DESIGN OF PD CONTROLLERS FOR UNSTABLE INFINITE DIMENSIONAL PLANTS WITH TIME-DELAY

A THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF

MASTER OF SCIENCE

IN

ELECTRICAL AND ELECTRONICS ENGINEERING

By

Deniz Varol

June, 2015

DESIGN OF PD CONTROLLERS FOR UNSTABLE INFINITE DIMENSIONAL PLANTS WITH TIME-DELAY

By Deniz Varol

June, 2015

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Prof. Dr. Hitay Özbay (Advisor)

Prof. Dr. Ömer Morgül

Prof. Dr. Mehmet Önder Efe

Approved for the Graduate School of Engineering and Science:

Prof. Dr. Levent Onural
Director of the Graduate School

ABSTRACT

DESIGN OF PD CONTROLLERS FOR UNSTABLE INFINITE DIMENSIONAL PLANTS WITH TIME-DELAY

Deniz Varol
M.S. in Electrical and Electronics Engineering
Advisor: Prof. Dr. Hitay Özbay
June, 2015

In real life, everything is in movement so there are always continuous transfers, transmissions and transports of people, materials, information and energy to other places. These transfers, transmissions and transports occur with a time delay. In control theory, time delays are encountered very commonly as in real life. By considering this fact, four unstable plants with time delays are considered from different application perspectives and five delay values are chosen. For stabilization of these infinite dimensional plants, PD controllers are designed by three different design methods: two of them give different least fragile PD controllers and the other design method gives a gain margin optimizing PD controller. Designed PD controllers and equivalent plants are put in feedback loop and robust performance test results and step responses are obtained and compared.

Keywords: Time Delay, Least Fragile PD Controller, Gain Margin Optimizing PD Controller, Robust Performance Test.

ÖZET

KARARSIZ SONSUZ BOYUTLU VE ZAMAN GECİKMELİ MODELLER İÇİN PD KONTROLCÜ TASARIMI

Deniz Varol
Elektrik ve Elektronik Mühendisliği, Yüksek Lisans
Tez Danışmanı: Prof. Dr. Hitay Özbay
Haziran, 2015

Gerçek hayatta herşey hareket halindedir, bu sebeple insanlar, eşyalar, bilgiler ve enerjiler daimi olarak başka yerlere taşınır, nakledilir ve iletilir. Bu taşınımlar, iletimler ve nakiller sırasında bir miktar zaman kaybedilir ve bu kaybedilen zaman zaman gecikmesi olarak tanımlanır. Gerçek hayatta olduğu gibi kontrol teorisinde de zaman gecikmeleri sıkça görülmektedir. Bu sebepten dolayı, dört adet zaman gecikmeli kararsız model oluşturulmuş ve beş tane zaman gecikme değeri seçilmiştir. Bu sonsuz boyutlu modellerin kararlı hale getirilebilmesi için üç farklı tasarım yöntemi seçilmiştir: bu yöntemlerden ikisi en az kırılgan olan kontrolcüyü, diğeri ise kazanç payı optimize eden kontrolcüyü vermektedir. Tasarlanan kontrolcüler ve karşılık gelen modeller geri besleme döngüsüne yerleştirilmiş ve gürbüz performans testi ve step girdisine olan tepkileri elde edilmiş ve karşılaştırılmıştır.

Anahtar sözcükler: Zaman Gecikmesi, En Az Kırılgan PD Kontrolcü, Kazanç Payı Optimize Eden Kontrolcü.

Acknowledgement

Firstly, I would like to express my deep gratitude to my supervisor, Prof. Dr. Hitay Özbay, for his patient guidance, effective suggestions and endless support throughout the master education and I would like to thank him for giving the opportunity of having this valuable education under his supervision.

I would like to thank Prof. Dr. Ömer Morgül and Prof. Dr. Mehmet Mete Önder for being on my thesis committee.

I would also like to thank Bilkent University EE Department for their financial support.

In addition, I would like to express my thanks to my office mate Buket Koyuncu for sharing all the valuable times with me during the graduate studies and Mustafa Akın for his technical support.

Last but not least, I am grateful to my family Ayhan, Fehmiye and Barış and my beloved companion Mustafa Mert Birincioğlu for their incredible love, support and faith in me. Without them, I won't be able to achieve this much.

Contents

List of Figures

List of Tables

Chapter 1

INTRODUCTION

In real life, everything is in movement so there are always continuous transfers, transmissions and transports of people, materials, information and energy to other places. These transfers, transmissions and transports occur with a time delay. In control theory, time delays are encountered very commonly. In this thesis, several unstable plants with time delays are investigated and stabilized by various PD controllers.

1.1 Scope of the Thesis

In this thesis, there are three stages of studies:
(i) selecting four different plants whose structures are frequently encountered in control area,
(ii) designing PD controllers with three design methods,
(iii) robustness and performance comparison of the feedback system under these controllers.

For the first step, commonly used and modelled plant structures in practical life are searched. After that, four plant structures are generated among encountered plant structures according to some criteria which are explained in Chapter 2.2.

These plants are unstable plants with five different chosen time delay values, so in total, twenty different version of plants are obtained.

For the second step, design of least fragile PD controllers with two different methods and design of gain margin optimizing PD contollers are investigated. These methods are chosen because of their functionality as fragility and gain margin are vital properties for control systems. Moreover, PD controllers are used in many practical applications for ease of their implementation.

Finally, for the last step robustness is checked by using robust performance condition as plants and controllers are fixed after the PD controllers are designed. Also, for comparing the time tracking performances of the designed feedback systems, step responses are obtained and compared.

1.2 Related Work

Significant amount of control systems include unstable plants and stabilization of these systems with controllers have a vital role in all areas, especially in engineering systems. Because of this fact, many researchers developed various controller design methods.

Some of the most frequently used controller design methods are P, PD, PI and PID controllers [2, 3, 4], LQR (Linear Quadratic Regulator) control design [5], Smith predictor design [6, 7], IMC (Internal Model Control) [8, 9], Taylor or Padé Approximation [10], H_∞ optimal control design [11] and so forth. However, many of these methods have some restrictions and cannot be applied to all types of plants. For example, LQR control design method can be only applied to the systems without time delays as in [5]. Its extension to infinite dimensional systems is possible but not very practical [12]. Other examples are the Smith predictor design and IMC, which are used with only stable open loop plants with time delays. So, some modifications are made on Smith predictor [7] and IMC [8, 9] in order to use them with unstable plants with time delays.

Design of P, PD, PI and PID controllers, Taylor approximation, Padé approximation and H_2/H_∞ optimal control design can be used with unstable plants with time delays without restrictions. Many of the approximation based controllers require relatively small time delays. Typically state space based methods require complex computations that may lead to fragile controllers. PI and PID controllers are used frequently with unstable systems with small or large time delays. But PD controllers are used less frequently (mainly when the plant contains an integral action).

After considering all of the related works, in this thesis, design of PD controllers for the unstable plants with time delay is selected between them for stabilization because of the simplicity of the control structure. In addition, in the literature there is relatively less research on PD controllers (compared to P, PI and PID controllers) on plants with large time delays.

1.3 On PD Controllers

Let us consider a controller with delayed feedback from the output as shown in the Figure 1.1

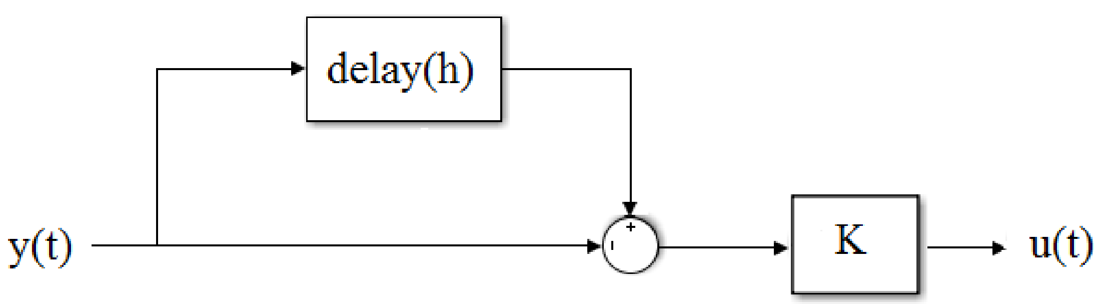

Figure 1.1: Controller graph with delayed feedback from the output.

This control scheme has a similarity with Smith predictor structure, repetitive and chaos control schemes and such a control scheme is studied in [13], [14] and [15]:

$$u(t) = -Ky(t) + Ky(t-h). \tag{1.1}$$

According to fundamental theorem of calculus

$$\dot{f}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}, \tag{1.2}$$

so for small h values

$$\dot{f}(a) = \frac{f(a+h) - f(a)}{h}. \tag{1.3}$$

After considering (1.3), (1.1) can be written as:

$$u(t) = -K(y(t) - y(t-h)) = -Kh(\tfrac{y(t)-y(t-h)}{h}) = -Kh\dot{y}(t).$$

It can be easily seen $u(t)$ is a derivative controller. As derivative controllers have rare usage, are not practical controllers and are not able to stabilize most of the plants, a new approach is investigated and a modification to $u(t)$ is decided.

For having a new approach, $u(t)$ is modified as:

$$\begin{aligned} u(t) &= -K_1 y(t) + K_2 y(t-h) \\ &= -K_1 y(t) + K_2 y(t-h) + K_2 y(t) - K_2 y(t) \\ &= -K_2[y(t) - y(t-h)] + (K_2 - K_1) y(t) \\ &= -K_2 h \dot{y}(t) + (K_2 - K_1) y(t). \end{aligned}$$

As can be seen $u(t)$ has the structure of a PD controller where:

$$K_d = -K_2 h \quad \text{and} \quad K_p = (K_2 - K_1).$$

It can be concluded that when K_1 and K_2 are used instead of one K in (1.1), PD controller can be obtained for small h values. As small h values must be used, h is selected between 0.1 sec and 0.5 sec.

By using different K_1 and K_2 values desired K_d and K_p pairs can be obtained so modified $u(t)$ can be directly accepted as PD controller. Because of this fact, K_1 and K_2 are not going to be mentioned after this section, i.e., instead K_d and K_p values are going to be calculated directly.

1.4 Organization of the Thesis

The thesis starts with Chapter 2 where problem definition is included. Problem definition is divided into two as feedback system stability and controller structure and plant types. The structure of PD controllers and feedback loop are explained and the chosen plants are stated. In Chapter 3, three PD controller design methods are defined, applied to the plants and PD controllers are obtained and listed. Chapter 4 includes all simulations and results where robust performance results and step responses are given. In Chapter 5, conclusion and future works are stated. Last but not least, Appendix 1 includes all Matlab codes which are used in the process of preparing the thesis.

Chapter 2

PROBLEM DEFINITION

2.1 Feedback System Stability and Controller Structure

The feedback system formed by the controller C and the plant P as in Figure 2.1 is stable if
(i) $S := (1 + PC)^{-1}$
(ii) CS
(iii) PS are stable, i.e., they are transfer functions in H_∞, [16].

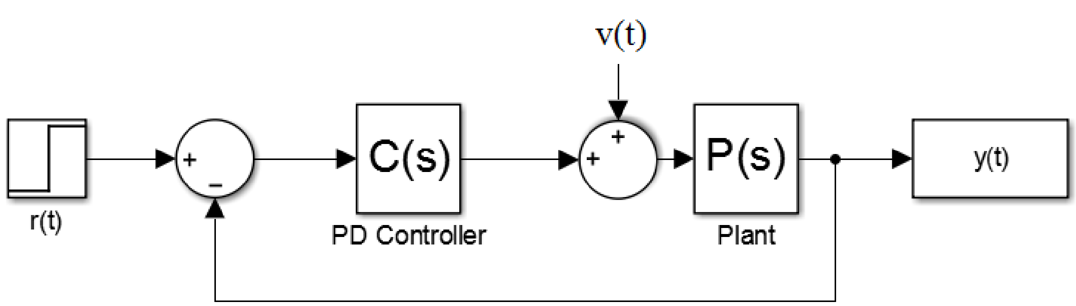

Figure 2.1: Feedback system.

If these conditions hold, then the controller C is said to stabilize the plant P. In order to achieve stabilization of the plants, PD controllers are used in this thesis; they are first order controllers in the form:

$C_{pd}(s) = K_d s + K_p$ where K_d is derivative gain and K_p is proportional gain.

This thesis aims to obtain the stabilizing K_d and K_p pairs, gain margin optimizing and least fragile PD controllers for four various plants by three design methods. The chosen plants are explained in the next section in detailed way.

2.2 Plant Types

Unstable processes with time delays have many different structures with different amount of orders. Most of the chemical systems, biological and engineering systems such as continuously stirred tank reactors, polymerization reactors, bioreactors and computer communication networks inherently have unstable behaviour and contain time delay because of measurement delay or the approximation of higher order dynamics of the process. Most of these systems are modeled as unstable first order processes with time delays (UFOPTD), unstable second order processes with time delays (USOPTD) and unstable third order processes with time delays (UTOPTD) [17, 18]. Some plant structures which have UFOPTD, USOPTD and UTOPTD are as follows.

Unstable first order processes with time delays mostly have the plant structure:

$$P(s) = \frac{Ke^{-hs}}{s-a} \tag{2.1}$$

where $h > 0$, $K > 0$ and $a \geq 0$.

Some processes which have this structure are control of the high frequency longitudinal dynamics (short period) of an aircraft [19] and batch chemical reactor which has a strong nonlinearity due to heat generation term in the energy balance

[20].

Unstable second order processes with time delays mostly have the plant structure:

$$P(s) = \frac{Ke^{-hs}}{(s-a)(s+b)} \tag{2.2}$$

where $h > 0$, $K > 0$, $a \geq 0$ and b can be both negative, positive or 0.

The temperature control in the heated tank of a pilot plant can be given as example where USOPTD are used [21].

Unstable third order processes with time delays mostly have the plant structure:

$$\frac{Ke^{-hs}}{(s-a)(s+b)(s+c)} \tag{2.3}$$

where $h > 0$, $K > 0$ and $a \geq 0$ and b, c can be both negative, positive or 0.

Third order unstable processes can be encountered in control of tank reactors. In stabilization of the continuously stirred tank reactor (CSTR), the transfer function between concentration of the product and cooling liquid temperature at the input is an example for UTOPTD [22].

After investigating most encountered unstable plant types with time delays four plants are chosen to be used in this thesis. One second order and three third order unstable plants with time delays are selected as there are many researches on first order plants [23, 24, 25, 26, 27]. In addition, when a process is higher order (more than third order), in many practical applications it can be reduced to first order, second order and third order process models by using the model reduction methods [28]. The chosen plants are:

1. $P_1(s) = \dfrac{e^{-hs}}{s(s-0.5)} = \dfrac{e^{-hs}}{s^2 - 0.5s}$

2. $P_2(s) = \dfrac{e^{-hs}}{(s-0.5)(s^2+s+1)} = \dfrac{e^{-hs}}{s^3+0.5s^2+0.5s-0.5}$

3. $P_3(s) = \dfrac{e^{-hs}}{s(s^2+s+1)} = \dfrac{e^{-hs}}{s^3+s^2+s}$

4. $P_4(s) = \dfrac{e^{-hs}}{(s-10^{-3})(s^2+s+1)} = \dfrac{e^{-hs}}{s^3+0.999s+0.999-0.001}$

where h is the time delay amount and $h > 0$. Different h values are used in order to see the effect of various time delays: $h = 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5$ sec.

The first plant $P_1(s)$ is USOPTD and have two unstable poles. The plant is designed as an integrating process with one pole at origin and one pole at right half plane because integrating processes are very frequently encountered in process industries and considerable numbers of chemical processes can be modeled as integrating processes with time delay [25].

The other plant models, $P_2(s)$, $P_3(s)$ and $P_4(s)$ only differ in the unstable pole. The plants are chosen in this way because the effect of the location of the unstable pole to PD controllers is desired to be investigated. The unstable pole of $P_3(s)$ and $P_4(s)$ are intentionally selected very close. The reason for this is going to be explained in Chapter 3.2.2.

Chapter 3

DESIGN OF PD CONTROLLERS

Three design methods are used to find the PD controllers (K_d and K_p values) which stabilize the plants. For each plant $P_1(s)$, $P_2(s)$, $P_3(s)$ and $P_4(s)$, separate PD controller sets are found, i.e., design methods are applied to each plant individually. First design method provides the least fragile PD controller via `allmargin` command of MATLAB, second design method provides gain margin optimizing PD controller and third design method provides another least fragile PD controller. Second and third design methods are taken from the "PD Controller Design" section of [1].

3.1 Design Method-I: Design of Least Fragile PD Controllers

Fragility of a controller refers to violation of the feedback system stability by perturbations in the controller parameters (see [29],[30]). In order to find the least fragile PD controller, one needs to find the set of all allowable control parameters for feedback system stability. For this purpose, design method-I uses `allmargin` command of MATLAB to find the stabilizing PD controller sets. The `allmargin` command computes gain margin, phase margin, delay margin and stability of a SISO open-loop model. This SISO open-loop model must be in the transfer function form and must not have time delay in the denominator so all four plants $P_1(s)$, $P_2(s)$, $P_3(s)$ and $P_4(s)$ are suitable for this design method. As `allmargin` command uses open-loop model, $P(s)C(s)$ is used as the SISO open-loop model. For different PD controllers (for different K_d and K_p pairs), `allmargin` command is used and the stability of the feedback system is examined. If the command results with "Stable:1", then it means feedback system is stable and that PD controller C stabilizes the plant P. Whereas, if the command results with "Stable:0", then it means that feedback system is not stable and that PD controller C does not stabilize the plant P.

In this design method, all PD controllers which stabilize each plant are recorded and indicated on the K_p-K_d plane. After that, the center point of each feasible region is found by drawing the largest circle which fits inside the stability region. The center point gives the least fragile PD controller parameters, because it is the farthest point to the stability-instability boundary. Also, the allowable ranges for K_p are observed for $P_2(s)$, $P_3(s)$, $P_4(s)$ and will be compared with the ranges which are obtained by design method-II. For finding the allowable ranges of K_p, the minimum and the maximum values of K_p are obtained from the graphs and the intervals between them gives the allowable ranges.

3.1.1 Least Fragile PD Controllers for $P_1(s)$

1. For $h = 0.1$ sec.

$$P_1(s) = \frac{e^{-0.1s}}{s(s-0.5)}. \tag{3.1}$$

The K_p and K_d pairs which stabilize $P_1(s)$ given in (3.1) are shown in Figure 3.1.

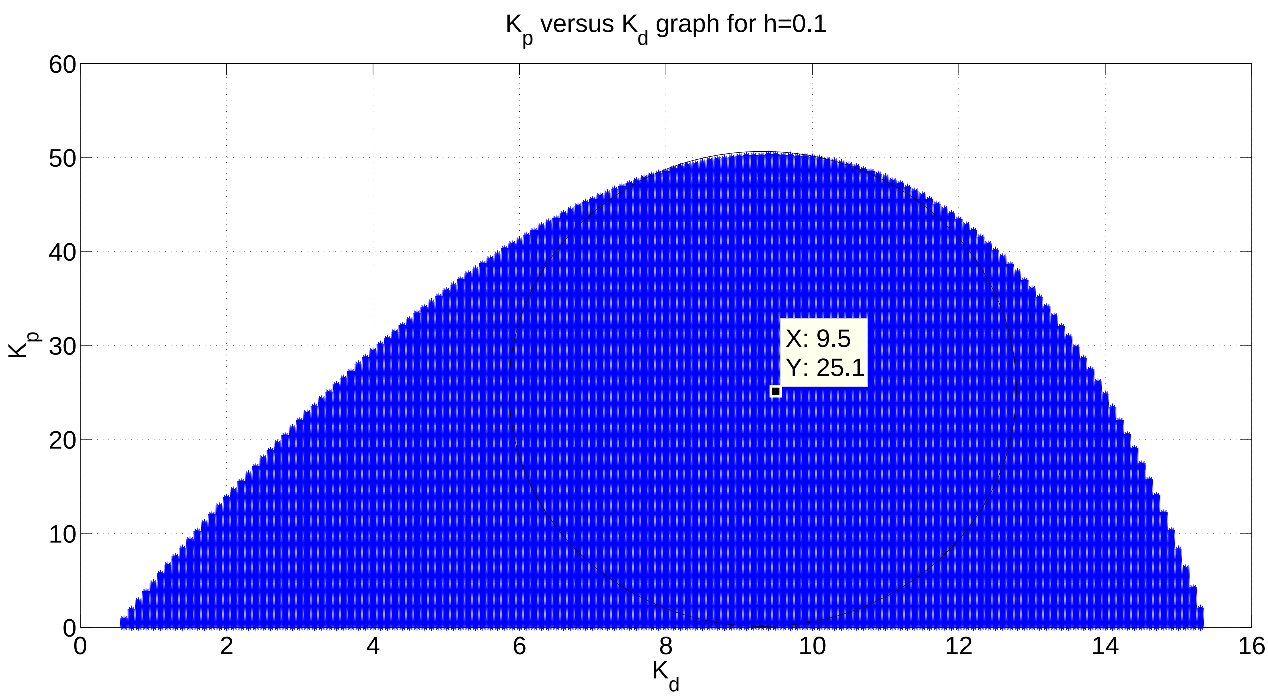

Figure 3.1: Stabilizing K_p and K_d pairs for $P_1(s)$ when $h = 0.1$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 9.5s + 25.1. \tag{3.2}$$

After finding the stabilizing K_d and K_p pairs, another stability condition is applied to check if the design method is giving the correct K_d and K_p pairs.

According to the stability condition, the phase margin should be positive, so the phase of $P(jw)C(jw)$ should satisfy:

$$tan^{-1}(\tilde{K}_d w_c) - \frac{\pi}{2} - \pi + tan^{-1}(\frac{w_c}{p}) - hw_c > -\pi \tag{3.3}$$

where $p = 0.5$, $\tilde{K}_d = \dfrac{K_d}{K_p}$ and w_c is the gain crossover frequency.

Note that (3.3) can be written as:

$$tan^{-1}(\tilde{K}_d w_c) + tan^{-1}(\frac{w_c}{p}) - hw_c > \frac{\pi}{2}. \tag{3.4}$$

The gain crossover frequency is computed from

$$\mid P(jw)C(jw) \mid = 1, \tag{3.5}$$

i.e.,

$$\frac{K_p\sqrt{(1+\tilde{K_d^2}{w_c}^2)}}{w_c\sqrt{w_c^2+p^2}} = 1. \tag{3.6}$$

By taking square of both sides we obtain:

$$K_p^2(1+\tilde{K}_d^2{w_c}^2) = w_c^2(w_c^2+p^2) \tag{3.7}$$

$$w_c^4 + (p^2 - K_p^2\tilde{K}_d^2)w_c^2 - K_p^2 = 0. \tag{3.8}$$

As w_c should be positive, the root which solves (3.8) is:

$$w_c = \sqrt{\frac{K_d^2-p^2}{2} + \sqrt{(\frac{K_d^2-p^2}{2})^2 + K_p^2}}. \tag{3.9}$$

Since w_c depends on K_d and K_p, it can be written as $w_c(K_d, K_p)$, then (3.4) means:

$$tan^{-1}(\frac{K_d w_c(K_d,K_p)}{K_p}) + tan^{-1}(\frac{w_c(K_d,K_p)}{p}) - hw_c(K_d,K_p) > \frac{\pi}{2}. \tag{3.10}$$

In conclusion, the positive phase margin condition is:

$$tan^{-1}\left(\frac{K_d\sqrt{\frac{K_d^2-p^2}{2}+\sqrt{(\frac{K_d^2-p^2}{2})^2+K_p^2}}}{K_p}\right)+tan^{-1}\left(\frac{\sqrt{\frac{K_d^2-p^2}{2}+\sqrt{(\frac{K_d^2-p^2}{2})^2+K_p^2}}}{p}\right)-h\sqrt{\frac{K_d^2-p^2}{2}+\sqrt{(\frac{K_d^2-p^2}{2})^2+K_p^2}}>\frac{\pi}{2}. \tag{3.11}$$

The pairs (K_d, K_p) which satisfy this inequality are recorded and the obtained region is shown in Figure 3.2:

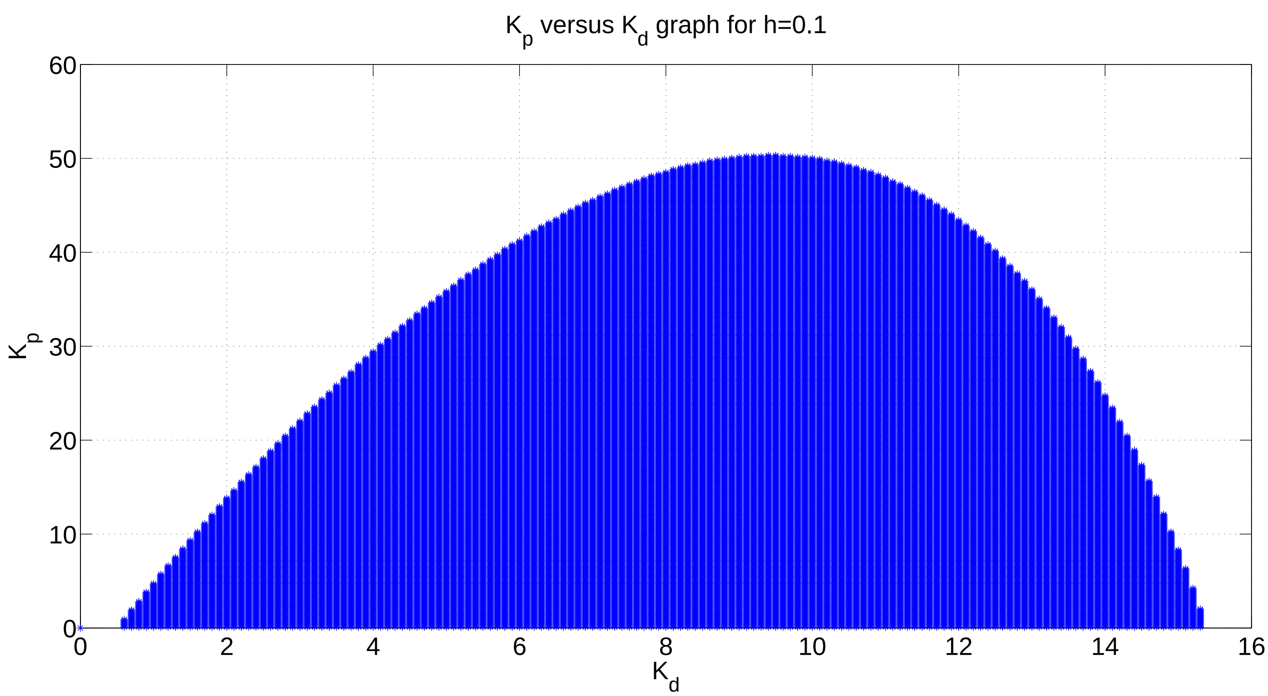

Figure 3.2: Stabilizing K_p and K_d pairs for $P_1(s)$ when $h = 0.1$ sec.

As can be seen all K_d and K_p pairs match exactly the ones which are found by `allmargin` command so it can be concluded that this design method is valid for finding stabilizing K_d and K_p pairs.

2. For $h = 0.2$ sec.

$$P_1(s) = \frac{e^{-0.2s}}{s(s-0.5)}. \tag{3.12}$$

The K_p and K_d pairs which stabilize $P_1(s)$ given in (3.12) are shown in Figure 3.3.

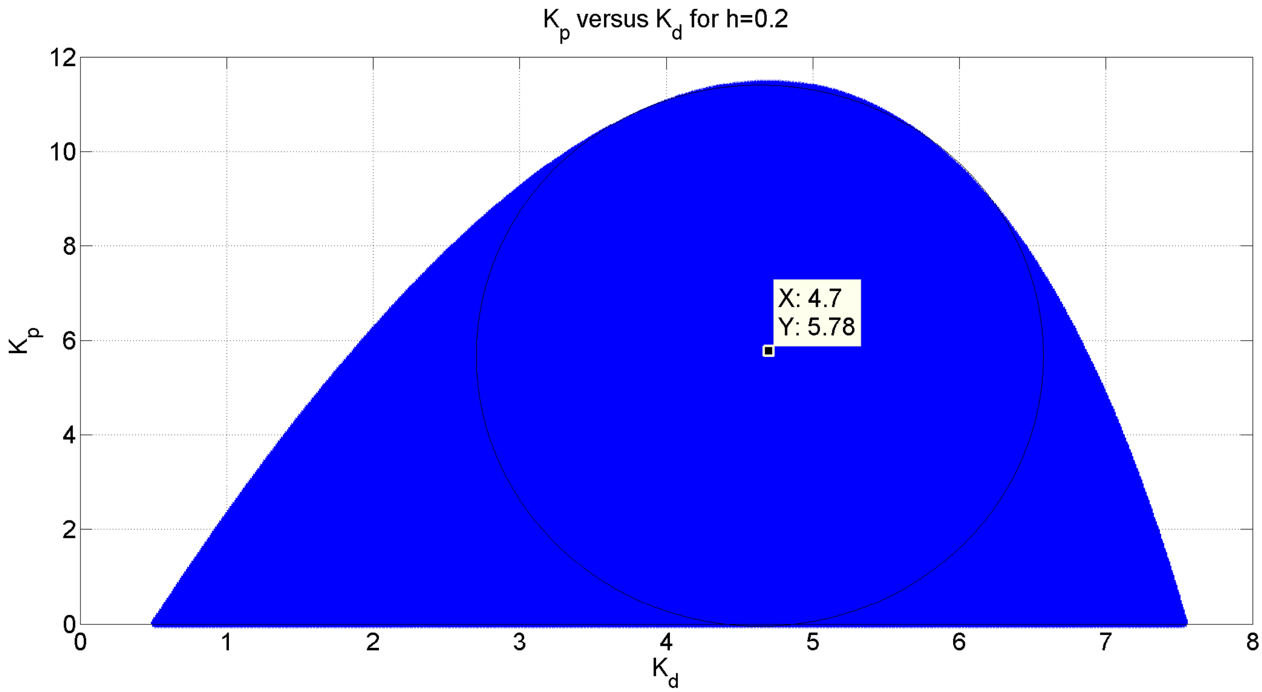

Figure 3.3: Stabilizing K_p and K_d pairs for $P_1(s)$ when $h = 0.2$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 4.7s + 5.78. \tag{3.13}$$

3. For $h = 0.3$ sec.

$$P_1(s) = \frac{e^{-0.3s}}{s(s-0.5)}. \tag{3.14}$$

The K_p and K_d pairs which stabilize $P_1(s)$ given in (3.14) are shown in Figure 3.4.

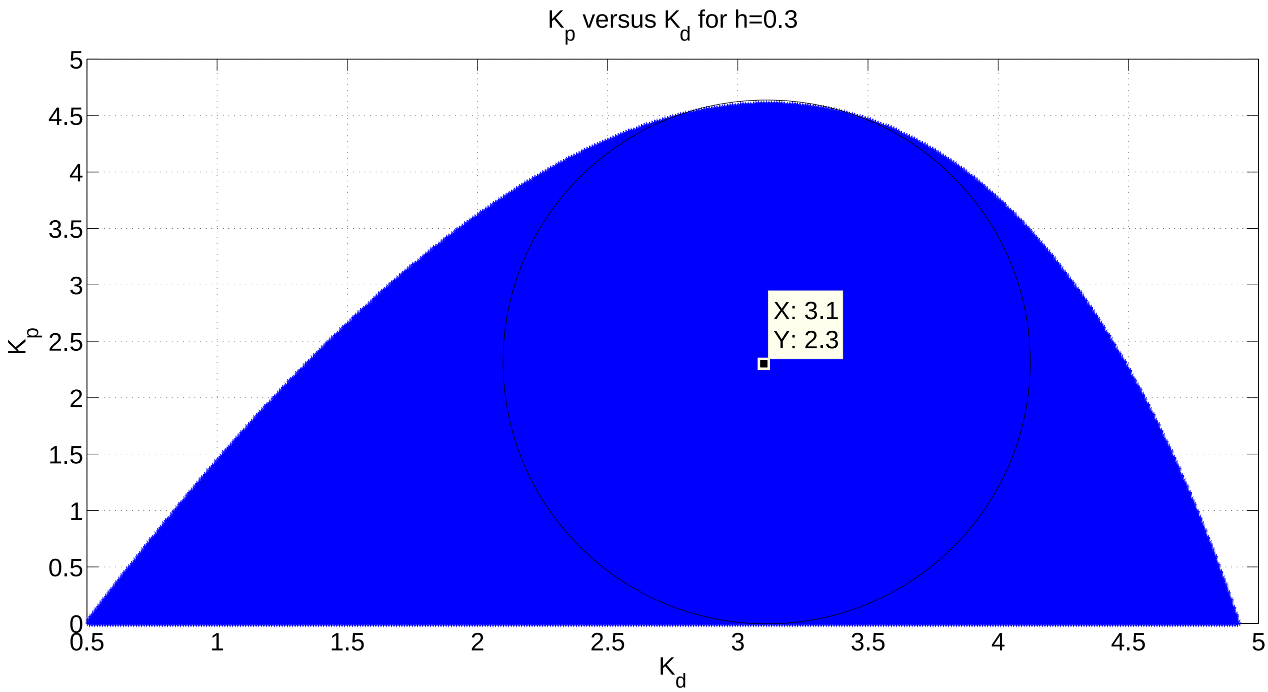

Figure 3.4: Stabilizing K_p and K_d pairs for $P_1(s)$ when $h = 0.3$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 3.1s + 2.3. \tag{3.15}$$

4. For $h = 0.4$ sec.

$$P_1(s) = \frac{e^{-0.4s}}{s(s-0.5)}. \tag{3.16}$$

The K_p and K_d pairs which stabilize $P_1(s)$ given in (3.16) are shown in Figure 3.5.

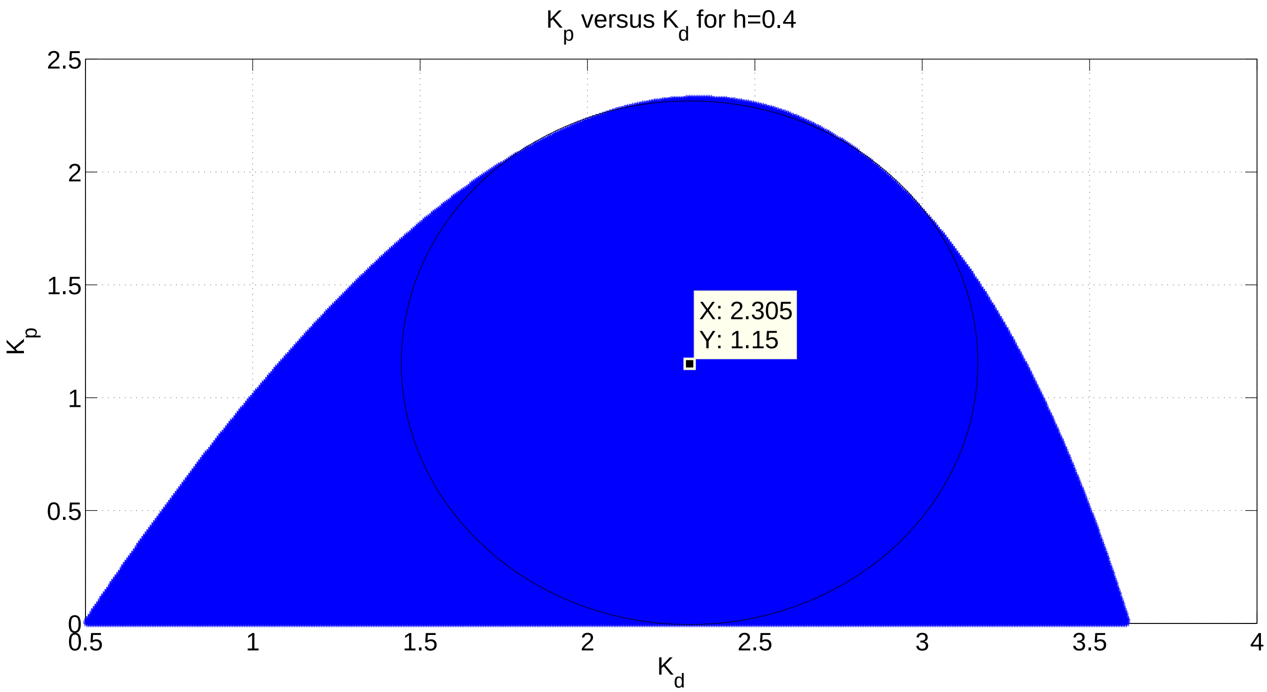

Figure 3.5: Stabilizing K_p and K_d pairs for $P_1(s)$ when $h = 0.4$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 2.305s + 1.15. \tag{3.17}$$

5. For $h = 0.5$ sec.

$$P_1(s) = \frac{e^{-0.5s}}{s(s-0.5)}. \tag{3.18}$$

The K_p and K_d pairs which stabilize $P_1(s)$ given in (3.18) are shown in Figure 3.6.

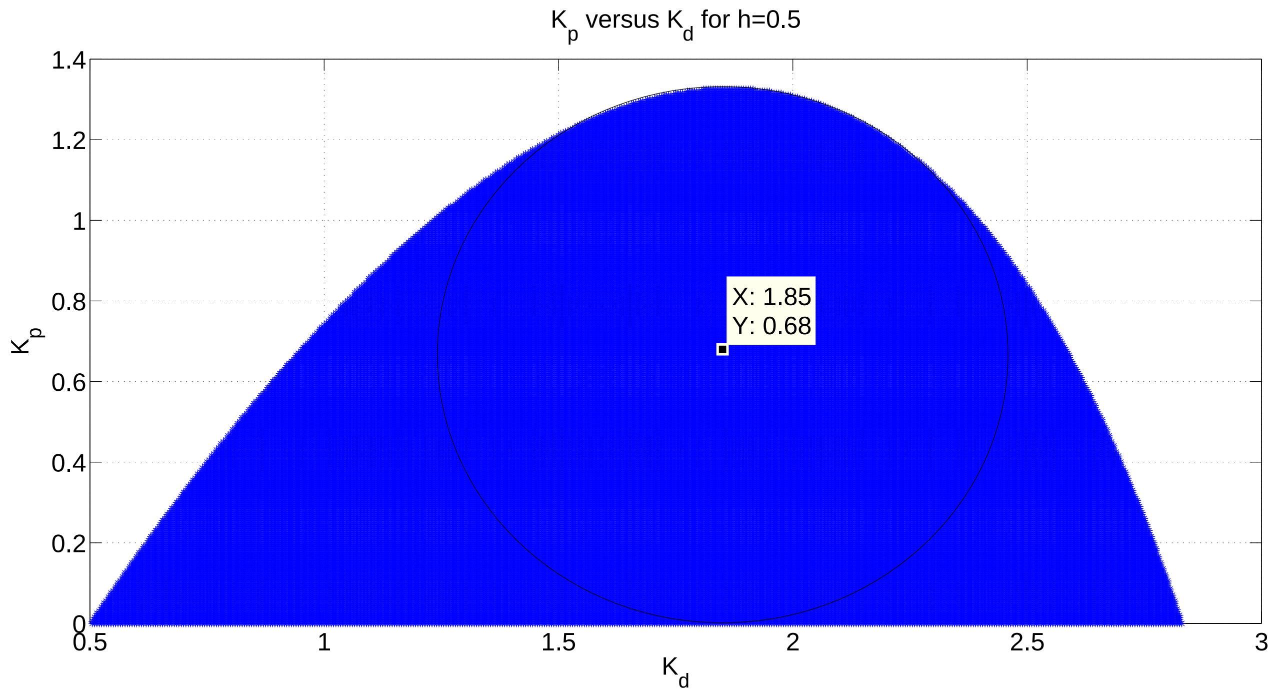

Figure 3.6: Stabilizing K_p and K_d pairs for $P_1(s)$ when $h = 0.5$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 1.85s + 0.68. \tag{3.19}$$

3.1.2 Least Fragile PD Controllers for $P_2(s)$

1. For $h = 0.1$ sec.

$$P_2(s) = \frac{e^{-0.1s}}{(s-0.5)(s^2+s+1)}. \tag{3.20}$$

The K_p and K_d pairs which stabilize $P_2(s)$ given in (3.20) are shown in Figure 3.7.

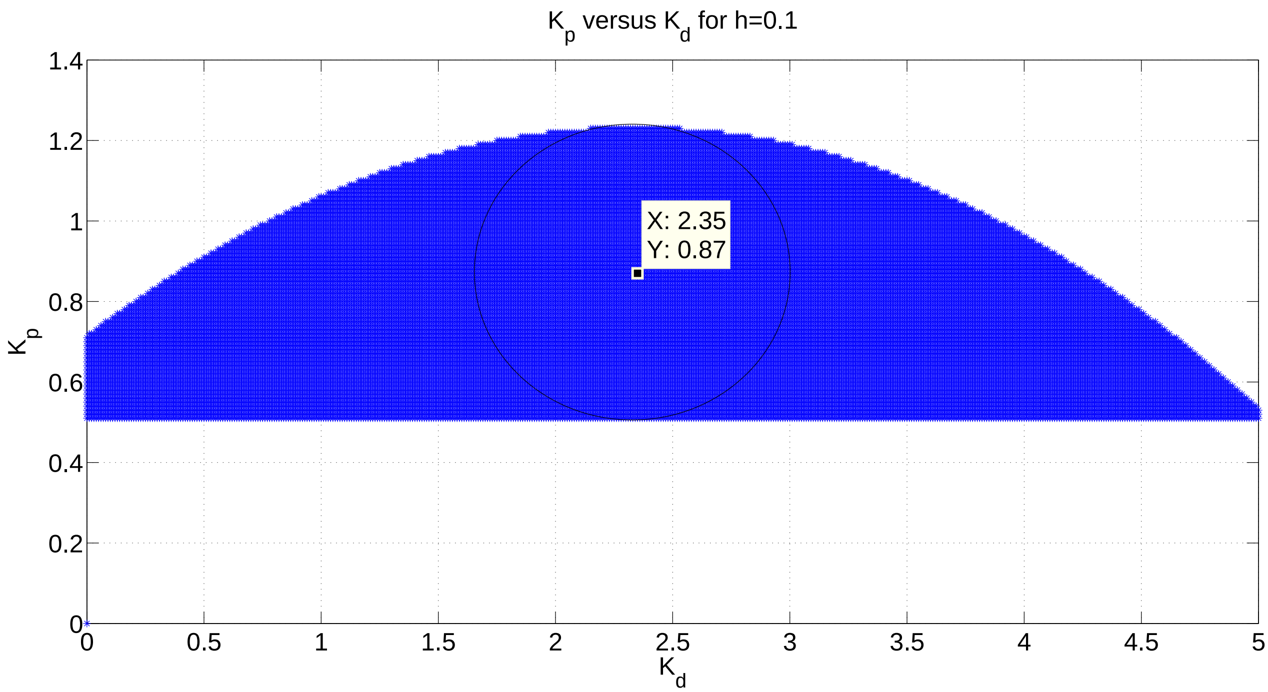

Figure 3.7: Stabilizing K_p and K_d pairs for $P_2(s)$ when $h = 0.1$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 2.35s + 0.87. \tag{3.21}$$

The allowable range for K_p:

$$0.5 < K_p < 1.23.$$

2. For $h = 0.2$ sec.

$$P_2(s) = \frac{e^{-0.2s}}{(s-0.5)(s^2+s+1)}. \tag{3.22}$$

The K_p and K_d pairs which stabilize $P_2(s)$ given in (3.22) are shown in Figure 3.8.

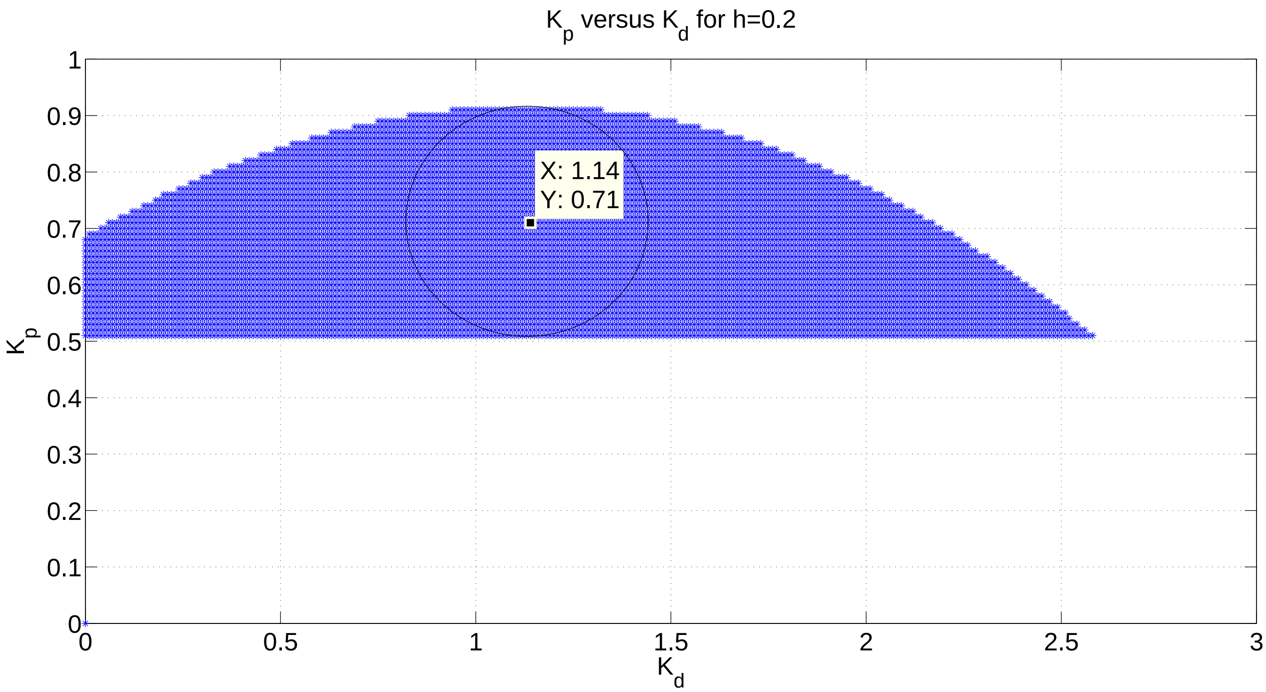

Figure 3.8: Stabilizing K_p and K_d pairs for $P_2(s)$ when $h = 0.2$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 1.14s + 0.71. \tag{3.23}$$

The allowable range for K_p:

$$0.5 < K_p < 0.91.$$

3. For $h = 0.3$ sec.

$$P_2(s) = \frac{e^{-0.3s}}{(s-0.5)(s^2+s+1)}. \tag{3.24}$$

The K_p and K_d pairs which stabilize $P_2(s)$ given in (3.24) are shown in Figure 3.9.

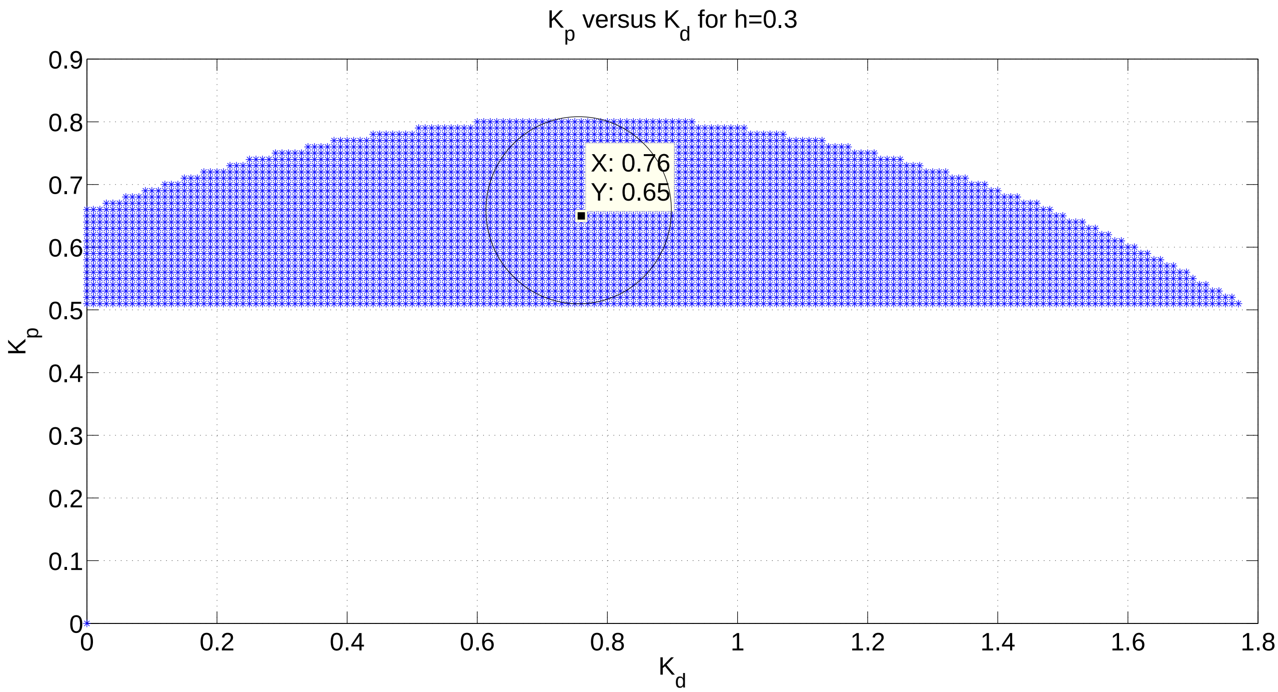

Figure 3.9: Stabilizing K_p and K_d pairs for $P_2(s)$ when $h = 0.3$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 0.76s + 0.65. \tag{3.25}$$

The allowable range for K_p:

$$0.5 < K_p < 0.8.$$

4. For $h = 0.4$ sec.

$$P_2(s) = \frac{e^{-0.4s}}{(s-0.5)(s^2+s+1)} \tag{3.26}$$

The K_p and K_d pairs which stabilize $P_2(s)$ given in (3.26) are shown in Figure 3.10.

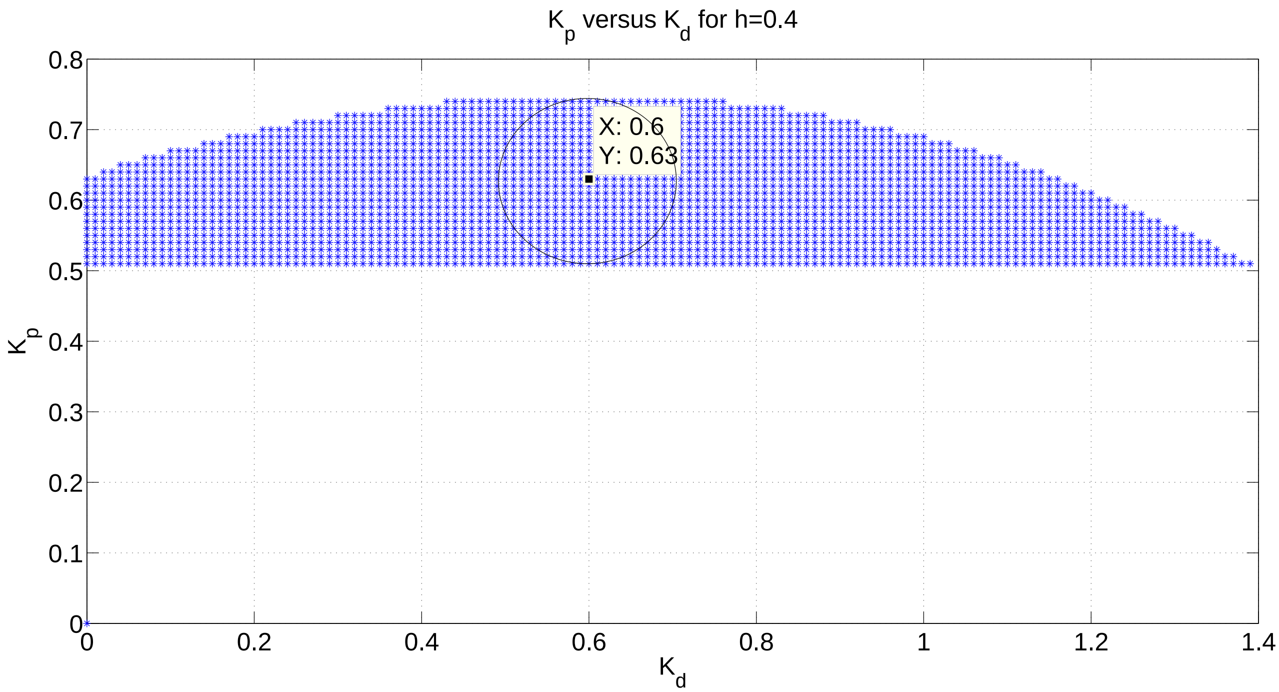

Figure 3.10: Stabilizing K_p and K_d pairs for $P_2(s)$ when $h = 0.4$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 0.6s + 0.63. \tag{3.27}$$

The allowable range for K_p:

$$0.5 < K_p < 0.74.$$

5. For $h = 0.5$ sec.

$$P_2(s) = \frac{e^{-0.5s}}{(s-0.5)(s^2+s+1)}. \tag{3.28}$$

The K_p and K_d pairs which stabilize $P_2(s)$ given in (3.28) are shown in Figure 3.11.

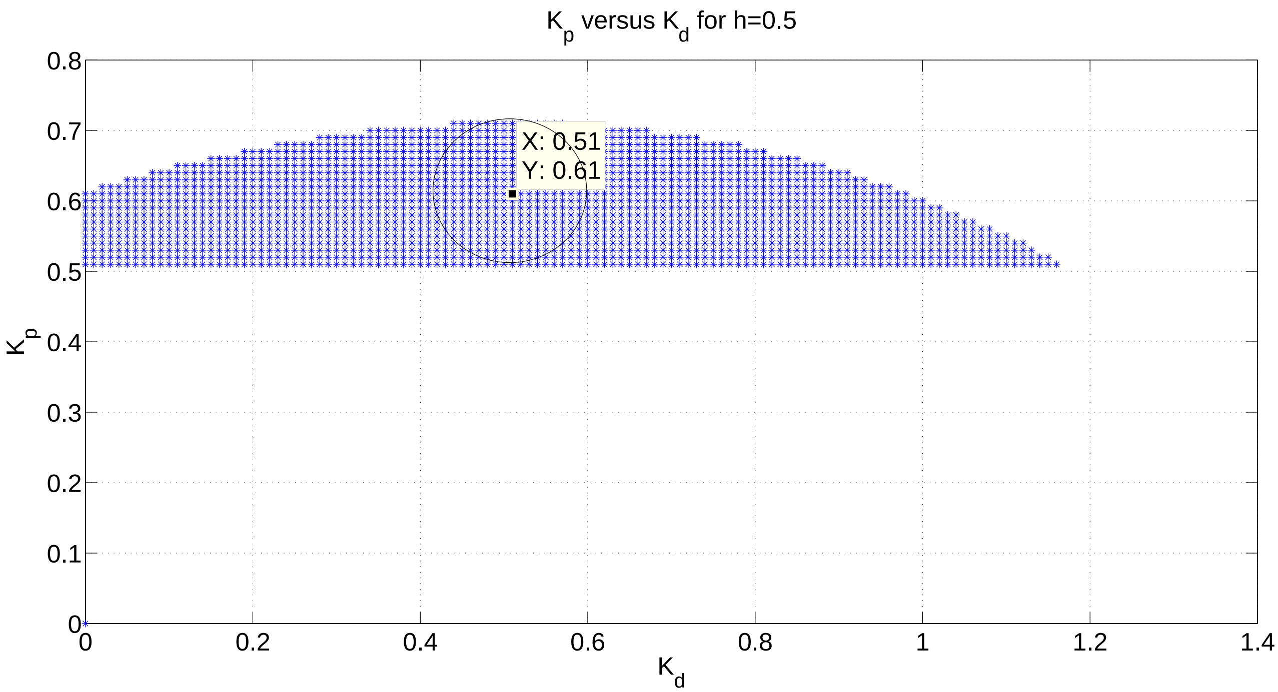

Figure 3.11: Stabilizing K_p and K_d pairs for $P_2(s)$ when $h = 0.5$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 0.51s + 0.61. \tag{3.29}$$

The allowable range for K_p:

$$0.5 < K_p < 0.71.$$

3.1.3 Least Fragile PD Controllers for $P_3(s)$

1. For $h = 0.1$ sec.

$$P_3(s) = \frac{e^{-0.1s}}{s(s^2 + s + 1)}. \tag{3.30}$$

The K_p and K_d pairs which stabilize $P_3(s)$ given in (3.30) are shown in Figure 3.12.

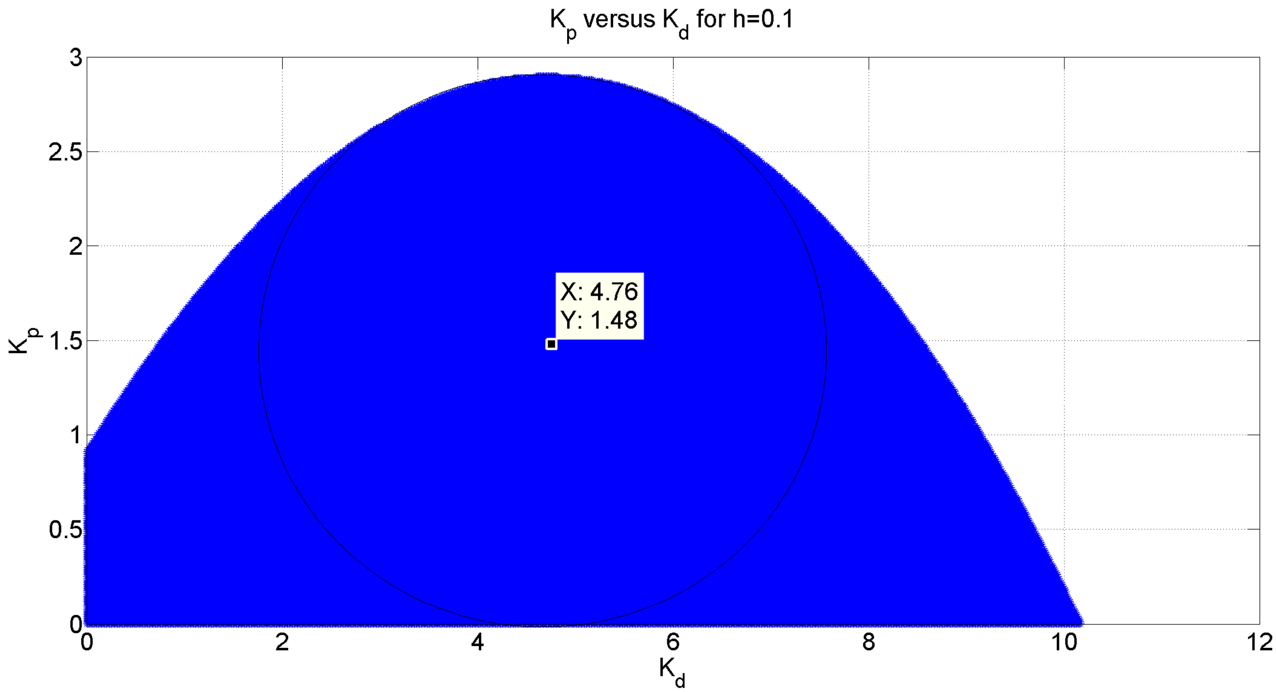

Figure 3.12: Stabilizing K_p and K_d pairs for $P_3(s)$ when $h = 0.1$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 4.76s + 1.48. \tag{3.31}$$

The allowable range for K_p:

$$0 < K_p < 2.9.$$

2. For $h = 0.2$ sec.

$$P_3(s) = \frac{e^{-0.2s}}{s(s^2 + s + 1)}. \tag{3.32}$$

The K_p and K_d pairs which stabilize $P_3(s)$ given in (3.32) are shown in Figure 3.13.

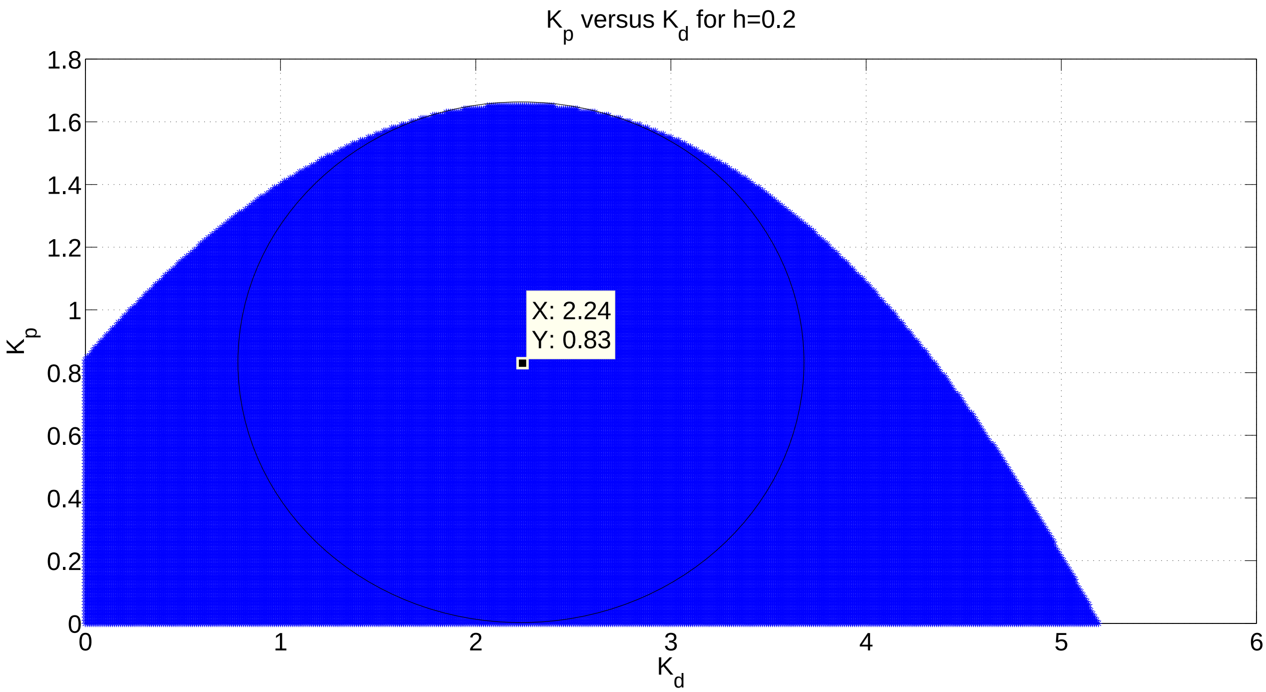

Figure 3.13: Stabilizing K_p and K_d pairs for $P_3(s)$ when $h = 0.2$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 2.24s + 0.83. \tag{3.33}$$

The allowable range for K_p:

$$0 < K_p < 1.65.$$

3. For $h = 0.3$ sec.

$$P_3(s) = \frac{e^{-0.3s}}{s(s^2 + s + 1)} \tag{3.34}$$

The K_p and K_d pairs which stabilize $P_3(s)$ given in (3.34) are shown in Figure 3.14.

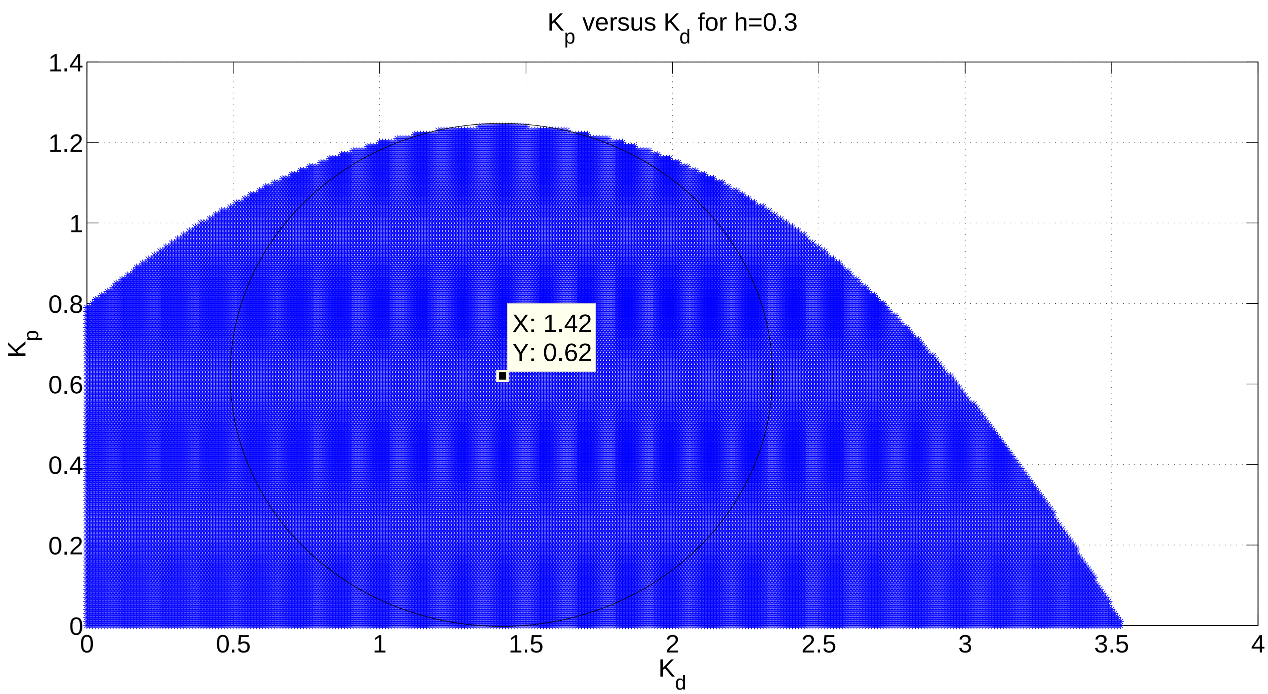

Figure 3.14: Stabilizing K_p and K_d pairs for $P_3(s)$ when $h = 0.3$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 1.42s + 0.62. \tag{3.35}$$

The allowable range for K_p:

$$0 < K_p < 1.24.$$

4. For $h = 0.4$ sec.

$$P_3(s) = \frac{e^{-0.4s}}{s(s^2 + s + 1)}. \tag{3.36}$$

The K_p and K_d pairs which stabilize $P_3(s)$ given in (3.36) are shown in Figure 3.15.

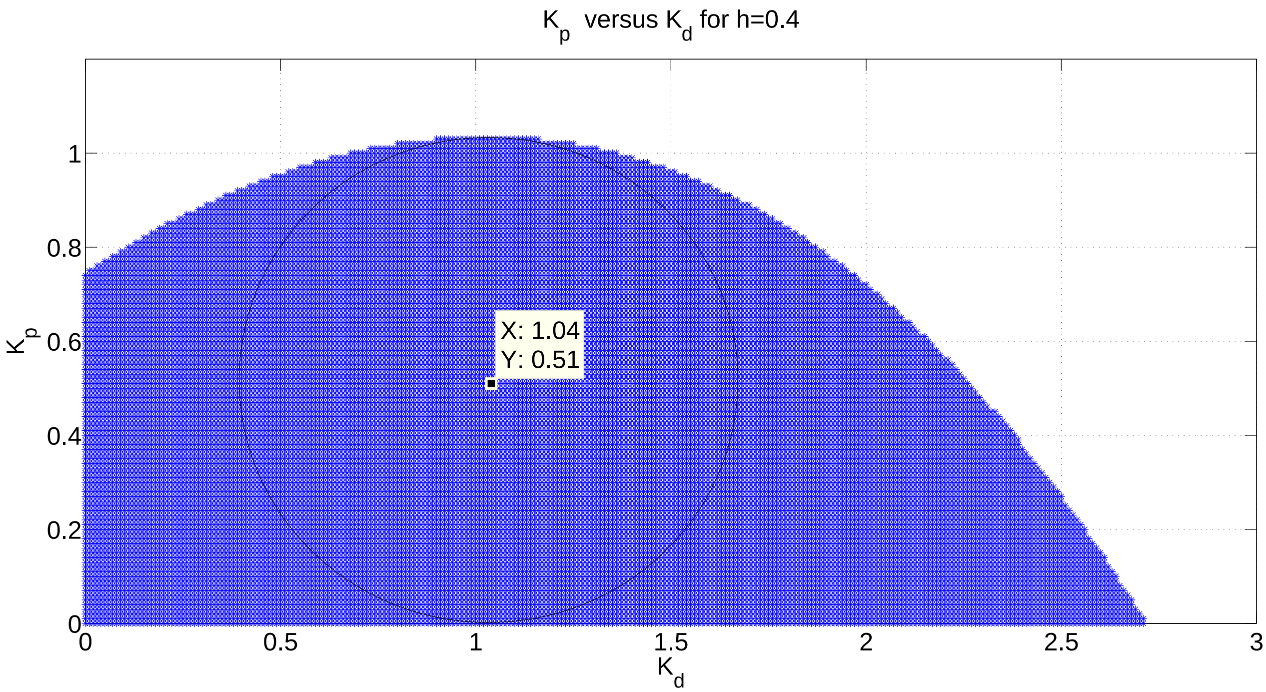

Figure 3.15: Stabilizing K_p and K_d pairs for $P_3(s)$ when $h = 0.4$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 1.04s + 0.51. \tag{3.37}$$

The allowable range for K_p:

$$0 < K_p < 1.03.$$

5. For $h = 0.5$ sec.

$$P_3(s) = \frac{e^{-0.5s}}{s(s^2 + s + 1)}. \tag{3.38}$$

The K_p and K_d pairs which stabilize $P_3(s)$ given in (3.38) are shown in Figure 3.16.

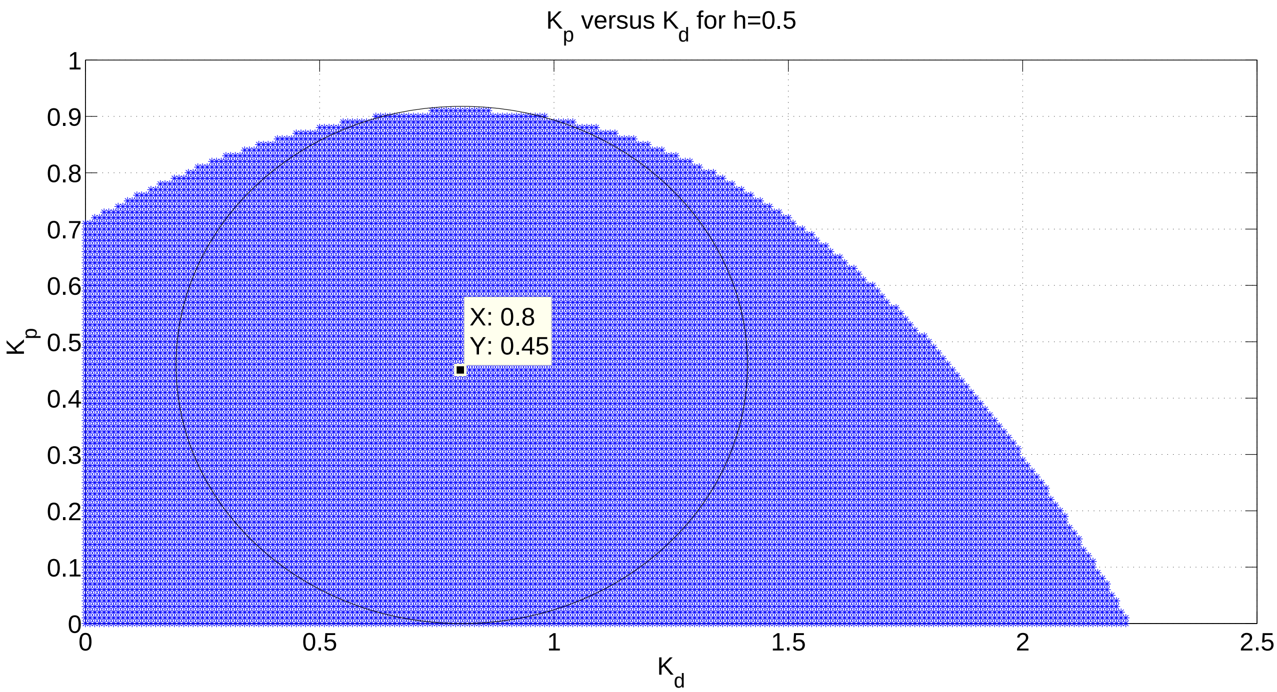

Figure 3.16: Stabilizing K_p and K_d pairs for $P_3(s)$ when $h = 0.5$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 0.8s + 0.45. \tag{3.39}$$

The allowable range for K_p:

$$0 < K_p < 0.91.$$

3.1.4 Least Fragile PD Controllers for $P_4(s)$

1. For $h = 0.1$ sec.

$$P_4(s) = \frac{e^{-0.1s}}{(s - 10^{-3})(s^2 + s + 1)}. \tag{3.40}$$

The K_p and K_d pairs which stabilize $P_4(s)$ given in (3.40) are shown in Figure 3.17.

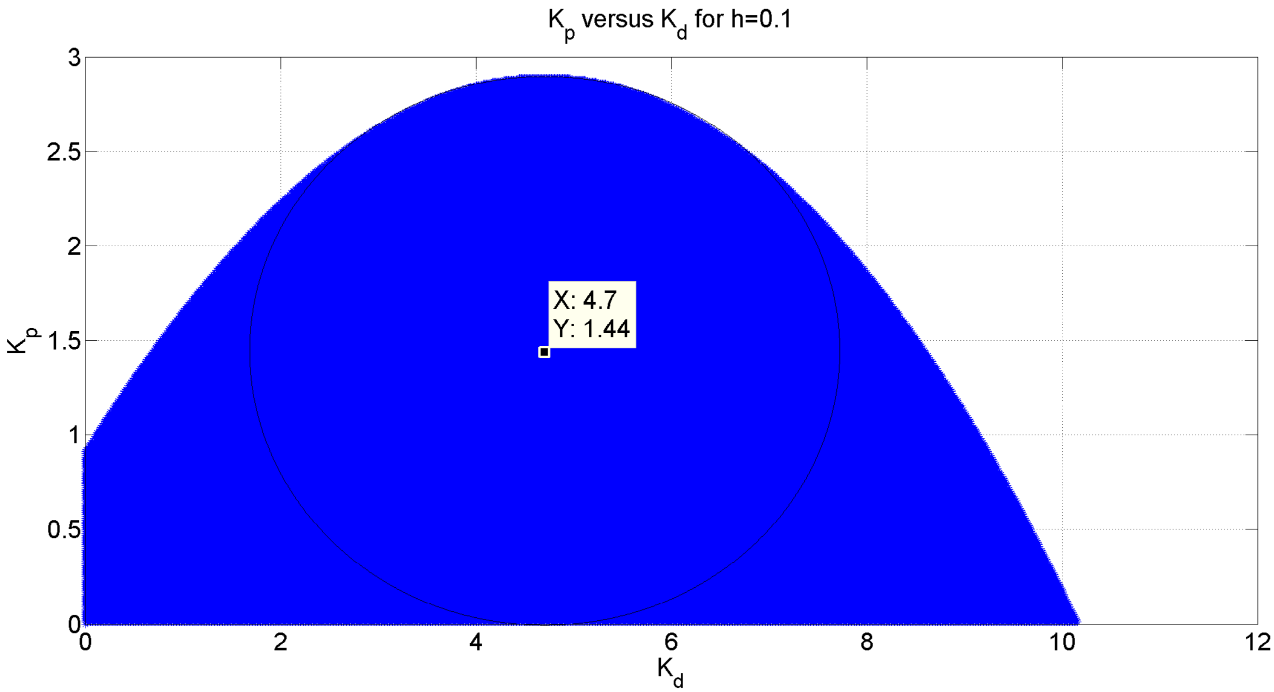

Figure 3.17: Stabilizing K_p and K_d pairs for $P_4(s)$ when $h = 0.1$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 4.7s + 1.44. \tag{3.41}$$

The allowable range for K_p:

$$0.001 < K_p < 2.89.$$

2. For $h = 0.2$ sec.

$$P_4(s) = \frac{e^{-0.2s}}{(s - 10^{-3})(s^2 + s + 1)}. \tag{3.42}$$

The K_p and K_d pairs which stabilize $P_4(s)$ given in (3.42) are shown in Figure 3.18.

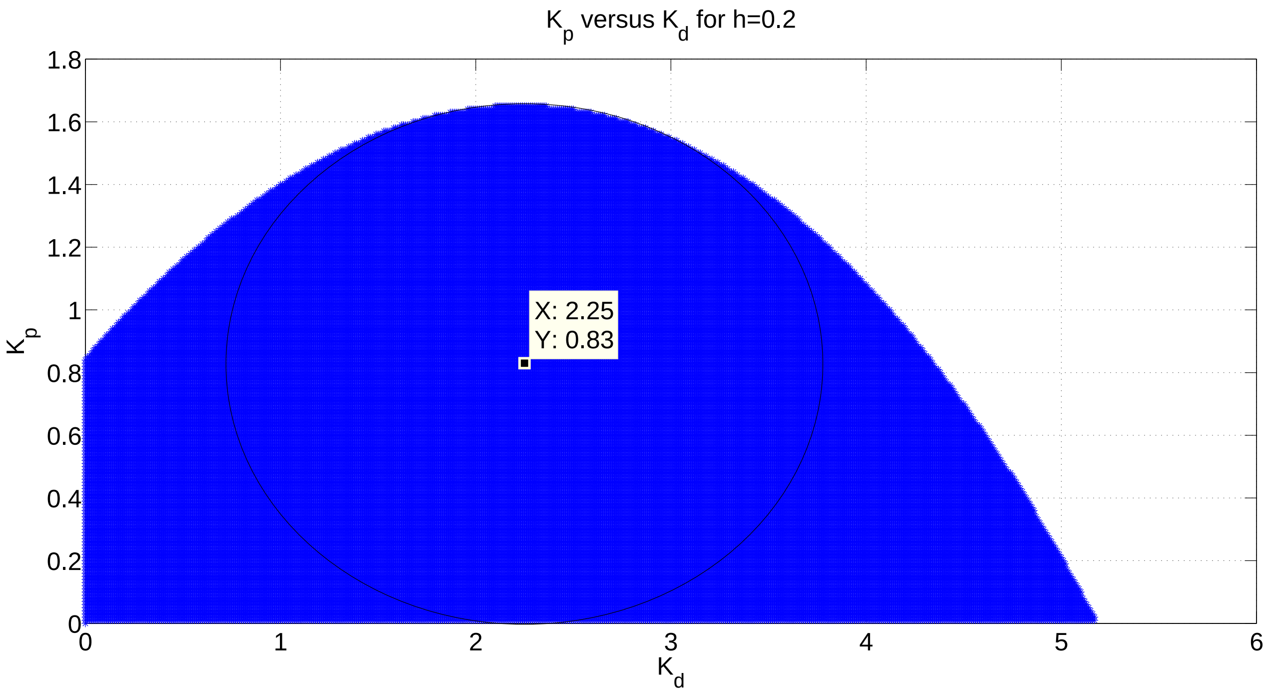

Figure 3.18: Stabilizing K_p and K_d pairs for $P_4(s)$ when $h = 0.2$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 2.25s + 0.83. \tag{3.43}$$

The allowable range for K_p:

$$0.001 < K_p < 1.65.$$

3. For $h = 0.3$ sec.

$$P_4(s) = \frac{e^{-0.3s}}{(s - 10^{-3})(s^2 + s + 1)}. \tag{3.44}$$

The K_p and K_d pairs which stabilize $P_4(s)$ given in (3.44) are shown in Figure 3.19.

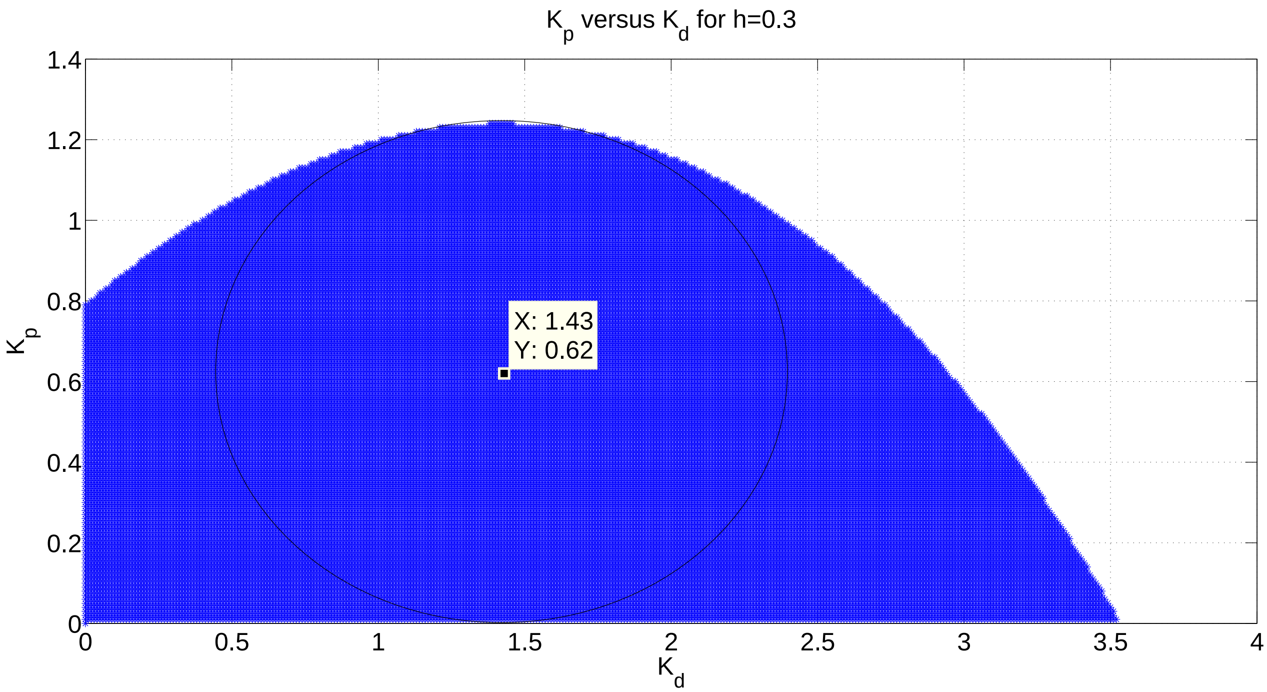

Figure 3.19: Stabilizing K_p and K_d pairs for $P_4(s)$ when $h = 0.3$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 1.43s + 0.62. \tag{3.45}$$

The allowable range for K_p:

$$0.001 < K_p < 1.24.$$

4. For $h = 0.4$ sec.

$$P_4(s) = \frac{e^{-0.4s}}{(s - 10^{-3})(s^2 + s + 1)}. \tag{3.46}$$

The K_p and K_d pairs which stabilize $P_4(s)$ given in (3.46) are shown in Figure 3.20.

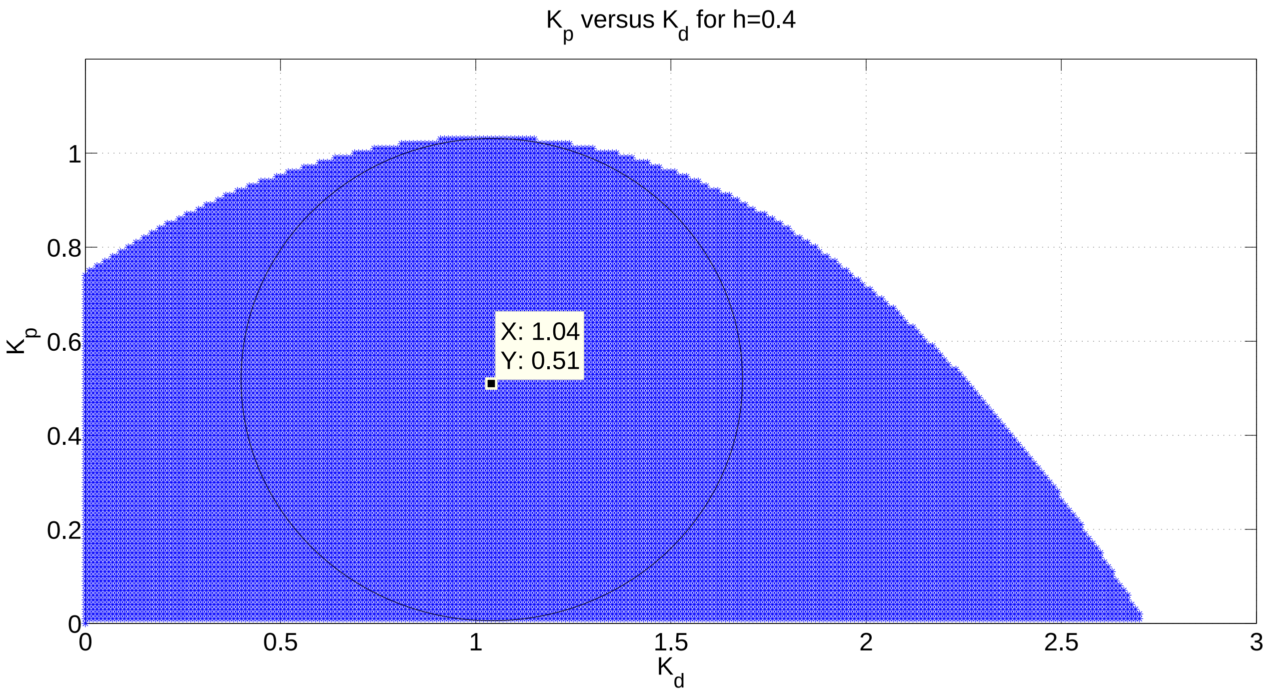

Figure 3.20: Stabilizing K_p and K_d pairs for $P_4(s)$ when $h = 0.4$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 1.04s + 0.51. \tag{3.47}$$

The allowable range for K_p:

$$0.001 < K_p < 1.03.$$

5. For $h = 0.5$ sec.

$$P_4(s) = \frac{e^{-0.5s}}{(s - 10^{-3})(s^2 + s + 1)}. \tag{3.48}$$

The K_p and K_d pairs which stabilize $P_4(s)$ given in (3.48) are shown in Figure 3.21.

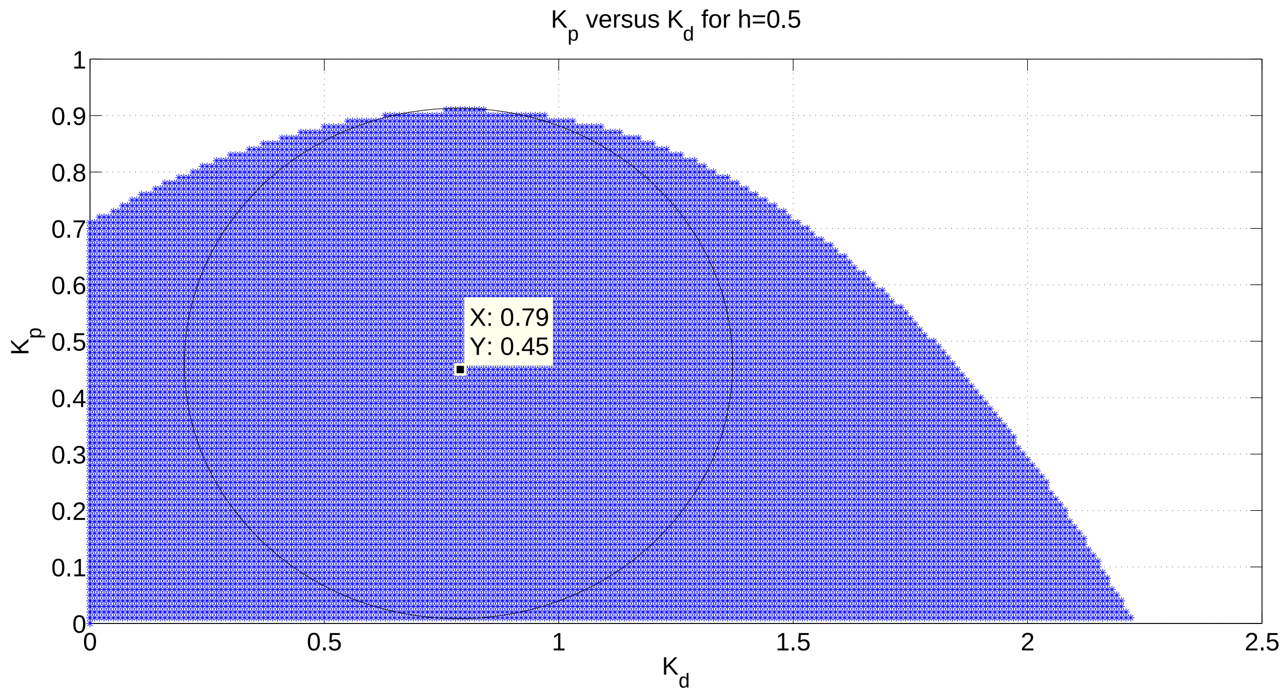

Figure 3.21: Stabilizing K_p and K_d pairs for $P_4(s)$ when $h = 0.5$ sec.

As can be seen from the figure, least fragile PD controller is:

$$C(s) = 0.79s + 0.45. \tag{3.49}$$

The allowable range for K_p:

$$0.001 < K_p < 0.91.$$

3.2 Design Method-II: Design of Gain Margin Optimizing PD Controllers Given in [1]

This design method which is defined in [1] is only applicable to the plants which have the form:

$P(s) = \dfrac{1}{s-p}G(s)$ where $p \geq 0$ is the unstable pole

and $G \in H_\infty$ is the stable part of the plant.

$G(s)$ can be irrational, i.e., $P(s)$ can be infinite dimensional.

As $P_1(s)$ has two unstable poles, this design method is not applicable to $P_1(s)$. The other plants $P_2(s)$, $P_3(s)$ and $P_4(s)$ satisfy this condition so gain margin optimizing PD controllers are found for only three of the plants.

According to [1], the structure of PD controller is defined as:

$$C_{pd}(s) = K_p C_0(s) \quad \text{where} \quad C_0(s) = (1 + \tilde{K}_d s) \quad \text{so}$$

$$C_{pd}(s) = (\tilde{K}_d K_p)s + K_p.$$

As $C_{pd}(s) = K_d s + K_p$ then $K_d = \tilde{K}_d K_p$. In the analysis below $\tilde{K}_d$ is denoted as a free parameter $Q \in \mathbb{R}$.

The first step of the design method is determining an admissable Q interval for stability:

$$Q \in [Q_{min}, Q_{max}].$$

After defining the range for Q, for each Q in this interval, the largest a value $(a_{max}(Q))$ should be found which satisfies the inequality:

$$(p+a) < \frac{1}{\gamma(Q,a)} \qquad \forall \quad a < a_{max}(Q)$$

where

$$\gamma(Q,a) := \| \frac{G_0(s)-1}{s+a} + Q\frac{s}{s+a}G_0(s) \|_\infty \text{ and } G_0(s) = G(s)G(0)^{-1}.$$

The next step is plotting $a_{max}(Q)$ versus Q and finding the maximum value of $a_{max}(Q)$ and defining:

$$Q_{opt} := arg\max\{a_{max}(Q)\}.$$

After finding the value of Q_{opt}, the allowable range of the controller gain K_p can be found by:

$$pG(0)^{-1} < K_p < (p+a_0)G(0)^{-1} \quad \text{ where}$$

$$a_0 := a_{max}(Q_{opt}). \tag{3.50}$$

In conclusion, for $p > 0$, gain margin optimizing PD controller parameters are defined as:

$$\tilde{K}_{d,opt} = Q_{opt}$$

$$K_{p,GMopt} = \sqrt{p(p+a_0)}G(0)^{-1}.$$

So as $K_d = \tilde{K}_d K_p$; K_d becomes

$$K_{d,GMopt} = Q_{opt}\sqrt{p(p+a_0)}G(0)^{-1}.$$

3.2.1 Gain Margin Optimizing PD Controllers for $P_2(s)$

Obtained $a_{max}(Q)$ versus Q graph for $P_2(s)$ for h values between 0.1 sec and 0.5 sec and maximum $a_{max}(Q)$ points for each h are shown in Figure 3.22.

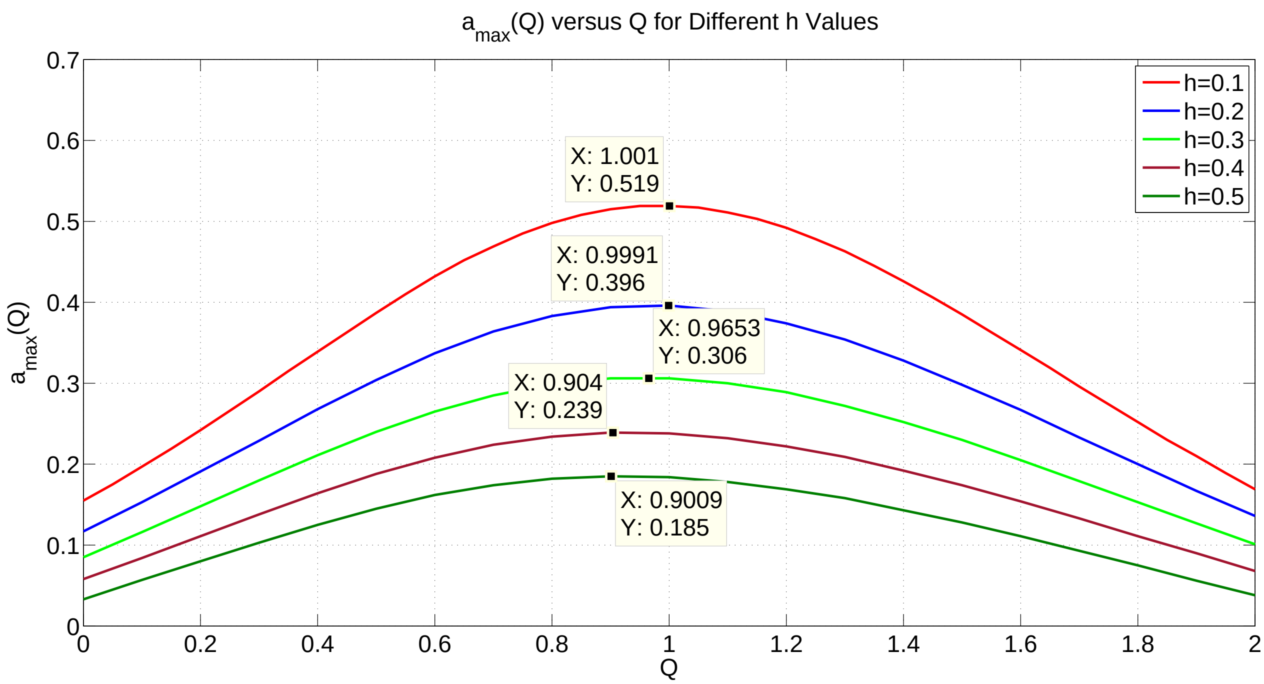

Figure 3.22: $a_{max}(Q)$ versus Q graph for $P_2(s)$ when h is between 0.1 sec and 0.5 sec.

As it is stated $P_2(s) = \dfrac{e^{-hs}}{(s-0.5)(s^2+s+1)}$ so

$$p = 0.5 \text{ and } G(s) = \frac{e^{-hs}}{(s^2+s+1)}$$

$$G(0) = 1.$$

1. For $h = 0.1$ sec.

$$Q_{opt} = 1.001 \text{ and } a_0 = 0.519.$$

The allowable range of the controller gain K_p is:

$$0.5 < K_p < 1.019.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.5(0.5 + 0.519)} = 0.714$$

$$K_{d,GMopt} = Q_{opt} K_{p,GMopt} = 1.001 \times 0.714 = 0.715.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.715s + 0.714. \tag{3.51}$$

2. For $h = 0.2$ sec.

$$Q_{opt} = 0.9991 \text{ and } a_0 = 0.396.$$

The allowable range of the controller gain K_p is:

$$0.5 < K_p < 0.896.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.5(0.5 + 0.396)} = 0.669$$

$$K_{d,GMopt} = Q_{opt} K_{p,GMopt} = 0.9991 \times 0.669 = 0.669.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.669s + 0.669. \tag{3.52}$$

3. For $h = 0.3$ sec.

$$Q_{opt} = 0.9653 \text{ and } a_0 = 0.306.$$

The allowable range of the controller gain K_p is:

$$0.5 < K_p < 0.806.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.5(0.5 + 0.306)} = 0.635$$

$$K_{d,GMopt} = Q_{opt}K_{p,GMopt} = 0.9653 \times 0.635 = 0.613.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.613s + 0.635. \tag{3.53}$$

4. For $h = 0.4$ sec.

$$Q_{opt} = 0.904 \text{ and } a_0 = 0.239.$$

The allowable range of the controller gain K_p is:

$$0.5 < K_p < 0.739.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.5(0.5 + 0.239)} = 0.608$$

$$K_{d,GMopt} = Q_{opt}K_{p,GMopt} = 0.904 \times 0.608 = 0.550.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.550s + 0.608. \tag{3.54}$$

5. For $h = 0.5$ sec.

$$Q_{opt} = 0.9009 \text{ and } a_0 = 0.185.$$

The allowable range of the controller gain K_p is:

$$0.5 < K_p < 0.685.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.5(0.5 + 0.185)} = 0.585$$

$$K_{d,GMopt} = Q_{opt} K_{p,GMopt} = 0.9009 \times 0.585 = 0.527.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.527s + 0.585. \tag{3.55}$$

3.2.2 Gain Margin Optimizing PD Controllers for $P_3(s)$

Obtained $a_{max}(Q)$ versus Q graph for $P_3(s)$ for h values between 0.1 sec and 0.5 sec and maximum $a_{max}(Q)$ points for each h are shown in Figure 3.23.

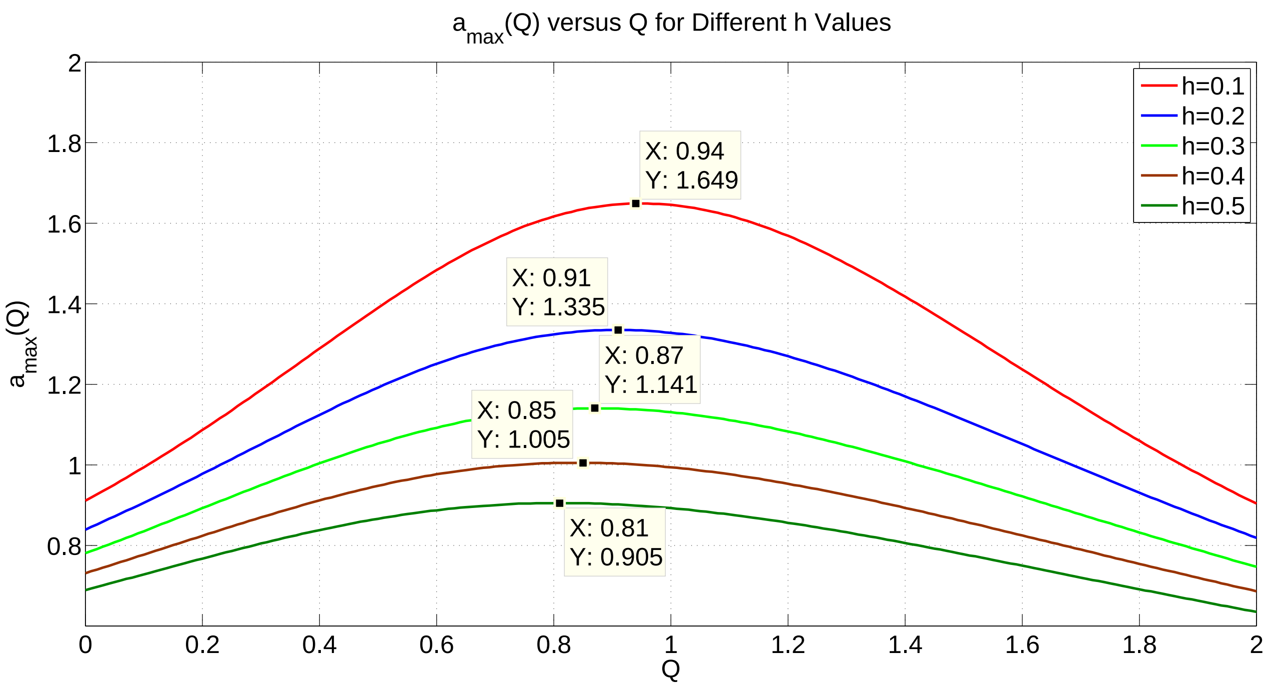

Figure 3.23: $a_{max}(Q)$ versus Q graph for $P_3(s)$ when h is between 0.1 sec and 0.5 sec.

As it is stated $P_3(s) = \dfrac{e^{-hs}}{s(s^2+s+1)}$ so

$$p = 0 \text{ and } G(s) = \frac{e^{-hs}}{(s^2+s+1)}$$

$$G(0) = 1.$$

As gain margin optimizing PD controller parameters are defined for $p > 0$ in [1], gain margin optimizing PD controllers cannot be found for $P_3(s)$ as $p = 0$. In this section, only the allowable ranges of the controller gain K_p are found. The plant $P_4(s)$ is chosen nearly equal to $P_3(s)$ in order to obtain gain margin optimizing PD controllers. Same gain margin optimizing PD controllers will be used for $P_3(s)$ and $P_4(s)$ in Chapter 5.

1. For $h = 0.1$ sec.

$$Q_{opt} = 0.94 \text{ and } a_0 = 1.649.$$

The allowable range of the controller gain K_p is:

$$0 < K_p < 1.649.$$

2. For $h = 0.2$ sec.

$$Q_{opt} = 0.91 \text{ and } a_0 = 1.335.$$

The allowable range of the controller gain K_p is:

$$0 < K_p < 1.335.$$

3. For $h = 0.3$ sec.

$$Q_{opt} = 0.87 \text{ and } a_0 = 1.141.$$

The allowable range of the controller gain K_p is:

$$0 < K_p < 1.141.$$

4. For $h = 0.4$ sec.

$$Q_{opt} = 0.85 \text{ and } a_0 = 1.005.$$

The allowable range of the controller gain K_p is:

$$0 < K_p < 1.005.$$

5. For $h = 0.5$ sec.

$$Q_{opt} = 0.81 \text{ and } a_0 = 0.905.$$

The allowable range of the controller gain K_p is:

$$0 < K_p < 0.905.$$

3.2.3 Gain Margin Optimizing PD Controllers for $P_4(s)$

Obtained $a_{max}(Q)$ versus Q graph for $P_4(s)$ for h values between 0.1 sec and 0.5 sec and maximum $a_{max}(Q)$ points for each h are shown in Figure 3.24

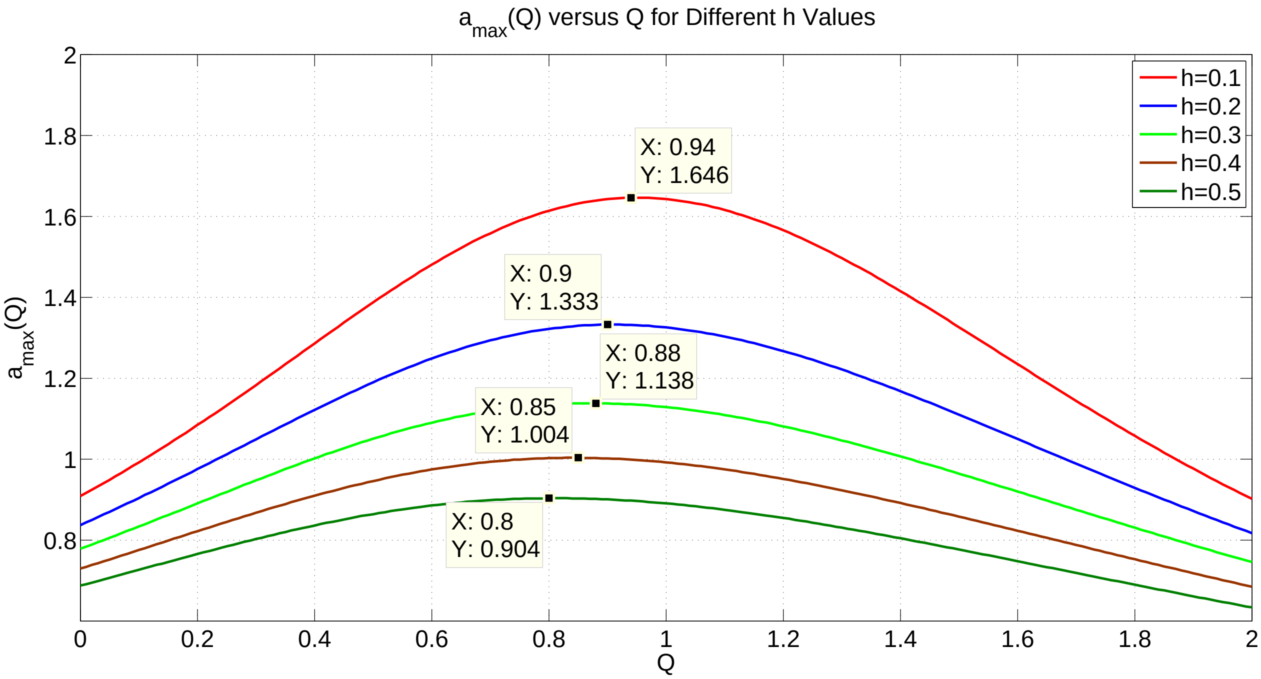

Figure 3.24: $a_{max}(Q)$ versus Q graph for $P_4(s)$ when h is between 0.1 sec and 0.5 sec.

As it is stated $P_4(s) = \dfrac{e^{-hs}}{(s - 10^{-3})(s^2 + s + 1)}$ so

$$p = 10^{-3} = 0.001 \text{ and } G(s) = \frac{e^{-hs}}{(s^2 + s + 1)}$$

$$G(0) = 1.$$

1. For $h = 0.1$ sec.

$$Q_{opt} = 0.94 \text{ and } a_0 = 1.646.$$

The allowable range of the controller gain K_p is:

$$0.001 < K_p < 1.647.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.001(0.001 + 1.646)} = 0.041$$

$$K_{d,GMopt} = Q_{opt} K_{p,GMopt} = 0.94 \times 0.041 = 0.038.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.038s + 0.041. \tag{3.56}$$

2. For $h = 0.2$ sec.

$$Q_{opt} = 0.9 \text{ and } a_0 = 1.333.$$

The allowable range of the controller gain K_p is:

$$0.001 < K_p < 1.334.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.001(0.001 + 1.333)} = 0.037$$

$$K_{d,GMopt} = Q_{opt} K_{p,GMopt} = 0.9 \times 0.037 = 0.033.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.033s + 0.037. \tag{3.57}$$

3. For $h = 0.3$ sec.

$$Q_{opt} = 0.88 \text{ and } a_0 = 1.138.$$

The allowable range of the controller gain K_p is:

$$0.001 < K_p < 0.139.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.001(0.001 + 1.138)} = 0.034$$

$$K_{d,GMopt} = Q_{opt} K_{p,GMopt} = 0.88 \times 0.034 = 0.03.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.03s + 0.034. \tag{3.58}$$

4. For $h = 0.4$ sec.

$$Q_{opt} = 0.85 \text{ and } a_0 = 1.004.$$

The allowable range of the controller gain K_p is:

$$0.001 < K_p < 1.005.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.001(0.001 + 1.004)} = 0.032$$

$$K_{d,GMopt} = Q_{opt} K_{p,GMopt} = 0.85 \times 0.032 = 0.027.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.027s + 0.032. \tag{3.59}$$

5. For $h = 0.5$ sec.

$$Q_{opt} = 0.8 \text{ and } a_0 = 0.904.$$

The allowable range of the controller gain K_p is:

$$0.001 < K_p < 0.905.$$

Gain margin optimizing PD controller parameters are:

$$K_{p,GMopt} = \sqrt{0.001(0.001 + 0.905)} = 0.03$$

$$K_{d,GMopt} = Q_{opt} K_{p,GMopt} = 0.8 \times 0.03 = 0.024.$$

So, the gain margin optimizing PD controller is:

$$C(s) = 0.024s + 0.03. \tag{3.60}$$

3.3 Design Method-III: Design of a PD Controller Minimizing Fragility in the Proportional Gain Given in [1]

In [1] derivative action $\tilde{K}_d = K_d/K_p = Q$ is optimized so that allowable region for the proportional gain is maximized, and the midpoint of this region is selected so that least fragile proportional gain is used in the controller. The following is a brief summary of the steps for calculating this controller.

All steps before calculating gain margin optimizing K_d and K_p pair in design method-II also hold for this design method. After finding a_0 in (3.50), by defining:

$$K_{p,LF} = (p + \frac{a_0}{2})G(0)^{-1}$$

$$K_{d,LF} = Q_{opt}(p + \frac{a_0}{2})G(0)^{-1},$$

the least fragile proportional gain $K_{p,LF}$ and the corresponding derivative gain $K_{d,LF}$ can be found.

3.3.1 PD Controllers for Minimizing Fragility in the Proportional Gain for $P_2(s)$

Recall that $P_2(s) = \dfrac{e^{-hs}}{(s-0.5)(s^2+s+1)}$.

So, define

$$p = 0.5 \text{ and } G(s) = \frac{e^{-hs}}{(s^2+s+1)}$$

$$G(0) = 1.$$

1. For $h = 0.1$ sec.

$$Q_{opt} = 1.001 \text{ and } a_0 = 0.519.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.5 + \frac{0.519}{2}) = 0.76$$

$$K_{d,LF} = 1.001 \times 0.76 = 0.76.$$

The resulting controller is:

$$C(s) = 0.76s + 0.76. \tag{3.61}$$

2. For $h = 0.2$ sec.

$$Q_{opt} = 0.9991 \text{ and } a_0 = 0.396.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.5 + \frac{0.396}{2}) = 0.698$$

$$K_{d,LF} = 0.9991 \times 0.698 = 0.697.$$

The resulting controller is:

$$C(s) = 0.697s + 0.698. \tag{3.62}$$

3. For $h = 0.3$ sec.

$$Q_{opt} = 0.9653 \text{ and } a_0 = 0.306.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.5 + \frac{0.306}{2}) = 0.653$$

$$K_{d,LF} = 0.9653 \times 0.653 = 0.630.$$

The resulting controller is:

$$C(s) = 0.630s + 0.653. \tag{3.63}$$

4. For $h = 0.4$ sec.

$$Q_{opt} = 0.904 \text{ and } a_0 = 0.239.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.5 + \frac{0.239}{2}) = 0.620$$

$$K_{d,LF} = 0.904 \times 0.620 = 0.560.$$

The resulting controller is:

$$C(s) = 0.560s + 0.620. \tag{3.64}$$

5. For $h = 0.5$ sec.

$$Q_{opt} = 0.9009 \text{ and } a_0 = 0.185.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.5 + \frac{0.185}{2}) = 0.593$$

$$K_{d,LF} = 0.9009 \times 0.593 = 0.533.$$

The resulting controller is:

$$C(s) = 0.533s + 0.593. \tag{3.65}$$

3.3.2 PD Controllers for Minimizing Fragility in the Proportional Gain for $P_3(s)$

Recall that $P_3(s) = \dfrac{e^{-hs}}{s(s^2 + s + 1)}$.

So, define

$$p = 0 \text{ and } G(s) = \frac{e^{-hs}}{(s^2 + s + 1)}$$

$$G(0) = 1.$$

1. For $h = 0.1$ sec.

$$Q_{opt} = 0.94 \text{ and } a_0 = 1.649.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (\frac{1.649}{2}) = 0.825$$

$$K_{d,LF} = 0.94 \times 0.825 = 0.775.$$

The resulting controller is:

$$C(s) = 0.775s + 0.825. \tag{3.66}$$

2. For $h = 0.2$ sec.

$$Q_{opt} = 0.91 \text{ and } a_0 = 1.335.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (\frac{1.335}{2}) = 0.668$$

$$K_{d,LF} = 0.91 \times 0.668 = 0.607.$$

The resulting controller is:

$$C(s) = 0.607s + 0.668. \tag{3.67}$$

3. For $h = 0.3$ sec.

$$Q_{opt} = 0.87 \text{ and } a_0 = 1.141.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (\frac{1.141}{2}) = 0.571$$

$$K_{d,LF} = 0.87 \times 0.571 = 0.496.$$

The resulting controller is:

$$C(s) = 0.496s + 0.571. \tag{3.68}$$

4. For $h = 0.4$ sec.

$$Q_{opt} = 0.85 \text{ and } a_0 = 1.005.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (\frac{1.005}{2}) = 0.503$$

$$K_{d,LF} = 0.85 \times 0.503 = 0.427.$$

The resulting controller is:

$$C(s) = 0.427s + 0.503. \tag{3.69}$$

5. For $h = 0.5$:

$$Q_{opt} = 0.81 \text{ and } a_0 = 0.905.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (\frac{0.905}{2}) = 0.453$$

$$K_{d,LF} = 0.81 \times 0.453 = 0.367.$$

The resulting controller is:

$$C(s) = 0.367s + 0.453. \tag{3.70}$$

3.3.3 PD Controllers for Minimizing Fragility in the Proportional Gain for $P_4(s)$

Recall that $P_4(s) = \dfrac{e^{-hs}}{(s-10^{-3})(s^2+s+1)}$.

So, define

$$p = 10^{-3} = 0.001 \text{ and } G(s) = \frac{e^{-hs}}{(s^2+s+1)}$$

$$G(0) = 1.$$

1. For $h = 0.1$ sec.

$$Q_{opt} = 0.94 \text{ and } a_0 = 1.646.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.001 + \frac{1.646}{2}) = 0.824$$

$$K_{d,LF} = 0.94 \times 0.824 = 0.775.$$

The resulting controller is:

$$C(s) = 0.775s + 0.824. \tag{3.71}$$

2. For $h = 0.2$ sec.

$$Q_{opt} = 0.9 \text{ and } a_0 = 1.333.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.001 + \frac{1.333}{2}) = 0.668$$
$$K_{d,LF} = 0.9 \times 0.668 = 0.601.$$

The resulting controller is:

$$C(s) = 0.601s + 0.668. \tag{3.72}$$

3. For $h = 0.3$ sec.

$$Q_{opt} = 0.88 \text{ and } a_0 = 1.138.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.001 + \frac{1.138}{2}) = 0.570$$
$$K_{d,LF} = 0.88 \times 0.57 = 0.502.$$

The resulting controller is:

$$C(s) = 0.502s + 0.570. \tag{3.73}$$

4. For $h = 0.4$ sec.

$$Q_{opt} = 0.85 \text{ and } a_0 = 1.004.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.001 + \frac{1.004}{2}) = 0.503$$

$$K_{d,LF} = 0.85 \times 0.503 = 0.428.$$

The resulting controller is:

$$C(s) = 0.428s + 0.503. \tag{3.74}$$

5. For $h = 0.5$ sec.

$$Q_{opt} = 0.8 \text{ and } a_0 = 0.904.$$

The PD controller for this design is obtained with the gains:

$$K_{p,LF} = (0.001 + \frac{0.904}{2}) = 0.453$$

$$K_{d,LF} = 0.8 \times 0.453 = 0.362.$$

The resulting controller is:

$$C(s) = 0.362s + 0.453. \tag{3.75}$$

3.4 Comparison of Obtained PD Controllers and Allowable K_p Ranges

3.4.1 Obtained PD Controllers for $P_1(s)$

For each h value, the least fragile PD controllers for $P_1(s)$ which are found by design method-I are listed in Table 3.1.

	Design Method-I
$h = 0.1$	$9.5s + 25.1$
$h = 0.2$	$4.7s + 5.78$
$h = 0.3$	$3.1s + 2.3$
$h = 0.4$	$2.305s + 1.15$
$h = 0.5$	$1.85s + 0.68$

Table 3.1: Obtained PD Controllers for $P_1(s)$ with Design Method-I

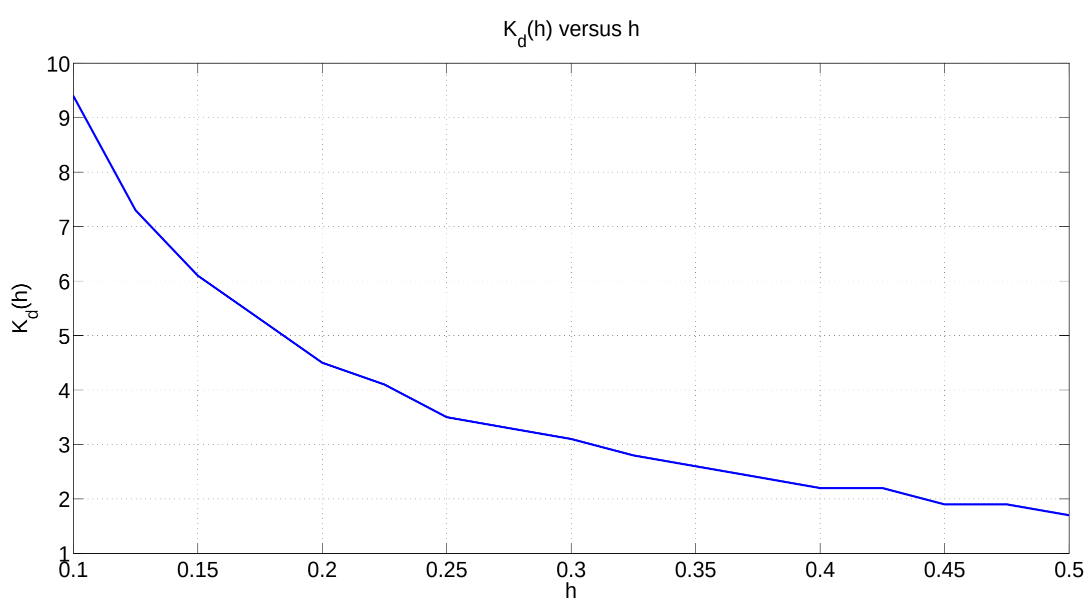

Figure 3.25: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-I.

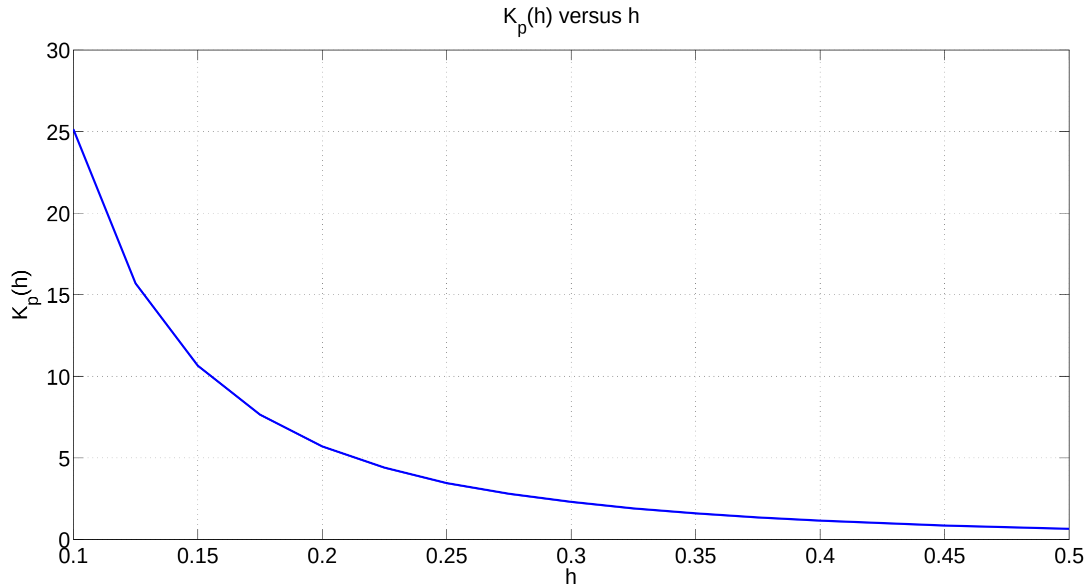

Figure 3.26: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-I.

As can be seen, both K_d and K_p values are getting smaller as h increases but K_p have a more significant reduction compared to K_d.

3.4.2 Obtained PD Controllers and Allowable K_p Ranges for $P_2(s)$

For each h value, the range for allowable K_p for $P_2(s)$ which are found by design method-I and design method-II are listed in Table 3.2. Also, for each h value, least fragile PD controllers which are found by design method-I, gain margin optimizing PD controllers which are found by design method-II and least fragile PD controllers which are found by design method-III for $P_2(s)$ are listed in Table 3.3.

	Design Method-I	Design Method-II
$h = 0.1$	$0.5 < K_p < 1.23$	$0.5 < K_p < 1.019$
$h = 0.2$	$0.5 < K_p < 0.91$	$0.5 < K_p < 0.896$
$h = 0.3$	$0.5 < K_p < 0.80$	$0.5 < K_p < 0.806$
$h = 0.4$	$0.5 < K_p < 0.74$	$0.5 < K_p < 0.739$
$h = 0.5$	$0.5 < K_p < 0.71$	$0.5 < K_p < 0.685$

Table 3.2: Allowable Ranges for K_p by Design Method I and II for $P_2(s)$

	Design Method-I	Design Method-II	Design Method-III
$h = 0.1$	$2.35s + 0.87$	$0.715s + 0.714$	$0.76s + 0.76$
$h = 0.2$	$1.14s + 0.71$	$0.669s + 0.669$	$0.697s + 0.698$
$h = 0.3$	$0.76s + 0.65$	$0.613s + 0.635$	$0.630s + 0.653$
$h = 0.4$	$0.6s + 0.63$	$0.550s + 0.608$	$0.560s + 0.620$
$h = 0.5$	$0.51s + 0.61$	$0.527s + 0.585$	$0.533s + 0.593$

Table 3.3: Obtained PD Controllers for $P_2(s)$

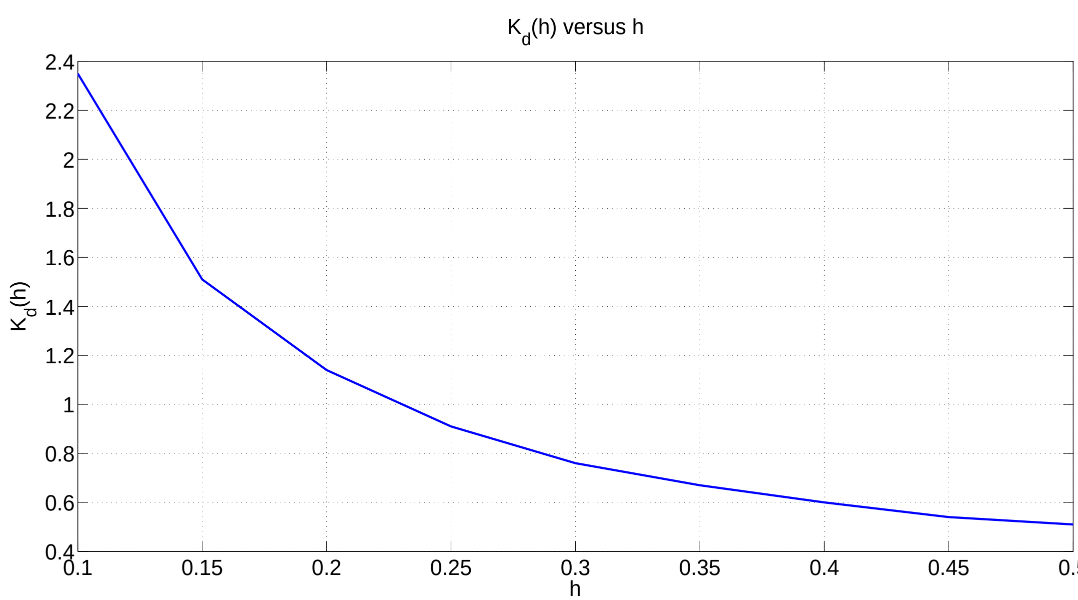

Figure 3.27: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-I.

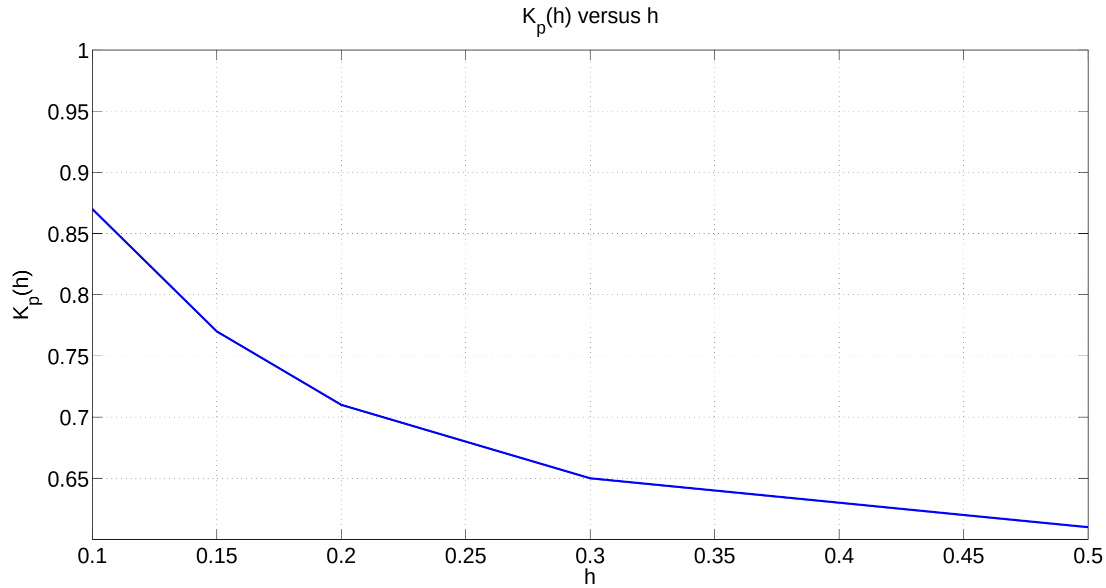

Figure 3.28: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-I.

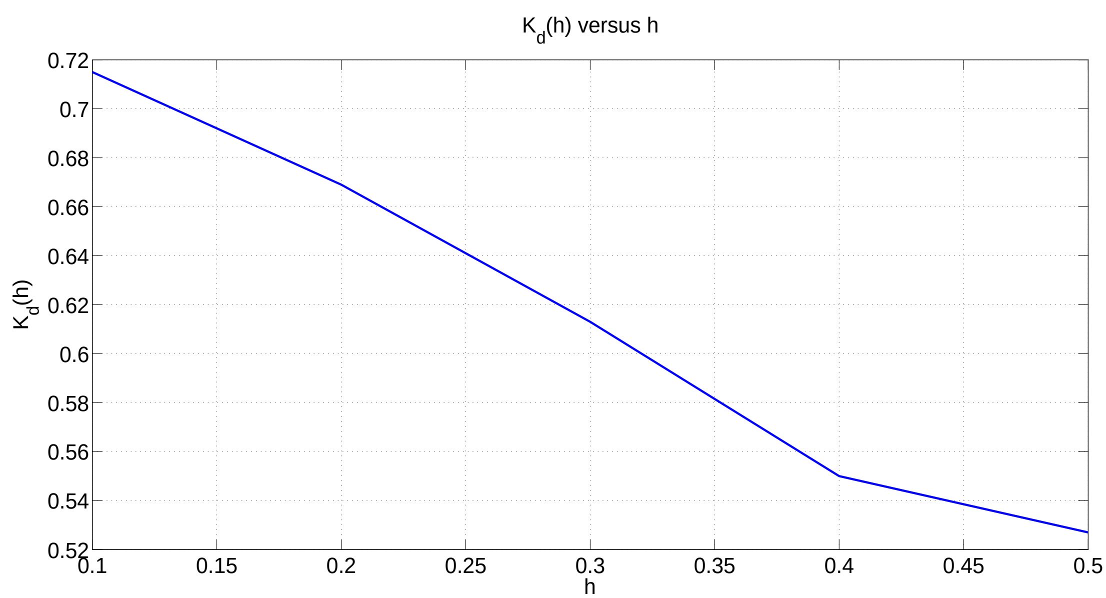

Figure 3.29: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-II.

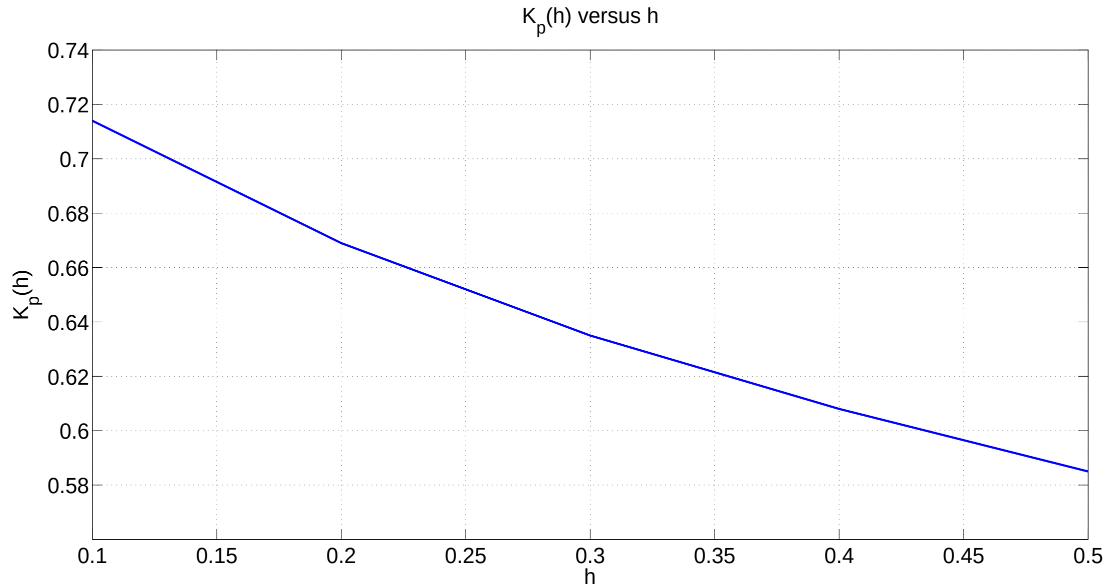

Figure 3.30: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-II.

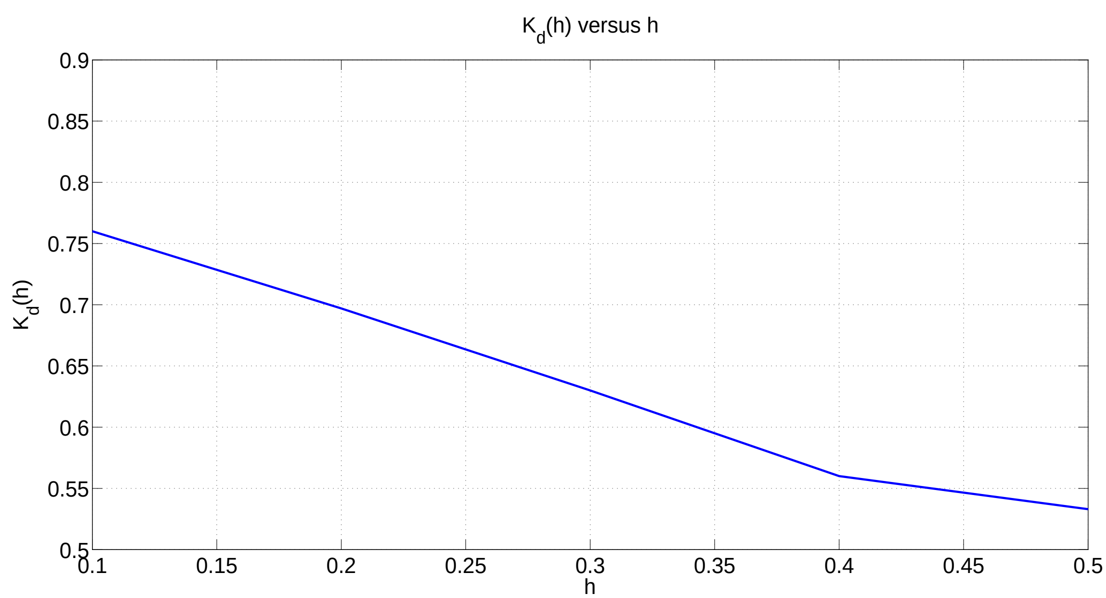

Figure 3.31: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-III.

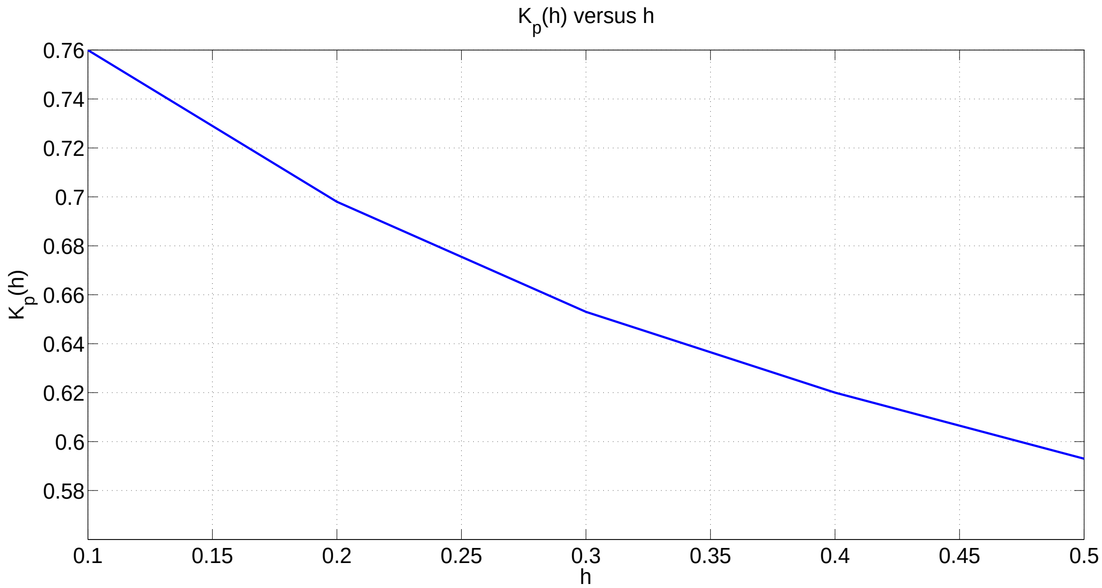

Figure 3.32: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-III.

From Table 3.2, it can be concluded that as h increases, the upper limits for K_p for design method-I and design method-II become closer to each other. The lower limits for K_p are equal to each other (unstable pole value) in two design methods and do not change with different h values.

When Table 3.3 is considered, it can be seen that as h increases, K_d and K_p values in all methods decrease. Also, K_d values in each design method become closer to each other as h increases. Same situation holds for K_p values.

3.4.3 Obtained PD Controllers and Allowable K_p Ranges for $P_3(s)$

For each h value, the ranges for allowable K_p for $P_3(s)$ which are found by design method-I and design method-II are listed in Table 3.4. Also, for each h value, least fragile PD controllers which are found by design method-I and least fragile PD controllers which are found by design method-III for $P_3(s)$ can be seen in

Table 3.5.

	Design Method-I	Design Method-II
$h = 0.1$	$0 < K_p < 2.90$	$0 < K_p < 1.649$
$h = 0.2$	$0 < K_p < 1.65$	$0 < K_p < 1.335$
$h = 0.3$	$0 < K_p < 1.24$	$0 < K_p < 1.141$
$h = 0.4$	$0 < K_p < 1.03$	$0 < K_p < 1.005$
$h = 0.5$	$0 < K_p < 0.91$	$0 < K_p < 0.905$

Table 3.4: Allowable Ranges for K_p by Design Method I and II for $P_3(s)$

	Design Method-I	Design Method-III
$h = 0.1$	$4.76s + 1.48$	$0.775s + 0.825$
$h = 0.2$	$2.24s + 0.83$	$0.607s + 0.668$
$h = 0.3$	$1.42s + 0.62$	$0.496s + 0.571$
$h = 0.4$	$1.04s + 0.51$	$0.427s + 0.503$
$h = 0.5$	$0.80s + 0.45$	$0.367s + 0.453$

Table 3.5: Obtained PD Controllers for $P_3(s)$

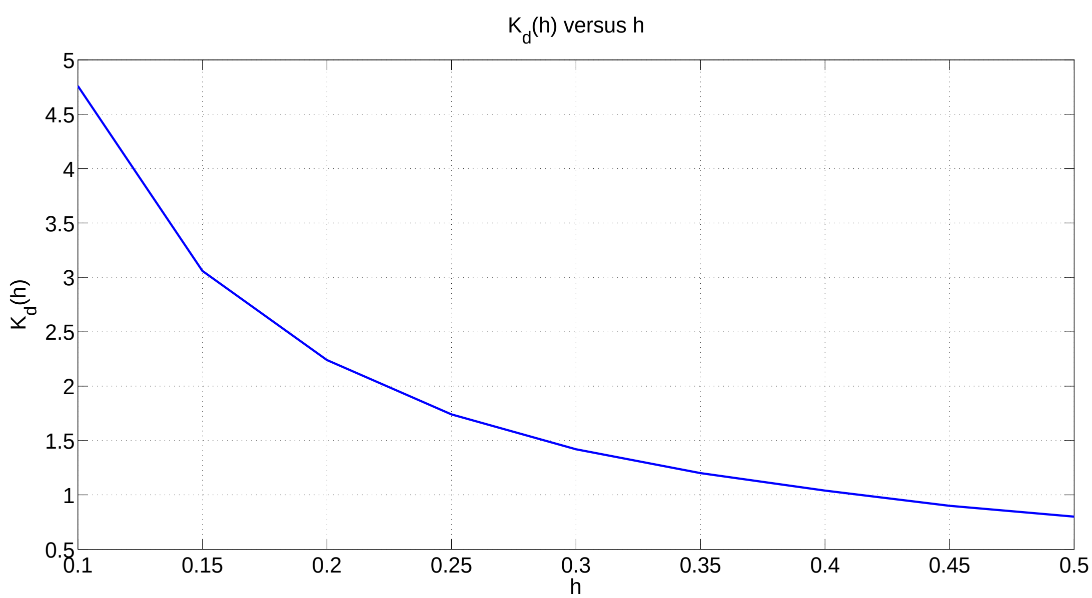

Figure 3.33: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-I.

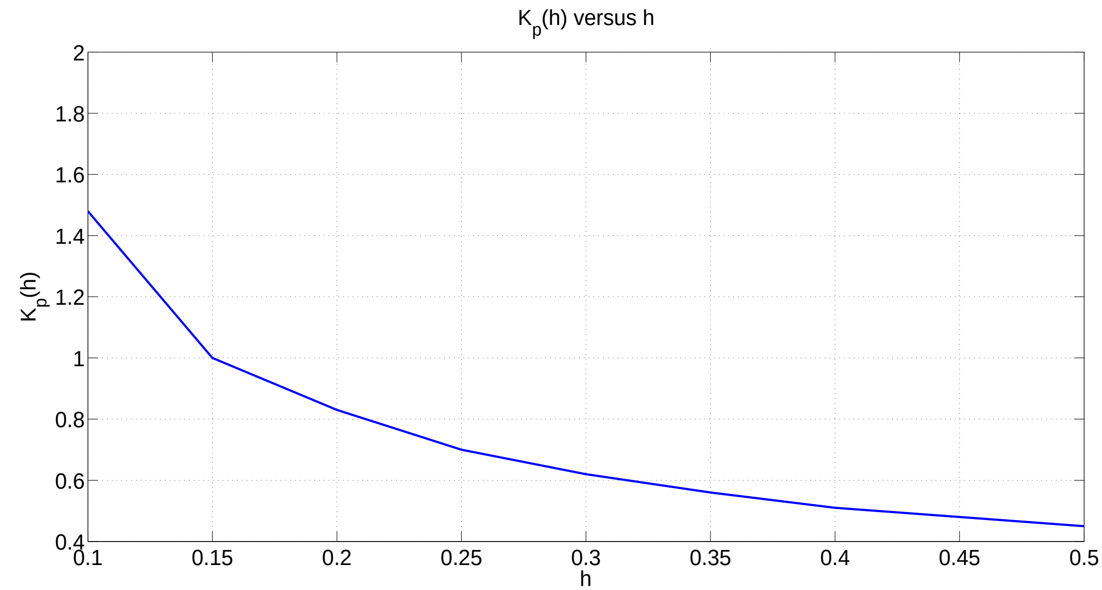

Figure 3.34: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-I.

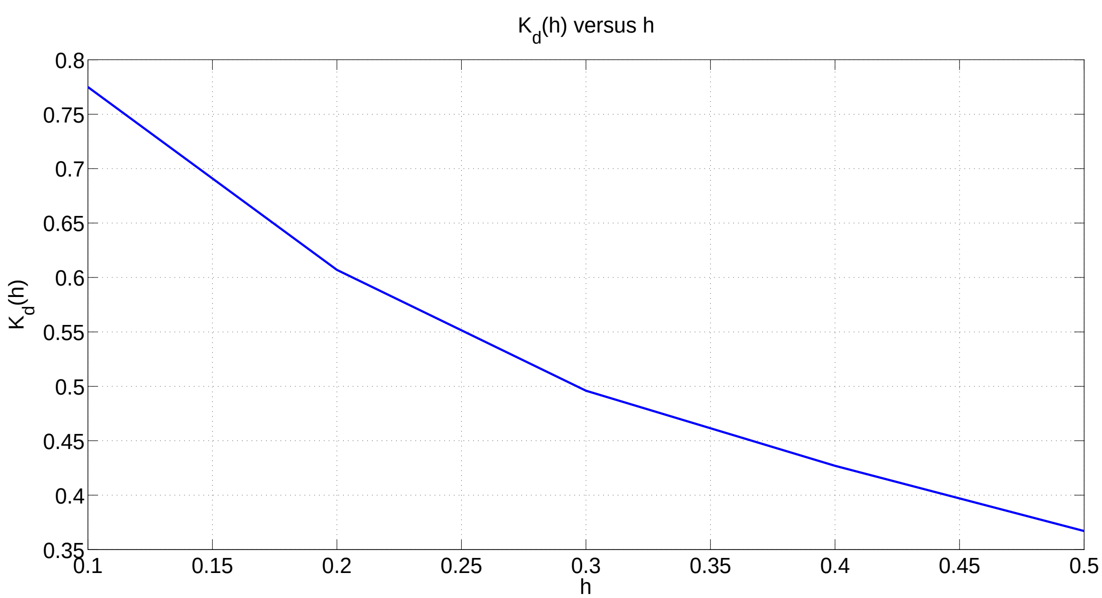

Figure 3.35: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-III.

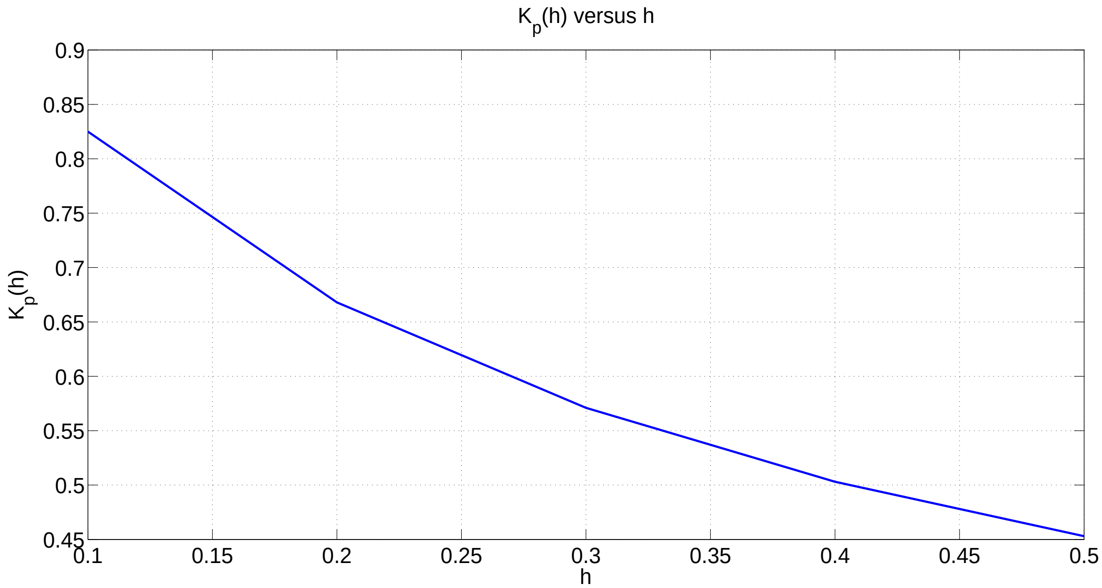

Figure 3.36: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-III.

From Table 3.4, again it can be concluded that as h increases, the upper limits for K_p for design method-I and design method-II become closer to each other and become nearly the same at $h = 0.5$ sec. The lower limits for K_p are equal to each other in two design methods and do not change with different h values, zero is the value of the unstable pole of $P_3(s)$.

When Table 3.5 is considered, it can be seen that as h increases, again K_d and K_p values in both methods decrease. In addition, K_p values in each design method become closer to each other as h increases.

If we compare Table 3.3 and 3.5, it can be concluded that as h increases, obtained PD controllers of $P_2(s)$ in each method becomes significantly equal to each other when compared with $P_3(s)$.

3.4.4 Obtained PD Controllers and Allowable K_p Ranges for $P_4(s)$

For each h value, the ranges for allowable K_p for $P_4(s)$ which are found by design method-I and design method-II are listed in Table 3.6. Also, for each h value, least fragile PD controllers which are found by design method-I, gain margin optimizing PD controllers which are found by design method-II and least fragile PD controllers which are found by design method-III for $P_4(s)$ are listed in Table 3.7.

	Design Method-I	Design Method-II
$h = 0.1$	$0.001 < K_p < 2.89$	$0.001 < K_p < 1.019$
$h = 0.2$	$0.001 < K_p < 1.65$	$0.001 < K_p < 0.896$
$h = 0.3$	$0.001 < K_p < 1.24$	$0.001 < K_p < 0.806$
$h = 0.4$	$0.001 < K_p < 1.03$	$0.001 < K_p < 0.739$
$h = 0.5$	$0.001 < K_p < 0.91$	$0.001 < K_p < 0.685$

Table 3.6: Allowable Ranges for K_p by Design Method I and II for $P_4(s)$

	Design Method-I	Design Method-II	Design Method-III
$h = 0.1$	$4.70s + 1.44$	$0.038s + 0.041$	$0.775s + 0.824$
$h = 0.2$	$2.25s + 0.83$	$0.033s + 0.037$	$0.601s + 0.668$
$h = 0.3$	$1.43s + 0.62$	$0.03s + 0.034$	$0.502s + 0.570$
$h = 0.4$	$1.04s + 0.51$	$0.027s + 0.032$	$0.428s + 0.503$
$h = 0.5$	$0.79s + 0.45$	$0.024s + 0.030$	$0.362s + 0.453$

Table 3.7: Obtained PD Controllers for $P_4(s)$

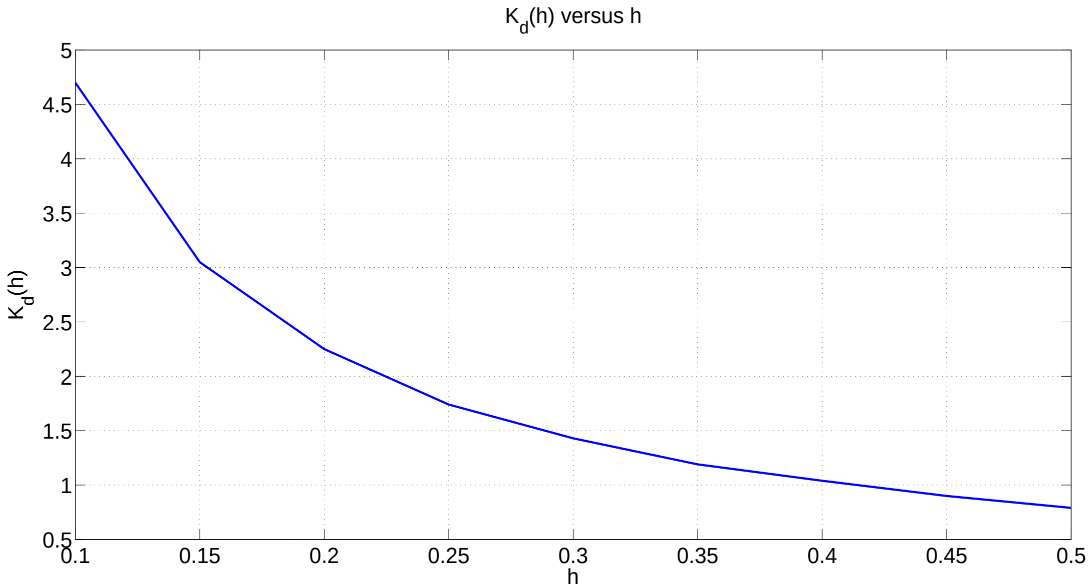

Figure 3.37: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-I.

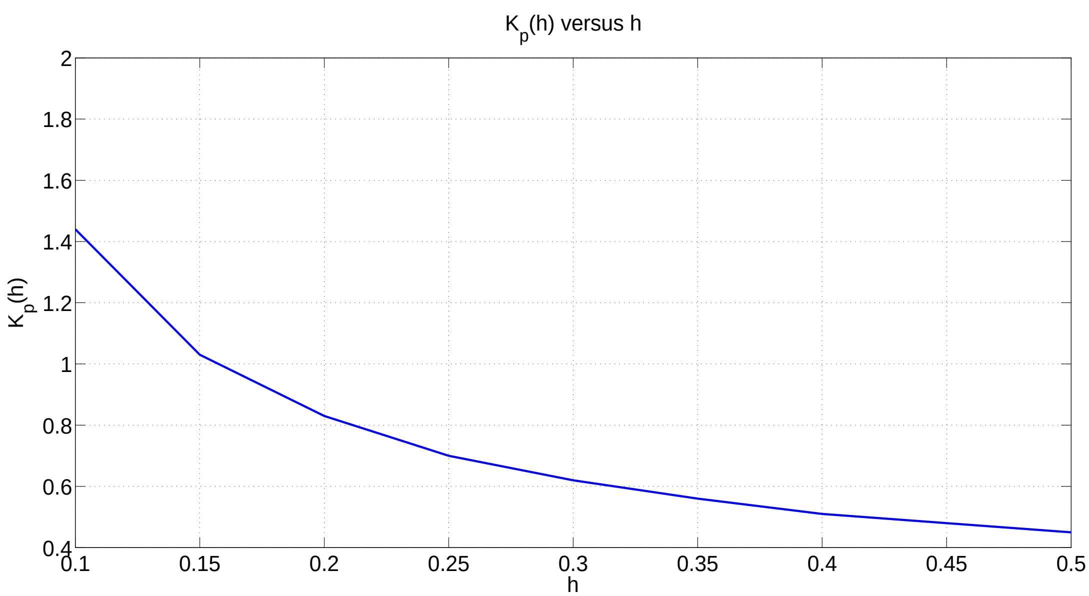

Figure 3.38: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-I.

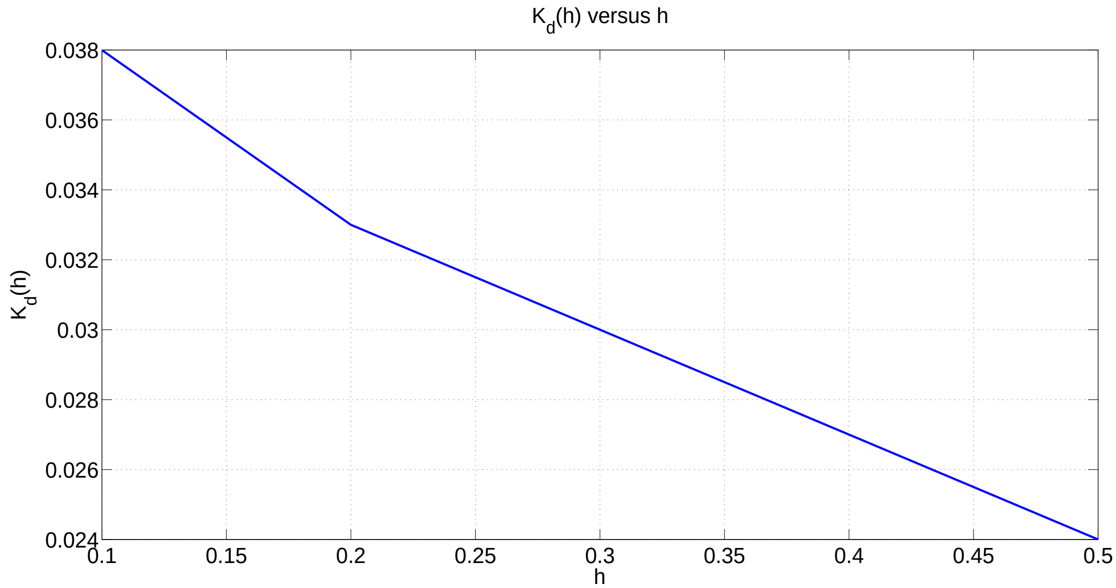

Figure 3.39: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-II.

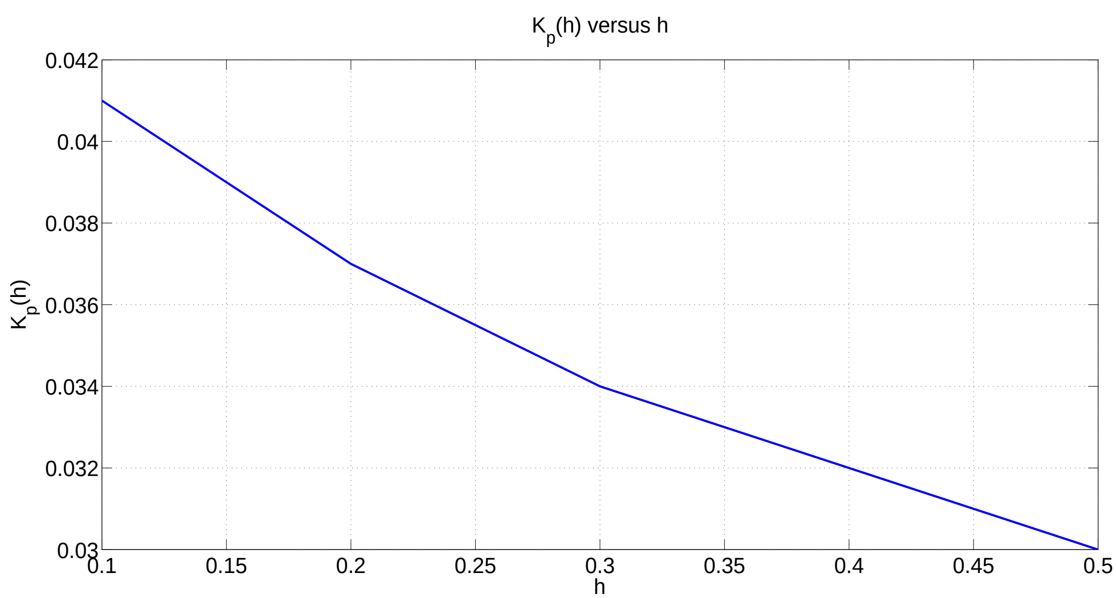

Figure 3.40: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-II.

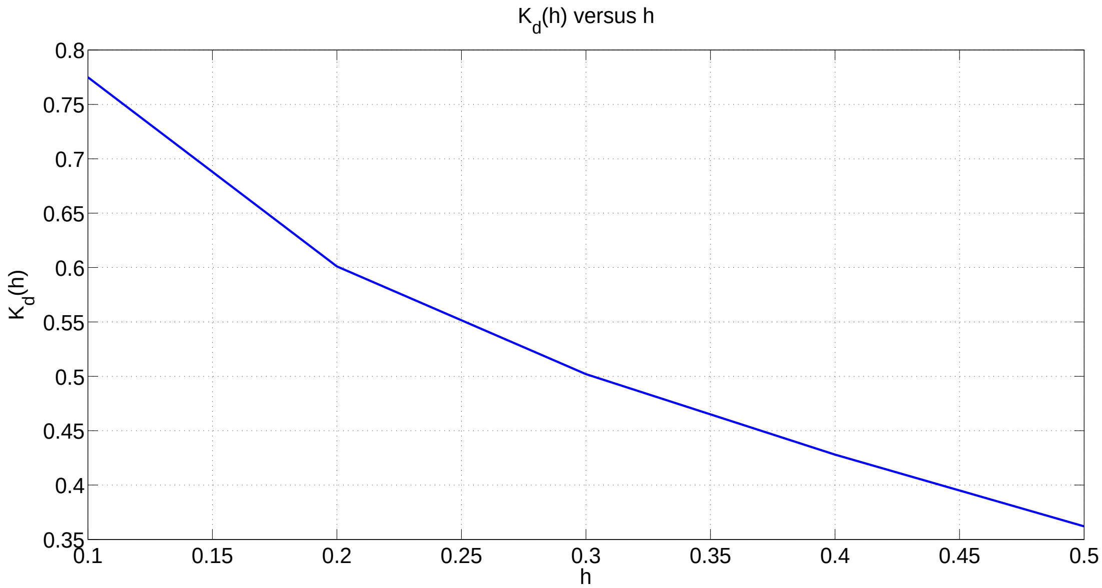

Figure 3.41: Stabilizing $K_d(h)$ versus h graph when h is between 0.1-0.5 sec with design method-III.

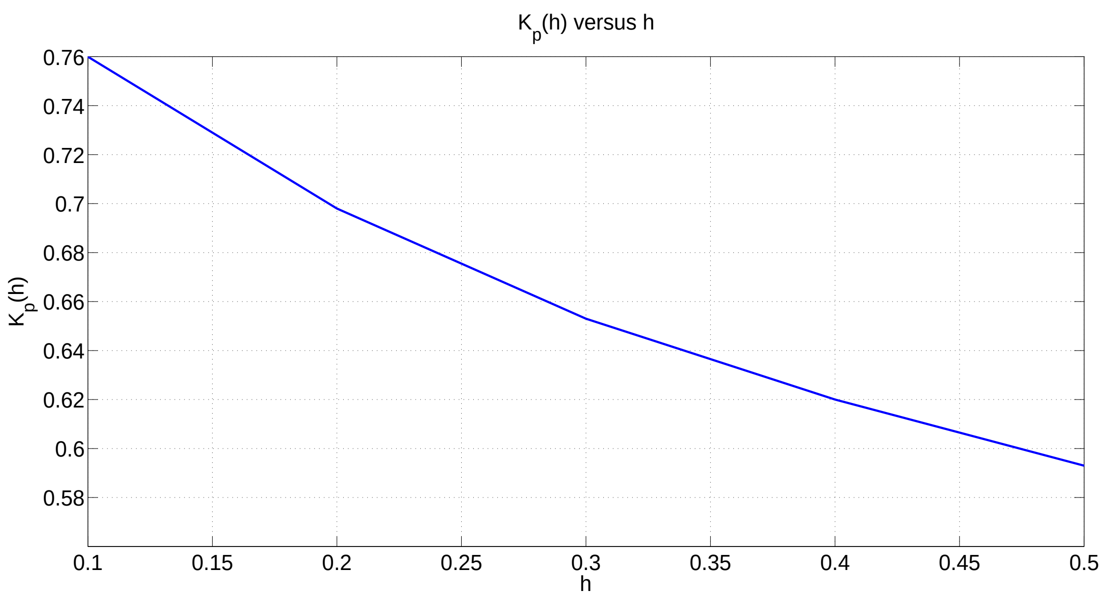

Figure 3.42: Stabilizing $K_p(h)$ versus h graph when h is between 0.1-0.5 sec with design method-III.

From Table 3.6, as in previous plants it can be concluded that as h increases, the upper limits for K_p for design method-I and design method-II become closer to each other. The lower limits for K_p are equal to each other in two design methods and do not change with different h values, 0.001 is the value of the unstable pole of $P_4(s)$.

When Table 3.7 is considered, it can be seen that as h increases, K_d and K_p values in all methods decrease. In addition, K_p values in design method-I and design method-III become closer to each other as h increases. K_d values also become closer but not as significant as K_p values. Design method-II has significantly small PD controller parameters compared to other methods because the unstable pole of $P_4(s)$ is very close to 0.

Chapter 4

TIME DOMAIN SIMULATIONS AND ROBUSTNESS RESULTS

4.1 Robust Performance Test Results with Obtained PD Controllers

It is known that the controller achieves robust performance if:

(i) The nominal feedback system (C, P) is stable, and
(ii) $\mid W_1(jw)S(jw) \mid + \mid W_2(jw)T(jw) \mid \leq 1$ for all w where $W_1(s)$ is the performance weight, $W_2(s)$ is the robustness weight, $S(s) = (1 + P(s)C(s))^{-1}$ is the sensitivity function and $T(s) = 1 - S(s)$ is the complementary sensitivity function.

According to [23], any stabilizing C satisfies the robust performance condition for some specially defined weights $W_1(s)$ and $W_2(s)$. In practice, $W_1(s)$ and $W_2(s)$ can be selected according to desire of assigning more weight to performance which means larger $W_1(s)$ and smaller $W_2(s)$ or assigning more weight to robustness which means smaller $W_1(s)$ and larger $W_2(s)$.

We want to investigate robust performance result of each PD controller derived for each plant. It is already known that first step for robust performance is satisfied as found controllers stabilize the plants. For the second condition, the inequality is modified as:

$$\rho \mid W_1(jw)S(jw) \mid +k \mid W_2(jw)T(jw) \mid \leq 1$$

and $W_1(s)$ and $W_2(s)$ are selected as:

$$W_1 = \frac{0.01}{s}$$

$$W_2 = 0.01s.$$

The reason for adding ρ and k to the inequality is that their values are going to show the performance quality of each feedback system. For each (C,P) the largest ρ (ρ_{max}) is found when $k = 1$. After that, the largest k (k_{max}) is found when $\rho = 1$. Obtained ρ_{max} and k_{max} values for $P_1(s)$, $P_3(s)$ and $P_4(s)$ are compared for each delay and design methods. As $P_2(s)$ and PD controllers don't have a pole at $s = 0$, the robust performance test does not applied to $P_2(s)$ in this thesis.

4.1.1 Robust Performance Test Results for $P_1(s)$

For each h, obtained ρ_{max} and k_{max} values for $P_1(s)$ are listed in Table 4.1. These values are found from the least fragile PD controllers of $P_1(s)$ which are obtained from design method-I.

	ρ_{max}	k_{max}
$h = 0.1$	123	2.1
$h = 0.2$	81	4.0
$h = 0.3$	55	5.9
$h = 0.4$	40	7.7
$h = 0.5$	28	8.6

Table 4.1: Robust Performance Test Results for $P_1(s)$ with PD Controllers Obtained from Design Method-I

Maximum $\mid W_1 S \mid$ values as h increases are 0.0043, 0.0093, 0.0150, 0.0215 and 0.0309, respectively. In addition, maximum $\mid W_2 T \mid$ values as h increases are 0.4712, 0.2439, 0.1651, 0.1266 and 0.1121, respectively. As can be easily seen, maximum values of $\mid W_1 S \mid$ and $\mid W_2 T \mid$ do not have a linear increase and decrease. So, the increase and decrease rate of maximum $\mid W_1 S \mid$ and maximum $\mid W_2 T \mid$ affects the increase or decrease of ρ_{max} and k_{max}. From Table 4.1, it can be seen that ρ_{max} decreases and k_{max} increases as h increases. When the increase and decrease rates of maximum $\mid W_1 S \mid$ and maximum $\mid W_2 T \mid$ are considered, this is an expected result.

4.1.2 Robust Performance Test Results for $P_3(s)$ and $P_4(s)$

For each h, obtained ρ_{max} and k_{max} values for $P_3(s)$ (also same for $P_4(s)$ as two plants are almost equal) are listed in Table 4.2. These values are found from the least fragile PD controllers of $P_3(s)$ which are obtained from design method-I, gain margin optimizing PD controllers of $P_4(s)$ which are obtained from design method-II and least fragile PD controllers of $P_3(s)$ which are obtained from design method-III.

	Design Method-I		Design Method-II		Design Method-III	
	ρ_{max}	k_{max}	ρ_{max}	k_{max}	ρ_{max}	k_{max}
$h = 0.1$	23	5	4.1	1600	44	38
$h = 0.2$	30	12	3.7	1800	46	43
$h = 0.3$	34	20	3.4	1900	41.8	54
$h = 0.4$	37	29	3.2	2100	41	62
$h = 0.5$	38	38	3	2300	40	85

Table 4.2: Robust Performance Test Results for $P_3(s)$ and $P_4(s)$

For design method-I, maximum $\mid W_1 S \mid$ values as h increases are 0.0349, 0.0302, 0.0276, 0.0261 and 0.0253 and maximum $\mid W_2 T \mid$ values as h increases are 0.1818, 0.0786, 0.0473, 0.0333 and 0.0250, respectively. As both maximum $\mid W_1 S \mid$ and $\mid W_2 T \mid$ decrease as h decreases, it is expected that ρ_{max} and k_{max} increase with the increase of h. From Table 4.2, it can be seen that expected results are obtained.

For design method-II, maximum $\mid W_1 S \mid$ values as h increases are 0.2439, 0.2703, 0.2941, 0.3125 and 0.3333 and maximum $\mid W_2 T \mid$ values as h increases are 6.1×10^{-4}, 5.4×10^{-4}, 5×10^{-4}, 4.6×10^{-4} and 4.2×10^{-4}, respectively. Maximum $\mid W_1 S \mid$ values increase but maximum $\mid W_2 T \mid$ values decrease as h increases. From Table 4.2, it can be seen that ρ_{max} decreases and k_{max} increases as h increases. When the increase and decrease rates of $\mid W_1 S \mid$ and $\mid W_2 T \mid$ are considered, this is an expected result. Also, as maximum $\mid W_2 T \mid$ values are relatively very small, k_{max} values are expected to be relatively high which is also an obtained result.

For design method-III, maximum $\mid W_1 S \mid$ values as h increases are 0.0217, 0.0227, 0.0233, 0.0240 and 0.0247 and maximum $\mid W_2 T \mid$ values as h increases are 0.0253, 0.0210, 0.0180, 0.0157 and 0.0138, respectively. When the increase and decrease rates of maximum $\mid W_1 S \mid$ and $\mid W_2 T \mid$ are considered, it is expected that ρ_{max} increases until $h = 2$ sec but starts to decrease after $h = 2$ sec but k_{max} is expected to increase as h increases. Table 4.2 proves that the expectations are corrrect.

When ρ_{max} and k_{max} values for each design method are compared, it can be concluded that best performance (largest ρ_{max}), i.e., the best time tracking performance is obtained with design method-III and best robustness (largest k_{max}) is obtained with design method-II.

4.2 Step Response Graphs for $P_1(s)$, $P_3(s)$ and $P_4(s)$ with Obtained PD Controllers

In this section, step responses of $P_1(s)$, $P_3(s)$ and $P_4(s)$ are obtained for different h values with PD controllers found by design methods. As $P_2(s)$ and PD controllers do not have a pole at $s = 0$, it is expected that the step responses of $P_2(s)$ will have a steady state error so the step responses of $P_2(s)$ are not included in the thesis. On the other hand, $P_1(s)$, $P_3(s)$ and $P_4(s)$ have zero (or negligible errors) steady-state errors as expected (due to a pole at $s = 0$ or nearby).

4.2.1 Step response Graphs for $P_1(s)$

1. For $h = 0.1$ sec.

The step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.2) is obtained as:

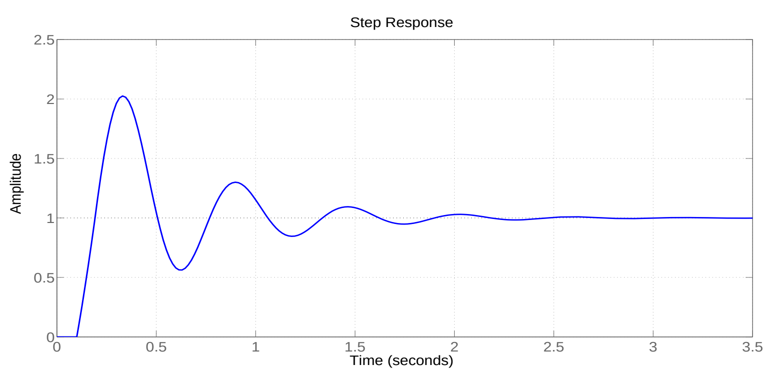

Figure 4.1: Step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.2) when $h = 0.1$ sec.

2. For $h = 0.2$ sec.

The step response of the feedback system with plant $P_1(s)$ and the controller

$C(s)$ in (3.13) is obtained as:

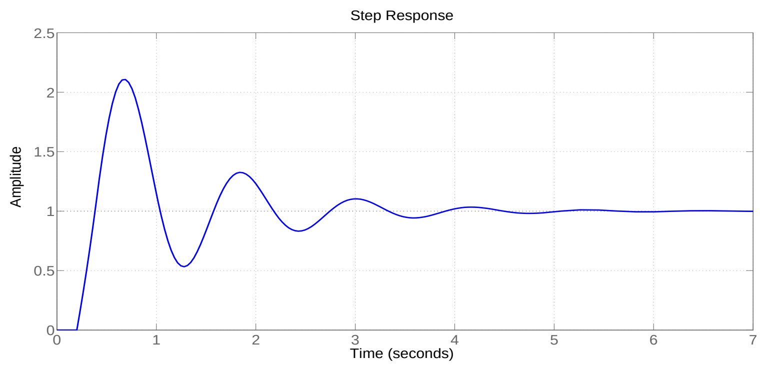

Figure 4.2: Step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.13) when $h = 0.2$ sec.

3. For $h = 0.3$ sec.

The step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.15) is obtained as:

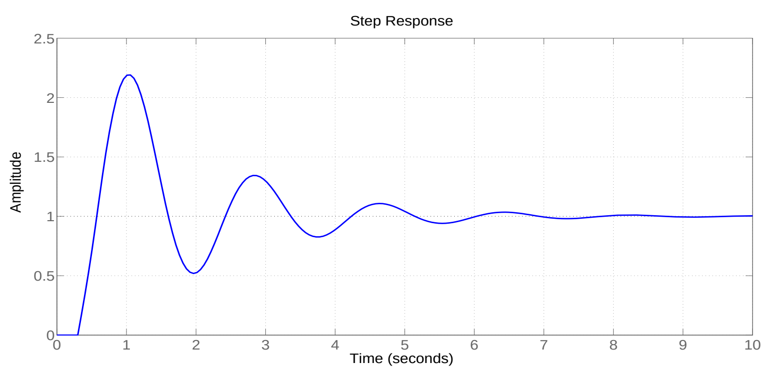

Figure 4.3: Step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.15) when $h = 0.3$ sec.

4. For $h = 0.4$ sec.

The step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.17) is obtained as:

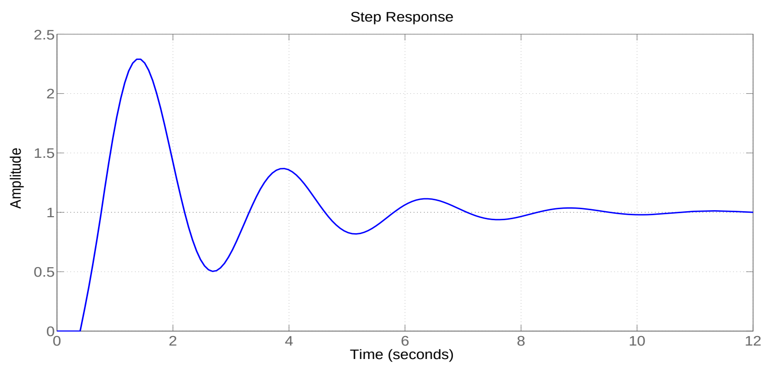

Figure 4.4: Step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.17) when $h = 0.4$ sec.

5. For $h = 0.5$ sec.

The step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.19) is obtained as:

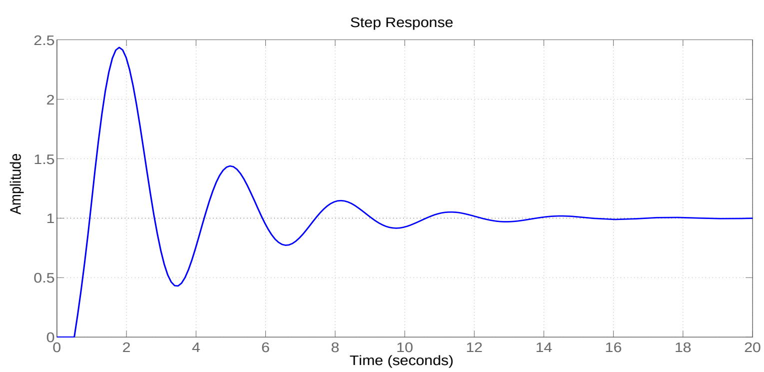

Figure 4.5: Step response of the feedback system with plant $P_1(s)$ and the controller $C(s)$ in (3.19) when $h = 0.5$ sec.

After examining the figures for $P_1(s)$, it can be concluded that when h is increasing, maximum overshoot and settling time also increase as expected which means performance of the system become worse with the increase of time delay. In addition, the oscillation amounts do not change with different h values.

4.2.2 Step response Graphs for $P_3(s)$

1. For $h = 0.1$ sec.

   - Design Method-I

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.31) is obtained as:

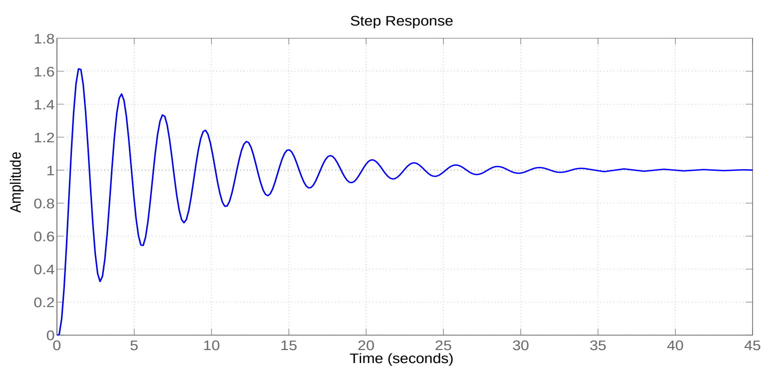

Figure 4.6: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.31) when $h = 0.1$ sec.

- Design Method-II

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.56) is obtained as:

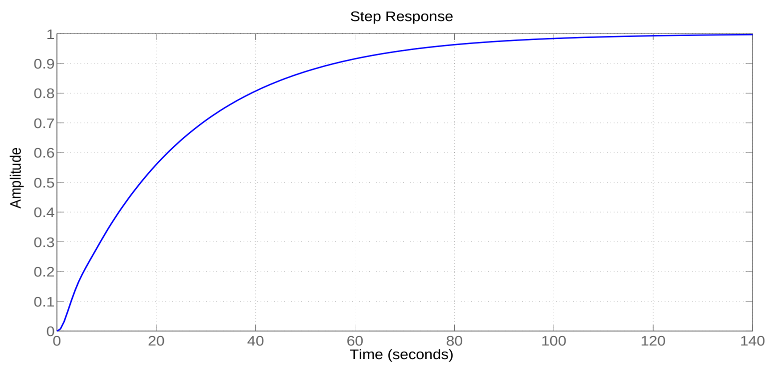

Figure 4.7: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.56) when $h = 0.1$ sec.

- Design Method-III

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.66) is obtained as:

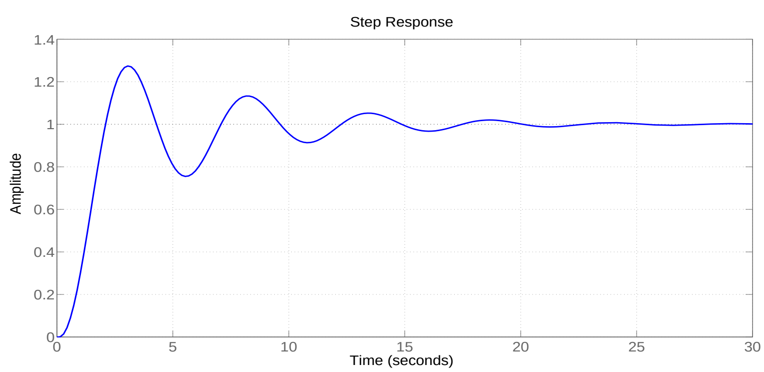

Figure 4.8: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.66) when $h = 0.1$ sec.

2. For $h = 0.2$ sec.

- Design Method-I

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.33) is obtained as:

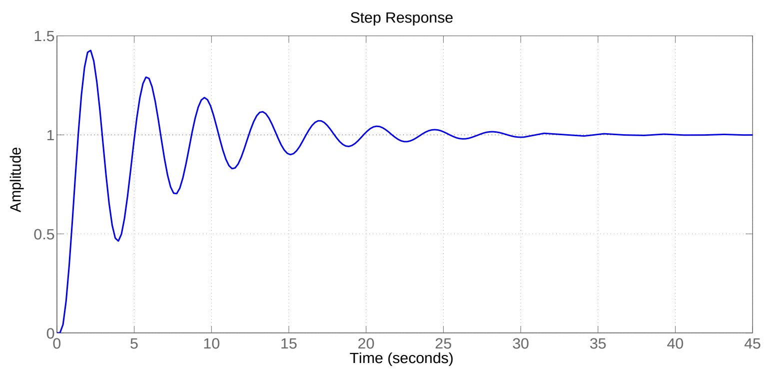

Figure 4.9: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.33) when $h = 0.2$ sec.

- Design Method-II

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.57) is obtained as:

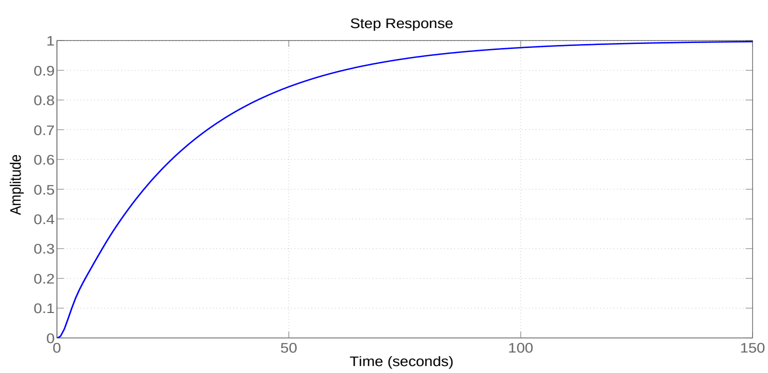

Figure 4.10: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.57) when $h = 0.2$ sec.

- Design Method-III

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.67) is obtained as:

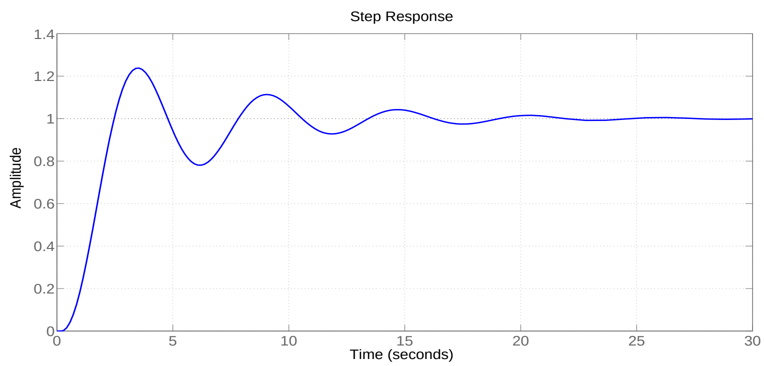

Figure 4.11: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.67) when $h = 0.2$ sec.

3. For $h = 0.3$ sec.

- Design Method-I

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.35) is obtained as:

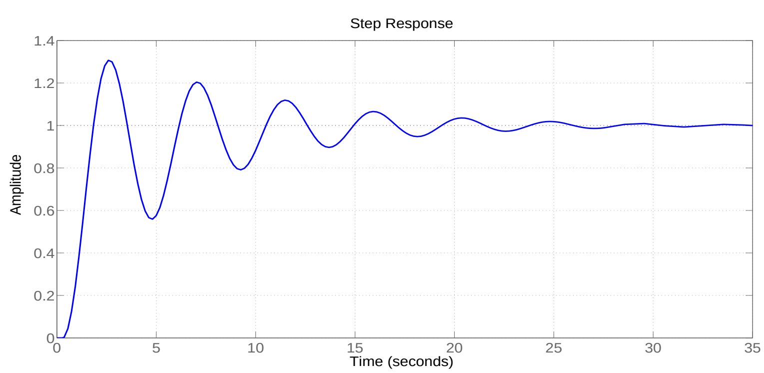

Figure 4.12: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.35) when $h = 0.3$ sec.

- Design Method-II

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.58) is obtained as:

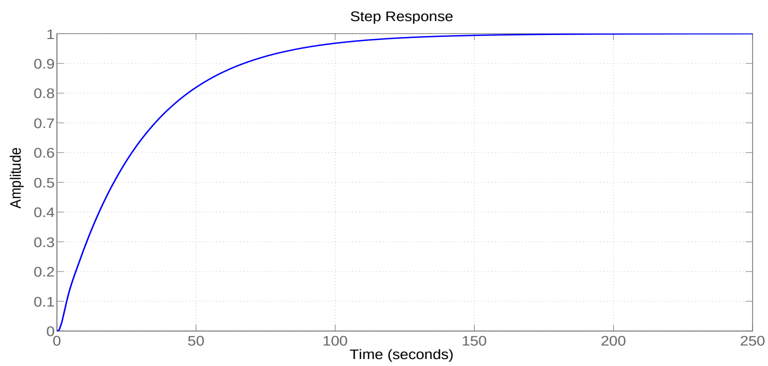

Figure 4.13: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.58) when $h = 0.3$ sec.

- Design Method-III

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.68) is obtained as:

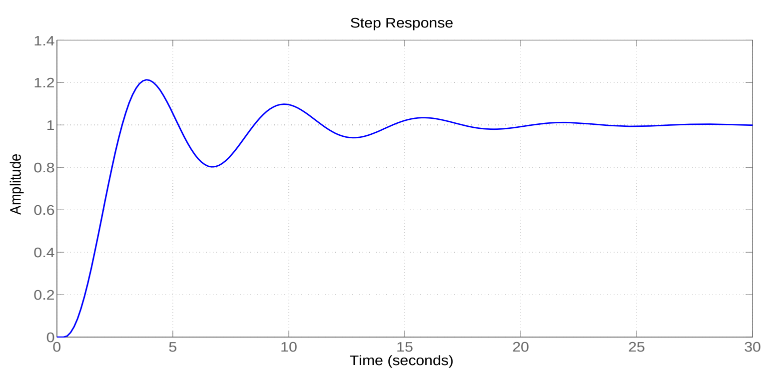

Figure 4.14: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.68) when $h = 0.3$ sec.

4. For $h = 0.4$ sec.

- Design Method-I

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.37) is obtained as:

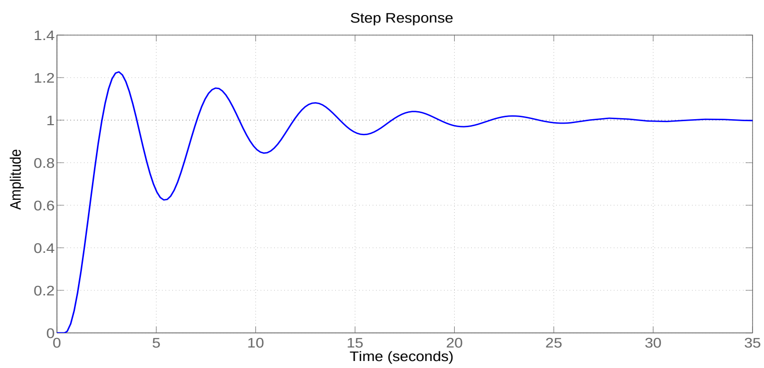

Figure 4.15: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.37) when $h = 0.4$ sec.

- Design Method-II

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.59) is obtained as:

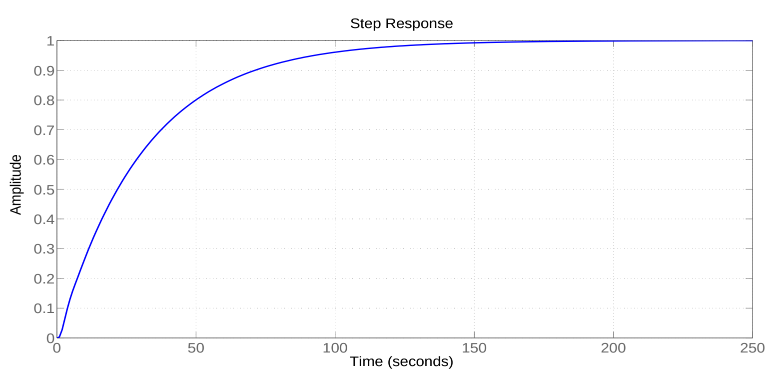

Figure 4.16: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.59) when $h = 0.4$ sec.

- Design Method-III

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.69) is obtained as:

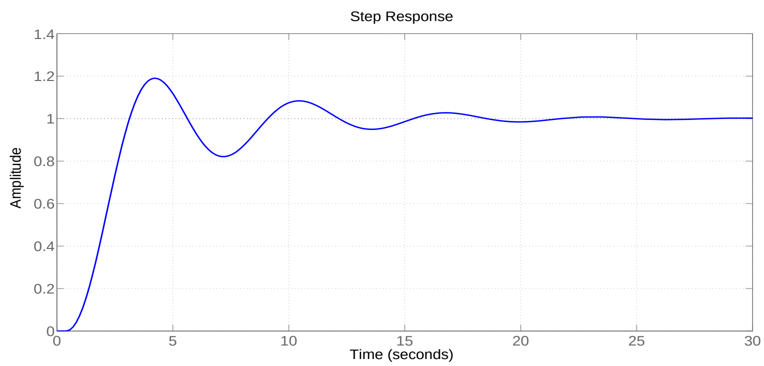

Figure 4.17: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.69) when $h = 0.4$ sec.

5. For $h = 0.5$ sec.

- Design Method-I

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.39) is obtained as:

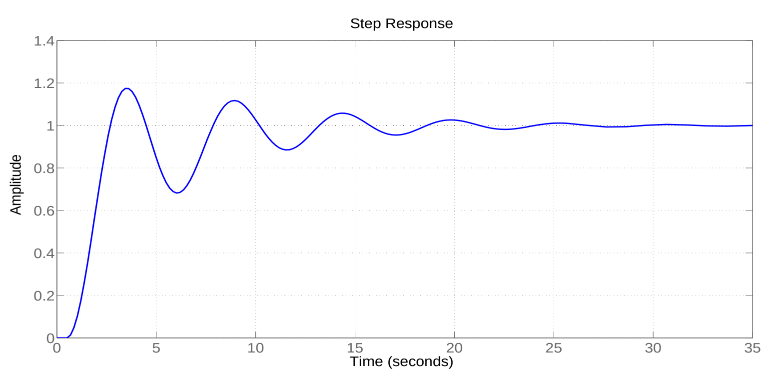

Figure 4.18: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.39) when $h = 0.5$ sec.

- Design Method-II

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.60) is obtained as:

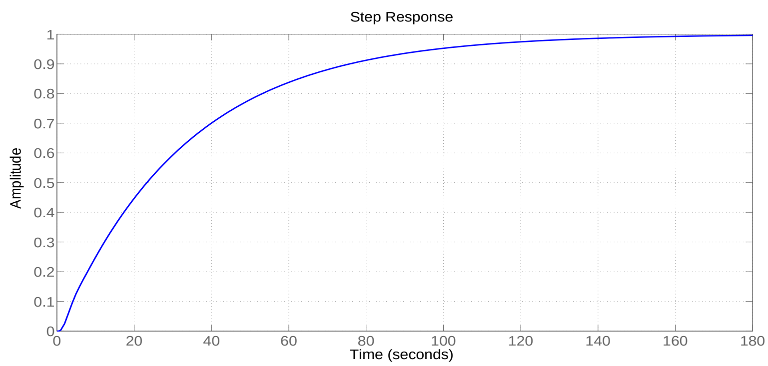

Figure 4.19: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.60) when $h = 0.5$ sec.

- Design Method-III

The step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.70) is obtained as:

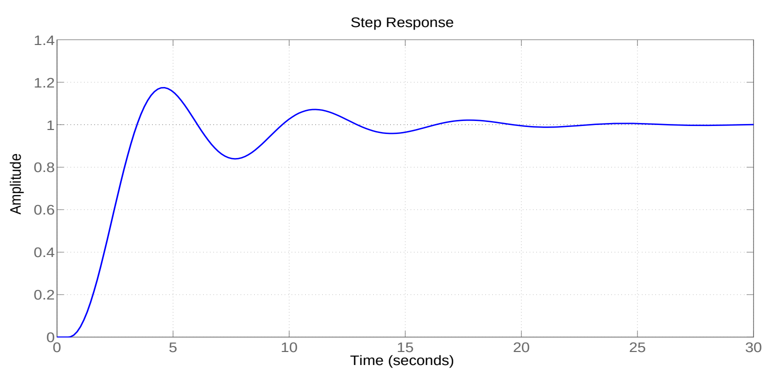

Figure 4.20: Step response of the feedback system with plant $P_3(s)$ and the controller $C(s)$ in (3.70) when $h = 0.5$ sec.

When step responses of $P_3(s)$ are compared, it can be concluded that in terms of percentage overshoot, oscillations and ideal step function shape, the best PD controller is found by design method-II for all time delay values. However, the step response of design method-II have relatively very high settling time (100-150 sec) and K_d and K_p pairs found by design method-II are significantly small compared to other design methods. In terms of settling time and lower amount of oscillations compared to design method-I, the best performance is obtained in design method-III. This fact matches with the results of robust performance condition tests as according to the robust performance condition tests, design method-III gives the best performance.

Also, as time delay value increases, the oscillations and maximum overshoot of design method-I decreases. Different time delay values do not have a significant effect on the oscillation amounts and maximum overshoot values of design method-II and design method-III and have a slight effect on settling time values for all design methods.

4.2.3 Step response Graphs for $P_4(s)$

1. For $h = 0.1$ sec.

- Design Method-I

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.41) is obtained as:

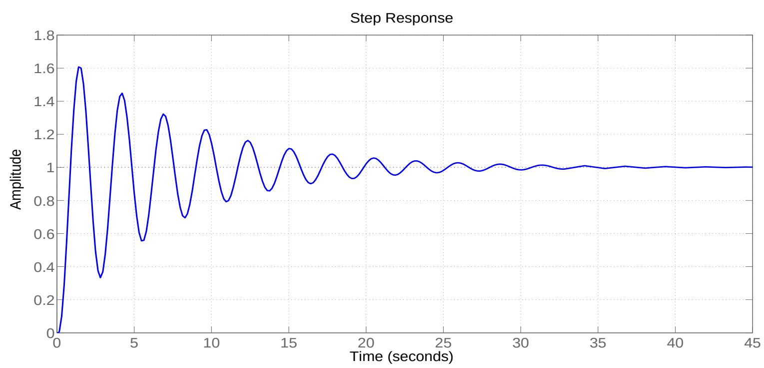

Figure 4.21: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.41) when $h = 0.1$ sec.

- Design Method-II

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.56) is obtained as:

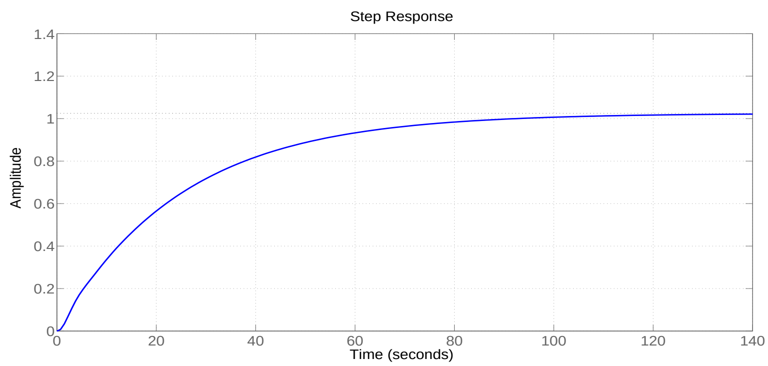

Figure 4.22: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.56) when $h = 0.1$ sec.

- Design Method-III

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.71) is obtained as:

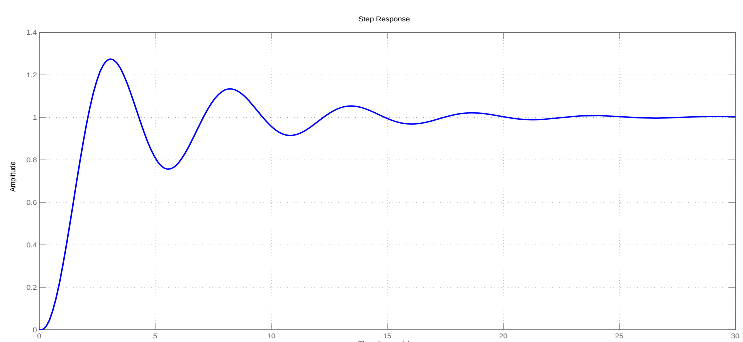

Figure 4.23: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.71) when $h = 0.1$ sec.

2. For $h = 0.2$ sec.

- Design Method-I

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.43) is obtained as:

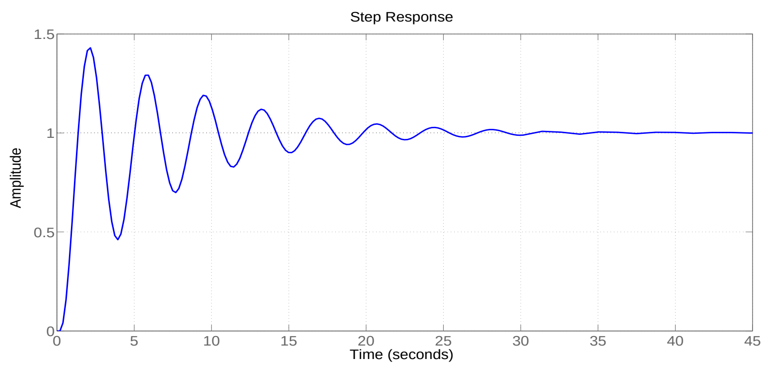

Figure 4.24: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.43) when $h = 0.2$ sec.

- Design Method-II

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.57) is obtained as:

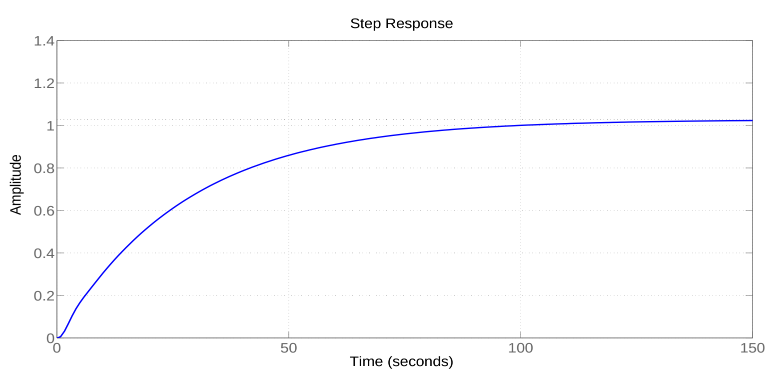

Figure 4.25: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.57) when $h = 0.2$ sec.

- Design Method-III

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.72) is obtained as:

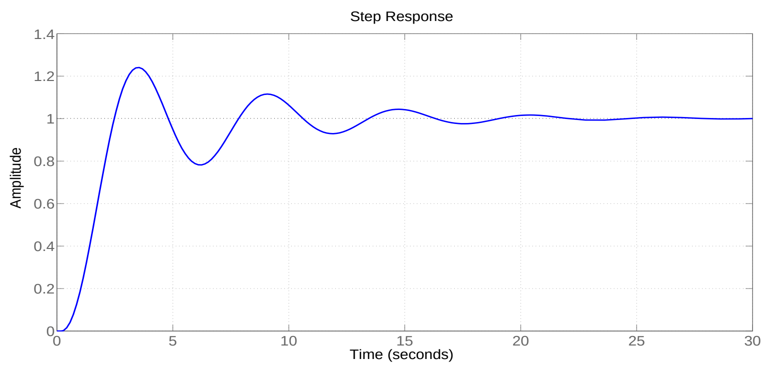

Figure 4.26: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.72) when $h = 0.2$ sec.

3. For $h = 0.3$ sec.

- Design Method-I

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.45) is obtained as:

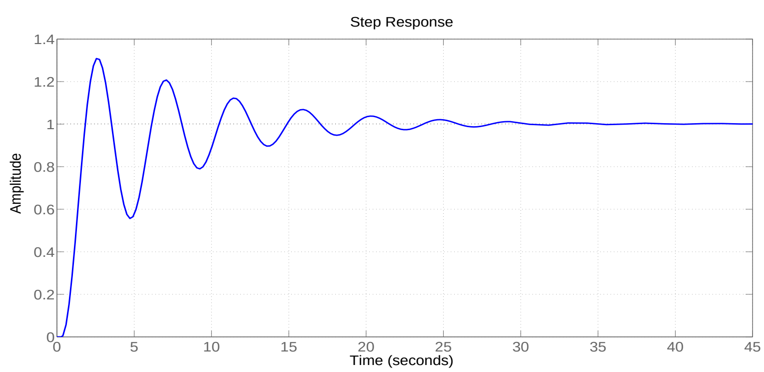

Figure 4.27: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.45) when $h = 0.3$ sec.

- Design Method-II

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.58) is obtained as:

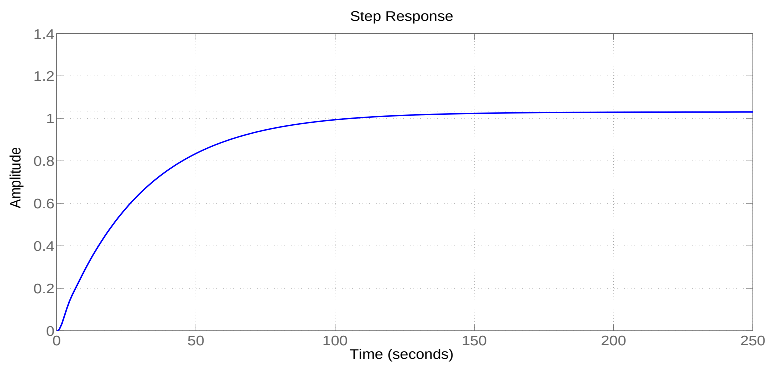

Figure 4.28: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.58) when $h = 0.3$ sec.

- Design Method-III

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.73) is obtained as:

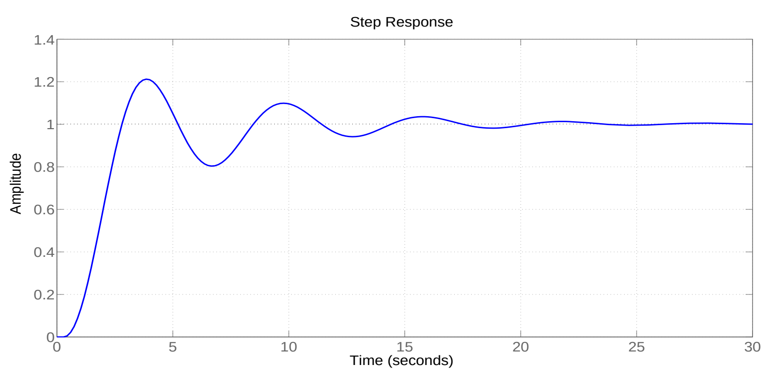

Figure 4.29: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.73) when $h = 0.3$ sec.

4. For $h = 0.4$ sec.

- Design Method-I

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.47) is obtained as:

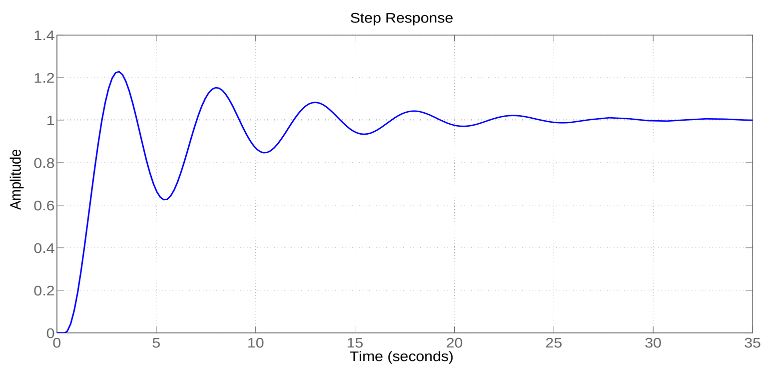

Figure 4.30: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.47) when $h = 0.4$ sec.

- Design Method-II

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.59) is obtained as:

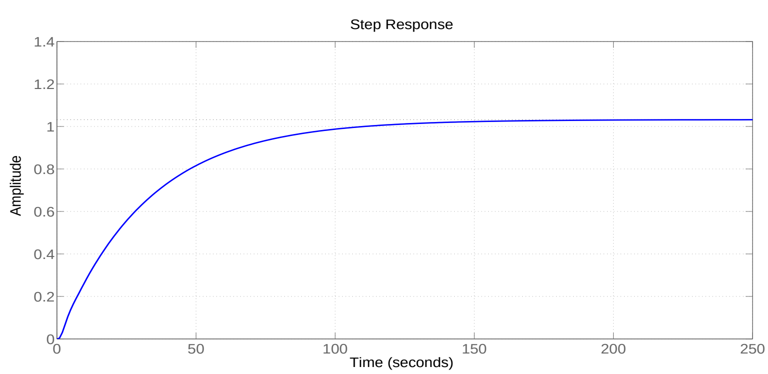

Figure 4.31: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.59) when $h = 0.4$ sec.

- Design Method-III

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.74) is obtained as:

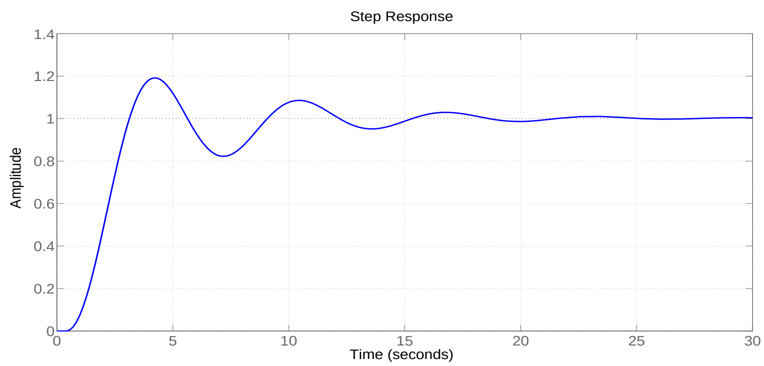

Figure 4.32: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.74) when $h = 0.4$ sec.

5. For $h = 0.5$ sec.

- Design Method-I

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.49) is obtained as:

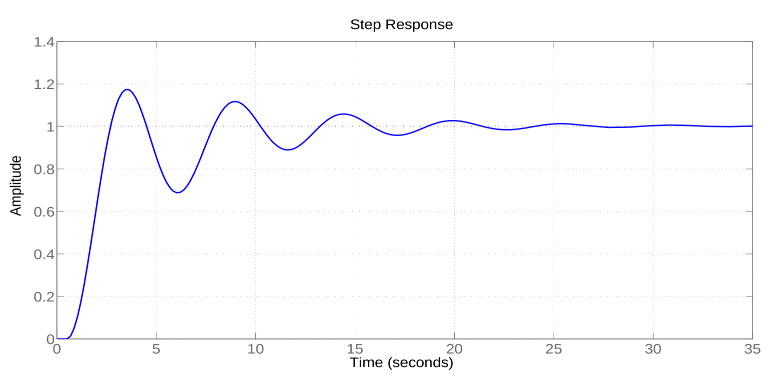

Figure 4.33: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.49) when $h = 0.5$ sec.

- Design Method-II

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.60) is obtained as:

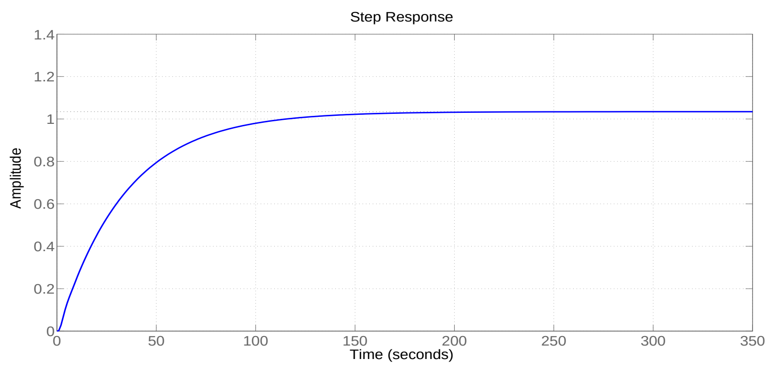

Figure 4.34: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.60) when $h = 0.5$ sec.

- Design Method-III

The step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.75) is obtained as:

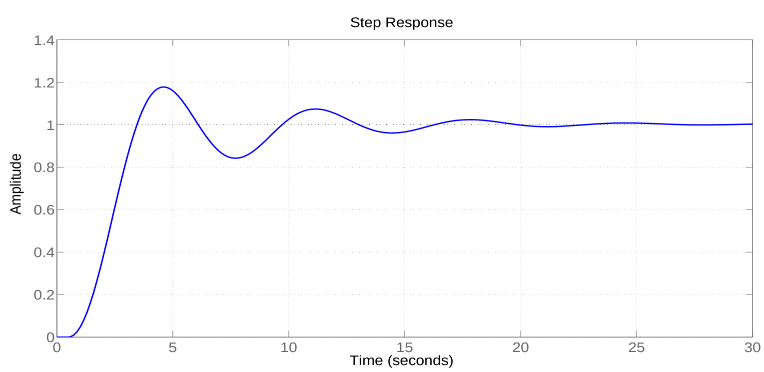

Figure 4.35: Step response of the feedback system with plant $P_4(s)$ and the controller $C(s)$ in (3.75) when $h = 0.5$ sec.

When step responses of $P_4(s)$ are compared, it is expected to have the same step responses with $P_3(s)$ as two plants are nearly equal to each other. The step responses shows that the observations for $P_3(s)$ also hold for step responses of $P_4(s)$ and when compared for each specific h value and design method only difference between two plants is a negligible steady-state error. The reason for this is because the unstable pole of $P_3(s)$ is 0 but unstable pole of $P_3(s)$ is nearly equal to 0 so it results a negligible steady-state error.

Chapter 5

CONCLUSION AND FUTURE WORK

In this thesis, commonly used unstable plant models with time delays are investigated and four plants are designed: one second order and three third order plants. In order to see the effect of different time delay values, five different time delays are chosen. Afterwards, three PD controller methods are applied to the plants and two least fragile and one gain margin optimizing PD controller is calculated for each plant and for each time delay value. In the end, by using these controllers robust performance test and step response graphs are obtained and compared for each time delay and each design method for each plant.

As can be seen in all plants, allowable regions of stabilizing K_d and K_p pairs get narrower as time delay increases which is an expected result as higher time delay means harder control of the stability for a feedback system. Also, as h increases, two least fragile and one gain margin optimizing PD controller (for fixed plant) become nearly equal so it can be claimed that as time delay increases all design methods result in nearly same PD controller and this PD controller is both least fragile and gain margin optimizing. Also, allowable K_p ranges become closer as time delay increases. These are very important results because for specifically high time delays, one can find allowable K_p range and least fragile and gain margin optimizing PD controller with only one method which is very practical.

When robust performance test is investigated, it can be concluded that design method-II has the best robustness and design method-III has the best performance. It is an expected result as for $P_3(s)$ (same for $P_4(s)$) gain margin optimizing PD controller parameters (K_p and K_d values) are relatively small compared to the other PD controller parameters which results in a worse performance but significantly better robustness. Also, ρ_{max} values of design method-I and design method-III are close to each other, i.e., the performance of design method-I is similar to the performance of design method-III. However, design method-III has a little higher ρ_{max} values than design method-I because the least fragile PD controllers are found more precisely in design method-III compared to design method-I so it is an expected result that design method-III has the best performance among all design methods.

Last but not least, step response graphs show that in terms of oscillation and maximum overshoot, design method-II gives the best performance as the step responses have zero oscillation and zero overshoot. However, design method-II has significantly high settling time with respect to other methods which is an important disadvantage. So, if settling time is concerned and as design method-III has lower amount of oscillations compared to design method-I, it can be concluded that design method-III gives the best performance among all design methods which matches with the results of robust performance test. Also, design method-I and design method-III have similar step response behaviour which again matches with robust performance test results. Different time delays do not effect step responses on contrary to robust performance test results.

As future work, effect of disturbances to the feedback systems are not investigated in this thesis so the effect of disturbances can be investigated in future.

Bibliography

[1] H. Özbay and A. Gündeş, "Design of first order controllers for unstable infinite dimensional plants," *Advances in Delays and Dynamics*, pp. 17–30, 2014.

[2] V. Venkatashankar and M. Chidambaram, "Design of P and PI controllers for unstable first-order plus time-delay systems," *International Journal of Control*, vol. 60, no. 1, pp. 137 – 144, 1994.

[3] X. Yang, C. Hua, J. Yan, and X. Guan, "PD control for teleoperation system with delayed and quantized communication channel," *Proceedings of the 10th World Congress on Intelligent Control and Automation*, pp. 2318 – 2323, 2012.

[4] G. Silva, A. Datta, and S. Bhattacharyya, *PID controllers for time-delay systems*. Boston: Birkhauser, 2005.

[5] C. Olalla, R. Leyva, A. E. Aroudi, and I. Queinnec, "Robust LQR control for PWM converters: An LMI approach," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 7, pp. 2548 – 2558, 2009.

[6] O. Smith, "A controller to overcome dead time," *ISA Journal*, vol. 6, no. 2, pp. 28 – 33, 1959.

[7] S. Majhi and D. Atherton, "Modified Smith predictor and controller for processes with time delay," *IEE Proceedings - Control Theory and Applications*, vol. 146, no. 5, pp. 359 – 366, 1999.

[8] M. Shamsuzzoha and M. Lee, "IMC-PID controller design for improved disturbance rejection of time-delayed processes," *Industrial and Engineering Chemistry Research*, vol. 46, no. 7, pp. 2077–2091, 2007.

[9] M. Morari and E. Zafiriou, *Robust process control*. N.J.: Prentice Hall, 1989.

[10] J. Marshall, *Control of time-delay systems*. Stevenage [England]: P. Peregrinus, 1979.

[11] P. Yan and H. Özbay, "On the robust controller design for hard disk drive servo systems with time delays," *Proc. of the 2013 European Control Conference (ECC)*, pp. 1286 – 1291, 2013.

[12] R. Datko, "The point spectra of some LQR problems," *Applied Mathematics and Optimization*, vol. 31, no. 1, pp. 85–99, 1995.

[13] A. Seuret and F. Gouaisbaut, "Integral inequality for time-varying delay systems and its application to output-feedback control," *Advances in Delays and Dynamics*, pp. 155–169, 2007.

[14] K.Pyragas, "Continuous control of chaos by self-controlling feedback," *Physics Letters A*, vol. 170, no. 6, pp. 421–428, 1992.

[15] S.Hara, Y.Yamamoto, T.Omata, and M.Nakano, "Repetitive control system: A new type servo system for periodic exogenous signals," *IEEE Transactions on Automatic Control*, vol. 33, no. 7, pp. 659–668, 1988.

[16] J.C.Doyle, B.A.Francis, and A.Tannenbaum, *Feedback control theory*. New York: Macmillan Pub. Co., 1990.

[17] J. Marshall, *PID-Like Controller Tuning for Second-Order Unstable Dead-Time Processes*. INTECH Open Access Publisher, 2012.

[18] I. Pandey and L. Dewan, "Stabilizing sets of PID controller for minimum phase integrating processes with dead time," *Recent Researches in Electrical Engineering*, pp. 79–88, 2014.

[19] D. Enns, H. Özbay, and A. Tannenbaum, "Abstract model and controller design for an unstable aircraft," *Journal of Guidance, Control, and Dynamics*, vol. 15, no. 2, pp. 498–508, 1992.

[20] Y. Lee, J. Lee, and S. Park, "PID controller tuning for integrating and unstable processes with time delay," *Chemical Engineering Science*, vol. 55, no. 17, pp. 3481–3493, 2000.

[21] J. Normey-Rico and E. Camacho, *Control of dead-time processes.* Berlin: Springer, 2007.

[22] M. Hernandez-Perez, B. D. M. Cuellar, D. Corts-Rodrguez, and I. Araujo-Vargas, "Stabilization of third-order systems with possible complex conjugate poles and time delay," *Revista Mexicana de Ingeniera Qumica*, vol. 12, no. 2, pp. 351–360, 2013.

[23] E. O. Olvera, B. D. M. Cuellar, J. S. Garcia, and G. D. Sanchez, "Stabilization of third-order systems with possible complex conjugate poles and time delay," *2011 8th International Conference on Electrical Engineering, Computing Science and Automatic Control*, 2011.

[24] G. Hassaan, "Tuning of a PD-PI controller used with an integrating plus time delay process," *International Journal of Scientific and Technical Research*, vol. 3, no. 9, pp. 309–313, 2014.

[25] M. Shamsuzzoha and M. Lee, "Analytical design of enhanced PID filter controller for integrating and first order unstable processes with time delay," *Chemical Engineering Science*, vol. 63, no. 10, pp. 2717–2731, 2008.

[26] M. Shamsuzzoha and M. Lee, "Stabilization region of PD controller for unstable first order process with time delay," *Int. J. Control Autom. Syst.*, vol. 12, no. 2, pp. 265–273, 2014.

[27] G.E.Arslan, "PID controller design for first order unstable time delay systems."

[28] D. Padhan and S. Majhi, "Modified Smith predictor and controller for time delay processes," *Electron. Lett.*, vol. 47, no. 17, p. 959, 2011.

[29] S.P.Bhattacharyya and L.H.Keel, “Robust, fragile, or optimal?,” *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 1098 – 1105, 1997.

[30] P.M.Makila and S.P.Bhattacharyya and L.H.Keel “Comments on ”Robust, fragile, or optimal?” [with reply],” *IEEE Transactions on Automatic Control*, vol. 43, no. 9, pp. 1265 – 1268, 1998.

Appendix A

Code

1. Code for finding stabilizing PD controllers by `allmargin` command

```
h=0.1;
D=0:0.1:17;
P=0:0.1:60;
n=1;


for i=1:length(D)
    for j=1:length(P)
        s=tf([1 0],1);
        OL=tf([D(i) P(j)],[1 -0.5 0])*exp(-h*s);
        x = allmargin(OL);
        if ( x.Stable == 1)
           a(n,1) = D(i);
           a(n,2) = P(j);
                 n=n+1;
        end
```

```
    end

end

plot(a(:,1),a(:,2),'*')
```

2. Code for obtaining $a_{max}(Q)$ versus Q graph for design method-II

```
function result = gama(s,a,Q)
    G=exp(-0.1*s)/(s^2+s+1);
    G0=G;
    result=((G0-1)/(s+a))+(Q*s*G0/(s+a));

end

rangeQ = 0:0.01:2;
rangea = 0:0.001:2;
w = logspace (-4,3,1000);
a_values = zeros(size(rangeQ)) - Inf;
for i=1:length(rangeQ)
    temp = zeros(size(rangea));
    for j=1:length(rangea)
        gamas = zeros(size(w));
        for k=1:length(w)
            Q = rangeQ(i);
            a = rangea(j);
            s = 1i * w(k);
            gamas(k) = gama(s,a,Q);
        end
        temp(j) = 1 / max(abs(gamas));
    end
        aMax =-Inf;
```

```
    for j=1:length(rangea)
        gama2 = temp(j);
        a = rangea(j);
        if ( a < gama2-(1e-3))
           aMax = a;
        end
    end
    a_values(i) = aMax;
end
plot(rangeQ, a_values)
```

3. Code for robust stability test

```
om=logspace(-3,3,10000);
h=0.1;
k=0:0.1:4;
n=1;
for p=1:length(k)

for j=1:length(om)
    s=1i*om(j);
    P=(exp(-h*s))/((s^2+s+1)*s);
    C=0.024*s+0.03;
    S(j)=1/(1+(P*C));
    T(j)=1-S(j);
    phi1(j)=0.01*abs(S(j)/s);
    phi2(j)=abs(0.01*(s)*T(j));
    phi(j)=k(p)*phi1(j)+phi2(j);

end
```

```
if (max(phi)<= 1)
        a(n)=k(p);
        n=n+1;
    end
end
max(a)
```

4. Code for obtaining step response

```
s=tf([1 0],1);
OL=tf([0.362 0.453],[1 0.999 0.999 -0.001])*exp(-0.1*s);
CL=feedback(OL,1);
step(CL)
```