

DESIGN-OBJECTIVE SPACE EXPLORATION AND MULTI-OBJECTIVE  
OPTIMIZATION OF INITIAL STRUCTURAL DESIGN ALTERNATIVES VIA  
MACHINE LEARNING

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OPTIMIZATION OF INITIAL STRUCTURAL DESIGN ALTERNATIVES  
VIA MACHINE LEARNING**

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## **ABSTRACT**

### **DESIGN-OBJECTIVE SPACE EXPLORATION AND MULTI-OBJECTIVE OPTIMIZATION OF INITIAL STRUCTURAL DESIGN ALTERNATIVES VIA MACHINE LEARNING**

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Master of Science, Building Science in Architecture

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Increasing implementations of digital workflows within design processes generate exponentially growing data in each phase. Therefore, decision making within a design space with growing complexity is expected to be a great challenge for designers in the future. Hence, this research aimed to seek the potentials of complex relations between data within design space and objective space of structural design problems for proposing a novel approach to augment capabilities of digital tools by artificial intelligence. As a method, a machine learning-based framework was proposed that can help designers to understand the trade-offs between initial structural design alternatives to make informed decisions. The proposed framework was tested in three stages: probabilistic, deterministic, and integrated; all of which allow users to conduct optimization studies with the help of different machine learning models. Finally, the proposed approaches were presented in case studies, and potentials/limitations of the models were discussed including future projections.

**Keywords:** Design Space, Objective Space, Structural Analysis, Multi-Objective Optimization, Artificial Intelligence, Machine Learning

## ÖZ

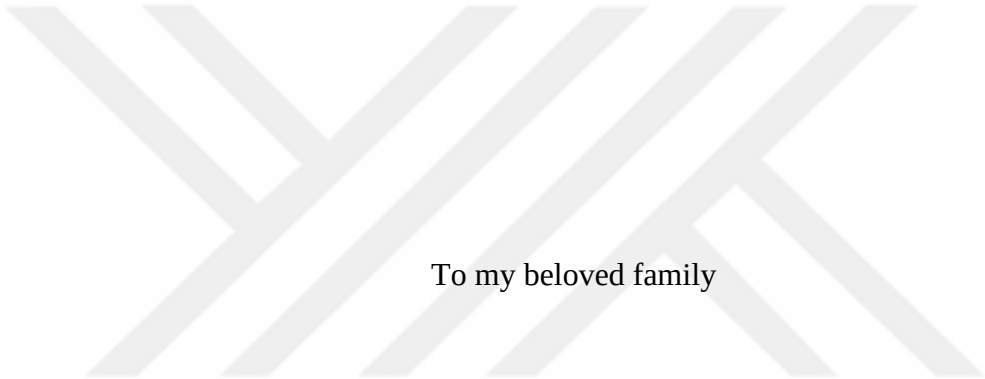
### **MAKİNE ÖĞRENMESİ İLE TASARIM-HEDEF UZAMININ ANALİZİ VE ERKEN TASARIM AŞAMASINDAKİ STRÜKTÜRLERİN ÇOKLU KRİTERLE OPTİMİZASYONU**

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Tasarım süreçlerinde dijital iş akışlarının artması, her aşamada katlanarak büyüyen veriler üretmektedir. Bu nedenle, karmaşıklığı giderek artan bir tasarım probleminde karar vermenin gelecekte tasarımcılar için büyük bir zorluk yaratması beklenmektedir. Bu araştırma, yapay zeka ile dijital araçların yeteneklerini artırmada yeni bir yaklaşım önererek tasarım uzamı içindeki veriler ile tasarım problemlerinin hedef uzamı arasındaki karmaşık ilişkileri kullanmayı amaçlamıştır. Yöntem olarak, tasarımcıların bilinçli kararlar vermek için ilk yapısal tasarım alternatifleri arasındaki ödünleşimleri anlamalarına yardımcı olabilecek makine öğrenimine dayalı bir çerçeve önerilmiştir. Önerilen çerçeve, farklı modellerle optimizasyon yapma imkanı tanıyan üç farklı yaklaşımla test edilmiştir: olasılıksal, belirlenimci ve tümleşik. Son olarak, önerilen yaklaşımlar vaka çalışmaları ile sunulmuş ve modellerin mevcut ve gelecekteki potansiyelleri / sınırlamaları tartışılmıştır.

Anahtar Kelimeler: Tasarım Uzamı, Hedef Uzamı, Yapısal Analiz, Çoklu-Kriter Optimizasyonu, Yapay Zeka, Makine Öğrenmesi



To my beloved family

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## **LIST OF ABBREVIATIONS**

### **ABBREVIATIONS**

AI: Artificial Intelligence

ML: Machine Learning

ANN: Artificial Neural Network

MOO: Multi-Objective Optimization

CAD: Computer Aided Design

GA: Genetic Algorithm

DT: Decision Tree

HCA: Hierarchical Cluster Analysis

KNN: K-Nearest Neighbors

NB: Naive Bayes

PCA: Principal Component Analysis

RF: Random Forest

SOM: Self Organizing Map

SVM: Support Vector Machine

## LIST OF SYMBOLS

### SYMBOLS

$S$  : Total span of the truss system

$t$  : Thickness of the cross-section

$\emptyset_{outer}$  : Outer diameter of the cross-section

$E_{steel}$  : Modulus of elasticity for steel

$d_{steel}$  : Density of the steel

$L_i$  : Length of the steel member

$F_j$  : Force applied to the node

$P_j(x_j, y_j)$  : Cartesian coordinates of nodes

$P_k(x_k, y_k)$  : Cartesian coordinates of supports

$h$  : Height of the truss system

$m$  : Total mass of the truss system

$N$  : Axial force of the truss member

$\Delta$  : Deflection of the truss



## CHAPTER 1

### INTRODUCTION

Digital media are being used in almost every phase of a design process from sketching to simulations, and consequently, each user generates a vast amount of data in each phase. Since the total amount of generated data grows exponentially (Figure 1.1), deriving information from this data will be a great challenge for the future of digital methods (Szalay & Gray, 2006). This research aims to propose a novel approach for design-space exploration by using machine learning. Multi-objective optimization methods are integrated in the proposed model as a decision support tool for the designer to make a choice in the design space which contains complex and huge number of data beyond our number crunching capacity. Alpaydın denoted this problem in his book as “Already we are exposed to more data than what our sensors can cope with, or our brains can process.” and proposes machine learning models to make sense of the complex data (2016).

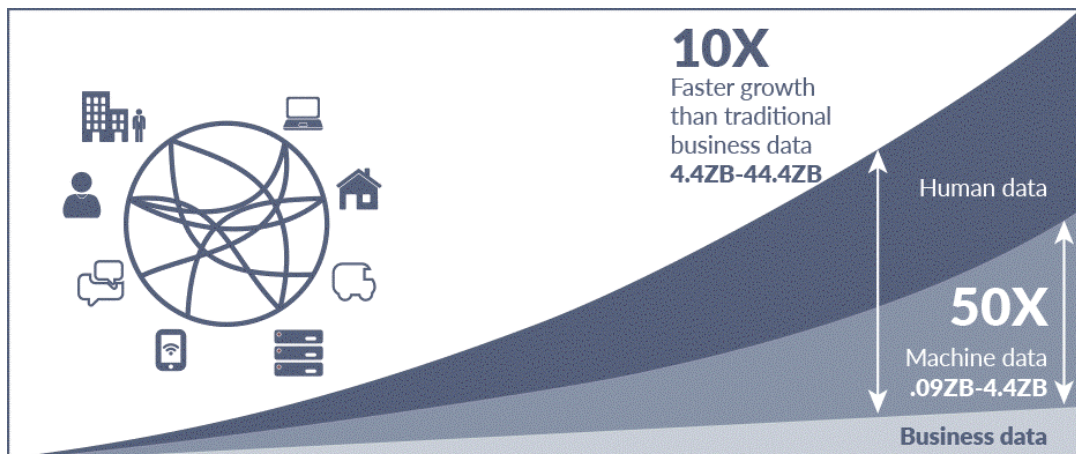


Figure 1.1. Exponential growth of data generated by humans (Ffoulkes, 2017)

In a design process, it may be a very challenging task for designers to fully understand the related variables within data having potentials to generate all possible solutions to a design problem. The very same challenge also exists to understand the outputs within data that define all the possible/expected outcomes for each design solution. This research focuses on the complexity of relations between data within the conceptual design phase of a structural design problem and the potentials of machine learning methods to extract information from that data.

## **1.1 Motivation**

As mentioned above, the primary motivation of this research is to propose a novel approach to handle the increasing complexity of design problems and to support decisions made in the initial stage of the design and modeling process. The proposed approach is exemplified on structural design for which major modeling decisions affect the physical performance of the building.

Besides, another incentive is the increasing number of data generated in the design processes and the importance of turning data into information. Hence, the machine learning approach was proposed to extract information from the data within the scope of this research in order to propose a generic framework that provides insight to the designer within the initial design process of structures.

## **1.2 Problem Statement**

The conceptual structural design phase of an architectural project is a very critical stage since decisions made during that stage affects the whole process. Thus, a designer must consider parameters (such as dimensions, topology, material, geometry) that control the possible instances of a design problem, namely the design space. However, the complexity of a design space increases exponentially as new parameters involved. Therefore, the complexity of data can exceed the limits of the human brain's data processing capacity since each parameter will add another

dimension to the design space, which can only be defined in n-dimensional hyperspace.

In addition to parameters that control the design space, performance outputs of possible alternatives (such as strength, cost, construction time) that defines the objective space of a design problem are also very critical for the designer to select the appropriate design solution with intended outcomes for the given design problem. Since the conceptual structural design phase involves multi-objective criteria and each criterion will also add another dimension to the objective-space, the complexity of the n-dimensional objective space of a design problem can also exceed the crunching capacity of human brain.

Hence, making sense of data within an initial design problem is a great challenge for designers. Since new parameters and objectives are getting involved as the process advances, the complexity will be more challenging. Therefore, extracting information from the data will be a crucial issue.

### **1.3 Disposition**

The aim of the research can be put as to improve the process of design space exploration simultaneously with objective space search. Design space exploration enables designers to see possible alternatives within an initial structural design problem. Similarly, objective space search was aimed to enable designers to get performance feedbacks of different design alternatives in the initial stage of the structural design. Therefore, this research proposes a methodology that enables users to derive correlations between design space and objective space and allow designers to understand the trade-offs to make informed decisions.

In order to discover the disposition of the research in a structured way, the following research questions were formulated as follows:

- Can parameters that control the design alternatives used to generate the design space of a structural design problem?

- Can criteria that control the performance of the design alternatives used to construct the objective space of a structural design problem?
- Can machine learning algorithms trained to extract features and derive correlations between design space and objective space to help the designer to explore trade-offs within the solution space of a structural design problem?

In the light of these research questions, the hypothesis of this research can be stated as, design and objective space data generated within initial structural design phase can be used to train machine learning models which can extract features and derive correlations in these data to make valid interpolations between design instances. Hence, it can guide the designer not to a single solution but to multiple solutions which can be explored together with trade-offs. Consequently, process of conceptual structural design and modeling will be assisted by artificial intelligence and led the designer to make informed decisions.

#### **1.4 Aim and Objectives**

The first objective of this research is to enhance the process of design space exploration and enable designers to see possible alternatives within a structural design problem in a fast and accurate way by using the computer's data crunching capacity.

Another objective of this study is to improve the process of objective space exploration and enable designers to get performance feedback on different design alternatives in the initial stage of structural design via simulations. Therefore, this research targets to derive correlations between design space and objective space so that complex data within the process can be transformed into analytical information via machine learning models. These models enable designers to understand the trade-offs of a multi-objective design problem and make informed decisions within the initial structural design stage.

## **CHAPTER 2**

### **LITERATURE REVIEW**

Initial design process is a very challenging stage in the field of architecture since it embraces highly complex solution spaces and objectives. Design process is described as an ill-structured problem-solving process by Herbert Simon (1973) since it depends on an assumption-based decision mechanism. On the other hand, structural design phase of an architectural project can be described as a well-structured problem-solving process since it depends on a rule-based and concrete decision mechanism. Therefore, this relation between an ill-defined and well-defined problem-solving lead designer to make assumptions first to reach a solution, and then validate the solution after it is achieved (Alfaris & Merello, 2008).

However, this approach may subject to change with recent developments in computational tools and performance analysis software that enable designers to extract information from both ill-defined and well-defined phases of a design which can guide them in decision making. Yet, insight obtained through these methods may still remain inadequate due to their strategy of searching the solution space. Additionally, since performance analysis tools are usually discipline specific, their ability to supply clear understanding of the possible tradeoffs between different objective requirements is also questionable. Therefore, proposing a plausible computational design framework requires the development of novel solution space search mechanisms and novel performance evaluation approaches for their different objective features (Figure 2.1).

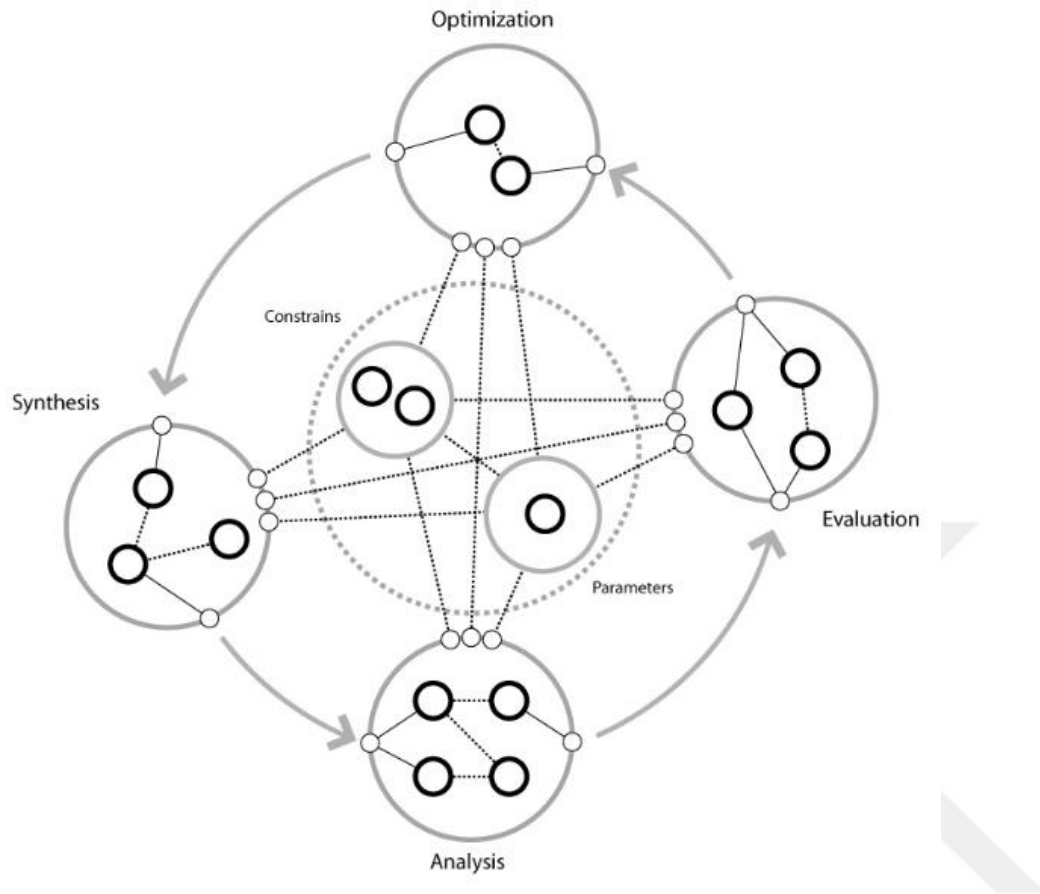


Figure 2.1. Generative multi-performance design system (Alfaris and Merello, 2008)

To propose a novel framework for design-space exploration and multi-objective optimization, understanding the complex relations between data generated during the process of exploration, optimization and decision making is a crucial step. Algorithms that can extract information from complex datasets cannot be implemented without analyzing and understanding the process. Importance of understanding the data before implementing the algorithm is stated in the book *Machine Learning: The New AI in a very neat way*: “Data starts to drive the operation; it is not the programmers anymore but the data itself that defines what to do next.” (Alpaydin, 2016). So, to implement machine learning methods in a design process necessitates the analysis of generated data in the design process. Hence, this literature review aims to understand the relations between design/objective space data, multi-objective optimization strategies using this data, and the potentials of

machine learning models to make sense of the data within design processes. In order to understand the mentioned key points, subject clusters are analyzed in the current literature using the Web of Science database and visualized by using VOSviewer (Van Eck and Waltman, 2010). Using the visual network map, sections under this chapter are structured on parametric/generative design, simulation/optimization, and artificial intelligence/machine learning since they represent most significant clusters within the current literature network (Figure 2.2).

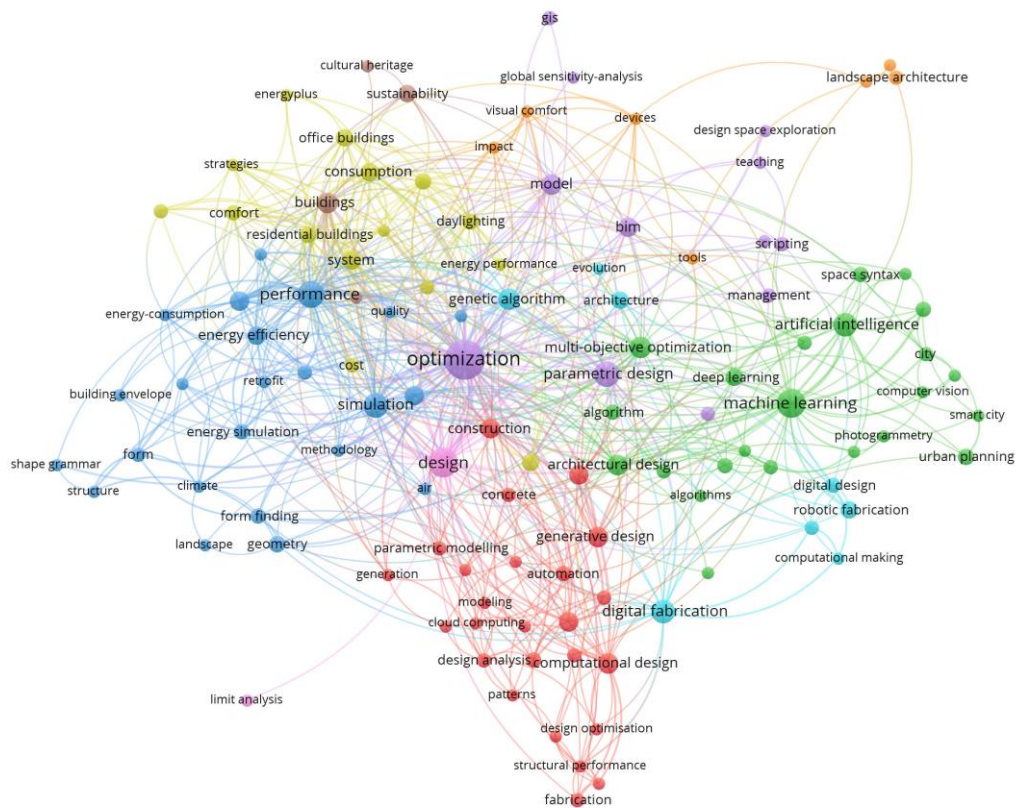


Figure 2.2. Network graph of the current literature of design, optimization, and AI (developed by the author)

## 2.1 Design Space and Objective Space Exploration

Design space in mathematics is defined as the set of all possible alternatives of a problem while objective space is defined as the set of all possible outcomes (Cámara

et al., 2012). Design and objective space of an initial structural design problem generally starts with simple geometry data and iteratively more data is added to the model such as material, physical properties, cost, time and more as the process advances. Therefore, it can easily get very complex and hard to process by human brain. Yet, contained data within a digital model can be processed via algorithms and it can improve the exploration process. Therefore, in this section of the literature review, the scope of the selected research is focused on studies related with improvement of the exploration process of generated alternatives in the architectural/structural design domain.

For instance, Brown and Mueller (2017), studied the design space catalogue (Figure 2.3) which is a collection of different options for designers to choose. They also show how the exploration can be enriched by designing with data. As a method, they reviewed the developments in four main categories that are interaction, automation, simplification, and visualization. Then they discussed that human-computer interaction should be integrated into early design stage and even in the initial brainstorming phase. Finally, they concluded that human-computer collaboration can harness the power of computational processes together with the creative capacities of designers to generate workflows that answer to the complexity of the architecture.

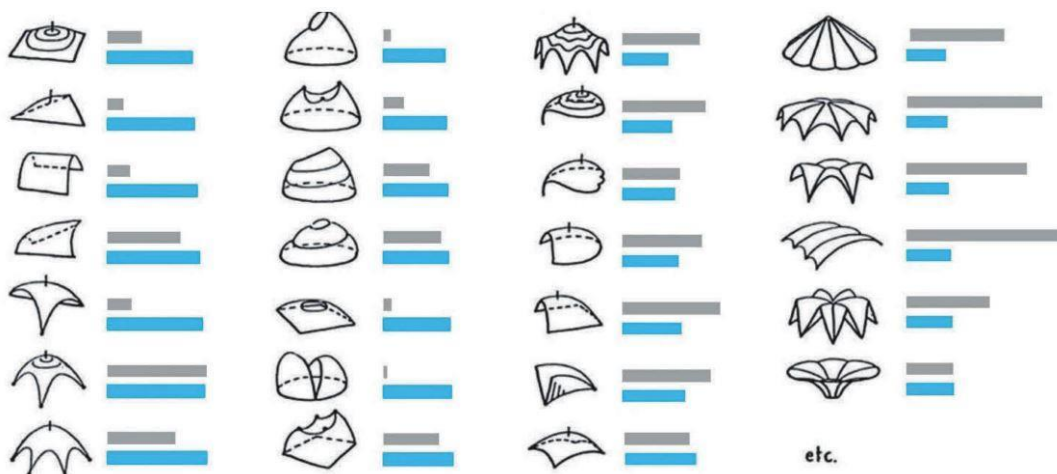


Figure 2.3. An example design space catalogue (Brown and Mueller, 2017)

The importance of their study is that it shows the variations within the solution space of a design problem and proposes a creative workflow via human-computer interaction. Yet, the workflow only proposes a design space catalogue that includes the design space parameters and objective space scores of the randomly generated design alternatives. In this case, exploration of the complex design space is not fully aided but generated by the computer.

In another study of Brown and Mueller (2017), they proposed an automated performance-based design space solution for parametric structural design where a large volume of data generated during the process of structural design and modelling. They proposed a two-step application to make the design process more viable and to perform with better outcomes. As the first step, they analyzed the design variables in the problem, ranked their importance and determined which ones should be eliminated or emphasized during the exploration process. In the second step, they clustered the design options into families so that designers can visit them anytime in their exploration. Finally, they developed components that demonstrate the benefits, limitations and future possibilities to enable designers to focus on high-performance regions within the structural solution space.

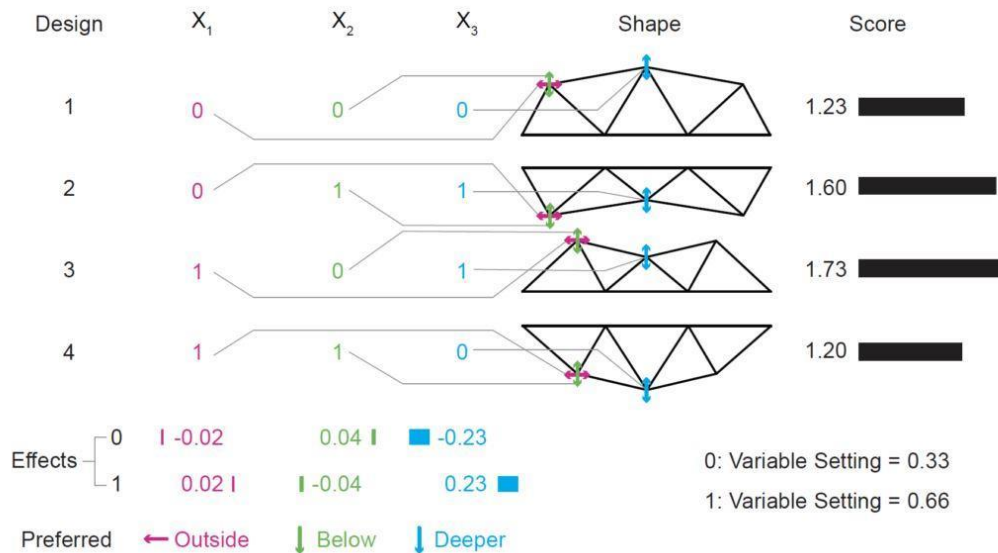


Figure 2.4. Generated trusses and their performance (Brown and Mueller, 2017)

Another research done by Villaggi et al. (2017), proposed a novel design space approach to be used in generative space planning in architecture. The model they developed is based on a data structure that allows subdivision and merge operations on a floor plan controlled by a smaller set of input parameters. Also, model includes evaluation for the performance of generated floor plans using a set of congestion metrics that allow them to be explored by a genetic algorithm (Figure 2.5). Researchers also presented a set of guidelines and methods for analyzing and visualizing the performance of the model in the solution space. Finally, authors concluded that the developed model could allow designers to break free of common rules and explore a wider range of design possibilities.

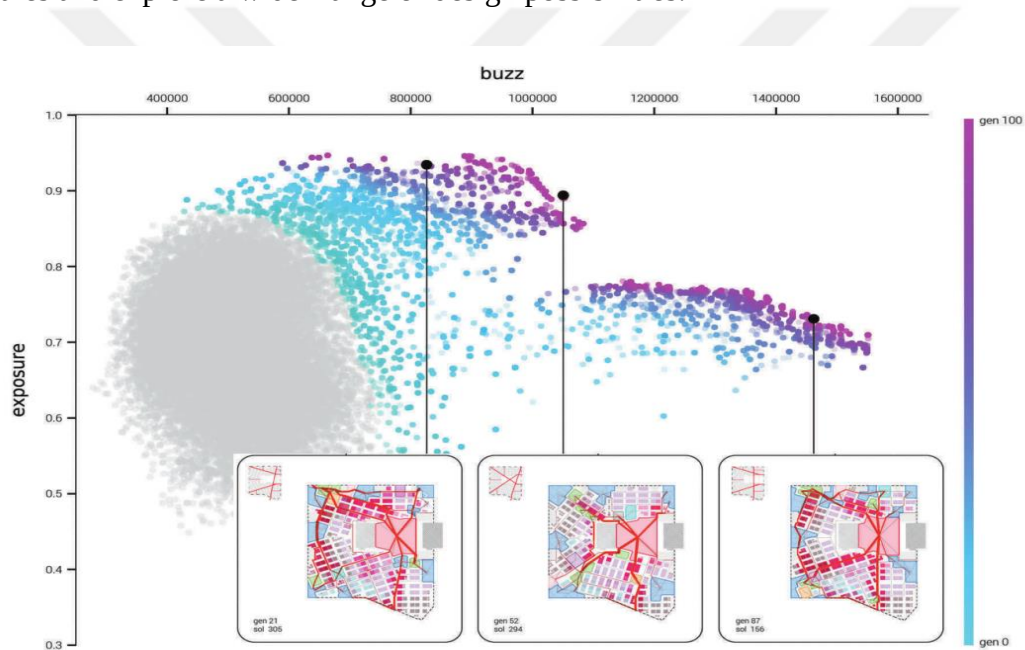


Figure 2.5. Results of alternatives by genetic algorithm (Villaggi et al., 2017)

In both cases, the proposed workflow is more integrated with algorithms and as a result, exploration of the design space and objective space is more correlated. Yet, the designer is still not aided in the process of selecting a design option that satisfies the multi-objective design criteria but left with the randomly generated set of design alternatives and their performances.

In a different study conducted by Loos and De Laet (2018), they proposed an interactive design process by the help of real-time data visualizations (Figure 2.6). They provided a tool for creating interactive graphics for each different design proposals to enable designers to make structurally informed decisions. They also presented the framework of the tool with a case study of different structural design alternatives for a stadium roof.

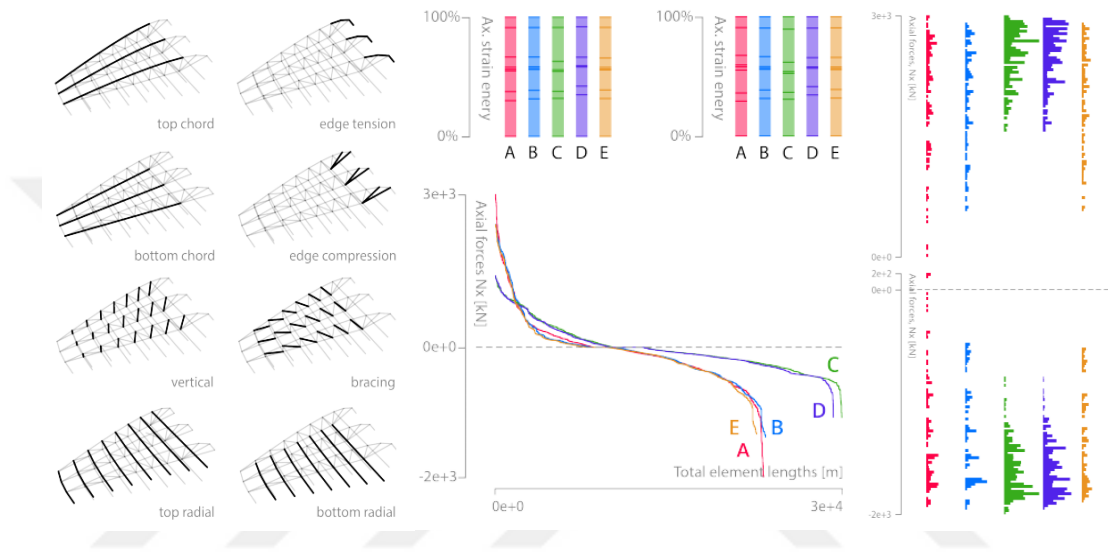


Figure 2.6. Data visualizations of structural elements (Loos and De Laet, 2018)

In this research, the interactive workflow for the performance feedback of design alternatives enables designers to explore design space freely since it is not constrained by randomly generated alternatives of a genetic algorithm. Yet, understanding the trade-offs among the design space alternatives and making an optimized multi-objective criteria decision remains as a big challenge for the designers.

## 2.2 Multi-Objective Optimization of Design Alternatives

Optimization is defined as the selection of the best element from a set of available alternatives (Wright, 2016). Within the field of architecture, it can be argued that optimization is a very hard process since many trade-offs, back-and-forth iterations

emerge during the process. Even Daniel Libeskind who had the chance to make most of his conceptually characteristic projects be built said in an interview that: "Architecture is a field of repression. You repress almost everything to produce a building. Everything is repressed because it has to fit into the context, it has to appeal the clients, it has to be normal." (Korody, 2015). However, as a result of the advances in technology, optimization process becomes more seamless and a new paradigm called cyber-physical emerged (Figure 2.7). Menges (2015) stated that this new paradigm in architecture will be a major catalyst for design innovation and interlinked, cross-monitored and feedbacked procedures will result in a fully integrated design process. Therefore, research reviewed in this section of the literature survey is limited with the studies that aims to achieve a seamless optimization process in early design stages and feedforward/feedback strategies between design/objective space are investigated.

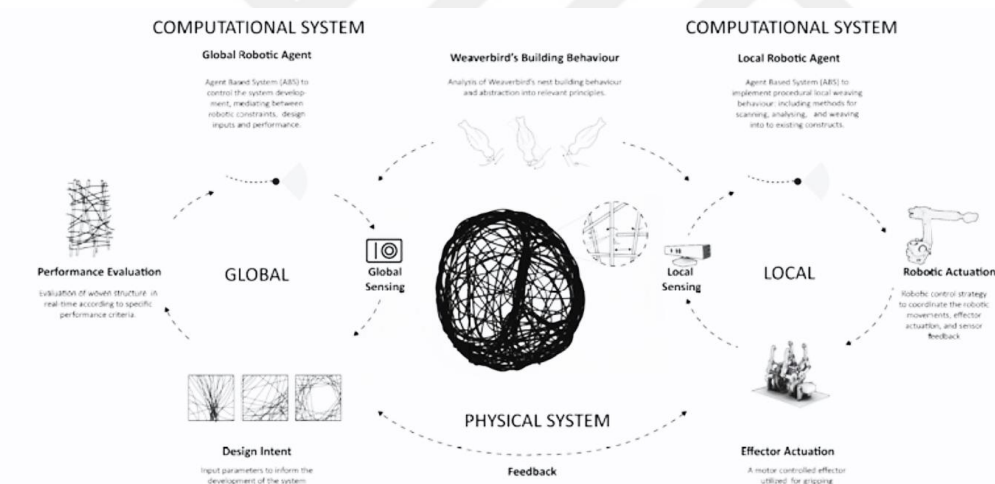


Figure 2.7. Cyber-physical making process in architecture (Menges, 2015)

For instance, in a research conducted by Kazi et al. (2017), authors proposed a novel interface called DreamSketch to integrate free-form and expressive sketching environment with the computational strength of generative algorithms. As a method, input was designated for the algorithm as coarsely defined problem via a user sketching the design with its context. After that, a generative algorithm produces

multiple design options which are augmented as 3-dimensional objects in the sketch. For the optimization process, they proposed an interface where users were allowed to interact with the generated options, navigate through, manipulate the variables and iteratively refine design solutions in the very early phases of the process (Figure 2.8). Finally, they conducted design study sessions with designers and engineers to test the DreamSketch interface and concluded that the developed hybrid environment helps the user in experiencing solution ideas that may not have been considered by the designer.

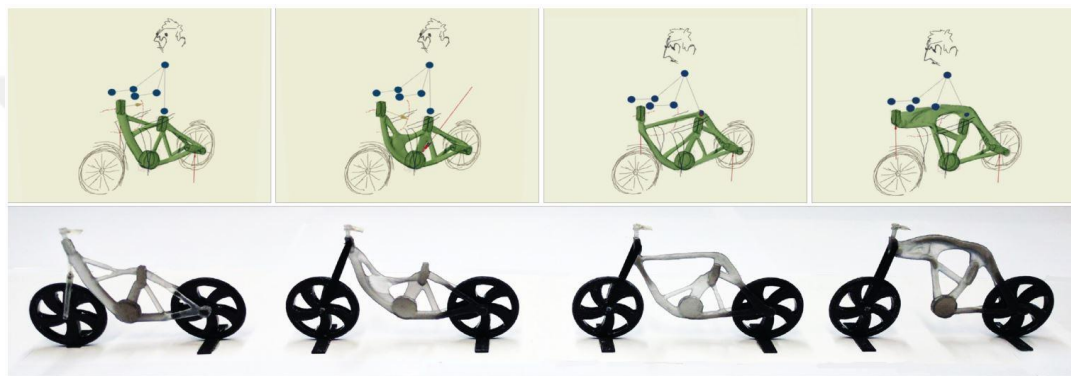


Figure 2.8. DreamSketch interface and interactive sketches (Kazi et al., 2017)

Pan et al. (2019) in their research, proposed an automated workflow that merges structural analysis and a multi-objective optimization engine for the exploration of an extensive design space which can help the informed decision-making process. The proposed workflow was composed of a flexible parametric model, a framework of interdisciplinary assessment criteria, and multi-objective optimization (MOO). The framework developed enables designer to use the multi-objective optimization engine. In the proposed engine, there are various indicators employed to assess the performance of the design regarding spectators, acoustics, and more. Results of the study were illustrated in real cases. The proposed method is implemented in the Barclay Centre and the O2 Arena sports center to validate the effectiveness of the process and the necessity of the broader design space (Figure 2.9).

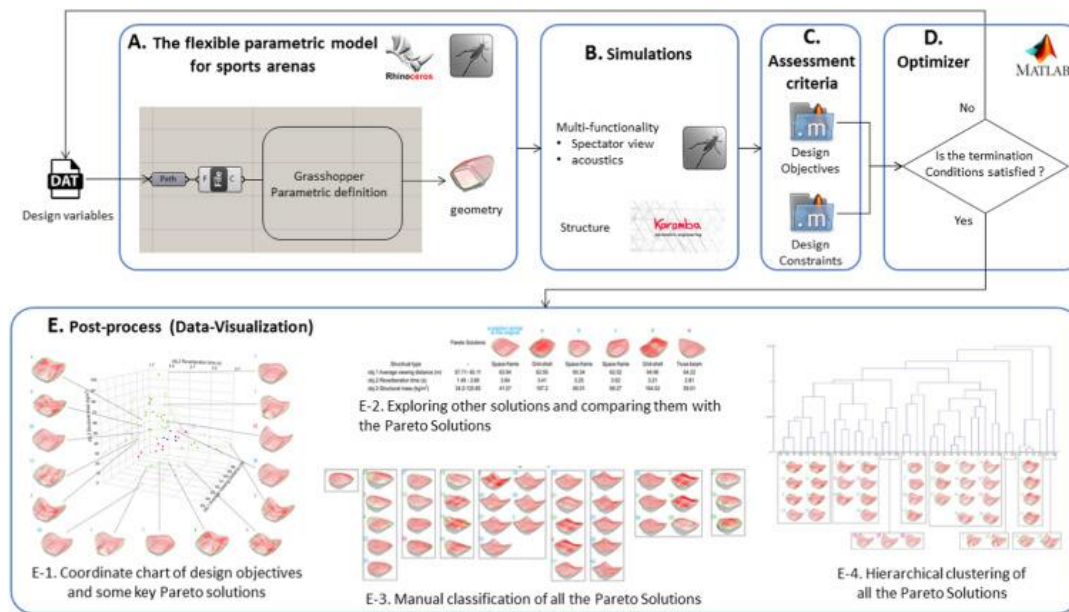


Figure 2.9. Design exploration and optimization process (Pan et al., 2019)

In another study conducted by Villaggi et al. (2018), they proposed a survey-based simulation of user satisfaction for generative design in architecture. Their novel approach is the measurement and simulation of user satisfaction of 270 people working in a 4550 square meter office space. As a method, they first conducted surveys to extract data about their adjacency preference and work style. Then, they calculated how different floor plan layouts match their preferences well. Finally, they generated a large variety of valid plan layouts and implemented a genetic algorithm to find a set of optimal designs according to the preferences of occupants (Figure 2.10).



Figure 2.10. Optimizing solution according to the survey results (Villaggi et al., 2018)

As these studies on optimization point out, evaluating design space instances to select an optimal solution for a given problem is critical in any design process. The necessity of optimization within the design process can also be seen in the result of a survey conducted with architects from 34 different countries by Chichocka et al. (2017). According to the survey results, 91 % of architects declared that optimization is a necessity in architectural practice and 78 % of architects declared that optimization process should be multi-objective. In addition, when the participants

were asked which aspects of their design they wanted to optimize, structure is selected more than any other aspect of a design by 20.95 % (Figure 2.11).

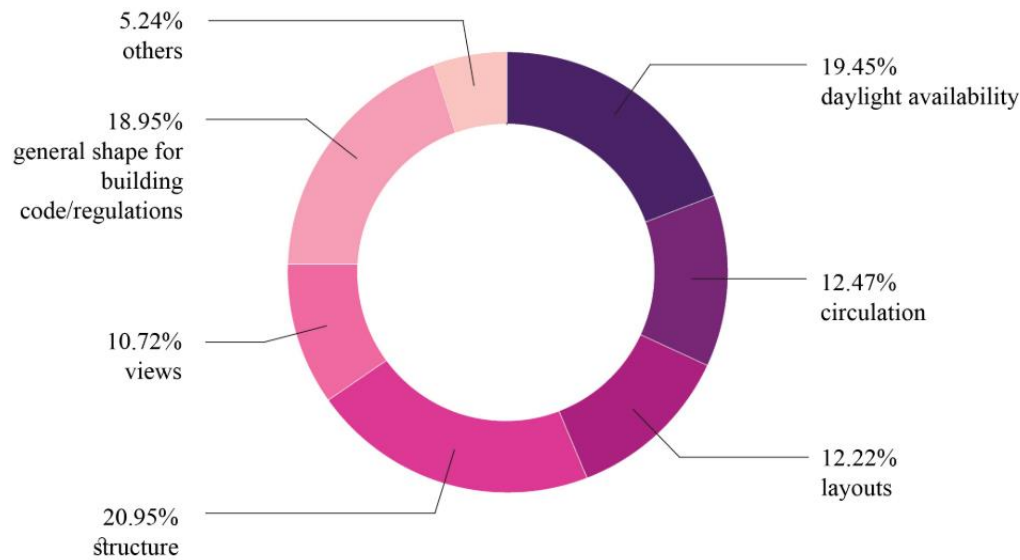


Figure 2.11. Features selected by participants to be optimized (Chichocka et al, 2017)

The term optimization is about finding the best solution but since every design process is unique, criteria to be computed by the algorithm should not dictate ‘the solution’ to a designer. For this problem, Minsky (1988) proposes the need for an additional mechanism which is called as the progress principle. It is an optimization system that leads or guides the search process rather than generating all possible solutions indiscriminately. Here, the interactivity and the adaptive behavior of the algorithm is important. Criteria set that evaluates the performance of design alternatives should allow the designer to control the trade-offs since the aim of this research is a multi-objective design space exploration rather than proposing a single optimization result. This is also supported by architects participated in the survey conducted by Chichocka et al. (2017) by declaring that they would like to influence outcomes by choosing promising designs during optimization process (90.7 %). It is also stated in the survey that architects would prefer to get a few high-quality solutions rather than one best design solution as an optimization result (82.2 %).

### **2.3 Potentials of Machine Learning to Extract Information from Data**

Machine learning is defined as “A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E.” by Mitchell (1997). In other words, machine learning is a model that has the capability of learning defined tasks and improve its performance with the experience. Since the experience of the algorithm is the only way to increase its performance measure, machine learning algorithms are often trained with large and complex datasets that would provide enough learning samples for the model.

Today, machine learning is one of the biggest research areas in the field of artificial intelligence since it includes various methods such as supervised learning, unsupervised learning, deep learning, reinforcement learning and more. Machine learning algorithms have the ability to learn many different tasks such as recognizing people from their faces, understanding speech, driving a vehicle, suggesting a movie and more as mentioned by Alpaydın (2016). In this part of the literature review, potentials of machine learning models to be trained with design and objective space data to extract information which can be used in multi-objective optimization of a structure will be discussed. Therefore, selected studies are not only limited with the architectural design domain as done in previous sections, but studies on machine learning to correlate design/objective space data of different fields and data design strategies are also included.

For example, in a research conducted by Tsanas and Xifara (2012), regression algorithms were used to estimate the energy performance of residential buildings. As a method, researchers first collected the input data from the Ecotect simulation software that includes 8 variables namely relative compactness, surface area, wall area, roof area, overall height, orientation, glazing area and glazing area distribution. After collecting the data, they conducted data analysis using non-parametric statistical analysis tools to investigate which of input variables are significantly related to the output values. Then, they trained a random forest algorithm with the

preprocessed data and estimated the heating load and cooling load of 768 different residential buildings (Figure 2.12). Finally, performance measures revealed that the chosen regression algorithm could predict heating load and cooling load of given buildings with an absolute error deviation of 0.51 and 1.42 from the ground truth which is quite successful.

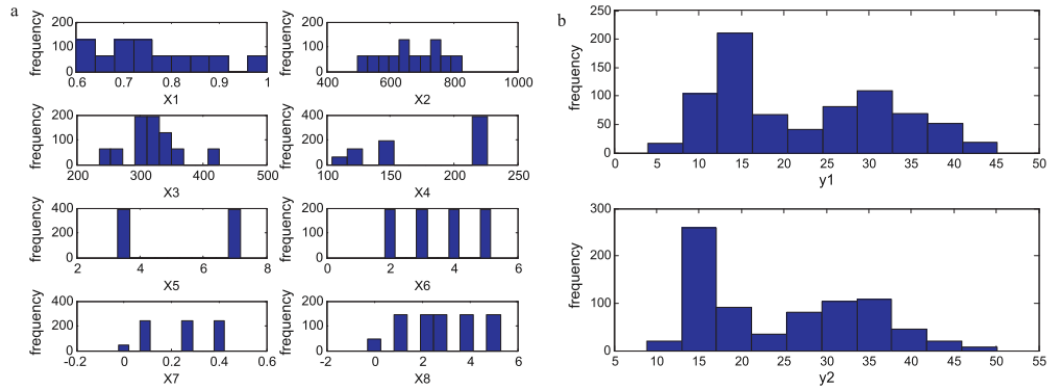


Figure 2.12. Estimation of the input and output variables (Tsanas & Xifara, 2012)

It is an important research that shows the data within design space parameters (relative compactness, surface area, wall area, roof area, overall height, orientation, glazing area and glazing area distribution) and data within objective space goals (heating load and cooling load) can be used to train machine learning models and correlation between design space and objective space can be learned by these algorithms.

In another research done by Umetani (2017), a machine learning algorithm was used to combine the orthogonal depth map computation and the shrink-wrapping approach to parameterize the triangle geometry of input meshes. Then, an autoencoder network was used to extract the topology of shapes in the same category to explore and synthesize a variety of shapes (Figure 2.13). Furthermore, a direct geometry manipulation user interface to navigate the geometry synthesis was presented with over one thousand generations of 3D car shapes as triangular meshes.

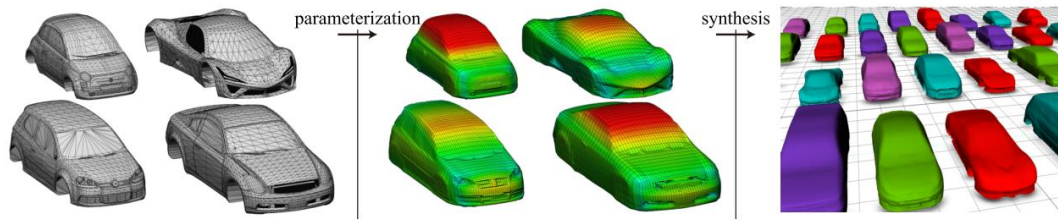


Figure 2.13. Parametrization and synthesis process of car shapes (Umetani, 2017)

In a similar research conducted by Yumer et al. (2015), they proposed an interface that allows users to create geometric modifications by using a set of semantic attributes which can help designers to explore shape variations. Method for applying machine learning in this research was crowdsourcing a large set of pairwise comparisons between the semantic attributes and geometry. It is aimed to be used for learning a continuous mapping from semantic attributed to geometry. Hence geometries can easily be modified by the interface without any expertise (Figure 2.14). Results of the research is a novel approach enabling users to manipulate 3D shapes by using a set of attribute sliders or by navigating a set of attribute maps which can be adapted to evolving shape preferences, different user bases, and new semantic attributes.

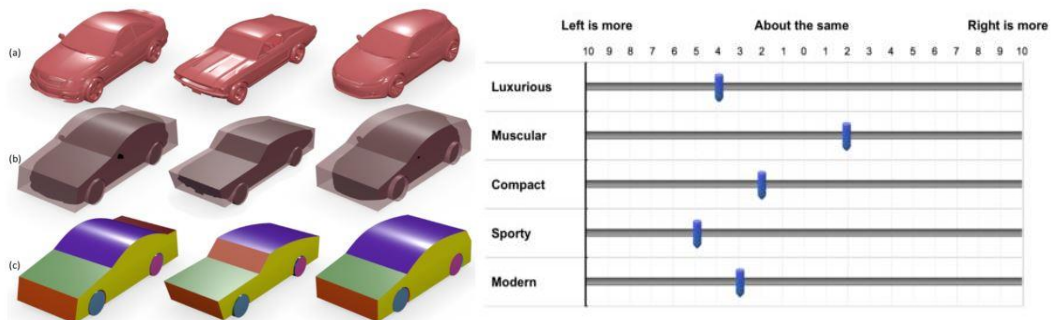


Figure 2.14. Modified geometries and semantic attributes of cars (Yumer et al., 2015)

As the literature survey indicates, machine learning is started to be implemented in the design processes recently, with studies having very promising results and therefore it offers great potentials to be studied. Since machine learning is an extensive research area that involves several different algorithms with the capability

of learning various tasks, it can be implemented in many disciplines including the architecture. Also, numerous features of data can be collected from the design processes even the process itself is very complex. To conclude, reviewed studies in the literature showed that Machine learning in design process offers great potentials to further improve design process and design decisions.

## **2.4 Critical Analysis of the Literature**

As presented in the literature review, previous studies conducted within the design and objective space exploration field were mainly focused on the parametrization of design problems in order to control instances and generating a catalogue in which design alternatives are accessible by designers to explore. Yet, since studies on design and objective space exploration proposed algorithms that use only cross products of design parameters and randomly generate a catalogue of design instances, the exploration process within such a huge number of design instances remains as a gap in the literature. Hence, this research positions itself within this gap in the literature to propose an adaptive framework that can give insights to the designer about the generation process rather than leaving designer with only a catalogue of randomly generated numerous design instances.

Secondly, previous studies on multi-objective optimization of design alternatives were also focused on the parametrization of design problems while the aim was not to generate different instances from these parameters but to tune the parameters to meet the objectives of the designer. Yet, since multi-objective optimization algorithms were only programmed to tune the parameters of a given design problem with respect to given objectives, it converges to a single solution. Therefore, while the optimization algorithms solve the problem of exploration algorithms regarding how to choose among numerous alternatives, it creates another problem which constrains the creative space of a designer. Regarding this fact, this research positions itself in the gap between exploration and optimization that aims to propose a framework for designers to understand the trade-offs between design alternatives

and make informed decisions rather than choosing intuitively among randomly generated instances or yielding to the decision of the optimization algorithm.

Finally, previous researches on machine learning that focus on deriving information from data were reviewed since they were mainly studies both in between the exploration and the optimization of a problem which stands on the scope of this research. Yet, even reviewed studies conducted within the field of machine learning proves the effectiveness and adequateness of solutions for exploration and optimization. They were aimed to propose generic solutions for geometry processing and data processing without any intervention to any design field including architecture.

In this part of the study, the exploration and optimization potentials of machine learning studies are exemplified in initial design process of architectural structures with the selected studies.



## CHAPTER 3

### MATERIALS AND METHOD

In this thesis, the research subject is selected as the process of structural design in the broadest context. The problem within the process of designing a structure comes from the dichotomy between the desire for freedom of a designer, and the requirements for strict rules i.e. constraints and objectives of a well-defined structural problem. Therefore, considering this opposition between designer's will for freedom in design decisions and the well defined requirements of the structural problem, a new methodology is proposed to allow designers to explore and choose the structural solutions as they wish, without sacrificing any structural requirements in a design space. This compromise is illustrated in Figure 3.1 as the in-between area.

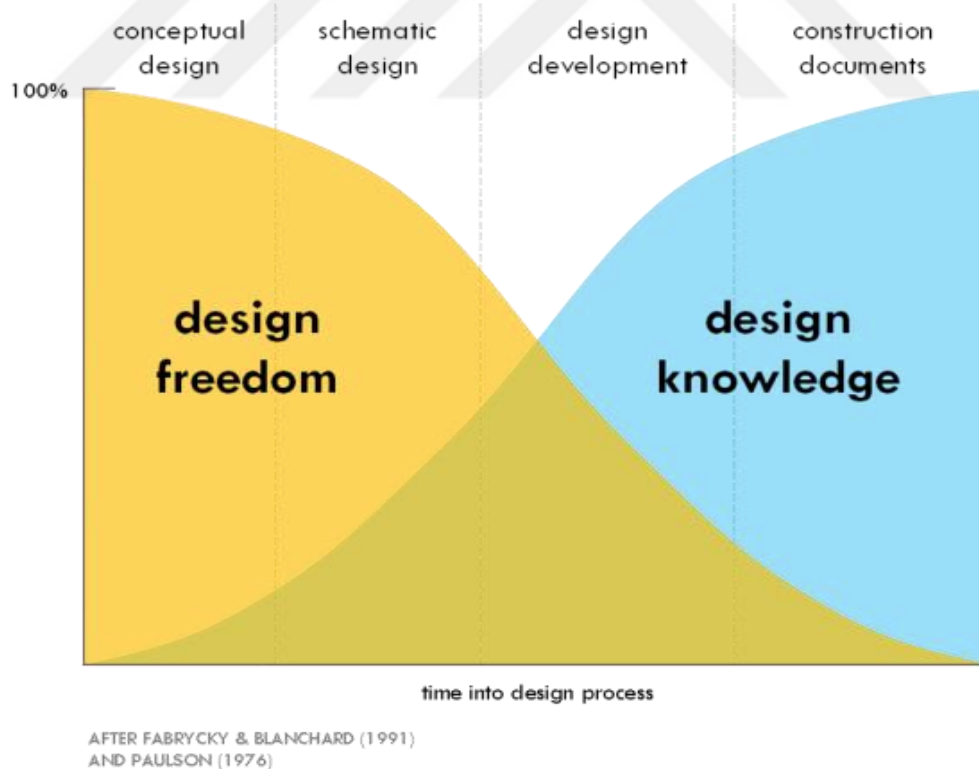


Figure 3.1. Dichotomy between freedom and knowledge (Rolvink et al., 2014)

In order to propose a generic framework, a literature survey for this dichotomous problem as shown in the figure above that integrates the freedom and knowledge is conducted. In the survey, methods that allows a designer to freely explore the design space with minimum constraints and the ones that allow designers to strictly search the objective space with maximum constraints are reviewed. It is observed that most of the methods aiming to expand the solution space to provide freedom in design, are probabilistic approaches (Brown and Mueller, 2017; Villaggi et al., 2017; Loos and De Laet, 2018; Yumer et al., 2015). Yet, as discussed in the critical analysis of the literature, studies that use probabilistic methods have limitations on narrowing the expanded solution space down to a set of optimal solutions.

On the other hand, it is observed that methods aiming to narrow down the solution space, to incorporate design knowledge are mostly formulated with deterministic approaches (Pan et al., 2019; Villaggi et al., 2018; Tsanas & Xifara, 2012; Umetani, 2017). However, it should be noted that studies that use deterministic methods have restrictions on expanding the solution space to enable designers to explore variety of different alternatives (Figure 3.2).

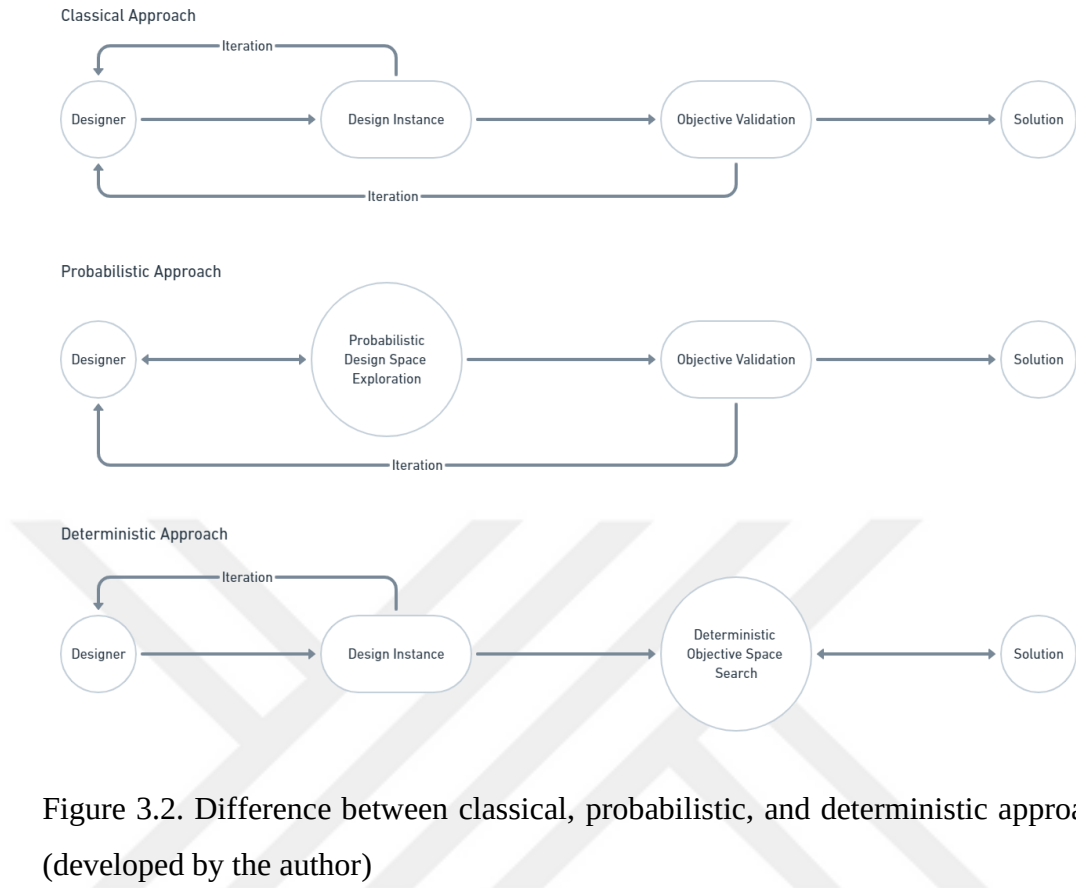


Figure 3.2. Difference between classical, probabilistic, and deterministic approach (developed by the author)

Based on previous studies, this thesis proposes a new generic framework bridging design and objective space without losing designer's freedom and structural constraints. In this realm an integrated hybrid approach including combining both probabilistic and deterministic approaches is developed.

Accordingly, a generic process diagram of design space construction from design instances and objective space deconstruction to a solution is formulated, as shown in the process diagram (Figure 3.3).

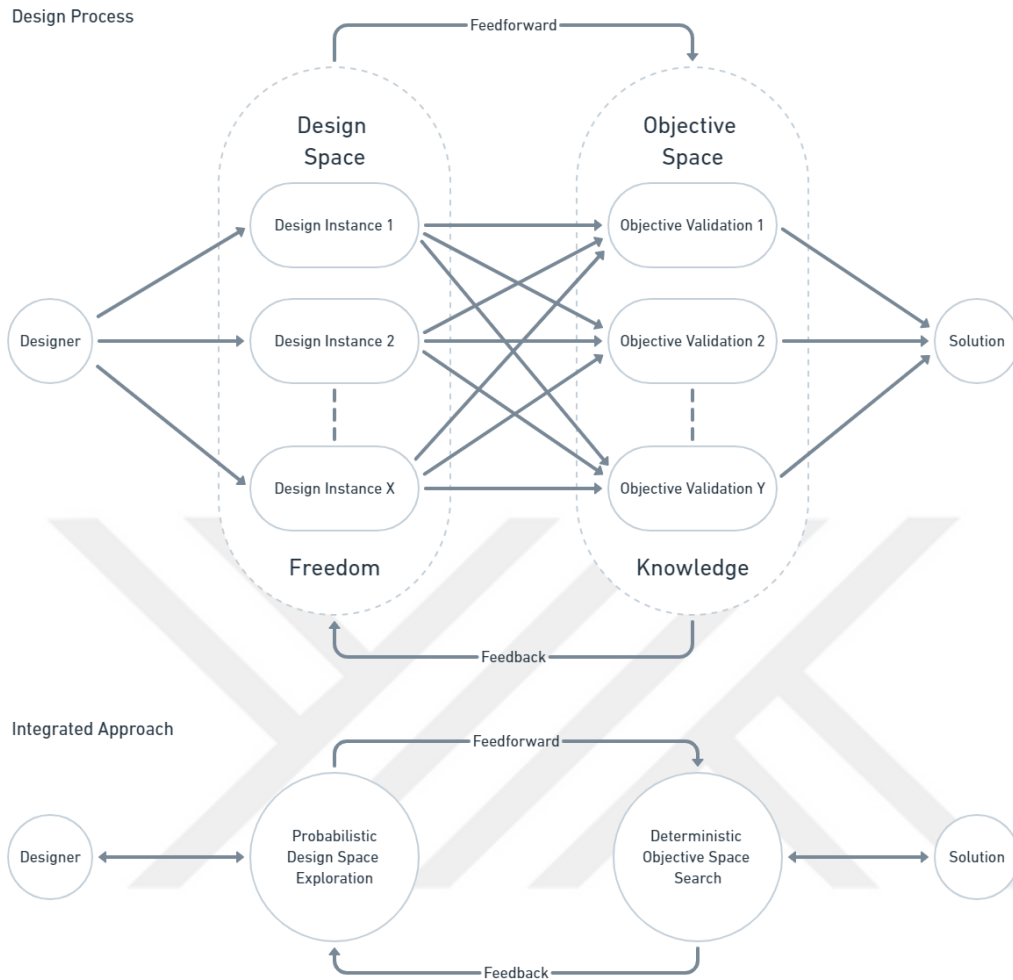


Figure 3.3. Design process workflow and integrated approach methodology (developed by the author)

Within the scope of this study, the proposed framework is demonstrated by three different structural design case studies, and results are analyzed. Firstly, a case study on probabilistic approach is conducted in order to test and assess its performance on a structural design problem. In the light of its performance, an integration strategy is proposed by analyzing the potentials and limitations of the probabilistic method.

Secondly, another case study is conducted employing a deterministic approach to test and show its performance on a structural design problem. Again, by considering

the performance, an integration schema is presented by examining pros and cons of the deterministic approach.

Finally, the methodology is extended to include probabilistic and deterministic approaches based on the information derived from the previous case studies. This integrated method is analyzed further in the following chapter with the first and second case studies, and the final discussion on the conclusion chapter is constructed on the potentials and limitations of the proposed generic framework.

### 3.1 Identification of Problem and Features

To illustrate proposed approach in three stages that are probabilistic, deterministic, and integrated; truss systems are selected as the main material of the case studies. In order to formulate and implement proposed methods, features of a simple truss system are defined in the first place (Figure 3.4).

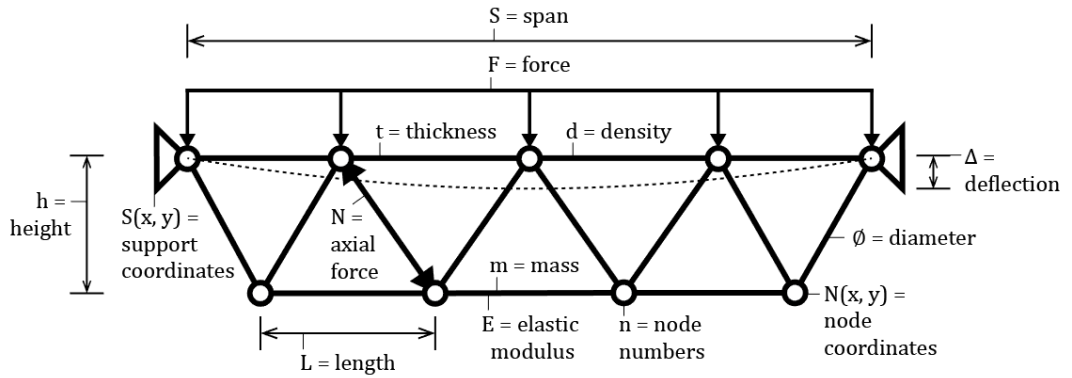


Figure 3.4. Features of a simple truss system to be used as the main material (developed by the author)

Following the feature extraction of a truss system, variables and constants to be included in the following analysis are determined. For the probabilistic study, only the horizontal location of diagonal nodes, and the length of truss members are selected as variables since those are the main variables that affects the main geometry and thus the structural performance of a truss. Remaining features that are total span

of the truss, support coordinates, number of nodes, height of the truss, force applied to truss, diameter and thickness of the cross section, density of steel, and elastic modulus are kept constant for the first step.

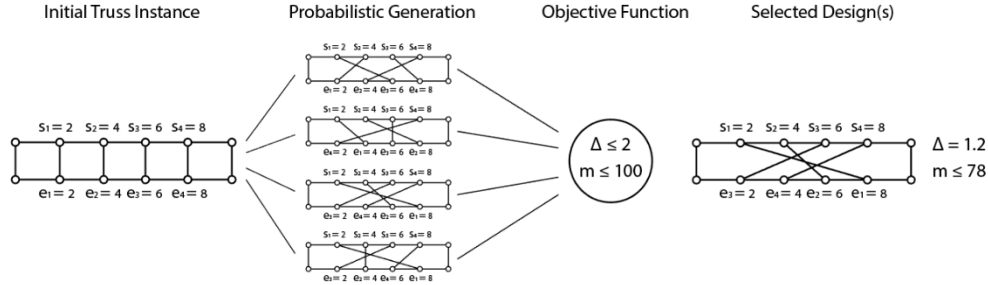


Figure 3.5. The process diagram of the first step (developed by the author)

For the second step, as the vertical position change of node members is found to be affecting the structural performance more than the horizontal relocation of truss diagonals in a truss with a constant topology; vertical position of node members on the lower part, correspondingly the height of the truss system, and the length of the truss members are selected as variables. Vertical position of the nodes on the top were kept as constant since it has dependency with the distributed load applied to truss (since the distributed load changes with the shape of the truss). Besides, total span, support locations, number of nodes, cross section, and steel properties are again preserved with the ones in the first step.

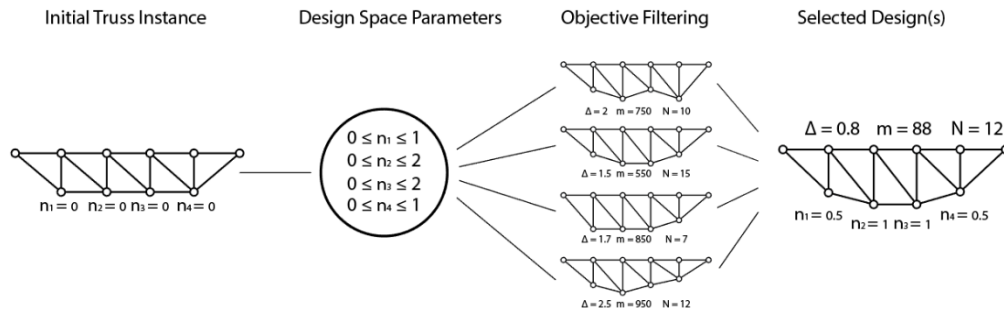


Figure 3.6. The process diagram of the second step (developed by the author)

Experiencing the first and the second step on probabilistic and deterministic approaches, the topology of the truss and cross section properties are conceded to be important features affecting the structural performance of the system. Consequently, the number of nodes, thickness and diameter of cross section are determined as variables along with the vertical location (i.e. y-coordinates) of nodes, height of the truss, and length of the steel members for the integrated approach employed in the last step.

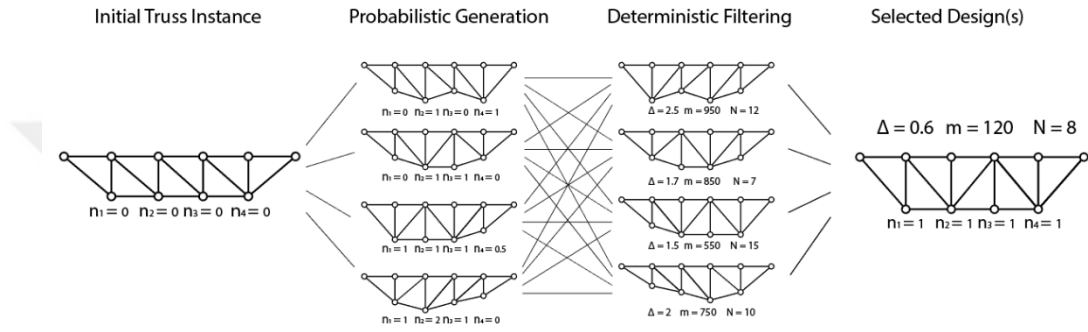


Figure 3.7. The process diagram of the third step (developed by the author)

Truss features preserved in this step are total span of the system, force applied to the truss nodes, and support locations (that has the dependency to the span), steel properties (that are density and elastic modulus).

To conclude, a methodology composed of three different approaches is developed and tested consecutively. It is seen that number of variables increased while number of constants decreased as the methodology is extended throughout the process (Figure 3.8).

|           | Step One:<br>Probabilistic Approach                                                                                                                                                                                                                                                                                                                                                                  | Step Two:<br>Deterministic Approach                                                                                                                                                                                                                                                                                                                               | Step Three:<br>Integrated Approach                                                                                                                                                                                                                                                          |
|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Variables | <ul style="list-style-type: none"> <li>• <math>N(x)</math> = x location of nodes</li> <li>• L = length of the member</li> </ul>                                                                                                                                                                                                                                                                      | <ul style="list-style-type: none"> <li>• <math>N(y)</math> = y location of nodes</li> <li>• h = height of the truss</li> <li>• L = length of the member</li> </ul>                                                                                                                                                                                                | <ul style="list-style-type: none"> <li>• <math>N(y)</math> = y location of nodes</li> <li>• h = height of the truss</li> <li>• L = length of the member</li> <li>• n = number of nodes</li> <li>• <math>\emptyset</math> = diameter of section</li> <li>• t = thickness of steel</li> </ul> |
| Constants | <ul style="list-style-type: none"> <li>• S = total span of the truss</li> <li>• <math>S(x, y)</math> = support locations</li> <li>• n = number of nodes</li> <li>• h = height of the truss</li> <li>• F = force applied to truss</li> <li>• <math>\emptyset</math> = diameter of section</li> <li>• d = density of steel</li> <li>• t = thickness of steel</li> <li>• E = elastic modulus</li> </ul> | <ul style="list-style-type: none"> <li>• S = total span of the truss</li> <li>• <math>S(x, y)</math> = support locations</li> <li>• n = number of nodes</li> <li>• F = force applied to truss</li> <li>• <math>\emptyset</math> = diameter of section</li> <li>• d = density of steel</li> <li>• t = thickness of steel</li> <li>• E = elastic modulus</li> </ul> | <ul style="list-style-type: none"> <li>• S = total span of the truss</li> <li>• <math>S(x, y)</math> = support locations</li> <li>• F = force applied to truss</li> <li>• d = density of steel</li> <li>• E = elastic modulus</li> </ul>                                                    |

Figure 3.8. Variables / constants for the proposed studies (developed by the author)

After the related features are extracted from a truss system, that corresponds to the problem definition and the main subject of this study, parameters are designated based on these selected features.

### 3.2 Identification of Case Studies and Parameters

In the initial step of the study which is the probabilistic approach, parameters are selected as start and end points of the 8 different truss diagonals based on the variable features that are location of nodes, and length of the members. Since the variable features of this step based on the study related with identification explained in the previous section are dependent to each other. Hence selected parameters are designated in such a way that it is possible to control both features and define two dimensions in the data (i.e. it was also possible to control features by the length parameter but it would define a single dimension, length, instead of two dimensions, start and end, within data).

For the objective space data in this step, two out of three objectives that are deflection, and total mass are selected since internal forces would introduce 8 more dimensions to the dataset (for each diagonal member). Therefore, strategy for the

data design of the first study is based on the idea of increasing the number of dimensions in the design space data while reducing the number of dimensions in objective space data. The reason of this decision is supported by the literature survey showing that probabilistic approaches are potent in exploring the design space and impotent in evaluating objective space.

On the other hand, for the second step of the study that is deterministic approach, strategy is completely contrary in selection of parameters. The reason of this is explained in the literature survey as the potentials and limitations of the deterministic approach is differing from the probabilistic approach (Rolvink et al., 2014). Hence, parameters selected for the deterministic study are limited to vertical position for 4 different nodes to control height of the truss and length of the members with respect to the selected case (Kassimali, 2009). For the objective space dataset of the second step, average value of all internal forces (for each member in the ground-truth truss that has 15 members) is added to the dataset along with the total mass, and deflection. Hence it is defined as a 17-dimensional objective space dataset in essence, without being subjected to dimensionality reduction as discussed in data preprocessing stage.

Finally, for the third step that is the integrated approach, which defines an extended methodology based on both probabilistic and deterministic approaches, data design is designated by the observations from previous case studies. Therefore, important features that are not included in the first two steps of the study that are number of nodes, diameter and thickness of the steel section; included to design space data as cross section index (that controls both the diameter and thickness of cross-section based on the EU standard library), number of division, and direction of diagonals (that controls the topology of the truss based on well-known truss type libraries).

For the objective space dataset, internal forces data with high dimensionality are reduced to its bounds (only the extremum values are taken), as the most important dimensions representing the internal force data is the maximum and minimum values for each truss instance. In conclusion, regarding the potentials and limitations for each approach explained in literature review, design space and objective space data

are designated as 16 by 2 for the probabilistic approach, 4 by 17 for the deterministic approach, and 9 by 4 for the integrated approach respectively.

|            | Step One:<br>Probabilistic Approach                                                                                    | Step Two:<br>Deterministic Approach                                                                                                                         | Step Three:<br>Integrated Approach                                                                                                                                                            |
|------------|------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Parameters | <ul style="list-style-type: none"> <li>• Start of truss diagonals (8)</li> <li>• End of truss diagonals (8)</li> </ul> | <ul style="list-style-type: none"> <li>• Vertical position of nodes (4)</li> </ul>                                                                          | <ul style="list-style-type: none"> <li>• Number of division (1)</li> <li>• Direction of diagonals (1)</li> <li>• Cross section index (1)</li> <li>• Vertical position of nodes (6)</li> </ul> |
| Objectives | <ul style="list-style-type: none"> <li>• <math>\Delta</math>: deflection (1)</li> <li>• m: total mass (1)</li> </ul>   | <ul style="list-style-type: none"> <li>• <math>\Delta</math>: max deflection (1)</li> <li>• m: total mass (1)</li> <li>• N: internal forces (15)</li> </ul> | <ul style="list-style-type: none"> <li>• <math>\Delta</math>: max deflection (1)</li> <li>• m: total mass (1)</li> <li>• N: internal force bounds (2)</li> </ul>                              |

Figure 3.9. Parameters and objectives for the three studies (developed by the author)

### 3.2.1 Step One: Probabilistic Approach

In order to exemplify the proposed method, simple truss systems which are having well defined requirements and well known topologies are chosen as described. A preliminary case study as a control study on probabilistic approach is conducted with the subset of proposed features as explained in the previous section. It is as an experimentation to explore input data, possible outcomes and more, in order to properly design the processes to integrate probabilistic and deterministic approaches as discussed above. Therefore, this section of the research aims to present both the proposed generic framework of the future research and the workflow of the completed initial experiment.

#### 3.2.1.1 Materials for the Probabilistic Approach

The main material of the study is the data generated by the designer which defines the design space alternatives correlated with the objective space. Within the scope of this study, design space data are constructed by 11 parameters i.e. data concerned here is 11 dimensional as: the location of node elements, the length of steel members, total span of the truss, location of supports, the number of node elements, the height of structural system, force applied to the system, the diameter of cross-section, the

thickness of cross section, density of the steel, and elastic modulus. With these 11 dimensions, data is assumed to be adequate to generate digital models and conduct initial structural simulations for all case studies as discussed in the previous section. For the objective space, data is to be 3 dimensional regarding three important structural features like maximum displacement, total mass of the structure, and internal axial force of steel members.

For the first part of the experiment, namely the probabilistic study, the material of the research is limited with three different components. In the first step, possible variations of simple trusses starting with the initial structural type is examined. Initial geometry data of the simple truss provided by the designer in the conceptual structural design phase and related features is then used to generate new design alternatives (Figure 3.10). For this research, Karamba's sample parametric structural truss design model is used (Preisinger, 2016).

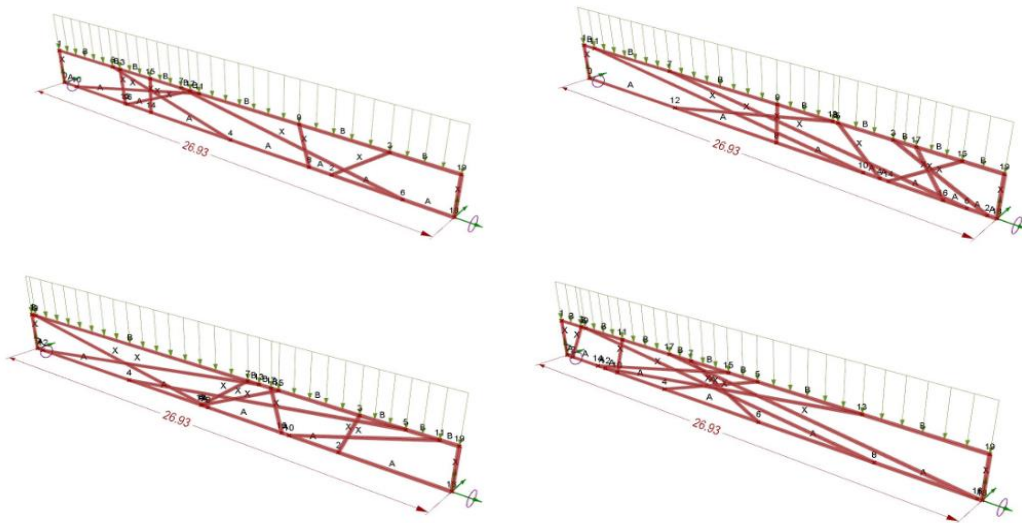


Figure 3.10. Different design alternatives generated by different parameters (developed by the author)

In the second part, variations in truss is determined for which total mass of the truss and displacement are the features for constraining the truss design and thus design

and objective space. In this part, maximum displacement is calculated by Karamba parametric engineering components (Preisinger and Heimrath, 2014).

The last part is to use artificial neural networks (ANN), which is fed with input data composed of design variables that are start and end locations of the 8 diagonal members on x-axis and output data composed of structural simulation data that are maximum deflection of the structure on y-axis, and total mass of the structure. In this part of the study, Crow artificial neural network component developed by Felbrich (2016) is used.

The preliminary study has shown that the selected design space and objective space data can be used to train artificial neural networks and results can be analyzed within these data. Yet, the initial study is also shown that the automated process of generating design instances without human interaction can cause problems such as generating invalid instances that exceeds the limits of intended design or objective space. In other words, automated process of generating design options may generate impossible, immeasurable, unanalyzable, or irresolvable instances.

Therefore, human-computer interaction steps are proposed within the iterations so that the generation process can be controlled by the designer to achieve valid design space instances rather than blindly generated random instances to be evaluated.

#### **3.2.1.2 Method of the Probabilistic Approach**

Aim of the method developed for the first step (probabilistic approach) is to generate instances within the design space using the selected variables mentioned. Then the structural performance of instances within the objective space using the selected objectives described in the previous section are evaluated. Finally, values of the variables using the intended values of the objectives are predicted via artificial neural networks. The generic method proposed within the realm of this study is started with the data collection process for both design and objective spaces. Data is collected via

generative methods using a simple genetic algorithm so that the framework can be tested in different design and objective space scenarios.

Design space parameters are used to randomly generate design options via generative digital models that defines the training dataset. Then, simulations are conducted for each design alternative to collect objective space data in accordance with design space data. After input (design space) and output (objective space) data are collected, preprocessing of the data is conducted in order to normalize the values that range extensively (between 4175 – 9138 for the mass data, and between 0.03 – 0.45 for the deflection data). The prepared dataset is proposed to be used to train the artificial neural networks with varying layer and node numbers to compare their performance on correlating the input and output data.

Finally, trained artificial neural network outputs that represent the design space data of the multi-objective design problem is obtained. These data is then compared to the generated dataset of design and objective spaces respectively and visualized together with digital models of design instances.

For the method of conducted initial study, all parameters (start and end locations of 8 truss diagonals) that control the initial structural geometry are initialized (with respect to constrained span and support locations) within a gene pool to generate new geometries in a randomized manner. After generating new geometries by using Galapagos genetic solver (Rutten, 2010) that shuffles input variables, two sets of data that are maximum displacement and the total mass of structure are calculated with finite element method via Karamba components for each design alternative.

Since two sets of recorded quantitative data for maximum displacement and the total mass have different ranges as mentioned, both are normalized and remapped between 0 to 1 just as input data in order each data feature to affect learning model equally. Then both input data from a gene pool that controls the geometry and output data from the simulation that gives the score for displacement and total mass were given to an artificial neural network engine to evaluate (Figure 3.11).

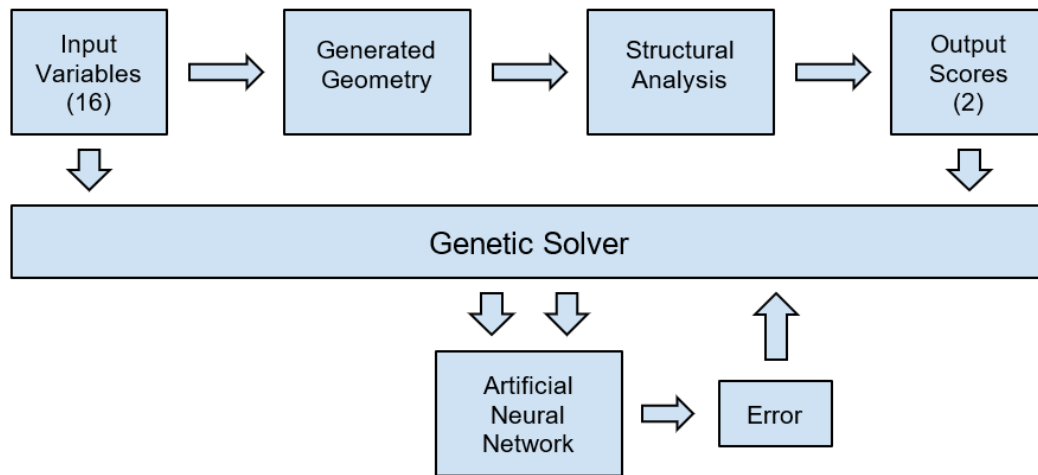


Figure 3.11. The flowchart of the initial probabilistic research experiment (developed by the author)

The preliminary study shows that correlations between design space and objective space data derived by an artificial neural network can be used to estimate approximate design space parameters when the objective space scores are given.

On the other hand, estimation of design space parameters is an automated process similar to generation of design instances in which designer has no freedom to interact with process to control. This fact is an obstacle for designers to explore more through further iterations within the design process.

Therefore, another human-computer interaction phase is proposed at the end of each iteration. This allows designers to regulate estimated design space parameters. It starts a new iteration that has a chance to achieve an instance not only controlled by an algorithm to find the instance closest to the given output scores dataset but also controlled by a designer who aims to find the instance satisfying other objectives which cannot be included in the dataset. Hence, by including additional human-computer interaction stages, a new flowchart is proposed for the future studies (Figure 3.12).

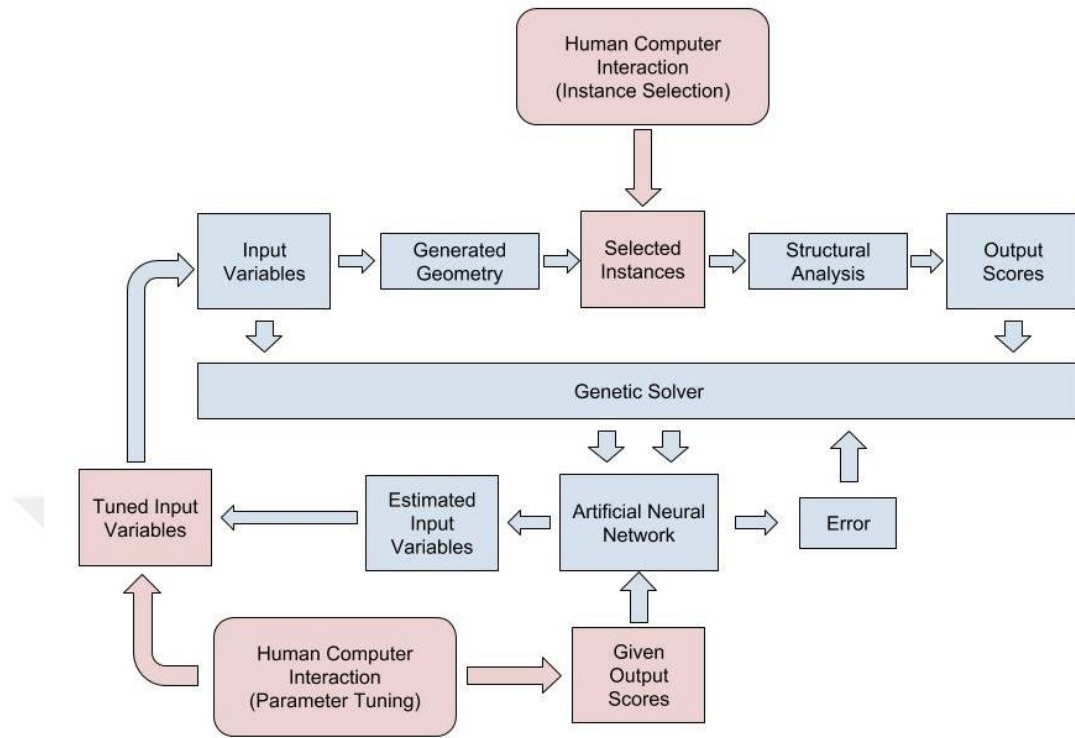


Figure 3.12. The flowchart proposed for future stages of the research experiment (developed by the author)

### 3.2.2 Step Two: Deterministic Approach

The case study for this research is structured in the light of previously conducted preliminary study. Therefore, potentials and limitations of the dataset are analyzed since the data is the most crucial part of the training and prediction process in the machine learning methods.

The most important potential of the first step is that; it has shown the data crunching capacity of the computational tools. Therefore, it has proven that it is possible for computers to process vast amount of data within a time period that would be impossible for any human to process. Another important potential of the preliminary study is that it demonstrates the possibility of simultaneous data processing. Hence,

the study has revealed that multi-feature data processing and multi-objective predictions are achievable via interoperable simulation tools and computational methods.

On the other hand, there are some limitations observed within the analysis of the process of the first step. The most important limitation is, it has verified that the predictions of the machine learning algorithms are only dependent to the input data. Thus, necessity of the data design becomes very critical since any uncontrolled and unfiltered data have a significant effect on overall accuracy and precision of the predicted results. Another important limitation is cross-check and validation between the behavior and dependency of the variables within the objective space. These data are used to train the machine learning models should be integrated since it affects the definition of the problem. Hence, importance of validating the results based on a ground-truth and cross-checking the dependent variables for a structural problem is realized after analyzing the results of the first study.

### 3.2.2.1 Materials for the Deterministic Approach

The first main material for the case study is selected amongst standard types of trusses (Ambrose, 1993), to generate the dataset in a controlled manner. Different from the preliminary study that is conducted with a truss system with arbitrary diagonals, selected standard truss system (Pratt Truss) is parametrized and translated into a generative model so that other instances of a standard Pratt Truss can be generated and simulated (Figure 3.13).

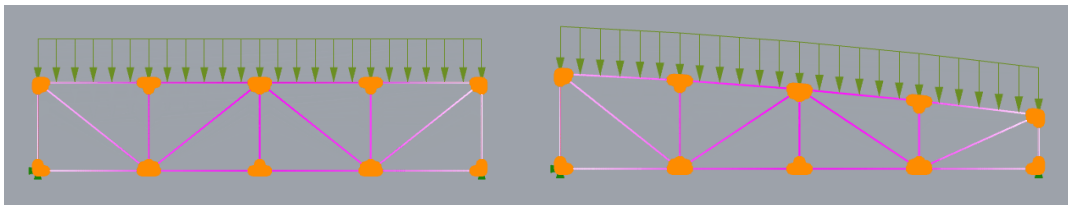


Figure 3.13. Different instances generated from a generative Pratt Truss model (developed by the author)

Data generated from the computational model of the selected truss system is controlled in a grid search system that computes all possible design space parameter combinations within a predefined range, rather than using a randomly generated instances that are used in the preliminary study. Therefore, design space dataset is generated in an evenly distributed way. In this assumption, start, middle, and end nodes of the truss system are defined to be moved in the Z-direction only with 10-centimeter intervals up to 100 centimeters. Hence,  $10^3$  combinations are generated in total and sent to the Karamba3D structural analysis software in order to understand the structural behavior of instances within the objective space. Yet, zero force members are emerged in the dataset creating fluctuations in the objective space data. Therefore, the main material of the case study is changed to a case that is known to be a statically determinate, structurally stable, and mathematically calculated example within the literature. That case is also selected to be the ground-truth which will be used as a benchmark to understand the convergence of the proposed method.

The revised main material for the case study was retrieved from a known example of a structural truss problem in the literature (Kassimali, 2009), to be able to converge to an instance based on ground-truth in the further processes. Unlike the preliminary study that is conducted with the randomly generated design space data, the problem selected for the case study can be described as reverse engineering first deriving the parameters and variables within the design space.

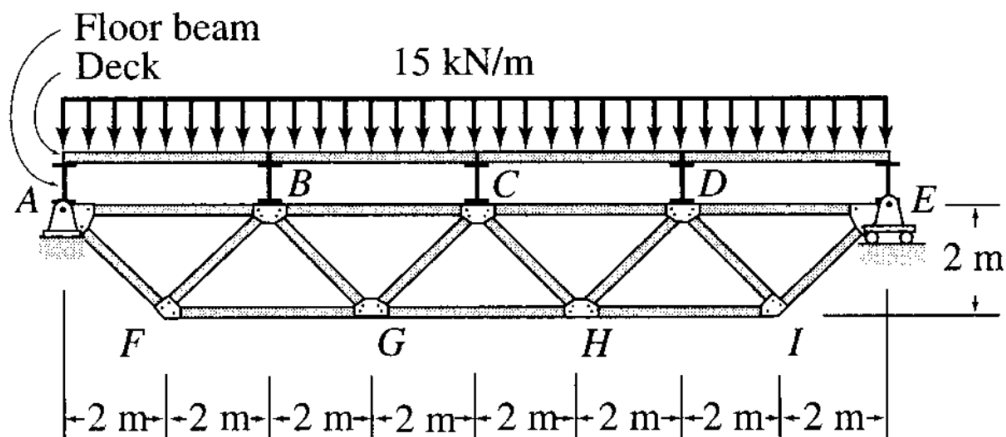


Figure 3.14. Example selected as the main material of case study (Kassimali, 2009)

The example selected as the main material of the case study is translated into a generative model, firstly by defining variables and constants within the given case. Since the main problem of this study is to understand how the geometry of an instance affects the structural performance of the system, only the position of bottom nodes of the truss (namely  $F$ ,  $G$ ,  $H$ , and  $I$ ) are defined as variables and all the other parameters that are cross section of the steel members, total span, number of steel members, load applied to the system, and topology are kept as constants. Also, Initial Cartesian coordinates of top nodes (namely  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$ ) are kept as they are since it also affects the distributed load case applied to the system.

Using the variables and constants, a new generative model is developed to obtain data for design space and geometry for the analysis. Design space is again generated with homogenous data structure since all variables (positions of all bottom nodes) are only allowed to change with a predefined interval (20-centimeter intervals in Z-direction, 100 centimeters maximum). Therefore, design space is composed of  $6^4 = 1296$  instances in total since each node can move to 6 different positions and there were 4 nodes that can change its position.

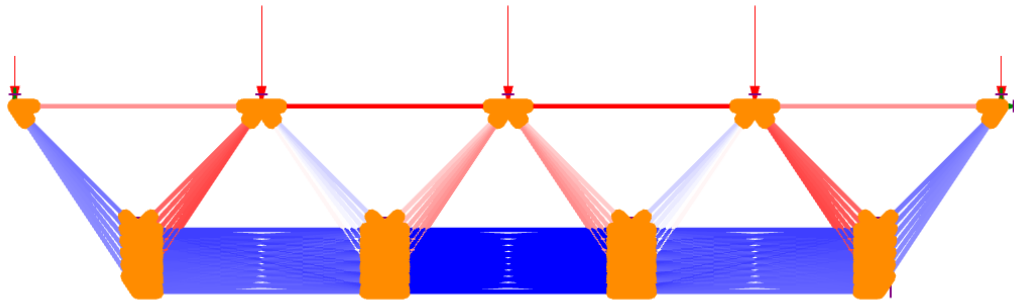


Figure 3.15. Juxtaposed image of all instances within the design space (developed by the author)

Generated design space data of the 1296 instance were recorded, and generated geometries are transformed into analytical models in order to conduct structural simulations. Since it would affect the objective space results, simulation parameters are defined based on the example case presented by Kassimali and kept as constant

for each instance in order not to change any parameter that is not included within the design space (*ceteris paribus* principle). Subjected under the same conditions, objective space data are generated from simulated analytical models. From the objective space data, three main features that are maximum displacement, the total mass of the structure, and average internal forces of steel members are recorded to analyze the structural performance and behavior of the selected truss system.

### **3.2.2.2 Method of the Deterministic Approach**

The method proposed for the case study is mainly based on overcoming limitations of the first study. Therefore, a data control mechanism for the training of machine learning methods is introduced upon the previous method. The grid search method which is a process that searches exhaustively through a specified subset of the parameter space of the targeted algorithm (Chan & Treleaven, 2015) is used to control the input data that was used to generate all instances within the design space. In other words, design space data is generated with all the possible combinations of all design space variables rather than using random search as in the first study (Figure 3.16). Hence, each truss instance generated within the design space is distributed equally and each variable is controlled with the same range and interval so that design space dataset was only dependent to the design space variables (without any dependency with a random seed that arbitrarily affects the behavior of the dataset as in the first step).

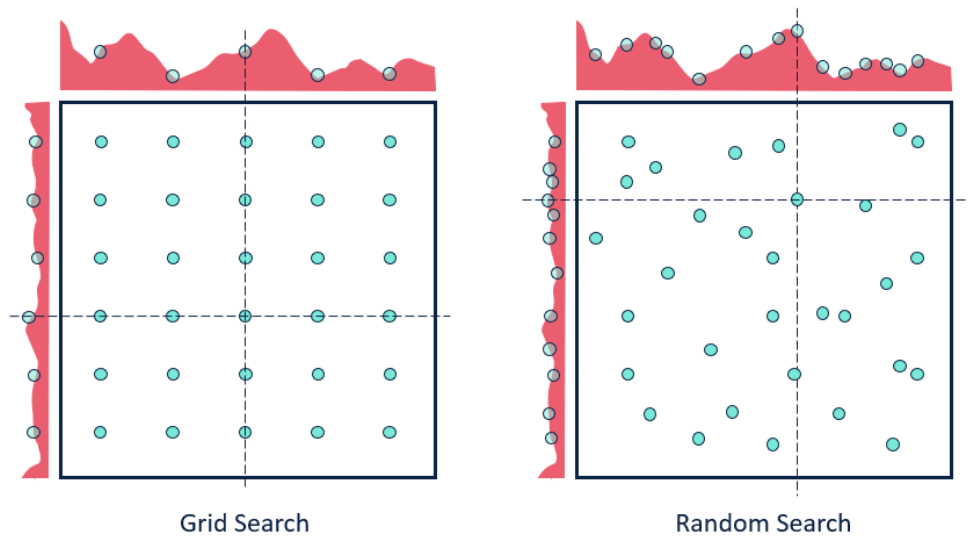


Figure 3.16. Comparison of grid search and random search methods (Sydney, 2019)

After recording the numerically controlled homogeneously distributed data of the design space, truss instances generated from this dataset are transformed into analytical models in order to conduct simulations on them as structural models. Also, in order not to rely on the results of a single software since each structural analysis software uses a different method (thrust line, limit state, or finite element) as a black-box and only returns numerical structural analysis results, various analysis programs are tested in this case study. Thus, data transfer and translation methods to maintain the interoperability of different applications are found and tested before conducting structural simulations. Software used for the case study are selected based on their automation capabilities since the proposed methodology is primarily based on computational methods and the ones which require manual interferences of each instance are not included. Accordingly, SAP2000, Robot Structural Analysis, and Tekla are selected for the case study in addition to Karamba3D which is already used in the first study. For the interoperability between generative models which are scripted on Grasshopper and SAP2000, a plug-in named as GeometryGym (Mirtschin, 2014) is used. Different from the direct data transfer to SAP2000, interoperability between Grasshopper and Robot Structural Analysis is indirect since

Robot Structural Analysis can connect and receive data directly from Dynamo but not from Grasshopper. Therefore, generated data is firstly transferred to Dynamo from Grasshopper via Speckle (Stefanescu, 2015), and data is then sent to Robot Structural Analysis. Lastly, for the interoperability between Grasshopper and Tekla, a generic standard data exchange format that is comma separated values (.csv) is used (Figure 3.17).

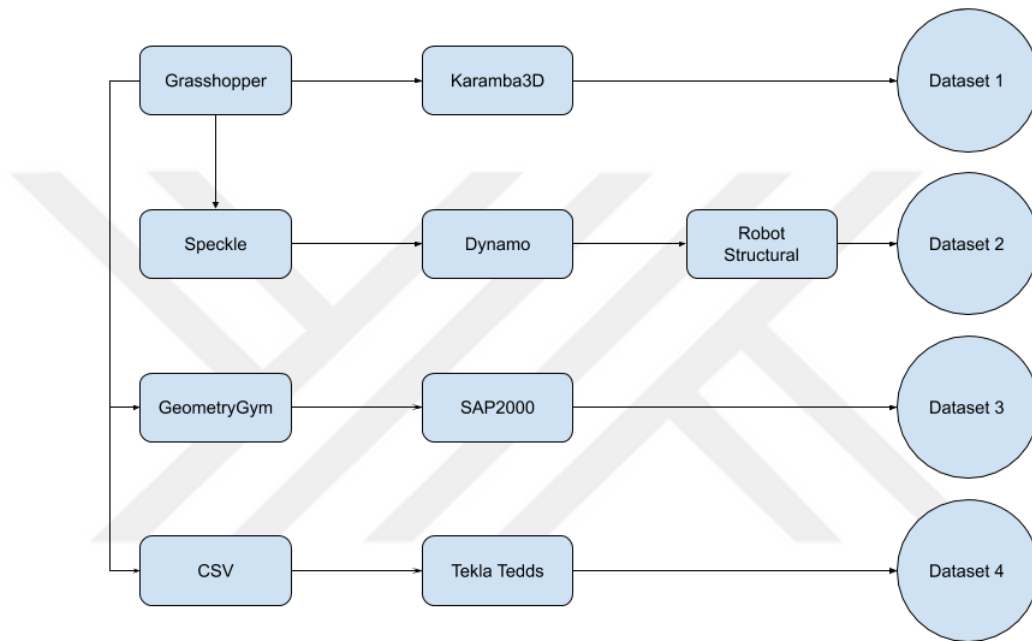


Figure 3.17. The data transfer and interoperability workflow of the case study (developed by the author)

Following the interoperability and data transfer methodology, cross-check and validation mechanisms are proposed to filter and control the different objective space datasets derived from different simulation software.

Instances are selected amongst the design space dataset which is the same for all simulated objective space datasets. Selection of instances within the design space is determined based on the critical points within the objective space data. In order to establish a benchmark that is valid for all objective space datasets, selected critical instances are solved using the Unit Load Method.

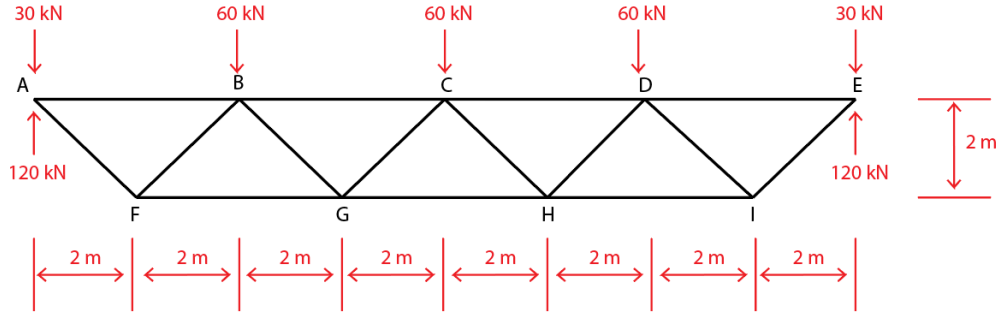


Figure 3.18. First instance selected to be calculated with Unit Load Method (developed by the author)

The first instance selected for the calculation is solved step by step in order to validate the simulation results returned from Karamba, Tekla and Robot Structural Analyses. In the first step, reaction forces of supports located in points A and E knowing that total force and moment values should be zero at those points in a stable truss system.

$$\sum F_Y = 0 \Rightarrow A_Y + E_Y = 30kN + 60kN + 60kN + 60kN + 30kN = 240 kN$$

$$\sum M_A = 0 \Rightarrow E_Y \times 16 m = 60kN \cdot 4m + 60kN \cdot 8m + 60kN \cdot 12m + 30kN \cdot 16m$$

$$E_Y = 120 kN \text{ and } A_Y = 120 kN$$

In the second step, internal forces of each truss member are calculated using the equilibrium equations of each joint. Considering the total force applied to each node should be equal to zero, each known and unknown forces applied to the nodes are analyzed in a free body diagram. Components of forces are calculated with respect to the Cartesian coordinate system using the sine and cosine rule and sum of forces were equalized in zero both in X-axis and Y-axis.

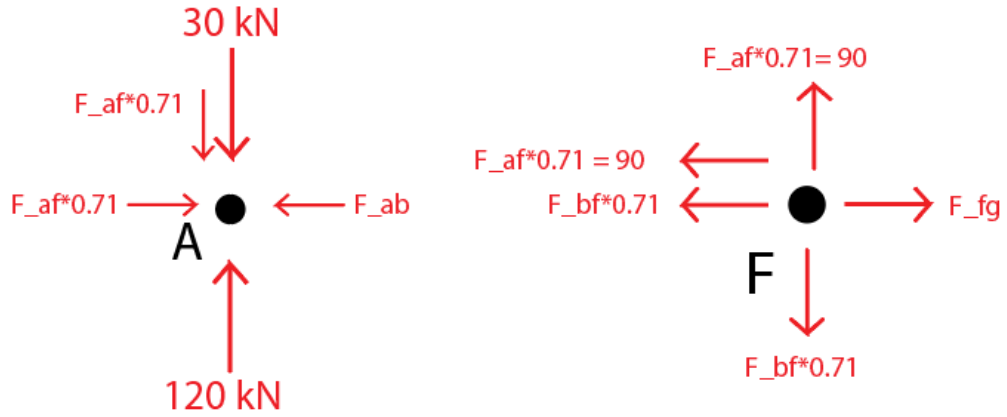


Figure 3.19. Free body diagrams of nodes A and F (developed by the author)

For node A:

$$\sum F_Y = 0 \Rightarrow 30kN + F_{AF} \cdot \sin 45^\circ = 120kN \Rightarrow F_{AF} = 127.3 kN$$

$$\sum F_X = 0 \Rightarrow F_{AF} \cdot \sin 45^\circ = F_{AB} \Rightarrow F_{AB} = 90 kN$$

For node F:

$$\sum F_Y = 0 \Rightarrow F_{AF} \cdot \sin 45^\circ = F_{BF} \cdot \sin 45^\circ \Rightarrow F_{BF} = 127.3 kN$$

$$\sum F_X = 0 \Rightarrow F_{AF} \cdot \sin 45^\circ + F_{BF} \cdot \sin 45^\circ = F_{FG} \Rightarrow F_{FG} = 180 kN$$

After calculating internal forces for each truss member using free body diagrams as formulated above, unit virtual loads are also calculated with the same method in the following step (Figure 3.20). Finally, calculated values for each member are inserted in a table that includes all variables that are  $N$  (internal force),  $n$  (unit virtual load), and  $L$  (length of the member) for the deflection calculation (Table 3.1).

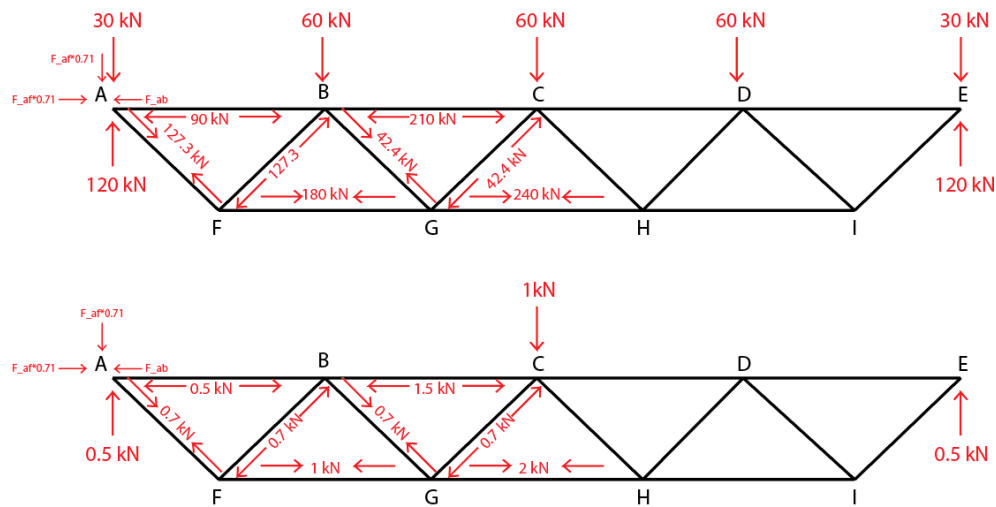


Figure 3.20. Calculated internal forces and unit virtual nodes of the first truss (developed by the author)

Table 3.1. Table including variables to calculate the deflection of the first truss (developed by the author)

| Member | N (kN) | n (kN) | L (m) | NnL (kN <sup>2</sup> /m) |
|--------|--------|--------|-------|--------------------------|
| AF     | -127.3 | -0.7   | 2.8   | 252.2                    |
| AB     | +90.0  | +0.5   | 4.0   | 180.0                    |
| BF     | +127.3 | +0.7   | 2.8   | 252.2                    |
| FG     | -180.0 | -1.0   | 4.0   | 720.0                    |
| BG     | -42.4  | -0.7   | 2.8   | 83.1                     |
| BC     | +210.0 | +1.5   | 4.0   | 1260.0                   |
| CG     | +42.4  | +0.7   | 2.8   | 83.1                     |
| GH     | -240.0 | -2.0   | 4.0   | 1920.0                   |

In the final step, deflection of the truss is calculated depending on the principle that external virtual work should be equal to internal virtual work. As a critical point where the deflection is calculated is selected as node C since it is located in the midpoint of the total span and it defines the symmetry axis of the truss instance. For

the steel members,  $E$  (modulus of elasticity) is assumed as  $2.10^8 \text{ kN/m}^2$  and cross section area is based on *Round HSS*  $76.2 \times 4.8$  steel section and deflection of the truss instance calculated accordingly.

$$\text{Int. virtual work} = \text{Ext. virtual work} \Rightarrow 1 \text{ kN} \cdot \delta_C = (\sum N \cdot n \cdot L) / A \cdot E$$

$$\sum N \cdot n \cdot L = 2 \times (252.2 + 180 + 252.2 + 720 + 83.1 + 1260 + 83.1) + 1920$$

$$A = \pi \cdot (0.762 \text{ m} / 2)^2 - \pi \cdot ((0.762 \text{ m} - 2 \cdot (0.048 \text{ m})) / 2)^2 = 0.001076 \text{ m}^2$$

$$\delta_C = 7581.2 \text{ kN}^2 \text{ m}^{-1} / (0.001076 \text{ m}^2) \cdot (2.10^8 \text{ kN/m}^2) = 0.0352 \text{ m} \cong 3.5 \text{ cm}$$

Another truss instance from the design space dataset is also calculated by using the same method (unit load method). Only difference between the first truss instance and the second truss instance is the truss depth that is changed from 2 meters to 3 meters. Calculated variables are again inserted in a table in order to make a comparison with the first instance (Table 3.2). Using the data entered to the table, deflection is calculated as 1.8 centimeters for this truss instance.

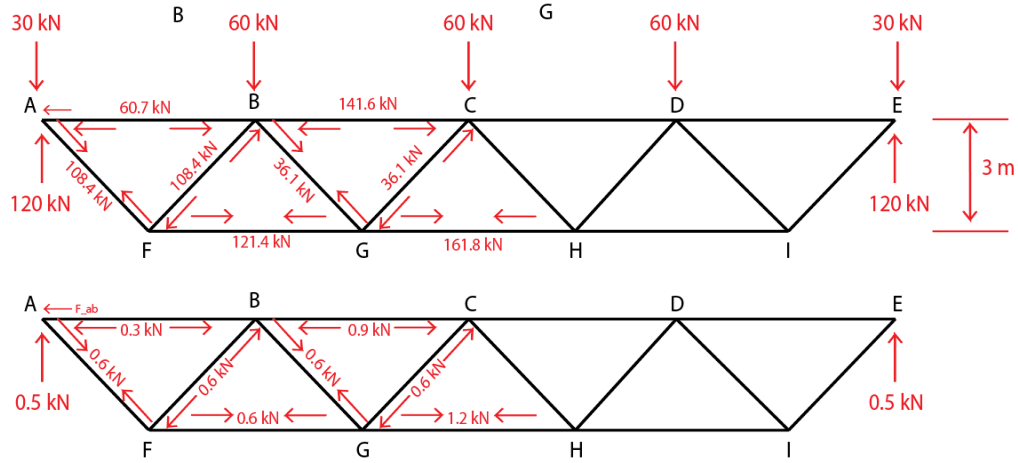


Figure 3.21. Calculated internal forces and unit virtual nodes of the second truss (developed by the author)

Table 3.2. Table including variables to calculate the deflection of the second truss (developed by the author)

| Member | N (kN) | n (kN) | L (m) | NnL (kN <sup>2</sup> /m) |
|--------|--------|--------|-------|--------------------------|
| AF     | -108.4 | -0.6   | 3.6   | 234.1                    |
| AB     | +60.7  | +0.3   | 4.0   | 72.8                     |
| BF     | +108.4 | +0.6   | 3.6   | 234.1                    |
| FG     | -121.4 | -0.6   | 4.0   | 291.4                    |
| BG     | -36.1  | -0.6   | 3.6   | 77.8                     |
| BC     | +141.6 | +0.9   | 4.0   | 509.8                    |
| CG     | +36.1  | +0.6   | 3.6   | 78.0                     |
| GH     | -161.8 | -1.2   | 4.0   | 776.7                    |

Finally, calculated deflection values for the selected truss instances are compared with the simulation results returned from Karamba, Tekla and Robot Structural Analysis with line charts. Moreover, in order not to rely on single objective result since the scope of this research is a multi-objective analysis, total masses of both instances are calculated as 407.5 and 457.5 kg respectively with the calculation below. After that, calculated mass values are also used to be compared with the simulation results in order to check the reliability of the software.

$$V_1 = L_{total} \cdot A_1, \quad L_{total} = 50.6 \text{ m}, \quad A_1 = 0.001076 \text{ m}^2$$

$$m_1 = d_{steel} \cdot V_1, \quad d_{steel} = 7480 \text{ kg/m}^3, V_1 = 0.55 \text{ m}^3 \Rightarrow m_1 = 407.473 \text{ kg}$$

$$V_2 = L_{total} \cdot A_2, \quad L_{total} = 56.8 \text{ m}, \quad A_2 = 0.001076 \text{ m}^2$$

$$m_2 = d_{steel} \cdot V_2, \quad d_{steel} = 7480 \text{ kg/m}^3, V_2 = 0.61 \text{ m}^3 \Rightarrow m_2 = 457.511 \text{ kg}$$

To sum up, in the case study using deterministic approach, the problem is tried to be handled as a structural engineering problem and results returned from the simulation software is used as a guide to find the optimal solution set.

Firstly, since both simulation software to analyze the instances of a truss structure and the analysis results varies, reliability of the software and analysis results are

questioned and subjected to a benchmark study within the analysis and results chapter based on calculations conducted in this chapter.

After the reliability issue is resolved and data is captured from the simulation software for multiple instances, data is analyzed with deterministic statistical methods to find optimal set of solutions only using deterministic methods. Analysis and results of the case study on deterministic approach have proven that deterministic method is more extensive in terms of design space search methodology and therefore more successful in terms of capturing the behavior of the objective space data. Yet, the computational cost of the grid search method used in this study is significantly higher than the random search method used in the first case study.

Also, control over the design space variables are very limited within the deterministic methods since user can only define the range of the design space parameters and grid search handles the interpolation between the user defined range.

In this regard, an integrated approach where deterministic approach and probabilistic approach can be used together in the following case study.

### **3.2.3 Step Three: Integrated Approach**

Following the two case studies conducted with a probabilistic approach and a deterministic approach, an integrated model is proposed as the main study of this research. Potentials and limitations discovered in both probabilistic and deterministic studies are analyzed and integrated into a hybrid workflow. The most important potential of probabilistic method is discovered to be the power of extensive design / objective space propagation capacity that comes from the computational and generative nature of the algorithms. Yet, it is also discovered that the same nature defines a limit when it comes to filtering and searching for optima through the solution space.

On the other hand, the most important potential of deterministic method is observed to be the ability of design / objective space backpropagation potential that comes

from the rule-based and systematic nature of the algorithms. Hence, the same nature sets a limit when generating and constructing a solution space. Therefore, the proposed hybrid method uses probabilistic methods to expand the solution space and deterministic methods to narrow down the solution space recursively.

### **3.2.3.1 Materials for the Integrated Approach**

Materials subjected to integrated approach are chosen with respect to the previous case studies. As mentioned in the previous sections, main materials for the probabilistic case study are composed of truss instances in which all the diagonal members are generated randomly where materials for the deterministic case study are composed of truss instances in which all the diagonal members are determined by a ground-truth case by Kassimali. Therefore, in the hybrid approach, an unlimited yet constrained set of truss instances in which number and direction of diagonal members are determined by the user. The main difference between the material of the hybrid study and materials of previous case studies is the interactive data generation that allow user to control truss instances with 9 parameters that are number of division, direction of the diagonals, cross section index, vertical position of upper and lower left / middle / right corners (Figure 3.22). Hence, the main material of the integrated approach is not a predefined set of truss instances either controlled with random generation or a specific ground-truth but a generative model where instances can be controlled by a user.

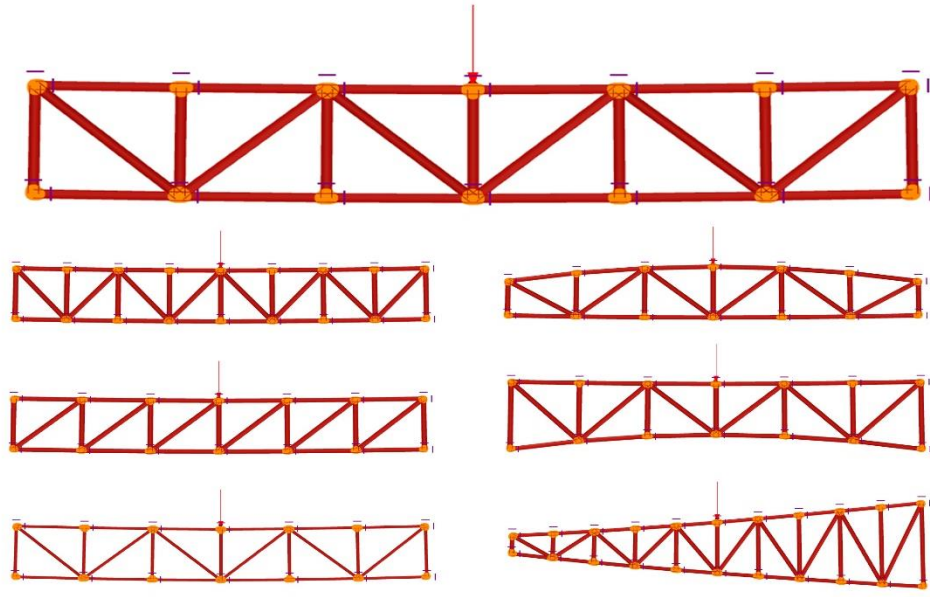


Figure 3.22. Parameters that can be controlled by user and their effect on instances (developed by the author)

Besides the main material collected from the generative truss model, other materials are also introduced to the integrated approach in order to capture and analyze the data. Firstly, data recorders are used in order to capture the input parameters that are generated by the user as the interaction proceeds. Similarly, they are also used to record the objective outputs that are generated by the simulation software simultaneously with the user interaction. Secondly, analytical models translated from the truss instances are also used as a material for the hybrid approach in order to deterministically calculate and simulate the structural performance of the generated trusses. Last but not least, a local database is also used as a material in the scope of this study in order to store the captured design space data and objective space data in correspondence (Figure 3.23).

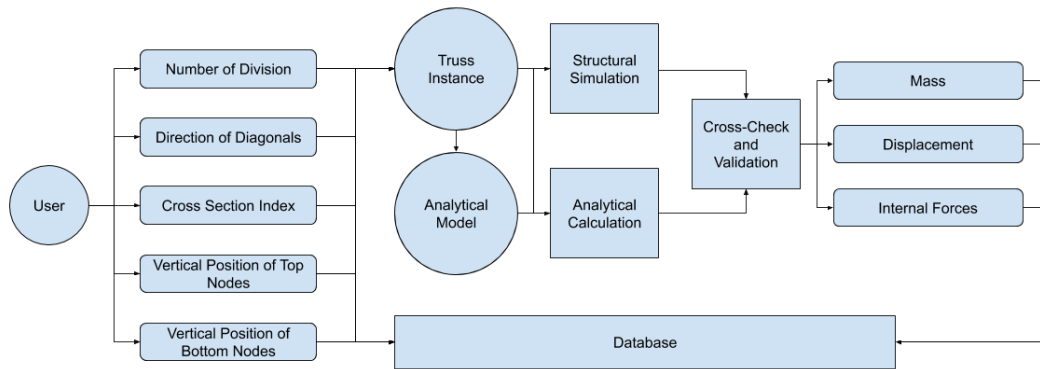


Figure 3.23. Materials used in the integrated hybrid study and their relations (developed by the author)

### 3.2.3.2 Method of the Integrated Approach

The method of the integrated approach is derived from the outcomes of previously conducted probabilistic and deterministic studies. Therefore, the main idea of the integrated approach methodology is to enhance the key potentials of probabilistic and deterministic approaches and reduce their limitations. In order to achieve a novel interlocking method that is more robust in terms of mathematical space search in a given design problem, the potentials and limitations of both studies are detected, and causalities are analyzed.

Firstly, the probabilistic approach is analyzed in terms of design space search and optimization strategy. The most significant potential of the probabilistic approach is found to be the extensive capacity of the algorithms that enables them to search through a broad solution space regardless of problem definition. Since probabilistic methods mostly depends on generative computational methods, it is possible to process infinitely many instances of solution space by simultaneously processing the variable data. Hence, probabilistic methods can search through multiple variables in any design problem, they are observed to be very limited when it comes to searching and filtering the optima solutions. It is evident that the optimization function of a design problem changes radically along parameter axes in general. Starting from a

random state and randomly changing the parameters to search for the optima solutions requires a lot of trial and error.

Secondly, the deterministic approach is analyzed with respect to solution space search and optimization strategies. Most importantly, the main advantage of the deterministic approach is found to be the controllability and inspectability of the algorithms enabling user to intervene and guide algorithm to search for the specific solution set that is intended for the design problem. In other words, deterministic methods depend on rule-based computational methods where it is possible to control the design space variables and generate intended instances. On the other hand even deterministic methods allow user to control over data to achieve intended objectives, they are found to be lean towards overfitting or local optima solutions. This shows that deterministic methods need to be controlled as well as they require a certain level of logical intuition and wise guess compared to probabilistic methods.

Results obtained in previous studies proves the need of a new extended approach. The integrated approach proposed in this study has all the potentials of both probabilistic and deterministic approaches. In the proposed method the limitations of these two methods are eliminated. In the light of these implications, probabilistic method is structured as a heuristic approach where limited intuitive user data is captured and used for training of algorithms. Analytical optimization matrix is returned to the user in each iteration to guide the user to the required solution. The main function of the probabilistic part integrated to the hybrid approach is to derive analytical correlations between the design space data generated intuitively by the user and the objective space data returned from initial structural analyses.

The first difference between the pure probabilistic method and the probabilistic approach integrated to the hybrid methodology is that, pure probabilistic method proceeds a single search without taking any intermediary guidance while probabilistic part of the hybrid approach proceeds a search in each iteration after every user input change.

The second difference between these two approaches is that, pure probabilistic method returns a set of solutions based on the objective function that user cannot decide on design space parameters but choose among the set of optima solutions, while probabilistic component of the hybrid method works on the principle that user can control each design space parameter. Therefore, heuristic approach is proposed to return an optimization matrix that will guide the user to a solution rather than returning a solution.

Method for the probabilistic component of the integrated approach is based on artificial neural networks since it is a generic stochastic model that can be adapted to different structural design problems and user input data which is not known beforehand. Therefore, a data structure is constructed based on the material decided to be used in this approach. Firstly, each input that controls the truss instances defined within a generative model are connected to a data recorder so that all parameters can be captured as user changes the input data.

Then, captured design space data are collected and matched in a local database so that number of divisions, direction of diagonals, cross section index, vertical positions of top nodes, and vertical positions of bottom nodes can be converted into a correspondent dataset. Also, objective space data are captured in correspondence with the user input data. For the objective space data, mass, displacement, and internal forces are captured from the results returned from Karamba which is selected as the main initial structural analysis software in benchmark studies.

Following the data capture and database management process, data normalization is conducted as done in the probabilistic study process. Design space and objective space data are remapped between 0 and 1 in order to eliminate bias between data features and make training process more robust.

Following the data preprocessing, remapped data are fed to artificial neural networks with  $9 \times n \times 3$  and  $9 \times n_1 \times n_2 \times 3$  fully connected network architecture where  $n$  is the number of neurons within the hidden layers that can be changed to increase the performance of the machine learning model.

Input and output neurons are fixed to 9 and 3 since there are 9 design space data (number of divisions, direction of diagonals, cross section index, vertical position of the first / middle / last top node, vertical position of the first / middle / last bottom node) for input and 3 objective space data (mass, displacement, average internal forces) for output (Figure 3.24). After the data frame is prepared, probabilistic part is tested with different data via discrete sessions.

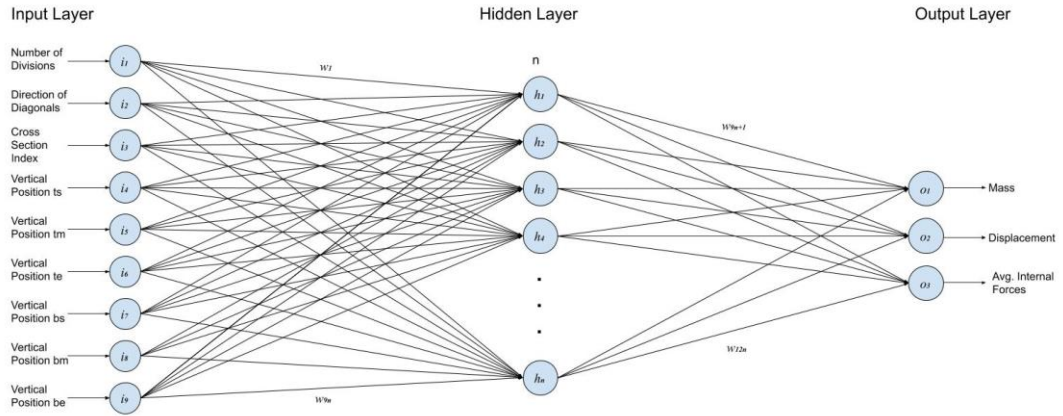


Figure 3.24. Elements of an ANN (9×n×3) used in the integrated hybrid study (developed by the author)

Regardless of the usage of different network architecture and learning models, methodology for the probabilistic part of the hybrid study is based on the idea of back-propagation of artificial neural networks (Rumelhart et.al., 1986). In this approach, firstly, inputs given to the neural network ( $i_n$ ) are multiplied with weight values assigned to connections between input layer and hidden layer ( $w_m$ ), and then transferred to the hidden layer ( $net_{h_n}$ ).

Then, hidden layer transforms the value ( $out_{h_n}$ ) with an activation function (a logistic function is used in most cases in order to calculate derivatives properly in the following steps). Calculated value is multiplied with weight values again assigned to connections between the hidden layer and output layer ( $w_l$ ), and initial outputs are calculated as formulated below:

$$net_{h_n} = \sum_{m,n=1}^{j,k} w_m \cdot i_n \Rightarrow out_{h_n} = \frac{1}{1 + e^{-net_{h_n}}} \Rightarrow o_i = \sum_{l,n=1}^{i,k} w_l \cdot out_{h_n}$$

Secondly, a backwards pass is conducted from outputs to inputs as the methodology suggests. In order to conduct a backpropagation, error ( $E_{total}$ ) is calculated that compares the targeted outputs and initial outputs with the formula above in order to observe how close the prediction of the neural network is to the ground-truth values ( $target_{o_i}$ ) in the dataset.

Then, partial derivatives ( $\delta$ ) of the weight values ( $w_{m,n}$ ) with respect to the error value ( $E_{total}$ ) are calculated in order to understand how each weight value affects the output of the neural network. Calculated partial derivative values ( $\delta_{o,h}$ ) are multiplied with learning rate ( $\mu$ ) which represents how fast the algorithm should learn in each iteration and weight values are updated accordingly with the formulation below.

Following update, rules derived from the backpropagation method (Rumelhart et.al., 1986) are used for a single iteration and weight values are updated until the performance criteria of the algorithms are met in order to obtain predictions close to the ground-truth values in the dataset:

$$E_{total} = \frac{1}{2} \sum (target_{o_i} - o_i)^2 \Rightarrow \delta_{o_i} = \frac{\partial E_{total}}{\partial o_i} = -(target_{o_i} - o_i) \cdot o_i \cdot (1 - o_i)$$

$$\frac{\partial E_{total}}{\partial w_l} = -(target_{o_i} - o_i) \cdot o_i \cdot (1 - o_i) \cdot out_{h_n} \Rightarrow w_l^+ = w_l - \mu \cdot \delta_{o_i} \cdot out_{h_n}$$

$$\frac{\partial E_{total}}{\partial w_m} = \left( \sum \delta_{o_i} \cdot w_l \right) \cdot out_{h_n} \cdot (1 - out_{h_n}) \cdot i_n = \delta_{h_n} \cdot i_n \Rightarrow w_m^+ = w_m - \mu \cdot \delta_{h_n} \cdot i_n$$

On the other hand, method for the deterministic part of the hybrid study is based on the idea of mathematical formulations and calculation methods used in the deterministic study. Calculations of the deterministic study are based on the analysis of specific critical instances chosen from the dataset. Calculations are reformulated

in a generic way so that they can be used for all instances generated in the scope of the integrated study.

In order to create generic formulations to be used in the unpredictable yet controlled hybrid study data, all variables that can be changed directly or indirectly with the user control are listed as outer diameter of the cross section, thickness of the cross section, length of the steel members, number of steel members, and cartesian coordinates of nodes.

Also, variables that can be changed via definition of the structural design problem but not be changed by the user (density of the steel, modulus of elasticity, force applied to the node, total span of the truss, cartesian coordinates of supports) are added to achieve a more generic formulation to calculate a truss system. All variables are listed and notated as shown below:

$t$  = thickness of the cross section

$S$  = total span of the truss system

$\emptyset_{outer}$  = outer diameter of the cross section

$E_{steel}$  = modulus of elasticity for steel

$d_{steel}$  = density of the steel

$L_i$  = length of the steel member

$F_j$  = force applied to the node

$P_j(x_j, y_j)$  = cartesian coordinates of nodes

$P_k(x_k, y_k)$  = cartesian coordinates of supports

Following the identification of all initial design space variables, dependent variables necessary for the objective space calculations are derived from the specified design space variables and equations are formulated in a manner that would not include any variable that is not identified as design space variable beforehand. Firstly, total mass of the structure calculations in deterministic study is reformulated in a generic equation. To achieve the generic equation for mass, dependent variables for mass calculations that are inner diameter of the cross section, inner and outer radius of the

cross section, area of the cross section, total length of steel members, and total volume of the steel members are derived respectively from the design space variables. After the dependent variables are derived, generic mass equation was formulated as below:

$$\begin{aligned}\phi_{inner} &= \phi_{outer} - t \Rightarrow r_{outer} = \phi_{outer}/2 \text{ and } r_{inner} = \phi_{inner}/2 \\ A_{steel} &= \pi \cdot (r_{outer}^2 - r_{inner}^2) \text{ and } L_i = \sqrt{(x_{j+1} - x_j)^2 + (y_{j+1} - y_j)^2} \\ L_{steel} &= \sum_{i=0}^n L_i \text{ and } V_{steel} = A_{steel} \cdot L_{steel} \Rightarrow m_{steel} = A_{steel} \cdot L_{steel} \cdot d_{steel}\end{aligned}$$

After formulating the mass equations, shear force equations are formulated since it should be calculated prior to deflection. Force equations are also reformulated based on the joint method calculations done on deterministic study. To obtain the generic equation for shear forces, dependent variables of force calculations that are angle of members with respect to the horizontal axis, and distances between supports and nodes are derived from cartesian coordinates of nodes and supports, and total span of the truss. With dependent variables derived from design space variables and force variable, generic shear force equation is formulated:

$$\begin{aligned}\text{for each } j \text{ and } k, \quad a_j &= \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2} \text{ and } b_j = S - a_j \\ \sum_{j=0}^n F_j &= V_1 + V_2 \Rightarrow V_j = \frac{\sum F_j \cdot b_j}{S} \text{ and } V_{j+1} = \frac{\sum F_j \cdot a_j}{S} \\ \text{for each } i \text{ and } j, \quad slope_i &= \frac{y_j - y_{j+1}}{x_j - x_{j+1}} = \tan \alpha_i \Rightarrow \alpha_i = \arctan\left(\frac{y_j - y_{j+1}}{x_j - x_{j+1}}\right) \\ \sum F_X &= \sum F_Y = 0 \Rightarrow \sum_{i,j=0}^{n,m} F_j + V_j + N_i \cdot \sin \alpha_i + N_i \cdot \cos \alpha_i = 0 \\ \text{for each } i \text{ and } j, \quad N_i &= -\frac{F_j + V_j}{\sin \alpha_i + \cos \alpha_i}\end{aligned}$$

Following the formulation of generic shear force equation, deflection is formulated as the final step of the deterministic part of the hybrid method. Deflection is formulated based on the unit load method which is also used in the deterministic study. To achieve the generic equation for the deflection, inner diameter of the cross section, inner and outer radius of the cross section, area of the cross section, and length of the steel sections are derived from the design space variables respectively. Together with dependent calculation results that are virtual force applied to the member, and actual force applied to the member, deflection is formulated as below:

$$\text{for each } i \text{ and } k, \quad n_i = -\frac{1 + 1/k}{\sin \alpha_i + \cos \alpha_i} \Rightarrow \Delta = \sum_{i=1}^{i=n} \frac{n_i \cdot N_i \cdot L_i}{A_{steel} \cdot E_{steel}}$$

Finally, both probabilistic and deterministic approach formulations are translated to computational environments in order to achieve a functional algorithm. For the probabilistic part of the hybrid algorithm, all design space parameters that are thickness of the cross section, total span of the truss, outer diameter of the cross section, modulus of elasticity, density of the steel, lengths of the steel members, force applied to the node, cartesian coordinates of the nodes, and cartesian coordinates of the supports are defined as variables in Grasshopper environment. After the initialization process, all variables are arranged to be captured using a data recorder which will collect, and cumulate data changed by a user to generate a truss instance together with simulation results returned from Karamba.

Following the setup of data capturing process, collected data samples are transferred to artificial neural network engine to observe whether they successfully return the intended outputs. After the artificial neural network run tests are successfully accomplished, different network architectures (Figure 3.25) are tested.

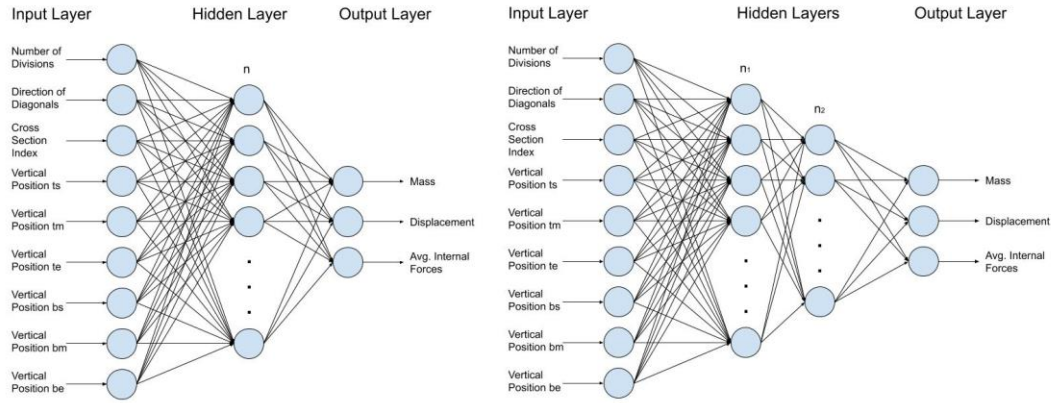


Figure 3.25. Artificial neural networks with different network architectures (developed by the author)

After the implementation of probabilistic part of the hybrid study, deterministic part is also implemented using the same variables that are thickness of the cross section, total span of the truss, outer diameter of the cross section, modulus of elasticity, density of the steel, lengths of the steel members, force applied to the node, cartesian coordinates of the nodes, and cartesian coordinates of the supports were initialized within a Python virtual environment. Following the initialization of all parameters, static and dynamic variables are separated in order to constrain any changes in the static ones (modulus of elasticity and density of the steel) while dynamic ones are left as changeable within the Python script.

Following the variable initialization, generic mathematical formulations used and described in the deterministic part of the hybrid study are written in Python language in order to calculate truss instances generated and controlled by the user. After the initial runs and calculations are deployed within the Python environment, AnaStruct package for 2D structural calculations in Python (Vink, 2016) is used to confirm and visualize the trusses (Figure 3.26). Calculated results are then returned to the probabilistic part of the hybrid study to be used as stopping criteria for the artificial neural networks.

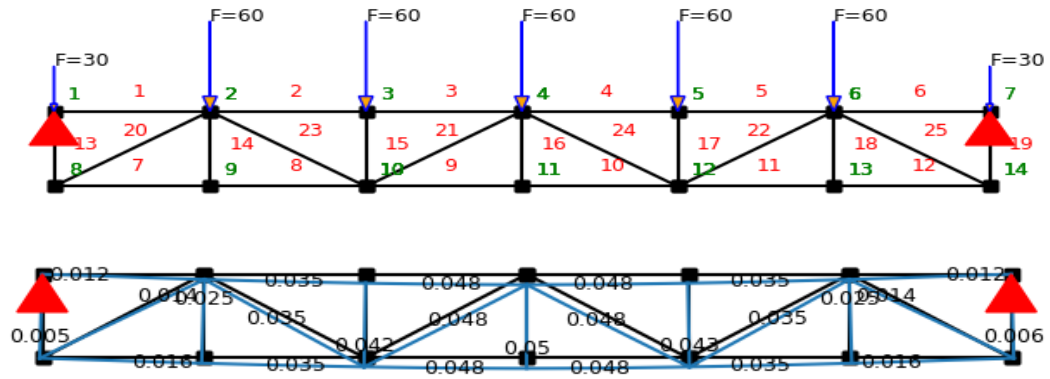


Figure 3.26. Visualization of force and deflection calculations of a truss in Python (developed by the author)

Finally, and most importantly; described probabilistic part that involves the prediction process by using artificial neural networks, and deterministic part that involves the analysis of structural performance of truss systems are integrated to enable the user to interact and collaborate. As described in the method of the probabilistic study and in the method of the deterministic study parts, both approaches have limitations in terms of enabling user to control the outcomes of the algorithm.

In the case study on probabilistic approach, it is observed that artificial neural networks are able to predict the design space parameters with a low error rate when given a sufficient amount of objective space data, as shown in the results of the probabilistic study part. Yet, this approach is not iterative and therefore predicted solution returned to user is only limited with the unique set of solution space, intuitively constructed by the user but randomly generated by a genetic algorithm.

Similarly, in the case study on deterministic approach, it is shown that analytical evaluations on truss instances will be used as a method to search through objective space and determine the design space parameters accurately as presented in the results of the deterministic study section. Hence, this approach is not also iterative and therefore rely on the ground-truth within the one and only set of solution space constructed by the user. To conclude, both approaches are inevitably remained as a

search method by themselves rather than proposing an exploration methodology as discussed in the conclusion chapter.

In order to overcome the problems discussed above, integrated method is formulated as a dynamic and iterative approach in which ground-truth of the structural design problem is reconstructed together with the dataset as the user interacts with the algorithm (Figure 3.27).

As a first step of the proposed methodology, design space parameters are allowed to be changed by the user and each change within the design space data is captured. After that, captured data (capta) are both used to construct the analytical model to be sent to structural simulation, and to construct the dataset to be sent to artificial neural network engines.

Then, simulated analytical model results are compared with deterministic structural analysis (as formulated and described previously) to determine whether the structure is within the safe region of the solution space via safety factors given by the user for each objective function (mass, force, and displacement). This returned information of being in the safe region or not (true / false) is used as a label for the captured user data and sent to artificial neural networks for each design space parameter. Therefore, a  $9 \times 3$  matrix of true / false values (booleans) are generated as shown below by artificial neural network engines representing the correlation between each design space parameter and each objective space function.

Generated matrix of true / false values is arranged to be returned to the user so that user can understand which parameters are responsible for which objective function failures (i.e. false values returned from the deterministic part since the instance is not classified as within the safe region). In the final step, user can determine the parameter to be changed to improve the performance of the truss for any of the objective functions within the matrix.

After each change is done by the user, it will define a new ground-truth and all the previous instances will be added to the dataset and therefore a new matrix will be

returned to the user. Therefore, user can change different parameters guided by a matrix returned in each iteration to achieve an instance where all parameters return true value for each objective function indicating that design space parameters satisfies the safe region within the solution space.

$$\left[ \begin{array}{l} \text{for each} \\ \text{objective} \\ \text{function} \\ f_o(x) \end{array} \right] \left[ \begin{array}{l} \text{Parameter } p_1 \rightarrow \text{True} | \text{False} \\ \text{Parameter } p_2 \rightarrow \text{True} | \text{False} \\ \text{Parameter } p_3 \rightarrow \text{True} | \text{False} \\ \text{Parameter } p_4 \rightarrow \text{True} | \text{False} \\ \vdots \\ \text{Parameter } p_n \rightarrow \text{True} | \text{False} \end{array} \right]_{n \times 1} \Rightarrow \left[ \begin{array}{l} 1|0 \\ 1|0 \\ 1|0 \\ 1|0 \\ \vdots \\ 1|0 \end{array} \right] \dots \left[ \begin{array}{l} 1|0 \\ 1|0 \\ 1|0 \\ 1|0 \\ \vdots \\ 1|0 \end{array} \right]_{n \times o}$$

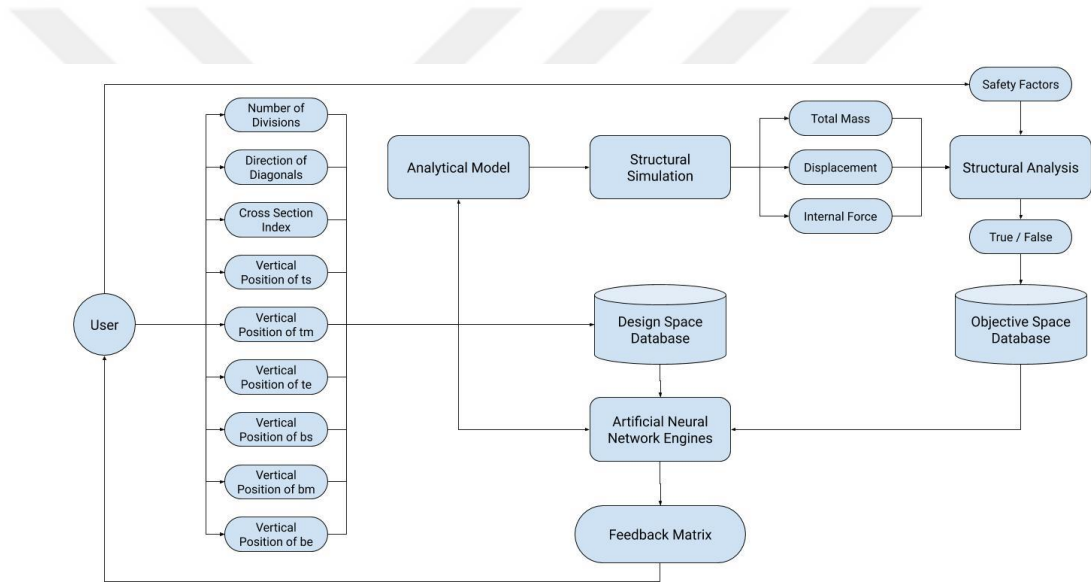


Figure 3.27. The flowchart of the iterative hybrid approach methodology (developed by the author)



## **CHAPTER 4**

### **ANALYSIS AND RESULTS**

In this chapter, proposed methods for probabilistic, deterministic, and integrated approaches are explained respectively and their potentials / limitations are compared. Also, results for all proposed methods are collected and analyzed in following sections. Finally, results are illustrated for each method to conclude the potentials / limitations discussed for each approach.

As potentials and limitations of each approach are discussed starting from literature review through materials and method, a statistical comparison in general is presented before analyzing each approach in detail. To compare the nature of each step, total solution space is compared with the explored solution space. For each step, size of the total solution space is calculated by multiplying each of the number of possible values that each parameter can take (i.e. interval of each parameter to the power of total parameter number).

On the other hand, explored space is calculated by the number of generated instances before the algorithm is terminated (i.e. number of iterations of each proposed methodology before finding a solution that satisfies all objective functions). Comparison of the size of total solution space with respect to explored portion of solution space have shown that, the probabilistic approach is potent in generating high number of instances and therefore exploring a wider design space since there is no ground-truth case. Yet, this nature also causes an exponential growth of total solution space which creates even more space to be explored.

It is seen that the deterministic approach is the most powerful approach in terms of exploring the highest ratio of solutions to the total number of all possible solutions. Yet, this nature is also caused impotency of exploring a wide variety of instances since instances are strictly based on the limited parametrizations on the selected

ground-truth. Finally, in the integrated approach, comparisons have shown that both the total size of solution space and the ratio of the explored space to all solution space is positioned itself in-between the probabilistic approach and the deterministic approach.

But most significantly, integrated approach also positioned as the approach that needs the least amount of iterations to converge to a solution that satisfies all objective functions. Therefore, even it defines a smaller solution space compared to the probabilistic approach, and explores a smaller portion of that space compared to the deterministic approach; it still ables to converge in fewer iterations compared to both approaches.

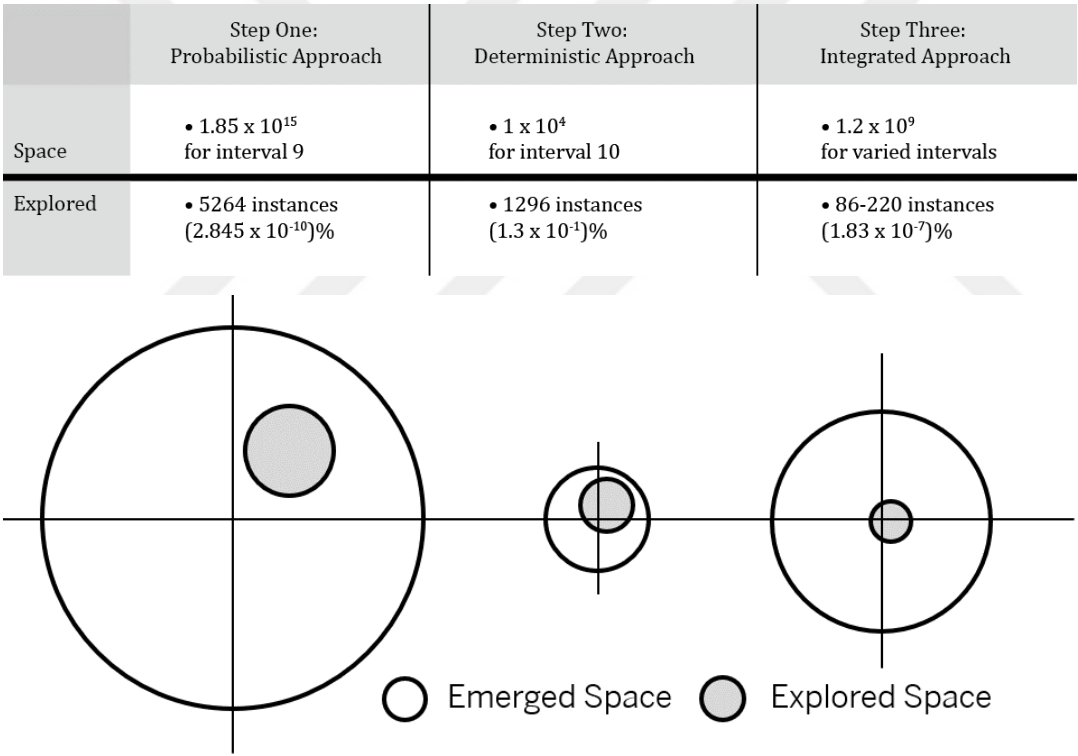


Figure 4.1. Total space and explored space for the studies (developed by the author)

#### 4.1 Analysis and Results of the Case Study on Probabilistic Approach

For analyzing the case study on probabilistic approach, data from sample parametric model for optimization of truss diagonals from Karamba is used as mentioned in the material of research. This sample parametric model has 16 input variables (start and end point of 8 truss diagonals) that control the whole geometry and 2 selected output variables that are the mass of the structure and maximum displacement.

Data collected by randomizing these 16 variables (Figure 4.2) within a gene pool in a randomized manner by using Galapagos genetic solver and recording 2 output data (Figure 4.3) for each design alternative. Finally, mean squared error of trained artificial neural network is calculated with the formula below where  $N$  is the number of data points,  $f_i$  is the value returned by the model and  $y_i$  is the actual value for data point. Mean squared error is recorded for each iteration to analyze how the learning process advances by the following formula  $i$ :  $MSE = \frac{1}{N} \sum_{i=1}^N (f_i - y_i)^2$ .

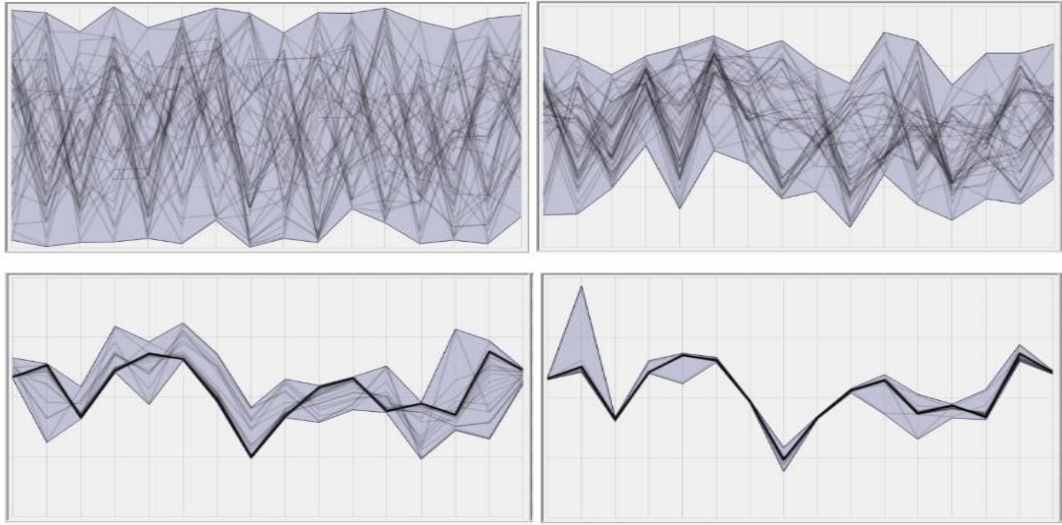


Figure 4.2. Area chart of 16 input variables for generations 1, 5, 25 and 50 (developed by the author)

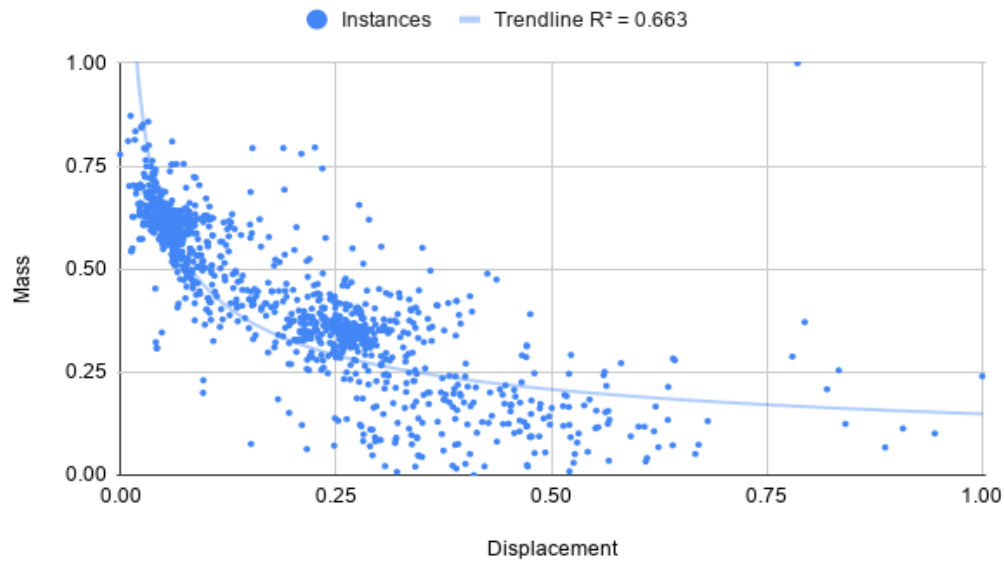


Figure 4.3. Scatter plot of 2 output variables: total mass and maximum displacement (developed by the author)

After data are collected, initial training of artificial neural network is conducted with 350 and 1150 different design alternatives and it is observed that the mean squared error of artificial neural network decreases as more data are given. Then, Galapagos evolutionary solver is used to generate different design alternatives by genetic shuffling and feed the artificial neural network with input and output in each iteration to minimize the mean squared error of the ANN. Following 5264 iterations, mean squared error rate of the artificial neural network is decreased to 0.11614 when ANN with 4 by 4 hidden layer architecture is used.

Then, same data of 5264 different design alternatives are given to an ANN with 8 by 8 hidden layers and mean squared error is decreased to 0.063562 (Figure 4.4). Finally, the trained artificial neural network with the mean squared error rate of 0.0636 is used to predict diagonal support positions of a truss when different scores for mass and displacement are given and it is observed that predicted positions are quite close to ground truth positions of truss diagonals.

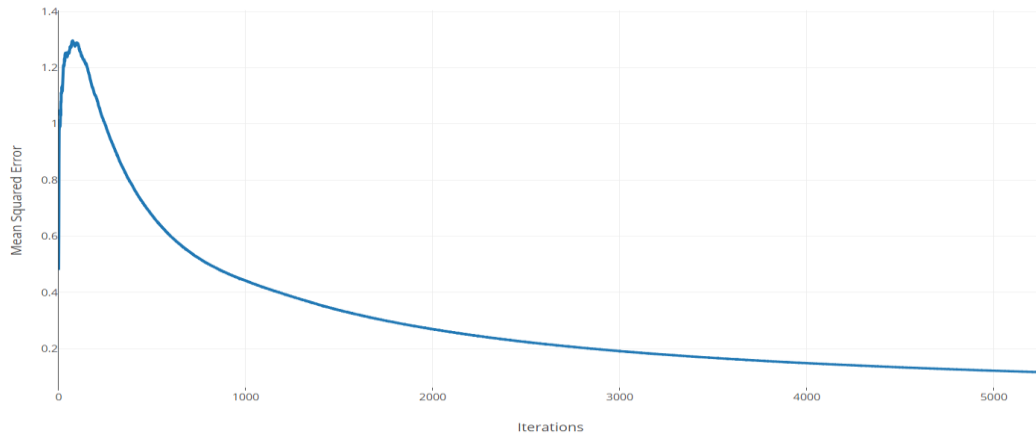


Figure 4.4. Line graph of mean squared error over trained model iterations (developed by the author)

## 4.2 Analysis and Results of the Case Study on Deterministic Approach

To analyze the results of the case study on deterministic approach, calculated and simulated results are compared both for deflection and mass in order to conduct a benchmark study for multi-objective data. Deflection value for the first instance is calculated as 3.51 cm with unit load method as shown previously. In comparison, simulated deflection values from Karamba, Tekla, SAP2000, and Robot software were resulted as 3.75 cm, 2.87 cm, 2.46 cm, and 2.10 cm respectively.

Similarly, deflection value for the second instance is calculated as 1.75 cm with unit load method where simulated deflection values from Karamba, Tekla, SAP2000, and Robot software are found to be as 1.97 cm, 1.63 cm, 1.19 cm, and 1.17 cm respectively (Figure 4.5). Likewise, mass value for the first instance is calculated as 407.47 kg with formulation shown above.

In comparison, simulated mass values from Karamba, Tekla, SAP2000, and Robot are returned as 395.0 kg, 361.0 kg, 419.6 kg, and 343.9 kg. In a similar way, mass value for the second instance is calculated as 457.51 kg where simulated mass values from Karamba, Tekla, SAP2000, and Robot are returned as 443.6 kg, 405.0 kg, 471.1 kg, and 385.7 kg (Figure 4.5).

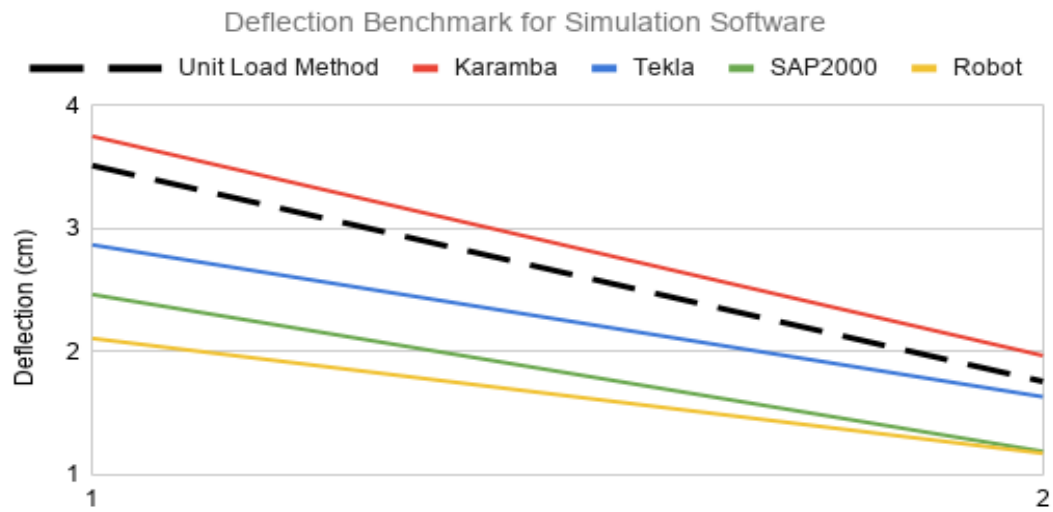


Figure 4.5. Visual comparisons of calculated and simulated results for deflection (developed by the author)

Table 4.1. Numerical comparisons of calculated and simulated results for deflection (developed by the author)

| # | Calculated | Karamba | Tekla | SAP2000 | Robot |
|---|------------|---------|-------|---------|-------|
| 1 | 3.51       | 3.75    | 2.87  | 2.46    | 2.10  |
| 2 | 1.75       | 1.97    | 1.63  | 1.19    | 1.17  |

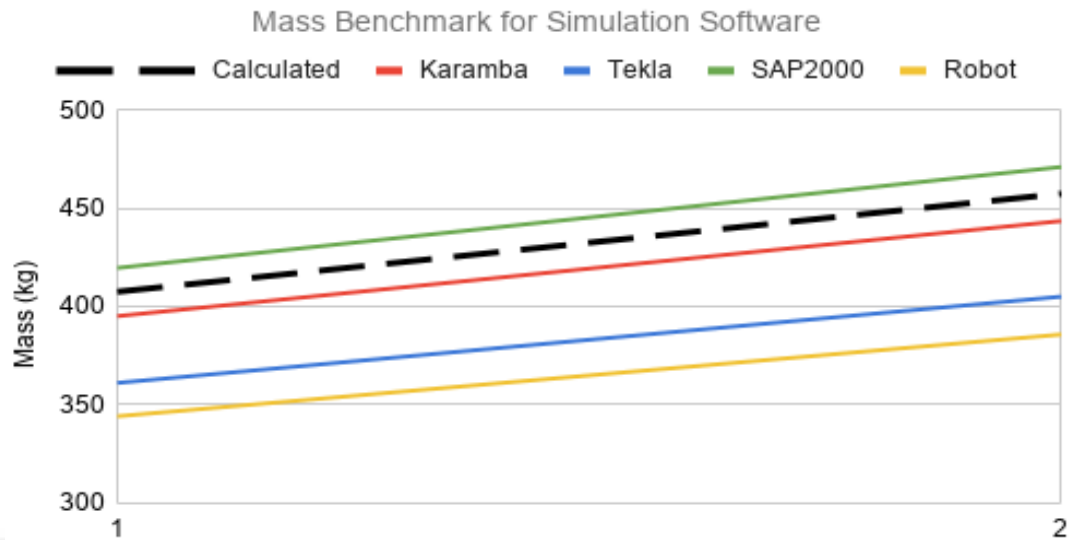


Figure 4.6. Visual comparisons of calculated and simulated results for mass (developed by the author)

Table 4.2. Numerical comparisons of calculated and simulated results for mass (developed by the author)

| # | Calculated | Karamba | Tekla | SAP2000 | Robot |
|---|------------|---------|-------|---------|-------|
| 1 | 407.5      | 395.0   | 361.0 | 419.6   | 343.9 |
| 2 | 457.5      | 443.6   | 405.0 | 471.1   | 385.7 |

Comparison results have shown that, most accurate results are returned from Karamba software since it defines the closest line to the calculated ground truth line both in deflection and mass benchmark tests. Results obtained from Tekla and Robot software is not as accurate as results obtained from Karamba due to several reasons. The first reason is that, Karamba is formulated to conduct initial structural simulations for the early stages of design which is more suitable for the case study while Tekla and Robot are designed for the advanced structural simulations which requires further details to be more accurate.

Another possible reason is the potential data loss throughout the process which is comparatively less since Karamba works directly within Grasshopper while Robot and Tekla needs intermediary processes to be connected with Grasshopper.

Last but not least, difference between working principles (i.e. finite element method, finite volume method, finite difference method, boundary element method) and materials (i.e. EU/US standards, metric/imperial system) of software can be another possible reason. Yet, since the scope of the study is not about interoperability and simulation methods but design and objective space exploration, Karamba is chosen as the main structural simulation software to conduct integrated case study as explained in previous sections.

Finally, calculated results obtained from the Karamba software are stored in a database and visualized using three-dimensional point construction method in order to enable user to visually explore the generated instances within the constructed solution space. Using the design space parameters mentioned in the material of the case study on deterministic approach, generated 1296 instances are enhanced to 19683 instances by including the position changes of top nodes in addition to bottom nodes and all instances are simulated to construct and visualize the solution space. Returned results of the total mass, average internal force and maximum displacement values are normalized and remapped between 0 and 100, in order to both eliminate the hierarchy between three data features and make simulation results visually understandable for the user.

After the remapping of three objective space data features, values are used to define points in three-dimensional space where each feature represents the value of a point in a different axis (for example, if an instance is composed of a mass value of 50, displacement of 30, and internal force of 80; then this instance will be represented by a point in which X, Y, Z coordinates are 50, 30, 80) (Figure 4.7).

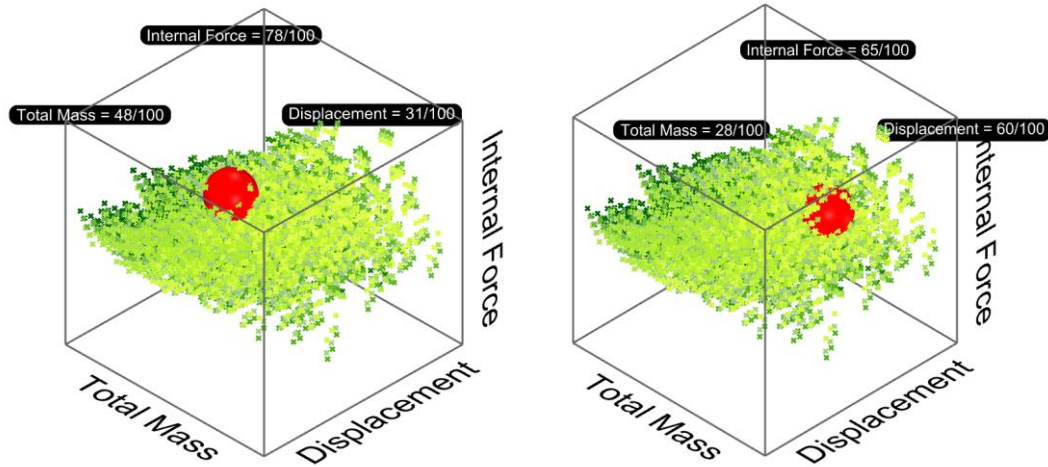


Figure 4.7. Visual exploration of the solution space for the deterministic study (developed by the author)

### 4.3 Analysis and Results of the Integrated Case Study

In order to analyze the results of the case study on integrated approach, proposed methodology is demonstrated and tested on numerous runs as a user. In each case, captured data for each initialized design space parameter that are number of divisions, direction of diagonal members, cross section index, vertical position of top nodes, and vertical position of bottom nodes are sent both to structural analysis and database for the training dataset.

Also, returned objective space data that are total mass of the structure, average internal forces of steel members, and maximum deflection are captured and accumulated in a different database in order to train the artificial neural network engines as it is proposed in the methodology.

Finally, predicted objective space matrix obtained from artificial neural networks is used as a feedback to reparametrize the design space in each iteration and truss instances are explored recursively (Figures 4.8, 4.9, and 4.10) until all indices within the matrix returned from artificial neural networks is equal to 1 (i.e. true).

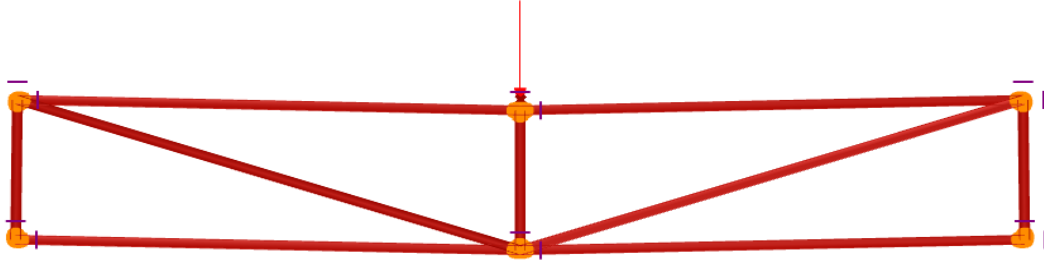


Figure 4.8. An initial optimization test study with hybrid approach matrices,  $i = 0$  (developed by the author)

$$\begin{bmatrix} x_0 & 2 \\ \text{for each} & 0 \\ \text{objective} & 10 \\ \text{function} & 0.0 \\ f_1 = \text{mass} & 0.0 \\ f_2 = \text{force} & 0.0 \\ f_3 = \text{delta} & 0.0 \end{bmatrix} \Rightarrow \begin{bmatrix} \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \end{bmatrix} f_1(x_0) \wedge \begin{bmatrix} \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \end{bmatrix} f_2(x_0) \wedge \begin{bmatrix} \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \\ \text{False} \end{bmatrix} f_3(x_0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

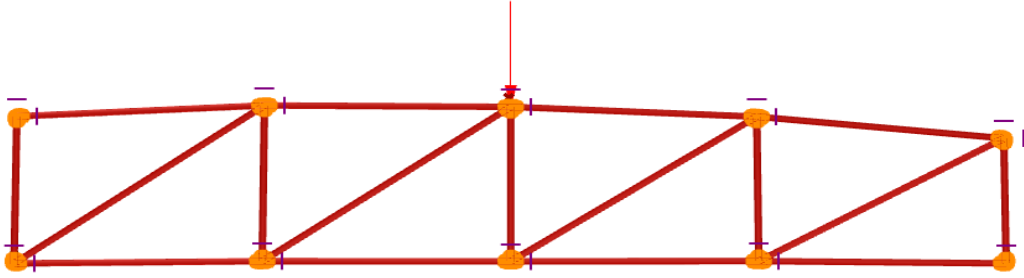


Figure 4.9. An initial optimization test study with hybrid approach matrices,  $i = 50$  (developed by the author)

$$\begin{bmatrix} x_{50} \forall f & 4 \\ & 2 \\ & 5 \\ & -0.3 \\ & 0.8 \\ & -0.8 \\ & -0.6 \\ & -0.1 \\ & -0.6 \end{bmatrix} \Rightarrow \begin{bmatrix} \text{True} \\ \text{False} \\ \text{True} \\ \text{False} \\ \text{False} \\ \text{True} \\ \text{True} \\ \text{True} \\ \text{True} \end{bmatrix} f_1(x_{50}) \wedge \begin{bmatrix} \text{True} \\ \text{False} \\ \text{True} \\ \text{False} \\ \text{False} \\ \text{True} \\ \text{True} \\ \text{True} \\ \text{True} \end{bmatrix} f_2(x_{50}) \wedge \begin{bmatrix} \text{True} \\ \text{True} \\ \text{True} \\ \text{True} \\ \text{False} \\ \text{True} \\ \text{True} \\ \text{True} \\ \text{False} \end{bmatrix} f_3(x_{50}) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

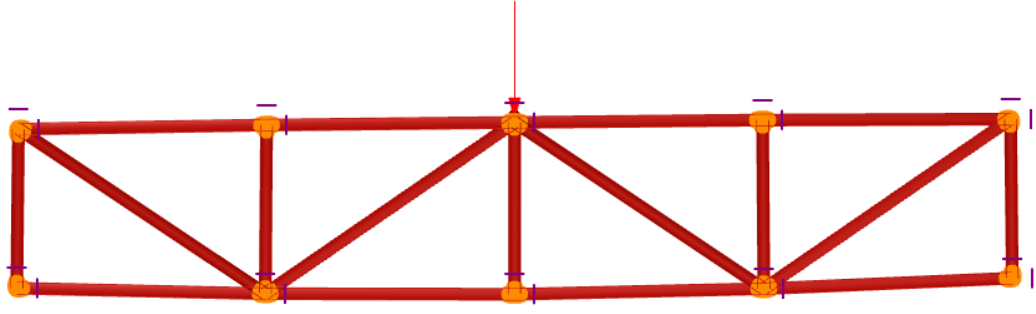


Figure 4.10. An initial optimization test study with hybrid approach matrices,  $i = 220$  (developed by the author)

$$\left[ \begin{array}{c} x_{220} \forall f \\ 4 \\ 0 \\ 30 \\ 0.6 \\ 1.0 \\ 0.8 \\ 0.5 \\ 0.4 \\ 0.7 \end{array} \right] \Rightarrow \left[ \begin{array}{c} f_1(x_{220}) \\ True \\ True \\ True \\ True \\ True \\ True \\ True \\ True \\ True \end{array} \right] \wedge \left[ \begin{array}{c} f_2(x_{220}) \\ True \\ True \\ True \\ True \\ True \\ True \\ True \\ True \\ True \end{array} \right] \wedge \left[ \begin{array}{c} f_3(x_{220}) \\ True \\ True \\ True \\ True \\ True \\ True \\ True \\ True \\ True \end{array} \right] = \left[ \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right]$$

Firstly, an initial optimization test is conducted with a  $9 \times 1$  matrix for a single objective to analyze the convergence of the neural network engines on one objective function. It is observed that neural network engines are able to guide the user for each design space parameter and eventually converge to a solution that satisfies the safe region for the single objective function. Yet, after conducting an initial optimization session using 3 different  $9 \times 1$  matrix representing whether each design space parameter is in the safe region (better than the average was used initially) for each objective function, it is observed that correlation between dependent design space parameters cannot be discovered by neural network engines since each dependent design space parameter is sent to a different artificial neural network independently.

It is also observed that the initial session tests required consecutive wise guesses from the user to be able to converge to a solution where each design space parameter was located in the safe region for each objective function since probability decreases

exponentially with the introduction of a new objective function  $1/(2^9)$  for single objective and  $1/(2^9)^3$  for three objectives), and eventually become exceedingly difficult to converge even with consecutive wise guesses on design space parameter changes.

Therefore, space complexity of the computational model was decreased by using all dependent design space parameters in the same artificial neural network for each objective function and returning a single boolean (true/false) value representing whether the generated instance was within the safe region for the objective function instead of returning a boolean for each design space parameter individually. As a result, probability of converging to a solution which satisfies all three objective functions was reduced from  $1/2^{27}$  to  $1/2^3$  when safety factor was set to  $1/2$  meaning that an instance which performs better than the average of all instances within the generated solution space will be labeled as true. Yet, even for a constrained solution space where each design space parameter was limited to bounds of a region where each instance would be analyzed further and fine-tuned accordingly; safety factors were reinitialized to search for an instance which would be more close to global optima. For the deflection objective, safety factor was set to the total span divided by 250 (O'Rourke, 2014); for the force objective, safety factor was set to yield and tensile strength of  $450 - 1700 \text{ MPa}$  (Pavlina, 2008) multiplied by the cross section area of the steel; and for the mass objective, safety factor was left to the user decision since dead load criteria could change according to the function of the truss. After reducing the space complexity of the algorithm, and reinitializing the safety factors, another optimization study was conducted (Figures 4.10, 4.11, 4.12) by using  $1 \times 1$  matrix for each of the 3 objectives, starting from the same ground-truth ( $x_0$ ) as the previous session.

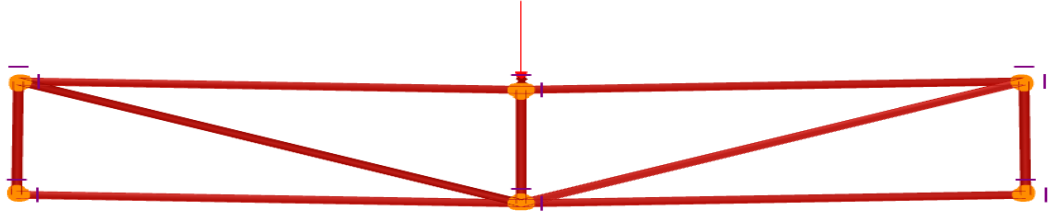


Figure 4.11. Optimization test study with matrices that has lower complexity,  $i=0$  (developed by the author)

$$[x_0 \forall f; f_1 = mass, f_2 = force, f_3 = delta \mid 2, 0, 10, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0] \\ [f_1(x_0) \mid False] \wedge [f_2(x_{50}) \mid False] \wedge [f_3(x_{50}) \mid False] = [False]$$

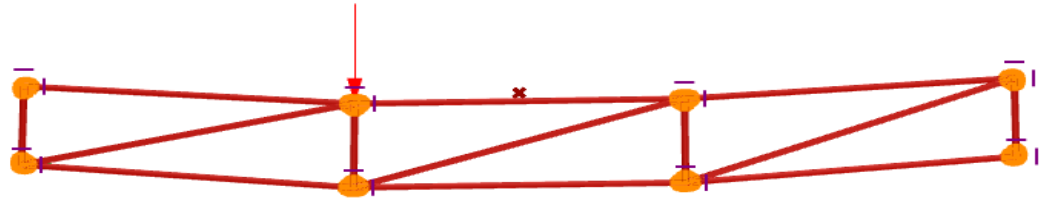


Figure 4.12. Optimization test study with matrices that has lower complexity,  $i=40$  (developed by the author)

$$[x_{40} \forall f; f_1, f_2, f_3 \mid 3, 2, 5, -0.6, -1.0, -0.4, 0.6, -0.1, 0.8] \Rightarrow \\ [f_1(x_{40}) \mid False] \wedge [f_2(x_{40}) \mid False] \wedge [f_3(x_{40}) \mid True] = [False]$$

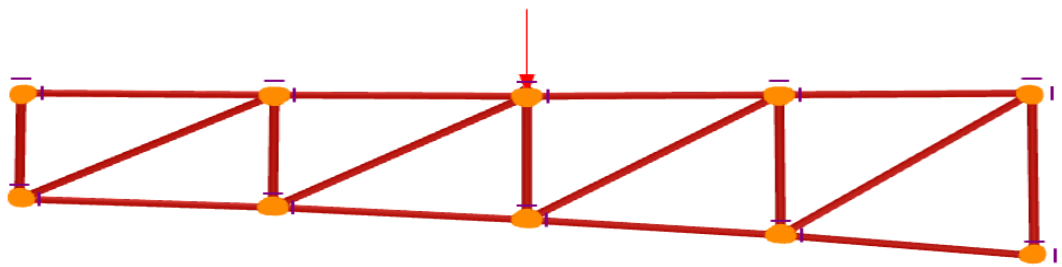


Figure 4.13. Optimization test study with matrices that has lower complexity,  $i=86$  (developed by the author)

$$[x_{86} \forall f; f_1, f_2, f_3 \mid 4, 2, 10, 0.7, 0.8, 0.7, 0.8, 0.7, -0.8] \Rightarrow \\ [f_1(x_{86}) \mid True] \wedge [f_2(x_{86}) \mid True] \wedge [f_3(x_{86}) \mid True] = [True]$$

Results from both optimization case studies have shown that space complexity reduction helped the algorithm to converge to a solution that satisfies all objective functions with less iterations (86 iterations in the second case compared to 220 iterations in the first case). Also, even the safety factors for objective functions were increased with reference to literature and the probability of instances to be located within the safe region of the solution space was decreased, it was observed that algorithm was able to converge in a robust manner. Hence, the comparison of two cases have proven one of the core principles of problem-solving, that is “Occam’s Razor” which suggests not to multiply entities without necessity (Schaffer, 2015). Therefore, solution that has the lower complexity was selected as the principle suggests and multiple optimization sessions were conducted with the algorithm to analyze its performance further. Data from the sessions were recorded separately and visualized to analyze the performance of the algorithms further.

#### Optimization Session 1 - Design Space Data

Stepped Area Chart with Polynomial Trendline

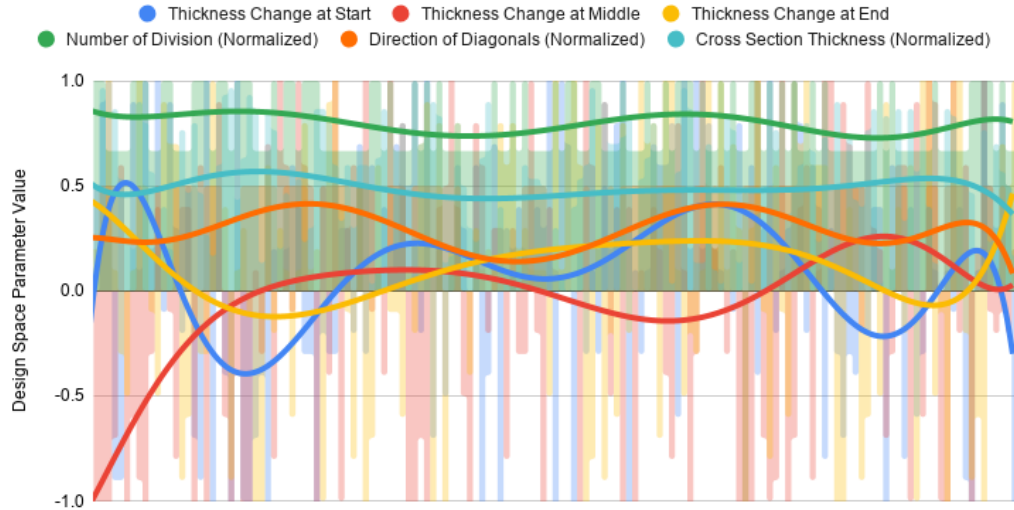


Figure 4.14. Visualization of design space data of an optimization study, session 1 (developed by the author)

### Optimization Session 1 - Objective Space Data

Stacked Stepped Area Chart with Regression Line

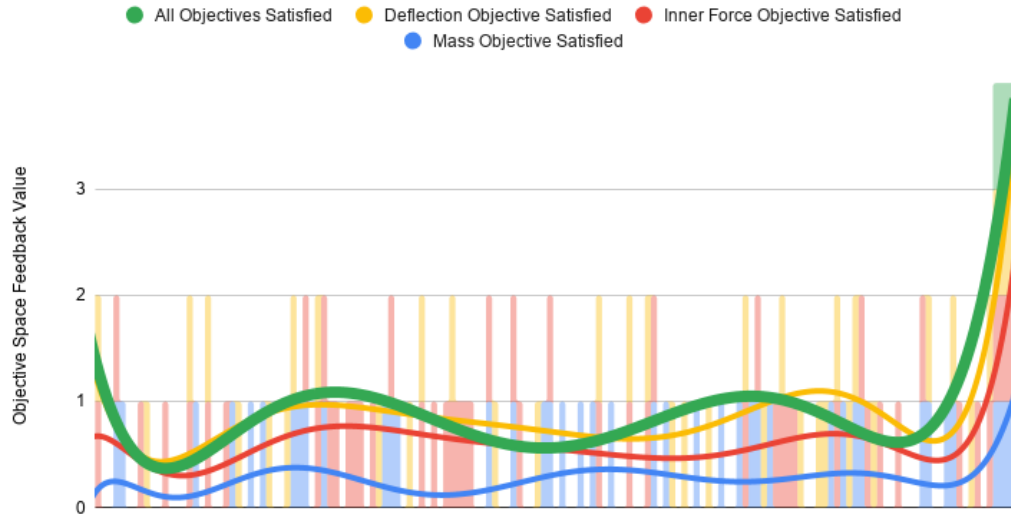


Figure 4.15. Visualization of objective space data of optimization study, session 1 (developed by the author)

In order to record and analyze data from the optimization studies, three different sessions were conducted starting from the same initial condition with design space parameters of  $[ 2, 0, 0, -1.0, -1.0, -1.0, 1.0, 1.0, 1.0 ]$  for the number of divisions, direction of diagonals, cross section index, position of top nodes at start, midpoint, end, and position of bottom nodes at start, midpoint, end respectively which would generate an instance that does not satisfy any of the objective functions of the structural design problem. The reason of starting from this initial condition is to observe whether the proposed integrated approach would converge to a solution that satisfies all the objective functions of the structural design problem, and to observe whether it would converge to different solutions within different sessions. Therefore, three different optimization sessions were conducted starting from the same initial condition and design space parameter values that are the number of divisions, direction of diagonals, cross section index, position of top nodes at start, midpoint, end, and position of bottom nodes at start, midpoint, end were captured along with objective space matrix values that are whether the deflection, inner force, and mass objective was satisfied. After that, data postprocessing was conducted to visualize

all design space parameters within the same chart. Hence, the number of divisions, direction of diagonals and cross section thickness index were remapped between 0 and 1 since they can take only positive values. Also, Euclidean distance between top nodes and bottom nodes were calculated from position of top and bottom nodes and visualized in the same chart as thickness change at start, midpoint, and end that ranges between -1 and 1. Finally, design space parameter values were visualized using a stepped area chart together with polynomial trendlines with a degree of 8 to make the value changes of a juxtaposed stacked area chart readable (Figure 4.14). For the objective space feedback values, data normalization was not applied since the returned values were only composed of booleans (either true or false). Therefore, objective space data were only turned into integer values (either 1 or 0) and visualized by stacked stepped area chart in order to visually represent where the algorithm was able to find a solution that satisfies more than one objective function. Polynomial trendlines were also shown in objective space data visualization to represent the convergence of the algorithm (Figure 4.15).

Visualizations of the first optimization study have proven that the integrated approach was successful in terms of searching variety of solutions within the design space using the objective space feedback values. Also, it was also proven that algorithm was able to converge to a solution set that satisfies all objective functions after 152 iterations. To prove the idea of converging to different solutions that also satisfies all objective functions and confirm the convergence of the algorithm, two more studies were added, and visualizations were compared to analyze the results of the algorithm further. Design space data were postprocessed in the same method and objective space data were encoded as integer as done in the first optimization study. Visualized data have shown that parallel optimization sessions were able to converge after 150 iterations and yet were also able to find different solutions that satisfies all three objective functions. Even it was possible to understand the algorithms ability to converge different truss solutions by analyzing the design space data charts, geometry of truss instances was also reproduced and added to the figure to represent the proficiency of the algorithm clearly (Figures 4.16, 4.17, and 4.18).

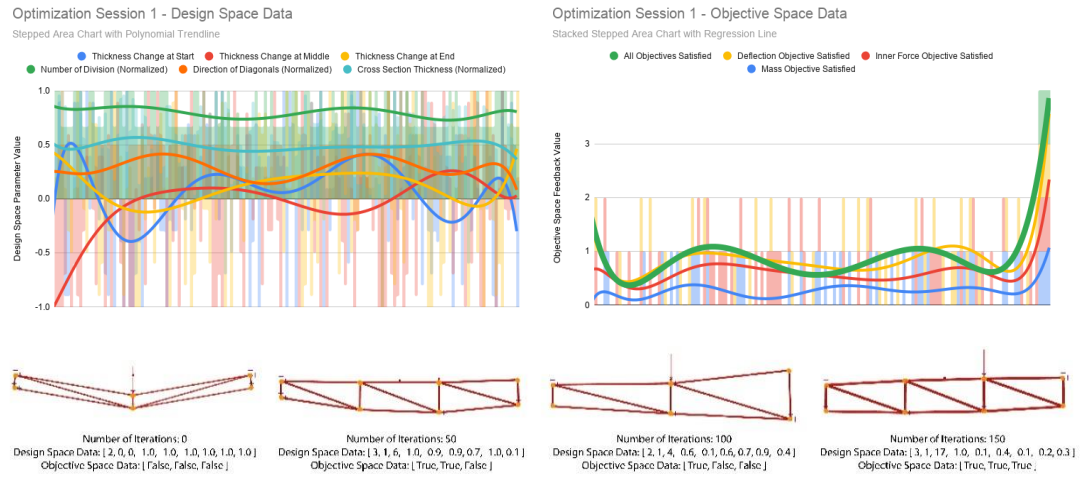


Figure 4.16. Design/objective space data and solutions over iterations, session 1 (developed by the author)

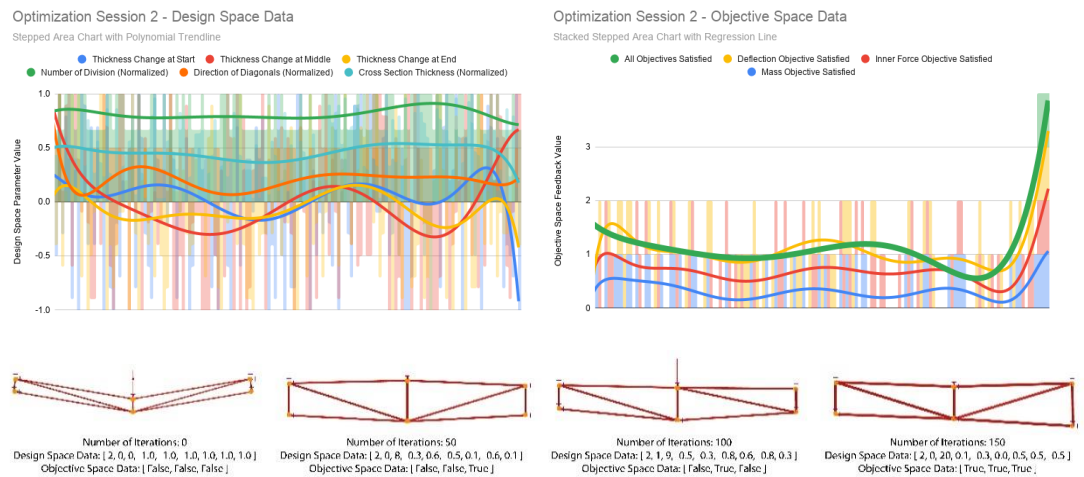


Figure 4.17. Design/objective space data and solutions over iterations, session 2 (developed by the author)

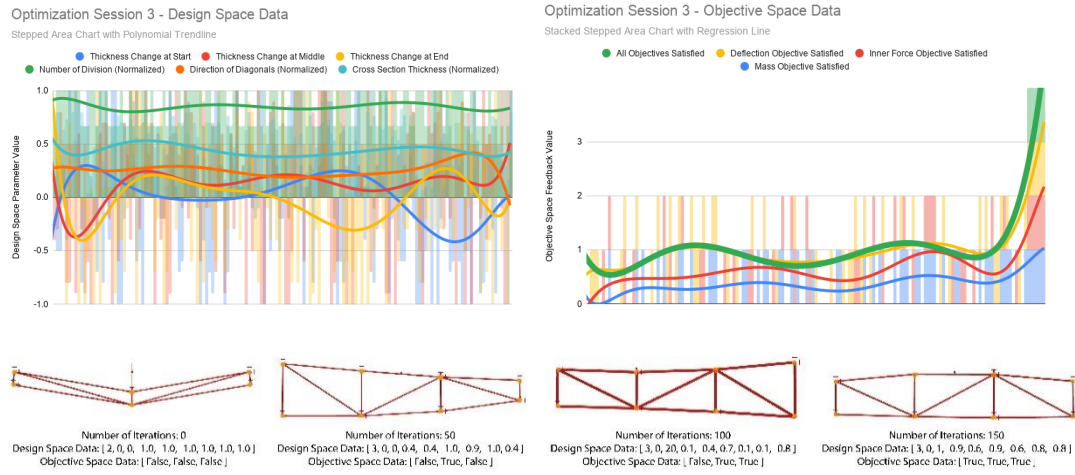


Figure 4.18. Design/objective space data and solutions over iterations, session 3 (developed by the author)

## **CHAPTER 5**

### **CONCLUSION**

To conclude, the three staged research experiment has shown that machine learning algorithms have a great potential to be implemented to initial structural design phase to catalyze design and objective space exploration. Also, results have proven that machine learning algorithms can be trained to extract features and derive correlations within multi-objective criteria to make valid and precise predictions. Yet, this research must be extended by implementing the proposed extensive methodology including different design cases to prove the hypothesis that the proposed learning model is adequate to learn solution space of different problems. Also, the study should be implemented with extensive variables within the design space such as material type, number of supporting elements, load type and quantity and so on to test if the learning model can still learn the design space by using correlations between inputs and outputs. Hence, an extended workflow was proposed to be studied and implemented in future research (Figure 5.1) and planned to be extended in the future.

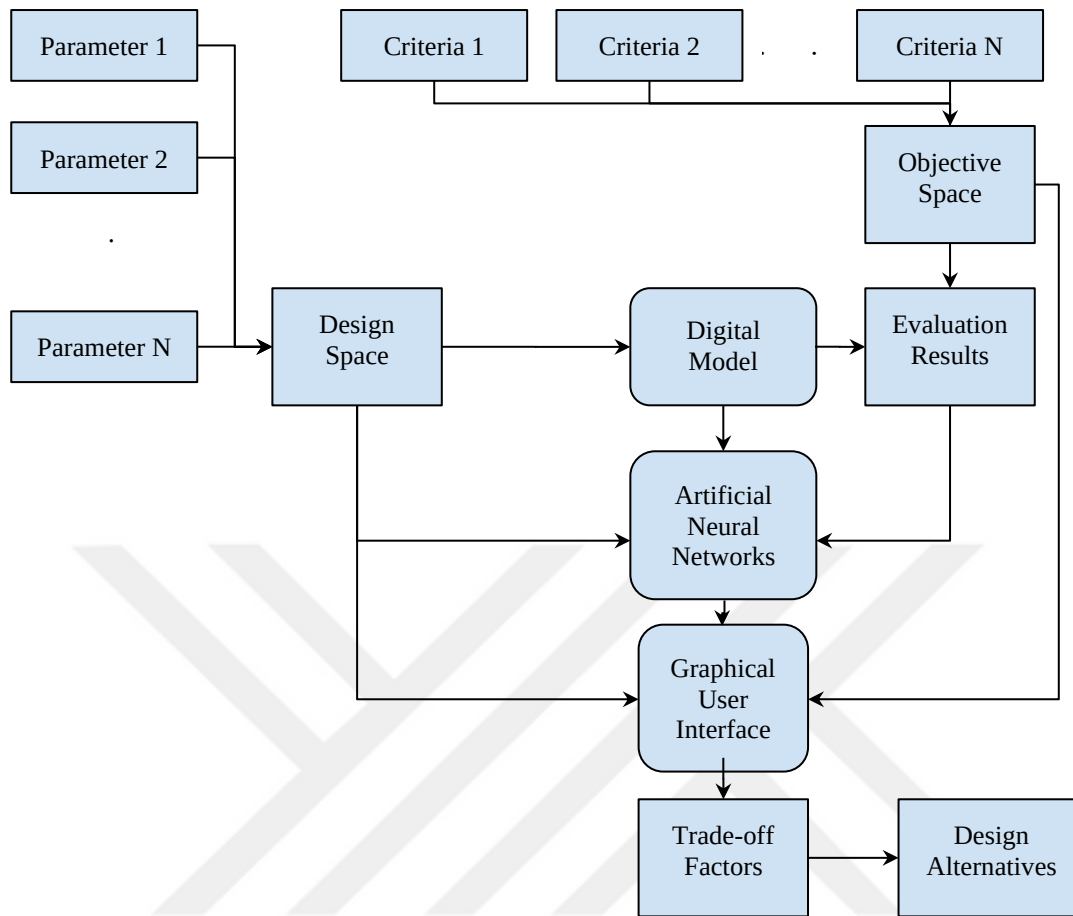


Figure 5.1. Initial extended workflow for the future research (developed by the author)

## 5.1 Discussion

Three staged research experiment has shown that, regardless of the approach to the problem that are probabilistic, deterministic, and integrated in this study; the most important aspect of the experiment is the problem itself. Even one of the most important assets of artificial intelligence models is the ability to adapt different problems because of the generic nature of the algorithms, results have shown that thinking about how to define the problem before trying to solve with different approaches is a crucial step to take. Furthermore, it is also crucial to determine which data is important to be able to solve the problem and therefore which data to be fed to the machine since it is highly associated with the problem definition.

For instance, in the probabilistic study, the structural design problem is defined as searching through a random solution space to find a viable solution that fits the given objective functions. Yet, results have shown that converged solutions do not define a ground-truth since the problem is not defined in a way that starts from a ground-truth case. On the other hand, in the deterministic case study, the problem was based on a ground-truth and it was defined as searching through a restricted solution space to find the solution that satisfies the same three objectives. But, even the converged solutions define a ground-truth, space exploration strategy was remained as questionable as the algorithm only fine-tunes the given ground-truth case instead of searching the global optima. Therefore, an integrated approach was proposed in the light of these previous studies with a problem definition based on a changing ground-truth and a dynamic dataset, controlled both by probabilistic and deterministic models. Results of the integrated approach have shown that it was possible to determine the degree of freedom, reliability of outputs, and user experience by iteratively re-understanding and re-defining the problem itself.

## **5.2 Potentials and Limitations**

Proposed methods to explore the given structural design problem have different potentials and limitations. Therefore, potentials and limitations of proposed approaches that are probabilistic, deterministic, and integrated are discussed in this section respectively. The first experiment conducted with the probabilistic approach is based on the extensive solution space search methods using random generative parameters. Therefore, potentials of the probabilistic approach are broad range of alternatives, freedom of introducing new parameters, and ease of implementation thanks to the chaotic nature of the algorithm. Yet, this chaotic nature also causes limitations that are uncontrolled alternative generation, unintended parameter changes, and unpredictable time for convergence. On the other hand, the second experiment led with the deterministic approach is structured on the formulated calculation methods using a ground-truth case as parameter tuner. Hence, potentials

of the deterministic approach are controlled design alternatives, intended parameter tuning, and predictable time domain for convergence thanks to the systematic nature of the algorithm. But, this systematic order also lead limitations which are narrow design space, constrained degree of freedom, and implementation complexity. Finally, the last experiment proposed as the integration of probabilistic and deterministic approaches is based on the idea of coupling the generativity of randomness with reliability of orderliness. Consequently, both the potentials and limitations of the integrated approach are positioned in-between the pros and cons of the probabilistic and deterministic approach (Table 5.1).

Table 5.1. Potentials and limitations of approaches (developed by the author)

|                    | <i>Probabilistic<br/>Approach</i>                                                                                                                             | <i>Deterministic<br/>Approach</i>                                                                                                                                | <i>Integrated<br/>Approach</i>                                                                                                                                          |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>Potentials</i>  | <ul style="list-style-type: none"> <li>• Variety of alternatives</li> <li>• Adaptability to new parameters</li> <li>• Simplicity of implementation</li> </ul> | <ul style="list-style-type: none"> <li>• Steadyness of alternatives</li> <li>• Controllability of parameters</li> <li>• Predictability of convergence</li> </ul> | <ul style="list-style-type: none"> <li>• Controllability of design freedom</li> <li>• Manageability of parameters</li> <li>• Customizability of optimization</li> </ul> |
| <i>Limitations</i> | <ul style="list-style-type: none"> <li>• Refractory of alternatives</li> <li>• Uncont of parameters</li> <li>• Imponderability of convergence</li> </ul>      | <ul style="list-style-type: none"> <li>• Inadequacy of alternatives</li> <li>• Disability of new parameters</li> <li>• Complexity of implementation</li> </ul>   | <ul style="list-style-type: none"> <li>• Responsibility of design freedom</li> <li>• Learning curve of parameters</li> <li>• Necessity of knowledge</li> </ul>          |

### 5.3 Future Projections

Developed and implemented methodology including probabilistic, deterministic, and integrated approach was presented in three staged experiment and illustrated with case studies on truss systems. Yet, the main idea of this research is to explore the potentials lie on the intersection of artificial intelligence and design domain. As discussed from literature review to results of the experimentations, research done in the intersection of artificial intelligence and architecture is still an unexplored yet growing scientific domain. Therefore, this research aimed to position itself in this intersection and continue to explore further, starting from this initial point. Therefore, even the case studies in this research are limited with two-dimensional simple truss systems, further studies can be done on many other well-defined design problems, using the proposed framework as an initial point. As the proposed system is based on the core principles of atomic design, which is a design system crafting methodology suggesting to think design systems both as a cohesive whole and a collection of parts at the same time (Frost, 2017), it is open to be developed both in atomic and holistic level. Therefore, not only the whole research, but also each microservice (Nadareishvili et al., 2016) of the research was open to be used, discovered, and developed. As Turing, the pioneer of theoretical computer science and artificial intelligence said: “We can only see a short distance ahead, but we can see plenty there that needs to be done.” (1950).



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## APPENDICES

### A. Dataset of the First Step: Probabilistic Approach

Data in this table were pruned due to its length (5264 rows).

Table below shows data in each 10th iteration.

Full data is available upon request.

| Input Vectors                                                                                   | Output Vectors    |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.43,0.562,0.336,0.72,0.472,0.4,0.547,0.659,0.578,0.359,0.595,0.559,0.679,0.336,0.693,0.396     | 0.780042,0.210288 |
| 0.561,0.647,0.661,0.369,0.619,0.853,0.468,0.166,0.67,0.346,0.487,0.958,0.219,0.848,0.879,0.614  | 0.195294,0.443698 |
| 0.096,0.108,0.533,0.362,0.543,0.194,0.637,0.075,0.377,0.324,0.273,0.174,0.647,0.277,0.669,0.995 | 0.554572,0.302958 |
| 0.602,0.209,0.675,0.365,0.81,0.485,0.949,0.646,0.695,0.561,0.162,0.71,0.918,0.264,0.605,0.989   | 0.156572,0.552376 |
| 0.625,0.288,0.39,0.843,0.066,0.767,0.89,0.897,0.593,0.226,0.191,0.454,0.586,0.581,0.663,0.031   | 0.119505,0.514194 |
| 0.037,0.058,0.747,0.501,0.271,0.579,0.089,0.696,0.955,0.753,0.36,0.144,0.09,0.888,0.448,0.779   | 0.162084,0.486854 |
| 0.281,0.174,0.498,0.21,0.789,0.267,0.986,0.053,0.685,0.957,0.818,0.182,0.9,0.05,0.611,0.959     | 1.0,0.785703      |
| 0.109,0.085,0.969,0.487,0.102,0.059,0.924,0.441,0.489,0.122,0.058,0.596,0.975,0.393,0.615,0.728 | 0.021216,0.472893 |
| 0.68,0.857,0.358,0.656,0.728,0.596,0.136,0.491,0.015,0.728,0.286,0.228,0.396,0.798,0.564,0.287  | 0.123191,0.402887 |
| 0.527,0.13,0.792,0.451,0.933,0.975,0.76,0.846,0.845,0.307,0.879,0.584,0.717,0.252,0.286,0.45    | 0.165588,0.386083 |
| 0.028,0.207,0.69,0.64,0.299,0.669,0.208,0.406,0.641,0.796,0.843,0.331,0.216,0.766,0.212,0.345   | 0.167118,0.339918 |
| 0.142,0.398,0.69,0.64,0.299,0.669,0.208,0.406,0.641,0.796,0.843,0.331,0.216,0.766,0.212,0.345   | 0.172624,0.358012 |
| 0.524,0.709,0.344,0.79,0.042,0.336,0.789,0.961,0.428,0.698,0.539,0.684,0.521,0.867,0.293,0.417  | 0.070003,0.289819 |
| 0.691,0.531,0.202,0.615,0.72,0.806,0.818,0.941,0.557,0.053,0.509,0.111,0.44,0.54,0.489,0.575    | 0.186391,0.274607 |
| 0.723,0.421,0.161,0.238,0.697,0.209,0.545,0.269,0.156,0.396,0.968,0.159,0.194,0.681,0.654,0.445 | 0.191698,0.517919 |
| 0.578,0.672,0.326,0.779,0.123,0.829,0.903,0.305,0.886,0.772,0.943,0.766,0.361,0.319,0.668,0.702 | 0.183455,0.375247 |
| 0.266,0.644,0.114,0.544,0.159,0.775,0.815,0.965,0.598,0.107,0.82,0.514,0.346,0.573,0.219,0.452  | 0.142382,0.49751  |
| 0.269,0.773,0.478,0.587,0.174,0.754,0.544,0.412,0.485,0.359,0.445,0.226,0.721,0.577,0.838,0.482 | 0.25489,0.342278  |
| 0.755,0.314,0.621,0.663,0.496,0.817,0.613,0.698,0.591,0.289,0.388,0.43,0.49,0.222,0.149,0.62    | 0.243548,0.30591  |
| 0.659,0.326,0.389,0.352,0.12,0.58,0.67,0.147,0.196,0.539,0.538,0.193,0.185,0.445,0.711,0.456    | 0.474666,0.436607 |
| 0.199,0.09,0.461,0.122,0.487,0.724,0.645,0.171,0.188,0.51,0.624,0.186,0.186,0.492,0.699,0.453   | 0.396944,0.408186 |
| 0.694,0.537,0.291,0.556,0.839,0.398,0.408,0.334,0.188,0.51,0.624,0.186,0.186,0.492,0.699,0.453  | 0.200596,0.357106 |
| 0.512,0.14,0.383,0.742,0.582,0.67,0.569,0.492,0.457,0.108,0.364,0.773,0.214,0.415,0.504,0.865   | 0.194888,0.352083 |
| 0.879,0.221,0.778,0.629,0.306,0.66,0.406,0.449,0.617,0.656,0.281,0.742,0.191,0.608,0.363,0.577  | 0.367858,0.394988 |
| 0.752,0.415,0.622,0.688,0.117,0.351,0.674,0.898,0.294,0.7,0.482,0.605,0.486,0.784,0.284,0.567   | 0.110822,0.290309 |
| 0.601,0.495,0.278,0.627,0.322,0.725,0.682,0.835,0.639,0.259,0.377,0.499,0.262,0.376,0.226,0.502 | 0.342726,0.274867 |
| 0.514,0.573,0.307,0.674,0.283,0.689,0.84,0.551,0.279,0.237,0.398,0.534,0.301,0.883,0.22,0.134   | 0.14823,0.305368  |
| 0.323,0.598,0.484,0.488,0.343,0.69,0.541,0.369,0.415,0.41,0.578,0.23,0.564,0.595,0.802,0.496    | 0.399517,0.207235 |
| 0.668,0.518,0.243,0.441,0.692,0.671,0.606,0.248,0.386,0.641,0.862,0.324,0.612,0.496,0.752,0.63  | 0.285454,0.25035  |
| 0.659,0.408,0.274,0.675,0.684,0.747,0.803,0.742,0.482,0.201,0.569,0.18,0.485,0.632,0.471,0.504  | 0.285571,0.222829 |
| 0.853,0.259,0.746,0.64,0.268,0.598,0.459,0.538,0.552,0.664,0.321,0.714,0.25,0.643,0.347,0.575   | 0.496266,0.359883 |
| 0.213,0.726,0.6,0.681,0.327,0.594,0.773,0.509,0.333,0.328,0.412,0.441,0.609,0.798,0.527,0.287   | 0.331118,0.221546 |
| 0.701,0.24,0.691,0.757,0.284,0.802,0.372,0.529,0.536,0.382,0.459,0.822,0.263,0.507,0.391,0.605  | 0.40703,0.342964  |
| 0.597,0.225,0.501,0.756,0.367,0.824,0.421,0.523,0.578,0.208,0.468,0.722,0.248,0.365,0.39,0.634  | 0.402982,0.337332 |
| 0.659,0.408,0.506,0.771,0.602,0.772,0.799,0.47,0.519,0.408,0.545,0.499,0.262,0.376,0.226,0.502  | 0.320524,0.196197 |
| 0.592,0.684,0.268,0.607,0.588,0.796,0.641,0.288,0.423,0.535,0.645,0.315,0.554,0.447,0.751,0.493 | 0.405519,0.249578 |
| 0.659,0.316,0.486,0.726,0.509,0.838,0.772,0.676,0.691,0.12,0.419,0.457,0.289,0.368,0.419,0.502  | 0.233144,0.259659 |
| 0.612,0.254,0.512,0.754,0.41,0.814,0.494,0.533,0.573,0.21,0.462,0.67,0.256,0.369,0.364,0.604    | 0.37766,0.293698  |

|                                                                                                 |                   |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.68,0.313,0.532,0.731,0.591,0.778,0.716,0.682,0.598,0.246,0.406,0.484,0.28,0.381,0.255,0.491   | 0.373378,0.199522 |
| 0.824,0.369,0.678,0.787,0.289,0.853,0.351,0.527,0.533,0.362,0.494,0.653,0.26,0.605,0.313,0.534  | 0.395024,0.345916 |
| 0.587,0.628,0.336,0.619,0.471,0.716,0.589,0.226,0.405,0.582,0.599,0.442,0.39,0.454,0.707,0.536  | 0.517127,0.18491  |
| 0.644,0.533,0.3,0.475,0.816,0.405,0.545,0.258,0.34,0.568,0.735,0.248,0.394,0.511,0.672,0.599    | 0.396704,0.274635 |
| 0.68,0.306,0.549,0.733,0.558,0.782,0.681,0.661,0.603,0.238,0.407,0.503,0.28,0.365,0.272,0.51    | 0.382053,0.209466 |
| 0.577,0.681,0.408,0.68,0.753,0.803,0.443,0.258,0.444,0.555,0.612,0.517,0.511,0.427,0.652,0.623  | 0.528176,0.061185 |
| 0.675,0.369,0.678,0.787,0.289,0.853,0.351,0.527,0.533,0.362,0.494,0.653,0.26,0.605,0.313,0.534  | 0.442078,0.308593 |
| 0.608,0.397,0.401,0.744,0.561,0.703,0.595,0.531,0.499,0.113,0.376,0.72,0.227,0.405,0.653,0.603  | 0.404257,0.236895 |
| 0.667,0.314,0.501,0.727,0.552,0.816,0.757,0.679,0.621,0.161,0.413,0.475,0.287,0.405,0.653,0.603 | 0.257668,0.216667 |
| 0.676,0.361,0.551,0.744,0.586,0.775,0.772,0.595,0.575,0.263,0.416,0.501,0.295,0.467,0.262,0.483 | 0.367535,0.213335 |
| 0.665,0.367,0.571,0.775,0.342,0.846,0.437,0.541,0.63,0.343,0.453,0.577,0.251,0.478,0.328,0.525  | 0.380083,0.282418 |
| 0.68,0.306,0.556,0.749,0.585,0.775,0.787,0.572,0.553,0.269,0.425,0.501,0.283,0.432,0.273,0.496  | 0.374493,0.224093 |
| 0.573,0.6,0.386,0.592,0.619,0.679,0.515,0.268,0.42,0.572,0.573,0.456,0.48,0.431,0.707,0.618     | 0.615459,0.065985 |
| 0.68,0.306,0.549,0.745,0.482,0.818,0.695,0.635,0.651,0.231,0.386,0.645,0.248,0.344,0.347,0.542  | 0.326634,0.28494  |
| 0.593,0.671,0.421,0.65,0.748,0.628,0.446,0.265,0.424,0.573,0.653,0.295,0.535,0.415,0.691,0.543  | 0.575922,0.151209 |
| 0.602,0.448,0.419,0.73,0.39,0.78,0.682,0.684,0.616,0.253,0.386,0.631,0.261,0.354,0.294,0.536    | 0.402282,0.261871 |
| 0.567,0.622,0.401,0.641,0.666,0.724,0.529,0.265,0.435,0.515,0.575,0.445,0.487,0.406,0.694,0.621 | 0.604617,0.071133 |
| 0.689,0.324,0.569,0.738,0.45,0.836,0.663,0.618,0.657,0.345,0.448,0.725,0.253,0.462,0.301,0.568  | 0.378248,0.30584  |
| 0.654,0.319,0.543,0.73,0.39,0.78,0.682,0.684,0.616,0.253,0.386,0.631,0.261,0.354,0.294,0.536    | 0.401882,0.275186 |
| 0.574,0.63,0.397,0.603,0.631,0.673,0.508,0.27,0.422,0.574,0.584,0.428,0.487,0.423,0.704,0.607   | 0.629583,0.076021 |
| 0.694,0.316,0.565,0.742,0.5,0.805,0.658,0.599,0.623,0.315,0.409,0.596,0.245,0.377,0.324,0.531   | 0.379657,0.238591 |
| 0.689,0.324,0.569,0.738,0.45,0.836,0.447,0.543,0.626,0.342,0.451,0.6,0.249,0.468,0.326,0.525    | 0.371283,0.262805 |
| 0.555,0.614,0.342,0.617,0.676,0.711,0.523,0.256,0.437,0.58,0.716,0.464,0.486,0.421,0.694,0.617  | 0.531658,0.116303 |
| 0.58,0.646,0.427,0.647,0.689,0.704,0.475,0.259,0.416,0.551,0.579,0.457,0.482,0.435,0.686,0.616  | 0.617853,0.056186 |
| 0.657,0.319,0.541,0.732,0.403,0.788,0.676,0.673,0.616,0.249,0.388,0.631,0.258,0.356,0.303,0.545 | 0.392015,0.277014 |
| 0.687,0.321,0.563,0.735,0.412,0.811,0.665,0.639,0.623,0.225,0.412,0.694,0.249,0.41,0.342,0.571  | 0.344337,0.311001 |
| 0.596,0.637,0.411,0.616,0.692,0.657,0.489,0.254,0.422,0.555,0.585,0.428,0.487,0.426,0.703,0.607 | 0.610888,0.068329 |
| 0.667,0.323,0.53,0.738,0.484,0.8,0.701,0.615,0.614,0.242,0.406,0.563,0.265,0.36,0.332,0.541     | 0.357471,0.246522 |
| 0.676,0.317,0.543,0.741,0.489,0.817,0.686,0.675,0.632,0.261,0.409,0.66,0.251,0.369,0.341,0.588  | 0.341356,0.277699 |
| 0.592,0.627,0.401,0.613,0.673,0.686,0.492,0.259,0.416,0.548,0.585,0.452,0.481,0.423,0.704,0.613 | 0.603573,0.061013 |
| 0.577,0.627,0.41,0.631,0.672,0.689,0.493,0.259,0.42,0.541,0.592,0.454,0.483,0.42,0.699,0.618    | 0.613442,0.062827 |
| 0.578,0.625,0.405,0.624,0.666,0.677,0.499,0.261,0.423,0.55,0.581,0.451,0.477,0.418,0.691,0.616  | 0.62993,0.06081   |
| 0.587,0.639,0.427,0.614,0.689,0.673,0.486,0.251,0.423,0.543,0.582,0.45,0.477,0.44,0.69,0.618    | 0.622075,0.049492 |
| 0.667,0.318,0.521,0.74,0.478,0.816,0.654,0.577,0.629,0.351,0.426,0.644,0.238,0.395,0.343,0.547  | 0.359878,0.256299 |
| 0.577,0.63,0.413,0.634,0.675,0.692,0.489,0.259,0.419,0.543,0.589,0.454,0.483,0.419,0.698,0.618  | 0.612325,0.062181 |
| 0.583,0.641,0.426,0.613,0.687,0.665,0.481,0.254,0.426,0.536,0.589,0.441,0.479,0.425,0.691,0.616 | 0.626427,0.052322 |
| 0.587,0.639,0.427,0.614,0.689,0.673,0.486,0.251,0.42,0.541,0.592,0.454,0.483,0.42,0.699,0.618   | 0.607948,0.055349 |
| 0.589,0.639,0.409,0.61,0.687,0.661,0.488,0.251,0.42,0.547,0.585,0.435,0.479,0.426,0.703,0.61    | 0.605776,0.063852 |
| 0.657,0.321,0.539,0.732,0.481,0.803,0.673,0.651,0.632,0.261,0.409,0.66,0.251,0.369,0.341,0.588  | 0.364452,0.26979  |
| 0.584,0.643,0.425,0.633,0.683,0.691,0.481,0.258,0.425,0.538,0.575,0.454,0.481,0.416,0.699,0.618 | 0.614015,0.053205 |
| 0.585,0.638,0.417,0.613,0.689,0.662,0.488,0.251,0.416,0.546,0.578,0.445,0.479,0.428,0.694,0.615 | 0.616569,0.057533 |
| 0.657,0.321,0.539,0.732,0.481,0.803,0.673,0.651,0.611,0.22,0.399,0.634,0.247,0.366,0.337,0.57   | 0.335558,0.267818 |
| 0.665,0.318,0.531,0.736,0.478,0.812,0.684,0.668,0.609,0.255,0.407,0.637,0.235,0.38,0.344,0.538  | 0.323682,0.261779 |
| 0.585,0.638,0.417,0.613,0.689,0.662,0.488,0.251,0.416,0.546,0.583,0.444,0.474,0.429,0.69,0.614  | 0.620603,0.057638 |
| 0.66,0.313,0.522,0.732,0.466,0.809,0.677,0.632,0.619,0.259,0.405,0.615,0.252,0.371,0.327,0.551  | 0.354766,0.265782 |
| 0.586,0.64,0.413,0.608,0.686,0.659,0.493,0.249,0.418,0.541,0.582,0.444,0.474,0.43,0.692,0.612   | 0.618477,0.058104 |
| 0.586,0.641,0.419,0.609,0.684,0.659,0.501,0.251,0.421,0.541,0.582,0.448,0.475,0.435,0.689,0.615 | 0.633061,0.055606 |
| 0.585,0.64,0.42,0.61,0.683,0.663,0.489,0.249,0.419,0.539,0.583,0.447,0.475,0.434,0.689,0.615    | 0.622385,0.053435 |
| 0.586,0.64,0.413,0.608,0.686,0.659,0.493,0.253,0.418,0.541,0.583,0.446,0.475,0.436,0.691,0.615  | 0.628167,0.055803 |
| 0.665,0.31,0.512,0.728,0.484,0.808,0.679,0.645,0.623,0.265,0.406,0.633,0.251,0.363,0.329,0.553  | 0.34976,0.265558  |
| 0.661,0.314,0.523,0.73,0.457,0.806,0.681,0.639,0.62,0.267,0.403,0.615,0.251,0.37,0.324,0.591    | 0.360729,0.275306 |
| 0.654,0.319,0.541,0.737,0.483,0.809,0.673,0.651,0.611,0.22,0.399,0.634,0.247,0.366,0.337,0.57   | 0.328188,0.269252 |
| 0.624,0.317,0.532,0.733,0.456,0.803,0.683,0.668,0.609,0.255,0.407,0.637,0.234,0.38,0.344,0.538  | 0.343539,0.25434  |
| 0.66,0.323,0.539,0.737,0.47,0.812,0.668,0.659,0.619,0.227,0.408,0.637,0.234,0.38,0.344,0.538    | 0.324424,0.268922 |

|                                                                                                 |                   |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.652,0.319,0.539,0.735,0.48,0.809,0.678,0.664,0.618,0.243,0.403,0.633,0.244,0.369,0.339,0.562  | 0.337222,0.262756 |
| 0.585,0.638,0.417,0.611,0.688,0.66,0.486,0.251,0.418,0.546,0.58,0.443,0.475,0.43,0.696,0.615    | 0.616349,0.056738 |
| 0.586,0.636,0.419,0.607,0.684,0.659,0.496,0.251,0.418,0.541,0.581,0.445,0.475,0.435,0.691,0.615 | 0.627845,0.054671 |
| 0.585,0.634,0.415,0.611,0.684,0.661,0.492,0.25,0.418,0.542,0.581,0.444,0.474,0.43,0.693,0.615   | 0.618826,0.056683 |
| 0.585,0.634,0.415,0.611,0.683,0.661,0.493,0.25,0.419,0.542,0.581,0.44,0.476,0.427,0.691,0.614   | 0.620274,0.057892 |
| 0.588,0.637,0.417,0.608,0.685,0.659,0.487,0.251,0.418,0.543,0.581,0.441,0.475,0.431,0.692,0.612 | 0.621532,0.055434 |
| 0.585,0.636,0.416,0.611,0.684,0.66,0.489,0.25,0.418,0.543,0.58,0.44,0.475,0.427,0.694,0.616     | 0.616401,0.057289 |
| 0.585,0.638,0.414,0.608,0.683,0.66,0.495,0.248,0.417,0.542,0.582,0.441,0.475,0.435,0.692,0.614  | 0.617621,0.058534 |
| 0.585,0.637,0.416,0.611,0.687,0.66,0.487,0.25,0.418,0.545,0.58,0.443,0.474,0.43,0.695,0.615     | 0.615431,0.056785 |
| 0.585,0.636,0.416,0.608,0.685,0.659,0.486,0.252,0.418,0.544,0.58,0.443,0.476,0.429,0.692,0.614  | 0.622317,0.054991 |
| 0.585,0.637,0.416,0.608,0.682,0.659,0.493,0.249,0.417,0.54,0.582,0.441,0.475,0.428,0.691,0.613  | 0.620405,0.057552 |
| 0.586,0.635,0.415,0.608,0.683,0.659,0.491,0.248,0.417,0.54,0.582,0.441,0.475,0.429,0.692,0.612  | 0.616047,0.057463 |
| 0.586,0.637,0.417,0.608,0.686,0.659,0.486,0.251,0.418,0.545,0.58,0.443,0.475,0.43,0.695,0.612   | 0.618405,0.055865 |
| 0.585,0.637,0.417,0.609,0.685,0.659,0.487,0.249,0.418,0.545,0.581,0.44,0.475,0.429,0.692,0.614  | 0.616858,0.057    |
| 0.585,0.637,0.414,0.608,0.682,0.659,0.493,0.247,0.417,0.54,0.581,0.442,0.476,0.428,0.691,0.614  | 0.615347,0.058023 |
| 0.639,0.38,0.514,0.71,0.519,0.776,0.637,0.575,0.576,0.293,0.435,0.592,0.292,0.377,0.406,0.575   | 0.467172,0.171773 |
| 0.585,0.637,0.416,0.608,0.685,0.659,0.486,0.25,0.418,0.545,0.58,0.442,0.475,0.429,0.693,0.614   | 0.617425,0.05605  |
| 0.585,0.634,0.415,0.608,0.683,0.659,0.679,0.658,0.619,0.221,0.406,0.628,0.241,0.372,0.339,0.555 | 0.484342,0.142117 |
| 0.656,0.319,0.536,0.735,0.477,0.807,0.676,0.656,0.618,0.223,0.402,0.628,0.243,0.368,0.336,0.56  | 0.329793,0.269258 |
| 0.655,0.32,0.539,0.737,0.478,0.805,0.675,0.657,0.616,0.235,0.399,0.627,0.249,0.366,0.331,0.566  | 0.3442,0.265665   |
| 0.585,0.636,0.428,0.61,0.683,0.659,0.493,0.249,0.418,0.541,0.581,0.441,0.475,0.427,0.69,0.615   | 0.623604,0.054635 |
| 0.643,0.39,0.486,0.71,0.582,0.755,0.59,0.398,0.568,0.282,0.527,0.592,0.347,0.402,0.53,0.583     | 0.582861,0.14022  |
| 0.599,0.57,0.438,0.633,0.639,0.689,0.53,0.328,0.457,0.485,0.531,0.48,0.292,0.377,0.406,0.575    | 0.625239,0.042005 |
| 0.585,0.637,0.415,0.608,0.685,0.659,0.486,0.25,0.418,0.545,0.58,0.441,0.474,0.428,0.694,0.614   | 0.615868,0.056651 |
| 0.585,0.635,0.415,0.608,0.683,0.659,0.487,0.249,0.417,0.542,0.58,0.44,0.475,0.427,0.692,0.612   | 0.615869,0.056793 |
| 0.585,0.636,0.414,0.608,0.683,0.659,0.488,0.249,0.418,0.544,0.579,0.44,0.474,0.426,0.697,0.614  | 0.610921,0.057862 |
| 0.654,0.317,0.537,0.734,0.476,0.805,0.675,0.628,0.616,0.227,0.4,0.628,0.243,0.367,0.332,0.561   | 0.336139,0.270701 |
| 0.585,0.637,0.417,0.607,0.683,0.659,0.494,0.249,0.421,0.541,0.581,0.44,0.475,0.427,0.691,0.614  | 0.621424,0.056744 |
| 0.584,0.635,0.415,0.607,0.682,0.659,0.491,0.249,0.419,0.542,0.58,0.44,0.474,0.427,0.692,0.613   | 0.618173,0.056903 |
| 0.584,0.635,0.414,0.608,0.682,0.659,0.491,0.251,0.42,0.541,0.58,0.44,0.474,0.427,0.691,0.612    | 0.622882,0.056483 |
| 0.584,0.634,0.414,0.608,0.682,0.659,0.491,0.223,0.417,0.541,0.58,0.441,0.474,0.427,0.691,0.612  | 0.564309,0.063511 |
| 0.588,0.616,0.42,0.613,0.665,0.672,0.495,0.265,0.463,0.508,0.539,0.473,0.369,0.388,0.573,0.582  | 0.626854,0.016051 |
| 0.584,0.635,0.415,0.608,0.682,0.659,0.489,0.249,0.418,0.543,0.58,0.44,0.474,0.426,0.695,0.614   | 0.613864,0.057492 |
| 0.64,0.38,0.513,0.708,0.517,0.775,0.638,0.558,0.581,0.285,0.428,0.595,0.28,0.378,0.394,0.571    | 0.447057,0.184176 |
| 0.584,0.634,0.414,0.609,0.682,0.659,0.491,0.251,0.42,0.541,0.58,0.44,0.474,0.427,0.691,0.614    | 0.62248,0.056279  |
| 0.585,0.628,0.417,0.613,0.681,0.659,0.494,0.25,0.42,0.535,0.58,0.443,0.473,0.426,0.686,0.613    | 0.628282,0.05421  |
| 0.584,0.634,0.414,0.608,0.682,0.659,0.49,0.249,0.417,0.541,0.58,0.44,0.474,0.427,0.69,0.613     | 0.618619,0.05689  |
| 0.584,0.635,0.415,0.608,0.683,0.659,0.489,0.249,0.417,0.541,0.58,0.44,0.471,0.426,0.691,0.614   | 0.61767,0.05638   |
| 0.599,0.571,0.442,0.632,0.638,0.688,0.529,0.317,0.458,0.477,0.544,0.477,0.427,0.446,0.331,0.566 | 0.687334,0.052362 |
| 0.611,0.41,0.486,0.682,0.597,0.777,0.645,0.452,0.523,0.398,0.47,0.625,0.244,0.366,0.328,0.562   | 0.426546,0.150904 |
| 0.587,0.606,0.42,0.621,0.673,0.674,0.516,0.28,0.431,0.525,0.571,0.454,0.436,0.496,0.635,0.609   | 0.716938,0.039299 |
| 0.585,0.618,0.417,0.613,0.68,0.659,0.494,0.25,0.418,0.532,0.576,0.449,0.471,0.426,0.68,0.612    | 0.632928,0.050016 |
| 0.584,0.635,0.414,0.608,0.682,0.659,0.491,0.251,0.426,0.541,0.58,0.44,0.474,0.427,0.691,0.614   | 0.623418,0.054979 |
| 0.631,0.478,0.47,0.676,0.607,0.735,0.63,0.412,0.561,0.444,0.489,0.522,0.314,0.46,0.44,0.586     | 0.671413,0.099761 |
| 0.584,0.634,0.414,0.608,0.682,0.659,0.487,0.249,0.417,0.541,0.579,0.433,0.474,0.426,0.691,0.612 | 0.615257,0.057875 |
| 0.586,0.616,0.42,0.614,0.673,0.662,0.499,0.259,0.422,0.527,0.573,0.449,0.468,0.428,0.651,0.608  | 0.670516,0.041995 |
| 0.584,0.634,0.414,0.608,0.683,0.659,0.489,0.249,0.417,0.543,0.58,0.44,0.474,0.426,0.693,0.612   | 0.615218,0.057867 |
| 0.584,0.634,0.414,0.608,0.682,0.659,0.488,0.249,0.417,0.541,0.58,0.44,0.474,0.426,0.691,0.604   | 0.617677,0.058163 |
| 0.584,0.633,0.415,0.608,0.682,0.659,0.487,0.249,0.417,0.542,0.579,0.44,0.474,0.426,0.692,0.612  | 0.615769,0.056409 |
| 0.584,0.634,0.414,0.608,0.683,0.659,0.489,0.248,0.417,0.541,0.579,0.44,0.474,0.426,0.69,0.612   | 0.615887,0.057008 |
| 0.583,0.633,0.416,0.608,0.681,0.659,0.492,0.25,0.42,0.526,0.577,0.448,0.469,0.408,0.65,0.609    | 0.63612,0.047835  |
| 0.588,0.614,0.423,0.614,0.676,0.663,0.513,0.26,0.437,0.51,0.546,0.454,0.443,0.403,0.553,0.593   | 0.676416,0.03236  |
| 0.584,0.634,0.412,0.608,0.682,0.659,0.487,0.248,0.417,0.541,0.579,0.44,0.474,0.426,0.69,0.612   | 0.614278,0.056931 |
| 0.584,0.629,0.415,0.61,0.68,0.661,0.494,0.25,0.421,0.533,0.575,0.444,0.467,0.438,0.678,0.612    | 0.6337,0.04952    |

|                                                                                                 |                   |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.584,0.634,0.414,0.608,0.683,0.659,0.488,0.249,0.417,0.541,0.579,0.434,0.474,0.426,0.691,0.612 | 0.615638,0.05796  |
| 0.584,0.633,0.414,0.608,0.682,0.659,0.487,0.249,0.417,0.541,0.579,0.435,0.474,0.426,0.691,0.612 | 0.615467,0.057334 |
| 0.584,0.634,0.415,0.608,0.681,0.665,0.497,0.255,0.422,0.536,0.576,0.442,0.467,0.438,0.676,0.611 | 0.642832,0.049868 |
| 0.588,0.614,0.423,0.614,0.676,0.663,0.513,0.26,0.437,0.51,0.48,0.518,0.367,0.445,0.612,0.596    | 0.628226,0.028162 |
| 0.583,0.633,0.416,0.608,0.681,0.659,0.491,0.25,0.42,0.524,0.577,0.448,0.469,0.411,0.647,0.605   | 0.636434,0.046923 |
| 0.584,0.635,0.415,0.608,0.683,0.659,0.492,0.305,0.42,0.534,0.575,0.443,0.468,0.435,0.68,0.612   | 0.733728,0.037806 |
| 0.583,0.633,0.414,0.608,0.68,0.659,0.488,0.248,0.417,0.541,0.579,0.439,0.474,0.426,0.69,0.612   | 0.615775,0.056809 |
| 0.584,0.634,0.409,0.608,0.682,0.659,0.488,0.248,0.417,0.54,0.579,0.44,0.473,0.425,0.69,0.613    | 0.612999,0.057504 |
| 0.584,0.628,0.417,0.609,0.678,0.659,0.492,0.253,0.422,0.517,0.569,0.459,0.416,0.448,0.612,0.588 | 0.651363,0.036013 |
| 0.583,0.633,0.414,0.608,0.681,0.659,0.487,0.248,0.417,0.54,0.579,0.44,0.473,0.425,0.69,0.608    | 0.615263,0.056859 |
| 0.583,0.634,0.414,0.608,0.682,0.662,0.487,0.248,0.417,0.541,0.579,0.44,0.473,0.426,0.65,0.609   | 0.627387,0.051461 |
| 0.583,0.631,0.413,0.607,0.679,0.659,0.492,0.252,0.419,0.53,0.573,0.445,0.456,0.435,0.661,0.605  | 0.642779,0.046052 |
| 0.584,0.626,0.417,0.609,0.677,0.661,0.494,0.252,0.427,0.52,0.562,0.45,0.458,0.434,0.64,0.607    | 0.652057,0.036503 |
| 0.584,0.624,0.416,0.613,0.675,0.675,0.5,0.26,0.423,0.532,0.573,0.447,0.458,0.466,0.636,0.608    | 0.66359,0.041341  |
| 0.584,0.618,0.417,0.607,0.673,0.665,0.502,0.263,0.429,0.52,0.564,0.453,0.448,0.458,0.633,0.607  | 0.682467,0.032684 |
| 0.583,0.633,0.414,0.608,0.682,0.659,0.487,0.255,0.417,0.54,0.579,0.44,0.477,0.425,0.689,0.611   | 0.629808,0.055061 |
| 0.584,0.629,0.415,0.608,0.677,0.662,0.496,0.252,0.426,0.526,0.562,0.449,0.464,0.434,0.613,0.608 | 0.651288,0.038    |
| 0.584,0.633,0.414,0.608,0.681,0.659,0.489,0.248,0.418,0.536,0.577,0.441,0.47,0.426,0.685,0.61   | 0.621308,0.054046 |
| 0.584,0.622,0.415,0.608,0.678,0.663,0.493,0.257,0.423,0.525,0.569,0.448,0.455,0.446,0.656,0.609 | 0.656474,0.038983 |
| 0.583,0.633,0.414,0.611,0.681,0.657,0.483,0.248,0.417,0.535,0.578,0.441,0.472,0.423,0.653,0.607 | 0.628324,0.050426 |
| 0.584,0.633,0.414,0.608,0.681,0.659,0.487,0.245,0.419,0.538,0.578,0.438,0.471,0.425,0.687,0.611 | 0.612087,0.055691 |
| 0.582,0.632,0.413,0.608,0.68,0.659,0.487,0.248,0.417,0.538,0.577,0.439,0.471,0.425,0.688,0.611  | 0.616824,0.055399 |
| 0.583,0.632,0.414,0.608,0.681,0.659,0.487,0.248,0.417,0.538,0.576,0.439,0.468,0.429,0.683,0.611 | 0.621964,0.053417 |
| 0.583,0.632,0.414,0.609,0.68,0.659,0.486,0.248,0.417,0.538,0.578,0.44,0.472,0.425,0.682,0.612   | 0.621875,0.054081 |
| 0.583,0.633,0.414,0.608,0.682,0.659,0.487,0.248,0.417,0.537,0.577,0.44,0.472,0.425,0.687,0.611  | 0.617834,0.054822 |
| 0.583,0.632,0.414,0.607,0.677,0.661,0.486,0.254,0.427,0.526,0.567,0.449,0.456,0.449,0.648,0.607 | 0.648824,0.03739  |
| 0.582,0.633,0.413,0.608,0.682,0.659,0.487,0.248,0.417,0.539,0.578,0.438,0.472,0.425,0.684,0.611 | 0.619559,0.055735 |
| 0.582,0.633,0.41,0.608,0.682,0.659,0.484,0.248,0.417,0.539,0.578,0.438,0.472,0.425,0.689,0.611  | 0.612647,0.056491 |
| 0.584,0.634,0.414,0.608,0.681,0.66,0.49,0.25,0.42,0.538,0.578,0.442,0.469,0.459,0.644,0.607     | 0.637481,0.046847 |
| 0.583,0.629,0.414,0.607,0.677,0.661,0.492,0.255,0.421,0.525,0.572,0.446,0.455,0.44,0.653,0.607  | 0.653275,0.041517 |
| 0.584,0.634,0.414,0.608,0.681,0.66,0.49,0.25,0.42,0.538,0.578,0.442,0.469,0.459,0.631,0.597     | 0.637756,0.046587 |
| 0.581,0.632,0.414,0.608,0.681,0.659,0.487,0.248,0.417,0.537,0.577,0.438,0.472,0.425,0.688,0.61  | 0.616974,0.055617 |
| 0.583,0.631,0.414,0.607,0.677,0.661,0.49,0.254,0.424,0.525,0.569,0.446,0.455,0.442,0.65,0.607   | 0.651071,0.039949 |
| 0.589,0.633,0.414,0.608,0.681,0.659,0.489,0.248,0.419,0.536,0.577,0.441,0.469,0.43,0.683,0.611  | 0.623407,0.052348 |
| 0.583,0.632,0.414,0.608,0.68,0.659,0.489,0.249,0.419,0.535,0.573,0.407,0.461,0.453,0.668,0.608  | 0.625293,0.055353 |
| 0.582,0.633,0.414,0.608,0.647,0.659,0.487,0.246,0.417,0.537,0.577,0.439,0.472,0.425,0.686,0.608 | 0.617762,0.055453 |
| 0.582,0.63,0.414,0.607,0.678,0.659,0.487,0.25,0.417,0.532,0.573,0.44,0.465,0.428,0.665,0.608    | 0.635384,0.048136 |
| 0.583,0.629,0.413,0.607,0.677,0.659,0.489,0.252,0.422,0.53,0.569,0.433,0.458,0.454,0.652,0.608  | 0.642416,0.044165 |
| 0.577,0.632,0.414,0.608,0.681,0.659,0.487,0.247,0.417,0.536,0.576,0.439,0.471,0.425,0.687,0.61  | 0.616168,0.055455 |
| 0.583,0.624,0.414,0.607,0.677,0.66,0.491,0.254,0.422,0.525,0.57,0.446,0.455,0.443,0.651,0.607   | 0.652448,0.040136 |
| 0.582,0.632,0.414,0.608,0.681,0.659,0.487,0.247,0.415,0.538,0.577,0.438,0.472,0.425,0.687,0.608 | 0.61641,0.0565    |
| 0.583,0.63,0.414,0.607,0.676,0.659,0.49,0.255,0.421,0.526,0.568,0.445,0.456,0.442,0.654,0.607   | 0.65282,0.040844  |
| 0.582,0.633,0.413,0.608,0.682,0.659,0.484,0.246,0.417,0.538,0.577,0.438,0.472,0.425,0.687,0.606 | 0.612101,0.056348 |
| 0.581,0.633,0.414,0.608,0.681,0.661,0.487,0.247,0.417,0.537,0.577,0.438,0.472,0.429,0.683,0.611 | 0.618683,0.054512 |
| 0.581,0.633,0.414,0.608,0.653,0.659,0.487,0.246,0.417,0.537,0.577,0.439,0.471,0.424,0.685,0.608 | 0.618418,0.055274 |
| 0.581,0.633,0.414,0.608,0.681,0.659,0.487,0.244,0.417,0.536,0.577,0.439,0.471,0.425,0.684,0.609 | 0.612497,0.055704 |
| 0.582,0.64,0.413,0.607,0.68,0.659,0.487,0.246,0.417,0.536,0.576,0.432,0.469,0.435,0.68,0.61     | 0.620276,0.055275 |
| 0.581,0.632,0.414,0.608,0.659,0.659,0.487,0.246,0.416,0.537,0.577,0.438,0.471,0.424,0.686,0.608 | 0.617507,0.055677 |
| 0.582,0.632,0.414,0.608,0.681,0.659,0.487,0.278,0.417,0.536,0.576,0.439,0.471,0.425,0.684,0.61  | 0.679222,0.047048 |
| 0.582,0.627,0.414,0.607,0.677,0.659,0.49,0.251,0.419,0.526,0.571,0.437,0.457,0.442,0.658,0.607  | 0.641988,0.044745 |
| 0.583,0.627,0.414,0.607,0.68,0.659,0.487,0.247,0.416,0.536,0.575,0.439,0.47,0.425,0.686,0.609   | 0.616556,0.053792 |
| 0.581,0.632,0.414,0.608,0.681,0.659,0.487,0.244,0.417,0.536,0.576,0.438,0.471,0.425,0.685,0.609 | 0.611432,0.055742 |
| 0.581,0.628,0.414,0.607,0.676,0.659,0.49,0.252,0.42,0.526,0.57,0.437,0.456,0.443,0.655,0.607    | 0.645592,0.043866 |
| 0.581,0.633,0.414,0.608,0.682,0.658,0.487,0.246,0.417,0.536,0.576,0.439,0.471,0.415,0.686,0.61  | 0.613334,0.056462 |

|                                                                                                 |                   |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.581,0.632,0.414,0.608,0.681,0.658,0.487,0.246,0.417,0.503,0.576,0.438,0.471,0.424,0.685,0.608 | 0.616484,0.049487 |
| 0.583,0.632,0.414,0.608,0.681,0.658,0.487,0.246,0.417,0.537,0.576,0.438,0.471,0.424,0.685,0.609 | 0.615785,0.055395 |
| 0.581,0.632,0.414,0.608,0.681,0.658,0.487,0.246,0.417,0.536,0.576,0.438,0.467,0.424,0.686,0.608 | 0.615291,0.055319 |
| 0.581,0.633,0.413,0.608,0.681,0.659,0.486,0.246,0.417,0.535,0.577,0.436,0.471,0.424,0.685,0.608 | 0.614701,0.056072 |
| 0.583,0.504,0.413,0.607,0.678,0.676,0.488,0.248,0.417,0.534,0.572,0.438,0.463,0.43,0.666,0.608  | 0.625471,0.053288 |
| 0.582,0.631,0.413,0.607,0.682,0.659,0.487,0.247,0.417,0.533,0.574,0.436,0.47,0.433,0.672,0.607  | 0.624258,0.051725 |
| 0.581,0.628,0.413,0.607,0.678,0.669,0.488,0.249,0.418,0.529,0.571,0.431,0.457,0.442,0.652,0.605 | 0.629678,0.046444 |
| 0.581,0.632,0.414,0.608,0.679,0.657,0.487,0.246,0.416,0.536,0.575,0.434,0.47,0.426,0.682,0.608  | 0.619078,0.05557  |
| 0.581,0.633,0.413,0.607,0.679,0.658,0.487,0.246,0.417,0.536,0.576,0.435,0.469,0.426,0.684,0.608 | 0.61694,0.055643  |
| 0.582,0.632,0.413,0.607,0.68,0.658,0.487,0.246,0.417,0.536,0.576,0.434,0.47,0.427,0.682,0.609   | 0.617893,0.055209 |
| 0.581,0.629,0.413,0.607,0.679,0.657,0.486,0.247,0.417,0.531,0.571,0.435,0.465,0.439,0.672,0.601 | 0.626629,0.050526 |
| 0.581,0.624,0.413,0.607,0.676,0.671,0.485,0.249,0.419,0.528,0.569,0.44,0.452,0.44,0.651,0.604   | 0.632117,0.042457 |
| 0.581,0.629,0.413,0.611,0.679,0.658,0.486,0.247,0.417,0.531,0.572,0.435,0.461,0.435,0.656,0.604 | 0.62968,0.049034  |
| 0.581,0.632,0.413,0.607,0.68,0.658,0.486,0.246,0.417,0.516,0.576,0.437,0.47,0.424,0.685,0.608   | 0.615106,0.051577 |
| 0.59,0.614,0.413,0.607,0.679,0.661,0.484,0.246,0.416,0.531,0.572,0.432,0.46,0.443,0.659,0.631   | 0.624393,0.043673 |
| 0.581,0.628,0.413,0.606,0.678,0.666,0.487,0.248,0.417,0.529,0.57,0.43,0.457,0.442,0.652,0.604   | 0.628233,0.046599 |
| 0.581,0.632,0.412,0.609,0.68,0.658,0.486,0.248,0.416,0.536,0.575,0.437,0.47,0.424,0.684,0.608   | 0.619417,0.055436 |
| 0.581,0.605,0.413,0.607,0.677,0.658,0.487,0.248,0.417,0.53,0.569,0.434,0.456,0.441,0.658,0.601  | 0.63411,0.045354  |
| 0.581,0.631,0.411,0.606,0.678,0.658,0.487,0.245,0.416,0.535,0.576,0.437,0.47,0.422,0.685,0.608  | 0.612708,0.056097 |
| 0.58,0.628,0.413,0.607,0.677,0.666,0.487,0.248,0.417,0.531,0.57,0.429,0.458,0.445,0.655,0.604   | 0.628431,0.047782 |
| 0.58,0.629,0.413,0.607,0.677,0.665,0.487,0.246,0.417,0.53,0.572,0.437,0.456,0.43,0.665,0.607    | 0.625553,0.048459 |
| 0.581,0.605,0.424,0.607,0.678,0.696,0.484,0.246,0.418,0.531,0.571,0.429,0.459,0.444,0.655,0.603 | 0.607266,0.0434   |
| 0.58,0.628,0.413,0.606,0.677,0.66,0.485,0.246,0.417,0.53,0.57,0.464,0.458,0.434,0.662,0.605     | 0.632589,0.041797 |
| 0.58,0.616,0.42,0.607,0.677,0.66,0.485,0.246,0.417,0.529,0.569,0.433,0.457,0.438,0.66,0.603     | 0.629925,0.044735 |
| 0.586,0.62,0.413,0.607,0.678,0.659,0.484,0.245,0.416,0.53,0.569,0.434,0.456,0.44,0.658,0.601    | 0.624957,0.046308 |
| 0.581,0.623,0.413,0.607,0.677,0.662,0.487,0.248,0.417,0.53,0.569,0.45,0.454,0.446,0.655,0.603   | 0.636607,0.042554 |
| 0.58,0.632,0.413,0.607,0.68,0.658,0.487,0.246,0.416,0.515,0.572,0.437,0.466,0.428,0.68,0.608    | 0.620645,0.049523 |
| 0.582,0.631,0.413,0.607,0.68,0.658,0.486,0.245,0.416,0.531,0.576,0.437,0.468,0.423,0.685,0.608  | 0.612936,0.054503 |
| 0.582,0.631,0.412,0.61,0.679,0.658,0.486,0.245,0.416,0.531,0.575,0.436,0.469,0.423,0.684,0.607  | 0.613253,0.05548  |
| 0.58,0.628,0.413,0.607,0.677,0.659,0.485,0.245,0.416,0.53,0.57,0.43,0.458,0.443,0.656,0.605     | 0.624917,0.048142 |
| 0.581,0.631,0.456,0.607,0.679,0.659,0.487,0.245,0.416,0.532,0.572,0.434,0.468,0.427,0.676,0.607 | 0.623027,0.04348  |
| 0.58,0.628,0.413,0.607,0.677,0.659,0.487,0.245,0.457,0.531,0.571,0.434,0.461,0.438,0.663,0.605  | 0.628269,0.042188 |
| 0.579,0.631,0.411,0.606,0.678,0.657,0.486,0.242,0.416,0.53,0.574,0.436,0.468,0.422,0.682,0.607  | 0.609127,0.055137 |
| 0.58,0.631,0.411,0.607,0.678,0.657,0.486,0.245,0.414,0.532,0.573,0.435,0.468,0.421,0.682,0.572  | 0.616787,0.061552 |
| 0.58,0.614,0.413,0.606,0.677,0.662,0.487,0.246,0.417,0.529,0.57,0.437,0.455,0.441,0.657,0.601   | 0.628928,0.045375 |
| 0.58,0.627,0.413,0.606,0.676,0.659,0.485,0.245,0.448,0.53,0.57,0.435,0.46,0.437,0.662,0.604     | 0.628832,0.04218  |
| 0.58,0.626,0.413,0.607,0.677,0.658,0.484,0.245,0.416,0.53,0.569,0.434,0.456,0.44,0.658,0.601    | 0.626092,0.047459 |
| 0.579,0.631,0.412,0.607,0.68,0.657,0.486,0.245,0.416,0.531,0.562,0.436,0.47,0.42,0.684,0.607    | 0.614051,0.053008 |
| 0.58,0.63,0.412,0.607,0.679,0.657,0.485,0.245,0.415,0.531,0.574,0.435,0.469,0.422,0.682,0.607   | 0.614496,0.054708 |
| 0.58,0.629,0.412,0.606,0.678,0.655,0.486,0.242,0.416,0.531,0.575,0.437,0.471,0.42,0.685,0.608   | 0.607238,0.055741 |
| 0.582,0.631,0.408,0.607,0.679,0.682,0.486,0.349,0.416,0.539,0.57,0.432,0.458,0.439,0.66,0.603   | 0.793932,0.027851 |
| 0.58,0.631,0.412,0.608,0.679,0.658,0.487,0.245,0.415,0.532,0.572,0.431,0.468,0.426,0.757,0.604  | 0.537492,0.07054  |
| 0.579,0.63,0.412,0.607,0.68,0.657,0.485,0.245,0.416,0.533,0.574,0.435,0.468,0.421,0.683,0.607   | 0.613667,0.055234 |
| 0.58,0.623,0.414,0.606,0.676,0.66,0.484,0.244,0.416,0.529,0.569,0.433,0.455,0.435,0.657,0.604   | 0.624437,0.046487 |
| 0.58,0.631,0.411,0.606,0.678,0.656,0.486,0.244,0.416,0.531,0.575,0.437,0.468,0.423,0.684,0.604  | 0.612633,0.055474 |
| 0.58,0.629,0.412,0.607,0.68,0.656,0.486,0.243,0.416,0.531,0.575,0.436,0.468,0.421,0.683,0.607   | 0.610717,0.055388 |
| 0.58,0.629,0.412,0.608,0.678,0.655,0.486,0.244,0.416,0.531,0.574,0.436,0.467,0.423,0.688,0.605  | 0.609455,0.055916 |
| 0.58,0.622,0.413,0.624,0.676,0.66,0.484,0.241,0.416,0.529,0.569,0.433,0.455,0.432,0.659,0.602   | 0.618,0.052177    |
| 0.58,0.622,0.413,0.607,0.676,0.659,0.484,0.242,0.416,0.529,0.569,0.431,0.456,0.43,0.662,0.602   | 0.618532,0.049025 |
| 0.58,0.626,0.413,0.607,0.677,0.66,0.484,0.244,0.416,0.529,0.569,0.433,0.455,0.435,0.657,0.604   | 0.622915,0.047305 |
| 0.579,0.63,0.412,0.607,0.679,0.657,0.485,0.244,0.415,0.532,0.574,0.435,0.468,0.42,0.683,0.607   | 0.611484,0.055528 |
| 0.58,0.622,0.413,0.606,0.676,0.659,0.484,0.237,0.416,0.532,0.569,0.433,0.456,0.438,0.658,0.601  | 0.609884,0.049324 |
| 0.579,0.629,0.412,0.606,0.678,0.656,0.485,0.242,0.415,0.531,0.574,0.435,0.468,0.42,0.682,0.607  | 0.60884,0.055307  |
| 0.579,0.629,0.412,0.606,0.676,0.656,0.485,0.243,0.415,0.531,0.574,0.434,0.468,0.42,0.682,0.607  | 0.611266,0.055172 |

|                                                                                                 |                   |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.58,0.629,0.412,0.606,0.678,0.655,0.485,0.243,0.416,0.531,0.52,0.435,0.457,0.436,0.658,0.602   | 0.623371,0.039504 |
| 0.58,0.612,0.413,0.605,0.676,0.662,0.484,0.244,0.416,0.53,0.569,0.431,0.455,0.44,0.655,0.602    | 0.623163,0.045802 |
| 0.579,0.629,0.412,0.606,0.678,0.656,0.485,0.244,0.415,0.531,0.574,0.435,0.467,0.422,0.686,0.605 | 0.610108,0.055456 |
| 0.579,0.629,0.412,0.606,0.677,0.656,0.485,0.243,0.417,0.531,0.574,0.436,0.468,0.42,0.667,0.607  | 0.619317,0.052243 |
| 0.58,0.629,0.411,0.606,0.678,0.655,0.486,0.243,0.416,0.531,0.574,0.331,0.467,0.419,0.684,0.605  | 0.585499,0.079642 |
| 0.58,0.656,0.412,0.649,0.679,0.656,0.485,0.243,0.416,0.531,0.574,0.436,0.469,0.42,0.683,0.607   | 0.609303,0.068663 |
| 0.579,0.629,0.412,0.606,0.677,0.655,0.485,0.243,0.415,0.525,0.574,0.436,0.467,0.42,0.683,0.606  | 0.611774,0.053896 |
| 0.579,0.629,0.413,0.605,0.676,0.662,0.486,0.244,0.418,0.53,0.569,0.431,0.42,0.468,0.656,0.602   | 0.622255,0.050314 |
| 0.58,0.616,0.413,0.605,0.653,0.659,0.484,0.243,0.415,0.529,0.569,0.431,0.456,0.436,0.657,0.601  | 0.636756,0.046764 |
| 0.579,0.629,0.412,0.605,0.676,0.654,0.485,0.243,0.415,0.531,0.574,0.434,0.468,0.415,0.682,0.606 | 0.611477,0.055787 |
| 0.579,0.629,0.411,0.606,0.678,0.655,0.485,0.243,0.415,0.531,0.573,0.439,0.468,0.42,0.682,0.606  | 0.612179,0.054466 |
| 0.579,0.627,0.411,0.605,0.677,0.655,0.481,0.242,0.415,0.531,0.572,0.434,0.465,0.423,0.677,0.605 | 0.611679,0.052815 |
| 0.579,0.629,0.411,0.606,0.677,0.655,0.484,0.243,0.415,0.531,0.573,0.435,0.468,0.42,0.682,0.606  | 0.610534,0.055004 |
| 0.579,0.629,0.411,0.605,0.677,0.656,0.485,0.243,0.415,0.531,0.598,0.435,0.467,0.417,0.682,0.605 | 0.61321,0.060736  |
| 0.579,0.629,0.412,0.605,0.676,0.655,0.484,0.123,0.415,0.531,0.574,0.435,0.467,0.42,0.684,0.606  | 0.45335,0.084377  |
| 0.579,0.629,0.411,0.605,0.677,0.656,0.485,0.243,0.415,0.531,0.574,0.434,0.467,0.42,0.682,0.601  | 0.611241,0.056068 |
| 0.582,0.629,0.411,0.606,0.677,0.655,0.485,0.243,0.415,0.526,0.574,0.436,0.467,0.419,0.683,0.557 | 0.613324,0.063065 |
| 0.579,0.629,0.412,0.606,0.676,0.655,0.484,0.243,0.415,0.531,0.574,0.438,0.467,0.42,0.683,0.605  | 0.611284,0.054506 |
| 0.579,0.629,0.411,0.606,0.676,0.654,0.484,0.243,0.415,0.54,0.573,0.434,0.467,0.419,0.681,0.606  | 0.612016,0.056852 |
| 0.579,0.623,0.411,0.606,0.675,0.656,0.484,0.242,0.415,0.53,0.57,0.432,0.462,0.426,0.666,0.603   | 0.61827,0.051113  |
| 0.579,0.624,0.411,0.605,0.677,0.657,0.484,0.24,0.415,0.531,0.574,0.439,0.466,0.421,0.663,0.606  | 0.613454,0.05116  |
| 0.579,0.629,0.411,0.606,0.676,0.654,0.483,0.242,0.415,0.531,0.573,0.434,0.468,0.419,0.682,0.606 | 0.60834,0.055327  |
| 0.579,0.627,0.411,0.605,0.676,0.654,0.481,0.242,0.415,0.53,0.569,0.447,0.456,0.47,0.658,0.601   | 0.625246,0.045296 |
| 0.579,0.622,0.411,0.605,0.676,0.655,0.484,0.243,0.415,0.53,0.573,0.434,0.467,0.419,0.681,0.601  | 0.61161,0.054607  |
| 0.579,0.627,0.411,0.605,0.676,0.654,0.482,0.242,0.416,0.529,0.572,0.434,0.465,0.421,0.679,0.606 | 0.611322,0.052825 |
| 0.579,0.629,0.411,0.602,0.676,0.654,0.484,0.242,0.415,0.529,0.574,0.434,0.467,0.42,0.682,0.605  | 0.609677,0.054399 |
| 0.579,0.629,0.411,0.605,0.676,0.655,0.484,0.243,0.415,0.531,0.574,0.434,0.467,0.42,0.683,0.605  | 0.610659,0.055345 |
| 0.579,0.629,0.411,0.606,0.676,0.654,0.484,0.268,0.415,0.53,0.572,0.434,0.467,0.42,0.681,0.606   | 0.662802,0.048606 |
| 0.579,0.629,0.411,0.606,0.676,0.654,0.484,0.243,0.415,0.531,0.574,0.435,0.467,0.418,0.683,0.6   | 0.61102,0.056455  |
| 0.579,0.623,0.411,0.605,0.676,0.654,0.484,0.243,0.415,0.53,0.573,0.434,0.467,0.419,0.679,0.625  | 0.611337,0.051034 |
| 0.579,0.629,0.411,0.606,0.677,0.655,0.485,0.245,0.415,0.535,0.574,0.435,0.467,0.418,0.682,0.606 | 0.615763,0.055879 |
| 0.579,0.629,0.411,0.605,0.674,0.655,0.483,0.242,0.415,0.531,0.574,0.434,0.467,0.418,0.682,0.605 | 0.609836,0.055309 |
| 0.579,0.628,0.411,0.605,0.678,0.654,0.482,0.242,0.415,0.529,0.573,0.434,0.465,0.42,0.679,0.605  | 0.611037,0.053717 |
| 0.579,0.628,0.411,0.605,0.657,0.653,0.483,0.226,0.415,0.53,0.573,0.434,0.466,0.418,0.682,0.605  | 0.576612,0.058082 |
| 0.579,0.628,0.411,0.605,0.676,0.653,0.484,0.243,0.415,0.53,0.573,0.434,0.467,0.419,0.679,0.604  | 0.614351,0.054574 |
| 0.579,0.628,0.411,0.605,0.676,0.654,0.479,0.242,0.415,0.528,0.573,0.434,0.465,0.421,0.679,0.605 | 0.610417,0.052584 |
| 0.579,0.628,0.411,0.554,0.676,0.654,0.483,0.242,0.415,0.529,0.573,0.434,0.467,0.419,0.681,0.605 | 0.610598,0.043137 |
| 0.579,0.628,0.411,0.613,0.676,0.655,0.483,0.242,0.415,0.529,0.573,0.434,0.466,0.419,0.679,0.605 | 0.610765,0.055853 |
| 0.579,0.628,0.411,0.605,0.675,0.654,0.482,0.242,0.415,0.531,0.573,0.432,0.467,0.418,0.681,0.604 | 0.609091,0.055267 |
| 0.579,0.628,0.411,0.605,0.676,0.654,0.48,0.242,0.415,0.529,0.573,0.434,0.484,0.419,0.679,0.605  | 0.608134,0.05561  |
| 0.579,0.629,0.411,0.605,0.692,0.654,0.483,0.242,0.415,0.529,0.574,0.432,0.467,0.418,0.682,0.604 | 0.598868,0.056841 |
| 0.579,0.628,0.411,0.605,0.676,0.654,0.484,0.243,0.415,0.53,0.573,0.434,0.467,0.415,0.675,0.481  | 0.619602,0.079142 |
| 0.579,0.628,0.411,0.605,0.676,0.654,0.484,0.243,0.415,0.53,0.573,0.434,0.467,0.418,0.681,0.606  | 0.611928,0.05463  |
| 0.579,0.628,0.411,0.605,0.675,0.654,0.483,0.242,0.415,0.541,0.572,0.434,0.466,0.418,0.681,0.605 | 0.609687,0.056601 |
| 0.579,0.628,0.411,0.605,0.675,0.654,0.483,0.242,0.415,0.53,0.573,0.433,0.466,0.418,0.681,0.604  | 0.609776,0.054987 |
| 0.579,0.629,0.411,0.605,0.675,0.654,0.482,0.242,0.415,0.532,0.573,0.434,0.466,0.418,0.682,0.605 | 0.609174,0.055044 |
| 0.579,0.628,0.384,0.605,0.675,0.655,0.483,0.242,0.415,0.571,0.573,0.434,0.466,0.418,0.679,0.605 | 0.592492,0.069084 |
| 0.579,0.628,0.411,0.605,0.675,0.654,0.482,0.242,0.415,0.531,0.573,0.432,0.467,0.419,0.676,0.603 | 0.613324,0.054533 |
| 0.579,0.627,0.411,0.605,0.675,0.652,0.482,0.242,0.414,0.536,0.572,0.432,0.466,0.418,0.679,0.603 | 0.611646,0.055974 |
| 0.579,0.628,0.41,0.605,0.675,0.654,0.481,0.242,0.415,0.533,0.573,0.432,0.466,0.418,0.681,0.604  | 0.608296,0.05554  |
| 0.579,0.628,0.41,0.605,0.674,0.654,0.482,0.242,0.415,0.53,0.572,0.432,0.467,0.418,0.681,0.635   | 0.604514,0.050862 |
| 0.579,0.628,0.411,0.626,0.675,0.654,0.487,0.242,0.415,0.529,0.573,0.432,0.466,0.417,0.681,0.604 | 0.611209,0.060964 |
| 0.579,0.628,0.41,0.605,0.674,0.654,0.482,0.242,0.415,0.53,0.573,0.432,0.466,0.418,0.681,0.604   | 0.608949,0.055137 |
| 0.579,0.628,0.41,0.605,0.674,0.654,0.481,0.242,0.415,0.53,0.572,0.436,0.466,0.418,0.681,0.604   | 0.609015,0.053848 |

|                                                                                                 |                   |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.579,0.627,0.411,0.605,0.675,0.654,0.483,0.242,0.415,0.531,0.573,0.432,0.467,0.416,0.675,0.534 | 0.616524,0.067709 |
| 0.579,0.628,0.411,0.605,0.675,0.654,0.482,0.242,0.415,0.53,0.572,0.433,0.465,0.418,0.678,0.604  | 0.611861,0.053958 |
| 0.579,0.628,0.41,0.604,0.675,0.657,0.482,0.242,0.415,0.53,0.576,0.433,0.466,0.417,0.681,0.604   | 0.608649,0.055347 |
| 0.579,0.628,0.41,0.605,0.674,0.654,0.481,0.242,0.415,0.53,0.573,0.436,0.465,0.418,0.706,0.604   | 0.582849,0.058253 |
| 0.579,0.628,0.41,0.604,0.674,0.656,0.482,0.242,0.415,0.53,0.573,0.432,0.465,0.417,0.681,0.604   | 0.608797,0.05481  |
| 0.577,0.628,0.41,0.605,0.674,0.654,0.482,0.242,0.415,0.532,0.578,0.432,0.466,0.418,0.68,0.604   | 0.610189,0.056667 |
| 0.579,0.628,0.411,0.605,0.674,0.654,0.482,0.242,0.415,0.53,0.563,0.434,0.47,0.418,0.679,0.604   | 0.610725,0.052558 |
| 0.576,0.628,0.411,0.605,0.675,0.654,0.482,0.242,0.415,0.532,0.573,0.433,0.466,0.418,0.681,0.604 | 0.609455,0.05549  |
| 0.579,0.628,0.411,0.605,0.675,0.654,0.482,0.242,0.415,0.532,0.578,0.432,0.466,0.418,0.68,0.604  | 0.610302,0.056257 |
| 0.579,0.628,0.41,0.604,0.674,0.654,0.482,0.242,0.415,0.53,0.574,0.432,0.465,0.417,0.682,0.604   | 0.608721,0.055277 |
| 0.577,0.628,0.41,0.603,0.674,0.654,0.482,0.242,0.415,0.531,0.572,0.432,0.466,0.418,0.681,0.604  | 0.609087,0.054893 |
| 0.576,0.628,0.41,0.605,0.675,0.654,0.482,0.242,0.415,0.532,0.576,0.433,0.465,0.427,0.68,0.604   | 0.612073,0.055177 |
| 0.578,0.628,0.41,0.605,0.674,0.656,0.482,0.242,0.415,0.53,0.573,0.435,0.465,0.418,0.702,0.604   | 0.587019,0.057987 |
| 0.579,0.628,0.41,0.604,0.674,0.654,0.482,0.241,0.415,0.53,0.572,0.433,0.465,0.427,0.68,0.604    | 0.609827,0.053546 |
| 0.579,0.627,0.41,0.607,0.674,0.654,0.482,0.241,0.415,0.53,0.572,0.432,0.465,0.418,0.678,0.603   | 0.609445,0.05509  |
| 0.579,0.627,0.412,0.628,0.674,0.653,0.481,0.242,0.414,0.532,0.572,0.433,0.465,0.417,0.68,0.607  | 0.610366,0.059277 |
| 0.577,0.628,0.41,0.603,0.674,0.654,0.482,0.242,0.414,0.532,0.572,0.433,0.465,0.578,0.68,0.609   | 0.611732,0.064501 |
| 0.579,0.627,0.41,0.604,0.674,0.654,0.482,0.241,0.415,0.53,0.572,0.432,0.465,0.483,0.649,0.603   | 0.621133,0.048235 |
| 0.577,0.628,0.41,0.604,0.672,0.654,0.481,0.242,0.415,0.53,0.573,0.432,0.465,0.417,0.688,0.604   | 0.602514,0.055939 |
| 0.578,0.627,0.41,0.615,0.674,0.654,0.482,0.242,0.414,0.53,0.572,0.433,0.465,0.45,0.677,0.604    | 0.614637,0.054072 |
| 0.578,0.626,0.411,0.604,0.676,0.654,0.482,0.241,0.415,0.53,0.572,0.432,0.465,0.417,0.679,0.601  | 0.608561,0.054857 |
| 0.579,0.628,0.41,0.875,0.678,0.654,0.482,0.242,0.415,0.53,0.573,0.432,0.465,0.417,0.681,0.604   | 0.472608,0.121862 |
| 0.579,0.627,0.41,0.603,0.677,0.654,0.482,0.241,0.415,0.53,0.572,0.432,0.465,0.417,0.68,0.603    | 0.60794,0.054755  |
| 0.578,0.628,0.411,0.604,0.674,0.655,0.482,0.242,0.414,0.53,0.572,0.432,0.465,0.418,0.678,0.603  | 0.611533,0.054302 |
| 0.578,0.626,0.412,0.605,0.674,0.654,0.482,0.242,0.415,0.53,0.572,0.432,0.465,0.419,0.678,0.603  | 0.612614,0.053819 |
| 0.578,0.626,0.411,0.604,0.674,0.654,0.482,0.241,0.414,0.529,0.572,0.432,0.465,0.417,0.679,0.601 | 0.60898,0.05475   |
| 0.579,0.627,0.41,0.604,0.674,0.654,0.484,0.242,0.415,0.53,0.572,0.432,0.465,0.417,0.68,0.604    | 0.610645,0.054904 |
| 0.578,0.626,0.41,0.606,0.674,0.654,0.482,0.241,0.415,0.53,0.572,0.432,0.465,0.417,0.679,0.603   | 0.608439,0.055126 |
| 0.579,0.627,0.41,0.604,0.674,0.654,0.484,0.242,0.415,0.53,0.573,0.432,0.465,0.417,0.681,0.603   | 0.609814,0.055431 |
| 0.577,0.627,0.409,0.604,0.674,0.653,0.481,0.242,0.414,0.53,0.576,0.431,0.465,0.417,0.68,0.604   | 0.608732,0.055953 |
| 0.578,0.627,0.41,0.607,0.674,0.652,0.482,0.241,0.414,0.53,0.572,0.432,0.465,0.417,0.68,0.603    | 0.608385,0.055929 |
| 0.577,0.627,0.409,0.604,0.674,0.654,0.481,0.242,0.414,0.529,0.572,0.436,0.461,0.418,0.684,0.604 | 0.605564,0.053906 |
| 0.577,0.628,0.41,0.603,0.674,0.654,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.68,0.603   | 0.607533,0.054586 |
| 0.579,0.626,0.41,0.605,0.674,0.654,0.482,0.24,0.414,0.53,0.572,0.432,0.465,0.417,0.68,0.603     | 0.605338,0.055377 |
| 0.578,0.626,0.41,0.603,0.674,0.654,0.482,0.241,0.415,0.53,0.572,0.433,0.464,0.418,0.68,0.603    | 0.608987,0.054161 |
| 0.577,0.626,0.409,0.604,0.674,0.654,0.481,0.241,0.414,0.529,0.572,0.431,0.464,0.417,0.679,0.603 | 0.60741,0.054869  |
| 0.577,0.627,0.41,0.604,0.674,0.653,0.481,0.242,0.414,0.529,0.575,0.432,0.465,0.417,0.681,0.603  | 0.608934,0.055417 |
| 0.578,0.626,0.41,0.604,0.679,0.654,0.482,0.241,0.414,0.529,0.572,0.433,0.464,0.417,0.679,0.603  | 0.606495,0.054627 |
| 0.578,0.627,0.41,0.604,0.674,0.653,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.68,0.603   | 0.607655,0.054637 |
| 0.577,0.626,0.41,0.604,0.675,0.654,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.678,0.603  | 0.608552,0.054323 |
| 0.578,0.626,0.41,0.604,0.674,0.653,0.482,0.241,0.414,0.53,0.57,0.432,0.464,0.431,0.669,0.602    | 0.61654,0.051683  |
| 0.577,0.629,0.41,0.603,0.674,0.653,0.482,0.241,0.415,0.53,0.571,0.432,0.464,0.417,0.679,0.603   | 0.610198,0.054628 |
| 0.577,0.626,0.41,0.603,0.674,0.653,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.68,0.603   | 0.608421,0.054406 |
| 0.577,0.625,0.409,0.603,0.674,0.654,0.481,0.268,0.414,0.529,0.572,0.432,0.464,0.417,0.68,0.603  | 0.661192,0.04807  |
| 0.577,0.623,0.409,0.604,0.674,0.648,0.481,0.241,0.413,0.529,0.572,0.431,0.464,0.417,0.678,0.603 | 0.610611,0.054923 |
| 0.575,0.627,0.41,0.604,0.674,0.654,0.481,0.242,0.414,0.529,0.571,0.432,0.463,0.417,0.68,0.603   | 0.609433,0.054376 |
| 0.577,0.627,0.409,0.604,0.674,0.654,0.482,0.241,0.414,0.54,0.572,0.432,0.537,0.417,0.679,0.603  | 0.605193,0.069041 |
| 0.577,0.626,0.409,0.604,0.672,0.653,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.679,0.601 | 0.608693,0.054924 |
| 0.577,0.626,0.409,0.603,0.674,0.653,0.481,0.241,0.414,0.529,0.572,0.432,0.465,0.417,0.68,0.603  | 0.607702,0.054748 |
| 0.577,0.626,0.41,0.604,0.672,0.654,0.481,0.241,0.413,0.53,0.572,0.432,0.476,0.417,0.678,0.601   | 0.607852,0.056333 |
| 0.577,0.626,0.409,0.604,0.672,0.676,0.481,0.241,0.413,0.529,0.572,0.432,0.464,0.417,0.678,0.601 | 0.597323,0.054519 |
| 0.577,0.626,0.409,0.603,0.673,0.653,0.481,0.242,0.414,0.529,0.572,0.432,0.464,0.417,0.897,0.604 | 0.429668,0.101873 |
| 0.576,0.625,0.409,0.602,0.673,0.653,0.481,0.241,0.414,0.529,0.57,0.432,0.453,0.417,0.678,0.603  | 0.609767,0.052531 |
| 0.577,0.626,0.409,0.603,0.674,0.653,0.481,0.241,0.414,0.531,0.571,0.431,0.478,0.467,0.679,0.603 | 0.610543,0.052313 |

|                                                                                                 |                   |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.576,0.625,0.409,0.603,0.673,0.653,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.679,0.603 | 0.608494,0.054427 |
| 0.577,0.625,0.409,0.603,0.674,0.653,0.487,0.241,0.413,0.529,0.572,0.431,0.463,0.417,0.679,0.602 | 0.610364,0.05626  |
| 0.577,0.625,0.409,0.604,0.673,0.653,0.481,0.241,0.412,0.529,0.572,0.433,0.464,0.417,0.678,0.601 | 0.609076,0.054884 |
| 0.567,0.626,0.409,0.603,0.674,0.653,0.481,0.241,0.413,0.529,0.572,0.431,0.464,0.416,0.678,0.601 | 0.609133,0.056383 |
| 0.577,0.626,0.409,0.604,0.673,0.653,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.678,0.603 | 0.609125,0.054502 |
| 0.576,0.625,0.409,0.602,0.673,0.653,0.481,0.241,0.414,0.528,0.569,0.432,0.463,0.417,0.678,0.603 | 0.609119,0.053107 |
| 0.576,0.626,0.409,0.603,0.673,0.653,0.481,0.241,0.414,0.535,0.572,0.432,0.463,0.417,0.679,0.603 | 0.608816,0.055619 |
| 0.577,0.626,0.409,0.603,0.674,0.653,0.481,0.238,0.414,0.529,0.57,0.432,0.461,0.417,0.678,0.603  | 0.60212,0.0543    |
| 0.577,0.625,0.409,0.603,0.674,0.654,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.678,0.602 | 0.60844,0.054314  |
| 0.577,0.626,0.409,0.603,0.674,0.654,0.481,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.679,0.603 | 0.608065,0.054427 |
| 0.577,0.626,0.409,0.603,0.673,0.653,0.481,0.241,0.414,0.529,0.572,0.431,0.464,0.417,0.678,0.603 | 0.609816,0.054482 |
| 0.576,0.624,0.409,0.603,0.673,0.653,0.481,0.241,0.413,0.528,0.571,0.431,0.463,0.417,0.679,0.603 | 0.607766,0.054205 |
| 0.576,0.626,0.409,0.603,0.673,0.653,0.473,0.241,0.414,0.529,0.572,0.432,0.464,0.417,0.679,0.603 | 0.604861,0.052633 |
| 0.577,0.626,0.409,0.603,0.673,0.653,0.481,0.242,0.414,0.529,0.572,0.43,0.464,0.367,0.679,0.603  | 0.597785,0.062072 |
| 0.577,0.626,0.409,0.603,0.673,0.653,0.481,0.241,0.414,0.528,0.57,0.432,0.461,0.417,0.678,0.626  | 0.605866,0.050153 |
| 0.575,0.625,0.409,0.602,0.673,0.653,0.48,0.241,0.413,0.528,0.571,0.432,0.463,0.417,0.678,0.606  | 0.608371,0.053137 |
| 0.576,0.625,0.409,0.603,0.674,0.652,0.481,0.241,0.414,0.529,0.572,0.43,0.464,0.416,0.678,0.602  | 0.608958,0.055067 |
| 0.576,0.624,0.409,0.603,0.673,0.653,0.481,0.241,0.413,0.529,0.572,0.431,0.463,0.416,0.677,0.601 | 0.609697,0.054721 |
| 0.576,0.625,0.409,0.602,0.673,0.653,0.481,0.241,0.413,0.528,0.571,0.432,0.463,0.417,0.678,0.698 | 0.580347,0.047764 |
| 0.577,0.625,0.409,0.603,0.673,0.653,0.481,0.241,0.414,0.529,0.572,0.431,0.464,0.417,0.683,0.602 | 0.604505,0.055325 |
| 0.575,0.625,0.409,0.602,0.673,0.652,0.48,0.241,0.414,0.528,0.571,0.43,0.463,0.416,0.678,0.603   | 0.608809,0.053985 |
| 0.576,0.625,0.409,0.589,0.672,0.653,0.481,0.241,0.41,0.529,0.572,0.431,0.464,0.417,0.678,0.601  | 0.609303,0.052334 |
| 0.577,0.626,0.409,0.603,0.673,0.653,0.481,0.241,0.413,0.528,0.571,0.431,0.463,0.417,0.678,0.601 | 0.609077,0.054475 |
| 0.576,0.625,0.409,0.602,0.673,0.653,0.48,0.241,0.414,0.528,0.57,0.432,0.461,0.416,0.645,0.622   | 0.619361,0.047176 |
| 0.577,0.627,0.409,0.603,0.671,0.653,0.481,0.241,0.405,0.529,0.572,0.43,0.464,0.416,0.678,0.602  | 0.606991,0.056767 |
| 0.575,0.625,0.407,0.602,0.673,0.653,0.481,0.241,0.414,0.528,0.57,0.431,0.461,0.439,0.678,0.603  | 0.610472,0.052114 |
| 0.576,0.625,0.409,0.603,0.673,0.653,0.481,0.241,0.41,0.529,0.572,0.431,0.463,0.416,0.678,0.601  | 0.60841,0.055583  |
| 0.577,0.626,0.409,0.603,0.673,0.653,0.481,0.242,0.414,0.529,0.572,0.43,0.457,0.411,0.679,0.611  | 0.608806,0.053292 |
| 0.575,0.624,0.409,0.602,0.673,0.652,0.479,0.241,0.413,0.528,0.571,0.435,0.463,0.416,0.678,0.602 | 0.608558,0.052931 |
| 0.576,0.624,0.409,0.603,0.671,0.653,0.481,0.241,0.413,0.529,0.572,0.43,0.463,0.416,0.676,0.602  | 0.610818,0.054526 |
| 0.576,0.624,0.409,0.603,0.673,0.653,0.481,0.241,0.409,0.52,0.572,0.43,0.463,0.416,0.678,0.601   | 0.606822,0.054118 |
| 0.576,0.625,0.409,0.603,0.672,0.653,0.481,0.241,0.236,0.528,0.571,0.43,0.463,0.416,0.678,0.601  | 0.517425,0.095246 |
| 0.576,0.625,0.409,0.604,0.673,0.653,0.48,0.241,0.412,0.528,0.572,0.431,0.464,0.416,0.678,0.633  | 0.604411,0.05081  |
| 0.575,0.625,0.409,0.603,0.672,0.653,0.481,0.241,0.412,0.529,0.572,0.43,0.464,0.416,0.676,0.601  | 0.61051,0.055263  |
| 0.576,0.625,0.409,0.603,0.673,0.653,0.481,0.241,0.41,0.523,0.572,0.431,0.463,0.416,0.678,0.601  | 0.608346,0.054406 |
| 0.576,0.641,0.409,0.603,0.672,0.653,0.48,0.241,0.411,0.528,0.57,0.43,0.463,0.416,0.677,0.6      | 0.609288,0.056735 |
| 0.576,0.624,0.409,0.603,0.673,0.653,0.479,0.241,0.41,0.529,0.572,0.43,0.463,0.417,0.676,0.601   | 0.609066,0.054781 |
| 0.576,0.624,0.409,0.602,0.673,0.653,0.482,0.241,0.413,0.528,0.571,0.43,0.463,0.416,0.676,0.601  | 0.610761,0.054379 |
| 0.576,0.625,0.409,0.603,0.672,0.653,0.481,0.241,0.192,0.528,0.571,0.43,0.463,0.416,0.678,0.601  | 0.490511,0.106115 |
| 0.576,0.625,0.409,0.602,0.671,0.646,0.479,0.241,0.412,0.528,0.571,0.43,0.463,0.416,0.675,0.601  | 0.613914,0.054099 |
| 0.576,0.624,0.409,0.508,0.672,0.653,0.48,0.241,0.411,0.528,0.572,0.43,0.463,0.416,0.677,0.601   | 0.609084,0.034748 |
| 0.576,0.624,0.409,0.602,0.672,0.653,0.48,0.241,0.412,0.528,0.572,0.43,0.462,0.415,0.677,0.601   | 0.609147,0.054414 |
| 0.576,0.624,0.409,0.603,0.673,0.653,0.481,0.241,0.394,0.528,0.571,0.43,0.463,0.416,0.677,0.601  | 0.602097,0.05836  |
| 0.576,0.623,0.408,0.603,0.673,0.653,0.479,0.241,0.412,0.522,0.575,0.43,0.462,0.41,0.677,0.602   | 0.606981,0.054466 |
| 0.576,0.624,0.409,0.602,0.672,0.653,0.48,0.241,0.412,0.528,0.571,0.43,0.463,0.416,0.677,0.602   | 0.609206,0.054046 |
| 0.576,0.624,0.409,0.603,0.672,0.653,0.477,0.241,0.393,0.527,0.571,0.43,0.463,0.416,0.677,0.601  | 0.600034,0.057355 |
| 0.576,0.627,0.409,0.602,0.672,0.653,0.48,0.241,0.373,0.526,0.57,0.43,0.462,0.416,0.676,0.6      | 0.58933,0.061903  |
| 0.576,0.624,0.409,0.602,0.672,0.653,0.479,0.241,0.412,0.528,0.571,0.43,0.463,0.416,0.675,0.601  | 0.610351,0.053653 |
| 0.576,0.623,0.408,0.603,0.672,0.653,0.479,0.241,0.409,0.526,0.571,0.43,0.462,0.414,0.677,0.602  | 0.607258,0.054465 |
| 0.576,0.624,0.409,0.602,0.672,0.653,0.479,0.241,0.412,0.528,0.571,0.43,0.463,0.416,0.675,0.621  | 0.608177,0.050869 |
| 0.576,0.624,0.409,0.602,0.671,0.396,0.477,0.241,0.396,0.527,0.571,0.43,0.463,0.416,0.676,0.601  | 0.622996,0.105794 |
| 0.576,0.624,0.409,0.539,0.672,0.653,0.479,0.248,0.41,0.528,0.57,0.43,0.462,0.416,0.675,0.605    | 0.624882,0.037625 |
| 0.576,0.624,0.409,0.602,0.672,0.653,0.479,0.241,0.41,0.525,0.57,0.43,0.462,0.416,0.676,0.6      | 0.609291,0.053447 |
| 0.576,0.628,0.409,0.602,0.672,0.653,0.48,0.241,0.388,0.526,0.57,0.43,0.462,0.416,0.676,0.6      | 0.599674,0.058819 |

|                                                                                                 |                   |
|-------------------------------------------------------------------------------------------------|-------------------|
| 0.576,0.623,0.409,0.602,0.672,0.653,0.479,0.241,0.41,0.528,0.57,0.43,0.462,0.416,0.675,0.605    | 0.609592,0.05301  |
| 0.575,0.624,0.408,0.602,0.672,0.653,0.479,0.241,0.41,0.528,0.571,0.43,0.461,0.416,0.676,0.607   | 0.608412,0.053421 |
| 0.576,0.623,0.409,0.602,0.671,0.651,0.479,0.243,0.41,0.528,0.571,0.43,0.462,0.416,0.676,0.6     | 0.614672,0.053709 |
| 0.576,0.623,0.408,0.602,0.672,0.653,0.479,0.241,0.411,0.527,0.57,0.43,0.462,0.415,0.676,0.6     | 0.60883,0.053871  |
| 0.576,0.623,0.409,0.602,0.671,0.65,0.479,0.797,0.411,0.531,0.57,0.43,0.461,0.415,0.676,0.601    | 0.755412,0.073764 |
| 0.576,0.623,0.409,0.602,0.671,0.65,0.479,0.241,0.405,0.527,0.57,0.43,0.462,0.416,0.675,0.6      | 0.610064,0.054667 |
| 0.576,0.624,0.409,0.601,0.671,0.653,0.479,0.241,0.41,0.527,0.57,0.43,0.461,0.415,0.675,0.6      | 0.610368,0.053406 |
| 0.573,0.623,0.408,0.602,0.672,0.653,0.479,0.241,0.411,0.528,0.571,0.43,0.461,0.412,0.677,0.604  | 0.607378,0.054378 |
| 0.576,0.623,0.409,0.602,0.671,0.651,0.479,0.243,0.392,0.526,0.57,0.405,0.462,0.416,0.68,0.6     | 0.598362,0.062794 |
| 0.575,0.623,0.408,0.602,0.671,0.653,0.479,0.241,0.411,0.528,0.57,0.43,0.472,0.414,0.675,0.606   | 0.608037,0.05439  |
| 0.576,0.624,0.409,0.601,0.671,0.653,0.479,0.241,0.41,0.528,0.571,0.43,0.462,0.42,0.675,0.6      | 0.611115,0.053398 |
| 0.576,0.623,0.408,0.602,0.671,0.652,0.479,0.241,0.41,0.528,0.571,0.43,0.462,0.416,0.675,0.604   | 0.610072,0.053601 |
| 0.575,0.623,0.425,0.605,0.672,0.653,0.479,0.241,0.41,0.528,0.57,0.43,0.461,0.415,0.676,0.601    | 0.612018,0.050912 |
| 0.575,0.623,0.408,0.602,0.672,0.653,0.479,0.242,0.411,0.528,0.571,0.43,0.472,0.414,0.675,0.606  | 0.609898,0.05441  |
| 0.575,0.623,0.408,0.602,0.672,0.652,0.479,0.241,0.411,0.528,0.572,0.43,0.512,0.387,0.675,0.617  | 0.597926,0.064902 |
| 0.575,0.623,0.408,0.602,0.672,0.653,0.479,0.241,0.413,0.528,0.571,0.427,0.461,0.415,0.093,0.6   | 0.451794,0.155931 |
| 0.575,0.623,0.408,0.602,0.671,0.652,0.479,0.241,0.41,0.527,0.573,0.43,0.461,0.414,0.675,0.601   | 0.610241,0.054503 |
| 0.575,0.622,0.408,0.602,0.671,0.653,0.479,0.241,0.41,0.528,0.638,0.43,0.461,0.414,0.675,0.602   | 0.613442,0.068979 |
| 0.575,0.623,0.408,0.602,0.672,0.65,0.484,0.205,0.411,0.528,0.57,0.428,0.461,0.414,0.676,0.6     | 0.556457,0.064668 |
| 0.576,0.623,0.409,0.601,0.671,0.652,0.479,0.241,0.409,0.527,0.604,0.43,0.461,0.415,0.675,0.607  | 0.612646,0.059909 |
| 0.576,0.623,0.408,0.602,0.671,0.653,0.479,0.241,0.409,0.527,0.604,0.43,0.461,0.415,0.675,0.607  | 0.611463,0.060322 |
| 0.575,0.623,0.408,0.602,0.672,0.653,0.482,0.241,0.41,0.528,0.57,0.428,0.461,0.415,0.674,0.601   | 0.611256,0.054944 |
| 0.575,0.623,0.411,0.602,0.671,0.652,0.479,0.241,0.409,0.527,0.595,0.439,0.461,0.415,0.675,0.604 | 0.613236,0.056245 |
| 0.576,0.623,0.408,0.602,0.671,0.653,0.479,0.241,0.41,0.528,0.57,0.405,0.462,0.415,0.675,0.6     | 0.605076,0.059476 |
| 0.577,0.623,0.408,0.602,0.671,0.653,0.479,0.241,0.41,0.527,0.57,0.429,0.46,0.414,0.675,0.6      | 0.609579,0.053858 |
| 0.575,0.623,0.408,0.601,0.672,0.605,0.479,0.241,0.415,0.527,0.57,0.432,0.461,0.415,0.676,0.6    | 0.62318,0.057252  |
| 0.575,0.623,0.408,0.602,0.654,0.654,0.479,0.24,0.407,0.527,0.57,0.429,0.464,0.414,0.675,0.602   | 0.607246,0.054635 |
| 0.576,0.622,0.408,0.601,0.672,0.652,0.479,0.241,0.41,0.527,0.572,0.429,0.461,0.415,0.674,0.6    | 0.610703,0.053993 |
| 0.576,0.623,0.408,0.602,0.671,0.652,0.479,0.241,0.41,0.528,0.571,0.43,0.461,0.415,0.676,0.611   | 0.608335,0.052727 |
| 0.575,0.622,0.409,0.602,0.671,0.652,0.479,0.241,0.409,0.527,0.597,0.437,0.461,0.414,0.675,0.605 | 0.612529,0.057429 |
| 0.577,0.624,0.408,0.602,0.669,0.653,0.479,0.241,0.41,0.528,0.57,0.428,0.461,0.415,0.674,0.601   | 0.610866,0.053981 |
| 0.575,0.622,0.408,0.601,0.671,0.652,0.479,0.241,0.41,0.527,0.571,0.432,0.461,0.415,0.674,0.6    | 0.611459,0.053207 |
| 0.575,0.622,0.408,0.602,0.671,0.652,0.479,0.24,0.407,0.527,0.565,0.429,0.463,0.414,0.675,0.602  | 0.60712,0.053822  |
| 0.575,0.623,0.408,0.602,0.671,0.652,0.479,0.241,0.41,0.527,0.57,0.428,0.463,0.412,0.683,0.608   | 0.600941,0.054901 |
| 0.576,0.623,0.408,0.602,0.671,0.652,0.479,0.241,0.41,0.527,0.57,0.429,0.46,0.413,0.673,0.6      | 0.611517,0.053815 |
| 0.575,0.623,0.408,0.602,0.671,0.652,0.479,0.241,0.41,0.524,0.6,0.431,0.461,0.414,0.675,0.605    | 0.61198,0.058979  |
| 0.551,0.622,0.408,0.601,0.669,0.652,0.479,0.24,0.409,0.527,0.569,0.428,0.459,0.414,0.675,0.6    | 0.609693,0.057165 |
| 0.648,0.623,0.408,0.601,0.676,0.652,0.479,0.241,0.409,0.527,0.578,0.43,0.461,0.415,0.675,0.603  | 0.609943,0.053365 |
| 0.575,0.621,0.408,0.603,0.671,0.651,0.479,0.24,0.407,0.526,0.567,0.427,0.465,0.414,0.675,0.603  | 0.607172,0.054713 |
| 0.575,0.622,0.408,0.601,0.673,0.652,0.479,0.24,0.407,0.526,0.576,0.429,0.46,0.414,0.675,0.601   | 0.606482,0.055642 |
| 0.575,0.623,0.408,0.602,0.671,0.652,0.479,0.241,0.41,0.519,0.57,0.412,0.458,0.414,0.677,0.604   | 0.6052,0.055713   |
| 0.575,0.621,0.408,0.602,0.671,0.651,0.473,0.24,0.407,0.524,0.567,0.424,0.463,0.414,0.675,0.603  | 0.603294,0.053058 |
| 0.575,0.621,0.408,0.601,0.671,0.652,0.479,0.241,0.41,0.527,0.57,0.429,0.461,0.297,0.675,0.6     | 0.589322,0.074916 |
| 0.575,0.616,0.408,0.601,0.671,0.652,0.479,0.241,0.409,0.526,0.572,0.429,0.461,0.414,0.675,0.594 | 0.610037,0.054647 |
| 0.575,0.622,0.408,0.601,0.671,0.651,0.479,0.241,0.41,0.526,0.57,0.428,0.46,0.413,0.673,0.6      | 0.61184,0.053649  |
| 0.575,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.423,0.526,0.57,0.428,0.46,0.413,0.674,0.6       | 0.610425,0.05134  |
| 0.575,0.62,0.407,0.601,0.671,0.652,0.479,0.241,0.41,0.526,0.57,0.428,0.46,0.413,0.674,0.6       | 0.609981,0.053767 |
| 0.575,0.62,0.408,0.601,0.671,0.652,0.479,0.241,0.41,0.527,0.57,0.429,0.461,0.412,0.673,0.6      | 0.611128,0.053592 |
| 0.575,0.623,0.408,0.602,0.671,0.652,0.479,0.241,0.41,0.526,0.561,0.428,0.461,0.413,0.688,0.601  | 0.596938,0.054429 |
| 0.575,0.622,0.44,0.601,0.671,0.652,0.479,0.241,0.411,0.526,0.57,0.428,0.459,0.413,0.675,0.601   | 0.613118,0.046233 |
| 0.575,0.622,0.408,0.601,0.671,0.652,0.423,0.241,0.41,0.526,0.57,0.435,0.46,0.413,0.675,0.6      | 0.594875,0.039349 |
| 0.575,0.62,0.408,0.601,0.671,0.651,0.479,0.241,0.409,0.526,0.57,0.428,0.46,0.413,0.674,0.603    | 0.610434,0.053327 |
| 0.573,0.621,0.408,0.601,0.671,0.651,0.479,0.237,0.41,0.526,0.57,0.428,0.46,0.413,0.673,0.6      | 0.603501,0.054718 |
| 0.574,0.62,0.404,0.601,0.671,0.652,0.479,0.24,0.409,0.526,0.572,0.429,0.46,0.414,0.674,0.6      | 0.606828,0.055105 |

|                                                                                                |                   |
|------------------------------------------------------------------------------------------------|-------------------|
| 0.574,0.62,0.408,0.601,0.671,0.651,0.479,0.238,0.407,0.526,0.569,0.428,0.461,0.412,0.673,0.6   | 0.604443,0.054862 |
| 0.575,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.41,0.526,0.57,0.43,0.46,0.413,0.675,0.6        | 0.608291,0.053557 |
| 0.573,0.62,0.408,0.601,0.671,0.651,0.479,0.236,0.409,0.526,0.57,0.428,0.455,0.413,0.77,0.6     | 0.49914,0.073537  |
| 0.574,0.62,0.407,0.6,0.671,0.651,0.479,0.24,0.409,0.526,0.569,0.428,0.46,0.398,0.674,0.6       | 0.605855,0.055772 |
| 0.575,0.621,0.408,0.601,0.671,0.652,0.479,0.241,0.41,0.526,0.57,0.428,0.46,0.385,0.616,0.6     | 0.616806,0.051413 |
| 0.574,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.569,0.428,0.46,0.413,0.674,0.6     | 0.608618,0.053918 |
| 0.572,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.57,0.428,0.455,0.413,0.673,0.6     | 0.609913,0.05367  |
| 0.574,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.562,0.428,0.46,0.413,0.685,0.603   | 0.597673,0.053724 |
| 0.574,0.62,0.407,0.601,0.671,0.65,0.478,0.212,0.407,0.526,0.569,0.428,0.461,0.412,0.675,0.6    | 0.564563,0.061481 |
| 0.575,0.622,0.408,0.601,0.671,0.652,0.479,0.241,0.409,0.526,0.57,0.428,0.455,0.413,0.673,0.6   | 0.611501,0.05327  |
| 0.574,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.569,0.428,0.459,0.413,0.674,0.6    | 0.608697,0.053808 |
| 0.575,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.57,0.428,0.46,0.413,0.673,0.601    | 0.609629,0.053717 |
| 0.573,0.62,0.407,0.601,0.671,0.651,0.479,0.238,0.409,0.505,0.569,0.427,0.458,0.413,0.673,0.6   | 0.604638,0.050617 |
| 0.574,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.569,0.428,0.458,0.413,0.673,0.595  | 0.610037,0.054328 |
| 0.574,0.62,0.408,0.601,0.671,0.651,0.479,0.234,0.409,0.526,0.569,0.428,0.459,0.413,0.673,0.6   | 0.596916,0.055093 |
| 0.574,0.62,0.407,0.601,0.671,0.65,0.478,0.217,0.407,0.526,0.569,0.428,0.46,0.412,0.674,0.6     | 0.571458,0.060001 |
| 0.574,0.62,0.407,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.569,0.428,0.458,0.413,0.673,0.601  | 0.608974,0.053626 |
| 0.569,0.62,0.407,0.601,0.671,0.926,0.478,0.233,0.41,0.526,0.569,0.428,0.459,0.412,0.673,0.6    | 0.401839,0.100346 |
| 0.574,0.62,0.407,0.601,0.671,0.651,0.479,0.238,0.409,0.523,0.569,0.425,0.46,0.413,0.52,0.6     | 0.614891,0.055623 |
| 0.574,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.568,0.423,0.46,0.413,0.674,0.598   | 0.607976,0.055078 |
| 0.574,0.62,0.407,0.601,0.671,0.651,0.479,0.236,0.409,0.525,0.569,0.428,0.458,0.433,0.674,0.6   | 0.602233,0.05299  |
| 0.574,0.62,0.407,0.601,0.669,0.926,0.478,0.233,0.41,0.526,0.569,0.428,0.459,0.412,0.673,0.6    | 0.402213,0.100264 |
| 0.574,0.62,0.408,0.601,0.671,0.651,0.479,0.24,0.409,0.526,0.568,0.424,0.46,0.413,0.673,0.598   | 0.608919,0.054709 |
| 0.573,0.62,0.407,0.601,0.671,0.66,0.479,0.239,0.409,0.525,0.569,0.427,0.459,0.413,0.673,0.6    | 0.602264,0.053893 |
| 0.574,0.62,0.407,0.593,0.671,0.651,0.479,0.24,0.407,0.526,0.57,0.428,0.46,0.407,0.657,0.6      | 0.614584,0.051207 |
| 0.574,0.62,0.407,0.601,0.671,0.651,0.479,0.24,0.407,0.526,0.57,0.428,0.459,0.407,0.658,0.6     | 0.612897,0.053051 |
| 0.578,0.62,0.407,0.593,0.671,0.651,0.479,0.24,0.407,0.526,0.57,0.428,0.46,0.407,0.657,0.6      | 0.614417,0.05079  |
| 0.574,0.62,0.407,0.601,0.671,0.651,0.479,0.239,0.407,0.525,0.569,0.427,0.459,0.413,0.673,0.6   | 0.605178,0.054534 |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.479,0.239,0.409,0.525,0.569,0.427,0.458,0.413,0.673,0.598  | 0.607612,0.054502 |
| 0.575,0.62,0.407,0.601,0.671,0.651,0.479,0.246,0.409,0.525,0.569,0.428,0.459,0.413,0.673,0.6   | 0.621403,0.052149 |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.478,0.238,0.409,0.525,0.568,0.426,0.458,0.413,0.673,0.599  | 0.604782,0.054347 |
| 0.579,0.62,0.407,0.601,0.671,0.651,0.479,0.24,0.409,0.525,0.569,0.428,0.458,0.413,0.724,0.6    | 0.554867,0.062257 |
| 0.573,0.62,0.407,0.601,0.671,0.651,0.479,0.239,0.409,0.525,0.568,0.426,0.454,0.413,0.673,0.599 | 0.60706,0.053886  |
| 0.573,0.626,0.407,0.601,0.671,0.65,0.479,0.239,0.409,0.525,0.568,0.426,0.458,0.413,0.673,0.6   | 0.607542,0.054888 |
| 0.575,0.62,0.407,0.601,0.667,0.651,0.479,0.246,0.409,0.525,0.569,0.428,0.459,0.413,0.673,0.6   | 0.622505,0.051966 |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.479,0.238,0.408,0.525,0.568,0.426,0.458,0.413,0.673,0.6    | 0.60493,0.054629  |
| 0.573,0.62,0.407,0.601,0.671,0.651,0.479,0.239,0.407,0.525,0.569,0.427,0.458,0.412,0.673,0.6   | 0.60541,0.054643  |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.478,0.237,0.409,0.525,0.568,0.427,0.458,0.413,0.673,0.598  | 0.602936,0.054529 |
| 0.573,0.623,0.407,0.601,0.67,0.65,0.478,0.239,0.408,0.525,0.568,0.426,0.46,0.413,0.673,0.6     | 0.606943,0.054654 |
| 0.573,0.62,0.407,0.601,0.671,0.648,0.479,0.24,0.408,0.525,0.575,0.428,0.458,0.413,0.673,0.6    | 0.61082,0.055327  |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.479,0.239,0.408,0.525,0.567,0.425,0.458,0.413,0.672,0.598  | 0.607734,0.054543 |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.479,0.138,0.408,0.521,0.568,0.426,0.458,0.412,0.672,0.6    | 0.489026,0.078288 |
| 0.573,0.62,0.407,0.601,0.671,0.651,0.479,0.239,0.408,0.525,0.565,0.427,0.458,0.412,0.673,0.6   | 0.606387,0.053602 |
| 0.573,0.62,0.407,0.601,0.689,0.651,0.479,0.239,0.407,0.525,0.569,0.426,0.458,0.411,0.67,0.6    | 0.592506,0.055805 |
| 0.573,0.62,0.407,0.6,0.671,0.65,0.478,0.238,0.407,0.525,0.568,0.427,0.458,0.412,0.672,0.599    | 0.604255,0.054253 |
| 0.573,0.62,0.407,0.601,0.67,0.65,0.479,0.24,0.408,0.525,0.568,0.427,0.459,0.413,0.683,0.6      | 0.6002,0.055586   |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.479,0.239,0.408,0.525,0.568,0.427,0.459,0.413,0.683,0.6    | 0.59803,0.055877  |
| 0.573,0.62,0.407,0.602,0.671,0.651,0.479,0.239,0.407,0.525,0.569,0.427,0.458,0.407,0.672,0.6   | 0.605349,0.055286 |
| 0.573,0.62,0.407,0.601,0.671,0.645,0.478,0.238,0.407,0.525,0.569,0.427,0.458,0.412,0.673,0.6   | 0.605699,0.05499  |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.479,0.239,0.408,0.525,0.568,0.426,0.458,0.413,0.695,0.6    | 0.585077,0.058046 |
| 0.573,0.62,0.407,0.601,0.67,0.65,0.478,0.238,0.408,0.525,0.568,0.426,0.46,0.413,0.673,0.6      | 0.604645,0.054554 |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.478,0.238,0.408,0.525,0.568,0.426,0.458,0.412,0.672,0.598  | 0.605288,0.054651 |
| 0.573,0.62,0.407,0.601,0.671,0.654,0.477,0.235,0.407,0.525,0.568,0.426,0.458,0.412,0.681,0.6   | 0.58769,0.056244  |
| 0.573,0.62,0.407,0.606,0.671,0.65,0.478,0.238,0.407,0.525,0.568,0.425,0.458,0.412,0.672,0.599  | 0.604462,0.056062 |

|                                                                                                |                   |
|------------------------------------------------------------------------------------------------|-------------------|
| 0.573,0.62,0.407,0.601,0.671,0.651,0.479,0.239,0.409,0.525,0.569,0.427,0.458,0.411,0.672,0.606 | 0.606932,0.053324 |
| 0.573,0.62,0.407,0.601,0.671,0.65,0.479,0.238,0.408,0.525,0.568,0.426,0.458,0.412,0.672,0.599  | 0.605641,0.054737 |
| 0.573,0.62,0.407,0.601,0.67,0.65,0.478,0.238,0.408,0.525,0.568,0.427,0.458,0.412,0.673,0.622   | 0.602385,0.051243 |



## B. Dataset of the Second Step: Deterministic Approach

Table below shows data of two instances side by side due to row length.

Data in this table is limited with objective space.

Full data is available upon request.

| displacement | mass   | internal forces | displacement | mass   | internal forces |
|--------------|--------|-----------------|--------------|--------|-----------------|
| 1,97         | 443,55 | 0,00            | 1,97         | 436,39 | 0,12            |
| 1,96         | 441,02 | 0,01            | 1,96         | 433,86 | 0,13            |
| 1,95         | 438,63 | 0,05            | 1,96         | 431,47 | 0,17            |
| 1,96         | 436,39 | 0,12            | 1,96         | 429,22 | 0,24            |
| 1,97         | 434,30 | 0,24            | 1,97         | 427,14 | 0,36            |
| 2,00         | 432,38 | 0,41            | 2,00         | 425,22 | 0,53            |
| 2,07         | 441,06 | 0,03            | 2,07         | 433,90 | 0,15            |
| 2,06         | 438,45 | 0,02            | 2,06         | 431,29 | 0,14            |
| 2,06         | 435,98 | 0,03            | 2,06         | 428,82 | 0,15            |
| 2,06         | 433,66 | 0,07            | 2,06         | 426,50 | 0,20            |
| 2,07         | 431,50 | 0,16            | 2,07         | 424,34 | 0,28            |
| 2,10         | 429,51 | 0,30            | 2,10         | 422,35 | 0,42            |
| 2,21         | 438,79 | 0,12            | 2,21         | 431,62 | 0,24            |
| 2,20         | 436,10 | 0,08            | 2,20         | 428,94 | 0,20            |
| 2,19         | 433,55 | 0,07            | 2,19         | 426,39 | 0,19            |
| 2,19         | 431,16 | 0,08            | 2,19         | 423,99 | 0,20            |
| 2,20         | 428,92 | 0,14            | 2,20         | 421,75 | 0,26            |
| 2,23         | 426,85 | 0,24            | 2,23         | 419,69 | 0,36            |
| 2,39         | 436,74 | 0,28            | 2,39         | 429,57 | 0,40            |
| 2,37         | 433,97 | 0,22            | 2,37         | 426,81 | 0,34            |
| 2,37         | 431,35 | 0,18            | 2,37         | 424,19 | 0,30            |
| 2,36         | 428,87 | 0,16            | 2,36         | 421,71 | 0,28            |
| 2,37         | 426,56 | 0,18            | 2,37         | 419,39 | 0,30            |
| 2,39         | 424,42 | 0,25            | 2,39         | 417,25 | 0,37            |
| 2,61         | 434,92 | 0,55            | 2,61         | 427,76 | 0,67            |
| 2,60         | 432,08 | 0,46            | 2,60         | 424,92 | 0,58            |
| 2,59         | 429,38 | 0,38            | 2,59         | 422,22 | 0,50            |
| 2,58         | 426,83 | 0,33            | 2,58         | 419,66 | 0,45            |
| 2,59         | 424,43 | 0,31            | 2,59         | 417,27 | 0,44            |
| 2,60         | 422,21 | 0,34            | 2,60         | 415,05 | 0,46            |
| 2,90         | 433,34 | 0,95            | 2,90         | 426,18 | 1,07            |
| 2,88         | 430,43 | 0,82            | 2,88         | 423,27 | 0,94            |
| 2,87         | 427,66 | 0,72            | 2,87         | 420,49 | 0,84            |
| 2,86         | 425,03 | 0,63            | 2,86         | 417,86 | 0,75            |
| 2,87         | 422,56 | 0,56            | 2,87         | 415,39 | 0,69            |
| 2,88         | 420,26 | 0,54            | 2,88         | 413,09 | 0,66            |
| 2,08         | 441,06 | 0,03            | 2,07         | 433,66 | 0,07            |
| 2,07         | 438,53 | 0,04            | 2,06         | 431,13 | 0,09            |
| 2,07         | 436,14 | 0,08            | 2,06         | 428,74 | 0,12            |
| 2,07         | 433,90 | 0,15            | 2,07         | 426,50 | 0,20            |
| 2,09         | 431,81 | 0,26            | 2,08         | 424,41 | 0,31            |
| 2,11         | 429,89 | 0,43            | 2,11         | 422,50 | 0,48            |
| 2,18         | 438,49 | 0,02            | 2,17         | 431,09 | 0,07            |
| 2,17         | 435,88 | 0,01            | 2,16         | 428,49 | 0,06            |
| 2,17         | 433,41 | 0,03            | 2,16         | 426,02 | 0,07            |
| 2,17         | 431,09 | 0,07            | 2,16         | 423,70 | 0,12            |
| 2,18         | 428,93 | 0,16            | 2,18         | 421,54 | 0,20            |
| 2,21         | 426,94 | 0,29            | 2,20         | 419,55 | 0,34            |
| 2,31         | 436,14 | 0,08            | 2,31         | 428,74 | 0,13            |
| 2,30         | 433,45 | 0,04            | 2,30         | 426,06 | 0,09            |
| 2,29         | 430,91 | 0,03            | 2,29         | 423,51 | 0,08            |
| 2,30         | 428,51 | 0,05            | 2,29         | 421,11 | 0,09            |
| 2,31         | 426,27 | 0,10            | 2,30         | 418,87 | 0,15            |
| 2,33         | 424,21 | 0,20            | 2,32         | 416,81 | 0,25            |
| 2,48         | 434,01 | 0,21            | 2,48         | 426,62 | 0,26            |
| 2,47         | 431,25 | 0,15            | 2,46         | 423,85 | 0,19            |
| 2,46         | 428,63 | 0,11            | 2,45         | 421,23 | 0,15            |
| 2,46         | 426,15 | 0,09            | 2,45         | 418,75 | 0,14            |
| 2,47         | 423,83 | 0,11            | 2,46         | 416,44 | 0,16            |
| 2,49         | 421,69 | 0,18            | 2,48         | 414,29 | 0,22            |
| 2,70         | 432,12 | 0,44            | 2,70         | 424,72 | 0,49            |
| 2,69         | 429,28 | 0,35            | 2,68         | 421,89 | 0,39            |
| 2,68         | 426,58 | 0,27            | 2,67         | 419,18 | 0,32            |
| 2,67         | 424,03 | 0,22            | 2,66         | 416,63 | 0,27            |
| 2,68         | 421,63 | 0,20            | 2,67         | 414,24 | 0,25            |
| 2,69         | 419,41 | 0,23            | 2,69         | 412,02 | 0,27            |
| 2,99         | 430,47 | 0,79            | 2,98         | 423,08 | 0,84            |

|      |        |      |      |        |      |
|------|--------|------|------|--------|------|
| 2,97 | 427,56 | 0,67 | 2,96 | 420,16 | 0,72 |
| 2,95 | 424,78 | 0,56 | 2,94 | 417,39 | 0,61 |
| 2,95 | 422,15 | 0,48 | 2,94 | 414,76 | 0,52 |
| 2,95 | 419,68 | 0,41 | 2,94 | 412,29 | 0,46 |
| 2,96 | 417,38 | 0,39 | 2,96 | 409,99 | 0,44 |
| 2,22 | 438,79 | 0,12 | 2,21 | 431,16 | 0,08 |
| 2,21 | 436,26 | 0,13 | 2,20 | 428,63 | 0,09 |
| 2,21 | 433,86 | 0,17 | 2,20 | 426,23 | 0,13 |
| 2,22 | 431,62 | 0,24 | 2,20 | 423,99 | 0,20 |
| 2,23 | 429,54 | 0,35 | 2,22 | 421,91 | 0,32 |
| 2,26 | 427,62 | 0,52 | 2,24 | 419,99 | 0,49 |
| 2,32 | 436,14 | 0,08 | 2,30 | 428,51 | 0,05 |
| 2,31 | 433,53 | 0,07 | 2,29 | 425,90 | 0,03 |
| 2,30 | 431,06 | 0,08 | 2,29 | 423,43 | 0,05 |
| 2,31 | 428,74 | 0,13 | 2,29 | 421,11 | 0,09 |
| 2,32 | 426,58 | 0,21 | 2,31 | 418,95 | 0,18 |
| 2,34 | 424,59 | 0,35 | 2,33 | 416,96 | 0,32 |
| 2,45 | 433,71 | 0,10 | 2,43 | 426,08 | 0,07 |
| 2,43 | 431,02 | 0,06 | 2,42 | 423,39 | 0,03 |
| 2,43 | 428,48 | 0,05 | 2,41 | 420,85 | 0,02 |
| 2,43 | 426,08 | 0,07 | 2,41 | 418,45 | 0,03 |
| 2,44 | 423,84 | 0,12 | 2,42 | 416,21 | 0,09 |
| 2,46 | 421,78 | 0,22 | 2,44 | 414,14 | 0,19 |
| 2,61 | 431,50 | 0,19 | 2,60 | 423,87 | 0,16 |
| 2,59 | 428,74 | 0,13 | 2,58 | 421,11 | 0,10 |
| 2,59 | 426,12 | 0,09 | 2,57 | 418,49 | 0,05 |
| 2,58 | 423,64 | 0,07 | 2,57 | 416,01 | 0,04 |
| 2,59 | 421,33 | 0,09 | 2,58 | 413,70 | 0,06 |
| 2,61 | 419,18 | 0,16 | 2,60 | 411,55 | 0,12 |
| 2,82 | 429,54 | 0,38 | 2,81 | 421,91 | 0,34 |
| 2,81 | 426,70 | 0,29 | 2,79 | 419,07 | 0,25 |
| 2,79 | 424,00 | 0,21 | 2,78 | 416,37 | 0,18 |
| 2,79 | 421,44 | 0,16 | 2,78 | 413,81 | 0,13 |
| 2,80 | 419,05 | 0,14 | 2,78 | 411,42 | 0,11 |
| 2,81 | 416,83 | 0,17 | 2,80 | 409,20 | 0,13 |
| 3,10 | 427,81 | 0,69 | 3,09 | 420,18 | 0,65 |
| 3,08 | 424,90 | 0,57 | 3,07 | 417,27 | 0,53 |
| 3,07 | 422,12 | 0,46 | 3,05 | 414,49 | 0,42 |
| 3,06 | 419,49 | 0,37 | 3,05 | 411,86 | 0,34 |
| 3,06 | 417,02 | 0,31 | 3,05 | 409,39 | 0,27 |
| 3,08 | 414,72 | 0,28 | 3,06 | 407,09 | 0,25 |
| 2,40 | 436,74 | 0,28 | 2,38 | 428,87 | 0,16 |
| 2,39 | 434,21 | 0,30 | 2,37 | 426,34 | 0,17 |
| 2,39 | 431,82 | 0,33 | 2,37 | 423,95 | 0,21 |
| 2,40 | 429,57 | 0,40 | 2,38 | 421,71 | 0,28 |
| 2,41 | 427,49 | 0,52 | 2,39 | 419,62 | 0,40 |
| 2,44 | 425,57 | 0,69 | 2,42 | 417,71 | 0,57 |
| 2,49 | 434,01 | 0,21 | 2,47 | 426,15 | 0,09 |
| 2,48 | 431,40 | 0,20 | 2,46 | 423,54 | 0,08 |
| 2,48 | 428,94 | 0,21 | 2,46 | 421,07 | 0,09 |
| 2,48 | 426,62 | 0,26 | 2,46 | 418,75 | 0,14 |
| 2,49 | 424,45 | 0,34 | 2,47 | 416,59 | 0,22 |
| 2,52 | 422,46 | 0,48 | 2,50 | 414,60 | 0,36 |
| 2,61 | 431,50 | 0,19 | 2,59 | 423,64 | 0,07 |
| 2,60 | 428,82 | 0,16 | 2,58 | 420,96 | 0,03 |
| 2,60 | 426,27 | 0,14 | 2,57 | 418,41 | 0,02 |
| 2,60 | 423,87 | 0,16 | 2,58 | 416,01 | 0,04 |
| 2,61 | 421,64 | 0,21 | 2,59 | 413,77 | 0,09 |
| 2,63 | 419,57 | 0,32 | 2,61 | 411,71 | 0,20 |
| 2,77 | 429,22 | 0,24 | 2,75 | 421,36 | 0,12 |
| 2,76 | 426,46 | 0,18 | 2,74 | 418,60 | 0,06 |
| 2,75 | 423,84 | 0,14 | 2,73 | 415,97 | 0,02 |
| 2,75 | 421,36 | 0,12 | 2,73 | 413,50 | 0,00 |
| 2,76 | 419,04 | 0,14 | 2,73 | 411,18 | 0,02 |
| 2,77 | 416,90 | 0,21 | 2,75 | 409,04 | 0,09 |
| 2,98 | 427,18 | 0,38 | 2,96 | 419,31 | 0,26 |
| 2,96 | 424,34 | 0,29 | 2,94 | 416,48 | 0,17 |
| 2,95 | 421,64 | 0,22 | 2,93 | 413,77 | 0,10 |
| 2,95 | 419,08 | 0,17 | 2,93 | 411,22 | 0,04 |
| 2,95 | 416,69 | 0,15 | 2,93 | 408,83 | 0,03 |
| 2,97 | 414,47 | 0,17 | 2,95 | 406,61 | 0,05 |
| 3,25 | 425,38 | 0,64 | 3,23 | 417,51 | 0,52 |
| 3,23 | 422,46 | 0,52 | 3,21 | 414,60 | 0,40 |
| 3,22 | 419,69 | 0,41 | 3,20 | 411,82 | 0,29 |
| 3,21 | 417,06 | 0,32 | 3,19 | 409,19 | 0,20 |
| 3,21 | 414,59 | 0,26 | 3,19 | 406,72 | 0,14 |
| 3,23 | 412,29 | 0,24 | 3,21 | 404,42 | 0,11 |
| 2,63 | 434,92 | 0,55 | 2,60 | 426,83 | 0,33 |
| 2,62 | 432,39 | 0,56 | 2,60 | 424,30 | 0,34 |
| 2,62 | 430,00 | 0,60 | 2,59 | 421,91 | 0,38 |
| 2,63 | 427,76 | 0,67 | 2,60 | 419,66 | 0,45 |
| 2,64 | 425,67 | 0,78 | 2,61 | 417,58 | 0,57 |
| 2,67 | 423,75 | 0,95 | 2,64 | 415,66 | 0,74 |
| 2,72 | 432,12 | 0,44 | 2,69 | 424,03 | 0,22 |
| 2,71 | 429,51 | 0,43 | 2,68 | 421,42 | 0,21 |

|      |        |      |      |        |      |
|------|--------|------|------|--------|------|
| 2,70 | 427,04 | 0,44 | 2,67 | 418,95 | 0,22 |
| 2,70 | 424,72 | 0,49 | 2,68 | 416,63 | 0,27 |
| 2,72 | 422,56 | 0,57 | 2,69 | 414,47 | 0,35 |
| 2,74 | 420,57 | 0,71 | 2,71 | 412,48 | 0,49 |
| 2,83 | 429,54 | 0,38 | 2,80 | 421,44 | 0,16 |
| 2,82 | 426,85 | 0,34 | 2,79 | 418,76 | 0,13 |
| 2,81 | 424,30 | 0,33 | 2,79 | 416,21 | 0,11 |
| 2,81 | 421,91 | 0,34 | 2,79 | 413,81 | 0,13 |
| 2,82 | 419,67 | 0,40 | 2,80 | 411,58 | 0,18 |
| 2,85 | 417,60 | 0,50 | 2,82 | 409,51 | 0,29 |
| 2,99 | 427,18 | 0,38 | 2,96 | 419,08 | 0,17 |
| 2,97 | 424,41 | 0,32 | 2,94 | 416,32 | 0,10 |
| 2,96 | 421,79 | 0,28 | 2,93 | 413,70 | 0,06 |
| 2,96 | 419,31 | 0,26 | 2,93 | 411,22 | 0,04 |
| 2,97 | 417,00 | 0,28 | 2,94 | 408,91 | 0,06 |
| 2,99 | 414,85 | 0,35 | 2,96 | 406,76 | 0,13 |
| 3,19 | 425,05 | 0,47 | 3,16 | 416,96 | 0,25 |
| 3,17 | 422,21 | 0,38 | 3,14 | 414,12 | 0,16 |
| 3,16 | 419,51 | 0,31 | 3,13 | 411,42 | 0,09 |
| 3,15 | 416,96 | 0,25 | 3,13 | 408,87 | 0,04 |
| 3,16 | 414,56 | 0,24 | 3,13 | 406,47 | 0,02 |
| 3,18 | 412,34 | 0,26 | 3,15 | 404,25 | 0,04 |
| 3,45 | 423,17 | 0,67 | 3,42 | 415,08 | 0,46 |
| 3,43 | 420,26 | 0,55 | 3,40 | 412,17 | 0,33 |
| 3,42 | 417,49 | 0,44 | 3,39 | 409,39 | 0,23 |
| 3,41 | 414,85 | 0,35 | 3,38 | 406,76 | 0,14 |
| 3,42 | 412,38 | 0,29 | 3,39 | 404,29 | 0,08 |
| 3,43 | 410,08 | 0,27 | 3,40 | 401,99 | 0,05 |
| 2,92 | 433,34 | 0,95 | 2,89 | 425,03 | 0,63 |
| 2,92 | 430,81 | 0,96 | 2,88 | 422,50 | 0,64 |
| 2,92 | 428,42 | 1,00 | 2,88 | 420,11 | 0,68 |
| 2,92 | 426,18 | 1,07 | 2,88 | 417,86 | 0,75 |
| 2,93 | 424,09 | 1,18 | 2,90 | 415,78 | 0,86 |
| 2,96 | 422,18 | 1,35 | 2,92 | 413,86 | 1,03 |
| 3,00 | 430,47 | 0,79 | 2,97 | 422,15 | 0,48 |
| 2,99 | 427,86 | 0,78 | 2,96 | 419,55 | 0,46 |
| 2,99 | 425,40 | 0,80 | 2,95 | 417,08 | 0,48 |
| 2,99 | 423,08 | 0,84 | 2,96 | 414,76 | 0,52 |
| 3,00 | 420,91 | 0,93 | 2,97 | 412,60 | 0,61 |
| 3,03 | 418,92 | 1,07 | 2,99 | 410,61 | 0,75 |
| 3,11 | 427,81 | 0,69 | 3,08 | 419,49 | 0,37 |
| 3,10 | 425,13 | 0,65 | 3,07 | 416,81 | 0,33 |
| 3,09 | 422,58 | 0,64 | 3,06 | 414,26 | 0,32 |
| 3,10 | 420,18 | 0,65 | 3,06 | 411,86 | 0,34 |
| 3,11 | 417,94 | 0,71 | 3,07 | 409,63 | 0,39 |
| 3,13 | 415,88 | 0,81 | 3,09 | 407,56 | 0,49 |
| 3,26 | 425,38 | 0,64 | 3,23 | 417,06 | 0,32 |
| 3,25 | 422,61 | 0,58 | 3,21 | 414,29 | 0,26 |
| 3,24 | 419,99 | 0,54 | 3,20 | 411,67 | 0,22 |
| 3,24 | 417,51 | 0,52 | 3,20 | 409,19 | 0,20 |
| 3,24 | 415,20 | 0,54 | 3,21 | 406,88 | 0,22 |
| 3,26 | 413,05 | 0,61 | 3,23 | 404,74 | 0,29 |
| 3,46 | 423,17 | 0,67 | 3,42 | 414,85 | 0,35 |
| 3,44 | 420,34 | 0,58 | 3,41 | 412,02 | 0,26 |
| 3,43 | 417,63 | 0,51 | 3,39 | 409,32 | 0,19 |
| 3,43 | 415,08 | 0,46 | 3,39 | 406,76 | 0,14 |
| 3,43 | 412,69 | 0,44 | 3,40 | 404,37 | 0,12 |
| 3,45 | 410,47 | 0,46 | 3,41 | 402,15 | 0,14 |
| 3,72 | 421,22 | 0,81 | 3,68 | 412,90 | 0,49 |
| 3,70 | 418,31 | 0,69 | 3,66 | 409,99 | 0,37 |
| 3,69 | 415,53 | 0,58 | 3,65 | 407,21 | 0,26 |
| 3,68 | 412,90 | 0,49 | 3,64 | 404,58 | 0,17 |
| 3,68 | 410,43 | 0,43 | 3,65 | 402,11 | 0,11 |
| 3,70 | 408,13 | 0,41 | 3,66 | 399,81 | 0,09 |
| 1,96 | 441,02 | 0,01 | 1,98 | 434,30 | 0,24 |
| 1,95 | 438,49 | 0,02 | 1,97 | 431,77 | 0,25 |
| 1,95 | 436,10 | 0,06 | 1,97 | 429,38 | 0,29 |
| 1,95 | 433,86 | 0,13 | 1,98 | 427,14 | 0,36 |
| 1,97 | 431,77 | 0,25 | 1,99 | 425,05 | 0,47 |
| 1,99 | 429,85 | 0,42 | 2,02 | 423,13 | 0,64 |
| 2,07 | 438,53 | 0,04 | 2,09 | 431,81 | 0,26 |
| 2,06 | 435,92 | 0,03 | 2,08 | 429,20 | 0,25 |
| 2,05 | 433,45 | 0,04 | 2,08 | 426,73 | 0,26 |
| 2,06 | 431,13 | 0,09 | 2,08 | 424,41 | 0,31 |
| 2,07 | 428,97 | 0,17 | 2,09 | 422,25 | 0,39 |
| 2,09 | 426,98 | 0,31 | 2,12 | 420,26 | 0,53 |
| 2,21 | 436,26 | 0,13 | 2,23 | 429,54 | 0,35 |
| 2,19 | 433,57 | 0,09 | 2,22 | 426,85 | 0,32 |
| 2,19 | 431,02 | 0,08 | 2,21 | 424,30 | 0,30 |
| 2,19 | 428,63 | 0,09 | 2,21 | 421,91 | 0,32 |
| 2,20 | 426,39 | 0,15 | 2,22 | 419,67 | 0,37 |
| 2,22 | 424,32 | 0,25 | 2,24 | 417,60 | 0,48 |
| 2,38 | 434,21 | 0,30 | 2,41 | 427,49 | 0,52 |
| 2,37 | 431,44 | 0,23 | 2,39 | 424,72 | 0,46 |
| 2,36 | 428,82 | 0,19 | 2,38 | 422,10 | 0,41 |

|      |        |      |      |        |      |
|------|--------|------|------|--------|------|
| 2,36 | 426,34 | 0,17 | 2,38 | 419,62 | 0,40 |
| 2,37 | 424,03 | 0,19 | 2,39 | 417,31 | 0,42 |
| 2,38 | 421,88 | 0,26 | 2,41 | 415,16 | 0,48 |
| 2,61 | 432,39 | 0,56 | 2,63 | 425,67 | 0,78 |
| 2,59 | 429,55 | 0,47 | 2,61 | 422,83 | 0,69 |
| 2,58 | 426,85 | 0,40 | 2,60 | 420,13 | 0,62 |
| 2,58 | 424,30 | 0,34 | 2,60 | 417,58 | 0,57 |
| 2,58 | 421,90 | 0,33 | 2,60 | 415,18 | 0,55 |
| 2,60 | 419,68 | 0,35 | 2,62 | 412,96 | 0,57 |
| 2,90 | 430,81 | 0,96 | 2,92 | 424,09 | 1,18 |
| 2,88 | 427,90 | 0,83 | 2,90 | 421,18 | 1,06 |
| 2,86 | 425,13 | 0,73 | 2,88 | 418,41 | 0,95 |
| 2,86 | 422,50 | 0,64 | 2,88 | 415,78 | 0,86 |
| 2,86 | 420,02 | 0,58 | 2,88 | 413,31 | 0,80 |
| 2,87 | 417,73 | 0,55 | 2,90 | 411,01 | 0,78 |
| 2,07 | 438,45 | 0,02 | 2,09 | 431,50 | 0,16 |
| 2,06 | 435,92 | 0,03 | 2,08 | 428,97 | 0,17 |
| 2,06 | 433,53 | 0,07 | 2,08 | 426,58 | 0,21 |
| 2,06 | 431,29 | 0,14 | 2,08 | 424,34 | 0,28 |
| 2,08 | 429,20 | 0,25 | 2,10 | 422,25 | 0,39 |
| 2,10 | 427,29 | 0,42 | 2,12 | 420,34 | 0,56 |
| 2,17 | 435,88 | 0,01 | 2,19 | 428,93 | 0,16 |
| 2,16 | 433,28 | 0,00 | 2,18 | 426,33 | 0,14 |
| 2,16 | 430,81 | 0,01 | 2,17 | 423,86 | 0,16 |
| 2,16 | 428,49 | 0,06 | 2,18 | 421,54 | 0,20 |
| 2,17 | 426,33 | 0,14 | 2,19 | 419,38 | 0,29 |
| 2,20 | 424,33 | 0,28 | 2,21 | 417,38 | 0,43 |
| 2,31 | 433,53 | 0,07 | 2,32 | 426,58 | 0,21 |
| 2,29 | 430,85 | 0,03 | 2,31 | 423,90 | 0,17 |
| 2,29 | 428,30 | 0,02 | 2,30 | 421,35 | 0,16 |
| 2,29 | 425,90 | 0,03 | 2,30 | 418,95 | 0,18 |
| 2,30 | 423,66 | 0,09 | 2,31 | 416,71 | 0,23 |
| 2,32 | 421,60 | 0,19 | 2,34 | 414,65 | 0,34 |
| 2,48 | 431,40 | 0,20 | 2,49 | 424,45 | 0,34 |
| 2,46 | 428,64 | 0,14 | 2,48 | 421,69 | 0,28 |
| 2,45 | 426,02 | 0,09 | 2,47 | 419,07 | 0,24 |
| 2,45 | 423,54 | 0,08 | 2,47 | 416,59 | 0,22 |
| 2,46 | 421,23 | 0,10 | 2,47 | 414,28 | 0,24 |
| 2,48 | 419,08 | 0,16 | 2,49 | 412,13 | 0,31 |
| 2,69 | 429,51 | 0,43 | 2,71 | 422,56 | 0,57 |
| 2,68 | 426,67 | 0,34 | 2,69 | 419,72 | 0,48 |
| 2,67 | 423,97 | 0,26 | 2,68 | 417,02 | 0,40 |
| 2,66 | 421,42 | 0,21 | 2,68 | 414,47 | 0,35 |
| 2,67 | 419,03 | 0,19 | 2,68 | 412,08 | 0,34 |
| 2,68 | 416,81 | 0,22 | 2,70 | 409,86 | 0,36 |
| 2,98 | 427,86 | 0,78 | 2,99 | 420,91 | 0,93 |
| 2,96 | 424,95 | 0,66 | 2,97 | 418,00 | 0,80 |
| 2,94 | 422,18 | 0,55 | 2,96 | 415,23 | 0,70 |
| 2,94 | 419,55 | 0,46 | 2,95 | 412,60 | 0,61 |
| 2,94 | 417,07 | 0,40 | 2,96 | 410,12 | 0,55 |
| 2,96 | 414,78 | 0,38 | 2,97 | 407,83 | 0,52 |
| 2,21 | 436,10 | 0,08 | 2,22 | 428,92 | 0,14 |
| 2,20 | 433,57 | 0,09 | 2,21 | 426,39 | 0,15 |
| 2,20 | 431,18 | 0,13 | 2,21 | 424,00 | 0,19 |
| 2,20 | 428,94 | 0,20 | 2,22 | 421,75 | 0,26 |
| 2,22 | 426,85 | 0,32 | 2,23 | 419,67 | 0,37 |
| 2,24 | 424,93 | 0,49 | 2,26 | 417,75 | 0,54 |
| 2,31 | 433,45 | 0,04 | 2,32 | 426,27 | 0,10 |
| 2,30 | 430,85 | 0,03 | 2,31 | 423,66 | 0,09 |
| 2,29 | 428,38 | 0,04 | 2,30 | 421,19 | 0,10 |
| 2,30 | 426,06 | 0,09 | 2,31 | 418,87 | 0,15 |
| 2,31 | 423,90 | 0,17 | 2,32 | 416,71 | 0,23 |
| 2,33 | 421,90 | 0,31 | 2,34 | 414,72 | 0,37 |
| 2,43 | 431,02 | 0,06 | 2,44 | 423,84 | 0,12 |
| 2,42 | 428,34 | 0,03 | 2,43 | 421,16 | 0,08 |
| 2,41 | 425,79 | 0,01 | 2,43 | 418,61 | 0,07 |
| 2,42 | 423,39 | 0,03 | 2,43 | 416,21 | 0,09 |
| 2,43 | 421,16 | 0,08 | 2,44 | 413,97 | 0,14 |
| 2,45 | 419,09 | 0,19 | 2,46 | 411,91 | 0,24 |
| 2,60 | 428,82 | 0,16 | 2,61 | 421,64 | 0,21 |
| 2,58 | 426,06 | 0,09 | 2,59 | 418,87 | 0,15 |
| 2,57 | 423,43 | 0,05 | 2,58 | 416,25 | 0,11 |
| 2,57 | 420,96 | 0,03 | 2,58 | 413,77 | 0,09 |
| 2,58 | 418,64 | 0,05 | 2,59 | 411,46 | 0,11 |
| 2,60 | 416,50 | 0,12 | 2,61 | 409,32 | 0,18 |
| 2,81 | 426,85 | 0,34 | 2,82 | 419,67 | 0,40 |
| 2,79 | 424,01 | 0,25 | 2,80 | 416,83 | 0,31 |
| 2,78 | 421,31 | 0,18 | 2,79 | 414,13 | 0,23 |
| 2,78 | 418,76 | 0,13 | 2,79 | 411,58 | 0,18 |
| 2,78 | 416,36 | 0,11 | 2,79 | 409,18 | 0,16 |
| 2,80 | 414,14 | 0,13 | 2,81 | 406,96 | 0,19 |
| 3,09 | 425,13 | 0,65 | 3,10 | 417,94 | 0,71 |
| 3,07 | 422,22 | 0,53 | 3,08 | 415,03 | 0,59 |
| 3,05 | 419,44 | 0,42 | 3,06 | 412,26 | 0,48 |
| 3,05 | 416,81 | 0,33 | 3,06 | 409,63 | 0,39 |

|      |        |      |      |        |      |
|------|--------|------|------|--------|------|
| 3,05 | 414,34 | 0,27 | 3,06 | 407,15 | 0,33 |
| 3,07 | 412,04 | 0,25 | 3,08 | 404,86 | 0,30 |
| 2,39 | 433,97 | 0,22 | 2,39 | 426,56 | 0,18 |
| 2,38 | 431,44 | 0,23 | 2,38 | 424,03 | 0,19 |
| 2,38 | 429,05 | 0,27 | 2,38 | 421,64 | 0,23 |
| 2,38 | 426,81 | 0,34 | 2,39 | 419,39 | 0,30 |
| 2,40 | 424,72 | 0,46 | 2,40 | 417,31 | 0,42 |
| 2,42 | 422,81 | 0,63 | 2,43 | 415,39 | 0,59 |
| 2,48 | 431,25 | 0,15 | 2,48 | 423,83 | 0,11 |
| 2,47 | 428,64 | 0,14 | 2,47 | 421,23 | 0,10 |
| 2,47 | 426,17 | 0,15 | 2,47 | 418,76 | 0,11 |
| 2,47 | 423,85 | 0,19 | 2,47 | 416,44 | 0,16 |
| 2,48 | 421,69 | 0,28 | 2,48 | 414,28 | 0,24 |
| 2,50 | 419,70 | 0,42 | 2,51 | 412,29 | 0,38 |
| 2,60 | 428,74 | 0,13 | 2,60 | 421,33 | 0,09 |
| 2,59 | 426,06 | 0,09 | 2,59 | 418,64 | 0,05 |
| 2,58 | 423,51 | 0,08 | 2,58 | 416,09 | 0,04 |
| 2,58 | 421,11 | 0,10 | 2,59 | 413,70 | 0,06 |
| 2,59 | 418,87 | 0,15 | 2,60 | 411,46 | 0,11 |
| 2,61 | 416,81 | 0,25 | 2,62 | 409,39 | 0,22 |
| 2,76 | 426,46 | 0,18 | 2,76 | 419,04 | 0,14 |
| 2,74 | 423,70 | 0,12 | 2,75 | 416,28 | 0,08 |
| 2,74 | 421,07 | 0,07 | 2,74 | 413,66 | 0,04 |
| 2,73 | 418,60 | 0,06 | 2,74 | 411,18 | 0,02 |
| 2,74 | 416,28 | 0,08 | 2,74 | 408,87 | 0,04 |
| 2,76 | 414,14 | 0,14 | 2,76 | 406,72 | 0,11 |
| 2,97 | 424,41 | 0,32 | 2,97 | 417,00 | 0,28 |
| 2,95 | 421,58 | 0,23 | 2,95 | 414,16 | 0,19 |
| 2,94 | 418,87 | 0,15 | 2,94 | 411,46 | 0,11 |
| 2,93 | 416,32 | 0,10 | 2,94 | 408,91 | 0,06 |
| 2,94 | 413,93 | 0,08 | 2,94 | 406,51 | 0,04 |
| 2,96 | 411,71 | 0,11 | 2,96 | 404,29 | 0,07 |
| 3,24 | 422,61 | 0,58 | 3,24 | 415,2  | 0,54 |
| 3,22 | 419,70 | 0,46 | 3,22 | 412,29 | 0,42 |
| 3,20 | 416,92 | 0,35 | 3,21 | 409,51 | 0,31 |
| 3,20 | 414,29 | 0,26 | 3,2  | 406,88 | 0,22 |
| 3,20 | 411,82 | 0,20 | 3,2  | 404,41 | 0,16 |
| 3,22 | 409,52 | 0,17 | 3,22 | 402,11 | 0,13 |
| 2,62 | 432,08 | 0,46 | 2,61 | 424,43 | 0,31 |
| 2,61 | 429,55 | 0,47 | 2,6  | 421,9  | 0,33 |
| 2,61 | 427,16 | 0,51 | 2,6  | 419,51 | 0,36 |
| 2,61 | 424,92 | 0,58 | 2,61 | 417,27 | 0,44 |
| 2,62 | 422,83 | 0,69 | 2,62 | 415,18 | 0,55 |
| 2,65 | 420,91 | 0,86 | 2,65 | 413,27 | 0,72 |
| 2,70 | 429,28 | 0,35 | 2,7  | 421,63 | 0,2  |
| 2,69 | 426,67 | 0,34 | 2,69 | 419,03 | 0,19 |
| 2,69 | 424,21 | 0,35 | 2,68 | 416,56 | 0,2  |
| 2,69 | 421,89 | 0,39 | 2,69 | 414,24 | 0,25 |
| 2,70 | 419,72 | 0,48 | 2,7  | 412,08 | 0,34 |
| 2,73 | 417,73 | 0,62 | 2,72 | 410,09 | 0,47 |
| 2,82 | 426,70 | 0,29 | 2,81 | 419,05 | 0,14 |
| 2,80 | 424,01 | 0,25 | 2,8  | 416,36 | 0,11 |
| 2,80 | 421,47 | 0,24 | 2,79 | 413,82 | 0,09 |
| 2,80 | 419,07 | 0,25 | 2,79 | 411,42 | 0,11 |
| 2,81 | 416,83 | 0,31 | 2,8  | 409,18 | 0,16 |
| 2,83 | 414,76 | 0,41 | 2,83 | 407,12 | 0,27 |
| 2,97 | 424,34 | 0,29 | 2,96 | 416,69 | 0,15 |
| 2,95 | 421,58 | 0,23 | 2,95 | 413,93 | 0,08 |
| 2,95 | 418,95 | 0,19 | 2,94 | 411,3  | 0,04 |
| 2,94 | 416,48 | 0,17 | 2,94 | 408,83 | 0,03 |
| 2,95 | 414,16 | 0,19 | 2,95 | 406,51 | 0,04 |
| 2,97 | 412,02 | 0,26 | 2,97 | 404,37 | 0,11 |
| 3,17 | 422,21 | 0,38 | 3,17 | 414,56 | 0,24 |
| 3,15 | 419,38 | 0,29 | 3,15 | 411,73 | 0,14 |
| 3,14 | 416,67 | 0,21 | 3,14 | 409,03 | 0,07 |
| 3,14 | 414,12 | 0,16 | 3,13 | 406,47 | 0,02 |
| 3,14 | 411,73 | 0,14 | 3,14 | 404,08 | 0    |
| 3,16 | 409,51 | 0,17 | 3,16 | 401,86 | 0,02 |
| 3,44 | 420,34 | 0,58 | 3,43 | 412,69 | 0,44 |
| 3,42 | 417,42 | 0,46 | 3,41 | 409,78 | 0,32 |
| 3,40 | 414,65 | 0,35 | 3,4  | 407    | 0,21 |
| 3,40 | 412,02 | 0,26 | 3,39 | 404,37 | 0,12 |
| 3,40 | 409,55 | 0,20 | 3,39 | 401,9  | 0,06 |
| 3,42 | 407,25 | 0,18 | 3,41 | 399,6  | 0,03 |
| 2,91 | 430,43 | 0,82 | 2,9  | 422,56 | 0,56 |
| 2,90 | 427,90 | 0,83 | 2,89 | 420,02 | 0,58 |
| 2,90 | 425,51 | 0,87 | 2,88 | 417,63 | 0,61 |
| 2,90 | 423,27 | 0,94 | 2,89 | 415,39 | 0,69 |
| 2,92 | 421,18 | 1,06 | 2,9  | 413,31 | 0,8  |
| 2,94 | 419,27 | 1,23 | 2,93 | 411,39 | 0,97 |
| 2,99 | 427,56 | 0,67 | 2,97 | 419,68 | 0,41 |
| 2,98 | 424,95 | 0,66 | 2,96 | 417,07 | 0,4  |
| 2,97 | 422,48 | 0,67 | 2,96 | 414,61 | 0,41 |
| 2,97 | 420,16 | 0,72 | 2,96 | 412,29 | 0,46 |
| 2,99 | 418,00 | 0,80 | 2,97 | 410,12 | 0,55 |

|      |        |      |      |        |      |
|------|--------|------|------|--------|------|
| 3,01 | 416,01 | 0,94 | 3    | 408,13 | 0,68 |
| 3,10 | 424,90 | 0,57 | 3,08 | 417,02 | 0,31 |
| 3,08 | 422,22 | 0,53 | 3,07 | 414,34 | 0,27 |
| 3,08 | 419,67 | 0,52 | 3,06 | 411,79 | 0,26 |
| 3,08 | 417,27 | 0,53 | 3,07 | 409,39 | 0,27 |
| 3,09 | 415,03 | 0,59 | 3,08 | 407,15 | 0,33 |
| 3,11 | 412,97 | 0,69 | 3,1  | 405,09 | 0,43 |
| 3,24 | 422,46 | 0,52 | 3,23 | 414,59 | 0,26 |
| 3,23 | 419,70 | 0,46 | 3,22 | 411,82 | 0,2  |
| 3,22 | 417,08 | 0,41 | 3,21 | 409,2  | 0,15 |
| 3,22 | 414,60 | 0,40 | 3,21 | 406,72 | 0,14 |
| 3,23 | 412,29 | 0,42 | 3,21 | 404,41 | 0,16 |
| 3,24 | 410,14 | 0,48 | 3,23 | 402,26 | 0,23 |
| 3,44 | 420,26 | 0,55 | 3,43 | 412,38 | 0,29 |
| 3,42 | 417,42 | 0,46 | 3,41 | 409,55 | 0,2  |
| 3,41 | 414,72 | 0,38 | 3,4  | 406,84 | 0,13 |
| 3,41 | 412,17 | 0,33 | 3,39 | 404,29 | 0,08 |
| 3,41 | 409,78 | 0,32 | 3,4  | 401,9  | 0,06 |
| 3,43 | 407,55 | 0,34 | 3,42 | 399,68 | 0,08 |
| 3,70 | 418,31 | 0,69 | 3,69 | 410,43 | 0,43 |
| 3,68 | 415,39 | 0,57 | 3,67 | 407,52 | 0,31 |
| 3,67 | 412,62 | 0,46 | 3,65 | 404,74 | 0,2  |
| 3,66 | 409,99 | 0,37 | 3,65 | 402,11 | 0,11 |
| 3,66 | 407,52 | 0,31 | 3,65 | 399,64 | 0,05 |
| 3,68 | 405,22 | 0,28 | 3,67 | 397,34 | 0,02 |
| 1,96 | 438,63 | 0,05 | 2,01 | 432,38 | 0,41 |
| 1,95 | 436,10 | 0,06 | 2    | 429,85 | 0,42 |
| 1,95 | 433,71 | 0,10 | 2    | 427,46 | 0,46 |
| 1,95 | 431,47 | 0,17 | 2,01 | 425,22 | 0,53 |
| 1,97 | 429,38 | 0,29 | 2,02 | 423,13 | 0,64 |
| 1,99 | 427,46 | 0,46 | 2,05 | 421,22 | 0,81 |
| 2,07 | 436,14 | 0,08 | 2,12 | 429,89 | 0,43 |
| 2,06 | 433,53 | 0,07 | 2,11 | 427,29 | 0,42 |
| 2,05 | 431,06 | 0,08 | 2,11 | 424,82 | 0,43 |
| 2,06 | 428,74 | 0,12 | 2,11 | 422,5  | 0,48 |
| 2,07 | 426,58 | 0,21 | 2,12 | 420,34 | 0,56 |
| 2,09 | 424,59 | 0,35 | 2,14 | 418,34 | 0,7  |
| 2,21 | 433,86 | 0,17 | 2,26 | 427,62 | 0,52 |
| 2,19 | 431,18 | 0,13 | 2,25 | 424,93 | 0,49 |
| 2,19 | 428,63 | 0,12 | 2,24 | 422,39 | 0,47 |
| 2,19 | 426,23 | 0,13 | 2,24 | 419,99 | 0,49 |
| 2,20 | 424,00 | 0,19 | 2,25 | 417,75 | 0,54 |
| 2,22 | 421,93 | 0,29 | 2,27 | 415,68 | 0,65 |
| 2,38 | 431,82 | 0,33 | 2,43 | 425,57 | 0,69 |
| 2,37 | 429,05 | 0,27 | 2,42 | 422,81 | 0,63 |
| 2,36 | 426,43 | 0,23 | 2,41 | 420,18 | 0,58 |
| 2,36 | 423,95 | 0,21 | 2,41 | 417,71 | 0,57 |
| 2,36 | 421,64 | 0,23 | 2,42 | 415,39 | 0,59 |
| 2,38 | 419,49 | 0,30 | 2,44 | 413,25 | 0,65 |
| 2,61 | 430,00 | 0,60 | 2,66 | 423,75 | 0,95 |
| 2,59 | 427,16 | 0,51 | 2,64 | 420,91 | 0,86 |
| 2,58 | 424,46 | 0,43 | 2,63 | 418,21 | 0,79 |
| 2,57 | 421,91 | 0,38 | 2,63 | 415,66 | 0,74 |
| 2,58 | 419,51 | 0,36 | 2,63 | 413,27 | 0,72 |
| 2,60 | 417,29 | 0,39 | 2,65 | 411,05 | 0,74 |
| 2,90 | 428,42 | 1,00 | 2,95 | 422,18 | 1,35 |
| 2,88 | 425,51 | 0,87 | 2,93 | 419,27 | 1,23 |
| 2,86 | 422,74 | 0,77 | 2,91 | 416,49 | 1,12 |
| 2,86 | 420,11 | 0,68 | 2,91 | 413,86 | 1,03 |
| 2,86 | 417,63 | 0,61 | 2,91 | 411,39 | 0,97 |
| 2,87 | 415,34 | 0,59 | 2,93 | 409,09 | 0,95 |
| 2,07 | 435,98 | 0,03 | 2,11 | 429,51 | 0,3  |
| 2,06 | 433,45 | 0,04 | 2,11 | 426,98 | 0,31 |
| 2,06 | 431,06 | 0,08 | 2,1  | 424,59 | 0,35 |
| 2,06 | 428,82 | 0,15 | 2,11 | 422,35 | 0,42 |
| 2,08 | 426,73 | 0,26 | 2,12 | 420,26 | 0,53 |
| 2,10 | 424,82 | 0,43 | 2,15 | 418,34 | 0,7  |
| 2,17 | 433,41 | 0,03 | 2,22 | 426,94 | 0,29 |
| 2,16 | 430,81 | 0,01 | 2,21 | 424,33 | 0,28 |
| 2,16 | 428,34 | 0,03 | 2,2  | 421,87 | 0,3  |
| 2,16 | 426,02 | 0,07 | 2,2  | 419,55 | 0,34 |
| 2,17 | 423,86 | 0,16 | 2,22 | 417,38 | 0,43 |
| 2,19 | 421,87 | 0,30 | 2,24 | 415,39 | 0,57 |
| 2,30 | 431,06 | 0,08 | 2,35 | 424,59 | 0,35 |
| 2,29 | 428,38 | 0,04 | 2,34 | 421,9  | 0,31 |
| 2,28 | 425,83 | 0,03 | 2,33 | 419,36 | 0,3  |
| 2,28 | 423,43 | 0,05 | 2,33 | 416,96 | 0,32 |
| 2,29 | 421,19 | 0,10 | 2,34 | 414,72 | 0,37 |
| 2,32 | 419,13 | 0,21 | 2,36 | 412,66 | 0,48 |
| 2,47 | 428,94 | 0,21 | 2,52 | 422,46 | 0,48 |
| 2,46 | 426,17 | 0,15 | 2,5  | 419,7  | 0,42 |
| 2,45 | 423,55 | 0,11 | 2,5  | 417,08 | 0,38 |
| 2,45 | 421,07 | 0,09 | 2,49 | 414,6  | 0,36 |
| 2,46 | 418,76 | 0,11 | 2,5  | 412,29 | 0,38 |
| 2,47 | 416,61 | 0,18 | 2,52 | 410,14 | 0,45 |

|      |        |      |      |        |      |
|------|--------|------|------|--------|------|
| 2,69 | 427,04 | 0,44 | 2,74 | 420,57 | 0,71 |
| 2,67 | 424,21 | 0,35 | 2,72 | 417,73 | 0,62 |
| 2,66 | 421,50 | 0,27 | 2,71 | 415,03 | 0,54 |
| 2,66 | 418,95 | 0,22 | 2,7  | 412,48 | 0,49 |
| 2,66 | 416,56 | 0,20 | 2,71 | 410,09 | 0,47 |
| 2,68 | 414,34 | 0,23 | 2,73 | 407,87 | 0,5  |
| 2,97 | 425,40 | 0,80 | 3,02 | 418,92 | 1,07 |
| 2,95 | 422,48 | 0,67 | 3    | 416,01 | 0,94 |
| 2,94 | 419,71 | 0,57 | 2,99 | 413,24 | 0,84 |
| 2,93 | 417,08 | 0,48 | 2,98 | 410,61 | 0,75 |
| 2,94 | 414,61 | 0,41 | 2,98 | 408,13 | 0,68 |
| 2,95 | 412,31 | 0,39 | 3    | 405,84 | 0,66 |
| 2,21 | 433,55 | 0,07 | 2,25 | 426,85 | 0,24 |
| 2,20 | 431,02 | 0,08 | 2,24 | 424,32 | 0,25 |
| 2,19 | 428,63 | 0,12 | 2,24 | 421,93 | 0,29 |
| 2,20 | 426,39 | 0,19 | 2,24 | 419,69 | 0,36 |
| 2,21 | 424,30 | 0,30 | 2,25 | 417,6  | 0,48 |
| 2,24 | 422,39 | 0,47 | 2,28 | 415,68 | 0,65 |
| 2,30 | 430,91 | 0,03 | 2,34 | 424,21 | 0,2  |
| 2,29 | 428,30 | 0,02 | 2,33 | 421,6  | 0,19 |
| 2,29 | 425,83 | 0,03 | 2,33 | 419,13 | 0,21 |
| 2,29 | 423,51 | 0,08 | 2,33 | 416,81 | 0,25 |
| 2,30 | 421,35 | 0,16 | 2,34 | 414,65 | 0,34 |
| 2,33 | 419,36 | 0,30 | 2,37 | 412,66 | 0,48 |
| 2,43 | 428,48 | 0,05 | 2,47 | 421,78 | 0,22 |
| 2,42 | 425,79 | 0,01 | 2,46 | 419,09 | 0,19 |
| 2,41 | 423,24 | 0,00 | 2,45 | 416,54 | 0,17 |
| 2,41 | 420,85 | 0,02 | 2,45 | 414,14 | 0,19 |
| 2,42 | 418,61 | 0,07 | 2,46 | 411,91 | 0,24 |
| 2,44 | 416,54 | 0,17 | 2,48 | 409,84 | 0,35 |
| 2,59 | 426,27 | 0,14 | 2,63 | 419,57 | 0,32 |
| 2,58 | 423,51 | 0,08 | 2,62 | 416,81 | 0,25 |
| 2,57 | 420,89 | 0,04 | 2,61 | 414,18 | 0,21 |
| 2,57 | 418,41 | 0,02 | 2,61 | 411,71 | 0,2  |
| 2,58 | 416,09 | 0,04 | 2,62 | 409,39 | 0,22 |
| 2,60 | 413,95 | 0,11 | 2,63 | 407,25 | 0,28 |
| 2,81 | 424,30 | 0,33 | 2,85 | 417,6  | 0,5  |
| 2,79 | 421,47 | 0,24 | 2,83 | 414,76 | 0,41 |
| 2,78 | 418,76 | 0,16 | 2,82 | 412,06 | 0,34 |
| 2,77 | 416,21 | 0,11 | 2,81 | 409,51 | 0,29 |
| 2,78 | 413,82 | 0,09 | 2,82 | 407,12 | 0,27 |
| 2,80 | 411,60 | 0,12 | 2,84 | 404,89 | 0,29 |
| 3,08 | 422,58 | 0,64 | 3,12 | 415,88 | 0,81 |
| 3,06 | 419,67 | 0,52 | 3,1  | 412,97 | 0,69 |
| 3,05 | 416,89 | 0,41 | 3,09 | 410,19 | 0,58 |
| 3,04 | 414,26 | 0,32 | 3,08 | 407,56 | 0,49 |
| 3,05 | 411,79 | 0,26 | 3,08 | 405,09 | 0,43 |
| 3,06 | 409,49 | 0,23 | 3,1  | 402,79 | 0,41 |
| 2,38 | 431,35 | 0,18 | 2,41 | 424,42 | 0,25 |
| 2,37 | 428,82 | 0,19 | 2,41 | 421,88 | 0,26 |
| 2,37 | 426,43 | 0,23 | 2,4  | 419,49 | 0,3  |
| 2,38 | 424,19 | 0,30 | 2,41 | 417,25 | 0,37 |
| 2,39 | 422,10 | 0,41 | 2,42 | 415,16 | 0,48 |
| 2,42 | 420,18 | 0,58 | 2,45 | 413,25 | 0,65 |
| 2,47 | 428,63 | 0,11 | 2,51 | 421,69 | 0,18 |
| 2,46 | 426,02 | 0,09 | 2,5  | 419,08 | 0,16 |
| 2,46 | 423,55 | 0,11 | 2,49 | 416,61 | 0,18 |
| 2,46 | 421,23 | 0,15 | 2,49 | 414,29 | 0,22 |
| 2,47 | 419,07 | 0,24 | 2,51 | 412,13 | 0,31 |
| 2,50 | 417,08 | 0,38 | 2,53 | 410,14 | 0,45 |
| 2,59 | 426,12 | 0,09 | 2,63 | 419,18 | 0,16 |
| 2,58 | 423,43 | 0,05 | 2,61 | 416,5  | 0,12 |
| 2,57 | 420,89 | 0,04 | 2,61 | 413,95 | 0,11 |
| 2,58 | 418,49 | 0,05 | 2,61 | 411,55 | 0,12 |
| 2,58 | 416,25 | 0,11 | 2,62 | 409,32 | 0,18 |
| 2,61 | 414,18 | 0,21 | 2,64 | 407,25 | 0,28 |
| 2,75 | 423,84 | 0,14 | 2,78 | 416,9  | 0,21 |
| 2,74 | 421,07 | 0,07 | 2,77 | 414,14 | 0,14 |
| 2,73 | 418,45 | 0,03 | 2,76 | 411,51 | 0,1  |
| 2,73 | 415,97 | 0,02 | 2,76 | 409,04 | 0,09 |
| 2,73 | 413,66 | 0,04 | 2,77 | 406,72 | 0,11 |
| 2,75 | 411,51 | 0,10 | 2,79 | 404,58 | 0,17 |
| 2,96 | 421,79 | 0,28 | 2,99 | 414,85 | 0,35 |
| 2,94 | 418,95 | 0,19 | 2,97 | 412,02 | 0,26 |
| 2,93 | 416,25 | 0,11 | 2,96 | 409,32 | 0,18 |
| 2,93 | 413,70 | 0,06 | 2,96 | 406,76 | 0,13 |
| 2,93 | 411,30 | 0,04 | 2,96 | 404,37 | 0,11 |
| 2,95 | 409,08 | 0,07 | 2,98 | 402,15 | 0,14 |
| 3,23 | 419,99 | 0,54 | 3,26 | 413,05 | 0,61 |
| 3,21 | 417,08 | 0,41 | 3,24 | 410,14 | 0,48 |
| 3,20 | 414,30 | 0,31 | 3,23 | 407,37 | 0,38 |
| 3,19 | 411,67 | 0,22 | 3,22 | 404,74 | 0,29 |
| 3,19 | 409,20 | 0,15 | 3,22 | 402,26 | 0,23 |
| 3,21 | 406,90 | 0,13 | 3,24 | 399,97 | 0,2  |
| 2,61 | 429,38 | 0,38 | 2,63 | 422,21 | 0,34 |

|      |        |      |      |        |      |
|------|--------|------|------|--------|------|
| 2,60 | 426,85 | 0,40 | 2,62 | 419,68 | 0,35 |
| 2,60 | 424,46 | 0,43 | 2,62 | 417,29 | 0,39 |
| 2,60 | 422,22 | 0,50 | 2,63 | 415,05 | 0,46 |
| 2,61 | 420,13 | 0,62 | 2,64 | 412,96 | 0,57 |
| 2,64 | 418,21 | 0,79 | 2,67 | 411,05 | 0,74 |
| 2,69 | 426,58 | 0,27 | 2,72 | 419,41 | 0,23 |
| 2,68 | 423,97 | 0,26 | 2,71 | 416,81 | 0,22 |
| 2,68 | 421,50 | 0,27 | 2,7  | 414,34 | 0,23 |
| 2,68 | 419,18 | 0,32 | 2,71 | 412,02 | 0,27 |
| 2,69 | 417,02 | 0,40 | 2,72 | 409,86 | 0,36 |
| 2,72 | 415,03 | 0,54 | 2,74 | 407,87 | 0,5  |
| 2,81 | 424,00 | 0,21 | 2,83 | 416,83 | 0,17 |
| 2,79 | 421,31 | 0,18 | 2,82 | 414,14 | 0,13 |
| 2,79 | 418,76 | 0,16 | 2,81 | 411,6  | 0,12 |
| 2,79 | 416,37 | 0,18 | 2,81 | 409,2  | 0,13 |
| 2,80 | 414,13 | 0,23 | 2,82 | 406,96 | 0,19 |
| 2,82 | 412,06 | 0,34 | 2,85 | 404,89 | 0,29 |
| 2,96 | 421,64 | 0,22 | 2,99 | 414,47 | 0,17 |
| 2,94 | 418,87 | 0,15 | 2,97 | 411,71 | 0,11 |
| 2,94 | 416,25 | 0,11 | 2,96 | 409,08 | 0,07 |
| 2,93 | 413,77 | 0,10 | 2,96 | 406,61 | 0,05 |
| 2,94 | 411,46 | 0,11 | 2,97 | 404,29 | 0,07 |
| 2,96 | 409,32 | 0,18 | 2,99 | 402,15 | 0,14 |
| 3,16 | 419,51 | 0,31 | 3,19 | 412,34 | 0,26 |
| 3,14 | 416,67 | 0,21 | 3,17 | 409,51 | 0,17 |
| 3,13 | 413,97 | 0,14 | 3,16 | 406,81 | 0,09 |
| 3,13 | 411,42 | 0,09 | 3,15 | 404,25 | 0,04 |
| 3,13 | 409,03 | 0,07 | 3,16 | 401,86 | 0,02 |
| 3,15 | 406,81 | 0,09 | 3,18 | 399,64 | 0,05 |
| 3,43 | 417,63 | 0,51 | 3,45 | 410,47 | 0,46 |
| 3,41 | 414,72 | 0,38 | 3,43 | 407,55 | 0,34 |
| 3,39 | 411,95 | 0,28 | 3,42 | 404,78 | 0,23 |
| 3,39 | 409,32 | 0,19 | 3,41 | 402,15 | 0,14 |
| 3,39 | 406,84 | 0,13 | 3,41 | 399,68 | 0,08 |
| 3,41 | 404,55 | 0,10 | 3,43 | 397,38 | 0,06 |
| 2,89 | 427,66 | 0,72 | 2,91 | 420,26 | 0,54 |
| 2,89 | 425,13 | 0,73 | 2,91 | 417,73 | 0,55 |
| 2,88 | 422,74 | 0,77 | 2,9  | 415,34 | 0,59 |
| 2,89 | 420,49 | 0,84 | 2,91 | 413,09 | 0,66 |
| 2,90 | 418,41 | 0,95 | 2,92 | 411,01 | 0,78 |
| 2,93 | 416,49 | 1,12 | 2,95 | 409,09 | 0,95 |
| 2,97 | 424,78 | 0,56 | 2,99 | 417,38 | 0,39 |
| 2,96 | 422,18 | 0,55 | 2,98 | 414,78 | 0,38 |
| 2,96 | 419,71 | 0,57 | 2,98 | 412,31 | 0,39 |
| 2,96 | 417,39 | 0,61 | 2,98 | 409,99 | 0,44 |
| 2,97 | 415,23 | 0,70 | 2,99 | 407,83 | 0,52 |
| 3,00 | 413,24 | 0,84 | 3,02 | 405,84 | 0,66 |
| 3,08 | 422,12 | 0,46 | 3,1  | 414,72 | 0,28 |
| 3,07 | 419,44 | 0,42 | 3,09 | 412,04 | 0,25 |
| 3,06 | 416,89 | 0,41 | 3,08 | 409,49 | 0,23 |
| 3,06 | 414,49 | 0,42 | 3,08 | 407,09 | 0,25 |
| 3,07 | 412,26 | 0,48 | 3,09 | 404,86 | 0,3  |
| 3,10 | 410,19 | 0,58 | 3,12 | 402,79 | 0,41 |
| 3,23 | 419,69 | 0,41 | 3,25 | 412,29 | 0,24 |
| 3,22 | 416,92 | 0,35 | 3,23 | 409,52 | 0,17 |
| 3,21 | 414,30 | 0,31 | 3,23 | 406,9  | 0,13 |
| 3,21 | 411,82 | 0,29 | 3,22 | 404,42 | 0,11 |
| 3,21 | 409,51 | 0,31 | 3,23 | 402,11 | 0,13 |
| 3,23 | 407,37 | 0,38 | 3,25 | 399,97 | 0,2  |
| 3,43 | 417,49 | 0,44 | 3,45 | 410,08 | 0,27 |
| 3,41 | 414,65 | 0,35 | 3,43 | 407,25 | 0,18 |
| 3,40 | 411,95 | 0,28 | 3,42 | 404,55 | 0,1  |
| 3,39 | 409,39 | 0,23 | 3,41 | 401,99 | 0,05 |
| 3,40 | 407,00 | 0,21 | 3,42 | 399,6  | 0,03 |
| 3,42 | 404,78 | 0,23 | 3,44 | 397,38 | 0,06 |
| 3,69 | 415,53 | 0,58 | 3,71 | 408,13 | 0,41 |
| 3,67 | 412,62 | 0,46 | 3,69 | 405,22 | 0,28 |
| 3,65 | 409,84 | 0,35 | 3,67 | 402,44 | 0,17 |
| 3,65 | 407,21 | 0,26 | 3,67 | 399,81 | 0,09 |
| 3,65 | 404,74 | 0,20 | 3,67 | 397,34 | 0,02 |
| 3,67 | 402,44 | 0,17 | 3,68 | 395,04 | 0    |

### C. Dataset of the Third Step: Integrated Approach

Tables below shows data of design and objective space data side by side.

Data is composed of three discrete optimization sessions.

Full data is available upon request.

| Division | Direction | Section | t_s  | t_m  | t_e  | b_s  | b_m  | b_e  | Mass | Force | Delta |
|----------|-----------|---------|------|------|------|------|------|------|------|-------|-------|
| 3        | 0         | 0       | -0,2 | 0,5  | -0,2 | 0,3  | -0,5 | 0    | 0    | 1     | 1     |
| 3        | 1         | 29      | -0,3 | 0,6  | -0,1 | 0,6  | -0,5 | 0,2  | 0    | 0     | 0     |
| 2        | 1         | 7       | -1   | 0,6  | -0,4 | 0    | -0,7 | 0,9  | 0    | 0     | 0     |
| 2        | 1         | 26      | 0,2  | 0,3  | -0,8 | -0,7 | -0,4 | -0,7 | 1    | 1     | 0     |
| 2        | 0         | 16      | 0,2  | 0    | 0,5  | -0,7 | 0,6  | 0,2  | 1    | 0     | 0     |
| 2        | 0         | 9       | -0,3 | 0,9  | 0,7  | -0,3 | -0,3 | 0,2  | 0    | 0     | 0     |
| 3        | 0         | 6       | -1   | 0,6  | 0,1  | -0,1 | 0    | 0,2  | 0    | 0     | 0     |
| 3        | 1         | 27      | -1   | 0,3  | -1   | -0,9 | -0,6 | 0,6  | 0    | 1     | 0     |
| 3        | 0         | 24      | -0,9 | 0,8  | 0,3  | 0,1  | -0,6 | 0,8  | 0    | 0     | 1     |
| 2        | 0         | 17      | 0    | -0,5 | 0,4  | 0,2  | -0,8 | 0,3  | 0    | 0     | 0     |
| 2        | 1         | 5       | -0,8 | 0,4  | -0,4 | 0,8  | 0,8  | -0,9 | 0    | 0     | 0     |
| 3        | 0         | 12      | 0,3  | 0,6  | -0,1 | 0,2  | 0,7  | 0,8  | 0    | 1     | 0     |
| 3        | 0         | 6       | -1   | -0,1 | -0,4 | -0,1 | -0,2 | 0,3  | 0    | 0     | 0     |
| 2        | 1         | 2       | -0,3 | 0    | -0,2 | 0    | -0,8 | 0,5  | 0    | 0     | 0     |
| 2        | 1         | 23      | -0,6 | 0    | 0    | -0,8 | 0,4  | -0,9 | 0    | 0     | 0     |
| 2        | 1         | 14      | -0,3 | -0,7 | 0,8  | 0,2  | -0,3 | 0,2  | 0    | 1     | 1     |
| 3        | 1         | 24      | 0,2  | 0,3  | -0,5 | -0,5 | -0,8 | -0,7 | 1    | 0     | 0     |
| 3        | 0         | 14      | 0,2  | 0    | 0,5  | -0,7 | 0,6  | 0,2  | 0    | 0     | 0     |
| 3        | 1         | 8       | -0,7 | 0,7  | -0,1 | 0,5  | -0,6 | -0,4 | 0    | 1     | 1     |
| 2        | 1         | 5       | 0,9  | -0,5 | -0,5 | -0,9 | -0,2 | 0,5  | 0    | 0     | 0     |
| 3        | 1         | 18      | -0,8 | -0,9 | -0,4 | -0,5 | 0    | -0,4 | 0    | 0     | 0     |
| 3        | 0         | 28      | 0,8  | 0,8  | -0,2 | 0,4  | 0,2  | 0,2  | 0    | 1     | 0     |
| 2        | 0         | 29      | 0,8  | 0,7  | 0,2  | -0,5 | -0,4 | -0,7 | 1    | 0     | 0     |
| 3        | 0         | 13      | -0,7 | -0,4 | -0,6 | 0,6  | -0,8 | -0,5 | 0    | 0     | 1     |
| 3        | 1         | 19      | 0,9  | 0,4  | -0,4 | -0,1 | -1   | -0,1 | 0    | 0     | 0     |
| 3        | 1         | 28      | 0,5  | 0,3  | -0,6 | -0,7 | -0,8 | -0,7 | 1    | 0     | 0     |
| 2        | 1         | 26      | -0,2 | -1   | 0,5  | -0,9 | -0,9 | -1   | 0    | 0     | 0     |
| 2        | 1         | 4       | 0,4  | 0,1  | 0,9  | 0    | -0,2 | -0,7 | 1    | 0     | 0     |
| 2        | 1         | 26      | 0,8  | 0,1  | 0,1  | -0,7 | 0,8  | 0,7  | 0    | 0     | 1     |
| 3        | 1         | 23      | 0    | -0,2 | 0,6  | 0,2  | -1   | 0,4  | 0    | 0     | 0     |
| 2        | 0         | 6       | 0,9  | -0,5 | 0,6  | 0,9  | 0,8  | 0,9  | 0    | 0     | 0     |
| 3        | 1         | 14      | -0,2 | -0,6 | -0,2 | 0    | 0    | -0,1 | 0    | 0     | 1     |
| 3        | 1         | 1       | -0,1 | -0,1 | -0,1 | 0,5  | -0,2 | -0,7 | 1    | 0     | 1     |
| 2        | 1         | 16      | -0,6 | -0,9 | 0,8  | -0,2 | 0,1  | 0,9  | 1    | 0     | 0     |
| 3        | 1         | 7       | 0,1  | -0,8 | -0,1 | -0,2 | -0,2 | 0,7  | 1    | 1     | 0     |
| 3        | 0         | 9       | 0,5  | -0,5 | 0,3  | 0,2  | 0    | 0,4  | 0    | 0     | 0     |
| 3        | 0         | 26      | 0,9  | 0,7  | 0,4  | -0,8 | -0,5 | 0,6  | 0    | 1     | 1     |
| 2        | 1         | 26      | 0,3  | -0,1 | 0,2  | 0,1  | 0,4  | -0,8 | 1    | 1     | 0     |
| 2        | 1         | 24      | -0,7 | 0,2  | -0,3 | -1   | -0,2 | 0,6  | 0    | 1     | 0     |
| 2        | 1         | 18      | -0,3 | -0,7 | -0,9 | -0,6 | 0,8  | 0,4  | 0    | 1     | 0     |
| 2        | 1         | 10      | -0,2 | 0,5  | -0,4 | 0,1  | -0,5 | 0    | 0    | 0     | 0     |
| 2        | 1         | 18      | -0,9 | -0,8 | -0,4 | -0,6 | -0,5 | -0,5 | 0    | 1     | 0     |
| 2        | 0         | 9       | 0,2  | -0,7 | 0,5  | 0    | -0,1 | -0,3 | 0    | 1     | 0     |
| 2        | 1         | 18      | -0,4 | -0,8 | -0,9 | 0,3  | -0,3 | -0,4 | 0    | 1     | 0     |
| 2        | 1         | 18      | -0,5 | -0,6 | 0,3  | -0,8 | 0,5  | -1   | 0    | 0     | 0     |
| 3        | 1         | 1       | -0,1 | -0,1 | -0,1 | 0,5  | -0,2 | -0,8 | 0    | 1     | 0     |
| 3        | 0         | 19      | 0    | -0,8 | -0,3 | 0,1  | -0,7 | 0,3  | 0    | 0     | 1     |
| 2        | 0         | 26      | 0,1  | 0    | 0    | 0,5  | -0,4 | 0    | 1    | 0     | 0     |
| 3        | 1         | 6       | -1   | -0,9 | -0,9 | -0,9 | 0,7  | -1   | 1    | 1     | 0     |
| 2        | 1         | 14      | -0,5 | -0,4 | 0,8  | 0,2  | -0,2 | 0,7  | 1    | 0     | 0     |
| 2        | 1         | 24      | -0,5 | -0,6 | 0,2  | 0    | 0,2  | -0,4 | 0    | 0     | 0     |
| 2        | 0         | 1       | -0,4 | 0,8  | 0,1  | 0,1  | -0,7 | 0,2  | 0    | 0     | 1     |
| 3        | 1         | 7       | -0,6 | 0,7  | -0,3 | 0,4  | -0,4 | -0,5 | 0    | 0     | 0     |
| 2        | 1         | 21      | 0    | 0,2  | -0,7 | -0,2 | -0,5 | 0    | 0    | 1     | 1     |
| 3        | 1         | 22      | -0,7 | 0,8  | -1   | 0,8  | -0,6 | -0,1 | 0    | 0     | 0     |
| 2        | 1         | 27      | -0,2 | -1   | 0,5  | -0,9 | -0,9 | -1   | 0    | 1     | 0     |
| 3        | 1         | 25      | -0,9 | -0,6 | -0,9 | -0,7 | -0,2 | -0,1 | 0    | 0     | 0     |
| 2        | 0         | 4       | -0,3 | -0,2 | 0,5  | -0,8 | 0,3  | -0,8 | 0    | 1     | 0     |
| 2        | 0         | 5       | 0,8  | -0,6 | 0,5  | 0,9  | 0,5  | 0,7  | 0    | 1     | 1     |
| 2        | 1         | 24      | -0,4 | 0    | -0,1 | 0,2  | 0,4  | 0,7  | 0    | 1     | 0     |
| 2        | 0         | 15      | -0,1 | -0,1 | 0,8  | 0,2  | -0,6 | 0,8  | 0    | 1     | 0     |
| 2        | 0         | 9       | -0,3 | 0,9  | 0,6  | -0,3 | -0,3 | 0,2  | 0    | 1     | 0     |

|   |   |    |      |      |      |      |      |      |   |   |   |
|---|---|----|------|------|------|------|------|------|---|---|---|
| 2 | 0 | 7  | 0    | -0,9 | -0,8 | 0,6  | -0,1 | -0,5 | 0 | 0 | 0 |
| 2 | 0 | 19 | 0,6  | 0    | 0,2  | -0,7 | 0,6  | 0,3  | 0 | 0 | 0 |
| 2 | 0 | 27 | 0,7  | 0,4  | -0,2 | 0,3  | 0,2  | 0    | 1 | 1 | 0 |
| 3 | 0 | 0  | -0,9 | 0,5  | -1   | -0,2 | 0,8  | -0,9 | 0 | 0 | 1 |
| 2 | 0 | 12 | 0,6  | -0,4 | -0,6 | -0,8 | -0,1 | 0,2  | 0 | 0 | 0 |
| 2 | 0 | 3  | 0    | 0,4  | -0,1 | 0,3  | -0,4 | 0,5  | 0 | 0 | 0 |
| 3 | 0 | 2  | -0,2 | 0,3  | -0,5 | 0    | 0,6  | 0,3  | 1 | 1 | 0 |
| 2 | 0 | 5  | -0,2 | 0,6  | -0,7 | 0,2  | 0,3  | -0,5 | 0 | 1 | 0 |
| 3 | 1 | 18 | -0,8 | 0,3  | -1   | -0,6 | 0    | -0,7 | 0 | 0 | 0 |
| 2 | 1 | 16 | -0,6 | -0,7 | 0,7  | -0,6 | 0,3  | 0,8  | 0 | 0 | 0 |
| 2 | 1 | 16 | -0,3 | 0,2  | -0,6 | 0,1  | 0    | -0,5 | 0 | 0 | 1 |
| 2 | 1 | 15 | -0,5 | 0,6  | -0,4 | -0,2 | -0,6 | -0,3 | 1 | 0 | 0 |
| 2 | 0 | 5  | -0,8 | 0,4  | -0,4 | 0,8  | 0,8  | -0,9 | 1 | 1 | 0 |
| 3 | 0 | 9  | 0,9  | -0,5 | -1   | -0,8 | -0,4 | 0,4  | 0 | 0 | 0 |
| 2 | 0 | 2  | -0,5 | -0,2 | -0,4 | 0,5  | -0,2 | -0,3 | 1 | 0 | 0 |
| 3 | 0 | 0  | -0,9 | 0,5  | -1   | -0,2 | 0,5  | -0,2 | 0 | 0 | 0 |
| 3 | 0 | 12 | 0,8  | 0    | -0,6 | -0,4 | 0    | 0,5  | 0 | 0 | 0 |
| 2 | 0 | 16 | -1   | 0,4  | 0    | -0,8 | -0,7 | 0,6  | 1 | 0 | 0 |
| 2 | 1 | 18 | -0,5 | -0,5 | 0,7  | -0,8 | 0,7  | -1   | 0 | 0 | 0 |
| 3 | 0 | 29 | -0,7 | -0,2 | -0,1 | -0,6 | 0,4  | -0,9 | 1 | 0 | 0 |
| 3 | 0 | 14 | 0,4  | 0,7  | 0    | 0,1  | 0,9  | 0,8  | 0 | 1 | 1 |
| 2 | 0 | 27 | 0,4  | -0,7 | 0,2  | 0,9  | 0,2  | -0,7 | 0 | 0 | 0 |
| 2 | 1 | 21 | -0,9 | -0,4 | -0,3 | -0,7 | -0,9 | -0,5 | 1 | 0 | 0 |
| 2 | 0 | 21 | 0    | -0,7 | -0,4 | 0,8  | -0,3 | -1   | 0 | 0 | 0 |
| 3 | 1 | 22 | -0,7 | 0,8  | -1   | 0,8  | -0,6 | -0,2 | 0 | 0 | 0 |
| 2 | 1 | 25 | 0,9  | -0,4 | 0,2  | -0,3 | 0    | 0    | 0 | 1 | 1 |
| 2 | 1 | 0  | -0,9 | 0,9  | 0,2  | -0,8 | -0,5 | 0,2  | 0 | 0 | 0 |
| 2 | 1 | 16 | 0,8  | -0,9 | -0,9 | 0,5  | -0,8 | 0,3  | 0 | 0 | 0 |
| 3 | 1 | 19 | 0,7  | -0,4 | -0,5 | 0,8  | -0,4 | 0,2  | 0 | 1 | 1 |
| 2 | 1 | 24 | -0,5 | 0    | -0,1 | 0,2  | 0,4  | 0,7  | 1 | 1 | 0 |
| 2 | 1 | 14 | -0,2 | -0,6 | 0,8  | 0,4  | -0,4 | 0,1  | 0 | 0 | 0 |
| 3 | 1 | 7  | 0,1  | 0,3  | -0,2 | 0,7  | -0,1 | 0,9  | 1 | 0 | 0 |
| 2 | 1 | 1  | -0,2 | 0,4  | -0,4 | 0,6  | -0,4 | 0,7  | 0 | 0 | 1 |
| 3 | 1 | 17 | -0,2 | -0,3 | 0,5  | -0,8 | 0,5  | 0,6  | 0 | 0 | 0 |
| 3 | 1 | 29 | -0,9 | 0,5  | 0,6  | 0,7  | 0,6  | 0,1  | 0 | 0 | 1 |
| 3 | 0 | 26 | 0,7  | 0,6  | 0,3  | 0,9  | -0,5 | 0,1  | 0 | 0 | 0 |
| 2 | 1 | 4  | -0,6 | -0,1 | 0,6  | 0,9  | -0,4 | 0,3  | 1 | 0 | 0 |
| 3 | 1 | 4  | 0,5  | -1   | -0,1 | 0,7  | 0,2  | 0    | 0 | 0 | 0 |
| 2 | 0 | 21 | -0,9 | -0,3 | -0,8 | -0,1 | 0,2  | 0,8  | 0 | 0 | 1 |
| 2 | 1 | 3  | -0,4 | -1   | 0,3  | -0,2 | -0,8 | 0,5  | 0 | 0 | 0 |
| 3 | 0 | 1  | 0    | 0,3  | -0,6 | 0,1  | 0,9  | 0,4  | 1 | 0 | 0 |
| 2 | 1 | 25 | -0,6 | 0,3  | -0,8 | -0,6 | 0    | -0,7 | 0 | 0 | 0 |
| 3 | 0 | 23 | 0,3  | -0,9 | -0,4 | 0,2  | -0,7 | 0,3  | 0 | 0 | 0 |
| 2 | 0 | 4  | -0,8 | 0,6  | 0,6  | 0    | -0,7 | 0,9  | 1 | 0 | 0 |
| 2 | 1 | 14 | 0,7  | 0,6  | 0,7  | -0,8 | -0,6 | -0,6 | 0 | 1 | 1 |
| 3 | 1 | 1  | -0,2 | 0,9  | -0,3 | 0,4  | -0,5 | 0    | 1 | 0 | 0 |
| 3 | 1 | 11 | -0,2 | -0,6 | -0,9 | -0,6 | 0,8  | 0,4  | 1 | 1 | 0 |
| 2 | 0 | 15 | -0,1 | -0,1 | 0,8  | 0,2  | -0,8 | 0,7  | 0 | 0 | 1 |
| 3 | 1 | 15 | -0,7 | 0    | 0,7  | 0,4  | -1   | -0,1 | 1 | 0 | 0 |
| 2 | 1 | 23 | -0,1 | 0,5  | -0,6 | -0,3 | -0,4 | 0    | 0 | 1 | 0 |
| 3 | 1 | 6  | -1   | -0,9 | -0,9 | -0,9 | 0,7  | -1   | 0 | 1 | 1 |
| 2 | 1 | 11 | -0,7 | 0,9  | 0,9  | -0,4 | -0,2 | 0,3  | 0 | 1 | 0 |
| 3 | 1 | 15 | -0,8 | 0,5  | -0,7 | 0,2  | -0,2 | 0,3  | 0 | 1 | 0 |
| 2 | 1 | 18 | -0,3 | -0,7 | -1   | 0,5  | -0,3 | -0,3 | 0 | 0 | 1 |
| 2 | 1 | 24 | -0,7 | 0,2  | -1   | -0,9 | 0,8  | 0,2  | 0 | 0 | 0 |
| 3 | 0 | 20 | 0,3  | 0,8  | -0,4 | 0,9  | -0,1 | 0,2  | 0 | 0 | 0 |
| 2 | 1 | 7  | -1   | 0,6  | -0,4 | -0,6 | 0,7  | 0,2  | 0 | 0 | 1 |
| 3 | 0 | 22 | 0,8  | -0,7 | -0,5 | 0    | 0,7  | 0,1  | 0 | 0 | 1 |
| 2 | 0 | 3  | 0,6  | -0,9 | -0,6 | 0,9  | 0,5  | -0,6 | 1 | 0 | 0 |
| 2 | 1 | 27 | 0,5  | 0,1  | 0,4  | 0,2  | 0,6  | -1   | 0 | 1 | 1 |
| 2 | 1 | 6  | -0,3 | -0,3 | 0,6  | -1   | 0,6  | -0,9 | 1 | 0 | 0 |
| 2 | 1 | 22 | -0,4 | -0,7 | -1   | -0,7 | 0    | -0,6 | 0 | 0 | 1 |
| 2 | 1 | 13 | 0,1  | -0,3 | 0,5  | -0,3 | 0,2  | 0,5  | 1 | 0 | 1 |
| 2 | 1 | 12 | 0    | -0,4 | 0,9  | 0,5  | -0,6 | 0    | 1 | 1 | 0 |
| 2 | 0 | 23 | 0,5  | -0,8 | 0,1  | 0,3  | 0,2  | -0,5 | 0 | 1 | 0 |
| 2 | 0 | 26 | -0,3 | -0,2 | 0,1  | 0,1  | 0,1  | 0,1  | 0 | 0 | 1 |
| 3 | 0 | 1  | 0,5  | -0,9 | 0,4  | 0,9  | -0,7 | -0,1 | 0 | 1 | 0 |
| 2 | 0 | 25 | 0,7  | 0,5  | -0,3 | -1   | -0,2 | 0,6  | 0 | 0 | 0 |
| 3 | 0 | 3  | -0,9 | -0,3 | 0,2  | -0,3 | -0,6 | 0,3  | 0 | 0 | 0 |
| 2 | 1 | 26 | -0,3 | -0,6 | -0,3 | -0,3 | 0    | 0,1  | 0 | 1 | 0 |
| 2 | 1 | 10 | 0,4  | 0    | 0,8  | -0,4 | 0,6  | 0,6  | 0 | 0 | 0 |
| 2 | 0 | 26 | 0,1  | 0    | 0    | 0,5  | -1   | 0,6  | 0 | 0 | 0 |
| 2 | 0 | 17 | 0,6  | -0,6 | 0,6  | 0    | 0,3  | -0,4 | 0 | 0 | 0 |
| 2 | 0 | 23 | -0,6 | 0,1  | -0,5 | -0,9 | -0,5 | 0,9  | 1 | 1 | 0 |
| 2 | 1 | 12 | 0,4  | -0,8 | -0,9 | 0,8  | 0,6  | 0,7  | 1 | 0 | 1 |
| 3 | 1 | 4  | 0,7  | -0,2 | 0    | 0,8  | 0,7  | -0,5 | 0 | 0 | 0 |
| 2 | 0 | 0  | 0    | 0,5  | 0,1  | 0    | -0,5 | 0    | 0 | 0 | 0 |
| 2 | 1 | 26 | 0,1  | 0,6  | 0,5  | 0,2  | -0,9 | -0,7 | 1 | 0 | 0 |
| 2 | 1 | 25 | 0,2  | -0,9 | -0,4 | 0,1  | 0,7  | 0,6  | 1 | 0 | 1 |
| 2 | 1 | 16 | -0,3 | 0,2  | -0,6 | 0,1  | 0    | 0,7  | 0 | 1 | 0 |
| 2 | 1 | 15 | 0,8  | -0,1 | 0,8  | -0,1 | 0,5  | 0,6  | 0 | 0 | 0 |
| 3 | 0 | 14 | 0,6  | 0,8  | 0,4  | -0,5 | -0,3 | -0,5 | 0 | 0 | 1 |

|   |   |    |      |      |      |      |      |      |   |   |   |
|---|---|----|------|------|------|------|------|------|---|---|---|
| 3 | 0 | 0  | 0    | -0,7 | 0,6  | 0,1  | -0,5 | 0    | 0 | 1 | 0 |
| 3 | 1 | 23 | -0,9 | -0,6 | 0,9  | 0,4  | 0,7  | -0,6 | 0 | 0 | 0 |
| 3 | 1 | 7  | -0,2 | 0,7  | -0,9 | 0,3  | 0,6  | -0,6 | 0 | 1 | 1 |
| 2 | 0 | 21 | -0,9 | -0,4 | -0,4 | -0,6 | -0,6 | -0,3 | 1 | 1 | 1 |
| 3 | 1 | 17 | -1   | -0,1 | -0,4 | -0,1 | -0,2 | 0,3  | 1 | 1 | 1 |
| 2 | 0 | 15 | -0,2 | -0,8 | -0,8 | 0,3  | -1   | -0,3 | 1 | 1 | 1 |
| 2 | 0 | 5  | 0,9  | -0,5 | -0,6 | -0,9 | -0,3 | 0,4  | 1 | 1 | 1 |

| Division | Direction | Section | t_s  | t_m  | t_e  | b_s  | b_m  | b_e  | Mass | Force | Delta |
|----------|-----------|---------|------|------|------|------|------|------|------|-------|-------|
| 2        | 1         | 13      | 0,7  | 0,6  | -0,4 | 0,2  | 0,7  | -0,8 | 1    | 0     | 0     |
| 3        | 1         | 20      | -0,3 | -0,7 | -0,8 | 0,8  | 0,7  | 0,3  | 0    | 1     | 1     |
| 2        | 1         | 6       | 0    | -0,4 | 0,3  | -0,3 | -0,3 | -0,3 | 0    | 0     | 0     |
| 3        | 0         | 18      | -0,6 | -0,4 | 0,2  | 0,2  | 0,5  | 0,3  | 1    | 1     | 0     |
| 3        | 0         | 26      | -1   | -1   | -0,6 | -0,6 | 0,2  | -0,6 | 0    | 1     | 1     |
| 3        | 1         | 15      | 0,1  | 0,1  | -0,7 | -0,8 | -0,5 | 0,3  | 0    | 1     | 0     |
| 2        | 0         | 12      | 0,6  | -1   | 0,5  | 0,9  | 0,6  | 0,6  | 1    | 0     | 0     |
| 3        | 0         | 2       | 0,4  | -1   | -0,2 | -0,8 | -0,6 | 0,8  | 1    | 0     | 1     |
| 3        | 0         | 25      | -1   | 0,5  | 0    | 0,9  | 0,4  | -0,7 | 1    | 0     | 1     |
| 3        | 0         | 10      | -1   | -0,3 | 0,7  | -0,1 | 0,2  | 0,8  | 0    | 0     | 1     |
| 2        | 0         | 23      | 0,6  | -0,1 | 0,5  | 0,2  | -0,3 | -0,6 | 1    | 0     | 1     |
| 2        | 0         | 20      | -0,3 | 0,3  | 0,3  | 0,4  | -0,6 | -0,2 | 1    | 0     | 1     |
| 2        | 1         | 8       | 0,4  | 0,3  | -0,8 | 0    | -0,6 | -0,9 | 0    | 0     | 0     |
| 2        | 0         | 17      | -0,2 | 0    | -0,9 | 0,5  | 0,5  | -0,8 | 1    | 1     | 0     |
| 2        | 0         | 11      | -0,3 | 0,5  | -0,6 | -0,3 | 0,1  | 0,5  | 0    | 1     | 0     |
| 2        | 0         | 7       | 0,3  | -0,5 | -0,8 | -0,9 | 0,5  | 0,7  | 0    | 0     | 0     |
| 3        | 0         | 26      | -0,5 | 0    | 0,3  | 0,5  | 0,7  | -0,8 | 1    | 1     | 0     |
| 3        | 1         | 8       | 0,4  | 0,2  | -0,5 | -0,4 | -0,4 | -0,9 | 1    | 0     | 1     |
| 2        | 1         | 7       | 0,4  | 0,9  | 0,9  | -0,7 | -0,1 | 0,1  | 0    | 0     | 0     |
| 3        | 1         | 26      | -0,7 | -0,3 | 0,3  | -0,9 | -0,7 | -0,2 | 0    | 0     | 1     |
| 3        | 1         | 8       | 0,8  | -0,4 | -0,7 | 0,1  | 0,9  | -0,8 | 0    | 1     | 1     |
| 3        | 0         | 10      | 0,4  | -1   | 0,8  | 0,6  | 0,2  | 0,7  | 1    | 0     | 0     |
| 2        | 0         | 13      | 0,8  | 0,7  | -0,6 | 0,8  | 0,8  | -0,4 | 0    | 0     | 0     |
| 2        | 1         | 17      | 0,4  | 0,3  | 0,8  | 0,9  | -1   | -0,2 | 0    | 0     | 0     |
| 2        | 1         | 24      | -0,7 | 0    | -0,7 | 0,8  | 0,1  | 0,8  | 1    | 1     | 0     |
| 2        | 1         | 2       | 0,1  | -0,1 | 0,9  | -0,7 | -0,2 | 0,6  | 1    | 0     | 1     |
| 3        | 0         | 20      | -0,7 | -0,3 | 0,3  | 0,5  | 0,7  | -0,8 | 0    | 1     | 0     |
| 2        | 0         | 8       | -1   | 0,3  | -0,1 | -0,4 | -0,8 | 0,1  | 1    | 0     | 0     |
| 2        | 1         | 1       | 0,7  | -0,5 | 0,4  | 0,4  | 0,1  | 0,9  | 0    | 0     | 0     |
| 2        | 0         | 15      | -0,4 | -0,3 | 0,1  | -1   | -0,5 | -0,3 | 0    | 0     | 1     |
| 3        | 0         | 24      | 0,9  | -0,9 | -1   | 0    | -1   | -0,7 | 0    | 1     | 1     |
| 3        | 1         | 29      | -1   | -0,6 | 0,1  | 0,7  | -0,5 | -0,6 | 1    | 1     | 0     |
| 2        | 1         | 1       | -0,1 | 0,5  | -0,2 | 0,3  | 0,1  | -0,2 | 1    | 0     | 0     |
| 2        | 1         | 1       | -0,3 | 0    | -0,4 | 0    | -0,2 | -0,9 | 0    | 0     | 0     |
| 3        | 0         | 1       | -0,8 | -0,8 | 0,9  | 0,6  | -0,5 | -0,8 | 0    | 0     | 0     |
| 2        | 1         | 17      | 0,8  | -0,4 | 0,4  | 0,6  | -0,3 | -0,4 | 0    | 0     | 1     |
| 2        | 1         | 20      | -0,3 | -0,7 | -0,8 | 0,8  | 0,7  | 0,3  | 1    | 0     | 1     |
| 2        | 0         | 14      | -0,5 | 0,4  | -0,5 | -0,1 | -0,6 | -0,4 | 0    | 1     | 1     |
| 3        | 0         | 9       | 0,7  | 0,8  | 0,2  | -0,2 | -1   | 0,3  | 0    | 0     | 0     |
| 2        | 1         | 25      | 0,3  | -0,4 | 0,6  | 0,2  | -0,9 | 0,5  | 0    | 0     | 0     |
| 3        | 0         | 23      | -0,5 | 0,4  | -0,3 | -0,4 | -0,9 | 0    | 0    | 1     | 0     |
| 2        | 0         | 3       | -0,6 | -0,8 | 0,1  | 0    | 0,1  | -0,2 | 0    | 0     | 0     |
| 3        | 1         | 7       | 0,4  | 0,2  | 0,5  | -0,8 | -0,1 | 0    | 1    | 1     | 0     |
| 2        | 1         | 15      | -0,4 | -0,3 | 0,1  | -1   | -0,5 | -0,3 | 0    | 1     | 0     |
| 2        | 0         | 19      | 0    | 0,2  | -1   | 0    | 0,1  | -0,6 | 0    | 0     | 0     |
| 2        | 0         | 19      | -0,2 | 0,2  | -0,5 | 0,3  | 0,1  | -0,5 | 0    | 1     | 0     |
| 3        | 0         | 20      | -0,4 | 0,7  | 0,1  | -0,1 | -1   | 0,1  | 0    | 0     | 1     |
| 2        | 0         | 3       | -1   | 0,5  | 0,9  | -0,6 | -1   | 0,2  | 0    | 1     | 0     |
| 2        | 0         | 8       | -0,3 | 0,6  | -0,5 | 0,1  | -0,6 | 0,1  | 0    | 0     | 1     |
| 2        | 0         | 13      | 0,1  | 0,1  | -0,7 | -0,7 | 0,1  | 0,2  | 0    | 0     | 1     |
| 2        | 1         | 23      | 0,6  | -0,1 | 0,4  | 0,1  | -0,6 | 0,1  | 1    | 0     | 0     |
| 2        | 1         | 3       | 0,4  | 0,9  | 0,9  | -0,7 | -0,1 | 0,1  | 0    | 1     | 1     |
| 3        | 0         | 10      | 0,4  | -1   | 0,8  | 0,8  | -0,7 | -0,2 | 0    | 0     | 1     |
| 3        | 0         | 24      | -0,3 | 0    | -0,6 | 0,1  | -0,7 | 0,2  | 0    | 0     | 0     |
| 3        | 0         | 7       | 0,8  | -0,4 | -0,5 | 0,2  | 0,8  | -0,7 | 0    | 0     | 1     |
| 2        | 0         | 11      | 0,9  | -0,4 | 0,5  | -0,6 | 0,2  | 0,5  | 1    | 0     | 0     |
| 3        | 0         | 25      | -0,5 | 0,2  | -0,4 | -0,5 | -0,9 | 0,2  | 0    | 0     | 0     |
| 3        | 0         | 23      | -0,4 | -1   | 0,3  | 0,3  | 0,4  | -0,1 | 0    | 0     | 1     |
| 3        | 0         | 13      | 0    | 0,8  | -0,5 | -0,6 | -0,6 | 0,5  | 1    | 1     | 0     |
| 2        | 0         | 1       | -0,6 | -0,1 | 0    | -0,3 | 0,7  | -0,4 | 0    | 1     | 0     |
| 2        | 0         | 9       | 0    | 0,5  | 0,7  | 0,4  | -0,7 | -0,2 | 0    | 1     | 0     |
| 2        | 0         | 9       | 0,1  | 0,4  | 0,7  | 0,5  | -0,4 | -0,2 | 1    | 1     | 0     |
| 2        | 0         | 1       | -0,8 | -0,8 | 0,3  | -0,9 | -0,5 | -0,9 | 0    | 0     | 0     |
| 2        | 0         | 16      | -0,8 | -1   | -0,7 | -0,1 | -0,5 | -0,9 | 0    | 0     | 1     |
| 2        | 0         | 1       | -0,7 | -0,1 | 0,1  | -0,3 | 0,4  | -0,4 | 1    | 0     | 0     |
| 2        | 0         | 7       | -0,5 | -1   | 0,4  | 0,3  | 0,4  | 0,1  | 0    | 0     | 1     |
| 2        | 0         | 7       | 0,2  | 0    | 0,7  | -0,1 | -1   | 0,4  | 0    | 0     | 1     |
| 3        | 0         | 2       | 0,4  | -0,9 | -0,1 | -0,7 | -0,5 | 0,8  | 1    | 1     | 0     |
| 2        | 1         | 17      | 0,3  | -0,2 | 0,1  | -0,1 | -0,3 | -0,2 | 0    | 0     | 0     |
| 3        | 0         | 13      | 0,8  | -0,3 | 0,4  | -0,1 | 0,3  | 0,9  | 0    | 0     | 1     |
| 2        | 1         | 3       | -0,6 | -0,1 | -0,2 | 0,4  | -0,2 | 0,3  | 0    | 1     | 1     |
| 3        | 0         | 6       | 0,8  | -0,3 | 0,7  | 0,3  | 0,4  | 0,2  | 0    | 1     | 0     |

|   |   |    |      |      |      |      |      |      |   |   |   |
|---|---|----|------|------|------|------|------|------|---|---|---|
| 2 | 1 | 25 | 0,7  | -0,6 | -0,4 | -1   | -0,8 | -0,1 | 1 | 0 | 1 |
| 2 | 1 | 3  | -0,1 | -0,9 | 0,1  | 0    | -0,3 | -0,1 | 0 | 0 | 1 |
| 2 | 1 | 16 | 0,2  | 0,3  | -0,6 | -0,6 | 0    | 0,1  | 1 | 0 | 0 |
| 2 | 1 | 11 | -0,1 | 0,6  | 0,3  | -0,9 | -1   | -0,8 | 0 | 1 | 0 |
| 2 | 0 | 2  | -0,7 | -0,5 | 0,6  | 0,3  | -0,5 | -0,9 | 0 | 0 | 1 |
| 3 | 0 | 17 | -0,8 | -1   | -0,7 | 0,9  | -0,3 | 0,1  | 0 | 0 | 1 |
| 2 | 0 | 14 | -0,4 | 0,8  | -0,2 | -0,2 | 0,7  | -0,2 | 0 | 0 | 0 |
| 2 | 0 | 3  | -1   | 0,5  | 0,9  | -0,6 | -1   | 0,2  | 1 | 1 | 0 |
| 3 | 1 | 13 | 0,4  | 0,8  | -0,5 | -0,9 | -0,9 | 0,5  | 0 | 1 | 1 |
| 2 | 1 | 19 | 0,5  | -0,9 | 0,2  | 0,8  | -0,7 | -0,2 | 1 | 0 | 1 |
| 2 | 0 | 7  | 0,3  | -0,1 | 0,8  | -0,2 | -1   | 0,6  | 0 | 1 | 1 |
| 2 | 0 | 8  | -0,2 | -0,9 | -0,8 | -0,3 | 0    | -0,6 | 0 | 0 | 1 |
| 3 | 0 | 7  | 0,9  | 0    | 0,7  | 0,6  | 0,5  | 0    | 1 | 1 | 0 |
| 3 | 1 | 6  | -1   | -0,9 | -0,9 | -0,9 | 0,7  | -1   | 0 | 0 | 0 |
| 3 | 0 | 27 | 0,5  | 0,6  | -1   | 0,3  | 0,1  | -0,5 | 1 | 1 | 0 |
| 3 | 1 | 22 | 0,9  | 0,1  | 0,2  | 0,3  | 0,5  | 0,9  | 1 | 0 | 0 |
| 2 | 1 | 25 | -0,6 | -0,1 | -0,2 | 0,4  | -0,2 | 0,3  | 0 | 0 | 0 |
| 3 | 1 | 27 | 0,3  | -0,1 | 0,7  | -0,7 | -1   | 0,1  | 0 | 1 | 1 |
| 2 | 0 | 16 | -0,2 | -0,2 | -1   | -0,1 | 0,2  | -0,7 | 0 | 1 | 1 |
| 3 | 0 | 29 | -0,4 | -0,2 | -0,9 | 0    | -0,2 | 0,5  | 0 | 0 | 0 |
| 2 | 0 | 20 | 0,8  | -0,3 | -0,1 | 0,5  | 0,2  | -0,1 | 0 | 1 | 0 |
| 2 | 1 | 21 | 0,6  | 0    | 0,2  | -0,1 | -0,4 | -0,2 | 0 | 1 | 0 |
| 2 | 1 | 27 | -1   | -0,3 | 0,7  | -0,1 | 0,2  | 0,8  | 0 | 0 | 0 |
| 3 | 0 | 26 | -0,5 | 0    | -0,3 | 0,4  | -0,6 | 0    | 0 | 0 | 0 |
| 2 | 0 | 5  | -0,6 | 0,9  | -0,7 | 0,7  | 0,4  | 0    | 0 | 0 | 0 |
| 3 | 0 | 10 | -0,6 | 0,3  | 0,1  | -0,6 | 0,8  | 0,7  | 0 | 0 | 0 |
| 2 | 1 | 9  | -0,5 | -0,3 | -0,8 | 0,6  | -0,8 | 0,3  | 0 | 1 | 0 |
| 2 | 0 | 5  | -0,5 | 0,7  | -1   | 0,5  | 0,2  | -1   | 0 | 0 | 0 |
| 3 | 1 | 12 | -1   | -0,5 | -0,3 | -0,6 | 0,8  | -0,5 | 1 | 0 | 0 |
| 3 | 1 | 10 | 0,2  | -0,6 | -0,3 | 0,1  | 0,7  | 0,7  | 0 | 1 | 1 |
| 2 | 0 | 1  | -0,8 | -0,8 | 0,3  | -0,9 | 0,9  | -0,4 | 1 | 1 | 0 |
| 2 | 1 | 2  | -0,6 | -0,7 | 0,1  | 0    | 0,1  | -0,2 | 0 | 0 | 0 |
| 3 | 0 | 20 | -0,7 | -0,3 | 0,4  | 0,3  | 0,5  | 0,3  | 0 | 1 | 0 |
| 3 | 1 | 10 | 0,6  | 0,4  | -0,8 | 0,1  | -0,7 | -0,9 | 0 | 0 | 1 |
| 3 | 0 | 23 | 0,6  | -0,1 | 0,5  | 0,3  | 0,3  | -0,7 | 1 | 0 | 1 |
| 2 | 1 | 11 | -0,4 | 0,7  | -0,8 | 0,2  | -0,7 | 0,1  | 1 | 1 | 0 |
| 2 | 1 | 24 | 0,6  | -0,1 | 0,5  | 0,1  | -0,8 | 0    | 0 | 1 | 1 |
| 2 | 0 | 10 | 0    | 0,5  | 0,7  | 0,5  | -0,6 | -0,2 | 0 | 0 | 1 |
| 3 | 0 | 29 | 0,9  | -0,7 | -0,9 | -0,3 | -0,3 | 0,6  | 0 | 0 | 0 |
| 2 | 0 | 0  | 0,8  | -0,9 | 0,9  | -0,9 | 0,7  | 0,9  | 0 | 0 | 0 |
| 3 | 0 | 7  | 0,9  | 0,8  | 0,7  | 0,2  | -1   | -0,1 | 0 | 0 | 0 |
| 3 | 0 | 25 | -0,8 | -1   | 0,2  | -0,9 | 0,2  | -0,5 | 0 | 0 | 0 |
| 3 | 1 | 15 | -0,6 | 0    | -0,4 | 0,5  | 0,5  | -0,8 | 1 | 0 | 0 |
| 3 | 1 | 25 | -0,5 | 0,4  | -0,2 | -0,5 | -1   | 0,2  | 0 | 1 | 0 |
| 3 | 1 | 22 | -0,4 | 0    | -0,1 | -0,9 | -0,2 | -1   | 1 | 0 | 0 |
| 3 | 1 | 18 | -0,2 | 0,6  | 0,4  | -0,5 | -0,8 | -0,3 | 1 | 1 | 0 |
| 3 | 1 | 7  | 0,4  | 0,2  | 0,5  | -0,8 | -0,1 | -0,9 | 1 | 1 | 0 |
| 2 | 0 | 7  | -0,2 | 0,6  | 0    | 0,1  | -0,5 | 0,2  | 1 | 0 | 0 |
| 3 | 0 | 26 | -0,8 | 0,8  | -0,4 | 0,2  | -1   | 0,7  | 0 | 0 | 0 |
| 3 | 1 | 23 | -0,2 | 0    | -0,9 | -1   | -0,7 | 0,4  | 0 | 0 | 0 |
| 2 | 0 | 16 | -0,8 | -1   | -0,7 | -0,1 | -1   | -0,4 | 0 | 0 | 0 |
| 3 | 1 | 20 | -0,9 | -0,1 | 0,1  | 0,2  | -0,4 | -0,8 | 0 | 0 | 1 |
| 3 | 0 | 17 | -0,2 | 0,7  | -0,7 | 0    | -0,3 | -0,6 | 0 | 1 | 1 |
| 3 | 0 | 16 | 0,2  | -0,4 | 0,9  | 0,5  | -0,8 | -0,6 | 0 | 0 | 1 |
| 2 | 1 | 29 | -0,8 | -0,3 | -0,7 | 0,2  | -0,4 | 0,1  | 0 | 0 | 0 |
| 3 | 0 | 0  | -0,2 | 0,7  | -0,2 | 0,4  | 0,9  | -0,4 | 0 | 0 | 1 |
| 3 | 0 | 11 | 0,9  | -0,4 | 0,5  | -0,6 | 0,2  | 0,5  | 0 | 1 | 0 |
| 2 | 1 | 18 | 0,2  | -0,6 | 0,9  | 0,8  | 0,4  | -0,2 | 0 | 0 | 0 |
| 3 | 0 | 28 | -0,8 | -0,3 | -0,8 | -0,8 | -0,1 | -0,6 | 0 | 1 | 0 |
| 3 | 0 | 22 | -0,8 | 0    | 0,5  | 0,9  | -0,5 | -0,3 | 0 | 0 | 1 |
| 2 | 1 | 14 | -0,5 | -0,3 | 0    | -1   | -0,1 | -0,8 | 0 | 0 | 0 |
| 3 | 1 | 13 | -0,3 | 0,8  | -0,5 | -0,1 | 0    | 0,5  | 0 | 0 | 0 |
| 2 | 0 | 3  | 0,1  | -0,5 | -0,8 | -0,9 | 0,5  | 0,7  | 1 | 0 | 1 |
| 2 | 0 | 19 | -0,2 | 0,2  | -0,5 | 0,1  | -0,4 | -0,9 | 0 | 0 | 1 |
| 2 | 0 | 17 | -0,4 | 0,1  | -1   | 0,5  | 0,6  | -0,9 | 1 | 0 | 0 |
| 3 | 0 | 10 | -0,4 | -1   | 0,3  | 0,3  | 0,4  | -0,1 | 1 | 0 | 1 |
| 2 | 0 | 20 | 0,2  | 0,4  | 0,6  | 0,4  | 0,6  | 0,8  | 0 | 1 | 0 |
| 3 | 1 | 3  | -0,1 | -0,1 | 0,4  | 0,5  | 0,9  | -0,3 | 0 | 0 | 0 |
| 2 | 1 | 9  | -0,4 | -0,5 | -0,8 | -0,3 | 0,3  | -0,9 | 0 | 1 | 1 |
| 2 | 0 | 1  | 0    | -0,3 | 0,2  | -0,8 | 0,3  | -0,1 | 0 | 1 | 1 |
| 2 | 1 | 26 | 0,6  | -0,4 | 0,9  | 0,2  | -1   | 0,6  | 1 | 0 | 0 |
| 2 | 0 | 7  | 0,3  | -0,1 | 0,8  | -0,2 | -1   | 0,7  | 1 | 1 | 1 |
| 3 | 0 | 24 | 0,5  | -0,8 | -1   | -0,2 | -0,9 | -0,7 | 1 | 1 | 1 |
| 2 | 1 | 4  | -0,6 | -0,9 | 0,7  | 0,4  | 0,4  | 0,4  | 1 | 1 | 1 |
| 2 | 0 | 0  | 0,8  | -0,9 | 0,9  | -0,9 | 0,7  | 0,5  | 1 | 1 | 1 |
| 3 | 1 | 1  | -0,4 | 0,1  | -0,7 | 0,9  | -0,2 | 0,2  | 1 | 1 | 1 |
| 2 | 0 | 20 | 0,1  | -0,3 | 0    | 0,5  | 0,5  | -0,5 | 1 | 1 | 1 |
| 3 | 0 | 6  | 0,7  | -0,3 | 0,7  | -0,3 | 0,3  | 0,9  | 1 | 1 | 1 |
| 3 | 0 | 24 | 0,3  | 0,8  | -0,2 | -0,8 | -0,3 | -1   | 1 | 1 | 1 |

| Division | Direction | Section | t_s | t_m | t_e  | b_s | b_m | b_e | Mass | Force | Delta |
|----------|-----------|---------|-----|-----|------|-----|-----|-----|------|-------|-------|
| 2        | 0         | 3       | 0,7 | 0,5 | -0,4 | 0,9 | 0,1 | 0,4 | 0    | 0     | 0     |

|   |   |    |      |      |      |      |      |      |   |   |   |
|---|---|----|------|------|------|------|------|------|---|---|---|
| 3 | 1 | 24 | 0,9  | 0,9  | 0    | 0,4  | 0,6  | 0,4  | 0 | 0 | 1 |
| 3 | 0 | 8  | -0,1 | -0,1 | -0,5 | -0,6 | 0,7  | -0,1 | 0 | 0 | 1 |
| 3 | 1 | 19 | -0,1 | -0,5 | 0,4  | 0    | -0,2 | 0,3  | 0 | 0 | 1 |
| 3 | 1 | 22 | 0    | 0,6  | 0,5  | -0,4 | -0,2 | -0,8 | 0 | 0 | 0 |
| 3 | 1 | 2  | -0,6 | 0,2  | -0,6 | 0,6  | -0,6 | 0,1  | 0 | 1 | 0 |
| 3 | 0 | 24 | -0,4 | -0,7 | -0,5 | -0,4 | -0,2 | -0,8 | 0 | 0 | 0 |
| 3 | 1 | 16 | -0,5 | 0,1  | -0,8 | 0,7  | -0,8 | -0,5 | 1 | 1 | 0 |
| 3 | 1 | 2  | -0,9 | 0,1  | 0,8  | 0    | 0,7  | -0,4 | 0 | 0 | 0 |
| 2 | 0 | 24 | 0,5  | 0,3  | 0,4  | -0,3 | 0    | 0,3  | 0 | 0 | 0 |
| 2 | 1 | 20 | -0,1 | 0,6  | 0,3  | -0,4 | -0,3 | -0,7 | 0 | 0 | 0 |
| 2 | 0 | 1  | -0,8 | 0,1  | 0,4  | 0,4  | -0,5 | -0,1 | 0 | 0 | 1 |
| 3 | 0 | 1  | -0,9 | 0,6  | -0,9 | -0,7 | -0,8 | -0,8 | 0 | 1 | 1 |
| 3 | 0 | 14 | -0,8 | 0,6  | 0,6  | -1   | 0,4  | -0,4 | 0 | 0 | 0 |
| 2 | 1 | 20 | 0,1  | 0,1  | -1   | -0,6 | -0,2 | 0,4  | 0 | 0 | 0 |
| 3 | 1 | 2  | -1   | 0,1  | 0,8  | 0    | 0,7  | -0,4 | 0 | 1 | 0 |
| 3 | 0 | 10 | 0,4  | -0,1 | 0,3  | -0,4 | 0,6  | 0,4  | 1 | 0 | 1 |
| 3 | 0 | 4  | -0,8 | -0,4 | 0,1  | 0,2  | 0,3  | 0,2  | 0 | 0 | 0 |
| 2 | 1 | 6  | 0,6  | 0,4  | -0,4 | 0,2  | 0,2  | -0,3 | 1 | 0 | 0 |
| 2 | 0 | 16 | -0,9 | 0,4  | -0,6 | 0,9  | -0,9 | -0,5 | 0 | 0 | 0 |
| 2 | 1 | 25 | -0,5 | 0,4  | 0    | 0,9  | 0,5  | -0,9 | 0 | 0 | 0 |
| 2 | 0 | 23 | 0,4  | 0,5  | 0,4  | -0,6 | -0,1 | 0    | 0 | 0 | 1 |
| 3 | 1 | 21 | 0,9  | 0,3  | 0,4  | -0,1 | 0,8  | 0    | 0 | 0 | 0 |
| 2 | 0 | 21 | 0,8  | -1   | 0,6  | 0,6  | -0,3 | -0,1 | 1 | 0 | 1 |
| 2 | 1 | 1  | -0,7 | 0,1  | -0,9 | -0,2 | 0,5  | -1   | 0 | 0 | 0 |
| 3 | 1 | 22 | -1   | 0,8  | -0,7 | 0,4  | 0,4  | 0,8  | 1 | 1 | 0 |
| 2 | 1 | 12 | -0,9 | 0,4  | -1   | 0    | 0,7  | 0,2  | 0 | 0 | 1 |
| 3 | 0 | 17 | 0,1  | -0,9 | 0,3  | -0,5 | 0,3  | -0,7 | 0 | 1 | 1 |
| 2 | 0 | 16 | 0,7  | 0,7  | -0,3 | 0,2  | 0,4  | 0,9  | 0 | 0 | 0 |
| 3 | 0 | 16 | 0,7  | -0,7 | -0,1 | -0,8 | 0    | 0,2  | 0 | 0 | 1 |
| 3 | 1 | 24 | -0,7 | -0,7 | -0,1 | -0,4 | 0,9  | 0,7  | 1 | 0 | 1 |
| 2 | 1 | 18 | -0,6 | 0,5  | -0,5 | -0,6 | 0,9  | 0,2  | 1 | 1 | 0 |
| 3 | 1 | 20 | 0,6  | -1   | -0,4 | 0,8  | -0,3 | -0,1 | 0 | 0 | 0 |
| 2 | 0 | 14 | -0,1 | 0,8  | 0,8  | 0,1  | -0,5 | 0,1  | 0 | 0 | 0 |
| 2 | 0 | 20 | 0,7  | -0,9 | 0,8  | 0,9  | -0,3 | 0,6  | 1 | 0 | 1 |
| 2 | 0 | 8  | 0,6  | 0,6  | -0,5 | 0,4  | 0,2  | 0,3  | 0 | 0 | 1 |
| 3 | 1 | 14 | -0,8 | 0,6  | 0,6  | -1   | 0,4  | -0,4 | 0 | 0 | 1 |
| 2 | 1 | 3  | -1   | 0,5  | 0,9  | -0,6 | -1   | 0,2  | 0 | 1 | 0 |
| 3 | 1 | 17 | -0,4 | 0,1  | -0,6 | 0,7  | -0,8 | -0,1 | 0 | 1 | 1 |
| 2 | 1 | 23 | -0,3 | 0    | 0,1  | 0    | 0,6  | 0,6  | 0 | 0 | 0 |
| 2 | 0 | 18 | -0,6 | -0,9 | -0,2 | -0,5 | 0,4  | -0,5 | 1 | 0 | 0 |
| 3 | 0 | 5  | 0,2  | 0,1  | -0,9 | -0,5 | 0,5  | -0,2 | 1 | 0 | 1 |
| 2 | 1 | 21 | 0,8  | 0,6  | 0    | 0,2  | 0,5  | 0,5  | 1 | 0 | 1 |
| 3 | 1 | 24 | -0,7 | -0,7 | 0,3  | -0,2 | 0,6  | 0,9  | 0 | 0 | 0 |
| 3 | 1 | 18 | 0    | 0,3  | -0,4 | 0,9  | 0,1  | 0,4  | 0 | 0 | 0 |
| 3 | 1 | 4  | -0,8 | -0,3 | -0,1 | -0,5 | 0,5  | -0,7 | 0 | 1 | 0 |
| 3 | 1 | 21 | 0,3  | -0,8 | -0,2 | 0,8  | -0,5 | -0,2 | 0 | 0 | 1 |
| 3 | 0 | 17 | 0,1  | -0,9 | 0,5  | 0    | -0,5 | -0,7 | 0 | 0 | 0 |
| 3 | 0 | 0  | 0,4  | -0,4 | -1   | -0,9 | -1   | 0,4  | 0 | 1 | 0 |
| 2 | 1 | 11 | -0,3 | -0,9 | -0,1 | -0,7 | 0    | -0,9 | 0 | 0 | 1 |
| 3 | 0 | 8  | 0,5  | -0,6 | -0,4 | -0,9 | -0,7 | 0,3  | 0 | 0 | 0 |
| 3 | 1 | 21 | 0,9  | 0,4  | 0,2  | 0,2  | 0,7  | 0,1  | 1 | 0 | 0 |
| 2 | 0 | 20 | 0,9  | 0    | 0,3  | 0    | 0,7  | -0,1 | 1 | 0 | 0 |
| 2 | 1 | 3  | -0,8 | 0,1  | 0,5  | 0,2  | -0,6 | -0,1 | 0 | 1 | 0 |
| 3 | 1 | 27 | 0    | -0,4 | -0,8 | 0,7  | -0,1 | 0,4  | 0 | 1 | 1 |
| 3 | 0 | 2  | -0,6 | -0,1 | -0,1 | -0,3 | 0,5  | 0,4  | 0 | 0 | 0 |
| 3 | 0 | 5  | 0,6  | 0,6  | 0,7  | -0,7 | -0,3 | 0    | 1 | 0 | 0 |
| 2 | 0 | 23 | 0,7  | -0,3 | 0,3  | 0    | -0,2 | 0    | 0 | 0 | 0 |
| 2 | 1 | 18 | -0,3 | -0,1 | 0    | 0,1  | 0,5  | 0,6  | 1 | 0 | 1 |
| 3 | 0 | 14 | -0,8 | 0,3  | 0,3  | -0,4 | 0    | 0,6  | 1 | 0 | 0 |
| 3 | 1 | 16 | -0,4 | -0,9 | -0,1 | 0,7  | 0,7  | 0,7  | 0 | 0 | 0 |
| 2 | 0 | 7  | -0,5 | 0    | -0,6 | -0,4 | 0,1  | -1   | 0 | 1 | 0 |
| 2 | 0 | 9  | -0,6 | -0,6 | -0,4 | -0,5 | -0,3 | -0,2 | 0 | 0 | 1 |
| 3 | 1 | 19 | -0,9 | 0,5  | -0,6 | -0,7 | -0,6 | -0,6 | 1 | 0 | 0 |
| 2 | 1 | 9  | -0,9 | 0,3  | -1   | -0,1 | 0,5  | -0,1 | 1 | 1 | 0 |
| 2 | 1 | 1  | -0,3 | 0,3  | -0,2 | -0,4 | -0,2 | 0,6  | 0 | 0 | 0 |
| 2 | 0 | 13 | -0,2 | -0,5 | -0,5 | -0,4 | -0,1 | -1   | 0 | 0 | 0 |
| 3 | 0 | 6  | 0,1  | -0,8 | 0,7  | 0    | 0,5  | -0,1 | 1 | 1 | 0 |
| 2 | 0 | 2  | -0,8 | 0,5  | -0,6 | 0,7  | -0,8 | -0,5 | 0 | 1 | 1 |
| 3 | 0 | 1  | 0    | 0,9  | -0,9 | -1   | -0,4 | -0,3 | 1 | 0 | 0 |
| 2 | 0 | 7  | -0,7 | 0,1  | -0,9 | -0,2 | 0,5  | -1   | 0 | 0 | 0 |
| 3 | 0 | 21 | 0,5  | -0,7 | 0    | 0    | 0,1  | -0,4 | 0 | 0 | 0 |
| 3 | 1 | 22 | 0,1  | -0,2 | -0,4 | 0,5  | 0,7  | 0,2  | 1 | 1 | 0 |
| 3 | 0 | 19 | -0,6 | -0,6 | -0,5 | -0,3 | -0,1 | -0,9 | 0 | 1 | 0 |
| 2 | 0 | 11 | -0,5 | -0,5 | 0,5  | -1   | -0,7 | -1   | 1 | 0 | 0 |
| 3 | 0 | 2  | -0,2 | -0,2 | -0,3 | -0,4 | 0,5  | 0,8  | 0 | 0 | 1 |
| 2 | 1 | 12 | -0,5 | 0,6  | -0,3 | -0,1 | -0,6 | 0,1  | 0 | 0 | 0 |
| 2 | 1 | 26 | -0,7 | 0,7  | 0,1  | -0,9 | -0,4 | -0,1 | 0 | 0 | 0 |
| 2 | 1 | 1  | -0,8 | 0    | 0,7  | 0,3  | -0,5 | -0,1 | 0 | 0 | 0 |
| 3 | 1 | 17 | -0,6 | -0,2 | -0,2 | -0,6 | 0,8  | 0    | 0 | 0 | 0 |
| 3 | 0 | 6  | 0,2  | -0,9 | 0,8  | 0,2  | 0,7  | 0    | 0 | 0 | 1 |
| 3 | 0 | 18 | -0,7 | -0,8 | -0,3 | -0,4 | 0,6  | -0,4 | 0 | 0 | 0 |
| 3 | 1 | 3  | 0,7  | 0,2  | 0,8  | -0,4 | -0,4 | 0,8  | 1 | 0 | 0 |
| 2 | 0 | 7  | -0,5 | 0    | -0,6 | -0,4 | 0,1  | 0,4  | 1 | 0 | 0 |
| 3 | 1 | 23 | -0,8 | -0,9 | 0,9  | -0,6 | -0,6 | -0,8 | 0 | 0 | 1 |
| 2 | 0 | 8  | 0,3  | -1   | 0,3  | -0,5 | -0,2 | -0,6 | 0 | 0 | 0 |

|   |   |    |      |      |      |      |      |      |   |   |   |
|---|---|----|------|------|------|------|------|------|---|---|---|
| 2 | 0 | 14 | 0,1  | 0,1  | 0,8  | 0,5  | 0,1  | 0,5  | 0 | 0 | 1 |
| 2 | 1 | 20 | -0,4 | -0,3 | 0,9  | -0,1 | 0,9  | -0,3 | 0 | 0 | 1 |
| 3 | 1 | 27 | 0    | -0,4 | -0,8 | 0,7  | 0,7  | 0,7  | 0 | 1 | 1 |
| 3 | 1 | 4  | -0,4 | -0,8 | -0,3 | 0,6  | 0,5  | 0,6  | 0 | 1 | 0 |
| 2 | 0 | 22 | -1   | -0,3 | -0,2 | -0,1 | 0,8  | -0,4 | 0 | 0 | 1 |
| 3 | 1 | 18 | 0    | 0,3  | 0,2  | 0,9  | -0,8 | 0,5  | 1 | 0 | 1 |
| 2 | 0 | 16 | 0,5  | 0,3  | 0,7  | 0,3  | -0,5 | 0    | 0 | 0 | 0 |
| 3 | 0 | 0  | 0,4  | -0,4 | -1   | -0,9 | -1   | 0,4  | 0 | 0 | 1 |
| 3 | 0 | 6  | 0,3  | 0    | -0,8 | -0,3 | 0,6  | -0,1 | 1 | 1 | 0 |
| 2 | 1 | 19 | 0,1  | -1   | 0,4  | -0,7 | 0    | -0,9 | 0 | 0 | 0 |
| 3 | 0 | 1  | -0,4 | 0,8  | 0,7  | -0,1 | -0,6 | 0,1  | 1 | 0 | 0 |
| 3 | 0 | 20 | 0,1  | -0,4 | 0,7  | 0,1  | 0,1  | -0,8 | 0 | 1 | 1 |
| 2 | 0 | 3  | -1   | 0,5  | 0,9  | -0,6 | -1   | 0,2  | 0 | 0 | 0 |
| 2 | 0 | 16 | -1   | -0,3 | 0,8  | 0,8  | -0,2 | -0,6 | 0 | 0 | 0 |
| 3 | 0 | 3  | -1   | 0,2  | -0,5 | 0,5  | 0,5  | -1   | 0 | 0 | 0 |
| 3 | 1 | 1  | 0,7  | -0,6 | 0,1  | 0,9  | -0,4 | -0,9 | 1 | 1 | 0 |
| 3 | 0 | 3  | 0    | -0,3 | -1   | -0,4 | -0,2 | -0,5 | 0 | 0 | 0 |
| 3 | 0 | 24 | 0,3  | -0,5 | 0,7  | 0,4  | 0,6  | -0,6 | 1 | 0 | 1 |
| 2 | 0 | 2  | -0,8 | 0,5  | -0,6 | 0,8  | -0,2 | 0,3  | 1 | 1 | 0 |
| 3 | 1 | 23 | -0,2 | 0,4  | -0,4 | -1   | 0,3  | -0,9 | 1 | 0 | 0 |
| 2 | 1 | 0  | -0,7 | 0,3  | 0,3  | -0,4 | 0    | 0,6  | 0 | 0 | 0 |
| 3 | 0 | 20 | -0,7 | 0,2  | -0,6 | 0,6  | 0,4  | 0,4  | 0 | 1 | 1 |
| 3 | 1 | 19 | -0,2 | -0,3 | 0,7  | -0,2 | -0,1 | 0,4  | 1 | 0 | 0 |
| 3 | 1 | 28 | -0,5 | 0    | 0,6  | 0,8  | -0,9 | -0,5 | 1 | 0 | 0 |
| 2 | 1 | 2  | 0,5  | 0,5  | -0,7 | -0,4 | -0,5 | 0,6  | 0 | 0 | 0 |
| 2 | 0 | 17 | 0,9  | -0,8 | -0,1 | 0,7  | 0,5  | -0,1 | 1 | 1 | 0 |
| 3 | 1 | 19 | -0,9 | 0,7  | -0,9 | -0,9 | -0,5 | -0,9 | 1 | 1 | 0 |
| 2 | 0 | 16 | 0    | -0,6 | -0,4 | -0,4 | -0,2 | -1   | 1 | 1 | 0 |
| 2 | 0 | 25 | 0,8  | -0,2 | 0,5  | 0,2  | 0,3  | 0,7  | 1 | 1 | 0 |
| 3 | 1 | 23 | 0,8  | -0,5 | 0,3  | 0,2  | -0,2 | 0    | 0 | 0 | 0 |
| 3 | 0 | 4  | -0,4 | -0,5 | -0,6 | -0,3 | 0,1  | -0,9 | 0 | 0 | 1 |
| 2 | 1 | 23 | -0,1 | -0,2 | 0    | 0,3  | 0,7  | 0,7  | 1 | 0 | 1 |
| 2 | 1 | 24 | 0,9  | -0,8 | -0,7 | 0,7  | 0    | 0,1  | 0 | 0 | 0 |
| 3 | 0 | 6  | 0,6  | 0,4  | 0,6  | -0,4 | 0,9  | 0,3  | 0 | 0 | 1 |
| 3 | 0 | 15 | 0,5  | 0,8  | 0,6  | -0,8 | 0,3  | -0,2 | 0 | 1 | 0 |
| 3 | 1 | 2  | 0,7  | 0,8  | -0,6 | -0,1 | -0,6 | -0,6 | 1 | 0 | 0 |
| 2 | 1 | 10 | -0,4 | 0,1  | -0,8 | -0,9 | 0,2  | -1   | 1 | 0 | 0 |
| 2 | 0 | 10 | -0,7 | -0,5 | 0,9  | -0,7 | 0,7  | 0,4  | 0 | 1 | 0 |
| 3 | 0 | 3  | 0,4  | 0,1  | 0,5  | -0,6 | 0,7  | 0,7  | 0 | 1 | 0 |
| 3 | 0 | 12 | -0,9 | -0,4 | 0,3  | 0,9  | 0,3  | 0    | 0 | 0 | 0 |
| 3 | 1 | 14 | 0,7  | -0,2 | -0,7 | -0,7 | 0,6  | 0,5  | 0 | 0 | 0 |
| 3 | 1 | 16 | -0,4 | -0,9 | -0,1 | -0,6 | 0,1  | -0,4 | 0 | 1 | 0 |
| 2 | 1 | 3  | -1   | 0,6  | 0,5  | -0,4 | 0,3  | 0,8  | 0 | 0 | 0 |
| 3 | 1 | 25 | -0,5 | 0,3  | 0,3  | -0,2 | 0,6  | 0,9  | 1 | 0 | 1 |
| 2 | 1 | 25 | 0,1  | 0,1  | -1   | -0,6 | -0,1 | 0,4  | 0 | 0 | 0 |
| 3 | 1 | 9  | -0,1 | -0,2 | 0,3  | -0,7 | -0,2 | -0,1 | 1 | 0 | 1 |
| 3 | 1 | 21 | -0,9 | 0,5  | 0,3  | -0,5 | -0,6 | 0,5  | 1 | 0 | 0 |
| 3 | 1 | 12 | 0,4  | -0,2 | 0,2  | -0,2 | 0,1  | 0,9  | 1 | 1 | 0 |
| 3 | 1 | 10 | 0,7  | -0,4 | -0,1 | -0,7 | 0,5  | 0,8  | 0 | 0 | 1 |
| 2 | 0 | 24 | -0,1 | 0,9  | 0,5  | -0,6 | -0,4 | 0    | 0 | 0 | 1 |
| 2 | 1 | 19 | -0,4 | -0,5 | -0,5 | -0,9 | 0,3  | -0,6 | 1 | 0 | 1 |
| 3 | 1 | 1  | 0    | 0,9  | -0,9 | -1   | -0,4 | -0,3 | 1 | 1 | 0 |
| 3 | 0 | 8  | -0,6 | 0,2  | -0,8 | 0,4  | 0,7  | -0,3 | 1 | 0 | 1 |
| 2 | 1 | 16 | 0,3  | -0,9 | 0,7  | 0,4  | -0,5 | 0,7  | 1 | 0 | 0 |
| 2 | 0 | 13 | 0,8  | 0,1  | -0,1 | -0,1 | -0,1 | -1   | 0 | 0 | 0 |
| 2 | 1 | 3  | 0,6  | -0,9 | -0,5 | -0,2 | 0,7  | -0,1 | 0 | 1 | 0 |
| 2 | 1 | 3  | -0,3 | -0,3 | 0,1  | 0,2  | -0,7 | -0,6 | 0 | 0 | 1 |
| 3 | 1 | 9  | -0,6 | -0,5 | -0,4 | -0,6 | 0,1  | -0,8 | 0 | 0 | 0 |
| 2 | 1 | 23 | 0,3  | -0,8 | 0,5  | 0    | -0,4 | -0,7 | 1 | 0 | 1 |
| 2 | 1 | 22 | 0,2  | 0    | -0,9 | -0,7 | 0    | 0,4  | 1 | 1 | 1 |
| 3 | 0 | 12 | -0,6 | -0,6 | -0,4 | -0,1 | -0,6 | 0    | 1 | 1 | 1 |
| 2 | 0 | 10 | -0,6 | -0,5 | 0,9  | -0,7 | 0,7  | 0,4  | 1 | 1 | 1 |
| 3 | 0 | 1  | -0,9 | 0,6  | -0,9 | -0,6 | -0,8 | -0,8 | 1 | 1 | 1 |
| 2 | 0 | 5  | 0,4  | -0,5 | 0,1  | -1   | 0,7  | -0,9 | 1 | 1 | 1 |
| 3 | 0 | 24 | -0,6 | 0,2  | -0,8 | 0,4  | 0,7  | -0,3 | 1 | 1 | 1 |