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LAPLACE RESIDUAL POWER SERIES TECHNIQUES FOR SOLVING FRACTIONAL PARTIAL NONLINEAR SYSTEMS WITH INDEPENDENT N DIMENSIONS

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OMAR MUAYAD JAMEEL ALTEKREETI

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LAPLACE RESIDUAL POWER SERIES TECHNIQUES FOR SOLVING FRACTIONAL PARTIAL NONLINEAR SYSTEMS WITH INDEPENDENT N DIMENSIONS

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June 2023

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science

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ABSTRACT

LAPLACE RESIDUAL POWER SERIES TECHNIQUES FOR SOLVING FRACTIONAL PARTIAL NONLINEAR SYSTEMS WITH INDEPENDENT N DIMENSIONS

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Master of Science in Mathematics

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In this study, the Laplace residual power series method is used to solve various systems of non-linear fractional partial problems to produce analysis and approximate solutions in quick convergent series representations. This method consumes less time and energy compared to the residual power series method through an idea of a limit. Several challenges are examined and replicated to see whether the suggested approach is straightforward, effective, or viable. A study of a collected findings demonstrates that aforementioned method is simple, precise, and appropriate to discover the answers to non-linear technical issues.

2023, 51 pages

Keywords: Nonlinear partial differential equations, Transform of Laplace, Laplace residual power series method, Residual power series.

ÖZET

BAĞIMSIZ N BOYUTLU KESİRLİ KISMİ LİNEER OLMAYAN SİSTEMLERİN ÇÖZÜMÜ İÇİN LAPLACE REZİDÜ KUVVET SERİSİ TEKNİKLERİ

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Bu çalışmada, Laplace rezidü kuvvet serisi yöntemi kullanılarak çeşitli lineer olmayan kesirli kısmi problemler çözülmekte ve analizler ile yaklaşık çözümler hızlı yakınsak seri temsilleri şeklinde üretilmektedir. Bu yöntem, bir limit fikri aracılığıyla, rezidü kuvvet serisi yöntemiyle karşılaştırıldığında daha az zaman ve enerji tüketimi sağlamaktadır. Önerilen yaklaşımın basit, etkili veya uygulanabilir olup olmadığını görmek için çeşitli zorluklar incelenir ve tekrarlanır. Toplanan bulguların bir çalışması, yukarıda bahsedilen yöntemin basit, kesin ve lineer olmayan teknik sorunların cevaplarını bulmak için uygun olduğunu göstermektedir.

2023, 51 sayfa

Anahtar Kelimeler: Kısmi lineer olmayan diferensiyel denklemler, Laplace dönüşümü, Laplace rezidü kuvvet serisi, Rezidü kuvvet serisi

PREFACE AND ACKNOWLEDGEMENTS

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CONTENTS

LIST OF SYMBOLS

$\mathcal{D}_t^{\alpha}\Omega(x,t)$	Caputo fractional derivative
$\zeta(x)$	Exponential Order
Γ(z)	Gamma function
$\Omega(\mathrm{x},t)$	Inverse Laplace transform
$LRes^{i}(x,s)$	Laplace residual power series
$\mathrm{H}(x,s)$	Laplace transform
$\mathcal{D}_t^{\alpha}\Omega(x,t)$	The Riemann-Liouville fractional derivative
$J_t^{\beta}\Omega(x,t)$	The Riemann-Liouville fractional integral
$LRes_k^{i}(x,s)$	The kth-Laplace residual power series

LIST OF ABBREVIATIONS

ADA	Adomian approach
DEs	Differential equations
FIVPs	Fractional initial value problems
FDEs	Fractional differential equations
FPDEs	Fractional partial differential equations
FPDTT	Fractional partial differential transform technique
FPS	Fractional power series
LRFs	Laplace residual function
LRPS	Laplace residual power series
ODEs	Ordinary differential equations
PDEs	Partial differential equation
RPS	Residual power series

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1. INTRODUCTION

Fractional derivative can be used to accurately describe a range of real-world occurrences in the fields of economics, biochemistry, systems engineering and other sciences (Kilbas *et al.* 2006, Bira *et al*. 2018, Al-Smadi and Arqub 2019, Abu Arqub 2019, Akdemir *et al*. 2020, Alabedalhadi *et al*. 2020). The main reason for this is that fractional calculus can be used to accurately simulate real-world situations when they depend on either the present time or a previous temporal date (Baleanu *et al*. 2009, Kumar 2013, Al-Smadi and Gumah 2014, Al-Smadi *et al.* 2015). As a result, many academics in the fields of science and technology have concentrated on FDEs in developing solutions for nonlinear problems and discussing nonlinear dynamics.

Furthermore, calculations and estimates were used to study FDE procedures (Momani *et al.* 2014, Al-Smadi *et al.* 2017, Al-Smadi 2018, Jleli *et al.* 2020). FIVPs, an expansion of traditional initial value calculations that can reflect several real-world properties more accurately than traditional DEs, have attracted the attention of numerous scholars. The idea underlying the availability and dependability of responses from FIVP systems has drawn a lot of interest (Talafha *et al.* 2020, Abdeljawad *et al.* 2020, Hasan *et al.* 2021, Al-Smadi *et al.* 2021 a, Chambré 1952, Boon *et al.* 1988, Atluri and Shen, 2002, Chawla 2002, Kiymaz and Mirasyedioglu 2005, Ramos 2005).

Assessment of nonlinear systems of single initial value problems (IVIs) with elaboration of their describe the different applications in cell genetics, future technologies, particle physics, and astrophysics, like the thermal behavior's of a spherical cloud of gas, isenthalpic gas groups, suppositions of thermoacoustic flows, electronics, and spectacular normalisation structure. This configuration gives us a strong incentive to search for methods to manage these organizations whilst considering the difficulties presented by uniqueness into account. Nevertheless, there is a dearth of study on the specific IVP system, particularly in relation to talks of remedies. Since these issues are frequently almost impossible to evaluate theoretically, numerous authors have become interested in attempting to address them using various assumptions and computations. This type of systems thus comes to the fore in research on numerical approaches (Rahman *et al.* 2021).

Since 16th century, fractional calculus (FC) was a field of study. It is a generalization of traditional calculus with regular differentiation and integration of the any rank as well. FC is presently a superb instrument for expressing the hereditary and storing elements of these occurrences. Lately, several academics used the FC to create a wide variety of nonlinear issues that exist in applied science and maths. Additionally, NPDEs are frequently generated using computational models used in real-world problems, including non-linear earthquake waves, optical, compressibility, gas velocity, kinetics, water transportation, and theory (Miller and Ross 1993, Baleanu *et al.* 2011, Oldham and Spanier 1974, El-Ajou *et al.* 2015, Al-Smadi 2018).

Since fractional order has a good memory over integer rank, the elastic behavior of polymer composites can be accurately described. Gement (Gement 1938) initially developed a modeling approach for viscoelastic materials utilizing fractional calculus. Using lower process parameters, this viscoelastic model can be utilized to accurately behavior represent the mechanical properties utilizing fundamental rheological formulations with fractional derivatives, a considerable amount of work was done to examine synthetic polymers' elastic characteristics examine the elastic characteristics of synthetic polymers (Bagley and Torvik 1983, Bagley and Torvik 1986). For instance, Leung et al. (Leung *et al.* 2013) investigated the fractional derivative-controlled non-linear consistent oscillations of elastic polymer structures. Throughout research has been done on many fractional forms to define their elastic characteristics. In order to explain the nonlinear oscillation of an elastic structural member made up of the elasticity, Lewandowski (Lewandowski and Wielentejczyk 2017) was using a fractional Zener approach. Investigations were done on how harmonics force affected nonlinear motion. Baum et al. description of the mechanical characteristics of a multi-layer hybrid beams used a multiple equation with fractional calculus (Lewandowski and Baum 2015). The characteristics of the structural member were ascertained using this new technique. In order to investigate the instantaneous kinematics of a beam structure with varied dampening levels at various loads, Cortes and Elejabarrieta (Cortés and Elejabarrieta

2007) employed a fractional derivative approach. To determine the huge displacement of the elastic shafts, Bahraini *et al.* (Bahraini *et al.* 2013) constructed the fractional 4 derivatives concept, and the associated nonlinear calculation was carried out using approach of finite element.

Referring to more studies on FDEs that appear in several scientific fields, see (Rashid *et al.* 2019, Atangana and Hammouch 2019, Rashid *et al.* 2020 a,b and c, Al-Qurashi *et al.* 2021, Li *et al.* 2021, Zhou *et al.* 2021). In addition, due to the mathematical complexity of the fractional derivatives involved, it might be challenging to determine the exact solutions for systems of FIVPs. In order to examine the solutions of various types of nonlinear systems of FIVPs, analytical and numerical methodologies were devised. In order to examine the solutions of various types of nonlinear structures of FIVPs, analytical and numerical methodologies were devised. To name just some, the Adomain decomposed method, the homotopy analysis convert method, the Wavelet coefficients interpolation method, the variational iteration method, the spectral collocation strategy, the differential converts method, the Sumudu transform method, the homotopy analysis method, the homotopy perturbation paired to sumudu transform approach, and the homotopy perturbation method (Momani *et al.* 2016, Al-Smadi *et al.* 2016, Atangana and Baleanu 2016, Ait Touchent *et al.* 2018, Gambo *et al.* 2018, Hasan *et al.* 2020, Kumar *et al.* 2021, Khader *et al.* 2021).

The majority of FPDEs lack appropriate mathematical results, hence scientists must find a suitable technique for these kinds of approximations answers (FPDEs). Various approaches are described in the literature for calculations; eigenvalues expansions, Adomian approach (ADA), fractional partial differential transform technique (FDTT) (Arikoglu and Ozkol 2007, Ertürk and Momani 2008) generalized operating matrix approach, and others are a few instances. The development of a practical matrix using fractional derivatives has recently received much attention a lot of attention recently. These operations matrices can be produced using of operational matrices can be produced utilizing wavelet techniques including wavelets of Legendre and wavelets of Chebyshev. Several approximation techniques for calculations to FDEs were created based on such operations matrix (Razzaghi and Yousefi 2001, Rehman and Khan 2011, Wang and Fan

2012). The researchers created a novel numerical method based on the Wavelet in (Yi and Chen 2012) to solve FPDEs. The researchers (Khalil and Khan 2014) produced many fresh findings about the Jacobi coefficients for operating matrix. FDE and FPDE problems are solved using all operating matrix techniques. As far as we are aware, both their study nor their application to the solution of linked PFDE system is done well. As far as we are aware, there aren't many studies that focus on the numerical results of linked systems of integrals and PDEs with fractional orders; for more information, check (Khalil and Khan 2015 a, Khalil and Khan 2015 b).

A numerical method for calculating many forms of PDEs and integro-differential formulas of fractional is known as the RPS method. Due to fact that it offers answers in a locked of well-known equations, it's an efficient strategy. With the use of the RFPS approach, a variety of FDEs, fuzzy FDEs, and fractional integral equation formulas were solved. As example, a fractional Newell-Whitehead-Segel formula (Saadeh *et al.* 2019) and time-fractional Fokker-Planck equations (Freihet *et al.* 2019). Solitary boundary - value issues (Komashynska *et al.* 2016) humongous Thirring formulas (Al-Smadi *et al.* 2020 b) combined fractional Schrödinger formulas (Al-Smadi *et al.* 2020 a), time-fractional Kundu-Eckhaus formulas (Al-Smadi *et al.* 2020 b), particular categories of fuzzy FDE (Alaroud *et al.* 2018, Alaroud *et al.* 2019 a and b, Al-Smadi *et al.* 2021 b), Fredholm fractional integro-differential formulas (Alaroud *et al.* 2019 c) and fractional differential equations (Al-Smadi *et al.* 2020 c).

The transform of Laplace technique is a powerful tool for resolving numerous complicated structures that are appearing in different branches of the scientific knowledge. When analytical methods are employed with Laplace transform operators, nonlinear issues may be solved more quickly and with greater precision.

The main purpose of this work is to examine the analytically and approximative solutions for systems of nonlinear FIVP by using LRPS technique, that was introduced and shown in (Eriqat *et al.* 2020).

The Laplace transform and RPS approaches are combined in the LRPS method, which provides both precise and approximative answers as quickly FPS solutions. The major issue is converted to Laplace space, and then the answers to the novel algebraic formulas are created. At last, the primary issue is solved by applying the Laplace inverse of results. In contrast to the FRPS approach, that focuses just on fractional derivative and it may require a while to compute the various fractional derivatives in stages of determining the solutions, the undetermined parameters in a novel Laplace expansions can be identified by employing the limit idea. The LRPS approach has fewer time- as well as accuracy-intensive small computing constraints.

The organization of this study is just as follows: Section 1 consists of an introduction. Section 2 revisits several important fundamental findings about the Laplace transform with novel kinds forms of FPS. In Section 3, the suggested method for getting results for a system of FDEs is explained. In Section 4, three examples of systems of FIVPs are solved in order to examine the accessibility and application of a LRPS technique. Finally, Section 5 contains conclusion observations about the findings and recommendation.

2. FUNDAMENTALS OF FRACTIONAL CALCULUS

We go through several concepts and ideas that are necessary to finish our study inside this part. We'll go into more specifics about some special functions, theorems related to fractional calculus, series of Taylor's, and concepts linked with them, as well as the idea of Laplace transform and principle surrounding it and series of Laurent and related concepts.

2.1 Function of Gamma

A factorial function expansion that works for any real and complex integer is known as the *Gamma Function* ($\Gamma(\mathrm{z})$, for short). With the exception of $z = 0, 1, 2, \ldots$, where it may be described in a variety of ways, it seems to encompass all numbers of z.

Definition 2.1 (Andrews 1998) There are several methods for defining $\Gamma(\mathrm{z})$, for example:

1. $\Gamma(\mathrm{z}) = \int_0^\infty \zeta^{\mathrm{z}-1} e^{-\zeta} d\zeta, \mathrm{z} \in \mathbb{R}, \mathrm{z} > 0.$
2. $\frac{1}{\Gamma(\mathrm{z})} = \mathrm{z} \lim_{n\to\infty} \left\{ n^{-\mathrm{z}} \prod_{k=1}^{n} \left(1 + \frac{z}{k}\right) \right\}.$
3. $\Gamma(\mathrm{z}) = \lim_{n\to\infty} \frac{n!n^z}{\mathrm{z}(\mathrm{z}+1)(\mathrm{z}+2)\ldots(\mathrm{z}+n)}, \mathrm{z} \in \mathbb{R}, \mathrm{z} \neq 0, -1, -2, \ldots$

Lemma 2.1 (Andrews 1998) Gamma function has the following characteristics:

1. $\Gamma(\mathrm{z}) = (\mathrm{z}-1)!, \mathrm{z} \in \mathbb{N}.$
2. $\Gamma(z) = \frac{\Gamma(\mathrm{z}+1)}{z}, \mathrm{z} < 0, z \neq -1, -2, -3, \ldots .$
3. $\Gamma(\mathrm{z}+1) = \mathrm{z}\Gamma(\mathrm{z}), \mathrm{z} > 0.$
4. $\Gamma\left(\frac{\mathrm{z}+1}{2}\right) = \frac{\sqrt{\pi}}{2^z}(2\mathrm{z}-1)!, \mathrm{z} \in \mathbb{N}.$
5. $\Gamma(\mathrm{z})\Gamma(1-\mathrm{z}) = \frac{\pi}{\sin(\pi z)}.$

2.2 Operators for Fractions

This section presents concepts and also some characteristics of derivatives and integrals of fractional in the meaning of Riemann-Liouville (R-L) and Caputo.

Definition 2.2.1 (Podlubny 1999) The multi variable function $\Omega(x,t)$ time-fractional derivative of rank $\beta \in (k-1,k]$ is given as

$$\mathcal{D}^{\beta}\Omega(x,t) = \begin{cases} \dfrac{\partial^k}{\partial t^k} J_t^{k-\beta}\Omega(x,t), & k-1<\beta<k, \\ \dfrac{\partial^k}{\partial t^k}\Omega(x,t) \qquad , & \beta = k. \end{cases}$$

Remark 2.2.2 (Andrews 1998) Assume $\beta > 0$ and $k, \lambda \in \mathbb{R}, t > 0$. Then

1. $\mathcal{D}^{\beta} k = \frac{k}{\Gamma(1-\beta)} t^{-\beta}, \beta \neq 1,2,3, \ldots$
2. $\mathcal{D}^{\beta} t^{\lambda} = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\beta+1)} t^{\lambda-\beta}, \lambda > -1.$

Definition 2.2.3 (Podlubny 1999) The multivariable function $\Omega(x,t)$ time-fractional integral of rank $\beta \in (k-1,k]$ is given as

$$J_t^{\beta}\Omega(x,t) = \frac{1}{\Gamma(\beta)}\int_0^t (t-\omega)^{\beta-1}\Omega(x,t)d\omega.$$

Lemma 2.2.4 (Podlubny 1999) In the event where $\Omega(x,t)$ and $\Phi(x,t)$ are continuous functions, $\xi, \kappa \in \mathbb{R}$, and $\gamma, \beta > 0$ implies

1. $J_t^{\beta}(\xi\Omega(x,t) + \kappa\Phi(x,t)) = \xi J_t^{\beta}\Omega(x,t) + \kappa J_t^{\beta}\Phi(x,t).$
2. $J_t^{\beta}J_t^{\gamma}\Omega(x,t) = J_t^{\beta+\gamma}\Omega(x,t) = J_t^{\gamma}J_t^{\beta}\Omega(x,t)$.

Definition 2.2.5 (El-Ajou 2021) The multi variable function $\Omega(x,t)$ Caputo time-fractional integral of rank $\beta \in (k-1,k]$ is given as

$$D^{\beta}\Omega(x,t)=\begin{cases} J_t^{k-\beta}\dfrac{\partial^k}{\partial t^k}\Omega\psi(x,t), & k-1<\beta<k, \\ \dfrac{\partial^k}{\partial t^k}\Omega(x,t), & \beta=\mathrm{k}. \end{cases}$$

Remark 2.2.6 (El-Ajou 2021) Let $\eta,\lambda \in \mathbb{R}$ and $D^{\beta}\Omega(x,t), D^{\beta}\Phi(x,t)$ exist, then:

If $\xi,\kappa \in \mathbb{R}$ are real and $D^{\beta}\Omega(x,t), D^{\beta}\Phi(x,t)$ exists, thus

1. $D_t^{\beta}(\xi\Omega(x,t)+\kappa\Phi(x,t))=\xi D_t^{\beta}\Omega(x,t)+\kappa D_t^{\beta}\Phi(x,t)$.
2. $D_t^{\beta}J_t^{\beta}\Omega(x,t)=\Omega(x,t)$.
3. $J_t^{\beta}D_t^{\beta}\Omega(x,t)=\Omega(x,t)-\sum_{\mathrm{n}=0}^{\mathrm{k}-1}\frac{\partial_t^n\Omega(x,t)}{\mathrm{n}!}t^n$.

Remark 2.2.7 (El-Ajou 2021) Let $k-1<\beta<k, k\in\mathbb{N}$ and $\Omega(x,t)$ be such that $D_t^{\beta}\Omega(x,t)$ exists. The following properties for the Caputo operator hold:

Assume that $k-1<\beta<k, k\in\mathbb{N}$,and $\Omega(x,t)$ are such that $D_t^{\beta}\Omega(x,t)$ occurs. The Caputo operation holds the following characteristics.

$$\lim_{\beta\to k}D_t^{\beta}\Omega(x,t)=\partial_t^k\Omega(x,t),$$

$$\lim_{\beta\to k-1}D_t^{\beta}\Omega(x,t)=\partial_t^{k-1}\Omega(x,t)-\partial_t^{k-1}\Omega(x,0).$$

2.3 Laplace Transformation and Power Series with a Fraction

The transform of Laplace of Fractional Derivative in the meaning of Caputo, is covered in detail in this section. Offering various hypotheses and traits associated with our findings in additions. Also, we'll go through a few fundamental ideas for applying a meaning of Caputo to generalizing power series to the fraction situation.

Definition 2.3.1 (Griffel 1991) If the improper integral shown below

$$\mathrm{H}(x,s)=\int_0^\infty e^{-st}\Omega(x,t)dt,$$

is referred to as the Laplace transform of $\Omega(x,t)$ and therefore is represented as $\mathcal{L}[\Omega(x,t)](x,s)$ if it occurs as all s in a region $\mathrm{E}\subseteq\mathbb{C}$.

Definition 2.3.2 (Griffel 1991) In the area $I\times[c,d]$, a function $\Omega(x,t)$ is piece - wise continuous if

(i) As long as Ω is continuous within every sub-region $I\times(t_i,t_{i+1})$, for $i=0,1,2,\dots,k-1$, the range $[c,d]$ may be split into a limited number of subintervals $c=t_0<t_1<\cdots<t_k=d$,.

(ii) The function exhibits Ω a leap discontinuity at (x,t_i), which means

$$\left|\lim_{t\to t_i^+}\Omega(x,t_i)\right|<\infty, \text{and}\left|\lim_{t\to t_i^-}\Omega(x,t_i)\right|<\infty, i\in\{1,2,\dots,k-1\}.$$

If a function is piece-wise continuous in $I\times[0,\infty)$ for any $k>0$, it is piecewise continuous on $I\times[0,K]$.

Definition 2.3.3 (Griffel 1991) If two positive functions $\mu(x)$ and $\zeta(x)$ exist such that $|\,\Omega(x,t)\;|\le\mu(x)e^{\zeta(x)t}$ for sufficiently big *t*, then the function $\Omega(x,t)$ is referred to as being of exponential order $\zeta(x)$.

Definition 2.3.4 **(**Griffel 1991) The below transform of inverse Laplace can be used to recover the function $\Omega(x,t)$ from the transform of Laplace $\mathrm{H}(x,s)$:

$$\Omega(x,\mathrm{t})= \mathcal{L}^{-1}[\mathrm{H}(x,s)] = \int_{-i\infty}^{\eta+i\infty} e^{st}\mathrm{H}(x,s)ds, \eta = \mathrm{Re}(s) > \eta_0,$$

where η_0 is located in the right half of the Laplace integral's actual closure plane.

Proposition 2.3.5 (El-Ajou 2021) Assume that $\Omega(x,t)$ and $\Phi(x,t)$ are piece - wise continuous functions on $I \times [0,\infty)$ and of exponential order, and that $\mathrm{H}(x,s) = \mathcal{L}[\Omega(x,t)]$, and $\mathrm{M}(x,s) = \mathcal{L}[\Phi(x,t)]$, respectively, with $\lambda, \xi, \delta, \varsigma$ are fixed. When this occurs, the ensuing characteristics are met:

1. $\mathcal{L}^{-1}[\lambda\mathrm{H}(x,s) + \xi\mathrm{M}(x,s)] = \lambda\mathcal{L}^{-1}[\mathrm{H}(x,s)] + \xi\mathcal{L}^{-1}[\mathrm{M}(x,s)] = \lambda\Omega(x,t) + \xi\Phi(x,t)$.
2. $\mathcal{L}[\lambda\Omega(x,t) + \xi\Phi(x,t)] = \lambda\mathcal{L}[\Omega(x,t)] + \xi\mathcal{L}[\Phi(x,t)] = \lambda\mathrm{H}(x,s) + \xi\mathrm{M}(x,s)$.
3. $\mathcal{L}[\Omega(\delta x, \delta t)] = \frac{1}{\delta}\mathrm{H}\left(\frac{x}{\delta}, \frac{s}{\delta}\right), \delta > 0$.
4. $\mathcal{L}[e^{\varsigma t}\Phi(x,t)] = \mathrm{H}(s-\delta)$.
5. $\mathcal{L}[\partial_t^k\Omega(x,t)] = s^k\mathrm{H}(x,s) - \sum_n^{k-1} s^{\mathrm{k-n-1}}\,\partial_t^n\Omega(x,0)$.
6. $\lim_{s\to\infty} s\mathrm{H}(x,s) = \Omega(x,0)$.

Remark 2.3.6 (El-Ajou 2021) Let $\Omega(x,t)$ be an exponentially ordered piece-wise continuous function on $I \times [0,\infty)$ where $\mathrm{H}(x,s) = \mathcal{L}[\Omega(x,t)]$. Next, we have

1. $\mathcal{L}\left[D_t^\beta\Omega(x,t)\right] = s^\beta\mathrm{H}(x,s) - \sum_i^{k-1} s^{\beta-\mathrm{i}-1}\,\partial_t^i\Omega(x,0)\,, \mathrm{k}-1 < \beta < \mathrm{k}$.
2. $\mathcal{L}\left[J_t^\beta\Omega(x,t)\right] = s^{\beta-1}\mathrm{H}(x,s), \beta > 0$.
3. $\mathcal{L}\left[D_t^{k\beta}\Omega(x,t)\right] = s^{k\beta}\Psi(x,s) - \sum_{\mathrm{n}=0}^{\mathrm{k}-1} s^{(\mathrm{k-n})\beta-1}D_t^{n\beta}\Omega(x,0), 0 < \beta < 1$,

   where $D_t^{k\beta} = D_t^\beta \cdot D_t^\beta \ldots D_t^\beta$ (k-times).

Definition 2.3.7 (El-Ajou *et al*. 2015) A power series illustration of the structure:

$$\sum_{i=0}^{\infty} h_i(x)(t-t_0)^{i\beta} = c_0 + c_1(t-t_0)^{\beta} + c_2(t-t_0)^{2\beta} + c_3(t-t_0)^{3\beta} + \cdots,$$

when t is a variable and $h_i(x)$ are functions of x that are referred to as the series parameters, but where $0 \le k-1 < \beta \le k$ and $t \ge t_0$ are referred to as FPS around t_0.

Proposition 2.3.8 (El-Ajou *et al*. 2015) (formula of Generalized Taylor's). Assume that at $t = t_0$, $\Omega(x,t)$ has an FPS expression of following.

$$\Omega(x,t) = \sum_{i=0}^{\infty} h_i(x)(t-t_0)^{i\beta}, 0 \le k-1 < \beta \le k, x \in I, t_0 \le t < t_0 + R. \quad (2.1)$$

If $i = 0,1,2,\ldots$, and $D_t^{i\Omega}\Omega(x,t)$ are continuous on $I \times (t_0, t_0 + R)$, therefore the parameters $h_i(x)$ of (2.1) are as follows:

$$h_i(x) = \frac{D_t^{i\Omega}\Omega(x,t)}{\Gamma(i\beta+1)}, i = 0,1,2,\ldots,$$

where $D_t^{i\beta} = D_t^{\beta}.D_t^{\beta}\ldots D_t^{\beta}$ (i-times).

Proposition 2.3.8 (El-Ajou 2021) Let $\Omega(x,t)$ be an exponentially-ordered piece-wise continuous on the interval $I \times [0,\infty)$. Assume that FLE of a function $\mathrm{H}(x,s) = \mathcal{L}[\Omega(x,\mathrm{t})]$ is as follows:

$$\mathrm{H}(x,s) = \sum_{\mathrm{i}=0}^{\infty} \frac{h_i(x)}{s^{1+\mathrm{i}\beta}}, 0 < \beta \le 1, s > 0. \quad (2.2)$$

Then $h_i(x) = \left(D_t^{i\beta}\Omega\right)(0)$.

Remark 2.3.9 (El-Ajou 2021) According to Proposition 2.3.8 of Equation (2.2), the FLE's transform of inverse Laplace takes on the following shape

$$\Omega(x,t)=\sum_{\mathrm{i}=0}^{\infty}\frac{D_t^{i\beta}\Omega(x,0)}{\Gamma(1+\mathrm{i}\beta)}t^{i\beta}, 0<\beta\leq 1, t\geq 0. \tag{2.3}$$

3. BUILDING THE LRPS APPROACH TO SYSTEM OF NONLINEAR FRACTIONAL PARTIAL DIFFERNTIAL EQUATIONS

The FIVPs may reconstruct a number of real-world events with in fields of science and technology for comprehend or reproduce certain of these phenomenon' advantages. However, a majority of FIVPs cannot be obtained analytically and have shuttered answers. Via establishing the parameters of FPS approximation result, this chapter seeks to introduce the fundamental idea of a LRPS method used to solve specific systems of non-linear FDEs analytically. Let's utilize the following systems of FDEs as in context of Caputo derivative in order to accomplish it easily.

$$D_t^{\alpha} y_i(x,t) = f_i\big((x,t), y_1(x,t), y_2(x,t), \dots, y_n(x,t)\big), \alpha \in (0,1], \mathrm{t} \geq 0, \quad x \in I \subseteq R. \tag{3.1}$$

and the initial circumstance

$$y_i(x,t) = y_{i,0}(x,0), i = 1,2,\dots,n. \tag{3.2}$$

It's going to be discovered that $y_i(x,t)$ is an undetermined analytic function and that f_i are nonlinear continuous-valued functions specified on $[0,\infty) \times R^n$.

Now, we apply the Laplace transform on the system's FPDEs on both sides as follows:

$$\mathcal{L}\{D_t^{\alpha} y_i(x,t)\} = \mathcal{L}\big\{f_i\big((x,t), y_1(x,t), y_2(x,t), \dots, y_n(x,t)\big)\big\} = 0. \tag{3.3}$$

According to the situation $\mathcal{L}\{D_t^{\alpha} y_i(x,t)\} = s^{\alpha} Y_i(x,s) - s^{\alpha-1} y_{i,0}(x,0)$, also by following

$$H(x,s) = \mathcal{L}\big\{f_i\big((x,t), y_1(x,t), y_2(x,t), \dots, y_n(x,t)\big)\big\} = F_i\big((x,s), Y_1(x,s), Y_2(x,s), \dots, Y_n(x,s)\big),$$

likewise by applying the original information $y_i(x,t) = y_{i,0}(x,0)$ and dividing both sides on s^{α}. Thus, Equation (3.3) may be rewritten as follows:

$$Y_i(x,s) - \frac{y_{i,0}(x,0)}{s} + \frac{1}{s^{\alpha}} H(x,s). \tag{3.4}$$

where, $Y_i(x,s) = \mathcal{L}\{y_i(x,t)\}, i = 1,2,\dots,n$.

When using the LRPS method, the series of Laplace result of (3.4) has the following form:

$$Y_i(x,s) = \sum_{n=0}^{\infty} \frac{y_{i,n}(x,n)}{s^{1+n\alpha}}, s > 0. \tag{3.5}$$

Through Proposition 2.3.8, the Equation (3.5) can be rewritten as:

$$Y_i(x,s) = \frac{y_{0,0}(x,0)}{s} + \sum_{n=1}^{\infty} \frac{y_{i,n}(x,n)}{s^{1+n\alpha}}, s > 0. \tag{3.6}$$

The kth-truncated series of $Y(x,s)$ in (3.6) is given by:

$$Y_i^k(x,s) = \frac{y_{0,0}(x,0)}{s} + \sum_{n=1}^{k} \frac{y_{i,n}(x,n)}{s^{1+n\alpha}}\ , s > 0, \tag{3.7}$$

The initial approximation is understood to be

$$Y_i^1(x,s) = \frac{y_{0,0}(x,0)}{s} + \frac{y_{1,0}(x,0)}{s^{1+\alpha}}\ , s > 0. \tag{3.8}$$

We must define the main LRPS procedures, such as the LRF of (3.4), which is presented as: in order to fix the value of the unknown parameter $y_{1,0}(x,0)$ in (3.8).

$$\mathcal{L}Res_i^1(x,s) = Y_i(x,s) - \frac{y_{0,0}(x,0)}{s} - \frac{1}{s^\alpha}H(x,s), s > 0, \tag{3.9}$$

as well as the kth-LRF of (3.9) as

$$Res_i^k(x,s) = Y_i^k(x,s) - \frac{y_{i,0}(x,0)}{s} - \sum_{n=1}^{k} \frac{y_{i,n}(x,n)}{s^{1+n\alpha}} - \frac{1}{s^\alpha}H(x,s), s > 0. \tag{3.10}$$

According to the following, it is obvious that $H(x,s)$ seems to have a Laurent series

$$H(x,s) = \sum_{i=0}^{k} \frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0))}{s^{1+i}}, s > 0, \tag{3.11}$$

for each $i = 0,1, \dots, k$, then Ω is a function for $y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0)$.

The next expanded shape of kth-LRF is obtained by replacing the extensions in Formulas (3.7) and (3.11):

$$\mathcal{L}Res_i^k(x,s) = \frac{y_{0,0}(x,0)}{s} + \sum_{n=1}^{k} \frac{y_{i,n}(x,n)}{s^{1+n\alpha}} - \sum_{i=0}^{k} \frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0))}{s^{1+\alpha+i}}, s > 0. \tag{3.12}$$

Hence, the first approximation obtained of (3.12) as:

$$\mathcal{L}Res_i^1(x,s) = \frac{y_{1,0}(x,0)}{s^{1+\alpha}} - \sum_{i=0}^{k} \frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0))}{s^{1+\alpha+i}}, s > 0. \tag{3.13}$$

When we multiply two sides of Equation (3.13) by $s^{1+\alpha}$, we obtain

$$s^{1+\alpha}\mathcal{L}Res_i^1(x,s) = y_{1,0}(x,0)$$

$$-\sum_{i=0}^{k}\frac{\Omega\left(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0)\right)}{s^i}, \qquad s>0, \tag{3.14}$$

Next, by resolving the formula of $y_{1,0}(x,0)$ and implementing the limits as s approach to ∞ from both sides of (3.14), together with reference (Eriqat et al. 2020), we may quickly establish the result of $y_{1,0}(x,0)$:

$$y_{1,0}(x,0) = \Omega(y_{1,0}(x,0)). \tag{3.15}$$

Likewise, to determine the value of $y_{2,0}(x,0)$ replace the 2nd-truncated of (3.7) and substituting in (3.10) we obtain:

$$Y_{2,0}(x,0) = \frac{y_{0,0}(x,0)}{s} + \frac{y_{1,0}(x,0)}{s^{1+\alpha}} + \frac{y_{2,0}(x,0)}{s^{2+\alpha}}$$

into 2nd to obtain the accompanying:

$$\mathcal{L}Res_i^2(x,s) = \frac{y_{2,0}(x,0)}{s^{2+\alpha}} - \sum_{i=0}^{k}\frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0))}{s^{2+\alpha+i}}, s>0. \tag{3.16}$$

When we multiply two sides of Formula (3.16) by $s^{2+\alpha}$, we obtain

$$s^{2+\alpha}\mathcal{L}Res_i^1(x,s) \\ = y_{2,0}(x,0) - \Omega(y_{1,0}(x,0)) - \Omega(y_{2,0}(x,0)) \\ - \sum_{i=1}^{k} \frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0))}{s^{i-1}}, s > 0. \tag{3.17}$$

Also, by resolving the formula of $y_{2,0}(x,0)$ and implementing the limits as s approach to ∞ from both sides of (3.17), together with reference (Eriqat et al. 2020), we may quickly establish the result of $y_{2,0}(x,0)$:

$$y_{2,0}(x,0) = \Omega\left(y_{1,0}(x,0)\right) + \Omega\left(y_{2,0}(x,0)\right). \tag{3.18}$$

Additionally, to determine the value of $y_{2,0}(x,0)$ in (3.18) replace the 2nd-truncated of (3.7)

$$Y_{3,0}(s) = \frac{y_{0,0}(x,0)}{s} + \frac{y_{1,0}(x,0)}{s^{1+\alpha}} + \frac{y_{2,0}(x,0)}{s^{2+\alpha}} + \frac{y_{3,0}(x,0)}{s^{3+\alpha}},$$

into 3nd to obtain the accompanying:

$$\mathcal{L}Res_i^3(x,s) = \frac{y_{3,0}(x,0)}{s^{3+\alpha}} - \sum_{i=0}^{k} \frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0))}{s^{3+\alpha+i}}, \tag{3.19}$$

where $s > 0$. When we multiply two sides of Formula (3.19) by $s^{3+\alpha}$, we obtain

$$s^{3+\alpha}\mathcal{L}Res_i^3(x,s)y = y_{3,0}(x,0) - \Omega\left(y_{1,0}(x,0)\right) - \Omega\left(y_{2,0}(x,0)\right) \\ -\Omega\left(y_{3,0}(x,0)\right) - \tag{3.20}$$

$$\sum_{i=2}^{k} \frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0))}{s^{i-2}}, s > 0.$$

Now, by resolving the formula of $y_{2,0}(x,0)$ and implementing the limits as s approach to infinity from both sides of (3.20), together with reference (Eriqat et al. 2020), we may quickly establish the result of $y_{2,0}(x,0)$:

$$y_{3,0}(x,0) = \Omega\left(y_{1,0}(x,0)\right) + \Omega\left(y_{2,0}(x,0)\right) + \Omega\left(y_{3,0}(x,0)\right). \tag{3.21}$$

The parameter $y_{k,0}(x,0)$ which is simply determined as (3.15), (3.18) and (3.21) by taking into account the structure of computed values, is as follows:

$$\begin{aligned} y_{k,0}(x,0) = \Omega\left(y_{1,0}(x,0)\right) + \Omega\left(y_{2,0}(x,0)\right) + \Omega\left(y_{3,0}(x,0)\right) + \cdots \\ + \Omega\left(y_{k,0}(x,0)\right). \end{aligned} \tag{3.22}$$

Because of this, the series format result to (3.4) and (3.22) is:

$$\begin{aligned} Y(x,s) = \frac{y_{0,0}(x,0)}{s} + \frac{y_{1,0}(x,0)}{s^{1+\alpha}} + \frac{y_{2,0}(x,0)}{s^{2+\alpha}} + \frac{y_{3,0}(x,0)}{s^{3+\alpha}} + \cdots \\ + \sum_{i=0}^{\infty} \frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \dots, y_{n,0}(x,0))}{s^{1+\alpha+i}}. \end{aligned} \tag{3.23}$$

The LRPS solution to formulas (3.1) and (3.2) are obtained by employing formula (2.3) for the inverse Laplace transform of (3.23) in the series arrangement like shown below:

$$y(x,t) = y_{0,0}(x,0) + \frac{y_{1,0}(x,0)}{\Gamma(\alpha)} t^{\alpha} + \frac{y_{2,0}(x,0)}{\Gamma(1+\alpha)} t^{2\alpha} + \frac{y_{3,0}(x,0)}{\Gamma(2+\alpha)} t^{3\alpha} + \cdots$$
$$+ \sum_{i=0}^{\infty} \frac{\Omega(y_{1,0}(x,0), y_{2,0}(x,0), \ldots, y_{n,0}(x,0))}{(i+\alpha)!} t^{i+\alpha}. \tag{3.24}$$

4. RESULTS AND DISCUSSION

To demonstrate the effectiveness and application of the suggested technique, two applications of nonlinear FPDEs are examined in this chapter.

Application 4.1 Take into account the system of nonlinear FPDEs below:

$$D_t^{\alpha} y_1(x,t) = -y_1(x,t) - y_2^2(x,t),$$
$$D_t^{\alpha} y_2(x,t) = y_1(x,t) + y_2(x,t) + y_2^2(x,t). \tag{4.1}$$

with initial conditions

$$y_1(x,0) = 0, \tag{4.2}$$
$$y_2(x,0) = x.$$

To use the starting input and also the transform of Laplace with both parts of system, in accordance with LRPS approach of (4.1) and (4.2), we obtain

$$Y_1(x,s) = \frac{1}{s^{\alpha}}(Y_1(x,s) + \mathcal{L}\{[\mathcal{L}^{-1}\{Y_2(x,s)\}]^2\}),, \alpha \in (0,1], s > 0.$$
$$Y_2(x,s) = \frac{x}{s} - \frac{1}{s^{\alpha}}(Y_1(x,s) + Y_2(x,s) + \mathcal{L}\{[\mathcal{L}^{-1}\{Y_2(x,s)\}]^2\}), \alpha \in (0,1], s > 0$$
$$\tag{4.3}$$

The solutions for k-th truncated series of (4.1), $Y_1^k(x,s)$ and $Y_2^k(x,s)$ are provided as:

$$Y_1^k(x,s) = \sum_{n=1}^{k} \frac{\lambda_n(x)}{s^{n\alpha+1}}, s > 0,$$

$$Y_2^k(x,s) = \frac{x}{s} + \sum_{n=1}^{k} \frac{\xi_n(x)}{s^{n\alpha+1}}, s > 0. \tag{4.4}$$

Then, for situation (4.3), construct the k-th truncated $\mathcal{L}\mathrm{Res}^{k}\left(Y_1(x,s)\right)$, and $\mathcal{L}\mathrm{Res}^{k}\left(Y_2(x,s)\right)$ as follows:

$$\mathcal{L}\mathrm{Res}^{k}\left(Y_1(x,s)\right)=Y_1^{k}(x,s)-\frac{1}{s^{\alpha}}\left(Y_1(x,s)+5\mathcal{L}\{[\mathcal{L}^{-1}\{Y_2^{k}(x,s)\}]^2\}\right),$$

$$\begin{aligned}\mathcal{L}\,\mathrm{Res}^{k}&\left(Y_2(x,s)\right)\\ &=Y_2^{k}(x,s)-\frac{1}{\mathrm{s}}\\ &+\frac{1}{s^{\alpha}}\left(Y_1^{k}(x,s)+Y_2^{k}(x,s)+\mathcal{L}\{[\mathcal{L}^{-1}\{Y_2^{k}(x,s)\}]^2\}\right).\end{aligned}\qquad(4.5)$$

The constants $\lambda_n(x)$ and $\xi_n(x), n=1,2,3,\cdots,k$ in the expanding (4.4) are obtained via writing the k-th Laplace consultative approach of (4.4) into k-th LRPS of (4.3), then using calculating the resultant equations $\lim_{s\to\infty}s^{k\alpha+1}\mathcal{L}Res^{\wedge}k\ (Y_1\,(x,s))=0$ and $\lim_{s\to\infty}s^{k\alpha+1}\mathcal{L}\mathrm{Res}^{k}\ (Y_2(x,s))=0$ for $k=1,2,3,\cdots$, we obtain

Thus, the first of five parameters of (4.5) are shown:

$$\lambda_1(x)=\mathrm{x},\xi_1(x)=-(x+x^2),$$

$$\lambda_2(x)=x-2x^2,\xi_2(x)=2x^2(1+x)+(x+x^2)-1,$$

$$\lambda_3(x)=-2+3x+\frac{x^2(1+x)^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}+4x^2(1+x),$$

$$\xi_3(x)=-1+x^2+2x^3+\frac{x^2(1+x)^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}+(2+4x)x^2(1+x),$$

$$\begin{aligned}\lambda_4(x)=-4+3x+2x^2+4x^3&+\frac{3x^2(1+x)^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}\\ &-\frac{2x(-1+2x^2+x^3)\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}\\ &-\frac{4(1+x)(-2\Gamma(1+\alpha)\Gamma(1+2\alpha)+x\Gamma(1+3\alpha))x^2[1+x]}{\Gamma(1+\alpha)\Gamma(1+2\alpha)},\end{aligned}$$

$$\xi_4(x) = 5 - 3x - 3x^2 - 6x^3 - \frac{4^{1+\alpha}x^2(1+x)^2\Gamma\left(\frac{1}{2}+\alpha\right)}{\sqrt{\pi}\Gamma(1+\alpha)} - 10x^2(1+x) - 12x^3[1+x] + \frac{2x(1+x)\Gamma(1+3\alpha)\left(-1+x+x^2+2x^2(1+x)\right)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}. \tag{4.6}$$

As a result, the following will constitute the series of Laplace results to (4.3) and (4.6):

$$Y_1(x,s) = \frac{x}{s^{\alpha+1}} + \frac{x-2x^2}{s^{2\alpha+1}} + \left(-2+3x+\frac{x^2(1+x)^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2} + 4x^2(1+x)\right)\frac{1}{s^{3\alpha+1}} + \left(-4+3x+2x^2+4x^3+\frac{3x^2(1+x)^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2} - \frac{2x(-1+2x^2+x^3)\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)} - \frac{4(1+x)(-2\Gamma(1+\alpha)\Gamma(1+2\alpha)+x\Gamma(1+3\alpha))x^2[1+x]}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}\right)\frac{1}{s^{4\alpha+1}} + \cdots$$

$$Y_2(\mathrm{x},s) = \frac{x}{s} + \frac{-(x+x^2)}{s^{\alpha+1}} + \frac{(2x^2(1+x)+(x+x^2)-1)}{s^{2\alpha+1}} + \left(-1+x^2+2x^3+\frac{x^2(1+x)^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2} + (2+4x)x^2(1+x)\right)\frac{1}{s^{3\alpha+1}} + \left(5-3x-3x^2-6x^3-\frac{4^{1+\alpha}x^2(1+x)^2\Gamma\left(\frac{1}{2}+\alpha\right)}{\sqrt{\pi}\Gamma(1+\alpha)} - 10x^2(1+x) - 12x^3[1+x] + \frac{2x(1+x)\Gamma(1+3\alpha)\left(-1+x+x^2+2x^2(1+x)\right)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}\right)\frac{1}{s^{4\alpha+1}} + \cdots \tag{4.7}$$

Lastly, the LFPS resolutions may be obtained by applying inverse of Laplace transform to an expanded series (4.7) we get

$$
\begin{aligned}
y_1(x,t) = {} & \frac{t^{\alpha}x}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}x}{\Gamma(1+2\alpha)} - \frac{2t^{2\alpha}x^2}{\Gamma(1+2\alpha)} - \frac{2t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{3t^{3\alpha}x}{\Gamma(1+3\alpha)} \\
& + \frac{4t^{3\alpha}x^2}{\Gamma(1+3\alpha)} + \frac{4t^{3\alpha}x^3}{\Gamma(1+3\alpha)} + \frac{t^{3\alpha}x^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} \\
& + \frac{2t^{3\alpha}x^3\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{t^{3\alpha}x^4\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} - \frac{4t^{4\alpha}}{\Gamma(1+4\alpha)} \\
& + \frac{3t^{4\alpha}x}{\Gamma(1+4\alpha)} + \frac{2t^{4\alpha}x^2}{\Gamma(1+4\alpha)} + \frac{4t^{4\alpha}x^3}{\Gamma(1+4\alpha)} + \frac{3t^{4\alpha}x^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+4\alpha)} \\
& + \frac{6t^{4\alpha}x^3\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+4\alpha)} + \frac{3t^{4\alpha}x^4\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+4\alpha)} \\
& + \frac{2t^{4\alpha}x\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} \\
& - \frac{4t^{4\alpha}x^3\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} \\
& - \frac{2t^{4\alpha}x^4\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} + \frac{8t^{4\alpha}x^2(1+x)}{\Gamma(1+4\alpha)} \\
& + \frac{8t^{4\alpha}x^3(1+x)}{\Gamma(1+4\alpha)} - \frac{4t^{4\alpha}\Gamma(1+3\alpha)x^3(1+x)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} \\
& - \frac{4t^{4\alpha}x^4\Gamma(1+3\alpha)(1+x)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} + \cdots \\
y_2(x,t) = {} & x - \frac{t^{\alpha}x(1+x)}{\Gamma(1+\alpha)} + \frac{t^{3\alpha}(x+x^2)^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} \\
& - \frac{4t^{4\alpha}(x+x^2)^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+4\alpha)} + \frac{t^{2\alpha}(-1+x+x^2+2x^2(1+x))}{\Gamma(1+2\alpha)} \\
& + \frac{2t^{4\alpha}\Gamma(1+3\alpha)x(1+x)(-1+x+x^2+2x^2(1+x))}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} \\
& + \frac{t^{3\alpha}(-1+x^2+2x^3+(2+4x)x^2(1+x))}{\Gamma(1+3\alpha)} \\
& - \frac{t^{4\alpha}(-5+3x(1+x+2x^2)+10x^2(1+x)+12x^3(1+x))}{\Gamma(1+4\alpha)} \\
& + \cdots
\end{aligned}
\tag{4.8}
$$

when $\alpha = 1$ in (4.8), then obtain the exact results as follows:

$$y_1(x,t) = -\frac{t^3}{3} - \frac{t^4}{6} + tx + \frac{t^2x}{2} + \frac{t^3x}{2} + \frac{3t^4x}{8} - t^2x^2 + t^3x^2 + \frac{2t^4x^2}{3} + \frac{4t^3x^3}{3} + \frac{t^4x^3}{2} + \frac{t^3x^4}{3} - \frac{t^4x^4}{2} - \frac{t^4x^5}{2} - \frac{t^8x^6}{6} - \frac{t^8x^7}{3} - \frac{t^8x^8}{6} + \cdots$$

$$y_2(x,t) = x + \frac{1}{24}t(-24x(1+x) + 12t(-1 + x(1+x)(1+2x)) + 4t^2(-1 + x^2(5 + 6x(2+x))) + t^3(5 + x(-9 + x(-21 + 2x(-10 + x(5+6x)))))) + \cdots \tag{4.9}$$

Table 4.1- Table 4.4 provides an overview of numerical solution again for derived resolutions of systems (4.1), (4.2) and (4.9), as well as the actual error $\left|y_i(x,t) - y_i^5(x,t)\right|$, which were calculated for t_i. Furthermore, Figure. 4.1 - Figure. 4.4 show the behavior of LFPS results with various constants of $\alpha = 1$ and 0.60 versus the exact results. The graphical findings demonstrate how the approximation and exact results are in agreement with one another. To demonstrate the effectiveness of our technique, just five items from LFPS approximation results are employed. Of course, the following our result is better. The findings obtained using the current method are identical to those obtained by the technique of Adomian decomposition (Ramana and Prasad, 2014, and varitional iteration technique (Odibat and Momani, 2009).

Table 4.1 Numerical comparisons between the exact value of $y_1(x,t)$ and the 5th-approximation of $y_1(x,t)$ at $\alpha = 1$

x	t	$y_1(x,t)$ – Approximation	$y_1(x,t)$ – exact	Abs. Err.
1	0.01	0.00995284	0.00995284	5.55112×10^{-12}
3		0.0293218	0.0293218	3.80826×10^{-11}
5		0.0481342	0.0481342	5.74627×10^{-10}
1	0.02	0.0198227	0.0198227	4.63496×10^{-10}
3		0.0575627	0.0575627	4.98733×10^{-9}
5		0.0939303	0.0939303	9.97466×10^{-8}
1	0.03	0.0296268	0.0296268	1.61275×10^{-8}
3		0.0851108	0.0851107	7.15573×10^{-7}
5		0.139155	0.139148	6.93889×10^{-6}
1	0.04	0.0393823	0.0393769	2.90024×10^{-5}
3		0.11232	0.11223	3.46945×10^{-4}
5		0.185145	0.185012	4.85723×10^{-4}

Table 4.2 Numerical comparisons between the exact value of $y_1(x,t)$ and the 5th-approximation of $y_1(x,t)$ at $\alpha = 0.60$

x	t	$y_1(x,t)$ – approximation	$y_1(x,t)$ –exact	Abs. Err.
1	0.01	0.0692685	0.0692685	5.93157×10^{-11}
3		0.201303	0.201303	2.22685×10^{-10}
5		0.345178	0.345178	3.71142×10^{-9}
1	0.02	0.97386	0.973848	1.15334×10^{-9}
3		0.106788	0.106788	3.46001×10^{-8}
5		0.459414	0.459414	5.76668×10^{-8}
1	0.03	0.139997	0.139997	1.50015×10^{-7}
3		0.460181	0.460181	4.50046×10^{-7}
5		0.20474	0.20474	7.50077×10^{-6}
1	0.04	0.172072	0.172071	1.81302×10^{-6}
3		0.578993	0.578985	5.43905×10^{-5}
5		-0.758279	−0.758192	9.06509×10^{-4}

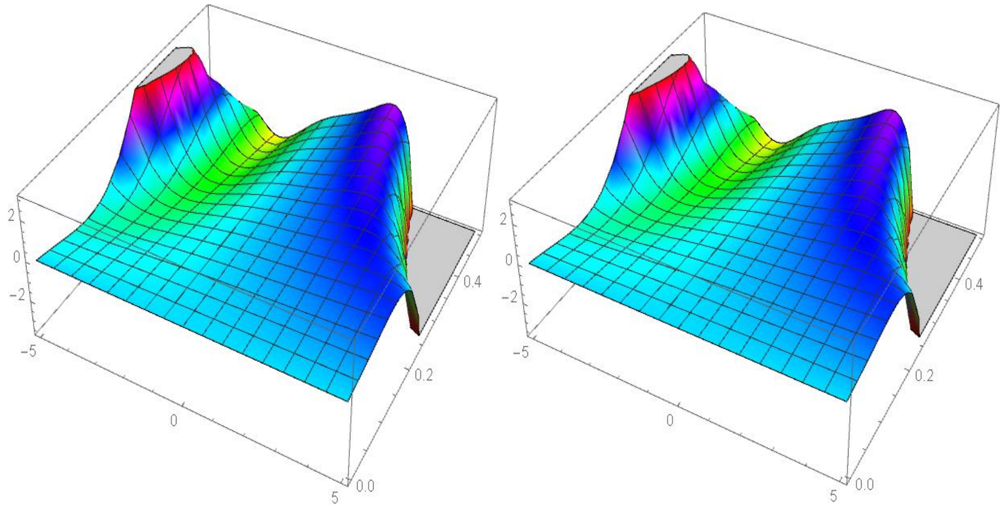
Table 4.3 Numerical comparisons between the exact value of $y_2(x,t)$ and the 5th-approximation of $y_2(x,t)$ at $\alpha = 1$

x	t	$y_2(x,t)$ – Approximation	$y_2(x,t)$ – exact	Abs. Err.
1	0.01	0.980254	0.980254	5.8701×10^{-13}
3		2.88429	2.88429	7.39531×10^{-11}
5		4.71736	4.71736	4.63993×10^{-10}
1	0.02	0.961029	0.961029	2.72669×10^{-9}
3		2.77776	2.77776	1.61924×10^{-9}
5		4.47324	4.47324	8.45339×10^{-8}
1	0.03	0.942348	0.942348	3.11701×10^{-7}
3		2.68129	2.68129	1.5173×10^{-6}
5		4.27361	4.27361	6.35945×10^{-6}
1	0.04	0.924232	0.924227	9.22427×10^{-5}
3		2.59583	2.59577	3.31708×10^{-4}
5		4.12486	4.12471	1.2759×10^{-4}

Table 4.4. Numerical comparisons between the exact value of $y_2(\text{x}.t)$ and the 5th-approximation of $y_2(\text{x}.t)$ at $\alpha = 0.60$

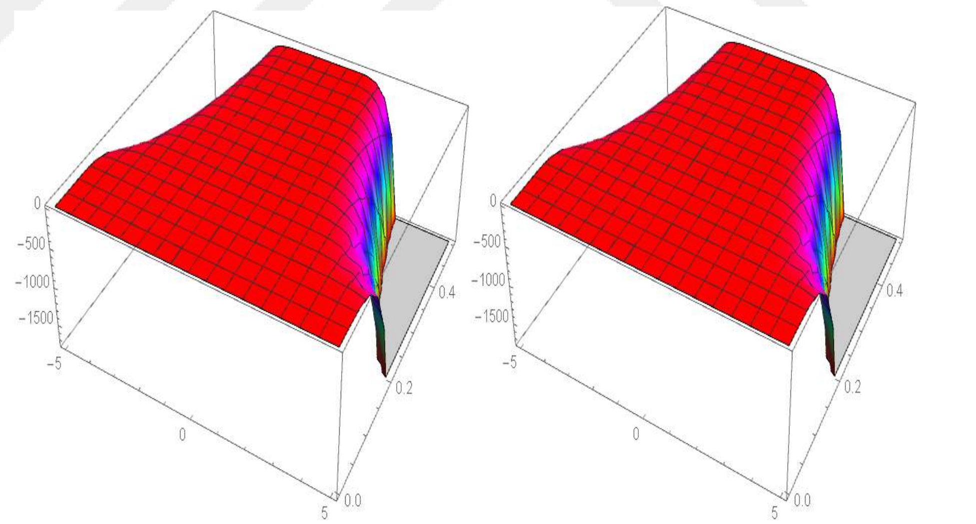
x	t	$y_2(x,t)$ – approximation	$y_2(x,t)$ –exact	Abs. Err.
0	0.01	0.783327	0.783327	7.820954×10^{-9}
2.5		0.854599	0.854599	2.160407×10^{-9}
5		−0.190859	-0.190859	6.323206×10^{-9}
0	0.02	0.97386	0.973848	1.577129×10^{-5}
2.5		0.124550	0.124454	3.241323×10^{-6}
5		−0.887517	-0.887567	1.352417×10^{-5}
0	0.03	0.428710	0.427380	1.329781×10^{-3}
2.5		-0.699696	-0.699875	1.784517×10^{-4}
5		−0.913789	-0.912582	1.206066×10^{-3}
0	0.04	-0.412125	-0.442520	3.039593×10^{-2}
2.5		-0.992669	−0.994553	1.883936×10^{-3}
5		-0.276064	−0.246974	2.908985×10^{-2}

Now, Figure. 4.1 – Figure 4.4 show the behavior of LFPS results with various constants of $\alpha = 1$ and 0.60 versus the exact results.



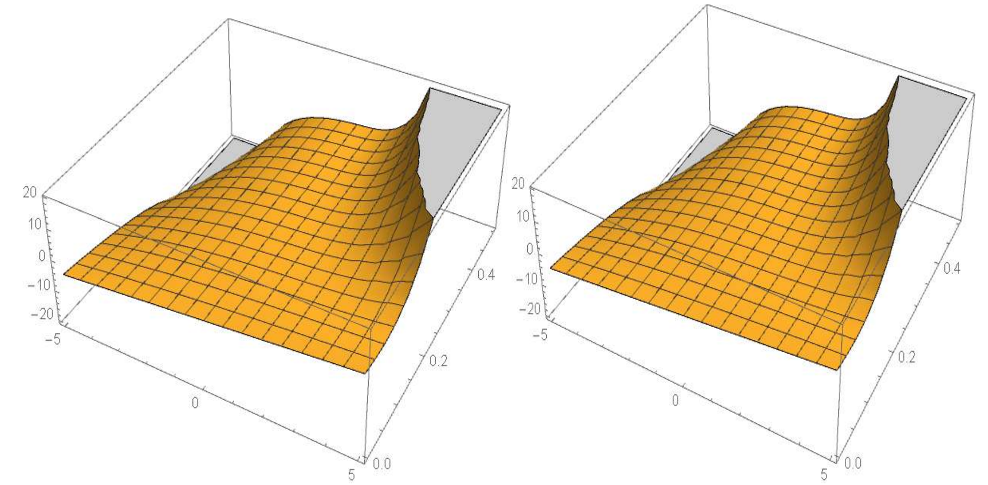
(a) exact solution at $\alpha = 1$ (b) approximat solution at $\alpha = 1$

Figure 4.1 The 5th estimations of $\boldsymbol{y_1(x, t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 1}$.



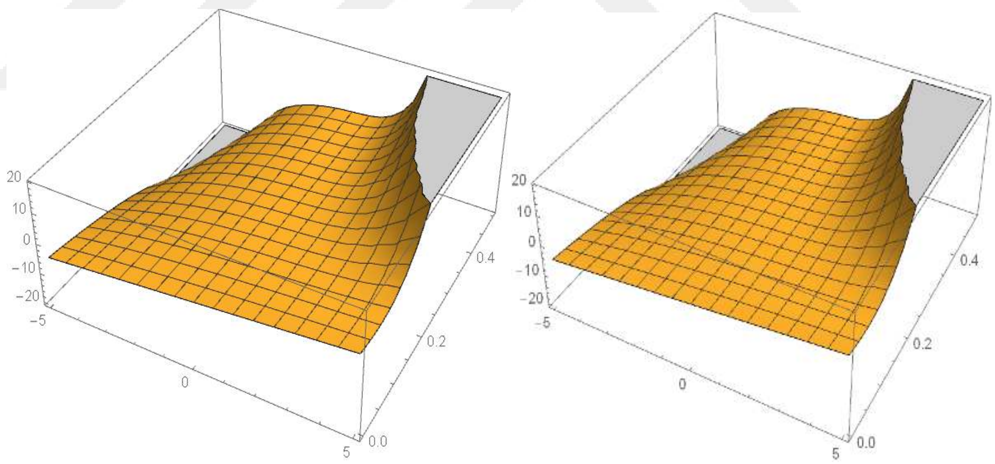
(c) exact solution at $\alpha = 0.60$ (d) approximat solution at $\alpha = 0.60$

Figure 4.2 The 5th estimations of $\boldsymbol{y_1(x, t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 0.60}$.



(a) exact solution at $\alpha = 1$ **(b) approximat solution at $\alpha = 1$**

Figure 4.3 The 5th estimations of $\boldsymbol{y_2(x,t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 1}$.



(c) exact solution at $\alpha = 0.60$ **(d) approximat solution at $\alpha = 0.60$**

Figure 4.4 The 5th estimations of $\boldsymbol{y_2(x,t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 0.60}$

Application 4.2 Take into account the system of nonlinear FPDEs below:

$$
\begin{aligned}
&D_t^{\alpha} y_1(x,t) = y_1(x,t),\\
&D_t^{\alpha} y_2(x,t) = y_1^2(x,t),\\
&D_t^{\alpha} y_3(x,t) = 3y_1(x,t)y_2(x,t).
\end{aligned} \tag{4.10}
$$

with initial conditions

$$
\begin{aligned}
&y_1(x,0) = x,\\
&y_2(x,0) = x,\\
&y_3(x,0) = x.
\end{aligned} \tag{4.11}
$$

To use the starting input and also the transform of Laplace with both parts of system, in accordance with LRPS approach of (4.10) and (4.11), we obtain

$$
\begin{aligned}
&Y_1(x,s) = \frac{x}{s} + \frac{1}{s^{\alpha}}\left(Y_1(x,s)\right),\\
&Y_2(x,s) = \frac{x}{s} + \frac{1}{s^{\alpha}}\left(\mathcal{L}\{[\mathcal{L}^{-1}\{Y_1(x,s)\}]^2\}\right),\\
&Y_3(x,s) = \frac{x}{s} + \frac{3}{s^{\alpha}}\left(\mathcal{L}\{\mathcal{L}^{-1}\{Y_1(x,s)\}\{\mathcal{L}^{-1}\{Y_2(x,s)\}\}\right), \alpha \in (0,1], s > 0.
\end{aligned} \tag{4.12}
$$

The solutions for k-th truncated series of (4.10), $Y_1^k(x,s)$, $Y_2^k(x,s)$ and $Y_3^k(x,s)$ are provided as:

$$
\begin{aligned}
&Y_1^k(x,s) = \frac{x}{s} + \sum_{n=1}^{k} \frac{\lambda_n(x)}{s^{n\alpha+1}}, s > 0,\\
&Y_2^k(x,s) = \frac{x}{s} + \sum_{n=1}^{k} \frac{\xi_n(x)}{s^{n\alpha+1}}, s > 0
\end{aligned} \tag{4.13}
$$

$$Y_3^k(x,s) = \frac{x}{s} + \sum_{n=1}^{k} \frac{\zeta_n(x)}{s^{n\alpha+1}}, s > 0.$$

Then, for situation (4.12), construct the k-th truncated $\mathcal{L}\mathrm{Res}^k\ (Y_1(x,s))$, $\mathcal{L}\mathrm{Res}^k\ (Y_2(x,s))$ and $\mathrm{LRes}^k\ (Y_3(x,s))$ as follows:

$$\mathcal{L}\mathrm{Res}^k\big(Y_1(x,s)\big) = Y_1^k(x,s) - \frac{x}{s} - \frac{1}{s^\alpha}\Big(Y_1^k(x,s)\Big),$$
$$\mathcal{L}\,\mathrm{Res}^k\big(Y_2(x,s)\big) = Y_2^k(x,s) - \frac{x}{s} - \frac{1}{s^\alpha}\big(\mathcal{L}\{[\mathcal{L}^{-1}\{Y_1^k(x,s)\}]^2\}\big),$$
$$\mathcal{L}\,\mathrm{Res}^k\big(Y_3(x,s)\big) = Y_3^k(x,s) - \frac{x}{s} - \frac{3}{s^\alpha}\big(\mathcal{L}\{\mathcal{L}^{-1}\{Y_1(x,s)\}\{\mathcal{L}^{-1}\{Y_2(x,s)\}\}\big). \tag{4.14}$$

The constants $\lambda_n(x)$, $\xi_n(x)$ and $\zeta_n(x), n = 1,2,3,\cdots,k$ in the expanding (4.13) are obtained via writing the k-th Laplace consultative approach of (4.13) into k-th LRPS of (4.13) and (4.14) then using calculating the resultant equations
$\lim_{s\to\infty} s^{k\alpha+1}\mathcal{L}\,\mathrm{Res}^k\big(Y_1(x,s)\big) = 0$ and $\lim_{s\to\infty} s^{k\alpha+1}\mathcal{L}\mathrm{Res}^k\ (Y_2\ (x,s)) = 0$ for $k = 1,2,3,\cdots$, we obtain:

$$\lambda_1(x) = \mathrm{x}, \xi_1(x) = x^2, \zeta_1(x) = x^2,$$
$$\lambda_2(x) = \mathrm{x}, \xi_2(x) = 2x^2, \zeta_2(x) = x^2 + x^3,$$
$$\lambda_3(x) = x, \xi_3(x) = 2x^2 + \frac{x^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}, \zeta_3(x) = x^2 + 2x^3 + \frac{x^3\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2},$$
$$\lambda_4(x) = x\ ,$$
$$\xi_4(x) = 2x^2 + \frac{2x^2\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}, \tag{4.15}$$
$$\zeta_4(x) = x^2 + 2x^3 + \frac{x^3\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2} + \frac{3x^3\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}.$$

As a result, the following will constitute the series of Laplace results to (4.12) and (4.15)

$$Y_1(x,s) = \frac{x}{s^{\alpha+1}} + \frac{x}{s^{2\alpha+1}} + \frac{x}{s^{3\alpha+1}} + \frac{x}{s^{4\alpha+1}} + \cdots$$

$$Y_2(x,s) = \frac{x}{s} + \frac{x^2}{s^{\alpha+1}} + \frac{2x^2}{s^{2\alpha+1}} + \frac{2x^2 + \frac{x^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}}{s^{3\alpha+1}} + \frac{2x^2 + \frac{2x^2\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}}{s^{4\alpha+1}} + \cdots$$

$$Y_3(x,s) = \frac{x}{s} + \frac{x^2}{s^{\alpha+1}} + \frac{x^2 + x^3}{s^{2\alpha+1}} + \frac{x^2 + 2x^3 + \frac{x^3\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2}}{s^{3\alpha+1}} + \frac{x^2 + 2x^3 + \frac{x^3\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2} + \frac{3x^3\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}}{s^{4\alpha+1}} + \cdots \tag{4.16}$$

Lastly, the LFPS resolutions may be obtained by applying inverse of Laplace transform of (4.16) to an expanded series (4.13) we get

$$y_1(x,t) = x + \frac{t^{\alpha}x}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}x}{\Gamma(1+2\alpha)} + \frac{t^{3\alpha}x}{\Gamma(1+3\alpha)} + \frac{t^{4\alpha}x}{\Gamma(1+4\alpha)} + \cdots$$

$$\begin{aligned} y_2(x,t) = x &+ \frac{t^{\alpha}x^2}{\Gamma(1+\alpha)} + \frac{2t^{2\alpha}x^2}{\Gamma(1+2\alpha)} + \frac{2t^{3\alpha}x^2}{\Gamma(1+3\alpha)} + \frac{t^{3\alpha}x^2\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} \\ &+ \frac{2t^{4\alpha}x^2}{\Gamma(1+4\alpha)} + \frac{2t^{4\alpha}x^2\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} + \cdots \end{aligned} \tag{4.17}$$

$$\begin{aligned} y_3(x,t) = x &+ \frac{t^{\alpha}x^2}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}x^2}{\Gamma(1+2\alpha)} + \frac{t^{2\alpha}x^3}{\Gamma(1+2\alpha)} + \frac{t^{3\alpha}x^2}{\Gamma(1+3\alpha)} + \frac{2t^{3\alpha}x^3}{\Gamma(1+3\alpha)} \\ &+ \frac{t^{3\alpha}x^3\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} + \frac{t^{4\alpha}x^2}{\Gamma(1+4\alpha)} + \frac{2t^{4\alpha}x^3}{\Gamma(1+4\alpha)} \\ &+ \frac{t^{4\alpha}x^3\Gamma(1+2\alpha)}{\Gamma(1+\alpha)^2\Gamma(1+4\alpha)} + \frac{3t^{4\alpha}x^3\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)} + \cdots \end{aligned}$$

when $\alpha = 1$ in (4.17), then obtain the exact results as follows

$$y_1(x,t) = \mathrm{t}x + x + \frac{t^2x}{2} + \frac{t^3x}{6} + \frac{t^4x}{24} + \cdots = xe^t$$

$$y_2(x,t) = x + \mathrm{t}x^2 + t^2x^2 + \frac{2t^3x^2}{3} + \frac{t^4x^2}{3} + \cdots = xe^{2t}$$

$$y_3(x,t) = x + \mathrm{t}\,x^2 + \frac{t^2x^2}{2} + \frac{t^3x^2}{6} + \frac{t^4x^2}{24} + \frac{t^2x^3}{2} + \frac{2t^3x^3}{3} + \frac{13t^4x^3}{24} + \cdots$$
$$= x(e^t + e^{2t}) \tag{4.18}$$

Table 4.5-Table 4.13 provides an overview of numerical solution again for derived resolutions of systems (4.10) (4.11) and (4.18), as well as the actual error $|y_i(x,t) - y_i^5(x,t)|$, which were calculated for t_i. Furthermore, Figure. 4.4- Figure 4.13 show the behavior of LFPS results with various constants of $\alpha = 1$, 0.60 and 0.90 versus the exact results. The graphical findings demonstrate how the approximation and exact results are in agreement with one another. To demonstrate the effectiveness of our technique, just five items from LFPS approximation results are employed. Of course, our following result is better.

Table 4.5 Numerical comparisons between the exact value of $y_1(x,t)$ and the 5th-approximation of $y_1(x,t)$ at $\alpha = 1$

x	t	$y_1(x,t)$ − Approximation	$y_1(x,t)$ − exact	Abs. Err.
1	0.01	1.01005	1.01005	8.34616×10^{-13}
3		3.03015	3.03015	2.50385×10^{-12}
5		5.05025	5.05025	4.17352×10^{-12}
1	0.02	1.0202	1.0202	2.67558×10^{-11}
3		3.0606	3.0606	8.02673×10^{-11}
5		5.10101	5.10101	1.33778×10^{-10}
1	0.03	1.03045	1.03045	2.03517×10^{-10}
3		3.09136	3.09136	6.10551×10^{-10}
5		5.15227	5.15227	1.01758×10^{-9}
1	0.04	1.04081	1.04081	8.59055×10^{-10}
3		3.12243	3.12243	2.57716×10^{-9}
5		5.20405	5.20405	4.29527×10^{-9}

Table 4.6 Numerical comparisons between the exact value of $y_1(x.t)$ and the 5th-approximation of $y_1(x.t)$ at $\alpha = 0.60$

x	t	$y_1(x,t)$ – Approximation	$y_1(x,t)$ –exact	Abs. Err.
1	0.01	1.07438	1.07438	1.71507×10^{-7}
3		3.22315	3.22315	5.14521×10^{-7}
5		5.37192	5.37192	8.57535×10^{-7}
1	0.02	1.11588	1.11588	1.39284×10^{-6}
3		3.34765	3.34765	4.17851×10^{-6}
5		5.57942	5.57942	6.96418×10^{-6}
1	0.03	1.15118	1.15117	4.75902×10^{-6}
3		3.45353	3.45352	1.42771×10^{-5}
5		5.75588	5.75586	2.37951×10^{-5}
1	0.04	1.18328	1.18327	1.14035×10^{-5}
3		3.54983	3.54980	3.42106×10^{-5}
5		5.91639	5.91633	5.70176×10^{-5}

Table 4.7 Numerical comparisons between the exact value of $y_1(x,t)$ and the 5th-approximation of $y_1(x,t)$ at $\alpha = 0.90$

x	t	$y_1(x,t)$ – Approximation	$y_1(x,t)$ – exact	Abs. Err.
1	0.01	1.01663	1.01663	4.71522×10^{-9}
3		3.04989	3.04989	1.41457×10^{-8}
5		5.08315	5.08315	2.35761×10^{-8}
1	0.02	1.03128	1.03128	5.71755×10^{-8}
3		3.09384	3.09384	1.71527×10^{-7}
5		5.15639	5.15639	2.85878×10^{-7}
1	0.03	1.04539	1.04539	2.46115×10^{-7}
3		3.13618	3.13618	7.38346×10^{-7}
5		5.22697	5.22697	1.23058×10^{-6}
1	0.04	1.05924	1.05924	6.93295×10^{-7}
3		3.17772	3.17772	2.07989×10^{-6}
5		5.2962	5.2962	3.46648×10^{-6}

Table 4.8 Numerical comparisons between the exact value of $y_2(x,t)$ and the 5th-approximation of $y_2(x,t)$ at $\alpha = 1$

x	t	$y_2(x,t)$ − Approximation	$y_2(x,t)$ − exact	Abs. Err.
1	0.01	1.0101	1.0101	3.33333×10^{-9}
3		3.09091	3.09091	3.0000×10^{-8}
5		5.25252	5.25252	8.33333×10^{-8}
1	0.02	1.02041	1.02041	5.33333×10^{-8}
3		3.18365	3.18365	4.8000×10^{-7}
5		5.51013	5.51013	1.33333×10^{-6}
1	0.03	1.03092	1.03092	2.7000×10^{-7}
3		3.27826	3.27826	2.4300×10^{-6}
5		5.77296	5.77295	6.7500×10^{-6}
1	0.04	1.04164	1.04164	8.53333×10^{-7}
3		3.37479	3.37478	7.6800×10^{-6}
5		6.04109	6.04107	2.13333×10^{-5}

Table 4.9 Numerical comparisons between the exact value of $y_2(x.t)$ and the 5th-approximation of $y_2(x.t)$at $\alpha = 0.60$

x	t	$y_2(x,t)$ − Approximation	$y_2(x,t)$ −exact	Abs. Err.
1	0.01	1.07438	1.07435	28.739×10^{-6}
3		3.70539	3.70513	25.8651×10^{-5}
5		6.95942	6.9587	71.8475×10^{-5}
1	0.02	1.12555	1.1254	15.1685×10^{-5}
3		4.12995	4.12858	13.6517×10^{-4}
5		8.13875	8.13495	37.9214×10^{-4}
1	0.03	1.16758	1.16718	40.1387×10^{-5}
3		4.50821	4.5046	36.1248×10^{-4}
5		9.18947	9.17944	10.0347×10^{-3}
1	0.04	1.20731	1.20651	8.006×10^{-4}
3		4.86583	4.85862	72.054×10^{-4}
5		10.1829	10.1628	20.015×10^{-3}

Table 4.10 Numerical comparisons between the exact value of $y_2(x.t)$ and the 5th-approximation of $y_2(x.t)$ at $\alpha = 0.90$

x	t	$y_2(x,t)$ − Approximation	$y_2(x,t)$ −exact	Abs. Err.
1	0.01	1.01678	1.01678	2.4393×10^{-8}
3		3.15104	3.15104	2.19537×10^{-7}
5		5.41956	5.41956	6.09825×10^{-7}
1	0.02	1.03182	1.03182	2.95783×10^{-7}
3		3.28637	3.28636	2.66205×10^{-6}
5		5.79546	5.79546	7.39458×10^{-6}
1	0.03	1.04653	1.04653	1.27322×10^{-6}
3		3.41878	3.41877	1.14589×10^{-5}
5		6.16327	6.16324	3.18304×10^{-5}
1	0.04	1.06118	1.06117	3.58659×10^{-6}
3		3.55058	3.55055	3.22793×10^{-5}
5		6.52939	6.5293	8.96647×10^{-5}

Table 4.11 Numerical comparisons between the exact value of $y_3(x,t)$ and the 5th-approximation of $y_3(x,t)$ at $\alpha = 1$

x	t	$y_3(x,t)$ − Approximation	$y_3(x,t)$ − exact	Abs. Err.
1	0.01	1.0101	1.0101	3.75×10^{-9}
3		3.09183	3.09183	1.0125×10^{-7}
5		5.25763	5.25763	4.6875×10^{-7}
1	0.02	1.02041	1.02041	6.000×10^{-8}
3		3.18743	3.18743	1.62×10^{-6}
5		5.53104	5.53104	7.5×10^{-6}
1	0.03	1.03093	1.03093	3.0375×10^{-7}
3		3.28698	3.28697	8.20125×10^{-6}
5		5.82104	5.82101	3.79687×10^{-5}
1	0.04	1.04168	1.04168	9.6×10^{-7}
3		3.39066	3.39064	2.592×10^{-5}
5		6.12844	6.12832	1.2×10^{-4}

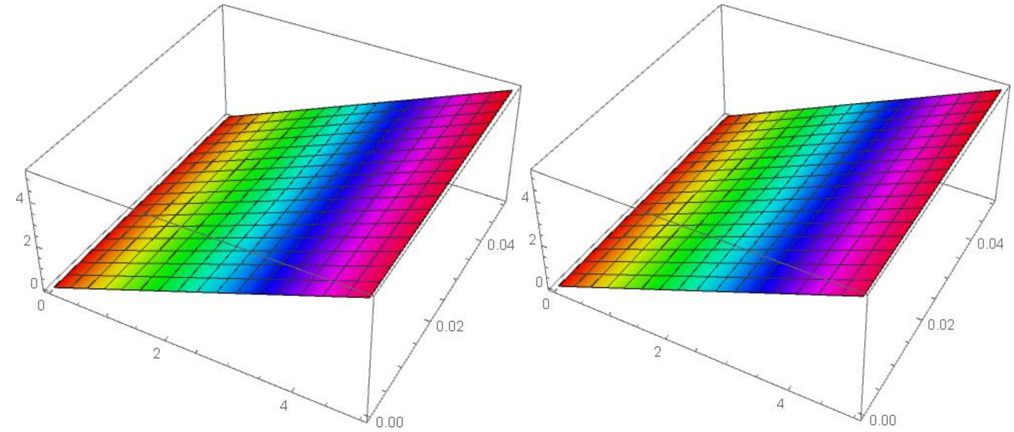
Table 4.12 Numerical comparisons between the exact value of $y_3(x.t)$ and the 5th-approximation of $y_3(x.t)$ at $\alpha = 0.60$

x	t	$y_3(x,t)$ – Approximation	$y_3(x,t)$ – exact	Abs. Err.
1	0.01	1.07857	1.07854	27.1597×10^{-6}
3		3.78247	3.78174	73.3311×10^{-5}
5		7.38282	7.37942	33.9496×10^{-4}
1	0.02	1.12626	1.12612	14.335×10^{-5}
3		4.3231	4.31923	38.7044×10^{-4}
5		9.19409	9.17617	17.9187×10^{-3}
1	0.03	1.16912	1.16874	37.9329×10^{-5}
3		4.84505	4.83481	10.2419×10^{-3}
5		11.0224	10.975	47.4161×10^{-3}
1	0.04	1.20999	1.20923	75.6604×10^{-5}
3		5.37095	5.35052	20.4283×10^{-3}
5		12.9222	12.8276	94.5755×10^{-3}

Table 4.13 Numerical comparisons between the exact value of $y_3(x.t)$ and the 5th-approximation of $y_3(x.t)$at $\alpha = 0.90$

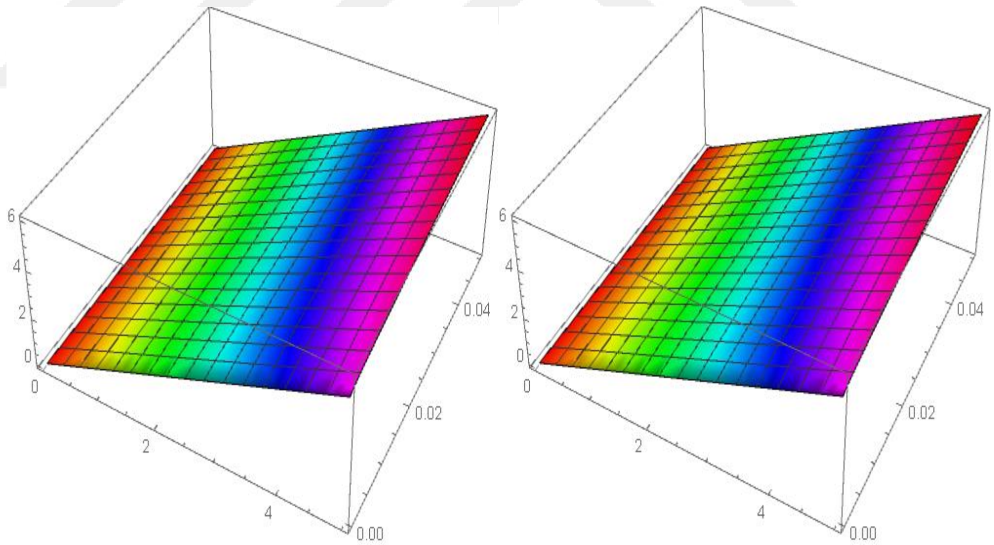
x	t	$y_3(x,t)$ – Approximation	$y_3(x,t)$ – exact	Abs. Err.
1	0.01	1.01678	1.01678	3.65895×10^{-8}
3		3.15381	3.15381	9.87917×10^{-6}
5		5.43494	5.43493	4.57369×10^{-5}
1	0.02	1.03182	1.03182	4.43675×10^{-7}
3		3.29625	3.29624	11.9792×10^{-6}
5		5.85023	5.85017	5.54594×10^{-5}
1	0.03	1.04655	1.04655	1.90982×10^{-6}
3		3.43976	3.43971	5.15652×10^{-5}
5		6.27937	6.27913	2.38728×10^{-4}
1	0.04	1.06122	1.06121	5.37988×10^{-6}
3		3.58659	3.58644	1.45257×10^{-4}
5		6.72833	6.72766	6.72485×10^{-4}

Now, the following Figure 4.5- Figure 4.13 show the behavior of LFPS results with various constants of $\alpha = 1,\ 0.60$ and 0.90 versus the exact results.



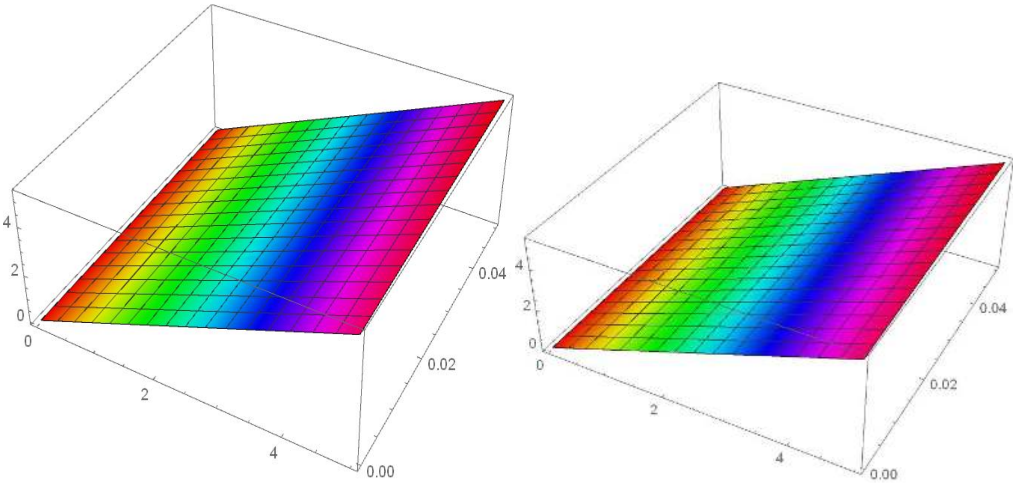
(a) exact solution at $\alpha = 1$ (b) approximat solution at $\alpha = 1$

Figure 4.5 The 5th estimations of $\boldsymbol{y_1(x,t)}$ surfaces graphing, as well as the exact result for value $\boldsymbol{\alpha = 1}$



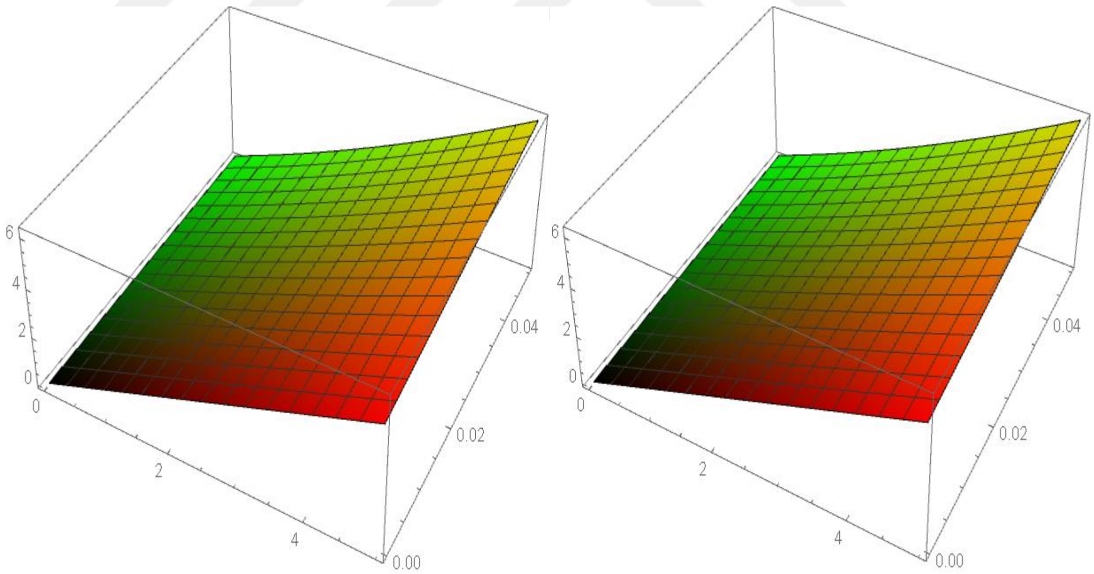
(c) exact solution at $\alpha = 0.60$ (d) approximat solution at $\alpha = 0.60$

Figure 4.6 The 5th estimations of $\boldsymbol{y_1(x,t)}$ surfaces graphing, as well as the exact result for value $\boldsymbol{\alpha = 0.60}$



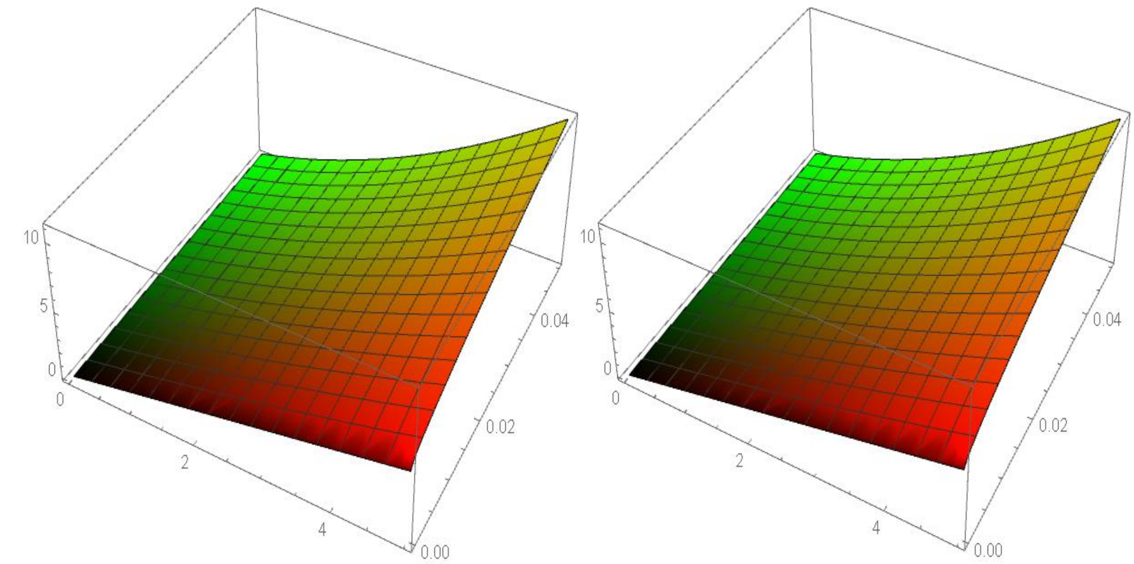
(e) exact solution at $\alpha = 0.90$ (f) approximat solution at $\alpha = 0.90$

Figure 4.7 The 5th estimations of $\boldsymbol{y_1(x,t)}$ surfaces graphing, as well as the exact result for value $\boldsymbol{\alpha = 0.90}$



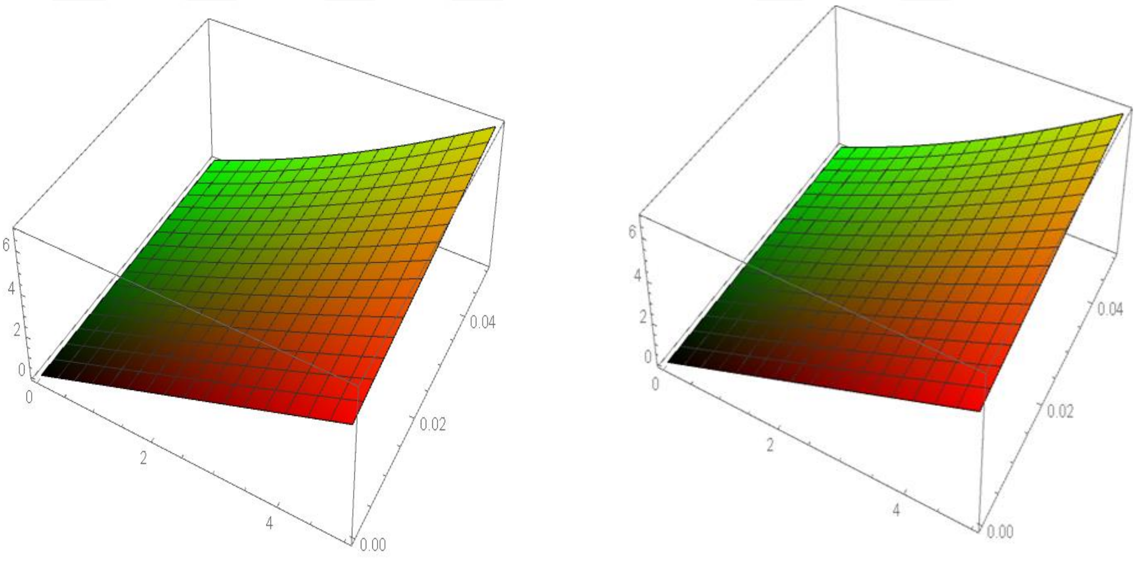
(a) exact solution at $\boldsymbol{\alpha = 1}$ **(b) approximat solution at $\boldsymbol{\alpha = 1}$**

Figure 4.8 The 5th estimations of $\boldsymbol{y_2(x,t)}$ surfaces graphing, as well as the exact result for value $\boldsymbol{\alpha = 1}$



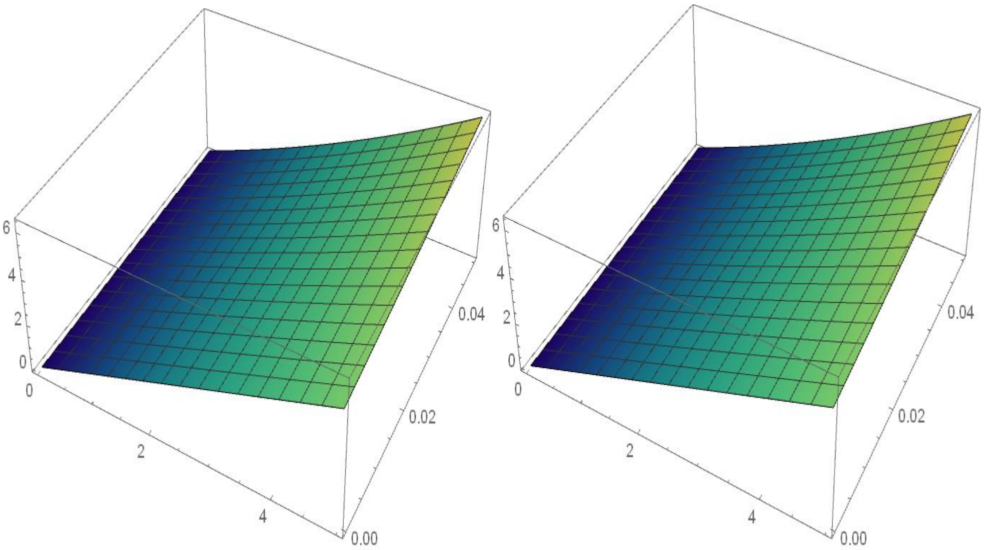
$(\mathbf{c})$ **exact solution at** $\boldsymbol{\alpha = 0,60}$ $(\mathbf{d})$ **approximat solution at** $\boldsymbol{\alpha = 0,60}$

Figure 4.9 The 5th estimations of $\boldsymbol{y_2(x,t)}$ surfaces graphing, as well as the exact result for value $\boldsymbol{\alpha = 0.60}$



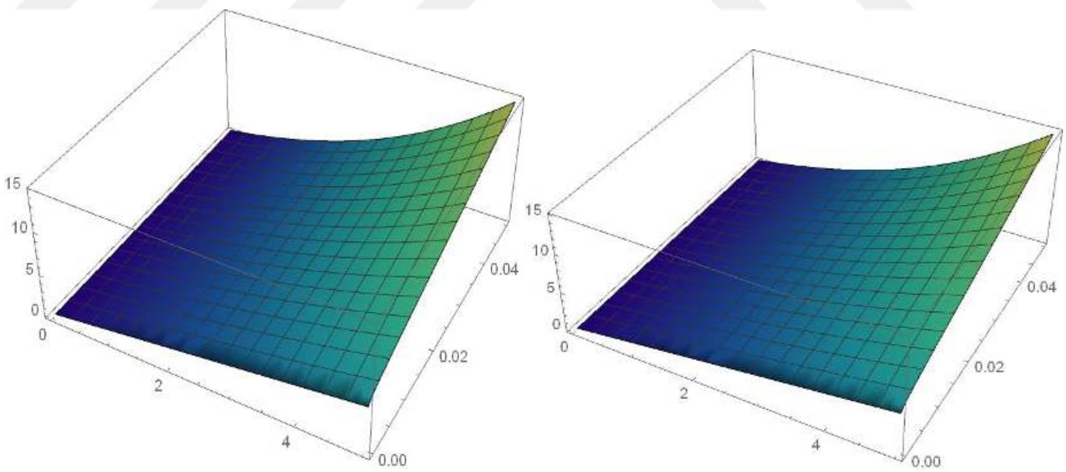
$(\mathbf{e})$ **exact solution at** $\boldsymbol{\alpha = 0.90}$ $(\mathbf{f})$ **approximat solution at** $\boldsymbol{\alpha = 0.90}$

Figure 4.10 The 5th estimations of $\boldsymbol{y_2(x,t)}$ surfaces graphing, as well as the exact result for value $\boldsymbol{\alpha = 0.90}$



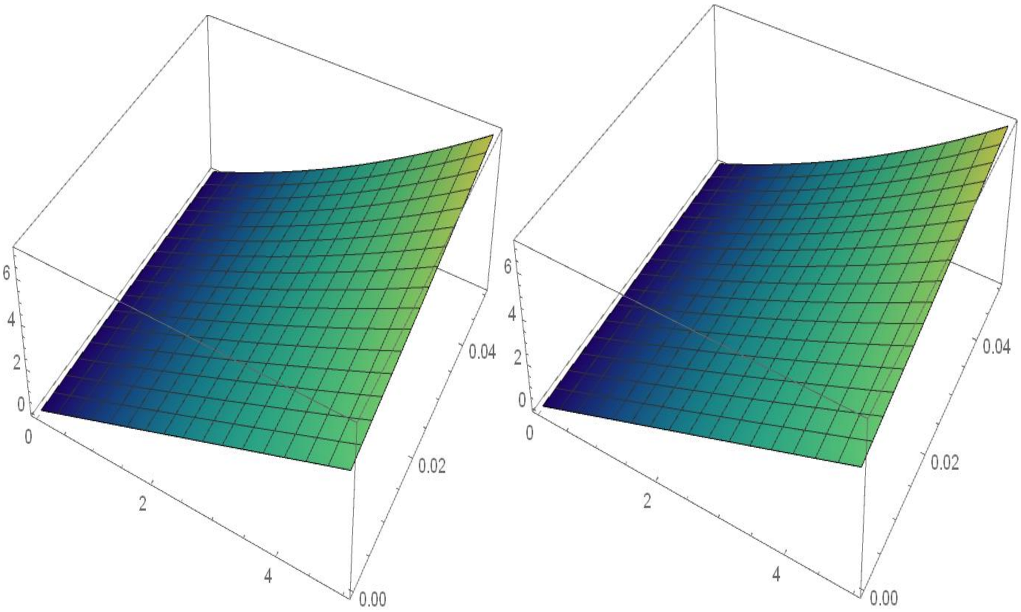
(a) exact solution at $\alpha = 1$ **(b) approximat solution at $\alpha = 1$**

Figure 4.11 The 5th estimations of $y_3(x, t)$ surfaces graphing, as well as the exact result for value $\alpha = 1$



(c) exact solution at $\alpha = 0.60$ **(d) approximat solution at $\alpha = 0.60$**

Figure 4.12 The 5th estimations of $y_3(x, t)$ surfaces graphing, as well as the exact result for value $\alpha = 0.60$



(e) exact solution at $\boldsymbol{\alpha = 0.90}$ **(f) approximat solution at $\boldsymbol{\alpha = 0.90}$**

Figure 4.13 The 5th estimations of $\boldsymbol{y_3(x,t)}$ surfaces graphing, as well as the exact result for value $\boldsymbol{\alpha = 0.90}$

5. CONCLUSIONS AND RECOMMENDATION

5.1 Conclusions

To examine the resolutions of standard PDEs and FPDEs, a variety of analytical and numerical techniques are employed; however, a few of these techniques get the drawback of just not offering accurate solution. This work underlined how the LRPS methodology, which was suggested, is a straightforward analytical technique for creating precise and approximative results for only certain types of FPDEs having suitable beginning circumstances. Also, with aid of the limiting notion at infinity, this aforementioned method made it simple to determine the parameters of the expansions series, giving us results in the Laplace transform space. Using less series components, the approximations are produced. A numeric and graphic discussion of the behaviour of derived results under various choices is being presented. The calculated results show that its approximate solutions are in perfect agreement with precise solutions for $\alpha = 1$ and are quite near to one another. Therefore, proves that aforementioned method is an appropriate and extraordinarily effective method for getting both approximative and analytically resolutions to a large number for issues of nonlinear fractional that arise in practical sciences.

5.2 Recommendation

1- All of the applications in this study were for initial value problems, but they may all be adapted to boundary value problems for both fractional linear and nonlinear equations.

2- All applications in this study about solutions of fractional linear and nonlinear equations can be generalized in later studies to solutions of systems of linear and nonlinear equations.

3- Other classes of fractional differential, integral, and integrodifferential equations can be solved using the LRPS approach.

REFERENCES

Abdeljawad, T., Amin, R., Shah, K., Al-Mdallal, Q. and Jarad, F. 2020. Efficient sustainable algorithm for numerical solutions of systems of fractional order differential equations by Haar wavelet collocation method. Alexandria Engineering Journal, 59(4): 2391-2400.

Abu Arqub, O. 2019. Computational algorithm for solving singular Fredholm time-fractional partial integrodifferential equations with error estimates. Journal of Applied Mathematics and Computing, 59(1): 227-243.

Ait Touchent, K., Hammouch, Z., Mekkaoui, T. and Belgacem, F. B. 2018. Implementation and convergence analysis of homotopy perturbation coupled with Sumudu transform to construct solutions of local-fractional PDEs. Fractal and Fractional, 2(3): 22.

Akdemir, A. O., Hemen, D. and Abdon, A. 2020. Fractional order analysis: theory, methods and applications. John Wiley & Sons, 336 Page, USA.

Alabedalhadi, M., Al-Smadi, M., Al-Omari, S., Baleanu, D. and Momani, S. 2020. Structure of optical solution for nonliear resonant space-time Schrödinger equation in conformable sense with full nonlinearity term. Physica Scripta, 95(10): 105215.

Alaroud, M., Al-Smadi, M., Ahmad, R. R. and Salma Din, U. K. 2018. Computational optimization of residual power series algorithm for certain classes of fuzzy fractional differential equations. International journal of differential Equations, 2018:1-11.

Alaroud, M., Al-Smadi, M., Rozita Ahmad, R. and Salma Din, U. K. 2019 a. An analytical numerical method for solving fuzzy fractional Volterra integro-differential equations. Symmetry, 11(2): 205.

Alaroud, M., Ahmad, R. R. and Salma Din, U. K. 2019 b. An efficient analytical-numerical technique for handling model of fuzzy differential equations of fractional-order. Filomat, 33(2): 617-632.

Alaroud, M., Al-Smadi, M., Ahmad, R. R. and Din, U. K. 2019 c. Numerical computation of fractional Fredholm integro-differential equation of order 2β arising in natural sciences. In Journal of Physics, IOP Publishing: Conference Series 1212(1):1-6.

Al-Qurashi, M., Rashid, S., Karaca, Y., Hammouch, Z., Baleanu, D. and Chu, Y. M. 2021. Achieving more precise bounds based on double and triple integral as proposed by generalized proportional fractional operators in the Hilfer sense. Fractals, 29(05): 2140027.

Al-Smadi, M. H. and Gumah, G. N. 2014. On the homotopy analysis method for fractional SEIR epidemic model. Research Journal of Applied Sciences, Engineering and Technology, 7(18): 3809-3820.

Al-Smadi, M., Freihat, A., Arqub, O. A. and Shawagfeh, N. 2015. A Novel Multistep Generalized Differential Transform Method for Solving Fractional-order. Lü Chaotic and Hyperchaotic Systems. Journal of Computational Analysis & Applications, 19(1): 713-724.

Al-Smadi, M., Freihat, A., Hammad, M. M. A., Momani, S. and Arqub, O. A. 2016. Analytical approximations of partial differential equations of fractional order with multistep approach. Journal of Computational and Theoretical Nanoscience, 13(11): 7793-7801.

Al-Smadi, M., Freihat, A., Khalil, H., Momani, S. and Ali Khan, R. 2017. Numerical multistep approach for solving fractional partial differential equations. International Journal of Computational Methods, 14(03): 1750029.

Al-Smadi, M. 2018. Simplified iterative reproducing kernel method for handling time-fractional BVPs with error estimation. Ain Shams Engineering Journal, 9(4): 2517-2525.

Al-Smadi, M. and Arqub, O. A. 2019. Computational algorithm for solving fredholm time-fractional partial integrodifferential equations of dirichlet functions type with error estimates. Applied Mathematics and Computation, 342: 280-294.

Al-Smadi, M., Arqub, O. A.and Momani, S. 2020 a. Numerical computations of coupled fractional resonant Schrödinger equations arising in quantum mechanics under conformable fractional derivative sense. Physica Scripta, 95(7): 075218.

Al-Smadi, M., Arqub, O. A. and Hadid, S. 2020 b. Approximate solutions of nonlinear fractional Kundu-Eckhaus and coupled fractional massive Thirring equations emerging in quantum field theory using conformable residual power series method. Physica Scripta, 95(10): 105205.

Al-Smadi, M., Arqub, O. A. and Hadid, S. 2020 c. An attractive analytical technique for coupled system of fractional partial differential equations in shallow water waves with conformable derivative. Communications in Theoretical Physics, 72(8): 085001.

Al-Smadi, M., Abu Arqub, O. and Gaith, M. 2021 a. Numerical simulation of telegraph and Cattaneo fractional-type models using adaptive reproducing kernel framework. Mathematical Methods in the Applied Sciences, 44(10): 8472-8489.

Al-Smadi, M., Arqub, O. A. and Zeidan, D. 2021 b. Fuzzy fractional differential equations under the Mittag-Leffler kernel differential operator of the ABC approach: Theorems and applications. Chaos, Solitons & Fractals, 146, 110891.

Andrews, L. C. 1998. Special functions of mathematics for engineers, Vol. 49. Spie Press, 512 page,New York.

Arikoglu, A. and Ozkol, I. 2007. Solution of fractional differential equations by using differential transform method. Chaos, Solitons & Fractals, 34(5): 1473-1481.

Atangana, A. and Baleanu, D. 2016. New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model. arXiv preprint arXiv:1602.03408.

Atangana, A. and Hammouch, Z. 2019. Fractional calculus with power law: The cradle of our ancestors. The European Physical Journal Plus, 134(9): 429.

Atluri, S. N. and Shen, S. 2002. The meshless method. Encino: Tech Science Press,700 page, USA.

Bagley, R. L. and Torvik, P. J. 1983. A theoretical basis for the application of fractional calculus to viscoelasticity. Journal of Rheology, 27(3): 201-210.

Bagley, R. L. and Torvik, P. J. 1986. On the fractional calculus model of viscoelastic behavior. Journal of Rheology, 30(1): 133-155.

Bahraini, S. M. S., Eghtesad, M., Farid, M. and Ghavanloo, E. 2013. Large deflection of viscoelastic beams using fractional derivative model. Journal of Mechanical Science and Technology, 27(4):1063-1070.

Baleanu, D., Golmankhaneh, A. K., Golmankhaneh, A. K. and Baleanu, M. C. 2009. Fractional electromagnetic equations using fractional forms. International Journal of Theoretical Physics, 48(11): 3114-3123.

Baleanu, D., Machado, J. A. T. and Luo, A. C. (Eds.). 2011. Fractional dynamics and control. Springer Science & Business Media, 310 page, London.

Bira, B., Raja Sekhar, T. and Zeidan, D. 2018. Exact solutions for some time-fractional evolution equations using Lie group theory. Mathematical Methods in the Applied Sciences, 41(16): 6717-6725.

Boon, J. P., Yip, S., Burgers, J. M., Van de Hulst, H. C., Chandrasekhar, S., Chen, F. F. and Smit, B. 1988. An introduction to fluid mechanics. Cambridge, UK: Cambridge University Press,371 page, London.

Chambré, P. L. 1952. On the solution of the Poisson-Boltzmann equation with application to the theory of thermal explosions. The Journal of Chemical Physics, 20(11):1795-1797.

Chawla, M. M. 2002. Generalized newmark schemes for singular second order initial-value problems. International Journal of Pure and Applied Mathematics, 1(3): 261-274.

Cortés, F. and Elejabarrieta, M. J. 2007. Finite element formulations for transient dynamic analysis in structural systems with viscoelastic treatments containing fractional derivative models. International journal for numerical methods in engineering, 69(10): 2173-2195.

El-Ajou, A., Arqub, O. A. and Momani, S. 2015. Approximate analytical solution of the nonlinear fractional KdV–Burgers equation: a new iterative algorithm. Journal of Computational Physics, 293: 81-95.

El-Ajou, O. A. Arqub and M. Al-Smadi, 2015. A general form of the generalized Taylor's formula with some applications. Applied Mathematics and Computation, 256: 851-859.

El-Ajou, 2021. Adapting the Laplace transform to create solitary solutions for the nonlinear time-fractional dispersive PDEs via a new approach. The European Physical Journal Plus, 136(2): 1-22.

Eriqat, T., El-Ajou, A., Moa'ath, N. O., Al-Zhour, Z. and Momani, S. 2020. A new attractive analytic approach for solutions of linear and nonlinear neutral fractional pantograph equations. Chaos, Solitons & Fractals, 138: 109957.

Ertürk, V. S. and Momani, S. 2008. Solving systems of fractional differential equations using differential transform method. Journal of Computational and Applied Mathematics, 215(1): 142-151.

Freihet, A., Hasan, S., Alaroud, M., Al-Smadi, M., Ahmad, R. R. and Salma Din, U. K. 2019. Toward computational algorithm for time-fractional Fokker–Planck models. Advances in Mechanical Engineering, 11(10): 1687814019881039.

Gambo, Y. Y., Ameen, R., Jarad, F. and Abdeljawad, T. 2018. Existence and uniqueness of solutions to fractional differential equations in the frame of generalized Caputo fractional derivatives. Advances in Difference Equations, 2018(1): 1-13.

Gement A.1938. On fractional differences [J]. Phil Mag, 25(1):92-96.

Griffel, D. H. 1991. Fourier series, transforms and boundary value problems , by JR Hanna and JH Rowland. Pp 354.£ 43· 65. 1990. ISBN 0-471-61983-3 (Wiley). The Mathematical Gazette, 75(473): 389-390.

Hasan, S., El-Ajou, A., Hadid, S., Al-Smadi, M. and Momani, S. 2020. Atangana-Baleanu fractional framework of reproducing kernel technique in solving fractional population dynamics system. Chaos, Solitons & Fractals, 133: 109624.

Hasan, S., Al-Smadi, M., El-Ajou, A., Momani, S., Hadid, S. and Al-Zhour, Z. 2021. Numerical approach in the Hilbert space to solve a fuzzy Atangana-Baleanu fractional hybrid system. Chaos, Solitons & Fractals, 143: 110506.

Jleli, M., Kumar, S., Kumar, R. and Samet, B. 2020. Analytical approach for time fractional wave equations in the sense of Yang-Abdel-Aty-Cattani via the homotopy perturbation transform method. Alexandria Engineering Journal, 59(5): 2859-2863.

Khader, M. M., Saad, K. M., Hammouch, Z. and Baleanu, D. 2021. A spectral collocation method for solving fractional KdV and KdV-Burgers equations with non-singular kernel derivatives. Applied Numerical Mathematics, 161: 137-146.

Khalil, H. and Khan, R. A. 2014. A new method based on Legendre polynomials for solutions of the fractional two-dimensional heat conduction equation. Computers & Mathematics with Applications, 67(10): 1938-1953.

Khalil, H. and Khan, R. A. 2015 a. The use of Jacobi polynomials in the numerical solution of coupled system of fractional differential equations. International Journal of Computer Mathematics, 92(7): 1452-1472.

Khalil, H. and Khan, R. A. 2015 b. Numerical scheme for solution of coupled system of initial value fractional order Fredholm integro differential equations with smooth solutions. Journal of Mathematical Extension, 9: 39-58.

Kilbas, A. A., Srivastava, H. M. and Trujillo, J. J. 2006. Theory and applications of fractional differential equations ,Vol. 204. Elsevier, 550 page, London.

Kiymaz, O. and Mirasyedioğlu, Ş. 2005. A new symbolic computational approach to singular initial value problems in the second-order ordinary differential equations. Applied mathematics and computation, 171(2): 1218-1225.

Komashynska, I., Al-Smadi, M., Arqub, O. A. and Momani, S. 2016. An efficient analytical method for solving singular initial value problems of nonlinear systems. Appl. Math. Inf. Sci, 10(2): 647-656.

Kumar, S. 2013. A new fractional modeling arising in engineering sciences and its analytical approximate solution. Alexandria Engineering Journal, 52(4): 813-819.

Kumar, S., Kumar, A., Samet, B. and Dutta, H. 2021. A study on fractional host–parasitoid population dynamical model to describe insect species. Numerical Methods for Partial Differential Equations, 37(2): 1673-1692.

Leung, A. Y. T., Yang, H. X., Zhu, P. and Guo, Z. J. 2013. Steady state response of fractionally damped nonlinear viscoelastic arches by residue harmonic homotopy. Computers & Structures, 121: 10-21.

Lewandowski, R. and Baum, M. 2015. Dynamic characteristics of multilayered beams with viscoelastic layers described by the fractional Zener model. Archive of Applied Mechanics, 85(12): 1793-1814.

Lewandowski, R. and Wielentejczyk, P. 2017. Nonlinear vibration of viscoelastic beams described using fractional order derivatives. Journal of Sound and Vibration, 399:228-243.

Li, Y. M., Rashid, S., Hammouch, Z., Baleanu, D. and Chu, Y. M. 2021. New Newton’s type estimates pertaining to local fractional integral via generalized p-convexity with applications. Fractals, 29(05): 2140018.

Miller, K. S. and Ross, B. 1993. An introduction to the fractional calculus and fractional differential equations. Wiley, 366 page, London.

Momani, S., Freihat, A. and Al-Smadi, M. 2014. Analytical study of fractional-order multiple chaotic FitzHugh-Nagumo neurons model using multistep generalized differential transform method. In Abstract and Applied Analysis, 2014:1-10, Hindawi.

Momani, S., Arqub, O. A., Freihat, A. and Al-Smadi, M. 2016. Analytical approximations for Fokker-Planck equations of fractional order in multistep schemes. Applied and computational mathematics, 15(3):319-330.

Odibat, Z. and Momani, S. 2009. The variational iteration method: an efficient scheme for handling fractional partial differential equations in fluid mechanics. Computers & Mathematics with Applications, 58(11-12), 2199-2208.

Oldham, K. and Spanier, J. 1974. The fractional calculus theory and applications of differentiation and integration to arbitrary order. Vol. 3, 322 page, Elsevier.

Podlubny, I. 1999. Fractional differential equations. Mathematics in science and engineering, 198: 41-119.

Rahman, H., Hasan, M. K., Ali, A. and Alam, M. S. 2021. Implicit methods for numerical solution of singular initial value problems. Applied Mathematics and Nonlinear Sciences, 6(1): 1-8.

Ramana, P. V. and Prasad, B. R. 2014. Modified Adomian decomposition method for Van der Pol equations. International Journal of Non-Linear Mechanics, 65, 121-132.

Ramos, J. I. 2005. Linearization techniques for singular initial-value problems of ordinary differential equations. Applied Mathematics and Computation, 161(2):525-542.

Rashid, S., Jarad, F., Noor, M. A., Kalsoom, H. and Chu, Y. M. 2019. Inequalities by means of generalized proportional fractional integral operators with respect to another function. Mathematics, 7(12): 1225.

Rashid, S., Jarad, F. and Chu, Y. M. 2020 a. A note on reverse Minkowski inequality via generalized proportional fractional integral operator with respect to another function. Mathematical Problems in Engineering, 2020(1):1-12.

Rashid, S., Kalsoom, H., Hammouch, Z., Ashraf, R., Baleanu, D. and Chu, Y. M. 2020 b. New multi-parametrized estimates having pth-order differentiability in fractional calculus for predominating $\hbar$-convex functions in Hilbert space. Symmetry, 12(2): 222.

Rashid, S., Hammouch, Z., Baleanu, D. and Chu, Y. M. 2020 c. New generalizations in the sense of the weighted non-singular fractional integral operator. Fractals, 28(08): 2040003.

Razzaghi, M. and Yousefi, S. 2001. The Legendre wavelets operational matrix of integration. International Journal of Systems Science, 32(4): 495-502.

Saadeh, R., Alaroud, M., Al-Smadi, M., Ahmad, R. R. and Salma Din, U. K. 2019. Application of fractional residual power series algorithm to solve Newell–Whitehead–Segel equation of fractional order. Symmetry, 11(12): 1431.

Talafha, A. G., Alqaraleh, S. M., Al-Smadi, M., Hadid, S. and Momani, S. 2020. Analytic solutions for a modified fractional three wave interaction equations with conformable derivative by unified method. Alexandria Engineering Journal, 59(5): 3731-3739.

Rehman, M. and Khan, R. A. 2011. The Legendre wavelet method for solving fractional differential equations. Communications in Nonlinear Science and Numerical Simulation, 16(11): 4163-4173.

Wang, Y. and Fan, Q. 2012. The second kind Chebyshev wavelet method for solving fractional differential equations. Applied Mathematics and Computation, 218(17): 8592-8601..052.

Yi, M. and Chen, Y. 2012. Haar wavelet operational matrix method for solving fractional partial differential equations. Computer Modeling in Engineering & Sciences(CMES), 88(3): 229-243.

Zhou, S. S., Rashid, S., Parveen, S., Akdemir, A. O. and Hammouch, Z. 2021. New computations for extended weighted functionals within the Hilfer generalized proportional fractional integral operators. AIMS Mathematics, 6(5): 4507-4525.

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