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INVESTIGATION OF THE STRUCTURES OF NUCLEI AROUND A~180 WITH THE COHERENT STATE APPROACH

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Investıgatıon Of The Structures Of Nucleı Around A~180 Wıth The Coherent State Approach

By Ismael Shukur Ismael AL-BAYATI

May 2023

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science

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Ismael Shukur Ismael AL-BAYATI

ABSTRACT

INVESTIGATION OF THE STRUCTURES OF NUCLEI AROUND A~180 WITH THE COHERENT STATE APPROACH

Ismael Shukur Ismael AL-BAYATI

Master of Science in Physics

Advisor: Prof. Dr. İlyas İNCİ

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Shape phase transitions, which show the structural changes observed due to the change in the number of nucleons in the atomic nucleus, is one of the important subjects of nuclear structure physics. One of the most successful models in this field, the Interactive Boson Model (IBM), gives analytical results only within the limits of dynamic symmetry. In this study, since nuclei exhibiting structures away from the dynamic symmetry limits are examined, the state functions of the structures are obtained using the Coherent State Approach (CSA). By using the state functions of various bands, the equations giving the excitation energy spectra of the nuclei were obtained in the Mathematica program. To test the reliability of the approximation, 20 even-even nuclei with a mass number of around 180 were selected. First, boson numbers were calculated for each isotope and parameters depending on the experimental data were determined. Then, the free interaction parameter that best gives the energies of the excited states in the ground state, beta and gamma bands was determined. Finally, the entire excitation energy spectrum of the selected nuclei was created and compared with the experimental data in the literature.

2023, 35 pages

Keywords: Even-even nuclei, Coherent state approach, Collective model, Shape phase transitions

ÖZET

A~180 CİVARINDAKİ ÇEKİRDEK YAPILARININ EŞEVRELİ DURUM YAKLAŞIMI İLE İNCELENMESİ

Ismael Shukur Ismael AL-BAYATI
Fizik, Yüksek Lisans
Tez Danışmanı: Prof. Dr. İlyas İNCİ
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Atom çekirdeğindeki nükleon sayısının değişmesine bağlı olarak gözlenen yapısal değişiklikleri gösteren şekil faz geçişleri, nükleer yapı fiziğin önemli konularından biridir. Bu alandan en başarılı modellerden biri Etkileşimli Bozon Modeli (IBM), sadece dinamik simetri limitlerinde analitik sonuçlar vermektedir. Bu çalışmada, dinamik simetri limitlerinden uzakta yapı sergileyen çekirdekler incelendiği için, yapıların durum fonksiyonları, Eşevreli Durum Yaklaşımı (CSA) kullanılarak elde edilmiştir. Çeşitli bandlara ait durum fonksiyonları kullanılarak, çekirdeklerin uyarılma enerji spektrumlarını veren denklemler Mathematica programında elde edilmiştir. Yaklaşımın güvenirliliğini test etmek için kütle numarası 180 civarında bulunan 20 tane çift-çift çekirdek seçilmiştir. Önce herbir izotop için bozon sayıları hesaplanmış ve deneysel verilere bağlı parametreler belirlenmiştir. Daha sonra taban durum, beta ve gama bantlarında bulunan uyarılmış durumların enerjilerini en iyi veren serbest etkileşim parametresi belirlenmiştir. Son olarak seçilen çekirdeklere ait tüm uyarılma enerji spektrumu oluşturulmuş ve literatürde yer alan deneysel verilerle karşılaştırılmıştır.

2023, 35 sayfa

Anahtar Kelimeler: Çift-çift çekirdekler, Tutarlı durum yaklaşımı, Kolektif model, Şekil faz geçişleri

PREFACE AND ACKNOWLEDGEMENTS

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Ismael Shukur Ismael AL-BAYATI
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LIST OF SYMBOLS

β	Deformation parameter Beta
ε	Boson energy
ε_d	d-Boson energy
ε_s	s-Boson energy
ε_v	Neutron-boson energy
ε_π	Proton-boson energy
$\hat{H}$	Hamiltonian
J	Total angular momentum quantum number
κ	Quadrupole-quadrupole interaction strength
κ''	Free parameter
$\hat{L}$	Angular momentum
$\mathcal{L}$	Laguerre polynomials
n_d	Number of d-boson
n_s	Number of s-boson
N_v	Neutron-boson number
N_π	Proton-boson number
$\hat{Q}$	Quadruple
$\hat{P}$	Pairing
$u(\beta)$	Reduced potential
V	Potential

LIST OF ABBREVIATIONS

CSA	Coherent State Approach
IBM	Interacting Boson Model
PSM	Projected Shell Models

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1. INTRODUCTION

Three dynamic symmetries involving the subalgebras U(5), SO(6), and SU(3) emerge from the model's algebraic structure, which is based on U(6). These symmetries may be connected to how nuclei are described geometrically (Iachello 2000).

The study of nuclear phase transitions was expanded once E(5) and X(5) were included as point symmetries. The structural characteristics of nuclei at critical may be analytically determined within the collective model's framework according to the bohr-hamiltonian solution collective operator with suitable potential (Iachello 2000).

Phase transitions can be researched using a variety of methods in addition to collective models. When it comes to how the model of interacting bosons (IBM) handle atoms outside of the closed shell of nucleons as bosons and, in its initial iteration, assumes that these bosons carry $\ell = 0\hbar$ (s-bosons) and $\ell = 2\hbar$ (d-bosons) angular momentum, is one of the most successful models. This results in an explanation using the unitary group, U(6). U(5), SU(3), and O(6), which stand for spherical vibrator, deformed-rotor, and γ-unstable rotor, respectively, are three dynamical symmetry restrictions, are produced by various reductions of U(6). The energy eigenvalues in the dynamic symmetry constraints are provided as casmir operator eigenvalues, while outside these constraints exact solutions no longer exist and every IBM Hamiltonian can be easily numerically diagonal (Arima and Iachello 2000).

An algebraic model called IBM and the geometric description by IBM is offered in the form of intrinsic states. Leviathan reviewed IBM's coherent state formalism technique for studying intrinsic states and related collective properties (Leviatan 1987). Atomic nuclei's static and dynamic characteristics are easily ascertained, in a fairly accurate manner (Schaaser and Brink 1984). The effectiveness of formality has been tested away from dynamic symmetry constraints, and it has been found to provide approximations to exact results to order 1/N (Inci *et al.* 2009). The traditional phase structure of boson systems with degrees of freedom for protons and neutrons are investigated using the

IBM-2 Hamiltonian limit (Caprio and Iachello 2007). More recently, a technique for building an IBM Hamiltonian by mapping from the skyrme energy density functional potential energy surface has been created (Skyrme 1958) onto the IBM Hamiltonian's coherent-state expectation value (Nomura *et al.* 2008).

For a rotating nucleus, the closed-form IBM's explanation of the moment of inertia is to obtain the first and higher order of the crank frequency ω from the solution of the self-consistent crank problem (Schaaser and Brink 1986). The general characteristic of the non-zero angular frequency quantum phase transition in finite nuclei has been studied using the IBM notion of rotational cranking (Cejnar 2003). For rotating nuclei, the fermion-boson mapping has been applied using the same method. On the other hand, several models have lately explored the Hf isotopes as well as some nearby isotopic chains. A comprehensive examination of the spectroscopic characteristics according to Ref, between the Pt and Yb isotopes nuclear structures, might serve as an example (Nomura *et al.* 2011), where the rotating bands of these nuclei were described using the same method as in the prior study, with the gogny effective interaction serving as the microscopic input. In addition, a more recent study utilising a selected collective potential and a phenomenological Bohr Hamiltonian method (Bonatsos 1988) been put out to investigate the spectroscopy of Hf nuclei.

Experimental observations of actual nuclei are obtained using coherent state structures. Hf isotopes were chosen because their rotating property predominates in nature. The moment of inertias and level energy are discussed in section 2 are calculated as functions using the IBM Hamiltonian, the deformation parameter for baseline state and excited states built for the SU(3) - O(6) transitional region. In section 3, intrinsic quadruple moments are derived in a closed form. Finally applying to the Hf isotopes and talking about the outcomes are covered in the final part.

2. LITERATURE REVIEW

Hamiltonian for interacting bosons, derived from microscopic and hartree-fock-bogoliubov calculations applying the gogny energy density functional D1M to the spectroscopic analysis of neutron-rich Yb, Hf, W, Os, and Pt (Nomura *et al.* 2011).

Using this spectroscopic analysis method, the transition rates and excitation energies of the associated deep calculating quadrupole collective states. The transition from elongated to flattened ground state shape is examined in relation to the number of neutrons. By computing excitation energies, B(E2) ratios, and N in a certain isotopic chain, and ground state related energies. It turns out that this transition tends to be faster for lower proton isotopes numbers Z when leaving Z = 82, the proton shell closed. It turns out that In discussing the, triaxial degrees of freedom are crucial range of masses under consideration. Predicted low-level spectra of exotic Hf and Yb isotopes that are neutron-rich are given. Ability to forecast various low-energy nuclear facts with a respectable level of accuracy (Nomura *et al.* 2011).

The coherent state method of the interaction the boson model (IBM) was examined in comparison to axial deformation and the γ-soft transition region. A straightforward IBM Hamiltonian created for this area yields the band's excitation energy (Inci 2014). Depending on the quantity of bosons in each band, the deformation parameter that minimises the state energy is discovered in closed form. The moment of inertia determined by solving the start-up issue is then used to define the state energy inside the band. The elemental matrix of the quadruple momentary operator, simplified electric quadruple capture give again. The coherent condition method was then tested on some selected Hf isotopes (Inci 2014).

The interaction degree of freedom relationship between several particles and a single particle is crucial, structural aspect to study. Nuclei in the A≈180 mass range are often cited as good examples for studying these issues, since these atoms are understood to have many spin based on bands multiquasiparticle K isomers.

A wealth of high-quality research findings on heterogeneous mass's high-K states range A≈180 has been accumulated. Their system description is a theoretical challenge because it requires a way beyond the typical mean-field with multi-quasiparticle shell-model setups. The K-isomer data offer an excellent platform for trying out theoretical ideas (Bohr and Mottelson 1998).

Projected shell models (PSM), recently extended according to the Pfaff method, are used for suitably big arrangement spaces with up to 10 quasi particles. Angular momentum projection is used to fix the distorted mean field's broken rotational symmetry. With axis in symmetry the underlying deformation, each multiuse particle condition, designated by the K quantum number, signifies the principal element of the rotating K-band. The diagonalization of the shell model based on this projection defines the K-mix, It is essential to the technique (Casten *et al.* 1987).

The quasi The even-even neutron-rich W174-186 isotope's high-K isomer's particle structure and rotational characteristics are detailed. Through repeated band crossings, the rotational development of the Yrast and Near-Yrast bands is explained. Using intricate quasi particle setups, the multi quasi particle K isomers and related rotational bands in each W isotope are studied. Investigation of the magnetic transition characteristics and comparison of the computed B (E2), B (M1), and g-factors were also done with experiments when data were available. Many nuclei in the mass range of A≈180 exhibit the property of axisymmetric shapes, and K is a good approximation quantum figure for these atoms, a PSM that is longer than normal an axisymmetric basis but incorporating K-mixing by diagonal zing the two-body Hamiltonian is a convenient way to study multi quasi-particle K isomers and K disruption the following states. For the specific example where strong K stoped transitions, further refinement of the current model is required (Iachello and Arima 1987).

Besides the methods discussed here, there are many other methods Attempt to understand microstructure far from nuclear excitation Closed shell. This section briefly summarizes some of them. Bes and co-workers (Bes 1959) described the macroscopic β and γ vibrations The Nilsson excitation conditions that generate them. Your method uses BCS theory as previously described, and the standard random phase technique Approximation (RPA) (Lane 1964) generates energy spectra and wave functions. One An alternative description using the same Woods-Saxon potential The excitation and Soloviev and associates conduct baseline state wave functions (Kishimoto and Tamura 1972).

Kishimoto has recently carried out large-scale numerical calculations Wadamura (Kishimoto and Tamura 1976). They study collective excitations emitted by bosons expand. For a number of nuclei, including 194Pt in Ref., calculations of energy spectra and transition probabilities were made (Dirac 1958). Of course, Not every tiny model is able to be shown here. summary contains Models only most relevant to Hf-W deformation nucleus.

2.1 Interacting Boson Model (IBM)

In the interacting boson model (IBM), protons and neutrons team together and effectively behave as a single particle with the characteristics of bosons with integer spins of 0, 2, or 4.

Both models are restricted to having protons and neutrons that are both even. Regions of differently shaped nuclei predicted by interacting boson approximations (Arima and Iachello 1975).

The model may be used to predict vibrational and rotational modes of non-spherical nuclei (Figure 2.1) (Iachello and Arima 1987).

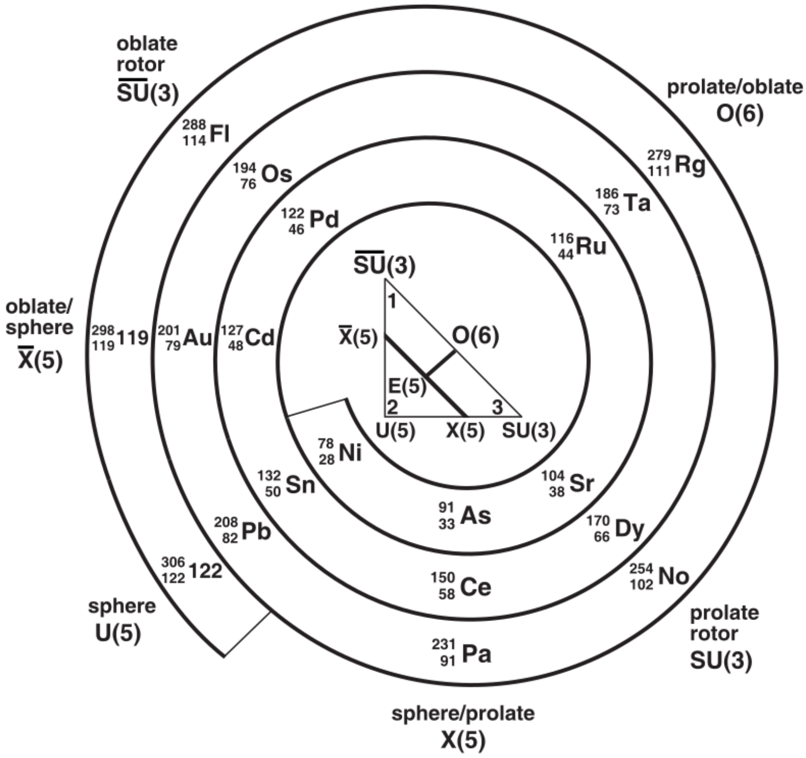

Figure 2.1 Regions of differently shaped nuclei predicted by interacting boson approximations (Iachello and Arima 1987)

2.2 Coherent State Formalism

The IBM-1 Hamiltonian in the SU(3) - O(6) region in the multiple form is given Equation (2.1).

$$\hat{H} = \kappa_3 \hat{Q} \cdot \hat{Q} - \kappa' \hat{L} \cdot \hat{L} + \kappa'' P^{\dagger} \cdot \tilde{P} \tag{2.1}$$

Where quadruple ($\hat{Q}$), the definitions of the angular momentum ($\hat{L}$) and pairing ($\hat{P}$) operators are Equation (2.2):

$$\begin{aligned} \hat{Q} &= [d^{\dagger} \times s + s^{\dagger} \times \tilde{d}]^{(2)} - \chi [d^{\dagger} \times \tilde{d}]^{(2)} \\ \hat{L} &= \sqrt{10} [d^{\dagger} \times \tilde{d}]^{(1)} \\ \tilde{P} &= \frac{1}{2} (\tilde{d} \cdot \tilde{d} - s \cdot s) \end{aligned} \tag{2.2}$$

χ products with spherical tensors are Bhose where is the control parameter Equation (2.3) (Inci 2014).

$$\left[T^{(\ell_1)} \times T^{\ell_2}\right]_M^{(L)} = \sum_{m_1,m_2} (\ell_1 m_1 \ell_2 m_2 \mid LM) T_{m_1}^{(\ell_1)} T_{m_2}^{(\ell_2)}$$
$$\left(U^{(k)} \cdot V^{(k)}\right) = \sum_m (-1)^m U_m^{(k)} V_{-m}^{(k)} \tag{2.3}$$

where *(ℓ*$_1$*m*$_1$ *ℓ*$_2$*m*$_2$*|LM)* is the Clebsch–Gordon coefficient. The states of the system are examined using the coherent state approach (CSA) may be created by using the boson generation operator on the boson condensate, |0>, in order to identify the energy eigenvalues. When N bosons are in the baseline state, the normalised intrinsic state is given by Equation (2.4):

$$|N; a\rangle^g = \mathcal{N}_N \left(B_g^\dagger\right)^N |0\rangle, B_g^\dagger = s^\dagger + \sum_\mu a_\mu d_\mu^\dagger \tag{2.4}$$

with normalization constant Equation (2.5) (Inci 2014).

$$\mathcal{N}_N = \left[N! \left(1 + \sum_\mu a_\mu^* a_\mu\right)^N\right]^{-1/2} \tag{2.5}$$

a_μ 's ($\mu = 0, \pm 1, \pm 2$) correspond to bohr's form parameters and consist of five actual variables. By substituting one of the condensate's bosons with an orthogonal combination, the boson production operators are achieved at high energies above the ground-state band. of $s^\dagger$ and $d_\mu^\dagger$. Then normalized β and γ-vibrational operators of bands are Equation (2.6):

$$B_\beta^\dagger = \sqrt{\frac{a_0^2 + 2a_2^2}{1 + a_0^2 + 2a_2^2}} \left(-s^\dagger + \frac{a_0}{a_0^2 + 2a_2^2} d_0^\dagger + \frac{a_2}{a_0^2 + 2a_2^2}\left(d_{-2}^\dagger + d_2^\dagger\right)\right)$$

$$B_\gamma^\dagger = \sqrt{\frac{2}{a_0^2 + 2a_2^2}} \left(-a_2 d_0^\dagger + \frac{a_0}{2}\left(d_{-2}^\dagger + d_2^\dagger\right)\right) \tag{2.6}$$

These definitions lead to the matrix components of the multipole operator being in Equation (2.7), Equation (2.8), and Equation (2.8).

$$ {}^{g}\langle N;a|\hat{Q}\cdot\hat{Q}|N;a\rangle^{g} = \frac{N}{7\cdot\mathcal{A}^2}\left[\begin{array}{c} 35+(\mathcal{C}_1+\mathcal{C}_3 a_2^2+\mathcal{C}_4 a_0^2)a_1^2+ \\ 4\mathcal{C}_5\left(\frac{a_0^4}{4}+a_1^4+a_0^2a_2^2+a_2^4\right)+\mathcal{C}_2(a_0^2+2a_2^2) \\ + \end{array}\right. \tag{2.7} $$
$$ \left.(N-1)\left(8\sqrt{6}\chi^2 a_0 a_1^2 a_2-4\chi\{\sqrt{14}a_0^3-2\sqrt{21}a_1^2a_2+\sqrt{14}a_0(a_1^2-6a_2^2)\}\right)\right] $$

$$ {}^{g}\langle N;a|\hat{L}\cdot\hat{L}|N;a\rangle^{g} = \frac{2N}{\mathcal{A}^2}[3a_0^2(1+a_0^2+4a_2^2)+6a_2^2(1+2a_2^2) $$
$$ +a_1^2\left(6+8\sqrt{6}a_0a_2(N-1)+12(a_1^2+a_2^2)+(12a_0^2+8a_2^2)N\right)\Big] $$
(2.8)

$$ {}^{g}\langle N;a|P^{\dagger}\cdot\tilde{P}|N;a\rangle^{g} = \frac{N(N-1)}{4\mathcal{A}^2}(-1+a_0^2-2a_1^2+2a_2^2)^2 \tag{2.9} $$

with the constants Equation (2.10) (Bohr and Mottelson 1998).

$$ \mathcal{A}=1+a_0^2+2a_1^2+2a_2^2,\ \mathcal{C}_1=14(\chi^2+6),\ \mathcal{C}_2=7(\chi^2+4N+2) $$

$$ \mathcal{C}_3=8(7-\chi^2(N-8)),\ \mathcal{C}_4=4(7+\chi^2(N+6)),\ \mathcal{C}_5=7+\chi^2(2N+5) \tag{2.10} $$

Using the aforementioned formulas, the energy of the baseline state band containing N bosons is then determined, $E^{g}(N;a) = {}^{g}\langle N;a|\hat{H}|N;a\rangle^{g}$. β and γ-band energies are simply determined using the Hamiltonian's component of a matrix in an excited state Equation (2.11) (Bohr and Mottelson 1998).

$$ E^{\beta}(N;a) = {}^{\beta}\langle N;a|\hat{H}|N;a\rangle^{\beta},\ E^{\gamma}(N;a) = {}^{\gamma}\langle N;a|\hat{H}|N;a\rangle^{\gamma} \tag{2.11} $$

With Equation (2.12):

$$|N;a\rangle^{\beta} = B_{\beta}^{\dagger}|N-1;a\rangle^{g} = \mathcal{N}_{N-1}B_{\beta}^{\dagger}\left(B_{g}^{\dagger}\right)^{N-1}|0\rangle$$
$$|N;a\rangle^{\gamma} = B_{\gamma}^{\dagger}|N-1;a\rangle^{g} = \mathcal{N}_{N-1}B_{\gamma}^{\dagger}\left(B_{g}^{\dagger}\right)^{N-1}|0\rangle \tag{2.12}$$

Energy of the states in the ground-state band for the rotating nuclei (Bohr and Mottelson 1998) are given by Equation (2.13):

$$E_{exc}^{g}(L) = \frac{L(L+1)}{2\mathcal{J}} \tag{2.13}$$

The self-consistent cranking problem's solution yields the moment of inertia (I) (Schaaser and Brink 1986), which rotates the system at a low frequency around any chosen axis. The operator for angular momentum is for rotation about the x-axis Equation (2.14) (Inci 2014).

$$\hat{L}_x = \frac{1}{2}\sum_{\alpha=-2}^{1}\sqrt{6-\alpha(\alpha+1)}\left(d_{\alpha+1}^{\dagger}\tilde{d}_{\alpha} + d_{\alpha}^{\dagger}\tilde{d}_{\alpha+1}\right) \tag{2.14}$$

where is the moment of inertia and the angular momentum matrix element Equation (2.15).

$$\ell_x(N;a) = \langle N;a|\hat{L}_x|N;a\rangle = \frac{2Na_1}{\mathcal{A}}\left(\sqrt{6}a_0 + 2a_2\right)$$
$$\mathcal{J}^{g}(N;a) = \left(\frac{\partial}{\partial a_1}\ell_x(N;a)\right)^{2}\left(\frac{\partial^2}{\partial a_1^2}E^{g}(N;a)\right)^{-1} \tag{2.15}$$

Using the following formulae, the state-specific energy contained within the stimulated bands is then determined Equation (2.16) and Equation (2.17).

$$E_{exc}^{\beta}(L,N;a) = \frac{L(L+1)}{2\mathcal{J}^{\beta}} + E^{\beta}(N;a) - E^{g}(N;a) \tag{2.16}$$

$$E_{exc}^{\gamma}(L,N;a) = \frac{L(L+1)}{2\mathcal{J}^{\gamma}} + E^{\gamma}(N;a) - E^{g}(N;a) \tag{2.17}$$

There has been no discussion of the parameters a_μ thus far. Bohr's form parameters are connected to these actual variables β and γ, this facilitates geometric interpretation of the data Equation (2.18).

$$a_{\pm2} = \frac{1}{\sqrt{2}}\beta\sin\gamma,\; a_{\pm1} = 0,\; a_0 = \beta\cos\gamma \tag{2.18}$$

For the SU(3)-O(6) region, these parameters are in their simplest form. Nuclear shape is prolate-deformed (γ=0) in this area, but γ-independent at the O(6) limit. Using these suppositions Equation (2.19) (Inci 2014).

$$a_{\pm2} = 0,\; a_{\pm1} = 0,\; a_0 = \beta \tag{2.19}$$

Then calculated energies are Equation (2.20), Equation (2.21) and Equation (2.22) (Inci 2014).

$$\begin{aligned} E^g(N;\beta) = \;& \frac{N}{7\mathcal{A}^2}\left[35 + \mathcal{C}_2\beta^2 + \mathcal{C}_5\beta^4 - (N-1)4\sqrt{14}\chi\beta^3\right]\kappa_3 \\ & - \frac{6N\beta^2}{\mathcal{A}^2}(1+\beta^2)\kappa' + \frac{N(N-1)}{4\mathcal{A}^2}(\beta^2-1)^2\kappa'' \end{aligned} \tag{2.20}$$

$$\begin{aligned} E^\beta(N;\beta) = \;& E^g(N;\beta) + \frac{6(\beta^4-1)}{\mathcal{A}^2}\kappa' - \frac{(N-1)(\beta^4-10\beta^2+1)}{2\mathcal{A}^2}\kappa'' \\ & + \frac{7(6-\chi^2)+8\sqrt{14}\chi\beta(2\beta^2-1)+4(2\chi^2-35)\beta^2+(3\chi^2-14)\beta^4}{7\mathcal{A}^2}\kappa_3 \\ & + \frac{2N(-7+4\sqrt{14}\chi\beta(1-\beta^2)+(70-4\chi^2)\beta^2+(2\chi^2-7)\beta^4)}{7\mathcal{A}^2}\kappa_3 \end{aligned} \tag{2.21}$$

$$\begin{aligned} E^\gamma(N;\beta) = \;& E^g(N;\beta) - \frac{6}{\mathcal{A}^2}\kappa' - \frac{(N-1)(\beta^2-1)^2}{2\mathcal{A}^2}\kappa'' \\ & + \frac{7(6-\chi^2)+8\sqrt{14}\chi\beta(2\beta^2+1)-4\chi^2\beta^4}{7\mathcal{A}^2}\kappa_3 \\ & + \frac{2N(-7-4\sqrt{14}\chi\beta(1+\beta^2)+21\beta^2+2\chi^2\beta^4)}{7\mathcal{A}^2}\kappa_3 \end{aligned} \tag{2.22}$$

where $\mathcal{A} = 1+\beta^2$. Deformation parameter β which reduces energy is discovered by solving the aforementioned functions for the relevant band. Prolate-deformed in the

$(\chi = -\sqrt{7/4}, \kappa'' = 0)$ and γ-unstable $(\chi = 0)$ limits, $\beta_{\min}$'s are obtained for the large N limit Equation (2.23).

$$\beta^{g}_{\min}\big(SU(3)\big) = \sqrt{2} \text{ for } \kappa_3 < 0 \text{ and } \beta^{g}_{\min}\big(O(6)\big) = 1 \tag{2.23}$$

In the Hamiltonian, κ_3 and κ' according to the ground state and γ-band 2^+energies. As the definition indicates κ_3 is always negative Equation (2.24) (Inci 2014).

$$\kappa_3 = \frac{E_\gamma(2^+) - E_g(2^+)}{3(1-2N)},\ \kappa' = -\frac{3}{8}\kappa_3 - \frac{1}{6}E_\gamma(2^+) \tag{2.24}$$

κ'' is the free variable. It is discovered that β-band energies may be calculated by fitting the β-band and are very sensitive to variations in this parameter. Then, using Equations (2.25), in a closed form, the ground state moment of inertia is calculated as a function of Equation (2.26).

$$\mathcal{J}^{g} = -\frac{42N\beta^2(1+\beta^2)}{\mathcal{D}_1\kappa' + \mathcal{D}_2\kappa'' + \mathcal{D}_3\kappa_3} \tag{2.25}$$

with:

$$\begin{aligned} \mathcal{D}_1 &= 42(1+\beta^2)(1+2\beta^2(N-1)) \\ \mathcal{D}_2 &= -7\beta^2(1-\beta^2)(N-1) \\ \mathcal{D}_3 &= -7(\chi^2-4) + (N-1)\chi\beta\big(2\sqrt{14} - 6\sqrt{14}\beta^2 + 2\chi\beta^3\big) \\ &\quad -\beta^2\big(28 + 5\chi^2 + 2N(\chi^2-28)\big) \end{aligned} \tag{2.26}$$

Once more, all bands have the same moment of inertia at the huge boson number limit (Inci 2014):

$$\begin{aligned} \mathcal{J}^{g} &= \mathcal{J}^{\beta} = \mathcal{J}^{\gamma} \\ &= \frac{-42\beta^2(1+\beta^2)}{84\beta(1+\beta^2)\kappa' - 7\beta(1-\beta^2)\kappa'' + \big(2\sqrt{14}\chi - 6\sqrt{14}\chi\beta^2 + 2\chi^2\beta^3 - 2\beta(\chi^2-28)\big)\kappa_3} \end{aligned} \tag{2.27}$$

In the $SU(3)$ limit where $\beta = \sqrt{2}, \chi = -\sqrt{7/4}$ and $\kappa'' = 0$

$$\mathcal{J}^{g} = \mathcal{J}^{\beta} = \mathcal{J}^{\gamma} = -\frac{4}{8\kappa' + 3\kappa_3} \tag{2.28}$$

and in the $O(6)$limit $\beta = 1$ and $\chi = 0$, (Inci 2014).

$$\mathcal{J}^{g} = \mathcal{J}^{\beta} = \mathcal{J}^{\gamma} = -\frac{3}{6\kappa' + 2\kappa_3} \tag{2.29}$$

2.3 Hamiltonian IBM-1

1974 Iachello and Arima (Iachello and Arima 1979). The first model of interacting bosons IBM-1 is a nuclear theory that disregards the levels of these bosons' freedom in order to outline the features of the positive parity and the energy levels in even nuclei of intermediate and heavy masses (Otsuka *et al.* 1978). The model was created by giving bosons degrees of freedom. New fundamental characteristics of the atomic nucleus were discovered as a result of this alteration, and IBM-2, a second model of interacting bosons, was created (Casten *et al.* 1987).

There is no distinction between proton bosons (s,d) and neutron bosons in the first interacting boson model (s_v,d_v). The total number of bosons (N) is equal to the logarithm of particles outside the closed shell, which is a fully conserved constant for each nucleus, but the total number of bosons (N) is the sum of (N) proton bosons and Nv neutron bosons) as particle pairs, starting from the nearest closed shell to the middle of the next shell. N_π and N_v take the logarithm of holes if it is greater than half of the shell (Iachello and Arima 1978).

We discover that s and d bosons can interact in this paradigm, so the generation effect (s+, d+) and annihilation effect (s, d) (annihilation operator) (Iachello and Arima 1974) are defined Equation (2.30) (Arima and Iachello 1976):

$$H = \varepsilon_s(s^+.\tilde{s}) + \varepsilon_d(d^+.\tilde{d}) + \sum_{L=0.2,4} \frac{1}{2}(2\,L+1)^{\frac{1}{2}} C_L\left[(d^+ \times d^+)^{(L)} \times (\tilde{d} \times \tilde{d})^{(L)}\right]^{(0)}$$
$$+\frac{1}{\sqrt{2}}\widetilde{V}_2\left[(\,d^+ \times d^+)^{(2)} \times (\tilde{d} \times \tilde{s})^{(2)} + (d^+ \times s^+)^{(2)} \times (\tilde{d} \times \tilde{d})^{(2)}\right]^{(0)}$$
$$+\frac{1}{2}\widetilde{V}_0\left[(\,d^+ \times d^+)^{(0)} \times (\tilde{s} \times \tilde{s})^{(0)} + (s^+ \times s^+)^{(0)} \times (\tilde{d} \times \tilde{d})^{(0)}\right]^{(0)} \quad (2.30)$$
$$+U_2\left[(\,d^+ \times s^+)^{(2)} \times (\tilde{d} \times \tilde{s})^{(2)}\right]^{(0)} + \frac{1}{2}U_0\left[(\,s^+ \times s^+)^{(0)} \times (\tilde{s} \times \tilde{s})^{(0)}\right]^{(0)}$$

Among them, the interaction between bosons is defined by the coefficients C_L(L=0,2,4), V_L(L=0,2), and U_L(L=0,2), which depend on the number of bosons N. Parentheses indicate angular momentum. General Hamilton's formula for IBM-1 is equivalent to a number of different formulae, including where $\varepsilon=\varepsilon_d+\varepsilon_s$ represents the energy of the boson. In short, assume that the energy of the boson is zero, that is: $\varepsilon=\varepsilon_d$

We can therefore rewrite Equation (2.30) using the multiple expansion Equation (2.31) (Casten and Warner 1988):

$$\hat{\mathrm{H}} = \varepsilon\hat{n}_d + a_0\hat{P}\cdot\hat{P} + a_1\hat{L}\cdot\hat{L} + a_2\hat{Q}\cdot\hat{Q} + a_3\hat{T}_3\cdot\hat{T}_3 + a_4\hat{T}_4\cdot\hat{T}_4 \cdots \quad (2.31)$$

Where $\varepsilon_s - \varepsilon_d = \varepsilon$ represents the energy of the boson, for the sake of simplicity, it is assuming that the boson ε_s energy is zero. The coefficients a_0, a_1, a_2, a_3, a_4 represent the duality, the sixteenth pole, the quadrupole, the octopole, and angular momentum interactions between bosons (Arima and Iachello 1978).

Where ε is the boson energy sum operator Equation (2.32) (Arima and Iachello 1976).

$$\hat{n}_s = \hat{s}^\dagger\tilde{s},\ \hat{n}_d = \hat{d}^\dagger\tilde{d} \quad , \quad \hat{P} = \frac{1}{2}(\tilde{d}\,\tilde{d}) - \frac{1}{2}(\hat{s}\hat{s})$$
$$\hat{L} = \sqrt{10}\left[\hat{d}^\dagger \times \tilde{d}\right]^{(1)} \ , \ \hat{Q} = \sqrt{5}\left[(\hat{d}^\dagger + \tilde{s}) + (\hat{s}^\dagger \times \tilde{d})\right]^{(2)} + \chi\left[\hat{d}^\dagger \times \hat{d}\right] \quad (2.32)$$

The phenomenological parameters $a_0, a_1, (a_2, \chi), a_3, a_4$ represent the paired quadrupole, and angular momentum octopole and hexadepole interaction strengths between bosons, respectively (Arima and Iachello 1978).

Will stand for the case involving two d-boson number variation terms.

Another the generic Hamiltonian's form can usually be written refers to specific interactions between bosons. In these cases Equation (2.33) (Arima and Iachello 1976).

$$H = \varepsilon \sum_m d_m^+ d_m - \kappa \sum_{ij} \overrightarrow{Q_\iota} \cdot \overrightarrow{Q_J} - \kappa' \sum_{i<j} L_{ij} - \kappa'' \sum_{i<j} P_{ij} \tag{2.33}$$

Where $\overrightarrow{Q_\iota}$ represents the moment of quadrupole of the i^{th} boson, $L_{ij} = 2\overrightarrow{\ell_\iota} \cdot \overrightarrow{\ell_J}$ with $\overrightarrow{\ell_\iota}, \overrightarrow{\ell_J}$ · angular momenta being the i^{th} and j^{th} boson, respectively, P_{ij} is an operator for pairing bosons, and $\kappa, \kappa', \kappa''$ are the various interactions' individual strengths, was set to 0 for simplicity, ensuring that only $\varepsilon = \varepsilon_d - \varepsilon_s = \varepsilon_d$ appears in Equation (2.34).

Transformation operators are assigned to aggregate states computed by IBM. In their most basic form, the conversion operators E0, M1, E2, M3, E4 are given in leading order (Arima and Iachello 1976).

$$T_m^{(\ell)} = \alpha_\ell \delta_{\ell 2} (d^+ s + s^+ d)_m^{(2)} + \beta_\ell (d^+ d)_m^{(\ell)} + \gamma_{l0} \delta_{l0} \delta_{m0} (s^+ s)_0^{(0)} \tag{2.34}$$

where l denotes multipolarity with projection m, and α, β, γ are the coefficients of various operators are used. Especially for E2 transitions Equation (2.35) (Arima and Iachello 1976).

$$T_m(E2) = \alpha_2 (d^+ s + s^+ d)_m^{(2)} + \beta_2 (d^+ d)_m^{(2)} \tag{2.35}$$

Which fulfills this meets the selection rule and the selection rule are the two components of this operator. The relevant limit or the criteria for choosing an acceptable intermediate structure determine the coefficients (Arima and Iachello 1978).

2.4 The Vibrational SU(5) Symmetry

The vibration limit is the first IBM limit symmetry to be explored (Arima and Iachello 1976). A system with a boson energy, as mentioned in the preceding section, gives birth to a very basic spectrum of collective states, as illustrated in Figure 2.2. This limit correspond to O(5), 5-dimensional orthogonal group. But that the IBM Hamiltonian can symmetric also solve SU(5) representations analytically Equation (2.36) (Iachello and Arima 1974).

The Hamiltonian's shape in this limit is given by (Iachello and Arima 1974):

$$H = \varepsilon \sum_m d_m^{+} d_m + \sum_J 1/2(2J+1)^{\frac{1}{2}} C_J \left[(d^{+}d^{+})^{(J)} \cdot (dd)^{(J)} \right]^{0)} \quad (2.36)$$

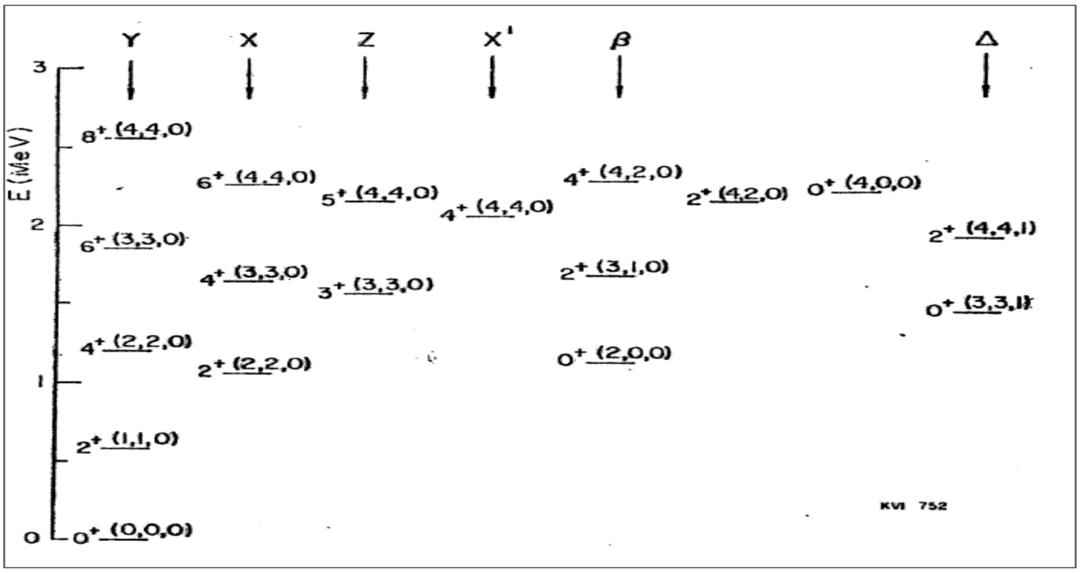

Figure 2.1 Typical nuclear spectrum showing the SU(5) symmetry. The quantum numbers label the states $J^{\pi}(n_d, v, n_\Delta)$. The spectrum is divided into several bands (Arima and Iachello 1976).

The eigenvalue Equation (2.37) may be expressed as:

$$H|n_d v n_\Delta J M\rangle = E|n_d v n_\Delta J M\rangle. \quad (2.37)$$

where H is determined by Equation 2.36 and the quantum numbers serve as labels for the states n_d, v, n_Δ, J, M. We are already aware with the number of d-bosons n_d, the angular momentum J, and its projection M. As was said before, n_d, is the quantity measuring how many triplets of d-bosons are linked to angular momentum zero, and v is the seniority, measuring how many triplets angular momentum zero is not related to any of the d-bosons Another form makes use of the quantum number n_β, It records how many d-boson pairs there are connected to angular momentum zero ; v and n_B are related by $v = n_d - 2n_\beta$. The distribution of all bosons is as follows (Arima and Iachello 1976):

$$n_d = 2n_\beta + 3n_\Delta + \lambda \quad (2.38)$$

where λ is the bosons in excess and controls the range of angular momentum (Arima and Iachello 1976):

$$J = 2\lambda, 2\lambda - 2, 2\lambda - 3, \dots, \lambda + 1, \lambda \quad (2.39)$$

The angular momentum $J = 2\lambda - 1$ is missing because bosons can only create symmetric states when they are linked (Bayman 1966). The Hamiltonian in Equation (2.36) may also be solved by rewriting considering the forces described in Equation (2.33). Because there can only be three angular momentum couplings, only three parameters are required to explain two d-bosons interacting with one another (Bayman 1966): $J = 0,2,4$. Therefore, the coefficients $C_J(J = 0,2,4)$ in Equation (2.36), or three different parameter options α, β, γ, Iachello and Arima have characterised the conversation as Equation (2.40) (Arima and Iachello 1976):

$$V = \sum_{i<j} V_{ij} = \sum_{i<j} \left(\alpha l_{ij} + \beta P_{ij} + \gamma L_{ij}\right). \quad (2.40)$$

where P_{ij} and arel_{ij}, the pairing and L_{ij} the anticipated values of these operators, as provided, and l_{ij} is the unit operator (Arima and Iachello 1976), are Equation (2.41):

$$
\begin{aligned}
&< \mathrm{l} >= \frac{1}{2} n_d (n_d - 1) \\
&< L >= J(J+1) - 6n_d \\
&< P >= (n_d - v)(n_d + v + 3)
\end{aligned} \tag{2.41}
$$

Consequently, the interacting d-boson Hamiltonian's eigenvalues are Equation (2.42) (Arima and Iachello1976).

$$
\begin{aligned}
E([N], n_d, v, n_\Delta, J, M) = \quad & \varepsilon n_d + \alpha \frac{1}{2} n_d (n_d - 1) \\
& + \beta (n_d - v)(n_d + v + 3) \; + \gamma [J(J+1) - 6n_d]
\end{aligned} \tag{2.42}
$$

Figure 2.2 shows a typical vibration limit spectrum. Since there are significant E2 matrix components between neighbouring members of a single band, the spectrum may be separated into a number of bands. Quantum numbers serve as the representation for the states in Figure 2.2 n_d, v, n_Δ. The "belt" is very reminiscent of the band that occurs in the rotating core. The Y band corresponds to the baseband, X and Z correspond to the gamma band, β corresponds to the beta band, and Δ corresponds to the 2-phonon gamma band , gives the state energies in some of these bands (Arima and Iachello 1976).

Provides the electric quadrupole transition operator's general form, T(E2) Equation (2.43). To the extent that analytical solutions are available. The transformation operator must be a generator of the basic group according to Arima and Iachello. Limits SU(5),T(E2) are characteristics are from (Arima and Iachello 1976):

$$
T_m(E2) = \alpha (d^+ s + s^+ d)_m^{(2)} \tag{2.43}
$$

for $\alpha = \langle d \parallel \vec{Q} \parallel s \rangle (1/5)^{(2)}$, where $\vec{Q}$ a quadripole operator. Using this operator type leads to the selection rule $\Delta n_d = \pm 1$.

Hence, in the case of vibrational determination, we can infer special relativity SU(5) Equation (2.44) (Arima and Iachello 1976):

$$R = \frac{B(E2{:}4_1^+ \to 2_1^+)}{B(E2{:}2_1^+ \to 0_1^+)} \tag{2.44}$$

2.5 The Rotational SU(3) Symmetry

Based on the SU(3) group, second restriction of the IBM model, leading to a nuclear structure that resembles a certain shape of a symmetrical rotor. As described in subsection. This symmetry develops when there are quadrupole-quadrupole interactions between bosons. The most general form of boson interaction also involves the notion of the form $L = \vec{l}_i \, . \vec{l}_j$.

Many years ago, the whole IBM Hamiltonian was presented (Elliott 1958) shown that the eigenvalue problem might be solved supposing a Hamiltonian may written in terms of the generators of a group, particularly SU(3), the three-dimensional special unitary group Equation (2.45) (Scholten 1979) :

$$H = -\kappa \sum \ \vec{Q}_i^{\alpha} . \vec{Q}_j^{\alpha} \tag{2.45}$$

where $\vec{Q}_i$ is the particle i quadrupole operator, and k is the quauadrupole-quadrupole interaction's strength.

The answer to Equation (2.68) is given (Arima and Iachello 1978). Here, some of the findings will be repeated. As a result, the eigenvalue Equation (2.46) (Scholten 1979):

$$H|[N](\lambda,\mu)KJM\rangle = E|[N](\lambda,\mu)KJM\rangle. \tag{2.46}$$

where $[N]$ identifies the SU representations that are completely symmetric O(6); (λ, μ) are two quantum numbers that identify SU(3) representations; and J, M are, angular momentum and its z-axis projection respectively. The extra K labels for the quantum number indicate the same λ, μ, J. The eigenvalues can be stated in this basis Equation (2.47) and Equation (2.48) (Scholten 1979):

$$E([N](\lambda,\mu)KJM) = K\big(J(J+1) - C(\lambda,\mu)\big) \tag{2.47}$$

Where $C(\lambda,\mu)$ is quadratic Casimir operator of $SU(3)$:

$$C(\lambda,\mu) = \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda+\mu). \tag{2.48}$$

As mentioned before, adding the diagonalization issue is unaffected by the L interaction. Consequently, in its most fundamental form, the Hamiltonian becomes in Equation (2.49) (Scholten 1979):

$$H = -\kappa \sum_{i,j} \vec{Q}_i \cdot \vec{Q}_j - \kappa' \sum_{i,j} \vec{\ell}_i \vec{\ell}_j. \tag{2.49}$$

With t he eigenvalues.

$$\begin{gathered} E([N](\lambda,\mu)KJM) = \alpha J(J+1) - \beta C(\lambda,\mu). \\ \alpha = \frac{3}{4}\kappa - \kappa', \beta = \kappa \end{gathered} \tag{2.50}$$

Because they're crucial for anticipating the horizontal spacing the parameters of distorted cores (λ,μ) are represented here by Young Tableaux for discussion. A box can be used to represent each particle, and boxes can be combined into a shape Symmetric or anti-symmetric state.

Where in the of rotational determination we can SU(3) Equation (2.51) (Medsker et al. 1977) (Medsker *et al.* 1977).

$$R = R_{4/2} = \frac{B\left(E2, 4_1^+ \to 2_1^+\right)}{B\left(E2, 2_1^+ \to 0_1^+\right)} = \frac{10}{7}\frac{(N-1)(N+5)}{2(N+4)} \leq \frac{10}{7} \tag{2.51}$$

2.6 The Gamma Unstable *O*(6) Symmetry

When the pairing force dominates the interboson interaction, the IBM model will experience its third limiting symmetry (Arima and Iachello 1978). The IBM Hamiltonian was diagonalized by Iachello and Arima. produced by SU(6) by discovering a subgroup of the Hamiltonian is invariant for SU(6), similar to the SU(5) and SU(3) symmetries. The subgroup in question is O(6), which also includes the subgroups O(5) and O(3). The IBM Hamiltonian in the O(6) limit may be expressed as follows by utilising the group chain SU(6)→ O(6) O(5) →O(3) Equation (2.52):

$$H = AP_6 + BC_5 + CC_3. \tag{2.52}$$

Where C_5 and C_3 are Casimir operators in O(5) and O(3), respectively, and P_6 is a pairing operator in O(6). The strengths of several components are A, B, and C. In accordance with the accompanying entry, the IBM Hamiltonian Equation (2.53):

$$v_0\left[(d^+d)^{(0)}(ss)^{(0)} + (s^+s^+)^{(0)}(dd)^{(0)}\right]^{(0)} \tag{2.53}$$

While C_5 and C_3 and correspond to the terms Equation (2.54):

$$\varepsilon \sum_m d_m^+ d_m + \sum_{J=0,2,4} \frac{1}{2}(2J+1)^{1/2} C_J\left[(d^+d^+)^{(J)}(dd)^{(J)}\right]^{0)} \tag{2.54}$$

The irreducible symmetric representation of O(6) is characterized by quantization number σ, where Equation (2.55) (Arima and Iachello 1978):

$$\sigma = N, N-2, N-4, \ldots, 0 \text{ or } 1 \text{ for N = even or odd} \tag{2.55}$$

The expected value of the $O(6)$ operating pair, P_6, may be expressed with σ as Equation (2.56) (Arima and Iachello 1978):

$$\langle P_6 \rangle = \frac{1}{4}(N-\sigma)(N+\sigma+4). \tag{2.56}$$

As in the quantum number τ is chosen to characterize the O(5) representation, where Equation (2.57).

$$\tau = \sigma, \sigma - 1, ,0 \tag{2.57}$$

The anticipated value of C_5 in the τ illustration of $\mathrm{O}(5)$ Equation (2.58) is given by (Arima and Iachello 1978):

$$\langle C_5 \rangle = \frac{1}{6}\tau(\tau+3). \tag{2.58}$$

Consequently, the states' eigenvalues that correspond to the Hamiltonian in are Equation (2.59) (Arima and Iachello 1978):

$$E([N]\sigma v_\Delta JM) = \frac{A}{4}(N-\sigma)(N+\sigma+4) + B\tau(\tau+3) + CJ(J+1). \tag{2.59}$$

where 1/6 in the Equation (2.57) is into the fixed value B. Since it is connected to counting the quantity of particles, the quantum number v_Δ bosonic triplets, it may be used to name states. coupled to zero angular momentum. The relationship between the quantum number τ and v_Δ is $\tau = 3v_\Delta + \lambda \ldots$..The value of λ determines the angular momentum of the above state Equation (2.60):

$$L = \lambda, \lambda+1, \ldots, 2\lambda-2, 2\lambda. \tag{2.60}$$

Arima and Iachello also managed to obtain analytical expressions for transition probabilities (Arima and Iachello 1978) . Like the E2 transformation operator T(E2) a generator of the underlying group structure must exist, in this instance O(6), in order to satisfy the SU(5) and SU(3) symmetries. The configuration of T(E2) that satisfies this need is Equation (2.61) (Arima and Iachello 1978):

$$T(E2) = \alpha(d^+s + s^+d)^{(2)} \tag{2.61}$$

Since T(E2) is an O(6) generator, which prevents it from joining states from various representations; thus the selection rule is Δσ=0. Since O(6) contains O(5) structures, the O(5) selection rule Δτ=±1 also applies. Several closed expressions for B(E2) values for transitions in $\sigma = \sigma_{\max} = N$, It ought to be emphasised that, as with other IBM B(E2) values, the system's finite dimensions are automatically taken into account. The branching ratio that takes place in the limit O(6) is not affected by the parameters A, B, and C due to the branch operator's form.

Consequently, in the situation of determination O(6), we can derive special relativity Equation (2.62) (Arima and Iachello 1978).

$$R = R_{\frac{4}{2}} = \frac{B(E2,4_1^+ \to 2_1^+)}{B(E2,2_1^+ \to 0_1^+)} = \frac{10}{7}\frac{(N-1)(N+5)}{2(N+4)} \approx 1.4 \tag{2.62}$$

2.7 U(6) Group

Considering the magnetic projections of the s and d bosons, six different cases are encountered, and this system is represented by the group *U(6)* in the language of group theory. The subgroups of *U(6)* are U(5), SU(3), and O(6) are those that are called circular vibrator, deformable rotor, and gamma rotates unsteadily structures, respectively. After determining the Hamiltonian of the physical system, it is desired to diagonalize the system. In order to do this, it is necessary to know the quantum phase states. The technique often used is to label quantum phase states (orbital energies = band energies) with quantum numbers that characterize various irreducible representations of a subalgebraic chain of algebra associated with the U(6) symmetry of the Hamiltonian. So finding every potential subalgebra of the complete algebra U is the next step (6). Using subalgebra is produced by a subset of the full algebra and closes under commutation. In the current situation, it will be revealed that there are possible sub-algebraic chains (Bonatsos 1988).

One may show that the three subalgebras I, II, and III are the only ones that can exist if one wants to add the rotation algebra, O(3), as a subalgebra. In reality, starting with U(6), one has considered U(5), U(3), and O(6). The algebra of O(6) is isomorphic to SU(4) since all routes from U(6) to the rotation algebra O(3), SU(2) have been investigated SU(4). There are only three conceivable outcomes as a result Equation (2.63).

$$\begin{aligned} &\mathrm{I}, \mathrm{U}(5) \supset \mathrm{O}(5) \supset \mathrm{O}(3) \supset \mathrm{O}(2) \\ &\mathrm{II}, \mathrm{U}(6) \rightarrow \mathrm{SU}(3) \supset \mathrm{O}(3) \supset \mathrm{O}(2) \\ &\mathrm{III}, \mathrm{O}(6) \supset \mathrm{O}(5) \supset \mathrm{O}(3) \supset \mathrm{O}(2) \end{aligned} \tag{2.63}$$

3. MATERIALS AND METHODS

Various methods have been proposed to explain the structural properties of the core. One such approach interacting boson model (IBM), a method of group theory due to Lie algebras. Upon which the model is based *U(6)* team and offers very successful results on medium and large mass uniform binaries. Today it is actively used with various models to explain the structural properties of atomic nuclei and continues to evolve (Inci *et al.* 2009). It was fairly successful in explaining atomic nuclei with even protons and even neutrons, and is still in use today, and there are no magic numbers. In this algebraic model, the boson interactions between the magic numbers of the examined nuclei are considered. The calculation of the boson number takes into account the closed shell.

The IBM Hamiltonian in terms of multiple moments is given as Equation (3.1):

$$\widehat{H} = \kappa_3 \hat{Q} \cdot \hat{Q} - \kappa \hat{L} \cdot \hat{L} + \kappa'' P^{\dagger} . \tilde{P} \tag{3.1}$$

Here, Q, L, and P are In terms of the s and d bosons, the quadruple moment, angular momentum, and coupling operators are defined, Equation (3.2).

$$\begin{aligned} \hat{Q} &= (d^{+}s + s^{+}d) + \chi \hat{T}_2 \\ \hat{L} &= \sqrt{10}\hat{T}_1 \\ \tilde{P} &= \frac{1}{2}\left(\tilde{d} \cdot \tilde{d} - s \cdot s\right) \\ \hat{T}_{\ell=} &\left(d^{\dagger}\tilde{d}\right)^{(\ell)}; \ell = 0,1,2,3,4 \end{aligned} \tag{3.2}$$

operators as κ_3 and κ' depends on the gamma band 2+ energies and the ground state band Equation (3.3) and Equation (3.4).

$$\kappa_3 = \frac{E_{\gamma}(2^{+}) - E_g(2^{+})}{3(1-2N)} \tag{3.3}$$

$$\kappa' = -\frac{3}{8}\kappa_3 - \frac{1}{6}E_\gamma(2^+) \tag{3.4}$$

It is determined by the expression κ'' is a completely free parameter. Layout parameter found in quadrupole operator χ it takes the value 0 in the *U(5)* limit and −√7/4 in the *SU (3)* limit.

baseline state of a system containing N valence bosons is defined as a boson condensation. Since, according to IBM, only s and d bosons exist in the system, the boson creation operator is given by the orthogonal combination of the operators corresponding to these bosons. The boson operator that produces the ground state is given in the following Equation (3.5):

$$b_g^\dagger(\beta,\gamma) = \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger \beta\cos\,\gamma d_0^\dagger + \frac{\beta}{\sqrt{2}}\sin\,\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right] \tag{3.5}$$

Here β and γ means the internal collective variable in the Bohr Hamiltonian.

Internal excitations will cause the ground state to change in the number of bosons. Thus, one of the ground state operators in the state function will be replaced by another operator that provides the excitation. The operator providing the first excitation β produces vibration. The operator below is the operator that provides the first excitation Equation (3.6).

$$b_{\beta v}^\dagger(\beta,\gamma) = \frac{1}{\sqrt{1+\beta^2}}\left[-\beta s^\dagger\cos\,\gamma d_0^\dagger + \frac{1}{\sqrt{2}}\sin\,\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right] \tag{3.6}$$

If the next state of arousal is$\gamma$the vibration band and the corresponding generating operator and the state of the system Equation (3.7):

$$\begin{aligned} b_{\gamma v}^\dagger(\beta,\gamma) &= \left[-\sin\,\gamma d_0^\dagger + \frac{1}{\sqrt{2}}\cos\,\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right] \\ |\beta,\gamma\rangle^\gamma = &= \frac{1}{\sqrt{N!}} b_{\gamma v}^\dagger(\beta,\gamma) b_{g.s}^\dagger(\beta,\gamma) I_g(\beta,\gamma) \end{aligned} \tag{3.7}$$

In order to obtain bands with higher energy than the ground state band, a ground state boson generation operator must be replaced by another operator orthogonal to this operator. β andγThe boson creation operators that will give their vibrations are, respectively Equation (3.8), Equation (3.9) and Equation (3.10):

$$E_{\gamma}\left(N, a_{\mu}\right) = {}^{\gamma}\langle N; a_{\mu}|\hat{H}|N; a_{\mu}\rangle^{\gamma} \tag{3.8}$$

$$E_{\beta}\left(N, a_{\mu}\right) = {}^{\beta}\langle N; a_{\mu}|\hat{H}|N; a_{\mu}\rangle^{\beta} \tag{3.9}$$

$$I = \frac{(\mathrm{dL}_{x}/\mathrm{da}_{1})^{2}}{\mathrm{d}^{2}E_{g}/\mathrm{da}_{1}^{2}} \tag{3.10}$$

The energy expression of the beta band is also defined this way. Here I is the nucleus's moment of inertia.The energies of the levels The ground state, having varying angular momentum in the same energy band respectively, β and γ for bands as follows Equation (3.11) and Equation (3.12):

$$E_{g}^{exc}(N, L) = \frac{L(L+1)}{2I}$$

$$E_{\beta}^{exc}(N, L) = \frac{L(L+1)}{2I} + \left(E_{\beta} - E_{g}\right)$$

$$E_{\gamma}^{exc}(N, L) = \frac{L(L+1)}{2\mathrm{I}} + \left(E_{\gamma} - E_{\beta}\right) \tag{3.11}$$

$$I_{g} = \frac{-42\mathrm{Nn}\beta^{2}(1+\beta^{2})}{A\cdot\kappa + B\kappa'' + C\cdot k_{3}}$$

$$A = 42(1+\beta^{2})(1+2(-1+\mathrm{Nn})\beta^{2})$$

$$B = 7(-1+\mathrm{Nn})\beta^{2}(-1+\beta^{2})$$

$$\mathrm{C} = \left[28 + 2\sqrt{14(-1+\mathrm{Nn})\beta\chi} - 6\sqrt{14(-1+\mathrm{Nn})\beta 3\chi} - 7\chi 2 + 2(-1 + \mathrm{Nn})\beta 4\,\chi 2 - \beta 2\left(28 + 5\chi 2 + 2\mathrm{Nn}(-28 + \chi 2)\right)\right] \tag{3.12}$$

4. RESULTS AND DISCUSSION

In this research ^{178}Yb , ^{187}Hf , ^{180}Hf , ^{178}W, ^{180}W , ^{182}W , ^{178}Os , ^{180}Os , ^{182}Os , ^{180}Pt , ^{182}Pt are choosed for examination of CSA outcomes. And use the first is that the thrilled bands' band-head energy should be high practically known that is vital to calculation some important parameter. The seconds is selection of isotopes have a structure that resembles spinning.

Another quantity is the number of bosons (N), which are different for each nucleus. Number of bosons found for each nucleus is given in column N of Table 4.1.

Table 4.1 The parameters that gives the energy of isotopes

Isotope	N	σ	$\kappa 3$	κ'	κ''
$^{178}_{70}Yb$	15	2.088	-13.069	-9.099	5.0
$^{178}_{72}Hf$	15	1.175	-12.436	-10.836	4.84
$^{180}_{72}Hf$	14	0.141	-13.666	-10.375	4.87
$^{178}_{74}W$	15	0.734	-10.793	-13.619	0.1
$^{180}_{74}W$	14	0.048	-12.506	-12.643	-0.1
$^{182}_{74}W$	13	0.667	-14.946	-11.061	5.5
$^{178}_{76}\mathrm{Os}$	15	0.745	-8.413	-18.844	-0.1
$^{180}_{76}Os$	14	0.774	-9.111	-18.633	-4.9
$^{182}_{76}Os$	13	0.135	-10.186	-17.346	-0.1
$^{180}_{78}Pt$	12	1.578	-7.608	-22.646	0.1
$^{182}_{78}Pt$	13	1.234	-6.826	-23.273	5.0

The examination of CSA research parameters is shown in Table 4.2, Table 4.3, and Table 4.4. It can be seen that the theoretical result increases faster as the value of the angular momentum of state increases. However, for all isotopes, the predictions in the band are excellent, and the observed rapid increase in this band is not observed in the ground state band. The experimental data is relatively low and difficult to compare (Der Mateosian and Goldhaber 1950).

Table 4.2 Comparison of the theoretical results with the experimental data (Firestone 1996) for Ground State Band. (KeV)

Isotope	$E(2_1^+)$		$E(4_1^+)$		$E(6_1^+)$		$E(8_1^+)$		$E(10_1^+)$	
	Theo	Exp	Theo	Exp	Theo	Exp	Theo	Exp	Theo	Exp
${}^{178}_{70}Yb$	80	84	265	278	558	576	956	--	1460	--
${}^{178}_{72}Hf$	90	93	300	307	630	632	1080	1059	1649	--
${}^{180}_{72}Hf$	90	93	300	309	629	641	1079	1084	1648	--
${}^{178}_{74}W$	103	106	344	303	723	695	1239	1143	1893	--
${}^{180}_{74}W$	100	104	335	338	703	688	1205	1138	1840	--
${}^{182}_{74}W$	96	100	322	329	675	681	1158	1144	1769	--
${}^{178}_{76}Os$	126	132	421	398	883	761	1514	1194	2313	1682
${}^{180}_{76}Os$	128	132	426	409	896	795	1536	1258	2347	--
${}^{182}_{76}Os$	123	127	409	400	860	794	1474	1278	2161	--
${}^{180}_{78}Pt$	150	153	498	411	1047	757	1794	1182	2741	--
${}^{182}_{78}Pt$	152	155	506	419	1063	744	1822	1205	2784	--

Table 4.3 Comparison of the theoretical results with the experimental data (Firestone 1996) for Beta-band (β). (KeV)

Isotope	$E(0^+)$		$E(2^+)$		$E(4^+)$		$E(6^+)$		$E(8^+)$	
	Theo	Exp	Theo	Exp	Theo	Exp	Theo	Exp	Theo	Exp
${}^{178}_{70}Yb$	1300	1315	1380	1387	1565	1564	1857	--	2256	--
${}^{178}_{72}Hf$	1080	1199	1170	1277	1380	1450	1709	1731	2159	--
${}^{180}_{72}Hf$	1078	1102	1168	1183	1377	1370	1708	--	2157	--
${}^{178}_{74}W$	866	1083	970	1276	1211	1557	1589	1917	2106	--
${}^{180}_{74}W$	939	--	1039	--	1274	--	1642	--	2144	--
${}^{182}_{74}W$	1082	1136	1179	1257	1404	1510	1758	--	2240	--
${}^{178}_{76}Os$	580	650	706	771	1001	1023	1463	1396	2094	--
${}^{180}_{76}Os$	664	--	792	831	1091	1053	1560	1379	2200	--
${}^{182}_{76}Os$	894	891	1018	1039	1181	1190	1386	1400	1632	1588
${}^{180}_{78}Pt$	437	479	586	861	935	1248	1483	1650	2231	--
${}^{182}_{78}Pt$	425	500	577	855	931	1238	1488	1649	2247	--

Table 4.4 Comparison of the theoretical results with the experimental data (Firestone 1996) for Gamma-band (γ).(KeV)

Isotope	$E(2^+)$		$E(3^+)$		$E(4^+)$		$E(5^+)$		$E(6^+)$	
	Theo	Exp	Theo	Exp	Theo	Exp	Theo	Exp	Theo	Exp
$^{178}_{70}Yb$	1195	1221	1274	--	1380	--	1513	--	1672	--
$^{178}_{72}Hf$	1160	1175	1250	1296	1370	1384	1520	1691	1700	--
$^{180}_{72}Hf$	1182	1200	1272	1291	1392	1409	1542	1557	1721	--
$^{178}_{74}W$	1040	1045	1143	1121	1281	1126	1453	1345	1660	--
$^{180}_{74}W$	1109	1117	1210	1233	1344	1361	1511	1536	1712	1703
$^{182}_{74}W$	1201	1221	1297	1331	1426	1443	1587	1624	1780	1770
$^{178}_{76}Os$	881	864	1008	1032	1176	1213	1386	1416	1639	--
$^{180}_{76}Os$	880	870	1008	1023	1179	1197	1392	1406	1648	1628
$^{182}_{76}Os$	687	--	809	--	1096	--	1547	--	2161	--
$^{180}_{78}Pt$	688	678	838	963	1037	1049	1286	1315	1585	1728
$^{182}_{78}Pt$	680	680	832	832	1034	1034	1287	1287	1591	1591

5. CONCLUSIONS

Preliminary information was available on the properties of the isotones nuclei, the subject of research, from knowing the locations of the primary irritation cases as well,The ratio between the second state of irritation to the first. The absence of back bending in the Os, W, and Hf isotones, which indicates that there is no the change of its properties, while in Pt the back bending appeared, and this is evidence of the change of its properties, and by using the ratio between the cases of the different excitations to the cases preceded show that the isotopic nuclei in question have rotational properties with the exception of Pt isotopes have interspecific transition properties SU(5) but are closer to the smooth gamma curve O(6). To make sure more of the properties of the nuclei subject the research and drawing the results show us that the Os, W, and Hf isotones have rotational properties. Whereas at Pt it is found to lie between the two rotational determinations and the soft kama, but it is closer to the soft kama curve, and thus we can say that Pt belongs to the two definitions, the vibrational determination and the soft gamma determination. From the swing, we can see that the properties of the isotones in question Pt, W, Hf do not change, which indicates stability of its properties. Calculations of the energy states using the Mathematica program showed that there is a good agreement between the values practical and experimental nuclei subject of research.

The practical energy levels of the isotopes Yb, Hf, , W, OS , Pt show the rotational nature of it. Therefore, the rotational determination SU(3) of the IBM-1 model was used to describe the nuclear structure of the energy levels of these isotopes, using the program Wolfram Mathematica. The coefficient values that gave the best match between the values of the theoretically calculated energy levels and values

• Referring to Table (1), which shows the coefficients The parameters that gives the energy of isotopes, due to the increase in the number of bosons outside the closed shell to increase the number of neutrons, and thus it will suffer from distortions due to the presence of a large number of Nucleons.

• As for the second coefficient (P.P), it remains constant because it has no effect due to the determination of SU (3) to which these two isotopes belong. The same is the case for the coefficient T3.T3)) and coefficient ((T4.T4). Regarding the third factor (L.L). It has variable values due to the nature of that determination, and it is clear from the values of the coefficients (Q.Q) and the distorted nature of the isotopes and their belonging to the determination SU(3).

• From the observation of Table (2) and in comparison with , we find that these isotopes, as we spoke earlier, belong to the rotational determination SU (3) through the typical values of the energy ratios and their comparison with the calculated practical and theoretical values.

• we find that the calculated energy levels are in good agreement with the practical energy levels and for the first package (Ground State-band only), while we find there is a clear difference in the levels at high energies, and this is what we conclude from that the boson model It gives inaccurate results for beams and high energies because it does not distinguish between proton bosons and neutron bosons, and thus it will lose some important nuclear properties such as magnetic and zero transitions.

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