SALES PLANNING IN CLOSED-LOOP SUPPLY CHAINS: RECYCLING AND REMANUFACTURING OPTIONS FOR EARLY-GENERATION RETURNS

A THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF

MASTER OF SCIENCE

IN

INDUSTRIAL ENGINEERING

By

Büşra Bayrak

August 2023

Sales planning in closed-loop supply chains: recycling and remanufacturing options for early-generation returns
By Büşra Bayrak
August 2023

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Emre Nadar(Advisor)

Özlem Karsu

Ece Zeliha Demirci

Approved for the Graduate School of Engineering and Science:

Orhan Arıkan
Director of the Graduate School

ABSTRACT

SALES PLANNING IN CLOSED-LOOP SUPPLY CHAINS: RECYCLING AND REMANUFACTURING OPTIONS FOR EARLY-GENERATION RETURNS

Büşra Bayrak
M.S. in Industrial Engineering
Advisor: Emre Nadar
August 2023

We consider a durable-good producer who optimizes its sales strategy for two successive generations of the same product and is able to remanufacture or recycle the first-generation product returns. The customer arrivals follow a multi-generation diffusion process that takes into account the word-of-mouth feedback spread within each customer population of successive product generations as well as the substitution effect among these product generations. We investigate the economic viability of deliberately slowing down the second-generation product diffusion to improve the first-generation remanufactured-item sales and the use of recycled content in the second-generation production in the long run. We prove that such a forward-looking approach is optimal if (i) the diffusion process is fast enough in the absence of any manipulation, (ii) the number of first-generation end-of-life returns and the recyclable-material amount from each such return are high enough, and (iii) the potential customer base of the first-generation product is sufficiently large. We also show that the same forward-looking approach is less likely to be optimal when the used items can only be acquired from the previous buyers of the first-generation product who return their used items to trade up to the second-generation product. We conjecture that our sales strategy has the potential not only to improve the profits in the long run but also to contribute to sustainable production and consumption by helping recover more used items via remanufacturing and recycling options.

Keywords: multi-generation product diffusion; sales planning; closed-loop supply chains; remanufacturing; recycling.

ÖZET

KAPALI DÖNGÜ TEDARİK ZİNCİRLERİNDE SATIŞ PLANLAMASI: ERKEN-NESİL İADELER İÇİN GERİ DÖNÜŞÜM VE YENİDEN ÜRETİM SEÇENEKLERİ

Büşra Bayrak
Endüstri Mühendisliği, Yüksek Lisans
Tez Danışmanı: Emre Nadar
Ağustos 2023

Aynı ürünün iki ardışık nesli için satış stratejisini eniyileyen ve geri getirilen birinci-nesil ürünleri yeniden üretebilen ya da geri dönüştürebilen dayanıklı ürün üreticisi ele alınmaktadır. Müşteriler, ardışık ürün nesillerinin her bir müşteri nüfusu içindeki sözlü iletim yayılımını ve bu nesiller arasındaki ikame etkisini göz önünde bulunduran çoklu-nesil yayılım sürecine göre gelmektedir. Uzun vadede birinci-nesil yeniden üretilmiş ürün satışını ve ikinci-nesil ürün üretiminde geri dönüştürülmüş içerik kullanımını iyileştirmek için bilinçli olarak ikinci-nesil ürün yayılımını yavaşlatmanın ekonomik açıdan kârlılığı incelenmektedir. Bu ileriye-dönük yaklaşımın (i) herhangi bir manipülasyon olmadan yayılım süreci yeterince hızlıysa, (ii) geri getirilen birinci-nesil ömür sonu ürün sayısı ve her bir geri getirilen üründen elde edilebilen geri dönüştürülebilir materyal miktarı yeterince büyükse ve (iii) birinci-nesil ürünün potansiyel müşteri kitlesi yeterince büyükse, eniyi olduğu kanıtlanmıştır. Aynı ileriye-dönük yaklaşımın, geri getirilen tüm ürünlerin kullanılmış birinci-nesil ürünlerini ikinci-nesil ürün almak için geri getiren müşterilerden elde edildiği durumda daha az olasılıkla eniyi olabileceği gösterilmiştir. Bu satış stratejisinin, uzun vadede kârlılığı arttırmaya yardımcı olurken yeniden üretim ve geri dönüştürme seçenekleriyle kullanılmış daha fazla ürünü değerlendirerek sürdürülebilir üretim ve tüketime de katkısının olabileceği beklenmektedir.

Anahtar sözcükler: çok nesilli ürün yayılımı; satış planlama; kapalı-devre tedarik zincirleri; yeniden imalat; geri dönüşüm.

Acknowledgement

First and foremost, I would like to express my endless gratitude to my advisor, Asst. Prof. Emre Nadar, for his invaluable support, encouragement, and guidance throughout my Masters. Under his mentorship, I not only acquired essential research skills but also gained invaluable life lessons. His insightful guidance has been pivotal in shaping my future path. I am truly fortunate to have completed this process under his supervision. I hope to do more research with him in the future.

I would like to thank to Assoc. Prof. Özlem Karsu and Asst. Prof. Ece Zeliha Demirci for their valuable time to read and review this thesis.

I extend my heartfelt gratitude to my mother, Hülya Bayrak, and father, Ali Bayrak, who have consistently shown me unconditional patience and support in every circumstance. Their consistent academic guidance and encouragement in my decisions have significantly eased my journey. I am genuinely grateful to my beloved sister, Burcu Öğütcü, whose opinions I value the most and who has always listened and supported me. I am also thankful to my cousin, Dilşad Yıldız Kaçar, who believes in my successful completion of this journey.

I cannot thank Atahan Bayır enough for demonstrating his unwavering presence by my side under all circumstances and offering unconditional support. No matter how much I express my gratitude, it seems inadequate. Sharing this journey with him has made the process more bearable. I hope we are together in every achievement.

Last but not least, I am deeply grateful to my valuable friend Büşra Güray for keeping her energy high, believing in me and motivating me throughout this process. I would like to extend my gratitude to Nilsu Uzunlar for her support and assistance throughout this process. I also express my deepest appreciation to Betül Özkaya, with whom I have grown together. I consider myself fortunate to have had her unwavering support and consistent guidance.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The United Nations 2030 Agenda for Sustainable Development Goals enforces governments and businesses to address the environmental issues raised by the world's finite resources. An important concern revolves around the issue of electronic waste (e-waste), arising from the substantial use of precious and rare earth materials in production that eventually become disposed of after consumption. To overcome this issue, many businesses now exert significant effort for transition to circular economy, in which "the value of products and materials is maintained for as long as possible; waste and resource use are minimised, and resources are kept within the economy when a product has reached the end of its life, to be used again and again to create further value" [1]. The closed-loop supply chains (CLSCs) have emerged from such efforts and currently play a pivotal role in extending the lifespan of materials via circularity. The CLSCs focus on recovering value from consumer product returns by reusing the entire product or some of its components/materials. The product returns may take different forms: end-of-use returns may occur when the consumers are willing to adopt a technological update for their old devices that have not been used so intensely, while end-of-life returns may occur when the product becomes technologically obsolete [2]. Many consumer-electronics producers not only develop new products that can be remanufactured/recycled at the end of their use/life, but also undertake used-item acquisition and disposition initiatives for their CLSCs.

For used-item acquisition and disposition, for example, Apple offers the Apple Trade In Program, the iPhone Upgrade Program, third-party trade-in platforms, and refurbishing and recycling initiatives. Their long-lasting product designs, despite the short technology life cycles, allow Apple to profit from refurbishing and remarketing the acquired used devices. Apple sells the refurbished versions of various product models on their official website with an Apple certification and a one-year warranty. Apple also collects the end-of-life iPhone devices by participating in e-waste recycling programs. Apple disassembles the iPhone devices into distinct components for material recovery, extracting materials such as rare earth elements, steel, tungsten, cobalt, and gold from components such as taptic engine and battery. Their recycling machine, Taz, can recycle the modules containing rare earth magnets more efficiently than the conventional shredders. Apple offers eight products with more than 20% recycled content, including the MacBook Air with M1 chip made with 44% recycled content. Apple produces the taptic engines of the iPhone 12 and later generations with 100% recycled tungsten and several components of the iPhone 13 with 100% recycled gold (Apple [3]). The other global leaders of consumer electronics also strive to transition to CLSCs (see Dell Technologies [4], Hewlett-Packard [5], Huawei [6], Microsoft [7], Samsung [8], Xiaomi [9]).

Despite such advances in practice, our theoretical knowledge of optimal sales strategies for CLSCs of durable goods is limited. Only a few studies to our knowledge have examined the sales planning problem by developing optimization models that capture the complex dynamics of durable-good sales. (e.g., Debo et al. [10], Akan et al. [11], Robotis et al. [12], Nadar et al. [13], Nadar [14], Uzunlar [15], and Güray [16]). All of these studies except Uzunlar [15] and Güray [16] construct single-generation optimization models for their CLSC settings by incorporating the impact of word-of-mouth feedback spread by previous buyers (within a finite customer population) on product demand. These studies specifically employ the Bass diffusion dynamics of Bass [17, 18]. Uzunlar [15] and Güray [16] distinguish their CLSC settings by reasonably allowing the successive generations of a durable good to exist together in the same marketplace. They construct multi-generation optimization models by also incorporating the impact of substitution among different product generations on product demand. They specifically utilize the Norton-Bass diffusion dynamics of Norton and Bass [19] and Jiang and Jain [20].

Although Uzunlar [15] and Güray [16] have made significant progress towards enhancing our understanding of multi-generation product diffusion in CLSCs, there are several limitations of their optimization models. Uzunlar [15] considers only the recycling option as their used-item disposition strategy: certain valuable materials can be recovered from the first-generation product returns and can be reused in the second-generation product manufacturing. Güray [16] considers only the remanufacturing option as their used-item disposition strategy: the first- and second-generation product returns can only be remanufactured and remarketed as imperfect substitutes of the first- and second-generation used items, respectively. In reality, however, different product returns may have different quality levels since different consumers have different usage patterns (see Nadar et al. [21]), potentially entailing the simultaneous consideration of recycling and remanufacturing options.

In this thesis, like Uzunlar [15] and Güray [16], we examine the sales decisions for two successive generations of a durable good that overlap in the same market. Our optimization model is similar to those of Uzunlar [15] and Güray [16] in that the potential customers demand the successive product generations according to the generalized Norton-Bass diffusion process developed by Jiang and Jain [20]. However, we depart from Uzunlar [15] and Güray [16] by allowing for both remanufacturing and recycling options for the first-generation returns to better reflect reality. Specifically, we assume that the first-generation returns are grouped into two classes: the high-quality returns are at the end of their use and the low-quality returns are at the end of their life. The first-generation end-of-use returns are remanufactured for resale as an imperfect substitute and the first-generation end-of-life returns are used for material recycling and reuse in the second-generation production.

We investigate whether reducing the in-store availability and thus the sales volume of the second-generation product in certain time periods might ever be appealing to the producer. Such a sales strategy slows down the second-generation product diffusion and may help sell more remanufactured items and integrate more recycled content into production in the long run. We label this sales strategy as the “forward-looking policy.” We label the alternative sales strategy (that fulfills all demand in each time period) as the “myopic policy.” The forward-looking policy may also help improve the total profit if the remanufactured item has a greater profit margin than the new item and/or the recycled content is cheaper to acquire and use than the virgin raw materials. These two conditions can indeed hold in certain environments (see Atasu et al. [22], Guide and Li [23], Gutowski et al. [24], Kitsara [25], Esenduran et al. [26], Wang et al. [27], and Matsui [28]).

For our investigation, we consider two different consumer-return scenarios: In the first scenario, the consumers’ timing of first-generation returns is independent of their timing of second-generation adoption and the consumers are heterogeneous in their return-timing decisions. In the second scenario, the consumers’ timing of first-generation returns is determined by their timing of second-generation adoption and thus is affected by any manipulation of the diffusion process via sales planning. In each scenario, we analytically derive sufficient conditions under which the forward-looking policy strictly outperforms the myopic policy. In the latter scenario, we also conduct extensive numerical experiments to evaluate the potential benefits of the forward-looking policy, in comparison with the myopic policy, in different environments.

We contribute to the CLSC literature as follows:

- This thesis is the first step towards exploring the optimal sales decisions of durable-good producers who offer two successive generations of the same product with remanufacturing and recycling capabilities.
- Our results imply that slowing down the second-generation product diffusion via the forward-looking policy can be more attractive than the myopic

policy. We establish sufficient conditions for the optimality of the forward-looking policy in each consumer-return scenario. We translate these conditions into managerial insights on the sales planning problem for consumer-electronics CLSCs.

- We have found that the forward-looking policy is optimal if (i) the diffusion process is fast enough in the absence of any manipulation, (ii) the number of first-generation end-of-life returns and the recyclable-material amount from each such return are high enough, and (iii) the potential customer base of the first-generation product is sufficiently large. The forward-looking policy is less likely to be optimal if the consumers return their old devices only at times when they switch to the next-generation device.

The rest of this thesis is organized as follows: Chapter 2 reviews the related literature. Chapter 3 formulates our sales planning problem in two different consumer-return scenarios. Chapter 4 contains the analytical results and their numerical illustrations. Chapter 5 constructs an exact solution algorithm for the optimal sales plan and offers a numerical study. Chapter 6 concludes. Detailed versions of our theorems and their proofs are relegated to Appendix A.

Chapter 2

Literature Review

The new-product diffusion model introduced by Bass [17, 18] has by far received the most attention in the marketing literature on consumers' adoption behavior for rapid technological developments. This diffusion model explains how a new product is spread and adopted in a market with a finite number of customers and provides some projections about the sales growth of the product. The Bass diffusion model splits the potential customer population into two groups: The *innovators* make their new-product purchase decisions independently from the other adopters, while the *imitators* make these decisions by taking into account the word-of-mouth feedback spread from the previous adopters. The Bass diffusion model formulates the new-product demand at any point in time by assuming that the purchase likelihood of any potential customer increases linearly with the number of previous adopters. Bass [17, 18] also presents empirical results to validate this diffusion model.

The consumers' adoption behavior becomes more complicated to predict as the newer generations of the same type of product are introduced in the market. This is because the consumers' adoption of one specific generation potentially influences the adoption rate of the other generations. In a major attempt to explain the consumer behavior in the presence of multiple generations, Norton and Bass [19] have extended the Bass diffusion model by incorporating the cross-generation substitution effect. The Norton-Bass diffusion model assumes that each subsequent generation attracts a new customer population that does not demand the previous generations, but may demand the current and later generations. Jiang and Jain [20] have generalized the Norton-Bass diffusion model by splitting the potential customer population of any specific generation into two groups: The *leapfroggers* purchase a newer generation of the product by skipping the current generation, while the *switchers* purchase both generations in different time periods. The existence of leapfroggers cannibalizes the current-generation demand and reduces the cross-generation repeat purchases. We implement the generalized Norton-Bass diffusion model of Jiang and Jain [20] into our problem setting.

Several papers have incorporated the Bass diffusion model into their sales planning problems for forward supply chains with single product generation. See, for examples, Ho et al. [29], Kumar and Swaminathan [30], Ho et al. [31], Shen et al. [32], Shen et al. [33], and Bilginer and Erhun [34]. Several other papers have allowed the customers with backlogged demands to spread positive or negative word-of-mouth feedback in their diffusion models. See, for examples, Jain et al. [35], Keith et al. [36], and Yan et al. [37]. Several papers have examined the impact of price in their diffusion models. See, for examples, Kalish [38], Jain and Rao [39], Bass et al. [40], Krishnan et al. [41], Li and Huh [42], Li [43], and Zhang et al. [44]. Unlike the above papers, we study the sales planning problem for CLSCs with successive product generations.

Several authors have focused on the problem of when to introduce new products in the presence of successive product generations. Wilson and Norton [45] establish the optimality of the so-called *Now-or-Never* policy for this problem if the unit profit margin of the early-generation product is higher than that of the new-generation product. Mahajan and Müller [46] show the optimality of the so-called *Now-or-at-Maturity* policy if the future profits are subject to discounting. Krankel et al. [47] reveal that delaying the new-product launch may be profitable if further technology improvements can be captured over time. Tang et al. [48] find that delaying the new-product launch may also be profitable if there is intense competition among producers. Ke et al. [49] show that the *Now-or-Never* policy is optimal if the inventory holding cost is low, and the sequential introduction policy is optimal otherwise. Several other authors have studied the product-launch timing problem by investigating the impact of different purchase options by strategic consumers [50], by allowing for different types of products (purchase-to-own vs. subscribe-to-use) under different business strategies [51], by taking into account supply restrictions [52], and by examining the impact of new-product launch time and diffusion speed on upstream investment planning [53]. Unlike the above papers, we consider a CLSC setting in which the new-product launch time is an exogenous model parameter. Several studies have considered different product-rollover strategies (single- vs. dual-roll strategies) and product-introduction frequency decisions. See, for examples, Druehl et al. [54], Liao and Seifert [55], Koca et al. [56], and Seref et al. [57]. We adopt the dual-roll strategy in our problem setting for CLSCs with successive product generations.

The new-product diffusion models have also received considerable attention in the CLSC literature. (We refer the reader to Atasu et al. [58], Guide and Wassenhove [2], Ferguson and Souza [59], Akçalı and Çetinkaya [60], Hassini et al. [61], Souza [62], Govindan et al. [63], and De Angelis et al. [64] for reviews of this literature.) Many papers in this research stream consider remanufacturing as the only used-product disposition option in their CLSC settings. Debo et al. [10] look into the pricing decisions for imperfectly substitutable new and remanufactured items by constructing a single-generation diffusion model that allows for repeat customer purchases, variable market sojourn time, and supply constraints. Robotis et al. [12] consider the leasing price and leasing duration decisions for perfectly substitutable new and remanufactured items of the same generation, which are leased to consumers in the presence of production/service capacity constraints. Akan et al. [11] examine the pricing and sales decisions for imperfectly substitutable new and remanufactured items of the same generation in the absence of any production capacity constraint. Gaur et al. [65] study the CLSC configuration problem for battery manufacturing by allowing for manipulation of the single-generation diffusion process. Nadar et al. [13] analyze the optimal sales plan for imperfectly substitutable new and remanufactured items in a setting where the Bass diffusion dynamics hold, any unmet demand can only be partially backordered, and different consumers adopting the product at the same time may return their used items at different times. Nadar et al. [13] derive sufficient conditions that ensure the optimality of the forward-looking policy in their setting. There are also papers that adopt the single-generation diffusion process as an exogenous model input. See, for examples, Georgiadis et al. [66], Georgiadis and Athanasiou [67], and Wang et al. [68]. Unlike this research stream, we consider two successive product generations offered in the same marketplace as well as two different quality levels for early-generation returns with the high-quality returns being remanufacturable and the low-quality returns being recyclable.

In the CLSC literature, the closest studies to ours are those of Uzunlar [15] and Güray [16]. These two studies look into the sales decisions for two successive generations of the same durable good that coexist in the same marketplace. These two studies utilize the generalized Norton-Bass diffusion model of Jiang and Jain [20]. Uzunlar [15] considers a CLSC setting in which certain valuable materials can be extracted from the early-generation product returns and can be used in the new-generation product manufacturing. Uzunlar [15] establishes conditions that ensure the optimality of the forward-looking policy applied to the customer population unique to the second-generation product. Unlike Uzunlar [15], we consider a CLSC setting in which the early-generation product returns are grouped into two classes based on their quality levels: The low-quality early-generation returns become available for material recovery and reuse in the new-generation manufacturing. The high-quality early-generation returns become available for remanufacturing and resale as an imperfect substitute of the early-generation new item. Although we also establish conditions for the optimality of a similar policy with the generalized Norton-Bass diffusion model, our proposed sales strategy benefits not only from increased use of recycled content (and reduced use of virgin raw materials) in production and improved cost savings, but also from higher profit margin and improved sales volume of remanufactured items.

Güray [16] considers a CLSC setting in which both early-generation and new-generation product returns can be remanufactured and resold as imperfect substitutes of early-generation and new-generation new items, respectively. Taking a similar path to that in Uzunlar [15], Güray [16] derives conditions for the optimality of the forward-looking policy again applied to the customer population unique to the second-generation product. Unlike Güray [16], we consider a CLSC setting in which the early-generation product returns can also become available for material recovery and reuse (depending on the quality level of the acquired used item) and the new-generation product returns are ignored for analytical convenience. The latter assumption may be benign if most returns of the new-generation product arrive later than the end of the planning horizon of the producer. Unlike Güray [16], we also consider an alternative consumer-return scenario in which the early-generation returns in any specific period can only arise from the switching adopters who have purchased the early-generation items in earlier periods and return their used items to the producer to trade up to the next-generation item. Furthermore, we conduct extensive numerical experiments to evaluate the potential benefit of our forward-looking policy in this consumer-return scenario.

Chapter 3

Problem Formulation

We adopt the sales planning model of Güray [16] that incorporates the second-generation sales decisions into the generalized Norton-Bass diffusion model of Jiang and Jain [20]. Therefore, our presentation and notation below closely follow the corresponding material in Güray [16]. Table 3.1 describes the notation of model parameters and variables.

The demand arising from the potential customer population (of size m_1) of the first-generation product is entirely met in each period. The key decision variable of our model in period t is the amount of demand arising from the customer population (of size m_2) unique to the second-generation product that is met in period t (denoted by s_{2t}^B). The Bass diffusion demands for the first- and second-generation products in period t are formulated as follows:

$$d_{1t}^B = \left(p_1 + \frac{q_1 D_{1t}^B}{m_1}\right)\left(m_1 - D_{1t}^B\right) \tag{3.1}$$

and

$$d_{2t}^B = \left(p_2 + \frac{q_2 S_{2t}^B}{m_2}\right)\left(m_2 - D_{2t}^B\right) \tag{3.2}$$

where $D_{11}^B = 0$; $D_{1t}^B = \sum_{i=1}^{t-1} d_{1i}^B$, $\forall t > 1$; $D_{2t}^B = 0$, $\forall t \leq \tau$; $D_{2t}^B = \sum_{i=\tau}^{t-1} d_{2i}^B$, $\forall t > \tau$; $S_{2t}^B = 0$, $\forall t \leq \tau$; and $S_{2t}^B = \sum_{i=\tau}^{t-1} s_{2i}^B$, $\forall t > \tau$. The Bass diffusion demand for the first-generation product is thus unaffected by our sales decisions. However, in each period t, the Bass diffusion demand for the second-generation product

Table 3.1: Summary of notation.

Parameters	Description
τ	Time period when the second-generation product is introduced to the market.
m_1	Size of the potential customer population of the first-generation product.
m_2	Size of the customer population unique to the second-generation product.
p_1, p_2	Innovation coefficients for the first- and second-generation products, respectively, $p_1, p_2 \in [0, 1]$.
q_1, q_2	Imitation coefficients for the first- and second-generation products, respectively, $q_1, q_2 \in [0, 1]$.
α	Backlogging fraction of unmet demand in any period, $\alpha \in [0, 1]$.
β_i^u, β_i^l	End-of-use and end-of-life return fractions of previously sold products after i periods, respectively, $\beta_i^u, \beta_i^l \in [0, 1]$.
γ	Fraction of the customers who prefer to buy the remanufactured items, $\gamma \in [0, 1]$.
θ_1	Fixed amount of material that can be recovered from one unit of the first-generation end-of-life product.
θ_2	Maximum amount of recycled material that can be used to produce one unit of the second-generation product.
λ	Fraction of the switchers who returns their first-generation products while buying the second-generation product.
δ^u, δ^l	End-of-use and end-of-life fractions of the switchers' product returns, respectively, $\delta^u, \delta^l \in [0, 1]$ and $\delta^u + \delta^l = 1$.
c_{1n}	Unit production cost of the first-generation new item.
p_{1n}	Unit selling price of the first-generation new item, $p_{1n} > c_{1n}$.
c_{2n}	Unit production cost of the second-generation item.
p_{2n}	Unit selling price of the second-generation item, $p_{2n} > c_{2n}$.
c_{1r}	Unit remanufacturing cost of the first-generation return.
p_{1r}	Unit selling price of the first-generation remanufactured item, $p_{1r} > c_{1r}$.
ρ_r	Cost savings enabled by the use of a unit amount of recycled material in the second-generation production.

Variables	Description
d_{1t}^B	Bass diffusion demand for the first-generation product in period $t \geq 1$.
D_{1t}^B	Total number of consumers who are initially attracted by the first-generation product and have bought the product up to period t, $D_{1t}^B = \sum_{i=1}^{t-1} d_{1i}^B$, $\forall t > 1$.
d_{2t}^B	Bass diffusion demand for the second-generation product in period $t \geq \tau$.
D_{2t}^B	Total number of consumers who are only attracted by the second-generation product and have bought the product up to period t, $D_{2t}^B = \sum_{i=\tau}^{t-1} d_{2i}^B$, $\forall t > \tau$.
d_{1t}^{NB}	Generalized Norton-Bass demand for the first-generation product in period $t \geq 1$.
d_{2t}^{NB}	Generalized Norton-Bass demand for the second-generation product in period $t \geq \tau$
s_{2t}^B	Sales volume for the customer population unique to the second-generation product in period $t \geq \tau$.
s_{2t}	Sales volume of the second-generation product in period $t \geq \tau$.
r_t	Sales volume of the first-generation remanufactured items in period t.
e_t^u	End-of-use return volume available at the beginning of period t.
e_t^l	End-of-life return volume available at the beginning of period t.
b_t	Number of backorders in period t from the customer population unique to the second-generation product.

changes with the cumulative total sales volume S^B_{2t}, and the Bass diffusion demands for the first- and second-generation products are jointly used to formulate the actual demands observed. Therefore, the observed demands for both product generations are affected by our sales decisions, taking the following forms in period $t \geq 1$:

$$d^{NB}_{1t} = d^B_{1t} - \frac{d^B_{1t} D^B_{2(t+1)}}{m_2} \tag{3.3}$$

and

$$d^{NB}_{2t} = d^B_{2t} + \frac{d^B_{1t} D^B_{2(t+1)}}{m_2} + \frac{d^B_{2t} D^B_{1t}}{m_2}. \tag{3.4}$$

The first demand term in equation (3.4) arises from the customer population unique to the second-generation product, the second demand term arises from the leapfroggers of the potential customer population of the first-generation product, and the last demand term arises from the switchers of the potential customer population of the first-generation product. These demand formulations were also adopted in the diffusion literature; see, for examples, Ho et al. [29], [31], Kumar and Swaminathan [30], Shen et al. [32], [33], Nadar et al. [13], and Nadar [14]. We note that these formulations become equivalent to those of Jiang and Jain [20] if all demand is met in each period. Figure 3.1 illustrates our diffusion model for two different sales plans.

We denote by s_{2t} the sales volume of the second-generation product in period $t \geq \tau$. Since all demand from the potential customer population of the first-generation product is met in each period, we obtain $s_{2t} = s^B_{2t} + \frac{d^B_{1t} D^B_{2(t+1)}}{m_2} + \frac{d^B_{2t} D^B_{1t}}{m_2}$, $\forall t \geq \tau$. We denote by b_t the accumulated number of backorders in period t from the customer population unique to the second-generation product. The sales volume of the second-generation product in period t must obey the following constraint:

$$\frac{d^B_{1t} D^B_{2(t+1)}}{m_2} + \frac{d^B_{2t} D^B_{1t}}{m_2} \leq s_{2t} \leq d^{NB}_{2t} + b_t. \tag{3.5}$$

The above constraint implies that $0 \leq s^B_{2t} \leq d^B_{2t} + b_t$ in period t. The accumulated number of backorders in any period $t \geq \tau$ can be calculated as follows:

$$b_{(t+1)} = \alpha(d^{NB}_{2t} + b_t - s_{2t}) \tag{3.6}$$

where $b_\tau = 0$. This formulation is also widely adopted in the diffusion literature;

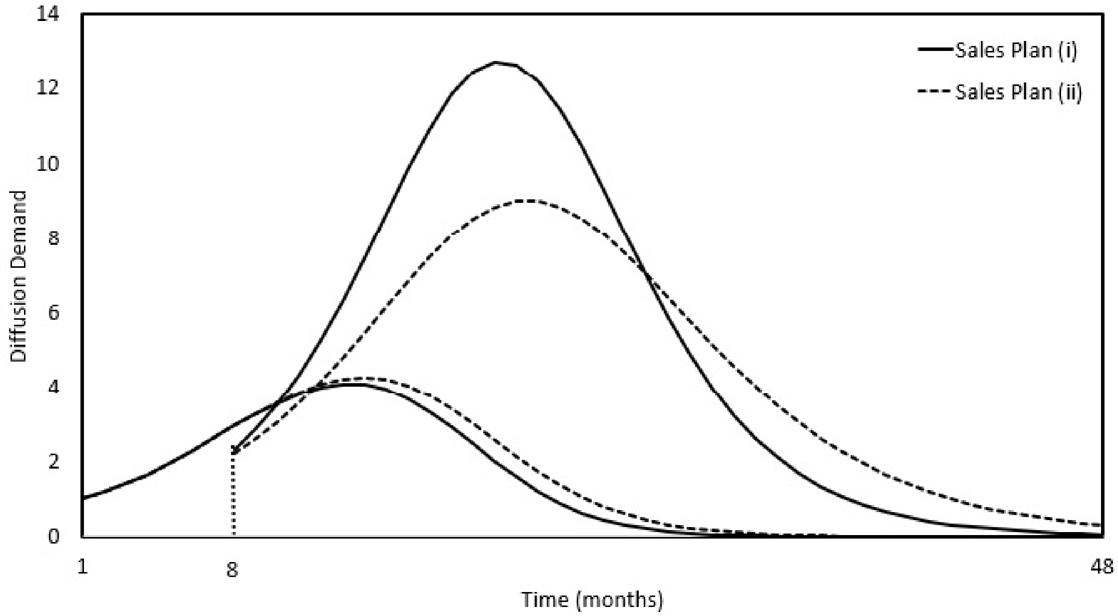

Figure 3.1 (Güray [16]) : Multi-generation product diffusion when $\tau = 8$, $p_1 = 0.01$, $p_2 = 0.02$, $q_1 = q_2 = 0.20$, and $m_1 = m_2 = 100$. Sales plan (i) meets all demand in each period and corresponds to the generalized Norton-Bass model. Sales plan (ii) meets all demand from the potential customer population of the first-generation product and 70% of the diffusion demand from the customer population unique to the second-generation product in each period.

see, for examples, Ho et al. [29], [31], Kumar and Swaminathan [30], Shen et al. [32], [33], Nadar et al. [13], and Nadar [14].

The consumer returns of first-generation products fall into two different categories: end-of-use vs. end-of-life returns. The end-of-use returns can be remanufactured and resold in the market, while the end-of-life returns can be recycled and reused in the second-generation production. This classification of returns results from different usage patterns of different customers.

The fraction γ of the potential customer population of the first-generation product purchases the remanufactured item if there are remanufactured items on hand and purchases the new item otherwise. Nadar et al. [13] state that this fraction of the population is the price-sensitive functionality-oriented customer segment. The remaining fraction $1 - \gamma$ of this customer population wants

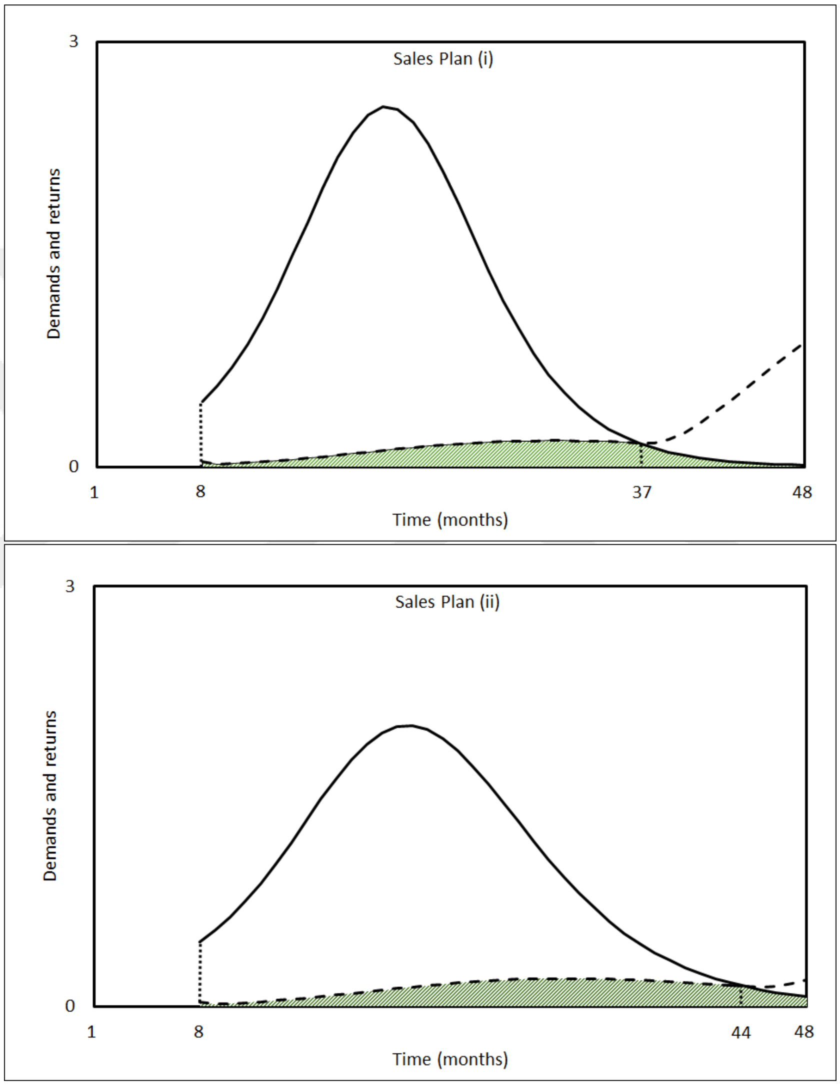

Figure 3.2: Second-generation recycled-material demands (solid lines) and accumulated recyclable-material amounts (dashed lines) for the sales plans in Figure 3.1. Both sales plans use as much recycled content as possible in manufacturing. Each shaded area represents the total recycled content used in manufacturing. The area in (ii) is larger by 28% than the area in (i). $\tau = 8$, $p_1 = 0.01$, $p_2 = 0.02$, $q_1 = q_2 = 0.20$, $m_1 = m_2 = 100$, $\theta_1 = 5\theta_2$, $\beta_i^l = 4\beta_i^u$, $\beta_i = (12\%)\mathbb{P}\{i - 0.5 \leq X \leq i + 0.5\}$, $X \sim \text{Weibull}(25, 2)$.

to purchase only the new item. Nadar et al. [13] state that this fraction of the population is the newness-conscious customer segment. The sales volume for remanufactured items is constrained by the maximum possible demand for remanufactured items in periods $t \geq 1$:

$$0 \leq r_t \leq \gamma \left(d_{1t}^B - \frac{d_{1t}^B D_{2(t+1)}^B}{m_2} \right). \tag{3.7}$$

The sales volume for remanufactured items is also constrained by the accumulated end-of-use return volume in periods $t \geq 1$:

$$0 \leq r_t \leq e_t^u. \tag{3.8}$$

The producer satisfies the remanufactured-item demand as much as possible in each period. Thus, by combining (3.7) and (3.8), we obtain

$$r_t = \min \left\{ \gamma \left(d_{1t}^B - \frac{d_{1t}^B D_{2(t+1)}^B}{m_2} \right), e_t^u \right\}. \tag{3.9}$$

We simplify our analysis by assuming that only a single type of material can be recovered from the first-generation end-of-life returns and the recycled material of only this type can be integrated into the second-generation new-item production. We consider 'cobalt' to exemplify such a material type and describe our notation. We denote by θ_1 the amount of cobalt that can be extracted from one unit of the first-generation end-of-life product. We also denote by θ_2 the maximum amount of recycled cobalt that can be used to produce one unit of the second-generation product. In some cases, the use of recycled content may not be acceptable in certain parts of a device, which require the use of only virgin raw materials. Consequently, the total amount of cobalt required to produce one unit of the second-generation product can exceed θ_2.

In Chapters 3.1 and 3.2, we consider two different scenarios to model the consumer behavior for their timing of early-generation returns. We formulate the producer's problem in each scenario.

3.1 Consumer Returns with Remanufacturing and Recycling Programs

In this scenario, the customers' timing of first-generation product returns is independent from their timing of second-generation product purchases. The fraction β_i^u (or β_i^l) of the first-generation products sold in period t are returned at the end of their use (or life) and become available for remanufacturing and resale (or recycling and reuse in second-generation production) in period $t+i$. Letting $\beta_i \triangleq \beta_i^l + \beta_i^u$ and $\beta \triangleq \sum_i \beta_i \leq 1$, we assume that the fraction $(1-\beta)$ of the first-generation products sold at any time cannot be collected by the producer. The end-of-use return volume available at the beginning of any period $t \geq 1$ can be calculated as follows:

$$e_{t+1}^u = e_t^u - r_t + \sum_{i=1}^{t} \beta_i^u d_{1(t+1-i)}^{NB} \tag{3.10}$$

where $e_1^u = 0$. The accumulated amount of material that can be recovered and used at the beginning of any period $t \geq 1$ (i.e., the end-of-life return volume available at the beginning of period t) can be calculated as follows:

$$e_{(t+1)}^l = \begin{cases} \theta_1 \sum_{i=1}^{t} \beta_i^l d_{1(t+1-i)}^{NB} & \text{if } e_t^l < \theta_2 s_{2t}, \\ e_t^l - \theta_2 s_{2t} + \theta_1 \sum_{i=1}^{t} \beta_i^l d_{1(t+1-i)}^{NB} & \text{if } e_t^l \geq \theta_2 s_{2t}, \end{cases} \tag{3.11}$$

where $e_1^l = 0$. We assume that $p_{1r} > c_{1r}$ and $p_{2n} > c_{2n} > \theta_2 \rho_r$. The producer seeks to maximize its total profit from the entire product line:

$$\max_{s_{2\tau},..,s_{2T}} \quad (p_{1n} - c_{1n}) \sum_{t=1}^{T} (d_{1t}^{NB} - r_t) + (p_{1r} - c_{1r}) \sum_{t=1}^{T} r_t + (p_{2n} - c_{2n}) \sum_{t=\tau}^{T} s_{2t} + \rho_r \sum_{t=\tau}^{T} \min\left\{e_t^l, \theta_2 s_{2t}\right\}$$

subject to (3.1)-(3.11). The first summation above is the total profit from the first-generation new item and the second summation is the total profit from the first-generation remanufactured item. The third summation is the total profit from the second-generation product in the absence of recycled content, while the last summation is the total cost savings in the second-generation manufacturing with the recycled content used. See Figure 3.2 for an illustration of how different sales plans affect the recycled content used in manufacturing: Partially satisfying the demand from the population of size m_2 may help raise the total amount of recycled content used.

3.2 Consumer Returns at Switching Times

In this scenario, unlike the previous scenario, the consumers' timing of first-generation product returns is governed by the second-generation product diffusion process. Recall that the switching adopters from the potential customer population of the first-generation product buy the first- and second-generation products in different time periods. We assume that these switching adopters are the only contributors of the first-generation product returns over the entire selling horizon. The fraction λ of the switching adopters returns their first-generation used items at times when they purchase the second-generation product. The fraction δ^u of these product returns can be remanufactured. The fraction $\delta^l = 1 - \delta^u$ of these product returns cannot be remanufactured but can be recycled. The former fraction represents the end-of-use items and the latter fraction represents the end-of-life items. This scenario may hold if the producer offers a trade-up program to acquire the used items from the consumers who are willing to bring their old devices for trade-up to the second-generation product.

We assume that the end-of-use returns of the switching adopters in any period can be used for remanufacturing only in subsequent periods. However, the end-of-life returns of the switching adopters can be used for second-generation manufacturing in the same period for fulfillment of these consumers' demand but in subsequent periods for fulfillment of the other consumers' demand. We also assume that the recyclable material available from the product return of any switching adopter can adequately meet the recycled material demand of the same switching adopter (i.e., $\lambda\delta^l\theta_1 \geq \theta_2$). Under these assumptions, the end-of-use return volume available at the beginning of any period $t \geq \tau$ can be calculated as follows:

$$e_{t+1}^u = e_t^u - r_t + \frac{\lambda\delta^u D_{1t}^B d_{2t}^B}{m_2} \tag{3.12}$$

where $e_t^u = 0$, $\forall t \leq \tau$. Under these assumptions, we also redefine e_t^l as the end-of-life return volume available for fulfillment of the second-generation demand, except the switching adopters' demand, at the beginning of any period $t \geq \tau$, which can be calculated as follows:

$$e_{(t+1)}^l = \begin{cases} \left(\lambda\delta^l\theta_1 - \theta_2\right)\frac{D_{1t}^B d_{2t}^B}{m_2} & \text{if } e_t^l < \theta_2\left(s_{2t} - \frac{D_{1t}^B d_{2t}^B}{m_2}\right), \\ e_t^l - \theta_2\left(s_{2t} - \frac{D_{1t}^B d_{2t}^B}{m_2}\right) + \left(\lambda\delta^l\theta_1 - \theta_2\right)\frac{D_{1t}^B d_{2t}^B}{m_2} & \text{if } e_t^l \geq \theta_2\left(s_{2t} - \frac{D_{1t}^B d_{2t}^B}{m_2}\right), \end{cases} \tag{3.13}$$

where $e_t^l = 0$, $\forall t \leq \tau$. The producer seeks to maximize its total profit from the entire product line:

$$\max_{s_{2\tau},..,s_{2T}} \quad (p_{1n} - c_{1n})\sum_{t=1}^{T}(d_{1t}^{NB} - r_t) + (p_{1r} - c_{1r})\sum_{t=1}^{T} r_t + (p_{2n} - c_{2n})\sum_{t=\tau}^{T} s_{2t}$$
$$+\theta_2\rho_r\sum_{t=\tau}^{T}\frac{D_{1t}^B d_{2t}^B}{m_2} + \rho_r\sum_{t=\tau}^{T}\min\left\{e_t^l, \theta_2\left(s_{2t} - \frac{D_{1t}^B d_{2t}^B}{m_2}\right)\right\}$$

subject to (3.1)-(3.9) and (3.12)-(3.13). The fourth summation above is the total cost savings in the second-generation manufacturing for fulfillment of the switching adopters' demand and the last summation is the total cost savings in the second-generation manufacturing for fulfillment of the other consumers' demand. All these cost savings are achieved with the recycled content used.

We note that the above objective function can be easily modified to capture the case with $\lambda\delta^l\theta_1 < \theta_2$ by removing the last summation and replacing θ_2 with $\lambda\delta^l\theta_1$ in the fourth summation. The total recycled content integrated into the second-generation manufacturing would then increase linearly with the total second-generation demand of the switching adopters. Since the total number of switching adopters increases with the slower second-generation product diffusion if the selling horizon is long enough (see Proposition 1 in Güray [16]), the forward-looking policy may be desirable in this case.

Chapter 4

Analytical Results

In this chapter, we examine whether the myopically optimal policy that always sells as much as possible might ever fail to yield the global optimal solution. For this purpose, we take a similar path to that in Güray [16] and introduce the following two classes of sales plans:

(i) The myopic policy: In each period, all demand for both product generations is met, demand for remanufactured items is met as much as possible, and the recycled content is integrated into production as much as possible. Our diffusion model under this policy reduces to the one in Jiang and Jain [20].

(ii) The forward-looking policy: Some demand from the customer population unique to the second-generation product is rejected in some period $t \in \{\tau, \tau{+}1, ..., T{-}1\}$, all demand from the potential customer population of the first-generation product is met in each period, demand for remanufactured items is met as much as possible in each period, and the recycled content is integrated into production as much as possible in each period.

We characterize sufficient conditions under which the myopic policy fails to be optimal in our sales planning problems defined in Chapters 3.1 and 3.2. Theorems 1 and 2 below present the abridged versions of these conditions. We refer to the reader to Appendix A for the unabridged versions of these conditions with lengthy mathematical expressions.

Theorem 1. *Suppose that the consumers' timing of first-generation product returns is unaffected by their timing of second-generation product adoption (as in Chapter 3.1). Then the forward-looking policy is optimal if, under the myopic policy, there exists a period $\kappa < T$ such that*

(i) the accumulated first-generation return volume exceeds the first-generation remanufactured-item demand in each period $t > \kappa$,

(ii) the second-generation recycled-material demand exceeds the accumulated recyclable-material amount in each period $t \leq \kappa$ whereas the reverse is true in each period $t > \kappa$,

(iii) the ratio of the recyclable-material amount available from one unit of the first-generation product to the recycled-material demand from one unit of the second-generation product is above a certain threshold (see Appendix A for the detailed expression), and

(iv) the rejection of a unit of the second-generation demand in period κ leads to a loss of diffusion demand in period $\kappa+1$ that is below a certain threshold (see Appendix A for the detailed expression) and a backlogged demand for the second-generation product in period $\kappa + 1$ that is above a certain threshold (see Appendix A for the detailed expression).

Suppose that the above conditions are met and the selling horizon is long enough. Then, if the forward-looking policy is initiated after period κ, it provides no improvement in the total recycled-material amount used in manufacturing.

Proof. See Appendix A.

For the problem setting in Chapter 3.1, Theorem 1 reveals that the producer can improve its total profit by rejecting some demand from the customer population unique to the second-generation product in some period (i.e., period κ) under four conditions that must hold together: Condition (i) in Theorem 1 implies that the forward-looking policy increases the first-generation remanufactured-item sales. We know from Proposition 1 of Güray [16] that the forward-looking policy improves the first-generation demand for both new and remanufactured items. In any future period, under condition (i), the first-generation returns will be sufficient to satisfy all of the larger remanufactured-item demand. The forward-looking policy also raises the second-generation demand in later periods. Conditions (ii) and (iii) in Theorem 1 ensure that each unit of the larger second-generation demand in these periods can still utilize the maximum possible recycled content that the product design permits. Condition (iv) in Theorem 1 ensures that the revenue gain from improved first-generation sales and improved use of recycled content in manufacturing outweighs the revenue loss from lower second-generation sales (that are reduced via the forward-looking policy). While conditions (i), (ii), and (iv) in Theorem 1 are analogous to conditions (i), (ii), and (iii) in Theorem 2 of Güray [16], respectively, condition (iii) in Theorem 1 is not applicable to the sales planning problem of Güray [16].

We conduct numerical experiments based on Theorem 1 to investigate the optimality of the forward-looking policy as well as the environmentally critical time period κ for the optimal initiation of the forward-looking policy in different environments. For our base scenario, we consider the parameter values calibrated for the smartphone industry: $T = 48$ months, $\tau = 12$, $p = p_1 = p_2 = 0.05$, $q = q_1 = q_2 = 0.35$, $m_1 = m_2$, $\theta_1 = \theta_2$, $\theta_2\rho_r/(p_{2n} - c_{2n}) = 0.2$, $(p_{1n} - c_{1n}) = (p_{2n} - c_{2n})$, $(p_{1n} - c_{1n})/(p_{1r} - c_{1r}) = 0.5$, $\alpha = 0.88$, $\gamma = 0.27$, $\beta_i^l = 4\beta_i^u = 0.8\beta_i$, $\beta_i = (0.12) \times \mathbb{P}\{i - 0.5 \leq X \leq i + 0.5\}$, $X \sim$ Weibull$(25, 2)$. Our experimental setup is similar to those used by Güray [16] and Nadar et al. [13] who focus on consumer electronics products in their experiments. We vary the parameter values of the base scenario to generate many other instances. Specifically, we consider instances in which $p, q \in [0, 1]$ s.t. $p + q \leq 1$, $\tau \in \{1, 2, ..., 40\}$, $m_1/m_2 \in (0, 5]$, $\theta_1/\theta_2 \in (0, 5]$, $\beta_i^l = \delta^l\beta_i$ and $\beta_i^u = (1 - \delta^l)\beta_i$ where $\delta^l \in [0, 1]$, and $\theta_2\rho_r/(p_{2n} - c_{2n}) \in (0, 5]$. Figures 4.1–4.2 exhibit our numerical results. We observe from Figures 4.1–4.2 that the optimality conditions of the forward-looking policy in Theorem 1 hold in a very large number of our instances.

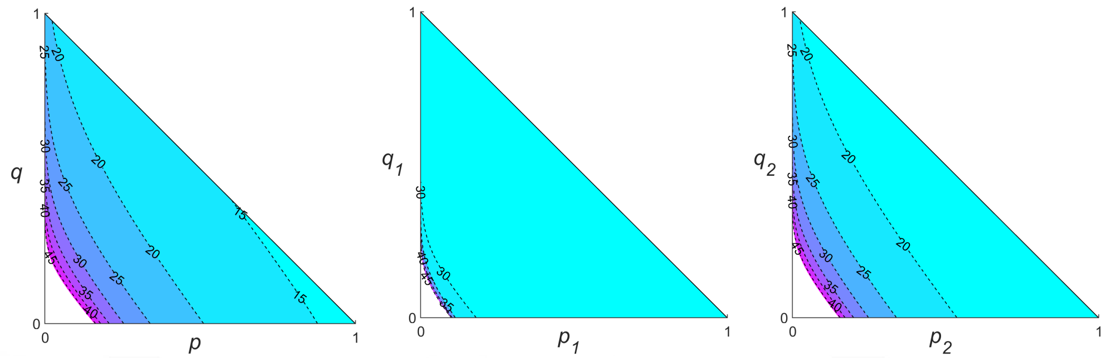

Figure 4.1: Diffusion parameters vs. environmentally critical period κ in Theorem 1. Colored regions indicate the optimality of the forward-looking policy.

Figure 4.1 indicates that the forward-looking policy is optimal when the diffusion parameters p and q are large: Large values of p and q lead to a faster diffusion process for the second-generation product. Therefore, the delayed demand via the forward-looking policy might still occur in large amounts during the selling horizon. In addition, the recyclable-material amount is likely to be adequate for meeting the delayed demand for the second-generation product in future periods. Figure 4.1 also shows that the environmentally critical time period κ decreases as the diffusion parameters p_2 and q_2 increase: The accumulated recyclable-material amount rises above the second-generation recycled-material demand earlier in the selling horizon when the diffusion process is faster. Another important observation is that the environmentally critical time period κ is more robust to changes in the diffusion parameters of the customer population unique to the second-generation product than in those of the potential customer population of the first-generation product. The forward-looking policy is optimal when the diffusion parameters p_1 and q_1 are sufficiently large: Low values of p_1 and q_1 lead to a slower diffusion process for the first-generation product. This escalates the leapfrogging behavior and lowers the first-generation sales. Hence, the first-generation returns decrease and arrive later in the selling horizon so that the accumulated recyclable-material amount may never reach the second-generation recycled-material demand.

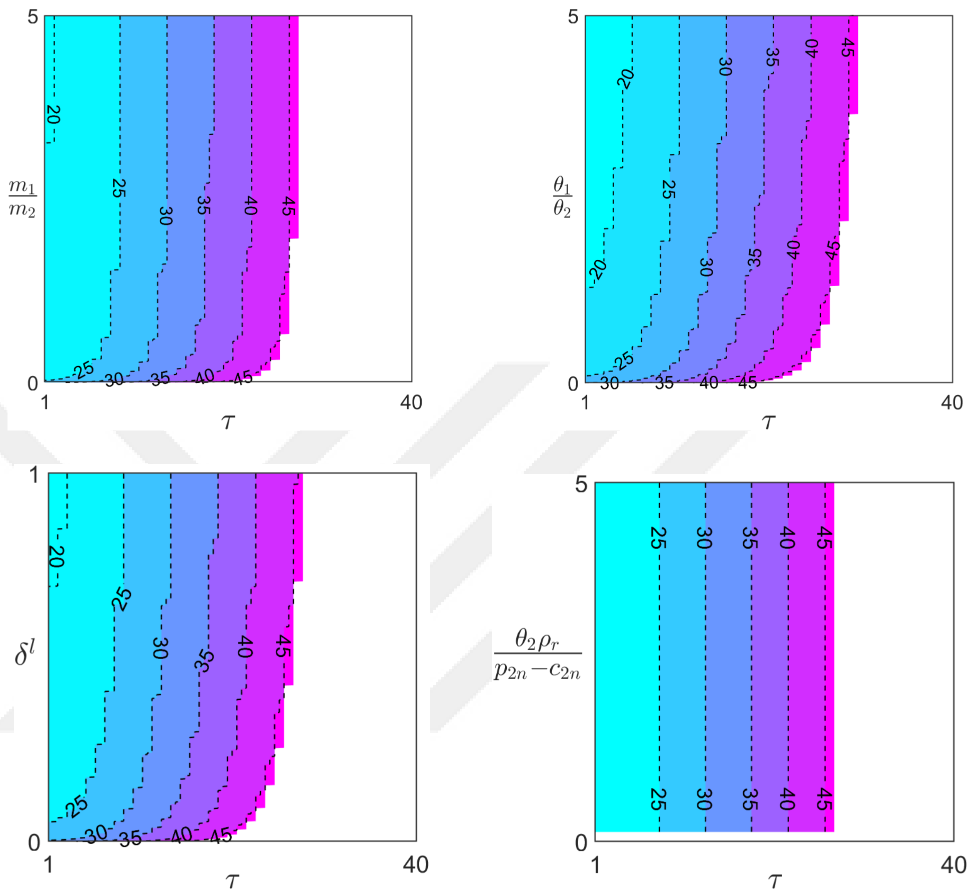

Figure 4.2: Population, return and recycling parameters vs. environmentally critical period κ in Theorem 1. Colored regions indicate the optimality of the forward-looking policy.

Figure 4.2 shows that the forward-looking policy is optimal, and the environmentally critical time period κ is sooner, when m_1/m_2, θ_1/θ_2, or δ^l is large: When m_1/m_2 is small, the second-generation product has a much greater demand base than the first-generation product. When θ_1/θ_2 is small, the recyclable-material amount available from one unit of the first-generation product is low whereas the recycled-material demand from one unit of the second-generation product is large. When δ^l is small, fewer returns become available for recycling and more returns become available for remanufacturing. Therefore, when m_1/m_2, θ_1/θ_2, and δ^l are small, the accumulated recyclable-material amount may never reach the second-generation recycled-material demand. Figure 4.2 also shows that the forward-looking policy is optimal when $\theta_2\rho_r/(p_{2n} - c_{2n})$ is not too low (larger than 14%): The forward-looking policy can provide a greater benefit if the use of recycled content in the second-generation manufacturing can substantially reduce the consumption of an expensive virgin raw material. Finally, we note that the forward-looking policy is optimal when τ is small enough: The selling horizon is long enough to observe most of the second-generation demand in this case. Therefore, slowing down the second-generation product diffusion via the forward-looking policy can only result in an insignificant amount of diffusion-demand loss when the selling horizon ends.

Theorem 2. *Suppose that the consumers' timing of first-generation product returns is determined by their timing of second-generation product adoption (as in Chapter 3.2). Then the forward-looking policy is optimal if, under the myopic policy, there exists a period $\kappa < T$ such that*

(i) the accumulated first-generation return volume exceeds the first-generation remanufactured-item demand in each period $t > \kappa$,

(ii) the second-generation recycled-material demand exceeds the accumulated recyclable-material amount available for fulfillment of the second-generation product demand, except the switching adopters' demand, in each period $t \leq \kappa$, whereas the reverse is true in each period $t > \kappa$,

(iii) the recyclable-material amount available from one unit of the first-generation product multiplied by the switching adopters' return fraction is larger than the recycled-material demand from one unit of the second-generation product, and

(iv) the rejection of a unit of the second-generation demand in period κ leads to a loss of diffusion demand in period $\kappa+1$ that is below a certain threshold (see Appendix A for the detailed expression) and a backlogged demand for the second-generation product in period $\kappa + 1$ that is above a certain threshold (see Appendix A for the detailed expression).

Suppose that the above conditions are met and the selling horizon is long enough.

Then, if the forward-looking policy is initiated after period κ, it provides no improvement in the total recycled-material amount used in manufacturing for fulfillment of the second-generation demand, except the switching adopters' demand.

Proof. See Appendix A.

For the problem setting in Chapter 3.2, Theorem 2 reveals that the producer can improve its total profit by rejecting some demand from the customer population unique to the second-generation product in some period (i.e., period κ) under four conditions that must hold together: Conditions (i) and (iv) in Theorem 2 are analogous to conditions (i) and (iv) in Theorem 1. Conditions (ii) and (iii) in Theorem 2 ensure that each unit of the larger second-generation demand in later periods (raised by the forward-looking policy) can still utilize the maximum possible recycled content that the product design permits. Although their implications are similar, conditions (ii) and (iii) in Theorem 2 are less likely to hold than those in Theorem 1. This is because the problem setting in Chapter 3.2 is more restrictive than the problem setting in Chapter 3.1: In Chapter 3.2, the first-generation products are returned by the switching adopters only at times when they purchase the second-generation product, and the vast amount of these returns are primarily used in manufacturing for fulfillment of these adopters' demand.

We conduct numerical experiments based on Theorem 2 by taking $\lambda = 0.8$, $\delta^u = 0.2$, $\delta^l = 0.8$, and $\theta_2 = (0.6)\lambda\delta^l\theta_1$ in our base scenario. We again vary the parameter values of the base scenario to generate many other instances. Specifically, we consider instances in which $p, q \in [0, 1]$ s.t. $p + q \leq 1$, $\tau \in \{1, 2, ..., 40\}$, $m_1/m_2 \in (0, 5]$, $\theta_2/(\lambda\delta^l\theta_1) \in (0, 1]$, and $\theta_2\rho_r/\left(p_{2n} - c_{2n}\right) \in (0, 5]$. Figures 4.3–4.4 exhibit our numerical results. We observe from Figures 4.3–4.4 that the optimality conditions of the forward-looking policy in Theorem 2 hold in a smaller number of our instances, in comparison with our results in Figures 4.1–4.2.

Unlike Figure 4.1, Figure 4.3 indicates that the optimality conditions in Theorem 2 may fail to hold even when the diffusion parameters p and q are large: The number of leapfrogging adopters increases and the number of switching adopters

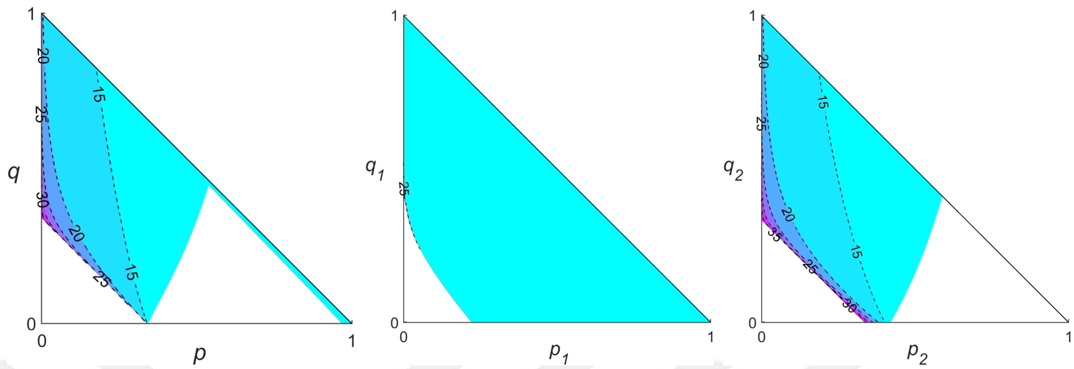

Figure 4.3: Diffusion parameters vs. environmentally critical period κ in Theorem 2. Colored regions indicate the optimality of the forward-looking policy.

decreases as the diffusion parameter p_2 increases. When p_2 is too large, since the switching adopters are the only contributors of the first-generation product returns, the accumulated first-generation return volume may always fall short of the first-generation remanufactured-item demand and the accumulated recyclable-material amount may always fall short of the second-generation recycled-material demand. Therefore, the optimality conditions in Theorem 2 may fail to hold and the forward-looking policy may bring no benefit to the producer. Similarly to Figure 4.1, Figure 4.3 shows that the environmentally critical time period κ is affected by changes in the diffusion parameters of the customer population unique to the second-generation product more than in those of the potential customer population of the first-generation product.

Figure 4.4 shows that the optimality conditions in Theorem 2 may fail to hold when m_1/m_2 is large or $\theta_2/(\lambda\delta^l\theta_1)$ is small: The accumulated recyclable-material amount is likely to exceed the second-generation recycled-material demand in a very early period when m_1/m_2 is too large or $\theta_2/(\lambda\delta^l\theta_1)$ is too small. Rejecting a demand in such an early period may induce a significant slowdown of the diffusion process and thus a significant amount of diffusion-demand loss when the selling horizon ends. Finally, we note that the optimality conditions in Theorem 2 may fail to hold even when τ is small: Introducing the first-generation product in the market too early increases the number of leapfrogging adopters and reduces the number of switching adopters. Therefore, when τ is too small, the accumulated

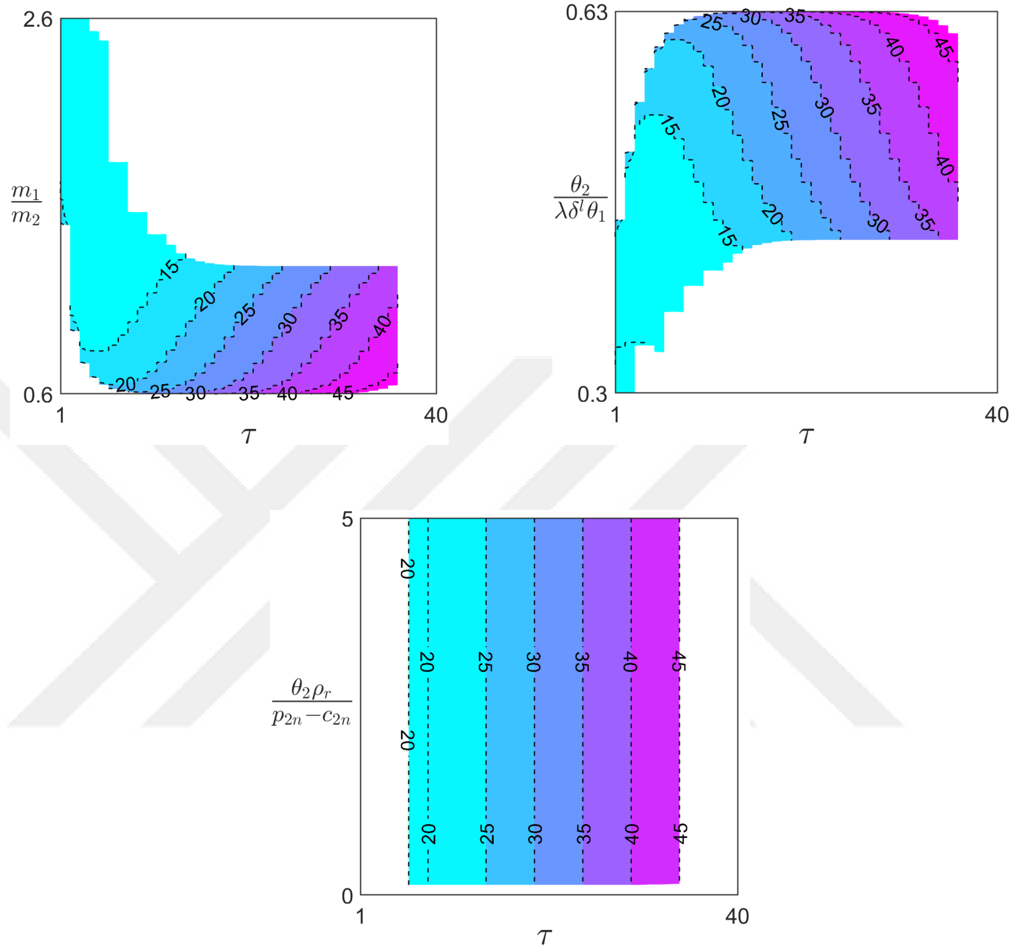

Figure 4.4: Population, return and recycling parameters vs. environmentally critical period κ in Theorem 2. Colored regions indicate the optimality of the forward-looking policy.

recyclable-material amount may never exceed the second-generation recycled-material demand in any future period, and the recycled content used in production cannot be improved via the forward-looking policy.

Chapter 5

Dynamic Programming Algorithm for the Optimal Sales Plan

We formulate a dynamic programming (DP) algorithm to calculate the optimal sales plan and its total profit for the problem setting described in Chapter 3.2. Although we have found in Chapter 4 that the optimality conditions of the forward-looking policy are less likely to hold in the problem setting of Chapter 3.2 than in the problem setting of Chapter 3.1, the consumer-return scenario in Chapter 3.2 reflects reality better than the one in Chapter 3.1 for consumer electronics such as smartphones. Therefore, we restrict our numerical study below to the problem setting of Chapter 3.2.

Recall from Chapter 3.2 that the fraction λ of the switching adopters returns their first-generation used items at the time of their purchases of the second-generation product. The fraction δ^u of these product returns are at the end of their use and can be remanufactured. The fraction $\delta^l = 1 - \delta^u$ of these product returns are at the end of their life and can be recycled. The state variables of our DP algorithm are as follows: Let E_t^u denote the accumulated end-of-use return volume at the beginning of period $t > \tau$. Let E_t^l denote the accumulated recyclable-material amount at the beginning of period $t > \tau$. Note:

$$E_t^u = E_{t-1}^u - r_{t-1} + \frac{\lambda \delta^u D_{1(t-1)}^B d_{2(t-1)}^B}{m_2}, \tag{5.1}$$

$$r_t = \min \left\{ \gamma \left(d_{1t}^B - \frac{d_{1t}^B D_{2(t+1)}^B}{m_2} \right), E_t^u \right\}, \tag{5.2}$$

and

$$E_t^l = \begin{cases} \left(\lambda \delta^l \theta_1 - \theta_2\right) \frac{D_{1(t-1)}^B d_{2(t-1)}^B}{m_2} & \text{if } E_{t-1}^l < \theta_2 \left(s_{2(t-1)} - \frac{D_{1(t-1)}^B d_{2(t-1)}^B}{m_2} \right), \\ E_{t-1}^l - \theta_2 \left(s_{2(t-1)} - \frac{D_{1(t-1)}^B d_{2(t-1)}^B}{m_2} \right) & \\ \quad + \left(\lambda \delta^l \theta_1 - \theta_2\right) \frac{D_{1(t-1)}^B d_{2(t-1)}^B}{m_2} & \text{if } E_{t-1}^l \geq \theta_2 \left(s_{2(t-1)} - \frac{D_{1(t-1)}^B d_{2(t-1)}^B}{m_2} \right), \end{cases} \tag{5.3}$$

where $E_t^u = E_t^l = r_t = 0$, $\forall t \leq \tau$. We also include the variables D_{2t}^B and S_{2t}^B in our state description. Note:

$$D_{2t}^B = D_{2(t-1)}^B + d_{2(t-1)}^B \tag{5.4}$$

and

$$S_{2t}^B = S_{2(t-1)}^B + s_{2(t-1)}^B \tag{5.5}$$

where $D_{2t}^B = S_{2t}^B = 0$, $\forall t \leq \tau$. Lastly, let B_t denote the accumulated number of backorders in period $t > \tau$ from the customer population unique to the second-generation product. Note:

$$B_t = \alpha \left(B_{t-1} + d_{2(t-1)}^{NB} - s_{2(t-1)} \right) \tag{5.6}$$

and

$$\frac{d_{1t}^B D_{2(t+1)}^B}{m_2} + \frac{D_{1t}^B d_{2t}^B}{m_2} \leq s_{2t} \leq d_{2t}^{NB} + B_t \tag{5.7}$$

where $B_\tau = 0$. With these state transitions, the DP recursion of our problem setting in Chapter 3.2 is given by

$$
\begin{aligned}
f_t\left(E_t^l, E_t^u, D_{2t}^B, S_{2t}^B, B_t\right) \;=\; & \max_{s_{2t}} \Bigg\{ (p_{1r}-c_{1r})r_t + (p_{1n}-c_{1n})\left(d_{1t}^{NB}-r_t\right) \\
& + (p_{2n}-c_{2n})s_{2t} + \rho_r \min\left\{E_t^l, \theta_2\left(s_{2t} - \frac{D_{1t}^B d_{2t}^B}{m_2}\right)\right\} \\
& + \frac{\theta_2 \rho_r D_{1t}^B d_{2t}^B}{m_2} + f_{t+1}\left(E_{t+1}^l, E_{t+1}^u, D_{2(t+1)}^B, S_{2(t+1)}^B, B_{t+1}\right) \Bigg\},
\end{aligned}
$$

$\forall t \in \{\tau, \tau+1, ..., T\}$, $f_t(0,0,0,0,0) = (p_{1n}-c_{1n})d_{1t}^{NB} + f_{t+1}(0,0,0,0,0)$, $\forall t \in \{1, 2, ..., \tau-1\}$, and $f_{T+1}(.) = 0$, subject to (3.1)-(3.4) and (5.1)-(5.7). Note that $f_1 = (0,0,0,0,0)$ is the optimal total profit over the T-period selling horizon. We also define $\breve{f}_t(.)$ as the profit function in period t under the myopic policy, which can be computed from the above DP recursion by replacing the action s_{2t} with the variable d_{2t}^{NB}, $\forall t \geq \tau$.

We have coded the discrete-state and discrete-action version of our DP algorithm in the Java programming language on a system with a 2.6 GHz CPU and 8 GB of RAM. We construct our experimental test bed by choosing the parameter values as follows: $T = 15$, $m_1 = 160$, $m_2 = 40$, $2(p_{1n}-c_{1n}) = 2(p_{2n}-c_{2n}) = p_{1r}-c_{1r} = 10$, $\alpha = 0.88$, $\gamma = 0.1$, $\lambda = 0.8$, $\theta_2 = 1$, $\delta^u = 1-\delta^l$, $\delta^l \in \{0.8, 0.9, 1\}$, $p = p_1 = p_2 \in \{0.03, 0.05, 0.07, 0.09\}$, $q = q_1 = q_2 \in \{0.30, 0.40, 0.50, 0.60\}$, $\tau \in \{4,5,6,7\}$, $\theta_1 \in \{2,3\}$, and $\rho_r \in \{1,2\}$. Notice that the assumption of $\lambda\delta^l\theta_1 \geq \theta_2$ made in Chapter 3.2 holds in each of our instances. We compute the percentage profit improvement achieved with the forward-looking policy by comparing the profits $f_\tau(0,0,0,0,0)$ and $\breve{f}_\tau(0,0,0,0,0)$. Tables 5.1–5.3 exhibit our numerical results. In these tables, the symbol * represents the instances for which the DP recursion could not be solved to optimality due to the memory limit. Figures 5.1–5.2 provide visual illustrations of the optimal sales plans in eight different instances.

Our results show that the forward-looking policy improves the profit by 1.28% on average in our instances. Notice that both remanufacturing and recycling options for early-generation returns exist in our instances with $\delta^l = 1-\delta^u \in \{0.8, 0.9\}$ while the remanufacturing option disappears in our instances with $\delta^l =$

$1 - \delta^u = 1$. The forward-looking policy appears to be more beneficial when both recycling and remanufacturing options exist. The average profit improvement is 0.81% when $\delta^l = 1$, it is 1.60% when $\delta^l = 0.9$, and it is 2.07% when $\delta^l = 0.8$. This result is consistent with the previous findings in the CLSC literature that examines the benefits of recycling and remanufacturing options separately with new-product diffusion models (see Nadar et al. [13], Uzunlar [15], Güray [16]). Our results, however, reveal the benefits of recycling and remanufacturing options that are jointly managed and optimized.

We observe that the forward-looking policy tends to become more attractive as τ decreases: When τ is small enough, the second-generation demand can be entirely observed during the selling horizon even if the diffusion process is decelerated with the forward-looking policy. We also observe that the benefit of the forward-looking policy tends to be lower when both p and q are small or when both p and q are large: The slowdown in the product diffusion via the forward-looking policy is less desirable for an already-slow diffusion process implied by small values of p and q. The numbers of switching adopters and their returns are too low so that the potential improvements in the recycled-content use and remanufactured-item sales are less significant when p_2 and q_2 are large. Finally, we note that the forward-looking policy tends to become less attractive as θ_1 increases: The accumulated recyclable-material amount is likely to exceed the second-generation recycled-material demand in very early periods of the selling horizon when θ_1 is large. Rejecting demand in significant amounts too early to improve the recycled-content use in production may lead to a substantial loss of diffusion demand at the end of the selling horizon. The high values of θ_1 may thus reduce the benefit of the forward-looking policy.

Table 5.1: Percentage profit improvements via forward-looking policy. $\delta^l = 1$.

p	$\delta^l = 1$	τ	$q = 0.3$	$q = 0.4$	$q = 0.5$	$q = 0.6$
0.03	$\rho_r = 1$ & $\theta_1 = 2$	4	0.53	1.11	1.28	3.42
		5	0.97	0.33	0.00	0.30
		6	0.53	0.51	0.00	0.70
		7	0.00	0.00	0.00	0.35
	$\rho_r = 1$ & $\theta_1 = 3$	4	0.37	0.71	0.96	2.97
		5	1.04	0.33	0.00	0.48
		6	0.44	0.43	0.00	0.76
		7	0.00	0.00	0.00	0.49
	$\rho_r = 2$ & $\theta_1 = 2$	4	0.63	1.13	1.21	3.49
		5	1.04	0.30	0.00	0.27
		6	0.56	0.59	0.00	0.69
		7	0.00	0.00	0.00	0.31
	$\rho_r = 2$ & $\theta_1 = 3$	4	0.34	0.69	0.60	2.67
		5	1.16	0.30	0.00	0.59
		6	0.39	0.45	0.00	0.79
		7	0.00	0.00	0.00	0.56
0.05	$\rho_r = 1$ & $\theta_1 = 2$	4	0.91	2.66	2.26	3.17
		5	0.73	2.01	1.60	1.21
		6	0.47	0.00	1.22	0.40
		7	0.00	0.00	0.00	0.44
	$\rho_r = 1$ & $\theta_1 = 3$	4	0.83	2.17	2.11	2.89
		5	0.73	1.92	1.46	1.14
		6	0.39	0.00	1.08	0.40
		7	0.00	0.00	0.00	0.44
	$\rho_r = 2$ & $\theta_1 = 2$	4	1.02	2.75	2.38	3.48
		5	0.66	2.04	1.72	1.31
		6	0.49	0.00	1.27	0.42
		7	0.00	0.00	0.00	0.45
	$\rho_r = 2$ & $\theta_1 = 3$	4	0.87	1.88	2.11	2.97
		5	0.65	1.95	1.47	1.18
		6	0.35	0	1.02	0.41
		7	0.00	0.00	0.00	0.45

p	$\delta^l = 1$	τ	$q = 0.3$	$q = 0.4$	$q = 0.5$	$q = 0.6$
0.07	$\rho_r = 1$ & $\theta 1 = 2$	4	1.37	2.74	1.61	2.38
		5	0.71	1.15	0.93	1.00
		6	0.37	1.29	0.42	0.50
		7	0.40	0.00	0.00	0.00
	$\rho_r = 1$ & $\theta_1 = 3$	4	1.22	2.58	1.59	2.30
		5	0.70	1.08	0.79	0.73
		6	0.44	1.07	0.35	0.43
		7	0.48	0.00	0.00	0.00
	$\rho_r = 2$ & $\theta_1 = 2$	4	1.55	2.83	1.74	3.07
		5	0.64	1.15	1.07	1.19
		6	0.00	1.33	0.43	0.57
		7	0.00	0.00	0.00	0.00
	$\rho_r = 2$ & $\theta_1 = 3$	4	1.27	2.54	1.70	2.39
		5	0.62	1.01	0.82	0.71
		6	0.46	0.95	0.31	0.44
		7	0.50	0.00	0.00	0.40
0.09	$\rho_r = 1$ & $\theta_1 = 2$	4	1.57	2.00	1.26	0.99
		5	1.21	0.99	0.48	0.49
		6	0.98	0.59	0.00	0.00
		7	0.40	0.95	0.00	0.00
	$\rho_r = 1$ & $\theta_1 = 3$	4	1.49	1.43	1.18	0.85
		5	1.20	0.77	0.41	0.42
		6	0.90	0.37	0.00	0.00
		7	0.40	0.95	0.00	0.00
	$\rho_r = 2$ & $\theta_1 = 2$	4	1.54	2.35	1.37	1.18
		5	1.21	1.12	0.55	0.56
		6	1.07	0.71	0.00	0.00
		7	0.35	0.97	0.00	0.00
	$\rho_r = 2$ & $\theta_1 = 3$	4	1.44	1.33	1.23	0.93
		5	1.19	0.74	0.42	0.43
		6	0.93	0.32	0.00	0.00
		7	0.35	0.97	0.00	0.00
Average			0.67	0.95	0.66	0.96

Table 5.2: Percentage profit improvements via forward-looking policy. $\delta^l = 0.9$.

p	$\delta^l = 0.9$	τ	$q = 0.3$	$q = 0.4$	$q = 0.5$	$q = 0.6$
	$\rho_r = 1$	4	0.92	1.97	1.59	*
	&	5	1.88	1.08	0.63	*
	$\theta_1 = 2$	6	0.89	1.23	0.33	*
		7	0.98	0.79	0.37	*
	$\rho_r = 1$	4	0.68	1.42	*	*
	&	5	1.60	0.99	*	*
	$\theta_1 = 3$	6	0.87	1.15	*	*
0.03		7	0.96	0.78	*	*
	$\rho_r = 2$	4	1.00	2.11	1.50	*
	&	5	1.96	1.04	0.57	*
	$\theta_1 = 2$	6	0.82	1.25	0.30	*
		7	0.90	0.72	0.33	*
	$\rho_r = 2$	4	0.54	1.39	*	*
	&	5	1.45	0.89	*	*
	$\theta_1 = 3$	6	0.79	1.10	*	*
		7	0.87	0.70	*	*
	$\rho_r = 1$	4	1.63	3.39	3.02	*
	&	5	2.00	3.03	2.25	*
	$\theta_1 = 2$	6	1.65	1.13	1.97	*
		7	1.04	0.85	0.37	*
	$\rho_r = 1$	4	1.39	2.95	*	*
	&	5	1.89	2.86	*	*
	$\theta_1 = 3$	6	1.55	1.05	*	*
0.05		7	0.86	0.76	*	*
	$\rho_r = 2$	4	1.68	3.50	3.19	*
	&	5	1.95	3.04	2.31	*
	$\theta_1 = 2$	6	1 56	1.07	2.00	*
		7	1.09	0.82	0.33	*
	$\rho_r = 2$	4	1.24	2.99	*	*
	&	5	1.76	2.73	*	*
	$\theta_1 = 3$	6	1.39	0.93	*	*
		7	0.77	0.67	*	*

p	$\delta^l = 0.9$	τ	$q = 0.3$	$q = 0.4$	$q = 0.5$	$q = 0.6$
0.07	$\rho_r = 1$ & $\theta1 = 2$	4	2.42	3.64	2.77	*
		5	1.57	2.32	1.60	*
		6	1.19	2.31	1.20	*
		7	1.63	0.77	0.38	*
	$\rho_r = 1$ & $\theta_1 = 3$	4	2.24	3.31	*	*
		5	1.48	1.95	*	*
		6	1.18	2.14	*	*
		7	1.69	0.77	*	*
	$\rho_r = 2$ & $\theta_1 = 2$	4	2.51	3.84	2.97	*
		5	1.54	2.32	1.67	*
		6	1.13	2.50	1.18	*
		7	1.44	0.68	0.33	*
	$\rho_r = 2$ & $\theta_1 = 3$	4	2.19	3.25	*	*
		5	1.38	1.67	*	*
		6	1.11	1.89	*	*
		7	1.57	0.68	*	*
0.09	$\rho_r = 1$ & $\theta1 = 2$	4	2.62	2.91	2.14	*
		5	2.51	2.12	1.39	*
		6	2.12	1.18	0.81	*
		7	1.37	1.59	0.39	*
	$\rho_r = 1$ & $\theta_1 = 3$	4	2.71	2.73	*	*
		5	2.26	1.89	*	*
		6	1.88	1.10	*	*
		7	1.20	1.35	*	*
	$\rho_r = 2$ & $\theta_1 = 2$	4	2.55	3.06	2.34	*
		5	2.57	2.20	1.54	*
		6	2.08	1.11	0.77	*
		7	1.34	1.74	0.34	*
	$\rho_r = 2$ & $\theta_1 = 3$	4	2.71	2.73	*	*
		5	2.13	1.79	*	*
		6	1.66	0.97	*	*
		7	1.05	1.31	*	*
Average			1.56	1.78	1.34	-

Table 5.3: Percentage profit improvements via forward-looking policy. $\delta^l = 0.8$.

p	$\delta^l = 0.8$	τ	$q = 0.3$	$q = 0.4$	$q = 0.5$	$q = 0.6$
0.03	$\rho_r = 1$ & $\theta_1 = 2$	4	1.63	2.39	*	*
		5	2.23	1.49	*	*
		6	1.79	1.54	*	*
		7	1.97	1.59	*	*
	$\rho_r = 1$ & $\theta_1 = 3$	4	1.50	*	*	*
		5	2.09	*	*	*
		6	1.83	*	*	*
		7	1.92	*	*	*
	$\rho_r = 2$ & $\theta_1 = 2$	4	1.58	2.57	*	*
		5	2.22	1.50	*	*
		6	1.66	1.47	*	*
		7	1.82	1.45	*	*
	$\rho_r = 2$ & $\theta_1 = 3$	4	1.37	*	*	*
		5	1.97	*	*	*
		6	1.75	*	*	*
		7	1.75	*	*	*
0.05	$\rho_r = 1$ & $\theta_1 = 2$	4	2.36	3.82	*	*
		5	2.39	3.33	*	*
		6	2.15	1.06	*	*
		7	1.76	1.16	*	*
	$\rho_r = 1$ & $\theta_1 = 3$	4	2.15	*	*	*
		5	2.41	*	*	*
		6	2.02	*	*	*
		7	1.72	*	*	*
	$\rho_r = 2$ & $\theta_1 = 2$	4	2.37	3.98	*	*
		5	2.32	3.27	*	*
		6	2.10	0.96	*	*
		7	1.59	1.04	*	*
	$\rho_r = 2$ & $\theta_1 = 3$	4	2.00	*	*	*
		5	2.35	*	*	*
		6	1.88	*	*	*
		7	1.53	*	*	*

p	$\delta^l = 0.8$	τ	$q = 0.3$	$q = 0.4$	$q = 0.5$	$q = 0.6$
0.07	$\rho_r = 1$ & $\theta 1 = 2$	4	2.65	4.01	*	*
		5	2.24	2.75	*	*
		6	1.66	2.54	*	*
		7	1.72	0.93	*	*
	$\rho_r = 1$ & $\theta_1 = 3$	4	2.52	*	*	*
		5	2.04	*	*	*
		6	1.55	*	*	*
		7	1.69	*	*	*
	$\rho_r = 2$ & $\theta_1 = 2$	4	2.62	4.22	*	*
		5	2.10	2.80	*	*
		6	1.63	2.59	*	*
		7	1.61	0.96	*	*
	$\rho_r = 2$ & $\theta_1 = 3$	4	2.08	*	*	*
		5	1.57	*	*	*
		6	1.44	*	*	*
		7	1.57	*	*	*
0.09	$\rho_r = 1$ & $\theta_1 = 2$	4	3.14	3.36	*	*
		5	2.39	*	*	*
		6	1.99	*	*	*
		7	1.30	*	*	*
	$\rho_r = 1$ & $\theta_1 = 3$	4	2.99	*	*	*
		5	2.55	*	*	*
		6	1.88	*	*	*
		7	1.20	*	*	*
	$\rho_r = 2$ & $\theta_1 = 2$	4	3.17	*	*	*
		5	2.35	*	*	*
		6	1.84	*	*	*
		7	1.22	*	*	*
	$\rho_r = 2$ & $\theta_1 = 3$	4	2.91	*	*	*
		5	2.64	*	*	*
		6	1.66	*	*	*
		7	1.05	*	*	*
Average			1.99	2.27	-	-

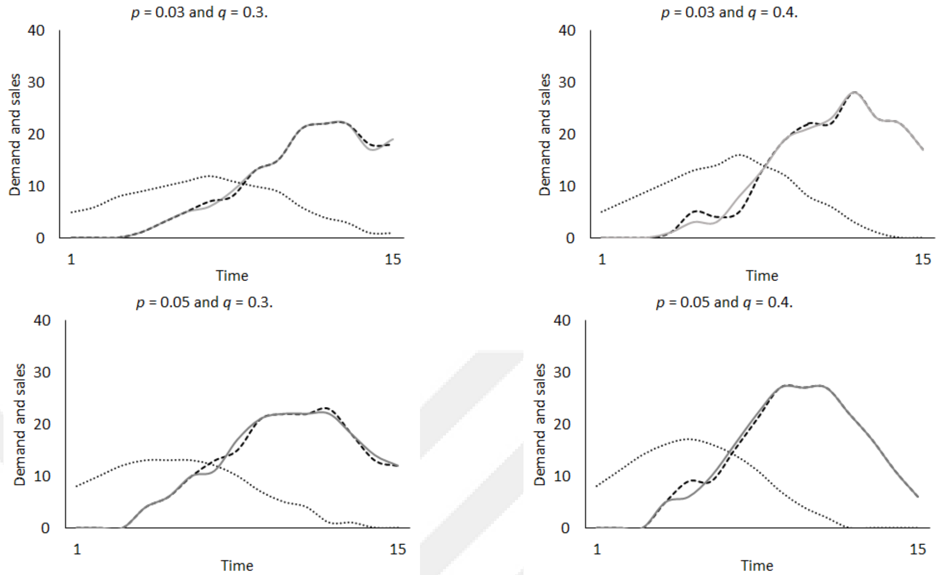

Figure 5.1: Dashed, gray, and dotted lines indicate the second-generation demand, the second-generation sales, and the first-generation demand, respectively. $T = 15$, $\tau = 4$, $m_1 = 160$, $m_2 = 40$, $\rho_r = 1$, $\theta_1 = 2$, $\theta_2 = 1$, $\delta^l = 1$, $\alpha = 0.88$, $\gamma = 0.1$ $\lambda = 0.8$, $p_{1n} - c_{1n} = p_{2n} - c_{2n} = 5$, $p_{1r} - c_{1r} = 10$.

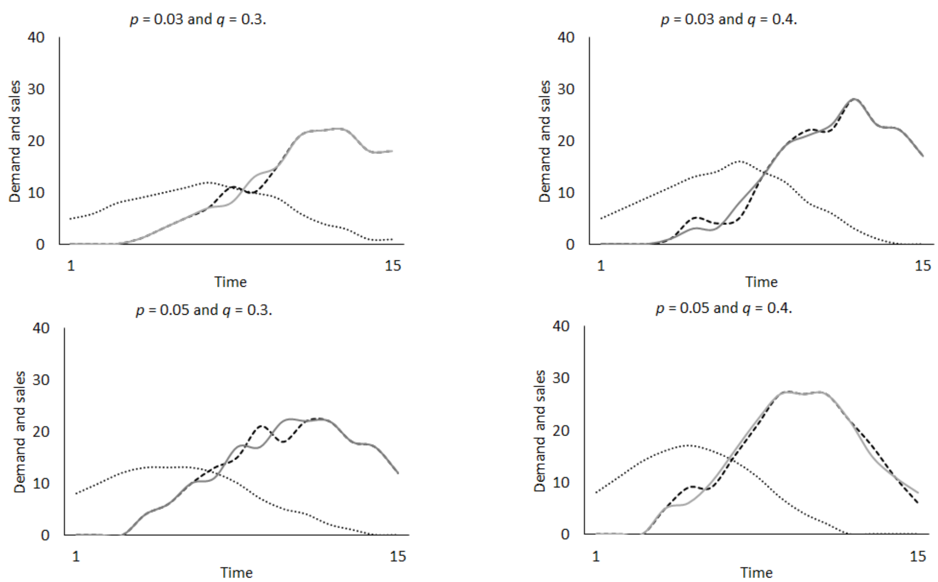

Figure 5.2: Dashed, gray, and dotted lines indicate the second-generation demand, the second-generation sales, and the first-generation demand, respectively. $T = 15$, $\tau = 4$, $m_1 = 160$, $m_2 = 40$, $\rho_r = 2$, $\theta_1 = 2$, $\theta_2 = 1$, $\delta^l = 0.8$, $\alpha = 0.88$, $\gamma = 0.1$, $\lambda = 0.8$, $p_{1n} - c_{1n} = p_{2n} - c_{2n} = 5$, $p_{1r} - c_{1r} = 10$.

Chapter 6

Conclusion

To our knowledge, our work is the first attempt to model and analyze the multi-generation diffusion process for CLSCs of durable goods by simultaneously considering the remanufacturing and recycling options. In the presence of two successive product generations, we identify conditions under which slowing down the second-generation product diffusion via our forward-looking policy improves the producer's total profit in the long run, despite the potential lost sales in the short term. The forward-looking policy is desirable if (i) the diffusion process is fast enough in the absence of any manipulation, (ii) the second-generation product is introduced to the market early enough, (iii) the number of first-generation end-of-life returns and the amount of recyclable material from each such return are high enough, and (iv) the potential customer base of the first-generation product is sufficiently large. The benefit of the forward-looking policy disappears in many instances, according to our numerical experiments, if the consumers bring their old devices only at times when they buy the next-generation device.

In their numerical study, Güray [16] have revealed that the forward-looking policy can only be profitable in a limited number of instances in a special case of our problem setting when the first-generation product returns can be remanufactured but cannot be recycled (i.e., when all product returns are at the end of their use). Our numerical results imply that the addition of the recycling option amplifies the potential benefit of the forward-looking policy so that it becomes optimal in many instances. Uzunlar [15], on the other hand, have found that the forward-looking policy can be optimal in many instances in another special case of our problem setting when the first-generation product returns can be recycled but cannot be remanufactured (i.e., when all product returns are at the end of their life). Our numerical results also imply that the addition of the remanufacturing option further raises the value of the forward-looking policy.

Future research may extend our optimization framework by taking a game theoretical approach and modeling the competition effect between multiple producers, remanufacturers, and/or recyclers. Several papers may inspire future work in this direction. See, for examples, Ferguson and Toktay [69], Atalay et al. [70], Matsui [28], and Esenduran et al. [71]. Future research may also endogenize the used-item acquisition decisions, unlike our sales planning model that accepts all returned items. Such an extension may benefit from Nadar et al. [21] who characterize the structure of the optimal procurement policy in the presence of different quality levels for the collected used items. Finally, future work may study the investment strategies for new product development, market penetration, and disposition options. In a recent study, Demirci and Erkip [72] explore the optimal budget allocation of an innovative firm for R&D investments and subsidies provided to a distributor for better product availability. Such a perspective might be extended to include the investments required for remanufacturing and/or recycling capabilities. Apple's recycling machine, for example, can recycle the modules containing rare earth magnets more efficiently than the conventional shredders [3]. High investment costs for such technological developments might also be taken into consideration for more realistic problem settings.

Bibliography

[1] EC, "Circular Economy Package: Questions & Answers." https://ec.europa.eu/commission/presscorner/detail/en/MEMO_15_6204, 2015. accessed June 30, 2023.

[2] V. D. R. Guide and L. N. V. Wassenhove, "The Evolution of Closed-Loop Supply Chain Research," *Operations Research*, vol. 57, no. 1, pp. 10–18, 2009.

[3] Apple, "Environmental Progress Report." www.apple.com/environment/pdf/Apple_Environmental_Progress_Report_2022.pdf, 2022. Accessed March 26, 2023.

[4] Dell Technologies, "FY22 Environmental, Social and Governance report." https://www.dell.com/en-us/dt/corporate/social-impact/esg-resources/reports/fy22-esg-report.htm#scroll=off, 2022. Accessed April 7, 2023.

[5] Hewlett-Packard, "Sustainable impact report." https://h20195.www2.hp.com/v2/GetDocument.aspx?docname=c08228880, 2021. Accessed March 26, 2023.

[6] Huawei, "Huawei Consumer Business Sustainability Progress Report." www-file.huawei.com/-/media/corp2020/pdf/sustainability/others/huawei-consumer-business-sustainability-progress-report-2022-en.pdf, 2022. accessed April 8, 2023.

[7] Microsoft, "2021 Environmental Sustainability Report." https://www.microsoft.com/en-us/corporate-responsibility/sustainability/report, 2021. accessed April 8, 2023.

[8] Samsung, "A Journey Towards a Sustainable Future." images.samsung.com/is/content/samsung/assets/global/ir/docs/sustainability_report_2022_en.pdf, 2022. Accessed March 26, 2023.

[9] Xiaomi, "ENVIRONMENTAL, SOCIAL AND GOVERNANCE REPORT." https://cdn.cnbj1.fds.api.mi-img.com/staticsfile/svhc/Xiaomi%20ESG%20REPORT%202022_EN.pdf, 2022. Accessed April 10, 2023.

[10] L. G. Debo, L. B. Toktay, and L. N. V. Wassenhove, "Joint life-cycle dynamics of new and remanufactured products," *Production and Operations Management*, vol. 15, no. 4, pp. 498–513, 2006.

[11] M. Akan, B. Ata, and R. C. Savaşkan-Ebert, "Dynamic pricing of remanufacturable products under demand substitution: A product life cycle model," *Annals of Operations Research*, vol. 211, pp. 1–25, 2013.

[12] A. Robotis, S. Bhattacharya, and L. N. Van Wassenhove, "Lifecycle pricing for installed base management with constrained capacity and remanufacturing," *Production and Operations Management*, vol. 21, no. 2, pp. 236–252, 2012.

[13] E. Nadar, B. E. Kaya, and K. Güler, "New-product diffusion in closed-loop supply chains," *Manufacturing & Service Operations Management*, vol. 23, no. 6, pp. 1413–1430, 2021.

[14] E. Nadar, "Tüketici davranışlarının zamanla değiştiği kapalı devre tedarik zincirlerinde satış planı eniyilemesi," *Gazi University Journal of Science Part C: Design and Technology*, vol. 7, no. 3, pp. 627–638, 2019.

[15] N. Uzunlar, "Diffusion control of successive product generations with recycling potential," Master's thesis, Bilkent Universitesi (Turkey), 2021.

[16] B. Güray, "Diffusion control in closed-loop supply chains: Successive product generations with remanufacturing potential," Master's thesis, Bilkent Universitesi (Turkey), 2023.

[17] F. M. Bass, "A new product growth for model consumer durables," *Management Science*, vol. 15, no. 5, pp. 215–227, 1969.

[18] F. M. Bass, "Comments on "a new product growth for model consumer durables the bass model"," *Management Science*, vol. 50, no. 12_supplement, pp. 1833–1840, 2004.

[19] J. A. Norton and F. M. Bass, "A diffusion theory model of adoption and substitution for successive generations of high-technology products," *Management Science*, vol. 33, no. 9, pp. 1069–1086, 1987.

[20] Z. Jiang and D. C. Jain, "A generalized Norton–Bass model for multigeneration diffusion," *Management Science*, vol. 58, no. 10, pp. 1887–1897, 2012.

[21] E. Nadar, M. Akan, L. Debo, and A. Scheller-Wolf, "Optimal procurement in remanufacturing systems with uncertain used-item condition," *Operations Research*, 2023.

[22] A. Atasu, V. D. R. Guide, Jr., and L. N. van Wassenhove, "So what if remanufacturing cannibalizes my new product sales?," *California Management Review*, vol. 52, no. 2, pp. 56–76, 2010.

[23] V. D. R. Guide, Jr. and J. Li, "The Potential for Cannibalization of New Products Sales by Remanufactured Products," *Decision Sciences*, vol. 41, no. 3, pp. 547–572, 2010.

[24] T. G. Gutowski, S. Sahni, A. Boustani, and S. C. Graves, "Remanufacturing and Energy Savings," *Environmental Science & Technology*, vol. 45, no. 10, pp. 4540–4547, 2011.

[25] I. Kitsara, "E-waste and innovation: unlocking hidden value." `https://www.wipo.int/wipo_magazine/en/2014/03/article_0001.html`. Accessed August 8, 2023.

[26] G. Esenduran, A. Atasu, and L. N. van Wassenhove, "Valuable e-waste: Implications for extended producer responsibility," *IISE Transactions*, vol. 51, no. 4, pp. 382–396, 2019.

[27] N. Wang, Q. He, and B. Jiang, "Hybrid closed-loop supply chains with competition in recycling and product markets," *International Journal of Production Economics*, vol. 217, pp. 246–258, 2019.

[28] K. Matsui, "Dual-recycling channel reverse supply chain design of recycling platforms under acquisition price competition," *International Journal of Production Economics*, vol. 259, p. 108769, 2023.

[29] T.-H. Ho, S. Savin, and C. Terwiesch, "Managing demand and sales dynamics in new product diffusion under supply constraint," *Management Science*, vol. 48, no. 2, pp. 187–206, 2002.

[30] S. Kumar and J. M. Swaminathan, "Diffusion of innovations under supply constraints," *Operations Research*, vol. 51, no. 6, pp. 866–879, 2003.

[31] T.-H. Ho, S. Savin, and C. Terwiesch, "Note: A reply to "new product diffusion decisions under supply constraints"," *Management Science*, vol. 57, no. 10, pp. 1811–1812, 2011.

[32] W. Shen, I. Duenyas, and R. Kapuscinski, "New product diffusion decisions under supply constraints," *Management Science*, vol. 57, no. 10, pp. 1802–1810, 2011.

[33] W. Shen, I. Duenyas, and R. Kapuscinski, "Optimal pricing, production, and inventory for new product diffusion under supply constraints," *Manufacturing & Service Operations Management*, vol. 16, no. 1, pp. 28–45, 2014.

[34] Ö. Bilginer and F. Erhun, "Production and sales planning in capacitated new product introductions," *Production and Operations Management*, vol. 24, no. 1, pp. 42–53, 2015.

[35] D. Jain, V. Mahajan, and E. Muller, "Innovation diffusion in the presence of supply restrictions," *Marketing Science*, vol. 10, no. 1, pp. 83–90, 1991.

[36] D. R. Keith, J. D. Sterman, and J. Struben, "Supply constraints and waitlists in new product diffusion," *System Dynamics Review*, vol. 33, no. 3-4, pp. 254–279, 2017.

[37] X. Yan, P. Cao, M. Zhang, and K. Liu, "The optimal production and sales policy for a new product with negative word-of-mouth," *Journal of Industrial and Management Optimization*, vol. 7, no. 1, p. 117, 2011.

[38] S. Kalish, "A new product adoption model with price, advertising, and uncertainty," *Management Science*, vol. 31, no. 12, pp. 1569–1585, 1985.

[39] D. C. Jain and R. C. Rao, "Effect of price on the demand for durables: Modeling, estimation, and findings," *Journal of Business & Economic Statistics*, vol. 8, no. 2, pp. 163–170, 1990.

[40] F. M. Bass, T. V. Krishnan, and D. C. Jain, "Why the bass model fits without decision variables," *Marketing Science*, vol. 13, no. 3, pp. 203–223, 1994.

[41] T. V. Krishnan, F. M. Bass, and D. C. Jain, "Optimal pricing strategy for new products," *Management science*, vol. 45, no. 12, pp. 1650–1663, 1999.

[42] H. Li and W. T. Huh, "Optimal pricing for a short life-cycle product when customer price-sensitivity varies over time," *Naval Research Logistics (NRL)*, vol. 59, no. 7, pp. 552–576, 2012.

[43] H. Li, "Optimal pricing under diffusion-choice models," *Operations Research*, vol. 68, no. 1, pp. 115–133, 2020.

[44] M. Zhang, H.-S. Ahn, and J. Uichanco, "Data-driven pricing for a new product," *Operations Research*, vol. 70, no. 2, pp. 847–866, 2022.

[45] L. Wilson and J. Norton, "Optimal entry timing for a product line extension," *Marketing Science*, vol. 8, no. 1, pp. 1–17, 1989.

[46] V. Mahajan and E. Muller, "Timing, diffusion, and substitution of successive generations of technological innovations: The ibm mainframe case," *Technological Forecasting and Social Change*, vol. 51, no. 2, pp. 109–132, 1996.

[47] R. M. Krankel, I. Duenyas, and R. Kapuscinski, "Timing successive product introductions with demand diffusion and stochastic technology improvement," *Manufacturing & Service Operations Management*, vol. 8, no. 2, pp. 119–135, 2006.

[48] W. Tang, T. Wang, and W. Xu, "Sooner or later? the role of adoption timing in new technology introduction," *Production and Operations Management*, vol. 31, no. 4, pp. 1663–1678, 2022.

[49] T. T. Ke, Z.-J. M. Shen, and S. Li, "How inventory cost influences introduction timing of product line extensions," *Production and Operations Management*, vol. 22, no. 5, pp. 1214–1231, 2013.

[50] Z. Guo and J. Chen, "Multigeneration product diffusion in the presence of strategic consumers," *Information Systems Research*, vol. 29, no. 1, pp. 206–224, 2018.

[51] J. Zhengrui, Q. Xinxue (Shawn), and D. C. Jain, "Optimal market entry timing for successive generations of technological innovations.," *MIS Quarterly*, vol. 43, no. 3, pp. 787 – A12, 2019.

[52] A. Negahban and J. S. Smith, "Optimal production-sales policies and entry time for successive generations of new products," *International Journal of Production Economics*, vol. 199, pp. 220–232, 2018.

[53] M. Dong, S. Li, and S. Mao, "Supplier's technology upgrading investment strategy considering product life cycle," *International Journal of Production Economics*, vol. 263, p. 108953, 2023.

[54] C. T. Druehl, G. M. Schmidt, and G. C. Souza, "The optimal pace of product updates," *European Journal of Operational Research*, vol. 192, no. 2, pp. 621–633, 2009.

[55] S. Liao and R. W. Seifert, "On the optimal frequency of multiple generation product introductions," *European Journal of Operational Research*, vol. 245, no. 3, pp. 805–814, 2015.

[56] E. Koca, G. C. Souza, and C. T. Druehl, "Managing product rollovers," *Decision Sciences*, vol. 41, no. 2, pp. 403–423, 2010.

[57] M. M. Şeref, J. E. Carrillo, and A. Yenipazarli, "Multi-generation pricing and timing decisions in new product development," *International Journal of Production Research*, vol. 54, no. 7, pp. 1919–1937, 2016.

[58] A. Atasu, V. D. R. Guide Jr, and L. N. Van Wassenhove, "Product reuse economics in closed-loop supply chain research," *Production and Operations Management*, vol. 17, no. 5, pp. 483–496, 2008.

[59] M. E. Ferguson and G. C. Souza, *Closed-loop supply chains: new developments to improve the sustainability of business practices*. Auerbach Publications, 2010.

[60] E. Akçalı and S. Çetinkaya, "Quantitative models for inventory and production planning in closed-loop supply chains," *International Journal of Production Research*, vol. 49, no. 8, pp. 2373–2407, 2011.

[61] E. Hassini, C. Surti, and C. Searcy, "A literature review and a case study of sustainable supply chains with a focus on metrics," *International Journal of Production Economics*, vol. 140, no. 1, pp. 69–82, 2012.

[62] G. C. Souza, "Closed-loop supply chains: a critical review, and future research," *Decision Sciences*, vol. 44, no. 1, pp. 7–38, 2013.

[63] K. Govindan, H. Soleimani, and D. Kannan, "Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future," *European Journal of Operational Research*, vol. 240, no. 3, pp. 603–626, 2015.

[64] R. De Angelis, M. Howard, and J. Miemczyk, "Supply chain management and the circular economy: towards the circular supply chain," *Production Planning & Control*, vol. 29, no. 6, pp. 425–437, 2018.

[65] J. Gaur, M. Amini, and A. Rao, "Closed-loop supply chain configuration for new and reconditioned products: An integrated optimization model," *Omega*, vol. 66, pp. 212–223, 2017.

[66] P. Georgiadis, D. Vlachos, and G. Tagaras, “The impact of product lifecycle on capacity planning of closed-loop supply chains with remanufacturing,” *Production and Operations Management*, vol. 15, no. 4, pp. 514–527, 2006.

[67] P. Georgiadis and E. Athanasiou, “The impact of two-product joint lifecycles on capacity planning of remanufacturing networks,” *European Journal of Operational Research*, vol. 202, no. 2, pp. 420–433, 2010.

[68] W. Wang, Y. Wang, D. Mo, and M. M. Tseng, “Managing component reuse in remanufacturing under product diffusion dynamics,” *International Journal of Production Economics*, vol. 183, pp. 551–560, 2017.

[69] M. E. Ferguson and L. B. Toktay, “The Effect of Competition on Recovery Strategies,” *Production and Operations Management*, vol. 15, no. 3, pp. 351–368, 2006.

[70] A. Atasu, M. Sarvary, and L. N. Van Wassenhove, “Remanufacturing as a marketing strategy,” *Management Science*, vol. 54, no. 10, pp. 1731–1746, 2008.

[71] G. Esenduran, Y.-T. Lin, W. Xiao, and M. Jin, “Choice of electronic waste recycling standard under recovery channel competition,” *Manufacturing & Service Operations Management*, vol. 22, no. 3, pp. 495–512, 2020.

[72] E. Z. Demirci and N. Erkip, “Integrating Efforts for Product Development and Market Penetration,” *European Journal of Operational Research*, 2023.

Appendix A

Detailed Versions of Theorems 1–2 and Their Proofs

Theorems A.1–A.2 are the unabridged versions of Theorems 1–2, respectively. In order to denote the fact that the myopic and forward-looking policies lead to different model variables, we include the breve in our notation under the myopic policy and the hat in our notation under the forward-looking policy.

Theorem A.1. *Suppose that the consumers' timing of end-of-life product returns is independent of their timing of second-generation product adoption (as in Chapter 3.1).*

(a) Suppose that $p_{1n} - c_{1n} > p_{2n} - c_{2n}$. *For optimality of the forward-looking policy, it is sufficient that* $\exists \kappa \in \{\tau, ..., T-1\}$ *s.t.* $\breve{e}_t^l < \theta_2 \breve{d}_{2t}^{NB}$ *for* $\tau \le t \le \kappa$, $\breve{e}_t^l > \theta_2 \breve{d}_{2t}^{NB}$ *and* $\breve{e}_t^u > \gamma \breve{d}_{1t}^{NB}$ *for* $t > \kappa$, $\frac{\theta_1}{\theta_2} \ge \frac{\breve{s}_{2t}}{\sum_{i=1}^{t-1} \beta_i^l \breve{d}_{1(t-i)}^{NB}}$ *for* $t \ge \kappa + 2$, $\alpha > \frac{(1+A)(1-2B)}{1-B}$, *and*

$$\begin{aligned}\alpha \quad > \quad & \left(\frac{p_{2n} - c_{2n}}{\theta_2\rho_r + p_{2n} - c_{2n}}\right)(1 + AE) - \left(\frac{p_{1n} - c_{1n}}{\theta_2\rho_r + p_{2n} - c_{2n}}\right)\frac{(1-\gamma)\, A\breve{d}_{1(\kappa+1)}^B}{m_2} \\ & - \left(\frac{p_{1r} - c_{1r}}{\theta_2\rho_r + p_{2n} - c_{2n}}\right)\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2} + \left(\frac{\theta_2\rho_r}{\theta_2\rho_r + p_{2n} - c_{2n}}\right)\left(AE + \sum_{t=\kappa+2}^{T} \frac{A\breve{d}_{1t}^B}{m_2}\right),\end{aligned}$$

where $A = q_2 - \frac{q_2 \breve{D}_{2(\kappa+1)}^B}{m_2}$, $B = p_2 + \frac{q_2 \breve{D}_{2(\kappa+1)}^B}{m_2}$, *and* $E = 1 + \frac{\breve{D}_{1(\kappa+2)}^B}{m_2}$.

(b) *Suppose that* $p_{1n} - c_{1n} < p_{2n} - c_{2n}$*. For optimality of the forward-looking policy, it is sufficient that* $\exists \kappa \in \{\tau, ..., T-1\}$ *s.t.* $\breve{e}^l_t < \theta_2 \breve{d}^{NB}_{2t}$ *for* $\tau \le t \le \kappa$, $\breve{e}^l_t > \theta_2 \breve{d}^{NB}_{2t}$ *and* $\breve{e}^u_t > \gamma \breve{d}^{NB}_{1t}$ *for* $t > \kappa$, $\frac{\theta_1}{\theta_2} \ge \frac{\breve{s}_{2t}}{\sum_{i=1}^{t-1} \beta^l_i \breve{d}^{NB}_{1(t-i)}}$ *for* $t \ge \kappa + 2$, $\alpha > \frac{(1+A)(1-2B)}{1-B}$, *and*

$$\begin{aligned}\alpha \quad > \quad & \left(\frac{p_{2n} - c_{2n}}{\theta_2 \rho_r + p_{2n} - c_{2n}}\right)(1 + AE) - \left(\frac{p_{1n} - c_{1n}}{\theta_2 \rho_r + p_{2n} - c_{2n}}\right) \frac{(1-\gamma) A \breve{d}^B_{1(\kappa+1)}}{m_2} \\ & - \left(\frac{p_{1r} - c_{1r}}{\theta_2 \rho_r + p_{2n} - c_{2n}}\right) \frac{\gamma A \breve{d}^B_{1(\kappa+1)}}{m_2} - \left(\frac{p_{1n} - c_{1n} - p_{2n} + c_{2n}}{\theta_2 \rho_r + p_{2n} - c_{2n}}\right) \sum_{t=\kappa+2}^{T} \frac{A \breve{d}^B_{1t}}{m_2} \\ & + \left(\frac{\theta_2 \rho_r}{\theta_2 \rho_r + p_{2n} - c_{2n}}\right)\left(AE + \sum_{t=\kappa+2}^{T} \frac{A \breve{d}^B_{1t}}{m_2}\right),\end{aligned}$$

where $A = q_2 - \frac{q_2 \breve{D}^B_{2(\kappa+1)}}{m_2}$, $B = p_2 + \frac{q_2 \breve{D}^B_{2(\kappa+1)}}{m_2}$, *and* $E = 1 + \frac{\breve{D}^B_{1(\kappa+2)}}{m_2}$.

(c) *Suppose that the conditions in part (a) or (b) hold and* T *is sufficiently large. Then the forward-looking policy, if initiated after period* κ*, leads to no improvement in the total amount of recycled material used in manufacturing of the second-generation product.*

Proof of Theorem A.1. Suppose that the conditions stated in Theorem A.1 hold. We will show that rejecting a demand of size $\epsilon > 0$ from the customers unique to the second-generation product in period $\kappa \ge \tau$ while meeting all the remaining demand for both generations in period κ and all demand for both generations in each period $t \ne \kappa$ (sales plan i) is more profitable than meeting all demand for both generations in each period (sales plan ii). We use the hat ($\widehat{\ }$) and the breve ($\breve{\ }$) to denote the variables of sales plans (i) and (ii), respectively. Let $\widehat{D}^{NB}_{1t} = \sum_{i=1}^{t-1} \widehat{d}^{NB}_{1i}$ for $t > 1$ and $\widehat{D}^{NB}_{11} = 0$, $\breve{D}^{NB}_{1t} = \sum_{i=1}^{t-1} \breve{d}^{NB}_{1i}$ for $t > 1$ and $\breve{D}^{NB}_{11} = 0$, $\widehat{D}^{NB}_{2t} = \sum_{i=\tau}^{t-1} \widehat{d}^{NB}_{2i}$ for $t > \tau$ and $\widehat{D}^{NB}_{2\tau} = 0$, and $\breve{D}^{NB}_{2t} = \sum_{i=\tau}^{t-1} \breve{d}^{NB}_{2i}$ for $t > \tau$ and $\breve{D}^{NB}_{2\tau} = 0$. We make the following observations:

(1) Periods $t < \kappa$: We note that $\widehat{d}^{NB}_{1t} = \breve{d}^{NB}_{1t}$, $\widehat{r}_t = \breve{r}_t$, $\widehat{s}_{2t} = \widehat{d}^{NB}_{2t} = \breve{d}^{NB}_{2t} = \breve{s}_{2t}$, and $\widehat{e}^l_t = \breve{e}^l_t$, $\forall t < \kappa$.

(2) Period κ: We note that $\widehat{d}^{NB}_{1\kappa} = \breve{d}^{NB}_{1\kappa}$, $\widehat{r}_\kappa = \breve{r}_\kappa$, and $\widehat{e}^l_\kappa = \breve{e}^l_\kappa$. Since $\breve{e}^l_\kappa < \theta_2 \breve{d}^{NB}_{2\kappa}$, $\exists \epsilon > 0$ s.t. a demand of size ϵ from the customers unique to the second-generation product cannot be met in period κ by using the recycled content. Rejecting a demand of size ϵ from these customers in period κ

reduces the new item sales in period κ by ϵ. Hence $\widehat{s}_{2\kappa} = \widehat{d}_{2\kappa}^{NB} - \epsilon = \breve{d}_{2\kappa}^{NB} - \epsilon = \breve{s}_{2\kappa} - \epsilon$.

(3) Period $\kappa+1$: We know from the proof of Theorem A.1 in Güray [16] that $\widehat{d}_{1(\kappa+1)}^{NB} = \breve{d}_{1(\kappa+1)}^{NB} + \frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}, \widehat{r}_{\kappa+1} = \breve{r}_{\kappa+1} + \frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}$, and $\widehat{s}_{2(\kappa+1)} = \widehat{d}_{2(\kappa+1)}^{NB} + \alpha\epsilon = \breve{d}_{2(\kappa+1)}^{NB} - \epsilon AE + \alpha\epsilon = \breve{s}_{2(\kappa+1)} - \epsilon AE + \alpha\epsilon$. Since $\breve{e}_{\kappa+1}^{l} = \theta_1\sum_{i=1}^{\kappa}\beta_i^l\breve{d}_{1(\kappa+1-i)}^{NB} > \theta_2\breve{d}_{2(\kappa+1)}^{NB}, \exists\epsilon > 0$ s.t. $\widehat{e}_{\kappa+1}^{l} = \theta_1\sum_{i=1}^{\kappa}\beta_i^l\widehat{d}_{1(\kappa+1-i)}^{NB} = \theta_1\sum_{i=1}^{\kappa}\beta_i^l\breve{d}_{1(\kappa+1-i)}^{NB} > \theta_2\breve{d}_{2(\kappa+1)}^{NB} - \theta_2\epsilon AE + \theta_2\alpha\epsilon = \theta_2\widehat{d}_{2(\kappa+1)}^{NB} + \theta_2\alpha\epsilon$.

(4) Periods $t \geq \kappa+2$: We know from the proof of Theorem A.1 in Güray [16] that $\widehat{d}_{1t}^{NB} \geq \breve{d}_{1t}^{NB}, \widehat{r}_t \geq \breve{r}_t, \widehat{d}_{2t}^{B} \geq \breve{d}_{2t}^{B}$, and $\widehat{s}_{2t} = \widehat{d}_{2t}^{NB} \geq \breve{d}_{2t}^{NB} - \left(\widehat{d}_{1t}^{NB} - \breve{d}_{1t}^{NB}\right) = \breve{s}_{2t} - \left(\widehat{d}_{1t}^{NB} - \breve{d}_{1t}^{NB}\right) \geq \breve{s}_{2t} - \frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}, \forall t \geq \kappa+2$. We note that $\exists t_r \geq \kappa$ s.t. the total cumulative demand for both generations in sales plan (i) is less than or equal to that in sales plan (ii) at the end of each period $t \leq t_r$ and the reverse is true at the end of each period $t \geq t_r + 1$. This is because $\widehat{d}_{1t}^{NB} + \widehat{d}_{2t}^{NB} = \breve{d}_{1t}^{NB} + \left(\widehat{d}_{1t}^{NB} - \breve{d}_{1t}^{NB}\right) + \widehat{d}_{2t}^{NB} \geq \breve{d}_{1t}^{NB} + \left(\widehat{d}_{1t}^{NB} - \breve{d}_{1t}^{NB}\right) + \breve{d}_{2t}^{NB} - \left(\widehat{d}_{1t}^{NB} - \breve{d}_{1t}^{NB}\right) = \breve{d}_{1t}^{NB} + \breve{d}_{2t}^{NB}, \forall t \geq \kappa+2$, and the total cumulative demand for both generations in sales plan (i) exceeds that in sales plan (ii) in the long run. Suppose that $\kappa+2 \leq t_r$:

$$\begin{aligned}
\breve{D}_{1(\kappa+3)}^{NB} + \breve{D}_{2(\kappa+3)}^{NB} &= \sum_{i=1}^{\kappa}\breve{d}_{1i}^{NB} + \breve{d}_{1(\kappa+1)}^{NB} + \breve{d}_{1(\kappa+2)}^{NB} + \sum_{i=1}^{\kappa}\breve{d}_{2i}^{NB} + \breve{d}_{2(\kappa+1)}^{NB} + \breve{d}_{2(\kappa+2)}^{NB} \\
&= \breve{D}_{1(\kappa+1)}^{NB} + \breve{d}_{1(\kappa+1)}^{NB} + \breve{d}_{1(\kappa+2)}^{NB} + \breve{D}_{2(\kappa+1)}^{NB} + \widehat{d}_{2(\kappa+1)}^{NB} + AE\epsilon + \breve{d}_{2(\kappa+2)}^{NB} \\
&\geq \widehat{D}_{1(\kappa+1)}^{NB} + \widehat{d}_{1(\kappa+1)}^{NB} + \widehat{d}_{1(\kappa+2)}^{NB} + \widehat{D}_{2(\kappa+1)}^{NB} + \widehat{d}_{2(\kappa+1)}^{NB} + \widehat{d}_{2(\kappa+2)}^{NB} \\
&= \sum_{i=1}^{\kappa}\widehat{d}_{1i}^{NB} + \widehat{d}_{1(\kappa+1)}^{NB} + \widehat{d}_{1(\kappa+2)}^{NB} + \sum_{i=1}^{\kappa}\widehat{d}_{2i}^{NB} + \widehat{d}_{2(\kappa+1)}^{NB} + \widehat{d}_{2(\kappa+2)}^{NB} \\
&= \widehat{D}_{1(\kappa+3)}^{NB} + \widehat{D}_{2(\kappa+3)}^{NB}.
\end{aligned}$$

Hence $\breve{d}_{2(\kappa+2)}^{NB} + \epsilon AE - \left(\widehat{d}_{1(\kappa+1)}^{NB} - \breve{d}_{1(\kappa+1)}^{NB}\right) - \left(\widehat{d}_{1(\kappa+2)}^{NB} - \breve{d}_{1(\kappa+2)}^{NB}\right) \geq \widehat{d}_{2(\kappa+2)}^{NB}$. This implies that $\breve{d}_{2(\kappa+2)}^{NB} + \epsilon AE \geq \widehat{d}_{2(\kappa+2)}^{NB}$. Proceeding similarly, it can be shown that $\breve{d}_{2t}^{NB} + ... + \breve{d}_{2(\kappa+2)}^{NB} + \epsilon AE \geq \widehat{d}_{2t}^{NB} + ... + \widehat{d}_{2(\kappa+2)}^{NB}, \forall t \in \{\kappa+2, \kappa+3, ..., t_r\}$. Since $\breve{e}_{\kappa+2}^{l} > \theta_2\breve{d}_{2(\kappa+2)}^{B} + \frac{\theta_2\breve{D}_{1(\kappa+2)}^{B}\breve{d}_{2(\kappa+2)}^{B}}{m_2} + \frac{\theta_2\breve{d}_{1(\kappa+2)}^{B}\breve{D}_{2(\kappa+3)}^{B}}{m_2} = \theta_2\breve{d}_{2(\kappa+2)}^{NB}, \exists\epsilon > 0$ s.t.

$$
\begin{aligned}
\widehat{e}^{l}_{\kappa+2} &= \widehat{e}^{l}_{\kappa+1} - \theta_2 \widehat{d}^{NB}_{2(\kappa+1)} + \theta_1 \sum_{i=1}^{\kappa+1} \beta^l_i \widehat{d}^{NB}_{1(\kappa+2-i)} \\
&= \widehat{e}^{l}_{\kappa+1} - \theta_2 \widehat{d}^{NB}_{2(\kappa+1)} + \theta_1 \beta^l_1 \widehat{d}^{NB}_{1(\kappa+1)} + \theta_1 \sum_{i=2}^{\kappa+1} \beta^l_i \breve{d}^{NB}_{1(\kappa+2-i)} \\
&= \breve{e}^{l}_{\kappa+1} - \theta_2 \widehat{d}^{NB}_{2(\kappa+1)} + \theta_1 \beta^l_1 \widehat{d}^{NB}_{1(\kappa+1)} + \theta_1 \sum_{i=2}^{\kappa+1} \beta^l_i \breve{d}^{NB}_{1(\kappa+2-i)} \\
&= \breve{e}^{l}_{\kappa+2} + \theta_2 \breve{d}^{NB}_{2(\kappa+1)} - \theta_2 \widehat{d}^{NB}_{2(\kappa+1)} - \theta_1 \beta^l_1 \breve{d}^{NB}_{1(\kappa+1)} + \theta_1 \beta^l_1 \widehat{d}^{NB}_{1(\kappa+1)} \\
&= \breve{e}^{l}_{\kappa+2} + \theta_2 \left(\breve{d}^{NB}_{2(\kappa+1)} - \widehat{d}^{NB}_{2(\kappa+1)} \right) + \theta_1 \beta^l_1 \left(\widehat{d}^{NB}_{1(\kappa+1)} - \breve{d}^{NB}_{1(\kappa+1)} \right) \\
&\geq \breve{e}^{l}_{\kappa+2} + \theta_2 \left(\breve{d}^{NB}_{2(\kappa+1)} - \widehat{d}^{NB}_{2(\kappa+1)} \right) \\
&= \breve{e}^{l}_{\kappa+2} + \theta_2 AE\epsilon \\
&> \theta_2 \breve{d}^{NB}_{2(\kappa+2)} + \theta_2 AE\epsilon \\
&\geq \theta_2 \widehat{d}^{NB}_{2(\kappa+2)}.
\end{aligned}
$$

Proceeding similarly, it can be shown that $\widehat{e}^l_t > \theta_2 \widehat{d}^{NB}_{2t}$, $\forall t \in \{\kappa+2, \kappa+3, ..., t_r\}$.

In order to prove part (a) of Theorem A.1, suppose that $p_{1n} - c_{1n} > p_{2n} - c_{2n}$. We will show that sales plan (i) is more profitable than sales plan (ii). As we assume $\frac{\theta_1}{\theta_2} \geq \frac{\breve{s}_{2t}}{\sum_{i=1}^{t-1} \beta^l_i \breve{d}^{NB}_{1(t-i)}}$, $\forall t \geq \kappa+2$, we obtain $\widehat{e}^l_t \geq \theta_1 \sum_{i=1}^{t-1} \beta^l_i \widehat{d}^{NB}_{1(t-i)} \geq \theta_1 \sum_{i=1}^{t-1} \beta^l_i \breve{d}^{NB}_{1(t-i)} \geq \theta_2 \breve{s}_{2t}$, $\forall t \geq \kappa+2$. Combining all of the above observations:

$$
\begin{aligned}
&(p_{1n} - c_{1n}) \sum_{t=1}^{T} \left(\widehat{d}^{NB}_{1t} - \widehat{r}_t \right) + (p_{1r} - c_{1r}) \sum_{t=1}^{T} \widehat{r}_t + (p_{2n} - c_{2n}) \sum_{t=\tau}^{T} \widehat{s}_{2t} + \rho_r \sum_{t=\tau}^{T} \min\left\{ \widehat{e}^l_t, \theta_2 \widehat{s}_{2t} \right\} \\
&= (p_{1n} - c_{1n}) \left[\sum_{t=1}^{\kappa-1} \left(\widehat{d}^{NB}_{1t} - \widehat{r}_t \right) + \left(\widehat{d}^{NB}_{1\kappa} - \widehat{r}_{1\kappa} \right) + \left(\widehat{d}^{NB}_{1(\kappa+1)} - \widehat{r}_{\kappa+1} \right) + \sum_{t=\kappa+2}^{T} \left(\widehat{d}^{NB}_{1t} - \widehat{r}_t \right) \right] \\
&\quad + (p_{1r} - c_{1r}) \left[\sum_{t=1}^{\kappa-1} \widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T} \widehat{r}_t \right] + (p_{2n} - c_{2n}) \left[\sum_{t=\tau}^{\kappa-1} \widehat{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T} \widehat{s}_{2t} \right] \\
&\quad + \rho_r \sum_{t=\tau}^{t_r} \min\left\{ \widehat{e}^l_t, \theta_2 \widehat{s}_{2t} \right\} + \rho_r \sum_{t=t_r+1}^{T} \min\left\{ \widehat{e}^l_t, \theta_2 \widehat{s}_{2t} \right\} \\
&= (p_{1n} - c_{1n}) \left[\sum_{t=1}^{\kappa-1} \left(\widehat{d}^{NB}_{1t} - \widehat{r}_t \right) + \left(\widehat{d}^{NB}_{1\kappa} - \widehat{r}_{\kappa} \right) + \left(\widehat{d}^{NB}_{1(\kappa+1)} - \widehat{r}_{\kappa+1} \right) + \sum_{t=\kappa+2}^{T} \left(\widehat{d}^{NB}_{1t} - \widehat{r}_t \right) \right] \\
&\quad + (p_{1r} - c_{1r}) \left[\sum_{t=1}^{\kappa-1} \widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T} \widehat{r}_t \right] + (p_{2n} - c_{2n}) \left[\sum_{t=\tau}^{\kappa-1} \widehat{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T} \widehat{s}_{2t} \right]
\end{aligned}
$$

$$
\begin{aligned}
&+\rho_r \Bigg[\sum_{t=\tau}^{\kappa} \widehat{e}^{l}_{t} + \theta_2 \widehat{s}_{2(\kappa+1)} + \theta_2 \sum_{t=\kappa+2}^{t_r} \widehat{s}_{2t} + \sum_{t=t_r+1}^{T} \min\left\{\widehat{e}^{l}_{t}, \theta_2 \widehat{s}_{2t}\right\}\Bigg] \\
&= (p_{1n}-c_{1n}) \Bigg[\sum_{t=1}^{\kappa-1}\left(\widehat{d}^{NB}_{1t}-\widehat{r}_t\right) + \left(\widehat{d}^{NB}_{1\kappa}-\widehat{r}_\kappa\right) + \left(\widehat{d}^{NB}_{1(\kappa+1)}-\widehat{r}_{\kappa+1}\right) + \sum_{t=\kappa+2}^{T}\left(\widehat{d}^{NB}_{1t}-\widehat{r}_t\right)\Bigg] \\
&\quad +(p_{1r}-c_{1r}) \Bigg[\sum_{t=1}^{\kappa-1}\widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T}\widehat{r}_t\Bigg] + (p_{2n}-c_{2n}) \Bigg[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\Bigg] \\
&\quad +\rho_r \Bigg[\sum_{t=\tau}^{\kappa} \widehat{e}^{l}_{t} + \theta_2 \widehat{s}_{2(\kappa+1)} + \theta_2 \sum_{t=\kappa+2}^{t_r} \widehat{s}_{2t} + \sum_{\substack{t:\widehat{e}^{l}_{2t}<\theta_2\widehat{s}_{2t} \\ T\geq t\geq t_r+1}} \widehat{e}^{l}_{2t} + \theta_2 \sum_{\substack{t:\widehat{e}^{l}_{2t}\geq\theta_2\widehat{s}_{2t} \\ T\geq t\geq t_r+1}} \widehat{s}_{2t}\Bigg] \\
&\geq (p_{1n}-c_{1n}) \Bigg[\sum_{t=1}^{\kappa-1}\left(\widehat{d}^{NB}_{1t}-\widehat{r}_t\right) + \left(\widehat{d}^{NB}_{1\kappa}-\widehat{r}_\kappa\right) + \left(\widehat{d}^{NB}_{1(\kappa+1)}-\widehat{r}_{\kappa+1}\right) + \sum_{t=\kappa+2}^{T}\left(\widehat{d}^{NB}_{1t}-\widehat{r}_t\right)\Bigg] \\
&\quad +(p_{1r}-c_{1r}) \Bigg[\sum_{t=1}^{\kappa-1}\widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T}\widehat{r}_t\Bigg] + (p_{2n}-c_{2n}) \Bigg[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\Bigg] \\
&\quad +\rho_r \Bigg[\sum_{t=\tau}^{\kappa} \widehat{e}^{l}_{t} + \theta_2 \widehat{s}_{2(\kappa+1)} + \theta_2 \sum_{t=\kappa+2}^{t_r} \left(\breve{s}_{2t} - \left(\widehat{d}^{NB}_{1t} - \breve{d}^{NB}_{1t}\right)\right)\Bigg] \\
&\quad +\theta_2\rho_r \Bigg[\sum_{\substack{t:\widehat{e}^{l}_{2t}<\theta_2\widehat{s}_{2t} \\ T\geq t\geq t_r+1}} \breve{s}_{2t} + \sum_{\substack{t:\widehat{e}^{l}_{2t}\geq\theta_2\widehat{s}_{2t} \\ T\geq t\geq t_r+1}} \left(\breve{s}_{2t} - \left(\widehat{d}^{NB}_{1t} - \breve{d}^{NB}_{1t}\right)\right)\Bigg] \\
&= (p_{1n}-c_{1n}) \Bigg[\sum_{t=1}^{\kappa-1}\left(\widehat{d}^{NB}_{1t}-\widehat{r}_t\right) + \left(\widehat{d}^{NB}_{1\kappa}-\widehat{r}_\kappa\right) + \left(\widehat{d}^{NB}_{1(\kappa+1)}-\widehat{r}_{\kappa+1}\right) + \sum_{t=\kappa+2}^{T}\left(\widehat{d}^{NB}_{1t}-\widehat{r}_t\right)\Bigg] \\
&\quad +(p_{1r}-c_{1r}) \Bigg[\sum_{t=1}^{\kappa-1}\widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T}\widehat{r}_t\Bigg] + (p_{2n}-c_{2n}) \Bigg[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\Bigg] \\
&\quad +\rho_r \Bigg[\sum_{t=\tau}^{\kappa} \widehat{e}^{l}_{t} + \theta_2 \widehat{s}_{2(\kappa+1)}\Bigg] + \rho_r \Bigg[\theta_2 \sum_{t=\kappa+2}^{T} \breve{s}_{2t} - \theta_2 \sum_{t=\kappa+2}^{t_r} \left(\widehat{d}^{NB}_{1t} - \breve{d}^{NB}_{1t}\right) - \theta_2 \sum_{\substack{t:\widehat{e}^{l}_{2t}\geq\theta_2\widehat{s}_{2t} \\ T\geq t\geq t_r+1}} \left(\widehat{d}^{NB}_{1t} - \breve{d}^{NB}_{1t}\right)\Bigg] \\
&= (p_{1n}-c_{1n}) \Bigg[\sum_{t=1}^{\kappa-1}\left(\breve{d}^{NB}_{1t}-\widehat{r}_t\right) + \left(\breve{d}^{NB}_{1\kappa}-\widehat{r}_\kappa\right) + \breve{d}^{NB}_{1(\kappa+1)} + \frac{A\epsilon\breve{d}^{B}_{1(\kappa+1)}}{m_2} - \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T}\breve{d}^{B}_{1t} \\
&\quad - \sum_{t=\kappa+2}^{T}\frac{\breve{d}^{B}_{1t}\widehat{D}^{B}_{2(t+1)}}{m_2} - \sum_{t=\kappa+2}^{T}\widehat{r}_t\Bigg] + (p_{1r}-c_{1r}) \Bigg[\sum_{t=1}^{\kappa-1}\widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T}\widehat{r}_t\Bigg] \\
&\quad +(p_{2n}-c_{2n}) \Bigg[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T}\widehat{d}^{B}_{2t}\left(1+\frac{\breve{D}^{B}_{1t}}{m_2}\right) + \sum_{t=\kappa+2}^{T}\frac{\breve{d}^{B}_{1t}\widehat{D}^{B}_{2(t+1)}}{m_2}\Bigg] \\
&\quad +\rho_r \Bigg[\sum_{t=\tau}^{\kappa} \widehat{e}^{l}_{t} + \theta_2 \widehat{s}_{2(\kappa+1)} + \theta_2 \sum_{t=\kappa+2}^{T} \breve{s}_{2t}\Bigg] - \rho_r \Bigg[\theta_2 \sum_{t=\kappa+2}^{t_r} \left(\widehat{d}^{NB}_{1t} - \breve{d}^{NB}_{1t}\right) + \theta_2 \sum_{\substack{t:\widehat{e}^{l}_{2t}\geq\theta_2\widehat{s}_{2t} \\ T\geq t\geq t_r+1}} \left(\widehat{d}^{NB}_{1t} - \breve{d}^{NB}_{1t}\right)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&= (p_{1n} - c_{1n}) \left[\sum_{t=1}^{\kappa-1} \left(\breve{d}_{1t}^{NB} - \widehat{r}_t \right) + \left(\breve{d}_{1\kappa}^{NB} - \widehat{r}_\kappa \right) + \breve{d}_{1(\kappa+1)}^{NB} + \frac{A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} - \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T} \breve{d}_{1t}^{B} - \sum_{t=\kappa+2}^{T} \widehat{r}_t \right] \\
&\quad - \left[(p_{1n} - c_{1n}) - (p_{2n} - c_{2n}) \right] \sum_{t=\kappa+2}^{T} \frac{\breve{d}_{1t}^{B} \widehat{D}_{2(t+1)}^{B}}{m_2} + (p_{1r} - c_{1r}) \left[\sum_{t=1}^{\kappa-1} \widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T} \widehat{r}_t \right] \\
&\quad + (p_{2n} - c_{2n}) \left[\sum_{t=\tau}^{\kappa-1} \breve{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T} \widehat{d}_{2t}^{B} \left(1 + \frac{\breve{D}_{1t}^{B}}{m_2} \right) \right] \\
&\quad + \rho_r \left[\sum_{t=\tau}^{\kappa} \widehat{e}_t^{l} + \theta_2 \widehat{s}_{2(\kappa+1)} + \theta_2 \sum_{t=\kappa+2}^{T} \breve{s}_{2t} \right] - \rho_r \left[\theta_2 \sum_{t=\kappa+2}^{t_r} \left(\widehat{d}_{1t}^{NB} - \breve{d}_{1t}^{NB} \right) + \theta_2 \sum_{\substack{t: \widehat{e}_{2t}^{l} \geq \theta_2 \widehat{s}_{2t} \\ T \geq t \geq t_r + 1}} \left(\widehat{d}_{1t}^{NB} - \breve{d}_{1t}^{NB} \right) \right] \\
&\geq (p_{1n} - c_{1n}) \left[\sum_{t=1}^{\kappa-1} \left(\breve{d}_{1t}^{NB} - \widehat{r}_t \right) + \left(\breve{d}_{1\kappa}^{NB} - \widehat{r}_\kappa \right) + \breve{d}_{1(\kappa+1)}^{NB} + \frac{A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} - \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T} \breve{d}_{1t}^{B} - \sum_{t=\kappa+2}^{T} \widehat{r}_t \right] \\
&\quad - \left[(p_{1n} - c_{1n}) - (p_{2n} - c_{2n}) \right] \sum_{t=\kappa+2}^{T} \frac{\breve{d}_{1t}^{B} \breve{D}_{2(t+1)}^{B}}{m_2} + (p_{1r} - c_{1r}) \left[\sum_{t=1}^{\kappa-1} \widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T} \widehat{r}_t \right] \\
&\quad + (p_{2n} - c_{2n}) \left[\sum_{t=\tau}^{\kappa-1} \breve{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T} \breve{d}_{2t}^{B} \left(1 + \frac{\breve{D}_{1t}^{B}}{m_2} \right) \right] \\
&\quad + \rho_r \left[\sum_{t=\tau}^{\kappa} \breve{e}_t^{l} + \theta_2 \widehat{s}_{2(\kappa+1)} + \theta_2 \sum_{t=\kappa+2}^{T} \left(\breve{s}_{2t} - \left(\widehat{d}_{1t}^{NB} - \breve{d}_{1t}^{NB} \right) \right) \right] \\
&\geq (p_{1n} - c_{1n}) \left[\sum_{t=1}^{\kappa-1} \left(\breve{d}_{1t}^{NB} - \widehat{r}_t \right) + \left(\breve{d}_{1\kappa}^{NB} - \widehat{r}_\kappa \right) + \breve{d}_{1(\kappa+1)}^{NB} + \frac{A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} - \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T} \breve{d}_{1t}^{B} - \sum_{t=\kappa+2}^{T} \widehat{r}_t \right] \\
&\quad - \left[(p_{1n} - c_{1n}) - (p_{2n} - c_{2n}) \right] \sum_{t=\kappa+2}^{T} \frac{\breve{d}_{1t}^{B} \breve{D}_{2(t+1)}^{B}}{m_2} + (p_{1r} - c_{1r}) \left[\sum_{t=1}^{\kappa-1} \widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T} \widehat{r}_t \right] \\
&\quad + (p_{2n} - c_{2n}) \left[\sum_{t=\tau}^{\kappa-1} \breve{s}_{2t} + \breve{s}_{2\kappa} - \epsilon + \breve{s}_{2(\kappa+1)} - \epsilon AE + \alpha\epsilon + \sum_{t=\kappa+2}^{T} \breve{d}_{2t}^{B} \left(1 + \frac{\breve{D}_{1t}^{B}}{m_2} \right) \right] \\
&\quad + \rho_r \left[\sum_{t=\tau}^{\kappa} \breve{e}_t^{l} + \theta_2 \breve{s}_{2(\kappa+1)} - \theta_2 \epsilon AE + \theta_2 \alpha\epsilon + \theta_2 \sum_{t=\kappa+2}^{T} \breve{s}_{2t} - \theta_2 \sum_{t=\kappa+2}^{T} \frac{A\epsilon \breve{d}_{1t}^{B}}{m_2} \right] \\
&\geq (p_{1n} - c_{1n}) \Bigg[\sum_{t=1}^{\kappa-1} \left(\breve{d}_{1t}^{NB} - \breve{r}_t \right) + \left(\breve{d}_{1\kappa}^{NB} - \breve{r}_\kappa \right) + \breve{d}_{1(\kappa+1)}^{NB} + \frac{A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} - \breve{r}_{(\kappa+1)} - \frac{\gamma A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} \\
&\quad + \sum_{t=\kappa+2}^{T} \breve{d}_{1t}^{B} - \sum_{t=\kappa+2}^{T} \breve{r}_t \Bigg] - \left[(p_{1n} - c_{1n}) - (p_{2n} - c_{2n}) \right] \sum_{t=\kappa+2}^{T} \frac{\breve{d}_{1t}^{B} \breve{D}_{2(t+1)}^{B}}{m_2} \\
&\quad + (p_{1r} - c_{1r}) \left[\sum_{t=1}^{\kappa-1} \breve{r}_t + \breve{r}_\kappa + \left(\breve{r}_{(\kappa+1)} + \frac{\gamma A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} \right) + \sum_{t=\kappa+2}^{T} \breve{r}_t \right] \\
&\quad + (p_{2n} - c_{2n}) \left[\sum_{t=\tau}^{\kappa-1} \breve{s}_{2t} + \breve{s}_{2\kappa} - \epsilon + \breve{s}_{2(\kappa+1)} - \epsilon AE + \alpha\epsilon + \sum_{t=\kappa+2}^{T} \breve{d}_{2t}^{B} \left(1 + \frac{\breve{D}_{1t}^{B}}{m_2} \right) \right]
\end{aligned}
$$

$$+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^l+\theta_2\breve{s}_{2(\kappa+1)}-\theta_2\epsilon AE+\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\breve{s}_{2t}-\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^B}{m_2}\right]$$

$$=(p_{1n}-c_{1n})\left[\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+\left(\frac{A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}\right)\right]+(p_{1r}-c_{1r})\left[\sum_{t=1}^{T}\breve{r}_t+\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}\right]$$

$$+(p_{2n}-c_{2n})\left(\sum_{t=\tau}^{T}\breve{s}_{2t}-\epsilon AE+\alpha\epsilon-\epsilon\right)$$

$$+\rho_r\left[\left(\sum_{t=\tau}^{\kappa}\breve{e}_t^l+\sum_{t=\kappa+1}^{T}\theta_2\breve{s}_{2t}\right)-\left(\theta_2\epsilon AE-\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^B}{m_2}\right)\right]$$

$$=(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\breve{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\breve{s}_{2t}+\rho_r\sum_{t=\tau}^{T}\min\left\{\breve{e}_t^l,\theta_2\breve{s}_{2t}\right\}$$

$$+(p_{1n}-c_{1n})\left(\frac{A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}\right)+(p_{1r}-c_{1r})\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}$$

$$-(p_{2n}-c_{2n})\left(\epsilon AE-\alpha\epsilon+\epsilon\right)-\theta_2\rho_r\left(\epsilon AE-\alpha\epsilon+\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^B}{m_2}\right).$$

In order to show that

$$(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\widehat{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\widehat{s}_{2t}+\rho_r\sum_{t=\tau}^{T}\min\left\{\widehat{e}_t^l,\theta_2\widehat{s}_{2t}\right\}\geq$$

$$(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\breve{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\breve{s}_{2t}+\rho_r\sum_{t=\tau}^{T}\min\left\{\breve{e}_t^l,\theta_2\breve{s}_{2t}\right\},$$

it suffices to show that

$$(p_{1n}-c_{1n})\left(\frac{A\breve{d}_{1(\kappa+1)}^B}{m_2}-\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}\right)-(p_{2n}-c_{2n})\left(AE-\alpha+1\right)+(p_{1r}-c_{1r})\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}$$

$$-\theta_2\rho_r\left(AE-\alpha+\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}\right)>0.$$

The above inequality holds as we assume

$$\begin{aligned}\alpha \quad > \quad & \left(\frac{p_{2n}-c_{2n}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)(1+AE)-\left(\frac{p_{1n}-c_{1n}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\frac{(1-\gamma)\,A\breve{d}_{1(\kappa+1)}^B}{m_2}\\ & -\left(\frac{p_{1r}-c_{1r}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}+\left(\frac{\theta_2\rho_r}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\left(AE+\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}\right).\end{aligned}$$

In order to prove part (b) of Theorem A.1, suppose that $p_{1n}-c_{1n}<p_{2n}-c_{2n}$. We will show that sales plan (i) is more profitable than sales plan (ii). With similar arguments to those used in the proof of part (a):

$$
\begin{aligned}
&(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\widehat{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\widehat{s}_{2t}+\rho_r\sum_{t=\tau}^{T}\min\left\{\widehat{e}_t^l,\theta_2\widehat{s}_{2t}\right\}\\
&=(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+(\widehat{d}_{1\kappa}^{NB}-\widehat{r}_\kappa)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)\right]\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^l+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{t_r}\widehat{s}_{2t}+\sum_{\substack{t:\widehat{e}_{2t}^l<\theta_2\widehat{s}_{2t}\\T\geq t\geq t_r+1}}\widehat{e}_{2t}^l+\theta_2\sum_{\substack{t:\widehat{e}_{2t}^l\geq\theta_2\widehat{s}_{2t}\\T\geq t\geq t_r+1}}\widehat{s}_{2t}\right]\\
&\geq(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\widehat{d}_{1\kappa}^{NB}-\widehat{r}_\kappa\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)\right]\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^l+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{t_r}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\quad+\theta_2\rho_r\left[\sum_{\substack{t:\widehat{e}_{2t}^l<\theta_2\widehat{s}_{2t}\\T\geq t\geq t_r+1}}\breve{s}_{2t}+\sum_{\substack{t:\widehat{e}_{2t}^l\geq\theta_2\widehat{s}_{2t}\\T\geq t\geq t_r+1}}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\geq(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\widehat{d}_{1\kappa}^{NB}-\widehat{r}_\kappa\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)\right]\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^l+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\geq(p_{1n}-c_{1n})\Bigg[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_\kappa\right)+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\widehat{r}_{\kappa+1}\\
&\quad+\sum_{t=\kappa+2}^{T}\left(\breve{d}_{1t}^{NB}+\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)-\widehat{r}_t\right)\Bigg]+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^l+\theta_2\breve{s}_{2(\kappa+1)}-\theta_2\epsilon AE+\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\breve{s}_{2t}-\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\right]
\end{aligned}
$$

$$
\begin{aligned}
&\geq (p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_\kappa\right)+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)\right]\\
&\quad+\left[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})\right]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\breve{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^{l}+\theta_2\breve{s}_{2(\kappa+1)}-\theta_2\epsilon AE+\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\breve{s}_{2t}-\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\right]\\
&\geq (p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\breve{r}_\kappa\right)+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\breve{r}_{(\kappa+1)}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right.\\
&\quad\left.+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^{NB}-\sum_{t=\kappa+2}^{T}\breve{r}_t\right]+\left[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})\right]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\breve{r}_t+\breve{r}_\kappa+\breve{r}_{(\kappa+1)}+\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}+\sum_{t=\kappa+2}^{T}\breve{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\breve{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^{l}+\theta_2\breve{s}_{2(\kappa+1)}-\theta_2\epsilon AE+\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\breve{s}_{2t}-\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\right]\\
&= (p_{1n}-c_{1n})\left[\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+\left(\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right)\right]\\
&\quad+\left[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})\right]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}+(p_{1r}-c_{1r})\left[\sum_{t=1}^{T}\breve{r}_t+\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right]\\
&\quad+(p_{2n}-c_{2n})\left(\sum_{t=\tau}^{T}\breve{s}_{2t}-\epsilon AE+\alpha\epsilon-\epsilon\right)\\
&\quad+\rho_r\left[\left(\sum_{t=\tau}^{\kappa}\breve{e}_t^{l}+\sum_{t=\kappa+1}^{T}\theta_2\breve{s}_{2t}\right)-\left(\theta_2\epsilon AE-\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\right)\right]\\
&= (p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\breve{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\breve{s}_{2t}+\rho_r\sum_{t=\tau}^{T}\min\left\{\breve{e}_t^{l},\theta_2\breve{s}_{2t}\right\}\\
&\quad+\left[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})\right]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\\
&\quad+(p_{1n}-c_{1n})\left(\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right)+(p_{1r}-c_{1r})\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\\
&\quad-(p_{2n}-c_{2n})(\epsilon AE-\alpha\epsilon+\epsilon)-\theta_2\rho_r\left(\epsilon AE-\alpha\epsilon+\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\right).
\end{aligned}
$$

In order to show that

$$(p_{1n}-c_{1n})\sum_{t=1}^{T}(\widehat{d}_{1t}^{NB}-\widehat{r}_t)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\widehat{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\widehat{s}_{2t}+\rho_r\sum_{t=\tau}^{T}\min\left\{\widehat{e}_t^l,\theta_2\widehat{s}_{2t}\right\}\geq$$

$$(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\breve{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\breve{s}_{2t}+\rho_r\sum_{t=\tau}^{T}\min\left\{\breve{e}_t^l,\theta_2\breve{s}_{2t}\right\},$$

it suffices to show that

$$\begin{aligned}&((p_{1n}-c_{1n})-(p_{2n}-c_{2n}))\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}+(p_{1n}-c_{1n})\left(\frac{A\breve{d}_{1(\kappa+1)}^B}{m_2}-\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}\right)\\&-(p_{2n}-c_{2n})\left(AE-\alpha+1\right)+(p_{1r}-c_{1r})\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}-\theta_2\rho_r\left(AE-\alpha+\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}\right)>0.\end{aligned}$$

The above inequality holds as we assume

$$\begin{aligned}\alpha\quad>\quad&\left(\frac{p_{2n}-c_{2n}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)(1+AE)-\left(\frac{p_{1n}-c_{1n}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\frac{(1-\gamma)\,A\breve{d}_{1(\kappa+1)}^B}{m_2}\\&-\left(\frac{p_{1r}-c_{1r}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}-\left(\frac{p_{1n}-c_{1n}-p_{2n}+c_{2n}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}\\&+\left(\frac{\theta_2\rho_r}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\left(AE+\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}\right).\end{aligned}$$

(c) Suppose that the conditions in part (a) or (b) hold and T is sufficiently large. Thus the forward-looking policy is optimal. Also, suppose that the forward-looking policy is initiated in period $\kappa'>\kappa$ at optimality. We use the tilde ($\widetilde{\ }$) to denote the variables of this sales plan. Note that $\widetilde{D}_{2(T+1)}^{NB}\cong\breve{D}_{2(T+1)}^{NB}\cong m_1+m_2$, and $\widetilde{D}_{2t}^{NB}=\breve{D}_{2t}^{NB}$, $\forall t\leq\kappa'+1$. Hence $\sum_{t=\kappa'+1}^{T}\widetilde{d}_{2t}^{NB}=\widetilde{D}_{2(T+1)}^{NB}-\widetilde{D}_{2(\kappa'+1)}^{NB}\cong\breve{D}_{2(T+1)}^{NB}-\breve{D}_{2(\kappa'+1)}^{NB}=\sum_{t=\kappa'+1}^{T}\breve{d}_{2t}^{NB}$. The total amount of recycled material used in manufacturing equals $\sum_{t=1}^{\kappa}\breve{e}_t^l+\sum_{t=\kappa+1}^{T}\theta_2\breve{d}_{2t}^{NB}$ under the myopic policy, while it cannot exceed $\sum_{t=1}^{\kappa}\widetilde{e}_t^l+\sum_{t=\kappa+1}^{T}\theta_2\widetilde{d}_{2t}^{NB}=\sum_{t=1}^{\kappa}\breve{e}_t^l+\sum_{t=\kappa+1}^{\kappa'}\theta_2\breve{d}_{2t}^{NB}+\sum_{t=\kappa'+1}^{T}\theta_2\widetilde{d}_{2t}^{NB}$ under the forward-looking policy. Since $\sum_{t=\kappa'+1}^{T}\theta_2\widetilde{d}_{2t}^{NB}=\sum_{t=\kappa'+1}^{T}\theta_2\breve{d}_{2t}^{NB}$, the total amount of recycled material used in manufacturing cannot be improved with the forward-looking policy. □

Theorem A.2. *Suppose that the consumers' timing of end-of-life product returns coincides with their timing of second-generation product adoption. Also suppose that* $\lambda\delta^l\theta_1>\theta_2$.

(a) *Suppose that* $p_{1n}-c_{1n}>p_{2n}-c_{2n}$. *For optimality of the forward-looking policy, it is sufficient that* $\exists\kappa\in\{\tau,...,T-1\}$ *s.t.* $\breve{e}_t^l<\theta_2\left(\breve{d}_{2t}^B+\frac{\breve{d}_{1t}^B\breve{D}_{2(t+1)}^B}{m_2}\right)$ *for* $\tau\leq t\leq\kappa$,

$\breve{e}^l_t > \theta_2 \left(\breve{d}^B_{2t} + \frac{\breve{d}^B_{1t}\breve{D}^B_{2(t+1)}}{m_2} \right)$ *and* $\breve{e}^u_t > \gamma \breve{d}^{NB}_{1t}$ *for* $t > \kappa$, $\alpha > \frac{(1+A)(1-2B)}{1-B}$, *and*

$$\begin{aligned} \alpha \quad > \quad & \left(\frac{p_{2n} - c_{2n}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) (1 + AE) - \left(\frac{p_{1n} - c_{1n}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \frac{(1-\gamma) A \breve{d}^B_{1(\kappa+1)}}{m_2} \\ & - \left(\frac{p_{1r} - c_{1r}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \frac{\gamma A \breve{d}^B_{1(\kappa+1)}}{m_2} + \left(\frac{\theta_2 \rho_r}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \left(AE + \sum_{t=\kappa+2}^{T} \frac{A \breve{d}^B_{1t}}{m_2} \right), \end{aligned}$$

where $A = q_2 - \frac{q_2 \breve{D}^B_{2(\kappa+1)}}{m_2}$, $B = p_2 + \frac{q_2 \breve{D}^B_{2(\kappa+1)}}{m_2}$, *and* $E = 1 + \frac{\breve{D}^B_{1(\kappa+2)}}{m_2}$.

(b) *Suppose that* $p_{1n} - c_{1n} < p_{2n} - c_{2n}$. *For optimality of the forward-looking policy, it is sufficient that* $\exists \kappa \in \{\tau, ..., T-1\}$ *s.t.* $\breve{e}^l_t < \theta_2 \left(\breve{d}^B_{2t} + \frac{\breve{d}^B_{1t}\breve{D}^B_{2(t+1)}}{m_2} \right)$ *for* $\tau \le t \le \kappa$, $\breve{e}^l_t > \theta_2 \left(\breve{d}^B_{2t} + \frac{\breve{d}^B_{1t}\breve{D}^B_{2(t+1)}}{m_2} \right)$ *and* $\breve{e}^u_t > \gamma \breve{d}^{NB}_{1t}$ *for* $t > \kappa$, $\alpha > \frac{(1+A)(1-2B)}{1-B}$, *and*

$$\begin{aligned} \alpha \quad > \quad & \left(\frac{p_{2n} - c_{2n}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) (1 + AE) - \left(\frac{p_{1n} - c_{1n}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \frac{(1-\gamma) A \breve{d}^B_{1(\kappa+1)}}{m_2} \\ & - \left(\frac{p_{1r} - c_{1r}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \frac{\gamma A \breve{d}^B_{1(\kappa+1)}}{m_2} - \left(\frac{p_{1n} - c_{1n} - p_{2n} + c_{2n}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \sum_{t=\kappa+2}^{T} \frac{A \breve{d}^B_{1t}}{m_2} \\ & + \left(\frac{\theta_2 \rho_r}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \left(AE + \sum_{t=\kappa+2}^{T} \frac{A \breve{d}^B_{1t}}{m_2} \right), \end{aligned}$$

where $A = q_2 - \frac{q_2 \breve{D}^B_{2(\kappa+1)}}{m_2}$, $B = p_2 + \frac{q_2 \breve{D}^B_{2(\kappa+1)}}{m_2}$, *and* $E = 1 + \frac{\breve{D}^B_{1(\kappa+2)}}{m_2}$.

(c) *Suppose that the conditions in part (a) or (b) hold and T is sufficiently large. Then the forward-looking policy, if initiated after period* κ, *reduces the total amount of recycled material used in manufacturing of the new-generation product for the customers who have not purchased the early-generation product.*

Proof of Theorem A.2. We consider sales plans (i) and (ii) described in the proof of Theorem A.1. We again show that sales plan (i) is more profitable than sales plan (ii). We make the following observations:

(1) Periods $t < \kappa$: We note that $\widehat{d}^{NB}_{1t} = \breve{d}^{NB}_{1t}$, $\widehat{r}_t = \breve{r}_t$, $\widehat{s}_{2t} = \widehat{d}^{NB}_{2t} = \breve{d}^{NB}_{2t} = \breve{s}_{2t}$, and $\widehat{e}^l_t = \breve{e}^l_t$, $\forall t < \kappa$.

(2) Period κ: We note that $\widehat{d}^{NB}_{1\kappa} = \breve{d}^{NB}_{1\kappa}$, $\widehat{r}_\kappa = \breve{r}_\kappa$, and $\widehat{e}^l_\kappa = \breve{e}^l_\kappa$. Since $\breve{e}^l_\kappa < \theta_2 \left(\breve{d}^B_{2\kappa} + \frac{\breve{d}^B_{1\kappa}\breve{D}^B_{2(\kappa+1)}}{m_2} \right)$, $\exists \epsilon > 0$ s.t. a demand of size ϵ from the customers unique to the second-generation product cannot be met in period κ by using the recycled

content. Rejecting a demand of size ϵ from these customers in period κ reduces the new item sales in period κ by ϵ. Hence $\widehat{s}_{2\kappa} = \breve{s}_{2\kappa} - \epsilon$.

(3) Period $\kappa + 1$: We know from the proof of Theorem A.1 that $\widehat{d}^{NB}_{1(\kappa+1)} = \breve{d}^{NB}_{1(\kappa+1)} + \frac{A\epsilon\breve{d}^{B}_{1(\kappa+1)}}{m_2}$ and $\widehat{s}_{2(\kappa+1)} = \widehat{d}^{NB}_{2(\kappa+1)} + \alpha\epsilon = \breve{d}^{NB}_{2(\kappa+1)} - \epsilon AE + \alpha\epsilon = \breve{s}_{2(\kappa+1)} - \epsilon AE + \alpha\epsilon$. Since $\breve{e}^{u}_{\kappa+1} = \breve{e}^{u}_{\kappa} - \breve{r}_{\kappa} + \frac{\lambda\delta^u\breve{D}^B_{1\kappa}\breve{d}^B_{2\kappa}}{m_2} > \gamma\breve{d}^{NB}_{1(\kappa+1)}$, $\exists\epsilon$ s.t. $\widehat{e}^{u}_{\kappa+1} = \widehat{e}^{u}_{\kappa} - \widehat{r}_{\kappa} + \frac{\lambda\delta^u\widehat{D}^B_{1\kappa}\widehat{d}^B_{2\kappa}}{m_2} = \breve{e}^{u}_{\kappa} - \breve{r}_{\kappa} + \frac{\lambda\delta^u\breve{D}^B_{1\kappa}\breve{d}^B_{2\kappa}}{m_2} > \gamma\breve{d}^{NB}_{1(\kappa+1)} + \frac{\gamma A\epsilon\breve{d}^{B}_{1(\kappa+1)}}{m_2}$. Thus $\widehat{r}_{\kappa+1} = \gamma\widehat{d}^{NB}_{1(\kappa+1)} = \gamma\breve{d}^{NB}_{1(\kappa+1)} + \frac{\gamma A\epsilon\breve{d}^{B}_{1(\kappa+1)}}{m_2} = \breve{r}_{\kappa+1} + \frac{\gamma A\epsilon\breve{d}^{B}_{1(\kappa+1)}}{m_2}$. Since $\breve{e}^{l}_{\kappa+1} > \theta_2\left(\breve{d}^B_{2(\kappa+1)} + \frac{\breve{d}^B_{1(\kappa+1)}\breve{D}^B_{2(\kappa+2)}}{m_2}\right)$, $\exists\epsilon > 0$ s.t. $\widehat{e}^{l}_{\kappa+1} = \frac{(\lambda\delta^l\theta_1-\theta_2)\widehat{D}^B_{1\kappa}\widehat{d}^B_{2\kappa}}{m_2} = \frac{(\lambda\delta^l\theta_1-\theta_2)\breve{D}^B_{1\kappa}\breve{d}^B_{2\kappa}}{m_2} = \breve{e}^{l}_{\kappa+1} > \theta_2\left(\breve{d}^B_{2(\kappa+1)} + \frac{\breve{d}^B_{1(\kappa+1)}\breve{D}^B_{2(\kappa+2)}}{m_2}\right) - \theta_2\epsilon A + \theta_2\alpha\epsilon \geq \theta_2\left(\widehat{d}^B_{2(\kappa+1)} + \frac{\widehat{d}^B_{1(\kappa+1)}\widehat{D}^B_{2(\kappa+2)}}{m_2}\right) + \theta_2\alpha\epsilon$.

(4) Periods $t \geq \kappa + 2$: We know from the proof of Theorem A.1 that $\widehat{d}^{NB}_{1t} \geq \breve{d}^{NB}_{1t}$, $\widehat{d}^{B}_{2t} \geq \breve{d}^{B}_{2t}$, and $\widehat{s}_{2t} \geq \breve{s}_{2t} - \left(\widehat{d}^{NB}_{1t} - \breve{d}^{NB}_{1t}\right) \geq \breve{s}_{2t} - \frac{A\epsilon\breve{d}^B_{1t}}{m_2}$, $\forall t \geq \kappa + 2$. Since $\breve{e}^{u}_{\kappa+2} = \breve{e}^{u}_{\kappa+1} - \breve{r}_{\kappa+1} + \frac{\lambda\delta^u\breve{D}^B_{1(\kappa+1)}\breve{d}^B_{2(\kappa+1)}}{m_2} > \gamma\breve{d}^{NB}_{1(\kappa+2)}$, $\exists\epsilon > 0$ s.t.

$$\begin{aligned}
\widehat{e}^{u}_{\kappa+2} &= \widehat{e}^{u}_{\kappa+1} - \widehat{r}_{\kappa+1} + \frac{\lambda\delta^u\widehat{D}^B_{1(\kappa+1)}\widehat{d}^B_{2(\kappa+1)}}{m_2} \\
&= \breve{e}^{u}_{\kappa+1} - \breve{r}_{\kappa+1} - \frac{\gamma A\epsilon\breve{d}^B_{1(\kappa+1)}}{m_2} + \frac{\lambda\delta^u\breve{D}^B_{1(\kappa+1)}\widehat{d}^B_{2(\kappa+1)}}{m_2} \\
&= \breve{e}^{u}_{\kappa+1} - \breve{r}_{\kappa+1} - \frac{\gamma A\epsilon\breve{d}^B_{1(\kappa+1)}}{m_2} + \frac{\lambda\delta^u\breve{D}^B_{1(\kappa+1)}\breve{d}^B_{2(\kappa+1)}}{m_2} - \frac{\lambda\delta^u A\epsilon\breve{D}^B_{1(\kappa+1)}}{m_2} \\
&= \breve{e}^{u}_{\kappa+2} - \frac{\gamma A\epsilon\breve{d}^B_{1(\kappa+1)}}{m_2} - \frac{\lambda\delta^u A\epsilon\breve{D}^B_{1(\kappa+1)}}{m_2} > \gamma\breve{d}^{NB}_{1(\kappa+2)} + \frac{\gamma A\epsilon\breve{d}^B_{1(\kappa+2)}}{m_2} \geq \gamma\widehat{d}^{NB}_{1(\kappa+2)}.
\end{aligned}$$

Thus $\widehat{r}_{\kappa+2} = \gamma\widehat{d}^{NB}_{1(\kappa+2)} > \breve{r}_{\kappa+2} = \gamma\breve{d}^{NB}_{1(\kappa+2)}$. We know from the proof of Theorem A.1 in Güray [16] that $\breve{d}^B_{2t} + A\epsilon \geq \widehat{d}^{NB}_{2t}$, $\forall t \geq \kappa + 2$. Since $\breve{e}^{l}_{\kappa+2} > \theta_2\left(\breve{d}^B_{2(\kappa+2)} + \frac{\breve{d}^B_{1(\kappa+2)}\breve{D}^B_{2(\kappa+3)}}{m_2}\right)$, $\exists\epsilon > 0$ s.t.

$$\begin{aligned}
\widehat{e}^{l}_{\kappa+2} &= \widehat{e}^{l}_{\kappa+1} - \theta_2\left(\widehat{d}^B_{2(\kappa+1)} + \frac{\widehat{d}^B_{1(\kappa+1)}\widehat{D}^B_{2(\kappa+2)}}{m_2}\right) + \frac{\left(\lambda\delta^l\theta_1 - \theta_2\right)\widehat{D}^B_{1(\kappa+1)}\widehat{d}^B_{2(\kappa+1)}}{m_2} \\
&= \breve{e}^{l}_{\kappa+1} - \theta_2\breve{d}^B_{2(\kappa+1)} + \theta_2\epsilon A - \frac{\theta_2\breve{d}^B_{1(\kappa+1)}\breve{D}^B_{2(\kappa+2)}}{m_2} + \frac{\theta_2\epsilon A\breve{d}^B_{1(\kappa+1)}}{m_2} \\
&\quad + \frac{\left(\lambda\delta^l\theta_1 - \theta_2\right)\breve{D}^B_{1(\kappa+1)}\breve{d}^B_{2(\kappa+1)}}{m_2} - \frac{\left(\lambda\delta^l\theta_1 - \theta_2\right)\epsilon A\breve{D}^B_{1(\kappa+1)}}{m_2}
\end{aligned}$$

$$
\begin{aligned}
&= \breve{e}^l_{\kappa+2} + \theta_2\epsilon A + \frac{\theta_2\epsilon A\breve{d}^B_{1(\kappa+1)}}{m_2} - \frac{\left(\lambda\delta^l\theta_1 - \theta_2\right)\epsilon A\breve{D}^B_{1(\kappa+1)}}{m_2} \\
&> \theta_2\left(\breve{d}^B_{2(\kappa+2)} + \frac{\breve{d}^B_{1(\kappa+2)}\breve{D}^B_{2(\kappa+3)}}{m_2}\right) + \theta_2\epsilon A \\
&= \theta_2\left(\breve{d}^B_{2(\kappa+2)} + \epsilon A + \frac{\breve{d}^B_{1(\kappa+2)}\left(\widehat{D}^B_{2(\kappa+2)} + \epsilon A + \breve{d}^B_{2(\kappa+2)}\right)}{m_2}\right) \\
&\geq \theta_2\left(\breve{d}^B_{2(\kappa+2)} + \epsilon A + \frac{\breve{d}^B_{1(\kappa+2)}\left(\widehat{D}^B_{2(\kappa+2)} + \widehat{d}^B_{2(\kappa+2)}\right)}{m_2}\right) \\
&= \theta_2\left(\breve{d}^B_{2(\kappa+2)} + \epsilon A + \frac{\breve{d}^B_{1(\kappa+2)}\widehat{D}^B_{2(\kappa+3)}}{m_2}\right) \\
&\geq \theta_2\left(\widehat{d}^B_{2(\kappa+2)} + \frac{\widehat{d}^B_{1(\kappa+2)}\widehat{D}^B_{2(\kappa+3)}}{m_2}\right).
\end{aligned}
$$

Proceeding similarly, it can be shown that $\widehat{r}_t \geq \breve{r}_t$ and $\widehat{e}^l_t > \theta_2\left(\widehat{d}^B_{2t} + \frac{\widehat{d}^B_{1t}\widehat{D}^B_{2(t+1)}}{m_2}\right)$, $\forall t \geq \kappa + 2$.

In oder to prove part (a) of Theorem A.2, suppose that $p_{1n} - c_{1n} > p_{2n} - c_{2n}$. We will show that sales plan (i) is more profitable than sales plan (ii). Combining all of the above observations:

$$
\begin{aligned}
&(p_{1n} - c_{1n})\sum_{t=1}^{T}\left(\widehat{d}^{NB}_{1t} - \widehat{r}_t\right) + (p_{1r} - c_{1r})\sum_{t=1}^{T}\widehat{r}_t + (p_{2n} - c_{2n})\sum_{t=\tau}^{T}\widehat{s}_{2t} \\
&+\rho_r\sum_{t=\tau}^{T}\min\left\{\widehat{e}^l_t, \theta_2\left(\widehat{s}_{2t} - \frac{\widehat{D}^B_{1t}\widehat{d}^B_{2t}}{m_2}\right)\right\} + \theta_2\rho_r\sum_{t=\tau}^{T}\frac{\widehat{D}^B_{1t}\widehat{d}^B_{2t}}{m_2} \\
&= (p_{1n} - c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}^{NB}_{1t} - \widehat{r}_t\right) + \left(\widehat{d}^{NB}_{1\kappa} - \widehat{r}_{1\kappa}\right) + \left(\widehat{d}^{NB}_{1(\kappa+1)} - \widehat{r}_{\kappa+1}\right) + \sum_{t=\kappa+2}^{T}\left(\widehat{d}^{NB}_{1t} - \widehat{r}_t\right)\right] \\
&\quad+(p_{1r} - c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t + \widehat{r}_\kappa + \widehat{r}_{\kappa+1} + \sum_{t=\kappa+2}^{T}\widehat{r}_t\right] + (p_{2n} - c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t} + \widehat{s}_{2\kappa} + \widehat{s}_{2(\kappa+1)} + \sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right] \\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}^l_t + \theta_2\widehat{s}_{2(\kappa+1)} - \frac{\theta_2\breve{D}^B_{1(\kappa+1)}\widehat{d}^B_{2(\kappa+1)}}{m_2}\right] + \rho_r\left[\theta_2\sum_{t=\kappa+2}^{T}\left(\widehat{s}_{2t} - \frac{\widehat{D}^B_{1t}\widehat{d}^B_{2t}}{m_2}\right) + \theta_2\sum_{t=\kappa+2}^{T}\frac{\breve{D}^B_{1t}\widehat{d}^B_{2t}}{m_2}\right] \\
&\quad+\rho_r\left[\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}^B_{1t}\widehat{d}^B_{2t}}{m_2} + \frac{\theta_2\breve{D}^B_{1(\kappa+1)}\widehat{d}^B_{2(\kappa+1)}}{m_2}\right] \\
&= (p_{1n} - c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}^{NB}_{1t} - \widehat{r}_t\right) + \left(\widehat{d}^{NB}_{1\kappa} - \widehat{r}_{1\kappa}\right) + \left(\widehat{d}^{NB}_{1(\kappa+1)} - \widehat{r}_{\kappa+1}\right) + \sum_{t=\kappa+2}^{T}\left(\widehat{d}^{NB}_{1t} - \widehat{r}_t\right)\right]
\end{aligned}
$$

$$+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]$$

$$+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^l+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^B\widehat{d}_{2t}^B}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]$$

$$\geq(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\widehat{d}_{1\kappa}^{NB}-\widehat{r}_{1\kappa}\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)\right]$$

$$+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]$$

$$+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^l+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^B\widehat{d}_{2t}^B}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]$$

$$=(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_{1\kappa}\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)\right]$$

$$+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]$$

$$+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{d}_{2t}^B\left(1+\frac{\breve{D}_{1t}^B}{m_2}\right)+\sum_{t=\kappa+2}^{T}\frac{\breve{d}_{1t}^B\widehat{D}_{2(t+1)}^B}{m_2}\right]$$

$$+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^l+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^B\widehat{d}_{2t}^B}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]$$

$$=(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_{1\kappa}\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^B\right.$$

$$\left.-\sum_{t=\kappa+2}^{T}\frac{\breve{d}_{1t}^B\widehat{D}_{2(t+1)}^B}{m_2}-\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]$$

$$+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{d}_{2t}^B\left(1+\frac{\breve{D}_{1t}^B}{m_2}\right)+\sum_{t=\kappa+2}^{T}\frac{\breve{d}_{1t}^B\widehat{D}_{2(t+1)}^B}{m_2}\right]$$

$$+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^l+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^B\widehat{d}_{2t}^B}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]$$

$$=(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_{1\kappa}\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^B-\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]$$

$$-\left[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})\right]\sum_{t=\kappa+2}^{T}\frac{\breve{d}_{1t}^B\widehat{D}_{2(t+1)}^B}{m_2}+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]$$

$$+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{d}_{2t}^B\left(1+\frac{\breve{D}_{1t}^B}{m_2}\right)\right]$$

$$+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^l+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^B\widehat{d}_{2t}^B}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]$$

$$
\begin{aligned}
&\geq (p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_{1\kappa}\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^{B}-\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad-[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})]\sum_{t=\kappa+2}^{T}\frac{\breve{d}_{1t}^{B}\breve{D}_{2(t+1)}^{B}}{m_2}+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\breve{d}_{2t}^{B}\left(1+\frac{\breve{D}_{1t}^{B}}{m_2}\right)\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\breve{d}_{2t}^{B}}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&=(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\breve{d}_{1\kappa}^{NB}-\widehat{r}_\kappa+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^{B}-\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad-[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})]\sum_{t=\kappa+2}^{T}\frac{\breve{d}_{1t}^{B}\breve{D}_{2(t+1)}^{B}}{m_2}+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\breve{d}_{2t}^{B}\left(1+\frac{\breve{D}_{1t}^{B}}{m_2}\right)\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\breve{d}_{2t}^{B}}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\geq (p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_\kappa\right)+\left(\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^{B}-\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad-[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})]\sum_{t=\kappa+2}^{T}\frac{\breve{d}_{1t}^{B}\breve{D}_{2(t+1)}^{B}}{m_2}+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\breve{d}_{2t}^{B}\left(1+\frac{\breve{D}_{1t}^{B}}{m_2}\right)\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\breve{d}_{2t}^{B}}{m_2}+\theta_2\breve{s}_{2(\kappa+1)}-\theta_2\epsilon AE+\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\breve{s}_{2t}-\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\right]\\
&\geq (p_{1n}-c_{1n})\Bigg[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\breve{r}_\kappa\right)+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\breve{r}_{(\kappa+1)}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\\
&\quad+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^{B}-\sum_{t=\kappa+2}^{T}\breve{r}_t\Bigg]-[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})]\sum_{t=\kappa+2}^{T}\frac{\breve{d}_{1t}^{B}\breve{D}_{2(t+1)}^{B}}{m_2}\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\breve{r}_t+\breve{r}_\kappa+\left(\breve{r}_{(\kappa+1)}+\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right)+\sum_{t=\kappa+2}^{T}\breve{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\breve{d}_{2t}^{B}\left(1+\frac{\breve{D}_{1t}^{B}}{m_2}\right)\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\breve{d}_{2t}^{B}}{m_2}+\theta_2\breve{s}_{2(\kappa+1)}-\theta_2\epsilon AE+\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\breve{s}_{2t}-\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\right]
\end{aligned}
$$

$$= (p_{1n} - c_{1n}) \left[\sum_{t=1}^{T} \left(\breve{d}_{1t}^{NB} - \breve{r}_t \right) + \left(\frac{A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} - \frac{\gamma A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} \right) \right] + (p_{1r} - c_{1r}) \left[\sum_{t=1}^{T} \breve{r}_t + \frac{\gamma A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} \right]$$

$$+ (p_{2n} - c_{2n}) \left(\sum_{t=\tau}^{T} \breve{s}_{2t} - \epsilon AE + \alpha\epsilon - \epsilon \right)$$

$$+ \rho_r \left[\left(\sum_{t=\tau}^{\kappa} \breve{e}_t^{l} + \theta_2 \sum_{t=\tau}^{\kappa} \frac{\breve{D}_{1t}^{B} \breve{d}_{2t}^{B}}{m_2} + \theta_2 \sum_{t=\kappa+1}^{T} \breve{s}_{2t} \right) - \left(\theta_2 \epsilon AE - \theta_2 \alpha\epsilon + \theta_2 \sum_{t=\kappa+2}^{T} \frac{A\epsilon \breve{d}_{1t}^{B}}{m_2} \right) \right]$$

$$= (p_{1n} - c_{1n}) \sum_{t=1}^{T} \left(\breve{d}_{1t}^{NB} - \breve{r}_t \right) + (p_{1r} - c_{1r}) \sum_{t=1}^{T} \breve{r}_t + (p_{2n} - c_{2n}) \sum_{t=\tau}^{T} \breve{s}_{2t}$$

$$+ \rho_r \sum_{t=\tau}^{T} \min\left\{ \breve{e}_t^{l}, \theta_2 \left(\breve{s}_{2t} - \frac{\breve{D}_{1t}^{B} \breve{d}_{2t}^{B}}{m_2} \right) \right\} + \theta_2 \rho_r \sum_{t=\tau}^{T} \frac{\breve{D}_{1t}^{B} \breve{d}_{2t}^{B}}{m_2} + (p_{1n} - c_{1n}) \left(\frac{A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} - \frac{\gamma A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} \right)$$

$$+ (p_{1r} - c_{1r}) \left(\frac{\gamma A\epsilon \breve{d}_{1(\kappa+1)}^{B}}{m_2} \right) - (p_{2n} - c_{2n}) (\epsilon AE - \alpha\epsilon + \epsilon) - \theta_2 \rho_r \left(\epsilon AE - \alpha\epsilon + \sum_{t=\kappa+2}^{T} \frac{A\epsilon \breve{d}_{1t}^{B}}{m_2} \right).$$

In order to show that

$$(p_{1n} - c_{1n}) \sum_{t=1}^{T} \left(\widehat{d}_{1t}^{NB} - \widehat{r}_t \right) + (p_{1r} - c_{1r}) \sum_{t=1}^{T} \widehat{r}_t + (p_{2n} - c_{2n}) \sum_{t=\tau}^{T} \widehat{s}_{2t}$$

$$+ \rho_r \sum_{t=\tau}^{T} \min\left\{ \widehat{e}_t^{l}, \theta_2 \left(\widehat{s}_{2t} - \frac{\widehat{D}_{1t}^{B} \breve{d}_{2t}^{B}}{m_2} \right) \right\} + \theta_2 \rho_r \sum_{t=\tau}^{T} \frac{\widehat{D}_{1t}^{B} \widehat{d}_{2t}^{B}}{m_2} \geq$$

$$(p_{1n} - c_{1n}) \sum_{t=1}^{T} \left(\breve{d}_{1t}^{NB} - \breve{r}_t \right) + (p_{1r} - c_{1r}) \sum_{t=1}^{T} \breve{r}_t + (p_{2n} - c_{2n}) \sum_{t=\tau}^{T} \breve{s}_{2t}$$

$$+ \rho_r \sum_{t=\tau}^{T} \min\left\{ \breve{e}_t^{l}, \theta_2 \left(\breve{s}_{2t} - \frac{\breve{D}_{1t}^{B} \breve{d}_{2t}^{B}}{m_2} \right) \right\} + \theta_2 \rho_r \sum_{t=\tau}^{T} \frac{\breve{D}_{1t}^{B} \breve{d}_{2t}^{B}}{m_2},$$

it suffices to show that

$$(p_{1n} - c_{1n}) \left(\frac{A \breve{d}_{1(\kappa+1)}^{B}}{m_2} - \frac{\gamma A \breve{d}_{1(\kappa+1)}^{B}}{m_2} \right) - (p_{2n} - c_{2n}) (AE - \alpha + 1) + (p_{1r} - c_{1r}) \frac{\gamma A \breve{d}_{1(\kappa+1)}^{B}}{m_2}$$

$$- \theta_2 \rho_r \left(AE - \alpha + \sum_{t=\kappa+2}^{T} \frac{A \breve{d}_{1t}^{B}}{m_2} \right) > 0.$$

The above inequality holds as we assume

$$\alpha \quad > \quad \left(\frac{p_{2n} - c_{2n}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) (1 + AE) - \left(\frac{p_{1n} - c_{1n}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \frac{(1 - \gamma) A \breve{d}_{1(\kappa+1)}^{B}}{m_2}$$

$$- \left(\frac{p_{1r} - c_{1r}}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \frac{\gamma A \breve{d}_{1(\kappa+1)}^{B}}{m_2} + \left(\frac{\theta_2 \rho_r}{\theta_2 \rho_r + p_{2n} - c_{2n}} \right) \left(AE + \sum_{t=\kappa+2}^{T} \frac{A \breve{d}_{1t}^{B}}{m_2} \right).$$

In order to prove part (b) of Theorem A.2, suppose that $p_{1n} - c_{1n} < p_{2n} - c_{2n}$. We will show that sales plan (i) is more profitable than sales plan (ii). With similar arguments to those used in the proof of part (a):

$$
\begin{aligned}
&(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\widehat{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\widehat{s}_{2t}\\
&+\rho_r\sum_{t=\tau}^{T}\min\left\{\widehat{e}_t^{l},\theta_2\left(\widehat{s}_{2t}-\frac{\widehat{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}\right)\right\}+\theta_2\rho_r\sum_{t=\tau}^{T}\frac{\widehat{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}\\
&=(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\widehat{d}_{1\kappa}^{NB}-\widehat{r}_\kappa\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)\right]\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^{l}+\theta_2\widehat{s}_{2(\kappa+1)}-\frac{\theta_2\breve{D}_{1(\kappa+1)}^{B}\widehat{d}_{2(\kappa+1)}^{B}}{m_2}+\theta_2\sum_{t=\kappa+2}^{T}\left(\widehat{s}_{2t}-\frac{\breve{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}\right)\right]\\
&\quad+\rho_r\left[\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}+\frac{\theta_2\breve{D}_{1(\kappa+1)}^{B}\widehat{d}_{2(\kappa+1)}^{B}}{m_2}+\theta_2\sum_{t=\kappa+2}^{T}\frac{\breve{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}\right]\\
&=(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\widehat{d}_{1\kappa}^{NB}-\widehat{r}_{1\kappa}\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)\right]\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]\\
&\geq(p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\widehat{d}_{1\kappa}^{NB}-\widehat{r}_{1\kappa}\right)+\left(\widehat{d}_{1(\kappa+1)}^{NB}-\widehat{r}_{\kappa+1}\right)+\sum_{t=\kappa+2}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)\right]\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\widehat{s}_{2t}+\widehat{s}_{2\kappa}+\widehat{s}_{2(\kappa+1)}+\sum_{t=\kappa+2}^{T}\widehat{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\geq(p_{1n}-c_{1n})\Bigg[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_\kappa\right)+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\widehat{r}_{\kappa+1}\\
&\quad+\sum_{t=\kappa+2}^{T}\left(\breve{d}_{1t}^{NB}+\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)-\widehat{r}_t\right)\Bigg]+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&\geq (p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\widehat{r}_\kappa\right)+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\left(\breve{d}_{1t}^{NB}-\widehat{r}_t\right)\right]\\
&\quad+[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\widehat{r}_t+\widehat{r}_\kappa+\widehat{r}_{\kappa+1}+\sum_{t=\kappa+2}^{T}\widehat{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\breve{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\geq (p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\breve{r}_\kappa\right)+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\breve{r}_{(\kappa+1)}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right.\\
&\quad\left.+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^{NB}-\sum_{t=\kappa+2}^{T}\breve{r}_t\right]+[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\breve{r}_t+\breve{r}_\kappa+\breve{r}_{(\kappa+1)}+\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}+\sum_{t=\kappa+2}^{T}\breve{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\breve{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\widehat{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\widehat{d}_{2t}^{B}}{m_2}+\theta_2\widehat{s}_{2(\kappa+1)}+\theta_2\sum_{t=\kappa+2}^{T}\left(\breve{s}_{2t}-\left(\widehat{d}_{1t}^{NB}-\breve{d}_{1t}^{NB}\right)\right)\right]\\
&\geq (p_{1n}-c_{1n})\left[\sum_{t=1}^{\kappa-1}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+\left(\breve{d}_{1\kappa}^{NB}-\breve{r}_\kappa\right)+\breve{d}_{1(\kappa+1)}^{NB}+\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\breve{r}_{(\kappa+1)}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right.\\
&\quad\left.+\sum_{t=\kappa+2}^{T}\breve{d}_{1t}^{NB}-\sum_{t=\kappa+2}^{T}\breve{r}_t\right]+[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\\
&\quad+(p_{1r}-c_{1r})\left[\sum_{t=1}^{\kappa-1}\breve{r}_t+\breve{r}_\kappa+\breve{r}_{(\kappa+1)}+\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}+\sum_{t=\kappa+2}^{T}\breve{r}_t\right]\\
&\quad+(p_{2n}-c_{2n})\left[\sum_{t=\tau}^{\kappa-1}\breve{s}_{2t}+\breve{s}_{2\kappa}-\epsilon+\breve{s}_{2(\kappa+1)}-\epsilon AE+\alpha\epsilon+\sum_{t=\kappa+2}^{T}\breve{s}_{2t}\right]\\
&\quad+\rho_r\left[\sum_{t=\tau}^{\kappa}\breve{e}_t^{l}+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^{B}\breve{d}_{2t}^{B}}{m_2}+\theta_2\breve{s}_{2(\kappa+1)}-\theta_2\epsilon AE+\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\breve{s}_{2t}-\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}\right]\\
&= (p_{1n}-c_{1n})\left[\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+\left(\frac{A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right)\right]\\
&\quad+[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^{B}}{m_2}+(p_{1r}-c_{1r})\left[\sum_{t=1}^{T}\breve{r}_t+\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^{B}}{m_2}\right]\\
&\quad+(p_{2n}-c_{2n})\left(\sum_{t=\tau}^{T}\breve{s}_{2t}-\epsilon AE+\alpha\epsilon-\epsilon\right)
\end{aligned}
$$

$$+\rho_r\left[\left(\sum_{t=\tau}^{\kappa}\breve{e}_t^l+\theta_2\sum_{t=\tau}^{\kappa}\frac{\breve{D}_{1t}^B\breve{d}_{2t}^B}{m_2}+\theta_2\sum_{t=\kappa+1}^{T}\breve{s}_{2t}\right)-\left(\theta_2\epsilon AE-\theta_2\alpha\epsilon+\theta_2\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^B}{m_2}\right)\right]$$

$$=(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\breve{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\breve{s}_{2t}$$

$$+\rho_r\sum_{t=\tau}^{T}\min\left\{\breve{e}_t^l,\theta_2\left(\breve{s}_{2t}-\frac{\breve{D}_{1t}^B\breve{d}_{2t}^B}{m_2}\right)\right\}+\theta_2\rho_r\sum_{t=\tau}^{T}\frac{\breve{D}_{1t}^B\breve{d}_{2t}^B}{m_2}$$

$$+\left[(p_{1n}-c_{1n})-(p_{2n}-c_{2n})\right]\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^B}{m_2}+(p_{1n}-c_{1n})\left(\frac{A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}-\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}\right)$$

$$+(p_{1r}-c_{1r})\frac{\gamma A\epsilon\breve{d}_{1(\kappa+1)}^B}{m_2}-(p_{2n}-c_{2n})\left(\epsilon AE-\alpha\epsilon+\epsilon\right)-\theta_2\rho_r\left(\epsilon AE-\alpha\epsilon+\sum_{t=\kappa+2}^{T}\frac{A\epsilon\breve{d}_{1t}^B}{m_2}\right).$$

In order to show that

$$(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\widehat{d}_{1t}^{NB}-\widehat{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\widehat{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\widehat{s}_{2t}$$

$$+\rho_r\sum_{t=\tau}^{T}\min\left\{\widehat{e}_t^l,\theta_2\left(\widehat{s}_{2t}-\frac{\widehat{D}_{1t}^B\widehat{d}_{2t}^B}{m_2}\right)\right\}+\theta_2\rho_r\sum_{t=\tau}^{T}\frac{\widehat{D}_{1t}^B\widehat{d}_{2t}^B}{m_2}\geq$$

$$(p_{1n}-c_{1n})\sum_{t=1}^{T}\left(\breve{d}_{1t}^{NB}-\breve{r}_t\right)+(p_{1r}-c_{1r})\sum_{t=1}^{T}\breve{r}_t+(p_{2n}-c_{2n})\sum_{t=\tau}^{T}\breve{s}_{2t}$$

$$+\rho_r\sum_{t=\tau}^{T}\min\left\{\breve{e}_t^l,\theta_2\left(\breve{s}_{2t}-\frac{\breve{D}_{1t}^B\breve{d}_{2t}^B}{m_2}\right)\right\}+\theta_2\rho_r\sum_{t=\tau}^{T}\frac{\breve{D}_{1t}^B\breve{d}_{2t}^B}{m_2},$$

it suffices to show that

$$((p_{1n}-c_{1n})-(p_{2n}-c_{2n}))\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}-\theta_2\rho_r\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}$$

$$+(p_{1n}-c_{1n})\left(\frac{A\breve{d}_{1(\kappa+1)}^B}{m_2}-\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}\right)+(p_{1r}-c_{1r})\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}$$

$$-(p_{2n}-c_{2n})\left(AE-\alpha+1\right)-\theta_2\rho_r\left(AE-\alpha\right)>0.$$

The above inequality holds as we assume

$$\alpha > \left(\frac{p_{2n}-c_{2n}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)(1+AE)-\left(\frac{p_{1n}-c_{1n}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\frac{(1-\gamma)A\breve{d}_{1(\kappa+1)}^B}{m_2}$$

$$-\left(\frac{p_{1r}-c_{1r}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\frac{\gamma A\breve{d}_{1(\kappa+1)}^B}{m_2}-\left(\frac{p_{1n}-c_{1n}-p_{2n}+c_{2n}}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}$$

$$+\left(\frac{\theta_2\rho_r}{\theta_2\rho_r+p_{2n}-c_{2n}}\right)\left(AE+\sum_{t=\kappa+2}^{T}\frac{A\breve{d}_{1t}^B}{m_2}\right).$$

(c) Suppose that the conditions in part (a) or (b) hold and T is sufficiently large. Thus the forward-looking policy is optimal. Also, suppose that the forward-looking policy is initiated in period $\kappa' > \kappa$ at optimality. We use the tilde ($\widetilde{\ }$) to denote the variables of this sales plan. Note that $\widetilde{D}^{NB}_{2(T+1)} \cong \breve{D}^{NB}_{2(T+1)} \cong m_1 + m_2$, and $\widetilde{D}^{NB}_{2t} = \breve{D}^{NB}_{2t}$, $\forall t \leq \kappa' + 1$. We know from Proposition 1 in Güray [16] that $\sum_{t=\kappa'+1}^{T} \widetilde{D}^{B}_{1t}\widetilde{d}^{B}_{2t} > \sum_{t=\kappa'+1}^{T} \breve{D}^{B}_{1t}\breve{d}^{B}_{2t}$. Hence $\sum_{t=\kappa'+1}^{T} \left(\widetilde{d}^{NB}_{2t} - \frac{\widetilde{D}^{B}_{1t}\widetilde{d}^{B}_{2t}}{m_2}\right) < \sum_{t=\kappa'+1}^{T} \left(\breve{d}^{NB}_{2t} - \frac{\breve{D}^{B}_{1t}\breve{d}^{B}_{2t}}{m_2}\right)$. For the customers who have not purchased the first-generation product, the total amount of recycled material used in manufacturing equals $\sum_{t=1}^{\kappa} \breve{e}^{l}_{t} + \sum_{t=\kappa+1}^{T} \theta_2 \left(\breve{d}^{NB}_{2t} - \frac{\breve{D}^{B}_{1t}\breve{d}^{B}_{2t}}{m_2}\right)$ under the myopic policy, while it cannot exceed $\sum_{t=1}^{\kappa} \widetilde{e}^{l}_{t} + \sum_{t=\kappa+1}^{T} \theta_2 \left(\widetilde{d}^{NB}_{2t} - \frac{\widetilde{D}^{B}_{1t}\widetilde{d}^{B}_{2t}}{m_2}\right) = \sum_{t=1}^{\kappa} \breve{e}^{l}_{t} + \sum_{t=\kappa+1}^{\kappa'} \theta_2 \left(\breve{d}^{NB}_{2t} - \frac{\breve{D}^{B}_{1t}\breve{d}^{B}_{2t}}{m_2}\right) + \sum_{t=\kappa'+1}^{T} \theta_2 \left(\widetilde{d}^{NB}_{2t} - \frac{\widetilde{D}^{B}_{1t}\widetilde{d}^{B}_{2t}}{m_2}\right)$ under the forward-looking policy. Since $\sum_{t=\kappa'+1}^{T} \theta_2 \left(\widetilde{d}^{NB}_{2t} - \frac{\widetilde{D}^{B}_{1t}\widetilde{d}^{B}_{2t}}{m_2}\right) < \sum_{t=\kappa'+1}^{T} \theta_2 \left(\breve{d}^{NB}_{2t} - \frac{\breve{D}^{B}_{1t}\breve{d}^{B}_{2t}}{m_2}\right)$, the total amount of recycled material used in manufacturing for the customers who have not purchased the first-generation product is lower under the forward-looking policy. $\square$