

# ESSAYS ON MACROECONOMICS

A Ph.D. Dissertation

by  
ZEYNEP KANTUR

Department of  
Economics  
İhsan Doğramacı Bilkent University  
Ankara  
September 2015

To my son, Ali

# ESSAYS ON MACROECONOMICS

Graduate School of Economics and Social Sciences  
of  
İhsan Doğramacı Bilkent University

by

ZEYNEP KANTUR

In Partial Fulfillment of the Requirements for the Degree  
of  
DOCTOR OF PHILOSOPHY

in

THE DEPARTMENT OF  
ECONOMICS  
İHSAN DOĞRAMACI BİLKENT UNIVERSITY  
ANKARA

SEPTEMBER 2015

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

---

Prof. Dr. Refet Soykan Gürkaynak  
Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

---

Assoc. Prof. Dr. Çağrı Sağlam  
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

---

Assist. Prof. Dr. Kağan Parmaksız  
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

---

Prof. Dr. Jülide Yıldırım Öcal  
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

---

Assist. Prof. Dr. Sang Seok Lee  
Examining Committee Member

Approval of the Graduate School of Economics and Social Sciences

---

Prof. Dr. Erdal Erel  
Director

# ABSTRACT

## ESSAYS ON MACROECONOMICS

KANTUR, Zeynep

Ph.D., Department of Economics

Supervisor: Prof. Dr. Refet Soykan Gürkaynak

September 2015

This dissertation consists of three essays on two topics in macroeconomics. The first essay focuses on the monetary policy implications in an aging society. The second and third essays revisit the famous Shimer puzzle in a theoretical and an empirical framework in a different perspective.

The first essay shows the impact of aging on effectiveness of monetary policy. To do so, it introduces an OLG-DNK framework where the demand side is represented by a two period overlapping generations setup and the supply side of the economy follows a New Keynesian framework. The model enables the study of the interaction of monetary policy with demographics in a coherent general equilibrium model. The main finding is that this merger of two basic strands of the macroeconomics literature implies monetary policy should be expected to be less effective as societies age since the interest rate sensitivity of real economic activity declines as the population ages.

The second essay studies the effect of employment-to-employment flows in a New Keynesian model with labor market frictions. Although New Keyne-

sian models with labor market frictions found an increase in unemployment and a decrease in labor market tightness in response to a positive technology shock (which appears to be in line with the recent empirical findings), the volatilities of these variables are not as high as their empirical counterparts. In that regard, we assume two types of firms which offer different wage levels, thereby incentivizing low-paid agents to search on-the-job. Differently from the literature, the main source of wage dispersion is the assumption of different bargaining powers of firms. The proposed model generates a higher volatility of unemployment and labor market tightness in response to a positive technology shock compared to the model without on-the-job search. Moreover, it is shown that bargaining power and on-the-job search intensity have an amplifying effect on the unemployment rate.

Finally, the last essay is an empirical application of the theoretical model proposed in Chapter 3. This essay revisits the Shimer (2005) puzzle by covering a longer period, 1951-2014, than Shimer's exercise. Firstly, essay shows some stylized facts on U.S. labor market by using raw data and a structural VAR model. Then, the study tests the performance of the model utilized in Chapter 3. The structural VAR models shows that there is a positive correlation between productivity and unemployment and negative correlation between productivity and labor market tightness conditional to technology shock. In addition, I show that the model with on-the-job search component adds more amplification to the standard New Keynesian model with labor market frictions and it is capable of generating both the magnitude and the sign of the fluctuations of labor market variables to productivity shocks.

*Keywords:* Aging, Monetary Policy, Overlapping Generations Model, New Keynesian Model, Employment-to-employment Flow, Unemployment Fluctuations, The Shimer puzzle, Search and matching, Structural VAR.

# ÖZET

## MAKROEKONOMİ ÜZERİNE MAKALELER

KANTUR, Zeynep

Doktora, İktisat Bölümü

Tez Yöneticisi: Prof. Dr. Refet Soykan Gürkaynak

Eylül 2015

Bu çalışma makroekonominin iki önemli konusu üzerine üç makaleden oluşmaktadır. İlk çalışma yaşlanan toplumlarda para politikası üzerine odaklanmaktadır. İkinci ve üçüncü çalışmalar ise sırasıyla teorik ve ampirik olmak üzere ünlü Shimer puzzle'ı farklı yönden tekrar gözden geçirmektedir.

Birinci makale yaşlanan toplumlarda para politikasının etkinliğini göstermektedir. Çalışma söz konusu soruyu ardışık nesiller ve yeni Keynesyen modellerinden oluşan bir sistemde incelemektedir. Modelin talep tarafı iki dönemli ardışık nesiller modeli ile arz tarafı ise yeni Keynesyen modelinden oluşmaktadır. Bu model para politikası ve demografik yapı arasındaki ilişkiyi genel denge modeli çerçevesinde incelememizi sağlamaktadır. Analiz sonuçlarına göre, para politikası, faiz oranına olan duyarlılıktaki azalma nedeniyle, yaşlanan toplumlarda etkinliğini yitirmektedir.

İkinci makale işten işe akışın sürtünmeli iş piyasası içeren yeni Keynesyen bir modeldeki etkilerini incelemektedir. Bahsedilen yeni Keynesyen modellerde pozitif teknoloji şokuna karşı işsizliğin arttığı, iş piyasası darlığının ise azaldığı

bulunmaktadır. Bu bulgu, ampirik çalışmalarla oynaklık yönü olarak tutarlı olsa da, ölçütsel olarak yeterli olmamaktadır. Bu makale, yeni Keynesyen modelin bu eksikliğini düşük maaşlı çalışanların iş aradığı ve farklı seviyelerde maaşlar sunan iki farklı tipte firma yapısı varsayarak gidermeyi amaçlamıştır. Literatürden farklı olarak bu çalışma maaşlar arasındaki farklılığı firmaların pazarlık güçlerinden kaynaklandığını varsaymıştır. Önerilen model, işsizlik ve iş piyasası darlığının oynaklıklarının işten işe akış varsayımı altında arttığını göstermektedir. Ayrıca, firmaların pazarlık gücünün ve çalışırken iş aramaya yönelimin işsizlik oranının oynaklığına arttırıcı etkisi olduğu gösterilmiştir.

Son olarak üçüncü makale ikinci bölümde sunulan teorik çalışmanın uygulamasıdır. Bu çalışma öncelikle Shimer (2005) puzzle'ı, makalede incelenen den daha uzun bir dönem için (1951-2014) tekrar gözden geçirmektedir. İlk olarak, ham veri ve yapısal vektör otoregresyon kullanarak A.B.D. iş piyasası dinamikleri gösterilmiştir. Daha sonra ise 3. bölümde kullanılan modelin performansı test edilmiştir. Çalışmanın sonuçları şöyledir: Yapısal vektör otoregresyon modeli teknoloji şoku altında üretkenlik ve işsizlik oranı arasında pozitif bir ilişki bulurken, üretkenlik ve iş piyasası darlığı için negatif korelasyon göstermektedir. Ayrıca, çalışma işten işe akış eklenmiş bir yeni Keynesyen modelin yapısal vektör otoregresyon modelin gösterdiği iş piyasası değişkenlerinin korelasyonlarının yönlerini replike etmekte yeterli olduğu gösterilmiştir.

*Anahtar Kelimeler:* Yaşlanma, Para Politikası, Ardışık Nesiller Modeli, Yeni Keynesyen Model, İşten İşe Akış, İşsizlik Dalgalanmaları, Eşleştirme Teorisi, Yapısal Vektör Otoregresyon.

# ACKNOWLEDGEMENTS

I just want to thank everyone who spends at least some part of their day contributing to my dissertation which wouldn't be possible otherwise.

First and foremost, I would like to thank my advisor, Refet Gürkaynak, for his persistent encouragement and support throughout my graduate study. His unique viewpoint on research and perception of the real world has continuously inspired my work and my understanding of economics as a science; not to mention his personal integrity and exceptional character.

During my two-semester visiting studies at the Universitat Pompeu Fabra, Barcelona, I was fortunate to work with Jordi Galí and Jaume Ventura to whom I must offer my profoundest gratitude. I especially thank Jordi Galí for his invaluable assistance and insight on the technical details particular to my work. His constructive and motivational approach provided me with the strength and enthusiasm that I lacked at times.

I am also indebted to Çağrı Sağlam and Kağan Parmaksız for their valuable comments throughout my dissertation. I would like to thank Jülide Yıldırım Öcal and Sang Seok Lee, who are the examining committee members, for their suggestions. I also wish to thank to all of the professors at the Department of Economics for their support and guidance throughout my graduate years at the department. I need to mention Özlem Eraslan, Nilgün Çorapçıoğlu and

Funda Yılmaz for their help with administrative matters.

The second chapter of this dissertation is a joint work with Kerim Keskin. I would like to thank him for working with me in harmony even though our approach to economics are apart. I would like to thank my friend Yıldız Akkaya for being there for me all the time. My friends, Tuğba Zeydanlı, Seda Köymen, Gülserim Özcan, Seçil Yıldırım and Anıl Taş who made my graduate life easier and cheerful.

I would also like to thank to my parents, Meral and Cihan and my brother Alp for their endless support and understanding.

Finally, I would like to thank to my husband, Yavuz. He was always there encouraging and supporting me and stood by me during the good and the bad times.

# TABLE OF CONTENTS

|                                                            |      |
|------------------------------------------------------------|------|
| ABSTRACT . . . . .                                         | iii  |
| ÖZET . . . . .                                             | vi   |
| TABLE OF CONTENTS . . . . .                                | x    |
| LIST OF TABLES . . . . .                                   | xiii |
| LIST OF FIGURES . . . . .                                  | xiv  |
| CHAPTER 1: INTRODUCTION . . . . .                          | 1    |
| CHAPTER 2: AGING AND MONETARY POLICY . . . . .             | 6    |
| 2.1 Motivation . . . . .                                   | 6    |
| 2.2 The Model . . . . .                                    | 11   |
| 2.2.1 Demographics . . . . .                               | 12   |
| 2.2.2 Households . . . . .                                 | 13   |
| 2.2.3 Firm side . . . . .                                  | 15   |
| 2.2.4 The Central Bank . . . . .                           | 18   |
| 2.2.5 Market Clearing and Equilibrium Conditions . . . . . | 19   |
| 2.2.6 Log-linearized Dynamics . . . . .                    | 23   |
| 2.3 Numerical Experiments . . . . .                        | 26   |

|       |                                                                                                           |    |
|-------|-----------------------------------------------------------------------------------------------------------|----|
| 2.3.1 | The Effects of a Monetary Policy Shock in a Model with<br>a Positive Old Age Dependency Ratio . . . . .   | 27 |
| 2.3.2 | The Comparison of The Effects of a Monetary Policy<br>Shock with Different Demographic Profiles . . . . . | 28 |
| 2.4   | Conclusion . . . . .                                                                                      | 32 |

## CHAPTER 3: A NEW KEYNESIAN MODEL WITH UNEMPLOYMENT: THE EFFECT OF ON-THE-JOB SEARCH . . . . .

|       |                                                                                                       |    |
|-------|-------------------------------------------------------------------------------------------------------|----|
| 3.1   | Motivation . . . . .                                                                                  | 34 |
| 3.2   | Model with On-the-job Search . . . . .                                                                | 40 |
| 3.2.1 | Households . . . . .                                                                                  | 42 |
| 3.2.2 | Firms . . . . .                                                                                       | 44 |
| 3.2.3 | Technological Process and Monetary Policy . . . . .                                                   | 54 |
| 3.2.4 | Market Clearing Conditions and Solving the Model . . .                                                | 54 |
| 3.2.5 | Calibration and the Steady State . . . . .                                                            | 55 |
| 3.3   | The Experiments . . . . .                                                                             | 57 |
| 3.3.1 | The Comparison of the Impulse Responses of the Models<br>with and without On-the-job Search . . . . . | 57 |
| 3.3.2 | The Effect of On-the-job Search Intensity on the Unemployment Rate . . . . .                          | 59 |
| 3.3.3 | The Effect of Bargaining Power on the Unemployment<br>Rate . . . . .                                  | 62 |
| 3.4   | Conclusion . . . . .                                                                                  | 63 |

|                                                     |           |
|-----------------------------------------------------|-----------|
| <b>CHAPTER 4: REVISITING THE SHIMER PUZZLE: THE</b> |           |
| <b>PERFORMANCE OF A NEW KEYNESIAN</b>               |           |
| <b>MODEL WITH ON-THE-JOB SEARCH . . . .</b>         | <b>65</b> |
| 4.1 More on Shimer Puzzle . . . . .                 | 66        |
| 4.2 Data and Methodology . . . . .                  | 67        |
| 4.2.1 U.S. Labor Market Data . . . . .              | 68        |
| 4.2.2 Structural VAR . . . . .                      | 69        |
| 4.2.3 Results . . . . .                             | 71        |
| 4.3 Model and Simulation . . . . .                  | 73        |
| 4.4 Conclusion . . . . .                            | 75        |
| <b>BIBLIOGRAPHY . . . . .</b>                       | <b>76</b> |
| <b>APPENDICES . . . . .</b>                         | <b>80</b> |
| A Derivation of the IS Equation . . . . .           | 80        |
| B Derivation of the Phillips Equation . . . . .     | 83        |
| C Simulation Values . . . . .                       | 86        |
| D Calibration . . . . .                             | 86        |
| E Log-linearized equations . . . . .                | 88        |
| F Impulse Responses . . . . .                       | 93        |

## LIST OF TABLES

|     |                                                        |    |
|-----|--------------------------------------------------------|----|
| 4.1 | Summary Statistics, Quarterly, 1951Q1-2014Q4 . . . . . | 68 |
| 2   | Preference Parameters . . . . .                        | 86 |
| 3   | Baseline calibration and steady state values . . . . . | 86 |
| 4   | Calibration values for experiments . . . . .           | 87 |

# LIST OF FIGURES

|     |                                                                                                                           |    |
|-----|---------------------------------------------------------------------------------------------------------------------------|----|
| 2.1 | Steady state of interest rate and old age dependency ratio . . . . .                                                      | 23 |
| 2.2 | Impulse Responses to a Tightening Monetary Policy Shock in an Economy<br>with 20% Old Age Dependency Ratio. . . . .       | 29 |
| 2.3 | Impulse Responses to a Tightening Monetary Policy Shock in Economies<br>with Different Old Age Dependency Ratios. . . . . | 31 |
| 2.4 | Responses to Contractionary Monetary Policy Shock . . . . .                                                               | 32 |
| 3.1 | The Impulse Responses to a Positive Technology Shock . . . . .                                                            | 58 |
| 3.2 | The Response of the Unemployment Rate to a Positive Technology Shock                                                      | 61 |
| 3.3 | The Response of the Unemployment Rate to a Positive Technology Shock                                                      | 63 |
| 4.1 | U.S. Labor Market Graphs . . . . .                                                                                        | 69 |
| 4.2 | Responses of Productivity and Unemployment to Technology<br>and Non-technology Shocks . . . . .                           | 72 |
| 4.3 | Responses of Productivity and Labor Market Tightness to Tech-<br>nology and Non-technology Shocks . . . . .               | 73 |
| 4   | Impulse Responses to a Positive Technology Shock . . . . .                                                                | 93 |

# CHAPTER 1

## INTRODUCTION

All developed countries are aging. The United Nations' latest demographic projections suggest that<sup>1</sup> the share of the old-aged population is anticipated to double on average in 40 years. For example, Japanese old-dependency ratio, defined as the ratio of adults aged 65 and above to the working-age population of adults aged 15 to 65, is expected to increase from three elderly persons to 10 working-age adults in 2010, to seven elderly persons to every 10 working-age adults by 2050. Similarly, German old-dependency ratio is expected to increase from three elderly persons to 10 working-age adults in 2010, to six elderly persons to every 10 working-age adults by 2050. Demographic structure of developed countries has been changing permanently due to decline in the fertility and increase in the longevity rates. In few years, baby boom generation will retire and reinforce the permanent effect on demographic structure by increasing the ratio of old to young people.

Economics literature has primarily been focusing on the sustainability of

---

<sup>1</sup><http://data.un.org>

the pension system or the labor market issues to address the concerns about aging population in developed countries. Related research have surprisingly kept the studies of monetary policy and demographics separate, effectively treating these as independent issues. The first essay of this thesis (Chapter 2) argues that demographics should be expected to have an impact on the effectiveness and hence conduct of the monetary policy. Effectively, this essay is attempting to answer the following questions: "How should the Central Bankers perceive a grayer society?" and "How should they react to it?" To answer these questions, this paper contributes to the theoretical literature by combining the standard New Keynesian framework and overlapping generations setup with aging. The overlapping generations structure of the demand side of the economy allows one to introduce aging into the model. On the supply side, a New Keynesian setup with nominal rigidities is required to analyze the effect of monetary policy. Main findings of the essay are as follows. First, as the demographic composition of the society ages, the natural rate of interest decreases monotonically. Secondly, the effectiveness of the monetary policy on output and inflation decreases due to decreasing interest rate sensitivity of the society. The model also suggests that it would not even be a surprise if the economy becomes old enough we may see positive response of output to a tightening monetary policy shock. Finally, as the old age dependency ratio increases we observe that the decrease in the response of output is larger compared to that of inflation after a tightening monetary policy shock. Therefore, the sacrifice ratio- the inflation that ensues from boosting the economy- between inflation and output increases as the economy ages. Hence, results of

the paper suggest that the policymakers should account for the demographic profile of the country when conducting monetary policies.

The second essay of the thesis (Chapter 3) revisits the famous Shimer puzzle by introducing employer-to-employer flows to the labor market of a New Keynesian model economy. The influential and the spurring study by Shimer (2005) argued that the search and matching model of Mortensen-Pissarides is incapable of generating observed fluctuations in the unemployment rate and labor market tightness in response to a positive productivity shock. Furthermore, in search and matching models a positive technology shock leads to a decrease in the unemployment rate and to an increase in labor market tightness which contradict with their empirical counterparts.

The aim of this study is to amplify the responses of the unemployment rate and labor market tightness in line with the related empirical literature. To do so, on-the-job search option is introduced to the New Keynesian model suggested by Clarida et al. (1999). The proposed model assumes two-tier sector including firms with different bargaining powers, namely *aggressive* and *passive* firms. We also assume that a fraction of workers are allowed to do on-the-job search. This assumption amplifies the flow of employment and increases the volatilities of the unemployment rate and labor market tightness. Additionally, the essay reveals the role of search intensity and bargaining power. In that regard, three experiments are conducted. First one compares the responses to a positive technology shock in the models with and without on-the-job search. Second experiment analyzes the effect of search intensity on model dynamics. Last one, by changing the bargaining power of aggressive firms, examines

the effectiveness of bargaining power on labor market dynamics. These three experiments lead to the following results: (1) The volatilities of the unemployment rate and labor market tightness in the model with on-the-job search are higher compared to the model without on-the-job search. Note that this constitutes a crucial part of the Shimer puzzle. (2) The response of the unemployment rate increases as the search intensity increases. (3) The response of the unemployment rate increases as the bargaining power of aggressive firms increases.

The third essay (Chapter 4) is an empirical application of the aforementioned criticism. In the first part of this essay, I estimate two bivariate structural VAR models with labor productivity and unemployment and labor productivity and labor market tightness to show that a positive technology shock increases unemployment and decreases labor market tightness and generates a positive correlation between unemployment and productivity and a negative correlation between labor market tightness and productivity by using U.S. data. This chapter also tests the implied dynamics of the proposed New Keynesian model with on-the-job search in Chapter 3. The simulation results suggest that the model with on-the-job search amplify the effect of the technology shock on unemployment. Moreover, the model is able to generate the accurate signs for elasticities of productivity-unemployment and productivity-labor market tightness in line with the findings of the structural VAR model.

Overall, Chapters 2 and 3 of this dissertation contributes to the theoretical literature by utilizing heterogeneous agents in the New Keynesian models whereas Chapter 4 attempts to quantitatively verify the findings of the theo-

retical model in Chapter 3.

# CHAPTER 2

## AGING AND MONETARY POLICY

### 2.1 Motivation

Aging populations are important policy concerns and a correspondingly important academic research area. The economic and the policy impact of aging have been extensively studied in the fiscal policy context, especially with respect to its social security implications. The literature, however, surprisingly kept the studies of monetary policy and demographics separate, effectively treating these as independent issues. This paper argues that demographics should be expected to have important bearing on the effectiveness and hence conduct of monetary policy.

The present paper contributes not only to address the proposed policy concern, but also to provide a tractable model for the qualitative analysis. This paper constructs a simple setup which merges two period overlapping generations (OLG) and dynamic new Keynesian (DNK) frameworks.<sup>1</sup> The demand side of the model assumes an OLG structure which enables us to

---

<sup>1</sup>A similar setup without population growth rate is utilized in Galí (2014).

introduce aging into the model economy. Furthermore, differently from the existing OLG literature with aging, I introduce a DNK setup to analyze the effectiveness of monetary policy which is measured by the impact of interest rate changes on output and inflation. Following the standard DNK setup of Galí (2008a), the introduction of price rigidities to the model allows monetary policy to influence interest rate and the real economy. This highly stylized model enables us to study the impact and transmission of monetary policy in economies with different demographic profiles.

A number of empirical and theoretical papers have focused on the relation between societal aging and monetary policy.<sup>2</sup> To the best of my knowledge, Imam (2013) is the first empirical study that attempts to show the empirical evidence of the decreasing effectiveness of monetary policy over time based on the demographic structure of the economy. Imam (2013) uses dynamic panel estimation technique for five advanced countries, U.S., Canada, Japan, U.K and Germany. First he shows the decreasing effect of monetary policy over time and then he attributes the weakening effect of monetary policy to the changing demographic profiles of these countries. He indicates that the effectiveness of monetary policy on unemployment and inflation decreases as the society ages. In the theoretical literature, the closest work to this study is Fujiwara and Teranishi (2005, 2008). Both<sup>3</sup> papers employ the same dynamic stochastic general equilibrium model by incorporating the life-cycle behavior a lá Gertler

---

<sup>2</sup>Miles (2002) is the first study that discusses the aging and monetary policy with various specifications for pension systems. He utilizes three different OLG models by assuming consumption smoothing over time and forward looking behavior of agents. He finds that aging has an ambiguous effect on the effectiveness of monetary policy. However, findings of the paper are highly sensitive to the existence and generosity of the pension system.

<sup>3</sup>Fujiwara and Teranishi (2005) is the welfare implication of the Fujiwara and Teranishi (2008)

(1997) to study whether structural shocks to the economy have asymmetric effects on workers and retirees. These papers suggest that the demographic structure of the society has an impact on the effectiveness of monetary policy. They find that in an older economy, tightening monetary policy shock has much larger negative effect on aggregate demand compared to younger and no cycle economies, which contradicts with the findings of this paper. Besides, the results of the studies depend on the assumption which allows retired agents to rejoin to the labor force. Moreover, due to the multidimensional aspect of the theoretical model that is used for analysis, the effects of aging on the transmission of monetary policy are not crystal clear.

In addition to the different structure of the model, the comparative advantage of the present paper is its simplicity. I believe that studying the proposed question in a basic and a tractable theoretical framework is an intermediate and also a required step for full understanding of the dynamics of such economy. This paper evaluates and analyzes the effects of integration of OLG setup and aging to key equations of the standard DNK framework: the dynamic IS equation, the forward looking Phillips equation and the Taylor rule. Differently from the Fujiwara and Teranishi (2008), this paper constructs the demand side of the model with two period OLG setup which is consistent with life cycle theory of Modigliani and Brumberg (1954). The life cycle theory suggests that individuals try to smooth their consumption over time and their savings follow a hump-shaped pattern, with higher savings during their working age. Moreover, in this simple framework, I assume that the retired agents do not rejoin to the labor force and do not participate actively in financial markets.

The proposed formulation in this paper suggests that the response of the society depends on the ratio of the number of old to young agents. Basically, the mechanism of the model can be summarized as follows: Young agents consume and save their labor income. On the other hand, old agents consume what they saved when they were young with its interest income. Furthermore, these agents respond differently to an unexpected monetary policy shocks according to their objectives. For instance, after a positive (tightening) monetary policy shock, young agents' level of consumption decreases due to *the substitution effect*, and old agents' increases due to *the wealth effect*. As the ratio of old agents increases, the dominance of the substitution level weakens over the wealth effect in the model. Hence, the effectiveness of monetary policy decreases. The main findings of the paper can be summarized as follows:

First, the natural rate of interest decreases monotonically as the ratio of population of old to young agents increases, and this result is consistent with the earlier applied studies. Bean (2004) suggests that both along the transition path and the steady state, societal aging may lead to a sharp decline in the saving behavior of agents, supply of labor and the natural rate of interest. Auerbach et al. (1989) and Auerbach et al. (1991) quantitatively show the negative impact of aging on the saving rate.<sup>4</sup> Miles (1999) points out that there will be fundamental changes in saving rates as a result of an increase in the old age dependency ratio which is the ratio of population of retired agents to workers. Results of the simulations of his study indicate that private saving rates are likely to fall in long term in line with aging populations. Besides,

---

<sup>4</sup>Rios-Rull (2001) finds a similar result by using Spanish data.

Krugman (1998) argues the depressing effect of aging on the natural rate of interest in Japan. Secondly, the effectiveness of monetary policy on output and inflation decreases due to decreasing interest rate sensitivity of the society as the population ages. The model also suggests that it is even possible to see positive response of output to a tightening monetary policy shock if the economy becomes old enough. Finally, the trade-off between inflation and output increases as the economy ages. To conclude, the results of the paper suggests that the policymakers should account for the demographic profile of the country when conducting monetary policies.

This highly stylized model may not provide an accurate quantitative responses to an unexpected monetary policy shock, since it assumes a two-period OLG setup on the demand side and an extreme case where retired agents are not active in the financial market and consume all their wealth (their marginal propensity to save is zero). However, the model utilized in this paper is enough to study the qualitative effects of aging on effectiveness of monetary policy and I believe it is an intermediate and also a required step to analyze and understand the dynamics of the monetary policy in an aging economy.

The rest of the paper is organized as follows: Section 2 sets up the model which incorporates two period OLG model into basic New Keynesian framework of Clarida et al. (1999). In section 3, I conduct experiments with the proposed theoretical model by assuming economies with different demographic profiles. Then, I interpret and compare the responses of these economies to an unexpected monetary policy shock. Finally, section 4 concludes.

## 2.2 The Model

This section introduces the model which incorporates a two period overlapping generations (OLG) setup to standard New Keynesian framework (DNK). OLG setup enables us to introduce aging to the model and DNK framework has the convenient environment to study the effectiveness of monetary policy.

The demand side of the economy, the households' problem, is modeled as an OLG setup, introduced by Samuelson (1958) and Diamond (1965). It contains two generations, workers and retirees, who are born at different dates and have finite lifetimes, even though the economy lasts forever. All agents in the economy are born as workers. In the first period of their lifetime, agents earn wage income by supplying labor and decide how much to consume and save. Workers can save in two types of assets: one-period nominally riskless discount bonds yielding a nominal return and equity shares of the firms which are infinite-lived assets. It is crucial to have stock market in this setup because it links the short-lived agents to infinitely lived firms. The ownership of the firms is transferred through equity market. All agents retire in the second period of their lifetime. In the retirement period, households stop supplying labor and consume all their wealth and at the end of the period they die. Supply side of the economy has the basic New Keynesian framework a la Clarida et al. (1999). Monetary policy follows a Taylor (1993) rule, where the central bank reacts to the output and inflation.

### 2.2.1 Demographics

This section describes how we introduce aging into the model. The number of workers and retired agents at time  $t$  are denoted by  $N_t^w$  and  $N_t^r$ , respectively. In this framework, only young agents can work and are fertile so that  $N_t^w = (1 + n)N_{t-1}^w$ , where  $n$  is the fertility rate of workers. The total population at time  $t$  is

$$\underbrace{N_{t-1}^w}_{\text{Number of Retirees}} + \underbrace{(1 + n)N_{t-1}^w}_{\text{Number of Workers}}$$

where  $N_{t-1}^w = N_t^r$ . Agents can work only in the first period of their lifetime. Hence, the labor supply at time  $t$  corresponds to the the number of workers at time  $t$ .

Finally, it is useful, to define an indicator for aging, which is the old-age dependency ratio, denoted by  $\varphi$ . It is the ratio of retired to employed agents in period  $t$ .

$$\frac{N_t^r}{N_t^w} = \frac{N_{t-1}^w}{(1 + n)N_{t-1}^w} = \frac{1}{1 + n} = \varphi$$

Old age dependency ratio decreases as the population growth rate increases. This study compares old and relatively younger societies at steady state, where the population growth rates are constant over time, but different across the economies.<sup>5</sup> In other words, the old age dependency ratio does not change across time in either economy.

---

<sup>5</sup>In the analysis, different  $\varphi$  considered at steady state for each type of economy.

### 2.2.2 Households

In the life cycle economy assumed in this model, there are two types of households; workers and retirees. Agents live for two periods.

#### Retirees

The representative retiree  $j$  consumes all his wealth and then dies. The budget constraint of retired agent  $j$  at time  $t + 1$  is given below.  $D_{t+1}(j)$  denotes consumption of a retired agent at time  $t + 1$ .  $P_{t+1}$  refers to price of a consumption good at time  $t + 1$ .  $Div_{t+1}(i)$  and  $Q_{t+1}(i)$  represent real dividend paid by the monopolistically competitive firm  $i$  and price of share of firm  $i$  at time  $t + 1$ , respectively.  $S_t(i, j)$  shows the amount of shares of firm  $i$  held by agent  $j$ . Formally,

$$P_{t+1}D_{t+1}(j) = B_t^n(j)(1 + i_t) + P_{t+1} \int_0^1 (Div_{t+1}(i) + Q_{t+1}(i)) S_t(i, j) di$$

where  $B_t^n(j)$  represents nominal bond holdings of agent  $j$  and  $i_t$  refers nominal interest rate at time  $t$ . Differently from the standard DNK model, we have an equity market in this setup, which enables us to combine the short-lived agents to infinite living firms. The ownership of the firm is transferred through the equity market, that is to say when agents are young they buy stocks of the firms and in the next period they become the owner of it.

## Workers

The representative worker  $j$  maximizes in period  $t$  the objective function,

$$V_t^j = \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \frac{L_t(j)^{1+\psi}}{1+\psi} + \beta E_t \left( \frac{D_{t+1}(j)^{1-\sigma}}{1-\sigma} \right)$$

subject to

$$P_t C_t(j) + B_t^n(j) + P_t \int_0^1 Q_t(i) S_t(i, j) di = W_t L_t(j)$$

and

$$P_{t+1} D_{t+1}(j) = B_t^n(j)(1+i_t) + P_{t+1} \int_0^1 (Div_{t+1}(i) + Q_{t+1}(i)) S_t(i, j) di$$

where  $C_t(j)$  and  $L_t(j)$ , respectively, denote consumption and labor supply of agent  $j$ . The parameters  $\sigma$  and  $\psi$  represent the inverse of intertemporal elasticity of substitution and labor supply, respectively. The representative worker faces the nominal wage rate  $W_t$  at time  $t$ . The saving occurs in form of equity shares of firms or one-period nominally riskless discount bonds. The first order conditions of the household's problem are

$$C_t(j)^{-\sigma} \frac{W_t}{P_t} = L_t(j)^\psi \tag{2.1}$$

$$C_t(j)^{-\sigma} = \beta E_t \left\{ (1+i_t) \frac{P_t}{P_{t+1}} D_{t+1}(j)^{-\sigma} \right\} \tag{2.2}$$

$$C_t(j)^{-\sigma} = \beta E_t \left\{ \left[ \frac{Q_{t+1}(i) + Div_{t+1}(i)}{Q_t(i)} \right] D_{t+1}(j)^{-\sigma} \right\}. \tag{2.3}$$

That is,

$$E_t\{\Lambda_{t,t+1}\} = \frac{1}{1+i_t} = \beta E_t \left\{ \left( \frac{C_t(j)}{D_{t+1}(j)} \right)^\sigma \frac{P_t}{P_{t+1}} \right\} \quad (2.4)$$

where  $\Lambda_{t,t+1}$  represents the stochastic discount factor.

Rearranging equations (2.2) and (2.3), we get the no arbitrage condition,

$$E_t \left\{ \left( \frac{D_{t+1}(j)}{C_t(j)} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} (1+i_t) - \left( \frac{Div_{t+1}(i) + Q_{t+1}(i)}{Q_t(i)} \right) \right) \right\} = 0 \quad (2.5)$$

which suggests that it does not matter whether workers save in riskless bonds or stocks in equity market. Both yield the same return in such a riskless economy.

### 2.2.3 Firm side

The supply side of the economy is modeled based on the basic New Keynesian framework following Clarida et al. (1999). There are two types of firms which are consumption and intermediate goods producers. There is imperfect competition in the intermediate goods market due to the assumption that each firm produces a differentiated good. We follow a staggered price setting a la Calvo (1983), in which every period a random fraction of firms is optimally setting prices.

#### Consumption Good Firm

There is a continuum of intermediate goods indexed by  $i \in [0, 1]$ , which are transformed into a homogenous consumption good according to the constant

returns to scale production function

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

where  $Y_t(i)$  is the quantity of the intermediate good  $i$  and  $\epsilon > 1$  denotes the elasticity of substitution. The consumption good sector is subject to perfect competition which determines the demand function for the representative intermediate good  $i$

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t$$

where  $P_t(i)$  and  $P_t$  denote the price of good  $i$  and the average price level, respectively. Reflecting the CES-structure of the technology in the final goods sector,  $P_t$ , is given by

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}.$$

### Intermediate Good Firms

In the intermediate good sector there is a continuum of firms indexed by  $i \in [0, 1]$ . Each firm produces a differentiated good,  $i$ , with the production function

$$Y_t(i) = N_t(i) \tag{2.6}$$

where  $Y_t(i)$  and  $N_t(i)$ , respectively, denote the output of firm  $i$  and the labor input. Labor market is competitive, i.e. the nominal wage rate  $W_t$  is taken as given in the production of good  $i$ . Intermediate firms are owned by the equity

holders (retired agents) and are managed to maximize the profit to the current owners. Through the final goods producing sector, intermediate firm  $i$  faces a downward sloping demand curve. At time  $t$  real profits (dividends) are:

$$Div_t(i) = Y_t(i) - \frac{W_t}{P_t} N_t(i)$$

Under flexible prices, after observing the shock, firms choose price  $P_t^*$ ,

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{P_t} P_t = \mathcal{M} MC_t$$

where  $MC_t$  and  $\frac{\epsilon}{\epsilon - 1} = \mathcal{M}$ , respectively, denote the marginal cost and the desired mark up value. In a symmetric equilibrium,  $P_t^* = P_t$  and following that  $\frac{W_t}{P_t} = \frac{1}{\mathcal{M}}$ .

Following Calvo (1983), nominal price rigidity is modeled by allowing random intervals between price changes. At each period a firm adjusts its price with a constant probability  $(1 - \theta)$  and keeps its price fixed with probability  $\theta$ . The reoptimizing firm solves

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \{ \Lambda_{t,t+k} (P_t^* Y_{t+k}(i) - W_{t+k} Y_{t+k}(i)) \}$$

subject to the demand function of the intermediate good. The first order condition is:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k}(i) \left( P_t^* - \frac{\epsilon}{\epsilon - 1} W_{t+k} \right) \right\} = 0$$

where  $\Lambda_{t,t+1}$  is the stochastic discount factor formalized in equation 2.4.

The reoptimizing firms owners set their price with the above equality considering the expected profit of the firm. The existence of equity market is crucial in this setup. If we do not have the equity market, then the owner of the firm will just maximize the profit of the current period because she dies at the end of the period. However, the equity market enables us to utilize the standard firm side problem as in the DNK setup. Following this argument, to maximize the price of the stocks today, owners of the firms (retired agents) should maximize the expected profit of the firm.

#### 2.2.4 The Central Bank

The monetary policy authority follows a standard Taylor (1993) type feedback rule:

$$i_t = \rho + \phi_\pi(\pi_t) + \phi_y(\hat{y}_t) + v_t$$

where  $\rho$  denotes the natural level of interest rate,  $\phi_\pi$  and  $\phi_y$  are feedback parameters,  $\pi_t$  is the deviation of rate of inflation from its steady state value, and  $\hat{y}_t$  is the deviation of the level of output from its steady state value. The exogenous component of the monetary policy is denoted by  $v_t$  and follows an AR(1) process

$$v_t = \rho_v v_{t-1} + \epsilon_t^v$$

where  $\epsilon_t^v$  denotes the monetary policy shock and  $\rho_v \in [0, 1)$  shows the persistence of the shock.

### 2.2.5 Market Clearing and Equilibrium Conditions

This section presents the market clearing conditions for the model economy. It is worth emphasizing that, in the standard DNK setup aggregate, per worker and per capita variables are all same. However, in this analysis they are all different and have to be kept track of.<sup>6</sup>

Goods market clearing condition requires that  $Y_t = N_t^w C_t(j) + N_t^r D_t(j)$  for all  $t$ . I normalize the equations in terms of per worker, that is:

$$Y_t^p = C_t(j) + \varphi D_t(j)$$

where  $Y_t^p$  refers to per worker output. Labor market clearing implies

$$L_t(j)N_t^w = \int_0^1 N_t(i)di = N_t.$$

In per worker terms, it is

$$L_t(j) = \frac{N_t}{N_t^w} = Y_t^p \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} di$$

where the term  $\left( \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} di \right)$  is the measure of price dispersion across firms. At equilibrium, agents do not trade bonds among themselves, therefore  $B_t = 0$ . The aggregate stock outstanding equity for each intermediate good producing firm must equal to the corresponding total amount of issued shares normalized to 1  $\forall i \in [0, 1]$ . Hence, the market clearing condition for shares at time  $t$  requires  $S_t(i) = 1$ .

---

<sup>6</sup>Per worker variables are utilized in order to attain stationarity in the model.

Combining the budget constraints of the two cohorts in period  $t$  at equilibrium we get,

$$\begin{aligned} Y_t^p &= C_t(j) + \varphi D_t(j) \\ &= \frac{W_t}{P_t} L_t(j) + Div_t^p \end{aligned}$$

where  $Div_t^p$  denotes the per worker real dividend payments.

Finally, real dividend payments by the intermediate firms and real stock price index are given:

$$Div_t = \int_0^1 Div_t(i) di \qquad Q_t = \int_0^1 Q_t(i) di. \quad (2.7)$$

In order to analyze the dynamics of the model, this paper focuses on the log-linearized system around steady state values.

## Steady State

The following equations describe the steady state values of the model's variables. The starred variables denotes the corresponding steady state values.

$$(y^p)^* = c^* + \varphi d^* \quad (2.8)$$

$$(w/p)^* = \frac{1}{\mathcal{M}} \quad (2.9)$$

$$(c^{-\sigma})^* = \mathcal{M}((y^p)^*)^\psi \quad (2.10)$$

$$d^* = (y^p)^* \left(1 - \frac{1}{\mathcal{M}}\right) \left(\frac{1 + i^*}{\varphi(1 + i^*) - 1}\right) \quad (2.11)$$

$$(div^p)^* = (y^p)^* \left( \frac{\mathcal{M} - 1}{\mathcal{M}} \right) \quad (2.12)$$

$$(q^p)^* = (div^p)^* \left( \frac{1}{\varphi(1 + i^*) - 1} \right) \quad (2.13)$$

$$\left( \frac{d^*}{c^*} \right)^\sigma = \beta(1 + i^*) \quad (2.14)$$

Equation (2.8) demonstrates the steady state of goods market clearing condition in per worker terms. Equation (2.9) shows the steady state value of real marginal cost, which is real wage in this setup and it equals to the desired markup in a frictionless environment. Steady state of the labor supply condition is given in Equation (2.10). Equations (2.11), (2.12) and (2.13) show, respectively, the steady state values of old age consumption, per worker real dividends and equity price, where all of them depends on the old age dependency ratio of the economy. The last equation is the steady state condition of the interest rate.

In standard DNK setup steady state interest rate depends on the discount factor,  $\beta$ . However, in this setup it is different. Equation (2.14) suggests that the steady state interest rate also depends on the demographic structure of the economy.<sup>7</sup> It is worth emphasizing the relation between steady state interest rate and the old age dependency ratio.

**Remark:** When  $\sigma = 1$ , the steady state interest rate equation is as follows:

$$1 + i = \frac{\mathcal{M} - 1 + \beta\mathcal{M}}{\varphi\beta}$$

---


$${}^7 \beta(1 + i) = \left( \frac{D}{C} \right)^\sigma = \left( \frac{\mathcal{M} - 1 + \beta\mathcal{M}}{\varphi\beta} \right)^\sigma$$

The above equation suggests that as the old age dependency increases, steady state interest rate decreases monotonically. For every  $\sigma > 0$ , this statement is still valid. Figure 2.1 shows the relation between old age dependency ratio, population growth rate and the steady state interest rate with the assumed parameter values.<sup>8</sup> The relation suggested by the model is illustrated in the above graph of Figure 2.1 is consistent with the applied literature and captures the fact that, a decrease in the number of workers implies scarcer labor compared to capital and this leads to a decrease in the interest rate. Krugman (1998) also points out that the aging population depresses the natural rate of interest in Japan. Furthermore, the second panel of Figure 2.1 shows that for every value of population growth rate, the model preserves the dynamic efficiency.

---

<sup>8</sup>The following parameter settings are assumed in constructing Figure 2.1:  $\beta = 0.99$ ,  $\mathcal{M} = 1.2$  and  $\sigma = 0.5, 1$  and  $1.1$

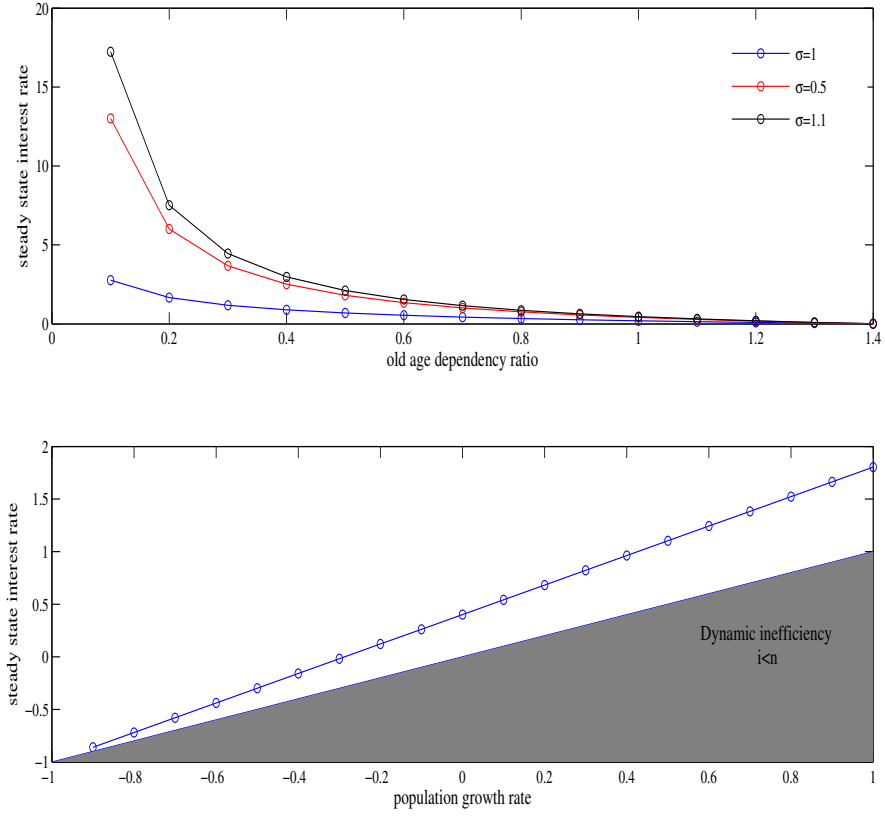


Figure 2.1: Steady state of interest rate and old age dependency ratio

### 2.2.6 Log-linearized Dynamics

This section provides the log-linearized equations around the zero inflation steady state. I use lower case letters to show the log of the variable and hat to indicate the percentage deviation from its steady state value. The demand side equations are as follows:

$$\hat{w}_t - \hat{p}_t = \psi \hat{l}_t + \sigma \hat{c}_t \quad (2.15)$$

$$\hat{c}_t = E_t\{\hat{d}_{t+1}\} - \frac{1}{\sigma}[\hat{i}_t - E_t\{\pi_{t+1}\}] \quad (2.16)$$

$$-\sigma \hat{c}_t(j) = -\sigma E_t\{\hat{d}_{t+1}\} + \frac{i^*}{1+i^*} E_t\{\widehat{div}_{t+1}^p\} + \frac{1}{1+i^*} E_t\{\hat{q}_{t+1}^p\} - E_t\{\pi_{t+1}\} - \hat{q}_t^p \quad (2.17)$$

$$\widehat{div}_t^p = \hat{y}_t^p - \frac{1}{\mathcal{M}-1}(\hat{w}_t - \hat{p}_t) \quad (2.18)$$

Equation (2.15) denotes the labor supply decision of a young agent at time  $t$ . Equation (2.16) is the linear Euler equation of young agent at time  $t$ . Differently from the standard DNK setup, Euler equation in this setup shows the relationship between consumption of a representative young agent at time  $t$  and her old age consumption at time  $t+1$ . Therefore, in this model one can not say Euler equation denotes the relation between total production (consumption) at time  $t$  and  $t+1$ , namely the *dynamic IS equation*. The total consumption (production) in this setup at time  $t$  is (in linearized terms):

$$\hat{y}_t^p = \frac{c^*}{(y^p)^*} \hat{c}_t(j) + \varphi \frac{d^*}{(y^p)^*} \hat{d}_t(j) \quad (2.19)$$

Equation (2.19) refers to *OLG-DNK IS equation*. To get the total consumption, additional calculations are made differently from the derivation of standard DNK-IS equation.<sup>9</sup> The OLG-IS equation not only depends on the current period's interest rate and expected inflation but also the previous period's interest rate and expected inflation rate and realized inflation. This is due to the fact that, at time  $t$  there are two types of agents, optimizing according to the available information to them. Young agents at time  $t$ , decide their consumption using the current period's information. However, retired agents at time  $t$ , have chosen their old age consumption in the previous period

---

<sup>9</sup>See appendix D for the detailed derivation.

using the information that is available in the previous period. Therefore, compared to the standard DNK–IS equation, a richer dynamic system is achieved. Finally, Equation (2.17) and (2.18) show the log-linear form of stock price and dividend equations. The log-linearized equations of the supply side of the model are production function and the forward looking Phillips equation.

$$\hat{y}_t^p = \hat{l}_t \quad (2.20)$$

$$\pi_t = \tilde{\beta} E_t\{\pi_{t+1}\} + \tilde{\kappa} \widehat{mc}_t \quad (2.21)$$

where  $\widehat{mc}_t = \hat{w}_t - \hat{p}_t$  is the real marginal cost,  $\tilde{\beta} = \beta J$ ,  $J = \frac{1}{\beta} \left( \frac{\mathcal{M}-1+\mathcal{M}\beta}{\varphi\beta} \right)^\sigma$  and  $\tilde{\kappa} = \frac{(1-\theta)(1-\theta\tilde{\beta})}{\theta}$ . We can express the inflation as the discounted sum of current and expected future deviations of marginal costs from steady state by solving the above equation forward.

Due to the OLG setup in the demand side, we have an unconventional Phillips equation<sup>10</sup>. Differently from the standard Phillips equation, both weights of the expected inflation and the marginal cost depend on the old age dependency ratio. The finite lifetime of agents leads society to value the expected inflation less compared to the standard DNK framework.<sup>11</sup> Inflation depends more on the anticipated cost conditions rather than the current ones in an aging society. Notice also that an older economy implies a higher weight on the expected inflation compared to a younger economy. Therefore, inflation

---

<sup>10</sup>See appendix E for the detailed derivation

<sup>11</sup>With quarterly calibration of the parameters  $\beta = 0.99$  and  $\theta = 0.75$ , we get  $\kappa = 0.085$  for the standard model.

becomes more sensitive to marginal cost changes in a younger society.

The equilibrium is characterized by equations (2.15)–(2.21), together with a description of monetary policy.

## 2.3 Numerical Experiments

This section compares the dynamics of younger and relatively older economies after an unexpected monetary policy shock. First, to provide a clear insight, I will be discussing the responses of the variables to a positive monetary policy shock in an economy with a positive old age dependency ratio. Then, I will compare and elucidate the responses of economies with different demographic profiles to the same monetary policy shock. Simulations in this section are carried out relying on the standard calibration of the model’s structural parameters. Table 2 summarizes the calibration values. In this paper, I use the conventional calibration values for the parameters and all parameter values are determined according to their quarterly values.

This paper attempts to study the responses of variables to an unexpected monetary policy shock in economies with different demographic structures and the utilized model in this paper is sufficient to examine the proposed question qualitatively. However, to analyze the effects quantitatively, the demand side of the model should be constructed in a way where agents live for more periods. But I believe that the way this paper handles the question is an intermediate and also a required step to analyze and understand the dynamics of the monetary policy in an aging economy.

Figures in this section illustrate the responses of young and old age con-

sumption, output per worker, real wage, labor, inflation, per worker asset prices and nominal and real interest rates to a tightening (positive) monetary policy shock.

### **2.3.1 The Effects of a Monetary Policy Shock in a Model with a Positive Old Age Dependency Ratio**

Before comparing the responses of variables to a monetary policy shock in economies with different old age dependency ratios, I display and explain the responses to a tightening monetary policy shock in the proposed model economy. For this analysis, I set the old age dependency ratio to be 20%.<sup>12</sup>

It is worth emphasizing that the evaluation of the impulse responses is different in this study than the ones in the standard infinite living agent models. In the standard setup, there is only one type of representative agent and the response of his/her consumption to the shock is reflected along one graph. However, in this paper, there is a representative young agent and a representative old agent, hence the consumption decisions of these agents demonstrated in two graphs. Besides, the consumption decision of a representative young agent to an unexpected monetary policy shock is displayed in both young and old agents' consumption graphs. Suppose that a shock hits the economy at time  $t$ . The first graph (young age consumption) demonstrates the response of an agent to the shock when he/she is young at time  $t$ , and the second graph (old age consumption) reflects the ongoing response to the shock when the agent becomes retired at time  $t + 1$ . Therefore, the response that we observe

---

<sup>12</sup>20% old age dependency ratio represents an economy where 100 workers take care of 20 retired agent.

at  $t + 1$  in young age consumption graph belongs to an agent who is born at time  $t + 1$ , and he/she perceives the continuing effect of a monetary policy shock as a new shock. Moreover, the heterogenous agents setup enables us to study the two effects of monetary policy separately: the substitution and the wealth effect of the interest rate change. We observe the substitution effect in the young agents' and the wealth effect in old agents' graphs.

In Figure 2.2 we see that a tightening monetary policy shock leads young agents at the time of the shock to decrease their consumption level due to the consumption smoothing behavior. Since old agents income rely on financial assets, at the time of the shock, interest rate hike leads to an increase in their wealth and accordingly their consumption level. Aggregate demand is the weighted average of young and old age consumption. In such a young economy, the substitution effect is stronger than the wealth effect. Hence, similar to the the standard NK framework's outcome, we observe a decline in the level of output after the shock.

### **2.3.2 The Comparison of The Effects of a Monetary Policy Shock with Different Demographic Profiles**

In this section, with an aim to analyze the impact of aging on the effectiveness of monetary policy, I carry out the model with different old age dependency ratios. The blue solid line with dot, the red solid line with star and the black solid line show the responses of a young ( $\varphi = 20\%$ ), relatively older ( $\varphi = 50\%$ ) and oldest economy ( $\varphi = 80\%$ ), respectively. Percentages refer to old age dependency ratios.

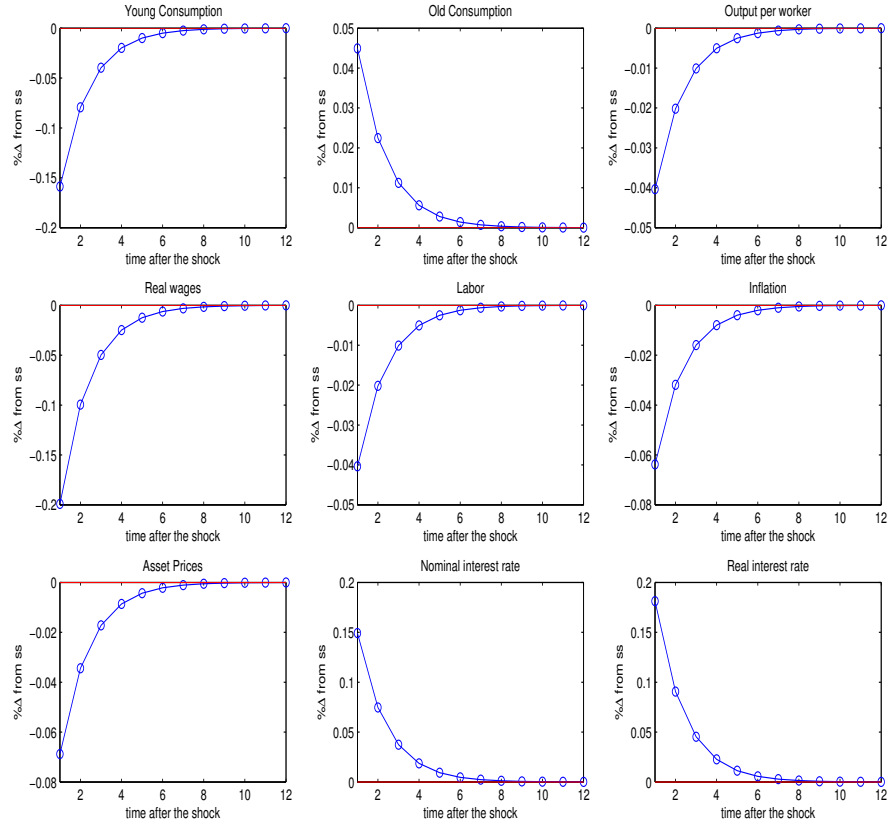


Figure 2.2: Impulse Responses to a Tightening Monetary Policy Shock in an Economy with 20% Old Age Dependency Ratio.

Figure 2.3 shows that in all types of economies a tightening monetary policy shock leads to a fall in the consumption level of the young agents due to the substitution effect and to an increase in the consumption level of old agents due to the wealth effect. The young agents' interest rate sensitivity is constant regardless of the demographic structure of the economy. On the other hand, we observe that the response of the level of consumption of old agents increases as the old age dependency increases. Since old agents income rely only on financial assets, an interest rate hike leads to an increase in their consumption level. We also notice that the real interest rate increases more as the society ages. This leads the income level, accordingly the consumption, of the old

agents to increase more in an older society.

On the supply side, output decreases due to a fall in total demand at the time of the shock. Depending on the demographic composition of the society, the magnitude of the decrease in output varies. As the population ages, the level of output decreases less. The reason is that, as the ratio of retired agents increases in the society, the dominance of the substitution effect decreases over the wealth effect and we observe a lower decrease in the level of output. In other words, the effectiveness of monetary policy on output decreases. Since, this is a demand-driven model and labor is the sole input in the consumption good production, demand for labor decreases following the magnitude of the decline in the level of output. Hence, real wages fall more in younger economy due to higher decline in the demand for labor. In DNK models, the source of inflation is marginal cost and the response of inflation follows the response of real wages. Accordingly, as the society ages, we observe a less decline in the response of inflation. Moreover, in older economies, the real per worker asset prices fall more compared to the younger economies due to the magnitude of the interest rate hike.

Results of the paper suggest that the monetary policy becomes less effective on controlling output and inflation. Moreover, the decrease in the effectiveness on output is more compared to inflation. Therefore, one can argue that the sacrifice ratio, which is the inflation ensued from a decline in the interest rate to stimulate the economy, increases as the society ages.<sup>13</sup> This result is consistent

---

<sup>13</sup>For example, when we normalize the responses of output for 20% and 80% old age dependency ratios, we get that 1% increase in the output leads to a 1.5% increase in inflation in an economy with 20% old age dependency ratio. However, the same increase in the inflation corresponds to 8% in an economy with 80% old age dependency ratio.

with the findings of Imam (2013).

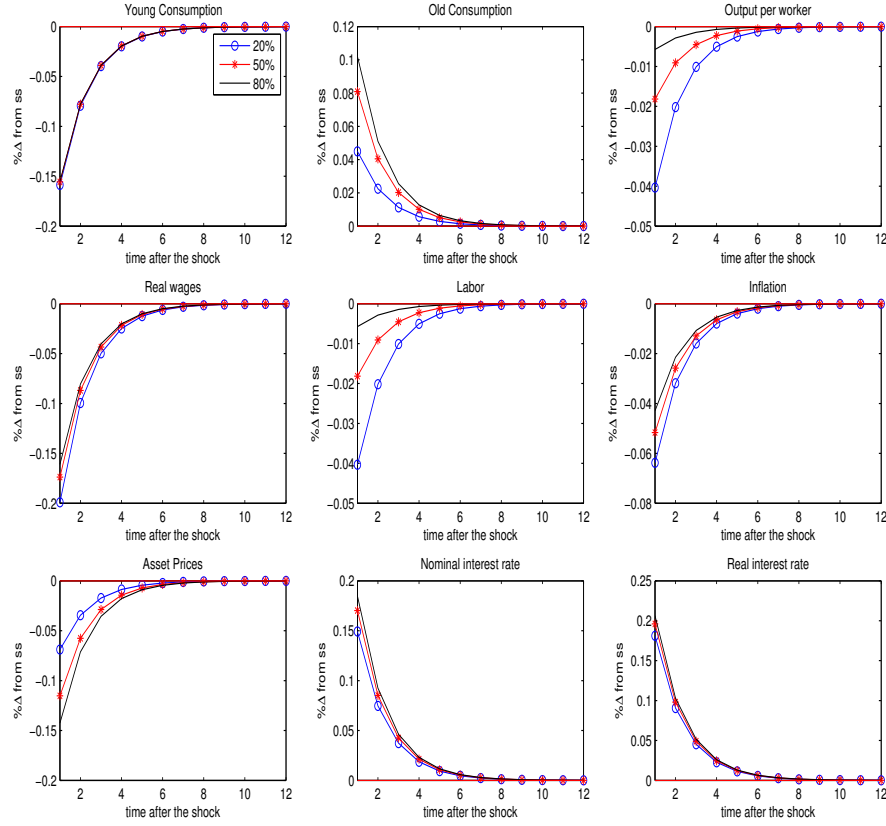


Figure 2.3: Impulse Responses to a Tightening Monetary Policy Shock in Economies with Different Old Age Dependency Ratios.

Figures 2.4a and 2.4b illustrate the attenuating effect of monetary policy on the level of output and inflation to a tightening monetary policy shock over time as the old age dependency ratio increases. In extreme cases, where old age dependency ratio is higher than 110%, we observe an increase in the response of the level of output to a tightening monetary policy shock.

To sum up, (1) the effectiveness of monetary policy on output and inflation decreases as the population ages. In other words, monetary policy authority becomes less effective in controlling output. To obtain the same effect on the economy, the policymaker should change the interest rates more as the old age

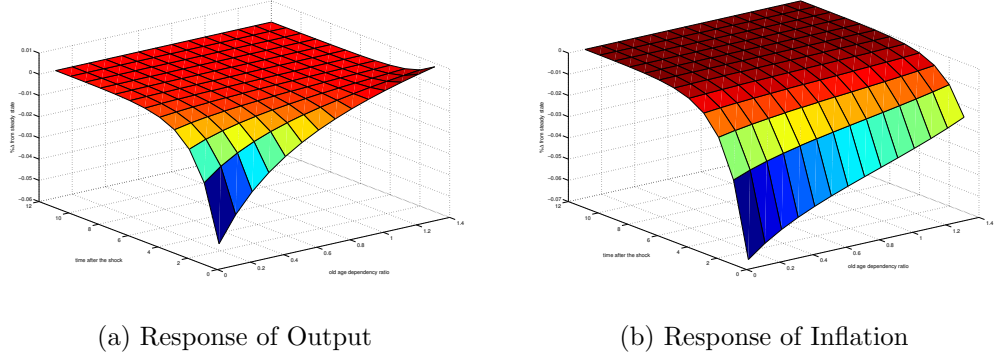


Figure 2.4: Responses to Contractionary Monetary Policy Shock

dependency ratio increases. (2) In extreme cases, tightening monetary policy shock may lead to an increase in output in an older economy due to the rise in the level of consumption of old agents. (3) The sacrifice ratio between inflation and output increases as the population ages.

## 2.4 Conclusion

All developed countries are aging. Demographic structure of developed countries has been changing permanently due to decline in the fertility and increase in the longevity rates. In few years, baby boom generation will retire and reinforce permanent the effect on demographic structure by increasing the ratio of old to young people. Aging can be related to economics through the saving behaviors of societies. Therefore, aging populations are important policy concerns and a correspondingly important academic research area. This paper argues that demographics should be expected to have an impact on the effectiveness and hence conduct of the monetary policy. In other words, this paper answers how should the Central Bankers perceive a grayer society and how should they react to it?

This paper provides a model which combines the standard New Keynesian framework of Clarida et al. (1999) and OLG setup of Samuelson (1958) and Diamond (1965) with aging. The overlapping generations structure of the demand side of the economy allows one to introduce aging into the model. On the supply side, a New Keynesian setup with nominal rigidities is required to analyze the effect of monetary policy. By utilizing the proposed model, this paper studies the effectiveness of monetary policy in an aging economy.

Main results of the experiments can be described as follows: First, as the population ages, the natural rate of interest decreases monotonically. Secondly, the effectiveness of the monetary policy on output and inflation decreases due to decreasing interest rate sensitivity of the society. The model also suggests that it would not even be a surprise if the economy becomes old enough we may see positive response of output to a tightening monetary policy shock. Finally, as the old age dependency ratio increases we observe that the decrease in the response of output is larger compared to that of inflation after a tightening monetary policy shock. Therefore, the sacrifice ratio- the inflation that ensued from boosting the economy- between inflation and output increases as the economy ages. Hence, results of the paper suggest that the policymakers should account for the demographic profile of the country when conducting monetary policies.

## CHAPTER 3

# A NEW KEYNESIAN MODEL WITH UNEMPLOYMENT: THE EFFECT OF ON-THE-JOB SEARCH

### 3.1 Motivation

Employer-to-employer flow is an important transition mechanism in the labor market and should not be disregarded in theoretical models. For instance, the empirical studies by Fallick and Fleischman (2001), Nagypal (2008), and Bjelland et al. (2011) showed that a significant part of the transitions in the labor market is employer-to-employer transitions.<sup>1</sup>

Before the influential and spurring study by Shimer (2005), the bulk of

---

<sup>1</sup>Fallick and Fleischman (2001) found that on average 2.6% of employed agents change their jobs each month. This number corresponds to the double of employment-to-unemployment flow in each month. It is also reported by Nagypal (2008) that almost half of the separations is job-to-job transitions. In a recent paper, two ratios are explicitly calculated by Bjelland et al. (2011) using the recently developed longitudinal linked employer and employee data from the Census Bureau's LEHD program. It is discovered that the ratio of the number of people experiencing an employer-to-employer transition to the total number of employees is 4.1% and the ratio of the same to the total number of people separating from their jobs is 27.3%. In earlier related studies, it is estimated by Diamond (1965) that hirings from other jobs is about 50% of hirings from unemployment, and it is pointed out by Pissarides (1994) that the fraction of new hires coming from other jobs is approximately 20% in the U.S. and 40% in the U.K..

relevant literature ignored employer-to-employer flow and mainly focused on the transitions from unemployment to employment. Shimer (2005) argued that the search and matching (SM) model of Mortensen-Pissarides is incapable of generating observed fluctuations in *the unemployment rate* and *labor market tightness* in response to a positive productivity shock. Stemming from this idea, some scholars concentrated on finding a way to amplify the impact of productivity shock on the unemployment rate. For example, in order to match the data, Shimer (2005) and Hall (2005) introduced real wage rigidities into the SM model. In another paper, Hagedorn and Manovskii (2008) analyzed the SM model using a different set of calibration values.

In SM-based models, a permanent positive productivity shock leads to a decrease in unemployment. Recently, this finding is challenged by several papers relying on an argument that the identification of productivity shock is problematic in SM-based models. Among these papers, Barnichon (2010) estimated the impact of a positive technology shock on labor market variables using structural VAR with long-run restrictions as in Galí (1999a), and found that a permanent increase in labor productivity (i.e., output per hour)<sup>2</sup> leads to an increase in unemployment and to a decrease in labor market tightness. In two other studies, Canova et al. (2009) utilized different labor market variables, whereas Clarida et al. (1999) employed structural VAR in a five-variable model. The findings of both papers are similar to those of Barnichon (2010). Following these results, we study a New Keynesian (NK) model in this paper. In particular, we formulate an extension to the model of Clarida et al. (1999)

---

<sup>2</sup>Since labor is the sole input of the production function, the terms labor productivity and output per hour can be interchangeably used.

by introducing *on-the-job search*.<sup>3</sup>

There is a bulk of theoretical studies focusing on the effect of job-to-job transitions on the real economy. For instance, Pissarides (1994) introduced on-the-job search which takes place only at short job tenures because of the accumulation of job-specific human capital, and found an increased jumpiness in vacancies and a dampened response of unemployment to changes in aggregate economic condition. Shimer (2006) focused on subgame perfect equilibria for the bargaining stage, noting that the standard strategic bargaining solutions are inapplicable due to the non-convexity of the set of feasible payoffs; and he characterized market equilibria in which more productive firms pay higher wages and analyzed the quantitative predictions of his model. Cahuc et al. (2006) formulated a model with strategic wage bargaining, on-the-job search, and counteroffers; and they estimated the influence of productivity, bargaining power, and between-firm competition on wages. Finally, Krause and Lubik (2007b) utilized a model with two types of firms and showed that on-the-job search is crucial for explaining the observed cyclical upgrading of workers to better employment opportunities in booms.

In a similar line of research, Van Zandweghe (2010) and Krause and Lubik (2007a) integrated on-the-job search into a business cycle model. It is assumed in both papers that there are two sectors and that the difference between these sectors lie within the firms' productivities and hiring costs. Van Zandweghe

---

<sup>3</sup>This is a highly tractable model allowing for a relatively simple and transparent analysis given that the related studies in the literature have richer models which are commonly solved through simulations. It is worth noting that a very similar model is used by Blanchard and Galí (2010). And, the reader is referred to Section 6 of Blanchard and Galí (2010) for a detailed literature review on studies that combine certain key elements of NK and SM models.

(2010) introduced price stickiness into the SM model of Krause and Lubik (2006), mainly concentrated on the monetary policy implications, and found that on-the-job search dampens the responses of real marginal cost and inflation after a tightening monetary policy shock. Afterwards, Krause and Lubik (2007a) built on the model of Van Zandweghe (2010) and assumed that matches become productive with a lag. Their main finding contradicts with the aforementioned results of Barnichon (2010) and Clarida et al. (1999).

To the best of our knowledge, the current model is the first business cycle model with on-the-job search which builds on the empirical finding that the levels of unemployment and productivity are positively correlated. We differ from the existing on-the-job search models not only in our motivation, but also in our way of model construction. The majority of earlier studies, including Van Zandweghe (2010) and Krause and Lubik (2007a), created wage dispersion by introducing different cost levels for different type of firms. Although we preserve this assumption in our model, there is an additional source of wage dispersion: the difference in bargaining powers of firms. As a matter of fact, we state this as the main source of wage dispersion. For this assumption, *labor unions* constitute a good motivation and support. It is well-known that individuals and firms are not the only actors in the labor market; but there are also intermediate associations whose aim is to protect their members' rights and to attain higher wage and lower unemployment levels for their members. In our model, we initially assume that there are two types of unions: *weak* union and *strong* union. Given this, a firm operating in a sector associated with the weak union would have relatively higher bargaining power in comparison

to that operating in a sector associated with the strong union. This *one-to-one* relation becomes helpful in simplifying the framework. In particular, it helps us to remove unions from the model considering the fact that unions will only have an indirect influence for which we do not have a particular interest. Accordingly, we argue that there are two types of firms; the ones facing a weak union and the ones facing a strong union. For the sake of simplicity, these types are referred to as *aggressive* and *passive*, respectively. As it turns out, this *simplified* model is enough to capture the fluctuations in the labor market.<sup>4</sup>

As shown by Barnichon (2010), a NK model with search frictions is capable of fulfilling two arguments of *the Shimer puzzle*: A positive technology shock leads to an increase in the unemployment rate and to a decrease in labor market tightness. Be that as it may, it is also discussed by the author that the Shimer puzzle is still visible because the magnitude of responses is below its empirical counterparts. In this paper, we aim to amplify the responses of the unemployment rate and labor market tightness. In the standard NK model, firms are demand constrained so that an increase in productivity leads to a sluggish adjustment in aggregate demand to the new productivity level due to nominal rigidities. Accordingly, firms employ less labor during this process. Hence, a positive change in technology leads to an increase in unemployment. When on-the-job search is introduced, a positive technology shock leads not only to a decrease in the flow of unemployment-to-employment, but also to a decrease in the flow of employment-to-employment. This is why posted vacancies decrease more compared to the model without on-the-job search. Accordingly,

---

<sup>4</sup>It is worth noting that although the model does not include unions *per se*, such an interpretation always exists due to the aforementioned one-to-one relation between the types of unions and the types of firms.

on-the-job search fundamentally amplifies the responses of unemployment.

We conduct three experiments. Firstly, we compare the responses of model parameters to a positive technology shock in the models with and without on-the-job search. Secondly, as we introduce on-the-job search intensity as the fraction of workers who are allowed to search for new jobs, we analyze the effect of an increase in search intensity on model dynamics. Finally, by changing the bargaining power of aggressive firms, we examine the effectiveness of bargaining power on labor market dynamics. These three experiments lead to the following results: (1) The volatilities of the unemployment rate and labor market tightness in the model with on-the-job search are higher compared to the model without on-the-job search. Consequently, since the volatilities of these two variables increase, it can be argued that the introduction of on-the-job search contributes to the related literature. Note that this constitutes a crucial part of the Shimer puzzle. (2) The response of the unemployment rate increases as on-the-job search intensity increases. (3) The response of the unemployment rate increases as the bargaining power of aggressive firms increases.

The structure of the paper is as follows. In the following section, we formulate the model specifying the differences between our model and that of Clarida et al. (1999). Moreover, we solve the model and present the calibration values which will be used in the following section. In Section 3, we report the results of the three experiments. Section 4 concludes.

## 3.2 Model with On-the-job Search

In this section, we describe the model structure. We extend the model of Clarida et al. (1999) by introducing on-the-job search for a particular group of households. In that regard, we assume two types of firms offering different wage levels. In other words, there is a wage dispersion in the economy. Accordingly, workers who earn relatively less would be willing to search for better-paid jobs. In this paper, unlike the existing literature, we capture wage dispersion through heterogeneity in bargaining powers of firms. We assume two types of firms: *aggressive* ( $A$ ) and *passive* ( $P$ ). The bargaining power of aggressive firms over workers is assumed to be greater than that of passive firms. As a result, passive firms offer higher wage levels compared to aggressive firms.

There are five assumptions about job search in this paper:

(i) Search is *indirect*: Job-seekers do not know the type of firms during job search.

(ii) The outside option of individuals is unemployment regardless of their prior-to-search state in the labor market. Put differently, if an on-the-job searcher is matched with a new firm, bargaining process does not start unless the individual resigns from his/her job.<sup>5</sup>

(iii) Wages are *flexible*: A worker's wage is updated every period as if he/she is newly matched. Hence, a worker at a passive firm has no incentive from on-the-job search. It then follows as a fact that an on-

---

<sup>5</sup>This assumption is, in fact, quite standard in the literature. A technical reason behind this assumption is that if the outside option of an on-the-job searcher is his/her current job, then there would be a continuum of wage levels which harms the simplicity of the model.

the-job searcher was a worker at an aggressive firm at the end of the previous period.

(iv) It is obvious that if an on-the-job searcher matches with a passive firm, then he/she prefers to resign from her existing job. If the new match is an aggressive firm, however, he/she would be indifferent. Here we assume that a matched on-the-job searcher resigns for sure to negotiate with the new match.<sup>6</sup>

(v) The job finding rates are different for on-the-job searchers (denoted by  $p_t$ ) and for unemployed agents (denoted by  $q_t$ ).<sup>7</sup>

The sequence of events is as follows: At the beginning of a period, firms announce vacancies. Unemployed agents and workers at an aggressive firm search for jobs. Both types of job search take place at the same time. Afterwards, matched parties start bargaining wages.<sup>8</sup> If both parties agree on a wage level, then the individual starts working at that particular period. Individuals work until the end of the period at which they may get separated by an exogenous rate of  $\delta \in [0, 1]$ . Those who are separated are unemployed at the beginning of the next period. If an individual does not get separated, he/she works at the same firm in the next period; unless he/she searches on-the-job and is matched with a new firm.

Finally, we assume that only a fraction  $\varphi \in [0, 1]$  of workers are allowed to search on-the-job. It is worth emphasizing here that this is not a restriction,

---

<sup>6</sup>Note that this assumption does not cause a qualitative difference in our results.

<sup>7</sup>In Section 3.2.5, we determine the steady state values of model parameters through calibration. It turns out that  $q$  is mostly over  $p$ . Furthermore, in Section 3.2, the sign of the response of the unemployment rate to a positive technology shock appears to be independent of this relation between  $p$  and  $q$ .

<sup>8</sup>It is worth noting that we determine wages via a simple surplus splitting rule.

but it enables us to examine labor market dynamics at any level of on-the-job search intensity.<sup>9</sup>

### 3.2.1 Households

We assume that the economy is populated by a continuum of households of measure one and that the representative household is a member of a large family.<sup>10</sup> The large family assumption enables us to assume full risk sharing within the family. This helps us avoid distributional issues which may arise due to heterogeneity of firms. The representative family maximizes the objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log(C_t) - \chi \frac{N_t^{1+\phi}}{1+\phi} \right)$$

where  $\beta \in [0, 1]$  is the discount factor and  $\phi$  indicates the inverse of the Frisch labor supply elasticity. The standard utility function shows that agents get utility from consumption and disutility from supplying labor, and that there is no burden of unemployed agents in terms of utility. The aggregate consumption level for different types of consumption goods are denoted by

$$C_t = \left( \int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

and the fraction of employed agents is shown by  $N_t$ . In this model, we assume full participation of households meaning that all agents are either employed or unemployed (but willing to work). Denoting the fraction of unemployed

---

<sup>9</sup>As a matter of fact, for our main result, we concentrate on the case  $\varphi = 1$  which means that any worker at an aggressive firm can search on-the-job. We specifically utilize on-the-job search intensity in Section 3.2 where we analyze its effect on the response of the unemployment rate to a positive technology shock.

<sup>10</sup>See Merz (1995).

agents by  $u_t$ , this implies that  $N_t + u_t = 1$ .

The budget constraint of the family is given by

$$\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + \int_0^1 W_t^A(j)N_t^A(j)dj + \int_0^1 W_t^P(j)N_t^P(j)dj + \Pi_t$$

where  $P_t(i)$  is the price of good  $i$  and  $B_t$  denotes one-period riskless nominal bond holdings of a family member at the price of  $Q_t$ . The nominal wage level paid by aggressive and passive firms are denoted by  $W_t^A$  and  $W_t^P$ , respectively.  $N_t^A$  and  $N_t^P$  are the fraction of employed agents working at aggressive and passive firms, respectively. The transfers and profit of final good firms are embedded in  $\Pi_t$ .

The optimal condition for individual goods is given by

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \quad (3.1)$$

where

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

is the price index for consumption goods. Consequently, the Euler equation is given by

$$Q_t = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\}.$$

It is worth noting that in this model, differently from the standard business cycle models, wages are not adjusted according to the labor supply decision of the household. As an attribute of the SM model, wages are demand-determined and set according to a bilateral bargaining between workers and firms.

### 3.2.2 Firms

There are intermediate goods and final goods sectors. Unlike the conventional NK models, price stickiness is introduced at the final good production stage.<sup>11</sup> Intermediate goods sector is perfectly competitive and the sole factor of production is labor. As we discussed earlier, to analyze the effect of on-the-job search, we assume two types of intermediate goods producers: aggressive and passive firms. These firms differ in their bargaining powers under the assumption that the (relative) bargaining power of aggressive firms over workers is higher compared to that of passive firms.

#### Final Goods Producers

The final goods sector is monopolistically competitive, and firms are indexed by  $i \in [0, 1]$ . They produce differentiated goods by utilizing an identical technology and use intermediate goods produced by aggressive and passive firms as inputs. The production function of a final good producer is

$$Y_t(i) = (Z_t^A(i))^\gamma (Z_t^P(i))^{1-\gamma}$$

where  $Z_t^A(i)$  and  $Z_t^P(i)$  denote the quantity of intermediate goods produced by aggressive and passive firms, respectively. The weight parameter  $\gamma \in (0, 1)$  is the share of intermediate goods produced by aggressive firms.

Under the assumption of flexible prices, final good producers would set the price of their good optimally subject to the demand equation (3.1) at each

---

<sup>11</sup>The assumption is first proposed by Walsh (2005) and used by Blanchard and Galí (2010), Clarida et al. (1999), and other papers with labor market frictions in DSGE models.

period. Therefore, the profit maximization condition suggests that  $P_t(i) = \mathcal{M}MC_t$  where  $MC_t$  denotes the real marginal cost and  $\mathcal{M} = \frac{\epsilon-1}{\epsilon}$  is the desired markup. The quantity demanded for intermediate goods  $Z_t^A$  and  $Z_t^P$  are given by

$$Z_t^A = \gamma \frac{P_t(i)}{P_t^A} Y_t(i) \quad Z_t^P = (1 - \gamma) \frac{P_t(i)}{P_t^P} Y_t(i)$$

where  $P_t^A$  and  $P_t^P$  are the price levels of intermediate goods produced by aggressive and passive firms, respectively. Accordingly, the real marginal cost of production is the weighted average of input prices:

$$MC_t = \left( \frac{P_t^A}{\gamma P_t} \right)^\gamma \left( \frac{P_t^P}{(1 - \gamma) P_t} \right)^{1-\gamma}. \quad (3.2)$$

Final good producers set the price of their goods to maximize the expected discounted profits due to Calvo (1983) type price-setting. At each period, a firm is able to reset its price with probability  $1 - \theta$ . This implies that the price levels of the  $\theta$  fraction of final good producers remain constant in any given period. The re-optimizing firms' price level is

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_0^\infty \theta^k \Lambda_{t,t+k} P_{t+k}^\epsilon Y_{t+k} MC_{t+k}}{\sum_0^\infty \theta^k \Lambda_{t,t+k} P_{t+k}^{\epsilon-1} Y_{t+k}} \quad (3.3)$$

where  $\Lambda_{t,t+k} = \beta^k (C_t/C_{t+k})$  denotes the stochastic discount factor.

### Intermediate Goods Producers

There are two types of intermediate goods firms in a perfectly competitive environment. The technology  $A_t$  is exogenous and common to all types of firms.

We assume that  $a_t = \log(A_t)$  follows an  $AR(1)$  process with an autoregressive

coefficient  $\rho_a$  and a variance  $\sigma_a^2$ . The production functions of aggressive and passive firms are, respectively, given by

$$Z_t^A = A_t N_t^A \quad Z_t^P = A_t N_t^P.$$

As in the model of Barnichon (2010), since labor is the sole input of the production functions, the terms labor productivity and output per hour can be interchangeably used in our model.

Employment in aggressive and passive firms evolve according to

$$N_t^A = (1 - \delta)N_{t-1}^A + H_t^A \quad N_t^P = (1 - \delta)N_{t-1}^P + H_t^P$$

where  $H_t^A$  and  $H_t^P$  are the newly hired agents in aggressive and passive firms at time  $t$ , respectively. Finally, as in Blanchard and Galí (2010) and Clarida et al. (1999), we assume that new hires start working in the period they are hired.

## Labor Market

The main difference between the two types of intermediate good producers is their bargaining powers over workers. In particular, the bargaining power of aggressive firms ( $\xi^A$ ) is greater than that of passive firms ( $\xi^P$ ). Given the above-defined production functions, the wage level in aggressive firms turns out to be less than the one in passive firms. As a consequence, individuals working at aggressive firms prefer searching for new jobs hoping to match with

a passive firm; but only a fraction of these workers is allowed.<sup>12</sup>

At the beginning of period  $t$ , there is a pool of on-the-job searchers and unemployed agents which are respectively denoted by  $OJS_t$  and  $U_t^0$ . We have

$$OJS_t = \varphi(1 - \delta)N_{t-1}^A \quad U_t^0 = 1 - (1 - \delta)N_{t-1}.$$

Accordingly, we can define job searchers at period  $t$  as  $Pool_t = U_t^0 + OJS_t$ . Total hiring at period  $t$  is denoted by  $H_t$ . Differently from Blanchard and Galí (2010) and Clarida et al. (1999), the number of posted vacancies is not equal to the number of newly hired workers in aggressive firms. The reason is that there is a reallocation of workers in aggressive firms. In particular, we can define the posted vacancies in aggressive firms as

$$V_t^A = H_t^A + \varphi p_t(1 - \delta)N_{t-1}^A$$

where  $H_t^A$  indicates the change in the number of workers in aggressive firms. The rest of the equation shows the reallocation of workers due to on-the-job search. The number of newly hired workers from the pool of on-the-job searchers is defined as  $H_t^o = p_t OJS_t$ . On the passive firm side, however, the number of posted vacancies is equal to the number of newly hired workers since there is no reallocation of workers. Therefore, we have  $V_t^P = H_t^P$ .

In our model, labor market tightness and the “average” job finding rate of

---

<sup>12</sup>Our model differs from those of Van Zandweghe (2010) and Krause and Lubik (2007b) at this point. To generate wage dispersion, those authors assumed a difference in firms’ productivity levels. Since high-productivity firms offer higher wage level, employed agents prefer switching to them.

job searchers can be interchangeably used. They are defined as

$$x_t = \frac{H_t}{Pool_t} = \frac{H_t^o + H_t^u}{U_t^0 + OJS_t}$$

where  $H_t^u = q_t U_t^0$  is the fraction of newly hired workers from the unemployment pool. Moreover, we define the end-of-period unemployment rate as  $U_t = 1 - N_t$  and assume that the relation between the job finding rate of unemployed agents and that of on-the job searchers is  $p_t = q_t^\zeta$ .<sup>13</sup>

Following Blanchard and Galí (2010) and Clarida et al. (1999), the cost for posting a vacancy is defined in terms of the CES bundle of final goods.<sup>14</sup> The cost per vacancy is an increasing function of the technology level and the corresponding ratio of posted vacancies to the pool of job searchers. In particular, we assume

$$G_t^A = A_t B \left( \frac{V_t^A}{Pool_t} \right)^\alpha \quad G_t^P = A_t B \left( \frac{H_t^P}{Pool_t} \right)^\alpha$$

where  $\alpha \geq 0$  and  $B$  is a positive constant.<sup>15</sup>

## Price Setting

Let  $P_t^A$  and  $P_t^P$  denote the price levels of the intermediate goods produced by aggressive and passive firms, respectively. These prices are taken as given.

---

<sup>13</sup>The motivation behind this assumption is that after a positive shock to the economy, the responses of both job finding rates are of the same nature.

<sup>14</sup>In Blanchard and Galí (2010) and Clarida et al. (1999), the number of vacancies is equal to the number of newly hired workers. However, in our model, they are different due to the reallocation of newly hired on-the-job searchers. Therefore, we use “cost for posting a vacancy” rather than the concept of cost for hiring.

<sup>15</sup>The reader is referred to Blanchard and Galí (2010) and Clarida et al. (1999) for a discussion on this issue.

Moreover, let  $W_t^A$  and  $W_t^P$  represent the nominal wage levels of aggressive and passive firms, respectively. Profit maximization requires the following conditions to be satisfied for all  $t$ :

$$\left(\frac{P_t^A}{P_t}\right) A_t = \frac{W_t^A}{P_t} + G_t^A - \beta(1 - \delta)E_t \left\{ \frac{C_t}{C_{t+1}}(1 - \varphi p_{t+1})G_{t+1}^A \right\} \quad (3.4)$$

$$\left(\frac{P_t^P}{P_t}\right) A_t = \frac{W_t^P}{P_t} + G_t^P - \beta(1 - \delta)E_t \left\{ \frac{C_t}{C_{t+1}}G_{t+1}^P \right\} \quad (3.5)$$

Note that the LHS of the equations (3.4) and (3.5) represent the real marginal product of labor, and their RHS denote the real marginal cost including vacancy posting costs. It is worth noting that the equation (3.5) resembles the findings of Clarida et al. (1999). However, our introduction of on-the-job search leads an additional term to appear in the equation (3.4). We can define the net vacancy posting costs of aggressive and passive firms as

$$B_t^A = G_t^A - (1 - \delta)E_t \{ \Lambda_{t,t+1}(1 - \varphi p_{t+1})G_{t+1}^A \}$$

$$B_t^P = G_t^P - (1 - \delta)E_t \{ \Lambda_{t,t+1}G_{t+1}^P \}$$

As a result, *ceteris paribus*, an increase in  $\varphi$  leads to an increase in the net vacancy posting cost of aggressive firms. We then rewrite the equations (3.4) and (3.5) as follows:

$$\left(\frac{P_t^A}{P_t}\right) A_t = \frac{W_t^A}{P_t} + B_t^A \quad (3.6)$$

$$\left(\frac{P_t^P}{P_t}\right) A_t = \frac{W_t^P}{P_t} + B_t^P \quad (3.7)$$

Finally, we describe the price dynamics. Plugging the log-linearized version

of the equation (3.3) into the (log-linearized) law of motion for the aggregate price level,<sup>16</sup> we have

$$\mathbf{p}_t = \theta \mathbf{p}_{t-1} + (1 - \theta) \mathbf{p}_t^*$$

where  $\mathbf{p}$  denotes the log-linearized aggregate price level.<sup>17</sup> The dynamic Phillips equation is derived as

$$\pi_t = \beta E_t \{\pi_{t+1}\} - \kappa \hat{\mu}_t$$

where  $\kappa = (1 - \theta)(1 - \theta\beta)/\theta$  and  $\hat{\mu}_t$  denotes the deviation of the average price markup from its steady state value. We write the marginal cost by log-linearizing the equation (3.2) and plugging it into  $\hat{\mu}_t = \mathbf{p}_t - mc_t - \mu$ , which is derived from the log-linearization of  $P_t(i) = \mathcal{M}MC_t$ . It then follows that

$$\hat{\mu}_t = \mu_t - \mu = \gamma \mu_t^A + (1 - \gamma) \mu_t^P - \mu$$

where  $\mu = \log \mathcal{M}$ .

## Wage Determination

When on-the-job search is introduced, the set of feasible payoffs is typically non-convex. Hence, the axiomatic Nash bargaining solution and the standard strategic bargaining solutions may be inapplicable (see Shimer, 2006, pg. 815). Keeping this in mind, we impose a surplus splitting rule.

In this model, wages are *flexible*. That is, each type of firm negotiates the wage level with its workers every period. For a representative family member,

---

<sup>16</sup>See Chapter 3 of Galí (2008b) for log-linearization and detailed derivations.

<sup>17</sup>In this paper, we use lower case letters for the log transformations of the corresponding variables. However, since  $\mathbf{p}$  denotes the job finding rate of on-the-job searchers, we make an exception for the log-linearized aggregate price level.

the value resulting from being a worker at an aggressive firm is given by

$$\begin{aligned}
V_t^{NA} = & \frac{W_t^A}{P_t} - \chi C_t N_t^\phi + E_t \left\{ \Lambda_{t,t+1} \left[ \delta(1 - q_{t+1}) V_{t+1}^U \right. \right. \\
& + \left( \delta q_{t+1} \tau_{t+1} + (1 - \delta)(\varphi(p_{t+1} \tau_{t+1} + (1 - p_{t+1})) + (1 - \varphi)) \right) V_{t+1}^{NA} \\
& \left. \left. + \left( \delta q_{t+1}(1 - \tau_{t+1}) + (1 - \delta)\varphi p_{t+1}(1 - \tau_{t+1}) \right) V_{t+1}^{NP} \right] \right\}.
\end{aligned}$$

where  $\tau$  is the probability of a job searcher matching with an aggressive firm.

For a representative family member, the value resulting from being a worker at a passive firm is given by

$$\begin{aligned}
V_t^{NP} = & \frac{W_t^P}{P_t} - \chi C_t N_t^\phi + E_t \left\{ \Lambda_{t,t+1} \left[ \delta(1 - q_{t+1}) V_{t+1}^U \right. \right. \\
& \left. \left. + \left( \delta q_{t+1} \tau_{t+1} \right) V_{t+1}^{NA} + \left( \delta q_{t+1}(1 - \tau_{t+1}) + (1 - \delta) \right) V_{t+1}^{NP} \right] \right\}.
\end{aligned}$$

Moreover, the value of an unemployed agent is

$$V_t^U = E_t \left\{ \Lambda_{t,t+1} \left[ (1 - q_{t+1}) V_{t+1}^U + q_{t+1}(\tau_{t+1} V_{t+1}^{NA} + (1 - \tau_{t+1}) V_{t+1}^{NP}) \right] \right\}.$$

Accordingly, we define the surplus of a representative family member resulting from an employment relationship with an aggressive firm as  $S_t^{HA} = V_t^{NA} - V_t^U$  and that with a passive firm as  $S_t^{HP} = V_t^{NP} - V_t^U$ , which in turn implies the following equations:

$$\begin{aligned}
S_t^{HA} = & \frac{W_t^A}{P_t} - \chi C_t N_t^\phi + (1 - \delta) E_t \left\{ \Lambda_{t,t+1} \left[ \left( \varphi p_{t+1}(\tau_{t+1} - 1) + 1 \right. \right. \right. \\
& \left. \left. \left. - q_{t+1} \tau_{t+1} \right) S_{t+1}^{HA} + \left( \varphi p_{t+1}(1 - \tau_{t+1}) - q_{t+1}(1 - \tau_{t+1}) \right) S_{t+1}^{HP} \right] \right\}.
\end{aligned}$$

$$\begin{aligned}
S_t^{HP} &= \frac{W_t^P}{P_t} - \chi C_t N_t^\phi + (1 - \delta) E_t \left\{ \Lambda_{t,t+1} \left[ \left( 1 - q_{t+1} (1 - \tau_{t+1}) \right) S_{t+1}^{HP} \right. \right. \\
&\quad \left. \left. - \left( q_{t+1} \tau_{t+1} \right) S_{t+1}^{HA} \right] \right\}.
\end{aligned}$$

The surpluses of aggressive and passive firms from the profit maximization conditions, which are respectively denoted by  $S_t^{FA}$  and  $S_t^{FP}$ , are given by

$$S_t^{FA} = MRP N_t^A - \frac{W_t^A}{P_t} + (1 - \delta) E_t \left\{ \Lambda_{t,t+1} (1 - \varphi p_{t+1}) S_{t+1}^{FA} \right\} \quad (3.8)$$

$$S_t^{FP} = MRP N_t^P - \frac{W_t^P}{P_t} + (1 - \delta) E_t \left\{ \Lambda_{t,t+1} S_{t+1}^{FP} \right\} \quad (3.9)$$

where  $MRPN_t^A = (P_t^A/P_t)A_t$  and  $MRPN_t^P = (P_t^P/P_t)A_t$  stand for the marginal productivity of labor in aggressive and passive firms, respectively. It follows from the equations (3.4) and (3.8) that  $S_t^{FA} = G_t^A$  and from the equations (3.5) and (3.9) that  $S_t^{FP} = G_t^P$ .

Recalling that  $\xi^A$  and  $\xi^P$  denote the bargaining powers of aggressive and passive firms, respectively, the surplus splitting rule stipulates that firms and workers determine the wage levels according to the following maximization problems:

$$\begin{aligned}
\max_{W_t^A} (S_t^{HA}(j))^{1-\xi^A} (S_t^F(j))^{\xi^A} & \quad \max_{W_t^P} (S_t^{HP}(j))^{1-\xi^P} (S_t^F(j))^{\xi^P}
\end{aligned}$$

subject to the corresponding value functions. The solutions to these maximization problems are as follows:

$$(1 - \xi^A) S_t^F(j) = \xi^A S_t^{HA}(j) \quad (1 - \xi^P) S_t^F(j) = \xi^P S_t^{HP}(j).$$

The real wage levels in aggressive and passive firms are

$$\begin{aligned}
\frac{W_t^A}{P_t} &= MRS_t + \eta^A \left( G_t^A - \beta(1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} (\varphi p_{t+1} (\tau_{t+1} - 1) + 1 \right. \right. \\
&\quad \left. \left. - q_{t+1} \tau_{t+1}) G_{t+1}^A \right\} \right) - \eta^P \left( \beta(1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} (\varphi p_{t+1} (1 - \tau_{t+1}) \right. \right. \\
&\quad \left. \left. - q_{t+1} (1 - \tau_{t+1})) G_{t+1}^P \right\} \right)
\end{aligned} \tag{3.10}$$

and

$$\begin{aligned}
\frac{W_t^P}{P_t} &= MRS_t + \eta^P \left( G_t^P - \beta(1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} (1 - q_{t+1} (1 - \tau_{t+1})) G_{t+1}^P \right\} \right) \\
&\quad + \eta^A \left( \beta(1 - \delta) E_t \left\{ \frac{C_t}{C_{t+1}} (q_{t+1} \tau_{t+1}) G_{t+1}^A \right\} \right)
\end{aligned} \tag{3.11}$$

where  $\eta^A = (1 - \xi^A)/\xi^A$  and  $\eta^P = (1 - \xi^P)/\xi^P$ .

In the standard models, the surplus splitting rule implies that the wage equation is a convex combination of marginal rate of substitution and marginal productivity of labor. However, due to firm heterogeneity and the introduction of on-the-job search, we have additional terms in the wage equations. In particular, the equation (3.10) shows that an increase in the ratio of on-the-job searchers leads to a decrease in the wage level in aggressive firms. The reason is that high ratio of on-the-job searchers causes an increase in the future cost of hiring in aggressive firms, thereby leading aggressive firms to decrease their current period's labor cost considering higher future hiring costs.

### 3.2.3 Technological Process and Monetary Policy

The monetary policy is assumed to follow Taylor type interest rule represented by

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

where  $i_t$  is the nominal interest rate and  $\rho = -\log(\beta)$ . We assume that  $a_t = \log A_t$  follows an  $AR(1)$  process

$$a_t = \rho_a a_{t-1} + \epsilon_t^a$$

where  $\epsilon_t^a$  denotes the technology shock and  $\rho_a$  is the autoregressive coefficient of the technological process.

### 3.2.4 Market Clearing Conditions and Solving the Model

In this section, we emphasize certain characteristics of the model. First, the steady state is independent of the monetary policy rule and the degree of price stickiness. Second, we assume that the level of technology  $A = 1$  at the steady state. Finally, following Blanchard and Galí (2010) and Clarida et al. (1999), we assume that hiring costs are paid in terms of final goods. This implies that the goods market clearing condition is

$$Y_t = C_t + G_t^A V_t^A + G_t^P V_t^P$$

at the equilibrium. To solve the model, we log-linearize the system of equations around zero inflation steady state. See the Appendix for the log-linearized

equations.

### 3.2.5 Calibration and the Steady State

In this section, the calibration of model parameters is illustrated. Here we use the conventional parameter values if they do not contradict with the model. All parameter values are determined according to quarterly values. We set the discount factor  $\beta$  to 0.99 and the inverse of the Frisch labor supply elasticity  $\phi$  to 5. The parameter for price stickiness is set to be its average duration in one year:  $\theta = 0.75$ . The gross markup of prices over marginal cost value  $\mathcal{M}$  is taken as 1.2. Following Blanchard and Galí (2010), we set the parameter  $\alpha$  in the hiring cost function to 1.<sup>18</sup> Moreover, having no evidence on the share of intermediate goods produced by aggressive firms in the production of final goods, we set  $\gamma$  to 0.5.

To calibrate the labor market parameters, we first pin down the steady state values of unemployment and the job finding rate of unemployed agents using the average values in the postwar U.S. economy. Accordingly, the average value for  $u$  is 0.05 and the approximate value for  $q$  is 0.7. Since we assume full participation of agents, we have  $N = 1 - u = 0.95$ . The separation rate  $\delta$  is calculated using  $\delta = qu/((1 - q)N) \cong 0.12$ . As for the coefficients in the Taylor rule, we take the calibration values from the literature. In particular, we set  $\phi_\pi = 1.5$  and  $\phi_y = 0.125$ .

We now turn to the calibration of the parameters related to on-the-job

---

<sup>18</sup>As argued by Blanchard and Galí (2010), one can relate their hiring cost function with the matching function of the standard SM model. Accordingly, setting  $\alpha$  to 1 corresponds to a case where the elasticity of the matching function is 0.5; which is the standard calibration value for this parameter.

search. Following Bjelland et al. (2011), we assume the steady state value of the ratio  $H^o/N$  to be 0.041.<sup>19</sup> This means that the employment-to-employment flow is approximately 4% of employment in a given quarter. Since  $H^o = \varphi p(1 - \delta)N^A = 0.041N$ , this leaves us to determine the values of  $\varphi, p$ , and  $N^A$  satisfying this equality. Following that, we will calculate the bargaining powers of firms which depend on the steady state value of the number of workers employed at aggressive firms,  $N^A$ .

We conduct several experiments with different calibration values for some of the model parameters. In the first experiment, we compare the responses of labor market dynamics in the models with and without on-the-job search. To do so, since there is no hard evidence on the number of aggressive firms, we assume that  $N^A = N^P = 0.475$ . For the case without on-the-job search, we assume that  $\varphi = 0$  and  $\xi^A = \xi^P = 0.5$ . As for the case with on-the-job search, we assume  $\varphi = \xi^A = 1$  and calculate the corresponding values for  $p$  and  $\xi^P$ . In the second experiment, we analyze the impact of on-the-job search intensity on the labor market by gradually changing the level of intensity between 0 and 1. We take the values of  $N^A$ ,  $N^P$ ,  $\xi^A$ , and  $\xi^P$  from the first experiment; and we recalculate the parameter  $p$ . In the third experiment, we investigate the effect of relative change in bargaining powers on the model dynamics. We set  $\xi^P$  to 0.5 and assign different values for  $\xi^A$  between 0.6 and 1.

Finally, the values for  $B$ ,  $\chi$ , and  $\zeta$  differ for each exercise. All of the above calibration values are summarized in Table 3 and Table 4.

---

<sup>19</sup>We once again note that Bjelland et al. (2011) used the recently developed longitudinal linked employer and employee data from the Census Bureau's LEHD program.

### 3.3 The Experiments

As discussed earlier, the aim of this paper is to examine the effect of on-the-job search on labor market dynamics, especially on unemployment and labor market tightness. To do so, we conduct three experiments. Firstly, we compare the responses in the models with and without on-the-job search. Secondly, by assigning different values to on-the-job search intensity, we examine the impact of search intensity on the unemployment rate. Finally, by changing the bargaining power of aggressive firms, we analyze the influence of bargaining powers on labor market dynamics in this model economy.

#### 3.3.1 The Comparison of the Impulse Responses of the Models with and without On-the-job Search

In the first part of the analysis, we compare the responses of labor market variables to a one percentage point increase in technology in the models with and without on-the job search. This positive technology shock dies out gradually according to  $AR(1)$  process with an autoregressive coefficient  $\rho_a$  of 0.9. Here, we use the calibration values reported in the first part of Table 4.

Figure 3.1 shows the dynamic responses of the unemployment rate and labor market tightness. The solid line with plus signs and the blue line with dots demonstrate the responses of the corresponding variable in the models with and without on-the-job search, respectively. In particular, when we set the level of on-the-job search intensity to 0%, the level deviation of the unemployment rate is 0.007 in the first period, and when we set its level to 100%, then

the same response increases to 0.008. These correspond to approximately 14% and 16% increases in the unemployment rate, respectively. Furthermore, when we set the level of on-the-job search intensity to 0%, the percentage deviation of the labor market tightness is  $-0.061$  in the first period, and when we set its level to 100%, then the same response decreases to  $-0.165$ .

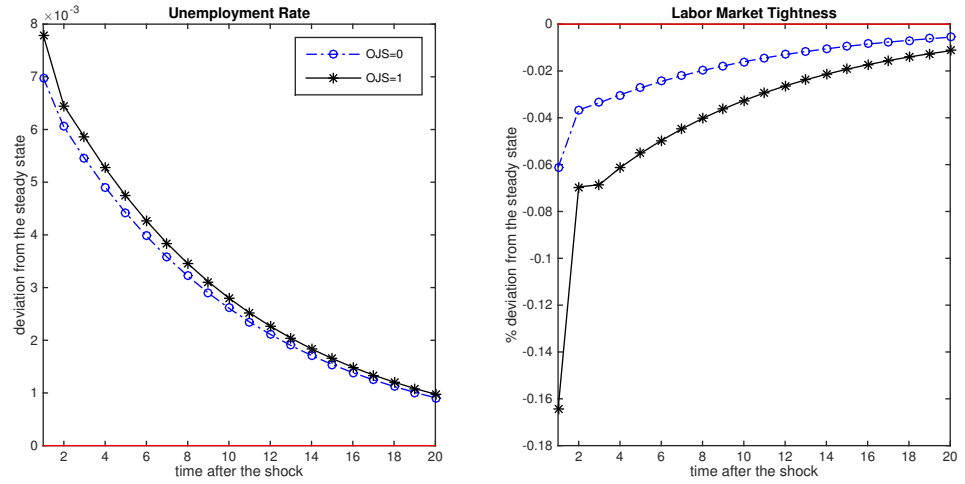


Figure 3.1: The Impulse Responses to a Positive Technology Shock

In the standard NK model, after a positive technology shock, we observe an increase in the unemployment rate. The mechanism behind this is as follows: After a positive technology shock, aggregate demand cannot adjust immediately due to nominal rigidities in the short run. Since firms become more productive, they decrease their demand for labor and post less vacancies. Consequently, the unemployment rate increases. Since employment-to-employment transitions is another channel affecting the flow of employment, the introduction of on-the-job search amplifies the increase in unemployment. More precisely, not only there is a decrease in vacancies posted by firms, but also a fraction of vacancies are filled by on-the-job searchers. Some of on-the-job

searchers rematches with aggressive firms, and some of them fills the positions posted by passive firms. The remaining vacancies are filled by unemployed agents. However, the number of vacancies filled by unemployed agents is less than the number of filled vacancies in the model without on-the-job search, which is even less than the number of separated agents. Moreover, in addition to the exogenous separation, the jobs left by these on-the-job searchers are destructed. Therefore, the unemployment pool expands, so that we observe a higher increase in the unemployment rate. Furthermore, the number of posted vacancies decreases more than the total number of job seekers after a productivity shock. We accordingly observe a higher decrease in labor market tightness compared to the model without on-the-job search.

Since we concentrate on the directions and the volatilities of the unemployment rate and labor market tightness, we just report the responses of these variables. Impulse response graphs of other variables are reported in Appendix F. For those variables, the differences between the responses in the models with and without on-the-job search are very small. The reason is that, following the calibration of Blanchard and Galí (2010), we assume a very small hiring cost to output ratio. Therefore, we do not observe significant differences in the responses of output, inflation, marginal cost, and average wage level.

### **3.3.2 The Effect of On-the-job Search Intensity on the Unemployment Rate**

The next analysis focuses on the effect of on-the-job search intensity on the unemployment rate. Here, we utilize the calibration values reported in the second

part of Table 4, which are consistent with the corresponding efficient steady state values. To perform the analysis, noting that on-the-job search intensity is exogenous to the system, we assign different values to the parameter (from  $\varphi = 10\%$  to  $\varphi = 100\%$ ). We then plot the responses of the unemployment rate to the above described one percent technology shock.

In this model, the response of the job finding rate of on-the-job searchers depends on the value of  $\varphi$ . To be more precise,  $\varphi$  determines the parameter  $\zeta$  which in turn influences the response of  $p_t$ . As it turns out, the calibrated value of  $\zeta$  is mostly greater than one,<sup>20</sup> and consequently  $p_t$  responds more compared to  $q_t$ . The intuition is that unemployed agents search for jobs independent of the state of the economy, whereas one would expect on-the-job searchers to be affected by the economic conditions. Accordingly, on-the-job search intensity indirectly determines the job destruction rate discussed in Mortensen and Pissarides (1994). All these effects can be interpreted as  $\varphi$  having an indirect endogenous effect on the model dynamics.

Figure 3.2 demonstrates the effect of on-the-job search intensity on the unemployment rate after a positive technology shock. As on-the-job search intensity increases, the volatility of the unemployment rate increases. In particular, when we set the level of on-the-job search intensity to 10%, the level deviation of the unemployment rate is 0.0072 in the first period, and when we set its level to 100%, then the same response increases to 0.0078. This follows from the fact that an increase in on-the-job search intensity leads the job finding rate of on-the-job searchers to respond more compared to the job finding

---

<sup>20</sup>For only one case,  $\zeta$  turns out to be less than one. Even in this case, we found a higher response of the unemployment rate compared to the model without on-the-job search.

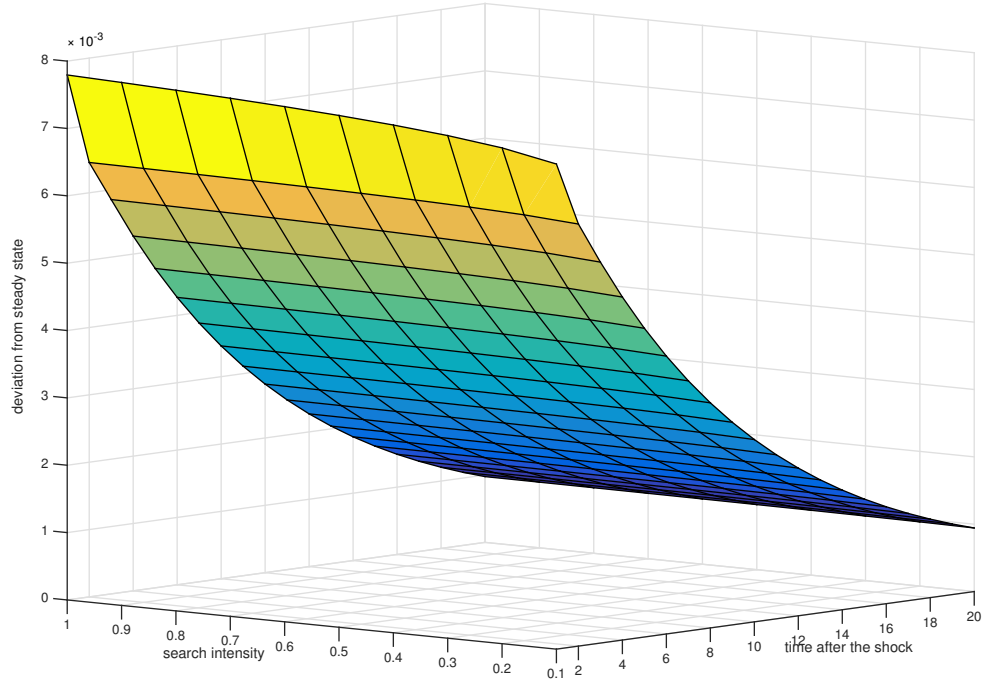


Figure 3.2: The Response of the Unemployment Rate to a Positive Technology Shock

rate of unemployed agents. More precisely, (i) the value of  $\zeta$  increases as  $\varphi$  increases; whereas (ii) the number of destructed jobs in response to a positive technology shock decreases as  $\varphi$  increases. Accordingly, not only there is a lower number of available vacancies, but also a fraction of these vacancies are refilled by on-the job searchers. Hence, the number of agents hired from the unemployment pool decreases. As a result, we observe a higher increase in the unemployment rate ensued from a positive productivity shock. The model is also capable of replicating a negative correlation between the job finding and job destruction rates (see Mortensen and Pissarides, 1994).

### 3.3.3 The Effect of Bargaining Power on the Unemployment Rate

Recently, Hagedorn and Manovskii (2008) proposed a different calibration strategy for bargaining power in the standard SM model, and they replicated the stylized facts of the labor market for very low values of workers' bargaining power. In this last exercise, we assume *more reasonable* values for bargaining power(s) and investigate the impact of bargaining power on the unemployment rate. The corresponding calibration values are reported in the last part of Table 4. To perform the analysis, we set the bargaining power of passive firms  $\xi^P$  to 0.5, which is the natural benchmark level in the literature. Restricted by the model assumption that the bargaining power of aggressive firms  $\xi^A$  is higher than  $\xi^P$ , we assign different values to  $\xi^A$  and plot the responses of the unemployment rate to a one percent technology shock.

Figure 3.3 illustrates that as the bargaining power of aggressive firms increases, the deviation of the unemployment rate from its steady state value increases after a positive technology shock. For example, when we set the bargaining power of aggressive firms to 0.6, the level deviation of the unemployment rate is 0.0070 in the first period, and when we set the bargaining power to 1, then the same response increases to 0.0077. This result can be interpreted as follows: If there is an increase in the bargaining power of aggressive firms, then aggressive firms get more share in the surplus-splitting phase. Accordingly, the responses of wages in aggressive and passive firms decrease due to higher  $\xi^A$ .<sup>21</sup> This leads to a further decrease in the levels of

---

<sup>21</sup>See the equations (3.10) and (3.11).

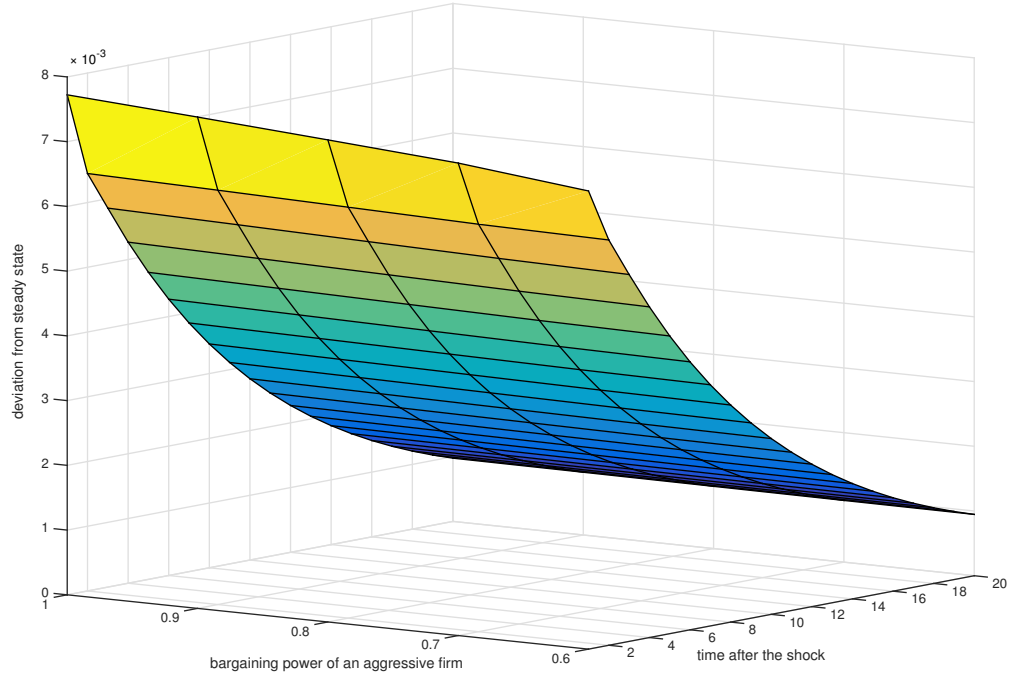


Figure 3.3: The Response of the Unemployment Rate to a Positive Technology Shock

consumption and demand. As a result, firms decrease their vacancies more, so that we observe a higher increase in the unemployment rate.

### 3.4 Conclusion

It is well-observed that a permanent increase in the productivity level leads to an increase in the unemployment rate and to a decrease in labor market tightness. Noting that SM-based models imply the opposite of these observations, Barnichon (2010) and Clarida et al. (1999) used a business cycle model with nominal rigidities and labor market frictions; and through their models, they were able to qualitatively replicate these empirical findings. Moreover, Barnichon (2014) recently replicated the magnitude of these responses.

In this paper, we introduce on-the-job search into the NK model suggested by Clarida et al. (1999). In that regard, we assume two-tier sector including firms with different bargaining powers, and we let a fraction of workers to do on-the-job search. This assumption amplifies the flow of employment and increases the volatilities of the unemployment rate and labor market tightness. Additionally, we conduct several experiments to demonstrate the roles of on-the-job search intensity and bargaining power. We found that both of them amplify the volatilities of the unemployment rate after a positive technology shock.

## CHAPTER 4

### REVISITING THE SHIMER PUZZLE: THE PERFORMANCE OF A NEW KEYNESIAN MODEL WITH ON-THE-JOB SEARCH

The search and matching (SM) model of Mortensen-Pissarides is considered as the backbone of the studies on labor market fluctuations. However, the seminal paper Shimer (2005) argues that the standard SM model is incompetent to generate the observed business cycle fluctuations in unemployment and vacancies, hence labor market tightness, in response to productivity shocks.

Following this criticism, some researchers focused on finding a way to amplify the impact of productivity shock on the unemployment rate. For instance, in order to match the data, Shimer (2005) and Hall (2005) introduced real wage rigidities into the SM model. On the other hand, Hagedorn and Manovskii (2008) argues that this problem can be solved by using different calibration values. These models are capable of amplifying the impact of productivity shocks on labor market variables. However, there is an additional shortcoming of the search and matching model which is discussed in the following section.

## 4.1 More on Shimer Puzzle

The search and matching models are challenged with several problems. The well known problem is that they undershoot the magnitude of the elasticities of unemployment and productivity and labor market tightness and productivity which is known as the Shimer (2005) puzzle. However, as stated by Barnichon (2014) a less emphasized issue of the model is the underlying signs of these elasticities. The Mortensen-Pissarides models uncover a positive correlation between unemployment and productivity. However, a structural VAR model displays a positive correlation conditional to technology shocks and a negative correlation to non-technology shocks. On the other hand, search and matching models find a positive correlation between labor market tightness and productivity. Whereas, a structural VAR model displays a negative correlation conditional to technology shocks and a positive correlation to non-technology shocks. It is also shown in Barnichon (2010) that since the mid 1980's the unconditional correlation between labor productivity and unemployment and labor productivity and labor market tightness have been positive and negative, respectively.

Barnichon (2010) states a New Keynesian model with search frictions fulfills the first argument of the Shimer (2005) puzzle: A positive technology shock leads to an increase in the unemployment rate and to a decrease in labor market tightness. However, it is also discussed by the author that the Shimer puzzle is still visible because the magnitude of responses is below its empirical counterparts. Chapter 3 of this dissertation qualitatively shows that introduction of on-the-job search to the standard New Keynesian (NK) model

with labor market frictions amplifies the impact of productivity shock. The aim of this essay is to revisit the Shimer (2005) puzzle. First, the stylized facts of U.S. labor market for the period 1951-2014 are presented both using raw data and empirical techniques. Then, the performance of the proposed model in Chapter 3 is tested.

The results of this essay are as follows. The bivariate structural VAR models suggest there is a positive correlation between productivity and unemployment and negative one between productivity and labor market tightness. Besides, the proposed model with on-the-job search component adds more amplification to the standard New Keynesian model with labor market frictions and the model is capable to generate the fluctuations of the labor market variables to productivity shocks.

The structure of the paper is as follows. The following section displays two bivariate structural VAR models to show the correlation between productivity and unemployment and productivity and labor market tightness. In Section 3 summarizes the strands of the search and matching model with on-the-job search and reports the simulation results of this theoretical model. Section 4 concludes.

## **4.2 Data and Methodology**

This section presents two bivariate long-run restricted structural VAR models of labor productivity and unemployment rate and labor productivity and labor market tightness.

### 4.2.1 U.S. Labor Market Data

The data used in the analyses are quarterly and cover the period 1951Q1-2014Q4. Both the labor productivity and the unemployment rate series are obtained from the Bureau of Labor Statistics (BLS). The labor productivity data is the seasonally adjusted real output per hour in the non-farm business sector. The unemployment rate is the quarterly average of the monthly unemployment rate series obtained from the Current Population Survey. The vacancies data is taken from the paper by Barnichon (2014), which constructs a composite Help-Wanted Index (HWI). He provides monthly data from 1951 to present in his personal website<sup>1</sup>. The vacancy data that is used in this paper is the quarterly average of the monthly series. All variables, except unemployment rate, are in logs and all data are detrended using HP filter with smoothing parameter  $\lambda = 1600$ . Figure 4.1 demonstrates labor productivity, unemployment rate, vacancies and their corresponding trends. Below right panel of the figure shows the Beveridge curve of U.S. labor market.

The labor market tightness is calculated using vacancy and unemployment rate data which is the ratio of vacancy to unemployment rate. The time series behaviors of labor productivity, unemployment rate, vacancies and labor market tightness in U.S. are summarized in Table 4.1.

Table 4.1: Summary Statistics, Quarterly, 1951Q1-2014Q4

|                           | $u$    | $v$    | $\theta$ | $p$    |
|---------------------------|--------|--------|----------|--------|
| Standard deviation        | 0.0079 | 0.1380 | 0.2621   | 0.0105 |
| Quarterly autocorrelation | 0.8964 | 0.9065 | 0.9034   | 0.7147 |

<sup>1</sup><https://sites.google.com/site/registbarnichon/data>

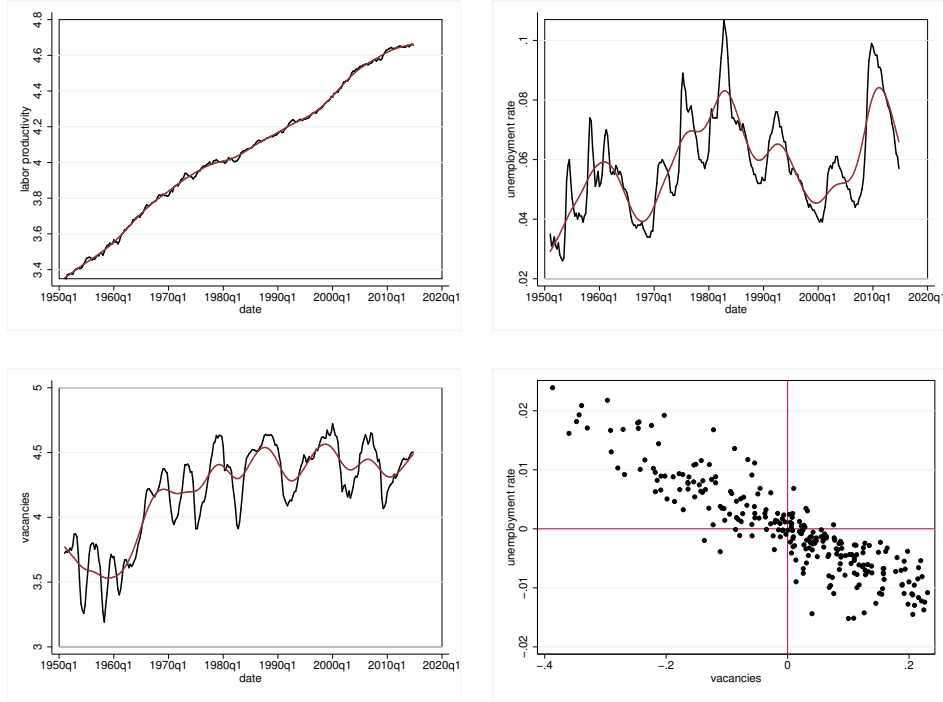


Figure 4.1: U.S. Labor Market Graphs

### 4.2.2 Structural VAR

Following the aforementioned criticism, this section aims to show the impact of exogenous changes in technology and to display the elasticities of productivity-unemployment and productivity-labor market tightness. Following Blanchard and Quah (1989) and Galí (1999b) and I utilize a structural VAR models with long run restrictions to identify technological disturbances. In line with the related literature, technological disturbances are the only shocks with permanent effect on productivity. The non-technology shocks explains the transitory movements in productivity .

The empirical model shows the observed variations in productivity ( $x_t$ ) and unemployment rate or labor market tightness ( $v_t = (u_t, \theta_t)$ ) as stemming from technology and non-technology shocks. These disturbances are orthogonal to each other. The general form of the system is as follows:

$$\begin{bmatrix} \Delta x_t \\ v_t \end{bmatrix} = \begin{bmatrix} C^{11}(L) & C^{12}(L) \\ C^{21}(L) & C^{22}(L) \end{bmatrix} \begin{bmatrix} \epsilon_t^z \\ \epsilon_t^m \end{bmatrix} \quad (4.1)$$

where  $\epsilon_t^z$  and  $\epsilon_t^m$  denote technology and non-technology shocks, respectively. The variable  $v_t = (u_t, \theta_t)$  denotes unemployment or labor market tightness, respectively. The orthogonality assumption suggests  $E\epsilon_t\epsilon_t' = I$ . Furthermore,  $C^{12}(1) = 0$ , meaning that only the technology disturbances have a permanent effect of productivity. Thus, the long-run multipliers are assumed to be lower triangular.

The estimates of the coefficients of  $C(L)$  in equation (4.1) are obtained following a technique proposed by Shapiro and Watson (1988). This method enables us to estimate the equations in the system one at a time. The productivity and the unemployment rate equations can be written as follows:

$$\Delta x_t = \sum_{j=1}^p \alpha_{xx,j} \Delta x_{t-j} + \sum_{j=0}^{p-1} \alpha_{xu,j} \Delta u_{t-j} + \epsilon_t^z. \quad (4.2)$$

$$u_t = \sum_{j=1}^p \beta_{ux,j} \Delta x_{t-j} + \sum_{j=1}^p \beta_{uu,j} u_{t-j} + \rho_{u,x} \epsilon_t^z + \epsilon_t^m \quad (4.3)$$

While regressing the equation (4.2), I use instrumental variables due to the contemporaneous effect of  $\epsilon_t^z$  on  $\Delta u_t$  at time  $t$ . Four lags, 1 to  $p = 4$ , of  $\Delta x_t$  and  $u_t$  are used as instruments. The residual of IV regression of equation (4.2) shows the estimated technological disturbance  $\epsilon_t^a$ .

Differently from the productivity equation, technology shock,  $\epsilon_t^z$ , enters into equation (4.3) in order to attain orthogonality between the non-technological and technological disturbances. Since there is no contemporaneous variable

that which is correlated with non-technology shock, I estimate equation (4.3) using ordinary least squares (OLS). In this regression, I use  $\epsilon_t^a$  that I obtained from the IV regression. Lastly, to allow heteroskedasticity of the structural innovations, I follow Francis and Ramey (2006) and estimate both equations jointly using GMM.

Similar estimation technique is applied to the labor productivity and the labor market tightness. I estimated the following equations similar to the aforementioned technique. The productivity and the unemployment rate equations are as follows:

$$\Delta x_t = \sum_{j=1}^p \alpha_{xx,j} \Delta x_{t-j} + \sum_{j=0}^{p-1} \alpha_{x\theta,j} \Delta \theta_{t-j} + \epsilon_t^z. \quad (4.4)$$

$$\theta_t = \sum_{j=1}^p \beta_{\theta,j} \Delta x_{t-j} + \sum_{j=1}^p \beta_{\theta\theta,j} \theta_{t-j} + \rho_{\theta,x} \epsilon_t^z + \epsilon_t^m \quad (4.5)$$

### 4.2.3 Results

Figure 4.2 displays the impulse response functions of labor productivity and unemployment. The above row shows the responses conditional to technology shock. Labor productivity responds with a hump shaped pattern and reaches to its new steady state level in a year. Unemployment rate, also displays a positive hump-shaped response and the effect of the shock dies out in 8 quarters. A 0.5% increase in labor productivity corresponds to 0.12 percentage point increase in the unemployment rate.

The below row of figure 4.2 demonstrates the responses to a non-technology shock. The labor productivity jumps as a first response and the impact of the

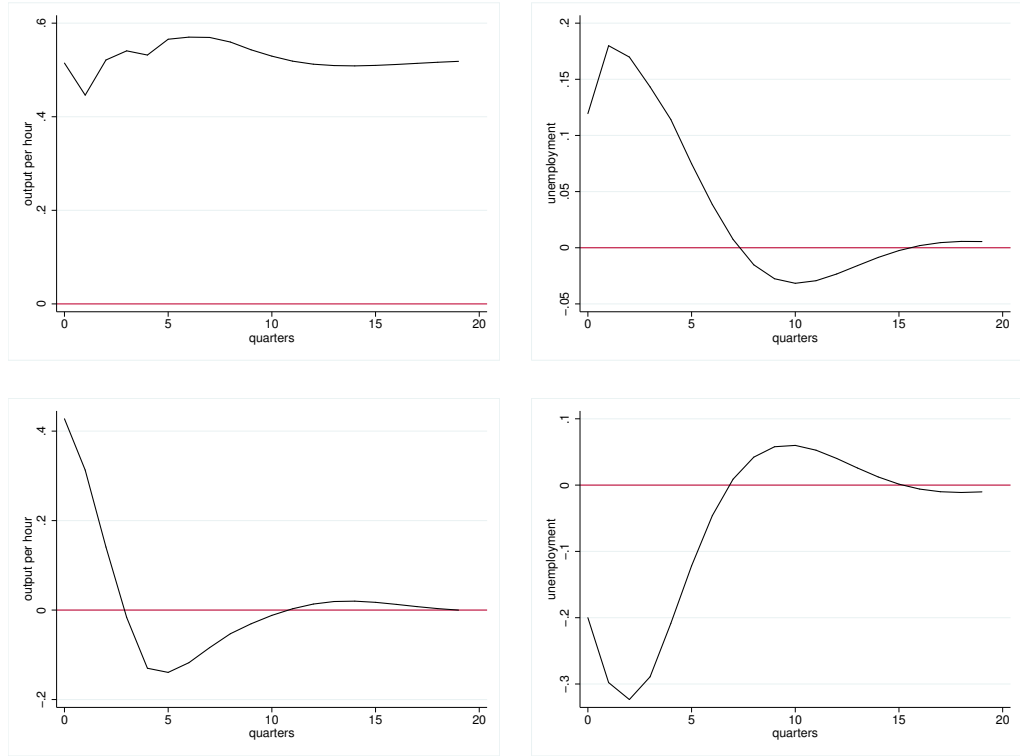


Figure 4.2: Responses of Productivity and Unemployment to Technology and Non-technology Shocks

shock dies out in one year. The unemployment rate decreases with hump shaped pattern and reaches to the lowest level after one year. Numerically, a 0.4% rise in labor productivity is associated with a 0.2 percentage point decrease in unemployment.

Figure 4.3 displays the impulse response functions of labor productivity and labor market tightness. The above row shows the responses to a technology shock. A technology shock leads to a negative correlation between labor market tightness and productivity. On the other hand, the elasticity is positive after a non-technology shock.

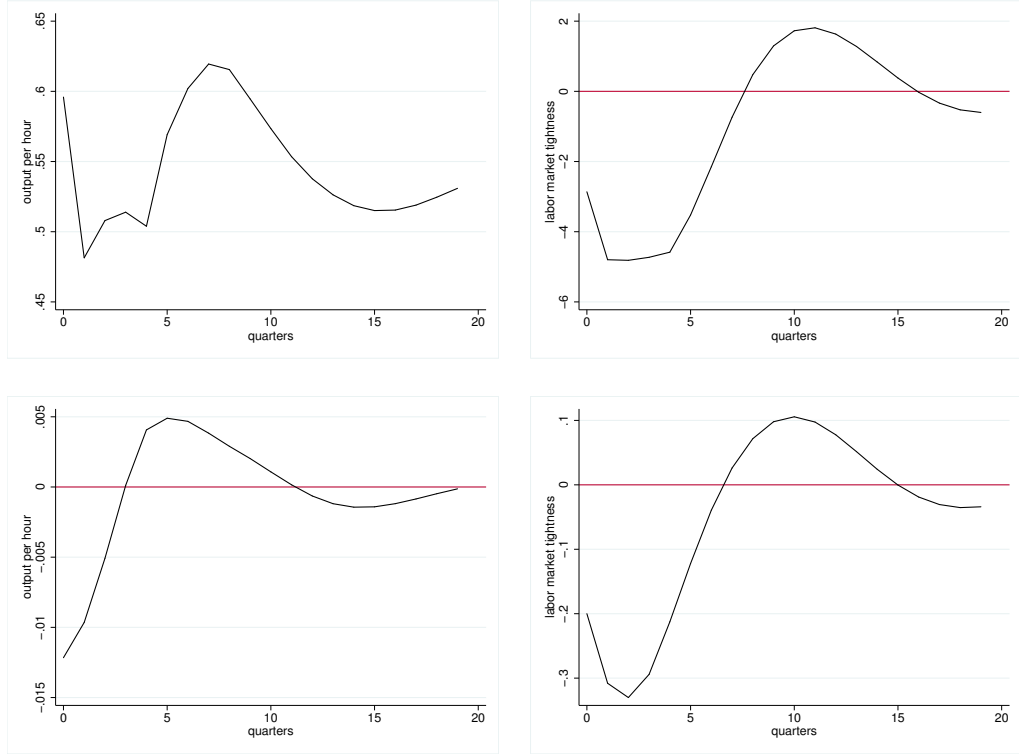


Figure 4.3: Responses of Productivity and Labor Market Tightness to Technology and Non-technology Shocks

### 4.3 Model and Simulation

The empirical analysis shows that technology and non-technology shocks are crucial in generating the business cycle fluctuations in labor market and productivity. This section tests the performance of the proposed new Keynesian model with on-the-job search<sup>2</sup>. The model integrates on-the-job search strand to a business cycle model with labor market frictions. The framework differs from the existing on-the-job search models by construction. Previous studies, Van Zandweghe (2010) and Krause and Lubik (2007a), generates wage dispersion by introducing different cost levels for different type of firms. Although this model preserves this assumption, we assume an additional source of wage dispersion: the difference in bargaining powers of firms. This setup

<sup>2</sup>See Chapter 3 for the detailed model.

assumes that there are two types of firms; the ones facing a weak union and the ones facing a strong union. For the sake of simplicity, these types are referred to as aggressive and passive, respectively. Mechanism of the model is as follows: In the standard NK model, firms are demand constrained so that an increase in productivity leads to a sluggish adjustment in aggregate demand to the new productivity level due to nominal rigidities. Accordingly, firms employ less labor during this process. Hence, a positive change in technology leads to an increase in unemployment. When on-the-job search is introduced, a positive technology shock leads not only to a decrease in the flow of unemployment-to-employment, but also to a decrease in the flow of employment-to-employment. This is why posted vacancies decrease more compared to the model without on-the-job search. Accordingly, on-the-job search fundamentally amplifies the responses of unemployment.

In order to show the mechanism quantitatively and to test the performance of the model, I run a simulation using the calibration values reported in the previous chapter. Before simulation, I describe a data generating process for both technology and non-technology shocks. I set the quarterly standard deviation of the technology shock to 0.0077 and the quarterly standard deviation of the monetary shock to 0.002 following the estimation results of the structural VAR exercise.

The proposed model is capable of generating accurate signs for the elasticities of the unemployment-productivity and the labor market tightness-productivity in line with the findings of the structural VAR. The simulation results also shows that the introduction of on-the-job search to the model am-

plifies the volatility of unemployment compared to the standard search and matching models. We observe that under a plausible calibration, the standard deviation of unemployment is 0.008 which is in line with its empirical counterpart reported in 4.1.

## 4.4 Conclusion

This essay revisits the Shimer (2005) puzzle by covering the period 1951-2014. First, by using a structural VAR model, I obtain the elasticities of unemployment-productivity and labor market tightness-productivity conditional to technology shocks and negative conditional to non-technology shocks. Secondly, by running a simulation for the model introduced in Chapter 3, this essay shows the amplifying effect of on-the-job search factor of the model.

The results of this study are as follows. The structural VAR models suggest that there is a positive correlation between productivity and unemployment and negative correlation between productivity and labor market tightness which contradicts with the standard SM model of Mortensen-Pissarides. The model with on-the-job search component adds more amplification to the standard New Keynesian model with labor market frictions and it generates both the magnitude and the sign of the fluctuations of labor market variables to productivity shocks.

## BIBLIOGRAPHY

- Auerbach, A. J., Cai, J., Kotlikoff, L. J., January 1991. U.s. demographics and saving: Predictions of three saving models. Carnegie-Rochester Conference Series on Public Policy 34 (1), 135–156.  
URL <http://ideas.repec.org/a/eee/crcspp/v34y1991ip135-156.html>
- Auerbach, A. J., Kotlikoff, L. J., Hagemann, R. P., Nicoletti, G., Jan. 1989. The economic dynamics of an ageing population: The case of four oecd countries. OECD Economics Department Working Papers 62, OECD Publishing.  
URL <http://ideas.repec.org/p/oec/ecoaaa/62-en.html>
- Barnichon, R., 2010. Productivity and Unemployment Over the Business Cycle. Journal of Monetary Economics 57 (8), 1013–1025.  
URL <http://ideas.repec.org/a/eee/moneco/v57y2010i8p1013-1025.html>
- Barnichon, R., 2014. The Shimer Puzzle and the Endogeneity of Productivity, cREI Working Paper.
- Bean, C. R., 2004. Global demographic change: some implications for central banks: overview panel. Federal Reserve Bank of Kansas City Proceedings (Aug), 447–454.
- Bjelland, M., Fallick, B. C., Haltiwanger, J., McEntarfer, E., 2011. Employer-to-employer Flows in the United States: Estimates Using Linked Employer–Employee Data. Journal of Business & Economic Statistics 29 (4), 493–505.  
URL <http://ideas.repec.org/a/bes/jnlbes/v29i4y2011p493-505.html>
- Blanchard, O. J., Galí, J., 2010. Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment. American Economic Journal: Macroeconomics 2 (2), 1–30.  
URL <http://ideas.repec.org/a/aea/aejmac/v2y2010i2p1-30.html>
- Blanchard, O. J., Quah, D., September 1989. The Dynamic Effects of Aggregate Demand and Supply Disturbances. American Economic Review 79 (4), 655–73.  
URL <http://ideas.repec.org/a/aea/aecrev/v79y1989i4p655-73.html>
- Cahuc, P., Postel-Vinay, F., Robin, J.-M., 2006. Wage Bargaining with On-the-job Search: Theory and Evidence. Econometrica 74 (2), 323–364.

- Calvo, G. A., September 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12 (3), 383–398.  
URL <http://ideas.repec.org/a/eee/moneco/v12y1983i3p383-398.html>
- Canova, F., Lopez-Salido, D., Michelacci, C., 2009. The Ins and Outs of Unemployment: An Analysis Conditional on Technology Shocks. *Economics Working Papers* 1213, Department of Economics and Business, Universitat Pompeu Fabra.  
URL <http://ideas.repec.org/p/upf/upfgen/1213.html>
- Clarida, R., Galí, J., Gertler, M., Dec. 1999. The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature* 37 (4), 1661–1707.
- Diamond, P. A., 1965. National debt in a neoclassical growth model. *The American Economic Review* 55 (5), pp. 1126–1150.  
URL <http://www.jstor.org/stable/1809231>
- Fallick, B. C., Fleischman, C. A., 2001. The Importance of Employer-to-employer Flows in the U.S. Labor Market. Tech. rep.
- Francis, N., Ramey, V. A., June 2006. The Source of Historical Economic Fluctuations: An Analysis Using Long-Run Restrictions. In: *NBER International Seminar on Macroeconomics 2004*. NBER Chapters. National Bureau of Economic Research, Inc, pp. 17–73.  
URL <http://ideas.repec.org/h/nbr/nberch/0084.html>
- Fujiwara, I., Teranishi, Y., 2005. Monetary policy in a life-cycle economy. *Federal Reserve Board Proceedings*.  
URL <http://ideas.repec.org/a/fip/fedgpr/y2005x3.html>
- Fujiwara, I., Teranishi, Y., August 2008. A dynamic new keynesian life-cycle model: Societal aging, demographics, and monetary policy. *Journal of Economic Dynamics and Control* 32 (8), 2398–2427.  
URL <http://ideas.repec.org/a/eee/dyncon/v32y2008i8p2398-2427.html>
- Galí, J., 1999a. Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? *American Economic Review* 89 (1), 249–271.  
URL <http://ideas.repec.org/a/aea/aecrev/v89y1999i1p249-271.html>
- Galí, J., March 1999b. Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? *American Economic Review* 89 (1), 249–271.  
URL <http://ideas.repec.org/a/aea/aecrev/v89y1999i1p249-271.html>
- Galí, J., September 2008a. Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. *Introductory Chapters*. Princeton University Press.  
URL <http://ideas.repec.org/h/pup/chapts/8654-1.html>

- Galí, J., 2008b. Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. Introductory Chapters. Princeton University Press.
- Galí, J., March 2014. Monetary Policy and Rational Asset Price Bubbles. *American Economic Review* 104 (3), 721–52.  
URL <http://ideas.repec.org/a/aea/aecrev/v104y2014i3p721-52.html>
- Gertler, M., Apr. 1997. Government debt and social security in a life-cycle economy. NBER Working Papers 6000, National Bureau of Economic Research, Inc.  
URL <http://ideas.repec.org/p/nbr/nberwo/6000.html>
- Hagedorn, M., Manovskii, I., 2008. The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. *American Economic Review* 98 (4), 1692–1706.  
URL <http://ideas.repec.org/a/aea/aecrev/v98y2008i4p1692-1706.html>
- Hall, R. E., 2005. Employment Fluctuations with Equilibrium Wage Stickiness. *American Economic Review* 95 (1), 50–65.  
URL <http://ideas.repec.org/a/aea/aecrev/v95y2005i1p50-65.html>
- Imam, P. A., Sep. 2013. Shock from Graying: Is the Demographic Shift Weakening Monetary Policy Effectiveness. IMF Working Papers 13/191, International Monetary Fund.  
URL <http://ideas.repec.org/p/imf/imfwpa/13-191.html>
- Krause, M. U., Lubik, T. A., 2006. The Cyclical Upgrading of Labor and On-the-job Search. *Labour Economics* 13 (4), 459–477.
- Krause, M. U., Lubik, T. A., 2007a. On-the-job Search and the Cyclical Dynamics of the Labor Market. Working Paper Series 0779, European Central Bank.  
URL <http://ideas.repec.org/p/ecb/ecbwps/20070779.html>
- Krause, M. U., Lubik, T. A., 2007b. The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions. *Journal of Monetary Economics* 54 (3), 706–727.  
URL <http://ideas.repec.org/a/eee/moneco/v54y2007i3p706-727.html>
- Krugman, P. R., 1998. It's Baaack: Japan's Slump and the Return of the Liquidity Trap. *Brookings Papers on Economic Activity* 29 (2), 137–206.  
URL <http://ideas.repec.org/a/bin/bpeajo/v29y1998i1998-2p137-206.html>
- Merz, M., 1995. Search in the Labor Market and the Real Business Cycle. *Journal of Monetary Economics* 36 (2), 269–300.  
URL <http://ideas.repec.org/a/eee/moneco/v36y1995i2p269-300.html>
- Miles, D., January 1999. Modelling the impact of demographic change upon the economy. *Economic Journal* 109 (452), 1–36.  
URL <http://ideas.repec.org/a/ecj/econj1/v109y1999i452p1-36.html>

- Miles, D., 2002. Should monetary policy be different in a greyer world? In: Auerbach, A. J., Herrmann, H. (Eds.), *Aging, Financial Markets and Monetary Policy*. Springer, Heidelberg.
- Modigliani, F., Brumberg, R., 1954. Utility Analysis and The Consumption Function: An interpretation of cross-section data, in kurihara, k.k. Edition. *Post-Keynesian Economics*. NJ Rutgers University Press.
- Mortensen, D. T., Pissarides, C. A., 1994. Job Creation and Job Destruction in the Theory of Unemployment. *Review of Economic Studies* 61 (3), 397–415.  
URL <http://ideas.repec.org/a/bla/restud/v61y1994i3p397-415.html>
- Nagypal, E., 2008. Worker Reallocation Over the Business Cycle: The Importance of Employer-to-employer Transitions, Unpublished Manuscript.
- Pissarides, C. A., 1994. Search Unemployment with On-the-job Search. *Review of Economic Studies* 61 (3), 457–475.  
URL <http://ideas.repec.org/a/bla/restud/v61y1994i3p457-75.html>
- Rios-Rull, J.-V., 2001. Population changes and capital accumulation: The aging of the baby boom. *The B.E. Journal of Macroeconomics* 0 (1), 7.  
URL <http://ideas.repec.org/a/bpj/bejmac/vadvances.1y2001i1n7.html>
- Samuelson, P. A., 1958. An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy* 66, 467.  
URL <http://ideas.repec.org/a/ucp/jpolec/v66y1958p467.html>
- Shapiro, M. D., Watson, M. W., May 1988. Sources of Business Cycle Fluctuations. NBER Working Papers 2589, National Bureau of Economic Research, Inc.  
URL <http://ideas.repec.org/p/nbr/nberwo/2589.html>
- Shimer, R., 2005. The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review* 95 (1), 25–49.  
URL <http://ideas.repec.org/a/aea/aecrev/v95y2005i1p25-49.html>
- Shimer, R., 2006. On-the-job Search and Strategic Bargaining. *European Economic Review* 50 (4), 811–830.  
URL <http://ideas.repec.org/a/eee/eecrev/v50y2006i4p811-830.html>
- Taylor, J. B., 1993. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy* 39 (1), 195–214.
- Van Zandweghe, W., 2010. On-the-job Search, Sticky Prices, and Persistence. *Journal of Economic Dynamics and Control* 34 (3), 437–455.  
URL <http://ideas.repec.org/a/eee/dyncon/v34y2010i3p437-455.html>
- Walsh, C. E., 2005. Labor Market Search, Sticky Prices, and Interest Rate Policies. *Review of Economic Dynamics* 8 (4), 829–849.  
URL <http://ideas.repec.org/a/red/issued/v8y2005i4p829-849.html>

# APPENDICES

## A Derivation of the IS Equation

This section of the paper provides the derivation of the modified dynamic IS equation step by step in detail. At time  $t$  the total consumption per worker equals to the total production per worker.

$$\hat{y}_t = \frac{c^*}{y^*} \hat{c}_t + \varphi \frac{d^*}{y^*} \hat{d}_t$$

The above equation is the dynamic IS equation of this setup. To provide a comprehensive understanding of the equation, we take a deeper look at the components of the equation.

Budget constraints of the workers' optimization problem are as follows:

$$P_t C_t(j) + B_t^n(j) + P_t \int_0^1 Q_t(i) S_t(i, j) di = W_t L_t(j)$$

and

$$P_{t+1} D_{t+1}(j) = B_t^n(j)(1 + i_t) + P_{t+1} \int_0^1 (Div_{t+1}(i) + Q_{t+1}(i)) S_t(i, j) di$$

Combining both yields the intertemporal budget constraint (IBC),

$$E_t\{D_{t+1}\} = \left[ \frac{W_t}{P_t} L_t(j) - C_t(j) \right] \frac{(1 + i_t)}{E_t\{1 + \pi_{t+1}\}}.$$

Log-linearize the IBC around steady state, we get

$$\hat{d}_{t+1} = (1 + i^*) \frac{1}{\mu} \frac{y^*}{d^*} (\hat{w}_t - \hat{p}_t + \hat{l}_t(j)) - (1 + i^*) \frac{c^*}{d^*} \hat{c}_t(j) + (\hat{i}_t - \pi_{t+1}).$$

Plug the log-linearized Euler equation into the IBC.

$$\hat{c}_t + \frac{1}{\sigma} (\hat{i}_t - E_t\{\pi_{t+1}\}) = (1 + i^*) \frac{1}{\mu} \frac{y^*}{d^*} (\hat{w}_t - \hat{p}_t + \hat{l}_t(j)) - (1 + i^*) \frac{c^*}{d^*} \hat{c}_t(j) + (\hat{i}_t - \pi_{t+1}) \quad (6)$$

The corresponding steady state conditions are as follows:

$$y^* = c^* + \varphi d^*$$

$$d^* = \left( \frac{1}{\mu} y^* - c^* \right) (1 + i^*)$$

Plug the steady state values into the equation (6). We have,

$$\hat{c}_{t-1} = \frac{\alpha_0}{\alpha_1} \hat{y}_{t-1} + \frac{1}{\alpha_0} (\hat{i}_{t-1} - \pi_t) - \frac{1}{\sigma \alpha_0} (\hat{i}_{t-1} - E_t\{\pi_t\}).$$

where

$$\alpha_0 = 1 - (1 + i^*) \frac{1}{\mathcal{M}} \frac{y^*}{d^*} \sigma + (1 + i^*) \frac{c^*}{d^*}$$

and

$$\alpha_1 = (1 + i^*) \frac{1}{\mathcal{M}} \frac{y^*}{d^*} (1 + \psi)$$

Hence, the equations for the young age and old age consumptions at time  $t$  are

$$\hat{c}_t = \frac{\alpha_1}{\alpha_0} \hat{y}_t + \frac{1}{\alpha_0} (\hat{i}_t - E_t\{\pi_{t+1}\}) - \frac{1}{\sigma \alpha_0} (\hat{i}_t - E_{t-1}\{\pi_t\})$$

and

$$\hat{d}_t = \hat{c}_{t-1} + \frac{1}{\sigma} (\hat{i}_{t-1} - \pi_t)$$

Recall the structure of the OLG-Dynamic New Keynesian IS curve.

$$\hat{y}_t = \frac{c^*}{y^*} \hat{c}_t + \varphi \frac{d^*}{y^*} \hat{d}_t$$

As discussed earlier in the paper, differently from the standard DNK IS curve, now we have a richer dynamic system. The derived IS equation not only depends on current periods nominal interest rate and next period's inflation expectation and also previous period's nominal interest rate and inflation expectation and the realized inflation.

## B Derivation of the Phillips Equation

Evolution of the aggregate price index is

$$P_t = [\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}.$$

Divide both sides by  $P_{t-1}$ .

$$\begin{aligned} \left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} &= \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \\ \Pi_t^{1-\epsilon} &= \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \end{aligned} \quad (7)$$

where  $\Pi_t = \frac{P_t}{P_{t-1}}$ . Log-linearize equation (7) around zero inflation steady state, it yields

$$\pi_t = (1-\theta)[p_t^* - p_{t-1}] \quad (8)$$

equivalently,

$$p_t = (1-\theta)p_t^* + \theta p_{t-1}.$$

Re-optimizers firms profit maximization objective function is

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \{ \Lambda_{t,t+k} (P_t^* Y_{t+k}(i) - W_{t+k} Y_{t+k}(i)) \}$$

subject to

$$Y_{t+k}(i) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}.$$

First order condition is

$$\sum_{k=0}^{\infty} \theta^k E_t \{ \Lambda_{t,t+k} (1-\epsilon)(P_t^*)^{-\epsilon} (P_{t+k})^{\epsilon} Y_{t+k} + \epsilon W_{t+k} (P_t^*)^{-\epsilon-1} (P_{t+k})^{\epsilon} Y_{t+k} \} = 0.$$

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k}(i) \left( P_t^* - \frac{\epsilon}{\epsilon-1} W_{t+k} \right) \right\} = 0$$

Now, divide both sides by  $P_{t-1}$ . We get

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k}(i) \left( \frac{P_t^*}{P_{t-1}} - \frac{\epsilon}{\epsilon-1} W_{t+k}^r \frac{P_{t+k}}{P_{t-1}} \right) \right\} = 0 \quad (9)$$

Perfect foresight, zero inflation steady state conditions are

$$\frac{P_t^*}{P_{t-1}} = 1, \quad \pi_{t-1,t+k} = 1, \quad Y_{t+k}(i) = y^*, \quad W^* = \frac{1}{\mathcal{M}}$$

$$\Lambda_{t,t+k} = \beta^k J^k, \quad J = \left( \frac{\varphi(1+i^*) - \mathcal{M}}{(\mathcal{M}-1)(1+i^*)} \right)^\sigma.$$

Log-linearize equation (9) around zero inflation steady state,

$$p_t^* - p_{t-1} = (1 - \theta\beta J) \sum_{k=0}^{\infty} (\theta\beta J)^k E_t \{ \widehat{mc}_{t+k} + p_{t+k} - p_{t-1} \}.$$

Rewriting the optimal price setting rule in recursive form, we get

$$p_t^* = (1 - \theta\beta J)(\hat{w}_t + p_t) + (1 - \theta\beta J)(\theta\beta J) E_t \{ \widehat{mc}_{t+1} + p_{t+1} \} + \dots$$

$$p_t^* = \theta\beta J E_t \{ p_{t+1}^* \} + (1 - \theta\beta J) \widehat{mc}_t + (1 - \theta\beta J) p_t. \quad (10)$$

Combining both (8) and (10) yields,

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1})$$

$$\pi_t = (1 - \theta)[\theta\beta J E_t \{ p_{t+1}^* \} + (1 - \theta\beta J) \widehat{mc}_t + (1 - \theta\beta J) p_t - p_{t-1}].$$

Plug  $p_{t+1}^*$  and rewrite the above expression.

$$\pi_t = (1 - \theta) \left[ \theta\beta J E_t \left[ \frac{\pi_{t+1}}{(1 - \theta)} + p_t \right] + (1 - \theta\beta J) \widehat{mc}_t + (1 - \theta\beta J) p_t - p_{t-1} \right]$$

Rearrange the terms to get the forward looking Phillips equation.

$$\pi_t = \tilde{\beta} E_t \{ \pi_{t+1} \} + \tilde{\kappa} \widehat{mc}_t$$

where  $\tilde{\beta} = \beta J$  and  $\tilde{\kappa} = \frac{(1-\theta)(1-\theta\tilde{\beta})}{\theta}$ .

## C Simulation Values

Table 2: Preference Parameters

| Parameter  | Name                                     | Value |
|------------|------------------------------------------|-------|
| $\sigma$   | intertemporal elasticity of substitution | 1     |
| $\psi$     | elasticity of labor supply               | 1     |
| $\beta$    | discount factor                          | 0.99  |
| $\epsilon$ | elasticity of substitution               | 6     |
| $\theta$   | price stickiness                         | 0.75  |
| $\phi_y$   | feedback parameter of output             | 0.125 |
| $\phi_\pi$ | feedback parameter of inflation          | 1.5   |
| $\rho_v$   | persistence of monetary policy shock     | 0.5   |

## D Calibration

|                                                    | Value | Description                                       |
|----------------------------------------------------|-------|---------------------------------------------------|
| <i>Preferences and Production</i>                  |       |                                                   |
| $\beta$                                            | 0.99  | discount factor                                   |
| $\phi$                                             | 5     | inverse of Frisch elasticity of labor supply      |
| $\theta$                                           | 0.75  | degree of price stickiness                        |
| $\mathcal{M}$                                      | 1.2   | desired markup                                    |
| $\gamma$                                           | 0.5   | share of aggressive good in final output          |
| <i>Labor market</i>                                |       |                                                   |
| $\delta$                                           | 0.12  | separation rate                                   |
| $u$                                                | 0.05  | unemployment rate                                 |
| $q$                                                | 0.695 | the job finding rate of unemployed agents         |
| $\alpha$                                           | 1     | weight of labor market tightness in cost function |
| <i>Technology, Monetary policy rule and Shocks</i> |       |                                                   |
| $\phi_\pi$                                         | 1.5   | feedback parameter of inflation                   |
| $\phi_y$                                           | 0.125 | feedback parameter of output                      |
| $\rho_a$                                           | 0.9   | persistence of the technology shock               |
| $\epsilon_a$                                       | 1     | technology shock                                  |

Table 3: Baseline calibration and steady state values

| Experiment I   |           |       |         |         |        |      |         |
|----------------|-----------|-------|---------|---------|--------|------|---------|
| $N^A$          | $\varphi$ | $p$   | $\xi^A$ | $\xi^P$ | $\chi$ | $B$  | $\zeta$ |
| 0.475          | 1         | 0.093 | 1       | 0.414   | 1.10   | 0.22 | 6.52    |
|                | 0         | N.A.  | 0.5     | 0.5     | 1.27   | 0.12 | N.A.    |
| Experiment II  |           |       |         |         |        |      |         |
| $N^A$          | $\varphi$ | $p$   | $\xi^A$ | $\xi^P$ | $\chi$ | $B$  | $\zeta$ |
| 0.475          | 0.1       | 0.932 | 1       | 0.414   | 1.10   | 0.07 | 0.19    |
|                | 0.2       | 0.466 |         |         |        | 0.09 | 2.10    |
|                | 0.3       | 0.311 |         |         |        | 0.11 | 3.21    |
|                | 0.4       | 0.233 |         |         |        | 0.13 | 4.00    |
|                | 0.5       | 0.186 |         |         |        | 0.14 | 4.61    |
|                | 0.6       | 0.155 |         |         |        | 0.16 | 5.12    |
|                | 0.7       | 0.133 |         |         |        | 0.17 | 5.54    |
|                | 0.8       | 0.116 |         |         |        | 0.19 | 5.91    |
|                | 0.9       | 0.104 |         |         |        | 0.20 | 6.23    |
| Experiment III |           |       |         |         |        |      |         |
| $N^A$          | $\varphi$ | $p$   | $\xi^A$ | $\xi^P$ | $\chi$ | $B$  | $\zeta$ |
| 0.241          | 1         | 0.183 | 0.6     | 0.5     | 1.24   | 0.13 | 4.66    |
| 0.295          |           | 0.150 | 0.7     |         | 1.17   | 0.16 | 5.21    |
| 0.338          |           | 0.131 | 0.8     |         | 1.14   | 0.17 | 5.59    |
| 0.373          |           | 0.119 | 0.9     |         | 1.13   | 0.19 | 5.86    |
| 0.403          |           | 0.110 | 1       |         | 1.12   | 0.20 | 6.07    |

Table 4: Calibration values for experiments

## E Log-linearized equations

1. Goods market clearing condition

$$\hat{y}_t = (1 - \theta_1 - \theta_2)\hat{c}_t + \theta_1(\hat{g}_t^A + \hat{v}_t^A) + \theta_2(\hat{g}_t^P + \hat{v}_t^P)$$

$$\text{where } \theta_1 = \frac{G^A V^A}{Y}, \theta_2 = \frac{G^P V^P}{Y}.$$

2. Aggregate production function

$$\hat{y}_t = \hat{a}_t + \gamma \hat{n}_t^A + (1 - \gamma) \hat{n}_t^P$$

3. Production function of intermediate goods firms

$$\hat{z}_t^A = \hat{a}_t + \hat{n}_t^A$$

$$\hat{z}_t^P = \hat{a}_t + \hat{n}_t^P$$

4. Net hiring in intermediate goods firms

$$\delta \hat{h}_t^A = \hat{n}_t^A - (1 - \delta) \hat{n}_{t-1}^A$$

$$\delta \hat{h}_t^P = \hat{n}_t^P - (1 - \delta) \hat{n}_{t-1}^P$$

5. Gross hiring in an aggressive firm

$$\hat{v}_t^A = (1 - \theta_3) \hat{h}_t^A + \theta_3 \hat{h}_t^O \quad \text{where } \theta_3 = \frac{H^O}{V^A}.$$

6. Hiring cost of intermediate goods firms

$$\hat{g}_t^A = \hat{a}_t + \alpha(\hat{v}_t^A - \widehat{pool}_t)$$

$$\hat{g}_t^P = \hat{a}_t + \alpha(\hat{v}_t^P - \widehat{pool}_t)$$

7. Labor market tightness

$$\hat{x}_t = \hat{h}_t - \widehat{pool}_t$$

8. The pool of job searchers

$$\widehat{pool}_t = (1 - \theta_4)\widehat{oj}s_t^0 + \theta_4\hat{u}_t^0 \quad \text{where } \theta_4 = \frac{U^0}{Pool}.$$

9. The unemployment pool (Beginning-of-period unemployment)

$$\hat{U}_t^0 = -\frac{(1 - \delta)N}{U^0}\hat{n}_{t-1}$$

10. End-of-period unemployment

$$\hat{u}_t = U^0\hat{U}_t^0 - qU^0\hat{q}_t = -N\hat{n}_t$$

11. Employment

$$\hat{n}_t = (1 - \theta_5)\hat{n}_t^A + \theta_5\hat{n}_t^P \quad \text{where } \theta_5 = \frac{N^P}{N}.$$

12. The job finding rate of unemployed agents

$$\hat{q}_t = \hat{h}_t^U - \hat{u}_t^0$$

13. The job finding rate of on-the-job searchers

$$\hat{p}_t = \hat{h}_t^O - \widehat{oj}s_t^O$$

14. The relation between the job finding rates

$$\hat{p}_t = \zeta\hat{q}_t$$

15. Hiring from the unemployment pool

$$\hat{h}_t^U = (1 - \theta_6)\hat{h}_t^A + \theta_6\hat{h}_t^P \quad \text{where } \theta_6 = \frac{H^P}{H^U}.$$

16. The pool of on-the-job searchers

$$\widehat{oj}s_t^0 = \hat{n}_{t-1}^A$$

17. Total hiring

$$\hat{h}_t = (1 - \theta_7)\hat{h}_t^O + \theta_7\hat{h}_t^U \quad \text{where } \theta_7 = \frac{H^U}{H}.$$

18. The Euler equation

$$\hat{c}_t = \hat{c}_{t+1} - \hat{r}_t$$

19. The Fisher equation

$$\hat{r}_t = \hat{i}_t - \pi_{t+1}$$

20. The forward-looking Phillips equation

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widehat{mc}_t$$

21. Optimal hiring condition of an aggressive firm

$$\hat{\mu}_t^A = \hat{a}_t - (1 - \phi_a)\hat{w}_t^A - \phi_a\hat{b}_t^A$$

22. Optimal hiring condition of a passive firm

$$\hat{\mu}_t^P = \hat{a}_t - (1 - \phi_p)\hat{w}_t^P - \phi_p\hat{b}_t^P$$

23. Net hiring condition of an aggressive firm

$$\begin{aligned} \hat{b}_t^A &= \frac{\hat{g}_t^A}{1 - (1 - \delta)\beta(1 - \varphi p)} - \frac{(1 - \delta)\beta(1 - \varphi p)}{1 - (1 - \delta)\beta(1 - \varphi p)}(\hat{g}_{t+1}^A - \hat{r}_t) \\ &\quad + \frac{(1 - \delta)\beta\varphi p}{1 - (1 - \delta)\beta(1 - \varphi p)}\hat{p}_{t+1} \end{aligned}$$

24. Net hiring condition of a passive firm

$$\hat{b}_t^P = \frac{\hat{g}_t^P}{1 - (1 - \delta)\beta} - \frac{(1 - \delta)\beta}{1 - (1 - \delta)\beta}(\hat{g}_{t+1}^P - \hat{r}_t)$$

25. Wage equation for an aggressive firm

$$\begin{aligned} W^A \hat{w}_t^A = & MRS(\hat{c}_t + \phi \hat{n}_t) + \eta^A G^A \hat{g}_t^A - \beta(1 - \delta) \Big( \eta^A G^A (\varphi p(\hat{p}_{t+1} \\ & + \hat{g}_{t+1}^A - \hat{r}_t) + \varphi p \tau(\hat{p}_{t+1} + \hat{\tau}_{t+1} + \hat{g}_{t+1}^A - \hat{r}_t) - (\hat{g}_{t+1}^A - \hat{r}_t) \\ & + q \tau(\hat{q}_{t+1} + \hat{\tau}_{t+1} + \hat{g}_{t+1}^A - \hat{r}_t)) - \eta^P G^P (\varphi p(\hat{p}_{t+1} + \hat{g}_{t+1}^P - \hat{r}_t) \\ & + \varphi p \tau(\hat{p}_{t+1} + \hat{\tau}_{t+1} + \hat{g}_{t+1}^P - \hat{r}_t) + q(\hat{q}_{t+1} + \hat{g}_{t+1}^P - \hat{r}_t) \\ & - q \tau(\hat{q}_{t+1} + \hat{\tau}_{t+1} + \hat{g}_{t+1}^P - \hat{r}_t)) \Big) \end{aligned}$$

26. Wage equation for a passive firm

$$\begin{aligned} W^P \hat{w}_t^P = & MRS(\hat{c}_t + \phi \hat{n}_t) + \eta^P G^P \hat{g}_t^P - \beta(1 - \delta) \Big( \eta^P G^P ((1 - q)(\hat{g}_{t+1}^P \\ & - \hat{r}_t) + q \hat{q}_{t+1} - q \tau(\hat{q}_{t+1} + \hat{\tau}_{t+1} + \hat{g}_{t+1}^P - \hat{r}_t)) \\ & + \eta^A G^A q \tau(\hat{q}_{t+1} + \hat{\tau}_{t+1} + \hat{g}_{t+1}^A - \hat{r}_t) \Big) \end{aligned}$$

27. Marginal cost

$$\widehat{mc}_t = \gamma(-\hat{\mu}_t^A) + (1 - \gamma)(-\hat{\mu}_t^P)$$

28. Markup of an aggressive firm

$$\hat{\mu}_t^A = \hat{\mathbf{p}}_t - \hat{p}_t^A$$

29. Markup of a passive firm

$$\hat{\mu}_t^P = \hat{\mathbf{p}}_t - \hat{p}_t^P$$

30. Rate of inflation

$$\pi = \hat{\mathbf{p}}_t - \hat{\mathbf{p}}_{t-1}$$

31. Weighted average wage level

$$\hat{w}_t = \gamma \hat{w}_t^A + (1 - \gamma) \hat{w}_t^P$$

32. Interest rate rule (Taylor rule)

$$\hat{i}_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t$$

33. Monetary shock process

$$v_t = \rho_v v_{t-1} + \epsilon_t^v$$

34. Technological progress

$$a_t = \rho_a a_{t-1} + \epsilon_t^a$$

35. Ratio of hirings in an aggressive firm

$$\tau_t = \hat{v}_t^A - \hat{h}_t$$

## F Impulse Responses

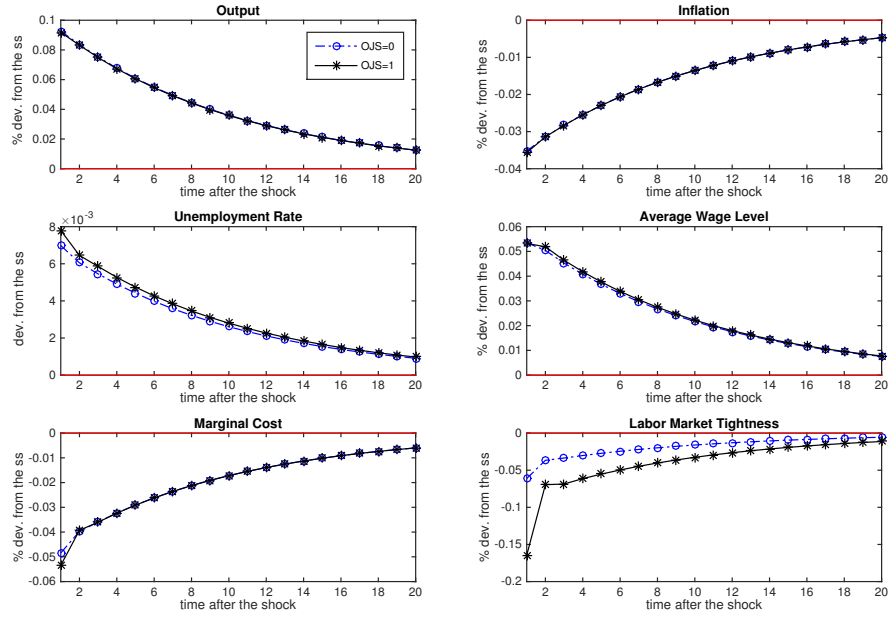


Figure 4: Impulse Responses to a Positive Technology Shock