

NONLINEAR STRUCTURAL MODELING OF HAGIA SOPHIA IN ISTANBUL

by

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ABSTRACT

NONLINEAR STRUCTURAL MODELING OF HAGIA SOPHIA IN ISTANBUL

Hagia Sophia in Istanbul is one of the most iconic architectural and engineering achievements in history. Constructed in the sixth century, the structure has remained standing for over 1500 years, despite being exposed to numerous destructive earthquakes that caused significant damage, including partial collapses. Its location in a high seismic hazard zone makes the evaluation of its structural performance essential. Since the 1990s, studies on Hagia Sophia have mostly relied on linear analyses and idealized geometries, largely neglecting long-term deformations. This thesis focuses on the evaluation of the level and extent of the present-day deformations on the structure; the creation of a finite element model based on the actual geometry rather than the idealized one; the assessment of the contribution of permanent deformations to the static and dynamic structural response; nonlinear static and dynamic modeling to understand the structure's present and future earthquake behavior; and the assessment of potential collapse mechanisms of the structure. The deformed geometry of the building was obtained using three-dimensional laser scanning and converted into a mesh. A model with ideal geometry was first developed in SAP2000 and then updated to reflect the observed deformations. Both deformed and undeformed models were transferred to ANSYS and subjected to static and dynamic analyses. Real earthquake records were used in the linear dynamic analyses. The nonlinear modeling was carried out using calibrated material definitions based on recordings from the Mw 6.1 Silivri earthquake on April 23, 2025. Additional analyses were performed using both real and synthetic ground motion records. The results were evaluated in terms of structural response and collapse mechanisms, with a particular emphasis on the influence of geometric deformation.

ÖZET

İSTANBUL'DA BULUNAN AYASOFYA'NIN DOĞRUSAL OLMAYAN YAPISAL MODELLEMESİ

İstanbul'daki Ayasofya, tarihin en ikonik mimarlık ve mühendislik başarılarından biridir. Altıncı yüzyılda inşa edilen yapı, kısmi göçmeler de dâhil olmak üzere ciddi hasara yol açan birçok depreme rağmen 1500 yılı aşkın süredir ayakta kalmıştır. Yüksek sismik tehlike bölgesinde yer alması, yapının yapısal performansının değerlendirilmesini zorunlu kılmaktadır. 1990'lardan bu yana yapılan çalışmalar, Ayasofya'nın davranışını çoğunlukla lineer analizler ve idealize geometriler kullanarak incelemiş, uzun dönemli deformasyonlar büyük ölçüde göz ardı edilmiştir. Bu tez, yapıdaki mevcut deformasyonların düzey ve kapsamının değerlendirilmesi; idealize değil, gerçek geometriye dayalı bir sonlu eleman modelinin oluşturulması; kalıcı deformasyonların statik ve dinamik yapısal tepkiye katkısının değerlendirilmesi; yapının mevcut ve gelecekteki deprem davranışını anlamak amacıyla doğrusal olmayan statik ve dinamik modellemeler yapılması; ve yapıdaki olası göçme mekanizmalarının değerlendirilmesi üzerine odaklanmaktadır. Yapının deforme olmuş geometrisi, üç boyutlu lazer tarama ile elde edilmiştir. SAP2000 yazılımında önce ideal geometriye sahip bir model oluşturulmuş, ardından deformasyonları yansıtacak şekilde güncellenmiştir. Deforme olmuş ve olmamış modeller Ansys yazılımına aktarılmıştır ve statik ve dinamik analizlere tabi tutulmuştur. Doğrusal dinamik analizlerde gerçek deprem kayıtları kullanılmıştır. Doğrusal olmayan modelleme, 23 Nisan 2025 tarihli Mw 6.1 Silivri depremi kayıtlarına göre kalibre edilen malzeme tanımlamaları ile gerçekleştirilmiştir. Daha sonra hem gerçek hem de sentetik deprem verileri kullanılarak ilave analizler yapılmıştır. Elde edilen sonuçlar, yapısal tepki ve göçme mekanizmaları açısından değerlendirilmiş, geometrik deformasyonun etkisi vurgulanmıştır.

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LIST OF SYMBOLS

R_{jb}	Joyner-Boore distance
R_{rup}	Rupture distance
S_{ae}	Elastic spectral acceleration
S_{D1}	Design spectral acceleration coefficient for the period of 1 second
S_{DS}	Design spectral acceleration coefficient for the short period region
S_{xx}	X directional normal stress
S_{yy}	Y directional normal stress
T	Vibration period of the structure
T_A	Lowerbound period for the constant acceleration plateau of the spectrum
T_B	Starting point of the flat region of the design spectrum
T_L	Period beyond which spectral acceleration decreases inversely with period
UX	Translational displacement in the X-direction
UY	Translational displacement in the Y-direction
V_{s30}	Average shear-wave velocity in the top 30 meters of soil
RZ	Rotation about the Z-axis

LIST OF ACRONYMS/ABBREVIATIONS

3D	Three Dimensional
APDL	Ansys parametric Design Language
CAD	Computer-Aided Design
C-A-H	Calcium Aluminate Hydrate
CPU	Central Processing Unit
C-S-H	Calcium Silicate Hydrate
DTA	Differential Thermal Analysis
E	East
FE	Finite Element
FEA	Finite Element Analysis
FRP	Fiber Reinforced Polymer
HDS	High-Definition Laser scanner
HPC	High Performance Computing
HSD	Hardening-Softening-Dilatation
N	North
NE	Northeast
NW	Northwest
RH	Rate of Humidity
S	South
SE	Southeast
SRSS	Square Root Sum of the Squares
SW	Southwest
TEM	Transmission Electron Microscopy
TGA	Thermogravimetry Analysis
TG-TDA	Thermogravimetric and Thermodilatometric Analysis
TSC	Turkish Seismic Code
W	West

1. INTRODUCTION

Hagia Sophia, illustrated in Figure 1.1, is often regarded as one of the greatest feats of construction ever accomplished by humans [1]. Its massive size and impressive architecture both date back to the sixth century when it was built. Anthemius of Tralles and Isidorus of Miletus designed an edifice that stood for 800 years as the world's biggest vaulted construction. Both Byzantine and Ottoman builders looked up to Hagia Sophia as an inspiration and a challenge due to the scale and intricacy of the structure.

The aims of this thesis are: evaluation of the level and extent of the present-day deformations on the structure; creation of a finite element model with the actual geometry but not the idealized one; assessment of the contribution of the permanent deformations to the static and dynamic structural response; non-linear static and dynamic modeling to understand the structure's present and future earthquake behavior; assessment of the failure mechanisms of the structure under earthquakes

In this introductory chapter brief background information will be given on the history, structural design, and past damages and restorations of Hagia Sophia, before proceeding forward.

1.1. Building History

In 1985, UNESCO added Hagia Sophia to its list of world historic sites. Isidore of Miletus and Anthemius of Tralles, the architects of Hagia Sophia, combined a centralized structure with a longitudinal basilica in their design [1]. Given its scale, structural complexity, and use of masonry as a construction material, the construction era (532–537) lasted just over 5 years, which is a remarkably short period of time. This is one of the main reasons that the structure suffered static problems. Since it is a highly complex structure, it is deeply necessary to understand its construction phases and the

restoration background.

The building's breadth is 69.5 m, and its longitudinal length is 100 m. The height of the main dome above the ground is 55.60 metres. The dome's diameter is 30.86 metres in the east-west direction and 31.87 metres in the north-south direction [1]. For more than 900 years, Hagia Sophia functioned as the Byzantine Empire's main church. It was turned into a mosque during the Ottoman invasion of Istanbul in 1453. Between 1935 to 2020, it served as a museum before transforming into a mosque again.



Figure 1.1. Hagia Sophia in İstanbul, Turkey.

Throughout its history, Hagia Sophia was subjected to numerous earthquakes that caused damage and partial collapses. The first destructive damage occurred on the eastern side of the main dome in 553 during an earthquake. Consequently, the main dome was raised six metres over its original height [2]. The second major earthquake, which struck in 989, damaged the west main arch and brought down nearby main dome sections. Then a thicker and wider arch was constructed [3]. The last time the east main arch and surrounding portions of the dome collapsed was in 1349, and they were reconstructed [1]. The building never collapsed as a whole.

1.2. Structural Design

Since the city was being threatened by earthquakes, the structural design considering the seismic behaviour was quite important, especially for that scale of construction. The primary structural system consists of four main piers, four main arches, pendentives, main dome and its ribs and two semidomes as can be seen in Figure 1.2.



Figure 1.2. Interior view of Hagia Sophia in Istanbul, Turkey.

A square set of main piers forms the center of the structure as can be seen in Figure 1.3. At that time, since Justinian asked to construct a huge inner space, Isidorus and Anthemius tried to find a solution to have a large inner space carrying a huge heavy dome without compromising the structural safety. Finally, they thought of making an illusion with the help of two semi-domes. The square area carrying the main dome would be enlarged with extra space under the semi-domes. The cathedral owes its large rectangular area to this idea [1].

Besides, it was quite important to use light material for the main dome. Another problem to face was how to build the dome on a square area. Justinian wished Hagia Sophia to be unique, different from any other cathedrals, and the huge dome symbolizing the sky and the God carried by a square area symbolizing the earth [4]. If the dome was built on a square area the corners would not have carried the load directly, therefore the edges would be overloaded [5]. Hence, the architects again found a solution by adding the pendentives to the support system. Pendentives, as can be seen in Figure 1.3, are linking the top of the arches to each other shaping the required circle for the dome and it was the first time that the pendentives were used in construction.



Figure 1.3. The semidomes (on the left) and the pendentives (on the right) [5].

However, even during the construction period, due to self-weight, instability problems occurred, and the base that will carry the dome was not circle anymore, but elliptic. Therefore, a circle base consisting of windows was added on that elliptic shape, which, after the first earthquake, was understood not to be a good idea [5]. As mentioned previously, even self-weight was a problem for Hagia Sophia, and after the dome was added, it pushed the supports outward. To solve this problem, the buttress piers were added to the north and south main piers: barrel vault and the semi-domes to the east and west parts. These elements, as shown in Figure 1.4, all together formed the first design of Hagia Sophia.

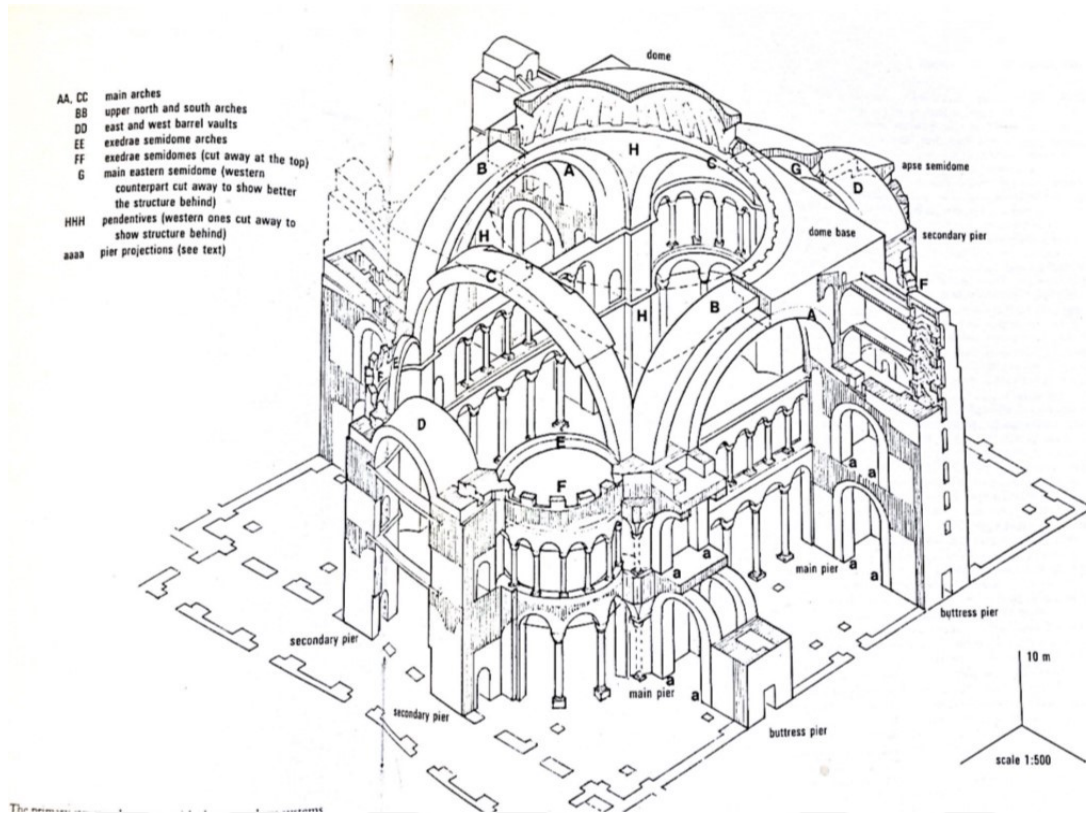


Figure 1.4. The structural elements of Hagia Sophia [1].

1.3. Damages and Restorations

There are 3 major earthquakes which remained damages on Hagia Sophia occurring in 557, 989 and 1346. Therefore in 6th, 10th and 14th centuries the structure has undergone major restorations [1]. The location of the mentioned restorations are illustrated in Figure 1.5.

557 Earthquake

The structure has experienced several earthquakes during its entire life and the first one was in 557, 20 years after the construction. After the earthquake, the eastern part of the main dome collapsed. Justinian hired the Young Isidoros, the nephew of

the first architect to build a new dome. He has designed the new dome considering several precautions in terms of stability [1].

The problems causing the collapse were the low height and the angle of the dome sphere pushing the substructure outwards and the weakness of the circle with the windows. Therefore, the new dome was constructed 7m higher than the older one and the window circle was not used as a support anymore, so the dome was connected to the structural elements directly [1].

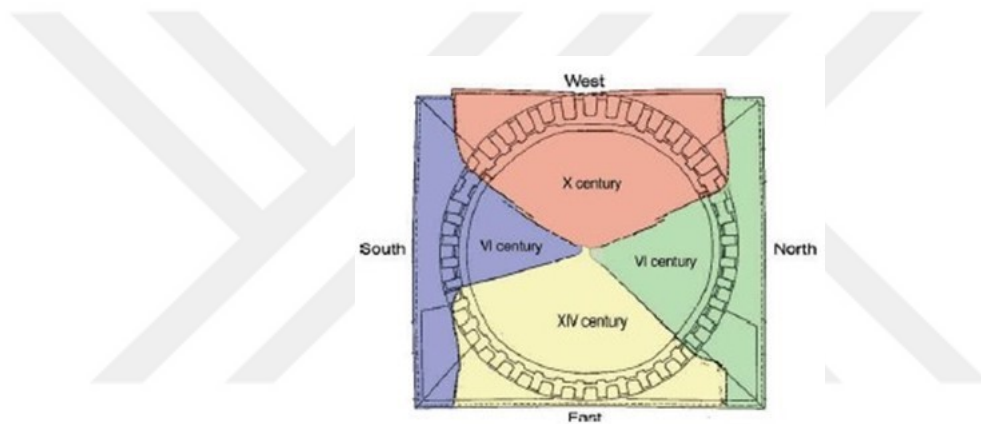


Figure 1.5. The restoration periods of Hagia Sophia [1].

989 Earthquake

In 989, Hagia Sophia experienced another earthquake that caused damage on the west arch and the west part of the dome. Then the west arch was widened and raised, the ribs in the damaged part of the main dome was deepened [1].

1343 Earthquake

This time, the eastern part of the main dome, the east arch, and part of the east semidome collapsed. Then these elements were repaired compatible with the old design [1].

2. LITERATURE REVIEW

This chapter attempts to present an overview of previous research efforts on structural analysis, material properties, earthquake monitoring, deformation monitoring, subsurface conditions, strengthening alternatives and shake table tests carried out for understanding Hagia Sophia's structural properties.

2.1. Structural Analysis

The primary objective of all past and present analytical and experimental studies has been to understand the static and dynamic behavior characteristics of Hagia Sophia. The structure's resilience in a future significant earthquake near Istanbul cannot be addressed without the comprehensive information obtained from such examinations. A series of analytical investigations on the dynamic behavior of Hagia Sophia have been carried. Static and dynamic linear structural models and analyses were conducted starting in the early 1990s. The first finite element model of Hagia Sophia was created by [6] for static analysis, under the supervision of Ahmet Cakmak from Princeton University. A linear finite element model was created using SAP90 software. The model considered the primary structural system. A second model combining primary and secondary structural systems was also developed. However the analyses used the model with the primary structural system which includes the main, secondary and buttress piers, the main dome and the semi-domes, the exedra domes, the main and secondary arches, barrel vaults, pendentives and surcharge [6]. The SAP90 model developed by Hill was expanded and refined by [7], [8], [9]. Durukal performed dynamic analysis and validated the model using real earthquake recordings under the supervision of Mustafa Erdik of Bogaziçi University [7]. Davidson examined the impact of different construction phases and stages [8].

Static behavior was observed through the stresses, forces, deformations due to various loading conditions by Davidson using the model created in SAP90 as given in

Figure 2.1. [8]. During the study, visual observations were also done to compare the analytical model results. Despite the fact that the linear deformations obtained by the model were quite small compared to the real ones, the model revealed the tension and compression zones, the crack susceptibility and the exciting forces. Additionally, the model forced further until the cracks occur and the cracking effects were examined.

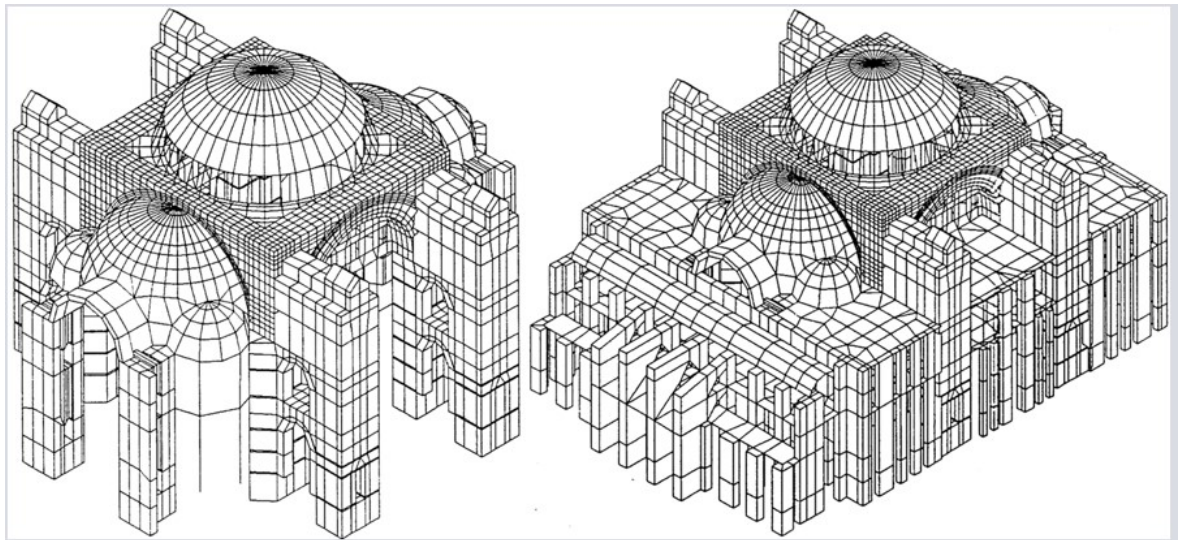


Figure 2.1. The finite element model of the primary structure (on the left) and the whole structure (on the right) [8].

Table 2.1. Comparison of the real and the analytical displacement results obtained by Davidson [8].

Cross section	FE Model (mm)	FE Direction	Measured (cm)	Measured Direction
Top of the main dome	7.3	Downward	25.8	Downward
East Arch Crown in Vertical dir.	8.9	Downward	13.7	Downward
East Arch Crown in E-W dir.	2.3	Inward	13.4	Inward
West Arch Crown in Vertical dir.	8.6	Downward	14.8	Downward
West Arch Crown in E-W dir.	1.8	Inward	8.9	Inward
North Arch Crown in Vertical dir.	5.9	Downward	13.7	Downward
North Arch Crown in N-S dir.	0.9	Outward	17.8	Outward
South Arch Crown in Vertical dir.	5.5	Downward	57.9	Downward
South Arch Crown in N-S dir.	0.9	Outward	40.5	Outward
Top of the N-E Pier in N-S dir.	1.6	Outward	8.5	Outward
Top of the N-E Pier in E-W dir.	0.5	Outward	4.5	Outward
Top of the N-W Pier in N-S dir.	1.6	Outward	8.5	Outward
Top of the N-W Pier in E-W dir.	0.7	Outward	40.5	Outward
Top of the S-E Pier in N-S dir.	1.6	Outward	7.4	Outward
Top of the S-E Pier in E-W dir.	0.5	Outward	45.5	Outward
Top of the S-W Pier in N-S dir.	1.7	Outward	7.4	Outward
Top of the S-W Pier in E-W dir.	0.7	Outward	25.8	Outward

The displacement obtained by the linear static analysis are given in Figure 2.2. Besides, comparison of them to the current deformations on the structure is given in Table 2.1. The high values of displacements should be the result of short construction period and long curing time of mortar. It is believed that the current deformation seen today occurred mostly while the construction was going on [8].

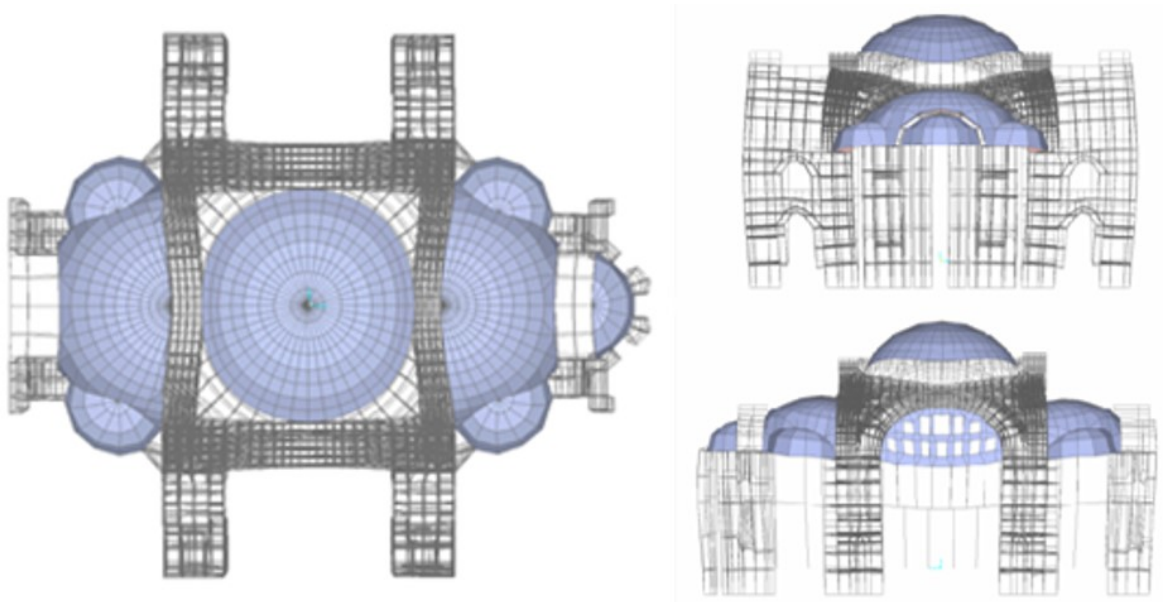


Figure 2.2. The deformation results of the linear static analysis [8].

After the static structural analysis, Davidson also studied the dynamic behaviour of the structure. The construction period was discretized into seven phases considering geometry and material properties varying within seven time steps [8]. In the model, for each construction phase, it was assumed to be deformed under the self-weight and due to the hardening the elasticity modulus increased then the next phase constructed on it; which means that elasticity modulus started with one value and got a higher value at the end, without hardening gradually. This method came out with more reliable results. Within the scope of nonlinear static analysis, cracking effect was also examined. Even though several previous studies revealed that the tensile strength of the mortar reaches upto 3.5 MPa, due to the complexity of the curing time and hardening, the regions with tensile stress exceeding 1.4 MPa were assumed to crack. First, the elements in the cracked regions were disconnected; however, instead of disconnecting them, reducing the elasticity modulus of the cracked members was found to be more realistic. The same concept was followed for the nonlinear dynamic analysis aiming to match the strong motion parameters of Hagia Sophia. In order to observe the prediction of the future behavior of the structure, response to produced accelerograms of magnitude 6.5

and 7.5 earthquakes were analyzed using the SAP90 nonlinear dynamic model. Even though the eigenvalue analyses were declared to be reliable, the deformation responses were determined to be underestimated.

Additionally, to inspect the dynamic behavior, an ambient vibration study was carried out using the monitoring data obtained from the accelerometer network located on the structure [10]. 9 locations were identified on the structure for the measurement. The natural frequencies and the mode shapes were obtained analyzing the Fast Fourier Transforms of the records. Another study was based on Karacabey Earthquake that occurred in 1992 with a magnitude of 4.8 and 120 km far from Hagia Sophia [11]. Durukal carried out dynamic analysis and validated the model with real earthquake recordings [7]. First the power spectral density and the transfer characteristics were inspected. Next, in order to observe the dependency on the input motion, the records were divided into three stationary intervals and the dominant frequencies were computed for each of them. Moreover, each of the three intervals revealed different dominant frequencies than the whole record. The second interval resulted in larger frequency which means lower stiffness. The reason might be the microcracking in mortar or the nonlinear behavior of the structure. Nevertheless, this low-level earthquake is probable to cause neither microcracking nor nonlinear behavior.

Besides, Çakmak and his colleagues have carried static and dynamic linear analysis and found that:

“The linear finite element models of the stone masonry portion of the structure are in fair agreement with measured responses, while the models underpredict the response of the brick masonry portion of the structure” [11].

Additionally, the elasticity modulus values used for the dynamic analysis must be different than the ones used in static analysis to match the real deformations and the natural frequencies. Also, it was observed that when the surrounding structures are included, the system gets stiffer and brings out more realistic frequencies; however, considering the computation time and difficulties the dynamic analysis was carried without

surrounding structures. Besides, they claimed that, nonlinearity must be incorporated in the study [11]. However, SAP90 software did not have the nonlinear analysis option; therefore, the nonlinearity effect was integrated to the analysis by nonlinear material properties and separated construction stages considering both dynamic and static models.

Researchers have also studied the inherent vulnerabilities of the structure. In 1993, Mark and his colleagues carried a study and concluded that the main east and west arches have a disposition to warp out of plane under gravity and since the first mode of the structure is the translational east-west direction, this might explain the collapse of adjacent parts in the east west portions throughout the history of Hagia Sophia [12].

According to Natsis, soil-structure interaction effect is quite important for stiff structures constructed on comparably softer soil, due to significant rocking motion. Therefore, the knowledge related to the ground condition that the structure lies on is remarkable. Hence he conducted an investigation consisting of a preliminary soil-structure interaction, cracking in the arches, and the effect of material anisotropy in the main piers [9]. As a preliminary study, soil-structure interaction was modeled using finite elements with addition of springs and varying material properties of the subsurface at the pier bases. The importance of soil-structure interaction is a result of its ability to change the fundamental frequency. The elasticity modulus of the soil was calculated using the soil density and the wave velocity which was estimated by the tomographic study . Then, together with the depth of bedrock, which was also determined by the tomographic survey, it was used to evaluate the stiffness of the soil. As a result, the natural frequency reduced to a value lower than the one measured by the ambient vibration test, getting closer to the frequency released under strong ground motion. Increased North-South motions at the southwest pier is captured, however there was no observed effect on arches response. Natsis concluded that:

“The proposed soil conditions at Hagia Sophia suggest that soil structure interaction, which is most effective when modeling stiff structures on soft soil,

may not be sufficient in itself to increase the model's response" [9].

Following Mainstone's suggestion, Natsis incorporated hinging cracks at east and west main arches; however, it did not make significant changes in the model response. Besides, varying combinations of cracking were applied on the model, however none of them managed to match the north-south or vertical responses at the crowns. In the structural analysis of Hagia Sophia, the modal responses were usually overestimated in terms of the east west motion, whereas the north south motion was underestimated. The model behavior was extremely symmetric, therefore anisotropy in the pier motion was incorporated to obtain the asymmetric response. The modified modulus of elasticity which reduced the stiffness of the cracked points, and the incorporation of the anisotropic material properties reduced the frequency and the response magnitudes [9].

Besides, according to the studies on the pre-1999 earthquakes, a difference in the southwest pier frequency characteristics were determined and the displacements of the east and west main arches during the earthquake were found to be significant implying the most susceptible locations on the structure under a major earthquake [10]. In 2003, Durukal and her colleagues carried another study on the dynamic behaviour of Hagia Sophia using 17 August 1999, Kocaeli (M_w 7.4) and 12 November 1999, Düzce (M_w 7.2), Turkey earthquake records applying the same technique and the similar results were obtained when the southwest pier and the east and west main arches were considered [13]. Besides, the study of Durukal resulted in different response characteristics of the southwest pier such as higher level of accelerations and different frequency content comparing to the other three main piers and high amplitude vibrations in order of lateral vibrations at the crowns of the east and west main arches in the vertical direction [13]. At this point, the knowledge related to the material properties, the construction technique and stages, soil structure interaction or the ground conditions become more of an issue.

Duppel carried non-invasive techniques on Hagia Sophia and observed the thickness of the main dome and concluded that accordingly with the reconstructions, the

thickness varies [14]. Regarding this result, Karlsruhe Institute created a 3D CAD model of Hagia Sophia considering the primary and secondary system together [15]. On that model, structural analyses are carried using parallel computers to overcome computation performance issues [16]. The first 3 model frequencies resulted in 1.92Hz, 2.08Hz and 2.35Hz in X, Y and rotational Z directions respectively. Except for this study, the other structural analyses in literature are carried on the primary system of Hagia Sophia.

In 2013, Kokkinos modeled Hagia Sophia using Abaqus software [17]. He created 2 models with different modulus of elasticity. One model was based on the results of Moroupulo's study [18] on the material of Hagia Sophia, whereas the other one considered the material results obtained by Davidson [8]. However, considering the linear static analysis none of them could figure the exact deformations observed on the real structure. Abaqus software incorporated nonlinearity applying the Newton-Raphson Method. The method employed incremental iteration on stiffness matrix, however the expected convergence was not succeeded. It was considered to be due to the excessive deformations obtained by linear static analysis. Besides, Abaqus software designed the elements separately and then combined including the constraints; hence the incorrect constraint definitions might have led to overestimated deformations. The dynamic analysis also approved that the problem is about the restraints, therefore the stiffness, rather than the material properties. Rhodes, Worcester, and Smith also explored structural behaviour utilizing software with nonlinear analytic capabilities [19], [20], [21]. In 1996, Davidson's model was recreated by Smith using ANSYS, which allows the incorporation of nonlinearity by unique stress-strain curves.

2.2. Material Properties

The evaluation of Hagia Sophia's material properties is essential for understanding present static deformations and previous collapses, as well as for predicting its future structural behaviour. The nave and narthexes are mostly covered with marble, mosaics, and plaster. However, significant portions of the main dome remain visible, revealing

the brick-and-mortar structure. The staircase- and passage-interiors, as well as the freshly renovated western exterior, are areas of the structure where one may examine the original building elements. To give an overview, the main piers and the secondary piers are constructed of stone masonry, while all structural elements springing from them or above them are of brick masonry Figure 2.3.

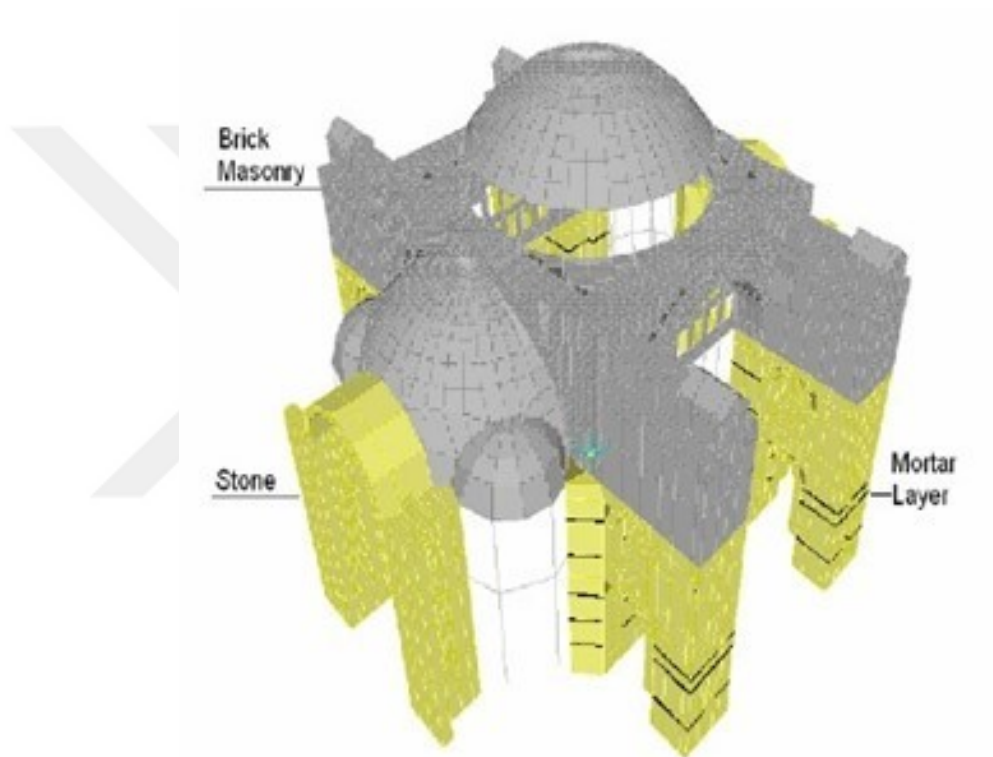


Figure 2.3. The material used in the structure [22].



Figure 2.4. The mortar layer between the bricks [18].

The brick masonry consists of brick and the mortar layers. The mortar height is at least the one of the brick above it and the main material to be tested is the mortar as illustrated in Figure 2.4. Physical studies on mortar samples, in-situ ultrasonic testing, and an inverse engineering method using structural models and analysis were employed to evaluate material properties. ([23], [5], [18]). The material analysis was carried out using X-Ray diffraction, thermal neutron diffraction and thermogravimetric analysis by Livingston [24]. It was found that the mortar has pozzolanic characteristics due to the ground brick dust content rather than being pure lime mortar. However, it consists both carbonated lime and C-S-H phase. When the effect of the material on the structural behavior is considered, the significant advantage of pozzolanic mortar over pure lime mortar is the higher tensile strength. Pozzolanic lime mortar can reach a tensile strength of 3.5 MPa after a year [24]. Besides the deformations occurred during the construction phases are attributed to the long curing time of pozzolanic mortar. It was assumed that “this slow strength development may account for the large plastic deformations of the structure” [24]. Furthermore, pozzolanic mortar is easily affected by the altering temperature when gaining its ultimate strength. Besides the composite material of Hagia Sophia is said to be lighter than today’s concrete. Since the large specimens of Hagia Sophia masonry are not available, it is not easy to obtain the results undoubtedly; therefore, non-destructive techniques are more appropriate to

apply. Besides, Çakmak and his colleagues carried out preliminary material tests on components of the supports, a rib of the main dome and main arches to identify the mechanical properties [25]. They also have uncovered that the mortar has pozzolanic characteristics, which probably is a result of dust brick content behaving as artificial pozzolanas. They have concluded that “the crushed brick phase of the arch mortar sample exhibits a transfer of pores to smaller radii from penetration of lime into the crushed brick phase and reaction with the crushed brick and the mortar matrix is very light with high porosity” [25]. Moropoulou and colleagues investigated the material properties of Hagia Sophia to observe the energy absorbing behavior under seismic load [18]. Following the instrumental methods such as Fiber Optics Microscopy, X-Ray diffraction Analysis, Optical Microscopy, Scanning Electron Microscopy, and Transmission Electron Microscopy. It was found that the calcium silicate hydrate (C-S-H) and calcium aluminate hydrate (C-A-H) content is remarkable, and the gel phase (C-S-H) is considered to be responsible for the nonlinear behaviour and self-healing effect. The experiments concluded that “the dated mortar samples examined proved to be resistant to continuous stresses and strains due to the presence of the amorphous hydraulic formations (CSH), investigated by transmission electron microscopy (TEM) at the crushed-brick powder/binder interfaces and at a sufficient content in the binder matrix, as proved by TG-TDA, which allows for greater energy absorption without initiation of fractures, let alone the transition of the gel to a higher order of formation” [18]. Owing to the chemical structure of the mortar matrix, as a result of the reactions, the pore size can be reformed contributing to the material durability against pollution and humidity as well as the high strength. Regarding these details about the original mortar, Moroupulo and colleagues carried out an experimental study to produce mortar compatible with the one used in Hagia Sophia [18]. Mechanical properties such as the compressive and flexural strength, the chemical evolution and the dynamic elasticity of modulus were considered regarding the produced four different types of mortars. Mechanical tests were applied for strength determination whereas the thermal analysis (Differential thermal analysis (DTA), thermogravimetric analysis (TGA)) and ultrasonic technique were used for the chemical evolution and the dynamic modulus of elasticity respectively. The four types of the produced mortar were: hy-

draulic lime mortar; mortar with lime, natural pozzolana and cement; mortar with lime; mortar with lime-natural pozzolanic addition. It was declared that “all the examined mortars seem to be in progress after 6 months of curing; the best mechanical behavior was observed in mortars with lime-natural pozzolanic addition” [18]. Besides, the low ratio of the compressive strength over flexural strength, which leads to low elasticity of modulus compromising the original mortar in Hagia Sophia was observed in the mortar with lime and mortar with lime-natural pozzolanic addition. Additionally, Cappa and colleagues carried out a thermographic survey on the structure and observed the construction methods and differences in the materials used in the construction and the restorations over the centuries [26].

2.3. Structural Monitoring

2.3.1. Accelerometer Network

Hagia Sophia earthquake monitoring network was first installed in August 1991 [10]. It was completely renewed in 2008 to meet the developments in communication and sensing technologies [27]. The network comprises nine three component accelerometers and four two-way tiltmeters. The sampling rate is 100 Hz. One accelerometer is positioned on the ground, four are installed at the tops of the four major load-bearing piers, and the other four are located at the crowns of the four main arches that support the main dome. Figure 2.5 depicts the position of the sensors and station codes on a schematic depiction of the building’s structural parts. The same figure also indicates the locations of the tiltmeters.

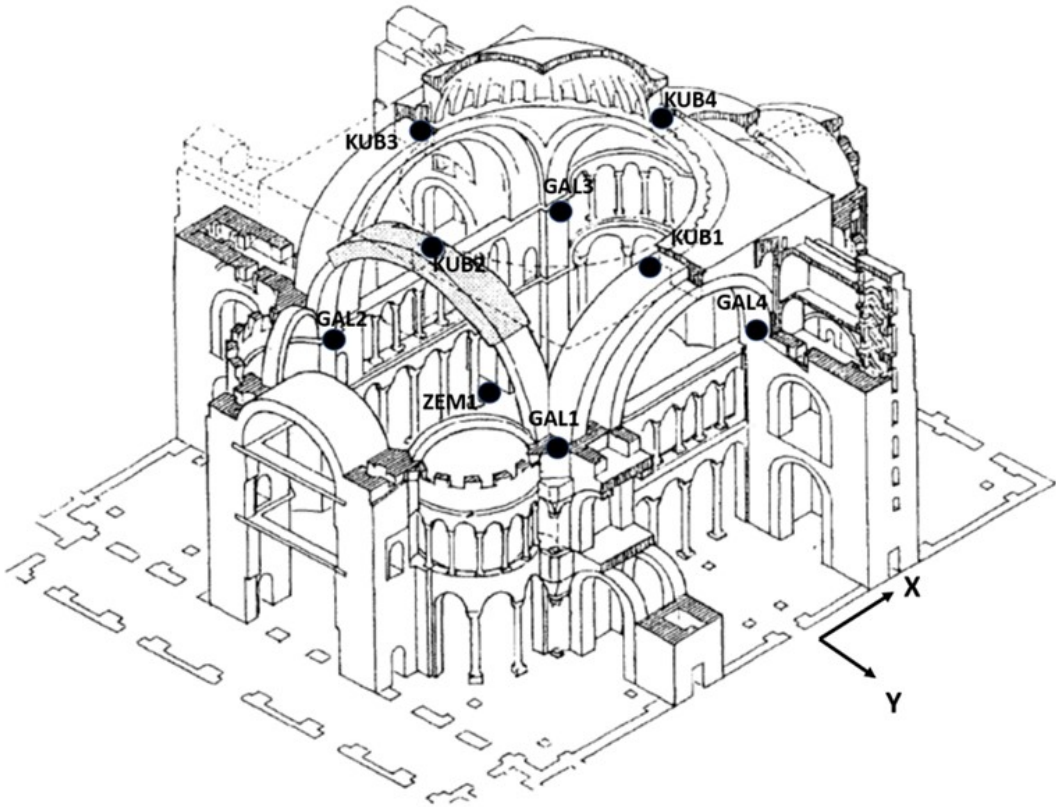


Figure 2.5. Accelerometric monitoring system in Hagia Sophia; Stations GAL1, GAL2, GAL3 and GAL4 include two-way tiltmeters in addition to accelerometers.

The isometric view is from Mainstone's book [1].

The network has been functioning in real-time mode since November 2008. Between 2008 and 2015, about a hundred earthquakes were registered at Hagia Sophia. The collected data are utilized to determine the structure's dynamic response characteristics, including modal shapes, modal frequencies, and modal damping. They are also used in the study of time-variation of these parameters, as well as in research on identification and characterization of damage-related structural parameters [13], [27].

The acceleration output varies both between the stations on the different levels and the stations on the same level. GAL1 (Southwest main pier) was found to have a higher level of acceleration in the y and z directions than the other stations at the

same level. Acceleration values recorded at KUB2 (west main arch) are significantly higher than the east main arch station in the x and z directions [13] as can be seen in Figure 2.6.

Drop in modal frequencies was identified in the structure as a common phenomenon [28]. Seventeen earthquakes during which largest accelerations were recorded in Hagia Sophia including Kocaeli and Düzce earthquakes were studied for the assessment of modal frequency variations. In the first two modal frequencies drops reaching up to 25% were found (Figure 2.6).

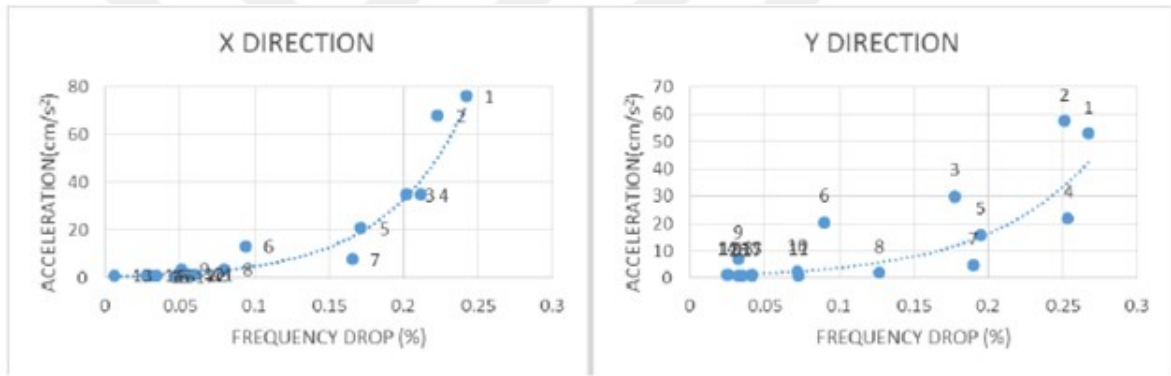


Figure 2.6. Reduction in modal frequencies during events with seventeen largest structural acceleration levels [28].

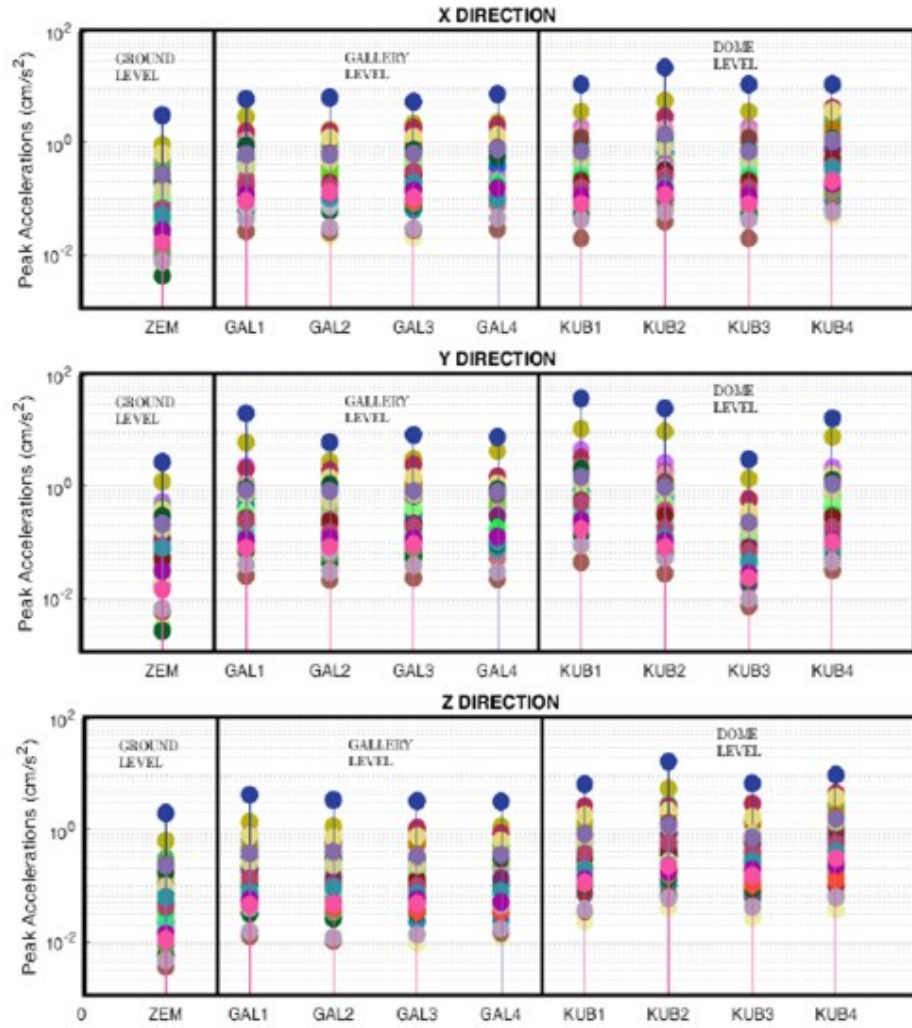


Figure 2.7. Observed peak accelerations at stations in three directions [28].

The building's modal vibration frequencies and damping coefficients are affected by atmospheric conditions. Temperature fluctuations on a daily and seasonal basis, for instance, create continual cyclic variation in the dynamic response. The Hagia Sophia is not an exception to this trend. Figure 2.7 illustrates the relationship between Hagia Sophia's first modal frequency and temperature in 2013.

2.3.2. Tilt-meters

Alongside acceleration sensors, four dual-component tiltmeters are positioned at stations GAL1, GAL2, GAL3, and GAL4. A tilt-meter is an instrument utilized to measure angular rotations of a structural component or the ground [28]. The measured tilt of a structural component results from the tilt induced by loads, the variation in the tiltmeter output caused by temperature fluctuations of the gauge, and the alteration in the element's tilt owing to temperature changes. Assuming uniform temperature exposure across all tiltmeter channels (with all sensors situated at the same elevation indoors) and considering event-specific data indicating that all sensors will register analogous tilt changes of the main piers due to temperature, we can pursue an inter-station comparative analysis of the tiltmeter data. In earthquakes above a magnitude of 5.0, the usual documented maximum rotations ranged from 0.0004 to 0.0006 degrees. The maximum documented tilt is approximately 0.005 degrees, observed during a magnitude 6.5 event. The rotations documented by station GAL1 are continuously elevated in both directions across all seismic events [28].

2.3.3. Long-term Deformation Monitoring

Laser technology has facilitated significant advancements in architectural and structural research. A study of Hagia Sophia commenced in 2016 as part of a broader initiative to monitor long-term deformations of several monumental structures in Istanbul utilizing laser technology. Measurements are conducted periodically over an extended duration. The objective is to evaluate the magnitude and progression of any structural deformations in Hagia Sophia, which may arise from geotechnical, structural, and/or material issues or be induced by dynamic factors such as earthquakes or traffic. Additional to the discussed dynamic analysis, the effect of atmospheric conditions on modal frequency was investigated [27]. The instrumentation on the structure was utilized to obtain the real time data from Hagia Sophia to observe the annual, monthly, and daily frequency variation as can be seen in Figure 2.8.

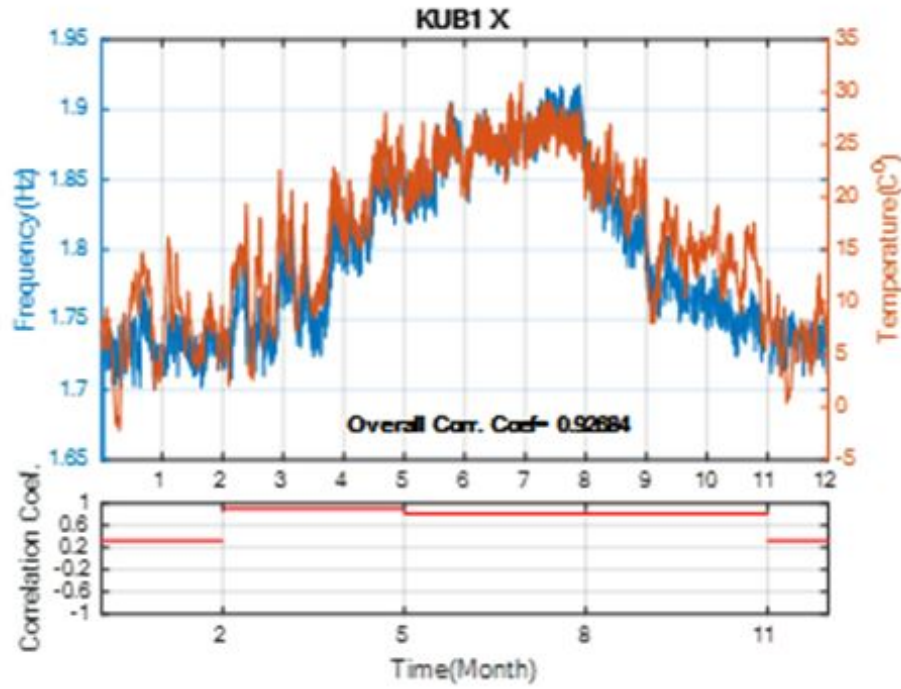


Figure 2.8. Variation of the first modal frequency of Hagia Sophia and the temperature in 2014 [27].

2.3.4. Subsurface Conditions

Hagia Sophia is situated on the first hill of the historical peninsula of Istanbul, where the geology is dominated by the so-called Trakya formation. In 1993, a seismic tomography study based on the recorded P wave velocities to a depth of 8 meters was carried out by Gürbüz [29]. It was observed that the foundations are located on stiff soil around the main piers except the southeast pier, whereas it is less stiff towards the center. As a second part of the study, a topographical map was generated [29]. According to the map shown in Figure 2.9, there is a cavity at the central and eastern parts, nevertheless piers stand on bedrock comparably near the surface; for instance, at 1 meter depth for the northern main piers, 2.5m for the southeast pier and 4m for the southwest pier.

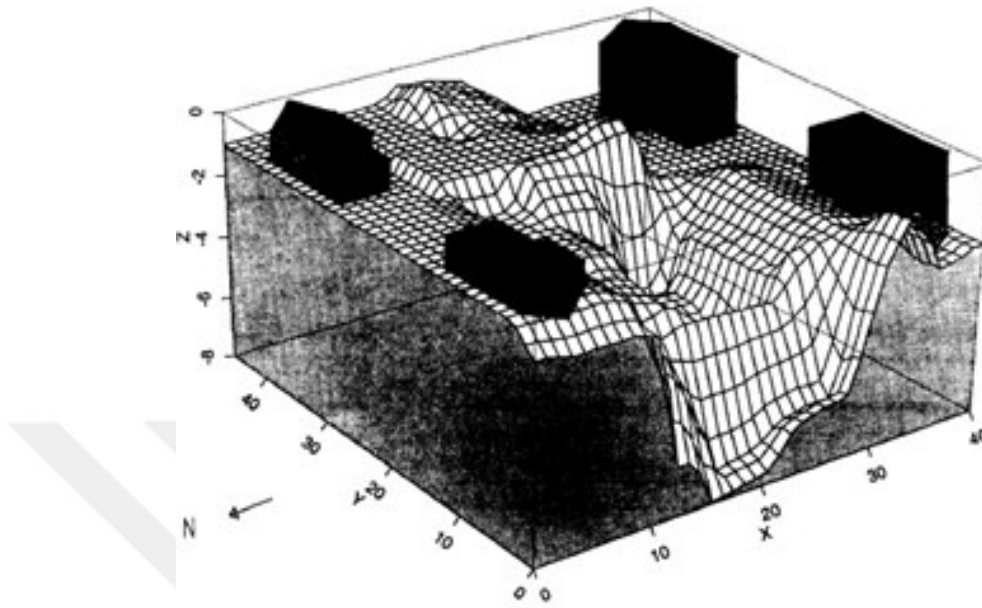


Figure 2.9. The tomographic map of the subsurface conditions [29].

More recently [28] observed the data obtained from the accelerometer on the ground level through the Fourier analysis. In all directions, the amplitudes tend to be stable between 1-6Hz values, therefore Fourier Amplitude Spectrum results pointed that the subsurface conditions were not significantly effective on the behaviour. It can be observed in Figure 2.10, where Dar plotted the transfer functions of the accelerometric data obtained from the gallery and dome base levels and the ground [30].

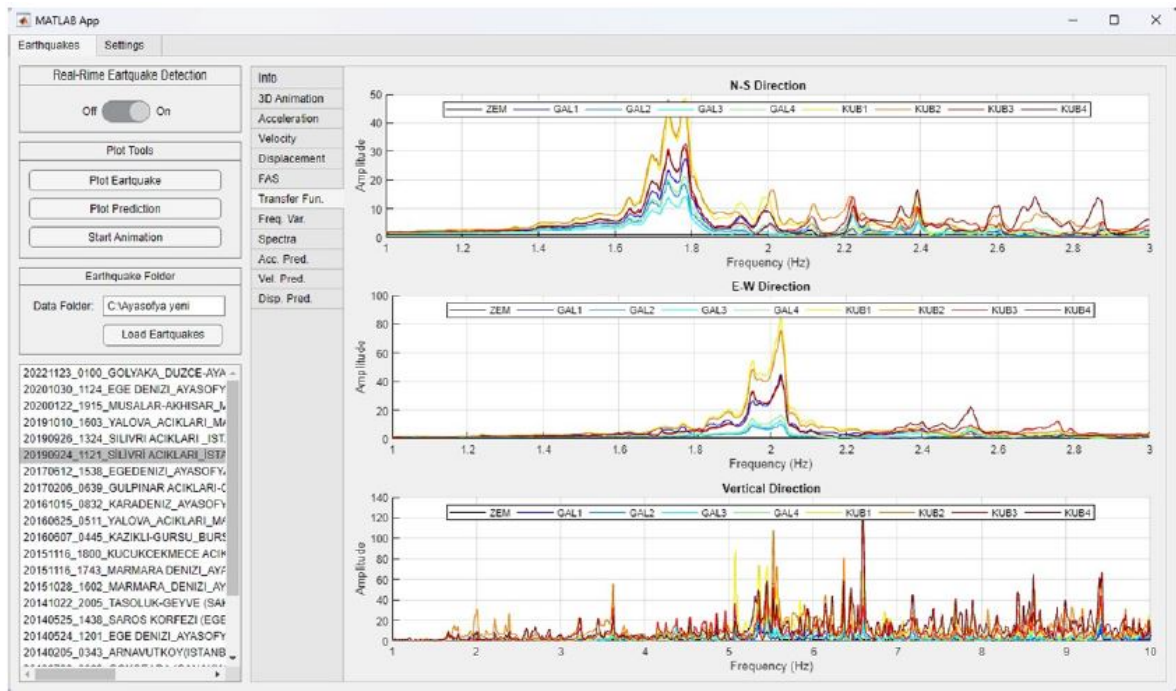


Figure 2.10. Transfer functions of the data obtained from the accelerometer on the gallery, dome base and ground level [30].

Anatolian Geophysical applied geo-radar and reflection techniques to inspect the subsurface conditions. They obtained shear wave velocity values. According to the study, the shear wave velocity at 10m depth varies between 750-1000 m/s whereas it is 900-1800 m/s at 20m depth [31].

2.4. Strengthening Alternatives

The studies suggesting strengthening alternatives for Hagia Sophia are limited. Numerous factors contribute to that. The primary problem pertains to our incomplete comprehension of Hagia Sophia's static and dynamic functioning. The vast majority of current studies depend on linear modeling, which is inadequate by itself. In the twenty-five years after the initiation of structural analyses of Hagia Sophia, substantial advancements in computational power and software have rendered analytical research

on reinforcement and behavior using non-linear analysis more attainable than ever before. Another reason is that reinforcing a structure of the magnitude of Hagia Sophia entails significant challenges in governance, decision-making, and stakeholder consensus. Current research primarily focuses on conceptual frameworks [32] or consists of first feasibility assessments of FRP applications and post-tensioning [22].

2.5. Shake Table Model

A scaled model of Hagia Sophia was created for a documentary film [33], as depicted in Figure 2.11. The model was constructed based on architectural blueprints and the material characteristics of the structure, adhering to the principles of similitude. It was built in phases, likely akin to the construction methodology employed by Anthemius of Tralles and Isidorus of Miletus. The scaled model underwent shake table testing in the shake table laboratory of Bogaziçi University’s Department of Earthquake Engineering. The experimental approach entailed sequential testing using progressively heightened acceleration recordings from the Kocaeli earthquake, continuing until failure occurred.



Figure 2.11. Scaled model of Hagia Sophia developed for the shake table test [33].

3. GEOMETRIC EVALUATION

Development of a model that reflects the present-day geometry of Hagia Sophia is one the aims of this thesis research. Such a model will include many details about the interior and the exterior of the structure and will provide important data and information not only for this study but also for many research groups working on different aspects of the structure as well. To serve this purpose 3D laser scanning technique is opted for due to its efficiency and reliability. First the structure is scanned by 3D laser scanner. The scans are carried out inside and outside of Hagia Sophia taking about two months to complete. After the data are collected, they are transferred to the computer. Cyclone Software is used to combine and register each scan data to form the whole shape. Then, to evaluate the current deformation state and characteristics of individual structural elements and of the structure in general, they are transferred to 3DReshaper software, which allows the user to access and measure every detail. Moreover creating ideal geometric shapes for each element is possible. The structure is first investigated in detail for each structural element and then as a whole. Hagia Sophia's structural system consists of primary and secondary elements. This identification is mainly to be able to separate them or to pinpoint them more easily. As the primary system, first the main piers are examined, followed by the main arches, the main dome and the dome base, the tympana and the pendentives. The secondary system covers the secondary piers, the semi-domes, the exedra domes, the barrel vaults and the buttress piers. Ideal geometrical shapes are created that observe the ideal geometry of each structural element. As an example, vertical planes are used for the piers, cylinders are created for the arches, an ideal sphere was used for the main dome. The comparison of the ideal geometrical shape and the actual geometry obtained from the survey reveals the deformation characteristics. Moreover the ideal geometric shapes, when merged, form a consistent general geometry for the whole structure. Once the shapes which are created separately are joined together, they come together at the center of the structure, which supplies a satisfying verification about the study and might therefore be evaluated as a representation of Hagia Sophia at its design stage centuries ago.

3.1. 3D Laser Scanning

Since it is one of the aims of this thesis to work on the deformed shape of Hagia Sophia, the first goal is determined as to observe the actual deformations of the structure. For this goal, several procedures were followed starting with 3D laser scanning. 3D Laser Scanning is a non-contact, non-destructive technology that digitally captures the shape of physical objects using a line of laser light [34]. Using the point and magnetic targets (3.1) a series of scans both inside and outside of the building are made. In this part of the study, two softwares are used: Cyclone and 3DReshaper of which references are [35] and [36] respectively. In the following sentences they are mentioned as Cyclone and 3DReshaper.



Figure 3.1. The point target (on the left) and the magnetic target (on the right).

32 scans from the inside and 35 scans from the outside, which have taken about 40 days, were necessary to capture the structure as a whole. Then the point clouds data are transferred to Cyclone, as separate folders for each scan as shown in Figure 3.2.

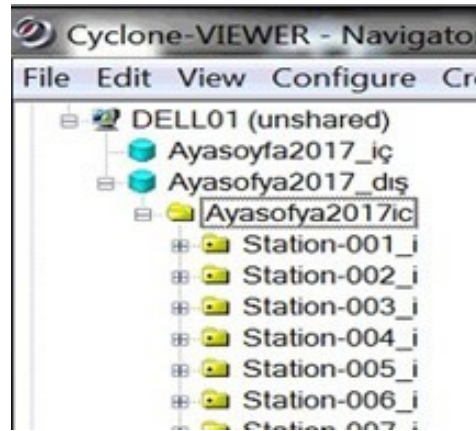


Figure 3.2. The transferred folders for each scan.

The next step is to combine data to obtain a representation of the whole structure. 3D laser scanning is ideal for the complex geometries to capture the fine details in a practical and accurate way. However, there are several limitations that needed to be considered while studying Hagia Sophia. The scaffold which covers the most part of the north side is one of them and had to be deleted from the data after combined. The large number of visitors and some close parts due to the maintenance works were the other obstacles to overtake. Therefore, the transferred and combined data required some more adoptions to be ready for the deformation measurements.

The high-definition laser scanner (HDS) utilized for this study is the ScanStation P20 from Leica Geosystems, Inc, which is shown in Figure 3.3.



Figure 3.3. The Leica Scan-station P20.

The principle of the technique is to measure distances using the travel duration of a short laser pulse sent from a source to an object and reflected back to a receiver and the laser scanner can also interpret the intensity of the reflected light that is closely correlated to the color or material type of the object [37].

3.2. Data Processing

After the data are obtained by the field study, it is time to transfer them to the computer (Cyclone) and combine them to have the point clouds of the whole structure. Even though the scanner detected the targets, on the computer file some scans might be unclear to contribute to the final result due to the distance or an unexpected object in front of the target. Therefore, some of the data must be eliminated. For each scan to be added on the previous one, they must have at least three common targets. It is not a problem not to use all the data, as long as redundancy is ensured. In this case, the data from the interior of the structure was combined first and then the scans from the exterior were added on one by one. This is an effective method to figure the exact shape of the structure with minimal error. While the error is 1 mm for inside, it is 5 mm for the whole structure, which are both acceptable for this study. The final shape can be seen in Figure 3.4 and in Figure 3.5.

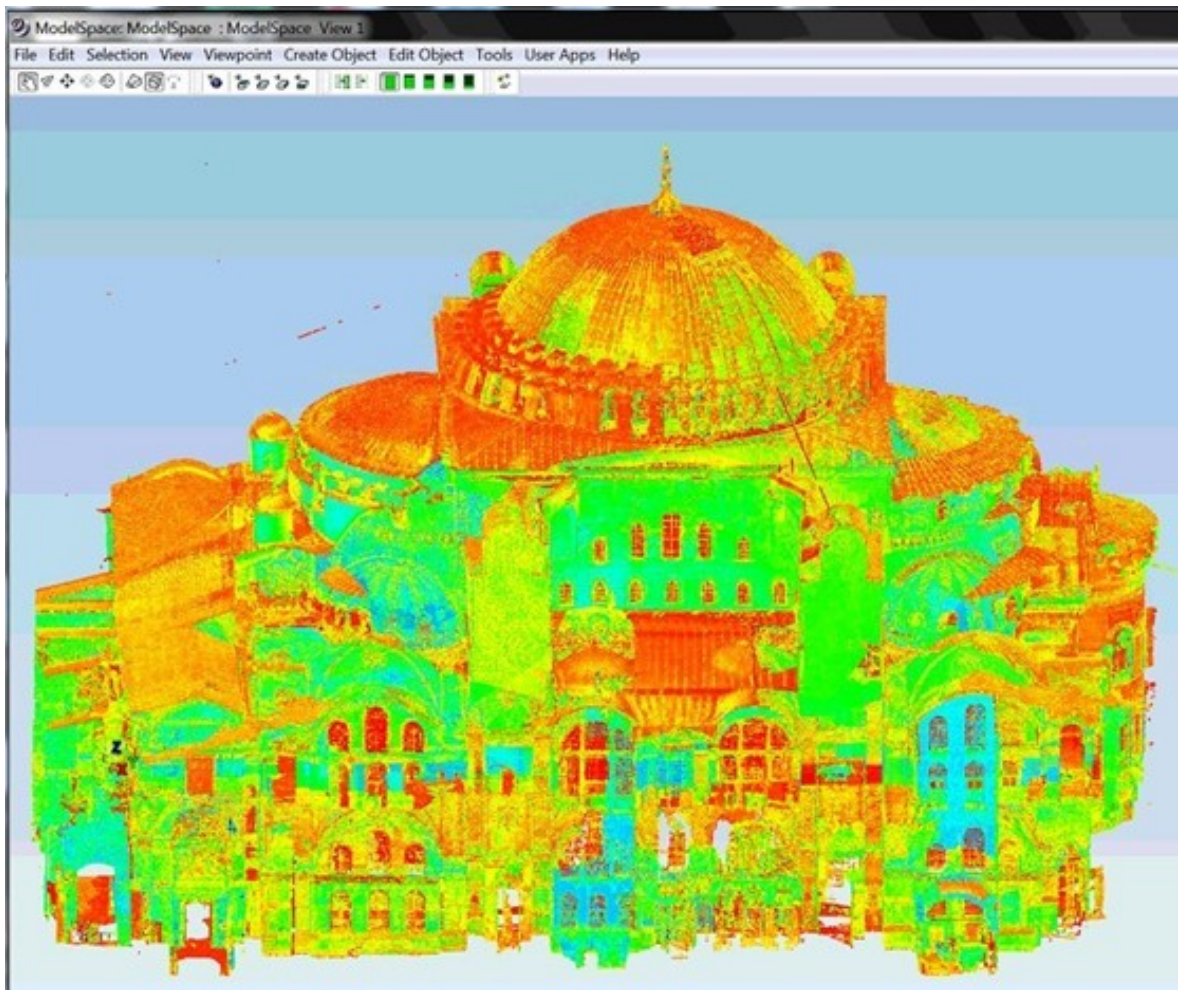


Figure 3.4. Registered point cloud data representing Hagia Sophia.

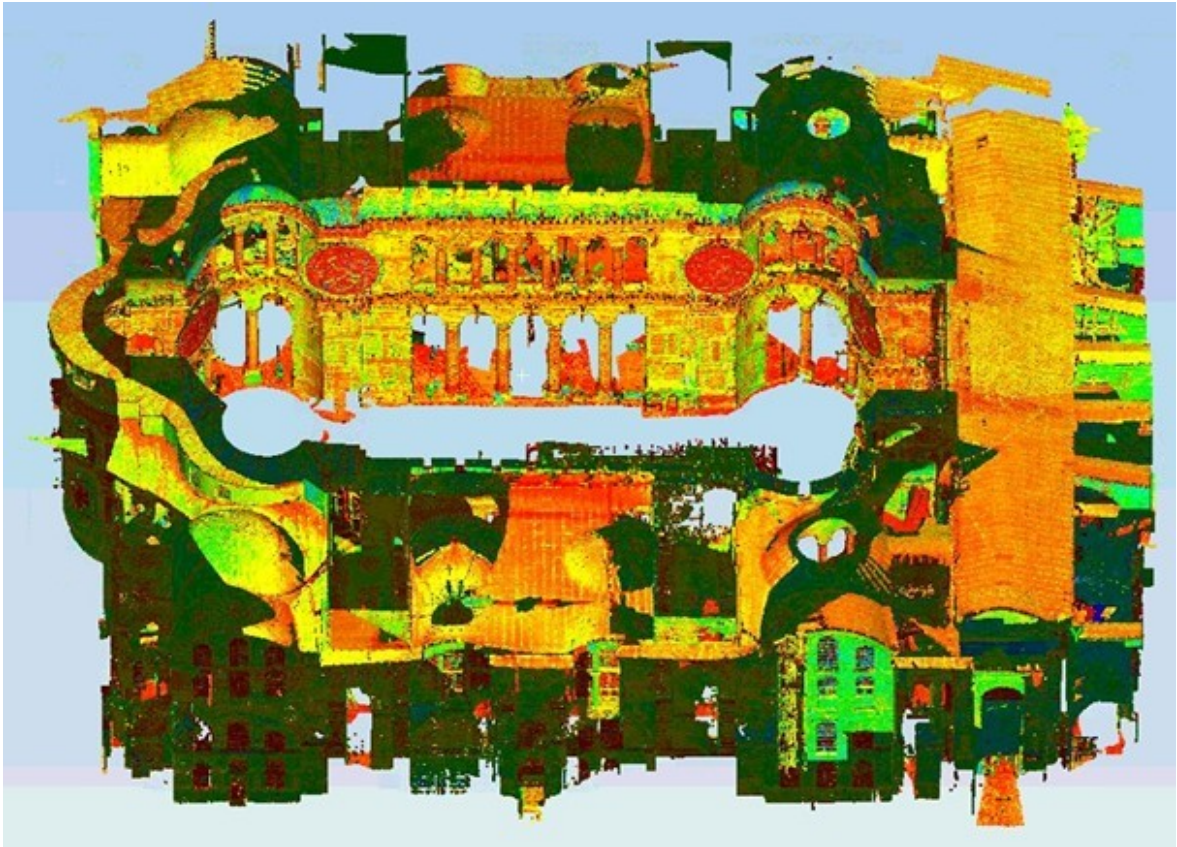


Figure 3.5. Registered point clouds representing the interior Hagia Sophia.

3.3. Geometry Definition Process

After the point cloud is created, in order to evaluate the current deformations on the structural elements, they are transferred to 3DReshaper. First the point cloud is converted to meshed elements. Then the ideal geometries such as cylinders, planes and spheres are defined. The meshed surfaces representing the real elements are compared with the ideal shapes in search for the extent and order of their deformations. Revealed deformations are compared to the ones mentioned in the survey [1].

The focus is put on the evaluation of deformation characteristics of the main piers, the main arches, the main dome, the dome base, the tympana, the pendentives,

the semidomes, the exedra domes, the barrel vaults. These elements can be seen in Figure 3.6. It should be pointed out that, due to the scaffolding on the northwest part of the structure at the time of the laser scanning campaign, there is a lack of data belonging to that area. An example of the meshed view created in 3DReshaper software is presented in Figure 3.7.

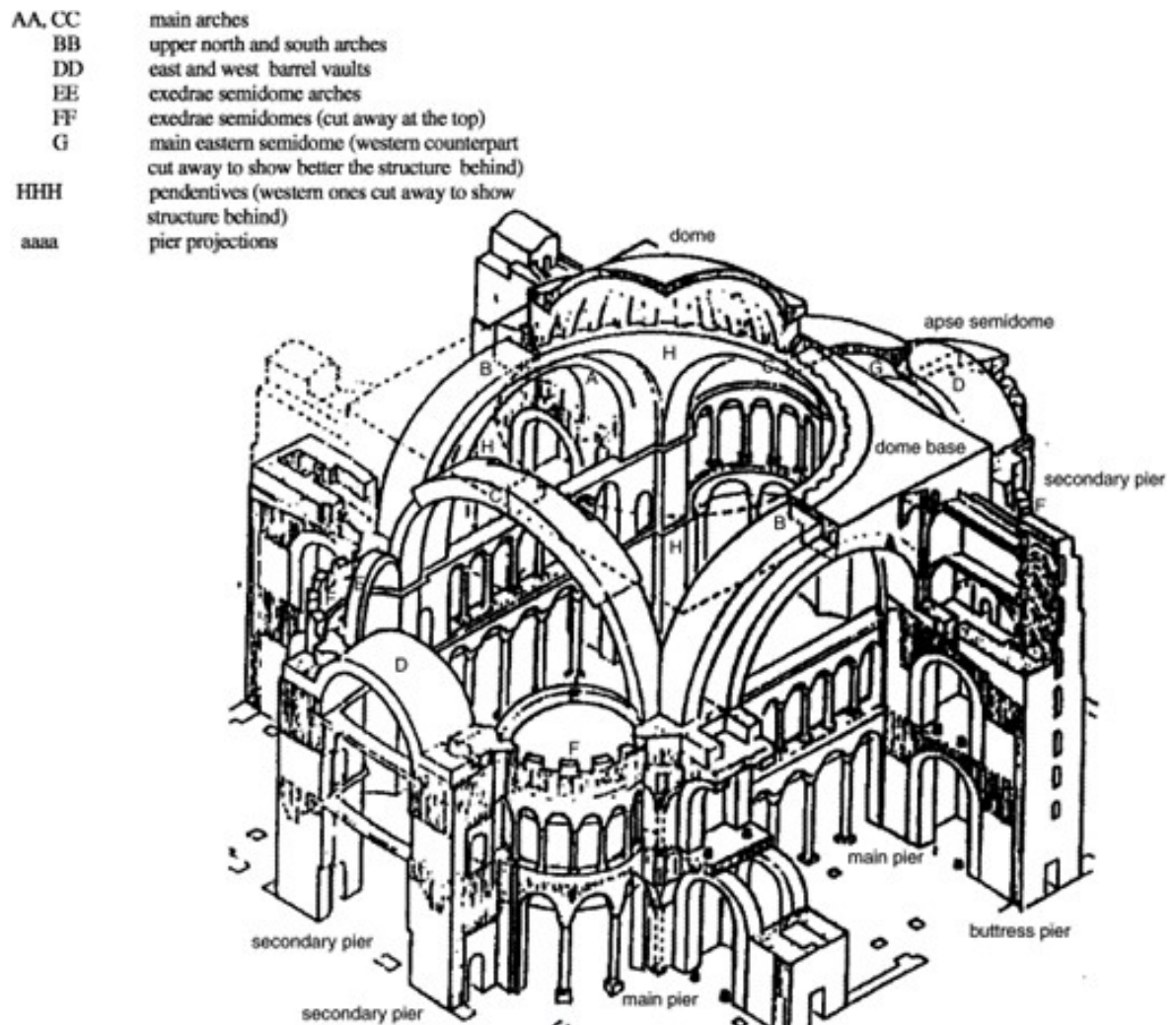


Figure 3.6. The structural elements of Hagia Sophia [1].

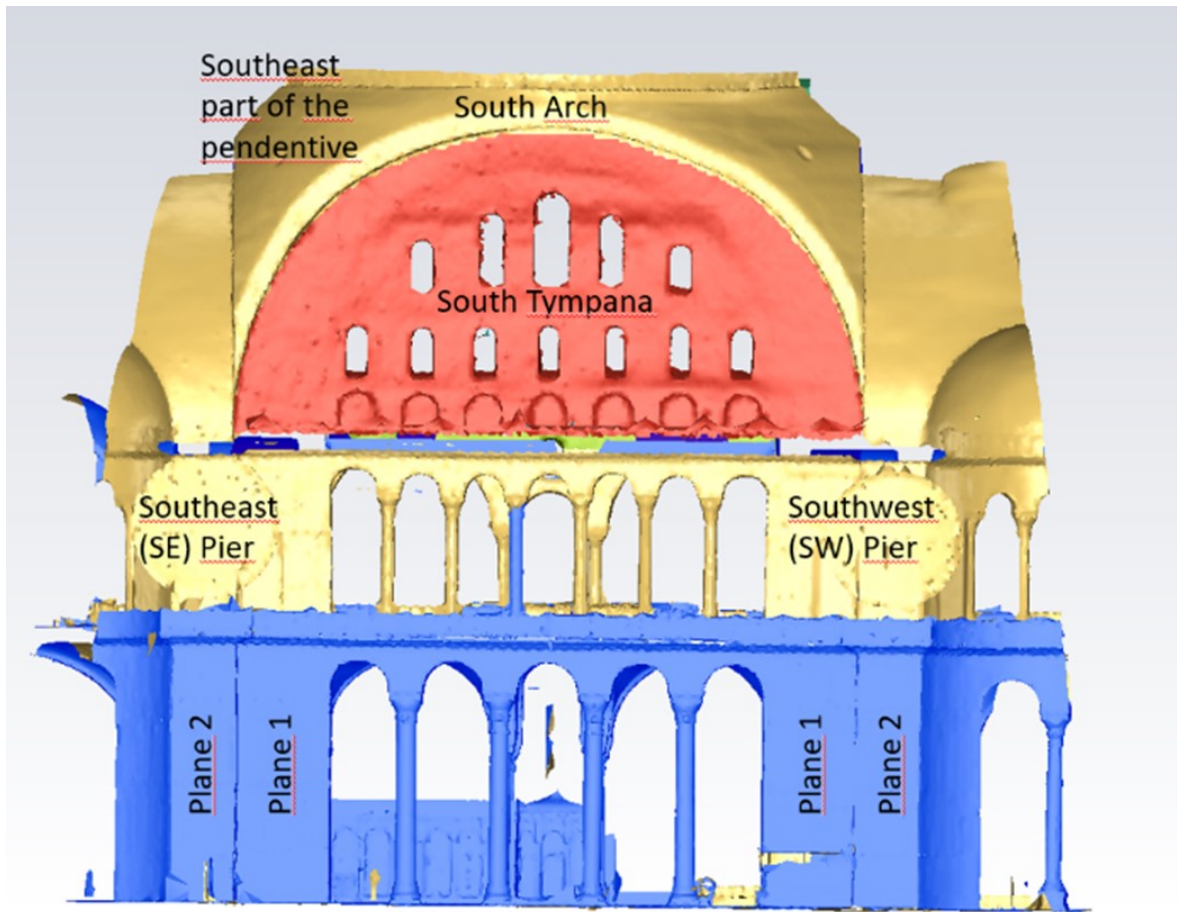


Figure 3.7. The 3DReshaper meshed view of the south part of the structure from inside.

3.3.1. The Main Piers

Ideal planes which are vertical with respect to the ground are created at each side of the main piers to assess their deformation. Sides with continuous faces named Plane 1, Plane 2 and Plane 3 at a main pier are compared to the ideal planes to evaluate the vertical inclination. Since plane 4 as shown in Figure 3.8 is not continuous and has a gap in the middle, an ideal plane is not created for it. Additionally, four contours with different colors are formed at the heights of 3.5m, 9m, 15m and 18.5 m from the ground level for each pier. The planes and the contour levels are shown in Figure 3.8

and in Figure 3.9. The inclination is revealed by using the difference between the top and the bottom contours, and by the comparison of the piers to the ideal planes. By this way, a verification of the assessments is ensured.

The color scale acquired from the comparison with the ideal plane gives approximate values since its sensitivity is at a level of 10cm as can be seen in Figure 3.11. Besides, the contours are not drawn on the real top and bottom points due to the rough surface of the piers. However, it is important for both methods to result in consistency with each other. It is normal to have some difference between the inclination obtained by the contours and those obtained by the color scale. The inclination found by the contours would appear lower. This is because the top contour does not pass through the exact top level of the pier to avoid unnecessary protrusions, so it does not actually represent the inclination at the very top level.

Hence, for neither of them it is easy to provide an exact value of inclination. In the former one, the results are given in the color scale and in the latter one the contours are not drawn on the real top and bottom points due to the rough surface of the piers. However, clearly both methods reveal compatible results with each other for each main pier.

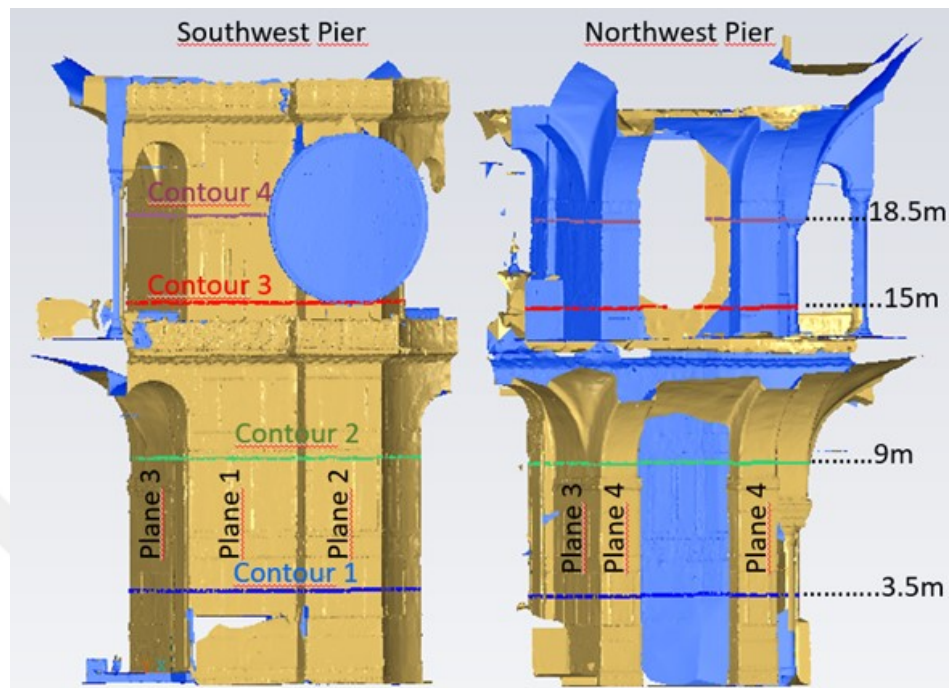


Figure 3.8. The surface names and the contour levels of the main piers.

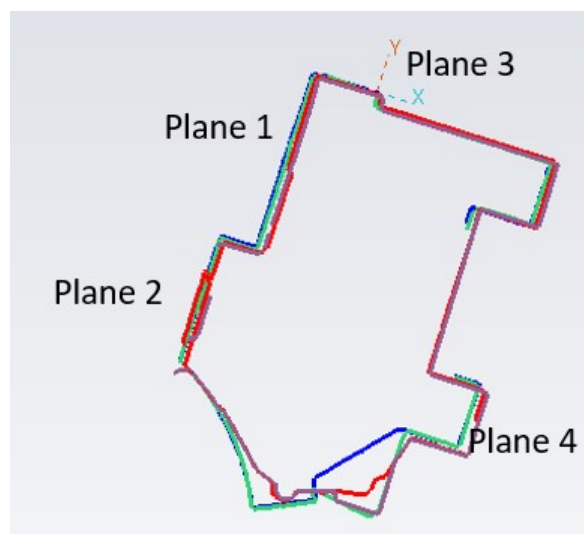


Figure 3.9. Plan view of the contours and the planes of a main pier.

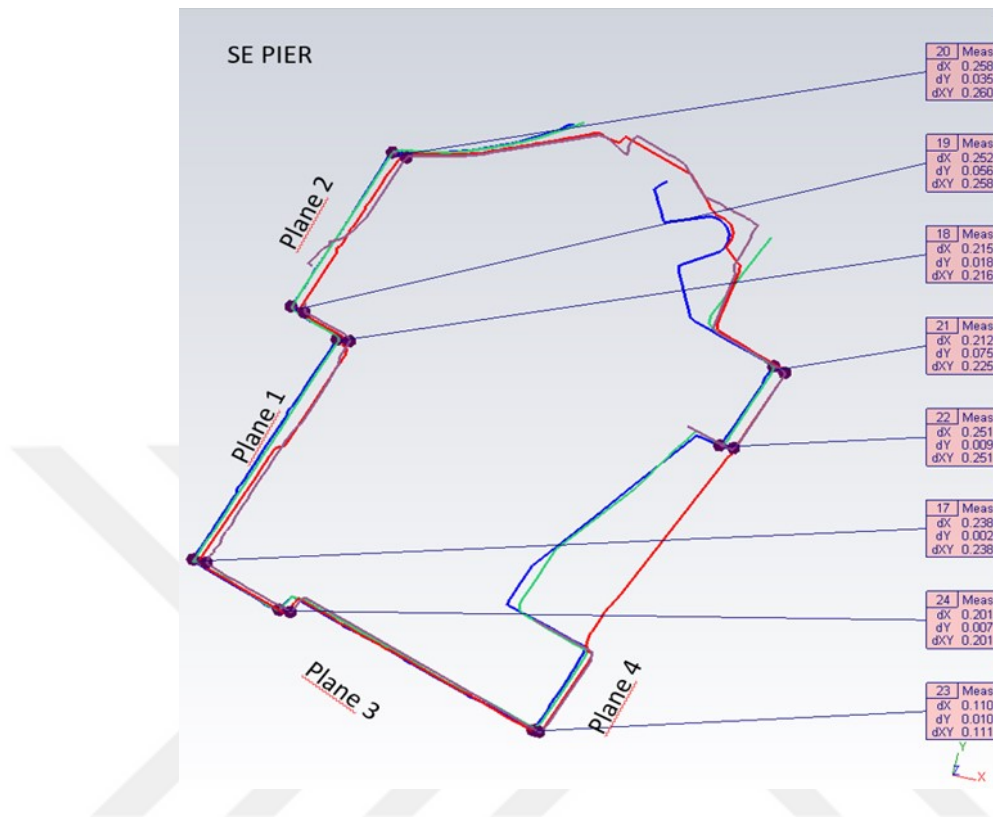


Figure 3.10. Plan view of the southeast (SE) main pier contours and the inclination measurement.

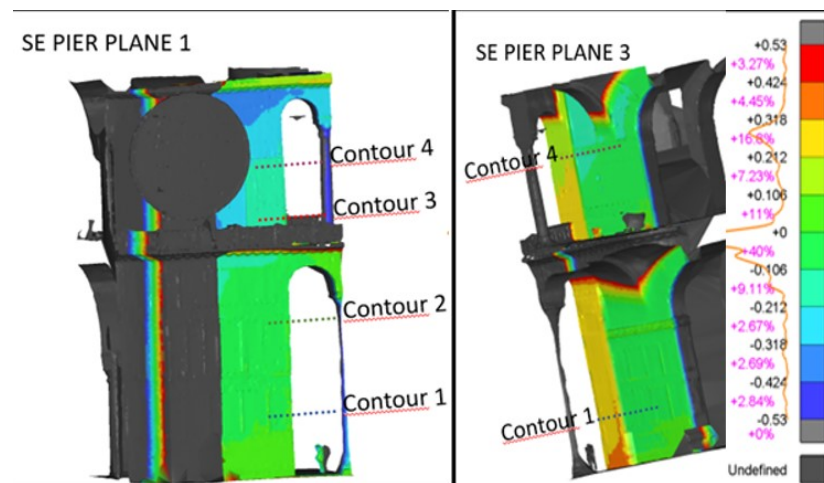


Figure 3.11. Inclination of the southeast (SE) pier surface (plane 1 and 3) obtained by the comparison of the pier and the ideal plane.

3.3.1.1. Southeast Main Pier. Regarding the contours in Figure 3.10, at Plane 1 and 2, there is an inclination around 22-23 cm towards the south; and in Plane 3, it is around 1 cm towards the east. In the scope of this geometric evaluation study, this low level of difference in the contour lines might be considered as irregularities on the surface coverings or any nonstructural reason, therefore it is regarded as ignorable.

When the color scale in Figure 3.11. is considered, in Plane 1, there is an inclination towards the south at the top level exceeding 31 cm in some areas. In Plane 3, a very low level of inclination is observed, which is consistent with the contours.

3.3.1.2. Southwest Main Pier. For the southwest main pier, looking at the contours, there is a southward inclination in Plane 1, 2, and 4; while in Plane 3, there is a negligible inclination towards the west as can be seen in Figure 3.12.

Considering the color scale, in Plane 1 and 2, there is a southward slope of more than 32 cm at the top level as can be seen in Figure 3.13. On Plane 3, a low slope of around 1 cm, is consistent with the contours.

3.3.1.3. Northwest Main Pier. Due to the ongoing restoration work at the time of the laser scanning campaign on the northwest facade of the structure, there is a scaffolding present. As can be seen in Figure 3. 15, normally the scaffolding would be in front of the NW main pier's Plane 1, but the scaffolding has been cleared and the rough surface behind it is made visible.

When looking at the contours in Figure 3.15, the NW main pier generally exhibits a northwardly slope in the north-south direction, which means it inclines towards the exterior of the structure. In the east-west direction, there is a negligible inclination towards the west side.

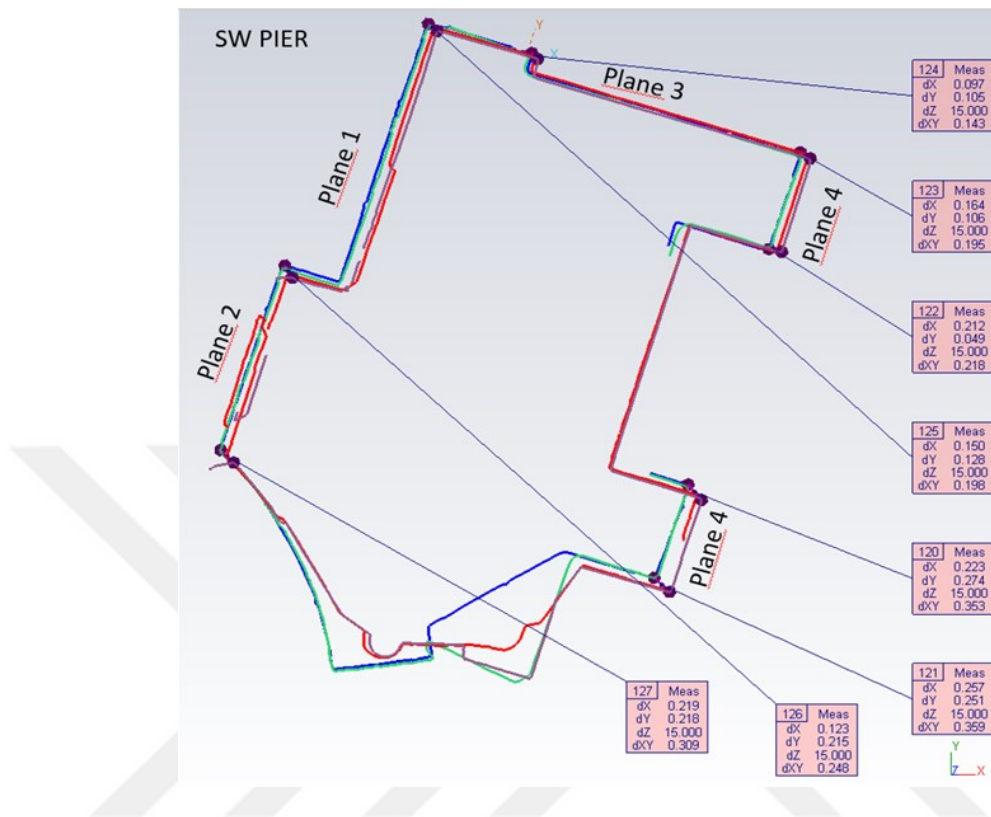


Figure 3.12. Plan view of the contours of the southwest (SW) main pier and the inclination measurements.

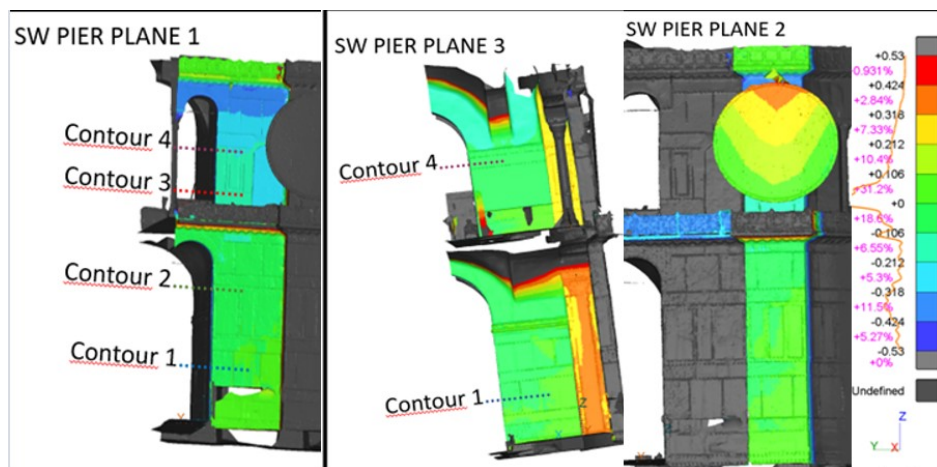


Figure 3.13. Inclination of the southwest (SW) main pier surfaces (plane 1, 2 and 3) obtained by the comparison of the main pier and the ideal planes.

When the color scale in Figure 3.15. and in Figure 3. 16. are considered, for Plane 1 and Plane 2, there is a northwardly slope of more than 30 cm at the top level. Besides, on Plane 3, a low level of inclination around 1 cm, consistent with the contours, is observed.

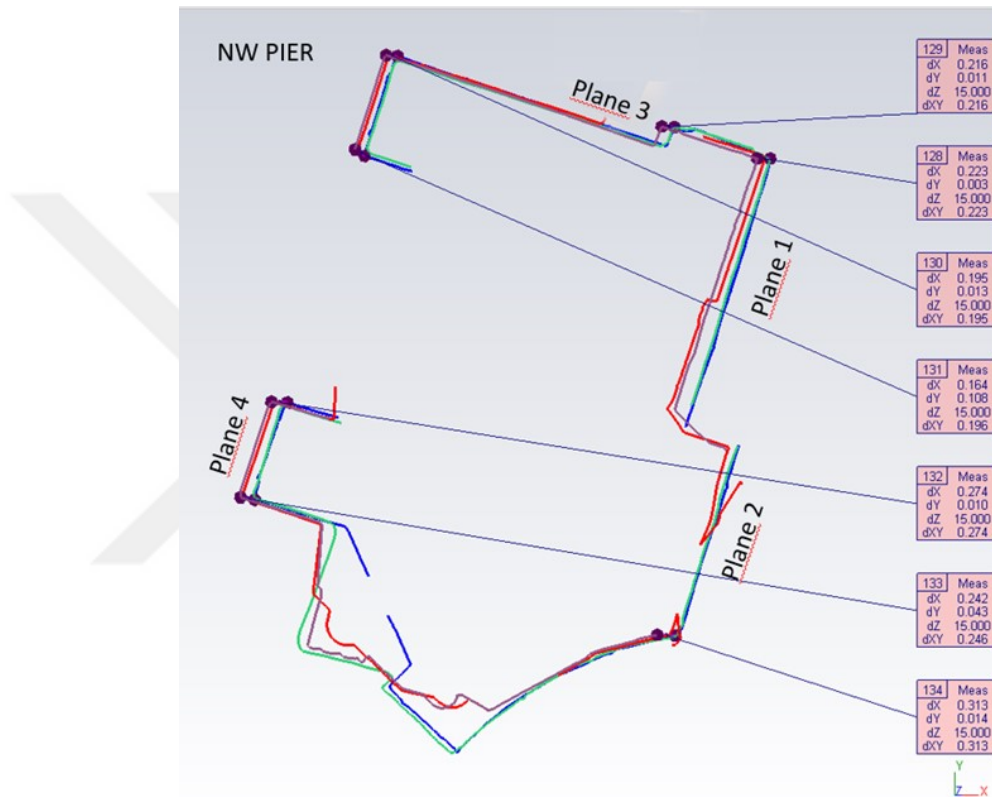


Figure 3.14. Plan view of the contours of the northwest (NW) main pier and the inclination measurements.

3.3.1.4. Northeast Main Pier. Similar to the other main piers, the northeast main pier also exhibits a northwardly inclination of above 30 cm towards the exterior of the structure in the north-south direction, and a negligible inclination about 1 cm towards the east in the east-west direction as can be seen in Figure 3.17, Figure 3.18 and Figure 3.19.

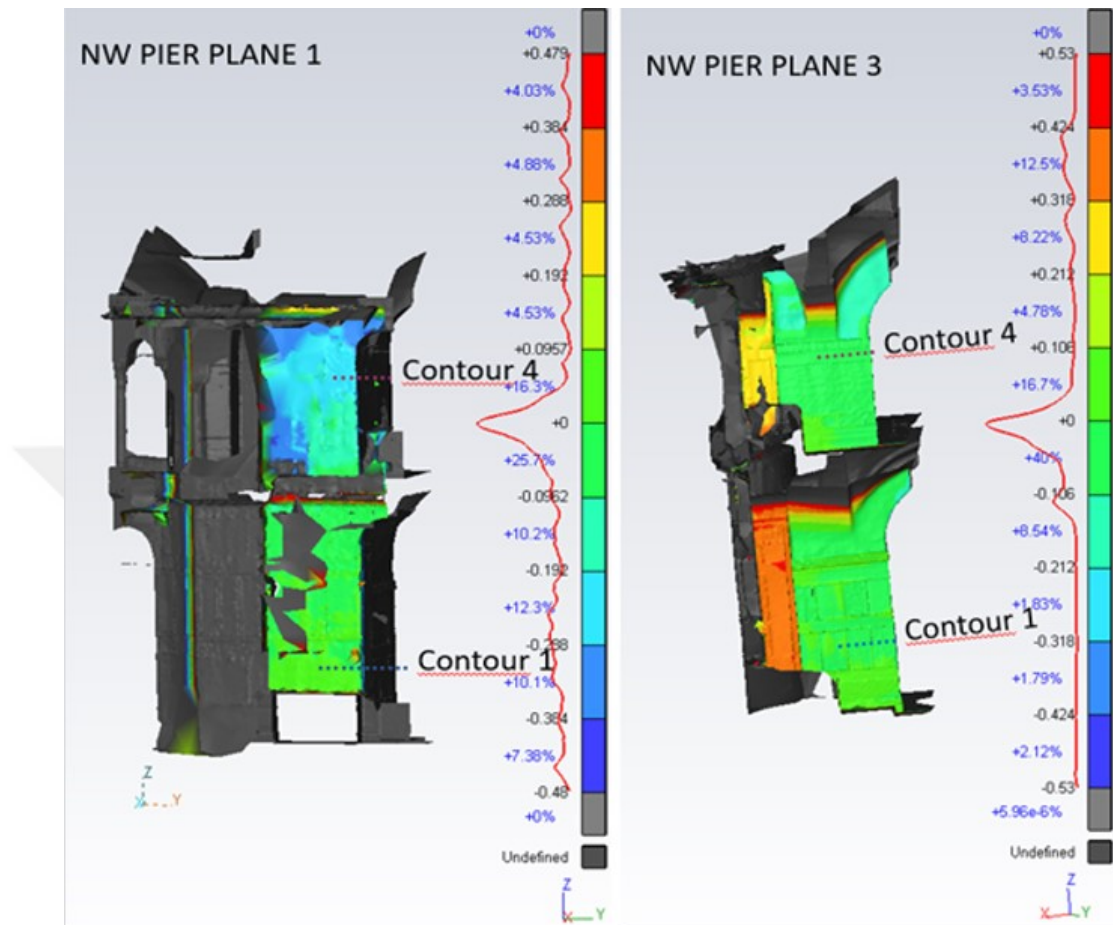


Figure 3.15. Inclination of the northwest (NW) main pier surfaces (Plane 1 and Plane 3) obtained by the comparison of the pier and the ideal planes.

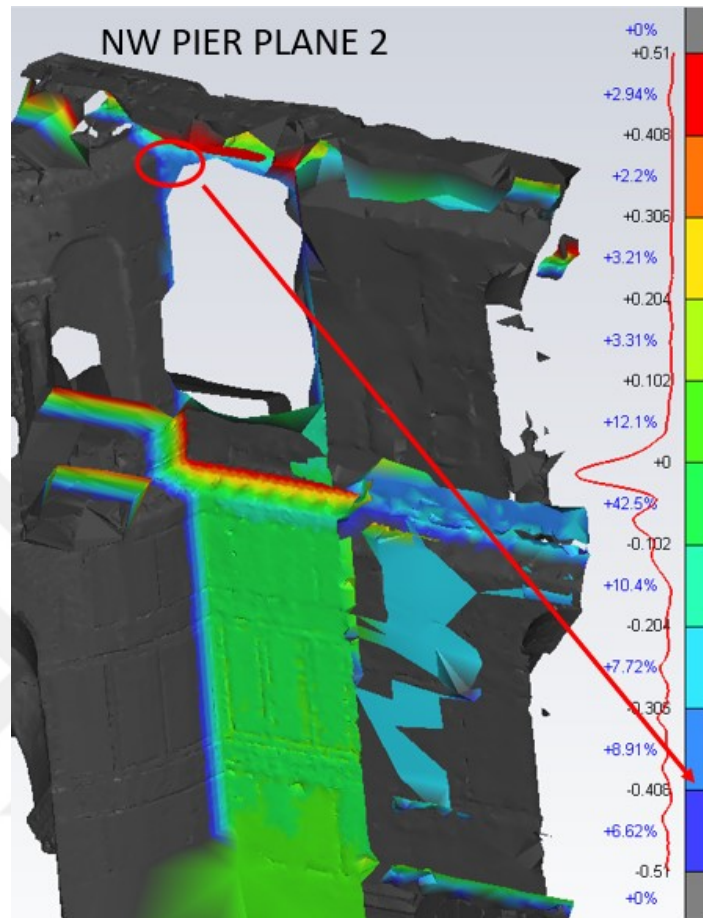


Figure 3.16. Inclination of the northwest (NW) main pier surface (plane 2) obtained by the comparison of the pier and the ideal plane.

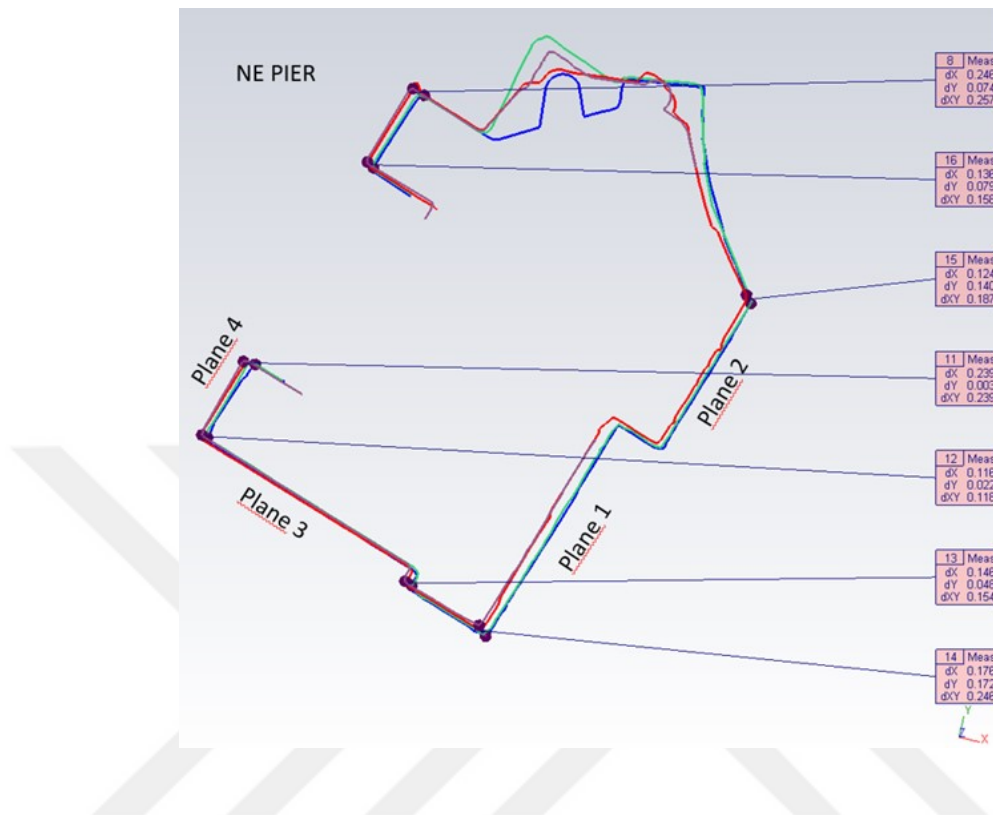


Figure 3.17. Plan view of the contours of the northeast (NE) main pier and the inclination measurements.

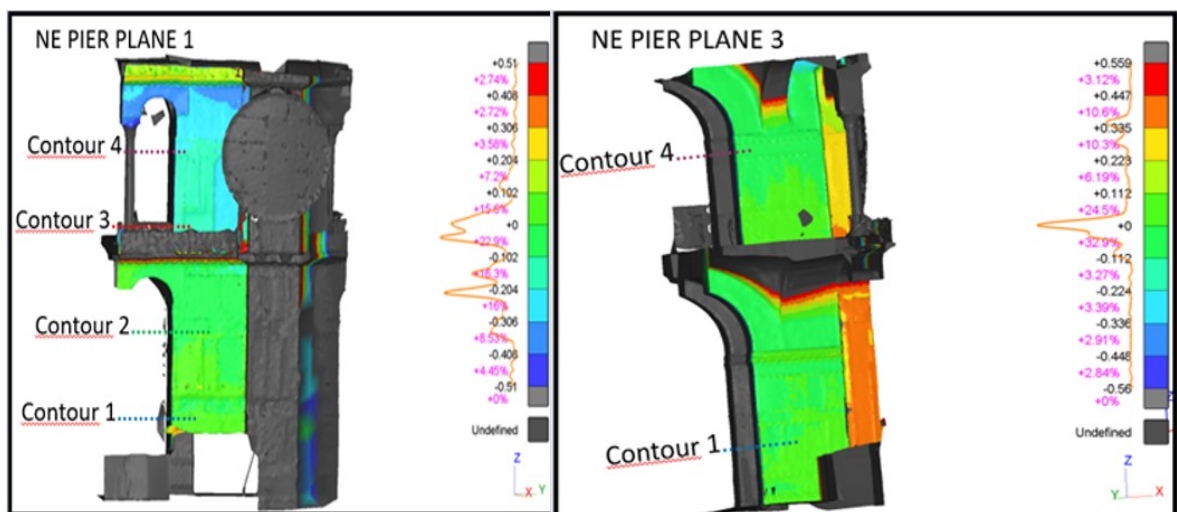


Figure 3.18. Inclination of the northeast (NE) pier surfaces (plane 1 and 3) obtained by the comparison of the pier and the ideal plane.

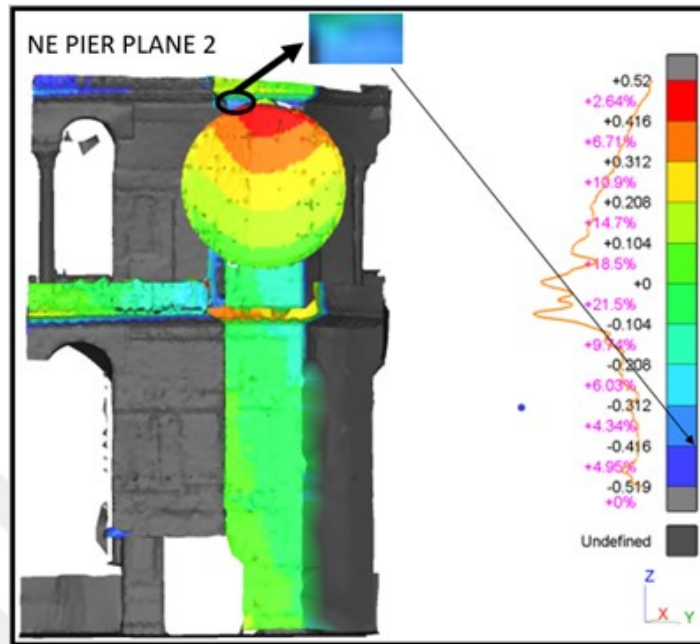


Figure 3.19. Inclination of the northeast (NE) pier surface (plane 2) obtained by the comparison of the pier and the ideal plane.

3.3.2. The Main Arches

Following the discussion about the piers, the study continues with the main arches. As mentioned previously, vertical planes were defined for the main piers. The ideal Plane 2 of the main piers are extended up to the starting point of the arches. It is observed that there are outwardly openings at the lower parts of the arches due to the inclinations in the main piers. Taking the ideal planes as a reference, ideal cylinders are defined for the main arches as illustrated in Figure 3.20. The existing deformations are examined by comparing the ideal cylinders with the current shape of the main arches. It is important to note that the north and south main arches, each one is composed of two arches, one higher and one lower. The arches that are at the same level as the west and east main arches are named the “upper north arch” and “upper south arch”, respectively. The ones at a lower level and located on the exterior side of the structure are called the “north arch” and “south arch”.

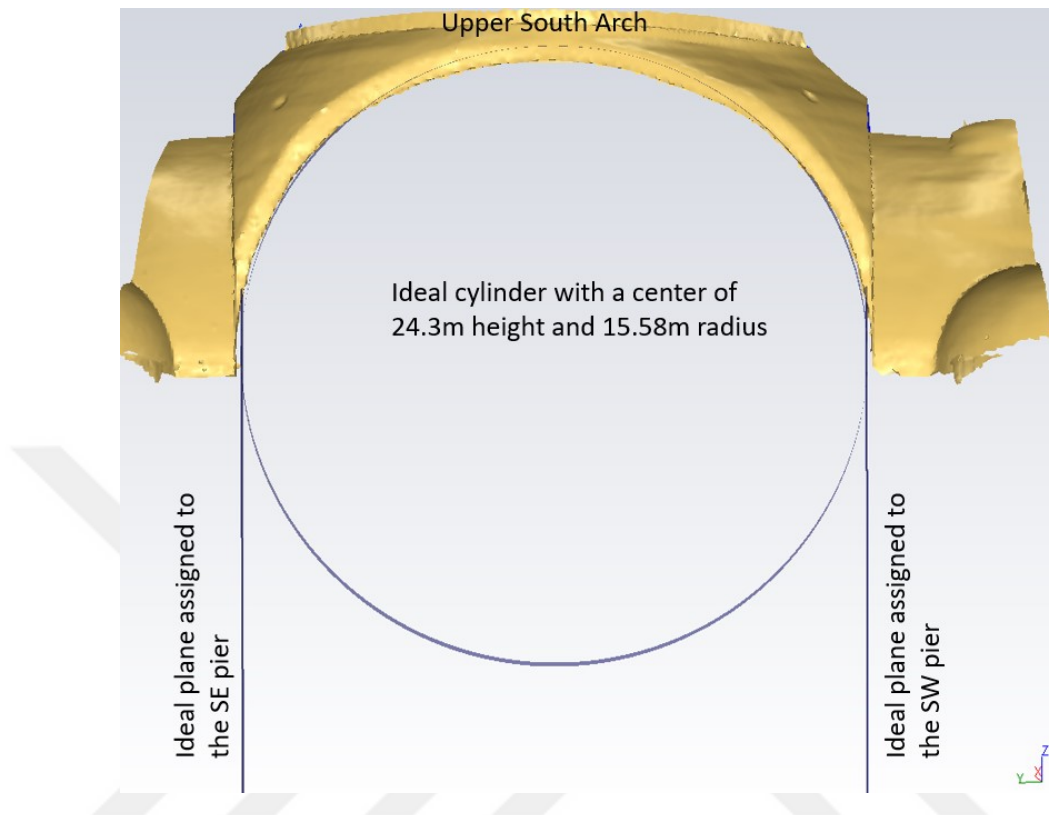


Figure 3.20. The ideal cylinder assigned to the upper south arch.

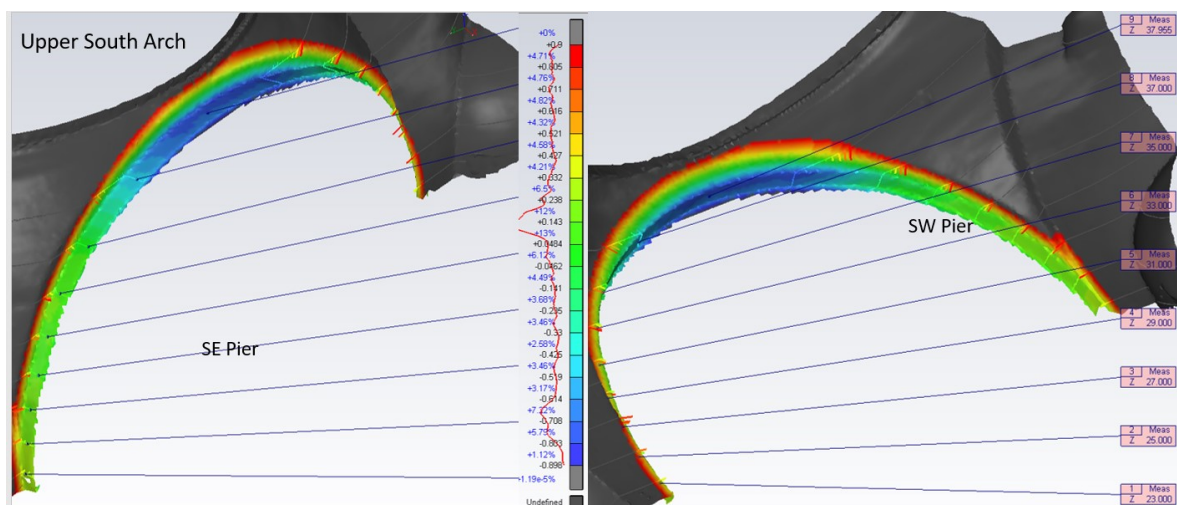


Figure 3.21. Upper south arch deformation results obtained by the comparison of the ideal cylinder and the arch.

In the color scale, the values shifting towards blue (negative values), indicate the settlement in the arch, while positive values indicate opening. When looking at 3.21, on the upper south arch, there is an outward opening of approximately 20 cm on each side (east and west), and a settlement of around 85 cm at the crown. This process is repeated for all arches. The height and radius of all ideal cylinders drawn for the arches are the same. Similar to the upper south arch, in all other arches, the center of the ideal cylinders are located at a height of 24.3 m above the ground, and their radius is 15.58 m. The main objective of this part is to determine at which points and to what extent the structure deviates from this designed ideal geometry.

In the upper north arch, there is an outward opening of approximately 20 cm at the point where it connects to the NE main pier, while there is a settlement of approximately 55 cm at the top point as illustrated in Figure 3.22.

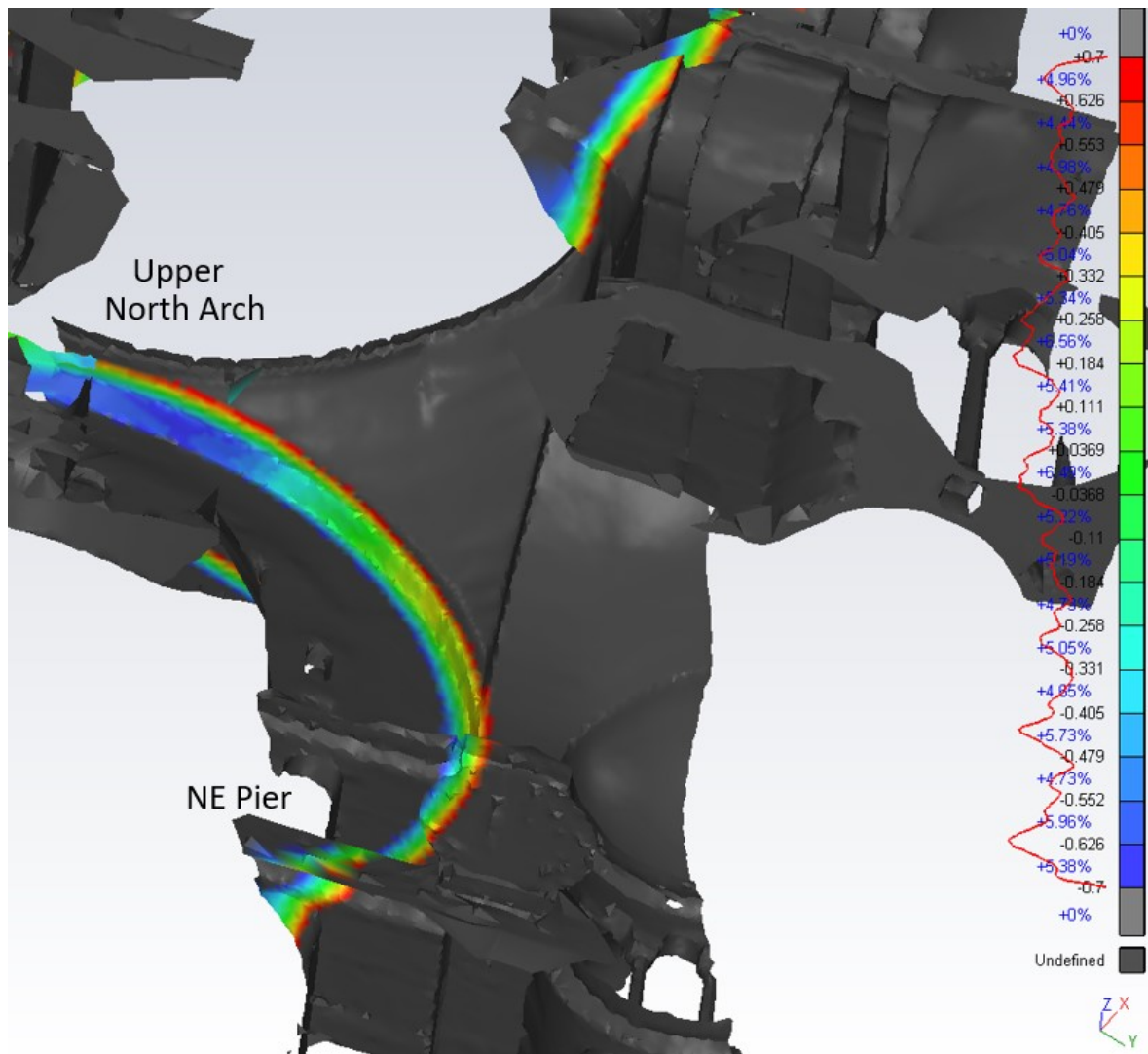


Figure 3.22. Upper north arch deformation results obtained by the comparison of the ideal cylinder and the arch.

As previously mentioned, the presence of scaffolding on the northwest side prevented the measurement of deformations in the arch of that part. Therefore, only the shape belonging to the SW side is observed for the west arch, where it connects to the SW main pier. There is no vertical deformation observed at the crown as depicted in Figure 3.24. Besides, it is pointed again in Figure 3.24 that there is an outward opening of approximately 50 cm.

Additionally, the east arch also has an approximately 50 cm outward deformation at the lower ends on both sides, while there is a settlement of 50 cm at the top point.

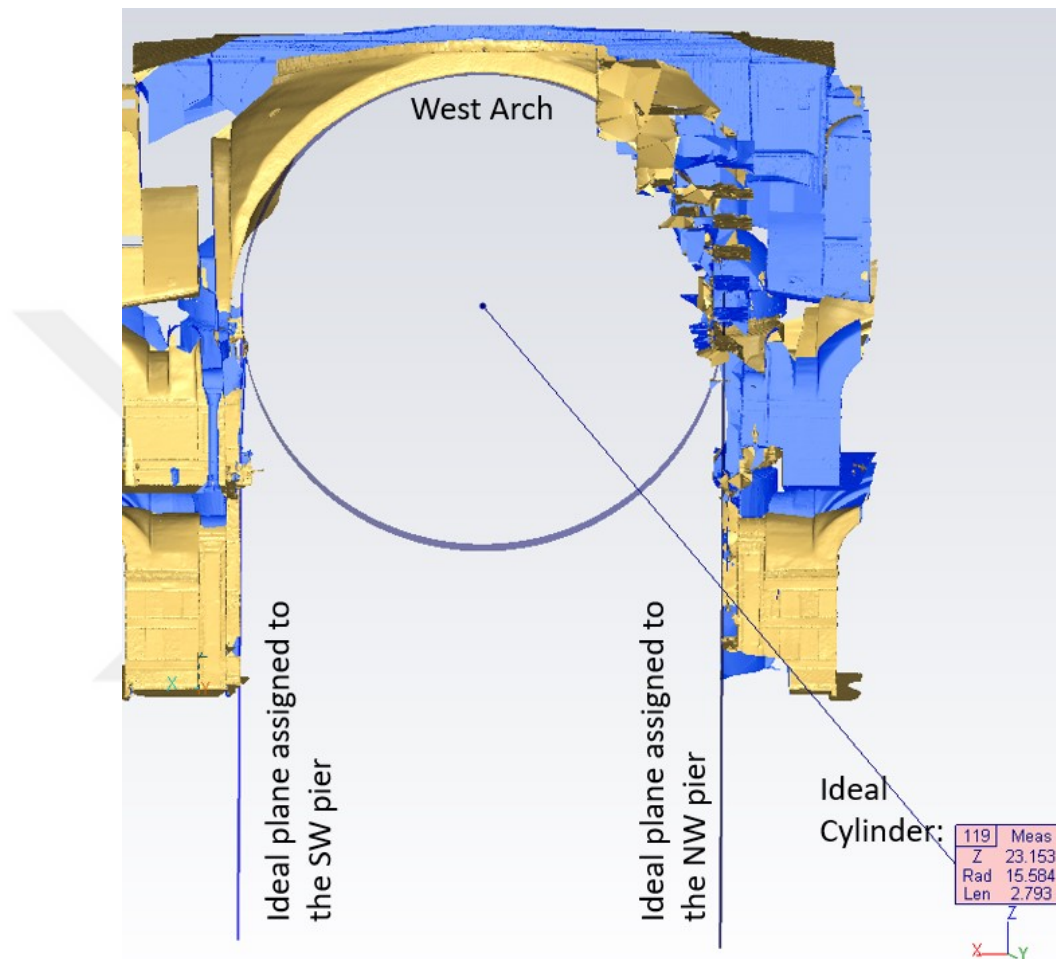


Figure 3.23. The ideal cylinder assigned to the west arch.

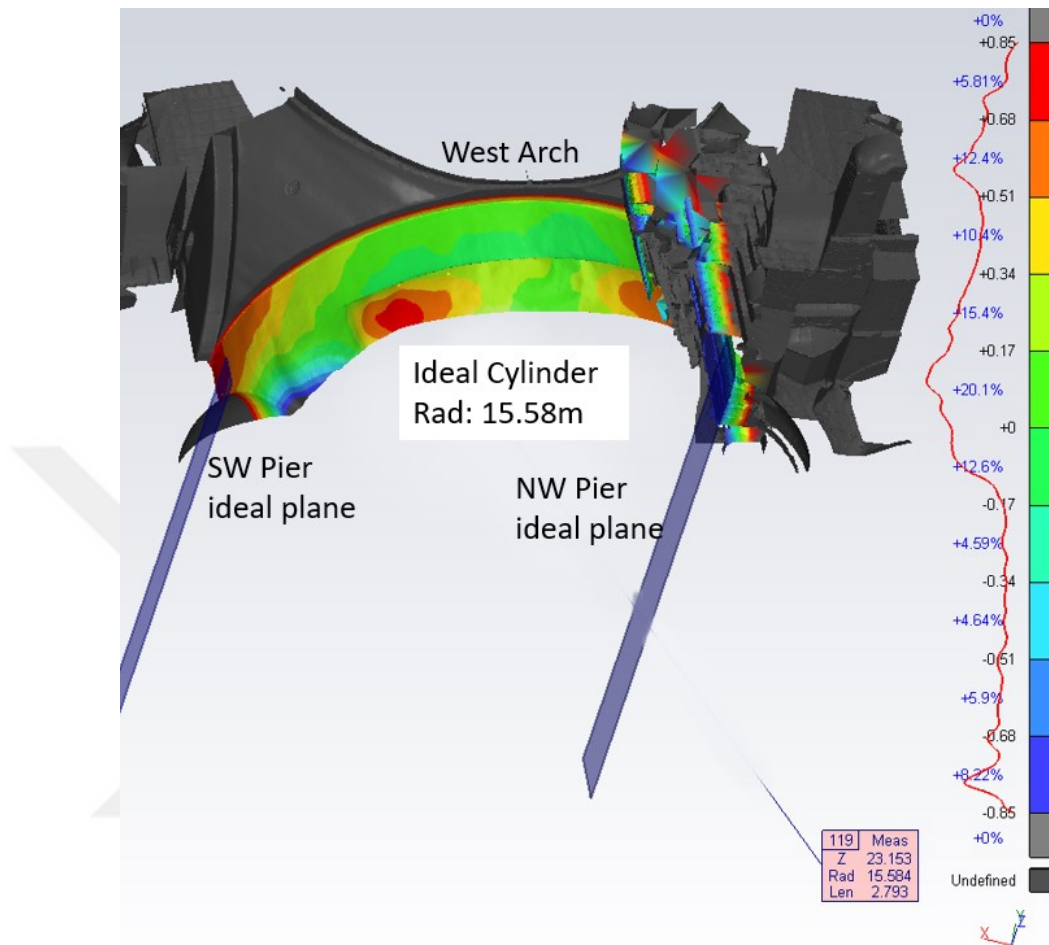


Figure 3.24. West arch deformation results obtained by the comparison of the ideal cylinder and the arch.

The assigned ideal cylinders to the north arch and south arch have a radius of 11.50 m. In Figure 3.25, the comparison results of them are provided. In both lower arches, there is a vertical settlement of approximately 10 cm at the crown. Besides, any remarkable outwards deformation is not detected at the springing of both arches. Thus, it can be concluded that the deformation of the lower arches, which have smaller radius, is less when compared to the deformation of the higher arches.

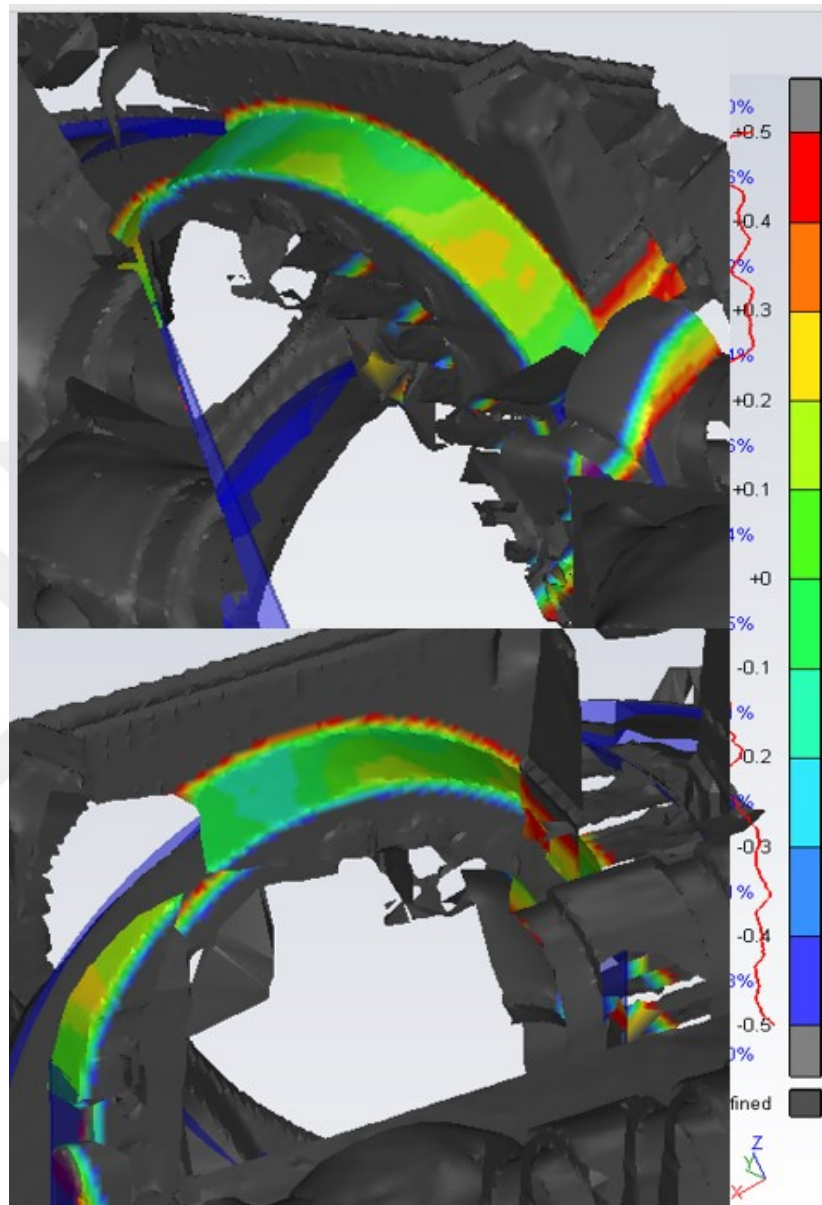


Figure 3.25. Lower arch deformation results obtained by the comparison of the ideal cylinder and the arch.

3.3.3. The Main Dome

An ideal sphere is defined to examine the deformation of the main dome. In order to display the deformation of the dome, the coordinates of the center and the radius

of the ideal sphere assigned to the main dome are shown in Figure 3.26. However, it should be noted that to determine the height of a point relative to the ground, 1.2m should be added to the Z-coordinate point that is determined by the program. Therefore, the coordinates of the center of the sphere are X: 28.96m; Y: 43.74m; Z: 18.05m, and the radius is 16.854 m.

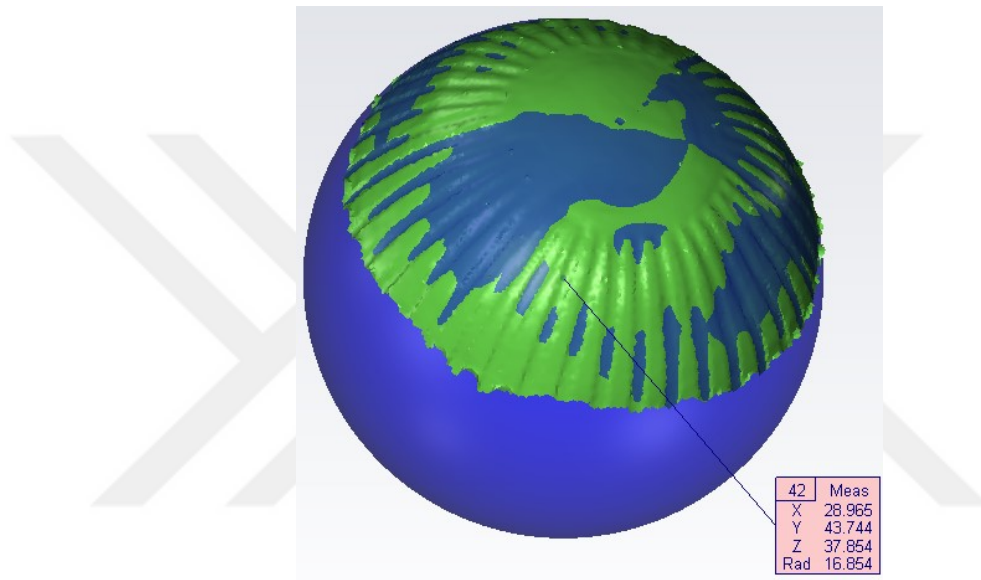


Figure 3.26. Main dome and the ideal sphere.

The comparison between the ideal sphere and the main dome is shown in Figure 3.27. Looking at the color scale, negative (blue) values represent deviations from the ideal sphere towards the interior, while positive (shifting towards red) values deformations towards the exterior.

Contour lines were drawn at 2 m intervals to determine the location of deformation. The most significant deformation is observed at the southern side, ranging from 52 m to 55 m in height, with approximately 40 cm settlement. Additionally, excluding the protrusions caused by dome ribs, settlements of around 15 cm is observed on the northern side again at the height between 52 to 55m.

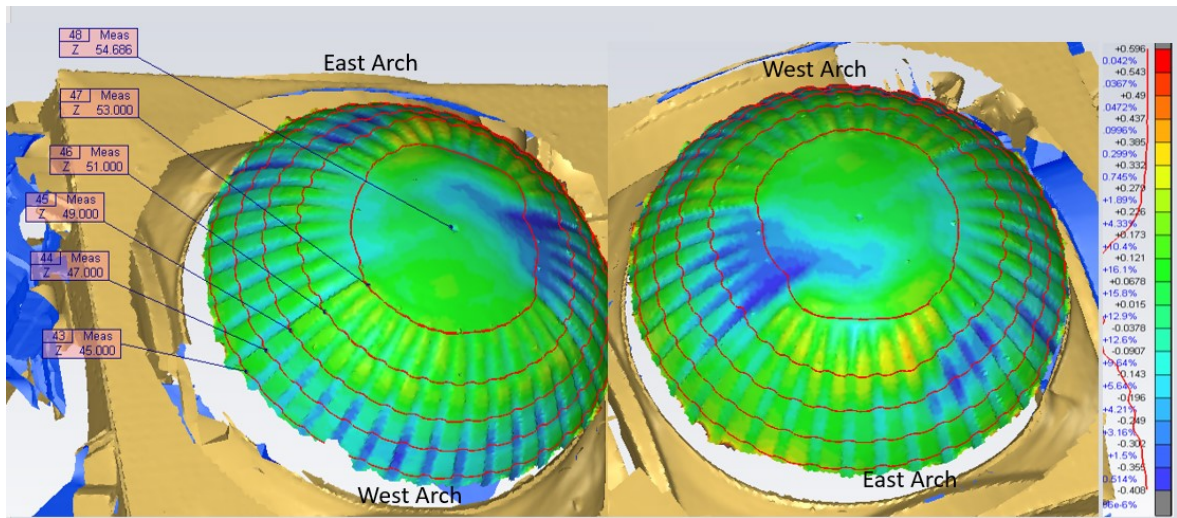


Figure 3.27. The deformation obtained by the comparison of the main dome and the ideal sphere.

3.3.4. The Dome Base

The bottom view of the main dome and the dome base is illustrated in Figure 3.28. An ideal circle is defined at the dome base and the deformations are studied by comparing the ideal circle and the dome base. The radius of the ideal circle is 16.8 m. Its center coordinates, X and Y, coincide with the center of the dome.

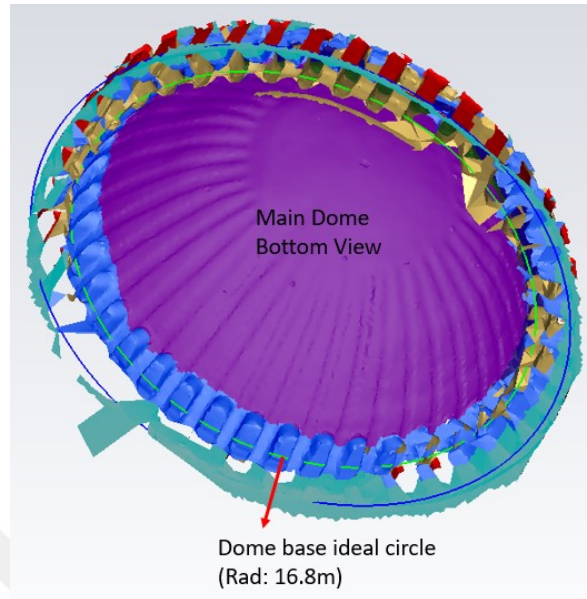


Figure 3.28. The bottom view of the main dome and the dome base with the ideal circle.

On the color scale, negative values (represented by blue) indicate inward deformations as compared to the ideal circle, while positive values indicate outward deformations. According to the observations, there is no noticeable deformation on the south side. However, when the north side is considered, there is a deformation of -10 cm on the north side and of -70 cm in the northeast direction as depicted in Figure 3.29. On the west side, there is a deformation of -55 cm at the top of the arch, but it decreases to 0 cm as we move towards the southwest. At the east side, deformations ranging from -45 cm to -30 cm are observed.

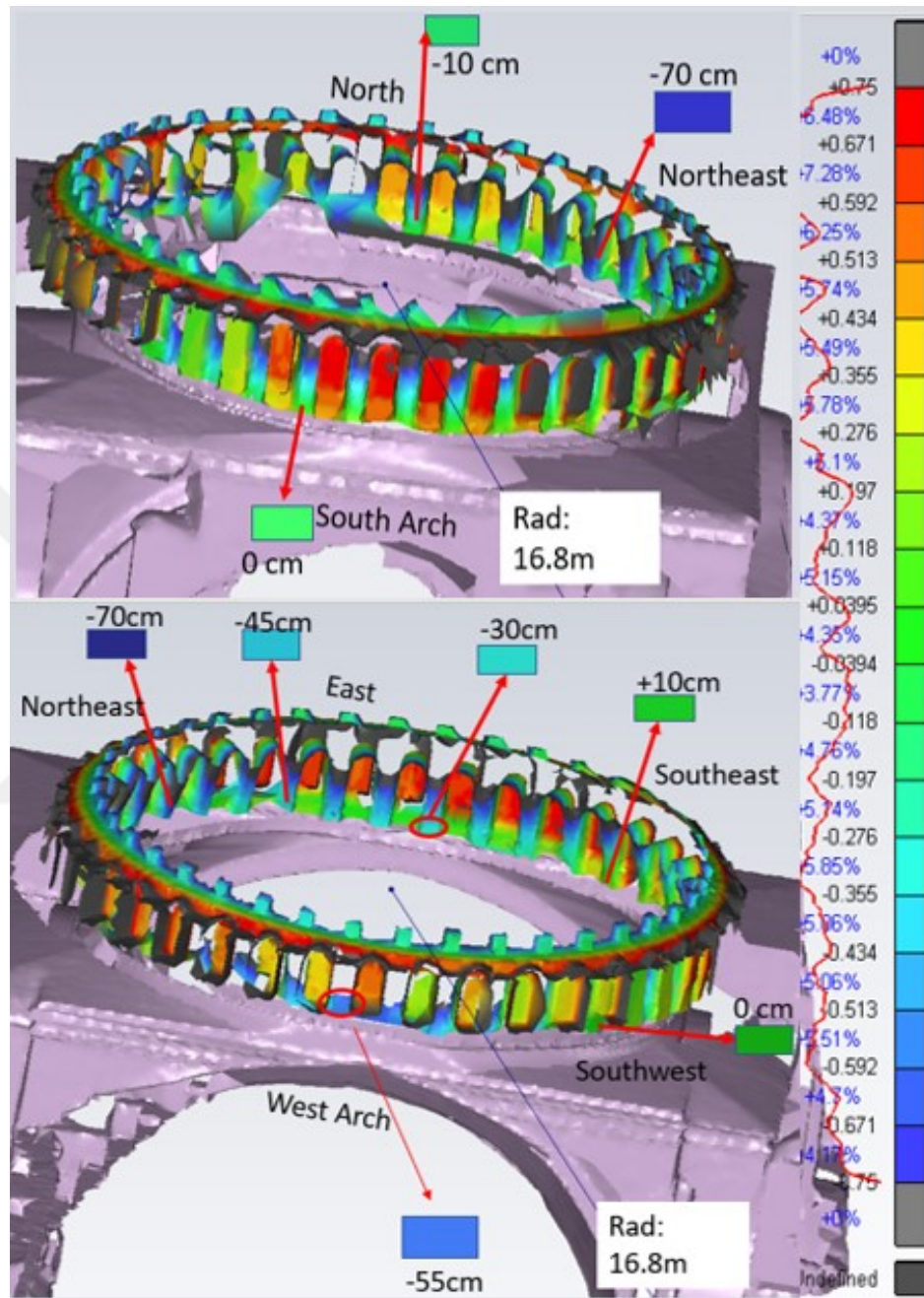


Figure 3.29. Deformation of the dome base obtained by the comparison with the ideal circle.

3.3.5. The Tympana

A plane has been assigned which is perpendicular to the ground, covering the width and height of each tympana, in order to examine the deformations of the north and south tympanum walls. The ideal planes and the tympana are compared and the deviations between them are studied.

Figure 3.30 illustrates the deformation of the north tympanum. Since the plane perpendicular to the ground starts from the bottom of the columns, there is a sudden jump at the point where the tympanum begins, as the tympanum is not aligned with the piers but recessed inside. Therefore, the tympanum should be evaluated within its own context. The positive values, represented by the direction towards red, indicate the outward inclination of the wall. At the bottom of the tympanum, there is an outward deformation of about 65 cm, and at the top it is 115 cm, resulting in a displacement of approximately 50 cm compared to the plane.

Figure 3.31 shows the deformation of the south tympanum. Here, negative values indicate the inclination of the structure towards outside. Similar to the north tympanum at the bottom of the south tympanum there is a -65 cm outward deflection, and at the top it is -115 cm; indicating an approximately 50 cm inclination.

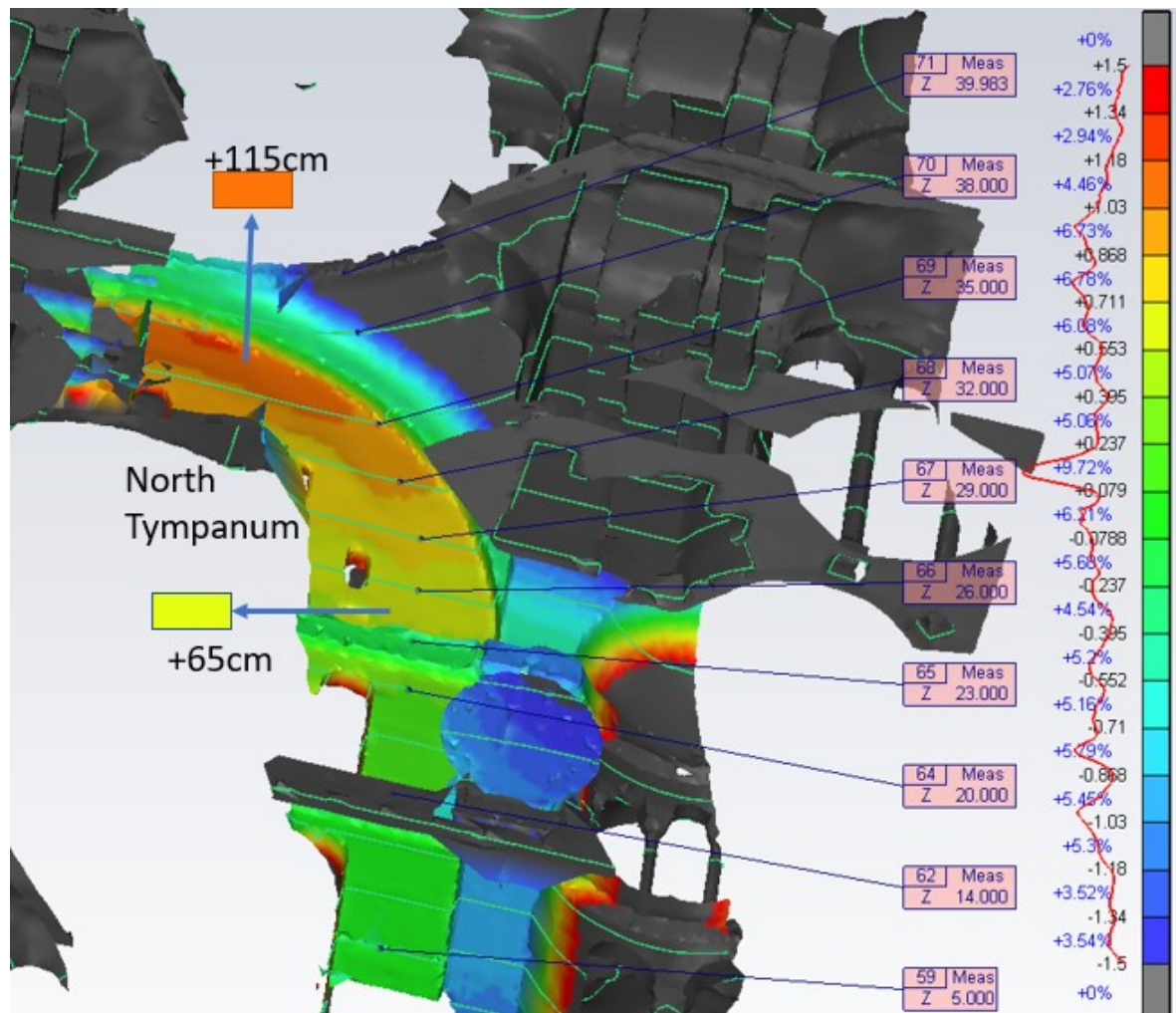


Figure 3.30. North tympanum deformation results obtained by the comparison of the ideal plane and the tympanum.

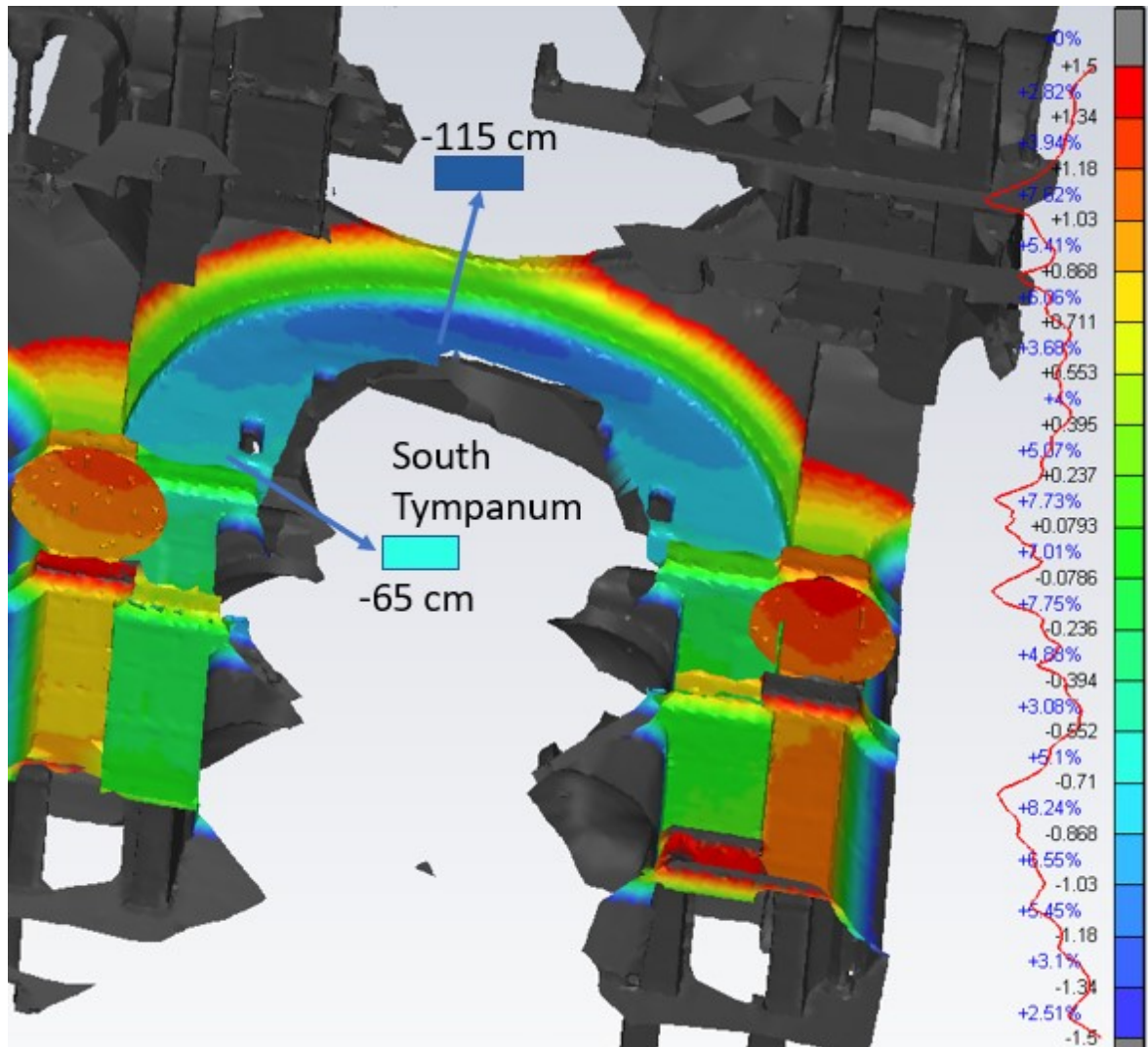


Figure 3.31. South tympanum deformation results obtained by the comparison of the ideal plane and the tympanum.

3.3.6. The Pendentives

As illustrated in Figure 3.32 and Figure 3.33, the center of the ideal sphere is located at a height of 23.6 m above ground, and its radius is 23.6 m. The X and Y coordinates of the center are the same as the exact midpoint of the dome and therefore the structure. This indicates that the sphere sits exactly on the ground and is tangent

to the structural elements. As a result, the designed ideal geometry of the structure is clear. Using this sphere, deformation occurred in the pendentives can be easily identified. Through these comparisons, the vertical distance between the ideal sphere and the pendentives is determined. The negative regions, indicated by the direction towards blue, represent settlement, while the positive regions represent the portions above the ideal sphere.

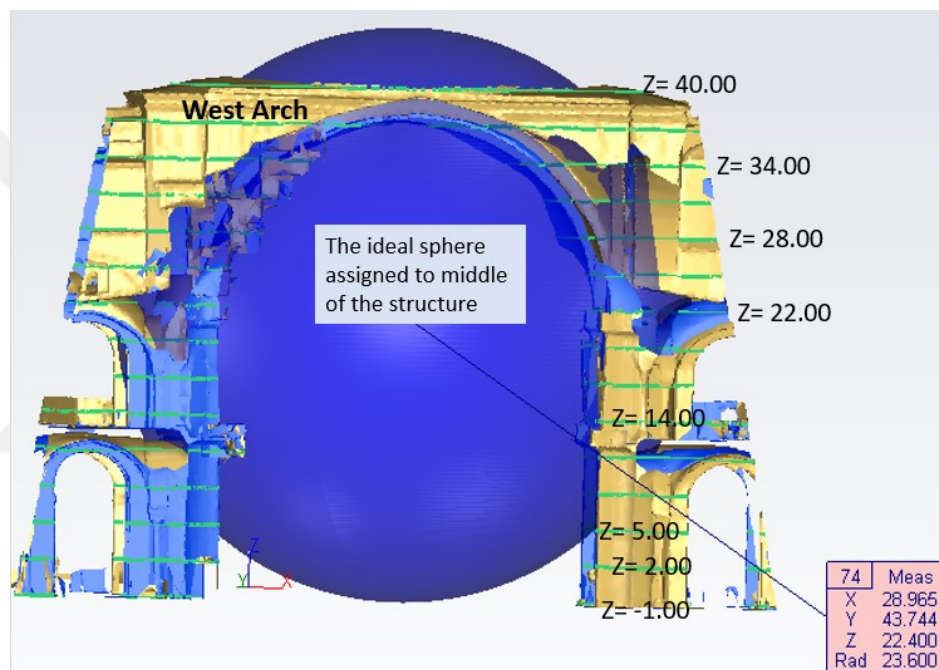


Figure 3.32. The ideal sphere assigned to the middle of the structure and the Z level contours.

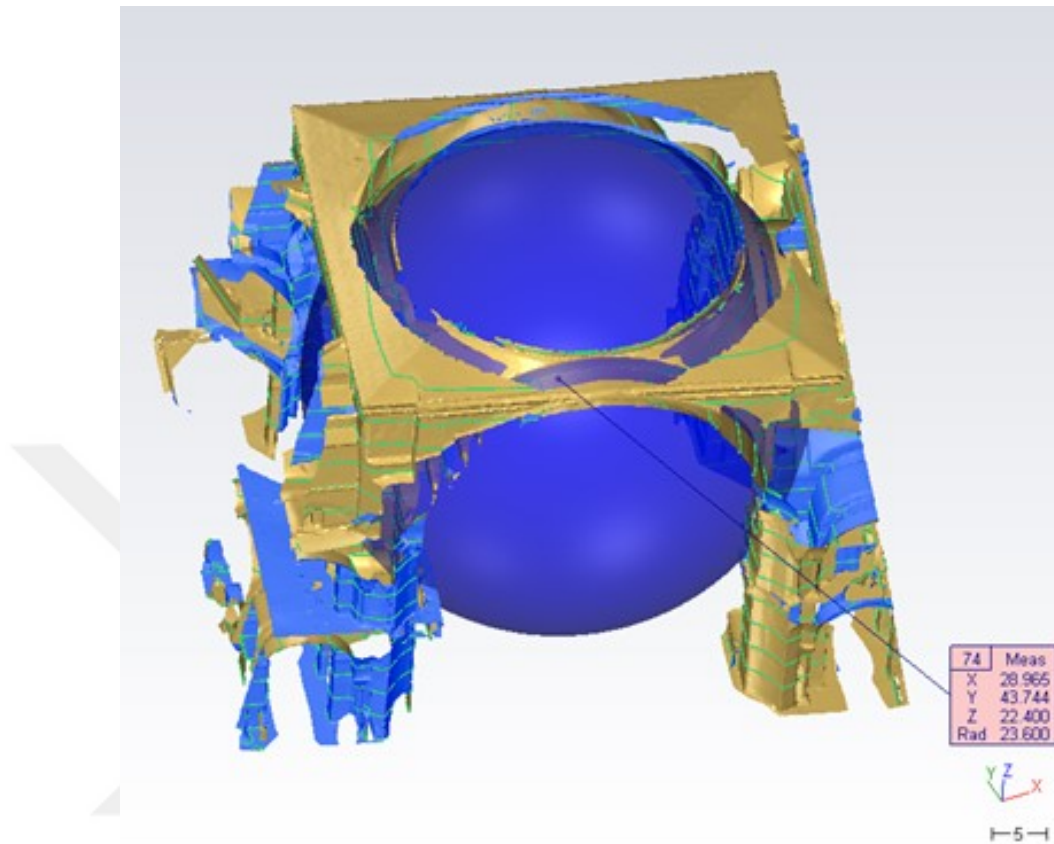


Figure 3.33. Plan view of the ideal sphere assigned to the middle of the structure and the contours.

As shown in Figure 3.34, at southwest pendentive, a noticeable settlement of -60cm is observed as the highest level of deflection, while there is no significant deformation in other areas. At the southeast pendentive, there is a difference of up to +50cm in the region slightly south of the central point, however this difference decreases as we move away from that area. At the northeast pendentive, there is generally a deflection of around 0 to +15cm, but between the Z=32 and Z=35m level contours, there is a +35cm deformation near the eastern arch. Besides, in the middle of the northeast pendentive, at the uppermost level, there is a roughness, that reaches upto 70cm shown with the blue color in Figure 3.34. Even with eye observation, it is visible on the structure. According to Mainstone:

“The chief irregularity here is at the northern pendentive. The half was built and the northern half was retained... and this led to a problem in smoothly reconciling the curve of the rebuilt half to that of the existing half” [1].

The northwest pendentive is not observed due to the presence of scaffolding erected for the restoration work.

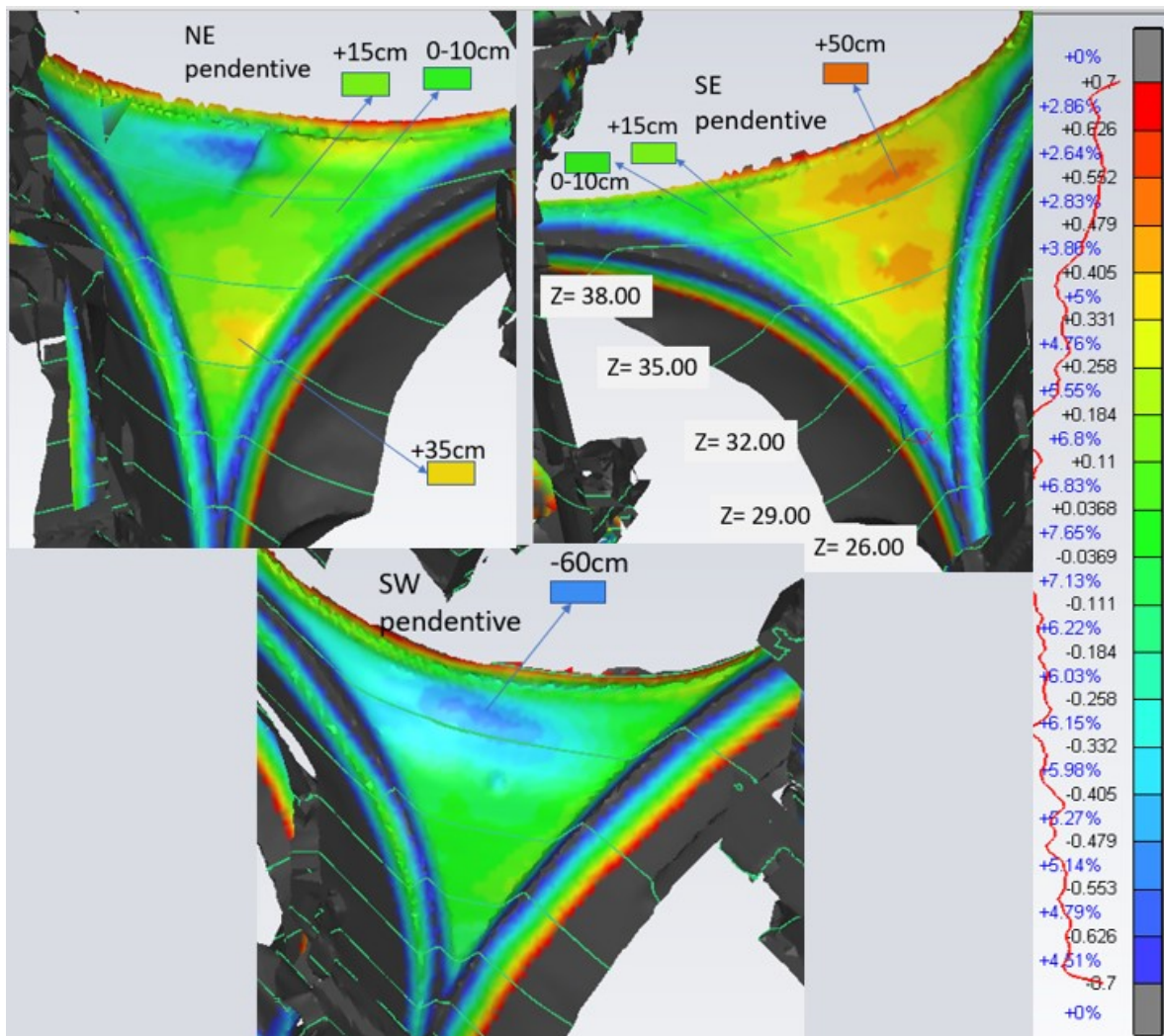


Figure 3.34. Southeast, northwest and southwest pendentives deformations obtained by the comparison of the ideal sphere and the pendentives.

3.3.7. Secondary Piers

Similar to the main piers, for the secondary piers, the contours at four levels are drawn as can be seen in Figure 3.35. Besides, ideal planes are created to be compared and assess the deformations of the secondary pier walls.

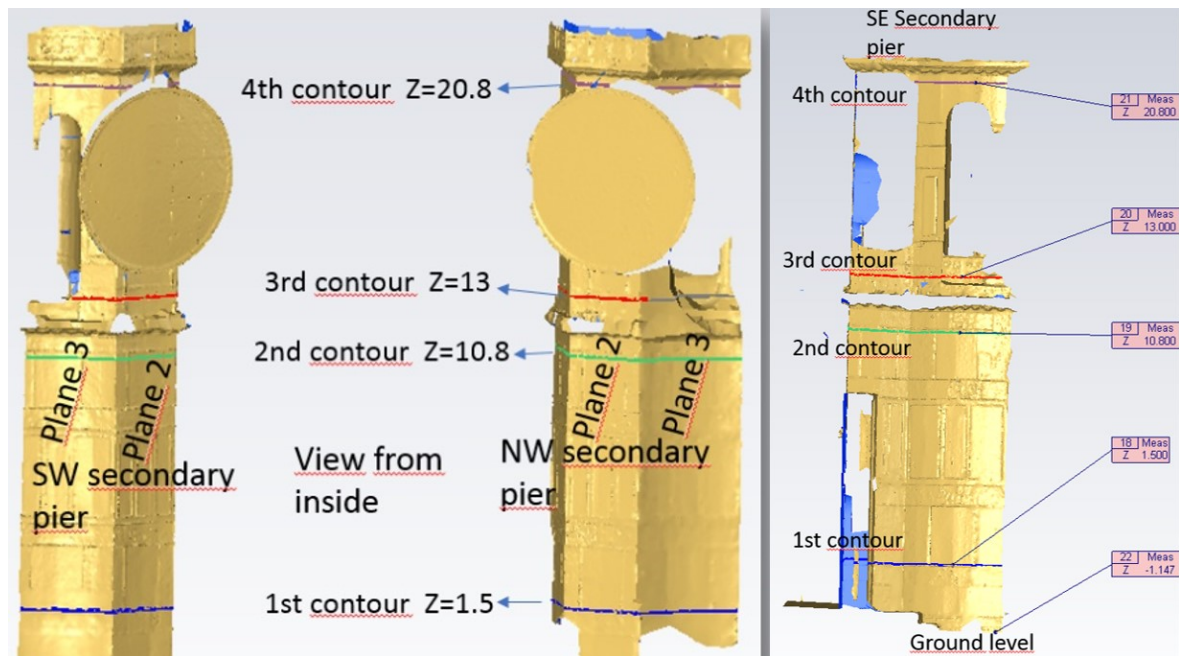


Figure 3.35. The four secondary piers and their contour levels.

As can be seen in Figure 3.36, the northeast secondary pier has a very limited deformation. On the other hand, at the top level, the southeast secondary pier seems to have a deformation of 8-10 cm which is not very clear to be the deflection or the roughness of the surface. Besides, these comparison data belong to Plane 1. Some difficulties hindered the remaining surfaces of the secondary piers from reliable evaluations. Hence it is not possible to verify the assessed inclination with similar observations on other sides. Similar to the secondary piers on the east side, the ones on the west side also have very limited deformation. The plan view of the contours created on the four secondary piers can be seen in Figure 3.37. They also point negligible inclination confirming the results obtained by the ideal planes.

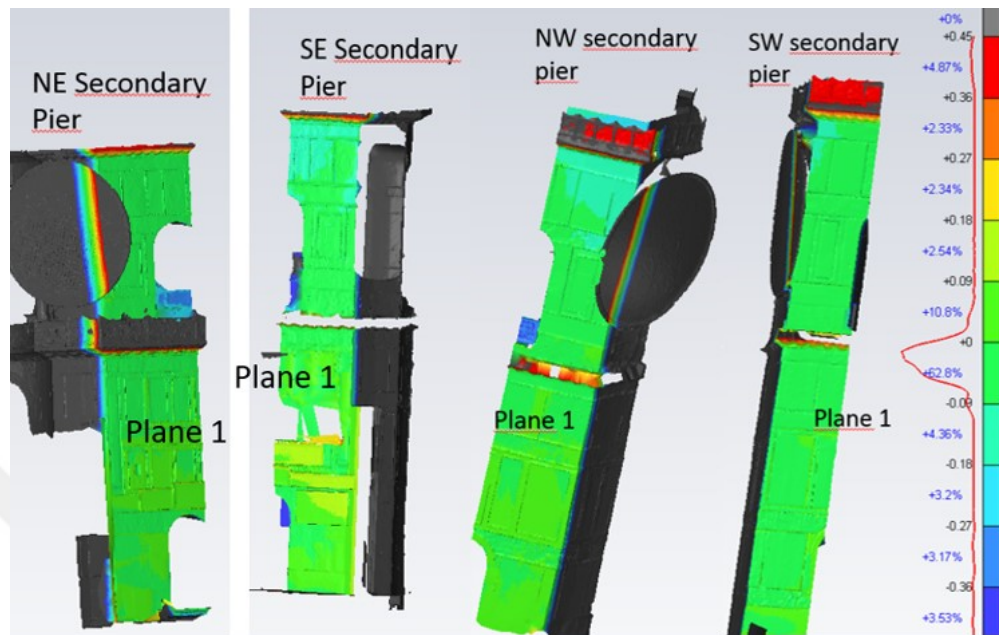


Figure 3.36. The four secondary piers deformations obtained by the comparison of the ideal planes.

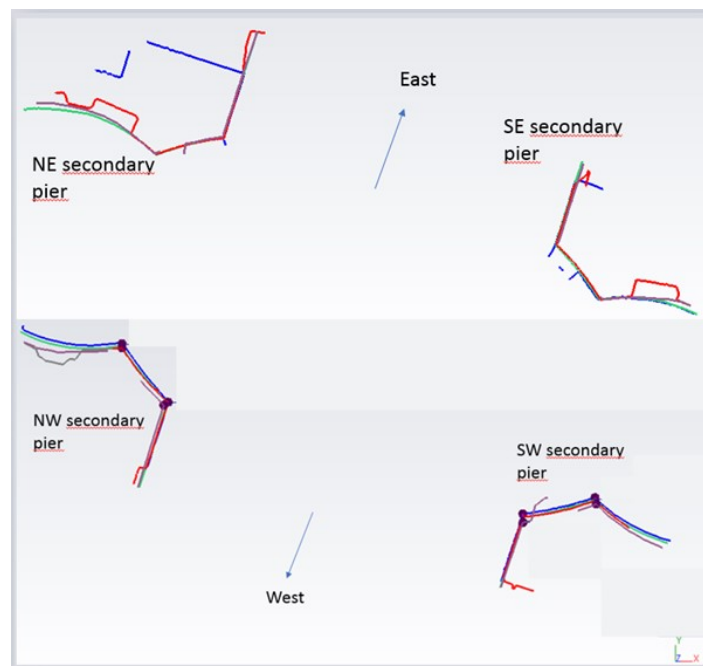


Figure 3.37. The plan view of the contours of the four secondary piers.

4. FINITE ELEMENT MODELLING

The finite element approach entails the use of finites, which are diminutive but linked components, to represent the structure. Each discrete element is correlated with a displacement function. All interconnected elements are connected to one another, either directly or indirectly, via shared interfaces, which may consist of nodes, boundary lines, surfaces, or both. One may ascertain the behaviour of a specific node by utilizing the stress-strain characteristics of the material comprising the structure, which are known to be associated with each other element in the structure. When represented in matrix notation, the whole set of equations describing the behaviour of each node becomes a sequence of algebraic equations [38].

One of the biggest problems facing structural engineers is numerical modeling and analysis of masonry structures. This challenge is ascribed to the existence of discontinuity and nonlinearity, joints as the primary source of weakness, and uncertainties in the geometrical and material qualities.

Two forms of structural behaviour need to be considered in an appropriate numerical model: (1) the behaviour of masonry units, or bricks; and (2) the behaviour of the joint material (i.e. mortar).

There are two main types of numerical modeling methodologies for masonry structures: macro-modeling and micro-modeling. A continuum homogenized model, often created with the finite element method, can be used in macro-modeling to depict a masonry element while implicitly taking mortar joint effects into account. Whereas the interface components are used in the micro-modeling of masonry elements with joint material to explicitly account for the interaction between masonry bricks along gaps [39].

In this study, macro-modeling is used, considering the large dimensions of the structure. In contrast to differentiating between individual units and joints, the macro-modeling technique considers masonry to be a homogeneous anisotropic continuum. When the structure consists of solid walls that are sufficiently vast in size to ensure that the stresses experienced along or across a macro-length are nearly uniform, then macro-models can be utilized. In addition to its user-friendly mesh creation and decreased time and memory needs, macro-modeling is undoubtedly more practical. When compromise between precision and efficiency is essential, this form of modelling is most beneficial [40].

In this study, four finite element models are created. The undeformed and deformed shapes of the structure are modeled on two widely used software platforms: SAP2000 and Ansys. The reference to these softwares are [41] and [42] respectively. In this study they are mentioned as SAP2000 and Ansys. Regarding the literature review on Hagia Sophia's dynamic behaviour, in the majority of previous studies the structure is modeled on SAP2000. Therefore, it is thought to be beneficial to create the models first on SAP2000 and then compare it to their counterparts on Ansys.

4.1. The Model in SAP2000

SAP2000, a robust analysis and design application, was initially developed by CSI more than three decades ago [43]. SAP2000 is capable of processing both straightforward 2D exercises and intricate 3D structures. Facilitating the engineer's computations pertaining to modelling, design, and optimization through the utilization of a robust analytical "engine" and an adaptable user interface is the program's inception.

Two numerical models that represent Hagia Sophia are created on SAP2000: undeformed and deformed models. They are both created using solid, shell and frame elements and are fixed at the ground level. Solid elements are used to model the four buttress piers, arches, four main piers, pendentives, dome base and four secondary piers. The main dome, the semidomes, the exedra domes, barrel vaults and tympana

are modeled as shell elements. Finally, the colonnades on the gallery and the ground levels and the dome ribs are modeled as frame elements as can be seen in Figure 4.1. Finite element model created in SAP2000. The model consists of 101340 joints, 47988 solid elements, 1306 shell elements and 142 frame elements. The elements up to $Z=23\text{m}$ level are modeled using stone masonry. Brick masonry material is used for the solid and the shell elements higher than that level.

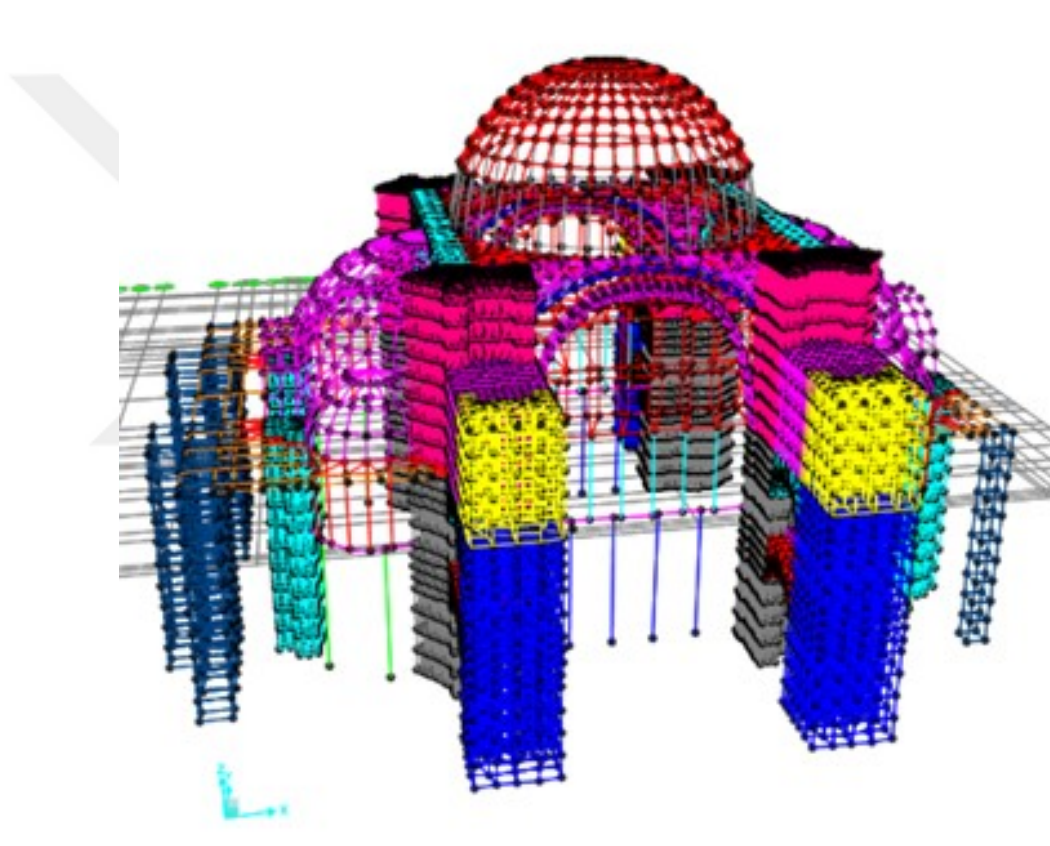


Figure 4.1. Finite element model created in SAP2000.

After creating the 3D model, the deformations discussed in the previous chapter are applied on it. Since there is no direct way currently available to import the deformed geometry from the point cloud data, several excel tables to evaluate the deformed joint coordinates of each element for X and Y directions on each Z level are created. Then, all deformed joint coordinates are applied on the model.

4.1.1. Modal Analysis and Calibrating the model

The main purpose of this study is to create a realistic numerical model of Hagia Sophia. The first step of it is to apply the deformed shape of the current structure on the 3D model and the second step is to calibrate the model using the information gained from the vibration monitoring system on the structure.

As mentioned earlier there is a vibration monitoring network installed in Hagia Sophia. The data provided by this monitoring system has been the subject of a series of studies on the identification of dynamic response properties and characteristics of the structure. According to Durukal and her colleagues, the first modal frequency of Hagia Sophia is 1.84 Hz [13]. It is a dominantly translational mode along the E-W direction. The second mode has a frequency of 2.09 Hz and is dominantly translational in the N-S direction. The third dominant mode is found to be rotational around the Z axis and has an estimated frequency of 2.35 Hz. There are other studies on the dynamic behaviour of Hagia Sophia as can be seen in Table 4.1. Cakmak studied the dynamic properties of the structure using a FE Model [11]. It revealed the first mode as 1.78 Hz. in the E-W translational direction and the second mode as 2.56 Hz in the N-S translational direction. The third dominant mode was found to be 2.43 Hz in Z axis rotational direction. Besides, Dar investigated the first two modes of Hagia Sophia varying with the environmental conditions such as temperature and humidity [30]. He concluded that during the whole year the first modal frequency changes between 1.65 and 1.95 Hz. And the second mode varies between 1.96 and 2.25 Hz depending on the changes in atmospheric conditions.

Table 4.1. Literature review findings on dynamic behaviour of Hagia Sophia (Modal frequencies in Hz.).

Mode	Observed (Durukal et al., 2003)	FE Model (Çakmak et al., 1993)	Observed (Dar, 2023)	Description
1	1.84	1.78	1.65–1.95	E-W Translational
2	2.09	2.16	1.96–2.25	N-S Translational
3	2.35	2.43		Torsional (Z axis rotation)

To verify the model with the observed results, modal analysis is carried out. The modal analysis technique is employed to do rudimentary dynamic analyses of structures. The dynamic features of a geometry undergoing vibrational stimulation may be investigated using modal analysis, which generates the relevant mode shapes from the natural frequencies at which the geometry oscillates [44]. In many cases, modal analysis serves as a preliminary step toward more sophisticated dynamic analysis methods, such as transient structural analysis or response spectrum analysis. It is important to acknowledge that the modal analysis operates in a linear fashion, disregarding non-linear characteristics of materials, geometries, and contacts. Modal analysis is a technique that utilizes the total mass and stiffness of the investigated structure to create the structure's resonance periods or natural frequencies. In contrast to loading input, modal analysis utilizes software to represent the model's oscillations at various natural frequencies. The dynamic reaction is contingent upon the stiffness and mass of the structures [45].

After the 3D actual geometry of Hagia Sophia is reflected to the numerical model, modal analysis is carried out. Several numerical models with varying material properties are considered with the one yielding dynamic parameter values as close as possible to those identified from the vibration data to be selected as the preferred one. Regarding the material properties used in previous studies [46], [25], [22], [17] the boundaries of the elasticity modulus of brick masonry is considered as 2500-5000 Mpa; for stone

masonry it is considered as 6000-8000MPa. Besides, the boundaries for the density are considered as 1.5-1.8kN/mm³ for brick masonry and 2-3kN/mm³ for stone masonry. The deformed model is considered when calibrating the model. The material properties of the brick and stone masonries which give out the best calibration are used in the model, and they are given in Table 4.2. The elasticity modulus of brick masonry is taken as 3000 MPa and that of stone masonry is taken as 7000 MPa. Besides, the mass of the brick masonry is 1.6 kN/mm³ and it is 3 kN/mm³ for the stone masonry.

Table 4.2. Linear material properties used for both undeformed and deformed FE model on SAP2000.

Materials	Elasticity modulus (N/mm ²)	Poisson's ratio	Mass (kN/mm ³)
Brick masonry	3000	1.67	1.6
Stone masonry	7000	1.67	3

The modal analysis results obtained by the undeformed and the deformed models with the material properties given in Table 4.2, can be seen in Table 4.3 and Table 4.4 respectively.

Table 4.3. The modal participating mass ratio results of the undeformed model on SAP2000.

StepType	Step Number	Frequency (Hz)	UX (Unitless)	UY (Unitless)	RZ (Unitless)
Mode	1	1.87	0.63	4.59E-09	1.02E-09
Mode	2	2.23	5.02E-09	0.72	8.09E-06
Mode	3	2.58	1.04E-08	1.24E-06	0.51

Table 4.4. The modal participating mass ratio results of the deformed model on SAP2000.

StepType	Step Number	Frequency (Hz)	UX (Unitless)	UY (Unitless)	RZ (Unitless)
Mode	1	1.79	0.64	3.29E-07	1.07E-05
Mode	2	2.19	3.87E-07	0.68	0.04997
Mode	3	2.32	2.36E-05	0.05009	0.59

This model needs to be calibrated with respect to modal frequencies and modal shapes previously identified from experimental data. However, this procedure is decided to be carried out on Ansys software due to some limitations of the SAP2000 Software. One of those problems is meshing capabilities of SAP2000 which is insufficient comparing to Ansys, which made the reflection of the deformed shape on the idealized model quite challenging. The deformed shape had to be applied node by node, manually. This resulted in a quite heavy model with a run time reaching up to 15-20 min for one modal analysis; whereas it takes less than 1 minute in Ansys. This made the calibration process almost impossible for the SAP2000 model, because it requires more than 200 analyses to find an optimum solution. Besides, the 3D geometry design interface of Ansys is more user-friendly and has a variety of options to design a geometry. At the beginning, it was decided to make the calibration and the linear analysis on SAP2000, then continue with Ansys for the nonlinear analysis. However, due to the reasons mentioned here, it is decided to transfer the geometry to Ansys and carry out the whole process there, since it is a much more sophisticated and capable engineering software.

4.2. The Model in Ansys

Ansys software is utilized to configure and analyze the numerical model. Utilizing the finite element approach, the program analyzes structures both statically and dynamically. Ansys is a commercially accessible computer program designed for numerical

modeling that is capable of doing a variety of investigations. This comprises fundamental vibration analysis, structural static studies, fluid dynamics, and heat transport. On Ansys, two-dimensional and three-dimensional geometries may be constructed [47].

The treatment of engineering issues consists of three fundamental steps: model development, problem resolution, and outcomes analysis. Similar to several other finite element analysis (FEA) tools, Ansys is structured around three primary components: the processors, known as the pre-processor, solution processor, and post-processor. Users are able to construct geometry, specify materials, and produce element mesh using the Ansys preprocessor. Users are able to address problems with the Ansys processor by applying loads and finding solutions. The output of Ansys post-processing jobs can be displayed and printed in tabular or enumerated formats [48].

4.2.1. 3D Geometry

4.2.1.1. Importing the 3D Geometry. This part of the study mainly focuses on the pre-processors of the software, since the next step is transferring the 3D geometry created in SAP2000 to Ansys. There are 3 add-ins in Ansys Software which allow to create or import 3D Geometry: APDL, Design Modeler and Space Claim.

APDL (ANSYS PARAMETRIC DESIGN LANGUAGE) can be used for creating the model and analyzing it both. It doesn't have a user-friendly interface, instead, it provides the users comfort in adapting specific arrangements allowing Python based coded scripts. Hence, instead of directly importing the geometry from SAP2000, a method based on creating a script that has the joint coordinates as input parameter is used. Thus, first, the joint coordinates and the design specification of each element are exported into excel tables from SAP2000. Then the coordinates are adapted in a script that can be read by APDL. The joints, lines, areas, and the volumes are required to be given as input respectively. However, this came out with a problem in creating the solid elements. The geometry creating logic of APDL works differently than the one of SAP2000. It requires surrounding areas (surfaces) to create volumes (solids), otherwise

it wouldn't know in which order to connect the joints to create that specific volume. Therefore, it requires several loops to be coded to process the joint coordinates and it would be time-consuming for such a complex and large geometry.

DESIGN MODELER is an add-in of Ansys that provides an interface to the user to create 3D geometry. It can only be used for the geometric design. The interface allows the user to import several kinds of CAD files. Therefore, I have exported the SAP2000 file into AutoCAD as .dxf file. After opening the .dxf file in AutoCAD software, I have exported it as .iges file which is an acceptable CAD file by Design Modeler. For a simple geometry file, this method works. However, the Hagia Sophia 3D model created in SAP2000 is quite complex and has large data due to the large number of joints and elements, hence the method did not work for it and the program stopped working at all attempts.

SPACE CLAIM is also an add-in included in Ansys package for the geometry design purposes. However, unlike Design Modeler, procedure is less time consuming and more convenient for the complex geometries since it directly opens the .iges files instead of importing. However, another problem is faced during the procedure: solid elements are created as surfaces (areas), not as volumes (solids).

As a whole, there is no way to import such a large 3D model as of Hagia Sophia directly into Ansys software, and it took remarkable time to come with this conclusion. Therefore, to avoid from much more time consumption, it is decided to go with the Space Claim since it is more user friendly and convenient for complex geometries. Besides, even though it transfers elements only as surfaces, it is the only one that can transfer the whole geometry. So, the model is transferred into Space Claim and only the surface boundaries remained, then the elements are recreated and connected to each other. After that, some geometrical modifications are applied to express the deformed shape more realistically.

4.2.1.2. Applied Deformed Shape. When the 3D geometry is recreated in Ansys, the deformed shape is rechecked with the undeformed one. This is done by comparing the elements of the deformed and undeformed models created in Ansys. The illustrations related to this comparison are shown in Figure 4.2 to Figure 4.9.

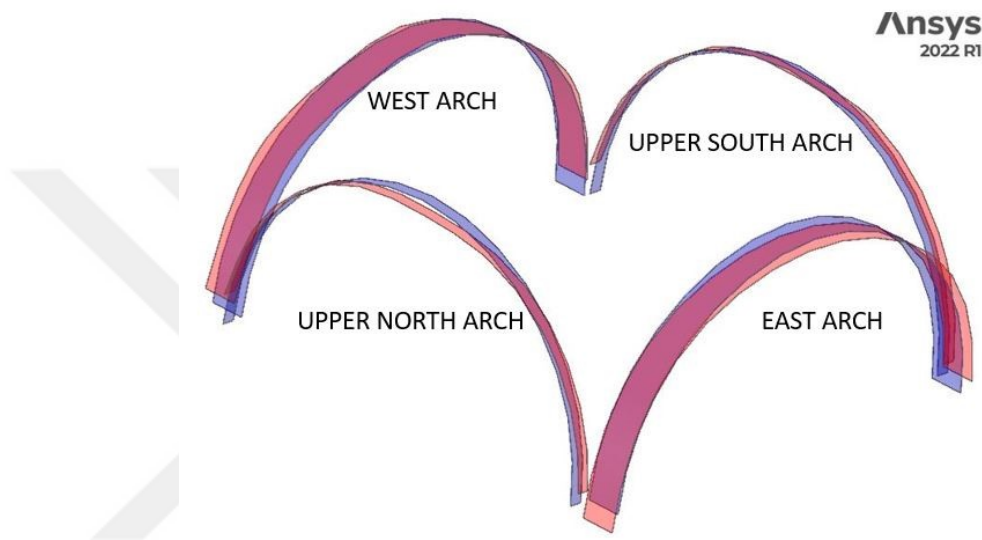


Figure 4.2. Deformed and undeformed interior surfaces of the four main arches created in Ansys.

Figure 4.2 shows the geometry of the interior surfaces of the main arches. The pink ones depict the deformed arches and the blue ones show the undeformed ideal arches. Using the deviation measurement tool of Ansys, deformed and undeformed surfaces are compared.

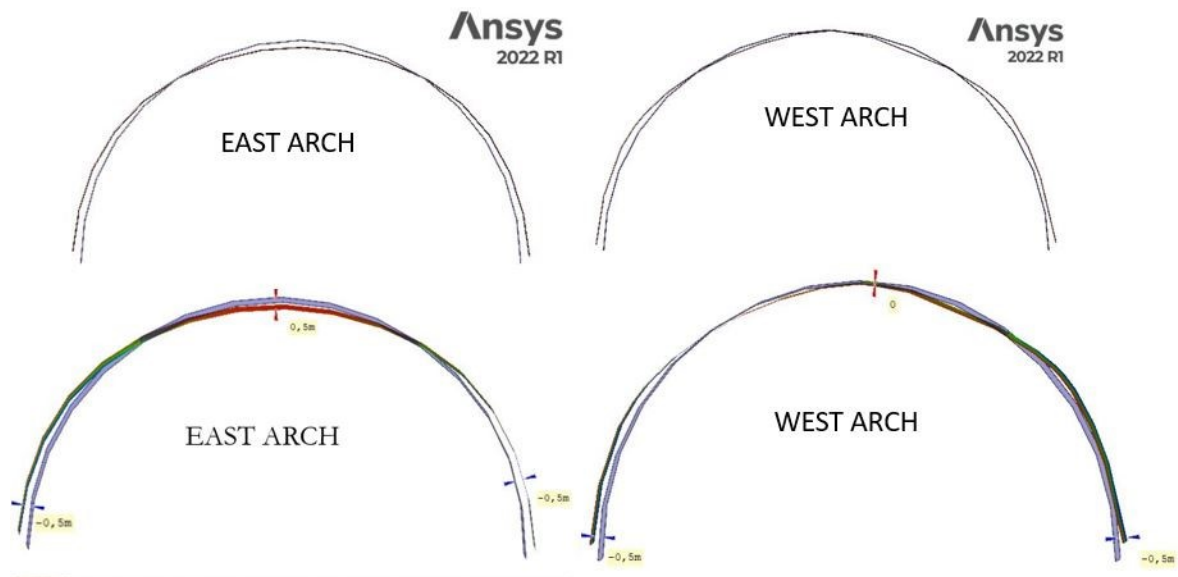


Figure 4.3. Comparison of deformed and undeformed east (left) and west (right) main arches created in Ansys.

Figure 4.3 shows that the crown of the east main arch has a vertical deformation of 0,5m while the west arch displays no deformation. Moreover, the outwards horizontal inclination at the springing points is 0.35m for both arches conforming to the geometry evaluation results. Additionally, Figure 4.4 illustrates the same procedure for the upper north and upper south arches. As mentioned earlier, since there are two arches on the north and south parts of the structure, they are called upper and lower arches. The one shown here are the upper arches.

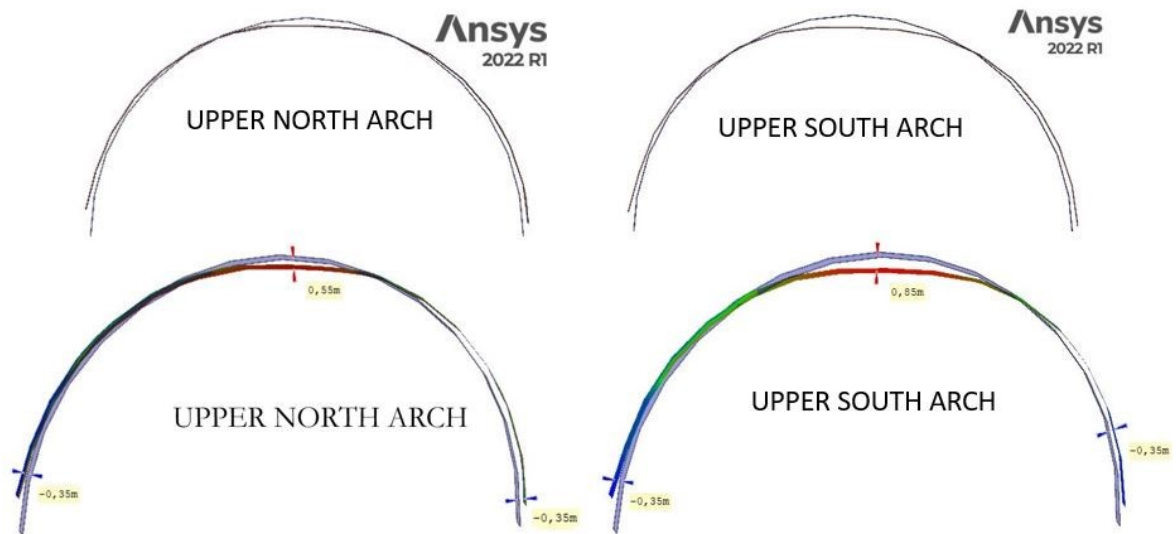


Figure 4.4. Deformed and undeformed interior surfaces of the upper north (left) and upper south (right) main arches created in Ansys.

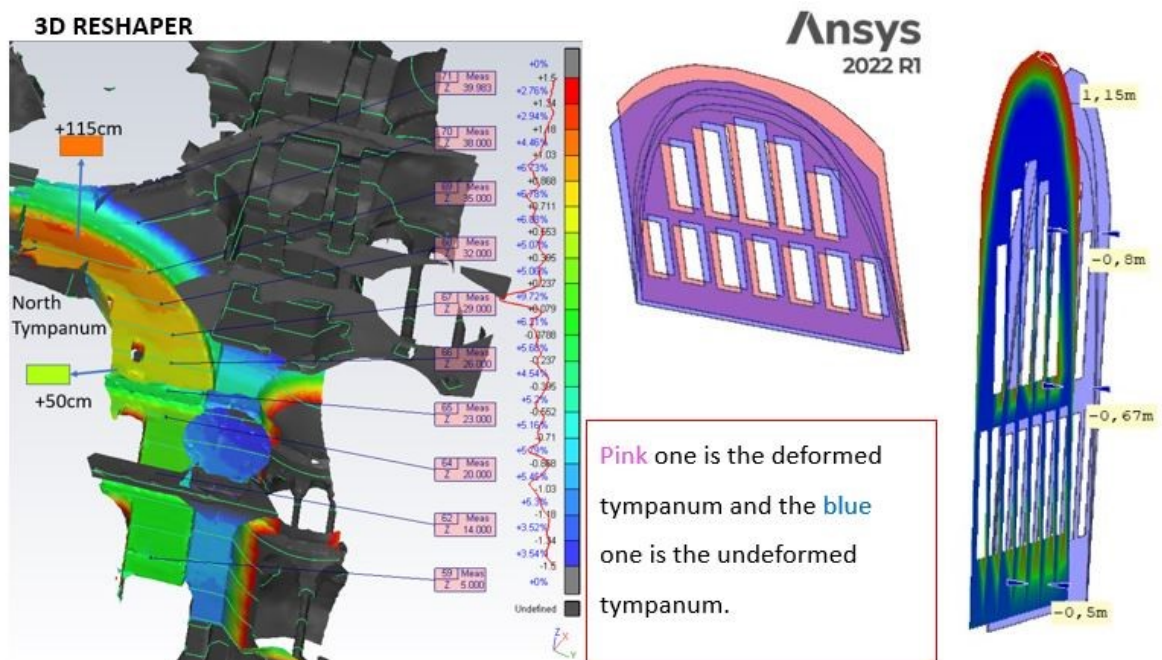


Figure 4.5. Comparison of deformed and undeformed tympanum in Ansys (right); tympanum in 3D Reshaper (left).

Figure 4.5 shows the deformed and undeformed tympanum created in Ansys, together with a reclaim of the results obtained by 3D Reshaper as part of geometrical evaluation part of the study. Figure 4.6-Figure 4.9 illustrate similar information for the dome base, main dome, east semidome and west semidome. In Figure 4. 6. comparison of the deformed dome base geometry is given. On the left, The real shape of the dome base and the ideal circle created in 3DReshaper is illustrated. The figure on the right side is taken from the Ansys model. It compares the dome base of the deformed model and the undeformed model created in Ansys. It shows that the deformations applied on the model and the ones obtained by 3DReshaper are compatible to eachother.

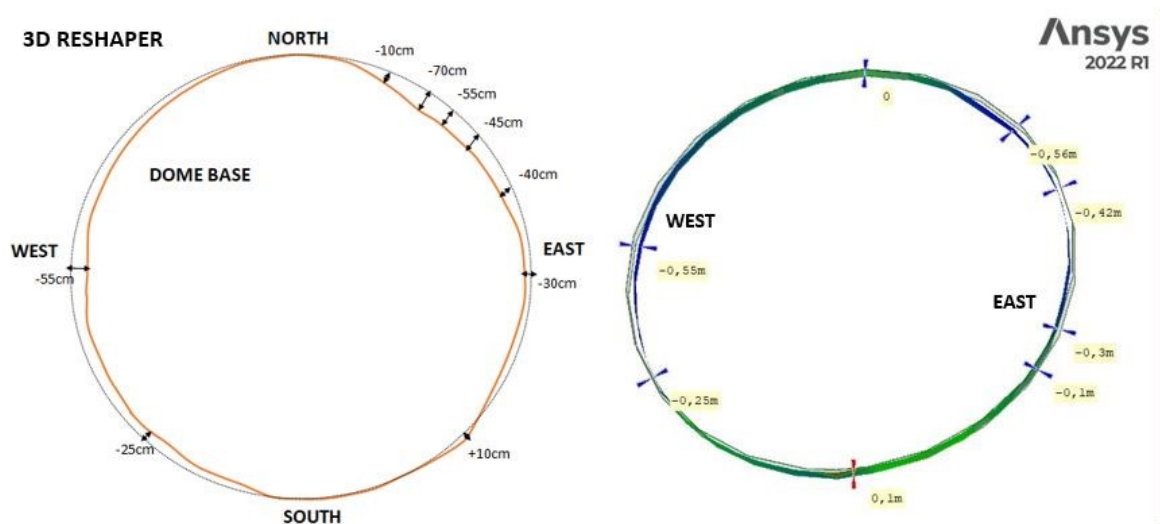


Figure 4.6. Comparison of deformed and undeformed dome base geometry in Ansys (right); dome base geometry in 3D Reshaper (left).

Figure 4.8 and Figure 4.9 illustrate the east and west semidomes, comparing their deformed and undeformed shapes in Ansys and also showing the results of the geometry analysis based on the data from laser scanning. The two semidomes are not identical. Due to the restorations on the west arch in the 10th century, the level of the west semidome is different than the east one. The crown of the west semidome is about 1m higher than the east one. Hence, the center and the radius of the ideal spheres are

different for the east and west semidomes. It is estimated that the settlement at the crown of the east semidome is 40 cm, and that of the west semidome is 25 cm. At their springing level the semidomes are deformed towards the exterior, thus deviating from the ideal spheres identified for them. This deviation is 10 cm for the east semidome and it is 25 cm for the west semidome. Moreover the general shapes of the semidomes are quite different from each other. The west semidome has a swelling as shown by the orange color in Figure 4.9 Mainstone also confirms the differences between the semidomes with the following sentences:

”At the west, one of the window rings is blocked and a heightened drum-like ring was built against the semidome to buttress its springings. Construction then continued almost vertically in the more westerly part well beyond the level at which the main arch started to close in towards the crown. Finally, it was therefore necessary to move inwards almost horizontally to close the gap at the top, leading to the heavy, high-shouldered external profile” [1].

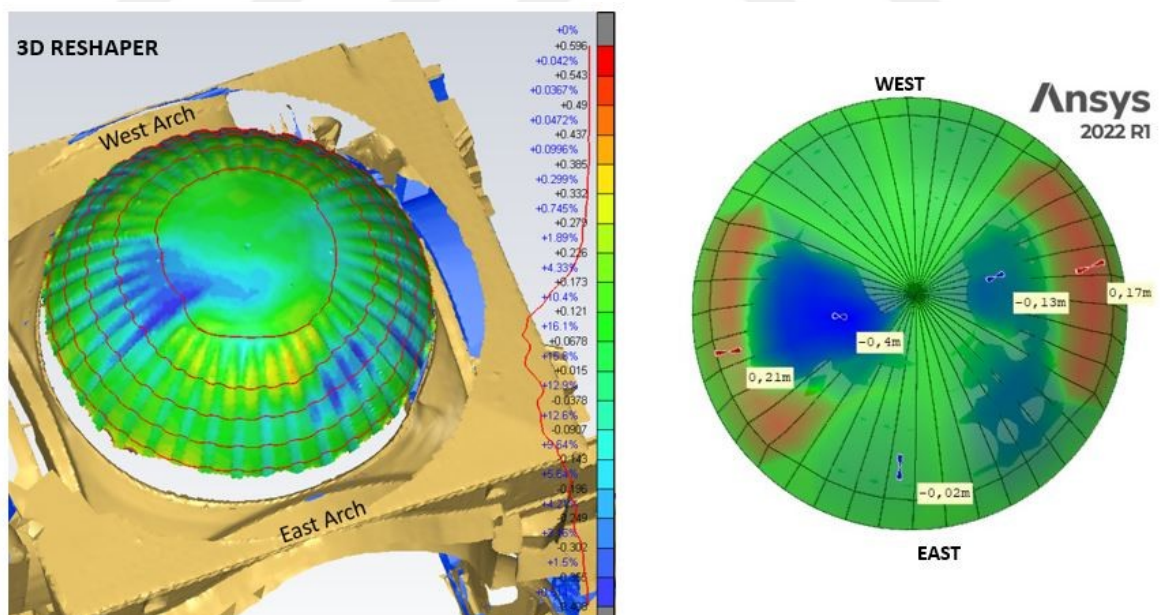


Figure 4.7. Comparison of deformed and undeformed main dome in Ansys (right); main dome in 3D Reshaper (left).

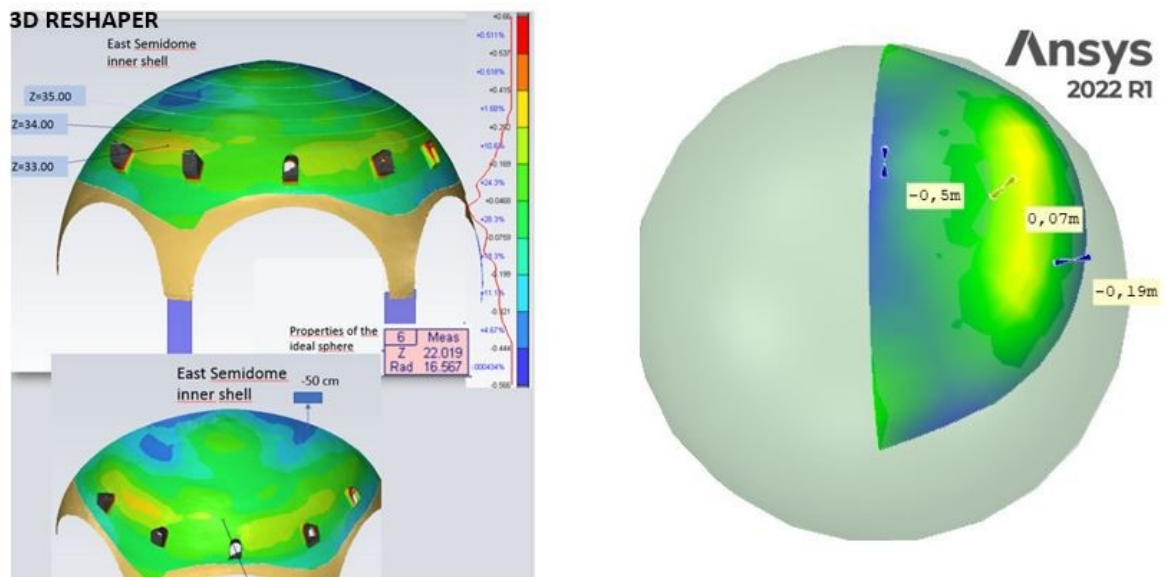


Figure 4.8. Comparison of deformed and undeformed east semidome in Ansys (right); east semidome in 3D Reshaper (left).

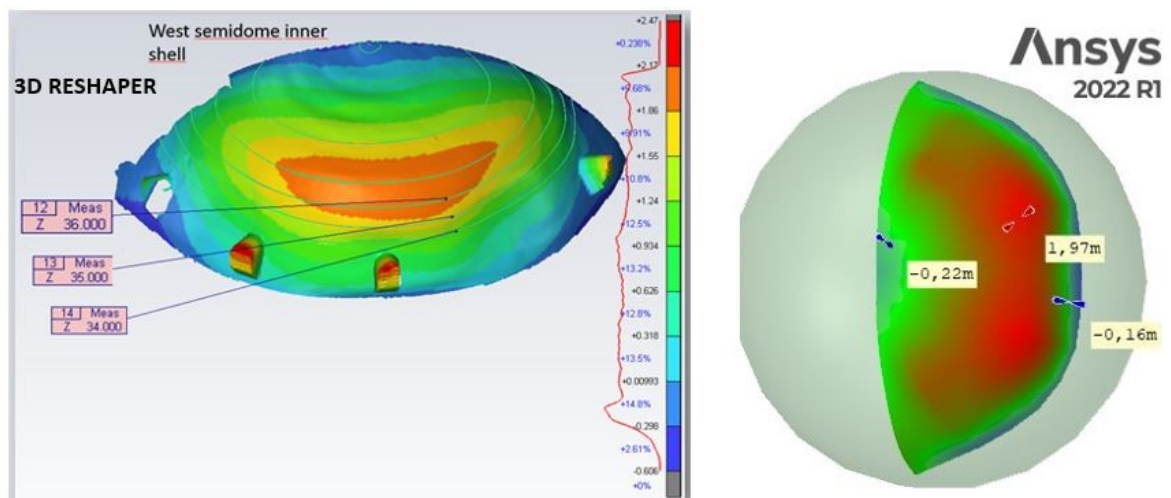


Figure 4.9. Comparison of deformed and undeformed west semidome in Ansys (right); west semidome in 3D Reshaper (left).

4.2.2. Calibrating the Model

The model to calibrate with the real data is the deformed 3D model created in Ansys. Since the contact, meshing and geometry modeling features are more advanced compared to SAP2000, the results obtained by Ansys can be considered more reliable for such a complex masonry structure.

When it comes to defining the model, it consists of solid, shell and frame elements. The solid elements are used for four buttress piers, four main arches, 2 lower arches on the south and north parts, four main piers, four secondary piers and 4 pendentives. The main dome, the semidomes, the exedra domes, barrel vaults and tympana are modeled as shell elements. Finally, the dome ribs are modeled as frames. The structure is fixed on the ground level. Initially the granite colonnades under the exedra domes and the tympana are included in the model as frame elements. Linear analyses are carried on this model. However, since the nonlinear analyses are quite vulnerable to convergence issues, the frame-shell and frame-solid joints in these critical regions resulted in non-convergence at the very beginning of time domain analyses. Therefore, these joints are replaced with restraints at the lower level of exedra domes and tympana. When the displacement and stress distribution results of the linear analysis are compared it is found that there is not any significant difference on the models with colonnades and without colonnades. Hence, to overcome the convergence issues occurring early stages of the analysis, the model without the colonnades is preferred. All analyses in this study are carried out this version of deformed and undeformed models.

The illustrations of the undeformed and deformed models created in Ansys are given in Figure 4.10 and Figure 4.11 X axis corresponds to the east-west direction whereas Y axis represent the north-south direction.

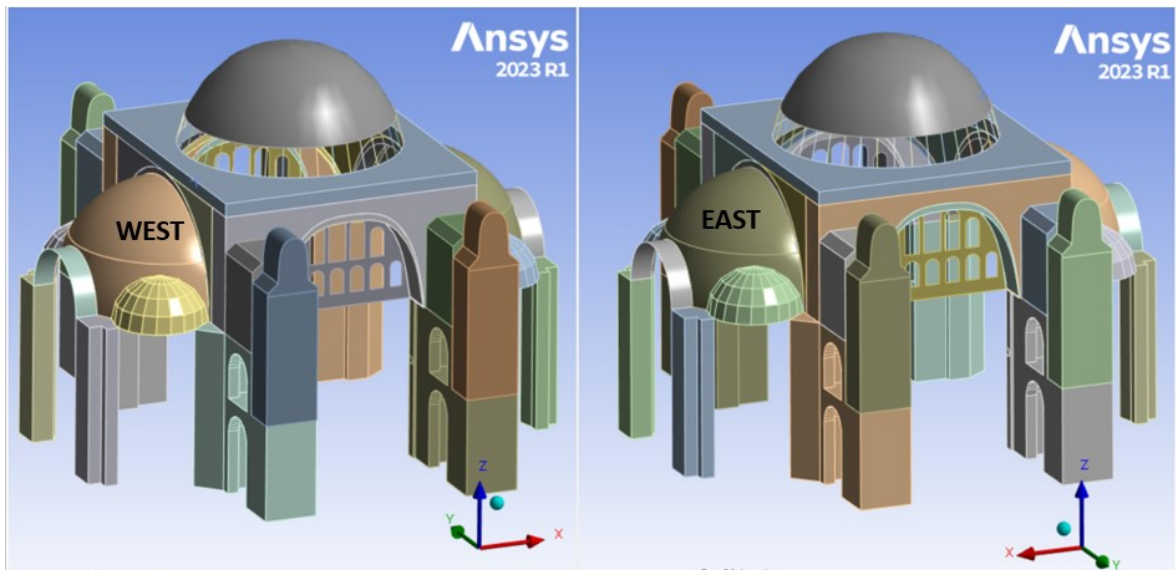


Figure 4.10. Illustration of the undeformed model created in Ansys from southwest (left) and northeast (right) point of view.

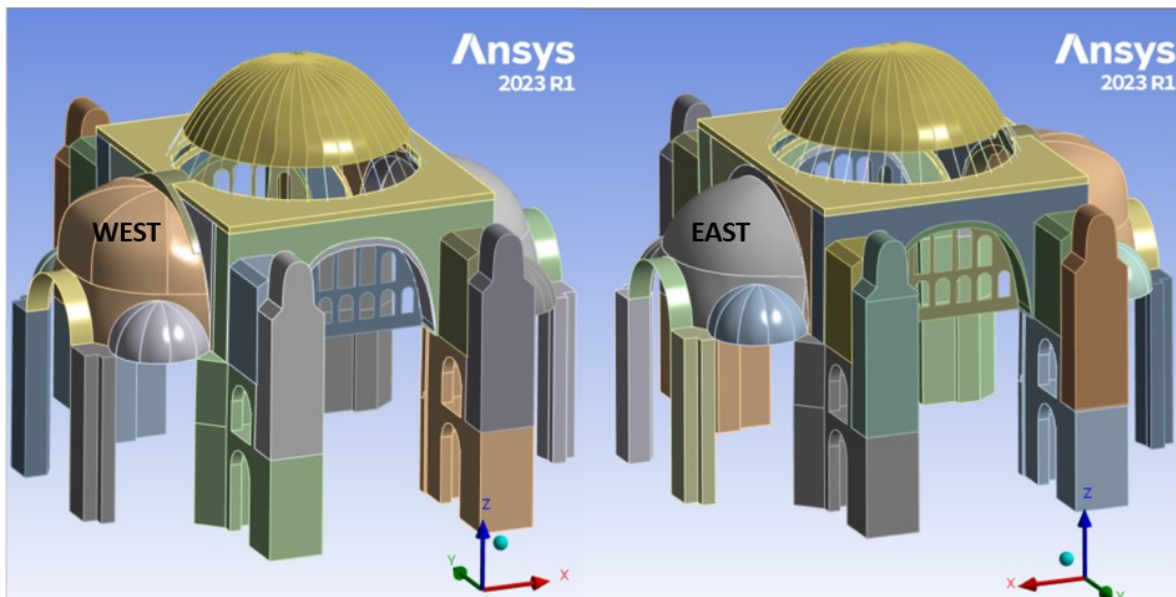


Figure 4.11. Illustration of the deformed model created in Ansys from southwest (left) and northeast (right) point of view.

There are two types of linear materials used in the model: Stone masonry, brick masonry. Brick masonry is used for the elements above $Z=23\text{m}$ level. Stone masonry is used for the elements below that level. The brick elements used for the pendentives are reported to be lighter than the bricks used elsewhere [1]. Therefore, another brick masonry material is created to use in the model for the pendentive elements. This material is called Brick Masonry-2. In this text, the materials are mentioned as Brick for brick masonry, Brick-2 for brick masonry-2 and Stone for stone masonry.

For calibration purposes, Ansys has an optimization tool under Ansys Design Explorer Module. To calibrate the model, first the parameters which are most effective on the results are decided. They are found as the densities and the elasticity modulus of the brick masonry, stone masonry and pendentive materials. And accordingly with the material property studies in literature, their upper and lower boundaries are determined and given in Table 4.5. These are given as the input parameters to the optimization tool. The boundaries for the first 3 modal frequencies as shown in Table 4.6 are given as the output parameters as a target for calibration. More than 200 analyses are carried out with the varying input parameters within the boundaries and the modal frequency results are obtained. And the optimum parameters are determined. Table 4.7 and Table 4.8 show the determined material properties of the solid and shell elements respectively; including the density, elasticity modulus and the Poisson's ratio.

Table 4.5. The boundaries of the material properties given as input parameters to the optimization tool of Ansys.

Material Name	Density Lower (kg/m ³)	Density Upper (kg/m ³)	E Lower (MPa)	E Upper (MPa)
BRICK	1600	1900	2500	5000
BRICK-2	1100	1400	2500	5000
STONE	2000	3000	6000	8000

Table 4.6. The boundaries of the modal frequencies given as output parameters to the optimization tool of Ansys.

FIRST 3 MODAL FREQUENCIES (Hz)						
Mode No	1		2		3	
Frequency Boundaries	1.70	1.95	2.00	2.25	2.30	2.45

Table 4.7. Linear material properties of the solid elements of the models created in Ansys.

SOLID ELEMENTS	Material Name	Density (kg/m ³)	E (N/m ²)	Poisson's Unitless
MAIN PIERS (Z>23m)	BRICK	1700	2.5E+09	0.1667
MAIN PIERS (Z<23m)	STONE	2000	6.2E+09	0.1667
BUTTRESSES (Z>23)	BRICK	1700	2.5E+09	0.1667
BUTTRESSES (Z<23)	STONE	2000	6.2E+09	0.1667
MAIN ARCHES & DOME BASE	BRICK	1700	2.5E+09	0.1667
SECONDARY PIERS	STONE	2000	6.2E+09	0.1667
PENDENTIVES	BRICK-2	1250	2.5E+09	0.1667

Table 4.8. Linear material properties of the shell elements of the models created in Ansys.

SHELL ELEMENTS	Material Name	Thickness (m)	Density (kg/m ³)	E (N/m ²)	Poisson's Unitless
TYMPANA	BRICK	0.8	1700	2.5E+09	0.1667
EXADRA DOMES	BRICK	0.4	1700	2.5E+09	0.1667
SEMIDOMES	BRICK	0.75	1700	2.5E+09	0.1667
MAIN DOME	BRICK	0.8	1700	2.5E+09	0.1667
BARREL VAULT	BRICK	1.0	1700	2.5E+09	0.1667

The 3D model is created in Ansys with the given properties and modal analysis is carried out for calibration purposes. As will be discussed in Section 5.1, modal analysis results are compared with those obtained from the system identification of real-life data. Table 4.9. illustrates the modal analysis of the first three modes of the deformed model. According to this table, the first mode is translational in the east-west (X) direction with a frequency of 1.75 Hz. The second one is translational in the north- south (Y) direction at 2.22 Hz. The third mode is rotational around Z axis with a frequency of 2.44 Hz.

Table 4.9. Modal frequency and the effective mass to total mass ratio results of the first three modes of the model with deformed geometry.

Mode	Frequency [Hz]	X Direction	Y Direction	Rotation Z
1	1.749	0.74659	1.682e-007	0.2845
2	2.2183	1.006e-007	0.76809	0.31741
3	2.4424	2.306e-003	1.3025e-005	0.14824
Sum	—	0.74896	0.7681	0.74868

Table 4.10 shows the comparison of these modal frequencies to the ones obtained in earlier studies. As can be seen, analytical modal frequencies are within the range of those reported by the analysis of real data obtained from the structure. This indicates that the model is calibrated reasonably well.

Table 4.10. Comparison of the modal results of this study with those reported previously.

Mode	Observed (Durukal et al., 2003)	FE Model (Çakmak et al., 1993)	Observed (Dar, 2023)	FE Model (ANSYS This Study)	Description
1	1.84	1.78	1.65–1.95	1.75	E–W Translational
2	2.09	2.16	1.96–2.25	2.22	N–S Translational
3	2.35	2.43		2.44	Torsional (Z axis rotation)

5. STRUCTURAL ANALYSIS

Structural analyses carried out within the scope of this thesis research include linear and nonlinear analyses. Linear analysis part consists of modal analysis, static analysis and dynamic analysis. Nonlinear analysis part of the study comprises definition of nonlinearity in the system, nonlinear static analysis and nonlinear time history analysis. All analyses are performed for both deformed and undeformed models created by Ansys software.

5.1. Linear Analysis

5.1.1. Modal Analysis

In this section, results of modal analysis of the deformed and undeformed models are presented and compared. The modal frequencies, mode shapes and the ratio of the effective mass to total mass are estimated for the first three modes and the first fifty modes. Figure 5.1 to 5.3 illustrate the mode shapes of the first three modes of the deformed model, and Figure 5.4 to 5.6 show those of the undeformed model. The dome ribs are modelled as frame elements and in the analysis results view of Ansys it is shown as the cross sections. Even though they seem as if only one point of the cross section is connected to the adjacent element, all the cross section is employed to transfer the load in the model actually. It is clear that in both models the first mode is translational along the X direction, the second mode is translational in the Y direction, and the third mode is in rotational around Z direction.

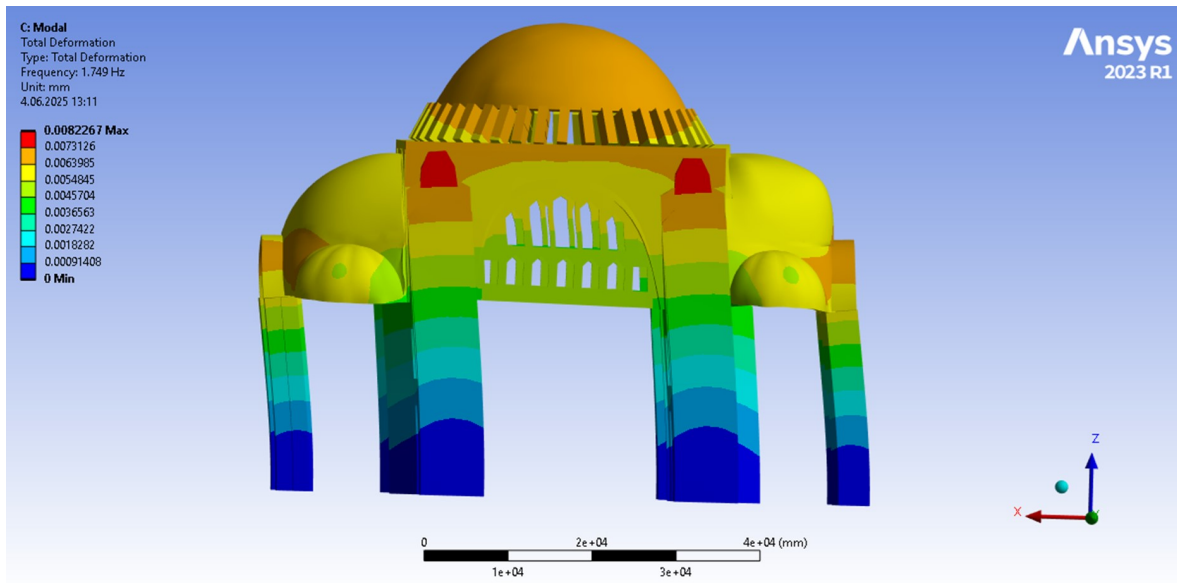


Figure 5.1. 1st mode shape of the deformed model (side view looking from north).

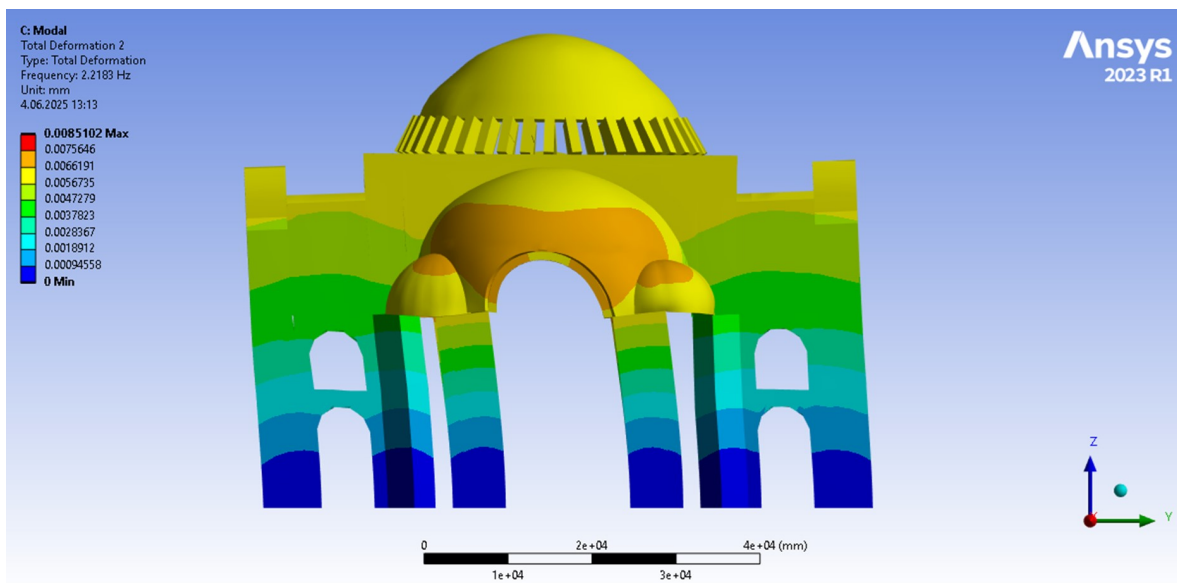


Figure 5.2. 2nd mode shape of the deformed model (side view looking from east).

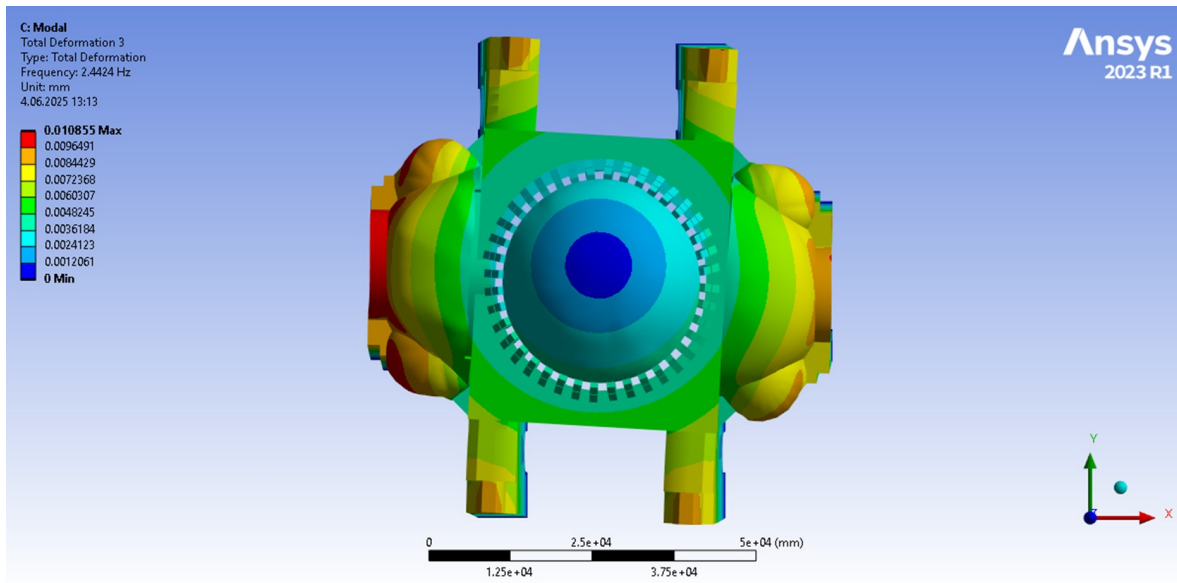


Figure 5.3. 3rd mode shape of the deformed model (top view).

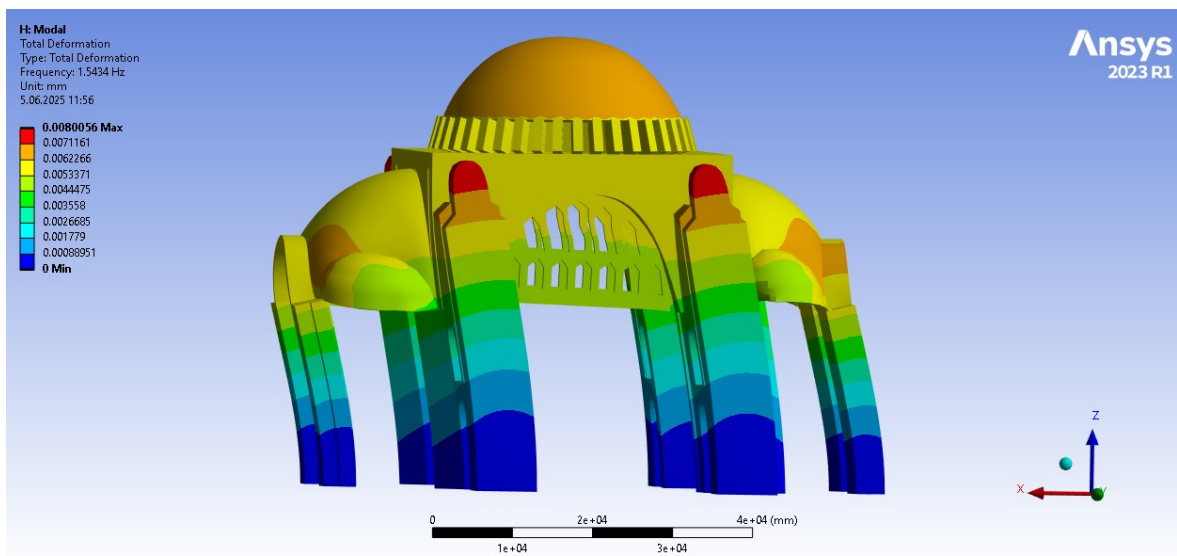


Figure 5.4. 1st mode shape of the undeformed model (side view looking from north).

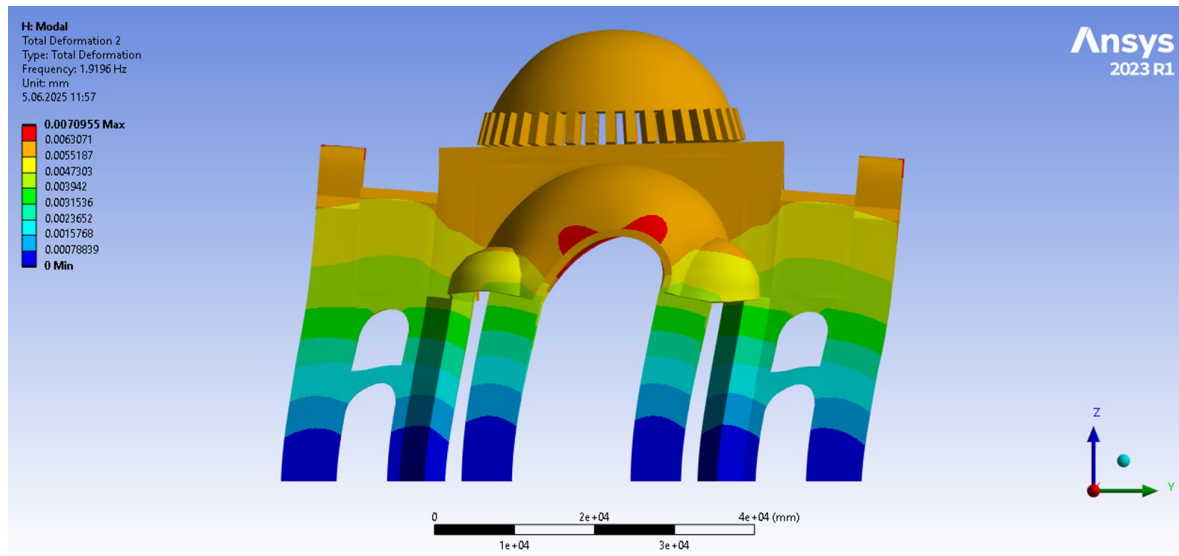


Figure 5.5. 2nd mode shape of the undeformed model (side view looking from east).

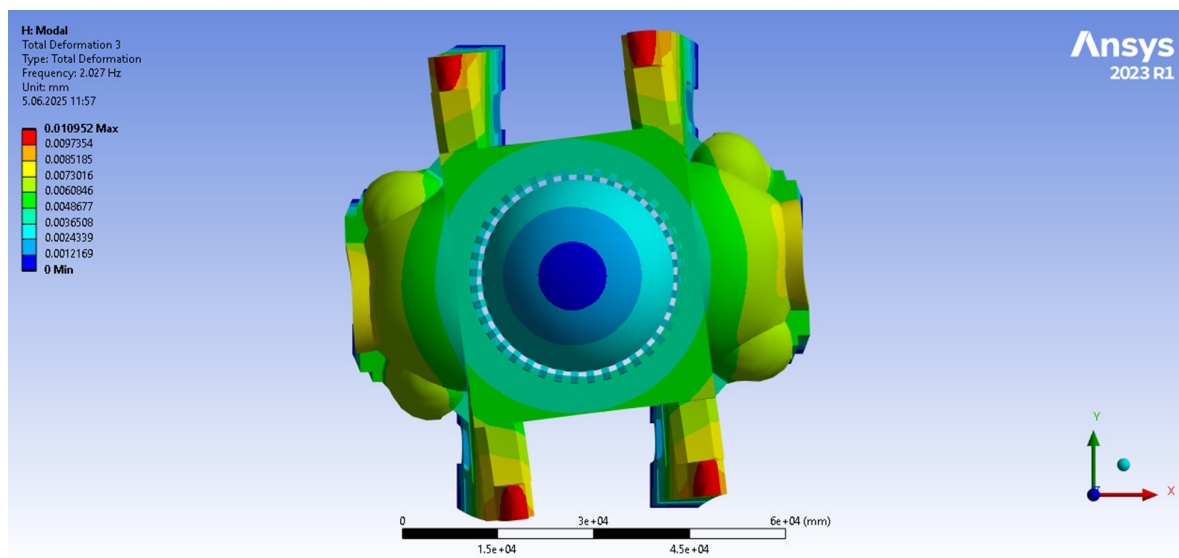


Figure 5.6. 3rd mode shape of the undeformed model (top view).

Table 5.1 and Table 5.2 illustrates the modal frequencies and the effective mass ratios of the first three modes of the deformed and undeformed models respectively.

The sum of the effective mass to total mass ratios for the first three modes are quite similar in both models being around 72% in X dominant direction, 75% in Y dominant direction and 74% rotational around Z direction.

Table 5.1. Modal frequency and the effective mass to total mass ratio results of the first three modes of the deformed model.

Mode	Frequency [Hz]	X Direction	Y Direction	Rotation Z
1	1.749	0.74659	1.682e-007	0.2845
2	2.2183	1.006e-007	0.76809	0.31741
3	2.4424	2.306e-003	1.3025e-005	0.14824
Sum	—	0.74896	0.7681	0.74868

Table 5.2. Modal frequency and the effective mass to total mass ratio results of the first three modes of the undeformed model.

Mode	Frequency [Hz]	X Direction	Y Direction	Rotation Z
1	1.9434	0.73653	3.1085e-007	0.26154
2	1.9196	2.954e-007	0.76583	0.30577
3	2.0227	3.2537e-006	7.0658e-004	0.17365
Sum	—	0.73653	0.76614	0.74098

Table 5.3 and Table 5.4 illustrate the modal frequencies and the effective mass ratios of the first hundred modes of the deformed and undeformed models respectively. The sum of the effective mass to total mass ratios for the first one hundred modes are also similar in both models. They are in the range of 86-90% in X and Y directions, and around Z direction.

Table 5.3. Modal frequencies and sum of the effective mass to total mass ratio of the first 100 modes of deformed model.

Mode	Frequency [Hz]	X Direction	Y Direction	Rotation Z
95	15.187	1.1841e-005	8.9533e-003	4.0536e-003
96	15.291	2.7134e-006	2.4113e-003	1.0936e-003
97	15.472	5.4375e-006	2.0272e-004	1.6548e-003
98	15.508	1.5476e-004	1.2659e-004	3.7525e-004
99	15.629	7.5964e-004	2.997e-005	9.1326e-005
100	15.694	1.6899e-004	7.5015e-005	2.8127e-007
Sum	—	0.86255	0.88854	0.86894

Table 5.4. Modal frequencies and sum of the effective mass to total mass ratio of the first 100 modes of undeformed model.

Mode	Frequency [Hz]	X Direction	Y Direction	Rotation Z
95	14.749	1.7453e-007	2.2681e-005	1.1372e-003
96	14.828	1.1319e-002	1.8994e-005	4.9434e-003
97	14.851	1.292e-004	4.4185e-005	5.6935e-004
98	14.873	3.0781e-003	9.742e-009	1.1294e-003
99	14.906	1.1194e-005	1.5452e-002	9.331e-003
100	14.938	1.0052e-005	1.8671e-003	1.1989e-005
Sum	—	0.89166	0.89641	0.88909

5.1.2. Linear Static Analysis

Static structural analysis is executed under the self-weight of the structure for both deformed and undeformed models. The stress distributions are examined in terms of equivalent stress, maximum and minimum principal stresses, and normal stresses in X (hoop) and Y (radial) directions. The stress distributions under self-weight are illustrated for the range of 0-1 MPa for shell and solid elements separately to investigate

the most vulnerable parts of the structure. Figure 5.7 to Figure 5.11 demonstrate the stress distributions on the shell elements of deformed and undeformed models.

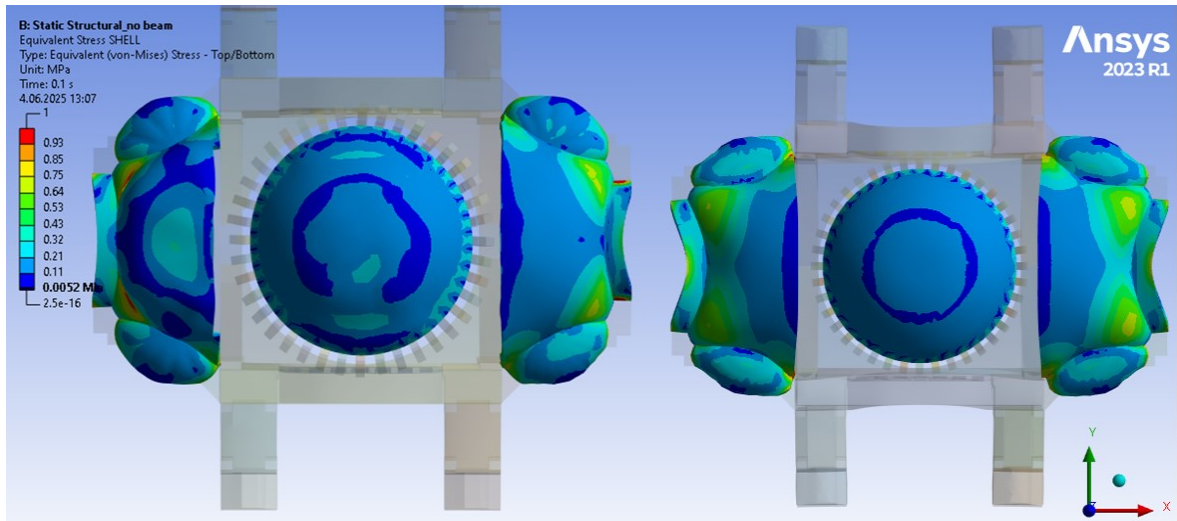


Figure 5.7. Von Mises stresses on the shell elements under self-weight (0-1 MPa): deformed model (left), undeformed model (right).

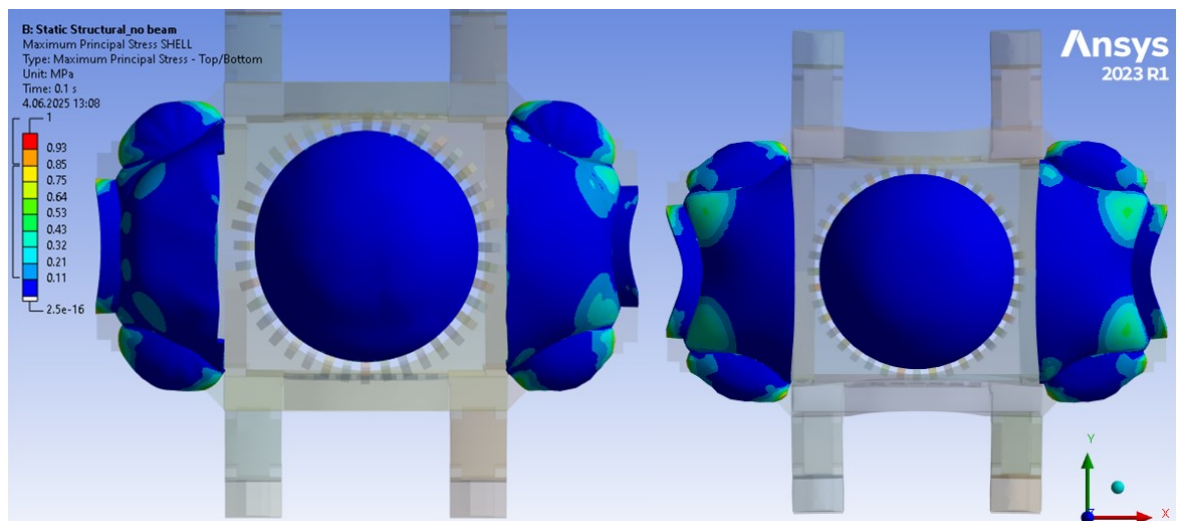


Figure 5.8. Maximum principal stresses on the shell elements under self-weight (0-1 MPa): deformed model (left), undeformed model (right).

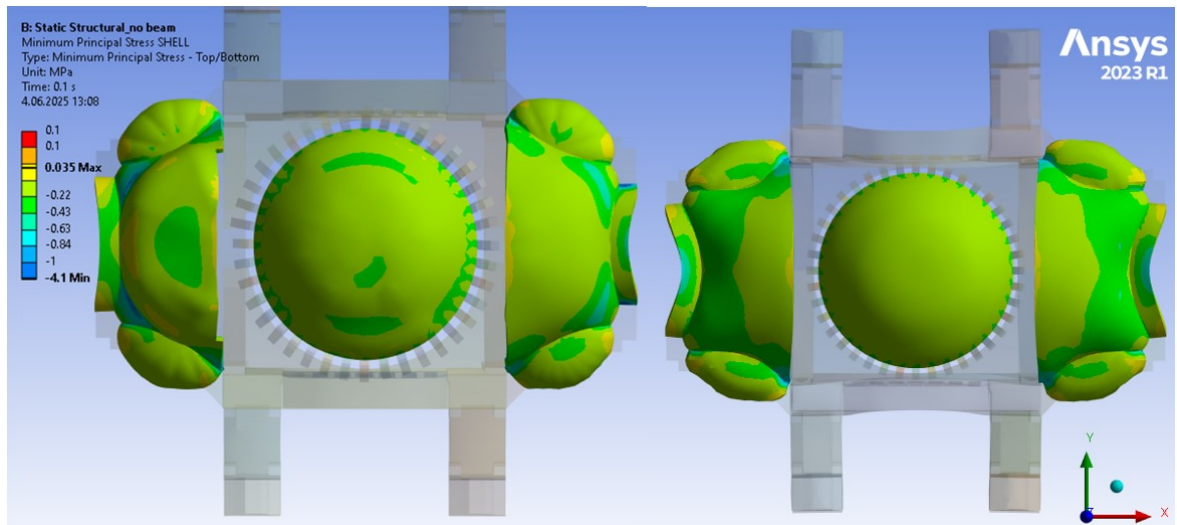


Figure 5.9. Minimum principal stresses on the shell elements under self-weight (0-1 MPa): deformed model (left), undeformed model (right).

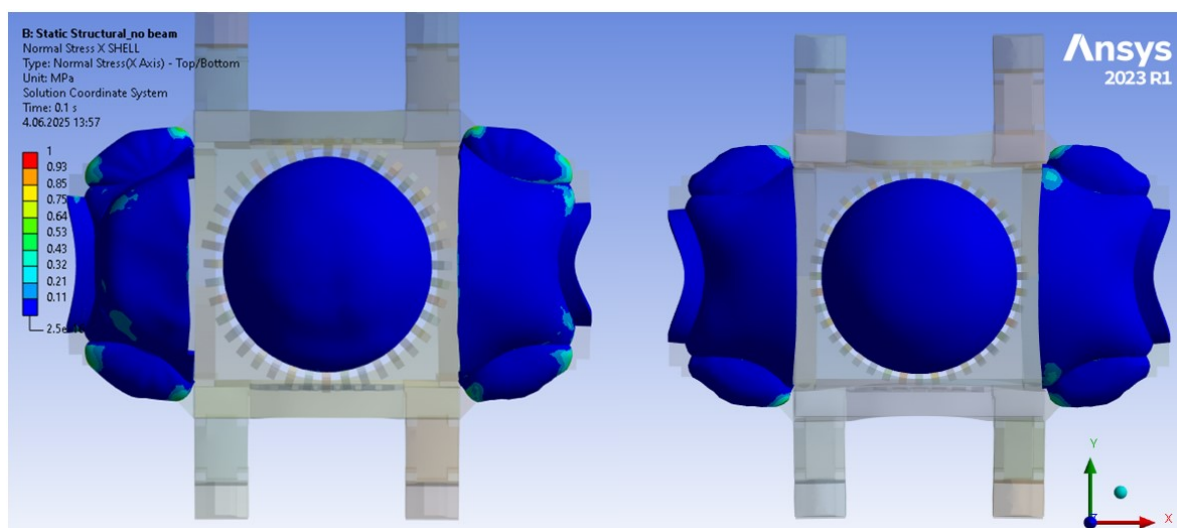


Figure 5.10. X direction normal (hoop) stresses on the shell elements under self-weight (0-1 MPa): deformed model (left), undeformed model (right).

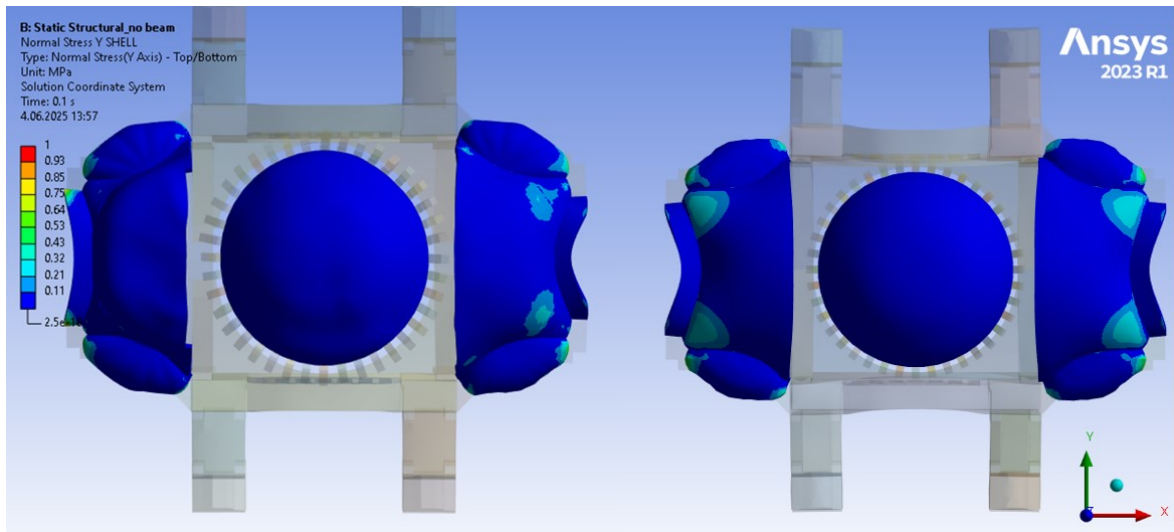


Figure 5.11. Y direction normal (radial) stresses on the shell elements under self-weight (0-1 MPa): deformed model (left), undeformed model (right).

Table 5.5. Peak principal stresses (MPa) on shell elements under self weight for deformed and undeformed models.

Location	DEFORMED		UNDEFORMED	
	MAX	MIN	MAX	MIN
Main Dome Top	0.00	-0.26	0.00	-0.10
Main Dome Springings	0.04	-0.30	0.02	-0.24
EAST SEMIDOME	0.40	-1.00	0.43	-0.94
WEST SEMIDOME	0.21	-0.90	0.43	-0.90
EAST EXEDRAE	0.57	-0.47	0.65	-0.51
WEST EXEDRAE	1.00	-1.21	0.65	-0.52

As indicated by Figure 5.7-5.11 and Table 5.5, the main dome it is mainly under compression. The semidomes are also mostly in compression except for some regions. The highest compressions are observed near the semidome - exedra dome connections and the semidome -barrel vault connections. Regions under tension are the semidomes and adjacent elements. In the deformed model, the maximum principal stresses reach

up to 1MPa, whereas in the undeformed model the maximum is about 0.65MPa. Besides, there is a difference between the size and extent of the areas under tension at the west semidome of the deformed and undeformed models. This is attributed to their differing geometries realized as a result of reconstructions/restorations following earthquakes. Areas under compressive stresses are near the connections of the semidomes with adjacent elements. The minimum principal stresses reach up to about -1.2MPa at the west exedra of the deformed model and to around -0.90MPa at both semidomes near the exedrae of the undeformed model. Hence it can be said that, in the undeformed model, east and west semidomes are stressed at similar levels which are however higher than the levels observed at the exedra domes. Similar to the areas under tension, there is also a difference in the size of areas under compression observed mostly at the west semidomes and the main domes of the deformed and undeformed models.

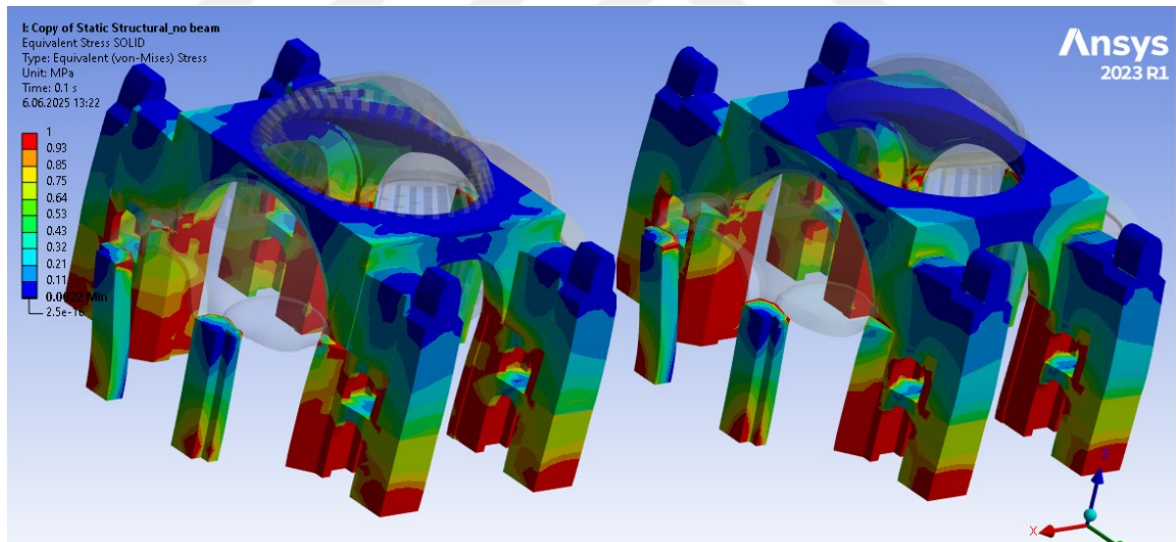


Figure 5.12. Von Misses stresses on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right), looking from northeast.

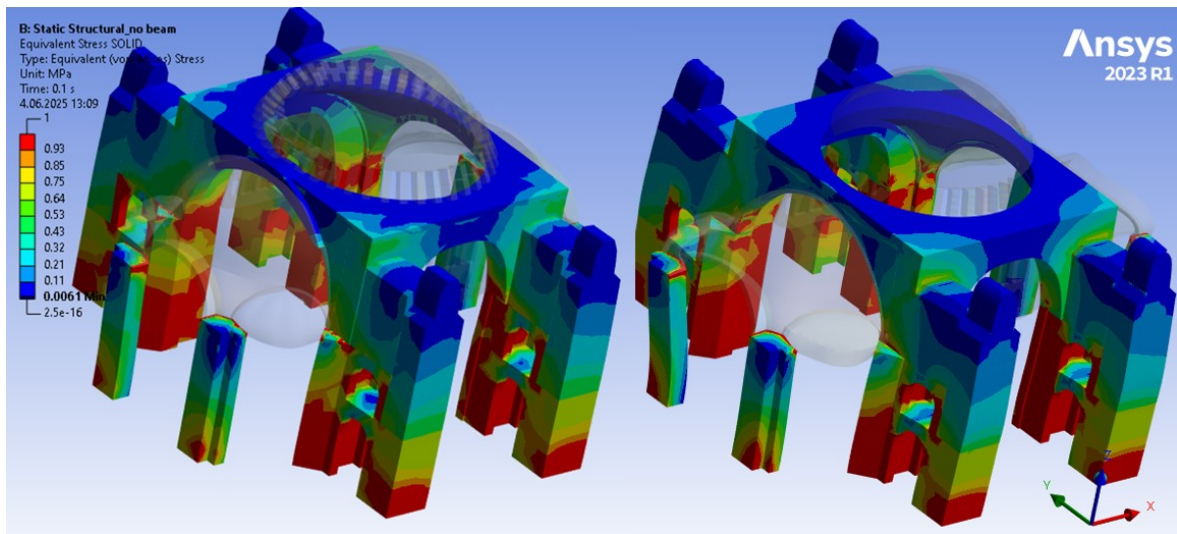


Figure 5.13. Von Mises stresses on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right), looking from southwest.

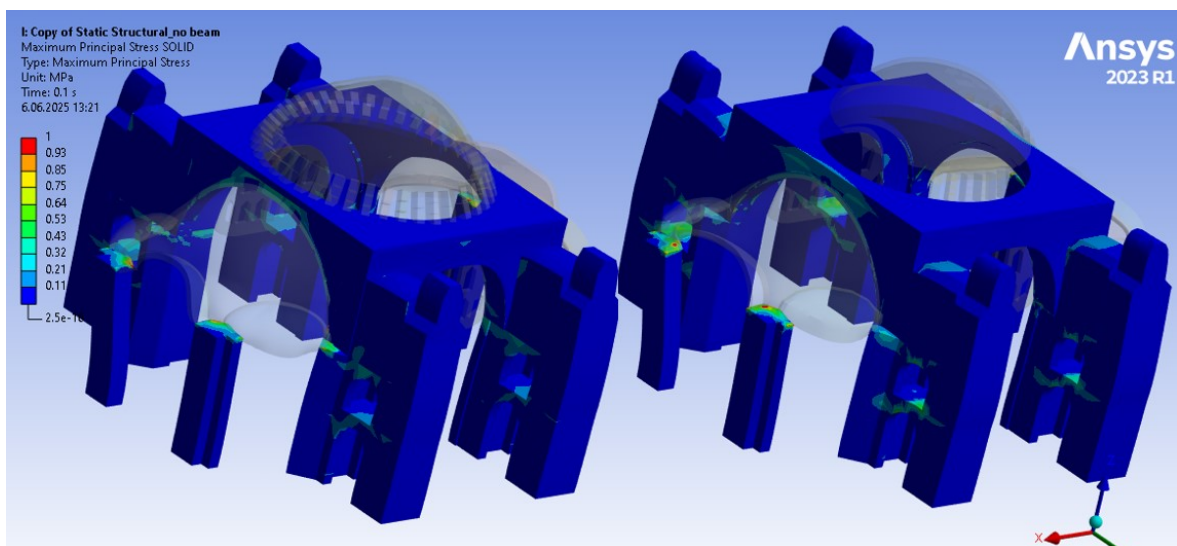


Figure 5.14. Maximum principal stresses on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right), looking from northeast.

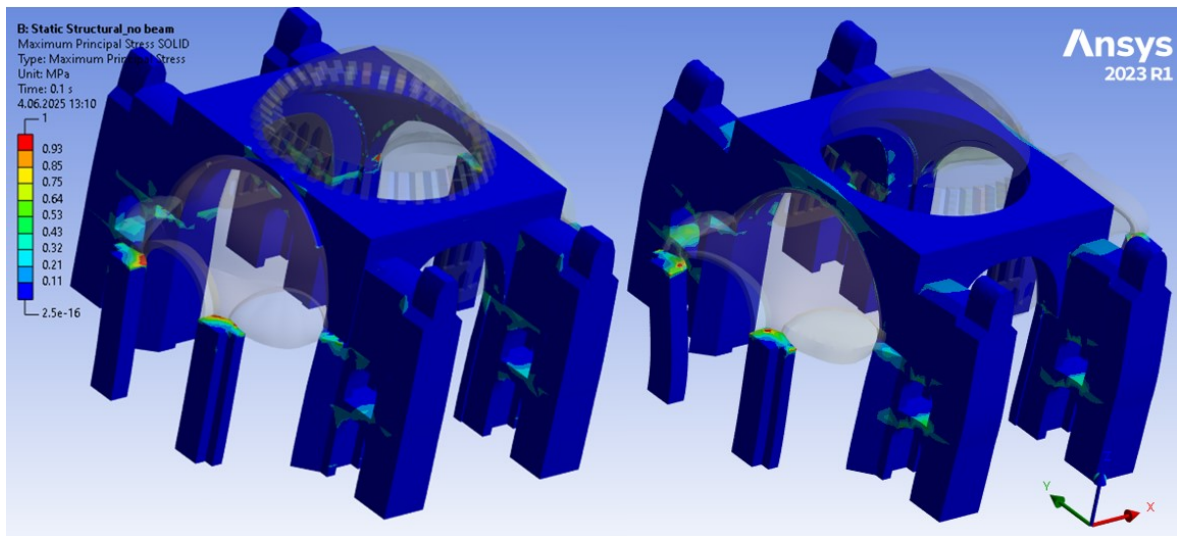


Figure 5.15. Max principal stresses on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right) looking from southwest.

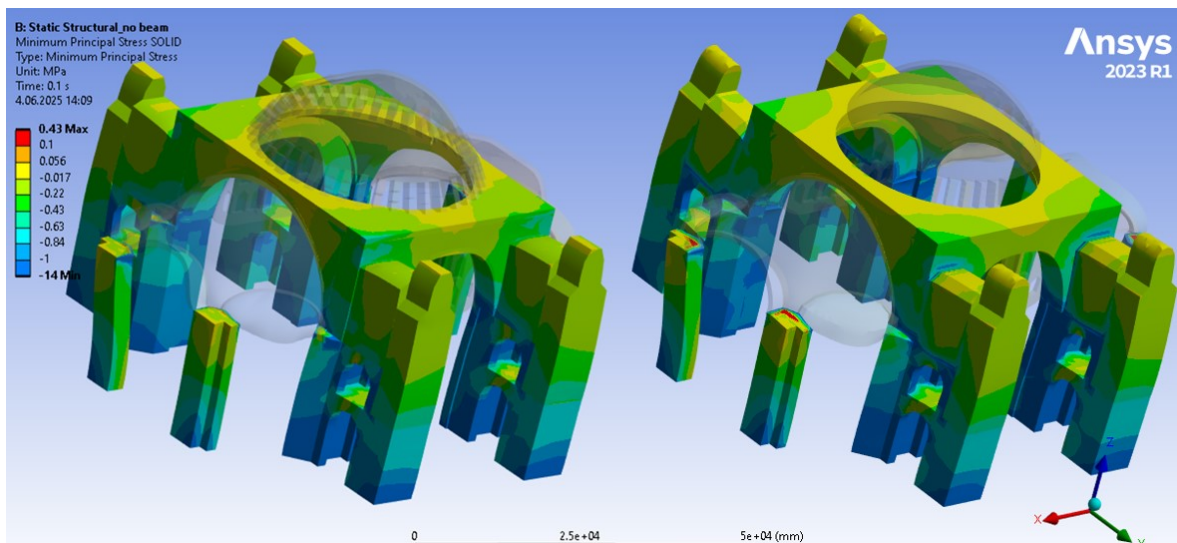


Figure 5.16. Min principal stresses on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right) looking from northeast.

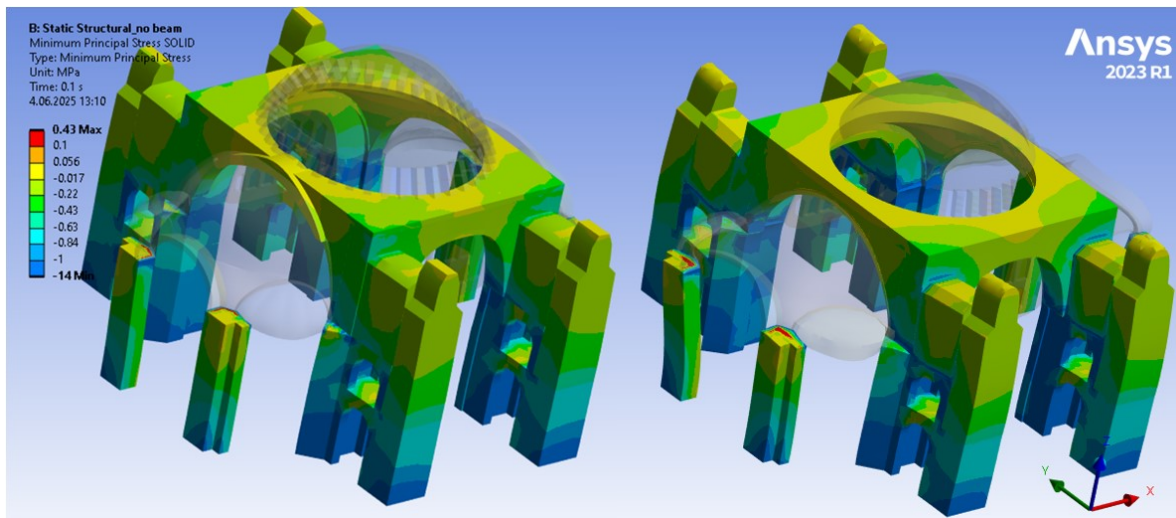


Figure 5.17. Min principal stresses on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right) looking from southwest.

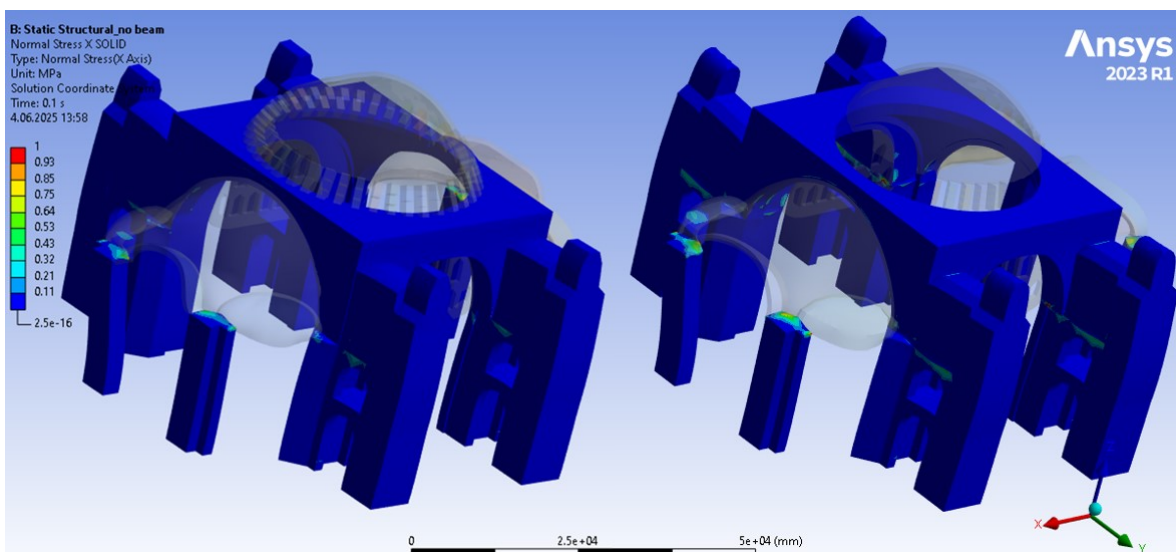


Figure 5.18. X direction normal stresses (S_{xx}) on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right) looking from northeast.

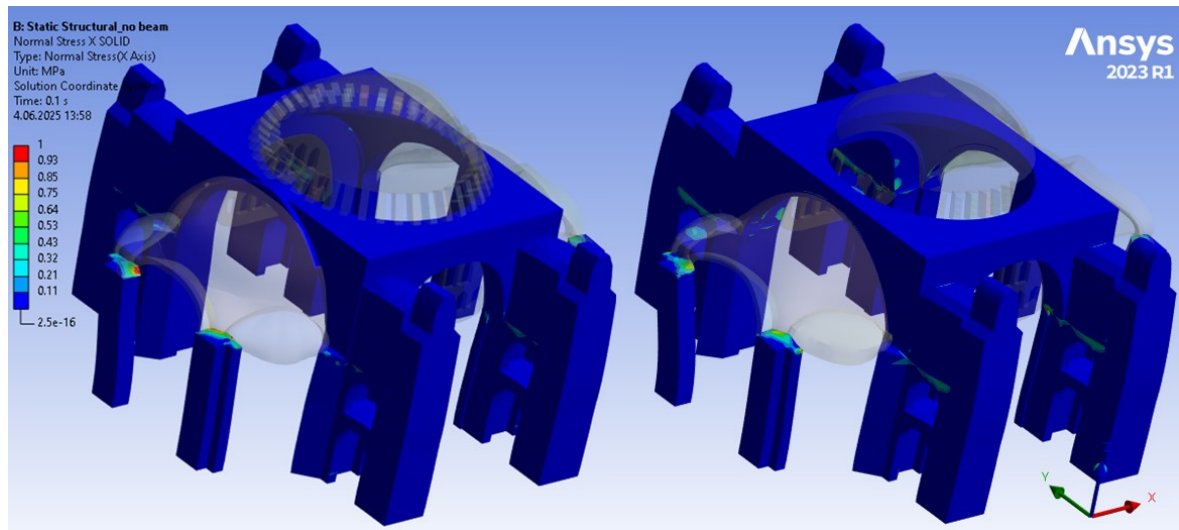


Figure 5.19. X direction normal stresses (S_{xx}) on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right) looking from southwest.

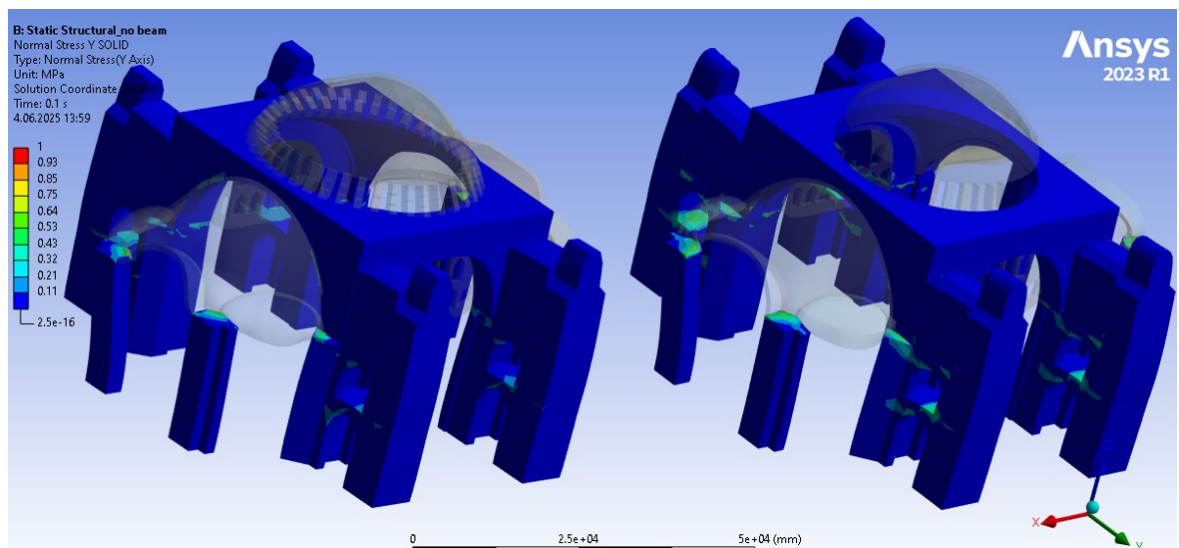


Figure 5.20. Y direction normal stresses (S_{yy}) on the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right) looking from northeast.

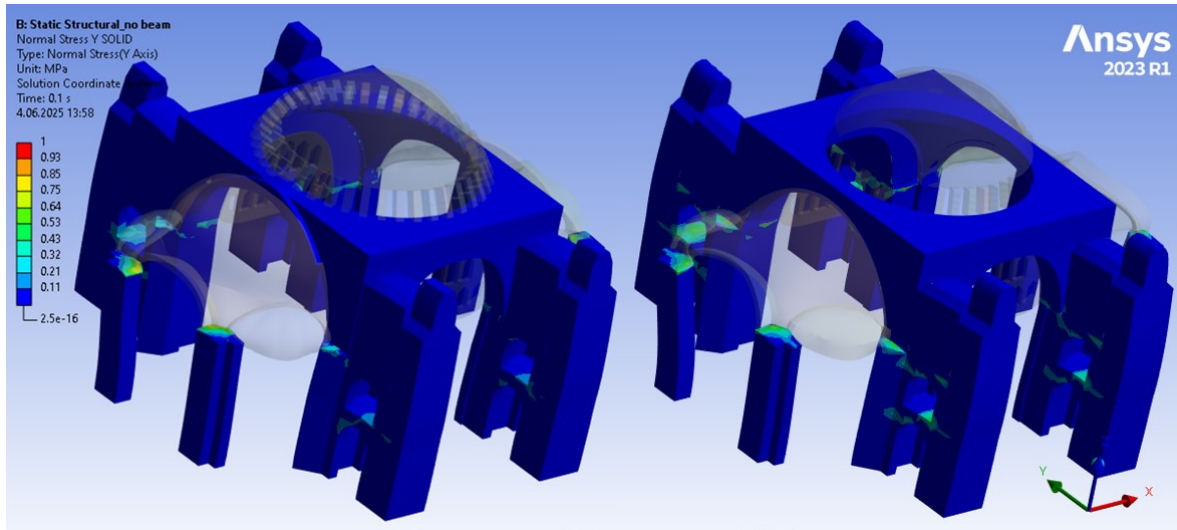


Figure 5.21. Y direction normal stresses (S_{yy}) of the solid elements under self-weight (0-1 MPa), deformed model (left), undeformed model (right) looking from southwest.

Table 5.6. Peak principal stresses (MPa) on solid elements under self weight for deformed and undeformed models.

Location	DEFORMED		UNDEFORMED	
	MAX	MIN	MAX	MIN
East Arch Springing	0.06	-0.50	0.03	-0.61
East Arch Crown	0.10	-0.04	0.07	-0.03
West Arch Springing	0.03	-0.45	0.04	-0.85
West Arch Crown	0.08	-0.02	0.07	-0.02
North Arch Springing	0.14	-0.45	0.09	-0.80
North Arch Crown	0.04	-0.02	0.01	-0.02
South Arch Springing	0.09	-0.50	0.06	-0.74
South Arch Crown	0.04	-0.02	0.02	-0.02
Buttress Arches	0.23	-0.80	0.50	-1.50

The stress concentrations at the solid elements are observed mostly at the main arches and the connecting elements between the main piers and the buttress piers, which are called “buttress arches” in the following sentences. The minimum principal

stresses are high at the springings of the arches as shown in Table 5.6 and Figure 5.22.

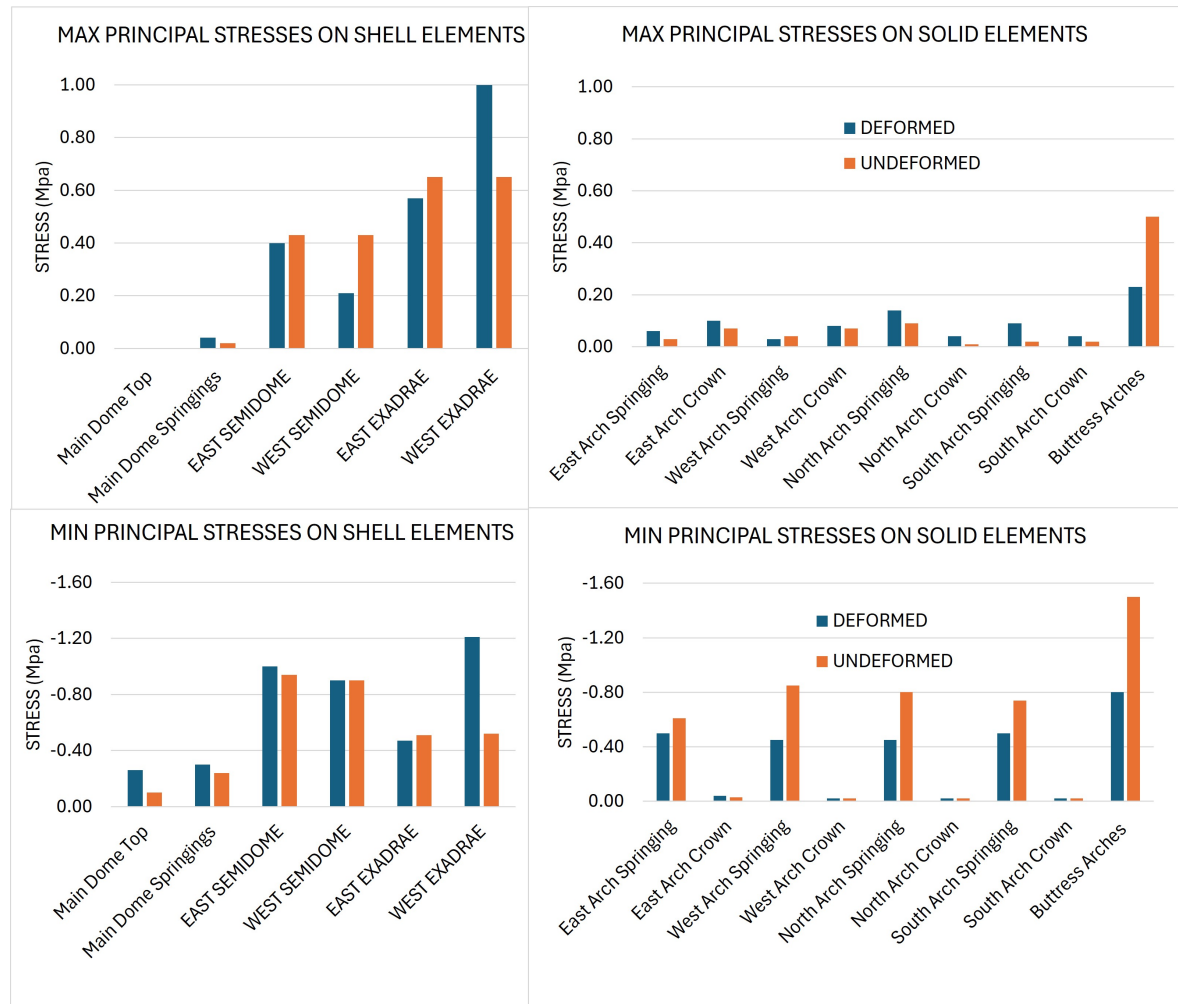


Figure 5.22. Comparison of the peak principal stresses on shell and solid elements of deformed and undeformed models under self weight.

Figure 5.22 illustrates the minimum and maximum principal stresses on shell and solid elements as bar charts. When the shell elements are considered, the deformed shape of the structure, especially the reconstructed shape of the west semidome, causes increased tensile and compressive stress levels at the west exedra, while it decreases the tensile stress on the west semidome. When it comes to the solid elements, for the main arches, downwards deformation at the crowns and the outwards deformation at

the arch springings of the structure's current deformed shape cause a bit relaxation on the arch springings in terms of compressive stresses. However, it slightly increases the tension at the crowns. Besides, deformed shape reduces the stress levels on the buttress arches as well.

When the stress distribution patterns and the peak stresses are compared between the deformed and undeformed models, the main differences are in the shell elements whereas on solid elements they differ slightly under self-weight.

Additionally, directional deformations are observed in both deformed and undeformed models in X, Y and Z directions under self weight as can be seen in Table 5.7. The largest displacements occur along the Z axis. They are slightly higher on the undeformed model, reaching up to 10mm. On the other hand, displacements in X and Y directions are less than 1mm. The deformed shape of both models under self weight are given in Figure 5.23 and 5.24.

Table 5.7. Displacements (mm) in X, Y, Z directions for deformed and undeformed (undef.) models.

Location	East-West (X axis)	East-West (X axis)	North-South (Y axis)	North-South (Y axis)	Vertical (Z axis)	Vertical (Z axis)
	Deformed	Undef.	Deformed	Undef.	Deformed	Undef.
GAL1	-0.49	-0.73	-0.76	-0.93	-4	-4
GAL2	-0.52	-0.74	0.82	0.94	-4	-4
GAL3	0.54	0.77	0.83	0.99	-4	-4
GAL4	0.46	0.76	-0.92	-0.94	-4	-4
KUB1	-0.06	0.01	-0.01	0.01	-7	-8
KUB2	-0.05	0.01	-0.01	-0.05	-7	-8
KUB3	-0.03	0.01	0.01	0.05	-7	-8
KUB4	-0.04	0.01	-0.03	-0.05	-7	-8
Main Dome	-0.03	-0.04	0.06	0.008	-9	-10

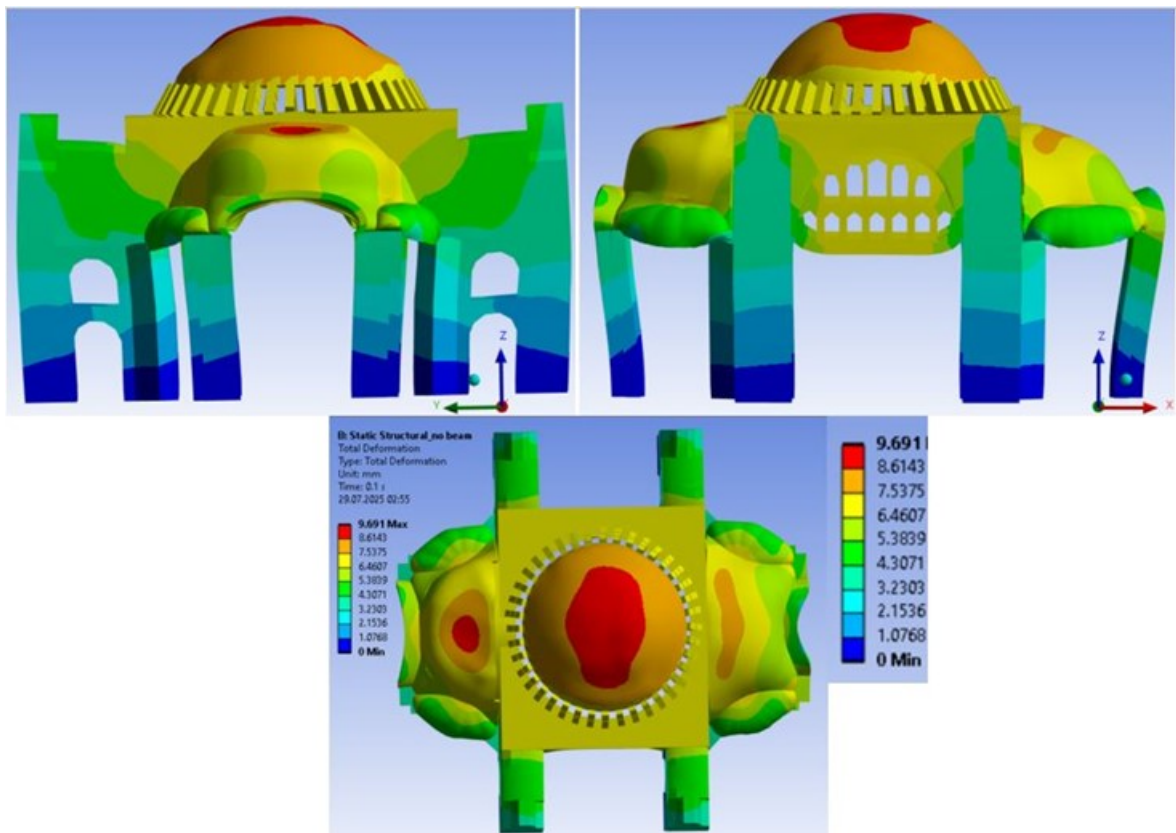


Figure 5.23. The view of the deformed shape of the deformed model under self weight from west side (upper left), east side (upper right) and top (bottom).

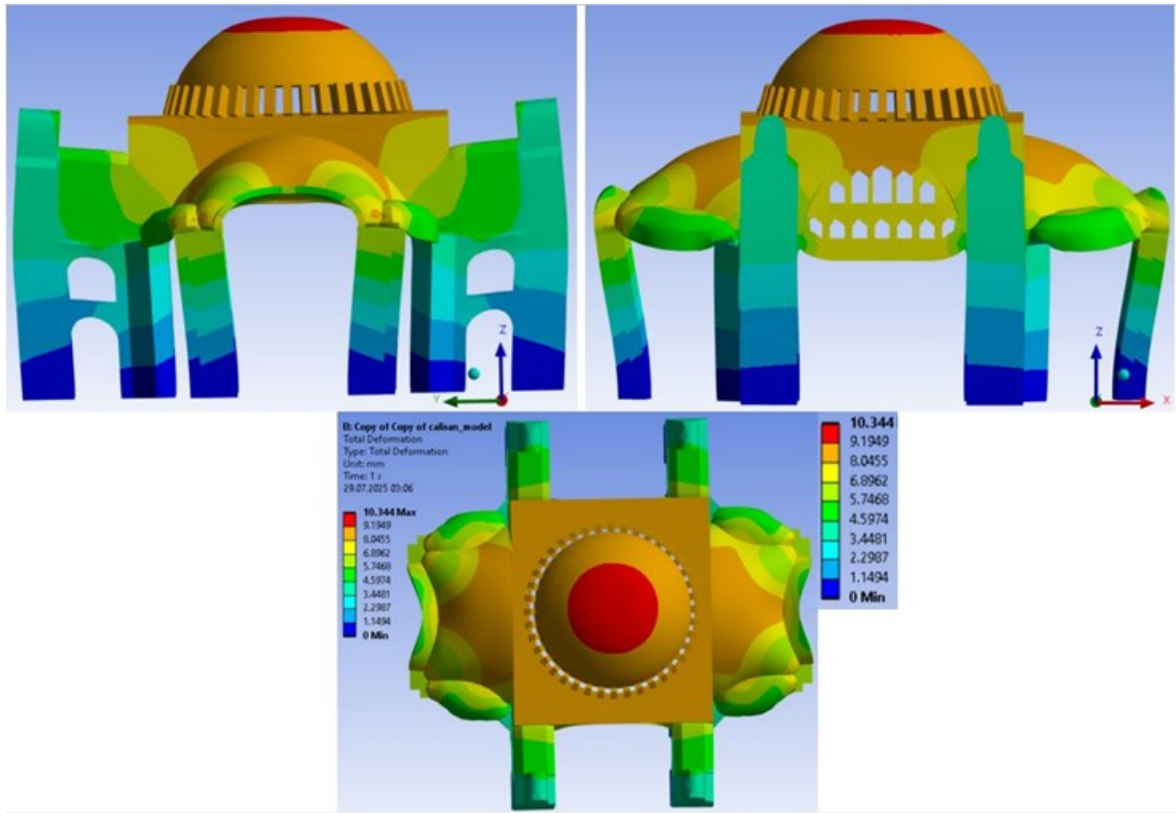


Figure 5.24. The view of the deformed shape of the undeformed model under self weight from west side (upper left), east side (upper right) and top (bottom).

5.1.3. Linear Dynamic Analysis

Static structural analyses are followed by linear dynamic analyses. Two types of linear dynamic analysis are performed: response spectrum analysis and time history analysis.

5.1.3.1. Response Spectrum Analysis. For the response analysis the Horizontal Elastic Design Acceleration Spectrum as per the Turkish Seismic Code 2018 (TSC-2018) [49] is used. The spectrum is shown in Figure 5.25. It is described by the following parameters: SDS - the design spectral acceleration coefficient for the short period region; SD1 - design spectral acceleration coefficient for the period of 1 second.; TA-The lower

bound period for the constant acceleration plateau of the spectrum; TB-The starting point of the flat region of the design spectrum; TL-The period beyond which spectral acceleration decreases inversely with period.

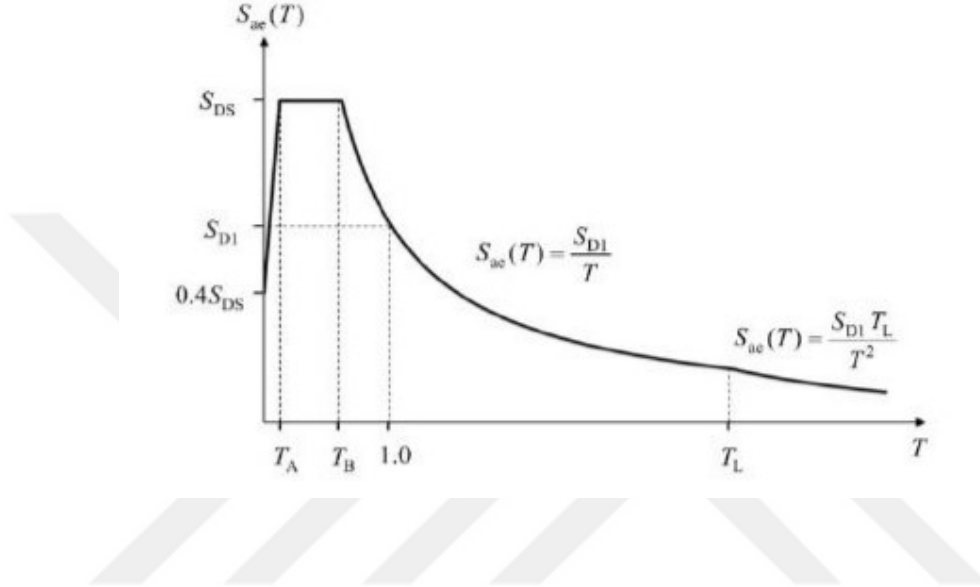


Figure 5.25. TSC-2018 horizontal elastic design acceleration spectrum [49].

TCS-2018 utilizes the seismic hazard map of Turkey to derive the site-specific spectral accelerations based on earthquake return period and site class. The site class of Hagia Sophia is specified as ZC (dense soil) [50]. Spectral acceleration coefficients, together with the soil specification, are transformed into design parameters complying with the formulations in TSC-2018. The obtained horizontal elastic design spectrum for Hagia Sophia for the return period of 475 years can be seen in Figure 5.26.

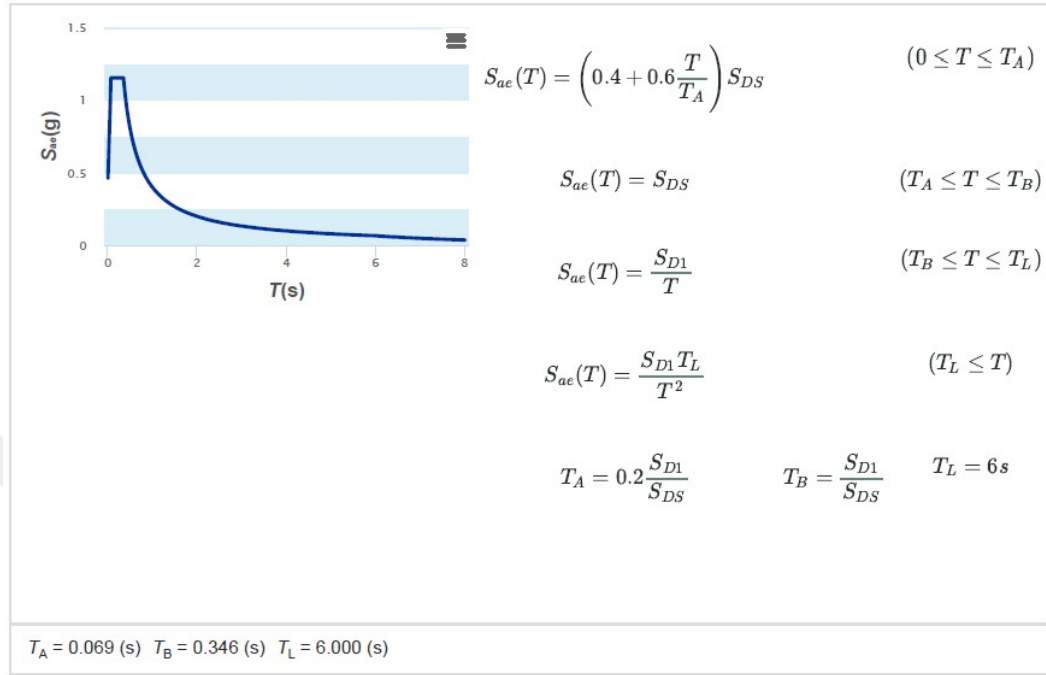


Figure 5.26. Horizontal elastic design acceleration spectrum of Hagia Sophia for the return period of 475 years.

Response Spectrum Analysis is performed using the modal analysis results for the first fifty modes for both deformed and undeformed models. The stress distributions under self-weight and the spectral forces are investigated in terms of normal stresses in X and Y directions and illustrated within a range of 0-1 MPa for shell and solid elements separately in Figure 5.27 - 5.32.

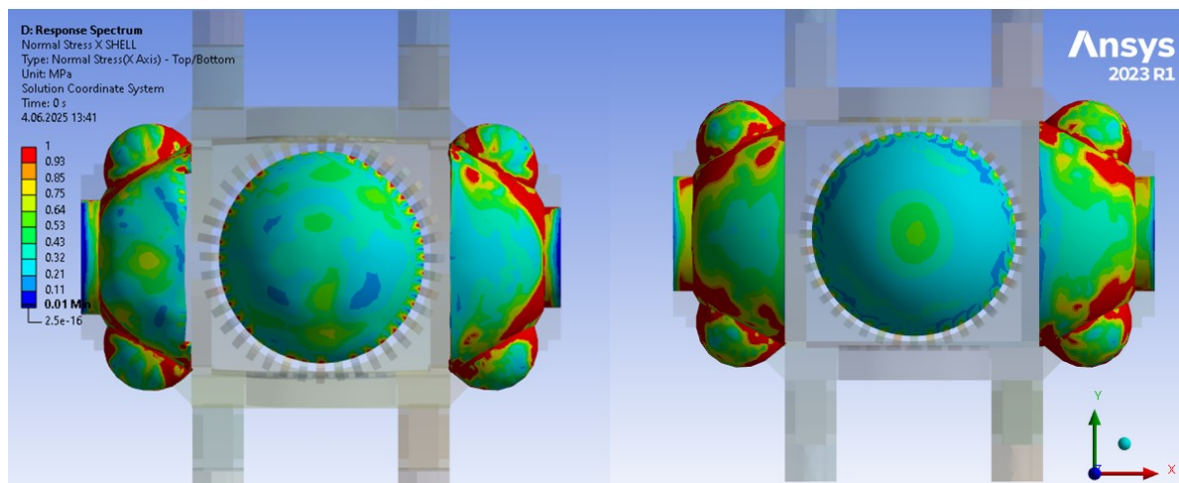


Figure 5.27. Distribution of hoop stresses on shell elements under self-weight and spectral forces: deformed model (left); undeformed model (right).

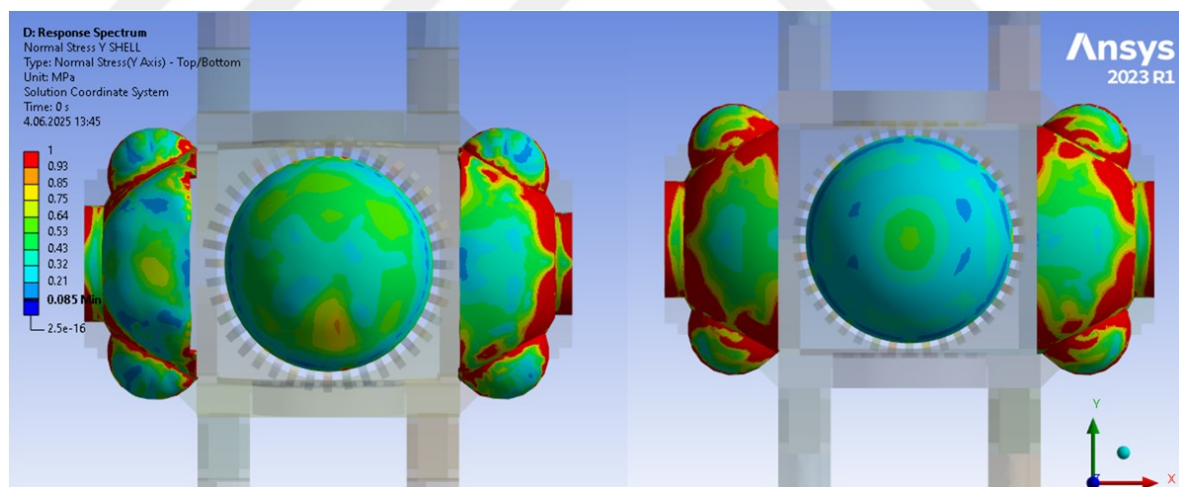


Figure 5.28. Distribution of radial stresses on shell elements under self-weight and spectral forces: deformed model (left), undeformed model (right).

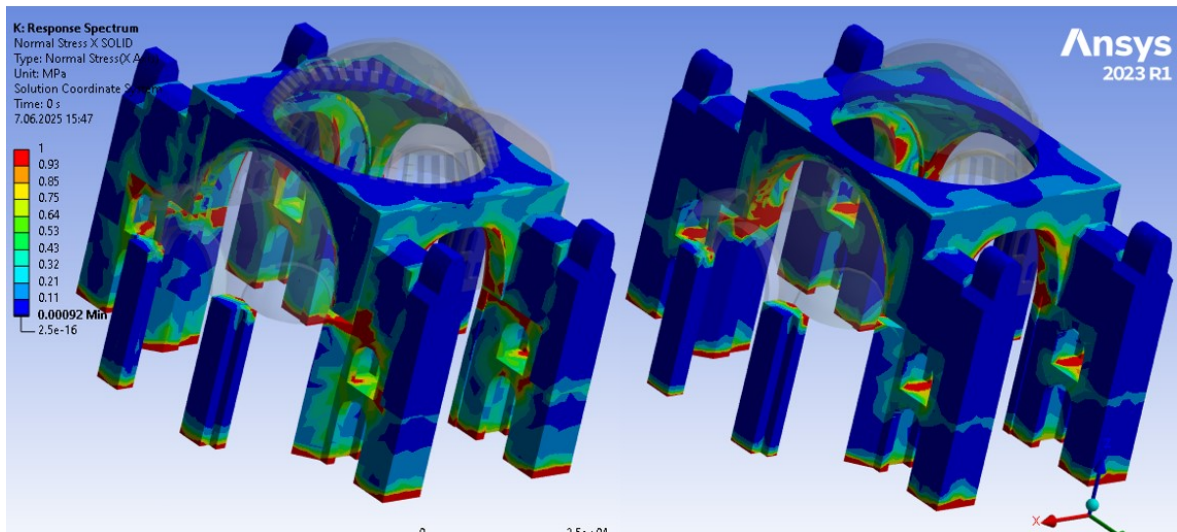


Figure 5.29. X direction normal stress (S_{xx}) distribution on the solid elements under self-weight and spectral forces: deformed model (left), undeformed model (right), looking from northeast.

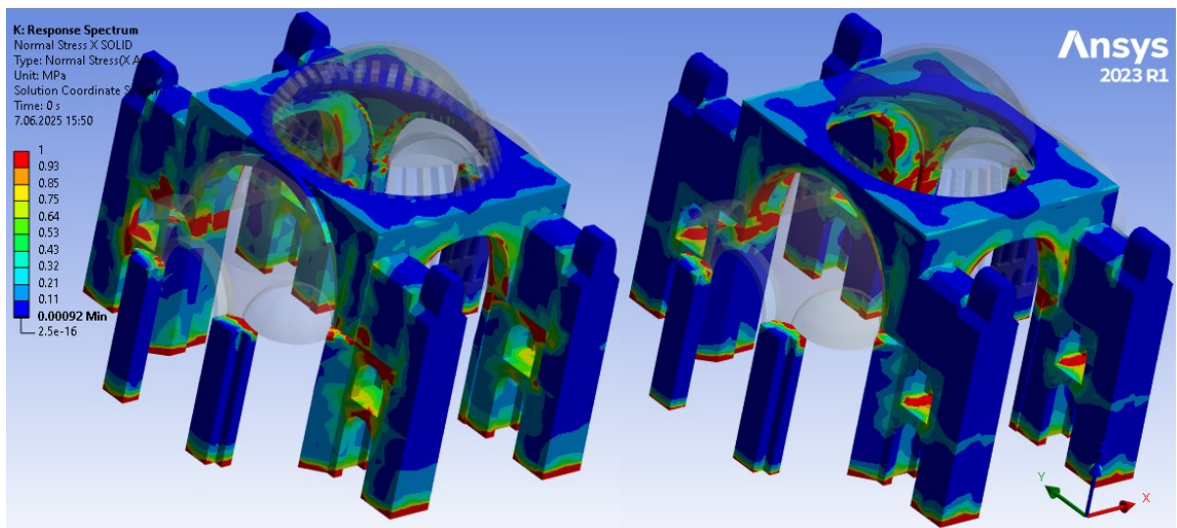


Figure 5.30. X direction normal stress (S_{xx}) distribution on the solid elements under self-weight and spectral forces: deformed model (left), undeformed model (right), looking from southwest.

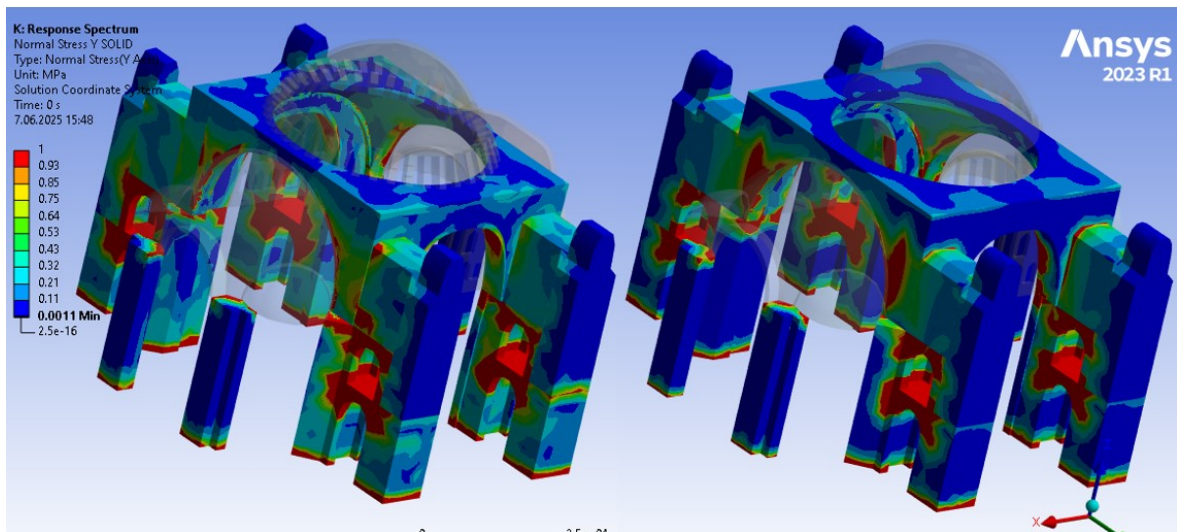


Figure 5.31. Y direction normal stress (S_{yy}) distribution on the solid elements under self-weight and spectral forces: deformed model (left), undeformed model (right), looking from northeast.

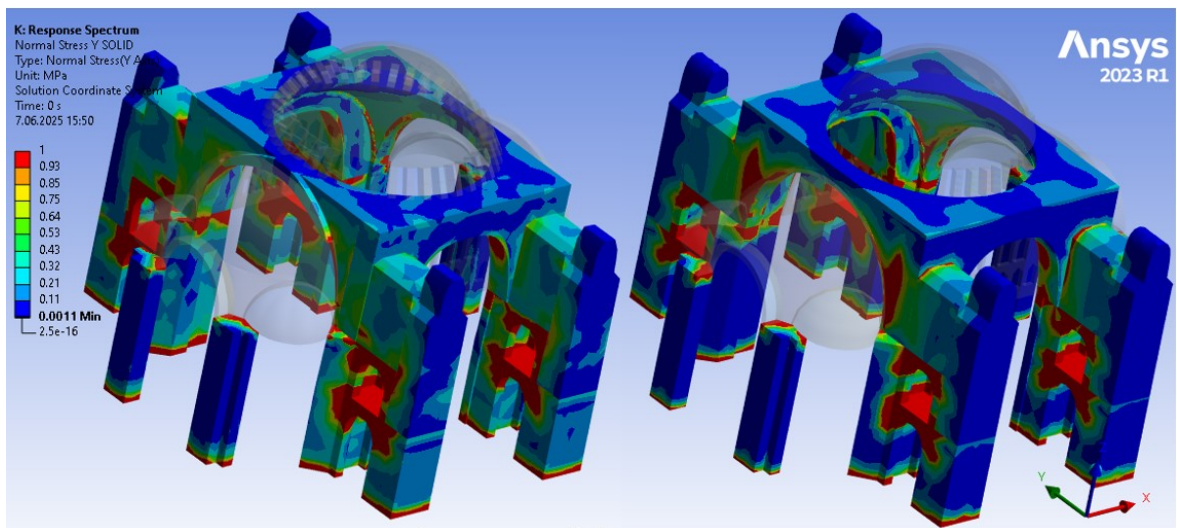


Figure 5.32. Y direction normal stress (S_{yy}) distribution on the solid elements under self-weight and spectral forces: deformed model (left), undeformed model (right), looking from southwest.

Table 5.8. Peak normal stresses (MPa) on shell elements in hoop (X) and radial (Y) directions for deformed and undeformed models under self weight and spectral forces.

Location	DEFORMED		UNDEFORMED	
	Hoop (X)	Radial (Y)	Hoop (X)	Radial (Y)
Main Dome Top	0.55	0.52	0.45	0.47
Main Dome Springings	0.45	0.41	0.28	0.40
EAST SEMIDOME	1.98	2.10	2.55	2.85
WEST SEMIDOME	2.39	2.20	2.80	3.10
EAST EXEDRAE	1.90	2.79	2.78	2.72
WEST EXEDRAE	2.95	3.06	2.51	2.87

Table 5.9. Peak normal stresses (MPa) on solid elements in global directions for deformed and undeformed models under self weight and spectral forces.

Location	DEFORMED		UNDEFORMED	
	Sxx	Syy	Sxx	Syy
East Arch Springing	0.78	0.71	0.99	0.70
East Arch Crown	0.11	0.19	0.50	0.22
West Arch Springing	0.63	0.95	0.88	0.75
West Arch Crown	0.12	0.20	0.16	0.28
North Arch Springing	0.26	1.05	0.92	1.52
North Arch Crown	0.17	0.20	0.22	0.23
South Arch Springing	0.33	0.77	0.81	1.40
South Arch Crown	0.22	0.23	0.20	0.43
Buttress Arches	0.81	3.25	0.97	3.80

Under the self-weight and spectral forces, large portions of the structure are under stresses exceeding 1 MPa. In the solid elements, springings of the main arches and the buttress arches exhibit higher stresses. In the shell elements, all the connections with the semidomes seem to be vulnerable, including the main arch-semidome connections as can be seen in Figure 5.26 - 5.31. In the undeformed model, the peak normal

stress (hoop stress) at the east and west semidomes and the exedrae are almost at the same level ranging from 2.5 to 2.8 Mpa. When the deformed model is considered, shell elements on the west side have higher levels of peak normal stress (hoop stress) however the size of the high stress distribution region seems to be smaller. The buttress arches have the highest stress levels within the solid elements exceeding 3MPa for both models.

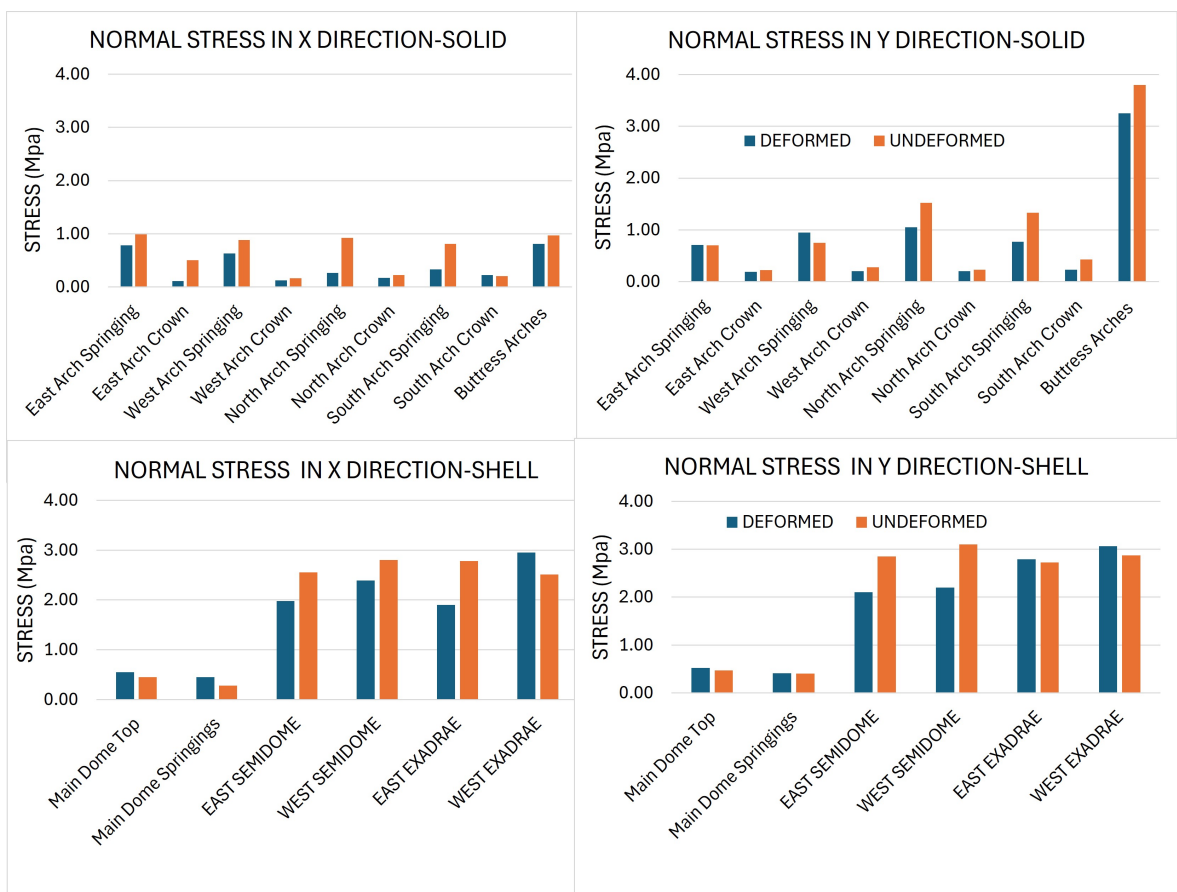


Figure 5.33. Comparison of the peak normal stresses in X and Y directions on shell and solid elements of deformed and undeformed models under self weight and spectral forces.

A bar chart which compares the maximum stresses on structural elements of deformed and undeformed models is presented in Figure 5.31. It is evident from Figure 5.31, that the peak normal stresses are higher on the shell elements than the solid

elements. Furthermore the stresses on undeformed model are higher than those on the deformed model. This is especially significant for the solid elements at the springings of the north and south main arches. For the shell elements it seems that the reconstructed shape of the west semidome reduces the stresses in that region whereas it leads to increased stresses in the west exedrae.

Table 5.10. Displacements (mm) in X, Y, Z directions for deformed and undeformed (Undef.) models under self weight and spectral forces.

Location	East–West (X axis)	East–West (X axis)	North–South (Y axis)	North–South (Y axis)	Vertical (Z axis)	Vertical (Z axis)
	Deformed	Undef.	Deformed	Undef.	Deformed	Undef.
GAL1	44	50	37	44	8	11
GAL2	47	50	36	44	9	11
GAL3	47	51	37	43	10	11
GAL4	43	51	37	43	9	11
KUB1	66	76	53	62	4	7
KUB2	68	76	53	64	5	7
KUB3	71	76	53	62	5	7
KUB4	69	76	54	61	4	6
Main Dome	74	80	56	62	7	9

The displacements of the main dome, the main arch crowns and the tops of the main piers in X, Y and Z directions are estimated under self-weight and spectral forces for both deformed and undeformed models. The displacements in X axis are higher than the ones in Y axis. The maximum displacements take place at the top of the main dome which is about 74 mm in the deformed model and 80 mm in the undeformed model in X direction; whereas they are 56 and 62 mm in the Y direction. In Z direction, the displacements reach up to 10-11 mm. Comparison of the deformations in three directions of deformed and undeformed models under self-weight and spectral forces as bar charts can be seen in Figure 5.34. It is important to note that, the displacement values given here are with respect to the ground.

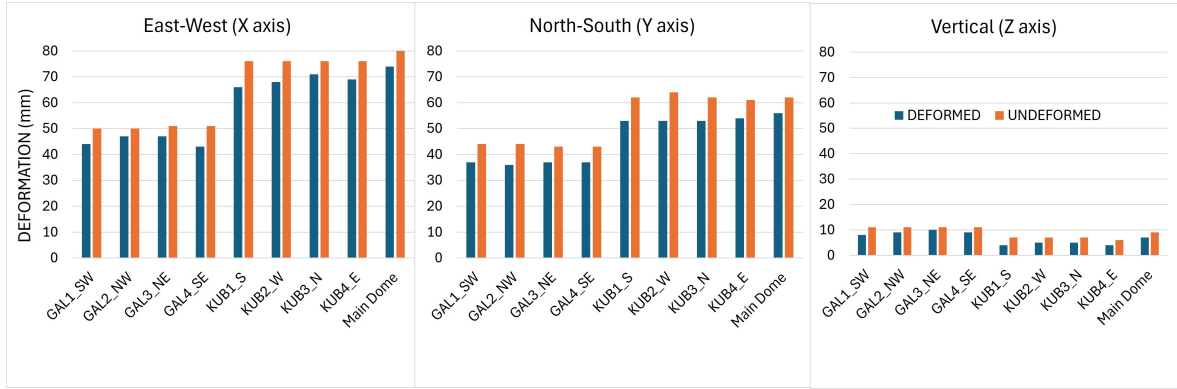


Figure 5.34. Comparison of the deformations in three directions of deformed and undeformed models under self weight and spectral forces.

5.1.3.2. Linear Time History Analysis. Linear time history analysis is carried out to evaluate the response of a structure to earthquake ground motion applied as a time-series. The dynamic response of the structure is computed at each time step over a specified duration. The procedure followed within the scope of this thesis includes evaluating the target spectra, selecting the appropriate ground motion records, matching them to the target spectra and running the analysis.

The target spectrum is defined in the previous chapter. For the selection of earthquake ground motion records the PEER database [51] is utilized. The target spectrum is given as an input to the database, the selection criteria are defined and the interface displays response spectra of selected unscaled records as shown in Figure 5.35. These criteria range are based on the magnitude of the scenario earthquake, distance of Hagia Sophia to the fault rupture and V_{s30} value of the soil where Hagia Sophia is located [50]. Then the records are scaled within a factor range of 0.5-2 as suggested by PEER Design Ground Motion Library User's Manual [52] and between the periods of 0.05s and 1s to cover all modal frequencies of the structure. When scaling, the data are also sorted regarding the minimum mean square error of the record by the database [51]. The scaling criteria and the scaled data are illustrated in Figure 5.36.

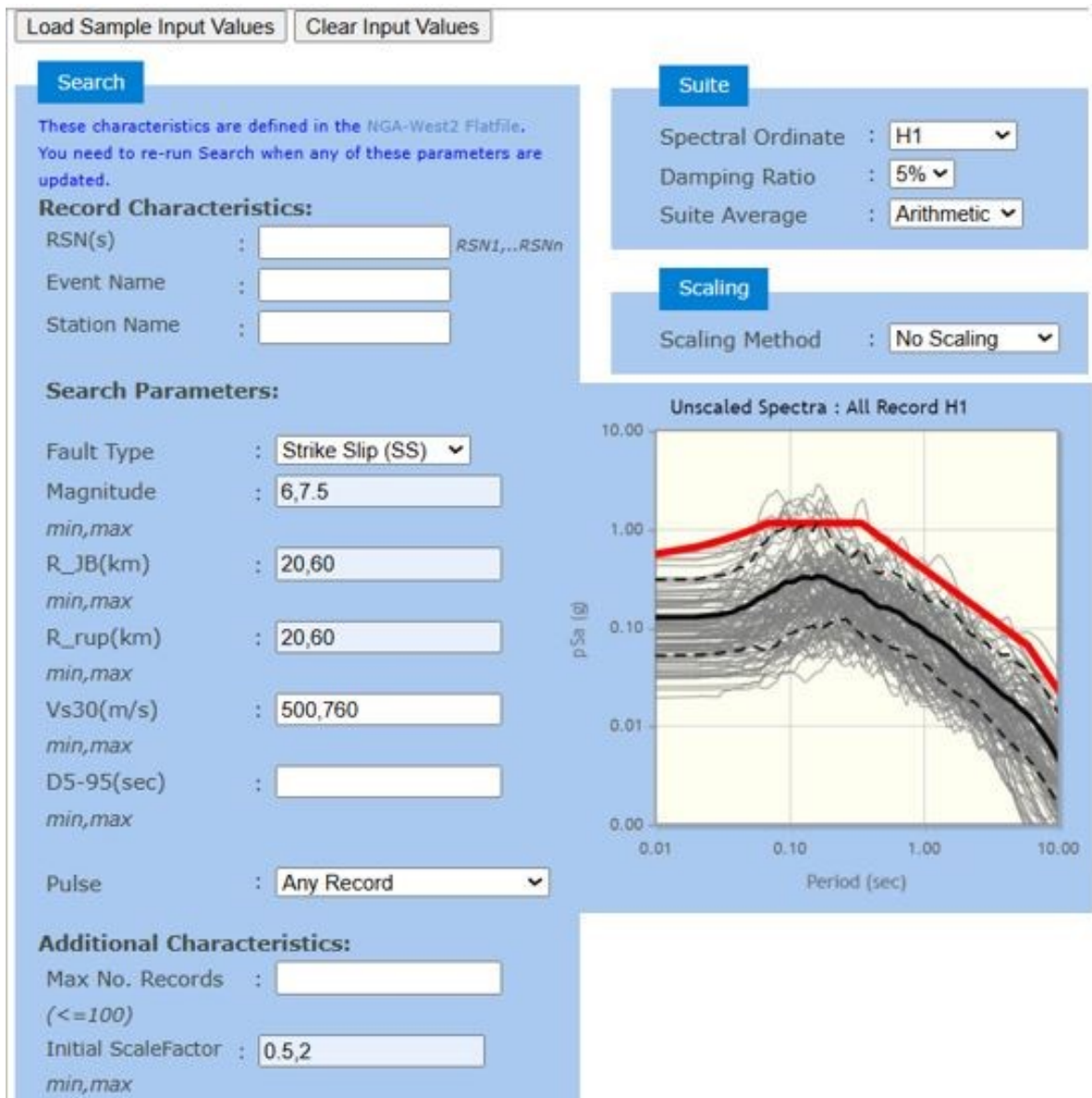


Figure 5.35. The selection criteria and the unscaled data on PEER Database.

As a common practice, after the scaling process it comes to matching the scaled data with the target spectrum. According to Finn and his colleagues, applying the spectral matching after linear scaling reduces the adjustment demand on spectral matching [53]. This procedure is carried on SeismoMatch software, which is a special program developed for the spectrum matching of the ground motion records [54]. The matched spectra can be seen in Figure 5.37. The records which have the best matching conver-

gence with the target spectrum are given in Table 5.11. Those records are chosen to be used in the analysis. The acceleration data of these records are also given in Figure 5.38.



Figure 5.36. The scaling criteria and the scaled data on PEER Database.

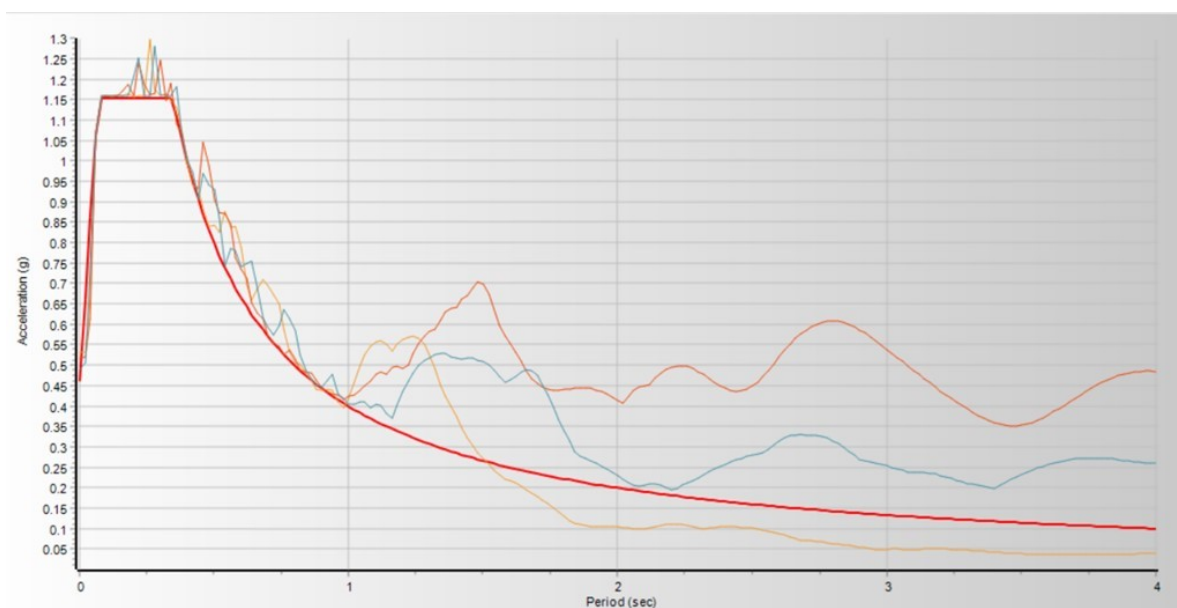


Figure 5.37. The matched spectra on Seismomatch Software.

Table 5.11. The selected records used in the analysis.

Record	Earthquake Name	Magnitude	Rjb (km)	Rrup (km)	Vs30 (m/s)
1102	Kobe_Japan	6.90	49.91	49.91	609.00
1164	Kocaeli_Turkey	7.51	49.66	51.95	595.20
1795	Hector Mine	7.13	50.42	50.42	686.12

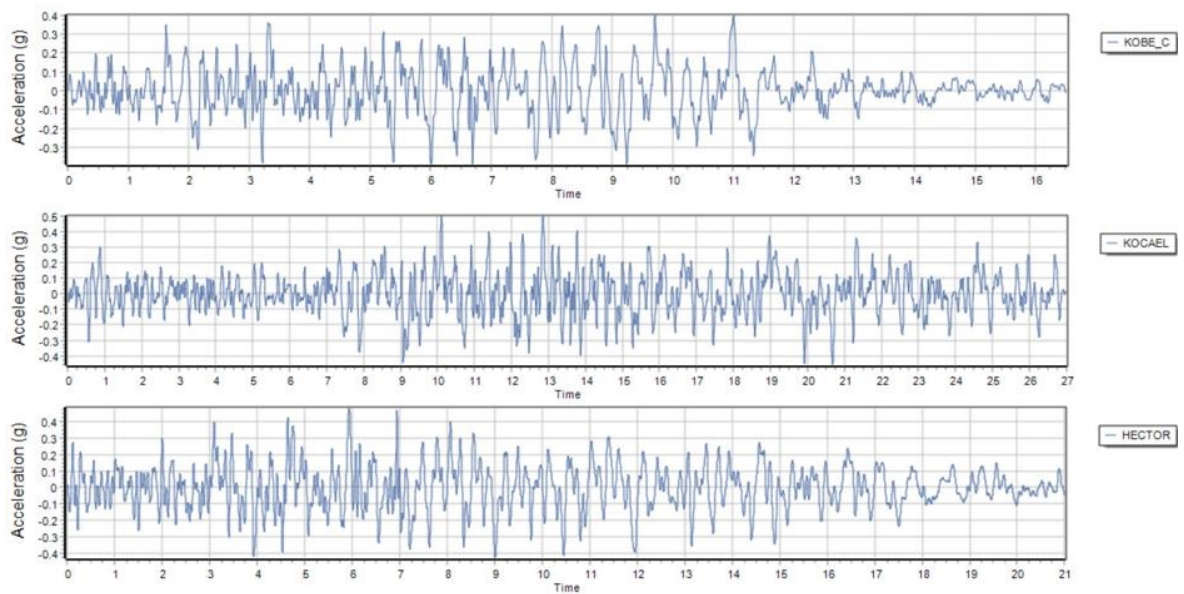


Figure 5.38. Acceleration data of the records used in time history analysis.

This study aims to reveal the nonlinear dynamic behavior of Hagia Sophia. The focus is more on the nonlinear time history analysis results carried out using both real records obtained on the structure and synthetic earthquake data simulated using by 3D physics based earthquake modelling approach. Therefore, the linear time history analysis part is considered as a prelude to nonlinear time history analysis. The findings associated with principal stresses and the deformations along three directions estimated using linear time history analyses are summarized for both deformed and undeformed models in Table 5.12 - 5.17.

Table 5.12. Peak principal stresses (MPa) on shell elements of the deformed model under selected ground motions.

Location	RSN1102 KOBE		RSN1795 HECTOR		RSN1164 KOCAELI	
	MAX	MIN	MAX	MIN	MAX	MIN
Main Dome Top	0.34	-0.31	0.40	-0.43	0.37	-0.41
Main Dome Springings	0.46	-0.40	0.62	-0.63	0.58	-0.56
EAST SEMIDOME	4.62	-4.91	4.87	-4.83	4.77	-4.90
WEST SEMIDOME	4.55	-4.23	4.83	-4.44	4.49	-4.31
EAST EXEDRAE	3.73	-3.35	4.51	-4.78	4.93	-4.92
WEST EXEDRAE	5.10	-5.52	5.70	-6.18	5.51	-5.94

Table 5.13. Peak principal stresses (MPa) on solid elements of the deformed model under selected ground motions.

Location	RSN1102 KOBE		RSN1795 HECTOR		RSN1164 KOCAELI	
	MAX	MIN	MAX	MIN	MAX	MIN
East Arch Springing	2.00	-1.80	2.40	-2.45	2.58	-2.64
East Arch Crown	0.21	-0.25	0.27	-0.26	0.30	-0.31
West Arch Springing	1.90	-2.34	2.73	-2.61	2.47	-2.52
West Arch Crown	0.27	-0.26	0.36	-0.36	0.42	-0.40
North Arch Springing	2.45	-2.60	2.74	-2.82	2.65	-2.78
North Arch Crown	0.60	-0.64	0.70	-0.77	0.73	-0.71
South Arch Springing	1.55	-1.37	1.64	-1.80	2.00	-2.14
South Arch Crown	0.70	-0.67	0.77	-0.68	0.70	-0.72
Buttress Arches	3.00	-3.60	3.80	-3.30	3.75	-4.00

Table 5.14. Max displacements (mm) in X, Y, Z directions on critical regions of deformed model under selected ground motions.

Location	RSN1102 KOBE			RSN1795 HECTOR			RSN1164 KOCAELI		
	X	Y	Z	X	Y	Z	X	Y	Z
GAL1	55	38	7.5	57	42	8.7	59	50	9.6
GAL2	58	36	12.3	60	40	11.4	60	47	11.6
GAL3	59	37	8.5	61	43	11.2	60	49	12.0
GAL4	53	39	11.0	55	44	12.5	56	50	10.6
KUB1	81	54	3.7	84	61	5.0	86	70	5.5
KUB2	84	53	4.0	87	59	4.5	89	69	4.8
KUB3	87	54	3.5	91	61	5.5	91	70	7.3
KUB4	85	55	3.5	88	63	4.8	89	71	4.8
Main Dome	90	56	4.2	95	64	5.2	95	71	6.0

Table 5.15. Peak principal stresses (MPa) on shell elements of undeformed model under selected ground motions.

Location	RSN1102 KOBE		RSN1795 HECTOR		RSN1164 KOCAELI	
	MAX	MIN	MAX	MIN	MAX	MIN
Main Dome Top	0.18	-0.16	0.22	-0.20	0.16	-0.17
Main Dome Springings	0.34	-0.35	0.42	-0.38	0.34	-0.33
EAST SEMIDOME	4.09	-4.03	4.68	-4.88	4.00	-4.18
WEST SEMIDOME	4.68	-4.63	5.43	-5.86	4.37	-4.74
EAST EXEDRAE	4.05	-4.10	4.05	-4.65	3.74	-3.85
WEST EXEDRAE	4.18	-4.30	4.75	-5.12	3.73	-4.18

Table 5.16. Peak principal stresses (MPa) on solid elements of undeformed model under selected ground motions.

Location	RSN1102 KOBE		RSN1795 HECTOR		RSN1164 KOCAELI	
	MAX	MIN	MAX	MIN	MAX	MIN
East Arch Springing	2.47	-2.24	2.37	-2.39	2.14	-2.23
East Arch Crown	0.53	-0.50	0.58	-0.61	0.52	-0.56
West Arch Springing	2.42	-2.77	2.78	-2.76	2.23	-2.33
West Arch Crown	0.63	-0.53	0.64	-0.65	0.60	-0.61
North Arch Springing	2.32	-2.22	2.28	-2.31	2.20	-2.17
North Arch Crown	0.39	-0.41	0.47	-0.47	0.36	-0.40
South Arch Springing	1.81	-1.57	2.04	-1.94	1.68	-1.56
South Arch Crown	0.39	-0.39	0.47	-0.45	0.39	-0.35
Buttress Arches	4.00	-4.20	4.42	-4.20	4.20	-3.90

Table 5.17. Max displacements (mm) in X, Y, Z directions on critical regions of undeformed model under selected ground motions.

Location	RSN1102 KOBE			RSN1795 HECTOR			RSN1164 KOCAELI		
	X	Y	Z	X	Y	Z	X	Y	Z
GAL1	58	47	11.0	74	52	11.0	58	51	11
GAL2	58	48	14.0	73	51	14.0	58	51	11
GAL3	59	48	11.0	75	51	12.0	60	52	11
GAL4	59	48	14.0	75	51	15.0	60	52	11
KUB1	89	69	5.0	112	74	6.0	90	73	5.4
KUB2	88	69	4.0	112	75	4.5	90	73	4.0
KUB3	89	69	5.0	112	74	6.0	90	73	5.2
KUB4	88	69	4.5	112	74	5.0	90	73	4.8
Main Dome	93	68	4.5	117	73	6.0	94	72	6.0

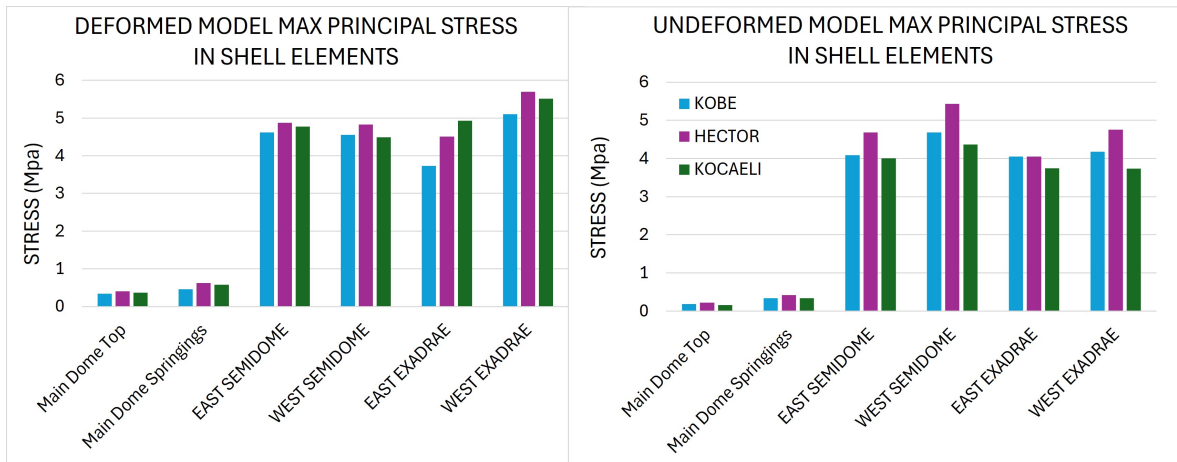


Figure 5.39. Maximum principal stresses on the shell elements of deformed (right) and undeformed (left) models under three records, linear time history analysis.

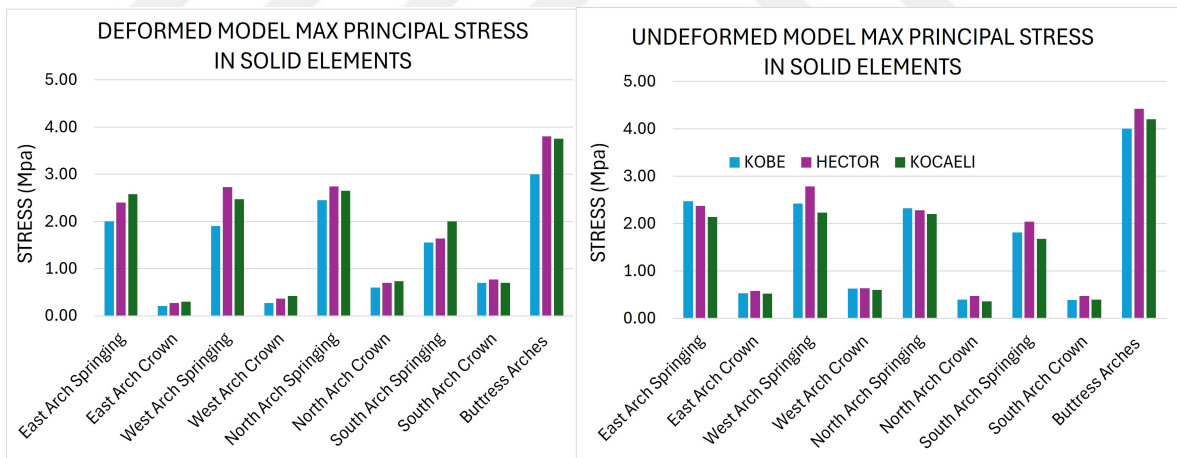


Figure 5.40. Maximum principal stresses on the solid elements of deformed (right) and undeformed (left) models under three records, linear time history analysis.

Figure 5.39 reveals that the largest maximum stresses are expected on the shell elements. Maximum principal stresses reach up to about 5 MPa at the semidomes and the exedra domes. The most significant difference between the deformed and undeformed model stress results is observed on the west exedra, where the stresses are

higher in deformed model. Maximum principal stress levels on the other shell elements exhibit variations dependent on the earthquake record. The main dome has a low level of stress in both models under each earthquake.

Figure 5.40 illustrates the maximum principal stresses on solid elements. According to the results, the springings of the main arches experience larger stress levels than their crowns.

5.2. Nonlinear Analysis

5.2.1. Nonlinearity

To conduct the nonlinear analysis, two types of nonlinearity are considered in the scope of this study: material nonlinearity and geometric nonlinearity.

5.2.1.1. Material Nonlinearity. Masonry components are classified as geo-materials. Numerous experimental results have been documented by various scholars in recent years about masonry constituents. Various multi-parameter yield functions have been presented to align with empirical data, resulting in a range of constitutive models that effectively reflect different elements of the behavior of geo-materials, including limestone, granite, concrete and ceramics. The principal characteristic of a material model is to ascertain the correlation between the stress tensor and the strain tensor, predicated on a limited number of factors that define the material's behavior [55].

The main construction material of Hagia Sophia is masonry. Masonry is an isotropic material which is dependent also on hydrostatic stress like other granular materials such as soil, rock, concrete etc. Hydrostatic pressure significantly influences the failure of these materials. Besides, many of them have different strengths in tension and compression [56].

Numerous yield criteria have been suggested to model the nonlinearity of these granular materials. One of those was developed by Drucker and Prager [57]. This is an isotropic hardening plasticity model and constitutes a linear combination of effective stress and hydrostatic stress. It may exhibit a yield stress contingent upon pressure, resulting in varying yield stresses under tension and compression. This is an appealing characteristic for geo-mechanical materials. Therefore the model has been utilized in many forms for soils, masonry, concrete, and rocks (Abrate, 2008).

In this study, Drucker-Prager plasticity is used to model the nonlinear behavior of masonry. The Drucker-Prager (DP) model is incorporated in ANSYS software, with distinct failure criteria for tension and compression. The Drucker-Prager (DP) model is included in the geo-mechanical toolbox of ANSYS. It can be used within static structural and transient structural analyses [42]. It requires physical properties such as elasticity modulus, Poisson's ratio, uniaxial compressive, biaxial compressive and tensile strengths.

The elasticity modulus and Poisson's ratio are determined in earlier chapters when calibrating the linear analysis results with the real data. Thus, ultimate compressive and tensile strengths are discussed in this chapter through the related studies in literature. A series of experiments are carried on reproduced examples of Byzantine masonries. Binda and his colleagues studied on the Basilica of St. Vitale in Ravenna which is a Byzantine structure constructed in the 5th century and has very similar properties with Hagia Sophia in terms of the materials used in the construction [46]. They inspected the mortar and the brick masonry of St. Vitale and reproduced them in laboratory to perform mechanical tests in terms of varying curing periods. They prepared twelve brick masonry specimens two of which are tested after 365 days curing time. The compressive stress-strain curves of those specimens which are called SB2.8 and SB2.9 can be seen in Figure 5.41. Besides the masonry unit they located devices on the specimen to measure the strain levels on the mortar layer alone and obtained the compressive stress-strain graph of mortar as shown in Figure 5.42.

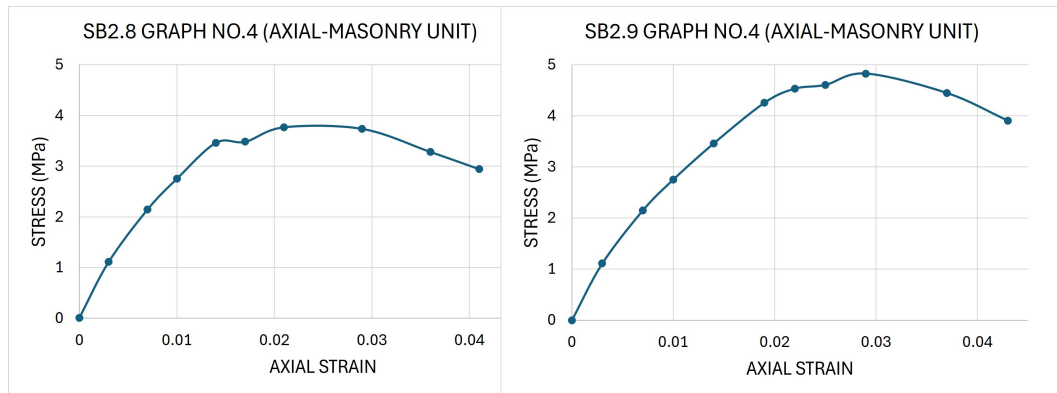


Figure 5.41. Stress-strain curves of the masonry units tested in the study of Binda and colleagues [46].

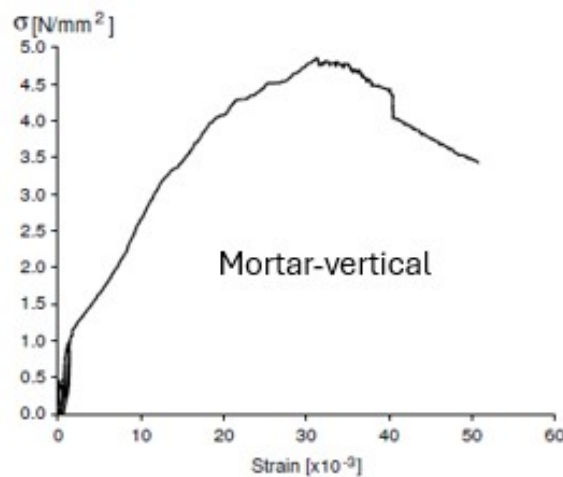


Figure 5.42. Stress-strain curves of the mortar tested by Binda and colleagues [46].

In a similar study Falter and Reinhardt investigated the brick masonry on a main column of the same structure, St. Vitale. They reproduced it in the lab [58]. The focus was on parameters such as environment (temperature and rate of humidity (RH)), consistency (“0”: as anticipated; “+”: a little softer; “-”: a little stiffer) and joint thickness as given in Table 5.18. The stress-strain graphs obtained are illustrated in Figure 5.43.

Table 5.18. Testing parameters that Falter and Reinhardt used in their study [58].

Specimen nr.	Joint thickness (mm)	Environment	Consistency
1	20	20°C/65% RH	0
2/1	40	20°C/65% RH	–
2/2	40	20°C/65% RH	+
2/3	40	20°C, wrapped	0
3/1	60	20°C/65% RH	–
3/2	60	20°C/65% RH	+

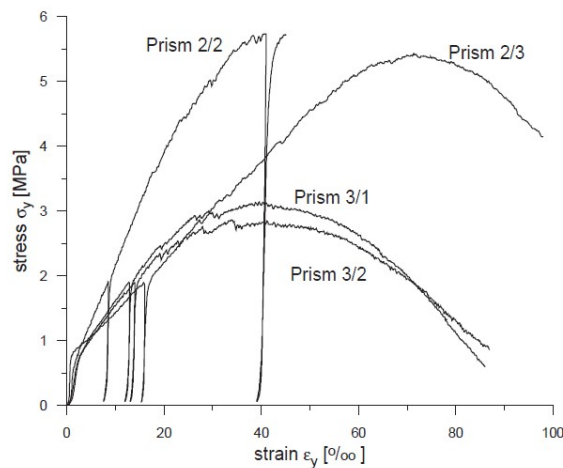


Figure 5.43. Stress-strain curves of brick masonry units tested by Falter and Reinhardt [58].

These two studies focus on the uniaxial compressive strength of the masonry units. Stavrakakis and colleagues studied the relationship between uniaxial and biaxial compressive strengths of Byzantine brick masonry [59]. They carried out an analytical analysis for modeling brick masonry units. The properties of the masonry units to be modeled are determined by lab and in-situ tests performed on the samples taken from Rotunda of Thessaloniki which has similar construction materials with Hagia Sophia and built in the 4th century in the Balkan Region. Their findings show that the ratio

of the biaxial compressive strength to the uniaxial compressive strength of the masonry can reach up to 1.5-2.

In this study, the nonlinear material mechanical properties are based on Binda and his colleagues [46]. The stress-strain graph is adopted according to linear hardening softening dilatation (HSD). The DP model and the required parameters are given as input to ANSYS. The compressive stress-strain graph of brick masonry used in this study is illustrated in Figure 5.44 together with those for samples SB2.9 and SB2.8 as defined by Binda and colleagues [46].

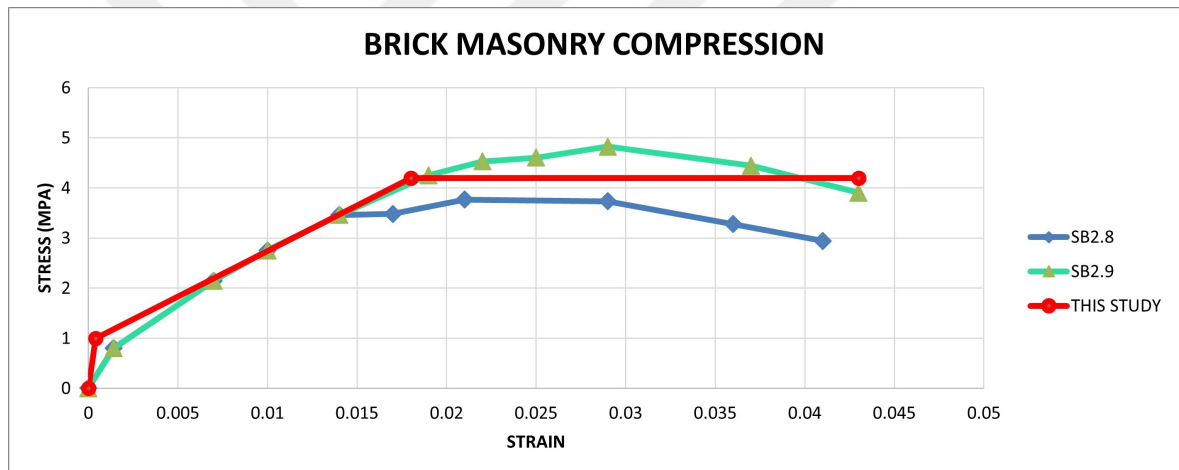


Figure 5.44. The compressive stress-strain graph of brick masonry adopted in this study. [58].

When it comes to the compressive strength of stone masonry, firstly the mechanical properties of limestone must be discussed, since the stone masonry used in Hagia Sophia is composed by mortar and limestone. Koçak investigated the structural response of Sts. Sergius and Bacchus in Istanbul (also known as Little Hagia Sophia) which is constructed in 536AD just before Hagia Sophia [60]. He carried out tests on the limestone specimens which are reproduced in the lab to represent the limestone used in the structure and concluded with the results shown in Table 5.19. It can be claimed that compressive strength of the specimen cured for 28 days is about 34 MPa.

Table 5.19. Mechanical test results of limestone [60].

Curing Time [days]	Unit Weight [N/cm ³ × 10 ²]	Tensile Strength [MPa]	Compressive Strength [MPa]
3	2.385	2.12	20.0
3	2.378	2.15	22.0
7	2.370	2.29	26.2
7	2.360	2.43	26.7
14	2.365	2.41	28.0
14	2.365	2.48	28.7
28	2.320	2.59	32.0
28	2.310	2.67	33.8

Studies on the compressive strength of stone masonry are hard to find in literature. However, information on the mechanical properties of mortar and limestone are available, as mentioned earlier. With this information at hand, a stone masonry cubic specimen is modeled in Ansys, and analyzed under compression to estimate its stress-strain curve. In Figure 5.45, the 3D model of the specimen and the stress-strain results can be seen. Due to the scarcity of experimental data on the biaxial compressive strength of historical or natural stone masonry, a rational assumption is required. Dong and colleagues report that the ratio of biaxial to uniaxial compressive strength for mature normal-strength concrete is approximately 1.14, based on several experimental studies [61]. Although this ratio is derived from concrete behavior, it is adopted in this study as a proxy for stone masonry, given they both reveal brittle and quasi-homogeneous mechanical characteristics with comparable compressive strength levels. While this introduces a degree of uncertainty, it provides a practical and physically reasonable approximation in the absence of more specific data for stone masonry.

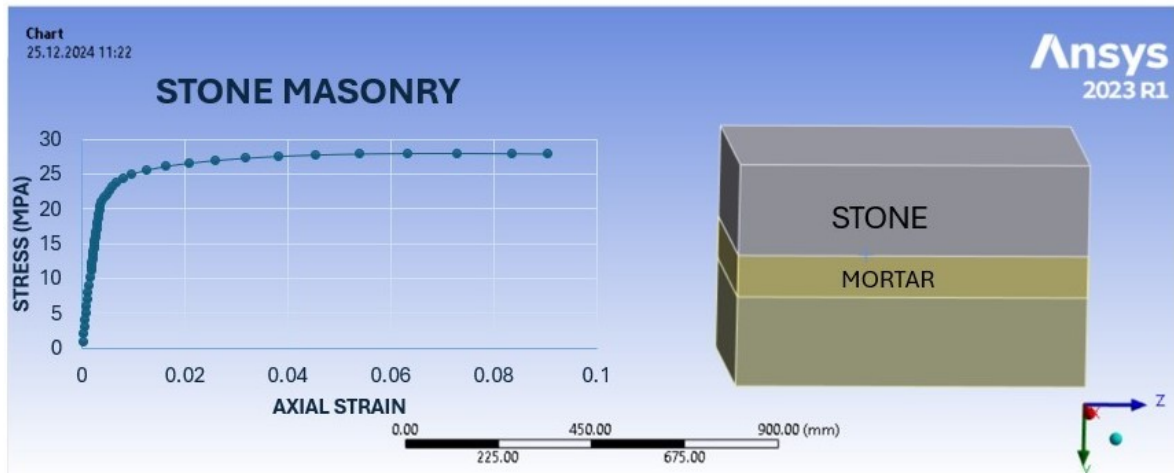


Figure 5.45. Stress strain curve of the stone masonry unit tested using Ansys.

After the compressive strength of the materials are decided, the tensile strength of the brick masonry and stone masonry are obtained from the literature. Mark and Çakmak performed laboratory tests on a brick masonry specimen taken from the main dome of Hagia Sophia and estimated its tensile strength is 3MPa [62]. Furthermore, Livingstone studied the mechanical properties of the masonry used in Hagia Sophia [24]. He claims that the mortar of Hagia Sophia has pozzolanic characteristics and its significant benefit over pure lime mortar is its superior tensile strength. It can achieve a tensile strength of up to 3-3.5 MPa in one year. Even though there are not many published papers on the tensile strength of limestone or stone masonry, it can be considered that the tensile strength of stone masonry is mostly dominated by that of the mortar rather than the limestone. Therefore, in this study it is assumed to be similar to the strength of the mortar. In the linear analyses, the regions with a tension higher than 1Mpa are considered as vulnerable to be on the safe side and also to make the visualisation clearer and easier. However, previous studies show that tensile strength of the materials used in Hagia Sophia can reach upto 3Mpa. Hence, nonlinear materials are modeled with an ultimate tensile strength of 3Mpa in this study.

When the nonlinear material properties are considered, it is suggested to decrease the elasticity modulus of brick masonry to the one used for the modal analysis [63]. This is claimed to be helpful to cover the behavior due to the micro-cracks. Thus, based on the literature review presented in this chapter and the linear parameters estimated during the calibration of the FE model, the material properties of the three types of construction materials adopted in nonlinear analysis are given in Table 5.20.

Table 5.20. Material properties used in non-linear finite element modelling.

Material Name	Density kg/m ³	Elastic Modulus MPa	Poisson's Ratio –	Uniaxial Compressive Strength MPa	Biaxial Compressive Strength MPa	Tensile Strength MPa
BRICK	1700	2000	0.17	4.2	6	3
BRICK-2	1250	2000	0.17	4.2	6	3
STONE	2000	6200	0.17	28	32	3

5.2.1.2. Geometric Nonlinearity. The second type of nonlinearity applied on the finite element model is geometric nonlinearity. It pertains to the nonlinear behavior in a structure or component resulting from alterations in geometry during deflection. The stiffness is dependent on the displacements. The stiffness varies due to alterations in shape and/or the rotation of the material. To engage the geometric nonlinearity on the finite element model, it is required to turn the large deflection on under the Analysis Settings [42], which is what was done in this study.

5.2.2. Nonlinear Static Analysis

Nonlinear analyses are performed on the deformed model. The nonlinear material model and properties which are introduced in the previous section, and geometric nonlinearity is applied in the analyses. Nonlinear static structural analysis is carried

out firstly. Stress distributions on shell and the solid elements under the self-weight of the structure as a result of non-linear modelling are shown in Figure 5.46 and 5.46 respectively. Besides, the peak displacements at the main structural elements along the three axes are given in Table 5.21.

Results of nonlinear static analysis indicate that the stresses caused by self-weight are lower than 1 MPa. They are therefore very similar to those of linear static analysis, which implies that the material mostly remains in the linear region under the self-weight. Hence, there are no significant differences between the peak values and distribution patterns of stresses obtained from linear static and nonlinear static analyses. These observations confirm that the structure can bear its self-weight safely without any reasonable expectation for further deformations.

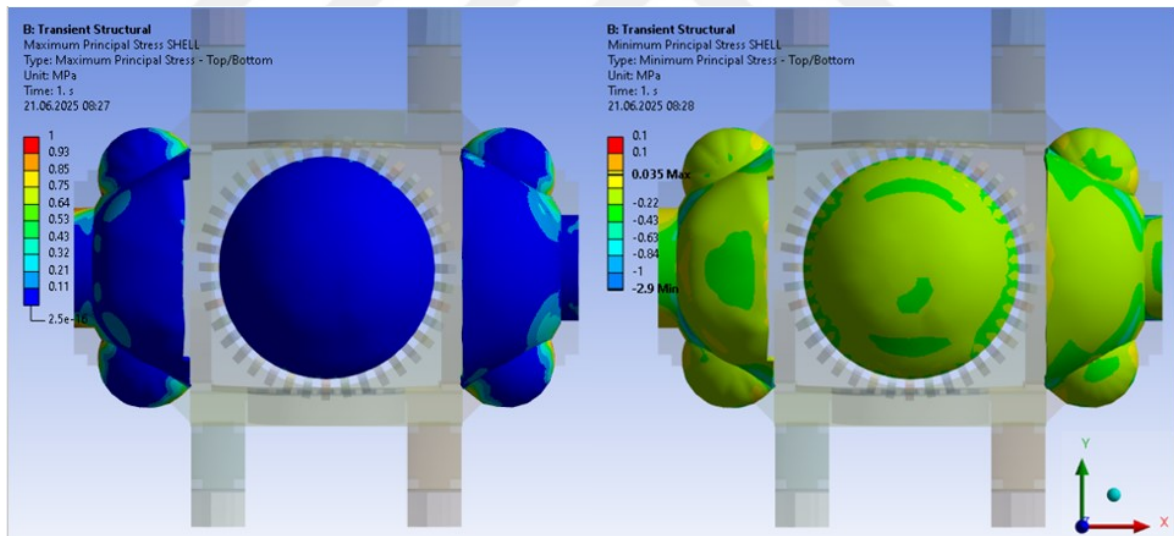


Figure 5.46. Maximum (left) and minimum (right) principal stresses on the shell elements under self-weight (0-1 MPa) of the deformed model.

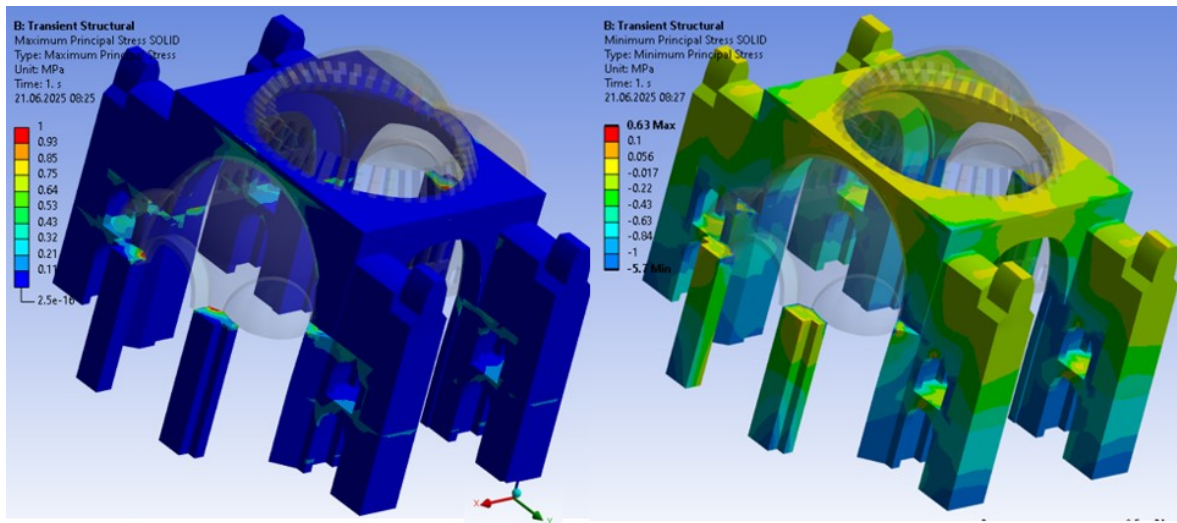


Figure 5.47. Maximum (left) and minimum (right) principal stresses on the solid elements under self-weight (0-1 MPa) of the deformed model.

Table 5.21. Max displacements (mm) in X,Y,Z directions of the nonlinear deformed model under self weight.

Location	East–West (X axis)	North–South (Y axis)	Vertical (Z axis)
GAL1	-0.6	-1.0	-4
GAL2	-0.6	0.9	-4
GAL3	0.6	0.9	-4
GAL4	0.6	-1.0	-4
KUB1	-0.1	-0.1	-7
KUB2	-0.1	-0.1	-8
KUB3	-0.1	-0.1	-7
KUB4	-0.1	-0.1	-7
Main Dome	-0.1	0.0	-10

5.2.3. Nonlinear Time History Analysis

5.2.3.1. Model Calibration. The linear finite element model is calibrated with the experimentally assessed modal frequencies. In this chapter, a further calibration will be introduced. There are earthquake records obtained by the structural monitoring system on Hagia Sophia. For each earthquake there exist twenty seven records obtained from nine accelerometers in three directions, which provide a chance to calibrate the model one more time, this time in time domain. The most remarkable earthquake recorded on Hagia Sophia occurred in 2025, the records of which are selected to be used to calibrate the model. The Mw 6.1 earthquake occurred on 23 April 2025 on one of the segments of the North Anatolian Fault under the Marmara Sea at about 28km south of Silivri/İstanbul. The fact that it is the largest earthquake recorded by the accelerometers on Hagia Sophia, is specifically helpful for the calibration of the nonlinear model under relatively high level of seismic loading.

The procedure starts with taking out the required part of the ground motion from the original data. The original record is much longer than what is optimal in terms of computer memory and analysis run time. Therefore, the 11s long part of the record that contains the main event is selected, which is prepared for analysis by detrending and band-pass filtering. This procedure is repeated for all twenty-seven records. Following the data preparation, the acceleration record obtained from the station at the ground level is given as input to the nonlinear model and the displacement time histories at the locations of the remaining eight accelerometers are determined. They are then compared to the real acceleration, velocity and displacement time histories taken from the accelerometers. The cut and filtered three component earthquake record obtained from the accelerometer on the ground of the structure and given as input to Ansys is illustrated in Figure 5.48.

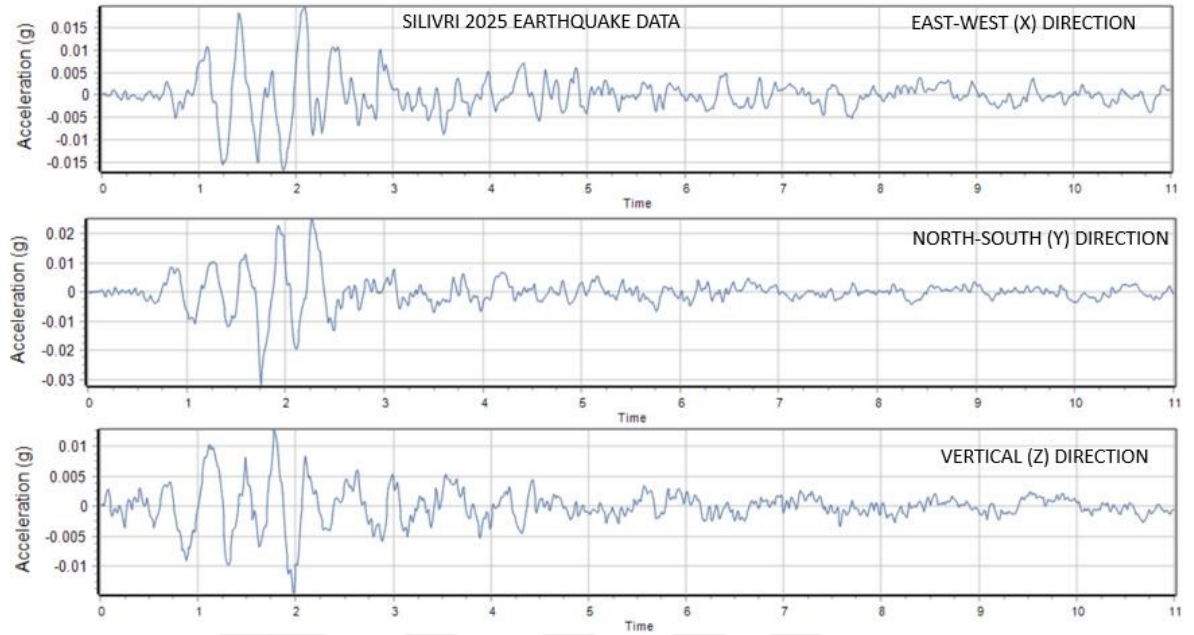


Figure 5.48. The 23 April 2025, Mw 6.1 Silivri earthquake as recorded at the ground level station in Hagia Sophia. The parts of the three-component record provided as input to the finite element model are shown.

For the analysis, full transient analysis method is used. Ansys employs Newton-Raphson method for the iterations of nonlinear analysis [42]. The same nonlinear material properties which are introduced in the material nonlinearity part are used and the geometric nonlinearity is implemented.

Table 5.22 shows the peak accelerations, velocities and displacements obtained from the accelerometers and from the nonlinear time history analysis at stations GAL3, KUB1, KUB2, KUB3 and KUB4. GAL1, GAL2 and GAL4 results are not included here, since the monitoring records seem to be problematic in all directions. The displacements of the records do not approach zero with the applied detrending and filtering processes. It is important to note that the same filtering and detrending process is applied on all twenty seven records obtained from Hagia Sophia during the earthquake for the sake of consistency. Acceleration and velocity time histories comparing Ansys

and monitoring results for the five locations in X and Y direction are given in Figure 5.49 - 5.53. Monitoring results graph is given in blue color and the Ansys results are in orange color. Additionally, displacement time histories are given in Figure 5.53.

Table 5.22. Displacements (mm) due to 2025 Silivri Earthquake obtained by structural nonlinear time history analysis and from the accelerometric data.

Location	ANSYS			MONITORING		
	East (X)	North (Y)	Up (Z)	East (X)	North (Y)	Up (Z)
GAL3	2.4	1.4	0.5	1.4	1.5	0.5
KUB1	3.7	2.2	0.2	4.3	4.3	0.7
KUB2	3.7	2.1	0.2	4.4	4.9	3.5
KUB3	3.7	2.2	0.2	2.9	2.9	0.6
KUB4	3.7	2.2	0.1	4.2	3.5	1.2

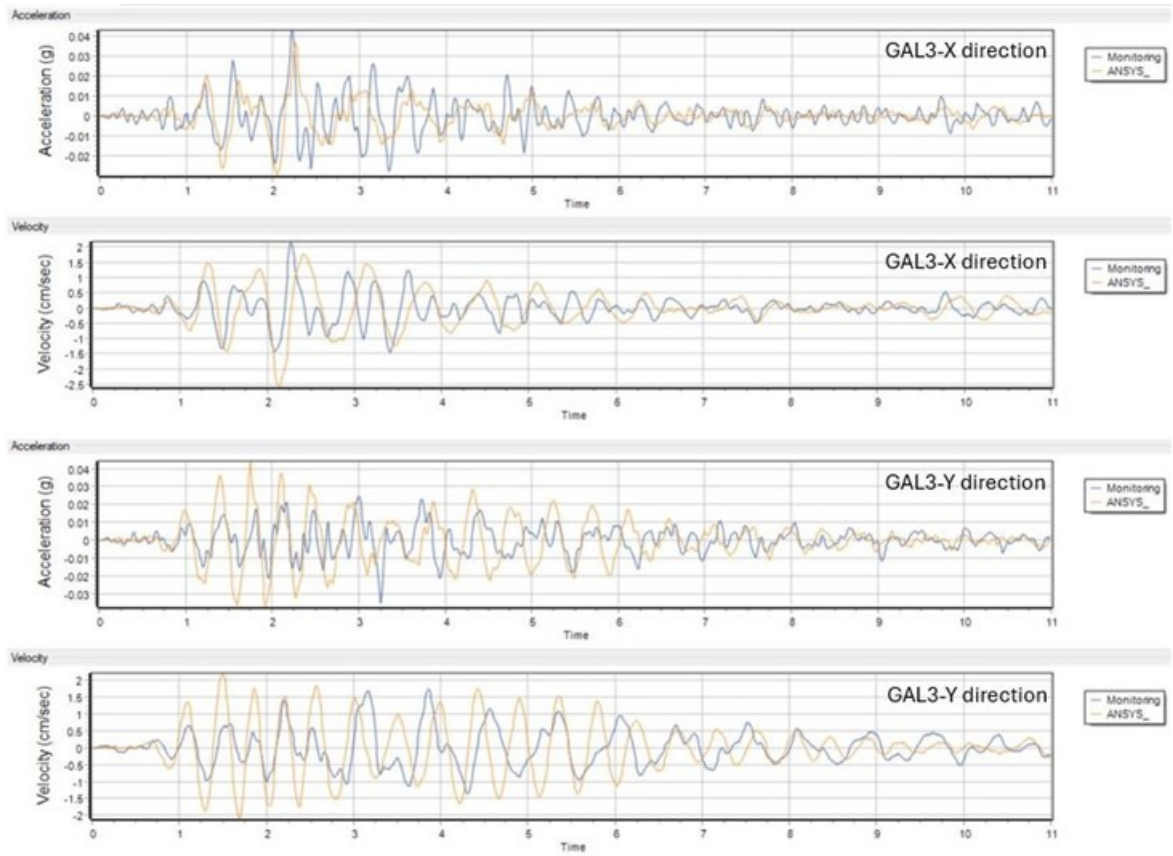


Figure 5.49. Comparison of the acceleration and velocity time histories under 2025 Silivri Earthquake obtained by the nonlinear time history analysis in Ansys and the accelerometer data at GAL3 location.

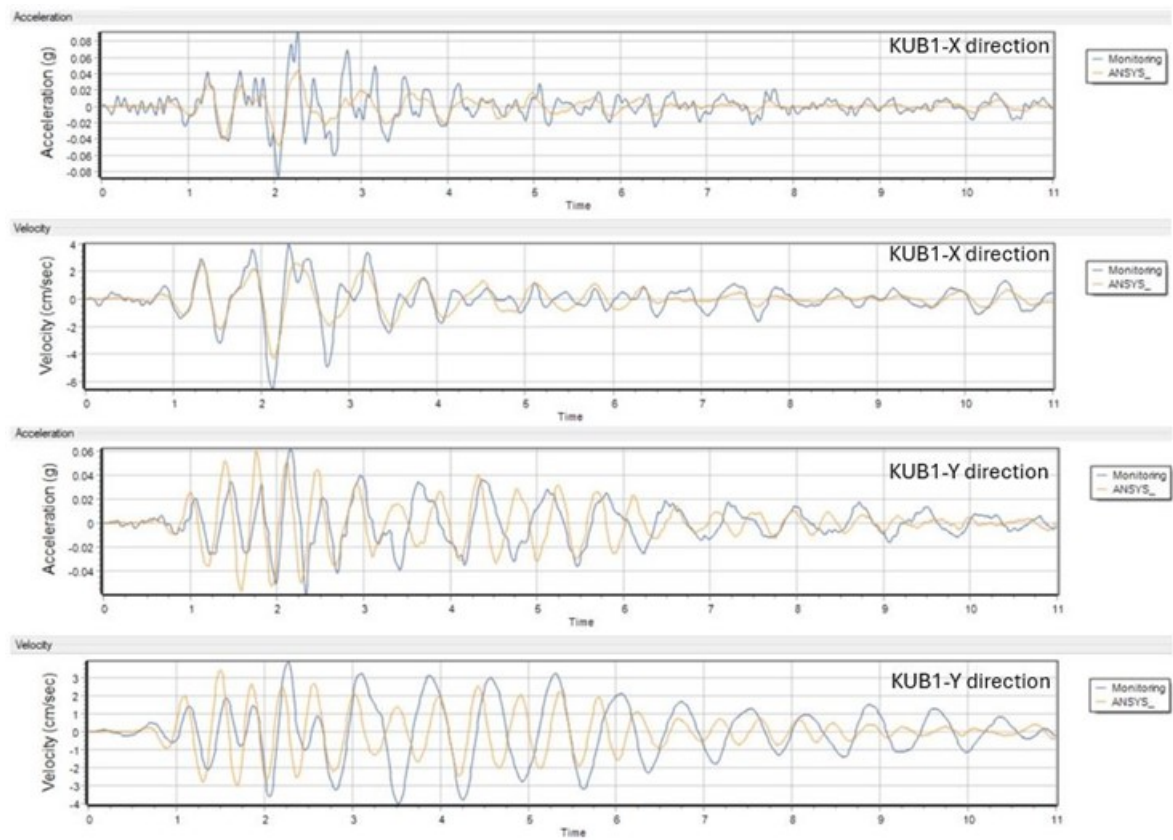


Figure 5.50. Comparison of the acceleration and velocity time histories under 2025 Silivri Earthquake obtained by the nonlinear time history analysis in Ansys and the accelerometer data at KUB1 location.

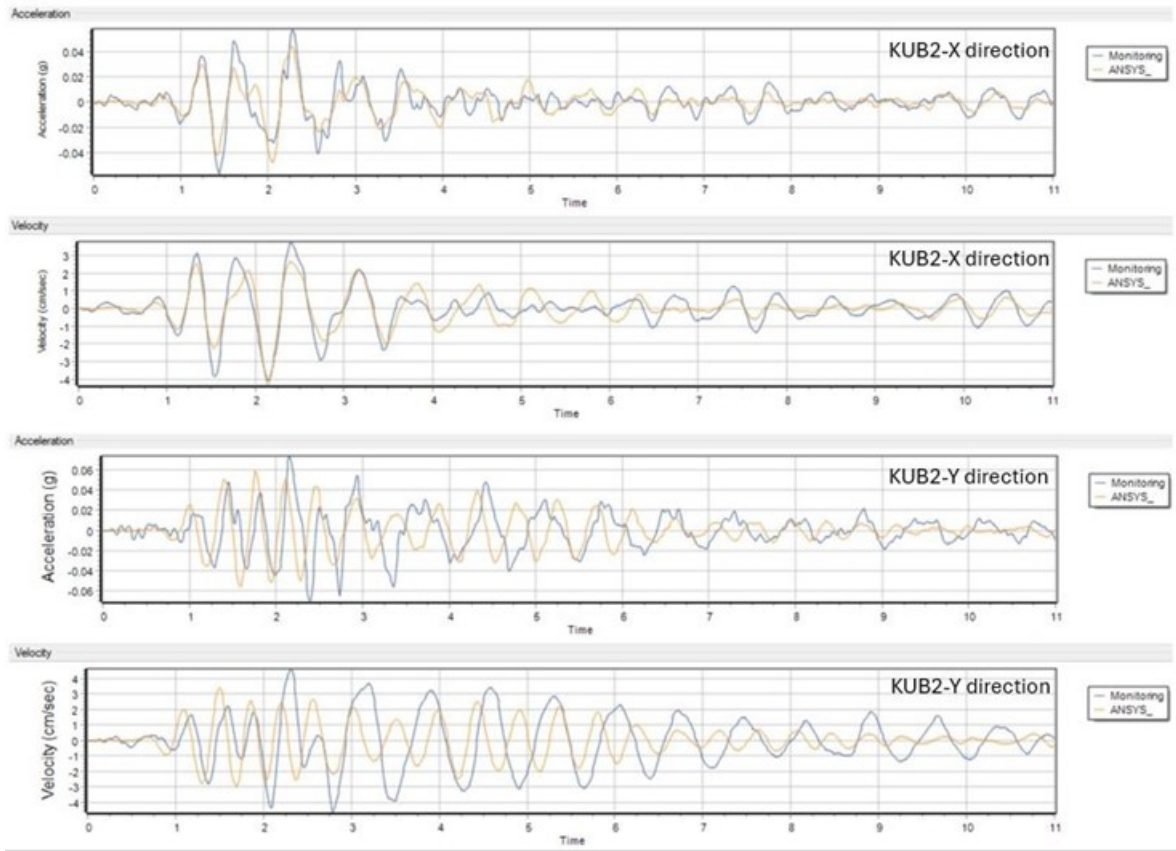


Figure 5.51. Comparison of the acceleration and velocity time histories under 2025 Silivri Earthquake obtained by the nonlinear time history analysis in Ansys and the accelerometer data at KUB2 location.

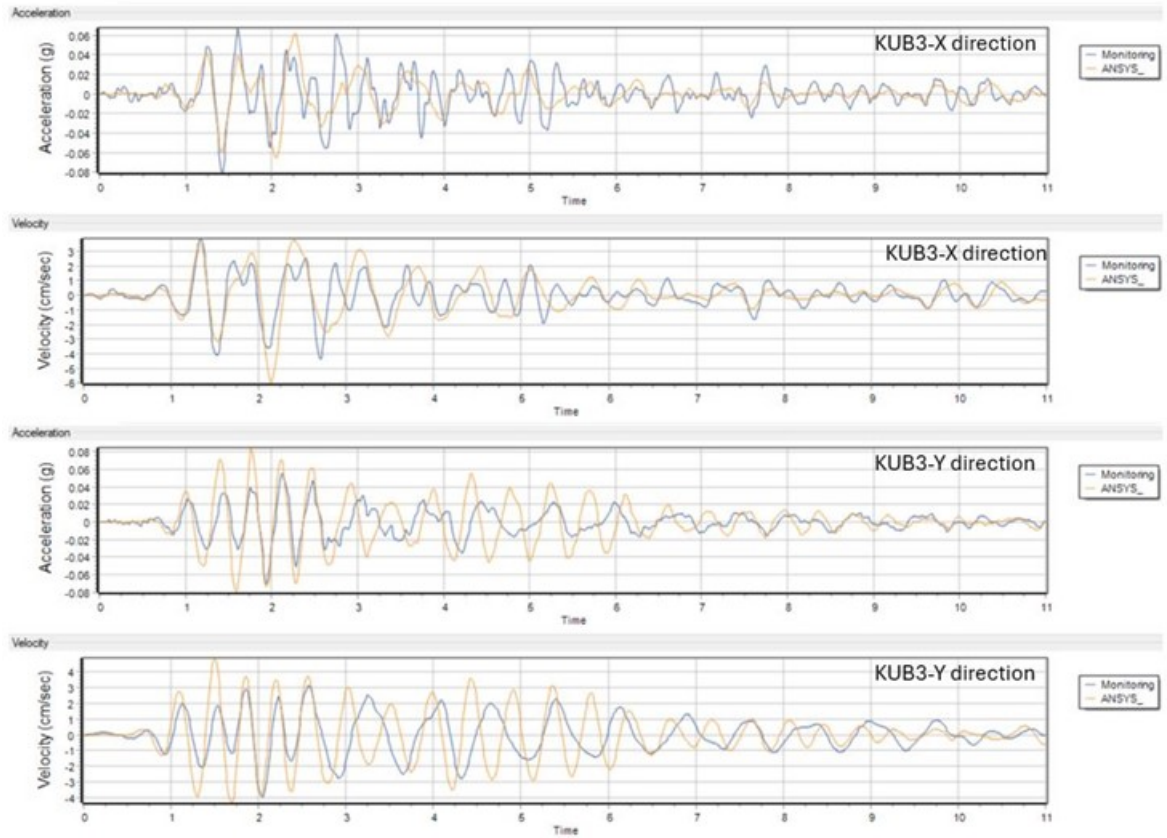


Figure 5.52. Comparison of the acceleration and velocity time histories under 2025 Silivri Earthquake obtained by the nonlinear time history analysis in Ansys and the accelerometer data at KUB3 location.

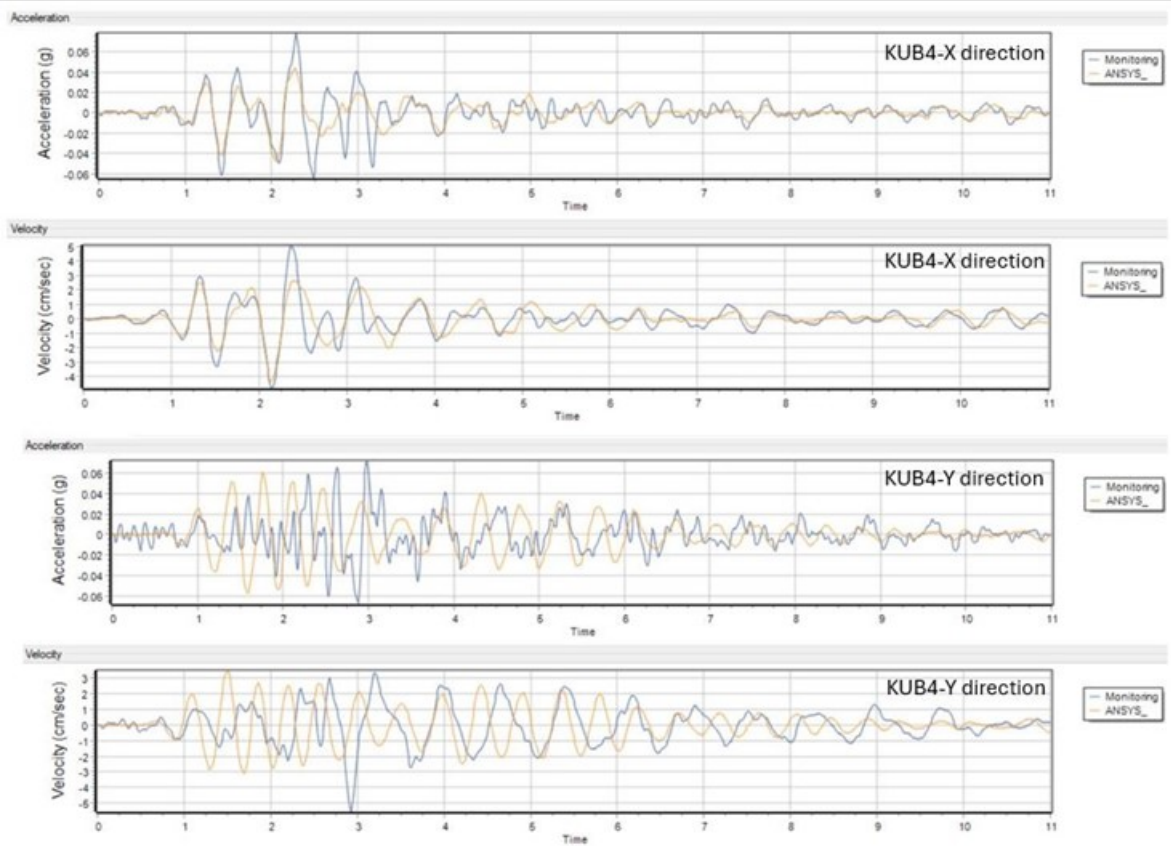


Figure 5.53. Comparison of the acceleration and velocity time histories under 2025 Silivri Earthquake obtained by the nonlinear time history analysis in Ansys and the accelerometer data at KUB4 location.

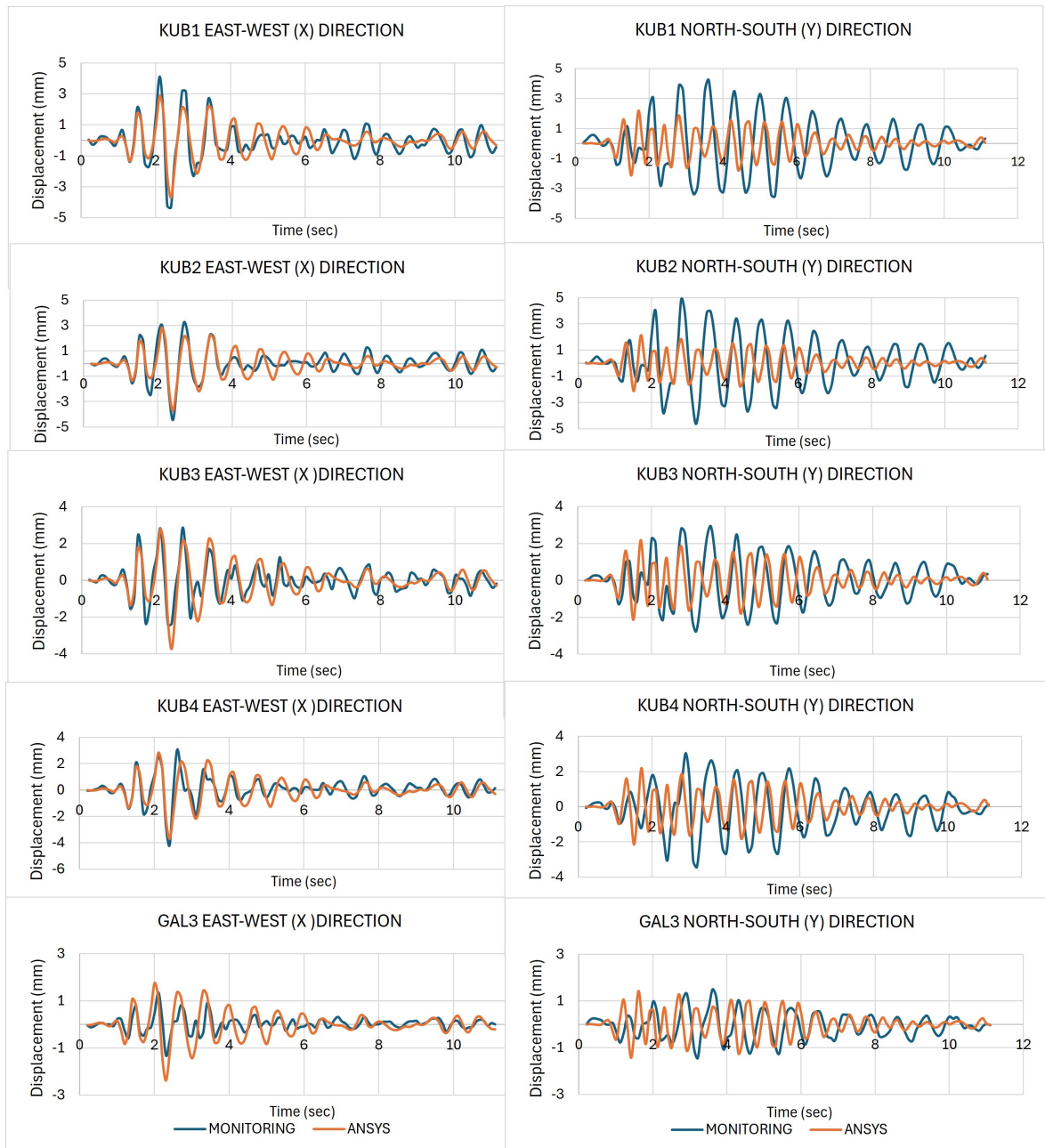


Figure 5.54. Comparison of the time histories under the 2025 Silivri earthquake obtained by the structural nonlinear time history analysis and the accelerometric data at five locations.

It can be claimed that, in the X direction the analytical results are in a very reasonable agreement with those from monitoring. However, in the Y direction, the

displacements, velocities and accelerations are underestimated at the dome base stations (KUB1, KUB2, KUB3, KUB4). On the other hand, at the gallery level (GAL3), even though the time history phase along the Y direction does not match properly that of the recording, the displacement, velocity and acceleration levels generally agree with each other. The difference between the analytical and experimental results in the displacement amplitudes in Y direction at the dome base level might be due the reflection of local problems from lower levels to the dome base level or they might directly be associated with the local problems at these locations. Besides, it is important to note that there are still many unknowns regarding the connections, microcracks or local material irregularities in Hagia Sophia. According to Almaç and his colleagues, the interaction between structural parts with varying construction periods is another significant concern [16]. Thus, further observations on the connections and/or connecting regions between structural parts and elements will be valuable to understand the role they play in the response of Hagia Sophia and most likely improve analytical models of the structure. Similar concerns are expressed by Natsis [9] and Mainstone [1] as well.

When the damping is considered, the best results are obtained when the damping ratio is taken as 3% in Ansys finite element model. Therefore, time history results shared in this study are obtained under 3% damping ratio.

5.2.3.2. Failure Mechanism. After the calibration of the nonlinear finite element model with the real data from a medium size earthquake, the model is analyzed under real and artificial large magnitude earthquake records. Two groups of earthquake data are used in nonlinear time history analyses. The first group is taken from the PEER Database [51] and includes the records of Kocaeli, Kobe and Morgan Hill earthquakes. Their properties are given in The database is searched for records similar to the target response spectrum of Hagia Sophia using the same parameters explained in subsection 5.1.3.2. However, the procedure differentiates in the number of horizontal components considered. For the linear analysis (Chapter 5.1), only one horizontal component of the records are regarded when searching the database, yet both horizontal components are considered for the nonlinear analyses. The records are scaled accordingly with

the target spectrum of Hagia Sophia. Square root sum of the squares (SRSS) method is used for scaling purposes and all three components are multiplied by the obtained scaling factors which are given in Table 5.23. The scaled response spectra and the scaled acceleration-time histories of the scaled records are given in Figure 5.55 - 5.58.

Table 5.23. Properties of the ground motions taken from PEER Database.

Record Sequence Number	Scale Factor	Earthquake Name	Magnitude	Mechanism	R_{jb} (km)	V_{s30} (m/sec)
476	5.36	Morgan Hill	6.19	strike slip	45.47	713.59
1102	4.14	Kobe, Japan	6.9	strike slip	49.91	609
1164	7.48	Kocaeli, Turkey	7.51	strike slip	49.66	595.2

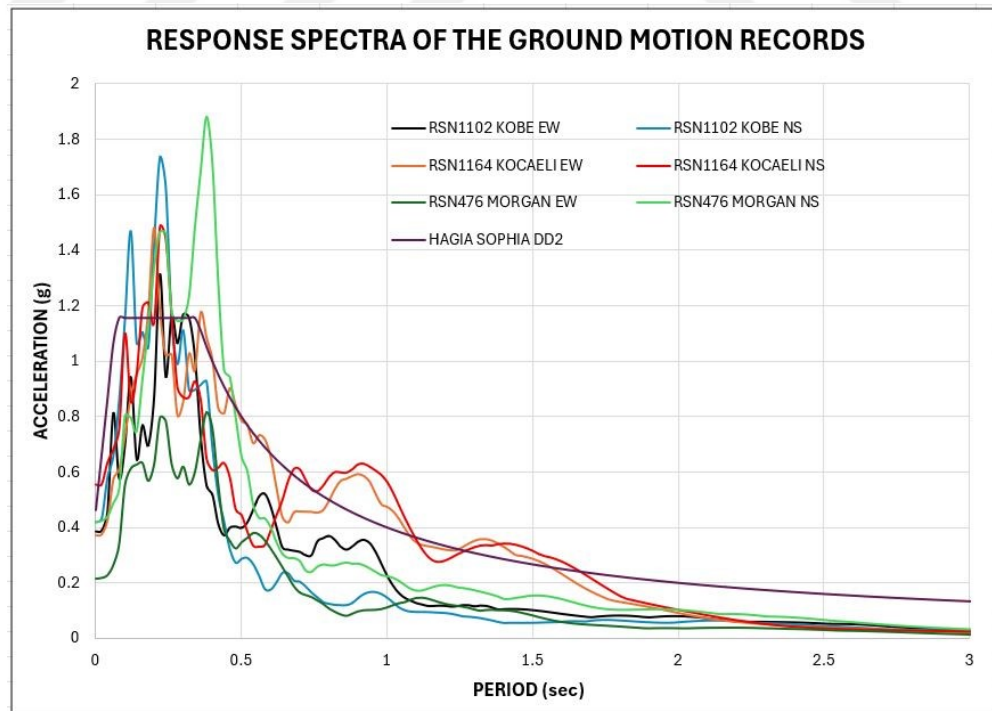


Figure 5.55. Response spectra of records selected from PEER Database.

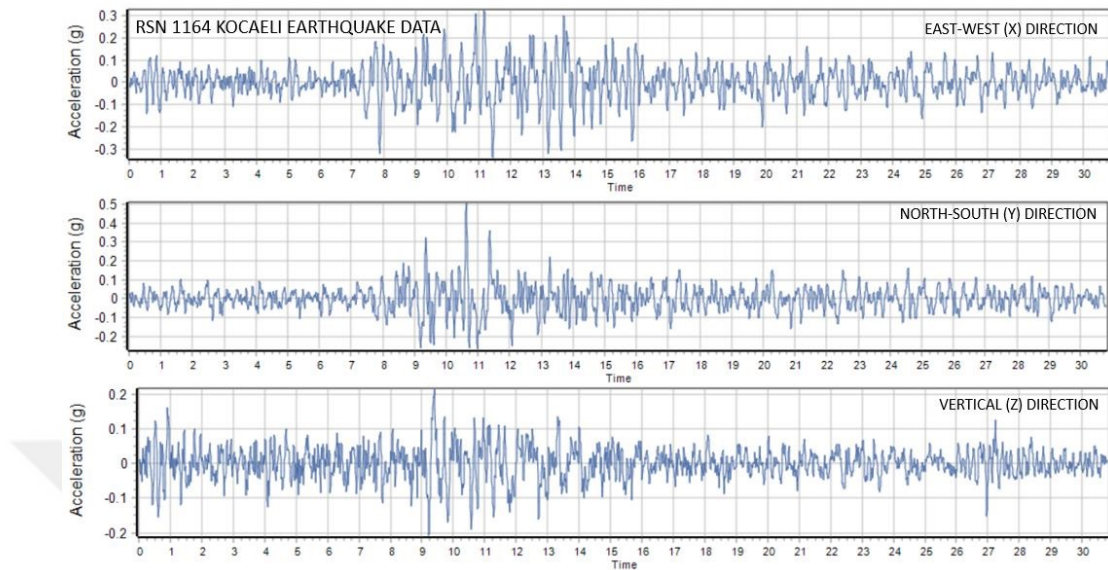


Figure 5.56. Acceleration-time history of the RSN1164 Kocaeli earthquake record in three components.

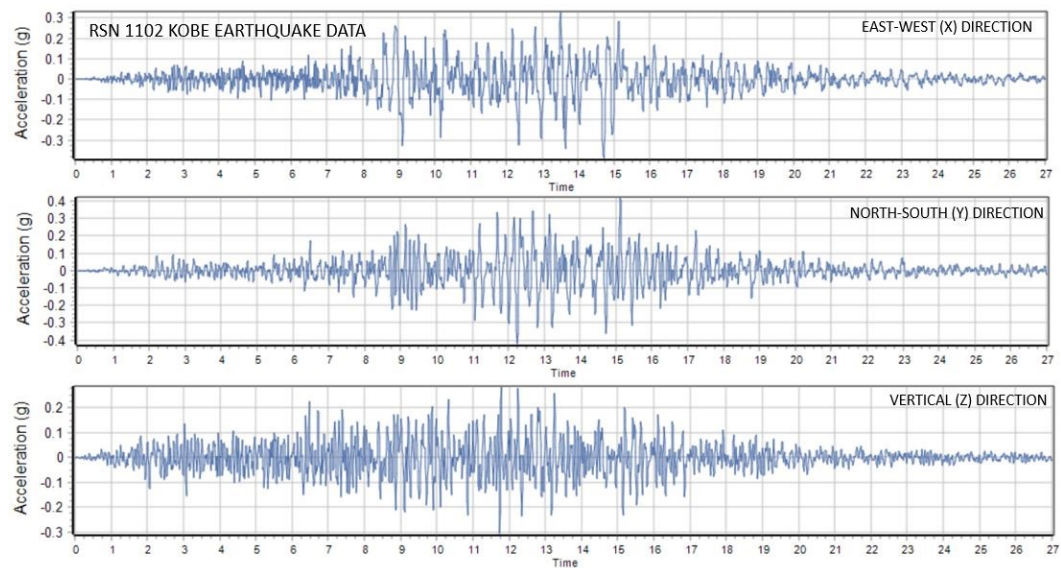


Figure 5.57. Acceleration-time history of the RSN1102 Kobe earthquake record in three components.

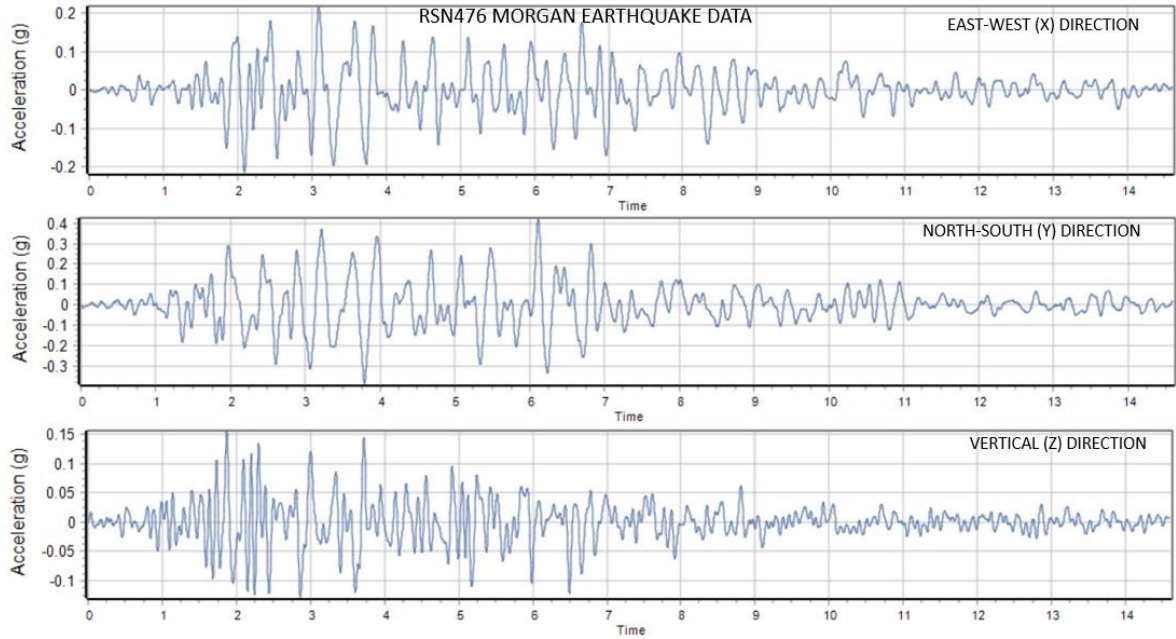


Figure 5.58. Acceleration-time history of the RSN476 Morgan earthquake record in three components.

The second ground motion data set consists of synthetic earthquakes. There is a recently finished TUBITAK (The Scientific and Technological Research Council of Türkiye) project that focuses on generating site specific earthquakes ground motions within Istanbul's municipal boundaries by 3D physics based simulation approach [64]. These synthetics are given as input in this study to investigate Hagia Sophia's response to scenario earthquakes. Researchers produced site specific simulated earthquake records emanating from the scenario earthquakes specific to the Marmara Sea. They developed a three-dimensional crustal velocity model for the Marmara region, which was validated by the real earthquake records. The source models of past large earthquakes with similar properties to the regional seismo-tectonics were adopted as scenario earthquakes. Moreover, the earthquake source models developed by Yenihayat for the historical Istanbul earthquakes were also used [65]. The simulations were performed for a grid system that represents Istanbul. For the purpose of this study, the records obtained from the closest grid to Hagia Sophia are used. The properties of

the selected earthquake scenarios are given in Table 5.24. The response spectra of the synthetic ground motions used in the analyses are given in Figure 5.59 together with the DD2 and DD3 level design elastic spectra for Hagia Sophia which have return periods of 475 and 72 years respectively as described in Turkish Seismic Code 2018 [49]. Besides, acceleration-time histories of the two horizontal components of each artificial record are given Figure 5.60 - 5.64.

Table 5.24. Properties of the synthetic ground motions.

ID	Event	Recording Source	V_{s30} (m/sec)	Magnitude	R_{jb} (km)
2	'Hector Mine (Calif.)'	Salichon et al. (2004)	554.77	7.09	18.03
3	'Izmit (Turkey)'	Reilinger et al. (2000)	554.77	7.35	18.26
10	'Morgan Hill (Calif.)'	Boore and Spudich (1988)	554.77	6.21	17.45
14	'Istanbul_7pt6'	Historical Marmara Eq.	554.77	7.60	17.57
15	'Istanbul_7'	Historical Marmara Eq.	554.77	7.00	17.72
16	'Istanbul_7pt1'	Historical Marmara Eq.	554.77	7.10	17.72

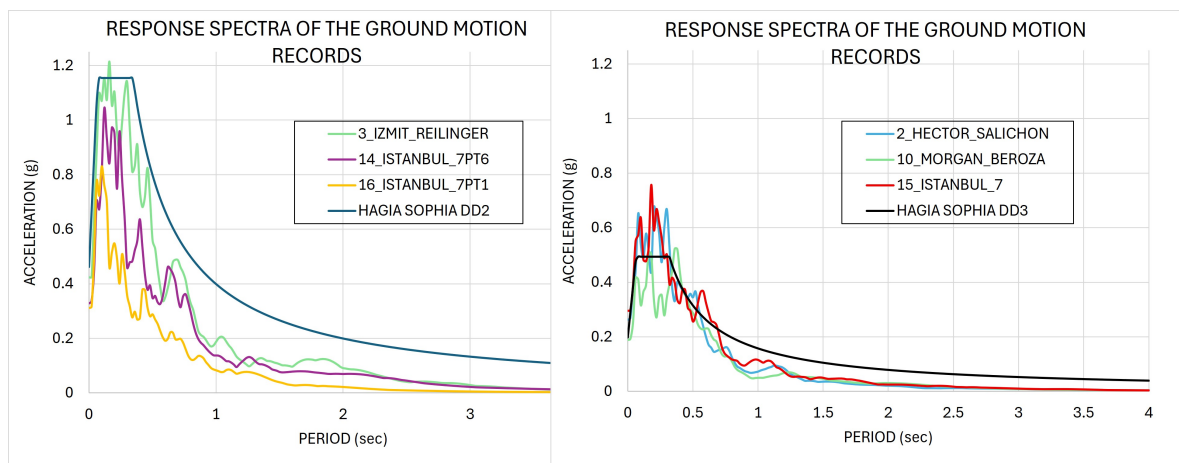


Figure 5.59. Response spectra of the synthetic records used in the analyses.

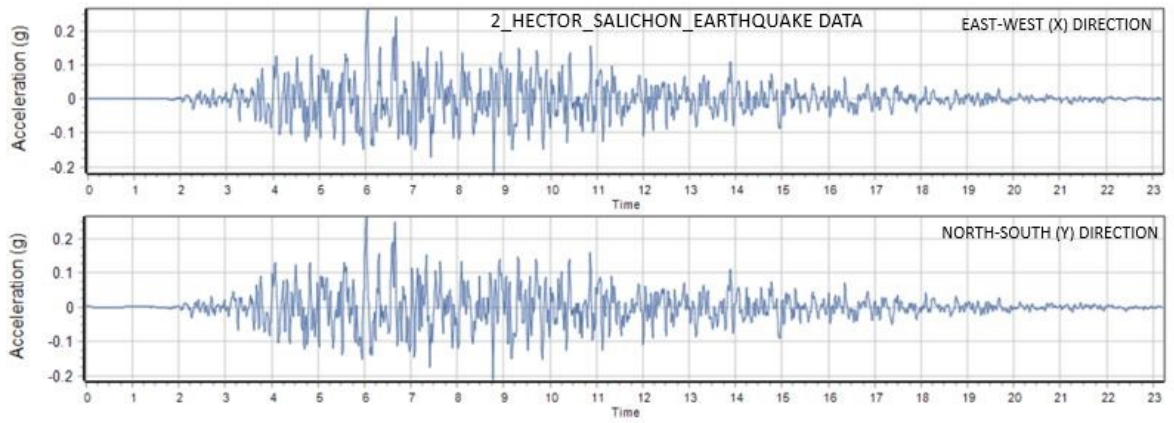


Figure 5.60. Acceleration-time history of the 2_Hector_Salichon record for two horizontal components.

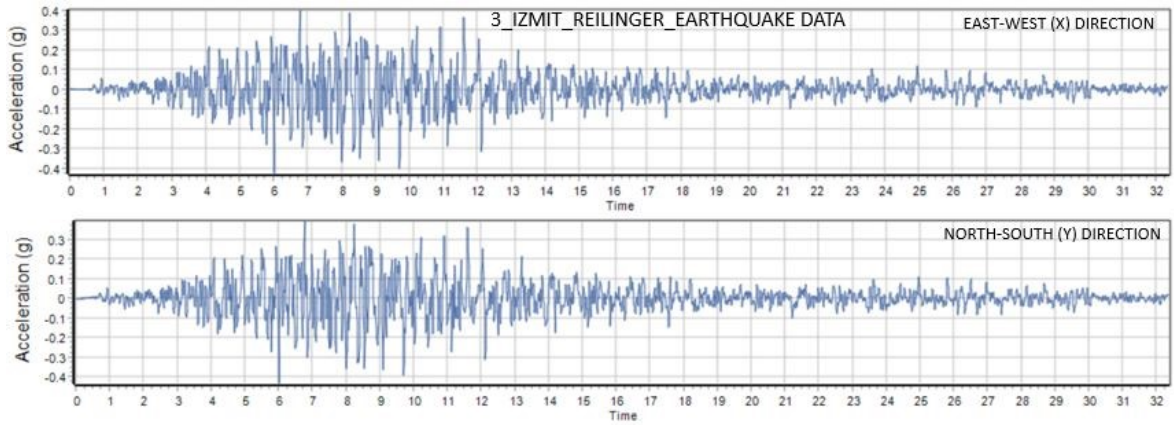


Figure 5.61. Acceleration-time history of the 3_Izmit_Reilinger record for two horizontal components.

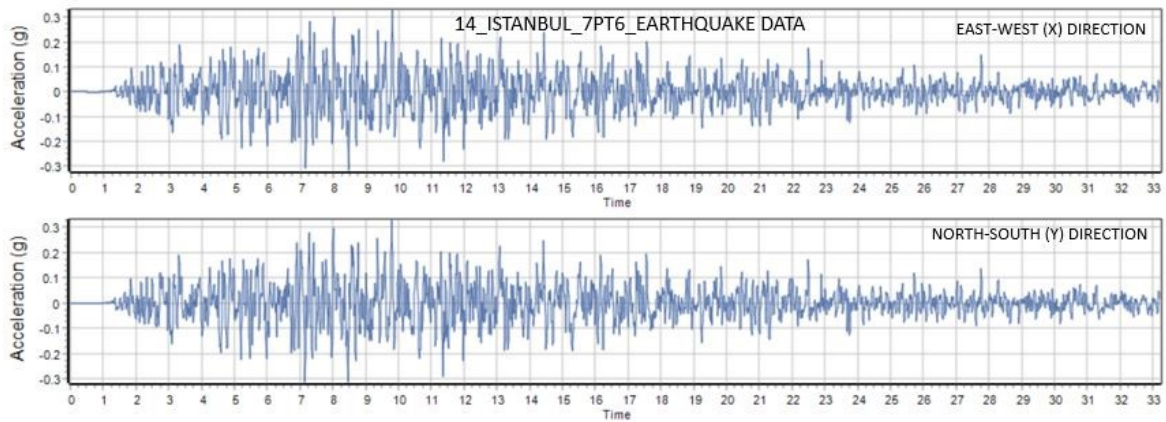


Figure 5.62. Acceleration-time history of the 14_Istanbul_7pt6 record for two horizontal components.

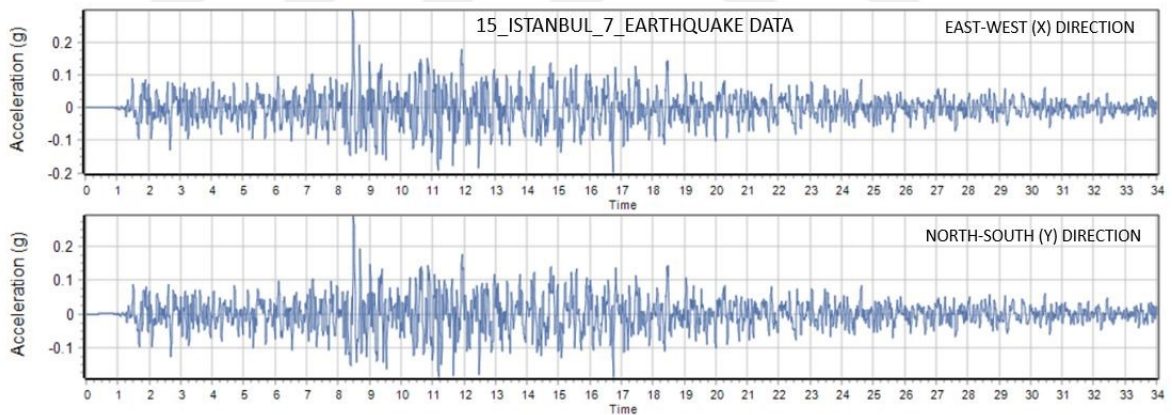


Figure 5.63. Acceleration-time history of the 15_Istanbul_7 record for two horizontal components.

In Ansys Workbench, the analysis does not always continue until the end of the record given as input. Under most of the earthquake records, the analysis stops before it is completed. In some cases, it is due to element failure, i.e. yielding, whereas in others, it is due to the convergence issues which might happen because of insufficient number of CPU cores according to the solver output report of Ansys. In any case however,

modelling of the complete failure is not possible with Ansys. On the other hand, it would be possible to animate the whole collapse of the structure on Ls-Dyna Software. For such model, in every iteration the program would search for the failed elements and erase them and continue analyzing. In that case the computation time and the demanded computer performance would increase, requiring an additional HPC (High Performance Computing) licensing to be able to use more CPU cores in the analysis. In this study, all analyses are carried out on Ansys Mechanical, and the current university licensing limits the maximum number of CPU cores to 4 to be used in the analysis.

Figure 5.65 illustrates the displacements on X and Y directions of the structure under given earthquakes. The order of displacements in X and Y directions at the locations of stations GAL1, GAL2, GAL3 and GAL4 at the gallery level are similar. It is the case also for KUB1, KUB2, KUB3, KUB4 at the dome base level. Therefore, displacements at the gallery and dome base levels are represented by only one location, namely GAL2 and KUB3, for each analysis, to be able to compare them. As mentioned above, under some of the large earthquake records the analysis stops before the end of record is reached. The time that the analysis stops is indicated under the graphs for each ground motion data.

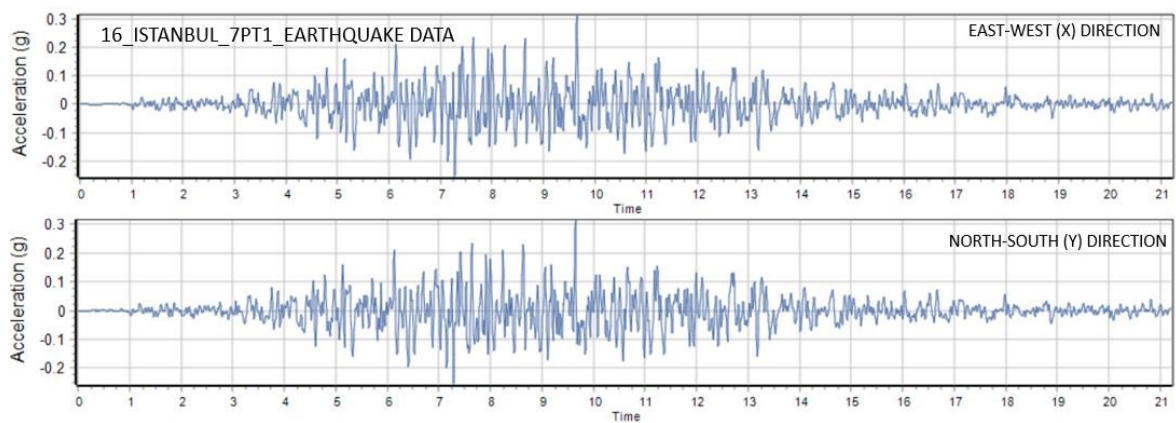


Figure 5.64. Acceleration-time history of the 16_Istanbul_7pt1 record for two horizontal components.

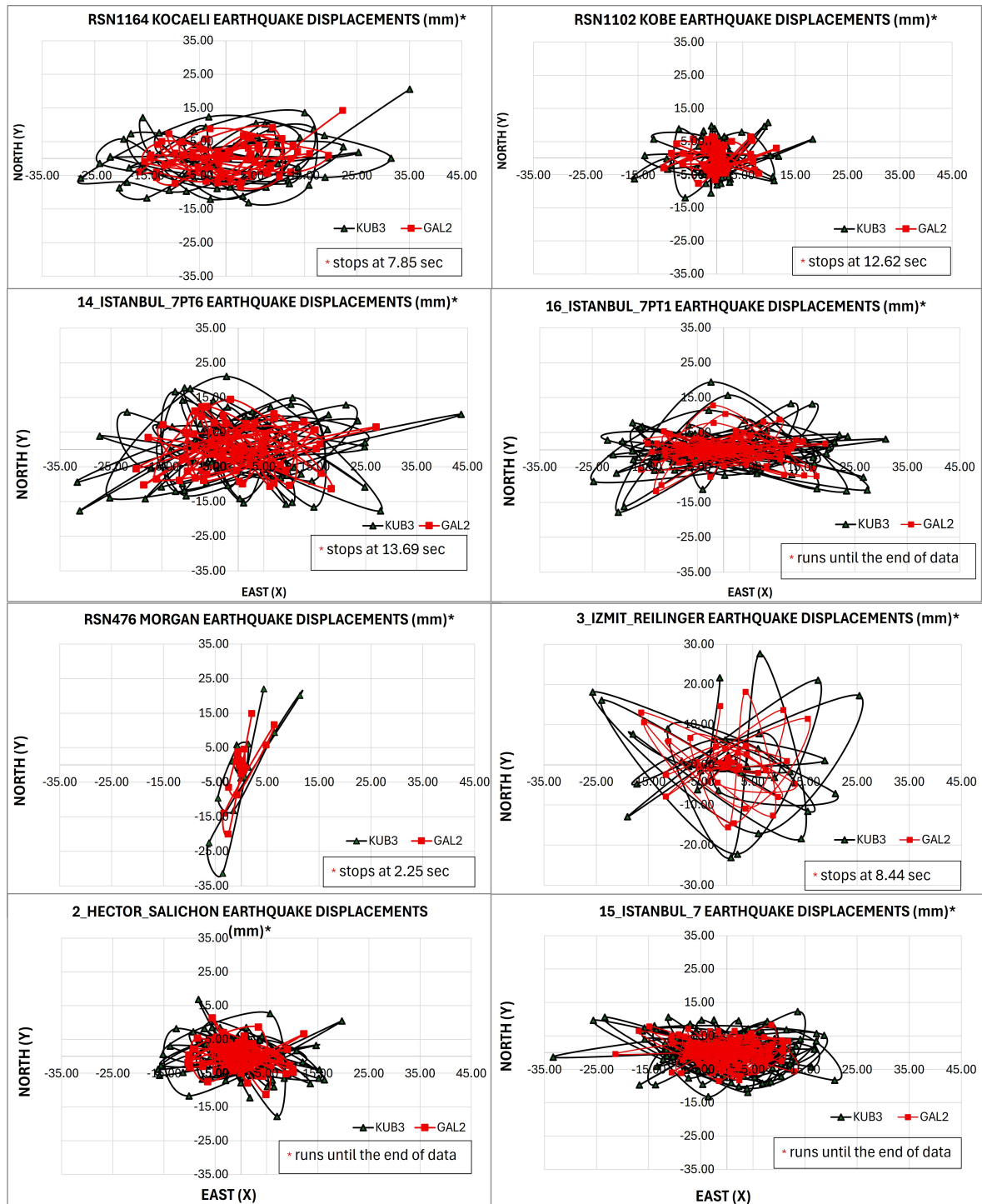


Figure 5.65. Displacements at the locations of GAL2 and KUB3 in X and Y directions under given earthquakes.

This part of the study aims to reveal the failure mechanism of the structure. However, In Ansys Mechanical Workbench, it is not possible to see the crack patterns, instead, when using Drucker Prager material model, it is advised by Ansys Help team to plot plastic strains to get a qualitative plot for cracking [66]. Besides, in the literature, the same method is used when observing the failure mechanism of masonry structures [67], [68]. Therefore, plastic strain contours are used to illustrate the crack pattern to overcome these limitations of the software. The related illustrations are given in Figure 5.66 - 5.72.

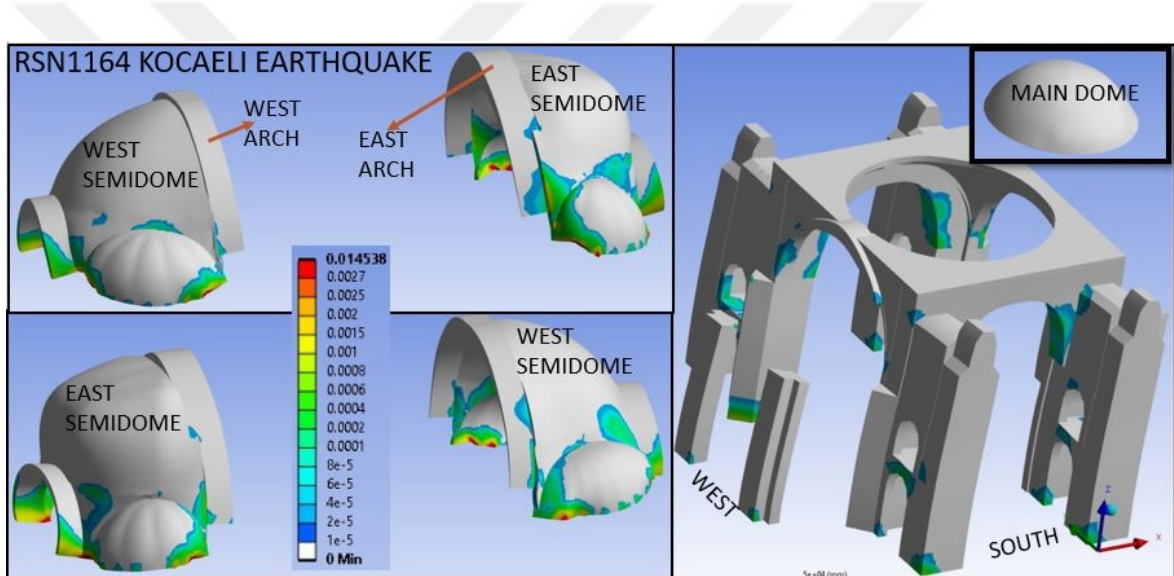


Figure 5.66. Plastic strain contours under RSN1164 Kocaeli earthquake record.

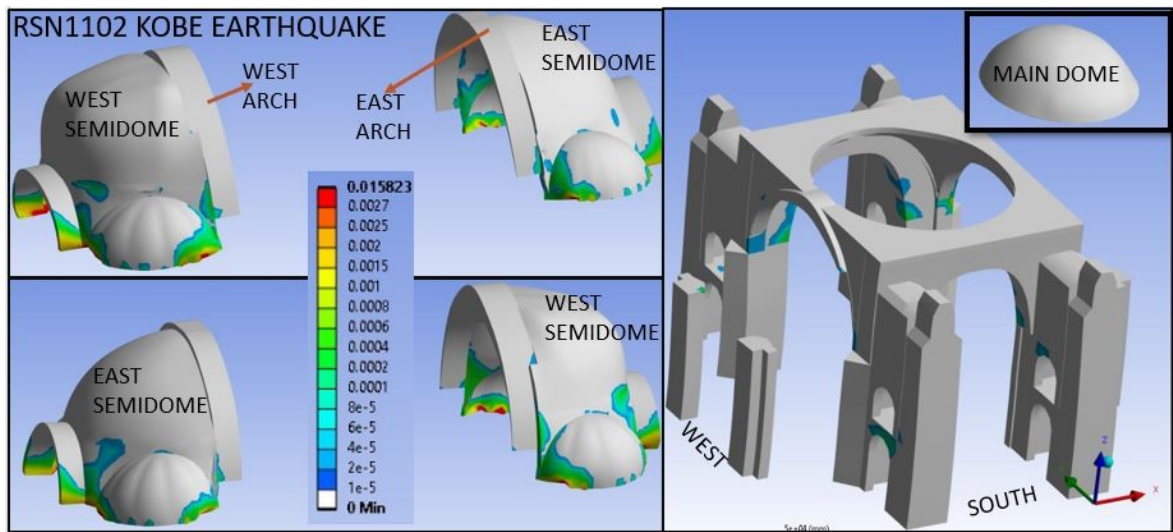


Figure 5.67. Plastic strain contours under RSN1102 Kobe earthquake record.

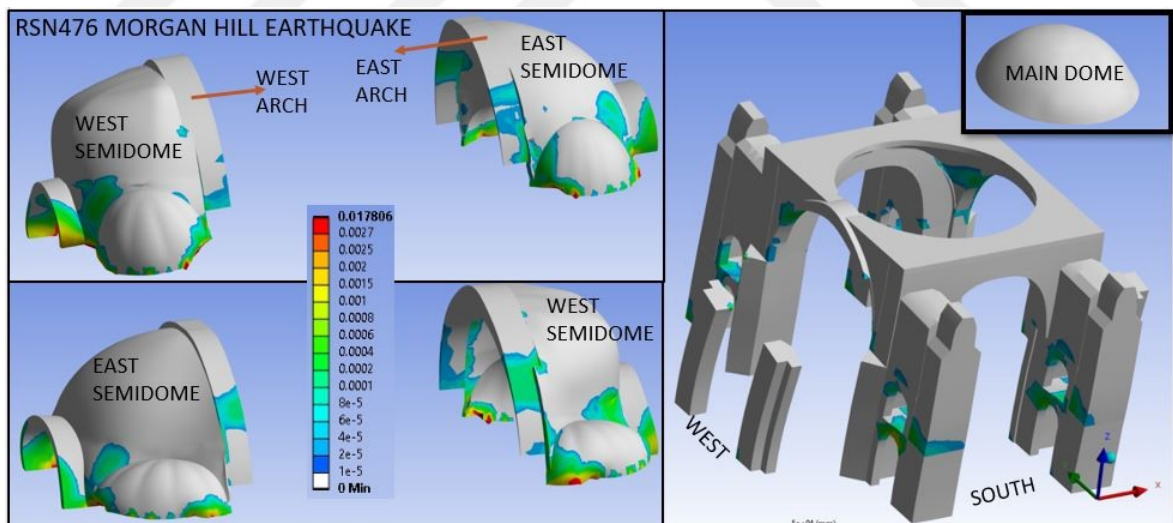


Figure 5.68. Plastic strain contours under RSN476 Morgan Hill earthquake record.

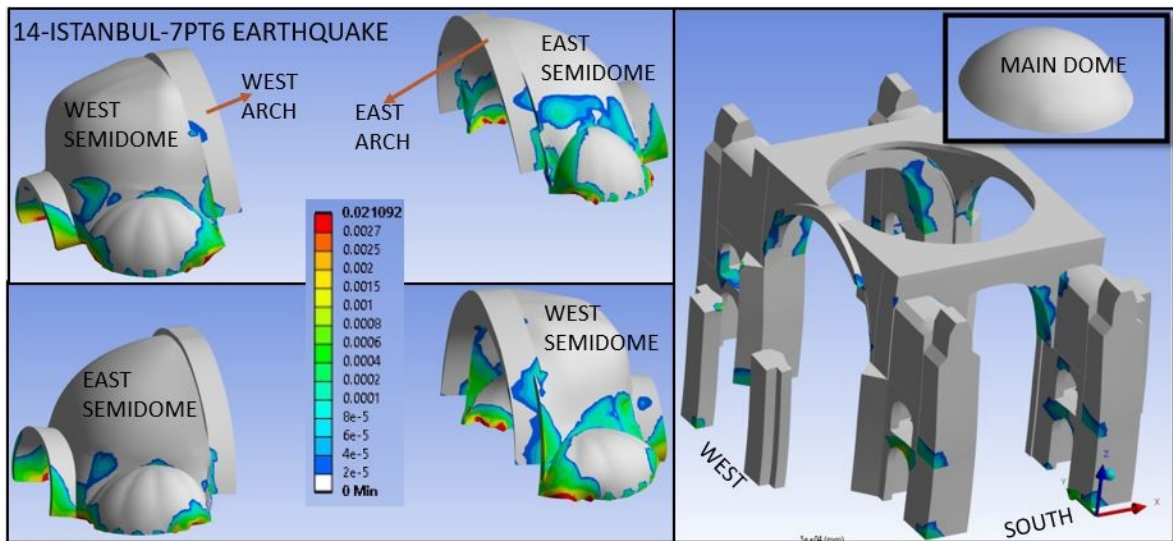


Figure 5.69. Plastic strain contours under 14_Istanbul_7PT6 earthquake record.

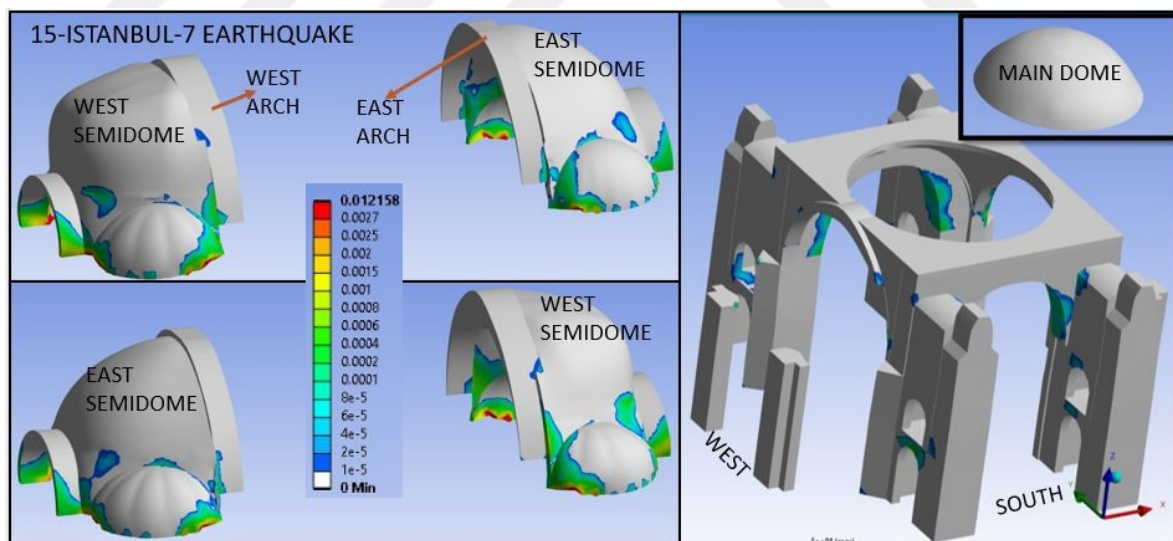


Figure 5.70. Plastic strain contours under 15_Istanbul_7 earthquake record.

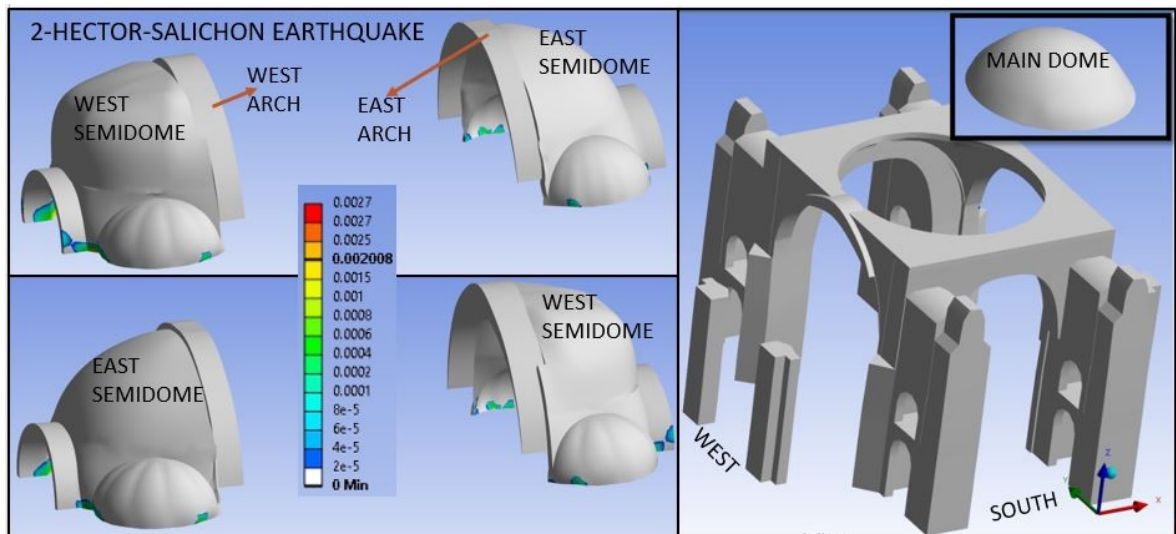


Figure 5.71. Plastic strain contours under 2_Hector_Salichon earthquake record.

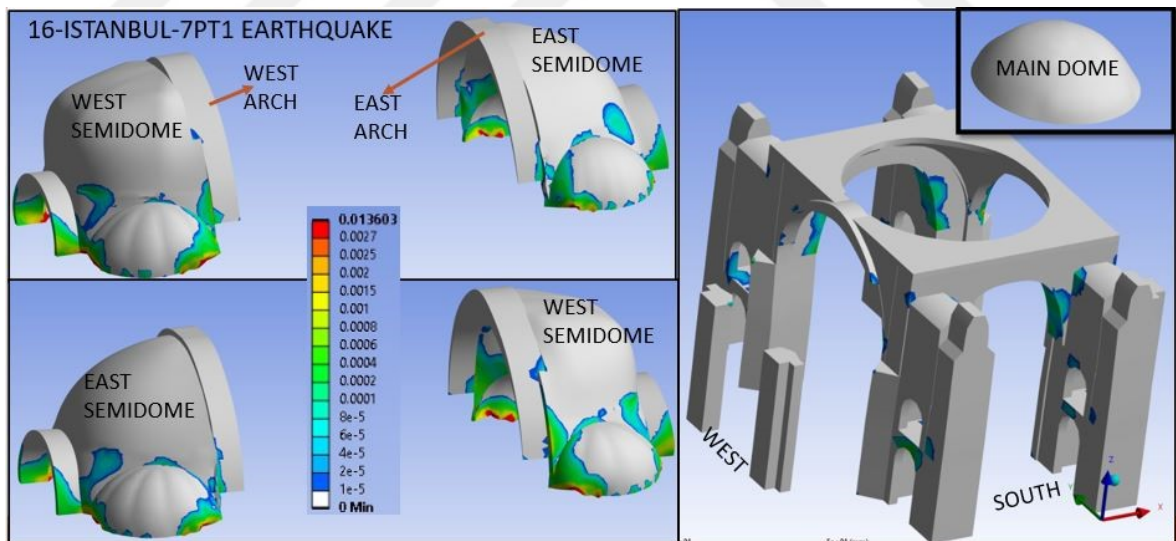


Figure 5.72. Plastic strain contours under 16_Istanbul_7PT1 earthquake record.

According to these results, the most vulnerable regions appear to be the exedra domes. The plastic deformations start to develop where the exedra connect to the east or west main arches. They then propagate towards the semidomes and spread up along

main arch -semi dome boundaries. The plastic deformations also propagate through the east and west main arches starting from their springings and leading to the upper parts of them, never however reaching the crown. They seem to be constrained at about halfway through the height of the main arches. In the majority of the runs, the analysis stops before the end of the input ground motion is reached. However, it seems plausible that the cracks developed across the width of the main arches may lead to the weakening and collapse of the upper half of these two arches, which in turn may lead to the collapse of the upper sections of the semi domes adjacent to the main arches and also adjacent parts of the main dome and the pendentives. Besides, it should be noted that some parts of the buttress arches also exhibit low levels of plastic strains. Plastic strains develop also at the north and south lower arches, however they are probably not as fatal as their counterparts in the eastern and western elements, due to the dual arch system consisting of upper and lower arches on the north and south sides. The collapses in the history of the structure support this idea as well. Hence, based on our findings, it can be proposed that the collapse will probably occur due to the east or west arch failures. This result is reasonable when the earlier partial collapses of the eastern and western parts of the structure due to the earthquakes mentioned in the literature review part of this study are considered. In his book, Mainstone compiles the information on these collapses from the written documents belonging to the era and states that

“Part of the first dome fell after an earthquake in 557 together with the part of the eastern semidome and arch. The subsequent collapses in 989 and 1346 were of similar extent, first of the corresponding parts at the west and then again at the east” [1].

Even though less is known and reported about the extent of the damages and collapse if any at the lower levels in the historical documents, analysis results of this study suggest that the failure starts from the edge of the exedra domes and spreads through the semidomes and main arches at the east and west sides. However, based on our results the cracks concentrating near the exedra - semidome margins seem to spread towards the apsis arch on the east and the barrel vault at the west as well. There-

fore, another possibility for the collapse mechanism which cannot be excluded is that the semidomes collapse following the cracks developed at these margins. The main arches losing the support of the failing semidomes, yield under the thrust exerted upon them by the maindome and collapse subsequently, followed by the adjacent portions of the maindome. Mainstone supports the vulnerability of the exedra domes with the following sentences:

“The most important are the shear failures which run through the exedra close to the terminations of the main piers. These are especially noticeable in the two western exedrae” [1]

As mentioned earlier, the analysis stops before the end of the input ground motion is reached except for 2_Hector_Salichon, 15_Istanbul_7, 16_Istanbul_7PT1 records. As indicated in Figure 5.59, the response spectra of these three ground motions are just above the DD3 level elastic design spectrum of Hagia Sophia. Therefore it may not be surprising for the analysis to run until the end. However, response spectra of the remaining input motions are very close or higher than the DD2 level elastic design spectrum of Hagia Sophia, hence they force the structure much more. It is remarkable to note that north-south (Y) horizontal component of the ground motion has an important role in the damage. As mentioned earlier, the most vulnerable regions are the east and west exedra and the main arches. In-plane direction of the east and west main arches is the Y direction. They tend to develop cracks earlier when forced with the Y component of the ground motion as can be seen in the case of RSN476 Morgan Hill and 3_Izmit_Reilinger records, both of which have larger amplitudes in Y direction. This study aims to investigate the failure mechanism of Hagia Sophia and therefore it deals with how the collapse will probably occur. A further study to look at the performance level of the structure will be valuable to estimate the levels of ground motion the structure will resist, get any level of damage or collapse.

6. DISCUSSION AND CONCLUSION

Hagia Sophia in Istanbul as one of the most remarkable pieces of architecture and engineering in human history was exposed to earthquakes of varying magnitudes over its existence spanning 1500 years. The structure experienced three combined partial collapses of the main dome, adjacent western or eastern principal arches and corresponding semi-domes, all of which were due to earthquakes. It has also received other types of earthquake damage of varying degrees to its structural elements which however are known not to be associated with collapse.

The aims of this research are copied below to assist the reader in following the discussions and conclusions that will be presented in this chapter.

- Evaluation of the level and extent of the present-day deformations on the structure
- Creation of a finite element model with the actual geometry but not the idealized one
- Assessment of the contribution of the permanent deformations to the static and dynamic structural response
- Non-linear static and dynamic modeling to understand the structure's present and future earthquake behavior
- Assessment of the failure mechanisms of the structure under earthquakes

The current deformed shape of Hagia Sophia was obtained from the three dimensional laser scanning data gathered during a recent, detailed field campaign. The point cloud data were processed and meshed to evaluate the present-day geometry of Hagia Sophia. It is a contribution to the literature with new data, and a new modeling and assessment of the structure. The results have not only revealed a complete, recent picture regarding the extent and level of deformations across the structural elements of Hagia Sophia, but the approach that was employed has also provided the complete

geometry of the structure which it is believed should be very similar to the one envisaged by Isidorus of Miletus and Anthemius of Tralles, a design known to come to life partially due to the deformations that started to take place while the construction was ongoing.

It is claimed that the deformations on the structure mostly occurred during the construction period. This is a view that is supported by our analytical results as well. Because when the nonlinear static analysis of the undeformed model is considered, the estimated displacements do not match the current deformation levels on the structure. Hence, if the mortar had set fully and gained its final strength, the self-weight of the structure would not lead to the deformations seen on today's structure. As Mainstone [1] and Livingstone [24] also mention, the mortar gains its full strength in one year. However, it was critical to complete the construction in a short time. So they did not wait for a sufficient amount of time and the self-weight of the structural elements became an issue for the structure causing the deformations.

One of the most challenging parts of analytical modeling of Hagia Sophia is to manage the convergence problems. Since the structure has a quite large and complex geometry, the finite element models have more than 100.000 nodes, different element types (solids, shells, frames), cross sections and different materials with varying stiffness values. Hence, the convergence problems have always been an issue especially when it comes to the nonlinear analysis as reported in the previous studies on Hagia Sophia as well. However, this issue is overcome in this study utilizing the correct restraints and constraints with the help of the calibration procedure.

This is the first analytical model of Hagia Sophia created following its actual deformed geometry. It therefore provides an opportunity to compare the analysis results of the deformed and undeformed models of the structure. The mode shapes of the two models are quite similar. However the modal frequencies slightly differ between the models especially towards higher modes. The dominant frequencies of the deformed model are higher than those of the undeformed model. It should be noted that the

calibration for material properties is carried out on the deformed model using the experimentally estimated modal frequencies obtained by the analysis of data from the Hagia Sophia vibration monitoring system. When the same material parameter values are used in the undeformed model, a reduction around 12 percent was observed in the modal frequencies. This implies that the previous analytical models, all based on Hagia Sophia's ideal geometry, with some calibrated with experimental modal frequencies, in fact tend to overestimate these values.

When stress distributions are considered, even though there are no significant differences in the peak stress levels between the deformed and undeformed models, the stress distribution patterns vary. The main difference in the principal stresses is on the west exedra domes and the buttress arches connecting the main piers and buttress piers. Buttress arches are less stressed in the deformed model. However, the maximum tensions and compressions are higher on west exedra domes in the undeformed model, whereas the west semidome is under less tension in the deformed model. This is due to the reconstructed shape of the west semidome and the widened and heightened west main arch. This observation is confirmed by the results of the static analyses, response spectrum analyses and time history analyses applied on both deformed and undeformed models. Based on our findings the most vulnerable regions appear to be the two western exedra and their parts close to the terminations of the western main arch, locations the vulnerabilities of which were pointed out by Mainstone as well [1].

The model adopted for defining nonlinearity of brick masonry and stone masonry is the Linear Hardening/Softening/Dilatation (Linear HSD) Drucker Prager nonlinear material constitutive model. The model parameters are either adopted from the literature or they are assessed from numerically modelling these materials. It is believed that this is the first finite element model introduced in which nonlinear material behaviour is specified for all elements.

Besides calibrating the model with respect to modal shapes and frequencies, a validation is performed by using the real earthquake records obtained from the struc-

ture during the 23 April 2025 Mw 6.1 Silivri earthquake. The deformed nonlinear model performed remarkably well in estimating the recorded accelerations, velocities and displacements in X direction in terms of amplitude and phase. However, in Y direction, the amplitudes on the dome base level are underestimated and phase-shifts are evident. This difference might be due to local conditions or may be a reflection of local issues transferred from lower levels. It should be remembered that there are still many unknowns regarding the connections, micro cracks and local material irregularities in Hagia Sophia. When calibrating the model with the recorded time histories in Hagia Sophia, the best results are obtained when the damping ratio is taken as 3%.

Nonlinear structural analyses are carried out under real and synthetic accelerations. Real records are selected from the PEER database and scaled for local hazard and site conditions of Hagia Sophia. Broadband synthetic accelerations are site specific, and produced by physics-based simulation approach involving a verified three-dimensional crustal velocity model of the Marmara Sea region and stochastic simulation using region specific models. Nonlinear time-domain analyses are carried out under these earthquake records and the results are interpreted for general response characteristics of Hagia Sophia and for its failure mechanisms.

The structural elements involved in the past partial collapses are known from the literature and are largely traceable on the structure. The underlying reason leading to these collapses, however, is not known and has therefore been the subject of previous investigations, which identified the connecting zones between the main arches and the semidomes near their crown on the east and west as the most vulnerable, attributed the start of failure to these locations and the collapse to the yielding of the main arch under the thrust of the main dome after losing the support of the semidome. The scenario proposed in this thesis is different.

According to the findings in this thesis, the most vulnerable regions appear to be the semidome – exedra dome boundaries and the exedra domes. The plastic deformations start on the exedra near the terminations of the west and east main arches.

They then spread along the exedra-semidome boundaries and along the main arch and semi-domes boundaries. Plastic deformations also penetrate the east and west main arches starting from their springings, and move towards their crowns. They never reach the crown and seem to be constrained to the lower half of the arches.

Observations on plastic deformations suggest that the damage pattern experienced by Hagia Sophia during the majority of earthquakes it was exposed to should be development of vertical cracks near the lower margins of the semidomes, along the exedrae-semidome boundaries and on the exedra domes. In the case of stronger events or in events having stronger Y components, the cracks that develop on the lower half of the main arches and cut along their width, may have led to the weakening of the main arches and under the uneven thrust of the main dome and the semi dome to the collapse of their upper half, followed by the collapse of the adjacent parts of the main and semi domes. A secondary scenario for collapse mechanism might be the weakening and collapse of the semidomes as a result of cracks that develop at their lower margins. The main arches losing the support of the failing semidomes, yield under the thrust exerted upon them by the main dome and collapse subsequently, followed by the adjacent portions of the main dome.

The finite element modelling and the analyses introduced in this thesis are performed on Ansys. The limitations encountered were associated with the allocated number of CPU cores defined in the university license and the adoption of plastic strain contours in assessing structural failures by the software. Assessment of collapse mechanism and its animation can probably be carried out more efficiently by software such as LS-Dyna or by the discrete element approach, both of which have their own limitations, given the size and complexity of a structure such as Hagia Sophia.

An improvement over the nonlinear, deformed finite element model presented in this thesis will be its calibration for the non-symmetrical earthquake response of Hagia Sophia at its gallery and dome-base levels, which is a work already ongoing.

A further study to look at the performance level of the structure will be valuable to estimate the levels of ground motion the structure will resist, get any level of damage or collapse.



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