ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE, ENGINEERING AND TECHNOLOGY

SCALAR FIELD POTENTIALS IN COSMOLOGY

M.Sc. THESIS

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Department of Physics Engineering

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

KOZMOLOJİDE SKALER ALAN POTENSİYELLERİ

YÜKSEK LİSANS TEZİ

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Fatma Kuzey EDEŞ HUYAL, a **M.Sc.** student of ITU **Graduate School of Science, Engineering and Technology** 509111110 successfully defended the thesis entitled **"SCALAR FIELD POTENTIALS IN COSMOLOGY"**, which she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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To Dad and my dear polar bear,

FOREWORD

In starting a journey, especially one that will last for your entire professional lifespan, those who are leading you and holding your hands, are of utmost importance in determining who you will be in the future. I am deeply indebted to my advisor Assoc. Prof. Dr. A. Savaş Arapoğlu who stood by my side in this journey with his outstanding patience and support and also showed me that progress in this journey can be made by taking independent steps. I am also grateful to Prof. Dr. Ömer Faruk Dayı for his invaluable lectures that allowed me to deepen my knowledge and for letting me believe in myself.

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May 2015 Fatma Kuzey EDEŞ HUYAL
(Physicist)

TABLE OF CONTENTS

Page

ABBREVIATIONS

CDM	**:** Cold Dark Matter
CMBR	**:** Cosmic Microwave Background Radiation
FRW	**:** Friedmann-Robertson-Walker
GR	**:** General Relativity
EE	**:** Einstein Equations
FE	**:** Friedmann Equations
WMAP	**:** Wilkinson Microwave Anisotropy Probe
SNIa	**:** Supernova Ia
HBB	**:** Hot Big Bang

LIST OF TABLES

LIST OF SYMBOLS

$(-,+,+,+)$	**:** The signature of the metric
∂_μ or subscript $,\mu$	**:** Ordinary or Partial Derivative
∇_μ or subscript $;\mu$	**:** Covariant Derivative
$\mathbf{G}$	**:** The Newtonian Gravitational Constant
$\mathbf{G}_{\mu\nu}$	**:** Einstein tensor
$\mathbf{R}$	**:** Ricci Scalar
$\mathbf{T}_{\mu\nu}$	**:** Energy-momentum tensor
$\mathbf{R}_{\mu\nu}$	**:** Riemann tensor
Λ	**:** Cosmological Constant
κ_5	**:** Gravitational Constant in 5-dimension

SCALAR FIELD POTENTIALS IN COSMOLOGY

SUMMARY

In this thesis we describe how to construct a scalar field potential $V(\phi)$ in a five-dimensional cosmological model with non-zero curvature for the three-dimensional spatial part, where we use the scale factor $a(t)$ for observed space and the additional scale factor $b(t)$ for the extra dimension. We want to verify whether or not scalar field potentials arise which are ruled out in the case of the effective four-dimensional observed universe.

Explicit expressions for the scalar field potential $V(\phi)$ and the scalar field ϕ in terms of cosmic time are derived for both non-interacting and interacting scalar fields. A standard method in cosmology is to obtain the scale factor $a(t)$ by solving Einstein's equations or modifications thereof. We, however, find the form of the potential and scalar field for non-interacting scalar fields using a given scale factor $a(t)$ and $b(t)$ by employing such a method of reverse engineering. One of the interesting results reached in this context is, in the case of de Sitter type expansion for the observed space, that there is a room for a universe with $k = -1$ for a certain contracting-type evolution of the extra dimension which was not possible in the context of effective four-dimensional model. There is also the possibility of getting a cosmological constant term in the potential which depends also on the size of the extra dimension. For the case of power-law-type expansion of observed space and for stabilized extra dimension, we get an exponential type potential which vanishes if the observed space evolves like non-relativistic matter, namely $a(t) \sim t^{2/3}$. This reminds us that these kind of models should further be investigated to get information about which kind of scalar field potentials can lead to which type of evolutions in these extra dimensional models.

With this thesis, the main concern is to investigate the implications of an extra dimension on the scalar field potential, $V(\phi)$, for the inflation phase because, it seems reasonable to take into account extra spatial dimensions for the early universe. Despite our focus on inflation, the method explained here is capable of finding other scalar field potentials in quintessence models, thus giving us a way of grasping effects of scalar field potentials in the late acceleration phase. But we have to stress that this toy-model approximation only equips us with a basic understanding of the shape of the potential under the consideration of an extra dimension, without going into details about the fundamental theory behind $V(\phi)$.

KOZMOLOJİDE SKALER ALAN POTENSİYELLERİ

ÖZET

Geride bıraktığımız yüzyıldan içinde bulunduğumuz ana süregelen yüksek hassasiyetli gözlemler, öncesinden teorinin yahut teorinin açmazlarının işaret ettiği "ivmelenen evren" kavramıyla bizleri baş başa bırakıyor. Üstelik bir değil iki adet ivmelen faza ihtiyaç duyuyor evren: karanlık enerji ve enflasyon.

Doksanlar öncesinde evrenin bilinen bileşenleri evrenin o an için yapılan hesapları ile uyuşmuyordu. Evren için tayin edilen madde kritik yoğunluğun yalnızca yüzde otuzuna tekabül ediyordu. Eğer evrendeki madde üzerine bir fikir sahibi olmak istiyorsanız yapılacak en makul şey evrenin genişleme hızının nasıl değiştiğini tayin etmek olacaktır. Beklenen şey evrenin genişlemesinin yavaşlamasıydı zira evren genişlerken bilinen madde buna sebebiyet verecektir.

Doksanların başında evrenin genişleme hızının değişimini incelemek amacıyla muhtelif araştırmalar başlatıldı. Bu maksatla süpernovalar üzerinden çalışmalar yürütüldü. 1996'da Perlmutter ve onun biraz sonrasında Schmitt birbirinden habersizce üzerinde çalıştıkları SNIa neticelerini açıkladılar. Bu ayrı ayrı yapılan iki çalışma aynı sonuca, evrenin ivmelenerek genişlediğine işaret ediyordu.

Çoğu zaman göz ardı edilen nokta evrenin ivmelenerek genişlemesi neticesi tek başına süpernova gözlemlerinin sonucunda elde edilmemiştir. CMB datasından yarlanıldıktan sonra evrenin ivmelenerek genişlediği sonucuna varılmıştır.

Daha önce revaçta olan CDM modelinin öne sürdüğü hiçbir bileşen böyle bir ivmelenmeyi açıklayamıyordu. Karanlık enerji adı verilen bu bileşen evrenin yüzde yetmişine hakim olup homojen olarak dağılmıştır. Karanlık enerjinin kendisiyle ilgili söylenecek ikinci önemli şey enerji yoğunluğunun zamana göre neredeyse değişmiyor olmasıdır. Yani evrenin radyasyon yahut madde bileşeni gibi zamanla seyrelmiyor. Böylelikle evrenin diğer bileşenleri ilerleyen zamanla etkisini kaybederken, karanlık enerji daha, biraz daha evrene hakim oluyor.

Karanlık enerji mekanizması için muhtelif öneriler olmuştur. En kuvvetli aday Einstein'ın statik bir evren elde etmek adına öne sürdüğü sonrasında kendisinden yüz çevirdiği kozmolojik sabit ile temsil edilen vakum enerjisidir. Ne talihsizliktir ki gözlemlerle oldukçu uyumlu bir görüntü çizen kozmolojik sabit bizi henüz bir çözüm üretilemeyen ağır bir problem ile karşı karşıya getirmiştir. Vakum enerjisi için yapılan teorik hesaplar, gözlemsel olarak elde edilen değerden yaklaşık on üzeri 121 kat daha büyüktür.

Kozmolojik sabit problemi bizi zamanla farklı arayışlara itmiştir. Vakum enerjisini neredeyse sıfır kabul etmek izlenebilecek en kolay yoldur. Fakat pratikte bu mümkün olmadığında yeni bir alan tanımlayıp gözlemlenenen karanlık enerjiyle tutarlı bir ifade elde edebiliriz. Quintessence modeli bize uzay zamanda oldukça yavaş değişen dinamik bir evren bileşeni vermiştir. Kanonik bir skaler alan ve onu temsil eden

potansiyel ile ifade edilen Quintessence modeli altında verilen ya da sizin elde edeceğiniz her bir potansiyel için evren farklı bir zaman evrimine sahip olabilir. Potansiyelin minimumuna inmeye çalışan skaler alan ivmelenmeye sebep olacaktır. Potansiyelin eğimi skaler alanın hareketi ve de modelin karanlık enerjiyi açıklaması açısında oldukça önemli bir rol oynamaktadır. Örneğin çok dik bir potansiyeliniz varsa skaler alanın hızı büyük olacaktır ve de bu mekanizma bize ivmelenen bir evren vermeyecektir.

Evrenin halihazırda ivmelenmesinin yanı sıra erken evrenin de enflasyon adını verdiğimiz ivmelenen bir faz geçirdiğini düşünüyoruz. Aslında düşünmenin de ötesinde enflasyon, BICEP-2 ekibinin geçtiğimiz yıl yayınladığı sonuçlar neticesinde evrenin inkar edilemeyecek bir parçası gibi görünüyor.

Büyük patlama teorisi evreni açıklamakta olağanüstü sayılabilecek bir başarı gösterdiyse de bazı ciddi problemlerle karşı karşıya kalmıştır. Esasında böylesine başlangıç koşullarına bağlı bir teorinin sıkıntı çıkarmamasını beklemek fazla naif bir tutum olurdu.

Büyük patlama teorisinin sıkıntılarını gidermek için ortaya muhtelif teoriler atılmıştır. Bu teorilerin ortak noktası başlangıç koşullarına bağlılığın mümkün mertebe azaltılmasıdır. Enflasyon da bu teorilerden biridir.

Gökyüzünde zıt yönlere odaklanalım. Muhtelif astronomik cisim ve bölgelerle karşılaşırız. Bu obje ya da bölgelerin evrenin herhangi bir zamanında birbirleri ile haberleşmemiş olmalarını bekleriz. Nedensel olarak irtibatta olmadıklarını düşündüğümüz bu yapılardan aynı fiziksel özelliklere sahip olmalarını pek tabi ki beklemeyiz. Fakat CMB için tayin edilen sıcaklık 10^{-5} mertebesinde bir uyum göstermektedir. Buna ufuk problemi diyoruz.

Büyük patlama teorisinin başını belaya sokan ikinci sıkıntı ise erken evrende kozmolojik sabit gibi bir bileşene sahip olmadığını düşündüğümüz, radyasyon ve madde muhteviyatına sahip olan evrende, eğriliğin git gide baskın olmasını bekleriz. CMB datasından evrenin halihazırda düz olduğunu biliyoruz. Sıkıntı evreni geriye sardığımızda evrenin neredeyse on üzeri eksi elli dört mertebesinde daha düz olduğunu görüyoruz. Bu durumu düzlük problemi olarak adlandırıyoruz.

Monopol problemi ise bir diğer cevapsız sorun olarak karşımıza çıkmaktadır. GUT teorileri ani sıcaklık düşmesinin topolojik kusurlar oluşturacağı ve böylelikle evrende manyetik monopoller gözlemleneceğini öngörür. Oysa evrende henüz bir manyetik monopole rastlamadık.

İşte yukarıda bahsi geçen iki büyük probleme eş zamanlı çözüm ilk olarak 1981 yılında Alan Guth tarafından verildi ve seksen buhranına atfen enflasyon olarak adlandırıldı. Enfasyon evrenin erken dönemde ivmelendiği kısa bir zaman dilimidir. Enflasyon için önerilen mekanizma evrenin içinde bulunduğu ivmelenen fazı açıklayan mekanizmayla oldukça benzerdir. Yine kozmolojik sabit önererek ya da inflaton dediğimiz skaler alanlardan yararlanarak ivmelen bir faz yaratırız.

Görüldüğü üzere skaler alanlar kozmolojinin önemli kilometre taşlarını oluşturan çözümler açısından oldukça işlevseldir. Ve skaler alan için seçilen her bir potansiyel ayrı bir evren tasvirine yer verecektir.

Kozmolojide genel ilerleyiş Einstein Denklemleri'ni çözüp ölçek parametresini bulmak yönündedir. Biz bu tezde zıt yönde ilerleyip verili bir ölçek parametresi için potansiyel elde etme yönünde çalışmalar yapacağız.

Bu tezde gözlemlenebilir evrene bir uzay boyutu daha ekleyerek, evrenin tasvir edildiği verili bir $a(t)$ durumu için skaler alan potansiyelini elde ediyoruz. Gözlemlenebilir evrenden kastımız 100 Mpc ve daha uzun mesafeler için homojen ve izotropik olduğu gözlemlerle gösterilmiş ve Friedmann-Robertson-Walker metriği ile tasvir edttiğimiz uzay-zamandır.

Bu çalışmada amacımız $3+1$ uzay-zamanının müsade etmediği, skaler alan potansiyellerinin fazladan bir boyut eklediğimizde işlevsellik kazanıp kazanmadıklarını kontrol etmek. Hem evrenin diğer bileşenleri ile etkileştiğini varsaydığımız, hem de bu etkileşmenin söz konusu olmadığı skaler alanlar için potansiyeli ve skaler alanın kendisini açıkça elde ediyoruz.

Gözlemlenen evrende şu an için ekstra boyutun bir etkisi görülmediğinden bu çalışmada evrene eklediğimiz yeni boyutun erken evrende sabitlendiğini düşünerek hareket ediyoruz. Gözönüne alınacak ikinci husus skaler alanın fiziksel olmasından yola çıkarak ortaya koyduğumuz tutarlılık koşuludur.

Kaydedilmeye değer sonuçlardan biri de de-Sitter modelinin izin verdiği skaler alan potansiyelinin bir boyut eklememizle birlikte $k=-1$ uzayına izin vermediğinin elde edilmesidir. Bununla birlikte beş boyutlu bir uzayda verili bir potansiyel için kozmolojik sabit elde etmenin mümkün olduğunu göstermiş olduk. Söz konusu kozmolojik sabit ekstra boyutun uzunluğu ile ilişkiili olup pozitif ya da negatif değerler alabilmektedir.

Evrenin genişlemesini üstel olarak ifade ettiğimizde ve de uzayı düz olarak ele aldığımızda, potansiyelin ikinci kısmının tamamıyla ekstra boyutun mevcudiyetine bağlı olduğunu görüyoruz. Ve gözlenen evrenin ifade eden fonksiyonun üstü iki bölü üç olduğunda, potansiyelimiz sıfır değerini alıyor ve gözlemlenen evren için tayin edilen ölçek parametresi madde gibi davraniyor.

Çalışmamızda uzayın eğriliğinin esasında beş boyutlu evren modelinde ne denli kısıtlayıcı olabileceğini gördük. Söz konusu eğriliğin izin vermediği potansiyeller, ekstra boyut tanımladığımız modellerde etkin olamayacak ve evrenin gelişimi üzerine söz sahibi olamayacaklardır.

Bu çalışma $3+1$ uzay-zamanının yasakladığı skaler alan potansiyellerinin, evrenin uzay bileşenine eklenecek bir boyut ile yeniden güncellik kazanacakları yönünde ışık tutacaktır.

1. INTRODUCTION

In cosmology the use of scalar fields is a common practice, when the observed/expected evolution of the universe is not in accordance with the known type of matter-energy content of the universe. What makes the scalar fields that much traditional, beyond the fact that it is the nature's simplest imaginable phenomenon, is the freely chosen potential $V(\phi)$, which can be 'devised' to obtain a certain type of evolution for the universe.

Furthermore, in considering an era of the universe as early as inflation, it is reasonable to take into account the presence and evolution of extra spatial dimensions. Even for the current late-acceleration [1,2], it is not non-sense to check whether there is any role played by the extra dimensions, even if they were already stabilized in earlier stages of the Universe. The models including extra dimensions applied to quintessence models, for example, are considered in Refs. [3, 4].

In this study, by combining the aforementioned instruments of the theoretical physics, we examine the dynamics of a five-dimensional toy-model universe, in the context of General Relativity containing a minimally coupled classical scalar field to gravity and matter, both relativistic and non-relativistic. The three-dimensional spatial part is taken as homogeneous and isotropically expanding with a scale factor $a(t)$ and has a non-zero curvature; but the extra spatial dimension, with the topology of S^1, is evolving with a different scale factor $b(t)$. Our aim is basically to get the scalar field potential for assuming specific evolutions for the space part of the universe; thus, this is in a sense a 'reverse engineering' for getting the scalar field potential, [5]. We consider both non-interacting and interacting scalar field-matter possibilities. The interacting case applied to scalar field-radiation can be a model for the dynamics of simple inflationary models, or applied to scalar field- non-relativistic matter, as in Refs. [6, 7], can be a model for the current accelerated expansion of the Universe. In the inflationary context, without the use of slow-roll approximation, we show how to obtain the scalar field potentials which lead to any desired scale factor behaviour. The method for prescribing initial data in numerical simulations in cosmology applied to late-acceleration of the universe is investigated in Ref. [8].

Our main concern in this thesis is, thus, to check whether there is a non-trivial effect of the existence of an extra dimension on the scalar field potential and on the effective low energy theory in this reverse engineering procedure. We seek, for example, a type of universe which is discarded in four-dimensional setting, but can exist in the framework of the toy-model considered in this note.

It is also interesting to see whether there is any relation with the curvature of the observed space and the allowed evolution of the universe in the setting of this thesis. Although the observations confirm a flat universe, closed spatial sections are preferred marginally by the data. Thus it is reasonable to consider the possibility of arbitrary curvature in inflationary and quintessential models, [9, 10].

2. A BRIEF OVERVIEW TO FRW COSMOLOGY

In response to the cosmological observations, roughly we need only three numbers to characterize current universe: expansion rate,energy density and curvature of space. General relativity both relate these quantities each others and tells how they change with time.

Although, large scale homogeneity and isotropy is cruelly significant feature of the universe, it couldn't go beyond being an physical assumption, almost the end of the twentieth century. Order of temperature fluctuations on the CMB shown that universe really has an extraordinary isotropy.

It could be difficult to have a direct evidence related homogeneity, but this high level isotropy is a strong evidence for our hopes on homogeneous cosmology. It should be noted that homogeneous-isotropic universe assumption is only useful scales larger than 100 Mpc.

Our expanding, homogeneous and isotropic universe is represented by the maximally symmetric FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right] \quad . \tag{2.1}$$

In the metric formulation curvature constant k tells about if universe flat,open or closed for values $0, +1, -1$. The expansion is governed by k which has specific way of changing as energy density.

The expansion rate of the universe as the Hubble parameter, defined as

$$H(t) = \frac{\dot{a}}{a} \quad . \tag{2.2}$$

Gravity is assumed to be a manifestation of space-time curvature induced by the presence of matter in GR and it is given by Einstein Equations which are in general complicated non-linear equations. Cosmological dynamics of the universe is given by solving the Einstein Equations, though EE have strict importance for milestones of cosmology. First cosmological solutions to EE is obtained by himself in the early 1917. Then, in 1922 and 1927, general solutions were independently given by Alexandre Friedmann and George Lemaître.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad , \tag{2.3}$$

where the Einstein Tensor is defined as,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad , \tag{2.4}$$

where $R_{\mu\nu}$ is the Riemann tensor and R is the Ricci scalar constructed via the metric and $T_{\mu\nu}$ is the energy momentum tensor. On large scales,we arrive at a universe which is modeled as a perfect fluid with covariant form of energy momentum tensor,

$$T_{\mu\nu} = (\rho + p) v_\mu v_\nu - p g_{\mu\nu} \quad , \tag{2.5}$$

where v_ν is the 4-velocity of the fluid. The energy momentum tensor is conserved by virtue of the Bianchi identities and the continuity equation is given as $\nabla_\mu T^{\mu\nu} = 0$ and the $\nu = 0$ component is,

$$\dot{\rho} + 3H(\rho + p) = 0 \quad . \tag{2.6}$$

Equation of state gives the relation between substances pressure and energy density,

$$\omega \equiv \frac{p}{\rho} \quad . \tag{2.7}$$

Non-relativistic cold dark matter corresponds $\omega = 0$ while radiation and dark energy are given respectively $\omega = \frac{1}{3}$ and $\omega = -1$ For a universe consisting of matter,radiation and cosmological constant we may define a total energy density as,

$$\rho_{tot} = \rho_{m,0} \left(\frac{a_0}{a}\right)^3 + \rho_{r,0} \left(\frac{a_0}{a}\right)^4 + \rho_{\Lambda,0} + \rho_{k,0} \left(\frac{a_0}{a}\right)^2 \quad . \tag{2.8}$$

First and second Friedmann equations are obtained by substituting metric and energy momentum tensor into Einstein equation,

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \quad , \tag{2.9}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad . \tag{2.10}$$

We may also derive continuity equation invoking both of two of the FE.

Recalling the metric competibility and energy momentum conservation, we may add any constant multiply of $g_{\mu\nu}$ to the left hand side of the EE,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad . \tag{2.11}$$

Then, new form of the FE,

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \quad , \tag{2.12}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad . \tag{2.13}$$

Λ is called as cosmological constant because its constant energy density. It was firstly presented by Einstein in 1917 to construct a static universe, but then dropped by him after Hubble's conclusion that universe should be expanding. As we mention later, it is still a crucial candidate for dark energy by present observations of the WMAP.

Assuming four component universe Hubble parameter is given by observable quantities below,

$$H^2(z) = H_0^2\left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\Lambda\right] \quad , \tag{2.14}$$

where H_0 is defined to be quantity of the Hubble parameter today. Ω_x is dimensionless density parameter and it is obtained by dividing energy density to critical energy density for each component.

Table 2.1: Cosmological Parameters.

Parameters	Recent Values
Hubble Constant (km/Mpc·s): H_0	71.0 ± 2.5
Physical baryon density: $\Omega_b \mathrm{h}^2$	0.02258 ± 0.00057
Physical dark matter density: $\Omega_c \mathrm{h}^2$	0.1109 ± 0.0056
Energy Density for Cosmological Constant: Ω_Λ	0.734 ± 0.029

We use dimensionless parameter $a(t)$ to show how distances grow or decrease with time. We can also define an observable parameter which is monotonically decreasing function of time, redshift parameter,

$$z \equiv \frac{1}{a(t)} - 1 \quad . \tag{2.15}$$

3. DYNAMICS OF SCALAR FIELD

Unlike Newtonian or quantum mechanics, dealing with classical or quantum field theory, we encounter with infinite degrees of freedom. Meanwhile, the position corresponding to each point of x which functions you be subject to an infinite degree of freedom. According to the theory of relativity, action which represents a flat space-time is expected to be Lorentz invariant.

We obtain [11] classic evolution of degrees of freedom referred to the action principle. Dimensionless action is given by,

$$S = \int L dt \quad , \tag{3.1}$$

where the lagrangian has dimensions of energy.

In the context of general relativity action in the flat space-time should be a scalar. To achieve this, lagrangian should have the form,

$$L = \int d^3 x \mathscr{L} \quad . \tag{3.2}$$

.

Physical theories should be generally covariant so we expect the action has to be a scalar under general coordinate transformations. Covariant volume element for non-flat space-time is taken as $dV = \sqrt{-g} d^4 x$, where g is the determinant of the metric and negative signature is related with which signature is used for the metric representation. The action is then,

$$S = \int \sqrt{-g} d^4 x \mathscr{L} \quad . \tag{3.3}$$

Evolution for the scalar field given by the action principle,

$$\delta S = \int \delta L dx^4 = \int \left[\frac{\partial \mathscr{L}}{\partial \phi} - \partial_\mu \left[\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \right] \right] d^4x \quad . \tag{3.4}$$

Then least action principle gives us EL equations for the single scalar field ,

$$\frac{\partial \mathscr{L}}{\partial \phi} - \partial_\mu \left[\frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} \right] = 0 \quad . \tag{3.5}$$

If varied functions in the action are not functions of metric tensor,derivation operator should be replaced by covariant derivation operator. [12] We find the EL equations in the form of,

$$\frac{\partial \mathscr{L}}{\partial \phi} - \nabla_\mu \left[\frac{\partial \mathscr{L}}{\partial(\nabla_\mu \phi)} \right] = 0 \quad . \tag{3.6}$$

Below lagrangian of single scalar field which evolves in a potential $V(\phi)$ is given,

$$\mathscr{L} = -\frac{1}{2} \nabla_\gamma \phi \nabla^\gamma \phi + V(\phi) \quad , \tag{3.7}$$

where $V(\phi)$ describes self interaction of the scalar field.

Varying the related action with respect to ϕ,

$$\frac{\delta S_\phi}{\delta \phi} = \Box \phi - \frac{dV}{d\phi} = 0 \quad . \tag{3.8}$$

Covariant derivative of energy momentum tensor for the minimally coupled homogeneous scalar field reduces to the following equation.

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad . \tag{3.9}$$

Energy-momentum tensor is defined by,

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_M)}{\delta g^{\mu\nu}} \quad . \tag{3.10}$$

The energy momentum tensor of the scalar field is then given as,

$$T_{\alpha\beta} = g_{\alpha\mu} g_{\beta\nu} T^{\mu\nu} = \nabla_\alpha \phi \nabla_\beta \phi - g_{\alpha\beta} \left[\frac{1}{2} \nabla_\gamma \phi \nabla^\gamma \phi + V(\phi) \right] \quad . \tag{3.11}$$

Then, we immediately find that in the RW background enegy density and pressure are respectively,

$$\rho_\phi = a^{-2} T_{00} = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad , \tag{3.12}$$

$$p_\phi = \frac{1}{3} a^{-2} T_{ik} g^{ik} = \frac{1}{2} \dot{\phi}^2 - V(\phi) \quad . \tag{3.13}$$

4. ACCELERATED PHASES OF THE UNIVERSE

4.1 Inflation

There is an inescapable puzzling aspect of a decelerating universe that increase serious questions about initial conditions. Inflation is an accelerating ($\ddot{a} > 0$) phase of the early universe, it is presented to be a solution of the HBB's problem.

The standard cosmological model is puzzled with some of its shortcomings since the observations has entered the era of high precision.

1) Why is the CMB temperature extremely smooth?

When we look in opposite directions as far as possible on the sky, we observe astronomical objects or regions which are never causally connected. This means they have no chance to share their information about their physical conditions. We measure CMB temperature is the same to 1 part in $\sim 10^5$ and this extraordinary smoothness called horizon problem.

2) Why is the critical density of the universe so consistent with the present density of the universe?

When the universe was dominated radiation and matter with nonzero curvature, its effects will tend to increase as the universe expands. But we know density parameter is extraordinarily close to one today. Standard cosmology fails to explain why the universe is close to spatially flat at the present epoch and this paradigm called as flatness problem.

In 1981, both flatness and horizon problems are simultaneously solved under the estimation that universe acquired a cosmological constant by Alan Guth. Early inflation phase may also be given by a basic physical mechanism which roots in matter described by scalar fields.

4.1.1 Slow-roll approximation

The easiest way to give rise the period of inflation is to assume that universe is filled with a scalar field which is slowly evolving in its potential. As we calculate before the equation of motion of the field,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad . \qquad \textbf{(4.1)}$$

The slow-roll approximation is related neglecting kinetic terms,

$$\dot{\phi}^2 \ll V(\phi) \quad . \qquad \textbf{(4.2)}$$

Slow roll also requires small acceleration. Then we obtain new form of first FRW equation and equation motion,

$$H^2 \approx \frac{8\pi G}{3} V(\phi) \quad , \quad 3H\dot{\phi} \approx -V'(\phi) \quad . \qquad \textbf{(4.3)}$$

Then new form of slow-roll conditions are given below,

$$\dot{\phi}^2 \ll V(\phi) \quad \Rightarrow \quad \frac{(V'^2)}{V} \ll H^2 \quad , \qquad \textbf{(4.4)}$$

$$\ddot{\phi} \ll 3H\dot{\phi} \quad \Rightarrow \quad V'' \ll H^2 \quad . \qquad \textbf{(4.5)}$$

There exists numerous scalar field models of inflation. Any chosen $V(\phi)$ gives us a novel scenerio in addition to get a new evolution of the universe. .

Generalized form of polynomial potential for small-field inflation,

$$V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p + \ldots\right], \quad 0 \ll \mu \ll m_{pl}, \quad p > 2 \quad . \qquad \textbf{(4.6)}$$

Generalized form of polynomial potential for quadratic inflation,

$$V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{\mu}\right)^2\right], \quad \phi \ll \mu \quad . \qquad \textbf{(4.7)}$$

Generalized form of polynomial potential for large-field inflation,

$$V(\phi) = \Lambda^4 \left(\frac{\phi}{\mu}\right)^4 \quad p > 1 \quad . \tag{4.8}$$

Exponential potential for large field inflation,

$$V(\phi) = \Lambda^4 \exp\left(\frac{16\pi\phi^2}{pm_{pl}^2}\right) \quad p > 0 \quad . \tag{4.9}$$

Generalized form for Hybrid Inflation

$$V(\phi) = \Lambda^4 \left[1 + \left(\frac{\phi}{\mu}\right)^p\right] \quad p > 0 \quad . \tag{4.10}$$

4.2 Late Acceleration

For cosmology the year of 1998 was the time of achievement: Riess et al [2][High Redshift Supernovae Team] and Perlmutter [1] [Supernovae cosmology project] independently reported that universe is accelerating by observing SN1a.

If a variety of astronomic object all has the same luminosity they will be processing to be standard candles. Supernovae observers discovered that all of the Type $1a$ reach the same maximum luminosity. They are the most luminous standard candles that observers have ever detected to determine the distances to distant galaxies.

Supernovae's redshift is obtained by measuring wavelength of light detected from them.SN1a observations gives us relation about luminosity distance and redshift. Then, observational data exerts expansion history of the universe.

It has to be kept in mind observations reveal neither Ω_m nor Ω_Λ alone. It gives us two relationship,

1) Ω_Λ + Ω_m=1

2) Ω_Λ - Ω_m=0,4

Then, we know 70 percent of the universe is responsible for acceleration. Since observational data has been confirmed current dominant component of the universe to be responsible for the acceleration of the universe, late time acceleration is mentioned with dark energy mechanism.

The second issue, the standart model fails could be denominated as “age crisis". Observations regarding the Type 1A supernova presents age submitted by the standart model is less than recent age. If we compare radiation and matter dominated of the universe , for a given expansion velocity radiation dominated universe need longer tie to be possessed of same age. To have a observational age, we need an repulsive physical aspect.Thus, the standart model should recess a new phase, “late time cosmic accelaration".

The theoretical solution to explain the accelerated universe would be represented by a new component of the universe which state parameter implies $\omega < -\frac{1}{3}$. The first solution that comes to mind is a cosmological constant with negative pressure. Thus, ΛCDM has been the most popular scenario of modern cosmology. For a new universe component, this is not an obstacle in respect to have a dynamical state parameter. Vacuum energy does not vary with space and time. Defining a new degree of freedom that a scalar field, vacuum energy gets effectively dynamic.

Proposals for the late acceleration include cosmological constant or a dynamical scalar field such as quintessence:

4.2.1 Cosmological constant

Cosmological constant is still most prominent candidate for dark energy.It is a stark contrast that this easiest method, has the most challenging problem of the the theoretical physics problems with scale of vacuum energy density.

In ΛCDM, it can be easily seen from the first Friedmann equation that the cosmological constant is of order square of cosmological constant. If we describe an energy density with H_0, this may be interpreted as vacuum energy density.

$$\rho_\Lambda = \frac{\Lambda m_{pl}^2}{8\pi} \approx 10^{-123} m_{pl}^4 \quad . \tag{4.11}$$

Another way to obtain vacuum energy to consontrate on a quantum field and summing over the zero point energies of this field up to a cut off scale k_{max} $(>> m)$. We obtain vacuum energy density,

$$\rho_{vac} = \int_0^{k_{max}} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2}\sqrt{k^2+m^2} \approx \frac{k_{max}^4}{16\pi^2} \quad . \tag{4.12}$$

As an assumption take k_{max} as m_{pl} , then you obtain $\rho_{vac} \simeq 10^{74} GeV^4$. It's about 10^{121} times larger then observed value.($(m_{pl} = 10^{19} GeV)$ This known as fine-tuning problem.

4.2.2 Quintessence

Quintessence is described by a canonical scalar field ϕ with a potential $V(\phi)$ that leads to late time cosmic acceleration. It is conceived as an alternative model to the dark energy mechanism by Caldwell et.al. [13]

The action of the Quintessence model is given by,

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu \phi - V(\phi) \right] \quad . \tag{4.13}$$

The variation of the action (**4.13**) with respect to the ϕ gives ,

$$\ddot{\phi} + 3H\phi + \frac{dV}{d\phi} = 0 \quad . \tag{4.14}$$

Then Friedmann Equations are given by,

$$H^2 = \frac{8\pi G}{3} \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) \right] \quad , \tag{4.15}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3} \left[\dot{\phi}^2 - V(\phi) \right] \quad . \tag{4.16}$$

In a a flat universe with the FRW backgground, energy density and pressure of the scalar field are given by,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad , \tag{4.17}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad . \tag{4.18}$$

The equation which gives the equation of state for the scalar field ϕ is,

$$w_\phi = \frac{p}{\rho} = \frac{\dot{\phi} - V(\phi)}{\dot{\phi}^2 + V(\phi)} \quad , \tag{4.19}$$

which satisfies $-1 \leq w_\phi \leq 1$. In this range we only take into account negative values of w. If we take kinetic term much smaller then potential energy, scalar field behaves just like a cosmological constant.

The representative potential and scalar field for the quintessence model becomes,

$$V(\phi) = \frac{3}{8\pi G}\left(H^2 + \frac{\dot{H}^2}{2}\right) \quad , \tag{4.20}$$

$$\phi = \int dt \left[-\frac{\dot{H}}{4\pi G}\right]^{\frac{1}{2}} \quad . \tag{4.21}$$

In addition to the fact that potentials behaviors emerges cosmic acceleration in the inflation case, they also play crucial role in the late accelerated phase. [14]

$$V(\phi) = M^{4+n}\phi^{-n} \quad , \tag{4.22}$$

$$V(\phi) = M^{4+n}\phi^{-n} exp(\alpha\phi^2/m_{pl}^2) \quad , \tag{4.23}$$

$$V(\phi) = V_0 + M^{4-n}\phi^n \quad , \tag{4.24}$$

$$V(\phi) = M^4 \cos^2(\frac{\phi}{f}) \quad . \tag{4.25}$$

Above representative potentials for the quintessence context are given and each potential gives another scenario for the evolution of late acceleration.

5. SCALAR FIELD POTENTIALS

5.1 Scalar Field Potential For Flat FRW

In cosmological context usual viewpoint is based on adopting the scale factor for any given potential. In this chapter, we consider inverse problem; we present related potential for the flat FRW universe which contains scalar field and non-interacting matter component. [6]

Assuming that equation of state for the scalar field id described by the same relation as other fourth component of the universe, we may define a new parameter, $f(t) \equiv (1+\omega)(1-\omega)^{-1} = \frac{\dot{\phi}^2}{2V}$. Then after some calculations, we obtain,

$$\frac{\dot{V}}{V} = -\frac{\dot{f}+6Hf}{1+f} \quad . \tag{5.1}$$

Integrating the formula we obtain respective potential,

$$V(t) = \frac{3H^2}{8\pi G}\left[1+\frac{\dot{H}}{3H^2}\right] \quad , \tag{5.2}$$

and scalar field as

$$\phi(t) = \int dt \left[-\frac{\dot{H}}{4\pi G}\right]^{\frac{1}{2}} \quad . \tag{5.3}$$

Thereby, last two equations give us the potential which is associated with well-known $a(t)$. We may define the new potential with any other kind of component of the universe by defining, $Q(t) \equiv \frac{8\pi G\rho_m}{3H^2}$. Reconstructed potential,

$$V(t) = \frac{1}{16\pi G}H(1-Q)\left[6H+\frac{2\dot{H}}{H}-\frac{\dot{Q}}{1-Q}\right] \quad , \tag{5.4}$$

and the new form of scalar field,

$$\phi(t) = \int dt \left[\frac{H(1-Q)}{8\pi G}\right]^{\frac{1}{2}} \left[\frac{\dot{Q}}{1-Q} - \frac{2\dot{H}}{H}\right]^{\frac{1}{2}} \quad . \tag{5.5}$$

5.2 Scalar Field Potential For Nonzero Curvature

In this section we follow the same road for the non-flat FRW universe. Difference is that equation of state seems more complicated. [7]

$$1 + \omega = -\frac{2}{3}\left[\frac{\dot{H}a^2 - k}{(Ha)^2 + k}\right] \quad . \tag{5.6}$$

Then, we define $f(t)$ and integrate the same formula as done before and optain the potential,

$$V(t) = \frac{3}{8\pi G}\left(H^2 + \frac{\dot{H}}{3} + \frac{2k}{3a^2}\right) \quad , \tag{5.7}$$

and scalar field

$$\dot{\phi} = \int dt \left[-\frac{1}{4\pi G}\left(\dot{H} - \frac{k}{a^2}\right)\right]^{\frac{1}{2}} \quad . \tag{5.8}$$

One may check $k = 0$ case is consistent with equations driven in other section. We may obtain potential and scalar field with any kind of new component ρ_m. Defining $r(t) = \frac{\rho_m}{\rho_\phi}$, we arrange the new potential

$$V(t) = \frac{3(1+r)^{-1}}{8\pi G}\left(H^2 + \frac{\dot{H}}{3} + \frac{2k}{3a^2} - \frac{\dot{r}}{6(1+r)}\right) \quad , \tag{5.9}$$

and scalar field equation,

$$\phi(t) = \int dt \left[\frac{(1+r)^{-1}}{4\pi G}\left(-\dot{H} - \frac{k}{a^2} + \frac{\dot{r}H}{2(1+r)}\right)\right]^{\frac{1}{2}} \quad . \tag{5.10}$$

6. EFFECT OF AN EXTRA DIMENSION ON SCALAR POTENTIALS

6.1 Set-up And The Method For Getting The Potential

We consider [20] a five-dimensional universe with the metric of the form

$$ds^2 = -dt^2 + a^2(t)\tilde{g}_{ij}dx^i dx^j + b^2(t)dy^2 \quad , \qquad \textbf{(6.1)}$$

where $a(t)$ is the scale factor of the three-dimensional observed part of the space (assuming homogeneous and isotropic expansion for these dimensions and a non-zero curvature), y and $b(t)$ are the coordinate and the scale factor of the extra spatial dimension, respectively, which has the topology of S^1. The three-dimensional spatial part of the Eq.(**6.1**) in comoving coordinates is explicitly

$$\tilde{g}_{ij}dx^i dx^j = \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \quad . \qquad \textbf{(6.2)}$$

The Einstein field equations in such a universe are of the form

$$3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} + 3\frac{k}{a^2} = \kappa_5 \rho_T \quad , \qquad \textbf{(6.3)}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} = -\frac{2\kappa_5}{3}(\rho_T + 2p_T) \quad , \qquad \textbf{(6.4)}$$

where

$$\rho_T = \rho_\phi + \rho_i \quad , \qquad \textbf{(6.5)}$$

$$p_T = p_\phi + p_i \quad , \qquad \textbf{(6.6)}$$

subscripts ϕ and i correspond to the scalar field and other matter types, respectively, and κ_5 is the gravitational constant in 5-dimension.

The equation of motion for the scalar field is, for a homogeneous and isotropic scalar field,

$$\ddot{\phi} + \left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)\dot{\phi} + \frac{dV}{d\phi} = 0 \quad , \qquad \textbf{(6.7)}$$

and for such a scalar field, the energy density and pressure are given as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad . \tag{6.8}$$

We define

$$H(t) \equiv \frac{\dot{a}}{a} \quad , \tag{6.9}$$

as the Hubble parameter of the 3-dimensional observed space,

$$h(t) \equiv \frac{\dot{b}}{b} \quad , \tag{6.10}$$

as that of extra dimension, and

$$w(t) \equiv \frac{\rho_\phi}{p_\phi} \quad , \tag{6.11}$$

as the equation of state parameter of the scalar field. Obviously, using Eq.(**6.8**), we have

$$V(\phi) = \frac{1}{2}(\rho_\phi - p_\phi) = \frac{1}{2}\rho_\phi(1 - w(t)) \quad . \tag{6.12}$$

To get ρ_ϕ, we can use Eq.(**6.3**), and in order to get $w(t)$, we can use Eq.(**6.7**) and Eq.(**6.8**), in a way similar in Refs. [5, 6]. Then the potential for the case of a scalar field as the only source is

$$\begin{aligned} V(t) = & \frac{3}{2\kappa_5}\frac{1}{3H+h}\left[2H^2\left(3H + 3Hh + h + \frac{h^2}{H} + \frac{\dot{H}}{H} + \frac{\dot{h}}{h}\right)\right] \\ & + \frac{3}{2\kappa_5}\frac{1}{3H+h}\left[\dot{H}h + \frac{4k}{a^2}H + \frac{2k}{a^2}h\right] \quad . \end{aligned} \tag{6.13}$$

Using the Eq.(**6.11**) and Eq.(**6.8**), one has

$$\dot{\phi}^2 = 2\left(\frac{1+w}{1-w}\right)V \quad , \tag{6.14}$$

and together with Eq. (**6.13**), this leads to the following integral expression for the scalar field ϕ:

$$\phi(t) = \int dt \left(-\frac{3}{\kappa_5}\frac{2H\dot{H} + \dot{H}h + H\dot{h} - 2kH/a^2}{3H+h}\right)^{1/2} \quad . \tag{6.15}$$

When we consider other components contributing to the energy density of the universe with known density $\rho_i(t)$, the previous equations for $V(t)$ and $\phi(t)$ become, applying the similar procedure,

$$V(t) = \frac{3}{2\kappa_5}\left[\frac{2\tilde{H}u(1+r) + \dot{u}(1+r) - u\dot{r}}{\tilde{H}(1+r)^2}\right] \quad , \tag{6.16}$$

and

$$\phi(t) = \int dt \left(-\frac{3}{\kappa_5} \frac{\dot{u}(1+r) - u\dot{r}}{\tilde{H}(1+r)^2} \right)^{1/2} \quad , \qquad \textbf{(6.17)}$$

where

$$r \equiv \frac{\rho_i(t)}{\rho_\phi(t)} \quad \text{and} \quad u \equiv H^2 + Hh + k/a^2 \quad . \qquad \textbf{(6.18)}$$

In what follows, requiring the positivity of the square-root in the integrals of $\phi(t)$ will be important to reach some interesting conclusions; thus this requirement is called, from now on, 'the consistency condition', [5].

6.2 Examples

To illustrate how the method works, two specific examples are considered in this section: de Sitter and power-law expansions. For simplicity, only a pure scalar field case is considered.

6.2.1 de sitter expansion

Consider the evolutions of the form $a(t) = a_0 \exp(\omega t)$ and $b(t) = b_0 \exp(-\gamma t)$, where a_0, b_0, ω, γ are all positive constants. For the extra dimension, we also assume exponential expansion but at a different rate than that of the three-dimensional space. For this type of evolutions, the consistency of ϕ-equation Eq.(**6.15**) implies two distinct cases: Either $3\omega - \gamma > 0$ and $k = 0$ or $k = +1$, or $3\omega - \gamma < 0$ and $k = -1$. The latter case implies that for a contracting extra dimension, $k = -1$ is possible which is not possible in four-dimensional space expanding exponentially, [5]. The form of the scalar field potential, using Eq.(**6.15**) and Eq.(**6.13**), is

$$V(\phi) = \Lambda + \frac{1}{3}\omega^2(\phi - \phi_0)^2 \quad , \qquad \textbf{(6.19)}$$

where

$$\Lambda = \frac{3}{\kappa_5} \frac{2\omega^2(3\omega - 3\omega\gamma - \gamma + \gamma^2/\omega)}{3\omega - \gamma} \quad , \qquad \textbf{(6.20)}$$

can be considered as the cosmological constant. An interesting point about this potential is that the scalar field dependent part of the potential seems to be independent of the evolution of the extra dimension - since it only contains the constant ω. On the other hand, the constant term of the potential (cosmological constant) depends on both ω and γ. One more issue related to this constant term is that for appropriate choices of ω and γ, one can get either positive or negative cosmological constants. Numerically, for example, taking $\omega = 10$, one can have a negative cosmological constant for $15 - 5\sqrt{949} < \gamma < 30$ and a positive cosmological constant for $0 < \gamma < 15 - 5\sqrt{949}$. The factor in front of this constant term can also be written in terms of the four-dimensional Newton's constant and the size of extra dimension as $1/\kappa_5 \sim 1/L\kappa_4$ where $\kappa_4 \simeq 2 \times 10^{-48}$ is the four-dimensional constant and L is the size of the extra dimension. Thus, in this setting, by adjusting the size of extra dimension one can get, on a phenomenological ground, the observationally confirmed cosmological constant.

An important point worth emphasizing here is the possibility of stabilization of the extra dimension. The naive substitution of $b(t) = b_0$ for the stabilization of extra dimension in this setting seems to make the expression $V(t)$, Eq.(**6.13**), ambiguous because of the $\dot{h}/h$ term. To avoid such a problem, one can choose $b(t) = b_0 \exp(-\varepsilon t)$ where $\varepsilon \ll 1$. In this case, this almost-stabilized behaviour for the extra dimension discards the possibility $k = -1$. This is in accordance with the expectation that one should recover effectively the four-dimensional case for the stabilized extra dimension where $k = -1$ is discarded on the ground of the mathematical consistency of the equations, [5].

6.2.2 Power-law expansion

Consider now the evolutions of the form $a(t) = a_0 t^n$ and $b(t) = b_0 t^m$, where a_0, b_0, m, n are all constants. In this case the integral expression giving $\phi(t)$ becomes

$$\phi(t) = \int dt \left\{ \frac{3}{\kappa_5} \left[\left(\frac{2n^2 + 2nm}{3n + m} \right) \frac{1}{t^2} + \left(\frac{2kn}{3n+m} \right) \frac{1}{a_0 t^{2n}} \right] \right\}^{1/2} \quad . \qquad (\mathbf{6.21})$$

One can reach the following conclusions from this expression before any further calculation: First of all, in all cases consistency requires $m > -n$. For $m > 0$, i.e. for an expanding extra dimension, $k = 0$ is always possible and $k = -1$ is possible for a restricted period of time. On the other hand, for $m < 0$, i.e. for a contracting extra dimension, and $m \neq -3n$, $k = 0$ and $k = -1$ is always possible but $k = +1$ is possible for a restricted period of time, which is not possible in four-dimensional case, [5]. This implies that the value of k is determined by whether the extra dimension is contracting or expanding.

For $k = 0$, the conditions for a contracting extra dimension are $m < -3n$ or $m > -n$.

Although this integral, for $k \neq 0$, can be done analytically, it cannot be converted to get $t(\phi)$. Thus, for simplicity, the scalar field potential only for the case $k = 0$ is considered: Using Eq.(**6.15**) and Eq.(**6.13**),

$$V(\phi) = \frac{3}{\kappa_5}\left[\frac{6n^3 + 2mn^2 + mn - 4n^2}{3n+m}\exp\left(-2\phi/\sqrt{\alpha}\right) + \frac{6n^3 m}{3n+m}\exp\left(-3\phi/\sqrt{\alpha}\right)\right] \quad , \tag{6.22}$$

where

$$\alpha = \frac{3}{\kappa_5}\frac{2n(n+m)}{3n+m} \quad . \tag{6.23}$$

For power-law expansion for the observed space, the scalar field potential takes the exponential form, as in Refs. [5, 6]. The interesting point about this potential is the second term which is directly related to the presence of the extra dimension because of the factor m. If the extra dimension is stabilized at some point, taking $m = 0$, the potential becomes

$$V(t) = \frac{2}{\kappa_5}(3n^2 - 2n)\exp\left(-2\phi/\sqrt{\alpha}\right) \quad , \tag{6.24}$$

where $\alpha = 2n/\kappa_5$ in this case. Note that $n = 2/3$ makes the potential zero and the expansion of the Universe is like a matter-dominated one. Thus one can reach the conclusion that a stabilized extra dimension with a scalar field mimics the effect of non-relativistic matter in the cosmic history. A similar result is reached in a four-dimensional universe for a tachyonic potential in Ref. [5], although the form of the potential is polynomial.

6.3 Scalar Field-Radiation Interaction

For a more realistic inflationary model, it is necessary to consider interaction between the scalar field and the radiation. The case of a non-interacting classical scalar field and radiation is considered in Ref. [5] to model the cosmic evolution from before inflation to the end of inflation. In a similar setting, the interaction between the scalar field and matter is considered in Ref. [7] to investigate the quintessence-matter interaction.

A simple way of introducing interaction between these two components is to modify the equation of motion for the scalar field as

$$\ddot{\phi} + (3H + h)\dot{\phi} + V'(\phi) = \zeta \quad , \tag{6.25}$$

and to modify the fluid equation of radiation as

$$\dot{\rho}_r + (3H+h)(\rho_r + p_r) = -\zeta\dot{\phi} \quad , \tag{6.26}$$

where ζ is the factor characterizing the interaction. This form actually constitutes many different type of interactions, [14, 17–19]. We will specifically consider a class of interaction of the form $\zeta = -\Gamma\dot{\phi}$ where Γ, in the context of theory of reheating, is the rate of particle decay from the inflaton field to radiation, and is determined from the physical parameters of underlying particle model for the interaction of particles.

Consider a simple model realizing the interaction form stated above: The fluid equation for radiation in five-dimensional space-time becomes

$$\dot{\rho}_r + (3H+h)(1+w_r)\rho_r = -\zeta\dot{\phi} = \Gamma\dot{\phi}^2 \quad , \tag{6.27}$$

where the pressure of radiation along the extra dimension is taken with the same equation of state parameter as that of the three-dimensional space. The fluid equation for the scalar field becomes

$$\dot{\rho}_\phi + (3H+h)(1+w_\phi)\rho_\phi = -\Gamma\dot{\phi}^2 \quad , \tag{6.28}$$

and from this equation one obtains

$$\frac{\dot{\rho}_\phi}{\rho_\phi} = -(3H+h)(1+w_\phi) - \Gamma\frac{\dot{\phi}^2}{\rho_\phi} \quad . \tag{6.29}$$

The Friedmann equation for the system is

$$3H^2 + 3Hh + 3\frac{k}{a^2} = \kappa_5\rho_T \quad , \tag{6.30}$$

where the total energy density is $\rho_T = \rho_r + \rho_\phi$. To proceed further, it is possible to take $\rho_r = r(t)\rho_\phi$ where $r(t)$ is a parameter quantifying the relative amount of radiation and the scalar field. Then, from the Friedmann equation Eq.(**6.30**) we get

$$\frac{\dot{\rho}_\phi}{\rho_\phi} = \frac{\dot{u}}{u} - \frac{\dot{r}}{1+r} \equiv \alpha(t) \quad , \tag{6.31}$$

where $u \equiv H^2 + Hh + k/a^2$. Combining Eq.(**6.29**) and Eq.(**6.31**), we have

$$\alpha(t) = -(3H+h)(1+w_\phi) - \Gamma\frac{\dot{\phi}^2 r}{\rho_r} \quad . \tag{6.32}$$

Using

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V \Rightarrow \dot{\phi}^2 = 2(\frac{\rho_r}{r} - V) \quad . \tag{6.33}$$

$$V = \frac{1}{2}\rho_\phi(1 - w_\phi) \Rightarrow 1 + w_\phi = 2(1 - \frac{rV}{\rho_r}) \quad . \tag{6.34}$$

Eq.(**6.32**) becomes

$$\alpha(t) = -2(3H + h + \Gamma)\left(1 - \frac{rV}{\rho_r}\right) \quad , \tag{6.35}$$

and from this we can get the potential

$$V(t) = \frac{\rho_r}{r}\left[\frac{\alpha(t)}{2(3H + h + \Gamma)} - 1\right] \quad . \tag{6.36}$$

The ρ_r dependence of this equation can also be eliminated using the Friedmann equation, Eq.(**6.30**)

$$V(t) = \frac{3}{\kappa_5}\frac{H^2 + Hh + k/a^2}{1 + r}\left[\frac{\alpha(t)}{2(3H + h + \Gamma)} - 1\right] \quad . \tag{6.37}$$

For the interacting case, it is more transparent to leave the scalar field in the form

$$\dot{\phi}^2(t) = -\frac{3}{\kappa_5}\frac{(H^2 + Hh + k/a^2)}{(1 + r)(3H + h + \Gamma)}\left[-(3H + h)(1 + \omega_\phi) - \Gamma\frac{r}{\rho_r}\dot{\phi}^2\right] \quad . \tag{6.38}$$

The similar equations for the case of interacting quintessence-matter are given in Ref. [7].

7. DISCUSSION

In this thesis we give a way of constructing a scalar field potential $V(\phi)$ in a five-dimensional cosmological model with a non-zero curvature for the three-dimensional spatial part, given a particular evolution for the observed space and the extra dimension. In this way, we aim to check whether one can obtain some scalar field potentials which seem not to be allowed in the effective four-dimensional case.

We derived the explicit expressions for the scalar field potential and for the scalar field itself as a function of cosmic time for the cases of both non-interacting and interacting scalar fields. For the case of non-interaction, for some specific evolutions of observed and internal spaces (including the possibility of stabilization), we give the form of the potential and the scalar field as an application of such a reverse engineering procedure. One of the interesting results reached in this context is, in the case of de Sitter type expansion for the observed space, that there is a room for a universe with $k = -1$ for a certain contracting-type evolution of the extra dimension which was not possible in the context of effective four-dimensional model considered in Ref. [5]. There is also the possibility of getting a cosmological constant term in the potential which depends also on the size of the extra dimension. For the case of power-law-type expansion of observed space and for stabilized extra dimension, we get an exponential type potential which vanishes if the observed space evolves like non-relativistic matter, namely $a(t) \sim t^{2/3}$. This reminds us that these kind of models should further be investigated to get information about which kind of scalar field potentials can lead to which type of evolutions in these extra dimensional models.

Actually, with the recipe given in this thesis, many other scalar field potentials used in quintessence models can also be obtained, though the main application here is for inflation. Obviously, in this toy-model approximation, the primary concern is not the microscopic origin of $V(\phi)$, but to see basically the results of bringing an extra dimension into the stage. Therefore, it is not claimed that one can get the exact shape of the scalar field. The reason to apply the method considered here is to extract some information about the shape of potential in the presence of an extra dimension.

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