

DESIGNING A MODULE TO SUPPORT DEVELOPMENT OF PRE-
SERVICE MATHEMATICS AND SCIENCE TEACHERS' SKILLS FOR
INTERDISCIPLINARY TEACHING

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ABSTRACT

DESIGNING A MODULE TO SUPPORT DEVELOPMENT OF PRE-SERVICE MATHEMATICS AND SCIENCE TEACHERS' SKILLS FOR INTERDISCIPLINARY TEACHING

Interdisciplinary education can help teachers in supporting their students in dealing with complex problems today. However, since teachers might not have sufficient competencies for interdisciplinary teaching, professional support is needed. So, preparing pre-service teachers as competent future teachers in interdisciplinary teaching is also important. Thus, this study aimed to design a module to support pre-service mathematics and science teachers' development of skills for interdisciplinary teaching. This study has adopted design-based research as a methodological framework. The study was conducted in three phases with two iterations to design a module on interdisciplinary teaching. Participants were pre-service mathematics and science teachers in a state university in Istanbul. In this study, qualitative data from different data sources such as participants' artifacts and responses to semi-structured interviews were collected and analyzed through open-coding, axial coding and thematic analysis. The findings indicated that the limited initial conceptualizations of the pre-service teachers changed to more elaborated conceptualizations after the implementation. Moreover, based on the findings, interdisciplinarity, mathematical modeling, and implementation were key dimensions of designing interdisciplinary tasks and problems. Based on the findings of this study, promoting interdisciplinarity, mathematical modeling and active teacher learning experiences were suggested as key aspects to design a module on interdisciplinary teaching for pre-service teachers.

ÖZET

MATEMATİK VE FEN BİLGİSİ ÖĞRETMEN ADAYLARININ DİSİPLİNERARASI ÖĞRETİM BECERİLERİNİN GELİŞİMİNİ DESTEKLEMELİK ÜZERE BİR MODÜLÜN TASARLANMASI

Disiplinlerarası eğitim, öğretmenlerin öğrencilerinin günümüzün karmaşık problemleriyle başa çıkmalarına yardımcı olabilir. Ancak, öğretmenler disiplinlerarası öğretim için yeterli yeterliliklere sahip olmayabileceğinden, profesyonel desteğe ihtiyaç duyulmaktadır. Bu nedenle, öğretmenler adaylarını, disiplinlerarası öğretimde yetkin geleceğin öğretmenleri olarak hazırlamak da önemlidir. Bu nedenle, bu çalışma, matematik ve fen bilimleri öğretmenler adaylarının disiplinlerarası öğretim için becerilerinin gelişimini desteklemek üzere bir modül tasarlamayı amaçlamıştır. Bu çalışma, tasarım tabanlı araştırmayı metodolojik bir çerçeve olarak benimsemiştir. Çalışma, disiplinlerarası öğretime yönelik bir modül tasarlamak amacıyla iki yinelemeli (iterasyonlu) üç aşamada yürütülmüştür. Katılımcılar, İstanbul'daki bir devlet üniversitesindeki matematik ve fen bilimleri öğretmen adaylarıdır. Bu çalışmada, katılımcıların ürünleri ve yarı yapılandırılmış görüşmelere verdikleri yanıtlar gibi farklı veri kaynaklarından nitel veri toplanmış ve açık kodlama, eksenli kodlama ve tematik analiz yoluyla analiz edilmiştir. Bulgular, öğretmen adaylarının sınırlı ilk kavramsallaştırmalarının, uygulama sonrasında daha ayrıntılı kavramsallaştırmalara dönüştüğünü göstermiştir. Ayrıca, bulgulara dayanarak, disiplinlerarasılık, matematiksel modelleme ve uygulama, disiplinlerarası görev ve problemlerin tasarlanmasında temel boyutlar olarak belirlenmiştir. Bu çalışmanın bulgularına dayanarak, disiplinlerarasılığı, matematiksel modellemeyi ve aktif öğretmen öğrenme deneyimlerini teşvik etmek, öğretmen adayları için disiplinlerarası öğretim üzerine bir modül tasarlanmasının temel unsurları olarak önerilmiştir.

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LIST OF ACRONYMS

BC	British Columbia
BC MoE	British Columbia Ministry of Education
CIdME	Course on Interdisciplinary Mathematics Education
CO ₂	Carbon dioxide
DBR	Design-based Research
DIFMT	Design and Implementation Framework for Mathematical Modeling Tasks
DP-V1	Design Principles-Version 1
DP-V2	Design Principles-Version 2
DP-V3	Design-Principles-Version 3
IMMTs	Interdisciplinary Mathematical Modeling Tasks
IMPT	Interdisciplinary Mathematical Problems for Teaching
IPCK	Interdisciplinary Pedagogical Content Knowledge
NAS	National Academy of Sciences
NGSS	Next Generation Science Standards
OECD	Organisation for Economic Co-operation and Development
P21	Partnership for 21 st Century Learning
PCK	Pedagogical Content Knowledge
PD	Professional Development
PDIMP	Program on Designing Interdisciplinary Mathematical Problems
PDM	Professional Development Module
SCED 352	Teaching Algebra and Pre-calculus for Secondary School
SCED 48F	Interdisciplinary Mathematics and Science Education
STEM	Science, Technology, Engineering, and Mathematics
STEAM	Science, Technology, Engineering, Arts, and Mathematics
TAGS	Task Analysis Guide in Science
TUIK/TÜİK	Türkiye İstatistik Kurumu

1. INTRODUCTION

Today rapid technological developments as well as emerging complex problems have led to an increasing need for raising citizens handling such global challenges (e.g., Maass et al., 2019; Organisation for Economic Co-operation and Development [OECD], 2019). *Interdisciplinarity* can help learners deal with new challenges in the twenty-first century such as pandemics or environmental issues (e.g., Maass et al., 2019). In the broad sense, *interdisciplinarity* refers to an approach of using different disciplines to focus on a phenomenon (e.g., Szostak, 2003).

The researchers (e.g., Boix Mansilla et al., 2000; Spelt, 2015) used different terms for referring *interdisciplinarity*. For example, the National Academy of Sciences (NAS) (2005) used the term of *interdisciplinary research*, which was defined as a type of research integrating necessary components from different disciplines to offer solutions to complex problems. This perspective considers interdisciplinarity in terms of research, therefore indicates a broader definition focuses on the process of interdisciplinarity. Instead, Boix Mansilla, Miller, and Gardner (2000) used the term of *interdisciplinary understanding*, which was defined as “modes of thinking” coming from several disciplines to explain phenomena, solve problems or generating a product (p. 17). Like the former mention on the cognitive aspect in interdisciplinarity, Spelt (2015) used the terms of *interdisciplinary thinking* or *interdisciplinary understanding* to refer to the concept of interdisciplinarity. Moreover, Lenoir and Hasni (2016) used the term of *school interdisciplinarity*, which was defined as an approach to curriculum that could connect different school disciplines in the curriculum for an aim of cooperation or interdependence. This latter definition is a more specific one that focuses on curricular connections between disciplines in a school and therefore, it implies a pedagogy including interdisciplinary learning goals coming from different disciplines. Although different terms might be used to refer to interdisciplinarity, the commonalities in the terms indicate that interdisciplinarity involves at least two different disciplines and requires cognitive aspects, focuses on an object (such as a problem, issue or a concept) that requires different insights, and produces a novel understanding to the object that has been investigated through different disciplines. Building on these different definitions, a cognitive perspective to interdisciplinarity was adopted in this study. Thus,

interdisciplinarity in this study is defined as a holistic way of using different disciplines to solve problems that go beyond using a single discipline (Boix Mansilla, 2010).

When the concept of *interdisciplinarity* is transferred to the education, *interdisciplinary education* is considered. *Interdisciplinary education* adopts an interdisciplinary approach to education such that learners could integrate different disciplinary insights to get an advanced level of understanding such as solving problems (e.g., Boix Mansilla et al., 2000; Song and Wang, 2021). The term of interdisciplinary education has often been used in undergraduate education that includes interdisciplinary programs (e.g., Boix Mansilla and Duraising, 2007; Song and Wang, 2021). This implies interdisciplinarity in terms of interdisciplinary research. Moreover, interdisciplinary education has been noted to offer students *interdisciplinary learning opportunities* that are kind of experiences that are expected to mediate “the construction of interdisciplinary thinking” (Nugraha et al., 2024, p. 1). Boix Mansilla (2010) also defined *interdisciplinary learning* as an integrated way of “information, data, techniques, tools, perspectives, concepts and/or theories from two or more disciplines to craft products, explain phenomena or solve problems, in ways that would have been unlikely through single-disciplinary means” (p. 289). This study adopts this definition of *interdisciplinary learning* (Boix Mansilla, 2010) since it explains the cognitive process of learning with more details, and this study focuses on the cognitive process of interdisciplinarity. By mentioning “information, data, techniques, tools, perspectives, concepts and/or theories”, Boix Mansilla (2010) focused on explicitly what are integrated in the process (p. 289). Moreover, in the definition, the focus on products, phenomena and problems indicates the outcomes which are expected by the learners (Boix Mansilla, 2010).

Concerning the teaching aspect of interdisciplinarity, a highly related concept is *interdisciplinary teaching*. Interdisciplinary teaching can be defined as “instructional activities that involve two or more disciplines in a way that all areas are meaningfully addressed simultaneously” (Hillman et al., 2000, p. 92). Interdisciplinary teaching could enhance students’ motivation and interest (e.g., Zhan et al., 2017), facilitate integrating knowledge and skills from different disciplines (e.g., Spelt, 2015; Nugraha et al., 2024), help students to make transfer from one context to others (OECD, 2019), enhance problem-solving skills (Williams et al., 2016) and develop interdisciplinary competence (Song and

Wang, 2021). Thus, interdisciplinary teaching could offer certain benefits for learners that are aligned with the twenty-first century skills such as collaboration, communication, problem-solving (e.g., the Partnership for 21st Century Learning [P21], 2019).

Interdisciplinary education can be implemented through different methods and techniques (Şahin, 2022). As an approach to interdisciplinary education, certain subjects can be regrouped into key learning areas for a specific purpose. *Science, Technology, Engineering, and Mathematics* (STEM) “(with some variations, e. g. STEAM- stem+ art and design)” is an example of regrouping subjects (OECD, 2019, Box 3, p. 8). STEM was especially prevalent in science education and was regarded as one of interdisciplinary approaches that emphasized scientific processes (e.g., Şahin, 2022; Siller et al., 2024). Particularly, *integrated STEM education* (as a version of STEM education) could be valuable for interdisciplinary education due to the integration of different disciplines (e.g., Akaygun and Aslan-Tutak, 2020; Nugraha et al., 2024). It is defined as a kind of integration for various contexts to arrive at solutions to problems of real-world (Mumcu et al., 2023). This is aligned with the definition of interdisciplinarity of Boix Mansilla (2010) since a coherent integration of disciplines has been needed to solve complex problems.

Although interdisciplinary education was more emphasized in science education (especially through integrated STEM) (e.g., Şahin, 2022), the role of mathematics in interdisciplinary education has gained an impetus in recent years (e.g., Williams et al., 2016; Maass et al., 2019). A similar interest in interdisciplinary approach for mathematics education was also mentioned in a review study on interdisciplinary approaches in Turkey (Ergün and Aydın, 2025). Concerning interdisciplinary education in mathematics, *interdisciplinary mathematics education* was mentioned to be relatively new focus of research (Williams et al., 2016). Based on their review, Williams and his colleagues (2016) have called for further research on interdisciplinary mathematics education since there is a need for empirical studies that would help to conceptualize interdisciplinarity in terms of mathematics education as well as develop pedagogical approaches to support teachers. Interdisciplinary *mathematics education* refers to teaching and learning mathematics by using one or more disciplines in a way to create a “cognitive advancement” (Boix Mansilla et al., 2000). Herein, *cognitive advancement* (Boix Mansilla et al., 2000) can be considered as an analogy of lichen. The relationship of disciplines can resemble the mutualism seen in

algae and fungi to produce lichen as a new form. Thus, a balanced contribution of each discipline is necessary to generate a profound relationship (e.g., An, 2017). However, interdisciplinary mathematics teaching could be challenging in implementing at different grade levels such as due to the teachers' lack of pedagogical knowledge for interdisciplinarity (e.g., Staats, 2014; Hasni et al., 2015; An, 2017; Weinberg and Sample McMeeking, 2017; Borromeo Ferri, 2019; Tytler et al., 2019; Santaolalla et al., 2020). As Staats (2014) also emphasized, teachers might have had difficulties in teaching other disciplines because authentic teaching in other disciplines was not familiar with them. Moreover, concerning conceptualizations of teachers about interdisciplinary teaching, Weinberg and Sample McMeeking (2017) found that mathematics and science teachers expressed the need for "interdisciplinary pedagogical skills" as well as "pedagogical content knowledge in their own disciplines" (p. 211). For further research and practice, to decrease the difficulties about the implementation of interdisciplinary teaching, researchers recommended to use different approaches such as collaboration, content training, and learning by doing (e.g., Weinberg and Sample McMeeking, 2017; Akaygun and Aslan-Tutak, 2020; Fitzpatrick and Leavy, 2025).

For effective interdisciplinary teaching, teacher preparedness was emphasized by the researchers (e.g., Ryu et al., 2019; Yang and Ball, 2024). In addition to professional development on in-service teachers, pre-service teacher education was recommended for interdisciplinary teaching (e.g., Ryu et al., 2019; Akaygun and Aslan-Tutak, 2020; Fitzpatrick and Leavy, 2025). There are also empirical studies on interdisciplinary teaching including pre-service teachers (e.g., An, 2017; Ryu et al., 2019; Akaygun and Aslan-Tutak, 2020; Pimthong and Williams, 2021; Fitzpatrick and Leavy, 2025). These studies mostly focused on pre-service primary school teachers (e.g., An, 2017; Pimthong and Williams, 2021; Fitzpatrick and Leavy, 2025) in integrated STEM education designs with a relatively low focus on secondary pre-service teachers (e.g., Ryu et al., 2019; Akaygun and Aslan-Tutak, 2020). An (2017) focused on the development of the pre-service elementary teachers' knowledge of interdisciplinary pedagogy. Mathematics and science integration was also the most frequent kind of interdisciplinary teaching approach that was used in certain studies about pre-service teachers (e.g., Krause et al., 2019; Akaygun and Aslan-Tutak, 2020) with a less focus on mathematics and social studies integration (An and Tillman, 2018; Santaolalla et al., 2020), STEAM integration (An, 2020), or mathematics and arts integration

(e.g., An, 2017). Moreover, concerning pre-service education on interdisciplinary teaching, some researchers prepared methods courses and expected the pre-service teachers develop interdisciplinary lesson plans (that is, integrated STEM education plans) (Ryu et al., 2019; Pimthong and Williams, 2021). Through design of authentic learning environments such as methods courses for STEM education, researchers investigated the pre-service teachers' experiences and practices of STEM integration (Ryu et al., 2019; Pimthong and Williams, 2021). Through authentic designs, the pre-service teachers could design lesson plans aligned with integrated STEM education, but they had difficulties in implementing the designed lesson plans in classroom environment related to "current school practices, limited interdisciplinary understandings, and a lack of role models" (Ryu et al., 2019, p. 493). Thus, different experiences were included in the designs to support pre-service teachers on interdisciplinary teaching. Interdisciplinary task design and the implementation of designed tasks in the classroom was a crucial part of investigating interdisciplinary experiences and practices of pre-service teachers. In this study, interdisciplinary task design was considered a core aspect of the design considering pre-service teachers for learning to teach interdisciplinary education.

In addition to STEM integration, *mathematical modeling* was also used as an approach to interdisciplinary education (e.g., English, 2022; Stillman et al., 2023; English, 2024; Gravemeijer, 2024; Jeon and Star, 2024; Siller et al., 2024). Especially, mathematical modeling was used to enhance the role of mathematics in interdisciplinary education (e.g., English, 2024). By using mathematics, *mathematical modeling* aims to solve complex real-world problems (Garfunkel and Montgomery, 2016). During this iterative process, a *mathematical model*, which is an idealized representation of phenomena in real-world to better understand and explore its characteristics, is produced (Palacios, 2022). *Mathematical modeling* is defined as an approach to utilize "mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (Garfunkel and Montgomery, 2016, p. 8). This definition was taken as a basis for the current study. According to Garfunkel and Montgomery (2016), mathematical modeling is a cyclical and iterative process composed of the following elements: "Identify the problem, make assumptions and identify variables, do the math, analyze and assess the solution, iterate, implement the model" (pp. 12-13). Since mathematical modeling is a kind of problem-solving including real-world problems, it can be a way to support interdisciplinary

approaches in education (e.g., Stillman et al., 2023). In this study, mathematical modeling was included as an approach to enhance the role of mathematics in interdisciplinary education.

Moreover, concerning studies on interdisciplinary education in Turkey, Ergün and Aydın (2025) pointed out the prevalence of opinion studies and a focus of elementary school students. Thus, the empirical studies on teachers and pre-service teachers concerning interdisciplinary education were limited (e.g., Ergün and Aydın, 2025). The opinion studies on interdisciplinary education could give information about the conceptualizations on interdisciplinary education in mathematics and might be beneficial to develop certain interventions to support interdisciplinary teaching practices. However, there is also a need for other types of research such as design-based research that involves investigation of complex educational settings with design because of the current state of limited design-based studies on interdisciplinary teaching in Turkey (e.g., Ergün and Aydın, 2025). Moreover, further research on developing skills of interdisciplinary teaching was called for the context of Turkey (e.g., Ergün and Aydın, 2025).

Based on the literature, although previous research about interdisciplinary education provided insights about interdisciplinary teaching in mathematics education, there is still a need for further research on interdisciplinary education with pre-service teachers, especially on investigating how pre-service mathematics and science teachers could be supported in interdisciplinary teaching. To address this problem, this study aimed to design a module on interdisciplinary teaching through a design-based research framework to support the development of the pre-service mathematics and science teachers on interdisciplinary teaching. Through the design of a module on interdisciplinary teaching, it was aimed to develop the pre-service mathematics and science teachers' skill in designing interdisciplinary mathematical problems for teaching.

Based on the gap identified in the literature, a module on interdisciplinary teaching was designed in this study. The module was implemented in three cycles. For Cycle 1 and Cycle 2, the module on interdisciplinary teaching was initially referred to the *Professional Development Module* (PDM) and then referred to the *Program on Designing Interdisciplinary Mathematical Problems* (PDIMP), respectively. The module on

interdisciplinary teaching aimed to support pre-service mathematics and science teachers' development of skills for interdisciplinary teaching. *Interdisciplinary teaching* in this study refers to teaching mathematics and science with an interdisciplinary approach integrating other disciplines to solve a mathematical modeling problem (Boix Mansilla, 2010; Garfunkel and Montgomery, 2019). The module on interdisciplinary teaching in this study focused on the investigation of the skill to *formulate interdisciplinary tasks*, which was conceptualized as a part of the skill for interdisciplinary teaching. To formulate interdisciplinary tasks refers to constructing mathematical modeling tasks focused on interdisciplinary teaching. For Cycle 2, in the module on interdisciplinary teaching (the *Professional Development Module*), to *formulate interdisciplinary tasks* referred to designing interdisciplinary mathematical modeling tasks. For Cycle 3, in module on interdisciplinary teaching (the *Program on Designing Interdisciplinary Mathematical Problems*), to *formulate interdisciplinary tasks* referred to designing interdisciplinary mathematical problems for teaching. The *module on interdisciplinary teaching* is defined as a module that has the core design elements of discussion, scaffolding, group work, resources on interdisciplinary teaching, and rich tasks aimed to help pre-service teachers learn to teach with an interdisciplinary approach.

1.1. The Purpose of Study and Research Questions

This study aims to develop the skill of pre-service mathematics and science teachers' development in interdisciplinary teaching through the design of a module on interdisciplinary teaching. To address this purpose, the conceptualizations of the pre-service teachers on interdisciplinary teaching, and how they designed interdisciplinary tasks or problems were investigated. Moreover, it was aimed to find out the design principles for a module on interdisciplinary teaching. This study used design-based research (DBR) as a methodological framework to address the research questions:

- RQ 1. What are the *conceptualizations* of the pre-service secondary mathematics teachers on interdisciplinary teaching within the context of a mathematics teaching course during Spring 2022? (Cycle 1)
- RQ 2. How did the *conceptualizations* of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Professional

Development Module (PDM) within the course of Interdisciplinary Mathematics and Science Education during Fall 2023? (Cycle 2)

- RQ 3. How did the pre-service mathematics and science teachers who participated in the Professional Development Module (PDM) design *interdisciplinary mathematical modeling tasks* (IMMTs)? (Cycle 2)
- RQ 4. What are the *design principles* of a module on interdisciplinary teaching? (Cycle 2)
- RQ 5. How did the *conceptualizations* of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) within the course of Interdisciplinary Mathematics and Science Education during Fall 2024? (Cycle 3)
- RQ 6. How did the pre-service mathematics and science teachers who participated in the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) design *interdisciplinary mathematical problems for teaching* (IMPTs)? (Cycle 3)
- RQ 7. What are the *design principles* of a module on interdisciplinary teaching? (Cycle 3)

1.2. Significance of the Study

The contribution to both literature and educational practice is the significance of this study. The current study has contributed to the literature and educational practice by offering a design to support the development of pre-service mathematics and science teachers' skills in interdisciplinary teaching. This study developed a *module on interdisciplinary teaching*, which has interdisciplinarity, mathematical modeling and active teacher learning experiences as key elements. This module could be used by teacher educators as practical guide for their administration of new university courses on interdisciplinary teaching for pre-service teachers.

This study may unpack how pre-service mathematics and science teachers collaboratively designed interdisciplinary tasks and problems. Concerning interdisciplinary education, since the literature mostly focused on integrated STEM task design, especially

lesson plan design (e.g., Fitzpatrick and Leavy, 2025), this study could offer a new focus on interdisciplinary tasks including connections of mathematics and another discipline.

Furthermore, this study defined a new construct of *interdisciplinary mathematical problems for teaching*: Such problems are those that include mathematical modeling and interdisciplinarity as key elements. This was a contribution on the literature that math-focused interdisciplinary task design by the pre-service teachers since this study combined interdisciplinarity and mathematical modeling for task design. Researchers suggested to improve interdisciplinary education programs (e.g., English, 2024; Yang and Ball, 2024). Consistent with this suggestion, this study used a math-focused perspective on task design that could increase the role of mathematics in interdisciplinary education.

Another contribution is about teacher education. This study developed a framework based on literature and data to analyze the pre-service teachers' interdisciplinary tasks. Thus, teacher educators could use the framework as a guideline for helping pre-service teachers in preparing and evaluating interdisciplinary tasks. Moreover, this framework could be further tested in other contexts by researchers.

Last but not the least important contribution is designing a module on interdisciplinary teaching for pre-service teachers through cycles of implementation. The use of design-based research to develop the module on interdisciplinary teaching was also a contribution since there were limited design-based studies on interdisciplinary teaching in Turkey (e.g., Ergün and Aydın, 2025). Teacher educators could use this module to support pre-service mathematics and science teachers in facilitating their skills in designing interdisciplinary problems. This module could also be adapted to in-service teacher education on interdisciplinary teaching and thus could offer new opportunities for design in teacher education. Overall, this study might be useful for teacher educators and policymakers in that new educational programs on interdisciplinary teaching could be offered.

1.3. Definitions of the Key Terms

A description of the meaning of certain key terms in this study is presented in this section. Design-based research, task, cognitive demand, mathematical modeling,

interdisciplinarity, interdisciplinary teaching, module on interdisciplinary teaching, and interdisciplinary mathematical modeling tasks are certain key terms.

Design-based research (DBR) is defined as a methodological framework having an advisory characteristic aiming to generate both innovative designs and theories (Bakker, 2019). A *cycle* is defined as “a step in the process of doing research and will include systematic reflection on the theoretical aspects or design principles in relationship to the status of the intervention ultimately resulting in *design principles* or theoretical statements” (Plomp, 2013, p. 31, italics added).

Task is defined as “any instructional or assessment-related unit of disciplinary work that is assigned to students to intellectually engage them in science or mathematics” (Tekkumru-Kisa et al., 2020, p. 607). The *cognitive demand* of a task is defined as the required thinking type and level of students when they experience a task (Stein et al., 2000). *Mathematical modeling* is defined as an approach to utilize “mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena” (Garfunkel and Montgomery, 2016, p. 8). *Mathematical modeling tasks* are defined as tasks including the mathematical modeling process such that a mathematical model is built (Garfunkel and Montgomery, 2016).

Interdisciplinarity is defined as a holistic way of using different disciplines to solve problems that go beyond using a single discipline (Boix Mansilla, 2010). *Interdisciplinary education* is defined as an interdisciplinary approach to education such that learners could integrate different disciplinary insights to get an advanced level of understanding such as solving problems (e.g., Boix Mansilla et al., 2000; Song and Wang, 2021). *Interdisciplinary teaching* in this study refers to teaching mathematics and science with an interdisciplinary approach integrating other disciplines to solve a mathematical modeling problem (Boix Mansilla, 2010; Garfunkel and Montgomery, 2019). *Formulating interdisciplinary tasks* is defined as designing interdisciplinary mathematical modeling tasks. The skill of formulating interdisciplinary tasks is considered as a component of the skill for interdisciplinary teaching.

Moreover, the *module on interdisciplinary teaching* is defined as a module that has the core design elements of discussion, scaffolding, group work, resources on interdisciplinary teaching, and rich tasks aimed to help pre-service teachers learn to teach with an interdisciplinary approach. The *Professional Development Module* (PDM) is the module on interdisciplinary teaching focusing on the design of interdisciplinary mathematical modeling tasks in Cycle 2. *Interdisciplinary mathematical modeling tasks* (IMMTs) are defined as mathematical modeling tasks including interdisciplinarity. Furthermore, the *Program on Designing Interdisciplinary Mathematical Problems* (PDIMP) is the module on interdisciplinary teaching focusing on the design of interdisciplinary mathematical problems for teaching in Cycle 3. *Interdisciplinary mathematical problems for teaching* (IMPT) are defined as problems that include mathematical modeling and interdisciplinarity as key elements with pedagogical purposes.

2. LITERATURE

The purpose of this study was to design a module to support pre-service mathematics and science teachers' development of skills for interdisciplinary teaching. Based on this purpose, literature on interdisciplinarity, mathematical modeling, and teacher education were examined. This section will be described in four parts: Interdisciplinary teaching, mathematical modeling for interdisciplinary teaching, teacher education for interdisciplinary teaching, and elements of pre-service teacher education on interdisciplinary teaching.

2.1. Interdisciplinary Teaching

The idea of interdisciplinarity is in fact an old one. According to Gunn (1992), interdisciplinarity can be traced back “in the West to classical antiquity, when Greek historians and dramatists drew on medical and philosophical knowledge, respectively, for clues to the reconception of their own material” (pp. 239-240). The term is commonly used for interdisciplinary research (e.g., Szostak, 2003), but the use of the term for approaches in education has been a recent trend based on the increase of interest in STEM education (e.g., Nugraha et al., 2024).

There are different definitions of *interdisciplinarity*. For instance, Szostak (2003) defined interdisciplinarity as “[o]penness to the application of *all* theories and *all* methods to *any* set of phenomena” (p. 30). Szostak’s (2003) definition can be considered as a broad one because it is allowed to contain all theories and methods to achieve a “collective understanding” about any type of phenomena (p. 34). This definition of interdisciplinarity is *research-focused* and aims to describe the process of interdisciplinarity in the sense of interdisciplinary research by analyzing *what* is integrated in the process. Szostak (2003) noted the deductive character of the definition by emphasizing the integration of “phenomena, theories, methods, and worldviews” (p. 47).

Another definition of interdisciplinarity was done by Klein and Newell (1996). Interdisciplinarity is associated with *interdisciplinary studies*. Like Szostak (2003), Klein and Newell (1996) considered interdisciplinarity as a complex process in the frame of

research, but this time emphasized the *integration* of disciplinary insights in interdisciplinary studies:

A process of answering a question, solving a problem, or addressing a topic that is too broad or complex to be dealt with adequately by a single discipline or profession . . . Interdisciplinary studies [draw] on disciplinary perspectives and integrates their insights through construction of a more comprehensive perspective. (Klein and Newell, 1996, p. 395)

Moreover, like Klein and Newell (1996), the National Academy of Sciences [NAS] (2005) associated interdisciplinarity with interdisciplinary research and defined *interdisciplinary research* as the following:

Interdisciplinary research is a mode of research by teams or individuals that integrates information, data, techniques, tools, perspectives, concepts, and/or theories from two or more disciplines or bodies of specialized knowledge to advance fundamental understanding or to solve problems whose solutions are beyond the scope of a single discipline or field of research practice. (p. 26)

The definitions of interdisciplinarity by both Klein and Newell (1996) and NAS (2005) emphasized the process of interdisciplinary research and took attention to cognitive elements of interdisciplinary research. Moreover, the definition by NAS (2005) also adds the human factor in research by indicating that interdisciplinary research could be conducted by an individual who is knowledgeable in different disciplines or by a team of experts in different disciplines. Besides, interdisciplinary thinking and interdisciplinary understanding were also used for describing interdisciplinarity (e.g., Boix Mansilla et al., 2000; Repko, 2007; Spelt, 2015). For example, Spelt (2015) interchangeably used the term of *interdisciplinary thinking* and *interdisciplinary understanding* like Repko (2007). Herein, the cognitive aspect of interdisciplinarity was emphasized such that certain skills were required to conduct interdisciplinary research or interdisciplinarity in the broader sense. By taking attention to the integration of different disciplinary ideas or tools for interdisciplinarity, Boix Mansilla, Miller, and Gardner (2000) preferred the term of *interdisciplinary understanding* defined as the following:

The capacity to integrate knowledge and modes of thinking in two or more disciplines or established areas of expertise to produce a cognitive advancement – such as explaining a phenomenon, solving a problem, or creating a product – in ways that would have been impossible or unlikely through single disciplinary means. (Boix Mansilla, Miller, and Gardner, 2000, p. 17)

Interdisciplinarity from a research study perspective can facilitate understanding *interdisciplinary learning* (particularly cognitive processes inherent in interdisciplinary research). Khadri (2014) attributed the “need for interdisciplinary education” to “the need for sophisticated approaches to difficult research questions, as well as the accelerating pace of change in both society and scholarship” (Khadri, 2014, p. 88). *Interdisciplinary instruction* can be defined as “instructional activities that involve two or more disciplines in a way that all areas are meaningfully addressed simultaneously, so that it is difficult to distinguish when one is doing mathematics or doing language arts” (Hillman et al., 2000, p. 92). Herein, interdisciplinary education was defined based on two disciplines, namely mathematics and language arts (Hillman et al., 2000) and the process was emphasized through an integration of two disciplines such that it would be difficult to understand the distinct characteristics of contributing disciplines when an outsider observed the education process. In the broadest sense, *interdisciplinary education* is an approach using interdisciplinarity and *interdisciplinary learning* is expected to be facilitated through interdisciplinary education.

Interdisciplinary learning can be defined as “an integrated, deliberate, and mindful approach that connects the disciplinary knowledge and competencies from more than one area of learning to examine a theme, experience, or topic and/or investigate an issue or problem” (British Columbia Ministry of Education [BC MoE], p. 7). Boix Mansilla (2010) also defined *interdisciplinary learning* as an integrated way of “information, data, techniques, tools, perspectives, concepts and/or theories from two or more disciplines to craft products, explain phenomena or solve problems, in ways that would have been unlikely through single-disciplinary means” (p. 289). The British Columbia curriculum advocates interdisciplinary learning in promoting deeper understanding by:

- expanding their capacity to understand multiple viewpoints on a given topic — students can acquire an appreciation of the differences between disciplines in how to approach and solve a problem
- extending structural disciplinary knowledge, both factual (declarative knowledge) and process-based (procedural knowledge), to solve complex problems
- integrating conflicting insights from alternative disciplines — different disciplines attempt to understand the same or related problems, but each discipline adopts different ways to analyze and evaluate the viability of their insights (BC MoE, p. 7)

For interdisciplinarity, there is a focus in *problem-solving*. This focus on problem-solving is also mentioned for interdisciplinarity (e.g., Boix Mansilla and Duraising, 2007;

Boix Mansilla, 2010; Weinberg and Sample McMeeking, 2017). Interdisciplinary education involves a focus on solving real-world problems that requires integration beyond a single discipline (Weinberg and Sample McMeeking, 2017). Interdisciplinarity is considered to solve complex problems that cannot be solved through a single discipline's perspective (Boix Mansilla and Duraising, 2007; Boix Mansilla, 2010). The complex problems in interdisciplinary courses are ill-structured problems, which involve challenges in defining due to incomplete data, uncertainty about solutions and have multiple solutions that necessitates critical thinking and reflective judgment (Lattuca et al., 2004, p. 33). Thus, this focus on problem-solving by interdisciplinarity is considered in this study. The design in this study included interdisciplinary problem development.

Interdisciplinary education is also associated with *curriculum* by considering curriculum integration and interdisciplinary standards (e.g., Dowden, 2014). Concerning interdisciplinarity in curriculum, Dowden (2014) chose the term curriculum integration among different terms such as interdisciplinary curriculum, integrative curriculum, or integrated curriculum and defined *curriculum integration* as the following:

A collective term for curricula where meaningful learning activities are designed by crossing discipline boundaries and/or utilising multiple disciplinary perspectives with the purpose of helping students to create and enhance knowledge and understanding. (Dowden, 2014, p. 2)

Herein, interdisciplinary education is emphasized for student learning through interdisciplinary connections beyond disciplinary borders. Curriculum integration is also highlighted in the standards for different disciplines such as science, English and mathematics by the Next Generation Science Standards (NGSS Lead States, 2013), the Common Core State Standards Initiative (2010), and the Partnership for 21st Century Learning [P21] (2019). For instance, the NGSS advocate integration in curriculum favoring teaching science and engineering practices and crosscutting concepts in an integrated way (NGSS Lead States, 2013, p. xiii). Furthermore, the integration of practice and content is recommended to help students understand science (NGSS Lead States, 2013, p. xiv). Similarly, linking content and practices in mathematics teaching is recommended for curriculum and assessment by the Common Core State Standards Initiative (2010).

“Life and career skills; information, media and technology skills; learning and innovation skills, mastery of key subjects and 21st century themes” are the 21st century student outcomes in the Partnership for 21st Century Learning (P21) framework for 21st century learning (P21, 2019, pp. 3-6). In addition to key subjects (mathematics, science, geography, arts, world languages, English, economics, history, government and civics), 21st century interdisciplinary themes (“global awareness; financial, economic, business, and entrepreneurial literacy; civic literacy, health literacy, environmental literacy”) are suggested to be integrated in the curriculum by P21 (P21, 2019, pp. 3-4). *Global awareness* includes using 21st century skills to make sense and focus on global problems, understanding and showing respect for different cultures, and working collaboratively in diverse communities and contexts (P21, 2019, p. 3). Such global issues might require interdisciplinary thinking because of the complexity of global problems; thus, global awareness can be considered as aligned with interdisciplinary teaching. To prepare students for the future complex life, as a part of 21st century learning, learning and innovation skills such as communication, critical thinking, creativity, and collaboration are expected to be developed (P21, 2019). To construct “understanding across and among key subjects as well as 21st century interdisciplinary themes” is stated among the 21st century standards for students (P21, 2019, p. 7). Institute of Museum and Library Services (n.d.) has added *cross-disciplinary thinking*, which requires students to “apply knowledge, attitudes, behaviors, and skills across disciplines in appropriate and effective ways” (lines 49-50), to the 21st century skills that were stated by P21. This can be considered to indicate the recent concern for interdisciplinary thinking needed today.

In addition to 21st century skills, interdisciplinary education is also associated with *authenticity*. This aspect is related to problem-solving dimension of interdisciplinarity since authenticity includes real-world problem solving (e.g., OECD, 2018). Concerning the real-world problem-solving aspect, *authenticity* and *inter-relation* are stated among process design principles of the overall “design principles for changes in curricula and education systems” (OECD, 2018, p. 7). The principles of authenticity and inter-relation emphasize *interdisciplinarity* in addition to a profound disciplinary basis:

- Authenticity. Learners should be able to link their learning experiences to the real world and have a sense of purpose in learning. This requires interdisciplinary and collaborative learning alongside mastery of discipline-based knowledge.

- Inter-relation. Learners should be given opportunities to discover how a topic or concept can link and connect to other topics or concepts within and across disciplines, and with real life outside of school. (OECD, 2018, p. 7)

Thus, this indicated that interdisciplinary connections between different disciplines and real-world problem-solving were highly related to interdisciplinary education, which was aligned with the recent reform-based education (e.g., OECD, 2019). Interdisciplinary knowledge has a crucial role in understanding and finding solutions to complex issues, and this necessitates interdisciplinary thinking (OECD, 2019).

In addition to authenticity and inter-relation that focus on interdisciplinary connections, disciplinary knowledge is necessary to make such interdisciplinary connections (e.g., OECD, 2019). Big ideas are associated with disciplinary knowledge. *Big ideas* include key concepts, principles, and generalizations crucial “in an area of learning” and illustrate what students are anticipated to *understand* at the end of a grade level and transfer to different contexts (Walt et al., 2017, p. 119). Since big ideas play an important role in conceptual understanding, they can strengthen “interdisciplinary approaches to teaching and learning” by providing a profound ground for interdisciplinary thinking (Walt et al., 2017, p. 119).

Besides, a highly related term about interdisciplinary education was the *cross-cutting concepts* since the cross-cutting concepts indicate interdisciplinary connections within science and with other disciplines including science (NGSS Lead States, 2013). It was noted that today *cross-cutting concepts* have been studied in science education but the revision of the curricula of British Columbia (BC) implied that crosscutting concepts could be extended to other areas of learning (Walt et al., 2017). Cross-cutting concepts can be considered in two categories: Those specific to one area of learning, and those that connect across different areas of learning (Walt et al., 2017). For an example to the first category, in the science curriculum of BC, cross-cutting (interdisciplinary) science concepts (“change, patterns, cycles, order, systems, form and function, matter and energy, interactions”, and “evolution”) comprise different disciplines such as biology, physics, earth/space science, and chemistry under the subject of science (Walt et al., 2017, p. 139). For example, to the second category, a crossing-cutting concept such as *change* may cover arts education, social studies, science, health and physical education, and mathematics, thus extending different areas of learning

(Walt et al., 2017, p. 137). Erickson (2007) called such cross-cutting concepts which “are broad in scope and allow for cross-disciplinary understanding” as “macro-concepts” (as cited in Walt et al., 2017, p. 116). Thus, in searching for interdisciplinary themes, examining cross-cutting concepts could be beneficial to find interdisciplinary relationships between science and another discipline. In the current study, considering standards and objectives in curricula was considered as a part of finding interdisciplinary themes, therefore a part of interdisciplinary teaching in the broader sense.

All in all, considering the literature on interdisciplinarity, this study conceptualizes *interdisciplinarity* as a holistic way of using different disciplines to solve problems that go beyond using a single discipline (Boix Mansilla, 2010). In this study, mathematics is one of the connected disciplines and other disciplines can change based on the nature of the problem. However, in the process of developing interdisciplinary tasks, pre-service teachers need to consider at least two disciplines to solve problems in the current study. This aspect of design was considered aligned with the definition of interdisciplinarity. Problem-solving and authenticity (e.g., Boix Mansilla and Duraising, 2007; OECD, 2018) were the aspects used in the conceptualization of this study in terms of interdisciplinarity. Thus, in this study, interdisciplinarity was conceptualized as a key component of problems.

Moreover, based on the general definition on interdisciplinary understanding coming from interdisciplinary research (e.g., Boix Mansilla et al., 2000) and interdisciplinary competence in science education (Song and Wang, 2021), in this study, *interdisciplinary education* is conceptualized as an interdisciplinary approach to education such that learners could integrate different disciplinary insights to get an advanced level of understanding such as solving problems (e.g., Boix Mansilla et al., 2000; Song and Wang, 2021). To enhance the role of mathematics in interdisciplinarity (e.g., Maass et al., 2019), mathematical modeling was used in the conceptualization of *interdisciplinary teaching* in this study. *Interdisciplinary teaching* is conceptualized as teaching mathematics with an interdisciplinary approach such that mathematics is integrated with other disciplines to solve a mathematical modeling problem (Boix Mansilla, 2010; Garfunkel and Montgomery, 2019). *Formulating interdisciplinary tasks* is defined as designing interdisciplinary mathematical modeling tasks, which are mathematical modeling tasks including interdisciplinary connections between mathematics and other disciplines.

Lastly, this study conceptualized interdisciplinarity based on a profound disciplinary basis (e.g., OECD, 2019) such that *big ideas* in mathematics and other disciplines (e.g., Walt et al., 2017), which would relate to mathematics, were parts of interdisciplinary tasks. In the current study, the pre-service teachers needed to consider big ideas in each discipline and how the connected concepts would contribute to the solution of the interdisciplinary problems. Different concepts and ideas from mathematics and other disciplines need to be integrated in developing interdisciplinary problems. Besides, the collaboration aspect coming from the interdisciplinary research (e.g., NAS, 2005) was used in this study. The participants in this study were expected to collaboratively develop interdisciplinary tasks and problems and aligned with the nature of interdisciplinary research and to reduce the work of complexity inherent in interdisciplinary problems. Two members with at least one mathematics pre-service teacher worked on developing interdisciplinary tasks or problems for their final projects in this study. To conclude, interdisciplinarity was considered as a crucial component of interdisciplinary task design by considering interdisciplinary thinking as a complex cognitive process.

2.2. Mathematical Modeling for Interdisciplinary Teaching

This section describes studies on mathematical modeling for interdisciplinary teaching. Initially, theoretical consideration for mathematical modeling is mentioned. Then, certain empirical studies including mathematical modeling were explained.

Mathematical modeling is considered as a process of applying mathematics to a real-world problem to understand the real-world problem or situation (Niss et al., 2007; Stillman, 2019; Durandt et al., 2022). The *modeling competence* was defined as the following:

the ability to identify relevant questions, variables, relations or assumptions in a given real-world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation, as well as the ability to analyse or compare given models by investigating the assumptions being made, checking properties and scope of a given model etc. (Niss et al., 2007, p. 12)

Therefore, the modeling competence is a complex notion that includes different dimensions and researchers continue to examine and assess different aspects of the modeling competence (that is, sub-competencies) (e.g., Blum, 2015; Greefrath and Vorhölter, 2016).

The definition of modeling competence (Niss et al., 2007) was conceptualized as a “global modelling competence in which certain partial processes can be identified by means of an atomistic perspective” (Wess et al., 2021, p. 7). Since the whole process of mathematical modeling was considered overall, this study also used mathematical modeling as global competence.

Modeling activities require high cognitive demand, and many studies have indicated that participants in the modeling process had difficulties in successfully completing the modeling tasks (e.g., Galbraith and Stillman, 2006; Blum, 2015; Wess et al., 2021). Based on a design experiment research study on identifying challenges of students of ages 14-15 in the modeling process, Galbraith and Stillman (2006) concluded that each step in the modeling process could pose cognitive challenges for students. Based on reviewed studies, Wess and his colleagues (2021) noted that the challenges of modeling activities could be described by the essential complexity of such activities, “measured by the necessary sub-competencies” (p. 15). As an example of difficulties in initial steps of modeling, students had difficulties in understanding the modeling activity and the real-world problem because of reading-related challenges such as a lack of language proficiency (Blum, 2015; Plath and Leiss, 2018). Moreover, the transition from the real-world aspect to the mathematics aspect (mathematization) in constructing mathematical models is also found as a cognitive obstruction for students because this stage necessitates sufficient prior mathematical knowledge (Galbraith and Stillman, 2006). The *simplifying/structuring* step (Blum and Leiß, 2007) also poses challenges for students in the modeling process because of misunderstanding of the problem or specifying assumptions about the real-world problem (Schukajlow et al., 2012).

Concerning research on pre-service teacher education for mathematical modeling, Anhalt and Cortez (2016) reviewed the related studies and concluded pre-service teachers had difficulties in mathematical modeling. In their own study, Anhalt and Cortez (2016) investigated the development of eleven pre-service “teachers’ understanding of mathematical modeling through the implementation of a modeling module within a curriculum course in a secondary teacher education program” (p. 523). The findings indicated that the intervention with the designed module positively affected participant pre-service teachers’ development of comprehending mathematical modeling “as an iterative

process involving making assumptions and validating conclusions connected to everyday situations” (Anhalt and Cortez, 2016, p. 523).

Another study using pre-service teachers in a mathematical modeling course showed that participants’ perceptions of modeling changed based on their experiences of modeling during the course (Doerr, 2007). The modeling-specific pedagogical content knowledge was also examined in research studies including pre-service teachers. For instance, a quasi-experimental study (Greefrath et al., 2022) investigated whether “the pedagogical content knowledge of mathematical modeling as part of the professional competence of pre-service teachers” could be supported in its various dimensions (particularly “knowledge about modeling tasks and about interventions”) through appropriate “university seminars” (p. 383). The results showed that treatment groups revealed a significant change in specific dimensions of their pedagogical content knowledge compared to the control group (Greefrath et al., 2022). To develop professional knowledge of pre-service teachers, Borromeo Ferri and Blum (2010) recommended specific modeling seminars in undergraduate education. Moreover, Blum (2015) suggested using the Model Eliciting Activities of Doerr and Lesh (2011) to support learning of both pre-service and in-service teachers. Blomhøj (2019) also emphasized the necessity of designing tools to facilitate teachers in viewing modeling tasks as an instructional medium to advance students’ learning of core mathematical concepts.

Mathematical modeling can be used to support interdisciplinarity. In the literature on mathematical modeling education, there are certain studies that also include interdisciplinary modeling, mostly integrating mathematics and science (e.g., Lesh and Lehrer, 2003; English, 2009; Guder and Gurbuz, 2017; Geiger et al., 2018; Maass et al., 2022). These studies together with the studies on education of mathematical modeling in general guided the development of design guidelines for the current study, particularly regarding the design of interdisciplinary modeling tasks. For instance, concerning “research on design principles for productive modeling activities for learning in school”, Lesh and Lehrer (2003) emphasized that “designing learning environments in which students develop deeper and higher-order understandings of powerful ideas in elementary mathematics and science” (p. 117). Moreover, research on interdisciplinary mathematical modeling search ways to enhance “conceptual development by putting students in situations where they repeatedly express-

test-and-revise their own current ways of thinking about “big ideas” in mathematics and science” (Lesh and Lehrer, 2003, p. 117).

Another study that used both mathematical modeling and interdisciplinarity was the study conducted by Guder and Gurbuz (2017). Guder and Gurbuz (2017) conducted a multi-tier teaching experiment with seventh graders in Turkey to examine how students construct models and how they learn certain scientific concepts through an interdisciplinary modeling problem. The researchers co-designed the interdisciplinary modeling problem with the science teacher, and the designed problem was implemented in small groups (three or four students) in the science class (Guder and Gurbuz, 2017). That study can be considered in the context of science as a main discipline since learning science concepts is put in the foreground. The findings indicated that students learnt certain science concepts as well as used mathematical knowledge and constructed connections among the learnt science concepts (Guder and Gurbuz, 2017).

Another study that used both mathematical modeling and interdisciplinarity was conducted by Geiger and his colleagues (2018). In that study, the design of an online learning module on mathematical modeling for secondary mathematics pre-service teachers was prepared as a part of a larger design study, namely “*Opening Real Science: Authentic Mathematics and Science Education for Australia*” project (Geiger et al., 2018, p. 218). The project aimed to develop and evaluate “eight mathematics and 17 science online learning modules for primary and secondary initial teacher education programs”, and these modules were based on “authentic contexts and enquiry-based pedagogical approaches” (Geiger et al., 2018, p. 218). The module reported by Geiger and his colleagues (2018) was co-designed by an interdisciplinary group of “mathematics and science educators, mathematicians, scientists, in-service and pre-service secondary mathematics teachers” (Geiger et al., 2018, p. 217). The module evaluation data revealed that the online module was effective in enhancing pre-service teachers’ “personal mathematics and science knowledge” by using mathematical models as well as in showing the power of the interdisciplinary collaboration in the design of the module (Geiger et al., 2018, p. 229).

To sum up, studies including mathematical modeling in mathematics education indicated that students and teachers could have difficulties in the mathematical modeling

process (e.g., Galbraith and Stillman, 2006; Blum, 2015; Wess et al., 2021). The difficulties in mathematical modeling were found to be related to the nature of complexity in mathematical modeling tasks (e.g., Wess et al., 2021), students' inadequate language proficiency (e.g., Blum, 2015; Plath and Leiss, 2018), lack of prior knowledge about certain sub-competencies such as mathematization (e.g., Galbraith and Stillman, 2006) and misunderstanding of the problem or inability to identify appropriate assumptions about the initial real-world problem (e.g., Schukajlow et al., 2012). Besides, to overcome difficulties associated with mathematical modeling, certain interventions were developed for pre-service teachers (e.g., Anhalt and Cortez, 2016). The findings of such interventions were promising on developing participants' mathematical modeling competencies (e.g., Anhalt and Cortez, 2016). It was found that the implementation of a modeling module in a teacher education course for secondary pre-service teachers supported pre-service teachers' development of comprehending mathematical modeling "as an iterative process involving making assumptions and validating conclusions connected to everyday situations" (Anhalt and Cortez, 2016, p. 523). Besides, negative perceptions of pre-service teachers on mathematical modeling could change after participating in a mathematical modeling course (e.g., Doerr, 2007). Moreover, pre-service teachers' PCK on mathematical modeling could be supported through university seminars (e.g., Borromeo Ferri and Blum, 2010; Greefrath et al., 2022). For instance, the quasi-experimental study of Greefrath and his colleagues (2022) study found significant change in the treatment group for certain aspects of their PCK (task dimension) (Greefrath et al., 2022).

In the literature on mathematical modeling education, there are certain studies that also include interdisciplinary modeling (e.g., Lesh and Lehrer, 2003; English, 2009; Guder and Gurbuz, 2017; Geiger et al., 2018; Maass et al., 2022). In certain interdisciplinary mathematical modeling studies, students could learn certain concepts in science and construct interdisciplinary connections with mathematics (e.g., Guder and Gurbuz, 2017). Online learning modules on mathematical modeling could be also useful in enhancing individual mathematics and science knowledge of pre-service mathematics teachers (e.g., Geiger et al., 2018). The findings of the study of Geiger and his colleagues (2018) showed the positive effect of the developed online module on supporting content knowledge of pre-service teachers and the importance of using interdisciplinary collaboration in designing the module. These studies together with the studies on education of mathematical modeling in

general guided the development of design elements for the current study, particularly regarding the design of interdisciplinary modeling tasks.

2.3. Teacher Education for Interdisciplinary Teaching

To adapt global changes and issues, teachers could need relevant competencies as teachers play a crucial role to prepare students for the future, and teacher quality is considered an important factor that influences the educational performance of a country in general (e.g., Tan et al., 2017). To help pre-service and novice teachers meet the complicated and challenging nature of teaching, a firm knowledge-grounded base for professional learning is noted to be necessary (Tan et al., 2017). Considering the importance of teacher education, *teacher learning* and *research on interdisciplinary teaching* including mathematics will be described in the following sections.

2.3.1. Teacher Learning

In this section, teacher learning will be described. Teacher learning from a situative perspective will be initially explained and then pedagogical content knowledge for interdisciplinary teaching will be described.

Learning to teach high school mathematics has a theoretical background from a situative perspective (e.g., Peressini et al., 2004). For instance, Peressini and his colleagues (2004) asserted a conceptual framework based on a situative lens to explore and analyze the process of learning to teach secondary mathematics. By using this framework, Peressini and his colleagues (2004) investigated pre-service teachers' learning to teach mathematics beginning in their "reform-based teacher preparation programs" to their initial two years of teaching (p. 67). According to the situative view, learning is considered to develop in various situations such as "mathematics and teacher preparation courses, pre-service field experiences, and schools of employment" (Peressini et al., 2004, p. 67). The participation to such different situations is important in clarifying mathematics teachers' conceptions about their profession ("the big ideas of mathematics, mathematics-specific pedagogy, and sense of self as a mathematics teacher") (Peressini et al., 2004, p. 67). A situated view of learning considers "learning as patterned participation in social contexts" (Buchbinder and McCrone,

2018, p. 3). Aligned with situative learning, teacher learning is considered “as a process of increasing participation in the practice of teaching, and through this participation, a process of becoming knowledgeable in and about teaching” (Adler, 2000, p.37). The situative perspectives contain a wide range of theoretical aspects grounded on different disciplines such as anthropology, psychology, and sociology (Greeno, 2003). Lave and Wenger (1991) regarded learning as a kind of participation in “socially organized activity”, which is a shared understanding of learning from situative perspectives (as cited in Peressini et al., 2004, p. 69). Situative views assert that since teacher learning develops in different contexts, research should consider such various “situations of practice” (Peressini et al., 2004, p. 69) or both personal and social contexts (Putnam and Borko, 2000). Concerning the unit of analyses, Cobb and Bowers (1999) have attributed a pragmatist view to the situative lens such that it involves “a range of units of analysis, the choice in any particular case being a pragmatic one that depends on the purposes at hand” (p. 6). Participated practices are critical in situative learning, and based on Cobb and Bowers (1999), *practices* are “the patterns of thought and action that have been established by participants in particular contexts or settings” (Peressini et al., 2004, p. 69). As Peressini and his colleagues (2004) have also emphasized, for teacher learning, it is important to examine “how practices learned in university courses can be recontextualized in elementary and secondary school classrooms” (p. 70). Putnam and Borko (2000) have emphasized that the situative view considers teacher learning in the context of distributed cognition including the role of individual and other people, social discourse, tools, and the context in the learning process. The current study adopts the situative perspective of learning to teach, and accepts the impact of the individual, social, and contextual aspects. In this study, aligned with the situative perspective about individual context, pre-service teachers were assigned individual homework such reflections on an online forum. Moreover, considering the social aspect, pre-service teachers were expected to collaboratively develop interdisciplinary tasks and problems for their final projects. Furthermore, for the social aspect, the pre-service teachers were expected to participate in class discussions on interdisciplinary teaching. For the contextual aspects, the pre-service teachers were supposed to participate in mathematical modeling tasks as learners to experience the process of solving mathematical modeling tasks before transition to task design.

The importance of pedagogical content knowledge for teacher learning is emphasized many times (e.g., Shulman, 1986; Grossman, 1990; Ball et al., 2001; Guerriero, 2014). Pedagogical content knowledge is important in making teacher knowledge used in the classroom. As Guerriero (2014) has noted, teacher knowledge in decision making in the classroom is affected by how teachers analyze and evaluate learning cases in certain contextual and situational factors, and the quality of their pedagogical content knowledge. In comparison of expert and novice teachers, Guerriero (2014) found that expert teachers had comprehensive pedagogical content knowledge, better problem-solving strategies, better adaptation to a variety of learners, better decision making, better grasp of classroom events, more sensitivity to context, and more respect for students compared to novice teachers.

For theoretical background of interdisciplinary pedagogical knowledge for pre-service teachers, Shulman's (1986) categorization of content knowledge and An's (2017) framework of interdisciplinary pedagogical knowledge for elementary teachers were considered. Shulman's (1986) descriptions of content knowledge were accepted as a foundation for teacher knowledge because succeeding models of teacher knowledge had their roots in that model and made extensions or modifications based on disciplines examined.

As a seminal paper on teacher knowledge, Shulman (1986) described three core categories of *content knowledge* as *subject matter content knowledge*, *pedagogical content knowledge* and *curricular knowledge*. The *subject matter content knowledge* refers to the type of knowledge that comprises of domain-specific facts and rules as well as teachers' justifications for the coherence of domain-specific facts and rules (Shulman, 1986). Thus, *subject matter content knowledge* requires teachers to understand key ideas *within* discipline. The *pedagogical content knowledge* (PCK) is the content knowledge for teaching including knowing different kinds of representation of content and preconceptions or alternative conceptions of students (Shulman, 1986). Thus, as instructional dimensions are dominant in PCK, PCK may be considered to create a way of communication of content between the teacher and students. Shulman's (1986) *curricular knowledge* contains knowledge of different curriculum resources, lateral curriculum (knowing connections with content in other disciplines in the same grade level), and vertical curriculum (knowing disciplinary connections across different grade levels). Thus, knowledge of vertical curriculum may

include knowing disciplinary key ideas in a developmental perspective (like making progressions in a metaphor of a grade level ladder). Moreover, the knowledge of lateral curriculum may form the basis for interdisciplinary education because it requires knowing how a disciplinary idea/concept is related with other disciplines at the same grade level.

The framework of *Interdisciplinary Pedagogical Content Knowledge* (IPCK) by An (2017) guided the design in this study. An (2017) proposed a new type of pedagogical knowledge called *interdisciplinary pedagogical content knowledge* to indicate the necessary set of a knowledge base for interdisciplinary teaching. The IPCK framework (An, 2017) was grounded on Shulman's (1986) *pedagogical content knowledge* (PCK) and Frykholm and Glasson's (2005) *pedagogical context knowledge*. The interdisciplinary content knowledge was constructed by the intersection between multiple content knowledge in different disciplines and the IPCK emerged out as an interaction between different PCKs. An (2017) formed four criteria of the IPCK that were expected from teachers in interdisciplinary education:

- (1) work with interdisciplinary considerations that include an understanding of the representation of concepts using themes across curriculum boundaries; (2) apply pedagogical methods and interdisciplinary themed activities in addressing content areas from multiple subjects simultaneously; (3) identify knowledge connections within and between particular subjects, and develop lessons based on such connections; and (4) employ knowledge of how interdisciplinary explorations can be developed as a part of an instructional process wherein students link existing knowledge across curricula, while presenting that new knowledge through contexts from multiple subjects. (p. 239)

In this study, aligned with the first criterion (An, 2017), pre-service teachers were expected to identify common themes or problems in mathematics and other disciplines to form interdisciplinary problems for their final projects. This aspect also considered that pre-service teachers would search for curriculum contents for mathematics and other disciplines. Moreover, this is also related to specifying interdisciplinary learning goals for students. This step can be regarded as determining a driving question or a large interdisciplinary problem. Moreover, determining which aspects from different disciplines would be combined with mathematics and finding big ideas for each discipline were expected from the pre-service teachers in this study. These can be related to the first and third criteria of An's framework (2017) and can be considered as sub-criteria. Aligned with the third criteria (An, 2017), pre-service teachers were expected to develop an interdisciplinary mathematics task. The fourth criterion (An, 2017) is related to assessing students' learning emerging out of the

interdisciplinary mathematics task. The pre-service teachers in this study were supposed to consider expected student answers to their developed problems. The next section will describe research on interdisciplinary teaching including mathematics.

2.3.2. Research on Interdisciplinary Teaching Including Mathematics

This section describes the relevant research on interdisciplinary teaching including mathematics. *Interdisciplinary mathematics education* refers to teaching and learning mathematics through the integration of one or more disciplines in a way to create a ‘cognitive advancement such as explaining a phenomenon, solving a problem, or creating a product’ (Boix Mansilla et al., 2000, p. 17). Herein, *cognitive advancement* (Boix Mansilla et al., 2000) indicates a balanced contribution of each discipline to generate an outcome.

There are studies on interdisciplinary education including mathematics. The empirical studies about interdisciplinary mathematics education are mostly on elementary pre-service teachers (e.g., An, 2017; An and Tillman, 2018; Krause et al., 2019; An, 2020; Santaolalla et al., 2020). Concerning integration of disciplines, the most prevalent use was on mathematics and science (e.g., Krause et al., 2019). There are also some studies that used the integration of mathematics with arts (e.g., An, 2017) or with social studies (e.g., An and Tillman, 2018; Santaolalla et al., 2020). Certain studies that include the integration of two disciplines for pre-service teachers (e.g., An, 2017; Krause et al., 2019) will be described in the following.

As a part of a broader study, An’s (2017) study aimed to improve pre-service elementary teachers’ interdisciplinary pedagogical content knowledge after a course based on interdisciplinary mathematics lessons. An (2017) used an intervention through eight weeks consisting of exemplary interdisciplinary activities on the integration of arts and mathematics during a mathematics teaching methods course for pre-service elementary teachers. Within a qualitative research approach, from pre-survey to post-survey, the change of pre-service teachers’ teaching strategies of an interdisciplinary math-science lesson was examined (An, 2017). The findings of An (2017) indicated that the intervention positively affected pre-service teachers’ interdisciplinary pedagogical content knowledge. According to the empirical study of An (2017), the pre-service teachers extended their repertoire of

pedagogical approaches on interdisciplinary mathematics lessons with a science contextualization and provided a more thorough connection between science and mathematics.

Another study was a collaborative project in continuum that aimed to design an interdisciplinary teaching module based on physics and mathematics (Krause et al., 2019). Its initial aim was to raise pre-service teachers' awareness of contributions of instructional theories of physics and mathematics to each other and then extend this to in-service teachers (Krause et al., 2019).

There can be different approaches to how disciplines are integrated for interdisciplinary education in mathematics (e.g., Williams et al., 2016; Maas et al., 2019). STEM and mathematical modeling are the commonly referenced approaches to interdisciplinary mathematics education (e.g., Williams et al., 2016; Borromeo Ferri and Mousoulides, 2017; Maas et al., 2019). Integrated STEM studies were more prevalent in the conceptualization of interdisciplinary mathematics education (e.g., English, 2016; Fan and Yu, 2017; Guzey et al., 2017).

As Williams and his colleagues (2016) have also highlighted, integrated STEM is the most favored means for interdisciplinary mathematics education. *Integrated STEM* can be described as “the approach to teaching the STEM content of two or more STEM domains, bound by STEM practices within an authentic context for the purpose of connecting these subjects to enhance student learning” (Kelley and Knowles, 2016, p.3). Based on the relevant literature, to guide researchers and practitioners of integrated STEM, Roehrig and her colleagues (2021) constructed an integrated STEM framework consisting of seven essential features: “(a) centrality of engineering design, (b) driven by authentic problems, (c) context integration, (d) content integration, (e) STEM practices, (f) twenty-first century skills, and (g) informing students about STEM careers” (Roehrig et al., 2021, p. 14). In this study, the *content integration* aspect of Roehrig and her colleagues (2021) was emphasized.

The *content integration* is critical to integrated STEM and “making connections between the disciplines and between the context and disciplines explicit to students” is essential for such content integration (English, 2016; Kelley and Knowles, 2016; Roehrig et

al., 2021, p. 4). Content integration is noted to be met by different approaches to interdisciplinarity such as multidisciplinary, interdisciplinary, or transdisciplinary approaches (e.g., Moore and Smith, 2014; Wang and Knobloch, 2018). Integrated STEM education can enhance learners' understanding of mathematics and science concepts (e.g., Fan and Yu, 2017; Guzey et al., 2017), but students could have difficulties in identifying content integration among different disciplines (e.g., English, 2016; Roehrig et al., 2021). Since content integration at higher levels can pose challenges to students, teacher support is necessary to facilitate this process. Based on relevant literature (e.g., Nathan et al., 2013), Roehrig and her colleagues (2021) suggested that "teachers need to model the connections for students, use interdisciplinary models and representations, and use purposeful facilitation and questioning to promote students' understanding of these connections" (p. 4). In this study, considering the suggestions of Roehrig and her colleagues (2021) to facilitate content integration, the researcher as an instructor acted as a facilitator in designing interdisciplinary problems within the class as well as outside the class. The researcher as an instructor modelled interdisciplinary connections by exemplifying certain connections between mathematics and other disciplines and allowed continuous class discussions to reveal possible interdisciplinary connections in the ideas of pre-service teachers. The pre-service teachers could directly ask questions through writing online and the researcher conducted additional scaffolding online sessions for some pre-service teachers when they had difficulties in content integration, especially in identifying interdisciplinary connections between mathematics and other disciplines.

The integration of mathematics within STEM education was noted to be challenging (e.g., Zhang et al., 2017; Little, 2019; Roehrig et al., 2021), and little effect on students' mathematical knowledge was found (e.g., Becker and Park, 2011). Nevertheless, mathematics is generally used "as a tool for data measurement and analysis with few or no conceptual learning goals for mathematics" (Roehrig et al., 2021, p. 9). For an effective mathematics integration within STEM, it is necessary to make the role of mathematics explicit to students (Roehrig et al., 2021) and going beyond considering "mathematics as the tool or language for solving science problems" or not just "using science as a set of examples for demonstrating math content" (Zhang et al., 2017, p. 358). The empowerment of the role of mathematics in STEM education has been emphasized in the literature to achieve a meaningful integration of mathematics within STEM (e.g., Maas et al., 2019; Swanson,

2019). Thus, the role of mathematics should go beyond to be instrumental (Williams and Roth, 2019). Considering this call to enhance the role of mathematics in interdisciplinary education (e.g., Maas et al., 2019; Swanson, 2019; Williams and Roth, 2019), this study used *mathematical modeling* in interdisciplinary teaching.

The literature on the *perceptions* about STEM education was also considered about conceptualizations of interdisciplinary education including mathematics (e.g., Yang and Ball, 2024; Yang et al., 2025). For instance, Yang and her colleagues' (2025) qualitative study including Taiwanese and Indonesian mathematics teachers investigated teachers' perceptions on integrated STEM education by focusing on the goals, values of mathematics, and teaching approaches in integrated STEM education (Yang et al., 2025). The study found that the mathematics teachers' perceptions concentrated on *content* and *context* as two primary aspects about task design in integrated STEM education. The content aspect included "considering mathematical content" and "other disciplines content. Furthermore, teachers who designed context first tended to initially give real-world situations for the task design whereas the teachers who designed content first tended to simplify real-world situations and to make the problems close-ended (Yang et al., 2025). This dissertation study also investigated how the conceptualizations of the pre-service teachers on interdisciplinary teaching changed during the implementation.

Research on *designing* integrated STEM tasks also informed this study (e.g., Akaygun and Aslan-Tutak, 2020; Mumcu et al., 2023; Fitzpatrick and Leavy, 2025). For instance, in the mixed-method study of Mumcu and her colleagues (2023) investigated "the effect of a professional development (PD) program designed for integrated STEM education on teachers' pedagogical design competencies and the contribution of the PD program to teachers' integrated STEM understanding" (Mumcu et al., 2023, p. 3443). The middle school teachers collaboratively prepared STEM lesson plans based on the 5E learning model in the PD program, which was based on collaboration and active practice (Mumcu et al., 2023). One mathematics teacher, one science teacher and two computer science teachers worked in a group for collaborative activities (Mumcu et al., 2023, p. 3448). The study of Mumcu and her colleagues (2023) found that the lessons plans were found to be at an acceptable level with respect to integrated STEM education, but some aspects needed to be improved (Mumcu et al., 2023, p. 3458). Moreover, according to the teachers' views, the PD program

enhances “their collaboration with colleagues, contributes to pedagogical design skills in integrated STEM lesson planning and integrating STEM disciplines, and improves their understanding of integrated STEM” (Mumcu et al., 2023, p. 3443). All in all, the studies on developing integrated STEM tasks indicated that collaboration and active participation were key aspects in designing interdisciplinary tasks. This study also considered collaboration and active teacher learning experiences as key aspects in the design of the module on interdisciplinary teaching for pre-service teachers.

To sum up, the empirical studies on interdisciplinary teaching including mathematics prevalently focused on the integration of two disciplines, which one of them was mathematics, and the investigation on pre-service elementary teachers (e.g., An, 2017; An and Tillman, 2018; Krause et al., 2019; An, 2020; Santaolalla et al., 2020). Moreover, mathematics was often integrated with science in these studies (e.g., Krause et al., 2019). Appropriate interventions could support pre-service teachers’ interdisciplinary pedagogical knowledge (e.g., An, 2017). Moreover, concerning the interdisciplinary teaching in integrated STEM studies, which included mathematics, content integration was revealed as a key aspect (e.g., Roehrig et al., 2021). Furthermore, teacher modeling of interdisciplinary connections, using “interdisciplinary models and representations” as well as using “purposeful facilitation” and scaffolding were found to be useful in facilitating effective content integration (e.g., Roehrig et al., 2021).

The integrated STEM studies also informed this study on conceptualizations on task design and key factors in designing interdisciplinary tasks. Concerning *perceptions* of teachers about STEM education, it was found that the mathematics teachers’ perceptions concentrated on *content* and *context* as two primary aspects about task design in integrated STEM education (e.g., Yang et al., 2025). Besides, the integrated STEM studies on pre-service teachers also informed this study on task design (e.g., Akaygun and Aslan-Tutak, 2020; Mumcu et al., 2023; Fitzpatrick and Leavy, 2025). Overall, the studies showed that collaboration and active learning were key aspects in designing interdisciplinary tasks. Thus, this study used collaboration and active teacher learning experiences as key aspects in the design of the module on interdisciplinary teaching for pre-service teachers.

2.4. Elements of Pre-service Teacher Education on Interdisciplinary Teaching

Aligned with the nature of a design-based study, this study used certain design elements derived from the literature and revised based on the implementations in cycles of the research. Initially, a course on interdisciplinary mathematics education was planned to be developed and relevant design elements were derived from the literature. Then, for the first implementation (Cycle 2), a module on interdisciplinary teaching for pre-service teachers was planned and relevant design elements were derived based on the literature. The following parts will examine the elements of pre-service teacher education on interdisciplinary teaching in the following two parts.

2.4.1. Elements of a Course on Interdisciplinary Mathematics Education

In this study, before the implementation of the module on interdisciplinary teaching in Cycle 2, a new course on interdisciplinary for pre-service mathematics teachers was initially designed based on literature (e.g., Putnam and Borko, 2000; Herrington and Herrington, 2006; Fishman et al., 2014; Capobianco et al., 2022). The purpose was to investigate how an undergraduate Course on Interdisciplinary Mathematics Education (CIdME) with interdisciplinary mathematical modeling tasks and a focus on communication and collaboration could support third-year and fourth-year pre-service secondary mathematics teachers' learning to teach interdisciplinary mathematics. The design of the Course on Interdisciplinary Mathematics Education (CIdME) for pre-service mathematics teachers was guided by the following design guidelines: *Provide authentic tasks for learning to teach interdisciplinary mathematics* and *facilitate communication and collaboration in the classroom community*. These design guidelines were derived based on the literature on authenticity and teacher learning e.g., Putnam and Borko, 2000; Herrington and Herrington, 2006; Fishman et al., 2014; Capobianco et al., 2022).

The new course design (CIdME) aimed to develop pre-service secondary mathematics teachers' learning to teach interdisciplinary mathematics. To develop the process of learning to teach, the literature on situative learning has suggested to use multiple contexts and practices of teaching (e.g., Putnam and Borko, 2000; Peressini et al., 2004). The course design was guided by the implication of the situative perspective such that pre-service

teachers were required to involve “in authentic teaching activities as both learners and teachers” (Capobianco et al., 2022, p. 750). Thus, pre-service teachers participating in this new course were supposed to experience interdisciplinary tasks as both learners and teachers. Based on Lave and Wenger (1991), learning to teach was also considered as social in nature and develops through “collaborative participation, discourse, and construction of knowledge by the community of learners” (Capobianco et al., 2022, p. 750). Designing, implementing, and reflecting on authentic tasks were used in pre-service teachers’ learning in the literature such as in the case of a study including elementary science pre-service teachers’ characterization of learning to teach “engineering design-based science” (Capobianco et al., 2022, p. 750). For learning to teach, the current course design was planned to develop, implement, and reflect on interdisciplinary mathematical modeling tasks as authentic tasks by pre-service mathematics teachers.

Aligned with situated perspectives (Putnam and Borko, 2000; Fishman et al., 2014), authentic activities are necessary for effective teacher learning. *Authentic activities* are considered those that develop thinking and problem-solving skills (Putnam and Borko, 2000). Authentic activities are ill-defined activities with a real-world relevance involving a complex task that requires an extensive commitment of time (Herrington and Oliver, 2000; Herrington, 2006; Herrington and Herrington, 2006). The authenticity in activities or tasks is also emphasized from a system approach by considering the elements of the learning environment. For instance, authenticity emerges in the interactive relations among the different elements such as “the learner, the task, or the environment” and “authenticity is manifest in the flow itself and is not an objective feature of any one component in isolation” (Barab et al., 2000, p. 38).

Moreover, Herrington (2006) derived key characteristics of authentic tasks for higher education for online courses based on a review of literature on authentic learning environments. The *real-world relevance* was emphasized as a characteristic of authentic tasks such that there should have been a high degree of correspondence between the activities and “real-world tasks of professional in practice” (Herrington, 2006, p. 5). Authenticity, particularly the real-world relevance of authentic tasks (Herrington, 2006) informed the design of this study. The *interdisciplinary mathematical modeling* considered in this study was based on real-world problems that was expected to provide real-world relevance of the

authentic tasks. Pre-service teachers were expected to design their own interdisciplinary tasks and to construct their own interdisciplinary problems or driving questions for their final projects. Furthermore, authentic tasks should have been *ill-defined* such that the problems inherent in an authentic task could not be solved by a single procedure, and students must have multiple perspectives and determine sub-components of the task to complete the tasks (Herrington and Oliver, 2000; Herrington, 2006). Being *complex* in nature, authentic tasks also create opportunities to *collaborate* and *reflect* (Herrington and Oliver, 2000; Herrington, 2006). The mathematical modeling involved in the interdisciplinary tasks in this study was considered complex in nature and ill-defined because pre-service teachers were expected to use multiple perspectives from both mathematics and science to solve the complex problem as well as to use the process of modeling cycle. High cognitive demand of mathematical modeling tasks also contributes to the complexity of the tasks by surpassing the superficial procedures. The *interdisciplinary* characteristic of authentic tasks was also emphasized such that different views and expertise could be combined going beyond separate domains (Herrington and Oliver, 2000; Herrington, 2006). In the current design, mathematics was expected to be combined with another discipline such as science for interdisciplinary tasks. As Cavalcanti (2014) has suggested in planning a lesson through a transdisciplinary STEM perspective, in the design of the new course, pre-service teachers were supposed to identify interdisciplinary themes and connections in mathematics and another discipline, identify applicable practices in mathematics and another discipline, and identify relevant big ideas for both mathematics and another discipline to be able to design their interdisciplinary tasks. These characteristics guided the design of the course on interdisciplinary mathematics education (CIdME). Based on the above literature on authenticity, the design guideline for CIdME was derived: *Provide authentic tasks for learning to teach interdisciplinary mathematics.*

Situative perspective also informed the design of the new course on interdisciplinary teaching. For effective teacher learning, research on teacher learning has benefited from the situated perspective (Putnam and Borko, 2000). The situated perspective presents a good theoretical framework for learning scientists to analyze complex real-world environments like classrooms in which teachers are involved because a “systems level of analysis” is used to understand the elements of a system and to specify the underlying processes of these elements (Greeno and Nokes-Malach, 2016, p. 67). The cognition is seen as situated, social,

and distributed such that learning is dependent on the context, interactions with other people, and tools in the context (Putnam and Borko, 2000; Greeno and Nokes-Malach, 2016). Thus, the situative perspective views learning in a system approach by considering the interactions of the individual with her/his learning environment.

As a social aspect of learning, learning from others is important in the situated perspective. Teacher learning is fostered as teachers can have opportunities to learn in a collaborative manner (Grossman et al., 2001; Fishman et al., 2014). Based on teacher learning research, as Putnam and Borko (2000) noted, face-to-face or online discourse communities via different technological tools created powerful teacher learning (especially when teaching was the emphasis of group discussions). Moreover, discourse communities are important for pre-service teachers to learn to “think, talk, and act as a teacher” (Putnam and Borko, 2000, p.10). Thus, in designing learning environments, constructing community-centered contexts is important as Bransford et al. (2000) have also emphasized. Based on this, another design guideline for the course on interdisciplinary teaching (CIdME) was derived: *Facilitate communication and collaboration in the classroom community.*

2.4.2. Elements of a Module on Interdisciplinary Teaching

The module on interdisciplinary teaching in the first implementation in Cycle 2 (the *Professional Development Module*) aimed to support pre-service mathematics and science teachers’ learning to design interdisciplinary mathematical modeling tasks (IMMTs). The design of the *Professional Development Module* in the first iteration was based on three pillars: *teacher education, interdisciplinary thinking, and teaching mathematics through mathematical modeling*. The design guidelines were derived from the literature as the following: *Use problem-based learning, use social constructivist pedagogy, increase communication, and support collaboration*. The elements of a module on interdisciplinary teaching will be described in the following parts: *Pedagogical content knowledge for interdisciplinary teaching, interdisciplinary thinking, and teaching mathematics through mathematical modeling*.

The literature on pedagogical content knowledge for interdisciplinary teaching offers insights on the elements of a module on interdisciplinary teaching. This part will describe *pedagogical content knowledge for interdisciplinary teaching*.

In the literature on teacher education, having a strong base of Pedagogical Content Knowledge (PCK) is extensively accepted as a crucial aim of teacher education (e.g., Peressini et al., 2004; Greefrath et al., 2022). PCK is considered an important factor on students' learning and achievement (e.g., Baumert et al., 2013; Guerriero, 2014). Teachers with high PCK apply tasks that can cognitively engage students and provide essential scaffolding for personal learning (Greefrath et al., 2022). PCK is the content knowledge for teaching including knowing different kinds of representation of content and preconceptions or alternative conceptions of students (Shulman, 1986). The emphasis of PCK is on the knowledge of teachers about how learners can learn a specific content, and teachers need awareness of students' difficulties "of certain subjects and involve foresight, prior experience and misconceptions in the learning process" (Wess et al., 2021, p. 24). Thus, PCK is a combination of understanding content and knowing how to teach content.

When two and more disciplines involved, the concept of PCK is broadened to the Interdisciplinary Pedagogical Content Knowledge (IPCK), which includes teachers' or pre-service teachers' competencies in designing interdisciplinary tasks and implementing these tasks (An, 2017). Since PCK is a broad concept and requires time to be developed, this study concentrates on the aspect of task design as an important dimension of PCK. *Task* is defined as "any instructional or assessment-related unit of disciplinary work that is assigned to students to intellectually engage them in science or mathematics" (Tekkumru-Kisa et al., 2020, p. 607). The *cognitive demand* of a task can be described as the required thinking type and level of students when they experience a task (Stein et al., 2000). As a part of PCK, an appropriate choice of tasks is also noted to be important to cognitively engage students, that is to facilitate students' learning (Baumert et al., 2013). Cognitively activating tasks could provide opportunities to relate learners' previous "knowledge by challenging and testing their preexisting ideas and beliefs" (Baumert et al., 2013, p. 9). Designing tasks with a high level of cognitive demand informed the design of the *Professional Development Module*.

Modeling-specific PCK (Greefrath et al., 2022) and teaching modeling literature (e.g., Fulton et al., 2019) also informed the design of this study, particularly designing modeling tasks. Modeling-specific pedagogical content knowledge includes four dimensions: knowledge about interventions, knowledge about modeling processes, knowledge about aims and perspectives, and knowledge about modeling tasks (Wess et al., 2021). The dimension of *knowledge about modeling tasks* includes characteristics of mathematical modeling tasks, “cognitive analyses of” mathematical modeling tasks, creation of modeling tasks and “multiple solutions of” mathematical modeling tasks (Wess et al., 2021, p. 29). This dimension guided the teaching of mathematical modeling tasks in this study. Moreover, in interdisciplinary education, cases or case-based tasks are helpful in supporting pre-service teachers to design their own tasks (Şahin, 2022). Thus, using case-based tasks as a part of problem-solving informed the design of the *Professional Development Module*.

The literature on interdisciplinary thinking also offers insights on the elements of a module on interdisciplinary teaching. This part will describe *interdisciplinary thinking* and relevant aspects for the design.

Interdisciplinarity involves a focus on solving real-world problems that requires integration beyond a single discipline (Weinberg and Sample McMeeking, 2017). *Interdisciplinarity* is considered as a continuum of the relationship between disciplines increasing in complexity (Williams et al., 2016). According to the classification of Williams and his colleagues (2016) based on interdisciplinary mathematics education literature, the continuum of interdisciplinarity changes from the level of mainly *monodisciplinary* to the level of mainly *multidisciplinary*, and then to the level of mainly *interdisciplinary* with increasing in the dependency of disciplines with each other. In the mainly *monodisciplinary* level of Williams and his colleagues (2016), one discipline is in the foreground or dominant. In the mainly *multidisciplinary* level of Williams and his colleagues (2016), there is an interaction between disciplines to some extent, but the disciplines are not totally integrated, and one discipline is considered as a context for the main discipline. The level of mainly *interdisciplinary* indicates a balanced contribution of different integrated disciplines through a problem- or project-based approach (Williams et al., 2016). This classification of interdisciplinarity informed the design, especially for characterizing interdisciplinary mathematical modeling tasks. Thus, interdisciplinary thinking is considered a significant

component underlying the design principle of *use problem-based learning* for the *Professional Development Module*. For the *Professional Development Module*, *interdisciplinary thinking* is defined as the following:

The capacity to integrate knowledge and modes of thinking in two or more disciplines or established areas of expertise to produce a cognitive advancement – such as explaining a phenomenon, solving a problem, or creating a product – in ways that would have been impossible or unlikely through single disciplinary means (Boix Mansilla, Miller, and Gardner, 2000, p. 17)

Besides, the literature on teaching mathematics through mathematical modeling also offers insights on the elements of a module on interdisciplinary teaching. This part will describe *teaching mathematics through mathematical modeling* and relevant aspects for the design.

Mathematical modeling is one of the effective ways of teaching mathematics (e.g., Bliss et al., 2016) and plays “an increasing role in teacher training” (Wess et al., 2021, p. 3). *Mathematical modeling* is considered as a process of applying mathematics to a real-world problem to understand the real-world problem or situation (Niss et al., 2007; Durandt et al., 2022). Moreover, *modeling competence* refers to being able to successfully go through the process of mathematical modeling in which multiple steps or phases are involved (Niss et al., 2007). Therefore, the modeling competence is a complex notion that includes different dimensions, and researchers continue to examine and assess different aspects of the modeling competence (e.g., Blum, 2015; Greefrath and Vorhölter, 2016). Related with modeling competence, modeling tasks require high cognitive demand, and many studies have indicated that participants in the modeling process had difficulties in successfully completing the modeling tasks (e.g., Blum, 2015; Wess et al., 2021). Thus, the role of teachers in teaching mathematical modeling is essential since learning mathematical modeling poses challenges to many learners. In teaching mathematical modeling, appropriate support given by teachers depending on students’ modeling competencies is important (Wess et al., 2021). Supporting students in independent group work through appropriate instruction could also be useful to develop students’ mathematical modeling competencies (Schukajlow et al., 2012).

Like the key characteristics of interdisciplinarity in tasks, there are key features of mathematical modeling tasks. The criteria for modeling tasks were asserted: “reality relation,

relevance, authenticity, and openness” (Wess et al., 2021, p. 14). The *reality relation* indicates referring to a fact outside of mathematics in defining the problem, and the *relevance* criterion indicates the problem to have a close connection to students’ lives (Wess et al., 2021). Thus, the reality relation criterion implies a real-world connection of mathematical modelling tasks. Namely, the context of the problem in the mathematical modeling task comes from real-world outside of mathematics, that is, another discipline or subject matter. Therefore, this reality relation suggests interdisciplinarity or interdisciplinary connections. Indeed, interdisciplinarity and mathematical modeling in this sense intersects each other and thus, are not disjoint constructs. The relevance criterion implies that mathematical modeling tasks should be motivating and attractive for students to engage students in the tasks. Thus, this can allow for personalization of tasks. *Authenticity* requires the problem definition to be authentic in terms of non-mathematical dimension, and mathematical inclusion in a concrete case (Wess et al., 2021). The authenticity implies meaningfulness of the problem in mathematical modeling tasks and going beyond superficial problems. The *openness* criterion requires the problem to be defined with various solutions and approaches (Wess et al., 2021). The openness criterion in mathematical modeling tasks implies that the problem must be generative and must create opportunities to investigate the problem in different ways through several methods. Thus, mathematical modeling tasks should be fruitful such that students should be able to examine the problem in a variety of ways.

Based on the above literature, interdisciplinary thinking, mathematical modeling and task design (especially tasks requiring high cognitive demand) informed the design of the *Professional Development Module*. Thus, the design element on *problem solving* was derived. Using problem-based learning included to use case-based tasks, and interdisciplinary mathematical modeling task criteria. The design element on *communication* included using feedback, discussion, reflection, and setting classroom norms. Another design element on *collaboration* included using small group work for collaborative activities. Moreover, the design element on *social constructivist pedagogy* included active learning, scaffolding, teacher being as a model, considering previous experiences of participants, community of learners, and social interaction.

3. METHOD

The study aimed to design a module to support pre-service mathematics and science teachers' development of skills for interdisciplinary teaching. For this research, design-based research (DBR) has adopted, that can be considered a methodological framework having an advisory characteristic aiming to generate both innovative designs and theories (Bakker, 2019). The following sections describe design-based research, research design, data collection, data analysis, and trustworthiness.

3.1. The Methodological Framework: Design-based Research

Educational design research can be defined as a kind of research that is characterized with “iterative development of solutions to practical and complex educational problems” and affords empirical inquiry that forms “theoretical understanding” (McKenney and Reeves, 2019, p. 6). Thus, educational design research has dual purposes of developing new designs and relevant theoretical insights into problems such as issues on teacher education (McKenney and Reeves, 2014). From now on, DBR will be used to refer to educational design research in this study.

DBR has been noted to be especially appropriate for studying teaching and learning because DBR aims to advance both instructional designs and learning theories (Fishman et al., 2014). In this study, pre-service teachers' theoretical knowledge and pedagogical knowledge for interdisciplinary teaching are aimed to be combined and situated in an interdisciplinary course concurrent with the nature of DBR combining theory and practice (Bakker, 2019). The course is aimed at innovation and its potential impact on supporting pre-service teachers' learning to teach interdisciplinary mathematics. Since teacher learning is a complex phenomenon, it is better to study it in its context without isolating certain variables as the situative nature of teacher learning implies. Thus, this is aligned with the nature of DBR, which includes messy situations and understanding the situation as Collins, Joseph, and Bielaczyc (2004) have emphasized.

As a methodological framework, DBR has certain key features. DBR has both theoretical and pragmatic purposes such that research-supported designs are developed to produce theories based on practice (Cobb et al., 2003; Bakker, 2019). The problems of DBR arise from practice (Cobb et al., 2017). Moreover, DBR is interventionist, that is novel interventions are developed, and these interventions are tested for optimization like engineering (Cobb et al., 2003; Bakker, 2019). DBR is also iterative, that is an intervention is revised in a cyclic manner (Bakker, 2019; Cobb et al., 2003). Furthermore, DBR aims for theoretical generalizability (Cobb et al., 2017). The following paragraphs will describe the key features of DBR for this study.

The concurrent focus of *theoretical and pragmatic purposes* is one of the key features of DBR studies. Cobb et al. (2017) emphasized the dual theoretical and practical goals of design studies such that the aim is “to develop both a practical design for supporting the development of increasingly sophisticated practices and a theory that comprises substantiated conjectures about both processes of development and the means of supporting those developments” (p. 209). Design and theory are intermingled in DBR studies. The theory aimed at DBR studies is local theories that relate to context instead of grand theories. The theory emerging out of DBR studies can be in the form of design principles, or theoretical frameworks that are closely related to developed designs and the research problems. The design researcher makes conjectures about design features and empirically tests these conjectures. In this study, initial design principles, conjectures related to design and support means were hypothesized and were tested to arrive at design principles about supporting pre-service mathematics and science teachers’ development in learning to teach interdisciplinary mathematics, especially in learning to design interdisciplinary mathematical problems for teaching.

Another key feature of DBR is that DBR studies deal with problems arising from *practice* since supporting practitioners’ learning (e.g., teachers’ or students’ learning) or enhancing instructional development in school or district (Cobb et al., 2017). In this study, it is aimed to support pre-service mathematics and science teachers’ development in designing interdisciplinary problems for teaching through a module on teaching interdisciplinary mathematics and science education. Students need to deal with complex problems today and as future teachers, pre-service teachers also need to learn how to support

students to solve such complex problems. Interdisciplinary teaching focuses on teaching how to solve complex problems that have interdisciplinary connections with several areas. Thus, it is important to support teachers, especially pre-service teachers as future teachers on interdisciplinary teaching. About support for pre-service teachers on teaching interdisciplinary teaching, there were no professional development or courses for this purpose. There were no undergraduate courses on developing pre-service teachers' skills for interdisciplinary teaching. Taking this gap, a new course as well as a module on interdisciplinary teaching was designed in this study.

Moreover, another feature of DBR studies is to be *interventionist* such that development of new kinds of practice is aimed at (Cobb et al., 2017). In the current study, development of pre-service teachers' learning to design interdisciplinary problems for teaching is intended through a new module using "iterative cycles of design and analysis" (Cobb et al., 2017, p. 209). The module on interdisciplinary teaching was designed to focus on supporting pre-service teachers to enhance their skills in developing interdisciplinary problems for teaching. The module was a new design aimed at novelty and optimization.

Another key feature of DBR studies is the *iterative* nature (Bakker, 2019). Testing, revising, or modifying conjectures of the study on educational improvement. The continual testing process of DBR studies contain "iterative cycles of design and analysis" (Cobb et al., 2017, p. 209). The product of DBR is optimized via the iterations (Wang, 2024). In this study, design principles and related conjectures were revised in iterative cycles for improving the design and supporting the research aim. This study was conducted in three phases. Moreover, there were two iterations of the interdisciplinary module (or two prototypes) in the context of the course. Cycle 1 was conducted for exploration and needs analysis. As a part of the design and construction phase, Cycle 2 was conducted for the implementation of the initial version of the module on interdisciplinary teaching (first iteration). Furthermore, Cycle 3 was administered for the second iteration of the module as a part of the design and construction phase. Then, overall reflection on findings was done to reach an overall theoretical understanding.

Another key feature of DBR studies is the goal for *generalizability* (Cobb et al., 2017). This generalizability is different from statistical generalizability because the number of

participants in DBR studies is generally limited to arrive at a statistical generalization, which needs a higher number of participants. There is a focus on theory in design studies and there is an aim of context-dependent theoretical generalization. The participants, school, or organization are particular cases to arrive at more generalized inferences by allowing empirical testing. The initial design and the aimed educational improvement together are considered “as an instance of a broader class of phenomena” (Cobb et al., 2017, p. 209), thus, allowing for theoretical investigation. In this study, revised design principles on supporting pre-service mathematics teachers’ development in learning to design interdisciplinary problems for teaching are aimed at contributing to the teacher education literature.

3.2. Research Design

The aim of this study was to design an intervention to support pre-service mathematics and science teachers’ development of skills for interdisciplinary teaching. This design-based study used a *qualitative research design* (Yin, 2016) to investigate deep insights into how the pre-service teachers’ conceptualizations on interdisciplinary teaching changed and how the pre-service teachers designed interdisciplinary tasks (interdisciplinary mathematical modeling tasks or interdisciplinary mathematical problems for teaching) during a designed module on interdisciplinary teaching.

3.2.1. Phases of the Design-based Study

DBR generally follows three phases although different terms are used to label them. For instance, Plomp (2013) proposed three phases of the research process: preliminary research, development or prototyping phase, and assessment phase. Moreover, McKenney and Reeves (2012) suggested a generic model consisting of three main phases of DBR process: Analysis and Exploration, Design and Construction, Evaluation and Reflection (p. 77). The current study follows the latter approach (McKenney and Reeves, 2012) in describing the research process. These phases also consist of certain cycles of research. A *cycle* is defined as “a step in the process of doing research and will include systematic reflection on the theoretical aspects or design principles in relationship to the status of the intervention ultimately resulting in *design principles* or theoretical statements” (Plomp,

2013, p. 31, italics added). The *retrospective analysis* is the analysis of the whole data set to help to construct products such as *design principles* (van den Akker, 1999). To clarify the DBR approach of this research, Figure 3.1 shows the design research process through iterative cycles based on McKenney and Reeves' (2012) generic model for DBR process.

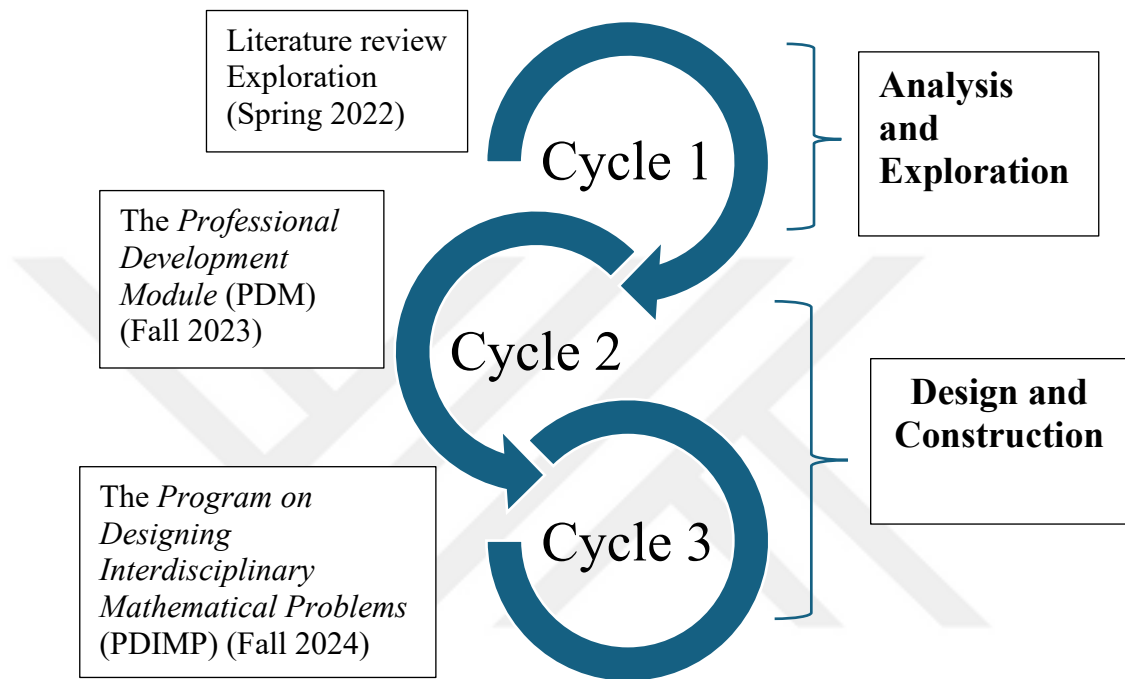


Figure 3.1. The design research process.

In the *Analysis and Exploration phase* of this DBR (McKenney and Reeves, 2012), the research gap was determined through the related literature review. As mentioned in the Literature Review, teachers had difficulties in interdisciplinary mathematics education (e.g., Tytler et al., 2019). Ways to support teachers, especially pre-service teachers in interdisciplinary mathematics education (e.g., An, 2020) still needed research. Before developing a design to support pre-service teachers on interdisciplinary teaching, conceptualizations on interdisciplinary teaching by pre-service secondary mathematics teachers in an undergraduate course were investigated to understand the need of such an intervention. This initial phase was preparation for the next phase of the study, which included the design of an intervention (a module on interdisciplinary teaching). In the first phase (Cycle 1), the purpose of research was to identify the conceptualizations of the pre-service secondary mathematics teachers on interdisciplinary teaching in the context of a

mathematics course (SCED 352) during Spring 2022. An's (2017) framework of interdisciplinary pedagogical content knowledge informed this phase such that to learn interdisciplinary content knowledge of pre-service teachers could give information about the need to design a module on interdisciplinary teaching. The mathematics teaching course had an aim at enhancing pre-service secondary mathematics teachers' mathematical content knowledge for teaching. During this course, an asynchronous module on interdisciplinarity was included and it focused on understanding conceptualizations of pre-service secondary mathematics teachers. The content for this asynchronous part was based on theoretical knowledge of interdisciplinarity and interdisciplinary education. Initial conceptualizations of the pre-service teachers were investigated through open-ended questions. The asynchronous module provided resources such as articles on interdisciplinarity and interdisciplinary education (NAS, 2005; Szostak, 2007; Williams et al., 2016; Maass et al., 2019), which were used as tools for discussion and reflection. The post-conceptualizations were investigated through open-ended questions based on the reflections on the given articles. Due to COVID-19 pandemics, the implementation of the course was administered as online. At the end of this cycle, the need to design a module on interdisciplinary teaching for pre-service teachers was established. This cycle led to the construction of initial design elements of the intervention in the second phase (Cycle 2).

In the Phase of *Design and Construction* (McKenney and Reeves, 2012), generating and implementing the design is of concern. For the Design part, innovative ideas are used to *explore* and *map* the design solutions to the problem at stake for the study (McKenney and Reeves, 2012, p. 110). For the Construction part, *prototypes* are created and revised (McKenney and Reeves, 2012, p. 110). This phase, which is named as “*development or prototyping phase*” by Plomp (2013, p. 19), aims to develop and refine the intervention iteratively through *prototypes* or *iterations* of the design. Aligned with the formative nature of DBR, the module on interdisciplinary teaching was designed iteratively in this study. The conjectured design principles were refined in this phase of the study.

In this study, there were two iterations in this phase (Cycle 2 and Cycle 3, see Figure 3.1). In *Cycle 2*, the *Professional Development Module* in the context of the course SCED 48F (Interdisciplinary Mathematics and Science Education) was designed as Prototype 1 and implemented in the first semester of 2022-2023 term (Fall 2023). In *Cycle 3*, the *Program*

on *Designing Interdisciplinary Mathematical Problems* (PDIMP) in the context of the course SCED 48F was designed as Prototype 2 and implemented in the first semester of 2023-2024 academic year (Fall 2024). At the end of each cycle, conjectured design principles were revised.

In the *Evaluation and Reflection phase* of this DBR (McKenney and Reeves, 2012), the “empirical testing” of a design (*Evaluation*) and holistic consideration of findings to get “theoretical understanding” (*Reflection*) were done (McKenney and Reeves, 2012, pp. 79-80). In this phase of the study, by making reflection on the findings, the final *design principles* were constructed as theoretical output of the study.

For this study, the research questions through cycles were shown in Table 3.1. Finding out the design principles for a module on interdisciplinary teaching for pre-service mathematics and science teachers was the intended outcome of this study. The initial phase of the study (Cycle 1) was an exploration phase based on literature review to identify the need for a module on interdisciplinary teaching for pre-service teachers. The first research question was intended to investigate the conceptualizations of the pre-service teachers on interdisciplinary teaching within Cycle 1. Cycle 2 and Cycle 3 are the two iterations during the second phase of the study. The second, third and fourth research questions were investigated through Cycle 2 while the fifth, sixth and seventh questions were investigated through Cycle 3. The research questions were similar in Cycle 2 and Cycle 3 because they were included in the same research phase, that is, the module designed in Cycle 3 was the second iteration of the module designed in Cycle 2. The second and fifth research questions focused on conceptualizations of the pre-service teachers while the third and sixth research questions focused on how the pre-service designed interdisciplinary tasks or problems in Cycle 2 and in Cycle 3, respectively. The fourth and seventh research questions aimed to investigate design principles on a module for interdisciplinary teaching in Cycle 2 and Cycle 3, respectively.

Table 3.1. The research questions through cycles.

Cycles	Cycle 1	Cycle 2	Cycle 3
Research questions	RQ 1. What are the <i>conceptualizations</i> of the pre-service secondary mathematics teachers on interdisciplinary teaching within the context of a mathematics teaching course during Spring 2022?	RQ 2. How did the <i>conceptualizations</i> of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Professional Development Module (PDM) within the course of Interdisciplinary Mathematics and Science Education during Fall 2023? RQ 3. How did the pre-service mathematics and science teachers who participated in the Professional Development Module (PDM) design <i>interdisciplinary mathematical modeling tasks</i> (IMMTs)? RQ 4. What are the <i>design principles</i> of a module on interdisciplinary teaching?	RQ 5. How did the <i>conceptualizations</i> of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) within the course of Interdisciplinary Mathematics and Science Education during Fall 2024? RQ 6. How did the pre-service mathematics and science teachers who participated in the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) design <i>interdisciplinary mathematical problems for teaching</i> (IMPTs)? RQ 7. What are the <i>design principles</i> of a module on interdisciplinary teaching?

3.2.2. The Implementation of the Second Phase

In the second phase (*Design and Construction*) (McKenney and Reeves, 2012), the *Professional Development Module* (PDM) was designed and implemented in the context of the course SCED 48F: Special Topics: Interdisciplinary Mathematics and Science Education. Pre-service mathematics and science teachers enrolled in the course. The implementation of the *Professional Development Module* (PDM) during Fall 2023 was considered as the first iteration (Cycle 2). Before the instruction, pre-surveys and focus group interviews were administered to document pre-service teachers' instructional starting points (Cobb et al., 2017). The flow for the first iteration (PDM) is summarized in Table 3.2. The course sessions are presented with the related activities and their goals.

Table 3.2. The PDM flow.

Sessions	Date	Activities	Focus
1	10 th November 2022	Pre-survey	To learn the current situation of the pre-service teachers' skills of designing interdisciplinary mathematical modeling tasks (IMMTs)
2	11 th -16 th November 2022	Focus group interviews	
3	17 th -18 th November 2022	Discussion on task design and cognitive demand of tasks	Introducing task design and cognitive demand of tasks
		Examining task frameworks	Analyzing tasks
4	24 th -25 th November 2022	Examining sample mathematical modeling tasks, student works	Understanding the mathematical modeling process through examples
		Engaging in mathematical modeling (e.g., Gas Problem)	
5	1 st -2 nd December 2022	Engaging in interdisciplinary mathematical modeling tasks as learners (e.g., The Midge Problem)	Relating mathematical modeling to interdisciplinarity
		Discussion on characteristics of interdisciplinary mathematical modeling tasks	Introducing interdisciplinary mathematical modeling tasks
		Searching for contexts of interdisciplinary mathematical modeling	Finding interdisciplinary themes for interdisciplinary mathematical modeling tasks
6	8 th -9 th December 2022	Ecological footprint	Working on a given interdisciplinary context (Preparation for the problem construction)
		Mid-evaluation feedback form	
		Rubric construction	Defining the criteria for good IMMTs
7	15 th -16 th December 2022	Rubric construction	Reflecting on the criteria for good IMMTs
		Designing IMMTs	Designing IMMTs
8	18 th -22 nd December 2022	Exit interviews	Post-conceptualizations
	22 nd December 2022	Wrapping up	Closure
9	29 th December 2022- 7 th January 2023	Online meetings	Scaffolding for final projects
10	9 th -10 th January 2023	Submitting reports on final projects	Designing IMMTs (final projects)
11	13 th January 2023	Peer feedback for final projects	Assessing IMMTs
12	20 th January 2023	Post-surveys	Accounting for the development of skills for designing IMMTs

The instruction for the PDM started on 17th November 2022 and was conducted by the researcher. Each daily instructional session took approximately 90 minutes since the lessons were conducted in blocks. On 17th November, discussion on task definition, characteristics of a good task, and cognitive demand was held in the session. Moreover, pre-service teachers analyzed the level of cognitive demand of tasks and engaged in Smith and Stein's (1998) sorting task activity based on cognitive demand. Since pre-service teachers were not familiar with task design, this session was planned to inform them on task design. On 18th November, the session continued to focus on task design and cognitive demand. Different task frameworks were examined, and a class discussion was held on how to increase the cognitive demand of examined tasks in that session. The Mathematical Analysis Task Guide (Smith and Stein, 1998) and the Task Analysis Guide in Science (TAGS) (Tekkumru-Kisa et al., 2015) were shared with pre-service teachers and certain tasks were analyzed based on these guides.

On 24th November and 25th November 2022, the focus of the sessions was mathematical modeling. Since pre-service teachers were not familiar with mathematical modeling, mathematical modeling was initially introduced. Sample mathematical modeling tasks (e.g., recycling example) were examined within a whole-class discussion focusing on the characteristics of the mathematical modeling tasks. The transition from textbook problems to mathematical problems was also exemplified by the researcher. Sample student work assessments of mathematical modeling tasks were also examined. Then, pre-service teachers engaged in different mathematical modeling tasks with varying mathematical content (algebra, geometry, statistics, etc.). The design goal herein was to help pre-service teachers experience mathematical modeling as learners of mathematics by investigating tasks relevant to their own experiences. Moreover, it was aimed at helping pre-service teachers understand the mathematical modeling process through examples. These design features were based on the relevant literature on the pedagogy of mathematical modeling (e.g., Geiger et al., 2022). Modeling tasks that were introduced in these sessions were not necessarily interdisciplinary because pre-service teachers were not accustomed to mathematical modeling tasks and they needed to be initially familiarized with mathematical modeling tasks.

On first and second December 2022, the focus of the sessions was interdisciplinary mathematical modeling tasks. In December first, pre-service teachers experienced the Midge Problem as learners through an online lesson. They worked in groups through Zoom breakout rooms and then participated in a whole class discussion facilitated by the researcher. Through this task, it was aimed to help pre-service teachers engage in interdisciplinary mathematical modeling tasks as learners. Particularly, understanding the problem and mathematization were expected from pre-service teachers in this session. Then, in December second, a discussion was held on the interdisciplinarity and pedagogical aspects of the Midge Problem. Moreover, different examples of interdisciplinary mathematical modeling tasks (Erbaş et al., 2016; Geiger et al., 2018; OECD, 2019) were examined within the class. Pre-service teachers also discussed characteristics of interdisciplinary mathematical modeling tasks, types of mathematical models (such as linear models, geometric models), the use of mathematical modeling problems (e.g., assessment through mathematical modeling tasks), and the levels of interdisciplinarity. Moreover, pre-service teachers discussed possible ideas for their projects. Namely, they searched for interdisciplinary contexts of an interdisciplinary mathematical modeling task. They were assigned to find interdisciplinary contexts for interdisciplinary mathematical modeling tasks till December first. However, they couldn't find the contexts till December first. Then, the researcher guided the pre-service teachers in finding possible contexts within the class on December second. As an example to the interdisciplinary context, ecological footprint was introduced to the pre-service teachers. Pre-service teachers were asked to consider the variables, assumptions and decide how to collect necessary data on the question of *how you could decrease the ecological footprint in our university* till eighth December 2022.

On eighth and ninth December 2022, the sessions focused on interdisciplinary mathematical modeling tasks, particularly finding interdisciplinary contexts for the tasks and discussing criteria for a good interdisciplinary mathematical modeling task. On December second, pre-service teachers were asked to develop models for reducing the ecological footprint on the campus. However, they did not come up with models, but they had some ideas on reducing ecological footprint. Then, on 8th December, pre-service teachers further searched for ideas on reducing ecological footprint and engaged in a whole class discussion on their ideas, possible variables and problem situations for ecological footprint. On ninth December 2022, a mid-evaluation feedback form was administered to get ideas of pre-

service teachers about the course and the module on interdisciplinary teaching. This form collected responses anonymously because pre-service teachers were intended to freely answer the questions. These responses were considered in the revision of the module for second iteration. Besides, a class discussion on defining criteria for good Interdisciplinary Mathematical Modeling Tasks (IMMTs) was held by the pre-service teachers. The researcher started the discussion and then took the role of mediator. The researcher wrote the ideas on the board and guided pre-service teachers on thinking about the main criteria and related sub-criteria for good IMMTs. It was aimed to determine the indicators of good IMMTs that would guide pre-service teachers in designing their own tasks for their final projects. Pre-service teachers named this activity of determining the criteria as the rubric construction.

On 15th and 16th December 2022, the sessions focused on the rubric construction and designing IMMTs. The rubric construction was completed on 15th December 2022. On 16th December 2022, pre-service teachers worked in groups for their final projects in the class under the guidance of the researcher. The researcher was a facilitator to provide scaffolding as needed. Pre-service teachers searched for interdisciplinary contexts and checked them for appropriateness to mathematical modeling. They especially needed help in constructing their main problems for their IMMTs. Then, they presented their initial ideas about their final projects at the end of the session on 16th December 2022.

Between 18th and 22nd December 2022, post-interviews were administered to learn pre-service teachers' conceptualizations on designing interdisciplinary mathematical modeling tasks after the module on interdisciplinary teaching. The interviews were conducted online. Moreover, on 22nd December 2022, the researcher met with the pre-service teachers in person to make a closure on the course. Besides, between 29th December 2022 and seventh January 2023, two groups needed scaffolding on their final projects, so the researcher met with these groups online to guide them on their final projects. Then, pre-service teachers submitted their final project reports on ninth and tenth January 2023. After the submission, pre-service teachers were assigned to two peers to give feedback on their friends' final projects (on 13th January 2023). The reason for including peer feedback was to support pre-service teachers in using different assessment techniques for tasks. On 20th January 2023,

post-surveys were administered online to account for pre-service teachers' development in designing interdisciplinary mathematical modeling tasks.

In the second phase (*Design and Construction*) (McKenney and Reeves, 2012), the *Professional Development Module* evolved into the *Program on Designing Interdisciplinary Mathematical Problems* (PDIMP) in the context of the course SCED 48F: Special Topics: Interdisciplinary Mathematics and Science Education. The course was opened in Fall 2024 and pre-service mathematics and science teachers enrolled in the course. The implementation of the PDIMP was considered Cycle 3. The revisions on design principles and classroom practices were done based on the implementation in Cycle 2. Pre-service teachers were not familiar with interdisciplinarity, mathematical modeling, task design and cognitive demand at the beginning of the course. Before the implementation of the module (PDIMP), consent forms and demographic forms were given to the participants.

On the first week of the course in Fall 2024, an introduction activity using marshmallows was conducted for ice-breaking and modeling an activity implementation in class. On sixth October 2023, initial surveys on interdisciplinarity and interviews were conducted. A discussion on interdisciplinarity was also started. The focus of the following three weeks was also interdisciplinarity. Pre-service teachers searched for interdisciplinary areas, reflected on assigned articles on interdisciplinarity, and searched for interdisciplinary experts to do interviews. Then, mathematical modeling was introduced with the chirping-temperature task (27th October 2023). In other classes, discussion of the previous mathematical modeling task was done, and additional mathematical modeling tasks were introduced. Like Cycle 2, pre-service teachers experienced mathematical modeling tasks as learners since they were not familiar with mathematical modeling. Different mathematical tasks were introduced and then cognitive demand as well as task design were discussed. The discussion on cognitive demand and task design started on ninth November 2023. Like Cycle 2, frameworks for cognitive demand of tasks were introduced. Pre-service teachers examined tasks based on The Mathematical Analysis Task Guide (Smith and Stein, 1998) and the Task Analysis Guide in Science (TAGS) (Tekkumru-Kisa et al., 2015). Moreover, the *tea cooling task* was a new mathematical modeling task introduced in Cycle 3 since pre-service teachers in Cycle 2 mentioned that more hands-on tasks could be added into the design of the course. Table 3.3 shows the flow of the PDIMP.

Table 3.3. The PDIMP flow.

Sessions	Date	Activities	Focus
1	24 th November 2023	Initial survey	<i>Pre-survey</i> on formulating interdisciplinary mathematical problems
2	30 th November 2023	Discussion on interdisciplinary mathematical tasks	<i>Interdisciplinary mathematical tasks</i>
3	1 st December 2023	The Midge Problem	Interdisciplinary mathematical tasks
4	7 th December 2023	Ecological footprint Working on a given context	Formulating interdisciplinary mathematical problems
5	8 th December 2023	Ecological footprint Curricula search	Formulating interdisciplinary mathematical problems
6	14 th December 2023	Discussion on criteria for a “good” Interdisciplinary Mathematical Problem for Teaching Searching interdisciplinary themes	<i>Assessing</i> interdisciplinary mathematical problems for teaching
7	15 th December 2023	Discussion on criteria for a “good” Interdisciplinary Mathematical Problem for Teaching Searching interdisciplinary themes Energy Problem	Interdisciplinary mathematical problems
8	21 st December 2023	Energy Problem Revising on criteria for a “good” Interdisciplinary Mathematical Problem for Teaching Formulating interdisciplinary mathematical problems for teaching	Formulating interdisciplinary mathematical problems for teaching (Working on final projects)
9	22 nd December 2023	Working on final projects Wrapping up	Formulating interdisciplinary mathematical problems for teaching
10	28 th -29 th December 2023	Meetings on formulating interdisciplinary mathematical problems for teaching	Scaffolding (Guideline for their final projects)
11	4 th -5 th January 2024	Meetings on formulating interdisciplinary mathematical problems for teaching	Scaffolding (Guideline for their final projects)
12	8 th January 2024	Final project report deadline	Formulating interdisciplinary mathematical problems for teaching
13	9 th -12 th January 2024	Peer feedback for final projects	Assessing interdisciplinary mathematical problems for teaching
14	13 th January 2024	Presenting final projects	Presenting problems
15	20 th -28 th January 2024	Post-surveys, exit interviews	Evaluation

Before the module (PDIMP) implementation, pre-surveys on formulating interdisciplinary problems were conducted on 24th November 2023. The following paragraphs give details on the implementation of the instruction on interdisciplinary mathematical tasks, which started on 30th November 2023 for the second iteration.

The instruction in PDIMP started with a class discussion on the meaning of interdisciplinary mathematical modeling tasks on 30th November 2023 (see Table 3.3). Then, in the next sessions, pre-service teachers engaged in interdisciplinary mathematical tasks as learners like in the first iteration. The Midge Problem and Ecological Footprint tasks were also administered as in the case of Cycle 2 since positive comments were made by the participants in Cycle 2. Different to Cycle 2, curricula search was added as an activity into the Cycle 3 since pre-service teachers in Cycle 2 had difficulties in finding interdisciplinary contexts for their tasks. Through the curricula search (eighth December 2023), pre-service teachers were aimed to support finding certain interdisciplinary contexts based on interdisciplinary connections identified in the curricular objectives. The researcher gave necessary scaffolding during the curricula search activity in the class. The search for interdisciplinary contexts continued in the next sessions, too. The discussion on criteria for an effective Interdisciplinary Mathematical Problem for Teaching was held on 14th and 15th December 2023. This was called the rubric construction as in the case of Cycle 2. In Cycle 3, a new problem (Energy Problem) was conducted on 15th and 21st December 2023. On 21st December 2023, the criteria for an effective Interdisciplinary Mathematical Problem for Teaching were revised and pre-service teachers worked on their final projects (tried to formulate Interdisciplinary Mathematical Problems for Teaching). They continued to work on their final projects on 22nd December 2023 and then, the course was wrapped up. Then, the next two weeks were separated for scaffolding to help pre-service teachers in their final projects. Online meetings were held with pre-service teachers to help them formulate interdisciplinary mathematical problems for teaching.

The final project reports in Cycle 3 were submitted till eighth January 2024. Then, peer feedback comments were submitted between ninth and 12th January 2024. Pre-service teachers were assigned to give feedback to two of their peers. Differently from Cycle 2, pre-service teachers presented their final projects to the class on 13th January 2024 in Cycle 3. In Cycle 2, pre-service teachers said that they wanted to hear their friends' ideas more. So,

this was considered in the design, and certain time was separated for the presentation of final projects. Finally, post-surveys and post-interviews were conducted between 20th and 28th January 2024.

3.2.3. Revisiting the Design Principles

The design principles that guide the design of the intervention can also change during a DBR study (Bakker, 2019). The design principles are revised at the end of each cycle. The evolution of the design principles in this study can be followed through Figure 3.2. The design principles were derived from related literature. After Cycle 1, the design principles were specified as focusing on authentic tasks, communication, and collaboration. These design principles initially aimed to design a course on interdisciplinary teaching (CIdME). Then, the design principles (Design Principles-Version 1) (DP-V1) were identified for Cycle 2. The research focus was changed from designing a course to a *module on interdisciplinary teaching* based on debriefing with the research team. The module was named as the *Professional Development Module* (PDM) for Cycle 2. Communication and collaboration were kept but were divided into two parts as design principles. The design principle of *increasing communication* included using feedback, discussion, reflection, and setting classroom norms. Another design principle of *supporting collaboration* included using small group work for collaborative activities. The new design principle of *using problem-based learning* was in fact a part of providing authentic tasks, but it was more specific that time. Using problem-based learning included to use case-based tasks, and interdisciplinary mathematical modeling task criteria. Another design principle of *using social constructivist pedagogy* included active learning, scaffolding, teacher being as a model, considering previous experiences of participants, community of learners, and social interaction. The second version of the design principles (Design Principles-Version 2) (DP-V2) were refined after Cycle 2 before transitioning to the Cycle 3. The design principle on collaboration was kept (*Facilitate collaboration*) since group work was important in the design of the module for interdisciplinary teaching. The design principles on social constructivist pedagogy and communication evolved into the design principle on active learning experiences for teacher preparation (*Enhance active learning experiences*). Furthermore, the design principle on problem-based learning evolved into a more specific design principle (*Enhance interdisciplinary problem-solving skills*). Moreover, after the second iteration (Cycle 3), the

third version of the design principles (Design Principles-Version 3) (DP-V3) were generated upon revision. The design principle on active teacher learning was saved (*Foster active teacher learning*). The focus on interdisciplinarity and mathematical modeling led to two separate design principles emphasizing these key aspects (*Promote understanding of interdisciplinarity* and *Enhance opportunities for learning to teach mathematical modeling in interdisciplinarity*). All revisions were based on literature and analyses of data in different cycles.

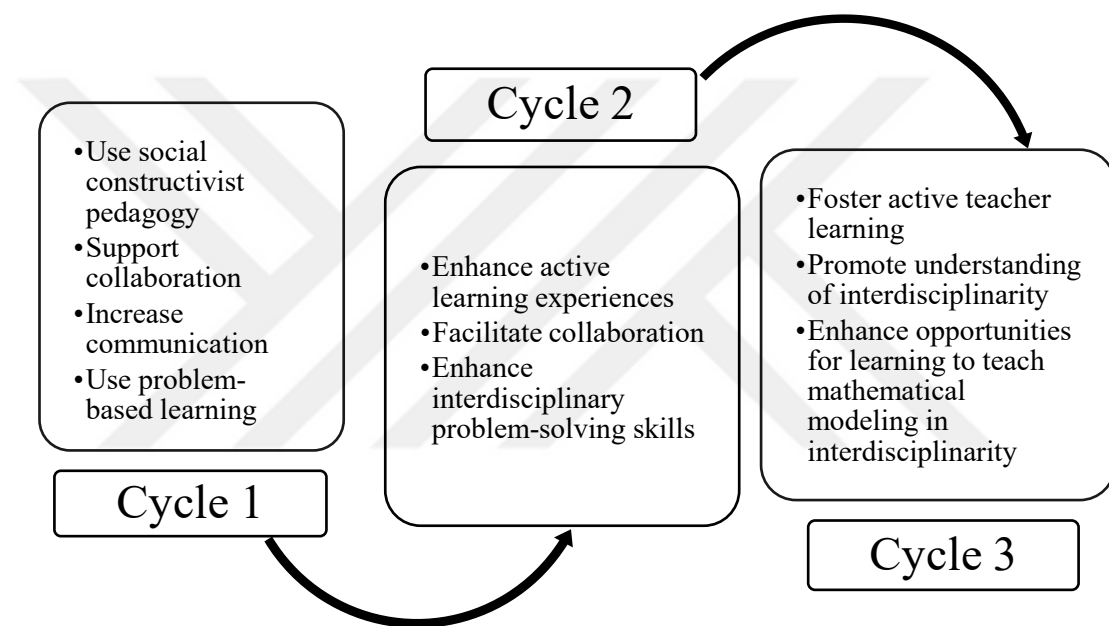


Figure 3.2. The evolution of design principles.

3.2.4. Conjecture Map as a Framework for Design

In this study, Sandoval's (2014) conjecture map was used as a framework to embody the design. *Conjecture maps* are considered as theoretical tools to organize research and guide the design of the research. Sandoval (2014) has recommended making a conjecture map to organize research with its six key tenets and their relationships:

Whatever the context, learning environment designs begin with some *high-level conjecture(s)* about how to support the kind of learning we are interested in supporting in that context. That conjecture becomes reified within an *embodiment* of a specific design. That embodiment is expected to generate certain *mediating processes* that produce desired *outcomes*. The ideas a research team has about how

embodied elements of the design generate mediating processes can be articulated as *design conjectures*. The ideas a team has about how those mediating processes produce desired outcomes are *theoretical conjectures*. (pp. 21-22)

A conjecture map consists of certain key elements such as embodiment, mediating process, and outcomes (Sandoval, 2014). The researcher starts with a tentative hypothesis about ways of supporting learning and Sandoval (2014) called this as *high-level conjecture*. The hypothesis in mind can be enacted through *embodiment* of the design, which can include “tools and materials, task structures, participant structures, and discursive practices” (Sandoval, 2014, p. 22). The embodiment, which includes design features, is expected to produce certain processes (“*mediating processes*”), and then in turn, the mediating processes are expected to generate intended “*outcomes*” (Sandoval, 2014, pp. 21-22). In the conjecture map, the arrows between the embodiment and the mediating processes represent *design conjectures*, and those between the mediating processes and the outcomes represent *theoretical conjectures* according to Sandoval (2014). These conjectures represent the relationships between the elements of the map. For instance, the if-then relationship between the embodiment and mediating processes represents design conjectures, and the directional relationship from the mediating processes to outcomes represents theoretical conjectures (Sandoval, 2014).

The design principles specified based on the literature shaped the initial version of the design on interdisciplinary teaching. The specific design principles were embodied through the design features that are tools and materials, task structures, and participant structures. These elements corresponded with the elements of the embodiment used in a conjecture map (Sandoval, 2014). *Tools and materials* refer to different resources used in the design such as “software programs, instruments, manipulable materials, media, and other resources” (Sandoval, 2014, p. 22). *Task structure* means “the structure of the tasks learners are expected to do- their goals, criteria, standards, and so on” (Sandoval, 2014, p. 22). *Participant structure* corresponds to how participants are supposed to engage in tasks, “the roles and responsibilities participants take on” (Sandoval, 2014, p. 22). Conjecture maps were revised based on data analyses and debriefings with experts (e.g., the thesis committee members). Since one of the purposes of this research is to develop an undergraduate course for interdisciplinary mathematics and science teaching, the evolution of the design principles

along with conjecture maps will be discussed under the chapter of Findings, 4.4. *The Findings for the Reflection on the Design Principles.*

3.2.5. Research Context and Participants

The participants (see Table 3.4) were chosen from pre-service teachers in a state university in Istanbul. The academic language of the university is English. Participants come to the undergraduate program in secondary school mathematics education by getting high scores in the University Entrance Exam after high school graduation. Pre-service secondary mathematics teachers in the undergraduate program in secondary mathematics education take predominantly mathematics content courses in the first four semesters (three calculus courses, one course on discrete mathematics, one course on mathematical structures, one course on differential equations, one course on matrix theory, and one course on group theory), three physics courses, one logic course, general education courses (introduction to education, and educational psychology) (The Department of Mathematics and Science Education, n.d.). In the fourth semester, department specific courses (SCED 282: Principles and Methods of Instruction in Mathematics and Science) are scheduled for the first time. In the fifth semester (beginning of the third year), pre-service secondary mathematics teachers take courses on teaching specific mathematics content (such as SCED 371: Teaching Methods in Mathematics, SCED 352: Algebra for Secondary School Teaching) (The Department of Mathematics and Science Education, n.d.). In the seventh semester of the program, there is a department specific course (SCED 370: Measurement and Assessment in Mathematics and Science Education) in addition to three mathematics elective courses. In the seventh semester (beginning of the fourth-year), the weight is on the department specific courses (pedagogical courses in mathematics and science) (courses on research methods, teaching methods, practice teaching in mathematics and science), and in the eight semester, a course on practice teaching in mathematics is included in addition to certain elective courses (The Department of Mathematics and Science Education).

The *purposeful sampling* (Creswell, 2013; Merriam and Tisdell, 2016) was used to select participants since this study aimed to develop pre-service teachers' skills for interdisciplinary teaching through the design of a module on interdisciplinary teaching. The first cohort of the study included ten pre-service mathematics teachers who enrolled in the

required course SCED 352 (Teaching Algebra and Pre-calculus for Secondary School) in the second semester of the 2021-2022 academic year. In Cycle 1, due to pandemics, lessons were mostly conducted online. The second cohort of the study consisted of nine pre-service mathematics and science teachers who enrolled in the course SCED 48F (Special Topics: Interdisciplinary Mathematics and Science Education) in the first semester of the 2022-2023 academic year. In Cycle 2, lessons were conducted both online and face-to-face. The course SCED 48F was offered for the first time in that period. Furthermore, the third cohort included four pre-service mathematics and science teachers who were enrolled in the course SCED 48F in the first semester of the 2023-2024 academic year. In Cycle 3, most lessons were conducted face-to-face. All the courses were opened under the name of the advisor. The ethical consent form was approved by the Ethical Committee of the Science and Engineering Human Research Council in the university before the study started. All participants gave the informed consent to attend the study and to get audio-recording.

Table 3.4. Participants during the study.

Cycles	Cycle 1	Cycle 2	Cycle 3
Cohorts	10 pre-service mathematics teachers enrolled in the course SCED 352	9 pre-service mathematics and science teachers enrolled in the course SCED 48F	4 pre-service mathematics and science teachers enrolled in SCED 48F

In the context of the initial phase (Analysis and Exploration), the first cohort of the study consisted of ten pre-service teachers (Cycle 1). This phase was a preparation phase for the design of the intervention (a module on interdisciplinary teaching). An asynchronous module on interdisciplinarity was administered as a part of the required course SCED 352 (Teaching Algebra and Pre-calculus for Secondary School) at a state university in Istanbul. The course was opened in the Spring 2022 semester (2021-2022 second semester) for pre-service secondary mathematics teachers to enhance their mathematical content knowledge for teaching. The course was a four-hour required course for teaching certain secondary school mathematics topics (real numbers, complex numbers, functions-definition of function, modeling and families of function, linear functions, trigonometry and

trigonometric functions, exponential function, logarithm function, definition of limit, derivative, and integral). During the course, pre-service teachers were expected to do mathematics by focusing on the concepts, definitions, and proofs as well as the historical background of the concepts. The course was conducted as both online and face-to-face, but mostly as online because of the pandemics based on COVID 19. The course consisted of 14 weeks of instruction but one week was separated for Spring Break and the Religious Festival. *Canvas* was selected as the learning management system for the course due to easy use of the system and its facility to allow modules for good organization. There were 20 secondary school mathematics pre-service teachers enrolled on the course at the beginning, but 17 of them were active. However, since data from ten of them were complete, these ten pre-service teachers were taken as participants. None of the pre-service teachers took a methods course before.

In the context of the second phase (Design and Construction), the second cohort of the study consisted of nine pre-service mathematics and science teachers (Cycle 2). The participants were chosen from the pre-service mathematics and science teachers who took the course SCED 48F: Special Topics: Interdisciplinary Mathematics and Science Education at a state university in Istanbul. Thus, *purposeful sampling* (Creswell, 2013; Merriam and Tisdell, 2016) was used to include both mathematics and science pre-service teachers. The reason for choosing science and mathematics pre-service teachers in the sample was the expected *collaboration* between different disciplines in designing interdisciplinary mathematical modeling tasks and engaging in relevant tasks planned in the course. Collaboration was also suggested in interdisciplinary teaching (e.g., Şahin, 2022) and teaching mathematical modeling (e.g., Bliss et al., 2016; Erbaş et al., 2016; Fulton et al., 2019).

The implementation of the Professional Development Module (PDM) was conducted as a part of the course SCED 48F, which was offered in Fall 2023 for the first time. The course was opened under the name of the advisor of this dissertation study. Initially, fifteen pre-service teachers enrolled in the course, but two of them have never attended the course. That is, actively ten undergraduate mathematics and science pre-service teachers and three graduate students (master) took the course. All the undergraduate pre-service teachers gave informed consent for participating in the study and were chosen as the participants. However,

one pre-service teacher has not attended the classes since the 17th of November class, which was the beginning of the module implementation. Thus, this pre-service teacher was left out of the participants, and nine pre-service teachers in total were taken as participants for Cycle 2.

The researcher was an observer at the first part of the course till the beginning of the PDM and has become active in the teaching process of the course (transitioning from the role of an observer to the role of an instructor) since the lessons on 17th November 2022. The lessons from 17th November were conducted by the researcher as the main instructor in the class. The purpose of the researcher being much an observer in the class at the first part of the course was to get familiarized with the participants, to build rapport with the participants, and to wait for the class norms to be settled down.

The Demographic Information Form (see Appendix F) was also completed by the participants before the intervention. It was given to get information about their demographics such as age, gender, and their departments, high school type, previous experiences such as teaching experience, mathematical modeling, and designing tasks. The pre-service teachers studied High School Mathematics Teaching (n=2), Middle School Mathematics Teaching (n=3), Science Teaching (n=1), Physics Teaching (n=2), and Chemistry Teaching (n=1). Besides, pseudo names such as P1 were given to the participants for anonymity. Four pre-service teachers (P1, P2, P3, P4) were in their fifth semesters of their undergraduate programs, two pre-service teachers (P5, P6) were in their fourth semesters of their programs, and one pre-service teacher (P8) was in sixth semester of the program. Furthermore, one pre-service teacher (P7) was in ninth semester and another one (P9) in 14th semester in their undergraduate programs. P9 gave a break to university and then returned to the program. Thus, most of the pre-service teachers were similar in their semesters and previously taken courses. Most of these pre-service teachers did not take any teaching practice courses. The pre-service teacher P7 noted to take a teaching practice course last year, so P7 was familiar with notions of teaching. Besides, P7 stated that P7 started to the Teaching Physics Program initially but changed the major to Mathematics Teaching Program after one year. P7 also mentioned making volunteer teaching in certain projects in the university, thus P7 had initial teaching experiences. Besides, the pre-service teacher P8 had an engineering degree before entering the Undergraduate Program in Mathematics Education. According to the

Demographics Information Form, none of the pre-service teachers attended a study or workshop about interdisciplinary mathematical modeling before. Besides, eight pre-service teachers have never designed a task themselves except one pre-service teacher (P9). The pre-service teacher (P9) noted that “I took lessons on designing web sites, simulations. I attended to these activities [lessons] in order to use ahead”. Moreover, all the pre-service teachers stated that they tutored as a kind of teaching experience.

In the context of the second phase (Design and Construction), the third cohort of the study consisted of four pre-service mathematics and science teachers (Cycle 3). Pre-service teachers enrolled in the course SCED 48F in Fall 2024. This course was the second iteration of the course in Fall 2023. Most of the classes were realized as face-to-face lessons. The asynchronous part of the course was managed by the researcher through the Learning Management System of the University Moodle and through a WhatsApp group formed at the beginning of the course. Moreover, the researcher taught the classes starting the implementation of the Program on Designing Interdisciplinary Mathematical Problems (PDIMP). Thus, from thereon, the researcher acted as an instructor in the class. There were seven pre-service teachers enrolled in the course at the beginning. However, since two of them dropped the course and another one did not attend the post-interviews, four pre-service teachers who regularly attended the course were considered as participants for Cycle 3. The pre-service teachers were called T1, T2, T3, T4 for anonymity. One pre-service teacher (T3) had a major in science teaching. Three pre-service teachers (T1, T2, and T4) had a major in primary school mathematics teaching. Two of the pre-service teachers (T1 and T2) were female, others were male. Three of the pre-service teachers (T1, T2, T3) graduated from an Anatolian High School, and one pre-service teacher (T4) graduated from a Science High School. Three of the pre-service teachers (T1, T2, T3) graduated from an Anatolian High School, and one pre-service teacher (T4) graduated from a Science High School. None of the pre-service teachers had a course or a kind of professional education on mathematical modeling or interdisciplinary mathematical modeling before. They did not design any tasks before either. Besides, they did not take any courses before. However, three of the pre-service teachers (T1, T3, and T4) had experience of giving private lessons. The pre-service teachers were similar in terms of their semesters. Three of the pre-service teachers (T2, T3 and T4) entered the university in 2020 and one pre-service teacher (T1) entered the university in 2019. Thus, they were similar in the courses taken.

3.3. Data Collection

In this study, *qualitative* data were collected to get an in-depth insight into how a module on interdisciplinary teaching could be designed to support pre-service teachers in interdisciplinary teaching. *Qualitative approach* (Creswell, 2012) was used to integrate various data to understand pre-service teachers' conceptualizations on interdisciplinary teaching and how pre-service teachers designed interdisciplinary tasks or problems.

Aligned with DBR, several types of data were collected in this study. Participants produced artifacts such as individual course assignments, class work emerging out engaging in interdisciplinary mathematical modeling tasks as learners during the course, group work assignment documents, designed interdisciplinary tasks/problems (final projects), and class-constructed criteria. Participants' individual assignments on Moodle included reflection notes for the weekly readings, and individual comments on the discussion forum. Feedback (instructor, researcher, and peer) comments and discussions during the course, class observations including field notes by the researcher, debriefing sessions, recordings of the course sessions, responses to pre-survey and post-survey were also collected as data.

As part of the initial phase, in *Cycle 1*, an *open-ended survey* and two *reflection papers* were used as data to investigate participants' conceptualizations on interdisciplinary mathematics education. As part of the second phase, in the *Cycle 2* and *Cycle 3*, participants' *artifacts* (mainly, designed interdisciplinary *tasks* in the *Cycle 2* and designed interdisciplinary *problems* in the *Cycle 3* as final projects), and semi-structured *interviews* (pre-interview and post-interview) were collected as main data sources. Interviews were held online through Zoom, recorded and were transcribed by the researcher. Participants' artifacts consisted of created tasks/problems and class-constructed rubric. Pre-surveys and post-surveys contributed as secondary data. The third phase of the study did not contain additional data collection but was considered as a reflection on findings from the second phase to arrive at design principles for a module on interdisciplinary teaching for pre-service teachers. For the third phase, interviews and artifacts were used primary data sources and the researcher notes were used as secondary data sources. To answer the research questions, data sources collected through cycles were described in Table 3.5. The detailed description of data sources through cycles will be given in the following sections.

Table 3.5. Data sources.

Research questions	Cycles	Data sources
RQ 1. What are the <i>conceptualizations</i> of the pre-service secondary mathematics teachers on interdisciplinary teaching within the context of a mathematics teaching course during Spring 2022?	Cycle 1	Open-ended survey Reflections
RQ 2. How did the <i>conceptualizations</i> of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Professional Development Module (PDM) within the course of Interdisciplinary Mathematics and Science Education during Fall 2023?	Cycle 2	Interviews Surveys Anonymous Feedback Forms
RQ 3. How did the pre-service mathematics and science teachers who participated in the Professional Development Module (PDM) design <i>interdisciplinary mathematical modeling tasks</i> (IMMTs)?	Cycle 2	Interviews Artifacts
RQ 4. What are the <i>design principles</i> of a module on interdisciplinary teaching?	Cycle 2	Interviews Artifacts Researcher notes
RQ 5. How did the <i>conceptualizations</i> of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) within the course of Interdisciplinary Mathematics and Science Education during Fall 2024?	Cycle 3	Interviews Surveys
RQ 6. How did the pre-service mathematics and science teachers who participated in the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) design <i>interdisciplinary mathematical problems for teaching</i> (IMPTs)?	Cycle 3	Interviews Artifacts
RQ 7. What are the <i>design principles</i> of a module on interdisciplinary teaching?	Cycle 3	Interviews Artifacts Researcher notes

3.3.1. Data Collection in Cycle 1

Cycle 1 was a preparation stage to identify the need for a module on interdisciplinary teaching. The data collected in Cycle 1 aimed to learn about pre-service secondary mathematics teachers' conceptualizations on interdisciplinary mathematics education. The data sources of the study for Cycle 1 were an *open-ended survey* (*SCED 352 Introduction*), *Reflection 1* and *Reflection 2* to answer the *second research question*. Initially, participants answered an open-ended survey on interdisciplinary mathematics education called *SCED 352 Introduction* before starting the implementation (on seventh March 2022). The survey consisted of four open-ended questions:

- How would you define the term interdisciplinary? You may give an example to explain your ideas.
- How does mathematics relate to other disciplines?
- How would you describe interdisciplinary mathematics education?
- How can a teacher implement interdisciplinary mathematics education with high school students? What would be the benefits and challenges of such implementation?

The questions asked for the definition of interdisciplinarity, the relationship of mathematics to other disciplines, the description of interdisciplinary mathematics education, and the implementation of interdisciplinary mathematics education in the classroom as well as the benefits and challenges of such an implementation. The aim of this initial survey was to get participants' initial ideas on interdisciplinarity and interdisciplinary mathematics education and to see their current experiences on interdisciplinary mathematics education. Since it was a relatively new topic, it was hypothesized that participants were not much familiar with interdisciplinarity at the beginning.

Reflection 1 and *Reflection 2* were given as assignments included in the asynchronous module on interdisciplinarity in the course. Pre-service teachers were assigned articles on interdisciplinarity and interdisciplinary mathematics education and then they were asked to write reflections to these given articles. Reflection 1 focused on the concept of interdisciplinarity and participants were given two related articles (NAS, 2005; Szostak, 2007). The due date for the peer feedback was determined as fourth April 2022 at the beginning, but since not everyone uploaded their individual reflections till fourth April, the due date for the feedback was changed to 18th April 2022. Participants were asked to think the following questions on interdisciplinarity for Reflection 1:

- What does interdisciplinarity mean?
- What are the defining features of interdisciplinarity common in different definitions in literature?
- What is interdisciplinary study? Why is interdisciplinary study conducted?
- What can be different levels in interdisciplinary understanding? Why are they considered?

- How are disciplines and interdisciplinarity related?
- How have your conceptualizations about interdisciplinarity changed after reflecting on these articles?

Reflection 2 focused on the meaning of interdisciplinary mathematics education, focusing more on the education part. Like the Reflection 1 assignment, two articles on interdisciplinary mathematics education were assigned (Maass et al., 2019; Williams et al., 2016). The due date for the Reflection 2 assignment was 15th May 2022. The following questions were suggested for participants in writing their second reflection (Reflection 2):

- What is interdisciplinary mathematics? How do authors define the concept? How do you conceptualize the concept? Could you compare your conceptualizations before and after reading these articles?
- Why is interdisciplinary mathematics necessary?
- What can be the benefits and challenges of interdisciplinary mathematics in the classroom?
- What approaches can be beneficial in implementing interdisciplinary mathematics?
- How can interdisciplinary research articles read in previous weeks relate to these week's readings?

Cycle 1 was conducted as an exploration phase to understand the need for a new design (module on interdisciplinary teaching). Cycle 1 aimed to investigate pre-service teachers' conceptualization on interdisciplinarity and interdisciplinary teaching. Thus, the data collection in Cycle 1 was based on pre-service mathematics teachers' self-responses through an open-ended survey and reflection papers (Reflection 1 and Reflection 2). The next sessions will describe the data collection for the second phase of the study including Cycle 2 and Cycle 3.

3.3.2. Data Collection in Cycle 2

This section presents data collection tools for the second phase. This phase consisted of two cycles for this study: Cycle 2 and Cycle 3. Primary data collection tools in Cycle 2 and Cycle 3 were *interviews* (pre-interview and post-interview), and pre-service teachers'

artifacts. Pre-surveys and post-surveys also contributed as secondary data. Anonymous feedback forms were also used as supplementary data in Cycle 2. The focus in Cycle 2 was on designing interdisciplinary mathematical modeling tasks and the focus in Cycle 3 was on designing interdisciplinary mathematical problems for teaching. The following sections give information about each data source.

In the second phase (Cycle 2 and Cycle 3), semi-structured *interviews*, which were conducted before and after the study (pre-interview and post-interview, respectively), were considered as data sources. The interview questions (open-ended) were created by the design researcher and experts in mathematics and science education (see Appendix D and Appendix E for initial and exit interviews, respectively). Interviews were audio-recorded based on the given informed consents. Interviews were planned to last approximately 45 minutes. The durations of interviews changed between approximately 30 minutes to 60 minutes. In Cycle 1, since the pre-service teachers' responses to reflection papers were limited, interviews were added as data collection tools for the second phase of the study to get more in-depth views of the pre-service teachers.

Focus group interviews are suggested when the interaction among interviewees might give the best information, when participants cooperate with each other and have similar characteristics, and when interviewees might be reluctant to talk in one-one-on interviewing (Creswell, 2013). In this study, collaboration among participants was important and informed the design of the study. Since the topic of the study (interdisciplinary mathematical modeling) was not familiar to the participants, they might have been reluctant to provide information when they were interviewed individually. It was expected that they might have been more comfortable talking in the group. Thus, focus group interviews were conducted in this study. For Cycle 2, the participants were initially required to determine their final project groups (till 10th November 2022) before the pre-interviews (conducted between 11th and 16th November 2022). The directions on how they would construct their groups were written on the Moodle. The groups were required to consist of two pre-service teachers (preferably but not as a must, one mathematics and one science). Hence, at the beginning, since there were ten people (who gave informed consent to take part in the study), five pairs were formed and interviewed. One pre-service teacher was dropped out at the exit interviews because that pre-service teacher did not attend any of the classes since the study began.

Hence, for post-interviews (conducted between 18th and 21st December 2022), the researcher conducted interviews with four pairs (P1-P8, P3-P4, P5-P6, P7-P9) and one individual (P2). By considering Creswell's (2013) warning about the possible power issues in focus group interviews, the researcher directed interviewees to talk in an order at the beginning of the interview, and encouraged interviewees to talk more by using expressions like "P1, what do you think about this question? What can you say about this? What can you add to your friend's thoughts?".

In the second phase, *surveys* were also used as data sources. The pre-survey and post-survey (see Appendices A and B) aimed to examine the change in participants' conceptualizations on interdisciplinary teaching. Individual assessment was the purpose for conducting pre-survey and post-survey. One and half hours were given for the pre-survey. The initial survey was conducted to get insight into participants' current conceptualizations on interdisciplinary teaching since documenting instructional starting points were important in DBR (Cobb et al., 2017). The questions focused on interdisciplinarity, mathematical modeling, and interdisciplinary mathematical modeling tasks. The post-survey, which was conducted online on 20th January 2023, (see Appendix B) was prepared as a parallel form to the pre-survey for reliability. Interdisciplinary thinking (the concept of interdisciplinarity, interdisciplinary math/science education), mathematical modeling, and interdisciplinary mathematical modeling tasks (characteristics, design, and teaching of such tasks) were the three components of the pre-survey and the post-survey. Two cases were given in both pre-survey and post-survey, but different cases were used in the pre-survey and post-survey. The cases were chosen from mathematical modeling tasks and adapted to be open-ended items asking for analyzing these tasks in terms of interdisciplinarity levels (Williams et al., 2016), ways of increasing interdisciplinarity levels of the given tasks by considering the continuum of interdisciplinarity (Williams et al., 2016), ways of assessing the given tasks with respect to certain criteria of interdisciplinary mathematical modeling tasks as well as ways of improving the given tasks to be interdisciplinary mathematical modeling tasks. Based on the results of Cycle 1, to get more in-depth views of the pre-service teachers, surveys were added as data collection tools.

Another primary data sources in second phase (Cycle 2 and Cycle 3) were participants' *artifacts*. Two types of artifacts were used as data: Constructed tasks and rubric. In the

context of the second phase, for Cycle 2, pre-service teachers were asked to prepare interdisciplinary mathematical modeling tasks (IMMTs) as their final projects until 9th January 2023. Before the submission of the final projects, pre-service teachers were initially expected to determine an interdisciplinary context for their final projects, which was assigned due second December 2022. After the rubric construction, pre-service teachers worked in their groups for their final projects in the class under the guidance of the researcher. The researcher acted herein just as a facilitator and provided scaffolding as needed. Pre-service teachers searched for interdisciplinary themes and checked for appropriateness to mathematical modeling during the class work. They especially needed help in constructing their main problems for their interdisciplinary mathematical modeling tasks. Some pre-service teachers changed their themes or ideas during the class work. At the end of the course, they presented their initial ideas about their final projects.

Pre-service teachers designed their tasks in groups. Only one pre-service teacher (P2) designed the task individually since another group member did not continue the course. There were four group tasks. Group 1 consisted of pre-service teachers P7 (high school mathematics teaching major) and P9 (physics teaching major). Group 2 consisted of pre-service teachers P3 (high school mathematics teaching major) and P4 (primary school science teaching major). Group 3 consisted of P5 and P6 (primary school mathematics teaching major). Moreover, Group 4 consisted of P1 (chemistry teaching major) and P8 (primary school mathematics teaching major).

In the context of class work, two class days were separated for the rubric construction. Pre-service teachers were asked how they would convince the audience that a designed task was appropriate for being an interdisciplinary mathematical modeling task and why the task could be worth applying in the classroom. A whole class discussion was conducted in these lessons, and the instructor was just a moderator of the discussion. The researcher (also the instructor) was responsible for writing pre-service teachers' ideas, for provoking the pre-service teachers to add more ideas, and for guiding them to categorize their ideas. The lessons were aimed at being exploratory such that pre-service teachers would share their own ideas and arrive at a shared understanding of the criteria and related categories. Pre-service teachers shortly named the criteria as *rubric*.

The constructed rubric, which is called “*Rubric for Interdisciplinary Mathematical Modeling Tasks*” (see Appendix K), consists of five dimensions: A. *Level of cognitive demand*, B. *Mathematical modeling*, C. *Organization/structure of task*, D. *Context of the problem*, and E. *Expected/possible student solutions*. These dimensions emerged as a result of the whole-class discussion by the pre-service teachers. Then, the researcher added levels for each dimension to finalize the constructed rubric. The levels changed from *Not Applicable* to *Outstanding*. In the rubric, the tasks were considered in ranks with an increasing order of appropriateness: *Not Applicable*, *Acceptable*, *Satisfactory*, and *Outstanding*. The rubric was shared with the pre-service teachers to guide them in their design of final projects. This revised rubric was also used to analyze pre-service teachers’ tasks.

Another secondary data source for Cycle 2 was the *anonymous feedback forms*. The mid-evaluation form was applied on ninth December 2022 as an anonymous feedback form (see Appendix C). The feedback answers were important on design decisions such as what would change and not change in the next iteration of the course (especially revising design principles). Pre-service teachers’ ideas were collected during the intervention to make sense their experiences of the course, especially those of designing interdisciplinary mathematical modeling tasks (IMMTs). In the anonymous forms, pre-service teachers were asked about evaluating the aspects of the task development in the course, whether they thought about changing such aspects, how the tasks in the course were helpful in their design of IMMTs, and their suggestions to improve the course.

The purpose of the second phase (Cycle 2 and Cycle 3) was to design a module on interdisciplinary teaching for pre-service teachers. In Cycle 2, the first iteration of the module was implemented. Interviews were used as primary data sources to examine pre-service teachers’ conceptualizations on interdisciplinary teaching. Surveys and anonymous feedback forms were used as secondary data sources to examine the conceptualizations. Moreover, participants’ artifacts and interviews were used as data sources to investigate how pre-service teachers designed interdisciplinary mathematical modeling tasks. Interviews, artifacts and researcher notes were also used as data sources to find out the design principles for the module on interdisciplinary teaching.

3.3.3. Data Collection in Cycle 3

Cycle 3 was a part of the second phase of the research. Data collection was similar in Cycle 2 because Cycle 2 and Cycle 3 were included in the same research phase. In Cycle 2, the first iteration of the module on interdisciplinary teaching was conducted. Moreover, in Cycle 3, the second iteration of the module on interdisciplinary teaching was implemented. Primary data collection tools in Cycle 3 were *interviews* (pre-interview and post-interview), and pre-service teachers' *artifacts* (formulated problems as final projects and constructed rubric). Pre-surveys and post-surveys also contributed as data. The focus in Cycle 2 was on designing interdisciplinary mathematical modeling tasks and the focus in Cycle 3 was on designing interdisciplinary mathematical problems for teaching. The following sections give more details on data tools for Cycle 3.

In Cycle 3, semi-structured *interviews* were applied to understand pre-service teachers' conceptualizations on interdisciplinarity, interdisciplinary education, mathematical modeling, and designing interdisciplinary problems for teaching. The pre-interviews lasted approximately 16 and 75 minutes. The post-interviews lasted approximately 60 and 75 minutes. Interviews were audio-recorded based on the given informed consents.

The initial four items in the pre-interview in Cycle 3 (see Appendix G) asked for defining the concepts of model, modeling, mathematical modeling, and teaching mathematical modeling. The fifth item in the pre-interview asked for designing a mathematical task and relevant design principles or criteria as well as general characteristics of designing a task. The sixth item required an example task design for an interdisciplinary mathematical modeling task and explaining how to write such a task. The seventh item gave a case on plastic use in Turkey and required to design an interdisciplinary mathematical modeling task based on this case.

The post-interview in Cycle 3 (see Appendix H) was parallel to the pre-interview. The post-interview was administered to understand pre-service teachers' conceptualizations on interdisciplinary teaching and their experiences of designing interdisciplinary problems. The first two items in the post-interview focused on interdisciplinarity. The other four items were about how to write an Interdisciplinary Mathematical Problem for Teaching. The seventh

and eight items were about evaluating the course and the Program on Designing Interdisciplinary Mathematical Problems (PDIMP). The ninth item focuses on the learning effect of the tasks used in the course.

In addition to interviews, *surveys* were used for triangulation. Surveys were used to get insight into pre-service teachers' conceptualizations on interdisciplinary teaching. Surveys consisted of open-ended items. The initial survey was conducted on 24th November 2023. Post-surveys were administered online between 20th and 28th January 2024. The pre-survey (see Appendix I) consisted of six main items on interdisciplinarity, mathematical modeling, and interdisciplinary mathematical task design. Like the pre-survey, the post-survey in Cycle 3 (see Appendix J) consisted of items about interdisciplinarity, mathematical modeling, and designing interdisciplinary mathematical problems for teaching. There were seven main items in the post-survey. Unlike the pre-survey, one item of the post-survey asked about evaluation of using interdisciplinary mathematical problems for teaching in the classroom. Namely, pre-service teachers were expected to write their ideas on the possible advantages and disadvantages of implementing such problems. Three pre-service teachers (T1, T3, T4) answered the post-survey although four pre-service teachers responded to the pre-survey.

In Cycle 3, artifacts were used as primary data sources to investigate how pre-service teachers designed interdisciplinary mathematical problems for teaching. Two types of artifacts were used as data: Formulated problems and constructed rubric. Pre-service teachers' *problems* as artifacts were used to get in-depth understanding about designing interdisciplinary problems for teaching. Pre-service teachers worked in peers to formulate their problems for the final project. There were two problems formulated as pre-service teachers' artifacts since there were two groups. One problem was about a triathlon race and prepared by the pre-service teachers T3 and T4. Another problem was about eco-friendly planning and prepared by the pre-service teachers T1 and T2.

In Cycle 3, as an artifact, class constructed *rubric* was used as a data source like in Cycle 2. Constructing rubric was used as data to reveal pre-service teachers' learning experiences of creating criteria for an effective interdisciplinary mathematical problem for teaching. Moreover, the rubric was used to assess their problems for their final projects. The

activity of constructing criteria for an effective Interdisciplinary Mathematical Problem for Teaching (shortly named *rubric*) was conducted as a whole-class discussion on 14th and 15th December 2023 with the researcher just as a facilitator. Constructing rubric was designed as a student-centered learning activity for pre-service teachers. It was an encompassing activity since it indicated main ideas of the intervention (module) and reflected how the key ideas were discussed in the classroom. Pre-service teachers themselves created indicators of an effective interdisciplinary mathematical problem for teaching based on their previous experiences through this course. The dimensions identified by the pre-service teachers were interdisciplinarity, context, mathematical modeling, teaching, and organization. On 21st December 2023, the constructed criteria were revised. After 21st December class, assessment/expectations were added as a new dimension by the pre-service teachers. The rubric was shared with pre-service teachers to guide them in designing their problems for the final project.

The purpose of the second phase was to design a module on interdisciplinary teaching for pre-service teachers. In Cycle 3, the second iteration of the module was implemented. Interviews and surveys were used as data sources to examine pre-service teachers' conceptualizations on interdisciplinary teaching. Moreover, participants' artifacts and interviews were used as data sources to investigate how pre-service teachers designed interdisciplinary mathematical problems for teaching. Interviews, artifacts and researcher notes were also used as data sources to find out the design principles for the module on interdisciplinary teaching.

3.4. Data Analysis

The purpose of this study was to design an intervention to support pre-service mathematics and science teachers in interdisciplinary teaching. The study was conducted in cycles. Table 3.6 shows the data analysis corresponding to research questions with relevant data sources.

Table 3.6. Data analysis corresponding to research questions.

Research questions	Cycles	Data sources	Data analysis
RQ 1. What are the <i>conceptualizations</i> of the pre-service secondary mathematics teachers on interdisciplinary teaching within the context of a mathematics teaching course during Spring 2022?	Cycle 1	Open-ended survey Reflections	Open coding Axial coding
RQ 2. How did the <i>conceptualizations</i> of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Professional Development Module (PDM) within the course of Interdisciplinary Mathematics and Science Education during Fall 2023?	Cycle 2	Interviews Surveys Anonymous Feedback Forms	Open coding Axial coding Thematic analysis
RQ 3. How did the pre-service mathematics and science teachers who participated in the Professional Development Module (PDM) design <i>interdisciplinary mathematical modeling tasks</i> (IMMTs)?	Cycle 2	Interviews Artifacts	Open coding Axial coding Thematic analysis Task analysis framework (for IMMTs)
RQ 4. What are the <i>design principles</i> of a module on interdisciplinary teaching?	Cycle 2	Interviews Artifacts Researcher notes	Coding for design elements
RQ 5. How did the <i>conceptualizations</i> of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) within the course of Interdisciplinary Mathematics and Science Education during Fall 2024?	Cycle 3	Interviews Surveys	Open coding Axial coding Thematic analysis
RQ 6. How did the pre-service mathematics and science teachers who participated in the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) design <i>interdisciplinary mathematical problems for teaching</i> (IMPTs)?	Cycle 3	Interviews Artifacts	Open coding Axial coding Thematic analysis Task analysis framework (<i>Interdisciplinary Mathematical Modeling Task Analysis Framework</i>)
RQ 7. What are the <i>design principles</i> of a module on interdisciplinary teaching?	Cycle 3	Interviews Artifacts Researcher notes	Coding for design elements

To answer the research questions in this study, qualitative data were mainly analyzed through *open coding* and *axial coding* (Vollstedt and Rezat, 2019; Leplaa et al., 2025). During *open coding*, the data are initially divided into “smaller parts that are deeply

analysed” and the goal of this analysis is to get the “core idea of each part and to develop a code to describe it” (Vollstedt and Rezat, 2019, p. 86). The aim of open coding is to specify and “list all topics presented in the data by allowing to let the data speak for itself” (Leplaa et al., 2025, p. 12). After open coding, *axial coding* was used to compare codes among participants and to connect themes (Vollstedt and Rezat, 2019; Leplaa et al., 2025). During *axial coding*, the aim is to organize the data by searching “relationships between the open codes” (Leplaa et al., 2025, p. 13). All interviews were transcribed and inductively analyzed. Initial open codes were constructed and repeated readings were done to revise codes. By constantly comparing data, similar open codes were compiled to construct codes and themes. In the following paragraphs, the qualitative analysis approach for each cycle is explained.

Cycle 1 served as an exploration stage for the need for a module on interdisciplinary teaching for pre-service teachers. In Cycle 1, it was aimed at investigating participants’ conceptualizations on interdisciplinarity. Data were *inductively* analyzed to understand pre-service mathematics teachers’ ideas on interdisciplinary mathematics education (to answer the *first research question*). *Open coding* (Corbin and Strauss, 1990, Vollstedt and Rezat, 2019) was used to analyze participants’ responses to an open-ended survey and reflection papers. Since ten pre-service teachers responded to all data tools, the responses of those were considered for the analysis. For data preparation, initially all responses of pre-service teachers were moved in a spreadsheet document and classified in terms of questions. Thus, each question of data tools (open-ended survey and reflection papers) had responses from ten pre-service teachers. The data were organized in a table with questions asked and responses to these questions with the rows under the questions. Before starting the analysis, all answers were read. Important parts were colored in the first reading, and a separate column was added to the analysis table to construct the initial open codes. Different colors were used to indicate different codes for each item. Initially, responses to the open-ended survey were analyzed. Then, responses to Reflection 1 and Reflection 2 were inductively analyzed to construct initial open codes. After reading several times of the data and constantly comparing the initial codes, similar codes were combined to form open codes. Then, all open codes were listed with relevant example quotes from participants in a separate document. After this, definitions of each code were made to inform the conceptualizations of the pre-service teachers on interdisciplinary teaching. Thus, analysis results of Cycle 1 were considered to take design decisions for the intervention (Cycle 2).

For Cycle 2, *open coding* (Corbin and Strauss, 1990, Vollstedt and Rezat, 2019) was used to analyze data. All *interviews* (pre-interview and post-interview) were initially transcribed and then read for constructing open codes. For data preparation, participants' responses to each question in the interviews were organized in a table. Before the analysis, all responses to the interviews were read. Interviews were conducted as focus group interviews, and analysis was organized with respect to focus groups. Each focus group interview was read and then analyzed separately. Initial open codes were written in a separate column in the analysis table next to a column including example quotes from participants. Different colors were used to represent the initial codes and then the refined codes. Initially, pre-interviews were inductively analyzed to form open codes. Then, similarly, post-interviews were inductively analyzed for open codes. In a similar manner, *surveys* (pre-survey and post-survey) were also initially inductively analyzed for open codes (Corbin and Strauss, 1990, Vollstedt and Rezat, 2019). Then, codes coming from interviews and surveys were combined to construct themes by categorizing similar codes under the same categories. Thus, to answer the *second research question* on conceptualizations, participants' responses to interviews and surveys were analyzed by *thematic analysis* (Braun and Clarke, 2021). Participants' responses to *anonymous feedback forms* were also used secondary data for investigating pre-service teachers' conceptualizations and were inductively analyzed for open codes.

To answer the *third research question* on designing interdisciplinary mathematical modeling tasks, interviews and participants' artifacts were used for analysis. Participants' artifacts were used as primary data sources to answer the *third research question*. Interviews were initially inductively analyzed through open coding (Corbin and Strauss, 1990, Vollstedt and Rezat, 2019) and then were thematically analyzed (Braun and Clarke, 2021). Pre-service teachers constructed rubric for an effective interdisciplinary mathematical modeling task in class discussion. The rubric (as an artifact) was used to understand pre-service teachers' learning experiences to design interdisciplinary tasks. It was inductively analyzed to form categories of criteria. Then, the rubric was revised by comparing the dimensions to the related literature. The revised rubric was used to analyze pre-service teachers' tasks as constructed for their final projects (artifacts). The finalized rubric served as a framework to deductively analyze the tasks. The tasks were analyzed based on the dimensions in the rubric. Four group tasks and one individual task were analyzed.

The tasks were analyzed by using the *Task Analysis Framework for Interdisciplinary Mathematical Modeling Tasks* (see Table 3.7). It was initially developed by the researcher and the advisor based on the relevant literature (Smith and Stein, 1998; Tekkumru-Kisa et al., 2015; Williams et al., 2016; Wess et al., 2021) and revised after the rubric construction in the class, in which pre-service teachers discussed the dimensions of effective interdisciplinary mathematical modeling tasks. Initially, there were three dimensions (Mathematical modeling, level of interdisciplinarity, and cognitive demand) in the framework. The revised framework has five dimensions: Dimension A (Level of Cognitive Demand), Dimension B (Mathematical Modeling), Dimension C (Organization), Dimension D (Context of the Problem), Dimension E (Expected Student Solutions) (see Table 3.7).

Table 3.7. Task analysis framework for interdisciplinary tasks.

Dimensions	Not Applicable	Acceptable	Satisfactory	Outstanding
A. Level of Cognitive Demand				
B. Mathematical Modeling				
C. Organization				
D. Context of the Problem				
E. Expected Student Solutions				

Dimension A refers to the level of cognitive demand for the tasks based on mathematics and other disciplines involved (e.g., science) (Smith and Stein, 1998; Tekkumru-Kisa et al., 2015). *Dimension B* refers to the characteristics of mathematical modeling tasks including open-endedness, reality, relevance, and authenticity (Wess et al., 2021). *Dimension C* refers to the organization of the task consisting of the structure of the task, content (e.g., objectives, grade level), implementation of the task (steps/procedure, tool use), and reporting. *Dimension D* refers to the context of the problem including the interdisciplinarity of the task. The levels of interdisciplinarity are based on the continuum of interdisciplinarity from monodisciplinary to interdisciplinary (Williams et al., 2016). The level of monodisciplinary refers to focusing on one discipline in the task. The multidisciplinary level refers to a higher level in which disciplines separately try to solve the problem without a common goal. The highest level (interdisciplinary level) refers to collaboratively solving a problem with a

common goal via a balanced contribution of involving disciplines. *Dimension E* refers to the expected student solutions to the problem. The range of rankings varies from *Not Applicable*, *Acceptable*, *Satisfactory*, to *Outstanding*, respectively.

In Cycle 2, interviews, artifacts, and researcher notes were used to answer the *fourth research question* on design principles. The data were analyzed by coding for design elements of a module for interdisciplinary teaching. Based on the findings on the implementation in Cycle 2, the design principles were identified. Since Cycle 2 was a part of the second phase of this research, the findings from this part contributed to the reflection on the evolution of design principles.

For Cycle 3, data preparation and data analysis were like Cycle 2 because Cycle 2 and Cycle 3 were included in the same research phase (second research phase). In Cycle 3, seven pre-service teachers attended pre-interviews at the beginning. However, since two of them dropped the course and another one did not attend the post-interviews, responses of four pre-service teachers who also attended the post-interviews were considered for the analysis. To answer the *fifth research question* on conceptualizations, pre-service teachers' responses to interviews and surveys were inductively analyzed. Initial open codes (Corbin and Strauss, 1990, Vollstedt and Rezat, 2019) were constructed, and then thematic analysis (Braun and Clarke, 2021) was used. To answer the *sixth research question* on designing interdisciplinary problems, pre-service teachers' responses to interviews and artifacts were analyzed. Artifacts consisted of the rubric which was constructed in the class and problems for the final projects. The rubric construction served as data to understand pre-service teachers' experiences to design interdisciplinary problems for teaching.

In Cycle 3, the rubric construction activity aimed to form the criteria for an effective Interdisciplinary Mathematical Problem for Teaching and participants constructed the criteria in class discussion. The researcher was just a facilitator of the whole-class discussion, which was done by participants. The participants constructed the criteria to guide their design of final projects. They determined key dimensions in writing Interdisciplinary Mathematical Problems for Teaching in this process. The researcher wrote participants' ideas on the computer and shared the screen with the class. The researcher also prompted the participants to add any different ideas by asking questions like "What else could you say

about this?'. Initially, the ideas were scattered, and the researcher asked the class how they could categorize them. Then, the participants grouped the ideas and constructed the main dimensions: interdisciplinarity, context, organization/planning/preparation, content, mathematical modeling, assessment/expectations, and teaching. These dimensions were not mutually exclusive, and participants had difficulties in classifying them. These dimensions were like those constructed in Cycle 2 except the dimension of *teaching*. *Teaching* was not stated as a separate dimension in Cycle 2. This might be related to the focus of the course on the pedagogical purposes of tasks.

Participants' ideas as criteria were considered indicators and contributed as the inductive analysis for the framework to be used in evaluating participants' problems. In addition to the class work (rubric construction), literature on interdisciplinarity, mathematical modeling, task design and teacher education were used to construct an analytical framework for Interdisciplinary Mathematical Problem for Teaching. There are three dimensions of this finalized framework: Interdisciplinarity, mathematical modeling, and implementation. The dimensions of the finalized framework were used to deductively analyze participants' problems.

In Cycle 3, interviews, artifacts, and researcher notes were used to answer the *seventh research question* on design principles. The data were analyzed by coding for design elements of a module for interdisciplinary teaching. Based on the findings on the implementation in Cycle 3, the design principles were identified. Since Cycle 3 was a part of the second phase of this research, the findings from this part contributed to the reflection on the evolution of design principles.

For the *third phase* of the study, no additional data were collected, but this phase was used for the reflection of the previous phases to finalize design principles for a module on interdisciplinary teaching. The second phase (*Design and Construction Phase*) aimed to design a module on interdisciplinary teaching for pre-service mathematics and secondary teachers. Initial design principles were identified based on the literature. Then, design principles were revised based on iterations. To reach the final design principles, data from the second phase were used for the analysis. Interviews, artifacts and researcher notes in

Cycle 2 and Cycle 3 were used as data and coded for design elements of a module for interdisciplinary teaching.

3.5. Trustworthiness

Validity and reliability are concepts that are used for evaluating research quality (Creswell, 2013; Bakker, 2019). Validity and reliability can take different names for qualitative studies. Lincoln and Guba (1985) suggested to use different names such as “credibility, transferability, dependability, and confirmability” instead of validity and reliability to evaluate the trustworthiness of a qualitative study (p. 300).

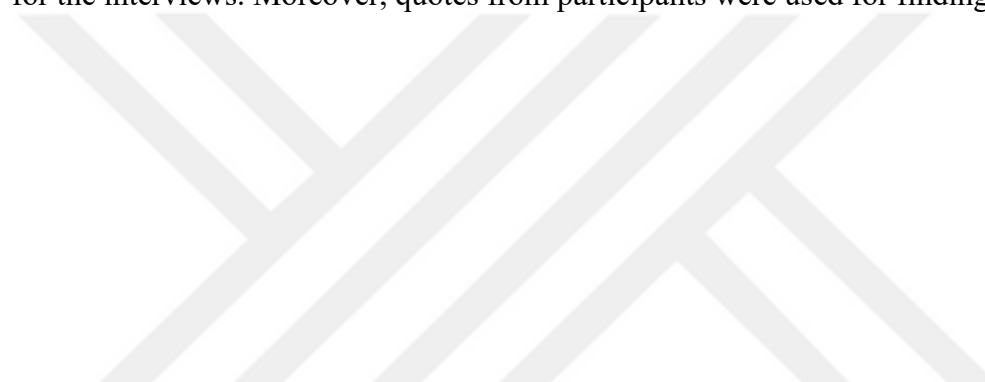
In qualitative study, *credibility* is considered an alternative to the term of internal validity that is used in quantitative studies (Lincoln and Guba, 1985). Lincoln and Guba (1985) suggested certain techniques for credibility: prolonged engagement, persistent observation, triangulation, peer debriefing, negative case analysis, referential adequacy, and member checking (p. 301). Prolonged engagement, persistent observation and triangulation were used to increase credibility in this study. *Prolonged engagement* refers to spending enough time with participants (Lincoln and Guba, 1985). In the context of the module implementation (the course on interdisciplinary mathematics and science education), the researcher and the advisor acted as co-instructors and the researcher attended all the classes. During the course, the researcher was an observer before the implementation started and then took a role as an instructor during the implementations of the first and second iterations of the module on interdisciplinary teaching. The researcher was in continuous interaction with participants throughout the implementation of the designed module such that trust was built. DBR studies also suggested using prolonged engagement with participants (Gravemeijer and Cobb, 2006). Thus, the prolonged engagement with participants was met in this study. Moreover, *persistent observation* refers to making “decisions about what is salient to study, relevant to the purpose of the study, and of interest for focus” (Creswell, 2013, p. 251). The researcher took field notes during the study and was involved in persistent observation to enhance credibility. *Triangulation* through sources refers to “corroborating evidence from different sources to shed light on a theme or perspective” (Creswell, 2013, p. 251). For triangulation, this study used multiple data sources such as interviews, surveys, and artifacts.

Transferability refers to analytic or theoretical generalization for qualitative studies (Lincoln and Guba, 1985; Bakker, 2019). In DBR, “*analytic or theoretical generalization*” was found to be appropriate (Bakker, 2019, p. 91). Certain features of the examined “learning ecology” could be transferred to other studies through theoretical generalization (Gravemeijer and Cobb, 2006, p. 43). In this study, design principles for the module on interdisciplinary teaching served for theoretical generalization. Thick description was suggested for enhancing transferability (Lincoln and Guba, 1985; Creswell, 2013). This study provided a thick description of the participants and research context, the design and the implementation of the design to facilitate transferability for other researchers.

Dependability means “virtual replicability, the ability to follow the whole research process so that the main ideas can be used in different situations with necessary adjustments” (Bakker, 2019, p. 93). In design studies, Bakker (2019) suggested “replication with variation” as feasible that could provide dependability by design ideas in different contexts (p. 93). Similarly, Gravemeijer and Cobb (2006) used the term of “ecological validity” as the purpose of design studies such that findings of the study would allow “for adaptation to other situations” (p. 45). Thick description was recommended for dependability (Gravemeijer and Cobb, 2006). In this study, the research design including how the design principles were derived, data collection, the design implementation process, and data analysis were described in detail. Using tapes for recording was also mentioned (Creswell, 2013). Thus, the study used audio recordings that could easily be transcribed. Interviews were transcribed from audio recordings.

Another criterion for trustworthiness is *confirmability*, which is associated with neutrality such that the findings of a study came from the participants without bias (Lincoln and Guba, 1985). Identifying researcher bias was critical for confirmability (Cresswell, 2013). The researcher’s positionality was considered for confirmability in this study. As a researcher and instructor, I attended the process of this study. In the role of an instructor, I presented the tasks and facilitated whole-class discussions. I have been a high school teacher in a state school since 2009. Although I have had teaching experiences with high school students for a long time, it was the first time that I instructed undergraduate students in a university. In the second iteration of the design, I was more familiar with teaching to undergraduate students. I adopted a situative perspective in learning such that learners need

different experiences for learning. I also believed in social constructivism such that social environment had a crucial role in constructing learning. Organizing a learning environment in this aspect was important for me. In the role of a researcher, I gathered data from the participants. I engaged in the whole research process. I initially gave information about the aim of the study to the participants and told them that the study would not influence their grades for the course. For confidentiality, I said that anonymous names would be used instead of their real names, and the raw data would not be shared. I emphasized that attending the study was voluntary and participants would drop out of the study whenever they wanted. For confirmability, the research process was audio recorded. I used verbatim transcription for the interviews. Moreover, quotes from participants were used for findings.



4. FINDINGS

The study aimed to design a *module on interdisciplinary teaching* to support pre-service mathematics and science teachers' development of skills for interdisciplinary teaching. Interdisciplinary teaching in this study referred to teaching mathematics with an interdisciplinary approach such that mathematics is integrated with other disciplines to solve a mathematical modeling problem (Boix Mansilla, 2010; Garfunkel and Montgomery, 2019). To develop a module on interdisciplinary teaching, this design-based research was conducted through cycles. This chapter is written to address findings for research questions written separately for each of the cycles. Cycle 1 was an exploration stage for setting the need for a module on interdisciplinary teaching for pre-service teachers. In addition to literature review, the conceptualizations of pre-service secondary mathematics teachers (in their third year of study) on interdisciplinary teaching were investigated in Cycle 1. Then, Cycle 2 was the first version of the module on interdisciplinary teaching administered as a first iteration. The module in Cycle 2 was called the *Professional Development Module* (PDM). Cycle 3 was the second version of the module on interdisciplinary teaching implemented as the second iteration. The module in Cycle 3 was called the *Program on Designing Interdisciplinary Mathematical Problems* (PDIMP). The module implementation in Cycle 3 was based on the revisions of that in Cycle 2 (based on the findings in Cycle 2). Finally, findings on the design principles for the module on interdisciplinary teaching will be presented. The findings that address the research questions will be presented based on cycles of research.

4.1. The Findings for Cycle 1

This section presents the findings for Cycle 1. Initially the findings on the conceptualizations of the pre-service secondary mathematics teachers on interdisciplinary teaching will be described. Then, the design decisions after Cycle 1 will be explained.

4.1.1. Findings on Participants' Conceptualizations in Cycle 1

The research question in Cycle 1 was investigated within the context of a course on mathematics teaching. In Cycle 1, the first research question was addressed to find pre-service secondary mathematics teachers' conceptualizations on interdisciplinary teaching:

- RQ 1. What are the *conceptualizations* of the pre-service secondary mathematics teachers on interdisciplinary teaching within the context of a mathematics teaching course during Spring 2022?

For Cycle 1, the core concept was interdisciplinarity as a theoretical focus. Table 4.1 shows the descriptions for the conceptualizations of pre-service mathematics teachers on interdisciplinarity in Cycle 1.

Table 4.1. Conceptualizations on interdisciplinarity (cycle 1).

Conceptualization	Description	Example Quote
Interdisciplinarity as an area	An area, a concept or a topic requiring different knowledge or comprehension from different disciplines	The term interdisciplinary involves two or more areas of learning.
Interdisciplinarity as connections	Connections between disciplines	I would define interdisciplinary as the relationship between two or more discipline[s] and how they are affecting each other.
Interdisciplinarity as a combination	Combination of two or more disciplines	Combination of two or more academic disciplines into one activity.
Interdisciplinarity as evolution of disciplines	An evolution of disciplines that are coming together	Also, I read that if discipline does not evolve then it has to disappear, so it needs interdisciplinary works to evolve.
Interdisciplinarity as working together	Working of different disciplines together on a common problem	To work together, to improve together, to help each other
Interdisciplinarity as problem solving	Solving a problem that requires more than one discipline	Interdisciplinary means that to solve a problem, there is [a] need of more than one disciplinary and these disciplinaries will turn into something from plural thing than to one.

Conceptualizations on interdisciplinarity refer to participants' perspectives of interdisciplinarity, namely how they consider the term of interdisciplinarity. In addition to these perspectives (see Table 4.1), some pre-service teachers had no idea about the meaning of interdisciplinarity. The perspectives of interdisciplinarity as an area and as connections were considered basic level of conceptualizations on interdisciplinarity indicating a simple description of relation between different disciplines. They are very broad descriptions that do not need deep conceptualizations of interdisciplinarity. Another perspective of interdisciplinarity as a combination implies a higher level of conceptualization since there is a kind of integration. Interdisciplinarity as evolution of disciplines indicates formation of a new entity such as a new discipline or new interpretation. Another perspective, interdisciplinarity as working together emphasizes collaborative work among people who make interdisciplinary research or interdisciplinary investigation. Finally, interdisciplinarity as problem solving focuses on solving a problem that is in the intersection of different disciplines. These perspectives emphasize different aspects of interdisciplinarity and can be considered as key indicators of interdisciplinarity in varying degrees.

As a sum, in Cycle 1, the pre-service teachers mostly noted general statements about interdisciplinarity at the beginning. Pre-service teachers' ideas about interdisciplinarity were not clear and the conceptualizations of interdisciplinarity were limited to the assigned articles for the reflection papers. In their reflection assignments, pre-service teachers did not state much individual comments but mostly summarized or rephrased the ideas given in the articles.

4.1.2. Design Decisions after Cycle 1

Certain *design decisions* were also taken based on Cycle 1. In one of the lessons, some pre-service teachers explicitly said that they couldn't understand why they covered interdisciplinarity in that course. Based on the class observations, they couldn't see the connection between interdisciplinarity and mathematics. For the design, this implied that pre-service teachers were not familiar with the concept of interdisciplinarity and did not even have much conceptual understanding for interdisciplinarity. In addition, considering the reflection assignments, it was found that reflections were not enough to understand pre-service teachers' conceptualizations of interdisciplinarity, and additional data collection

methods would be necessary. Thus, *interviews* were added as data collection tools to the next implementation. Moreover, since the implementation focused on the theoretical part and reflections were not enough to see the conceptualizations, it was decided that additional elements such as applications or *examples* of interdisciplinary mathematics education would be needed.

The course (SCED 352) was conducted online due to the COVID-19 pandemic, and the implementation was limited to being online. Based on class observations through Zoom sessions, pre-service teachers did not participate much in the class discussions unlike face-to-face lessons, and they had difficulties in mathematics content. There were individual and peer interaction through online assignments such as article reflections, peer feedback, and asynchronous discussions on the Canvas module, but it was found out that this online communication was limited. Thus, it was planned to include more *face-to-face communications* to the implementation.

Since this implementation was included in the required course, there was less time for conceptualization of interdisciplinarity. Thus, it was decided that a new course on interdisciplinary mathematics education might be better for a longer time implementation. A *new course*, focusing just on interdisciplinary mathematics teaching, was planned to be opened in the next semester for this purpose. The new course was initially named as *Course on Interdisciplinary Mathematics Education* (CIdME). For this course, *developing pre-service teachers' learning to teach interdisciplinary mathematics* was identified as an outcome. Based on situative perspective of teacher learning, the concept of learning to teach is dynamic and experiential. Engaging in multiple teaching experiences is the core of such approach to teacher learning. Thus, different teaching experiences were included in the design of the course to develop the process of learning to teach interdisciplinary mathematics. Since learning to teach interdisciplinary mathematics included the knowledge dimension, developing interdisciplinary mathematical pedagogical content knowledge of participants was also expected. The interdisciplinary pedagogical content knowledge (IPCK) was defined as a special kind of pedagogical content knowledge (PCK) based on the criteria of the framework of An (2017): “an understanding of the representation of concepts using themes across curriculum boundaries”; the ability to “apply pedagogical methods and interdisciplinary themed activities”; the ability to “identify knowledge connections within

and between particular subjects, and develop lessons based on such connections”; and enacting interdisciplinarity through instruction “wherein students link existing knowledge across curricula” (p. 239). Developing interdisciplinary pedagogical content knowledge refers to understanding what interdisciplinary mathematics education means from a theoretical perspective, knowing how to plan and teach interdisciplinary mathematics lessons, knowing possible student conceptions and difficulties related to interdisciplinary mathematics tasks. Thus, based on the literature and findings on Cycle 1, the design principles for a new course (CIdME) were identified as: (i) *Provide authentic tasks for learning to teach interdisciplinary mathematics* and (ii) *facilitate communication and collaboration in the classroom community*.

After Cycle 1, the purpose of the new planned course (CIdME) was to investigate how an undergraduate *Course on Interdisciplinary Mathematics Education* with interdisciplinary mathematical modeling tasks and a focus on communication and collaboration could support pre-service secondary mathematics teachers’ learning to teach interdisciplinary mathematics. The conjecture map that guided the design of the course was shown in Figure 4.1. In the conjecture map, the arrows between the embodiment and the mediating processes represent *design conjectures*, and those between the mediating processes and the outcomes represent *theoretical conjectures* according to Sandoval (2014). For instance, a design conjecture in the map is “If learners engage in *interdisciplinary mathematical modeling tasks in a small group*, then *analyzing interdisciplinary mathematics tasks* will emerge”. Besides, a theoretical conjecture in the map is “*If pre-service secondary mathematics teachers identify key features of interdisciplinary thinking, then they will develop the process of learning to teach interdisciplinary mathematics.*” These were conjectures because the statements were provisional that were hypothesized before the implementation of the design. They had the potential to change based on the findings for the implementation of the design. However, they served as a theoretical guide for the course initially planned after Cycle 1.

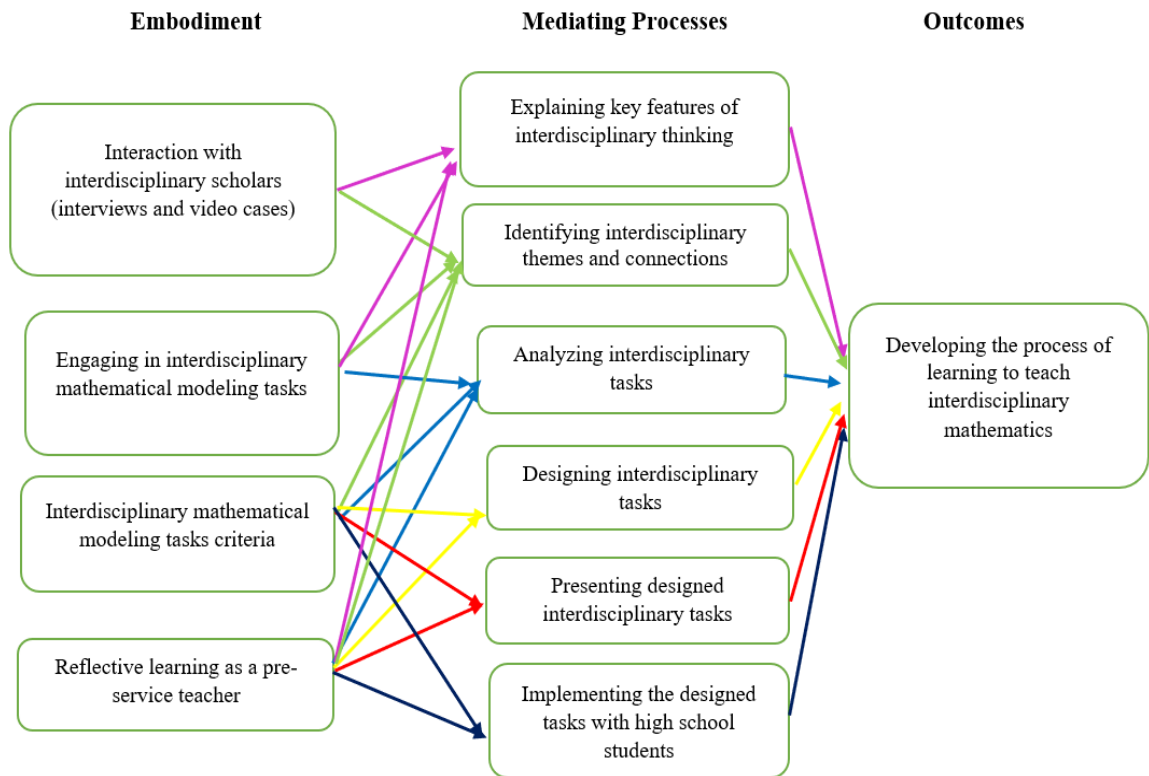


Figure 4.1. The conjecture map for course design.

The design of the Course on Interdisciplinary Mathematics Education (CIdME) for pre-service mathematics teachers was guided by the following design principles: (i) *Provide authentic tasks for learning to teach interdisciplinary mathematics* and (ii) *facilitate communication and collaboration in the classroom community*. Initially, the embodiment of the design principles was explained in terms of embodiment elements: Tools and materials, task structure and participant structure. In the design of the course after Cycle 1 (CIdME), the Moodle Learning Management System (LMS) (as part of *tools and materials*) was planned to be the main tool of *online courseware* to make the learning environment confidential for the course as other researchers (e.g., Parker et al., 2013) used for online authentic course design. The Moodle LMS was planned to include course materials such as weekly articles, announcements for the course, discussion forum, assignment submission interfaces with assignment instructions, and Zoom link for synchronous online communication. Therefore, the design principle on communication and collaboration (*Facilitate communication and collaboration in the classroom community*) was considered to reify in this tool since in addition to face-to-face communication, it was the basic online

communication tool between the instructor, researcher, and pre-service teachers. Moreover, Moodle was chosen because it has been regularly used by the university in which participants are studying. The online tool was an important medium for focusing on interdisciplinary content knowledge for pre-service teachers as the teacher learning research has also emphasized (e.g., Putnam and Borko, 2000; Fishman et al., 2014). Thus, Moodle was expected to provide experiences for understanding interdisciplinary mathematics education by setting the theoretical background via articles, discussion forums, and assignments.

Discussion (as a task structure) was one of the fundamental components of this course design. The weekly assigned articles were planned to be discussed by pre-service teachers in the course through first individual comments added to the discussion forum via Moodle, then through classroom community in course hours. In literature, discussion spaces or boards as online supports were used to construct communities of practice for teacher learning (e.g., Putnam and Borko, 2000; Herrington et al., 2010; Parker et al., 2013; Fishman et al., 2014). Discussion was involved in each envisioned lesson as an essential element to sustain communication. Discussion was a way to get active participation in the classroom community aligned with the literature on learning to teach (e.g., Borko et al., 2000; Buchbinder and McCrone, 2018).

Furthermore, *small group work (as a participant structure)* was important in the design of the course because authentic tasks required collaboration among participants (e.g., Herrington, 2006). Participants were expected to use group work to deal with interdisciplinary mathematics tasks implemented during the course hours as well as to perform other course tasks. Pre-service mathematics teachers were expected to collaborate with pre-service science teachers in small groups to complete their group work. In the course, by co-teaching, the instructor and/or researcher was planned to act as a facilitator of discussions and course tasks because participants' active participation in the tasks and via sharing ideas was expected. The instructor being a coach, or a mentor (providing routine, continuous, and individualized communication around practice) has also been effective in teacher learning as noted in literature (e.g., Fishman et al., 2014). Effective professional development (PD) opportunities for teacher learning included teachers' collaborative work (Borko et al., 2010). Pre-service teachers were planned to be grouped such that each group contains at least one mathematics and one science pre-service teacher. The rationale for such

grouping was based on the necessity of collaboration between mathematics and science teachers, especially on the contents of mathematics and science. Since pre-service mathematics teachers might not have had a strong content knowledge in science, the inclusion of pre-service science teachers aimed to complement the lack of content knowledge in science. According to the framework of *Interdisciplinary Pedagogical Content Knowledge* (IPCK) (An, 2017), the interdisciplinary content knowledge in mathematics and science requires having content knowledge in both mathematics and science. Thus, combining the content knowledge of pre-service mathematics and science teachers in the designed course in Cycle 2 was expected to set on the development of the interdisciplinary content knowledge in both disciplines. Then, this interdisciplinary content knowledge was expected to be the basis for the transition to develop the interdisciplinary pedagogical content knowledge of mathematics and science.

The design feature of *interaction with interdisciplinary scholars* involves interviews with interdisciplinary scholars and examining video cases of interdisciplinary research. This was considered a *task structure* to embody the second design principle (*Facilitate communication and collaboration in the classroom community*) because expert knowledge was important for pre-service teachers to understand the theoretical basis of a new area (interdisciplinary research). Learning from knowledgeable others was also prevalent in pre-service teacher learning (e.g., Fishman et al., 2014) aligned with the social aspect of learning or situated learning (e.g., Greeno and Nokes-Malach, 2016). Through the interviews with interdisciplinary scholars in the first phase of the course, interdisciplinary scholars, who made interdisciplinary studies, were expected to model how interdisciplinary studies were done as well as to exemplify interdisciplinary themes. Pre-service teachers were expected to identify key features of interdisciplinary thinking, and therefore, to understand interdisciplinary thinking and the process of interdisciplinary research. Thus, communication based on knowledge-sharing could be facilitated by this activity structure.

Besides, the *video cases* (as a *task structure*) are expected to reify the first design principle (*Provide authentic tasks for learning to teach interdisciplinary mathematics*). Research on video pedagogy in teacher learning indicated that videos were helpful in supporting teacher learning in various situations (Orland-Barak and Maskit, 2017). Videos were considered as a crucial tool for pre-service teacher learning such that videos created

observation experiences of teaching, and videos could help pre-service teachers to “react and construct knowledge collaboratively while and after observing video lessons” (Orland-Barak and Maskit, 2017, p. 59). As a crucial means for facilitating the development of noticing (Borko et al., 2008), videos could support continuous teacher reflection (Sherin, 2004). Moreover, videos could provide recurrent watching opportunities to examine lesson interactions from several perspectives, and therefore, develop novel approaches for teachers to notice what occurs in the classroom (Sherin and Russ, 2014). Thus, in the designed course, videos were initially planned as a means for pre-service teachers to create reflective opportunities for them on the meaning of interdisciplinarity and interdisciplinary thinking involving interdisciplinary research. Initially, pre-service teachers were planned to examine video cases of interdisciplinary research by individually reflecting on the key features of such tasks, and then, they were supposed to engage in a class discussion about their examined video cases. The interdisciplinary thinking component of the design was aimed through this design feature. This was part of the content knowledge on interdisciplinary thinking.

As another *task structure*, since *interdisciplinary mathematical modeling tasks* were authentic tasks that aimed to develop learning to teach interdisciplinary mathematics, this design feature was expected to reify the first design principle (*Provide authentic tasks for learning to teach interdisciplinary mathematics*). Moreover, pre-service teachers were planned to engage in interdisciplinary mathematical modeling tasks within small groups, the second design principle (*Facilitate communication and collaboration in the classroom community*) was also expected to be reified in this design feature. Pre-service teachers were expected to experience the tasks as learners within class.

Interdisciplinary mathematical modeling tasks criteria were considered another *task structure* to embody the design principle on authentic tasks. *Interdisciplinary mathematical modeling tasks criteria* referred to the combination of the criteria for interdisciplinarity and criteria for mathematical modeling. *Interdisciplinary mathematical modeling tasks* are expected to reify the first design principle (*Provide authentic tasks for learning to teach interdisciplinary mathematics*). For the course design, the interdisciplinary mathematical modeling tasks were defined as tasks that included mathematical modeling, and mathematics was integrated with science for interdisciplinarity. As Savard and Samson (2014) have also emphasized for interdisciplinary mathematics and science tasks, “complex interdisciplinary

learning situations” can prepare a basis “for tasks of high cognitive demand” (Savard and Samson, 2014, p. 147). Including mathematical modeling in the interdisciplinary tasks aimed at sustaining high cognitive demand of the tasks as a crucial element of the tasks in this design. In the categorization of levels of cognitive demand for tasks in mathematics, Stein and her colleagues (2009) considered tasks containing *connections* and *doing mathematics* as high cognitive demand, which helped to develop “a deeper understanding of mathematics by making connections or by exploring some concepts or processes” (Savard and Samson, 2014, p. 148). Pre-service teachers were expected to find and analyze interdisciplinary tasks including mathematics and science with mathematical modeling. Pre-service teachers were also planned to engage in interdisciplinary tasks as learners in the classroom. This was based on the literature on learning to teach and the associated situative perspective to learning (e.g., Peressini et al., 2004). Experiencing interdisciplinary tasks as learners was assumed to help pre-service teachers understand student thinking in interdisciplinary tasks, and therefore, to facilitate their interdisciplinary teaching. The instructor of the course and the researcher were initially planned to co-teach interdisciplinary tasks as a class application. This was expected to model teaching interdisciplinary tasks for pre-service teachers, therefore, to increase their experiences for learning to teach interdisciplinary tasks. Since teaching interdisciplinary tasks posed challenges, scholars suggested such exemplification of teaching in the classroom (e.g., Martins, 2012). Moreover, aligned with situative learning, pre-service teachers were expected to design their own interdisciplinary mathematical modeling tasks within groups as an opportunity for multiple contexts for learning to teach. Designing interdisciplinary tasks was a part of their final project assignments for the course. Pre-service teachers were expected to submit the final project part by part because the parts of the projects would be revised based on feedback, reflection, and discussion aligned with the process-oriented assessment (formative assessment) and the development-oriented nature of DBR. Furthermore, pre-service teachers were expected to find interdisciplinary themes and connections between mathematics and science to find an interdisciplinary problem as a driving question for their final projects. The revised interdisciplinary tasks of the pre-service teachers would be shared with their peers through presenting as posters and then taught to high school students. Not only knowing but also doing was important in teaching experience so that pre-service teachers needed to know both theory of interdisciplinary learning and practices of interdisciplinary teaching (Wu and Cai, 2022).

The criteria for interdisciplinarity (as a part of the *interdisciplinary mathematical modeling tasks criteria*) in the course design referred to the criteria in the framework for interdisciplinarity by Boix Mansilla and Duraising (2007). As a criterion for interdisciplinary mathematics tasks, cognitive advancement (Boix Mansilla et al., 2000) needed to occur. Boix Mansilla and Duraising (2007) defined a framework for interdisciplinary understanding in students' works based on three dimensions as criteria: Disciplinary grounding, advancement through integration, and critical understanding. These three criteria initially guided the design of the course (for Cycle 2) in assessing pre-service mathematics teachers' interdisciplinary tasks. The first criterion "disciplinary grounding" contains which disciplines are chosen ("*selection* of disciplines and insights") and how the chosen disciplines are used ("*appropriateness* in the use of knowledge and modes of thinking") (Boix Mansilla and Duraising, 2007, p. 225). Each group of pre-service teachers were expected to have a sufficient level of expertise in their disciplines. The second criterion "advancement through the integration" could be considered as the core of assessing interdisciplinarity and was defined as "identifying points of disciplinary integration and articulating the cognitive advantage enabled by the combination of perspectives" (Boix Mansilla and Duraising, 2007, p. 227). It is like the gestalt idea that the whole is greater than the parts when the advancement through integration is considered (Boix Mansilla and Duraising, 2007, p. 228). During the course, advancement through integration was expected in solving interdisciplinary mathematical modeling tasks by integrating mathematics and science. The third criterion "critical awareness" involves assessing "the degree to which a piece of work exhibits clarity of purpose" and searching for "evidence of reflective self-critique" (Boix Mansilla and Duraising, 2007, p. 228). In the context of mathematical modeling problem, students' articulation of the rationale for the selected disciplinary aspects for integration was planned to be assessed under this criterion. The third criterion involves meta-cognition about the interdisciplinary task and provides a bird's-eye view on the task. As a sum, the three criteria can be considered as the responses to the questions of *what* (What are integrated? Which disciplines/disciplinary ideas are integrated to make the task interdisciplinary?), *how* (How cognitive advancement is achieved in the task?), and *why* (Why is such interdisciplinary task created?), respectively. These three criteria of Boix Mansilla and Duraising (2007) are general criteria for assessing interdisciplinary student tasks in higher education but can be adapted for assessing pre-service mathematics teachers' interdisciplinary tasks. For the conjectured course, one of the integrated disciplines was

mathematics, and the guiding assessment questions given in the framework of Boix Mansilla and Duraising (2007) were used by considering mathematics as one of the disciplines chosen for the interdisciplinary tasks. Thus, for instance, for disciplinary grounding, it was considered whether the selected disciplinary ideas of science would be appropriate to make the task interdisciplinary as well as whether disciplinary ideas would be appropriately combined with mathematics.

The criteria for mathematical modeling (as a part of the *interdisciplinary mathematical modeling tasks criteria*) in the course design referred to the mathematical modeling task criteria defined by Wess and his colleagues (2021): “reality relation, relevance, authenticity, openness, promoting sub-competencies” (p. 14). The *reality relation* criterion requires a real-world connection, and the *relevance* criterion necessitates a close link to students’ personal lives (Wess et al, 2021). This implies the inclusion of another discipline different from mathematics, thus suggests indeed interdisciplinarity. The criterion of *relevance* suggests that the tasks should be engaging for students. The *authenticity* criterion refers to meaningfulness of the problem whereas the *openness* criterion requires the problem with various solutions and approaches, and *promoting sub-competencies* necessitates the problem definition to develop cognitive parts related with sub-dimensions of mathematical modeling competencies (Wess et al., 2021). Pre-service teachers in the designed course were expected to apply these criteria for appropriateness of their tasks with mathematical modeling.

Reflective learning as a pre-service teacher was considered a participant structure for embodying the design. Gasparič and Pečar (2016) have highlighted the importance of reflection in teaching such that teaching practice involves reflective practices on profession in addition to content knowledge. Reflection relates to active learning and continuous professional development (Ryan and Ryan, 2013). Particularly, academic or professional reflection, which is usually used in higher education, consists of a certain aim (Moon, 2006) and requires “evidence of learning and a growing professional knowledge” (Ryan and Ryan, 2013, p. 247). Reflection can be defined as “an active process of witnessing one’s own experience in order to take a closer look at it” (Amulya, 2004, p. 1). Amulya (2004) considered reflection as the base for purposeful learning and stated the aim of reflection as to create “the possibility of learning through experience” (p. 1). Thus, reflection is closely related with experiential learning. Experiences are also important in learning to teach

mathematics from a situative perspective, which focuses on generating multiple experiences pertaining to learning and teaching.

The second design principle (*Facilitate communication and collaboration in the classroom community*) was expected to be reified in the element of *reflective learning as a pre-service teacher*. Reflective practices encompassed practices such as regular feedback from the instructor, the researcher, and peers on course assignments, individual reflection on weekly articles, and reflection on designed tasks, or implementation of tasks with high school students. Feedback was planned to be a part of assessing the assignment of finding and examining interdisciplinary mathematics and science tasks, the task of finding an interdisciplinary problem, interdisciplinary task application within the class, the developed interdisciplinary mathematics task, and presenting the developed tasks. Draft versions of the reports and tasks were planned to be revised after feedback. Pre-service teachers were expected to state strengths and necessary areas for improvement in their peers' assignments while giving feedback. Feedback is especially useful when students can have a chance to improve by revising their work of a project (Bransford et al., 2000). Peer feedback, instructor's "highly targeted feedback" (Fishman et al., 2014, p. 714), and self-assessment (Bransford et al., 2000) all support pre-service teachers' learning. Moreover, regular reflections also play an important role in learning to teach. For instance, Capobianco et al. (2022) used regular reflections to create experiences for pre-service teachers "to analyze their own learning (e.g., reading reflections) and teaching (e.g., teaching reflections)" (p. 751). Moreover, the first design principle (*Provide authentic tasks for learning to teach interdisciplinary mathematics*) was expected to be reified in the *reflective learning as a pre-service teacher* because authentic tasks would require participants to reflect on their learning and reflection was integrated within the authenticity of the tasks. Opportunities for feedback and reflection could help develop teachers' learning (e.g., Putnam and Borko, 2000).

Another component of a conjecture map was the mediating process. *Mediating processes* in the design of the course for Cycle 2 were planned by considering the literature on interdisciplinary education and teacher education. Practice-based pre-service teacher education (e.g., Grossman, Compton, et al., 2009; Grossman, Hammerness, and McDonald, 2009) has indicated that it is essential to provide pre-service teachers "opportunities to engage in both pedagogies of investigation and enactment" which centered on intended

“instructional practices (e.g., eliciting and building on student thinking to accomplish an instructional agenda)” (Cobb et al., 2017, p. 220). “Pedagogies of investigation” include “analyzing and critiquing representations of practice” (Cobb et al., 2017, p. 220). Representations of teaching practice may include student work and video cases of teaching (e.g., Kazemi and Franke, 2004; Sherin and Han, 2004; Borko et al., 2008). In the design of the course for Cycle 2, the mediating processes of explaining key features of interdisciplinary thinking, identifying interdisciplinary themes and connections, and analyzing interdisciplinary tasks were considered as pedagogies of investigation. “Pedagogies of enactment” contain dimensions of “planning for, rehearsing, and enacting” for a sequential progression in “complex settings (e.g., teaching other teachers who play the role of students, working with a small group of students, teaching an entire class)” (Cobb et al., 2017, p. 220). In the current design, designing interdisciplinary modeling tasks, presenting the designed tasks to peers, and implementing the designed tasks with high school students were considered as pedagogies of enactment.

Explaining key features of interdisciplinary thinking was identified as a key mediating process in the course design. Interaction with interdisciplinary scholars (interviews and video cases), engaging in interdisciplinary mathematical modeling tasks as learners, and reflective learning as a pre-service teacher were expected to support the mediating process of *explaining key features of interdisciplinary thinking*. Interdisciplinary research implies that there are certain characteristics of interdisciplinary thinking.

Interdisciplinary thinking is a complex construct, which includes complex sub-processes as elements (Nikitina, 2005). Based on an empirical study, Nikitina (2005) asserted that there are three core cognitive processes in interdisciplinary thinking: “overcoming monodisciplinarity, provisional integration, and questioning integration” (p. 2). Overcoming monodisciplinarity is the realization that one discipline is not enough for the problem at hand and Nikitina (2005) noted three steps of overcoming monodisciplinarity containing “1) appreciation of alternative disciplinary views; 2) identifying strengths and weaknesses inherent in one’s disciplinary position; and 3) making a decision to accept or reject a given disciplinary tool or method as useful or relevant” (Nikitina, 2005, p. 7). Thus, for overcoming monodisciplinarity, pre-service teachers needed to understand that certain problems (namely, interdisciplinary problems) required using different disciplines and one

discipline was not enough for the solution. As a key characteristic of interdisciplinary thinking, integration is seen as provisional such that integration is “open to other disciplinary inputs and in need of refinement” (Nikitina, 2005, p. 11). The provisional integration and questioning integration indicate the dynamic nature of integration. The cognitive process of provisional integration, which is a key part of interdisciplinary thinking, accepts that integration is incomplete because there is a continual quest for “a better fit of ideas” (Nikitina, 2005, p. 15). By questioning integration, the integration is aimed to be developed through extending the connections between disciplinary aspects, and finding additional disciplines as needed (Nikitina, 2005). Understanding how interdisciplinary researchers have used interdisciplinary research was considered important for pre-service teachers to make sense of the processes of interdisciplinary thinking in the current design. Course materials such as articles on interdisciplinary research, interviews with interdisciplinary scholars, or video cases of interdisciplinary research aimed to help pre-service teachers understand the interdisciplinary process, and therefore interdisciplinary thinking that was involved in. Identifying key features of interdisciplinary thinking was considered as a theoretical basis for interdisciplinary learning and teaching.

Identifying interdisciplinary themes and connections was specified as another key mediating process in the current course design. Interaction with interdisciplinary scholars (interviews and video cases), engaging in interdisciplinary mathematical modeling tasks as learners, and reflective learning as a pre-service teacher were expected to facilitate the mediating process of *identifying interdisciplinary themes and connections*. Identifying interdisciplinary themes and connections requires pre-service teachers to find big ideas in each discipline that are identified in a common theme across different curricula. Thus, participants would need to determine relevant mathematical big ideas that could be combined with big ideas in science. Initially, pre-service teachers were expected to understand the theoretical background of interdisciplinary mathematics education. The resources on the online courseware formed the content of the course and together with the expert knowledge (via interviews with interdisciplinary scholars or video cases). Moreover, shared knowledge that was expected to emerge during classroom and online discussion, collaboration through small group work were supposed to support pre-service teachers in identifying interdisciplinary themes and connections.

Analyzing interdisciplinary mathematics tasks was identified as another mediating process in the course design. Engaging in interdisciplinary mathematical modeling tasks as learners, interdisciplinary mathematical modeling tasks criteria, and reflective learning as a pre-service teacher were expected to facilitate the mediating process of *analyzing interdisciplinary mathematics tasks*. Pre-service teachers would experience the interdisciplinary mathematics tasks during the course via thinking like a student in these tasks. Pre-service teachers' active participation as learners in the tasks in small groups during the course was important because this could facilitate their understanding of student learning related to the tasks and how collaborative work in small groups (two or three people) could be helpful to complete the complex interdisciplinary tasks. Pre-service teachers were expected to analyze the interdisciplinary tasks based on the interdisciplinary mathematical modeling tasks criteria integrating the level of cognitive demand (Stein et al., 2009), the interdisciplinary assessment criteria (e.g., Boix Mansilla and Duraising, 2007), and mathematical modeling task criteria (Wess et al., 2021). Moreover, to analyze interdisciplinary mathematics tasks, participants needed to identify which disciplinary ideas in science were integrated with mathematics, to find out the interdisciplinary theme (common problem across disciplines, the driving question for the task), the big ideas, and learning goals within mathematics and science, to determine instructional strategies used in the applied tasks and reflect on how the tasks and their implementation could be improved. Pre-service teachers were also expected to write a group report for their analysis of interdisciplinary tasks.

Another mediating process that was identified in the course design was *designing interdisciplinary tasks*. Interdisciplinary mathematical modeling tasks criteria, and reflective learning as a pre-service teacher were expected to facilitate the mediating process of *designing interdisciplinary tasks*. The interdisciplinary task to be developed by the participants was expected to include interdisciplinary themes, the general aim of the lesson, learning goals and big ideas in combined disciplines (namely, in science and mathematics), explanation of learning activities and assessment. The interdisciplinary assessment criteria of Boix Mansilla and Duraising (2007) were initially regarded as the basis for assessing interdisciplinarity in pre-service teachers' tasks. Moreover, the interdisciplinary mathematical modeling tasks criteria synthesized the level of cognitive demand (Stein et al.,

2009), the interdisciplinary assessment criteria (e.g., Boix Mansilla and Duraising, 2007), and mathematical modeling task criteria (Wess et al., 2021).

Presenting the designed interdisciplinary tasks was another specified mediating process in the course design. Interdisciplinary mathematical modeling tasks, and reflective learning as a pre-service teacher were expected to facilitate the mediating process of *presenting the designed interdisciplinary tasks to peers*. Presenting the designed interdisciplinary tasks to peers as posters was a way to construct shared knowledge in classroom and a preparation of pre-service teachers for the real classroom implementation. Conducting lessons is the practical side of having interdisciplinary pedagogical content knowledge (An, 2017). How knowledge is enacted or applied in practice is important to observe the teacher's knowledge in action. As Putnam and Borko (2000) also noted, making practice is essential for pre-service teachers to “act as a teacher” (p.10).

Implementing the designed tasks with high school students was another mediating process that was specified in the course design. Interdisciplinary mathematical modeling tasks criteria, and reflective learning as a pre-service teacher were expected to facilitate the mediating process of *implementing the designed tasks with high school students*. Aligning with situative perspectives to teacher learning, providing multiple contexts for pre-service teachers to learn to teach interdisciplinary mathematics is important. Applying the designed tasks with high school students was considered as one of such teaching experiences for pre-service teachers as classroom implementation was suggested for developing professional knowledge of teachers (e.g., Putnam and Borko, 2000). The ability to perform developed interdisciplinary lessons was conceptualized under the teaching dimension of interdisciplinary pedagogical content knowledge like Greefrath et al. (2022) conceptualized in their framework for modeling-specific PCK for pre-service secondary teachers.

As Ball and her colleagues (2008) emphasized, “being able to use the mathematics required inside the work of teaching” is as crucial as knowing the content of mathematics (p. 404). Based on Shulman's (1986) model of teachers' subject matter knowledge, Ball and her colleagues (2008) developed a new model specific to mathematics, that is Mathematical Knowledge for Teaching (MKT). MKT is practice-based and defined as “the mathematical knowledge needed to carry out the work of teaching mathematics” (Ball et al., 2008, p. 395).

MKT suggests that how tasks are chosen and implemented in classroom are important for understanding teachers' mathematical knowledge for teaching. Thus, it was necessary to see how pre-service mathematics teachers in the current design would apply their designed interdisciplinary mathematics tasks with high school students. The atmosphere of the actual classroom could differ that of the university classroom, and since pre-service teachers would be future teachers soon, it was important for them to experience the dynamics of a real classroom in order to avoid practice-shock, which indicated the challenges of novice teachers in early years of teaching in the classroom and the lack of transfer of the university teacher education into the classroom practices (Korthagen, 2010). Concerning the situative nature of teacher learning, *recontextualization*, which is defined as “what is taken up during a teacher education course and enacted during one's classroom practice”, is important in studying how learned practices in university programs of teaching are recontextualized (Gomez Marchant et al., 2021, p. 673). Thus, the recontextualization of interdisciplinary teaching into the high school classroom was included in the initial design of the course for learning to teach. Overall, high school implementation of the designed tasks was hypothesized to develop pre-service mathematics teachers' process of learning to teach interdisciplinary mathematics.

As a sum, certain design decisions were taken based on the findings on Cycle 1 and literature. In Cycle 1, the conceptualizations of the pre-service teachers on interdisciplinary education were found to be limited. Considering this and the literature on interdisciplinary education, a new course on interdisciplinary education (the Course on Interdisciplinary Mathematics Education) for pre-service teachers was planned. The design of the new course (CidME) was guided by the following design principles: *Provide authentic tasks for learning to teach interdisciplinary mathematics and facilitate communication and collaboration in the classroom community.*

4.2. The Findings for Cycle 2

This section presents the findings for Cycle 2. Cycle 2 was a part of the Design and Construction Phase in which the first iteration of the implementation was developed. Initially, findings on conceptualizations of the pre-service teachers on interdisciplinarity, interdisciplinary education and mathematical modeling will be described. Then, findings on

designing interdisciplinary tasks and design principles of a module on interdisciplinary teaching will be presented. The following research questions were investigated in Cycle 2:

- RQ 2. How did the *conceptualizations* of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Professional Development Module (PDM) within the course of Interdisciplinary Mathematics and Science Education during Fall 2023?
- RQ 3. How did the pre-service mathematics and science teachers who participated in the Professional Development Module (PDM) design *interdisciplinary mathematical modeling tasks* (IMMTs)?
- RQ 4. What are the *design principles* of a module on interdisciplinary teaching?

4.2.1. Findings on Conceptualizations in Cycle 2

In Cycle 2, it was investigated how the pre-service mathematics and science teachers' conceptualizations on interdisciplinary teaching changed during the implementation of the module on interdisciplinary teaching (The Professional Development Module). To investigate the *second research question*, findings on the conceptualizations based on *interviews* will be presented initially. Based on the *interviews*, conceptualizations of pre-service teachers on characteristics of interdisciplinary mathematical modeling tasks were presented in Table 4.2.

Table 4.2. Conceptualizations on characteristics of interdisciplinary tasks.

Themes	Codes (Pre-interview)	Codes (Post-interview)
Context	Real-life connection (P8)	Real-life problems (P3, P5, P7, P8, P9)
Interdisciplinarity	Product (P5)	Integration (P3, P4, P7)
	Multiple perspectives (P3)	Multiple perspectives (P5, P6)
Modeling	Open-ended (P4, P7, P9)	Open-ended (P5, P6, P7, P9)
Modeling cycle		
	Points of math modeling (P4)	Assumptions (P9)
		Variables (P3, P8)
		Results (P1, P3, P4, P5, P8)
	Data (P1, P7, P4)	Data (P3, P4, P7, P9)
Organization	Meaningfulness (P5, P6)	Understandable (P1, P5)
	Inquiry (P3, P4)	Researchable (P4)
		Solvable (P4)
		Reporting (P9)
		Scaffolding (P9)
		Rubric (P5)
		Applying concepts or methods (P1)
		Tools (P3, P4, P7)
Level	Cognitive level (P7)	Cognitive demand (P1, P7, P8, P9)
		Student level (P1, P7, P9)

These conceptualizations were grouped under three themes as *context*, *modeling*, and *organization*. *Context* refers to the context of the problem containing a reference to *real-life* and *interdisciplinarity*. In the pre-interview, *real-life connection* was mentioned by just one participant while *real-life problems* were emphasized by five participants in the post-interview. *Interdisciplinarity* refers to *integration* of disciplines with *multiple perspectives* in the post-interview whereas it refers to a *product* addressing all involved disciplines with *multiple perspectives* in the pre-interview. Besides, *modeling* refers to a mathematical *modeling cycle* with *open-ended* problems that lead to multiple solutions. Implicit mathematics (situations in which mathematics is not seen at first sight but have underlying mathematical modeling) was mentioned by one participant (P8) in the pre-interview compared to the prevalent mention of appropriateness for mathematical modeling in the post-interview (P1, P3, P4, P7, P8). In the pre-interview, modeling cycle was limited to the generic mention of *data* (deducing inferences by combining given data) and *points of mathematical modeling*. In the post-interview, modeling cycle referred to a systematic combination of identifying *assumptions*, *variables*, *results* and *data* (allowing data collection or use of data, finding the best model fitting the data). *Results* referred to assessable results having students make inferences. As another theme, *organization* refers to the structure of

tasks about implementation including *levels* of tasks. *Meaningfulness* refers to an *understandable* problem with *meaningful models* or *graphs*. *Inquiry* (researchable in the post-interview) refers to tasks prompting students to make inquiries. New codes such as *scaffolding* were added in the post-interview. Besides, concerning *level*, *cognitive level* was mentioned in the pre-interview while high *cognitive demand* (challenging tasks) and *student level* were emphasized in the post-interview.

Based on interviews, for *initial conceptualizations*, the participants (P1, P8, P9) mostly referred to class examples. For instance, the pre-service teacher P9 focused on the process of mathematical modeling by exemplifying a task applied in the class (chirping-temperature task):

Like in there, data exist at first, how to form a meaningful question from this [given data], students themselves deciding what to get there [from the given data], namely firstly making interpretations, then deciding on which variables to be assessed based on data, then translating the relationship to a graph, then tried to infer a meaningful result from this graph, made an analysis. (Pre-interview, 4th question, P9)

Based on interviews, the *post-conceptualizations* were dominantly on mathematical modeling and organization. For instance, all pre-service teachers described the *process* of mathematical modeling (P1, P3, P4, P5, P6, P7, P8, P9) while trying to explain the characteristics of interdisciplinary mathematical modeling tasks:

Not giving all data to students at once and then requiring them to do operations, finding x and/or y but bringing something to the table that allows children to produce ideas, find their own variables, construct their own models, solve their constructed models, and then come to the beginning and comment/evaluate how to adapt the solution to the model or problem (Post-interview, 5th question, P8)

Moreover, the pre-service teacher P3 also emphasized that to do mathematical modeling, it is necessary to *know* or investigate the *areas* affecting the problem well. P3 exemplified their problem for the final project such that the problem involved different concepts such as work, power, energy. Thus, P3 indeed focused on sufficient content knowledge for different disciplines, and if the content knowledge was not enough, investigation or research was suggested to deal with this situation.

In summary, the findings on *interviews* indicated that the conceptualizations of pre-service teachers on interdisciplinary teaching were gathered under three themes of *context*,

modeling and *organization*. For initial conceptualizations, there were less codes in all themes implying that the initial conceptualizations of the pre-service teachers were limited. The findings on post-conceptualizations based on interviews showed that there were more codes in the themes of modeling and organization after the module implementation compared to the initial case. Although there were not changes in the number of codes for the theme of context, there were also some changes in some codes about the theme of context as revealed in the post-interviews.

For Cycle 2, *surveys* were also used to get information about pre-service teachers' conceptualizations on interdisciplinarity, interdisciplinary education, mathematical modeling and interdisciplinary mathematical modeling tasks. There were questions on interdisciplinarity, mathematical modeling, and interdisciplinary mathematical modeling tasks.

Concerning *interdisciplinarity*, one question on the pre-survey was: "How would you define interdisciplinary research? State and explain four drivers of interdisciplinary research.". The four *drivers of interdisciplinary research* could be the complexity of nature and society, the drive to explore, the need to solve societal problems, and the stimulus of generative technologies (National Academy of Sciences, National Academy of Engineering, and Institute of Medicine [NAS], 2005). Only one of the pre-service teachers (P7) could write all four drivers of interdisciplinary research ("inherent nature of science, solving societal problems, generative technologies, solving basic problems related to interfaces fields"), and other pre-service teachers (P1, P2, P3, P4, P5, P6, P9) did not write any answers to the drivers of interdisciplinary research. According to the NAS (2005), *interdisciplinary research* is defined as:

a mode of research by teams or individuals that integrates information, data, techniques, tools, perspectives, concepts, and/or theories from two or more disciplines or bodies of specialized knowledge to advance fundamental understanding or to solve problems whose solutions are beyond the scope of a single discipline or field of research practice. (NAS, 2005, p. 26)

Pre-service teachers' conceptualizations on interdisciplinarity based on pre-surveys was described with example quotes. For instance, the pre-service teacher P1 defined the interdisciplinary research like the definition given by NAS (2005):

Interdisciplinary means that more than one area come together to find a solution for a problem. I want to say that there is a problem, and some scientific area [which] share[s] and integrate[s] their ideas, methods and knowledge to ask [a] question: How [is the] problem related to other area and how [can] we find a solution by integrating each area's method and ideas? (Pre-survey, P1)

The pre-service teacher P2 did not make a direct definition of the interdisciplinary research, but gave some features connected to it in a more general way: A person who work[s] for interdisciplinary [research] should know at least two areas for focusing on the area. The pre-service teacher P2 also mentioned the aims of interdisciplinary research as “to make it easier, to define it better ways, while our area is not sufficient, we try to use different areas”. Besides, the pre-service teacher P3 described the process of the interdisciplinary research instead of giving a direct definition:

When a researcher handle[s] a problem, one must evaluate this inquiry for every aspect. A problem should be investigated from different disciplines since it is related to it. Therefore, a researcher first need[s] to split a problem into different cases.... a problem might be an interest area of a discipline or maybe many of disciplines at the same time. Thus, a researcher must identify each discipline separately at first. Then, a researcher should find a relation between disciplines that one can find and make connections between them. In [the] third case, one must collect data about what one researched about the disciplines which are in interest area of the problem. After that, one could find the best possible way considering all disciplines together and finish the interdisciplinary research. (Pre-survey, P3)

The pre-service teacher P4 wrote the main role of interdisciplinary research as connection and explanation without giving a direct definition: Interdisciplinary research basically connect[s] disciplines and want[s] to explain [a] subject deeply. The pre-service teacher P5 gave a similar definition to NAS (2005):

Interdisciplinary research is the collaboration of individuals or teams to solve problems whose solutions are the scope of more than one discipline by using data, knowledge, tools, perspectives. At the end of the research, a common solution on a common product is handled. (Pre-survey, P3)

Besides, the pre-service teacher P6 focused on solving problems in describing interdisciplinary research: “Interdisciplinary research means being [in] relationship [of] more [than one] discipline[s]. As integrating disciplines, [a] problem or subject which is related [to] one more discipline can be solved and worked”. *Interdisciplinary research* was defined by the pre-service teacher P7 as “It is integration of more than one discipline to solve a problem with the help of theories, methods, tools from each discipline”. This definition is parallel with the definition of NAS (2005). Furthermore, the pre-service teacher P9 gave a very broad definition of interdisciplinary research: Interdisciplinary research is the research that hybridize two or more areas for. In short, pre-surveys indicated that some pre-service

teachers (e.g., P2, P3, P4, P6) initially did not define interdisciplinary research or mentioned it in general statements, but others gave certain definitions focusing on different aspects of interdisciplinary research (e.g., P1, P5, P7).

Another question of the pre-survey required to explain *mathematical modeling* together with the process steps, and to state the role of mathematical modeling in interdisciplinary tasks: “a) Why does a science/math teacher who wants to incorporate interdisciplinary education need to know mathematical modeling? b) What could be the role of mathematical modeling in interdisciplinary tasks?”. The pre-service teacher P1 described mathematical modeling as in the case that “we can formulate a problem and convert [it] to [a] mathematical problem”. P1 also emphasized the close connection of mathematics and physics: “...Physics mostly include[s] mathematic[s] and we generally solve math and equation. The content of physics or other area[s] can be back side. ...Math is in every field”. The pre-service teacher P2 emphasized the role of mathematical modeling in interdisciplinary tasks “to show the results in better way, to make easy, and imagine in our mind.”. Moreover, P2 wrote the aim of incorporating mathematical modeling: “...to use a model will be better understand[ing] for other area students”. The pre-service teacher P3 explained mathematical modeling and its process steps detailly. For instance, P3 wrote: “It [Mathematical modeling] is about identifying/determining variables and making connections between them, we try to formulize what the question wants and asks for us. For example, one could define a function on two variables (like temperature x chirping)”. For the process of mathematical modeling, P3 also emphasized the necessity of other disciplines and for the role of mathematical modeling in interdisciplinarity, identifying variables was emphasized:

Mathematical modeling requires lots of disciplines that are related to the problem...most of the disciplines are directly related to each other (science and math and zoology in our example). To evaluate the problem, one must take these variables and the disciplines that interest these topics into consideration to understand the problem properly. In interdisciplinary tasks, we need to identify variables and try to find which disciplinary area interests these variables. In mathematical modeling...identifying variables and making connections between them is significant. Moreover, after discussion about variables or disciplines we're working on in interdisciplinary tasks, we need to interpret variables mathematical modeling would be an important role for this context. (Pre-survey, P3)

Besides, the pre-service teacher P4 gave a broad answer on the mathematical modeling process and its role in interdisciplinarity: Because mathematical modeling help[s] student[s] how to solve all problems briefly. It helps to imagine the problem. Interdisciplinary areas

want to solve problems, explain things and produce new things. Mathematical modeling help[s] this area because this model [is] so useful [to] solve problems. Furthermore, the pre-service teacher P5 defined mathematical modeling as “the process of transformation of a real-world problem into mathematics to make the problem visual and more understandable and to solve it”. Moreover, concerning mathematical modeling, P5 emphasized the visibility and accessibility of the results of mathematical models in interdisciplinary tasks:

[U]sing mathematical modeling techniques can help the teachers to show the topic more visual and some graphs, or some results which can be interpreted with real life problems will make the topic more understandable, meaningful and easier for students. Thanks to results which is handled by using mathematical modeling, the topic will be covered successfully. In an interdisciplinary task, there are different disciplines and all of them try to get a common result...In this situation, mathematization [of] such results and put[ting] them together in a mathematical model makes the results more visible and easier to interpret. Also, thanks to such interpretations, evaluation of the results and connection them with the main problem will be easier. (Pre-survey, P5)

Concerning mathematical modeling, the pre-service teacher P6 focused students' enhanced understanding due to mathematical modeling: “When real life and mathematical world connect, students understand subjects deeply. Students make connections between them, then understanding mathematics can be more sensible for students. So, students gain problem solving skills and understand cause-effect relationships”. Moreover, P6 emphasized the role of mathematical modeling in interdisciplinary tasks as facilitating solving problems:

The role of mathematical modeling in interdisciplinary area[s] is evaluating problems according to disciplines and deciding to which tools are used in modeling to solve [the] problem. When we don't know disciplines related to the problem, we can't mathematize the problem. (Pre-survey, P6)

Concerning mathematical modeling, the pre-service teacher P7 emphasized the variety of methods and procedures in mathematical modeling: “There is not one way to solve a mathematical modeling question. That's why students can improve their mathematical thinking and analyzing skills”. Furthermore, P7 added on mathematical modeling: “In mathematical modeling, ...they [students] also gather data and analyze their findings. For gathering data and interpreting it (using softwares etc.), they need other disciplines”. Besides, the pre-service teacher P9 focused on the need to understand the real-world through mathematical modeling and to motivate students to do science or mathematics: “We live in a real world that means our student needs a motivation for doing science/math... We need to teach them [students] also how to investigate real world situation by using science/math. This is why mathematical modeling is important”.

In short, the findings on the pre-survey indicated that pre-service teachers (e.g., P1, P2, P4) usually gave broad descriptions on mathematical modeling, especially on the relationship of mathematical modeling and interdisciplinarity. Some pre-service teachers focused on the mathematical modeling process in general (e.g., P3, P5, P7). Others focused on the connection with real-world and solving problems (e.g., P6 and P9).

In the pre-survey, concerning the design of an interdisciplinary problem (“Write an example of interdisciplinary mathematical modeling task”), one pre-service teacher (P9) gave no answer. Two pre-service teachers (P3 and P4) wrote a task with an interdisciplinary theme but without mathematical modeling. P1 tried to give an interdisciplinary example about photosynthesis and temperature but the problem was not clear enough and needed revision to fit mathematical modeling. P2 also focused on interdisciplinarity but not mathematical modeling. There was not a clear statement of problem, but only a potential for an interdisciplinary topic (“frequency of notes in logarithmic graph”): “That is a model for music and math, physics relationships”. P5 wrote general statements about making a cake for the topic of ratio and proportions, which could be regarded as a routine word problem. Thus, it was not a mathematical modeling problem. P6 wrote a word problem about finding frequencies in a pendulum. Only P7 wrote a problem that mostly aligned with an appropriate interdisciplinary mathematical modeling problem. After giving a table of energy consumptions of different washing machines, the problem asked for “the most efficient machine”.

In short, the findings on the pre-survey indicated that pre-service teachers could not write an example of interdisciplinary mathematical modeling task. One pre-service teacher (e.g., P9) did not even write anything about such asked example. Others (e.g., P3 and P4) tried to write some problems but without any mathematical modeling. Besides, some pre-service teachers could not write a problem but just some topics instead (e.g., P2). Moreover, some pre-service teachers (e.g., P5) wrote problems that were routine word problems.

For Cycle 2, *post-surveys* were also used to get information about pre-service teachers’ post-conceptualizations on interdisciplinarity, interdisciplinary education, mathematical modeling and interdisciplinary mathematical modeling tasks. There were questions on

interdisciplinarity, mathematical modeling, and interdisciplinary mathematical modeling tasks in the post-survey.

Concerning *interdisciplinarity*, one question in the post-survey was about interdisciplinarity and asked for defining interdisciplinarity: How would you define interdisciplinarity/interdisciplinary research? Give at least one example to interdisciplinarity/interdisciplinary research and explain how this example fits with interdisciplinarity. The post-conceptualizations of pre-service teachers on interdisciplinarity based on post-surveys will be described in the following paragraphs.

The pre-service teachers P8 and P9 did not fill out the post-survey. Although P9 attended most of the lessons, P8 did not attend most of them. The pre-service teacher P7 focused on a *common theory, research or product* to solve a problem in describing interdisciplinarity:

Interdisciplinarity can be defined as the *work of more than one field on the same question/problem in an integrated manner. Expertise in one area* may not be sufficient to solve a problem. In these cases, different disciplines come up with a *common theory, research or product to solve a problem*. For example, both engineering and business fields work together in research on *supply chain management*. While doing research in this field, some metrics and data are represented with the help of mathematics and engineering, and the business deals with the interpretation of these metrics. (Post-survey, P7, Italics added)

The pre-service teachers P1 and P7 both give the same examples of interdisciplinary research from their course assignments. For defining interdisciplinarity, the pre-service teacher P1 highlighted the work of two or more scientific areas to solve a problem but did not mention a common product counter to the answer of the pre-service P7:

Interdisciplinary can be defined as the *combination of two or more scientific areas* working together to *solve and identify the problem* and find a solution by *searching and evaluating* together. For example, the *behavioral economy* is the most related to the area of the economy and psychology. Both economy and its theories affect this area. Not only the economy but also human behaviors and decisions are affected by their psychological situations. Scientists work together to find the relationship between these two areas. (Post-survey, P1, Italics added)

Like the pre-service P1, P2 also emphasized combination of two or more different areas to define interdisciplinarity: “I define [it] as *combining* two or more different area[s], it is *more than helping each other*, those are *inside each of them*. So, the research process should have different parts”. P2 described the relationship of disciplines in interdisciplinarity

as stronger than just helping each other. The disciplines are described to be *inside* each other, which can be resembled to be *integrated* as P7 wrote. On the other hand, the pre-service teacher P3 focused on variables and factors that could affect a problem, and relevant expertise in different disciplines to solve an interdisciplinary problem. P3 did not give a specific definition of interdisciplinarity but described certain key points in the process of interdisciplinary research. The example given by the pre-service teacher P3 was about design of vehicles, which was a different example from previous ones. In general, the focus of P3 was on the problem itself (nature of interdisciplinary problem):

A researcher who wants to evaluate a problem must correctly *determine the variables and determining factors* that affect the problem, and for this, he must have a *command of the disciplines* that affect the problem. In other words, I will say that a project designed on a problem should examine and consider the *factors* affecting the problem in detail. An example of this is *vehicle design*. While designing a vehicle, a researcher should consider the fuel consumption of the vehicle, the effect of the gases it emits on nature, its cost and many other variables. (Post-survey, P3, Italics added)

Besides, the pre-service teacher P4 made a general description of interdisciplinarity, mentioning the joint work of more than one discipline. The pre-service teachers P2 and P4 gave the same example of biochemistry for interdisciplinary research. The relationship between disciplines in interdisciplinarity was described as a *key-lock relationship* by the pre-service teacher P4:

It can be said that interdisciplinary research is the *work of more than one discipline* together. These areas are *connected* to each other like a *key-lock relationship* and act together. To give an example, *biochemistry* is an interdisciplinary study. Biology and chemistry disciplines work together in this field. Thanks to this work they have done together; a lot of new information has been revealed. (Post-survey, P4, Italics added)

Like P4, the pre-service teacher P5 broadly defined interdisciplinarity as an area of more than one discipline. The nature of the relationship of contributing disciplines in interdisciplinarity was not mentioned by P5. Moreover, P5 gave the same example (archaeometry) for interdisciplinarity as in the course assignment: “Interdisciplinary is an area of *more than one discipline* work together. For example, *archaeometry* is one of such areas because history, geology, chemistry, archaeology work together in an excavation field”.

Besides, the pre-service teacher P6 emphasized the relationship of disciplines in interdisciplinarity, and defined interdisciplinarity as disciplines in correlation. Furthermore,

sufficient knowledge of disciplines was highlighted. Cognitive science was given as an example for interdisciplinarity by P6:

Interdisciplinary means being [a] *correlation* with disciplines. Interdisciplinary means that the fields are *in correlation with each other*. When examining a field, we must have *sufficient knowledge* to understand the field we are examining, and it needs to examine of many fields. If we want to do a study on *cognitive science*, we need to examine and research many fields such as biology, psychology, linguistics. It is not possible for us to do our research without having *enough knowledge* about these issues because we need to have *sufficient and correct information*. (Post-survey, P6, Italics added)

In short, the findings on the post-survey indicated that pre-service teachers gave more detailed descriptions on interdisciplinarity compared to their answers in the pre-survey. In the post-surveys, some pre-service teachers (e.g., P1, P3, P7) focused on the goal of solving problems for interdisciplinarity. The combination of two or more disciplines and working together were other prominent aspects for interdisciplinarity mentioned by the pre-service teachers in the post-survey (e.g., P1, P2, P4, P5).

Another question of the post-survey was about increasing the level of *interdisciplinarity* of an interdisciplinary mathematical modeling task. About increasing the level of interdisciplinarity, the pre-service teacher P1 recommended *adding more related disciplines*. As an example, a possible task could relate to mathematics, climate science, and chemistry. However, it was a general suggestion and did not include a detailed description:

We can increase the level of task by *including more related areas*. For example, if our task includes only mathematics and climate science, we can add a chemistry area. For instance, we ask about other reasons for the solution outside the *climate effect*. By doing this, we can find a *chemistry-related solution*. (Post-survey, P1, Italics added)

Besides, the pre-service P2 recommended to include videos of graphs, or formulas of graphs as a mathematical language. Physics of music was given as an example in which models such as graphs of the sine function or formulas were used:

I can add *videos of graphs*, and a *mathematical language* can help me. The formulas are better than saying increase or decrease, so *formulas of graphs* always help us for modeling. In my *music* theory lessons, I have talked about *physics of music*, I send videos of graphs, and my students can learn the different situations. I have drawn graphs, and I wrote formulas of a sine graph. Those basic models help me while telling the music. (Post-survey, P2, Italics added)

Moreover, the pre-service teacher P3 suggested adding new perspectives through new variables as well as diversifying and developing the initial problem: *Diversifying and*

developing the interdisciplinary problem and *adding perspectives* that include *new variables* to the question makes the assignment more thought-provoking, increasing the interdisciplinary level and allowing the problem to have a *higher cognitive demand*. As mentioned, these were parallel with the *cognitive demand level*. But P3 did *not* mention *how to do* this or give any specific example for this. Besides, the pre-service teacher P4 recommended making students *more active* such as by asking them to find data or infer formulas.

By *making the students more active* in the question, for example, we can write questions that are created by asking them to *find the data* or waiting for the *formula inference* from them. They can be asked questions that do not have a single result, in which they can use their own perspective, interpret the questions, and research them. (Post-survey, P4, Italics added)

Furthermore, the pre-service teacher P5 suggested using different graphic drawing *applications* and different *topics* of science and math: “We can have the model, give *as many science and math subjects as possible*. In addition, different *graphic drawing applications* on the internet can be used to make the model more visual”. Besides, the pre-service teacher P6 emphasized *daily life* connections and high-level of *cognitive demand*: “The task must have a *high level of cognitive demand*. The more the task is from *daily life*, the more interesting and logical it is to make sense of and examine it”. Lastly, the pre-service teacher P7 wrote a general statement, and emphasized asking students to use their interdisciplinary knowledge to solve an interdisciplinary problem:

In some tasks, although the subject of the problem and some of the information provided are from the field of science, we only use mathematics to solve the problem. For example, we talked about the *midge problem* in one of our lectures. In this task, biology was used for the characteristics of midge species and giving the collected data. However, we did *not need biology knowledge to solve the problem*. To increase the interdisciplinary level, students should be asked to *use their knowledge from different fields*, and they should be asked to understand *how disciplines can be integrated* to solve the problem. (Post-survey, P7, Italics added)

In short, concerning interdisciplinarity levels, the findings on the post-survey indicated that most of the pre-service teachers (e.g., P1, P3, P6, P7) recommended to add new disciplines, perspectives, knowledge or daily life connections for interdisciplinarity. Some pre-service teachers also emphasized using tools (e.g., P2, P5). Finding data (e.g., P4) and a focus on cognitive demand (e.g., P6) were other aspects mentioned to increase interdisciplinarity.

Moreover, another question of the post-survey was about the description of *mathematical modeling* and its process steps as well as the role of mathematical modeling in interdisciplinary tasks. The pre-service teacher P1 did not explain mathematical modeling or its process steps but emphasized *comprehensiveness* of the problem for the need of mathematical modeling: “a) to understand and evaluate the problem and its solution *more comprehensively*. b) Student will *improve their thinking and interpreting skill* in detail. They think and find a solution by including more than one area *deeply*.” For the role of mathematical modeling in interdisciplinary tasks, P1 highlighted to improve students’ *skill of interpreting and thinking*.

The pre-service teacher P2 did *not* mention what mathematical modeling was and what the process steps were but wrote that mathematical modeling was needed for *drawing graphs*. For the role of mathematical modeling in science, P2 noted that it was for *modeling data* and facilitating the solution process of the problem:

a) The science teacher needs to know a bit [of] math for making *graph[s]* of the formulas so she can add more examples and draw [necessary] graphs for different situations. The teacher can start [a] project about *daily life* as we do in [the] final project, and it can help [us] not to forget the course. b) The role is *show[ing] the idea* and the *thesis*, in different parts we use math statistics and *modeling of data*, the role of modeling in science class is help[ing] and mak[ing] it easier. (Post-survey, P2, Italics added)

Moreover, the pre-service teacher P3 explained mathematical modeling as a *system* focusing on the variables and their interrelationships for the problem. According to P3, mathematical modeling helps students to find the searched *connections*. For the role of mathematical modeling in interdisciplinary tasks, P3 stated that mathematical modeling was helpful to make a more *comprehensive* solution to the problem:

a) Mathematical modeling is a *system* that explains how the *variables* that affect and determine the problem we are dealing with, *evaluate* together and how their relations with each other affect the problem and enable students to [do] *visual modeling*. Teachers should know this because they ask the student to clearly demonstrate with mathematical modeling the *connections* they expect them to find in the task they present. In this context, he/she should be an *expert* in the concept of mathematical modeling and use it effectively in his/her class to determine whether the homework they presented fits the interdisciplinary concept and to solve the problem correctly. b) Mathematical modeling is also frequently used in interdisciplinary research, as it clearly states the *variables* that affect the problem and their *relationships* with each other. In this context, evaluating the problem with mathematical modeling *makes the solution more comprehensive*. (Post-survey, P3, Italics added)

The pre-service teacher P4 emphasized the need for *more research* and therefore associated *effective learning* through mathematical modeling. Moreover, for the role of

mathematical modeling in interdisciplinary tasks, P4 focused on *different perspectives* and *visualization* of the problem:

a) In line with the work done by the teachers in this area, students are encouraged to do *more research for the lessons* and *more efficient learning* can be achieved. b) Mathematical modeling provides *another perspective* on the question and allows [us] to *visualize* the question and the answers. Since it is a type used in mathematics, we can see the separation of mathematics and other fields thanks to this modeling. (Post-survey, P4, Italics added)

Like P4, the pre-service teacher P5 also focused on the *visualization* of the problem through tables and graphs as a need for mathematical modeling. According to P5, mathematical modeling was also useful in *interpreting and understanding the results*. It was also noted to clarify the subject matter. For the role of mathematical modeling, P5 emphasized that the mathematical model as a *visual template* made the data and results *more understandable*:

A teacher who does interdisciplinary work and has his students do it needs *tables and graphics* to make the steps of his/her work *more visual and understandable*. Students can benefit from these tables and graphics that they have created so that their students can *interpret the results*. Students can easily *interpret the values* they read on tables and graphs, and they can *understand the subject in a better way*. b) The most important part of an interdisciplinary working process is the part where the mathematical model is created because having a *visual template* where all the collected data are processed makes the whole process and the collected data *more understandable*. Also, it makes the *results* obtained as a result of the collected data *easier to read and understand*. (Post-survey, P5, Italics added)

Like P5, the pre-service teacher P6 stressed *understanding* the subject matter for the need of including mathematical modeling. Again, for the role of mathematical modeling, P6 noted that mathematical modeling helped to *understand* the problem through a *model*:

a) In order to talk about a subject, we need to think about it, and if we do not include information from different fields in this reflection period, we do *not really understand that subject*. For this reason, it is necessary to make lectures and teaching by including many disciplines in education. b) Mathematics modeling helps us *understand* the task by showing it on a *model* in the task. Someone who does not know the disciplines of the task allows him to interpret and understand by looking at the model. (Post-survey, P6, Italics added)

Besides, the pre-service teacher P7 only defined mathematical modeling and briefly explained the steps of a mathematical modeling process. P7 also focused on understanding the connection between disciplines about the role of mathematical modeling in interdisciplinary tasks:

Mathematical modeling is an *open-ended, research and analysis related problem* that does *not* have a *procedural solution*...students not only construct a model but also analyze the model in mathematical modeling tasks. For this reason, they better understand the relationship between related disciplines. (Post-survey, P7, Italics added)

In short, about mathematical modeling, the findings for the post-survey indicated that most of the pre-service teachers (e.g., P1, P2, P3, P5 and P6) focused on problem solving. Particularly, the role of mathematical modeling to provide comprehensive solutions and understanding the problem was emphasized. Visualization was another prominent aspect (e.g., P2, P4, P5). Some pre-service teachers also emphasized data visualization and modeling data (e.g., P2 and P5).

Another question of the post-survey was about *writing an interdisciplinary mathematical modeling task* and describing that process. The sub-questions also asked for the criteria for designing interdisciplinary mathematical modeling tasks, application methods in the classroom, and the effect of such tasks on students' learning math/science. The post-conceptualizations on *designing interdisciplinary problems* based on post-surveys will be described in the following paragraphs (Cycle 2).

The pre-service teacher P1 *just* noted the *topic* of the asked problem as water footprint but did *not* give a detailed expression of the problem. For the criteria of writing an interdisciplinary mathematical modeling task, P1 emphasized the level of the task, multiple solutions, using different tools, allowing experiments and inquiry-based learning. For application methods, P1 generally mentioned to do activities in class such that students experienced the task themselves. Possible benefits or challenges were *not* mentioned by P1. For students' learning in science or mathematics, P1 did *not* write a *specific* statement, but only mentioned in a general manner that students' thinking, evaluation and interpretation skills could improve:

Water footprint a) First of all, I define the *level* of the interdisciplinary task. Then, design a task according to the level. I limited some knowledge about the cases according to the level of the task. Then, I present some questions with *more than one solution*. They could find the solution by using *different tools*. In addition, I asked extra questions to allow students to *experiment*, so students experienced and interpreted the task clearly by doing *inquiry-based learning*. b) I would have students design an activity *to do in class*. They apply the task by seeing and *experiencing it* so they can evaluate the task more clearly. c) They improve their *thinking, evaluating and interpreting skills*. They find a solution by *evaluating more than one area*. (Post-survey, P1 Italics added)

Besides, the pre-service teacher P2 pointed out the need for sufficient data, ethical considerations for lab work, and understanding relevant content (for the criteria). For application method in class, P2 shared an experience of using shorter sentences or simplifying the task as a suggestion. Moreover, for the benefits as well as the learning effects of such tasks, P2 focused on *understanding*:

a) The *data* should be *enough*, and the *lab* part should be *ethical* like all research. Sometimes I can ignore something for making [it] easy. The topic should be known well. For example, if I need to work on mutation and genetics, I must know *data* and I must *understand* biology so I can work the biostatistics, epidemic. b) The benefits of using math part of topics make students get. But in my music theory lessons I had students from history, and they don't know math like science students. *Some models* of music [in which] I used scales as series, the formulas can be *difficult* for social science students, [and] I wrote *short sentences for them*. c) The modeling and formulas help to get chemistry and biology, too. *Graphs and formulas* help [us] to *understand* and instead of memorizing we can *get the logic*. (Post-survey, P2, Italics added)

The pre-service teacher P5 did *not* write any problem for this question but mentioned the final project assignment as the task. For the criteria, the pre-service teacher P5 focused on the level of interdisciplinarity (ensuring at least multidisciplinary), clear steps, and an understandable model. For the benefits, P5 wrote that students could use certain *knowledge* of science (such as representation of elements), improve *skills of research* through *technological tools*, convert and evaluate data. For challenges, P5 noted that *finding* appropriate *data* could be difficult, but could be handled by teacher support. For learning effects, P5 reported that students' learning could be *permanent*, and their *interest* levels could be increased. Associating mathematics and science with daily life was also mentioned by P5:

a) We must ensure that it is a *multidisciplinary* field of study, and *adequate constraint steps* must be *clearly stated*. The resulting *model* should be *easy to read and understand*, even for people who are not related to the subject. b) I think that it is an area that will *attract the attention* of students, since *farming and agricultural practices* are quite a part of *daily life*. Thanks to this assignment, students use the *symbols* of the elements they learned from the science lesson and improve their *research skills* by using *technological tools*. They can convert the *data* they have obtained into other units, evaluate these data, and make profit and loss calculations. They may have *difficulty in finding some data*, but they can obtain data as a result of the *teacher's guidance* with effective questions. c) Students associate both disciplines with the problems of *daily life*, and their *interest* in both courses increases. In addition, using the information they have learned at every opportunity makes this information *more permanent*. (Post-survey, P5, Italics added)

The pre-service teacher P6 wrote an interdisciplinary theme that could allow physical experiments but did *not* write it as a specific *problem*. It was about melting ice-cubes, but it was *not* a mathematical modeling problem. For the criteria, P6 focused on daily life

connection and the inclusion of several disciplines. For application methods, P6 drew a general picture by noting that students would discuss the change of variables, would derive equations, and could do experiments. P6 did *not* mention related benefits or challenges. For learning effects, P6 wrote that students could understand the reasoning behind graphs (plots) and see the interrelationships of the disciplines. The need to be proficient in different disciplines by the students was also mentioned by P6:

There are ice cubes of different sizes, and the melting times of these ices are given in different environments. And then the student calculates the melting time depending on which variables. Finally, air-melting time and surface measurement-melting time graphs are drawn. 5a) It is one of my primary criteria [such] that it includes *many disciplines* and is a problem from *daily life*. b) A few questions are asked about which parts of the students should focus on the question, then the student discussed *how the variables change* the duration. If necessary, a *physical experiment* is done so that they can blacken the event in a concrete way. Based on these variables, an *equation* is derived, and this is plotted. c) Students here need to be *proficient in many disciplines* such as physics and mathematics, and they do *research* accordingly in this task. Finally, they identify the *interrelationships* of the disciplines and understand the logic of the plot thanks to the example from *daily life*. (Post-survey, P6, Italics added)

The pre-service teacher P7 posed a problem of mathematical modeling on designing milk boxes. While describing the process of writing the problem, P7 highlighted aligning involved concepts with the curriculum objectives, using data from reliable sources, and giving clear instructions for students. For the sub-question on the criteria for writing interdisciplinary mathematical modeling tasks, the pre-service teacher P7 wrote cognitive demand level of tasks, open-endedness, using different mathematical tools and real-life context, high level of integration of different disciplines, and appropriate determination of variables. For the sub-question on how to apply the designed interdisciplinary mathematical task in the classroom, the pre-service teacher just noted that the written task aimed for the concepts area and volume in mathematics and toughness in physics. However, P7 did not write any details of *implementation* of the task in class. For the benefits of using interdisciplinary mathematical modeling tasks in the classroom, P7 emphasized the real-life application of mathematics and then that could motivate students. For the challenges of using interdisciplinary mathematical modeling tasks in the classroom, P7 focused on the lack of time and finding an appropriate task theme to attract students' attention. For the sub-question on how using interdisciplinary mathematical modeling tasks affect students' *learning* of mathematics/science, P7 wrote in a general manner that it was useful for conceptual understanding, and especially to reify the abstractness in mathematics through science:

Interdisciplinary mathematical modeling task: A dairy company wants to redesign its 1-liter carton boxes to reduce costs and waste consumption. For this purpose, you are asked to create a model about how you can use a shape in the design of 1-liter milk boxes. While designing an interdisciplinary mathematical modeling task, we should define the *concept of the problem* according to the *objectives*. Then, necessary information and data about the problem should be obtained from *reliable sources*. Students should be given *clear instructions*. a) While designing a task on interdisciplinary mathematical modeling, we can pay attention to the *cognitive demand* level of the task, its *open-endedness*, the lack of procedural solutions, the ability of students to use different *mathematical tools*, the *real-life context*, the *high level of integration* of the disciplines required for the solution, and the *determination of the variables*. b) It can be used in mathematics lessons to provide the relationship between the *area and volume of 3D objects*. In addition, students should also understand and use the concept of *toughness* that they learned in *physics* class. Modeling questions are useful for students to explore the *application areas* of the concepts they learn in mathematics. It also helps to *reduce* the frequency of asking [the] following question, "*Where will we use them in daily life?*" On the other hand, sometimes the designed tasks may *not be implemented in just one lesson*, or the application may take longer than we planned. In addition, it can be a *challenge to determine the task subject* in a way that will *attract the attention* of each student. c) It is important to use interdisciplinary mathematical modeling tasks for students to *understand both mathematical concepts and science-related applications*. Mathematics can be integrated with science to *break the abstract structure in students' minds*. In addition to this, mathematical operations, graphics, and mathematical modeling are used to solve any science-related problem. (Post-survey, P7, Italics added)

In short, the findings on *post-survey* about interdisciplinary mathematical modeling tasks indicated that pre-service teachers mostly conceptualized interdisciplinarity as one of the criteria for writing interdisciplinary mathematical modeling tasks (e.g., P5, P6, P7). Mathematical modeling was another prominent aspect mentioned as criteria (e.g., P2, P5, P7). Concerning mathematical modeling, use of sufficient data, identification of variables and building an understandable model were emphasized. Tool use and considering level of tasks such as cognitive demand (e.g., P1 and P7) as well as using inquiry-based learning and experiments were other aspects emerged.

In Cycle 2, the *anonymous feedback forms* were also used to investigate pre-service teachers' conceptualizations on interdisciplinary teaching. The feedback forms were administered during the implementation of the module on interdisciplinary teaching. Concerning the tasks applied in the course, in the anonymous feedback forms, pre-service teachers emphasized aspects of interdisciplinarity, mathematical modeling, level of tasks, and using tools. For instance, one pre-service teacher focused the aspect of multiple perspectives about interdisciplinarity:

I understand better the necessity of this subject in the example about *buying gasoline*. I knew that one could look at a problem from *many perspectives* but seeing it in a single example made things much clearer. In fact, I learned that it is an *important and frequently used thing in our daily life*. (Feedback form, italics added)

Similarly, another pre-service teacher emphasized interdisciplinarity as expanding perspectives. The level of tasks was also emphasized by noting the complexity. Moreover, mathematical modeling was also mentioned by emphasizing the variables of a problem:

In the past, we have approached some basic mathematics questions in a *narrow perspective*, but in real life, the problem concerns *much more disciplines* that we need to take into consideration when we try to solve. Mathematical modeling tasks enable us to investigate the *variables* of the problem and make *connections* between them. Moreover, IMMT provides us to realize that a problem may need *different kinds of proficiencies to evaluate* correctly. The tasks might be *more complex* to make students think *higher order*. (Feedback form, italics added)

Another pre-service teacher also focused on interdisciplinarity and mathematical modeling. The weight of other discipline than mathematics was mentioned to be elevated for interdisciplinarity. Furthermore, mathematical modeling was emphasized through data collection and analysis:

We can say that biology and mathematics were generally together, and math was more at the forefront. For example, I think *biology has been used just to utilize data* (the chirping of crickets, wings of midges). In *gas example, several topics were handled, and searching, collecting data, and evaluating about all of them were done*. However, we did not actively use biology in this way in areas concerning biology. If math and science were integrated as *interdisciplinary* areas, then different assignments in which *science was active* could be chosen. (Feedback form, translation, italics added)

Other pre-service teachers also emphasized mathematical modeling, tool use, and level of tasks. For level of tasks, one pre-service teacher explicitly noted cognitive demand level of tasks. Moreover, mathematical modeling was focused through assumptions. Tool use was emphasized by mentioning internet sites. Internet sites were seen as resources of knowledge for tasks:

In lessons, we learnt *cognitive demand levels* of a task, *what kind of characteristics* a mathematical modeling task has, and *how to develop* interdisciplinary mathematical modeling questions. In those aspects, I think that I have *enough background* for the final project. (Feedback form, translation, italics added)

While developing such a task, *assumptions* and whether our *reasoning aligns* with the modeling question are very important. While developing such a task, the sites we use and how we use such sites are very important. It would be better to exemplify *how these sites are used* through a task. (Feedback form, translation, italics added)

Furthermore, in the feedback forms, some pre-service teachers described the *mathematical modeling process* in the implemented tasks with more details. The key aspects in the mathematical modeling process were emphasized. For instance, collecting information about the problem to understand and make relevant assumptions, listing variables, relating

variables, collecting data, and building a model were written as important part of modeling process:

In mathematical modeling tasks, we initially *gather information* about the presented task. This makes us *understand and conceptualize the task* as much as possible. In modeling, we make *necessary assumptions*. While deciding on these assumptions, it is very important *how many and what kind of assumptions* are. We can develop ourselves in this point, and we should evaluate which approach is reasonable when we put them[assumptions] into the model. (Feedback form, translation, italics added)

In the first step, one should make a *list of variables* that concerns the problem. Secondly, one could make *relations of variables* and *collect data* via observations. Since each variable may affect the problem separately, one should evaluate *connections* with each variable [in] pair, and because of this, one might conclude a solution. After that, one can *formulize* what one found and try to obtain a *graph, line equation or table* to show the relation between each variable and the connection of these variables with the problem. (Feedback form, italics added)

I can now *better analyze data* when confronted with them since we have done *many examples* about modeling not just one or two. *I don't have the concern that I felt at the beginning. I think that I could generate a model inspired by the applied modeling tasks* based on the *gathered data* aligned with our *assumptions*. (Feedback form, translation, italics added)

The *anonymous feedback forms* were also considered a mid-evaluation of the process by getting anonymous feedback on the course and the module on interdisciplinary teaching. Certain aspects were found to be useful such as focusing on task design, engaging in several mathematical modeling tasks, and group work. In the anonymous feedback forms, two pre-service teachers explicitly wrote that it was challenging at the beginning. The awareness in the lack of knowledge about technological tools such as applications of drawing graphs was also stated. The two-dimensional aspect of designing mathematical modeling tasks was noted as conceptual and practical. Discussing what a task and what a mathematical task were stated to help conceptualization. The variety of emerging models was also recorded to be attractive. The group work was also emphasized to be enjoyable during the course. What learnt on the course was thought to be helpful in analyzing daily life problems. The task development part of the course was thought to be useful to design tasks for the final projects. Particularly focusing on details of posing such interdisciplinary tasks was mentioned in the feedback forms.

As a sum, the findings on the *anonymous feedback forms* indicated that pre-service teachers conceptualized certain aspects such as focusing on task design, engaging in several mathematical modeling tasks, and group work as helpful. Moreover, pre-service teachers shared their ideas on the mathematical modeling tasks implemented during the course. About the tasks implemented in the course, pre-service teachers emphasized aspects such as

interdisciplinarity, mathematical modeling, level of tasks, and using tools. Concerning task characteristics, allowing different perspectives, involving real-life connections and different disciplines, being interdisciplinary, complex and higher order, requiring different skills, including collection and evaluation of data were emphasized. Moreover, cognitive demand level of tasks and assumptions were also noted as important to take into consideration in designing *interdisciplinary mathematical modeling tasks*. In general, pre-service teachers believed that they could pose interdisciplinary mathematical modeling tasks.

As a summary, the findings on *interviews* showed that the pre-service teachers' initial conceptualizations were limited and grounded on the examples used in the course. At first, most of the pre-service teachers (P1, P3, P5, P6, P7, P9) could not explicitly define interdisciplinary mathematics or science education. Initial conceptualizations focused on mathematical modeling while reference to interdisciplinarity was rare. The findings on *surveys* also supported the findings on interviews. The findings on pre-surveys indicated that most of the pre-service teachers could not define interdisciplinarity and the pre-service teachers often focused on the mathematical modeling process in general. Concerning mathematical modeling, the emphasis was on real-world connection and problem solving as the findings on the pre-surveys showed.

As a summary for the post-conceptualizations, the findings on *interviews* also indicated that some pre-service teachers (e.g., P7 and P9) could give a more explicit definition of interdisciplinary mathematics or science education after the implementation of the module. Moreover, for post-conceptualizations, pre-service teachers were able to give different examples about interdisciplinarity or interdisciplinary mathematics/science education (e.g., P9's example of his brother's area: geophysics). Finally, pre-service teachers gave rich descriptions for characteristics of interdisciplinary mathematical modeling tasks as reflected in the post-interview. The findings also indicated that the post-conceptualizations were concentrated on mathematical modeling and organization. Like the findings on interviews, the findings on *surveys* indicated that pre-service teachers' conceptualizations were focused on mathematical modeling and aspects such as using tools and level of tasks related to organization. Like the findings on interviews, the findings on the post-surveys indicated that the pre-service teachers' descriptions on interdisciplinarity were more detailed and focused on problem solving, combination of different disciplines

and working together of disciplines. Moreover, like the findings on the interviews, the findings on the post-survey indicated that the pre-service teachers focused on interdisciplinarity, mathematical modeling and organization among the characteristics of interdisciplinary mathematical modeling tasks. Especially, mathematical modeling was the predominant aspect as revealed in both interviews and post-surveys. The findings on *anonymous feedback forms* also supported the findings on interviews and surveys such that mathematical modeling was the prominent aspect. Particularly, identification of assumptions as well as collection and evaluation of data were the dominant aspects about mathematical modeling as revealed in anonymous feedback forms. Like the findings on the surveys, using tools and considering the level of tasks (including cognitive demand) were emphasized in the anonymous feedback forms. Moreover, like the interviews, aspects about context were other aspects that were focused on the anonymous feedback forms. Real life connections, different perspectives and interdisciplinarity were aspects about context mentioned in the anonymous feedback forms. All in all, the findings indicated that pre-service teachers' conceptualizations focused on three themes context, mathematical modeling and organization, especially mathematical modeling being the dominant aspect.

4.2.2. Findings on Designing Interdisciplinary Tasks based on Interviews

In Cycle 2, it was also investigated how the pre-service teachers designed interdisciplinary mathematical modeling tasks. To answer the *third research question*, interviews were used to understand how the pre-service teachers designed interdisciplinary mathematical modeling tasks. The conceptualizations of pre-service teachers on *designing* interdisciplinary mathematical modeling tasks were shown in Table 4.3. The conceptualizations on designing interdisciplinary mathematical tasks can be grouped under three themes: Context, modeling, and organization.

Table 4.3. Conceptualizations on designing interdisciplinary tasks.

Themes	Codes (Pre-interview)	Codes (Post-interview)
Context	Connections (P1, P4, P6, P8, P9)	Interdisciplinarity (P1, P3, P4, P5, P6, P7, P8, P9)
	Real-life (P3, P5, P6, P7, P8, P9)	Real-life (P3, P5, P6, P9)
		Open-ended (P3, P7, P9)
Modeling	Model construction (P1, P3, P4, P9)	Model construction (P1, P3, P5, P6, P7, P9)
	Variables (P3)	Variables (P1, P3, P4, P7, P8)
		Assumptions (P7)
	Data (P8, P9)	Data (P6, P9)
	Results (P3)	Results (P3, P7, P8)
		Narrowing down (P3, P4, P7)
Organization	Active class (P5, P6)	Ways of work (P1, P6, P7, P8, P9)
		Resources (P9)
		Duration (P9)
		Level (P1, P7, P8, P9)

As a theme, the *context* refers to the context of the problem. Initially, pre-service teachers conceptualized context as general connections between disciplines or as real-life connections. After the module implementation on interdisciplinary teaching, pre-service teachers conceptualized context as interdisciplinary connections, real-life connections or open-ended problem situations. Initially, modeling was based on general mentions on model construction, variables, data or results. After the module implementation on interdisciplinary teaching, pre-service teachers conceptualized modeling as mathematical modeling, and they focused on the mathematical modeling with varying aspects. For post-conceptualizations, mathematical modeling was composed of model construction, variables, assumptions, data, results or narrowing down of the problem. Initially, pre-service teachers conceptualized organization as active class in teaching tasks, but after the module implementation, they conceptualized organization in more aspects such as ways of work, resources, duration or level.

Interdisciplinarity involved in different degrees in the interviews. In the pre-interview, participants mentioned generic connections between disciplines but used more specific terms about interdisciplinarity in the post-interview (such as interdisciplinary). Only one participant talked about the borders of disciplines in the pre-interview while most of the participants concentrated on interdisciplinarity with different aspects in the post-interview. For instance, identifying themes came out as an important code in addition to the prevalent reference of the interdisciplinary. Besides, participants mentioned building connections with different disciplines in the pre-interview. This was a very broad description about interdisciplinarity, and this code did not emerge in the post-interview.

There were more codes about the theme of *modeling* in the post-interview compared to that in the pre-interview. More aspects were added to this theme. Variables, assumptions, and narrowing down were the new codes compared to pre-interviews. Besides, the emphasis on certain codes changed. For instance, *results* meant interpretable results allowing inferences in the pre-interview while results meant applicable and reasonable results in the post-interview. Furthermore, the pattern in *data* was the focus in the pre-interview while real data and not giving all data at once were so in the post-interview.

Concerning *organization*, there are less codes in the pre-interview compared to those in the post-interview. For instance, *active class* (keeping the class active with interesting tasks) was the focus in the pre-interview while *ways of work* including inquiry, experiments, and group work were added in the post-interview. Concerning *level*, the *mathematical level* of the task and the *student level* were considered in the post-interview. Besides, additional codes about implementation such as *resources* or *duration* were mentioned in the post-interview.

This part will describe the findings on the *initial conceptualizations* of the pre-service teachers on designing interdisciplinary problems in Cycle 2. For the pre-interviews, the pre-service teachers were asked how *Interdisciplinary Mathematical Modeling Tasks* could be designed. The *initial conceptualizations* of the pre-service teachers on designing interdisciplinary problems in Cycle 2 were investigated based on pre-interview data (see Table 4.4). The initial conceptualizations of pre-service teachers on designing interdisciplinary problems for Cycle 2 with example quotes were presented in Table 4.4.

Table 4.4. Initial conceptualizations on designing interdisciplinary problems.

Themes	Codes	Excerpts
Context	Connections (P1, P4, P6, P8, P9)	For example, we looked at the relationship between these gas particles and temperature, and their relationship with volume. Maybe we can examine the relationship between the same type of gas in different closed containers in cold environments and hot environments, and by modeling this, by casting it into a visual model, we can ensure that it is better understood and connected. (P1, 5 th item)
	Real-life (P3, P5, P6, P7, P8, P9)	Because the results obtained are also connected to <i>real life</i> . Since they can connect it to <i>real life</i> in the Evaluate section, it should be interesting, and I think students want to work on it. (P5, 5 th item)
Modeling	Model construction (P1, P3, P4, P9)	Every question can be solved by buying chocolate from the grocer, but they can all be solved differently, the writings are different, and the children get confused. When I take them to the grocer, they really go to <i>life</i> because they don't set up an equation at the grocer. But if they have 10 liras in their head, they know what to buy. Using that logic, we can turn it into the one in the question and set up an equation, formulate it and set up that equation. From there, I solve the question. And I think I design activities like this. I always bring it back to <i>life</i> so that they can think. From there, I <i>switch back to mathematics</i> . But I don't think I design an activity right now. (P4, 5 th item)
	Variables (P3) Results (P3)	First, I would try to choose a topic from <i>real life</i> because in mathematical modeling, we start from real life first. I would try to do something that I can <i>easily transition to the mathematical world</i> , that is, something that I can <i>easily formulate</i> . Then I try to choose a topic that can be <i>interpreted</i> and from which I can make inferences. For example, I try to choose a topic with known <i>variables</i> , I try to handle a problem whose variables I can easily determine. After determining the variables, I formulate them and try to handle a problem from which I can easily make inferences or reach a definite <i>conclusion</i> . (P3, 5 th item)
	Data (P8, P9)	The context of the course could be creating a <i>graph</i> or creating a <i>connection</i> . It was graph creation that I took in the lab course last year. How are these <i>data</i> connected? This is an activity that will allow students to transfer this and what they have learned there. Do they know how to create a graph? Accordingly, we then asked about the relationship between the data. We asked them to write the function. (P9, 5 th item)
Organization	Active class (P5, P6)	There must be a <i>real problem</i> , an example from <i>life</i> , first because it keeps the <i>class active</i> and makes it more interesting. At least something happens, it makes <i>connections</i> . The first stage can be a <i>real-life</i> problem. (P6, 5 th item)

This part will describe the findings on the *post-conceptualizations* of the pre-service teachers on *designing interdisciplinary problems* in Cycle 2. For the post-interviews, the pre-service teachers were asked *how they designed* their final projects and what they considered in designing their *Interdisciplinary Mathematical Modeling Tasks*. The *post-conceptualizations* of the pre-service teachers on designing interdisciplinary problems in Cycle 2 with example quotes were presented in Table 4.5.

Table 4.5. Post-conceptualizations on designing interdisciplinary problems.

Themes	Codes	Excerpts
Context	Interdisciplinarity (P1, P3, P4, P5, P6, P7, P8, P9)	First, <i>interdisciplinary</i> . That is, <i>more than one discipline comes together</i> . In other words, when you get into an event that seems very simple, you can think of so many things together. (P8, 7 th item)
	Real-life (P3, P5, P6, P9)	Another thing is that it is an everyday event, that is, it is taken from <i>real life</i> . (P6, 7 th item)
	Open-ended (P3, P7, P9)	It should be <i>open-ended</i> and a little not obvious. Then, we need to be careful that this question is <i>not too endless</i> . (P3, 7 th item)
Modeling	Model construction (P1, P3, P5, P6, P7, P9) Assumptions (P7) Results (P3, P7, P8)	I thought about the <i>design process</i> . For example, first we determine a <i>concept</i> , we determine a <i>theme</i> . This becomes a <i>very broad question</i> . We need to <i>narrow down</i> this a little bit. We pay attention to many things. Does this have the <i>characteristics</i> of <i>mathematical modeling</i> activities, which <i>assumptions</i> do I need to make. Can a <i>result</i> be obtained? I need to <i>solve the question myself</i> , or can the student produce <i>several results</i> ? (P7, 7 th item)
	Variables (P1, P3, P4, P7, P8)	First, we want students to find the <i>variables</i> (P3, 7 th item)
	Data (P6, P9)	When collecting <i>data</i> , we tried to benefit from real data like this. (P6, 7 th item)
	Narrowing down (P3, P4, P7)	We thought that it should neither go to infinity nor remain so limited within a <i>certain limit</i> . (P4, 7 th item)
	Ways of work (P1, P6, P7, P8, P9)	We could determine <i>as a group</i> of two that we had a lot of <i>variables</i> . (P1, 7 th item)
Organization	Resources (P9) Duration (P9)	I would give them a two-week period for this; I would give them some resources that could be key. (P9, 7 th item)
	Level (P1, P7, P8, P9)	the <i>level of the student</i> or the person (P1, 7 th item)

As a summary, the findings based on interviews indicated that pre-service teachers initially conceptualized *context* as general connections between disciplines or as real-life connections. After the module implementation on interdisciplinary teaching, pre-service teachers conceptualized context as interdisciplinary connections, real-life connections or open-ended problem situations. Besides, *modeling* was initially conceptualized based on general mentions on model construction, variables, data or results. After the module implementation on interdisciplinary teaching, pre-service teachers conceptualized modeling as mathematical modeling, and they highlighted mathematical modeling with varying aspects. For post-conceptualizations, mathematical modeling was composed of model construction, variables, assumptions, data, results or narrowing down of the problem. Furthermore, pre-service teachers initially conceptualized *organization* as active class in teaching tasks, but after the module implementation, they conceptualized organization in more aspects such as ways of work, resources, duration or level.

4.2.3. Findings on Designing Interdisciplinary Tasks based on Artifacts

This section presented the findings on the *third research question*. In Cycle 2, in addition to the investigation of conceptualizations, it was also investigated how the pre-service teachers design interdisciplinary mathematical modeling tasks based on their artifacts. The artifacts consisted of the *rubric* constructed during the class and the *tasks* that the pre-service teachers designed for their final projects.

The *rubric construction* was conducted as a class discussion on constructing a guideline in designing interdisciplinary mathematical modeling tasks. Pre-service teachers themselves constructed the indicators for interdisciplinary mathematical modeling tasks, and the researcher was a facilitator of the discussion by noting pre-service teachers' ideas on the board, by saying to share more ideas and by asking them to categorize their ideas to form a rubric. The findings on the rubric construction indicated which aspects the pre-service teachers focused on designing interdisciplinary mathematical modeling tasks.

The rubric construction in Cycle 2 revealed the *Rubric for Interdisciplinary Mathematical Modeling Tasks* (see Appendix K) which consisted of five dimensions: Level of cognitive demand, mathematical modeling, organization/structure of task, context of the

problem, and expected/possible student solutions. Pre-service teachers initially called the rubric as *Criteria for Assessment of Interdisciplinary Mathematical Modeling Tasks*. These dimensions emerged based on the whole-class discussion by the pre-service teachers. Then, the researcher added levels for each dimension to finalize the constructed rubric. In the rubric, the tasks were considered in levels with an increasing order of appropriateness: *Not Applicable*, *Acceptable*, *Satisfactory*, and *Outstanding*. The rubric was shared with the pre-service teachers to guide them in their design of final projects. Then, the rubric was revised by comparing the dimensions to the related literature. The revised rubric was used to analyze pre-service teachers' tasks as constructed for their final projects (artifacts). The finalized rubric served as a framework to analyze the tasks.

The tasks were analyzed by using the *Task Analysis Framework for Interdisciplinary Mathematical Modeling Tasks* (see Table 4.6). It was initially developed by the researcher and the advisor based on the relevant literature (Smith and Stein, 1998; Tekkumru-Kisa et al., 2015; Williams et al., 2016; Wess et al., 2021) and revised after the class work to construct a rubric in Cycle 2, in which pre-service teachers discussed the dimensions of interdisciplinary mathematical modeling tasks. Initially, there were three dimensions (Mathematical modeling, level of interdisciplinarity, and cognitive demand) in the framework. The revised framework has five dimensions: Dimension A (Level of Cognitive Demand), Dimension B (Mathematical Modeling), Dimension C (Organization), Dimension D (Context of the Problem), Dimension E (Expected Student Solutions). *Dimension A* refers to the level of cognitive demand for the tasks based on mathematics and other disciplines involved (e.g., science) (Smith and Stein, 1998; Tekkumru-Kisa et al., 2015). *Dimension B* refers to the characteristics of mathematical modeling tasks including open-endedness, reality, relevance, and authenticity (Wess et al., 2021). *Dimension C* refers to the organization of the task consisting of the structure of the task, content (e.g., objectives, grade level), implementation of the task (steps/procedure, tool use), and reporting. *Dimension D* refers to the context of the problem including the interdisciplinarity of the task. The levels of interdisciplinarity are based on the continuum of interdisciplinarity from monodisciplinary to interdisciplinary (Williams et al., 2016). The level of monodisciplinary refers to focusing on one discipline in the task. The multidisciplinary level refers to a higher level in which disciplines separately try to solve the problem without a common goal. The highest level (interdisciplinary level) refers to collaboratively solving a problem with a

common goal via a balanced contribution of involving disciplines. *Dimension E* refers to the expected student solutions to the problem. The range of rankings varies from *Not Applicable*, *Acceptable*, *Satisfactory*, to *Outstanding*, respectively (see Table 4.6).

Table 4.6. Task analysis framework for interdisciplinary tasks.

Dimensions	Not Applicable	Acceptable	Satisfactory	Outstanding
A. Level of Cognitive Demand				
B. Mathematical Modeling				
C. Organization				
D. Context of the Problem				
E. Expected Student Solutions				

As participant *artifacts*, pre-service teachers were asked to prepare their final projects on designing interdisciplinary mathematical modeling tasks in groups. There were four tasks designed by four focus groups. The tasks were analyzed by using the Task Analysis Framework for Interdisciplinary Mathematical Modeling Tasks (see Table 4.6). Based on the analysis, the levels of the tasks for Groups 1 and 2 were ranked higher than those for Groups 3 and 4. Each dimension had a different focus. Dimension D was about interdisciplinarity. Dimension B was about mathematical modeling. Dimensions E and C were about teaching tasks. Lastly, Dimension A was about the level of cognitive demand. The analysis was presented in the following order in terms of dimensions: D, B, E, C, and A. The overall description of the tasks was presented in Table 4.7.

Table 4.7. Task description based on focus groups.

Groups	Tasks	Levels
Group 1: P7 and P9	<i>Growing Wheat with Renewable Energy Sources:</i> Suppose you have 40 decare of agricultural land, and you will grow wheat. Your aim is to create a model in which renewable energy sources are used for energy use in agriculture.”	Dimension A: Acceptable
		Dimension B: Satisfactory
		Dimension C: Satisfactory
		Dimension D: Satisfactory
		Dimension E: Satisfactory
		Overall, the task was considered satisfactory.
Group 2: P3 and P4	<i>An Efficient Architectural Design:</i> A construction company has land of 900m ² . He is meeting with an engineer to make the most efficient use of this land. He asks the engineer to evaluate this land in two ways. In the first of these shapes, a multi-storey building can be erected on this land, and in the second, one-storey houses in the slum style can be built on this land. The engineer is asked to evaluate the efficiency of the two projects, provided that the number of floors of the building is equal to the number of single-storey houses. If you were this engineer, how would you prepare these projects? Can you model the project you see fit? Which project do you think provides more efficient use?	Dimension A: Satisfactory
		Dimension B: Satisfactory
		Dimension C: Satisfactory
		Dimension D: Satisfactory
		Dimension E: Satisfactory
		Overall, the task was considered satisfactory.
Group 3: P5 and P6	<i>Which One is more Profitable? Capia Pepper or Banana Pepper?</i> A farmer owns two acres of land in Mersin. The farmer plants banana pepper on one acre of his two-acre field; he will plant Capia peppers on one acre. The following information is known about peppers: Planting was done at appropriate distance intervals. It is irrigated regularly with the field irrigation system. Appropriate amounts of N, P, and K fertilizers are used. When the harvest period comes, both products will be collected and sold at market value for the city of Mersin. Examine the obtained profit and loss situation.	Dimension A: Satisfactory
		Dimension B: Satisfactory
		Dimension C: Satisfactory
		Dimension D: Acceptable
		Dimension E: Acceptable
		Overall, the task was mostly satisfactory but needed to develop certain aspects.
Group 4: P1 and P8	<i>Water Footprint:</i> Wheat is one of the products with the highest blue-green water footprint in Turkey. When we consider the Aksaray, Konya, and Kayseri, a) Do a mathematical model [choosing] the relationship between blue and green water footprint and create a formula. b) Design an experiment to observe a relationship of types of water. Write your variables and their effect of growing wheat/corn. Then, make [a] list of the factors affecting the yield. Briefly describe the implications of these views.	Dimension A: Satisfactory
		Dimension B: Acceptable
		Dimension C: Acceptable
		Dimension D: Satisfactory
		Dimension E: Acceptable
		Overall, the task was considered acceptable.
<i>Note.</i> The levels of the tasks were determined based on the <i>Task Analysis Framework for Interdisciplinary Mathematical Modeling Tasks</i> . The framework has five dimensions: Dimension A (Level of Cognitive Demand), Dimension B (Mathematical Modeling), Dimension C (Structure of Task), Dimension D (Context of the Problem), Dimension E (Expected Student Solutions). The range of ranking is changing from <i>Not Applicable</i> , <i>Acceptable</i> , <i>Satisfactory</i> , to <i>Outstanding</i> .		

For the *dimension D (Context of the Problem)*, tasks of the three groups were ranked as satisfactory and another (Group 3) as acceptable. As a part of Dimension D, the theme of *interdisciplinarity* was explained based on the groups. The interdisciplinary levels of the tasks were mostly multidisciplinary whereas one task (Group 3) was monodisciplinary based on the framework of Williams et al. (2016). None of the tasks were regarded as the highest level of interdisciplinarity (interdisciplinary level). Thus, the tasks could be said to remain at an intermediate level of interdisciplinarity (transitioning to interdisciplinary). Pre-service teachers were still trying to develop their conceptions of interdisciplinarity in the process of designing such tasks.

For the task of *Group 1*, in addition to mathematics, physics and agriculture was chosen. The theme was appropriate for an interdisciplinary mathematics theme. However, for the level of interdisciplinarity, considering the framework of Williams et al. (2016), the level was at most *multidisciplinary* since there was an interaction between disciplines physics, agriculture, and mathematics, but the disciplines were not totally integrated. There were key concepts from different disciplines and some of them (such as renewable energy, clean energy, agri-food systems, sustainability, agricultural land, energy source, efficiency, mathematical model, agricultural machines, preparing the soil, sowing seeds, fertilizers, irrigation, harvesting) were explicitly written. However, teaching these concepts as a holistic manner was not the central point, the concepts of other disciplines except mathematics mainly served as a *context* for building a mathematical model to solve the introduced. Herein, other disciplines (or concepts from those) were especially needed to understand the constructed problem. Particularly, they were necessary for determining key variables in the mathematical model. Besides, *interdisciplinary learning goals* were *not* specified in this project.

For the task of *Group 2*, in addition to mathematics, physics was chosen for the theme of *interdisciplinarity*. The theme was appropriate for an interdisciplinary mathematics problem. However, for the level of interdisciplinarity, according to the framework of Williams et al. (2016), the level fit to the level of *multidisciplinary* since there was an interaction between physics and mathematics, but they were not integrated. The key concepts of work, force, energy from physics were explicitly written in the report. Nevertheless, these concepts were used just as a tool to construct a mathematical model. There was not a balance

between the disciplines in terms of interdisciplinarity (mathematics was at the forefront). The concepts of physics were used to specify key variables in the mathematical model. There was not a direct aim to teach the concepts concerning physics, therefore *interdisciplinary learning goals* were not determined.

For *interdisciplinarity*, in addition to mathematics, science (agriculture and chemistry) was chosen for the task of *Group 3*. The theme was appropriate for an interdisciplinary mathematics problem. However, for the level of interdisciplinarity, according to the framework of Williams et al. (2016), the level fit to the level of *monodisciplinary* since the problem was a mathematical problem that science was used only as a tool for mathematics. The chemistry concepts (e.g., Phosphorus, Nitrogen, Potassium) just served as a context for the mathematical problem. Science helped to make sense of the problem. Certain agricultural concepts such as growing peppers, irrigation systems, harvest periods, and how to seed plants were needed to solve the problem. However, mathematics was at the center and science was in the background, therefore, disciplines were not balanced in the problem. The objectives were not stated for related disciplines. Besides, *interdisciplinary learning goals* were not specified.

Considering the theme of *interdisciplinarity*, for the task of *Group 4*, in addition to mathematics, science was chosen. Connected areas were explicitly written: In this part, students will think and evaluate other areas like weather which includes climate science, the kind of soil which is plant science and chemistry, and the kind of wheat with its germination times which is related to molecular biology and genetics. There was an emphasis on interdisciplinarity by noting related disciplines: Then, they [students] analyze it together while thinking and evaluating the different areas like chemistry, science, and biology. Moreover, in the *context of the problem*, crosscutting concepts in different fields were emphasized:

They [Students] think about other related factors, and this makes the task interdisciplinary. Because maybe the kind of soil with its minerals and its structure will be related to chemistry, the amount of rain will be related to climates, and what part of the growing time this product needs more water/rain is related to plant science in biology.

For *Group 4*, the theme was appropriate for an interdisciplinary mathematics problem. However, for the level of interdisciplinarity, according to the framework of Williams et al.

(2016), the level fit to the level of at most *multidisciplinary* since there were crosscutting concepts that helped to identify key variables, but the weight of the contributing disciplines was not equal. Other disciplines were used as a context, and disciplines were not totally integrated. In the problem statement, there were separate questions for mathematics (the a) part) and for science (the b) part) implying separate examination of the disciplines by different disciplines. Although there was mathematics within science experiments, science was more dominant in this part. Besides, *interdisciplinary learning goals* were not specified in the report.

For the *dimension B (Mathematical Modeling)*, three of the tasks were ranked as Satisfactory, but one of them (Group 4) was Acceptable. Thus, most of the tasks were appropriate for mathematical modeling. Pre-service teachers took attention to their tasks in terms of mathematical modeling. The theme of *mathematical modeling* was more dominant than interdisciplinarity for the tasks. For the task of *Group 1*, it met the criterion of *reality relation* (Wess et al., 2021) for modeling tasks since there was a real-world connection (renewable energy) outside of mathematics. This criterion was also mentioned by the pre-service teachers in the class during the rubric construction. However, it was named differently as *Real-life connection* and generated as a sub-criterion for the *Context of the Problem*. The problem also met the criterion of *relevance* (Wess et al., 2021) since energy consumption and food were relevant for every student's life. In addition, it met the *authenticity* criterion (Wess et al., 2021) because it went beyond routine problems, and it did not have one direct solution. However, the problem met the *openness* criterion (Wess et al., 2021) to some extent because there was a specification given in the problem statement that limited its openness ("Below are the steps to grow wheat"). Instead of giving the steps to the students at the beginning, students could be required to freely explore the problem and find relevant variables themselves. There was a certain level of openness because students had a chance to choose certain things in the problem (such as asking students to "Select a province in Turkey and find data for renewable energy capacity of the selected province"). However, since there was a specific guide for students to follow deliberate procedures, the openness criterion was not totally met.

According to the *rubric* constructed in the class, *interpretation and evaluation* of initial findings were emphasized for mathematical modeling in the task of *Group 1*. A separate

section called “Analysis and Evaluation” was separated for this purpose. However, the *limitations* of the model were *not* stated. *Appropriate assumptions* were also written in the report. *Use of mathematical models/mathematical data* was also included in the report. However, *different tools* (e.g., GeoGebra, Desmos) were *not* mentioned. *Variables* were emphasized, but the discussion of independent/dependent variables or the reason for including the variables were *not* written. The task appeared to be mostly *implementable* (usable by others), but the related discussion was *not* made up of the report. For *data collection*, links were given to support students. It is good to scaffold students, but this also limits the openness of the task, so the teacher needs to implement the task in a balanced way. There could be a student version of the task and teacher version that would guide teachers in the implementation of the task.

Considering *mathematical modeling*, for the task of *Group 2*, it met the *reality relation* criterion (Wess et al., 2021) for modeling tasks since there was a real-world connection (work and energy in constructing a building) outside of mathematics. This criterion was also mentioned by the pre-service teachers during the rubric construction (“Real-life connection” as a sub-criterion for the “Context of the Problem”). The problem also met the *relevance* criterion, (Wess et al., 2021) since considering one’s budget in building or buying a house could be relevant for students’ lives and could be interesting and motivating for students. In addition, it met the *authenticity* criterion (Wess et al., 2021) because it went beyond routine problems, and it did not have one direct solution. Moreover, the problem met the *openness* criterion (Wess et al., 2021) because it allowed different ways to solve the problem. There were several variables to consider, and the problem might have *multiple solutions* based on choosing different key variables as well as setting different assumptions. Hence, the problem could be said to be fruitful. As the pre-service teachers P3 and P4 noted, the problem was a mathematical modeling problem: This question is a mathematical modeling question designed for grades 11-12.

According to the rubric constructed in the class, concerning *mathematical modeling*, *variables* were highly emphasized in the task of *Group 2* since there was a section (“*Variables Expected to be Evaluated*”) that explained the variables in a detailed manner by including reasons for choosing the specified variables. For the *interpretation and evaluation* of initial findings, there was an interpretation of key variables in the report; however, not all

physics-related variables were explained with their limitations. For the *specification of the problem*, certain *assumptions* were made; but there were many variables, and not all of them were considered in terms of their *limitations*. For instance, about an assumption of the model, it is written in the report as the following: To limit the problem, we will assume that the acceleration caused by the force applied to the objects is neglected. *Use of mathematical models/mathematical data* was also included in the report. A model construction, use of graphs and geometric figures as well as mathematical calculations were contained. Moreover, the use of *different tools* (e.g., AutoCAD) was also mentioned in the introduction of the problem: You can benefit from the AutoCAD program. GeoGebra was also used to draw graphs related to the mathematical model. The task appeared to be mostly *implementable* (usable by others), but the related discussion was *not* made up of the report. A teaching guide in addition to the student version of the task could be prepared to facilitate the implementation of the task in a classroom. No information about *data collection* was included.

Considering *mathematical modeling*, for the task of *Group 3*, it met the *reality relation* criterion (Wess et al., 2021) for modeling tasks since there was a real-world connection (growing crops, using fertilizers and appropriate irrigations systems) outside of mathematics. This criterion also matched the criterion of “Real-life connection”, which was determined as a sub-criterion for the “Context of the Problem” by the pre-service teachers during the rubric construction. The problem also met the criterion of *relevance* (Wess et al., 2021) since growing crop was highly relevant for students’ lives. Agriculture was a part of our society and considering food from the beginning of the growing process to the product on the market could grasp students’ attention easily. Moreover, it met the *authenticity* criterion (Wess et al., 2021) since it went beyond routine problems, and it did not have one direct solution. Furthermore, the problem met the *openness* criterion (Wess et al., 2021) because it allowed different ways to solve the problem. There were different variables to consider, and the problem could have multiple solutions based on selecting certain key variables as well as specifying different assumptions.

According to the rubric constructed in the class, concerning *mathematical modeling*, *variables* were mentioned in the task of *Group 3*, especially in the solution part consisting of multiple steps. For instance, in the fourth step of the solution, the amount of fertilizer was

a key variable: Students search for the appropriate amount of fertilizer for 1 acre and calculate the cost of fertilization for each type of fertilizer. For the *interpretation and evaluation* of initial findings, variables were used to get a mathematical solution, but there was not an evaluation of variables considering their limitations. Only one solution was given for the problem, thus, there was not a comparison of different solutions for the problem by evaluating their efficiency. Moreover, certain *assumptions* were made, but the reasons for assumptions were not explained. In the fifth step of the solution, there was a note to teachers about assumptions: To make the question solvable, the teacher encourages students to make appropriate assumptions and walks around the groups to check the assumptions made and examine the solutions. *Use of mathematical models/mathematical data* was also included in the report. There was a mathematical model calculating profit by subtracting cost from revenue. Mathematical calculations were explicitly shown to reach the mathematical model. Moreover, rectangles were drawn to represent the field where seeds would be grown. However, the use of *different tools* was not mentioned. No explicit information was given about *data collection*, but students were expected to search for appropriate data to solve the problem as mentioned in the solution of the problem: Students find the price of the seeds from the internet, and they calculate the total price for seeds. The task was *implementable* (usable by others) as the steps of the possible solution indicate. The report gave hints for the teacher to implement the task, but a separate more detailed teaching guide could be added to facilitate the implementation of the task in a classroom:

The teacher gives students time to think about what data might be needed to solve the question and asks them indirect questions to guide them. For example, has anyone grown anything in the field before, what factors should we consider? discussed on many topics. After that, students start to try to solve the problem.

For *mathematical modeling*, the task of *Group 4* met the *reality relation* criterion (Wess et al., 2021) for modeling tasks since there was a real-world connection outside of mathematics (water footprint, water use). This criterion was also aligned with the *Real-life connection* in the rubric. The real-life connection was also emphasized in *context of the problem* part of the report:

The growing wheat and its yield are affected by some variables and students make a relationship with real life. For example, in rainy areas which can be the Mediterranean and Black Sea regions, they will think [to] produce the product on the farm.

The problem also met the criterion of *relevance* (Wess et al., 2021) because the use of water was an everyday activity that was also relevant for every student's life. Furthermore, it met the *authenticity* criterion (Wess et al., 2021) since mathematics was used in a real-life situation. In addition, the problem met the *openness* criterion (Wess et al., 2021) because it allowed different solutions.

According to the rubric constructed in the class, concerning *mathematical modeling*, *variables* were explicitly written in the task of *Group 4*: Other possible factors that affect the yield are soil, weather (temperature, moisture...), and types of kinds of wheat seeds that have different germination times. Especially, for different experiment designs, variables (independent, dependent, and conferring variables) were emphasized: "Keep kind of water constant. Select one type of water which can be green. Then change the type of wheat seeds. Constant variables: type of water, soil, weather". For the *interpretation and evaluation* of initial findings, there was an interpretation of variables in the report: "In conclusion: For this experiment, we all see much more green water decrease the yield of wheat, but have you considered that other factors may also affect the yield". A comment on results was also done: "They [Students] inferred that using much green water decreased the yield of wheat by graph and by their formulas". For the *specification of the problem*, certain *assumptions* were *implicitly* made such as taking the relationship between variables as linear; but assumptions were not addressed in terms of their *limitations*. The *use of mathematical models/mathematical data* was also included in the report. In the mathematical modeling part, *data collection* was explained: "These data are taken from the graph. The amount of blue and green water". For a mathematical model, equations (such as an equation of a line) and graphs were included. Furthermore, the use of *different tools* (e.g., Desmos, math lab, GeoGebra) was stated in the report: "If they [students] want to use different tools, they use for example GeoGebra or Desmos to show the relationship between the types of water and yield". There were certain notes to teachers on how to apply the task, so the task could be *implementable* (usable by others): "Teacher gives tips to direct the students", and "Teacher: Do you think that is the only factor that affects the yield of wheat". However, more details could be better to guide teachers in the implementation.

For *Dimension E (Expected Student Solutions)*, two tasks were ranked as Satisfactory (Groups 1 and 2) and the other two were ranked as Acceptable (Groups 3 and 4). All groups had difficulties in writing this dimension.

For the task of *Group 1*, two solutions were written, therefore *different ways of solving the problem* and *expected/possible student solutions* were noted. However, *assessment of solution ways* by students were not added. For the task of *Group 2*, there were expected students' solutions in terms of efficiency/cost and energy, respectively. Therefore, *different ways of solving the problem* and *expected/possible student solutions* were included. However, understanding the solutions were difficult, since the readability of the images was low. There was a short assessment of the solution about efficiency, however, it would be better if an extended evaluation of solution(s) had been done. For the task of *Group 3*, one *expected/possible student solution* was shown, but *different ways of solving the problem* were not included. An *expected/possible student solution* was mentioned in the task of *Group 4*, but *different ways of solving the problem* were not discussed except for various experimental design choices connected to the problem.

For *Dimension C (Organization of the Task)*, one task was ranked as Acceptable (Group 4) and three tasks were ranked as Satisfactory. For *Group 1*, concerning *organization of the task*, parts of the *introduction, development, and conclusion* were explicitly emphasized in the report. For instance, there was a *Conclusion* section at the end of the report: "If we add 2 solar panels for our farm, then we can produce adequate energy for daily energy needs. In addition, a wind turbine can provide sufficient energy". For *introducing the problem*, the problem statement was specifically written in the introduction of the report. However, concerning the *content of the problem*, the related *objectives* and *grade level of students* were not specified in the report. The *implementation* of the task was also not explained in the report. Neither the *procedure* of the task nor the *steps/phases of the task* were added. This part was related to teaching the task, and since pre-service teachers did not have sufficient teaching experience with students in the class, information on the implementation of the task was missing. Since the *tool use* was not mentioned, the concerning *justification of used tools* was also missing. The *reporting* criterion concerns the way of presentation of the task and can be matched with the "*Writing style and organization*"

dimension of the generic rubric (Garfunkel and Montgomery, 2019). The problem was presented well with good organization.

For *Group 2*, understanding the physics concepts of work, force, and energy was the main objective of the task concerning physics, but the *objectives* of physics for this task were not explicitly written. Moreover, concerning the dimension of the organization/structure of task, parts of the *introduction* and development were explicitly emphasized in the report. However, the *conclusion* part was not explicitly stated. For *introducing the problem*, the problem statement was specifically written in the introduction of the report. However, concerning the *content of the problem*, the related *objectives* were not specified in the report, but the *grade level of students* was explicitly written in the introduction (designed for grades 11-12). The *implementation* of the task was also not explained in the task of *Group 2*. Neither the *procedure* of the task nor the *steps/phases of the task* were added. This part was related to the pedagogy of the task, and since pre-service teachers did not have sufficient teaching experience with students in the class, information on the implementation of the task was insufficient. The *use of tools* such as AutoCAD was mentioned, but the *justification of used tools* was missing. The *reporting* criterion concerns the way of presentation of the task and can be matched with the “*Writing style and organization*” dimension of the generic rubric (Garfunkel and Montgomery, 2019). In general, the problem was presented well. However, the images were hard to read although graphs were easily identified. It was also good to have a reference section at the end of the paper. There were certain writing errors such as copying a section into a different place or incorrect writing of a tool. Furthermore, the pronoun of “he” was used in referring to an engineer without considering the gender emphasis. Thus, the language of the paper needed certain rewritings for proper academic language.

For *Group 3*, the *objectives* concerning science for this task were not written. Moreover, concerning the dimension of the organization/structure of task, parts of the *introduction*, *development* or *conclusion* were not explicitly emphasized in the report. However, after introducing the problem at the beginning of the report, possible steps of a solution were given in titles of first step, second step, etc. This could be regarded as the development part of the report. After the “last step”, there was a result sentence, which could be considered as a conclusion part: “At the end of the model they obtained, it is clearly seen that planting [capia] peppers is more than four times as profitable as planting [banana]

peppers”. Besides, concerning the *content of the problem*, the related *objectives* (science or mathematics) as well as the *grade level of students* were not written. The *implementation* of the task of *Group 3* was explained to a certain extent in the report. *Steps* of a possible solution were written and some hints for teachers were given therein: “In order to make the question solvable, the teacher encourages students to make appropriate assumptions and walks around the groups to check the assumptions made and examine solutions”. Neither the *use of tools* nor the *justification of used tools* was written. The *reporting* criterion is the way of presentation of the task (aligned with the “*Writing style and organization*” dimension of the generic rubric of Garfunkel and Montgomery, 2019). In general, the problem was presented well since it was easy to follow the steps given in titles and the language of the paper was understandable. It was also good to have a reference section at the end of the paper. There were also certain spelling errors of some words such as banana peppers.

Concerning the dimension of the *organization/structure of task*, parts of the *introduction*, *development* or *conclusion* were not explicitly emphasized in the report for the task of *Group 4*. However, in the introduction part, the problem statement was given. In the development part, possible solutions and different experiment designs were given. There was not an explicit conclusion part, but a possible student solution section could be considered as such. Besides, concerning the *content of the problem*, the related *objectives* were not given explicitly for the contributing disciplines as well as for the interdisciplinary part. In the organization part of the report, the general aim (as part of the criterion *clear definition of topic/task*) was noted: The aim of the question is to determine the factors that affect the yield of wheat. Moreover, for the *content*, it was explicitly noted: “The content will be introduced to the student and the needed information represented to evaluate the graph. The content is to identify the effect of the yield which is a type of water”. The *grade level of students* was written as middle school or high school. Furthermore, since the problem was given with two sub-questions, *task sequence* was used. About *referencing*, there were no references stated. Although there were certain writing errors, the report was easy to read. Since there were sections in the report, it was good at organization that made it more fluent. The class experience of constructing the rubric highly affected the organization of this report. The report explicitly examined the task based on the criteria generated in the previous lessons. The *implementation* of the task of *Group 4* was mostly explained in the report. The *steps/phases of the task* were explicitly written. As part of the *reporting* criterion, the

procedure of the task was clearly expressed: “The blue and green water are clearly explained. After the first question, students create a formula according to the graph. Then [they] evaluate the relationship between the blue and green water”. The *use of tools* such as Desmos, math lab, and GeoGebra was mentioned, but the *justification of used tools* was missing.

For *Dimension A (Level of Cognitive Demand)*, three tasks were ranked as Satisfactory, and one task was ranked as Acceptable (Group 1). The *Mathematical Analysis Task Guide* (Smith and Stein, 1998) and the *Task Analysis Guide in Science* (Tekkumru-Kisa et al., 2015) were used to analyze the level of cognitive demand of the tasks. Thus, the level of cognitive demand of the tasks was high in terms of mathematics, but lower in science in general.

Concerning the *level of cognitive demand*, the problem for the task of *Group 1* was a high-level mathematical task based on the *Mathematical Analysis Task Guide* (Smith and Stein, 1998), but presenting steps in the directions of the problem limited its openness, therefore decreasing its cognitive demand level. However, the level in science was lower since students were expected to use given tables about growing wheat. The level of cognitive demand in science could be considered as *Scripted Content* according to the *Task Analysis Guide in Science* (Tekkumru-Kisa et al., 2015). Facts and knowledge in science were asked to be used by given certain guidelines. Thus, the rank for this task (Acceptable) was categorized as lower than those of others.

Concerning the *level of cognitive demand*, the problem for the task of *Group 2* was a high-level mathematical task because it was an open-ended real-life problem based on the *Mathematical Analysis Task Guide* (Smith and Stein, 1998). However, in terms of physics, its level of demand could be higher because it was identified as *Guided Content* in terms of cognitive demand level according to the *Task Analysis Guide in Science* (Tekkumru-Kisa et al., 2015). It was the level of *Guided Content* since there was guidance for students to use work, force, and energy in the introduction of the problem: If you were to evaluate the project to be designed in terms of work force and energy, how would you change your project? Considering this, create a new project.

Concerning *the level of cognitive demand*, the problem for the task of *Group 3* was a high-level mathematical task because it was an open-ended real-life problem based on the *Mathematical Analysis Task Guide* (Smith and Stein, 1998). However, in terms of science (chemistry or agriculture), its level of demand was low since reproducing definitions, representations or formulas were required. The level of cognitive demand in science could be considered as *Memorized Content* according to the *Task Analysis Guide in Science* (Tekkumru-Kisa et al., 2015).

Concerning *the level of cognitive demand*, the problem of *Group 4* was a high-level mathematical task because it was a mathematical modeling problem based on the *Mathematical Analysis Task Guide* (Smith and Stein, 1998). However, giving directions for students decreased the openness of the problem, therefore decreased the level of cognitive demand. In terms of science, it was also not a low level of cognitive demand because designing experiments was expected from students. The level of cognitive demand in science could be considered as *Guided Integration* according to the *Task Analysis Guide in Science* (Tekkumru-Kisa et al., 2015) since there was a guidance for both understanding practices and specific content. As emphasized in the problem statement, students were required to “[d]esign an experiment to observe a relationship of types of water”. The dimensions of the rubric constructed in the class were used in the report to analyze their report. These dimensions were given in the main titles of *level of cognitive demand*, *mathematical modeling*, *organization*, *context of the problem*, and *expected/possible student solutions*. The pre-service teachers P1 and P8 wrote the reasons for the task having a high level of cognitive demand: “Then we decided on our problem’s level of cognitive which is high since the question includes mathematical data containing graphics, the only question is about yield by using graph, to determine the relationship between yield and its factors”. However, only these reasons do not guarantee the high level of cognitive demand. A better justification needs to be written in the report. The *objectives* concerning science were not written. Moreover, *data presentation* was limited to a given graph.

As a summary, the findings on the artifacts (constructed rubric and tasks) informed how the pre-service teachers designed interdisciplinary mathematical modeling tasks. The constructed rubric was considered one of the participant artifacts to address how the pre-service teachers designed interdisciplinary mathematical modeling tasks. As participant

artifacts in Cycle 2, the pre-service teachers constructed a guideline for designing interdisciplinary mathematical modeling tasks. This guideline was regarded as a *rubric*, which was named as the *Rubric for Interdisciplinary Mathematical Modeling Tasks* (see Appendix K). The findings about the rubric showed the key dimensions the pre-service teachers considered in designing interdisciplinary mathematical modeling tasks. These dimensions were level of cognitive demand, mathematical modeling, organization/structure of task, context of the problem, and expected/possible student solutions. The context of the problem focused on the interdisciplinarity of the tasks in general. The dimension of mathematical modeling emphasized mathematical modeling process and relevant sub-dimensions about mathematical modeling. Other dimensions (organization, level of cognitive demand and expected student solutions) were about teaching and focused on task structure and planning of tasks for the implementation in class.

Moreover, pre-service teachers' tasks as *artifacts* were analyzed to address how the pre-service teachers designed interdisciplinary mathematical modeling tasks. The *Task Analysis Framework for Interdisciplinary Mathematical Modeling Tasks* (see Table 4.6) was used for analyzing the tasks. The dimensions in the framework were context of the problem, mathematical modeling, expected student solutions, organization of the task, and the level of cognitive demand.

Concerning the *Context of the Problem*, tasks of the three groups were specified as satisfactory and another (Group 3) as acceptable. Based on the framework of Williams and his colleagues (2016), the interdisciplinary levels of the tasks were mostly multidisciplinary whereas one task (Group 3) was monodisciplinary. Besides, about the *Mathematical Modeling*, three of the tasks were specified as Satisfactory, but one of them (Group 4) was Acceptable. Thus, most of the tasks were appropriate for mathematical modeling.

About *Expected Student Solutions*, two tasks were specified as Satisfactory (Groups 1 and 2) and the other two were ranked as Acceptable (Groups 3 and 4). All groups had difficulties in this dimension. This was also aligned with the findings of the *post-interview*. Finding multiple solutions was noted to be a challenge for some pre-service teachers. Besides, concerning *Organization of the Task*, one task was specified as Acceptable (Group 4), and three tasks were ranked as Satisfactory. Besides, about *Level of Cognitive Demand*,

three tasks were specified as Satisfactory, and one task was ranked as Acceptable (Group 1). The *Mathematical Analysis Task Guide* (Smith and Stein, 1998) and the *Task Analysis Guide in Science* (Tekkumru-Kisa et al., 2015) were used to analyze the level of cognitive demand of the tasks. So, the level of cognitive demand of the tasks was identified as high in terms of mathematics, but lower in science in general.

In conclusion, the findings on the artifacts indicated that the designed tasks were mostly satisfactory overall with only one task that was identified as acceptable. According to the analysis based on the *Task Analysis Framework for Interdisciplinary Mathematical Modeling Tasks* (see Table 4.6), the task of Group 1 was identified as satisfactory in most of the dimensions but as acceptable in only one dimension (Dimension A: Level of Cognitive Demand). The task of Group 2 was specified as satisfactory in all dimensions. Moreover, the task of Group 3 was identified as satisfactory in most of the dimensions but as acceptable in some dimensions (Dimension D: Context of the Problem and Dimension E: Expected Student Solutions). Besides, the task of Group 4 was identified as satisfactory in some dimensions and as acceptable in other dimensions (Dimension B: Mathematical Modeling, Dimension C: Organization of the Task and Dimension E: Expected Student Solutions). All in all, pre-service teachers could design interdisciplinary mathematical modeling tasks although some aspects could be improved.

4.2.4. Findings in Cycle 2 on Design Principles

This section presented the findings for the research question regarding the design principles. Initially how the design principles were identified after Cycle 1 was mentioned. Then, to answer the *fourth research question* in Cycle 2, the findings based on the analysis of interviews, artifacts and researcher notes will be described.

After debriefing with the research team (thesis committee) based on the design of a course on interdisciplinary mathematics (after Cycle 1), the research focus was changed from designing a new course to designing a new *module* on interdisciplinary mathematical modeling tasks in Cycle 2. The change was based on the manageability of the research. The module on interdisciplinary mathematical modeling tasks intended to support pre-service mathematics and science teachers' development of skills for interdisciplinary teaching. The

module on interdisciplinary mathematical modeling tasks was called the *Professional Development Module* (PDM). The design in Cycle 2 aimed to examine how a Professional Development Module on interdisciplinary mathematical modeling tasks could support pre-service secondary mathematics and science teachers' learning to design interdisciplinary mathematical modeling tasks. The design principles were identified as the following: i) *Use problem-based learning*, ii) *use social constructivist pedagogy*, iii) *increase communication*, and iv) *support collaboration*. These design principles were called *Version 1* (V1).

The embodiment of the design principles was described through a conjecture map (Sandoval, 2014). The conjecture map that guided the Professional Development Module was shown in Figure 4.2. The embodiment in the conjecture map (see Figure 4.2) consists of course materials, different kinds of tasks, group work, scaffolding, and discussion. *Group work, scaffolding and discussion* were the core participant structures of the design that were kept the same through the different cycles but were named differently. In the initial conjecture map (for the initial course design) (see Figure 4.1), group work, scaffolding and discussion were considered under the participant structure of reflective learner as a pre-service teacher, but in the second conjecture map (see Figure 4.2), they were taken as separately to show the emphasis on each element. Moreover, *different kinds of tasks* were embodied as task structure in the design. Different kinds of tasks referred to tasks such as mathematical modeling tasks and interdisciplinary mathematical modeling tasks that would be applied in the classroom with pre-service teachers. Tools and materials are also important in this design and *course materials* were included as the design element. The course materials included materials such as articles, cognitive demand frameworks, and online learning environment. Different from the initial consideration of video cases as embodiment elements, the revised conjecture map (see Figure 4.2) did not include video cases. The reason for this exclusion was time management such that the initial planned tasks would take more time during the implementation. Similarly, the mediating process of *implementing the designed tasks with high school students* was excluded in the new design (see Figure 4.2). Based on debriefings with the research team, it was considered that pre-service teachers could not find an opportunity to administer their designed tasks with high school students during the module implementation. Ongoing analysis during the implementation also informed the decisions on design elements. Moreover, new mediating processes such as constructing rubric for assessing IMMTs was included in the revised design (see Figure 4.2).

The design principle of *increasing communication* would be reified in using feedback, discussion, reflection, and setting classroom norms. Another design principle of *supporting collaboration* would be embodied in using small group work for collaborative activities. The new design principle of *using problem-based learning* was in fact a part of providing authentic tasks, but it was more specific that time. Using problem-based learning would be embodied in using case-based tasks, and interdisciplinary mathematical modeling task criteria. Another design principle of *using social constructivist pedagogy* was expected to be embodied in active learning, scaffolding, teacher being as a model, considering previous experiences of participants, community of learners, and social interaction.

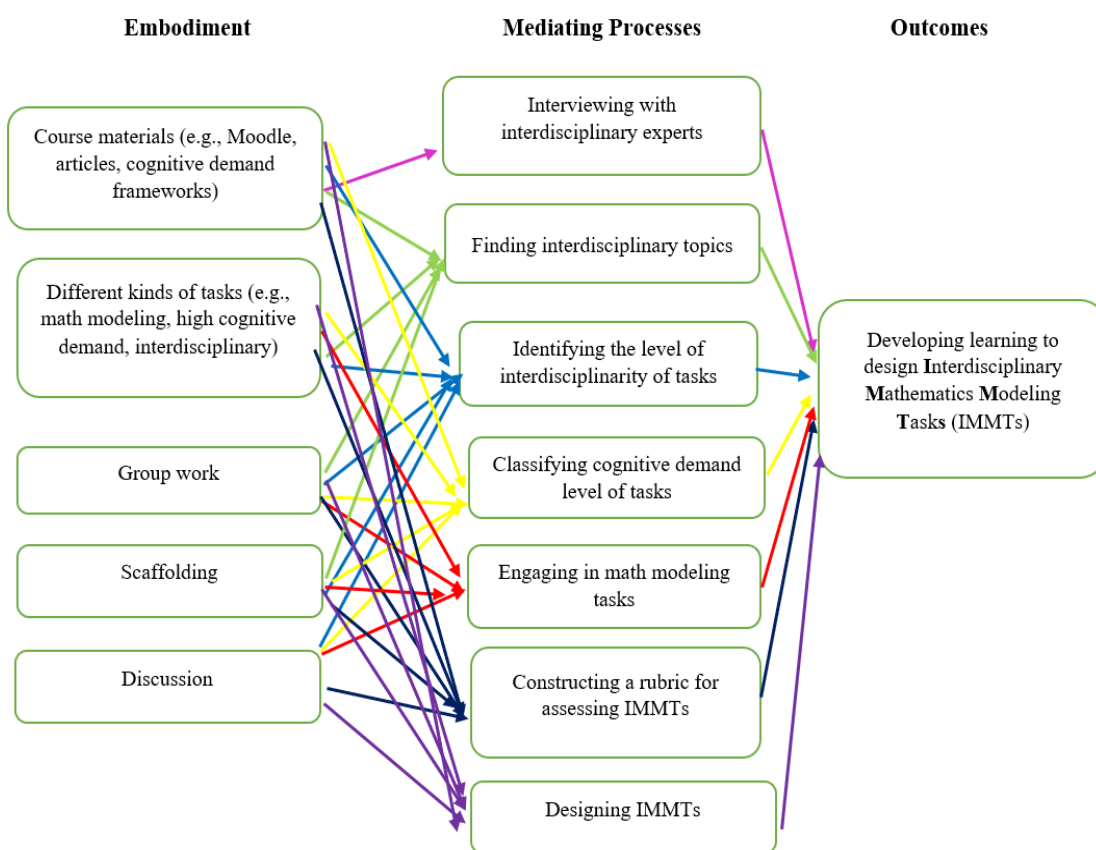


Figure 4.2. The revised conjecture map (cycle 2).

The design principles for the *Professional Development Module* were revised by considering analyses before the second iteration. These design principles were named

Version 2 (V2) and were identified as the following: *Enhance interdisciplinary problem-solving skill, facilitate collaboration, and enhance active learning experiences.*

In this design, understanding the cognitive demand level of tasks, interdisciplinarity, and mathematical modeling tasks were considered key elements of interdisciplinary mathematical modeling. Thus, the mediating processes of finding interdisciplinary topics, identifying the level of interdisciplinarity of tasks, classifying cognitive demand level of tasks, engaging in mathematical modeling tasks, constructing a rubric for assessing IMMTs, and designing IMMTs intended to reify the design principle on interdisciplinarity (*Enhance interdisciplinary problem-solving skills*). *Interdisciplinary competence* referred to the ability to consider different disciplinary viewpoints, to utilize different disciplinary viewpoints for solving interdisciplinary problems by constructing connections, to integrate knowledge across academic fields, and the ability to recognize the necessity to rethink on the orientation of a person's thinking and problem-solving methods (Brassler and Dettmers, 2017). As a part of interdisciplinary competence, considering different disciplinary viewpoints and utilizing different disciplinary aspects to solve problems and integrating content from different disciplines (Brassler and Dettmers, 2017) were taken as indicators for *interdisciplinary problem-solving*. In the broad sense, the identified mediating processes above aimed to develop interdisciplinary competence (Brassler and Dettmers, 2017). The outcome (*developing learning to design interdisciplinary mathematical modeling tasks*) in the revised Conjecture Map (see Figure 4.2) could also be considered as a sub-dimension to support interdisciplinary problem-solving skill since interdisciplinary competencies in mathematical modeling problems were needed to design IMMTs. The pre-service teachers needed to understand the characteristics of IMMTs and internalize the mathematical modeling process to design IMMTs.

The design principle on collaboration (*Facilitate collaboration*) was expected to be reified through the design feature of group work. Pre-service teachers would design their interdisciplinary mathematical modeling tasks in collaboration with their peers. By requiring working in groups with disciplinary majors, pre-service teachers were expected to work interdisciplinary and support each other in designing their tasks. During the class, pre-service teachers were also expected to work in groups with different majors (such as mathematics and science) to complete mathematical modeling tasks. Besides, the design principle

(*Enhance active learning experiences*) was expected to reify through the design features of group work, scaffolding, and discussion. The researcher as an instructor had the role of facilitator during the implementation. Scaffolding would be provided by the researcher in the class during the enactment of tasks and outside the class through extra sessions to help pre-service teachers design their final projects.

The design in Cycle 2 aimed to examine how a Professional Development Module on interdisciplinary mathematical modeling tasks could support pre-service secondary mathematics and science teachers' learning to design interdisciplinary mathematical modeling tasks. The design principles as the first version (DP-V1) were as the following: *Use problem-based learning, support collaboration, increase communication, and use social constructivist pedagogy*. After Cycle 2 before transitioning to the Cycle 3, the design principle on collaboration was saved (*Facilitate collaboration*) since group work was important in the design of the module for interdisciplinary teaching. The design principles on social constructivist pedagogy and communication evolved into the design principle on active learning experiences for teacher preparation (*Enhance active learning experiences*). Furthermore, the design principle on problem-based learning evolved into a more specific design principle (*Enhance interdisciplinary problem-solving skills*). The second version of the design principles (DP-V2) were as the following: *Enhance interdisciplinary problem-solving skills, facilitate collaboration, and enhance active learning experiences*. The evolution of the design principles from V1 to V2 was described based on the findings of Cycle 2 to answer the *fourth research question*.

The designed *tasks* as participant *artifacts* in Cycle 2 informed the change in the design principles. The design principle (DP-V1) of *using problem-based learning* was expected to embody in the design feature of *interdisciplinary mathematical modeling task criteria*. This design feature was a part of course materials. The criteria consisted of interdisciplinarity framework (Williams et al., 2016), mathematical modeling task criteria (Wess et al., 2021), and level of cognitive demand (Smith and Stein, 1998; Tekkumru-Kisa et al., 2015). The designed tasks were analyzed based on this design feature.

Based on the interdisciplinarity framework (Williams et al., 2016), the designed tasks in Cycle 2 were identified as mostly interdisciplinary level and only one task (of Group 3)

was identified as monodisciplinary level. In terms of the interdisciplinary framework, the lowest level was monodisciplinary and then the next level was multidisciplinary. Thus, although none of the tasks were identified as higher levels in the interdisciplinarity continuum, most of the tasks were at an intermediate level of interdisciplinarity.

Moreover, based on the mathematical modeling task criteria (Wess et al., 2021), most of the designed tasks in Cycle 2 met the criteria of mathematical modeling. Three tasks met all the criteria (reality relation, relevance, authenticity, and openness) although one task (of Group 1) did not totally meet the criterion of openness. The criterion of openness for the task of Group 1 was identified as partially met.

In addition, based on the cognitive demand frameworks (Smith and Stein, 1998; Tekkumru-Kisa et al., 2015), the designed tasks were identified as high in terms of mathematics. However, the level of cognitive demand in science showed variance, and mostly lower compared to mathematics. The level of cognitive demand in science for the examined tasks was identified as scripted content, guided content, memorized content, and guided integration (Tekkumru-Kisa et al., 2015). Guided content and guided integration were higher levels of cognitive demand in science compared to the other identified levels. Using mathematical modeling and satisfying the mathematical modeling task criteria indicated a high level of cognitive demand in mathematics. In terms of science, although the levels of some tasks were lower, those of others (two tasks) were identified as higher for cognitive demand.

Thus, considering all the three criteria for the embodiment of the design principle, since the tasks mostly met the criteria, this design feature (*interdisciplinary mathematical modeling task criteria*) could be said to *mostly support* the design principle (*Use problem-based learning*).

The design principle of *using problem-based learning* (DP-V1) was also considered to be embodied in the design feature of *different kinds of tasks* (especially case-based tasks, namely example tasks). As revealed in the *post-interviews*, most of the pre-service teachers conceptualized (e.g., P3, P4, P5, P8 and P9) the class *examples* and tasks as helpful and powerful aspects of the design. For instance, the pre-service teacher P4 highlighted the use

of examples in the post-interview: We believed that what we read was true. Of course it was true, but it was limited information. Then you have developed all of them with *examples*. That was very nice (10th question). Another pre-service teacher (P8) also emphasized the use of examples in the post-interview: I mean, it's good to have examples like this, *examples from different fields* (10th question). In the post-interview, another pre-service teacher (P9) added that the tasks implemented in the course had to be kept the same: Right, there was the task of reducing the ecological footprint. No, we did it, we did some *good tasks*. I think the *tasks* should stay the same (10th question). Thus, the analysis of the interviews revealed that the design feature of *different kinds of tasks* facilitated *using problem-based learning*.

Moreover, as the *researcher notes* indicated that engaging in different mathematical modeling *tasks* facilitated pre-service teachers' skills in problem-solving. Being able to solve mathematical modeling problems (especially by producing mathematical models) was considered an indicator of interdisciplinary mathematical modeling problem-solving. Based on the class observations, the pre-service teachers were unfamiliar with the mathematical modeling tasks at the beginning and could not understand where they had to start to solve the problems since they thought that the problems were lack of information for solution. The pre-service teachers were not accustomed to solving open-ended problems at the beginning. Through scaffolding, the researcher guided the pre-service teachers to think on assumptions to narrow down the mathematical modeling problem. Trying to solve the problems as learners, the pre-service teachers could produce mathematical models as solutions for the tasks implemented in the class. As the examples increased in the number during the class, the pre-service teachers got familiar with the mathematical modeling process and solved the mathematical modeling tasks easier. Thus, the analysis of the *researcher notes* revealed that the design feature of *different kinds of tasks* facilitated *using problem-based learning*.

All in all, the *interviews* and *researcher notes* indicated that the design feature of *different kinds of tasks* facilitated *using problem-based learning*. Thus, it was decided to continue using different kinds of tasks in the next iteration. Moreover, based on the *task* analysis in terms of interdisciplinarity for Cycle 2, since the levels of the tasks were mostly multidisciplinary, the design principle of *using problem-based learning* (DP-V1) was changed to the design principle of *enhancing interdisciplinary problem-solving skills* (DP-

V2). To increase the focus on interdisciplinarity and to support the pre-service teachers in designing interdisciplinary tasks, the change in the former design principle was done.

Another design principle on collaboration was *supporting collaboration* (DP-V1). This design principle was expected to be embodied in the design feature of using *small group work*. The design feature of small group work was searched in the post-interviews. The findings of the *post-interviews* about small group work indicated that some pre-service teachers emphasized the benefit of group work and collaboration (e.g., P1, P2 and P8). For example, in the post-interview, the pre-service teacher P2 emphasized working in groups: So, *working in different ways, in groups*, and then telling each other, oh, look, there's something wrong or this could be like that, it was good for me (11th question). In the post-interview, the pre-service teacher P8 also highlighted the importance of collaboration as a life skill: It's something that will last a lifetime. It helped me in that respect. It showed me the importance of working in *collaboration* (11th question). Learning from others and exchanging ideas were seen as helpful by the pre-service teacher P1 as expressed in the post-interview: That's why being *together with our friends* who are in different fields, both physics, chemistry, science and mathematics, will be beneficial for us both now and in the future (11th question). Especially, working with peers of different disciplines was considered important to conduct the tasks. Moreover, sharing ideas in a small group like that of at most three members was regarded easier as expressed by the pre-service teacher P1 in the post-interview: I think there should be *at most three. Two or three* would be good. For example, if there were four people, I think it wouldn't have happened at all (11th question). Thus, the interviews indicated that using small group work could facilitate pre-service teachers' learning to design interdisciplinary mathematical modeling tasks.

The *researcher notes* also supported the use of collaboration in helping pre-service teachers to solve mathematical modeling tasks as well as to design tasks. Based on the researcher notes, the pre-service teachers could design tasks as their final projects by working in small groups such as groups with two members. Moreover, the researcher notes (based on classroom observations) indicated that when the pre-service teachers were required to individually solve mathematical modeling problems in the class, they had difficulties in solving the problems and it took more time for them to arrive at a conclusion. Thus, the findings on the interviews and researcher notes in Cycle 2 supported the design

principle: *Support collaboration* (DP-V1). This design principle was kept the same for the next iteration with only some wording change: *Facilitate collaboration* (DP-V2).

The design principle of *increasing communication* (DP-V1) was expected to be reified in the design feature of *discussion*. Based on the *researcher notes*, pre-service teachers could be able to construct a *rubric* for assessing the IMMTs as a product of whole-class discussion. The researcher was a facilitator during the whole-class discussion. The rubric (as an *artifact*) facilitated the design of the interdisciplinary mathematical modeling tasks since the pre-service teachers used the dimensions in the constructed rubric in their designed *tasks*. Besides, in the *post-interviews*, some pre-service teachers (e.g., P5 and P6) also emphasized constructing the rubric as a helpful experience. For example, in the post-interview, the pre-service teacher P6 described the rubric construction activity occurred in the class: When we last created *rubric*, for example, even though you didn't say anything, we came up with so many ideas that we could never do this if we thought of it alone, in my opinion (10th question).

Another design principle of *using social constructivist pedagogy* (DP-V1) was expected to reify through the design features of *group work*, *scaffolding*, and *discussion*. The researcher as an instructor had the role of a facilitator during the course. Social constructivism underlies the instruction of the course. Thus, students were expected to be *active* in their learning processes. Based on the *researcher notes*, the pre-service teachers negotiated their ideas and actively performed the tasks in the class while they were working on the mathematical modeling tasks in *groups*. The pre-service teachers were very active during the course activities such that they did not even mention the need for breaks without completing the tasks (based on the researcher notes). Moreover, based on the *researcher notes*, the researcher provided *scaffolding* during pre-service teachers' engagement in class tasks as well as designing their final projects. The researcher also conducted some additional online sessions as scaffolding to help some of the pre-service teachers design their tasks. Moreover, the researcher notes indicated that the *discussions* were mediators to help the pre-service teachers freely express their ideas in the class.

Besides, in the *post-interviews*, most of the pre-service teachers (P3, P4, P5, P6) emphasized the *student-centered* instructional method as a strength of the course. For

example, in the post-interview, the pre-service teacher P4 focused on the active learning experience in the class: I think it was very enjoyable. Because we did everything ourselves. So, it was educational (10th question). Moreover, in the post-interview, another pre-service teacher P6 stressed on constructing their own ideas, which was aligned with social constructivism: Yes, I think that thing is very nice. You know, we are given time to think. We think for ourselves, we come up with *our own ideas* (10th question). In the post-interview, the pre-service teacher P1 also emphasized the interactive nature of the course and the relevant learning experience:

It is very beneficial for us to be constantly *interactive* in the lesson, especially if it is a lesson where students are more *active* and play a role. Because we can hear, *discuss* and evaluate not only our own ideas but also the ideas of others. We could also see how useful or useless, right or wrong the criteria we were evaluating were. For example, I learned how to evaluate the words and sentences of each friend I evaluated. (11th question)

In the *post-interviews*, the comfortable class climate was also emphasized. The post-interviews indicated that some pre-service teachers (e.g., P2 and P5) viewed the comfortable class climate as beneficial. As the *researcher notes* also indicated the pre-service teachers easily talked and expressed their ideas in the course environment. For example, in the post-interview, the pre-service teacher P5 focused on the active learning in a comfortable learning environment and emphasized working in a *small group*:

I think there is a very *comfortable environment* in the class. Because there are already 8-10 of us and everyone knows each other. Everyone is extremely comfortable. We can go out and make presentations easily and we can express our opinions directly. When creating rubrics, I mean everyone says something. And for example, when placing them, especially when placing them under the headings, if there are too many people, I don't know, this time there could be a lot of confusion, I think. Because it was a multi-faceted picture and I think a common decision could be reached easily. I think there wouldn't be so many comments when there were more people. Silence would prevail more. I think the quota should have been this much if this course were opened again. (10th question)

In the post-interview, the pre-service teacher P2 also emphasized the active learning experiences of the pre-service teachers during the class: I mean, instead of learning something theory, learning this, memorizing that, we felt *more active*, that's how I think. My friends probably think that, too (11th question).

Thus, based on these findings, the design principle of *increasing communication* (DP-V1) and the design principle of *using social constructivist pedagogy* (DP-V1) were supported. However, they were combined in the new design principle of *Enhance active*

learning experiences (DP-V2) in the next iteration due to their close connection with each other.

As a summary, after the implementation of the *Professional Development Module* in Cycle 2, the findings indicated that using problem-based learning, using social constructivist pedagogy, increasing communication and collaboration were important in designing a module on interdisciplinary teaching. However, although the key aspects were not changed, certain revisions on the design principles were done to increase the specificity and help the pre-service teachers design interdisciplinary tasks. The design principles for the *Professional Development Module* were revised by considering analyses before the second iteration. Thus, based on Cycle 2, the revised design principles were named Version 2 (V2) and were identified as the following: *Enhance interdisciplinary problem-solving skill, facilitate collaboration, and enhance active learning experiences.*

4.3. The Findings for Cycle 3

This section presents the findings for Cycle 3. Cycle 3 was a part of the Design and Construction Phase in which the second iteration of the implementation was developed based on the findings of the previous cycles (particularly Cycle 2). Initially, findings on conceptualizations of the pre-service teachers on interdisciplinarity, interdisciplinary education and mathematical modeling will be explained. Then, the findings on designing interdisciplinary mathematical problems for teaching and design principles for a module on interdisciplinary teaching will be described. The following research questions were examined in Cycle 3:

- RQ 5. How did the *conceptualizations* of the pre-service mathematics and science teachers on interdisciplinary teaching change during the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) within the course of Interdisciplinary Mathematics and Science Education during Fall 2024?
- RQ 6. How did the pre-service mathematics and science teachers who participated in the Program on Designing Interdisciplinary Mathematical Problems (PDIMP) design *interdisciplinary mathematical problems for teaching* (IMPTs)?

- RQ 7. What are the *design principles* of a module on interdisciplinary teaching?

Initially, participants will be introduced. Then, to answer the research questions, conceptualizations of the pre-service teachers on interdisciplinarity, interdisciplinary education and mathematical modeling will be described. After this, findings on designing interdisciplinary problems and design principles will be explained.

Participants in Cycle 3 consisted of four pre-service mathematics and science teachers who enrolled in the course on interdisciplinary mathematics and science teaching (SCED 48F) in Fall 2024. This course was the second version of the course in Fall 2023. The module on interdisciplinary teaching (The Program on Designing Interdisciplinary Mathematical Problems-PDIMP) in the context of this course was the second iteration. There were less participants in this iteration because the course was elective and there were conflicts with other courses. Pre-service teachers anonymously named as T1, T2, T3, and T4. Based on the Demographic Information Form, the major of the pre-service teachers T1, T2 and T3 was Primary School Mathematics Teaching. The major of the pre-service teacher T3 was Science Teaching. Three of the pre-service teachers (T1, T2, T3) graduated from an Anatolian High School, and one pre-service teacher (T4) graduated from a Science High School. None of the pre-service teachers had a course or a kind of professional education on mathematical modeling or interdisciplinary mathematical modeling before. They did not design any tasks before either. However, three of the pre-service teachers (T1, T3, and T4) had experience of giving private lessons. The following paragraphs explain the findings on *conceptualizations* of pre-service teachers before and after the implementation.

4.3.1. Findings for Conceptualizations in Cycle 3

In Cycle 3, it was investigated how the pre-service mathematics and science teachers' conceptualizations on interdisciplinary teaching changed during the implementation of the module on interdisciplinary teaching (The Program on Designing Interdisciplinary Mathematical Problems). To investigate the *fifth research question*, findings on the conceptualizations on interdisciplinarity, interdisciplinary education, and mathematical modeling will be presented initially. The conceptualizations of the pre-service teachers based on interviews and surveys were described in the following paragraphs.

Pre-service teachers' *conceptualizations* on interdisciplinarity were defined as pre-service teachers' perspectives about interdisciplinarity. Surveys were used to get individual information about pre-service teachers' conceptualizations on interdisciplinarity. Pre-service teachers were asked to define interdisciplinarity in the initial surveys. Some pre-service teachers initially had basic conceptualizations on interdisciplinarity. For instance, interdisciplinarity was considered basic relations between different disciplines without mentioning the nature of such relations by the pre-service teacher T2 in initial surveys: "Interdisciplinarity is about putting together the information about more than one discipline and showing the connections between them". This conceptualization is aligned with the perspective of *interdisciplinarity as connections* derived from Cycle 1. *Interdisciplinarity as connections* is defined as connections between disciplines (based on data in Cycle 1). This is a very broad definition just focusing on the connections. Another basic conceptualization is interdisciplinarity as an area. *Interdisciplinarity as an area* is defined as an area, a concept or a topic requiring different knowledge or comprehension from different disciplines (based on data in Cycle 1). The pre-service teacher T4 conceptualized interdisciplinarity as an area because the areas of science, technology, engineering, and mathematics were taken as related disciplines for interdisciplinarity:

T4: There are four disciplines in education such as science, technology, engineering, mathematics. These four disciplines teach students different areas. But these areas are not separate from each other, therefore, there is a need for an education system which is interdisciplinary.

T4 indeed mentioned the disciplines for STEM and associated interdisciplinarity with STEM. The connection of disciplines was emphasized by noting the disciplines were not separate from each other, that is the disciplines were related. Interdisciplinarity was also associated with the educational system in a broader sense. However, T4 could not give a specific definition of interdisciplinarity. This conceptualization of interdisciplinarity was broad.

Initially, pre-service teachers mostly considered interdisciplinarity to solve problems (T2, T4). This is aligned with the interdisciplinary perspective revealed in Cycle 1 (first reflection) as interdisciplinarity as *problem-solving*. *Interdisciplinarity as problem solving* is defined as solving a problem that requires more than one discipline. This definition was

derived based on the data on Cycle 1. For instance, the pre-service teacher T2 noted that “It’s usually about doing research in interdisciplinary areas to find solutions to problems” (pre-survey). The pre-service teacher T4 also considered interdisciplinarity as problem solving, especially focusing on the purpose of solving complex problems in real-world: “Interdisciplinary research is research using methods, techniques, theories, [and] tools of two or more disciplines instead of a single discipline to solve complex problems of nature and society” (pre-survey). The definition of interdisciplinarity by the pre-service teacher T4 was like the definition of interdisciplinary research by National Academy of Sciences, National Academy of Engineering, and Institute of Medicine [NAS] (2005):

a mode of research by teams or individuals that integrates information, data, techniques, tools, perspectives, concepts, and/or theories from two or more disciplines or bodies of specialized knowledge to advance fundamental understanding or to solve problems whose solutions are beyond the scope of a single discipline or field of research practice. (NAS, 2005, p. 26)

Benefiting from different disciplinary insights to solve hard problems which cannot be dealt by one discipline has been emphasized by T4 as in the definition. Working together is another feature of the definition for interdisciplinarity. Interdisciplinary research is usually done by a research team (NAS, 2005). This feature was also mentioned in the pre-survey by one pre-service teacher (T1) in describing interdisciplinarity in the context of interdisciplinary research:

T1: Interdisciplinary research refers [to] disciplines to work together. Specifically, some disciplines come together and work on the study collaboratively. They don’t do their work separately. Sometimes, at the end of the study, a new discipline may be created by the other disciplines.

Collaboration or working together to do interdisciplinary research was also mentioned in Cycle 1. *Interdisciplinarity as working together* is defined as working together of different disciplines on a common problem (based on data from Introduction and First Reflection in Cycle 1). There is a mutual relationship between disciplines working together, therefore implying mutual benefits of contributing disciplines. The perspective of the pre-service teacher T3 in the pre-survey was also like interdisciplinarity as working together since the metaphor of blood could be represented as a necessary entity for disciplines: “If we describe disciplines as organs, interdisciplinary will be blood”. This indicates an interdependent relationship with disciplines (where organs are represented as disciplines in this metaphor). As a result of working together interdependently, a *new discipline* can be produced as also

emphasized by T1. This new discipline creation is also aligned with the perspective of *evolution of disciplines* revealed in Cycle 1. In this perspective, interdisciplinarity is defined as an evolution of disciplines that come together to become one from several disciplines. This definition was derived from the data from the First Reflection in Cycle 1. Similarly, the creation of a new discipline through mutual interaction between disciplines was emphasized by the pre-service teacher T1 in the initial surveys on interdisciplinarity:

T1: We can say that different branches of science support each other. In other words, a new discovery in one branch of science can lead to another discovery in another field. Cumulatively accumulated information can lead to the formation of new branches of science in the future, while feeding existing science even more and leading to further development.

In short, *initial* conceptualizations of the pre-service teachers on interdisciplinarity were often broad conceptualizations such as interdisciplinarity as *connections* or interdisciplinarity as an *area* (e.g., T1, T2, and T4). Moreover, interdisciplinarity was initially conceptualized as *working together* of disciplines (e.g., T1 and T3). Herein, the collaborative nature of interdisciplinarity was highlighted. Other initial conceptualizations on interdisciplinarity were interdisciplinarity as *evolution of disciplines* (e.g., T1) and interdisciplinarity as *problem solving* (e.g., T2 and T4). The generation of a new disciplines out of interdisciplinarity was emphasized in the conceptualization that viewed interdisciplinarity as evolution of disciplines. Moreover, the goal of solving a complex problem through interdisciplinarity was the focus of the conceptualization that viewed interdisciplinarity as problem solving.

Pre-service teachers' conceptualizations on interdisciplinarity *after* the implementation were investigated through the post-interviews and post-surveys. The following parts described the findings on conceptualizations through the post-surveys and post-interviews. The post-conceptualizations on interdisciplinarity through the post-surveys were shown in Table 4.8.

Table 4.8. Post-conceptualizations on interdisciplinarity (post-survey).

Participants	Excerpts	Conceptualization
T1	T1: Interdisciplinarity refers to the <i>integration</i> and <i>collaboration</i> of knowledge, methods, and perspectives from different academic disciplines. Let's consider a project focused on addressing environmental sustainability. In this interdisciplinary approach, experts from different fields such as ecology, environmental science, economics, sociology, and engineering might collaborate to develop <i>comprehensive solutions</i> .	• Interdisciplinarity as problem solving • Interdisciplinarity as working together • Interdisciplinarity as a combination
T3	T3: For me, interdisciplinarity means combining different fields <i>to solve a problem</i> or approach a topic from <i>multiple perspectives</i> . For example, solving an environmental pollution problem can involve biology, chemistry, and mathematics. This way, the problem is addressed in a more <i>comprehensive</i> manner.	• Interdisciplinarity as problem solving • Interdisciplinarity as a combination • Multiple perspectives
T4	T4: Interdisciplinarity can be defined as the <i>integration</i> and <i>collaboration</i> of different disciplines or fields of study <i>to solve complex problems</i> that cannot be solved with single discipline. It combines <i>multiple disciplines</i> to solve problems. For example, sustainability is [an] interdisciplinary area. Because it is a complex real-life problem with ecological, economic, social and other different sides.	• Interdisciplinarity as problem solving • Interdisciplinarity as a combination • Multiple perspectives • Interdisciplinarity as working together

In the *post-survey*, pre-service teachers were asked to answer the question “How would you define interdisciplinarity? Give at least one example to interdisciplinarity and explain how this example fits with interdisciplinarity” to learn about their conceptualizations on interdisciplinarity after the implementation. Three pre-service teachers (T1, T3, and T4) answered the question. Most of the pre-service teachers (T1, T3, and T4) conceptualized *interdisciplinarity as problem solving* as revealed in their responses to post-surveys (see Table 4.8). The purpose of interdisciplinarity to solve complex problems was emphasized in their responses. Moreover, most of the pre-service teachers (T1, T3, and T4) also conceptualized *interdisciplinarity as a combination*. Interdisciplinarity was considered as

combining different disciplines by integrating different aspects of the disciplines. This is aligned with the literature on defining interdisciplinarity through the concept of integration. *Integration* has been viewed as the key feature of interdisciplinarity (e.g., Boix Mansilla et al., 2000; Repko, 2007). For instance, Boix Mansilla, Miller, and Gardner (2000) defined interdisciplinarity by using integration of certain cognitive elements to solve a problem:

The capacity to integrate knowledge and modes of thinking in two or more disciplines or established areas of expertise to produce a cognitive advancement – such as explaining a phenomenon, solving a problem, or creating a product – in ways that would have been impossible or unlikely through single disciplinary means. (Boix Mansilla et al., 2000, p. 17)

Integration and problem solving are taken together in this definition of interdisciplinarity like the pre-service teachers' conceptualizations (T1, T3 and T4) of interdisciplinarity. Integration represents the process and problem solving represents the purpose with an interdisciplinary solution to the intended problem serving as an outcome. Moreover, *interdisciplinarity as working together* was more emphasized by some pre-service teachers (T1 and T4). The collaboration focused on the module implementation in Cycle 3 could be influential in this conceptualization. The collaboration in interdisciplinarity was also noted in the literature (e.g., Spelt, 2015). Collaboration was considered an interaction dimension which includes “the multiple interactions with disciplinarians sharing disciplinary knowledge in an open manner in order to learn from each other and to come to a shared understanding (social processes)” (Spelt, 2015, p. 86).

Moreover, some pre-service teachers (T3, T4) conceptualized interdisciplinarity as *multiple perspectives*. This perspective of interdisciplinarity was not emphasized before the implementation. Thus, this is a new perspective that has been explicitly highlighted by some pre-service teachers. This is also parallel to the description of integrated elements in interdisciplinarity. Perspectives were regarded as integrated elements for interdisciplinarity as also explained by Szostak (2003) by noting the integration of “phenomena, theories, methods, and worldviews” (p. 47).

The post-conceptualizations of the pre-service teachers on *interdisciplinarity* were also investigated through the post-interviews (see Table 4.9). In the *post-interview*, pre-service teachers were asked for the definition of interdisciplinarity, interdisciplinary thinking (with

relevant examples if possible), interdisciplinary mathematics/science education as well as ways of implementing such education in class as a teacher.

Table 4.9. Post-conceptualizations on interdisciplinarity (post-interview).

Participants	Excerpts	Conceptualization
T1	T1: When we deal with an event or a concept, interdisciplinary approach is a <i>process</i> in which we search for <i>solutions</i> or analyze or think over not just one discipline but many disciplines. Interdisciplinary is an area in which we benefit from different disciplines and their applications if needed (but not just one discipline) to explain something or to look for solutions.	• Interdisciplinarity as problem solving
T2	T2: The concept of interdisciplinarity is knowledge which one discipline cannot sustain. We don't just benefit from disciplines, [but] disciplines <i>develop together</i> to form a common discipline. Such an <i>area</i> can be defined as interdisciplinary. Interdisciplinary thinking is thinking such that developing knowledge for a common goal and thinking how to reach that common goal.	• Interdisciplinarity as a combination • Interdisciplinarity as evolution of disciplines
T3	T3: Interdisciplinarity was a thing that we have known but also was a concept that we did not know. [Interdisciplinarity] is the <i>relation</i> among disciplines within the <i>solution of a problem</i> or definition.	• Interdisciplinarity as problem solving
T4	T4: Interdisciplinarity is an important concept for education because some work in real life could have difficulties in explaining by one discipline. So, there are works on which more than one discipline needs to <i>work together</i> . Namely, several <i>areas</i> can be interdisciplinary (e.g., Sustainability, climate change).	• Interdisciplinarity as working together

Based on the post-interview, pre-service teachers' conceptualizations were like those revealed in the post-survey (see Table 4.9). The conceptualizations of interdisciplinarity as problem solving (T1, T3), interdisciplinarity as combination (T2), and interdisciplinarity as working together (T4) were also revealed in the post-survey. In the post-interview, interdisciplinarity as evolution of disciplines (T2) emerged. As a summary, Table 4.10 shows the comparison of conceptualizations on interdisciplinarity before and after the module.

Table 4.10. Conceptualizations on interdisciplinarity.

Participants	Pre-conceptualizations	Post-conceptualizations
T1	• Interdisciplinarity as working together • Interdisciplinarity as evolution of disciplines • Interdisciplinarity as an area	• Interdisciplinarity as working together • Interdisciplinarity as a combination • Interdisciplinarity as problem solving
T2	• Interdisciplinarity as problem solving • Interdisciplinarity as connections	• Interdisciplinarity as a combination • Interdisciplinarity as evolution of disciplines
T3	• Interdisciplinarity as working together	• Interdisciplinarity as problem solving • Interdisciplinarity as a combination • Multiple perspectives
T4	• Interdisciplinarity as problem solving • Interdisciplinarity as an area	• Interdisciplinarity as problem solving • Interdisciplinarity as a combination • Multiple perspectives • Interdisciplinarity as working together

To summarize, some pre-service teachers (T1, T2, T4) had basic pre-conceptualizations on interdisciplinarity such as interdisciplinarity as an area or interdisciplinarity as connections. In these initial conceptualizations, interdisciplinarity was viewed as interrelated areas. Interdisciplinarity as problem solving was emphasized more in post-conceptualizations. Namely, pre-service teachers focused on the aim of interdisciplinarity as problem solving more for post-conceptualizations. Furthermore, new conceptualizations such as multiple perspectives emerged in the post-conceptualizations. Different elements of interdisciplinarity emerged as different perspectives on interdisciplinarity and different perspectives came together in the conceptions. For example, the collaborative character of interdisciplinarity was considered as working together and the purpose of solving complex problems was considered as problem solving. Such key aspects of defining interdisciplinarity revealed together in the post-conceptualizations of some pre-service teachers (T1, T4). The post-conceptualizations were mostly aligned with the reflections and discussions on the articles about interdisciplinarity (e.g., NAS, 2005). These

materials were a part of the design. As pre- and post-conceptualizations were compared, it was observed that there was not a sharp change in the conceptualizations. This might be due to the gradual development of understanding interdisciplinarity, which is a complex concept to theoretically internalize. Moreover, it might be due to the short duration of the implementation to make a deeper conceptualization.

Pre-service teachers were also asked about *interdisciplinary teaching*. To get information about their initial conceptualizations, pre-service teachers were required to reflect on “Why do you think a mathematics teacher needs to have interdisciplinarity thinking?” and “How would you use interdisciplinarity in your teaching?” before the implementation. Table 4.11 shows initial conceptualizations on interdisciplinary education.

Table 4.11. Initial conceptualizations on interdisciplinary education.

Participants	Excerpts	Conceptualization
T1	As a math teacher, we need to have interdisciplinary thinking because many disciplines feed each other. There are also <i>connections</i> with each other. Sometimes, for teaching some topics, we need to mention other topics from <i>other disciplines</i>. We need to teach math [in] different aspects. Then, students can <i>apply</i> the math topic to other lessons such as science class. Since my field is mathematics, we can provide teaching on how mathematics and physics support each other.	• Connections • Use of mathematics
T2	Mathematics as a lesson is always a <i>hard</i> subject for students to understand. Students usually cannot connect math with [the] <i>real world</i> and that affects their attention to class. A math teacher needs to have interdisciplinary thinking so that s/he can show and teach the <i>importance of math</i> in other areas. Interdisciplinary areas are that different disciplines come together to <i>create new disciplines</i> and show mathematics is essential in many of them a math teacher needs to have interdisciplinary thinking to teach. I would use the connections between mathematics and science as a reasoning to how people use the knowledge we learn in class and why it is necessary.	• Connections • Use of mathematics
T4	I would use different disciplines together. I will try to do an activity in class that shows examples of these disciplines being used together in <i>real-life</i>.	• Connections

Connections and *use of mathematics* were the prevalent initial conceptualizations on interdisciplinary teaching. *Connections* refer to using connections between disciplines for teaching (description based on the data). This conceptualization resembles the perspective of *Interdisciplinarity as Connections*, which was the conceptualization on defining interdisciplinarity (mentioned in the previous part and Cycle 1) because they both focus on relations between different disciplines. However, for teaching, there is some differentiation because the connections between disciplines was indeed meant to be between school subjects. The *use of mathematics* refers to noticing the value of mathematics in other disciplines and applying mathematics in real-life and in other disciplines (description based on the data). The following paragraphs mention the details on the initial conceptualizations on interdisciplinary teaching.

The pre-service teachers T1, T2, and T4 reflected on interdisciplinary teaching (see Table 4.11). The pre-service teacher T1 mentioned interdisciplinarity in a general manner focusing on *connections* between different disciplines, especially focusing on the relationship between mathematics and science. However, how to enact interdisciplinary education in mathematics classes was not specifically mentioned. Another pre-service teacher (T2) also mentioned interdisciplinarity in a broad way by focusing on the *connection* of mathematics with real-life. Abstractness of mathematics was implied by stating the difficulty of mathematics to understand by students. The real-world connection through interdisciplinary thinking might facilitate students to better understand mathematics. The *use of mathematics* in other lessons such as science was emphasized by T1. Concerning the use of mathematics, T2 mostly highlighted the real-world *use of mathematics* and mathematics as being fundamental to other disciplines. The definition of interdisciplinarity from a research perspective, which includes generating new disciplines, was mentioned. Similarly, the pre-service teachers superficially mentioned the relation of mathematics to other disciplines and real-life without giving details on how to teach mathematics (*connections*). Thus, the pre-service teachers broadly described the importance of interdisciplinarity in mathematics education without mentioning specific aspects.

Concerning *conceptualizations on interdisciplinary teaching* after the implementation, pre-service teachers' responses to post-interviews were considered (see Table 4.12). In the first and second items of the post-interview, pre-service teachers were asked to define

interdisciplinarity, interdisciplinary education, how to implement interdisciplinary education in future and how they relate interdisciplinarity to mathematical modeling. Post-conceptualizations of pre-service teachers on interdisciplinary education were inferred from these responses in the post-interview. Three themes were revealed in the post-conceptualizations of pre-service teachers: *interdisciplinarity*, *level*, and *mathematical modeling*. The following paragraphs give details on the themes.

Table 4.12. Post-conceptualizations on interdisciplinary education.

Participants	Excerpts	Conceptualization
T1	Mathematics is indeed everywhere. By integrating interdisciplinarity into mathematics, this is somewhat related to increasing the level of <i>cognitive demand</i> . We should construct a plot by putting different disciplines into play. Namely, we should not just apply mathematics but should analyze these with different information in question. It needs to think more, have more information, or maybe make prior search. (Post-interview, 1 st item)	• Connections • Cognitive demand • Mathematical modeling
T2	Teaching mathematics through science shows the student what mathematics works for. We need to continuously present <i>real-life connections</i> to students as we did in <i>mathematical modeling</i>. It will be more reasonable and <i>valuable</i> for the student. While teaching, always mentioning <i>connections</i> of the topic with other disciplines will make students value the topic. (Post-interview, 1st item) Mathematical modeling and interdisciplinarity are <i>interwoven</i> like parts of the same whole. (Post-interview, 2nd item)	• Use of mathematics • Mathematical modeling • Connections
T3	I see <i>mathematical modeling as a bridge</i> between two disciplines. Interdisciplinarity is <i>more connected with science</i> because science contains physics, chemistry, and biology. (Post-interview, 1st item) Two different disciplines have distinct values and topics, but they have <i>common things</i>, too. This <i>common language</i> cannot be other than <i>mathematics</i>. (Post-interview, 2nd item)	• Mathematical modeling • Connections
T4	Tests could be <i>high cognitive demand</i> by adding mathematics into interdisciplinary education through lessons, lesson plans, or tests. This may help students to better understand <i>real life problems</i>. (Post-interview, 1st item) We can explain <i>real-life problems</i> by mathematical terms. [Experts from different disciplines] might have to do interdisciplinary mathematical modeling to <i>solve real-life problems</i>. (Post-interview, 2nd item)	• Cognitive demand • Mathematical modeling • Use of mathematics

In the post-conceptualizations based on the post-interview, one theme revealed was *interdisciplinarity*. The theme of *interdisciplinarity* referred to the interdisciplinary connections of mathematics with other disciplines. All pre-service teachers focused on *connections* or the *use of mathematics* by describing interdisciplinary education. The conceptualizations of *connections* and the *use of mathematics* also emerged as revealed in initial conceptualizations. The *use of mathematics* refers to valuing mathematics and focusing on the application of mathematics in other areas. In the perspective of the *use of mathematics*, pre-service teachers T2 and T4 especially focused on real-life connections. For instance, T4 considered interdisciplinary mathematics education as a tool to understand real-life problems. Thus, mathematics was viewed as helpful for other disciplines. Similarly, T2 thought teaching mathematics through science was a useful approach to show students the use of mathematics, especially the value of mathematics. Indeed, interdisciplinary teaching was seen as a tool to enhance the value of mathematics. Moreover, *connections* refer to relations with other disciplines. T1 focused on the general connections of mathematics with other disciplines while T2 and T3 focused on more specific connections of mathematics such as with science. Connections within science were also mentioned by one pre-service teacher (T3). Herein, T3 highlighted interdisciplinarity within science while talking about interdisciplinary education. This explicit remark might stem from the major of T3 as being science teaching. Natural sciences are interdisciplinary because they contain other disciplines, and this allows interdisciplinarity per se.

In the post-conceptualizations, *level* was another theme revealed about interdisciplinary education. Some pre-service teachers (T1 and T4) focused on *cognitive demand* and associated it with interdisciplinary mathematics education. For instance, the pre-service teacher T1 claimed that interdisciplinary mathematics education has been a way to increase the level of cognitive demand of tasks. Furthermore, the pre-service teacher T4 asserted that instruments could have a high level of cognitive demand through interdisciplinary mathematics education.

In the post-conceptualizations, *mathematical modeling* was also a prevalent theme focused by all the pre-service teachers. Mathematical modeling was viewed as a key element for interdisciplinarity by the pre-service teachers. Mathematical modeling was seen as a *tool* to connect disciplines. The bridge was used as a metaphor to emphasize the role of

mathematical modeling in interdisciplinary education by one of the pre-service teachers (T3). Furthermore, mathematical modeling was considered as setting a *common ground* between disciplines through mathematics as a common language. Mathematical modeling was also conceived to solve *real-life problems* by one pre-service teacher (T4). Moreover, mathematical modeling and interdisciplinarity were perceived as having intertwined nature and interdisciplinarity was regarded as an important element to empower the reality of the mathematical modeling problem (T2). Herein, although the intertwined relationship between mathematical modeling and interdisciplinarity, a reverse relationship between them was mentioned compared to other pre-service teachers' conceptualizations. In others' views, mathematical modeling was a tool for interdisciplinarity, but according to the view of the pre-service teacher T2, interdisciplinarity was a tool for mathematical modeling. This may imply the barely indistinguishable nuance between pre-service teachers' conceptualizations on interdisciplinary education containing mathematical modeling.

In short, the conceptualizations of the pre-service teachers on *interdisciplinary teaching* (based on interviews and surveys) initially focused on connections (e.g., T1 and T4) and the use of mathematics (e.g., T2). For post-conceptualizations on interdisciplinary teaching, in addition to the pre-conceptualizations, the pre-service teachers focused on mathematical modeling (e.g., T1, T2, and T3) and cognitive demand (e.g., T1 and T4). The post-conceptualizations on interdisciplinary teaching were grouped as the three themes: Interdisciplinarity, mathematical modeling, and level. Interdisciplinarity was composed of connections (e.g., T1, T2, and T3) and the use of mathematics (e.g., T2).

Pre-service teachers' conceptualizations on *mathematical modeling* were defined as pre-service teachers' perspectives about mathematical modeling. Pre-interviews were used to get information about pre-service teachers' conceptualizations on mathematical modeling. The third and fourth items in the pre-interview were used to inform initial conceptualizations on mathematical modeling. These items asked the definition of mathematical modeling and how to teach mathematical modeling, respectively. The initial conceptualizations on mathematical modeling revealed in the pre-interviews were presented in Table 4.13. The findings based on the pre-interviews showed that the initial conceptualizations of the pre-service teachers on mathematical modeling were concrete model, representation and real-life situation.

Table 4.13. Initial conceptualizations on mathematical modeling.

Participants	Excerpts	Conceptualization
T1	When I think of mathematical modeling, this comes to mind. I took a geometry teaching course. There, we showed through an <i>activity</i> that the sum of the interior angles of a triangle is 180 degrees. Here, we showed where the rule comes from by cutting the edges and drawing a triangle on a piece of paper, cutting the edges and putting them side by side to form a straight line. This immediately reminded me of mathematical modeling. So, I can say that modeling is a facilitating step for us on the path to proof. (Pre-interview, 3 rd item)	Concrete model
T2	So, this time the <i>purpose</i> of our model is to be able to <i>learn and teach mathematics</i>. For example, Thomson's raisin pie was something related to science. But this time it was about the cell. I think these are like the <i>materials</i> we use to <i>represent</i> mathematical information, things related to mathematics, to be able to <i>explain</i> this. (Pre-interview, 3rd item) Since the subject is teaching modeling in class, we are generally <i>unfamiliar</i> with modeling. But I don't have much knowledge or anything about how to teach it exactly. I have <i>no idea</i>. (Pre-interview, 4th item)	Concrete model
T3	When I think of mathematical modeling, something <i>short</i> like this comes to mind; you know, there is a problem with a long solution, but it is <i>shortened</i> and <i>concise</i> by doing mathematical modeling. (Pre-interview, 3rd item) I mentioned modeling, when I say modeling, it reminds me of something <i>short like a representation</i>. I guess it's for the child to remember. I mean, I can write a long version of it. It's a bit of <i>proof</i>. This is where this model comes from. The child uses that model, but if he knows where it comes from, maybe he can learn it more easily. (Pre-interview, 4th item)	Representation
T4	For example, we can explain this topic by taking something from <i>real life</i> and modeling it. For example, a child is going to throw a ball into a <i>basketball</i> hoop. If we express the trajectory of the ball as a parabola with a function, I think we are doing mathematical modeling. (Pre-interview, 3rd item) I think it can be something that can be used in many subjects. I think the important thing is to use it according to <i>real life situations</i>. (Pre-interview, 4th item)	Real-life situation

Pre-service teachers initially could not define mathematical modeling, but they focused on certain aspects to explain mathematical modeling. Some pre-service teachers (T1

and T2) initially conceptualized mathematical modeling as *concrete models*. Although a definition of mathematical modeling was not specified, mathematical modeling was understood to provide reification through concrete models like manipulatives (e.g., T2). Mathematical modeling was viewed as materials to teach and learn mathematics. So, mathematical modeling was described through its purpose on mathematics education. Like T2, T1 also viewed mathematical modeling through concrete models. In this case, mathematical modeling was perceived as a helpful step toward proof. One pre-service teacher (T3) conceptualized mathematical modeling as *representation*. Mathematical modeling was understood as a language or a kind of symbol to represent certain expressions. For instance, exponential numbers were given as examples to models (T3). Another pre-service teacher (T4) regarded mathematical modeling as *a real-life situation*. A real-life situation was used as an example to describe mathematical modeling although mathematical modeling was not defined. Mathematical modeling was explained through a basketball path as a quadratic function. Moreover, although some descriptions about mathematical modeling were stated, there were no explicit statements on teaching mathematical modeling. Some pre-service teachers even explicitly stated that they had no idea how to teach mathematical modeling (e.g., T2). Since they were unfamiliar with mathematical modeling, teaching about it was also unfamiliar.

Post-conceptualizations on *mathematical modeling* were revealed based on the data from the post-surveys (see Table 4.14). The second question of the post-survey required to explain mathematical modeling together with the process steps, and to state the role of mathematical modeling in interdisciplinary tasks. The findings on the post-surveys indicated that the post-conceptualizations of the pre-service teachers on mathematical modeling concentrated on problem solving.

Table 4.14. Post-conceptualizations on mathematical modeling.

Participants	Excerpts	Conceptualization
T1	Mathematical modeling in interdisciplinary tasks serves as a <i>common language</i> , allowing experts from different fields to <i>collaborate</i> , gain insights, and make informed decisions about <i>complex real-world problems</i> . It facilitates a <i>systematic</i> and <i>quantitative</i> approach to problem-solving across diverse domains. (Post-survey, 2 nd item)	• Problem solving
T3	Mathematical modeling helps to <i>quantify</i> and <i>analyze</i> problems in interdisciplinary tasks. It acts as a <i>common language</i> that connects different fields, making it easier to find solutions <i>collaboratively</i> . (Post-survey, 2 nd item)	• Problem solving
T4	Mathematical modeling is a very important concept for interdisciplinary problems. Because the essence of interdisciplinary problems is that they are taken from <i>real life</i> . Interdisciplinary problems are problems that have <i>more than one variable</i> and even <i>more than one solution</i> and need to be solved by modeling. A problem taken from real life has mathematical aspects. Therefore, using mathematical modeling would be an excellent way to solve this <i>complex</i> problem. (Post-survey, 2 nd item)	• Problem solving

For post-conceptualizations on mathematical modeling, the pre-service teachers gave similar steps of mathematical modeling process although they used different terms to explain the process. One pre-service teacher (T4) made more detailed descriptions for the process steps by explaining the specific steps for the given example. For instance, T4 emphasized making necessary assumptions, which was an important aspect of mathematical modeling process. Besides, pre-service teachers used different examples for mathematical modeling: epidemic (T1), traffic jam (T3), and chirping crickets (T4). The chirping crickets' example was also used in the class. Thus, one pre-service teacher (T4) preferred using one of the course examples. Besides, for the role of mathematical modeling in interdisciplinary tasks, they conceptualized mathematical modeling as a tool for *problem solving* (see Table 4.14). The pre-service teachers similarly emphasized mathematical modeling as a common language (T1, T3) and as a quantitative feature (T1, T3) to solve complex, real-life problems (T1, T4). Collaboration (T1, T3) was also focused on by considering experts' collaboration to solve interdisciplinary problems. One pre-service teacher (T4) also highlighted the multiple variables and solutions of a real-life problem.

In short, conceptualizations of the pre-service teachers on *mathematical modeling* (based on interviews and surveys) initially were limited to mathematical modeling as concrete model (e.g., T1 and T2), mathematical modeling as representation (e.g., T3) and mathematical modeling as a real-life situation (e.g., T4). Initially, the pre-service teachers could not define mathematical modeling but specified certain aspects of mathematical modeling. For the conceptualization of concrete model, it was focused on the generation of the model as a product of mathematical modeling process. For the conceptualization of representation, representing non-mathematical conditions through mathematical terms was the key aspect. For the conceptualization of real-life situation, the initial real-world problem that would be solved through mathematical modeling was the focus. Thus, the initial steps on mathematical modeling were considered for initial conceptualizations on mathematical modeling by the pre-service teachers. For the post-conceptualizations on mathematical modeling, the pre-service teachers agreed on the view of mathematical modeling as problem solving. For this conceptualization, other aspects of mathematical modeling process (such as assumptions, variables, solutions) were emphasized. To sum, mathematical modeling was viewed as a tool for problem solving in the post-conceptualizations.

All in all, for Cycle 3, the findings indicated that the initial conceptualizations of the pre-service teachers on *interdisciplinarity* were interdisciplinarity as an area (e.g., T1 and T4), interdisciplinarity as connections (e.g., T2), interdisciplinarity as working together (e.g., T1 and T3), interdisciplinarity as an evolution of disciplines (e.g., T1), and interdisciplinarity as problem solving (e.g., T2 and T4). The post-conceptualizations on interdisciplinarity were interdisciplinarity as working together (e.g., T1 and T4), interdisciplinarity as a combination (e.g., T1, T2, T3 and T4), interdisciplinarity as problem solving (e.g., T1, T3 and T4), interdisciplinarity as an evolution of disciplines (e.g., T2), and multiple perspectives (e.g., T3 and T4). Although there were similar conceptualizations (such as interdisciplinarity as working together, interdisciplinarity as problem solving) in both pre- and post-conceptualizations, new aspects were added in the post-conceptualizations. Compared to the initial conceptualizations, interdisciplinarity as a *combination* and *multiple perspectives* were new aspects in the post-conceptualizations on interdisciplinarity. Interdisciplinarity as a combination focused on the integrative nature of interdisciplinarity. Besides, multiple perspectives highlighted different disciplinary insights that came together through interdisciplinarity. Thus, there were some changes in the pre- and post-conceptualizations of

the pre-service teachers on interdisciplinarity. The broad descriptions on interdisciplinarity (such as interdisciplinarity as an area and interdisciplinarity as connections) in the initial conceptualizations did not take place in the post-conceptualizations anymore.

Moreover, the findings on the conceptualizations of the pre-service teachers on *interdisciplinary teaching* indicated that the initial conceptualizations were connections (e.g., T1 and T4) and the use of mathematics (e.g., T2). Like the broad descriptions of interdisciplinarity for initial conceptualizations on interdisciplinarity, the pre-service teachers broadly described the importance of interdisciplinary education without giving details. For instance, the conceptualization of *connections* broadly referred to the connections of mathematics with other disciplines and real-life without specific mentioning on how to teach. Besides, the initial conceptualizations on interdisciplinary teaching were also mentioned in the post-conceptualizations. The conceptualizations of connections (e.g., T1, T2, and T3) and the use of mathematics (e.g., T2) were grouped as a theme of interdisciplinarity in the post-conceptualizations. Besides, there were new aspects added in the post-conceptualizations such as mathematical modeling (e.g., T1, T2, and T3) and cognitive demand (e.g., T1 and T4). This was like the post-conceptualizations on interdisciplinarity. New aspects were added in conceptualizations on both interdisciplinarity and interdisciplinary teaching. Interdisciplinarity, level, and mathematical modeling were the themes that revealed in the post-conceptualizations on interdisciplinary teaching.

Furthermore, the findings on the conceptualizations of the pre-service teachers on *mathematical modeling* indicated that the initial conceptualizations were mathematical modeling as concrete model (e.g., T1 and T2), mathematical modeling as representation (e.g., T3) and mathematical modeling as a real-life situation (e.g., T4). Besides, mathematical modeling as problem solving was viewed as the post-conceptualization on mathematical modeling. Although there were differences in their initial conceptualizations on mathematical modeling, all the pre-service teachers focused on problem solving as their post-conceptualizations on mathematical modeling. Although the change in the conceptualizations on interdisciplinarity and interdisciplinary teaching were both on the quantity and the meaning of aspects, the change in the conceptualizations on mathematical modeling was just on the meaning (but not on the quantity) of aspects.

4.3.2. Findings on Designing Interdisciplinary Problems based on Interviews

In Cycle 3, it was also investigated how the pre-service teachers designed Interdisciplinary Mathematical Problems for Teaching. To answer the *sixth research question*, interviews were used to understand how the pre-service teachers designed Interdisciplinary Mathematical Problems for Teaching. This part will describe the findings on the *conceptualizations* of the pre-service teachers on designing Interdisciplinary Mathematical Problems for Teaching. The *initial conceptualizations* of the pre-service teachers on designing interdisciplinary problems were investigated based on pre-interview data (see Table 4.15).

Table 4.15. Initial conceptualizations on designing interdisciplinary problems.

Participants	Excerpts	Conceptualization
T2	For example, in <i>another lesson</i> of ours, we both design games. In other words, here we both use <i>technology</i> and write a <i>scenario</i> ourselves; we deal with different things. I think a mathematical activity can be created using different things, those disciplines. I <i>don't know</i> exactly. In this regard, I am a bit <i>confused</i> about the <i>interdisciplinary</i> part. Because our aim here is to <i>develop mathematics</i> , to teach mathematics, and to explain mathematics, I cannot fully understand how it can be interdisciplinary. (Pre-interview, 6 th item)	• Previous experiences • Interdisciplinarity
T3	The child can say DNA is just about science, there is <i>no mathematics</i> in it. But its sequences, replications, proteins that go inside, etc. these are all numerical things. I can say to the student, <i>mathematics</i> can be learned <i>in every field of science</i> . (Pre-interview, 6 th item)	• Previous experiences
T4	I had a fifth-grade student. They had an assignment. This was an interdisciplinary mathematical activity. The student [was] asked about <i>water consumption</i> in the family or at home. Ask your family one by one, examine your bills. They were written down in detail one by one, such as eating, drinking, washing, bathing, dental, etc. Then there was a mathematical activity. With this given <i>data</i> , there was something like their own <i>water consumption index</i> . They were calculating it and comparing it with the Turkish mean. It was related to the environment. It was also mathematical. In other words, it was an activity where <i>two disciplines were used together</i> . (Pre-interview, 6 th item)	• Previous experiences

For the pre-interviews, the pre-service teachers responded the question “How can you design an interdisciplinary mathematical task? Write an example task design” about designing interdisciplinary problems. Some pre-service teachers explicitly noted that they could not give an example for an interdisciplinary mathematical task at the beginning of the module. For instance, the pre-service teacher T1 stated that “I don’t think I could give an example right now” (Pre-interview, 6th item). Other pre-service teachers’ initial conceptualizations focused on their *previous experiences* (see Table 4.15). For instance, based on the prior university courses, one pre-service teacher (T2) explained that an educational game that included technology might have been designed, but could not describe how to do this. Mathematics and technology were considered the connected disciplines for interdisciplinarity. Besides, the conceptual difficulties about *interdisciplinarity* were also noted by one pre-service teacher (T2). The confusion in the meaning of interdisciplinarity affected the pre-service teacher’s inability to give an example of an interdisciplinary mathematical task. Besides, although examples of interdisciplinary mathematical tasks were not given, certain science examples that contained mathematics were given based on previous experiences (e.g., T3). For instance, cell division and DNA were exemplified topics that included mathematical calculations. The focus was finding mathematics in science. Task examples were not given although some possible connections between disciplines were stated. Moreover, based on *previous experiences* in private tutoring, one pre-service teacher (T4) gave an example that could be regarded as an interdisciplinary mathematical task. That example was about water consumption and there were interdisciplinary connections between mathematics and environmental sciences. Overall, although most of the pre-service teachers initially conceptualized designing interdisciplinary problems based on their previous examples, they could not exemplify interdisciplinary problems.

This part will describe the findings on the *post-conceptualizations* of the pre-service teachers on designing interdisciplinary problems. For the post-interviews, the pre-service teachers were asked *how they designed* their final projects and what they considered in designing their *Interdisciplinary Mathematical Problems for Teaching*. The *post-conceptualizations* of the pre-service teachers on designing interdisciplinary problems were investigated based on post-interview data (see Table 4.16).

Table 4.16. Post-conceptualizations on designing interdisciplinary problems.

Themes	Codes	Excerpts
Interdisciplinarity	Choosing disciplines (T2, T3, T4)	Initially we <i>chose disciplines</i> while we were preparing the problem. (Post-interview, 5 th item, T2) We said that <i>there should be another discipline in the solution</i> . (Post-interview, 5 th item, T3)
	Authenticity (T1, T2, T4)	We wanted to write a problem where students had to do <i>research</i> , do <i>mathematical modeling</i> , and use <i>multiple sciences</i> together. Within the context of our task, we tried to determine the <i>interests</i> of our students and something that would interest them. (Post-interview, 5 th item, T4)
Mathematical Modeling	Mathematization (T1, T2)	We set up the relation to mathematics over the <i>transition</i> between ratio and proportion, percentages, and units. Namely, we focused on how other disciplines are appropriate to ask the question because <i>mathematical modeling</i> has been revealed in any <i>calculation</i> . (Post-interview, 5 th item, T2)
	Assumptions (T1, T2, T4)	Our aim was to have the student learn in an artificial area by <i>attracting the student completely</i> into the question, totally leaving the <i>assumptions</i> in the hands of students. (Post-interview, 5 th item, T1)
	Data (T3)	The reason that we gave up the first problem was that we could <i>not collect data, proper data</i> . We chose another topic where we could reach <i>more concrete</i> data. (Post-interview, 5 th item, T3)
	Solution (T2, T3)	We often notice <i>mathematics in the solution</i> . Indeed, we identify <i>which mathematical topic</i> therein. (Post-interview, 5 th item, T2)
Implementation	Ways of presentation (T2, T3)	We used technology in the solution to the problem, and by thinking about how much information could be <i>presented</i> to students. (Post-interview, 5 th item, T2)
Sub-theme 1: Level	Cognitive demand (T1)	We have the design principle of raising the <i>level</i> , namely making <i>high cognitive demand</i> . (Post-interview, 5 th item, T1)
	Student level (T1, T4)	Even, we adapted to the <i>level</i> and <i>grade</i> parts like this. (Post-interview, 5 th item, T1)
	Difficulty level (T4)	This could be <i>extremely difficult</i> for students. (Post-interview, 5 th item, T4)
Sub-theme 2: Tools	Internet (T1, T2, T3, T4)	So, we used the <i>internet</i> to create the route. (Post-interview, 5 th item, T4)
	Mobile applications (T3)	We used an application on the phone, like a bicycle map. (Post-interview, 5 th item, T3)

The post-conceptualizations of the pre-service teachers were grouped under three themes: *Interdisciplinarity*, *mathematical modeling*, and *implementation*. The theme of implementation has also sub-themes of *level* and *tools*. Table 4.16 shows each theme with its related codes and example quotes from the pre-service teachers. The pre-service teachers' acronyms (T1, T2, T3, and T4) are added into the parenthesis next to the name of the codes. For instance, the code of *data* (T3) means that the most explicit reference to that code was mentioned by the pre-service teacher T3. The codes were constructed inductively based on the data from post-interviews. The definition of themes and codes were also inferred based on the data. The following paragraphs will explain the themes.

Concerning the post-conceptualizations, the theme of *interdisciplinarity* refers to considering interdisciplinary relations between the disciplines in designing interdisciplinary problems for teaching (see Table 4.16). *Choosing disciplines* and *authenticity* were the codes about interdisciplinarity. *Choosing disciplines* refers to identifying which disciplines will relate to mathematics in the interdisciplinary problem. Choosing disciplines was stated as the first step to design the problem (e.g., T2). Indeed, pre-service teachers chose disciplines to relate to mathematics herein. Mathematics was considered the inherent discipline in the problem and to make the problem interdisciplinary, there was a search for other disciplines. Furthermore, while choosing disciplines, the pre-service teacher T3 explicitly emphasized that the solution of the problem needed to be interdisciplinary. There should have been other disciplines in addition to mathematics in the expression of the solution. Thus, the pre-service teacher T3 emphasized the necessity of an interdisciplinary solution while choosing disciplines for the problem. *Authenticity* refers to the characteristic of the problem being complex, realistic and relevant for the students. Pre-service teachers (e.g., T4) emphasized that the problem should have been interesting for students and complex enough that require inquiry. It was also noted that the problem should have been realistic enough (e.g., T2). Thus, interdisciplinarity was considered important by pre-service teachers in designing interdisciplinary problems for teaching. The following paragraphs will explain other themes. Initially, the theme of mathematical modeling will be described.

Another theme on post-conceptualizations of pre-service teachers on designing interdisciplinary problems was *mathematical modeling*. *Mathematical modeling* is the cyclical process of using mathematical models to solve problems in the real world. This

theme is composed of the codes *mathematization*, *assumptions*, *data* and *solution* based on data on post-interviews (see Table 4.16). Mathematical modeling was seen at the center in designing the problems (e.g., T1). *Mathematization* refers to the transition to mathematics, namely expressing something outside from mathematics through mathematical terms. Some pre-service teachers (e.g., T2) conceptualized mathematical modeling as inherent in any calculation involved in, so that other disciplines were interrogated with respect to the appropriateness for an interdisciplinary problem. Besides, *assumptions* refer to the certain specifications on the problem situation that helps to narrow down the problem at the beginning and helps to express the problem in mathematical terms. The open-ended nature of mathematical modeling problems necessitates to set certain assumptions to make the problems solvable. Some pre-service teachers (e.g., T1, T4) explicitly highlighted the need to make assumptions in mathematical modeling and therefore, in designing interdisciplinary problems for teaching. Moreover, *data* refer to the appropriate data collection during mathematical modeling. Some pre-service teachers (e.g., T3) emphasized that the availability of proper data was important in the design process of interdisciplinary problems for teaching. It was also pointed out that there should have been opportunities for students to collect appropriate data to make mathematical modeling. Furthermore, the *solution* refers to mathematical solutions of the problem with different possible solutions to the interdisciplinary problem. This code also includes the solution process based on pre-service teachers' answers (e.g., T2). Moreover, getting different solutions were also important in design. For instance, T3 and T4 described that they initially solved the problem separately, then compared their solutions and reached the conclusion that the solutions were different from each other. In summary, mathematical modeling was conceptualized as a key element in designing interdisciplinary problems for teaching. The next paragraphs will explain another theme of *implementation* (see Table 4.16).

Another theme on post-conceptualizations of pre-service teachers on designing interdisciplinary problems was *implementation*. The *implementation* refers to the preparation of the task for application in the class including the way of presentation of the problem to students, the level of the task as well as the students, and the necessary tools for application. *Ways of presentation* refers to how the problem is presented to the students. For example, how the problem could be asked and how much information would be presented to students were emphasized by some pre-service teachers (e.g., T2). Besides, this theme of

implementation has two sub-themes of *level* and *tools*. The *level* refers to the *cognitive demand* level of a task, *difficulty level* of a problem, and *student level* to enable the problem. Pre-service teachers meant *cognitive demand* of a problem as the high level of cognitive demand (e.g., T1). This indicates keeping the level of the problem high during the implementation, therefore implies complexity of the problem. The difficulty level of the problem is also connected with this. The *difficulty level* of the problem refers to the appropriateness for students such that they can continue to solve it. So, the difficulty level of the problem is linked with enough complexity of the problem. Some pre-service teachers (e.g., T4) emphasized that the problem should not have been too difficult to solve. Other pre-service teachers (e.g., T1) also paid attention to the appropriate *student level* that includes appropriate grade level. Besides, the *tools* refer to the necessary materials such as *internet* and *mobile applications* to solve the problem. The *Internet* was seen as a useful tool in designing interdisciplinary problems by the pre-service teachers. The Internet was used to generate initial ideas for creating the context of the problem (e.g., T4), to search for certain data about the problem (e.g., T3), and to write the solution part of the problem (e.g., T1). Moreover, different *mobile applications* were used to understand the relationship between the variables concerned (e.g., T3). Overall, pre-service teachers conceptualized certain elements on the implementation of the problem as important while designing interdisciplinary problems for teaching. Indeed, this part was related to teaching of a problem, particularly preparation of the problem for teaching. Thus, pre-service teachers paid attention to certain aspects such as ways of presentation, level of the problem and students, and tools in designing interdisciplinary problems.

As a summary, the findings on the *conceptualizations* of the pre-service teachers on designing interdisciplinary problems were based on the interviews. The findings indicated that the initial conceptualizations on designing Interdisciplinary Mathematical Problems for Teaching were focused on interdisciplinarity (e.g., T2) and previous experiences (e.g., T2, T3 and T4) such as prior university courses. Besides, the pre-service teachers could not initially exemplify interdisciplinary problems. On the other hand, the post-conceptualizations on designing Interdisciplinary Mathematical Problems for Teaching were grouped as three themes: Interdisciplinarity, mathematical modeling, and implementation. The theme of *interdisciplinarity* consisted of choosing disciplines (e.g., T2, T3 and T4) and authenticity (e.g., T1, T2 and T4). The theme of *mathematical modeling* consisted of

mathematization (e.g., T1 and T2), assumptions (e.g., T1, T2 and T4), data (e.g., T3), and solution (e.g., T2 and T3). Moreover, the theme of *implementation* consisted of ways of presentation (e.g., T2 and T3) and two sub-themes of *level* and *tools*. Cognitive demand (e.g., T1), student level (e.g., T1 and T4) and difficulty level (e.g., T4) formed the sub-theme of *level*. Besides, internet (e.g., T1, T2, T3, T4) and mobile applications (e.g., T3) formed the sub-theme of *tools*. Compared to the initial conceptualizations on designing interdisciplinary problems, the post-conceptualizations included more aspects and more details on designing interdisciplinary problems such that these latter conceptualizations could draw a richer portrait on how to design Interdisciplinary Mathematical Problems for Teaching.

4.3.3. Findings on Designing Interdisciplinary Problems based on Artifacts

In Cycle 3, in addition to the investigation of conceptualizations, it was also investigated how the pre-service teachers designed Interdisciplinary Mathematical Problems for Teaching based on their *artifacts*. The artifacts consisted of the *rubric* constructed during the class and the *problems* that the pre-service teachers designed for their final projects.

Initially, the findings on the *rubric construction* were described. The rubric construction was conducted as a class discussion on constructing a guideline in designing interdisciplinary mathematical problems for teaching. As a result, pre-service teachers constructed the key dimensions for the rubric: *interdisciplinarity*, *context*, *organization*, *content*, *mathematical modeling*, *expectations*, and *teaching* (see Table 4.17). Pre-service teachers could not further combine the dimensions although the dimensions were not completely distinctive. These dimensions were like those constructed in Cycle 2 except the dimension of *teaching*. *Teaching* was not stated as a separate dimension in Cycle 2.

Table 4.17. Rubric by pre-service teachers in cycle 3.

Dimensions	Indicators	Description
Interdisciplinarity	Which disciplines selected	Specification of disciplines
	Balance among disciplines	Interdisciplinarity with equal weight
	Connections among disciplines	Explicit/implicit connections among disciplines
	Big ideas on each discipline	Important disciplinary ideas
	Knowledge of other disciplines	Discipline-specific knowledge contribution
Context	Choosing an appropriate theme	Theme allowing interdisciplinarity
	Concepts	Different concepts used in the problem
	Interest area, relevance	Students' interest areas, meaningful for students
Organization	Authentic, realistic	Appropriate to real-life
		Planning/preparation, lesson design
	Time	Duration of solving
	Materials	How to get/provide
	Working style	Group work or individual work
	Grade level/ student level	Level of students for the task
	How much information is given	The extent of information presented to students
	Understandability/clarity	Problem understanding
	Goal/ objectives	Solving the problem aligned with the objectives
	Content on contributing disciplines	Deciding on which content
Content	Prior knowledge	How much prior knowledge is necessary
	Difficulty level	Solvable based on students' prior knowledge
	Cognitive demand level	High level of cognitive demand for a task
	Modeling cycle	Writing the steps of mathematical modeling
	Math model production	Producing mathematical models
Mathematical modeling	Open-ended	Giving space for exploration
	General solution	Generalization of the math model
	Problem specification	Identifying the problem
	Different solutions	Multiple solutions of the problem
	Assumptions	Assumptions: the solution length and complexity
	Reasoning on assumptions	Kind of assumptions that students have formed
	Decision for data collection	Checking whether data collection is necessary
	Student activity	Solving the problem according to the procedures
	Pre-search	Making search for problems
	Reporting mathematical modeling	Writing a report for mathematical modeling
	Sources of data	Multiple sources of data
	Data collection	How to collect data
	Student learning	Learning outcomes
Expectations	Mathematical accurateness	Correctness in mathematics
Teaching	Scaffolding	Scaffolding level
	Noting student possible difficulties	Identifying possible student challenges
	Teacher and student role	Expectations from students and teacher
	Not allowing misconceptions	Avoiding misconceptions

The rubric construction was designed as an authentic learning activity for pre-service teachers such that they themselves generated indicators for interdisciplinary problems for teaching via a whole-class discussion. It was also an exemplifying learning activity for the course because it was student-centered and discussion-based, and this design feature was kept during the whole course. The rubric construction lasted three lessons. Initially, pre-service teachers used brainstorming to freely discuss their ideas on finding appropriate indicators for the rubric. Then, in the next lessons, they tried to organize their ideas and formed categories for the emerging indicators. They called the indicators as criteria. The rubric was shared with pre-service teachers to guide them in designing their problems for the final project. Besides, the researcher was a mediator facilitating the whole-class discussion for the rubric construction. During the discussion, the researcher simultaneously recorded pre-service teachers' ideas as a document on the computer, shared the document on the screen with the class, and asked prompting questions to further express their ideas. The researcher also prompted the class to categorize their ideas in dimensions.

Participants' ideas as criteria were considered indicators and contributed as the inductive analysis for the framework to be used in evaluating participants' problems. In addition to the rubric construction, literature on interdisciplinarity, mathematical modeling, task design and teacher education were used to construct an analytical framework for Interdisciplinary Mathematical Problem for Teaching (see Table 4.18). There are three dimensions of this finalized framework: *Interdisciplinarity*, *mathematical modeling*, and *implementation*. The dimensions of the finalized framework were used to deductively analyze participants' problems.

Table 4.18. Interdisciplinary mathematical modeling task analysis framework.

Dimensions	Indicators	Description
Interdisciplinarity		A holistic way of integrating mathematics with other discipline(s) to solve a problem (Boix Mansilla, 2010) with three key elements: Integration, interdisciplinary content, and authenticity.
	Integration	The extent of the contribution by the connected disciplines to solve the problem.
	Interdisciplinary content	<i>Interdisciplinary content</i> refers to the content of mathematics and another discipline that are used to solve the problem.
	Authenticity	The problem of the task is authentic with respect to mathematics and another discipline. Authenticity refers to meaningfulness in all contributing disciplines including openness of the task (OECD, 2018; Hwang and Han, 2025). <i>Openness</i> refers to openness of problem context, answer, and problem-solving process (Hwang and Han, 2025, p. 10).
Mathematical modeling		A cyclic process that is composed of defining the problem, model building, getting a solution, and explaining results (Schukajlow et al., 2015; Garfunkel and Montgomery, 2019).
	Defining the problem	A question from the <i>real-world</i> is identified and specified.
	Model building	A math model is produced by <i>making assumptions</i> (explicitly specifying key assumptions) and <i>defining variables</i> (stating and justifying important quantities for the model construction).
	Getting a solution	Mathematical ideas are organized to solve the problem.
	Explaining results	The mathematical results are interpreted and validated in the real-world context.
Implementation		Determining the duration, level, and goals of the task with necessary materials and students' working ways on the task, scaffolding, and expected students' answers and difficulties (Geiger et al., 2022; Tekin Dede and Bukova Güzel, 2023).
	Preparation	Preparation of the task for implementation consisting of the duration, level, materials, and ways of task.
	Goals/objectives	The intended goals and objectives for the task are stated.
	Scaffolding	Decision by the teacher on the level and time of possible scaffolding is stated.
	Expected students' answers and difficulties	Expected students' answers to the problem and associated possible students' difficulties in the task implementation are stated.

The relation between the dimensions in the framework (*Interdisciplinary mathematical modeling task analysis framework*) (see Table 4.18) and the *rubric* constructed by the pre-service teachers (see Table 4.17) was described in the following paragraphs. The

rubric constructed by the pre-service teachers in Cycle 3 (see Table 4.17) had some interrelated elements that could be combined. The ideas of the pre-service teachers for the rubric still needed to be organized. In all dimensions, there were combined indicators in the revised version, but the most change was observed in the *interdisciplinarity* dimension. Indicators from different dimensions such as content and context were added into those of the interdisciplinarity dimension. For instance, which disciplines selected, big ideas, choosing an appropriate theme/topic, concepts, connections between/among disciplines were all related and were combined into one indicator as interdisciplinary content in the revised rubric (see Table 4.18). Moreover, certain dimensions such as interdisciplinarity and context were also associated with each other. These two dimensions collapsed into one dimension as *interdisciplinarity* in the revised rubric. Certain indicators from the content dimension in the constructed rubric (see Table 4.17) were also combined into the interdisciplinarity dimension in the new rubric (see Table 4.18). In other words, *content of contributing disciplines* and *prior knowledge* were contained into the interdisciplinarity dimension. Moreover, the indicator of *open-ended* in mathematical modeling for the rubric constructed by the pre-service teachers was put into the expanded indicator of *openness* in the revised rubric. Openness contained open-endedness and added new aspects such as *openness in context* (e.g., Hwang and Han, 2025). In addition, openness was included in the indicator of authenticity for the dimension of interdisciplinarity.

Concerning *mathematical modeling*, similar aspects in the rubric constructed by the pre-service teachers were again combined in the revised rubric. During the revision of these indicators, the literature on mathematical modeling, especially literature on mathematical modeling competencies (e.g., Garfunkel and Montgomery, 2019) also contributed. For instance, different aspects about assumptions (such as *assumptions* and *reasoning on assumptions*) in the initial rubric were combined into the indicator of *model building* in the revised rubric. This new indicator of model building contained both *making assumptions* and *defining variables*. Although pre-service teachers generally focused on the modeling cycle and modeling process during the rubric construction, they did not explicitly mention *variables* in the rubric. They mostly emphasized assumptions during the rubric construction. Another focus of pre-service teachers were aspects about *data* during the rubric construction. These aspects were also integrated into the dimension of mathematical modeling in the revised rubric. Besides, the pre-service teachers did not mention validation or interpretation

of the results during the rubric construction. As a new indicator, these aspects were contained in *explaining results* in the revised rubric.

The dimensions of *content*, *assessment/expectations*, *organization* and *teaching* in the rubric constructed by the pre-service teachers were also related. The related elements were integrated into one dimension as *implementation* in the revised rubric. Time, materials, working style, grade level/student level, difficulty level, cognitive demand level were merged into the indicator of *preparation* in the revised rubric since these were all about planning and preparation for class implementation of a task. The aspects of teaching were mostly incorporated into the dimension of implementation with some change in wordings or combination of some similar elements again into one indicator. For instance, *noting students' possible difficulties* and *not allowing misconceptions* were coupled into one indicator as *expected students' answers and difficulties* in the revised rubric.

To sum up, the dimensions in the *rubric* constructed by the pre-service teachers in Cycle 3 (see Table 4.17) were as the following: *Interdisciplinarity*, *context*, *organization*, *content*, *mathematical modeling*, *expectations*, and *teaching*. The dimension of *teaching* was new in the rubric for Cycle 3 compared to that for Cycle 2. In Cycle 3, the dimensions in the rubric constructed by the pre-service teachers were not distinct but some of them were related and could be combined. Thus, the rubric was revised and there were less dimensions in the revised rubric. In the revised rubric, there were three dimensions: *Interdisciplinarity*, *mathematical modeling*, and *implementation*. The revised rubric was an output of this study that could contribute the literature on helping pre-service teachers design interdisciplinary problems. This revised rubric was named as the *Interdisciplinary Mathematical Modeling Task Analysis Framework* (see Table 4.18) and was used to analyze the problems of the pre-service teachers that were designed as their final projects (in Cycle 3).

In addition to the rubric constructed by the pre-service teachers, the *designed problems* were also used as participant *artifacts* to address how the pre-service teachers designed Interdisciplinary Mathematical Problems for Teaching (*sixth research question*). The problems were analyzed through the *Interdisciplinary Mathematical Modeling Task Analysis Framework*. The first problem was designed by the pre-service teachers T3 and T4 as a group and was described in the final project that was called *Triathlon Race*

Interdisciplinary Modeling Task. The second problem was designed by the pre-service teachers T1 and T2 as a group and was described in the final project that was called *Mathematical Modeling with Eco-friendly Planning*. Initially, *Triathlon Race Interdisciplinary Modeling Task* was presented.

As an artifact, the first problem was the *Triathlon Race Interdisciplinary Modeling Task*. The task was about a triathlon race for high school. After giving a short description of a triathlon, the task started with a scenario in which a triathlon was organized in Istanbul. The designed route of the triathlon race started from Küçüksu Pier, continued to Aşiyan Pier and finished at the July 15 Martyrs Bridge. There was also a map of the route given at the beginning of the task. Then, the goal of the task was stated: Calculate a racer's total finishing time by mathematical modeling based on the given triathlon route with considering real-life factors that affect the competitors (Triathlon race interdisciplinary modeling task- teacher version, p. 4). For the dimension of *interdisciplinarity*, there were three indicators: *Interdisciplinary content*, *integration*, and *authenticity*. Each indicator was explained in the following paragraphs.

For *interdisciplinarity*, a triathlon in Istanbul was chosen as an interdisciplinary theme. The theme allowed connections between physics and mathematics. It was stated that mathematics related to physics in the project report. Thus, on the macro level for *interdisciplinary content*, which disciplines were integrated were specified. On the micro level, big ideas within mathematics and physics were also stated. Thus, disciplinary big ideas were given, but interdisciplinary learning goals were not explicitly stated. However, in the problem statement, it was required to find a triathlon racer's duration for completing the race through mathematical modeling. This implied an implicit mention of interdisciplinary learning goal. Moreover, concerning the indicator of *integration*, it could be said that the problem was *mostly interdisciplinary*. The weight of physics was more than mathematics, so it was not totally balanced. However, the contribution of mathematics was also very important to solve the problem.

Concerning interdisciplinarity, for the indicator of *authenticity*, the triathlon problem was highly connected to the *real world* in which students could relate their learning experiences in mathematics and physics to the context of the triathlon. The problem was

meaningful in terms of physics and mathematics. To solve the problem, students needed to use physics concepts such as velocity, position, displacement, relative motion, or constant acceleration. Moreover, concerning mathematics, important mathematical ideas such as equations, inequalities, functions-trigonometric functions, and Pythagorean Theorem were needed to solve the problem. Thus, the problem was authentic with respect to both physics and mathematics considering its *real-world connection*. Concerning *openness*, as another component of authenticity, the problem could be said to be open overall:

Calculate a racer's average total finishing time by mathematical modeling based on the given triathlon route with considering real-life factors that affect the competitors. The homework period is 1 day. You are free to use various resources including internet. You will work individually. (Triathlon race interdisciplinary modeling task- teacher version, p. 4)

The problem was *open* in terms of *problem context* since students had to identify the factors affecting the finishing time of the racers (such as sea current, incline of the road, and transition time between the phases of the triathlon). In the problem report, its context as a triathlon was mentioned: The context: A triathlon is an endurance multisport race consisting of swimming, cycling, and running over various distances (Triathlon race interdisciplinary modeling task- teacher version, p. 2). Since the identified factors or variables could change, the problem allowed different answers. Two possible answers based on varying assumptions were given in the problem report. Thus, the *openness of answer* was also satisfied. However, according to the *openness of problem-solving process* (Hwang and Han, 2025, p. 10), only one way of solution was presented in the report.

Concerning *mathematical modeling*, there were four indicators examined: *Defining the problem*, *model building*, *getting a solution*, and *explaining results*. In the problem report, there was a description about what mathematical modeling was and its process was schematized in a figure. The steps of mathematical modeling were explicitly written in the task description. There was also a scoring rubric for assessment, including the mathematical modeling process. In the report, there was information about modeling and its cyclic process via a diagram showing its components as problem, formulate, compute, interpret, validate and report based on Common Core Standards (Triathlon race interdisciplinary modeling task- teacher version, p. 4). Moreover, the teacher's role in implementing the modeling task was emphasized in the report. The next paragraphs gave details about indicators of mathematical modeling for the mentioned problem.

For *defining the problem*, a concise statement was given from the real-world (about triathlon) early in the report. Students needed to search for necessary factors on the asked dependent variable. Concerning *model building*, the problem also allowed different solutions by setting different *assumptions*. Moreover, explicit assumptions were written for two different solutions. Some justifications for the assumptions were made (but not for all). Thus, justifications of the assumptions could be said to be limited to a certain extent. For instance, on page 11, there was a justification about one assumption set: There's nothing else we have to consider because there is no inclination on this coastal road. On page 14, an explanation about an assumption was given: Since the race will be held on a windless sunny day, we can assume that there is no other factor affecting swimming speed. On page 17, based on an article and the discipline of physics, a justification was made: The cycling speed is 24.999 km/h and the difference between the initial speed of 25 km/h and 24.999, which is extremely small, therefore neglectable. On page 18, including the transition time as a key variable was built upon an article: According to Haggit (2010), average total transition time, which is sum of transition time T1 (swim-cycle) and T2 (cycle-run), is 2 minutes and 45 seconds. For *defining variables* to build models, key variables were stated. They were also written as dimensions in the assessment rubric given in the task report. Some variables (such as transition time) were justified, but not all. Concerning *getting a solution*, two different solutions were presented based on some changes in assumptions. The mathematical approach has not changed in the solutions. The mathematics used was meaningful and seemed to be aligned with reality. However, no validation was done. Besides, mathematical results were accessible to the audience since they were understandable. Pictures showing the route of the race were presented but no visual aids were used to communicate the results. About *explaining results*, solutions were clearly presented, but they were not explained in the context of the real-world. Namely, mathematical results were not interpreted in the triathlon context.

Concerning the dimension of *implementation*, there were four indicators: Preparation, goals/objectives, scaffolding, and expected students' answers and difficulties. About *preparation*, the expected *duration* for the task was explicitly written (as one day). About the *level*, the grade level for students was explicitly written (high school). In the section of the teacher's role, the necessity of checking the prior knowledge of students was also emphasized: Before giving the task to students, briefly review the subjects and ensure that

students have *prior knowledge* to solve and model (Triathlon race interdisciplinary modeling task- teacher version, p.5, italics added). The difficulty level or cognitive demand was not directly written, but it was implied that the level of the problem could be high since the complexity of the problem was high due to the expectations from the students.

Moreover, for the Triathlon problem, the *materials* were explicitly written in the report. The *ways of work* referred to how many people the task was implemented with (group or individual) (Tekin Dede and Bukova Güzel, 2023). The way of work was noted as individual work in this task. Students were expected to work individually for this task. It could be better to include group work to reduce the cognitive load.

For the Triathlon problem, concerning another indicator of implementation-*goals/objectives*, the related objectives of mathematics and physics from Turkish curricula were written. Some objectives of the task were given as the following in the *Triathlon Race Interdisciplinary Modeling Task* (teacher version): Students understand the concepts of location, distance taken, displacement, speed and speed to each other. Students find the solution sets of equations and inequalities with first-order unknown (p. 1). However, one objective about the analytical plane was irrelevant with the task: Students solve problems by obtaining the relation that gives the distance between two points in the analytical plane (Triathlon race- teacher version, p. 1). On the other hand, in general, the problem could allow students to solve the problem aligned with the objectives. On page 6, the role of the student was explicitly written: The student must create a report or explanation explaining the Solution process step by step. The mathematical model used, methods and results obtained should be explained in the report. Moreover, learning goals for physics and mathematics were added:

Students will demonstrate a comprehensive understanding of fundamental physics concepts related to motion, including the interrelation of position, distance, displacement, speed, and velocity. Additionally, they will be able to apply these concepts to analyze the motion of objects undergoing uniform linear motion, interpret the movement of objects with constant velocity in different frames of reference, and perform calculations related to relative motion. Furthermore, students will explain the concept of average speed and analyze one-dimensional uniformly accelerated motion, applying mathematical calculations to solve problems in this context. In mathematics, they will acquire the ability to obtain expressions that describe the distance between two points in the analytical plane. They will be able to solve equations and functions. They will be able to do mathematization for given context. They will be able to construct and solve the mathematical model. (Triathlon race interdisciplinary modeling task- teacher version, pp. 1-2)

Besides, interdisciplinary learning objectives were not explicitly stated for the Triathlon problem. However, in the section of learning goal, it was emphasized that students were expected to apply certain mathematics and physics concepts to solve the problem of finding a triathlon racer's duration for completing the race. The ideas of two disciplines needed to be used in relation to each other to bring students to the possible solutions.

For the Triathlon problem, about *scaffolding*, in the teacher's role part of the task, teachers were required to briefly review the topic and help students to have the necessary preliminary knowledge. This could be considered as scaffolding, but no explicit time and level for scaffolding were stated. Moreover, in the task description on page five, the steps of mathematical modeling were given, and this could help students do the task: "Before giving the task to students, briefly review the subjects and ensure that students have prior knowledge to solve and model" (Triathlon race interdisciplinary modeling task- teacher version, p. 5).

For the Triathlon problem, concerning *expected students' answers and difficulties*, in the section of the teacher's role, the importance of avoiding *misconceptions* was written by clarifying the given task worksheet: "Give the task as a worksheet paper to students and clarify everything on the worksheet to avoid any misconceptions" (Triathlon race interdisciplinary modeling task- teacher version, p. 5). Moreover, discussing the difficulties and misconceptions as well as the possible answers to the problem was suggested: After the task, bring the class together to share and discuss the task. Discuss the common challenges, misconceptions, difficulties, and solution strategies (Triathlon race interdisciplinary modeling task- teacher version, p. 6). However, students' possible difficulties were *not* stated. According to the rubric given in the report, it was implicitly implied that students could have difficulties in identifying the variables for the model construction. A *scoring rubric* for assessing the task was added to the task, which indicated expected solutions for the task. The rubric included dimensions as "modeling of triathlon race for calculating average total finishing time, route-distance measurement, average speed of triathletes at different steps, sea current and relative motion, incline change, transition time, mathematical accurateness, report" (Triathlon race interdisciplinary modeling task- teacher version, pp. 6-7). The rank of scores for each dimension changed from zero to four. The maximum total point for the rubric was given as 32. Moreover, among the teacher's roles, the importance of

having explicit expectations for the task was emphasized: Provide clear expectations. Tell them [the] time for this task is 1 day, each student will work individually, they can use any resource for searching (Triathlon race interdisciplinary modeling task- teacher version, p.5).

As another *artifact* in Cycle 3, the findings for another *designed problem* were presented to address how the pre-service teachers designed Interdisciplinary Mathematical Problems for Teaching. This problem was designed by the pre-service teachers T1 and T2 as a group for their final projects: *Mathematical Modeling with Eco-friendly Planning*.

The *Mathematical Modeling with Eco-friendly Planning* was another artifact and it was about mathematics and environmental science for ninth grade students with an expected time of two lesson hours. The problem was described after stating the topic, grade level, time required, selected disciplines, materials needed, learning outcomes, and students' needs. The problem asked to reduce CO₂ emissions by changing fossil fuelled vehicles with electrical ones. At the beginning of the problem, there was a map showing total greenhouse gas emissions by road vehicles of big cities in Turkey for the year of 2019. The problem was analyzed based on the *Interdisciplinary Mathematical Modeling Task Analysis Framework* (see Table 4.18). For the dimension of *interdisciplinarity*, there were three indicators: *Interdisciplinary content*, *integration*, and *authenticity*. The following paragraphs gave details on each indicator for interdisciplinarity about the concerned problem on eco-friendly planning.

For the eco-friendly planning problem, an environmental theme on reducing CO₂ emissions was chosen as an *interdisciplinary* theme. The theme had potential to allow connections between environmental science (especially sustainability) and mathematics. Thus, concerning *interdisciplinary content*, on the macro level, it was given which disciplines were integrated. On the micro level, mathematical concepts and key ideas (basic equations, concept of ratio and proportion, percentage, converting units, and constructing relations between units) were stated in the report although environmental ones were not added. Gaining environmental awareness was noted as a learning outcome, which was a very broad environmental outcome. Moreover, reducing CO₂ emissions was specified as an outcome: Since it is a more environmentally friendly plan, students should achieve lower CO₂ emissions as a result (Mathematical modeling with eco-friendly planning task, p. 2).

Reducing CO₂ emissions could be considered an implicit *interdisciplinary learning goal* although there were no explicit interdisciplinary learning goals. Hence, it could be said that environmental science and mathematics were connected in this problem although they were not totally integrated.

For the eco-friendly planning problem, concerning the indicator of *integration*, mathematics related to environmental science, but the weight of mathematics was more than environmental science since environment was used just as a context. Among the learning outcomes, there was only one outcome concerning the environment that aimed for environmental awareness. The learning outcomes of mathematics were stated as the following: Students will be able to use mathematical modelling to reach accurate solutions. Students will be able to convert and compare the products of resources (Mathematical modeling with eco-friendly planning task, p. 1). Therefore, the problem could be said to be *mostly interdisciplinary* (like the multidisciplinary level of Williams et al., 2016).

For the eco-friendly planning problem, concerning *authenticity*, the problem was authentic in terms of mathematics and environment with its connection to *real-world* because it was meaningful as a real-world problem. The real-world connection was also emphasized in the report: It is a topic that is based on reality, and there is no visible project in the area, so it's authentic (Mathematical modeling with eco-friendly planning task, p. 2). Reducing (carbon dioxides) CO₂ emissions was an important key issue on sustainability all over the world: CO₂ emissions are a problem for daily life and the future. Different cautions are needed; therefore, the topic gives students a sense of responsibility and makes sense in the relevant area (Mathematical modeling with eco-friendly planning task, p. 2). Electrical vehicles were also an increasing trend topic for sustainability. Moreover, students could relate their learning experiences to the problem since carbon dioxide emissions and the transition to electrical vehicles were the topic of the whole world today: The question that allows the creation of a variety of scenarios gives the question a mystery and *makes it interesting for students* (Mathematical modeling with eco-friendly planning task, p. 2, italics added).

For the eco-friendly planning problem, concerning *openness*, the problem was open in terms of *problem context* since students had to identify the factors affecting the average

change in the amount of CO₂ emissions (such as fuel type, number of vehicles): Enabling students to choose the city and the different options for electricity production makes them more involved with the topic and do more research (Mathematical modeling with eco-friendly planning task, p. 2). Since the identified factors or variables could change, the problem could allow different answers: The openness of the problem depends on the choice of cities and relevant data for those. Thus, there were different entry-points for the problem: The question can be solved by starting with different steps, but in general, it has similar operations (Mathematical modeling with eco-friendly planning task, p. 2). Two possible answers based on varying assumptions were given in the problem report. Thus, the *openness of answer* was also satisfied. However, according to the *openness of problem-solving process* (Hwang and Han, 2025, p. 10), only one way of solution has been presented in the report.

For the eco-friendly planning problem, concerning the dimension of *mathematical modeling*, there were four indicators examined: Defining the problem, model building, getting a solution, and explaining results. The modeling process was considered in the task, but some parts needed improvement. Particularly, defining the problem, making assumptions, identifying variables, and getting a solution were considered. Model assessment and verification, interpreting results based on real-life were not focused. The report especially highlighted defining the problem, making assumptions, and getting different answers:

In our problem, the problem is clearly stated, and it is an open-ended question that does not have a single answer. Students have enough space to explore the problem. Students are restricted in which years of data they can use. They have to collect data about the question themselves, and they can easily access this data via the Internet. There are different assumptions in many steps, and since these assumptions will mostly be determined by the students, different answers will be received from each group. Because the students determine assumptions and can find a variety of solutions, and the task is open-ended, the cognitive level of the task is high. (Mathematical modeling with eco-friendly planning task, p. 2).

In the eco-friendly planning problem, for *defining the problem*, a question from the real word (CO₂ emissions) was identified. General information about the context was given at the beginning, and then the problem was specified by requiring students to choose a specific city to analyze the problem:

Within the scope of a project, in 1 year, the total amount of greenhouse gas emissions caused by highways in Türkiye's provinces is being examined...Within the scope of this project, it is desired to examine how the change in CO₂ emissions will occur if road vehicles switch from fuel-powered vehicles

to fully electric vehicles in order to reduce the amount of emissions in provinces with high emission. According to this project, select any city with high CO₂ emissions from the highway and specify a year. Calculate the total CO₂ emissions from road vehicles during this one year and both analyze and explain that if road vehicles were electric vehicles and not fueled vehicles, how would the average change in the amount of CO₂ emissions be? (Please use the data for the last 5 years) (Mathematical modeling with eco-friendly planning task, p.3).

For the eco-friendly planning problem, concerning *model building*, students had to make certain assumptions to solve the problem due to its openness. The necessity of *making assumptions* was explicitly written: This modeling task requires many assumptions for solutions and has a variety of solutions (Mathematical modeling with eco-friendly planning task, p.1). There were some explicit and clear assumptions. Rarely were their justifications noted, but generally they were not made. There were also implicit assumptions, but their justifications were not included. For instance, there was an implicit assumption on taking the average CO₂ emissions, but it was not identified why it was chosen: The average CO₂ emissions per vehicle is 4,6 ton (Mathematical modeling with eco-friendly planning task, p.6). Moreover, there was an assumption on the equal distribution of vehicles with respect to their fuel types (e.g., diesel, gasoline, or gas), but it was not explained why equality is chosen. In fact, it would be better to take a weighted average based on the fuel type of vehicles. It would have been more realistic in that case.

For the eco-friendly planning problem, the variety of assumptions was stated in the report. On different pages of the report, certain assumptions were emphasized: On the fourth page, there was an explicit assumption on the (equal) distribution of the vehicles based on the fuel type: To make an average calculation, let's assume that all vehicles run 1 in 3 with gasoline, 1 in 3 with diesel, 1 in 3 with LPG (Mathematical modeling with eco-friendly planning task, p. 4). On the sixth page, there was an explicit assumption on the percentage of charging stations using renewable energy sources: Let's assume that 25% of the total stations generate electrical energy using fossil fuels, 75% will use renewable energy. There is very little CO₂ emission when electricity is produced from renewable energy sources, let's ignore this emission (Mathematical modeling with eco-friendly planning task, p.6). On the seventh page, an explicit assumption was written but without its justification: To make an average calculation, we assume that all vehicles run 1 in 3 with gasoline, 1 in 3 with diesel, 1 in 3 with LPG (Mathematical modeling with eco-friendly planning task, p. 7). On the eighth page, there was an explicit assumption: Assume that 50% of the total stations generate electrical energy using fossil fuels while the other 50% will use renewable energy. CO₂

emissions are little as ignorable when electricity is produced from renewable energy sources (Mathematical modeling with eco-friendly planning task, p. 8).

In the eco-friendly planning problem, for *defining variables*, important variables were identified and considered in solving the problem. An article (Dündar, 2021) on greenhouse gas emissions from the big cities in Turkey was used as a resource for identifying variables. Thus, this could be considered a kind of justification to choose such variables in the problem. The average amount of CO₂ emissions per vehicle, number of vehicles, type of fuel, amount of fuel used, distance travelled by a vehicle based on fuel type, average energy consumption per 100 km of road, energy produced by an electrical charging station, and change in CO₂ emissions were key variables in this problem.

In the eco-friendly planning problem, for *getting a solution*, two different solutions were presented based on some changes in assumptions. The mathematical approach has not changed in the solutions. It could be better if the approximations were explained in the paper. Verbal expressions and numbers were used to present the results, but no visual aids were used to communicate the results.

For the eco-friendly planning problem, concerning *explaining results*, solutions were mostly clearly presented, but they were not explained in the context of the real-world, namely in the context of the environment. Besides, mathematical operations and procedures could have been written in more detail. The clearness of presenting results was emphasized in the report:

Students are expected to study CO₂ emissions and help research for eco-friendly planning. They will do this by converting fuel-powered vehicles to electric vehicles in one of the cities with the highest CO₂ emissions in Türkiye. The problem is clear to understand, enough information is provided, and it is open to assumptions. (Mathematical modeling with eco-friendly planning task, p. 1).

Concerning the dimension of *implementation*, there were four indicators: *Preparation*, *goals/objectives*, *scaffolding*, and *expected students' answers and difficulties*. Each indicator of the dimension implementation was described based on the analyzed problem (the eco-friendly planning problem).

For the eco-friendly planning problem, about *preparation*, the expected *duration* for the task was explicitly written (two lesson hours). More time might be better based on the necessary calculations and interpretations. If certain searches for the internet were done at home, then the expected time for the task could be feasible. Concerning *level*, the grade level for students was explicitly written (ninth grade). Due to its openness and authenticity, the task could be said to have a high level of cognitive demand. Students had to make inquiries and take decisions on data collection. They were also expected to be involved in the process of mathematical modeling. The difficulty level was not explicitly stated, but the cognitive level of the task was noted to be high: Because the students determine assumptions and can find a variety of solutions, and the task is open-ended, the cognitive level of the task is high (Mathematical modeling with eco-friendly planning task, p. 2). Besides, *materials* needed were also explicitly written to include pencil, paper, and the internet. Moreover, the *ways of working* were noted. Students were required to work in pairs.

For the eco-friendly planning problem, concerning *goals/objectives*, intended disciplinary learning outcomes were stated, but interdisciplinary learning goals were not explicitly written. Reducing CO₂ emissions could be considered an implicit interdisciplinary learning goal: Since it is a more environmentally friendly plan, students should achieve lower CO₂ emissions as a result (Mathematical modeling with eco-friendly planning task, p.2). The needs of students and learning outcomes were explicitly stated in the report. On the first page of the project report, the learning outcomes for the eco-friendly problem were stated as the following: Students will be able to use mathematical modeling to reach accurate solutions. Students will be able to convert and compare the products of resources. Students will gain environmental awareness. Besides, student needs were listed in the eco-friendly problem on the first page of the project report:

- A student should be able to solve basic mathematical equations
- A student should be able to know the concept of ratio and proportion
- A student should be able to know how to calculate percentage
- A student should be able to convert units to each other
- A student should be able to know how to establish relationships between units (Mathematical modeling with eco-friendly planning task, p. 1)

Moreover, the eco-friendly planning problem had the potential to allow students to solve the problem aligned with the objectives. Mathematical accuracy, logical reasoning and environmental awareness were expected from students. Students were also expected to

collect data and work in pairs to solve the problem. Students had to present their solutions to the class at the end of the task.

For the eco-friendly planning problem, concerning *scaffolding*, at the beginning of the problem, students could have difficulties starting the task due to unfamiliarity with open tasks and mathematical modeling. The teacher could scaffold students at the beginning. This initial need for help was written in the report:

If there is a student who does not know what to do or does not know where to start solving, the teacher can offer small tips to the student on how to follow a path without telling the student what to do step by step. (Mathematical modeling with eco-friendly planning task, p.1).

For the eco-friendly planning problem, the teacher was expected to scaffold based on constructivism such that students themselves would find what to do in the task. However, there were no other *scaffolding* issues mentioned. Moreover, concerning *expected students' answers and difficulties*, two expected solutions were written based on changing some assumptions and some students' difficulties were noted. For instance, students could have difficulties in data collection. This was noted in the report:

If there is a student who does not know what to do or does not know where to start solving, the teacher can offer small tips to the student on how to follow a path without telling the student what to do step by step. (Mathematical modeling with eco-friendly planning task, p. 1).

Moreover, for the eco-friendly planning problem, there were no mentions about possible misconceptions. However, some resources could be recommended by the teacher (e.g., TUIK/TÜİK (Türkiye İstatistik Kurumu) website):

Students will be able to use more than one resource. If they do not have a lot of relevant information, some preliminary research may be required first. They can access key data via the Internet using the TUIK website and some PDF files. (Mathematical modeling with eco-friendly planning task, p. 2).

In summary, in Cycle 3, both *designed problems* (artifacts as final projects) were generally similar in terms of the analysis based on *Interdisciplinary Mathematical Modeling Task Analysis Framework*. Both problems were considered satisfactory in general. The pre-service teachers considered the rubric constructed in the class and took this as a guideline to design their final projects. They paid attention to many points in the rubric in designing their

problems, but there were certain aspects of development according to the *Interdisciplinary Mathematical Modeling Task Analysis Framework*.

In Cycle 3, for the dimension of *interdisciplinarity*, both designed problems had similar points that could be improved. For the indicator of *interdisciplinary content*, both problems did not have explicit interdisciplinary learning goals, but stated implicit ones. The pre-service teachers were familiar with objectives or goals in different subjects, but these objectives or goals were usually presented as separately in disciplinary tasks, even in STEM tasks. Moreover, for the indicator of *integration*, the level of interdisciplinarity were identified as *mostly interdisciplinary*. This level was not the highest level of interdisciplinarity because there was not a total balance of contributing disciplines in the problems. Besides, concerning the indicator of authenticity, both problems were open overall. However, they were not open in terms of problem-solving process since one way of solution was stated.

Like the dimension of interdisciplinarity, for the dimension of *mathematical modeling*, both problems in Cycle 3 had similar points that could be improved. The pre-service teachers mostly followed the indicators of the rubric constructed in the class on mathematical modeling. Based on the *Interdisciplinary Mathematical Modeling Task Analysis Framework*, for the indicators of *getting a solution* and *explaining results*, both problems did not state validation or interpretation of results. Besides, concerning the dimension of *implementation*, pre-service teachers had difficulties in the indicators of *scaffolding* and *expected students' answers and difficulties* as revealed in both designed problems in Cycle 3. These aspects needed to be improved in both problems.

4.3.4. Findings in Cycle 3 on Design Principles

This section presented the findings for the research question regarding the design principles. Initially how the design principles were identified after Cycle 2 was mentioned. Then, to answer the *seventh research question* in Cycle 3, the findings based on the analysis of interviews, artifacts and researcher notes will be described.

After Cycle 2, revisions were made on the design of the module on interdisciplinary teaching, and the module was named as the *Program on Designing Interdisciplinary Mathematical Problems* for Cycle 3. The aim in Cycle 3 was to develop pre-service teachers' skills in designing interdisciplinary mathematical problems for teaching. This skill is a part of a broader concept of learning to teach mathematics. The revised design principles after the second iteration were the following: (i) *Promote understanding of interdisciplinarity*, (ii) *enhance opportunities for learning to teach mathematical modeling in interdisciplinarity*, and (iii) *foster active teacher learning*.

The conjecture map (Sandoval, 2014) (see Figure 4.3) was revised after the second iteration (Cycle 3). The embodiment in the conjecture map for Cycle 3 (see Figure 4.3) contains the design features: *Resources on teaching interdisciplinary mathematics, rich mathematical tasks, group work, scaffolding, and reflection*.

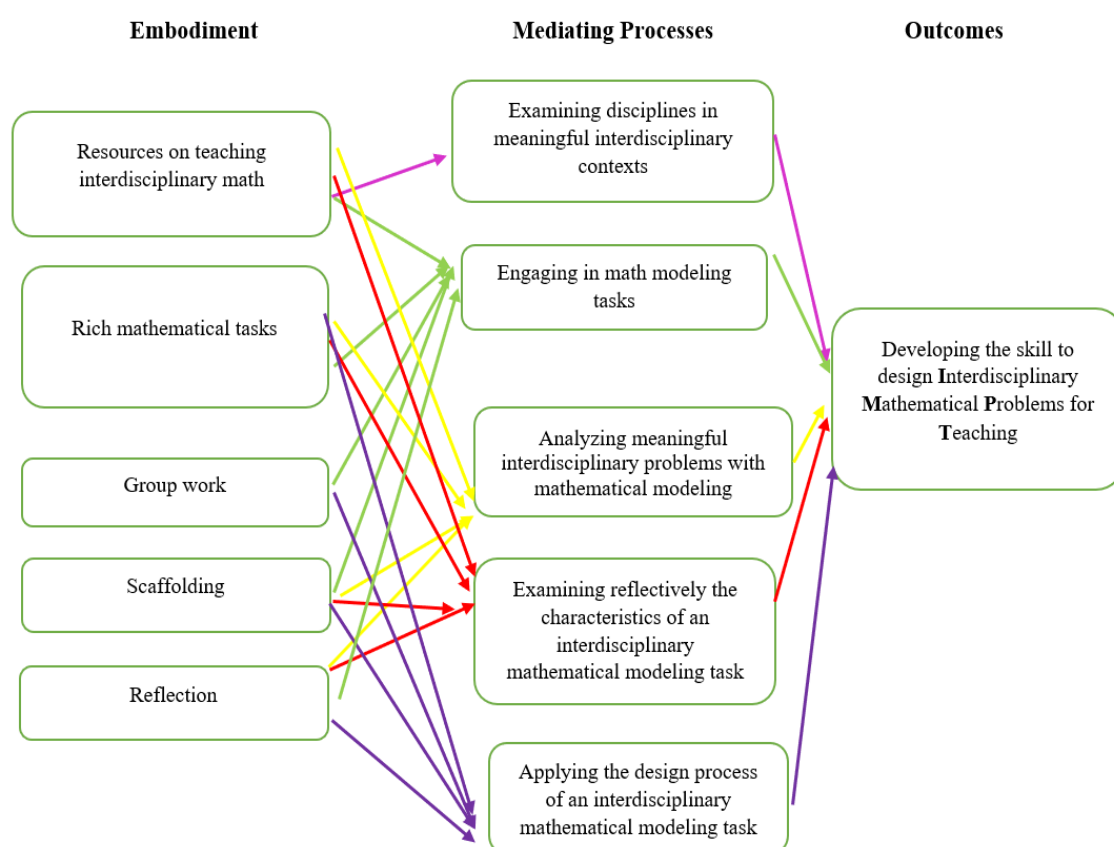


Figure 4.3. The revised conjecture map (cycle 3).

The *resources on teaching interdisciplinary mathematics* included course presentations, readings on interdisciplinary research and interdisciplinary mathematics education. *Rich mathematical tasks* were challenging tasks that aimed for teacher learning. Pre-service teachers were expected to experience mathematical modeling tasks as learners with pedagogical reflection on the experience of doing tasks. Thus, it was aimed to develop “*retrospective abilities* such as reflecting on results” (Durandt, 2018, p. 264) of pre-service teachers concerning mathematical tasks. *Group work* as another design feature was conceptualized to be helpful not just for interdisciplinary thinking but also for teacher active learning. Mathematical modeling tasks are often recommended to be conducted within small groups due to the complex nature of such tasks (e.g., Geiger et al., 2022). *Scaffolding* referred to “targeted assistance that is provided to students as they perform a task” (Rachamim et al., 2022, p. 4). Concerning scaffolding, the concept of “*measured responsiveness*” guided this design (Geiger et al., 2022, p. 326). This refers to sufficient scaffolding, that is a “balance between too much guidance by the teacher and sufficient guidance to help learners to be autonomous in learning” (Geiger et al., 2022, p. 326). The importance of using scaffolding in helping teachers in task design was also emphasized in the literature (e.g., Rachamim et al., 2022). *Reflection* can be defined as “providing opportunities for introspection of the task” (Rachamim et al., 2022, p. 4). Reflecting on the engaged tasks for teaching was the basis of this design since it was important for pre-service teachers to learn how to teach, particularly how to design tasks. Moreover, *reflective activities* (Maaß, 2006) such as “in-class discussions”, and “comparing different solutions” were noted to promote pre-service teachers’ mathematical modeling skills (Durandt, 2018, p. 147).

The design principle on interdisciplinarity (*Promote understanding of interdisciplinarity*) was expected to be embodied in resources on teaching interdisciplinary mathematics. Especially, the readings on interdisciplinary research and interdisciplinary mathematics education as well as example interdisciplinary tasks were planned to reify this design principle. The course resources and tasks concerning interdisciplinarity can be thought as the *pre-engagement* part of “*pedagogical architecture*” (in Geiger et al.’s terms, 2022) to support pre-service teachers learn how to design intended interdisciplinary tasks.

To enhance *modeling-specific pedagogical content knowledge* (Greefrath et al., 2022) or *instructional modeling competency* (Geiger et al., 2022) of pre-service teachers, pre-

service teachers needed to know mathematical modeling both theoretically and practically. Since mathematical modeling was conceptualized as a pre-requisite for developing pre-service teachers' skill of formulating interdisciplinary mathematical problems for teaching, pre-service teachers were expected to experience a variety of mathematical modeling tasks as learners to better understand how to teach mathematical modeling to their prospective students. The design principle on mathematical modeling (*Enhance opportunities for learning to teach mathematical modeling in interdisciplinarity*) was expected to reify through rich mathematical tasks and resources on teaching interdisciplinary mathematics.

The design principle on teacher learning (*Foster active teacher learning*) was expected to be reified through reflection, group work, scaffolding, resources on teaching interdisciplinary mathematics, and rich mathematical tasks. Learning to teach mathematics requires a contextual perspective that makes emphasis on learning context (Peressini et al., 2004, p. 68). This perspective implies that pre-service teachers need to have various learning experiences within different contexts in the course of time (Durandt, 2018). Thus, in this design, providing varying experiences for pre-service teachers was considered. Pre-service teachers needed to experience the tasks as learners themselves before they were expected to teach such tasks as a teacher.

The design in Cycle 3 aimed to examine how a module on interdisciplinary teaching (Program on Designing Interdisciplinary Mathematical Problems) could support pre-service mathematics and science teachers' learning to design interdisciplinary mathematical problems for teaching. The second version of the design principles (DP-V2) guided the design in Cycle 3: *Enhance interdisciplinary problem-solving skills, facilitate collaboration, and enhance active learning experiences*. After Cycle 3, the design principles of the third version (DP-V3) were identified as the following: *Promote understanding of interdisciplinarity, enhance opportunities for learning to teach mathematical modeling in interdisciplinarity, and foster active teacher learning*. The evolution of the design principles from V2 to V3 was described based on the findings of Cycle 3 to address the *seventh research question*.

In the broad sense, *interdisciplinary problem-solving* referred to the problem-solving skill in the context of interdisciplinary problems (Mercan and Tekbıyık, 2022).

Interdisciplinary problem-solving is the ability to utilize different disciplinary viewpoints for solving interdisciplinary problems by constructing connections, to integrate knowledge across academic fields, and the ability to recognize the necessity to rethink on the orientation of a person's thinking and problem-solving methods (Brassler and Dettmers, 2017). The *resources on teaching interdisciplinary mathematics* included course presentations, readings on interdisciplinary research and interdisciplinary mathematics education. *Rich mathematical tasks* were challenging tasks that aimed for teacher learning. To *enhance interdisciplinary problem-solving skills* (DP-V2), the design features of *resources* and *tasks* were utilized.

The designed problems in Cycle 3 were used as *artifacts* to inform about the design principle of *enhancing interdisciplinary problem-solving skills*. For this purpose, the designed problems were analyzed based on the *interdisciplinary mathematical modeling task analysis framework* (see Table 4.18) in terms of *interdisciplinarity*. The analysis on the artifacts indicated that the two designed problems were similar in terms of interdisciplinarity. For the indicator of *integration*, the two problems (triathlon and eco-friendly planning) were identified as mostly interdisciplinary. For the indicator of *interdisciplinary content*, the two problems included no explicit statements for interdisciplinary learning goals. For another indicator of interdisciplinarity in the framework (see Table 4.18), namely *authenticity*, the two problems related to real-world. Moreover, for authenticity, the two problems were identified as open overall but not open in terms of *problem-solving process* since the problems included only one method of solution.

In Cycle 3, in the *post-interviews*, the *interdisciplinary* focus in problem-solving was also mentioned by some pre-service teachers. Some pre-service teachers (e.g., T1 and T3) emphasized that identifying different perspectives and disciplines was a contribution on learning during the course. For example, in the post-interview, the pre-service teacher T1 mentioned the awareness of different disciplines in problems:

I think I gained *very different perspectives* in these used activities. I was only going to a question with a *solution focus*. But now when I see a question, I go to this. *What discipline* was used here? Or what does this question *teach* me other than solving mathematics? I go directly to this now. This was the biggest thing it taught me. This was its *biggest contribution*, and I can no longer look at questions normally. I really think about the design phase, sometimes when a very *complicated* question comes, I say, *why* did they write this thinking or, for example, this has a point that will surprise me, for example, at what point will it surprise me, I think about the answer on the one hand, and on the other hand, I think

about a different one that may surprise me, I could actually choose this one here, it would lead me astray. But what kind of a clue did it give me so that I wouldn't go there, etc., I started to *think very deeply* now. This was the biggest contribution the activities made to me. I can say that it gained *different perspectives* when looking at a question. It's not just doing mathematics or physics, or both, or biology, but two, three, four, now I can see *how many disciplines* there are. This is a huge plus. (9th item)

In the post-interview, the pre-service teachers (e.g., T2, T3 and T4) mentioned the *rubric* construction experience in the class. For example, in the post-interview, the pre-service teacher T2 focused on the rubric constructed by themselves in the class: Because we could have been given *rubric* directly and told to do something similar but studying it and writing it *ourselves* allowed us to have a *better grasp* of the information, and in that sense it was useful (8th item). In the post-interview, another pre-service teacher (e.g., T4) mentioned the rubric as a checkpoint to write the interdisciplinary problem: I think those *criteria* were useful for us. I can say that we understood better *how to formulate* the question. We examined it, and we paid attention to the *key steps* there (8th item). Moreover, based on the *researcher notes*, the pre-service teachers could produce the rubric themselves through a class discussion to guide their final projects in designing interdisciplinary problems for teaching. Like in Cycle 2, the rubric was considered as participant artifacts that informed the embodiment in the design principles in Cycle 3. The pre-service teachers considered the aspects in the constructed rubric in designing their problems for their final projects. Besides, based on the researcher notes, the pre-service teachers referred to resources used in the class. Moreover, in the post-interviews, the pre-service teachers (e.g., T2) emphasized the *resources* such as readings in the course were helpful: But I think that there should be individual and group work, I think it should remain the same in that sense. Other than that, these *readings* were nice (8th item). In the post-interviews, the pre-service teachers (e.g., T2) also referred to the variety of *example tasks* administered in the class as helpful for their learning:

I really enjoyed the course. Especially the group work. The fact that we were constantly working on a *different problem* every week. I really liked it. Other than that, I was very unfamiliar with all these terms. I think there is a big difference between the forum and interview you gave at the beginning and the current forum and interview. At least, there is a big difference from my perspective in terms of answering the questions. I don't know how the answers came about, but I can say that I really *learned* in that sense. (7th item)

All in all, based on these findings, the design principle *Enhance interdisciplinary problem-solving skills* (DP-V2) was *mostly supported*. Since the task analysis indicated that the interdisciplinarity in the designed problems in Cycle 3 were mostly interdisciplinary, the

focus on interdisciplinarity was decided to be specified in the revised design principle as *Promote understanding of interdisciplinarity* (DP-V3). Thus, the design principle became more specific. Moreover, mathematical modeling was another key dimension in interdisciplinary mathematical problems for teaching, so the design principle *Enhance interdisciplinary problem-solving skills* (DP-V2) also evolved to a new design principle focusing on mathematical modeling: *Enhance opportunities for learning to teach mathematical modeling in interdisciplinarity* (DP-V3). The analysis on design principles was harder when the design principles were compact since they contained many components to be evaluated. When the key aspects were separated, it would be better to understand their effectiveness. When interdisciplinarity and mathematical modeling were separated in the design principles, it was better to reflect on them.

The design principle *Facilitate collaboration* (DP-V2) was expected to be reified in the design feature of *group work*. Like in Cycle 2, the *researcher notes* showed that the pre-service teachers in Cycle 3 could design problems for their final projects through working in small groups (in pairs). Furthermore, based on the researcher notes, as the pre-service teachers were asked to individually solve mathematical modeling problems in the class, it was harder for them to solve the problems compared to group work. In the *post-interviews*, the pre-service teachers (e.g., T1) also emphasized peer learning and group work: I think our *group work* was very useful. Because, in my opinion, it was a course where we did a lot of *peer learning*. *Group assignments* and *group work* were very strong in this sense as well (7th item).

Another design principle *Enhance active learning experiences* (DP-V2) was expected to reify through design elements of *group work*, *scaffolding*, and *discussion*. In the *post-interviews*, the pre-service teachers (e.g., T2) emphasized group work and the active learning environment:

I think *doing group work is more useful* and productive in this lesson than doing individual work. I think it was more suitable for this because it was a class where we *constantly talked* about something and were *active*. I really liked it in that sense. Apart from that, I *learned* a lot of new information. It was a chatting and chatting environment where we were *constantly active*. (7th item)

In the post-interviews, the pre-service teachers (e.g., T1) also emphasized the student-centered pedagogy applied during the course and the open-ended nature of the tasks administered in the class:

I was wondering why it was left *so open-ended*, but now I am just beginning to realize how *instructive* it is. In other words, leaving it open-ended is not a bad thing. I also realized this when I was designing the question. In fact, it is very broad, and you can do it according to your own wishes. It was a process that was completely left to you, completely *student-centered* and completely in our hands. In this case, it was very, in general, very instructive in every way. In other words, I can say that preparing its content was a separate instructiveness and managing that process was a separate instructiveness. (7th item)

In the post-interviews, the pre-service teachers mentioned the contribution of the tasks in the course as contribution for teacher learning. For example, the pre-service teacher T4 noted the use of the active learning experiences in the class as informative for them as future teaching practices:

The activities we used in class were both *fun and interesting*. In other words, I can say that we approach mathematics in *different ways*. I think I will use this in my future education and teaching life. In other words, I would like to use the concept of interdisciplinary and interdisciplinary mathematical problems with my students in my future education and teaching life. Because I think it will make students *think better*. It is very *open-ended*, and they *need to model*. They need to use *real life*. These activities were also nice in this way. I can say that they contributed to me. (9th item)

Furthermore, based on the *researcher notes*, like Cycle 2, the researcher as an instructor in Cycle 3 provided *scaffolding* while pre-service teachers were dealing with the class tasks and they were preparing their final projects. In Cycle 3, the researcher also implemented additional online sessions as scaffolding sessions to facilitate the process of designing problems. Besides, the researcher notes indicated that the *discussions* facilitated the pre-service teachers comfortably share their ideas in the class.

Based on these findings, the design principles *facilitate collaboration* (DP-V2) and *enhance active learning experiences* (DP-V2) were supported. Since these design principles were highly connected and group work was also used to embody the design principle on active learning experiences, it was decided to combine these two design principles into the new design principle: *Foster active teacher learning* (DP-V3). Moreover, since the active learning experiences in the class intended for teacher learning, the emphasis of the teacher learning was added in the expression of the design principle: *Foster active teacher learning* (DP-V3). Thus, after Cycle 3, the *design principles* were identified as the following:

- Promote understanding of interdisciplinarity
- Enhance opportunities for learning to teach mathematical modeling in interdisciplinarity
- Foster active teacher learning

To sum up, after the implementation of the Program on Designing Interdisciplinary Mathematical Problems in Cycle 3, the findings indicated that interdisciplinarity, mathematical modeling, and active teacher learning experiences were three key aspects in supporting the pre-service teachers' skills in designing interdisciplinary mathematical problems for teaching. The design principles after the implementation in Cycle 3 had minor changes compared to Cycle 2. Thus, the transition from the second version to the third version of the design principles (from V2 to V3) was done to emphasize the key aspects with more specificity on design elements. The second version of the design principles (Version 2) after Cycle 2 were as the following: *Enhance interdisciplinary problem-solving skill, facilitate collaboration, and enhance active learning experiences*. Thus, the revised design principles after Cycle 3 (Version 3) were as the following: *Promote understanding of interdisciplinarity, enhance opportunities for learning to teach mathematical modeling in interdisciplinarity, and foster active teacher learning*.

4.4. The Findings for the Reflection on the Design Principles

The *third phase* of the study (*Evaluation and Reflection Phase*) aimed to reflect on the findings of the second phase (*Design and Construction Phase*) to suggest finalized version of design principles on a module on interdisciplinary teaching for pre-service mathematics and science teachers. In the second phase, the purpose was to design a module on interdisciplinary teaching for pre-service mathematics and secondary teachers. Initial design principles were specified based on the literature and then these were revised based on iterations. To reach the final design principles, data from the second phase were utilized for the analysis. For this purpose, interviews, artifacts and researcher notes in the second phase (Cycle 2 and Cycle 3) were coded for design elements.

The reflection on the evolution of the design principles in this study led some insights into the design of a module on interdisciplinary teaching. Based on the findings of Cycle 2

and Cycle 3, resources on interdisciplinarity such as the articles for reflection on interdisciplinarity were helpful in designing interdisciplinary tasks or problems. Thus, it was suggested to use *resources* on interdisciplinarity such as articles on interdisciplinarity and interdisciplinary mathematics education allowing reflection in future designs of the module on interdisciplinary teaching. Moreover, based on the findings in Cycle 2 and Cycle 3, the pre-service teachers initially could have difficulties in identifying interdisciplinary themes. To overcome such difficulties, continuous *scaffolding* by the instructor was suggested in the design. Moreover, to enhance understanding of interdisciplinarity, additional curriculum search in both mathematics and other disciplines as well as more examples on interdisciplinary tasks could be added in the design.

Concerning mathematical modeling, the pre-service teachers in Cycle 2 and Cycle 3 experienced different mathematical modeling tasks as learners during the course. Based on the findings, it was found to be useful in designing interdisciplinary tasks or problems including mathematical modeling. The pre-service teachers used the mathematical modeling task characteristics in their design of tasks or problems. Thus, it was suggested to include different *mathematical modeling tasks* such that the pre-service teachers would experience as learners themselves in the future designs to support the pre-service teachers designing interdisciplinary modeling tasks or problems.

Moreover, about *mathematical modeling*, some pre-service teachers (e.g., P7, T3, T4) noted that they might need to know how to collect appropriate *data*. Some pre-service teachers (e.g., T3 and T4) stated that they had to change their initial ideas for the problems since they could not manage to find data. Some pre-service teachers (e.g., P8) also stated that identifying *variables* could be challenging and exchange of ideas might help to overcome such a challenge. To overcome the challenge of data collection for designing tasks, tutorials or certain links to tutorials could be shared with pre-service teachers. Moreover, search for extra resources on collecting data or choosing appropriate data for tasks could be given as assignment. Thus, it was suggested to use additional resources or assignments on data collection in mathematical modeling in the future designs in the case of possible difficulties in mathematical modeling process. Especially, when the pre-service teachers were unfamiliar with the mathematical modeling process, it would be beneficial to focus on the steps on mathematical modeling.

Moreover, *collaboration* with different majors such as mathematics and science was found to be useful based on the findings of Cycle 2 and Cycle 3. In this study, the pre-service teachers designed their tasks/problems in small groups that included two members. It was suggested to use *small group* work in which different majors collaborate with each other in the future design to support the pre-service teachers design interdisciplinary tasks/problems. Moreover, concerning *teacher learning*, *discussion* was important for teacher learning in the class. The pre-service teachers constructed a rubric (a guideline) through a whole-class discussion in which the researcher was just a facilitator. Student-centered activities including reflection and discussion were at the center of the design in this study. Thus, it was suggested to use discussion and reflection in student-centered activities for teacher learning in the future designs. Overall, since active teacher learning experiences were found to be useful, it was suggested to use a participant structure that would encourage pre-service teachers to share their ideas in a comfortable classroom climate without a dictating instructor. The focus should be on teacher learning for the pre-service teachers.

To conclude, in this study, certain key aspects were found to be useful in designing interdisciplinary tasks or problems. Interdisciplinarity, mathematical modeling and active teacher learning experiences were critical in the design of a module on interdisciplinary teaching. Although the design principles during the study evolved based on the implementations (see Figure 3.2), the core aspects of interdisciplinarity, mathematical modeling and active learning did not change. The design principles became more specific, and some aspects were combined. For example, communication and collaboration were conceptualized together as components of fostering active teacher learning experiences. Moreover, discussion and scaffolding were also key design aspects to foster active teacher learner experiences. Interdisciplinary problem-solving was important in the design but this was separated into two main components of interdisciplinarity and mathematical modeling based on the findings. All in all, based on the reflections, as a design principle for a module on interdisciplinary teaching, it was suggested to *promote understanding of interdisciplinarity* with appropriate interdisciplinary resources. Besides, as another design principle for a module on interdisciplinary teaching, it was suggested to *enhance opportunities for learning to teach mathematical modeling*. Lastly, as a design principle for a module on interdisciplinary teaching, it was suggested to *foster active teacher learning* with a variety of experiences including collaboration, discussion and scaffolding.

5. DISCUSSION AND CONCLUSION

This study aimed to design a module to support pre-service mathematics and science teachers' development of skills for interdisciplinary teaching. Initially, as a theoretical outcome of the study, the development of a new construct (skill to formulate *Interdisciplinary Mathematical Problems for Teaching*) will be described. As another theoretical outcome of the study, the development of the framework on analyzing interdisciplinary tasks (the *Interdisciplinary Mathematical Modeling Task Analysis Framework*) will be presented. Then, the findings on conceptualizations of the pre-service teachers and how they designed interdisciplinary tasks or problems will be discussed. Finally, after the discussion on the development of the module on interdisciplinary teaching, the limitations and implications of the study will be mentioned.

5.1. Interdisciplinary Mathematical Problems for Teaching

Even though the main purpose of this study was to design and implement a module on interdisciplinary teaching, as a result of careful analysis through iterations also resulted into a theoretical outcome, a new construct to be defined: The skill to formulate *Interdisciplinary Mathematical Problems for Teaching* (IMPT). Before designing a task for interdisciplinary teaching, pre-service teachers needed to think about problems for the task. *Interdisciplinary Mathematical Problems* (IMPT) are defined as interdisciplinary problems that can be solved through mathematical modeling. Based on findings from Cycle 2 and 3, this skill to formulate IMPT is composed of elements: *Content knowledge of each discipline, knowledge for interdisciplinary tasks, mathematical modeling competency, and interdisciplinary thinking*. A pre-service teacher, a learner of interdisciplinary teaching has a certain level of understanding for these elements, and then the pre-service teacher could develop how s/he learns to formulate IMPT. These elements need to be developed together to support the development of this skill. With an aim of developing a rich mathematical teaching task with interdisciplinary aspect, a teacher should be able to formulate a mathematical modeling problem. Solving this problem should require addressing mathematics with interdisciplinary thinking. Critical thinking, design thinking, and problem solving are also required for pre-

service teachers to develop this skill. Each of the elements of IMPT that emerged from this study will be discussed below by relating them with existing literature.

5.1.1. Interdisciplinary Thinking

The literature on interdisciplinarity, interdisciplinary teaching and learning informed to define the skill to formulate IMPT. As an element of this skill, *interdisciplinary thinking* is conceptualized as composed of *finding an interdisciplinary theme*, *determining interdisciplinary learning goals*, *deciding on an interdisciplinary problem*, and *solving an interdisciplinary problem*. In the post-interviews and class discussions, the pre-service teachers in both Cycle 2 and Cycle 3 often referred to finding a theme as a critical aspect of designing interdisciplinary tasks and problems. Sometimes the pre-service teachers had difficulties in finding an interdisciplinary theme that they considered as an initial step for preparing interdisciplinary tasks and problems. Moreover, although they emphasized interdisciplinary goals in class discussions, they could not explicitly write the goals in their artifacts (final projects). Solving an interdisciplinary problem was also considered by the pre-service teachers in class discussions by focusing on the need for appropriate results and solutions for interdisciplinary problems. They wrote possible solutions to their prepared problems for their final projects. Deciding on an interdisciplinary problem was highly affected by their interests such as triathlon mentioned by one of the pre-service teachers in Cycle 3. Moreover, content knowledge of other disciplines was influential in deciding on an interdisciplinary problem. Some pre-service teachers in Cycle 2 mentioned this in the class discussions and post-interviews. These pre-service teachers stated that they had to change their initial ideas for the problem because they could not manage the advanced level of knowledge required such as knowledge in medicine. The components of interdisciplinary thinking are explained in the following paragraphs.

Finding an interdisciplinary theme is considered one of the components of interdisciplinary thinking. *Finding an interdisciplinary theme* refers to finding a theme that has connections with at least two disciplines (mathematics and other disciplines). The *theme* should allow opportunities to find *commonalities* (common topic, context, concepts) in different curricular subjects/contents (An, 2017). Some pre-service teachers in this study had difficulties in finding an interdisciplinary theme and they associated this component with

creativity. This component is regarded as the initial key step because it necessitates an awareness in connections between disciplines. This step is an exploration phase in which certain commonalities are looked for. For example, in Cycle 2, some pre-service teachers stated that they had to change their initial themes for the problem because they could not find appropriate data for the theme. Similarly, in Cycle 3, some pre-service teachers mentioned that they had difficulties in finding an interdisciplinary theme, so they conducted further internet search, even asked for the artificial intelligence applications for help. A profound content knowledge in each discipline is necessary to make a transition to interdisciplinary connections (e.g., An, 2017; Boix Mansilla, 2007).

Another component of interdisciplinary thinking is conceptualized as *determining interdisciplinary learning goals*. Since aim of pre-service teachers is to formulate interdisciplinary problems for teaching, they needed to take consideration of learning goals. It is defined as specifying intended interdisciplinary learning goals (Wu et al., 2019). It includes determining *big ideas* in contributing disciplines. In other terms, it includes clarifying disciplinary big ideas or objectives to be taught with a justification that explains why they are included (Boix Mansilla, 2007; Tripp and Shortlidge, 2020). Determining interdisciplinary learning goals requires learning how to design goals as a part of designing an interdisciplinary problem as mentioned in the context of “backward STEM learning design” (Wu et al., 2019, p. 2274). This component is related to teaching (interdisciplinary teaching). Designing problems is a preparation for teaching. To design an interdisciplinary problem, goals are necessary to specify the use of the problem with students. Determining goals for each discipline separately is useful but not enough to determine goals in an interdisciplinary manner. Determining separate disciplinary goals is like the multidisciplinary level in the continuum of interdisciplinarity by Williams and his colleagues (2016). In the multidisciplinary level (Williams et al., 2016), each discipline separately works on the same problem without interaction. However, in determining interdisciplinary learning goals, the interrelation between disciplines should be considered. Thus, this component is important in empowering interdisciplinarity in teaching.

Moreover, *deciding on an interdisciplinary problem* is another component of interdisciplinary thinking. It is defined as *constructing* a real-life problem that includes at least two different disciplines (e.g., science and mathematics) appropriate to mathematical

modeling. It requires checking the problem with the appropriateness to the characteristics of mathematical modeling. This part was the focus of this study in which the pre-service teachers were required to design interdisciplinary problems

Solving an interdisciplinary problem is another component of interdisciplinary thinking. It refers to reaching a mathematical and realistic solution to the concerned interdisciplinary problem by analyzing the problem and then benefiting from different disciplines. *Analyzing the interdisciplinary problem* contains making sense of the problem, and determining its components, that is determining “several factors or variables across disciplines” (Song and Wang, 2021, pp. 1044-1045). Analyzing the interdisciplinary problem requires to identify which content in different disciplines are necessary to solve the problem. It is like breaking the building blocks of an entity to understand functions of each block. The variables or factors in each discipline could be resembled to the building blocks metaphor. As mathematical modeling is involved in the interdisciplinary problem, analyzing the problem refers to identifying variables in each discipline and setting appropriate assumptions to build a mathematical model. Besides, *benefiting* from different disciplines towards a solution to an interdisciplinary problem requires *integrating* different disciplinary ideas “as a coherent whole” to find potential solutions for the specified interdisciplinary problem (Song and Wang, 2021, pp. 1045-1046). Concerning the former building block metaphor, benefiting from different disciplines is like using different types of building blocks to form a strong building. When mathematical modeling is inherent in the interdisciplinary problem, benefiting from different disciplines includes using different disciplinary insights such as methods, theories or concepts to arrive at a solution to the problem. The solution to the problem should be meaningful in all disciplines, namely mathematics and other disciplines that relate to mathematics. For example, in the interviews, one pre-service teacher in Cycle 3 highlighted the necessity of an interdisciplinary solution. Analyzing the interdisciplinary problem and benefiting from different disciplines correspond to the analysis and synthesis of solving a problem.

To sum up, the components of interdisciplinary thinking are related to each other: *Finding an interdisciplinary theme, determining interdisciplinary learning goals, deciding on an interdisciplinary problem, and solving an interdisciplinary problem*. Finding an interdisciplinary theme can be regarded as the initial step for interdisciplinary thinking. This

step is based on searching for links between disciplines (e.g. mathematics and another discipline). After finding commonalities between disciplines, interdisciplinary learning goals are determined. Setting goals is highly associated with teaching and determining interdisciplinary learning goals requires transforming the interdisciplinary connections to intended common goals for interdisciplinary teaching (designing interdisciplinary problems). Deciding on an interdisciplinary problem and solving an interdisciplinary problem are more intermingled for this study. To formulate an interdisciplinary problem with mathematical modeling, pre-service teacher needs to solve the problem before its presentation to the students. Since an interdisciplinary problem in this study is considered an interdisciplinary problem *for teaching*, the possible solutions to the formulated. Hence, interdisciplinary thinking is a complex process that involves different components.

5.1.2. Knowledge for Interdisciplinary Tasks

The literature on *task design* also contributed to defining the skill to formulate IMPT. Particularly, the element on tasks (*knowledge for interdisciplinary tasks*) was of concern herein. Tasks are important elements of teaching and learning, and teachers' design of tasks affects students' learning. Teachers' selection, alteration and formulation of tasks are the essence of preparation for teaching (Riard and Kaur, 2022). As future teachers, pre-service teachers also need to learn how to design effective tasks.

According to Henningsen and Stein (1997), a *mathematics task* is “a classroom activity, the purpose of which is to focus students' attention on a particular mathematical concept, idea or skill” (p. 528). A mathematics task can be an important tool to develop learners' mathematical understanding. Henningsen and Stein (1997) referred to such mathematical tasks as *high-level tasks*. Like Henningsen and Stein's (1997) high-level tasks, a *rich task* is defined as a task which “is complex, non-algorithmic, and non-routine, allowing for multiple strategies and representations and no single pathway to a solution” (Knott et al., 2013, p. 600). As Stephan and Akyuz (2013) have also emphasized, such tasks are “problematic situations that are experientially real for students” and are suitable for mathematization by students (p. 509). However, there are differences in rich tasks for pre-service teachers and high school/middle school students. The rich tasks for high school or middle school students aim to promote student learning. However, they can be regarded as

a two-sided coin in the case of pre-service teachers because they include both the purpose of teacher learning as well as that of experiencing the tasks as learners like students. In this study, the teacher learning perspective of a rich mathematical task was considered in the design. The pre-service teachers had to consider the characteristics of interdisciplinary tasks that they prepared by keeping in mind how the students would interact with the tasks and how the tasks would contribute student learning. The knowledge of high-level tasks (rich tasks) needs to be considered in the design of interdisciplinary tasks.

When the characteristics of rich tasks are thought (Knott et al., 2013), mathematical modeling tasks can be also considered rich tasks. Mathematical modeling tasks are complex, non-routine tasks that have the potential for multiple solution strategies (e.g., Wess et al., 2021). Based on Maaß' (2010) aspects of the *quality* of a task to develop modeling competency, Geiger and his colleagues (2022) focused on identifying “objectives of a task”, the “nature of the relationship to reality, type of model used, type of representation, openness of a task, cognitive demand, and mathematical content” in task design and implementation to enhance mathematical modeling competencies (p. 318). Moreover, Geiger and his colleagues (2022) developed a Design and Implementation Framework for Mathematical Modeling Tasks (DIFMT), and in the design part, the principles for modeling task design were derived as the following: “nature of problem, relevance and motivation, accessibility, feasibility of approach, feasibility of outcome, and didactical flexibility” (p. 324). The *nature of the problem* refers to the open-endedness and connections within mathematics and with other disciplines (Geiger et al., 2022). The feature of *relevance and motivation* refers to the connection of the tasks with students' lives (Geiger et al., 2022). The tasks should engage students and motivate them to arrive at a conclusion. *Accessibility* as another task characteristic means that the problem allows “a mathematical approach accessible to students” (Geiger et al., 2022, p. 324). That is, students could be able to develop mathematical methods to let them solve the given problem. Another feature is the *feasibility of approach* that allows using mathematics in an appropriate way, setting assumptions, and collecting data to solve the problem (Geiger et al., 2022). Moreover, the *feasibility of outcome* refers to the possibility of a mathematical solution with necessary interpretations (Geiger et al., 2022). Lastly, the *didactical flexibility* refers to the structure of presenting the problem and checking the need for “sequential questions” to facilitate the instruction (Geiger

et al., 2022, p. 324). Altogether, these features of mathematical modeling contribute to the design of interdisciplinary tasks as guiding principles.

Similarly, Tekin Dede and Bukova Güzel (2023) developed a framework to help teachers to plan and implement mathematical modeling tasks. This framework also informs certain task characteristics in designing tasks for teaching. The *planning* part of this framework contains “*identifying the goals, deciding the modeling task, and making preparations*” as three main parts (Tekin Dede and Bukova Güzel, 2023, p. 191). Identifying the goals is a part of designing tasks and an initial step for the construction of tasks. “Identifying the grade, subject, objectives/goals, concept and duration” (Tekin Dede and Bukova Güzel, 2023, p. 191) were the components of the aspect of *identifying the goals*. These aspects are especially associated with teaching, namely the pedagogical dimension of tasks. These are also the characteristics of tasks that should be considered in designing interdisciplinary tasks.

To sum up, considering the characteristics of tasks is a crucial aspect of designing interdisciplinary tasks. The characteristics of rich tasks, particularly those of mathematical modeling tasks guide the design process of interdisciplinary tasks. Mathematical modeling is an essential component of interdisciplinary mathematical problems for teaching, so it is necessary to meet the characteristics of mathematical modeling tasks.

5.1.3. Mathematical Modeling Competency for Interdisciplinary Teaching

The literature on mathematical modeling also contributed to defining *mathematical modeling competency* for the skill to formulate IMPT. *Mathematical modeling* is defined as an iterative process of real-world problem solving by using mathematics to make sense the real-world problem (Niss et al., 2007; Bliss et al., 2014). Moreover, *modeling competence* refers to being able to successfully go through the cyclical process of mathematical modeling in which multiple steps or phases are involved. *Modeling competence* is defined as the following:

the ability to identify relevant questions, variables, relations or assumptions in a given real-world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation, as well as the ability to analyse or compare

given models by investigating the assumptions being made, checking properties and scope of a given model etc. (Niss et al., 2007, p. 12)

This definition of modeling competence implies the complexity of the notion. Such a definition of mathematical modeling competence is called *global competence* (e.g., Wess et al., 2021) because the mathematical modeling process is considered as a whole process. There are different dimensions in the notion, and some researchers have called these dimensions as *sub-competencies* (e.g., Blum, 2015; Greefrath and Vorhölter, 2016). Based on Borromeo Ferri's (2006) cognitive perspective on mathematical modeling, *modeling competence/competency* is composed of understanding the problem, simplifying, mathematizing, working mathematically, interpreting, and validating, which are considered as sub-competencies of mathematical modeling competency. Understanding the problem refers to how students make sense of the given situation (Greefrath and Vorhölter, 2016). Simplifying refers to identifying relevant aspects of a real-world situation. Mathematizing means the transformation of the real-world situation into a mathematical model, namely expressing the given information in mathematical terms. Working mathematically includes the solution approaches using mathematical methods. Moreover, interpreting means considering the mathematical results in terms of the real-world situation and trying to make sense the mathematical results in the original problem situation. Lastly, validating refers to questioning the results in terms of sufficiency such that the obtained results would offer adequate solutions for the initial problem (Wess et al., 2021). Thus, the sub-competencies in mathematical modeling draw a portrait of specific student expectations that are necessary to complete the mathematical modeling process.

To sum up, while keeping in mind the sub-competencies in mathematical modeling, this study considered global competence of mathematical modeling. In this study, the focus was not directly on the sub-competencies of mathematical modeling but there was an emphasis on the whole process overall. This is aligned with the design principle on mathematical modeling in this study because mathematical modeling is conceptualized for interdisciplinarity. Mathematical modeling is considered a component of interdisciplinary tasks or interdisciplinary problems in the design of the module on interdisciplinary teaching in this study. The pre-service teachers in this study engaged in the mathematical modeling tasks as learners to experience interdisciplinary tasks with mathematical modeling problems introduced during the class. Moreover, the pre-service teachers had to envisage the whole

process of mathematical modeling during the design process of interdisciplinary mathematical problems for teaching.

5.1.4. Content Knowledge of Each Discipline

Moreover, teacher learning contributed to including the element of *content knowledge of each discipline*. Content knowledge is related to the notion of *subject matter content knowledge* (Shulman, 1986). The *subject matter content knowledge* refers to the type of knowledge that comprises of domain-specific facts and rules as well as teachers' justifications for the coherence of domain-specific facts and rules (Shulman, 1986). Thus, *subject matter content knowledge* requires teachers to understand key ideas *within* a discipline. In this study, *content knowledge of each discipline* refers to disciplinary knowledge base including key disciplinary ideas. To formulate IMPT, content knowledge of each connected discipline for the interdisciplinarity is needed. This is related to the framework of *Interdisciplinary Pedagogical Content Knowledge* (IPCK) (An, 2017). IPCK is developed to indicate the necessary set of a knowledge base for interdisciplinary teaching (An, 2017). The IPCK framework (An, 2017) has been grounded on Shulman's (1986) *pedagogical content knowledge* (PCK) and Frykholm and Glasson's (2005) *pedagogical context knowledge*. The *pedagogical content knowledge* (PCK) is the content knowledge for teaching including knowing different kinds of representation of content and preconceptions or alternative conceptions of students (Shulman, 1986). The interdisciplinary content knowledge was constructed by the intersection between multiple content knowledge in different disciplines and the IPCK emerged out as an interaction between different PCKs (An, 2017).

Like the framework of interdisciplinary teaching by An (2017) which was developed based on an empirical study on pre-service teachers, Weinberg and Sample McMeeking (2017) also proposed a model on interdisciplinary teaching (IPCK: *interdisciplinary pedagogical content knowledge*). For the IPCK, based on the interdisciplinarity of mathematics and biology, Weinberg and Sample McMeeking (2017) focused on the two components as subject matter knowledge and pedagogical methods in the intersection of different disciplines. The subject matter knowledge refers to the content knowledge to teach through an interdisciplinary approach. This component corresponds to the interdisciplinary

content knowledge in the framework of An (2017). The pedagogical methods especially emphasize the interdisciplinary concepts that provide links between disciplines. Moreover, this component about pedagogical methods corresponds the interdisciplinary pedagogical content knowledge. The model of Weinberg and Sample McMeeking (2017) can be considered as a subset of the framework by An (2017) since the former model is based on two disciplines in terms of interdisciplinary teaching and specific to the interdisciplinarity between mathematics and biology while the latter one is broader such that there can be more than one discipline.

To sum up, content knowledge of each discipline is an essential component for the skill to formulate IMPT since disciplinary content knowledge forms the base for setting interdisciplinary connections. Identifying key ideas in each connected discipline is important for designing IMPT. Moreover, the two elements of the skill to formulate IMPT can be associated with IPCK (An, 2017; Weinberg and Sample McMeeking, 2017). The content knowledge of each discipline and interdisciplinary thinking are the two elements related to IPCK. The content knowledge forms the foundational base to initiate interdisciplinary connections. Furthermore, interdisciplinary thinking necessitates considering interdisciplinary connections between different disciplines that is expected in interdisciplinary content knowledge.

5.2. The Interdisciplinary Mathematical Modeling Task Analysis Framework

The *Interdisciplinary Mathematical Modeling Task Analysis Framework* (see Table 4.18) was developed based on Task Analysis Dimensions from Cycle 2 and the rubric constructed in Cycle 3. The dimensions were improved based on literature review to extend the coverage of dimensions. Initially, the dimensions were deductively developed based on the literature on interdisciplinarity and mathematical modeling. Then, the dimensions were revised based on the data analysis. Thus, the task analysis framework was finalized.

The rubric constructed by the pre-service teachers in Cycle 3 had the following key dimensions: *interdisciplinarity*, *context*, *organization*, *content*, *mathematical modeling*, *expectations*, and *teaching* (see Table 4.17). However, the pre-service teachers could not further combine the dimensions although the dimensions were not completely distinctive.

These dimensions were like those constructed in Cycle 2 except the dimension of *teaching*. *Teaching* was not stated as a separate dimension in Cycle 2. This might stem from the explicit focus of the course on the pedagogical purposes of tasks in Cycle 3. Moreover, the scattered indicators of interdisciplinarity observed in the rubric might stem from their lack of deep conceptualization of interdisciplinarity. Learning a new complex concept and applying this concept is challenging for pre-service teachers. This is also connected with teacher learning and implies the complexity of teacher learning. Especially for learning complex notions, pre-service teachers may need more support and different learning opportunities aligned with a *situated learning perspective* that requires learners engage in different learning experiences (e.g., Peressini et al., 2004).

In the *Interdisciplinary Mathematical Modeling Task Analysis Framework*, the first dimension, *interdisciplinarity*, is composed of the three indicators as *integration*, *interdisciplinary content*, and *authenticity*. Boix Mansilla (2010) has defined interdisciplinarity as *integrating* different disciplines to make products, solve problems or describe events. In the definition of interdisciplinarity, *integration* of disciplines is a key concept. To solve the problem, disciplinary ideas from each contributing discipline are connected “as a coherent whole” (Song and Wang, 2021, pp. 1045-1046). As Song and Wang (2021) emphasized, *knowledge integration* from different disciplines is necessary to solve an interdisciplinary problem. Through the *integration* of various “disciplinary insights”, it is possible to get an *enrichment* in understanding or solving the problem (“Advancement through integration”) (Boix Mansilla and Duraising, 2007, p. 233). Integration can also be defined as “not only collecting the appropriate disciplinary pieces of information and placing them in a central repository, but also proficiency in *integrating*-mixing, connecting, and applying them to discover new insights or ideas” (Tripp and Shortlidge, 2019, p. 6). In this study, *integration* refers to the extent of the contribution by the connected disciplines (mathematics and another discipline) to solve the problem. As the balance of contribution is settled down, the level of integration is defined to be *interdisciplinary* (highest level). There is a full balance in this level. However, as the contributions of all disciplines are important although not equal, it can be said that the level is *mostly interdisciplinary*. When the weight of the contribution of the disciplines was not balanced and focused on just one discipline, the level of integration is defined to be low (*mostly monodisciplinary*). These levels are similar with the continuum of interdisciplinarity

(Williams et al., 2016). In the continuum of interdisciplinarity, the lowest level corresponds to the *mostly monodisciplinary* (one discipline is on the focus to solve the problem, each discipline separately tries to solve the problem), the *intermediary level* corresponds to the *mostly multidisciplinary* (disciplines come together to solve the same problem but still within their disciplinary limits), and the next upper level corresponds to the *mostly interdisciplinary* (disciplines jointly try to solve the problem on a joint theme without considering the disciplinary limits) (Williams et al., 2016). Thus, these levels could be considered different approaches to interdisciplinarity.

Interdisciplinary content is another indicator of interdisciplinarity. It refers to the content of contributing disciplines that are used to solve the interdisciplinary problem. Discipline specification, interdisciplinary theme, and interdisciplinary learning goals are considered herein. Discipline specification is addressing which disciplines are integrated to solve the problem. *Big ideas* (Walt et al., 2017) of each contributing discipline are the key element of the interdisciplinary content. Identifying *interdisciplinary learning goals* includes clarifying disciplinary big ideas to be taught with a justification that explains why they are included (Boix Mansilla and Duraising, 2007; Wu et al., 2019; Tripp and Shortlidge, 2020). An *interdisciplinary theme* has *connections* with *at least two* disciplines (mathematics and another one). The *theme* should allow opportunities to find *commonalities* (common topic, context, concepts) in different curricular subjects/contents (An, 2017).

Authenticity is another indicator of interdisciplinarity. According to the Organisation for Economic Co-operation and Development [OECD] (2018), authenticity is a task characteristic that allows learners to have a *real-world connection* with “their learning experiences” and an insight of *goal* in learning (p. 7, italics added). For this study, authenticity contains the connection of the task to the *real-world* and the *openness* of the task. The problem should be authentic with respect to mathematics and another discipline. Authenticity refers to meaningfulness in all contributing disciplines.

Concerning authenticity, *openness* refers to the openness of problem context, answer, and problem-solving process (Hwang and Han, 2025, p. 10). *Openness of problem context* refers to ill-defined situations that are required to solve the problem and associated numbers are not given (Hwang and Han, 2025). In the *openness of the context*, additional search for

data or information is necessary to solve the open problem. The *openness of answers* mean that the problem offers different answers. Open tasks lead to different solutions to the problem (Wess et al., 2021). The *openness of problem-solving* is about the variety of approaches to the problem and refers to the opportunity of different solution methods (Wess et al., 2021; Hwang and Han, 2025).

In the framework, mathematical modeling is composed of four indicators: *Defining the problem*, *model building*, *getting a solution*, and *explaining results*. Mathematical modeling is defined as an approach to utilize “mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena” (Garfunkel and Montgomery, 2019, p. 8). According to Garfunkel and Montgomery (2019), mathematical modeling is a cyclical and iterative process composed of the following elements: “Identify the problem, make assumptions and identify variables, do the math, analyze and assess the solution, iterate, implement the model” (pp. 12-13). For this study, *mathematical modeling* is a cyclic process that is composed of defining the problem, model building (making assumptions and defining variables), getting a solution, and explaining results (Schukajlow et al., 2015; Garfunkel and Montgomery, 2019). These components correspond to sub-competencies of mathematical modeling.

Modeling cycle refers to the iterative process of mathematical modeling. The modeling process is iterated if needed to “refine and extend” the model (Garfunkel and Montgomery, 2019, p. 12). The modeling cycle is considered including different phases or steps corresponding to different competencies. For the analysis framework, the task should allow students to go through the different phases of modeling cycle during solving the problem. In literature, the most common referenced modeling cycles are those of Borromeo Ferri (2006) and Blum and Leiss (2007). The number of steps used in the modeling cycle changes with respect to the purpose of usage. Learners who are novel to mathematical modeling could have difficulties in differentiating the steps of simplifying and mathematizing as well as the steps of interpreting, validating, and exposing (Schukajlow et al., 2015, p. 1244). As Schukajlow, Kolter and Blum (2015) have used a simplified model that merged certain phases in the modeling cycle to reduce the complexity of representation, a simplified model (combining certain phases) in this study is used to analyze the steps of modeling cycle. Since the pre-service teachers did not know mathematical modeling before the course, they might

have difficulties in proceeding with the steps like interpreting and validating. For analyzing the designed tasks, the steps are *defining the problem*, *model building*, *getting a solution*, and *explaining results*.

For mathematical modeling, *defining the problem* means that a *real-world* question is written, and a *concise* problem statement is *specified*. The *concise* problem statement is given early in the report (Garfunkel and Montgomery, 2019). The problem should be *clear* such that students could understand the problem and then work on it. Moreover, *model building* refers to the process of producing a mathematical model by *making assumptions* and *defining* necessary *variables*. Model building is also called *mathematising*, which is the process of translating a real model to a mathematical model, in which knowledge of other disciplines is necessary (Borromeo Ferri, 2006). *Making assumptions* refers to explicitly specifying key assumptions to build a mathematical model. Namely, reasoning on assumptions is mentioned in the report such as appropriate assumptions that can lead to different solutions, or the existence and complexity of assumptions with their justification if needed. *Defining variables* refers to identifying important quantities for the mathematical model construction. The justification of including certain variables could be added and presented in a clear way.

As another indicator, *getting a solution* refers to using *reasonable* and *understandable mathematical method(s)* to get a solution to the problem (Garfunkel and Montgomery, 2019, p. 218). Getting a solution also corresponds to Borromeo Ferri's (2006) competency, *working mathematically*, in which mathematical results are got from the mathematical model. *Mathematical competencies* are needed to make a "transition from the mathematical model to mathematical results" (Borromeo Ferri, 2006, pp. 92-93). The model also utilizes *significant mathematics* to make sense of the real-world situation. Certain *mathematical ideas* are combined to depict the real-world *situation* and solve the problem. Mathematical results are *precisely presented* aligned with the initial problem statement to be "accessible to the audience" (Garfunkel and Montgomery, 2019, p. 219). Appropriate *visual aids/graphics* are used to communicate the results if needed (Garfunkel and Montgomery, 2019, p. 219).

As another indicator of mathematical modeling, *explaining results* refers to interpreting and validating mathematical results in real-world context. The modeling process and mathematics used are explained in the context of the problem and with respect to the audience (Garfunkel and Montgomery, 2019, p. 211). The mathematical solution is transformed back into the real situation (Schukajlow et al., 2015, p. 1245). The mathematical results are checked whether they are meaningful in another discipline (real-life context). When they are not reasonable in the real world, the process is iterated to get reasonable results (Borromeo Ferri, 2006; Schukajlow et al., 2015).

In the framework, *implementation* is related to teaching the task. It is defined as determining the duration, level, and goals of the task with necessary materials and students' working ways on the task, scaffolding, and expected students' answers and difficulties (Geiger et al., 2022; Tekin Dede and Bukova Güzel, 2023). The indicators of implementation in this framework are *preparation*, *goals/objectives*, *scaffolding*, and *students' answers and difficulties*. These indicators will be described in the following paragraphs.

The first indicator of implementation is *preparation*. It refers to the preparation of the task for implementation consisting of the *duration*, *level*, *materials*, and *ways of task*. These components are also mentioned in the literature on designing and teaching mathematical modeling tasks (e.g., Tekin Dede and Bukova Güzel, 2023). *Duration* is the expected duration for the task. *Level* means the level of the task with associated student level and grade level. Moreover, the task should be solvable based on students' prior knowledge. The cognitive demand level of the task should also be at a high level (Smith and Stein, 1998; Tekkumru-Kisa et al., 2015). Relevant *materials* for the task implementation are noted. Besides, how the task would be implemented in terms of students' "*ways of work*" (group work or individual work) is noted (Tekin Dede and Bukova Güzel, 2023, p. 191). Mathematical modeling tasks are often recommended to be conducted within small groups due to the complex nature of such tasks (e.g., Geiger et al., 2022).

Another indicator of implementation is *goals/objectives*. Identifying goals is a part of planning for (mathematical modeling) tasks (Tekin Dede and Bukova Güzel, 2023). The intended disciplinary and interdisciplinary goals/objectives for the task are stated. The problem is checked whether it creates opportunities for students to solve it as aligned with

the objectives. Besides, *scaffolding* is also an indicator of implementation. In general, *scaffolding* can be defined as “targeted assistance that is provided to students as they perform a task” (Rachamim et al., 2022, p. 4). Concerning teaching mathematical modeling tasks, scaffolding is viewed as a “balance between too much guidance by the teacher and sufficient guidance to help learners to be autonomous in learning” (Geiger et al., 2022, p. 326). The importance of using scaffolding in helping teachers in task design has also been emphasized in the literature (e.g., Borromeo Ferri and Blum, 2010; Schukajlow et al., 2015; Rachamim et al., 2022). For this analysis framework, *scaffolding* consists of the *extent* and *time* of help that would be offered to students during the implementation of the task. Considering the teaching of the task (implementation), possible scaffolding by the teacher is noted in the report. The decision by the teacher on the level and time of possible scaffolding is stated (how much help and *when to scaffold*). Moreover, *expected students’ answers and difficulties* are also considered as an indicator for implementation. Expected students’ answers to the problem and associated possible students’ difficulties in the task implementation are stated. Recognizing difficulties and mistakes are considered among competencies for teaching mathematical modeling (Borromeo Ferri and Blum, 2010).

5.3. The Conceptualizations on Interdisciplinary Teaching

The conceptualizations of the pre-service teachers on interdisciplinary teaching in this study had certain similarities with other studies (e.g., Weinberg and Sample McMeeking, 2017; Akaygun and Aslan-Tutak, 2020; Akbal, 2024).

Some initial conceptualizations of the pre-service teachers about interdisciplinary teaching in this study were similar in certain aspects to the study of Akbal (2024). Akbal (2024) examined how in-service mathematics teachers used interdisciplinarity in the International Baccalaureate Diploma Programme in their instruction and assessment. Akbal (2024) investigated the conceptions of the in-service teachers on interdisciplinarity about teaching through semi-structured interviews in a qualitative study. *Interdisciplinary thinking* was one of the themes revealed for the conceptions of the teachers in the study of Akbal (2024). This theme consisted of codes of *multiple perceptiveness*, *relationship with other disciplines*, and *critical analysis* (Akbal, 2024). *Multiple perceptiveness* refers to considering “a multiplicity of perspectives” and interpreting within different views (Akbal,

2024, p. 38). This was similar to the initial conceptualizations of the pre-service teachers on interdisciplinarity in this study such that some pre-service teachers referred to *multiple perspectives* in describing the characteristics of interdisciplinary mathematical modeling tasks (in Cycle 2). *Multiple perspectives* in this study was contained in the theme of context for the initial conceptualizations on the characteristics of interdisciplinary mathematical modeling tasks (in Cycle 2). Such conceptualizations were broad conceptualizations on interdisciplinarity aligned with the conceptions of interdisciplinary thinking in the study of Akbal (2024). Like this general conceptualization, some initial conceptualizations on interdisciplinarity resemble those about the *relationship with other disciplines* (Akbal, 2024). The *relationship with other disciplines* referred to the “intersection of mathematics with other disciplines” (Akbal, 2024, p. 38). This conception resembles conceptualizations such as *interdisciplinarity as connections* and *interdisciplinarity as an area* as well as *connections* in the context of interdisciplinary education in this study. In this study, initial conceptualizations of the pre-service teachers on interdisciplinarity were mostly basic such as the conceptualizations of *interdisciplinarity as connections* and *interdisciplinarity as an area* (in Cycle 3). Interdisciplinarity was considered as interrelated areas in these initial conceptualizations. Moreover, in this study, the conceptualization of *connections* was one of the initial conceptualizations on interdisciplinary education (in Cycle 3). This conceptualization means using connections between disciplines for teaching. This is like the perspective of *interdisciplinarity as connections*, which is the conceptualization on defining interdisciplinarity as they both emphasize on relations between different disciplines. However, for teaching, there is a nuance such that the connections between disciplines are expected to be between school subjects. Namely, the conceptualization on interdisciplinarity was broader when compared to that on interdisciplinary education. Thus, overall, these initial conceptualizations were general descriptions on interdisciplinarity and interdisciplinary education.

Another initial conceptualization on interdisciplinary teaching in this study was the *use of mathematics* (in Cycle 3). The *use of mathematics* is defined as noticing the value of mathematics in other disciplines and applying mathematics in real-life and in other disciplines. This is like the conception on interdisciplinarity in instruction (that is, *mathematical usage*) of the in-service teachers in the study of Akbal (2024). The *mathematical usage* belongs to the theme of *conceptual understanding and procedural*

fluency and refers to the use of mathematical concepts with a purpose of conceptual understanding (Akbal, 2024). However, in this study, the *use of mathematics* was viewed to be more comprehensive such that the application of mathematics in real-life situations was of concern.

Moreover, certain conceptualizations on interdisciplinary teaching in this study showed similarities with the pre-service teachers' definitions of STEM education in the study of Akaygun and Aslan-Tutak (2020). After a module implementation focusing on collaboration in integrative STEM activities, the pre-service mathematics and chemistry teachers' views on the definition of STEM education changed from definitions with no integration to those with more integration such as "teaching subjects together" and "integration of subject areas" (Akaygun and Aslan-Tutak, 2020, p. 156). In this study, the pre-service teachers initially conceptualized interdisciplinary education as *connections* (using connections between disciplines for teaching) and the *use of mathematics* (applying mathematics in real-life and in other disciplines) (in Cycle 3). In addition to these initial conceptualizations on interdisciplinary education, *cognitive demand* and *mathematical modeling* were other new conceptualizations revealed in the post-conceptualizations in this study. The conceptualization of *connections* is like the "teaching subjects together" (Akaygun and Aslan-Tutak, 2020, p. 156) because both focus on teaching different disciplines together. However, the conceptualization of connections is not just teaching disciplines separately but also involves interdisciplinary relationships between mathematics and another discipline. In this respect, this conceptualization resembles the "integration of subject areas" due to the interdisciplinarity focus, but it is still not a direct mention on the integration. Besides, the *use of mathematics* implies a more explicit focus on mathematics concerning the role of mathematics in teaching with other disciplines. Moreover, the post-conceptualizations of *cognitive demand* and *mathematical modeling* directly emphasize the method of interdisciplinary teaching. Especially, mathematical modeling tasks experienced during the module implementation could be influential on this conceptualization by offering a model of instruction. Aligned with situative perspective, the pre-service teachers engaged in different learning experiences in the process of learning to teach with an interdisciplinary approach. Cognitive demand was also continuously mentioned in the class discussions during the module implementation on interdisciplinary teaching (in both Cycle 2 and Cycle 3). Questioning the level of tasks presented for reflection or those tasks administered during

the class became the routine of the class. All in all, the post-conceptualizations on interdisciplinary education of the pre-service teachers in this study evolved to more explicit mentions on the method of interdisciplinary education (including mathematical modeling) compared to the initial conceptualizations.

In their qualitative study, Weinberg and Sample McMeeking (2017) aimed to explain “how and to what extent” high school biology and mathematics teachers integrate science and mathematics curricula in their lessons after involvement in an intervention to enhance science and mathematics integration (p. 204). The study intended to encourage interdisciplinary instruction by setting collaborations between mathematics and biology teachers (Weinberg and Sample McMeeking, 2017). Based on semi-structured interviews, Weinberg and Sample McMeeking (2017) examined teachers’ descriptions of integration approaches. They found that integration approaches were described as mainly cross-disciplinary (using real-world examples which address different disciplines to make context for a specific content topic) but also “intradisciplinary, multidisciplinary, and interdisciplinary approaches” (Weinberg and Sample McMeeking, 2017, p. 210). *Intradisciplinary* instruction refers to the approach specific to one discipline and consists of teaching subjects separately; *multidisciplinary* instruction refers to using themes that cross across different disciplines without an explicit focus on problems, and *interdisciplinary* instruction refers to using different ways from different disciplines to solve a problem (Weinberg and Sample McMeeking, 2017). Moreover, Weinberg and Sample McMeeking (2017) found that mathematics and science teachers differed in their descriptions on integration approaches (curriculum integration) such that mathematics teachers mainly focused on cross-disciplinary approaches in which the use of mathematics (“applications”) was exemplified (Weinberg and Sample McMeeking, 2017, p. 210). The descriptions of science teachers were found to be more parallel with interdisciplinary instruction such that the science instruction “went beyond the use of applications” and to solve problems in science with the contributions of mathematics (Weinberg and Sample McMeeking, 2017, p. 210). The *use of mathematics* as a conceptualization in this study was like the description of “applications” in the study of Weinberg and Sample McMeeking (2017). They both focused on the applications of mathematics in real world or other disciplines. However, mathematical modeling as one of the post-conceptualizations of the pre-service teachers in this study focused on the use of mathematics to solve real world problems. This conceptualization was

more like the description of the science teachers on integration with a more emphasis on problem solving. Thus, the post-conceptualizations of some pre-service teachers in this study approximated the interdisciplinary approach depicted by the science teachers in the study of Weinberg and Sample McMeeking (2017). However, unlike the differentiation of descriptions based on majors of teachers in the study of Weinberg and Sample McMeeking (2017), the conceptualizations of the pre-service teachers on interdisciplinary education in this study did not explicitly differ based on their majors. This might have stemmed from that both majors of the pre-service teachers in this study were unfamiliar with interdisciplinary instruction and were just introduced interdisciplinary education through the module on interdisciplinary teaching.

Based on interviews, it was found that mathematics and biology teachers needed “the interdisciplinary pedagogical skills in addition to the pedagogical content knowledge in their own discipline to successfully move toward interdisciplinary instruction” (Weinberg and Sample McMeeking, 2017, p. 211). Similarly, the pre-service teachers in this study initially expressed the need for professional development on interdisciplinary education in the interviews. Moreover, some pre-service teachers in this study mentioned that they felt comfortable in designing interdisciplinary tasks, but they could probably have difficulties in interdisciplinary teaching in the future implementation with students when they became a teacher. As Weinberg and Sample McMeeking (2017) also suggested, to overcome such difficulties “content training, modeling of interdisciplinary pedagogy and collaboration, and ongoing supports” could be used (p. 211) in future designs on interdisciplinary teaching.

Besides, in this study, some pre-service teachers could not initially give an explicit definition of mathematical modeling and the initial conceptualizations about *mathematical modeling* were mostly concentrated on mathematical modeling as concrete model and representation. Mathematical modeling was also viewed as a real-life situation by a few of the pre-service teachers at the beginning of the module implementation on interdisciplinary teaching, but this was less compared to other conceptualizations. The post-conceptualizations on mathematical modeling in this study mostly focused on mathematical modeling as problem solving (in both Cycle 2 and Cycle 3). Moreover, mathematical modeling was also viewed as *visualization* (especially for data visualization) by some pre-service teachers (Cycle 2).

The conceptualizations about mathematical modeling of the pre-service teachers in this study were aligned with certain studies (e.g., Jung and Newton, 2018). For instance, Jung and Newton (2018) conducted a study with eighteen secondary mathematics pre-service teachers in the context of a secondary teacher preparation program which consisted of three courses on mathematics education. Discussions and problem solving in small groups “followed by instructor-led, whole-class discussions” as well as peer feedback were used in the courses (Jung and Newton, 2018, p. 172). That study investigated how secondary mathematics pre-service teachers conceptualized and enacted mathematical modeling standards (Jung and Newton, 2018, p. 170). The pre-service teachers’ definitions of mathematical modeling were examined before and after the initial methods course (Jung and Newton, 2018, p. 173). Jung and Newton (2018) found that the modeling definitions were mostly aligned with the notion of *modeling mathematics* (Cirillo et al., 2016) such that the pre-service teachers emphasized using “representations or manipulatives to represent mathematics without considering real-life contexts” (p. 173). The definitions using the real-world were very few at the beginning (Jung and Newton, 2018). After one modeling session, the definitions of the pre-service teachers were mostly like *mathematical modeling* (that is, including real world reference) (Jung and Newton, 2018).

Like the initial definitions of mathematical modeling in the study of Jung and Newton (2018), the pre-service teachers in this study mostly conceptualized mathematical modeling as a *concrete model* or *representation* (in Cycle 3). This conceptualization resonates with the definition focusing on modeling mathematics such that representations were the focus for mathematics without reference to real-world. Some pre-service teachers in this study also initially conceptualized mathematical modeling as *real-life situation*, which was more like the definition of the pre-service teachers after the modeling session in the study of Jung and Newton (2018). Real-world reference was an important nuance factor for mathematical modeling in the study of Jung and Newton (2018). The post-conceptualization of mathematical modeling as *problem solving* in this study was like *teaching mathematics via problem solving* (Schroeder and Lester, 1989). Teaching mathematics via problem solving means that a problem context (usually real-world) is used to teach mathematics and problem solving is viewed as a main goal (Jung and Newton, 2018, p. 172). Problem-based learning through real-world problems is considered in this aspect of teaching mathematics and this is aligned with using mathematical modeling since mathematical modeling focuses on complex

problems from real-world. The pre-service teachers in this study were more aware of the problem-solving aspect of mathematical modeling as revealed in their post-conceptualizations.

5.4. Designing Interdisciplinary Tasks and Problems for Teaching

The *conceptualizations* of the pre-service teachers on designing interdisciplinary tasks or problems were investigated before and after the implementation of the module on interdisciplinary teaching. Concerning how to design *interdisciplinary mathematical modeling tasks*, the pre-service teachers focused on *context*, *modeling* and *organization* as their conceptualizations that revealed in the interviews (Cycle 2). These themes were common for both pre- and post-conceptualizations on designing interdisciplinary mathematical modeling tasks but there were more codes in the post-conceptualizations compared to the pre-conceptualizations in each theme. Although the pre-conceptualizations were more limited, the post-conceptualizations included more detailed descriptions on designing interdisciplinary mathematical modeling tasks. For the pre-conceptualizations, the pre-service teachers viewed context as connections between disciplines or as real-life connections. For the post-conceptualizations, the pre-service teachers viewed context as interdisciplinary connections, real-life connections or open-ended problem context. Moreover, in addition to the initial codes of model construction, variables, data and results, the pre-service teachers also focused on assumptions and narrowing down of the problem related to mathematical modeling conceptualizations after the module implementation. There was a transition from modeling to mathematical modeling for conceptualizations. Initially, modeling was considered as general aspects related to model construction without specifically focusing on the steps of the mathematical modeling process. For the post-conceptualizations, the reference to different aspects of mathematical modeling was more prevalent. For the initial conceptualizations, the pre-service teachers mainly focused on active class about the theme of organization, which was the aspect about teaching interdisciplinary mathematical modeling tasks. Moreover, for the post-conceptualizations, the descriptions on organization were richer and included aspects such as ways of work, resources, duration, and level of tasks.

Besides, the initial conceptualizations on designing *interdisciplinary mathematical problems for teaching* were limited and mostly based on previous experiences of the pre-service teachers such as previous university courses or tutoring experiences (Cycle 3). For the initial conceptualizations, interdisciplinarity was revealed as a rare mention. Moreover, the post-conceptualizations on designing interdisciplinary mathematical problems for teaching concentrated on interdisciplinarity, mathematical modeling and implementation with more detailed descriptions compared to the initial conceptualizations. For interdisciplinarity, the post-conceptualizations referred to choosing disciplines and authenticity of the tasks. For mathematical modeling, the post-conceptualizations referred to mathematization, assumptions, data and solution. In addition, for the implementation, the post-conceptualizations included ways of presentation, level and tools.

The conceptualizations on designing interdisciplinary mathematical modeling tasks or interdisciplinary mathematical modeling problems for teaching had similarities in certain aspects with literature on the perceptions about STEM education and designing realistic mathematics problems (e.g., Sevinc, 2024; Yang et al., 2025). For instance, a qualitative study conducted with Taiwanese and Indonesian mathematics teachers examined teachers' perceptions on integrated STEM education by focusing on the goals, values of mathematics, and teaching approaches in integrated STEM education (Yang et al., 2025). The study found that the mathematics teachers' perceptions concentrated on *content* and *context* as two main aspects about task design in integrated STEM education. The content aspect included "considering mathematical content" and "other disciplines content" while the context aspect included "considering real-world context", students' real-life context, and "simplified context" (Yang et al., 2025, p. 334). In this study, the pre-service teachers also focused on context concerning task design. In this study, the pre-service teachers mentioned *choosing disciplines* for interdisciplinarity in designing interdisciplinary tasks and this could be associated with *considering other disciplines* in the study of Yang and her colleagues (2025) since both focused on connecting mathematics with other disciplines. Moreover, the *authenticity* aspect for interdisciplinarity could be associated with *considering real-world context* in the study of Yang and her colleagues (2025) since both aspects emphasized real-world problems. Similarly, the reference to *real-life* in designing interdisciplinary problems in this study was linked to considering students' *real-life* (Yang et al., 2025) context due to the focus on real-life in both views. Moreover, *technology usage* was perceived as important

in integrated STEM education by the mathematics teachers in the study of Yang and her colleagues (2025). This aspect was also like the use of *tools* that emerged in the post-conceptualizations of the pre-service teachers in this study (Cycle 3). The pre-service teachers focused on the *use of the internet* and *mobile applications* in designing interdisciplinary problems for teaching. In that respect, there was a focus on technological tools in designing tasks by both the pre-service in this study and the mathematics teachers in the study of Yang and her colleagues (2025).

In addition to the similarities, this study also differed in certain aspects from the study of Yang and her colleagues (2025) on the perceptions of integrated STEM education. For instance, one difference on the conceptualizations on task design between this study and the study of Yang and her colleagues (2025) could be the conceptualization of *open-ended* situations revealed in the context for the post-conceptualizations in this study (Cycle 2). The open-ended emphasis could be stemmed from the nature of mathematical modeling problems implemented during the module on interdisciplinary teaching. Mathematical modeling problems did not have one answer and were open such that multiple approaches to solve the problems as well as different solutions were possible. Moreover, *considering mathematical content* in the task design for integrated STEM education in the study of Yang and her colleagues (2025) was expressed differently in this study. The mathematical aspect in this study revealed itself in the emphasis on *mathematical modeling*. Modeling and mathematical modeling explicitly emerged in the conceptualizations of the pre-service teachers on designing interdisciplinary tasks or problems in this study.

Besides, this study had certain similarities with studies on creating problems (e.g., Sevinc, 2024). As a part of a larger design-based study, Sevinc (2024) examined pre-service middle school mathematics teachers' *personal meanings* of realistic mathematics problems through training sessions including "small group discussions, writing realistic mathematics problems, and whole-group discussions" (p. 1). Personal meanings refer to "what a person finds meaningful and relevant about an action, concept, or issue" (Sevinc, 2024, p. 3). Through content analysis, Sevinc (2024) investigated the realistic mathematics problems created by the pre-service mathematics teachers to understand the personal meanings as well as the characteristics of realistic mathematics problems conceptualized by the pre-service teachers. Based on the analysis, the *aspects of personal meanings* in the study of Sevinc

(2024) were “involving mathematics topics, using mathematics in context, using mathematics for optimization”, and the “nature of the mathematical answer” (p. 15). In addition to the written artifacts, the analysis of discussions in the study of Sevinc (2024) showed that the pre-service teachers considered *curriculum* in designing problems, especially considered the *grade-level* of the students. Furthermore, *cognitive demand level*, *mathematical content* of the designed problems, and the *connection of mathematics with other disciplines* were taken into consideration by the pre-service teachers (Sevinc, 2024). All in all, Sevinc (2024) found that the pre-service teachers conceptualized realistic situations containing “multiple information” with possible *multiple answers* through “multiple solution methods” (p. 1). *Multiple information* referred to the togetherness of relevant and irrelevant information to solve the problem; “*multiple ways of solutions*” required determining priorities and using them in calculating, and “*multiple correct answers*” referred to reasonable different solutions (Sevinc, 2024, p. 20). Based on the findings, Sevinc (2024) concluded that the pre-service teachers’ personal meanings about realistic mathematics problems revealed four dimensions: “mathematical content/curriculum, mathematics in meaningful contexts, goal-directed use of mathematics”, as well as “reasonable mathematical answers” (p. 23).

Sevinc (2024) also noted that the “*previous experiences* of the pre-service teachers as learners and teachers of mathematics” influenced their personal meanings and became “a starting point for developing new personal meanings of realistic mathematics problems” (p. 20). This aspect was similar in this study. Initial conceptualizations of the pre-service teachers on designing interdisciplinary problems were based on *previous experiences* in this study. For example, prior university courses (such as a course on teaching through games) were used to give examples on designing interdisciplinary problems prior to the module implementation on interdisciplinary teaching. Moreover, the pre-service teachers emphasized *real-life* context in designing interdisciplinary mathematical modeling tasks and *authenticity* in designing interdisciplinary problems for teaching in this study. These conceptualizations were similar with the personal meaning aspect of *using mathematics in context* for the study of Sevinc (2024) since real-life context was also emphasized in *using mathematics in context*. Moreover, using *multiple acceptable answers* as the nature of the mathematical answer was aligned with the nature of mathematical modeling component of the interdisciplinary tasks and problems in this study. The multiple answers were also

considered by the pre-service teachers as revealed in interviews and designed tasks and problems in this study. In the theme of mathematical modeling about conceptualizations, *solution* (in Cycle 3) and *results* (in Cycle 2) referred to possible multiple answers in this study. Furthermore, as the pre-service teachers considered curriculum, grade-level, cognitive demand, and interdisciplinary connections in their design of realistic mathematics problems (Sevinc, 2024), the pre-service teachers in this study considered cognitive demand, student level, interdisciplinarity in designing interdisciplinary problems and considered level, interdisciplinarity in designing interdisciplinary tasks. Since mathematical modeling problems are also regarded as realistic mathematics problems (e.g., Sevinc), this aspect is not surprising. The core aspect inherent in interdisciplinary tasks and problems was mathematical modeling. Besides, the aspect of involving multiple information (Sevinc, 2024) could be associated with *identifying the problem* in mathematical modeling because the person engaged in mathematical modeling problems would need to select relevant information at the beginning of the mathematical modeling process. The broad context of the mathematical modeling problem is narrowed down to solve the problem in the step of *simplifying* (e.g., Greefrath and Vorhölter, 2016; Wess et al., 2021). Through *simplifying*, the relevant and irrelevant information on the real-situation were identified (e.g., Greefrath and Vorhölter, 2016; Wess et al., 2021).

The findings on the pre-service teachers' *artifacts* (interdisciplinary mathematical modeling tasks and interdisciplinary mathematical problems for teaching) indicated that the designed artifacts were mostly satisfactory in many aspects but needed improvement in certain aspects.

In Cycle 2, the pre-service teachers' tasks were analyzed based on the *Task Analysis Framework for Interdisciplinary Mathematical Modeling Tasks* (see Table 4.6), which included dimensions of *context of the problem*, *mathematical modeling*, *expected student solutions*, *organization of the task*, and the *level of cognitive demand*. About the *context of the problem*, most of the tasks were satisfactory (three of the four tasks). The interdisciplinary levels of the tasks were identified as mostly multidisciplinary based on the framework of the interdisciplinary continuum by Williams and his colleagues (2016). The tasks did not include any explicit interdisciplinary learning goals. For the dimension of *mathematical modeling*, most of the tasks (three of them) were specified as satisfactory since

they were mostly aligned with the criteria of mathematical modeling tasks (Wess et al., 2021). Only one task did not meet the criterion of *openness* for the criteria of mathematical modeling tasks. Besides, for another dimension, that is *expected student solutions*, two tasks were identified as satisfactory whereas other two tasks were identified as acceptable. This implied that the pre-service teachers somewhat had difficulties in this aspect during the design as also revealed in their post-conceptualizations on designing interdisciplinary tasks. Considering student thinking and finding multiple solutions aligned with student thinking was expressed as a challenge in the design process for some pre-service teachers. Moreover, for the dimensions of the *organization of the task* and the *level of cognitive demand*, most of the tasks were identified as satisfactory (three of them). Based on the *Mathematical Analysis Task Guide* (Smith and Stein, 1998) and the *Task Analysis Guide in Science* (Tekkumru-Kisa et al., 2015), the analysis of the tasks in terms of cognitive demand indicated that all tasks were identified as high level of mathematics whereas two tasks were identified as low level of science.

In Cycle 3, the pre-service teachers' problems were analyzed based on the *Interdisciplinary Mathematical Modeling Task Analysis Framework*, which included *interdisciplinarity*, *mathematical modeling*, and *implementation* as dimensions (see Table 4.18). The problems for the final projects were designed in small groups (pairs). There were two problems designed by the pre-service teachers. The findings on the analysis of the problems indicated that both problems were satisfactory overall but certain aspects could be improved.

For the dimension of *interdisciplinarity*, both problems were mostly aligned with the indicators of *integration*, *interdisciplinary content*, and *authenticity*. For *interdisciplinary content*, both problems did not include explicit interdisciplinary learning goals, but implicitly mentioned them. Besides, for *integration*, the levels of interdisciplinarity for both problems were mostly interdisciplinary. This indicated that the level of interdisciplinarity in designed problems was not at the top level since a complete balance was not achieved in the problems. Moreover, concerning *authenticity*, although both problems were open overall, they were not open in terms of problem-solving process since one way of solution was noted in the reports for designing the problems. Besides, for the dimension of *mathematical modeling*, both problems met the criteria overall. However, for the indicators of *getting a solution* and

explaining results, both problems did not include validation or interpretation of results. Moreover, for the dimension of *implementation*, both problems needed improvement in the indicators of *scaffolding* and *expected students' answers and difficulties*.

The findings for the artifacts in this study showed that pre-service teachers had certain challenges in designing interdisciplinary tasks or problems. Concerning *interdisciplinarity*, pre-service teachers had difficulties in enhancing interdisciplinary levels in their designed tasks. Moreover, they had challenges in satisfying the openness of tasks. These findings were like the findings of a recent study on designing mathematical modeling tasks by pre-service mathematics teachers (Hwang and Han, 2025). Hwang and Han (2025) required the pre-service teachers to design mathematical modeling tasks and evaluated the tasks based on the criteria of *reality*, *complexity*, and *mathematical model usage*. The pre-service teachers in the study of Hwang and Han (2025) especially had challenges in meeting the reality dimension. This criterion focused on the inclusion of a real-world situation that was highly relevant for students. This reality aspect resembles the interdisciplinarity dimension of this study. Pre-service teachers in this study needed to use a non-mathematics context and to connect it with mathematical modeling. The interdisciplinarity dimension is related to integrating this non-mathematical context as an interdisciplinary theme in which learners would find interesting for their lives. Increasing the weight of discipline other than mathematics (namely, the real-world context) was the aim for interdisciplinarity. Like Hwang and Han (2025), the pre-service teachers in this study had difficulties in the real-world inclusion reflected in their designed tasks. Hwang and Han (2025) explained the result of unrealistic tasks with the limited conceptualization of reality, mostly based on word problems. Similarly, pre-service teachers in this study did not experience open-ended, authentic or interdisciplinary tasks before and their previous experiences could influence their challenges in interdisciplinarity in designing problems.

The findings of this study also showed that some pre-service teachers had certain challenges about *mathematical modeling* in designing interdisciplinary tasks or problems. For their designed tasks in this study, pre-service teachers did not focus on dimensions such as explaining results in real-world. Besides, about the design process, the pre-service teachers stated their difficulties about data collection and adapting assumptions for the mathematical modeling process. This could be related to their lack of certain mathematical

modeling competencies. This finding was consistent with the literature on pre-service teachers and new learners on mathematical modeling (e.g., Hwang and Hang, 2025; Schukajlow et al., 2015). Especially, since pre-service teachers have been usually new learners for mathematical modeling, they may lack mathematical competencies such as “interpreting, validating, and exposing” (e.g., Schukajlow et al., 2015, p. 1244). Besides, difficulties of pre-service teachers on data collection during mathematical modeling in this study were parallel with the pre-service mathematics teachers’ challenges in dealing with real data in designing mathematical modeling tasks in the study of Hwang and Han (2025). It was connected to the lack of mathematical competencies in solving complex mathematical modeling tasks (e.g., Hwang and Han, 2025). This can be also related to managing the cognitive demand level of tasks (Stein et al., 2000), in turn pedagogical competencies specific to mathematical modeling. Setting appropriate assumptions for mathematical modeling tasks was also related to the challenges about the mathematical modeling process. Learners could have difficulties in identifying about the real-world problem (e.g., Schukajlow et al., 2012).

For the development of the skill in designing interdisciplinary mathematical problems for teaching, *mathematical modeling* was a core aspect. The key role of mathematical modeling in interdisciplinary learning, particularly in mathematics and science education was also emphasized in the literature (e.g., Sokolowski, 2018; English, 2024). Mathematical modeling was considered a tool to develop interdisciplinarity (e.g., English, 2024; Nugraha et al., 2024). In mathematics education, mathematical modeling was viewed as a bridge connecting mathematical reasoning and “reasoning about a situation in the real world” (Sokolowski, 2018, p. 34).

Concerning the aspect of *implementation* in the design, pre-service teachers had difficulties about *scaffolding* and *expected students’ answers and difficulties*. This is expected since these aspects are specifically related to teaching and pedagogical content knowledge (PCK). This is also aligned with the pedagogical aspects of task design (e.g., Greefrath et al., 2022). Pre-service teachers lack teaching experience, so they could have challenges in estimating student difficulties and in turn acting in response to such difficulties. Adapting scaffolding level was also emphasized in literature (e.g., Geiger et al., 2022). Considering expected student answers and related possibilities was also noted as a challenge

in the literature connected to noticing student thinking about mathematical modeling (e.g., Cai et al., 2022).

The empirical studies on designing STEM tasks also had certain similarities with the findings of this study about designing interdisciplinary tasks or problems (e.g., Mumcu et al., 2023). On *designing interdisciplinary tasks*, one mixed-method study examined “the effect of a professional development (PD) program designed for integrated STEM education on teachers’ pedagogical design competencies and the contribution of the PD program to teachers’ integrated STEM understanding” (Mumcu et al., 2023, 3443). The participants were computer science, mathematics, and science middle school teachers and they were asked to collaboratively prepare STEM lesson plans based on the 5E learning model in the PD program consisted of five modules (Mumcu et al., 2023). The PD program included “active practice” via “collaborative group work” such that one mathematics, one science and two computer science teachers formed a group for collaborative activities (Mumcu et al., 2023, p. 3448). The findings on the analysis of the lesson plans showed that the lessons plans were found to be at an acceptable level with respect to integrated STEM education but some aspects such as the *explain* and *evaluate* phases in the 5E learning model needed to be improved (Mumcu et al., 2023, p. 3458). In the *explain* phase, it is expected to give elaborative descriptions “in terms, definitions, and explanations for a particular topic or concept” and in the *evaluate* phase, it is expected to evaluate “the skills acquired by the students in the learning and teaching process” (Mumcu et al., 2023, p. 3459). At the *explain* and *evaluate* phases, an *interdisciplinary problem* expression was supposed to be elicited (Mumcu et al., 2023). Mumcu and her colleagues (2023) found that in the *explain* phase, the teachers in the PD program had challenges in providing integrated STEM content and in the *evaluate* phase, the teachers had difficulties in the assessment of “interdisciplinary learning outcomes” (p. 3459). Mumcu and her colleagues (2023) associated this situation with the lack of such experiences as the teachers have taught their disciplines separately aligned with the curriculum structure.

The difficulties of the teachers for the *explain* phase in the study of Mumcu and her colleagues (2023) had similarities with the design experiences of the pre-service teachers in this study. The pre-service teachers in this study also showed difficulties in certain aspects in designing interdisciplinary tasks and problems. For instance, for interdisciplinarity, the

pre-service teachers in this study did not include interdisciplinary learning goals in their designed interdisciplinary mathematical modeling tasks and interdisciplinary mathematical modeling problems for teaching. This finding was like the study of Mumcu and her colleagues (2023) since both participants had difficulties in integrating content from different disciplines. Although the teachers in the study of Mumcu and her colleagues (2023) had certain STEM education experiences before the PD program, the pre-service teachers in this study did not have any experience with interdisciplinary education before the intervention. Designing interdisciplinary problems requires integration of different disciplinary knowledge (e.g., Mumcu et al., 2023; Nugraha et al., 2024), and therefore, it might need an explicit focus on developing skills in designing interdisciplinary tasks or problems. Moreover, concerning *collaboration*, this study had certain similarities with the study of Mumcu and her colleagues (2023). Different majors were used to support each other in terms of content. In the study of Mumcu and her colleagues (2023), two computer science teachers were included in the groups to support mathematics and science teachers in the process of designing STEM lesson plans. This study included collaboration as a key design feature of the module on interdisciplinary teaching. In designing interdisciplinary tasks or problems for their final project assignments, the pre-service teachers often worked in collaboration in small groups. One mathematics pre-service teacher and one science pre-service teacher were expected to be involved in the groups for their final project assignments. The different majors fed each other in terms of content as the pre-service teachers also expressed in the post-interviews about designing their final projects (interdisciplinary tasks or problems).

In their review study on “STEM education programs for secondary mathematics teachers”, Yang and Ball (2024) examined how STEM tasks were “designed and implemented to support secondary mathematics teachers’ learning of STEM teaching” (p. 190). The review focused on STEM *task design* for both pre-service and in-service secondary mathematics teachers’ learning (Yang and Ball, 2024). Based on the review, Yang and Ball (2024) concluded that “goals, principles and implementation” were considered as “the phases of STEM task development” (p. 203). As an initial phase, identification of *goals* (“the intended goals for students’ learning through completion of the tasks”) necessitates to make mathematical content and practices explicit and associated with other disciplines (Yang and Ball, 2024, p. 197). In the next phase, Yang and Ball (2024) suggested to use the

principles for task design, which referred to “rules of design to effectively achieve the intended goals of a task” (p. 197). In the third phase, the *implementation* meant “the ways employed to conduct the tasks” (Yang and Ball, 2024, p. 197). Yang and Ball (2024) recommended using these three phases as guidelines to design “mathematics-based STEM tasks” (p. 204). In this study, the pre-service teachers considered *goals* to develop their interdisciplinary tasks. Especially, they focused on the learning goals aligned with curriculum objectives for mathematics and the connected discipline. As Yang and Ball (2024) suggested using explicit mathematical content and practices and relative interdisciplinary connections, the pre-service teachers took into consideration to make the mathematical goals explicit in the interdisciplinary tasks. However, although they considered goals separately for each contributing discipline, the pre-service teachers had difficulties in making the interdisciplinary learning goals explicit. The inclusion of mathematical modeling in designing interdisciplinary tasks in this study could be associated with the *principles for task design* (Yang and Ball, 2024) since mathematical modeling provides a way to achieve goals related to mathematical content and practices. On the other hand, the implementation aspect in this study was different from the phase of *implementation* for STEM task design (Yang and Ball, 2024) because the implementation in this study referred to planning for teaching the designed task without including applications with students in the classroom. However, the implementation phase in the study of Yang and Ball (2024) included a reference to the administration of the task with students.

5.5. The Module on Interdisciplinary Teaching

As a practical outcome, a new module on interdisciplinary teaching was designed and developed through iterative cycles. Indeed, the module was developed in the context of a new course on interdisciplinary mathematics and science education. This course and module on interdisciplinary teaching can provide teacher educators with useful insights for their implementation on interdisciplinary teaching, especially interdisciplinary mathematics and science education. The *design principles* on the module on interdisciplinary teaching were specified after Cycle 3 as the following: (i) *Promote understanding of interdisciplinarity*, (ii) *Enhance opportunities for learning to teach mathematical modeling in interdisciplinarity*, and (iii) *Foster active teacher learning*.

In this study, to promote understanding of interdisciplinarity, the inclusion of resources on interdisciplinarity such as articles on interdisciplinary research and interdisciplinary education (e.g., Williams et al., 2016) was found to be useful. The pre-service teachers reflected on these articles in this study. The aim of the inclusion of these *resources* in this study was to familiarize the pre-service teachers in theory about interdisciplinarity and interdisciplinary education since the pre-service teachers were not familiar with such content. This was aligned with the study of Fitzpatrick and Leavy (2025) on integrated STEM education. Fitzpatrick and Leavy (2025) conducted a qualitative study (as a part of larger action research) to investigate “evolving understandings” of pre-service primary teachers of integrated STEM education during a 12-week intervention and to specify “the types of learning experiences that contributed to these changes” (p. 24). The researchers used certain principles to design a module on integrated STEM education for pre-service primary teachers (Fitzpatrick and Leavy, 2025). Engaging “in theory of STEM education” focusing on “integration, STEM pedagogies and frameworks” was one of the principles that framed the design of the module on integrated STEM education by the researchers (Fitzpatrick and Leavy, 2025, p. 7). Aligned with this guiding principle, the researchers found that it was necessary to provide a balance in disciplines for integrated STEM education (Fitzpatrick and Leavy, 2025, p. 24). This was also parallel with the findings of this study on the interdisciplinary thinking for designing interdisciplinary tasks. The findings revealed that a need on interdisciplinary focus on designing interdisciplinary tasks was evident such that a balance between the connected disciplines (mathematics and other disciplines) was considered. In future implementations, the use of such resources on interdisciplinarity for reflection was suggested to be included.

To enhance opportunities for learning to teach mathematical modeling in interdisciplinarity, the pre-service teachers experienced different *mathematical modeling tasks* as learners during the lessons. The pre-service teachers used the mathematical modeling task characteristics in their design of tasks or problems (e.g., Wess et al., 2021). Experiencing mathematical modeling tasks as learners and engaging in a variety of mathematical modeling tasks were also suggested in the literature on teaching mathematical modeling and integrated STEM education (e. g., Garfunkel and Montgomery, 2019; Geiger et al., 2022; Tekin Dede and Bukova Güzel, 2023; Fitzpatrick and Leavy, 2025). For instance, Fitzpatrick and Leavy (2025) designed a module on integrated STEM education

for pre-service teachers with a guiding principle of engaging the pre-service teachers “in STEM activities as a learner” (p. 7). Fitzpatrick and Leavy (2025) found that the pre-service teachers benefited from “learning by doing” in integrated STEM education experiences (p. 19). Thus, it was suggested to include a variety of *mathematical modeling tasks* such that the pre-service teachers would experience as learners themselves in the future designs to support the pre-service teachers designing interdisciplinary modeling tasks.

Learning to teach in an interdisciplinary approach was the broader aim of this study. This is related to *active teacher learning*. To support the pre-service teachers in learning to teach in an interdisciplinary approach, the pre-service teachers were required to actively engage in learning experiences during the module on interdisciplinary teaching. This study has adopted a situative perspective of learning to teach such that learning of the pre-service teachers were a combination of experiences distributed in different contexts such as individuals, social discourse, tools or learning environment (Putnam and Borko, 2000). Aligned with this perspective, the pre-service teachers had individual assignments such as reflections on articles shared in the online learning environment. This intended the individual aspect of learning. Moreover, “participation in social contexts” (Buchbinder and McCrone, 2018) was considered in participating in collaborative activities such as designing interdisciplinary tasks in small groups as well as participating in discussions through small-group and whole-group discussions. In addition, the interaction with resources in the course (the setting in which the research was conducted) was a part of situated learning in this study. The pre-service teachers also used different technological tools during mathematical modeling tasks to represent their mathematical models in this study. As revealed in the post-interviews, the pre-service teachers used the Internet as a common source of knowledge to design their interdisciplinary tasks. Similarly, other researchers used situative perspectives in pre-service education (e.g., Ryu et al., 2019). For example, a methods course on integrated STEM was developed using situative perspectives (e.g., Putnam and Borko, 2000) as theoretical framework for secondary pre-service teachers (Ryu et al., 2019). The pre-service teachers in the study of Ryu and her colleagues (2019) were asked to develop integrated STEM lessons and the study of Ryu and her colleagues (2019) aimed to investigate “how STEM teacher education” pre-service teachers experienced “teaching STEM in an integrated way” (p. 494). The participants benefited from different “resources and ideas from their own experiences, classroom observations, and the Internet” in designing their STEM lesson plans

(Ryu et al., 2019, p. 501). Ryu and her colleagues (2019) concluded that this finding indicated “the situatedness of learning and importance of learning contexts” (p. 502). Similarly, this study also found that the Internet, classroom resources, and collaboration in small groups were aspects that contributed to the pre-service teachers’ learning to design interdisciplinary tasks and this implied the situatedness of learning by the pre-service teachers as well as the variety of different experiences provided by the module on interdisciplinary teaching.

In addition, as a part of *learning to teach*, the pre-service teachers actively involved in activities during the class. This feature is like the study of Akaygun and Aslan-Tutak (2024). *Learning to teach integrative STEM* requires active involvement of the pre-service teachers as learners during the implementation of STEM education with relevant reflection (Akaygun and Aslan-Tutak, 2020, p. 160). Moreover, like the *integration* of STEM disciplines focused on interdisciplinarity and solving real life problems through *modeling* (Akaygun and Aslan-Tutak, 2020), this study focused on interdisciplinary teaching through *mathematical modeling*. The pre-service teachers in this study engaged in mathematical modeling tasks as learners and they prepared interdisciplinary mathematical tasks/problems such that interdisciplinarity and mathematical modeling were key aspects.

To foster active teacher learning, *collaboration* with another major and working in small groups were found to be useful in this study. This is aligned with literature on teaching mathematical modeling and STEM education (e.g., Akaygun and Aslan-Tutak, 2020; Geiger et al., 2022; Mumcu et al., 2023; Fitzpatrick and Leavy, 2025). In this study, collaboration between different majors of pre-service teachers was found to be useful in supporting pre-service teachers in designing interdisciplinary tasks. This was aligned with the study of Akaygun and Aslan-Tutak (2020) on integrative STEM education including collaboration between chemistry and mathematics teachers. Different majors of pre-service teachers were used to collaboratively engage in the STEM activities in the study of Akaygun and Aslan-Tutak (2020) and they found that most of the pre-service teachers mentioned collaborative activities as positive learning experiences such that group members fed each other in terms of knowledge. Moreover, the lack of subject matter knowledge of a pre-service teacher in the group work “diminishes the interdisciplinary work of the group” while “that knowledge of the other discipline improved their work” (Akaygun and Aslan-Tutak, 2020, p. 158).

Similarly, in this study, some pre-service teachers mentioned that they needed a profound content knowledge of their own areas since they had sometimes difficulties in conducting interdisciplinary tasks. Moreover, in this study, the pre-service teachers (in both Cycle 2 and Cycle 3) emphasized the facilitating effect of group work in designing interdisciplinary tasks or problems. They added that they could not be able to design such an interdisciplinary task/problem without their group member. Moreover, Fitzpatrick and Leavy (2025) also found collaboration and reflection as key learning experiences in integrated STEM education including pre-service primary teachers. Group work is useful in interdisciplinary learning and in mathematical modeling tasks (e.g., Garfunkel and Montgomery, 2016; Geiger et al., 2022). Thus, it was recommended using collaboration through small groups with different majors of pre-service teachers in the future implementations.

To foster active teacher learning, this study used *discussion* as a core element in the design. The pre-service teachers used discussion in different ways during the module implementation. For instance, through a whole-class discussion, the pre-service teachers developed a rubric as a guideline to design interdisciplinary tasks. Moreover, the pre-service teachers also used discussion during the class work while engaging in mathematical modeling tasks in small groups. This is aligned with certain studies on pre-service teacher education about mathematical modeling, realistic mathematics problems or integrated STEM education (e.g., Jung and Newton, 2018; Pimthong and Williams, 2021; Sevinc, 2024). Jung and Newton (2018) used discussions and problem solving in small groups as well as whole-group discussions on their study with pre-service secondary mathematics teachers. Furthermore, Pimthong and Williams (2021) designed a STEM methods course for pre-service primary teachers to support the pre-service teachers' STEM understanding. Student-centered activities guided the design of the STEM course and discussion was among the core principles that guided the design of the STEM methods course (Pimthong and Williams, 2021). Small-group and whole-class discussions were used to introduce "the concept, purposes, and importance of STEM education" (Pimthong and Williams, 2021, p. 7). Moreover, Sevinc (2024) used small group and whole group discussions in her study investigating personal meanings of pre-service middle school mathematics teachers on realistic mathematics problems.

In this study, *scaffolding* was also a core design feature on the module on interdisciplinary teaching to foster *active teacher learning*. In the current study, the researcher provided timely scaffolding for within-class tasks as well as conducted additional online scaffolding sessions to help the pre-service teachers design interdisciplinary tasks. Scaffolding was found to be useful in mathematical modeling pedagogy, especially on task design (e.g., Borromeo Ferri and Blum, 2010; Schukajlow et al., 2015; Geiger et al., 2022). For scaffolding, Geiger and his colleagues (2022) have focused the concept of *measured responsiveness*, in which teachers should give “prompts that focused on the modeling process or ask questions structured to promote exploration of a range of possible solution pathways without directing them to a single pre-determined response” (p. 330). Such scaffolding indicates that the instructor’s help during the mathematical modeling process (engaging in mathematical modeling tasks) should not impose the pre-service teachers to solve the mathematical problem with one solution but instead should encourage the pre-service teachers to solve the problem in different ways. Thus, to foster active teacher learning, it was suggested to include scaffolding for the design of a module on interdisciplinary teaching.

To conclude, this study found certain key aspects on designing a module on interdisciplinary teaching for pre-service teachers. Concerning the design of the module on interdisciplinary teaching, one key aspect was using resources on interdisciplinarity that allowed the pre-service teachers reflect on was beneficial for promoting understanding of interdisciplinarity. Theoretical engagement with interdisciplinary resources was found to be a key aspect to support pre-service teachers’ understanding of interdisciplinarity. Thus, as a design principle for a module on interdisciplinary teaching, it was suggested to *promote understanding of interdisciplinarity* with appropriate interdisciplinary resources.

In this study, other key aspects on designing the module on interdisciplinary teaching was including a variety of mathematical modeling tasks and engaging the pre-service teachers in mathematical modeling tasks as learners. Thus, as a design principle for a module on interdisciplinary teaching, it was suggested to *enhance opportunities for learning to teach mathematical modeling*. Moreover, in this study, collaboration in small groups, discussion, and scaffolding were found as other key aspects in designing the module on interdisciplinary teaching. Thus, as a design principle for a module on interdisciplinary teaching, it was

suggested to *foster active teacher learning* with a variety of experiences including collaboration, discussion and scaffolding.

5.6. Limitations and Implications

This study designed and implemented a module on interdisciplinary teaching for pre-service mathematics and science teachers. The focus of the study was on pre-service teachers' skills on designing interdisciplinary tasks. This study has certain limitations and implications.

This study has certain limitations. This study is limited within its context of the study, including participants. The study was conducted in iterative cycles so that different cohorts of pre-service teachers were included as participants. In the first iteration, a variety of majors of pre-service teachers participated in the study: secondary and primary education mathematics teachers, chemistry and physics teachers as well as science education pre-service teachers. The pre-service teachers in the first iteration were mostly in their third year of study implying that they have taken certain courses on teacher education. The variety of majors were not considered as a disadvantage for the design but considered as an opportunity for using collaboration. In the second iteration, the majors of the pre-service teachers were less varied: mathematics and science education pre-service teachers. However, the common point in two iterations was that the pre-service teachers did not take an education on interdisciplinary teaching before this study. Moreover, before the study, the pre-service teachers did not have any experience of designing interdisciplinary tasks which included mathematics and other disciplines. The lack of experience on task design and mathematical modeling by the pre-service teachers led the design of the course on interdisciplinary mathematics and science education to include aspects of task design and engaging in mathematical modeling tasks. This was a preparation stage for the transition to the development of interdisciplinary tasks. Thus, when participants were ready in terms of task design and mathematical modeling, the implementation of the module on interdisciplinary teaching would take a less focus on these aspects and instead a more emphasis on interdisciplinary aspect of creating interdisciplinary tasks. For this, in future implementations, pre-service teachers in their third-year or more could be included as participants based on their readiness in terms of task design and teaching courses. Additional

investigation on curriculum objectives for interdisciplinary connections could be added to increase the focus on interdisciplinarity in further research on the implementation of a module on interdisciplinary teaching with pre-service teachers.

One limitation of this study is the lack of a research team in terms of data collection. Design-based studies are usually conducted with research teams to facilitate the research process (e.g., Cobb et al., 2017). Other researchers in the research team could have observed the sessions during the implementation of the module on interdisciplinary teaching and could have taken field notes to make additional data including observations from an outsider view. The thesis advisor had observations time to time in addition to weekly meetings with the researcher before and after each instruction. To strengthen this aspect of the research the researcher took audio-recordings of each session during the implementation.

Another limitation is concerning data analysis. Due to the nature of design-based studies (e.g., Bakker, 2019), many sources of data were collected that led large data. Dealing with data analysis of large qualitative data within the limited time of a dissertation study was challenging. To address this, the researcher focused on a certain amount of data sources from different iterations. A research team that would include peer researchers could have facilitated the analysis process of qualitative data. Follow-up studies could focus on other data sources such as the artifacts produced by the pre-service teachers during the class work to investigate the development of mathematical modeling competencies from one session to other or audio-recordings to examine the discourse among the pre-service teachers to understand the interaction dynamics during the group work.

This study considered one aspect of pedagogical content knowledge as task design. Another limitation is the lack of classroom implementation with secondary school students. The pre-service teachers designed interdisciplinary tasks or formulate such problems for their final projects, but the tasks were not implemented in a school environment with students. This study focused on the task dimension in terms of development as a part of pedagogical competencies for interdisciplinary teaching. For modeling-specific PCK (Greefrath et al., 2022), there is also another dimension, knowledge about interventions, indicating an implementation phase with students. In addition to the task dimension, the instructional dimension focuses on conducting lessons with mathematical modeling tasks as

a part of competencies related to teaching mathematical modeling (Borromeo Ferri and Blum, 2010). Further studies on interdisciplinary teaching with pre-service teachers could add the dimension of implementation with students to further examine the effect of the tasks developed by the pre-service teachers on student learning.

This study focused on the development of interdisciplinary tasks or formulate problems by the pre-service teachers in groups. Although individual data were collected during the class work or other classroom assignments, the focus of analysis for tasks was the artifact produced in groups because of the collaboration feature in group work that was expected to facilitate the development of the tasks. The lack of focus on the individual could be a limitation to understand the individual development of designing interdisciplinary tasks. Further studies could plan research to include both the individual and social development of designing interdisciplinary tasks. To increase the focus on the analysis on the individual level, more assignments could be given to the pre-service teachers on designing interdisciplinary tasks before they would develop their tasks in groups. So, comparison could be made between the analysis on the individual level and social level to investigate further insights on designing interdisciplinary tasks.

This study used the global modeling competency (e.g., Niss et al., 2007), therefore the use of the whole modeling process was the aim for the pre-service teachers' engagement in mathematical modeling tasks. Evaluating the sub-competencies of mathematical modeling for the pre-service teachers was not intended in this study. Further studies could investigate the development of sub-competencies concerning mathematical modeling during the implementation of the module on interdisciplinary teaching. Moreover, different types of mathematical modeling tasks could be chosen in further studies to understand the effect of different types of modeling tasks on interdisciplinary teaching.

This study has certain implications for teacher education and task design. As an implication, this study contributed to the literature on teaching mathematical modeling in a context of interdisciplinary task design. In this study, interdisciplinarity and mathematical modeling were integrated in the new construct of interdisciplinary mathematical problems for teaching. Such problems included mathematical modeling and interdisciplinarity as core elements. This was a contribution on the literature that math-focused interdisciplinary task

design by the pre-service teachers. Researchers suggested to improve math-focused STEM education programs by increasing the role of mathematics in STEM education, therefore in interdisciplinary education (e.g., English, 2024; Yang and Ball, 2024). Aligned with this suggestion, in the broader sense, the study provided contribution to the literature on integrated STEM education with a math-focused perspective on task design such that mathematics was integrated with other disciplines through mathematical modeling.

This study has implications for task design in teacher education. This study used a framework developed based on literature and data to analyze the pre-service teachers' tasks. As an implication of the study, the framework could offer a new way to guide the researchers and teacher educators in providing new opportunities for pre-service teacher education. Thus, teacher educators could use the framework as a guideline for helping pre-service teachers in the design process of interdisciplinary tasks as well as in the evaluation of the interdisciplinary tasks. This framework could be tested further by different researchers to understand its practicality for the analysis. Moreover, the developed tasks by the pre-service teachers could be analyzed based on different frameworks on integrated STEM education or combining different frameworks on mathematical modeling, interdisciplinary education and STEM education (e.g., Akaygun and Aslan-Tutak, 2020; Tripp and Shortlidge, 2020; Geiger et al., 2022; Nugraha et al., 2024) in further research.

This study has also implications for in-service teacher education. This study developed and implemented a module on interdisciplinary teaching based on iterative cycles of design. As an implication, the design developed in this study could be iterated with certain changes with in-service teachers. To further investigate the relevant design principles on interdisciplinary teaching, interdisciplinarity, mathematical modeling and active teacher learning experiences could be investigated within a series of module implementation. A series of modules could be designed in further research on interdisciplinary teaching. In these series of modules, one module could focus on interdisciplinarity, another module on mathematical modeling, and the other one on task design. For a longer intervention, these modules could be adapted to teacher PD sessions on interdisciplinary teaching as series of workshops for in-service mathematics and science teachers. Moreover, in the adaptation of this module on interdisciplinary teaching for PD for in-service teachers, class enactment of designed tasks could be added to increase the focus on student learning. Moreover, like the

collaboration between different majors in this study, the adapted PD could include collaboration between teachers with different majors such as mathematics and science.



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APPENDIX A: INITIAL SURVEY 1

1. How would you define interdisciplinary research? State and explain to the four drivers of interdisciplinary research?
2. Discuss levels of interdisciplinary (Williams et al., 2016)? For each, give an example and explain why this example suits the level.
3. You have completed a task on interdisciplinarity by investigating an area, interviewing an expert. Please answer the following questions based on your experiences of this task:
 - a. What was the area that you explored? Why do you think this is an interdisciplinary area?
 - b. Choose one area, one of your classmates explored. What was the area, how is this area an interdisciplinary area? What is the interdisciplinary level based on your friend's report? Explain why this level suits this area.
4. Explain mathematical modeling. Explain the process steps.
 - a. Why does a science/math teacher who wants to incorporate interdisciplinary education, need to know mathematical modeling?
 - b. What could be the role of mathematical modeling in interdisciplinary tasks?
5. You are given two cases. For each case, you are expected to read it and answer the following questions.
 - a. Case 1 (Water Tank): A digital company, SciMath Animations, has just been assigned to do an animation showing a variety of water tanks that will be filled with water. To do the animation, the SciMath Animations need a graph that denotes the height of the water given the amount of water in the tank (a height/volume graph). As a part of the team, you are expected to approximately draw graphs for the given water tanks below (see Figure A.1) and write a strategy guide or manual that tells how to make any possible graph for any different shaped water tank (Erbaş et al., 2024). How would you categorize the given task according to the interdisciplinarity levels of Williams et al. (2016)?



Figure A.1. Some water tank shapes (Erbaş et al., 2024, p. 103).

Note: The figure A.1 was taken from Erbaş, A. K., B. Çetinkaya, C. Alacacı, E. Çakıroğlu, A. Aydoğan Yenmez, A. Şen Zeytun, H. Korkmaz, M. Kertil, M. G. Didiş Kabar, S. Baş Ader, and Z. Şahin, 2024, *Lise Matematik Konuları için Günlük Hayattan Modelleme Problemleri*, Second Edition, Türkiye Bilimler Akademisi (TÜBA), Ankara, p. 103.

- b. Case 2: Early this morning, the police discovered that, sometime late last night, some nice people rebuilt the old brick drinking fountain in the park. The mayor would like to thank the people who did it. However, nobody saw who it was. All the police could find were lots of footprints. One distinctive, very large footprint was found and is shown below. The police feel this is the person they want to find first because whoever made the footprint seems to be very big. To find this person and his friends, it would help if we could figure out how big he/she really is. Your job is to make a “how to” toolkit, a step-by-step procedure, the police can use to figure out how big people are by looking at their footprints

(Case Studies for Kids). How would you categorize the given task according to the interdisciplinarity levels of Williams et al. (2016)?

6. Write an example of interdisciplinary mathematical modeling task.



APPENDIX B: POST-SURVEY 1

Instructions: The post evaluation consists of the following questions. Please express *your* own ideas as frankly as possible.

1. How would you define interdisciplinarity/interdisciplinary research? Give at least one example to interdisciplinarity/interdisciplinary research and explain how this example fits with interdisciplinarity.
2. Discuss levels of interdisciplinarity concerning mathematics/science education. For each, give an example and explain why this example suits the level.
3. How could you increase the level of an interdisciplinary mathematical modeling task?
4. Explain mathematical modeling. Explain the process steps.
 - a. Why does a science/math teacher who wants to incorporate interdisciplinary education need to know mathematical modeling?
 - b. What could be the role of mathematical modeling in interdisciplinary tasks?
5. Write an example of an interdisciplinary mathematical modeling task. Describe the process of designing an interdisciplinary mathematical modeling task.
 - a. Indicate your criteria for designing an interdisciplinary mathematical modeling task.
 - b. Write how you would apply the interdisciplinary mathematical task (you wrote) in the classroom. Discuss benefits and challenges of using interdisciplinary mathematical modeling tasks in the classroom.
 - c. How could using interdisciplinary mathematical modeling tasks affect students' learning of math/science?
6. You are given two cases. For each case, you are expected to read it and answer the following questions.
 - a. Case 1 (Intercontinental Eurasia Marathon): Why did the July 15 Martyrs Bridge oscillate during the Intercontinental Eurasia Marathon? Does the Intercontinental Eurasia Marathon Public Run damage the structure of the July 15 Martyrs Bridge? The high number of participants for the public run of the marathon was claimed to damage the structure of the bridge and argued that it must have been cancelled (Kesler, 2010): Some experts explained that the bridge could have been

fallen due the overload on the bridge and said that this oscillation was caused by the *resonance*, and the bridge could have been collapsed. How could you decide whether the public run of the marathon was dangerous or not for the bridge if you are asked to write a strategy report for the municipality as a part of the expert committee of sports (Mercan and Tekbıyık, 2022)? Please form a strategy based on a mathematical model considering different characteristics such as the capacity of carrying load and vibration characteristics.

- How would you categorize the given task according to the interdisciplinarity levels of William et al. (2016)?
 - How could you increase the level of interdisciplinarity of the given task?
 - How would you assess the given task considering your assessment criteria of interdisciplinary mathematical modeling tasks? How would you change the task to fit your criteria of assessing interdisciplinary mathematical modeling tasks?
- b. Case 2 (Ancient Theater Problem): A group of tourists visited Aspendos Ancient Theater in Antalya. You see a photograph taken during this visit (see Figure B.1). Assume that the location of the person B is constant (Bukova Güzel et al., 2016).
- How can you find the real distance between the marked people in the photograph?
 - How can you find the real height of the ancient theater?



Figure B.1. Ancient theater (Bukova Güzel et al., 2016, p. 103).

Note: The figure was taken from Bukova Güzel, E., A. Tekin Dede, Ç. N. Hidiroğlu, S. Kula, and A. Özaltun Çelik, 2016, *Matematik Eğitiminde Matematiksel Modelleme Araştırmacılar Eğitimciler ve Öğrenciler için*, E. Bukova Güzel (Ed.), Pegem Akademi, Ankara, p. 103.

- How would you categorize the given task according to the interdisciplinarity levels of Williams et al. (2016)?
- How could you increase the level of interdisciplinarity of the given task?
- How would you assess the given task considering your assessment criteria of interdisciplinary mathematical modeling tasks? How would you change the task to fit your criteria of assessing interdisciplinary mathematical modeling tasks?

APPENDIX C: ANONYMOUS FEEDBACK FORMS

We have been working on developing interdisciplinary mathematical modeling tasks (IMMTs) for a couple of weeks. Before we finalize this part of the course, I would like to hear from you to make possible changes to the rest of the course. Please do not write your name, let's keep it anonymous. But please write your suggestions and give your feedback freely. I value your feedback.

Sevil Sezer

PhD Candidate

Mathematics and Science Education Department

1. Which aspect(s) of the *task development part of the course* did you like? What worked well, why?
2. Which aspects of this part of the course might be improved/changed and how?
3. We have been completing some applied mathematical modeling tasks in the course. How were the applied tasks helpful for developing/designing Interdisciplinary Mathematical Modeling Tasks (IMMTs)? How could the tasks be changed to make them more useful? Please give specific task names when explaining your thoughts.
4. Your final project is to develop an IMMT. What would be a suggestion to support you to develop such a task? What else do you like to experience as part of this course to be able to develop an IMMT?

APPENDIX D: INITIAL INTERVIEW 1

1. Disiplinlerarası/disiplinlerarası yaklaşım kavramını nasıl tanımlarsınız? Disiplinlerarası düşünme size göre nedir? Düşüncelerinizi açıklamak için örnekler ve çizimler kullanabilirsiniz.
2. Disiplinlerarası eğitim sizce nedir? Disiplinlerarası matematik/fen eğitimini nasıl tanımlarsınız?
3. Disiplinlerarası matematik/fen eğitimini bir öğretmen sınıfta nasıl uygulayabilir?
4. Disiplinlerarası matematiksel modelleme etkinliklerinin özellikleri sizce neler olmalıdır? Disiplinlerarası matematiksel modelleme etkinliklerini nasıl değerlendirirsiniz?
5. Bir disiplinlerarası matematiksel modelleme etkinliğini nasıl tasarlıyorsunuz? Örnek bir etkinlik tasarımı yazınız.
6. Disiplinlerarası matematiksel modelleme etkinliğini sınıfta nasıl uygularsınız?
7. Disiplinlerarası matematiksel modelleme etkinlikleri, öğrencilerin matematik/fen öğrenmelerini nasıl etkileyebilir?
8. Disiplinlerarası matematiksel modelleme etkinlikleri kavramada, geliştirmede, uygulamada ve değerlendirmedeki yeterliliğinizi nasıl tanımlarsınız? Bu hususlarda bir eğitime/bilgilendirmeye ihtiyaç duyuyor musunuz?

Not: Etkinliklere, özellikle disiplinlerarası matematiksel modelleme etkinliklerine yönelik eklemek istediğiniz başka hususlar varsa lütfen belirtiniz.

APPENDIX E: POST-INTERVIEW 1

Disiplinlerarası üzerine biraz sohbet edelim mi? Bazı konular konuştuk derste onlar üzerine düşüncelerinizi merak ediyorum, soruları cevaplarken örnekler vererek düşüncelerinizi açıklayabilirsiniz:

1. Disiplinlerarası/disiplinlerarası yaklaşım kavramını nasıl tanımlarsınız?
2. Disiplinlerarası düşünme sizce nedir?
3. Matematik ve fen eğitiminde disiplinlerarası eğitimi nasıl tanımlarsınız?
4. Öğretmen olduğunuzda, disiplinlerarası matematik/fen eğitimini sınıfta siz nasıl uygulardınız?
5. Disiplinlerarası matematiksel modelleme etkinliklerinin özellikleri sizce neler olmalıdır?
6. Genel olarak dersi nasıl değerlendirirsiniz? Derste ne tür zorluklar yaşadınız? Dersi geliştirmek için önerileriniz nelerdir?
7. Bir disiplinlerarası matematiksel modelleme etkinliğini nasıl tasarlıyorsunuz? Bu tarz bir etkinlik tasarlarırken nelere dikkat edersiniz? Örneğin; final projenizi tasarlarırken neler yapıyorsunuz? Bahseder misiniz?
8. Disiplinlerarası matematiksel modelleme etkinliklerini nasıl değerlendirirsiniz?
9. Disiplinlerarası matematiksel modelleme etkinlikleri kavramada, geliştirmede, uygulamada ve değerlendirmedeki yeterliliğinizi nasıl tanımlarsınız? Disiplinlerarası matematiksel modelleme etkinlikleri tasarlarırken ne tür desteğe/yardıma ihtiyaç duyarsınız?
10. Bu ders ile ilgili nelerin aynı kalmasını ve değiştirilmesini isterdiniz? Özellikle, disiplinlerarası matematiksel modelleme etkinlikleri tasarlama ile ilgili bölümde aynı kalmasını ve değiştirilmesini/geliştirilmesini önerdiğiniz yanlar neler olabilir?
11. Derste kullandığımız etkinliklerin sizin öğrenmenize nasıl katkı sağladığını düşünüyorsunuz?

Not: Etkinliklere, özellikle disiplinlerarası matematiksel modelleme etkinliklerine yönelik başka eklemek istedikleriniz var mı?

APPENDIX F: DEMOGRAPHIC INFORMATION FORM

1. Adınız-soyadınız:
2. Yaşınız:
3. Cinsiyet:
4. Mezun olunan lise türü:
5. Mezun olunacak/olunan program/bölüm (branş):
6. Disiplinlerarası matematiksel modellemeye yönelik önceden bir eğitime/bilimsel çalışmaya katılmış mıydınız? Yanıtınız evet ise katılmış olduğunuz eğitimi/çalışmayı kısaca açıklayınız.
7. Önceden ne tür öğretmenlik deneyimleriniz olmuştur? (Özel ders, dersane/etüt merkezi, gönüllü etkinlikler, proje çalışmaları vd.) Açıklayınız.
8. Öğretmenlik deneyim süresi:
9. (Eğer varsa) Çalıştığınız okul türü-sınıf düzeyi:
10. Tamamen kendinizin geliştirdiği etkinlik(ler) var mı? Açıklayınız.

APPENDIX G: INITIAL INTERVIEW 2

1. Sizce model nedir?
2. Sizce modelleme nedir?
3. Sizce matematiksel modelleme nedir?
4. Sizce matematiksel modelleme sınıfta nasıl öğretilir?
5. Matematik müfredatından bir konu/kavram/önemli fikir seçiniz.
 - a. Buna göre bir etkinlik tasarlayınız.
 - b. Bu etkinliği nasıl tasarladınız?
 - c. Bu etkinliği tasarlarken nelere dikkat ettiniz?
 - d. Sizce etkinlik nasıl tasarlanır?
6. Disiplinlerarası matematiksel etkinliği nasıl tasarlıyorsunuz? Örnek bir etkinlik tasarımı yazınız.
7. Aşağıda Türkiye’de plastik durumu ile ilgili rapordan bir bölüm sunulmuştur (TMMOB Kimya Mühendisleri Odası, 2021):

Türkiye kişi başına plastik tüketimini artan hazır ve paketlenmiş gıda tüketiminin artışına paralel olarak Batı Avrupa’nın seviyesi olan kişi başına 70 kilografa çıkacağını ve TÜİK’in nüfus tahminine göre 2030 yılında nüfusun 100 milyona ulaşacağını düşünürsek on yıl içerisinde 7 milyon ton plastik atığın bertaraf ve geri kazanımı problemiyle baş başa kalacağız. Ayrıca 2020 yılında hayatımıza giren COVID-19 pandemisi ile tek kullanımlık plastiklerin kullanımı daha da artmıştır. Sadece tek kullanımlık 4 gramlık polipropilen dokumasız kumaştan imal edilen cerrahi maskelerin dahi Türkiye’de 120 bin ton ekstra plastik atığa sebep olduğu tahmin edilmektedir (p. 3).

Yukarıda verilen bölümden yararlanarak nasıl bir disiplinlerarası matematiksel etkinlik tasarlıyorsunuz?

APPENDIX H: POST-INTERVIEW 2

Soruları yanıtlarken örnekler vererek düşüncelerinizi açıklayabilirsiniz:

1. Disiplinlerarası yaklaşım ile ilgili aşağıdaki soruları yanıtlayınız.
 - a. Disiplinlerarası/disiplinlerarası yaklaşım/disiplinlerarasılık kavramını nasıl tanımlarsınız? Disiplinlerarası düşünme sizce nedir? *Örnek* vererek açıklayınız.
 - b. Matematik/fen eğitiminde disiplinlerarası *eğitimi* nasıl tanımlarsınız? Öğretmen olduğunuzda, disiplinlerarası matematik/fen eğitimini sınıfta siz *nasıl uygulardınız?*
2. Disiplinlerarasılık ve matematiksel modellemeyi *nasıl ilişkilendiriyorsunuz?*
3. Avrasya Maratonu ve Boğaziçi Köprüsü ile ilgili bir gazete haberine göre maraton sırasında köprü sallandı (Kesler, 2010): Bu duruma “rezonans” denen titreşimlerin neden olduğuna dikkat çeken uzmanlar, “Büyük bir tehlike atlatıldı, köprü darmadağın olabilirdi” dedi. Burada verilen bölümden yararlanarak *nasıl* bir Öğretim için Disiplinlerarası Matematiksel Problem *yazarsınız?*
4. Sizce Öğretim için Disiplinlerarası Matematiksel Problemi *nasıl tasarlanmalı?*
 - a. Bu tarz bir problem tasarlarlarken *nelere dikkat edersiniz?*
 - b. Öğretim için Disiplinlerarası Matematiksel Problemlerinin *özellikleri* sizce neler olmalıdır? Bu tür problemleri *nasıl değerlendirirsiniz?*
5. Ders kapsamında grup arkadaşınla hazırladığınız etkinlik için problem tasarlama sürecini düşünürsen
 - a. Bu tür problemleri *yazma prensipleriniz* nedir?
 - b. Disiplinlerarasılık ve matematiksel modellemeyi nasıl ilişkilendirip bir probleme dönüştürdünüz?
 - c. Problem yazarken ne gibi kaynak veya araçlardan yararlandınız?
 - d. Öğretim için Disiplinlerarası Matematiksel Problemleri tasarlarlarken ne tür desteğe ihtiyaç duydunuz?
6. Bir arkadaşınıza Disiplinlerarası Matematiksel Problemleri tasarlamadan bahsettiğinizi hayal etseniz, ona bir öğretmen adayı bu süreçte ne gibi *bilgi ve becerilere* ihtiyaç duyacağını söylediniz?

7. Biraz da dersimiz hakkında görüşlerinizi almak isterim. Genel olarak dersi *nasıl değerlendirirsiniz?* Dersin *güçlü ve gelişime açık yönleri* nelerdir?
8. Bu ders ile ilgili nelerin aynı kalmasını ve değiştirilmesini isterdiniz? Özellikle, disiplinlerarası matematiksel problemleri tasarlama ile ilgili bölümde aynı kalmasını ve değiştirilmesini/geliştirilmesini önerdiğiniz yanlar neler olabilir?
9. Derste kullandığımız etkinliklerin sizin *öğrenmenize* nasıl katkı sağladığını düşünüyorsunuz?

Not: Etkinliklere, özellikle Öğretim için Disiplinlerarası Matematiksel Problemlere yönelik başka eklemek istedikleriniz var mı?

APPENDIX I: INITIAL SURVEY 2

1. How would you define interdisciplinary research? State and explain four *drivers* of interdisciplinary research.
2. Discuss levels of interdisciplinarity (Williams et al., 2016). For each, give an example and explain why this example suits the level.
3. Considering the mathematical modeling, and its steps,
 - a. Explain the process steps with an example.
 - b. What could be the role of mathematical modeling in interdisciplinary tasks?
4. Explain the process of task design. How do you know that a mathematical/science task is a *good* task?
5.
 - a. How could you formulate an interdisciplinary mathematical *problem* for teaching? Give an example of an interdisciplinary mathematical problem.
 - b. Describe how you could design an interdisciplinary mathematical *task*. Explain the process and criteria that you consider in the design process.
6. Give an example of an interdisciplinary area and explain why it is an interdisciplinary area.

APPENDIX J: POST-SURVEY 2

Instructions: The post-survey consists of the following questions. Please express *your* own ideas as frankly as possible.

1. How would you define interdisciplinarity? Give at least one example to interdisciplinarity and explain how this example fits with interdisciplinarity.
2. Considering the mathematical modeling, and its steps,
 - a. Explain the process steps with an example.
 - b. What could be the role of mathematical modeling in interdisciplinary *tasks*?
3. How could you increase the level of interdisciplinarity of an interdisciplinary mathematical modeling *task*?
4. Explain the process of task design. How do you know that a mathematical/science task is a *good* task?
5. How could you formulate an interdisciplinary mathematical *problem* for teaching? Explain the *process* and *criteria* that you consider in the design process.
6.
 - a. Give an example of an interdisciplinary mathematical problem.
 - b. Write how you would *apply* the interdisciplinary mathematical problem for teaching in the classroom.
7. Discuss benefits and challenges of using interdisciplinary mathematical problems for teaching in the classroom.

APPENDIX K: RUBRIC IN CYCLE 2

1. Level of cognitive demand (*high-low*, Mathematical and Science Framework, data presentation)
2. Mathematical modeling
 - 2.1. Interpretation and evaluation of initial findings
 - 2.2. Use of mathematical models (model construction, different tools)
 - 2.3. Variables (Identifying the relationship between variables, range of variables)
 - 2.4. Different ways of solving the problem (Solution ways/methods)
 - 2.5. Implementable (The task can be usable by others)
 - 2.6. Data collection (Source of data, appropriate data, usable data)
3. Organization/structure of task (Introduction, development, and conclusion parts)
 - 3.1. Introducing the problem (Problem that can be specified)
 - 3.2. Content of the problem (Alignment with objectives and grade level)
 - 3.3. Reporting (The way of explanation, justification of used tools, task sequence)
4. Context of the problem (Interdisciplinary level, real-life connection)
5. Expected/possible student solutions (e.g., one solution is given two points, all possible solutions are given three points)

APPENDIX L: INSTITUTIONAL ETHICAL APPROVAL

Evrak Tarih ve Sayısı: 14.11.2022-97050



T.C.
BOĞAZİÇİ ÜNİVERSİTESİ REKTÖRLÜĞÜ
Fen Bilimleri ve Mühendislik Alanları İnsan Araştırmaları Etik Kurulu
(FMİNAREK)

Sayı : E-84391427-050.01.04-97050
Konu : 2022/30 Kayıt no'lu başvurunuz hakkında

14.11.2022

Sayın Doç. Dr. Fatma ASLAN TUTAK
Matematik ve Fen Bilimleri Eğitimi Bölüm Başkanlığı - Öğretim Üyesi

"Öğretmenlerin Disiplinlerarası Matematiksel Modelleme Etkinlikleri Tasarlama Yetkinliklerinin Geliştirilmesi" başlıklı projeniz ile Boğaziçi Üniversitesi Fen Bilimleri ve Mühendislik Alanları İnsan Araştırmaları Etik Kurulu (FMİNAREK)'e yaptığınız 2022/30 kayıt numaralı başvuru 07.11.2022 tarihli ve 2022/11 No.lu kurul toplantısında incelenerek etik onay verilmesi uygun bulunmuştur.

Bu karar tüm üyelerin toplantıya on-line olarak katılımıyla ve oybirliği ile alınmıştır. Onay mektubu tüm üyeler adına Komisyon Başkanı tarafından e-imzalanmıştır.

Saygılarımızla bilginize sunarız.

Prof. Dr. Tınaz EKİM AŞICI
Başkan

Figure L.1. Institutional ethical approval.

APPENDIX M: INFORMED CONSENT FORM

Sayın öğretmen adayı/öğretmen,

Boğaziçi Üniversitesi Matematik ve Fen Bilimleri Eğitimi Bölümü öğretim üyesi Doç. Dr. Fatma Aslan-Tutak ve bölümün doktora öğrencilerinden Sevil Sezer'in tez araştırması olan "Disiplinlerarası Matematiksel Modelleme Etkinlikleri Tasarlama Yetkinliklerinin Geliştirilmesi" adlı bilimsel bir araştırma projesi gerçekleşecektir. Bu araştırmanın amacı katılımcıların disiplinlerarası matematiksel modelleme etkinlikleri hazırlama yeterliklerini geliştirmektir. Bu amaca ulaşmak için mesleki gelişime yönelik tasarım tabanlı bir araştırma (TtA) (Bakker, 2019) yürütülmektedir. Bu çalışmada bize yardımcı olmanız için siz öğretmen adaylarınızı/öğretmenleri de projemize davet ediyoruz. Kararınızdan önce araştırma hakkında sizi bilgilendirmek istiyoruz. Bu bilgileri okuduktan sonra araştırmaya katılmak isterseniz lütfen bu formu imzalayıp kapalı bir zarf içinde bize ulaştırınız.

Bu araştırmaya katılmayı kabul ettiğiniz takdirde, çalışmanın başında isim, yaş, bölüm gibi bilgilerin sorulduğu demografik bilgi formunu doldurmanız ve disiplinlerarası matematiksel modelleme ile ilgili açık uçlu soruların olduğu bir anketi tamamlamanız istenecektir. Bu anketler disiplinlerarası matematiksel modelleme ile ilgili görüşlerinizi daha iyi anlamamıza yardımcı olacaktır. Bu anketlerin her birini tamamlamanın yaklaşık 60 dakikanızı alacağı beklenmektedir.

Ayrıca, mesleki gelişim modülü uygulaması öncesi ve sonrasında sizlerle gönüllü olduğunuz takdirde mülakatlar yapılacaktır. Mülakatların yaklaşık 45 dakika sürmesi öngörülmektedir. Mesleki gelişim öğretim süreci ve mülakat sırasında görüntü kaydı ve ses kaydı alınacaktır. Anket ve mülakat uygulamaları mesleki gelişim oturumları dahilinde yürütülecektir. Bu çalışmanın 10 hafta sürmesi planlanmaktadır. Mesleki gelişim modülü uygulaması sırasında oluşturulan yazılı ve/veya görsel dokümanlar analiz edilecektir. Bu çalışmanın her bir oturumunun yaklaşık 90 dakika sürmesi planlanmaktadır.

Bu araştırma bilimsel bir amaçla yapılmaktadır ve katılımcı bilgilerinin gizliliği esas tutulmaktadır. Çalışma içerisinde, katılımcıların kimliğini açığa çıkaracak hiçbir ifade kullanılmayacaktır. Katılımcı isimleri için takma isimler kullanılacaktır. Toplanan yazılı ve

görsel dokümanlar, video/ses kayıtları proje ekibi dışında kimsenin erişimine açık olmayacaktır. Seçilmiş video kayıtları katılımcıların kimliği belli olmayacak şekilde gerekli ayarlamalar yapılarak öğretmen eğitiminde veya bilimsel nitelikte gerçekleşen sunumlarda kullanılabilir. Elektronik veriler harici hard disklerde saklanacak ve harici diskler ile basılı dokümanlar araştırma projemiz süresince sadece proje ekibinin erişimindeki kilitli bir dolapta muhafaza edilip, araştırma sona erdiğinde silinecek ve yok edilecektir. Çalışmanın çıktıları, sonraki senelerde yapılacak araştırmalar için de yol gösterici olacaktır.

Bu araştırmaya katılmak tamamen isteğe bağlıdır. Katıldığınız takdirde çalışmanın herhangi bir aşamasında herhangi bir sebep göstermeden onayınızı çekmek hakkına da sahipsiniz ve bu durumdan olumsuz etkilenmeyeceksiniz. (Çalışmaya katılmak öğretmen adaylarının ders notlarını etkilemeyecektir.) Çalışmadan çekilen katılımcıların verileri silinip hiçbir şekilde kullanılmayacaktır. Çalışmaya katılım sonucunda ücret verilmeyecek olup, çalışmaya katılım belgesi verilecektir.

Araştırma projesi hakkında ek bilgi almak istediğiniz takdirde lütfen Boğaziçi Üniversitesi Matematik ve Fen Bilimleri Eğitimi Bölümü Öğretim Üyesi Doç. Dr. Fatma Aslan-Tutak veya araştırmacı Sevil Sezer ile temasa geçiniz.

Araştırmayla ilgili haklarınız konusunda Boğaziçi Üniversitesi Fen Bilimleri ve Mühendislik Alanları İnsan Araştırmaları Etik Kurulu'na (FMINAREK'e) danışabilirsiniz.

Eğer bu araştırma projesine katılmasını kabul ediyorsanız, lütfen bu formu imzalayıp kapalı bir zarf içerisinde bize geri yollayın.

Ben, (katılımcının adı), yukarıdaki metni okudum ve katılmam istenen çalışmanın kapsamını ve amacını, gönüllü olarak üzerime düşen sorumlulukları tamamen anladım. Çalışma hakkında soru sorma imkânı buldum. Bu çalışmayı istediğim zaman ve herhangi bir neden belirtmek zorunda kalmadan bırakabileceğimi ve bıraktığım takdirde herhangi bir olumsuzluk ile karşılaşmayacağımı anladım.

Bu kořullarda söz konusu arařtırmaya kendi iřteęimle, hiębir baskı ve zorlama olmaksızın katılmayı kabul ediyorum.

Çalıřma süresince ses kaydı yapılmasını onaylıyorum/onaylamıyorum.

Çalıřma süresince görüntü kaydı yapılmasını onaylıyorum/onaylamıyorum.

Formun bir örneęini aldım/almak istemiyorum (bu durumda arařtırmacı bu kopyayı saklar).

Katılımcının Adı-Soyadı:.....

İmzası:.....

Tarih (gün/ay/yıl):...../...../.....

Varsa Katılımcının Vasisinin Adı-Soyadı:.....

İmzası:.....

Tarih (gün/ay/yıl):...../...../.....

Arařtırmacının Adı-Soyadı:.....

İmzası:.....

Tarih (gün/ay/yıl):...../...../.....

APPENDIX N: PERMISSION FOR FIGURE A.1

The permission for the use of water tanks in Figure A.1 was given in the Figure N.1 (Copyright 2024 by Türkiye Bilimler Akademisi). The figure was taken from the book *Lise Matematik Konuları için Günlük Hayattan Modelleme Problemleri* by Erbaş and his colleagues (2024) on page 103.

Sevil Hanım merhabalar tekrar,
Mutlaka kaynak belirterek (belirttiğiniz notu bizimle de paylaşmanızı bekleriz baskı öncesi) kullanabilirsiniz.
İyi çalışmalar, kolaylıklar dileriz.

Saygılarımla,

Asiye Komut
Basın Sorumlusu
Basın-Yayın Halkla İlişkiler Birimi



Figure N.1. Permission for figure A.1.

APPENDIX O: PERMISSION FOR FIGURE B.1

The permission for the use of Figure B.1 was given in the Figure O.1 (Copyright 2016 by Pegem Akademi Yayıncılık). The figure was taken from the book *Matematik Eğitiminde Matematiksel Modelleme Araştırmacılar Eğitimciler ve Öğrenciler için* by Bukova Güzel and her colleagues (2016) on page 103.

Hocam Merhaba,

İlginiz için teşekkür ederiz.

Kaynak göstererek bahsi geçen görseli çalışmanızda kullanabilirsiniz.

Çalışmalarınızda kolaylıklar dileriz.

Saygılarımla

Selcan Durmuş

Department of Project & Publication
Pegem Akademi Yayıncılık
Eğitim Danışmanlık Hizmetleri Tic. A.Ş.

Figure O.1. Permission for figure B.1.