A Matheuristic for Sustainable Logistics for Food Banks

by

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Graduate School of Sciences and Engineering

This is to certify that I have examined this copy of a master's thesis by

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and that any and all revisions required by the final

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ABSTRACT

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Sustainable food system is important in many regards including minimizing food waste, decreasing carbon emission and efficient use of natural resources. Food banking operations are significant in achieving a sustainable food system. Motivation of this thesis is derived from three of the Sustainable Development Goals of United Nations, namely "Zero Hunger", "Responsible Consumption and Production"and "Climate Action". In this regard, providing a decision framework that entails approaches to solve routing and allocation problems in food banking operations is aimed by this study. The novel problem of Multi Depot-Periodic Unpaired Pickup and Delivery Vehicle Routing Problem (MD-PUPDVRP) is introduced to mimic day to day supply chain operations of a major player in food banking industry. Involvement of various objectives, including minimizing carbon emissions, fuel costs and unfairness in food distribution, maximizing total amount of rescued food requires dedicated functions to quantify them. For that purpose, a dedicated function for fairness is presented and an adaptation of a carbon emission function is made to transporter type vehicles using empirical data. As solution methodologies, a Mixed Integer Linear Programming (MILP) model is developed. In MILP model, binary variables are used to represent routing scheme, whereas linear variables are utilized to model allocation. A Variable Neighborhood Search (VNS) based matheuristic is implemented to compete with MILP model. In the matheuristic, an exact method called Allocation Subroutine (AS) is used to take advantage of decomposable structure of the problem. To generate upper bounds in a very short amount of time, another exact method called Allocation Upper Bound model (ALUB) is developed. Instances from real life scenarios are created for experimentation along with a set of synthetic instances. It has been observed that the matheuristic performs better than the MILP model for large instances where the number of nodes and number of vehicles are both high. Yet in small scale instances, mathematical model gener-

ates near optimal solutions. ALUB provides very close dual bounds to best bounds found by MILP, which enables practical interpretations as well as potential future approaches that utilize dual bound.

ÖZETÇE

Yüksek Lisans Tez Başlığı
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Sürdürülebilir gıda sistemi, gıda israfını en aza indirmek, karbon emisyonlarını azaltmak ve doğal kaynakların verimli kullanımını sağlamak gibi birçok açıdan önemli bir rol oynamaktadır. Gıda bankacılığı operasyonları, sürdürülebilir bir gıda sistemine ulaşmada kritik bir pozisyona sahiptir. Bu tezin motivasyonu, Birleşmiş Milletler'in üç Sürdürülebilir Kalkınma Hedefi'nden, yani “Açlığa Son”, Sorumlu Üretim ve Tüketim”ve “İklim Eylemi”hedeflerinden türetilmiştir. Bu bağlamda, gıda bankacılığı operasyonlarında rotalama ve paylaştırma problemlerini çözmeye yönelik yaklaşımlar içeren bir karar mekanizması oluşturmak bu çalışmanın amacıdır. Çok Depolu - Periyodik Eşlenmemiş Toplamalı Dağıtımlı Araç Rotalama Problemi (ÇD-PETDARP) adlı yeni bir problem, gıda bankacılığı sektöründe büyük bir oyuncunun günlük tedarik zinciri operasyonlarından esinlenerek oluşturulmuştur. Probleme karbon emisyonlarını, yakıt maliyetlerini ve gıda dağıtımındaki adaletsizliği en aza indirirken kurtarılan gıda miktarını maksimize etmek gibi çeşitli hedeflerin dahil edilmesi, bu hedefleri matematiksel ifadelere dönüştürüp çözümlemeyi gerektiren özel fonksiyonlar oluşturulmasını zorunlu kılmaktadır. Bu amaçla, adalet için özel bir fonksiyon sunulmuş ve transporter tip araçlar için karbon emisyonu fonksiyonunun uyarlanması empirik verilerle yapılmıştır. Çözüm yöntemi olarak, bir Karışık Tam Sayılı Doğrusal Programlama modeli geliştirilmiştir. Bu modelde, rotalama şemasını temsil etmek için ikili değişkenler kullanılırken, paylaştırmayı modellemek için doğrusal değişkenler kullanılmaktadır. Karışık Tam Sayılı Doğrusal Programlama modeline karşı rekabet edebilmek için Değişken Komşuluk Arama tabanlı bir matsezgisel uygulanmıştır. Matsezgiselde, problemin polinom zamanda çözülebilir yapısından yararlanmak için kesin çözüm yaklaşımı olan Paylaştırma Altprogramı kullanılmaktadır. Çok kısa bir sürede üst sınırları oluşturabilmek için başka bir kesin çözüm yaklaşımı olan Paylaştırma Üst Sınır Modeli geliştirilmiştir. Deneyler için gerçek hayat senaryolarından örnekler ile birlikte sentetik örneklerden

bir küme oluşturulmuştur. Yapılan gözlemler, matsezgiselin, düğüm sayısının ve araç sayısının yüksek olduğu büyük örneklerde Karışık Tam Sayılı Doğrusal Programlama modelinden daha iyi performans sergilediğini göstermektedir. Ancak küçük ölçekli örneklerde, matematiksel model neredeyse optimal çözümler üretmektedir. Paylaştırma Üst Sınır Modeli, Karışık Tam Sayılı Doğrusal Programlama modeli tarafından bulunan en iyi sınırlara çok yakın üst sınırlar sağlamaktadır. Bu durum da günlük operasyonlardaki uygulamaya yönelik değerlendirmeleri kolaylaştırırken potansiyel gelecekteki üst sınır kullanan yaklaşımların önünü açmayı amaçlar.

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ABBREVIATIONS

ALUB	Allocation Upper Bound Model
AS	Allocation Subroutine
DAFR	Deviation from average fill rate function
FB	Food Bank
MD-PUPDVRP	Multi Depot - Periodic Unpaired Pickup and Delivery Vehicle Routing Problem
MILP	Mixed Integer Linear Programming
PUPDVRP	Periodic Unpaired Pickup and Delivery Vehicle Routing Problem
SDG	Sustainable Development Goal
UN	United Nations
UNDP	United Nations Development Programme
VNS	Variable Neighborhood Search
VRP	Vehicle Routing Problem

Chapter 1

INTRODUCTION

According to the Sustainable Development Goals Report by United Nations (UN), around 9.2% of the world population (which translates to about 735 million people) faced chronic hunger in 2022 (UNDP, 2023b). As described by United Nations' (UN) Sustainable Development Goals (SDGs), ensuring a sustainable food system is crucial in order to secure sustainable development at global scale. Sustainable food systems focus on bringing nutritious and secure food to everyone. One of the most critical elements in achieving a sustainable food system are food banks. Food banks help reduce food loss by recovering surplus food and delivering them to individuals in need in an intact form.

This thesis is inspired by three of the Sustainable Development Goals of United Nations, namely "Zero Hunger", "Responsible Consumption and Production"and "Climate Action". Therefore, the main objective of this thesis is to support the decision making in food bank logistic operations by maximizing the amount of food rescued from being wasted while allowing fair distribution of intact food and cutting the CO_2 emissions. Consequently, this study introduces a novel problem, namely Multi Depot - Periodic Unpaired Pickup and Delivery Vehicle Routing Problem (MD-PUPDVRP) and aims to tackle it with a mixed integer mathematical model and a matheuristic approach.

1.1 United Nations Sustainable Development Goals

All of the United Nations member states have agreed to follow the universal call to actions to end poverty, protect the planet and ensure that all people enjoy peace and prosperity. That agreemenent was materialized under the Sustainable Development

Goals (UNDP, 2023a). SDGs, also known as the Global Goals, include 17 specific goals that UN member states have ensured to reach by 2030. In this thesis it is aimed to develop approaches to tackle issues that are related to SDGs 2, 12 and 13, namely “Zero Hunger”, “Responsible Consumption and Production”, “Climate Action”.

The 13 SGD of UN, “Climate Action”, attempts to keep the rise in global mean temperature below two degrees Celsius compared to pre-industrial times. Greenhouse gas emissions which is 50% above 1990 level fuels the increase in the temperature (UNDP, 2023a). Given these circumstances, one of the main targets of this thesis is to contribute to the environmental dimension by reducing carbon emissions. Therefore, the methods developed in this study minimizes carbon emissions to tackle with the environmental challenges that need to be addressed by building sustainable supply chains.

The 12 Sustainable Development Goal of the UNDP, “Responsible Consumption and Production”, argues that it is essential to reduce global food waste per capita by 50% by increasing the efficiency of production and supply chains. This facilitates a transition to a more resource-efficient economy while boosting food security (UNDP, 2023a). Cultivating sustainable food systems is one of the most vital components of maintaining food security. Sustainable food systems, like many other areas, take into account economic, social and environmental impacts. In this thesis, social impact of sustainable food system is considered by defining a fairness function that keeps track of whether allocation of food is equitable. Solution methodologies aim to maximize fairness in distribution, while minimizing the total amount of food.

The 2 Sustainable Development Goal of United Nations Development Programme, “Zero Hunger”, aims to eradicate hunger and all forms of malnutrition by 2030 (UNDP, 2023a). As mentioned in 12 Sustainable Development Goal of the UNDP one of the main objectives to achieve a sustainable food system is to minimize food loss and food waste. Food loss can occur at any stage of food systems whereas food waste results from suppliers or consumers throwing intact food away. Food waste is especially dreadful given that a substantial segment of the world population experiences severe malnutrition, undernourishment, and hunger. This study aims to minimize food waste by developing methodologies that help decision makers efficiently allocate and deliver intact food to those in need.

1.2 Food Banking System

One specific source of food waste is supermarkets setting the shelf life (best before date) of goods to a date before expiration date. The motivation behind such configuration is to offer high quality products to high income consumers. There are solutions that has been established to prevent these practices from wasting of intact food by delivering them to individuals or companies/associations/foundations in need before the expiration date. Food banking is one of the systems that provides solutions to cease the waste. Food banking system which started as an individual initiative in the USA has served more than 39 million people by numerous means including storing and delivering food (The Global FoodBanking Network, 2023b). In order to achieve a sustainable food system in which all people would have reliable and affordable access to food, food banks bear a significant role (The Global FoodBanking Network, 2023a).

Food banks differ by whether or not their contact with the end consumer is direct. The food banking structure in which food banks give the products to intermediary organizations will be considered in this thesis. In this structure, the donation of products is made by companies, organizations and individuals. The matching between the donated products and particular food banks is the main decision problem. There are digital platforms and companies that engage with this problem by means of simple ad-hoc rules. The data for the thesis is provided by one of these companies which enable a static and deterministic consideration of the problem within a time horizon (Lahyani et al., 2015). In addition to matching donated food and food banks, collection and delivery of food is in focus. Consequently, pick-up and delivery routes of food is considered along with pairing food with food banks.

1.3 Introduction to Problem Structure and Methodology

The base decision problem in this study involves two main sub-components, namely allocation and routing of the food. The routing subproblem is based on the widely known Vehicle Routing Problem (VRP) that is introduced to the literature by Dantzig and Ramser (1959). VRP is a problem that has a numerous applications in the literature and it is relevant to many different fields and sectors. VRP can be defined as a problem where the goal is to find optimal routes for a fleet of vehicles to traverse a set of customers. It is a computationally complex problem such that it belongs to NP-Hard problem class.

Although the problem centering this thesis is based on VRP, many aspects that increase the complexity of problem are embodied in the problem structure. The definition of the problem involves multiple depots. Furthermore, delivery and pick-up of the food is also included in the decision making. Food banks and donors are not matched in advance for the delivery and pick-up operations. Hence, the problem is unpaired. Vehicles do not have to receive (or fulfill) 100% of the donation (or demand). Thus, nodes can be visited by multiple vehicles. The problem is also periodic, as new demand and supply (donation) are considered. Perishability of food at each period is also an element that further complicates the problem. The novel problem in this study can be defined as Multi Depot - Periodic Unpaired Pickup and Delivery Vehicle Routing Problem (MD-PUPDVRP).

The objective of MD-PUPDVRP in this paper involves various sub-components. Allocation of the distribution is optimized through different methodologies while minimizing routing related factors. In the allocation, maximization of the amount of distributed food is targeted while considering the fairness in the distribution. In order to quantify fairness, a dedicated fairness function is developed and compared with alternatives. In routing related factors, fuel cost is considered along with carbon emission costs. In order to estimate carbon emission, an emission function used in the literature is extended to transporter type vehicles by analyzing the empirical distribution.

As solution methodology, a Mixed-Integer Linear Programming (MILP) model is developed to generate optimal solutions to problem instances. In MILP, a set of binary variables are used to represent the routing is utilized. Moreover, complex allocation related constraints and objectives are handled by linear variables. Some constraints that bind the integer and linear variables are added to the model to represent the dependency of allocation to the route of vehicles. In order to challenge MILP model, a Variable Neighborhood Search (VNS) based matheuristic is developed. Matheuristic makes use of VNS while generating the routes. Linear Programming (LP) based Allocation Subroutine (AS) provides optimal allocation schemes for each solution generated by VNS. Another LP based method, Allocation Upper Bound Model (ALUB) is presented to generate dual bound to problem very fast to ease the decision making in practical applications. Practical applications include cases where stakeholders can simply run ALUB to decide whether it would provide satisfactory amount of social benefit to invest further (for example rent more vehicles) to generate better routes. Furthermore, ALUB can potentially serve as an enabler for future algorithms that rely on dual bounds.

In order to assess the performance of methodologies, a detailed experimentation is carried out. During the experimentation phase, data of a company in İstanbul is cleaned, aggregated and formatted as instances. In addition to real life instances, a set of synthetic instances are generated by inferring the statistical distributions of parameters from real life data. Effect of different demand scenarios on performance of methodologies are investigated and analyzed thoroughly along with many other factors.

Chapter 2

LITERATURE REVIEW

The problem in this thesis has two sub-components, namely allocation and routing of the food. The routing subproblem is based on Vehicle Routing Problem (VRP) that is introduced by Dantzig and Ramser (1959). VRP is a problem that is relevant to various industries and it has many applications in the literature. Despite its wide application area, VRP and its variants has emerged in the food banking context decades later than the introduction of VRP. As discussed in section 1.3, the complex structure of the problem in this study renders it more difficult than the basic VRP. Therefore, taking into consideration that the problem entails complex features such as periodicity, and the importance of pairing, pickup, and delivery, the problem can be categorized as Periodic Unpaired Pickup and Delivery Vehicle Routing Problem (PUPDVRP). In this regard, Parragh et al. (2008) underlines that PUPDVRP has not been studied in depth in the literature. One of the examples of VRP applications in food banking context is provided by Solak et al. (2012). Yet, in Solak et al. (2012) pickup and distribution aspect is not incorporated into the decision framework. Thus, the distinguishing factor of this study is consideration of both collection and distribution decisions while constructing the routing schedules.

A key target of this study is to identify strategies for decreasing carbon based greenhouse gas emissions in pickup and delivery routes while simultaneously minimizing food waste and ensuring equitable distribution of donated food. In this regard, carbon emissions of routes need to be quantified. Demir et al. (2014) argues that two types of approaches can be relevant in estimating carbon emissions, namely microscopic and macroscopic models. In microscopic models, instantaneous emissions are considered and in macroscopic models average speed is assumed. This study assumes the latter case for the vehicles used in delivery operations. Various functions to calculate carbon emissions have been used in the literature. Koç et al.

(2016) provide a widely accepted function suitable for the problem centering this thesis. Kumar et al. (2022) and Bektaş and Laporte (2011) use similar functions to calculate emissions. Yet, the function in Koç et al. (2016) cannot be directly used in this thesis as only three types of vehicles (light duty 1, light duty 2 and medium duty) with respective curb weights of 3500, 4500 and 5500 kilograms are considered in that function. As the vehicles in real life last mile operations of food banks consider transporter type vehicles which are much lighter (1000 to 1500 kilograms) than light duty vehicles, an adaptation is made to the function by a correction factor via fitting the function to empirical data of a service center in Turkey.

Another layer of this study is the differentiation of food types. Furthermore, multiple vehicle compartments, such as refrigerated compartment and room temperature compartment, to carry different types of food is also integrated to problem structure. One of the studies done in the literature that resembles the core problem of this study is Gunes Corlu et al. (2010). However, while solving the problem via Column Generation and Constraint Programming, Gunes Corlu et al. (2010) deals with only one product type. Assumption of single product type relaxes the necessity of multiple compartments in vehicles. Additionally, this study solves the problem for a single time period which decreases the complexity. One other similar study, namely Lien et al. (2014), incorporates the concept of fair allocation through maximizing expected minimum fill rate. Although this study models fairness differently, the main difference of the studies lie in using multiple vehicles. Balcik et al. (2014) extends the problem in Lien et al. (2014) with multiple vehicles. Yet, problem structure in Balcik et al. (2014) involves collection followed by distribution. The problem centering this thesis distinguishes itself via allowing to perform collection and distribution in a mixed sequential order. The envy based fairness function in Rey et al. (2018) is investigated in this thesis. However, a different function is used for quantifying inequity in the distribution after careful analyses of theoretical and practical cases with domain experts.

Nair et al. (2018) tries to tackle with PUPDVRP in food banking operations via a mathematical model and Tabu Search based metaheuristic algorithm. However, they do not incorporate the multi-depot structure that this study considers. Moreover, the vehicles with which the collection is made are owned by food banks as opposed to a central authority in this thesis. Eisenhandler and Tzur (2019a) decomposes a similar problem and solves the allocation subproblem by a dedicated Robin Hood algorithm. Then, Eisenhandler and Tzur (2019b) extends the solution methodology to multiple vehicles. However, since this study allows multiple vehicles to visit the same site and partial pick-ups and deliveries, optimal allocations do not show the same properties that are assumed by Robin Hood based algorithm. Thus, the same methodology to solve allocation subproblem cannot be implemented.

The problem also entails the feature of being multi-depot which makes the problem Multi Depot - Periodic Unpaired Pickup and Delivery Vehicle Routing Problem (MD-PUPDVRP). To the best of author's knowledge, this problem has never been studied in the literature. A separate contribution of this study is the analysis of cooperation between food banks. In this thesis, the scenario where food banks that own vehicles collect products for every other food bank is analyzed.

As a solution methodology to tackle with the novel problem (MD-PUPDVRP), a MILP based mathematical model is developed. Yet, as MD-PUPDVRP belongs to NP-Hard class, the developed MILP model cannot provide solutions in polynomial time. In the literature there are several metaheuristic methodologies that has been utilized to come up with polynomial time algorithms to NP-Hard problems. Variable Neighborhood Search (Mladenović and Hansen, 1997) is one of these methodologies that is known to be successful in routing problems. Polacek et al. (2004) presents a VNS that is implemented for a different variant of VRP that includes time windows and multiple depots. The results show that VNS outperforms adapted Tabu Search algorithm. Imran et al. (2009) provides a designated VNS implementation for a VRP variant with heterogeneous vehicles. Designated VNS provides competitive results and generates the new best solutions for some of the instances. Kytöjoki et al. (2007) shows a VNS structure that works well for large scale VRP instances that have up to 20000 customers. Gansterer et al. (2017) tackles a variant of VRP with option to select customer requests according to profitability with general variable neighborhood search. The general variable neighborhood search approaches outperforms guided local search procedures. As many studies in literature report that VNS produces sufficiently high-quality solutions for routing problems, a VNS based matheuristic approach is also implemented as a solution methodology although (unlike MILP) it does not offer a performance guarantee even for small instances.

Chapter 3

PROBLEM DEFINITION

This study aims to investigate a novel problem while considering three SGD's of UN by minimizing the food loss, inequity in distribution of food and carbon emissions in food banking operations. The novel problem centering this thesis is a variant of the infamous VRP. Due to some complicating practical constraints in the day to day operations of food banking logistics and the concurrence of complex decisions (such as routing of the vehicles, fair allocation of food) involved in the problem setting, it can be defined as an extension of VRP, namely, MD-PUPDVRP. To the best of author's knowledge the MD-PUPDVRP problem centering this study has never been investigated in the literature. The detailed explanation of the problem structure is given in section 3.1.

3.1 Detailed Description of the Problem Structure

MD-PUPDVRP is a problem that entails routing and allocation aspects of food banking logistics simultaneously. MD-PUPDVRP is created such that the problem structure resembles real life operations of a key player in food banking sector. To that end, details of constraints are shaped by a range of business rules applied by various domain experts. Moreover, objectives of the problem and the demand scenarios are also framed in close collaboration with domain experts.

Problem is formulated to consider multiple depots. The depots where vehicles start and end the period are geographically located at the food banks that own that vehicle. As opposed to classical VRP, food needs to be picked up and delivered. Although the plan of routes and allocation is done by a central planner, pick-up and delivery operations are handled by vehicles of food banks. Yet, all food banks may not own a vehicle. Thus, each vehicle can pick-up from any donor and deliver to any food bank, for the sake of humanitarian act towards more people in need. The food banks and donors are not matched in advance which makes the problem unpaired. Partial pick-ups and deliveries are also allowed in the problem structure. Furthermore, each of the food banks can be visited by many vehicles. Yet, a vehicle can visit a node at most once except for the depot food bank which is visited at the start and end of the tour. Additionally, vehicles must deliver all of the donations to food banks, meaning inter-vehicle food transfer is prohibited. Periodicity is also considered in the problem. Therefore, at each period there can be new donations and new demands. It is assumed that the supply and demand quantities are deterministic and known in advance. At each period some of the food donated earlier can be spoiled. The time at which each product will be spoiled is known by the planners. Thus, some of the donations must be picked-up and delivered until the end of a predefined period. Similarly, food banks can demand a specific quantity of a product until the end of an predefined period. It should be noted that all the food picked-up by a vehicle must be delivered until the end of that period. Consequently, vehicles start their routes at each period with no food. Multiple products are considered in the problem as the nature of food bank operations require different food types. Since some of the products must be maintained in refrigerated conditions, multiple compartments (namely refrigerated compartment and room temperature compartment) with their corresponding capacities are considered in the study. The limited working hours that vehicles can operate at each period is another factor that adds a level of complexity to the problem. In Figure 3.1 visual representation of the problem for single period is provided.

The objective function of the problem considers multiple factors. Maximizing the distributed food, minimizing the inequity in allocation, minimizing traveling cost and carbon emissions related tax. To quantify the first objective (maximizing the distributed food), the total value of all intact food that is picked up from (or equivalently distributed to) all donors (all of the food banks) at all periods is considered. Although, it is not straightforward to define and quantify the inequity in the allocation of food, a dedicated function is developed to be used in the methodology. The detailed explanation of definition of fairness function is given in section 3.2.

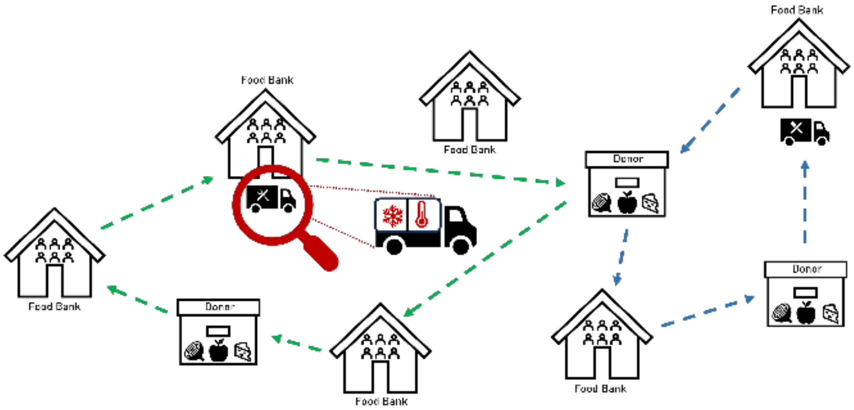
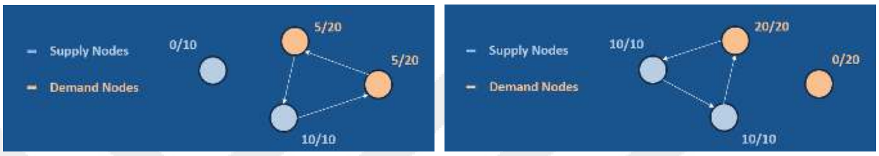


Figure 3.1: Visual representation of the problem for single period.

Traveling cost is the total price paid for fuel while traveling all the nodes defined by routes. For carbon related tax, 20% of the fuel price is assumed. To estimate the fuel consumption, the formula in Koç et al. (2016) is adapted to transporter type vehicles by analyzing the empirical fuel consumption data that was obtained from a service center in Turkey.

3.2 Fairness in Allocation of Food

The 2 SDG "Zero Hunger"is concentrated on eliminating the malnutrition and hunger, and food banks are carrying out the necessary actions to provide healthy food to those in need. However, in some cases the total amount of donated food can be much less than the total amount of food demanded by all food banks. Consequently, the food has to be allocated in a way that not all of the demand can be satisfied. The allocation decision should be made fairly such that no food bank (and consequently those in need which are served by that food bank) faces a systematic injustice. In order to incorporate fairness into the solution methodology, fairness should be quantified and measured properly. However, one should be aware of the trade-off between fairness and effectiveness (distributing maximum amount of food). The trivial solution of distributing no food is perfectly equitable. However, it cannot achieve any success in eliminating hunger. Once the fairness function is incorporated into the objective function of the mathematical model, in many cases it will not find the perfectly effective solution. In Figure 3.2, two different cases are presented for a total of 20 units of products. In the first case, allocation is totally fair while 10 units of product is distributed to each demand point. In the second case, 20 units of product is distributed to only one of the demand nodes. It is not straightforward to judge which of these cases is better in food banking domain. Therefore, the right balance between effectiveness and fairness must be put forward in collaboration with domain experts.



(a) (b)

Figure 3.2: Tradeoff between effectiveness and fairness: (a) Fair Allocation, (b) Effective Allocation.

One approach to deal with maximizing the total distributed food and minimizing the unfairness can be maximizing a non-convex increasing utility function. Three different non-convex increasing functions are analyzed as potential utility functions. In the functions, N represents the set of demand nodes and X_i stands for the amount of food that node $i \in N$ receives.

1.

$$f(x) = \sum_{i \in N} X_i^{(1/2)} \tag{3.1}$$

2.

$$f(x) = \sum_{i \in N} \frac{X_i}{(X_i + 1)} \tag{3.2}$$

3.

$$f(x) = \sum_{i \in N} log(X_i + 1) \tag{3.3}$$

Once these functions are compared with each other in terms of trade-off between effectiveness and fairness, the first function dominates the others with respect to effectiveness and the second function dominates with respect to fairness. These functions are not used in the solution methodology due to their non-linear structure.

One other approach to minimize the unfairness is to have a dedicated function for unfairness and incorporating the function into the objective function. Some well-known indicators such as Gini Index can be used to quantify fairness. However, non-linear structure of such indices prevents commercial solvers of mathematical optimization models to handle it efficiently and robustly. Therefore, two different functions are investigated to quantify the inequity in the distribution.

1. Envy based inequality function

2. Deviation from average fill rate function (DAFR)

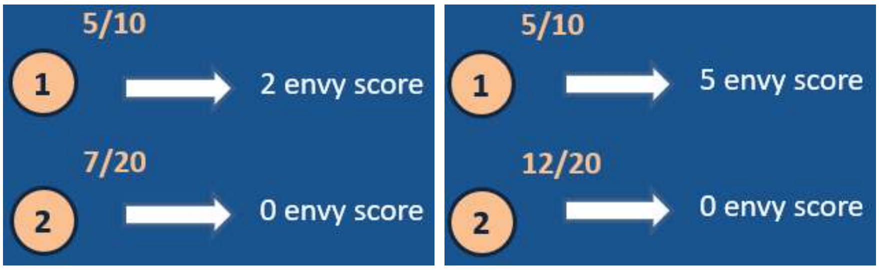
In the envy based function, all food banks envy each other as long as the other food bank receives more than them (Rey et al., 2018). If the other food bank receives more than the demand quantity, the envy score does not increase. Figure 3.3 shows the total envy scores in a 2-player scenario. More formal definition of envy score of player $i \in N$ (where N is the set of demand nodes) towards player $j \in N$ is

$$envy_{ij} = max(min(R_j - Ri, d_i - Ri), 0) \tag{3.4}$$

where d_i is the demand, R_i is the received amount of player $i \in N$. Then total envy score can be defined as $\sum_{i \in N} \sum_{j \in N} envy_{ij}$.

In the DAFR function, demand percentage of a food bank in the total demand of all food banks is calculated. Then, percentage of food received by that food bank in the total received food amount across all food banks is calculated. The absolute difference between these two percentages is argued to represent the unfairness in allocation. Two cases of a 2-player scenario is depicted in Figure 3.4. The formal definition of DAFR is

$$DAFR = \sum_{i \in N} (R_i - dp_i \sum_{j \in N} R_j) \tag{3.5}$$



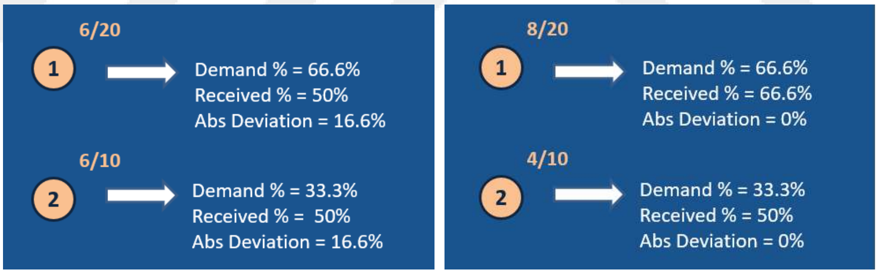
(a) (b)

Figure 3.3: Envy based function in a 2-player scenario: (a) First configuration, (b) Second configuration.

where R_i is the received amount, dp_i is the demand percentage of of player $i \in N$

$$dp_i = \frac{d_i}{\sum_{j \in N} d_j} \tag{3.6}$$

and d_i is the demand of player $i \in N$.



(a) (b)

Figure 3.4: DAFR in a 2-player scenario: (a) First configuration, (b) Second configuration.

After in depth analyses conducted by the domain experts, DAFR function is found to be more suitable for real life scenarios. Therefore, DAFR is used as the baseline function for fairness across this study including the solution methodologies. Although not distinguishing food banks that own vehicles in the fairness has the risk of not delivering to that food bank, it is an highly unlikely case since there are also costs associated with routing.

Chapter 4

SOLUTION METHODOLOGY

In the previous chapters, the aim of this thesis has been defined as providing decision making support to players in the food banking sector. To that end, MD-PUPDVRP, which reflects the daily operations of a key player in the food banking industry is put forward. MD-PUPDVRP simultaneously considers routing of the vehicles that are owned by food banks and allocation of the food. The complex problem structure requires sophisticated approaches to deal with multiple objectives constituted in MD-PUPDVRP. Thus, two alternative approaches are developed, namely a Mixed Integer Linear Programming (MILP) based mathematical model and a VNS based matheuristic. The formulation of MILP model is given in section 4.1. Moreover, the detailed explanation (along with the pseudocode) of VNS based matheuristic approach is presented in section 4.2. Additionally, an LP based approach to generate dual bound to MD-PUPDVRP, namely Allocation Upper Bound Model (ALUB), is presented in section 4.3. This method generates bound by assuming only allocation part of the problem. Although it is not used in any part of the matheuristic or MILP model, it is presented as an approach to provide decision making support since it can obtain upper bounds in very short amount of time.

4.1 Mixed Integer Linear Programming Model

Mixed Integer Linear Programming (MILP) based mathematical model is the first solution methodology to maximize the objective function of MD-PUPDVRP. Four different objectives are handled by linear scalarization. Concurrent consideration of routing related constraints, allocation related constraints and business related constraints (such as limited working hours) are presented in the formulation. Binary decision variables are used to handle routing related constraints and objectives, whereas linear decision variables handle the allocation related constraints and objectives. As discussed in the previous chapters, this thesis considers the scenario where each vehicle can deliver to (and pick-up from) any food bank (donor). This cooperation enables the food banks that do not own a vehicle to participate in humanitarian ecosystem. Notation, mathematical formulation and detailed description of objectives and constraints are presented throughout this section.

Index Sets

$N = F \cup G$: Set of all nodes (union of demand and supply nodes)

H : Set of all vehicles

P : Set of all products

U : Set of all compartments

T : Set of all periods in planning horizon

Parameters

a_{pu} : 1 if product $p \in P$ should be stored in compartment $u \in U$, 0 otherwise

dp_i : $\frac{\sum_{t \in T} \sum_{p \in P} q_{ip}^t}{\sum_{j \in N} \sum_{t \in T} \sum_{p \in P} q_{jp}^t}$: demand proportion of demand node $i \in F$

c_{ku} : Total capacity of compartment $u \in U$ in vehicle $k \in H$

d_{ij} : Distance between node $i \in N$ and node $j \in N$

t_{ij} : Travel time between node $i \in N$ and node $j \in N$

e_{ij} : The amount of carbon emission between node $i \in N$ and node $j \in N$

q_{ip}^t : The amount of product $p \in P$ that can be picked-up from (dropped to) node $i \in N$ which will be spoiled at the end of period $t \in T$

L: Total allowable work time per period

h_k : The number of periods vehicle $k \in H$ can be used in the planning horizon

$WAPrice$: Weighted (with respect to demand quantity) average price

$price_p$: Price per kg of product type $p \in P$

fp : Fuel price per liter

fc : Fuel consumption of each vehicle per km

ct : Carbon tax percentage

Decision Variables

X^t_{ijk}: 1 if vehicle $k \in H$ travels directly from node $i \in N$ to $j \in N$ in period $t \in T$, 0 otherwise

R^t_{kip}: The amount of product $p \in P$ that vehicle $k \in H$ received from (delivered to) node $i \in N$ in period $t \in T$

T^t_{kip}: The total amount of product $p \in P$ that vehicle $k \in H$ carries when it arrives at $i \in N$ in period $t \in T$

S^t_{ki}: The order vehicle $k \in H$ visits node $i \in N$ in period $t \in T$

$\hat{R}_i$: The dummy variable that represents the amount of disruption on fair allocation caused by the delivery to food bank $i \in F$

Objective Function and Constraints

$$\text{Max } f_1 w_1 + f_2 w_2 + f_3 w_3 + f_4 w_4 \tag{4.1}$$

subject to

$$X^t_{0ik} \leq o_{ik} \qquad \forall k \in H, i \in N, t \in T \tag{4.2}$$

$$X^t_{0ik} = X^t_{i0k} \qquad \forall k \in H, i \in N, t \in T \tag{4.3}$$

$$|N|(\sum_{i\in N} X^t_{0ik}) \geq \sum_{i\in N}\sum_{\substack{j\in N\\ j\neq i}} X^t_{ijk} \qquad \forall k\in H, t\in T \tag{4.4}$$

$$\sum_{i\in N}\sum_{\substack{j\in N\\ j\neq i}} X^t_{ijk}t_{ij} \leq L \qquad \forall k\in H, t\in T \tag{4.5}$$

$$\sum_{\substack{j\in N\\ j\neq i}} X^t_{ijk} \leq 1 \qquad \forall k\in H, i\in N, t\in T \tag{4.6}$$

$$\sum_{\substack{j\in N\\ j\neq i}} X^t_{ijk} = \sum_{\substack{j\in N\\ j\neq i}} X^t_{jik} \qquad \forall k\in H, i\in N, t\in T \tag{4.7}$$

$$S^t_{kj} + (1-X^t_{ijk})|N| \geq S^t_{ki} + X^t_{ijk} \qquad \forall k\in H, t\in T, i\in N, j\in N, j\neq i, o_{ik}\neq 1 \tag{4.8}$$

$$T^t_{kjp} + (1-X^t_{ijk})(\sum_{u\in U} a_{pu}c_{ku}) \geq T^t_{kip} + R^t_{kip} \qquad \forall k\in H, t\in T, p\in P, i\in N, j\in N, j\neq i \tag{4.9}$$

$$T^t_{kip} + R^t_{kip} + (1-X^t_{ijk})(\sum_{u\in U} a_{pu}c_{ku}) \geq T^t_{kjp} \qquad \forall k\in H, t\in T, p\in P, i\in N, j\in N, j\neq i \tag{4.10}$$

$$\sum_{p\in P} a_{pu}T^t_{kip} \leq c_{ku} \qquad \forall k\in H, t\in T, i\in N, \forall u\in U \tag{4.11}$$

$$T^t_{kip} \geq 0 \qquad \forall k\in H, t\in T, i\in N, \forall p\in P \tag{4.12}$$

$$R^t_{kip} \leq 0 \qquad \forall k\in H, t\in T, i\in F, p\in P \tag{4.13}$$

$$R^t_{kip} \geq 0 \qquad \forall k\in H, t\in T, i\in G, p\in P \tag{4.14}$$

$$\sum_{\substack{j\in N\\ j\neq i}} X^t_{jik}(\sum_{t'=t}^{T} q^{t'}_{ip}) \geq R^t_{kip} \qquad \forall k\in H, t\in T, i\in G, p\in P \tag{4.15}$$

$$\sum_{\substack{j\in N\\ j\neq i}} X^t_{jik}(\sum_{t'=t}^{T} q^{t'}_{ip}) \geq -R^t_{kip} \qquad \forall k\in H, t\in T, i\in F, p\in P \tag{4.16}$$

$$\sum_{k\in H}\sum_{t'=t}^{T} R^{t'}_{kip} \leq \sum_{t'=t}^{T} q^{t'}_{ip} \qquad \forall i\in G, p\in P, t\in T \tag{4.17}$$

$$\sum_{k\in H}\sum_{t'=t}^{T} -R^{t'}_{kip} \leq \sum_{t'=t}^{T} q^{t'}_{ip} \qquad \forall i\in F, p\in P, t\in T \tag{4.18}$$

$$\sum_{i\in N}\sum_{t\in T} X^t_{0ik} \leq H_k \qquad \forall k\in H \tag{4.19}$$

$$\sum_{i\in N} R^t_{kip} = 0 \qquad \forall k\in H, t\in T, p\in P \tag{4.20}$$

$$\sum_{t\in T}\sum_{k\in H}\sum_{p\in P}(R^t_{kip} - dp_i\sum_{j\in N} R^t_{kjp}) \leq \hat{R}_i \qquad \forall i\in F \tag{4.21}$$

$$-(\sum_{t\in T}\sum_{k\in H}\sum_{p\in P}(R^t_{kip} - dp_i\sum_{j\in N} R^t_{kjp})) \leq \hat{R}_i \qquad \forall i\in F \tag{4.22}$$

$$X^t_{ijk} \in \{0,1\} \qquad \forall i\in N, j\in N, k\in H, t\in T \tag{4.23}$$

$$S^t_{ki} \in \{0,1,..,|N|\} \qquad \forall i\in N, k\in H, t\in T \tag{4.24}$$

4.1.1 Description of the MILP Model

In the mathematical model, four different functions were multiplied by the weights w_1, w_2, w_3, w_4 in the objective function. The first function

$$f_1 = \sum_{k \in H} \sum_{i \in G} \sum_{p \in P} \sum_{t \in T} R^t_{kip} price_p \tag{4.25}$$

aims to maximize the total value of products collected. The second function

$$f_2 = -WAPrice \sum_{i \in F} \hat{R}_i \tag{4.26}$$

aims to minimize social injustice in distribution.

$$f_3 = -(fp)(fc) \sum_{i \in N} \sum_{j \in N} \sum_{k \in H} \sum_{t \in T} X^t_{ijk} d_{ij} \tag{4.27}$$

minimizes the total distance traveled by all vehicles. Finally,

$$f_4 = -(fp)(fc)(ct) \sum_{i \in N} \sum_{j \in N} \sum_{k \in H} \sum_{t \in T} X^t_{ijk} e_{ij} \tag{4.28}$$

aims to minimize the carbon emission consumed during distribution. Constraint set (4.2) ensures that vehicles can only start their daily movement from the depot (food bank) with which they have an ownership relationship. Constraint set (4.3) requires that the depot where a vehicle starts its daily movement and the depot where it finishes its daily movement is the same. Constraint set (4.4) guarantees that a vehicle that has not started its daily movement cannot move between two nodes. Constraint set (4.5) ensures that the total time spent by a vehicle during the day does not exceed the working hour limit. Constraint set (4.6) ensures that a vehicle can make only one visit in a period to any node except its depot node. Constraint set (4.7) is also known as the flow constraint and states that the vehicle must leave its visiting node. Constraint set (4.8) is the sub-tour elimination constraints known in the literature as Miller-Tucker-Zemlin constraints (Miller et al., 1960). Constraint sets (4.9) and (4.10) update the amount of product carried by the compartment in the vehicle after pick up (or delivery) at a node. Constraint sets (4.11) and (4.12) ensure that the total amount of product in a compartment is between 0 and the capacity of the compartment. Constraint sets (4.13) and (4.14) specify that only delivery can be made at demand nodes, and only pick-up can be made from supply nodes. Constraint sets (4.15) and (4.16) prevent distribution and collection from non-visited nodes. Constraint sets (4.17) and (4.18) guarantee that distribution and collection cannot exceed the total amount of intact product. Constraint set (4.19) prevents a vehicle from exceeding the limit of days it can be used in the planning horizon. Constraint set (4.20) ensures that the total quantity of product distributed is equal to the total quantity of product collected. Constraint sets (4.21) and (4.22) are used to linearize the function used in the fair allocation calculation. In the fair distribution function, the percentage of the demand in total demand (dp_i) is initially calculated for each demand point. Then, the ratio of the total product received by the demand point to the total product received from all demand points is calculated. The difference between the demand percentage and the realized percentage is argued to constitute injustice and all differences are summed. Constraint sets (4.23) and (4.24) are the domain constraints for binary and integer variables.

4.2 Matheuristic Algorithm

Since MD-PUPDVRP is in the NP-hard problem class, the MILP-based mathematical model may fail to provide decent solutions within a short amount of time for large data sets. Therefore, a matheuristic method based on Variable Neighborhood Search (VNS) algorithm, which is known to be successful in routing problems, is designed. Unlike the MILP, VNS does not offer a performance guarantee. On the other hand, studies that has been referred to in chapter 2 argue that the VNS produces sufficiently high-quality solutions for routing problems. Pseudocode of the VNS based matheuristic is given in Algorithm 1.

In the VNS based matheuristic developed for MD-PUPDVRP, the solutions are encoded using permutation representation. In this encoding, there are as many permutations as the number of vehicles multiplied by the number of periods, such that each vehicle's route in each period is a permutation. Each permutation starts at the node that owns the vehicle, visits each node at most once, and ends at the node that owns it. This representation does not contain any information about the amount

Algorithm 1: Matheuristic

input : s_{init} (initial solution), z_{init} (objective value of initial solution), k_{max} (number of move operators)

output: s (incumbent solution), z (objective value of incumbent solution)

$iteration \leftarrow 0$
$s \leftarrow s_{init}$
$z \leftarrow z_{init}$
repeat
    $k \leftarrow 1$
    repeat
        Check which shaking operators N_i $(i = 1, .., k_{max})$ are applicable to any subtour of s
        repeat
            $k \leftarrow (k \bmod k_{max}) + 1$ **until** *shaking operator* N_k *is applicable*
            s', z', r_{veh}, r_{per} $\leftarrow$ `ApplyShaking`(s, z, k)
            s'',z'' $\leftarrow$ `MathEnhancedLocalSearch`$(s', z', r_{veh}, r_{per})$
            if $z'' \geq z$ **then**
                $s \leftarrow$s$''$
                $z \leftarrow$z$''$
                $k \leftarrow 1$
            else
                $k \leftarrow k + 1$
            $iteration \leftarrow iteration + 1$
        until $k > k_{max}$ *or time limit is reached*
    until *time limit is reached*

of each product picked up from or delivered to each node. This information is obtained by solving a Linear Programming Model called the Allocation Subroutine (AS). AS optimally decides appropriate collection and distribution quantities for a given permutation representation. Five different neighborhood relations are defined in the VNS. In the “shaking”(Algorithm 2) step of the algorithm “food bank insertion”(Algorithm 3), “donor drop”(Algorithm 4), “donor insertion”(Algorithm 5), “food bank drop”(Algorithm 6), and “swap”(Algorithm 7) are used, whereas “node insertion ”is used in the local search (Algorithm 8). The food bank insertion neighborhood adds a random food bank to a random location of the permutation. The donor drop operator subtracts a random donor from the permutation. The donor insertion neighborhood adds a random donor to a random location of the permutation. Similarly, the food bank drop operator subtracts a random food bank from the permutation. The swap operator exchanges the location of two randomly selected nodes from a given permutation that represents the route of a specific vehicle in a specific period. Figure 4.1 shows enumerated visuals depicting each of the random shaking operators and how they can alter the original route displayed above. The

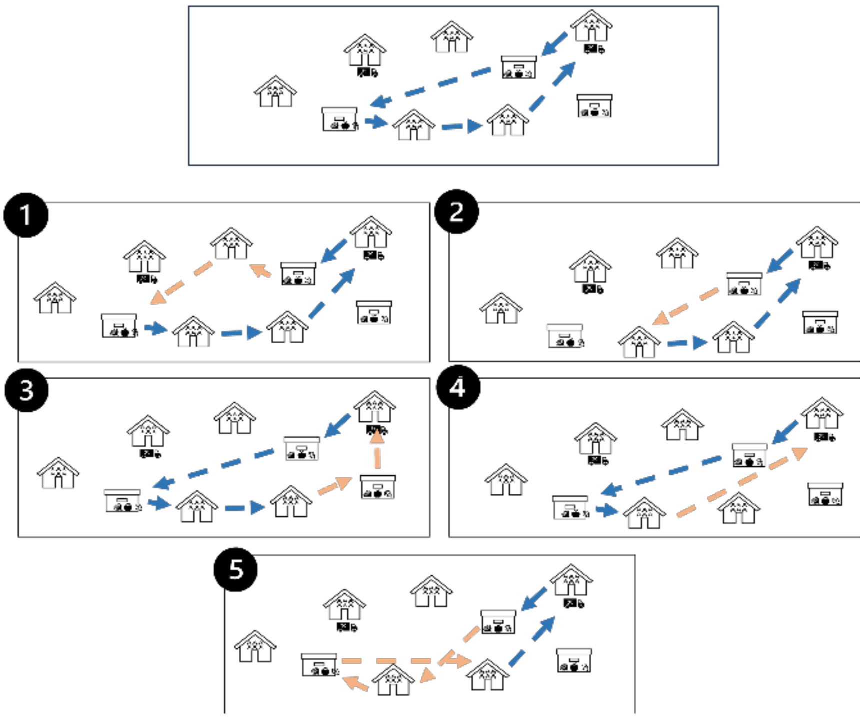


Figure 4.1: Shaking move operators.

node insertion used in local search relocates a node of the permutation to another index of the same permutation. Initially, in each of the neighborhood structures used in the shaking step, a period and vehicle are randomly selected. The move operators are executed on the permutation that was pointed by the selected period and vehicle. Then a local search is performed on the shaken solution, using node insertion. The solution found is compared with the incumbent solution, and the incumbent solution is updated if necessary. When a new permutation representation appears in each of these steps, AS must be rerun to calculate the objective function value and to obtain a complete solution.

Algorithm 2: ApplyShaking

input : s, z, k

output: s', z'

if $k = 1$ **then**
- r_{veh}, r_{per}, s_t $\leftarrow$ Randomly select a vehicle and period (r_{veh}, r_{per}) whose tour (s_t) have the property of $F \backslash s_t \neq \emptyset$
- s', z' $\leftarrow$ `ApplyAddFShaking`(s, z, r_{veh}, r_{per})

else
- **if** $k = 2$ **then**
  - r_{veh}, r_{per}, s_t $\leftarrow$ Randomly select a vehicle and period (r_{veh}, r_{per}) whose tour (s_t) have the property of $|s_t \cap G| \geq 2$
  - s', z' $\leftarrow$ `ApplyDropGShaking`(s, z, r_{veh}, r_{per})
- **else**
  - **if** $k = 3$ **then**
    - r_{veh}, r_{per}, s_t $\leftarrow$ Randomly select a vehicle and period (r_{veh}, r_{per}) whose tour (s_t) have the property of $G \backslash s_t \neq \emptyset$
    - s', z' $\leftarrow$ `ApplyAddGShaking`(s, z, r_{veh}, r_{per})
  - **else**
    - **if** $k = 4$ **then**
      - r_{veh}, r_{per}, s_t $\leftarrow$ Randomly select a vehicle and period (r_{veh}, r_{per}) whose tour (s_t) have the property of $|s_t \cap F| \geq 2$
      - s', z' $\leftarrow$ `ApplyDropFShaking`(s, z, r_{veh}, r_{per})
    - **else**
      - r_{veh}, r_{per}, s_t $\leftarrow$ Randomly select a vehicle and period (r_{veh}, r_{per}) whose tour (s_t) have the property of $|s_t| \geq 4$
      - s', z' $\leftarrow$ `ApplySwapShaking`(s, z, r_{veh}, r_{per})

Algorithm 3: ApplyAddFShaking

input : s, z, r_{veh},r_{per}

output: s', z'

$s_t \leftarrow$ Subtour of s on period r_{per} for vehicle r_{veh}

$N_i \leftarrow$ Randomly select node N_i which has the properties of $N_i \in F$ and $N_i \notin s_t$

$s'_t \leftarrow$ Add N_i to a random location of s_t

$s' \leftarrow$ Change s by substituting s_t with s'_t

$z' \leftarrow$ `AllocationSubroutine(`s'`)`

Algorithm 4: ApplyDropGShaking

input : s, z, r_{veh},r_{per}

output: s', z'

$s_t \leftarrow$ Subtour of s on period r_{per} for vehicle r_{veh}

$N_i \leftarrow$ Randomly select node N_i which has the properties of $N_i \in G$ and $N_i \in s_t$

$s'_t \leftarrow$ Drop N_i from of s_t

$s' \leftarrow$ Change s by substituting s_t with s'_t

$z' \leftarrow$ `AllocationSubroutine(`s'`)`

Algorithm 5: ApplyAddGShaking

input : s, z, r_{veh},r_{per}

output: s', z'

$s_t \leftarrow$ Subtour of s on period r_{per} for vehicle r_{veh}

$N_i \leftarrow$ Randomly select node N_i which has the properties of $N_i \in G$ and $N_i \notin s_t$

$s'_t \leftarrow$ Add N_i to a random location of s_t

$s' \leftarrow$ Change s by substituting s_t with s'_t

$z' \leftarrow$ `AllocationSubroutine(`s'`)`

Algorithm 6: ApplyDropFShaking

input : s, z, r_{veh},r_{per}

output: s', z'

$s_t \leftarrow$ Subtour of s on period r_{per} for vehicle r_{veh}

$N_i \leftarrow$ Randomly select node N_i which has the properties of $N_i \in F$ and $N_i \in s_t$

$s'_t \leftarrow$ Drop N_i from of s_t

$s' \leftarrow$ Change s by substituting s_t with s'_t

$z' \leftarrow$ `AllocationSubroutine(`s'`)`

Algorithm 7: ApplySwapShaking

input : s, z, r_{veh},r_{per}

output: s', z'

$s_t \leftarrow$ Subtour of s on period r_{per} for vehicle r_{veh}

N_i, $N_j \leftarrow$ Randomly select node two nodes N_i, N_j from s_t which are not the depot of vehicle r_{veh}

$s'_t \leftarrow$ Exchange the locations of N_i, N_j in s_t

$s' \leftarrow$ Change s by substituting s_t with s'_t

$z' \leftarrow$ `AllocationSubroutine(`s'`)`

Algorithm 8: MathEnhancedLocalSearch

input : s, z, r_{veh}, r_{per}

output: s'', z''

Initialize a list s_{list} with s, z being the only entry

$s_t \leftarrow$ Subtour of s on period r_{per} for vehicle r_{veh} in the form of $(d, ..., d)$ where d is the depot

for *all nodes* $N_i \in s_t$ **do**

  for *all nodes* $N_j \in s_t$ *which comes after the successor of* N_i **do**

    if N_j *is not the depot or its predecessor* **then**

      $s_{t-new} \leftarrow$ Exchange the arc between N_i and its successor and N_j and its successor

      $s_{new} \leftarrow s$ with subtour on period r_{per} for vehicle r_{veh} being changed to s_{t-new}

      $z_{new} \leftarrow$ `AllocationSubroutine(`s_{new}`)`

      Add s_{new}, z_{new} to s_{list}

$s'', z'' \leftarrow \arg\max_{i \in s_{list}} z_i$

4.2.1 Allocation Subroutine

In the matheuristic, routing schemes are explored trough VNS. Although VNS provides a sophisticated search procedure in the routing solution space, VNS design in this methodology lacks the ability to generate allocation plans. Therefore, a functionality to come up with allocations (for each visited solution) is added to the matheuristic. Solving allocation subproblem for a given routing scheme is carried out by Allocation Subroutine (AS). Since routing is predefined for AS, it only considers allocation related objectives (and constraints) and solves them to optimality. AS is a mathematical formulation that uses the same allocation variables, parameters, and constraints of the MILP. Since there are no integer variables in the underlying mathematical model of AS, it is based on a linear programming (LP) model. As opposed to MD-PUPDVRP, which is an NP-hard problem, the allocation subproblem is in class P.

Index Sets

$N = F \cup G$: Set of all nodes (union of demand and supply nodes)

H : Set of all vehicles

P : Set of all products

U : Set of all compartments

T : Set of all periods in planning horizon

Parameters

a_{pu} : 1 if product $p \in P$ should be stored in compartment $u \in U$, 0 otherwise

X^t_{ijk} : 1 if vehicle $k \in H$ travels directly from node $i \in N$ to $j \in N$ in period $t \in T$, 0 otherwise

dp_i : $\frac{\sum_{t \in T} \sum_{p \in P} q^t_{ip}}{\sum_{j \in N} \sum_{t \in T} \sum_{p \in P} q^t_{jp}}$: demand proportion of node $i \in F$

c_{ku} : Total capacity of compartment $u \in U$ in vehicle $k \in H$

q^t_{ip} : the amount of product $p \in P$ that can be picked-up from (dropped to) node $i \in N$ which will be spoiled at the end of period $t \in T$

$WAPrice$: Weighted (with respect to demand quantity) average price

$price_p$: Price per kg of product type $p \in P$

Decision Variables

R^t_{kip}: The amount of product $p \in P$ that vehicle $k \in H$ received from (delivered to) node $i \in N$ in period $t \in T$

T^t_{kip}: The total amount of product $p \in P$ that vehicle $k \in H$ carries when it arrives at $i \in N$ in period $t \in T$

$\hat{R}_i$: The dummy variable that represents the amount of disruption on fair allocation caused by the delivery to food bank $i \in F$

Objective Function and Constraints

$$\text{Max } f_1 w_1 + f_2 w_2 \tag{4.29}$$

subject to

$$T^t_{kjp} + (1 - X^t_{ijk})(\sum_{u \in U} a_{pu} c_{ku}) \geq T^t_{kip} + R^t_{kip} \qquad \forall k \in H, t \in T, p \in P, i \in N, j \in N, j \neq i \tag{4.30}$$

$$T^t_{kip} + R^t_{kip} + (1 - X^t_{ijk})(\sum_{u \in U} a_{pu} c_{ku}) \geq T^t_{kjp} \qquad \forall k \in H, t \in T, p \in P, i \in N, j \in N, j \neq i \tag{4.31}$$

$$\sum_{p \in P} a_{pu} T^t_{kip} \leq c_{ku} \qquad \forall k \in H, t \in T, i \in N, \forall u \in U \tag{4.32}$$

$$T^t_{kip} \geq 0 \qquad \forall k \in H, t \in T, i \in N, \forall p \in P \tag{4.33}$$

$$R^t_{kip} \leq 0 \qquad \forall k \in H, t \in T, i \in F, p \in P \tag{4.34}$$

$$R^t_{kip} \geq 0 \qquad \forall k \in H, t \in T, i \in G, p \in P \tag{4.35}$$

$$\sum_{\substack{j \in N \\ j \neq i}} X^t_{jik} (\sum_{t'=t}^{T} q^{t'}_{ip}) \geq R^t_{kip} \qquad \forall k \in H, t \in T, i \in G, p \in P \tag{4.36}$$

$$\sum_{\substack{j \in N \\ j \neq i}} X^t_{jik} (\sum_{t'=t}^{T} q^{t'}_{ip}) \geq -R^t_{kip} \qquad \forall k \in H, t \in T, i \in F, p \in P \tag{4.37}$$

$$\sum_{k \in H} \sum_{t'=t}^{T} R^{t'}_{kip} \leq \sum_{t'=t}^{T} q^{t'}_{ip} \qquad \forall i \in G, p \in P, t \in T \tag{4.38}$$

$$\sum_{k\in H}\sum_{t'=t}^{T} -R_{kip}^{t'} \leq \sum_{t'=t}^{T} q_{ip}^{t'} \qquad \forall i \in F, p \in P, t \in T \qquad (4.39)$$

$$\sum_{i\in N} R_{kip}^{t} = 0 \qquad \forall k \in H, t \in T, p \in P \qquad (4.40)$$

$$\sum_{t\in T}\sum_{k\in H}\sum_{p\in P}(R_{kip}^{t} - dp_i \sum_{j\in N} R_{kjp}^{t}) \leq \hat{R}_i \qquad \forall i \in F \qquad (4.41)$$

$$-(\sum_{t\in T}\sum_{k\in H}\sum_{p\in P}(R_{kip}^{t} - dp_i \sum_{j\in N} R_{kjp}^{t})) \leq \hat{R}_i \qquad \forall i \in F \qquad (4.42)$$

In the objective function of AS, f_1 and f_2 are the same as equation (4.25) and equation (4.26), respectively. Constraint sets (4.30) and (4.31) are the correspondents of (4.9) and (4.10) in MILP model. Constraint set (4.32) represents constraint set (4.11). Constraint sets (4.33), (4.34) and (4.35) define the domains of decision variables as in (4.12), (4.13) and (4.14) of MILP. Constraint sets (4.36) and (4.37) are the equivalents of (4.15) and (4.16) of MILP. Constraint sets (4.38) and (4.39) are represented by (4.17) and (4.18) in MILP. Constraint set (4.40) is identical to (4.20) of MILP. Constraint sets (4.41) and (4.42) are parallel to (4.21) and (4.22) in MILP.

4.2.2 Generation of Initial Solutions

Since VNS is a well-known trajectory based metaheuristic which requires an initial solution, there is a need to construct a feasible solution to the MD-PUPDVRP. Although VNS is known to be an initial solution independent method, it is not able to execute many iterations in some of the very large instances. Thus, selection of the initial solution plays a critical role on the quality of the incumbent solution. The complex structure of the problem renders it difficult to find a high a quality solution in short amount of time. Therefore, an ad-hoc rule is implemented to construct a feasible solution. According to that rule, all of the vehicles start from their owner food banks and visit a random supply node and return to their owner node at each period. This rule is guaranteed to be feasible as long as the total allowable work time is not exceeded. If an infeasible solution is created, then the same procedure is applied until a feasible solution is found. Once the feasible solution is generated, AS is called to find the objective function value. Although this rule generates a feasible solution quickly, the solutions are usually of low quality. Moreover, it can provide worse solutions than the trivial solution of not visiting any node in any of the periods. Therefore, starting with the heuristic solutions of the commercial solver (Gurobi) is also implemented. Yet, due to the complex structure of the problem, commercial solver provides the trivial solution with the objective value of 0 as the initial solution. The third method to generate initial solutions is very similar to nearest node insertion for Travelling Salesman Problem. In this method, only supply nodes are selected to be inserted to the solution. However, since the total traveling time cannot exceed parameter L, the algorithm does not add all donation nodes to the tour. Before inserting nodes to a tour it checks whether adding it would be infeasible with respect to L and if so, it stops adding nodes to that tour. This procedure is repeated for every vehicle-day combination. Third approach is found to be the best in the toy examples. Hence, it is used in the extensive computational study. This method and the order of shaking moves in the VNS is designed to be in sync as food banks are inserted in between the donors of initial solution.

4.3 Allocation Upper Bound Model

An LP based model is used to come up with upper bounds to the maximization problem MD-PUPDVRP. Allocation Upper Bound Model (ALUB) disregards the routing subproblem of MD-PUPDVRP, and focuses on the allocation subproblem. As opposed to AS, it is agnostic of the current route (or any other route). Therefore, it assumes that every product can be assigned to any food bank without incurring any routing related costs such as emission cost or distance cost. It generates a fair allocation while distributing maximum amount of products. The solution of the ALUB can rarely be feasible with respect to routing constraints of MILP such as working hour limit or truck capacity. Even when there exists a feasible MILP solution with allocation provided by ALUB, in some cases it is a suboptimal solution due to high routing costs. Therefore, ALUB is not used in the initial solution generation or in any part of VNS. Nevertheless, it is presented to be used as a tool to come up with optimality gap in a short amount of solution time. ALUB makes use of some of the variables and parameters of AS.

Index Sets

$N = F \cup G$: Set of all nodes (union of demand and supply nodes)

H : Set of all vehicles

P : Set of all products

U : Set of all compartments

T : Set of all periods in planning horizon

Parameters

$dp_i : \frac{\sum_{t \in T} \sum_{p \in P} q^t_{ip}}{\sum_{j \in N} \sum_{t \in T} \sum_{p \in P} q^t_{jp}}$: demand proportion of node $i \in F$

q^t_{ip} : the amount of product $p \in P$ that can be picked-up from (dropped to) node $i \in N$ which will be spoiled at the end of period $t \in T$

$WAPrice$: Weighted (with respect to demand quantity) average price

$price_p$: Price per kg of product type $p \in P$

Decision Variables

R^t_{kip}: The amount of product $p \in P$ that vehicle $k \in H$ received from (delivered to) node $i \in N$ in period $t \in T$

$\hat{R}_i$: The dummy variable that represents the amount of disruption on fair allocation caused by the delivery to food bank $i \in F$

Objective Function and Constraints

$$\text{Max } f_1 w_1 + f_2 w_2 \tag{4.43}$$

subject to

$$R^t_{kip} \leq 0 \qquad \forall k \in H, t \in T, i \in F, p \in P \tag{4.44}$$

$$R^t_{kip} \geq 0 \qquad \forall k \in H, t \in T, i \in G, p \in P \tag{4.45}$$

$$R^t_{kip} \leq (\sum_{t'=t}^{T} q^{t'}_{ip}) \qquad \forall k \in H, t \in T, i \in G, p \in P \tag{4.46}$$

$$-R^t_{kip} \leq (\sum_{t'=t}^{T} q^{t'}_{ip}) \qquad \forall k \in H, t \in T, i \in F, p \in P \tag{4.47}$$

$$\sum_{k \in H} \sum_{t'=t}^{T} R^{t'}_{kip} \leq \sum_{t'=t}^{T} q^{t'}_{ip} \qquad \forall i \in G, p \in P, t \in T \tag{4.48}$$

$$\sum_{k \in H} \sum_{t'=t}^{T} -R^{t'}_{kip} \leq \sum_{t'=t}^{T} q^{t'}_{ip} \qquad \forall i \in F, p \in P, t \in T \tag{4.49}$$

$$\sum_{i \in N} R^t_{kip} = 0 \qquad \forall k \in H, t \in T, p \in P \tag{4.50}$$

$$\sum_{t \in T} \sum_{k \in H} \sum_{p \in P} (R^t_{kip} - dp_i \sum_{j \in N} R^t_{kjp}) \leq \hat{R}_i \qquad \forall i \in F \tag{4.51}$$

$$-(\sum_{t \in T} \sum_{k \in H} \sum_{p \in P} (R^t_{kip} - dp_i \sum_{j \in N} R^t_{kjp})) \leq \hat{R}_i \qquad \forall i \in F \tag{4.52}$$

The objective function of ALUB (4.43) is the same as the objective function of AS (4.29). Constraint sets (4.44) and (4.45) define the domains of decision variables for allocation, similar to (4.34) and (4.35) of AS. Constraint sets (4.46) and (4.47) are parallel to (4.36) and (4.37) of AS. Constraint sets (4.48) and (4.49) of ALUB and (4.38) and (4.39) of AS are identical. Constraint set (4.50) is portrayed by (4.40) in AS. Fairness constraints (4.51) and (4.52) of ALUB are corresponding to (4.41) and (4.42) in AS.

Chapter 5

COMPUTATIONAL STUDIES

5.1 Generation of Synthetic Instances

While developing the MILP model and matheuristic, computer-generated instances played a significant role in verifying the accuracy of the methods and gaining small-scale experimental insights. The computer-generated data is based on knowledge from the literature and insights from the real data of a company in Turkey. In synthetic instances, parameters and sets H, P, U, T, h_k, L, c_{ku}, a_{pu}, q^t_{ip} were generated considering the company data. For H, two sets were created with the number of vehicles being 33% and 100% of the number of demand nodes (food banks). For T, the number of periods is set as 3. For P and N the number of products is 10 where the cardinality of set of all nodes is 20 and 5 where there are 50 nodes. The percentage of demand nodes (F) in all nodes is determined as 33 and 66. For U, the number of vehicle compartments is always fixed to 2, namely room temperature and refrigerated compartments. L is fixed to 8 hours considering real-life working hours, while h_k is fixed to be equal to the number of periods. a_{pu} is generated with a uniform distribution according to 5-95 parity, which is the percentage of products in the company data that need to be transported in the cold and room temperature compartments. c_{ku} is generated by dividing the total capacity of the Fiat Doblo vehicle, 750 kg, which is frequently used in company operations, again according to 5-95 parity.

It is inferred from the company data that q^t_{ip} where $i \in G$ is exponentially distributed with different parameters for each product. Thus, the amount of each product that supply nodes are willing to donate is randomly generated with their corresponding parameters. Then, two different methods, namely, Infinite Demand (ID) and Total Equality (TE) are used to reproduce the demand of food banks. ID is the case where all food banks are willing to be fulfilled with all of the donations. Therefore, in ID all the donations are aggregated for each product and q_{ip}^t where $i \in F$ is set to that amount for the corresponding product. In TE scenario, q_{ip}^t where $i \in F$ is randomly distributed from the same distributions of donations and normalization is applied such that $\sum_{i \in G} \sum_{t \in T} q_{ip}^t = \sum_{i \in F} \sum_{t \in T} q_{ip}^t \; \forall p \in P$. Prices of each product $price_p$ is taken from a local grocery delivery app in Turkey. Then weighted (with respect to demand quantities) price average is calculated for the second objective function. The coordinates of the nodes are randomly generated, as in literature (Rey et al., 2018). In the random assignment of coordinates uniform distribution between -100 and 100 is utilized. Using the coordinates, the distance matrix d_{ij} is calculated according to euclidean distance. For the speed of the vehicles, using insights about Istanbul traffic, it is assumed that the first quarter of the road segment between each node i and j will be traveled at a random speed with a uniform distribution between 30-50. Similarly, the constant speed is assumed to be random uniform between 50-80 in the second and third quarters and 30-50 in the last quarter, as depicted in Figure 5.1. In this approach, it is assumed that the first and last quarters will be on the streets where the nodes are located, and the second and third quarters will be on the routes such as freeways, highways, or avenues. t_{ij} is simply calculated using coordinate and speed data. Since the formula

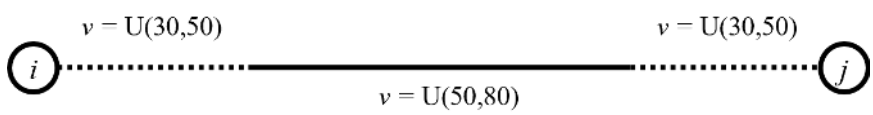


Figure 5.1: Speed of vehicles between node i and node j.

in Koç et al. (2016) to calculate e_{ij} is not applicable to transporter type vehicles (due to less weight than light duty vehicles), an adaptation is made by multiplying with a correction factor using empirical fuel consumption data that was obtained from a service center in Turkey. Fuel price is assumed to be 43.5 Turkish Liras and the vehicle consumes around 6.4 liters of fuel per 100 kilometers. Carbon tax of %20 is also assumed. Size of the instance is influenced highly by parameters N, K,

FB and T as they determine the number of routes to be dealt with in the solution methodologies. P is also a critical parameter that impacts the number of variables in the allocation.

5.2 Experiments for Mathematical Model on Synthetic Instances

The experiments of the mathematical model on synthetic instances were undertaken to determine the behavior of the model. During the experimentation, aspects such as the sensitivity of the model to demand scenarios, the effect of the number of products and vehicles on solution quality were taken into consideration. The model was developed in Python using the Gurobipy library Gurobi Optimization (2024), and the results were acquired using version 9.5.2 of the Gurobi commercial solver tool. No constraint was set on the number of cores the solver can employ, except for the default value. Time limit of 900 seconds is given to solve each instance which was the realized case by the company. In the tables presented, each row represent a unique instance. The weights of the objective function indicated by parameters w_1, w_2, w_3 and w_4 were set as 0.3, 0.2, and 0.3, 0.2, respectively. N denotes the total number of nodes, while FB ratio reflects the proportion of nodes belonging to a food bank. The vehicle ratio (Veh) shows the percentage of food banks possessing a vehicle, and in each experiment, there is at least one vehicle. The parameter P represents the number of various product types. DS indicates the demand scenario. $RandID$ is the index of the random instance with which the same parameters are used with different random seed. From the model outputs, *Obj* specifies the objective function value of the best solution discovered by the commercial solver within the provided time. BB represents the best dual bound, while *Gap*(%) measures the percentage gap between BB and the *Obj* which was taken directly from the solver. The solver gap is calculated using $|z_p - z_d|/z_p$ where z_p denotes the objective value of the primal solution and z_d is the objective value of dual solution. $HeurObj$ is the initial heuristic solution of the solver.

From Table 5.1 and Table 5.2, independent from the demand scenario, it can be observed that as N increases from 20 to 50, gap values tend to increase. Therefore,

Table 5.1: Mathematical Model Metrics for Synthetic Instances in Total Equality Demand Scenario.

N	FB	Veh	P	DS	RandID	Obj	BB	Gap (%)	HeurObj
20	0.33	1	10	TE	1	137651.3	151254.5	9.9	135563.2
20	0.33	1	10	TE	2	123398.8	140287.3	13.7	123144.0
20	0.33	1	10	TE	3	140461.3	153192.9	9.1	135761.8
20	0.33	1	10	TE	4	192509.7	195433.6	1.5	180331.2
20	0.33	0.33	10	TE	1	134161.9	153087.6	14.1	133758.6
20	0.33	0.33	10	TE	2	97587.4	109085.4	11.8	97175.5
20	0.33	0.33	10	TE	3	125016.4	134766.0	7.8	124967.9
20	0.33	0.33	10	TE	4	138761.2	142666.4	2.8	137553.1
20	0.66	1	10	TE	1	45679.3	51173.8	12.0	28043.6
20	0.66	1	10	TE	2	6795.8	45191.8	565.0	662.9
20	0.66	1	10	TE	3	26763.8	102123.0	281.6	116.7
20	0.66	1	10	TE	4	4586.2	78518.8	1612.1	0.0
20	0.66	0.33	10	TE	1	54079.1	55525.9	2.7	54060.6
20	0.66	0.33	10	TE	2	77888.1	79093.7	1.5	75837.6
20	0.66	0.33	10	TE	3	58295.3	70214.7	20.4	57085.9
20	0.66	0.33	10	TE	4	76409.7	88797.0	16.2	68888.9
50	0.33	1	5	TE	1	139392.1	284275.2	103.9	0.0
50	0.33	1	5	TE	2	168941.6	255966.8	51.5	0.0
50	0.33	1	5	TE	3	113350.8	285874.1	152.2	0.0
50	0.33	1	5	TE	4	67950.4	293078.3	331.3	0.0
50	0.33	0.33	5	TE	1	180514.9	251770.4	39.5	156823.1
50	0.33	0.33	5	TE	2	194215.6	326224.5	68.0	153963.6
50	0.33	0.33	5	TE	3	192759.6	292570.2	51.8	152036.8
50	0.33	0.33	5	TE	4	195896.6	278960.3	42.4	19150.0
50	0.66	1	5	TE	1	0.0	10675591.0	inf	0.0
50	0.66	1	5	TE	2	3724.3	146901.7	3844.4	0.0
50	0.66	1	5	TE	3	5103.8	135063.5	2546.3	0.0
50	0.66	1	5	TE	4	3248.8	138836.0	4173.4	0.0
50	0.66	0.33	5	TE	1	18975.2	136030.3	616.9	0.0
50	0.66	0.33	5	TE	2	17213.1	141188.6	720.2	0.0
50	0.66	0.33	5	TE	3	33366.9	119006.8	256.7	0.0
50	0.66	0.33	5	TE	4	1287.2	143744.1	11067.0	0.0

when N is 50, instances get significantly harder for mathematical model to solve, despite the decrease in the number of products. For many of the instance sets with 50 nodes, the heuristic of commercial solver rarely came up with any non-trivial solution. When Veh and FB both have high values, gap percentages are higher due to additional complexity. Since the number of donors (and consequently the total

Table 5.2: Mathematical Model Metrics for Synthetic Instances in Infinite Demand Scenario.

N	FB	Veh	P	DS	RandID	Obj	BB	Gap (%)	HeurObj
20	0.33	1	10	ID	1	134253.9	135242.4	0.7	134138.7
20	0.33	1	10	ID	2	141296.6	141963.2	0.5	141141.3
20	0.33	1	10	ID	3	112442.1	113070.5	0.6	110057.5
20	0.33	1	10	ID	4	168771.5	169304.9	0.3	168654.0
20	0.33	0.33	10	ID	1	153261.2	160242.6	4.6	152470.4
20	0.33	0.33	10	ID	2	113235.0	133395.2	17.8	110477.0
20	0.33	0.33	10	ID	3	150322.9	152878.4	1.7	149724.3
20	0.33	0.33	10	ID	4	153678.6	154286.8	0.4	153261.0
20	0.66	1	10	ID	1	46215.7	47168.7	2.1	46006.2
20	0.66	1	10	ID	2	82330.3	84742.8	2.9	82140.1
20	0.66	1	10	ID	3	74875.9	75291.3	0.6	74657.6
20	0.66	1	10	ID	4	88120.4	89012.8	1.0	70393.7
20	0.66	0.33	10	ID	1	62126.1	66637.3	7.3	61672.3
20	0.66	0.33	10	ID	2	61654.2	62620.6	1.6	60855.8
20	0.66	0.33	10	ID	3	55310.1	55646.7	0.6	41623.1
20	0.66	0.33	10	ID	4	45600.0	52203.2	14.5	42273.4
50	0.33	1	5	ID	1	169451.0	262036.9	54.6	0.0
50	0.33	1	5	ID	2	163639.7	278006.2	69.9	0.0
50	0.33	1	5	ID	3	230550.9	312290.5	35.5	0.0
50	0.33	1	5	ID	4	117438.9	264226.4	125.0	0.0
50	0.33	0.33	5	ID	1	112222.3	277986.6	147.7	71195.6
50	0.33	0.33	5	ID	2	200911.2	261660.3	30.2	153396.6
50	0.33	0.33	5	ID	3	229899.1	314205.2	36.7	103822.4
50	0.33	0.33	5	ID	4	172296.6	245280.6	42.4	136497.3
50	0.66	1	5	ID	1	125502.2	158380.4	26.2	0.0
50	0.66	1	5	ID	2	118781.4	130544.5	9.9	0.0
50	0.66	1	5	ID	3	0.0	121935.9	inf	0.0
50	0.66	1	5	ID	4	0.0	137489.2	inf	0.0
50	0.66	0.33	5	ID	1	89361.7	178257.3	99.5	0.0
50	0.66	0.33	5	ID	2	35060.7	158998.1	353.5	0.0
50	0.66	0.33	5	ID	3	65279.3	147635.6	126.2	0.0
50	0.66	0.33	5	ID	4	47582.5	172316.6	262.1	0.0

amount of donation) decreases once FB is decreased while all other parameters are held constant, the objective function values tend to decrease as expected. A counter-intuitive result from managerial perspective is obtained, once Veh is increased and other parameters are kept constant. Although reduction in routing related costs and potential improvements in allocation efficiencies are expected, the objective function values tend to worsen according to Table 5.1. As implied by gap values in Table 5.1 and Table 5.2, in none of the 64 instances the optimality gap has been closed by the solver. Therefore, all of the models were terminated by the time limit of 900 seconds.

5.3 Experiments for Matheuristic on Synthetic Instances

To examine the behavior of the model, some experiments using the proposed matheuristic were carried out on the same set of computer generated instances. A variety of different factors were taken into account during the experimentation, including the sensitivity of matheuristic, the effect of the vehicle ratio (Veh) on solution quality, the significance of iteration count in the outputs, cumulative build and solution times of AS models. The experiment tables show the number of iterations (IC), the number of iterations that did not improve from the step where a better solution was discovered to the algorithm's termination (UIC), and the total number of iterations for each of the shaking move operators (S_i). The cumulative percentage of times spent on the solution ($TASS\%$) and building ($TASB\%$) of the AS model and relative to the total duration are also shown. For performance assessment, the deviation of the matheuristic solution from the mathematical model solution and gap with the best dual bound (DMM, GBB) are also indicated. Deviation and gap are calculated using $(z_{mm} - z_{mh})/z_{mm}$ and $|z_{bb} - z_{mh}|/z_{bb}$, respectively, for comparison of the matheuristic with mathematical model and the best dual bound. The comparison between ALUB and the best bound of the mathematical model is also documented.

Table 5.3 and Table 5.4 show the number of total iterations and the number of iterations since the last improving iteration, along with the number of iterations where each of the shaking moves took place. As expected, there is a very clear relation between IC and instance parameters that define the number of possible routes such as N, FB and Veh. Since build and solve time of each AS model (for each visited solution during local search phase) is dependent on P, it is safe to claim that P also effects the scalability of the matheuristic algorithm. Yet, this

Table 5.3: Matheuristic Iteration Metrics for Synthetic Instances in Total Equality Demand Scenario.

N	FB	Veh	P	DS	RandID	IC	UIC	S1	S2	S3	S4	S5
20	0.33	1	10	TE	1	234	1	75	63	38	31	27
20	0.33	1	10	TE	2	519	0	126	118	102	87	86
20	0.33	1	10	TE	3	596	5	162	142	116	94	82
20	0.33	1	10	TE	4	294	3	95	83	49	35	32
20	0.33	0.33	10	TE	1	1269	107	281	262	255	239	232
20	0.33	0.33	10	TE	2	1564	20	338	323	316	296	291
20	0.33	0.33	10	TE	3	1147	175	248	236	228	218	217
20	0.33	0.33	10	TE	4	1202	4	269	255	242	220	216
20	0.66	1	10	TE	1	187	5	74	61	20	19	13
20	0.66	1	10	TE	2	106	0	52	41	6	5	2
20	0.66	1	10	TE	3	359	23	100	88	62	56	53
20	0.66	1	10	TE	4	206	0	74	62	28	23	19
20	0.66	0.33	10	TE	1	360	11	115	82	64	55	44
20	0.66	0.33	10	TE	2	822	10	206	170	160	149	137
20	0.66	0.33	10	TE	3	919	67	226	199	179	163	152
20	0.66	0.33	10	TE	4	1113	49	249	233	221	208	202
50	0.33	1	5	TE	1	38	0	20	14	2	2	0
50	0.33	1	5	TE	2	43	4	18	17	5	1	2
50	0.33	1	5	TE	3	12	0	6	4	1	1	0
50	0.33	1	5	TE	4	17	0	9	6	2	0	0
50	0.33	0.33	5	TE	1	34	0	13	9	4	4	4
50	0.33	0.33	5	TE	2	78	0	42	25	7	2	2
50	0.33	0.33	5	TE	3	129	19	50	36	21	13	9
50	0.33	0.33	5	TE	4	27	0	14	9	2	1	1
50	0.66	1	5	TE	1	20	0	12	8	0	0	0
50	0.66	1	5	TE	2	14	0	8	4	1	0	1
50	0.66	1	5	TE	3	7	1	3	1	1	1	1
50	0.66	1	5	TE	4	20	0	10	8	1	0	1
50	0.66	0.33	5	TE	1	67	0	32	21	6	4	4
50	0.66	0.33	5	TE	2	25	2	13	9	1	1	1
50	0.66	0.33	5	TE	3	90	0	44	33	5	3	5
50	0.66	0.33	5	TE	4	59	1	32	19	3	3	2

factor is not investigated in synthetic instances. However, when N is set to 50, despite the decrease in P, the number of iterations are significantly higher. That is due to the growth in the number of neighbor solutions during local search phase. Furthermore, since parameters FB and Veh directly impact the number of available vehicles, the number of neighbor solutions during local search decrease when they

Table 5.4: Matheuristic Iteration Metrics for Synthetic Instances in Infinite Demand Scenario.

N	FB	Veh	P	DS	RandID	IC	UIC	S1	S2	S3	S4	S5
20	0,33	1	10	ID	1	734	54	177	166	145	126	120
20	0,33	1	10	ID	2	834	72	195	191	163	141	144
20	0,33	1	10	ID	3	486	0	123	116	90	81	76
20	0,33	1	10	ID	4	627	72	144	142	122	110	109
20	0,33	0,33	10	ID	1	1824	57	387	378	370	346	343
20	0,33	0,33	10	ID	2	1272	34	277	264	257	240	234
20	0,33	0,33	10	ID	3	1027	378	227	218	203	190	189
20	0,33	0,33	10	ID	4	716	59	171	155	142	129	119
20	0,66	1	10	ID	1	365	29	100	97	60	55	53
20	0,66	1	10	ID	2	442	0	124	121	86	34	77
20	0,66	1	10	ID	3	98	0	54	42	1	1	0
20	0,66	1	10	ID	4	313	4	93	91	45	43	41
20	0,66	0,33	10	ID	1	765	34	175	163	150	140	137
20	0,66	0,33	10	ID	2	637	69	140	133	126	120	118
20	0,66	0,33	10	ID	3	654	64	165	145	125	115	104
20	0,66	0,33	10	ID	4	1007	213	221	210	196	191	189
50	0,33	1	5	ID	1	18	0	12	6	0	0	0
50	0,33	1	5	ID	2	35	1	16	11	4	2	2
50	0,33	1	5	ID	3	12	0	7	5	0	0	0
50	0,33	1	5	ID	4	24	0	14	9	1	0	0
50	0,33	0,33	5	ID	1	113	0	49	36	11	9	8
50	0,33	0,33	5	ID	2	53	0	26	16	6	3	2
50	0,33	0,33	5	ID	3	134	0	49	39	21	14	11
50	0,33	0,33	5	ID	4	70	0	31	23	6	5	5
50	0,66	1	5	ID	1	14	0	8	6	0	0	0
50	0,66	1	5	ID	2	22	0	12	10	0	0	0
50	0,66	1	5	ID	3	12	1	7	5	0	0	0
50	0,66	1	5	ID	4	25	0	13	12	0	0	0
50	0,66	0,33	5	ID	1	65	0	30	21	6	5	3
50	0,66	0,33	5	ID	2	55	2	31	21	1	1	1
50	0,66	0,33	5	ID	3	75	0	40	23	5	4	3
50	0,66	0,33	5	ID	4	58	3	34	23	1	0	0

have lower values. Consequently, similar to the case with N, FB and Veh have a direct relationship with IC. Although the number of iterations are relatively low, the best performing shaking moves are prioritized in alignment with the structure of VNS. That can be inferred from the difference between consecutive shaking moves S_i's, as they are indications of number successful shakings that led to an improving solution. UIC values are usually low when Veh is equal to 1. In ID demand scenario, number of converging instances are relatively higher as more of the instances with 20 nodes converged and IC values are higher in general.

Table 5.5: Matheuristic Performance Comparison Metrics for Synthetic Instances in Total Equality Demand Scenario.

N	FB	Veh	P	DS	RandID	Obj	DMM(%)	GBB(%)	TASS(%)	TASB(%)
20	0.33	1	10	TE	1	132980.5	3.4	12.1	50.4	32.3
20	0.33	1	10	TE	2	105129.3	14.8	25.1	46.2	35.1
20	0.33	1	10	TE	3	133302.3	5.1	13.0	41.9	38.1
20	0.33	1	10	TE	4	179543.8	6.7	8.1	43.7	36.9
20	0.33	0.33	10	TE	1	116107.3	13.5	24.2	46.7	35.1
20	0.33	0.33	10	TE	2	84910.2	13.0	22.2	45.7	36.1
20	0.33	0.33	10	TE	3	107285.2	14.2	20.4	46.0	35.6
20	0.33	0.33	10	TE	4	118568.0	14.6	16.9	45.8	36.0
20	0.66	1	10	TE	1	41867.8	8.3	18.2	45.5	34.7
20	0.66	1	10	TE	2	27356.2	-302.5	39.5	44.1	35.7
20	0.66	1	10	TE	3	79361.3	-196.5	22.3	43.3	36.3
20	0.66	1	10	TE	4	68299.9	-1389.3	13.0	43.8	35.9
20	0.66	0.33	10	TE	1	43609.8	19.4	21.5	47.6	34.8
20	0.66	0.33	10	TE	2	72807.8	6.5	7.9	49.0	33.9
20	0.66	0.33	10	TE	3	62414.5	-7.1	11.1	46.7	35.5
20	0.66	0.33	10	TE	4	57972.9	24.1	34.7	45.9	35.9
50	0.33	1	5	TE	1	126302.7	9.4	55.6	38.3	38.2
50	0.33	1	5	TE	2	119348.6	29.4	53.4	41.8	36.0
50	0.33	1	5	TE	3	207117.2	-82.7	27.5	64.4	22.2
50	0.33	1	5	TE	4	160816.1	-136.7	45.1	40.6	36.6
50	0.33	0.33	5	TE	1	68216.1	62.2	72.9	41.0	36.9
50	0.33	0.33	5	TE	2	144302.1	25.7	55.8	39.2	38.2
50	0.33	0.33	5	TE	3	92810.1	51.9	68.3	39.9	37.8
50	0.33	0.33	5	TE	4	108232.4	44.8	61.2	41.4	36.7
50	0.66	1	5	TE	1	87567.4	-inf	99.2	36.1	38.8
50	0.66	1	5	TE	2	95758.1	-2471.1	34.8	35.2	39.1
50	0.66	1	5	TE	3	75864.8	-1386.4	43.8	55.4	27.7
50	0.66	1	5	TE	4	75793.7	-2233.0	45.4	34.7	39.6
50	0.66	0.33	5	TE	1	43244.8	-127.9	68.2	38.7	37.5
50	0.66	0.33	5	TE	2	23082.2	-34.1	83.7	39.3	37.8
50	0.66	0.33	5	TE	3	25827.2	22.6	78.3	37.5	38.9
50	0.66	0.33	5	TE	4	40455.1	-3042.8	71.9	40.4	37.2

Table 5.5 and Table 5.6 depict the objective values of matheuristic, their comparison with MILP's best primal and dual bounds, and the percentage of cumula-

Table 5.6: Matheuristic Performance Comparison Metrics for Synthetic Instances in Infinite Demand Scenario.

N	FB	Veh	P	DS	RandID	Obj	DMM(%)	GBB(%)	TASS(%)	TASB(%)
20	0.33	1	10	ID	1	122084.9	9.1	9.7	42.9	37.4
20	0.33	1	10	ID	2	138950.6	1.7	2.1	42.5	37.7
20	0.33	1	10	ID	3	109766.7	2.4	2.9	42.0	38.0
20	0.33	1	10	ID	4	158028.6	6.4	6.7	44.3	36.5
20	0.33	0.33	10	ID	1	121263.6	20.9	24.3	43.7	37.5
20	0.33	0.33	10	ID	2	105449.3	6.9	20.9	44.9	36.7
20	0.33	0.33	10	ID	3	106908.2	28.9	30.1	43.9	37.1
20	0.33	0.33	10	ID	4	145310.0	5.4	5.8	47.6	34.5
20	0.66	1	10	ID	1	41856.3	9.4	11.3	40.3	38.2
20	0.66	1	10	ID	2	67269.8	18.3	20.6	40.8	37.9
20	0.66	1	10	ID	3	68147.5	9.0	9.5	45.5	34.8
20	0.66	1	10	ID	4	85255.5	3.3	4.2	40.7	38.0
20	0.66	0.33	10	ID	1	48373.9	22.1	27.4	46.5	35.5
20	0.66	0.33	10	ID	2	41614.7	32.5	33.5	48.7	34.0
20	0.66	0.33	10	ID	3	53837.3	2.7	3.3	45.1	36.4
20	0.66	0.33	10	ID	4	49070.1	-7.6	6.0	45.5	36.2
50	0.33	1	5	ID	1	176629.0	-4.2	32.6	37.4	37.5
50	0.33	1	5	ID	2	162322.3	0.8	41.6	38.5	38.1
50	0.33	1	5	ID	3	190082.5	17.6	39.1	40.3	35.5
50	0.33	1	5	ID	4	160368.0	-36.6	39.3	37.4	37.5
50	0.33	0.33	5	ID	1	92277.5	17.8	66.8	38.5	38.6
50	0.33	0.33	5	ID	2	124435.3	38.1	52.4	39.9	37.8
50	0.33	0.33	5	ID	3	112453.1	51.1	64.2	38.6	38.6
50	0.33	0.33	5	ID	4	76954.1	55.3	68.6	39.5	38.0
50	0.66	1	5	ID	1	132187.8	-5.3	16.5	35.3	39.4
50	0.66	1	5	ID	2	85555.2	28.0	34.5	36.6	38.5
50	0.66	1	5	ID	3	87179.4	-inf	28.5	37.0	38.4
50	0.66	1	5	ID	4	95903.7	-inf	30.2	34.7	39.6
50	0.66	0.33	5	ID	1	60482.8	32.3	66.1	40.0	37.3
50	0.66	0.33	5	ID	2	45743.8	-30.5	71.2	38.3	38.4
50	0.66	0.33	5	ID	3	79599.5	-21.9	46.1	40.1	37.5
50	0.66	0.33	5	ID	4	74807.7	-57.2	56.6	38.7	38.2

tive time that VNS based matheuristic has spent to build and solve the allocation subproblem for a given tour. When N is set to 50 matheuristic is more likely to outperform the mathematical model. Clearly, once Veh is increased and other parameters are held constant, objective function values of matheuristic solutions tend to increase. Thus, the counter-intuitive result from managerial perspective is not present in matheuristic outputs as opposed to mathematical model outputs. As complexity of the problem increases by increasing N, FB and Veh values (in other words, growing the number of possible routes), $GBB(\%)$ also tend to increase. That would lead to an insight about unsatisfactory performance of matheuristic for the cases where $DMM(\%)$ is positive. However, when $DMM(\%)$ is negative, it is not possible to infer whether matheuristic performed inadequately or best bounds provided by mathematical model are not strict enough. From $TASS(\%)$ and $TASB(\%)$ values, it is obvious that computational burden in the algorithm design is not on performing the move operations, but to solve allocation subproblem for each visited solution. From Table 5.3 and Table 5.4 the iteration counts for each instance was also obtained. In future studies, replacing the AS based optimal allocation method with a heuristic approach can be further investigated to increase the iteration counts at the expense of generating suboptimal allocations. Careful analysis over $DMM(\%)$ reveals that matheuristic approach came up with more robust solutions in general.

A detailed comparison of the best solutions found by two different algorithms for synthetic instances are provided. Yet, one of the contributions of this thesis is ALUB to find upper bounds. ALUB exploits the linear structure of allocation subproblem. It disregards the routing part and relaxes all routing related constraints. Due to the linear structure of ALUB, it is anticipated that solutions will be reached in a rapid manner. Comparison between two methods to find upper bounds is also provided in this study.

Objective function values of ALUB, solution times are given in Table 5.7 and Table 5.8. Since objective function values of ALUB serve as an upper bound to the problem, comparison of ALUB and the best bounds provided mathematical model in 900 seconds are also shown in tables. It can be observed that independent from the demand scenario, ALUB takes a few seconds (which is practically trivial) to solve since the underlying model is based on an LP. Despite being very fast, ALUB has been able to generate bounds that are very close to dual bounds MILP based mathematical model can achieve in 900 seconds. Therefore, methods that rely on upper bounds can make use of ALUB in future studies. Furthermore, decisions regarding practical business operations can be guided by ALUB instead of mathematical

Table 5.7: ALUB Performance Comparison Metrics for Synthetic Instances in Total Equality Demand Scenario.

N	FB	Veh	P	DS	RandID	ALUB Obj	ALUB Time (s)	Dev(%) BB-ALUB
20	0.33	1	10	TE	1	151794.5	0.2	0.4
20	0.33	1	10	TE	2	140994.6	0.2	0.5
20	0.33	1	10	TE	3	153866.6	0.2	0.4
20	0.33	1	10	TE	4	196014.2	0.2	0.3
20	0.33	0.33	10	TE	1	153866.6	0.1	0.5
20	0.33	0.33	10	TE	2	109810.5	0.1	0.7
20	0.33	0.33	10	TE	3	135558.8	0.1	0.6
20	0.33	0.33	10	TE	4	143507.3	0.1	0.6
20	0.66	1	10	TE	1	51639.9	0.6	0.9
20	0.66	1	10	TE	2	45765.5	0.6	1.3
20	0.66	1	10	TE	3	103363.6	0.6	1.2
20	0.66	1	10	TE	4	79331.4	0.6	1.0
20	0.66	0.33	10	TE	1	57292.0	0.2	3.1
20	0.66	0.33	10	TE	2	79687.8	0.2	0.7
20	0.66	0.33	10	TE	3	71085.7	0.2	1.2
20	0.66	0.33	10	TE	4	89477.0	0.2	0.8
50	0.33	1	5	TE	1	285167.4	0.7	0.3
50	0.33	1	5	TE	2	256792.0	0.7	0.3
50	0.33	1	5	TE	3	286662.8	0.7	0.3
50	0.33	1	5	TE	4	293843.4	0.7	0.3
50	0.33	0.33	5	TE	1	252651.1	0.3	0.3
50	0.33	0.33	5	TE	2	327178.1	0.3	0.3
50	0.33	0.33	5	TE	3	293546.8	0.3	0.3
50	0.33	0.33	5	TE	4	279814.1	0.3	0.3
50	0.66	1	5	TE	1	150123.9	3.6	7011.2
50	0.66	1	5	TE	2	147568.1	3.6	0.5
50	0.66	1	5	TE	3	135803.2	3.7	0.5
50	0.66	1	5	TE	4	139602.6	3.6	0.5
50	0.66	0.33	5	TE	1	136706.4	1.2	0.5
50	0.66	0.33	5	TE	2	142020.7	1.2	0.6
50	0.66	0.33	5	TE	3	119882.5	1.2	0.7
50	0.66	0.33	5	TE	4	145059.4	1.2	0.9

model bounds.

5.4 Generation of Real Instances

Real instances are derived from real life cases of a company for İstanbul, Turkey. The provided data is cleaned and reformatted in close collaboration with domain

Table 5.8: ALUB Performance Comparison Metrics for Synthetic Instances in Infinite Demand Scenario.

N	FB	Veh	P	DS	RandID	ALUB Obj	ALUB Time (s)	Dev(%) BB-ALUB
20	0.33	1	10	ID	1	135747.3	0.2	0.4
20	0.33	1	10	ID	2	142505.4	0.2	0.4
20	0.33	1	10	ID	3	113554.2	0.2	0.4
20	0.33	1	10	ID	4	169821.7	0.2	0.3
20	0.33	0.33	10	ID	1	160944.1	0.1	0.4
20	0.33	0.33	10	ID	2	151022.6	0.1	11.7
20	0.33	0.33	10	ID	3	153450.1	0.1	0.4
20	0.33	0.33	10	ID	4	154856.4	0.1	0.4
20	0.66	1	10	ID	1	47492.5	0.6	0.7
20	0.66	1	10	ID	2	85169.5	0.6	0.5
20	0.66	1	10	ID	3	75582.7	0.6	0.4
20	0.66	1	10	ID	4	89364.6	0.6	0.4
20	0.66	0.33	10	ID	1	68602.6	0.2	2.9
20	0.66	0.33	10	ID	2	63089.0	0.2	0.7
20	0.66	0.33	10	ID	3	56033.0	0.2	0.7
20	0.66	0.33	10	ID	4	52709.8	0.2	1.0
50	0.33	1	5	ID	1	262708.6	0.7	0.3
50	0.33	1	5	ID	2	278586.6	0.7	0.2
50	0.33	1	5	ID	3	312864.4	0.7	0.2
50	0.33	1	5	ID	4	264892.7	0.7	0.3
50	0.33	0.33	5	ID	1	278689.7	0.3	0.3
50	0.33	0.33	5	ID	2	262450.1	0.3	0.3
50	0.33	0.33	5	ID	3	315037.4	0.3	0.3
50	0.33	0.33	5	ID	4	245982.9	0.3	0.3
50	0.66	1	5	ID	1	158703.3	3.6	0.2
50	0.66	1	5	ID	2	130930.2	3.5	0.3
50	0.66	1	5	ID	3	122245.9	3.6	0.3
50	0.66	1	5	ID	4	137840.9	3.6	0.3
50	0.66	0.33	5	ID	1	178714.7	1.1	0.3
50	0.66	0.33	5	ID	2	159475.3	1.1	0.3
50	0.66	0.33	5	ID	3	148151.4	1.2	0.3
50	0.66	0.33	5	ID	4	172734.7	1.1	0.2

experts in the company. Three days from each week of the year are considered while defining the planning horizon ($T = 3$). Three different weeks from a year is arbitrarily selected for experimentation. The products that can substitute each other are grouped to form a product type. This grouping is done at different hierarchical levels so that instances with $P = 5$ and $P = 10$ can be generated. Then the experimental results can be served to domain experts to be analyzed which of the hierarchical levels are more suitable in real life scenarios. As depicted in Figure 5.2, İstanbul is divided to three regions and data of each are used to generate instances. N is the number of nodes and F is the number of food banks. H is the number of total vehicles, which is considered to be 33% and 100% of the total number of demand nodes. U is always 2, as in the case of synthetic instances. L is 8 hours according to working hours. h_k is fixed to be equal to the number of periods. a_{pu} is defined with respect to refrigeration requirements of the product types. c_{ku} is calculated by dividing the total capacity of Fiat Doblo vehicle (750 kg) according to 5-95 parity. q_{ip}^t where $i \in G$ is taken directly from the company data. Similar to synthetic instances q_{ip}^t where $i \in F$ is generated using TE and ID scenarios. Coordinates of supply demand nodes are from İstanbul data and euclidean distance is used as distance metric to generate d_{ij} matrix. Speed, emission and time between two nodes are calculated similar to synthetic instances. The same prices are used for product types.

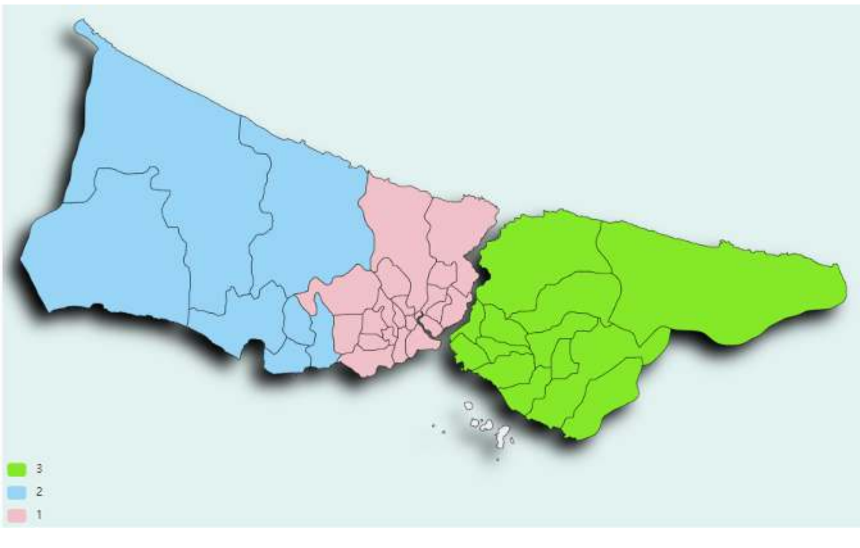


Figure 5.2: Three regions of Food Banks.

5.5 Experiments for Mathematical Model on Real Instances

The performance and behavior of the MILP model is tested on real life instances. The quality of solutions are analyzed across different neighborhoods, weeks and instance parameters. The model settings and objective function weights (w_1, w_2, w_3 and w_4) are kept the same as experiments for synthetic instances. Since instances are from real life cases, each of the rows represent a unique instance in experiment tables. Performance metrics are given for one solution replica. *Reg* denotes the region for which the instance is created. $Week$ represents the week of the year. TPP is the total potential products, which is an indicator of product grouping hierarchy. P is the total number of product types that is actually donated by food banks for the given week. DS is the demand scenario that indicates whether TE or ID is used in generation of demands (q^t_{ip} where $i \in F$). Since demand scenario is an artificial parameter that is not taken from company data, a unique instance is created with 2 different demand scenarios for each real life configuration. In the experiment tables, N is the total number of nodes and F is the demand nodes (food banks). Veh is the number of vehicles and *Obj* is the best solution found by MILP. BB is the best bound provided by mathematical model and *Gap*(%) is the gap between *Obj* and BB. $HeurObj$ is heuristic solution of Gurobi commercial solver.

Table 5.9 and Table 5.10 depict the mathematical model metrics for the first region across varying times of the year. In TE scenario, solver heuristic has been able to generate nontrivial solutions only for instances with less vehicles. In ID scenario such behavior is also apparent with a few exceptions. Mathematical model performance (gap percentage) seem to be highly dependent on the solver heuristic outputs. It can be concluded from objective values (where gap is relatively small) and best bounds that having more vehicle has a little impact on the optimal solution. However, as more vehicles complicate the problem structure, mathematical model is likely to generate worse solutions.

Table 5.11 and Table 5.12 show the mathematical model metrics for the second region for the same times with Table 5.9 and Table 5.10. Gap percentages in both tables are negligible from practical point of view. Therefore, it can be inferred that

Table 5.9: Mathematical Model Metrics for First Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	BB	Gap(%)	HeurObj
1	5	7	TE	36	21	5	21	17396.9	132170.7	659.7	0.0
1	5	7	TE	36	21	5	7	131899.0	132141.6	0.2	129830.7
1	5	26	TE	36	21	5	21	5151.2	86246.1	1574.3	0.0
1	5	26	TE	36	21	5	7	86116.6	86235.8	0.1	85947.7
1	5	47	TE	39	21	5	21	16839.4	185222.4	999.9	0.0
1	5	47	TE	39	21	5	7	165675.5	185210.7	11.8	58947.0
1	10	7	TE	36	21	8	21	12002.1	114622.4	855.0	0.0
1	10	7	TE	36	21	8	7	113089.1	114595.6	1.3	51526.9
1	10	26	TE	34	21	9	21	3506.7	66227.9	1788.6	0.0
1	10	26	TE	34	21	9	7	24631.3	66212.5	168.8	10822.0
1	10	47	TE	37	21	9	21	0.0	183656.8	inf	0.0
1	10	47	TE	37	21	9	7	31027.5	183647.0	491.9	1089.4

Table 5.10: Mathematical Model Metrics for First Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	BB	Gap(%)	HeurObj
1	5	7	ID	36	21	5	21	131712.6	131931.7	0.2	110804.6
1	5	7	ID	36	21	5	7	131705.7	131904.3	0.2	131687.3
1	5	26	ID	36	21	5	21	66526.2	86044.2	29.3	25149.6
1	5	26	ID	36	21	5	7	85908.7	86037.5	0.1	85900.6
1	5	47	ID	39	21	5	21	142219.3	184771.5	29.9	4083.0
1	5	47	ID	39	21	5	7	184530.6	184767.3	0.1	120222.0
1	10	7	ID	36	21	8	21	64772.1	114286.7	76.4	0.0
1	10	7	ID	36	21	8	7	114070.9	114270.2	0.2	114031.5
1	10	26	ID	34	21	9	21	34788.8	66125.1	90.1	0.0
1	10	26	ID	34	21	9	7	66011.1	66124.9	0.2	65946.5
1	10	47	ID	37	21	9	21	61010.3	183184.4	200.3	0.0
1	10	47	ID	37	21	9	7	170968.1	183178.1	7.1	121906.6

instances of the second region are computationally easier to solve. The reason of performance discrepancy across regions is the amount of nodes being less (or higher)

Table 5.11: Mathematical Model Metrics for Second Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	BB	Gap(%)	HeurObj
2	5	7	TE	14	6	5	6	91294.8	91347.4	0.1	91286.6
2	5	7	TE	14	6	5	2	91277.7	91286.3	0.0	91270.7
2	5	26	TE	12	6	5	6	73573.4	73584.2	0.0	73570.1
2	5	26	TE	12	6	5	2	73571.9	73579.2	0.0	73521.2
2	5	47	TE	13	6	5	6	77141.5	77159.4	0.0	77140.8
2	5	47	TE	13	6	5	2	74508.4	74515.7	0.0	74500.4
2	10	7	TE	14	6	8	6	84251.3	84273.2	0.0	84243.1
2	10	7	TE	14	6	8	2	84249.2	84259.5	0.0	84238.9
2	10	26	TE	12	6	9	6	36242.7	36265.2	0.1	36216.2
2	10	26	TE	12	6	9	2	30581.2	30584.1	0.0	30579.8
2	10	47	TE	12	6	9	6	69344.9	69372.8	0.0	69341.7
2	10	47	TE	12	6	9	2	69349.4	69356.1	0.0	69343.9

Table 5.12: Mathematical Model Metrics for Second Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	BB	Gap(%)	HeurObj
2	5	7	ID	14	6	5	6	91291.8	91306.6	0.0	91284.1
2	5	7	ID	14	6	5	2	91263.8	91272.9	0.0	91260.6
2	5	26	ID	12	6	5	6	73531.9	73540.6	0.0	73510.1
2	5	26	ID	12	6	5	2	73530.7	73536.3	0.0	73522.4
2	5	47	ID	13	6	5	6	77104.2	77120.5	0.0	77102.8
2	5	47	ID	13	6	5	2	75193.7	75201.2	0.0	75188.6
2	10	7	ID	14	6	8	6	84225.6	84256.9	0.0	84212.9
2	10	7	ID	14	6	8	2	84230.2	84241.1	0.0	84226.7
2	10	26	ID	12	6	9	6	37513.6	37525.3	0.0	37502.3
2	10	26	ID	12	6	9	2	34761.4	34764.8	0.0	34746.8
2	10	47	ID	12	6	9	6	69311.9	69332.0	0.0	69311.0
2	10	47	ID	12	6	9	2	69312.0	69322.3	0.0	69308.6

such that a routing scheme that maximizes the allocation is much easier (or harder) to obtain. Even solver heuristic has provided decent outputs for all instances. It can be observed that different demand scenarios (thus, varying equality function structures) had a little influence over the optimal objective function values.

Table 5.13: Mathematical Model Metrics for Third Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	BB	Gap(%)	HeurObj
3	5	7	TE	33	20	5	20	18340.3	97793.5	433.2	4844.4
3	5	7	TE	33	20	5	7	97680.3	97776.0	0.1	97660.2
3	5	26	TE	35	20	5	20	35512.3	85939.3	142.0	421.0
3	5	26	TE	35	20	5	7	85775.1	85929.5	0.2	85756.1
3	5	47	TE	32	20	5	20	2047.3	92290.1	4407.8	0.0
3	5	47	TE	32	20	5	7	73001.0	73232.7	0.3	71148.6
3	10	7	TE	30	20	8	20	1501.2	56587.0	3669.4	0.0
3	10	7	TE	30	20	8	7	56293.7	56572.3	0.5	56257.0
3	10	26	TE	34	20	9	20	0.0	23737.6	inf	0.0
3	10	26	TE	34	20	9	7	21070.8	23702.0	12.5	18900.8
3	10	47	TE	32	20	9	20	0.0	105272.5	inf	0.0
3	10	47	TE	32	20	9	7	104224.5	105259.2	1.0	98387.9

Mathematical model metrics for the third region are given in Table 5.13 and Table 5.14. For TE scenario, instances with more vehicles have very high gap percentages while for instances with less vehicles mathematical model has been able to generate near-optimal solutions. Additionally, gap values are not sensitive to the number of vehicles in neither of the demand scenarios. Relation between the best found solution and heuristic objective value is visible in third region instances as well.

5.6 Experiments for Matheuristic on Real Instances

Performance and robustness of matheuristic over real life scenarios are examined through experiments on real instances. While analyzing effects of regions, weeks, product counts, number of vehicles on matheuristic performance, the abbreviations and factors (and their calculation methodologies) shown in tables are the same as synthetic instances.

Table 5.14: Mathematical Model Metrics for Third Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	BB	Gap(%)	HeurObj
3	5	7	ID	33	20	5	20	90849.0	98042.5	7.9	49044.2
3	5	7	ID	33	20	5	7	97957.2	98023.1	0.1	97934.1
3	5	26	ID	35	20	5	20	84947.3	86228.2	1.5	62432.9
3	5	26	ID	35	20	5	7	86118.3	86218.9	0.1	86062.8
3	5	47	ID	32	20	5	20	56893.2	91256.4	60.4	37153.2
3	5	47	ID	32	20	5	7	77138.6	77268.9	0.2	77109.8
3	10	7	ID	30	20	8	20	56480.4	56768.4	0.5	34378.3
3	10	7	ID	30	20	8	7	56644.9	56749.6	0.2	56626.3
3	10	26	ID	34	20	9	20	21954.4	23807.7	8.4	6182.8
3	10	26	ID	34	20	9	7	23144.3	23788.3	2.8	20906.3
3	10	47	ID	32	20	9	20	30274.0	105632.8	248.9	0.0
3	10	47	ID	32	20	9	7	105450.4	105624.8	0.2	105417.0

Table 5.15: Matheuristic Iteration Metrics for First Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	IC	UIC	S1	S2	S3	S4	S5
1	5	7	TE	36	21	5	21	2	0	2	0	0	0	0
1	5	7	TE	36	21	5	7	4	0	4	0	0	0	0
1	5	26	TE	36	21	5	21	2	0	2	0	0	0	0
1	5	26	TE	36	21	5	7	4	0	4	0	0	0	0
1	5	47	TE	39	21	5	21	1	0	1	0	0	0	0
1	5	47	TE	39	21	5	7	2	0	2	0	0	0	0
1	10	7	TE	36	21	8	21	1	0	1	0	0	0	0
1	10	7	TE	36	21	8	7	3	0	3	0	0	0	0
1	10	26	TE	34	21	9	21	1	0	1	0	0	0	0
1	10	26	TE	34	21	9	7	3	0	3	0	0	0	0
1	10	47	TE	37	21	9	21	1	0	1	0	0	0	0
1	10	47	TE	37	21	9	7	2	0	2	0	0	0	0

Table 5.15 and Table 5.16 depict the number of total iterations and the number of iterations since the last improving iteration, along with the number of iterations

Table 5.16: Matheuristic Iteration Metrics for First Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	IC	UIC	S1	S2	S3	S4	S5
1	5	7	ID	36	21	5	21	2	0	2	0	0	0	0
1	5	7	ID	36	21	5	7	4	0	4	0	0	0	0
1	5	26	ID	36	21	5	21	2	0	2	0	0	0	0
1	5	26	ID	36	21	5	7	4	0	4	0	0	0	0
1	5	47	ID	39	21	5	21	1	0	1	0	0	0	0
1	5	47	ID	39	21	5	7	2	0	2	0	0	0	0
1	10	7	ID	36	21	8	21	1	0	1	0	0	0	0
1	10	7	ID	36	21	8	7	3	0	3	0	0	0	0
1	10	26	ID	34	21	9	21	1	0	1	0	0	0	0
1	10	26	ID	34	21	9	7	3	0	3	0	0	0	0
1	10	47	ID	37	21	9	21	1	0	1	0	0	0	0
1	10	47	ID	37	21	9	7	2	0	2	0	0	0	0

where each of the shaking moves took place for the first region. Obviously, the number of iterations are really low for all of the instances. As UIC and S_2 are 0 for all instances, the first shaking had been successful in all iterations. Furthermore, dependency of the matheuristic on initial solution is apparent across all instances.

Table 5.17 and Table 5.18 present the same iteration metrics with Table 5.15 and Table 5.16 for the second region. As discussed in mathematical model results, second region instances are easier than the first region instances. Since N, F and Veh are much lower than the first region instances, iteration counts are higher than that of first region results. It is apparent that IC and P are negatively correlated as time to build and solve each AS takes longer. For some of the instances matheuristic has shown a converging behavior whereas some of the instances with high number of vehicles have low UIC values.

Same iteration metrics as Table 5.15 and Table 5.16 are shown by Table 5.19 and Table 5.20 for third region instances. As in the first region, matheuristic has not performed many iterations to make use of its full potential. Therefore, outputs are highly dependent on the initial solutions.

Table 5.17: Matheuristic Iteration Metrics for Second Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	IC	UIC	S1	S2	S3	S4	S5
2	5	7	TE	14	6	5	6	134	1	66	52	9	6	1
2	5	7	TE	14	6	5	2	297	126	83	68	51	50	45
2	5	26	TE	12	6	5	6	1329	511	297	292	248	247	245
2	5	26	TE	12	6	5	2	569	336	133	120	107	108	101
2	5	47	TE	13	6	5	6	541	11	151	140	88	84	78
2	5	47	TE	13	6	5	2	547	251	131	119	103	100	94
2	10	7	TE	14	6	8	6	75	0	41	29	2	2	1
2	10	7	TE	14	6	8	2	177	44	55	44	29	28	21
2	10	26	TE	12	6	9	6	292	21	92	80	43	40	37
2	10	26	TE	12	6	9	2	324	85	77	71	59	59	58
2	10	47	TE	12	6	9	6	325	86	102	92	47	46	38
2	10	47	TE	12	6	9	2	320	180	86	70	58	58	48

Table 5.18: Matheuristic Iteration Metrics for Second Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	IC	UIC	S1	S2	S3	S4	S5
2	5	7	ID	14	6	5	6	155	3	74	59	12	10	0
2	5	7	ID	14	6	5	2	355	83	101	81	63	62	48
2	5	26	ID	12	6	5	6	1969	1561	524	514	466	9	456
2	5	26	ID	12	6	5	2	929	555	210	194	179	178	168
2	5	47	ID	13	6	5	6	627	52	185	178	123	31	110
2	5	47	ID	13	6	5	2	536	94	129	118	101	97	91
2	10	7	ID	14	6	8	6	78	1	46	32	0	0	0
2	10	7	ID	14	6	8	2	320	77	80	73	56	56	55
2	10	26	ID	12	6	9	6	243	5	81	69	34	34	25
2	10	26	ID	12	6	9	2	504	141	118	106	93	96	91
2	10	47	ID	12	6	9	6	678	74	197	185	138	32	126
2	10	47	ID	12	6	9	2	396	73	99	87	71	73	66

For the first region, Table 5.21 and Table 5.22 depict the objective values of matheuristic, their comparison with MILP's best primal and dual bounds, and the

Table 5.19: Matheuristic Iteration Metrics for Third Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	IC	UIC	S1	S2	S3	S4	S5
3	5	7	TE	33	20	5	20	2	0	2	0	0	0	0
3	5	7	TE	33	20	5	7	6	0	5	1	0	0	0
3	5	26	TE	35	20	5	20	2	0	2	0	0	0	0
3	5	26	TE	35	20	5	7	4	0	4	0	0	0	0
3	5	47	TE	32	20	5	20	3	0	3	0	0	0	0
3	5	47	TE	32	20	5	7	7	0	7	0	0	0	0
3	10	7	TE	30	20	8	20	3	0	2	1	0	0	0
3	10	7	TE	30	20	8	7	8	0	6	2	0	0	0
3	10	26	TE	34	20	9	20	1	0	1	0	0	0	0
3	10	26	TE	34	20	9	7	3	0	3	0	0	0	0
3	10	47	TE	32	20	9	20	2	0	1	1	0	0	0
3	10	47	TE	32	20	9	7	4	0	4	0	0	0	0

Table 5.20: Matheuristic Iteration Metrics for Third Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	IC	UIC	S1	S2	S3	S4	S5
3	5	7	ID	33	20	5	20	3	0	2	1	0	0	0
3	5	7	ID	33	20	5	7	6	0	5	1	0	0	0
3	5	26	ID	35	20	5	20	2	0	2	0	0	0	0
3	5	26	ID	35	20	5	7	4	0	4	0	0	0	0
3	5	47	ID	32	20	5	20	3	1	3	0	0	0	0
3	5	47	ID	32	20	5	7	7	0	7	0	0	0	0
3	10	7	ID	30	20	8	20	3	0	2	1	0	0	0
3	10	7	ID	30	20	8	7	7	1	6	1	0	0	0
3	10	26	ID	34	20	9	20	1	0	1	0	0	0	0
3	10	26	ID	34	20	9	7	3	0	3	0	0	0	0
3	10	47	ID	32	20	9	20	2	0	1	1	0	0	0
3	10	47	ID	32	20	9	7	4	0	4	0	0	0	0

percentage of cumulative time that VNS based matheuristic has spent to build and solve the allocation subproblem for a given tour. As discussed in mathematical

Table 5.21: Matheuristic Performance Comparison Metrics for First Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	DMM(%)	GBB(%)	TASS(%)	TASB(%)
1	5	7	TE	36	21	5	21	125349.7	-620.5	5.2	41.0	34.4
1	5	7	TE	36	21	5	7	25486.0	80.7	80.7	45.6	34.3
1	5	26	TE	36	21	5	21	80826.1	-1469.1	6.3	42.1	34.0
1	5	26	TE	36	21	5	7	23775.7	72.4	72.4	45.2	34.8
1	5	47	TE	39	21	5	21	179858.4	-968.1	2.9	43.4	33.6
1	5	47	TE	39	21	5	7	45157.2	72.7	75.6	46.4	33.9
1	10	7	TE	36	21	8	21	107792.5	-798.1	6.0	39.9	36.1
1	10	7	TE	36	21	8	7	19437.6	82.8	83.0	43.3	35.5
1	10	26	TE	34	21	9	21	61012.0	-1639.9	7.9	41.5	35.5
1	10	26	TE	34	21	9	7	20634.4	16.2	68.8	44.9	34.5
1	10	47	TE	37	21	9	21	178589.1	-inf	2.8	40.4	35.6
1	10	47	TE	37	21	9	7	45381.7	-46.3	75.3	42.9	35.9

Table 5.22: Matheuristic Performance Comparison Metrics for First Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	DMM(%)	GBB(%)	TASS(%)	TASB(%)
1	5	7	ID	36	21	5	21	125102.0	5.0	5.2	41.6	34.2
1	5	7	ID	36	21	5	7	21280.8	83.8	83.9	45.8	34.4
1	5	26	ID	36	21	5	21	80588.7	-21.1	6.3	42.5	34.0
1	5	26	ID	36	21	5	7	20903.0	75.7	75.7	45.7	34.4
1	5	47	ID	39	21	5	21	179359.7	-26.1	2.9	43.9	33.3
1	5	47	ID	39	21	5	7	47401.0	74.3	74.3	47.7	33.2
1	10	7	ID	36	21	8	21	107435.5	-65.9	6.0	40.8	35.6
1	10	7	ID	36	21	8	7	32093.1	71.9	71.9	44.5	34.8
1	10	26	ID	34	21	9	21	60881.2	-75.0	7.9	38.9	37.0
1	10	26	ID	34	21	9	7	22651.9	65.7	65.7	42.5	36.0
1	10	47	ID	37	21	9	21	178035.6	-191.8	2.8	40.6	35.4
1	10	47	ID	37	21	9	7	69019.7	59.6	62.3	43.5	35.5

model results, for instances with less vehicles, mathematical model has achieved nearly optimal solutions. Therefore, $DMM(\%)$ and $GBB(\%)$ are very close for such instances. However, for instances with higher number of vehicles matheuristic outperformed mathematical model with relatively low $GBB(\%)$ values. Despite best bound not being sensitive to the number of vehicles, matheuristic objective function values are positively correlated on available vehicles. $DMM(\%)$ figures uncover that matheuristic performance is more robust in general. As in the case with synthetic instances, building and solving AS is the most computationally expensive part of the matheuristic for all of the instances.

Table 5.23: Matheuristic Performance Comparison Metrics for Second Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	DMM(%)	GBB(%)	TASS(%)	TASB(%)
2	5	7	TE	14	6	5	6	90851.2	0.5	0.5	45.5	34.1
2	5	7	TE	14	6	5	2	91247.2	0.0	0.0	41.4	36.6
2	5	26	TE	12	6	5	6	73476.7	0.1	0.1	42.6	36.3
2	5	26	TE	12	6	5	2	73509.2	0.1	0.1	38.9	39.3
2	5	47	TE	13	6	5	6	76893.0	0.3	0.3	44.0	35.5
2	5	47	TE	13	6	5	2	73894.3	0.8	0.8	37.1	39.5
2	10	7	TE	14	6	8	6	83571.3	0.8	0.8	47.0	34.9
2	10	7	TE	14	6	8	2	84150.2	0.1	0.1	45.0	34.3
2	10	26	TE	12	6	9	6	34617.1	4.5	4.5	45.0	36.3
2	10	26	TE	12	6	9	2	30571.4	0.0	0.0	40.6	37.4
2	10	47	TE	12	6	9	6	69307.6	0.1	0.1	43.2	37.3
2	10	47	TE	12	6	9	2	69315.2	0.0	0.1	42.2	36.0

Table 5.24: Matheuristic Performance Comparison Metrics for Second Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	DMM(%)	GBB(%)	TASS(%)	TASB(%)
2	5	7	ID	14	6	5	6	90948.7	0.4	0.4	45.0	34.5
2	5	7	ID	14	6	5	2	91228.1	0.0	0.0	39.2	38.0
2	5	26	ID	12	6	5	6	73479.3	0.1	0.1	42.4	36.7
2	5	26	ID	12	6	5	2	73487.1	0.1	0.1	36.7	40.4
2	5	47	ID	13	6	5	6	76864.9	0.3	0.3	42.6	36.4
2	5	47	ID	13	6	5	2	75161.1	0.0	0.1	35.7	40.3
2	10	7	ID	14	6	8	6	83568.6	0.8	0.8	46.6	35.1
2	10	7	ID	14	6	8	2	84180.8	0.1	0.1	43.1	35.8
2	10	26	ID	12	6	9	6	37373.2	0.4	0.4	43.2	37.3
2	10	26	ID	12	6	9	2	34706.5	0.2	0.2	40.0	38.0
2	10	47	ID	12	6	9	6	69276.2	0.1	0.1	42.3	38.2
2	10	47	ID	12	6	9	2	69298.7	0.0	0.0	39.6	37.8

Table 5.23 and Table 5.24 present the same metrics as Table 5.21 and Table 5.22 for the second region. As discussed in mathematical model results, MILP has been able to come up with solutions that have practically negligible gap percentages.

Moreover, deviation values are always positive but very close to zero in both demand scenarios. This signals that matheuristic can provide reliable results for small instances. Furthermore, objective values tend to increase as products are aggregated in hierarchical fashion.

Table 5.25: Matheuristic Performance Comparison Metrics for Third Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	DMM(%)	GBB(%)	TASS(%)	TASB(%)
3	5	7	TE	33	20	5	20	93099.0	-407.6	4.8	41.1	34.7
3	5	7	TE	33	20	5	7	25973.1	73.4	73.4	44.5	35.5
3	5	26	TE	35	20	5	20	79277.9	-123.2	7.8	40.9	35.0
3	5	26	TE	35	20	5	7	33049.0	61.5	61.5	44.8	35.0
3	5	47	TE	32	20	5	20	69535.3	-3296.4	24.7	41.8	34.3
3	5	47	TE	32	20	5	7	26066.0	64.3	64.4	45.7	34.8
3	10	7	TE	30	20	8	20	53293.0	-3450.0	5.8	38.0	37.0
3	10	7	TE	30	20	8	7	13409.0	76.2	76.3	42.9	36.2
3	10	26	TE	34	20	9	20	16067.3	-inf	32.3	37.9	37.2
3	10	26	TE	34	20	9	7	2332.9	88.9	90.2	40.9	36.6
3	10	47	TE	32	20	9	20	101151.4	-inf	3.9	39.6	36.2
3	10	47	TE	32	20	9	7	31825.5	69.5	69.8	43.1	35.9

Table 5.26: Matheuristic Performance Comparison Metrics for Third Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	Obj	DMM(%)	GBB(%)	TASS(%)	TASB(%)
3	5	7	ID	33	20	5	20	93305.4	-2.7	4.8	41.1	34.6
3	5	7	ID	33	20	5	7	25405.8	74.1	74.1	45.0	35.0
3	5	26	ID	35	20	5	20	79502.4	6.4	7.8	41.3	34.6
3	5	26	ID	35	20	5	7	23398.6	72.8	72.9	45.2	34.8
3	5	47	ID	32	20	5	20	86998.4	-52.9	4.7	41.8	34.3
3	5	47	ID	32	20	5	7	32960.7	57.3	57.3	45.9	34.7
3	10	7	ID	30	20	8	20	53442.5	5.4	5.9	38.1	36.9
3	10	7	ID	30	20	8	7	22573.5	60.1	60.2	42.7	36.2
3	10	26	ID	34	20	9	20	16128.9	26.5	32.3	36.7	38.0
3	10	26	ID	34	20	9	7	5202.9	77.5	78.1	40.2	37.0
3	10	47	ID	32	20	9	20	101482.0	-235.2	3.9	38.8	36.6
3	10	47	ID	32	20	9	7	46372.9	56.0	56.1	42.8	36.0

Same performance comparison metrics as Table 5.21 and Table 5.22 are given in Table 5.25 and Table 5.26 for third region instances. Similar to the first region results, the proposed matheuristic outperformed the mathematical model for the instances where number of vehicles are higher. For those instances, gap percentages with best dual bound of mathematical model are relatively low. As in the case of synthetic instances, it is not straightforward whether best bounds are not tight enough or matheuristic performance is not satisfactory. Cumulative percentage of AS solve and build times add up to a very high percentage of the total time of matheuristic for both demand scenarios.

As in synthetic instances, two different approaches to generate solutions for real life instances are analyzed. Yet, a seperate contribution of this study to generate dual bounds (namely, ALUB) need to be analyzed. Therefore, outputs of ALUB are compared with the dual bounds obtained by MILP. As in the case of synthetic instances, it would be expected that ALUB provides solutions in short amount of time due to the linear structure. An analysis of the comparison is provided in two different dimensions, namely, tightness and time.

Table 5.27: ALUB Performance Comparison Metrics for First Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	ALUB Obj	ALUB Time (s)	Dev(%) BB-ALUB
1	5	7	TE	36	21	5	21	132237.3	1.0	0.1
1	5	7	TE	36	21	5	7	132237.3	0.3	0.1
1	5	26	TE	36	21	5	21	86321.4	1.0	0.1
1	5	26	TE	36	21	5	7	86321.4	0.4	0.1
1	5	47	TE	39	21	5	21	185315.3	1.0	0.1
1	5	47	TE	39	21	5	7	185315.3	0.3	0.1
1	10	7	TE	36	21	8	21	114683.7	1.5	0.1
1	10	7	TE	36	21	8	7	114683.7	0.5	0.1
1	10	26	TE	34	21	9	21	66298.6	1.7	0.1
1	10	26	TE	34	21	9	7	66298.6	0.6	0.1
1	10	47	TE	37	21	9	21	183739.6	1.8	0.0
1	10	47	TE	37	21	9	7	183739.6	0.6	0.1

Table 5.27 and Table 5.28 depict the objective values and solution time of ALUB along with deviation of ALUB objective from best dual bound found by mathematical model in 900 seconds. LP based ALUB came up with solutions within a few seconds for all instances. With one exception in each demand scenario, deviation

Table 5.28: ALUB Performance Comparison Metrics for First Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	ALUB Obj	ALUB Time (s)	Dev(%) BB-ALUB
1	5	7	ID	36	21	5	21	131982.5	0.9	0.0
1	5	7	ID	36	21	5	7	131982.5	0.3	0.1
1	5	26	ID	36	21	5	21	86081.9	0.9	0.0
1	5	26	ID	36	21	5	7	86081.9	0.3	0.1
1	5	47	ID	39	21	5	21	184827.8	1.0	0.0
1	5	47	ID	39	21	5	7	184827.8	0.4	0.0
1	10	7	ID	36	21	8	21	114327.0	1.5	0.0
1	10	7	ID	36	21	8	7	114327.0	0.6	0.0
1	10	26	ID	34	21	9	21	66159.1	1.7	0.1
1	10	26	ID	34	21	9	7	66159.1	0.6	0.1
1	10	47	ID	37	21	9	21	183226.2	1.8	0.0
1	10	47	ID	37	21	9	7	183226.2	0.6	0.0

values are practically negligible. Therefore, ALUB provides relatively tight bounds within a very short amount of time.

Table 5.29: ALUB Performance Comparison Metrics for Second Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	ALUB Obj	ALUB Time (s)	Dev(%) BB-ALUB
2	5	7	TE	14	6	5	6	91525.0	0.0	0.2
2	5	7	TE	14	6	5	2	91525.0	0.0	0.3
2	5	26	TE	12	6	5	6	73658.4	0.1	0.1
2	5	26	TE	12	6	5	2	73658.4	0.0	0.1
2	5	47	TE	13	6	5	6	77250.2	0.0	0.1
2	5	47	TE	13	6	5	2	77250.2	0.0	3.5
2	10	7	TE	14	6	8	6	84382.9	0.1	0.1
2	10	7	TE	14	6	8	2	84382.9	0.0	0.1
2	10	26	TE	12	6	9	6	37981.6	0.1	4.5
2	10	26	TE	12	6	9	2	37981.6	0.0	19.5
2	10	47	TE	12	6	9	6	69444.4	0.1	0.1
2	10	47	TE	12	6	9	2	69444.4	0.0	0.1

Table 5.29 and Table 5.30 show the same metrics as Table 5.27 and Table 5.28 for the second region. Clearly, Veh has no impact on the objective values as solving allocation to optimality without considering any routing related constraints and costs is independent from the number of vehicles. According to objective values,

Table 5.30: ALUB Performance Comparison Metrics for Second Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	ALUB Obj	ALUB Time (s)	Dev(%) BB-ALUB
2	5	7	ID	14	6	5	6	91470.8	0.1	0.2
2	5	7	ID	14	6	5	2	91470.8	0.0	0.2
2	5	26	ID	12	6	5	6	73608.1	0.0	0.1
2	5	26	ID	12	6	5	2	73608.1	0.0	0.1
2	5	47	ID	13	6	5	6	77203.3	0.0	0.1
2	5	47	ID	13	6	5	2	77203.3	0.0	2.6
2	10	7	ID	14	6	8	6	84354.2	0.1	0.1
2	10	7	ID	14	6	8	2	84354.2	0.0	0.1
2	10	26	ID	12	6	9	6	37969.6	0.1	1.2
2	10	26	ID	12	6	9	2	37969.6	0.0	8.4
2	10	47	ID	12	6	9	6	69401.8	0.1	0.1
2	10	47	ID	12	6	9	2	69401.8	0.0	0.1

breaking down product groups into various subgroups (hence, increasing the number of product types) decreases the optimal allocation objective function values.

Table 5.31: ALUB Performance Comparison Metrics for Third Region of Real Instances in Total Equality Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	ALUB Obj	ALUB Time (s)	Dev(%) BB-ALUB
3	5	7	TE	33	20	5	20	97865.4	0.8	0.1
3	5	7	TE	33	20	5	7	97865.4	0.4	0.1
3	5	26	TE	35	20	5	20	86041.7	0.9	0.1
3	5	26	TE	35	20	5	7	86041.7	0.3	0.1
3	5	47	TE	32	20	5	20	92470.0	0.9	0.2
3	5	47	TE	32	20	5	7	92470.0	0.3	20.8
3	10	7	TE	30	20	8	20	56676.5	1.3	0.2
3	10	7	TE	30	20	8	7	56676.5	0.5	0.2
3	10	26	TE	34	20	9	20	23828.4	1.5	0.4
3	10	26	TE	34	20	9	7	23828.4	0.6	0.5
3	10	47	TE	32	20	9	20	105340.7	1.6	0.1
3	10	47	TE	32	20	9	7	105340.7	0.6	0.1

For the third region, Table 5.31 and Table 5.32 illustrate the same metrics with Table 5.27 and Table 5.28. As discussed in the first region results, ALUB takes a very low amount of time to solve, yet rarely come up with significantly looser bounds than mathematical model best bounds. Since demand scenario alters demand percent-

Table 5.32: ALUB Performance Comparison Metrics for Third Region of Real Instances in Infinite Demand Scenario.

Reg	TPP	Week	DS	N	F	P	Veh	ALUB Obj	ALUB Time (s)	Dev(%) BB-ALUB
3	5	7	ID	33	20	5	20	98079.3	0.9	0.0
3	5	7	ID	33	20	5	7	98079.3	0.3	0.1
3	5	26	ID	35	20	5	20	86298.1	0.8	0.1
3	5	26	ID	35	20	5	7	86298.1	0.4	0.1
3	5	47	ID	32	20	5	20	92765.2	0.9	1.6
3	5	47	ID	32	20	5	7	92765.2	0.3	16.7
3	10	7	ID	30	20	8	20	56824.6	1.3	0.1
3	10	7	ID	30	20	8	7	56824.6	0.5	0.1
3	10	26	ID	34	20	9	20	23885.2	1.5	0.3
3	10	26	ID	34	20	9	7	23885.2	0.5	0.4
3	10	47	ID	32	20	9	20	105674.9	1.5	0.0
3	10	47	ID	32	20	9	7	105674.9	0.6	0.0

ages, it has an influence on the structure of equality objective. However, although the optimal allocations vary across the demand scenarios, the difference between optimal objective values are relatively low.

Table 5.33: Analysis of Various Weight Configurations Across Three Instances in Three Regions.

Reg	TPP	Week	DS	N	F	P	Veh	w1	w2	w3	w4	Obj
1	5	47	ID	39	21	5	7	0.3	0.2	0.3	0.2	184530.5
1	5	47	ID	39	21	5	7	0.5	0	0.3	0.2	309535.7
2	5	47	TE	13	6	5	6	0.3	0.2	0.3	0.2	77141.5
2	5	47	TE	13	6	5	6	0.3	0.3	0.4	0	77081.0
3	5	26	ID	35	20	5	7	0.3	0.2	0.3	0.2	86118.3
3	5	26	ID	35	20	5	7	0.1	0.05	0.45	0.4	28473.4

The extensive computational study on both real and synthetic instances has been presented so far. In all of the experiments, weights of each function in the objective function has been set to 0.3, 0.2, 0.3, 0.2 in the respective order. Yet as a separate case study, the effect of different weight configurations on the objective function (with trivial optimality gap) is given in Table 5.33. It can be observed that objective functions are highly impacted by the weight of the first function. The influence of other functions on the objective function value is not as high as the first function since altering their weights do not change the objective function value as much as the first function. This is also relevant for the real life insights as the main target of food banking operations are to rescue as much food as possible.

Chapter 6

CONCLUSION

In this thesis, the aim was to maximize the amount of food rescued from being wasted while enabling fair allocation of nutritious food and reducing the carbon emissions in food banking operations. This aim is motivated by the issues that are related to three Sustainable Development Goals (SDG) of UN, namely "Zero Hunger", Consumption and Production"and Action". To achieve these goals simultaneously, a novel problem structure, namely Multi Depot - Periodic Unpaired Pickup and Delivery Vehicle Routing Problem (MD-PUPDVRP) is raised in the study. In this problem, multiple product types along with multiple vehicle compartments with different temperatures are considered. A detailed analysis on different functions to quantify fairness is presented. The appropriate fairness function is developed in alignment with real life scenarios. In order to come up with an estimated fuel consumption (thus carbon emission), an adaptation of a widely used carbon emission function to transporter type vehicles is made after analyzing empirical data.

As solution methodologies, a Mixed-Integer Linear Programming based mathematical model is developed. To compete with mathematical model, a matheuristic based on a Variable Neighborhood Search which is in sync with a Linear Programming based Allocation Subroutine is implemented. In the matheuristic, VNS conducts a systematic search in routing solution space while AS solves allocation to optimality (and generates objective value) at every solution that is visited by VNS. Since the developed matheuristic methodology is a trajectory based method, a construction heuristic is also devised to generate the initial solutions. To generate dual bounds immediately, another LP model that takes advantage of the decomposable structure of the problem is introduced.

To conduct the experiments, both real life and synthetic instances are created. The real life data is cleaned, aggregated and formatted as instances to generate real instances. Synthetic instances are constructed by inferring the statistical distributions of parameters from real life data. In the experimentation phase, it is presented that when solver heuristic generates a solution of high quality, the optimality gap decreases a lot. However, solver heuristic often fails to provide nontrivial solutions for large instances. In the large instances, mathematical model generates worse solutions when there are more vehicles. For small instances (second region) where mathematical model closes the optimality gap, the matheuristic is also able to provide solutions of very high quality. For large instances where the number of nodes and number of vehicles are both high, the matheuristic outperforms the MILP model. An analysis on the cumulative time matheuristic spent on each step, it is revealed that building and solving AS takes very high percentage of time. Thus, in some of the instances number of iterations are not at a satisfactory level and initialization independency cannot be assumed. It is also presented that although ALUB never outperforms mathematical model's best bound by providing tighter dual bounds, it generates bounds of similar tightness in a few seconds (as opposed to 900 seconds). A case study on the effects of different objective function weights on the total output has been presented. According to the results, maximizing the total amount rescued food has the most impact on the objective function. As high percentage of cumulative times are obtained while building and solving AS, another heuristic methodology can be implemented in the future studies at the expense of optimality in allocation. In future work, another scenario (leading to a slightly different problem structure) where cooperation between food banks is disregarded can be studied. Moreover, difference between solutions of scenarios would highlight the impact of cooperation towards humanitarian act. While this thesis provides a range of solution methodologies to the novel problem of MD-PUPDVRP, further research may uncover different approaches that result in further improvements.

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