INTEGRATED PRODUCTION TRANSPORTATION AND STORAGE POLICIES WITH CARBON COSTS

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Başak Erman

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Nesim K. Erkip(Advisor)

Ülkü Gürler(Co-Advisor)

Zeynep Pelin Bayındır

Emre Nadar

Approved for the Graduate School of Engineering and Science:

Ezhan Karaşan
Director of the Graduate School

ABSTRACT

INTEGRATED PRODUCTION TRANSPORTATION AND STORAGE POLICIES WITH CARBON COSTS

Başak Erman
M.S. in Industrial Engineering
Advisor: Nesim K. Erkip
Co-Advisor: Ülkü Gürler
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Governments adopt carbon policies such as taxation in order achieve carbon reduction targets set by global agreements. In this study, we analyze a two-echelon supply chain model where production and dispatching decisions are made in an integrated way. We allow the use of multi-modes of transportation under carbon taxation. We consider a system with a single retailer and a manufacturer and assume that demand is deterministic and constant. We first assume that lead time is negligible, which is then extended to a case where lead time is a stochastic random variable. For transportation, we focus on a system where both in-house and outsource transportation options are available. Our objective is to minimize average total cost which is composed of operational costs and carbon tax by determining optimal production interval, number of dispatches, backorder level and number of vehicles to be used. We provide structural results and solution algorithms to select the optimal policy. We also present different integration scenarios to discuss the benefits of integrated supply chains. The proposed methods are applied in a numerical study and the results are explained in detail. Finally, we discuss benefits of integrating system and introducing carbon tax and show that significant carbon reductions can be achieved with manageable increase in cost. It is also concluded that although integration and allowing outsource option provides a decrease in total cost of the system, it can yield to a solution with greater carbon emissions.

Keywords: carbon reduction, integrated supply chain, transportation, EOQ .

ÖZET

KARBON VERGİLENDİRMESİ ALTINDA ENTEGRE ÜRETİM ULAŞIM VE DEPOLAMA POLİTİKALARI

Başak Erman
Endüstri Mühendisliği, Yüksek Lisans
Tez Danışmanı: Nesim K. Erkip
İkinci Tez Danışmanı: Ülkü Gürler
Temmuz 2018

Uluslararası anlaşmalarla belirlenen karbon salınımı hedeflerini sağlayabilmek için hükümetler karbon vegilendirmesi gibi politikalar uygulamaya başlamıştır. Bu uygulama altında firmalar sorumlu oldukları karbon salınımını maliyet analizlerinde göz önünde bulundurmaya ve salınımı azaltıcı önlemler almaya başlamıştır. Bu çalışmada distribütör ve üretici firmanın koordineli kararlar verdiği ve çoklu ulaşım opsiyonlarının olduğu bir sisteme karbon vergilendirmesinin etkileri analiz edilmiştir. Çalışılınan sistemde talebin sabit olduğu varsayılmıştır. Çalışmanın başında teslim süresinin sabit ve sıfır olduğu varsayılmış, ilerleyen kısımlarda bu varsayım esnetilerek değişken teslim süresinin sistemdeki etkileri analiz edilmiştir. Mevcut ulaşım opsiyonları firma içi ve dış kaynak kullanımı olarak belirlenmiştir. Tüm bu koşullar altında sistemin maliyetini en aza indirgeyecek en iyi üretim süresi, ürünlerin distribütöre tedarik sıklığı, kullanılacak ulaşım tipi ve araç sayısı ve bakiye siparişinin miktarına karar verilmiştir. En iyi çözüme ulaşmak için problemin matematiksel özellikleri ve geliştirilen çözüm algoritması kullanılmıştır. Koordineli karar verilmesinin etkilerini görebilmek adına farklkı koordinasyon senaryoları üretilmiştir. Önerilen çözüm yöntemleri sayısal bir çalışma ile desteklenmiştir. Çalışma sonucunda maliyetteki karşılanabilir artış sonucunda ciddi karbon salınımı azalımlarının elde edilebilineceği görülmüştür. Aynı zamanda distribütör ve üretici firma koordinasyonunun ve ulaşımda dış kaynakların kullanılmasının maliyeti azaltacağı fakat bazı durumlarda karbon salınımını arttıracak etkilerde bulunabileceği sonuçlarına ulaşılmıştır.

Anahtar sözcükler: karbon salınımı, envanter politikaları, ulaşım.

To polar bears
and all those who affected from global warming

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Chapter 1

Introduction

Carbon reduction has gained importance in recent years as increased greenhouse gas (GHG) emissions have begun to threaten the earth's climate system. Governments have enacted legislation to limit temperature rise by restricting the amount of released emissions. Since 1997 different target levels have been introduced for carbon cut. The G8 climate agreement is one of the recent compromises which aims for a 50% emissions cut by 2050 [1]. The agreement has been signed by the EU, Canada and Japan. Since the initial agreement, different target levels such as a 20% cut by 2020 and a 30% cut by 2030 have also been introduced and pledged by different countries [2]. Governments embrace various policies to reach the desired carbon reductions such as carbon cap, cap-and-trade and taxation. Cap policy sets strict upper bounds on carbon emissions for companies and the upper bound is called the carbon cap. Selection of the carbon cap depends on the company's sustainability policies or it can be predetermined by governments. The cap does not reflected on the profit or the costs of the company. The cap-and-trade policy uses both the constraint and the price of carbon, under this policy carbon is tradable so that the company can decide to sell or buy carbon according to usage and the cap defined. On the other hand, tax policy charges the company for the amount of carbon that is emitted. In this case, the company is free to consume carbon in any amount as long as the required price is paid. The most common policy is carbon tax which is currently in place in more than

15 countries including Canada, France, Norway and Sweden. The tax rates used in these countries range from 10$ to 200$ per ton. Besides these countries, carbon taxation is under consideration in many others, including Turkey. Figure 1.1 shows the countries which are either currently using or considering taxation [3].

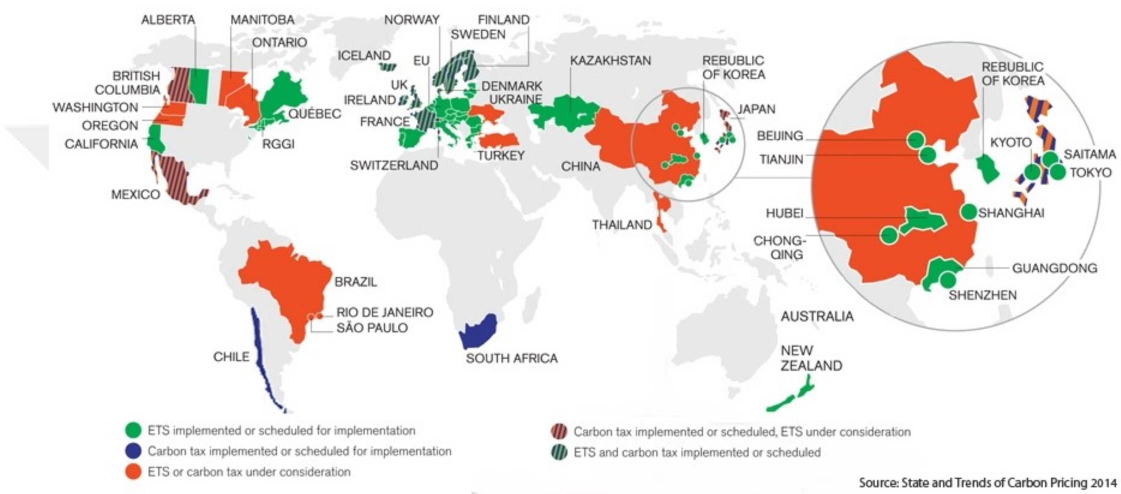


Figure 1.1: Carbon taxation

Under carbon taxation, companies must include emissions in their cost analysis and seek ways to reduce the amounts of carbon emitted. One way to achieve reduction is to consider carbon costs in production and inventory management [4]. In classical inventory management problems the aim is to minimize cost by determining optimal lot sizes. The main components of the system are production set-up, inventory carrying and backorder costs. However, when companies are charged for the carbon amounts that are emitted, minimizing only operational costs will not be sufficient since carbon tax must also be incorporated into the objective function. It is possible to revise the inventory management problems to minimize carbon emission by exchanging cost terms with carbon terms. Under carbon taxation, rather than directly minimizing carbon emission it is also possible to minimize total cost by adding emission components to cost components. In both cases the resulting policy favours emission reduction.

Transportation is another area which contributes in great amounts to increasing carbon emissions. In Europe, 23% of carbon emissions are a result of transportation operations [5]. Using different modes for transportation of items results in variations in both cost and carbon emissions. In road transportation, various options are available with different capacities and different technologies. Vehicles with different capacities in a fleet provides a better chance to minimize cost and emissions as there is more room for selection of the order size. Furthermore, logistic firms such as FedEx have begun to use electric vehicle fleets in order to reduce GHG emissions [6]. When all options are considered, it is possible to have vehicles with different capacities or technologies using in-house fleets. Using outsource options may also provide opportunities for benefiting from technology investments of logistic firms.

It is possible to reduce carbon emissions by changing the strategies on the scope of the company, by deciding more beneficial production, inventory or transportation policies. To generalize further, since supply chain decisions can be used to reduce carbon emission of companies they are open areas for improvement. By adding carbon constraints to decision models, it is possible to maintain a more sustainable organization without completely restructuring ongoing systems. Another area which can lead to improvements is working on an integrated system of production, inventory and transportation operations rather than focusing on separate systems. In integrated systems the objective is to minimize the manufacturer's, the retailer's and transportation costs simultaneously. Integrated systems provide a more balanced solution as opposed to minimizing cost's of a single player. While including manufacturer's, retailer's and transportation costs in objective represents the complete integration, different integration scenarios can also be considered. These scenarios are obtained by integrating the combination of separate players such as manufacturer and retailer or manufacturer and transportation.

The main focus of this study is the relation between production and transportation policies under the presence of carbon reduction considerations with multiple transportation modes. By including both integration of operations, reduction of carbon emission and multi-mode transportation options, it is aimed to reach improved results in both environmental and economic areas. The main objective of the study is to obtain the optimal integrated solution for the manufacturer, retailer and different transportation modes. We prove structural properties of the problem and propose a solution algorithm to find optimal solution. In order to compare the results of the integrated system and determine whether integration always favors carbon reduction or not, benchmark models with different integration combinations are also solved. For transportation, the economic and enviromental effects of allowing different modes and specifically in-house fleet and outsource option is analyzed. As a result of the study we show that it is possible to reach significant carbon reduction at low tax rates with an affordable rise in the cost. Further increases in carbon tax results in worsening of service level and rapid increase in total cost of the system. We also observe that although the integration and outsource option always provides a less costly solution, it is not possible to generalize this result for carbon emissions and that it is possible to obtain better solutions in terms of carbon emissions with different benchmark models.

The remainder of this study is organized as follows: Chapter 2 reviews relevant studies on carbon constrained supply chains, transportation systems and integrated supply chains. A comparison of literature is provided and contributions of this study are given explicitly. Chapter 3 defines the specifications of the considered problem and assumptions. The derivation of objective function and solution methodology that includes technical details of the problem and solution algorithm is also given in this chapter. In Chapter 4, benchmark models which define objective functions and solution methods under different integration sceneries are presented. In Chapter 5, numerical analysis is provided on parameter setting, effect of transportation options, and integration are discussed.

All chapters up to Chapter 5 consider the case where lead times are deterministic and known. In Chapter 6 an extension of the problem for stochastic lead time is given. We start the chapter by reviewing appropriate literature for the stochastic lead time problem. For the study conducted we consider only cases where there are at most one or two outstanding orders. We construct two Markov chain models to evaluate the expected cost of a given policy. At the end of the chapter numerical analysis is also provided. Finally, in Chapter 7 an overview of the study and directions for further studies are given.

Chapter 2

Literature Review

In this chapter we review the literature with deterministic lead times under three main headings: sustainable supply chains, transportation problems and integrated supply chains. The literature related to stochastic lead times used for various supply chain problems are studied in the beginning of Chapter 6, for the sake of completeness of the problem presented there.

2.1 Sustainable Supply Chains

In recent years the effect of carbon emission policies has been studied in many different areas such as production, inventory holding, selection of plant location and reducing waste. To begin with the inventory holding problems, the economic order quantity (EOQ) model is solved under different carbon policies. Absi et al. [7], Hua et al. [8] and Chen et al. [9] can be given as examples where carbon cap is used while obtaining optimal replenishment policies. To give more detail, Absi et al. [7] focuses on a single-item uncapacitated lot-sizing problem under carbon sensitive environment over a planning horizon of multi-periods. They consider four different carbon caps as the constraints of the system with different sourcing options. The objective is to minimize fixed and variable supplying and holding costs. They formulate mixed integer programming model and dynamic programming model and prove that an optimal solution can be found in polynomial time. Hua et al. [8] work on sustainable EOQ model and obtain a solution for the optimal order size when a carbon cap exists in the traditional EOQ environment. They focus on a system with set-up and inventory holding costs under single product and deterministic demand assumptions. They prove analytical properties on the relation between problem parameters and optimal lot size selection. The effects of adding carbon cap to the model is analyzed as well. Chen et al. [9] work under the same setting and extend the results found by Hua et al. [8]. They conclude that it is possible to significantly reduce carbon emissions without significantly increasing cost by only changing the lot size decisions.

Benjaafar et al. [4] also focus on carbon cap but provide solutions for the cases when carbon tax and cap-and-trade is used. They work on simple models which focus on procurement, production and inventory decisions on single and multi firm settings. They show ways to include different carbon policies in models and discuss the effect of each policy on carbon reduction. The main objective is to minimize the cost while carbon is included either as a constraint, or in objective as carbon price or both. A multi-period integer programming model is solved to find optimal inventory and backorder levels at each period. They also compare the operational cost under optimal policy with and without carbon considerations. Similar to Chen et al. [9], they conclude that by only adjusting operational decisions it is possible to obtain significant carbon reductions.

Jiang and Klabjan [10] work on GHG reductions by allowing emission reduction investments in a setting which is used in standard replenishment problems. They solve a joint production capacity and investment problem under carbon cap, cap-and-trade and carbon tax. The objective of the study is to minimize total cost. They find expected profit, total emissions, and investment amount using a setting with single and two period models where demand is uncertain. As a result of the study, they provide managerial insights on how to include carbon policies and emission reduction investments in current systems. Toptal et al. [11] also focus on a similar system and discuss how production planning decisions or investments in green technologies can be used to meet carbon regulations. They work on the same three carbon policies with Jiang and Klabjan [10] but under deterministic demand and single period considerations. They provide an analytical comparison between different investment alternatives.

Rather than using mixed policies for carbon reduction it is also possible to minimize carbon directly and compare the results obtained by cost and emission minimization problems. Battini et al. [12] focus on a single product replenishment problem where external costs, such as delivery and waste disposal, are also included. For the transportation option, rail, road and ship are provided as different modes. The problem is jointly solved for economic and environmental concerns. The optimal order size is found by a solution procedure which uses a model based on the standard EOQ model. They analyze changes in operational costs which result from introducing carbon emission factors and how the optimal policy differs when compared to the cost minimization problem. Bozorgi et al. [13] work on a single product replenishment problem specialized for cold items. Production, inventory and transportation costs are considered for the problem together with carbon emission costs. They determine optimal order sizes for cold items which are transported via capacitated and refrigerated vehicles. They focus on a single product, multi-period replenishment problem with set-up, inventory holding and transportation costs under deterministic demand. The two main objectives are cost and emission minimization. They provide a solution algorithm in order to determine optimal lot sizes. Fichtinger et al. [14] work on a single product replenishment problem in a carbon sensitive environment and use a EOQ based model to decide order sizes. However, different than other works in literature, the importance of warehouse management is emphasized in the paper and factors such as warehouse climate, energy usage and material handling are also included in the model. They consider different sourcing scenarios and warehouse types and analyze their effects of those on carbon emission.

2.2 Transportation Problems

In the papers discussed in the previous section, transportation of items are mostly assumed to be carried out by a single vehicle which is either capacitated or uncapacitated. However, as transportation is responsible for a significant percentage of the total cost and carbon emissions, it is beneficial to consider more realistic and possibly more complex structures for transportation. Hoen et al. [15] focus on transportation operations and find mode selections which prevent carbon emissions exceed the predetermined cap. They consider weight, volume and density of products and assume that logistic firms charge companies according to these properties of the products. The objective of the problem is minimizing total cost which also includes emission related costs. Transport mode and order-up-to level are jointly optimized and conditions for a mode to be selected are derived. They provide analysis on how the solution changes with respect to emission parameters and conclude that it is possible to reduce carbon emissions significantly by adapting logistic policies. Hoen et al. [5] extend this problem to multi-item case and include pricing decisions as well. They found optimal mode selections and pricing decisions under varying carbon policies.

Konur [16] works on an inventory control problem in a setting with carbon cap and multiple transportation options which are defined as truck load and less than truck load with different types of trucks. He assumes that demand is deterministic and constant and delivery lead time is fixed. The EOQ model is revised separately to include truck load and less than truck load transportation options. It is shown that the problem with truck load transportation option is NP-hard, hence a heuristic method is presented to solve the problem which performs a local search to select order size. A numerical analysis is also given which discusses the relation between carbon cap and selection of the transportation option. Schaefer and Konur [17] extend this work by adding demand variability and defining delivery speed as a decision variable. They propose a sustainable (Q,r) model where truck load and less than truck load options are available for transportation. The effects of demand and lead time variation on costs and carbon emissions are analyzed throughout the paper.

Rieksts and Ventura [18] focus on a single product replenishment problem with constant demand and two mode transportation option. In a classical EOQ setting without any environmental considerations, they allow the use of truck load and less than truck load transportation options at the same time. They provide a solution algorithm to determine optimal transportation and replenishment policy.

2.3 Integrated Supply Chains

Studies on carbon management operations usually focus on production, transportation and various different processes separately. The interest of this study originated from the idea of reducing carbon emissions in an integrated system. In the literature various integration methods were studied without introducing carbon costs or constraints. Calvete et al. [19] and Mula et al. [20] review the studies on supply chain integration. Calvete et al. [19] focuses on papers where different ways of integration are studied. Examples of these integrations include location and routing, supplier selection and inventory holding, and routing and inventory. Mula et al. [20] reviews mathematical models on integrated production and transportation planning problems.

Saglam and Banerjee [21] study integrated systems and focus on integration of production lot scheduling problems and outbound shipment decisions. They consider a setting with multi products and a single retailer. They allow using either truck load or less than truck load transportation options separately. They solve mixed integer non-linear programming models to determine optimal policy. Feng et al. [22] focus on a similar integrated system with production and transportation operations. They consider heterogeneous vehicle fleet with different capacities and formulate a mixed integer non-linear programming model. They state that the model is unsolvable due to size and propose two heuristic methods to solve the coordinated production transportation problem. Kaya et al. [23] study a single supplier, single retailer problem under deterministic demand and finite production capacity. They consider two models, where in both models the retailer does not hold inventory but backorders items. In the first model which is refereed as to coordinated problem, supplier makes all decisions and selects production and transportation intervals according to her profit. In the second model, which is referred to as decentralized, retailer selects transportation intervals and the supplier adjusts production cycles accordingly.

This study focuses on the integration of inventory holding, production and transportation where both manufacturer and retailer hold inventory. Hahm and Yano [24] and Axsäter [25] work on integration of production and dispatches which is referred as two-echelon supply chain under single product and finite and infinite production rate assumptions, respectively. In both studies a setting with a single manufacturer and a single retailer is considered and it is assumed that demand is deterministic and lead time is zero. A nested policy is considered in the studies where the production cycle is a positive integer multiple of transportation cycle. Optimal order sizes and number of dispatches are selected as a result of the work.

The studies that are given above do not include carbon reduction considerations. However, there are also other studies which focus on carbon reduction in integrated systems. Bouchery et al. [26] focus on a two-echelon supply chain under carbon constraints. Together with production set-up and inventory, transportation costs are also included in the problem. It is assumed that a single transportation mode is available and production rate is infinite. Rather than focusing on a carbon policy they solve the multi-objective problem for cost and carbon minimization. Bouchery et al. [26] provide optimal decision for order size and number of dispatches. It is concluded that centralized and decentralized systems have different benefits in terms of sustainability concerns. Ghosh et al. [27] focus on a two-echelon supply chain and find optimal order sizes and dispatch numbers in a deterministic environment. Three different carbon policies are considered which are strict cap, cap-and-trade, and carbon tax and effects of each policy in operational decisions are discussed. Both Bouchery et al. [26] and Ghosh et al. [27] provide an analysis of how the optimal policy changes with the carbon reduction targets and discuss its effects on the operational costs. Xu et al. [28] work on two-echelon supply chains as well, but under cap-and-trade regulation. Jaber et al. [29] also work on an integrated supply chain and discuss the applications of the European Union Emissions Trading Scheme in the system. Production rate and manufacturer's lot size decisions are made in the study. It is assumed that carbon emissions depend on the production rate. The results of using different trading schemes are analyzed in detail.

In Table 2.1 the most relevant studies which are given throughout the chapter are compared according to different considerations. In the first column the selected carbon policy is given if a carbon consideration exists. The second and third columns indicate means of integration and the benchmark model that is used for comparing integrated model. The transportation options that are allowed are given in the fourth column. The option *capacitated* refers to the case where a single capacitated vehicle is used. The *LT or LTL* and *LT and LTL* options refer to cases where truck load (TL) and less than truck load (LTL) options are allowed to be used separately or simultaneously, respectively.

Table 2.1: Literature comparison

	Carbon policy	Integration	Benchmark models	Transportation	Production rate
Absi et al. [7]	Cap	✗	✗	✗	Infinite
Hua et al. [8]	Cap	✗	✗	✗	Infinite
Chen et al. [9]	Cap	✗	✗	✗	Infinite
Benjaafar et al. [4]	Cap & trade, tax	✗	✗	✗	Infinite
Battini et al. [12]	Carbon min.	✗	✗	Rail, road and ship	Infinite
Bozorgi et al. [13]	Carbon min.	✗	✗	Capacitated	Infinite
Konur [16]	✗	✗	✗	TL or LTL	Infinite
Schaefer and Konur [17]	Carbon min.	✗	✗	TL or LTL	Infinite
Rieksts and Ventura [18]	✗	✗	✗	TL and LTL	Infinite
Kaya et al. [23]	✗	Manufacturer and retailer	Manufacturer makes decision	✗	Infinite
Hahm and Yano [24]	✗	Manufacturer and retailer	✗	✗	Finite
Axsater [25]	✗	Manufacturer and retailer	✗	✗	Infinite
Bouchery et al. [26]	Carbon min.	Manufacturer, retailer and transportation	Non-integrated	Capacitated	Infinite
Ghosh et al. [27]	Cap, tax	Manufacturer and retailer	✗	✗	Finite
Jaber et al. [29]	Tax and trading	Manufacturer and retailer	Non-integrated	✗	Finite
This study	Tax	Manufacturer, retailer and transportation	All possible integration scenarios	TL and LTL	Finite

To summarize some of the most relevant studies, Chen et al. [9] solve the sustainable EOQ problem in a single echelon system with an infinite production rate and transportation capacity and Battani et al. [12] add transportation capacity to this problem. Although the main problem is similar in this study we focus on a much more specific problem and solve it with multiple transportation options. Ghosh et al. [27] work on a two-echelon supply chain but in a simpler setting where transportation is not capacitated. Schaefer and Konur [17] work on transportation options with different modes and a heterogeneous fleet. However, they do not focus on inventory decisions. On the other hand, Bouchery et al. [26] designs a multi-objective problem where cost and carbon emission minimization are the two objectives. They only consider a single vehicle and provide a solution methodology to find efficient solutions of the problem. In this study we provide a multi-mode transportation option which incorporates the technical difficulties of the problem, as well as the benchmark problems that are designed for different leader-follower scenarios. We propose a solution algorithm which gives the exact solution when a carbon tax rate exists and present extensive analysis on the effects of various integration models and transportation options on operational cost and carbon emissions.

Chapter 3

Model Formulation

3.1 Model Description

A supply chain with a single manufacturer and a retailer with a transportation activity in between is considered in this study. There is a single item under consideration with carbon emissions and operational costs resulting from manufacturing, inventory carrying, backordering and transportation. It is assumed that, in the environment considered, there is a tax to be paid for carbon emissions, the rate is shown by p. Retailer demand is assumed to be deterministic and constant. The demand rate is denoted by D. In this chapter, the lead-time for transportation is assumed to be deterministic, and without loss of generality, known and zero, as the demand is assumed to be constant. Deterministic lead time assumption is relaxed later, in Chapter 6. Figure 3.1 depicts the material flow for this system.

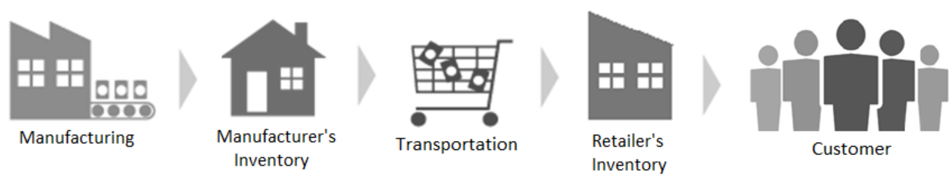


Figure 3.1: Process description

Manufacturer operates with a finite production rate, denoted by P. Items produced can be placed at the manufacturer's inventory and transported to the retailer. Transportation of items from the manufacturer to retailer can be handled in various different ways, using only in-house fleet with similar vehicles (single mode) or two vehicles (multi-mode) and outsourcing, note that the vehicles in in-house fleet differ in capacities. Finally, after items are moved to the retailer, retailer keeps those units in the inventory to satisfy the demand.

It is assumed that a nested coordination policy is followed in the supply chain where the production interval of the manufacturer is a positive integer multiple of retailer's reorder interval. In the system, manufacturer makes the production in an interval denoted by T and holds inventory if necessary. The inventory of the manufacturer decreases upon the dispatches. The number of dispatches that are scheduled during a production cycle is denoted by m. Backorders are allowed in retailer level and the maximum number of units backordered is represented by b. Retailer's inventory decreases by the sales of items. The production interval and number of dispatches are the two main decision variables of the problem while the length of transportation cycle can be found by T/m. The inventory graphs in Figure 3.2 shows manufacturer's and retailer's inventory levels.

For the transportation of items two main cases are analyzed. The first case focuses on using only in-house fleet and the second case provides an outsource option together with in-house option. It is assumed that transportation option does not affect lead time. For both cases the problem is solved when single and two type of vehicles are available. Numbers of different vehicle types that are used are specified by vector $\mathbf{x}$.

Two different parameter categories are considered and given in Table 3.1. The first category is operational costs that are incurred for manufacturing, inventory and transportation and the second set of parameters pertains to the carbon emissions resulting from the same operations. To begin with operational costs, K_M refers to the fixed production set up cost incurred per production interval. Inventory holding costs per unit per unit time for manufacturer and retailer are

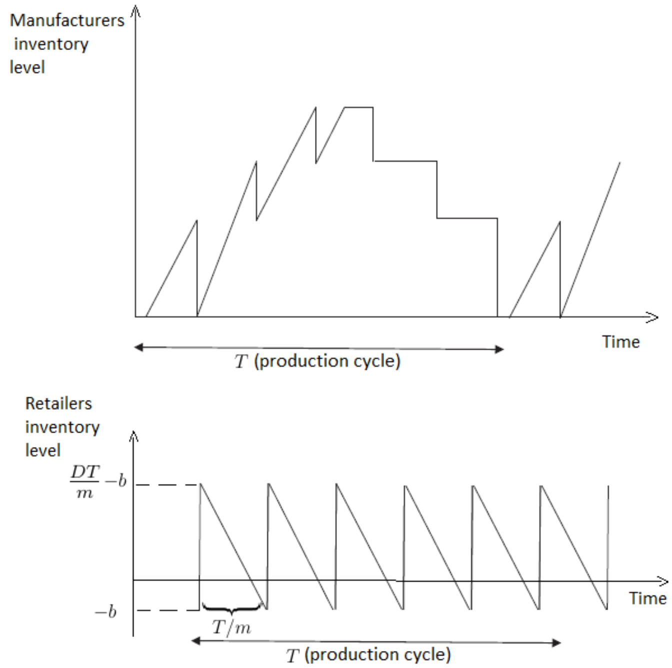


Figure 3.2: Inventory levels

denoted as h_M and h_R, respectively. Backorder cost per unit per unit time is denoted as c_B. Carbon amounts emitted during production and inventory holding are denoted as E_M and E_Z, respectively. The variable carbon emission is denoted by e_Z per unit per unit time for inventory. Without loss of generality the variable emission for manufacturing is taken as zero.

For the transportation, a fixed cost per vehicle is defined which depends on the mode selection but not capacity utilization. The fixed cost of vehicle is given as K_{Vi} for vehicle type i and capacity is denoted as C_i. The carbon emitted during transportation is calculated in the same way with a fixed emission quantity per vehicle and denoted by E_{Vi}. The cost of using outsource option and resulting emission is introduced as k_o and e_o per unit item, respectively. For the time being we assume that when the integrated system uses the outsource option, there is no additional carbon tax paid for the outsourced units. However, in Chapter 5, we remove this assumption and analyze its effect on results. Note that, when there is a single type vehicle in the fleet, there is no need to use index i. Detailed explanation on the transportation options are given in the following subsection.

Table 3.1: Notation

Symbol	Description
D:	Demand[units/unit time]
C_i:	Capacity of vehicle i [units]
K_M:	Setup cost [$/cycle]
K_{Vi}:	Cost of transportation for vehicle i [$/vehicle]
k_o:	Cost of outsource transportation per unit item [$/unit]
h_M:	Inventory holding cost for the manufacturer [$/unit /unit time]
h_R:	Inventory holding cost for the retailer [$/unit /unit time]
c_B:	Backorder cost [$/unit /unit time]
p:	Carbon price [$/kg]
E_M:	Carbon emission of production [kg/cycle]
E_{Vi}:	Carbon emission for trucks [kg/vehicle]
E_Z:	Fixed carbon emission for inventory holding [kg/cycle]
e_Z:	per unit carbon emission during inventory holding [kg/unit]
e_o:	per unit carbon emission during outsourcing an item [kg/unit]

3.1.1 Transportation Cost

Here we summarize the average transportation cost and emissions resulting from different transportation options. Four options are considered which includes in-house fleet with single and two vehicle types and outsource option. All the results are summarized in Table 3.2.

A single vehicle type without outsource option

The first option is the case where only in-house transportation is possible and a single type vehicle is available in the fleet. In order to carry all of the items enough vehicles must be assigned from the fleet. Under this case it is possible to have at most one vehicle that has an idle capacity. Notice that the vehicle can either be fully loaded or has an idle capacity depending on the order size to be carried. Both cost of transportation and emission amount are linear with the number of vehicles used. Let transportation cost and emission quantity be defined by $\Lambda(x)$ and $\Lambda_E(x)$, respectively where $\mathbf{x}$ is the number of vehicles used for operations, then, we can write $\Lambda(x)$ and $\Lambda_E(x)$ as follows:

$$\Lambda(x) = xK_V$$
$$\Lambda_E(x) = xE_V$$

Two vehicle types without outsource option

Approaching the problem when different vehicle types exist is the next aspect that is focused. Under this case it is possible to utilize both types of vehicles simultaneously, in different quantities. Both operational costs and carbon emissions can be defined the same way with the single type problem, but separately for each vehicle type. In order to prevent trivial solutions, the following are assumed: (1) Fixed cost of the larger vehicle is higher while the marginal cost is less; (2) Using a fully loaded vehicle with larger capacity is assumed to be less costly compared to splitting the quantity to multiple vehicles of smaller sizes; (3) It is always cheaper to use a small vehicle than a large one, if the size of the quantity is less than capacity of small vehicle; (4) Regarding the emissions, emission per unit capacity is smaller for the large vehicle, whereas the larger vehicle has a larger fixed emission quantity. When the number of vehicles from each type given as x_1 and x_2 the related total cost and emission quantity can be written as following.

$$\Lambda(x_1, x_2) = x_1K_{V1} + x_2K_{V2}$$
$$\Lambda_E(x_1, x_2) = x_1E_{V1} + x_2E_{V2}$$

A single vehicle type with outsource option

It is also possible to use outsource option in transportation operations together with in-house fleet. Under this assumption the strategy will be to use in-house transportation for the number of items which will exactly fill the greatest number of vehicles that will be used without an idle capacity. For the remaining lot it is possible to use either outsource option or another vehicle from in-house fleet which will have an idle capacity. It is assumed that for the in-house transportation companies are charged for the amount of carbon that was emitted while under outsource option it is under the responsibility of the third party. Transportation equation is revised in order to include outsource amount and related cost and emission are given below.

$$\Lambda(x) = xK_V + k_o\left(\frac{DT}{m} - xC\right)^+$$
$$\Lambda_E(x) = xE_V$$

Two vehicle types with outsource option

In this part outsource option is also added to the problem where two types of vehicles are available and options are enlarged to using the truck with two different capacity types and outsourcing. The assumptions that were previously made for two type vehicles problem are still valid. It is possible to use all of the options with different proportions during one dispatch. The transportation cost is revised in order to include the outsource option over the one that is used for in-house fleet case. Operational cost and emission values defined as follows.

$$\Lambda(x_1, x_2) = x_1K_{V1} + x_2K_{V2} + k_o\left(\frac{DT}{m} - (x_1C_1 + x_2C_2)\right)^+$$
$$\Lambda_E(x_1, x_2) = x_1E_{V1} + x_2E_{V2}$$

Table 3.2: Transportation cost function

		Without outsource option	With outsource option
Operational Costs	A single vehicle type $\Lambda(x)$	xK_V	$xK_V + k_o\left(\frac{DT}{m} - xC\right)^+$
	Two vehicle types $\Lambda(x_1, x_2)$	$x_1K_{V1} + x_2K_{V2}$	$x_1K_{V1} + x_2K_{V2} + k_o\left(\frac{DT}{m} - (x_1C_1 + x_2C_2)\right)^+$
Carbon Emission	A single vehicle type $\Lambda_E(x)$	xE_V	xE_V
	Two vehicle types $\Lambda_E(x_1, x_2)$	$x_1E_{V1} + x_2E_{V2}$	$x_1E_{V1} + x_2E_{V2}$

The objective function of the problem is defined as the total of average operational costs resulted from manufacturing, inventory carrying and transportation activities and average carbon tax paid for the carbon emitted by those activities. To make the exposition simpler, we track average operational costs and average emissions separately, and then combine. We present a methodology for deciding length of production cycle; T, number of dispatches in each cycle; m, number of backorders; b and number of vehicles; $\mathbf{x}$. As a result of the study an inventory policy is proposed in which DT items are produced in each cycle with length T and sent to the retailer in m dispatches with $\mathbf{x}$ vehicles.

3.2 Derivation of Objective Function

In order to come up with the average operational cost function, we take into account production set-up cost and transportation cost per unit time together with the average inventory holding cost per unit per unit time. The average operating cost function is represented by $G(T, b, m, \mathbf{x})$ and is given in Equation 3.1. In the equation transportation cost is given as $\Lambda(\mathbf{x})$ which is derived for each transportation option in Section 3.1.1.

$$\begin{aligned} G(T, b, m, \mathbf{x}) = & \frac{K_M + m\Lambda(\mathbf{x})}{T} + \frac{mD\left[(\frac{T}{m} - \frac{b}{D})^2 h_R + (\frac{b}{D})^2 c_B\right]}{2T} \\ & + h_M \left[\frac{DT}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2 T}{Pm} - \frac{DT}{2m}\right] \end{aligned} \tag{3.1}$$

In the equation, K_M/T and $m\Lambda(\mathbf{x})/T$ represent average production setup and transportation cost per unit time, respectively. The average inventory costs are computed by using standard arguments and retailer's average inventory holding level is represented as:

$$\frac{mD\left[(\frac{T}{m} - \frac{b}{D})^2 h_R + \frac{b}{D}^2 c_B\right]}{2T}$$

In order to interpret manufacturer's average inventory level the production interval can be analyzed in two cases such that manufacturer either produces items and supplies the demand at the same time or does not make production but supplies the demand from inventory. Then, $DT/2\,(1 - D/P)$ is used to represent inventory of the standard EPQ model while D^2T/Pm represents the extra inventory that is accumulated before the dispatch. This amount includes inventory both at the supplier and the retailer. Hence, in order to find inventory of the manufacturer, retailer's inventory: $DT/2m$ is subtracted. Note that this model is a standard one and analyzed by several papers including Hahm and Yano [24] and well-known text-books such as Axsäter [25]. In the model, as P reaches infinity, manufacturer does not keep inventory. Manufacturer's average inventory holding cost is represented as follows:

$$\frac{DT}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2T}{Pm} - \frac{DT}{2m}$$

For the emission function, the carbon emission resulting from production, transportation and inventory holding are calculated. Fixed emissions for production, transportation and inventory are found by E_M/T, $m\Lambda_E(\mathbf{x})/T$ and $(m+1)E_Z/T$, respectively. For the variable carbon emission in inventory, it is assumed that emission resulting from holding a unit item in retailer's or manufacturer's inventory is indifferent, hence the resulting average emission per unit per unit time is formulated as following:

$$e_Z\left[\frac{mb^2}{2DT} + \frac{DT}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2T}{Pm} - b\right]$$

As a result, $E(T, b, m, \mathbf{x})$ is defined as the total quantity for carbon emission resulting from the operations.

$$E(T, b, m, \mathbf{x}) = \frac{E_M + E_Z(m+1) + m\Lambda_E(\mathbf{x})}{T} + e_Z\left[\frac{mb^2}{2DT} + \frac{DT}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2T}{Pm} - b\right] \tag{3.2}$$

In this study our aim is to find optimal production interval, backorder level, number of dispatches and number of vehicles for each type, when there exists carbon tax. The objective function of the problem, $TC(T, b, m, \mathbf{x})$, is given in Equation 3.3 and includes minimization of both the operational costs, which is defined by Equation 3.1, and the carbon tax that will be paid for each unit of carbon that is emitted which is equal to multiplication of carbon tax and total carbon emitted which is given in Equation 3.2.

$$TC(T, b, m, \mathbf{x}) = G(T, b, m, \mathbf{x}) + p * E(T, b, m, \mathbf{x}) \tag{3.3}$$

3.3 Structural Properties

In this section, structural properties of the problem are given. The transportation option with two vehicles and outsource option is considered while proposing structural properties and solution algorithm. The solution methods when other transportation options are used are given in Appendix A.

It is possible to find optimal value of b which minimizes Equation 3.3 as a function of problem parameters. In Proposition 1 the unique minimizer of b is given.

Proposition 1. *The unique minimizer b^* of $TC(T, b, m, \mathbf{x})$ for given T and m is*

$$b^* = \frac{DT\phi}{m}$$

where $\phi = \frac{h_R + pe_Z}{h_R + c_B + pe_Z}$.

Proof. The first order condition of $TC(T, b, m, x)$ with respect to b results in:

$$\frac{\partial TC(T, b, m)}{\partial b} = -D(h_R + pe_Z) + \frac{mb(h_R + c_B + pe_Z)}{T} = 0$$

The optimal value of b is then given as the following ratio.

$$b^* = \frac{DT\phi}{m} \tag{3.4}$$

In order to check uniqueness of b, the second condition of the cost function is checked and found to be positive.

$$\frac{\partial^2 TC(T,b,m)}{\partial b^2} = \frac{m(h_R + c_B + pe_Z)}{T} \geq 0$$

$\square$

Now, it is possible to represent the objective function for $b = b^*$ and denote it as $TC(T, m, \mathbf{x})$ as b^* does not depend on transportation mode selection and hence valid for all transportation options.

$$TC(T,m,\mathbf{x}) = \frac{K_M + p(E_M + E_z)}{T} + T\gamma + \frac{m(\Lambda(\mathbf{x}) + p(\Lambda_E(\mathbf{x}) + E_z))}{T} + \frac{T\lambda}{m} \quad (3.5)$$

where $\gamma = h_M \frac{D}{2}\left(1 - \frac{D}{P}\right) + e_Z p \frac{D}{2}\left(1 - \frac{D}{P}\right)$

and $\lambda = h_R \frac{D}{2}(1 - 2\phi + \phi^2) + \frac{Dc_B\phi^2}{2} + h_M\left(\frac{D^2}{P} - \frac{D}{2}\right) + e_Z p\left(D\phi(\frac{\phi}{2} - 1) + \frac{D^2}{P}\right)$

In Proposition 2 the maximum number of Type 1 and Type 2 vehicles, where Type 1 vehicle have a greater capacity than Type 2, are given. The resulting terms are used as the upper bound on the decision variable $\mathbf{x}$.

Proposition 2. *(i) For given m optimal number of Type 2 vehicles is bounded above by $x_{u2} = \lceil DT_0/mC \rceil$ which is the nearest integer up to the solution of DT_0/mC where*

$$T_0 = \sqrt{\frac{K_M + pE_M + pE_Z(m+1)}{\gamma + \frac{\lambda}{m}}} \quad (3.6)$$

is the solution of the problem when transportation costs are excluded from the total cost function defined in Equation 3.5.

(ii) The upper bound for the Type 1 vehicles is denoted by x_{u1} and found by the ratio $\lfloor (K_{V1} + pE_{V1})/(K_{V2} + pE_{V2}) \rfloor$ which gives the greatest number of Type 1 vehicles that is less costly to operate than a single Type 2 vehicle.

Proof. (i) Follows from Rieksts and Ventura [18]

(ii) The number of smaller capacitated vehicles that is less costly to operate than a single vehicle with greater capacity is found by the solution of $K_{V1} + pE_{V1} = x_{u1}(K_{V2} + pE_{V2})$.

□

In order to find optimal m value a finite search is performed from 1 to upper bound of m; m_u which is given in Proposition 3.

Proposition 3. *The upper bound of number of dispatches; m under uncapacitated transportation assumption and when $D/P \geq 1/2$ is;*

$$m_u = \sqrt{\frac{(K_M + pE_M)(2D - P)}{(K_V + pE_V)(P - D)}}$$

Proof. The factors that restricts m to increase are the inventory costs of retailer and transportation costs. In order to find an upper bound on m let $h_R = 0, \quad e_z = 0, \quad E_z = 0, \quad K_V = min\{K_{V1}, K_{V2}\}, \quad E_V = min\{E_{V2}, E_{V2}\}$ when $C = \infty$. Hence, there will not be any factor that restricts m to increase. Under this assumption, total cost function and T value minimizing it can be given as below:

$$TC(T, m) = \frac{K_M + pE_M}{T} + T\gamma_0 + \frac{m(K_V + pE_V)}{T} + \frac{T\lambda_0}{m} \tag{3.7}$$

where $\gamma_0 = h_M \frac{D}{2}\left(1 - \frac{D}{P}\right)$ and $\lambda_0 = h_M D\left(\frac{D}{P} - \frac{1}{2}\right)$.

It is possible to obtain T value that minimizes Equation 3.7 by checking first condition of equation. The resulting value is given as a function of m below.

$$T(m) = \sqrt{\frac{K_M + pE_M + m(K_V + pE_V)}{\gamma_0 + \frac{\lambda_0}{m}}} \tag{3.8}$$

Similarly, it is also possible to find the optimal m value from the first condition of Equation 3.7 with respect to m as a function of T:

$$\frac{\partial TC(T,m)}{\partial m} = \frac{K_V + pE_V}{T} - \frac{T\lambda_0}{m^2} = 0$$

$$m(T) = \sqrt{\frac{T^2\lambda_0}{K_V + pE_V}}$$

Under the condition $D/P \geq 1/2$ Equation 3.7 is convex in m hence, $m(T)$ is the unique minimizer of Equation 3.7:

$$\frac{\partial^2 TC(T,m,b^*,x)}{\partial m^2} = \frac{T\lambda_0}{m^3} \geq 0 \quad if \quad \frac{D}{P} \geq \frac{1}{2}$$

When T ie equal to the solution given in Equation 3.8, it is possible to find optimal m value independent from other decision variables which gives an upper bound for the problem under uncapacitated transportation assumption.

$$m_u = \sqrt{\frac{(K_M + pE_M)(2D-P)}{(K_V + pE_V)(P-D)}}$$

$\square$

Please note that, m is unbounded under zero tax rate or if outsourcing transportation of all items is less costly and hence no vehicles are used. As if all items are transported with outsource option then total cost of the system is found as below:

$$TC(T,m) = \frac{K_M}{T} + T\gamma + Dk_o + \frac{T\lambda}{m}$$

Total cost function takes the minimum value as m goes to infinity.

In order to find upper bound on m when vehicles are capacitated or outsourcing transportation of all items is less costly or $D/P < 1/2$, m_u is used as the upper bound for the initial step and a search algorithm is used to find optimal solution. If the initial solution $\hat{m}$ is less than m_u; $\hat{m} < m_u$ then the search is completed and optimal solution m^* is selected as $\hat{m}$, else if $\hat{m} = m_u$ then m_u is increased incrementally; $m_u = m_u + 1$ and search region is enlarged, this step is repeated until the solution is less than the upper bound; $\hat{m} < m_u$.

For the next step optimal production interval, T, is found. Due to the capacity limit of the vehicles, cost function is not continuous over T. In Figure 3.3 total cost is plotted against production interval when m is fixed and equal to one and single vehicle without outsource option is used for transportation. From the plot, it can be seen that total cost function is not continuous over T. Instead, there exists discontinuity points at the production intervals where number of items produced is greater than the capacity of the vehicles that are assigned. In Figure 3.3, each continuous region corresponds to a solution when a specific number of vehicle is used. A local optima is maintained as one of the solution of the problem with given x or discontinuity points which are given as $\hat{T}, j_1$ and j_2, respectively in Figure 3.3. After a local optima is found for each continuous region, global optima is selected as the local optima which minimizes the objective.

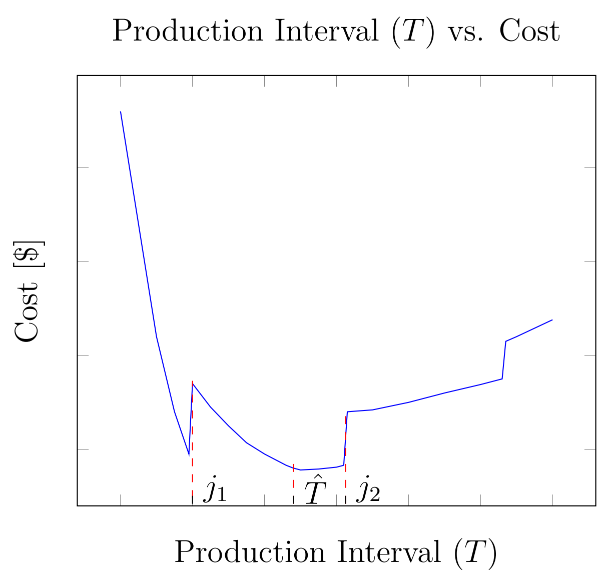


Figure 3.3: Production interval (T) vs. cost

Note that when two vehicles with outsource option is considered, then, rather than two, four discontinuity points exists which are denoted as j_1, j_2, j_3 and j_4 and explained in detail later in this section.

Under this transportation option, the combination of transportation modes can be represented in three different cases which are explained below. For given T let $x_1 \in [0, x_{u1}]$ and $x_2 \in [0, x_{u2}]$ be the number of loaded vehicles from each type and $C_2 > C_1$. The optimal combination of vehicles is the one that minimizes the transportation cost:

$$\Lambda(x_1, x_2) + p\Lambda_E(x_1, x_2) = \min\{x_1(K_{V1} + pE_{V1}) + x_2(K_{V2} + pE_{V2}) + k_o\left(\frac{DT}{m} - (x_1C_1 + x_2C_2)\right), x_2(K_{V2} + pE_{V2}) + (x_1 + 1)(K_{V1} + pE_{V1}), (x_2 + 1)(K_{V2} + pE_{V2})\} \quad (3.9)$$

In the first case, after x_1 Type 1 vehicles and x_2 Type 2 vehicles are fully loaded, rest of the items can be outsourced. In this case the transportation cost can be found as follows:

$$x_1(K_{V1} + pE_{V1}) + x_2(K_{V2} + pE_{V2}) + k_o\left(\frac{DT}{m} - (x_1C_1 + x_2C_2)\right) \quad (3.10)$$

For a second case, rather than outsourcing the remaining lot it is possible to use another Type 1 vehicle which will have an idle capacity. In this case outsourcing option will not be used and the transportation cost can be found as follows:

$$x_2(K_{V2} + pE_{V2}) + (x_1 + 1)(K_{V1} + pE_{V1}) \quad (3.11)$$

As a third case, it is also possible to use $x_2 + 1$ Type 2 vehicles directly rather than using neither another vehicle type or outsourcing for which the following cost function is used.

$$(x_2 + 1)(K_{V2} + pE_{V2}) \quad (3.12)$$

Note that if the marginal cost of using Type 2 vehicle is greater than per unit outsourcing cost, then only outsourcing will be used. Similarly, if outsourcing cost is greater than marginal cost of using Type 2 vehicle but less than Type 1, then the problem will be simplified to single type with outsource problem.

Otherwise, mode selection will be done as the solution with the minimum cost among the three cases suggests which is given in Equation 3.9.

For a given m and $\mathbf{x}$ the T value that minimizes cost can be found by optimizing the objective function as if it is feasible in all regions and referred as $T(m,x)$. Note that if the $T(m,x)$ value found is feasible, then it is a local optimal point. Otherwise $T(m,x)$ is adjusted to the value that gives the minimum cost among discontinuity points which are explained below. We show this result in Proposition 4.

Proposition 4. *The local minimizer of the objective function for given m and $\mathbf{x}$ is defined as T(m,x) and can be found by the following equation:*

$$T(m,x) = argmin\{TC(T_1,m,x), \quad TC(T_2,m,x), \quad TC(T_3,m,x)\}$$

where argmin function is defined as over the arguments T_1, T_2 or T_3 which are defined below.

- *Let $\hat{T}_1$ be defined as*

$$\hat{T}_1 = \sqrt{\frac{K_M+m(x_1(K_{V1}-k_oC_1)+x_2(K_{V2}-k_oC_2))+p(E_M+E_Z(m+1)+m(x_1E_{V1}+x_2E_{V2}))}{\gamma+\frac{\lambda}{m}}}$$

$$T_1 = \begin{cases} \hat{T}_1, & \text{if } j_1 \leq \hat{T}_1 \leq j_2 \\ argmin\{TC(j_1,m,x) \quad TC(j_2,m,x)\}, & \text{otherwise} \end{cases} \tag{3.13}$$

- *Let $\hat{T}_2$ be defined as*

$$\hat{T}_2 = \sqrt{\frac{K_M+m((x_1+1)K_{V1}+x_2K_{V2})+p(E_M+E_Z(m+1)+m((x_1+1)E_{V1}+x_2E_{V2}))}{\gamma+\frac{\lambda}{m}}}$$

$$T_2 = \begin{cases} \hat{T}_2, & \text{if } j_2 \leq \hat{T}_2 \leq j_3 \\ argmin\{TC(j_2,m,x), \quad TC(j_3,m,x), & \text{otherwise} \end{cases} \tag{3.14}$$

- *Let $\hat{T}_3$ be defined as*

$$\hat{T}_3 = \sqrt{\frac{K_M+m(x_2+1)K_{V2}+p(E_M+E_Z(m+1)+m(x_2+1)E_{V2})}{\gamma+\frac{\lambda}{m}}}$$

$$T_3 = \begin{cases} \hat{T}_3, & \textit{if } j_3 \leq \hat{T}_3 \leq j_4 \\ argmin\{TC(j_3, m, x), \quad TC(j_4, m, x), & otherwise \end{cases} \tag{3.15}$$

where $j_1 = \frac{(x_1C_1+x_2C_2)m}{D}$
$j_2 = \frac{m(K_{V1}+pE_{V1})}{Dk_o} + \frac{(x_1C_1+x_2C_2)m}{D}$
$j_3 = \frac{x_2C_2m}{D} + \frac{(x_1+1)C_1m}{D}$
$j_4 = \frac{(x_2+1)C_2m}{D}$

Proof. For the first step the production interval that minimizes total cost function under a specific transportation option selection is found by checking the first condition of Equation 3.5. Let us consider the first case that is defined by Equation 3.10, and let

$$\hat{T}_1 = \sqrt{\frac{K_M + m(x_1(K_{V1} - k_oC_1) + x_2(K_{V2} - k_oC_2)) + p(E_M + E_Z(m+1) + m(x_1E_{V1} + x_2E_{V2}))}{\gamma + \frac{\lambda}{m}}}$$

which is the solution of first condition of Equation 3.5:

$$\frac{\partial TC(T, m, \mathbf{x})}{\partial T} = -\frac{K_M + p(E_M + E_Z)}{T^2} + \gamma -$$
$$\frac{m((x_1(K_{V1} + pE_{V1}) + x_2(K_{V2} + pE_{V2}) - k_o(x_1C_1 + x_2C_2) + pE_Z)}{T^2} + \frac{\lambda}{m}$$

The equality above is found under the assumption that x_1 Type 1 vehicles and x_2 Type 2 vehicles used with full capacity, hence it is necessary to check whether the assumption holds when T_1 is used as production interval. If $j_1 \leq \hat{T}_1 \leq j_2$ then assumption holds, $T_1 = \hat{T}_1$.

The lower bound of the inequality gives the time necessary to produce minimum number of items that will fill desired number of vehicles fully. The upper bound is found by finding the production interval at which it is equally costly to outsource remaining lot or using another vehicle with idle capacity. If $\hat{T}_1$ is not in between these bounds, than it is set as the bound which gives the minimum total cost; $T_1 = argmin\{TC(j_1, b^*, m, x), \quad TC(j_2, b^*, m, x)\}$. Note that if (x_1, x_2) vehicles are fully loaded and T_1 is selected as the optimal production interval then the optimal number of vehicles from each type are (x_1, x_2). For finding T_2 and T_3 same arguments are used with the difference if T_2 or T_3 gives the optimal production interval then the number of vehicles is updated as $(x_1 + 1, x_2)$ or $(0, x_2 + 1)$, respectively.

$\square$

3.4 Solution Algorithm

We have T, b, m and $\mathbf{x}$ as decision variables, and hence solution is not straightforward and the structural properties that are discussed in Section 3.3 are used to solve the problem. We consider the approach as summarized below for the transportation option two vehicle types with in-house fleet. We refer to the algorithms given in A for the solution of other transportation options.

(i) We proposed and proved a structural property that allows the decision variable on the backorder level; b removed by its unique minimizer, written in terms of the other decision variables in Proposition 1.

(ii) We showed that an upper bound can be found for the integer decision variables, $\mathbf{x}$ and m. The upper bounds are given in Proposition 2 and 3, respectively. As a result, a finite search on the range is performed and the solution which gives the least cost is selected as optimal when all other variables are either given or at their optimal values.

(iii) We showed that T optimal can be found for given $\mathbf{x}$ and m in Proposition 4. A finite search is executed with the upper bounds that are previously found for $\mathbf{x}$ and m and for each combination optimal T is found.

(iv) Finally, we present steps of the solution algorithm.

In the solution algorithm, firstly it is checked whether it is less costly to use only outsource option or only single type vehicle. If it is, then the related problem should be solved. Note that solution algorithms for the problems with other transportation options are given in Appendix A. Otherwise, in steps 9 through 11, m

and $\mathbf{x} = (x_1, x_2)$ are set to a fixed value such that $m \in (0, m_u)$, $x_1 \in (0, x_{u1})$, $x_2 \in (0, x_{u2})$ to perform a finite search. In step 12 the optimal production interval is found for given m and $\mathbf{x}$ values, as it is described in Proposition 4 and in step 14, the optimal number of vehicles are selected by using Proposition 4 again. The procedure is repeated for each combination of x_1, x_2 and m, and the solution which gives minimum cost is returned as optimal. Finally, the solution of b that gives the minimum cost is given when all other variables are at their optimal values. Please note that, this solution can not be generalized for the problems in which the structural properties are not valid. Under this case, total enumeration can be used to solve the problem.

Algorithm 1 Minimize Total Cost for Two Vehicle Types with Outsource Option

1: **procedure** COMPUTE THE BEST COMBINATION FOR $(T, b, m, \mathbf{x})$
2: **if** Using outsource option is cheaper than using both vehicles **then**
3:         $T^* \leftarrow T_0$, skip steps 11-22.
4: **else if** Using Type 1 vehicle is cheaper than using Type 2 **then**
5:         Solve single type with outsource problem
6: **else**
7:         Continue
8: **end if**
9: **for** $m = 1 : m_u$ **do**
10:    **for** $x_2 = 0 : x_{u2}$ **do**
11:       **for** $x_1 = 0 : x_{u1}$ **do**
12:          $T^* \leftarrow argmin\{TC(T_1, m, x), \quad TC(T_2, m, x), \quad TC(T_3, m, x)\}$
13:          by Propostion 4
14:          **if** $T^* = TC(T_1, m, x)$ **then**
15:             $(x_1^*, x_2^*) \leftarrow (x_1, x_2)$
16:          **else if** $T^* = TC(T_2, m, x)$ **then**
17:             $(x_1^*, x_2^*) \leftarrow (x_1 + 1, x_2)$
18:          **else**
19:             $(x_1^*, x_2^*) \leftarrow (0, x_2 + 1)$
20:          **end if**
21:       **end for**
22:    **end for**
23: **end for**
24: **return** T^*, m and (x_1^*, x_2^*) that yields minimum total cost.
25: **if** $m = m_u$ **then**
26:     $m_u \leftarrow m_u + 1$, return to step 9.
27: **else**
28:             Continue
29: **end if**
30: $b^* = \frac{DT^*\phi}{m}$

Chapter 4

Benchmark Models

In this chapter, we introduce a number of models which are likely to be used if integration of all activities for planning purposes is not possible. We name these models as "benchmark" and compare them with the integrated policy to investigate numerically and answer some of the research questions, as well as understand performance differences when such integrated policies may not be applicable.

Although, cooperation of manufacturer, retailer and transporter is necessary for integration of operations, under many cases it may not be possible to maintain such a unity. The difficulties of cooperation mainly rise due to the differences in objectives of supply chain players and hardness of communication. Under the cases when it is highly unlikely to maintain integration, it is possible to work under non-integrated or partially integrated models. In literature there exists studies which focuses on partial integration of manufacturer and retailer, and manufacturer and transportation which are Jaber et al. [29] and Battini et al. [12], respectively. In this section all possible partial integration and non-integration scenarios are analyzed in order to enable selection of best model when not all supply chain players are in favor of integration.

We start with two straightforward benchmark models where in the first we do not take into account the effect of emission tax and in the latter only consider emissions. We then proceed with a number of decentralized models where we assume one of the entities of the system considered (manufacturer or retailer) to be the leader, make decisions and followed by the other. The notation used in Table 4.1 is used to refer benchmark models in the rest of this study. We refer the average cost functions used for the decentralized models with superscripts as defined in Table 4.1 referring to the model utilized.

Table 4.1: Notation for benchmark models

Notation	Problem
IC	Cost minimization
IE	Emission minimization
M-R	Manufacturer is the leader and transportation is exogenous
MT-R	Manufacturer is the leader and transportation is integrated with the manufacturer's decision
R-M	Retailer is the leader and transportation is exogenous
RT-M	Retailer is the leader and transportation is integrated with the retailer's decision

Integrated Model with No Emissions - IC

We consider an integrated model where emissions are not assumed to be a part. Hence, this is a more traditional model without carbon concerns. The objective function of this model is defined by Equation 3.1; $G(T, m, b, \mathbf{x})$. To solve this optimization problem, we follow the solution method discussed in Chapter 3, depending on transportation options. In Chapter 3 the objective is given by the Equation 3.3; $TC(T, m, b, \mathbf{x})$, for the solution of this problem the same steps are repeated with Equation 3.1. Appendix B.1 gives explanation on the solution of the model. Optimal objective function of this model gives a lower bound for the cost of the optimal integrated system. We can compute emissions resulted by this partially integrated model and can obtain an upper bound for the carbon emission amount with respect to the optimal integrated model. The total actual cost of this system can be found by sum of operational costs and the carbon tax paid as a result of the chosen policy.

Integrated Model with Only Emission Concerns - IE

It is also possible to consider an integrated model where the only concern is carbon emission rather than operational costs. The objective function of this model is given by Equation 3.2; $E(T, m, b, \mathbf{x})$. Note that to solve this optimization problem, we follow the cases discussed in Section 3.3 again, depending on transportation options. An explanation on the solution method can be found in Appendix B.1. Solution of this model gives the lowest carbon emission value that can be achieved and hence becomes lower bound for solution of integrated system. On the other hand, when the resulting total cost is calculated with optimal solution of emission minimization problem, an upper bound is obtained for integrated problem.

Decentralized Models with Manufacturer as the Leader - M-R and MT-R

In this case, we assume that manufacturer is the leader, and hence decisions made by the manufacturer are followed by the retailer given the problem environment. Note that retailer may also solve an optimization problem if there is room for her decision without violating the leader's decisions. We consider two sub-cases.

In the first case we assume that two main functions (manufacturing and retailing) make hierarchical decisions starting with manufacturer determining optimal cycle length by taking into account operational costs and emission tax of her part only without transportation cost. Equation 4.1 depicts the objective of the manufacturer.

$$C_M^{M-R}(T) = \frac{K_M + p(E_M + E_Z)}{T} + (h_M + pe_Z)\left[\frac{DT}{2}\left(1 - \frac{DT}{P}\right) + \frac{D^2T}{P} - \frac{DT}{2}\right] \tag{4.1}$$

Transportation is not under manufacturer's responsibility and hence becomes part of the system cost. It is possible to use any of the transportation options that are available for the order size given by manufacturer. Finally, retailer decides on maximum backorder level given the decision of the manufacturer. Equation 4.2 depict the objective function of retailer.

$$C_R^{M-R}(T,b) = \frac{D\left[(T-\frac{b}{D})^2(h_R+pe_Z)+\frac{b}{D}^2 c_B\right]}{2T} + \frac{pE_Z}{T} \tag{4.2}$$

In order to find optimal cycle length and backorder level first order condition of Equation 4.1 and 4.2 is used, respectively. Algorithm 6 in Appendix B.2 gives the solution method that is used. The total cost is obtained as the sum of manufacturer's, retailer's and transportation cost when the solution obtained by Algorithm 6 is used.

In the second decentralized model, we assume that manufacturer is also responsible of transportation activities, hence, solves those two functions in an integrated way. Retailer, on the other hand, solves for the optimal value of the maximum backorder level given the decisions of the manufacturer. Equations 4.3 depicts the optimization problem solved by the manufacturer:

$$\begin{aligned} C_M^{MT-R}(T,\mathbf{x}) = \frac{K_M + p(E_M+E_Z+\Lambda_E(\mathbf{x}))+\Lambda(\mathbf{x})}{T} &+ \\ (h_M+pe_Z)\left[\frac{DT}{2}\left(1-\frac{DT}{P}\right)+\frac{D^2T}{P}-\frac{DT}{2}\right]& \end{aligned} \tag{4.3}$$

The cost function of the retailer is the same with the previous case and given in Equation 4.2 rather than Equation 3.3. We follow the solution steps outlined in Chapter 3 to solve the above optimization problem with the objective defined in Equation 4.3. Solution of the problem is given in Appendix B.3. The total cost is find as sum of costs of all operations.

Decentralized Models with Retailer as the Leader - R-M and RT-M

In this model we assume that retailer is the leader and manufacturer follows the decisions made by the leader. It is again possible for manufacturer to solve an optimization problem unless it depends on retailer's decision. Similar to the decentralized models with manufacturer as the leader, we again focus on two cases.

First, we assume transportation is not under responsibility of retailer, hence there is no room for decisions considering it but available transportation options can be used for the given order size. The aim of this problem is deciding the retailer's reorder intervals such that retailer's inventory cost and carbon tax incurred for emissions resulting from inventory will be minimized. Once retailer makes the decision, manufacturer can determine the production interval as a multiple of retailer's reorder interval. Equations 4.4 gives the objective of the retailer.

$$C_R^{R-M}(T,b) = \frac{D\left[(T-\frac{b}{D})^2(h_R+pe_Z)+\frac{b}{D}^2 c_B\right]}{2T} + \frac{pE_Z}{T} \tag{4.4}$$

Equation 4.5 gives the objective of manufacturer.

$$C_M^{R-M}(T,m) = \frac{K_M+pE_m}{mT} + \frac{pE_Z}{mT} + (h_M+pe_Z)\left[\frac{DTm}{2}\left(1-\frac{DTm}{P}\right)+\frac{D^2T}{P}-\frac{DT}{2}\right] \tag{4.5}$$

The problem is solved for retailer's objective which is given in Equation 4.4. For the solution of optimal cycle length and backorder level first condition of Equation 4.4 is checked with respect to T and b, respectively. In order to find optimal number of dispatches first condition of Equation 4.5 is used. The solution algorithm can be found in Appendix B.4.

As a second case, it is also possible to include transportation mode selection to the decisions of the leader. As a consequence, retailer will solve the problem for her inventory and transportation activities in an integrated way. In this case the objective of the problem is minimizing retailers cost together with transportation cost which is given in Equation 4.6.

$$C_R^{RT-M}(T,b,\mathbf{x}) = \frac{D\left[(\frac{T}{m}-\frac{b}{D})^2(h_R+pe_Z)+\frac{b}{D}^2 c_B\right]}{2T} + \frac{p(E_Z+\Lambda_E(\mathbf{x}))+\Lambda(\mathbf{x})}{T} \tag{4.6}$$

Manufacturer can still decide the production interval as a multiple of retailers reorder interval and her objective is given in Equation 4.5. In order to solve the model, solution algorithm that is defined in Appendix B.5 is used.

Chapter 5

Numerical Analysis

The previously defined problems are analyzed through a numerical study and behaviors of optimal production interval, dispatch number and transportation policies under different scenarios are evaluated. Through the first section of the numerical study, solution of the problem with different parameter values are discussed, following this, benefits provided by allowing outsource option for transportation is studied and lastly, benchmark problems are compared and improvements maintained by integrating the system rather than solving separately are shown. In order to refer the different transportation options, the following notation is used. We refer to the benchmark models with the notation previously defined in Table 4.1.

Table 5.1: Notation for models

Notation	Problem
S	Single vehicle without outsource
D	Two vehicles without outsource
SO	Single vehicle with outsource
DO	Two vehicles with outsource

5.1 Parameter Setting

Our parameter choice is based on the numerical studies provided in Cachon [30], Benhelal et al. [31], Ravanshadnia and Ghanbari, [32] and Ali et al. [33]. In Cachon [30], transportation and inventory cost is calculated as a combination of emission related term and operational costs. While emission related cost corresponds to E_V, operational cost corresponds to K_V for transportation and e_z, h_R and h_M corresponds to emission and cost terms for inventory holding. For the carbon emitted during manufacturing processes we focused on cement industry and refer to Benhelal et al. [31], Ravanshadnia and Ghanbari [32] and Ali et al. [33]. They state that during production of 1 ton cement 800-900 kg carbon is emitted. In the numerical analysis we assumed that demand is 600 units however, the order size that will be produced in a single interval is a decision variable. We used an average order size by considering vehicle capacities and set E_M accordingly. The parameters that are taken from existing studies are given in Table 5.2, rest of the parameters are chosen by us are given in Table 5.3, for these parameters sensitivity analysis is also done and provided in this section. Unless otherwise stated the same parameter selection is used throughout the chapter.

Table 5.2: Parameter values taken from literature

parameter	K_{V1}	K_{V2}	h_M	h_R	E_{V1}	E_{V2}	E_Z	e_Z
input data	12	20	1.00	1.25	10	15	12.90	0.12

Table 5.3: Remaining parameter values

parameter	D	P	C_1	C_2	K_M	E_M	c_B	k_o
input data	600	700	80	250	56	77.50	2.25	0.33

In this part we provide an analysis on the parameter setting and discuss how the optimal policy changes with respect to shifts on backorder cost; c_B, carbon emitted during manufacturing E_M and production set-up cost; K_M. The problem is solved for transportation option single vehicle without outsource. The plots for tax vs. cost and carbon emission under different parameter values are given in Figure 5.1.

Sensitivity analysis with respect to backorder cost

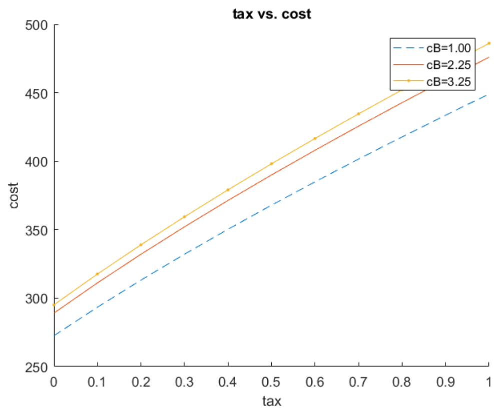
The results of sensitivity analysis of c_B and details are given in Appendix C.1, Table C.5, and summarized in Figures 5.1a and 5.1b. When backorder cost is decreased, backorder levels are increased as expected and time intervals between two dispatches are also increased due to ability to backorder more items in a cycle. Similar to the solutions under different h_R values, this is either maintained by an increase in T or decrease in m. On the opposite case, when backorder cost is increased the interval is shortened in order to decrease number of backordered items. The increase in c_B gives rise to total cost and carbon emission which can be observed via Figures 5.1a and 5.1b.

Sensitivity analysis with respect to carbon emitted during manufacturing

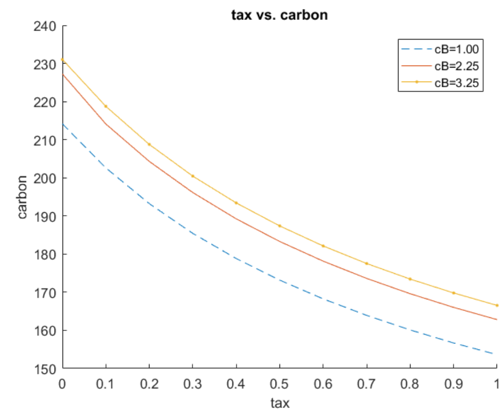
Different technology selections can lead to variations in amounts of carbon emissions in manufacturing processes. In order to analyze possible changes in optimal policy and objective, the problem is solved with different E_M values which are given in, Appendix C.1 Table C.6. In the solutions it is observed that length of production interval increases with E_M as expected. The Figures 5.1c and 5.1d shows the changes in cost and carbon emission under different tax rates for all E_M values that are given. Under tax rate 0.5$ per kg., when an improved technology is used with $E_M = 60$ kg. rather than $E_M = 76$ kg. , carbon emission is reduced to 173.48 kg. from 183.30 kg. which results with a decrease in cost from 389.92$ to 383.92$. On the other hand, when a technology with $E_M = 77.5$ kg. is used the same carbon emission level is reached when tax rate is increased to 0.7$ per unit with a resulting total cost of 425.57$. These results can be interpreted in favor of a better technology selection in terms of carbon emission amounts.

Sensitivity analysis with respect to production set-up cost

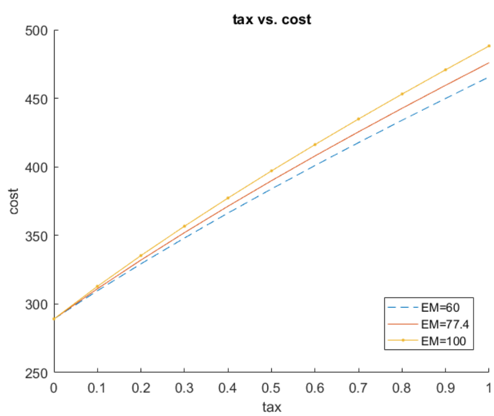
Similar results are maintained when problem is solved with different K_M values.



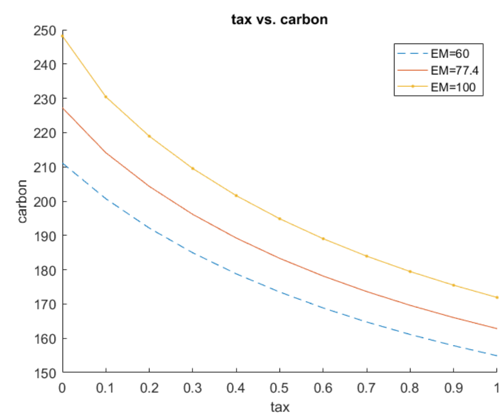
(a) tax vs. cost with different c_B values



(b) tax vs. carbon with different c_B values



(c) tax vs. cost with different E_M values

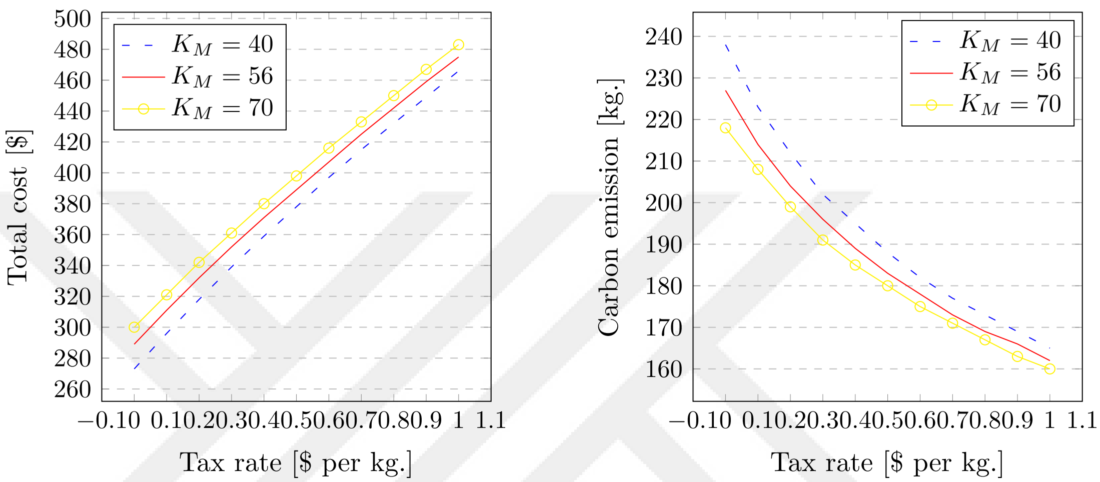


(d) tax vs. carbon with different E_M values

Figure 5.1: Sensitivity analysis - 1

Soluitons to problems with different K_M are given in Appendix C.1, Table C.7. Carbon emission levels decreases as K_M increases while for total cost of the system opposite case holds as expected. The results can be observed from Figure 5.2a and 5.2b.

Sensitivity analysis show that total cost of the system changes from 272$ to 300$ at zero tax rate and from 449$ to 487$ when tax is equal to one, according to the different parameter selections. On the other hand, carbon emission changes from 211 kg. to 248 kg. and from 154 kg. to 171 kg. when tax rate is zero and one, respectively. As a result, we can conclude that total cost and carbon emission may change up to 9 % and 14 % with respect to the parameter settings. It is also observed that at low tax rates total cost is most sensitive to production set-up cost while at high tax rates backorder cost effects total cost more.



(a) Tax rate vs. total cost (b) Tax rate vs. carbon emission

Figure 5.2: Sensitivity analysis - 2

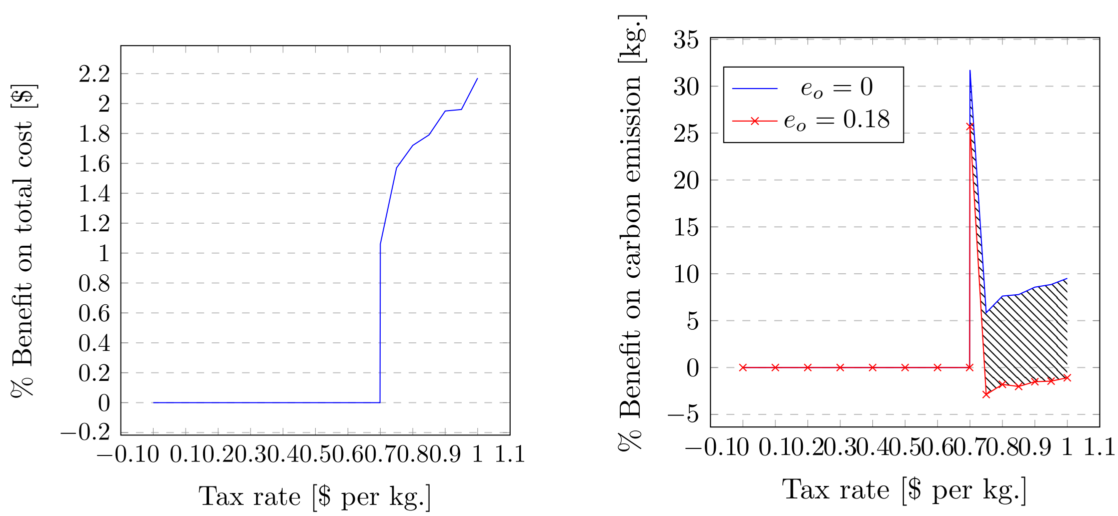
5.2 Effects of Multiple Transportation Modes

5.2.1 Constant Outsourcing Price

For the next analysis, benefits of providing different transportation options are discussed when outsourcing price does not depend on carbon tax. This assumption is relaxed in the following subsection. The problem is solved when following three transportation options are available: single vehicle with outsource, two vehicles with and without outsource. The parameters that were defined in Table 5.2 and 5.3 are used for all problems. For the single type vehicle problem, we consider only the vehicle with larger capacity.

Optimal solution of the problems with transportation options two vehicles with and without outsource are given in Tables C.1 and C.3 in Appendix C for different tax rates. From the table it is seen that when tax rate is relatively low, only in-house fleet is used in optimal policy but when the tax increases, outsource option is considered as cost of outsourcing a unit item does not depend on carbon tax. From Table C.3 it is observed that both production interval, T and the number of dispatches, m increases, until outsource option began to be used in order to decrease the effect of carbon cost incurred for manufacturing operations. After that point, m decreases as a further increase ends up with rise in the amount of inventory and leads to a less profitable solution. Another observation is, as tax rate increases number of backorders also increase in order to reduce inventory at retailer's level. Although, the solution yields a decrease in cost of retailer, for customers it represents a worse service level.

Figure 5.3 gives the benefit of including outsource option on total cost and carbon emission in percentage with respect to the tax rate. While plotting a precision of 0.05 is used for the tax rate. The percent improvement in the total cost is found by $100(TC_D^* - TC_{DO}^*)/TC_D^*$ where TC_D^* and TC_{DO}^* are optimal solutions of the problems of two vehicles without and with outsource option, respectively. Similarly, in order to find improvement in carbon emission $100(E_D^* - E_{DO}^*)/E_D^*$ is used where E_D^* and E_{DO}^* are resulting emissions when total cost is minimized. While improvement on total cost maintained by allowing outsource option is always positive, it can be negative for carbon emission. From Figure 5.3a, it is seen that when tax rate is relatively high, outsource option is preferred, and as a result total cost is decreased up to 2%. In Figure 5.3b, percent reduction in carbon emission vs. tax rate is given. In the figure, the two plots correspond to the cases where carbon emission resulted from outsourcing transportation is considered and not considered. As the carbon emitted during transportation highly depends on technology selection, total emission can vary according to the vehicle type that is chosen by the third party. In order to compare total emissions, we firstly considered the best technology selection in which vehicles do not emit carbon. An example to such logistic firms can be given as FedEx [6] which uses an electric car fleet. The opposite case will be using worst technology selection. It is assumed that under the worst technology the combination of smallest vehicle; $C = 80$ units and highest carbon emission per vehicle value; $E_V = 15$ kg. that is defined in parameter setting is used. In order to come up with the resulting per unit emissions, ratio of carbon emission to capacity is found. To represent different technology selections, we consider any per unit emission level that lies between worst selection and ideal. With this aim, in Figure 5.3b the upper and lower plots give the improvements on carbon emission level when it is assumed that carbon emitted per unit during transportation; e_o is 0 and 0.18, respectively. According to the technology selection, carbon emission can lie in between two plots; inside the shaded region. As a result, it is observed that when tax rate is relatively high and carbon emission resulted from outsourcing is considered allowing outsource option decreases total emission up to 30% according to the technology used. However, the percent improvement decreases after a further increase in tax as the number of vehicles increased in the problem with only in-house fleet option in order to adjust emission level. As a consequence it is possible to comment that, allowing outsource option when vehicles with different capacities exists in the in-house fleet does not necessarily gives a better solution in terms of carbon reduction.



(a) Tax rate vs. % improvement on total cost (b) Tax rate vs. % improvement on emission

Figure 5.3: Benefit of outsourcing

Tables 5.4 and 5.5 summarizes the improvements on total cost and operational costs and emissions in percentage when outsource transportation option is added to in-house fleet with two vehicle types. From Table 5.4 it can be seen that allowing outsource option never gives a less profitable solution. On the other hand, percent benefit may become negative for operational costs and carbon emissions which means that introducing outsource option ends up with an increase in operational costs and emissions. At high tax rates, outsourcing transportation gives more profitable solutions as the carbon tax is not under the responsibility of manufacturer or retailer. However, more carbon is emitted when the whole system is considered. Please note that in Table 5.5a there exists two carbon emission results for solution of the problem with two vehicles with outsource option which corresponds to the cases where carbon emission resulting from outsourcing is included and not included. In the tables, $\Delta\%$ gives the percent improvement and found by $100(TC_D^* - TC_{DO}^*)/TC_D^*$, $100(G_D^* - G_{DO}^*)/G_D^*$ and $100(E_D^* - E_{DO}^*)/E_D^*$ for total cost, operational costs and carbon emission, respectively where for operational costs and emission level we refer to Equations 3.1 and 3.2.

Table 5.4: Percent benefit of outsource on total cost

p[$/kg]	D [$]	DO[$]	Δ [%]
0	248.81	248.81	0
0.5	374.87	374.87	0
1	475.94	465.50	2.19

Table 5.5: Percent benefit of utilizing outsource option on operational costs and carbon emissions

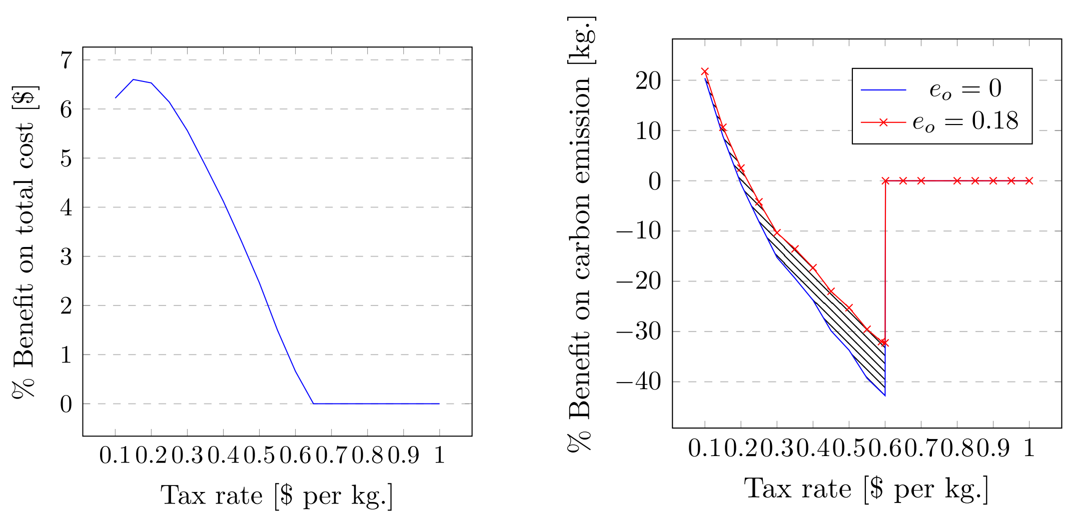
(a) Percent benefit of utilizing outsource on carbon emission

p[$/kg]	D [kg]	DO [kg]	Δ [%]
0	258.58	258.58	0
0.5	246.22	246.22	0
1	162.81	164.60‖147.32	-1.51‖9.59

(b) Percent benefit of outsource on operational costs

p[$/kg]	D [$]	DO [$]	Δ[%]
0	248.81	248.81	0
0.5	251.77	251.77	0
1	313.14	318.18	-1.59

Similarly, it also possible to compare problems with single and two vehicles with outsource. Tables C.2 and C.3 in Appendix C gives the optimal solution of these problems under different tax rates. From the tables it is seen that at low tax rates the vehicle with smaller capacity is preferred while at higher tax rates only outsource option is used. Figure 5.4 summarizes the improvements on total cost and carbon emission when a vehicle with a smaller capacity is added to the in-house fleet with outsource option. It is observed that percent benefit can have a decreasing or increasing pattern as at different tax rates, optimal solutions of the two problems vary and hence the difference between them can increase or decrease. From Figure 5.4a it is seen that adding a second vehicle type decreases total cost up to 6%. At high tax rates the optimal solution suggests using outsource option, hence, no improvements maintained. From Figure 5.4b it is seen that in the problem with single vehicle with outsource less carbon emitted compared to two vehicles with outsource as improvement rates are negative. From Table C.2, it is seen that only outsource option is used in when a single vehicle is available which gives a more costly solution compared with two vehicles but with less carbon emission. As tax rate rises, significance of emission also rises, and hence, the policy which emits less emission, outsourcing items, is chosen as optimal. This results also shows that less costly solution does not result in less emission at all times.



(a) Tax rate vs. % improvement on total cost (b) Tax rate vs. % improvement on emission

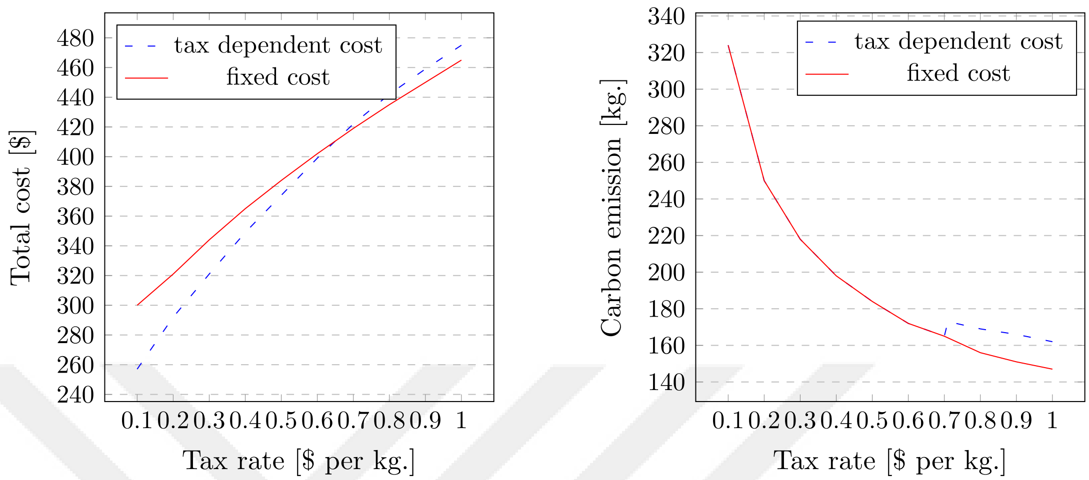
Figure 5.4: Benefits of adding second vehicle type to the in-house fleet

When all problems are considered, it is observed that when tax is increased from 0$ per kg. to 0.7$ per kg., 37 % reduction in carbon emission is maintained while the decrease when tax is increased further becomes meager. From these results it is observed that increasing tax rates above a point will not end up with significant emission reductions but decrease in service level. In this section we assumed that outsource price does not depend on carbon tax, hence as tax rate increases optimal solution shifts towards selecting only outsource option in order to prevent extra cost charged for emission resulting from transportation. As a result, optimal policy does not always favor emission considerations when outsource option is used. In following section we consider the case when outsourcing cost is a function of emission related variables.

5.2.2 Outsourcing Price as a Function of Emission Related Variables

Under the tax policy, as well as applying a constant per unit price for outsourcing transportation, third party firms can also prefer to use tax dependent price. In this case outsource cost can be written as sum of a constant price plus per unit tax rate: $k_o = \alpha K_v/C + \beta E_v p/C$ where α and β are constants which are used to balance the effect of fixed and emission related costs and taken as $\alpha = 2.00$ and $\beta = 1.80$. As a consequence, at high carbon tax rates outsourcing may not be preferred when tax dependent outsourcing cost is applied.

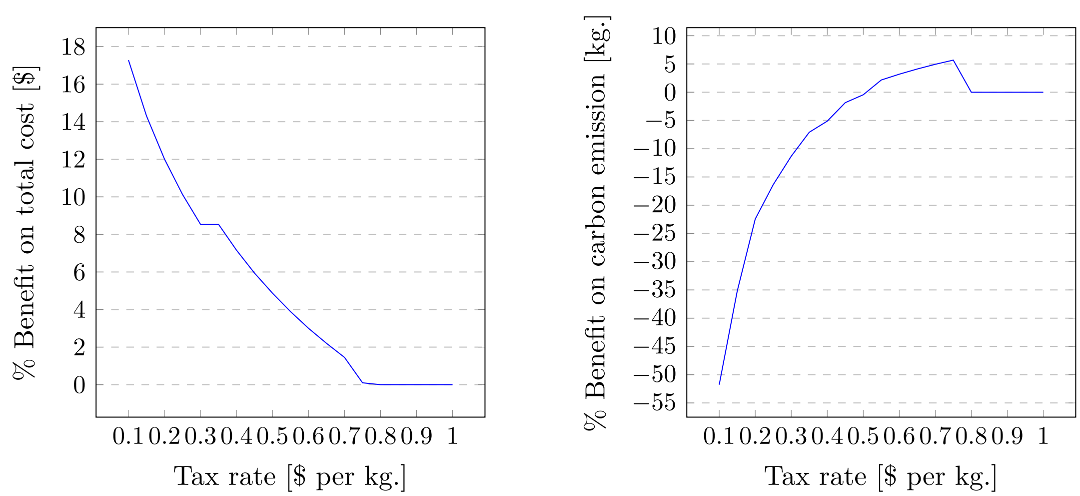
The results and details are given in Table C.4 in Appendix C and summarized in Figure 5.5 and 5.6. In Figure 5.5 total cost and carbon emission levels obtained under optimal solutions of fixed and tex dependent outsource problems are given. From the figure and the table, it can be seen that when tax rate is below 0.7$ per kg. the solution of the problem is same with fixed outsource price problem, however, total cost slightly changes due to the rise in outsource cost. Another observation is that, under high tax rates using outsource option becomes costly hence, policy is changed to in-house fleet which ends up with a slight increase in emission levels.



(a) Tax rate vs. total cost (b) Tax rate vs. carbon emission

Figure 5.5: Comparison of fixed and variable outsource price

In Figure 5.6a the percent benefit reached by allowing outsource option with variable price is given. From the figure, it is observed that the percent reduction in total cost decreases with the increase in tax as expected. Figure 5.6b shows that using outsource option yields benefit in terms on carbon reduction only at high tax rates, just before the optimal policy is switched to using in-house fleet. On the other hand, when tax rate increases further, outsource option is no longer used in optimal policy, hence, percent improvements declines to zero.

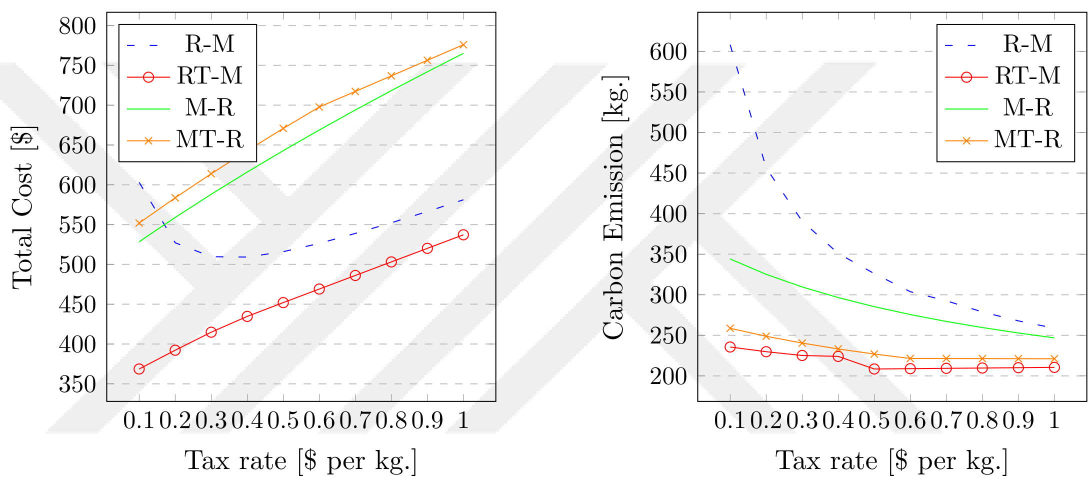


(a) Tax rate vs. % improvement on total cost (b) Tax rate vs. % improvement on carbon emission

Figure 5.6: Benefits of outsource option when price depends on carbon tax

5.3 Comparison of Benchmark Models

In this section, solutions of benchmark problems were compared with each other. The same parameter values are used in all benchmark problems. Figure 5.7 gives total cost and carbon emission values for each benchmark problem.



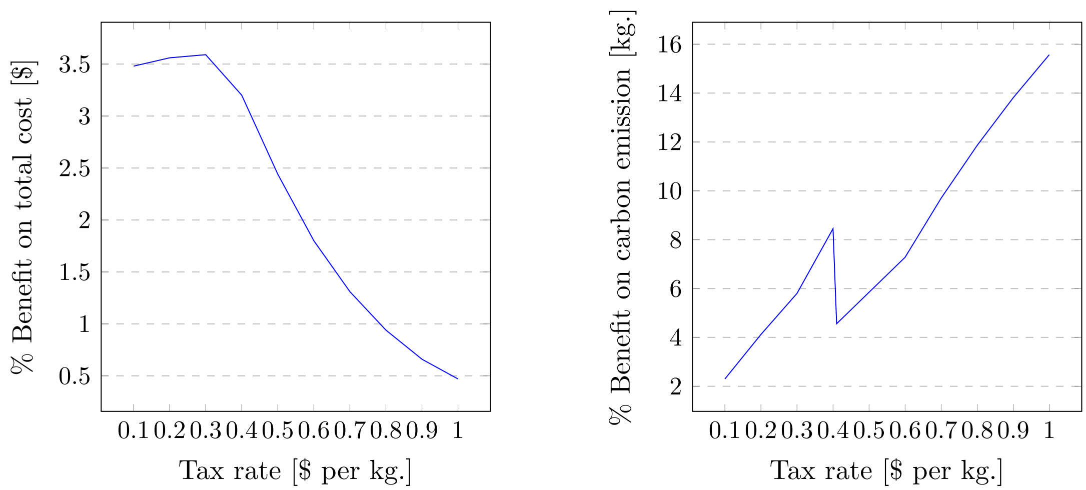
(a) Tax rate vs. Total cost (b) Tax rate vs. Carbon Emission

Figure 5.7: Solutions of benchmark problems

Comparison of the systems reveals that the benchmark model where retailer is the leader and makes the transportation (RT-M) gives a better solution in terms of total cost. The reason behind this result is that the decision is made by considering more operations; both retailers inventory and transportation. In addition to these, after setting optimal solutions, manufacturer can still decide its production intervals as multiples of retailers transportation intervals. On the other hand, at low tax rates that the model where retailer is the leader (R-M) gives the worst solution however, as tax rate increases it is found under the benchmark manufacturer is the leader and makes the transportation (MT-R) as manufacturer's decisions when he also considers transportation highly contradicts with retailer's objective. To continue with carbon emissions, transportation highly effects amount of carbon emitted, hence, when it is removed from the objective emission levels increase as expected. Furthermore, comparison of the models where manufacturer is the leader (M-R) and R-M shows that when manufacturer is the leader number of backorders increases which ends up with a decrease in carbon emission. As a result, the worst carbon emission levels are reached under R-M.

5.4 Effects of Integration

The solution of best benchmark problem; RT-M is compared with original problem in order to see the improvement maintained by integrating the system under single vehicle without outsource transportation option. In Figure 5.8, improvements maintained by using integrated system rather than the best decentralized system is given in percentage. The percent improvements are found by $100(TC^*_{RT-M} - TC^*_{integrated})/TC^*_{RT-M}$ and $100(E^*_{RT-M} - E^*_{integrated})/E^*_{RT-M}$ for total cost and carbon emission, respectively. From the figure it can be seen that integrated system gives the best solution with the minimum cost value under all tax rates. In the figure, the sudden decrease in the emission level results from increase in optimal number dispatches.



(a) Tax rate vs. % improvement on total cost (b) Tax rate vs. % improvement on carbon emission

Figure 5.8: Benefits of integrating system

In Table 5.6 and 5.7 the exact cost and carbon emission differences are shown for the integrated problem and decentralized case when retailer is responsible from the transportation. It is seen that by integrating the system the total cost is reduced with 1-4% for different tax rates, however, operational cost is increased up to 9% at high tax rates. Although, total cost will always be less under integrated system, it is not possible to generalize this result for carbon emission. A parameter setting where opposite case holds is given in following example.

Table 5.6: Percent benefit of integrated system to total cost

p[$/kg]	integrated [$]	RT-M [$]	Δ [%]
0.1	355.79	368.64	3.48
0.5	440.96	452.02	2.44
1	534.64	537.18	0.47

Table 5.7: Percent benefit of integrated system to carbon emission and operational cost

(a) Percent benefit of integrated system to carbon emission

p[$/kg]	integrated [kg]	RT-M [kg]	Δ [%]
0.1	230.15	235.59	2.36
0.5	198.99	208.5	4.50
1	177.71	210.5	15.57

(b) Percent benefit of integrated system to operational cost

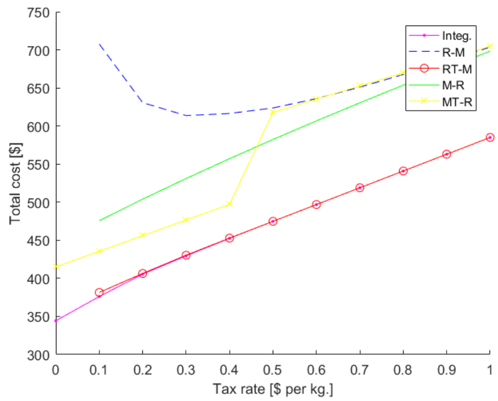
p[$/kg]	integrated [$]	RT-M [$]	Δ[%]
0.1	332.77	345.08	3.76
0.5	341.46	347.77	1.72
1	356.92	326.68	-9.20

Furthermore, carbon emission is reduced by 2-16 % by the integration. As a consequence integration can be given as a way to reduce both operational costs and carbon emission of the whole system.

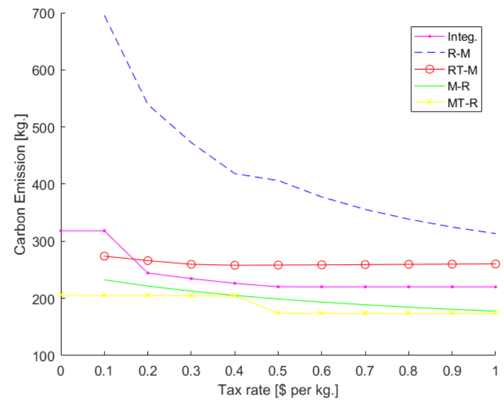
5.5 Effects of Production System

It is possible to evaluate results of integrated and decentralized models under different production systems. We consider two different settings, one with a relatively high production rate such as $P = 2000 \quad units/period$ and one with a higher production set-up cost such as $K_M = 70\$$. Both problems solved for the integrated model and all benchmark models and in Figure 5.9 plots of tax rate vs. total cost and carbon emission are given.

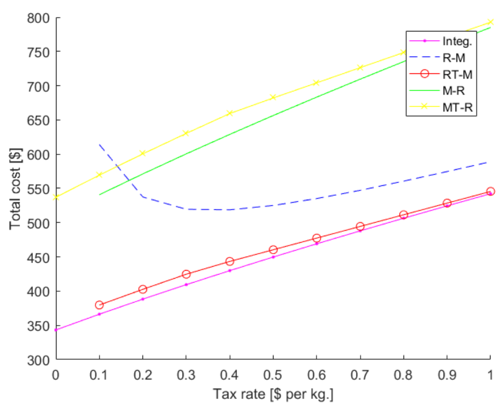
From the Figure 5.9 it is observed that integrated model gives a less costly solution compared to benchmark case even under high production rates and set-up costs. However, benchmark models may give a better solution in terms of carbon emissions. In the setting with high production rates, carbon emission level reached under MT-R and M-R is less than the solutions of integrated model. The reason under this behavior can be the increase in backorder levels; as retailer can only adjust costs by changing backorder levels so he prefers to keep less inventory to decrease cost and the increase in production interval. A similar result is also obtained when production set-up cost is higher. At low tax rates the solutions reached under RT-M results in less carbon emission as well. Although, the previous comparisons of integrated problem and benchmark models favors integration, this examples avoids generalizing these results when carbon emission objectives are considered.



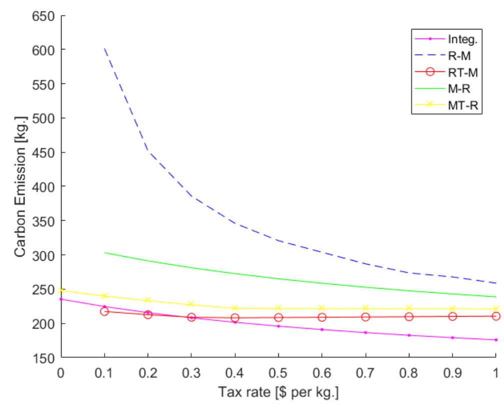
(a) tax vs. cost when $P = 2000$



(b) tax vs. carbon emission when $P = 2000$



(c) tax vs. cost when $K_M = 70$



(d) tax vs. carbon emission when $K_M = 70$

Figure 5.9: Solutions under different production systems

5.6 Main Findings

There are some main findings that can be concluded from the numerical study and summarized in this section. First of all the effects of carbon tax on an integrated system in which production, transportation and inventory included is analyzed throughout the study. It is observed that significant carbon reductions are reached under low tax rates with a manageable increase in total cost of the system. However, a further decrease in emission level becomes harder as the increase in carbon tax below a point results in a great rise in total cost. It is also shown that under such a situation backorder levels increase and hence, service level decreases which ends up with dissatisfaction of customers. In order to decrease carbon levels further, different technology selections can be used. It is shown that the differences in emissions resulting from production and transportation highly affect solution of the problem and provide reduction in carbon levels that can not be maintained with policy selection.

In the system, both the use of in-house fleet and outsource transportation is allowed. Although, providing more options always favors total cost and gives more areas for cost reductions, it is observed that outsourcing regimes may affect carbon emission levels negatively. Under the pricing regime where outsource cost does not depends on carbon tax, companies are willing to choose this option rather than using in-house fleet as the resulting tax is not reflected on their cost. However, when the total carbon emissions are considered, a worse solution can be reached depending on the technology selection of third parities.

Finally, we discussed the effects of integration and show that integration always gives a less costly solution however, it is possible to reach better carbon emission levels with benchmark models. In the original setting that has been considered, the improvement in total cost is found as 3.5% when the best benchmark model is considered. The improvement levels goes up to 70% when the worst benchmark model is considered. Although, in the initial parameter setting the best carbon emissions reached under integrated model, it is shown that under different production systems different benchmark models may result in less carbon emissions.

Chapter 6

Extension to Stochastic Lead Time

In previous chapters it is assumed that transportation lead time is constant, however, in practice there can be many reasons which effects lead time. In order to model this variability, deterministic lead time assumption is relaxed and stochastic lead time is introduced to the problem. One of the key issues with stochastic lead time is order crossovers which happens when orders are not received in the same order that they have been issued. Crossover can occur when lead time is larger than replenishment cycle. In that case, at the time a second order is issued the previous may not have been delivered yet and hence the orders of two successive deliveries may cross.

In this chapter we firstly work under the assumption lead time can not be greater than retailer's replenishment cycle, so at most one order can be outstanding. For this problem we propose a solution algorithm which enables determining optimal policy. Next, we relax this assumption and allow lead time to be at most as large as two replenishment cycles. In this case two orders can be outstanding and hence order crossovers are allowed. For this problem it is not possible to find optimal solution, instead, we use a Markov Chain (MC) model to evaluate results of selected policies. In order to check the precision, we firstly model one outstanding order problem by MC and compare the solutions obtained by using solution algorithm and MC model. After showing that MC model fulfills the precision targets, we use it to evaluate the problem when two orders are outstanding.

The objective function of this problem is defined as minimizing expected average costs resulted from manufacturing, inventory carrying and transportation activities. In the following parts of this chapter firstly the related literature is given in Section 6.1, then in Section 6.2 the solutions of problems with one orders are given. Lastly the MC model is given in Section 6.3.

6.1 Literature Review

The literature on stochastic lead time and order crossover are reviewed by Dolgui et al. [34], Riezebos [35] and Robinson et al. [36]. The related literature can be classified according to the decision of allowing order crossover or not. To begin with the studies who do not allow order crossover, Liberetore [37] is one of the first studies in the area. In the study, classical EOQ model is studied with stochastic lead time extension. It is assumed that demand is deterministic and there exists a single product. The objective is to minimize production setup cost and inventory holding cost. In order to eliminate order crossover problem, it is assumed that orders are non-interchangeable. The decision variables are defined as time units of demand satisfied in each order and time differential between placing an order and start of supplying the demand. Although, a closed form solution can not be maintained, it is shown that optimal solution can be found by solving a system of equations. Sphicas [38] focuses on the same problem with Liberatore [37] and uses the same system and assumptions. However, the two cases where placing an order before and after the start of a cycle are optimal, are discussed separately which ends up with differences in analytical derivations. He also provide a system of equations to find the optimal policy. Chang [39] algebraically derives the optimal cycle time and the optimal reorder time for the EPQ model with shortages and variable lead-time. They assume that order crossover is not possible and explain the conditions under which the assumption holds. The system and solutions follows the one used by Liberatore [37] and Sphicas [38]. Ramasesh et al. [40] focus on an inventory system where it is possible to replenish items from two different sources. It is assumed that demand is deterministic while lead time is stochastic and orders are non-interchangeable. Objective of the problem is to minimize inventory holding and replenishment costs. They both solve the single vendor and two vendors problem. While single vendor problem follows the solution of Liberatore [37], for the two vendor problem they suggest an (s, Q) policy as the optimal solution where s and Q are reorder level and order quantity, respectively. They conclude that dual sourcing provides cost reductions in terms of inventory holding and backorder costs under stochastic lead time. Hayya et al. [41] focus on a system with one retailer and a number of identical suppliers. They extend the study of Ramasesh et al. [40] by assuming that both demand and lead time are stochastic. He et al. [42] formulate a heuristic where the objective is minimizing inventory and production set-up costs. The decision variables are defined as reorder level and order quantity. It is assumed that demand is deterministic while lead time is stochastic and follows a truncated distribution. By using a truncated distribution, the complexity of order crossovers is excluded from the problem. They discuss effects of lead time variance and mean on total cost and provide an alternative to Liberatore [37].

Next we continue with the studies in which order crossover is allowed. Zalkind [43] studies an inventory system where order crossover is possible. Objective of the study is to minimize unit carrying, shortage and replenishment costs in a system with infinite time horizon. Both demand and lead time is considered to be stochastic. He suggests that optimal policy is a tZ policy where Z is the optimal order level and t is the scheduling period. In the study, probability distribution of outstanding orders are deduced and the problem is modeled accordingly. He et al. [44] provide a heuristic which focuses on a continuous review inventory system where order crossovers are allowed. In the study they follow (s, Q) system and consider the cases where only pairwise crossovers are possible and there exists an exchange in physical inventory levels between the crossed orders. Under these assumptions they found an upper bound on the total cost. Wensing and Kuhn [45] aim to minimize holding and set-up costs in a periodic-review inventory system. Lead time is assumed to be discrete uniform and demand is deterministic. They allow order crossovers and found solution for order size and order-up-to level.

In this chapter, our aim is to introduce stochastic lead time to the two-echelon supply chain problem with capacitated transportation and model the problem both when order crossovers are allowed and not allowed.

6.2 Stochastic lead time case with at most one outstanding order

Firstly, we focus on the case where lead time is less than the length of replenishment cycle and hence, only one order can be outstanding. To summarize the system, manufacturer makes production and holds inventory according to the number of dispatches that will be made. Upon dispatches, manufacturer's inventory decreases and items are sent to the retailer. In the deterministic lead time case, exact delivery time of the retailer is known, however, when stochasticity is introduced, time spent during transportation becomes a random variable. Hence, depending on the realized lead time, there can be cases where retailer either holds excess inventory or backorders items. To explain further, there will be excess inventory when lead time demand is less than reorder level and there will be backorder when lead time demand is greater than it, where reorder level is defined as the level of inventory at which retailer will place an order.

In the problem reorder level is introduced as a new decision variable and denoted as γ. Furthermore, realized lead time is introduced as a new parameter and denoted by L and maximum lead time as L_u. The formulations are given for an arbitrary lead time distribution where probability density function and cumulative distribution function are denoted as $f(l)$ and $F(l)$, respectively. The problem is solved for specific distributions after defining general problem as well.

6.2.1 Derivation of Objective Function

Several changes are made in the model in order to include stochastic lead time. The formulations associated with operations of manufacturer such as production set-up cost: K_M/T and manufacturer's inventory holding cost:

$$\left(\frac{DT}{2}\left(1-\frac{D}{P}\right)+\frac{D^2T}{Pm}-\frac{DT}{2m}\right)$$

remains same as stochasticity of lead time does not effect manufacturer's operations directly. However, due to variability in lead time, inventory hold at retailer's level and number of backorders are revised. In order to calculate amount of excess inventory and backorder, plots of time against retailer's inventory level are used and given below. The plots represents two cases where the lead time demand is greater than and less than reorder level in Figure 6.1 and 6.2, respectively.

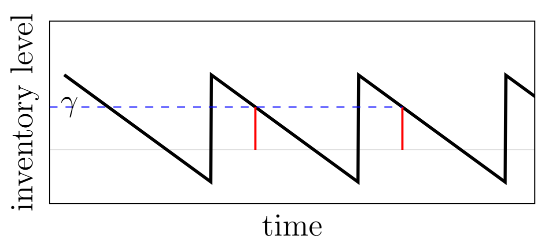


Figure 6.1: Inventory level of the retailer when $L > \gamma/D$

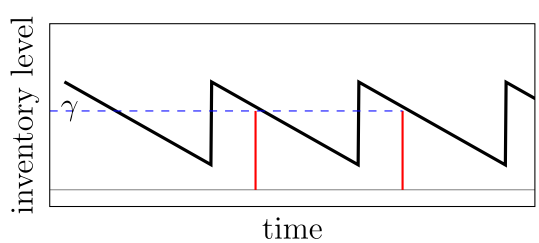


Figure 6.2: Inventory level of the retailer when $L < \gamma/D$

From Figure 6.1 total inventory hold during one interval when lead time demand is greater than reorder level is calculated as below:

$$\frac{\gamma^2}{2D} + \frac{(\frac{T}{m} - L)(\frac{DT}{m} - (L - \frac{\gamma}{D})D + \gamma)}{2}$$

Similarly, total amount of backordered items are given below.

$$\frac{(L - \frac{\gamma}{D})^2 D}{2}$$

On the other hand, when lead time demand is less than reorder level, there will be no backorders and the total inventory is found as below:

$$\frac{L(2\gamma - LD)}{2} + \frac{(\frac{T}{m} - L)\left(\frac{DT}{m} + 2\gamma - LD\right)}{2}$$

In order to find expected inventory level of retailer, total inventory and backorder levels that are found previously are multiplied with probability mass function of lead time distribution and given in Equation 6.1 in simplified form.

$$\begin{aligned}
&\mathbb{E}U_{retailer} = \\
&h_R \int_0^{\frac{\gamma}{D}} \left(\frac{DT}{2m} + \gamma - lD\right) f(l)dl \quad + \quad h_R \int_{\frac{\gamma}{D}}^{L_u} \left(\frac{mD}{2T}\left(l - \frac{\gamma}{D}\right)^2 + \frac{DT}{2m} + \gamma - lD\right) f(l)dl \\
&\quad + \quad c_B \int_{\frac{\gamma}{D}}^{L_u} \frac{mD}{2T}\left(l - \frac{\gamma}{D}\right)^2 f(l)dl \\
&= h_R \left[\frac{DT}{2m} + \gamma - \mathbb{E}(L)D\right] + (h_R + c_B) \int_{\frac{\gamma}{D}}^{L_u} \frac{mD}{2T}\left(l - \frac{\gamma}{D}\right)^2 f(l)dl \qquad (6.1)
\end{aligned}$$

For the stochastic lead time problem only it is assumed that only a single capacitated vehicle is available. The only change in the cost of transportation is including the expected cost incurred for the time that is spend during transportation: $\mathbb{E}(L)K_l$.

$$\mathbb{E}\Lambda(x) = xK_V + \mathbb{E}(L)K_l$$

As a result, expected cost per unit time of the whole system is formulated as follows:

$$\mathbb{E}G(T,m,x,\gamma) = \frac{K_M + m\mathbb{E}\Lambda(x)}{T} + h_M\left(\frac{DT}{2}\left(1-\frac{D}{P}\right)+\frac{D^2T}{Pm}-\frac{DT}{2m}\right)$$
$$+ h_R\left[\frac{DT}{2m}+\gamma-\mathbb{E}(L)D\right] + (h_R+c_B)\int_{\frac{\gamma}{D}}^{L_u}\frac{mD}{2T}\left(l-\frac{\gamma}{D}\right)^2 f(l)dl \quad (6.2)$$

In order to keep the notation simpler and focus on technical details we work under zero carbon tax rate in this chapter. However, it is straightforward to include carbon tax in the objective function as the only difference will be the additional carbon cost terms without changing order of the problem. The expected total cost function with carbon tax is given below. Although, Equation 6.2 is used in following parts of this chapter the results apply for Equation 6.3 as well.

$$\mathbb{E}TC(T,m,x,\gamma) = \frac{K_M + pE_M + m(\mathbb{E}\Lambda(x)+pE_V)+(m+1)pE_Z}{T}$$
$$+ (h_M+pe_Z)\left(\frac{DT}{2}\left(1-\frac{D}{P}\right)+\frac{D^2T}{Pm}-\frac{DT}{2m}\right)$$
$$+ (h_R+pe_Z)\left[\frac{DT}{2m}+\gamma-\mathbb{E}(L)D\right]+(h_R+c_B+pe_Z)\int_{\frac{\gamma}{D}}^{L_u}\frac{mD}{2T}\left(l-\frac{\gamma}{D}\right)^2 f(l)dl$$
$$(6.3)$$

6.2.2 Structural Properties

Firstly, it is shown that optimal value of γ can be found when all other variables are given.

Proposition 5. *The unique minimizer $\gamma(T,m)$ of $\mathbb{E}G(T,m,x,\gamma)$ for given T and m is the solution of*

$$\frac{h_R}{(h_R+c_B)} = \int_{\frac{\gamma}{D}}^{L_u}\frac{m}{T}\left(l-\frac{\gamma}{D}\right)f(l)dl \quad (6.4)$$

Proof. In order to solve the problem the first condition of Equation 6.2 with respect to γ is found by using Leibniz integral rule:

$$\frac{\partial \mathbb{E}G}{\partial \gamma} = h_R - (h_R + c_B) \int_{\frac{\gamma}{D}}^{L_u} \frac{m}{T} \left(l - \frac{\gamma}{D}\right) f(l)dl = 0$$

$$\Rightarrow \frac{h_R}{(h_R + c_B)} = \int_{\frac{\gamma}{D}}^{L_u} \frac{m}{T} \left(l - \frac{\gamma}{D}\right) f(l)dl$$

When the second condition of Equation 6.2 is checked it is seen that the function is non-negative which proves uniqueness of γ for given all other variables.

$$\frac{\partial^2 \mathbf{E}G}{\partial \gamma^2} = (h_R + c_B) \int_{\frac{\gamma}{D}}^{L_u} \frac{m}{DT} f(l)dl \geq 0$$

$\square$

On Equation 6.4 right hand side gives the portion of unmet demand as a multiple of m/T or in other words the unmet demand per cycle and left hand side gives the ratio of inventory holding cost and backorder cost. With this interpretation it is possible to relate the equality to the news-vendor model with the difference that in news-vendor model critical ratio is used to determine the optimal order quantity whereas in our model it is used to find the time to reorder.

It is also possible to define $\left(l - \frac{\gamma}{D}\right)^+ = Y$ as the total backorder time in a given cycle. With this representation Equation 6.4 can be re-written as follows:

$$\frac{h_R}{h_R + c_B} = \frac{m}{T}\mathbb{E}(Y)$$

It is also possible to find unique minimizer of T for given other variables.

Proposition 6. *The unique minimizer $T(\gamma, m, x)$ of $\mathbb{E}G(T, m, x, \gamma)$ for given m and x is*

$$T(\gamma, m, x) = min\{\sqrt{\frac{K_M + m\mathbb{E}\Lambda(x) + (h_R + c_B)\int_{\frac{\gamma}{D}}^{L_u} \frac{mD}{2}\left(l - \frac{\gamma}{D}\right)^2 f(l)dl}{h_M \frac{D}{m}\left[\frac{D}{P}(1 - \frac{m}{2}) + \frac{m-1}{2}\right] + h_R \frac{D}{2m}}}, \quad \frac{xCm}{D}\} \tag{6.5}$$

Proof. In order to find the optimal value of T the first condition of Equation is checked with respect to T.

$$\frac{\partial \mathbf{E}G}{\partial T} = -\frac{K_M + m\mathbb{E}\Lambda(x)}{T^2} + h_M \frac{D}{m}\left[\frac{D}{P}(1 - \frac{m}{2}) + \frac{m-1}{2}\right] + h_R \frac{D}{2m}$$
$$- (h_R + c_B)\int_{\frac{\gamma}{D}}^{L_u} \frac{mD}{2T^2}\left(\frac{\gamma}{D} - l\right)^2 f(l)dl = 0$$

$$T(\gamma, m) = \sqrt{\frac{K_M + m\mathbb{E}\Lambda(x) + (h_R + c_B)\int_{\frac{\gamma}{D}}^{L_u} \frac{mD}{2}\left(l - \frac{\gamma}{D}\right)^2 f(l)dl}{h_M \frac{D}{m}\left[\frac{D}{P}(1 - \frac{m}{2}) + \frac{m-1}{2}\right] + h_R \frac{D}{2m}}}$$

The second derivative of Equation 6.2 with respect to T always gives a non-negative value which proves uniqueness:

$$\frac{\partial^2 \mathbf{E}G}{\partial T^2} = 2\frac{K_M + m\mathbb{E}\Lambda(x)}{T^3} + (h_R + c_B)\int_{\frac{\gamma}{D}}^{L_u} \frac{mD}{T^3}\left(l - \frac{\gamma}{D}\right)^2 f(l)dl \geq 0$$

$T(\gamma, m)$ value is defined to be realizable if $\frac{DT}{m}$ falls within the interval that corresponds to the vehicle number that is used to compute T value. Otherwise maximum number of items that can be transported with given vehicle number is found and defined as the local optima of order quantity: $\frac{DT}{m}$.

□

Proposition 7. *Let $T(m, x, \gamma)$ be the production interval minimizing $\mathbb{E}G(T, m, x, \gamma)$ for given m and x and γ. Then, $T(m, x, \gamma)$ takes its maximum value; T_u when*

$\gamma = 0$ and minimum value; T_l when $\gamma = L_u D$.

$$T_u = \sqrt{\frac{K_M + m\mathbb{E}\Lambda(x) + (h_R + c_B)\mathbb{E}(L^2)}{h_M \frac{D}{m}\left[\frac{D}{P}(1+\frac{m}{2}) + \frac{m-1}{2}\right] + h_R \frac{D}{2m}}}$$

$$T_l = \sqrt{\frac{K_M + m\mathbb{E}\Lambda(x)}{h_M \frac{D}{m}\left[\frac{D}{P}(1+\frac{m}{2}) + \frac{m-1}{2}\right] + h_R \frac{D}{2m}}}$$

Proof. For given m and x, T_m value depends only on the following integral:

$$\int_{\frac{\gamma}{D}}^{L_u} \frac{mD}{2}\left(l - \frac{\gamma}{D}\right)^2 f(l)dl$$

The γ values 0 and $L_u D$ are the ones at which the integral takes its maximum and minimum value respectively. Hence, at the points production interval takes it's maximum and minimum value as well. □

Proposition 8. *T decreases as γ increases.*

Proof follows from convexity of T for given γ and Proposition 7.

Proposition 9. *If*

$$\frac{2\left[\frac{D}{P}\left(1 - \frac{m}{2}\right) + \frac{m-1}{2}\right](h_M h_R + h_M c_B) + h_R c_B}{h_R^2} > \frac{F(\frac{\gamma}{D})}{\bar{F}(\frac{\gamma}{D})}$$

then, T and γ are jointly convex.

Proof. T and γ are jointly convex if and only if the Hessian is positive semi-definite.

$$H = \begin{bmatrix} (h_R + c_B)\frac{m}{DT}\int_{\frac{\gamma}{D}}^{L_u} f(l)dl & (h_R + c_B)\frac{m}{T^2}\int_{\frac{\gamma}{D}}^{L_u}\left(l - \frac{\gamma}{D}\right) f(l)dl \\ (h_R + c_B)\frac{m}{T^2}\int_{\frac{\gamma}{D}}^{L_u}\left(l - \frac{\gamma}{D}\right) f(l)dl & \frac{2K}{T^3} + (h_R + c_B)\frac{mD}{T^3}\int_{\frac{\gamma}{D}}^{L_u}\left(l - \frac{\gamma}{D}\right)^2 f(l)dl \end{bmatrix}$$

Which can be simplified as follows:

$$H = \begin{bmatrix} (h_R + c_B)\frac{m}{DT}\left(1 - F(\frac{\gamma}{D})\right) & (h_R + c_B)\frac{m}{T^2}\mathbb{E}(Y) \\ (h_R + c_B)\frac{m}{T^2}\mathbb{E}Y & \frac{2K}{T^3} + (h_R + c_B)\frac{mD}{T^3}\mathbb{E}(Y^2) \end{bmatrix}$$

H is positive semi-definite if and only if all principal minors are non-negative:

1. $(h_R + c_B)\frac{m}{DT}\left(1 - F(\frac{\gamma}{D})\right) \geq 0$
2. $(h_R + c_B)\frac{m}{DT}\left(1 - F(\frac{\gamma}{D})\right)\left[\frac{2K}{T^3} + (h_R + c_B)\frac{mD}{T^3}\mathbb{E}(Y^2)\right] - \left((h_R + c_B)\frac{m}{T^2}\mathbb{E}Y\right)^2$

Although the first principal minor; (1) is always non-negative, (2) is non-negative if and only if:

$$h_M h_R 2B\left(1 - F(\frac{\gamma}{D})\right) + h_M c_B 2B\left(1 - F(\frac{\gamma}{D})\right) + (h_R c_B + h_R^2)\left(1 - F(\frac{\gamma}{D})\right) - h_R^2 > 0$$

$$\Rightarrow -h_R^2 F(\frac{\gamma}{D}) + h_R\left((h_M 2B + c_B)\bar{F}(\frac{\gamma}{D})\right) + h_M c_B 2B\bar{F}(\frac{\gamma}{D}) \geq 0$$

$$\Rightarrow \frac{(h_M h_R + h_M c_B)2B + h_R c_B}{h_r^2} > \frac{F(\frac{\gamma}{D})}{\bar{F}(\frac{\gamma}{D})}$$

where B is $\left[\frac{D}{P}\left(1 - \frac{m}{2}\right) + \frac{m-1}{2}\right]$.

□

6.2.3 Solution Algorithm and examples

In the solution algorithm we follow a similar procedure that is given for deterministic lead time problem. To give a brief summary of the algorithm, we firstly define upper bounds on m and x to be able to perform a finite search. Please note that the upper bounds defined for m and x are still valid with the only difference that in Proposition 2, T_0 becomes solution of Equation 6.2 when transportation costs are excluded. Next, we use Proposition 5 and define an equality to remove the decision variable on γ by it's unique minimizer with respect to other variables. Similarly, we also use an equality defined by Proposition 6 to find unique minimizer of T depending on other decision variables and solve these two equations jointly. If the solution of T is realizable then it is set as the local optimal solution, otherwise it is adjusted upwards to the time interval that is necessary to produce maximum number of items that can be transported with given number of vehicles. According to the selection of T, γ is revised. The procedure is repeated for each combination of x and m and the solution which gives minimum cost is chosen as optimal. In Algorithm 3 given in Appendix A, which is defined for deterministic lead time problem with single vehicle type, it is possible to express unique minimizer of backorder level in a closed form and hence we do not need to jointly solve two equations to find optimal T and b which is the only significant difference between Algorithms 2 and 3.

Algorithm 2 Minimize Total Cost when Lead Time is Stochastic

1: **procedure** COMPUTE THE BEST COMBINATION FOR (T, γ, m, x)
2: **for** $m = 1 : m_u$ **do**
3:     **for** $x = 1 : x_u$ **do**
4:         Jointly solve $\gamma(T, m)$ and $T(\gamma, m, x)$ and find $\hat{T}$, $\hat{\gamma}$ which are the solutions
5:             of the system of equations.
6:         $T^* \leftarrow min\{\hat{T}, \frac{xCm}{D}\}$ by Propostion 6
7:         **if** $T^* = \hat{T}$ **then**
8:             $\gamma^* = \hat{\gamma}$
9:         **else**
10:             $\gamma^* = \gamma(\frac{xCm}{D}, m)$
11:         **end if**
12:     **end for**
13: **end for**
14: **return** T^*, γ^*, m and x that yields minimum total cost.
15: **if** $m = m_u$ **then**
16:       $m_u \leftarrow m_u + 1$, return to step 2.
17: **else**
18:             Continue
19: **end if**

We continue with numerical examples and explain the results of stochastic lead time problem. The parameter selection is the same with the one used in Chapter 5 and given in Tables 5.2 and 5.3 except those related to carbon emissions. We analyzed two cases where lead time has uniform or beta distributions. For all distributions it is assumed that maximum lead time is 0.20 and expected lead time is 0.10. Under both uniform and beta distribution, maximum lead time is less than the length of replenishment cycle, hence, one outstanding order assumption is valid in the provided examples. When lead time is uniformly distributed variance is 0.0033, on the other hand, for beta distribution, problem is solved for two different variances which are 0.0075 and 0.0457, hence the coefficient of variation is 0.57, 0.86 and 2.12 for the distributions, respectively. For deterministic lead time problem it is assumed that lead time is constant and equal to 0.10.

In Table 6.1 the selected optimal policies when lead time has uniform or beta distribution is given. In the table, columns represent optimal production interval, reorder level, number of dispatches, vehicles and implied backorder level and resulting cost and emissions, respectively. In the last column, expected cost for the deterministic solution is given which is computed by using Equation 6.2. In order to compare results of the two models, implied γ value is found as the difference between expected lead time demand and backorder level for the deterministic solution. Similarly, in stochastic problem, T, m, x and γ are decision variables and implied b is found later. The cost value that is given in the table is obtained by using cost function that is defined by Equation 6.2; for stochastic lead time problem. In order to make a fair comparison for the two lead time considerations we only consider the effect of cost coming from operations, anything additional incurring during lead time is omitted.

In order to compare the results, we firstly use the optimal solution found under deterministic lead time to find expected cost by using the stochastic cost function defined by Equation 6.2. The results indicates that modeling the problem under deterministic lead time assumption results in underestimation of total cost and prevents maintaining expected gains from the optimization problem. On the other hand, using the solutions found for stochastic lead time problem yields about 5% reduction in cost.

When the problem is solved under uniformly distributed lead time assumption, production interval is enlarged compared to deterministic problem in order to reduce the effect of uncertainty in each cycle. When beta distribution is used instead of uniform, reorder level is decreased as beta is a left skewed distribution and it is a greater probability for lead time time to be less than expected value compared to uniform distribution. It is also observed that expected backorder levels are reduced compared to deterministic lead time problem under both distributions.

Table 6.1: Solutions of stochastic lead time problem

	Optimal Values*				Implied	Expected	Exp. Cost
	T	γ	m	x	b	Cost	(Det. Solution)**
Uniform[1]	1.13	11.42	5	1	13.23	304.86	307.38
Beta[2]	1.15	0.00	6	1	3.23	251.66	269.41
Beta[3]	1.13	0.00	6	1	2.54	244.88	257.83

* Solution of stochastic lead time problem is found by Algorithm 2.

** Solution of the deterministic problem is found by Algorithm 3: $T = 1.07, m = 5, x = 1, b = 47$ and implied $\gamma = 13$.

[1] *lower bound* $= 0$, *upper bound* $= L_u$

[2] $\alpha_1 = 0.1, \alpha_2 = 0.9$

[3] $\alpha_1 = 1.1, \alpha_2 = 9.9$

6.3 Markov Chain Model for Discrete Stochastic Lead Time Distributions

We use Markov Chain (MC) model to compute approximately the expected cost of a given set of decision variables. In this chapter our aim is to understand effect of randomness in lead time and discuss the differences in the results found in previous chapters. In order to see the resultant expected cost effects, we use an evaluative tool, MC and consider two instances, one with at most one outstanding order and another MC model for at most two outstanding orders.

We firstly present our results for the first MC model, which uses a discretized lead-time demand version of the model we have used in the previous sections and show that it is possible to maintain approximately the same expected cost results by MC model. The second MC model considers up to two outstanding orders and it is formulated and analyzed in Section 6.3.2.

6.3.1 Markov Chain Model for at Most One Outstanding Order

By defining these two instances, we make sure that exact computations can be done. Under one outstanding order assumption, lead time must be less than length of a replenishment cycle, hence our MC model can only compute the expected cost exactly if maximum lead time is less than T/m that is selected. In the MC model a discrete probability mass function is defined for the lead time and it is confirmed that MC model yields same results up to a computational error with the results obtained using the algorithm defined in Section 6.2.3.

The states of the MC represent the discrete values the lead-time is allowed to take. We define the lead time as L_t, a random variable taking values in between zero and the maximum value, L_u. Let j define the state of this Markov Chain and assume that $j = 0, .., J$ where $J+1$ is the total number of states defined. Next, we define w_j; the value lead time will take at state j such that $\sum_j \mathbf{P}\{L_t = w_j\} = 1$ where $w \in [0, L_u]$. Note that as J increases the representation will be closer to the continuous problem solved in the beginning of this section, and hence expected cost computed by the MC using the optimal decision values will be very close (if not the same) as the cost computed by the Algorithm presented in Section 6.2.3 for the optimal solution. It is possible to obtain average inventory level; I_j and backorder level; B_j at state j by Equations 6.6 and 6.7, and find total inventory cost for given T, m and γ.

$$I_j = \begin{cases} \frac{\gamma^2}{2D} + \frac{(\frac{T}{m}-w_j)\left[\frac{DT}{m}-(w_j-\frac{\gamma}{D})D\right]+\gamma}{2}, & \text{if } \gamma/D \leq w_j \\ \frac{w_j(2\gamma-w_jD)}{2} + \frac{(\frac{T}{m}-w_j)\left(\frac{DT}{m}+2\gamma-w_jD\right)}{2}, & \text{otherwise} \end{cases} \tag{6.6}$$

$$B_j = \begin{cases} \frac{(w_j-\frac{\gamma}{D})^2D}{2}, & \text{if } \gamma/D \leq w_j \\ 0, & \text{otherwise} \end{cases} \tag{6.7}$$

Transition probabilities of the MC model are defined as p_{ij}, $i, j \in (0, J)$; probability of moving from state i to j. Transition probabilities are shown in Table 6.2 and derived as in the following equality.

$$p_{ij} = \mathbb{P}\{L_t = w_j | L_{t-1} = w_i\} = \mathbb{P}\{L_t = w_j\} \quad \forall i, j \in (0, J)$$

Table 6.2: Transition matrix - 1

lead time	0	1	2	...	J
0	$p_{0,0}$	$p_{0,1}$	$p_{0,02}$	...	$p_{0,J}$
1	$p_{1,0}$	$p_{1,1}$	$p_{1,2}$	...	$p_{1,J}$
.	.	.	.	...	.
.	.	.	.	...	.
J	$p_{J,0}$	$p_{J,1}$	$p_{J,2}$	...	$p_{J,J}$

In order to derive expected cost, the steady state probabilities; probability of being in various states, are found as follows by using the standard procedure.

$$\pi_j = \mathbb{P}\{L_t = w_j\} \quad i, j \in (0, J)$$

Finally, the expected inventory cost of retailer is found as sum of inventory cost at state j times probability of being in each state and expected total cost is written as in Equation 6.8. The resulting cost will be an approximation to Equation 6.2 and converge to the same value (with an error) when a high number of states is used.

$$\mathbb{E}U_{retailer} = \sum_{j=0}^{J} \pi_j * (h_R I_j + c_B B_j)$$

$$\mathbb{E}G(T,m,x,\gamma) = \frac{K_M + m\mathbb{E}\Lambda(x)}{T} + h_M \left[\frac{DT}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2 T}{Pm} - \frac{DT}{2m} \right] + \sum_{j=0}^{J} \pi_j * (h_R I_j + c_B B_j) \quad (6.8)$$

It is possible to provide a numerical example and compare the MC model with the model defined in previous section. The optimal solution found by Algorithm 2 is used as the input of MC chain model and resulting expected cost is compared with the original solution found by Equation 6.2. Please note that the only difference in expected cost functions defined by Equation 6.2 and 6.8 is retailers cost, hence it is also enough to only track change in retailer's cost.

We use the same parameter setting defined in Chapter 5 and assume that lead time is uniformly distributed. We define maximum lead time as 0.20 time units and expected lead time as 0.10 as well. While defining maximum lead time we ensure that there will be one outstanding order or in other words, maximum lead time will be less than the length of replenishment cycle and hence, T/m value under optimal solution is greater than 0.20. While the optimal value found from Equation 6.2 is 304.86$, when 500 states are used to define MC model the resulting cost is very close and found as 304.92$. Cost values found with different state numbers are given in Table 6.3.

Table 6.3: MC model costs

Number of States	Expected Cost [$]
10	308.11
50	305.42
100	305.11
250	304.93
500	304.92

6.3.2 Markov Chain Model for at Most Two Outstanding Orders

In this section we allow lead time to be at most as long as two replenishment cycles and hence, two orders can be outstanding at the same time. Note that it is difficult if not impossible to find optimal solution due to the complexity of the problem, hence, we model the system as a MC to approximate expected cost for a given set of variables. In the previous model, states are defined as L_t, lead time at period t, however, in this model it is necessary to keep track of the last two orders as the inventory and backorder levels depends on lead time of last two periods rather than one. As result, the state of MC model is defined as lead time of two consecutive periods; (L_{t-1}, L_t), where L_{t-1} and L_t can take values between zero and L_u. The states of MC are defined as j, k under the assumption that $j, k = 0, .., J$ and $J + 1$ is the total number of states defined. The values of lead time at states j, k are defined the same way with the MC model with one outstanding orders as w_j, w_k.

For the next step, inventory and backorder levels at each state are derived in order to find expected cost. In this model, orders are classified into four according to time intervals they have received. A visual representation is provided for describing cases in Figure 6.3. In the first case both orders received at the first cycle that they have been issued. This setting corresponds to the case where only one outstanding orders exist. In a second case first issued order can arrive at the first cycle while second order arrives at second cycle. In a third case first order arrives at second cycle and next order arrives at first cycle, hence order crossovers are possible. Lastly in a fourth case both orders arrive at second cycle. While calculating inventory and backorder levels the method that is used when only one orders can be outstanding is followed with the difference that there exists much more possibilities. In that model there are only two possibilities which depends on the relation between lead time demand and reorder level. However, when two orders can be outstanding both the relation between reorder level and lead time demand and order of the two consecutive deliveries are considered. Inventory and backorder levels at each state are represented by I_{jk} and B_{jk}, respectively and given in Appendix D.

In Table 6.4 an example of transition matrix is given for visual representation. In the example, three states are used for a single period which are (0, 1, 2).

Table 6.4: Transition matrix - 2

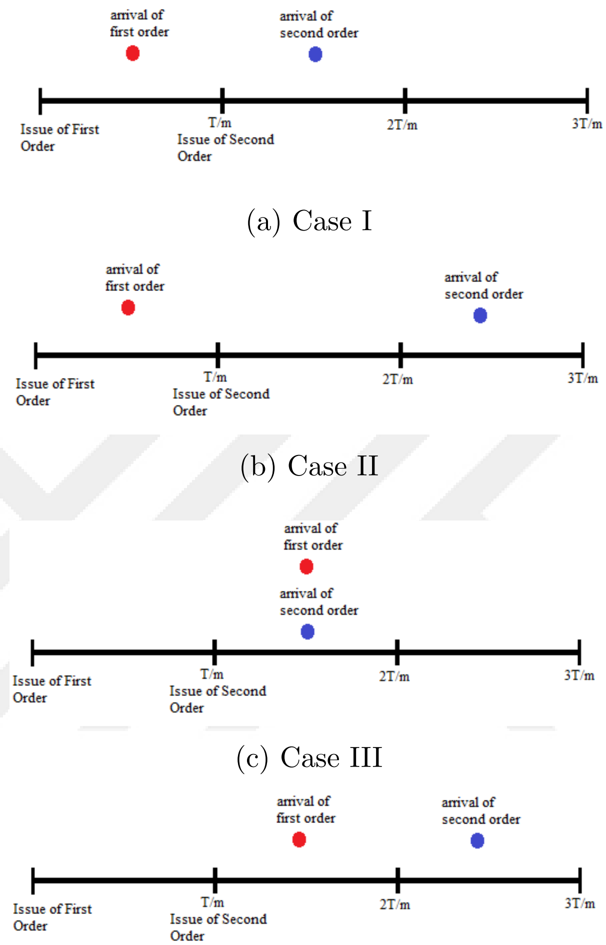
lead time	(0,0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)	(2,0)	(2,1)	(2,2)
(0,0)	$p_{(0,0),(0,0)}$	$p_{(0,0),(0,1)}$	$p_{(0,0),(0,2)}$						
(0,1)				$p_{(0,1),(1,0)}$	$p_{(0,1),(1,1)}$	$p_{(0,1),(1,2)}$			
(0,2)							$p_{(0,2),(2,0)}$	$p_{(0,2),(2,1)}$	$p_{(0,2),(2,2)}$
(1,0)	$p_{(1,0),(0,0)}$	$p_{(1,0),(0,1)}$	$p_{(1,0),(0,2)}$						
(1,1)				$p_{(1,1),(1,0)}$	$p_{(1,1),(1,1)}$	$p_{(1,1),(1,2)}$			
(1,2)							$p_{(1,2),(2,0)}$	$p_{(1,2),(2,1)}$	$p_{(1,2),(2,2)}$
(2,0)	$p_{(2,0),(0,0)}$	$p_{(2,0),(0,1)}$	$p_{(2,0),(0,2)}$						
(2,1)				$p_{(2,1),(1,0)}$	$p_{(2,1),(1,1)}$	$p_{(2,1),(1,2)}$			
(2,2)							$p_{(2,2),(2,0)}$	$p_{(2,2),(2,1)}$	$p_{(2,2),(2,2)}$

The probability of moving from a state (i, j) to (j, k) are defined by $p_{(i,j),(j,k)}$ and given below.

$$p_{(i,j),(j,k)} = \mathbb{P}\{L_t = w_k | L_{t-2} = w_i, L_{t-1} = w_j\} = \mathbb{P}\{L_t = w_k\} \quad \forall i, j, k \in (0, L_u)$$

One can come up with the steady-state probabilities defined by $\pi_{(j,k)}$ for such a system computationally.

The above expected value is used to compute an approximation of expected cost



(d) Case IV

Figure 6.3: Order arrivals

using a set of given decision variables for settings where the chances of having more than one outstanding order is positive, and more than two outstanding orders is zero. Hence, in this way, we can see the expected cost error we incur when we use the "optimized" decision variables using Section 6.2.3 (which assumes one outstanding order, at most). Equation 6.9 is written similar to the previous MC model and can be used to approximate expected cost when at most two orders are outstanding for given decision variables when lead time is coming from a discrete distribution.

$$\mathbb{E}U_{retailer} = \sum_{j=0}^{J}\sum_{k=0}^{J}\pi_{j,k} * (h_R I_{j,k} + c_B B_{j,k})$$

$$\mathbb{E}G(T,m,x,\gamma) = \frac{K_M + m\mathbb{E}\Lambda(x)}{T} + h_M\left[\frac{DT}{2}\left(1-\frac{D}{P}\right) + \frac{D^2T}{Pm} - \frac{DT}{2m}\right] + \sum_{j=0}^{J}\sum_{k=0}^{J}\pi_{j,k} * (h_R I_{j,k} + c_B B_{j,k}) \quad (6.9)$$

It is also possible to provide an example where using MC model can be beneficial. In order to use MC model the decision variables found by Algorithm 2 is used. The solution of problem can violate assumption of one outstanding order when length of replenishment cycle is less than length of lead time. Hence, the resulting expected value is not correctly reflected and can lead to wrong interpretations. Even if it the optimal solution can not be found when two orders can be outstanding, MC model gives an insight on the amount of underestimation of the cost. In Table 6.5 the optimal solutions found via Algorithm 2 is given together with the resulting MC cost. The problem is solved when maximum lead time is 0.23, 0.24 and 0.25 for uniform and beta distributions with mean 0.10. The solution shows that the assumption of one outstanding order is violated as the length of replenishment cycle is less than the length of lead time. Hence, the MC model with 50 states is formulated for two outstanding orders is used to evaluate expected cost. It is observed that the assumption of one outstanding order ends up with underestimation of the cost and expected cost found by MC model can be about 25% higher than the cost found by Equation 6.2.

Table 6.5: Results of MC model

	L_u	T	m	T/m	γ	x	Implied b	Expected cost*	MC cost**
uniform[1]	0.23	1.14	5	0.23	21.18	1	14.45	309.58	368.22
	0.24	1.15	5	0.23	24.66	1	15.05	311.24	384.56
beta[2]	0.23	1.17	6	0.19	0.00	1	4.39	261.76	349.20
	0.24	1.18	6	0.20	0.00	1	4.69	265.31	349.23
beta[3]	0.23	1.13	6	0.19	0.00	1	2.59	250.72	342.54
	0.24	1.14	6	0.19	0.00	1	2.73	254.12	342.56

* Expected cost is found by Equation 6.2.

** MC cost is found by Equation 6.9.

[1] *lower bound* $= 0$, *upper bound* $= L_u$

[2] $\alpha_1 = 0.1, \alpha_2 = 0.9$

[3] $\alpha_1 = 1.1, \alpha_2 = 9.9$

Chapter 7

Conclusion

In this thesis, we study the problem of determining optimal policy for production, transportation and inventory operations of a two-echelon supply chain in a setting where carbon tax is applied. There exists a single manufacturer and a single retailer in the system and demand is deterministic. For the lead time, we firstly consider that it is fixed and zero, then provide an extension for the case where lead time is a random variable. We consider four transportation options, using in-house fleet with single or two vehicles and with or without outsource option. We showed that it is possible to find optimal production interval, number of dispatches and vehicles and retailer's backorder level by the proposed solution algorithm. In the solution algorithm we firstly defined upper bounds on the variables that we performed a search and for the rest of the variables we used mathematical properties of the problem. We also present benchmark models where we consider different scenarios of integration.

In a numerical study we discussed the advantages of considering multi-mode transportation and integration. For the parameter setting of the numerical study we refer to the literature and provide sensitivity analysis for the parameters that we come up with. As a result of the numerical study, firstly, we observe that significant carbon reductions achieved at low tax rates without increasing total cost in great amounts. As tax rates increases further, total cost begins to increase rapidly and determined optimal solutions result in a decrease in service level.

We also discuss effects of allowing outsource option and conclude that although it results in reduction in cost, it may cause an increase in carbon emission levels. At this point, the pricing policy and technology selection of the third party highly effects the results. When outsource cost does not depend on carbon tax and third party uses a technology with high carbon emission levels, total carbon emission increases. However, if tax is included in the price or carbon friendly vehicles are used, than both cost and emissions are decreased by allowing outsource option. Similar results are also observed when a second vehicle is added to the in-house fleet as depending on the tax rates total carbon emissions may increase.

Effects of integration is another issue that is discussed. We compare several benchmark models and present ones result in least cost and carbon emission. Although, integrated problem always offers a less costly solution, under different production systems benchmark problems may provide solutions with less emissions. When a high production rate is used, benchmark models where manufacturer is the leader results in less carbon emission and when a high production set-up cost is used, the benchmark model retailer is the leader and is responsible from transportation may give less emissions depending on the tax rate. The comparison of benchmark models can be used to select a partially integrated model or non-integrated model under the cases when a full integration is not possible.

We also analyze the problem under stochastic lead time and consider cases with one and two outstanding orders. For the one outstanding order problem we present an algorithm to find optimal solution of the problem and also Markov Chain model to evaluate determined solution. However, we only provide Markov Chain model to evaluate the resulting expected cost for two outstanding orders case. We conclude that deterministic lead time assumption results in underestimation of the cost.

Several areas can be identified to extend this study. First of all, the selection of emission technology in production and transportation can be detailed and differences in the resulting solutions can be discussed. It is also possible to include investment decision to the problem and allow selection of technology according in the trade off between carbon emission levels and investment costs. Another extension can be including different ways of transportation such as rail or shipment rather than allowing only road transportation.

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Appendix A

Solutions of Other Transportation Options

For the solution of the problem with different transportation options the same steps are used. The upper bounds and structural properties are valid for this problem as well. The only difference is the derivation of optimal T value which is given in Proposition 4 previously. In the following part the propositions that need to be used for other transportation options are given together with solution algorithms.

A.1 Single Type Vehicle without Outsource Option

Rather than Proposition 4 the following proposition is used in order to find optimal value of T. The proof is omitted as it is very similar to the proof of Proposition 4.

Proposition 10. *$T(m)= min\{\hat{T}, \frac{xCm}{D}\}$ where $T(m)$ is called the local minimizer of the objective function given m and x and $\hat{T}$ =*

$\sqrt{\frac{K_M+mxK_V+pE_M+pE_Z(m+1)+mxpE_V}{\gamma+\frac{\lambda}{m}}}$ *is the unique minimizer of* $TC(T,m,x)$.

The solution algorithm that is given for two type vehicles with outsource option is revised to exclude extra transportation options.

Algorithm 3 Minimize Total Cost for Single Vehicle Type

```
1: procedure COMPUTE THE BEST COMBINATION FOR (T, b, m, x)
2: for m = 1 : m_u do
3:     for x = 1 : x_u do
4:         T* ← min{T̂, xCm/D}
5:             by Propostion 10
6:     end for
7: end for
8: return T*, m and x that yields minimum total cost.
9: if m = m_u then
10:        m_u ← m_u + 1, return to step 2.
11: else
12:            Continue
13: end if
14: b* = DT*φ/m
```

A.2 Two Type Vehicles without Outsource Option

Rather than Proposition 4 the following proposition is used in order to find optimal value of T.

Proposition 11. *The local minimizer of objective function for given* m *and* $\mathbf{x}$ *is* $T(m) = argmin\{TC(T_1,m,x), \quad TC(T_2,m,x)\}$
where T_1 *and* T_2 *find as below.*

- $$\hat{T_1} = \sqrt{\frac{K_M + m(x_1K_{V1} + x_2K_{V2}) + p(E_M + E_Z(m+1) + m(x_1E_{V1} + x_2E_{V2}))}{\gamma + \frac{\lambda}{m}}}$$

$$T_1 = \begin{cases} \hat{T}_1, & if\ j_1 \leq \hat{T}_1 \leq j_2 \\ argmin\{TC(j_1, m, x), \quad TC(j_2, m, x)\}, & otherwise \end{cases}$$

-
$$\hat{T}_2 = \sqrt{\frac{K_M + m(x_2+1)K_{V2} + p(E_M + E_Z(m+1) + m(x_2+1)E_{V2})}{\gamma + \frac{\lambda}{m}}}$$

$$T_2 = \begin{cases} \hat{T}_2, & if\ j_2 \leq \hat{T}_2 \leq j_3 \\ argmin\{TC(j_2, m, x), \quad TC(j_3, m, x), & otherwise \end{cases}$$

$$where\ j_1 = \frac{(x_1C_1+x_2C_2)m}{D}, \quad j_2 = \frac{x_2C_2m}{D} + \frac{(x_1+1)C_1m}{D}, \quad j_3 = \frac{(x_2+1)C_2m}{D}$$

The Solution Algorithm 4 is used to find optimal solution.

A.3 Single Vehicle Type with Outsource Option

Rather than Proposition 4 the following proposition is used in order to find optimal value of T. For the solution of the problem Solution Algorithm 5 is used.

Proposition 12. *The local minimizer of the objective function for given m and* **x** *is*
$T_m = argmin\{TC(T_1, m, x), \quad TC(T_2, m, x)\}$
where T_1 and T_2 find as below.

-
$$\hat{T}_1 = \sqrt{\frac{K_M + m(xK_V - k_oC) + p(E_M + E_Z(m+1) + m(xE_V))}{\gamma + \frac{\lambda}{m}}}$$

$$T_1 = \begin{cases} \hat{T}_1, & if\ j_1 \leq \hat{T}_1 \leq j_2 \\ argmin\{TC(j_1, m, x), \quad TC(j_2, m, x)\}, & otherwise \end{cases}$$

-
$$\hat{T}_2 = \sqrt{\frac{K_M + m(x+1)K_V + p(E_M + E_Z(m+1) + m(x+1)E_V)}{\gamma + \frac{\lambda}{m}}}$$

$$T_2 = \begin{cases} \hat{T}_2, & if\ j_2 \leq \hat{T}_2 \leq j_3 \\ argmin\{TC(j_2, m, x), \quad TC(j_3, m, x), & otherwise \end{cases}$$

$where\ j_1 = \frac{xCm}{D}, \quad j_2 = \frac{m(K_V + pE_V)}{Dk_o} + \frac{xCm}{D}, \quad j_3 = \frac{(x+1)Cm}{D}$

Algorithm 4 Minimize Total Cost Two Vehicle Types

1: **procedure** COMPUTE THE BEST COMBINATION FOR $(T, b, m, \mathbf{x})$
2: **if** Using Type 1 vehicle is cheaper than using Type 2 **then**
3: Solve Single type without outsource problem
4: **else**
5: Continue
6: **end if**
7: **for** $m = 1 : m_u$ **do**
8: **for** $x_2 = 0 : x_{u2}$ **do**
9: **for** $x_1 = 0 : x_{u1}$ **do**
10: **if** $x_2 = 0$ **then**
11: skip $x_1 = 0$
12: **else**
13: Continue
14: **end if**
15: $T^* \leftarrow argmin\{TC(T_1, m, x), \quad TC(T_2, m, x)\}$
16: by Propostion 12
17: **if** $T^* = TC(T_1, m, x)$ **then**
18: $(x_1^*, x_2^*) \leftarrow (x_1, x_2)$
19: **else**
20: $(x_1^*, x_2^*) \leftarrow (0, x_2 + 1)$
21: **end if**
22: **end for**
23: **end for**
24: **end for**
25: **return** T^*, m and (x_1^*, x_2^*) that yields minimum total cost.
26: **if** $m = m_u$ **then**
27: $m_u \leftarrow m_u + 1$, return to step 7.
28: **else**
29: Continue
30: **end if**
31: $b^* = \frac{DT^*\phi}{m}$

Algorithm 5 Minimize Total Cost for Single Vehicle Type with Outsource Option

1: **procedure** COMPUTE THE BEST COMBINATION FOR $(T, b, m, \mathbf{x})$
2: **for** $m = 1 : m_u$ **do**
3:     **for** $x = 0 : x_u$ **do**
4:         $T^* \leftarrow argmin\{TC(T_1, m, x), \quad TC(T_2, m, x)\}$
5:             by Propostion 11
6:         **if** $T^* = TC(T_1, m, x)$ **then**
7:                 $x^* \leftarrow x$
8:         **else**
9:                 $x^* \leftarrow x + 1$
10:         **end if**
11:     **end for**
12: **end for**
13: **return** T^*, m and x^* that yields minimum total cost.
14: **if** $m = m_u$ **then**
15:       $m_u \leftarrow m_u + 1$, return to step 2.
16: **else**
17:             Continue
18: **end if**
19: $b^* = DT^*\phi$

Appendix B

Solution of Benchmark Models

B.1 Solution of IC and IE Models

In order to solve cost minimization problem the same methodology and algorithm defined in Chapter 3 is used when all emission parameters has zero value. Similarly, in emission minimization model the same methods are used when operational parameters has zero value and carbon tax rate is equal to one.

B.2 Solution of M-R Model

In M-R model, in the first step manufacturer decides optimal production interval which minimizes his profits. According to the selected policy retailer can chose backorder level which will minimize his costs. First order condition of Equation 4.1 and 4.2 is used to find optimal cycle length; T^* and backorder level; b^*, respectively. In the last step, the necessary number of vehicles will be selected according to the order size that is decided previously. Algorithm 6 is used to solve the problem.

Algorithm 6 Minimize Total Cost for M-R Model

1: **procedure** COMPUTE THE BEST COMBINATION FOR (T, b)

2: $T^* = \sqrt{\frac{K_M+p(E_M+E_Z)}{(h_M+pe_Z)\left[\frac{D}{2}\left(1-\frac{D}{P}\right)+\frac{D^2}{P}-\frac{D}{2}\right]}}$

3: $b^* = DT^*\phi$

4: $\frac{DT}{C}$

B.3 Solution of MT-R Model

In order to solve the MT-R model, the solution methodology that is defined in Chapter 3 is revised for the objective defined in Equation 4.3. Updated propositions and Algorithm 7 is given below for this model.

Proposition 13. *(i) Optimal number of Type 2 vehicles is bounded above by $x_{u2} = \lceil DT_0/mC \rceil$ which is the nearest integer up to the solution of DT_0/mC where*

$$T_0 = \sqrt{\frac{K_M + p(E_M + E_Z)}{(h_M + pe_Z)\left[\frac{D}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2}{P} - \frac{D}{2}\right]}} \tag{B.1}$$

is the solution of the problem when transportation costs are excluded from the total cost function defined in Equation 4.3.

(ii) The upper bound for the Type 1 vehicles is denoted by x_{u1} and found by the ratio $\lfloor (K_{V1}+pE_{V1})/(K_{V2}+pE_{V2}) \rfloor$ which gives the greatest number of Type 1 vehicles that is less costly to operate than a single Type 2 vehicle.

Proposition 14. *The local minimizer of the objective function for given* **x** *is defined as T(x) and can be found by the following equation:*

$$T(x) = argmin\{C_M^{MT-R}(T_1, x), \quad C_M^{MT-R}(T_2, x), \quad C_M^{MT-R}(T_3, x)\}$$

where argmin function is defined as over the arguments T_1, T_2 or T_3 which are defined below.

- *Let $\hat{T}_1$ be defined as*

$$\hat{T}_1 = \sqrt{\frac{K_M + x_1(K_{V1} - k_o C_1) + x_2(K_{V2} - k_o C_2) + p(E_M + E_Z + x_1 E_{V1} + x_2 E_{V2})}{(h_M + pe_Z)\left[\frac{D}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2}{P} - \frac{D}{2}\right]}}$$

$$T_1 = \begin{cases} \hat{T}_1, & \text{if } j_1 \le \hat{T}_1 \le j_2 \\ argmin\{C_M^{MT-R}(j_1, x) \quad C_M^{MT-R}(j_2, x)\}, & otherwise \end{cases} \tag{B.2}$$

- *Let $\hat{T}_2$ be defined as*

$$\hat{T}_2 = \sqrt{\frac{K_M + (x_1+1)K_{V1} + x_2 K_{V2} + p(E_M + E_Z + (x_1+1)E_{V1} + x_2 E_{V2})}{(h_M + pe_Z)\left[\frac{D}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2}{P} - \frac{D}{2}\right]}}$$

$$T_2 = \begin{cases} \hat{T}_2, & \text{if } j_2 \le \hat{T}_2 \le j_3 \\ argmin\{C_M^{MT-R}(j_2, x), \quad C_M^{MT-R}(j_3, x)\}, & otherwise \end{cases} \tag{B.3}$$

- *Let $\hat{T}_3$ be defined as*

$$\hat{T}_3 = \sqrt{\frac{K_M + (x_2+1)K_{V2} + p(E_M + E_Z + (x_2+1)E_{V2})}{(h_M + pe_Z)\left[\frac{D}{2}\left(1 - \frac{D}{P}\right) + \frac{D^2}{P} - \frac{D}{2}\right]}}$$

$$T_3 = \begin{cases} \hat{T}_3, & \text{if } j_3 \le \hat{T}_3 \le j_4 \\ argmin\{C_M^{MT-R}(j_3, x), \quad C_M^{MT-R}(j_4, x)\}, & otherwise \end{cases} \tag{B.4}$$

where $j_1 = \frac{(x_1C_1 + x_2C_2)}{D}$
$j_2 = \frac{(K_{V1} + pE_{V1})}{Dk_o} + \frac{(x_1C_1 + x_2C_2)}{D}$
$j_3 = \frac{x_2C_2}{D} + \frac{(x_1+1)C_1}{D}$
$j_4 = \frac{(x_2+1)C_2}{D}$

B.4 Solution of R-M Model

Solution of R-M model is similar to the M-R model in the sense that firstly the replenishment interval and backorder level that minimizes retailers objective, which is defined in Equation 4.4, is found. According to the selected policy, manufacturer can chose the production interval as a positive integer multiple of selected policy. The solution algorithm is given in Algorithm 8.

Algorithm 7 Minimize Total Cost of MT-R Model for Two Vehicle Types with Outsource Option

```
1: procedure COMPUTE THE BEST COMBINATION FOR (T, b, x)
2: if Using outsource option is cheaper than using both vehicles then
3:         T* ← T_0, skip steps 9-22.
4: else if Using Type 1 vehicle is cheaper than using Type 2 then
5:         Solve single type with outsource problem
6: else
7:         Continue
8: end if
9: for x_2 = 0 : x_u2 do
10:     for x_1 = 0 : x_u1 do
11:         T* ← argmin{C_M^{MT-R}(T_1, x),    C_M^{MT-R}(T_2, x),    C_M^{MT-R}(T_3, x)}
12:           by Propostion 14
13:         if T* = C_M^{MT-R}(T_1, x) then
14:             (x_1*, x_2*) ← (x_1, x_2)
15:           else if T* = C_M^{MT-R}(T_2, x) then
16:             (x_1*, x_2*) ← (x_1 + 1, x_2)
17:         else
18:             (x_1*, x_2*) ← (0, x_2 + 1)
19:         end if
20:     end for
21: end for
22: return T* and (x_1*, x_2*) that yields minimum total cost.
23: b* = DT*φ
```

Algorithm 8 Minimize Total Cost for R-M Model

1: **procedure** COMPUTE THE BEST COMBINATION FOR (T, m, b)
2: $T^* = \sqrt{\frac{pE_Z}{(h_R+pe_Z)\frac{D}{2}(1-2\phi+\phi^2)+\frac{Dc_B\phi^2}{2}}}$
3: $b^* = DT^*\phi$
4: m^* is the solution of $\frac{\partial C_M^{R-M}(T,m)}{\partial m}$
5: Select number of vehicles according to transportation options that are available

B.5 Solution of RT-M Model

Solution of RT-M model follows the methodology given in Chapter 3 with the retailer's objective defined in Equation 4.6. The propositions and solution algorithm that is used for the integrated problem are revised for this objective and given below. Algorithm 9 is used to solve the model.

Proposition 15. *(i) Optimal number of Type 2 vehicles is bounded above by $x_{u2} = \lceil DT_0/mC \rceil$ which is the nearest integer up to the solution of DT_0/mC where*

$$T_0 = \sqrt{\frac{pE_Z}{(h_R + pe_Z)\frac{D}{2}(1 - 2\phi + \phi^2) + \frac{Dc_B\phi^2}{2}}} \tag{B.5}$$

is the solution of the problem when transportation costs are excluded from the total cost function defined in Equation 4.3.

(ii) The upper bound for the Type 1 vehicles is denoted by x_{u1} and found by the ratio $\lfloor (K_{V1}+pE_{V1})/(K_{V2}+pE_{V2}) \rfloor$ which gives the greatest number of Type 1 vehicles that is less costly to operate than a single Type 2 vehicle.

Proposition 16. *The local minimizer of the objective function for given* $\mathbf{x}$ *and b^* given in Equation 3.4 is defined as $T(x)$ and can be found by the following equation:*

$$T(x) = argmin\{C_R^{RT-M}(T_1, x), \quad C_R^{RT-M}(T_2, x), \quad C_R^{RT-M}(T_3, x)\}$$

where argmin function is defined as over the arguments T_1, T_2 or T_3 which are defined below.

- *Let $\hat{T}_1$ be defined as*

  $\hat{T}_1 = \sqrt{\frac{x_1(K_{V1}-k_oC_1)+x_2(K_{V2}-k_oC_2)+p(E_Z+x_1E_{V1}+x_2E_{V2})}{(h_R+pe_Z)\frac{D}{2}(1-2\phi+\phi^2)+\frac{Dc_B\phi^2}{2}}}$

$$T_1 = \begin{cases} \hat{T}_1, & \text{if } j_1 \leq \hat{T}_1 \leq j_2 \\ argmin\{C_R^{RT-M}(j_1,x) \quad C_R^{RT-M}(j_2,x)\}, & \text{otherwise} \end{cases} \tag{B.6}$$

- *Let $\hat{T}_2$ be defined as*

  $\hat{T}_2 = \sqrt{\frac{(x_1+1)K_{V1}+x_2K_{V2}+p(E_Z+(x_1+1)E_{V1}+x_2E_{V2})}{(h_R+pe_Z)\frac{D}{2}(1-2\phi+\phi^2)+\frac{Dc_B\phi^2}{2}}}$

$$T_2 = \begin{cases} \hat{T}_2, & \text{if } j_2 \leq \hat{T}_2 \leq j_3 \\ argmin\{C_R^{RT-M}(j_2,x), \quad C_R^{RT-M}(j_3,x)\}, & \text{otherwise} \end{cases} \tag{B.7}$$

- *Let $\hat{T}_3$ be defined as*

  $\hat{T}_3 = \sqrt{\frac{K_M+(x_2+1)K_{V2}+p(E_M+E_Z+(x_2+1)E_{V2})}{(h_R+pe_Z)\frac{D}{2}(1-2\phi+\phi^2)+\frac{Dc_B\phi^2}{2}}}$

$$T_3 = \begin{cases} \hat{T}_3, & \text{if } j_3 \leq \hat{T}_3 \leq j_4 \\ argmin\{C_R^{RT-M}(j_3,x), \quad C_R^{RT-M}(j_4,x)\}, & \text{otherwise} \end{cases} \tag{B.8}$$

where $j_1 = \frac{(x_1C_1+x_2C_2)}{D}$
$j_2 = \frac{(K_{V1}+pE_{V1})}{Dk_o} + \frac{(x_1C_1+x_2C_2)}{D}$
$j_3 = \frac{x_2C_2}{D} + \frac{(x_1+1)C_1}{D}$
$j_4 = \frac{(x_2+1)C_2}{D}$

Algorithm 9 Minimize Total Cost of RT-M Model for Two Vehicle Types with Outsource Option

1: **procedure** COMPUTE THE BEST COMBINATION FOR $(T, b, \mathbf{x})$
2: **if** Using outsource option is cheaper than using both vehicles **then**
3: $\quad\quad$ $T^* \leftarrow T_0$ defined in Proposition 15, skip steps 9-22.
4: **else if** Using Type 1 vehicle is cheaper than using Type 2 **then**
5: $\quad\quad$ Solve single type with outsource problem
6: **else**
7: $\quad\quad$ Continue
8: **end if**
9: **for** $x_2 = 0 : x_{u2}$ **do**
10: $\quad$ **for** $x_1 = 0 : x_{u1}$ **do**
11: $\quad\quad$ $T^* \leftarrow argmin\{C_R^{RT-M}(T_1, x), \quad C_R^{RT-M}(T_2, x), \quad C_R^{RT-M}(T_3, x)\}$
12: $\quad\quad\quad$ by Propostion 16
13: $\quad\quad$ **if** $T^* = C_R^{RT-M}(T_1, x)$ **then**
14: $\quad\quad\quad\quad$ $(x_1^*, x_2^*) \leftarrow (x_1, x_2)$
15: $\quad\quad\quad$ **else if** $T^* = C_R^{RT-M}(T_2, x)$ **then**
16: $\quad\quad\quad\quad$ $(x_1^*, x_2^*) \leftarrow (x_1 + 1, x_2)$
17: $\quad\quad$ **else**
18: $\quad\quad\quad\quad$ $(x_1^*, x_2^*) \leftarrow (0, x_2 + 1)$
19: $\quad\quad$ **end if**
20: $\quad$ **end for**
21: **end for**
22: **return** T^* and (x_1^*, x_2^*) that yields minimum total cost.
23: $b^* = DT^*\phi$
24: m^* is the solution of $\frac{\partial C_M^{R-M}(T^*, m)}{\partial m}$

Appendix C

Results

In the tables provided the optimal values for the decision variables that were defined through the study were given for various carbon tax rates. While operational cost value represents the cost resulting from manufacturing, transportation and inventory holding, total cost includes both operational costs and the tax paid for carbon emission. In Table C.2 and C.3, emission value includes the carbon emitted during manufacturing, inventory holding and transportation if and only if in-house fleet is used. On the other hand, total emission value also includes the per unit carbon emission for items which were transported via outsource option.

Table C.1: Two type vehicles without outsource option

p	T	m	x_1	x_2	Q	b	*emission*	*oper. cost*	*total cost*
0	1.20	9	1	0	80	28	258.58	248.81	248.81
0.1	1.20	9	1	0	80	28	258.58	248.81	274.67
0.2	1.33	10	1	0	80	29	251.72	249.86	300.2
0.3	1.33	10	1	0	80	29	251.72	249.86	325.37
0.4	1.46	11	1	0	80	29	246.22	251.76	350.25
0.5	1.46	11	1	0	80	29	246.22	251.77	374.87
0.6	1.6	12	1	0	80	29	241.75	254.31	399.36
0.7	1.6	12	1	0	80	30	241.75	254.32	423.53
0.8	1.73	6	0	1	173	64	169.6	307.04	442.73
0.9	1.77	6	0	1	178	67	166.02	310.08	459.51
1.0	1.82	6	0	1	183	69	162.81	313.14	475.94

Table C.2: Single type vehicle with outsource option

p	T	m	x	*outsource*	Q	b	*emission*	*total emission*	*oper. cost*	*total cost*
0	-	∞	-	-	-	-	-	-	-	-
0.1	1.17	20	0	36	36	13	324.96	330.70	293.25	306.06
0.2	1.29	17	0	46	46	16	250.16	258.33	271.16	321.19
0.3	1.37	15	0	55	55	20	218.36	228.20	279.04	344.55
0.4	1.45	14	0	62	62	23	198.92	210.00	285.74	365.31
0.5	1.50	13	0	70	70	26	184.13	196.60	292.28	384.34
0.6	1.54	12	0	77	77	29	172.45	186.60	298.68	402.15
0.7	1.62	12	0	81	81	30	165.04	179.44	303.48	419.01
0.8	1.63	11	0	89	89	33	156.95	172.79	309.57	435.12
0.9	1.70	11	0	93	93	35	151.79	168.35	313.95	450.55
1.0	1.76	11	0	96	96	37	147.32	164.42	318.18	465.50

Table C.3: Two type vehicles with outsource option

p	T	m	x_1	x_2	*outsource*	Q	b	*emission*	*total emis.*	*oper. cost*	*total cost*
0	1.20	9	1	0	0	80	28	258.58	-	248.81	248.81
0.1	1.20	9	1	0	0	80	28	258.58	-	248.81	274.67
0.2	1.33	10	1	0	0	80	29	251.72	-	249.86	300.2
0.3	1.33	10	1	0	0	80	29	251.72	-	249.86	325.37
0.4	1.46	11	1	0	0	80	29	246.22	-	251.76	350.25
0.5	1.46	11	1	0	0	80	29	246.22	-	251.77	374.87
0.6	1.60	12	1	0	0	80	29	241.75	-	254.31	399.36
0.7	1.62	12	0	0	81	81	30	165.04	171.52	303.48	419.01
0.8	1.63	11	0	0	89	89	33	156.95	164.05	309.57	435.12
0.9	1.70	11	0	0	93	93	35	151.79	159.18	313.95	450.55
1.0	1.76	11	0	0	96	96	37	147.32	155.00	318.18	465.50

Table C.4: Single type vehicle with outsource option- variable k

p	T	m	x	*outsource*	Q	b	*emis.*	*total emis.*	*oper.cost*	*total cost*
0	-	∞	-	-	-	-	-	-	-	-
0.1	1.19	23	0	32	32	12	324.96	327.51	224.89	257.38
0.2	1.28	17	0	46	46	17	250.16	253.8	242.12	292.15
0.3	1.36	15	0	55	55	20	218.36	222.73	256.48	321.99
0.4	1.45	14	0	62	62	23	198.92	203.88	269.66	349.23
0.5	1.51	13	0	70	70	26	184.13	189.68	282.68	374.74
0.6	1.54	12	0	77	77	29	172.45	178.6	295.56	399.03
0.7	1.62	12	0	81	81	31	165.04	171.52	306.84	422.37
0.8	1.73	6	1	0	173	65	169.6	-	307.04	442.73
0.9	1.77	6	1	0	178	67	166.02	-	310.08	459.51
1	1.82	6	1	0	183	69	162.81	-	313.14	475.94

C.1 Solutions with Different Parameter Values

Table C.5: Solutions with different c_B values

p	c_B	T	m	x	Q	b	*emis.*	*oper. cost*	*total cost*
0	1.00	1.14	5	1	138	77	214.15	272.55	272.55
	2.25	1.08	5	1	130	47	227.24	289.09	289.09
	3.25	1.18	6	1	120	34	231.03	295.11	295.11
0.5	1.00	1.46	5	1	177	101	173.16	281.28	367.86
	2.25	1.57	6	1	157	58	183.3	298.27	389.92
	3.25	1.52	6	1	154	44	187.37	304.41	398.10
1.0	1.00	1.72	5	1	207	120	153.62	295.46	449.08
	2.25	1.81	6	1	183	69	162.81	313.14	475.94
	3.25	1.77	6	1	179	53	166.53	319.53	486.06

Table C.6: Solutions with different E_M values

p	E_M	T	m	x	Q	b	*emis.*	*oper. cost*	*total cost*
0	60.0	1.08	5	1	130	47	211.11	289.09	289.09
	77.5	1.08	5	1	130	47	227.24	289.09	289.09
	100.0	1.08	5	1	130	47	248.18	289.09	289.09
0.5	60.0	1.36	5	1	164	61	173.48	297.18	383.92
	77.5	1.55	6	1	157	58	183.3	298.27	389.92
	100.0	1.58	6	1	160	59	194.86	299.65	397.08
1.0	60.0	1.57	5	1	190	72	154.89	310.68	465.57
	77.5	1.81	6	1	183	69	162.81	313.14	475.94
	100.0	1.87	6	1	187	71	171.9	316.29	488.19

Table C.7: Solutions with different K_M values

p	K_M	T	m	x	Q	b	*emis.*	*oper. cost*	*total cost*
0	40	1.02	5	1	123	44	238.32	273.89	273.89
	56	1.08	5	1	130	47	227.24	289.09	289.09
	70	1.26	6	1	135	49	218.79	300.42	300.42
0.5	40	1.29	5	1	162	60	188.36	284.35	378.53
	56	1.56	6	1	157	58	183.3	298.27	389.92
	70	1.60	6	1	160	59	180.06	308.74	398.77
1.0	40	1.58	5	1	191	73	165.65	300.8	466.45
	56	1.82	6	1	183	69	162.81	313.14	475.94
	70	1.85	6	1	186	71	160.92	322.65	487.57

Appendix D

Inventory and Backorder Levels of Two Outstanding Orders Model

Inventory and backorder levels of the retailer is found for combination of two successive lead times. Inventory plots are used while deriving the equations. In Case 1 the setting where two successive orders arrived in one cycle is considered. In the setting of Case 2, while first order arrives at first cycle, second orders did not. In Case 3 it is possible to receive second issued order first, hence order crossovers are allowed. While calculating inventory and backorder levels we consider the possibilities that first order arrives before and after the second order. Lastly, in Case 4 the setting where both orders arrive at second cycle is addressed. These levels are used in Markov Chain model in order to calculate expected cost function.

Case 1: $w_j \leq T/m$ **and** $L_t \leq T/m$

$$I_{j,k} = \begin{cases} \frac{\gamma^2}{2D} + \frac{(\frac{T}{m}-w_k^2)(\frac{DT}{m}-w_k^2-\frac{\gamma}{D})D)+\gamma}{2}, & \text{if } \gamma/D \leq w_k^2 \\ \frac{w_k^2(2\gamma-w_k^2D)}{2} + \frac{(\frac{T}{m}-w_k^2)\left(\frac{DT}{m}+2\gamma-w_k^2D\right)}{2}, & \text{otherwise} \end{cases}$$

$$B_{j,k} = \begin{cases} \frac{(w_k - \frac{\gamma}{D})^2 D}{2}, & \text{if } \gamma/D \leq w_k \\ 0, & \text{otherwise} \end{cases}$$

Case 2: $L_{t-1} \leq T/m$ **and** $T/m \leq L_t \leq 2T/m$

$$I_{j,k} = \frac{\gamma^2}{2D}$$

$$B_{j,k} = \frac{(\frac{T}{m} - \frac{\gamma}{D})^2 D}{2}$$

Case 3: $T/m \leq L_{t-1} \leq 2T/m$ **and** $L_t \leq T/m$

$$I_{j,k} = \begin{cases} \frac{\left(w_k - w_j + \frac{T}{m}\right)\left(\frac{DT}{m} + 2\gamma - D(w_j + w_k)\right)}{2} + \\ \frac{\left(\frac{T}{m} - w_k\right)\left(\frac{DT}{m} + 2\gamma - Dw_k\right)}{2}, & \text{if } w_j \leq T/m + w_k, \quad w_k \leq \gamma/D \\ \frac{\left(w_j - w_k - \frac{T}{m}\right)\left(\frac{DT}{m} + 2\gamma - D(w_j + w_k)\right)}{2} + \\ \frac{\left(\frac{2T}{m} - w_j\right)\left(\frac{2DT}{m} + 2\gamma - Dw_j\right)}{2}, & \text{if } w_j \geq T/m + w_k, \quad w_k \leq \gamma/D \\ \frac{\left(\frac{\gamma}{D} - (w_j - \frac{T}{m})\right)^2 D + \left(\frac{T}{m} - w_k\right)\left(\frac{DT}{m} + 2\gamma - Dw_k\right)}{2}, & \text{if } w_j \leq T/m + \gamma/D, \quad w_k \geq \gamma/D \\ \frac{\left(\frac{\gamma}{D} - w_k\right)^2 D + \left(\frac{2T}{m} - w_j\right)\left(\frac{2DT}{m} + 2\gamma - Dw_j\right)}{2}, & \text{if } w_j \geq T/m + \gamma/D, \quad w_k \leq \gamma/D \\ \frac{\left(\frac{T}{m} - w_k\right)\left(\frac{DT}{m} + 2\gamma - Dw_k\right)}{2}, & \text{if } w_j \leq T/m + w_k, \quad w_k \geq \gamma/D \\ \frac{\left(\frac{2T}{m} - w_j\right)\left(\frac{2DT}{m} + 2\gamma - Dw_j\right)}{2}, & \text{otherwise} \end{cases}$$

$$B_{j,k} = \begin{cases} \frac{\left(w_j - \frac{T}{m}\right)\left(\frac{DT}{m} - 2\gamma - Dw_j\right)}{2}, & \text{if } w_j \leq T/m + w_k, \quad w_k \leq \gamma/D \\ \frac{w_k\left(\frac{2DT}{m} - 2\gamma - Dw_k\right)}{2}, & \text{if } w_j \geq T/m + w_k, \quad w_k \leq \gamma/D \\ \frac{\left(w_j - \frac{T}{m}\right)\left(\frac{DT}{m} - 2\gamma + Dw_j\right) + \left(w_k - \frac{\gamma}{D}\right)^2 D}{2}, & \text{if } w_j \leq T/m + \gamma/D, \quad w_k \geq \gamma/D \\ \frac{w_k\left(\frac{2DT}{m} - 2\gamma + Dw_k\right) + \left(w_j - (\frac{T}{m} + \frac{\gamma}{D})\right)^2 D}{2}, & \text{if } w_j \geq T/m + \gamma/D, \quad w_k \leq \gamma/D \\ \frac{\left(w_j - \frac{T}{m}\right)\left(\frac{DT}{m} - 2\gamma + Dw_j\right)}{2} + & \text{if } w_j \leq T/m + w_k, \quad w_k \geq \gamma/D \\ \frac{\left(w_k - w_j + \frac{T}{m}\right)\left(\frac{-DT}{m} - 2\gamma + D(w_j + w_k)\right)}{2} \\ \frac{w_k\left(\frac{2DT}{m} - 2\gamma + Dw_k\right)}{2} + & \text{otherwise} \\ \frac{\left(w_j - w_k - \frac{T}{m}\right)\left(\frac{-2DT}{m} - 2\gamma + D(w_j + w_k)\right)}{2} \end{cases}$$

Case 4: $T/m \leq L_{t-1} \leq 2T/m$ **and** $T/m \leq L_t \leq 2T/m$

$$I_{j,k} = \begin{cases} \frac{\left(\frac{T}{m}+\frac{\gamma}{D}-w_j\right)^2 D}{2}, & \text{if } L_{j,k} \leq T/m + \gamma/D \\ 0, & \text{otherwise} \end{cases}$$

$$B_{j,k} = \begin{cases} \frac{\left(w_j-\frac{T}{m}\right)\left(\frac{DT}{m}-2\gamma+Dw_j\right)+\left(\frac{T}{m}+\frac{\gamma}{D}\right)^2 D}{2}, & \text{if } w_j \leq T/m + \gamma/D \\ \frac{\left(w_j-\frac{T}{m}\right)\left(\frac{DT}{m}-2\gamma+Dw_j\right)+\left(\frac{2T}{m}-w_j\right)(w_j D-2\gamma)}{2}, & \text{otherwise} \end{cases}$$