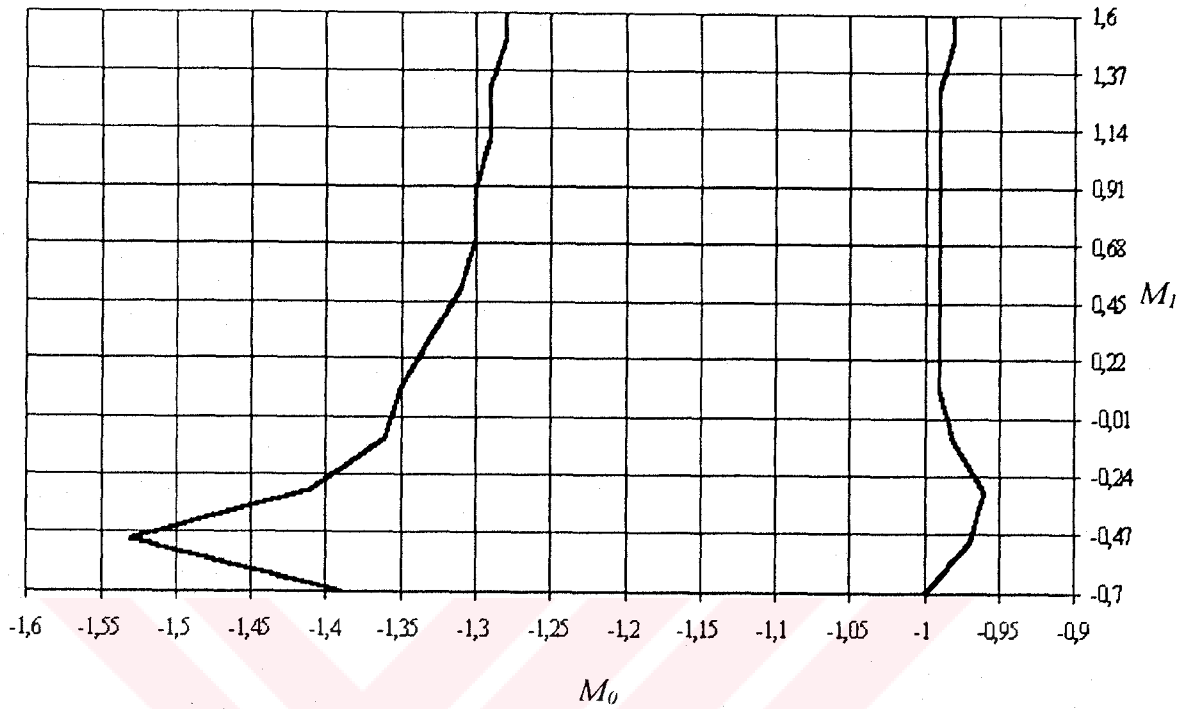

Fig. 4.8 Two parameter bifurcation diagram in the (M_0 - M_1) plane for b = 0.01 and m=0.5

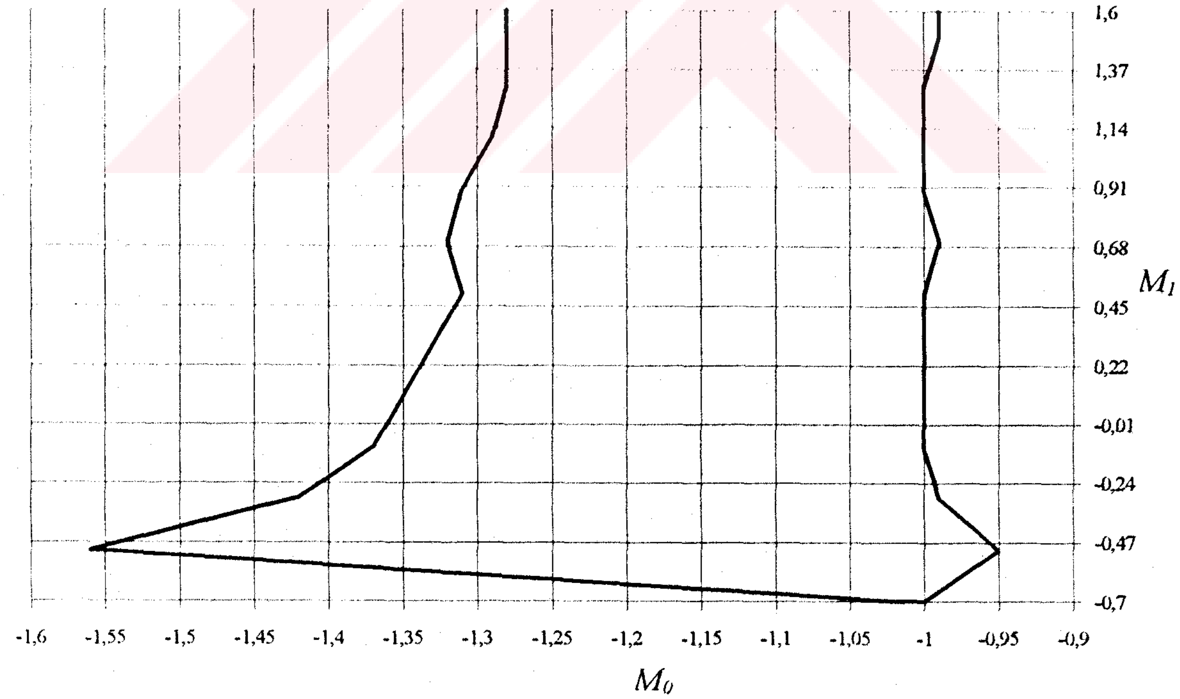

Fig. 4.9 Two parameter bifurcation diagram in the (M_0 - M_1) plane for b = 0.02 and m=0.5

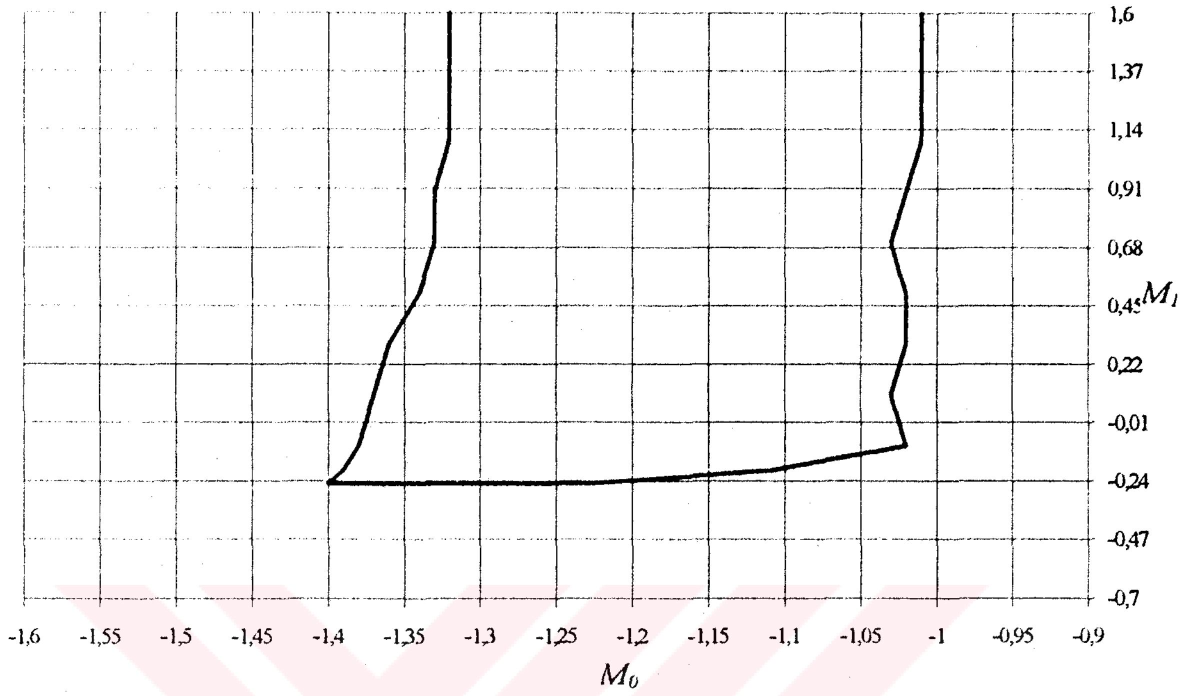

Fig. 4.10 Two parameter bifurcation diagram in the (M_0 - M_1) plane for b = 0.1 and m=0.5

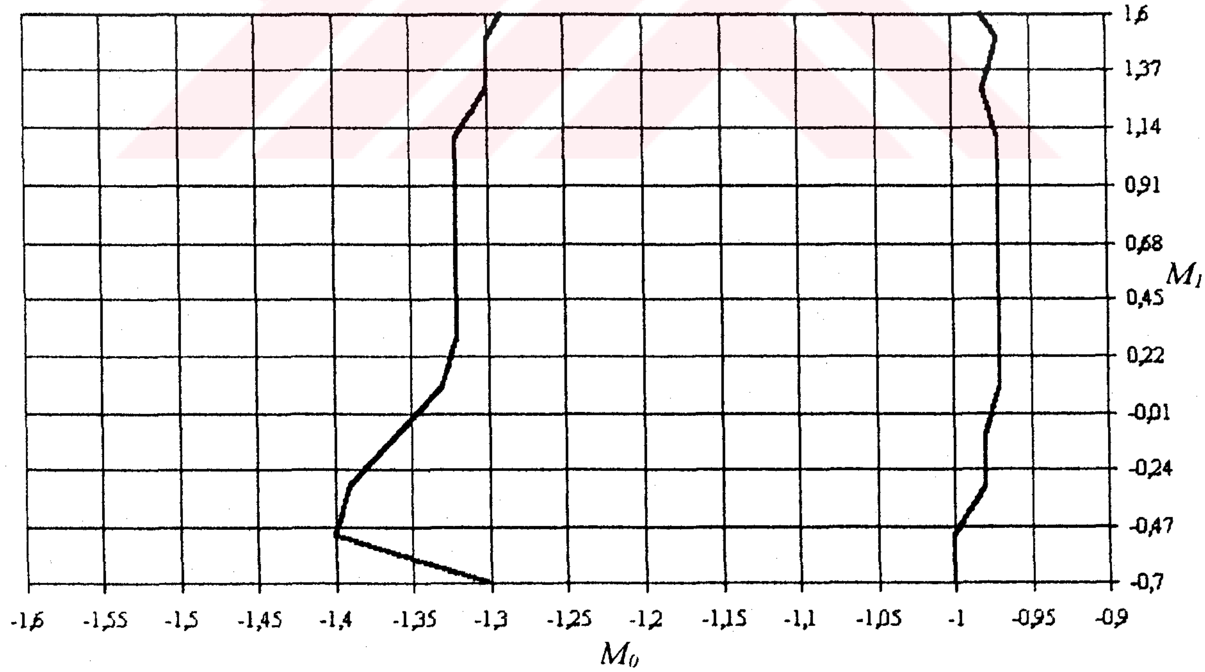

Fig. 4.11 Two parameter bifurcation diagram in the (M_0 - M_1) plane for b = -0.01 and m=0.5

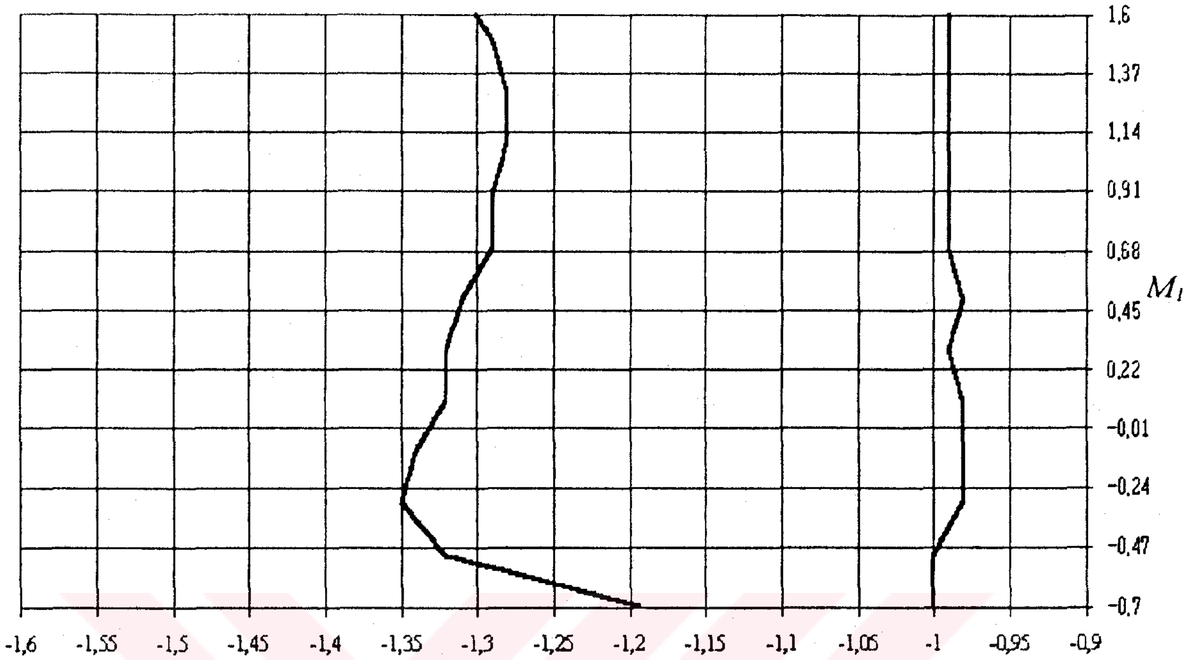

Fig. 4.12 Two parameter bifurcation diagram in the (M_0 - M_1) plane for b = -0.02 and m=0.5

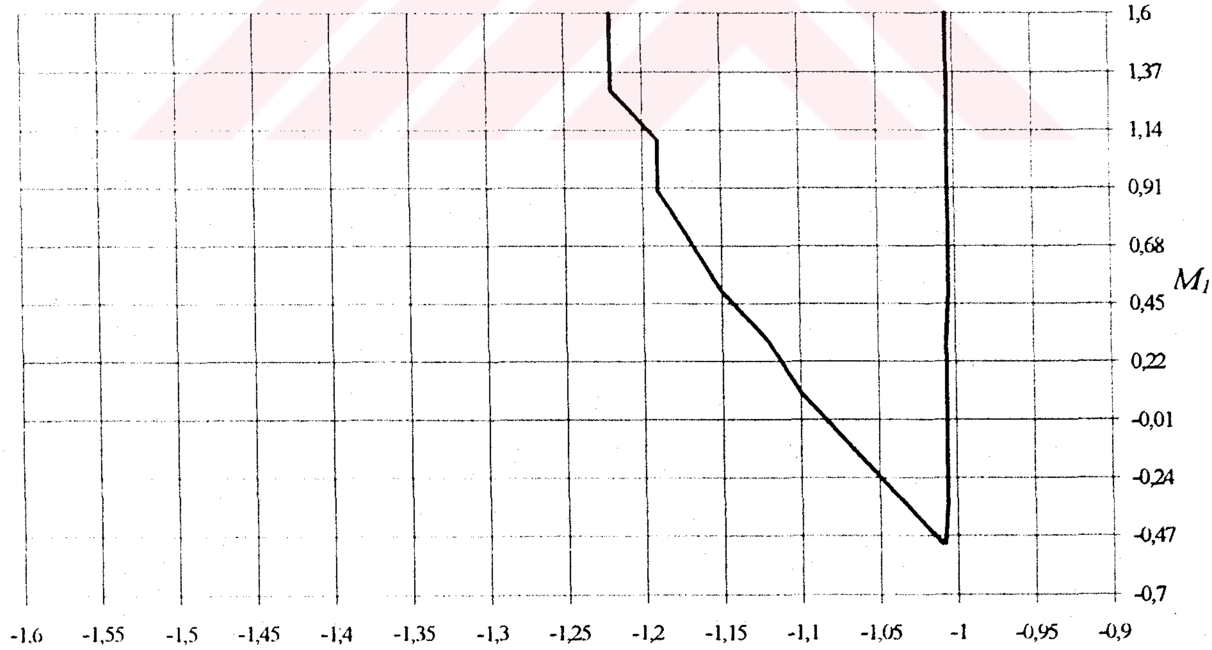

Fig. 4.13 Two parameter bifurcation diagram in the (M_0 - M_1) plane for b = -0.1 and m=0.5

CHAPTER 5

INTERACTION OF INDUCTIVE AND RESISTIVE NONLINEARITIES IN THE CIRCUIT

Interaction of capacitive and resistive nonlinearities in the circuit has been studied in Chapter 4 and the effects of nonlinear capacitor on the circuit was discussed. In this chapter, we numerically investigate the interaction of inductive and resistive nonlinearities in the circuit and discuss the effects of nonlinear inductor on the chaotic behavior of the circuit. The original simple nonautonomous circuit (shown in Fig. 2.1) is modified by changing the linear inductor in circuit with a nonlinear one

5.1 ANALYSIS

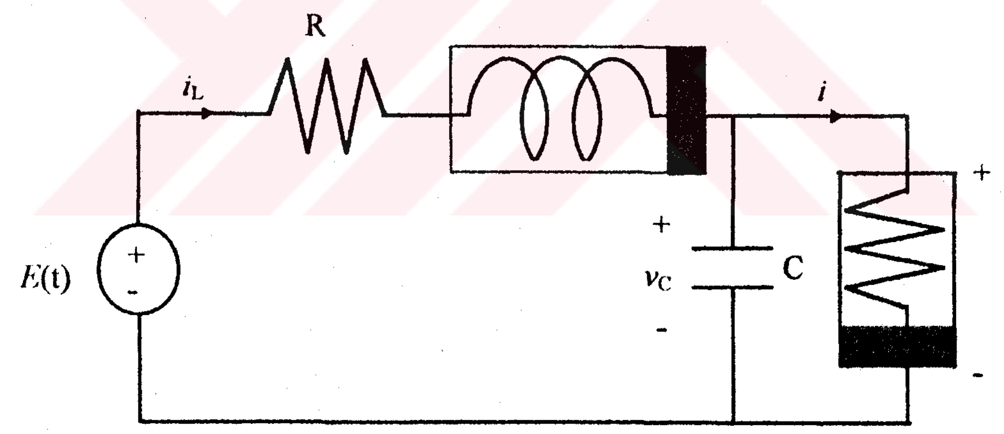

Fig. 5.1 Modified circuit realization

The circuit realization is shown in Fig. 5.1. It contains a linear capacitor, a linear resistor, a nonlinear resistor (Chua's diode), an external forcing, and a nonlinear inductor. This nonlinear inductor is magnetic flux controlled and its $i-\Phi$ characteristics is given by

$i_L = a_1\Phi + a_3\Phi^3$. By applying Kirchoff's laws to this circuit, the governing equations for the voltage v_C across the capacitor C and the current i_L through the nonlinear inductor L are given by the following:

$$E(t) = v_R + v_L + v_C \qquad 5.1$$

$$i_L = C\frac{dv_C}{dt} + g(v_C) \qquad 5.2$$

$$E(t) = Ri_L + \frac{d\Phi}{dt} + v_C$$

$$C\frac{dv_C}{dt} = i_L - g(v_C)$$

Here $\frac{d\Phi}{dt}$ is the voltage across the nonlinear inductor and Φ is magnetic flux. The term

$$g(v_C) = G_1 v_C + 0.5(G_0 - G_1)\left[\left|v_C + B_p\right| - \left|v_C - B_p\right|\right]$$

is the mathematical representation of characteristic curve of Chua's diode, G_0, G_1 and B_p are , respectively, the slopes and breakpoint, and the current i_L is;

$$i_L = a_1\Phi + a_3\Phi^3$$

$E(t)$ is the external signal and it will be a periodic signal and multi-frequency signal according to two cases. Let us now assume that the input signal is amplitude modulated signal. We again write above equations;

$$C\frac{dv_C}{dt} = i_L - G_1 v_C - 0.5(G_0 - G_1)\left[\left|v_C + B_p\right| - \left|v_C - B_p\right|\right] \qquad 5.3$$

$$\frac{d\Phi}{dt} = -R(a_1\Phi + a_3\Phi^3) - v_C + E_m(1 + mCos\omega_1 t)Cos\omega_2 t \qquad 5.4$$

At this point, let us use new variables as, $v_C = xx_0$, $\Phi = y_0 y$, $\tau = \omega_0 t$. Here ω_0, x_0, y_0 are arbitrary frequencies voltage and magnetic flux scaling factors, respectively, and x, y are the new respective nondimensional variables. If we substitute these variables in (5.3), (5.4), then

$$C\frac{d(x_0x)}{d\left(\frac{\tau}{\omega_0}\right)} = a_1y_0y + a_3y_0^3y^3 - \left(G_1x_0x + 0.5(G_0 - G_1)\left[\left|x_0x + B_p\right| - \left|x_0x - B_p\right|\right]\right)$$

$$\frac{d(y_0y)}{d\left(\frac{\tau}{\omega_0}\right)} = -R(a_1y_0y + a_3y_0^3y^3) - x_0x + E_m(1 + mCos(\omega_1/\omega_0)\tau)Cos(\omega_2/\omega_0)\tau$$

$$Cx_0\omega_0\frac{dx}{d\tau} = a_1y_0y + a_3y_0^3y^3 - \left(G_1x_0x + 0.5(G_0 - G_1)\left[\left|x_0x + B_p\right| - \left|x_0x - B_p\right|\right]\right)$$

$$y_0\omega_0\frac{dy}{d\tau} = -R(a_1y_0y + a_3y_0^3y^3) - x_0x + E_m(1 + mCos(\omega_1/\omega_0)\tau)Cos(\omega_2/\omega_0)\tau$$

If we divide both of two equations by $Cx_0\omega_0$ and $y_0\omega_0$ first and second differential equations, respectively;

$$\frac{dx}{d\tau} = \frac{a_1y_0}{Cx_0\omega_0}y + \frac{a_3y_0^3}{Cx_0\omega_0}y^3 - \frac{1}{Cx_0\omega_0}\left(G_1x_0x + 0.5(G_0 - G_1)\left[\left|x_0x + B_p\right| - \left|x_0x - B_p\right|\right]\right)$$

$$\frac{dy}{d\tau} = -\frac{R}{y_0\omega_0}(a_1y_0y + a_3y_0^3y^3) - \frac{x_0}{y_0\omega_0}x + \frac{E_m}{y_0\omega_0}(1 + mCos(\omega_1/\omega_0)\tau)Cos(\omega_2/\omega_0)\tau$$

$$\frac{dx}{d\tau} = \frac{a_1y_0}{Cx_0\omega_0}y + \frac{a_3y_0^3}{Cx_0\omega_0}y^3 - \frac{G_1x_0}{Cx_0\omega_0}x - 0.5\frac{(G_0 - G_1)}{Cx_0\omega_0}\left[\left|x_0x + B_p\right| - \left|x_0x - B_p\right|\right]$$

$$\frac{dy}{d\tau} = -\frac{Ra_1y_0}{y_0\omega_0}y - \frac{Ra_3y_0^3}{y_0\omega_0}y^3 - \frac{x_0}{y_0\omega_0}x + \frac{E_m}{y_0\omega_0}(1 + mCos(\omega_1/\omega_0)\tau)Cos(\omega_2/\omega_0)\tau$$

If we divide and multiply first equation by R and we divide and multiply *y* variable in the second equation by x_0, then;

$$\frac{dx}{d\tau} = \frac{Ra_1y_0}{RCx_0\omega_0}y + \frac{Ra_3y_0^3}{RCx_0\omega_0}y^3 - \frac{RG_1x_0}{RCx_0\omega_0}x - 0.5\frac{R(G_0 - G_1)}{RCx_0\omega_0}\left[\left|x_0x + B_p\right| - \left|x_0x - B_p\right|\right]$$

$$\frac{dy}{d\tau} = -\frac{Ra_1y_0x_0}{x_0y_0\omega_0}y - \frac{Ra_3y_0^3x_0}{x_0y_0\omega_0}y^3 - \frac{x_0}{y_0\omega_0}x + \frac{E_m}{y_0\omega_0}(1 + mCos(\omega_1/\omega_0)\tau)Cos(\omega_2/\omega_0)\tau$$

$$\frac{dx}{d\tau}=\frac{1}{RC\omega_0}\left[\frac{a_1y_0R}{x_0}y+\frac{Ra_3y_0^3}{x_0}y^3-RG_1x-0.5R(G_0-G_1)\left[\left|x+\frac{B_p}{x_0}\right|-\left|x-\frac{B_p}{x_0}\right|\right]\right]$$

$$\frac{dy}{d\tau}=\frac{x_0}{y_0\omega_0}\left[-\frac{Ra_1y_0}{x_0}y-\frac{Ra_3y_0^3}{x_0}y^3-x\right]+\frac{E_m}{y_0\omega_0}(1+mCos(\omega_1/\omega_0)\tau)Cos(\omega_2/\omega_0)\tau$$

Now, we again change the variables as;

$\beta=\frac{x_0}{y_0\omega_0}$, $b=\frac{a_3y_0^3R}{x_0}$, $M_0=G_0R$, $\varepsilon=\frac{B_p}{x_0}$,

$\alpha=\frac{1}{RC\omega_0}$, $f=\frac{E_m}{y_0\omega_0}$, $\Omega_1=\frac{\omega_1}{\omega_0}$,

$a=\frac{a_1y_0R}{x_0}$, $M_1=RG_1$, $\Omega_2=\frac{\omega_2}{\omega_0}$,

Hence, the system of (5.3) and (5.4) are transformed into new dimensionless system;

$$\frac{dx}{d\tau}=\alpha[g(y)-g(x)] \qquad 5.5$$

$$\frac{dy}{d\tau}=\beta[-g(y)-x]+f(1+mCos\Omega_1\tau)Cos\Omega_2\tau \qquad 5.6$$

where, $g(y)=ay+by^3$, and $g(x)=M_1x+0.5(M_0-M_1)[|x+\varepsilon|-|x-\varepsilon|]$

The differential equations (5.5) and (5.6) now depend on the parameters β, α, M_0, M_1, Ω_1, Ω_2, m, f, a, and b. Here, b is the coefficient of cubic nonlinearity for the nonlinear inductor. If we choose b =0.0, the differential equations (5.5) and (5.6) will be same as (2.3) and (2.4), thus the nonlinear inductor will be omitted from the circuit given in Fig. 5.1 and the circuit will be same as given in Fig. 2.1.

5.1.1 EQUILIBRIUM POINT ANALYSIS

We will find the equilibrium points of the differential equations (5.5) and (5.6). For this case, we assume that the input external signal is zero and write $a = 1$, $\beta = 1$, $\alpha = 1$, $\varepsilon = 1$, where the obtained equations will be as follows.

$$\frac{dx}{d\tau} = [g(y) - g(x)] \qquad 5.7$$

$$\frac{dy}{d\tau} = [-g(y) - x] \qquad 5.8$$

The equilibrium points of (5.7) and (5.8) are identified by setting $\frac{dy}{d\tau} = \frac{dx}{d\tau} = 0$, and obtained equations are;

$$-g(y) - x = 0$$
$$g(y) - g(x) = 0$$

If we add both sides of two equations above, then obtained equation is ;

$$-g(x) - x = 0 \qquad 5.9$$

where, $g(x) = M_1 x + 0.5(M_0 - M_1)\left[|x+1| - |x-1|\right]$, $g(y) = by^3 + y$. If we rewrite *g(x)* clearly, then

$$g(x) = \begin{cases} M_1 x + (M_0 - M_1) & x \ge 1 \\ M_0 x & -1 < x < 1 \\ M_1 x - (M_0 - M_1) & x \le -1 \end{cases}$$

As shown above, there are three cases of *g(x)* . The equilibrium points will be found for each cases of *g(x)*. First, $x \ge 1$ then $g(x) = M_1 x + (M_0 - M_1)$ and if we substitute *g(x)* in the equation (5.9), then we can obtain the below equation:

$$-M_1 x - (M_0 - M_1) - x = 0$$

We can find the equilibrium point *x* from this equation and it is;

$$x_{eq1} = -\frac{M_0 - M_1}{1 + M_1} \qquad 5.10$$

For finding y equilibrium points, we use the below equation;

$$g(y) = -x$$

or

$$by^3 + y = -x$$

and if we substitute x_{eq1} at above equation, then

$$by^3 + y - \frac{M_0 - M_1}{1 + M_1} = 0 \qquad 5.11$$

If we look at the equation (5.11), we can see that there are three equilibrium points for y This equation is solved numerically according to M_0, M_1, b .

Second case is $g(x) = M_0 x$ when $-1 < x < 1$ and if we substitute g(x) in (5.9), then we obtain;

$$-x - M_0 x = 0$$

The equilibrium point for x is equal to;

$$x_{eq2} = 0 \qquad 5.12$$

The equilibrium points for y can be found by using below equation;

$$g(y) = -x$$

or

$$by^3 + y = 0 \qquad 5.13$$

In this case, the equilibrium points for y are;

$$y_{eq1} = 0, \qquad y_{eq2} = -\sqrt{-\frac{1}{b}}, \qquad y_{eq3} = \sqrt{-\frac{1}{b}}$$

Third case is $g(x) = M_1 x - (M_0 - M_1)$ when $x \le -1$ and if we substitute g(x) in (5.9), then the obtained equation is;

$$-M_1 x + (M_0 - M_1) - x = 0$$

We can find x equilibrium point from this equation and the equilibrium point is;

$$x_{eq1} = \frac{M_0 - M_1}{1 + M_1} \qquad 5.14$$

For finding y equilibrium points, we use the below equation;

$$g(y) = -x$$

or

$$by^3 + y = -x$$

and if we substitute x_{eq1} at the above equation, then

$$by^3 + y + \frac{M_0 - M_1}{1 + M_1} = 0 \qquad 5.15$$

If we look at the equation (5.15), we can say that there are three equilibrium points for y. This equation is solved numerically according to M_0, M_1, b.

We will find the eigenvalues of the system. For this case, we calculate det(λI-J) where J is Jacobean matrix and I is identity matrix. Jacobean matrix is defined as derivative of (5.7) and (5.8) with respect to x an y at the equilibrium point. We can write it as follows.

$$J = \begin{bmatrix} \frac{df_1(x,y)}{dx} & \frac{df_1(x,y)}{dy} \\ \frac{df_2(x,y)}{dx} & \frac{df_1(x,y)}{dy} \end{bmatrix}$$

where, $f_1(x,y) = -g(y) - x$, $f_2(x,y) = g(y) - g(x)$

If we substitute $f_1\ (x,y)$ and $f_2\ (x,y)$ in Jacobean matrix, then we obtain;

$$J = \begin{bmatrix} -1 & -g'(y) \\ -g'(x) & g'(y) \end{bmatrix}$$

Now, we calculate det(λI-J)

$$\det(\lambda I - J) = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & -g'(y) \\ -g'(x) & g'(y) \end{bmatrix} = \begin{bmatrix} \lambda + 1 & g'(y) \\ g'(x) & \lambda - g'(y) \end{bmatrix}$$

$$\det(\lambda I - J) = \lambda^2 + \lambda(1 - g'(y)) - [g'(y) + g'(x)g'(y)]$$

If we make the right side of this equation zero, we can find the roots of it;

$$\lambda^2 + \lambda(1-g'(y)) - [g'(y) + g'(x)g'(y)] = 0 \tag{5.16}$$

where $g'(y) = by^3 + y$ and $g'(x)$ is;

$$g'(x) = \begin{cases} M_1 & x \geq 1 \\ M_0 & -1 < x < 1 \\ M_1 & x \leq -1 \end{cases}$$

If we look at $g'(x)$ function, we can see that there are three cases of it, when $x \geq 1$ and $x \leq -1$, $g'(x)=M_1$. If we substitute $g'(x)$ in (5.16), then the obtained equation is;

$$\lambda^2 - \lambda 3by^2 - [(1+3by^2)(1+M_1)] = 0 \tag{5.17}$$

The roots of this equation are;

$$\lambda_{1,2} = \frac{3by^2 \pm \sqrt{(-3by^2)^2 + 4(1+M_1)(1+3by^2)}}{2} \tag{5.18}$$

If we look at the equation, we can say that when b is positive and M_1>-1, there are three cases of the system. Since the system has saddle points, the equilibrium points (y) must equal to;

$$-\sqrt{\frac{1}{3b}} < y_{eq1} < \sqrt{\frac{1}{3b}}$$

For the other cases, the system is stable.

If b is negative and M_1 >-1, the system always has saddle point when the equilibrium points are real. The system will have one equilibrium point at least. For the other cases, the system will be stable.

When -1< x <1, $g'(x)=M_0$ and if we substitute $g'(x)$ in (5.16), then obtained equation is;

$$\lambda^2 - \lambda 3by^2 - [(1+3by^2)(1+M_0)] = 0 \tag{5.19}$$

The roots of this equation are;

$$\lambda_{1,2} = \frac{3by^2 \pm \sqrt{(-3by^2)^2 + 4(1+M_0)(1+3by^2)}}{2} \tag{5.20}$$

We have found that one of the equilibrium points is zero (y_{eq1} = 0) for this case. If we substitute this equilibrium point in (5.20), we obtain;

$$\lambda_{1,2} = \pm\sqrt{1+M_0} \qquad 5.21$$

From this equation, the system will have one positive real eigenvalue and one negative real eigenvalue when M_0 > -1. When M_0 < -1, the system will have two complex conjugate eigenvalues.

The other equilibrium points are equal to $y_{eq2,3} = \pm\sqrt{-\frac{1}{b}}$ and if we substitute this equilibrium points in (5.16), then we obtain,

$$\lambda_{1,2} = \frac{-3\pm\sqrt{1-8M_0}}{2} \qquad 5.22$$

If we look at the above equation, we can see that the system will have two complex eigenvalues when $M_0 > \frac{1}{8}$. In this case, the system will be stable. When $-1 < M_0 < \frac{1}{8}$, the system will have two negative real eigenvalues. When $M_0 < -1$, the system will have one positive eigenvalue and one negative eigenvalue.

5.1.2 NUMERICAL ANALYSIS

The system of equations is given in (5.5) an (5.6). For the purpose of numerical simulations we write $\varepsilon = 1$, $\Omega_1 = 0.11$, $\Omega_2 = 0.6$, $a = 1$, $\beta = 1$, $\alpha = 1$, $f = 0.25$. These values of system parameters are the same with the values given in Chapters 2, 3 and 4. M_0, M_1, m and b as the system parameters and by changing these parameters, we investigate the effects of these parameters on the chaotic behavior of the circuit. If we now choose $m = 0$, $b = 0$ then the input signal will be a periodic signal (cosine signal) and the nonlinear inductor will be a linear one. In this case, two parameter bifurcation diagram in the (M_0 -M_1) plane was obtained in Chapter 2 and it was given in Fig. 2.4.

Two parameter bifurcation diagram in the (M_0 - M_l) plane for values of b as, 0.01, 0.02, 0.1, 0.5, -0.01, -0.02, -0.1 are given in Figs. 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8. The regions between the darker lines in each diagram show the chaotic region of the circuit.

If we choose b=0.01, for this case the bifurcation diagram is shown in Fig.5.2. If we increase b (b=0.02), for this case, the bifurcation diagram is shown in Fig 5.3. When we compare these bifurcation diagrams, we can see that the chaotic regions are decreased. When M_1 <-0.7 for b=0.02 , there is not any chaotic region. If we increase b (b=0.1), for this case the bifurcation diagram is shown in Fig.5.4. If we compare this bifurcation diagram with Fig. 5.3, we can say that the chaotic regions of system are decreased. If we again increase b (b=0.5) for this case the bifurcation diagram is shown in Fig. 5.5. From this bifurcation diagram, we can see that there are two chaotic regions which are very small and there is not any chaotic region when M_1 < -0.4 according to other bifurcation diagrams.

If we choose b=-0.01, for this case the bifurcation diagram is given in Fig.5.6. If we compare this bifurcation diagram with Fig 2.4, we can see that there are three chaotic regions which are large with respect to Fig.2. 4. The chaotic regions are the largest when -0.47<M_1 <1. If we decrease b (b=-0.02), for this case, the bifurcation diagram is shown in Fig. 5.7. If we compare two bifurcation diagrams (Fig. 5.6 and Fig. 5.7), we can say that the chaotic regions are increased a little bit, but the chaotic regions are pulled towards increasing values of M_1 . When M_1 <-0.6, there is not any chaotic region. If we decrease b (b=-0.1) , for this case, the bifurcation diagram is shown in Fig.5.8. As shown in Fig. 5.8, there are three chaotic regions,. one of them is large with respect to others and there is not a chaotic region when M_1 <0.20.

When b is decreased, another problem occurs except the chaotic behavior. This problem is the operating range of the system which is affected. For instance; for b = -0.1 and M_1

is less than 0, the circuit doesn't operate until M_0 =-0.07, and up to M_l= 0.2, there is no chaotic region of system and we can say for the decreasing negative values of b operating region of the system is pulled towards the positive values of M_1.

If we choose m =0.5 and b =0.0 then the input signal will be amplitude modulated signal. For this case, two parameter bifurcation diagram in the (M_0 -M_l) plane was given in Fig.3.4. If we fix m =0.5 and change the values of b then two parameter bifurcation diagram in the (M_0 -M_l) plane for different values of b as, 0.01, 0.02, 0.1, 0.5, -0.01, -0.02, -0.1 are given in Figs. 5.9, 5.10, 5.11, 5.12, 5.13, 5.14, 5,15. If we compare this bifurcation diagrams with Figs. 3.4, we can obtain same results with the obtained results for m=0.0, but the chaotic region of the circuit for m =0.5 is larger than for m =0.0 and there is one chaotic region.

As a result, we can say that the chaotic region of the system enlarges independent on other parameters when $m \neq 0$. When b is increased positively, the chaotic region of system decrease. When b is decreased negatively, the chaotic region is increased and the operating range of the system is pulled towards the positive values of M_1 . Thus, addition of the inductive nonlinearity plays effective role on chaos.

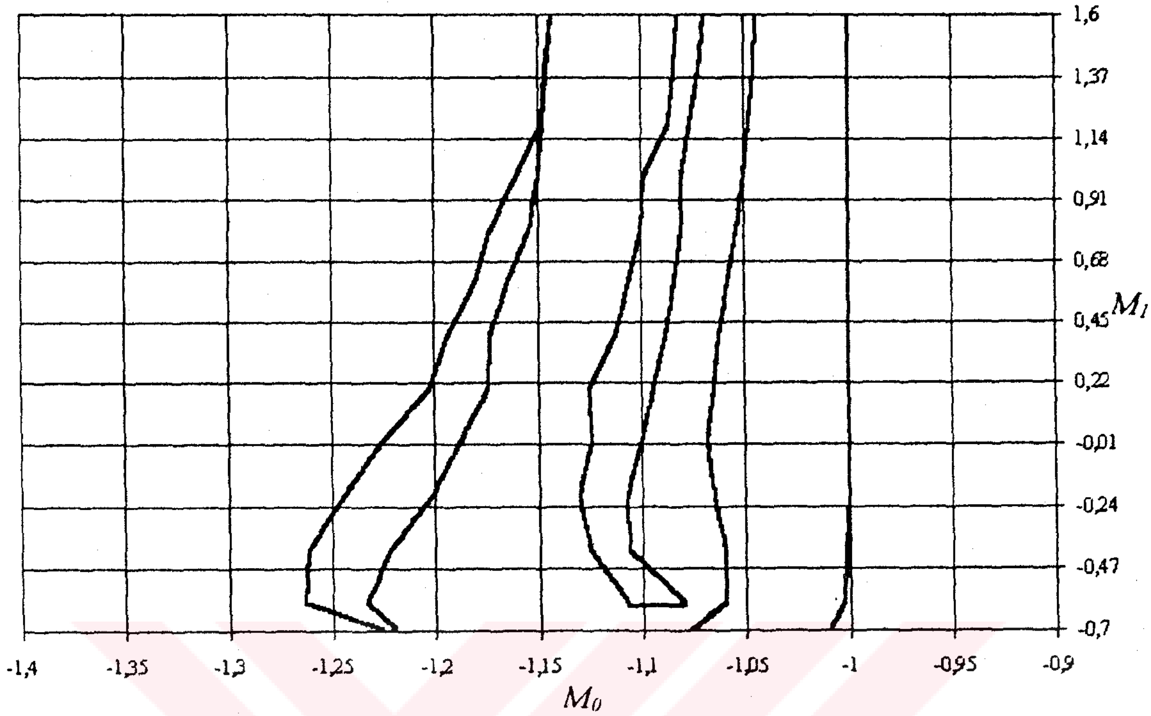

Fig. 5.2 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b =0.01

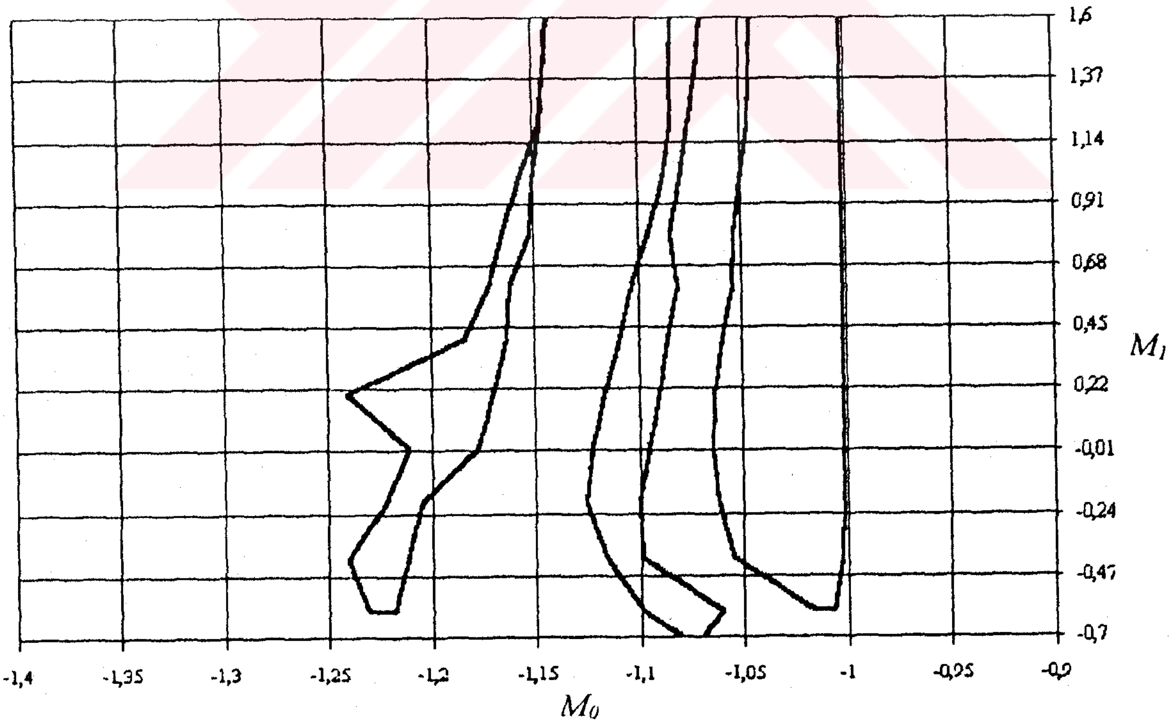

Fig. 5.3 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b =0.02

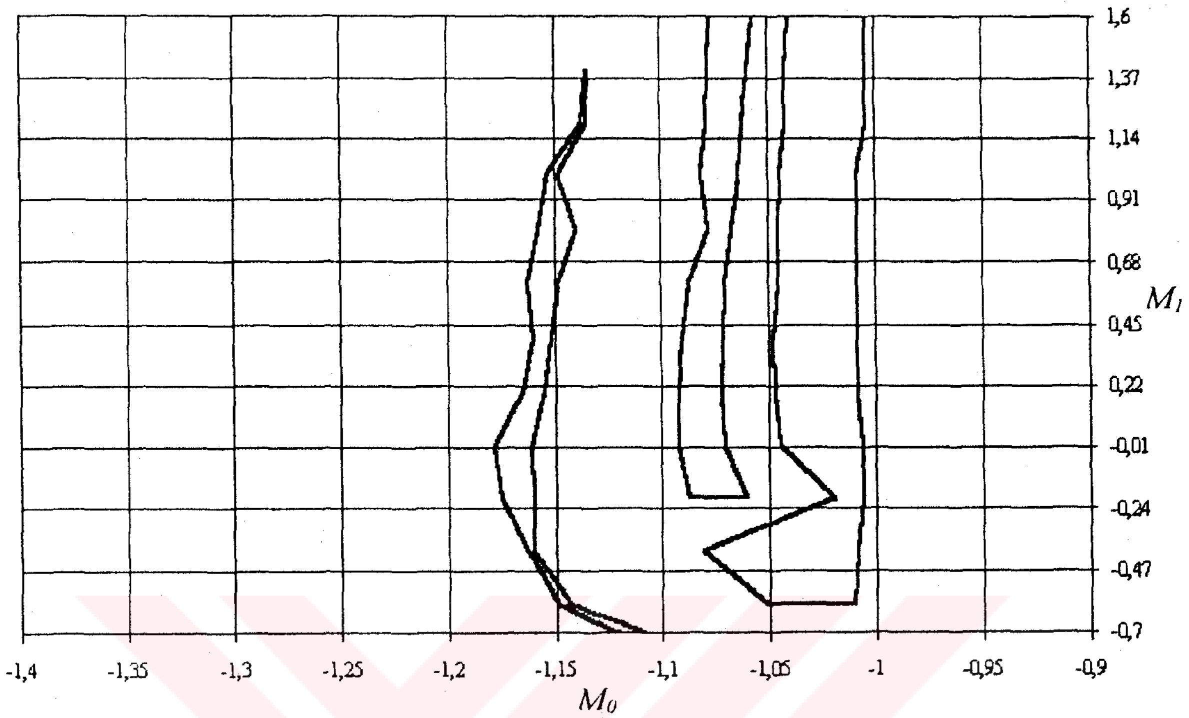

Fig. 5.4 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b =0.1

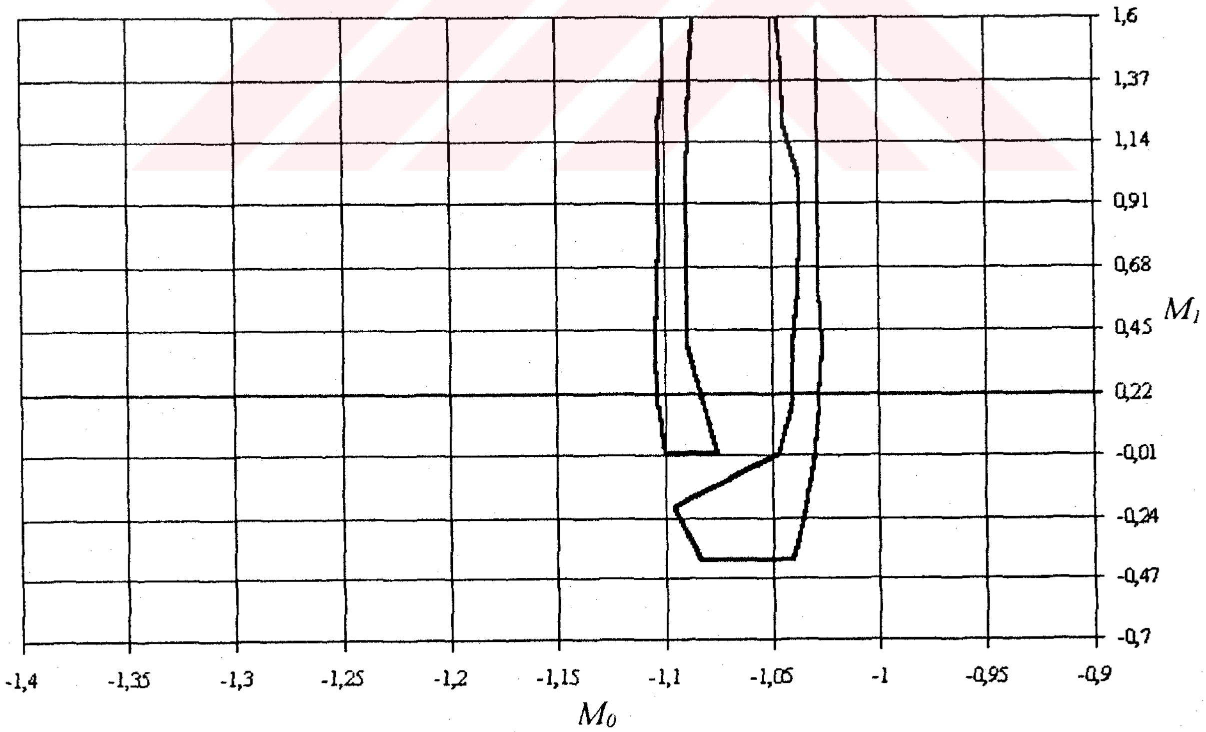

Fig. 5.5 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b =0.5

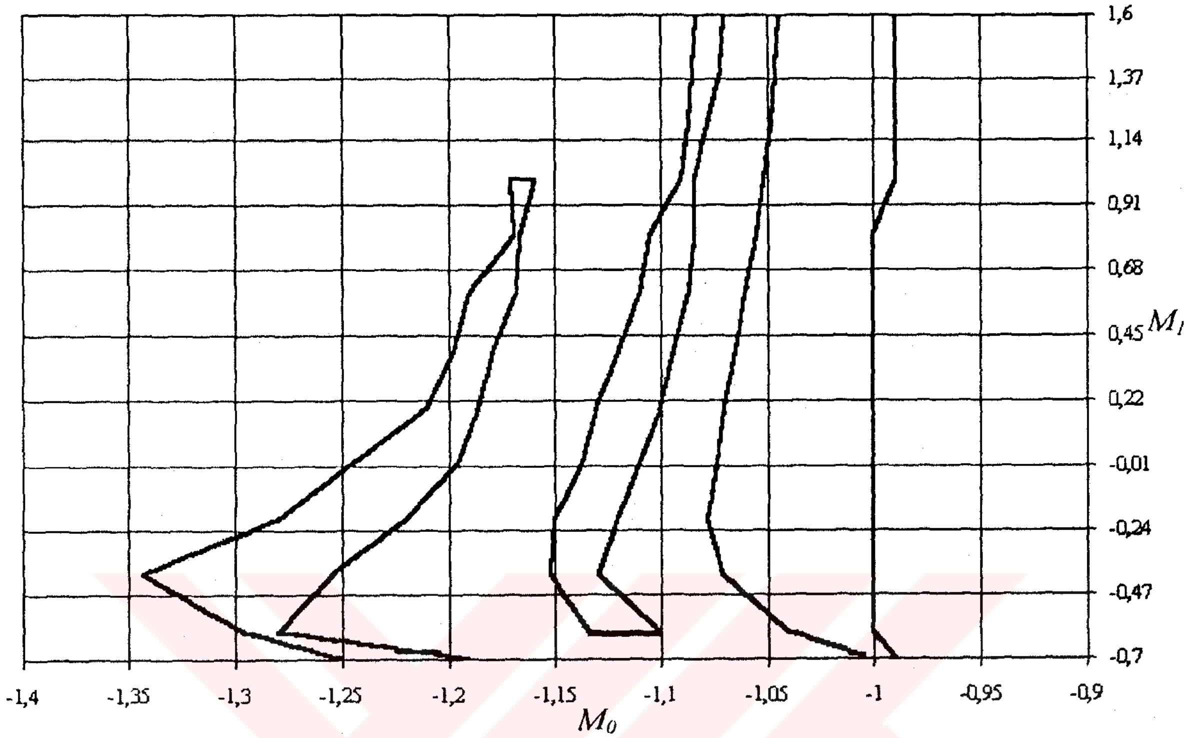

Fig. 5.6 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = -0.01

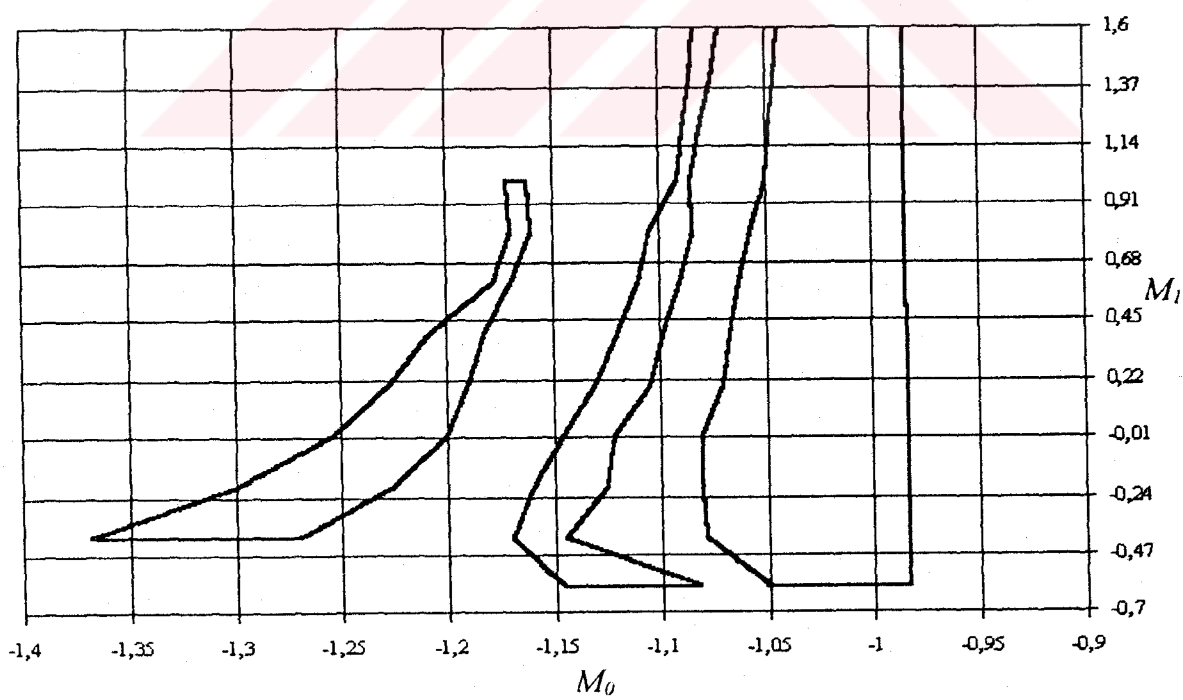

Fig. 5.7 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = -0.02

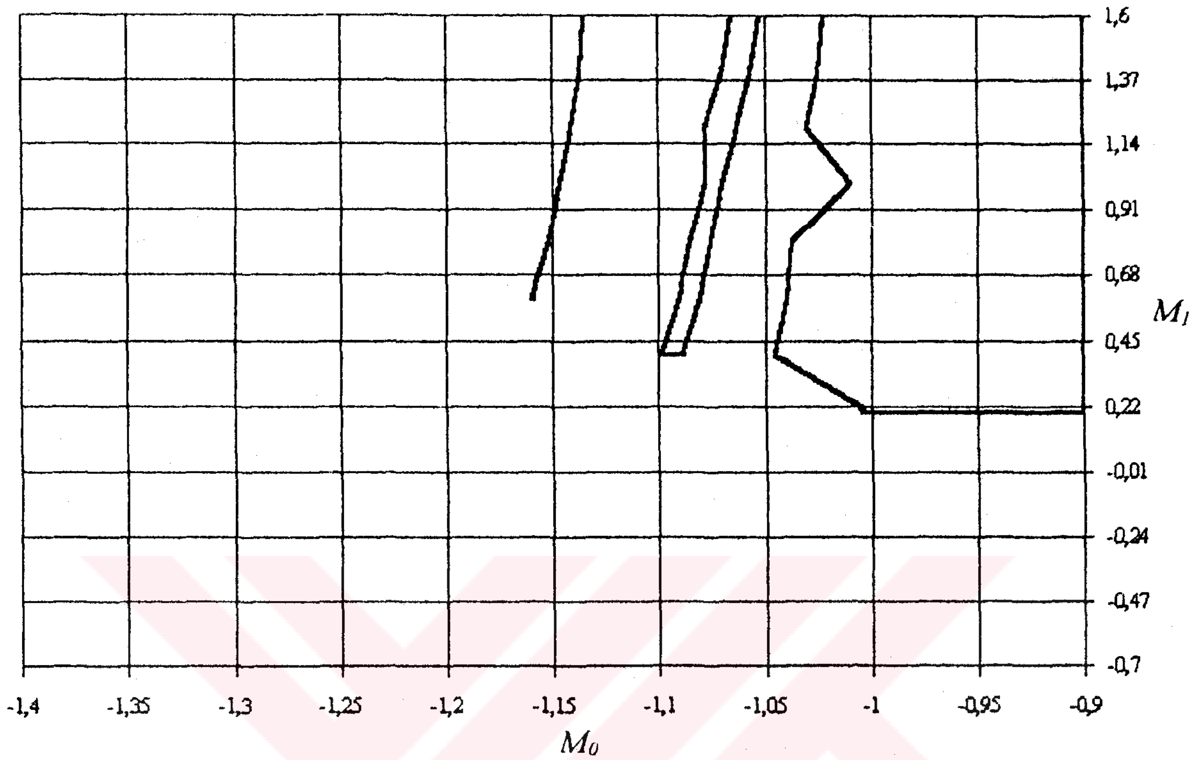

Fig. 5.8 Two parameter bifurcation diagram in the (M_0 -M_1) plane for b = -0.1

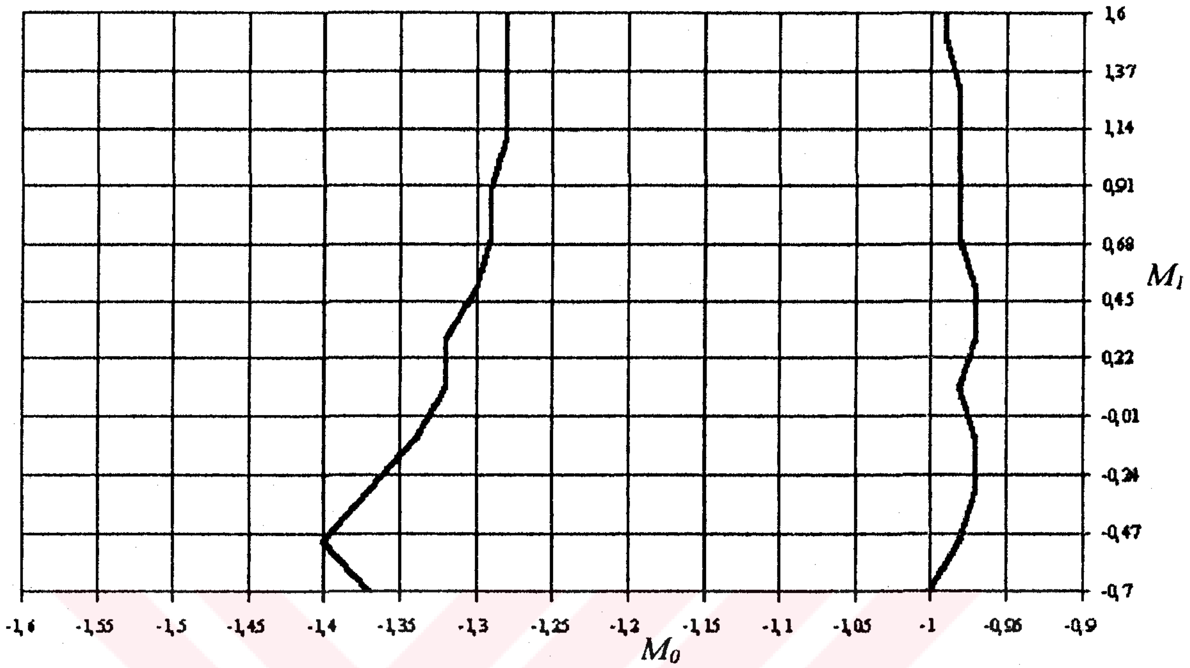

Fig. 5.9 Two parameter bifurcation diagram in the (M_0 -M_0) plane for b=0.01 and m =0.5

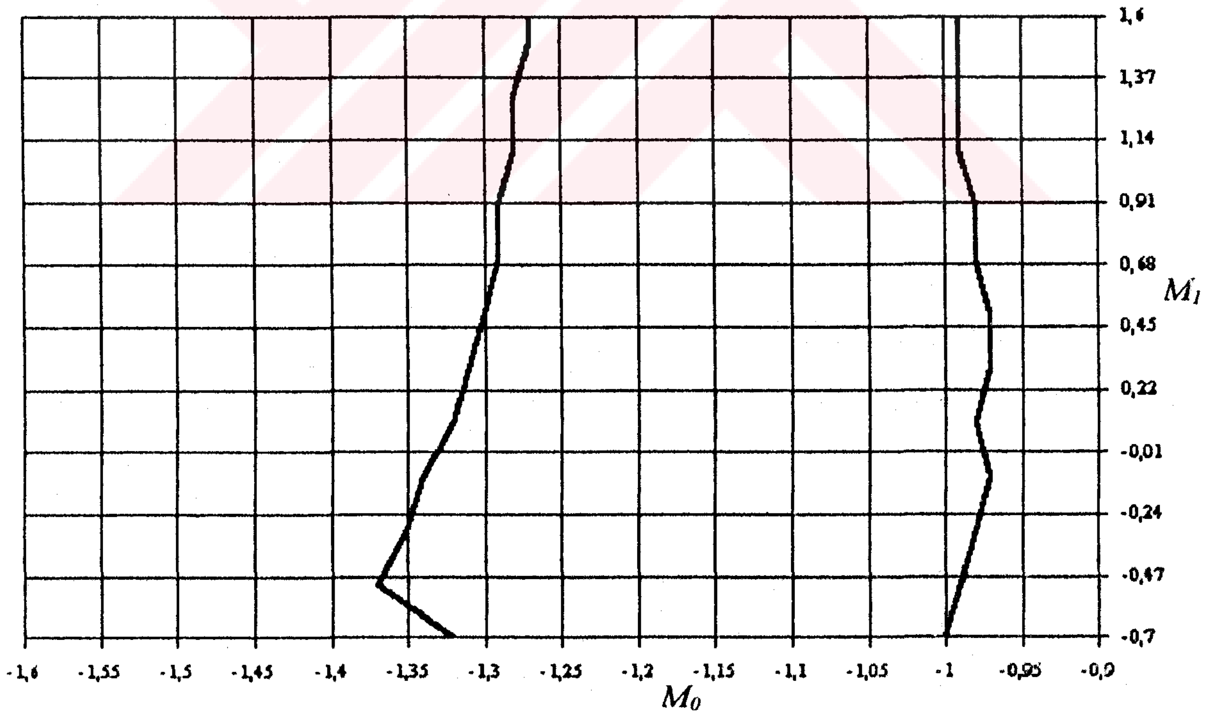

Fig. 5.10 Two parameter bifurcation diagram in the (M_0 -M_0) plane for b=0.02 and m =0.5

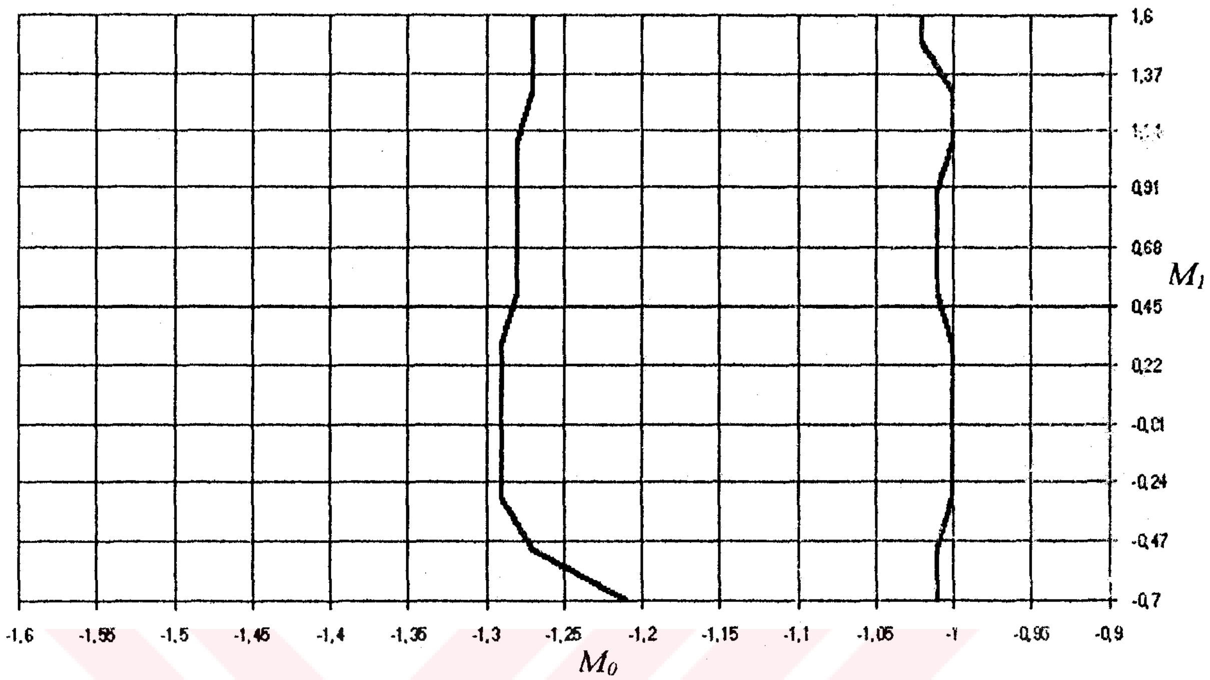

Fig. 5.11 Two parameter bifurcation diagram in the (M_0 -M_0) plane for b=0.1 and m =0.5

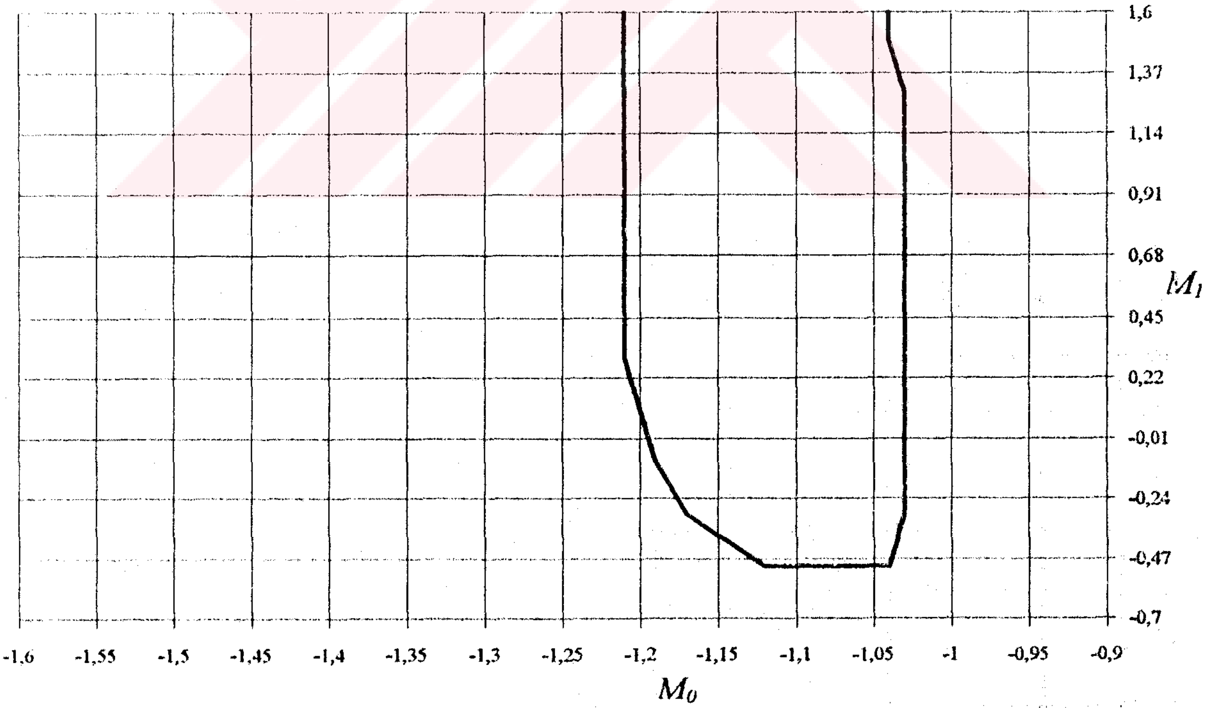

Fig. 5.12 Two parameter bifurcation diagram in the (M_0 -M_0) plane for b=0.5 and m =0.5

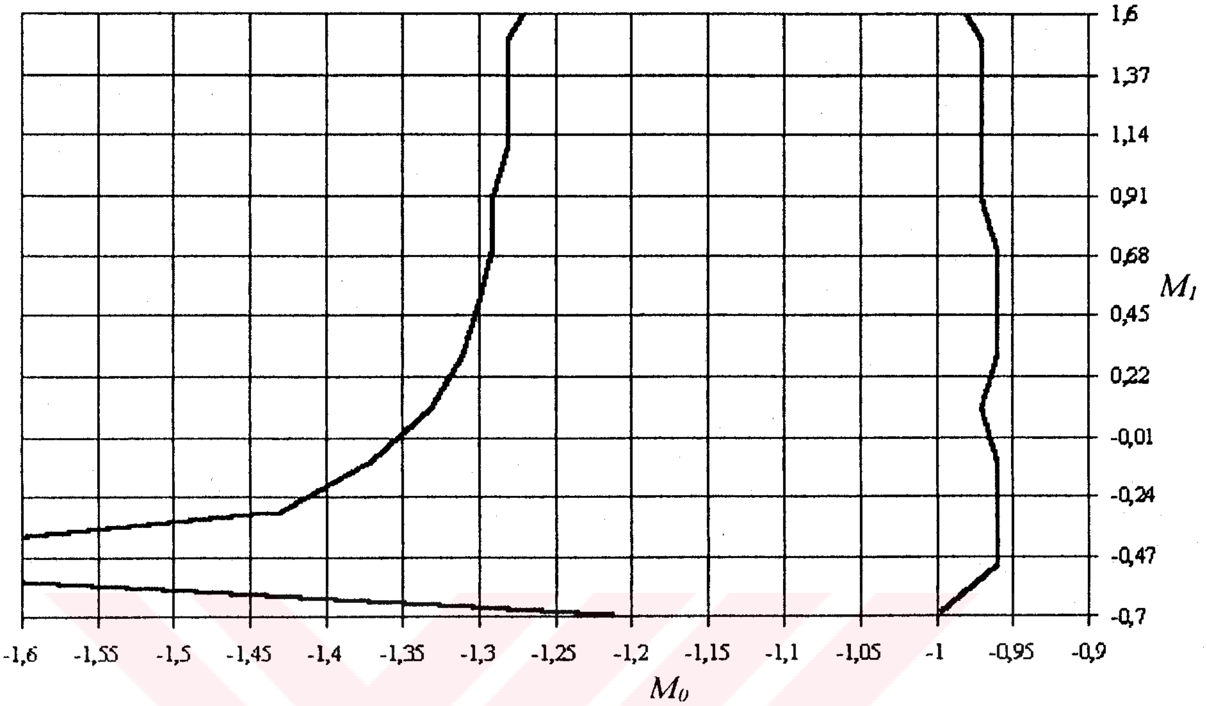

Fig. 5.13 Two parameter bifurcation diagram in the (M_0 -M_0) plane for b= -0.01 and m =0.5

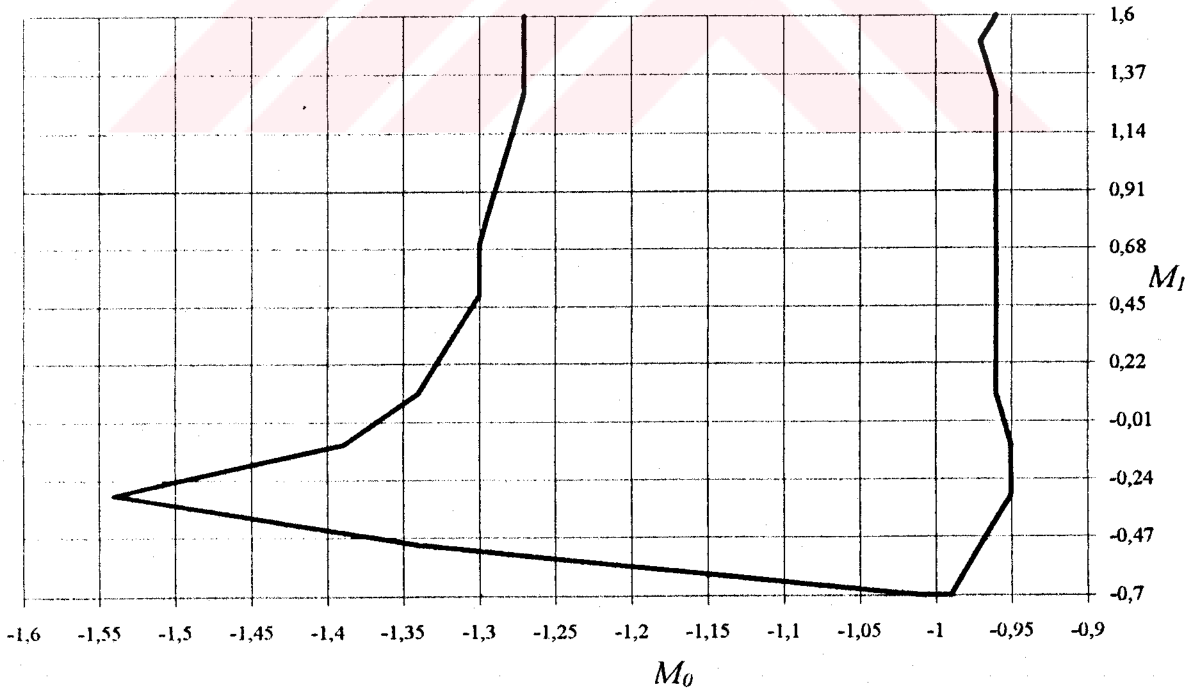

Fig. 5.14 Two parameter bifurcation diagram in the (M_0 -M_0) plane for b= -0.02 and m =0.5

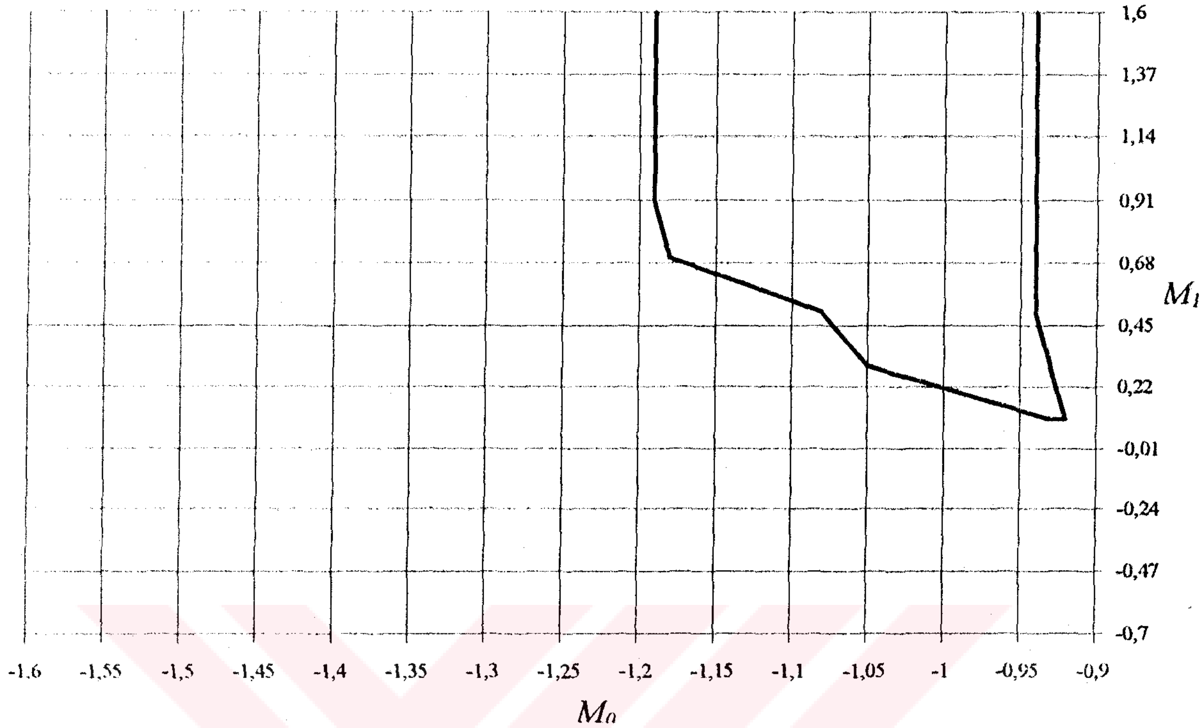

Fig. 5.15 Two parameter bifurcation diagram in the (M_0 -M_0) plane for b= -0.1 and m =0.5

CONCLUSION

We have numerically analyzed the behavior of a very simple second-order dissipative nonlinear circuit under single frequency and multi frequency input and chaotic instability for this circuit was shown in Chapter 2 and 3. The parameters which are affected on chaotic behavior of the system was investigated. The numerical results obtained show that the chaotic dynamics demonstrated by even a very simple system in the case of multi frequency forcing ($m \neq 0$) can differ from those of the harmonic ($m=0$) input case. Under multi frequency excitation the range of the system parameters for chaotic behavior is larger than the case of single frequency input.

In Chapter 4, we have presented a two dimensional bifurcation diagrams to show the effects of the interaction of the resistive and the capacitive nonlinearities in the circuit for the two cases of single frequency input case and multi frequency input case. The bifurcation diagrams in the M_0-M_1 plane for the different values of b was given in Figures. Note that b is the coefficient of the cubic nonlinearity for the nonlinear capacitor. From this bifurcation diagrams, we have seen that for the decreasing negative values of b (b=-0.01, -0.02, -0.1), the chaotic region of the system is reduced. When the values of b was increased (b=0.01, 0.02, 0.3) , the chaotic region of the system enlarged, and the operating range of the system was affected. Thus we can say that the additional of the capacitive nonlinearity in this case plays a limiting role on chaos.

We have numerically investigated the interaction of inductive and resistive nonlinearities in the circuit and discussed the effects of nonlinear inductor on the chaotic behavior of the circuit for two cases of single frequency and multi frequency input case. The bifurcation diagrams in the M_0-M_1 plane for different values of b was given in Figures. Here, b is the coefficient of the cubic nonlinearity for the nonlinear inductor. From this bifurcation diagrams, we have seen that for the increasing positive values of b (b=0.01, 0.02, 0.1, 0.5), the chaotic regions of the system was decreased. If the values of b was decreased negatively (b=-0.01, -0.02, -0.1), the chaotic region of the system is enlarged and the operating range of the system was affected.

SUMMARY

In this thesis, a second - order nonautonomous dissipative nonlinear electrical circuit is studied with respect to its chaotic instability under single frequency and multi-frequency cases. Interaction of capacitive and resistive and, inductive and resistive nonlinearities in the circuit are investigated with due regard in the effect of capacitive and inductive nonlinearities on its various chaotic behavior. The results of the study are presented in the form of graphics. The effective parameters on the chaotic behavior of the circuit are emphasized in the discussion of, ways of controlling and synchronizing chaos .

ÖZET

Bu tezde, ikinci dereceden lineer olmayan ve nonotonom bir elektirik devresinin tek frekans ve multi frekans altındaki kaotik kararsızlığı çalışılmıştır. Ayrıca devrenin rezistiv ve kapasitiv, rezistiv ve endüktiv etkileşimi ve bu etkileşimin devre üzerindeki etkisi incelenmiştir. Bu çalışmanın sonuçları grafiksel olarak verilmiş ve karşılaştırılmıştır. Devrenin kaotik davranışı üzerinde etkili olan parametreler bulunmuştur. Böylece kaosun değişik yollarla kontrolü ve senkranizasyonu tartışıldı.

REFERENCES

[1] K. Murali, M. Lakshamanan and L. O. Chua, "The simplest dissipative nonautomous chaotic circuit, " IEEE Trans. Circuits Syst., vol. 41, no. 10, pp. 462-463, 1994.

[2] D. M. Vavriv and A. Okşaşoğlu, "Stability of varactor circuits," Electron. Lett, vol.30, no.6, pp. 462-463, 1994.

[3] T. Matsumoto, "Chaos in electronic circuits," IEEE Proceedings, vol. 75, no.8, pp.1033-1057, 1987.

[4] M. P. Kennedy, "Three steps to chaos. Part I: Evolution," IEEE Trans. Circuits Syst., vol.40, no. 10, pp. 640-656, 1993.

[5] M. P. Kennedy, "Three steps to chaos. Part II: A Chua's circuit primer," IEEE Trans. Circuits Syst., vol. 40, no. 10, pp. 657-674, 1993.

[6] T. S. Parker and L. O. Chua, "Chaos: A tutorial for Engineers," IEEE Proceedings, vol. 75, no. 8, pp. 982-1008, 1987.

[7] A. Okşaşoğlu and D. Vavriv, "Interaction of low and high frequency oscillations in a nonlinear RLC circuit," IEEE Trans. Circuit Syst. vol., 41, no. 10, pp. 669-672, 1994.

[8] A. Mees and C. Sparrow, " Some tools for analyzing chaos," IEEE Proceedings, vol. 75, no.8, pp. 1058-1070, 1987.

[9] L. O. Chua and F. Ayrom, "Designing nonlinear single op-amp circuits: A cook book approach," Circuit Theo. and Appl., vol. 13, pp. 235-268, 1985.

[10] M.Ohnishi and N. Inaba, "A single bifurcation into instant chaos in a piecewice linear circuit," IEEE Trans. Circuit Syst., vol. 41, no. 6, pp. 443-441, 1994.

ACKNOWLEDGMENTS

I want to express my deep thanks to Asst. Prof. Dr. Ali Okşaşoglu for his extra ordinary help and motivation on this thesis. I would like to thank also to the members of commission that control and correct the thesis. I also want to thank Prof. Dr. A. Hamit Serbest, Asst. Prof. Dr. Aslan Yazıcı and Asst. Prof. Dr. Z. Gökalp Altun individually for their encouraging and close interests.

BIOGRAPHY

I was born in Dörtyol, on 29 th. May 1969. I graduated as an Electronics Engineer from Erciyes University in 1992.

I have been attending as a MS student at Çukurova University Faculty of Engineering and Architecture, Department of Electrical and Electronics Engineering since October 1992, and working as a research assistant at Mustafa Kemal University Faculty of Engineering and Architecture since October 1993.