

DESIGNING A PARTICIPATORY LEARNING ENVIRONMENT
TO FOSTER MATHEMATICS TEACHER AGENCY:
FROM DILEMMAS TO COLLECTIVE ACTION



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Designing a Participatory Learning Environment
to Foster Mathematics Teacher Agency: From Dilemmas to Collective Action

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- . I am the sole author of this thesis and that I have fully acknowledged and documented in my thesis all sources of ideas and words, including digital resources, which have been produced or published by another person or institution;
- . this thesis contains no material that has been submitted or accepted for a degree or diploma in any other educational institution;
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ABSTRACT

Designing a Participatory Learning Environment

to Foster Mathematics Teacher Agency: From Dilemmas to Collective Action

Recent mathematics education reforms demand teachers develop responsive practice for students' ambitious and equitable learning, calling for professional development (PD) models that position teachers as active agents of their own learning. This study responds to this call by designing a participatory learning environment that supports teachers' agency in professional learning. The Participatory Mathematics Teacher Learning Environment (PMTLE) was developed through a design research study and implemented with in-service mathematics teachers affiliated with a PD organization in Turkey. Informed by formative interventions method and conceptualizations of agency from sociocultural theory perspective, PMTLE supported teachers in identifying and analyzing shared problems and co-designing novel professional learning activities. Qualitative analysis employed Vasilyuk's theory of experiencing and Bateson's theory of Learning III to examine agency formation and explore how design elements supported or hindered it. Findings indicate that individually, teachers surfaced internal conflicts, expanded perspectives on PD needs, attributed personal meaning to collective activities, and developed personal disposition to engage in them. Collectively, they identified systemic contradictions underlying their problems and co-constructed and enacted novel professional learning activities in response. Design features, such as participant-driven reflection and responsive facilitation, enabled teachers' active participation. This study proposes a theoretically grounded, empirically supported participatory PD model that promotes teacher agency and offers principles for design.

ÖZET

Matematik Öğretmeni Aktörlüğünü Teşvik Etmek İçin Katılımcı Bir Öğrenme

Ortamı Tasarlamak: İkilemlerden Ortak Eyleme

Günümüzde matematik eğitimi reformları, öğretmenlerden öğrencilerin iddialı ve adil öğrenmelerine duyarlı öğretim pratikleri geliştirmelerini talep etmekte; bu da öğretmenleri öğrenmenin etkin öznesi olarak konumlandıran mesleki gelişim (MG) modellerine ihtiyaç doğurmaktadır. Çalışma, öğretmenlerin mesleki öğrenmelerinde aktörlük geliştirmelerini destekleyen bir öğrenme ortamı tasarlayarak yanıt vermektedir. Katılımcı Matematik Öğretmeni Öğrenme Ortamı (PMTLE), tasarım araştırması yöntemiyle geliştirilmiş ve Türkiye’de bir MG organizasyonuna bağlı hizmet içi matematik öğretmenleriyle uygulanmıştır. Biçimlendirici müdahale ve sosyokültürel kuram perspektifinden aktörlük kavramsallaştırmalarına dayanarak geliştirilen PMTLE, öğretmenlerin ortak sorunları tanımlayıp analiz etmelerini ve özgün mesleki öğrenme etkinlikleri tasarlamalarını desteklemiştir. Nitel veri analizi, öğretmen aktörlüğünün oluşumunu incelemek ve tasarım unsurlarının süreci nasıl destekleyip engellediğini anlamak için Vasilyuk’un yaşantılama kuramı ile Bateson’un Öğrenme III kuramını kullanmıştır. Analiz, öğretmenlerin bireysel düzeyde içsel çatışmalarını açığa çıkardığını, MG ihtiyaçlarına dair bakış açılarını genişlettiğini, kolektif etkinliklere kişisel anlamlar yükleyip katılım için eğilim geliştirdiğini göstermiştir. Kolektif düzeyde ise sorunlarının ardındaki sistemsel çelişkileri tanımlayarak özgün mesleki öğrenme etkinlikleri tasarlayıp uygulamışlardır. Katılımcı temelli yansıtma ve duyarlı kolaylaştırıcılık öğretmenlerin etkin katılımını sağlamıştır. Çalışma, öğretmen aktörlüğünü teşvik eden, kuramsal ve ampirik destekli katılımcı bir MG modeli önermekte ve ilkeler sunmaktadır.

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CHAPTER 1

INTRODUCTION

In-service teachers' professional development (PD) has been a long-standing concern of various stakeholders as it is linked to enhanced student learning and considered an essential component of school improvement and broader system-level change efforts (McLaughlin & Marsh, 1978; Randi & Zeichner, 2004; Villegas-Reimers, 2003; Fishman et al., 2003). Mathematics education reforms in many countries that emphasize student-centered pedagogies have further motivated mathematics teacher educators to better understand and improve the field (Borko et al., 2008; Ministry of National Education [MEB], 2005; NCTM, 1989, 2000; Simon et al., 2000; Stein et al., 1999).

As efforts to improve the field intensified, studies evidenced that it is challenging for teachers to embrace the ambitious goals of reforms, as this is an issue of cultural transformation, requiring teachers to enact agency by critically questioning their firmly held beliefs and habitual practices and taking deliberate, conscious actions to shift away from practices that restrict students' ambitious and equitable learning (e.g., Borko, 2004; Clarke & Hollingsworth, 2002; Gresalfi & Cobb, 2011; Guskey, 1986, 2002; Heaten, 2000; Horn & Garner, 2022; Little, 1993; Koellner et al., 2007; Stigler & Hiebert, 1999/2009). Additionally, improvements in the field of Learning Sciences have highlighted the complexity and situated nature of teaching practice, underscoring the importance of teachers' developing responsive teaching, by maintaining a critical eye on students' situated needs for their equal and meaningful participation in their own learning (Horn & Garner, 2022; Servage, 2008). All these pointed out teachers' being conscious actors of their own

professional learning and development rather than "passive recipients of information generated by others" (Hardy, 2012, p. 112).

Such developments increased the calls for considering "teacher agency" as a learning outcome of teacher education programs as it has been seen as a pathway to empower teachers in creating more democratic classrooms, schools, and societies (e.g., Cochran-Smith et al., 2022). Despite these, PD programs remain dominated by the in-service training models from the old paradigm, "where teachers are expected to learn a clearly defined body of skills through a well-specified process" leading to teachers' alienation from their own professional learning and development (Borko et al., 2010, p. 548). This ultimately contributes to the deprofessionalization of teaching practice, expecting teachers to enact standardized procedures rather than using professional knowledge for developing responsive practice (Milner, 2013).

In Turkey, empirical studies on teachers' satisfaction with PD programs indicate predominantly negative perceptions, particularly toward programs provided by the Ministry of National Education (MEB), which serves approximately 84.6 % of the country's teachers (MEB, 2023). These negative perceptions stem from limited opportunities for interaction among colleagues, a lack of feedback mechanisms, and a misalignment with teachers' contextual needs, all of which suppress teachers' agency over their professional learning. (Abbasioglu, 2017; Altun & Yengin Sarpkaya, 2021; Düşkün, 2021; Güven, 2011). Consequently, there is a growing need to propose alternative pedagogies for teacher education that position teachers as active participants and agents in their own professional development and teacher agency as a significant learning outcome.

In this study, following a Vygotskian theoretical perspective, teacher agency is understood as teachers consciously making decisions and taking action to deal

with problems of practice, transform and expand their teaching, support their own learning, and influence policies and procedures within their departments or schools by employing social, material, and conceptual resources within the environment or by creating new ones (Edwards, 2010; Engeström et al., 2020; Eteläpelto et al., 2013; Günal-Aggül et al., 2023; Vygotsky, 1997).

This perspective acknowledges the dialectical relationship between the context and the individual in influencing each other. Humans have the capacity to shape their conditions and environments, while they are also shaped by the circumstances in which they enact agency (Lasky, 2005). Rather than being a fixed trait or innate capability, one's agency is dynamic and context-dependent, varying across different situations (Haapasaari et al., 2016; Lipponen & Kumpulainen, 2011). Building on this perspective, this study operates on the premise that teachers can enact agency to lead their own professional learning and change—provided they are given the necessary space and support.

Building on this understanding of teacher agency, research has largely explored how teachers enact agency in real-world educational settings, yet there is a growing need to design professional learning environments that explicitly foster and enhance teacher agency. Most studies have examined teachers' enactment of agency in naturalistic settings, such as schools or classrooms, within the context of curriculum reform (e.g., Campbell, 2012), the implementation of innovative pedagogical approaches (e.g., Eteläpelto et al., 2015), or the development of responsive teaching practices for inclusion and social justice (e.g., Cochran-Smith & Reagan, 2022; Min et al., 2021). While these studies illuminate the conditions that enable teachers to exercise agency, there is a pressing need to design teacher learning environments that intentionally provide opportunities to enhance teacher agency

(Morselli & Sannino, 2021; Orland-Barak, 2017; Toom et al., 2015). Orland-Barak (2017) underscores this in her commentary in the *Sage Handbook of Research on Teacher Education*, calling for the development of design-based research agendas "whereby teaching-learning environments geared to promoting agency are formatively developed and dynamically revised and assessed over time by the actors/participants themselves" (p. 251). In other words, pedagogical approaches, which move beyond traditional, structured programs, adopting a formative approach that continuously incorporates teachers' input, need to be intentionally designed to support teacher agency within research-practice partnerships.

This study is particularly focused on designing participatory professional learning environments—structured as a research-practice partnership (Farrell et al., 2021)—that foster teachers' agency in consciously and strategically directing their own learning processes. Indeed, studies have shown that PD models developed under the influence of the advancements in teacher learning—those that position teachers as active constructors of knowledge within communities (e.g., video clubs [Van Es & Sherin, 2010], Lesson Study [Lewis & Perry, 2013]; Professional Learning Communities [Stoll & Louis, 2007])—along with the century old approach of action research, have strong potential to support teacher agency for professional learning (Emam et al., 2023; Kim, 2021; Lewis et al., 2022; Pyhältö et al., 2015). However, rather than explicitly treating teacher agency as a targeted learning outcome grounded in a theoretical perspective and designing these models accordingly, empirical studies have often framed them merely as "affordances," "venues," or "contexts" for teacher agency (Emam et al., 2023; Kim, 2021; Philpott & Oates, 2016). This lack of a systematic design process has limited opportunities to refine

and enhance these models in ways that could more effectively support teacher agency in PD settings.

This study aims to address this gap by designing a participatory teacher professional learning environment that supports teachers' agency in leading their own professional learning by developing novel learning activities tailored to their contextual needs. With teacher agency as a core analytical focus, the study examines how teachers enact agency within PD contexts—specifically, how they take ownership of and co-design their own learning processes in conscious and strategic ways within a research-practice partnership. To achieve this goal, I conducted a design research study (Bakker, 2020; Cobb et al., 2003; Plomp & Nieveen, 2013) aimed at supporting teachers' collective agency in identifying their professional learning needs based on an expanded understanding of their problems of practice and in consciously and strategically organizing their own professional learning processes in response.

The participants of this study are in-service mathematics teachers who teach mathematics at various schools across different cities in Turkey and at different grade levels (1-12). They are all affiliated with a non-profit PD organization, Teachers Network (TN), where they hold the additional role of change agents due to their participation in specific PD activities. This role grants them the opportunity to take initiative in forming communities based on their interests. I partnered with TN because of their desire to support the development of a teacher community focused on mathematics education.

Based on the gap identified in the literature and the needs of the partner organization, I designed a teacher learning environment—referred to in this dissertation as Participatory Mathematics Teacher Learning Environment (PMTLE).

PMTLE aims to support teachers in transitioning from a group of individuals to a collective that consciously and strategically co-models professional learning activities, grounded in their expanded understanding of their contextual challenges and professional learning needs as mathematics teachers in Turkey.

The design principles for the PMTLE were derived from the literature on socio-cultural notions of agency, with equal emphasis placed on the collective, relational, and individual aspects of agency formation. To support teachers' collective agency formation in co-designing and engaging in PD processes, I first drew on the notion of collective transformative agency. The notion of collective transformative agency highlights the development of collective agency to engage in joint efforts aimed at transforming or creating collective activities to address systemic problems (Haapasaari & Kerosuo, 2015; Virkkunen, 2006). To identify principles for supporting this form of agency, I incorporated perspectives and tools from formative interventions, a research-practice partnership approach designed to support transformative agency of collectives in developing novel activity structures as solutions to their contextual systemic problems (Engeström, 2011; Sannino et al., 2016). In formative interventions, the process is structured according to Engeström's (1987/2015, 2016) cultural-historical activity theory-based theory of expansive learning, which outlines seven learning actions that collectives engage in while developing novel activity models and practices as solutions to systemic problems—questioning and analyzing problems of practice, modeling a new solution, examining and implementing the new model, reflecting on the process, and consolidating and generalizing the new practice (Engeström, 2016).

PMTLE primarily aims to support teachers' collective agency in carrying out the first three actions—questioning, analysis, and modeling—each of which is

conceptualized as a professional learning process in its own right. PMTLE aims to support a group of mathematics teachers from different schools in exercising agency as they transition from questioning their teaching practices to consciously and strategically designing and executing the modeling activities, ultimately leading to the development of a PD activity model for their future community activities.

While the theory of expansive learning and the method of formative interventions guided the design to support teachers' collective agency, two additional perspectives from the agency literature were also incorporated to address other dimensions of agency formation in collective change and development processes. One perspective focuses on the individual dimension of agency formation in work-related change processes, emphasizing individuals' contribution to collective agency through their identities, past experiences, current conditions, and future prospects. To integrate this dimension into the design, I drew on subject-centered approaches to agency (Eteläpelto et al., 2013; Priestley et al., 2012). In this study, while the focus is on collective action for co-designing professional learning processes, particular emphasis is placed on teachers finding space within these collective activities as individuals—by incorporating the individual dimension of agency formation.

The other perspective enriches the design by addressing the relational dimension of agency formation, which involves building relationality among practitioners, researcher-facilitators, and external collaborators. For this, I drew on the concept of relational agency (Edwards, 2010) and insights from the formative interventions literature on building relations within learning environments that aims to support learners' agency.

Based on the principles derived from this literature, I designed the initial version of the PMTLE with a constant focus on the formative co-design of the

process. These principles were embodied through design elements (Sandoval, 2014), including tools and materials (e.g., a book for reflection [Stigler & Hiebert, 1999], video clips from teachers' discussions for analysis), task structures (e.g., writing reflection papers, revising Our Problems Map), participant structures (e.g., "all participants contribute to the process by sharing their resources and expertise as equal partners"), and discursive practices (e.g., "dialogic discourse practices that encourage the hybridization of ideas and solutions").

For the analysis, PMTLE was taken as the case under analysis (Stake, 2005; Yin, 2018). Aligned with the purposes of design research, the qualitative analysis of the case has two goals (Cobb et al., 2003). First, it illuminates how teachers exercise agency in organizing their own professional learning processes and activities within a participatory professional learning environment designed using principles from the agency literature. This first dimension illuminates the "learning" aspect, offering insights into teachers' learning through the lens of teacher agency. The second goal is to come up with lessons for designing teacher professional learning environments that support teacher agency.

For the first analysis, I focused on agency formation at two levels within PMTLE: the individual and the collective. Two theoretical frameworks guided this analysis. First, I draw on Vasilyuk's (1988) theory of "experiencing," utilizing Sannino's (2008, 2010) interpretation for agency formation, to analyze teachers' agency formation at the individual level for professional learning. This theory conceptualizes agency formation as the process of resolving internal critical conflicts, which is understood as enhancing one's "psychological possibilities to carry on the activity required for the realization of" personal needs and developing an "individual disposition to act" (Sannino, 2008a, p. 241). This theory provides a

foundational understanding of how individuals navigate internal challenges to increase their capacity for action.

Second, to analyze agency formation at the collective level within PMTLE, I draw on Bateson's (1987) theory of Learning III, along with Engeström's (1987/2014) interpretation of collective action for change and transformation in human activities. This framework conceptualizes agency formation as the resolution of collectively experienced dilemmatic situations—occurring outside the individual psychological plane and reflecting systemic contradictions—through their recognition and the development of contextually relevant solutions, which manifest as new forms of activity.

For the second analysis, which focuses on design, I inductively examined how the design elements embodying the principles guiding the PMTLE design supported or inhibited the formation of each agency indicator addressed in the first analysis.

All in all, the research questions of this study are formulated as follows:

- i. How did a community of mathematics teachers exercise agency to collaboratively question and analyze their shared problems of practice, and model professional learning activities in response within PMTLE?
 - a. How did teachers exercise agency at the individual level to develop a disposition to act for their professional learning within PMTLE?
 - b. How did teachers exercise agency at the collective level to model their professional learning processes within PMTLE?
- ii. How did the PMTLE design support or inhibit teachers' agency in engaging with these processes?

CHAPTER 2

LITERATURE REVIEW

The goal of this study is, first, to design a participatory professional learning environment (PMTLE) that supports mathematics teachers' agency in collectively designing and leading their own professional learning processes and second, to assess the formation of agency through the implementation of the design. In general, this chapter reviews the literature to which this study contributes and the theoretical approaches it draws on to design for and examine teacher agency.

This chapter is composed of five sections. In the first section, I will discuss the new paradigm of teacher PD to which this study aims to contribute by centering teacher agency in the design and analysis of the professional learning environments.

In the second section, the discussion will be followed by an overview of theoretical perspectives on teacher agency, as well as a review of pedagogical approaches and empirical studies focused on supporting teacher agency for professional learning.

The third section will focus on the theoretical frameworks that guided the PMTLE design as well as the design principles derived from these frameworks.

The fourth section will examine approaches to analyzing agency formation in designed environments that support teacher agency and will detail the theoretical approaches followed in this study.

In the fifth section, I will conclude the chapter by summarizing each section and elaborating on their relevance and contributions to this study.

2.1 The new paradigm of teacher professional development

For the past three decades, the design of PD for teachers has become an increasingly prominent area of inquiry among teacher educators, driven by reforms aimed at promoting cultural transformation in teaching toward student-centered pedagogies. According to Stein et al. (1999), this shift represents an effort to move away from the old paradigm of teacher PD—characterized by traditional, short-term, individually focused, and standardized programs—toward a new paradigm that positions teachers as active constructors of teaching knowledge, grounded in the specificities of their contexts. This section reviews the core premises of this new paradigm, reviews relevant literature, and underscores the importance of positioning teacher agency as a central construct in teacher professional learning to realize the goals of the new paradigm.

As Borko et al. (2010) highlight, the shift away from traditional PD approaches was driven not only by empirical studies revealing their limitations in supporting teachers' implementation of reforms but also by developments in research on learning, cognition, and teaching. A significant transition occurred in the Learning Sciences—beginning in the late 1980s—with a shift in focus from behavioral to cognitive, and then to situative, distributed, and sociocultural perspectives (Borko et al., 2010; Lerman, 2000; Nathan & Sawyer, 2014). These evolving perspectives provided strong support for the move toward more context-sensitive and participatory models of professional development.

According to Borko et al. (2005), the new paradigm synthesizes constructivist and sociocultural theories, emphasizing that learning and cognition involve subjects' active knowledge construction and are inherently embedded in social and cultural contexts (Cobb, 1994; Lave, 1988; Vygotsky, 1978). Following Cobb's (1994) call

for overcoming the "forced choice" between constructivist and sociocultural perspectives, Borko et al. (2005) maintain that teachers, like children, learn through a process of both active construction of knowledge on an individual psychological plane and enculturation through the social contexts—the complex ecosystems of teaching (e.g., the classroom, collegial and school communities, PD activities, or the broader education system).

From a constructivist perspective, learning is conceptualized as subjects' active construction of new knowledge in a pursuit of reorganizing their existing knowledge, which is triggered through new encounters during individual experience that create cognitive conflicts (von Glasersfeld, 2003; Piaget, 1970). Under the influence of constructivist theories of learning and pedagogy, PD programs developed within the new paradigm center teachers as active builders of the knowledge of teaching, rather than passive recipients of a pre-defined, fixated knowledge of teaching. Teacher-centered activities prioritize providing time, space, and inquiry-based experiences for teachers' active reflection on their current conceptions and sensemaking of alternative conceptions related to their practice (Simon et al., 2000).

In addition to constructivist accounts, the new paradigm of teacher PD also grounds itself in sociocultural theories, which highlight the "socially and culturally situated nature of mental activity" (Borko et al., 2005, p. 43). These theories propose that social and cultural contexts are not external variables that can be controlled to observe and support cognitive processes; instead, cognition is deeply embedded in the contexts and can develop through social activity instead of isolated processes occurring in individuals' heads (Cobb, 2001; Lave, 1991; Lave & Wenger, 1991; Rogoff, 1994). In Lave's (1998) words, "cognition is in fact a complex social

phenomenon... ‘Cognition’ observed in everyday practice is distributed—stretched over, not divided among—mind, body, activity, and culturally organized settings (which include other actors)" (p. 1).

This perspective shifted the focus to everyday practices and the tools used within them when designing learning environments, placing authentic practices at the center of the learning activities. Aligning with this perspective, PD programs developed under the new paradigm brought the complexity of teachers’ own classroom contexts—teaching and learning situations and interactions—to the PD settings (Ball & Cohen, 1999; Putnam & Borko, 2000; Wilson & Berne, 1999). Artifacts of practice, such as lesson plans, curricular materials, student work (e.g., Kazemi & Franke, 2004), and video clips from actual classroom teaching (e.g., Borko et al., 2008; Hollingsworth & Clarke, 2017; van Es & Sherin, 2010), have been utilized to foster productive conversations and to build teacher capacity to notice, interpret, and reflect on actual classroom practices (Borko et al., 2008; Van Es & Sherin, 2010).

Furthermore, under the influence of the sociocultural perspective, learning has been conceptualized as a process of enculturation, that is, a process of participation in socially organized activities of communities of practice (Borko et al., 2005; Lave & Wenger, 1991). In this view, discourse around a practice is produced by and through communities and utilized to pursue their activities. Enculturation corresponds to subjects' involvement in a community’s discourse and activities. With the shift to viewing learning as enculturation, PD activities in the new paradigm often focus on enhancing teacher learning by fostering a supportive professional learning community (PLC) (Koellner et al., 2007). In time, teacher communities have been validated to be a component of effective (Darling-Hammond et al., 2017) and

meaningful (Horn et al., 2023) PD. Compiling different definitions developed for teacher PLCs by different scholars, Stoll and Louis (2007) define a teacher PLC as "a group of teachers sharing and critically interrogating their practice in an ongoing, reflective, collaborative, inclusive, learning-oriented, growth promoting way" (p. 2).

To support the development of professional learning communities, PD programs were conducted with communities of teachers and included collective activities that foster the development of shared pedagogical orientations, discourse, goals, and practices, such as collaborative planning and implementation of lessons based on collectively experienced pedagogical frameworks (e.g., Palincsar et al., 1998). This expansion in the teacher PD area reflected on the policy scene in many countries and in time, PLCs started to be seen as a vital component of ground-up educational change and instructional improvement (Little, 2002; McLaughlin & Tilbert, 2007; Altun and Yenigün Sarpkaya, 2021; Düşkün, 2021; Elçiçek and Yaşar, 2016). The effects of this world-wide popularity of teacher professional learning communities as a part of educational reforms were seen in Turkey, too. The School-Based Professional Development Model (Okul Temelli Mesleki Gelişim, OTMG) which aimed to empower teachers and administrators to conduct their activities according to their local needs, foregrounded collaboration within departments and among school community, began in 2005 under the influence of wide-spread curriculum reform in Turkey that initiated transition from behaviorist program to a program based on a student-centered pedagogy¹ (Deveci & Aykaç, 2020; Ergün et al., 2015).

¹ Unfortunately, the School-Based Professional Development Model program was not followed-up after its pilot implementation and faded away over time (Altun and Yenigün Sarpkaya, 2021; Düşkün, 2021; Elçiçek and Yaşar, 2016).

Teacher PD models developed within the new paradigm integrated its premises: positioning teachers as active sensemakers and constructors of teaching knowledge, using artifacts to capture the complexity of teaching situations within the PD setting, engaging with authentic practices, and providing support for community development. For example, a well-known example in mathematics teacher education, Lesson Study includes teachers' collaborative research on a teaching problem they face in practice, planning a lesson based on this background, and implementing it in a real classroom setting. In this collaborative reflective process, teachers actively use data—such as video recordings of lessons and student work—collected from the implementation as a tool for joint professional development, continuing through further cycles of Lesson Study to refine the designed lesson (Dudley, 2015; Lewis & Perry, 2013; Stigler & Hiebert, 1999).

In another example, called the Problem Solving Cycle, designed and implemented by Hilda Borko and her colleagues (Borko et al., 2008; Koellner et al., 2007), teachers collectively work on a single rich mathematical task, engaging in discussions on the related mathematical content and student thinking and develop a shared pedagogical vision, plan a lesson to implement this task in their classrooms, videotape the lessons, and video clips are used in a follow-up workshop to situate teachers' professional learning in their classroom practice (Borko et al., 2008).

Another PD model developed within this paradigm, the video clubs (Van Es & Sherin, 2010), used video clips from teachers' classrooms selected according to their potential to foster collective discussion. Video is used to help teachers notice the classroom interactions that may go unnoticed, enabling the development of a teacher community. In another teacher community building endeavor, Kazemi and Franke (2004) utilized student artifacts produced as a result of each participant

teacher's implementation of a similar mathematical problem in their classrooms. Student artifacts provided a means for the development of a shared language, negotiation of meaning, within monthly PD workshops where the researchers introduced the principles and terms from specific pedagogical approaches (Cognitively Guided Instruction [Carpenter et al., 1999]) in an emerging fashion as teachers examined their students' mathematical thinking.

In a recent model called Video Formative Feedback, Horn and her team utilized both video clips from a focal mathematics teacher's video-recorded teaching, which were selected according to the teacher's inquiry question (Horn & Garner, 2022), and recently a visual representation of teacher movement during teaching using a tool called Interactive Geography Slicer as representations of practice (Shapiro et al., 2024) to foster a collective discussion by the focal teacher, her colleagues, and the research team members during a debrief session.

With these qualities, PD approaches developed within the new paradigm depart from the tendency of teacher educators to take control of the learning processes with fixed agendas of seminars that are mainly focused on the transfer of pedagogical techniques, ideas, and materials in a unidirectional manner in short durations (Stein et al., 1999). Instead, they center teachers as agents of change, focusing on problems and questions that emerge from real teaching situations as the foundation of professional learning activities (Cochran-Smith et al., 2022; Zeichner et al., 2015). They encourage teachers to employ artifacts coming from their own teaching and classrooms (e.g., videos from classroom interactions, student work reflecting student thinking) as authentic tools to direct their own professional learning within their collegial communities, ultimately influencing the course of events in their practice and in their schools. Although these characteristics point out

the importance of teachers' individual and collective agency for teacher learning from the new paradigm's perspective, teacher agency in teacher professional development as a construct has been understudied (Brodie, 2021), with a focus on different constructs, such as content and pedagogical content knowledge (e.g., Koellner et al., 2007; Shulman, 1986), teacher noticing (van Es & Sherin, 2002, 2010; Sherin et al., 2011), shifts in ways of participation in a teacher community (e.g., Kazemi & Franke, 2004), to design supports for and/or assess learning in these PD models.

While aiming to contribute to the new paradigm of teacher PD from a design research perspective, this study centers teacher agency in both the design of a teacher professional learning environment and the examination of their implementation outcomes.

2.2 Teacher agency: Definitions, pedagogical approaches, and empirical studies

2.2.1 Teacher agency: A sociocultural conceptualization

As the following discussion will detail, from a sociocultural theoretical perspective, teacher agency can be defined as teachers consciously making decisions and acting to deal with problems of practice, transform and expand their teaching practice, support their own learning by employing social, material, and conceptual resources within the environment or by creating new ones (Edwards, 2010; Engeström et al., 2020; Eteläpelto et al., 2013; Günal-Aggül et al., 2023; Vygotsky, 1997).

From a dialectical point of view, a central premise of Vygotsky's theory is that "we are both shaped by our cultures and we shape them" (Edwards, 2010, p. 67). Based on such a stance, the starting point of the discussions on agency in Vygotskian socio-cultural tradition is that humans have the capacity to shape their conditions and

environments, while they are also under the influence of social circumstances (Lasky, 2005). Dialectics between social structures and the individual characterize agency formation.

First, human beings have the capacity to manage the social context for their own purposes (Eteläpelto et al., 2013). This perspective is based on the Vygotskian notion of “volitional action and will,” that is, human beings have an active role in capitalizing on the resources in the environment for their own purpose, and in turn take control of their own actions. In Vygotsky’s (1997) words,

the person, using the power of things or stimuli, controls his own behavior through them, grouping them, putting them together, sorting them... makes them serve his own purposes and controls that power as he wants. He changes the environment with the external activity and in this way affects his own behavior, subjecting it to his own authority. (p. 121)

Individuals can actively manage and transform the context in which they act to serve their own purposes by creating or utilizing conceptual, social, and material resources (Billett, 2006; Edwards, 2010; Eteläpelto et al., 2013; Lasky, 2005; Vygotsky, 1997). They are not passive recipients of their surrounding circumstances and “socially determined beings;” they have the capacity to create goals and carry out deliberate acts that surpass and alter the established norms and circumstances of the activity and system they are a part of (Engeström & Sannino, 2020, p. 87). As Engeström and Sannino (2020) highlight, Vygotsky’s perspective goes back to Marxist notion of practical-critical activity (Marx, 1976).

At the same time, in the sociocultural perspective, agency is not seen as a fixed quality, disposition, or capability residing solely in the individual mind (Haapasaari et al., 2016; Lipponen & Kumpulainen, 2011). Agency is achieved according to the constraints and resources of the social environment and as a result, one’s agency performance might shift from one situation to another (Lipponen &

Kumpulainen, 2011). In individualistic-psychological approaches which sociocultural approaches oppose, agency is understood as a capacity people have in different degrees due to their personal characteristics (Priestley et al., 2015). A definition from Calhoun (2002) reflects the individualistic perspective: agency is “(t)he capacity for autonomous social action” and “the ability of actors to operate independently of the determining constraints of social structure” (cited in Biesta & Tedder, 2006, p. 5). Countering this view, Edwards (2017) highlights that describing agency as a capacity risks reducing it to an individual trait, once again overlooking the dialectical processes through which it emerges and evolves.

If we interpret this perspective for teachers’ agency, agency depends not only on the capacity of teachers, but also on the resources in different forms -cultural, relational and material- that are within the reach of teachers (Drew et al., 2016, p. 2). As examples of various types of resources, Drew et. al. (2016) speak of “constructive collegial and external support for innovation, conceptual framings for educational practice, research findings and intelligently framed educational policy (i.e. curriculum regulation)” (p. 2).

Sociocultural view also differs from social-cognitive approaches to agency (e.g., Bandura, 2001) which emphasize “intentionality as the core aspect of agency,” with its premise that “humans can only exert agency by internal goal setting, independent of external input, and that this can be expressed through self control and beliefs as one’s self-efficacy” (Damşa et al., 2010, p. 148). According to Stetsenko (2019), overemphasis on individual capacity shifts the attention away from structural issues and disseminates the discourse that individuals are free to take control of their lives despite many structural factors.

Teachers' work is also framed by various structural, relational, and cultural conditions, which can be overlooked with an overemphasis on individual teacher capacity for agency formation, which some policy initiatives pursue as a goal (Drew et al., 2016). At the same time, tying back to Vygotsky's notion of "volitional action and will," and emphasizing the mutual influence between agents and structures, within Vygotskian sociocultural and specifically cultural historical activity theory traditions, human beings are seen to be able to transform the conditions in which they enact agency. However, this requires developing collective consciousness about systemic contradictions and a transformative stance for systemic change, rather than a primary focus on individual capacity development (Engeström et al., 2014; Stetsenko, 2019). Still, there are debates in sociocultural approaches regarding how to position individuals' roles in collective processes (Eteläpelto et al., 2013; Stetsenko, 2019) which will be briefly mentioned in Section 2.2.1.3.

Various forms of agency have been proposed within this tradition. One way to categorize these forms is based on whether they take collectives, relations, or individuals as their unit of analysis. In the following sections, these forms will be discussed following this categorization, ending with brief notes on their relevance for teacher agency.

2.2.1.1 Collectivist approaches to agency

At the collective level, the notion of "collective transformative agency" was proposed within the cultural-historical activity theory (CHAT) tradition (Haapasaari & Kerosuo, 2015) with a focus on how people develop agency to engage in joint work to transform collective activities by developing critical consciousness about their activities and problems and creating new practices as solutions. Transformative

agency is defined by Virkkunen (2006) as agents' "breaking away from the given frame of action and taking the initiative to transform it" (p. 49).

Except Sannino's studies (2008, 2010) where individuals' contribution to the formation of collective transformative agency is analyzed (Virkkunen & Newnham, 2013), the main unit of analysis for transformative agency is collective. According to Haapasaari et al. (2016), transformative agency "goes beyond the individual as it seeks possibilities for collective change efforts" (p. 33). This emphasis on collective agency for change and development in activities stems from the premises of second-generation Cultural-Historical Activity Theory (CHAT). This approach takes collectives and collective activity systems as the primary units of analysis, extending Vygotsky's focus on culturally mediated individual action to the collective level (Engeström & Sannino, 2021). Activity is constructed within collectively operated systems and these activities can only be transformed—or new ones created—through collective action (Engeström & Sannino, 2021).

From CHAT perspective, transformation in activity systems is triggered by the systemic contradictions and subjects' utilizing these contradictions to take agentic action to deal with them. As Haapasaari and Kerosuo (2015) state,

Agency appears in actions, and it develops in historically shaped collective activity systems, tightly connected to the motives and contradictions within them. Transformative agency is about working with the contradictions of an activity system. It includes actions of questioning the status quo and of search for new possibilities. (p. 4)

Furthermore, transformation can only be achieved and pursued through collective agentic action, albeit initiated by individuals:

Although initiated by individuals, agentic actions gain their meaning, their consequences and their continuity in the interplay between individuals and their collective. In this sense, agency is collectively produced and maintained. (p. 4)

In that regard, scholars proposing the notion of transformative agency mainly focus on how collectives form agency by identifying and analyzing tensions, disruptions, and contradictions within local activity and work practices, as well as envisioning future alternatives and initiating changes to transform the activity. Virkkunen (2006) argues that through these processes, “a set of individuals” becomes “a collective subject and agent of transformation” (p. 44).

Transformative agency, as a construct to understand collective action, has been interpreted for teacher agency in cases of instructional and school-level change and development processes (e.g., Diao et al., 2022; Wei & Sannino, 2024). For example, in Wei and Sannino (2024), mentors and preservice teachers exercised collective transformative agency to redesign the preservice teacher internship program from the bottom up through a formative intervention study by addressing existing contradictions in the mentoring activity. Contributing to this line of inquiry, this study investigates how a group of teachers from different organizations enact collective transformative agency by identifying contradictions within their shared activity system and by consciously and strategically organizing their own professional learning activities in response.

2.2.1.2 A relational approach to agency

At the relational level, Edwards (2010) brings our attention to relational action as "a middle layer... located between the system and the individual" (p. 140)" as another unit of analysis for agency formation. It refers to the individual's capacity to act flexibly to address unpredictable aspects of their practice by engaging with different professionals working on the same problems of practice, drawing on others'

perspectives and specialized knowledge, and making their own perspective and expertise visible to others in approaching it. In Edwards' (2017) words:

Relational agency (Edwards, 2005) refers to the agency that arises in collaborations, which: (i) expand interpretations of the problem being worked on, for example the child's trajectory towards social exclusion, so that its complexity is recognised; and (ii) allow more than one practitioner to work on features of the problem, drawing on their specialist expertise in concert with others from different practices. (p. 281)

It is a form of agency that can be exercised during relational action. It goes beyond individuals but at the same time requires individuals' "capacity to offer support and to ask for support from others" (Edwards, 2005, p. 168).

Relational agency differs from individual agency since the relational plane is where teachers and other professionals act agentically with resources that emerge only in interaction (Edwards, 2005, 2010). According to Edwards, complex problems require relational work from professionals who bring various resources to these efforts, rather than relying on the individual work of "heroic individuals who are given status through their ability to work autonomously" (Edwards, 2010, p. 64). In this way, within collective problem-solving environments where relational agency is enacted, each professional's interpretation and reaction to problems become visible, becoming tools to be used by others and collectives as a whole to develop enriched understandings of shared problems and produce solutions.

This conceptualization has significantly to do with teachers' professional agency, which "is not a fixed disposition of an individual teacher, but is highly relational and thus embedded in professional interactions between teachers, pupils, and their parents, as well as with other members of the school community" (Pyhältö et al., 2015, p. 306-307). That is, while approaching problems of practice, teachers are required to exercise relational agency and vice versa, schools should provide an environment where relational agency is promoted within the school community.

2.2.1.3 Subject-centered approaches to agency

Conceptualizations that focus on the subject level in agency formation, while adhering to a socio-cultural perspective that rejects an individualistic view of agency as a human capacity, still acknowledge the role of subject-level variables in the enactment of agency, with an emphasis on their interaction with social conditions. Subject-centered approaches to agency enrich the socio-cultural view of agency by revealing the importance of individual factors and identities in work-related change and development processes.

One such conceptualization is proposed by Eteläpelto et al. (2013, 2015) as the "subject-centered sociocultural view of professional agency." From this perspective, professional agency describes the situations "when professional subjects and/or communities exert influence, make choices and take stances in ways that affect their work and/or their professional identities" (Eteläpelto et al., 2013, p. 61). While acknowledging the role of social condition, this conceptualization highlights that individual history, prior experiences, and identities shape one to exercise agency "in choosing problems they will engage in, and do so with different degrees of engagement" (Eteläpelto et al., 2013, p. 56). That is, a professional's identity (commitments, ideals, motivations, interests, goals), professional knowledge and competencies, and work history and experience constitute individual resources to exercise professional agency. Social conditions influencing the enactment of professional agency include material circumstances, physical artifacts, power relations, or work cultures and discourses (Eteläpelto et al., 2013).

By evaluating teachers' professional practice from a subject-centered perspective, Eteläpelto et al. (2015) define "teacher agency" as having influence and negotiation opportunity on teaching practices, including "core pedagogical and

instructional practices (including applying new ideas at work), making decisions on one's ways of teaching, and developing one's work" (p. 663). Teacher agency is shaped by both professional identities, which include pedagogical ideals, interests, competencies, and work-related past experiences, and social conditions, which include the curriculum, the professional tasks, school norms, and the available material and social resources (Eteläpelto et al., 2015).

A similar perspective is followed by Mark Priestley, Gert Biesta, and their colleagues in defining teacher agency, taking the individual teacher as the unit of analysis. Priestley et al. (2012) explain their theoretical view as the "ecological approach to teacher agency," which conceptualizes agency as an emergent phenomenon—"as something that is achieved by individuals, through the interplay of personal capacities and the resources, affordances and constraints of the environment by means of which individuals act." (Priestley et al., 2015, p. 20). Similar to Vygotskian approaches, they reject the view that agency is something people own as a capacity, classifying individuals as more or less agentic. However, their conceptualization has been criticized by other scholars within the Vygotskian tradition for ignoring the dialectical relationship between the agent and the social context. That is, while the ecological view acknowledges that subjects' actions are influenced by the social conditions, it is criticized for not emphasizing the role of subjects in transforming those conditions (Edwards, 2017).

Still, from a Vygotskian perspective, I find the ecological approach to teacher agency useful in identifying the individual-level factors at play as individuals engage in a dialectical relationship with their environments. Priestley and colleagues complement their conceptualization by adding the temporal aspect of agency formation, drawing on Emirbayer and Mische's (1998) temporal-relational

perspective on agency by proposing that "achievement of agency should be understood as a configuration of influences from the past, orientations towards the future, and engagement with the present" (Priestley et al., 2015, p. 23). Agency should be perceived as a temporally embedded process of social engagement, influenced by the past, directed toward the future with subjects' ability to envision alternative possibilities, and enacted in the present by contextualizing habits and experiences from the past and future endeavors with the contingencies of the moment. This points out three temporal dimensions underpinning the achievement of agency: iterative (i.e., past experiences from life histories and professional histories), practical-evaluative (i.e., cultural, structural, and material conditions in the present), and projective (i.e., short- and long-term objectives, values, and aspirations). While emphasizing both individual and structural contexts in agency formation, the ecological view of agency unpacks the subject-centered dimension by parsing it into temporal units (Figure 1).

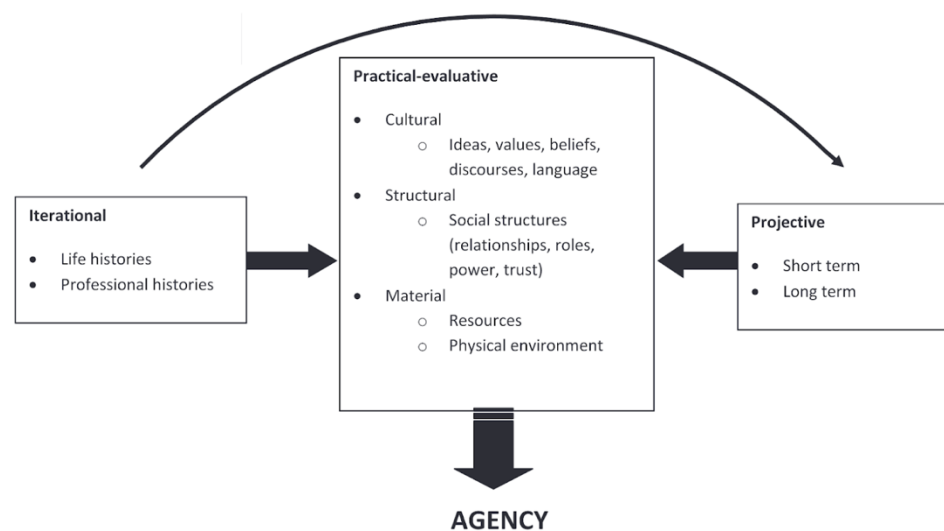


Figure 1. Three temporal dimensions behind the achievement of agency (Priestley et al., 2015, p. 30).

All in all, conceptualizations of agency developed within the sociocultural tradition help to account for an aspect of teacher agency as defined in the introduction of this section, i.e., consciously making decisions and acting to influence the course of events in their professional settings by employing resources, collectively or individually. Accordingly, each contributes to the perspective followed in this design research.

In this study, these conceptualizations provided a background to derive principles to design a participatory teacher professional learning environment to support teacher agency. Before elaborating on this, I will review the literature on (1) different pedagogical approaches developed to support teacher agency (2) empirical studies that employ such approaches as well as analyze the formation of teacher agency within such settings. This literature review will also inform the development of design principles.

2.2.2 Pedagogical approaches supporting teacher agency in professional development

Pedagogical approaches that support teacher agency can be classified into three categories: new paradigm professional development (PD) approaches, the nearly century-old approach called action research (Lewin, 1946), and collaborative design. Particular emphasis will be placed on the third category, as formative interventions—a specific approach within this category—are specifically designed to support agency and provided a foundational background for this design study.

2.2.2.1 New paradigm PD approaches and teacher agency

Various pedagogical approaches that position teachers as drivers of their own professional learning were developed within the new paradigm of teacher PD. These approaches are based on this paradigm's central premises, including the development of teacher communities, the use of authentic artifacts to ground PD activities in real teaching problems, and teachers' active participation in PD activities. These models include PLCs as an umbrella approach, the widely implemented Lesson Study model, and other locally designed and implemented models, such as the Problem-Solving Cycle. Although these models do not explicitly design pedagogical processes with the formation of teacher agency as a primary outcome, some empirical studies have demonstrated their potential to support teacher agency in professional development.

PLCs have been highlighted in the literature as mediums for teachers to enact and develop agency, while teacher agency is also recognized as crucial for enhancing the effective functioning of PLCs (e.g., Masuda, 2010; Philpott & Oates, 2016; Pyhältö et al., 2015; Riveros et al., 2012). In one of the empirical studies exploring the relationship between PLCs and teacher agency, Philpott and Oates (2016) suggest that PLCs can offer "affordances" for teacher agency. Their analysis of data from a PLC program in Scotland reveals several barriers that limit its potential as an "affordance" for teacher agency, such as policy-driven discourse that restricts the diversity of perspectives in community discussions and the absence of academic resources to enrich participants' viewpoints.

Some scholars have explored the connection between Lesson Study and teacher agency. According to Kim (2021), Lesson Study is a democratic PD model that aligns with the new paradigm of PD by centering teacher agency. Within this practice, teachers make sense of the content they are teaching and the methods to do

this by consciously exploring the curriculum during the research step and design lessons with a deep understanding of their students' needs and the content they teach. They are encouraged and empowered to contribute to the collective work of the community. All in turn contribute to the emergence of "knowledge democracy" and fosters "collaboration, communication, and community" among colleagues and experts, creating the necessary context and conditions for achieving teacher agency (Kim, 2021, p. 204). From a policy perspective, Collet (2019) argues that Lesson Study has the potential to counteract the restrictive impact of reforms that often limit teacher agency. She points out that the emphasis on teacher agency in Lesson Study facilitates teachers' motivation to participate in this activity, where they are provided with space and empowered to be instructional decision-makers sensitive to their students' needs and flexibly use resources different from fixed agendas of PDs.

Despite the strong connections between Lesson Study and teacher agency, Kitada (2022) highlights the lack of empirical studies that focus on teacher agency as the main construct in the investigation of this link. Lewis et al.'s (2022) study is significant in this regard, as it explores the role of teacher agency in the sustainability of Lesson Study. By examining the implementation of school-wide Lesson Study in three U.S. schools, the authors questioned the factors that enable schools to create sustained and effective Lesson Study. They found teacher agency as a major theme related to its sustainability. Through the analysis of videos, observation notes, and artifacts from Lesson Study cycles, they identified that teachers exercised agency by being "key agents and co-designers of the work" (p. 49). This included practices such as crafting the research theme based on long-term goals, selecting curriculum units based on their authentic problems and inquiries, and designing these units with consideration of their students' needs. Lewis et al.'s study suggests that Lesson

Study projects should prioritize teacher agency as a primary outcome for teacher learning. Resonating with Lewis et al., Uştuk and De Costa (2021) revealed that the reflective practice embedded in Lesson Study supports a transformative form of teacher agency (Mezirow, 1997).

All in all, PD models developed within the new paradigm of teacher PD have been linked to teacher agency as they are seen as "affordances" for teacher agency (Philpott & Oates, 2016) or they provide conditions to achieve agency in professional development (Kim, 2021). On the other hand, the literature review revealed a lack of a design research perspective—specifically, the intentional design of supports to facilitate teacher agency in professional development—which could strengthen models as pedagogical approaches that help teachers enact agency in their professional learning.

2.2.2.2 Action research and teacher agency

Action research, and its variations such as participatory action research (Stapleton, 2021) and teacher research (Cochran-Smith & Lytle, 1993; Pine, 2009), have also been evidenced to support teacher agency in different contexts of teacher learning and development (e.g., Edwards, 2019; Xin & Brion-Meisels, 2024).

Kurt Lewin (1946) describes action research as "a spiral of steps each of which is composed of a circle of planning, action, and fact-finding about the result of the action" (p. 38). A typical action research cycle includes (1) problem-posing by reflecting on current practices, (2) planning for solutions, (3) acting to implement solutions, and (4) collecting data, (5) reflecting on data collected from the implementation, asking new questions and revising the plan for another cycle (Baum, et al., 2006; Dodman, 2023; Emam et al., 2023; Lambirth et al., 2021; Morales,

2015). Action research in schools involves research-practice partnerships in which teachers resolve situated problems at the classroom or school level.

According to Emam et al. (2023), action research supports teachers' agency in driving their own learning since it "emphasizes the role of practitioners to take ownership of their learning and the improvement of their schools; where schools [...] become learning institutions guided by professional inquiry" (p. 139). Other studies, based on their empirical research, identify key characteristics of action research that make it particularly suitable for helping teachers develop and maintain agency for change and development in their practices and schools: (1) offering space for teachers to engage with their own problems, questions, and uncertainties (Dikilitaş & Griffiths, 2017), supporting them in "understanding their context and status quo" (Emam et al., 2023, p. 139), and guiding their own professional development (Keay et al., 2019); (2) providing tools that teachers can adapt and craft to meet their own needs (e.g., cyclical AR framework) (Edwards, 2019); (3) fostering collaboration for collective change initiatives in educational settings (Chochran-Smith & Lytle, 2009, cited in Lambirth et al., 2021); (4) offering opportunities for teachers to plan and take action within their own settings.

Similar to how scholars describe PLC environments within the new paradigm of teacher PD as "affordances" for teacher agency, action research is also seen as "a venue for teachers to exercise agency" or "a context where teachers can achieve agency" (Emam et al., 2023, p. 138-139). There is a lack of a design-based research perspective, which may hinder the development of action research practices that better support teachers in achieving agency in professional development. While action research offers valuable insights and experiences for designing participatory

teacher learning environments, it does not address the gap left by design studies specifically focused on teacher agency.

2.2.2.3 Growing attention to supporting teacher agency within design-based approaches

Although a few studies have shown that the pedagogical approaches mentioned above support teacher agency, their design processes are not guided by principles from the literature on agency. As a result, they do not explicitly position teacher agency—particularly in leading their own professional learning and development—as a primary learning outcome. These models are often seen as contexts in which teachers may achieve agency, rather than being structured from the outset to foster teacher agency as a learning outcome, as would be the case in a design-based research study. This resonates with Morselli and Sannino's (2021) finding: "not much progress seems to take place in offering meaningful alternatives that can be actually implemented in support of agency formation" (p. 2).

Recently, there have been efforts in the learning sciences literature to address the lack of models that set teacher agency as its primary learning outcome. Voogt et al. (2015) framed collaborative design for teacher PD as a model supporting teacher agency by identifying three theoretical principles for a co-design process: situatedness, agency, and the cyclical nature of learning. This approach is different from co-design approaches that include teachers in the researcher-led design processes as end-users of the educational innovations to provide insider view for more contextually relevant designs (e.g., Cober et al., 2015; Matuk et al., 2016; Penuel et al., 2007; Reiser et al., 2000; Shrader et al., 2001). In teacher learning by collaborative design, teachers have a more central role and the emphasis is on co-

design team's becoming a professional learning community and teachers' active involvement in the process from the identification of the problem situation, design, enactment, and the refinement of the designed products. Voogt et al. specifically highlight a method called formative interventions (Engeström, 2011) as a way to organize the collaborative design for teacher learning with a focus on fostering participants' agency (see also Penuel, 2014).

2.2.2.4 Formative interventions

Formative interventions (FIs), traditionally, do not only have to do with the co-design of curricular materials and tangible tools but also the co-design of new activity structures as contextually relevant novel solutions to the problems in collective activities that are identified by the communities themselves within research-practice partnerships (Engeström, 2011; Sannino, 2016). According to Hopwood (2024), FIs are a distinctive approach to action research that is gaining growing attention for their potential to enhance the field by introducing "theoretical rigor and criticality" (p. 21).

FIs are organized based on Engeström's (1987/2014) theory of expansive learning and the method of double stimulation (Sannino et al., 2016; Vygotsky, 1997). Expansive learning is a process theory of learning and accounts for transformation in activities as a collective problem-solving process through seven learning actions: questioning, analysis, modeling the new solution, examining and testing the new model, implementing the new model, reflecting on the process, and consolidating and generalizing the new practice (Engeström, 2016). Based on the theory of expansive learning, in FIs, researchers support participants with necessary tools and activities in questioning their ongoing practices to surface problems and

tensions in the collective activities, identifying and analyzing historically formed systemic contradictions behind these problems, and eventually resolving them by creating novel, contextually relevant solutions, where the whole process is considered as the enactment of collective transformative agency (Engeström, 1987/2014; Engeström, 2011; Haapasaari et al., 2016).

Double stimulation is a method used in FIs that researchers draw on while making decisions about the mediating tools and activities that participants can draw on while developing agency in engaging in expansive learning in a FI. Double stimulation occurs through the introduction of two types of stimulating artifacts, respectively—one set of tools reflecting the problematic situation (first stimuli) to provoke collective discussions on shared problems and one set of tools (second stimuli) to mediate a better understanding/analysis of the problem situation and the design of a solution to the identified problems (Sannino, 2015; Vygotsky, 1997).

First stimuli trigger "conflicts of motives," meaning "a clash between opposite aspirations or tendencies which occur in situations involving uncertainty ... requiring the courage of deliberate choice" (Sannino, 2015, p. 8) which are overcome through the use of mediators called second stimuli. The double stimulation method was utilized in Vygotsky's waiting experiments² to examine the emergence of

² Vygotsky's waiting experiment is explained by Engeström and Sannino (2016) as follows: "A subject is invited to participate in an experiment. The experimenter leaves the subject in a room without explanation and without a task. The experimenter observes from a separate room how the subject deals with the situation. The problematic situation is the first stimulus. It is turned into a conflict of motives—to stay in the room or to leave. The subject hesitates, oscillating between the two options. The subject then looks at the clock on the wall and decides to leave when the hands of the clock reach a certain position. The clock functions as a second stimulus. When it is invested with meaning ('at a certain point I will leave'), it becomes a sign that enhances the will of the subject and allows breaking out of the paralysing situation [...] [S]econd stimuli, help them undertake volitional actions and transform the situation" (p. 404).

volitional action of subjects under situations of uncertainty. Subjects develop agency/volitional action to deal with problematic situations by employing mediating tools, first to face the problematic situation and then to develop solutions to it.

Until now, formative interventions have been applied in various contexts with different collectives, including community members, practitioners, researchers, and other stakeholders, to co-design solutions to contextual problems. These have included efforts such as eradicating homelessness in a city (Sannino, 2020), developing new collaboration models to facilitate technology integration in workplaces (Engeström et al., 2013; Virkkunen & Newnham, 2013), creating culturally responsive disciplinary support systems in schools (Bal et al., 2019), reorganizing homecare work for elderly in a city (Engeström et al., 2015), and improving productivity in a post-office setting (Engeström et al., 1996). Researchers' role in a formative intervention process is defined as "to intervene by provoking and supporting the process led and owned by the learners," i.e., to support participants' agency in leading their own learning and change processes (Sannino et al., 2016, p. 2).

Change Laboratory is a widely implemented method under the family of FIs, specifically designed to support collaborative transformation, redesign, and development of collective work activities and foster workplace learning through these processes, during which double stimulation is put into practice with a specified toolkit (Engeström, 2007; Engeström et al., 1996; Virkkunen & Newnham, 2013). In a Change Laboratory intervention organized to support practitioners' expansive learning and agency, ethnographic data collected from the activity setting, such as video-recorded situations (e.g., Virkkunen et al., 2012), interviews with the actors (e.g., Engeström et al., 2013), or statistics (e.g., Bal et al., 2021) which helps to make

disturbances, critical incidents, troubles, or problems in the workplace "visible" also functions as first stimuli. Mirror materials are used during initial intervention sessions for discussion. Such first stimuli support subjects in noticing problems that might otherwise remain unnoticed in their routine activities. They serve as "evidence and examples of recurring problems and disturbances in their activity," providing an opportunity for collective reflection on their workplace problems—particularly the pressing ones—by making them visible for further elaboration (Sannino et al., 2016, p. 5).

To develop a solution to an identified problem, second stimuli are introduced by the interventionists or developed by the practitioners. A second stimulus also functions as an "anchoring device" to guide the design process. However, the second stimulus is not the solution itself; rather, it is a tool that helps practitioners develop solutions for themselves (Engeström, 2011). Additionally, the second stimulus can serve as a mechanism for introducing research-based ideas and practices from other cultures or countries, allowing practitioners to utilize those to create novel solutions to their contextual problems (Engeström & Sannino, 2010).

In most CL studies, the implementation phase is autonomously put into practice between the sessions or after the sessions are completed by the practitioners. Researcher-interventionists often opt for collecting data about the implementation process and conduct follow-up meetings to gain information about the implementation of the new practices and ideas (e.g., Engeström et al., 2002; Haapasaari et al., 2016; Kajamaa & Hyrkkö, 2022).

2.2.3 Empirical studies supporting teacher agency

Empirical studies on teacher agency can be classified in two ways. The first category includes studies that investigate agency and the conditions that limit or support these processes without any researcher intervention. Most of the examples include situations where educational change and transformation are ongoing in teachers' settings and teachers are expected to adapt to these changes, such as when teachers enact prescribed curricula as part of school- or country-level curriculum reform (e.g., Campbell, 2012; Priestley et al., 2012; Tao & Gao, 2017; Yang, 2015; Paris, 1993), novice teachers try to apply their pedagogical ideals in classrooms and departmental or school-level social settings (e.g., Eteläpelto et al., 2015), or teachers implementing alternative/innovative pedagogical approaches by adapting to their contexts amidst the complexities of teaching practice (Brevik et al., 2019; Eteläpelto et al., 2015; Ketelaar et al., 2012; Pappa et al., 2017; Toom et al., 2015), or teachers' developing culturally responsive teaching practices for inclusion and social justice (Hamilton & Dwyer, 2018; Luong, 2024, Min et al., 2021; Nguyen & Huynh, 2021); a teacher's agentic solutions to increase engagement of silent students during distance education during COVID-19 (Aardalen & Aagaard, 2024). In these studies, there is not any collaboration with teachers to support their agency in dealing with these challenging situations, but their agency is followed in naturalistic settings.

In this section, given the specific focus of this study on designing participatory professional learning environments to support teacher agency, I will focus on the second category which include empirical studies that include an intervention focused on supporting teachers' agency in leading their professional learning and development and analyze teachers' agency formation in these processes. This literature review on empirical studies discusses four key dimensions: what kind

of an intervention method was adopted, how the intervention process was designed and embodied, what kinds of analysis methods and frameworks were followed for the analysis of agency formation, and the striking findings.

First dimension is related to the design of the interventions to support agency formation in specific contexts. Formative interventions are a widely implemented, systematic methodological approach to support teachers' agency in these efforts. The interventions varied according to how these processes were embodied using different types of artifacts. In most of these studies, the intervention process was framed through the double stimulation method. Across these studies, the types of first stimuli varied depending on the context of the study. Examples include a proposed framework for K-12 science education used to surface teachers' concerns about its implementation (Severance et al., 2016), videos from teachers' classrooms to help them identify problems they might not notice in regular practices (Rainio & Hoffman, 2021), and teachers' autobiographical reflection papers to help them articulate concerns about student evaluation practices (Sannino, 2010). These first stimuli triggered teachers to surface the problems, disturbances, or concerns arising from their own contexts, which were often expressed as "conflicts of motives."

The second stimuli were either presented by the interventionist-researchers (e.g., Severance et al., 2016; Yang, 2021) or found by the teachers or other practitioners (e.g., Barma et al., 2015; Kajamaa & Hyrkkö, 2022). These second stimuli included various artifacts, such as a lesson design framework proposed in the science education literature (Severance et al., 2016), a new pedagogical approach promoted through a curriculum reform (Rainio & Hoffman, 2021), a teacher's article on an alternative evaluation strategy she implemented in her classroom (Sannino, 2010), or video clips from an experienced teacher's teaching of Chinese characters

(Yang, 2021). In all cases, the focus was on teachers' meaningful use of these artifacts to create novel practices, products, and activities to address the contradictions arising from their own contexts.

Unlike traditional design-based research approaches, the design in these studies has been an unfolding process, explained as part of the process of agency formation. Moreover, in studies that mentioned the implementation of co-designed products, researchers did not design specific interventions to guide the practitioners' implementation. Instead, this phase was largely carried out autonomously by the practitioners. In some studies, researchers visited the sites to observe and collect data on the implementation (e.g., Barma et al.) or conducted interviews with teachers who reflected on their implementation during follow-up intervention meetings (e.g., Kajamaa & Hyrkkö, 2022).

Based on the literature review focusing on the first dimension—the design of intervention processes—I identified a lack of clarity regarding the nature of the relationship between interventionist researchers and practitioners that enables teachers to develop ownership of the process. This represents a gap that needs to be addressed.

In terms of the analytical frameworks used, some studies analyzed the double stimulation process using qualitative approaches to trace how agency is formed step by step during the interventions (Barma et al., 2015; Yang, 2021). Additionally, scholars integrated various analytical frameworks to deepen their analysis. For example, Rainio and Hoffman (2021) used Engeström's (2007) concepts of possibility and impossibility knowledge to reveal teachers' transition from stabilizing to destabilizing discourse while evaluating students' learning capabilities. Similarly, Sannino (2010) applied Vasilyuk's (1988) notion of "experiencing" to analyze how a

single teacher transitioned from resistive to constructive discourse over the course of a Change Laboratory intervention. Some studies specified their approach as discourse analysis (Rainio & Hoffman, 2021), while others explained their analytical process using qualitative deductive and/or inductive coding to trace the formation of agency (Barma et al., 2015; Yang, 2021). Additionally, the taxonomy to analyze the formation of collective transformative agency, proposed by Engeström (2011) and developed by Haapasaari et al. (2016), was also used in several studies conducted with teachers (Kajamaa & Hyrkkö, 2022; see also Diao et al., 2022).

Based on the literature review focusing on analysis methods, I identified several gaps. First, in formative interventions, agency is supported to foster collective progress through the expansive learning cycle by resolving contradictions in activity systems and developing novel practices as solutions (Engeström & Sannino, 2016; Haapasaari et al., 2016; Kajamaa & Kumpulainen, 2019; Kerosuo, 2017). Although some studies involving teachers have highlighted the relationship between agency formation through double stimulation and the expansive learning process (Kajamaa & Hyrkkö, 2022; see also Hopwood, 2023), this relationship remains underexplored and warrants further investigation. Second, while the formation of mediating processes and tools was analyzed as part of the agency formation process, the studies did not systematically analyze how these designed or co-designed aspects specifically supported the formation of agency. Third, in all studies reviewed, the interventions were conducted with collectives. Regarding the units of analysis, most studies focused on agency at the collective level, specifically in relation to change and development in collective activities, with an emphasis on the construct of transformative agency. The formation of agency at the relational level in a formative intervention process was analyzed using the concept of relational

agency by Sannino and Engeström (2017). While this study is distinctive in emphasizing the relational nature of such learning environments, other studies do not elaborate enough on the relational aspects, such as how researchers supported teachers' agency while also facilitating and presenting mediating tools with their own expertise.

In the analyses, with an extensive focus on collectives, the contribution of individual teachers and the formation of agency at the individual level have not been sufficiently explored. Sannino's (2010) work is illuminating and provides guidance in this regard. This study unpacked the formation of agency at the individual level and how individuals contribute to the formation of collective agency during formative interventions. However, studies did not follow integrated analytical frameworks to analyze the formation of agency at the collective and individual levels at the same time. Understanding how the formation of agency at both the individual and collective levels supports each other requires more comprehensive approaches to analysis.

Fourth dimension focused on this literature review revealed that the key findings of these studies can be grouped under two main themes. First, interventions help teachers expand their understanding of problems in their activities collectively, particularly those related to their students' (the object of their activity) capabilities (Barma et al., 2015; Rainio & Hoffman, 2021; Sannino & Engeström, 2021). Transitioning from a closed to an open conceptualization of their students' abilities prompted teachers to take alternative initiatives in their practice. A similar pattern is observed in Sannino's (2010) study, where a teacher transitioned from resistive to constructive dialogue about improving evaluation practices in her classroom.

The second important finding is that, in interventions aimed at supporting teachers' agency for their professional learning and development, teachers develop a conscious and strategic attitude toward the use of mediating tools to resolve the problems identified in their practices. These tools may include teaching materials or frameworks proposed by experts, curriculum developers, and others. The studies showed that through interventions, teachers proposed changes to these mediating artifacts to better adapt them to their needs, which served as an indicator of agency (Severance et al., 2016; Yang, 2021).

In the following paragraphs, I will discuss some of these to illustrate how these dimensions (intervention method, process design and embodiment, analysis methods and frameworks, and findings) come together in an intervention.

Rainino and Hoffman (2021) analyzed the evolution of teachers' collaborative discourse as evidence of agency formation during a whole-school intervention in Finland, employing the Change Laboratory method to enhance student learning and engagement. The process began with researchers using selected video excerpts from teachers' classrooms as mirror data to highlight dilemmas or problematic situations. This was followed by teachers co-designing pilot teaching units based on a new pedagogical approach, "phenomenon-based learning," promoted through the Finnish curriculum reform.

Drawing on Engeström's (2007) framework of possibility and impossibility knowledge, the authors conducted a close discourse analysis, focusing on the discursive devices that either maintained or challenged teachers' conceptualizations of their students' capabilities during problem talk, before designing solutions to increase student engagement. The analysis involved a coding process to refine categories such as "surprise talk" and "dilemma talk" and to trace the shift from

stabilization to possibility discourse. The findings revealed that the intervention created a space for what the authors termed "reflexive noticing." Through this process, teachers developed a "discursive consciousness" of their stabilization discourse, which had previously constrained them to rigid classifications of their students' capabilities. This shift facilitated a transition to a "destabilization" or "possibility" discourse, enabling teachers to view students' learning challenges as "the object of, not impediment to" their work (p. 35). The authors identified the sustainment of possibility discourse in teachers' conversations as evidence of teacher agency. This discourse empowered teachers to construct themselves as "agents of change" and to envision alternative futures for their teaching practices (p. 35).

Barma et al. (2015) examined two science teachers' agentic actions in developing a novel instructional strategy during a formative intervention within a school setting. The intervention was initiated by the teachers, who sought collaboration with the research team to implement a prescribed science curriculum using an open-ended instructional approach. Initial interviews with the teachers served as the first stimulus, revealing conflicts of motives that arose during the implementation of the new curriculum. These conflicts included challenges related to teaching strategies and differing interpretations of the topic of environment, particularly its controversial aspects.

As the intervention progressed, the teachers identified and addressed these conflicts by reviewing available resources and selecting an existing teaching strategy. The authors conducted a qualitative analysis, focusing on the teachers' processes of articulating, identifying, and resolving conflicts of motives using artifacts in a double stimulation to analyze agency formation. Data sources included transcripts of teacher meetings, ethnographic field notes, teaching documents, student discourse and

artifacts. The study concluded that collective discussions about their conflicts enabled the teachers to derive meaning from their struggles, such as recognizing that consensus is not always achievable when discussing environmental issues. This realization led them to plan a new teaching sequence aimed at engaging students in debates on controversial environmental topics. By moving away from a lecture-based approach, the teachers adopted an open-ended teaching model. The authors interpreted the teachers' ability to make sense of their struggles, consciously and strategically select a second stimulus to resolve conflicts, and take action to implement the designed teaching sequence as evidence of transformative agency formation at the collective level.

In another study, Yang (2021) employed the method of double stimulation in a university language teaching course to support pre-service language teachers' agency in creating resources for teaching Chinese characters. A survey and pre-discussion served as the first stimuli, helping pre-service teachers identify their conflicts of motives regarding teaching Chinese characters. Subsequently, researchers presented video clips of an experienced teacher's instruction as a second stimulus, supplemented by additional activities, such as discussion worksheets with questions like, "Which parts of the teaching in the video do you like most? Which parts do you think need further improvement? Why and how?" These activities encouraged students to analyze the methods in the videos and develop novel ideas based on the demonstrated strategies. Qualitative analysis—including thematic coding, a pre-designed coding scheme for classroom dialogue, and comparative analysis grounded in the theoretical framework—focused on the emergence of epistemic agency (Damşa et al., 2010; Scardamalia, 2002) and the double stimulation process. The findings revealed that pre-service teachers created resources by both

directly adopting strategies from the videos and innovatively transforming them (e.g., incorporating embodied practices) for their lesson planning, providing evidence of their enactment of epistemic agency.

In a formative intervention study conducted by Severance et al. (2016) as part of a larger project aimed at supporting teachers in adopting student-centered approaches in the US, a team comprising teachers, administrators, community stakeholders, curriculum experts, and researchers collaborated to develop a new unit on ecosystems. This unit was designed to align with the requirements of *A Framework for K-12 Science Education* (National Research Council, 2012).

As the first stimulus, the research team asked teachers to design a curriculum unit that embodied the principles outlined in the *Framework*, anticipating that this would highlight teachers' concerns regarding various aspects of the vision and its implementation. Unlike other formative intervention studies, the second stimulus—a framework for designing project-based curriculum units called Storyline (Reiser, 2014)—was predetermined by the researchers. For the analysis, an ethnographic approach was used to trace how teachers exercised collective transformative agency in the development of new curriculum materials. Data included field notes from co-design sessions, reflective essays from seven participating teachers, feedback surveys, and the lessons created by the team. The researchers examined evidence of teachers' agency by identifying "instances of breaking away from previous patterns of activity to achieve new forms of activity," as well as how teachers adapted, resisted, or modified the tools and activities (p. 544). The analysis revealed that teachers exercised agency by adapting the second stimulus to better meet their needs. They created "in-between tools," such as enhanced tables and practical templates for

designing lesson sequences, to bridge the gap between the abstract framework of the Storyline and the concrete requirements of lesson planning (p. 553).

In Sannino and Engeström (2017), a formative intervention was conducted in an elementary school in a rural area in Southern Italy for the co-design and implementation of extracurricular activities for first graders in collaboration with in-service teachers and pre-service teachers. The intervention followed the Fifth Dimension (5D) method (Cole, 1996; Cole & the Distributed Literacy Consortium, 2006), promoting learning through mediation for student-driven interests and initiative, beyond curricular boundaries. Over three months, teachers implemented weekly extracurricular activities with first-grade pupils to complete self-selected tasks at different levels in the school's multimedia lab, completed with follow-up reflective discussions by teachers. Intervention, functioning as a first stimulus, helped teachers surface their conflict "between the dominant predefined age-appropriate curriculum and pedagogy on the one hand and the possibilities offered by the horizontal pedagogy of the 5D on the other hand" (p. 73).

As the intervention progressed, students' demonstrating that they could go beyond the age-appropriate curriculum functioned as a second stimulus, helping teachers overcome their conflicts and triggering them to create a novel activity. The proposed activity was transitioning the club activity to the classroom, where two project pupils taught fractions to their peers. The data included ethnographic fieldnotes, video recordings of 5D, reflective sessions, classroom implementation, and various artifacts produced during the process, and audio recorded interviews with teachers. Without specifying any method for analysis, the authors utilized "relational agency" as the main construct for their analysis and provided various evidence on how teachers dealt with their conflicts by enacting relational agency.

The authors found out that relational agency is exercised through many interactions between pre-service and in-service teachers. In these interactions, as a mediating tool for their dialogue, teachers used students' work which demonstrated that students can challenge the curricular boundaries. This has been found to be a significant underlying process supporting teachers' expansion in their conceptualization of their students as learners.

Kajamaa and Hyrkkö (2022) conducted a Change Laboratory in an elementary school to transform local practices as part of a curriculum reform. A video recording of the first meeting that focused on the problems within the school, with a presentation slide listing the problem topics discussed in this session served as the first stimulus for the next meeting. During the intervention, in line with the major conflict they identified—centralized leadership in the school community—the school principal introduced "a list of workgroups currently operating in the school" as a stimulus. This led the teams to create numerous template models for the organizational structure and roles, and these preliminary models later served as the second stimulus for the creation of a final version. Transcripts of the videotaped meeting discussions were analyzed deductively using Haapasaari et al.'s (2016) framework of discursive expressions of transformative agency. In developing this taxonomy, Haapasaari et al. (2016) identified six expressions that can emerge specifically in Change Laboratory interventions: resisting, criticizing, explicating, envisioning, committing to actions, and taking actions (Table 1).

Table 1. Types of Expressions of Transformative Agency

Types of expressions	Identification criteria
Resisting	Resisting the change, new suggestions or initiatives. Directed at management, co-workers or the interventionist
Criticizing	Criticising the current activity and organisation. Change-oriented and aiming at identifying problems in current ways of working
Explicating	Explicating new possibilities or potentials in the activity. Relating to past positive experiences or former well-tried practices
Envisioning	Envisioning new patterns or models in the activity. Future-oriented suggestions or presentations of a new way of working
Committing to actions	Committing to taking concrete, new actions to change the activity. Commissive speech acts are tied to time and place
Taking actions	Reporting having taken consequential actions to change the activity in between or after the laboratory sessions

Source: Haapasaari et al. (2016), page 242.

Kajamaa and Hyrkkö (2022) presented the results of the deductive coding of the data, sharing the number and percentage of expressions across the seven Change Laboratory meetings. In interpreting these results, they also integrated concepts from CHAT, such as double stimulation, contradictions, and the object of activity. At the same time, they traced the manifestation of transformative agency indicators alongside the community's progress in the expansive learning cycle—from questioning the current problems in the school activity system to modeling, implementing, and reflecting on the new activity structure as a solution. The authors concluded that the intervention helped participants articulate the "uncontextualized ('abstract') disturbances in their current work practices," leading to the identification of an underlying systemic contradiction and the creation of novel practices (p. 552). Additionally, they concluded that expressions of transformative agency, such as resisting and criticizing, were not obstacles, but rather driving forces for professional learning and development.

Distinct from empirical studies focused on the formation of collective or relational agency, Sannino (2010) concentrated on the formation of agency at the individual level in a Change Laboratory. The intervention, conducted in collaboration with teachers and administrators at a high school in Italy, aimed at developing teaching practices with a focus on changing student evaluation procedures at the school. To trigger the articulation of teachers' conflicts of motives, autobiographical texts were written by the participating teachers, serving as the first stimulus. As the intervention progressed, one of the participating teachers' article on an alternative evaluation strategy she had previously implemented in her classroom was used as a second stimulus to model alternative evaluation strategies schoolwide. The author drew on Vasilyuk's (1988) concept of experiencing—the process of surfacing and resolving internal critical conflicts—as a method to trace the formation of agency at the individual level in a formative intervention (Sannino, 2008a). With this perspective as the analytical focus, Sannino (2008a) examined the formation of a single teacher's agency, tracking the change in the teacher's discourse "from critical and disruptive to constructive and agentive" (p. 838). The process of experiencing started with the teacher's externalization of her critical conflict underlying her resistance to the change efforts and experiencing could be achieved through the support of the intervention. The analysis revealed a positive connection between teachers' resistance and agency, highlighting resistance's potential as a driving force, rather than an obstacle, to teacher professional development. Agency formation occurred through the teacher's surfacing of personal internal conflicts, an underlying factor of resistance (Kindred, 1999), beginning to recognize alternative possibilities in her practice, and ultimately engaging in mutually supportive collective and individual initiatives for creative change in the school.

2.3 Deriving principles to design a participatory professional learning environment that supports teachers' agency

This study aimed to design a participatory professional learning environment for mathematics teachers in which teachers were supported with mediating tools and activities to enact agency to collaboratively design their own professional learning processes in conscious and strategic ways while becoming a collective subject from being a set of individuals (Virkkunen, 2006). Through the implementation of this design, the study seeks to provide insights into the formation of teacher agency as a process of professional learning. While the formative interventions method guided me in designing the learning environment because of its primary emphasis on the formation of agency during collaborative design, I enriched the design by deriving design principles from other socio-cultural conceptualizations of teacher agency discussed in section 2.2.1.

2.3.1 Supporting agency formation at the collective level

The method of formative interventions aims to support the formation of agency at the collective level while collectives transform their activities or create new forms of activity to develop solutions to systemic problems (Engeström, 2011). In the literature, the type of agency supported through formative interventions is referred to as collective transformative agency (Haapasaari et al., 2016). The relationship between expansive learning and transformative agency has been described as a mutually supportive process by many scholars (e.g, Kajamaa & Hyrkkö, 2022; Haapasaari et al., 2016). As Kajamaa and Hyrkkö (2022) elaborate,

expansive learning and transformative agency are theoretically and conceptually closely connected, even inseparable: they both require and enable each other in the sense that 'expansive learning within the CL [Change Laboratory] is indeed a process of formation of transformative agency'

(Haapasaari et al., 2016, 243). Furthermore, both expansive learning and transformative agency are always connected to a specific contradiction in a specific historically evolved activity system. They enable overcoming the contradiction by introducing new, more adequate mediator(s) (e.g., new cultural tool or practice), to transform the collective activity and its local sociomaterial context. (p. 542)

The theory of expansive learning accounts for how subjects identify contradictions in their activities, using these as catalysts for transforming their activities and creating new activity forms (Engeström, 2015). Actors go through seven learning actions as they engage in expansive learning (Figure 2).

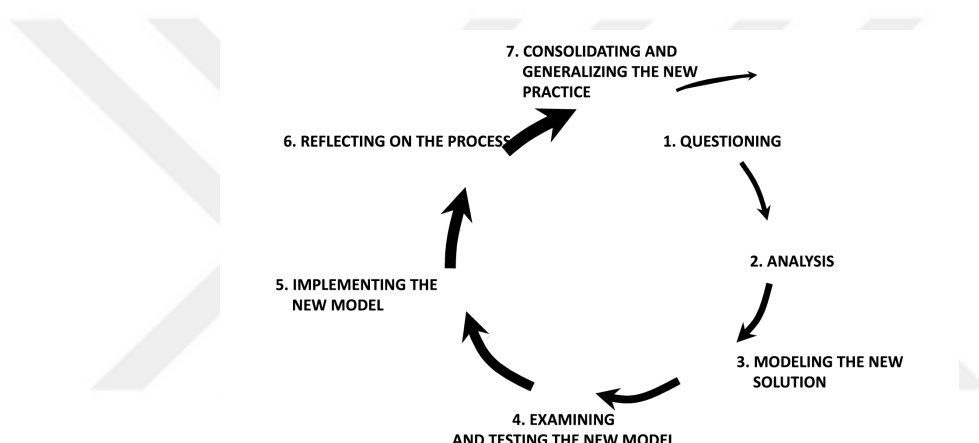


Figure 2. The prototypical sequence of expansive learning actions (Engeström and Sannino, 2021, p. 10).

The process begins with "questioning," defined as "questioning, criticizing, or rejecting certain aspects of accepted practice and existing wisdom" (Engeström, 2016, p. 25). Subjects question the taken-for-granted problems, conflicts, or disturbances in their ongoing activities. First stimuli in double stimulation support the questioning action in a formative intervention, helping subjects become aware of a "paralyzing conflict of motives" in their activities (Sannino et al., 2016, p. 5).

The questioning action is followed by the "analysis" action to investigate the causes of the identified problematic situation. Two types of analysis are conducted: one at the historical-genetic level and the other at the actual-empirical level. At the

historical-genetic level, the origins and evolution of the problematic situation are examined. At the actual-empirical level, the current systemic causes of the problem are analyzed. Through the analysis action, subjects uncover systemic contradictions in their activity system that underlie the problems initially identified in their practices through the questioning action. In their journey from questioning to analyzing, they develop perspectives on their problems, seeing different aspects that help them decipher the systemic issues that "time and again" lead to the same disturbances in their activities (Engeström, 2000, p. 153).

According to Engeström (2000), workplace cultures of problem solving might inhibit practitioners from seeing the connections among seemingly unconnected disturbances in everyday work practices. Formative interventions aim to provide practitioners space and support for the mental transition from dealing with these disturbances to working out systemic contradictions. In Engeström's (2000) words, in formative interventions:

The task is first, to make these disturbances visible to the practitioners, and second, to engage practitioners in analyses in which these seemingly random incidents are connected with and interpreted in the light of contradictions in the activity system that time and again give rise to such disturbances and innovations. (p. 153)

Through all these processes, interventionists aim to draw on the energy of the contradictions to support practitioners' agency development. In that regard, the problem identification through questioning and analysis is considered crucial for an agentic and conscious transition to the modeling a solution phase.

After questioning and analyzing, subjects work to resolve the identified contradiction(s) through the "modeling the new solution" action. In a formative intervention, as a part of the double stimulation, second stimuli support the modeling process and also help the deepening of the analysis of the systemic contradiction. In

the modeling step, subjects transform their activities or model new activity forms as solutions.

The fourth action is "examining and testing the new model" by "running, operating, and experimenting on it to fully grasp its dynamics, potentials, and limitations" (Engeström et al., 2014, p. 109). That is, collectives test how this model could function in practice to make necessary changes before the implementation. Examining and testing the new model is followed by "implementing the new model." After implementation, the collective reflects on the process and the model itself; this reflection consolidates the model and initiates a new cycle of expansive learning aimed at addressing the newly emerged problems in the modeled solution.

PMTLE was designed primarily to support teachers' collective agency in questioning, analyzing, and modeling expansive learning actions, with the goal of consciously and strategically shaping their own professional learning processes. Based on this goal, I derived three design principles from the discussion above:

- DP 1: Support teachers in collectively questioning their teaching practices and identifying their shared problems by providing first stimuli.
- DP 2: Facilitate collective identification of the systemic contradictions underlying their problems and the analysis of their historical and actual-empirical causes.
- DP 3: Support teachers in co-designing novel solutions to the identified contradictions, as well as in co-creating or choosing mediating artifacts, concepts, or models to use as second stimuli in this process.

2.3.2 Organizing relationality to support the agency formation

Supporting agency requires a different type of relationality and social infrastructure between practitioners and researchers, one that contrasts with the traditional hierarchical structures often emphasized in professional development settings (Bang & Vossoughi, 2016; Jurow et al., 2018).

Some principles underlying formative interventions, which were not systematically documented elsewhere, help researchers organize relationships in the intervention setting to support agency formation. According to Engeström (2011), unlike typical design studies, "the contents and goals of the intervention" are not strictly determined by the interventionists. During the problem identification phase, the role of the interventionist is "to help participants (whether children, adult practitioners, or both) confront a problematic and contradictory object embedded in their vital life activity (p. 606). In other words, rather than directing participants to work on a problem situation pre-identified by researchers or through a standard needs analysis process where participants have limited opportunities to discuss their problems with colleagues, formative interventions support participants in collectively identifying problem situations and developing perspectives on them. In expansive learning, both researchers and participants learn "something that is not yet there" (Engeström, 2015). That is, neither the problem situation nor the solution outcome is fixed ahead of time. It is a "formative" process through which both researchers and participants learn together.

In this context, the researcher's role in the problem identification process involves paying careful attention to practitioners' discussions and how they make use of mediating tools and activities. The aim is to support them in progressing through the expansive learning cycle. Practitioners are not expected to work on problems pre-

identified by the researchers or their managers (Engeström, 2011). Instead, the focus is on helping participants develop perspectives on their problems and refine their understandings.

In addition, in Engeström's (2011) words "the contents and course of the intervention are subject to negotiation and the shape of the intervention is eventually up to the participants" (p. 606). To facilitate this, the researcher should encourage the articulation of diverse perspectives, ask open-ended questions to stimulate dialogue, support teachers in challenging or building on each other's ideas, and engage in joint work to alleviate their lack of knowledge about these problems (Damşa et al., 2010). Additionally, the facilitator-researcher should use teachers' discussions as data to adaptively design activities that support not only the problem identification but also the co-design of the solutions process. For example, as Kajamaa and Hyrkkö (2022) elaborate, Change Laboratory meetings "generate rich discursive data (including video data) which the research team can analyze between the meetings to prepare for subsequent meetings" (p. 539). This aspect is important to be able to support participants in a responsive manner not only through facilitation during the meetings but also through the design of the sessions.

It is essential that researchers remember that this process is "led and owned by the learners," with their role being "to intervene by provoking and supporting" this process through mechanisms like double stimulation (Sannino et al., 2016, p. 2). Resistance of participants is not seen as a failure of the design but as an opportunity for learning and agency formation (Sannino, 2010). Collectives are guided to model a solution "the contents of which are not known ahead of time to the researchers" (Engeström, 2011, p. 606). This calls for an alternative form of relationality compared to typical design studies where the targeted solution is predefined, often

hindering learners from developing genuine ownership over the process. In that regard, researcher-facilitators need to carefully observe participants' discussions to support their co-modeling and co-design processes.

To organize the relationality in the designed participatory professional learning environment in a way that it supports teachers' agency for identifying their problems, expanding their perspectives, and designing novel solutions, I extracted the following design principles from the discussion above:

- DP4: Facilitate participant-led problem identification and collective design through responsive facilitation and open-ended inquiry.

In addition to the perspective provided by formative interventions, Edwards' (2005, 2010) concept of relational agency contributed to the development of principles for relationality. I interpreted the insight drawn from the concept of relational agency in the context of formative interventions as a way to facilitate boundary-crossing between researcher-interventionists, practitioners, and other experts (Akkerman & Bakker, 2011). This aspect is specifically important since how researchers can contribute to the process with their own expertise, as well as what kind of an attitude they can adopt to support teachers in leveraging their expertise in problem identification and modeling processes, are not specified in formative interventions.

According to Engeström and Sannino (2010), the utilization of the second stimulus is the time when practitioners can draw on knowledge and practices developed by others to design solutions that are novel and tailored to their local needs. Utilization of other resources which provide alternative perspectives can also be important for the problem identification process, as many studies revealed that formative interventions can support practitioners' expanding their problem framings (e.g., Rainino & Hoffman, 2021; Sannino, 2008a).

For the modeling a solution phase, by making their expertise visible as equal partners, researchers help participants envision alternatives beyond their immediate environment, thus fostering their imagination (Power et al., 2023). Similarly, spaces for mutual negotiation and hybridization encourage practitioners to bring their professional histories into the process, using these resources for imagination and problem-solving—not just for the practitioners themselves, but also for the researchers. In this regard, a formative intervention process should support relational agency by facilitating the visibility of expertise and resource-sharing among all participants as equals. Sannino and Engeström (2017) also highlight this link by saying, "relational agency may be deliberately mobilized by means of formative interventions and can play a key role in disclosing the dynamics of collective and distributed engagement in transforming human activities" (p. 59). Interactions during a formative intervention process, which may involve conflicts and differences in perspective, can lead to creative learning and the development of novel solutions. Researcher-interventionists need to create space for these novel outcomes by maintaining dialogue (Bakhtin, 1981; Gutiérrez et al., 1999).

Accordingly, another principle for designing relationality in a participatory teacher professional learning environment where teachers' agency is supported can be formulated as follows:

- DP5: Facilitate the visibility of expertise and resource-sharing in the professional learning environment; share your expertise, resources, and perspectives as an equal participant.

2.3.3 Supporting agency formation at the individual level

According to Eteläpelto et al. (2013), researchers studying agency from a sociocultural perspective differ in how they frame the contribution of individual agency in collective change and development processes. Some scholars do not consider individual contributions in the formation of collective agency, taking collectives as the main unit of analysis (Eteläpelto et al., 2013). Another perspective acknowledges the role of individual agency, suggesting that researchers should consider the role of the agency of individuals, their professional histories, and future prospects when supporting and analyzing the transformation of collective activities (Eteläpelto et al., 2013; Vähäsantanen et al., 2017). Some scholars conducting formative interventions also highlight the role of individuals in initiating and maintaining collective transformative agency (e.g., Kerosuo, 2015; Sannino, 2008a). According to Haapassari et al. (2016), “agentive actions gain their meaning, their consequences, and their continuity in the interplay between individuals and their collective” (p. 235).

This design research study follows the premise that for the formation of teachers’ collective agency to model their own professional learning processes in conscious and strategic ways, each teacher need to find a space to develop personal meaning of these activities. In other words, teachers' individual identities—comprising their histories, current practical conditions, and future prospects—must be recognized. Their individual agency, which allows them to draw on their past as a resource, envision alternative futures for their practices, and make sense of collective activities, should be supported. To identify a design principle for supporting individual agency in PMTLE, I drew on Eteläpelto et al.’s (2013) conceptualization

of the "subject-centered sociocultural view of agency" and Priestley et al.'s (2012) ecological view of agency:

- DP 6: Support teachers in bringing their personal and professional histories into the setting, connecting their unique work contexts to the learning environment, and envisioning the learning environment as a resource for achieving their teaching ideals.

How these principles were embodied in the designed professional learning environment will be detailed in the Methods chapter.

2.4 Theoretical frameworks to analyze agency formation in design-based interventions supporting agency

In the literature review on empirical studies supporting teacher agency, which is primarily dominated by formative interventions (see Section 2.2.3), I provided an overview of the analytical approaches and frameworks used to analyze the formation of teachers' agency. This review revealed that most studies focused on the collective as the unit of analysis, predominantly utilizing two frameworks: Double Stimulation, which examines how its stepwise occurrence in a specific intervention serves as evidence of agency formation, and the taxonomy for collective transformative agency, with seven types of identifiers (Engeström, 2011; Haapasaari et al., 2016). While these frameworks are insightful, they do not facilitate the analysis of agency formation as collectives undergo the process of expansive learning—i.e., generating novel activities as solutions to identified systemic contradictions—particularly when participants come from different organizations and transition from being a group of individuals to a collective subject.

Expansive learning is grounded in the CHAT idea that contradictions are sources of change and development within activity systems (Engeström, 1996). Through the expansive learning actions of "questioning" and "analysis," subjects identify systemic contradictions in activities, which then trigger change and development, often manifesting in the "modeling" action. However, as Engeström (2016) highlights, contradictions are not "an automatism" (p. 27); for them to drive change, subjects must enact agency—specifically, a transformative form of agency—in each learning action of expansive learning and throughout the transitions between them. In Haapasaari et al.'s (2016) words, "transformative agency is about working with the contradictions in an activity system" (p. 235) and FIs support these processes. There is a need for frameworks to analyze agency formation for change and development in activities in relation to the identification and resolution of systemic contradictions, which are not directly addressed through dominantly utilized analytical frameworks in empirical studies. To address this gap, I employed two alternative frameworks in this study: experiencing (Sannino, 2008a, 2008b, 2010; Vasilyuk, 1991) and Learning III (Bateson, 1987; Engeström, 2015), to analyze agency formation at the individual and collective levels, respectively.

According to Engeström and Sannino (2011), systemic contradictions are experienced by subjects as personal or interpersonal conflicts during workplace interactions. Based on their analysis of data collected through formative interventions in workplace settings, they identify various forms of such conflictual situations. One of these forms is critical conflicts, which are experienced by individuals as internal struggles that occur at the personal level, often paralyzing them and preventing progress in their conditions. This notion was introduced by Vasilyuk (1991), along with the concept of experiencing—the process of working through critical conflict

situations, which culminates in the development of a conscious personal disposition to act in resolving internal conflicts, often accompanied by feelings of liberation and empowerment.

Sannino (2008, 2010) frames experiencing as a process of agency formation at the individual level, particularly within work-related change and development processes. She emphasizes its significance for practitioners' engagement with the systemic contradictions underlying such conflicts—that is, addressing critical conflicts "is an avenue into an engagement with the actual systemic contradictions of the given activity" (p. 843). By utilizing this framework for agency formation at the individual level, Sannino addresses a gap in the formative interventions literature—namely, the lack of attention to the role of individuals, their need to resolve internal conflicts, and the development of a personal sense of collective activity as a means of overcoming systemic contradictions within collective change efforts (Virkkunen & Newnham, 2013). In that regard, formulating individual agency formation during collective change processes through expansive learning as resolution of inner conflicts contributes to the literature. This framework will be elaborated in the next section as it will be drawn on in this study.

Another form in which contradictions manifest in practice and discursive interactions, as identified by Engeström and Sannino (2011), is the double bind—a term originally coined by Bateson (1987) along with the concept of Learning III. In double bind situations, "the individual is caught in a scenario where another person in the relationship conveys two conflicting messages, with one denying the other" (Bateson, 1987, p. 157). Learning III is the process of confronting and gaining control over double bind situations. In Bateson's conceptualization, the double bind is an individual psychological situation, and Learning III is an individual-centered

process. In their interpretation for organizational change processes, Engeström and Sannino (2011) frame double binds as collectively experienced dilemmatic situations in workplaces, arising from systemic contradictions. Furthermore, Engeström (2014) redefines Learning III as a collective process of generating new forms of activity aimed at "the mastery of double binds," positioning collectives as agents of transformation rather than victims of these contradictions.

A few studies in the existing literature have conceptualized the resolution of double bind situations as the formation of collective transformative agency (Frankel et al., 2024; Gutiérrez et al., 2019; Meléndez, 2021). For example, Meléndez (2021) utilized the concept of double bind to describe the forms that systemic contradictions are faced by Latino participants in a participatory budgeting process in Chicago. A formative intervention was organized to help participants overcome the double binds they faced in these processes (e.g., lack of the confidence to participate in decision-making). New tools and practices designed through the intervention re-mediated their roles and enhanced participants' agency for expressing their needs in the public sphere. In his analysis, the author focused on the unfolding of the process of expansive learning as a process through which collectives resolve double binds through the development of a new activity (Spanish-Language Committee).

All in all, in both experiencing and Learning III, individuals or collectives empower and agentially develop new perspectives and conscious and strategic actions to deal with these contradictory situations by utilizing social, material, and conceptual tools. This study will draw on these frameworks to analyze agency formation at both individual and collective levels through the implementation of PMTLE. The following sections will elaborate on these processes of agency formation.

2.4.1 Experiencing: Agency formation at the individual level

Experiencing is the process of one's dealing with critical conflicts. In critical conflict situations, individuals are confronted with internal doubts that render them unable to move forward (Engeström & Sannino, 2011; Vasilyuk, 1991). Engeström and Sannino (2011) define *critical conflicts* as "situations in which people face inner doubts that paralyze them in front of contradictory motives unsolvable by the subject alone" (p. 374). Conflicting motives often lead subjects to passivity or develop superficial solutions that deviate from her or his values. Subjects cannot develop new perspectives to reframe the critical situation and cannot create solutions to change it.

Engeström and Sannino (2011) maintain that one can find critical conflicts in discourse by looking for certain linguistic cues, such as "personally, emotionally, and morally charged accounts that have narrative structure and frequently employ strong metaphors" and expressions that show "feelings of being violated or guilty" (p. 374).

Overcoming critical conflict situations requires agency that can only be mobilized with the support of others and the use of mediating tools and activities (Sannino, 2008a). Addressing and dealing with critical conflicts is called experiencing, which "refers to a process through which an individual, supported by others, engages in a quest to overcome critical situations" (Sannino, 2008b, p. 272). It is a process of struggle against feelings of impossibility and the effort to realize one's internal necessities. This process starts with subjects perceiving the situation as a problem of internal conflict and articulating it, which is a courageous act (Vasilyuk, 1991). This is followed by the subjects' search for a new meaning of the initial internal conflict situation, i.e. seeing the situation through a different lens as a first step, which is a difficult task, requiring "material support and dialogic processes" (Sannino, 2008b, p. 289).

A successful experiencing does not necessarily end with taking concrete actions to resolve the critical conflict, but the subject overcomes impossibility feelings, gains motivation, and starts seeing alternative possibilities in their activities. In Sannino's words, "the psychological possibilities to carry on the activity required for the realization of" one's needs and an "individual disposition to act is prepared" (Sannino, 2008a, p. 241). This step can be seen as an intermediate resolution of critical conflicts, leading to feelings of "personal liberation or emancipation" (Engeström & Sannino, 2011, p. 374). An individual gains a new personal sense, negotiates "a new meaning" for the initial situation, and reconnects with own values that motivate her/him to envision possibilities for practical change (Engeström & Sannino, 2011; Sannino, 2008a).

The notion of experiencing was coined by Vasilyuk to describe patients' psychotherapeutic practice experiences. A successful psychotherapy treatment is a process of experiencing (Sannino, 2008a). Sannino (2008a) exemplifies experiencing by sharing a narrative from Vasilyuk's psychotherapy sessions. In one of the cases, Vasilyuk's patient visited him complaining of being unable to solve her family problems. The married woman found herself caught between her husband, who forbade her from seeing her mother, and her mother, with whom she acted as if there were no restrictions on her part regarding their meetings. Vasilyuk explained the critical conflict that his patient faced as follows: "she had not been able to stand up to her husband regarding the value (not just the importance) to her of her own mother, while betrayal of that value was making her feel that she was disintegrating as a personality" (p. 195-196, quoted in Sannino, 2008a, p. 241). The counseling occurred as an instance of experiencing when the patient gained "a clear-cut, conscious recognition of the value involved, and an understanding of the need to defend it and

embody it in real behavior" (p. 195-196, quoted in Sannino, 2008a, p. 241). Based on this narrative, Vasilyuk describes experiencing as a process of expanding the possibilities for resolving internal conflicts—one that involves the subject perceiving the difficulty of the situation, yet becoming empowered to face the challenge and reunite with her own values:

... experiencing, in the shape of value-development of consciousness, did not in itself solve the patient's life problems, but it did transform a conflict that was causing torment owing to its insolubility into a complication of life, itself far from simple, but potentially resolvable and therefore no longer causing psychotrauma. The experiencing did not make the choice, it made choice subjectively possible. (Vasilyuk, 1988, pp. 195-196, quoted in Sannino, 2008a, p. 241)

In experiencing, "success" does not correspond to the realization of the practical transformation of the critical situation by subjects. Instead, success refers to a subject's gaining a new perspective on her critical conflicts that enables her to envision possibilities for change:

The process of experiencing does not lead the participant directly to realize his or her needs. It leads to restoring the psychological possibilities to carry on the activity required for the realization of these needs. In other words, experiencing may be seen as a process through which individual disposition to act is prepared. (Sannino, 2008a, p. 241)

Experiencing gives rise to subjects' gaining a new perspective not only on their current problems but also on their own needs and motives for future actions to change the situation. For subjects to engage in transformation of their situations, they need to remember their motives and needs and start seeing the conflict situation from a new, possibility perspective. Experiencing is a process at the end of which individuals start envisioning possibilities of practical transformation in their practices.

In this study, this framework will guide the analysis of the data to address the study's first sub-research question: How did teachers exercise agency at the

individual level to develop a disposition to act for their professional learning within the PMTLE? The operationalization of this framework for analytical purposes will be detailed in the Methods chapter.

2.4.2 Learning III: Agency formation at the collective level

To analyze agency formation at the collective level, I utilized the notion of Learning III coined by Bateson (1987) and later interpreted by Engeström (1987/2014) to develop his theory of Expansive Learning. Learning III is the process during which subjects deal with a double bind situation, which is, similar to critical conflicts, another discursive manifestation of contradictions in activities as identified by Engeström and Sannino (2011).

Double bind is defined as a situation in which individuals are continually faced with "pressing and equally unacceptable alternatives in their activity system, with seemingly no way out" (Engeström & Sannino, 2011, p. 374). The concept of double bind was originally defined by Bateson (1987) as situations in which "the individual is caught in a scenario where another person in the relationship conveys two conflicting messages, with one denying the other" (Bateson, 1987, p. 157). In double-bind situations, individuals are unable to comment on the contradictory messages or commands from a source with which they share an intense relationship (Bateson, 1987; Engeström, 2015).

A double bind is a communication dilemma where a person is presented with two or more conflicting messages, where responding to one negates the response to the other. For example, in a romantic relationship, a double bind situation arises when a partner says, "I want you to be honest with me, but don't ever say anything that would hurt my feelings." The person is trapped in a contradictory situation as

being honest sometimes involves saying things that might hurt the other's feelings. This is a double bind situation since each option negates the other and has negative consequences.

In Engeström and Sannino's (2011) framework of discursive manifestations of contradictions, double bind is another type of manifestation of systemic contradictions in discourse. This idea roots back to Engeström's conceptualization of double bind as a collectively experienced dilemma by expanding Bateson's idea into the arena of organisational and collective learning within Activity Theory. Instead of considering double binds as individual psychological traps, Engeström sees them as manifestations of structural contradictions that occur within collective human activities, particularly in workplaces, institutions, and larger social systems. That is, double binds occur outside the individual psychological plane and it needs to be recognized by the group who experiences the consequences of this dilemma that occurs at the social level as workplace problems. In Engeström's (2015) words, when a double bind is recognized by a group, it goes beyond being an individual psychological state and gains the characteristics of "a social, societally essential dilemma that cannot be resolved through separate individual actions alone-but in which joint cooperative actions can push a historically new form of activity into emergence" (p. 131). That is, when a double bind is addressed by collectives whose members experience the conclusions of the same dilemma in their activities, they develop novel activity structures as innovative solutions.

Accordingly, as Engeström and Sannino (2011) maintain, a double bind is usually a situation that cannot be resolved by the individuals alone. They are dilemmas experienced at the collective level, and by expressing a double bind in their discourse, subjects demonstrate their demand for collaboration to overcome it to

which they were all exposed as subjects of the activity system whose contradictions manifest as double bind. Focusing on organizational collective change processes, they identify the following linguistic cues of double bind in discourse: "rhetorical questions indicating a cul-de-sac, a pressing need to do something and, at the same time, a perceived impossibility of action," such as "What can we do?" (Engeström & Sannino, 2011, p. 374). Accordingly, they argue that reflection of a double bind situation in discourse includes a transition from the individual "I" to the collective "we," expressed through phrases such as "we must," "we have to," filled with a sense of urgency.

Learning III is the process during which subjects deal with a double bind situation. Learning III is one form of learning defined in Bateson's (1972) theory on the hierarchy of learning processes. Learning I includes simple forms of learning such as Pavlovian conditioning or rote learning. Learning II corresponds to learning to learn, how one learns and adapts to new situations, shaping the character or habits of a person. One acquires habits regarding how to learn and perspectives about how to interpret the world through Learning II, without "having to examine the abstract, philosophical, aesthetic, and ethical aspects of many sequences of life" each time (Bateson, 1972, p. 303). On the other hand, Learning II gives rise to double bind situations: "The outcomes of Learning II, the unconscious habits, frequently and necessarily lead the individual to double bind situations" (Engeström, 2015, p. 113). Learning III occurs when subjects question these "unexamined premises" of learning and thinking "open to question and change" (Bateson, 1972, p. 303). That is,

On Level III, the individual learns to control, limit, and direct his or her Learning II. One becomes conscious of one's habits and their formation. "Certainly it must lead to a greater flexibility in the premises acquired by the process of Learning II – a freedom from their bondage" (Bateson, 1972, p. 304). (Engeström, 2015, p. 112)

Contrary to Learning III, in which subjects produce their own novel problems or the tasks, in Learning II, the subject is given a problem and attempts to solve it. In Learning III, subjects construct a novel problem "from the materials of problematic situations that are puzzling, troubling, and uncertain" (Schön, 1983, p. 40, cited in Engeström, 2015, p. 119-120). Additionally, if the problem is presented, the participant inquires: "What is the meaning and sense of this problem in the first place? Why should I try to solve it? How did it emerge? Who designed it, for what purpose and for whose benefit?" (Engeström, 2015, p. 120). Learning III is the mechanism through which a double bind is controlled by the subjects, who are no longer the victims of contradictory messages but actors in designing tasks for themselves and resolving the problems that they develop out of these contradictory situations.

In Bateson's theory, Learning III is an explosive conclusion of double bind situations. According to Engeström (2015), it does not have to be in this form. The form Engeström highlights is the gradual creation of new forms of societal activities out of the contradictions. He conceives Learning III as generating something new and novel as a part of human development, which has to be collective from his theoretical perspective. Engeström (2015) calls Learning III "the mastery of double binds," which involves "first of all historical analysis or historical intuition of the inner contradictions of the activity system in which the subject is a part" (p. 131). That is, subjects engage in an analysis/research process to grasp the roots of the double bind that they are faced with, which makes them engage with a historical as well as actual empirical examination of the systemic contradictions giving rise to the double bind situation. Following this, the resolution of double bind, i.e., Learning III, comes and it includes "collective, conscious action in the present" for the "expansive

generation of new activity structures" (Engeström, 2015, p. 131). For this, new instruments are either invented or borrowed from already existing artifacts for new uses while developing novel solutions. Engeström (1987/2015) connects Learning III with his theory of expansive learning, which is a more specific form of Learning III within activity theory, highlighting the occurrence of double binds within activity systems and their resolution as a process of collaborative problem-solving.

In this study, this framework will guide the analysis of the data to address the study's second sub-research question: How did teachers exercise agency at the collective level to model their professional learning processes within PMTLE? The operationalization of this framework for analytical purposes will be detailed in the Methods chapter.

2.5 Summary of the chapter

The purpose of this study is to design a participatory professional learning environment for teachers, where they are supported by mediating tools and activities to exercise agency in collaboratively designing their professional learning processes in conscious and strategic ways. Additionally, the study seeks to explore the emergence and evolution of teachers' agency through the implementation of this designed learning environment.

This chapter began by outlining the relevant literature to which this study contributes. It then derived design principles for a participatory professional learning environment based on insights from the literature.

The first section highlighted that the premises of the new paradigm of PD essentially provide a foundation for supporting teacher agency in directing their own professional learning. However, teacher agency in the context of teacher PD, as both

a construct for designing and assessing teacher learning, has not been sufficiently emphasized. This study aims to address this gap by contributing to the new paradigm of teacher PD, centering teacher agency as the primary construct in designing professional learning environments and examining teacher learning within them.

The second section situates the study within a sociocultural theoretical framework on teacher agency, outlining three categories of agency from this perspective: collectivist, relational, and subject-centered approaches. It emphasizes the complementary nature of these concepts.

Following this theoretical discussion, a review of pedagogical approaches to support teacher agency revealed that formative interventions method is a widely used pedagogical method intentionally designed to support practitioners' agency as a standalone construct. This approach helps practitioners transform and develop their practices and activities within their settings. While empirical studies indicate that other models, such as PLCs or action research, can support teacher agency, these models were not explicitly designed with that purpose in mind. Instead, the literature suggests that the underlying premises of these models create conditions for teachers to enact agency. However, the absence of a design research perspective, along with a theoretical grounding for agency, limits the potential of these models to effectively support teachers in developing agency for leading professional learning and change.

Due to its strong theoretical foundations, FIs offer a significant alternative to these models. On the other hand, a review of empirical studies focused on supporting teacher agency—predominantly through FIs—highlighted several gaps in the literature. First, there is a lack of detailed explanations of design processes, particularly concerning relationality. Second, the studies often fail to systematically analyze how designed or co-designed elements, such as the relational infrastructure,

specifically foster the formation of agency. Third, there is insufficient attention to the simultaneous formation of agency at both individual and collective levels, with most studies focusing on collectives as the primary unit of analysis. Finally, there is a scarcity of fine-grained analyses that illuminate the processes of agency formation, particularly in transitions between expansive learning actions, such as moving from problem talk to modeling solutions.

While following the theoretical premises coming from the FIs literature, this study aims to fill the gaps in the literature by shedding light on teachers' agency formation at the individual and collective levels, and the interaction between them, while transitioning from the questioning to the modeling a solution stage, i.e., collaboratively designing their professional learning processes consciously and strategically. Additionally, this study aims to contribute to the literature on teacher PD environment designs that support teachers' agency in leading their own professional learning. To enrich both the relational and individual dimensions of agency formation, design principles were derived not only from the collectivist-transformativist conceptualization of agency that foregrounds FIs but also from other sociocultural conceptualizations of agency.

The last section focused on two theoretical frameworks for analyzing agency formation in interventions that promote agency at both the individual and collective levels. These frameworks address a gap in the literature concerning the formation of agency in relation to expansive learning and the resolution of contradictions. They will be operationalized in the Methods chapter and used to analyze the data collected in this study.

CHAPTER 3

METHODS

In this chapter, I first describe educational design research as the methodological framework of the study. This will be followed by the research design. Under the research design section, first, I will share the phases of the design process of PMTLE. Then, I will describe PMTLE as a single-case study (Stake, 2005; Yin, 2018). This section will also include information about the research contexts and participants. The research design will be followed by the data collection and data analysis procedures. The chapter will conclude with a description of the researcher positionality.

3.1 Methodological framework: Educational design research

This study aims to (1) design and implement a participatory teacher learning environment that supports teachers' agency in collaboratively designing and leading their own professional learning processes in conscious and strategic ways while becoming a collective subject from being a set of individuals, and (2) assessing how this learning environment supports teachers' agency in designing and leading their professional learning processes. Because of its emphasis on developing theories on both the processes of learning and the means that support a specific kind of learning, educational design research, which will be referred to as design research in the remainder of the text, is a suitable methodological framework for this study (Bakker, 2020; Cobb et al., 2003; Plomp, 2013).

In design research, theoretical questions are addressed in real learning contexts rather than controlled conditions of laboratories. This characteristic is rooted

in a fundamental premise of the learning sciences, which has motivated educational researchers to adopt design research that goes beyond the limitations of experimental research designs: "learning, cognition, knowing, and context are irreducibly co-constituted and cannot be treated as isolated entities and processes" (Barab & Squire, 2004, p. 1). This perspective recognizes that learning environments are complex ecosystems (Brown, 1992; Collins, 1992; Collins et al., 2004; Confrey, 2006). Likewise, in this study, teacher learning is conceptualized as contextual, distributed, and a phenomenon that can only be understood under the complex ecosystem of teacher learning (Horn & Garner, 2022). Accordingly, the issue of teacher learning is addressed in the real context of teacher professional development.

Design research is interventionist in the sense that theoretical questions on learning are addressed through educational designs to provide solutions to the real problems of practice (Barab & Squire, 2004; Cobb et al., 2003). The major problem identified in this study was the inadequacy of participatory teacher learning environment models, which prioritize teacher agency in directing their learning processes. In addition, the non-profit organization I partnered with needed a teacher community focused on mathematics education, which is driven by teachers. The project was also a solution to this practical problem of the organization.

Designers' prospective plans in design research are open to change as they reflect on the actual consequences of the implementation in real settings (Bakker, 2020). In PMTLE, I consistently reflected on each previous session while planning the next, which led to modifications in the initial plan based on the input and changing needs of the participating teachers. Throughout the process, the participants were included in the design process as equal partners, and constant negotiation between me as the interventionist and the participants shaped the content of the

intervention process (the activities, tools, and goals) (Engeström & Sannino, 2010). As Barab (2014) points out, because of not only their previous experience with the setting but also their function as “an essential system component,” practitioners should be positioned as partners in the design research process, not “as subjects to be analyzed” (p. 162).

In design research, researchers test their designs in real context multiple times with a cyclic approach, each time with a revised version based on the evaluation of a past cycle (Collins et al., 2004). Through this progressive refinement approach, not only designs but also theories are developed and improved (Collins, 1992; Barab, 2014). Although this dissertation includes an analysis only from the first iteration of the PMTLE design, many lessons derived from this analysis will illuminate future studies on designing participatory teacher learning environments. The goal of this study is not to generalize the PMTLE as a standardized solution for participatory professional development across contexts, but to propose it as a germ cell idea—one with the generative potential to inspire new implementations in future studies with similar aims, conducted in different contexts, at different times, and with different participants (Sannino et al., 2016).

3.2 Research design

PMTLE was designed as a participatory professional learning environment that aimed to support a group of mathematics teachers who came from different schools in exercising agency for transitioning from questioning their teaching practices to modeling their professional learning processes. Since the learning outcome of each expansive learning action is the development of practitioners' agency, and each such action simultaneously requires the agency of the subjects (Engeström, 2011;

Haapasaari et al., 2016; Kajamaa & Hyrkkö, 2022; Kerosuo, 2017), every activity designed or co-designed throughout the process served as an opportunity to support and examine the formation of teacher agency. The following sections will detail the research design by presenting the phases of the design research study, description of the PMTLE as a case study, research context and participants, and the details of the PMTLE design.

3.2.1 Phases of the Design Research Study

Design research projects typically comprise three phases: preparation and design, implementation, and retrospective analysis and redesign (Bakker, 2020).

The preparation and design phase starts with the identification of the design problem. For this, initially, I identified the research gap by reviewing the relevant literature. As discussed in the Literature Review chapter, there is a scarcity of PD models that center on teacher agency. I have also consulted with the facilitators of the non-profit professional development organization I partnered with, Teachers Network, to identify the practical problem. The facilitators mentioned that they had long been in an effort for the emergence of subject-specific communities. They introduced an ongoing community-building endeavor in Network and suggested that I follow the activities of this community for a possible partnership. I conducted a preliminary fieldwork study by participating in this initiative as an observer, which faded away without a chance to start a partnership between me, the teachers, and the organizers of this endeavor. Based on my observation, I identified what kind of support Network teachers needed to build a community focused on mathematics education.

To support teachers in building a subject-specific professional learning activity model for their community I determined design principles based on the literature. Since the whole process would be designed in a formative manner, I created an initial draft of the processes designed to support questioning action, rather than designing the whole process ahead of time. Supports provided for the analyzing action, and then for the modeling action were designed or co-designed with participants in a formative manner through evaluating the past sessions. For example, supports for their transitioning to the analyzing action were designed based on my analysis of the past session. Teachers' discussions were intensified on a specific systemic contradiction as an underlying factor behind their shared problems.

PMTLE was implemented between April 2022 and June 2023.³ The supports for the questioning action were designed before the implementation. Supports for the subsequent expansive learning actions were designed in a formative manner. This formative design approach was realized through my constant analysis of the previous sessions, which will be detailed in Section 3.2.5. Data collection occurred during the implementation phase.

In the retrospective analysis and redesign phase, I conducted a detailed analysis of PMTLE with two primary goals: (1) to generate theoretical insights into teacher learning within participatory teacher learning environments by focusing on the formation of teachers' agency for professional learning (2) to evaluate how the design supports or inhibits teachers' agency in identifying and analyzing professional learning problems and modeling their professional learning processes accordingly (Figure 3).

³ After June 2023, teachers put the activity model that they designed for their community into action through the examining and implementing the new model expansive learning action.

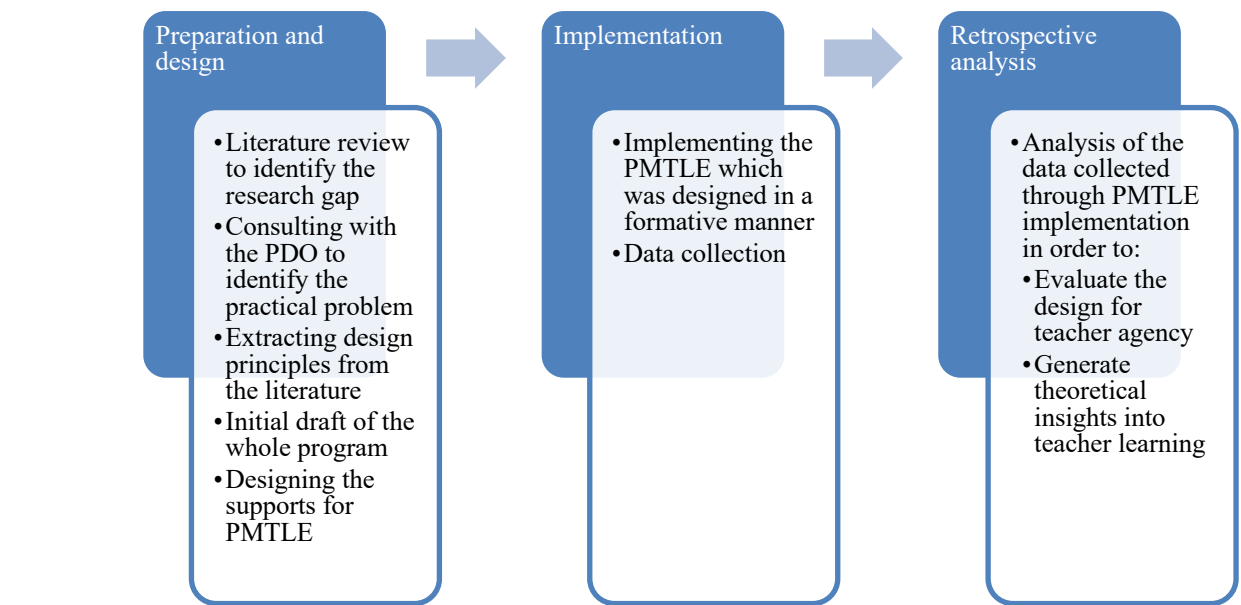


Figure 3. Phases of the PMTLE design research project

3.2.3 PMTLE as a single-case study

This case is bounded as a particular PD program (PMTLE) implemented between April 2022 and June 2023 to support a group of mathematics teachers to design a professional learning activity model for their community, organized under a non-profit professional development organization in Turkey with a researcher-facilitator and eight participant teachers who teach mathematics at different schools but are affiliated with the PDO. The larger phenomenon of interest in this case study is the design of participatory teacher learning environments through which teachers lead their own professional learning and development, as well as the way teachers enact agency during these processes. To provide insights into this broader interest, I have selected PMTLE as a case —how teachers enact agency to question and analyze their problems of practice and design their professional learning processes in conscious and strategic ways toward the goal of designing a novel professional learning activity prototype to address their needs.

3.2.4 Research Context and Participants

3.2.4.1 Partnership with the PDO

For this study, I partnered with a non-profit professional development organization in Turkey called the Teachers Network (TN). TN is "a sharing and collaboration network that supports teacher empowerment through creating a sustainable learning environment in which teachers can collaborate with their colleagues and stakeholders in education from various disciplines" (Teachers Network, 2023). By organizing this study with TN, I became a stakeholder. To achieve their goals, TN organizes different PD programs for teachers (e.g., Thinking Skills, Creative Problem Solving, and Teacher as Action Researcher). Teachers who teach various subjects at different types of schools (e.g., public or private; primary or secondary) in various cities in Turkey can join these activities voluntarily. Currently, there are approximately 10,000 teachers affiliated with TN, and 719 of them were granted a special role called "change agents." Change agents can launch teacher communities around their shared interests and needs and get support from the TN for their activities.

Partnering with TN was appropriate for realizing the goals of this study because of their emphasis on teacher agency, empowerment, and support for the formation and functioning of teacher communities under the network. In May 2021, I contacted the network facilitators and explained my intention to support teachers in building a professional learning community focused on mathematics education. In response, they mentioned an ongoing initiative to create a subject-specific community of mathematics teachers. From May to November 2021, I joined this group's meetings as a participant observer and inquired if a partnership with this emerging community could be possible. Over time, the community lost its momentum because it couldn't find a focus for its activities and dissolved as the

number of participants who attended the meetings decreased. Following this observation, I consulted with network facilitators, who suggested that I initiate a new community by calling on mathematics teachers who are change agents.

The call, titled "Whatever it is, let's build together!" outlined the key steps teachers should take to design their community over the course of three semesters as follows: (1) Understanding our needs (Spring 2022); (2) Exploring solutions and building our own community model (Fall 2022); (3) Implementation and evaluation of the model (Spring 2023); (4) Disseminating the model (embedded in the whole process)⁴ (see Appendix A). PMTLE corresponded to the first and second stages, while the third and fourth stages were implemented by the group autonomously, with my role as a consultant.

Eight in-service mathematics teachers, each with over five years of teaching experience, responded to my research call, resulting in the community launch on April 6, 2022. Five months after the study launched, when the community transitioned to the modeling action, an elementary school teacher and change agent in TN, Ali, joined the community upon the community's invitation, and the number of participant teachers became nine.

3.2.4.2 Participants

All participants of this study are affiliated with TN and change agents. At the start of the study, each participant had 5–30 years of teaching experience, having taught

⁴ Two reasons caused a deviation from the initially planned timescale indicated in the call. First, this study is responsive to the emerging needs of the community, which made us take the process slower than expected. The second reason is the February 6, 2023, earthquakes in Turkey that tremendously affected ten big cities, which terminated all educational activities in the country and traumatized every citizen, leading to the need for more emotional support for each other in the community. Accordingly, the second stage took place during Fall 2022 and Spring 2023.

mathematics in various schools, at various levels, and in various cities across Turkey. Despite differences and local needs, these teachers share many common points. First and foremost, they are committed to their professional development. Furthermore, because of their involvement in TN, they share some common values derived from TN's principles, such as "create change with the teachers, not on behalf of the teachers" (Ayan, 2020). The third commonality is that the education system in Turkey is highly centralized with a nationwide standard curriculum and a high-stakes testing system for high-school and college enrollments, giving rise to similar challenges in their practices.

During the first interviews I conducted before the implementation started, I collected information about teachers' work and professional profiles. Five of the participants teach at public schools, being employed by the National Ministry of Education, while four teach at private schools. Two teachers teach mathematics at the high school level (9th to 12th grade), five teach mathematics at the middle school level (5th to 8th grade), and the only elementary school teacher, Ali, teaches mathematics and other subjects to 1st–4th graders. Açelya, who had an additional role in her school, has taught at middle and high school levels and currently works as a curriculum designer for a special program for gifted students in grades 1–12. The profiles of the teachers are summarized in Table 2, with all names anonymized for confidentiality.

Table 2. Participant Teachers' Work Profiles

Name	Exp.	School	Duties in their schools	Grade	Graduation
Defne	5	Public	Mathematics teacher	5-8	Primary mathematics teaching, Faculty of Education
Lale	6	Private	Mathematics teacher	5-8	Primary mathematics teaching, Faculty of Education
Yonca	7	Minority	Mathematics teacher	9-12	Mathematics department + Teaching certificate
Demet	14	Public	Mathematics teacher	5-8	Primary mathematics teaching, Faculty of Education
Açelya	16	Private	Curriculum designer (Gifted children program)	1-12	Mathematics department + Teaching certificate
Gonca	20	Public	Mathematics teacher	5-8	Primary mathematics teaching, Faculty of Education
Gül	23	Public	Mathematics teacher	9-12	Mathematics department + Teaching certificate
Yaprak	25	Private	Mathematics teacher	5-8	Mathematics department + Teaching certificate
Ali	27	Public	Teaches literacy, mathematics, social sciences, and science	1-4	Educational Higher Institute

I additionally collected information about the participants' professional profiles in terms of their entrance to the teaching profession, their professional development background, their engagement with TN, and their expectations from this study. Some distinguishing highlights from the participants' professional profiles, which

contributed to their identities as teachers, were as follows: Defne's first place of duty was a disadvantaged city in Turkey with a predominantly Kurdish population; Lale led the establishment of a measurement and evaluation unit at the private school where she began her teaching career; Yonca has only worked at an Armenian minority school, from which she also graduated; and Ali and Yaprak worked in various sectors prior to their teaching careers.

Most teachers joined TN by voluntarily attending one or more professional development programs organized by TN or its stakeholders. For example, Defne's first engagement was through a program on gender equity in education called the Purple Certificate Program. Yonca, Aelya, and Yaprak attended the Thinking Skills Program and its follow-up activities. Demet joined the Teacher as a Learning Leader program organized by a stakeholder of TN, RAV (The Teachers Academy Foundation). Appendix B provides further details about each participant's professional profiles.

The research received approval from the Ethics Committee for Master's and PhD Theses in Social Sciences and Humanities (SOBETİK) at the Social Sciences Institute of Boğaziçi University. Approval was granted for the main study (Document Date and No: 13.01.2022 – 47877) and for the extended fieldwork involving new data collection instruments (Document Date and No: 29.09.2022 – 90011) (see Appendix C and Appendix D).

3.2.5 The design of the PMTLE

3.2.5.1 Revisiting the principles for designing a participatory teacher learning environment

I determined the principles for designing a participatory mathematics teacher learning environment in the Literature Review chapter through a review of relevant literature and theories. These principles were as follows:

- DP 1: Support teachers in collectively questioning their teaching practices and identifying their shared problems by providing first stimuli.
- DP 2: Facilitate collective identification of the systemic contradictions underlying their problems and the analysis of their historical and actual-empirical causes.
- DP 3: Support teachers in co-designing novel solutions to the identified contradictions, as well as in co-creating or choosing mediating artifacts, concepts, or models to use as second stimuli in this process.
- DP4: Facilitate participant-led problem identification and collective design through responsive facilitation and open-ended inquiry.
- DP5: Facilitate the visibility of expertise and resource-sharing in the professional learning environment; share your expertise, resources, and perspectives as an equal participant.
- DP 6: Support teachers in bringing their personal and professional histories into the setting, connecting their unique work contexts to the learning environment, and envisioning the learning environment as a resource for achieving their teaching ideals.

3.2.5.2 Embodying the design principles

The identified design principles shaped the initial version of the PMTLE design.

Specific principles were embodied through the design elements, which are tools and materials, task structures, participant structures, and discursive practices. These concepts come from Sandoval's (2014) conjecture mapping framework. In this framework, a design is embodied through:

- "Tools and materials," which include resources, such as software programs, instruments, manipulable materials, media, and other resources.
- "Task structure," which specifies the tasks that learners are expected to engage in and "their goals, criteria, standards, and so on" (Sandoval, 2014, p. 22).
- "Participant structure," which corresponds to how participants, not only learners but also instructors, facilitators, etc., are expected to participate in tasks and their roles and responsibilities in the process.
- "Discursive practices," which are "ways of talking," and "desirable discursive modes," which can be designed to some extent.

According to Sandoval, the last three elements of embodiment—task structure, participant structure, and discursive practices—inform the "social infrastructure" of a learning setting in which tools function. In the PMTLE, tools, materials, and the tasks in which they were used embodied the first three design principles. The last three principles primarily influenced the participant structure and discursive practices, which determined "the social infrastructure" of the design. That is, the last three principles established the social infrastructure through which the first three principles were enacted.

3.2.5.3 Embodying the design principle one: Supporting teachers' questioning expansive learning action through tools, materials, and tasks

The first design principle is: "support teachers in collectively questioning their teaching practices and identifying their shared problems by providing first stimuli."

The tools and materials and the tasks designed based on this principle informed the program of the first five sessions.

In formative interventions, the first stimuli to support questioning is often collected as data through ethnographic fieldwork from practitioners' settings—called mirror material. However, since participant teachers were from different schools, this method was not relevant to this project. The first stimuli included these tools: a book called *The Teaching Gap* (Stigler & Hiebert, 1999), video recordings of sample mathematics lessons from other countries, and teachers' reflection papers.

In the first session, I introduced the discussion question: "What are your thoughts on our shared problems as mathematics teachers? What obstacles prevent you from performing your ideal practice? Under what conditions do you feel at an impasse?" These questions served as a tool, prompting a collective discussion on teachers' problems of practice. After this discussion, I introduced *The Teaching Gap* (Stigler & Hiebert, 1999) and asked them to read the first six chapters of the book until the next meeting, sharing the reasons that I suggested reading this book for problem identification.

In the second session, I introduced an "Our Problems Map," which I created by analyzing the teachers' discussion from the first session using the MURAL interactive online workspace. This tool served as a mediating artifact for problem identification throughout the first stage. I asked the teachers for their feedback on the map and whether they had any suggestions for revisions. After introducing the "Our

Problems Map" and making the revisions based on a collective discussion, I facilitated the discussion on the first six chapters of *The Teaching Gap*. In this book, James Stigler and James Hiebert conduct a comparative analysis of the mathematics teaching cultures in Japan, Germany, and the United States, drawing on typical mathematics lesson videos from the TIMSS Video Study (1995). From chapters three to six, the analysis focused on variables such as the "level and nature of content" and "who does the work in the classroom" (whether it is the teacher or the students, individually or in groups), presenting detailed case studies from each country, while also discussing the variations within each case. In chapters five and six, the authors contrasted Japanese and U.S. mathematics classrooms. Japanese lessons emphasized students' collective work on challenging problems and sharing their results, fostering student agency. In contrast, U.S. lessons relied on rapid question-and-answer cycles between the teacher and students, with students practicing solution methods on similar problems until they became routine. This difference highlighted how U.S. teaching culture often limited students' learning opportunities by reducing the challenges they faced (Stigler & Hiebert, 1999). These findings were supported by raw data, including lesson plans, outlines of lesson structures, and video recordings of the lessons.

Although both *The Teaching Gap* and the video lessons in the archive featured data from countries other than Turkey, I anticipated that they would serve as effective mirror materials in the PMTLE case. To reiterate, mirror material serves as "evidence and examples of recurring problems and disturbances" in practitioners' activities which can be taken-for-granted, providing an opportunity for collective reflection—particularly the pressing ones—by making them visible for further elaboration (Sannino et al., 2016, p. 5). In their classification of video use in teacher

education, Hollingsworth and Clarke (2017) highlight the power of "cross-cultural video cases" as reflective tools, which encourage questioning the taken-for-granted aspects of teaching practice:

When teachers view videotapes of classrooms, the familiarity of the classroom setting can reduce the power of the video clip to catalyse teacher reflection. However, if the video-taped lessons are taken from a very different culture, the teacher's assumptions about accepted and expected practice no longer apply. In this situation, teachers are more inclined to interrogate the videotape and, by implication, their own practice. The unfamiliarity of what they are viewing challenges their assumptions about what is acceptable, competent teaching practice. (p. 460)

Accordingly, I expected the book which included the comparative analysis of mathematics teaching cultures from different countries and the video recordings of the actual mathematics classrooms from these countries, to provide various examples of problems and assumptions from their ongoing practices. In the end, they could function as the first stimuli that trigger "conflicts of motives" experienced by the participant teachers in their practices. Another reason that I selected this book was that by acknowledging the cultural and systemic issues framing teachers' practices, the book refrains from a simplistic interpretation of teachers' "failure" in implementing reforms in their practice, helping teachers to examine the reasons for their problems in a more detailed way.

As a task for preparation for the second session, teachers read the first six chapters of the book and watched the videos of the analyzed lessons, which were available as open resources on the websites⁵. They were asked to reflect on the following question: "What kind of questions did this book make you ask, what issues did you identify, and what critiques did you raise?" During the second session, the task involved a collective discussion centered around these reflections.

⁵ The video archive of mathematics lessons can be accessed via <http://www.timssvideo.com/australia-mathematics-lessons-1>

In the third session, selected clips from the sample lessons were collectively examined to further discuss the analysis shared in the book, based on teachers' needs. The clips were chosen collaboratively; each teacher was asked to fill out a Google document, indicating which sections of the video clips they were interested in discussing and providing their reasons for selecting them. I designed the session around both their selections and my own.

In the fourth session, to support teachers in further articulating their reflections and identifying their shared problems, teachers' reflection papers served as tools. As a task, teachers were asked to write their reflection papers in preparation for the session, using the following prompting question:

- What types of questions, issues, and critiques did you raise about your practice and the mainstream mathematics teaching practices in our country after reading *The Teaching Gap*, watching the lesson videos, and participating in our discussions?

During the session, collective discussions on their reflections in sub-groups of three were held using these questions:

- What kinds of common critiques did you raise, and what issues did you identify and infer based on your sharings?
- If you have any observations regarding the causes of these problems, please note them down.

Teachers were also asked to use the Our Problems Map during this discussion as a tool and make necessary changes in the copies created for each subgroup after their discussions.

- Which parts of the Problems Map do your sharings correspond to?

- Examine the current version of the Problems Map and make the changes you need. You can make additions, removals, change in the problem designations, or make new connections between problem titles.

In the fifth session, the task involved a community discussion on the 7th chapter of *The Teaching Gap*, in which Japanese Lesson Study is introduced as a PD model for transforming teaching practices in various cultures. To prepare for the session, teachers were asked to read the chapter. During the session, we held a collective discussion on teachers' general reflections regarding the Japanese Lesson Study model. I then invited the teachers to explore the connections between the first six chapters and the Japanese Lesson Study model.

After the whole group discussion, teachers continued working on Our Problems Map in their subgroups, making necessary revisions based on the insights from the discussion. Overall, the tools and tasks throughout the five sessions were designed to support teachers in making the disturbances in their practices visible to themselves in the community while they were becoming a "collective subject" from being "a set of individuals" (Virkkunen, 2006, p. 44).

Table 3 summarizes the elements that embodied the design principle 1.

Table 3. Design Principle 1 and the Corresponding Embodiment Elements

Sessions	Tools and materials	Task structure
Session 1 (April 6, 2022)	• Discussion question: "What are your thoughts on our shared problems as mathematics teachers? What obstacles prevent you from performing your ideal practice? Under what conditions do you feel at an impasse?"	• A collective discussion on the pressing common problems of mathematics teachers in Turkey
Session 2 (April 20, 2022)	• Our Problems Map • First six chapters of <i>The Teaching Gap</i> and mathematics lesson samples from other cultures	• Reading the first six chapters of the book considering the reflection question • A collective discussion on teachers' reflections based on their readings
Session 3 (April 27, 2022)	• Video clips from Japanese and US classrooms selected by the participants and the facilitator • First six chapters of <i>The Teaching Gap</i>	• Collective watching of classroom videos from other cultures focusing on the findings provided in the book
Session 4 (May 11, 2022)	• Teachers' individual reflection papers • Discussion questions • Our Problems Map	• Writing reflection papers (Between sessions 3 and 4) • Sharing individual reflections for collective discussions in small groups • Revising Our Problems Map to refine ideas about common problems
Session 5 (May 25, 2022)	• Seventh chapter of <i>The Teaching Gap</i> • Our Problems Map	• A collective discussion on the Japanese Lesson Study model • Revising Our Problems Map to refine ideas about common problems

3.2.5.4 Embodying the design principle two: Supporting teachers' analyzing expansive learning action through tools, materials, and tasks

The second design principle of the PMTLE is "Facilitate collective identification of the systemic contradictions underlying their problems and the analysis of their historical and actual-empirical causes." I prepared the embodiment of this principle based on an analysis of teachers' discussions on their problems during all previous sessions (Table 4). Teachers' discussions on their problems, using different mediating tools (e.g., *The Teaching Gap*, reflection papers, or *Our Problems Map*), were saturated enough to transition to the analysis action. For this, I designed an activity akin to the stimulated recall interview for the sixth session. Dempsey (2010) defines a stimulated recall interview as a technique that "involves interviewing individuals by playing them audio or audiovisual recordings of their own behavior in social situations and discussing different aspects of those recorded interactions" (Dempsey, 2010, p. 349). I reviewed all previous sessions' video recordings, conducted a content log analysis to identify different problem themes, and selected exemplary video clips.

While selecting the video clips, I aimed to highlight the similarities and differences between teachers' ideas on the same problem. This approach could trigger them to investigate further information about the problem, address their knowledge gaps, clarify terms (e.g., curriculum or framework) they used to refer to the same issue and question the reasons for facing this problem in their practices. From this analysis, three major problem categories emerged: curriculum/framework; mathematics learning environments and their designs; and collaboration and professional learning of mathematics teachers. Among these categories, curriculum/framework emerged as a distinct theme, identified as a pressing problem

in the fourth session by all three subgroups. In the sixth and seventh sessions, the discussion focused on this theme.

I also prepared a set of questions to ask after teachers had freely discussed the recorded interactions. These questions included prompts such as, "What type of research would you like to conduct, and whom would you like to talk to and consult with to learn more about what is happening in our country regarding this problem?" The task for teachers during the session was to watch the selected clips and engage in a collective discussion using the provided discussion questions as tools. At the end of these discussions, the community made several decisions regarding professional learning activities through which they could pursue further analysis, such as consulting with curriculum design experts to inquire about the Mathematics Teaching Program of Turkey or examining curricula from other countries to compare with the Teaching Program they are mandated to implement in Turkey. Another task for the community was to fill out a Google Document to consolidate the outcomes of these discussions. By the end of the seventh session, teachers had identified a systemic contradiction underlying the disturbances in their activities. Furthermore, they decided on activities for further analysis of the identified contradiction.

Table 4. Design Principle 2 and the Corresponding Embodiment Elements

	Tools and materials	Task structure
Session 6 (June 1, 2022) & Session 7 (June 15, 2022)	• Categorized video clips from previous sessions on teachers' discussions on their problems for the stimulated recall interview • Discussion questions about future community activities to trigger further analysis of the shared pressing problem	• Collective discussion during the stimulated recall interview activity • Filling out a document to consolidate the discussion outcomes and to come to a shared understanding of the problems

3.2.5.5 Embodying the design principle three: Supporting teachers' modeling expansive learning action through tools, materials, and tasks

The third design principle is: "Support teachers in co-designing novel solutions to the identified contradictions, as well as in co-creating or selecting mediating artifacts, concepts, or models to use as second stimuli in this process." This principle relates to "modeling the new solution" action in the expansive learning cycle. This transition occurred organically in the eighth session. My initial goal for the eighth session was to support teachers in discussing the other problem themes that emerged from my analysis. However, during the session, the teachers subtly shifted from my intended focus, leading me to realize that the discussion on the curriculum/framework was an overarching theme that encompassed the other problem areas. It became clear that the curriculum/framework problem was a systemic contradiction underlying the problems related to mathematics learning environments and teachers' collaboration that teachers had mentioned throughout their discussions. By the eighth session, I observed that this systemic contradiction began to serve as an "energizing resource" for their transition into modeling a solution action. In other words, it encouraged teachers to view their community's activities as a potential solution to the identified contradiction and, ultimately, to address other problems in a more conscious and agentic way.

As a result, in the eighth session, when I felt that the teachers were ready to discuss potential solutions to the contradiction they had identified, I facilitated a discussion on planning their future community activities. I framed my follow-up question, "What type of actions can we as a community take in response to this problem?" based on their discussion. As will be detailed in the Findings chapter, the teachers proposed establishing a shared mathematics teaching perspective for the

community before engaging in activities like Lesson Study. While Lesson Study appeared to address many of their problems, the systemic contradiction they identified related to the curriculum led them to explore an alternative approach to organizing Lesson Study, one better suited to their local needs. This realization prompted them to suggest alternative professional learning activities that they could engage in the subsequent stages. Teachers began conducting professional learning activities as a part of the modeling a solution action, with the identified contradiction serving as an energizing resource.

The first meeting of the opening semester (the ninth session) served as a preparation meeting for modeling activities. During the seventh and eighth sessions, the community made several decisions, some focused on further analyzing the systemic problem they had identified, and others on modeling professional learning activities as local solutions. However, these plans were not extensively discussed, consolidated, or finalized during these sessions.

Before the preparation meeting, I asked the teachers to watch the recordings of the seventh and eighth sessions to prepare for the session. During this meeting, the community selected mediating resources as second stimuli to support further analysis of the identified problem and to model their community activities as solutions. I supported their selection of these resources. Examples of these resources included articles on the history of the Mathematics Teaching Program in Turkey (Ministry of National Education, 2018); NCTM's (2000) *Principles and Standards for School Mathematics*; an expert who had played an active role in designing the reform curriculum in Turkey; and *Elementary and Middle School Mathematics: Teaching Developmentally* by Van de Walle et al. (2020). These resources served as second stimuli to deepen their understanding/analysis of the problem situation, take control

of it, and model potential solutions. I supported the community in selecting these resources.

During the following nine sessions (September 28th - January 19th), the community carried out the activities decided upon during the preparation meeting. These sessions, during which modeling action dominated, were co-designed and co-facilitated by rotating subcommittees of three members, with me participating in all subcommittees. For each session, subcommittees prepared a PowerPoint presentation to facilitate the discussions. During these sessions, the community explored fundamental concepts in mathematics teaching and learning (e.g., mathematical problem-solving, conceptual understanding, teaching through problem-solving) using NCTM's *Principles and Standards for School Mathematics* and Van de Walle et al. (2020) as resources. They documented their definitions and examples in a running document titled "Our Playground" to document their takeaways from their discussions for their community model. In some sessions, teachers also brought materials from their classrooms to examine as part of their discussions.

During twelve sessions conducted between March and June 2023, teachers designed an activity prototype using Lesson Study as a model. This work built on their prior efforts to develop the community's pedagogical and mathematics teaching principles. In this context, Lesson Study served as an existing artifact repurposed for the community's needs. I introduced a university research website on Lesson Study (lessonresearch.net) to the group as a resource for their modeling of the activity prototype of their community.

The community modeled all steps of their activity prototype in a simple, publicly observable, and transmittable format using the Canva online tool. They also

included concise representations of their pedagogical principles, mathematics teaching perspective, and overall community goals.

Table 5 provides a summary of the embodiment elements of the third principle.

Table 5. Design Principle 3 and the Corresponding Embodiment Elements

Tools and materials		Task structure
Session 8 (June 29, 2022)	Discussion questions about future activities related to modeling solutions to the identified contradiction	Collective discussions for planning mediating tools and activities for modeling
Session 9 (September, 7, 2022)	Video recordings of the seventh and eighth sessions	Selecting and employing the mediating resources for further analysis of the contradictions and modeling solutions
Sessions 10-18 (September 2022 to January 2023)	• Van de Walle et al. (2020)-first three chapters • Mathematics Teaching Programs of Turkey • Scientific resources on Mathematics Teaching Program's history (e.g., article, book chapter) • NCTM 's (2000) Principles and Standards of School Mathematics • Materials from teachers' classrooms • Our Playground document	• Collective analysis (historical and empirical) of the contradictions by employing new resources • Mutual negotiation for the selection of these resources • Sub-committees' design of meetings • Collective discussions on fundamental mathematics teaching concepts for designing the community's general, pedagogical, and mathematics teaching principles
Sessions 19-28 (March 2023 to June 2023)	• A university research website sharing resources and guidelines on Lesson Study • Our Playground document • Canva online tool	• Step-by-step construction of the community activity prototype drawing on the Lesson Study model • Documenting the solution into a publicly observable and transmittable medium

3.2.5.6 Embodying the design principles four and five: Designing relationality to support teachers' agency

Two principles guided the relational dynamics among the teachers, me as the facilitator-researcher, and outside experts during the implementation of the tasks and enactment of the tools. The first of this is the design principle four: "Facilitate participant-led problem identification and collective design through responsive facilitation and open-ended inquiry."

In line with this principle, particularly to support questioning and analyzing actions, I carefully observed participants' discussions during and after meetings, using their input as resources to support their progression toward expansive learning. Throughout these meetings, I formatively designed the sessions based on the teachers' contributions, focusing on how they used mediating tools to articulate their problems rather than steering them toward pre-identified problems. While I shared my perspective as an equal participant when necessary—particularly to help refine problem understandings—I minimized these interventions, especially in the initial sessions. Instead, I prioritized asking open-ended questions to encourage teachers to critically examine their current perspectives, surface their needs, and identify further activities for a deeper understanding of their problems. I also consistently evaluated whether deviations from the intended plans presented a learning opportunity for the community and myself.

The second principle on relationality was particularly useful when sharing my expertise during both stages: "Facilitate the visibility of expertise and resource-sharing in the professional learning environment; share your expertise, resources, and perspectives as an equal participant." To embody this principle through participant structure and discursive practices, I first shared my expertise and resources as a

mathematics educator and teacher educator, while intentionally avoiding direct influence over the teachers as they shaped their own professional learning processes. For instance, when selecting resources as second stimuli for their analysis and modeling activities, the teachers chose Van de Walle et al.'s (2020) book since they recognized it as a widely used resource among mathematics educators in the country for accessing research-based knowledge on teaching and learning. However, after reading *The Teaching Gap*, they began to view Van de Walle et al. (2020) from a new perspective, identifying how its vision aligned with the pedagogical approach they had started developing. Although they had the book on their bookshelves before this project started, they hadn't had the opportunity to read it thoroughly. They expressed a desire to read the book, so I supported their selection process by asking open-ended questions such as, "What are the reasons you picked this book? Why do you consider it a good resource for your community activities?"

I facilitated the teachers' discussions on ideas from the resources they read, supporting their integration of practice-based knowledge with research-based knowledge. By helping them make sense of the research, I encouraged them to generate novel ideas. This approach enabled them to develop hybrid solutions, such as making some revisions to the proposed Lesson Study model, though not major ones.

Table 6 provides a summary of the embodiment of the design principle four and five in terms of the participant structure and discursive practices.

Table 6. Embodiment Elements for Design Principles 4 and 5

Participant structure	Discursive practices
• The researcher designs sessions formatively based on participants' input to support their progress in expansive learning. • All participants contribute to the mutual negotiation in selecting resources and co-designing the modeling activities in a formative manner. • Researcher facilitates teachers' discussions in problem identification and solution modeling. • All participants contribute to the process by sharing their resources and expertise as equal partners.	• Open-ended questioning to support teachers: ○ in challenging or building on each others' ideas ○ in selecting and employing mediating resources for their activities • Deviations from intended plans are not devalued but are evaluated to determine if they present learning opportunities. • Dialogic discourse practices that encourage the hybridization of ideas and solutions

3.2.5.7 Embodying the design principle six

Design principle six is: "Support teachers in reflecting their individual histories in the setting, connecting their work context to the learning environment, and envisioning this context as a resource for realizing their teaching ideals." This principle guided the participant structure and discourse practices, contributing to the formation of the social infrastructure of the PMTLE design. Throughout the process, I, as the facilitator-researcher, encouraged teachers to contribute to community development by sharing their diverse professional histories and experiences.

To provide a few examples, each teacher wrote individual reflection papers, and this activity allowed them to share their unique experiences in practice and teaching challenges within the community setting while also situating themselves within the community through their personal and professional histories. Additionally,

during the modeling activities, a subgroup with expertise in a program called Thinking Skills—an influential study they had participated in and referred to throughout the PMTLE implementation—prepared a presentation for their peers, combining NCTM principles with their Thinking Skills knowledge.

Through such activities, which allowed them to reflect on their commitments, ideals, motivations, interests, goals, professional knowledge, competencies, and work-related experiences, teachers were able to bring their individual identities into the community.

Table 7 provides a summary of the embodiment of the design principle six in terms of the participant structure and discursive practices.

Table 7. Embodiment Elements for Design Principle 6

Participant structure	Discursive practices
- Encouraging teachers to contribute to the community development with their diverse professional histories and experiences - Designing session activities (e.g., individual reflections, stimulated recall) to bring diverse perspectives to the light	Teachers' personal histories and diverse needs and goals are acknowledged and not suppressed in discursive interactions.

3.3 Data collection

This study focuses on the data collected between April 2022 and June 2023, encompassing the sessions during which questioning, analyzing, and modeling actions were exercised during the PMTLE implementation.⁶ Three types of data were collected during this period: Interviews, video recordings of the meetings, and teacher artifacts.

⁶ A follow-up focus group interview was conducted in May 2024 after teachers implemented the activity model that they developed.

3.3.1 Interviews

- First interviews (pre-intervention one-to-one interviews): Before the PMTLE implementation, I conducted semi-structured individual interviews to understand teachers' pedagogical ideals for teaching mathematics, collegial relations within their schools, professional histories, previous professional development (PD) experiences, relationships with TN, and expectations from this study. Also, I introduced the details of the project, received their consent, and established rapport during these interviews. The interview questions were added as Appendix E (English Version) and Appendix F (Turkish Version).
- Second interviews (within the intervention one-to-one interviews): After the eighth session was completed, during the summer vacation time of teachers, I conducted semi-structured individual interviews to gain insight into the teachers' experiences with the intervention until then and their expectations for future activities. The interview questions were added as Appendix G (English Version) and Appendix H (Turkish Version).
- Third interview (post-intervention focus group interview): After teachers implemented the designed activity prototype that they modeled throughout the PMTLE implementation, I conducted a focus group interview to reflect on the entire process. The interview questions were added as Appendix I (English Version) and Appendix J (Turkish Version).

3.3.2 Video recordings of the meetings

During the time scale the analyses focus on, the community held 28 online or hybrid meetings, biweekly or weekly, lasting between 72 and 172 minutes.⁷ Transcripts of the video recordings of these sessions constituted the primary data sources for analyzing teachers' agency formation as the community evolved.

3.3.3 Teacher artifacts produced during the intervention

- Our Problems Map: I initially created “Our Problems Map” based on teachers' discussions during the first session. Teachers worked on this map throughout the first stage, making the changes they needed as they discussed their problems.
- Teachers' reflection papers: After reading the first six chapters of *The Teaching Gap* (Stigler & Hiebert, 1999), each teacher wrote one reflection paper based on the open-ended discussion questions that I provided. Later on, they enhanced their reflection papers to prepare them for publication as blog posts on Teachers Network's blog platform.
- Online shareable tools to document ideas: The ideas produced as teachers modeled their community (community pedagogical principles, math teaching perspective, activity prototype) on Google documents. Additionally, we co-created posters and presentations using Canva and Prezi interactive design tools to transmit these ideas into a shareable format.

⁷ The first eight biweekly community meetings were held between April and June 2022, lasting from 129 to 172 minutes. Between September 2022 and January 2023, the community conducted ten meetings, each lasting between 72 and 168 minutes. From March 2023 to June 2023, ten meetings were conducted, each lasting between 86 and 155 minutes.

3.4 Data analysis

The goal of this study is to design a participatory professional learning environment that supports teachers' individual and collective agency in co-designing and leading their own professional learning processes in conscious and strategic ways.

Research questions are:

- i. How did a community of mathematics teachers exercise agency to collaboratively question and analyze their shared problems of practice, and model professional learning activities in response within PMTLE?
 - a. How did teachers exercise agency at the individual level to develop a disposition to act for their professional learning within PMTLE?
 - b. How did teachers exercise agency at the collective level to model their professional learning processes within PMTLE?
- ii. How did the PMTLE design support or inhibit teachers' agency in engaging with these processes?

To answer the research questions, I conducted the analysis in two rounds. The first round involved coding all twenty-eight session transcripts for expansive learning actions. I deductively coded the transcripts based on definitions of expansive learning actions from the literature to examine whether teachers exercised the expansive learning action that was intended for a specific session (Engeström, 1987/2015, 2016). I parsed the sessions according to the dominant expansive learning action exercised during a specific session.

While coding for expansive learning actions, I used both speaker turns and conversation episodes as units of analysis (Engeström et al., 2013). Expansive learning was exercised individually and collectively, respectively, within these units. Coded speaking turns mostly included long explanations of personal ideas by a

single teacher, providing meaningful units to code for any expansive learning action.

For example, the following speaking turn, shared by Gül in response to my prompting question about how they felt after reading the book during the second session, was coded as stand-alone for the questioning expansive learning action:

When I started teaching... because I used to emphasize the importance of making sense of something ... Then, from time to time, we detach and come back to the teaching culture applied to us. I've experienced, lived through exactly that. I went back to the past, I came back. I stopped in between... That's how I experienced the book.

After Gül's answer to my question, I facilitated the discussion by giving the turn to Yonca, who requested to speak by raising her hand during this online meeting. She began her account by saying, "When I was reading the book, I first felt this. I mean, you had given us a set of questions to pay attention to or think about while reading," and she continued:

So at first, I started comparing myself, wondering if the way I teach or the methods I use with the kids are more German, American, or Japanese in style. Then I realized that there's actually a bit of everything in it. After that, my mind kind of drifted. We do so many things, but why can't we still achieve success?

Then, the whole session continued in a similar way.

Differently, conversation episodes were those that began with one person starting a certain type of expansive learning action, others joining in the conversation immediately by sharing their opinion or question on the raised topic, and the conversation growing until it changed to a different topic. These episodes, as a whole, constitute meaningful units to code for expansive learning actions. An example of conversation episode from the sixth session, which was coded for the analysis expansive learning action, is as follows:

Defne: When I say "curriculum," I think of what I fill out in the class book. But what I mean by "guide" is actually something more comprehensive... something that should actually be in the teacher's file. I only learned this last

year because when my principal asked for it, no teacher had it in their file... It explains how we are supposed to teach the learning objectives.

Demet: Isn't that the yearly plan, Defne?

Gonca: I think it is called the Teaching Program. I believe it's available on the Ministry of Education's website. [...] I mean, the Ministry has a booklet—it's detailed, but not excessively detailed.

Demet: I've looked at it too; at the beginning, it includes teaching methods and techniques. Then it starts right away—from the fifth grade onward [...] Is this all there is in what we call the curriculum?

Açelya: In the past, the National Education Program included teacher guides for grades one through eight. These guides contained learning objectives, sample activities, and classroom applications for teachers. [...] However, when the curriculum was changed and took its current form, it became a 48-page document that only includes learning objectives, general aims, and the number of hours allocated to each objective.

YGA⁸: The name for this is Teaching Program, right?

Gonca: Yes, it's the Teaching Program.

YGA: So, [currently] we only have the Teaching Program, nothing else, right?

Açelya: I don't really know.

YGA: Could the guide that Defne mentioned be the answer to this? Could it be? Should we put that question out there? Should we check it out? Should we examine it? What do you think?

[The group]: I think we should examine it.

I used MAXQDA's "Code Coverage" function to analyze the distribution of codes across sessions. This function helps analysts assess the extent to which a document has been coded with a specific code by comparing the number of coded characters to the total characters in the document. This method helped to deal with the mismatch between the length of the coding units. Reporting the frequency of codes by counting how many times a code was assigned in a document—which is the method used in studies reporting expansive learning during formative interventions (e.g., Engeström

⁸ Acronym for Yeliz Günel Aggöl.

et al., 2013)—does not accurately represent the distribution of these codes within and across sessions.

After the initial coding, I conducted a peer debriefing with a colleague who is a doctoral student at the Department of Teaching and Learning at Vanderbilt University and speaks Turkish to increase the credibility of the analysis (Lincoln & Guba, 1985). For this, I randomly selected raw data that were coded for expansive learning actions and discussed their consistency with her.

The first round of coding helped me to immerse myself in the data and parse the sessions according to the expansive learning actions. Then, I conducted a detailed analysis for the agency formation in these blocks of sessions, when teachers were engaged in specific expansive learning action and transitioning from one to another.

3.4.1 Analysis to answer the first research question

After the initial coding for expansive learning actions, to answer the first research question "How did a community of mathematics teachers exercise agency to collaboratively question and analyze their shared problems of practice, and model professional learning activities in response within PMTLE?" I thoroughly read the transcripts of the sessions and simultaneously watched the videotapes for the analysis of the agency formation, concentrating on the data coded in the first round. This is because I was interested in how teachers enacted agency at a more fine-grained level while exercising and transitioning between the expansive learning actions. The speaker turns and conversation episodes coded for expansive learning actions were the starting point for the analysis of agency formation. I also checked the uncoded data not to skip any important processes of agency formation that supported the

emergence of expansive learning actions. I deductively coded the data for the indicators of agency formation derived from the literature.

For this analysis, I drew on two concepts from the Cultural-Historical Activity Theory framework: Experiencing and Learning III. These two concepts help to account for the formation of agency at the individual and collective levels, respectively. Both frameworks have been elaborated in Section 2.4 in the Literature Review chapter.

From the theoretical discussion on these concepts, I identify indicators of agency formation for operationalization in the following sections.

3.4.1.1 Operationalizing experiencing to analyze agency formation at the individual level

I identified the indicators of agency formation at the individual level based on the theoretical discussion on Vasilyuk's (1991) notions of critical conflict and experiencing and Sannino's interpretation for Change Laboratory interventions.

The first indicator (E1) is "sharing internal critical conflicts in the collective space." To identify this in teachers' discursive interactions, I utilized the linguistic indicators of critical conflicts identified by Engeström and Sannino (2011).

The second indicator (E2) is "quest to negotiate a new meaning for the internal conflict." This step is where teachers, individually, in their discursive interactions, express an interest in developing a new explanation of the initial internal conflict situation following or accompanying their rearticulation of their individual critical conflict. Accordingly, when I saw this link—articulation of critical conflict and quest to develop new meaning—I coded it for the E2 indicator.

The last indicator for experiencing (E3), i.e., agency formation at the individual level, is "developing individual disposition to act consciously to resolve the internal conflict." To identify this indicator in teachers' discourse, I focused on discursive expressions reflecting feelings of liberation such as "for the first time in a long time, I can think of ideas you can call crazy" (Defne, second interview) as well as restored psychological capacity needed to engage in the activities required to fulfill one's needs. The second focus corresponds to a subject's expressions regarding an expanded perception of the problem space (restored psychological capacity) on which she or he could take action to resolve her or his critical conflicts (to engage in the activities required to fulfill one's needs).

Table 8 includes these indicators, detailing the criteria used to identify them in the data during analysis.

Table 8. Indicators of Agency Formation at the Individual Level

Experiencing	Identification criteria	Exemplifying quote
E1. Sharing internal critical conflicts in the collective space	Linguistic indicators of critical conflicts: - personally, emotionally, and morally charged accounts - expressions reflecting feelings of being paralyzed/unable to move, violated or guilty - expressions of inner doubts and superficial solutions that deviate from one's values.	Gül: When I started teaching... because I used to emphasize the importance of making sense of something ... Then, from time to time, we detach and come back to the teaching culture applied to us. I've experienced, lived through exactly that. I went back to the past, I came back. I stopped in between... That's how I experienced the book (Session 2)
E2. Quest to negotiate a new meaning for the internal conflict	Teachers engage in a quest to develop a new explanation of the initial internal critical conflict situation following or accompanying a rearticulation of their critical conflict.	Gül: Demet and I have talked about the same thing. I mean, we've actually talked about loneliness. We start something. We've gotten so much training. Hmm. "Let's implement it." Then we leave it. Demet: Actually, I don't see it exactly as loneliness. Gül: Not loneliness, but we're not sustaining it. Demet, what's the reason we're not sustaining it? (Session 4)
E3. Developing individual disposition to act consciously to resolve the internal conflict	Discursive expressions reflecting - restoring the psychological possibilities needed to engage in the activities required to fulfill one's needs, - feelings of liberation.	Defne: The problem we need to take first is the teaching method. How are we going to teach this [mathematics]? How should mathematics be taught? I think we have to move forward beginning here. (Session 8) Defne: For the first time in a long time, I can think of ideas you can call crazy (Second interview)

3.4.1.2 Operationalizing Learning III to analyze agency formation at the collective level

To analyze agency formation at the collective level, I utilized the notion of Learning III coined by Bateson (1987) and later interpreted by Engeström (1987/2014) to develop his theory of Expansive Learning.

Following Engeström's conceptualization of Learning III, I identified the indicators for the step by step formation of Learning III, i.e., the formation agency at the collective level, as presented in Table 9. The first indicator (LIII-a) is the "group-wise identification and articulation of the collectively experienced double bind situation." To identify this indicator in teachers' discursive interactions, I utilized the linguistic indicators of double binds identified by Engeström and Sannino (2011). However, some nuances were added due to the characteristics of the spoken Turkish. One indicator from Engeström and Sannino's taxonomy is transitioning from the individual "I" to the collective "we," such as "we must" or "we have to," loaded with a sense of urgency in discourse. While analyzing the data, I identified that use of "you" as a pronoun in the generalized sense corresponds to "we" in Turkish. For example, when Gül stated, "Therefore, after a while, you can't keep the wheel turning," she used "you" to generalize her own difficulty to all mathematics teachers in Turkey—the professional group she identifies with. Accordingly, she transitioned from the individual to the collective plane, by rephrasing the challenge by using "you" instead of "I" as a pronoun.

Second, this transition needs to be coupled with "we must" or "we have to," loaded with a sense of urgency in discourse, according to Engeström and Sannino's (2011) framework. On the other hand, in the context of this study, the collectively identified double bind situation reflected a systemic contradiction at a broader activity system level—i.e., national education—participant teachers couldn't easily use "we must do" as a phrase. Instead, they continued to use passive voice when expressing their sense of urgency, as seen in Demet's words: "The need of all classrooms is to be covered with a structured new understanding of teaching... There

should be a sustainable system, a structured and sustainable system that needs to be presented to us..."

Table 9 includes these indicators, detailing the criteria used to identify them in the data during analysis.



Table 9. Indicators of Agency Formation at the Collective Level

Learning III	Explanation/indicator	Exemplifying quote
LIII-a. Group-wise identification and articulation of the collectively experienced double bind	Linguistic cues of double bind: • Transitioning from the individual "I" to the collective "we," such as "we must" or "we have to," loaded with a sense of urgency in discourse; rhetorical questions indicating a cul-de-sac, a pressing need to do something and, at the same time, a perceived impossibility of action (Engeström & Sannino, 2011, p. 374). • Linguistic markers specific to Turkish language and the context of the study: Use of generalized "you" as a pronoun instead of "we"; use of passive voice to indicate the urgency of solutions. • expressions reflecting conflicting messages subjects collectively receive	Gül: As you said, since there is no systemic transformation, you [we] are implementing it alone in one classroom. It's not like this in other classrooms. Therefore, after a while, you [we] can't keep the wheel turning. Demet: Yes. Gül: When I say loneliness, I mean this [lack of a unified perspective for cultural transformation in teaching]
LIII-b. Collective construction of novel questions based on the double bind situation and conducting research to address these questions	Collective discussion on the double bind situation and underlying systemic problems followed by the posing novel questions and making decisions and taking action for further analysis of the problem by using resources	YGA: So, is there such a guide in Turkey that matches what you're envisioning? Lale: I don't have any information about whether it exists or not. But last week, Gül mentioned that the Ministry of Education guide also covers methods and techniques. Based on that, I started wondering—could there be something that includes teaching methods as well? Does such a guide actually exist? [...] Demet: I've looked at it too; at the beginning, it includes teaching methods and techniques. Then it starts right away—from the fifth grade onward [...] Is this all there is in what we call the curriculum?
LIII-c. Collective action for the generation of new activity forms as a response to the systemic problem	• Teachers linked the identified systemic problem to the solution ideas • Teachers' planning for modeling activities by proposing resources • Executing the planned modeling activities • Teachers' documenting their takeaways from their collective discussions onto a shareable medium for the step by step construction of the new activity.	Exemplifying quote for the first criteria: Gül: Putting the Lesson Study into practice alone will not bring about any change. There is also a risk of implementing Lesson Study to transfer content [to students], as we have done for years. This only means that we develop another way to maintain our cultural codes... The groups that will implement the Lesson Study first need to review and change their beliefs about the roles of teachers and students in the classroom. Exemplifying quote for the last criteria: Lale: How could we phrase it? Instead of saying "includes components," can we say: "A student who understands the problem, decides what the correct operation is, and reaches the result using numbers flexibly. This is a student who is flexible in operations." It sounds like poetry. Gonca: We could write Defne's sentence too, I think. After that, we can use mine as an example. Lale: Sure.

3.4.2 Analysis to answer the second research question

To address the second research question "How did the PMTLE design support or inhibit teachers' agency formation?", I analyzed the transcripts of the second and third interviews. The second interview was conducted after the eighth session to gather teachers' experiences with the design of PMTLE, before they began working on the modeling expansive learning action. I directed questions related to each design element to each teacher. The third interview, on the other hand, was a focus group interview conducted after the teachers implemented the new activity prototype they had modeled. It aimed to gather their retrospective evaluations of the entire process. For this analysis, I focused on sections where teachers reflected on their experiences with the PMTLE, which was designed to support the questioning, analysis, and modeling expansive learning actions.

While analyzing data from both sources, I examined how the design elements connected to indicators of agency formation at both individual and collective levels. For this, first, I coded the transcripts for design elements by identifying sections where teachers mentioned specific elements. Tools and materials, along with corresponding task structures (e.g., The Teaching Gap or collective discussions on teachers' reflection papers), were coded individually. I then reviewed these coded sections to match teachers' accounts with agency indicators. For example, I initially coded Gül's following account for "reflection paper activity" and then coded it as the reduced data for the agency indicators. It was coded for E1 as well as "E2 → LIII-a" as it showcased both the support of reflection papers to articulate their individual critical conflicts as well as to be a means to express their emotions in the collective space to transition from the individual to the collective plane in teachers' pursuit of understanding their problems:

Gül: [...] what makes a community is our shared struggle with the same problem. Those reflection papers actually describe very well what we go through in our professional processes. It [the reflection paper activity] also expresses our emotions very well... In such environments, while our minds are more active, when we write, our hearts are more active. I saw a lot of heart there. I saw a lot of heartfelt sentences. (Second interview)

In this way, I identified the details of how a specific design element supported or inhibited the emergence of a specific agency indicator.

I repeated this process for the social infrastructure, including task structure, participant structure and discourse practices. While coding for social infrastructure, I followed a broader approach rather than focusing only on the design elements that initially informed the design, which were shared in Sections 3.2.5.4 and 3.2.5.5 of this chapter. I coded for social infrastructure each time teachers referred to their relationships with me and other teachers in the community, as well as my role as a researcher-facilitator. For example, I coded the following account of Lale for social infrastructure and then re-coded it for the E1 indicator, as it reflects that the characteristics of the relationship established between me, as the researcher-facilitator, and the participant teachers mediated the teachers' expression of their critical conflicts in the collective space:

Lale: The atmosphere of the meetings was also very important. I think it should have been a type of meeting where comfortable sharing and a sharing environment were provided. Because it already needed to be a process nourished by our teaching experiences, that is, our individual teaching troubles... Now, that comfort—the feeling that sharing a problem or challenges related to teaching is not a weakness... In a way, the type and nature of these meetings also evolved us to this point. Not holding meetings in a dictating manner, but in a more organic structure; designing them as a sharing environment where you [researcher] wouldn't say things like, "No, not like this," or intervene directly. (Second interview)

Then, I interpreted this finding to draw lessons for design, comparing and contrasting them with the initial design principles and embodiment elements of social infrastructure, which I present in Chapter 5.

To triangulate the data and increase the validity of the analysis, I also utilized the session transcripts, which helped me identify disconfirming evidence (Creswell & Miller, 2000).

Table 10 provides a summary of the data sources and analysis for each research question.

Table 10. Research Questions, Data Sources, Data Analysis

Research question	Data sources	Data analysis
How did a community of mathematics teachers exercise agency to collaboratively question and analyze their shared problems of practice and model new professional learning activities in response within PMTLE? a) How did teachers exercise agency at the individual level to develop an individual disposition to act for their professional learning within PMTLE? b) How did teachers exercise agency at the collective level to model their professional learning processes within PMTLE?	• Transcripts of the video recordings of 28 sessions • For triangulation: ○ Interviews ○ Artifacts	1. Coding for expansive learning actions 2. Parsing the sessions according to the dominant action 3. Coding for the indicators of agency formation
How did the PMTLE design support or inhibit teachers' agency in engaging in these processes?	• Second interviews • Third interview (focus group) • For triangulation: ○ Transcripts of the video recordings of 28 sessions ○ Artifacts	1. Coding for design elements 2. Coding for the agency indicators

3.4.3 Thrustworthiness

To establish the trustworthiness of the study, I employed multiple strategies recommended in the literature (Lincoln & Guba, 1985; Merriam & Tisdell, 2016). In my roles as researcher, facilitator, and co-designer, I engaged in prolonged fieldwork

and made persistent observations. This engagement allowed me to mitigate potential credibility issues arising from limited contextual understanding or distortions from participants or myself (Lincoln & Guba, 1985). It also helped build trust with participants. Persistent observation during the two-year fieldwork enriched the data and added depth to the findings.

Additionally, I used multiple data sources to triangulate the data and enhance the credibility of the findings. Data triangulation was particularly important in identifying disconfirming evidence for this qualitative analysis (Creswell & Miller, 2000). For the analysis of the first research question and its subquestions, I used interviews and teacher artifacts as additional sources for triangulation. For the second research question, I utilized transcripts of 28 video-recorded sessions and teacher artifacts, alongside the interview data.

After the initial coding, I conducted a peer debriefing with a doctoral student from the Department of Teaching and Learning at Vanderbilt University who is fluent in Turkish. This process aimed to enhance the credibility of the analysis (Lincoln & Guba, 1985). I randomly selected raw data coded for expansive learning actions and discussed their consistency with her.

Member checking was also employed to ensure the reliability of the analysis. A focus group interview was conducted at the end of the study. Additionally, a stimulated recall interview—designed to support teachers' agency in transitioning from questioning to analysis as an expansive learning action—also served as a form of member checking.

Rich and thick descriptions of the context, participants, and research process are provided throughout this dissertation. This approach reduces the risk of

superficial interpretation, increases transparency, and supports reflexivity in the research process.

3.5 Researcher's positionality

As a former mathematics teacher, I worked in various private schools in Turkey for eight years before starting the PhD program. I hold a master's degree in sociology, following a bachelor's degree in mathematics education. While studying sociology, I explored how individuals become—or fail to become—agents of social change for the emancipation of their communities and social classes, and the creation of a more democratic and just society. This exploration was situated within the context of engineers as a professional middle-class occupational group, and their relationship with neoliberal policies that have impacted their social rights in Turkey (Günel, 2013). This background has significantly influenced my approach to my experiences as a mathematics teacher.

During my teaching experience, I not only observed but also personally felt the detrimental consequences of teachers' exclusion from decision-making processes in schools. Reducing the teacher's role to that of a mere implementer of knowledge generated by experts (such as research and development teams in the school or teacher educators conducting school visits for PD activities) led to teachers' alienation from their professional practice, hindering their professional learning, and reinforcing the culture of individualism among teachers (Little, 2010).

As a result, my background and identity—shaped by my experience as a teacher, my training as a sociologist, and the questions arising from the interaction between these two significant facets of my personal history—directly influence my interest in the topic of this study.

In this study, I had multiple positions and roles, including those of a researcher, a former mathematics teacher, a facilitator, and a community member. Additionally, I have developed friendships with the participants.



CHAPTER 4

INDIVIDUAL AND COLLECTIVE AGENCY IN INTERACTION FOR PARTICIPATORY PROFESSIONAL LEARNING

This section is organized as follows: First, I will remind the reader of the case under analysis. Then, I will present the initial perspectives of the participant teachers before implementation, focusing on their expectations from the study and the support they needed to become the leading actors in their professional learning as a collective.

Following this introduction, I will present the findings related to the study's first research question and its subquestions, which are as follows:

How did a community of mathematics teachers exercise agency to collaboratively question and analyze their shared problems of practice, and model new professional learning activities in response within PMTLE?

- a. How did teachers exercise agency at the individual level to develop a disposition to act for their professional learning within PMTLE?
- b. How did teachers exercise agency at the collective level to model their professional learning processes within PMTLE?

4.1 Case description: Mathematics teachers seeking tangible outcomes while needing support to define their problems

This case is bounded as a particular PD program (PMTLE) implemented between April 2022 and June 2023 to support a group of mathematics teachers to design a professional learning activity model for their community, organized under a non-profit professional development organization in Turkey with a researcher-facilitator

and eight participant teachers who teach mathematics at different schools but are affiliated with the PDO. The PMTLE program was developed in response to the gap in the literature on designing participatory professional learning environments that support teacher agency for leading their own professional learning, the need within the partner PDO for teacher-driven, subject-specific professional learning communities, and my participation as an observer in an ongoing but unsustainable initiative under the PDO to meet this demand.

There were 28 sessions in the program, excluding additional meetings held to address the emerging needs of teachers. All sessions were conducted online, except for the 6th and 7th sessions, which were hybrid. The first session was conducted online on April 6, 2022. The participating teachers taught mathematics in private or public schools across various cities in Turkey and had a wide range of experience, from 5 to 30 years. All participants were affiliated with a non-profit professional development organization called Teachers Network. The PMTLE program lasted approximately one and a half years, concluding in June 2023, when teachers co-developed an activity prototype for their future professional learning activities within their community. Teachers joined the study in response to a call that highlighted a participatory and open-ended professional development process.

In the first individual interviews, conducted in the two weeks before the intervention began, teachers were asked about the activities they hoped to pursue within the community. Four out of eight teachers (Defne, Açılya, Gül, and Yaprak) demanded collaborative lesson design to be the community's primary activity. Defne described this as "designing, implementing, and disseminating lessons to show other teachers that mathematics teaching and learning can be enjoyable," while Yaprak

envisioned it as "bringing to life the mathematics lesson designs she had always imagined within a community setting."

Three teachers (Gonca, Yonca, and Demet) did not specify particular activities but emphasized the need for clear organization and well-defined boundaries. Additionally, three teachers (Defne, Lale, and Aelya) expressed a desire to contribute not only to their own teaching but also to the broader improvement of education in the country through community activities (Table 11).

Table 11. Participant Teachers' Expectations from the Study

Participant	Expectation from the study
1 Defne	Showing other teachers that mathematics teaching and learning can be enjoyable. Designing, implementing, and disseminating lessons for this goal.
2 Lale	Improving her teaching and contributing to nationwide efforts to enhance the education system.
3 Yonca	Engaging in discussions on how mathematics can be taught to students in more meaningful ways.
4 Demet	Engaging in collaborative activities rather than merely sharing problems.
5 Aelya	Creating pedagogically well-grounded designs and taking a leadership role in sharing her knowledge and experience with fellow teachers through community activities.
6 Gonca	Establishing a community dedicated to mathematics education, where various activities can be conducted systematically, guided by planned steps and clear boundaries.
7 Gl	Experience-based professional development activities by collectively designing, implementing, reflecting, and refining as a process.
8 Yaprak	Realizing the "mathematics lesson designs" that she always envisioned within a community setting.

Similarly, during the introductory session, participants shared various concerns when asked about their challenges in practice but ultimately expressed a strong desire to

collaborate in producing tangible outcomes. However, they struggled to define the specific problem situations they aimed to address through these activities. Lale's account reflected both her need for "producing together" and her difficulty in articulating the specific problems that motivated her desire for this type of collaborative work:

Lale: I had always imagined it. You know, finding an environment where mathematics teachers who are concerned with similar problems come together and say, "We are concerned about this, we need to do something related to this, we need to find something related to this, at least share something, and think about what we can do," I truly feel happy because I found such a place.

The difficulty of identifying a specific problem was accompanied by a tendency to suppress their need to talk about their problems in favor of quickly producing something collaboratively. For instance, Demet experienced confusion over her inability to transfer learning from professional development activities into her practice despite her efforts to draw on professional development opportunities provided to them. However, she emphasized her immediate need to "produce something right now:"

Demet: You know, I follow things like you, like a hunter. The [teacher] trainings on National Education IT Systems⁹ and such, what's available, I join all the groups, but you already mentioned a few too, right? The confusion never ends; I guess it won't ever end, we're constantly bombarded. Failing to adapt it to the classroom, I mean not being able to sustain it in individual [practice], creates even more enthusiasm but can also cause more confusion. I am in that mood now: Let's do something, let's produce something right now. (First session)

As Demet's quote reflects, they tended to suppress the challenging situations that they individually encountered both in practice and in their professional development

⁹A pseudonym for a centralized digital information system developed by Turkey's Ministry of National Education, which delivers administrative and educational services online. It includes tools for staff management, student records, performance tracking, and other features that support the country's educational infrastructure.

efforts, prioritizing quick action to address problems that were not yet fully defined. Consequently, these teachers sought a community where they could collectively define their shared problems and design solutions. Their need for collaboration was not only a reflection of their inability to find a community to take action on solving problems but also of their lack of support in articulating their problems.

All in all, the analysis of the initial individual interviews and the introductory session discussion helped to describe the case at hand at the outset of the process. The participant teachers expressed a desire to use the community space for collective activities to produce concrete outcomes, while simultaneously tending to suppress their need to define their problems in favor of pursuing immediate results. This highlighted their need for support in collaboratively identifying their problems and formulating questions before engaging in activities aimed at developing tangible solutions. The following analysis demonstrates how the PMTLE addressed these needs within this group of mathematics teachers.

4.2 An overview of the findings

The analysis of the sessions following the introductory first session through the lens of expansive learning actions revealed that questioning was the dominant action from the second to the fifth session, analysis prevailed in the sixth, seventh, tenth, and eleventh sessions, and modeling became prominent from the eighth to the twenty-eighth session (Table 10).

In the second round of analysis, which focused on agency formation and built upon the first round findings, indicators for experiencing and Learning III were considered as markers of agency formation at the individual and collective levels,

respectively (Table 8 and Table 9). Different indicators were identified within each learning action:

- During questioning, E1 (Sharing internal conflicts in the collective space), E2 (Quest to negotiate a new meaning for the internal conflict), and LIII-a (Group-wise identification and articulation of the collectively experienced double bind) were observed.
- During analysis, LIII-b (Collective construction of novel questions based on the double bind situation and conducting research to address these questions) was identified.
- During modeling, E3 (Developing an individual disposition to act consciously to resolve the internal conflict) and LIII-c (Collective action for the generation of new activity forms as a response to the systemic problem) were present (Table 12).

Table 12. An Overview of the Findings

Session parsing based on expansive learning actions	Dominant expansive learning actions	Agency indicators
2-5	Questioning	E1, E2, LIII-a
6, 7, 10, 11, 16	Analysis	LIII-b
8, 9, 12-28	Modeling	LIII-c, E3

The diagram shared in Figure 4 summarizes the process of teachers developing agency for becoming conscious and strategic actors of their professional learning within the PMTLE, answering the first research question and its subquestions.

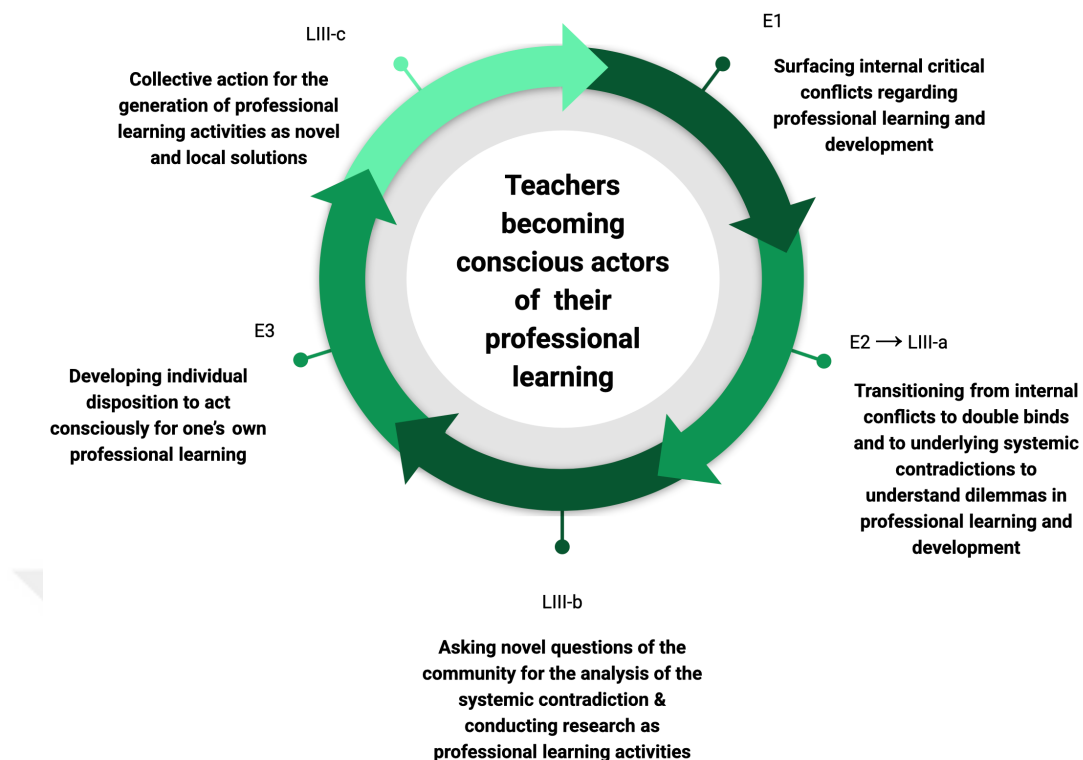


Figure 4. Agency formation in teachers becoming conscious actors of their professional learning

First, teachers surfaced their internal critical conflicts regarding their efforts of professional learning and development, which was being torn between two teacher identities, "teacher as I was taught to be/expected/compelled to be" on one hand, and "teacher as I aspire to be" on the other, and shared by most of the teachers in the context of this study (E1). Second, this shared critical conflict served as a background for their transition from the individual to the collective plane during their quest to negotiate a new meaning of it (E2), by reframing their internal conflicts as a double bind—societal dilemma—situation, which pointed to a systemic contradiction (LIII-a). In the context of this study, the double bind situation, i.e., the conflicting messages that they continuously received, that teachers' accounts pointed out was "transform your teaching" and "keep up with the system." An underlying systemic

contradiction that fueled these dilemmas at both the internal and societal levels was identified as the lack of consistent, well-grounded, well-articulated, and unifying framework that could support teachers in transforming the teaching culture in their settings.

Engaging in discussions on this contradiction facilitated teachers developing first novel questions of the community and planning and executing research activities to analyze and better understand the systemic contradiction (LIII-b). Based on this improved perspective regarding their internal and societal dilemmas and the underlying systemic contradiction, teachers developed individual disposition to act consciously for their own professional learning (E3) and collective action for the generation of professional learning activities as novel and local solutions (LIII-c). Teachers developed individual disposition by expanding the problem space on which they could perform their professional learning actions, helping them to envision the community as a space to conduct these activities to deal with their critical conflicts (E3). In the context of this study, teachers achieved this by expanding the problem space of their professional learning activities, beginning to view the teaching culture itself—including the roles of teachers, students, and the content in the classroom—as a meaningful entry point for their professional development. Simultaneously, they developed collective action for the generation of professional learning activities as novel and local solutions to the identified systemic contradiction. In the context of this study, this occurred as teachers first linked systemic contradictions with potential solutions, then planned and executed modeling professional learning activities to develop the community's unifying framework for teaching mathematics, subsequently creating a local model of Lesson Study from the ground up as the community's activity prototype.

4.3 Sessions 2- 5: Agency formation within the questioning action

From the second to the fifth session, teachers dominantly exercised "questioning" expansive learning action. Coverage of questioning code in the transcripts were 22.41%, 4.80%, 28.80%, and 3,91% from the second to fifth sessions, respectively. In none of the sessions, another expansive learning action was found except 0.61% analysis action in the fourth session.

In these sessions, teachers critically examined various aspects of their practice. During the second session, they specifically focused on the teaching culture in their classrooms and the country (e.g., "Teachers want to control every step of students' learning;" "We ignore the nature of mathematics and promote popular approaches"). In the fourth session, their discussions centered on the curriculum and related reforms (e.g., "The curriculum is not holistic and does not emphasize connections among subjects;" "Fundamental mathematical thinking processes, such as proof, are disregarded in the curriculum"). Other themes of questioning included teacher education, professional development systems, and centralized exams.

In the second round of analysis, in the second and fourth sessions, I identified:

- The first two indicators of agency formation at the individual level, "sharing internal conflicts in the collective space" (E1), "quest to negotiate a new meaning for the conflict" (E2) and,
- The first indicator of the agency formation at the collective level, "group identification of the double bind experienced collectively" (LIII-a).

In the second session teachers surfaced their internal critical conflicts within the collective space. In the fourth session, they noticed that their internal critical

conflicts are shared and this triggered them to seek a new personal understanding of their conflicts and for this, they interacted with their peers in the community. This engagement in meaning-making within the community triggered their raising attention to the double bind situation that they collectively experience as mathematics teachers in Turkey, signifying a shift from the individual to the collective plane in agency formation.

4.3.1 Sharing internal critical conflicts in the collective space (E1): Articulating being caught in between two teacher identities and recognizing own pedagogical values (Session 2)

As preparation for the second meeting, teachers read the first six chapters of *The Teaching Gap* (Stigler & Hiebert, 1999), keeping the following questions in mind: "What kind of questions did this book make you ask? What issues did you identify, and what critiques did you raise regarding your teaching practices?" In the first six chapters of *The Teaching Gap*, the authors compare mathematics teaching cultures in Japan, Germany, and the U.S. using TIMSS Video Study data, analyzing variables like "level and nature of content" and "who does the work in the classroom" (whether it is the teacher or the students, individually or in groups). They highlight how Japanese classrooms foster student agency through problem-solving, while U.S. classrooms focus on routine practice, ultimately limiting learning opportunities.

During the session, I prompted the discussion by asking how the book made them feel. Teachers shared a common internal conflict they encountered while striving to achieve their pedagogical goals in practice. The teachers expressed feelings caught between two teacher identities: "teacher as I was taught to be/expected/compelled to be" on one hand, and "teacher as I aspire to be" on the

other. This often led them to a mindset of impossibility and emotional frustration regarding their professional development. For example, Gül began her account by expressing her emotions and reflecting on her past experiences as both a student and a teacher:

Gül: Now, I've been experiencing the same problem since the days I was a student anyway... When I started teaching, at different times, because I used to learn this way, I used to emphasize the importance of making sense of something ... Then, from time to time, we detach and come back to the teaching culture applied to us. I've experienced, lived through exactly that. I went back to the past, I came back. I stopped in between... That's how I experienced the book.

Her critical conflict emerged in her discourse through "personally, emotionally, and morally charged" expressions (Engeström & Sannino, 2011, p. 374), such as "I went back to the past, I came back. I stopped in between..." Throughout her teaching career, Gül made significant efforts to become the teacher she aspired to be—one who prioritizes all students' sensemaking in mathematics. However, as she shared in the following quote, due to various factors, such as the pressure to meet her students' needs regarding centralized exams and the dominant teaching culture that shapes her students' expectations, she found herself in a state of "paralysis" between these two teacher identities:

Gül: Here's how it goes with me. Let's say, I start like the Japanese [...] Then when the kids say, "But isn't there a shortcut for this?" I continue like the Americans. I can't get out of it, because they say, "My tutor at the exam prep school said this has a quick fix." When it's said, "We can handle this question much more practically," and this is shared across the class, actually, we can't get out of just continuing from a version of solving puzzles. I realize that I continue like the Americans.

The data showed that six out of eight teachers expressed the same individual internal conflict of being torn between two contrasting teacher identities: Gül, Yaprak, Lale, Defne, and Yonca during the second session, and Demet during the fourth session (Table 5).

Teachers' surfacing of their individual internal conflicts in the collective space also emerged through their engagement with a process of reconnecting with their own values. They experienced feelings of guilt upon realizing that they had deviated from these values, which were strongly represented by the Japanese teacher prototype described in the book. For example, Yaprak expressed her guilt by labeling instances where she deviated from her values as "mistakes:"

Yaprak: While reading, I also sometimes felt like a teacher in the United States, just like Yonca. Sometimes like in Japan, sometimes in Germany. So, all of them lead you to look back at your own teaching. Of course, it's not just my teaching that gave rise to this... What the system expects from us provided it. Ultimately, it's what the parents want. The fact that our assignments are parent-oriented, meaning they want more at times, necessitated us to do these things, these mistakes, well, for me at least. When I look back at my own teaching, I also did all these things from time to time... sometimes we really can forget... I read the book experiencing all these emotions too.

Yaprak expressed feeling the tension between different teacher identities, finding temporary ways to manage these internal conflicts by compromising her values "from time to time." While dedicated to maintaining her values, the expectations of parents, the school, and the system pulled her into uncertainty about the pedagogical values she upheld and her teaching identity. Postponing genuine solutions to this crisis and distancing herself from her values—such as by increasing the homework load—only intensified her internal tension. Similarly, Defne and Lale shared how they recognized their pedagogical values aligned with the Japanese teacher prototype, and both expressed feelings of guilt about moments when they deviated from these values (Table 13).

Table 13. Teachers' Expressions of Their Internal Critical Conflict in the Collective Space in Sessions 2-5

Participant teacher	Exemplary quote for critical conflict expressions	Linguistic cues
Gül	I used to emphasize the importance of making sense of something ... from time to time, we detach and come back to the teaching culture applied to us. I've experienced, lived through exactly that.	Emotionally charged account: I've experienced, lived through exactly that. I went back to the past, I came back. I stopped in between
Yaprak	I also sometimes felt like a teacher in the United States, sometimes like in Japan, sometimes in Germany. So, all of them lead you to look back at your own teaching. I also did all these things from time to time... sometimes we really can forget... I read the book experiencing all these emotions too.	Emotionally charged account; feelings of being violated or guilty: I also did all these things from time to time... sometimes we really can forget... I read the book experiencing all these emotions too.
Lale	I thought a lot about my own teaching, "Oh my God, which one is mine?" Mine is soup. There is also a Japanese part. I was happy to see that. I was really happy to say I am doing this. There is also an American part... It made me feel good to see that I am doing a little less of the practices that America does. I ... realized that I mostly do not like the parts like America, where I try to teach everything myself.	Emotionally charged account; feelings of being guilty: I mostly do not like the parts when I teach like Americans, where I try to teach everything myself.
Defne	I got muddled... When it comes to what I implement, actually, yes, what I want to implement is very parallel to Japan, but we can only implement a part of it... the times when I say, "Yes, I'm doing this in class, but it feels like I don't really want to," are always parallel to the USA.	Emotionally charged account; feelings of being guilty: I got muddled... The times when I say, "Yes, I'm doing this in class, but it feels like I don't really want to," are always parallel to the USA.
Yonca	So at first, I started comparing myself, wondering if the way I teach or the methods I use with the kids are more German, American, or Japanese in style. Then I realized that there's actually a bit of everything in it. After that, my mind kind of drifted. We do so many things, but why can't we still achieve success? That's when I started thinking about it a little.	Note: I have identified no visible linguistic markers of critical conflict. Still, Yonca's expressions satisfy the operational definition of critical conflict. Yonca feels like being in a situation in which she faces inner doubts regarding which teaching style she is following. She made a critical reflection and expressed how she couldn't still pursue her values and achieve her goals as an individual teacher.
Demet ¹⁰	I mostly admit this to my students, I am not happy with what I do. I don't see it as teaching. We give information as if we have to do this and want them to remember it immediately in the exam... After watching those videos... It was actually really bothering me before. I started to see it more clearly. ... How can I create a lesson plan in a systematic way, connecting it to the topic as well as what comes before and after (for students)? ... And if I had done that, if I had been able to, I think we could have engaged the students more actively—similar to how it's done in Japan—by getting to know them better. (Fourth session)	Emotionally charged account; feelings of being guilty: I mostly admit this to my students, I am not happy with what I do.

¹⁰ Since the community watched and discussed the lesson videos in the third session, Demet also referenced the videos while expressing her critical conflict in the fourth session.

In this session, the process of experiencing—i.e., agency formation at the individual level—began with teachers identifying their personal critical conflicts regarding their professional development, which were "unsolvable" by the teachers alone within a supportive environment (Engeström & Sannino, 2011, p. 374). It was followed by their engagement in a process of reuniting with their values as mathematics teachers. This marked the first step in agency formation at the individual level—i.e., their experiencing process.

This session set the stage for teachers to continue working on their internal conflicts. Additionally, they needed support to transition from the individual to the collective plane, which prompted me to organize the subsequent sessions.

4.3.2 Quest to negotiate a new meaning for the internal conflict (E2) and group identification of the double bind experienced collectively (LIII-a) (Session 4)

In the fourth session, to support teachers in further articulating their reflections and identifying their shared problems, teachers' reflection papers served as tools. As a task, teachers were asked to write their reflection papers in preparation for the session, using the following prompting question: "What types of questions, issues, and critiques did you raise about your practice and mainstream mathematics teaching practices in our country after reading *The Teaching Gap*, watching the lesson videos, and participating in our discussions?" During the session, collective discussions on their reflections in sub-groups of three were held. To guide their collective discussions, I provided some prompting questions, such as: "What common critiques did you raise, and what issues did you identify and infer based on your shared reflections?" However, not all subgroups required these prompts to engage in

discussions about the commonalities between their internal conflicts and their underlying causes.

Teachers in each group began their discussions by taking turns presenting their reflection papers to one another. This process helped them rearticulate their internal conflicts, most of which had initially emerged in the second session. As they rearticulated these conflicts, they engaged in an effort to negotiate a new meaning for their internal conflicts with the support of their community. More specifically, they sought an explanation for their failure of sustaining their ideal teaching in their classrooms. This marked the continuation of the experiencing process with the second indicator (E2) of agency formation at the individual level.

Furthermore, this process triggered a transition to agency formation at the collective level (LIII-a), as teachers' discussions led to the group identification of the double bind experienced collectively—the contradictory messages that mathematics teachers receive in Turkey from external sources which pull them into internal critical conflicts. In this context, once teachers recognized that their internal conflicts, arising from being caught between different teacher identities, were shared, they began to question their origins and identified a prominent double bind they needed to challenge in their practice: "transform your teaching" while simultaneously "keep up with the system."

I will illustrate the findings presenting how this transition happened in each subgroup.

4.3.2.1 First subgroup: Gül, Demet, and Açelya

In the first subgroup, Gül began by sharing her reflection, rearticulating her internal critical conflict and her struggle to maintain her ideal teacher identity in alignment with her values despite her efforts.

Gül: Well, I really felt that way. I mean, about my own practices. Since 2016, I've been chasing something... sometimes, as the programs change, even though the program booklets say there is no proof in ninth and tenth [grades], I still did small things for them to discover, you know... A few questions to arouse curiosity... [But] sometimes, I see myself in a rush, like an American teacher.

To uphold her values in practice, Gül independently participated in professional development programs and even deviated from the mandatory teaching program when necessary. Despite her efforts, she found herself compromising on her ideal of allowing students to actively explore mathematical ideas. Similarly, Demet articulated her own critical conflict of struggling to sustain her ideal teacher identity, drawing connections between her situation and Gül's:

Demet: I also started from myself here, actually, just like Gül. You know, with this kind of hunger for a long time... Just as you mentioned, I've always been rushing from one training [program] to another, because of not being able to sustain it in the classroom...

After articulating their internal critical conflicts, they embarked on a quest to develop a new meaning for them. Gül and Demet's realization that their internal conflict was shared encouraged them to actively engage in this questioning:

Gül: Demet and I have talked about the same thing. I mean, we've actually talked about loneliness. We start something. We've gotten so much training. Hmm. "Let's implement it." Then we leave it.

Demet: Actually, I don't see it exactly as loneliness.

Gül: Not loneliness, but we're not sustaining it. Demet, what's the reason we're not sustaining it?

In this conversation, they began supporting each other in their experiencing processes. Both sought to develop an explanation for their failure to sustain their

ideal teaching in the classroom despite their efforts. In response to Gül's question, Demet developed an explanation, drawing on her reflection paper:

Demet: For example, I wrote the last sentence here like this... The need of all classrooms is to be covered with a structured new understanding of teaching. I mean, there should be a sustainable system, a structured and sustainable system that needs to be presented to us... There needs to be such a plan. At least in the form of a framework.

Demet's response prompted Gül to elaborate on the dilemmatic situation that they all had to face in their practices, at the societal level rather than at the individual internal level this time:

Gül: As you said, since there is no systemic transformation, you are implementing it alone in one classroom. It's not like this in other classrooms. Therefore, after a while, you can't keep the wheel turning.

Demet: Yes.

Gül: When I say loneliness, I mean this.

Gül's metaphor of 'turning the wheel' highlighted a dilemmatic situation: on one hand, teachers were expected to transform their practices to make mathematics learning more enjoyable and meaningful—motivated by both professional development programs and their own pedagogical values. On the other hand, they were expected to conform to a system that failed to coherently support their efforts, due to the lack of a sustainable and consistent shared pedagogical perspective and goals. This situation is a double bind for these teachers because they are caught between two conflicting demands, denying each other. If they go after the first option, they risk facing pushback from the system (e.g., they do not get institutional or collegial support for these practices or their performance is evaluated through standardized outcomes). However, if they choose the second option, they end up reproducing the old, ineffective methods of teaching mathematics, which reinforces their students' sense of alienation and demand for more engaging mathematics classrooms, while also making them feel that their professional values are not being

honored. The dialogue between Demet and Gül represented a transition from the individual experiencing process to the collective Learning III process, showing the first indicator of collective agency formation—group identification of the double bind experienced collectively.

The double bind emerged in Gül's discourse, expressed metaphorically as a "cul-de-sac" situation (Engeström & Sannino, 2011): "You can't keep the wheel turning." Demet's discourse reflected a sense of urgency: "There should be a sustainable system, a structured and sustainable system that needs to be presented to us."

While both teachers agreed on the need for systemic transformation to collectively uphold and pursue their pedagogical goals and values in mathematics teaching practices, Demet emphasized the necessary changes at the national level, while Gül focused on the local level (school or department). Overall, the data showed that agency formation at the collective level began in this subgroup with teachers' identification and articulation of the contradictory messages, i.e., "transform your teaching" and "keep up with the system", they collectively receive as mathematics teachers in Turkey.

4.3.2.2 Second subgroup: Yaprak, Defne, and Gonca

In the second subgroup, Yaprak began sharing her reflection by rearticulating her critical conflict. Like Gül and Demet, she mentioned the considerable effort she put into maintaining her values and ensuring enjoyable mathematics learning for her students. One approach she explored was gamification as a teaching method. However, she still found herself trapped in the same vicious cycle of reverting to traditional teaching, even when she applied these alternative practices. She expressed

frustration over her inability to implement what she had learned from PD activities in a way that aligned with her values in her school, while reiterating her commitment to upholding those values:

Yaprak: Now I look, what has been bothering me for a long time? Always do something, let's do this activity. I joined gamification groups. So I tried to do it by asking how we can apply gamification in mathematics. After that, let's do this another activity. Now I look. For example, our school publishes something weekly on Instagram, an activity [we conduct in the school]. This time we started doing it just to do it. So come up with an activity, take a picture of it and put it there... It started to become very meaningless. I mean, it's also tiring... "You have to go by standing behind the things you are sure of and believe in," I tell myself.

Before her articulation of her internal critical conflict, Yaprak engaged in a quest to negotiate a new meaning of her initial situation by sharing the historical process that drove her to this critical conflict situation. Unlike Gül and Demet, who emphasized the need for systemic transformation through reforms, Yaprak viewed the reforms themselves as the starting point of the problems she faced. When a reform curriculum was introduced in Turkey in 2005, resources like textbooks were updated, and teachers were provided with guidebooks to help them adapt to the new student-centered teaching philosophy. Teachers abandoned older materials as they adjusted to the new program. However, at the policy level, these reforms were not implemented persistently. One example Yaprak shared was the eventual withdrawal of resources such as the "Teaching Guidelines". Yaprak describes her experience with this process as follows:

Yaprak: When we first started teaching, we used to make daily plans ... We were three teachers [in our department] then. So, even though we didn't think about it like a Japanese [teacher], we had a regular program and it was transforming and improving day by day. At least we were acting with a plan and a schedule. Then the curriculum changed. You know, a Teacher's Guidebook came out... During that process, we started to adapt to it. We set aside daily plans and such because we had a guidebook in front of us. After that [was withdrawn], we probably continued on our way with the accumulation of years [of experience]

Before the reform curriculum was introduced, Yaprak was able to pursue a shared pedagogical perspective with her colleagues in her department, and they could sustain this at the school level by shaping their practice coherently. The lack of persistence in implementing newly introduced reforms, which she tried to adapt, resulted in arbitrariness. As her initial quote reveals, due to the absence of a shared perspective that genuinely communicated the meaning of student-centered classrooms, activities could easily deviate from their intended aim, leading Yaprak to feel as though she was shifting away from her values. The arbitrariness in terms of unifying pedagogical principles in her school made it more difficult for Yaprak to stand firm in her values, often bringing her into critical conflict situations. Ultimately, Yaprak experienced a sense of being "lost" and "alone" in her efforts to implement cultural transformation.

Similarly, Defne was critical of her use of games in instruction, particularly designing lessons focused solely on students' enjoyment without considering how this approach failed to meaningfully change the classroom culture. She explained that this led to a lack of opportunities for meaningful mathematics learning, using an example from a video recording of one of her lessons in which she implemented a game using balloons to teach digits in decimal numbers: "I didn't design a game by adapting the teaching part of the work... In other words, playing games, as you mentioned earlier, my teacher Yaprak, just for the sake of doing it. That's not it, I realized it's not like that."

Defne recognized that even in situations where she thought she was teaching in line with her values, she wasn't. In other words, she couldn't escape the teaching culture she was trying to move away from in order to offer her students meaningful mathematics learning opportunities. However, after reading *The Teaching Gap* and

designing and individually implementing a lesson inspired by one of the Japanese lessons presented in the book, she shared how she regained her confidence in overcoming her critical conflict.

While responding to one of the prompting questions I prepared for the session ("What kinds of common critiques did you raise, and what issues did you identify and infer based on your sharings?"), their quest for a new meaning of their shared internal conflict—the second step in agency formation at the individual level (E2)—continued in the dialogic space:

Yaprak: I always likened it to something, that's how I started the title [of my reflection paper]. I said that being a teacher, like being a mother, is shaped by hand. I think it is like that. Because even though we graduated from a university—since I didn't study teaching, I graduated from the Mathematics department—I couldn't use anything I learned there in my teaching. In other words, I developed on my own.

Defne: I am a graduate of a faculty of education, but it doesn't work like that. As you said, when you get involved, like motherhood, you experience it a little more when you have a child.

Yaprak: I always think that we are alone, and that you can do as much research and develop yourself as you want. That's what I see, what I observe, what I experience in myself. You can improve yourself as much as you want. If you don't want to, you go in the morning and come back in the evening without changing your standards.

Yaprak's point led them to a discussion on the roots of their shared critical conflict when they addressed the following prompt: "If you have any observations regarding the causes of these [common] problems, please note them down:"

Yaprak: So, actually, not having a guide is our lack of common ground, in my opinion. As I said, everyone is going their own way, based on their knowledge and experience. We are in an education system that is not standardized.

Defne: I mean, like, you graduate from university, whatever you do.

Yaprak: Yes, so you're thrown in alone and you [don't] have a framework in your hand. Okay, there are objectives, but what are the limitations? What studies have been done about it? There is nothing.

When they transitioned from expressing their internal conflicts to questioning the reasons behind, teachers in the second subgroup pointed out the same pressing dilemma at the social level—a double bind—that was mentioned in the first subgroup. This transition reflected in their discourse as a shift from individual "I" to collective, generalized "you," meaning "we" in Turkish. Even though they had achieved some level of change in their classroom practices, their "loneliness"—stemming from the lack of unifying frameworks and discourse—was giving rise to the conflictual situations they continuously faced at the individual level. As explained in Yaprak's accounts, through reforms, on the one hand, a nation-wide goal of transforming the teaching culture was set, and expectations toward this goal increased; on the other hand, teachers were left without meaningful resources, as well as unifying principles and discourse articulated through robust frameworks and lack of persistence behind these reforms, with frameworks becoming more superficial from one version to another throughout the years.

This created a double bind situation. Teachers were continuously receiving "conflicting messages, with one denying the other" from those with whom they had intense relationships, such as colleagues, school administrators, employers, parents, and students (Bateson, 1987, p. 157). The double bind situation expressed by the first subgroup, as noted on page 140, can be repeated for this group: "On one hand, they were expected to transform their practices to make mathematics learning more enjoyable and meaningful for students, driven also by their own expectations as teachers upholding such pedagogical values. On the other hand, they were expected to adhere to a system that failed to coherently support their efforts to sustain cultural transformation in their practice due to the absence of a sustainable and consistent pedagogical perspective, which was communicated through frameworks."

Identification and articulation of this double bind marked the initial step toward agency formation at the collective level in the second subgroup (LIII-a).

4.3.2.3 Third subgroup: Lale and Yonca

A similar discussion took place in the third group. Yonca and Lale reiterated their critical conflict between maintaining their values and following mainstream teaching practices. Yonca shared how she has difficulty maintaining her pedagogical values, which prioritize teaching "the logic" behind mathematical topics rather than teaching "tactics" to solve routine problems. She began her account by sharing her reflection paper, which included an analysis she had conducted a couple of years ago on one of the newly introduced curricula. What she noticed in this change was the simplification of the curriculum content, which came at the expense of removing "key topics" that help teachers connect subjects (e.g., polygons and trigonometric calculations), thus supporting students' mathematical reasoning.

Yonca gave an example from one of the challenges she faced while teaching the applications of trigonometry on polygons to students preparing for the university entrance exams. The topic of diagonals and area calculations of polygons had been removed from the curriculum, and students learned "tactics" to solve such problems from other sources (e.g., university preparation courses). Due to the pressure of preparing students for these exams, Yonca found it difficult to create alternative ways of teaching the "logic" behind the topic, which led her into an internal critical conflict between maintaining her values and satisfying the expectations of those to whom she was accountable:

Yonca: In order to calculate these, I will either need to divide them into very new triangular regions or we will need to come up with something using specific tactics, formulas, and other methods to solve them. But the logic behind it is never explained. It's like there's a formula that just fell from the

sky [...] I told how I am having difficulty with this. The expectation of students, and inevitably the school administration and parents as well, is to proceed with tactics that enable quick problem-solving in exams.

Lale also rearticulated the internal conflict she faces when deviating from the mandatory curriculum in order to uphold her pedagogical values:

Lale: After reading the articles and watching those lesson videos, ... I noticed that the lessons I enjoyed the most were the ones where I helped children discover things, asked them guiding questions, and appreciated the moments when they made their own discoveries. But sometimes... my colleague and I check where we should be according to the curriculum—what week we are in, where we actually are—and we see that we've fallen behind. Then, there's this rush to catch up with the curriculum. And when that happens, we automatically switch modes. Instinctively, I guess, I slip into "pilot mode." I start delivering information non-stop [...] My most active students take a step back, becoming more passive, and I shift into a lecture-heavy teaching style. This really bothers me. Because when I see the students' faces, I can tell they haven't fully understood. And then I feel guilty...

Accompanying their rearticulation of critical conflicts and pedagogical values, Yonca and Lale engaged in a quest to negotiate a new meaning for their internal conflicts, with each other's support. This signified the second step in the formation of agency at the individual level. Yonca invited Lale to contribute to her quest to negotiate a new meaning for her internal conflict by saying, "honestly, I would love to hear your thoughts on this matter:"

Yonca: I also see that these proofs are removed from textbooks from year to year. When these are not available, I sometimes have difficulty, frankly. "Where did it come from, teacher? Why is this formula like this?" In other words, we encounter such questions. This time, what the student learns does not make much sense [to them], I think. Honestly, I would love to hear your thoughts on this matter.

Building on Yonca's accounts, Lale pointed to the lack of a coherent framework that highlights the standards for meaningful mathematical learning and provides guidance for teachers in Turkey. She identified this as an underlying systemic problem leaving everything to "teachers' discretion":

Lale: I don't know—maybe it's the same in other countries—but in Turkey, education seems to depend on the teacher's conscience. There is no real structure. Sure, there are topics to be covered, but our teaching methods vary

drastically. Having different methods isn't necessarily a bad thing, but there needs to be a solid foundation—a well-structured guiding framework. Not the current one, but an effective guide. Right now, because there isn't one, everything is left to the teacher's discretion. And I think that's deeply unfair.

With her words "our teaching methods vary drastically," Lale highlighted the presence of teachers who want to break away from traditional methods and improve their teaching to support students' meaningful learning. However, under the circumstances of lack of clear framework, it was challenging to maintain practices that went against the grain and at the same time, it was "unfair" for students as they might not be able to be taught by teachers who could take initiative to enhance their teaching.

This uncertainty and arbitrariness led to the conflicting messages teachers continuously received: make mathematics enjoyable and meaningful for students, while simultaneously keeping up with the current system, as identified and articulated by the first and second groups, too. Similar to other groups, they identified the lack of a system-wide effort, which left teachers feeling isolated in their attempts to transform their teaching practices, as a systemic problem underlying this double bind they were collectively facing. This group also began the process of agency formation at the collective level by identifying the double bind—contradictory messages that they, as mathematics teachers in Turkey, collectively received.

4.3.2.4 Summary of the fourth session findings

When the discussions from all groups are evaluated together, this session demonstrates the continued process of teachers' experiencing grappling with internal critical conflicts as they engaged in a quest to negotiate a new meaning for their internal conflict. This marked the second critical moment for agency formation at the

individual level (E2). By recognizing that their critical conflict was shared and engaging in dialogue to explore its causes, the teachers identified and articulated a double bind situation, initiating the process of collective agency formation (LIII-a). During their discussions, they highlighted the contradictory messages they received, reflected in their shift from using the individual "I" to the collective "we" or generalized "you" meaning "we" in Turkish. Interestingly, in all subgroups, teachers identified how they were caught between two conflicting messages they continuously received: transform your teaching to make mathematics learning more meaningful for students, but simultaneously keep up with the system that continuously pulls them back to teaching methods that oppose these pedagogical values. Responding to one of these messages negates the response to the other for mathematics teachers in Turkey.

Since this double bind reflected a systemic contradiction at a broader activity system level—i.e., national education—they couldn't easily use "we must do" as a phrase. Instead, they continued to use passive voice when expressing their sense of urgency, as seen in Demet's words: "The need of all classrooms is to be covered with a structured new understanding of teaching... There should be a sustainable system, a structured and sustainable system that needs to be presented to us..." Despite this, the situation marked the first step toward the formation of agency at the collective level (LIII-a), as they collectively identified and articulated the contradictory messages they, as mathematics teachers in Turkey, had received.

In a nutshell, during the fourth session, teachers began to recognize the commonality in their individual internal conflicts. Their need to analyze and understand the reasons behind this shared conflict led them to question its roots. This triggered a transition to the collective formulation of a double-bind situation—a

societal dilemma in which they, as mathematics teachers in Turkey, were exposed to conflicting messages that hindered their ability to pursue their values and professional development in a sustainable manner.

Their discussions also highlighted the systemic contradiction that time and again give rise to such dilemmatic situations in practice (Engeström, 2000, p. 153): Lack of a consistent, well-grounded, well-articulated, and unifying framework that could support teachers in transforming the teaching culture in their settings.

This session laid the groundwork for teachers to continue working on the identified double bind situation and its underlying systemic contradiction, without falling into the trap of making hasty decisions about its causes. This necessitated a transition from the questioning phase of expansive learning to the analysis phase, which prompted me to organize the subsequent session accordingly.

4.4 Sessions 6, 7, 10, 11, 16: Agency formation within the analysis action

During the sixth, seventh, tenth, and eleventh sessions, teachers predominantly engaged in the "analysis" expansive learning action.

In the sixth and seventh sessions—two sessions which were evaluated together since they built upon one another—analysis expansive learning action represented 36.69% of the entire transcript. In comparison, questioning action comprised 27.05%, while modeling action made up 3.32% of the total. In the eleventh session, the analysis action increased to 49.50%, with modeling action rising to 32.2%. In this session, where teachers engaged in analysis activities more systematically, their analysis action focused on both the actual-empirical (33.6%) and historical (15.9%) analysis of the Mathematics Teaching Program—commonly referred to as the curriculum—in Turkey.

Although modeling was the dominant action in the sixteenth session, accounting for 24% of the coverage, the analysis action, with 11% coverage, was also significant and therefore included in the analysis to address the research questions.

In the second round of analysis, the second indicator of agency formation at the collective level was identified across these sessions. The first part of this indicator—collective construction of novel questions based on the double bind situation—was identified in the transcripts of the sixth and seventh session discussions. In the subsequent sessions (sessions 10, 11, and 16), the second part of this indicator—conducting research to address these questions—emerged.

4.4.1 Collective construction of novel questions based on the double bind situation (LIII-b-first step) (Sessions 6 and 7)

I formatively designed the activity for the sixth session based on my analysis of the teachers' discussions during the fourth session. There, I identified that each subgroup had mentioned a similar dilemmatic situation they were collectively facing—the double bind. My goal was to support their agentic transition to analyzing the double bind and the underlying systemic problem. To achieve this, I followed a specific method: I noticed that the teachers used different terms to refer to the same problematic issue related to the curriculum when articulating the double bind. I identified the following terms: "Structure; systemic transformation; framework" (first subgroup); "Common ground; standardization; framework; curriculum" (second subgroup); and "Curriculum; standard; framework; backbone; skeleton; guide" (third subgroup). Focusing on these keywords, I identified representative video clips from teachers' discussions in the earlier sessions and designed a stimulated recall interview

by categorizing them (Dempsey, 2010). This activity continued in the seventh session as well.

In the sixth session, I began by having the teachers watch video clips, explaining each time why I had selected a specific episode. After summarizing the different views presented in the videos, I asked the teachers to elaborate on the meanings behind their expressions in these episodes. Defne's quote reflects the shared view that emerged after a 25-minute discussion:

Defne: When I say guide, it actually evokes something like this for me: a common guide based on teaching methods, of course, the content can vary greatly depending on where we are, even within different classes in the same school, because students' connections to daily life can vary greatly. But, for example, what I mean by guide is this: "teach by letting the student discover." There should be a guide related to this.

Similar elaborations came from Lale, Aelya, and Demet. By sharing their vision of a guide, the teachers highlighted the need for a common goal and discourse among mathematics educators that lead a cultural transformation in teaching, as well as guidance on teaching methods to implement in practice, while preserving teacher autonomy to address the varying needs of their students.

Teachers conducted this discussion on their experience-driven assumption that no clear guideline or framework exists in Turkey that articulates the principles for teaching mathematics. Then, Gl took the floor and initiated an important discussion to counter this hypothesis, leading to an organic transition to the analysis of the systemic problem—lack of a consistent, well-grounded, well-articulated, and unifying framework. This was a hybrid meeting, with Gl participating virtually. She opened The Mathematics Teaching Program (MEB, 2018) individually, which she referred to as "program booklets," to see if it provided the type of framework they were collectively seeking:

Gül: We just talked about the fundamental principles of learning mathematics, right? They actually exist. They are in the booklets, in the program booklets... Now I opened it, I looked to see if I remembered correctly. There are also specific goals among what we just talked about. Like what? "[Students] will be able to develop mathematical literacy skills and use them effectively" ... Actually, the predetermined general goals, specific goals, if we discuss them, we'll get to that point...

According to Gül, the general and specific goals outlined in the Teaching Program shared a common discourse that was coherent with their pedagogical vision. Gül's initial step in the analysis action served as a foundational background for the transition to the second indicator of agency formation at the collective level—the collective construction of novel questions based on the double bind situation and conducting research to answer these questions.

This occurred first through the subsequent discussion in the seventh session, which built on Gül's counter-hypothesis. This counter-hypothesis gave rise to novel questions, produced by the community for the first time. Inviting the teachers to clarify the concepts they were using, I asked, "What exactly do we mean by 'curriculum'? Is this really what we always refer to as the scope or the curriculum? Is this what comes to your mind?" Lale responded by equating the curriculum with The Teaching Program, which included the learning objectives teachers used to create their annual plans, while the teaching guide she envisioned was a more comprehensive version of it:

Lale: Yes. Isn't there just one curriculum? The one determined by the Ministry of Education? The National Education Curriculum, which shows what we will teach and in what order in fifth, sixth, seventh, and eighth-grade mathematics. I don't recognize any other curriculum. When I hear "curriculum," I think of the learning outcomes I will teach, their timeline, and the objectives I include in my annual plan. But when it comes to something like a teaching guide, I see it as something more detailed—it includes both the methods and the spiral [curriculum] structure. So, to me, a guide is more comprehensive, including instructional methods, whereas the curriculum is just about the learning objectives.

Linking this opening to the discussion in the previous session, I asked "So, is there such a guide in Turkey that matches what you're envisioning?" Lale admitted her lack of knowledge about it and reminded the community of Gül's statements from the previous session and posed a question:

Lale: I don't have any information about whether it exists or not. But last week, Gül mentioned that the Ministry of Education guide also covers methods and techniques. Based on that, I started wondering—could there be something that includes instructional methods as well? Does such a guide actually exist?

Lale's question, "Could there be something that includes instructional methods as well? Does such a guide actually exist?" sparked a lengthy discussion in which the teachers mentioned various resources released by the Ministry of National Education, along with their goals, names, and whether any of these corresponded to the guide they needed. Lale's question soon became the central question for the community to answer. In other words, Lale transformed the teachers' assumptions into a novel question, which was subsequently adopted by the other teachers, making the task of answering it a collective effort. At the same time, this process reflected the teachers' confusion regarding the documents released by the Ministry of Education:

Defne: When I say "curriculum," I think of what I fill out in the class book. But what I mean by "guide" is actually something more comprehensive... something that should actually be in the teacher's file. I only learned this last year because when my principal asked for it, no teacher had it in their file... It explains how we are supposed to teach the learning objectives.¹¹

Demet: Isn't that the yearly plan, Defne?

Gonca: I think it is called the Teaching Program. I believe it's available on the Ministry of Education's website. [...] I mean, the Ministry has a booklet—it's detailed, but not excessively detailed.

¹¹ The resource Defne shared turned out to be a resource for formative evaluation in mathematics classrooms when Defne shared with the community, as a guide not mandatory for teachers to consider. While embarking on research activities, teachers focused on The Teaching Program instead of this guide to evaluate the guidance it provided to teachers.

Demet: I've looked at it too; at the beginning, it includes teaching methods and techniques. Then it starts right away—from the fifth grade onward [...] Is this all there is in what we call the curriculum?

Açelya: In the past, the National Education Program included teacher guides for grades one through eight. These guides contained learning objectives, sample activities, and classroom applications for teachers. [...] However, when the curriculum was changed and took its current form, it became a 48-page document that only includes learning objectives, general aims, and the number of hours allocated to each objective.

YGA: The name for this is Teaching Program, right?

Gonca: Yes, it's the Teaching Program.

YGA: So, [currently] we only have the Teaching Program, nothing else, right?

Açelya: I don't really know.

YGA: Could the guide that Defne mentioned be the answer to this? Could it be? Should we put that question out there? Should we check it out? Should we examine it? What do you think?

[The group]: I think we should examine it.

In this way, Lale's question became a novel question for the community to explore.

Açelya proposed to investigate whether there are other similar guidelines in addition to the guideline mentioned by Defne; Gonca proposed another document on out-of-school learning environments published and disseminated by the Ministry of Education to address this question.

As the discussion progressed, more specific questions related to the curriculum emerged, leading to decisions about different research activities for further analysis. Gül wondered why certain topics (e.g., modular arithmetic) were removed from the curriculum and whether this was the same in other countries. Her inquiry was triggered by a previous conversation with a curriculum expert. This question was also raised by other teachers in different sessions. After reminding them that this was a shared concern, I asked, "Whom would you like to consult with to learn more about what is happening in our country regarding this?" One suggestion

came from Lale, who proposed inviting a curriculum expert she met before—also a mathematics education professor at a university and a member of the curriculum development committees in Turkey. Another suggestion, from Açıelya, was to invite other stakeholders from a board of the Ministry of Education responsible for curriculum design.

Referring to Gül's other question—whether the topics covered are the same in other countries—I asked the teachers if they would like to conduct a comparative analysis to better understand their own curriculum. Lale supported this idea by proposing a general comparison approach: "So, what kind of things exist in [the curricula of] different countries? What kind of applications are there? What is prominent? What can we take inspiration from or use as an example? How can it be adapted for us? If we take a general look at these?" Agreeing with Lale, Açıelya asked which countries that they should prioritize in this research, whether they should focus on the successful ones determined based on international comparisons such as TIMSS.

When the teachers planned and implemented the activities they had proposed during the seventh session in the following months, their questions became more refined with the contributions of all community members. The sixth and seventh sessions were crucial for the second step of agency formation at the collective level, as the teachers collectively constructed novel questions based on the double bind and the underlying systemic problem. By posing their own novel questions, rather than receiving ready-made questions and answers, they initiated a collective effort to explore the broader context of the double bind they had initially identified in the fourth session. In doing so, they created tasks for their collective professional learning.

4.4.2 Conducting research to address community's novel questions: Uncovering different dimensions of the systemic problem (LIII-b-second step) (Sessions 10, 11, and 16)

Implementing the tasks that teachers set for themselves took time and was realized over the following months, becoming evident as analysis expansive learning action in the 10th, 11th, and 16th sessions. Throughout these sessions, the teachers conducted research to address the novel questions they developed based on the double bind situation. Specifically, they analyzed the underlying systemic problem they hypothesized—a lack of a consistent, well-grounded, well-articulated, and unifying framework—by answering these questions. To do so, they drew on the material, conceptual, and relational resources they had selected.

The ninth session, held after a summer break, served as a check-in to organize the community's activities for the new semester. The group revisited the decisions made during the seventh session, refining them and creating action plans. Açelya took the lead in reminding the community of the activities they had decided to pursue for further research. Based on the discussion in this session, the teachers and I co-organized three research activities.

The first activity was a comparative analysis of two teaching programs: NCTM's (2000) *Principles and Standards for School Mathematics* and the Mathematics Teaching Program in Turkey. NCTM (2000) was chosen both upon my suggestion and because it served as the guideline for another resource that the community teachers had selected for their modeling activities (Van de Walle et al., 2020). They used NCTM (2000) as a point of reference for their analysis of the Teaching Program. Alongside this, as a second analysis activity, the teachers read scientific texts on the history of the Mathematics Teaching Program in Turkey,

which they discovered during their literature review (e.g., Ergün et al., 2015; Memnun, 2013). The third research activity involved inviting a mathematics education researcher who was part of the author team for the 2005 reform curriculum. This meeting was organized over the following two months and took place in December 2022. These activities enabled teachers to gain insight into different dimensions of the systemic problem's background.

Sessions 10—co-designed by Defne, Açelya, and YGA—and 11—co-designed by Gül, Ali, and YGA—were dedicated to the comparison activity. In preparation for this analysis, the subcommittees asked the teachers to read the first three chapters of NCTM (2000), which I translated for the community's use, as well as the introductory sections of the Mathematics Teaching Programs of Turkey (both primary [grades 1-8] and secondary [grades 9-12]).

The organizing subcommittee for the 10th session extracted the guiding questions for the session from the discussions in the sixth and seventh sessions. These questions were: "Do the curriculum and frameworks provide guidance on teaching methods and how to teach mathematics?" "How is the sequence, spirality, and scope of mathematical content in the curriculum determined?" "Is this clearly defined?" "Does the curriculum offer teachers a holistic perspective on mathematical topics?" "Is the big picture—the ultimate goal we want students to achieve—clearly established?" (Figure 5)

During the tenth and eleventh sessions, teachers collectively discussed key takeaways from their reading of both resources (MEB, 2018; NCTM, 2000) in light of these questions. These discussions represented moments of developing awareness about the curriculum and its influence on their practice and professional development. For example, in response to the last two discussion questions, in the eleventh session, Gonca noted that, unlike NCTM's support for teachers in identifying their goals and principles in teaching, the Mathematics Teaching Program in Turkey assumes that mathematics teachers already share common perspectives and methods. As a result, it refrains from providing details that could create opportunities for teachers to develop a critical perspective on their teaching:

Gonca: I'm specifically referring to the first chapter of the book [NCTM (2000)] I read—there's something entirely about the teacher. I mean, what is the teacher aiming for? Is the teacher acting in line with that goal? Does the teacher know what they should be expecting? [...] [our] curriculum doesn't even provide this kind of perspective. It assumes that we all already know everything very well. It's about how we deliver what we already know, in what order, to whom, what, and when. But there's nothing that questions us or makes us question ourselves.

Building on Gonca's observations, other teachers highlighted similar issues. Through this comparative analysis, the assumptions embedded in the Mathematics Teaching Program became more visible to the community teachers.

Açelya provided a historical perspective on the development of mathematics teaching programs in Turkey, drawing from an article on curriculum changes since the establishment of the Republic of Turkey (Ergün et al., 2015). She explained how these changes were shaped by political shifts in the country, which directly influenced teachers' practices:

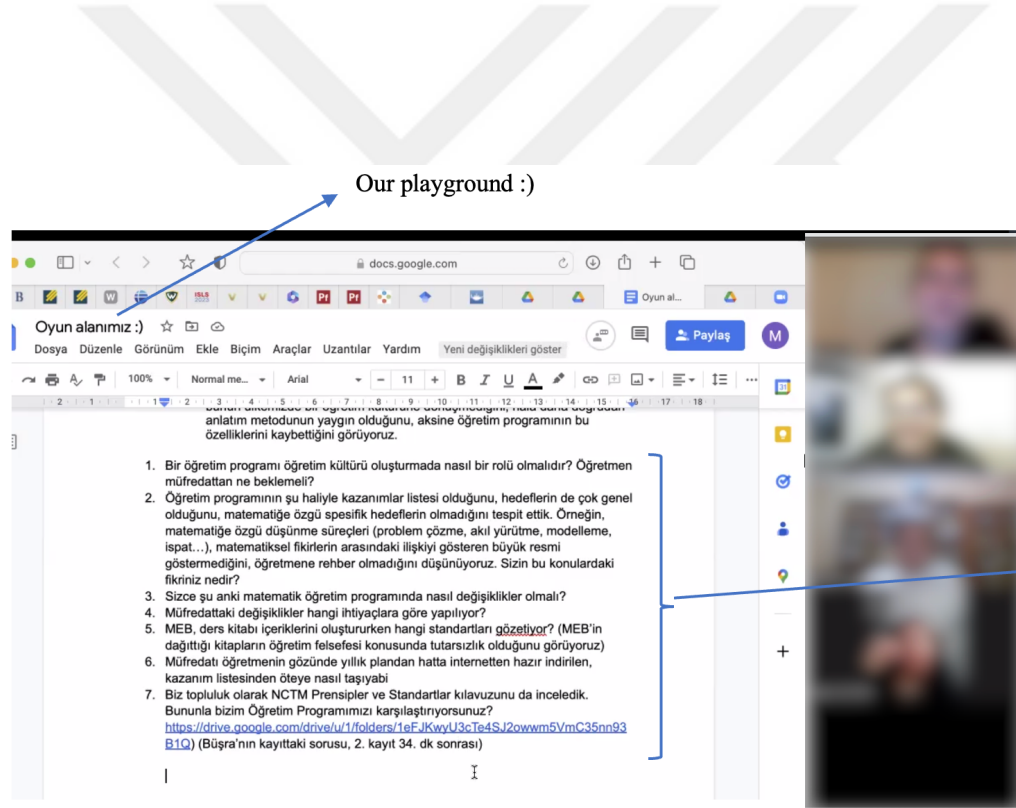
Açelya: When I was reading that chapter of the book [Ergün et al., 2015], I was excited about understanding the strong foundations behind the curriculum changes. But when I realized that the changes were driven by political shifts and military influences, I felt a deep sense of disappointment.

Gül: What we are trying to understand now is that in 2013, a very different curriculum guide was published regarding mathematics education. But by 2018, it had completely regressed. Of course, in between, there was a coup attempt.

Açelya: Let me remind you of something. In the 2004 curriculum change, fractal geometry was introduced for eighth-grade students. I never understood why it was included. Then, after about five years, in either 2008 or 2013, it was removed again. [The topic of] histograms were also added and later removed. There are so many good things, yet changes happen so rapidly—and the most painful part is that they happen without any real educational foundation.

This analysis activity revealed another dimension of the systemic problem related to the curriculum. The curriculum issue was interconnected with other aspects of the system: frequent changes were not always driven by scientific developments in mathematics education but rather by political shifts in the country.

The third analysis activity that the teachers collectively engaged in involved inviting a mathematics education researcher who had been part of the author team for the 2005 reform curriculum. This gave the community an opportunity to ask questions about the Mathematics Teaching Program in Turkey. Drawing from their discussions since the sixth and seventh sessions, the community collaboratively refined their questions to explore the systemic problem related to the curriculum. These included questions such as "What should a mathematics teacher expect from the curriculum?" "On what basis was the curriculum changed?" "What role should the curriculum play in shaping a teaching culture?" (Figure 6)



1. What role should a teaching program play in fostering a teaching culture? What are reasonable expectations for teachers to have from the curriculum?
2. We have identified that, in its current form, the curriculum is essentially a list of learning outcomes, and that its goals are too general, lacking mathematics-specific objectives. We believe that it does not guide teaching in terms of mathematical thinking processes—such as problem solving, reasoning, modeling, and proof—nor does it present the big picture that shows the connections between mathematical ideas. It also fails to serve as a guide for teachers. What are your thoughts on these issues?
3. In your opinion, what changes should be made to the current mathematics curriculum?
4. According to which needs are changes in the curriculum made?
5. What standards does the Ministry of National Education consider when developing textbook content? (We have observed inconsistencies in the instructional philosophy across the textbooks distributed by the Ministry.)
6. How can the curriculum go beyond being just a list of learning outcomes, or even something downloaded from the internet, in the eyes of the teacher?
7. As a community, we also examined the NCTM Principles and Standards guide. How do you compare this with our curriculum? (Lale's question in the recording, Second recording, after the 34th minute)

Figure 6. The community prepares their questions to address to the guest

Teachers posed these questions to the guest during an additional session held on December 13, 2022. One key insight that emerged from this interview, later articulated in a follow-up session (16th session), was that the Teaching Program extends beyond a booklet. Referring to the expert's words, Açelya noted, "We can interpret the curriculum as encompassing all the materials and individuals involved in the teaching process. In other words, course resources, materials used in lessons, supplementary materials, and even the teacher's experience, perspective, and collaboration with their department are all crucial factors in the curriculum."

Building on this expanded perspective, Açelya emphasized that while the Teaching Program plays a role in transforming teaching culture, the activities that take place during implementation—such as professional communities—are equally significant. Defne reinforced this point by reflecting on her own experience within the community: "Putting together such a guide is great. But if I hadn't been part of this community since April... how meaningful would that guide be to me if I received it without ever being in the community?"

As a result, the third analysis activity—the interview with a curriculum expert—revealed that the systemic problem teachers had identified was not solely tied to the Teaching Program as a published document. Instead, it was interconnected with various other elements of the broader activity system in which teachers operate.

Consequently, in the tenth, eleventh, and sixteenth sessions, teachers engaged in analysis activities to explore the questions that they posed as a collective, drawing on both material, conceptual, and relational resources. These activities allowed them to uncover different dimensions of the system that produce the systemic problem underlying the dilemmas (i.e., critical conflicts and the double binds) they were experiencing.

In this way, this process set the stage for the next expansive learning action—modeling—where they moved beyond problem identification and analysis to co-designing solutions tailored to their professional community’s needs with new understandings of their problems. In the following section, I will explore how the teachers collectively exercised agency by engaging in modeling action to create novel professional learning activities and processes responsive to their needs.

4.5 Sessions 8, 9, and 12-28: Agency formation within the modeling action

Modeling action was dominant in the 8th, 9th, and 12th to 28th sessions. Most of these sessions were intentionally co-designed and implemented to focus on modeling activities. The coding for modeling action ranged from 22% (Session 17) to 56% (Session 15) of the transcripts for these sessions. Additionally, although it was not the dominant action, modeling constituted a significant portion of the transcript for the 11th session, with 32% devoted to modeling and 50% to analysis.¹²

In the second round of analysis during which I focused on the parts of the transcripts coded for the modeling action, the third indicator of agency formation at the individual level (E3)—developing individual disposition to act consciously—and the third indicator of agency formation at the collective level—collective action for the generation of new activity forms as a response to the systemic problem (LIII-c-c)—were identified in these sessions.

¹² In most of these sessions, questioning and analysis actions were marginal and, therefore, not meaningful enough to include in the coding scheme. I made this decision during a second round of coding, which focused on portions of the sessions where modeling activities were conducted for the analysis of agency formation. An exception was the 16th session, where analysis accounted for 11% of the transcript. This session was a follow-up meeting to one with the guest. However, modeling remained the dominant action, covering 24% of the transcript. Additionally, I excluded discussions where experience sharing among teachers dominated but did not contribute to community modeling. Conversely, I included extended discussions on any idea or teaching concept when they directly contributed to constructing the model.

In the eighth session, the teachers' discourse revealed that they overcame their internal critical conflicts by expanding the problem space of their professional development. This expansion led to the development of their individual disposition to act consciously in organizing their professional learning, both within and outside the community. This is the point at which the "individual disposition to act is prepared" (Sannino, 2008a, p. 241), and discourse reveals feelings of "personal liberation or emancipation" (Engeström & Sannino, 2011, p. 374). I used second individual interviews to triangulate the data for the analysis of this indicator.

The last indicator of agency formation at the collective level (LIII-c), which is the collective action for the generation of new activity forms as a response to the systemic problem, was identified throughout the sessions where the modeling action was exercised. In these sessions, teachers proposed solution ideas by linking them with the systemic problem that they collectively identified, they planned modeling activities by proposing relevant resources, and documented their ideas onto a shareable medium to be used during the execution of the developed model.

4.5.1 Linking the identified systemic problem with solution ideas and planning modeling activities (LIII-c first step) (Sessions 8 and 9)

In the eighth and ninth sessions, the teachers initiated collective practical action to generate local professional learning activities as a solution to the systemic problem they had collectively identified and begun to analyze. A first step to the final indicator of agency formation at the collective level is demonstrated in these sessions by connecting the identified systemic problem with the solution models—that is, rationalizing the solution models in light of the identified problem.

Initially, Japanese Lesson Study¹³— which became a shared tool among the community teachers following their reading and discussion of the seventh chapter of *The Teaching Gap* during the fifth session—began to be reimagined by the teachers in the context of the systemic problem of the lack of a unifying framework for transforming teaching. This reimagining emerged as early as the fifth session during a sub-group activity:

Defne: Wouldn't we also be finding the systemic support we're looking for if we implement this?

Lale: So, a teaching culture, a teaching framework would be formed, right? Should I say creating a teaching culture?

That is, this model could create the conditions for developing a teaching culture among teachers, while fostering collaboration among colleagues and contextually relevant continuous professional development.

This idea became more localized to their community during the eighth and ninth sessions. Specifically, teachers shifted their focus to finding ways to address the systemic issues hindering their meaningful collaboration and professional development, rather than immediately engaging in lesson design, despite the appeal of conducting Lesson Study within their community.

This process began with Defne sharing her analysis of new materials proposed to them, informed by a new understanding of teachers' problems. This analysis reinforced her recognition of the need for a guiding and unifying framework for their professional learning and collaboration. Defne explained how a set of

¹³Japanese Lesson Study is a school-based professional development model characterized by a group of teachers' long-term engagement in a cyclical process of collaborative research on a teaching issue. This process typically involves studying the selected teaching issue using scientific and curricular resources, lesson planning, implementation in a real classroom setting, reflection on the implementation using data collected during the lesson, refinement of the lesson, and re-teaching of the revised lesson (Dudley, 2015; Lewis & Perry, 2013; Stigler & Hiebert, 1999).

classroom activities—recently published on the *Teachers' Material Hub* (pseudonym), a central online platform for sharing teaching resources by the Ministry of National Education (MEB)—reproduced the very systemic problem that hinders teachers from transforming classroom culture. According to Defne, these materials lacked a focus on teaching processes, offering alternative activities without addressing this critical issue:

Defne: Now, these types of activities have recently started to be [offered], but there is still nothing about the teaching methods... Today I watched the video [shared] on Teachers' Material Hub. It always emphasized ... that the activities carried out should be related to daily life. It even includes ... taking students to the museum, etc. [It says] the teacher briefly explains the subject of ratio and proportion before going to the museum. The teacher explains it, but how does he explain it? ... The activities are very nice, really fun... but that's only up to there.

The summer school materials that Defne analyzed proposed fun activities; however, with her new perspective, she became critical of the tendency to prioritize making mathematics lessons "fun" in a superficial way, at the expense of effective teaching. Accordingly, the lesson designs permeating the current teaching culture was lacking a reconsideration of teaching methods so that these activities can support meaningful mathematics learning.

Following the discussion on Defne's example—during which other teachers also shared similar analyses from their daily practice (e.g., Demet shared lesson plans from pre-service intern teachers at their schools)—I asked, "What kind of solutions can we offer and produce regarding these?" My question sparked a 43-minute conversation about various activities the community could undertake. This discussion continued in the ninth session, after the teachers rewatched and reflected on the discussion from the eighth session to organize activities for the new semester. Defne summarized their approach in the eighth session, emphasizing their priority to "get mature" before engaging in collective lesson planning:

Defne: Like, you know, we talked a lot about, especially in the eighth session. You know, when we say planning for learning outcomes, lesson planning... before we take action on these, let's mature and reach a certain level, of course, that's how it should be.

Despite its appeal for teachers, teachers put off their plan for enacting a collaborative lesson planning process through a model like Lesson Study based on their identification of their need to create a teaching culture within the community, intending to fill the gap emerged from the lack of a consistent, well-grounded, well-articulated, and unifying framework that could support teachers in transforming the teaching culture in their settings.

Immediately, Defne suggested reading a book on mathematics teaching and learning (Van de Walle et al., 2020) as a resource to guide them in this process, reinforcing the connection between the systemic problem and the idea of reading this book as a community professional learning activity:

Defne: As we progressed through this process, I mean, everyone had it. I also obtained it. This "primary and secondary school mathematics" [...] I thought maybe some readings could be done again. Maybe not all of them, but before moving on to the lesson design for the [learning] objectives. I guess most people have this book anyway. It says foundations and perspectives. Maybe we can structure our own maturity a little more from here.

Lale supported this idea by linking the emerging pedagogical perspective, or teaching culture, of the community—through reading *The Teaching Gap*—with the ideas in this new resource:

Lale: Here, it directly said, you know, teaching mathematics but teaching it through problem solving. In other words, it focused on this, just like in Japanese lessons. I don't know, I personally want to read this book, I don't know what other friends will say, but... I think we can combine it [with *The Teaching Gap*] and build [our work] on a really solid skeleton.

Following this, other teachers (Yaprak, Gül, Açılya, Ali, Gonca, Demet) participated in the conversation, focusing on how to execute the plan—deciding which edition of the book to use and which chapters to read, considering feasibility and their needs. The connection the community established between the identified systemic problem

and the proposed solution ideas, as well as the activities they were undertaking to put these ideas into practice, was later articulated by Gül in a blog post¹⁴ she published in Teachers Network's blog account about two months later, with these words:

Gül: The Lesson Study model excited me and made me think that a change in education in our country is possible with this model... Putting the Lesson Study into practice alone will not bring about any change. There is also a risk of implementing Lesson Study to transfer content [to students], as we have done for years. This only means that we develop another way to maintain our cultural codes... The groups that will implement the Lesson Study first need to review and change their beliefs about the roles of teachers and students in the classroom. The path is a process of discovery. What can we do to realize and transform our cultural codes that hinder us on this path? For this, as the Teachers Network Mathematics Teaching community, we continue our work by reading and discussing about mathematics education and reflecting on our own practices.

Consequently, in the eighth and ninth sessions, the third indicator of agency formation at the collective level emerged as the teachers linked the identified systemic problem with solution ideas and began planning modeling activities as a community. This included proposing a resource that gained new meaning for the community through their modeling a solution process. This resource started to function as a second stimulus—a tool that subjects utilize to develop solutions to their problems.

4.5.2 Developing individual dispositions to act consciously to resolve the critical conflict situations (E3) (Session 8)

Before sharing the findings from the remaining sessions regarding agency formation at the collective level, I will first share another finding related to agency formation that contributed to the modeling process, revealed in the eighth session. In this

¹⁴ Gül's blog post can be accessed via <https://ogretmenagi.medium.com/ogretmenin-degisimi-uzerine-kulturel-olarak-miras-aldigimiz-ogretme-kodlarimizin-degisimi-mumkun-92ffa7f2ad46>. Permission to release Gül's actual name was obtained from Gül.

session, I identified the final indicator of agency formation at the individual level—developing an individual disposition to act consciously to resolve internal conflicts. This indicator was significant because it showed how teachers maintained their motivation to remain part of the community as individual teachers during the collective modeling activities, attributing personal meaning to the collective professional learning activities.

This became evident during a discussion in the eighth session, when I prompted teachers to reflect on the emphasis they had placed on centralized exams in the first introductory session as a primary obstacle before performing their ideal practice and why this focus had later disappeared from community discussions. In this session, teachers answered my questions—"What are your thoughts on our shared problems as mathematics teachers? What obstacles prevent you from performing your ideal practice? Under what conditions do you feel at an impasse?"—with "centralized exams" being mentioned by 14.85% of respondents, compared to 9.13%, 4.56%, and 6.24% for issues related to the teaching and learning culture, curriculum and related reforms, and the profession and teacher education, respectively. As an answer to my question, Defne initiated the conversation, reflecting on how her understanding of the initial problem had evolved over time as she engaged in PMTLE activities:

Defne: During this process, I realized that, yes, this is one of our problems; but that's not the problem we should have taken on in the first place. The problem we need to take first is the teaching method. How are we going to teach this [mathematics]? How should mathematics be taught? I think we have to move forward beginning here.

Defne reconceptualized the teaching itself as the fundamental dimension of her problem space of professional learning and development through a core question she had not deeply considered before the PMTLE study: "How should mathematics be

taught?" As the analysis of the second session revealed, the teachers articulated a shared critical conflict that had caused them to deviate from their values in practice and hindered their progress in professional learning. Throughout the PMTLE sessions, they began to treat *mathematics teaching* itself as a site of inquiry for their professional development, thereby expanding their problem space. Not engaging in this question limited the possibilities for them to overcome their critical conflicts taking control of their own professional learning processes. Açelya referred to the collective analysis of the systemic problem as a resource for them to expand the problem space by centering the mathematics teaching:

Açelya: We thought of this as the first problem situation, but then we read, thought about and discussed... As we researched the causes and causes of the causes, and examined them, we concluded that the real problem is not the exam but the mathematics teaching or the curriculum, framework.

Similarly, Yonca emphasized that the work conducted within the community helped her develop a new perspective on her challenges, positioning "mathematics teaching" as the central element of her problem space:

Yonca: I think in a similar way as Defne and Açelya. At first, I thought that the exam system was a very big problem, but as time went on and our studies increased, I saw that there was actually a serious problem in mathematics teaching ... and here the exam system turned into a very, very small building block of this big picture for me...

In the following quote, Lale also provided how shifting the focus from exam system to teaching itself expands the possibilities for a teachers' improvement of her practice compared to the situation in which they focused extensively on the exam system as an obstacle:

Lale: the exam system is not something that we can change alone with a few teachers. It is not directly connected to us. It is no longer a situation that directly affects our practice in the classroom, at least after understanding the book [...] As my teacher Yaprak said, everyone thought it was the exam system at first, it seemed like a big problem, but as I read it, I thought the same way. You know, the exam system is a small part of it.

While decreasing the role of the exam system as a factor shaping her teaching, she developed a perspective about the parts of her teaching that can be under her control. The most prominent quote illustrating the feelings of liberation from this situation came from Defne's second individual interview, a couple of months after the eighth session, in response to my question: "Do you think there was room for your opinions and contributions in this study?"

Defne: Definitely... I keep coming up with ideas, ideas, ideas, and maybe, you know, extreme ideas... For the first time in a long time, I can think of ideas you can call crazy ... I think it [the study] is a catalyst that reopens my imagination or my creativity. This made me extremely happy... I reproached the environment and society, "How, how did they kill my creativity?" But it's nice ... to be able to say, "Yes, I can do those so-called crazy ideas here, and from here [the community], I can get support, we can feed each other, and we can do it together."... It feels safe to be able to think about it.

According to Vasilyuk (1991), successful experiencing involves "transforming a conflict that was causing torment owing to its insolubility into a complication of life, itself far from simple, but potentially resolvable" (pp. 195-196, quoted in Sannino, 2008a, p. 241). The task for teachers was now clearer compared to the centralized exams problem, though still ambitious. The teaching culture, including the roles of teachers, students, and the content in the classroom, expanded the problem space within which teachers could engage in their professional learning actions, which was an ambitious task.

The feeling of impossibility, stemming from their inability to address the critical conflict they faced, was replaced by a readiness to tackle the challenge, recognizing that, although it was difficult, they could confront it with the support of the community, as it was explicit in Defne's words, "How should mathematics be taught? I think we have to move forward beginning here." Teachers were ready to create tasks for themselves onto this expanded problem space with the shared consciousness of their needs. This new tool—the teaching itself and the surrounding

teaching culture—along with the professional learning activities that could be built around this core issue, emerged as a means to overcome their critical conflict and reconnect with their values.

By freeing themselves from the idea that centralized exams are the most prominent systemic obstacle standing in the way of realizing the teacher they aspired to be—and drawing them back to the teacher they had been taught to be—teachers restored their psychological capacities needed to engage in professional learning activities required to fulfill their need of overcoming such internal conflicts in their practice.

This session demonstrated that teachers envisioned the space as an opportunity to pursue their individual professional learning goals, thereby supporting their agency in shaping their own professional learning processes. While this expanded problem space enabled them to reimagine their individual development, it also evolved into a shared space through collective processes of problem identification. This collective dimension became especially evident in the ninth session, when the community chose the name Mathematics Teaching Community.

4.5.3 Executing the modeling activities (LIII-c second step) (Sessions 11-28)

Teachers executed the planned modeling activities, as mentioned in Section 4.3.1, between Sessions 11 and 28. During sessions 11 to 15, they primarily focused on reading the initial chapters of NCTM (2020) in session 11 and Van de Walle et al. (2020) during sessions 12 to 15. Through reading NCTM (2020), they examined principles such as equity, assessment, and curriculum in mathematics education. By reading Van de Walle et al. (2020), they explored key concepts in mathematics teaching and learning, including problem solving, teaching through problem solving,

conceptual and procedural understanding, as well as the use of manipulatives and tools in mathematics education.

The sessions were co-designed by subcommittees: teachers read chapters from the resources, discussed questions presented by organizing committees, and documented key takeaways in a shared document called "Our Playground." This document served as a record of the "objectified version" of the community.

Discussions around these tools allowed teachers to articulate the definitions, concepts, and ideas related to mathematics teaching in their own words, including their interpretation and exemplifying based on their experiences and shared pedagogical perspective, as a collective (Figure 7).

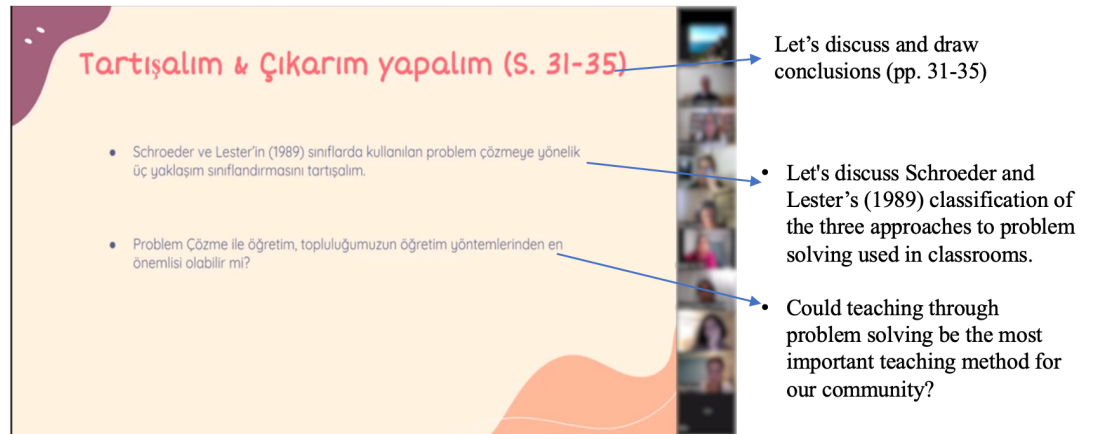


Figure 7. Community teachers are discussing three different approaches to problem solving followed in classrooms and making inferences for their community.

I will share a portion of a typical conversation that led to an addition to the community's pedagogical framework from the thirteenth session which was co-designed by Defne, Yonca, and me. Prior to concluding their discussion with an inference, teachers conducted a 105-min discussion (10 minutes in the 13th session and 95 minutes in the 12th session) on the notions of "conceptual understanding" and

"operational fluency." During this time, they had the opportunity to discuss the examples shared in the book, explore the definitions of the concepts, share their related experiences from their classrooms, and freely ask questions and advice from one another. Using an example that Defne shared from her classroom (which discursive representation of a fraction is more appropriate for fifth graders: calling $\frac{6}{8}$ as "six out of eight" or "six over eight"?) and the following discussion that teachers conducted about it as a point of departure, I asked whether they would like to add the definitions of these concepts to the "Our Playground" document. Below is a brief portion of the 23-minute discussion from where they constructed definitions and examples of these concepts:

YGA: Specifically, Defne, do you have any conclusions related to conceptual and procedural knowledge here? Or how would you relate your experience to those definitions?

Defne: Well, I think conceptual knowledge really needs to be fully grasped here. I realized that, actually, it hasn't fully clicked for us either. I mean, for me personally.

Gül: Yeah, it hasn't fully clicked.

Defne: Since we started this study, I've realized how many things I knew mathematically just by rote. Honestly, it feels like I'm learning math from scratch again—like I'm reinterpreting it from the beginning. [...] For example, I say "six over eight" or "six out of eight" without really thinking about it. I only think of it procedurally, without considering the conceptual meaning.

[...]

Defne: I think Lale raised her hand.

Lale: I had very similar thoughts when I read this part. I started questioning, How deeply do I actually know this?. [...]

[...]

YGA: Would you like to add these concepts, such as conceptual understanding and procedural understanding, to our "Our Playground" document? [...] Defne, would you like to open the document? What do you think about conceptual understanding?

Ali: For example, "Conceptual understanding is a flexible connection and network of relationships between ideas and mathematical concepts, including

their images and interpretations." If we think of this like a concept map or a mind map, it's about the connections built between concepts, as I understand it.

[...]

Yaprak: So, friends, when reading a unit, how do we read it? Kilometer per hour. If we think of it holistically, could this cause anything [negative] if we always teach a child [reading] from the bottom to the top?

Lale: Great question. I love this example. Because yes, when I taught ratios, I remember that we had these conversions in sixth grade. We converted from kilometer per hour to meter per minute, for example. Yes, I think so too.

Yaprak: After all, it is still a ratio, right?

Lale: Yes. Correct.

Defne: I just read something here and I'm jumping in. In the conceptual understanding part, it says: "Students with a conceptual understanding integrate what they know about division and numbers to make sense of scaling, unit prices, and similar concepts." This is on page 22 at the very end. "Notice how much is needed to understand ratios in a relational sense." I think it's telling us directly.

[...]

Gonca: There's also procedural flexibility and fluency there.

[...]

Gonca: Well, the sentence that I think explains it best—though I don't know if you all would agree—is this: "A flexible student looks at a problem, considers the given numbers, takes into account which method would be most efficient, and selects an appropriate method," even though they know the standard procedure.

Defne: I think there's a sentence here that defines it—I marked this part: "Flexibility includes four components: efficiency, accuracy, flexibility, and selecting an appropriate method."

Gonca: Mm-hmm.

[...]

Lale: There's another nice concept added there: Flexibility. So, actually, a child who can think flexibly using numbers can do this. It's not just about adding straightaway, like "where's the tens place? It's here, I'll add the tens," but immediately converting the numbers, like in the example here, adding thirty-seven to twenty-eight. For example, they make thirty-seven into forty, reduce the other accordingly, and add them up. The result won't change. They need to have that flexibility. How could we phrase it? Instead of saying "includes components," can we say: "A student who understands the problem, decides what the correct operation is, and reaches the result using numbers flexibly. This is a student who is flexible in operations." It sounds like poetry.

Gonca: We could write Defne's sentence too, I think. After that, we can use mine as an example.

Lale: Sure.

While the teachers demonstrated ownership by developing definitions for the community, they still required my facilitation to stay focused on the task and draw conclusions by integrating their classroom experiences with concepts shared in the resources. This highlights the tension between fostering agency and providing necessary support. In this regard, while the teachers developed a sense of ownership over generating these principles as part of forming their collective professional learning processes community as a novel activity form, their collective action still required my facilitation to make progress.

Over time, the dimensions that teachers modeled through these activities were categorized into three groups: the principles of the community (e.g., focusing on mathematics teaching), general pedagogical principles (e.g., equity, technology), and the mathematics teaching approach (e.g., "Problem solving is not a separate learning objective; it is the essence of doing mathematics itself"). Each principle was developed through extensive discussions and revisited when they were documented in a shareable medium between sessions 18 and 22 and teachers came up with a shareable document including all these principles. Figure 8 illustrates this work, and Table 14 presents the English version of the content used during the activity.

1	matematik öğretiminin gelişimi
2	Evrlerle ilerlemek ve sürdürülebilirlik
3	Küçük ama emin adımlarla
3	Öğretmenleri süreçlere dahil etmek
4	Kartopu gibi büyükmek
5	Saha ile teori el ele
6	(Okul matematiği için) Öğrenme ortamları tasarımı
7	Uygulamadan öğrendiklerimizi yaygınlaştırmak
8	İşbirliği

4. SAHA İLE TEORİ EL ELE

Bir yandan bilimsel literatür ve (materyal veya sosyal) kaynaklardan beslenip, bir yandan sahada deneyimlemenin ve pratiğimize yansıtma yapmanın mesleki gelişim için temel olduğuna inanıyoruz.

5. İŞBİRLİĞİ

Farklı uzmanlık alanlarından kişilerle zaman zaman ilişkilenecek.

KAYNAKLAR

İnfografikte kullanılan kaynaklardan alıntı yapın

Figure 8. The Community transfers their notes on the Our Playground document to a shareable medium

Table 14. English Version of the Content in Figure 8

English version of the content in the left column of Figure 8.		English version of the content in the right column of Figure 8.
1. Development of Mathematics Education	Our community's goal is to contribute to the development of mathematics education in our country and to the formation of a teaching culture.	4. THEORY AND PRACTICE HAND IN HAND We believe that drawing from scientific literature and (material and social) resources, while simultaneously gaining field experience and reflecting it in our practice, is fundamental for professional development.
2. Progressing through evolving and Sustainability	Our goal is to use the Lesson Study model as a starting point, and through practices and improvements, to develop our own sustainable activity model.	
3. Small but Sure Steps	We want to move forward with small but sure steps.	
4. Involving Teachers in the Processes	We aim to involve teachers outside the community in the processes through various channels. Before spreading the activity model we have developed, we want to experience it ourselves.	
5. Growing Like a Snowball	When it comes to spreading the model we have reached, we plan to start with highly motivated teachers and gradually expand it.	
6. Theory and Practice Hand in Hand	We believe that drawing from scientific literature while simultaneously gaining field experience is essential for professional development.	
7. Designing Learning Environments (For School Mathematics)	Design bridges theory with practice and provides us with a space to test what we've learned; therefore, we want to design math learning environments together and apply and improve them in the field.	5. COLLABORATION Engaging from time to time with individuals from different fields of expertise.
8. Spreading What We Have Learned from Practicing our Model	While implementing our activity model, we want to share the experiences we've accumulated regarding mathematics teaching and learning with our colleagues through various channels.	
9. Collaboration	Community principle: Engaging from time to time with individuals from different fields of expertise. Açelya listed people who can be connected. This will be shaped according to need. For example, with a mathematician...	
Ecological validity		

Between sessions 22 and 28, the community worked on modeling their activity prototype from the ground up, using the Lesson Study model as its anchor. This marked their transition to collaborative lesson design, a goal they had envisioned from the beginning of the process. Throughout their work, teachers reimaged Lesson Study in the context of the systemic problem they had identified in their country. For instance, in the 17th session¹⁵, first, teachers revisited their reasons to implement Lesson Study. when the community revisited multiple reasons for implementing Lesson Study. Some of the reasons that Lesson Study attracted teachers were as follows: Well-grounded lessons (Defne), detailed discussion of the content, "freeing the lesson from randomness" (Açelya), being proactive about students' emerging needs (Lale), increasing the "ecological validity" of the scientific ideas (Demet), teachers acting as "researchers" (Lale), "holistic approach" to lesson planning (Yaprak). Teachers emphasized that in such a structure, teachers act as researchers, empowering them in teaching methods and that it provides a medium to deepen in how teaching can be conducted.

Once the community solidified their desire to adopt an activity prototype similar to Lesson Study, I introduced a website (lessonresearch.net) containing guidelines for groups to implement Lesson Study. This functioned as another second stimulus for teachers to anchor their modeling work. The sessions followed each other to complete the task at hand. Subcommittees were organized upon need (e.g., subcommittee to improve the design of the shareable document/poster, to refine the

¹⁵ The sixteenth and seventeenth sessions were transition sessions, during which I prepared short video clips from previous discussions where participants shared their ideas on how Lesson Studyteachers should work and collaborate. These ideas mostly corresponded to their discussions on lesson study (primarily in the fifth session), highlighting their reasons for viewing it as an ideal model for teacher collaboration and professional learning. Consequently, starting from the twenty-second session, the teachers focused on modeling their activity prototype based on the Lesson Study model.

language of each element). While the steps of Lesson Study and the details of these steps described on the guiding website were mostly kept, novelty was added in details of its parts. For example, in this guideline, research theme for a Lesson Study practice focused on a group of students' learning needs, however, at some point of their 27-min discussion, teachers added a component for teachers' learning as well with Lale's contribution to Ali's offer:

Ali: So, you know, we are going to identify a problem, right? Our biggest problem is: What would a lesson design that belongs to the students look like? I mean, we release [the control of the lesson]; activate students' thinking, cognitive processes.

Lale: In fact, we're also discussing creating space for the teacher. Because what lies behind this planning [process] is that the teacher finds space for themselves so that they can reflect, design, and share these plans. By creating space for themselves, they are actually trying to create space for the student. In fact, this is a highly interconnected process, isn't it?

Following these conversations, they formulated their ideas as research questions to explore within a Lesson Study practice, such as, "How can space be created for students' thinking in teaching through problem-solving?" (with multiple sub-questions) and "How does the Lesson Study process affect the teacher's learning process?"

At the end of the process, which included the sessions between the 22nd and 28th, the teachers concluded with an activity prototype and a set of activities for its implementation in the community's subsequent activities, with some parts to be detailed during implementation.

As a result, the solution developed throughout the sessions dominated by the modeling action involved the teachers' collective action to generate shared pedagogical principles and goals, along with an activity prototype. This solution, which deliberately refrained from the prompt implementation of Lesson Study, represented a novel, locally produced response to the systemic problem. It reflected

the final indicator of agency formation at the collective level within the scope of the PMTLE study.

According to Engeström (2015), resolving a dilemma through a new instrument "demands experimentation, borrowing, or 'conquering' already existing artifacts... for new uses" (p. 132). In the case of PMTLE, by bringing different tools together over an extended process, teachers created their own novel process of professional learning to respond to the systemic contradiction underlying the dilemmas that they face in practice, resulting in outcomes for which they developed genuine ownership.

Let me remind the reader of the dilemmatic situation—double bind—that mathematics teachers in Turkey collectively faced, as identified through the analysis of the fourth session (p. 140):

On one hand, they were expected to transform their practices to make mathematics learning more enjoyable and meaningful for students, driven also by their own pedagogical values. On the other hand, they were expected to adhere to a system that failed to coherently support such efforts, due to the absence of a sustainable and consistent pedagogical perspective communicated through frameworks.

Caught in this dilemma, the teachers struggled to find sustainable ways to pursue meaningful professional development. However, through a locally developed professional learning process within PMTLE—including the creation of a shared pedagogical perspective and an activity prototype—they established alternative approaches to sustaining cultural transformation in their practices and beyond. This structure addressed the lack of supportive frameworks necessary to unite teachers in pursuing their transformation goals. In doing so, they became conscious and strategic drivers of their own professional learning, enabling them to reconnect with their values and pursue their goals.

An important finding under this indicator is that my facilitation, as the researcher-facilitator, was crucial in keeping the teachers on track and sustaining their progress toward their goals. However, this can be considered adverse evidence for agency formation among teachers, highlighting the need for further research on leadership mechanisms and the nature of research-practice partnerships to address this issue. The next research question will explore the process from the teachers' perspective.



CHAPTER 5

TOOLS, ACTIVITIES, AND SOCIAL INFRASTRUCTURE TO SUPPORT TEACHER AGENCY IN PROFESSIONAL LEARNING

The findings of the analysis shared in this chapter provide nuanced insights into how to design a participatory professional learning environment for teachers, with the goal of supporting teachers' agency in co-designing their professional learning processes in conscious and strategic ways. The design principles, derived from the literature, were identified at the beginning of the process and shared in the Methods chapter. Based on these principles, the initial PMTLE design was developed, which was also outlined in the Methods chapter.

The design elements were categorized according to Sandoval's (2014) conjecture mapping framework. The first category includes tools and materials. The second category encompasses task structure, participant structure, and discourse practices, which Sandoval refers to as the social infrastructure of the design.

I analyzed the PMTLE design by examining how it supported or inhibited the emergence of each indicator of agency formation at both the individual and collective levels. The findings revealed that, during the implementation of the design, various emerging features also contributed to the process, offering insights into how the PMTLE design could be improved for future implementations. These reflections will be shared at the end of each section which answers the research question for each agency indicator. At the end of the chapter, a summary table will be shared.

5.1 Design features for E1 indicator: Creating safe spaces for expressing vulnerable feelings and providing comparative tools for reflection

The analysis of the second individual interviews revealed that two specific aspects of the design facilitated teachers' articulation of critical conflicts in the collective space, i.e., the first indicator of the formation of agency at the individual level. First, the social infrastructure that informed the relational dynamics within the PMTLE created a safe space where teachers could openly express vulnerable feelings. From the first session, encouraging teachers to share their reflections, rather than imposing a specific direction—focusing on listening and prioritizing an understanding of their struggles and motivations—helped establish this space. Lale described the "atmosphere" of the meetings and the organization behind them, which fostered the comfortable sharing of "individual teaching troubles":

Lale: The atmosphere of the meetings was also very important. I think it should have been a type of meeting where comfortable sharing and a sharing environment were provided. Because it already needed to be a process nourished by our teaching experiences, that is, our individual teaching troubles... Now, that comfort—the feeling that sharing a problem or challenges related to teaching is not a weakness... In a way, the type and nature of these meetings also evolved us to this point. Not holding meetings in a dictating manner, but in a more organic structure; designing them as a sharing environment where you [researcher] wouldn't say things like, "No, not like this," or intervene directly. (Second interview)

Second, *The Teaching Gap* and the lesson videos from other cultures in the TIMSS Video Study archive (timssvideo.com) played a significant role in helping teachers articulate their critical conflicts, initiating the agency formation at the individual level. That is, they functioned as the mirror material by triggering teachers' critical conflicts and making disturbances, critical incidents, and problems in their practices visible. The data shared in the book provided "evidence and examples of recurring problems and disturbances in their activity," offering an opportunity for reflection on a pressing critical conflict by making it visible for further elaboration (Sannino et al.,

2016, p. 5). The videos presented teaching situations that either contrasted with or aligned with the dominant teaching practices they had witnessed in the system and implemented in their own classrooms. This initiated the experiencing process for them, not only by identifying critical conflicts but also by reminding them of the pedagogical values they needed to shift from in practice. Yaprak specifically referred to the Japanese videos of mathematics classrooms. She shared that they reminded her of the lesson plans she had once developed in alignment with her values years ago:

Yaprak: What affected me the most in this process were those videos. I mean, the attitudes of the teachers in them. They made me think again that the things I wanted to do should be those. They took me back to my plans from years ago. (Second interview)

As a less experienced teacher compared to Yaprak (25 years of experience), the book helped Lale (5 years of experience) reevaluate her own teaching practices and pedagogical values and align with one that coheres with her perspective:

Lale: We were already influenced by the lesson in Japan... we were impressed as teachers. At one point, I felt something like, "I want to be a teacher like that too." Why did I feel that way then? I think I saw a part of the teacher I want to be in that example. [...] I don't think I will ever overlook what I saw there from now on—I really don't. Even when I tried to apply it, I felt better and thought, "Yes, yes, I'm on the right path. Let me work on this a bit more." (Second interview)

The data coming from the transcript of the second session also supports this finding, adding dimensions regarding how this could be possible. Focusing on the moments when teachers shared their internal conflicts in the collective space, I found that they used the conceptual tools from the book to express their critical conflicts. Notably, the concept of "teaching culture" helped them articulate their conflicts and reconsider their pedagogical values. Gül's accounts during the second session reveal that the Japanese teaching prototype, along with the contrasting example of the U.S. teaching prototype shared in the book, reminded her of her priority to help students "make sense of" mathematical ideas. She also reflected on her struggle to develop a teacher

identity distinct from the dominant "teaching culture," another concept frequently discussed in the book:

Gül: When I started teaching, at different times, because I used to learn this way, I used to emphasize the importance of making sense of something ... Then, from time to time, we detach and come back to the teaching culture applied to us. I've experienced, lived through exactly that.

The analysis of the interviews also revealed that the approach authors followed while presenting their study and their ideas was an important variable helping teachers use it as a tool for critical reflection. The book presented a scientific study along with raw data, rather than prescribing solutions. The authors invited the reader to become partners in the analysis process by transparently sharing the research behind the book's findings:

Lale: The book doesn't dictate anything to us. When we had discussions about it, none of us said, "it says this here, and this wouldn't work in reality," because there are videos of the system they tried or studied. We could see ourselves that it worked.

All in all, the social infrastructure of the PMTLE, which prioritized understanding each other's struggles and motivations in teaching, the data shared and conceptual tools utilized in the book, and the way the book presented a research study helped teachers comfortably share their internal critical conflicts in the collective space. It reminded them of the values they attached to teaching and initiated the experiencing process, i.e., agency formation at the individual level.

Based on these findings, I will reflect on the initial design supporting the E1 indicator.

The first design principle of PMTLE was "support teachers in collectively questioning their teaching practices and identifying their shared problems by providing first stimuli." To embody this principle, I utilized *The Teaching Gap* and videos from other cultures, which were intended to serve as first stimuli. The analysis

confirmed that they fulfilled this role. Data also revealed nuances in how these stimuli supported agency formation during the questioning process. Lessons learned regarding the design and suggested embodiments can be summarized as follows:

- First stimuli mediates not only collective questioning, but also surfacing individual critical conflicts. Unlike traditional formative interventions, such as Change Laboratory interventions, this study did not utilize ethnographic data from practitioners' settings as mirrors of problematic situations.

However, the findings showed that groups from different institutions can still be supported in articulating their critical conflicts by using tools such as *The Teaching Gap*, which:

- provide practitioners with opportunities for reflection by presenting alternative cases they can compare and contrast with their own practices,
 - offer conceptual tools to help practitioners reflect on and articulate their critical conflicts, and
 - do not dictate solutions, but instead provide data—compensating for the lack of ethnographic insights from practitioners' settings—and a scientifically grounded analysis that participants can use during reflection activities.
- First stimuli should be presented within a social infrastructure that encourages teachers to share their reflections, rather than imposing a specific direction. The focus should be on listening and prioritizing an understanding of their struggles and motivations.

5.2 Design features for transitioning from E2 to LIII-a indicator: Writing reflection papers and subgroup discussions for inquiry on dilemmas, sincerity, and maintaining identities

As revealed by the analysis addressing the first research question, during the fourth session, teachers sought to develop a new explanation for their internal conflicts—representing the second indicator of agency formation at the individual level. They also extended this inquiry to the collective space, leading them to identify a double-bind situation experienced by mathematics teachers in Turkey, which became the first indicator of agency formation at the collective level.

The data analysis addressing the second research question revealed that reflection papers, sharing them in small groups, and engaging in discussions without my facilitation supported these processes. Specifically, writing reflection papers allowed teachers to genuinely express their emotions. The sincerity of these shared reflections helped teachers recognize the commonalities in their emotions as a result of the "difficulties" they faced in practice, giving them a sense of "relief," in Yonca's words: "Of course, we realized that the difficulties we each individually experienced were shared by all of us. That gave us some relief" (Second interview).

This sense of relief encouraged them to seek a redefinition of their internal conflict rather than blaming themselves for being unable to overcome it. For example, once Lale realized that other teachers—including the most experienced ones—still faced similar challenges and conflictual situations in practice, she shifted her focus from asking "is there something wrong with me as a teacher or with my students?" to embracing a more open-ended vision of professional development. This shift gave her a sense of relief and motivation, similar to Yaprak:

Lale: For example, here we talk about "Well, this is what happens [in my teaching]. Does this happen to you too? I wonder if there is something wrong with my children or me?" and in the parts where we share our troubles with each other, we say "I am not alone, yes, these things happen. It is not specific to just my students, there are people, teachers, who have the same concerns as me." For example, Yaprak's experiences, for example, how many years older than us she is but she still worries about things too... You know, in life we cannot say that we have become human. Human being, that continuity, continuity continues, I think, the "teacher being" part also continues, in other words, there is no such thing as I have become a teacher, that is a process. It is a process that will always continue.

Yonca and Lale's accounts showed that reflection papers and the subgroup work supported the second indicator of agency formation at the individual level. These teachers mainly described the psychological transition as a sense of "relief" and a process of self-inquiry to better understand their situation. On the other hand, other teachers' accounts provide evidence regarding how this process was linked to agency formation at the collective level. Teachers' accounts revealed that this activity supported the transition from the individual to the collective plane through emotions. That is, critical conflict expressions served as materials supporting this transition. Gül's account, for instance, expressed how this psychological transition at the individual level was connected to the community's development. According to Gül, the realization that they experienced similar conflicts—which could only be uncovered through emotional openness made possible by the reflection paper activity—was crucial for the community's development:

Gül: [...] what makes a community is our shared struggle with the same problem. Those reflection papers actually describe very well what we go through in our professional processes. It [the reflection paper activity] also expresses our emotions very well... In such environments, while our minds are more active, when we write, our hearts are more active. I saw a lot of heart there. I saw a lot of heartfelt sentences. (Second interview)

The subgroup activity enriched with teachers' reflection papers have been a first step helping to transform a group of individuals into a collective by making the problems

they were trying to handle individually public. It represented a transition from individual to collective problem identification.

However, this transition also highlighted individual differences and personal viewpoints, which teachers valued because it allowed them to remain present in the community with their unique identities:

Gül: What gives me a sense of community is our gathering around a common purpose. Everyone is focused on the same... Well, not the same problem, but of course, we have different perspectives, and we reach an agreement by discussing them.

Açelya shared similar accounts about how the transition from the individual to the collective plane helped surface different viewpoints. From her accounts, it becomes clearer how this transition was crucial for teachers to remain present in the collective space while preserving their individual identities:

Açelya: Those reflection papers were actually very valuable. While doing group work, reflection papers helped us become aware of the points we were focusing on from our perspectives. For example, it was very valuable in terms of giving insight into what concepts or problems were most important for my friends. For instance, sustainability is a key issue for Demet. For Gül, it's the part that directly needs to be applied to real practice... This is why it was so valuable. In fact, the reflection papers had a huge impact on us getting to know each other as a group. (Second interview)

Açelya referred to the various viewpoints on defining the double-bind situation.

While both of her group mates, Demet and Gül, agreed that the lack of systemic effort to collectively uphold and pursue pedagogical goals and values in mathematics teaching practices was an underlying cause of their shared conflicts and dilemmas, they described the system at different levels—Demet at the national level and Gül at the local level (school or department). These different viewpoints did not change the community's focus on shared problems, as Gül emphasized in her quote above. On the contrary, they helped the group get to know each other better and remain present

with their differing identities, while working toward organizing their collective professional learning processes around these collectively defined problems.

As a result, reflection papers, along with sharing and discussing them in subgroups, supported the transition from the individual to the collective plane in identifying the shared problems. Writing reflection papers allowed teachers to express the emotional outcomes of their internal conflicts, while sharing them in the community space helped them engaging in a quest to redefine their internal conflict situation and transition to the collective plane in reformulating these internal conflicts as a collectively experienced societal dilemma—i.e., a double bind. This process also highlighted individual differences and contributed to developing a sense of community, where teachers could remain present with their differing identities, contributing to the agency formation at the individual level.

Based on these findings, I will reflect on the initial design supporting the emergence of the E2 and LIII-a indicators and the transition between them.

The activities conducted in the fourth session also aimed to embody the first design principle: "Support teachers in collectively questioning their teaching practices and identifying their shared problems by providing first stimuli." Unlike the previous sessions, the goal of the fourth session activities was to support teachers in further articulating their reflections on their problems within the community, thereby fostering the formation of collective agency through shared problems.

The analysis revealed that these activities fulfilled this role, but the nuances of how it supported the agency formation during the questioning process emerged from the data:

- Writing is a powerful tool for expressing internal conflicts and maintaining identities in collective activities. Accordingly, it can be used to support teachers in further articulating their internal conflicts.
- Sharing these reflections in the collective space—specifically in sub-groups without an external facilitation, which encourages more sincerity—should also be included in this activity. This approach supports, first, the teachers’ process of redefining their initial internal conflict situation and, second, their transition from the individual to the collective plane in problem identification, all while maintaining their individual identities. In this process, their expressions of internal conflicts served as materials supporting for this transition.

Based on these lessons, I revise the design principle one as "support teachers in individually surfacing their critical conflicts in the collective space providing first stimuli and transitioning to collective questioning of the underlying systemic contradictions by using individual critical conflicts as materials."

5.3 Design features for the LIII-b indicator: Using stimulated recall to develop a shared language and identify initial novel questions to guide community activities

The stimulated recall interview activity, implemented in the sixth and seventh sessions, was designed to support teachers’ agency in transitioning from the questioning to the analysis expansive learning action. The analysis of the interview data provided partial evidence how this activity mediated this transition by supporting teachers’ agency formation at the collective level—specifically, the collective construction of novel questions based on the double bind situation and researching to address them.

First, through this activity, teachers engaged in collective construction of novel problems as a step toward taking control of the double bind situation. The activity supported this by first triggering a reflection on their use of taken-for-granted concepts such as "curriculum" or "teaching guideline" and helping them develop a shared, clear language. This clarification elevated their discussions to a new level, making their conversations about their problems more systematic, "goal-oriented," "organized," and "rhythmic," in Demet's words:

Demet: I think the most impactful thing here was you re-watching the recordings and analyzing them. That was an incredible feedback for all of us. Because you collected them and presented them to us again. We were just talking in the natural flow, as if we were in the teachers' room... But as a result of your summaries, the categories that emerged in a systematic way were very meaningful. It was as if the things we talked about started to evolve on their own afterward... Even the things [terms] we were used to [took for granted], we started questioning and trying to name and clarify them. After they emerged—after you brought them out from our previous conversations—our discussions began to become more meaningful and goal-oriented. We progressed in a more organized, rhythmic way. (Second interview)

Through this activity, the community began to evolve in a direction that was meaningful for teachers. Their previous discussions resembled those held in the "teachers' rooms," in Demet's words, by which she meant conversations that were arbitrary, unfocused, and lacked a clear goal. The analytical perspective used to structure the stimulated recall interview helped them reflect on their thinking and integrate into the collective thinking of the whole community. Echoing Demet, Lale shared that these discussions became "more structured" through this activity, improving their "quality." Furthermore, the discussions became more "productive," helping them ask, "what's next?" after clarifying the common points of their problem discussions—making them more "compact":

Lale: You were analyzing the meetings we had, going through them meticulously behind the scenes until the next meeting... Then, you would show clips or play audio recordings to identify common points... This process

was the intermediate form where those discussions began to take shape and become more structured. It was a process that increased the quality and productivity of our discussions... and here, making things more compact helped us move on to the next stage, the part where we could say, "Yes, we did this, here's what we got from it, now what's next?" (Second interview)

As the findings of the first research question revealed, the community answered the question, "What's next?" by posing new questions to make progress as a community. Instead of repeatedly discussing the same systemic problem situation—lack of a unifying framework— the community formulated new questions related to this problem, which provided a foundation for their future activities. These activities were either for further analysis of the problem or for developing local solutions in the form of consciously and strategically co-designed professional learning activities.

In addition to clarifying concepts and making discussions more systematic, the following quotes show that revisiting everyone's ideas through this analytical lens made participants feel that their ideas were important, providing the actual foundation for the community to move forward. For Defne, nothing that was said was "actually wasted," while for Demet, the activity didn't take for granted what everyone said and contributed:

Defne: Nothing is overlooked. That macro perspective makes me feel really good... Yes, we are doing something, but, for instance, it didn't escape your notice. Then we come back to it again. The idea of revisiting had a big impact on me, for example. Because nothing we say is actually wasted, everything gains value, everything has significance... That macro perspective... honestly, it satisfies me a lot. (Second interview)

Demet: ... We get the opportunity to look at the community from a broader perspective. For instance, what everyone said, what they did, how they contributed, and how feedback was provided—all of this. For me, this was a different and very beautiful experience that I learned and gained here. (Second interview)

Through this activity, teachers also began to better realize and adapt to the social infrastructure underlying the PMTLE design. It reinforced my role in the community—not as someone dictating a specific direction, but as someone helping

them find their own way. The analytical work I did to support this process particularly impressed the teachers, as it demonstrated that I valued their ideas. This, in turn, reinforced an alternative approach to the researcher-teacher relationship, leading Lale to describe my attitude as something beyond the typical role of a researcher, as it differed from mainstream approaches to researcher-teacher interactions:

Lale: Because actually, it was a meeting like this... Because there were statements that even I had forgotten. I said them in a [previous] meeting, but I didn't remember after a long time. To flash back to them... and the fact that you carried out all this work in the background was an incredible effort. But this inclusive and understanding [attitude of yours] toward all of us... I don't know if this is a bit personal. It's not because of your role as a researcher—I think it's because of your character. But I think it was very good for all of us. (Third interview)

Furthermore, this activity helped the community meaningfully identify their problems before moving on to the stage of modeling a solution. Defining the problem collectively provided a sense of relief—which began in the fourth session—and a feeling that the community had control over these issues. This, in turn, helped teachers shed the sense of unease, as Defne put it, that might lead them to take hasty steps toward solutions. This approach justified taking things slowly, rather than rushing to prescribed solutions:

Defne: So, we put something clear in front of us... To solve a problem, you need to define it. And this, I mean, it's like the problems we solve in mathematics. What is the problem? That's why I think it's the most important stage. Yes, this is the problem. These, these, these... What can be done about them? ... Because this has been thought collaboratively and shaped by the experiences of many people... Yes, these are the problems, and of course, influenced by what has been discussed, and we need time to solve them. So, you have to be patient, you have to persevere... It's actually like this: it's something psychological, like you feel uneasy, because you can't define the problem. And therefore, you can't solve it. But when you define the problem, you can take a step towards solving it (Second interview)

Defne described the entire process as a problem-solving process, placing extra importance on problem identification activities, such as the stimulated recall interview, in helping the community move forward with confidence.

On the other hand, from the analysis of the interview data, I didn't identify any clear evidence that teachers were aware that this activity helped them develop novel research questions for further analysis of the problem situation—except Lale's quote shared above, which highlighted that the activity helped them ask "What's next?" and Demet's quote, which pointed out that the community "evolved on its own afterward." However, the analysis conducted to address the first research question through the examination of the transcripts of the sixth and seventh sessions revealed this transition. Still, this could be interpreted as a failure in the implementation of the design, stemming from the timewise mixing of activities focused on the further analysis of the systemic problem (e.g., the curriculum [NCTM vs. MEB] comparison activity) with activities aimed at developing novel and local solutions (e.g., reading and discussing NCTM and Van de Walle et al. [2020] to develop the community's pedagogical principles).

The interview data also did not reveal teachers' reflection on the PMTLE design in terms of how it supported their conducting research to answer their questions regarding the systemic problem, despite the analysis conducted for the first research question revealed that this occurred during the 10th, 11th, and 16th sessions. This discrepancy may be due to the limitations of the data for this analysis, as the second interviews were conducted before these sessions took place, and the third interview occurred a long time afterward, shifting the teachers' focus to other activities.

Based on these findings, I will reflect on the initial design supporting the emergence of the LIII-b indicator.

The stimulated recall activity, the prominent activity conducted in the sixth and seventh sessions, was organized to embody the design principle two: "Facilitate collective identification of the systemic contradictions underlying their problems and the analysis of their historical and actual-empirical causes." Additionally, this activity was conducted within a social infrastructure informed by the fourth design principle derived from the literature: "facilitate participant-led problem identification and collective design through responsive facilitation and open-ended inquiry." To embody this principle, I framed my participation (participant structure) as the formative design of sessions, shaped by participants' input, to support their agentic progress in expansive learning. Also, as a discursive practice, open-ended questioning to support teachers in challenging or building on each others' ideas was adopted.

The findings presented in this section revealed that the activities organized to embody these principles supported the second indicator of agency formation at the collective level for analysis expansive learning action, namely, teachers' collective construction of novel questions based on the double bind situation and conducting research to answer these questions—albeit partially evinced. The lessons derived from this analysis are as follows:

- Reflection activities that organize teachers' previous discussions on their problems, while incorporating each unique idea from an analytical perspective, can be used to support teachers' collective agency in developing questions to analyze the systemic problem. One of these activities is the stimulated recall interview technique, which, within PMTLE:

- helped to develop a shared language, triggering a reflection on their use of concepts such as curriculum or teaching guideline,
 - elevated teachers' discussions about their problems to a more systematic form, and
 - supported the community in moving forward in a more organized way, based on teachers' ideas and input.
- These activities should be conducted within a social infrastructure that emphasizes the researcher's role in using analytical tools and methods to support teachers in shaping their unique process, rather than dictating a specific direction.
 - The sequential organization of the analysis and modeling activities can be more effective in helping teachers become more autonomous in initiating and carrying out both types of activities.

Based on these lessons, I revise the design principle 2 as "facilitate the transition from questioning to collective analysis of contradictions by supporting the organization of problem discussions and the development of novel questions within the community."

5.4 Design features for E3 indicator: Tools for reflection, discourse on expansive views of good teaching, and peer experiences as resources for reuniting with personal values and identifying PD needs

The analysis of the interview data revealed that teachers' developing individual disposition to act consciously to resolve their internal conflict, the last indicator of agency formation at the individual level that I focused on in this study, became possible

through the social infrastructure of the PMTLE design in combination with the tools, such as the book and the videos.

First of all, the readings and teachers' experiences mediated teachers' reflection on their own pedagogical values and gain self-awareness about them.

Yaprak expresses clearly how she reunited with her own values through PMTLE:

Yaprak: The work we did in this group actually helped me become more self-aware. It felt like it allowed me to discover myself. I attend trainings, but you can't engage in such deep sharing there... The things I've encountered here, whether in books or through the contributions of others in the group, made me feel like, "Yes, this is exactly what I've been looking for." Perhaps the greatest discovery for me was myself—realizing the value of what I do....
(Third interview)

Yaprak continued with my role as a researcher, which she describes as being a role-model for a teaching practice that cohere with their values having features of "without pressure," "giving gentle nudges," "asking plenty of questions" in one's self-discovery process, in Yaprak's words:

Yaprak: You also served as a guide for teaching, demonstrating us what teaching could be—without pressure, but by shedding light, giving gentle nudges, and asking plenty of questions, creating people who believe in their ability to succeed in the classroom. That belief in me has grown even stronger. I mean, I am on the right path. I am valuable with what I am doing.
(Third interview)

In her quote above, with the phrase "I am on the right path," Yaprak didn't mean that she developed over-confidence about what she was doing (this was validated with her accounts during the sessions) but that she reunited with her values and gained confidence to go after these values by engaging in professional learning activities in a more self-confident and conscious ways.

The discursive interactions that I encouraged helped them in this self-discovery process. I followed an "asset-oriented" perspective and encouraged an "expansive view of good teaching" (Horn & Garner, 2022) while teachers were questioning their practices. This view emphasizes evaluating teaching from an

exploratory and interpretative perspective, offering enriched opportunities for professional learning, as this stance "makes teaching choices discussable" rather than simply categorizing them as good or bad from a binary perspective (Horn & Kane, 2019; Horn & Garner, 2022, p. 125). An example of this general attitude can be found in my feedback to their reflection papers. Figure 8 and its English version in Table 15 show my feedback to Gül's reflection paper while she tended to blame herself for not being able to pursue their values in practice.

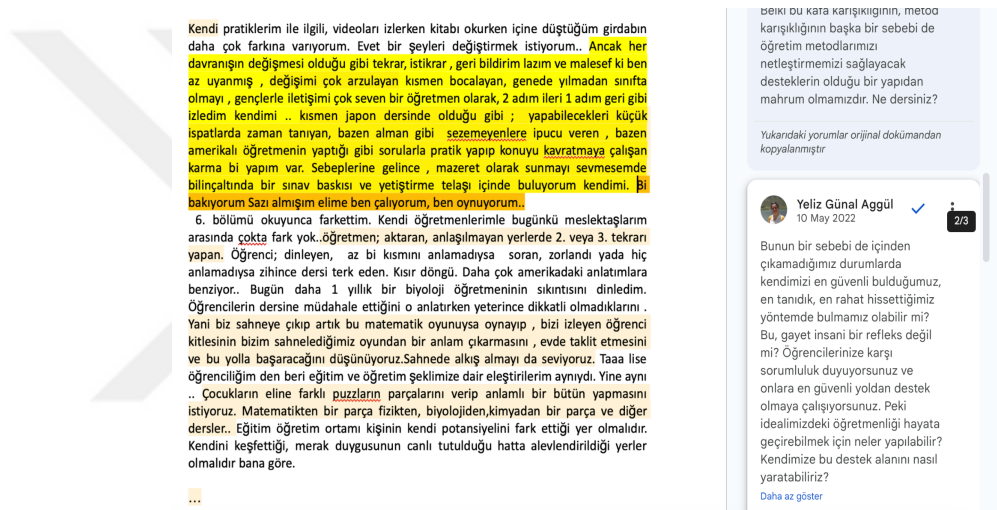


Figure 8. Gül's reflection paper and YGA's comment showing the enactment of the social infrastructure based on an expansive view of good teaching

Table 15. English Version of the Content of Figure 8

The content of Gül's reflection paper	Comment from YGA
Regarding my own practices, I became more aware of the vortex I fell into while watching the videos and reading the book. Yes, I want to change something.. However, as in every behavioral change, repetition, stability and feedback are needed. And unfortunately, as a teacher who is a little awakened, who desires change very much, who is partly struggling, who still loves to be in the classroom without giving up and who loves communicating with young people, I watched myself as walking two steps forward, one step back.. Partly like in Japanese lessons; I have a mixed character [as a teacher] that allows time for small proofs that they can do, sometimes gives clues to those [student] who cannot make sense like the German, sometimes tries to make them understand the subject by practicing with questions like the American teacher does. As for the reasons, although I do not like to present it as an excuse, I find myself under a pressure of exams subconsciously and rush to catch up. Suddenly I see that I have picked up the saz [a traditional Turkish long-necked stringed instrument] and I am playing it, I am dancing.. I realized this when I read the 6th chapter [of <i>The Teaching Gap</i>]. There is not much difference between my own teachers and my colleagues today... The one who conveys, who repeats the content second or third time when it is not understood. The student; listens, asks if he does not understand a little part, leaves the lesson mentally if he has difficulty or does not understand at all. A vicious cycle. It is more like the explanations in America. Today, I listened to the troubles of a biology teacher who has only been teaching for one year. The students interfere in his lesson, they are not careful enough while explaining. In other words, we go on stage and play this math game, we think that the student group watching us will make sense of the game we staged, imitate it at home and succeed in this way. We also like to be applauded on stage. My criticisms about our education and teaching method have been the same since I was a high school student. It is the same again. We want children to make a meaningful whole by giving them pieces of different puzzles. A piece of math, a piece of physics, biology, chemistry and other courses. The education and teaching environment should be a place where the person realizes his own potential. In my opinion, it should be a place where he discovers himself, where his sense of curiosity is kept alive and even inflamed.	Could one reason for this be that in situations we cannot get out of, we find ourselves in the method we find most secure, most familiar and most comfortable in? Isn't this a very human reflex? You feel responsible for your students and you try to support them in the safest way. So what can be done to bring our ideal of teaching to life? How can we create this support area for ourselves?

Similarly, Lale emphasized how this study differed from the ones she participated previously in helping to reunite with her values. Similar to helping them to surface their internal conflicts, *The Teaching Gap*, by inviting the reader to be a part of the analysis process and combining the lesson videos as raw data with the reading

process helped teachers to reunite with their values. The way these resources were utilized was unique, rather than the direct transfer of ideas presented in the book. Reading *The Teaching Gap* and analyzing the lesson videos in the light of the book helped teachers to engage in the analysis process rather than adopting some ready-made answers. This facilitated their conscious and deliberate engagement with a professional learning process mediated by the book:

Lale: It helped me discover the strength within me, to summarize. Like Yaprak Hoca said, "There was always something inside me, but I needed to discover myself." I realized that I was actually thinking correctly and confirmed that I learn well this way. All of us had a shared goal: to have students who question, to encourage them to question and discover... For this, yes, there were a few methods suggested to us, weren't there? .. But [here] we figured this out on our own. We watched videos, and we got to see examples of different teaching styles. We analyzed and evaluated them... And this directly reflected in our classrooms. (Third interview)

Lale discovered that these tools and the way they were utilized within PMTLE helped them to be the main actors of their professional learning and engage in professional development in conscious ways, and encouraged them to implement new ideas in their practice.

On the other hand, teachers' accounts of increased self-confidence in their existing practices do not imply that the PMTLE merely reinforced what they were already doing. Instead, it broadened the problem space within which they engaged in professional learning activities. They began to view teaching—including the roles of teachers, students, and classroom content—as problematized in *The Teaching Gap* (Stigler & Hiebert, 1999), as a gateway to meaningful professional learning. For this, not only the book and the videos, but also the teachers' experiences, served as mediating resources for each other to develop new perspectives and restore their beliefs, enabling them to find numerous entry points for meaningful professional development. The PMTLE design supported teachers to draw on not only their own

but also each other's experiences as tools by opening them space for this sharing. For example, Gül explains how the problem space she used to conduct activities aimed at improving her practice expanded through the contribution of her friend Defne's experience. Defne taught fractions by presenting a traditional flatbread ('tandır ekmeği' in Turkish) instead of pizza, responding to the needs of her students who live in a culturally underrepresented and economically disadvantaged region of Turkey, based on their cultural identities. This shifted Gül's perspective and increased her curiosity about improving this aspect of her teaching—specifically, being responsive to students' needs for more inclusive and equitable teaching practices:

Gül: Everyone approached things from a different perspective, and this allowed others to voice things I couldn't see on my own, which also changed my perspective. In this way, we nurtured each other... For example, the exam system has always been a key issue for me, like a recurring problem. But through these discussions, this view of mine was pushed to the background... In this space, expressing it gave me the push I needed to take action. It became a reason for me to move forward. For instance, inclusivity wasn't a topic I thought about much—it wasn't something I saw as an important issue. Later, subconsciously, I realized that this topic indeed holds significance. Here, I discovered that even the way we frame a [mathematics] problem has to do with inclusivity. For example, Defne mentioned something—a specific example about using things like flatbreads or pizzas to teach fractions. Isn't that such a unique approach? It's a very effective approach, and I hadn't even realized it before. (Second interview)

By problematizing such aspects of teaching, she expanded her problem space, pushing other constraints that limit opportunities for professional development (PD) to the background.

Based on these findings, I will reflect on the initial design supporting the emergence of the E3 indicator.

The book and the videos, the way I facilitated teachers' process of self-discovery of their values and PD needs, and their experiences mediated teachers to develop individual disposition to engage in activities toward resolving their internal conflict, as the last indicator of agency formation at the individual level.

The lessons derived from the data analysis can be used to refine these design features and to improve the PMTLE design:

- The mediating resources, such as mirror material, can also function as resources that provide teachers a space to reunite with as well as reevaluate their values and identify their own PD needs, if they are approached as tools for analysis of teaching actions rather than instructive resources. *The Teaching Gap* and the accompanying lesson videos are one sample set of tools that mathematics teacher educators can utilize for this goal.
- Teachers' individual experiences should be considered as one significant resource for each other to identify their PD needs, expanding the problem space on which they conduct their professional learning activities consciously.
- This should be accompanied by a social infrastructure (participant structure, discursive practices) that is grounded on a researcher or teacher educator attitude that foregrounds an expansive view of good teaching, helping teachers to evaluate their current practices from an expansive rather than a "good/bad binary" view of teaching, encouraging an interpretative perspective regarding their practices (Horn & Garner, 2022).

The third design principle was related to supporting teachers in enacting agency for transition to the "modeling a solution" stage: "support teachers in co-designing novel solutions to the identified contradictions, as well as in co-creating or selecting mediating artifacts, concepts, or models to use as second stimuli in this process."

This analysis revealed the importance of supporting teachers' developing individual dispositions for a conscious transition to the modeling phase. Accordingly, the design principle can be revised as "facilitate teachers in developing both an individual

disposition and collective action to co-design professional learning activities as solutions to identified systemic contradictions and to co-create or select mediating tools that support these activities."

In addition, the sixth design principle has to do with the agency formation at the individual level: "support teachers in reflecting their individual histories in the setting, connecting their work context to the learning environment, and envisioning the learning environment as a resource for realizing their teaching ideals." To embody this principle, participant structure included researchers' encouraging teachers to contribute to community development with their diverse professional histories and experiences. This was supported by discursive practices that bring to light teachers' personal histories and diverse needs and goals in discursive interactions. The data revealed that these design features supported the agency formation at the individual level. I will keep this principle as it is.

5.5 Design features for LIII-c indicator: Flexibility in timing and the open-endedness for a novel process of co-construction of solutions, with a need of an "authentic" facilitation to stay focused on task

The analysis of the interview data revealed that teachers' engagement in collective action to generate and execute novel professional learning activities, responding to the collectively identified systemic problem—the final indicator of agency formation at the collective level within the PMTLE— was made possible primarily through the PMTLE's formative and flexible design approach. This approach included the integration of diverse expertise in selecting mediating resources and activities which contributed to the authentic development of both the partnership and the process.

The analysis of the data from the third interviews showed that dedicating considerable time to problem identification and analysis in PMTLE allowed teachers to gradually engage in the expansive learning process. This process facilitated their conscious and strategic transition to the solution-modeling stage. Demet described the process as having "spiral-like dynamics," beginning with asking questions and seeking answers, which made them feel as conscious and deliberate—or in Demet's words, "capable, thoughtful"---learners. By saying, "Together, we grew and expanded with that knowledge," Demet highlights the collective agency driving these professional learning processes:

Demet: I really felt the spiral-like dynamics within this group. We started from one point, there was a question in the middle, and then all of us suddenly became students—we questioned it, explored it, became learners, and then brought what we had learned back to the group. Together, we grew and expanded with that knowledge. This experience felt very, very different to me, truly... I think we all became capable, thoughtful learners together. (Third interview)

Similarly, Lale describes the dynamics of becoming thoughtful professional learners by highlighting the flexibility of the PMTLE design, which differs from mainstream PD programs. This flexibility allowed teachers time to process each stage, a fundamental factor in developing ownership over their professional learning journey. By using "we" as the pronoun, Lale emphasized how this process empowered them as collective agents of their professional learning:

Lale: We didn't move on to any stage without fully internalizing it. And there was no rush. I realized how incredibly important that is. You know, in a typical training program, it's usually something like six weeks maximum... it can't be extended. Here, though, it's more flexible, and... having the chance to discuss things at length when something doesn't quite make sense to me—and to truly internalize it—was really important, I think. (Third interview)

In addition to flexibility in timing, each session was formatively designed taking teachers' needs into consideration. Adapting sessions to address these needs fostered a sense of freedom, belonging, and ownership within the community. This approach

moved away from the traditional division of labor in such settings, motivating teachers to contribute to the co-modeling process. I shared my expertise when necessary, acting as an equal partner while maintaining the open-ended nature of the process. I supported teachers in bringing their experiences and expertise into the co-modeling setting. Lale describes this process as "a snowball gathering momentum," a metaphor illustrating how it started small but grew in size and impact, accumulating ideas and participation over time:

Lale: Your role as a researcher wasn't about sharply defining our path and saying, "We're going this way." Instead, it was more like your insights and what you brought from your research helped us grow together, like a snowball gathering momentum. Our path was never predetermined, like, "This is where we're heading." It was more about seeing where it would take us. We became a system where experiences were shared, discussions were held, and we grew together—a truly organic structure emerged. Your role in this... was never prescriptive. It wasn't about dictating anything, but rather, during our conversations, you'd share something like, "Oh, I read something about this." It was about sharing, brainstorming, and contributing what we all had in our pockets to enrich the process. (Third interview)

The third interview was a focus group interview conducted after teachers implemented the prototype. Based on this experience, they concluded that the professional learning activities in the PMTLE—which supported questioning, analysis, and modeling—were essential for developing a shared perspective. This shared perspective, in turn, contributed to the meaningful design and healthy implementation of the activity prototype. Açelya refers to the phase in which modeling activities took place as the "second phase," as we had named it, and highlights how this foundational work helped teachers take collective practical action with confidence:

Açelya: Everything we talked about and discussed in the second phase provided a framework for the subsequent steps—particularly in the prototype implementation process and lesson planning stages... we were moving forward with the support of the science and the sources we had studied in the background. This gave us strength and clarity as we progressed. (Third interview)

Regarding the emergence of the third indicator of agency formation at the collective level—collective action for generating new activity forms in response to a systemic problem—Açelya's accounts reveal her perception of the process as the community's conscious and strategic collective action toward a shared goal with confidence.

Similarly, Yonca described the process of building the Lesson Study as the community's activity prototype, characterizing it as a process of "gradually refining and building together." This process, shaped by all community members, culminated in their developing ownership over the final product:

Yaprak: If you had directly told us everything, maybe we wouldn't have taken ownership of it to this extent. It felt like we progressed step by step, gradually refining and building together. And in the end, it feels like we can say, "Everything we've done represents, in our view, the way we carry out Lesson Study." (Third interview)

In this focus group interview, I provoked the discussion by asking:

YGA: So, you didn't have a feeling like, 'Was all of this for nothing? Why didn't we just do this from the beginning? Why did we waste so much time—with all these discussions and problem-solving? If we were going to do this anyway, we should have done it from the start.'

Lale answered my question elaborating on why taking this process slowly, enriched with various types of activities, helped them digest the logic behind adopting Lesson Study as the community's activity. This collective process of building Lesson Study as the community's activity dispelled the feeling that they were "copying and pasting" a ready made solution:

Lale: What difference would it have made back then? It would have just been another copy-paste work. I mean, we're all already uncomfortable with popular culture [...] so we understood why this made sense. Putting everything else aside—even if we had copy-pasted the whole thing, it still wouldn't have been a problem for this Lesson Study. Because after researching it so deeply, we can now explain why it makes sense and view it from different perspectives. But of course, reading different sources, comparing them, and blending the insights we gained to create something new was a different experience. There was this sense of 'We did this.' That alone made a difference. Instead of just grabbing something as-is, we all felt a sense of contribution in creating something new.

Similar interpretations came from Açıya and Ali. Both emphasized the importance of the background work done before designing and implementing Lesson Study as the community's activity prototype. This work, developed as a novel response to the systemic contradiction they identified—lack of a unifying framework—was crucial. Açıya described this process as one that helped develop a common pedagogical vision, connecting different aspects of teaching together, like "puzzle pieces coming together," in her words. Ali described the process of determining the principles as "filling in the gaps by progressing with small and solid steps," a strategic and conscious process of reaching their goals. The full accounts of both teachers in response to my question were as follows:

Açıya: If we had come together without going through the first and second phases and simply said, "Let's start collectively and create a lesson plan," we would have ended up with something like this: a lesson plan resembling a Wednesday market. Because we wouldn't have reached a common understanding. [...] Naturally, by the end of the first and second phases, we knew what we were doing, we had reached a consensus, and at the same time, we had the power of science and the sources we had read backing us up. [...] In the end, if the process had started directly with the Lesson Study and we had just implemented it, we would have been focusing on the end without seeing the process itself. It wouldn't have been very meaningful, and it wouldn't have fully settled in our minds. In a way, this approach actually helped some puzzle pieces in our minds come together in terms of teaching.

Ali: If we had done as you said—starting with the Lesson Study and implementing it—we definitely wouldn't have built it together. That's for sure. Maybe it would have sounded like an appealing approach. "Look, we are doing Lesson Study." [...] But we wouldn't have been able to fill in the gaps by progressing with small and solid steps, as we had determined in our principles, or by growing like a snowball. Instead, we would have either failed to reach our goals or ignored them. In that sense, I believe that starting and progressing in this way is more valuable. [...] Of course, we don't all think the same; we have different perspectives. However, through our discussions and debates, what we have accumulated has brought us—this sentence is important—closer to a common language. We have started to reach a consensus on mathematics, or more precisely, on mathematics education.

This validated that, although teachers primarily adopted the general structure of the Lesson Study and the guidance provided by collectives who had experienced this

practice before (e.g., lessonresearch.net), the approach of taking things slow—starting with problem identification, followed by developing community principles using scientific resources in response to the collectively identified systemic problem, and discussing each step of the Lesson Study with the flexibility to make necessary adaptations—made this process entirely unique and novel for these teachers. In this regard, Lesson Study, along with the scientific resources and activities that teachers engaged with during the modeling process, acted as second stimuli—tools to mediate the design of a solution to the identified problems.

Although these resources were not crafted from scratch, the way teachers structured the process and adopted these resources and the Lesson Study made the professional learning process wholly unique. Thus, PMTLE supported teachers in exercising collective action to generate a new activity form in response to the systemic problem.

At first, the open-endedness of the process did not make sense to the teachers. Over time, however, they began to enjoy it, feeling that they were contributing to a novel process of co-construction. Ali describes the process as one of "democratic functioning, not directing":

Ali: At first, the call's content—especially the phrase "building together"—felt somewhat unconventional to us. We wondered, "What are we going to do? What are we going to build together?" In particular, I kept asking myself, "Where is Yeliz leading us? What is she trying to achieve?" I couldn't find many clues at the beginning. However, in the end, we realized something—we learned this through experience, as they say. We truly engaged in a process of learning by doing, and that was significant. Of course, as a researcher, she had a certain stance, a question in mind, or a research inquiry she had formulated. But instead of imposing it on us, she maintained a non-directive and discussion-oriented approach, which was crucial for us. It was a completely democratic process and mindset—not a governance, but rather an operation, a democratic way of functioning. In this sense, we genuinely felt, as we often say, valued.

All in all, with these supportive mechanisms, teachers developed a collective consciousness and the confidence to strategically organize their own collective professional learning processes. In collaboration with me, they took collective practical action to pave a new path toward realizing their dream—designing lessons together that would share the joy of mathematics with others.

On the other hand, as the analysis of the first research question revealed, teachers needed my facilitation to work toward collectively determined goals while conducting the community activities, which I interpreted as a limitation on their agency. Although these goals were set collectively, they still required my facilitation to keep them on track. Yaprak and Yonca's subsequent accounts support this finding while also adding another dimension to this relationship. They described my attitude as patient—maintaining a distant eye, opening space for their unique contributions, but also encouraging them to re-center their focus. They emphasized that this type of support was essential for their progress:

Yaprak: Okay. I have always seen you as someone who brings things together and keeps everything organized. Even when we lost focus at times, I saw you as the one who brought us back on track. I was deeply influenced by what you did. Thinking of myself as a student here, I realized how valuable that effort is, even as a teacher. I truly felt that from you. Your ability to observe everything from an external perspective was valuable. You both gave us space to grow and helped us move forward in that space, but at the same time, when we felt lost, you guided us back to the center. I believe that approach was truly valuable. (Third interview)

Yonca: I'd also like to thank you. You patiently listened to all of our ideas, and that meant a lot. Sometimes, we would drift away from the main topic and go in different directions. But even then, you patiently listened and gently guided us back, encouraging us to stay on track and keep the discussion focused. You were really successful in that, and we needed it. When we start talking, one idea leads to another, and before we know it, we've strayed far from what we were originally supposed to discuss. That might also be valuable in its own way, but it doesn't help us move our work forward. Maybe those thoughts [discussions] are meant for another time. (Second interview)

Gül designates the role that I assumed, based on these features of facilitation, as "authentic leadership" rather than "authoritarian leadership" in supporting teachers as they collaborate with each other:

Gül: I've started to believe more in collaboration. I had already believed in the power of working with teachers. [...] I really liked the way this was supported together with you. [...] But this leadership, in our definition, was much more of an authentic leadership rather than an authoritarian one. This felt good to me. [...] I feel like authentic leadership works better in these kinds of structures. Because a horizontal hierarchy is needed in these situations. And you made that work beautifully. (Third interview)

In this regard, while my leadership in its authentic form might initially be interpreted as limiting teachers' agency, teachers' accounts reveal a need for alternative research-practice partnerships to help them stay on track in advancing their communities. These partnerships also allow for using the community space to brainstorm ways to navigate the complexities of teaching, which sometimes cause the discussion to go 'off track' from the task at hand. That is, within their communities, they require open spaces where they can freely discuss daily struggles, share experiences, seek advice, and brainstorm without strict constraints—while still maintaining a focus on progress.

The analysis revealed that one dimension of this "authentic leadership" was established through my careful listening of conversations and helping teachers translate or connect their ideas to both the specific and broader goals of the community. This, in turn, contributed to a second dimension of "authentic leadership": creating opportunities to integrate my expertise as a mathematics education researcher and learning scientist with their field-based experiences and prior professional development. Furthermore, our collaborative work—such as jointly examining mathematics teaching practices from different cultures—provided yet another medium for fostering what Gül described as "authentic" leadership and

Lale referred to as facilitation respecting the "organic" emergence of the process. Lale previously described in detail as follows:

Lale: It was like this—rather than using a top-down tone or attitude [...] we all read things related to our profession, follow the news, and have our own classroom experiences. Your direct field of work was already this, but I never encountered any attitude or statement from you like "This is how it should be" or "It must be done this way." In none of our meetings did I feel like we were being dictated to. Instead, it was more like, "Oh, I read this too," as if we were brainstorming together. Your role as a researcher wasn't about sharply defining our path and saying, "We will go this way." Instead, through what you shared and the things you researched, we grew like a snowball. Our path was never rigidly defined as leading to a specific outcome—we were exploring where it would take us. We had our experiences, our readings, our research, and knowledge of how other countries implement education. There are many different education systems worldwide, and in our country, we also navigate the distinctions between public and private education. What emerged was an organic structure where all these experiences blended, were discussed, and helped us grow together. The outcome wasn't predetermined; it was all about the process. Yes, we would eventually reach an outcome, but that wasn't our main concern. We learned so much along the way. [...] instead of adopting a prescriptive tone, you shared things in a conversational manner—like, "Oh, I read something about this too." It was about sharing, not directing. The atmosphere wasn't one of instruction or enforcement, but of collaboration—where we pooled our knowledge and ideas together. (Second interview).

Lale's description of my facilitation as respectful of the 'organic' development of the process provides further insight into the type of researcher–teacher relationship necessary to prioritize the development of agency. Teacher educators sharing their expertise does not hinder the development of teachers' agency in leading their professional learning, as long as teacher educators position their expertise in relation to the teachers' experiences, prioritizing "an organic structure where all experiences are blended," rather than prescribing solutions, in Lale's words.

Based on these findings, I will reflect on the initial design supporting the emergence of the LIII-c indicator.

The co-designed modeling activities and my role as a facilitator in this process supported teachers in developing collective action to generate new activity

forms in response to the systemic problem they identified together. This served as the final indicator of agency formation at the collective level that I traced in PMTLE.

The data-driven findings reveal how the design supported or inhibited agency formation, as well as provide insights on improving the design to better support this indicator:

- Considerable time should be dedicated to problem identification and analysis before moving to the "modeling a solution" stage, as this allows teachers to pose collective questions, process each stage before transitioning to the next, and helps them make a strategic and conscious shift to this stage. In one of the participants' words, they become "capable, thoughtful learners together."
- Flexibility in both the timing and formative design of the process should be combined. This approach motivates teachers to contribute to the co-modeling process, and enhances their sense of belonging, rather than merely following a predetermined content, schedule, and goal.
- Previously developed and tested solutions, such as Lesson Study, can be utilized by teachers as responses to their problems. However, to ensure these models are adopted as collectively developed, relevant solutions—rather than being "copied and pasted"—, teachers should be given the time and flexibility to craft their own processes for developing the necessary background to address the gap created by their contextual problems.
- While my facilitation style, which Gül described as "authentic" leadership, kept them motivated to take ownership of the process, researchers' establishing a leadership role can be seen as a limitation on their agency. Authentic leadership in this context is built through careful listening, integrating expertise with practitioners' experiences, and fostering organic

collaboration, rather than enforcing a rigid trajectory. Although teachers' reflections suggest that this leadership style motivated them to continue the work, further research is needed to determine the extent to which this facilitation model supports or inhibits teachers' agency.

5.6 Reflection on the PMTLE design

Tables 16, 17, and 18 summarize the lessons learned from the analyses conducted to answer research questions one and two, based on the design principles. Design principles one, two, and three were embodied through tools and materials, and task structures while design principles four, five, and six were embodied through participant structure and discourse practices.

It has been identified that design principle one is related to the E1 indicator, as well as the emergence and transition between the E2 and LIII-a indicator. Design principle two is related to the LIII-b indicator, while design principle three is related to the E3 and LIII-c indicators. The analysis revealed that the embodiment elements mostly supported these indicators and highlighted the nuances of how this occurred. In Tables 16 and 17, the "lessons learned" column presents major lessons derived from the analysis for each specific principle, while the "suggested embodiments" column outlines the qualities of the embodiment that can support each indicator, based on the findings of the analysis. These principles were revised based on the prominent lessons derived from the analysis.

Design principles four, five, and six informed the social infrastructure of the design, primarily through participant structure and discourse practices, and applied to all indicators of agency formation. Lessons learned from design principles one, two, and three included insights related to the social infrastructure in which tools and

materials, and tasks need to be enacted. In this regard, embodiment suggestions for design principles four, five, and six emerged from these lessons. I did not conclude with any major findings that required revisions to these principles.



Table 16. Lessons Learned, Suggested Embodiments, and Revised Design Principles for the Design Principles One and Two

Initial design principle	Relation to agency indicators	Lessons learned from both analyses (RQ1 & RQ2)	Suggested embodiments	Revised design principle
DP 1: Support teachers in collectively questioning their teaching practices and identifying their shared problems by providing first stimuli.	E1 E2 and LIII-a	- First stimuli mediates not only collective questioning, but also surfacing individual critical conflicts. - Individual critical conflicts serve as materials for collective questioning.	- Alternative to ethnographic material from the subjects' settings, tools to support teachers in articulating their critical conflicts and then transition to the collective questioning can be materials (e.g., <i>The Teaching Gap</i>) that have the following features: - provide alternative cases to facilitate reflection on their own teaching - offer conceptual tools to articulate critical conflicts - do not dictate solutions, but provide data and robust analysis that mediate reflection - First stimuli need to be presented within a <i>social infrastructure</i> that encourages teachers to share their reflections on their individual practices. The focus should be on listening and prioritizing an understanding of each other's struggles and motivations. - Second level activities to articulate critical conflicts, such as writing reflection papers, and use of subgroups that provide more sincerity can be utilized to: - mediate transition from the individual to the collective plane in problem identification. - maintain identities in collective activities.	DP1 (revised): Support teachers in individually surfacing their critical conflicts in the collective space providing first stimuli and transitioning to collective questioning of the underlying systemic contradictions by using individual critical conflicts as materials.
DP 2: Facilitate collective identification of the systemic contradictions underlying their problems and the analysis of their historical and actual-empirical causes.	LIII-b	- Structuring conversations to explore problems systematically and encouraging teachers to generate new questions based on their discussions are crucial for helping them enact agency to transition from questioning to analysis. - The sequential organization of the analysis and modeling activities can be more effective in helping teachers become more autonomous in initiating and carrying out both types of activities.	- Reflection activities that organize teachers' previous discussions on their problems (e.g., stimulated recall interview), while considering each unique idea from an analytical perspective, can be used to support: - developing a shared language - elevating the discussions about problems to a more systematic form, moving forward in a more organized way - developing novel questions of the community regarding the collectively identified systemic problem. - A social infrastructure that leverages researchers' analytical tools (e.g., the stimulated recall interview technique, Maxqda for systematically categorizing teachers' discussions) is needed to support teachers in shaping their own processes rather than prescribing a specific direction.	DP2 (revised): Facilitate the transition from questioning to collective analysis of contradictions by supporting the organization of problem discussions and the development of novel questions within the community.

Table 17. Lessons Learned, Suggested Embodiments, and Revised Design Principles for the Design Principle Three

Initial design principle	Relation to agency indicators	Lessons learned from both analyses (RQ1 & RQ2)	Suggested embodiments	Revised design principle
DP 3: Support teachers in co-designing novel solutions to the identified contradictions, as well as in co-creating or choosing mediating artifacts, concepts, or models to use as second stimuli in this process.	E3	Transitioning to the solution modeling stage involves teachers developing not only collective action to generate new activity forms in response to the systemic problem (LIII-c) but also an individual disposition to act consciously in resolving their internal conflicts through collective PD activities (E3).	• Mirror material (e.g., <i>The Teaching Gap</i> and the accompanying lesson videos) can also be used as resources to support teachers in reevaluating their values and identifying their own PD needs. • These resources should be approached as tools for analysis of teaching actions rather than instructive resources. • Each other's individual experiences can also function as such resources, so space should be ensured for sharing them. • Social infrastructure that is grounded on an expansive view of good teaching, rather than a binary one should be followed.	DP 3 (revised): Facilitate teachers in developing both an individual disposition and collective action to co-design professional learning activities as solutions to identified systemic contradictions and to co-create or select mediating tools that support these activities.
	LIII-c		• Previously developed and tested solutions can be utilized as second stimuli, ensuring these models are adopted as novel solutions through teachers' crafting the adoption and adaptation process based on their contextual needs. • Considerable time dedicated for problem identification and analysis before passing to the modeling a solution stage • Flexibility in timing and formative design of the sessions • Alternative facilitation mechanisms should be developed to minimize the facilitator-researcher's role as a "leader."	

Table 18. Suggested Embodiments for the Design Principles Four, Five, and Six

Initial design principle	Relation to agency indicators	Suggested embodiments
DP4: Facilitate participant-led problem identification and collective design through responsive facilitation and open-ended inquiry.	Social infrastructure applied to all agency indicators. The analysis revealed that the social infrastructure	• First stimuli need to be presented within a <i>social infrastructure</i> that encourages teachers to share their reflections on their individual practices. The focus should be on listening and prioritizing an understanding of each other's struggles and motivations. • Considerable time dedicated for problem identification and analysis before passing to the modeling a solution stage • Alternative facilitation mechanisms should be developed to minimize the facilitator-researcher's role as a "leader."
DP5: Facilitate the visibility of expertise and resource-sharing in the professional learning environment; share your expertise, resources, and perspectives as an equal participant.		• A social infrastructure that leverages researchers' analytical tools (e.g., the stimulated recall interview technique, Maxqda for systematically categorizing teachers' discussions) is needed to support teachers in shaping their own processes rather than prescribing a specific direction.
DP 6: Support teachers in reflecting their individual histories in the setting, connecting their work context to the learning environment, and envisioning the learning environment as a resource for realizing their teaching ideals.		• Resources utilized as tools for analysis rather than instructive resources, helping to identify professional development needs at the individual level and expanding the problem space for professional learning • Second level activities to articulate critical conflicts, such as writing reflection papers, and use of subgroups that provide more sincerity can be utilized to: ○ maintain identities in collective activities. ○ Each other's individual experiences can also function as such resources, so space should be ensured for sharing them. • An expansive view of good teaching, which foregrounds an exploratory and interpretative perspective to teaching should be followed.

CHAPTER 6

DISCUSSION AND CONCLUSION

PMTLE was designed to support a group of mathematics teachers in exercising agency for expansive learning and transitioning between expansive learning actions while co-modeling their professional learning processes. Analysis of the data collected through the PMTLE implementation revealed that teachers became agentic in collaboratively designing and leading their own professional learning in conscious and strategic ways through two mutually supportive and intricately developed agency formation processes—at the individual and collective levels—as teachers progressed through the expansive learning cycle through PMTLE’s mediation. Analysis of each step in both processes provided nuanced insights into the formation of teacher agency. Additionally, the data analysis addressing the second research question offered lessons for designing participatory teacher learning environments that foster teacher agency.

In the following sections, I will discuss prominent findings in relation to the previous literature.

6.1 Mutually supportive nature of agency formation at the individual and collective levels in participatory professional learning

To understand how teachers become conscious actors of their professional learning, the first research question of this study was formulated as "How did a community of mathematics teachers exercise agency to collaboratively question and analyze their shared problems of practice, and model professional learning activities in response within PMTLE?"

This question was answered through two sub research questions. One was related to agency formation at the individual level: "How did teachers exercise agency at the individual level to develop a disposition to act for their professional learning within PMTLE?" The other one was related to the agency formation at the collective level: "How did teachers exercise agency at the collective level to model their professional learning processes within PMTLE?"

The findings reveal the mutually reinforcing nature of agency formation at both the individual and collective levels. The lessons derived from the analysis contribute to the existing literature on teacher learning, as well as on pedagogical approaches—such as formative interventions—designed to support practitioners' agency in leading work related change and development processes within research-practice partnerships.

Detailed findings related to this main result will be presented in the following subsections.

6.1.1 Surfacing emotionally dense internal conflicts initiates authentic professional learning and facilitates a transition from individual to collective problem identification

Within PMTLE, teachers found space and support to share emotionally challenging critical conflicts (Vasilyuk, 1991) related to their professional development, thereby initiating agency formation at the individual level.

Initially, teachers prioritized taking quick actions in collective professional learning activities, often overlooking the need to confront and address emotionally dense internal conflicts. As Zembylas (2004) notes, this tendency is common among teachers and stems from their identification themselves as knowledge carriers, which

can make them reluctant to share or express vulnerable emotions. This reluctance is rooted in traditional Western thought, which separates emotion from rationality and frames emotional expression as irrational—threatening the ideal of "disembodied, detached, and neutral knower" (p. 187). Through professional practice, teachers learn to suppress their emotions to align with institutional and cultural expectations and to conform to the sanctioned emotional norms of their environment (Hochschild, 2012; Zembylas, 2004).

In this study, teachers broke away from this tendency and expressed emotions such as guilt due to shifting from their own pedagogical values, confusion about their teaching style, and regret for not achieving their ideal teacher identity despite their efforts, often accompanied by feelings of paralysis regarding their professional development. Teachers' shared critical conflict, which they expressed with emotionally charged accounts, was expressed as "teacher as I was taught to be/expected/compelled to be" on one hand, and "teacher as I aspire to be" on the other hand, in this study's context. Teachers' feeling being in trapped between these identities and experiencing them with strong emotions resonate with the literature that highlights that when teachers feel obliged to shift from their values in practice due to various structural factors (e.g., standardized tests, accountability measures, or expectations based on the dominant culture of teaching), they experience strong emotions of anger, frustration, disempowerment, and vulnerability (Yoo & Carter, 2017).

In typical PD settings, such emotions—which were identified as guilt, confusion, or regret in this study—are considered signs of weakness and low agency and are not generally expected to be expressed. However, once teachers break away from this traditional thinking and develop awareness of their emotions, they

experience a sense of freedom. Emotions, as integral to their professional development and transformation rather than being barriers to them, start to function as tools for empowerment and change, allowing teachers to engage in more authentic and agentic forms of professional learning (Zembylas, 2003). In this study, emotions helped teachers express their internal conflicts related to their professional development. As Vasilyuk (1991) highlights, genuinely expressing one's internal conflicts is a courageous and agentic act for individuals, allowing subjects to confront with surface-level solutions that they took due to not being able to deal with them alone (Sannino, 2008a; Vasilyuk, 1991). Furthermore, it opens space to engage more deeply with the roots of their conflicts, providing an opportunity to resolve these conflicts in a conscious and meaningful way with the support of collective activities (Sannino, 2008a, 2008b).

Teachers' emotionally dense expressions of critical conflicts provided rich data for formatively shaping the process, supporting their elaboration of the roots of their conflicts and their transition from a set of individuals into a collective starting with the problem identification. This transition to the collective plane occurred as teachers exercised agency to engage in a quest to redefine the meaning of their internal conflicts (E2), a process specifically triggered by their realization that these conflicts were shared. What Zembylas (2003) calls emotional affinities—solidarity and emotional connection formed through shared struggles and collective reflection—that they developed through openly expressing their emotions provided the conditions for this. It also allowed teachers to maintain their identities within the collective space with their emotions ensuring sincerity and differing views within the collective.

While teachers' critical conflict stemmed from being torn between two teaching identities, the double bind connected to it was the contradictory messages they received: "transform your teaching" vs. "keep up with the system." This shift in problem framing created a fruitful foundation for teachers to become a collective agent, and further interrogate the systemic roots of these dilemmas—both critical conflicts and double binds (L1).

These findings contribute to both the teacher professional development and formative interventions literature by revealing the role of emotions in teachers' authentic engagement with their professional learning processes as well as agentic transition from the individual to the collective plane. While the teacher learning literature revealed teachers' emotional struggle with change in practices (e.g., Yoo & Carter, 2017), this study provided insights regarding how these emotions can be used as productive resources for teachers' authentic engagement with their professional development as well as in collective processes of problem identification.

Second, in the formative interventions literature, which is grounded in the cultural-historical view of contradictions as resources for change and development, dilemmas in practice are understood as manifestations of systemic contradictions. These are referred to as "action-level contradictions" (Sannino, 2005). For these contradictions to serve as triggers for change and development within communities, they need to be reframed as "activity-level contradictions" (Engeström & Sannino, 2011; Sannino, 2005). This study examined how the transition from action-level contradictions—those initially noticed by practitioners—to activity-level contradictions can occur without suppressing teachers' needs to discuss and make sense of the conflicts experienced at the action level. This transition unfolded as a process of moving from internal conflicts to double binds, and ultimately to systemic

contradictions, through teachers' gradual yet genuine engagement with these contradictions and their collective modeling processes grounded in this work.

6.1.2 From passive "victims" to conscious collective "responders" of systemic contradictions: Asking novel questions and acting for analysis

Participant teachers identified an underlying systemic contradiction fueling both dilemmas (critical conflict and double bind)—the lack of a consistent, well-grounded, well-articulated, and unifying framework guiding mathematics educators toward a shared goal of transforming the country's mathematics teaching culture.

This study unraveled the processes of how teachers transitioned from being "victims" of systemic contradictions to empowered agents who acted with a consciousness of these constraints. This shift occurred as teachers began first by formulating novel questions of the community related to the systemic contradiction—questions they had not previously articulated—and then pursuing professional learning activities to answer these questions. At this stage, PD activities were not designed to immediately resolve systemic contradictions. Instead, teachers focused on understanding the roots of these contradictions, helping them gain agency over them rather than remaining passive recipients of conflicting expectations arising from this systemic contradiction (Engeström, 2015).

Teachers' transition from individually framing problems to recognizing them within the context of broader system level contradictions also marks a significant shift—from self-blame to an effort to better understand the underlying systemic contradictions. On the one hand, identifying systemic problems as the root of their dilemmas could be interpreted by others as "blame avoidance"—a tendency to evade responsibility for negative outcomes, especially when individuals feel their public

image is at risk (Cohen-Zamir & Vedder-Weiss, 2022). However, this study revealed that rather than fostering apathy or resistance—reactions that might be expected given the scale of systemic problems—teachers’ initiative to articulate and analyze these contradictions within a supportive environment strengthened their sense of collective agency to deal with them by shaping their actions in a more conscious way.

This finding aligns with research emphasizing the role of articulating and interrogating systemic contradictions in collective spaces, going beyond an individual focus, as a key factor in teachers’ agency development (Durrant, 2019; Elmberger et al., 2019; Lee & Tan, 2014; Sannino, 2010). On the other hand, this study extends the literature by highlighting the importance of an individual's own processes of dealing with internal conflicts for a conscious transition to a focus on systemic contradictions, going beyond "blame avoidance." This relationship was previously highlighted by Sannino (2010), who conducted a formative intervention study with teachers. She found that "working out conflicting motives is an avenue into an engagement with the actual systemic contradictions of the given activity" (p. 483). That is, teachers’ shifting the focus from individual conflicts to systemic contradictions is not an effort to escape from the demanding work of dealing with their internal conflicts, to the contrary, an initiative taken to deal with them. In her study, being in the process of experiencing—dealing with critical conflicts—while collective activities are held helped teachers make sense of their collective work of examining and resolving systemic contradictions (Sannino, 2010).

The collective activities that teachers engaged in to analyze the systemic contradictions did not eliminate the systemic problem that "time and again give rise to" the double bind and internal conflicts (Engeström, 2000, p. 153). However, they

enabled teachers to develop a broader perspective and gain a clearer understanding of the systemic problem and to analyze and address it collectively as part of their professional learning process. Ultimately, contributing to the literature on teacher professional development, this environment supported teachers in adopting an "inquiry stance," as described by Cochran-Smith and Lytle (1999). An inquiry stance in professional development helps teachers challenge "the purposes and underlying assumptions of educational change efforts rather than simply helping to specify or carry out the most effective methods for predetermined ends" (p. 295). With this broadened perspective and new analytical tools, teachers became more conscious and strategic in responding to systemic contradictions and organizing their professional learning.

6.1.3 Framing collective professional development as a space for internal conflict resolution

Building on this foundation of problem identification and analysis, teachers developed a collective consciousness and strategic practical action to model PD activities as novel, localized solutions to the systemic problems they had identified. This collective process was supported by individual dispositions to act consciously in addressing their internal conflicts through PD activities conducted within the community. The findings revealed that while teachers engaged in collective problem identification and analysis following the articulation and questioning of their internal conflicts, they continued their experiencing process at the individual level. In other words, throughout their collective activities, teachers positioned themselves within the community while maintaining their identities and continuously considering their

individual professional learning needs as part of their experiencing process—their ongoing process of resolving critical conflicts.

This became particularly evident in how they expanded the problem space in which they could exercise professional learning actions, creating new possibilities for working through their critical conflicts. By sharing their individual perspectives, teachers collectively agreed on this expanded problem space as the object of their collective activities within PMTLE. An important implication of this finding is that such collective formation processes include subjects' development of personal meaning regarding their actions and presence within the community. Specifically, the expansion of the object of activity/problem space, on which the collective exercises their actions, should be perceived by individual subjects as a space in which they can continue to be part of, with their identities which compose the collective, and act to meet their needs to resolve their critical conflicts.

With this finding, this study contributes to the literature on agency at work and organizational learning by extending the focus on collective agency in formative intervention studies through revealing the role of individual agency formation in collective change processes (Goller & Paloniemi, 2017; Engeström, 2011). According to Engeström (2014), "the expansive cycle begins with individual subjects questioning the accepted practice, and it gradually expands into a collective movement or institution" (p. xx). Aligned with this perspective, formative intervention studies and work transformation processes typically focus on the formation of collective agency, often overlooking individual agency (e.g., Brevik et al., 2019; Ko et al., 2021; Haapasari et al., 2016). This study validates Engeström's emphasis on the importance of progression from the individual to the collective in collective change processes but expands on it by revealing how subjects' perception

of their professional learning evolves throughout such transition. They develop agency at the individual level as the collective moves forward in the expansive learning process. Individual agency is formed as subjects develop personal dispositions to act not only for their collectives but also for their own individual practices.

The importance of agency formation at the personal level in cultural transformation and creation processes has been highlighted by scholars following sociocultural and cultural-historical activity theory traditions (e.g., Billett, 2006, 2008; Eteläpelto et al., 2013; Sannino, 2008a). According to Billett (2008), individuals whose identities are shaped by social experiences throughout their life histories, mediate and contribute to the transformation of work activities as cultural practices. Furthermore, individuals need to cultivate a "sense of self" to adapt to changing work activities (Billett, 2006). In this sense, work-related change and creation processes are the result of "negotiations between both individual and social agency" (Billett, 2008, p. 41). Through intra-psychological processes, individuals re-define their identities and experiences while collective activities are re-made (Billett, 2008).

This study validates these arguments by observing the co-evolution of "collective motives and personal goals" (Stetsenko, 2005, p. 70) during the formation of collective professional learning activities as a novel form of cultural activity. In this study, each individual teacher entered into the process of experiencing through which they developed a personal sense of the collectively formed activities and the object of activity (Leont'ev, 1978; Sannino, 2008a). Within the context of teacher professional development, "sense of self" was created through teachers' recognizing

their professional development needs, and considering the PD activities as a medium to meet these needs.

Teachers' seeing the collective PD activities developed as a response to the collectively identified systemic contradiction as a site for their resolving their critical conflicts also demonstrates how they expanded their perspectives on their problems of practice through the process of experiencing. This finding resonates with the literature on formative interventions conducted with teachers, which showed that interventions help teachers expand their understanding of problems in their activities, particularly those related to their students' (the object of their activity) capabilities (Barma et al., 2015; Rainio & Hoffman, 2021; Sannino & Engeström, 2017). These studies showed that transitioning from a closed to an open conceptualization of their students' abilities prompted teachers to take alternative initiatives in their practice. This study extends this literature by showcasing how teachers' expanding their understanding of their own professional development problems contributed to their developing perspectives about themselves as learners. This supported their imagination about alternative practices of professional development, and becoming ready to take alternative initiatives related to their professional learning. That is, teachers got rid of closed conceptualizations about their problems which led them to frustration regarding their professional development. Conceptualizing teaching itself as an entry point for professional learning, they expanded possibilities to deal with individual dilemmas related to their professional development which initially dragged them to the feeling of impossibility. This discursive change empowered teachers to construct themselves as "agents of change" and to envision alternative futures for their professional development practice (Rainio & Hoffman, 2021, p. 35).

6.1.4 Breaking away from the normative PD practices gives rise to conscious and strategic use of preexisting models through co-designed modeling activities

Teachers developed a new activity as a response to the systemic problem that they identified by becoming more conscious in directing their own professional learning processes. As evidence of Learning III, teachers learned to "control, limit, and direct" their professional learning processes, rather than following their habits of following traditional forms of professional development where teacher educators lead teachers within structured activities (Engeström, 2015, p. 112). They broke away from the normative ways of teachers organizing their relationship with the curriculum, PD, and teacher educators in PD settings.

This happened first through their developing collective activities to analyze the systemic contradiction related to the curriculum as a novel professional development process. Second, teachers co-designed their professional development activities as local contextually relevant solutions by "experimentation, borrowing, or conquering already existing artifacts for new uses" (Engeström, 2015, p. 131).

Japanese Lesson Study was an artifact already in use by mathematics educators in other parts of the world and in Turkey, albeit scarce, as a part of small-scale research projects (e.g., Abazoğlu, 2014; Günay, 2016). However, the way participant teachers in this study utilized Lesson Study was unique, shaped by a deeper understanding of their problems, rather than implementing a pre-designed model with the directions of a teacher educator.

Teachers in this study used the systemic contradiction of lacking a unifying teaching perspective as a resource to strategically approach the Lesson Study model. They deliberately "redesigned the context into which Lesson Study is being imported" by deciding to build the teaching perspective of their community as a first

step (Stigler & Hiebert, 2016, p. 5). In that regard, the novel solution that they developed as a response to the identified systemic problem involved not only their modeling and implementing Lesson Study but also other modeling activities that provided a background for their work related to Lesson Study. Their path toward creating a professional development activity prototype was unique, co-designed to address their professional development needs. With such an approach, this study contributes to the current scholarship on Lesson Study which inquires for more democratic ways of implementing it out of the contexts that it has been culturally born (Kim, 2021; Stigler & Hiebert, 2016).

This resonates with previous literature on formative interventions with teachers, which identified that formative intervention process supports teachers' agency to develop a conscious and strategic attitude toward the use of mediating tools such as teaching materials or frameworks proposed by experts or curriculum developers (Barma et al., 2015; Severance et al., 2016; Yang, 2021).

On the other hand, this study contributes to the literature by conceptualizing the "modeling a solution" expansive learning action as a form of professional development process in itself—novelly co-designed in response to the identified contradictions. Traditionally, in formative interventions, the end product is seen as the novel solution modelled as a response to the systemic contradictions, while the contribution of the modeling process to practitioners' professional development remains implicit. This study revealed that modeling a solution action can also be framed as a solution formatively co-designed by participants, during which they enact agency in selecting mediating resources—second stimuli—, co-designing and executing modeling activities consciously and strategically while working toward developing novel activity structures for their future activities.

6.2 PMTLE design supports the formation of both collective and individual agency for conscious and strategic co-modeling of professional learning activities

The second research question of this study was "How did PMTLE design support or inhibit teachers' agency in engaging in the agency formation processes?" The findings shared in Chapter 5 revealed significant insights regarding the design elements supporting or inhibiting the agency indicators, letting me to revise some design principles as well as propose design lessons for future studies.

The findings contribute to the existing literature on designing professional development as well as the formative interventions processes. The most prominent findings will be discussed below in relation to the previous literature.

6.2.1 Mirror material helps not only collective discussions on problems but also surfacing individuals critical conflicts

As discussed in section 6.1.1, surfacing emotionally challenging dilemmas, which corresponded to the E1 indicator of agency formation, is significant for teachers to engage in an authentic professional learning experience.

The analysis revealed how the PMTLE design supported these. In Change Laboratory interventions, as a widely implemented version of formative interventions, based on Vygotsky's method of double stimulation, ethnographic material from participants' settings are used to provoke collective discussions on problems and disturbances in the practitioners' activities to serve as first stimuli (Sannino et al., 2016). These materials are also called mirror material. As Sannino et al. (2016) highlights, "in CL interventions aimed at supporting expansive learning, the conflict of motives is triggered and brought into the open by presenting to the participants evidence and examples of recurring problems and disturbances in their

activity as first stimuli, often in the form of videotaped critical situations and encounters" (p. 5). In these studies, the articulation of conflict of motives is approached as a collective process, rather than practitioners' individual surfacing of emotionally challenging critical conflict situations as means for their meaningful engagement with the process of collective questioning and change. That is, the meaning of expressing critical conflicts at the individual level is not elaborated.

For example, in a school intervention, ethnographic material mainly including "interviews and videotaped interactions inside and outside classrooms" were presented to the participants during the sessions, which showed recurring problems such as students' sitting on the floors of the school corridors, making teachers interpret the situation as students' apathy as well as an "opposite image of students as energetic and active" (Sannino et al., 2016, p. 10) This triggered a conflict of motives regarding their students' capabilities: "Should students be trusted or controlled" (p. 23). Following this, researcher-interventionists facilitated the collective discussion to analyze the roots of this problem, which led the collective to identify the systemic contradiction—socioeconomic changes in the neighborhood—giving rise to this recurring problem. Extending this literature, a nuanced insight regarding the function of first stimuli came from this analysis. That is, first stimuli—the problematic situation which is made visible through mirror material—also help practitioners surface their internal critical conflicts by triggering emotional openness, which supports teachers' meaningful engagement into the collective processes.

For this, these materials need to encourage practitioner reflection by presenting alternative cases of workplace situations, offering conceptual tools for articulating critical conflicts, and inviting data analysis through scientifically grounded insights—without imposing solutions. Such artifacts enable teachers to

explicitly discuss their personal histories and teaching perspectives, opening space to articulate their concerns regarding shifting from their pedagogical values as well as reconnecting with them—elements they may have overlooked while trying to keep up with systemic demands (Edwards, 2017). Necessarily, to function in these ways, these materials should be introduced within a social infrastructure that fosters reflections without any concern of sharing emotions and active listening to each other, promoting a deep understanding of practitioners' struggles and motivations.

6.2.2 Writing supports emotional affinity and an agentic transition from the individual to the collective plane for problem identification

As elaborated on in Section 6.1.1, teachers' individual critical conflicts served as materials for an agentic transition from the individual to the collective plane for problem identification. Teachers' written reflections and their subsequent sharing within subgroups supported this, fostering sincerity among participants.

Yoo and Carter (2017) conceptualize writing as praxis, drawing on Aristotle's definitions of poiesis—"actions that lead to a tangible outcome"—and praxis—"actions that hold inherent value in itself...which refreshes individuals through reviving their energy as opposed to burdening them with the pressure to achieve results" (p. 42). In writing as praxis, the focus is not on the written text as the final product but on the writing activity itself, which serves as a "shared space of vulnerability and learning" by portraying teachers' "lived experiences" and cultivating new perspectives (Yoo & Carter, 2017, p. 42).

In this study, writing functioned as praxis by supporting teachers' sharing of vulnerable emotions in the collective space. Furthermore, expanding on this perspective, the findings of this study reveals its additional role in facilitating the

transition from the individual to the collective plane in problem identification.

Writing, as a reflection activity and subgroup discussions reinforced the "emotional affinity"---solidarity and emotional connection formed through shared struggles and collective reflection---among teachers (Zembylas, 2003). Such affinity supported their engagement in a quest to negotiate a new meaning of their internal conflicts. In addition, it helped teachers establish their presence in the professional community as individuals while also externalizing individual dilemmas for collective inquiry into their root causes.

Writing as a tool for reflection has long been utilized in teacher PD (e.g., Hoover, 1994; Yoo & Carter, 2017; Walker, 2005). Resonating with this literature, writing supported emotional processing helping teachers explore how their feelings affect their professional identity (Yoo & Carter, 2017); functioned as a dialogic tool, leading to deeper collegial dialogue on problems (Yoo & Carter, 2017); made the often-tacit experiences visible, helping teachers externalize their thoughts and emotions; slowed down the thinking process; structured their thoughts; allowed teachers to acknowledge, explore, and reframe emotional experiences—turning emotionally challenging situations into sources of insights rather than burnout (Walker, 2005). While confirming these qualities, this study revealed another function of writing in collective change and development processes. It helped teachers transition from the individual to collective space, while facilitating individual critical conflicts becoming materials for collective questioning, helping to establish sincerity among participants as well as protect identities within the collective space.

This study also contributes to the formative interventions literature, by proposing another tool for an immediate step in teachers' agentic transition from

being a set of individuals to a collective while progressing forward in expansive learning. While the function of autobiographical texts as first stimuli to trigger the articulation of teachers' conflicts of motives within a Change Laboratory setting was found by Sannino (2010), this study extends on this by deciphering details regarding another function of writing in a formative intervention, which is being a mediator for the transition from the individual to collective plane in problem identification.

6.2.3 The stimulated recall interview facilitates the transition from questioning to the analysis of systemic contradictions

The design features that facilitated the significant transition from problem identification to analysis, rather than teacher apathy because of the scale of the systemic problem surpassing teachers' ability to resolve, included reflection activities that presented teachers' previous discussions on their challenges in a structured form, specifically stimulated recall interviews. By organizing these discussions and considering each unique idea from an analytical perspective, this activity fostered collective agency, enabling teachers to formulate novel questions about the systemic problems they identified to be answered through collective analysis activities. Such analytical tools used by researchers or teacher educators in professional development settings helped dismantle traditional boundaries, allowing teachers to shape their own professional learning processes rather than following externally imposed directions.

Stimulated recall interview has been a widely used activity in PD settings to support individual teacher reflection (e.g., Gazdag et al., 2019; Geiger et al., 2016; Williams & Nell, 2023). For example, Gazdag et al.'s (2019) systematic review on the use of Video Stimulated Recall in teacher education reveals that this method

promotes reflective thinking, provides insights to teacher educators to follow teachers' developmental trajectories and learning needs, enriches data collection to assess teacher education programs' effectiveness.

While this study validated its function as a tool for reflection, it revealed other features of it in teacher professional development, specifically when teachers' collective agency is fostered to consciously organize their own professional learning. Its use in this study showed its effectiveness to help to develop a shared language while providing a chance to discuss different viewpoints, elevate discussions into a systematic form, and move forward in a more organized way based on teachers' input. All these provided a background for teachers' agentic transition to the analysis expansive learning action, asking critical and novel questions as well as planning and executing analysis activities in partnership with the teacher educators. In a more general sense, use of stimulated recall interview to help a group of teachers identify their own learning paths and design further professional development activities is unique to this study.

Additionally, its use in formative interventions as a mechanism for an agentic transition from the questioning to the analysis action in expansive learning is a novel contribution. Which processes triggered teachers' agency to transition between these actions remains an implicit aspect of formative intervention studies (e.g., Engeström et al., 2013). This study revealed that stimulated recall interview can be a good way for teachers to structure their problem discussions and an agentic and authentic transition to the analysis of systemic contradictions by practitioners with researcher-interventionists' facilitation and partnership.

6.2.4 An expansive view of good teaching supports teachers in identifying their professional development needs

In terms of the design that supported teachers' individual engagement with collective co-design processes, it has been identified that mediating resources functioning as tools for analysis of teaching actions rather than instructive resources helped teachers to identify their PD needs and expand the problem space of their professional learning, ultimately supporting their situating their PD needs into the collective PD activities that they planned for their community. Teachers' experiences also functioned as resources for each other. In addition, a social infrastructure that is based on an expansive view of good teaching, rather than a binary one, foregrounded these processes (Horn & Garner, 2022).

These findings shed light on, to some extent, how teachers can be supported to enact agency to expand their understanding of their professional learning needs, ultimately giving meaning to their presence as individuals within collective activities. With such insights, this study contributes to studies that interrogate the means of supporting agency formation at the personal level in work-related transformation processes (Billett, 2006, 2008; Eteläpelto et al., 2013; Sannino, 2008a). That is, this study makes the means of support for "negotiations between both individual and social agency" during such processes of change and development to some extent visible (Billett, 2008, p. 41).

6.2.5 An authentic form of leadership facilitates the organic development of the process

A prominent finding regarding the PMTLE design in supporting teachers' conscious collective action for generating and implementing professional learning activities as

novel, local solutions is the importance of its social infrastructure. Findings revealed three key, not exclusive, features of design related to this: dedicating substantial time to problem identification and analysis before transitioning to the modeling a solution phase; ensuring flexibility in timing and formative design of the sessions; and my facilitation as called by the participants of this study as an "authentic leadership" or facilitation respecting the "organic" development of the process.

The third dimension requires specific attention as the review of literature revealed that there is a lack of explicit attention to the relational aspects of a formative intervention design, such as how researchers support teachers' agency while also facilitating the process and presenting mediating tools with their own expertise, without taking the ownership of the process over. Findings indicated that teachers perceived my role in this study as a form of "authentic" leadership or facilitation supporting the "organic" development of the process rather than an authoritarian one.

While this leadership style might initially seem to limit teachers' agency, this gave rise to an alternative form of research-practice partnership supporting teachers to stay on track to advance their communities while using the community space to freely brainstorm ways to navigate the complexities of teaching. That is, within these communities, teachers need open spaces where they can freely discuss daily struggles, share experiences, seek advice, and brainstorm while still maintaining a focus on progress. The analysis identified two key dimensions of the kind of "authentic leadership" mentioned in teachers' accounts. First, it was established through my careful listening of their conversations, helping teachers translate their ideas into both the specific and broader goals of the community. This, in turn, contributed to the second dimension: integrating my expertise as a mathematics

education researcher and learning scientist with their field-based experiences and prior professional development.

This second dimension was alternatively interpreted by some participants not as leadership but as being a part of the "organic" development of the community with my own expertise, without forcing the group to a pre-defined endpoint. This interpretation of my sharing expertise as part of my facilitation, rather than as a form of leadership, offers important insights into designing teacher professional learning environments that support teachers' agency in leading their own professional development while collaborating with teacher educators or researchers.

According to Engeström and Sannino (2010), the utilization of the second stimulus is the time when practitioners can draw on knowledge and practices developed by others to design solutions that are novel and tailored to their local needs. For example, in a Change Laboratory intervention at a university library in Finland to redesign the organization of the activity of librarians, the idea of knotworking—a heuristic for a distributed partly spontaneously created cooperative performance among loosely connected actors around a define problem situation (Engeström et al., 2012)—was proposed by the researcher-interventionists as a potential solution (Engeström et al., 2013; Sannino et al., 2016). In their analysis, the authors identified as a significant finding that "the librarians worked out and began to implement in practice their own version of the idea of knotworking as a concept to guide the long-term development of the library's organization and way of working" (p. 103).

The findings of this study coheres with Engeström et al. (2013) by showing that teachers created their novel route of adopting the Lesson Study model, representing a novel, locally produced response to the systemic problem. Teachers

were introduced with the model of Lesson Study by *The Teaching Gap* (Stigler & Hiebert, 1999) and by me, sharing the book with teachers. Through the social infrastructure that prioritized responsive facilitation and open-ended inquiry, we achieved what one of the participants described as "organic structure where all these experiences blended" (Lale) rather than teachers feeling that they were directed to a pre-identified direction and an end-goal. On the other hand, previous studies which highlight similar findings (e.g., Barma et al., 2015; Engeström et al., 2013; Severance et al., 2016; Yang, 2021) do not explicitly reveal how researcher-practitioner relationship is established so that such novel processes can emerge. This study contributes to the literature by providing insights regarding that type of facilitation.

6.3 Implications for research, practice, and policy

One implication of this study for research on teacher learning is that teacher agency can be positioned as a learning outcome in the design of professional learning environments, countering the mainstream view that agency, compared to knowledge of specific content, is too vague to serve as a measurable outcome. Specifically, designing supports for teacher agency—fostering teachers’ conscious and strategic leadership of their own professional learning processes—emerges as an important site of inquiry within the new paradigm of teacher professional development (PD). As calls for more democratic forms of PD grow stronger, there is increasing emphasis on teacher-centered pedagogies that position teachers as agents of change in their teaching, classrooms, and schools. This study invites researchers to further explore how the accumulated knowledge in the new paradigm of teacher PD—such as the use of authentic tools reflecting the complexity of teaching practice and the

fostering of collegial communities—can be integrated with pedagogies that explicitly support the development of teacher agency.

In addition, while incorporating teacher agency into professional learning, a broader perspective is needed, as agency is often understood narrowly in terms of “action,” overlooking the agentic processes that prepare teachers to take intentional action to address their problems. Accordingly, this study invites researchers to further inquire into the processes of teacher agency formation across different contexts.

Another implication concerns teacher education policies: to prevent teachers from becoming alienated from their own professional development, they need to be more actively involved in organizing their professional learning processes within research-practice partnerships. This study highlights the importance of teachers making sense of professional learning activities by deeply understanding their own development needs and connecting those needs to broader systemic problems.

In educational practice, teachers are expected to be lifelong learners, responding to the emerging needs of their profession and updating their practices in line with developments in educational policy and research. To achieve this, teachers must be in the driver’s seat of their own professional learning: they need to continuously question their ongoing practices, take ownership of problematic situations, analyze their root causes, and take conscious, strategic actions to address them. This requires the capacity to access and mobilize tools, resources, and relationships. High levels of collective agency are essential for this process, and, as this study demonstrated, such agency can be fostered through professional development programs that prioritize teacher empowerment. Accordingly, both schools and central organizations responsible for organizing professional

development programs should engage in research-practice partnerships that center teacher agency as a foundation for professional learning.

6.4 Limitations of the study

One limitation of this study is that it did not collect data from teachers' real teaching contexts to observe how their participation in the PMTLE study influenced their enactment of agency in their classrooms, schools, and other professional development sites. Insights regarding this aspect remained limited to teachers' self-reports about changes in their practices. While such data could have provided further opportunities to examine agency formation at the individual level, its absence did not prevent answering the research questions of this study, as they relate to agency formation for activities that they conduct within PTLE.

One limitation in terms of data collection is the absence of a research group. Specifically, in a study where the design formatively evolves in response to data from implementation, many decisions must be made based on ongoing data analysis. The presence of a research group could have eased this process. While the participant teachers were active contributors, the researcher side of this research-practice partnership could have been stronger, facilitating a more effective use of labor during the study. Future follow-up studies or iterations will be organized with a research group, on the condition that researchers adopt the philosophy underlying a participatory teacher learning environment that supports teacher agency. Additionally, other researchers could have taken field notes regarding my facilitation, providing an outsider perspective. I compensated for this by reviewing the recordings of the sessions and maintaining a critical eye on my facilitation practices to improve them. I also maintained continuous informal contact with

participants to assess my facilitation, prioritizing the goal of not dominating the sessions but instead creating space for teachers' voices.

Another limitation arises from the large amounts of data collected through a design-based study, which made it difficult to manage during both the fieldwork and data analysis processes within the scope of a Ph.D. study. To address this, I focused on a portion of the collected data to answer the specific research questions of this study and followed a data reduction strategy, which led to the exclusion of a significant portion of the data that can be utilized in future research. In addition, although data were collected during teachers' implementation of the activity model, these data were not included in the analysis within the scope of this dissertation. Follow-up studies can be conducted using this existing data and by collecting additional data regarding teachers' experiences with this process.

6.5 Epilogue

As an end note to this dissertation, I would like to highlight the activities of the Teachers Network Mathematics Teaching Community, which developed within the scope of the PMTLE study but was excluded from the analyses presented here. Following the development of the activity prototype based on the Lesson Study model, along with the establishment of community principles and perspectives supported by the PMTLE design, teachers implemented their activity prototype during the 2023–2024 school year. During this period, I participated more as a member of the community rather than as its facilitator.

The process began in the summer of 2023, when the community planned for the implementation of the activity prototype. Implementation started with teachers executing the Research step of the Lesson Study cycle, during which they split into

three groups to examine the mathematical objective they selected from various resources. The objective chosen was from the fifth-grade geometry curriculum: "Identifies and draws the basic elements of a rectangle, parallelogram, rhombus, and trapezoid." Two groups reviewed curricular resources—namely, the MEB Teaching Program, the MEB-sanctioned textbook, and NCTM's *Principles and Standards*—focusing both on the specific objective and its horizontal and vertical connections to other geometry topics. The third group, which I joined as a member, examined propositions from Euclid's *Elements, Book 1*.

Following this research phase, the community held an in-person meeting at Teachers Village, a space designed for educational activities by a non-profit organization near İzmir, to share their findings with each other. At this meeting, held in November 2023, the community worked collaboratively on selected propositions from *Elements* to inform the development of a lesson plan for fifth graders. I attended this meeting in person.

In May 2024, the community implemented the lesson plan with a group of fifth-grade students from a public school in İzmir, again at Teachers Village. Açıyla taught the lesson while other members observed. The lesson was recorded, and immediately afterward, the community conducted a reflection session. All of these activities were organized autonomously by the community teachers, with support from Teachers Network facilitators.

The semester closed with a collective reflection on the activity prototype they had developed. The community began the 2024–2025 season by planning new activities and decided to expand their membership. Currently, the community includes fifteen additional members and is planning to implement the activity prototype once again with the new participants. New members are introduced to the

community's approach by reading *The Teaching Gap*, selected chapters from Van de Walle et al. (2020), and reviewing key community documents, such as the Community Principles and Teaching Approach. Throughout this process, the community has operated autonomously, with my role shifting to that of a consultant, providing support only when requested.



APPENDIX A

THE CALL FOR THE STUDY

Dear mathematics teacher friends,

I would like to make a call to you to build a professional learning community together as a group of volunteer mathematics teachers gathered under the Teachers' Network. In this study, which will also serve as my doctoral thesis research, we will collaborate to establish a distinct community where we discuss the teaching and learning of mathematics and produce materials.

What is the purpose?

Within the scope of this study, we, as mathematics teachers, will first discuss the common problems in our own practice. We will then explore different professional learning community models and the solutions offered by mathematics education researchers (specific mathematics learning environment design approaches and learning theories in mathematics education). Finally, we will decide what kind of community model we want to design, drawing inspiration from these models and approaches. Later, we will implement this model, share it with our colleagues, and disseminate it. In this process, we will both develop our knowledge and application infrastructure that can be a solution to our common problems and reach outputs that can be shared.

How long will it take?

The study will be divided into four stages:

1. Understanding our needs (April-June 2022)
2. Exploring solutions and building our own community model (2022-2023)

First term, as of October)

3. Implementation and evaluation of the model (2022-2023, second semester)
4. Disseminating the model (which will be spread in the process)

We will proceed by clarifying the contents of the stages together. We will conduct our meetings online every two weeks, lasting approximately two hours each. We will decide on the day of the study together with the participating teachers.

In order for the study to be efficient, the number of participating teachers will be limited, and equality will be observed in the representation of private schools and public schools.

If you decide to participate in the study or want more detailed information, you can contact me by e-mail at xxxxxxxxxx or by phone at xxxxxxxxxxxxxx until Saturday, March 26; or you can fill out the Google Form and let me reach out to you.

I wish you healthy days...

Yeliz Günel Aggöl

APPENDIX B

DETAILED INFORMATION ABOUT EACH PARTICIPANT TEACHER

Defne: After completing her degree in primary school mathematics teaching at a faculty of education, she initiated her teaching career at a private tutoring center. A year and a half later, she transitioned to a public school in the disadvantaged city of X, situated in the south-eastern region of Turkey, where the Kurdish population is predominant. This presented challenges, as most students couldn't receive education in their mother tongue. Following one and a half years in city X, Defne was then appointed to a school in another south-eastern region of Turkey. This new location was economically more affluent and culturally cosmopolitan, primarily receiving immigrants from the disadvantaged eastern cities of Turkey.

She participated in TN in 2020 in a teachers' summer camp organized by TN. She described the role of TN in her life as a motivator for preserving her belief that teachers will be the transformative agents of the problems of education: "So, there are issues; as teachers, we will be the transformative force here." She joined a TN program called "Purple Certificate" on gender equality and designed and implemented a project in her school as an output. Defne is living in a city that was affected by the February 2023 earthquakes that occurred in the southern-eastern region of Turkey. Since then, she has been actively working on TN's activities directed toward rehabilitation in that region.

Her primary motivation for participating in the PMTLE project was to create tangible lesson materials that make mathematics enjoyable for students. She aimed to collaboratively implement these materials and share the outcomes with other teachers

nationwide. Her long-term vision was to inspire and encourage other teachers to believe that mathematics teaching and learning can indeed be enjoyable.

Lale: Following her graduation from a primary school mathematics teaching department at a faculty of education, Lale immediately started teaching at a private school in western Turkey. During that time, she was pursuing her master's degree in the field of measurement and evaluation. Given her expertise in this area, she took on the role of leading the establishment of the measurement and evaluation unit at the school. Lale was teaching at a private school in a metropolitan city in Turkey when the project started.

She connected with the Teachers Network through the previous mathematics community organized within the network. Her motivation for joining the PMTLE study was rooted in the aspiration to refine her teaching practices and actively contribute to nationwide educational enhancement, driven by a sense of responsibility toward the 'children of the country,' as she put it. She highlighted that even if systemic contributions appeared challenging, simply engaging with people who shared similar concerns could serve as a meaningful starting point for making a contribution.

Yonca: Yonca graduated from a mathematics department and later acquired her teaching certificate from a different university. Upon completing her degree in 2015, she commenced her teaching career at the high school from which she graduated. Notably, this high school is an Armenian minority institution, classified as a 'minority school' according to Turkish regulations. Schools operated by minority groups in Turkey, such as Armenian, Greek, and Jewish schools, adhere to specific regulations determined by the National Ministry of Education, allowing them to implement their own curriculum.

Yonca joined TN through the Thinking Skills program. Subsequently, a group of mathematics teachers who had participated in the program continued their engagement by establishing a follow-up mathematics teachers' group. They organized diverse activities, such as 'writing mathematics stories with students,' to integrate the insights gained from the Thinking Skills program into their teaching practices. This community eventually faced challenges and was unable to sustain its ongoing activities.

Yonca's initial motivation for joining this study was to engage in activities that foster discussions on teaching mathematics in more meaningful ways. She aimed to explore how mathematics could be presented as a tool for solving real-world problems rather than solely as a means to pass university enrollment exams.

Demet: Since graduating from the primary school mathematics teaching department of a faculty of education in 2008, Demet has been teaching mathematics at public middle schools. The first school she taught was located in a village in Central Anatolia. She currently teaches at a public middle school in a metropolitan city located in western Turkey.

Demet first participated in a program called Teacher as a Learning Leader (Öğrenen Lider Öğretmen) organized by a non-profit professional development organization called Teachers Academy Foundation (Öğretmen Akademisi Vakfı, ÖRAV). Then, her connections in this program invited her to TN, and she participated in the Creative Problem Solving program of TN. When I asked Demet her expectations from this study, she answered, "Instead of expectations, I believe in... This time, we can do something other than say that 'there are people who have

similar problems. I mean, this time, I feel like we would be able to say, ‘Yes, this is the problem, but what can we do?’”¹⁶

Açelya: Açelya obtained her undergraduate degree in mathematics and received her teaching certificate separately. Then, she completed a master’s program in educational programming and management. She had been teaching dominantly at private schools for sixteen years when the study started. She is currently the director of the "gifted students program" at her school. For this program, with her team, she designed a holistic curriculum from 1st to 8th grade. She actively participated in the trainings organized in her school by an external researcher with expertise in gifted education to contribute to the design of this program. She worked with this expert in close contact throughout the years. Currently, she also conducts teacher training activities for teachers who implement "gifted students program" in her school.

Açelya joined TN through the Thinking Skills program. She had been in the same follow-up group with Yonca. Açelya’s aspiration for this study was to collaborate with teachers who were eager to produce scientifically well-grounded pedagogical applications and implement them in the classrooms. Like most of the participants, she was motivated to take on a leadership role in disseminating her knowledge and experience with other teachers in the country.

Gonca: Gonca had twenty years of teaching experience when the PMTLE project started. She teaches at the middle school level. After graduating from the primary school mathematics teaching department of a faculty of education, she immediately started working at public schools in a Western industrial city in Turkey.

¹⁶ “Ya nasıl bir şey... Yani beklentiden ziyade şöyle inanıyorum... Hani bu sefer gene ‘Ya bak, aynı sorunları yaşayan insanlar var’dan öte bir şeyler yaparız diye düşünüyorum.”

Gonca was a part of the mathematics community previously organized under TN, which I joined as a participant observer. According to Gonca, the previous community couldn't sustain itself because the boundaries were not clear. For that reason, Gonca expected from the PMTLE study to create a community where various activities could be conducted but with planned steps and clear boundaries.

Gül: Gül earned her undergraduate degree in Mathematics. Following college, she pursued a master's degree in educational management. With twenty-three years of teaching experience at the project's inception, Gül currently teaches at a public high school (9th to 12th grades) in a metropolitan city in Western Turkey. Like many other teachers, Gül has actively participated in various professional development programs, including workshops on mathematical modeling, STEM,¹⁷ and flipped learning, throughout her career. Alongside a group of mathematics teachers, she initiated the establishment of a professional community in her city; however, it couldn't be sustained. During this community endeavor, Gül actively sought resources to support and nurture the community, leading her to discover Teachers Network (TN). She took part in the Creative Problem Solving program conducted in her city by TN.

Her motivation for participating in PMTLE was to conduct experience-based professional development activities in which she could experiment with her colleagues by collectively designing, implementing, reflecting, and refining as a process. She highlighted that seminars based on one-way experience sharing by experts from universities or by teachers are not satisfactory for her anymore.

¹⁷ In teachers' talk, STEM refers to classroom activities that integrate diverse STEM disciplinary contents. The purpose of these activities is to engage students and apply learning objectives specific to each STEM subject.

Yaprak: Yaprak received her undergraduate degree in Mathematics. After her graduation in 1990, she worked for a computer company that planted computers in schools, then worked for a gold mining company as a computer expert, and then worked as a computer teacher. After six years of experience in these fields, she started working as a mathematics teacher in a western metropolitan city in Turkey. She currently teaches at the middle school level at a private school.

Similar to other teachers, Yaprak has attended many professional development seminars (e.g., use of Web 2.0 tools in education) throughout her career. She joined TN through the Thinking Skills program. She was in the same follow-up group with Yonca and Açelya.

When she initially came across the call for this study, she pondered the possibility of collectively realizing the 'mathematics lesson designs' she had always envisioned. Considering this program falls under Teachers Network, she anticipated it to be a 'meaningful' initiative, reflecting the strong rapport she holds with TN and serving as a motivating factor for her to engage in this open-ended project.

Ali: Ali graduated from an educational higher institute. When he participated in this study, he taught at a primary public school in a city located in northern Turkey. He started teaching after his fourteen years of experience in different sectors. Previously, he taught in villages in Turkey, where there were limited opportunities to organize teaching and learning activities in schools.

Ali has joined various professional development projects previously on topics such as "employing children's literature in mathematics teaching" and wrote two books as the product of this project with a group they created in TN.

Ali received an invitation to join TN after one of the leading facilitators encountered him while presenting his project at a teachers' conference. He became

involved in the PMTLE study at the onset of the second phase. The community had ongoing discussions about including elementary school teachers during the initial phase. Ultimately, at the end of the first phase, the community decided to invite elementary school teachers they had connected with through TN. Accepting the invitation, Ali became a part of the community in August 2022. His motivation for participating in the study stemmed from its participatory structure, contrasting with hierarchical professional development programs that view teachers merely as implementers. Ali also expressed that my role as a researcher from Boğaziçi University was an additional motivating factor for him.

APPENDIX C

ETHICS COMMITTEE APPROVAL FOR THE MAIN RESEARCH

Evrak Tarih ve Sayısı: 13.01.2022-47877

T.C.
BOĞAZİÇİ ÜNİVERSİTESİ
SOSYAL VE BEŞERİ BİLİMLER YÜKSEK LİSANS VE DOKTORA TEZLERİ ETİK İNCELEME
KOMİSYONU
TOPLANTI KARAR TUTANAĞI

Toplantı Sayısı : 27
Toplantı Tarihi : 13.01.2022
Toplantı Saati : 10:00
Toplantı Yeri : Zoom Sanal Toplantı
Bulunanlar : Prof. Dr. Ebru Kaya, Prof. Dr. Fatma Nevra Seggie, Dr. Öğr. Üyesi Yasemin Sohtorik İlkmen
Bulunmayanlar :

Yeliz Günel Aggöl
Öğrenme Bilimleri

Sayın Araştırmacı,
"Co-Design in Professional Development for Supporting Teachers' Agency: A Design-Based Study" başlıklı projenizin ilk adımı ile ilgili olarak yaptığımız SBB-EAK 2022/05 sayılı başvuru komisyonumuz tarafından 13 Ocak 2022 tarihli toplantıda incelenmiş ve uygun bulunmuştur.

Bu karar tüm üyelerin toplantıya çevrimiçi olarak katılımı ve oybirliği ile alınmıştır. COVID-19 önlemleri kapsamında kurul üyelerinden ıslak imza alınamadığı için bu onay mektubu üye ve raportör olarak Fatma Nevra Seggie tarafından bütün üyeler adına e-imzalanmıştır.

Saygılarımızla, bilgilerinizi rica ederiz.

Prof. Dr. Fatma Nevra SEGGIE
ÜYE

e-imzalıdır
Prof. Dr. Fatma Nevra SEGGIE
Raportör

SOBETİK 27 13.01.2022

Bu belge 5070 sayılı Elektronik İmza Kanununun 5. Maddesi gereğince güvenli elektronik imza ile imzalanmıştır.

APPENDIX D

ETHICS COMMITTEE APPROVAL FOR THE EXTENDED FIELDWORK

Evrak Tarih ve Sayısı: 29.09.2022-90011

T.C.
BOĞAZİÇİ ÜNİVERSİTESİ
SOSYAL VE BEŞERİ BİLİMLER YÜKSEK LİSANS VE DOKTORA TEZLERİ ETİK İNCELEME
KOMİSYONU
TOPLANTI KARAR TUTANAĞI

Toplantı Sayısı : 34
Toplantı Tarihi : 28.09.2022
Toplantı Saati : 16:00
Toplantı Yeri : Zoom Sanal Toplantı
Bulunanlar : Prof. Dr. Feyza Çorapçı, Doç. Dr. Arhan S. Ertan, Doç. Dr. Senem Yıldız, Dr. Öğr. Üyesi
Yasemin Sohtorik İlkmen, Dr. Öğr. Üyesi Ayşegül Metindoğan
Bulunmayanlar : Dr. Öğr. Üyesi Harun Muratoğulları

Yeliz Günel Aggöl
Öğrenme Bilimleri

Sayın Araştırmacı,

"Co-Design in Professional Development for Supporting Teachers' Agency: A Design-Based Study" başlıklı projenizin ikinci kısmı ile ilgili olarak yaptığımız SBB-EAK 2022/05 sayılı başvuru komisyonumuz tarafından 28 Eylül 2022 tarihli toplantıda incelenmiş ve uygun bulunmuştur.

Bu karar tüm üyelerin toplantıya çevrimiçi olarak katılımı ve oy birliği ile alınmıştır. Onay mektubu üye ve raportör olarak Yasemin Sohtorik İlkmen tarafından bütün üyeler adına e-imzalanmıştır.

Saygılarımızla, bilgilerinizi rica ederiz.

Dr. Öğr. Üyesi Yasemin
SOHTORİK İLKMEN
ÜYE

e-imzalıdır
Dr. Öğr. Üyesi Yasemin Sohtorik
İlkmen
Öğretim Üyesi
Raportör

SOBETİK 34 28.09.2022

Bu belge, güvenli elektronik imza ile imzalanmıştır.

APPENDIX E

FIRST (PRE-INTERVENTION) INTERVIEW QUESTIONS (ENGLISH
VERSION)

Thank you for taking the time to meet with me. In this interview, I will ask you some questions to understand your perspectives on learning and teaching, your experiences with professional collaboration, and the support you feel you need for your professional development.

Before we begin, I would like to remind you that participation in this study is entirely voluntary, and all the information you provide will be kept strictly confidential. If at any point during the interview you would like me to turn off the recording device, please let me know.

Before we start, is there anything you would like to ask me about the interview?

Introductory Questions

I will start with a few questions to get to know you as a mathematics teacher.

1. Could you tell me about your educational background?
2. How many years have you been teaching mathematics?
3. At which level are you currently teaching?
4. Do you have any responsibilities at your school other than teaching mathematics? If so, what are they?

Perspectives on Learning and Teaching

Now, I would like to ask you some questions to understand your views on mathematics learning and teaching.

5. If you were observing another teacher's mathematics class, what would you look for to determine whether it was a good lesson?

a. Would you want to see what types of math problems or activities the teacher uses?

Could you give an example?

b. What kind of classroom interactions would you expect to see?

c. What else would you want to observe?

d. Would you compare this with your own teaching practices?

6. How do you plan your lessons?

- What is the purpose of your planning?
- Are there principles you rely on while designing your lessons? What are they?
- What types of materials do you use when planning your lessons? (e.g., textbooks, teacher guidebooks, national curriculum plans, yearly plans, online resources, lesson plans, student notebooks or portfolios, academic publications)

(For each mentioned resource) Could you provide an example of how you typically use it?

- What types of resources/supports assist you in planning your lessons? What elements help you the most?
- What constraints do you encounter when planning your lessons?
- To what extent do these lesson plans allow you to realize your ideal teaching practices?
- How closely do you think you follow your lesson plans in the real classroom setting? In which situations do you feel the need to deviate from the plan?

How free and competent do you feel in making such deviations?

Teacher Collaboration and Departmental Work (Institutional Context)

I will now ask you some questions about the work you do with other mathematics teachers at your school.

7. Under what circumstances do the mathematics teachers at your school come together? (e.g., school meetings, grade-level meetings, committees, various internal workgroups)

Which of these are administratively expected, and which are initiated by the teachers themselves?

The following questions apply to all types of meetings mentioned:

- a. How often do you meet, and what is generally done during these meetings? What is their purpose?
- b. What kinds of sharing take place during these meetings? (e.g., sharing about pacing in the curriculum, logistical discussions about how to proceed, tips on teaching specific topics, sessions where student work is analyzed together)
- c. What tools/resources do you use during these meetings? (e.g., curriculum framework, student work, assessment data, textbooks, lesson plans)
- For what purposes and how do you use these tools?
 - Do you have sessions where you collaboratively evaluate student work?
- d. Are future instructional decisions usually made during these meetings, or are they more about status updates?
- e. How do these meetings impact your mathematics teaching practices and professional development?
- f. What aspects of these meetings would you like to see continue as they are, and what aspects would you like to see change?

Teacher Agency – Individual and Collective

Now, I would like to ask a few questions to understand the possibilities and needs you perceive in realizing your intended teaching practices.

8. Do you see a gap between your perspective on learning and teaching mathematics and your actual teaching practices?
 - a. What are the reasons for this gap?
 - b. What resources, supports, and tools (inside or outside your school) do you have to bridge this gap?
 - What additional supports/tools would you like to have?
9. Do you notice any differences between your perspectives on learning and teaching mathematics and those of other teachers and administrators at your school? If so, in what ways? In which situations do you observe or experience these differences?
10. The following question will be asked separately for each of these three levels:
 - Departmental level (mathematics team practices)
 - School-level policies
 - National education policies

What aspects of the work, practices, or policies at each level do you find supportive or limiting for your mathematics teaching practices?

- a. What would you like to change and what would you like to continue as is?
- b. If you see problems, what do you think their sources are?
 - Historical, longstanding reasons?
 - Current organizational structures/practices?
- c. What kind of efforts do you think are needed to address these problems?
- d. What resources, supports, and tools do you currently have to address these problems?
 - What additional ones would you like to have?

e. Have you ever individually faced and resolved a problem at any of these levels?

Could you give an example? f. Have you ever collectively (as a group of teachers)

faced and resolved a problem at any of these levels? Could you give an example?

11. How would you describe your relationship with your colleagues? How do your relationships with your colleagues impact you as a mathematics teacher?

12. Is there a space within your school community or department where you can share your opinions? How much influence do you feel you have? How much do you think your opinions impact decisions? Do you feel there is openness to your suggestions for change in your school?

13. To what extent do you feel (a) free and (b) competent in the following areas:

- Practicing your profession (e.g., designing course content, establishing classroom culture)
- Innovating and transforming your professional practices

14. Reflecting on your past experiences, what have been the turning points/situations/resources that supported your progress as a mathematics teacher?

15. Currently, what resources/situations/conditions do you think support your professional development?

16. What professional goals would you like to achieve in the future? What kind of work environment do you envision for realizing these goals?

Professional Development

Now, I would like to ask you some questions about your professional development experiences both inside and outside of school.

17. What are the most challenging areas in mathematics teaching where you feel you need support?

18. Are there any resources, either within or outside your institution, that you turn to when facing challenges in your teaching career? If so, what are they?
19. Could you describe a professional development activity that you have participated in which you believe had a positive impact on your mathematics teaching practices?
- a. Who organized it? b. What was its purpose and content? c. What types of activities were conducted during the program? (e.g., presentations, workshops, video-lesson analyses) d. How would you evaluate its impact on your mathematics teaching practices (lesson design, classroom interactions, etc.)? If it had an impact, in what way? Could you provide an example?

Boundaries Between Educational Research and Practice

Now, I would like to ask your opinions on the relationship between educational research and practice.

20. Do you use educational research in your professional practices? If so, in what way? Could you give an example?
21. In your opinion, what should be done to make educational research more helpful to practitioners?
22. What kinds of collaborations do you think could happen between academics in mathematics education and practitioners?
23. What kinds of research projects should educational researchers conduct in schools?
24. What contributions do you think practitioners can make to the advancement of educational research?
25. Have you ever participated in a research study as a participant? Could you describe your experience?

- What was the purpose of the study?
- What contribution do you think you made?
- How did participating in the research make you feel?

Motivation for Joining ERG (Teachers Network)

26. Could you tell me about your reasons for joining the Teachers Network?
27. What have you found here that you couldn't find when working with your colleagues at your school?
28. Are you able to apply what you learn in the Teachers Network to your own workplace practices?
29. How do you transfer the experience and knowledge you gain from ERG and other professional development channels into your work at your school? How might sharing what you have learned in this community impact the teaching and learning of mathematics at your school? Do you have any thoughts or plans regarding this?
30. What is your purpose for participating in the work we are planning together?
 - Professional development?
 - Contributing to the public good?
 - Developing practical materials?
31. What kind of activities do you envision for the professional learning and practice community we aim to build together?
32. In your opinion, who should be the members of this working group?

APPENDIX F

FIRST (PRE-INTERVENTION) INTERVIEW QUESTIONS (TURKISH VERSION)

Zaman ayırdığınız için teşekkür ederim. Bu görüşmede size, öğrenme ve öğretmeye dair bakış açınızı, mesleki iş birliği tecrübenizi ve mesleki gelişiminiz konusunda ihtiyaç duyduğunuz desteği anlamak için bazı sorular yönelteceğim.

Mülakata başlamadan önce bu çalışmaya katılımın gönüllülük esaslı olduğunu ve vereceğiniz bilgilerin tamamen gizliliğinin gözetileceğini hatırlatmak isterim.

Mülakat sırasında herhangi bir aşamada kayıt aletini kapatmamı isterseniz lütfen belirtin. Başlamadan önce görüşme ile ilgili bana sormak istediğiniz bir şey var mıdır?

Giriş soruları

Sizi matematik öğretmeni olarak tanımak için bazı sorular yönelteceğim.

1. Eğitim geçmişinizden bahseder misiniz?
2. Kaç yıldır matematik öğretmenliği yapıyorsunuz?
3. Şu anda hangi düzeyde ders vermektесiniz?
4. Okulunuzda matematik öğretmenliği dışında bir göreviniz var mı? Varsa, nelerdir?

Öğrenme ve öğretmeye dair bakış açısı:

Şimdi sizin matematik öğrenme ve öğretme ile ilgili bakış açınızı anlamak için birkaç soru yöneltmek istiyorum.

5. Başka bir öğretmenin matematik dersini izliyor olsanız, bu matematik dersinin iyi bir ders olduğuna karar vermek için nelere bakarsınız?

- a. Öğretmenin ne tip matematik problemleri kullandığını ya da etkinlikler uygulandığını görmek istersiniz? Bir örnek verir misiniz?
- b. Sınıftaki etkileşimin nasıl olmasını beklersiniz?
- c. Başka görmek istediğiniz neler vardır?
- d. Kendi öğretmenlik uygulamanız ile bunu karşılaştırır mısınız?

6. Derslerinizi nasıl planlıyorsunuz?

- a) Bu planların amacı nedir?
- b) Derslerinizi tasarlarken dayandığınız ilkeler var mı? Nelerdir?
- c) Derslerinizi planlarken ne tür materyallerden faydalaniyorsunuz? (Örn., Ders kitabı, öğretmen yardımcı kitapları, Milli Eğitim müfredat planı, yıllık plan, web kaynakları, ders planları, öğrenci defterleri veya portfolyoları, bilimsel yayınlar) (Bahsedilen her kaynak için) genellikle nasıl kullandığınıza dair örnek verir misiniz?
- d) Derslerinizi tasarlarken ne tip kaynaklardan/desteklerden faydalaniyorsunuz? Sizin derslerinizi planlamanızda yardımcı unsurlar nelerdir?
- e) Derslerinizi planlarken karşılaştığınız kısıtlayıcı faktörler nelerdir?
- f) Bu tasarımlar sizin idealinizdeki öğretmenliği gerçekleştirmenizi ne ölçüde sağlıyor?
- g) Gerçek sınıf ortamında bu plana ne kadar uyduğunuzu düşünüyorsunuz? Bu plandan hangi durumlarda sapma ihtiyacı hissediyorsunuz? Bu sapmaları yapma konusunda ne kadar özgür ve yetkin hissediyorsunuz?

Öğretmen işbirliği ve zümre çalışmaları (kurumsal bağlam)

Çalıştığınız okuldaki diğer matematik öğretmenleri ile yaptığınız çalışmalara dair bazı sorular yönelteceğim.

7. Okulunuzdaki matematik öğretmenleri hangi durumlarda bir araya gelmektedir? (Örn., Okul toplantıları, düzey toplantıları, komiteler, çeşitli zümre içi çalışma takımları) Bunların hangileri idari olarak beklenmektedir, hangileri öğretmenlerin kendi inisiyatifleri ile gerçekleştirdikleri toplantılardır?

* Aşağıdaki sorular tüm toplantı çeşitleri için yöneltilecektir.

- a. Ne sıklıkta toplanıyorsunuz ve bu toplantılarda genel olarak ne yapılıyor? Amacı nedir? Kısaca açıklayınız?
- b. Bu toplantılarda genel olarak nasıl paylaşımlar gerçekleştiriliyor? (Örn., müfredatta nerede olduğu gibi hız ile ilgili paylaşımlar, müfredatta nasıl ilerleneceği gibi lojistik konular, bir konunun öğretimi ile ilgili tüyolar, öğrenci çıktılarının birlikte yorumlandığı oturumlar)
- c. Bu toplantıları gerçekleştirmek için ne gibi araçlar/kaynaklar kullanıyorsunuz? (Örn., müfredat çerçevesi, öğrenci çalışmaları, ölçme-değerlendirme verisi, ders kitapları, ders planları)
 - i. Bu araçları ne amaçla ve nasıl kullanıyorsunuz?
 - ii. Öğrenci ürünlerini değerlendirerek bir çalışma yapıyor musunuz?
- d. Bu toplantılarda genel olarak ileriye dönük uygulama kararları alınıyor mu yoksa genellikle durum tespiti mi yapılıyor?
- e. Bu toplantıların sizin matematik öğretmenliği uygulamalarınıza ve mesleki gelişiminize nasıl bir etkisi oluyor?
- f. Bu toplantılarda neyin aynen devam etmesini, neyin değişmesini isterdiniz?

Öğretmenin bireysel ve kolektif aktör olma hali

Şimdi size öğretmenlik uygulamalarınız ile ilgili yapmak istediklerinizi gerçekleştirmede sahip olduğunuz ve olmadığınız imkanlar ile ihtiyaçlarınızı anlamak için bazı sorular yönelteceğim.

8. Matematik öğretmeni olarak öğrenme ve öğretmeye dair bakış açınız ile öğretmenlik uygulamalarınız arasında bir fark görüyor musunuz?

- a. Bu farkın nedenleri nelerdir?
- b. Bu farkı kapatmak için elinizdeki okul içi veya dışı imkan/kaynak, destek ve araçlar
 - i. Nelerdir?
 - ii. (Başka) neler olmasını isterdiniz?

9. Matematik öğrenme ve öğretme ile ilgili bakış açısı yönünden okulunuzdaki diğer öğretmen ve idarecilerle aranızda farklılık görüyor musunuz? Eğer öyle ise, hangi açılardan? Hangi durumlarda bu farklılıkları gözlemliyor ya da yaşıyorsunuz?

10. Aşağıdaki soru üç düzey için ayrı ayrı sorulacaktır:

- Matematik zümresi olarak yapılan çalışma ve uygulamalar
- Okul düzeyindeki politikalar
- Milli eğitim düzeyindeki politikalar

Bu düzeydeki çalışma, uygulama ya da politikaların sizin matematik öğretmenliği uygulamalarınızı gerçekleştirmede destekleyici ve sınırlayıcı olduğunu düşündüğünüz tarafları nelerdir?

- a. Neyi değiştirmek ve neyin aynen devam etmesini istersiniz?
- b. Eğer problemler görüyorsanız, sizce bu problemlerin kaynağı nedir?
 - i. Tarihsel, geçmişten gelen nedenler?

- ii. Şu anda halihazırdaki organizasyon yapınızdan/uygulamalardan kaynaklanan nedenler?
- c. Sizce bu problemleri çözmek için nasıl çalışmalar yapılması gerekir?
- d. Bu problemleri çözmek için elinizdeki imkan, destek ve araçlar
- i. nelerdir?
- ii. (başka) neler olmasını isterdiniz?
- e. Bireysel olarak bugüne kadar bu düzeyde karşılaştığınız ve çözüm geliştirdiğiniz bir sorun oldu mu? Bir örnek verebilir misiniz?
- f. Kolektif (öğretmen grubu) olarak bugüne kadar bu düzeyde karşılaştığınız ve çözüm geliştirdiğiniz bir sorun oldu mu? Bir örnek verebilir misiniz?
11. Çalışma arkadaşlarınızla ilişkinizi nasıl tanımlarsınız? Çalışma arkadaşlarınızla ilişkileriniz sizi matematik öğretmeni olarak nasıl etkiliyor?
12. Okul topluluğu ve zümre içinde görüşlerini paylaşman için bir alan var mı? Ne kadar etki alanının olduğunu, görüşlerinin kararlar üzerinde ne kadar etkili olduğunu düşünüyorsun? Okulunuzda sizin değişiklik önerilerinize alan açıldığını düşünüyor musunuz?
13. Aşağıdaki konularda kendinizi ne kadar a) özgür b) yetkin hissediyorsunuz?
- Kendi mesleğinizi icra etmek (ders içerikleri hazırlama, sınıf içi kültürünü oluşturma, vb..)
 - Kendi meslek pratiğinizde değişim ve dönüşüm yapmak
14. Geçmiş yaşantınızı düşündüğünüzde matematik öğretmeni olarak ilerlemenizi sağlayan dönüm noktaları/ durumlar/kaynaklar nelerdir?
15. Şu anda içinde bulunduğunuz durumda mesleki olarak gelişiminizi sağlayan/ sizi destekleyici olduğunu düşündüğünüz kaynaklarınız/durumlar/koşullar nelerdir?

16. Gelecekte mesleki açıdan gerçekleştirmek istediğiniz hedefleriniz nelerdir?

Bunları gerçekleştirmek için nasıl bir çalışma ortamı hayal ediyorsunuz?

Mesleki gelişim:

Şimdi okul içi ve okul dışındaki mesleki gelişim tecrübeleriniz ile ilgili bazı sorular yönelteceğim.

17. Matematik öğretimi konusunda en zorlandığınız ve destek beklediğiniz konular nelerdir?

18. Öğretmenlik hayatınızda zorlandığınız konularda yardımlaştığınız, kurum içi ya da dışı kaynaklar var mıdır? Varsa nelerdir?

19. Bugüne kadar katıldığınız ve sizin matematik öğretmenliği uygulamalarınıza olumlu anlamda etkisinin olduğunu düşündüğünüz bir mesleki gelişim çalışmasından bahseder misiniz?

- a. Kim tarafından organize edildi?
- b. Amacı ve içeriği ne idi?
- c. Bu mesleki gelişim çalışması sırasında ne tip etkinlikler gerçekleşti?
(Sunum, atölye çalışması, video-ders analizi vb.)
- d. Bu çalışmanın matematik öğretmenliği uygulamalarınıza (ders tasarımı, sınıf içi etkileşim vb. pratiklerine) katkısını nasıl değerlendirirsiniz? Eğer katkısı olduysa, ne yönde? Örnek verir misiniz?

Eğitim araştırmaları ve uygulamaları arasındaki sınırlar

Şimdi size eğitim araştırmaları ve uygulamaları arasındaki ilişki ile ilgili görüşlerinizi almak için bazı sorular yönelteceğim.

20. Mesleki uygulamalarınızda eğitim arařtırmalarından faydalaniyor musunuz? Eęer öyle ise, ne řekilde? Örnek verebilir misiniz?
21. Sizce, eğitim arařtırmacılarının eğitim uygulayıcılarına daha çok yardımcı olabilmesi için neler yapılması gerekir?
22. Sizce matematik alanından akademisyenlerin matematik alanındaki uygulayıcılar ile ne tür iş birlikleri yapabilirler?
23. Sizce, eğitim arařtırmacıları okullarda ne tür çalışmalar gerçekleřtirmeli?
24. Sizce, eğitim uygulayıcılarının eğitim arařtırmalarının ilerlemesi için ne tür katkıları olabilir?
25. Daha önce katılımcı olarak yer aldığınız bir arařtırma oldu mu? Bu deneyiminizden bahseder misiniz?
- a) Çalışmanın amacı ne idi?
 - b) Çalışmaya nasıl bir katkınızın olduğunu düşünüyorsunuz?
 - c) Arařtırmaya katılmak size nasıl hissettirdi?

ERG'ye katılma motivasyonu

26. Öğretmen Aęı'na katılma nedenlerinizden bahseder misiniz?
27. Okulunuzdaki meslektaşlarınız ile gerçekleřtiremeyip, burada bulduğunuz neler vardır?
28. Öğretmen Aęında öğrendiklerinizi kendi iş yerinizdeki uygulamalarınıza yansıtabiliyor musunuz?
29. ERG'de ve dięer mesleki gelişim kanallarından edindiğiniz tecrübe ve bilgiyi okulunuzdaki çalışmalarınıza nasıl aktarıyorsunuz? Bu toplulukta öğrendiklerinizi okulunuzdaki dięer matematik öğretmenleri ile paylaşma ve okuldaki matematik

eđitim đretimini iyileřtirme konusunda nasıl bir etkisi olabilir? Bununla ilgili bir dřnceniz ve planınız var mı?

30. Birlikte yapmayı planladığınız alıřmaya katılma amacınız nedir?

- Mesleki gelişim?
- Kamuoyu faydasına bir řeyler yapmak?
- Kullanışlı materyal geliřtirmek?

31. Burada birlikte kuracağımız profesyonel đrenme ve uygulama topluluđu ile ilgili nasıl bir aktivite řekli hayal ediyorsunuz?

32. Bu alıřma grubunun yeleri sizce kimler olmalı?

APPENDIX G

SECOND INTERVIEW QUESTIONS

(ENGLISH VERSION)

Thank you for taking the time to meet with me. In this interview, I will ask you a series of questions to understand your experiences related to the first phase of this study.

Before we begin, I would like to remind you that participation in this study is entirely voluntary, and the information you provide will be kept strictly confidential.

If at any point during the interview you would like me to stop recording, please feel free to let me know.

Before we start, is there anything you would like to ask me regarding the interview?

1. How would you compare your experience in this study to your previous professional development and community experiences?
2. During this time, as the researcher, I provided some supportive mechanisms to help you build this community. What are your thoughts on the contribution of the following supports to this aim?
 - a) The book *The Teaching Gap* we read together and the accompanying lesson videos we watched
 - b) Our "Problem Mapping" activity
 - c) The reflective writings you produced and the presentations you gave to each other
 - d) Our sessions at the end of the term, where we clarified our problems through excerpts from our meetings
 - e) The preparation process for the Teacher Network Summer Gathering

3. Throughout this process, what questions did you start asking about your own practice, and what observations or critiques did you make?
4. We are surrounded by many structural problems in both our schools and the education system. These issues can sometimes hinder our ability to improve our practice. During this process, has your perspective on these challenges shifted? Has your view of the impact you can have on your own practice changed?
5. During our community work, what individual questions regarding your teaching practice have you found answers to, or are you in the process of answering?

Does our community serve as a resource for developing solutions to the problems you face in your school or department?
6. Throughout this process, do you feel that space was created for your opinions and unique contributions within the community? What event or moment do you think was particularly influential in shaping the formation and direction of the community?
7. Do you believe this community can be a resource to help you achieve your future professional goals and to influence your colleagues?
8. Do you think we, as community members, have come together around a common problem?

What gives you the feeling that we are a professional learning community?
9. Do you think our community provides or could provide a model for interaction between educational research and practice?
10. What are your hopes and dreams for the future of this community?

How have your aspirations and dreams evolved throughout this process?

APPENDIX H
SECOND INTERVIEW QUESTIONS
(TURKISH VERSION)

Zaman ayırdığınız için teşekkür ederim. Bu görüşmede size, bu çalışmanın birinci fazı ile ilgili deneyimlerinizi anlamak için birtakım sorular yönelteceğim. Mülakata başlamadan önce bu çalışmaya katılımın gönüllülük esaslı olduğunu ve vereceğiniz bilgilerin tamamen gizliliğinin gözetileceğini hatırlatmak isterim. Mülakat sırasında herhangi bir aşamada kayıt aletini kapatmamı isterseniz lütfen belirtin. Başlamadan önce görüşme ile ilgili bana sormak istediğiniz bir şey var mıdır?

1. Bu çalışmadaki deneyiminizi önceki mesleki gelişim ve topluluk deneyimleriniz ile kıyaslar mısınız?

2. Bu süre zarfında ben araştırmacı olarak sizlerin bu topluluğu oluşturmanızda bazı destekleyici mekanizmalar sundum. Bu süreçte sunulan aşağıda saydığım desteklerin bu amaca katkısı hakkında ne düşünüyorsunuz?

- a) Birlikte okuduğumuz “Öğretim Açığı” adlı kitap ve eşliğinde izlediğimiz ders videoları
- b) “Problem Haritası” çalışmamız
- c) Yazdığınız yansıtma yazıları ve birbirinize sunumlarınız
- d) Dönem sonunda kendi toplantılarımızdan kesitler eşliğinde problemlerimizi netleştirme çalışmalarımız
- e) Öğretmen Ağı Yaz Buluşmasına hazırlık süreci

3. Süreçte yaptığımız çalışmalar size kendi pratiğinize dair hangi soruları sordurdu

ve hangi tespit ve eleştirileri yaptırdı?

4. Hem okullarımızda hem de eğitim sisteminde yapısal bir çok sorun ile çevreleniyoruz. Bu sorunlar bizim pratiğimizi iyileştirmemizde engelleyici olabiliyor. Süreçte, bunlarla ilgili bakış açınızda bir değişiklik oldu mu? Kendi pratiğiniz üzerinde sizin sahip olduğunuz etki ile ilgili bakış açınızda bir dönüşüm oldu mu?

5. Topluluk çalışmalarımız süresince öğretmenlik pratiğiniz ile ilgili hangi bireysel sorularınıza yanıt buldunuz ya da buluyorsunuz? Topluluğumuz okulunuzda ve zümrenizde yaşadığınız problemlere dair çözüm geliştirmede bir kaynak oluşturuyor mu?

6. Süreç boyunca topluluğumuzda görüşlerinize ve özgün katkınıza alan açıldığını düşünüyor musunuz? Topluluğun oluşumu ve gidişatı ile ilgili etkin olduğunu düşündüğünüz olay veya an nedir?

7. Topluluğun sizin gelecekteki mesleki hedeflerinizi gerçekleştirmeye ve meslektaşlarınızı

etkilemeye dönük bir kaynak olduğunu düşünüyor musunuz?

8. Topluluk üyeleri olarak ortak bir problemde buluştuğumuzu düşünüyor musunuz? Size bir

mesleki öğrenme topluluğu olduğumuz hissini veren nedir?

9. Topluluğumuzun eğitim araştırmaları ve uygulamaları arasındaki etkileşime bir model oluşturduğunu veya oluşturabileceğini düşünüyor musunuz?

10. Topluluğun geleceğine dair nasıl hayalleriniz var? Bu istek ve hayalleriniz süreç içinde nasıl dönüştü?

APPENDIX I

THIRD (POST-INTERVENTION FOCUS GROUP) INTERVIEW

QUESTIONS (ENGLISH VERSION)

This study aimed to design participatory professional learning environments where teachers are the subjects of their own learning processes. Now, I am writing my doctoral thesis with the goal of contributing to future studies by analyzing what this process, which we developed together, has taught us.

We can think of our study in general as consisting of four phases: discussing problems (from April to June 2022); modeling solutions (from September 2022 to June 2023); implementing the solution (from August 2023 to February 2024); and currently, reflection.

This interview has two main purposes: 1.To ask questions about your experience on certain topics. 2.To learn how my findings resonate with you and listen to your perspective. This will be a two-hour focus group interview. I would appreciate it if we could proceed by giving each other the floor and allowing everyone to voice their opinions one at a time, so we can hear everyone's voice.

1. How would you generally evaluate my attitude as a researcher during this process? How would you evaluate the researcher-teacher relationship?
 - a. Can we talk about how this relationship has affected your own professional learning process and how you have learned as a teacher?
 - b. Can we discuss how this has influenced your view of your profession and your professional learning process?
 - c. Can we talk about how this has affected your perspective on educational research?

- d. Can you describe how this relationship differs from and resembles other professional development experiences you have had?
2. Now, I would like to focus on our discussions in the second phase, which took place from September 2022 to June 2023. In the second phase, we made various readings and discussions that we decided on together, in line with your need for deeper collaboration around math teaching. We also had discussions to build the infrastructure of our community. For this, we:
- Read sections from Van de Walle,
 - Examined the principles and standards of the NCTM,
 - Discussed fundamental teaching concepts like equity, problem-solving, and conceptual learning,
 - Had discussions in the sessions we designed together and at the end of these discussions, I asked, “What insights can you draw for our community from this discussion?”
 - Filled out a document called the "Playfield" together, where we defined key concepts,
 - Later, we examined the steps of "Lesson Reciprocity" and created our activity prototype based on this inspiration.

I would especially like to hear your views on the discussions we had here. How would you define the nature of these discussions?

3. Now, I would like to share some conclusions I drew from the discussions we had in the first phase: We started by talking about our problems. To briefly remind you, we read the *Teaching Gap* book, created a problem map, and carried out various activities so that you could discuss your problems in

groups. Based on this groundwork, we planned the next steps of our community. It might be difficult to recall your experience from that time, but we can give it a try. I would like to present this to you in writing as well and hear your opinions.

Here's my finding related to this process: At the very beginning of the process, the *Teaching Gap* book and the reflections you made on it served as a mirror to identify your individual crises. It helped you understand and name your individual crises. The most dominant issue that made you feel stuck as teachers open to development was the tension between two identities: the teacher identity you wanted to be and the teacher identity you inherited and that the system forced you to embody. Seeing that you were not alone in these individual crises and discussing, talking, and sharing about them provided an opportunity for you to identify the systemic issues behind your individual crises, both as a community and as individuals. In other words, these conflicting messages were not only experienced by you as individuals, but by teachers across the country, and this encouraged you to learn more about these issues. Many of you pointed out that the lack of a clear pedagogical vision and teaching approach for math educators in our country, along with the uncertainty of collective goals, appeared as a systemic crisis that we had hoped would help you overcome this crisis, but it could not.

At the same time, realizing that you were not united under a single pedagogical vision or teaching approach led you to focus on defining the pedagogical principles and approach to mathematics for the community through readings and discussions before moving on to lesson designs. In

other words, it made you transition more consciously into the solution-building phase.

Additionally, in this process, you expanded your problem area. You began to see more clearly the areas of problems you could collectively solve—teaching culture and the systemic dimensions affecting it—and this brought you closer to becoming the subjects of the process.

In summary, our study showed the importance of discussing problems in making sense of professional learning processes. What do you think?

4. Finally, how did it feel to build the third phase of the study on this infrastructure?
5. In the third phase of the study, we implemented the activity prototype we developed with you. This section is not yet analyzed in my study. To conduct a deeper analysis, I am focusing on the first two phases in my work.

How did it feel for you to build the third phase on this infrastructure?
How does it feel to start this step after the previous work, compared to starting at this step without the prior work?

APPENDIX J

THIRD (POST-INTERVENTION FOCUS GROUP) INTERVIEW

QUESTIONS (TURKISH VERSION)

Bu çalışmanın öğretmenlerin kendi öğrenme süreçlerinin öznesi olduğu katılımcı mesleki öğrenme ortamları tasarlamak gibi bir amacı vardı. Şimdi birlikte geliştirdiğimiz bu süreç bize ne öğretti, bunun analizini yaparak ileriki çalışmalara katkıda bulunmak gibi bir amaçla doktora tezimi yazıyorum.

Çalışmamızı genel olarak 4 faz olarak düşünebiliriz. Problemlerin konuşulması (2022 Nisan-Haziran arası); Çözüm modelleme (2022 Eylül-2023 Haziran arası); Çözümün uygulanması (2023 Ağustos-2024 Şubat) ve şu anda yansıtmı.

Bu görüşmenin iki amacı var: 1. Bazı konularda deneyiminiz ile ilgili sorular sormak 2. Bazı bulgularımın sizde nasıl karşılık bulduğunu öğrenmek, dinlemek. Bu iki saatlik bir odak grup görüşmesi olacak. Hepinizin sesini duyabilmek için sırayla ve birbirimize söz hakkı tanıyarak ilerlersek sevinirim.

Şu soru ile başlamak istiyorum:

1. Genel olarak bu süreçte araştırmacı olarak benim tutumumu nasıl değerlendirirsiniz? Araştırmacı-öğretmen ilişkilenmesini nasıl değerlendirirsiniz?
 - a. Peki bu ilişkilenme biçiminin sizin kendi mesleki öğrenme sürecinize ve öğretmen olarak nasıl öğrendiğinize etkisinden bahsedebilir miyiz?
 - b. Sizin kendi mesleğe ve mesleki öğrenme sürecinize bakışına etkisinden bahsedebilir miyiz?

- c. Sizin eğitim arařtırmalarına olan bakıřınıza etkisinden bahsedebilir miyiz?
- d. Bu iliřkilenme biçiminin diğerk mesleki gelişim deneyimlerinizden farklı ve benzer yönlerini tanımlar mısınız?
2. řimdi Eylül 2022-Haziran 2023 arası yaptığımız ikinci fazdaki tartışmalarımıza odaklanmak istiyorum. İkinci fazda sizlerin matematik öğretimi üzerine daha fazla ortaklaşma, derinleşme ihtiyacınız doğrultusunda birlikte karar verdiğimiz çeşitli okumalar ve tartışmalar yaptık.
- Topluluğumuzun alt yapısını oluşturmak için tartışmalar yaptık. Bunun için:
- Van de Walle'dan belirlediğimiz bölümleri okuduk, NCTM'in prensip ve standartlarını inceledik. Eşitlik, problem çözme, kavramsal öğrenme gibi öğretim ile ilgili temel kavramları tartıştık.
 - Birlikte tasarladığımız oturumlarda tartışmalar yaptık ve tartışmaların sonunda sizlere "bu tartışmadan topluluğumuz için nasıl çıkarımlar yaparsınız?" şeklinde sorular sordum.
 - Oyun Alanımız denen bir dökümanı birlikte doldurduk, temel kavramları birlikte tanımladık.
 - Daha sonra Ders İmecesi adımlarını inceledik ve buradan ilhamla kendi aktivite prototipimizi oluşturduk.
 - Özellikle burada yaptığımız tartışmalarla ilgili görüşlerinizi duymak isterim. Bu tartışmaların doğasını nasıl tanımlarsınız?
3. řimdi birinci fazda yaptığımız tartışmaları dönüp incelediğimde çıkardığım bazı sonuçları sizinle paylaşmak istiyorum. İlk olarak problemlerimizi

konusarak başladık. Kısaca hatırlatmak gerekirse, ğretim aıęı kitabını okuduk, problem haritası oluřturduk, sizlerin grupa problemlerinizi konuřabilmeniz iin eřitli aktiviteler yaptık. Ve bu altyapı zerine topluluęumuzun ilerleyen alıřmalarını kurguladık. O tarihteki deneyiminizi hatırlamak zor olabilir, fakat bir deneyebiliriz. Sizlere yazılı olarak da sunmak ve grřlerinizi duymak isterim. Bu srele ilgili bulgum řyle. Srecin en bařında ğretim aıęı kitabının ve zerine yaptığınız yansıtmanın size bireysel krizlerinizi tespit etmeye dnk bir ayna iřlevi grdę. Bireysel krizlerinizi anlamlandırmanız ve adını koymanızda yardımcı oldu. En baskın, sizleri kendini geliřtirmeye aık ğretmenler olarak sıkıřmıř hissettiren konunun iki kimlik arasında sıkıřmıř olmak olduęunu tespit ettim: Olmak istedięiniz ğretmen kimlięi ile miras aldıęınız ve sistemin de sizi olmaya zorladıęı ğretmen kimlięi.

Topluluk olarak bu bireysel krizlerde yalnız olmadığınızı grmek ve bunlar zerine tartıřmak, konuřmak, paylařmak sizlerin hem topluluk olma hem de bireysel krizlerinizin arkasındaki sistemsel sorunları tespit etme olanaęı saęladı. Yani benzer eliřkili mesajları yalnızca sizler bireyler olarak deęil, lkedeki ğretmenler olarak deneyimliyormuřsunuz ve bunlarla ilgili daha ok ğrenme konusunda teřvik etti. Bir oęunuz lkemizde matematik eęitimcilerinin altında birleřtięi pedagojik vizyonun ve ğretim yaklařımının net olmaması, kolektif hedeflerimizin belirsizlięi sizin bu krizi ařmanızda yardımcı olmasını beklediğimiz fakat olamayan bir sistemsel kriz olarak ortaya ıktı.

Aynı zamanda belirli bir pedagojik vizyon ve ğretim yaklařımında birleřmiyor olduęunuzu fark etmek sizleri ders tasarımları yapmaya

geçmeden önce okumalar ve tartışmalar yoluyla topluluğun pedagojik prensiplerini ve matematiği yaklaşımını belirlemeye itti. Yani sizlerin daha bilinçli bir şekilde çözüm üretme fazına geçmenizi sağladı. Aynı zamanda bu süreçte problem alanınızı genişlettiniz; dokunabileceğiniz, kolektif olarak çözüm üretebileceğiniz problem alanlarını daha net görmeye başladınız—öğretim kültürü ve bunu etkileyen sistemsel boyutlar—ve sizi sürecin öznelere olmaya daha çok yaklaştırdı.

Özetle problemleri konuşmanın mesleki öğrenme süreçlerini anlamlandırma anlamında önemini gösterdi bu çalışmamız. Ne dersiniz?

4. Son olarak, çalışmanın üçüncü fazını bu alt yapı üzerine kurmak size nasıl hissettirdi?
5. Çalışmanın üçüncü fazında sizlerle geliştirdiğimiz aktivite prototipinin uygulamasını yaptık. Bu kısım şimdilik benim çalışmamda analiz ettiğim bölüm değil. Daha derinlikli analizini yapabilmek için sizinle birlikte geçtiğimiz öğrenme süreçlerinin, ilk iki faza odaklanıyorum kendi çalışmamda.

Üçüncü fazı böyle bir alt yapı üzerine kurmak size nasıl hissettirdi? Önceki çalışmaları yapmadan bu adımdan başlamak ile arasında nasıl bir fark hissediyorsunuz.

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