

PRODUCT INNOVATION IN DURABLE GOODS MONOPOLY WITH PARTIAL PHYSICAL OBSOLESCENCE

A Master's Thesis

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PRODUCT INNOVATION IN DURABLE GOODS MONOPOLY WITH PARTIAL PHYSICAL OBSOLESCENCE

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ABSTRACT

PRODUCT INNOVATION IN DURABLE GOODS
MONOPOLY WITH PARTIAL PHYSICAL
OBSOLESCENCE

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In the literature on planned obsolescence, it has always been assumed that the durable goods monopolist is able to limit the durability of the whole of the product. However, usually it is a component of the product rather than the whole unit that becomes physically obsolete. In this paper, we analyze R&D incentives of a durable goods monopolist when he is able to engage in partial physical obsolescence. We showed that under these circumstances competition in component goods market causes inefficient R&D decisions in the primary market.

Keywords: Planned Obsolescence, Innovation, Component Goods.

ÖZET

KİSMİ FİZİKSEL AŞINMALI DAYANIKLI MALLAR TEKELİNDE ÜRÜN YENİLİĞİ

Atıl, Aysun

Yüksek Lisans, Ekonomi Bölümü

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Planlanmış amortisman literatüründe, her zaman varsayılmıştır ki dayanıklı mallar üreten tekелci ürünün tamamının dayanıklılığını sınırlandırabilir. Ancak ürünün tamamından ziyade genelde ürünün bir parçası fiziksel olarak kullanılmaz hale gelir. Bu tezde, kısmi fiziksel aşınma uygulayabilen dayanıklı mallar üreten tekелcinin AR-GE eğilimlerini inceledik. Bu koşullar altında, parça mallar piyasasındaki rekabetin orijinal ürün piyasasında verimsiz AR-GE kararlarına neden olduğunu gösterdik.

Anahtar Kelimeler: Planlanmış amortisman, Yenilik, Parça mallar.

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CHAPTER 1

INTRODUCTION

Durability choice and the related issue of planned obsolescence are main concerns of the analyses in durable goods theory. Planned obsolescence is defined as the production of goods with uneconomically short useful lives (Bulow, 1986). In the literature on planned obsolescence by a durable-good monopoly, it has always been assumed that the monopolist is able to limit the durability of the whole of the product. However, for the most durable goods such as personal computers, automobiles, cellphones, stereo systems, it is not the whole unit that becomes completely obsolete, but a component of a durable good. For example, it is the battery not the cellphone, the toner fluid not the photocopier, the car parts not the car itself that die first. This requires analyzing the incentives of a durable good monopolist with regard to the component durability. Recently, components and component related service markets gained the attention within the context of pricing decisions without any possible links to R&D incentives.

In this paper, we study the R&D incentives of a durable good monopolist when he is able to engage in partial physical obsolescence, which refers to the obsolescence of components. We use the model in Fishman and Rob (2000) and adopt it to our setting of planned partial physical obsolescence.

Fishman and Rob (2000) investigate a durable good monopolist's R&D

incentive in a model with identical consumers under the assumption that innovations are recurrent and knowledge builds up cumulatively. They find that the monopolist innovates less frequently and invests less than the efficient level. The reasoning is as follows: When a new model appears, old models in consumers' possession are still physically functional. Consumers are willingly to pay for the incremental flow of services provided by the new model—until it's replaced by an upgraded model. However, the introduction of a new model increments consumers' utility in perpetuity since the new model forms the technological base for the subsequent models. This implies that the monopolist receives less than the social value of an innovation. In order to compensate this loss, the monopolist waits longer between product innovations, thereby lengthening the period which consumers pay for the services provided by the new model. Fishman and Rob (2000) also suggest that if the monopolist is able to design each model to last until the new one is introduced—in other words engage in planned obsolescence, the monopolist gets the social value of each model and is induced to innovate at the socially optimal pace.

However, as we state above usually it is a component that becomes physically obsolete. The competitive environment that these components are produced become important for a durable good monopolist's R&D incentives. With regard to this, we extend Fishman and Rob's analysis.

We incorporate the component durability by allowing the monopolist to engage in partial physical obsolescence. The monopolist introduces the new model as the component of the previous model obsolesces. In that case, the monopolist is able to charge consumers both for the incremental utility and the replacement cost of the component. For example, consider a Xerox printer. The price for one of the models of a Xerox printer is \$189, while its component is sold at a price of \$100.55. Moreover, repair costs related to Xerox machines are quoted at least \$50 and there is also an extra cost depending on the

damage. This implies replacing the component costs as much as buying a new product. Under these circumstances, we find that if the monopolist controls the component market, he innovates at the socially optimal pace. However, if the component market is perfectly competitive, like in Fishman and Rob (2000) the monopolist innovates less frequently and invests less than the socially optimal level.

The main concern of this paper is to analyze R&D incentives in the presence of partial physical obsolescence; nevertheless the insights of this study also contribute to the aftermarket literature. Aftermarkets are markets for spare parts, repairs, services and upgrades, etc. The aftermarket good or service is used together with the original equipment, but is sold after the purchase of the equipment (Chen, 1998). In this context, components market can be considered as an aftermarket.

Recently, a number of important court cases in USA, Canada and Europe has indicated the importance of the aftermarkets. For example, in the Kodak and Xerox cases, the original equipment manufacturers were sued for the attempts to monopolize their aftermarket. One of the questions arising out of these court cases is that “Does the monopolization of the aftermarket by the original equipment manufacturers cause an efficiency loss?”.

Borenstein et. al. (2000), point out that many independent service providers for high technology products have sued equipment manufacturers for allegedly excluding them from providing maintenance services. They show that price in the services and parts market (aftermarket) will exceed marginal cost despite the competition in the equipment market. Chen and Ross (1998) observe that both in USA and Europe a number of anti-trust cases have involved allegations that manufacturers of durable goods have refused to supply parts to independent service organizations, apparently to monopolize the market for repairs of their products. They study such refusals in a competitive market and its connected aftermarket and find that refusals help to support

higher prices for high value users but at the same time permit the recovery of higher costs incurred during an initial warranty period. Since full prices reflect full marginal costs at equilibrium, the refusals permit the attainment of a first-best outcome. Accordingly, an attempt by anti-trust authorities to force supply would be welfare reducing. Similarly in terms of R&D decisions, we find that if the component good market becomes more competitive, the R&D investments depart more from its socially optimal level. Hence, the monopolization of the components market is welfare inducing.

The distinct feature of our model is the presence of partial physical obsolescence- namely a component becomes obsolete. We analyze the effects of the market structure in the components market on R&D decisions of a durable good monopolist and find that if the monopolist engages in partial physical obsolescence, then the competition in the component market causes innovations to depart from socially optimal level. When there is perfect competition in the component market, our analysis of monopoly equilibrium is same as Fishman&Rob's (2000) analysis of monopoly equilibrium without obsolescence. We should note that although much of our discussion concerns durable goods monopolists, this does not mean that the insights from our analysis only apply to such settings. Even though most durable goods producers are not monopolists, most do have market power so that monopoly analysis should provide useful insights.

In the next chapter, we present relevant literature in planned obsolescence. In Chapter 3, we set up the basic framework of our model and in Chapter 4, we analyze the social optimum. Chapter 5 contains the analysis of monopoly equilibrium when the monopolist controls the component market. Chapter 6 and 7 contain respectively analyses of monopoly equilibrium, first with perfect competitive component market and second with imperfect competitive component market. In Chapter 8, we present our results.

CHAPTER 2

LITERATURE REVIEW

Recently two issues have generated controversy in durable goods literature: (i) Do firms have an incentive to reduce durability below socially optimal level? (ii) To what extent do firms have an incentive to introduce new products that make old units obsolete? The main concern of these issues is actually to explore whether firms engage in "planned obsolescence" or not.

Analyses related to the planned obsolescence in durable goods theory can be grouped in two categories. First group, which includes Coase (1972), Bulow (1986) and Swan (1972) accept the definition that planned obsolescence is the production of goods with uneconomically short useful lives. Second group leading by Waldman (1993; 1996), Choi (1994) and Kumar (2002) suggest that planned obsolescence is about how often a firm will introduce new products that make old units obsolete. Therefore, the first approach to planned obsolescence can be considered in terms of durability choice whereas the second approach to planned obsolescence can be considered in terms of R&D decisions. Within this context, several authors analyze the incentives of a durable goods monopolist who sells its products.

Coase (1972) considers the dynamic pricing problem of a monopoly selling a durable good to consumers with different valuations. He suggests that durable goods monopolist faces a time inconsistency problem. The problem

arises since durable goods sold in the future affect the value of units sold today. In the absence of ability to commit to future prices, the monopolist does not internalize this effect. Coase argues that under these circumstances the price must eventually fall as the market clears high-valuation consumers. In Coase's analysis, planned obsolescence restores the monopoly's ability to charge monopoly prices.

Following Coase's conjecture (Coase, 1972), Bulow (1982; 1986), considers a durable good monopolist who sells output in each of two periods. If the monopolist is unable to commit to future production levels, in the second period he will choose the production level which maximizes its second period profits. However, additional units in the second period reduces the second period value of the units previously sold. Rational consumers who anticipate this reduction are not willing to pay higher prices in the first period and as a result overall monopoly profits fall. Bulow argues that by reducing durability of its output, the durable goods monopolist can solve its time inconsistency problem.

Swan (1972) handles the issue of planned obsolescence in terms of durability choice incorporating secondhand markets to the analysis. In the context of automobile market, he examines the optimal durability choice of a durable goods monopolist when new and used cars are perfect substitutes. Given a flexible production policy, the monopolist would like to minimize the cost of any given service flow from a stock of durable goods. Consumers are willing to pay higher prices for new goods if they also receive higher prices for their trade-ins. Swan argues that given the inelastic demand for used cars limiting durability enables the monopolist to reduce the flow of services provided by used cars, thereby increasing used car prices. However, a monopolist can restrict the supply of future used cars by varying the price of new goods rather than varying the durability level. Swan shows that for the durable goods monopolist there is no distortion in terms of durability.

Related to Swan's analysis (Swan, 1972), Rust (1986) also considers the optimal durability choice of a durable goods monopolist when there is a secondary market for used durable goods. He suggests that secondary market provides close substitutes for new durable goods limiting profits of the monopolist in the primary market. By limiting product durability, the monopolist ensures that used goods are worse substitutes for new goods. Contrary to Swan, Rust also argues that it is not the existence of secondary markets, but the endogenous scrappage of durables which provides consumers with a substitution possibility that constraints monopoly profits. He considers a Stackelberg game between the monopolist and consumers and shows that for some specific values of parameters the selling monopolist prefers to kill off the secondary market by reducing durability.

Samuelson and Bond (1987) analyze the effects of durability on the incentives to innovate. They argue that since future price of a durable good affects the current demand for that good, durability creates incentives to innovate. They consider a monopoly that sells a durable good in two periods, with the cost of production in each period depending on investments in R&D. Since some costs of increasing output is not internal to the firm, the monopolist can increase its output in order to exploit the residual demand he faces in the second period. Expanding output increases the marginal profitability of innovation. However, the standard incentive of the monopolist to reduce output below its socially optimal level has a diverse effect on innovation. Therefore, depending on which one of these conflicting effects dominates, the selling monopolist invests less or more on innovation than is socially optimal.

The common approach accepted in planned obsolescence (in terms of durability choice) literature is that the monopolist is able to limit the durability of whole of the product. Within this context, the economic motives for and welfare consequences of limiting product durability in durable goods monopoly are analyzed. However, usually a component of a product rather than the

whole good becomes obsolete. In this paper, different than the previous literature, we handle the issue of planned obsolescence in terms of component durability, which we refer as "partial physical obsolescence".

Waldman (1993; 1996), Choi (1994), and Kumar (2002) have also considered that the introduction of a new product can lower the value of used units. Waldman (1996) demonstrates that a similar result to that in Coase (1972) and Bulow (1986) holds within the context of the monopolist's R&D expenditures. Since the monopolist does not internalize in the second period how its behavior affects the value of units sold previously, the monopolist's incentive to invest in R&D that makes past production "technologically obsolete" is too high. Hence in that paper the term planned obsolescence is used to mean that the monopolist has an incentive to engage in R&D decisions and new products introductions and thereby make the past production technologically obsolete. Waldman finds that although time inconsistency causes overinvestment in R&D from the standpoint of the monopolist's own profitability, from the standpoint of social welfare the time inconsistency problem is in fact beneficial. He finds that in the case where the monopolist can commit to a future value for R&D, the firm is unable to capture all the societal benefits from the improved quality of its output. As a result the private incentive to invest in R&D is less than the incentive that is social welfare maximizing.

Like Waldman (1996), Choi (1994) examines the economic incentives for inducing incompatibilities between generations of products and explores the welfare consequences of the product differentiations. He considers a durable goods monopolist who offers products in each of two periods and determines to introduce a compatible or an incompatible product in the second period. He argues that if the monopolist is able to price discriminate between old and new customers, the monopolist prefers to sell an incompatible product to both type of consumers whereas social efficiency requires to sell a compatible product only to newcomers. If the monopolist is not able to price discriminate,

the society suffers an extreme underconsumption in the first period. In that case, the monopolist does not offer any product in the first period and social inefficiency arises due to no product availability.

Kumar (2002) analyzes the effects of resale trading on the price and quality decisions of a durable goods monopolist. He considers a durable goods monopolist who varies price and quality of an infinitely durable product over time in a market of heterogeneous, but rational consumers. He suggests that time inconsistency problem arises due to intertemporal quality discrimination. The reason is that quality upgrades may induce high-valuation consumers to delay purchase, which leads a constraint on monopoly prices. However, by resale trading the monopolist can price discriminate between high-valuation and low-valuation consumers, thereby overcomes its time inconsistency problem. Kumar observes that the monopolist's optimal price and quality offers in new goods market may have complex dynamic patterns. He shows that because of future resale trading, the monopolist may introduce a product of inefficient quality. However, initial quality distortions are followed by steady-state quality allocations that are always efficient for high-valuation consumers.

Both Waldman (1996) and Choi (1994) have considered planned obsolescence as how often a firm will introduce a new product and how compatible the new product will be with older products. In both analyses, costs of innovation incurred by firms are ignored. We incorporate investment expenditures on R&D into the analysis.

Fishman and Rob's study (2000) explained in Chapter 1 is the benchmark analysis we adopt our model. In the setting of Fishman and Rob, product durability limits market power by increasing the value of the consumers' outside option and thereby reducing their willingness to pay for new models. Hence limiting product durability reduces the value of consumers' outside options. However, we should note that while the ability to precommit to future sales completely resolves the Coasian monopoly's problem a similar

ability to precommit to future introduction dates does not accomplish the same purpose in the context of recurring innovations. It is also important to note that in the Coasian setting (Coase, 1972), market efficiency is improved as prices fall, but in Fishman and Rob a monopolist that can not charge for the social value of an innovation innovates less than the socially efficient amount.

We analyze R&D incentives of a durable goods monopolist when a component that is complementary to the original product obsolesces. Therefore, the structure of the component market becomes crucial in the analysis of innovation activity in the primary market in which original goods are sold. If we consider components market as an aftermarket, the insights of our study also contribute to the aftermarket literature.

Chen et al. (1998) analyze the court cases such as Kodak, Chrysler and Xerox in which original equipment manufacturers are accused of refusing supply parts to independent service organizations. Chen et al. state that these refusals by original equipment manufacturers involve attempts to monopolize their aftermarkets. Furthermore, they provide a summary of the economic theories of aftermarkets which try to explain the motives of aftermarket monopolization. One of the aftermarket theories cited in Chen et al. (1998) is "Consumer Surprise Theory", which suggests that switching costs may prevent a customer from switching to a different brand even if the prices of the aftermarket products and services raised substantially. In this way, original equipment manufacturers earn abnormal profits by their installed base of customers. However, this theory is criticized since it does not involve "reputation effects", which implies higher aftermarket prices induce potential new consumers to purchase other brands.

Borenstein et al. (2000) analyze firms' two goals- exploiting lock-in customers by raising aftermarket prices and limiting its aftermarket prices due to reputation effects, in a differentiated duopoly model. They examine whether

competition in durable goods market prevents manufacturers from exercising market power over aftermarket products and services. They show that regardless of the structure of the equipment market, the price in the aftermarket exceeds marginal cost of production. In their model, original equipment manufacturers monopolize their associated aftermarket goods and firms are not able to commit to future prices. There is a crucial restriction on demand side that consumers are always prefer using old units by acquiring services to purchasing new goods. Borenstein et al. show that if the option to scrap and buy new good is binding at the margin, the aftermarket price is increasing in the degree of firm's market power in the equipment market. This analysis does not include the connection between intensity of use and switching costs.

Chen and Ross suggest that aftermarket prices can be used to discriminate between high intensity- high value users and low intensity- low value users. They state that higher aftermarket prices are needed to recover the costs of warranty protection. The reason is that more frequent servicing is required for high intensity consumers in the post warranty period and if the manufacturer can not identify higher intensity customers before purchase, it can not imbed higher expected costs into their original equipment price. In a competitive market and connected aftermarket where firms are able to commit future prices, by charging a low price for the primary product, providing a warranty for the first period and then supracompetitive price for repairs in the second period, each firm is able to set an expected full price for each customer equal to marginal costs of serving that customer. Hence, forcing original equipment manufacturers to supply parts to independent service organizations is welfare reducing. Chen and Ross's analysis is an example to "The Price Discrimination Theory" cited in Chen et. al, which suggests that aftermarket prices as a metering device to discriminate between high intensity- high value users and low intensity- low value users.

Marinosa also analyzes endogenous switching costs that reduce competi-

tion in aftermarkets. He suggests that in order to monopolize their aftermarket, firms may use technological incompatibility which creates endogenous switching costs. In a two period duopoly model, firms introduce a system which consists of various components in the first period whereas in the second period they introduce both the system and the complement that is broken. Endogenous switching costs are driven by second period equilibrium prices of complements and durables. Marinoso states that if firms are able to price discriminate between old consumers and newcomers, switching costs do not affect rational consumers' initial purchasing decisions. Hence, there is no connection between aftermarket prices and initial equipment sales. In these circumstances, introduction of compatible technology in the second period depends on the costs incurred by firms to achieve compatibility. Marinoso shows that with homogeneous products and small costs of reaching compatibility, endogenous switching costs induce firms to prefer compatible products.

The studies mentioned above and other aftermarket theories generally focus on the incentives of original equipment manufacturers to raise aftermarket prices and its welfare consequences. There has been little research concerning efficiency losses in terms of R&D decisions due to the structure of the component market. Within this context, our study presents a different approach to the aftermarket literature.

Fishman and Rob (2000) is related to the Coase Conjecture (Coase, 1972) in the sense that planned obsolescence is a business strategy that help a durable good monopolist to maintain its market power. Coase (1972) considered the dynamic pricing problem of a monopoly selling a durable good (of fixed quality) to consumers with different valuations (Bulow, 1982; 1986). Coase argued that if the monopoly is unable to commit to future prices (or equivalently, commit not to sell any more units), the price must eventually fall as the market clears of high valuation buyers in the second period. In that setting, planned obsolescence restores the monopoly's ability to charge

monopoly prices. In the setting of Fishman and Rob (2000), product durability limits market power by increasing the value of the consumers' outside option and thereby reducing their willingness to pay for new models. Hence limiting product durability reduces the value of consumers' outside options. However, we should note that while the ability to precommit to future sales completely resolves the Coasian monopoly's problem a similar ability to precommit to future introduction dates does not accomplish the same purpose in the context of recurring innovations. It is also important to note that in the Coasian setting, market efficiency is improved as prices fall, but in Fishman and Rob a monopolist that can not charge for the social value of an innovation innovates less than the socially efficient amount.

Waldman (1993; 1996), Choi (1994), Kumar (2002) have also considered that the introduction of a new product can lower the value of used units. Waldman (1996) demonstrates that a similar result to that in Coase (1972) and Bulow (1972) holds within the context of the monopolist's R&D expenditures. Since the monopolist does not internalize in the second period how its behavior affects the value of units sold previously, the monopolist's incentive to invest in R&D that makes past production "technologically obsolete" is too high. Hence in that paper the term planned obsolescence is used to mean that the monopolist has an incentive to engage in R&D decisions and new products introductions and thereby make the past production technologically obsolete. Waldman finds that although time inconsistency causes overinvestment in R&D from the standpoint of the monopolist's own profitability, from the standpoint of social welfare the time inconsistency problem is in fact beneficial. He finds that in the case where the monopolist can commit to a future value for R&D, the firm is unable to capture all the societal benefits from the improved quality of its output. As a result the private incentive to invest in R&D is less than the incentive that is social welfare maximizing.

CHAPTER 3

THE MODEL

We consider a monopolist that introduces infinitely durable products periodically. Every period starts with an introduction of a new model and at the beginning of each period, the monopolist decides on its R&D investment. The quality level of the current product is $q \in \mathbb{R}^+$. As Fishman and Rob (2000) state, introduction of a new model is preceded by an R&D stage, called a “gestation period”. The monopolist has to decide the length of the gestation period, which is denoted by t and the per period R&D expenditures in it, which is denoted by x . Quality increment between two consecutive products is determined by g , which is a function of x and t . Then the quality of the new model is $q + g(x, t)$.

Like in Fishman and Rob (2000), we allow recurrent introductions and assume that quality improvements are cumulative¹. When the monopolist introduces a new product, it incurs in addition to R&D expenditures a fixed cost of F , which is a lump-sum payment. F can be considered an implementation cost, the amount the monopolist must pay for augmenting the present product. There is a constant marginal cost of production, c . We assume for simplicity the production cost is invariant to the quality of the product. We

¹Fishman and Rob define recurrent introductions such that the introduction of one model triggers the development of the next one. They also assume quality improvements are cumulative: If two models are introduced in sequence, and if their R&D inputs are (x_1, t_1) and (x_2, t_2) , then the quality of the second-generation model is $q + g(x_1, t_1) + g(x_2, t_2)$ and similarly for later generations.

impose the following assumption to ensure that innovation is not too costly relative to the benefit:

Assumption 1 g is strictly concave, bounded, increasing in x and t , twice continuously differentiable, $g(x, 0) = g(0, t) = 0$, and there exists an $\{x_p, t_p\}$ so that $r(F + c) < \delta g(x_p, t_p) - (e^{rt_p} - 1)x_p$ where $\delta \in (0, 1]$.

r denotes the positive discount rate, which is common for consumers and firms. We have the first five restrictions on g in order to simplify the analysis. The last restriction guarantees that the monopolist always invests in R&D and introduces new models ($t < \infty, x > 0$).

Different than Fishman and Rob (2000), we have the additional assumption for g in order to achieve concavity around the optimal solution, thereby obtaining differentiability of the objective function.

Assumption 2 For $\lambda \in (0, 1]$,

$$\lambda^2 g_{xx} g_{tt} \geq (\lambda g_{xt} - r e^{rt})^2$$

It is just a stronger concavity condition than what is stated in Assumption 3.

There is a finitely durable component good that complements the primary product. The life of the component coincides with the gestation period. As the new generation of the primary product is introduced, the component that is complementary to the current product obsolesces. We assume the component good is produced at a cost of αc , where $\alpha \in (0, 1)$. Thus, the production cost of the component is just a constant fraction of the marginal cost of the primary product.

On the demand side, there is a continuum of identical infinitely lived consumers of measure 1. Each consumer may consume at most one durable good. A representative consumer derives a flow utility of $\$q$ from product of quality,

q . When the new generation is introduced, consumers must decide whether to purchase a new model or to replace the component of the current product. If the component is replaced, then the flow utility is the same as if they had bought a new good of the same generation. The price of the component, which is denoted by p_c , is crucial for the monopolist's R&D decisions since p_c affects the opportunity cost of buying a new model. We assume p_c depends on $g, q, \alpha c$. We have the following restrictions on $p_c(g, q, \alpha c)$:

Assumption 3 $p_c(g, q, \alpha c)$ is concave, bounded, increasing in q and twice continuously differentiable. Moreover, $p_c \in \{\alpha c, q\}$.

The first four assumptions are fairly standard. The last assumption ensures that the price of the component can not exceed the flow utility of the primary product that the component is complementary to and also it can not be below its production cost.

Throughout our analysis the monopolist sells its products rather than rent them and there is no secondhand market for the primary product.

CHAPTER 4

THE SOCIAL OPTIMUM

We first derive the socially optimal outcome for the innovation activity in order to detect whether the monopolist's R&D decisions are efficient or not. Given the initial quality, q_0 , the social planner chooses R&D expenditures, $\{x_1, x_2, x_3, \dots\}$ and gestation periods, $\{t_1, t_2, t_3, \dots\}$. We can denote the calendar dates at which new models are introduced as T_i , where $\forall i, T_i = \sum_{j=1}^i t_j$. The quality of the model introduced at T_i is $\sum_{j=1}^i g(x_j, t_j)$ and the social planner delivers benefits from this model over $[T_i, T_{i+1})$. Thus, as of date T_i , it generates a discounted benefit of $\left(\sum_{j=1}^i g(x_j, t_j) \right) \left(\frac{1 - e^{-rt_{i+1}}}{r} \right)$. Hence, the sequential problem for the social planner can be written as follows:

$$\begin{aligned}
 (SP) \quad & \sup_{\{x_i, t_i\}_{i=1}^{\infty}} W(q_0, x, t) \text{ s.t.} \\
 W(q_0, x, t) = & \frac{q_0}{r} + (-x_1) \frac{1 - e^{-rt_1}}{r} \\
 & + \sum_{i=1}^{\infty} e^{-rT_i} \left[-(F + c) + \left[\sum_{j=1}^i g(x_j, t_j) - x_{i+1} \right] \frac{1 - e^{-rt_{i+1}}}{r} \right]
 \end{aligned}
 \tag{4.1}$$

or

$$W(q_0, x, t) = \frac{q_0}{r} + \sum_{i=1}^{\infty} e^{-rT_{i-1}} \left[-x_i \frac{(1 - e^{-rt_i})}{r} + e^{-rt_i} \left(\frac{g(x_i, t_i)}{r} - (F + c) \right) \right] \quad (4.2)$$

Let W be the social welfare function. Our first result states that the social welfare function in equation 4.2 is bounded.

Lemma 1 *For any given path $\{x_i, t_i\}_{i=1}^{\infty}$, $W(q_0, x, t) < \infty$. Furthermore, if $\{x_i, t_i\}_{i=1}^{\infty}$ is an optimal path, then $\frac{q_0}{r} < W(q_0, x, t)$ and $\forall i \in \mathbb{N}, t_i < \infty$.*

Proof Suppose that for a given path $\{x_i, t_i\}$, $W(q_0, x, t) = \infty$.

In equation 4.2, W is infinite only if for infinitely many periods $e^{-rT_{i-1}}$ is nearly 1. Since $e^{-rT_{i-1}} = e^{-r(T_{i-2} + t_{i-1})}$, that is equivalent to $e^{-rT_{i-1}} \rightarrow 1$ as $I \rightarrow \infty$. $T_{I-1} \rightarrow 0$ iff $\forall i \in \mathbb{N}, t_i \rightarrow 0$. This implies $\forall i \in \mathbb{N}, g(x_i, t_i) \rightarrow 0$. Hence, we get

$$\begin{aligned} & \lim_{T_{I-1} \rightarrow 0} \sum_{i=1}^{I-1} e^{-rT_{i-1}} \left[-x_i \frac{(1 - e^{-rt_i})}{r} + e^{-rt_i} \left(\frac{g(x_i, t_i)}{r} - (F + c) \right) \right] \\ & \rightarrow - (I - 1) (F + c) \end{aligned}$$

As $I \rightarrow \infty$, $W(q_0, x, t) \rightarrow -\infty$. However, we assumed that $W(q_0, x, t) = \infty$. Thus we have a contradiction.

Now, suppose that $\{x_i^*, t_i^*\}$ solves (SP). Let W_p be the value of the constant stream, where $\forall i \in \mathbb{N}, \{x_i, t_i\} = \{x_p, t_p\}$. Hence,

$$W_p(q_0, x_p, t_p) = \frac{q_0 - x_p}{r} + \frac{e^{-rt_p}}{1 - e^{-rt_p}} \left(- (F + c) + \frac{g(x_p, t_p)}{r} \right) \quad (4.3)$$

If we substitute $r(F + c) < g(x_p, t_p) - (e^{rt_p} - 1)x_p$ into equation 4.3, we get $W_p(q_0, x_p, t_p) > \frac{q_0}{r}$.

Since $\{x_i^*, t_i^*\}$ solves (SP), the value generated by the stream $\{x_i^*, t_i^*\}$ must be at least equal to any other stream. This implies that $W^*(q_0, x^*, t^*) \geq W_p > \frac{q_0}{r}$.

Finally we need to show that $\forall i \in \mathbb{N}, t_i < \infty$. Suppose that there exists an optimal path $\{x_i, t_i\}$ such that for some period k , $t_k = \infty$. The value of the stream after the introduction of the product of quality q_{k-1} ,

$$W(q_{k-1}, x, t) = \frac{q_{k-1}}{r} + (-x_k) \frac{1 - e^{-rt_k}}{r} + \sum_{i=k}^{\infty} e^{-rT_i} \left[-(F + c) + \left[\sum_{j=1}^i g(x_j, t_j) - x_{i+1} \right] \frac{1 - e^{-rt_{i+1}}}{r} \right]$$

Second line in the above expression is zero since $\forall i \geq k, e^{-rT_i} \rightarrow 0$ as $t_k \rightarrow \infty$. Hence,

$$\begin{aligned} W(q_{k-1}, x, t) &= \frac{q_{k-1}}{r} + (-x_k) \frac{1}{r} \\ &\leq \frac{q_{k-1}}{r} \end{aligned}$$

However, we proved that $W(q_{k-1}, x, t) > \frac{q_{k-1}}{r}$. Thus, $\forall i \in \mathbb{N}, t_i < \infty$. $\square$

We are now going to show that non-degenerate R&D only occurs if the next generation will be introduced. Otherwise, there is no innovation.

Lemma 2 *If there exists a solution to (SP) in equation 4.2, then $\forall i \in \mathbb{N}, t_i < \infty \implies x_i > 0$ for the optimal path.*

Proof Suppose that there exists an optimal path $\{x_i, t_i\}$ s.t. $\forall i \in \mathbb{N}, t_i < \infty$, but $\exists s \in \mathbb{N}$ s.t. $x_s = 0$. The value generated by this stream after the introduction of the product at generation $s - 1$:

$$W(q_{s-1}, x, t) = -(F + c) + \frac{1 - e^{-rt_s}}{r} \sum_{j=1}^{s-1} g(x_j, t_j) + e^{-rt_s} W(q_s, x, t)$$

Take an another stream s.t. $\forall i/s \in \mathbb{N}, x'_i = x_i, t'_i = t_i$ and for period s , $x'_s = 0, t'_s = 0$. The value of this path at time $s - 1$:

$$W(q'_{s-1}, x', t') = -(F + c) + \frac{1 - e^{-rt'_s}}{r} \sum_{j=1}^{s-1} g(x'_j, t'_j) + e^{-rt'_s} W'(q'_s, x', t')$$

Since $t'_s = 0$,

$$\frac{1 - e^{-rt'_s}}{r} \sum_{j=1}^{s-1} g(x'_j, t'_j) = 0.$$

Note that $q_{s-1} = \sum_{i=1}^{s-1} g(x_i, t_i) = \sum_{i=1}^{s-1} g(x'_i, t'_i) = q'_{s-1}$. Also we know that $g(x_s, t_s) = 0$ when $x_s = 0$. That is $q'_s = q_s$, which implies $W(q_s, x, t) = W'(q'_s, x', t')$. Since $\{x_i, t_i\}$ is optimal, $W(q_{s-1}, x, t) \geq W'(q'_{s-1}, x', t')$. Thus,

$$\frac{1 - e^{-rt_s}}{r} \sum_{j=1}^{s-1} g(x_j, t_j) + (e^{-rt_s} - 1) W(q_s, x, t) \geq 0 \quad (4.4)$$

It is satisfied when either $t_s = 0$ or $W(q_s, x, t) \leq \frac{q_s}{r}$.

However, by Lemma 1 above, $W(q_s, x, t) > \frac{q_s}{r}$. Hence, $t_s = 0$ and

$$\begin{aligned} W(q_{s-1}, x, t) &= -(F + c) + e^{-rt_s} W(q_s, x, t) \\ &< W(q_s, x, t) \end{aligned}$$

However, when $g(x_s, t_s) = 0$, $W(q_{s-1}, x, t) = W(q_s, x, t)$. Thus, we have a contradiction. So, if $\{x_i, t_i\}$ is an optimal path and $\forall i \in \mathbb{N}$, $t_i < \infty$, then $\forall i \in \mathbb{N}$, $x_i > 0$. $\square$

Our next result shows that the social welfare function, W attains a global maximum.

Proposition 1 *There exists a global maximum to the planner's problem.*

Proof The objective function for the social planner is

$$W(q_0, x, t) = \frac{q_0}{r} + \sum_{i=1}^{\infty} e^{-rt_{i-1}} \left[-x_i \frac{(1 - e^{-rt_i})}{r} + e^{-rt_i} \left(\frac{g(x_i, t_i)}{r} - (F + c) \right) \right]$$

Note that $\forall i \in \mathbb{N}$, $g(x_i, t_i)$ is bounded. So, for some $j \in \mathbb{N}$, $W(q_0, x, t) \rightarrow -\infty$ as $x_j \rightarrow \infty$. That is for some X , $W(q_0, x, t) < 0$, $\forall x_i > X$. However, we established in Lemma 1 that $W(q_0, x, t) > \frac{q_0}{r}$. Hence, x_i 's must be chosen from $[0, X]$. Also, for some $j \in \mathbb{N}$, as $t_j \rightarrow \infty$, $W(q_0, x, t) <$

$W(q_0, x, t) + W(q_{j-1}, x_p, t_p) = W'$ where $W(q_{j-1}, x_p, t_p)$ is the profit generated by a constant stream $\{x_p, t_p\}$ after the introduction of the product of quality q_{j-1} . We proved before $W(q_{j-1}, x_p, t_p) > \frac{q_{j-1}}{r}$. This implies for some T , $W(q_0, x, t) < W'$, $\forall t_i > T$. So, t_i 's must be chosen from $[0, T]$. The maximum must be within the compact domain $[0, X] \times [0, T]$.

Since g is continuous, W is continuous. W is continuous within the compact domain $[0, X] \times [0, T]$. So, it has a maximum. By the Lemma 2, we have an interior solution. $\square$

We have showed that there exists a solution to the planner's problem. Let W^* denote a solution to (SP) in equation 4.1. However, we want to proceed our analysis with dynamic programming methods. In order to do that, we have to establish the functional equation for the social planner's problem. Let q denote the quality of the current product and $V(q)$ denote the value function of the social planner after the introduction of the current product. Then, we can write the following Bellman equation:

$$(FE) \quad V(q) = \max_{x,t} \left\{ (q-x) \frac{1-e^{-rt}}{r} - e^{-rt}(F+c) + e^{-rt}V(q+g(x,t)) \right\} \quad (4.5)$$

Hereafter, we refer equation 4.5 as the functional equation for the social planner's problem. In the following lemma, we will prove that the solutions to (SP) in equation 4.1 coincide with the solutions to (FE) in equation 4.5.

Lemma 3 W^* solves (FE) .

Proof Let Q be the set of possible values for the state variable, in case the quality of the current product, q . $\top : Q \rightarrow Q$ denotes the correspondence such that for each $q \in Q$, $\top(q)$ is the set of the feasible values for the quality of the product next period. Let $A = \{(q_i, q_{i+1}) \in Q \times Q : q_{i+1} \in \top(q_i)\}$ be the graph of $\top$, and $H : A \rightarrow \mathbb{R}$ be the per period return function. The

objective function in equation 4.2 can also be written as follows:

$$\begin{aligned} W(q_0, x, t) &= \frac{q_0}{r} + \sum_{i=1}^{\infty} \beta_i H_i \text{ where} \\ \beta_i &= e^{-rT_{i-1}} \text{ and} \\ H_i &= -x_i \frac{(1 - e^{-rt_i})}{r} + e^{-rt_i} \left(\frac{g(x_i, t_i)}{r} - (F + c) \right) \end{aligned} \quad (4.6)$$

Since $\top(q_i) = q_i + g(x_{i+1}, t_{i+1}) = q_{i+1}$ for each $i \in \mathbb{N}$, Q is a convex subset of $\mathbb{R}$, and the correspondence $\top : Q \rightarrow Q$ is nonempty, compact-valued and continuous. For each $i \in \mathbb{N}$, $0 < t_i < \infty$, which implies $0 < \beta_i < 1$. Note also that g is bounded and continuous, and $\forall i \in \mathbb{N}$, $(x_i, t_i) \in [0, X] \times [0, T]$. Hence, H_i is bounded and continuous for each $i \in \mathbb{N}$. With conditions above satisfied, solutions to (FE) coincide exactly to solutions of (SP) (Stokey, 1989). $\square$

Now, we point out an interesting fact about the derivative of this value function with respect to q .

Lemma 4 *V is continuously differentiable at q . Moreover, if $V_q(q) < \infty$, then at the optimal solution $V_q(q) = \frac{1}{r}$.*

Proof First, we will show the differentiability of V . By Assumption 3, H is strictly concave (see the appendix). g is continuously differentiable, so is H . Also, by construction, $\top$ is convex. With these conditions satisfied, V is continuously differentiable at q (Stokey, 1989).

Now, assume that $V_q(q) < \infty$. Since $V_q(q) = \frac{1-e^{-rt}}{r} + e^{-rt}V_q(q+g)$, V_q is increasing in $V_q(q+g)$. Let G be the greatest lower bound and U be the least upper bound for V_q . Then,

$$G \leq \frac{1 - e^{-rt}}{r} + e^{-rt}V_q(q+g). \quad (4.7)$$

Since the right hand side of equation 4.7 is increasing in $V_q(q+g)$, it achieves

its minimum when $V_q(q+g) = G$. Thus,

$$G = \frac{1 - e^{-rt}}{r} + e^{-rt}G$$

or $G = \frac{1}{r}$. Likewise, $U = \frac{1}{r}$. Since the least upper bound and the greatest lower bound are equal, $G \leq V_q(q) \leq U$ implies $V_q(q) = \frac{1}{r}$. $\square$

V satisfies the following first order conditions:

$$-\frac{(1 - e^{-rt})}{r} + e^{-rt}V_q(q)g_x = 0 \quad (4.8)$$

$$e^{-rt}((q - x) + r(F + c) - rV(q + g) + V_q(q)g_t) = 0 \quad (4.9)$$

Substituting $V_q(q) = \frac{1}{r}$,

$$-\frac{(1 - e^{-rt})}{r} + e^{-rt}\frac{1}{r}g_x = 0 \quad (4.10)$$

$$(q - x) + r(F + c) - rV(q + g) + \frac{1}{r}g_t = 0 \quad (4.11)$$

These first order conditions will be useful when we analyze the market equilibrium.

CHAPTER 5

THE EQUILIBRIUM WHEN THE MONOPOLIST CONTROLS THE COMPONENT MARKET

In this chapter, we will analyze the market equilibrium, where all generations are introduced by an infinitely lived monopolist and the monopolist also controls the component market. We find that this leads to the socially optimal amount of investment and gestation periods.

When consumers decide whether to buy the newly introduced product or not, they will compare the quality increment they observe in the new product versus the price of the component good. Thus, when the monopolist sells its primary product, the price it charges is affected by the following factors:

- the quality of the product consumers have in their possession,
- the quality of the new product,
- consumers' expectations regarding the length of time the new generation would be on the technological frontier before a superior product is introduced
- the price of the component.

Consumers with a quality q good in their possession will buy a quality q' good at a price of p if

$$\begin{aligned}
q' \frac{1 - e^{-rt^e}}{r} - p &\geq (q - p_c) \frac{1 - e^{-rt^e}}{r} \\
q' \frac{1 - e^{-rt^e}}{r} - p &\geq 0
\end{aligned}$$

where p_c is the price of the component good¹ and t^e is the consumers' expectation about how long they are going to use the new generation. Thus the price of the primary product, p must satisfy

$$p \leq \frac{1 - e^{-rt^e}}{r} \min \{q', (q' - q) + p_c\}$$

Therefore, $p_c = q$ is the optimal price for the component and $p = \frac{1 - e^{-rt^e}}{r} q'$ is the optimal price for the primary product. If consumers are using several different generations of the good, then p_c is the maximum of the various quality levels. In a rational-expectations equilibrium the consumers' expectations are fulfilled, which implies t^e at the i^{th} introduction equals t_{i+1} for all $i \in \mathbb{N}$. Hence, the sequential problem for the monopolist can be written as follows:

$$(SP) \sup_{\{x_i, t_i\}_{i=1}^{\infty}} \pi(q_0, x, t)$$

where

$$\begin{aligned}
\pi(q_0, x, t) &= \frac{q_0}{r} + (-x_1) \frac{1 - e^{-rt_1}}{r} \\
&+ \sum_{i=1}^{\infty} e^{-rT_i} [g(x_i, t_i) + p_c(g, q, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\
&+ \sum_{i=1}^{\infty} e^{-rT_i} \left[-(F + c) - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right] \quad (5.1)
\end{aligned}$$

In the following lemma, we will show that the objective function in equation 5.1 is bounded.

¹for simplicity it is in flow terms

Lemma 5 For any given path $\{x_i, t_i\}_{i=1}^{\infty}$, $\pi(q_0, x, t) < \infty$. Furthermore, if $\{x_i, t_i\}_{i=1}^{\infty}$ is an optimal path, then $\frac{q_0}{r} < \pi(q_0, x, t)$ and $\forall i \in \mathbb{N}$, $t_i < \infty$.

Proof We can rewrite the objective function in equation 5.1 as follows:

$$\begin{aligned} \pi(q_0, x, t) &= \frac{q_0}{r} \\ &+ \sum_{i=1}^{\infty} e^{-rT_{i-1}} e^{-rt_i} [g(x_i, t_i) + p_c(g, q, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\ &+ \sum_{i=1}^{\infty} e^{-rT_{i-1}} \left[-e^{-rt_i} (F + c) - x_i \frac{1 - e^{-rt_i}}{r} \right] \end{aligned} \quad (5.2)$$

Now suppose that $\pi(q_0, x, t) = \infty$. Under Assumption 3 and Assumption 3, π is infinite only if for infinitely many periods $e^{-rT_{i-1}}$ is nearly 1. This implies $e^{-rT_{I-1}} \rightarrow 1$ as $I \rightarrow \infty$. $T_{I-1} \rightarrow 0$ iff $\forall i \in \mathbb{N}$, $t_i \rightarrow 0$. This implies $\forall i \in \mathbb{N}$, $g(x_i, t_i) \rightarrow 0$. We look for rational expectations equilibrium. So, $\forall i \in \mathbb{N}$, $t_i^e \rightarrow 0$. This yields

$$\begin{aligned} &\lim_{T_{I-1} \rightarrow 0} \sum_{i=1}^{I-1} e^{-rT_{i-1}} e^{-rt_i} [g(x_i, t_i) + p_c(g, q, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\ &+ \lim_{T_{I-1} \rightarrow 0} \sum_{i=1}^{I-1} e^{-rT_{i-1}} \left[-e^{-rt_i} (F + c) - x_i \frac{1 - e^{-rt_i}}{r} \right] \\ \rightarrow &- (I - 1) (F + c) \end{aligned}$$

As $I \rightarrow \infty$, $\pi(q_0, x, t) \rightarrow -\infty$. However, we assumed that $\pi(q_0, x, t) = \infty$.

Thus we have a contradiction.

Now, suppose that $\{x_i^*, t_i^*\}$ solves (SP). Let π_p be the value of the constant stream, where $\forall i \in \mathbb{N}$, $\{x_i, t_i\} = \{x_p, t_p\}$. Hence,

$$\pi_p = \frac{q_0 - x_p}{r} + \frac{g(x_p, t_p) (1 - e^{-rt_p^e})}{r(e^{rt_p} - 1)} + \sum_{i=1}^{\infty} \frac{e^{-rit_p} (1 - e^{-rt_p^e}) p_c(g, q_i, \alpha c)}{r} - \frac{(F + c)}{e^{rt_p} - 1} \quad (5.3)$$

Since the monopolist controls the component market, for each $i \in \mathbb{N}$, $p_c(g, q_i, \alpha c) = q_{i-1} > 0$. If we take $\delta = (1 - e^{-rt_p^e})$ in Assumption 3, we have $\pi_p(q_0, x_p, t_p) > \frac{q_0}{r}$.

$\{x_i^*, t_i^*\}$ solves (SP), thereby implying $\pi^*(q_0, x^*, t^*) \geq \pi_p > \frac{q_0}{r}$.

In order to prove that $\forall i \in \mathbb{N}, t_i < \infty$, suppose there exists an optimal path $\{x_i, t_i\}$ such that for some period k , $t_k = \infty$. The value of the stream after the introduction of the product of quality q_{k-1} ,

$$\begin{aligned} \pi(q_{k-1}, x, t) &= \frac{q_{k-1}}{r} + (-x_k) \frac{1 - e^{-rt_k}}{r} \\ &\quad + \sum_{i=k}^{\infty} e^{-rT_i} [g(x_i, t_i) + p_c(g, q, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\ &\quad + \sum_{i=k}^{\infty} e^{-rT_i} \left[-(F + c) - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right] \end{aligned}$$

Again, second line in the above expression is zero since $\forall i \geq k, e^{-rT_i} \rightarrow 0$ as $t_k \rightarrow \infty$. Hence,

$$\begin{aligned} \pi(q_{k-1}, x, t) &= \frac{q_{k-1}}{r} + (-x_k) \frac{1}{r} \\ &\leq \frac{q_{k-1}}{r}. \end{aligned}$$

However, we find that $\pi(q_{k-1}, x, t) > \frac{q_{k-1}}{r}$. This leads to a contradiction.

Thus, $\forall i \in \mathbb{N}, t_i < \infty$. □

We are now going to show that the monopolist invests in R&D in every gestation period.

Lemma 6 *If there exists a solution to (SP), then $\forall i \in \mathbb{N}, t_i < \infty \implies x_i > 0$ for the optimal path.*

Proof Suppose that there exists an optimal path $\{x_i, t_i\}$ s.t. $\forall i \in \mathbb{N}, t_i < \infty$, but $\exists s \in \mathbb{N}$ s.t. $x_s = 0$. The value generated by this stream after the introduction of the product at generation $s - 1$:

$$\begin{aligned} \pi(q_{s-1}, x, t) &= -(F + c) + \frac{1 - e^{-rt_s}}{r} [g(x_{s-1}, t_{s-1}) + p_c(g, q, \alpha c)] \\ &\quad + e^{-rt_s} \pi(q_s, x, t) \end{aligned}$$

Take an another stream s.t. $\forall i/s \in \mathbb{N}$, $x'_i = x_i$, $t'_i = t_i$ and for period s , $x'_s = 0$, $t'_s = 0$. The value of this path at time $s - 1$:

$$\begin{aligned} \pi(q'_{s-1}, x', t') &= -(F + c) + \frac{1 - e^{-rt'_s}}{r} [g(x'_{s-1}, t'_{s-1}) + p_c(g, q', \alpha c)] \\ &\quad + e^{-rt'_s} \pi(q'_s, x', t') \end{aligned}$$

Note that we substitute $t'_{s-1} = t_s$ in above equations. Since $t'_s = 0$,

$$\frac{1 - e^{-rt'_s}}{r} [g(x'_{s-1}, t'_{s-1}) + p_c(g, q', \alpha c)] = 0.$$

Again we have $q_{s-1} = \sum_{i=1}^{s-1} g(x_i, t_i) = \sum_{i=1}^{s-1} g(x'_i, t'_i) = q'_{s-1}$. We know that $g(x_s, t_s) = 0$ when $x_s = 0$. That is $q'_s = q_s$, which implies $\pi(q_s, x, t) = \pi(q'_s, x', t')$. Also, at time $s - 1$, $p_c(g, q, \alpha c) = q_{s-1} = p_c(g, q', \alpha c)$. By optimality, $\pi(q_{s-1}, x, t) \geq \pi(q'_{s-1}, x', t')$. It is satisfied when

$$\frac{1 - e^{-rt_s}}{r} [g(x_{s-1}, t_{s-1}) + p_c(g, q, \alpha c)] + (e^{-rt_s} - 1) \pi(q_s, x, t) \geq 0$$

That is either

$$t_s = 0$$

or

$$\pi(q_s, x, t) \leq \frac{g(x_{s-1}, t_{s-1}) + p_c(g, q, \alpha c)}{r}. \quad (5.4)$$

Remember that at time $s - 1$, $p_c(g, q, \alpha c) = q_{s-2}$. Substituting $q_s = q_{s-1}$ in equation 5.4 yields

$$\pi(q_s, x, t) \leq \frac{q_s}{r} \quad (5.5)$$

However, by Lemma 5 above, $\pi(q_s, x, t) > \frac{q_s}{r}$. Thus, $t_s = 0$ and

$$\begin{aligned} \pi(q_{s-1}, x, t) &= -(F + c) + e^{-rt_s} \pi(q_s, x, t) \\ &< \pi(q_s, x, t) \end{aligned}$$

However, $g(x_s, t_s) = 0$ implies $\pi(q_{s-1}, x, t) = \pi(q_s, x, t)$. Thus we have a contradiction. Hence, if $\{x_i, t_i\}$ is an optimal path and $\forall i \in \mathbb{N}, t_i < \infty$, then $\forall i \in \mathbb{N}, x_i > 0$. $\square$

The following proposition states that there is a solution to monopolist's problem in equation 5.1.

Proposition 2 *There exists a global maximum to the monopolist's problem.*

Proof The objective function for the selling monopolist is

$$\begin{aligned} \pi(q_0, x, t) = & \frac{q_0}{r} + (-x_1) \frac{1 - e^{-rt_1}}{r} \\ & + \sum_{i=1}^{\infty} e^{-rt_i} [g(x_i, t_i) + p_c(g, q, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\ & + \sum_{i=1}^{\infty} e^{-rt_i} \left[-(F + c) - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right] \end{aligned}$$

Note that $\forall i \in \mathbb{N}, g(x_i, t_i)$ and $p_c(g, q, \alpha c)$ are bounded. So, for some $j \in \mathbb{N}, \pi(q_0, x, t) \rightarrow -\infty$ as $x_j \rightarrow \infty$. That is for some $X, \pi(q_0, x, t) < 0, \forall x_i > X$. However, we established in Lemma 5 that $\pi(q_0, x, t) > \frac{q_0}{r}$. Hence, x_i 's must be chosen from $[0, X]$. Also, for some $j \in \mathbb{N}$, as $t_j \rightarrow \infty, \pi(q_0, x, t) < \pi(q_0, x, t) + \pi(q_{j-1}, x_p, t_p) = \pi'$ where $\pi(q_{j-1}, x_p, t_p)$ is the profit generated by a constant stream $\{x_p, t_p\}$ after the introduction of the product of quality q_{j-1} . However, we have proved that $\pi(q_{j-1}, x_p, t_p) > \frac{q_{j-1}}{r}$. This implies for some $T, \pi(q_0, x, t) < \pi', \forall t_i > T$. So, t_i 's must be chosen from $[0, T]$. Hence, the maximum must be within the compact domain $[0, X] \times [0, T]$.

Since g and p_c are continuous, π is continuous. π is continuous within the compact domain $[0, X] \times [0, T]$. So, it has a maximum. By Lemma 5, we have an interior solution. $\square$

Let π^* denote a solution to (SP) in equation 5.1. We now establish the functional equation for the monopolist's problem as we did in Chapter 4,

$$(FE) \quad \Pi(q) = \max_{x,t} \left\{ -\frac{1}{r}x + e^{-rt} \left(-(F+c) + \frac{x}{r} + p(q, x, t) + \Pi(q') \right) \right\} \quad (5.6)$$

$$\text{where } p(q, x, t) = (g + p_c(g, q, \alpha c)) \frac{1 - e^{-rt^e}}{r}.$$

We again show that the sequential problem in equation 5.1 and functional equation in 5.6 are equivalent.

Lemma 7 π^* solves (FE).

Proof Again, let Q represents the set of possible values for the state variable, q and $\top : Q \rightarrow Q$ be the correspondence s.t for each $q \in Q$, $\top(q)$ is the set of the feasible values for the quality of the product next period. Let $A = \{(q_i, q_{i+1}) \in Q \times Q : q_{i+1} \in \top(q_i)\}$ be the graph of $\top$, and $H : A \rightarrow \mathbb{R}$ be the per period return function. The objective function in equation 5.1 can also be written as follows:

$$\begin{aligned} \pi(q_0, x, t) &= \frac{q_0}{r} + \sum_{i=1}^{\infty} e^{-rT_{i-1}} e^{-rt_i} [g(x_i, t_i) + p_c(g, q_{i-1}, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\ &\quad + \sum_{i=1}^{\infty} e^{-rT_{i-1}} \left[-e^{-rt_i} (F+c) - x_i \frac{1 - e^{-rt_i}}{r} \right] \end{aligned} \quad (5.7)$$

Then, we have following form for the sequential problem:

$$\begin{aligned} \pi(q_0, x, t) &= \frac{q_0}{r} + \sum_{i=1}^{\infty} \beta_i H_i \text{ where} \\ \beta_i &= e^{-rT_{i-1}} \text{ and} \\ H_i &= e^{-rt_i} [g(x_i, t_i) + p_c(g, q_{i-1}, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\ &\quad - e^{-rt_i} (F+c) - x_i \frac{1 - e^{-rt_i}}{r} \end{aligned}$$

As it is stated before, Q is a convex subset of $\mathbb{R}$, and the correspondence $\top : Q \rightarrow Q$ is nonempty, compact-valued and continuous. Moreover, for each $i \in \mathbb{N}$, $0 < t_i < \infty$ implies that $0 < \beta_i < 1$. Now, g and p_c are bounded and

$\forall i \in \mathbb{N}, (x_i, t_i) \in [0, X] \times [0, T]$. Hence, H_i is bounded for each $i \in \mathbb{N}$. Since g and p_c are continuous, H_i is continuous. With conditions above satisfied, solutions to (FE) coincide exactly to solutions of (SP) (Stokey, 1989). $\square$

We can now continue our analysis with solving functional equation. At first, we go through with the differentiability of the functional equation in order to compare the market equilibrium with the social optimum.

Lemma 8 Π is continuously differentiable at q . Moreover, the greatest lower bound for $\Pi_q(q)$ is less than $\frac{1}{r}$.

Proof The proof of differentiability is similar to the one in Chapter 4. By Assumption 3, H is strictly concave (see the appendix). Since g and p_c are continuously differentiable, so is H . Moreover, it can be easily shown that $\mathbb{T}$ is convex. With these conditions satisfied, Π is continuously differentiable at q (Stokey, 1989).

$$\begin{aligned}\Pi_q(q) &= \Pi_{q_1}(q_1) = e^{-rt_1} (p_{q_1}(q, x, t) + \Pi_{q_2}(q_2)) \\ &= \sum_{i=1}^{\infty} e^{-r\left(\sum_{j=0}^i t_j\right)} p_{q_i}(q, x, t) \text{ where } t_0 = 0.\end{aligned}\quad (5.8)$$

Since $p(q, x, t) = (g + p_c(g, q, \alpha c)) \frac{1-e^{-rt^e}}{r}$ and $p_c(g, q, \alpha c) = q$, $p_{q_i}(q, x, t) = \frac{1-e^{-rt_i^e}}{r}$ for each $i \in \mathbb{N}$. If $\Pi_q(q) = K$ where K is the greatest lower bound, K is achieved by constant stream of t_i 's. Hence,

$$\begin{aligned}K &= \sum_{i=1}^{\infty} e^{-r\left(\sum_{j=0}^i t\right)} \frac{1 - e^{-rt^e}}{r} \\ &= \frac{1 - e^{-rt^e}}{(e^{rt} - 1)r}\end{aligned}$$

Since $0 < t < \infty$, $K < \frac{1}{r}$.

Thus, the greatest lower bound for $\Pi_q(q)$ is less than $V_q(q)$. $\square$

We have showed that when the quality of a product increases, the social planner benefits more than the monopolist. The reason is that in order to get

the social value of an innovation as the social planner does, the monopolist charges the flow utility of the current product for the price of the component. However, while an increase in the quality of a product affects the social planner in the current period, the monopolist realizes this change in the next period when consumers have to replace the component. Therefore, the monopolist waits longer than the social planner does in order to capture the gains from quality change.

Π satisfies the following first order conditions:

$$\begin{aligned}\Pi_x = & -\frac{(1 - e^{-rt})}{r} + e^{-rt} \frac{\partial p(q, x, t)}{\partial x} \\ & + e^{-rt} \frac{\partial p(q, x, t)}{\partial q} g_x + e^{-rt} \frac{\partial \Pi(q')}{\partial q'} g_x\end{aligned}\quad (5.9)$$

$$\begin{aligned}\Pi_t = & -re^{-rt} \left(-(F + c) + \frac{x}{r} + p(q, x, t) + \Pi(q') \right) \\ & + e^{-rt} \left(\frac{\partial p(q, x, t)}{\partial t} + \frac{\partial p(q, x, t)}{\partial q} g_t + \frac{\partial \Pi(q')}{\partial q'} g_t \right)\end{aligned}\quad (5.10)$$

Here, we will assume that $\frac{\partial p(q, x, t)}{\partial x} = 0$ and $\frac{\partial p(q, x, t)}{\partial t} = 0$. This is reasonable since the optimum price the monopolist charges for the primary product is $q' \frac{1 - e^{-rt^e}}{r}$. Hence, R&D inputs x and t affect the price of the new generation through their effects on quality. If we substitute $p_q(q, x, t) = \frac{1 - e^{-rt^e}}{r}$, $\Pi_q(q) = \frac{1 - e^{-rt^e}}{(e^{rt} - 1)r}$ and $t^e = t$ in equations 5.9 and 5.10, we obtain

$$-\frac{(1 - e^{-rt})}{r} + e^{-rt} \frac{1}{r} g_x = 0 \quad (5.11)$$

$$r(F + c) - x - rp(q, x, t) - r\Pi(q') + \frac{1}{r} g_t = 0 \quad (5.12)$$

Note that equation 4.10 is same as equation 5.11. We can not know exact forms of functional equations, V and Π so that we can not say equations 4.11 and 5.12 are also equal. However, we expect their equivalence since the

monopolist's problem is same as the social planner's when $p_c(q, x, t) = q$ and $t^e = t$. By setting $p_c = q$, the monopolist gets the social value of the innovation. Thus, the monopolist chooses the level of R&D investment and the gestation periods at social optimum. It is somewhat different than Fishman&Rob's result. They find that the monopoly implements the socially optimal rate of innovation by designing old models to expire just as new ones are introduced (Fishman, Rob, 2000). However, we find that the monopolist can achieve the socially optimal rate of innovation by partial physical obsolescence if it has full monopoly power in the component market.

CHAPTER 6

THE EQUILIBRIUM WHEN THE COMPONENT MARKET IS PERFECTLY COMPETITIVE

We now consider the case where replacement components are supplied by competitive firms. This means that $p_c = \alpha c$, and that $p = (q' - q) + \alpha c$. Since every consumer will always buy the latest generation, this implies that $p = g + \alpha c$. Given this, we can write the sequential problem for the monopolist as follows:

$$(SP) \quad \sup_{\{x_i, t_i\}_{i=1}^{\infty}} \pi(q_0, x, t)$$

where

$$\begin{aligned} \pi(q_0, x, t) = & \frac{q_0}{r} + (-x_1) \frac{1 - e^{-rt_1}}{r} \\ & + \sum_{i=1}^{\infty} e^{-rT_i} (g(x_i, t_i) + \alpha c) \frac{1 - e^{-rt_i^e}}{r} \\ & + \sum_{i=1}^{\infty} e^{-rT_i} \left[-(F + c) - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right] \end{aligned} \quad (6.1)$$

We first establish that this payoff function is bounded, and that new generations are introduced.

Lemma 9 *For any given path $\{x_i, t_i\}_{i=1}^{\infty}$, $\pi(q_0, x, t) < \infty$. Furthermore, if*

$\{x_i, t_i\}_{i=1}^{\infty}$ is an optimal path, then $\frac{q_0}{r} < \pi(q_0, x, t)$ and $\forall i \in \mathbb{N}, t_i < \infty$.

Proof Suppose that for a given path $\{x_i, t_i\}$, $\pi(q_0, x, t) = \infty$.

π is infinite only if for infinitely many periods e^{-rT_i} is nearly 1. Since $e^{-rT_i} = e^{-r(T_{i-1}+t_i)}$, that is equivalent to $e^{-rT_I} \rightarrow 1$ as $I \rightarrow \infty$. $T_I \rightarrow 0$ iff $\forall i \in \mathbb{N}, t_i \rightarrow 0$. This implies $\forall i \in \mathbb{N}, g(x_i, t_i) \rightarrow 0$. We assume rational expectations, so $\forall i \in \mathbb{N}, t_i^e \rightarrow 0$. Hence, we obtain

$$\begin{aligned} & \lim_{T_I \rightarrow 0} \sum_{i=1}^I e^{-rT_i} \left[-(F+c) + (g(x_i, t_i) + \alpha c) \frac{1 - e^{-rt_i^e}}{r} - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right] \\ \rightarrow & -I(F+c) \end{aligned}$$

Above equation implies that as $I \rightarrow \infty$, $\pi(q_0, x, t) \rightarrow -\infty$. However, we assumed that $\pi(q_0, x, t) = \infty$. This is a contradiction.

Now, suppose that $\{x_i^*, t_i^*\}$ solves (SP) in equation 6.1. Let $\bar{\pi}$ be the value of the constant stream, which is $\forall i \in \mathbb{N}, \{x_i, t_i\} = \{x_p, t_p\}$. Hence,

$$\bar{\pi}(q_0, x_p, t_p) = \frac{q_0 - x_p}{r} + \frac{[g(x_p, t_p) + \alpha c] (1 - e^{-rt_p^e})}{r(e^{rt_p} - 1)} - \frac{(F+c)}{e^{rt_p} - 1}$$

By Assumption 3, $\bar{\pi}(q_0, x_p, t_p) > \frac{q_0}{r}$. Since $\{x_i^*, t_i^*\}$ solves (SP), the value generated by the stream $\{x_i^*, t_i^*\}$ must be at least equal to any other stream. That is $\pi^*(q_0, x^*, t^*) \geq \bar{\pi}(q_0, x_p, t_p) > \frac{q_0}{r}$.

In order to prove the final step, suppose that there exists an optimal path $\{x_i, t_i\}$ such that for some period k , $t_k = \infty$. The value of the stream after the introduction of the product of quality q_{k-1} ,

$$\begin{aligned} \pi(q_{k-1}, x, t) &= \frac{q_{k-1}}{r} + (-x_k) \frac{1 - e^{-rt_k}}{r} \\ &+ \sum_{i=k}^{\infty} e^{-rT_i} (g(x_i, t_i) + \alpha c) \frac{1 - e^{-rt_i^e}}{r} \\ &+ \sum_{i=k}^{\infty} e^{-rT_i} \left[-(F+c) - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right] \end{aligned}$$

Again, the second line in the above expression is zero. Hence,

$$\begin{aligned}\pi(q_{k-1}, x, t) &= \frac{q_{k-1}}{r} + (-x_k) \frac{1}{r} \\ &\leq \frac{q_{k-1}}{r}.\end{aligned}$$

However, we proved that $\pi(q_{k-1}, x, t) > \frac{q_{k-1}}{r}$. Thus, $\forall i \in \mathbb{N}, t_i < \infty$. $\square$

Our next result states that the monopolist invests in R&D in every gestation period eventhough the components are supplied by competitive firms.

Lemma 10 *If there exists a solution to (SP) in equation 6.1, then $\forall i \in \mathbb{N}, t_i < \infty \implies x_i > 0$ for the optimal path.*

Proof Suppose that there exists an optimal path $\{x_i, t_i\}$ s.t. $\forall i \in \mathbb{N}, t_i < \infty$, but $\exists s \in \mathbb{N}$ s.t. $x_s = 0$. The value generated by this stream after the introduction of the product at time $s - 1$:

$$\pi(q_{s-1}, x, t) = -(F + c) + \frac{1 - e^{-rt_s^e}}{r} [g(x_{s-1}, t_{s-1}) + \alpha c] + e^{-rt_s} \pi(q_s, x, t)$$

Take an another stream s.t. $\forall i/s \in \mathbb{N}, x'_i = x_i, t'_i = t_i$ and for period s , $x'_s = 0, t'_s = 0$. The value of this path at time $s - 1$:

$$\pi(q'_{s-1}, x', t') = -(F + c) + \frac{1 - e^{-rt_s'^e}}{r} [g(x'_{s-1}, t'_{s-1}) + \alpha c] + e^{-rt_s'} \pi(q'_s, x', t')$$

Again, we substitute $t_{s-1}^e = t_s$ in above equations. So, $t'_s = 0$ implies

$$\frac{1 - e^{-rt_s'}}{r} [g(x'_{s-1}, t'_{s-1}) + \alpha c] \frac{1 - e^{-rt_s'}}{r} = 0.$$

Note that $q_{s-1} = \sum_{i=1}^{s-1} g(x_i, t_i) = \sum_{i=1}^{s-1} g(x'_i, t'_i) = q'_{s-1}$. Also we know that $g(x_s, t_s) = 0$ when $x_s = 0$. That is $q'_s = q_s$, which implies $\pi(q_s, x, t) = \pi(q'_s, x', t')$. Since $\{x_i, t_i\}$ is optimal, $\pi(q_{s-1}, x, t) \geq \pi(q'_{s-1}, x', t')$. It is sat-

isfied when

$$\frac{1 - e^{-rt_s}}{r} [g(x_{s-1}, t_{s-1}) + \alpha c] + (e^{-rt_s} - 1) \pi(q_s, x, t) \geq 0$$

That is either

$$t_s = 0$$

or

$$\left(\pi(q_s, x, t) - \frac{[g(x_{s-1}, t_{s-1}) + \alpha c]}{r} \right) \leq 0. \quad (6.2)$$

By Lemma 9 above, $\pi(q_s, x, t) > \frac{q_s}{r}$. Moreover, $q_s = q_{s-2} + g(x_{s-1}, t_{s-1}) + g(x_s, t_s)$ where $g(x_s, t_s) = 0$. If we substitute this into equation 6.2, we obtain:

$$\frac{\alpha c}{r} > \frac{q_{s-2}}{r}$$

However, by Assumption 3, $p_c(g, q, \alpha c) \leq q$, in case $\alpha c \leq q_{s-2}$. Hence, $t_s = 0$.

$$\begin{aligned} \pi(q_{s-1}, x, t) &= -(F + c) + e^{-rt_s} \pi(q_s, x, t) \\ &< \pi(q_s, x, t) \end{aligned}$$

Since $g(x_s, t_s) = 0$, $\pi(q_{s-1}, x, t) = \pi(q_s, x, t)$. Thus we have a contradiction.

If $\{x_i, t_i\}$ is an optimal path and $\forall i \in \mathbb{N}$, $t_i < \infty$, then $\forall i \in \mathbb{N}$, $x_i > 0$. $\square$

Given the above we can now show there is a solution to the monopolist's problem when there is perfect competition in the component market.

Proposition 3 *There exists a global maximum to the monopolist's problem.*

Proof The profit function for the selling monopolist is

$$\begin{aligned}\pi(q_0, x, t) &= \frac{q_0}{r} + (-x_1) \frac{1 - e^{-rt_1}}{r} \\ &\quad + \sum_{i=1}^{\infty} e^{-rT_i} (g(x_i, t_i) + \alpha c) \frac{1 - e^{-rt_i^e}}{r} \\ &\quad + \sum_{i=1}^{\infty} e^{-rT_i} \left[-(F + c) - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right]\end{aligned}$$

Note that $\forall i \in \mathbb{N}$, $g(x_i, t_i)$ is bounded. So, for some $j \in \mathbb{N}$, $\pi(q_0, x, t) \rightarrow -\infty$ as $x_j \rightarrow \infty$. That is for some X , $\pi(q_0, x, t) < 0$, $\forall x_i > X$. However, we established in Lemma 9 that $\pi(q_0, x, t) > \frac{q_0}{r}$. Hence, x_i 's must be chosen from $[0, X]$. Also, for some $j \in \mathbb{N}$, as $t_j \rightarrow \infty$, $\pi(q_0, x, t) < \pi(q_0, x, t) + \pi(q_{j-1}, x_p, t_p) = \pi'$ where $\pi(q_{j-1}, x_p, t_p)$ is the profit generated by a constant stream $\{x_p, t_p\}$ after the introduction of the product of quality q_{j-1} . However, we know that $\pi(q_{j-1}, x_p, t_p) > \frac{q_{j-1}}{r}$. This implies for some T , $\pi(q_0, x, t) < \pi'$, $\forall t_i > T$. So, t_i 's must be chosen from $[0, T]$. Hence, the maximum must be within the compact domain $[0, X] \times [0, T]$.

Since g is continuous, π is continuous. π is continuous within the compact domain $[0, X] \times [0, T]$. So, it has a maximum. By Lemma 10, we have an interior solution. $\square$

Let π_C^* be a solution to the sequential problem when the component market is perfectly competitive. We establish again the corresponding functional equation,

$$(FE) \quad \Pi(q) = \max_{x, t} \left\{ -\frac{1}{r}x + e^{-rt} \left(-(F + c) + \frac{x}{r} + p(q, x, t) + \Pi(q') \right) \right\} \quad (6.3)$$

$$\text{where } p(q, x, t) = (g + \alpha c) \frac{1 - e^{-rt^e}}{r}.$$

As we did before, we will show now the sequential problem and the functional equation have the same solutions.

Lemma 11 π_C^* solves (FE) in equation 6.3.

Proof $Q, A, H, \top$ are defined as before. The objective function in equation 6.1 can also be written as follows:

$$\pi(q_0, x, t) = \frac{q_0}{r} + \sum_{i=1}^{\infty} e^{-rT_{i-1}} e^{-rt_i} (g(x_i, t_i) + \alpha c) \frac{(1 - e^{-rt_{i+1}})}{r} \quad (6.4)$$

$$+ \sum_{i=1}^{\infty} e^{-rT_{i-1}} \left[-e^{-rt_i} (F + c) - x_i \frac{1 - e^{-rt_i}}{r} \right] \quad (6.5)$$

Then, we have following form for the sequential problem in equation 6.4:

$$\begin{aligned} \pi(q_0, x, t) &= \frac{q_0}{r} + \sum_{i=1}^{\infty} \beta_i H_i \text{ where} \\ \beta_i &= e^{-rT_{i-1}} \text{ and} \\ H_i &= -x_i \frac{1 - e^{-rt_i}}{r} + e^{-rt_i} \left((g(x_i, t_i) + \alpha c) \frac{(1 - e^{-rt_{i+1}})}{r} - (F + c) \right) \end{aligned}$$

Again, since $\top(q_i) = q_i + g(x_{i+1}, t_{i+1}) = q_{i+1}$ for each $i \in \mathbb{N}$, Q is a convex subset of $\mathbb{R}$, and the correspondence $\top : Q \rightarrow Q$ is nonempty, compact-valued and continuous. For each $i \in \mathbb{N}$, $0 < \beta_i < 1$. Also, g is bounded and $\forall i \in \mathbb{N}$, $(x_i, t_i) \in [0, X] \times [0, T]$. Hence, H_i is bounded for each $i \in \mathbb{N}$. Since g is continuous, H_i is continuous. With conditions above satisfied, solutions to (FE) coincide exactly to solutions of (SP) (Stokey, 1989). $\square$

We can now continue our analysis with functional equation. In order to analyze how R&D incentives are affected when the market structure in the component market changes, we should look at the first order conditions.

Lemma 12 Π is continuously differentiable at q . Moreover, $\Pi_q(q)$ is zero.

Proof The proof of differentiability is similar to the one in Chapter 5. By Assumption 3, H is strictly concave (see the appendix). Since g and p_c are continuously differentiable, so is H . As we stated before, $\top$ is convex. Hence,

Π is continuously differentiable at q (Stokey, 1989).

$$\begin{aligned}\Pi_q(q) &= \Pi_{q_1}(q_1) = e^{-rt_1} (p_{q_1}(q, x, t) + \Pi_{q_2}(q_2)) \\ &= \sum_{i=1}^{\infty} e^{-r\left(\sum_{j=0}^i t_j\right)} p_{q_i}(q, x, t) \text{ where } t_0 = 0.\end{aligned}$$

However, since $p(q, x, t) = (g + \alpha c) \frac{1 - e^{-rt^e}}{r}$, $p_{q_i}(q, x, t) = 0$ for each $i \in \mathbb{N}$. That is $\Pi_q(q) = 0$. $\square$

Above lemma states that when the component market is perfectly competitive, the quality of a product does not affect monopolist's profits. This is different than what we find in the case where the monopolist controls the component market. Since there is now competition in the component market, the monopolist can not charge the flow utility of the current product for the price of the component. Therefore, the price of the new generation is now determined by three factors; g — quality increment between two consecutive products, t^e — consumers' expectations and αc — the marginal cost of the component.

Π satisfies the following first order conditions:

$$\begin{aligned}\Pi_x &= -\frac{(1 - e^{-rt})}{r} + e^{-rt} \frac{\partial p(q, x, t)}{\partial x} \\ &\quad + e^{-rt} \frac{\partial p(q, x, t)}{\partial q} g_x + e^{-rt} \frac{\partial \Pi(q')}{\partial q'} g_x\end{aligned}\tag{6.6}$$

$$\begin{aligned}\Pi_t &= -re^{-rt} \left(-(F + c) + \frac{x}{r} + p(q, x, t) + \Pi(q') \right) \\ &\quad + e^{-rt} \left(\frac{\partial p(q, x, t)}{\partial t} + \frac{\partial p(q, x, t)}{\partial q} g_t + \frac{\partial \Pi(q')}{\partial q'} g_t \right)\end{aligned}\tag{6.7}$$

If we substitute $p_q(q, x, t) = 0$ and $\Pi_q(q) = 0$ into equations 6.6 and 6.7, we obtain

$$-\frac{(1 - e^{-rt})}{r} + e^{-rt} \frac{1 - e^{-rt}}{r} g_x = 0 \quad (6.8)$$

$$r(F + c) - x - rp(q, x, t) - r\Pi(q') + \frac{1 - e^{-rt}}{r} g_t = 0 \quad (6.9)$$

First order conditions for the monopoly are different when there is perfect competition in the component market. There is an extra term $(1 - e^{-rt})$ multiplied by $e^{-rt} \frac{g_x}{r}$ and $\frac{g_t}{r}$. This implies marginal productivities of R&D inputs are less effective on monopoly profits if the environment in the component market becomes competitive. The reason is as follows: R&D inputs affect the prices of the later generations in two ways. The direct effect is that R&D investments determine the level of the quality increment, g , thereby affecting immediately the price of the next generation. The indirect effect is that since knowledge builds up cumulatively, they determine the quality of the next product, which affects the price of the component in later generations. When the monopolist controls the component market, these two effects are active. However, if the component market is perfectly competitive, the indirect effect is eliminated as the price of the component is always set up at its marginal cost.

We are not able to analyze the optimal choices of x and t just solving these first order conditions. However, in the next chapter we will also be able to determine how gestation periods and R&D investments change if the competition arises in the component market.

Our analysis in this chapter coincide with Fishman and Rob's monopoly analysis for slightly different costs. They find the monopolist innovates less frequently and invests less in R&D than the social planner. In the next chapter, we are going to show that the market structure of the component market changes these results even the monopolist can engage in partial physical obsolescence.

CHAPTER 7

THE EQUILIBRIUM WHEN THE COMPONENT MARKET IS IMPERFECTLY COMPETITIVE

We now consider the case where the component market is imperfectly competitive. In Chapter 5, we find that if the monopolist has full market power in the component market, the optimal price for the component good is the quality of the current product in flow terms. In Chapter 6, if the component market is perfectly competitive, the price of the component good is set up at its marginal cost, αc . Accordingly, we assume that if the component market is imperfectly competitive, the price of the component lies between these two values. Since there is now competition in the component good market, the monopolist can not charge q for the component good and also we rule out the case where $p_c \leq \alpha c$, thereby preventing firms making loss in the component market. Therefore, $p_c \in (\alpha c, q)$ and the corresponding sequential problem for the monopolist is

$$(SP) \quad \sup_{\{x_i, t_i\}_{i=1}^{\infty}} \pi(q_0, x, t) \text{ s.t.}$$

$$\begin{aligned}
\pi(q_0, x, t) &= \frac{q_0}{r} + (-x_1) \frac{1 - e^{-rt_1}}{r} \\
&+ \sum_{i=1}^{\infty} e^{-rt_i} [g(x_i, t_i) + p_c(g, q_i, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\
&+ \sum_{i=1}^{\infty} e^{-rt_i} \left[-(F + c) - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right] \quad (7.1)
\end{aligned}$$

Note that when for each $i \in \mathbb{N}$, $p_c(g, q_i, \alpha c) = q_i$, we have the same problem when the monopolist controls the component market. When for each $i \in \mathbb{N}$, $p_c(g, q_i, \alpha c) = \alpha c$, we ended up with the monopolist's problem in Chapter 6, where the monopolist faces perfect competition in the component market.

We are going to follow the same steps as we did in previous chapters.

Lemma 13 *For any given path $\{x_i, t_i\}_{i=1}^{\infty}$, $\pi(q_0, x, t) < \infty$. Furthermore, if $\{x_i, t_i\}_{i=1}^{\infty}$ is an optimal path, then $\frac{q_0}{r} < \pi(q_0, x, t)$ and $\forall i \in \mathbb{N}$, $t_i < \infty$.*

Proof *Since for each $i \in \mathbb{N}$, $p_c(g, q_i, \alpha c) < q_i$, for any path $\{x_i, t_i\}_{i=1}^{\infty}$*

$$\pi(q_0, x, t) < W(q_0, x, t).$$

We proved in Lemma 1 that for any given path $\{x_i, t_i\}_{i=1}^{\infty}$, $W(q_0, x, t) < \infty$. Hence, $\pi(q_0, x, t) < \infty$.

Now, suppose that $\{x_i^, t_i^*\}$ solves (SP). Let π_p^{IM} be the value of the constant stream in imperfect component market, which is $\forall i \in \mathbb{N}$, $\{x_i, t_i\} = \{x_p, t_p\}$. Hence,*

$$\begin{aligned}
\pi_p^{IM} &= \frac{q_0 - x_p}{r} + \frac{g(x_p, t_p) (1 - e^{-rt_p^e})}{r(e^{rt_p} - 1)} \\
&+ \sum_{i=1}^{\infty} \frac{e^{-rit_p} (1 - e^{-rt_p^e}) p_c(q_i, \alpha c)}{r} - \frac{(F + c)}{e^{rt_p} - 1}
\end{aligned}$$

Now, for each $i \in \mathbb{N}$, $\alpha c \leq p_c(g, q_i, \alpha c)$. Thus, $\pi_p^C \leq \pi_p^{IM}$, where π_p^C be

the value of the constant stream in perfectly competitive component market. By Lemma 9, $\pi_p^{IM} > \frac{q_0}{r}$. Since $\{x_i^*, t_i^*\}$ solves (SP), the value generated by the stream $\{x_i^*, t_i^*\}$ must be at least equal to any other stream. That is $\pi^*(q_0, x^*, t^*) \geq \pi_p^{IM} > \frac{q_0}{r}$.

In order to prove the final step, suppose that there exists an optimal path $\{x_i, t_i\}$ such that for some period k , $t_k = \infty$. The value of the stream after the introduction of the product of quality q_{k-1} ,

$$\begin{aligned} \pi(q_{k-1}, x, t) &= \frac{q_{k-1}}{r} + (-x_k) \frac{1 - e^{-rt_k}}{r} \\ &\quad + \sum_{i=k}^{\infty} e^{-rT_i} [g(x_i, t_i) + p_c(q_i, \alpha c)] \frac{1 - e^{-rt_i^e}}{r} \\ &\quad + \sum_{i=k}^{\infty} e^{-rT_i} \left[-(F + c) - x_{i+1} \frac{1 - e^{-rt_{i+1}}}{r} \right] \end{aligned}$$

As before, second line in the above expression is zero. Hence,

$$\begin{aligned} \pi(q_{k-1}, x, t) &= \frac{q_{k-1}}{r} + (-x_k) \frac{1}{r} \\ &\leq \frac{q_{k-1}}{r}. \end{aligned}$$

However, we know that $\pi(q_{k-1}, x, t) > \frac{q_{k-1}}{r}$. Thus, $\forall i \in \mathbb{N}$, $t_i < \infty$. $\square$

Until now we have established that independent of the market structure of the component market, R&D investments are positive. We will show that this result holds even the component market is imperfectly competitive.

Lemma 14 *If there exists a solution to (SP), then $\forall i \in \mathbb{N}$, $t_i < \infty \implies x_i > 0$ for the optimal path.*

Proof Again, suppose that there exists an optimal path $\{x_i, t_i\}$ s.t. $\forall i \in \mathbb{N}$, $t_i < \infty$, but $\exists s \in \mathbb{N}$ s.t. $x_s = 0$. The value generated by this stream after the

introduction of the product at generation $s - 1$:

$$\begin{aligned}\pi(q_{s-1}, x, t) = & -(F + c) + \frac{1 - e^{-rt_{s-1}^e}}{r} [g(x_{s-1}, t_{s-1}) + p_c(g, q, \alpha c)] \\ & + e^{-rt_s} \pi(q_s, x, t)\end{aligned}$$

Take an another stream s.t. $\forall i/s \in \mathbb{N}$, $x'_i = x_i$, $t'_i = t_i$ and for period s , $x'_s = 0$, $t'_s = 0$. The value of this path at time $s - 1$:

$$\begin{aligned}\pi(q'_{s-1}, x', t') = & -(F + c) + \frac{1 - e^{-rt_{s-1}^{e'}}}{r} [g(x'_{s-1}, t'_{s-1}) + p_c(g, q', \alpha c)] \\ & + e^{-rt'_s} \pi(q'_s, x', t')\end{aligned}$$

Substituting $t_{s-1}^e = t_s$ and $t'_s = 0$, we obtain

$$\frac{1 - e^{-rt'_s}}{r} [g(x'_{s-1}, t'_{s-1}) + p_c(g, q', \alpha c)] = 0.$$

Again, $q'_s = q_s$ implies $\pi(q_s, x, t) = \pi(q'_s, x', t')$. Since $\{x_i, t_i\}$ is optimal, $\pi(q_{s-1}, x, t) \geq \pi(q'_{s-1}, x', t')$. It is satisfied when

$$\frac{1 - e^{-rt_s}}{r} [g(x_{s-1}, t_{s-1}) + p_c(g, q, \alpha c)] + (e^{-rt_s} - 1) \pi(q_s, x, t) \geq 0$$

That is either

$$t_s = 0$$

or

$$\pi(q_s, x, t) \leq \frac{g(x_{s-1}, t_{s-1}) + p_c(g, q, \alpha c)}{r}. \quad (7.2)$$

We know that at time $s - 1$, $p_c(g, q, \alpha c) < q_{s-2}$. Equation 7.2 is satisfied when

$$\pi(q_s, x, t) < \frac{q_s}{r}$$

However, by Lemma 13 above, $\pi(q_s, x, t) > \frac{q_s}{r}$. Thus, $t_s = 0$ and

$$\begin{aligned}\pi(q_{s-1}, x, t) &= -(F + c) + e^{-rt_s} \pi(q_s, x, t) \\ &< \pi(q_s, x, t)\end{aligned}$$

Since $g(x_s, t_s) = 0$, $\pi(q_{s-1}, x, t) = \pi(q_s, x, t)$. Thus, we have a contradiction.

□

Proposition 4 *There exists a global maximum to the monopolist's problem.*

Proof See the proof of Proposition 2. □

Let π_{IM}^* be a solution to (SP). We now establish the functional equation for the monopolist's problem as we did in previous chapters,

$$(FE) \quad \Pi(q) = \max_{x,t} \left\{ -\frac{1}{r}x + e^{-rt} \left(-(F + c) + \frac{x}{r} + p(q, x, t) + \Pi(q') \right) \right\} \quad (7.3)$$

$$\text{where } p(q, x, t) = (g + p_c(g, q, \alpha c)) \frac{1 - e^{-rt^e}}{r}.$$

Lemma 15 π_{IM}^* solves (FE) in equation 7.3.

Proof See the corresponding lemma when the monopolist controls the component market. □

In order to prove our main proposition, we need the following lemma.

Lemma 16 Π is continuously differentiable at q .

Proof Similar to the corresponding lemma in Chapter 5. Note that

$$\begin{aligned}\Pi_q(q) &= \Pi_{q_1}(q_1) = e^{-rt_1} (p_{q_1}(q, x, t) + \Pi_{q_2}(q_2)) \\ &= \sum_{i=1}^{\infty} e^{-r \left(\sum_{j=0}^i t_j \right)} p_{q_i}(q, x, t) \text{ where } t_0 = 0.\end{aligned}$$

Since $p(q, x, t) = (g + p_c(g, q, \alpha c)) \frac{1 - e^{-rt^e}}{r}$, $p_{q_i}(q, x, t) = p_{cq}(g, q, \alpha c) \frac{1 - e^{-rt_i^e}}{r}$ for each $i \in \mathbb{N}$. Hence,

$$\Pi_q(q) = \sum_{i=1}^{\infty} e^{-r \left(\sum_{j=0}^i t_j \right)} p_{cq}(g, q, \alpha c) \frac{1 - e^{-rt_i^e}}{r} \quad (7.4)$$

□

Π satisfies the following first order conditions:

$$\begin{aligned} \Pi_x &= -\frac{(1 - e^{-rt})}{r} + e^{-rt} \frac{\partial p(q, x, t)}{\partial x} \\ &\quad + e^{-rt} \frac{\partial p(q, x, t)}{\partial q} g_x + e^{-rt} \frac{\partial \Pi(q')}{\partial q'} g_x \end{aligned} \quad (7.5)$$

$$\begin{aligned} \Pi_t &= -re^{-rt} \left(-(F + c) + \frac{x}{r} + p(q, x, t) + \Pi(q') \right) \\ &\quad + e^{-rt} \left(\frac{\partial p(q, x, t)}{\partial t} + \frac{\partial p(q, x, t)}{\partial q} g_t + \frac{\partial \Pi(q')}{\partial q'} g_t \right) \end{aligned} \quad (7.6)$$

Now, we are going to analyze the direction of changes in R&D decisions when the price of the component good increases. In this way, we will be able to figure out how the market structure of the component market affects the innovation activity.

Proposition 5 *If the price of the component increases, the selling monopolist innovates more frequently and invests more in R&D.*

Proof We know from first order conditions in equations 7.5 and 7.6,

$$\Pi_t(q) = 0$$

$$\Pi_x(q) = 0$$

If we take derivatives with respect to p ,

$$\begin{aligned}\Pi_{tt}(q) \frac{\partial t}{\partial p} + \Pi_{tx}(q) \frac{\partial x}{\partial p} + \Pi_{tp} &= 0 \\ \Pi_{xt}(q) \frac{\partial t}{\partial p} + \Pi_{xx}(q) \frac{\partial x}{\partial p} + \Pi_{xp} &= 0\end{aligned}$$

In matrix form, we have

$$\begin{pmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{pmatrix} \begin{pmatrix} \frac{\partial t^*}{\partial p} \\ \frac{\partial x^*}{\partial p} \end{pmatrix} = - \begin{pmatrix} \Pi_{tp} \\ \Pi_{xp} \end{pmatrix}$$

If we take derivative of (FE) in equation 7.3 with respect to p , $\Pi_p = e^{-rt}$.

Hence,

$$\begin{aligned}\Pi_{tp} &= -re^{-rt} \\ \Pi_{xp} &= 0\end{aligned}$$

Now we apply Cramer's Rule.

$$\begin{aligned}\frac{\partial x^*}{\partial p} &= \frac{\det \begin{vmatrix} \Pi_{tt} & re^{-rt} \\ \Pi_{xt} & 0 \end{vmatrix}}{\det \begin{vmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{vmatrix}} = \frac{-re^{-rt}\Pi_{xt}}{\det \begin{vmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{vmatrix}} \\ \frac{\partial t^*}{\partial p} &= \frac{\det \begin{vmatrix} re^{-rt} & \Pi_{tx} \\ 0 & \Pi_{xx} \end{vmatrix}}{\det \begin{vmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{vmatrix}} = \frac{re^{-rt}\Pi_{xx}}{\det \begin{vmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{vmatrix}}\end{aligned}$$

We know that the denominators are positive by concavity. In order to establish how the price of the component affects R&D decisions, we need to

figure out the signs of Π_{xt} and Π_{xx} . Since

$$\begin{aligned}
\Pi_x &= -\frac{(1-e^{-rt})}{r} + e^{-rt} \frac{\partial p(q, x, t)}{\partial x} + e^{-rt} \frac{\partial p(q, x, t)}{\partial q} g_x + e^{-rt} \frac{\partial \Pi(q')}{\partial q'} g_x \\
&= -\frac{(1-e^{-rt})}{r} + e^{-rt} \frac{1-e^{-rt^e}}{r} g_x \\
&\quad + e^{-rt} \frac{1-e^{-rt^e}}{r} \frac{\partial p_c(g, q, \alpha c)}{\partial q} g_x + e^{-rt} \frac{\partial \Pi(q')}{\partial q'} g_x \\
\Pi_{xx} &= e^{-rt} \left[\frac{1-e^{-rt^e}}{r} g_{xx} + \frac{1-e^{-rt^e}}{r} p_{cq}(g, q, \alpha c) g_{xx} + \frac{1-e^{-rt^e}}{r} p_{cqq}(g, q, \alpha c) (g_x)^2 \right. \\
&\quad \left. + \Pi_{q'}(q') g_{xx} + \Pi_{q'q'}(q') (g_x)^2 \right]
\end{aligned}$$

Under Assumptions 3 and 3, we have $p_{cq}(g, q, \alpha c) > 0$, $p_{cqq}(g, q, \alpha c) \leq 0$ and $g_{xx} \leq 0$. Moreover, equation 7.4 implies $\Pi_{q'q'}(q') \leq 0$ and $\Pi_{q'}(q') > 0$. Thus, $\Pi_{xx} < 0$.

$$\begin{aligned}
\Pi_{xt} &= -e^{-rt} r \left(\frac{1}{r} + \frac{1-e^{-rt^e}}{r} g_x + \frac{1-e^{-rt^e}}{r} p_{cq}(g, q, \alpha c) g_x + \Pi_{q'}(q') g_x \right) \\
&\quad + e^{-rt} \left(\frac{1-e^{-rt^e}}{r} g_{xt} + \frac{1-e^{-rt^e}}{r} p_{cq}(g, q, \alpha c) g_{xt} \right. \\
&\quad \left. + \frac{1-e^{-rt^e}}{r} p_{cqq}(g, q, \alpha c) g_x g_t \right) \\
&\quad + e^{-rt} (\Pi_{q'}(q') g_{xt} + \Pi_{q'q'}(q') g_x g_t)
\end{aligned}$$

If we rearrange the terms, we have

$$\Pi_{xt} = e^{-rt} \left(-1 + \frac{1-e^{-rt^e}}{r} (g_{xt} - r g_x) + \frac{1-e^{-rt^e}}{r} p_{cq}(g, q, \alpha c) (g_{xt} - r g_x) \right. \\
\left. + \Pi_{q'}(q') (g_{xt} - r g_x) \right. \\
\left. + \frac{1-e^{-rt^e}}{r} p_{cqq}(g, q, \alpha c) g_x g_t + \Pi_{q'q'}(q') g_x g_t \right)$$

If x and t are substitutes- $g_{xt} < 0$ for all (x, t) –or if x and t are complements but not strong complements- $0 < g_{xt} < r g_x$, the term in parenthesis would be negative. As Fishman and Rob (2000) state some conventional

productions functions, such as Cobb-Douglas or the constant elasticity of substitution satisfy this condition. Hence, we assume this restriction on g holds.

Thus, $\Pi_{xt} < 0$.

Therefore,

$$\frac{\partial x^*}{\partial p} \frac{1 - e^{-rt^e}}{r} = \frac{\partial x^*}{\partial p_c} > 0 \quad (7.7)$$

and

$$\frac{\partial t^*}{\partial p} \frac{1 - e^{-rt^e}}{r} = \frac{\partial t^*}{\partial p_c} < 0 \quad (7.8)$$

□

From Proposition 5, one can easily show the following results.

Corollary 1 *The monopolist innovates less frequently and invests less in R&D than the efficient level unless he controls the component market.*

Corollary 2 *Competition in the component good market causes efficiency loss in terms of R&D investments.*

The results state that if the price of the component is higher, the monopolist innovates more frequently and invests more in R&D. The reason is that consumers are willingly to pay for the incremental quality improvement when a new generation is introduced. This implies the monopolist receives less than the social value of an innovation. However, if the monopolist is able to determine the durability of the component, he receives the social value of an innovation by increasing the price of the component, which consumers have to replace. It can charge as high as possible for the price of the component when he controls the component market. On the other hand, when the component market is imperfectly competitive, he is not able to compensate his loss by changing the price of the component, thus he waits longer between two product introductions. Thereby, consumers pay for the present model in a longer period, which leads innovations to be delayed.

Different than Fishman and Rob (2000), our results depend on the market structure of the component market. We find that the monopolist innovates less frequently and invests less in R&D than the efficient level unless he controls the component market. Incorporating the component durability enables us to conclude that competition in the component market causes departures from socially optimal level of R&D investments. Therefore, a monopolist in a primary market should also have market power in its component market.

We examine now how current R&D decisions change when the price of the component good increases in the future.

Proposition 6 *If the price of the component increases in the future, the monopolist innovates less frequently and invests less in R&D today.*

Proof Assume that at time $s - 1$, the monopolist has to decide its R&D inputs, x_s and t_s . Let the price of the component be p_k after k generations.

$$\Pi_{t_s}(q) = 0$$

$$\Pi_{x_s}(q) = 0$$

If we take derivatives with respect to p_k ,

$$\begin{aligned} \Pi_{tt}(q) \frac{\partial t_s}{\partial p_k} + \Pi_{tx}(q) \frac{\partial x_s}{\partial p_k} + \Pi_{tp_k} &= 0 \\ \Pi_{xt}(q) \frac{\partial t_s}{\partial p_k} + \Pi_{xx}(q) \frac{\partial x_s}{\partial p_k} + \Pi_{xp_k} &= 0 \end{aligned}$$

In matrix form, we have

$$\begin{pmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{pmatrix} \begin{pmatrix} \frac{\partial t_s}{\partial p_k} \\ \frac{\partial x_s}{\partial p_k} \end{pmatrix} = - \begin{pmatrix} \Pi_{tp_k} \\ \Pi_{xp_k} \end{pmatrix}$$

Now, if we take derivative of Π with respect to p_k at time $s - 1$, we obtain

$$\Pi_{p_k} = \Pi_{i=s}^k e^{-rt_i}$$

Hence,

$$\begin{array}{rcl} \Pi_{tp_k} & = & -r\Pi_{i=s}^k e^{-rt_i} \\ \Pi_{xp_k} & = & 0 \end{array}$$

Again we apply Cramer's Rule.

$$\begin{aligned} \frac{\partial x_s}{\partial p_k} &= \frac{\det \begin{vmatrix} \Pi_{tt} & -r\Pi_{i=s}^k e^{-rt_i} \\ \Pi_{xt} & 0 \end{vmatrix}}{\det \begin{vmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{vmatrix}} = \frac{r\Pi_{i=s}^k e^{-rt_i} \Pi_{xt}}{\det \begin{vmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{vmatrix}} \\ \frac{\partial t_s}{\partial p_k} &= \frac{\det \begin{vmatrix} -r\Pi_{i=s}^k e^{-rt_i} & \Pi_{tx} \\ 0 & \Pi_{xx} \end{vmatrix}}{\det \begin{vmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{vmatrix}} = \frac{-r\Pi_{i=s}^k e^{-rt_i} \Pi_{xx}}{\det \begin{vmatrix} \Pi_{tt} & \Pi_{tx} \\ \Pi_{xt} & \Pi_{xx} \end{vmatrix}} \end{aligned}$$

We have $\Pi_{xt} < 0$ and $\Pi_{xx} < 0$. Therefore,

$$\frac{\partial x_s}{\partial p_k} \frac{1 - e^{-rt^e}}{r} = \frac{\partial x_s}{\partial p_k} < 0 \quad (7.9)$$

and

$$\frac{\partial t_s}{\partial p_k} \frac{1 - e^{-rt^e}}{r} = \frac{\partial t_s}{\partial p_k} > 0$$

□

CHAPTER 8

CONCLUSION

Early work in durable goods theory has always suggested that planned obsolescence is an incentive to reduce the durability of the whole of the product. However, usually it is a component that becomes physically obsolete rather than the whole unit.

In this paper, we analyze the R&D investments and the frequency of product innovations of a durable good monopoly under the assumption that the monopolist determines the durability of the component. We study the effects of the market structure in the component market on R&D decisions of a durable good monopolist. We extend Fishman and Rob's analysis by incorporating partial physical obsolescence to the model. The main assumptions of the model are; innovations are recurrent, the knowledge builds up cumulatively, consumers are homogenous and the monopolist sells rather than rent its products. We showed that under these circumstances if the monopolist has market power in the component market, he innovates at the socially optimal pace. However, if the component market is perfectly competitive, the monopolist innovates less frequently and invests less than the efficient level. Our results in the case of perfectly competitive component market is similar to Fishman and Rob's.

We should note that the price structure different than the one in our

analysis may change the results. However, incorporating more complex price structure to the model will complicate the analysis. In addition, allowing the monopolist to rent its products or relaxing the consumer homogeneity assumption may strengthen or weaken our results. The analysis of these issues is going to be the topic of future research.

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APPENDIX

Proof [Proof of Lemma 5] In order to prove V is differentiable, we need to prove first per period return function H is strictly concave. So, we are going to show $D^2H(x, t)$ is a negative definite matrix.

$$D^2H(x, t) = \begin{pmatrix} H_{xx} & H_{xt} \\ H_{tx} & H_{tt} \end{pmatrix}$$

$D^2H(x, t)$ is negative definite if and only if $H_{xx} < 0$ and $H_{xx}H_{tt} - H_{xt}H_{tx} > 0$. Now, we will derive second order derivatives.

$$\begin{aligned} H &= -x \frac{(1 - e^{-rt})}{r} + e^{-rt} \left(\frac{g(x, t)}{r} - (F + c) \right) \\ H_x &= -\frac{(1 - e^{-rt})}{r} + e^{-rt} \frac{g_x}{r} = 0 \\ H_{xx} &= e^{-rt} \frac{g_{xx}}{r} \end{aligned}$$

We assume that g is strictly concave. Hence, $H_{xx} < 0$.

$$\begin{aligned} H_t &= -xe^{-rt} - re^{-rt} \left(\frac{g(x, t)}{r} - (F + c) \right) + e^{-rt} \frac{g_t}{r} = 0 \\ H_{tt} &= rxe^{-rt} + r^2e^{-rt} \left(\frac{g(x, t)}{r} - (F + c) \right) - re^{-rt} \frac{g_t}{r} \\ &\quad - re^{-rt} \frac{g_t}{r} + e^{-rt} \frac{g_{tt}}{r} \\ H_{tx} &= -e^{-rt} - e^{-rt} g_x + e^{-rt} \frac{g_{tx}}{r} \end{aligned}$$

Similarly,

$$H_{xt} = -e^{-rt} - e^{-rt} g_x + e^{-rt} \frac{g_{xt}}{r}$$

We assume that $g_{xt} = g_{tx}$. This implies $H_{xt} = H_{tx}$. We want to have differentiability around the optimal solution so that we substitute first order conditions into the second order conditions. This yields:

$$\begin{aligned} H_{xx}H_{tt} - H_{xt}^2 &= e^{-rt} \frac{g_{xx}}{r} \left(-re^{-rt} \frac{g_t}{r} + e^{-rt} \frac{g_{tt}}{r} \right) - \left(-1 + e^{-rt} \frac{g_{xt}}{r} \right)^2 \\ &= -e^{-2rt} \frac{g_{xx}}{r} g_t + e^{-2rt} \frac{g_{xx}}{r} \frac{g_{tt}}{r} - \left(-1 + e^{-rt} \frac{g_{xt}}{r} \right)^2 \end{aligned}$$

By Assumption 3, we have $g_{xx}g_{tt} \geq (g_{xt} - re^{rt})^2$. If we multiply both side with $\frac{e^{-2rt}}{r^2}$, we obtain

$$e^{-2rt} \frac{g_{xx}}{r} \frac{g_{tt}}{r} \geq \left(e^{-rt} \frac{g_{xt}}{r} - 1 \right)^2$$

Since g is increasing in t , $-e^{-2rt} \frac{g_{xx}}{r} g_t > 0$. Thus, $H_{xx}H_{tt} - H_{xt}^2 > 0$. So, H is strictly concave and V is differentiable. $\square$

Proof [Proof of Lemma 10] Again, we need to prove per period return function H is strictly concave.

$$D^2H(x, t) = \begin{pmatrix} H_{xx} & H_{xt} \\ H_{tx} & H_{tt} \end{pmatrix}$$

$D^2H(x, t)$ is negative definite if and only if $H_{xx} < 0$ and $H_{xx}H_{tt} - H_{xt}H_{tx} > 0$. We derive second order conditions.

$$\begin{aligned} H &= -x \frac{(1 - e^{-rt})}{r} + e^{-rt} \left((1 - e^{-rt^e}) \frac{g(x, t) + q}{r} - (F + c) \right) \\ H_x &= -\frac{(1 - e^{-rt})}{r} + e^{-rt} (1 - e^{-rt^e}) \frac{g_x}{r} = 0 \\ H_{xx} &= e^{-rt} (1 - e^{-rt^e}) \frac{g_{xx}}{r} \end{aligned}$$

Since g is strictly concave, $H_{xx} < 0$.

$$\begin{aligned}
H_t &= 0 = -xe^{-rt} - re^{-rt} \left((1 - e^{-rt^e}) \frac{g(x, t) + q}{r} - (F + c) \right) \\
&\quad + e^{-rt} (1 - e^{-rt^e}) \frac{g_t}{r} \\
H_{tt} &= -rH_t + -re^{-rt} (1 - e^{-rt^e}) \frac{g_t}{r} + e^{-rt} (1 - e^{-rt^e}) \frac{g_{tt}}{r} \\
H_{tx} &= -e^{-rt} - e^{-rt} (1 - e^{-rt^e}) g_x + e^{-rt} (1 - e^{-rt^e}) \frac{g_{tx}}{r}
\end{aligned}$$

Similarly,

$$H_{xt} = -e^{-rt} - e^{-rt} (1 - e^{-rt^e}) g_x + e^{-rt} (1 - e^{-rt^e}) \frac{g_{xt}}{r}$$

We assume that $g_{xt} = g_{tx}$. This implies $H_{xt} = H_{tx}$. We again substitute first order conditions into the second order conditions. This yields:

$$\begin{aligned}
H_{xx}H_{tt} - H_{xt}^2 &= e^{-2rt} (1 - e^{-rt^e})^2 \frac{g_{xx}}{r} \left(-g_t + \frac{g_{tt}}{r} \right) \\
&\quad - \left(e^{-rt} (1 - e^{-rt^e}) \frac{g_{xt}}{r} - 1 \right)^2 \\
&= -e^{-2rt} (1 - e^{-rt^e})^2 \frac{g_{xx}}{r} g_t + e^{-2rt} (1 - e^{-rt^e})^2 \frac{g_{xx}}{r} \frac{g_{tt}}{r} \\
&\quad - \left(e^{-rt} (1 - e^{-rt^e}) \frac{g_{xt}}{r} - 1 \right)^2
\end{aligned}$$

By Assumption 3, we have $(1 - e^{-rt^e})^2 g_{xx}g_{tt} \geq ((1 - e^{-rt^e}) g_{xt} - re^{rt})^2$. If we multiply both side with $\frac{e^{-2rt}}{r^2}$, we get

$$e^{-2rt} (1 - e^{-rt^e})^2 \frac{g_{xx}}{r} \frac{g_{tt}}{r} \geq \left(e^{-rt} (1 - e^{-rt^e}) \frac{g_{xt}}{r} - 1 \right)^2$$

Thus, $H_{xx}H_{tt} - H_{xt}^2 > 0$. H is strictly concave and Π is differentiable. $\square$

Proof [Proof of Lemma 17] The proof is similar to the one above. The only difference is in the formation of per period function H , q is replaced with αc .

$\square$