A STUDY ON SHELL STRUCTURES THROUGH A COMPARATIVE CASE STUDY ANALYSIS

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By

Khawaja Talha Ejaz

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We certify that we have read this thesis and that, in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

A STUDY ON SHELL STRUCTURES THROUGH A COMPARATIVE CASE STUDY ANALYSIS

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M.Sc. in Architecture

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August 2023

Shell structures in architecture have been a topic of interest for both architects and engineers. As they do not only provide aesthetic architectural spaces but also provide efficient engineering solutions to cover larger areas. This thesis examines the historical development of shell structures and analyzes the classification of contemporary shell forms as pure shells. A comparative study of form-finding methodologies and structural systems based on fundamental shell principles is conducted, considering advancements in design techniques. The research investigates the differences in form-finding methods between historical and contemporary shell structures and their impact on design and construction. The role of auxiliary support systems in contemporary shells and their effect on classification as pure forms are explored. A categorization based on structural systems is proposed to distinguish between historical and contemporary interpretations, accounting for complexity, integration of non-shell elements, and auxiliary support systems.

The analysis of structural systems focuses on load-bearing mechanisms, load paths and shell type in terms of original shell behavior in each case study. Findings highlight the shared characteristics, disparities, and patterns in form-finding methods and structural systems. The implications of deviations from historical shell principles in contemporary cases are assessed in relation to their classification as pure shell forms. Furthermore, the degree of adherence to fundamental principles is discussed. The primary objective was to gain a deeper understanding of form-finding techniques, structural systems, and functionality in shell structures over the years. Through this investigation, the research aims to determine how closely contemporary shell structures align with the fundamental principles of historical shell structures. Through an extensive literature review and examination of historical and contemporary case studies, this thesis offers a comprehensive understanding of shell structure evolution.

Keywords: *Shell Structures, Form finding methods, structural system*

ÖZET

KABUK YAPILARI ÜZERİNE KARŞILAŞTIRMALI BİR VAKA İNCELEMESİ ANALİZİ İLE İLGİLİ BİR ARAŞTIRMA

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Mimarlık, Yüksek Lisans

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Kabuk yapıları, hem mimarlar hem de mühendisler için ilgi konusu olmuştur. Çünkü bunlar sadece estetik mimari mekanlar sağlamakla kalmaz, aynı zamanda daha geniş alanları kaplamak için verimli mühendislik çözümleri sunarlar. Bu tez, kabuk yapılarının tarihsel gelişimini incelemekte ve çağdaş kabuk formlarını kabuklar olarak sınıflandırmaktadır. Temel kabuk prensiplerine dayalı olarak form bulma yöntemlerinin ve yapısal sistemlerin karşılaştırmalı bir çalışması, tasarım tekniklerindeki ilerlemeler göz önünde bulundurularak yapılmaktadır. Araştırma, tarihsel ve çağdaş kabuk yapıları arasındaki form bulma yöntemlerindeki farklılıkları ve bunların tasarım ve inşaat üzerindeki etkilerini incelemektedir. Çağdaş kabuklardaki yardımcı destek sistemlerinin rolü ve form olarak sınıflandırılmalarına etkisi araştırılmaktadır. Bu çalışmada, yapısal sistemlere dayalı bir sınıflandırma önerilmektedir. Böylece yapıdaki karmaşıklığı, kabuk olmayan unsurların entegrasyonunu ve yardımcı destek sistemlerine dikkat çekmektedir.

Yapısal sistemlerin analizi, her bir vaka çalışmasında orijinal kabuk davranışı açısından yük taşıma mekanizmaları, yük yolları ve kabuk tipine odaklanmaktadır. Bulgular, form bulma yöntemlerinde ve yapısal sistemlerde paylaşılan özellikleri, farklılıkları ve desenleri vurgulamaktadır. Çağdaş vakalarda tarihsel kabuk prensiplerinden sapmaların kabuk formu olarak sınıflandırılmalarına etkileri değerlendirilmektedir. Ayrıca, temel prensiplere ne kadar uyulduğu tartışılmaktadır. Temel amaç, yıllar içinde kabuk yapılarında form bulma teknikleri, yapısal sistemler ve işlevselliği daha derinlemesine anlamaktır. Bu araştırma, çağdaş kabuk yapılarının temel tarihsel kabuk prensiplerine ne kadar yakın durduğunu belirlemeyi amaçlamaktadır. Kapsamlı bir literatür taraması ve tarihsel ile çağdaş vaka çalışmalarının incelenmesi aracılığıyla bu tez, kabuk yapı evriminin kapsamlı bir anlayışını sunmaktadır.

Anahtar kelimeler: *Kabuk Yapılar, Form bulma yöntemleri, yapı sistemleri*

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CHAPTER 1

INTRODUCTION

The discipline of architecture has been historically fascinated by the alluring design and remarkable efficacy of shell structures. Throughout history, architects and engineers have consistently endeavored to explore novel methods for designing shell structures that possess both a lightweight nature and structural durability. The appeal of shell structures is not solely limited to their visually captivating aesthetics, but also encompasses their capacity to offer practical resolutions to a diverse range of architectural obstacles.

The shell behavior examined in this study pertains to slender, curved structural components that primarily distribute the applied loads through in-plane membrane forces, namely tension or compression. The observed behavior can be attributed to the significant increase in mobilized membrane stiffness, surpassing the corresponding bending stiffness. Historical shell structures possess a thickness that is smaller than one-twentieth of their curvature radius. (Ramasway & kreiger, 1984)

Historical shell structures serve as a prime illustration of how the state of equilibrium is attained by means of form, thereby diminishing the reliance on additional material consumption and strength. This phenomenon facilitates the increase of load-carrying capacity in materials, even when there is a decrease in material usage (Pedreschi & Macdonald, 2000).

Throughout the course of history, a multitude of notable examples of shell structures have arisen, showcasing the limitless ingenuity and technical proficiency of architects and engineers. From the renowned Pantheon in ancient Rome to the innovative architectural creations of Félix Candela and Heinz Isler, these typologies have captivated designers and engineers through their elegant design, resourceful utilization of form, and capacity to cover substantial spans without additional internal reinforcements.

Gaining a comprehensive understanding of the fundamental principles and form-finding methodologies utilized in the design and construction of shell structures holds significant importance in the progression of the architectural discipline. Through the examination of these principles, architects acquire a deeper understanding of the structural dynamics exhibited by shells, enabling them to conceive novel designs that not only satisfy aesthetic aspirations but also effectively address functional necessities. In addition, a comprehensive understanding of form-finding techniques empowers architects and engineers to enhance the structural efficacy and efficiency of shell structures, thereby fostering sustainable and ecologically mindful architectural resolutions (De Souza, 2017).

The examination of shell structures is inherently interdisciplinary, encompassing various fields including structural engineering, mathematics, material science, and computational design. Through a comprehensive exploration of the historical progression of shell structures and a meticulous examination of the various form-finding techniques utilized, architects and researchers have the opportunity to extract significant insights and fundamental principles that can effectively help the classification of contemporary shell structures in context of pure shell form (historical) and point out any differences from the historical shells.

As the architectural field continues to push the limits of what is possible, there is a growing imperative to delve into and understand the complexities inherent in contemporary shell structures. Shells possess inherent qualities that render them highly efficient and visually appealing, thereby making them a desirable choice for architects in search of sustainable, visually captivating, and structurally sound solutions. Through the exploration and expansion of form-finding methodologies, professionals in the fields of architecture and engineering have the potential to discover novel design opportunities and make significant contributions to the advancement of the built environment.

1.1 Problem and Research Statement:

The objective of this study is to examine the changes of the fundamental principles of conventional shell structures in the contemporary shell design examples. The primary emphasis lies on the increased complexity, incorporation of non-shell components, and dependence on supplementary support systems, potentially posing a challenge to the categorization of these structures as purely shell forms. This study aims to investigate the historical and contemporary interpretations of shell structures by analyzing the characteristics and design strategies employed over the years.

Contemporary shell structures have made notable progress in design methodologies and material utilization. However, it is imperative to evaluate the extent to which these advancements align with the fundamental principles of a historical shell. The growing intricacy, incorporation of non-shell components, and dependence on supplementary support systems in modern designs need to be studied for the classification or lack thereof of these structures as pure shell forms. The ambiguous demarcation between conventional and modern interpretations of shell structures gives rise to the need for further research regarding the similarities and differences of contemporary shell designs to the historical ones.

The importance of this study lies in the lack of comprehensive analysis of contemporary shell structures questioning the fundamental principles of a shell structure. Through an analysis of basic principles of historical shell design in conventional and contemporary shell structures, professionals in the field of architecture and engineering can acquire valuable perspectives on the progressive development of shell design and its implementation. The study aims to make a contribution to the advancement of comparative knowledge in the historical evolution of shell structures.

With their minimal use of material, efficient load-bearing capacity, and their ability to provide large column-free spaces, Shell structures have been useful for both architects and engineers. From the earliest examples of thin-shell structures to contemporary works, shells‟ design process and construction have evolved over time. The form-finding approach is one of the most crucial aspects of shell design, enabling the creation of aesthetically pleasing and structurally stable shapes. Advanced materials, such as reinforced concrete and composite steel materials, have enabled the construction of complex shell geometries that were once thought impossible.

Shell structures are classified as 3D curved structures in which generally one dimension of the structure is significantly smaller than the other two. The shell's inherent three-dimensional shape provides the required stiffness to support external loads (Paoli, 2007). Shell structures resist forces predominantly through membrane action. The loads on shells are transferred to the ground through in-plane stresses that are commonly compressive, but could be a combination of both compressive and tensile stresses (Adriaenssens et al., 2014). The key to the optimum performance of shell structures is to have all the stresses act in the shell‟s curved plane. That way, the shell does not undergo any out-of-plane stresses such as bending. Thus, undergoing only membrane stresses allows shell structures to be built slender over larger spans (C. Meyer and M. Sheer, 2005).

The success of the structural efficiency of shells depends predominantly upon their geometric forms rather than the materials used in their design (Michiels and Adriaenssens, 2017). The phenomenon of finding optimal geometric shell forms to have the structure in static stability is referred to as *form-finding*. Form-finding is one of the most crucial factors while designing shell structures. To keep all the forces acting in the shell plane, a functional geometric shell form needs to be generated, keeping in mind the aesthetic requirements of the structure.

Many parameters, such as boundary conditions, supports, etc., are directly controlled in the form-finding process. The possible outcomes of the final shell form could be infinite as it is an iterative process (Adriaenssens et al. 2012). However, some commonly used forms for shell structures are hyperboloids (used as water storage), dome structures, hyperbolic paraboloids, and cylindrical shells. All these forms present their own respective strengths and advantages for shell structures (Dailamani, 2010).

Most of the pioneers of RC shell structures realized their famous shell structures without the help of numerical modelling software that are available today. 1925 -1975 is considered to be the most prominent period for shell structures because of their popularity in the construction realm and the number of shell structures built during that time (Tang G, 2012). That era saw the pioneer shell builders deploy numerous techniques to find the form of structurally sound and aesthetical shells. However, due to limited or no digital help, they relied upon analytical and experimental methods to find the optimum forms of the shells. This resulted in the structural aspects of the shells pre-dominantly governing their final form over their architectural features. Recent advances in computational design and analysis have made it possible to design and construct even more complex and intricately thin concrete shell structures as well as other types of shells. These structures utilize the latest computer-aided design and analysis techniques to create unique and striking forms that would not have been possible with historical design methods.

This dissertation investigates and compares the structural identity of shell structures designed using digital tools, relative to historical shell behavior that are commonly designed using mechanical and analytical form-finding methods. It also compares the structural systems of the contemporary shell structures with the traditional shell structures and identifies additional structural elements in contemporary shell structures.

1.2 Aim and Scope of Thesis

The objective of this thesis is to examine the development of shell structures in the last 100 years and analyze shell behavior through a comparative examination of their form-finding methodologies and structural systems. This research assesses the degree to which contemporary shell structures conform to the fundamental principles and characteristics of historical shell structures, taking into account advancements in design techniques and materials.

The study will center on the assessment of the form-finding methods and structural systems of shell structures, encompassing both historical and modern examples. This will provide insight into the potential deviations and challenges encountered when classifying contemporary shell structures as authentic shell forms, through a comparative and contrasting analysis of these aspects.

The primary objective of this thesis is to make a scholarly contribution to the existing knowledge regarding the historical development of shell structures. Additionally, it aims to initiate academic discussions regarding the categorization and understanding of contemporary shell structures within the framework of conventional understanding of shells in architectural and engineering design.

This study carries significance within the field of architecture due to its objective of investigating the historical progression of shell structures and evaluating the classification of contemporary shell structures by means of a comparative analysis of their form-finding methodologies and structural systems with regards to pure shell behavior. This research enhances the current body of knowledge in architectural history by examining the distinctions between historical and contemporary shell structures. It illuminates the progression of design methodologies and the utilization of structural systems over time. Gaining a comprehensive understanding of the form-finding techniques utilized in historical and contemporary shell structures, as well as their consequential influence on the design, structural systems and construction, offers significant insights for professionals in the disciplines of architecture, engineering, and academia. Furthermore, through the evaluation of the form-finding, construction techniques, and structural systems of shell structures, this research enhances our comprehension of the capacities and constraints of this architectural typology. In conclusion, this research serves as a catalyst for scholarly comparative analysis and encourages additional investigation into the correlation between historical principles, technological progress, and the interpretation of shell structures in modern architectural practices.

The research presented in this study is distinguished by its comprehensive methodology, which investigates the historical evolution and classification of shell structures. Previous research has focused on examining isolated elements of shell structures, such as form-finding methodology. However, this thesis aims to provide a more comprehensive analysis that considers various factors, including form-finding methods together with structural systems. Through the implementation of a comparative analysis encompassing historical and contemporary case studies, this study amalgamates a diverse array of examples, thereby facilitating a holistic comprehension of the development and diversification within the field of shell structures. This study aims to provide an analysis of the variations encountered in classifying contemporary shell structures as traditional shell forms.

1.3 Research Questions and Hypothesis:

- What are the differences in form-finding methods used in early examples and contemporary shell structures, and how do these differences impact the design and construction of the shells?
- What role do auxiliary support systems play in contemporary shell structures, and how do they affect the shell forms and pure shell behavior?

The hypothesis of this study is as follows: Shell structures of the digital era exhibit notable progress in design methodologies and material utilization, albeit potentially deviating from the fundamental principles inherent in historical shell structures. The hypothesis posits that the growing complexity causes incorporation of non-shell components, and dependence of supplementary support systems in these shell structures which may pose a challenge to their identity as shell structures.

1.4. Structure of Thesis:

This dissertation is organized into six chapters. Chapter 1 introduces the problem statement, research questions, aims, and scope of the thesis.

Chapter 2 provides an overview of the literature on shell structures, the historical evolution in the form-finding methods of shell structures, common geometries, and surfaces of shells. It lays the ground for understanding different type of shell structures and their different form-finding methods over the years.

Chapter 3 explains the methodology and steps undertaken to assess the structural behavior of shell structures designed with different form-finding approaches. It describes the case studies selected for this thesis and the justification behind the selection of those case studies.

In Chapter 4, case studies are introduced and analyzed according to the defined methodology. The main emphasis is to determine whether the shells in the case studies act as the sole load-bearing elements or do they have additional structural systems and the functionality with regards to original shell behavior.

Chapter 5 discusses the results and classification of contemporary shell structures in the scope of this thesis. It highlights how the shells once built as a sole load-carrying system over large spans evolved into complex hybrid structures.

Chapter 6 summarizes the thesis and outlines the limitations of the thesis.

CHAPTER 2

SHELL STRUCTURES

2.1 Origin of Shell Structures:

Shell structures are characterized by their use of a structural system characterized by their relatively slender, curved surface. These structures are known for their ability to span large distances with minimal material, making them popular for architects and engineers looking to create visually striking and lightweight buildings. The earliest examples of shell structures in architecture mainly consist of roof structures of churches, temples, and mosques. One of the oldest structures in Europe with a dome roof is the Pantheon, built in 126 AD. Another ancient example of a dome structure is Hagia Sofia in Istanbul, constructed approximately 1500 years ago. More recently, numerous roof shells have been built worldwide thanks to the advancement of reinforced concrete, space framing systems and a greater understanding of structural dynamics. The construction of the Carl Zeiss spherical planetarium, only 30mm thin reinforced concrete shell with a 16-meter diameter in Jena, Germany, in 1922, can be regarded as the start of a new era for the building of thin RC roof shells. This building"s design resulted from the joint efforts of Dischinger and Walter Bauersfield (Hineset al., 2004). According to Bechthold (2008), there have been two distinct periods in the history of rigid structural surfaces in architecture. The first, from 1912 to 1939, that saw the design and construction of structures derived from vault and dome construction. The pioneering efforts of Dyckerhoff and Widman AG (Dywidag) are recognized for initiating this movement, as they constructed their inaugural concrete shell in 1922 for the Carl Zeiss spherical planetarium. (Bechthold. M, 2008). Introducing their design philosophy and Construction method, in 1936, Tedesko constructed the first large-scale thin roof shell in the U.S in the shape of a barrel vault shell with a large 222 ft (71 m) span for an ice hockey stadium in Hershey, Pennsylvania (Saliklis and Billington, 2007). Two significant advancements in thin shell design and construction were made possible by Tedesko's efforts to export the idea of thin roof shells to the United States. They are the ribless shells and the short, wide-spanning barrel shells (Hines, Billington, et al., 2004).

In the 1930s and 1940s, the use of thin concrete shells in architecture gained popularity, particularly in the work of architects such as Felix Candela and Eduardo Torroja. Candela, a Mexican architect and structural designer, developed a technique for constructing thin concrete shells using a technique known as "shells of revolution," which involved rotating a two-dimensional curve about an axis to create a three-dimensional form. This technique was used to construct several of his iconic structures, such as the Los Manantiales Restaurant in Mexico City. Candela also developed the “hyperbolic paraboloid” method, which was characterized by a double curvature and an efficient way to distribute the load over the structure (Cassinello et al, 2010). Due to Félix Candela's comprehension of the influence of geometric shape on structural behavior, especially in the use of new material such as reinforced concrete, he was able to enhance the design of shell structures. Consequently, the resulting structural components are lightweight and slim, with dynamic and flowing shapes. The development and construction of hypars, or hyperbolic paraboloids, is considered a significant advancement in conventional architectural shell forms. Additionally, Candela used this surface most frequently, with hypars comprising 90% of his constructed projects (Cassinello, 2010).

The hyperbolic paraboloid is generated through the displacement of one parabola along another parabola. Notably, the curvature of the second parabola is characterized by an opposite sign compared to the first parabola. The planes that encompass the parabolas must exhibit orthogonality to one another, thereby rendering them as surfaces of translation. Translation surfaces are generated through the process of displacing a planar curve along another planar curve while maintaining parallel form with the original plane. The angle between the two entities remains invariant, and the planes encompassing them are mutually orthogonal (Figure 2.1).

Hyperbolic paraboloid shells are classified within the category of shells characterized by surfaces exhibiting negative or anticlastic Gaussian curvature, thereby presenting a wide range of morphological possibilities. The initial implementation of this particular shell variant was carried out on a significant scale in 1933 by Aimond, a French engineer (Cassinello, P. (2010). The contributions of Félix Candela have played a significant role in the utilization of hyperbolic paraboloids in expansive dimensions.

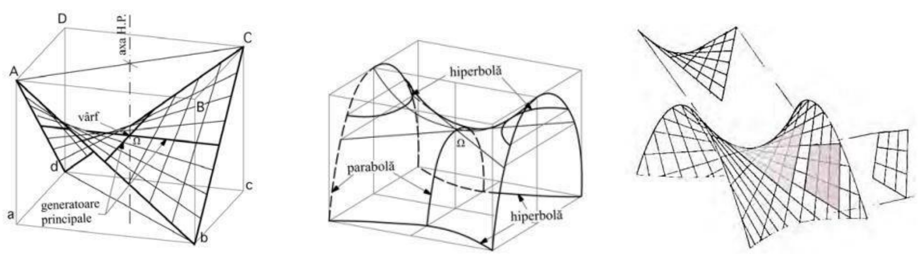


Figure 2.1 Generation of Form through Hyperbolic Paraboloid (Rusu, 2015)

From 1925 to 1975, the majority of masonry and concrete shells were built. As shell structures gained popularity, Innumerable industries, warehouses, metro stops, grandstands, theaters, cinemas, churches, restaurants, and homes were covered in concrete shells over the five golden decades. Eduardo Torroja, a Spanish engineer and architect, also made significant contributions to the development of thin concrete shell structures (Cassinello et al, 2010). He designed several notable structures, including the covered market in yecla (1933) and the roof of the Spanish Pavilion at the 1937 Paris World Fair. According to Cassinello, Schlaich, and Torroja (2010), this growth in the popularity of masonry and reinforced shell structures was mainly because of the pioneer architects/engineers, better referred to as shell builders. Some of the most notable names include Felix Candela, Pier Luigi Nervi, Heinz Isler, Anton Tedesko and Ulrich Muther.

Numerous roof shells constructed in various parts of the world demonstrate the simultaneous contribution of architectural and engineering work. Examples include the work of the roof shells created by Isler (Switzerland), Candela (Mexico), and Nervi (Italy) (Ortega 2005). Concerning the quantity and variety of concrete shells constructed, the period from the 1920s to the early 1960s is regarded as the golden age for shell structures (A.C. Scordelis, 1985). Subsequently, there has been a decline in the construction of concrete shells, and a decrease in the number of research papers exploring their analysis and design methods. This decline in the popularity of shell structures can be partly attributed to the complex construction techniques required during the construction phase (E.M. Hines, D.P. Billington, et al., 2004).

The work of the pioneers opened new doors of possibilities and shapes in terms of shell designs and construction. The Los Manantiales restaurant's construction was completed in 1958 in Mexico City. It featured a hyperbolic paraboloid reinforced concrete roof, 16 mm thick, which had been designed by Candela and built-in 1951 for the Cosmic Ray Pavilion in the same city. Both structures revolutionized the concept of slenderness, establishing new benchmarks in the field (R. Bradshaw et al., 2002). The new architectural potential of shells was also demonstrated by Candela in his design of the Los Manantailes restaurant, where an eight-sided groined vault composed of four connecting hyperbolic paraboloid saddles (Burger and Billington, 2006). Another example is Pier Luigi Nervi‟s dome-shaped roof structure of Plazetto Dello Sport, completed in 1957 in Rome for the summer Rome Olympics (Lori and Poretti, 2005). 1960s was an essential decade for shell structures. This decade saw new design theories and construction processes due to designers like Pier Luigi Nervi, Eduardo Torrja, Vasilii Vlassov, and Felix Candela experimenting with various shell forms. Many shell theories were also put forth at this time to explain the behavior of shell formations better. Many iconic double-curved shell structures were built during this period all across the world. However, by the end of the 1960s, the concrete shells had started to lose their charm. This was also reflected in the loss of popularity of concrete shells and the subsequent decline in the technical papers published regarding concrete shells in the years that followed. A survey was conducted among engineers, architects, and people concerned with the construction industry that revealed that the two main reasons behind the decline in the popularity of shell structures were; the high construction costs and the highly trained labor required to avoid any small errors (Meyer and Sheer, 2005). Minor errors during the construction phase could make a huge difference in the shells' geometric stability and internal forces. That is why expert labor was preferred for the job concerning shell structures.

Shell Surface:

Shells surfaces are characterized by their minimal thickness relative to their other two dimensions. The geometrical quality known as "developability" is a crucial factor determining the performance of shell geometries such as hypars etc. The figure shows that shell surfaces can be classified as either developable or non-developable in this regard. Developable surfaces, exemplified by the barrel vault in Figure 2.2, can be easily transformed into a flat plane without cutting or stretching. Monoclastic shells are an example of developable surfaces. On the other hand, synclastic and anticlastic surfaces fall under the category of non-developable surfaces, as they cannot be deformed into a flat plane without alterations, as explained further in the next topic. The significant advantage of undevelopable surfaces over their developable counterparts is their superior strength and stability due to the need for more external energy to cause any deformation. (Farshad, 1992).

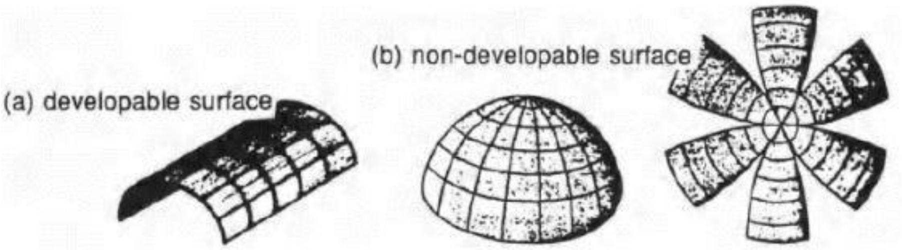


Figure 2.2 Developable and non-developable surface (Farshad, M 1992).

2.2 Shell Curvature

Shells are made up of curved surfaces. These surfaces can be classified into three types: singly curved, doubly curved, or free-form. A singly curved surface, like a barrel vault, has a singular type of curve. Doubly curved surfaces can be further divided into two types: synclastic and anticlastic. A synclastic surface, like a dome, has the same type of curve in every direction and has positive Gaussian curvature at all points. An anticlastic surface, like a hyperbolic paraboloid, has opposite curvature in different directions and has negative Gaussian curvature at all points (Bagneris et al., 2008). Free-form surfaces cannot be defined by curvature and are not described by simple mathematical equations.

A scalar value called the Gaussian curvature describes how curved a two-dimensional surface is in three dimensions. It is named after the mathematician Carl Friedrich Gauss, who extensively studied curvature in the 19th century. The Gaussian curvature at a specific point is defined as the product of the maximum and minimum curvatures in two perpendicular directions on the surface, known as the principal curvatures. The Gaussian curvature can be interpreted as measuring how much a small surface patch deviates from being flat, either curving inward or outward relative to its surroundings. A positive Gaussian curvature indicates a locally spherical surface, while a negative one indicates a locally saddle-shaped surface. The Gaussian curvature has significant applications in various fields, such as differential geometry, topology, and physics. It can be utilized to study the shape of surfaces in different contexts, such as the geometry of space-time in general relativity or the properties of 3D objects in computer graphics.

2.2.1 Singly Curved / Monoclastic Surface

A singly curved surface has curvature around only one axis with zero Gaussian curvature. If one of the principal curvatures of the surface is zero, then such surfaces are called monoclastic surfaces .These types of curved shells are commonly seen in barrel vaults shown in Figure 2.3 (a). Singly curved shells are typically suitable for covering large spaces owing to their light weight. However, when compared to doubly curved shells, they do not perform as well against seismic loadings (Michiels &Adreaenssens, 2017). As one of the most common fundamental shell surfaces, it has been widely used as roof shells in many structures, notably in Louis I. Kahn"s Kimbell Art Museum in Texas, USA, as shown in Figure 2.3 (b).

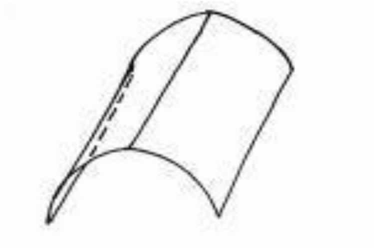


Figure 2.3 (a) Singly Curved/Monoclastic shell (b) The Kimbell Art Museum Louis I. Kahn (Moussavi, 2009)

2.2.2 Doubly Curved Surface

(a) Synclastic

If a doubly curved surface has a positive Gaussian curvature, it is known as synclastic. In its basic form, a doubly curved surface with a positive Gaussian curvature takes the geometry of an elliptic paraboloid (Figure 2.4). In this type of surface, both principal curvatures have the same sign, and the surface usually experiences in-plane meridional and circumferential stresses to bear loads.

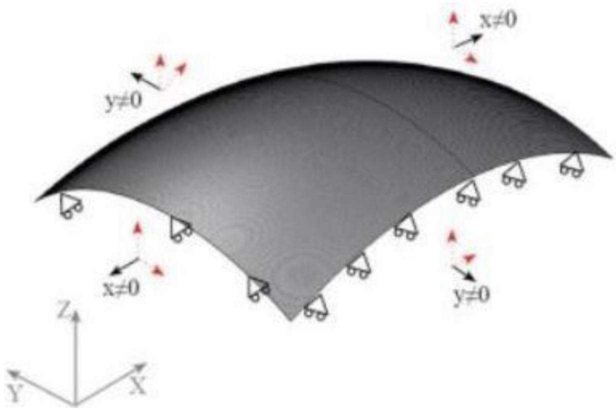
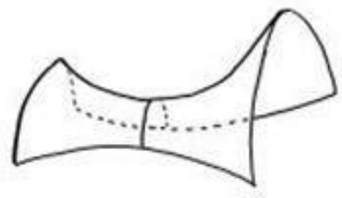


Figure. 2.4 Doubly curved synclastic shell (Michiels and Adriaenssens, 2017)

(b) Anticlastic

Anticlastic surfaces are doubly curved surfaces with negative Gaussian curvature, with principal curvatures having an opposite sign. When the principal curvatures of a surface have opposite signs, the surface can exhibit compressive and tensile arch behaviors when subjected to perpendicular loads (Peerdeman, 2008). These types of surfaces can be constructed using straight lines, which is why they are referred to as ruled surfaces. F. Candela has employed these surfaces frequently in his designs because of thesimple construction procedure owing to straight form work, particularly for reinforced concrete shells (Figure 2.5). The construction procedure is based on two straight lines acting as the main guides, with another straight line resting on these two to generate hyperbolic paraboloid surfaces. If the generating line is inclined and rests on two circles, it will result in a single-sheet hyperboloid of revolution (Figure 2.6). While designing shell structures, hyperboloids and their components are commonly utilized.



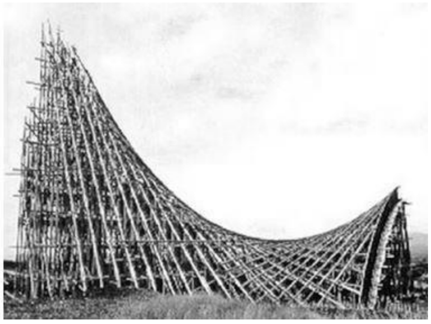


Figure. 2.5 (a) Anticlastic curvature (b) Form-work for Chapel Lomas de Palmira at Cuernavaca by Felix Candela

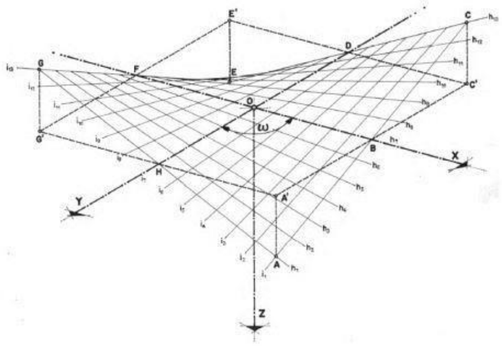


Figure. 2.6 Two straight generator lines system that forms a hyperbolic paraboloid (Faber, C. 1963)

2.3 Form-finding:

Form finding is a design process that involves both physical and digital methods to determine the shape of a structure, achieving a state of static equilibrium while considering specific loads and boundary conditions. It highlights that the structure's form is the ultimate outcome of the design procedure (Veenendaal & Block, 2012). The overall design and form of a shell's geometry determine its ability to hold weight and cover large areas. When engineered correctly, shells that function as membranes can be lightweight structures supporting a significant load (Ramm & Wall, 2004). There are various methods for creating a shell's shape, including developing two-dimensional shapes, using mathematical analysis, applying mechanical or cable-based designs, utilizing inflatable membranes, experimenting with freeform designs, and utilizing numerical optimization techniques.

2.3.1 Analytical Approach

While the historical theory of structures treats geometry as a predetermined input, focusing on evaluating and verifying internal stresses as the primary objective of the analytical process, Musmeci, 1979 challenges this perspective. He argues that, in the realm of structural design, the form should be seen as the true unknown variable. This innovative insight marks a paradigm shift that introduced fresh possibilities in structural design. Consequently, analytical methods transform into powerful design tools that enable control over the form of the structure and the utilization of materials. Musmeci's approach emphasizes the importance of considering form as a fundamental element in the design process, prompting a reevaluation of conventional methodologies in structural engineering (D‟Acunto & Ingold, 2016). Numerous shell forms can be determined and represented as mathematical functions. This includes simple geometries such as cylinders and spherical structures and more complex shapes such as a hyperbolic paraboloids (hypar). These types of shell forms that can be represented as a mathematical function are known as analytical (Bagneris et al., 2008). These analytical shapes were more prevalent among architects and engineers before the advent of finite element methods (FEM). They only required hand calculations to solve their intrinsic load calculation issues (Meyer & Sheer, 2005). Some examples of analytical shells and their analytical expressions are shown in Figure 2.7

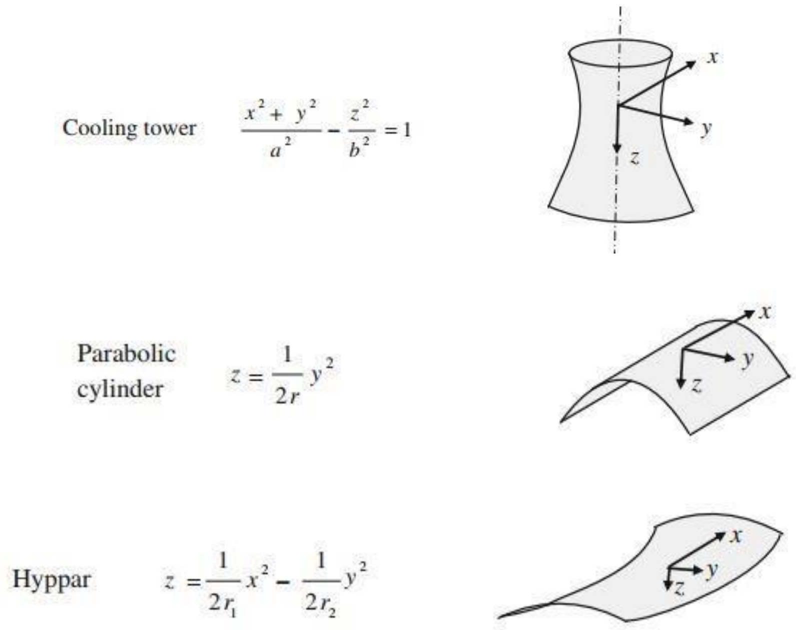


Figure. 2.7 Analytical shell geometries

2.3.2 Mechanical Approach

The process of determining a structure's shape through mechanical constraints such as hanging the fabric under gravity from fixed ends is referred to as "mechanical" in literature (Bagneris et al., 2008). For mechanically restrained shells, various form-finding methods can be used, focusing on determining the geometry that ensures a state of static equilibrium. The main aim of these methods is to ensure a *funicular* form for the shell structure. The form-finding methodologies employed by Otto, Isler, and Musmeci are primarily influenced by natural phenomena, albeit from distinct perspectives. Otto, for instance, examined "nature" as a biological process and closely collaborated with German natural scientists. On the other hand, Isler's fascination with the natural world indicates a more formal connection to its materialized physical phenomena (Neri, 2014). Isler aimed to recreate natural shell shapes by engaging in a physical form-finding process. While the designer sets the experiment's parameters, such as boundary conditions and material selection, the process itself unfolds organically. Isler's approach shares similarities with Otto's experiments, but their objectives differ. While Otto focused on studying the process's evolution, Isler‟s aim was to achieve a specific form through his static intuition. His concept of the pneumatic membrane pertains to a surface under pressure, while the form attained through the hanging membrane is based on extending the catenary curve into three-dimensional space (Boller et al, 2021). Heinz Isler was among the most prominent designers deploying experimental form-finding to build his concrete shells. One of the experimental form-finding methods used by Isler was using the reverse hanging technique, which was based on suspending fabric in a polymerized resin, and then inverting it once it had hardened (Larson, 2003) (Figure 2.8)

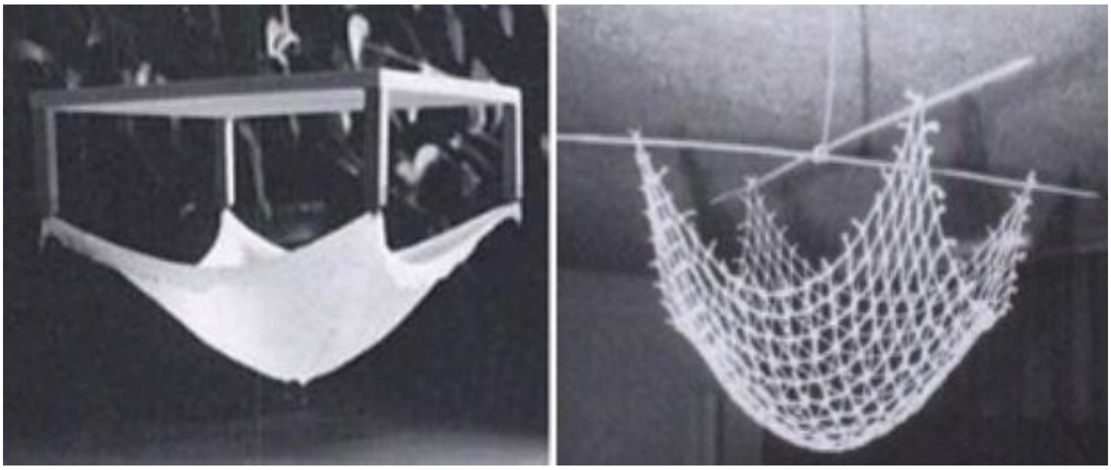


Figure 2.8 Heinz Isler‟s hanging fabric models

(https://blog.buildllc.com/2009/04/heinz-isler-a-few-important-things/)

This technique is particularly intriguing due to the correlation between a fabric's ability to handle tension and a concrete's ability to withstand compression. Using this method, the most optimal structural shapes were determined by hanging the fabric, with the fabric primarily experiencing tension - a strength of the said material. To transfer this same curvature to concrete, the model was solidified using epoxy resins and then turned upside down, putting the material under compression. Several architects have incorporated hanging chains and cloth in their structural designs. When natural sources of form inspiration are examined, one of which is hanging chains, it demonstrates that a square-mesh chain net hanging naturally can take on the shape of conventional Asian roofs. Alternatively, by suspending inverted chain nets in various ways, architects can create the perfect framework for arches, domes, and vaults. The same core principles apply to a hanging cloth (Frei Otto and Bodo Rasch, 1995). Another experimental technique, known as the inflated membrane method, ensured a funicular shape under internal pressure. Having numerous construction benefits, one issue with this approach is that the optimal shape, known as the funicular shape, can limit the design options for the shell. While the funicular shape may be structurally efficient, it may not necessarily be the best option in terms of other factors, such as freedom in architectural design. Furthermore, well-known for his vast model testing methods is Pier Luigi Nervi. He held the view that the value of intuition is immense for an engineer as it leads to innovation. This is demonstrated by his design for the 1935 Italian Air Force competition (Chao, 2005). Owing to a dearth of theoretical understanding and analytical tools, he developed an experimental celluloid model to understand structural stresses better. He added ribs and stiffeners to the second design's model to improve it. He was among the very first designers to employ prefabricated components.

It is evident that each set of physical models represents a specific type of static force equilibrium, adhering to the principle of "form follows force." However, all the groups of physical models, whether hanging models or pneumatic form-finding, maintain a state of pure tension under specific loads and boundary conditions. By appropriately modifying them (such as inversion, proportional scaling, and section design) and constructing them as shell structures, these models can retain a high level of structural efficiency (Li et al, 2017).

2.4 Structural Behavior of Shells:

For effective shell design, a solid understanding of the structural behavior of shells is essential. Depending on the types of loads the shell is required to handle, its behavior can vary greatly. Shells are form-based structures, and their shape affects how much load they can support. Their structural geometry substantially determines the evolution of stresses in the shell elements. The structural behavior of the shells must be considered to prevent any unintended deformations and structural collapse.

2.4.1 Membrane Theory:

It is crucial to understand that the resistance of a shell to loads primarily relies on the generation of forces within the plane, referred to as membrane forces, while minimizing the development of bending and shear forces. In the case of in-plane forces, the stresses exhibit a nearly uniform distribution across the entire thickness of the material. This implies that the entire cross-sectional area is effectively utilized and subjected to uniform stress in order to maintain equilibrium. The presence of bending forces significantly reduces the efficiency of this behavior, as it leads to non-uniform utilization of the surface and generally higher levels of stress (Sewell &Keller, 1987).

The predominant equilibrium of membrane forces is a key characteristic that substantiates the structural efficiency in shell structures when compared to flat slab structures. The membrane theory postulates the presence of negligible bending moments and is required to adhere to the following conditions (Montoya, et al , 1981)

1. The shell should possess a slender profile, indicating that the ratio between its thickness and the radius of curvature of its average surface should be exceedingly minute. This condition is characterized by a diminished capacity to produce bending forces, which are only observed following substantial deformations.
2. Ideally, the mean surface needs to exhibit a state of continuous curvature. Alternatively, the presence of discontinuous zones may lead to the occurrence of bending and shear forces.
3. It is recommended that the loads be distributed instead of being concentrated. This condition guarantees the absence of localized bending forces that is necessary for the equilibrium of concentrated loads.

4. The supports and edge elements must exhibit compatibility with the free edge deformations of the membrane

According to the membrane theory, the inability of shells to support out-of-plane loads is due to their behavior acting like a membrane. They differ from plates in that they transfer these loads by producing in-plane membrane forces. Unlike plates, shells can expand and contract without significantly bending or altering the local curvature (Maten, 2011). The shell should ideally solely be designed for membrane forces because doing so makes it possible for it to be thin and more cost-effective. This is generally applicable to RC shells. However, the presence of undesirable loadings and boundary conditions make this an impossible task. Under these circumstances, the membrane stress is insufficient to achieve equilibrium, leading to disturbed patches with added stresses. Hence, a bending theory is necessary to thoroughly study shell behavior (Blaauwendraad and Hoefakker, 2014).

2.4.2 Bending Behavior:

The shell's bending behavior is a complex phenomenon limited to areas where the stresses on the membrane are inadequate to support the external loads. In such regions, the bending moments that arise only compensate for the insufficiency of the membrane's behavior and do not contribute to carrying the load (Peerdeman, 2008). Interestingly, the concentrated forces that cause the bending moment in the shell create an area affected by the bending moment. In contrast, the rest of the shell continues to behave under membrane behavior. This behavior of shells provides an advantage for designers as it demonstrates the structural efficiency of shells. With this capability, designers have the freedom to create slender shells, even in situations where membrane-incompatible conditions are present. They can then implement specific optimizations or structural remedies to handle these conditions, while the majority of the shell retains its true membrane functionality (Kanta, 2015)

2.5 Free Form Design

The shell structures not constrained by any geometric regularity or repetition are known as free-form shells. These structures are not bound by the aforementioned design approaches (Haakonsen, 2013). Free-form shell structures consist of mainly complex architectural design that utilizes advanced computer-aided design and manufacturing techniques and structural systems to create aesthetically pleasing and structurally efficient structures. These structures are characterized by their unique, flowing shapes that are often inspired by nature. Free-form shell structures are well known for their aesthetic appeal. The unique, flowing shapes of these structures can create a sense of flow and fluidity that is not found in historical structures. This can make these structures stand out as unique and visually unique architectural features. While it is difficult to encapsulate such structures under a curvature umbrella, free-form shells are often identified by the sections of their doubly curved surfaces (Bagneris et al. 2008).

One of the major differences between free-form shell structures from the previously-mentioned shells is that they allow for a great deal of flexibility in design. Unlike historical structures, which are typically limited by the design constraints discussed in the previous section, these structures provide endless possibilities in design that are visually striking (Fig 2.9).

Figure. 2.9 Typical conceptual view of a free form shell (Kedilaya B, 2018)

However, these complex structures may require complex load calculations (using digital methods) and additional structural systems. The introduction of 3D computer programs and architects utilizing these programs to design irregular curved surfaces has led to the majority of shell theories no longer providing analytical solutions for these complex curved surfaces. Instead, numerical methods must be utilized to calculate these structures. The shape design method of free-curved RC shells, which employs sensitivity analysis, is a prime example of a structural design approach that maximizes the benefits of modern engineering practices (Sasaki, 2005). This method is based on the principle of determining a rational shape for the shell structure using computer technology to minimize strain energy, particularly bending strain energy (Ebata et al., 2003). Essentially, the method uses numerical algorithms to simulate the processes of physical model experiments that are typically used to define the shape of such structures.

Complex curved designs do not always carry the load optimally to the foundation (without any additional support system) from a structural standpoint. To tackle this challenge, Sasaki employs sensitivity analysis, which begins by establishing an initial digital shape based on an envisioned curved surface within a study model. The goal is then to find an engineered shape that closely resembles the initial envisioned shape while optimizing it structurally to minimize strain energy. This is achieved by computing the strain energy gradient with respect to changes in the vertical coordinate (z) for each node. The z-values are then adjusted in the direction that reduces strain energy, and this process iterates until the strain energy reaches a point where no significant change is observed (Sasaki, 2005).

Furthermore, the advancements in computer technology have given architects more freedom in their design process. With the help of new digital design techniques, almost any shape can be created both in the digital and physical worlds. This evolution in computer technology has led many architects to investigate the potential of new, non-Cartesian surface arrangements (Durmazoglu 2008). Another addition to the study of shell structures has been the usage of unorthodox reinforcement in the shells. For example, fiber-reinforced polymers have been utilized in elastic grid shells because of their notable elasticity (Xiang et al., 2021). A more contemporary investigation showcased the advantages of incorporating fiber-reinforced polymers in freeform structures. The study employed force density and topological mapping techniques to design the shell, highlighting the benefits of utilizing fiber-reinforced polymers in this context (Moskaleva et al, 2023).

CHAPTER 3

METHODOLOGY

This chapter presents an overview of the methodology used in this study. The chapter provides the overview of the data collection procedure and case study selections, emphasizing the diverse sources utilized and the methodologies employed to acquire relevant information. In addition, the parameters employed for the analysis of the gathered data are highlighted, thereby guaranteeing coherence in the interpretation of the findings. This chapter lays the groundwork for the following chapters and offers a framework for effectively carrying out the research study by establishing a precise and clearly defined methodology.

This study aims to conduct a comprehensive review of the existing literature in order to gain a deeper understanding of the topic by employing the following:

- Performing a comprehensive examination of scholarly literature, books, research papers, and technical report publications pertaining to shell structures, encompassing both historical and contemporary examples of shell structures.
- Analyzing case studies offering comprehensive information on form-finding methods and structural systems.

- Retrieving information observations related to the parameters of concern (form-finding methods and structural systems) from the existing body of scholarly works.

3.1 Selection of Case Studies:

It is important to choose a comprehensive collection of case studies that encompass various time periods and demonstrate a diverse array of attributes in terms of form-finding and structural systems, across both the historical shell structures and the contemporary shell structures. It is imperative to verify that the chosen case studies possess an ample amount of information readily accessible in the existing body of literature. This is crucial in order to enable a comprehensive analysis and facilitate meaningful comparisons.

For a consistent research analysis in relation to the scope of the thesis, case studies are mainly selected based on their different form-finding methods i.e., mechanical, analytical, or freeform using digital tools. In total, 18 case studies are selected that are further classified under two groups i.e., Historical/classical case studies and contemporary case studies.

3.2 Classification of case studies:

The two groups of case studies are discussed below.

1. **Historical/Classical case studies:**

The shell structures that employ mechanical and/or analytical approaches (as discussed in the previous chapter) in their design and were constructed in the 20th century are structured within this category of case studies. These cases studies are selected based on different form-finding methods that cover the entire spectrum of form-findings methods in shell structures designed during the golden age of shell structures. The case studies in this category include:

Table 3.1 Historical Case Studies

Structure	**Architect**	**Date**
Chapel Lomas de Palmira at Cuernavaca	Felix Candela	1958
The Kimbell Art Museum	Louis I.Kahn	1972
Deitingen Service Station	Heinz Isler	1968
Coop center	Heinz Isler	1960
Palazetto Dello Sport	Pier Luigi Nervi	1960
Thompson Arena	Pier Luigi Nervi	1975

2. Contemporary case studies:

Shell structures that were not designed using analytical or mechanical approaches but instead contain a combination of form-finding methods, computer-aided design and advanced structural solutions to achieve their complex form, and were constructed in the 21st century is grouped within this category of case studies. This diverse group of cases include wide range of different types of shell structures including RC shell structures, Steel space-frame structures, wooden grid shell etc. The list is shown in the table .

Table 3.2 Contemporary Case Studies

Structure	Architect	Date
Meiso no Mori Crematorium	Toyo Ito & Associates	2006
Centre Pompidou, Metz	Shigeru Ban Architects	2006
Rolex Learning Center	SANAA	2010
Heydar Aliyev Center	Zaha Hadid Architects	2012
The Sage Gateshed Auditorium	Mott MacDonald	2004
L"Oceanografic Restaurant	Felix Candela, Alberto Domingo	2003
Downland Gridshell Building	Tedd Cullinan	2002
Bosjes Chapel	Steyn Studio	2016
Auditorio de Tenerife	Santiago Calatrava	2003
Teshima Art Museum	Ryue Nishizawa	2010
Savill Building	Glen Howells Architects	2006
Harbin Opera House	Ma Yansong	2015

Collection of comprehensive data for the case studies, including architectural drawings, photographs, technical specifications, design documentation, historical records, and any accessible reports is realized. Data is gathered related to the form-finding methods utilized in each case study, encompassing graphical or physical methodologies, experimental techniques, and innovative design approaches.

The study of historical case studies involves a methodical investigation and evaluation of previously published research, articles, books, and other pertinent sources of information related to the chosen historical constructions. The procedure entails the collection and integration of data from sources that provide valuable insights into the form-finding methods and structural systems utilized in those shell structures. The analysis of historical case studies establishes a basic framework of pure shell behavior on the basis of which contemporary case studies are compared.

The analysis of contemporary case studies involves systematically searching, evaluating, and synthesizing similar sources used for historical case studies and other relevant sources of information related to the form-findings and structural systems of the case studies to gain insights into the shell design and the inherent need for additional structural systems that deems the structure‟s behavior away from the historical shell behavior.

In this study the case studies were selected from a wide variety of recent and historical architectural projects, each of which features distinctive design principles and engineering solutions. The investigation aimed to identify the presence of conventional shell behavior within the structures through a thorough assessment of their form, load distribution, and structural systems. The examination also sought to locate any additional structural components that were incorporated into the designs, clarifying how they supported the total load-bearing capability.

3.3 Key Parameters for the Comparative Analysis:

As discussed earlier, the two key parameters to methodically compare and analyze the case studies are form-finding methods and structural systems.

Form finding:

This study examines the form-finding methodologies employed in each case study, taking into account their historical backdrop, design principles, and influence on the overall configuration and geometry of the structure. It involves the mechanical and analytical form-finding methods used in the historical case studies as well as the digital tools used in the design of contemporary case studies.

Structural systems:

Case studies are analyzed by comparing the structural systems utilized in each case study, focusing on their load-bearing mechanisms, material selections, and load paths in terms of original shell behavior.

Finally, the results obtained from the comparative analysis by identifying shared characteristics, disparities, and patterns in form-finding methods and structural systems across the various case studies are materialized. The findings are presented within the framework of the research questions and hypotheses outlined in the thesis. The deviations from historical shell structure principles are studies in contemporary shell cases and the implications of these deviations are analyzed on their classification as pure shell forms. The degree to which contemporary shell structures adhere to or deviate from the fundamental principles of historical shell structures is discussed.

3.4 Categorization of Contemporary Case Studies:

The contemporary case studies are then further classified into three groups based on the resemblance of their structural systems, or lack thereof to traditional shell behavior.

Category 1 encompasses case studies that exemplify traditional shell behavior, characterized by structures that derive their strength and stability primarily from their shell-like geometry. These designs showcase the inherent strength and efficiency of shell structures, demonstrating their ability to support substantial loads without the need for additional structural elements.

In contrast, Category 2 incorporates case studies that retain traditional shell behavior while incorporating supplementary load-bearing elements. These additional structural elements, though not deviating significantly from conventional design principles, enable engineers to address specific challenges or enhance the overall performance of the structure. This category reflects a subtle fusion of traditional and modern approaches, showcasing the adaptability of shell structures to meet evolving design requirements.

Category 3 represents a departure from traditional shell behavior, featuring case studies that adopt entirely different structural systems. These designs challenge conventional notions of shell structures and explore novel engineering solutions to achieve their intended functions. By deviating from the established norms of traditional shell behavior, Category 3 case studies push the boundaries of structural engineering, offering fresh perspectives on how shell structures can be conceived and realized.

In the end, a timeline is presented for the contemporary case studies to show whether the traditional shell behavior in the contemporary case studies or the divergence of these case studies from traditional shell behavior follows a linear timeline.

CHAPTER 4

CASE STUDY ANALYSIS

This chapter presents a detailed analysis of the case studies through existing literature and technical design documents. The analysis focuses on the form finding methods and structural systems employed in the case studies. The chapter is divided into two main sections of historical and contemporary case studies. Firstly, through the analysis of historical case studies, the basis of the pure/historical shell behavior is established and in the second part of the chapter, the contemporary cases‟ conformity to the historical shell behavior is analyzed through the exploration of the structural systems.

4.1 Historical Case Studies

This section deals with the historical/Classical case studies of shell structures that have been selected for this study. This set of case studies contain shell structures designed by analytical and mechanical/experimental form finding methods. The six case studies in this section cover the majority of form finding methods that were used before the advent of computer aided design in architecture. In the scope of this thesis, historical case studies include The Chapel of Lomas de Cuernavaca, Deitingen Service Station, Kimbell Art Museum, Coop Distribution Center, Palazetto Dello Sport, and Thompson Arena.

4.1.1 The Chapel of Lomas de Cuernavaca:

The Chapel of Lomas de Cuernavaca, also known as Cuernavaca chapel is located in the town of Cuernavaca, Morelos Mexico (Figure 4.1). In collaboration with architects Guillermo Rosell and Manuel Larrosa, Candela created a hyperbolic paraboloid with a saddle shape that was significantly modified for the structure in 1958. The front free edge which is the taller side rises to an impressive height of 21.9 meters while the widest point of the shell spans 31 meters. (Figure 4.2)

Figure. 4.1 The Chapel of Lomas de Cuernavaca (Chris J.K Williams)

Félix Candela recognized a fundamental principle which emphasized the importance of minimizing bending forces through the careful selection of an appropriate structural form (Cassinello, 2010). This principle is especially well-suited for structures made of concrete or masonry, considering the inherent limitations in terms of tensile strength. The utilization of the hyperbolic paraboloid form was a prominent feature in the works of Félix Candela, signifying the form-finding aspect of his architectural designs. The utilization of this particular form was attributed to its capacity to effectively fulfill significant structural and construction prerequisites in a simultaneous manner. Similar approach was utilized by Candela in the design of the Chapel.

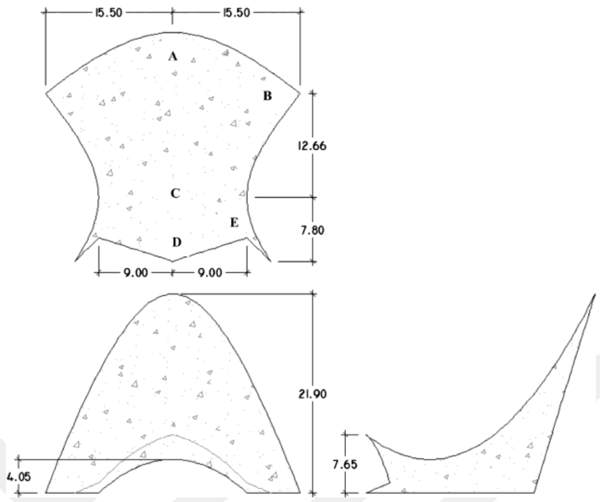


Figure 4.2 Geometrical Properties of The Chapel (Draper 2008)

To address the lack of symmetry, Candela incorporated an edge beam on the opposite rear side of the shell. Although the shell's thickness remains mostly 4 centimeters (1.6 inches) throughout, it is increased along the sides. Specific information regarding these thicker regions is limited, apart from personal observations by Draper in 2008. The width of the taller opening's base was measured to be 52 centimeters. The equilibrium of the hyperbolic paraboloid form is primarily achieved through membrane action. This form is characterized by its composition of linear elements, which greatly facilitates the construction process in terms of formwork (Garlock & Billington, 2008)

The examination of Candela's various structures indicates that each structure exhibited a progressive development from previous designs, such as the reduction in support quantity and the augmentation of spans. Therefore, it is conceivable to assert that each structure served as a comprehensive prototype from which knowledge was obtained. An example of this phenomenon, which holds significant pertinence, can be observed in the accomplished endeavor to construct thin-shell structures like the Chapel of Lomas Cuernavaca featuring exceedingly slender and with non-reinforced free edges (Garlock & Billington, 2008). This development represents a notable progress in the field of thin shell construction, potentially stemming from the recognition that manipulating the form can effectively redirect and mitigate the concentration of force pathways along the edges. As a result, this approach helps to alleviate stress concentrations and bending forces. Furthermore, the selection of extremely slender dimensions, including the periphery, results in the elimination of structural stiffness that could give rise to unwanted forces.

The primary benefits associated with the use of hyperbolic paraboloid as the shell form in the Chapel design are as follows (Cassinello, P. (2010):

- The primary mechanism by which a hypar counteract loads is through the generation of forces within the plane.
- The load-bearing system employed by the subject exhibits a high level of efficiency, enabling the creation of aesthetically valuable, lightweight, and versatile solutions while achieving substantial material savings.
- The structural analysis for shells of this nature has been extensively developed since it is an analytical form
- It demonstrates a high level of resistance to various forms of instability phenomena.

With the design of the Chapel, Felix Candela showcased that the hyperbolic paraboloid's fundamental shape doesn't confine designers to preconceived geometries. He revealed that within the realm of hyperbolic paraboloids, there are opportunities for imaginative and innovative designs. By strategically maximizing the impact of a tall opening, he emphasized the captivating effect of the free edge, unlocking new avenues for creative expression (Draper et al., 2008).

Hyperbolic paraboloid structures can be found in many construction and architectural projects, particularly in the design of roof systems or when multiple shell surfaces are required. The surface can bear compression or stress in any direction or at any point, tangential to its curvature, and can be supported at the two lower points (Beles, 1964). Double-curvature shells offer significant advantages in terms of their mechanical behavior compared to other forms (Domingo et al., 1999).

To effectively transfer the primary load of a roof, which is the self-load, a design approach that utilizes suspended parabolas with downward curvature is favored for tension forces, while parabolas with upward curvature are preferred for compression forces. It is advantageous that these parabolas align precisely with the pressure lines and possess the capability to support their own weight. As a result, the deviation tendencies of the forces from the thin surface's parabolic curvature are minimized from the initial stages, ensuring a highly efficient load distribution system (Rusu, 2015).

The highlighted section of the structure represents the shell working entirely under membrane action. As the shell is symmetrical with respect to one axis, the opposite side to the highlighted region is also under membrane action without any out of plane bending forces. No columns or additional load bearing system is present in the shell. The entire shell works under the principles of a traditional thin shell, providing unrestricted space underneath it (Figure 4.3).

Figure 4.3 Shell under membrane action (Chris J.K Williams)

These shells can avoid the presence of bending moments, making them highly desirable. This behavior is attributed to the arch effect in two planes, which enables them to support various load configurations primarily through internal membrane forces, minimizing the risk of bending. Additionally, these surfaces maintain their form almost unchanged and remain in equilibrium regardless of the load distribution, within certain limits. The inherent mechanical characteristics of this geometry make it an efficient structure, even without the need for stiffening elements like edge beams. Consequently, shell structures designed to function as membranes are inherently optimal in terms of their performance. The hyperbolic paraboloid shape of the shell effectively reduces bending and eliminates the necessity for ribs or additional reinforcement to support its thinness. Despite its substantial size, the structure maintains low stress levels, enabling it to maintain an impressive thinness and free edges (Draper et al., 2008).

4.1.2 Deitingen Service Station:

Deitingen service station is a thin RC shell that serves as a service and gas station that was designed in 1968 by Heinz Isler in Solothurn, Switzerland (figure 4.4). Isler, renowned for his expertise in constructing concrete shell structures since the 1950s, employed a systematic approach that involves the deliberate use of physical models for form-finding. By utilizing various mechanical and physical models, such as funicular forms, pneumatic forms, and floating forms, Isler developed stable and aesthetically appealing shell shapes (RAMM et al., 1989).

Figure 4.4 Deitingen Service station (Nicolas Janberg)

In contrast to many other shell structures that derive their shapes from basic Euclidean geometric objects, Isler's free-form shells exhibit exceptional structural behavior. These shells are characterized by their optimal shapes, which maximize their performance and integrity. Deitingen service station is a prime example of Isler's design philosophy. Notably, this structure features free edges, eliminating the need for additional support from edge beams or other structural elements in the facade.

Isler's meticulous methodology, which revolves around the use of physical models to pursue optimal shapes, has significantly influenced the field of shell construction. His approach not only pushes the boundaries of what is conventionally achievable but also demonstrates the potential for seamlessly integrating structural and architectural considerations. Through his innovative techniques, Isler has established himself as a visionary in the realm of concrete shell structures, leaving a lasting impact on the discipline.

Heinz Isler had a notable skepticism towards technology, and it is safe to assert that during the early stages of his career, there were limited finite element programs available to analyze his shell structures. Therefore, similar to his mentor and professor Lardy, he relied on physical models as a means to demonstrate the structural stability and durability of his designs. According to Isler, it was when he observed the configuration of jute fabric draped over steel reinforcement mesh (which was being used damp to assist in concrete drying) that he initially recognized the potential for form-finding using suspended cloth (Chilton J, 2000).

Isler used hanging models to physically portray the structural shapes he wanted his concrete shell structures to have. His aim was to produce shapes that automatically centered themselves in response to gravity by suspending cloth or other materials that could later be hardened, such as polyester or frozen water. He was able to investigate and analyze the stable shapes that developed thanks to these hanging models, which revealed the shells' innate structural activity. A similar approach was used in the form-finding process of the Deitingen service station (Baghdadi et al, 2019).

Figure 4.5 Stage 1: Dipping the cloth into a hardening substance. Stage 2: Hanging the model Stage 3: Inverting the hardened model (John Chilton 2000)

The utilization of inverted suspended membrane configurations in shell structures enables the development of highly efficient structures that exhibit robust resistance against buckling. The shell structures designed by Heinz Isler can be regarded as exemplary instances of this approach to the design of shell structures. The design of these structures was based on a qualitative approach, prioritizing shape and stability assessment through extensive experimental testing, rather than relying on intricate calculations.

The utilization of scaled physical models facilitated the opportunity to observe and manipulate said models within a controlled laboratory setting, thereby enabling analysis without the necessity of on-site construction. By conducting tests on these models, it is likely that a comprehensive understanding of the design and structural characteristics of this particular type of structure was attained.

The influence of critical loads and buckling modes is contingent upon the proportions of dimensions in the plan. Consequently, the selection of an initial model that maximizes load capacity should was guided by this factor. Isler, as a designer, exercised his choice in determining the size and shape of the fabric surface used for form-finding. However, similar to a simple catenary chain, the horizontal force exerted on the shell and supports amplifies proportionately to the degree of sag exhibited by the three-dimensional hanging fabric or membrane surface. Conversely, as the sag increases, the force decreases but the surface area of the roof expands, thereby augmenting the self-weight and vertical stress that must be countered at the supports. Hence, it is evident that the initial size and shape of the surface significantly influence the stresses and support reactions occurring within the shell (Chilton J, 2012). Plaster models were utilized to produce epoxy resin models, which were subsequently subjected to load tests within specially designed experimental rigs (Figure 4.6).

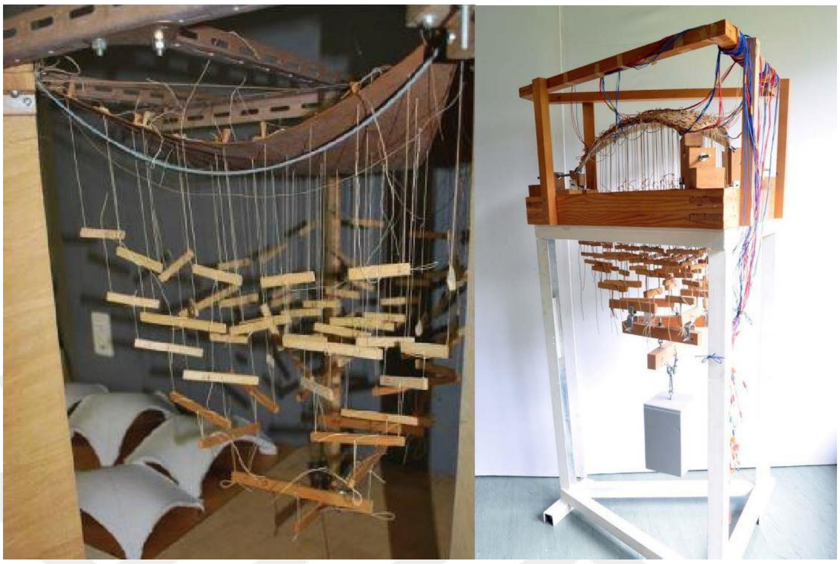


Figure 4.6 Isler"s Experimental Rig (Chilton 2012)

These tests aimed to assess the internal forces within the models and their resistance to shell buckling. The models and rigs themselves were crafted with meticulous artistry and served the purpose of recreating various loading scenarios, including scenarios involving differential movement of the supports (John Chilton, 2012)

As this structure is a result of the form finding using inverted hanging fabric models, it consists primarily of in-plane membrane forces that are represented as highlighted shell in the figure 4.7. No additional load bearing elements can be seen as the shell distributes the effectively on its own.

Figure 4.7 In Plane Membrane Forces in the Shell (Highlighted in Blue) (Nicolas Janberg)

The properties of the structure that exhibits form resulting from the inversion of membranes or fabrics play a crucial role in determining their behavior. The analyses presented in the study indicate that the influence of these properties is associated with the distribution of axial or membrane forces. These forces play a crucial role in determining the required geometry for achieving equilibrium under applied vertical loads (De souza, 2017).

One of the limitations associated with thin reinforced concrete shells is their vulnerability to buckling instability. This susceptibility is closely influenced by the curvature and distribution of membrane forces generated in relation to the structure's form. In the study conducted by Ramm 1982, various instances of shell structures are examined, with a particular focus on designing structural configurations that exhibit reduced vulnerability to buckling instability.

The dimensions of each shell are 31.6 meters in length and 26 meters in width. In the proposed plan, the two structures were positioned at a distance of 7 meters from each other. The block structure, which is situated between the two supports, contributes to the overall spacing, resulting in a total distance of 68.24 meters between the outer supports. The vertical distance from the ground to the highest point is measured to be 11.5 meters. The structure exhibits a uniform thickness of 9 centimeters across its surface, but experiences a greater thickness in the support area to accommodate the intensified stresses (Chilton, 2008) (figure 4.8).

Figure 4.8 Increased thickness at the edge(Chilton, 2008)

The two structures effectively distribute and equalize the horizontal forces by means of the central structure, which serves as a conduit for force transfer (Chilton, 2000). The equilibrium of horizontal forces is achieved through the implementation of prestressed cables that are strategically positioned between the supports, as depicted in the figure 4.9.

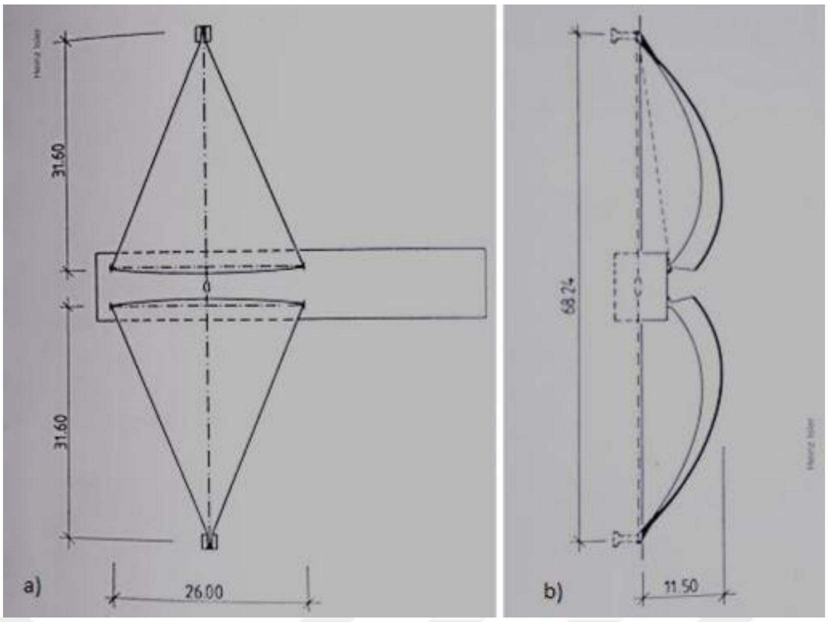


Figure 4.9 (a) The plan of the shell structures in Deitingen (b) Elevation (Chilton, 2000)

4.1.3 The Kimbell Art Museum:

The Kimberly Art Museum, designed by Louis I. Kahn in association with structural engineer August Komendant, is recognized as one of the most remarkable examples of modern American architecture. Kahn was known for his utilization of straightforward geometric shapes and spatial arrangements in large concrete constructions. His work highlights his deep respect for exposing the significance of clear structural systems, and materials. Pushing structural systems to their limits is one of the intrinsic components of his architectural design. (Figure 4.10)

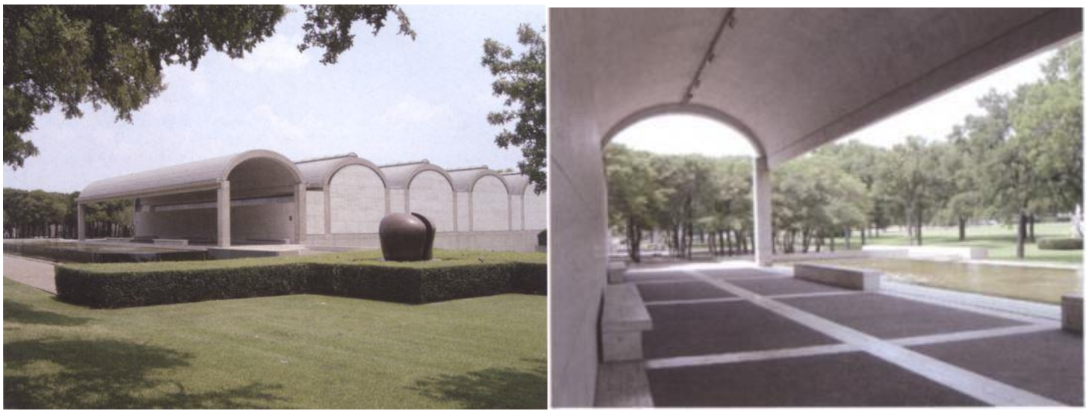


Figure 4.10 Louis I. Kahn's Kimbell Art Museum (Park et al, 2007)

August Komendant, a structural engineer of Estonian and American descent, made significant contributions to the field of reinforced concrete and the utilization of post-tensioning and pre-stressing techniques. In conjunction with the architect Louis Kahn, the individual served as the structural engineer for notable architectural projects including the Kimbell Art Museum in Fort Worth, Texas. The construction of this project was the result of a strong partnership between the architect and the structural engineer, a collaboration that is not commonly observed in contemporary architectural projects (Charleson and Pirie 2009)

The form of the structure is a singly curved/mono-clastic shell, which falls under the umbrella of analytical shells in the scope of this thesis. The museum's most notable design feature consists of sixteen parallel cycloid vaults, which were not initially part of the design process but were arrived at through a series of trials and experiments by Kahn. Initially, Kahn had proposed a semicircular ceiling in his presentation, but Brown, who was a collaborator in the project, found it to be excessively showy and impractically high due to its shape. Later, Kahn attempted to address Brown's concerns with a flattened arch, which satisfied the requirements but lacked elegance. Marshall D. Meyers, the project manager, who was had also tried several options such as a flat roof with quarter-round edges, ellipses, and segmented circles, but none of them proved suitable for the purpose (Leslie, 2005). A raised semi-circular arch was later agreed upon as the design.

According to the membrane theory, the state of stress that a specific element is in a barrel vault support along its longitudinal length as well as supported just at the corners is shown in figure 4.11. As the barrel shell in the museum is supported at the corners only, the stresses generated are longitudinal in-plane stresses throughout the shell. The highlighted section of the shell in figure 4.12 depicts the part of the shell following membrane action along with the load path of the structure. The entire barrel shaped shell behaves like a membrane while transferring the load to the ground through the columns at the edges.

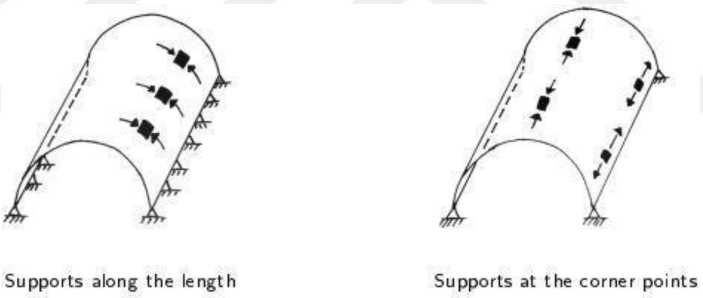


Figure 4.11 Stresses in a semicircular arch with different support conditions

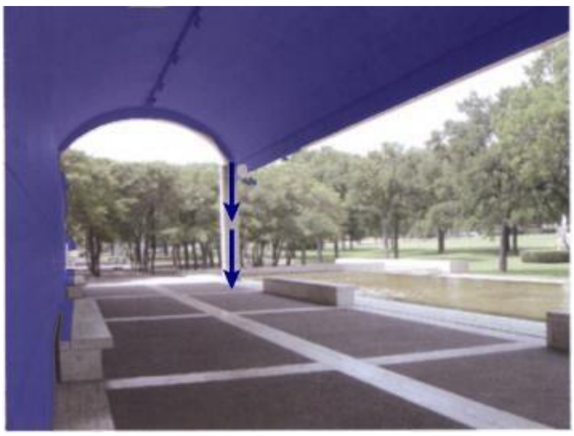


Figure 4.12 Membrane action and load path in the museum shell (Park et al, 2007)

Cylindrical shells are frequently employed as structural systems, offering a wide range of solutions that span from individual, uncomplicated cylindrical shells to collections of interconnected or neighboring cylindrical shells. These objects are classified as surface shells characterized by having zero or monoclastic Gaussian curvature. They are formed through the process of translating a straight line, known as the generatrix, along a flat curve referred to as the directrix. (Figure 4.13)

The resulting shell of the museum is a cycloid that is a geometric figure that can be described as the areas of points generated by a fixed point on a circle as it rotates along a straight path.

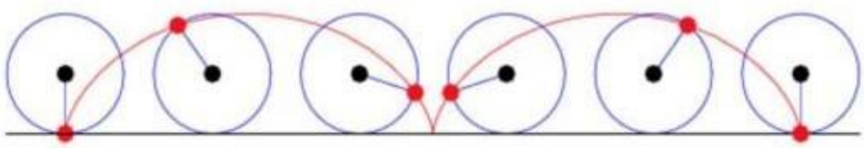


Figure 4.13 Formation of a cycloid by a fixed point of a rotating circle (Wolfram Mathworld, 2014)

When it comes to supporting a barrel roof, it can historically be done in two ways: along the curved ends or the longitudinal edges. If a barrel roof is supported along its longitudinal edges, it behaves like a series of adjoining arches that generate horizontal stresses pushing outward, which are countered by buttress walls. On the other hand, a barrel vault operates like a beam, if it is supported on its curved ends, transferring the load to the end walls and the ground (Salvadori, 1980). However, Louis I. Kahn's design for the roof structure of the Kimbell Art Museum surpasses what is typically known about conventional vault systems. Through the incorporation of intricate longitudinal post-tensioning cables and diaphragms, Kahn's design makes use of an unusual combination of approaches.

Osanov, 2014 conducted a thorough structural analysis which demonstrated the feasibility of eliminating ring and edge beams within a structure through the utilization of pre-stressed elements. Nevertheless, when examining the matter from an aesthetic perspective, architects may perceive them as more appropriate compared to inflexible walls, as evidenced by the architectural composition of the Kimbell Art Museum. Although Komendant's design may not have been the most economically efficient, it effectively achieved the creation of a structure that harmonized with Louis Kahn's monumental and robust architectural aesthetic. The convergence of Komendant's design methodology and Kahn's advocacy for imposing architectural forms resulted in a historically significant shell structure.

Ring and edge beams were included in the design of the shells of the museum. The structural element located at the terminations of a system is commonly known as a diaphragm or partition, and it can exhibit various configurations (Figure 4.14). The most prevalent configurations in structural engineering include solid diaphragms, simple arches, and trussed arches. Diaphragms exhibit rigidity within their own plane, while demonstrating flexibility outside of it. The decision to include or exclude edge beams in cylindrical shells can be determined (Ramasway & kreiger, 1984).

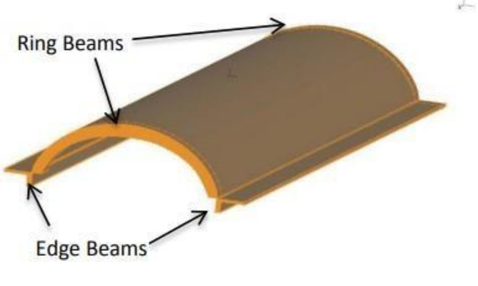


Figure 4.14 Model of the Museum shell (Osanov, 2014)

Circular cylindrical shells are frequently utilized in this particular category of structures. However, it is also possible to incorporate parabolic, elliptical, cycloidal, or catenoidal directrices in their design. Cylindrical shells possess a multitude of advantages in their practical implementation, which include:

- The primary mechanism through which they respond to loads is by generating forces within the plane.
- Prefabricated cylindrical structures possess the potential to provide economic benefits.
- Reinforced and pre-stressed concrete shells exhibit practicality in the context of cylindrical shells, primarily owing to the straightforwardness of formwork assembly and concrete casting processes involved in constructing such structures.

4.1.4 Coop Storage and Distribution Center:

The coop storage and distribution building in Wangen, Switzerland, is a storage center with a concrete shell roof designed by Heinz Isler in 1960 (figure 4.15). Isler's proficiency in developing inventive and effective form-finding techniques in shell structures is demonstrated by this endeavor. The storage and distribution facility has concrete curved roof that covers a considerable area and offers both a practical and attractive solution. Isler's design strategy was centered on using the inherent stability and strength of shell roof. His expertise in shell structures and his ability to harmoniously and sustainably combine form and function can be seen in the Coop storage and distribution center in Wangen that was designed by the pneumatic form-finding method.

Figure 4.15 Coop storage and Distribution center (Chilton 2000)

The use of elliptical paraboloid shells has been found to be advantageous in specific roof structures. These elements are classified as belonging to the group of shells characterized by surfaces exhibiting positive or synclastic Gaussian curvature. The depicted curves in figure 4.16 are obtained by translating a convex parabola over another convex parabola, both of which are situated within mutually orthogonal planes(Cardozo, 2008)

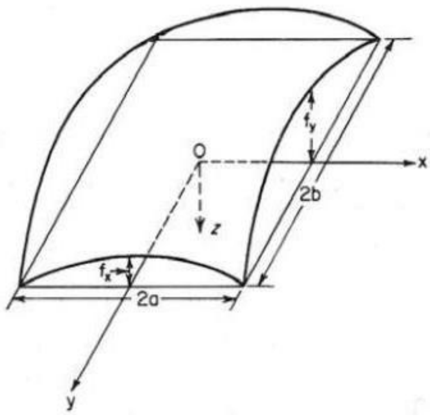


Figure 4.16 Elliptical Paraboloid Shell Element Cardozo, 2008)

There are several notable advantages associated with this particular type of shells(Cardozo, 2008).

- The primary mechanism by which they respond to loads is through the generation of forces within the plane.
- The reduced material usage of these structures is attributed to their ability to exhibit an arch effect in both directions of the principal curvatures. This characteristic contributes to their notable efficiency in responding to external forces.
- The shells offer a significant degree of rigidity to any deformation of the shell which leads to stiffness within the plane in two orthogonal directions.

Heinz Isler conducted experimental investigations involving a series of test shells. These shells were subjected to various treatments, such as the application of clay, plaster, concrete, mortar, water (in frozen state), or plastics, with the support of pneumatic systems. He later,fabricated concrete shells using pneumatically supported formwork, which were subsequently employed. Similar technique was used in the form-finding method of coop storage and distribution center‟s roof shell. Additionally, there is an office building located in Langenthal, Switzerland, characterized by two intersecting domes, each boasting a diameter of 7.6 meters (Sobek, 1987).

Similar to Neff, Heifetz, and Isler, Horral I Harrington in the United States also engages in the production of shells through the application of concrete onto pneumatic-supported formwork (figure 4.17). To date, the individual in question has successfully constructed more than 65 shells utilizing this particular system. Notably, one of these shells holds a record for its impressive span of 57.30 meters in diameter, accompanied by a wall thickness measuring 8.9 cm

Figure 4.17 The first layer of concrete being shotcreted (Sobek,1987)

Pneumatic components were not new to Heinz Isler because he had used them earlier in his career to build industrial shells. One such component was the pressure membrane, which took its design cues from a pillow. It was composed of a membrane that was inflated and held by a wooden frame; when the air was pushed into the membrane, the membrane assumed the shape of a shell. The desired shell form may be created by adding a gypsum coating that has solidified and followed the contours of the inflated part. A grid was drawn on the surface of the small-scale model to measure it correctly. Then, to create technical drawings for the building process, the numerous portions that dictated the shape's curvature were transferred onto paper (Beckh, 2019).

From a structural perspective, Isler's gravity-based shells behaved better than the Analytical Forms of the 1930s. The cause of this is that Isler's structures, when supported by their own weight, followed the natural rules of pure compression or pure tension. Analytical shapes, on the other hand, were only rough approximations of these "natural" shapes. Isler's designs were difficult to analyze, so the construction process was thought to be a little more difficult than the production of Analytical Forms (Huijben et al., 2011).

These models utilize a membrane or soap film that is either airtight or has minimal air permeability. The film is inflated into a particular form, which may include a predetermined enclosed boundary. The internal pressure caused by inflation results in the creation of a balanced shape (figure 4.21). Pneumatic models were valuable for constructing molds and determining the optimal configuration for shells, that is why Heinz Isler had a preference for employing pneumatic models. The resulted form obtained using pneumatic form-finding behaves as a pure shell under membrane action. Typical load path for such a form is demonstrated in the figure 4.18.

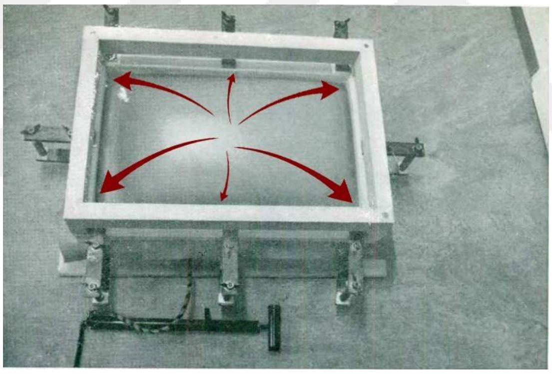


Figure 4.18 Load path on a Pneumatic form finding set-up (Chillton 2000)

Isler acknowledged the significance of utilizing pressure on soap films to generate novel structures, as they possess the graceful qualities of shapes found in nature. He asserted that, statistically, these shells exhibit a considerable level of quality. However, he noted that when dealing with larger dimensions, irregularities may arise due to the difference in the direction of forces. The gas pressure responsible for shaping the structure acts perpendicular to the surface, while the force of gravity acts parallel to it. (Chilton, 2009)

4.1.5 Palazetto Dello Sport:

Palazetto Dello Sport, constructed for the 1960 Rome Games, was Pier Luigi Nervi's most renowned example of rotated lamella domes. While the larger enclosed area, known as the Palazzo dello Sport, utilized a prefabricated trough system, the Palazetto dello Sport represented Nervi's purest embodiment of the radial pan lamella configuration. From an external perspective, the dome's overarching structural behavior is distinctly evident. The dome's beams are consolidated along the periphery of the framework to form an array of fan vaults, which are, in turn, buttressed by thirty-six Y-shaped supports that transmit gravitational and thrust loads to the foundational base (Leslie, 2003). (Figure 4.19)

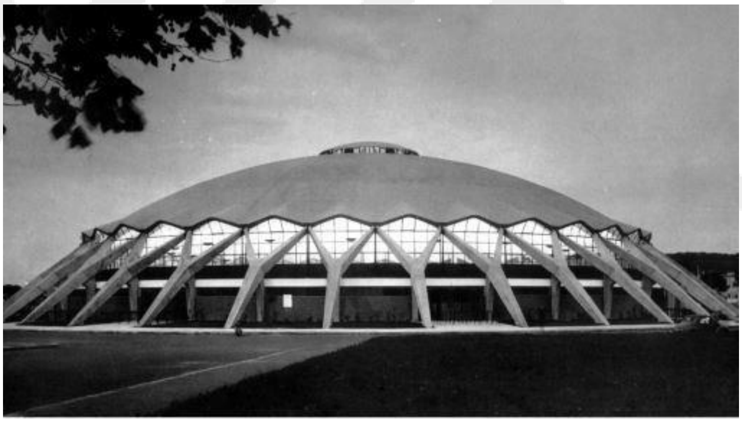


Figure 4.19 The Palazetto dello Sport, Rome. Pier Luigi Nervi, 1958 (New York: Praeger, 1963)

The outer shell is constructed using pre-fabricated modules that are wedge-shaped in design. The pre-fabricated concrete ribbed shell dome is 61m in diameter supported by concrete buttresses (figure 4.20). The precast ribs of the shell, along with the Y-shaped columns, align with the isostatic lines of principal stresses. This alignment theoretically eliminates shear stress, leading to a reduction in the overall consumption of concrete material. The dome behaves like a membrane (highlighted) while the perfectly aligned Y-shaped columns transfer the loads to the ground (Figure 4.21).

Figure 4.20 The Palazetto dello Sport exterior view (New York: Praeger, 1963)

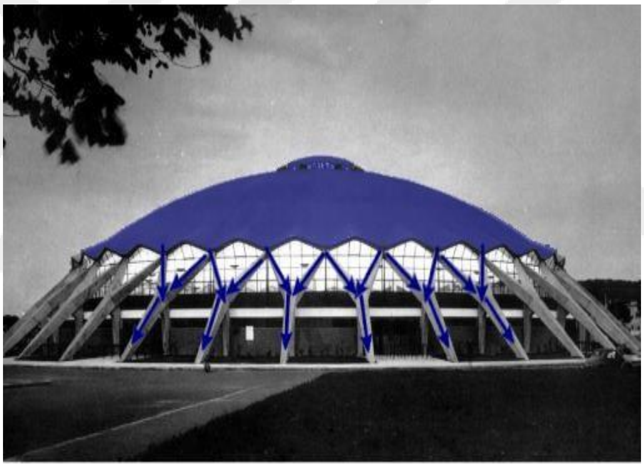


Figure 4.21 Membrane Action and the Load paths of the dome shell (New York: Praeger, 1963)

The ribbed structure of the dome serves the dual purpose of enhancing lateral load support through the maximization of the cross-sectional area and accommodating asymmetric loading conditions. Nervi's innovative utilization of steel-reinforced concrete not only conveys the lines of principal stresses but also achieves a harmonious blend of aesthetic appeal and engineering effectiveness. When dissecting the Palazetto Dello Sport structure, one can discern primary load-bearing elements. These components convey their loads to the foundation along the tangent plane originating from the dome's thrust, encompassing seating banks and the surrounding gallery (figure 4.22).

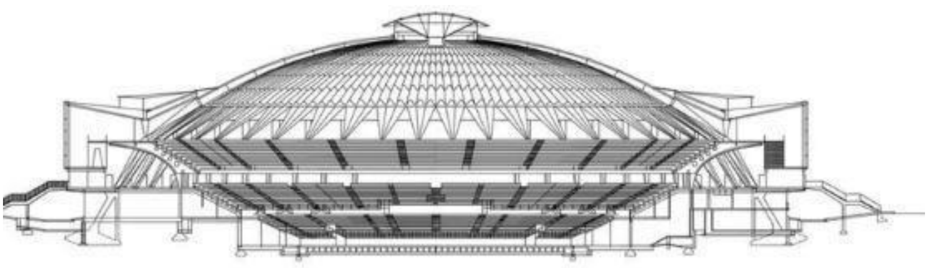


Figure 4.22 Section View of Palazetto Dello Sport (ArchDaily)

4.1.6 Thompson Arena:

The design process for the Thompson Arena commenced in 1965. Upon its completion, this ice arena stood as the largest facility of its type among American colleges for hosting varsity hockey games, as noted by Dartmouth College in 1962. Construction initiated in 1973, and by 1975, the arena was ready for its inaugural game. While it captured some attention within hockey programs, it received limited recognition in the field of structural design due to being one of the final projects by Nervi. This project marked his last completed work in the United States

The Thompson Arena's relative obscurity might stem from its obscured street view or its resemblance to the Leverone Field House, another project of Nervi, causing it to go unnoticed. Even publications discussing Nervi's works tend to rarely mention the Thompson Arena and sometimes omit it from lists of his accomplished projects.

Figure 4.23 provides an illustration of the Thompson Arena, showcasing its parabolic vault measuring 54m by 90m, composed of 1,024 triangular precast concrete units, each weighing one ton. The only publication to extensively cover the Thompson Arena is "Pier Luigi Nervi negli Stati Uniti in 2013" by Bologna.

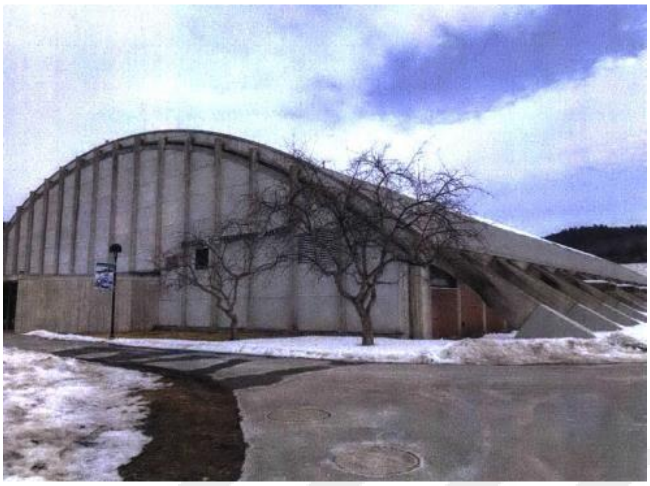


Figure 4.23 Thompson Arena (Sun, 2017)

This structure showcases two of Nervi's distinctive styles: columns with variable cross-sections that twist and taper, and a ribbed shell featuring precast geometric components. External Y-shaped buttresses, depicted in Figures 4.24, play a role in transmitting both gravity and thrust loads to the foundation while rest of the shell remains under membrane action as highlighted (figure 4.25). These buttresses resemble those found at the renowned Palazzetto dello Sport.While the rest o

Figure 4.24 Thompson Arena's exterior side view of buttresses (Sun, 2017)

Figure 4.25 Membrane action and Load path for Thompson Arena (Sun, 2017)

As depicted in Figure 4.26, the arena's interior grandeur surpasses its external appearance. The building is excavated to position the ice surface and stadium seats below ground level, creating a substantially more spacious area than anticipated from its square footage. Despite its lack of widespread recognition, it continues to stand as one of the premier venues for college hockey games.

Figure 4.26 Interior view of Thompson Arena (Sun, 2017)

The Thompson Arena bears striking similarities to the Palazzetto dello Sport, with Nervi incorporating numerous design elements from the latter. He drew heavily from the details of the buttresses used in the Palazzetto dello Sport, applying these features to the Thompson Arena. These buttresses, which had previously been employed in a more subtle manner, gained a newfound structural and artistic prominence in the Thompson Arena. Positioned on the exterior and intentionally exposed, they evolved into a distinctive and expressive characteristic of the arena's design. Notably, the Y-shaped buttresses at the Thompson Arena bear an uncanny resemblance to those found at the Palazzetto dello Sport in Rome, further highlighting Nervi's deliberate and skillful integration of design elements from his earlier work

4.2 Contemporary Cases

This set of case studies deal with the contemporary shell structures that have been completed in the 21st century using digital form finding methods. A total of 12 case studies have been included. The first four case studies in this section contain a more detailed overview of the structural systems. The rest of the case studies have comparatively less detailed structural systems, as these shell structure types are already covered in the first four case studies (Space-frame, RC shell, wooden Grid shell etc.)

4.2.1 Heydar Aliyev Cultural Center:

The Heydar Aliyev Centre in Azerbaijan is a modern curvilinear shell structure that is a national cultural hub encompassing a museum, an auditorium/opera house, and various cultural amenities. Situated in the city‟s heart, the center showcases a distinctive and iconic architectural design crafted by Zaha Hadid Architects in 2012.

Figure 4.27 Heydar Aliyev Cultural Center (Tarboush, 2019)

When optimizing a structural network on an arbitrary surface like the Baku cultural center, achieving a significant level of regularity and uniformity in lengths of individual members, node angles, and panel types is a common objective. The aforementioned phenomenon not only affects the visual appeal, but also carries economic ramifications for the production and assembly of the architectural elements. Nevertheless, when it comes to free-form designs, there are often other architectural and functional considerations that take precedence over the exclusive emphasis on geometric optimization. It is imperative to integrate these supplementary factors into the geometric modeling and optimization procedure, while duly acknowledging their significance, interplay with other design variables, and overall influence on the design outcome (Alvarez, 2009).

The proposed topological network for the space frame geometry of the Baku cultural center consisted of a grid configuration with square-on-offset with two layers. The aforementioned design provides significant adaptability in accommodating geometric distortions while simultaneously maintaining its structural integrity and feasibility of construction.

A geometric optimization exercise was undertaken to examine the external networks of the space frames. The structural quadrangular grid lines were aligned parallel to a specific direction, with a regular spacing of 3.0 meters and a consistent 9.0-meter gap between adjacent building axes. To analyze the external layer of the structure, horizontal straight lines were projected onto the corresponding carrier NURBS surface to obtain the corresponding curves in 3D space (figure 4.28).

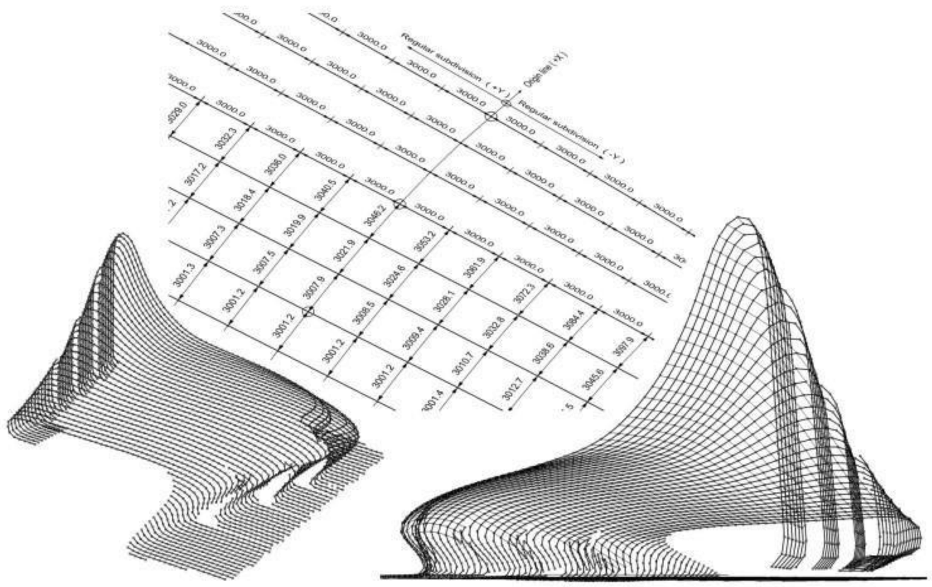


Figure 4.28 Optimization of initial line-length and orientation for the external grid of the Library-Auditorium space frame (Alvarez, 2009)

The further optimization strategy for the initial geometry and target for both the external and internal layers of the space structures in the cultural center were established through the initial line-length and orientation optimization, resulting in an ideal geometric approach. Nevertheless, a range of architectural and constructional demands arose, which compelled the adjustment and alteration of the originally optimized grids. Additionally, these demands restricted the implementation of the geometric optimization procedure to particular areas. The refinement of the geometric design and adjustments to the initially envisioned optimization approach were influenced by these more significant requirements. The most significant among these demands were the support positions and interaction of the grid shell with internal architectural elements as shown in fig.4.29 (Alvarez, 2009).

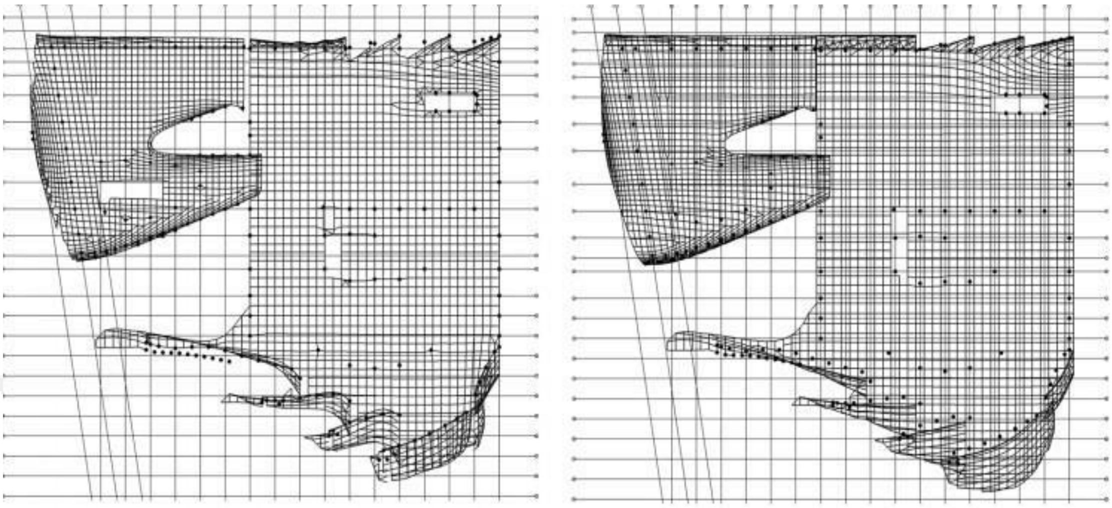


Figure 4.29 Modified to accommodate the placement of supports and architectural elements, the figures show the internal (left) and external (right) structural grids (Alvarez, 2009)

The evolution of the roof's space frame followed a sequential process, commencing with the definition of a structural zone that delineates the architectural free-form shapes of both the inner and outer skins. Subsequently, the space frame was oriented in accordance with the structural configuration of the primary framework.

Initially, the top and bottom chord grid-lines were established along the numerical axes, while the structural grid-lines at 9 m intervals were determined as multiples of these axes. In the subsequent phase, a three-dimensional grid of diagonals was introduced, with continuous monitoring of its compatibility with the architectural form to avoid any conflicts. A finite element (FE) model was subsequently created and analyzed. Ultimately, the auxiliary forces and specific particulars were effectively synchronized with the primary framework ((Winterstetter et al., 2015).

In situ, reinforced concrete (RC) and a composite steel space structure comprise the basic building structure. The slabs in the museum and library portions are cast on a metal deck and supported by long-span composite beams and reinforced concrete (RC) slabs. The theater and multipurpose hall, on the other hand, have historical RC slabs supported by RC beams. The design process of the roof's space frame involved multiple stages. The initial step involved identifying a distinct structural zone positioned between the distinctive architectural free-form shapes of the inner and outer skins. Following this, a careful coordination process was undertaken to integrate the space frame seamlessly with the main structure's structural design, ensuring precise alignment and harmonious integration. This iterative process allowed for the seamless integration of the space frame within the overall structural framework, resulting in a cohesive and robust roof design. The coordination of the structural interfaces at the supports, where the space frame connects to the main structure and the foundations, was a critical component of the design process (figure 4.30). Ensuring the seamless integration and the best possible structural performance, it required great attention to details and a thorough study of the load transfer systems. The overall stability and integrity of the structure were improved by skillfully coordinating these interconnections, enabling optimal load distribution and ensuring the long-term longevity of the building. This iterative process made sure that the space frame met the project's structural needs while also complementing the overall architectural design. In the computerized stress analysis, it was of utmost significance to incorporate an accurate representation of the collaborative influence exerted by both the space frame and the underlying main concrete-steel frame structure (Winterstetter et al., 2015).

Figure 4.30 Space Frame and Core RC structure (Tarboush, 2019)

The coordination of the space frame design with other architectural elements, including the solid skin, curtain wall, and interior 3D ceilings, was a crucial aspect to consider. The task involved the coordination of the structural interfaces between the supports of the space frame, the main structure, and the foundations. The comprehensive analysis and design of the steel columns and steel frames that provide support for the space frame roof and glazed curtain wall was thoroughly examined through the verification and synchronization of the 2D and 3D design submittals that have been furnished by MERO.

The structural calculations provided by MERO were verified using two separate, comprehensive 3D finite element models in Etabs and Rstab. These models incorporated both the primary structure and the space frame, ensuring a thorough analysis.

The riveting steel boot columns and steel flooring platforms were crucial in raising the space frame's stiffness. In addition to improving structural stability, these columns and beams made it easier to incorporate glass walls and connect the steel and concrete structures. They served as steel bridges, providing a method to intermittently and strategically connect the internal utilities on the top floors of the building (figure 4.31). This configuration preserved the space frame design's overall structural integrity while ensuring effective circulation and accessibility (Arc space 2014).

Figure 4.31 Steel bridge of Heydar Aliyev center (Tarboush, 2019)

The lateral stiffening system is made to handle two crucial issues: the architectural design's spatial restrictions and the potentially significant horizontal forces brought on by seismic events. A 3D free-form shear wall with a concrete core is used to offer lateral stability in the museum area. Several reinforced concrete (RC) shear walls are built into the multipurpose hall and auditorium for this reason. In order to provide the necessary stiffness and structural integrity in the library, which is located in the uppermost portion of the structure, a combination of a slender concrete core and a steel section frame system is used. The building's ability to endure both design limitations and seismic stresses is ensured by the integration of these various lateral stiffening elements (Tarboush 2019). The load paths for the interconnected outer and inner elements are shown in the figure 4.32. It can be seen that the RC core plays a vital role as an additional load bearing system to provide overall stability to the system. The load path flows from the outer steel grid towards the RC core through the lateral stiffening bridges and then from the RC core, it goes to the ground.

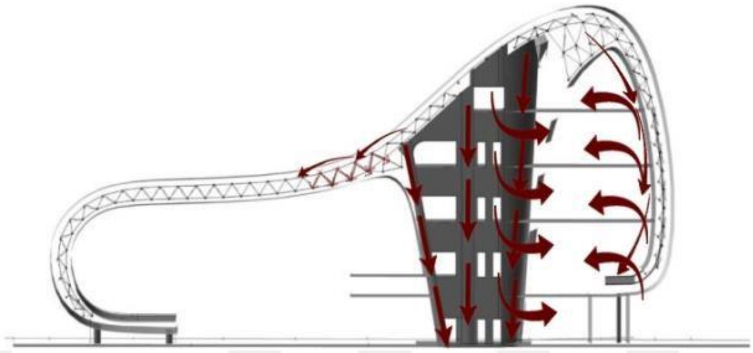


Figure 4.32 Load Paths and the additional element (RC Core) (Alvarez 2009)

Figure 4.31 illustrates the alterations carried out on the interior structural grid of the Museum's space frame. These modifications were implemented to create constructive interfaces, ultimately serving as support consoles at the distinctive, free-form apex of the Museum's concrete core walls. The interior core and the space frame had to be in complete coherence interacting with each other to ensure the structural support of the core towards the steel frame roof. The section on the right side of the image demonstrates substantial modifications carried out on the double-layered grid. These adjustments were necessary to establish a structurally seamless connection with a sequence of curved, steel H-beams that provide support to the concrete slabs along the building's wall. It is crucial in this situation that the space frame components and the vertical H-beams can coexist harmoniously within a very restricted inter-skin space of approximately 1.0 m. Their modeling ensures that they only structurally interact for an acceptable horizontal stabilizing interaction (Alvarez 2009) (figure 4.33).

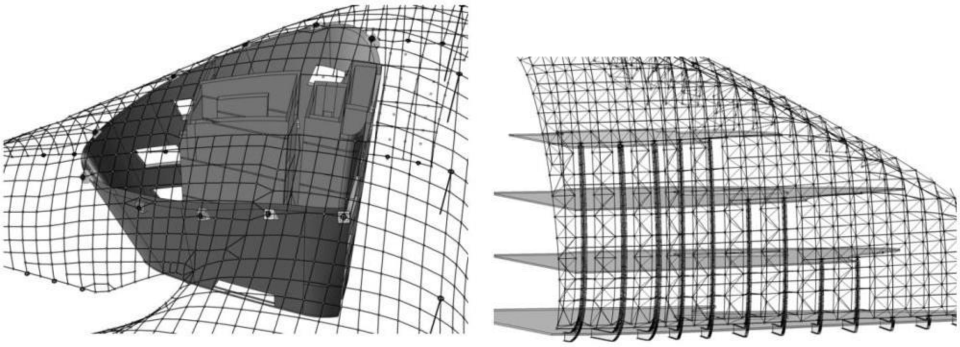


Figure 4.33 The Museum's interior architectural elements interact with a double-layered structural grid. (Alvarez 2009)

4.2.2 The Meiso No Mori Crematorium:

The Meiso No Mori Municipal Funeral Hall, situated in Kakamigahara, Japan, is a contemporary and architecturally notable establishment dedicated to offering funeral services to the local community. Designed by the renowned architect Toyo Ito, the facility was completed in 2006 and stands as a testament to its unique and modern architectural design. Notably, the reinforced concrete shell design of the Meiso No Mori Municipal Funeral Hall successfully captures the essence of both nature and spirituality, creating a harmonious and calm atmosphere.

Toyo Ito created an innovative vision for the funeral hall in collaboration with engineer Mutsuro Sasaki, attempting to fuse symbolic significance with a logical structural approach. The hall's architecture incorporates a continuous, ethereal layer of white concrete that is reminiscent of the billowing smoke from cremation ovens. The ground, sky, lake, and hill are all connected visually and conceptually by this flowing concrete shell, which also smoothly blends into the surrounding landscape. The primary objective of Ito's design was to achieve a delicate balance between the ephemeral and the enduring. Despite the inherent weightiness of concrete as a material, Ito aimed to create an impression of weightlessness by crafting an ultra-thin concrete shell that appears to float effortlessly. The design of the roof canopy and the columns are integrated in such a way that the loads are concentrated towards the columns and through the columns to the ground. Due to irregular shape of the roof, there is a frequent areas of load concentrations and hence the frequently placed columns throughout the plan (figure 4.34).

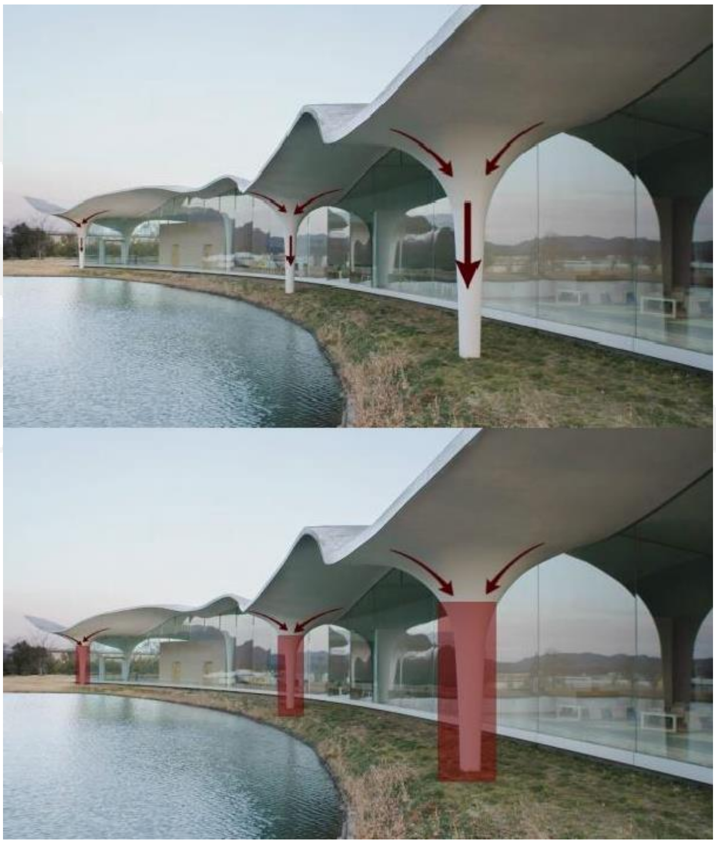


Figure 4.34 Top: Load Path Bottom: Additional elements in Meiso No Mori Hall, (OpenBuildings)

The design process exemplifies a composite design approach. In this case, the design of the building's roof was developed through a collaborative effort between the architect and the structural engineer, Mutsuro Sasaki. The design process commenced with rudimentary sketches, which were derived from Ito's intention to fabricate a complex form resembling a cloud that would hover above the interior spaces.

Subsequently, the sketches were digitized in order to facilitate the implementation of structural analyses. The structural analyses programs conducted numerous calculations to alter the shape. The shape of the roof was altered through the utilization of a refinement algorithm, which involved multiple iterations, in order to achieve the most optimal structural efficiency for the roof. A final optimal form emerged as a result of structural considerations occurring within the digital medium. The final form of the project emerged unexpectedly as architects and engineers solely established the criteria and executed the optimization and analysis programs without any intervening actions (Demir, 2013).

The unique curves of the undulating roof canopy posed challenges in terms of reinforcement requirements. The varying degrees of curvature and shape demand different levels of reinforcement throughout the surface of the roof (figure 4.36). The irregular and undulating shape of the roof created uneven stress distribution across its surface, leading to the need for additional steel meshes at specific locations. Despite the requirement for reinforced support, great care was taken to preserve the thinness of the overall shell structure. Once the concrete had set, final layers were meticulously applied to achieve a sleek and polished surface, enhancing the visual appeal of the completed building (Windeck 2016).

Numerous columns were placed strategically around the area to support the large roof of the shell structure. Incorporating multiple columns to uphold the slender shell roof brings about a deviation from the fundamental feature of uninterrupted space beneath, a characteristic historically associated with shell structures. The presence of columns introduces vertical elements that partition the space, affecting the sense of openness and continuity typically attributed to shells (Figure 4.35). While the columns fulfill the crucial role of providing structural support and stability, they unavoidably introduce a visual and spatial differentiation within the interior setting. This modification in the spatial experience necessitates meticulous consideration during the design process to achieve a seamless fusion of the columns with the overarching architectural vision.

Figure 4.35 Column Supports throughout the plan. [Pugnale (2017)]

4.2.3 Rolex Learning Center:

The Rolex Learning Center is a well-known shell structure in Lausanne, Switzerland. One of Europe's top colleges for science and technology, École Polytechnique Fédérale de Lausanne (EPFL), uses it as its main library and educational hub. The facility was designed by the SANAA architectural firm, under the direction of Kazuyo Sejima and Ryue Nishizawa, and it became accessible to the public in 2010.

Figure 4.36 Rolex Learning Center. (Alain Herzog,EPFL)

For everyone involved, the architectural design of the Rolex Learning Center in Lausanne posed a distinctive and difficult challenge. Instead of the normal partition of rooms by horizontal and vertical slabs and walls, the design deviates from conventional building methods by envisioning an architectural landscape inside the building. This was made possible by designing two shells that had different spans and flat sections that were situated in the spaces between the shells (figure 4.37). These shells, in contrast to conventional shell constructions, served as the inner space's floor rather than the building's roof. The interior, which had a single story, had a steel roof and translucent glass facades, which made the area feel airy and bright (Weilandt, et al 2009).

Figure 4.37 ROLEX Learning Center, Lausanne, architectural model of the main floor by SANAA (Grohmann et al, 2009)

The structural design of the shells demanded both high-performance concrete and a substantial amount of steel reinforcement, amounting to approximately 470 kg/m³. This was primarily due to the low curvature of the shells and their asymmetrical geometry. This significant reinforcement was necessary for both minimizing long-term deflections brought on by creep and shrinkage effects as well as successfully transferring tension in regions with large bending moments. The design boosted structural strength and load-bearing capacity by using a significant amount of steel reinforcement (Grohmann et al, 2009).

The optimization procedure for the geometry was initiated by conducting an analysis of the geometries proposed by the architects. During this particular stage of the project, the geometric aspects were still susceptible to substantial modifications as a result of user demands and various influencing factors. For example, the flat regions situated between the two shells were expanded in order to enhance the surface areas that could be readily equipped for the convenience of the occupants. However, significant advancements towards the ultimate geometry were already made during this phase of the design process in such a way that the out of plane stresses were minimized. The load path for the big shell, resembling the load path for a traditional shell is shown in figure 4.38.

Figure 4.38 Load Path on the big shell (Weilandt 2009)

Throughout the subsequent design process, the development of the shell geometries entailed an interactive procedure that necessitated the harmonization of structural requirements, user requirements, and design aspects. The objective was to incorporate the following elements into the shell geometries (Grohmann et al, 2009).

- The modification of the positioning and layout of patios to facilitate the installation of load-bearing arches connecting them.
- To optimize the geometric configuration of the arches, with a focus on achieving parabolic and symmetric cross-sectional profiles.
- To minimize the presence of flaws in the shell structures.
- The prevention of counter-curvatures in shell bearings. In order to ensure a smooth and uninterrupted connection between the shell structure and the flat surfaces, a suggestion was made to the architects to establish a distinction between the geometric properties of the shell and the finished floors at the shell bearings.

In order to refine and finalize the geometry of the shells, the design team engaged in an iterative process that aimed to find a harmonious equilibrium between structural considerations, user requirements, and aesthetic design principles.

The central emphasis of the formwork planning revolved not exclusively around the production, generation and continuous refinement of a three-dimensional model. The 3D model was utilized as the basis for the automated production of the formwork components. To proactively address and mitigate a certain degree of deflection before the commencement of construction, it was determined that a curved geometry would be integrated into the shells. Consequently, an additional three-dimensional model was created in conjunction with the architectural model, with the specific purpose of accurately depicting the curved geometry of the shell structures. The distinction between the architectural model, which was un-deformed, and the structural model, which exhibits a slightly deflected geometry, proved to be crucial for the subsequent phases of the design process (weilandt, 2009). To account for the asymmetry of the curvature of the shells, numerous steel arches were incorporated within the shells for structural stability (Figure 4.29).

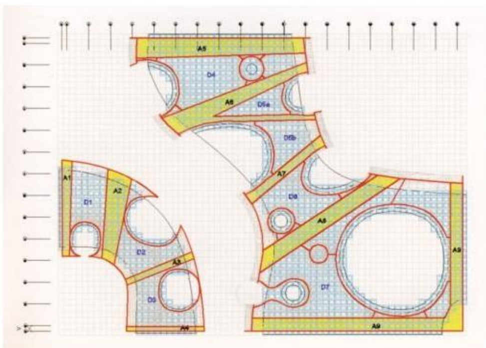


Figure 4.39 Arches and Slab areas of the shells in Rolex Learning Center (Bollinger 2009)

The lower part of the Rolex Learning Center features a basement area that is enclosed by a concrete slab. Within the framework of the building, this slab fulfills two functions.

It serves as the main level's floor first, as it is located between the center's two curved shells. This creates a useful area for a variety of tasks and improves the building's general usability.

Second, by acting as a horizontal support for the shells, the concrete slab plays a vital structural role. It is intended to support the horizontal loads placed on the building, and post-tensioning cables placed inside the slab are used to do this task. In order to establish a balanced and stable structural structure, these cables are carefully positioned and tensioned within the concrete slab. By incorporating the concrete slab with the underlying cables as a horizontal load-bearing element, the Rolex Learning Center guarantees the efficient transfer of horizontal loads, such as wind and seismic stresses, throughout the structure post-tensioning technology. This helps maintain the building's overall stability and integrity while preserving the shells' distinctive architectural form. The load paths in the Rolex Learning Center shells are similar to that of a traditional shell in a way that it follows in-plane membrane action. However the addition of columns and post tensioning cables are critical in the design. The load path for the shell and the additional elements (highlighted) are shown in the figure 4.40.

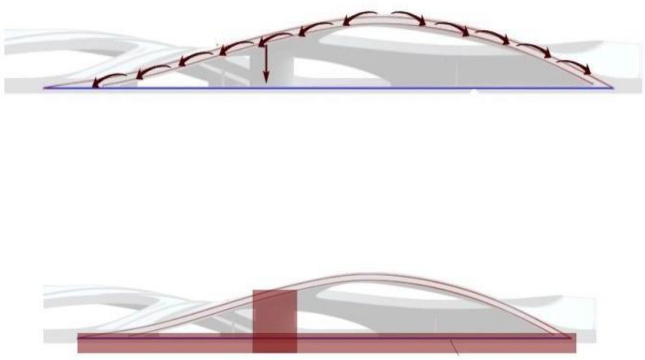


Figure 4.40 Top: Load paths Bottom: Additional support (column)and (tension cables) (Bollinger 2009)

4.2.4 The Metz Pompidou Center:

The Metz Pompidou Center, a captivating architectural feat representing innovation, creativity, and cultural relevance, is situated in Metz, France. The center was created by famous architects Shigeru Ban and Jean de Gastines, together with Philip Gumuchdjian. It is a remarkable blend of strong geometric shapes with a translucent roof made of fiberglass and Teflon-coated fabric. The building"s distinctive shell roof was inspired by the design of a historical Chinese hat.

Figure 4.41 The Metz Pompidou Center (Didier Boy De La Tou)

According to Ban, the idea of using a Chinese hat in the design came when he found it in an antique shop in Paris in 1999 while collaborating with Frei Otto to design a Japan pavilion for Hannover expo.The use of Frei Otto's wire-mesh technology, which used few materials, allowed for the creation of an intriguing three-dimensional interior space. But because the wire was a linear feature, it was necessary for Ban to build a timber shell over the wire mesh to create a historical roof structure. Realizing this, the idea of creating a grid framework out of two-dimensional, easily flexible laminated lumber developed. The idea involves putting the roof right on top of the grid construction. It was envisioned that this construction might be used as a compressive shell, improving its functionality beyond that of a simple tensile mesh structure, by utilizing the versatility of timber as both a tensile and compressive component (figure 4.42).

Figure 4.42 Historical Chinese hats and the Centre Pompidou Metz‟s roof system (Bulut, 2018)

Individual volumes are gently covered by a hexagonal timber roof structure, which combines them into a single, cohesive whole. This method avoids the requirement for large surfaces and unusual element lengths while yet maintaining the desired structural integrity and providing a solution that is both aesthetically pleasing and economically viable.

The imposing metal ring and a cone-shaped part that serves as a distinctive ten-branched node are located at the top of the hexagonal tower. This prominent steel structure, which rises to a height of 37 meters, offers crucial support for the project's large and wavy roof. A tripod building and spire that soars an astounding 77 meters above the ground are also located on top of the steel structure. These components help the overall architectural composition's vertical prominence and visual impact.

As per the statement made by Ben Lewis, a senior structural engineer affiliated with Arup, it has been determined that the roof consists of multiple layers of intersecting glulam timbers. These timbers are formed by the process of bonding together layers of wood, resulting in a grid pattern comprising hexagonal and triangular cells. The grid exhibits a subtle undulating and rippling effect as it takes shape to conform to the irregular contours of the roof. The geometry of the structure is designed in a manner that revolves around the presence of arched and circular openings, which serve as the primary architectural elements. These openings allow for the eminence of box-shaped gallery spaces, thereby contributing to the overall design and layout of the structure. In order to safeguard the structures and foyer from environmental conditions, a roofing system is employed, consisting of a rigid membrane. The roof is supported by two primary elements: the steel tower located in the center of the foyer, and the vortex-like columns created by the roof structure itself (Reid, 2005).

The tower has a vertical extension of 77 meters, starting from the floor slab and reaching up to the narrow peak of the roof. The tower's steel framework, which is visible, takes the shape of a hexagon and consists of steel tubes with a diameter of 510 mm (Arch Daily). In order to provide structural reinforcement for the vertical loads exerted on the roof, the framework is connected to a steel ring positioned at an elevation of 37 meters. The connection between the ring and the tower is established through the use of both horizontal and inclined struts. The tower gradually narrows towards a steel spire, situated above the roof. The remaining loads are borne by the columns within the roof's structural system. The vortex-like columns are situated along the perimeter of the complex and are connected to pile caps in the foundation.

The laminated timber consists of three layers that are oriented in different directions, resulting in the formation of a hexagonal lattice structure (figure 4.43). Timber blocks are incorporated within the layers, thereby augmenting the depth of the structure and consequently enhancing the structural efficacy of the system (Silver, et al 2013).The roofing structure of Centre Pompidou in Metz is constructed using glulam wood and was designed utilizing the parametric modeling method (Ban & McQuaid, 2003). The load from the timber roof shell is distributed towards the core ring structure which helps to transfer it to the ground with the load path shown in figure 4.44.

Figure 4.43 Wooden frame and roof system (Ban, 2010)

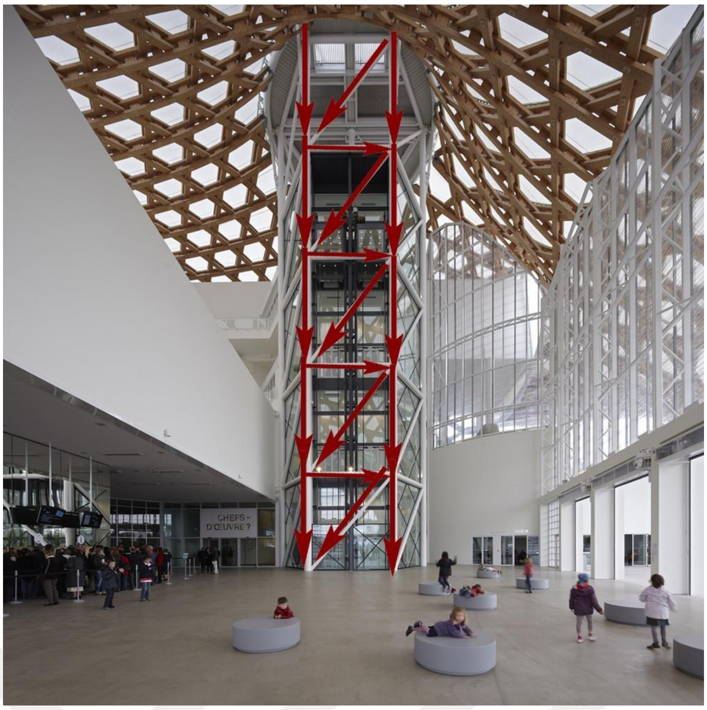


Figure 4.44 Load path on the Core Steel Ring Structure (ArchDaily)

The Metz Pompidou Center‟s metal ring structure is necessary in supplying the timber roof with crucial structural support. The roof's weight is dispersed by this metal ring, which also serves as a substantial load-bearing component, assuring the stability and integrity of the roof. The metal ring construction is necessary for the roof structure to function properly; otherwise, it would not be able to sustain its load without an end support from the metal structure (figure 4.45). The metal ring serves as a supporting structure for the form and function of the roof, which results in a harmonic and interdependent interaction between it and the timber roof. The combination of these two components demonstrates how the Metz Pompidou Center's architecture carefully considered both aesthetics and structural performance.

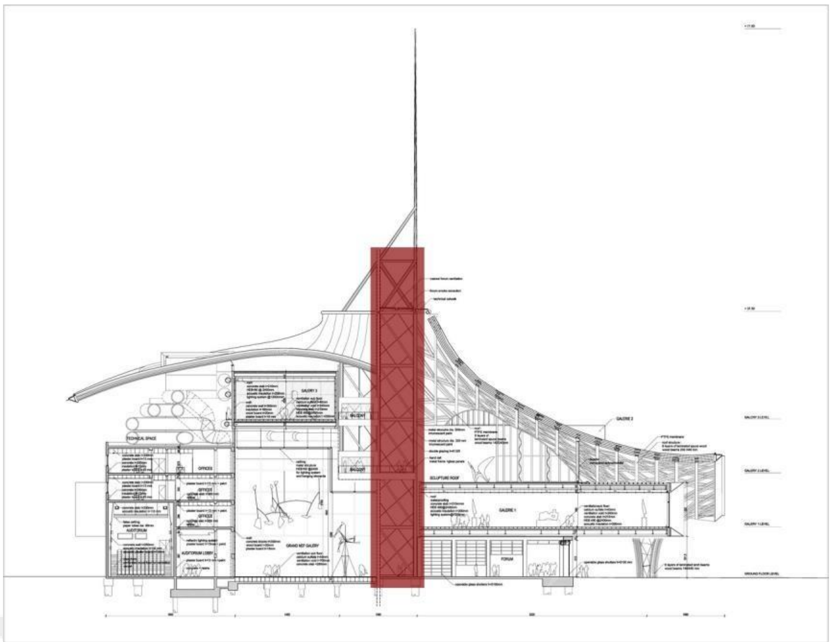


Figure 4.45 Additional Structural element (ArchDaily)

4.2.5 The Sage Gateshed Auditorium

The Sage Gateshead Music Center is situated on the industrial neighborhood of Gateshead's quayside. It acts as an essential catalyst for the regeneration of Newcastle and Gateshead's quaysides and offers the general public a recognizable and dynamic cultural center. With its conception in 1997 it became the first performing arts building by Foster & Partners**.** According to Chris and David, 2009, the curved silver shell around the three halls is the most alluring feature of this architectural masterpiece.

Figure 4.46 The Sage Gateshead (Sargent & Buchanan, 2010).

The four giant steel arches that span 80 meters from north to south support the roof shell, which is a separate structure. It comprises of a 720 tons steel grid-shell that is coated with 3,000 linen-finished stainless steel panels and 280 glass cladding panels. The complex toroid shape of the enclosure was simplified using specialist parametric modeling software to allow repetition and standardization of construction components. Underneath this horizontal steel and glass structure, supported by the four steel arches, there are three separate concrete structures. Each area within the structure is isolated from the others to prevent noise transfer. The complexity of the interface between the concrete layers of the rooms and the steel components of the roof remains concealed under the shell covering (figure 4.47).

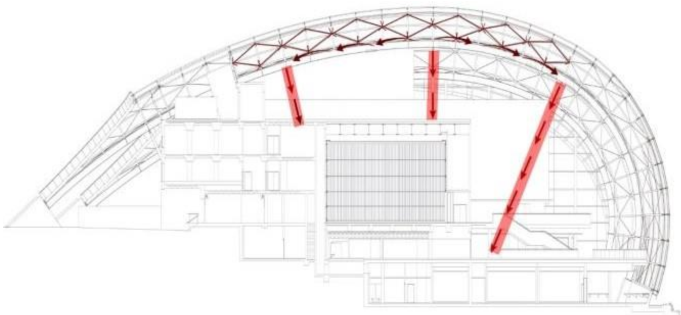


Figure 4.47 Additional Structural Elements in section view (Stephens, S, 2005)

250 flat glass panels with a trapezoidal pattern and 3,000 stainless steel panels make up the distinctive curved roof, which weighs 750 tones. The building's walls, floors, and foundations cleverly conceal the 3,858 tons of steel that makes up the entire construction. The steel arches are also supported by tilted steel rings that help transfer the load to the ground (figure 4.48).

Figure 4.48 Load transfer through Steel Arch and steel rings (Sargent & Buchanan, 2010).

4.2.6 L'Oceanografic Restaurant

Situated in Valencia, Spain, this structure is a polar array that forms a single shell with hyperbolic paraboloids makes up the building's distinctive design (Barrallo, 2011). Candela has used this specific form and equation in earlier designs and they have shown to be a great option for a shell structure, evenly distributing all compression stresses. Additionally, this design deems it possible to cover a large area with only eight supports, leaving an open area inside the building where no extra roof supports are needed. A glass and steel curtain wall is employed to partition the area below the lobes.

Figure 4.49 L"Oceanografic Restaurant (Barrallo, 2011)

The building's shell can be built to be extremely thin because of the use of the hyperbolic paraboloids.The outer perimeters are 47 mm by 47 mm in size. (Domingo, Lázaro, & Serna, 2004). Surprisingly, the shell's greatest thickness is less than 60 mm. As the hyperbolic paraboloid shells transfer loads through membrane action, there is minimal bending stresses in the shells and no additional load bearing elements are used. The load path follows the double curvature of the hyperbolic paraboloid as shown in the figure 4.50 in the plan view.

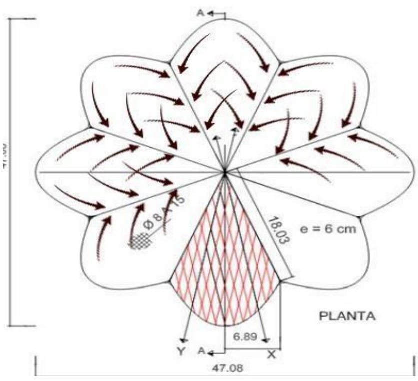


Figure 4.50 Load Paths of the shell (Domingo et al, 2004)

In order to build this shell, it was essential to manually place the steel reinforcement in accordance with the design requirements (figure 4.51). Due to the manual procedure used on-site, it took longer than usual to finish installing the steel reinforcement, which was necessary to alleviate the structural tension.

Figure 4.51 Temporary structure underneath the shell (CompetitionLine, 2012)

4.2.7 Downland Gridshell Building

This unique curved building was made by an integrated team of Architects, carpenters and structural engineers. The main space, a workshop for timber framing, was built using thin green oak laths joined together with bolts to form a square lattice of roughly 50 x 35 meters. After being put together, the lattice was bent using gravity into a characteristic shape like a shell of a three-nut peanut. The end product was a mix of conventional, and unique construction techniques, with design from the twenty-first century (figure 4.52).

Figure 4.52 The organic form of the Downland Gridshell. Image: Buro Happold/ Adam Wilson

The researchers used an interactive computer analysis approach that took on a set of nonlinear equations, a technique frequently used for modeling and analyzing tensile constructions, to carry out the analysis. With measurements of 50 meters in length, 12 meters in breadth, and 10 meters in height, the finished form resembles a massive peanut shell.

In order to create a spinning grid plane, the gridshell's architecture used a network of intersecting and twisted straight lines. To get this unique and outstanding result, the team had to create new methodologies that were especially suited to this project (figure 4.53).

Figure 4.53 Gridshell Detail (Buro Happold/ Adam Wilson)

4.2.8 Bosjes Chapel

The Bosjes chapel's roof is made of a thin concrete shell that has been cast, sustaining itself with remarkable undulations that cascade down to the earth. Glass panels that are organized in the shape of a cross decorate the facade at the crest of each wave on the roof structure. Other building functions are deftly hidden within the plinth or subtly incorporated into the outside corners of the surrounding garden to preserve the clean and uncluttered form of the roof and assembly space.

Figure 4.54 Bosjes Chapel under construction

(https://www.peri.co.za/projects/cultural-buildings/bosjes-chapel.html)

The chapel's elaborate ceiling measures 20 meters long by 12 meters wide and rises 6 meters at its tallest point. Shotcrete was chosen by the professional team and project team for the construction material, and a 1:50 scale concrete model was built in the PERI Cape Town yard to serve as a reference. The objective was to retain structural efficiency while producing an aesthetically pleasing result that seamlessly connected the entire structure.

Six peaks are present on the roof, one at each corner and one in the center of each long side. It is made of a thin shell of cast concrete. At four locations where it drastically lowers to the ground, it is supported. It was necessary to divide the surface into eight mirror portions for the assembly process, with a minimum of 130 sections being designed as a guide. Even though the chapel gives the impression of being weightless, its support relies on four concealed buttresses, each capable of bearing a vertical load of over 50 tons. Due to the shell's unique and unconventional shape, every rebar (both top and bottom) had to be separately sliced and shaped to fit precisely (figure 4.58). The application of concrete was achieved using shotcrete, a method that involves using mobile platforms while pneumatically projecting concrete at high speed through a hose. The final design consists of numerous dome shells (full and portions). The load transfer is similar to that of a typical dome shell in which the load path takes the meridian and hoop lines to transfer the load to the ground. The load path in both these lines are in-plane paths and avoid any out of plane stresses (figure 5.55).

Figure 4.55 Load Path (https://www.peri.co.za/projects/cultural-buildings/bosjes-chapel.html)

4.2.9 Teshima Art Museum

The Teshima Art Museum is located on Teshima Island in Japan. It features contemporary art and emphasizes the connection between art and nature. The architectural design offers a unique experience for visitors. The building's structure is a column-less concrete shell with a maximum height of 4.3 meters, measuring roughly 40 by 60 meters in size (figures 4.58). Concrete is used primarily in the construction of the museum. The Teshima Art Museum is constructed as a shell using a layer of concrete that is 25 cm thick and has no extra supporting structures. A formwork made of earth and mortar was built during construction to contain the amount of earth that would serve as the base. Then, a concrete shell 25 cm thick was cast over the formwork. The inside formwork was taken out once the outside shell had fully set. This resulted in the shell being in compression, without any out of plane bending stresses. The shell is not supported by any additional elements such as columns or edge beams and it transfers the self-load to the ground on its own. The load path for the Teshima Art Museum is shown in the figure 4.57.

Figure 4.57 Teshima Art Museum (left) Load Path for the shell (Right) (Ryue Nishizawa Archdaily)

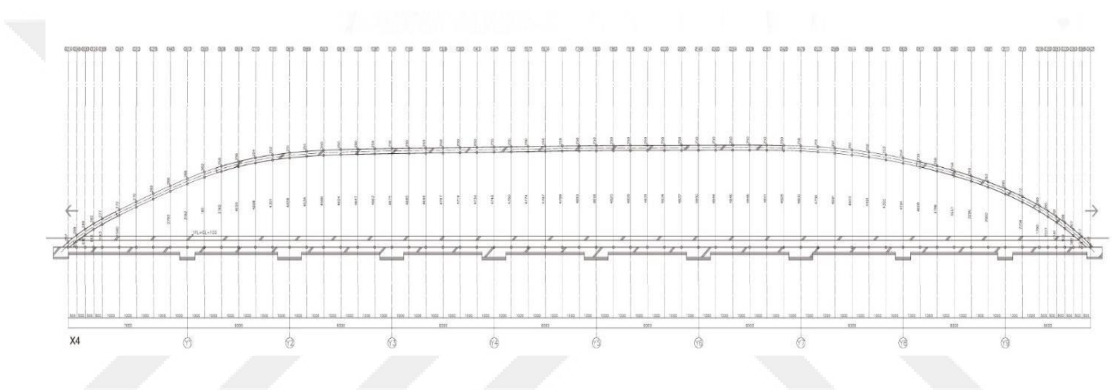


Figure 4.58 Shell"s section view (Arcdaily)

4.2.10 Savill Building at Windsor Great Park

The gridshell of the Savill Building is an interesting work of contemporary engineering that was inspired by the tough and fluid shape of a seashell. This structure has four layers and a consistent, 1 m-wide grid made of larch that is 80 x 50 mm in size. A tubular steel beam that surrounds the circumference and is lifted by slender paired legs supports the three-domed design. The stresses are sent down these legs to massive dumbbell pad foundations in the garden and a concrete bunker construction on the side facing the parking lot. A system of tie beams linking both sides below ground stabilizes the entire structure.

Figure 4.59 Savill Building (https://www.burohappold.com/projects/savill-building/)

The gridshell roof's timbers were malleable during construction, enabling them to be precisely bent in accordance with the intricate 3-dimensional model. This gridshell building method imitates the strength and flexibility present in natural shells and can be carried out using timber, steel, or concrete. This design's meticulous use of the stresses produced by the bent wood, which successfully distributes them through the steel ring beam and down to the foundations, is of utmost importance (figure 4.60).

Figure 4.60 Load Path in Savill Building through the Steel ring beams and columns (https://www.burohappold.com/projects/savill-building/)

4.2.11 Auditorio de Tenerife

The Auditorio de Tenerife stands as a notable cultural facility situated in Tenerife, the largest of the Canary Islands in Spain. It functions as a platform for diverse artistic performances and events, playing a significant role in enriching the island's cultural landscape. The architect behind its design is Santiago Calatrava. The complex's indoor and outdoor areas of this structures include natural patterns that resemble waves, wings, or moons. The tallest point of the roof rises to a height of 58 meters above the main auditorium. The structure's shape is based on platform terraces and includes a variety of intriguing geometrical designs. Two cone segments that intersect one other make up the geometric design of the roof. Contrarily, the concert hall's symmetrical inner shell, which rises to a height of 50 meters, resembles a rotational body created by spinning a curve to represent an ellipse (calatrava.com).

Figure 4.61 Auditorio de Tenerife (www.designingbuildings.co.uk)

For the construction, concrete is preferred over steel and glass, and a mixture of sand and titanium dioxide is used to achieve the flawless white tint. The use of white concrete has a number of benefits, including longevity, the elimination of finish painting requirements, and the capacity to reflect light, resulting in lower lighting expenses.

As seen in the Auditorium of Santa Cruz de Tenerife, Calatrava's inventive use of reinforced concrete in this project defies gravity and enables him to create forms that appear to be falling from the sky. A substantial concrete structure that resembles a cresting, crashing wave is the building's most noticeable feature. This exquisite shell, which is made up of two cone-shaped segments, rises elegantly, reaching its pinnacle at 58 meters, then gracefully narrows and thins as it lowers, finally coming to a spear-like tip pointed in the direction of the auditorium's entrance. The arch like shell of the structure transfers self-load to the ground via membrane action. That is the reason it could be built extremely slender without any additional supporting element. The load path for the shell is shown in figure 4.62.

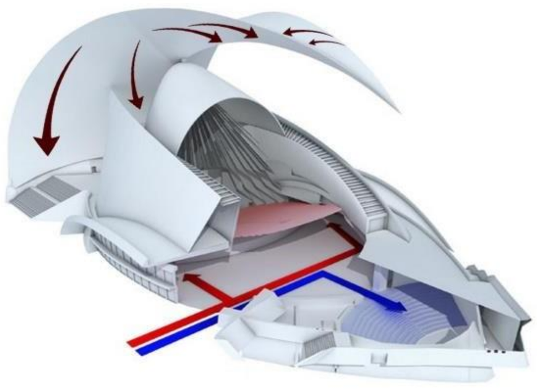


Figure 4.62 Load Path for Auditorio de Tenerife (www.designingbuildings.co.uk)

4.2.12 Harbin Opera House

The Harbin Opera House is an architectural structure located in Harbin, the capital city of Heilongjiang Province in northeastern China. It was designed by the architect Zaha Hadid and was opened to the public in 2015. Despite the building's extensive use of curvilinear curves, the structural elements of the theater have a more historical aspect.

By increasing the cross-sectional areas, using robust welded joints, and arranging structural components in a way that enhances the moment of inertia, the engineers were able to ensure strength and stability. As a result, the theater displays large, heavy constructions that individually weigh a few hundred tons. The exterior cladding is made up of smooth white aluminum panels giving it shell-like smooth surfaces.

The theater's architectural design presents an intriguing contrast between curvilinear curves and historical structural elements. While the exterior boasts a visually striking appearance with white aluminum panel, the underlying structural framework follows a more conventional approach to ensure strength and stability. The engineers employed ingenious techniques to enhance the theater's structural integrity, such as increasing the cross-sectional areas of key load-bearing components, effectively bolstering its capacity to withstand heavy loads and external forces (figure 4.63).

Alongside the increased cross-sectional areas, the engineers opted for robust welded joints, which significantly contributed to the overall strength of the theater's structural framework. These welds provided exceptional durability, minimizing the risk of structural failure and ensuring the safety of its occupants. Furthermore, the strategic arrangement of structural elements played a crucial role in enhancing the moment of inertia, reinforcing the building's resistance to bending and flexing.

Figure 4.63 Harbin Opera House exterior (left) Structural components underneath the shell (Right)

The form finding methods and the structural systems of the case studies have been explored in this chapter. The case studies from both historical and contemporary shells were analyzed within the scope of the study. The comparative analysis contained eighteen case studies in total. The structural systems were analyzed with a particular focus on shell behavior and additional elements. The next chapter deals with the results and classifications of the case studies under various contexts.

CHAPTER 5

RESULTS AND DISCUSSION

In the preceding chapter, an analysis was conducted on the selected case studies through literature and technical reports, with a specific focus on scrutinizing their form finding methods and structural systems. The primary objective was to determine how much these structures are incorporated with any additional structural systems in conjunction with their shell behavior. Building upon this groundwork, this chapter interprets the research findings obtained by the case studies in chapter 4. The aim of this chapter is to process the results, presenting an analysis that answers the research objectives and hypotheses.

The aim of the analysis was to investigate the historical evolution of shell structures and evaluate how much contemporary shell structures work is pure shell behavior. This was achieved through a comparative analysis of their form-finding approaches and structural systems.

5.1 Form–Finding Analysis

The analysis of both historical and contemporary case studies on shell structures revealed the use of multiple form-finding techniques within some cases. Figure 5.1 shows that the form-finding of contemporary shell structures is not completely independent of the mechanical/analytical methods as these methods may be combined with the modern design software to achieve the desired form and structural performance. These architectural solutions blend aesthetics, functionality, and structural integrity by combining multiple form-finding methodologies. The use of several form-finding techniques allows for a deeper understanding of design options, ultimately pushing the limits of what is possible with shell construction. The case studies and their form-findings classification is summarized in figure 5.1

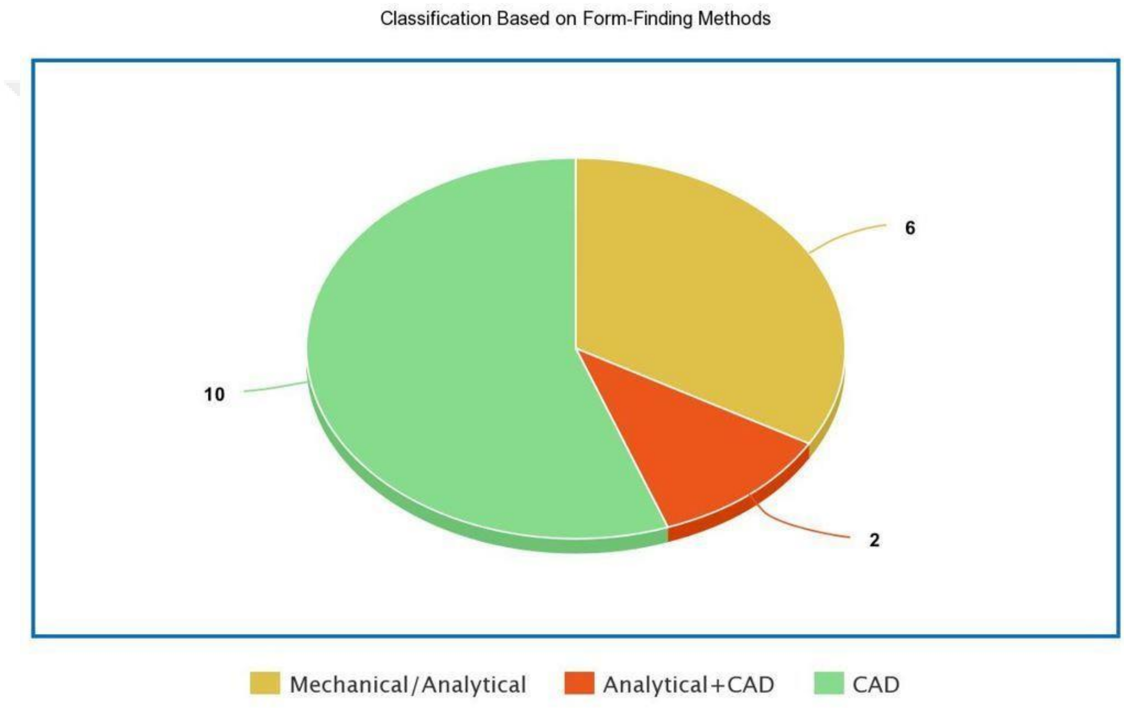


Figure 5.1 Form finding classification

Among the case studies, all six belonging to historical case studies expectedly employ mechanical/analytical form finding methods. Ten structures among contemporary case studies were designed using advanced computer design software while two i.e. Teshima Art Museum and La‟Oceanografic Restaurant utilized a mixture of both analytics/mechanical form finding methods together with computer aided design. A noteworthy observation is the correlation of the form findings method and the subsequent structural system seen in the case studies. The aforementioned two case studies that use both historical and contemporary from finding methods display pure shell behavior due to the funicularity achieved from the mechanical/analytical from findings method.

5.2 Analysis of Structural Systems:

The results of the analysis of the case studies are summarized here forth, with an emphasis on the demonstration of original shell behavior, the identification of additional structural systems, and the degree to which the case studies follow pure shell principles. The case studies are then grouped into certain classifications. The examination also sought to locate any additional structural components that were incorporated into the designs.

The category of analyzed case studies encompassed six traditional shell structures having both analytical and physical/mechanical form-finding approaches and twelve contemporary shell structures. Through an analysis of their structural systems and load paths it is evident that all the structures in historical case studies exhibited characteristic shell behavior. Notably, no supplementary structural support systems were employed for load transfer within these shell structures.

The results of these analyses revealed a common trend among the historical case studies. It is apparent that all of the structures exhibited structural behavior governed by the principles of shell structures. The intrinsic strength and stability of the shell forms allowed them to efficiently bear and distribute the applied loads without the need for any additional structural support systems. This finding emphasizes the inherent structural efficiency and load-bearing capacity of historical/historical shell structures that constitute the fundamentals of shell behavior.

Meanwhile, the structural systems in the contemporary case studies varies from one shell to another. Four contemporary cases have membrane action similar to the traditional shell behavior. These cases employ no additional elements as they depend solely on the membrane action for the load transfer. Two shell structures among the contemporary case studies display traditional shell behavior, however, with the support of additional structural elements. The remaining six structures showcase entirely different structural systems to traditional shell behavior. These include steel space frames, wooden grid shells, ring beams etc. The analysis of the structural systems of the case studies is summarized in the table 5.1.

Table 5.1 Structural Systems

Case Study	Type of Shell	Function	Structural System
Deitingen Service Station	Reinforced Concrete Shell	Roof	Membrane Action
The Chapel of Lomas de Cuernavaca	Reinforced Concrete Shell	Roof & Envelope	Membrane Action
Kimberly Art Museum	Reinforced Concrete Shell	Roof	Membrane Action
Coop Distribution Center	Reinforced Concrete Shell	Roof	Membrane Action
Palazetto Dello Sport	Reinforced Concrete Shell	Roof	Membrane Action
Thompson Arena	Reinforced Concrete Shell	Roof	Membrane Action
La"Oceanografic Restaurant	Reinforced Concrete Shell	Roof	Membrane Action
Bosjes Chapel	Reinforced Concrete Shell	Roof	Membrane Action
Teshima Art Museum:	Reinforced Concrete Shell	Roof	Membrane Action
Auditorio de Tenerife	Reinforced Concrete Shell	Arc	Membrane Action
Rolex Learning Center	Reinforced Concrete Shell	Floor	Membrane Action + Arches, Columns, and Post-tensioning Cables
Saijo Crematorium	Reinforced Concrete Shell	Roof	Membrane Action + Shells supported by structural columns
Heydar Aliyev Center	Steel Space-frame	Envelope	Integrated Core RC+ Space-frame Structure
Pompidou Center-Metz	Wooden-gridshell	Roof	Central steel ring structure supporting the grid roof shell
The Sage Gateshed Auditorium	Steel -Gridshell	Envelope	Steel Arches + RC structures
Downland Gridshell Building	Wooden-gridshell	Roof	Bolted Grid shell System
Savill Building	Wooden-gridshell	Roof	Steel Rings for Vertical Support
Harbin Opera House	Steel Space-frame	Envelope	Integrated Core RC+ Space-frame Structure

The membrane action in the historical cases is shown in the figure 5.2.

Historical Case Studies

- **The Highlighted region shows the shell area under membrane action.**
- **There are no additional load-bearing elements under these regions of the shell.**
- **Presence of negligible out of plane forces allows these shells to be built over larger spans without additional elements.**

References: (Chris J.K Williams) (Nicolas Janberg) (Park et al, 2007) (New York: Praeger, 1963) (Sun, 2017)

Figure 5.2 Membrane action in historical case studies

5.3 Categorization of Contemporary Case Studies

After analyzing the structural systems of contemporary case studies through a series of technical reports and scientific journals. It can be inferred that incorporating additional structural elements to support the shells diminishes the primary structural characteristics of shell behavior. In these structures, the shells are not the sole load-bearing elements and the additional structural systems are employed for these complex shells to be structurally sound.

The contemporary curvilinear architectural design revolves around the seamless integration of buildings within their surrounding environment, achieving a sense of harmony and cohesion. Architects and engineers have found inspiration in the graceful curves and forms observed in nature, such as arches, vaults, and domes, utilizing them to create expansive and impressive structures. Through the incorporation of these curved elements, architects have been able to craft extraordinary architectural compositions that transcend the limitations of historical design methods. The term "curvilinear" has been coined to capture the essence of these innovative and dynamic forms. Nevertheless, the construction of curvilinear structures has presented unique challenges for professionals due to the inherent complexities involved. These more complex and fluid curved shell designs require more robust structural systems to fulfill the design requirements. Hence, deviating from the classification of a traditional shell.

The contemporary case studies are classified into three different categories based on their shown behavior after conducting an analysis of their structural systems. These three categories are as follows:

5.3.1 Category 1: (Original Shell Behavior)

The first group of contemporary structures includes those that firmly adhere to pure shell behavior, which is distinguished by an effective load distribution that is primarily mediated by membrane action. Due to their self-supporting shell geometries, these case studies show an innate ability to sustain loads. These formations exhibit considerable curvature, symmetrical patterns, and shell curves, closely resembling conventional shell arrangements. Additionally, the efficient way in which their load-distribution systems transmit outside forces across their surfaces effectively taps into the inherent strength of shell structures. These case studies demonstrate the pure shell form's ongoing relevance in modern architectural practice. This category includes La‟Oceanografic restaurant Bjoses Chapel, Teshima Art Museum and Auditorio de Tenerife among our case studies.

La’Oceanografic Restaurant:

One of the very few designs of Felix Candela that was designed by using both analytical form-finding together with computer aided design. The use of Hypar sections in the design of the structure signifies the effieciency of this form as a pure shell even when designed with the assistance of computer aided softwares. The structure does not incorporate any supplementary load bearing system and the self load of the shells is primarily transfered to the ground through membrane action.

Bosjes Chapel:

Although designed using modern design tools, Bosjes Chapel displays inherent shell behavior as it does not employ any additional load bearing element. The self supporting shell behavior eleminiates the need of any interior columns and enhances the opnenness of the structure, another innate characteristic of the shells.

Teshima Art Museum:

The Teshima Art Museum, with its distinctive structural structure that personifies the idea of a pure shell, stands as an illustration of architectural ingenuity. Visitors can experience a flowing and uninterrupted environment because there are no internal columns or other supporting structures, which heightens the feeling of openness and unity inside the area, fulfilling the structural and functional characteristic of original shell structures.

Auditorio de Tenerife:

The Auditorio de Tenerife is characterized by its distinctive arc structure, which functions as a self-supporting shell without the requirement of supplementary load-bearing components. This architectural feature demonstrates a structural efficiency that eliminates the necessity for internal columns or beams.

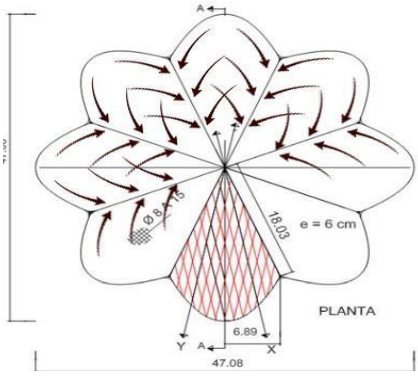
Category 1 and its load path is summarized in Table 5.2 & 5.3.

Table 5.2 Category 1

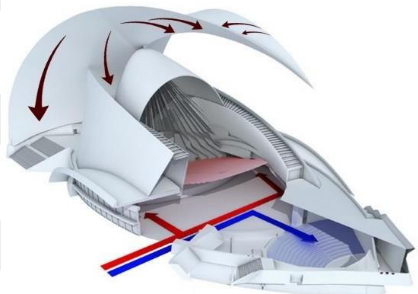
Case Study	Type of Shell	Function	Structural System
La"Oceanografic Restaurant	Reinforced Concrete Shell	Roof	Membrane Action
Bosjes Chapel	Reinforced Concrete Shell	Roof	Membrane Action
Teshima Art Museum:	Reinforced Concrete Shell	Roof	Membrane Action
Auditorio de Tenerife	Reinforced Concrete Shell	Arc	Membrane Action

Table 5.3 Load Paths for Category 1

Category 1



Left: Load path in the hypar shells , Right: Load Path in Bosjes Chapel



Left: Load Path for Teshima Art Museum Right: Auditorio de Tenerife

- Load path in the Case studies of category 1 follow the historical shell load path without the need of any additional load bearing system.
- The load path acts as in-plane membrane forces.
- Category 1 includes structures with the load paths of a hyperbolic paraboloid section (Top left) and Dome structure (Top right) in addition to doubly curved shells.

References: (Ryue Nishizawa Archdaily)
(https://www.peri.co.za/projects/cultural-buildings/bosjes-chapel.html) (www.designingbuildings.co.uk)
Domingo et al, 2004, p. 1144)

5.3.2 Category 2 (Shell Behavior with Additional Elements)

Case studies that utilize the addition of structural systems beyond the conventional shell behavior are included in the second category called Hybrid Shells. Even while these structures might have certain shell-like features, they also incorporate extra components like trusses, columns, or beams to increase their load-bearing capability. These added structural systems are used to handle problems brought on by longer spans, intricate geometries, or increased load requirements. By adding these components, structures become more stable and secure overall, allowing architects and engineers to fulfill ambitious and avant-garde architectural designs that go beyond the bounds of simple shell behavior. These case studies, which combine shell shapes with extra supports to push the boundaries of architectural design, serve as an example of the hybrid structural solutions trend in contemporary shell architecture.

This category includes the following structures from the case studies:

The Meiso No Mori Crematorium:

The Meiso No Crematorium deviates from the typical perception of shells as buildings that offer a large, open space without columns. To support the roof shell and transfer the weight through the shells, it instead features several structural columns. The idea of an uninterrupted and open interior environment provided by a historical shell is tested by this divergence from the conventional understanding of shells. Despite the fact that the presence of columns is crucial for supporting the structure and guaranteeing the stability of the roof, it also introduces vertical features that divide the space and break the impression of continuity generally found in shell buildings. The use of columns calls for a careful balancing of the space's structural needs with its desired aesthetic and practical aspects.

Rolex Learning Center:

Similarly, The Rolex Learning Center's structural plan takes into account the shell's components as well as the existence of structural arches, columns and post-tensioning cables below the shell. The shells themselves are not the only load-bearing structures and are well supported by these additional structural elements. The arches distribute the loads and stabilize the entire shells by providing the necessary bending stiffness to the shells while the columns provide vertical support to the shells in Rolex learning center. The post-tensioning cables improve the structural integrity by acting as a lateral load-bearing element. These components operate cohesively to maintain the Rolex Learning Center's structural integrity and functionality. The integration of these complimentary structural components demonstrates the care taken in the design and construction of this structure to address both aesthetics and functionality (Table 5.4 & 5.5).

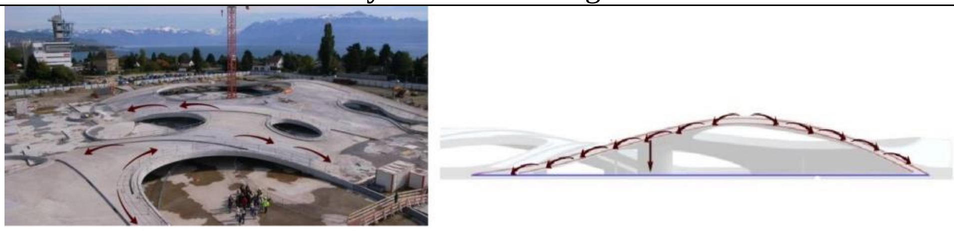
Table 5.4 Category 2

Case Study	**Type of Shell**	**Function**	**Structural System**
Rolex Learning Center	Reinforced Concrete Shell	Floor	Combination of Shells, Arches, Columns, and Post-tensioning Cables
Saijo Crematorium	Reinforced Concrete Shell	Roof	Shells supported by structural columns

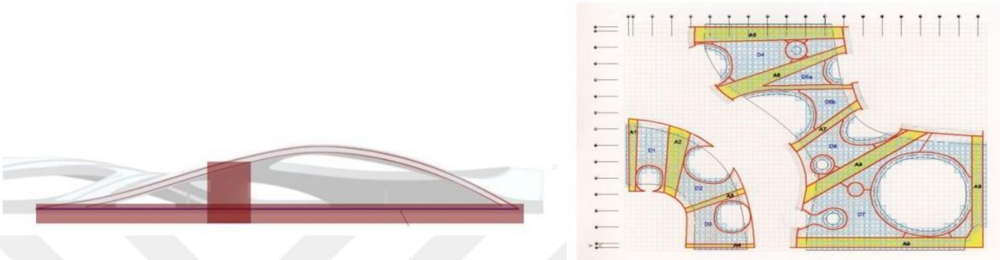
Table 5.5 Load Path for Category 2

Category 2

Case Study- Rolex Learning Center



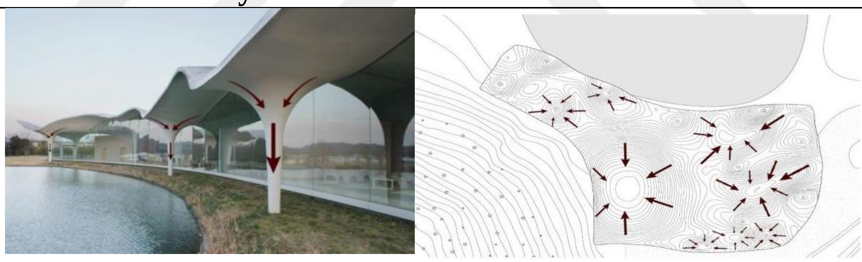
Load path for the Big shell



(Highlighted) Additional Load bearing systems (Column, arches and horizontal post-tension cable)

- The Load path for the Rolex learning center resembles that of a historical shell.
- The additional Loadbearing elements include columns, post-tensioned cables and steel arches.
- This combination makes up an efficient hybrid system.

Case Study: The Meiso No Mori Crematorium



Left: Load path for the roof shell, Right: Plan View)

Additional Structural system (columns) and the Frequency of columns" placement in the plan

- The Load path of the roof shell concentrates around the columns through which they are transferred to the ground.
- The frequent placement of columns under the shell takes away from the column-free area i.e. a property of historical shells.

Refernces: Pugnale (2017), Windeck (2016), (OpenBuildings), Weilandt (2009), Bollinger (2009)

5.3.3 Category 3 (No Shell Behavior)

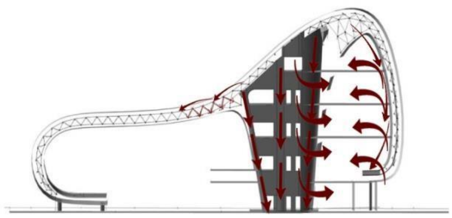
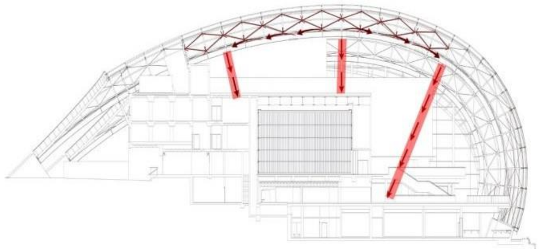
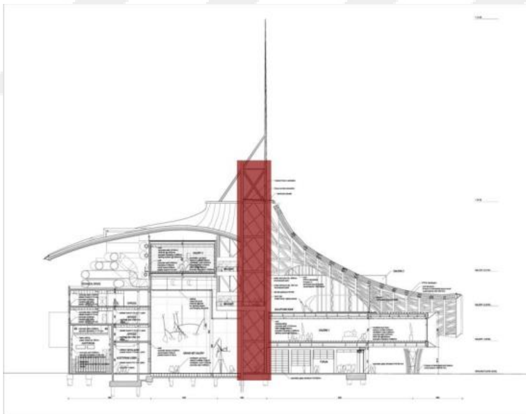
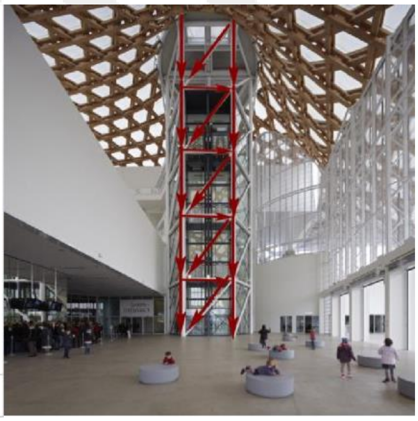
Structures that differ greatly from pure shell behavior and have no resemblance to conventional shell setups fall into this category. These case studies mostly rely on other structural systems for primary load-bearing support, such as space frames, grids shear walls, trusses etc. Even though they might integrate some shell-like components for aesthetic reasons, their load distribution algorithms and architecture do not follow the basic rules of pure shell behavior. Instead, they adopt alternate structural techniques that are frequently affected by technical specifications, aesthetics in architecture, and practical factors. These case studies demonstrate the variety of architectural strategies used in contemporary shell constructions and emphasize the adaptability of the design options accessible to architects and engineers in solving particular difficulties and design goals.

The remaining of the contemporary case studies in this thesis fall under category 3. They display advanced structural systems that do not adhere to the characteristics of pure shell behavior. These systems include steel space frame, wooden grid structures and a combination of other structural solutions. The case studies are summarized in the table 5.6 & 5.7.

Table 5.6 Category 3

Case Study	**Type of Shell**	**Function**	**Structural System**
Heydar Aliyev Center	Steel Space-frame	Envelope	Integrated Core RC+ Space-frame Structure
Pompidou Center-Metz	Wooden-gridshell	Roof	Central steel ring structure supporting the grid roof shell
The Sage Gateshed Auditorium	Steel -Gridshell	Envelope	Steel Arches + RC structures
Downland Gridshell Building	Wooden-gridshell	Roof	Bolted Grid shell System
Savill Building	Wooden-gridshell	Roof	Steel Rings for Vertical Support
Harbin Opera House	Steel Space-frame	Envelope	Integrated Core RC+ Space-frame Structure

Table 5.7 Load Path examples for Category 3

Category 3
  Left: Load Path from space frame to RC core, Right: Load path in steel grid   Load path of steel ring beam system   Left: Additional Structural Element (Steel core) Right: Load Path for the Steel Core • Category 3 have three type of structural systems, Grid shells, Ring beams and core Rc structure. • The Load paths suggests a completely different and independent structural system compared to historical shells. • Load path follows additional load bearing elements such as columns, ring beams and integrated RC core. References: https://www.burohappold.com/projects/savill-building/) (Alvarez 2009) (Stephens, S, 2005) (Sargent & Buchanan, 2010) (ArchDaily)

The summary of the classification of contemporary case studies on the basis of adherence to primary shell behavior is shown in the figure 5.3.

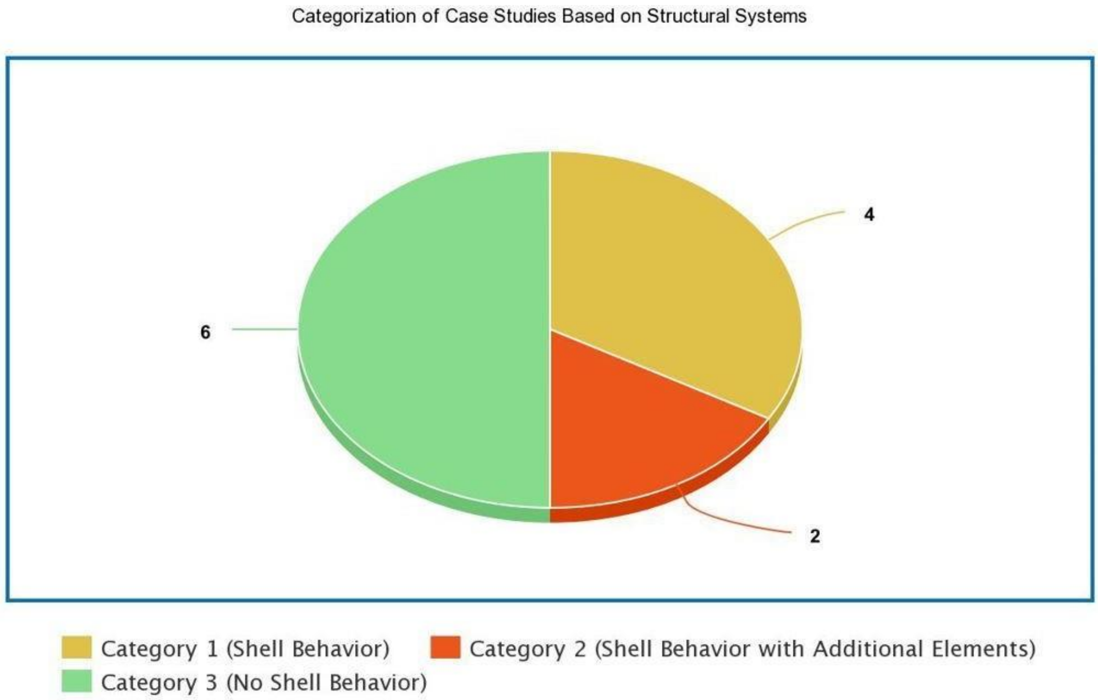


Figure 5.3 Categorical classification of Contemporary Cases

From the aforementioned categories, it can also be seen that type of shell i.e. RC shell, Steel Space frame, wooden grid shell etc. is also correlated with the structures" adherence to historical shell behavior. The four case studies among contemporary cases that demonstrate membrane action as their structural system are all Reinforced concrete shells. Meanwhile the case studies that use wooden-grid shell, steel space frame or the ring beams system do not display traditional shell behavior.

Timeline of the contemporary case studies and changes in shell behavior:

The evolution of the structural systems of contemporary shell structures and the shift from membrane action to additional structural systems is not found to be a year-specific phenomena among case studies. It is evident that the shift from membrane action to additional structural systems is not confined to a specific time period among the analyzed case studies. In examining the data presented in Table 5.5 it becomes apparent that pure shell behavior is observed sporadically across the contemporary case studies. This finding suggests that despite the advancements and availability of complex structural systems, the historical shell behavior remains relevant and is not rendered obsolete in contemporary shell structures. The coexistence of both historical shell forms and innovative structural approaches highlights the versatility and adaptability of shell structures as an architectural solution. The evolution of shell behavior over the years in contemporary case studies is shown in figure 5.4.

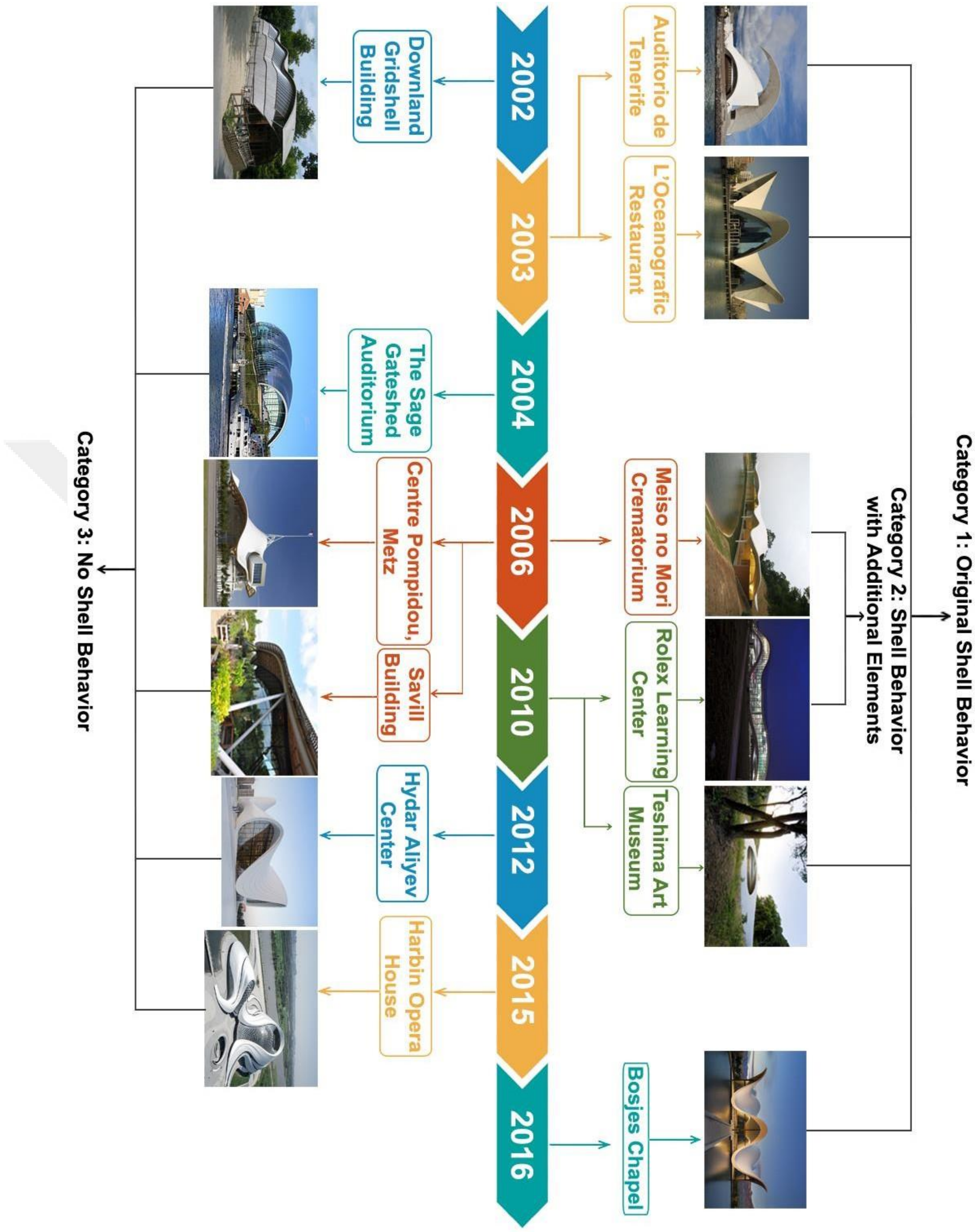


Figure 5.4 Evolution of shell behavior in contemporary case studies

To conclude, this chapter presents classification of the case studies based on different parameters such as form finding methods and structural systems. It presents how the historical shell behavior or membrane action evolves within different shell types. It also summarizes any correlation of timeline of the contemporary case studies and changes in shell behavior.

CHAPTER 6

CONCLUSION

The structural systems of shell structures in architecture have undergone significant evolution, marked by notable advancements. Shell structures, renowned for their curved and self-supporting nature, have captivated the imaginations of architects and engineers throughout history. Through continuous exploration and experimentation, architects have pushed the boundaries of design and construction techniques, resulting in the emergence of diverse structural systems that have redefined the field of shell structures.

In the early stages of shell structures, exemplified by iconic structures like the Pantheon in Rome, the potential of utilizing a single curved surface to create expansive interior spaces was showcased. During this period, the structural integrity of shell structures relied primarily on the inherent strength of the shell itself, allowing it to bear the imposed loads and distribute forces efficiently. Despite the limited technological advancements and availability of construction materials, ancient builders demonstrated exceptional skill in harnessing the compression and tension forces within the shell to achieve stability and durability.

However, as architectural and engineering knowledge progressed, new materials and construction techniques paved the way for more intricate and sophisticated shell structures. The mid-20th century witnessed a revolutionary shift in the evolution of shell structures with the advent of computer-aided design and analysis. The integration of advanced computational tools enabled architects and engineers to simulate and optimize the structural behavior of shell forms. This led to the development of innovative structural systems, such as the space frame and grid systems, which combines the inherent strength of the shell with an underlying framework of interconnected members. These advancements in computational analysis and structural engineering allowed for more complex structures, setting new benchmarks in shell structure design.

By performing an intensive assessment of the available literature and examining a carefully chosen group of historical and contemporary case studies, this thesis set out on a thorough analysis of the evolution shell structures. The main goal was to comprehend form-finding techniques and structural systems in shell structures better. Through this analysis, it was aimed to determine the degree to which contemporary shell structures abide by or stray from the fundamental shell structure principles.

For the initial stage of the investigation, sixteen case studies were chosen that included both classic and contemporary examples of shell structures. Based on their form-finding strategies, that include mechanical, analytical, or freeform employing digital tools, the case studies were chosen. The selection criteria made sure that distinct form-finding strategies and structural systems from various historical eras and building approaches were represented.

When analyzing historical case studies, the form-finding processes and structural systems used by well-known shell structures created in the 20th century were analyzed which laid the groundwork for the comparative investigation of the contemporary case studies by offering insightful information about the evolution of conventional shell behavior.

After the analysis, this study categorizes contemporary shell structures into three main groups. The first category comprises structures that strictly adhere to pure shell behavior, showcasing effective load distribution through membrane action and inherent strength. Examples of this category among case studies include La'Oceanografic restaurant, Bjoses Chapel, and Teshima Art Museum. The second category, Hybrid Shells, incorporates additional structural systems like trusses and columns to enhance load-bearing capabilities, allowing for ambitious designs. The third category encompasses structures that deviate from conventional shell setups and rely on alternative structural techniques such as space frames and grids for load support, showcasing the diversity and adaptability of contemporary shell architecture. The remaining case studies in this thesis belong to category 3, showcasing various advanced structural systems that depart from pure shell behavior.

The contemporary case studies present an impressive variety of intricate shell forms made possible by the combination of form-finding techniques, computer-aided design, and cutting-edge structural solutions. A variety of shell constructions, including reinforced concrete, steel space frames, and timber grid shells, were included in this varied collection of case studies. Due to the incorporation of additional structural elements to maintain their load-bearing capacities, the investigation of these modern structures allowed this study to spot variations from conventional shell behavior.

The two main factors in the comparative analysis were structural systems and form-finding techniques. Examining these components carefully throughout the case studies aided to identify common traits, discrepancies, and patterns. The research elaborated the historical development of shell design and emphasized the transition to novel form-finding methods and the use of cutting-edge structural systems in contemporary shell architecture.

This thesis offers a clearer grasp of how form-finding techniques and structural systems have evolved within this architectural typology as a result of comparative analysis of shell structures. Comparing historical and contemporary case studies has shed light on the development of shell design principles that may give architects, engineers and researchers a better understanding of the evolution of shell structures.

This study also shows that a combination of form–findings methods is still incorporated in some instances of contemporary shell structures. The incorporation of supplementary structural components, such as space frames, arches, columns, and steel ring structures, led to improved structural reinforcement and functionality, albeit at the expense of deviating from the inherent simplicity and fundamental principles of conventional shell structures. The contemporary interpretations presented in this study have questioned the historical understanding of shells as the exclusive load-bearing elements, instead adopting novel approaches that challenge established classifications. The examination and incorporation of diverse structural systems underscore the fluidity of shell design and its ability to adapt to the changing architectural environment. The exploration of historical and contemporary shell structures offers valuable insights for the future development and classification of complex architectural typologies, as architects strive to expand the limits of form and function.

In conclusion, the evolution of shell structures in architecture has been marked by remarkable advancements in their form-finding methods and structural systems. Architects have continually pushed the boundaries of design and construction techniques, harnessing the potential of shell forms to create awe-inspiring spaces. From the ancient wonders to the modern marvels, the evolution of shell structures reflects the change in the historical function of the shell structures; being sole load bearing elements and providing unrestricted column-free spaces over large spans. As we move towards more contemporary cases of shell structures that display a complex geometry and a combination of multiple structural systems to support that geometry, the shells are no longer seen as a structural solution to cover large spaces, but a more aesthetics solution for the built environment and hence, cannot be explicitly classified as the pure shell forms.

It is important to acknowledge the limitations of this study. The scope of this thesis has focused on a select set of case studies, and further investigations could expand the analysis to include more diverse examples from different regions and cultural contexts. Furthermore, this thesis uses the existing body of literature to make a comparative analysis. Further investigations may include finite element analysis of shell structures from historical and contemporary eras to have a deeper understanding of the structural systems in shell structures from different eras.

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