

A NOVEL VERSE OPTIMAL RF PULSE DESIGN METHOD FOR PARALLEL TRANSMISSION IN MAGNETIC RESONANCE IMAGING

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by

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September, 2009

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ABSTRACT

A NOVEL VERSE OPTIMAL RF PULSE DESIGN METHOD FOR PARALLEL TRANSMISSION IN MAGNETIC RESONANCE IMAGING

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A novel radio frequency (RF) pulse design method for magnetic resonance imaging (MRI) and an improvement to an existing method that reduces specific absorption rate (SAR) in MRI are presented. The new RF pulse design method, variable rate selective excitation optimal RF pulse design method for parallel transmission (VERSEp), is developed for parallel transmission and aim of the method is to design RF pulses with lowest SAR after SAR reduction with variable rate selective excitation (VERSE) method. This is achieved by modifying the SAR optimal RF pulse design method for parallel transmission. Performance of the VERSEp method is tested by comparing VERSE-SAR reduced SAR of the RF pulses designed with SAR optimal RF pulse design method and VERSE-SAR reduced SAR of the RF pulses designed using VERSEp. In the simulations, SAR reductions up to 47% are obtained. Different aspects of VERSEp are also shown with simulations. The second contribution of this work is an improvement made to an existing constrained VERSE-SAR reduction method. The existing VERSE-SAR reduction method uses a peak RF constraint for SAR reduction. In this work, peak square root power constraint is used instead of peak RF constraint in the VERSE-SAR reduction method. In the simulation results, the SAR of the RF pulses designed using the improved method were compared with SAR of the RF pulses designed using the method before improvement. SAR reductions up to 50% are obtained by using peak square root power constrained SAR reduction instead of peak RF constrained SAR reduction.

Keywords: SAR reduction, RF power, Parallel Transmission, Transmit SENSE, VERSE

ÖZET

MANYETİK REZONANS GÖRÜNTÜLEMEDE DEĞİŞKEN HIZLI SEÇİCİ UYARIM İÇİN İYİLEŞTİRİLMİŞ RADYO FREKANS DARBE TASARLAMA YÖNTEMİ

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Bu çalışmada, manyetik rezonans görüntüleme (MRG) kullanılan radyo frekans (RF) darbelerin tasarımı için yeni bir yöntem geliştirilmiştir ve MRG için tasarlanmış mevcut bir özgül soğurma hızı (ÖSH) düşürme yöntemine bir geliştirme yapılmıştır. Bu çalışmada geliştirilen değişken hızlı seçici uyarım için iyileştirilmiş paralel darbe tasarlama yöntemi (DHSUp) paralel iletim için tasarlanmıştır ve yöntemin amacı değişken hızlı seçici uyarım (DHSU) yöntemi ile ÖSH'si düşürüldükten sonra en az ÖSH'ye sahip olan RF darbeleri tasarlamaktır. DHSUp yönteminin getirisi paralel iletim için ÖSH düşürme yöntemi ile tasarlanmış ve DHSU yöntemi ile ÖSH'si düşürülmüş RF darbelerin ÖSH'leri ile DHSUp yöntemi ile tasarlanmış ve DHSU yöntemi ile ÖSH'si düşürülmüş RF darbelerin ÖSH'lerinin karşılaştırılmasıyla incelenmiştir. Yapılan simülasyonlarda paralel iletim için ÖSH düşürme yöntemi yerine DHSUp kullanarak %47'ye varan ÖSH düşümleri elde edilmiştir. Bu tezde anlatılan diğer bir çalışma ise mevcut bir koşullu DHSU ile ÖSH düşürme yöntemine yapılan geliştirmedir. Mevcut yöntemde ÖSH düşürmek için bir en yüksek RF koşulu kullanılmaktadır. Bu çalışmada mevcut yöntemdeki en yüksek RF koşulu yerine en yüksek kare kök güç koşulu kullanılarak daha iyi ÖSH düşürümü sağlanmıştır. Yapılan geliştirmenin getirisi en yüksek RF koşulu kullanılarak ÖSH'si düşürülmüş RF darbelerin ÖSH'leri ile en yüksek kare kök güç koşulu kullanılarak ÖSH'si düşürülmüş RF darbelerin ÖSH'leri karşılaştırılarak incelenmiştir. Yapılan simülasyonlarda en yüksek RF koşulu yerine en yüksek kare kök güç koşulu kullanılarak %50'ye varan ÖSH düşümleri sağlanmıştır.

Anahtar Sözcükler: özgül soğurma hızı, ÖSH, manyetik rezonans görüntüleme, MRG, paralel iletim

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Chapter 1

Introduction

Due to the high tissue contrasts obtained, magnetic resonance imaging (MRI) is a very important imaging modality. However, low signal-to-noise ratio and low resolution problems are the main drawbacks of MRI. Using high main field strengths such as 3T or 7T in MRI, images with higher signal-to-noise ratio can be obtained. For this reason, solution of the problems that arise with high field is critical for development of MRI. The most important problem in high field MRI is the magnetic field inhomogeneity because the frequency of operation of transmit coils is directly proportional with the main field strength in use and in higher frequencies, it is not possible to design coils with homogeneous magnetic field distribution. To overcome this problem, new techniques such as RF shimming and parallel transmission are developed [1, 2]. While both methods uses more than one transmit coils, in the RF shimming method the pulse shape kept constant and only amplitude and phase of the applied currents to each of the coil is modified to achieve a uniform field. However, the degree homogeneity that can be obtained using RF shimming is limited and RF shimming yields unacceptable high specific absorption rate (SAR) in high fields. To obtain homogeneous flip angles using coils with non-uniform magnetic field sensitivities, 3-D k-space trajectories should be used but covering k-space in three dimensions takes too long time due to gradient hardware limits. To overcome this problem, parallel transmission can be used. Parallel transmission uses multiple coils with independent transmit channels to enable usage of under-sampled k-space trajectories. One other advantage of parallel transmission is SAR reduction.

SAR is a measure of amount of RF power deposition inside the imaged body during magnetic resonance imaging operation and it is measured in watts per kilogram. There are two different limits on SAR. The first one is the whole-body SAR limit and it is related with the amount of total heating human body can handle by sweating etc. without being harmed. The other limit is the local SAR limit and it is for preventing tissue burning. In high field MRI, local SAR is a more important limit but this work focuses on whole-body SAR reduction because of the simpler formulations obtained for whole-body SAR. For the rest of

this text, the whole-body power deposition that RF pulses causes for a single excitation is denoted as SAR. When SAR of RF pulses is reduced, successive RF pulses can be applied in shorter time intervals i.e. repetition time can be reduced and this yields shorter scan times.

By using multiple radio frequency (RF) coils for data acquisition in MRI, acquisition time can be reduced. This is achieved by using the sensitivity encoding (SENSE) method [3]. The ideas used in SENSE are adapted to multiple coil transmission in 2002 independently by Zhu and Katscher *et. al.* [4, 5]. Using multiple coils for transmission, excitation time and/or SAR can be reduced [2, 6].

SAR reduction methods for parallel transmission other than the one developed in [6] are also developed [7, 8]. The method developed in [8] adapts variable rate slice-selective excitation (VERSE) to parallel transmission. VERSE is an SAR reduction method first developed for single channel transmission [9]. VERSE is a method that modifies the designed RF pulses for SAR reduction and/or RF pulse duration reduction.

In this work, a novel RF pulse design method for SAR reduction in parallel transmission is developed. The goal of the RF pulse design method developed in this work is to design RF pulses that have the lowest SAR after VERSE-SAR reduction. This is achieved by modifying the RF pulse design method developed by Zhu in [6] and the new method is called VERSEp (VERSE optimal RF pulse design in parallel transmission).

The second contribution of this work is an improvement made to the VERSE-SAR reduction method developed by Lee *et. al.* in [8]. The VERSE-SAR reduction method in [8] is developed for single channel transmit pulses and a generalization of the method for multiple channel transmit pulses is proposed. Here, direct application of the VERSE-SAR reduction method defined in [8] to parallel transmission is explained and a modification to the method is given where the modified method yields better SAR reduction.

In Chapter 2, RF pulse design for parallel transmission using small-tip-angle approximation is explained and various SAR reduction methods are given. Chapter 3 explains the new methods developed in this thesis. The methods are compared with previous methods with simulations. Feasibility and performance of the methods are shown with simulation results in Chapter 4. Conclusions and future work are presented in Chapter 5.

Chapter 2

Background

This chapter explains some of the important RF pulse design techniques that are used for SAR reduction. First, small-tip-angle approximation will be explained. Using the small-tip-angle approximation, the RF pulse waveform that excites the arbitrary desired excitation pattern can easily be calculated [10]. Later, a method for designing parallel transmission RF pulses under small-tip-angle approximation will be explained [37]. The SAR reduction method developed by Zhu will also be introduced [6]. Finally, two other SAR reduction techniques, VERSE and phase variation based SAR reduction, will be summarized [7, 9]. The information in this chapter is critical in understanding the contribution of this thesis.

2.1 RF Pulse Design by Using Small-Tip-Angle Approximation

In the excitation phase of MRI, magnetization vectors at different positions are tilted at different angles by applying RF magnetic field at resonance frequency. When relaxation effects are neglected, the effect of applied RF magnetic field on magnetization vectors is a rotation, and this rotation is formulated by the Bloch equation [10]:

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma \mathbf{M} \times \mathbf{B}$$

where, $\mathbf{M}$ is the three dimensional magnetization vector and $\mathbf{B}$ is the three dimensional magnetic field vector and γ is the gyromagnetic ratio.

Rotation is a non-linear process. Therefore, designing the current waveforms that should be applied to the RF and gradient coils to achieve the desired excitation pattern is a sophisticated task. In the past, different design methods were developed for the non-linear problem of excitation assuming single RF coil with uniform magnetic field distribution [11, 12]. The

relation between the current waveform applied to the RF coil and the excitation pattern reduces to a simple Fourier-Transform like linear relation when tilt angles of the magnetization vectors are assumed to be small (small-tip-angle approximation) [10]. This relation is given by the following formula

$$M_{xy}^+(\mathbf{x}) = i\gamma M_0 \int_0^T B_1(t) \exp^{i\mathbf{x} \cdot \mathbf{k}(t)} dt. \quad (2.1)$$

Here, $M_{xy}^+ = M_x + iM_y$ and M_x and M_y are respectively the x and y components of the magnetization vector in rotating frame at resonance frequency. $B_1(t) = B_{1,x}(t) + iB_{1,y}(t)$ where $B_{1,x}(t)$ and $B_{1,y}(t)$ are the x and y components of the RF magnetic field in rotating frame. $\mathbf{x}$ is the three dimensional position vector, T is the total duration of the RF pulse and M_0 is magnitude of the initial magnetization vector. The tilt angle is given by

$$\alpha(\mathbf{x}) = \sin^{-1} \frac{|M_{xy}^+(\mathbf{x})|}{M_0}$$

where $\alpha(\mathbf{x})$ is the tilt angle as a function of position. $\mathbf{k}(t)$, the k-space trajectory, is given by the following formula

$$\mathbf{k}(t) = -\gamma \int_t^T \mathbf{g}(s) ds. \quad (2.2)$$

Here, $\mathbf{g}(s)$ is the three dimensional gradient vector in time $(G_x(s), G_y(s), G_z(s))$. Gradient waveforms should be designed to give a k-space trajectory that does not cause aliasing and covers a sufficient part of k-space so that desired resolution is achieved in the spatial domain.

If magnetization vectors are tilted with small angles, the desired excitation profile can be achieved by designing the RF waveform according to the equation in 2.1. Furthermore, it is shown in [13] that this relation works well for large-tip-angles for certain k-space trajectories.

2.1.1 Sprial K-Space Trajectory

In this section, the spiral k-space trajectory for two dimensional excitations will be explained. In 2-D excitation, magnetization vectors inside a plane are considered. Therefore, a plane in k-space is covered. One way of doing this is to sample k-space linearly. In rectangular trajectory, the spacing between k-space lines should be selected carefully to avoid aliasing inside the field-of-view (FOV).

Other very useful k-space trajectory is the spiral k-space trajectory. Spiral k-space trajectory is faster than the linear k-space trajectory. Usually, most of the energy of an RF pulse is located in the center of the k-space. Sprial k-space trajectory covers the center of the k-space slowly and this results in RF pulses with lower SAR than SAR of the RF pulses designed for linear k-space trajectory [14]. With these advantages, sprial k-space trajectory is usually preferred over linear k-space trajectory.

There are many types of spiral trajectories [15, 16], among which Archimedian spiral

k-space trajectory is given in [17] by

$$\mathbf{k} = n_r a \theta \exp^{i\theta}$$

where $\mathbf{k} = k_x + k_y$. This formula defines the path of Archimedian spiral. "a" should be chosen correctly to avoid aliasing in the spatial domain when $n_r = 1$. Increasing n_r results in radial under-sampling of k-space and this is used in excitations with multiple transmit coils. An example of Archimedian spirial k-space trajectory versus time is given by the following equations in [13]:

$$k_x(t) = \frac{At}{T} \cos \frac{2\pi nt}{T}$$

$$k_y(t) = \frac{At}{T} \sin \frac{2\pi nt}{T}$$

and the gradient waveforms for this this k-space trajectory are given by

$$g_x(t) = \frac{A}{\gamma T} \left[-\frac{2\pi nt}{T} \sin \frac{2\pi nt}{T} + \cos \frac{2\pi nt}{T} \right]$$

$$g_y(t) = \frac{A}{\gamma T} \left[\frac{2\pi nt}{T} \cos \frac{2\pi nt}{T} + \sin \frac{2\pi nt}{T} \right].$$

Here, A determines the area of the k-space to be covered, n determines the number of spirals and T is the total duration of the spiral trajectory. Distances between spirals and angular sampling intervals should be chosen as explained in [17] to avoid aliasing. A spiral k-space trajectory designed with $n_r = 1$ and the corresponding gradient waveforms are shown in Figures 2.1 and 2.2 respectively.

2.2 Parallel Transmission

2.2.1 Theory of Parallel Transmission

By using multiple RF coils for transmission, RF pulse duration and SAR can be reduced. These benefits of parallel transmission can be clearly seen by adapting 2.1 to the case when multiple RF coils with non-uniform circularly polarized magnetic field distribution are used [2, 6]. This formulation is independently presented in the 2002 ISMRM conference by Zhu [5] and Katscher *et. al.* [4]. Here, a simpler derivation of the same idea developed by Grissom *et. al.* is explained [18, 19].

Number of RF transmit coils will be denoted by R . Sensitivity pattern or circularly polarized magnetic field of the r 'th coil will be denoted by $S_r(\mathbf{x})$. Also, normalized current waveform applied to the r 'th coil will be given by $b_r(t)$. Since the problem will be formulated

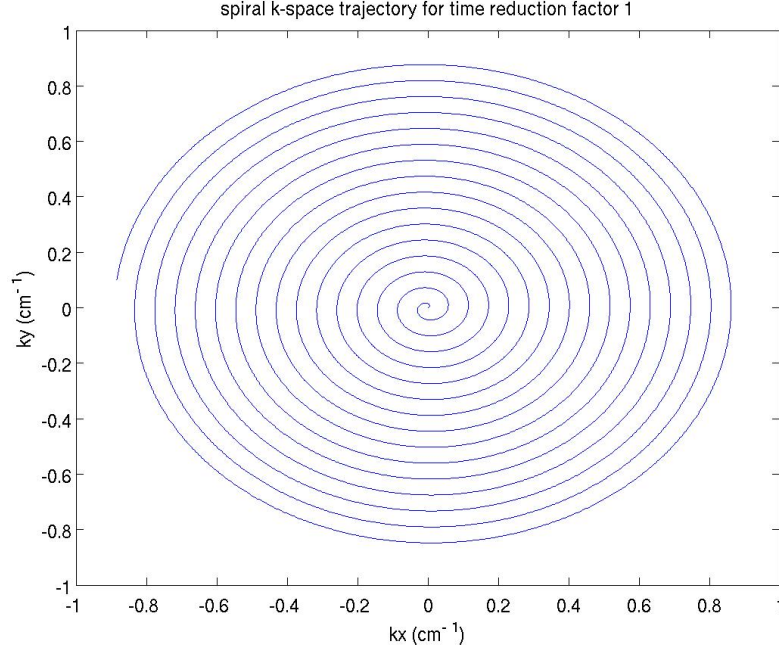


Figure 2.1: A spiral k-space trajectory designed with a time reduction factor of 1.

for transmit coils with non-uniform sensitivity, the variable $B_1(t)$ in equation 2.1 is now a function of $\mathbf{x}$ and t and it is given by the following formula:

$$B_1(\mathbf{x}, t) = \sum_{r=1}^R S_r(\mathbf{x}) b_r(t). \quad (2.3)$$

The following equation is obtained by plugging equation 2.3 into equation 2.1.

$$M_{xy}^+(\mathbf{x}) = i\gamma M_0 \sum_{r=1}^R S_r(\mathbf{x}) \int_0^T b_r(t) \exp^{i\mathbf{x} \cdot \mathbf{k}(t)} dt \quad (2.4)$$

The sensitivity profiles of the coils are usually obtained before the scan. Therefore, analytic solution of the equation 2.4 is not possible in most cases. For this reason, integral in 2.4 and the spatial domain are discretized to find the RF current waveforms with computation. As explained in [20], after discretization, 2.4 can be written in the following form.

$$\mathbf{m}(\mathbf{x}_i) = \sum_{r=1}^R \text{diag}\{S_r(\mathbf{x}_i)\} \mathbf{A} \mathbf{b}^r \quad (2.5)$$

Here, $\mathbf{m}(\mathbf{x}_i)$ is the desired transverse magnetization value of the i 'th sample in discretized spatial domain and $\mathbf{b}^r$ is a size M column vector that contains samples of $b_r(t)$ in time where M is the number samples in time. $\mathbf{A}$ is a size $N \times M$ matrix and $a_{ij} = i\gamma M_0 \Delta t \exp^{-i\gamma \mathbf{x}_i \cdot \mathbf{k}(t_j)}$. Δt is the sampling interval in time and N is the number of samples in the spatial domain. $\text{diag}\{S_r(\mathbf{x}_i)\}$ is a size $N \times N$ diagonal matrix with i 'th diagonal element equal to $S_r(\mathbf{x}_i)$. The linear relation in 2.5 can be written in the following single matrix vector multiplication form.

$$\mathbf{m}_{full} = \mathbf{A}_{full} \mathbf{b}_{full} \quad (2.6)$$

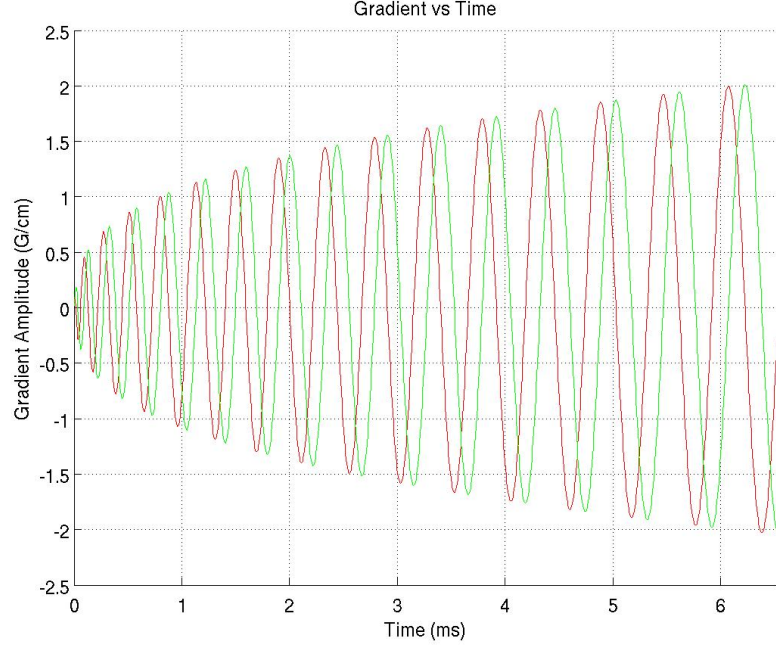


Figure 2.2: Gradient waveforms corresponding to this k-space trajectory given in Figure 2.1. x-gradient waveform is shown with red and y-gradient waveform is shown with green.

Here, $\mathbf{m}_{full}$ a size N column vector that contains the desired transverse magnetization values at spatial domain samples. $\mathbf{b}_{full}$ is a column vector of size RM which contains the current waveforms of all coils at time samples and $\mathbf{A}_{full}$ is a matrix of size $N \times RM$. Here, $\mathbf{b}_{full}$ is in the following form.

$$\mathbf{b}_{full} = \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \\ \vdots \\ \mathbf{b}_M \end{pmatrix} \quad (2.7)$$

where the elements in the form $\mathbf{b}_i$ are column vectors of size R and j 'th element of $\mathbf{b}_i$, is $b_i(t_j)$. We can write the same element as $b_i(\mathbf{k}(t_j))$ which represents the sample of current waveform of i 'th transmit coil in the j 'th k-space sample.

To guarantee a solution for the linear system of equations in 2.6, we need $RM \geq N$ or $M \geq N/R$. Now it is clear that we can decrease the number of k-space samples by using more than one coils. In other words, k-space can be under-sampled up to the number of coils used and still the desired excitation pattern in the resolution defined by number of spatial samples N can be achieved. This is one of the important advantages of using parallel transmission. The other important advantage of using parallel transmission is SAR reduction.

Since all these derivations are made under small-tip-angle approximation, excitation accuracy of the pulses designed with this methodology decreases as the desired flip angles increases. However, small-tip-angle design method is still important because of the simple linear relations obtained. With the linear equations obtained, it is easier to make further improvements to the methodology [21]. Some other applications of parallel transmission with this methodology can be found in the literature [22, 23]. Different methods to design large-tip-angle parallel transmit pulses have been developed [24, 25].

2.2.2 SAR Reduction Using Parallel Transmission

During excitation, electromagnetic field induced inside the imaged body causes heating of the imaged body. SAR is used as a measure of this heating. To find the relation between RF pulses and the heating, whole-body average SAR (SAR_{ave}) for a single excitation is defined with the following formula [6]:

$$SAR_{ave} = \frac{\Delta t}{TRm} \sum_{i=0}^{M-1} \frac{1}{M} \int \sigma(\mathbf{x}) |\mathbf{E}(\mathbf{x}, t_i)|^2 dv \quad (2.8)$$

where TR , P , m and $\sigma(\mathbf{x})$ are the repetition time, total number of time samples, object mass and conductivity as a function of position respectively. $\mathbf{E}(\mathbf{x}, t_i)$ is the three dimensional electric field vector as a function of position and time and the integral is taken over the imaged volume. By considering the multiple coils, $\mathbf{E}(\mathbf{x}, t_i)$ can be written in the following form:

$$\mathbf{E}(\mathbf{x}, t_i) = \sum_{j=1}^R b_i^j \mathbf{E}_j(\mathbf{x}) \quad (2.9)$$

where $\mathbf{E}_j(\mathbf{x})$ is the three dimensional electric field vector of j 'th coil as a function of position. b_i^j , j 'th element of the vector $\mathbf{b}_i$, is the i 'th current sample of j 'th coil. $|\mathbf{E}(\mathbf{x}, t_i)|^2$ is as a dot product of the electric field vector with itself. By using the dot product notation and equation 2.9 the following equations are obtained:

$$\begin{aligned} & \int \sigma(\mathbf{x}) |\mathbf{E}(\mathbf{x}, t_i)|^2 dv \\ &= \int \sigma(\mathbf{x}) \left(\sum_{j=1}^R b_i^j \mathbf{E}_j(\mathbf{x}) \right) \cdot \left(\sum_{k=1}^R b_i^k \mathbf{E}_k(\mathbf{x}) \right) dv \\ &= \sum_{k=1}^R \sum_{j=1}^R b_i^{j*} b_i^k \int \sigma(\mathbf{x}) \mathbf{E}_j(\mathbf{x}) \cdot \mathbf{E}_k(\mathbf{x}) dv. \end{aligned}$$

Where $*$ is the complex conjugate operator. With these modifications, the relation between current waveforms of the coils and average SAR can be reduced to a simple quadratic form given by:

$$SAR_{ave} = \sum_{i=1}^M \mathbf{b}_i^H \mathbf{S}_{sub} \mathbf{b}_i \quad (2.10)$$

where H is the conjugate transpose operator. $\mathbf{S}_{sub}$ is a positive-definite matrix and the element in j 'th row and k 'th column of $\mathbf{S}_{sub}$, denoted as s_{sub}^{jk} , is given by:

$$s_{sub}^{jk} = \frac{\Delta t}{TRMm} \int \sigma(\mathbf{x}) \mathbf{E}_j(\mathbf{x}) \cdot \mathbf{E}_k(\mathbf{x}) dv. \quad (2.11)$$

To reduce the relation to a simpler form, a block diagonal matrix $\mathbf{S}$ is defined where each block of $\mathbf{S}$ is $\mathbf{S}_{sub}$. The equation reduces to

$$SAR_{ave} = \mathbf{b}_{full}^H \mathbf{S} \mathbf{b}_{full}. \quad (2.12)$$

If number of k-space samples are chosen to be sufficiently high so that the inequality $M > N/R$ is satisfied, solution of the equation 2.6 is not unique, which means that different RF waveforms can be used to achieve the desired excitation pattern at the given resolution. The SAR reduction method explained in [6] finds the RF pulses with minimum SAR among the pulses that achieve the desired excitation profile. This method is formulated by the following

$$\text{minimize} \quad SAR_{ave} = \mathbf{b}_{full}^H \mathbf{S} \mathbf{b}_{full} \quad (2.13)$$

$$\text{subjected to} \quad \mathbf{m}_{full} = \mathbf{A}_{full} \mathbf{b}_{full} \quad (2.14)$$

and the solution of the optimization problem is

$$\mathbf{b}_{opt} = \mathbf{S}^{-1} \mathbf{A}_{full}^H (\mathbf{A}_{full} \mathbf{S}^{-1} \mathbf{A}_{full}^H)^{-1} \mathbf{m}_{full}$$

$$SAR_{opt} = \mathbf{b}_{opt}^H \mathbf{S} \mathbf{b}_{opt}.$$

A derivation of the solution can be found in [26]. Zhu reported an SAR reduction of %38 by using this method [6].

2.3 Variable-Rate Selective Excitation (VERSE)

The pulse design method explained in section 2.2.2 assumes a given k-space trajectory. The velocity in which k-space is traced is also determined before the pulse design. On the other hand, it is possible to modify the k-space trajectory and the RF pulses without changing the excitation profile if the modifications are made properly. A simple procedure for such modifications is called variable rate selective excitation (VERSE) [9]. By using VERSE, the path of the k-space trajectory is kept unchanged but the velocity in which k-space is traced and RF pulse waveforms are changed coherently to reduce the SAR and/or RF pulse duration without changing the excitation profile. VERSE is not a method for designing new pulses, it is a method for modifying the designed pulses for SAR reduction and/or pulse duration reduction.

The applied magnetic field causes a rotation of the magnetization vector around the axis of the magnetic field in an angular velocity proportional to magnitude of the magnetic field.

Now consider rotation of the magnetization vector around an applied field. If we double the duration of the rotation and reduce the magnitude of the magnetic field to it's half, rotation of magnetization vector will be the same. This is the idea used in VERSE and it is proved in a formal way in [8] for the single coil and multiple coil cases.

VERSE is formulated for parallel transmission by Lui *et. al.* in 2008 [27]. VERSE pulse design starts with an RF pulse designed for a given desired excitation profile by any design technique. Based on the VERSE method, for a sample point in k-space, if the gradient amplitude and the RF pulse magnitude for all transmit coils at that k-space point are multiplied by a constant $\alpha(\mathbf{k}(t_i))$ and the duration of the RF pulse sample is multiplied by $1/\alpha(\mathbf{k}(t_i))$ then the excitation profile does not change. This procedure is applied to all k-space samples to reduce the SAR and/or the RF pulse duration. Without gradient constraints, VERSE-SAR reduction technique gives a uniform RF amplitude in time, with constraints, this may change. So we have constrained VERSE and unconstrained VERSE. Second one is not suitable for MRI implementation since all practical coils have constraints on their amplitude and slew rates.

2.3.1 Time Optimal Design Using VERSE

The modified RF waveforms and gradient waveforms are given by

$$\mathbf{B}_i = \alpha(\mathbf{k}(t_i))\mathbf{b}_i \quad (2.15)$$

$$\mathbf{G}(\mathbf{k}(t_i)) = \alpha(\mathbf{k}(t_i))\mathbf{g}(\mathbf{k}(t_i)) \quad (2.16)$$

and the relation between new durations of the RF pulse samples and the durations before modification are given by

$$\Delta T(\mathbf{k}(t_i)) = \Delta t(\mathbf{k}(t_i))/\alpha(\mathbf{k}(t_i)). \quad (2.17)$$

The new total pulse duration is

$$T = \sum_{i=1}^M \Delta T(\mathbf{k}(t_i)). \quad (2.18)$$

Purpose of the time optimal design becomes finding $\alpha(\mathbf{k}(t_i))$'s such that T is minimized under the maximum gradient amplitude, maximum gradient slew rate and maximum RF current constraints. When maximum gradient slew-rate constraint is neglected, $\alpha(\mathbf{k}(t_i))$ is given by

$$\alpha(\mathbf{k}(t_i)) = \max \left\{ \frac{G_x(\mathbf{k}(t_i))}{G_{max}}, \frac{G_y(\mathbf{k}(t_i))}{G_{max}}, \frac{G_z(\mathbf{k}(t_i))}{G_{max}}, \frac{\max\{\mathbf{b}_i\}}{b_{max}} \right\}. \quad (2.19)$$

Gradient slew-rate constraints become dominant when parallel transmission is used. However, VERSE optimization with slew-rate constraint requires usage of more sophisticated techniques [8]. Technique developed in [8] uses a time-optimal path planing method to implement VERSE optimization with gradient slew rate constraint.

2.3.2 SAR Reduction Using VERSE

SAR reduction is another important beneficial outcome of VERSE. SAR reduction method of VERSE keeps total duration of RF pulses constant and reduces total SAR of the pulses. The expression for the VERSE SAR reduction is

$$\text{minimize} \quad \sum_{i=1}^M \mathbf{b}_i^H \mathbf{S}_{sub} \mathbf{b}_i \alpha(\mathbf{k}(t_i)) \quad (2.20)$$

$$\text{subjected to} \quad \sum_{i=1}^M T(\mathbf{k}(t_i)) = \sum_{i=1}^M t(\mathbf{k}(t_i)). \quad \Leftrightarrow \quad \sum_{i=1}^M \frac{1}{\alpha(\mathbf{k}(t_i))} = M. \quad (2.21)$$

The equation in 2.20 is the expression for SAR after VERSE optimization. When the optimization problem is solved without any gradient hardware constraints, VERSE optimized RF pulse becomes a pulse with fixed power over time. For example for the single transmit coil case, VERSE optimized RF pulse has a constant amplitude over time. For the unconstrained multiple coil case, optimum $\alpha(\mathbf{k}(t_i))$'s are

$$\alpha(\mathbf{k}(t_i)) = \frac{\left(\sum_{k=1}^M \sqrt{\mathbf{b}_k^H \mathbf{S}_{sub} \mathbf{b}_k} \right)}{\sqrt{\mathbf{b}_i^H \mathbf{S}_{sub} \mathbf{b}_i} M}. \quad (2.22)$$

Therefore, power associated with each k-space sample, given by $\mathbf{B}_i^H \mathbf{S}_{sub} \mathbf{B}_i$, are equal after unconstrained VERSE optimization.

A VERSE-SAR reduction method with constrained design capability is explained in [8]. This method modifies the constrained time optimal design explained in [28] to obtain a peak RF constrained VERSE time optimal design. Afterwards, the new time-optimal VERSE design method is modified to obtain a VERSE-SAR reduction method capable of handling maximum gradient slew-rate and maximum gradient amplitude constraints [8]. The SAR reduction method in [8] will be explained in more detail.

2.4 SAR Reduction in Parallel Transmission by Allowing Spatial Phase Variation In the Magnetization Profile

In the SAR reduction method developed in [6], SAR of shortened pulses can not be reduced significantly. In [7], an SAR optimization method based on releasing phase constraints of desired excitation samples is developed. This method is developed for pulses designed based on small-angle approximation. This method benefits the fact that phase variations causes less error in the final image than magnitude variations (i.e. there is a higher degree of freedom in phase variations). The method allows minimization of SAR in time reduced pulses (time

reduction factor up to the number of coils). Phase optimization problem is non-convex, therefore method uses a search based optimization algorithm. Such search algorithms can be found in [29]. A phase variation based SAR reduction method that uses the optimization algorithm particle swarm optimization (PSO) [30] is explained by Bayindir *et. al.* in [31].

In this chapter, parallel RF pulse design using small-tip-angle approximation, SAR reduction using parallel transmission and VERSE-SAR reduction method are explained. In the following chapter, a new method for SAR reduction in parallel transmission that is developed for designing RF pulses according to VERSE-SAR reduction is explained and an improvement to the VERSE-SAR reduction method in [8] is given.

Chapter 3

SAR Reduction Method

3.1 SAR Reduction by Combining VERSE and Parallel Transmission

3.1.1 Analytic Expression of SAR of VERSE Optimized RF Pulses

VERSE is a very important method that reduces SAR of a given RF pulse without changing excitation profile of the pulse. To combine different SAR reduction methods with VERSE, it is crucial to understand how SAR of an RF pulse will change after VERSE is applied to it. In most cases, a VERSE algorithm should incorporate gradient hardware constraints so that maximum gradient amplitude and maximum slew-rate constraints are not violated and such algorithms use optimization methods of time optimal path planning etc [8]. Therefore when gradient constraints are imposed, there is no analytic expressions for the SAR of an RF pulse after VERSE-SAR reduction. On the other hand, SAR expressions become simple when gradient hardware constraints are not imposed. Relation between a given RF pulse and SAR of the RF pulse after VERSE-SAR reduction can be expressed with a simple equation. Substituting the $\alpha(\mathbf{k}(t_i))$'s from 2.22 into the expression of the SAR after verse optimization given in 2.20 yields

$$\sum_{i=1}^M \mathbf{b}_i^H \mathbf{S}_{sub} \mathbf{b}_i \frac{\left(\sum_{k=1}^M \sqrt{\mathbf{b}_k^H \mathbf{S}_{sub} \mathbf{b}_k} \right)}{\sqrt{\mathbf{b}_i^H \mathbf{S}_{sub} \mathbf{b}_i} M} \quad (3.1)$$

and after simplifications, the following is obtained

$$SAR_{V_{opt}} = \frac{\left(\sum_{i=1}^M \sqrt{\mathbf{b}_i^H \mathbf{S}_{sub} \mathbf{b}_i} \right)^2}{M}. \quad (3.2)$$

Here, $\mathbf{S}_{sub}$ is a positive definite matrix. Therefore, the expression inside the square root is square of a norm which can be designated as $\|\mathbf{b}_i\|_2^2$. Hence, we can write the equation in a sum-of norms form.

$$SAR_{Vopt} = \frac{\left(\sum_{i=1}^M \|\mathbf{b}_i\|_2\right)^2}{M} \quad (3.3)$$

Obviously we expect SAR of the RF pulses after VERSE-SAR reduction to be less than SAR before VERSE-SAR reduction. In other words, we expect $SAR_{Vopt} \leq SAR_{ave}$. Expression of SAR_{ave} is given in section 2.10. A proof of the inequality is given in chapter 6.

3.1.2 Combining VERSE and Parallel Transmit SAR reduction

Earlier works on adapting VERSE to parallel transmission introduced methods that reduce the SAR of a pulse designed for parallel transmission. However in those methods, degree of freedom obtained in using parallel transmit channels is not benefited for SAR reduction using VERSE. Here SAR reduction method introduced by Zhu [6] and VERSE are combined. Method developed in [6] was based on designing the RF pulse that has the minimal SAR among the pulses that give the desired excitation pattern. However, that pulse may not be the pulse that has the minimal SAR after VERSE-SAR reduction. The method developed in this work is based on designing the RF pulse that excites the desired excitation pattern and has the minimal SAR after SAR reduction with VERSE. This idea is formulated by the following

$$\text{minimize} \quad \frac{\left(\sum_{i=1}^M \|\mathbf{b}_i\|_2\right)^2}{M} \quad (3.4)$$

$$\text{subjected to} \quad \mathbf{m}_{full} = \mathbf{A}_{full}\mathbf{b}_{full}. \quad (3.5)$$

There is no analytical solution of this optimization problem but it is a convex optimization problem and the problem is in the form of a sum of norms minimization with linear equality constraint. An extensive work for such problems can be found in [32]. This optimization problem is also in the form of second-order conic constraint optimization with a linear constraint. Therefore, the global optimum for this optimization problem can be computed using second-order cone programming. The reformulation of the problem for second-order cone programming is given by the following

$$\text{minimize} \quad \sum_{i=1}^M t_i \quad (3.6)$$

$$\text{subjected to} \quad \|\mathbf{b}_i\|_2 \leq t_i \quad \text{for all} \quad 1 \leq i \leq M \quad (3.7)$$

$$\text{and} \quad \mathbf{m}_{full} = \mathbf{A}_{full}\mathbf{b}_{full}. \quad (3.8)$$

The last two optimization problems are identical, and the reformulation is only necessary to reduce the optimization problem to a form which can be solved by second order-cone programming.

Pulses designed with this optimization problem guarantee to give lower SAR than pulses designed using Zhu's method [6] after VERSE-SAR reduction without any gradient constraints. However when unconstrained VERSE-SAR reduction is applied to parallel transmit RF pulses, violation of gradient constraints becomes significant. Therefore pulses designed with the optimization problem in 3.4 and 3.5 should go into a constrained VERSE-SAR reduction so that gradient constraints are not violated. We call this overall method VERSEp.

In this work, the second-order cone programming is implemented by using a free-ware package called SeDuMi which is designed for Matlab and the package can be found on the web [34]. SeDuMi is implemented by using an easy-to use Matlab interface YALMIP which is also free and available on the web [33].

3.1.3 Adding SAR Reduction Capability for Parallel Transmit Pulses to the VERSE-SAR Reduction Method in [8]

Gradient constraints are important limiting factors for VERSE-SAR reduction of parallel transmission RF pulses. Here, the constrained VERSE-SAR reduction method given in [8] will be explained. The constrained VERSE-SAR reduction method of [8] is tested for single channel transmit pulses and application of the method is proposed for multi channel transmission. Moreover, direct application of the constrained VERSE-SAR reduction method of [8] to parallel transmit pulses is also explained and an improvement to the method that yields better SAR reduction is introduced.

As explained in section 2.3.2, VERSE-SAR reduction methods keep the RF pulse duration constant and reduces the SAR. Furthermore, unconstrained VERSE-SAR reduced pulses have constant power over time which corresponds to an RF pulse with constant amplitude for single coil transmission and this is approximately true for pulses designed with gradient hardware and peak RF constraints. By using these ideas, the method given in [8] modifies a time optimal VERSE design method into a VERSE-SAR reduction method. First, the peak RF constraint is imposed by modifying the peak gradient constraint. Equations 2.16 and 2.15 show that the peak RF can be constrained to a maximum value if appropriate maximum gradient constraints are imposed to different k-space points. This is given by the following formula

$$|\mathbf{G}(\mathbf{k}(t_i))| \leq \max \left\{ \frac{B_{max} |\mathbf{g}(\mathbf{k}(t_i))|}{\max\{\mathbf{b}_i\}}, G_{max} \right\} \quad (3.9)$$

where B_{max} and G_{max} are the maximum RF and gradient limits respectively and $\max\{\mathbf{b}_i\}$ is the absolute value of the element of $\mathbf{b}_i$ with the highest absolute value. If B_{max} is reduced,

duration of the pulses designed using time optimal VERSE increases but peak amplitude over time becomes closer to constant. This is the idea used in [8] for SAR reduction of single channel transmit pulses. B_{max} is reduced in iterations so that the duration of the time optimal pulse becomes the desired duration and $\max\{\mathbf{B}_i\}$ is close to being constant over time. The iterative method is summarized in the following algorithm.

```

[1]  $B_{max}^- = 0, B_{max}^+ = \max\{\mathbf{b}_i\}$ 
[2]  $B_{max} = (B_{max}^- + B_{max}^+)/2$ 
[3] Compute the time-optimal VERSE using the method developed in [28] with
    gradient constraints as explained in equation 3.9 and obtain the new pulse duration  $T^V$ 
    and the new gradient waveforms  $\mathbf{G}^V(t)$ 
[4] if  $|T^V - T| \leq \epsilon$ : return  $T^V$  and  $\mathbf{G}^V(t)$ 
    else
    if  $T^V \leq T - \epsilon$ :  $B_{max}^+ = B_{max}$ 
    else:  $B_{max}^- = B_{max}$ 
    goto 2

```

Figure 3.1: Constrained VERSE-SAR reduction method developed in [8]

The variable denoted as T in the algorithm given in Figure 3.1 is the pulse duration before VERSE-SAR reduction.

The goal of achieving pulses with constant RF amplitude is not consistent with the VERSE-SAR optimal pulses with constant power over time but not constant RF amplitude over time. The goal should be achieving pulses with constant square root power. For this reason, a different constraint $\text{sqr}tP_{max}$ (the maximum square root power constraint) for VERSE design is introduced in this work. Like the maximum RF constraint, maximum square root power constraint can be imposed by modifying the gradient amplitude constraint for every k-space space sample. This is given by

$$|\mathbf{G}(\mathbf{k}(t_i))| \leq \max\left\{\frac{\text{sqr}tP_{max}|\mathbf{g}(\mathbf{k}(t_i))|}{\|\mathbf{b}_i\|_2}, G_{max}\right\}. \quad (3.10)$$

After modifying the gradient constraints, rest of the iterative method given in Figure 3.1 should be applied by searching $\text{sqr}tP_{max}$ instead of B_{max} . By using the peak square root power constraint instead of the peak RF constraint, pulses with the same duration but lower SAR can be designed. Furthermore, extra computation time introduced as a result of using peak square root power constraint is insignificant compared to the computation time of the method in [8].

3.1.4 Combining Excitation Profile Modification based SAR Reduction Techniques with VERSEp

When the desired excitation profile is modified properly, significant SAR reduction can be obtained with an insignificant distortion on the obtained image. Methods that benefits

this fact can be found in the literature [7, 21, 31, 35]. One of such methods is explained in section 2.4. These SAR reduction methods can be used together with VERSE-SAR reduction methods such as VERSEp method explained in section 3.1. An excitation profile modification based SAR reduction method can be used before VERSE optimization. When the pulses are designed for the modified excitation profile, a VERSE optimization can be performed on the designed RF pulses. By this procedure, a chain of SAR reduction methods can be obtained. Moreover in the SAR reduction method explained in [21], the excitation profile modification can be reformulated for VERSE-SAR reduction like the method explained in 3.1.2.

In [21], the RF pulse design is realized by a trade-off between the amount of distortion on the excitation profile and the reduction in SAR.

$$\hat{\mathbf{b}}_{full} = \arg \min_{\mathbf{b}_{full}} \left\{ \|\mathbf{A}_{full} \mathbf{b}_{full} - \mathbf{m}\|_2^2 + \lambda \mathbf{b}_{full}^H \mathbf{S} \mathbf{b}_{full} \right\}.$$

Here, the trade-off between the reduction of SAR and the amount of image distortion is adjusted by the regularization parameter λ . If the pulse design problem is reformulated to find the lowest SAR RF pulse after VERSE-SAR reduction, the following is obtained.

$$\hat{\mathbf{b}}_{full} = \arg \min_{\mathbf{b}_{full}} \left\{ \|\mathbf{A}_{full} \mathbf{b}_{full} - \mathbf{m}\|_2 + \lambda \frac{(\sum_{i=1}^M \|\mathbf{b}_i\|_2)^2}{M} \right\}.$$

This reformulation is an example of combining VERSEp with other SAR reduction methods.

This chapter introduced new methods for SAR reduction in parallel transmission. Feasibility and performance of these new methods are shown with simulation results in the following chapter.

Chapter 4

Simulations

In this chapter, results and methods of the simulations performed for checking feasibility and performance of VERSEp method are presented. Performance improvement obtained by the modification, explained in Section 3.1.3, made to the method in [8] is also shown with simulations.

Simulations are performed for two different cases. In the first case, VERSEp is compared with pulse design method introduced by Zhu [6]. SAR of the pulses designed with Zhu's method in [6] are reduced with VERSE and the final SAR is compared with the SAR of the pulses designed using VERSEp. The used VERSE-SAR reduction method is capable of gradient constrained design. In the second case, improvement obtained by using peak square root power constrained SAR reduction in VERSE-SAR reduction method of [8] instead of peak RF constrained SAR reduction is checked by comparing the SAR of the pulses designed with square root power constrained method and the peak RF constrained method. Simulations are performed with a twelve-channel transmit array and results are obtained for different time reduction factors.

4.1 Methods

Here in the first part, the RF coil configuration used in the simulations will be explained and in the second part, parameters used in RF pulse designs will be explained.

The transmit coil array used for the simulations consists of twelve coils. Each coil is a rectangular loop and the coils are placed around a cylindrical phantom. Circularly polarized magnetic field distributions and electric field distributions of transmit coils are obtained from the electromagnetic simulation software FEKO by Yiğitcan Eryaman. Figure 4.2 shows the simulation model for the rectangular coils placed next to the cylindrical. To calculate the

whole-body SAR, electric fields are obtained for all of the voxels inside the phantom where magnetic fields are obtained only inside the center plane. The center plane is the plane perpendicular to the z-axis (as shown in Figure 4.2) and equidistant to the top and bottom of the cylinder. Electric and magnetic fields of only one coil is obtained from FEKO. Fields of other coils are obtained by rotating the fields of the simulated coil around the z-axis. The phantom used is a head phantom. Phantom and coils parameters are given in Table 4.1. Conductivity and relative permittivity values are obtained by averaging over conductivity

Table 4.1: Phantom and coil parameters for the simulations

Main field strength 3T
 Phantom length 20 cm
 Radius of the phantom 8.5 cm
 Conductivity 0.49 S/m
 Relative permittivity 65
 Coil length 22 cm
 Coil width 5.86 cm
 Coil-phantom separation 5.5 cm

and relative permittivity values of different tissues of the head. Conductivity and relative permittivity values of tissues can be found in [36].

Circularly polarized magnetic field and electric field distributions of one of the coils inside the center plane are given in Figure 4.1.

Now the details of the RF pulse designs used in the simulations will be explained. Archimedean k-space trajectory, explained in Section 2.1.1, is used. Gradient waveforms and k-space trajectory are designed using the code developed by Hargreaves (<http://mrsrl.stanford.edu/~brian/mritools.html>). Explanations for the design method can be found in [17]. Parameters used for the k-space trajectory design are shown in Table 4.2. For the k-space trajectory design, S_{max} is selected as 200 T/m/sec for time reduction factor

Table 4.2: K-space trajectory design parameters

FOV 17.4 cm
 Maximum gradient amplitude (G_{max}^k) 4.5 G/cm
 Resolution 5.61 mm
 Maximum gradient slew-rate (S_{max}^k) 140 T/m/sec (unless otherwise stated)

of 1, whereas for simulations with other time reduction factors, $S_{max} = 140$ T/m/sec is used. The time reduction factor is denoted as n_r . n_r determines the factor of undersampling in radial direction. Therefore, increasing n_r decreases duration of the k-space trajectory. Results are obtained for the integer values of n_r from 1 to 10. For the constrained VERSE-SAR reduction, parameters in Table 4.3 were used for gradient hardware limits.

These are the constraints of one of the available fast insert head gradients [22].

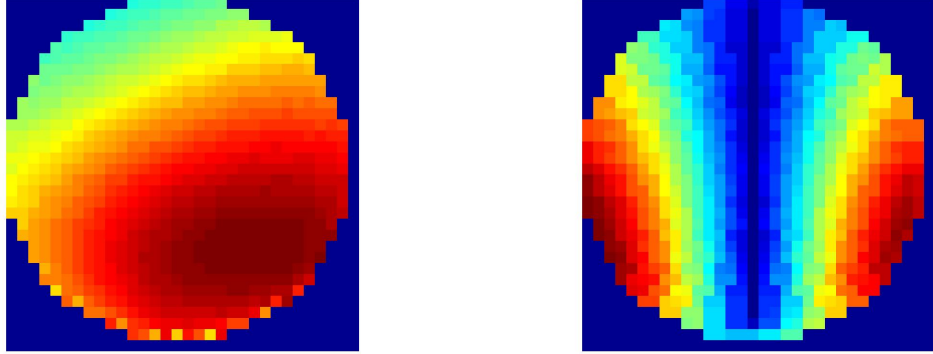


Figure 4.1: Circularly polarized magnetic field (left) and electric field (right) distributions of one of the coils.

Table 4.3: Gradient hardware limits used for constrained VERSE-SAR reduction

G_{max}	7 G/cm
S_{max}	700 T/m/sec

Waveform of square root power vs k-space samples is used in the analyses of the results. Square root power for the i 'th k-space sample is given by $\sqrt{\mathbf{b}_i^H \mathbf{S}_{sub} \mathbf{b}_i}$ where the column vector $\mathbf{b}_i$ and the matrix $\mathbf{S}_{sub}$ are explained in Sections 2.2.1 and 2.2.2 respectively.

The maximum local SAR to whole-body SAR ratio is given in the results. The maximum local SAR is given by the following formula.

$$\max\{SAR_{local}\} = \max\left\{\frac{\Delta t}{MTRm} \sum_{j=0}^{M-1} \sigma(\mathbf{x}_i) |\mathbf{E}(\mathbf{x}_i, t_j)|^2\right\} \quad (4.1)$$

where M , $\sigma(\mathbf{x}_i)$ and $\rho(\mathbf{x}_i)$ are the total number of time samples, conductivity as a function of position and density as a function of position respectively. $\mathbf{E}(\mathbf{x}_i, t_i)$ is the three dimensional electric field vector as a function of position and time. The $\max\{\cdot\}$ operator is the maximum value of the expression inside $\max\{\cdot\}$ for all voxels inside the phantom. The maximum local SAR to whole-body SAR ratio is given by

$$\frac{\max\{SAR_{local}\}}{SAR_{ave}} \quad (4.2)$$

where SAR_{ave} is the whole-body SAR as explained in Section 2.2.2.

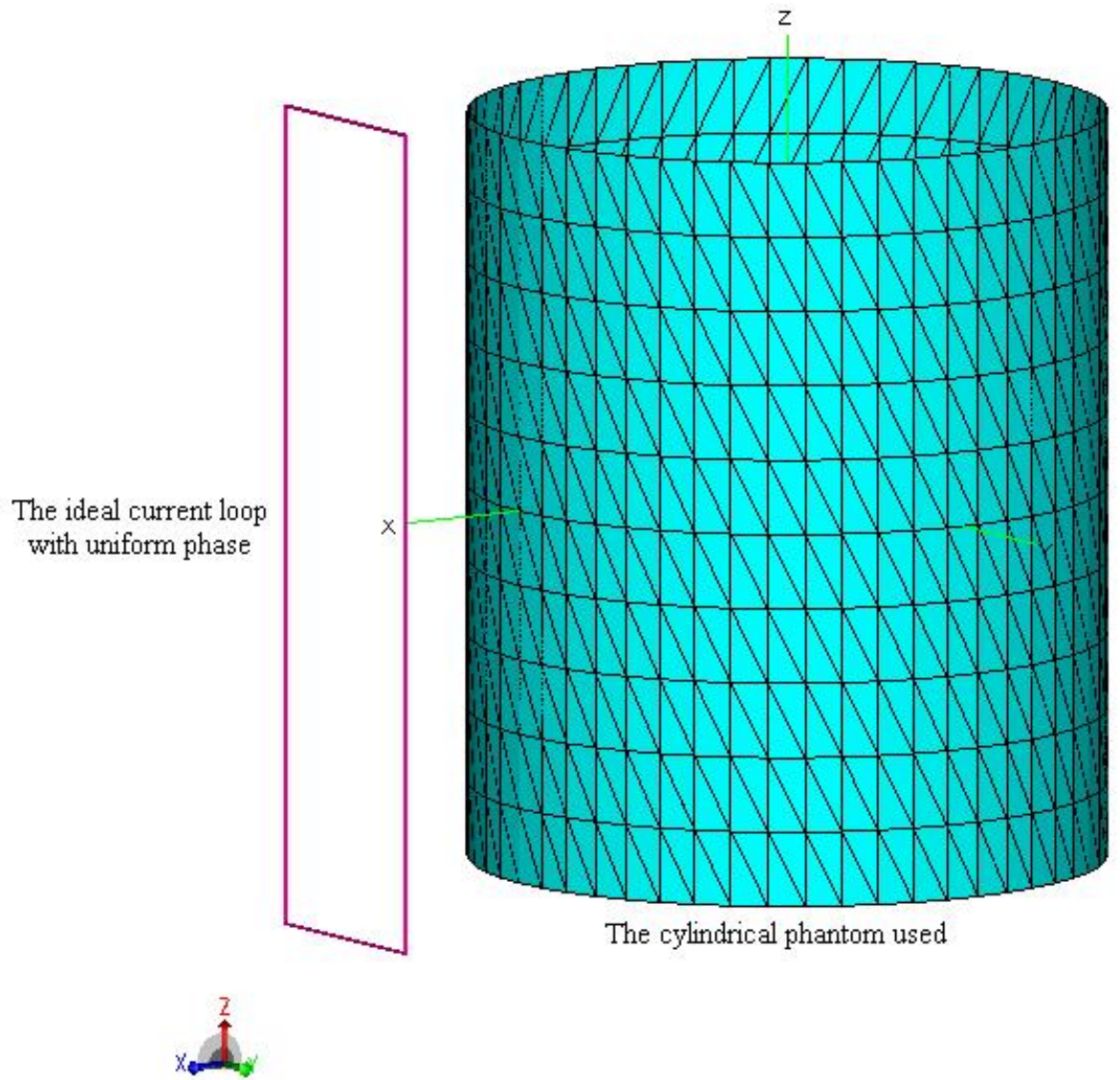


Figure 4.2: The electromagnetic simulation model. The model is shown as a screenshot from the simulation software. In this model, the rectangular loop had dimensions of 22X5.86 cm and the cylindrical phantom a height of 20 cm and a radius of 8.5 cm. Conductivity of 0.49 s/m and a relative dielectric constant of 65 were assumed and simulations were carried out for 127.28 MHz.

4.2 Feasibility and Performance of VERSEp

Here, feasibility and performance of the VERSEp method is tested. First, parallel transmit pulses are designed using the SAR reduction method of Zhu [6]. Zhu's method is explained in Section 2.2.2. SAR of these pulses after unconstrained and constrained VERSE-SAR reduction are calculated. These SAR values are compared with the SAR values of RF pulses designed with constrained and unconstrained VERSEp. For the constrained VERSE-SAR reduction, the improved method of [8] is used. The improvement to the constrained VERSE SAR reduction method in [8] is explained in Section 2.2.2. Here, the SAR variables shown in Table 4.4 are used to define the values of percentage improvements.

Table 4.4: Percentage improvement definitions

SAR^{Zhu} : SAR of the RF pulses designed with Zhu's method [6] after unconstrained VERSE-SAR reduction

SAR_C^{Zhu} : SAR of the RF pulses designed with Zhu's method [6] after constrained VERSE-SAR reduction

SAR^{VERSEp} : SAR of the pulses designed with unconstrained VERSEp

SAR_C^{VERSEp} : SAR of the pulses designed with constrained VERSEp

$$\%IMP = \left(1 - \frac{SAR^{VERSEp}}{SAR^{Zhu}}\right) \times 100 \quad \%IMPC = \left(1 - \frac{SAR_C^{VERSEp}}{SAR_C^{Zhu}}\right) \times 100$$

4.2.1 Results

Table 4.5 summarizes the percentage improvements.

Table 4.5: Percentage SAR improvements and pulse durations for different time reduction factors

n_r	% IMP	% IMPC	Pulse Duration (ms)
1	80	53	6.65
2	81	66	4.01
3	75	56	2.69
4	66	47	2
5	56	36	1.64
6	45	27	1.39
7	33	14	1.2
8	24	4	1.06
9	15	0	0.948
10	7	0	0.84

Table 4.5 shows that SAR reductions up to 47% are obtained for RF pulses with shorter duration than 2 ms. For example with $n_r = 4$, 47% SAR reduction is obtained by using VERSEp method with 2 ms long RF pulses. Some results for time reduction of 4 are given in the Figures.

Figure 4.3 shows the four-times radially undersampled k-space trajectory obtained with time reduction of 4. Corresponding gradient and slew-rate waveforms before VERSE-SAR reduction are shown in Figure 4.4.

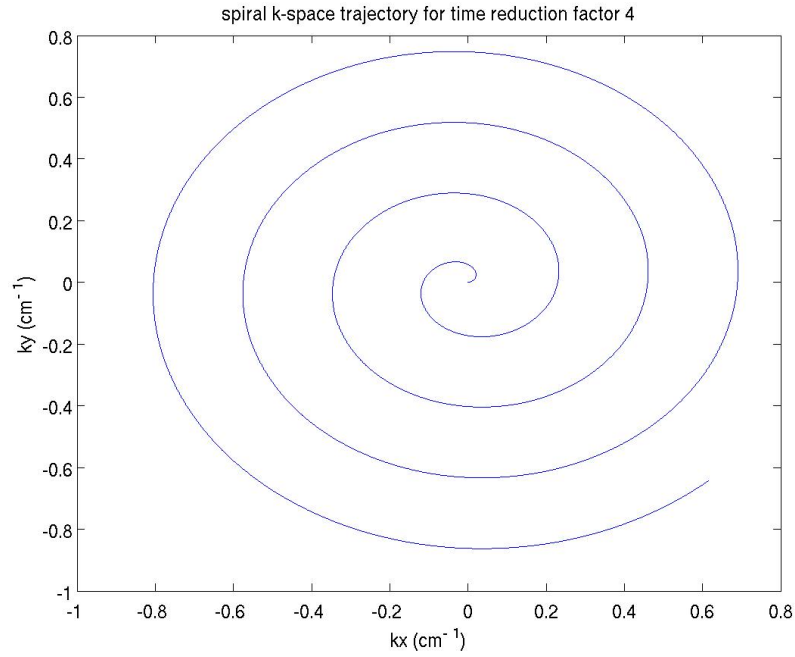


Figure 4.3: The k-space trajectory for $n_r = 4$.

Figures 4.5 and 4.6 show the VERSE-SAR optimized gradient waveforms for the pulses designed with VERSEp and the corresponding slew-rate waveforms respectively. Figure 4.7 shows the waveforms of square root power for the RF pulses designed with VERSEp before VERSE-SAR reduction and after VERSE-SAR reduction. In Figure 4.8, waveforms of square root power for the RF pulses designed with Zhu's method [6] are shown for the same cases.

The local SAR distributions are given for constrained VERSE-SAR reduced RF currents calculated with VERSEp for time reduction factor of 4. In Figures 4.9 and 4.10, the local SAR distributions inside planes parallel to the center plane are given. The local SAR values were higher for voxels closer to the bottom or top of the cylindrical phantom. Therefore, the local SAR values of pixels closer to the center plane are scaled up for better demonstrations in the Figures. The maximum local SAR to whole-body SAR ratio was calculated as 42.

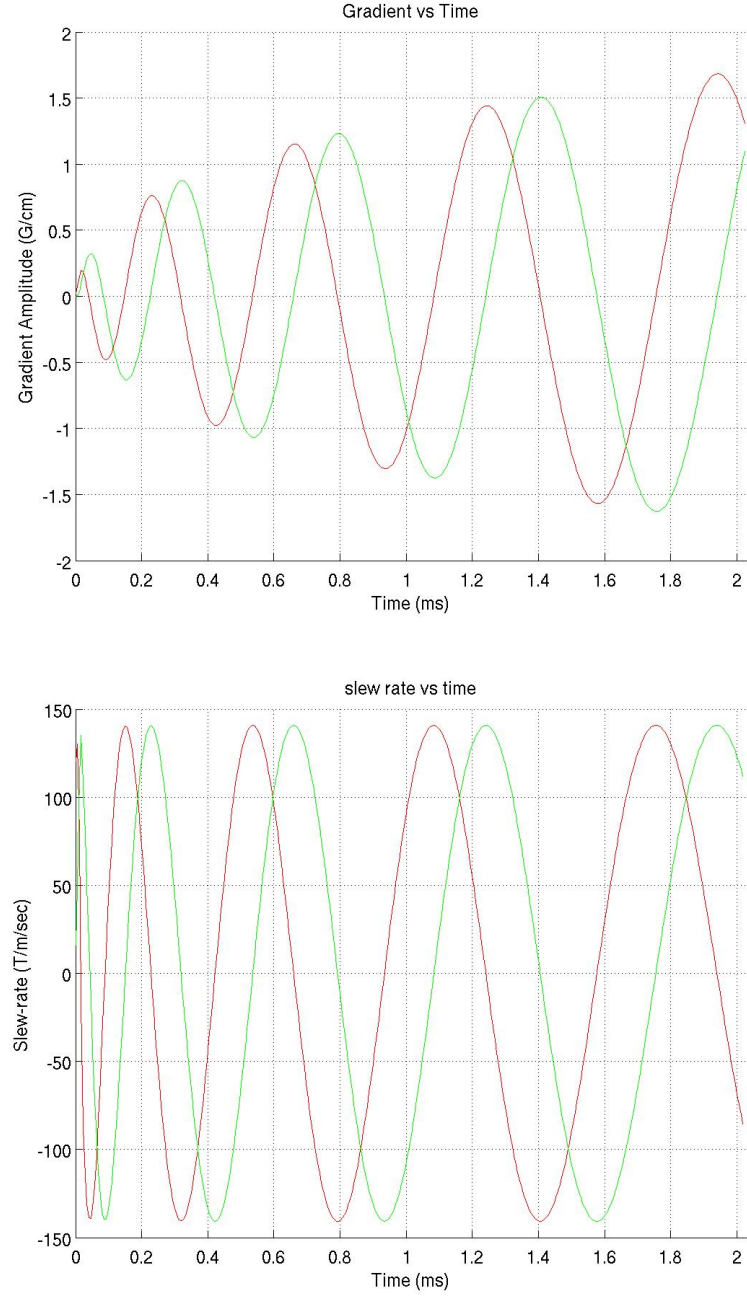


Figure 4.4: Gradient (top) and slew-rate (bottom) waveforms for $n_r = 4$ before VERSE-SAR reduction. x-gradient and x-slew rate waveforms are shown with red. y-gradient and y-slew rate waveforms are shown with green.

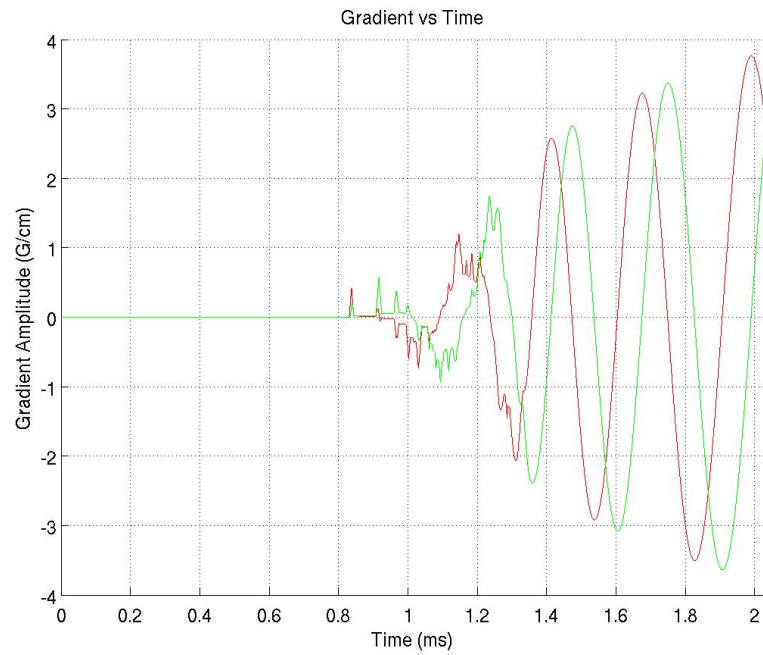


Figure 4.5: Constrained VERSE optimized gradient waveforms for the pulse designed with VERSEp. x-gradient (green), y-gradient (red) are shown separately. The maximum gradient amplitude constraint of 7 G/cm was not a limiting factor.

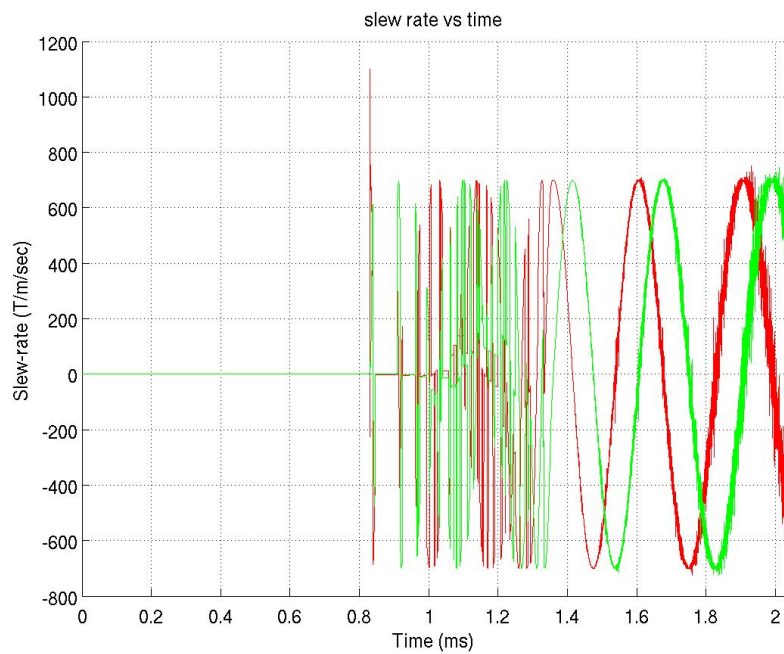


Figure 4.6: Slew rates of the constrained VERSE optimized gradient waveforms for the pulse designed with VERSEp. x-gradient slew rate (green) and y-gradient slew rate (red) are shown separately. The maximum slew rate constraint of 700 T/m/sec was exceeded for a short time interval but this is a problem that can easily be dealt with programming.

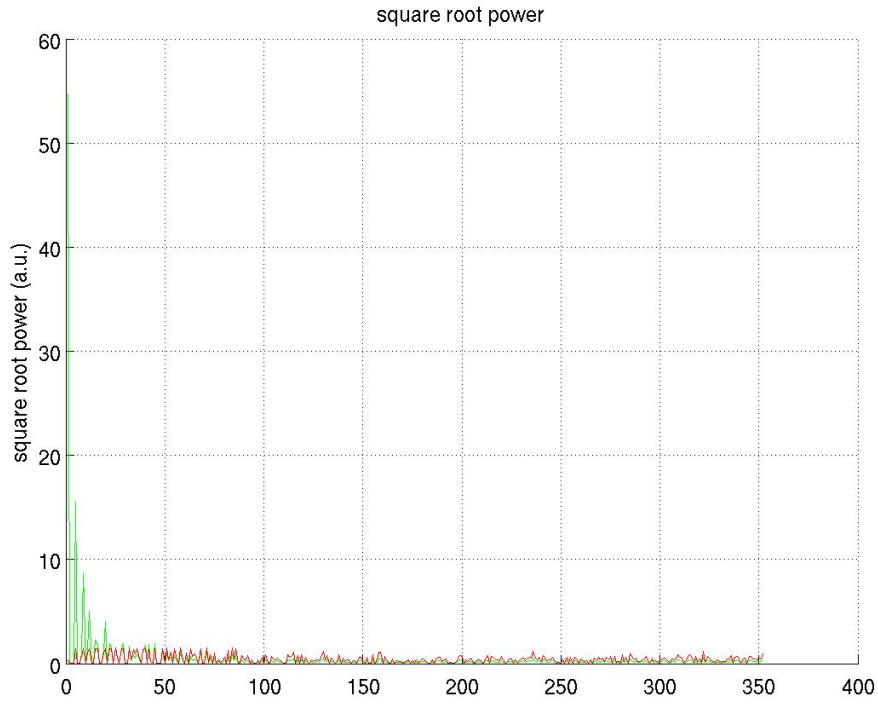


Figure 4.7: Square root power of the currents designed with VERSEp vs k-space samples. Square root power before VERSE optimization (green) and after VERSE optimization (red) are shown.



Figure 4.8: Square root power of the currents designed with Zhu's method [6] vs k-space samples. Square root power before VERSE optimization (green) and after VERSE optimization (red) are shown.

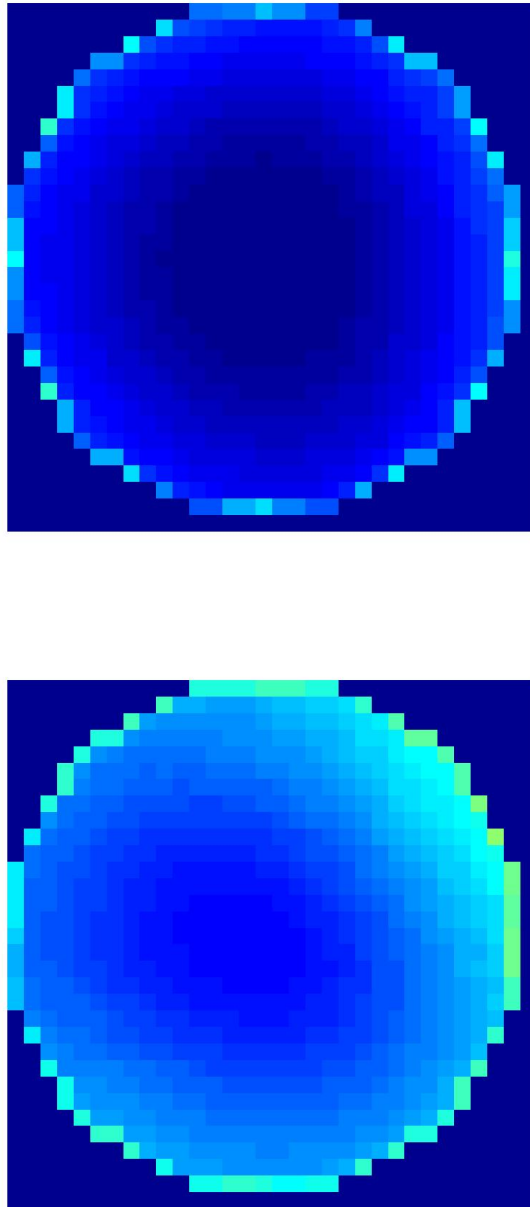


Figure 4.9: Local SAR distribution inside the center plane (top) scaled up with 180 and local SAR distribution inside the plane 3.4 cm below the center plane (bottom) scaled up with 90 are shown

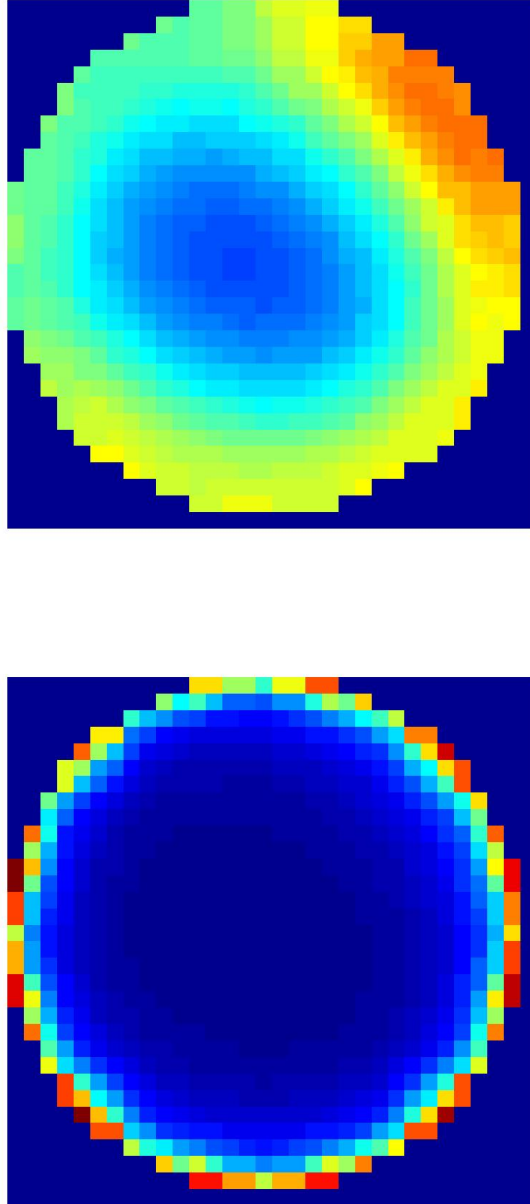


Figure 4.10: Local SAR distribution inside the plane 6.6 cm below the center plane plane (top) scaled up with 30 and local SAR distribution inside the plane 10 cm below the center plane (bottom) without a scaling are shown.

4.2.2 Discussion

Constrained VERSE-SAR reduced SAR of RF pulses designed with VERSEp were up to % 66 lower than the constrained VERSE-SAR reduced SAR of the RF pulses designed with Zhu's method [6]. The 66% SAR reduction was obtained with time reduction factor of 2, where duration of the pulses was 4 milliseconds. This is an impractically long RF pulse duration. However, the 47% percent SAR reduction obtained for 2 milliseconds RF pulses designed for $n_r = 4$ show that VERSEp can be used for the SAR reduction of RF pulses with acceptable durations. The percentage improvements shown for the unconstrained VERSE-SAR reduction case are significantly higher than the percentage improvements of constrained VERSE-SAR reduction case. This shows that VERSEp will become more beneficial with the development of gradient hardwares with higher slew-rate and maximum gradient constraints.

When VERSE-SAR reduction is not performed, the SAR of the RF pulses designed with Zhu's method [6] is always less than the SAR of the pulses designed using VERSEp. However, the RF pulses designed with VERSEp have more room for improvement using VERSE-SAR reduction. This can be better understood by comparing Figures 4.8 and 4.7. The square root power waveform of the pulses designed with VERSEp have a more non-uniform shape than the square root power waveform of the RF pulses designed with Zhu's method [6]. As a result of this, impact of VERSE-SAR reduction on SAR is more significant for RF pulses designed using VERSEp.

The SAR reduction obtained by using VERSEp tends to decrease as the time reduction factor increases. This is due to the fact that the dimension of solution space of $\mathbf{b}_{full}$ in equation 2.6 decreases when time reduction factor is increased. As a result of the decreased freedom in the choice of $\mathbf{b}_{full}$, the optimization defined in equations 3.4 and 3.5 becomes less effective.

In Figure 4.6 there is an over-shoot of slew-rate in a 4 μs interval at 0.83 milliseconds, where the peak of the overshoot is at 1100 T/m/sec. The choosen maximum slew-rate constraint was 700 T/m/sec. The overshoot is a result of the optimal path planning method used in constrained VERSE-SAR reduction method developed by Lee *et. al.* The integrals in this method may not converge at some points when sufficiently small sampling intervals are not used. When shorter sampling intervals are used, the peak of the overshoot of the slew-rate becomes lower however, the sufficiently short sampling intervals could not be used in the simulations due to memory problems. The divergence of the algorithm only occurred in RF pulses designed with VERSEp becuase VERSE-SAR reduction of RF pulses designed with VERSEp requires very low gradients to be applied and this causes divergence of the constrained VERSE-SAR reduction method.

The local SAR distributions given in Figures 4.9 and 4.10 show that local SAR increases significantly towards top and bottom of the cylindrical phantom. This problem is associated with the sharp edges of the used rectangular coil geometry which causes an increase in the

electric -field and local SAR. This problem can be solved by using coils with smoother shapes and lower maximum local SAR to whole-body SAR ratios can be achieved.

The divergence problem is not associated with the improvement, introduced in Section 3.1.3, that is made in the constrained VERSE-SAR reduction method developed by Lee *et. al.* The slew-rate waveforms obtained using the constrained VERSE-SAR reduction method developed by Lee *et. al.*, without applying modification as explained in Section 3.1.3, are shown in Figure 4.11 where it is observed that an overshoot in slew-rate still occurs.

The main drawback of the VERSEp method is the computation time. For the simulations a desktop computer with 2.4 GHz Intel(R) core(TM)2 Quad CPU is used. The computation time of the VERSEp pulse design using second order cone programming was 50 min for $n_r = 4$. This is an impractically long computation time for RF pulse design. The computation time problem can be solved by using another optimization method which is designed to solve the specific optimization problem given by equations 3.4 and 3.5. One such method can be found in [32]. The method explained in [32] is developed for sum of norms minimization problems with linear equality constraint and the optimization problem of VERSEp pulse design is in this category.

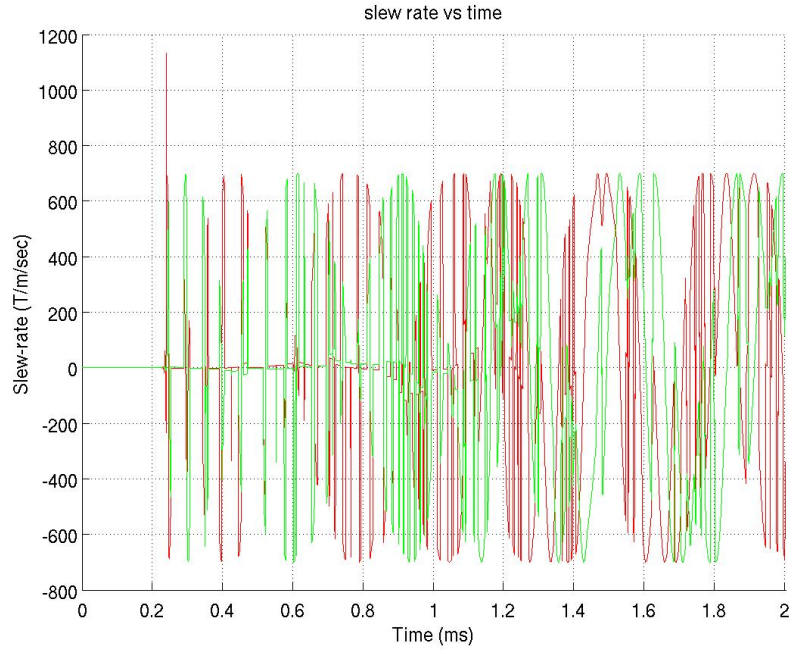


Figure 4.11: Slew-rates of the gradient waveforms designed with the constrained VERSE-SAR reduction method developed by Lee *et. al.* x-gradient slew-rate (green) and y-gradient slew-rate (red) are shown separately. The slew-rate limit of 700 T/m/sec is exceeded for a short time interval.

4.3 Comparison of Peak Square Root Power Constrained SAR Reduction with Peak RF Constrained SAR Reduction by Lee *et. al.* [8]

VERSE-SAR reduction method developed by Lee *et. al.* uses peak RF constraint for SAR reduction [8]. Here, improvement obtained by using peak square root power constrained SAR reduction, introduced in Section 3.1.3, in Lee's method [8] instead of peak RF constrained SAR reduction is given. The improvement is tested with RF pulses designed using Zhu's method [6] and RF pulses designed using VERSEp. Percentage improvements are evaluated based on the formulas in Table 4.6. In the analyses of the designed pulses, waveform of

Table 4.6: Percentage improvement definitions

SAR_{PP}^{Zhu} : SAR of the RF pulses designed with Zhu's method [6] and VERSE-SAR reduced by using peak square root power constraint.

SAR_{PRF}^{Zhu} : SAR of the RF pulses designed with Zhu's method [6] and VERSE-SAR reduced by using peak RF constraint.

SAR_{PP}^{VERSEp} : SAR of the pulses designed with VERSEp and VERSE-SAR reduced by using peak square root power constraint.

SAR_{PRF}^{VERSEp} : SAR of the pulses designed with constrained VERSEp and VERSE-SAR reduced by using peak RF constraint.

$$\%IMPZ = \left(1 - \frac{SAR_{PP}^{Zhu}}{SAR_{PRF}^{Zhu}}\right) \times 100$$

$$\%IMPVp = \left(1 - \frac{SAR_{PP}^{VERSEp}}{SAR_{PRF}^{VERSEp}}\right) \times 100$$

maximum of currents plays an important role. In Section 2.2.1, the currents applied to the transmit coils in the i 'th k-space sample was denoted as $\mathbf{b}_i$. $\max\{\mathbf{b}_i\}$ was defined as the absolute value of the element of $\mathbf{b}_i$ with highest absolute value. Here, $\max\{\mathbf{b}_i\}$, a function of i , is called the maximum of currents.

4.3.1 Results

The design parameters are identical with parameters used in Section . Improvements are summarized in Table 4.7.

Significant improvements are obtained for both pulses designed with Zhu's method [6] and VEREp in time reduction factor of 3. Some results for pulses designed using Zhu's [6]

Table 4.7: Percentage SAR improvements and pulse durations for different time reduction factors

n_r	% IMPVp	% IMPZ	Pulse Duration (ms)
1	34	64	6.65
2	36	57	4.01
3	27	59	2.69
4	12	50	2
5	0	35	1.64
6	0	35	1.39
7	0	19	1.2
8	1	16	1.06
9	0	1	0.948
10	0	2	0.84

method with $n_r = 3$ are shown in the figures.

Figures 4.12 and 4.13 show the results obtained by the VERSE-SAR reduction method that uses peak RF constraint. Figure 4.12 shows the maximum of currents vs k-space samples and figure 4.13 shows the square root power vs k-space samples.

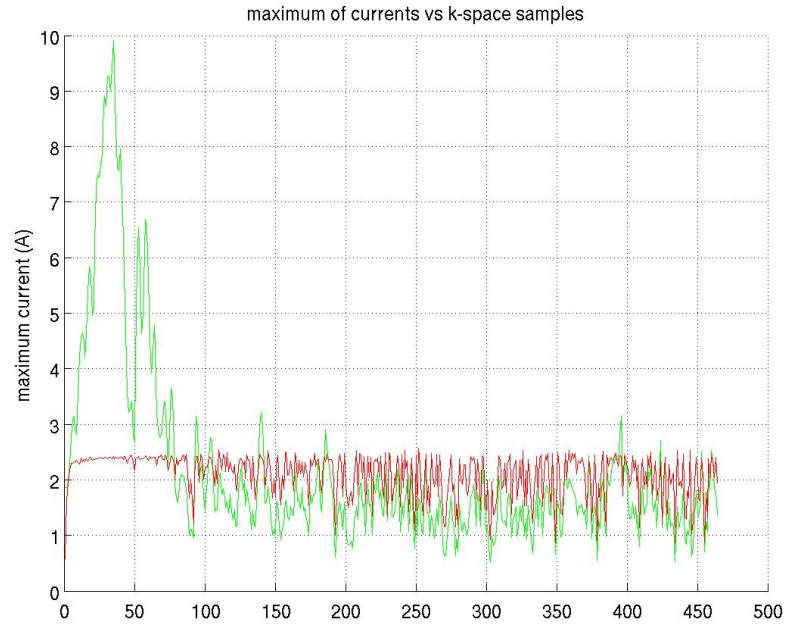


Figure 4.12: Maximum of currents vs k-space samples. Waveform of maximum of currents before peak RF constrained VERSE-SAR reduction (green) and after peak RF constrained VERSE-SAR reduction (red) are shown separately.

Maximum of currents and peak square-root power vs k-space samples for pulses designed with the peak square root power constrained VERSE-SAR reduction method are given in Figures 4.14 and 4.15 respectively.

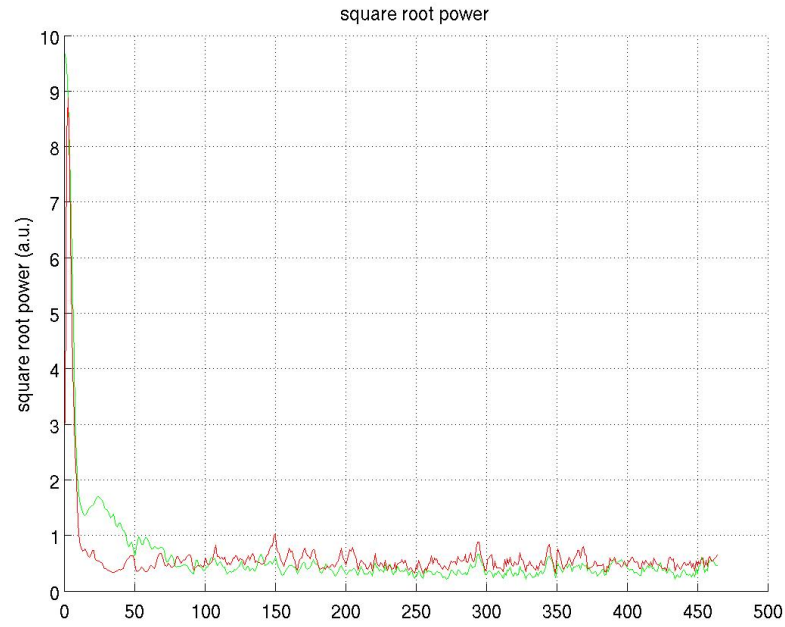


Figure 4.13: Square root power vs k-space samples. Square root power waveform before peak RF constrained VERSE-SAR reduction (green) and after peak square root power waveform after peak RF constrained VERSE-SAR reduction (red) are shown with separately.

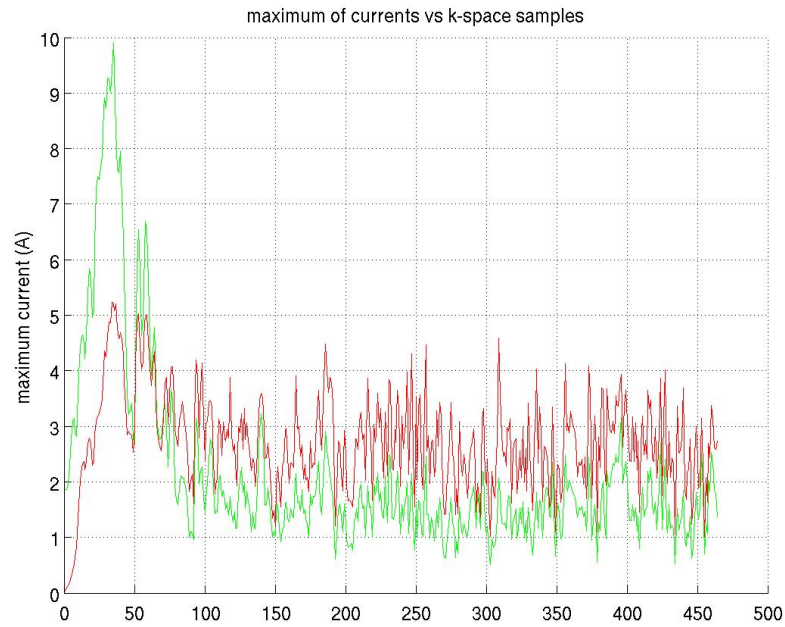


Figure 4.14: Maximum of currents vs k-space samples. Waveform of maximum of currents before peak square root power constrained VERSE-SAR reduction (green) and after peak square root constrained VERSE-SAR reduction (red) are shown separately.

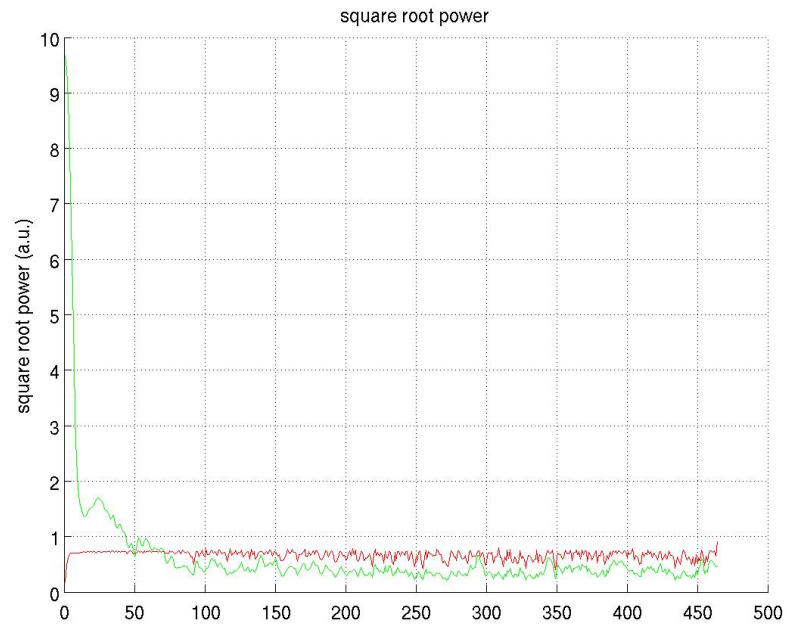


Figure 4.15: Square root power vs k-space samples. Square root power waveform before peak square root power constrained VERSE-SAR reduction (green) and after square root power waveform after peak square root power VERSE-SAR reduction (red) are shown separately.

4.3.2 Discussion

When VERSE-SAR reduction is performed without any gradient constraints, the obtained parallel transmit pulses have constant square root power over time. Therefore, the goal of gradient constrained VERSE-SAR reduction is to obtain RF pulses with square root power waveform as close to constant as possible. When peak RF constrained VERSE-SAR reduction method explained in [8] is used, RF pulses with maximum of currents close to a constant waveform are obtained as shown in Figure 4.12 and shape of the square root power waveform was closer to a constant waveform than the square root power waveform of the initial pulses. However before VERSE-SAR reduction, the peak of the waveform of maximum of currents (shown in Figure 4.12) and the peak of the square root power waveform (shown in Figure 4.13) are placed at different k-space samples and as a result of this, the peak RF constrained VERSE-SAR reduction could not suppress the peak of the square root power at the first k-space sample. The square root power waveform obtained from the peak square root power constrained method was closer to a constant waveform than the square root power waveform obtained by the peak RF constrained VERSE-SAR reduction. Furthermore, the peak of the square root power waveform at the first k-space sample is reduced to a lower value successfully with the peak square root power constrained VERSE-SAR reduction. The percentage improvements obtained by using peak square root power constraint in VERSE-SAR reduction method of Lee *et. al.* [8] instead of peak RF constraint, proposed in [8], are shown in Table 4.7.

Chapter 5

Conclusions and Future Work

A new method developed for the design of parallel transmit RF pulses before VERSE-SAR reduction is presented (VERSEp). Performance of this method is illustrated by comparing SAR of the gradient constrained VERSE-SAR reduced pulses designed using Zhu's method [6] and SAR of the gradient constrained VERSE-SAR reduced pulses designed using VERSEp. SAR reductions up to 66% is obtained by using VERSEp instead of Zhu's method [6] in RF pulse design. The 47% SAR reduction obtained in 2 ms RF pulses designed for a time reduction factor of 4 shows that significant SAR reduction can be obtained in practical pulse durations by using VERSEp. When VERSE-SAR reduction was performed without any gradient constraints, significantly higher SAR improvements were obtained. This shows that VERSEp will perform even better when gradient coils with looser constraints are used.

An improvement to the VERSE-SAR reduction method developed in [8] is explained. The improvement is obtained by using peak square root power constraint instead of peak RF constraint for SAR reduction. The further SAR reduction obtained by using peak square root power constraint instead of peak RF constraint in SAR reduction shows that it is highly beneficial to use the improved method. Furthermore, there was no drawback of using the peak square root power constraint.

In the future work, the problems pointed out in section 4.2.2 should be solved. The most important problem is the long computation time required to calculate the RF pulses using the second-order cone optimization method. To solve the computation time problem, an optimization algorithm specialized for the optimization problem of VERSEp should be used. Such an algorithm can be found in [32]. The high local SAR to whole-body SAR ratio is a result of the rectangular coil geometry used for the simulations. The electric field induced by this coil geometry around top and bottom of the cylindrical phantom was very high. This causes a high local SAR because the local SAR is related with square of the magnitude of the electric field. To overcome this problem, circular loop coils can be used. Another problem

was the overshoot of the slew-rate shown in Figure 4.6. The slew-rate overshoot problem can be solved by using short time steps in the constrained VERSE-SAR reduction method or by using a different constrained VERSE-SAR reduction method. To demonstrate the feasibility of VERSEp, the design method should be tested by experiments. The off-resonance effects become a more important problem in VERSE optimized pulses. The off-resonance effects should also be analysed in the future work.

Chapter 6

Appendix

SAR of RF pulses after unconstrained VERSE SAR optimization is allways lower than or equal to SAR of the RF pulses before VERSE SAR optimziation. Here, this fact will be proved by proving the following inequality

$$\frac{(\sum_{i=1}^M \|\mathbf{b}_i\|_2)^2}{M} \leq \sum_{i=1}^M \|\mathbf{b}_i\|_2^2. \quad (6.1)$$

Where $\|\mathbf{b}_i\|_2 = \sqrt{\mathbf{b}_i^H \mathbf{S}_{sub} \mathbf{b}_i}$. Since norm operation gives a positive real number, we can write the following.

$$\sum_{i=1}^M \sum_{j=1}^M (\|\mathbf{b}_i\|_2 - \|\mathbf{b}_j\|_2)(\|\mathbf{b}_i\|_2 - \|\mathbf{b}_j\|_2) \geq 0$$

$$2 \sum_{i=1}^M \sum_{j=1}^M \|\mathbf{b}_i\|_2^2 - 2 \sum_{i=1}^M \sum_{j=1}^M \|\mathbf{b}_j\|_2 \|\mathbf{b}_i\|_2 \geq 0$$

$$M \sum_{i=1}^M \|\mathbf{b}_i\|_2^2 - \left(\sum_{i=1}^M \|\mathbf{b}_i\|_2 \right)^2 \geq 0$$

The inequality in 6.1 is proved.

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