

A CAPACITATED AND DISTANCE-LIMITED CONTINUOUS LOCATION-ALLOCATION PROBLEM FOR SPATIAL PLANNING OF DECENTRALIZED DISTRIBUTION SYSTEMS

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A Capacitated and Distance-Limited Continuous Location-Allocation
Problem for Spatial Planning of Decentralized Distribution Systems

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September 2017

We certify that we have read this thesis and that in our opinion it is fully adequate,
in scope and in quality, as a thesis for the degree of Master of Science.



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ABSTRACT

A CAPACITATED AND DISTANCE-LIMITED CONTINUOUS LOCATION-ALLOCATION PROBLEM FOR SPATIAL PLANNING OF DECENTRALIZED DISTRIBUTION SYSTEMS

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We introduce a new continuous location-allocation problem with capacitated and distance-limited facilities for spatial planning of decentralized distribution systems to contribute the greenfield development. Our aim is to determine the number, location and allocation configurations of facilities to be opened as stand-alone systems such as solar panels, windmills, diesel generators, water pumps and etc. The problem minimizes the sum of fixed facility opening costs and distribution costs such as cable, pipeline or transportation costs which are linearly dependent to their physical lengths. To be able to model some physical facts such as energy loss, voltage drop, pressure drop and etc., facilities to be opened are assumed to be distance limited.

Due to the fact that greenfield areas are undeveloped regions and wide open spaces, where generally no previous structures exist, facilities can be located at almost any site in the field. Therefore, the domain of the problem is not restricted by the discrete space and it is subject to the continuous space. We provide a quadratically constrained mixed integer programming model and a heuristic approach for the problem. In the heuristic, firstly discrete counterpart of the problem is solved with augmented set of candidate location points. Alternative augmented sets of candidate locations are formed in accordance with the spatial distribution of demand points. After solving the discrete counterpart of the problem with augmented domain, Cooper's well known heuristic that alternates between allocation and location steps is followed to find better allocation configuration of customers to the facilities.

Since the facilities considered in the problem are capacitated, some of the demand points may not be assigned to their nearest facility. Therefore, allocation steps are performed with respect to the priority measure of each demand point based on a regret function. After each allocation step, to determine the new locations of the facilities, Weiszfeld's iterative convergence algorithm is applied with some modifications and adjustments to conform the distance limitation constraints.



Keywords: Distribution network planning, continuous multi-facility location, Greenfield development, decentralized distribution systems, multi-stage heuristic, quadratic mixed integer programming.

ÖZET

MERKEZİ OLMAYAN DAĞITIM SİSTEMLERİNİN SÜREKLİ UZAYDA KONUMSAL PLANLAMASI İÇİN KAPASİTE VE MESAFE KISITLI YER SEÇİMİ PROBLEMİ

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Gelişmemiş bölgelerin kalkınmasına katkıda bulunmak amacıyla, merkezi olmayan dağıtım sistemlerinin konumsal planlaması için sürekli uzayda kapasite ve mesafe kısıtlı tesislerle yer seçimi ve ataması problemi sunulmuştur. Güneş paneli, rüzgar gülü, dizel jeneratör, su devir daim pompası gibi şebeke bağımsız olarak açılacak tesislerin sayısı, yerseçimi ve atama yapılarılarına karar verilmesi amaçlanmaktadır. Problem sabitleştirilmiş tesis kurma maliyetleri ile taşıma, nakliye, kablo, boru hattı gibi fiziksel uzunluklara doğrusal bağımlı olan dağıtım maliyetlerinin toplamını en küçükmektedir. Enerji kaybı, voltaj düşümü, basınç düşümü gibi fiziki gerçeklerin modellenebilmesi için, açılacak tesisler mesafe kısıtlı olarak addedilmiştir.

Söz konusu gelişmemiş bölgeler, fiziksel yapıların mevcut olmadığı işlenmemiş geniş açık alanlar olduğu için, tesisler sahada bulunan noktaların neredeyse tamamında konumlandırılabilirler. Bu sebeple problemin tanım kümesi kesikli uzayla kısıtlı değildir ve sürekli uzaya tabidir. Problem için karesel sınırlı karışık tam-sayı programlama modeli ve sezgisel yaklaşım önerilmiştir. Bu sezgisel yaklaşımda ilk olarak, problemin kesikli uzayda tanımlı eş benzeri, arttırılmış aday yerleşim noktaları kümesi ile çözülmüştür. Alternatif yöntemlerle arttırılan aday yerleşim noktaları kümesi, talep noktalarının konumsal dağılımına göre oluşturulmuştur. Problemin kesikli uzaydaki eş benzeri, büyütülmüş tanım kümesi ile çözüldükten sonra; daha iyi atama yapıları elde etmek için Cooper'in atama ve yer seçimi

adımlarını deęişimli olarak ve sırayla yaptığı, tanınmış yinelemeli sezgisel yaklaşımı uygulanmıştır. Tesisler kapasite kısıtlı olarak ele alındıkları için bazı talep noktaları kendilerine en yakın uzaklıktaki tesise atanamayabilirler. Bu sebeple atama adımları pişmanlık fonksiyonuna dayalı öncelik ölçütlerine istinaden yürütölmektedir. Her bir atama adımından sonra, tesislerin güncel konumlarına karar vermek için, Weiszfeld'in yinelemeli yakınsama algoritması, mesafe sınırlı kısıtlara uymak için bir takım uyarlama ve düzenlemelere tabi olarak uygulanmıştır.



Anahtar sözcükler: Dağıtım ağı planlaması, sürekli uzayda çoklu tesis yerseçimi, gelişmemiş bölgelerin kalkındırılması, merkezi olmayan dağıtım sistemleri, çok seviyeli sezgisel, karesel karışık tamsayı programlama.

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Chapter 1

Introduction

Location analysis is a fundamental branch of Operations Research concentrated on determining the best placement configuration of facilities in the defined domain of the problem. Objective function to be minimized generally consists of transportation and facility costs.

Location and allocation problems are crucial for spatial planning of source distribution systems to have cost effective infrastructures. The domain of most of the problems is basically restricted to discrete space where the candidate set of location points are subject to the set of known demand (customer) points of the problem. However, in the problems studied in the continuous space, new sites are generated for locating the facilities in addition to the set of existing points. In fact, facilities can be located anywhere in the continuous space as far as the location is physically appropriate and empty, whereas in the discrete site selecting problems, candidate sites are determined in advance. Thus, the continuous location problems are referred as “site generating” problems [1].

In this thesis, we study on a site generating location and allocation problem that

enables the distribution and transportation of any product, resource or data from capacitated facilities to the end points (customers) for the spatial planning of resource distribution systems. Location, number and allocation configuration of facilities are to be determined where a facility can be thought as a stand-alone system such as solar panel, wind turbine, diesel generator and etc. Our objective is to minimize the sum of distribution and fixed facility opening costs. Additional insights are associated with the ordinary characteristics of common site generating (continuous space) problems to find out better estimates for contributing greenfield development. Hence, the problem we propose is newly introduced.

Unlike the most of the problems in the literature, in addition to the distribution costs, a major cost component of fixed opening costs of facilities are inserted to the objective function of our problem to make better cost effective assessment. Thereby, the number of facilities to be opened is not assumed to be known a priori and subject to decision as well. Accordingly, a trade-off occurs between distribution (transportation) costs and fixed opening costs. For this reason, in addition to location and allocation configurations, the number of facilities needed, accordingly the best trade-off between the connection and fixed opening costs is also left to the decision maker with the inclusion of fixed opening costs.

Connections starting from the capacitated facilities enable the distribution of data, information, resource, energy or whatever needed to the end demand points. Distribution scheme is subject to the star configuration. Although similar studies on facility location and network design problems are present in the literature, a good part of them are performed on discrete site-selecting problems. In these problems, the candidate set of location points forms a set having finite number of elements. However, since facility location on continuous space would relax this constraint, the feasible region is enlarged and the solution would be improved accordingly. For instance, unlike the urban environment, greenfield contains so many wide open spaces and undeveloped regions where facilities can be located at almost any site in the field.

Working on a wider domain would certainly enable to enlarge the feasible region to set up more cost effective infrastructures.

Another point to consider is that in order to be able to model some physical facts such as energy loss, voltage drop, pressure drop and etc., facilities to be opened are assumed to be distance limited. That is, in addition to the upper bound associated with the amount of service/source it could provide (capacitated facilities), there is another limitation associated with the spatial coverage of a facility and it can only serve the demand points within a specified distance. For instance, it is possible to find distance limitation of electricity distribution when the amount of periodic demand, load factor, line voltage, permitted voltage drop and cable type is known. This limitation can be obtained by the straightforward electrical engineering calculations and linear operations.

Beyond the fact that there is only limited number of problems studied in the continuous facility location and network design literature, there is not any similar problem studied with capacitated and unknown number of facilities where physical loss conditions are modeled with coverage distance constraints.

In this context, our motivation is to find out strong estimates for the cost effective spatial planning of decentralized distribution systems in site generating environment to contribute the development of greenfield infrastructures. In the light of these, our aim is to determine the number, location and allocation configurations of facilities to be opened as stand-alone systems such as solar panels, windmills, diesel generators or water pumps [2]. Definitely, location and allocation decisions are made in accordance with the spatial distribution of demand points spread over the plane. Assuming the fact that energy or resource availability is adequate, our objective is to minimize the sum of fixed facility opening costs and distribution costs such as cable, pipeline costs which are linearly dependent to their physical lengths.

To consider the mentioned trade-off and to make a cost effective network design, problem is mathematically modeled subject to the specified constraints. Due to the fact that the formulated model for the problem which is quadratically constrained mixed integer, can be reduced to well-known NP Hard problems, heuristic approaches are developed and the solutions obtained by these approaches are compared. In order to attain strong estimates, we propose multi stage heuristic framework with three different approaches by combining the well accepted tools of location literature with appropriate tailoring. Firstly, to determine the domain of the problem, we either augment or keep the set of candidate locations. To augment the set of candidate locations forming the domain of the problem, we either insert smart points or circular points which are determined in accordance with the spatial distribution of demand points. Hence, the approaches differ in the augmentation stage. Secondly, discrete counterpart of the problem is solved with augmented or non augmented set of candidate location points. After solving the discrete counterpart of the problem with augmented or original domain, in the final stage, Cooper's well known heuristic [3], [1] that alternates between allocation and location steps is followed adaptively to find better allocation configuration of customers to the facilities. Considering the fact that facilities are assumed to be capacitated and some of the demand points could not be assigned to their nearest facility, allocation steps are performed with respect to the priority value of each demand point. After each allocation step, to determine the new locations of the facilities, Weiszfeld's iterative convergence algorithm [4], [5] is applied with some modifications and adjustments to conform the distance limitation constraints. Location and allocation steps of Cooper's heuristic is revisited with the same adaptations until no further improvement is possible and no further change occurs at least in the location of one facility.

To our best knowledge, a capacitated continuous location and allocation problem with distance limited service constraints and fixed facility opening costs has not been considered in the literature yet. Hopefully, developing heuristic algorithms that could enable cost effective distribution network design, would also contribute

to the solution of other continuous location problems aiming to determine the number, service allocation configuration and location decisions in communication, fiber optic, energy distribution, computer, electrical, transportation and military logistics networks/grids.

The rest of this thesis is organized as follows. In Chapter 2, we review the most related studies present in both discrete and continuous location literature. In Chapter 3, we introduce the “Capacitated Single Source Multifacility Continuous Location Allocation Problem With Distance Limited Service Constraints For Spatial Planning Of Decentralized Distribution Systems”. The problem is mathematically formulated as a quadratically constraint mixed integer model. In Chapter 4, we propose our multi-stage heuristic framework with three different approaches to solve the problem. We performed numerical analysis along with the computational results in Chapter 5 and compare the approaches with respect to their cost effectiveness. In Chapter 6, we summarize the contributions and propose future work to be performed as extensions of this study.

Chapter 2

Literature Review

The problem that we introduce to the literature is related some well known problems studied in both discrete and continuous facility location and distribution planning literature. Because Capacitated p -median Problem (CPMP) and Capacitated Clustering Problem (CCP) are strongly related and nearly define the same set of characteristics, the works conducted upon these problems are clustered under the same section. In the latter section, proposed works specifically discussing continuous location and allocation problems are surveyed. In the last section, papers discussing the different extensions of capacitated continuous location and allocation problem are presented.

2.1 Survey of Discrete Location & Allocation Problems

The studies conducted on CPMP and CCP constitute the large part of the capacitated facility location literature. Both problems are discrete site-selecting problems as they are defined in the discrete domain and the candidate set of location points are restricted to the set of demand points.

p -median problem is one of the widely studied problems in the facility location literature, where p facilities are to be located among the set of n given candidate points. The aim is to minimize the total distance between each customer and the corresponding facility that it is assigned to. When each candidate facility to be opened has a determined capacity; that is, if there is an upper bound for the service it can supply, the problem is called capacitated p -median problem (CPMP).

Another related combinatorial optimization problem to consider is the Capacitated Clustering Problem (CCP) in which a set of objects are to be clustered in accordance with their similarities. Basically, a set of n objects are to be partitioned in k distinct groups, which are subject to a known capacity. As in CPMP, each object has to be assigned to a single cluster. Each cluster has a predetermined capacity that should not be exceeded by the overall weight of the objects assigned to it. The center (facility) which has a limited service capacity is one of the objects (customers) that belong to the cluster. The problem is to determine the set of centers (facilities) to fulfill the known weight (demand) of objects (customers) while minimizing the total sum of dissimilarities (distances) between each object and the center of the cluster that it belongs. As Osman and Christofides summarized in [6], clustering of the objects are needed in diverse fields such as biology, economics, marketing, pattern recognition, signal processing and etc. CPMP is the equivalent of CCP in location literature as the candidate set of location points are restricted to the demand point

set.

The Capacitated Plant Location Problem (CPLP) has similar characteristics with the CPMP and CCP. CPLP is to determine the best placement configuration of the facilities and the assignment configuration of the demand of each customer ensuring the capacity constraints of the facilities while minimizing the total transportation and fixed costs. Demand to be fulfilled for each customer can be split among the facilities in CPLP, whereas it must exactly be supplied by a single facility in CPMP and CCP. If there is a restriction in CPLP that each customer must be served by only one facility, Capacitated Plant Location Problem with Single Source Constraints (CPLPSS) is obtained [7]. When the capacity constraint is relaxed, so called Simple Plant Location Problem (SPLP) is obtained. Clearly, CPMP and CCP are special forms of CPLP as fixed opening costs are not regarded and single source constraints are added.

Stefanello et al. for solving CPMP in [8], proposed matheuristic approaches that eliminate some variables possibly do not contribute much to good solutions to reduce the size of the mathematical models. “Iterated Reduction Matheuristic Algorithm” (IRMA) is a three stage hybridized algorithm in which so called size-reduced models are iteratively exposed to partial optimization algorithm and solved as subproblems.

An hybrid algorithm is proposed to solve CPMP by Kaveh et al. in [9] where K -means algorithm is followed by fixed neighborhood search (FNS) algorithm or its variants. Basically, FNS algorithm improves the solutions of K -means algorithm with facility location exchanges and exclusion of unsuitable candidate locations. After removing specific number of facilities from the solution of K -means algorithm, the same number of new facilities are generated and random local search is performed to obtain better solutions. Proposed algorithm is compared with other algorithms such as variable neighborhood search (VNS) algorithm of Fleszar and Hindi [10], in which lower bounds are easily found without any computational burden, scatter

search (SS) and path relinking (PR) of Diaz and Fernandez [11].

A column generation approach is presented for solving CPMP by Lorena and Senne [12]. New columns are generated to solve 1-median capacitated subproblems which consider the dual variables of the master problem and cluster capacities. Lagrangean and surrogate relaxation is carried out on master problem which provides new bounds and new columns via individual and modified knapsack problems. Computational results are presented for the real data of Sao Jose dos Campos City collected via geographic information systems.

A set partitioning approach is proposed by Baldacci et al. in [13] as exact algorithm for solving CPMP. Two different heuristics are associated to obtain reasonable lower bounds for the LP relaxation of the original model. Dual solution of LP relaxation is used to reduce problem to set partitioning problem that can be solved by an IP solver.

Mulvey and Beck, firstly introduced CCP in [14] by adding capacity constraints to the uncapacitated clustering problem (UCP). They developed two heuristic algorithms. In the first method, customers are assigned to p randomly generated initial facility configuration in the decreasing order of their regret values. Regret value is defined as the absolute difference in distances of each customer to the first and second nearest facilities that are not fully loaded. The customers having higher regret values are more privileged in being assigned to their closest facility. After allocation of each customer, location of facilities are recalculated so that allocation step is repeated for the newly calculated locations of the facilities. This procedure continues until no further change occurs in the locations of the facility configuration. In subgradient algorithm, the lagrange relaxation of CCP is solved to obtain facility locations. When assignments are not feasible to the original problem, heuristic attempts to find feasible assignment of objects by solving series of integer knapsack problems, considering all candidate points and solutions of these knapsack problems to obtain

improved lower bounds.

K -means algorithm is proposed by Geetha et al. in [15] to solve CCP. Basically, objects (demand points) are clustered based on the Euclidean distance that determines the dissimilarity among objects. Analogous with Cooper's [3] popular heuristic alternating between location and allocation steps; in this algorithm k points are randomly generated, each being a center of cluster. Each demand point i , is assigned to the closest possible center with a priority value $P_i = cost_{ij}$, where $cost_{ij}$ represents the distance between i^{th} customer and j^{th} center. Locations of the centers of the clusters are recalculated and location and allocation steps are repeated until no further improvement is possible according to a convergence criterion. K -Means Clustering Algorithm is then improved with a new assignment criterion in a way that assignment priority of the requesters is updated as $P_i = cost_{ij}/D_i$ to sort them based on maximum demand and minimum distance order, where D_i is the demand of the i^{th} customer.

Another hybrid algorithm is developed by Osman and Christofides [6] to solve CCP. After generating initial solution with K -means algorithm, an interchange mechanism is operated where a neighborhood search is applied for better solutions. A hybrid simulated annealing (SA) and tabu search (TS) algorithm is developed with systematic neighborhood search and new non-monotonic cooling schemes such as stepwise reduction, continuous reduction, non-monotonic reduction of temperature, whereas classical SA applications use monotonic cooling schemes and random neighborhood search.

A population based scatter search heuristic for the CCP is proposed by Scheuerer and Wendolsky in [16]. Algorithm basically stores solutions in the reference set and generate solutions by combining these. According to the flow of the algorithm, initial solutions are generated and improved. Considering these promising solutions as seeds, new diversified solutions are created. The solutions obtained so far are

improved or eliminated from the replicas, to form the reference set. After reference set is formed, search is carried out with respect to a stopping criterion. If the reference can reach a state that it could not have a new solution anymore, algorithm converges. Results obtained from benchmark instances show that the heuristic is one of the best known methods developed so far.

An adaptive tabu search (TS) algorithm is applied to solve CCP by França et al. in [17]. Initial solutions are generated by four different constructive heuristics and two neighbor generation methods are used for local search. To increase the performance of the TS, two main strategies, intensification and diversification, are used. Intensification enables to direct movements towards the regions where historically better solutions are yielded whereas diversification enables to direct movements towards unexplored regions. The adaptive TS algorithm makes the search to automatically adapt the required one of these phases. Basically, TS implementation adjusts itself whether to carry out intensification for the promising regions if ongoing improvement is present or diversification to explore unattended regions if an improvement does not appear.

A clustering based approach is developed by Liao and Guo in [18] for a variant of capacitated facility location problem to determine location of facilities and allocation of large number of demands to the facilities. Unlike the classical capacitated problems in the literature, the allocation of demand points is constrained by a different capacity limitation; that is, allocation is subject to the capacities of different type of facilities. Besides, it considers not only the sufficient capacity problems but it also make assessment for insufficiently capacitated problems. Even if the overall capacity is not sufficient for the total demand, a solution is suggested with a maximum demand coverage scenario. The proposed approach is basically established upon the common K -means algorithm with some newly introduced modifications and presented as a suite of heuristics. They compared this algorithm against adapted genetic algorithm and observed that comparable and even better solutions are obtained.

Berman and Yang considered two medi-centre location problems in [19] where the first problem is the extension of p -median problem with additional distance constraints and the second problem is another extension of p -median problem in which they also include fixed costs of opening the facilities in addition to distance constraints. Thus in the second problem they considered, number of facilities is unknown and left to decision maker as well. Basically, the objective of the medi-centre problem is to locate a given number of facilities so as to minimize the total distance of demand points to the facilities they are assigned, subject to the limitation that maximum distance of demand points to the facility they are assigned could not exceed a certain threshold. For both problems they presented heuristic based algorithms. An iterative relaxation algorithm was developed for the first problem they proposed. For the second problem where the number of facilities is also a decision variable, they presented an algorithm that repeatedly uses the algorithm developed for the first problem.

2.2 Survey of Continuous Location and Allocation Problems

The problems in the literature mentioned so far are mostly restricted to discrete space where candidate set of location points are subject to the set of demand points of the problem. However, in problems studied in the continuous space, new sites are generated for locating the facilities. In fact, facilities can be located anywhere in site generating environment, whereas the former problems are discrete site-selecting.

Weber Problem is one of the most famous problems in location theory where the aim is finding a point on the continuous plane to minimize the sum of distances from this point to n different end points. Each end point could have different cost/weight

factors to be multiplied with the corresponding distances. Multisource Weber Problem; also known as continuous location-allocation problem, is the most recognized and prominent problem present in the continuous location literature. Briefly, problem is to find given number (m) of sites (locations) in $\mathbb{R}^2$ to supply the known demands of n customers in a way that the total weighted distance between each customer and the corresponding facility is minimized. Explicitly, Weber Problem is the special form of multisource Weber Problem with $m = 1$. When the associated weights of each end point are assumed to be the same, Weber Problem reduces finding the *geometric median* of the set of points. If there is an upper bound on the amount that a single facility can serve, the Capacitated Weber Problem is obtained.

In the elaborated survey of Brimberg et al. [1], wide literature of multi facility Weber problem (MFWP) that is also known as continuous location-allocation problem and its capacitated extension are examined. Exact methods, heuristic approaches and metaheuristics developed to solve the Weber problem and its capacitated extension are classified in the paper as well.

Exact algorithms are developed by Ostresh in [20], and by Drezner in [21] for two facilities ($m = 2$). Computational results are presented by Drezner [21] for at most 100 customers ($n = 100$). Chen et al. presented the similar results for up to 1000 customers with two facility in [22]. Branch and bound algorithm is developed by Kuenne and Soland in [23] to obtain exact solutions but only small instances could be shown to be solved such as $(m = 4), (n = 15)$ and $(m = 3), (n = 50)$. Improvements are obtained by Rosing to solve larger instances exactly as $(m = 5), (n = 30)$ and $(m = 6), (n = 25)$ in [24]. Further drastic improvements are reported by Krau in [25], where 287 customer case could exactly be solved up to 100 facilities. Hansen et al. in [26] were able to solve 1000 customer problem with 100 facilities with column generation approach where starting solutions drastically effect the solutions obtained finally. Although there are exact methods to solve the uncapacitated problem, most

of them can only succeed in very small instances. Thereby, for large problem instances appropriate heuristic methods are required.

The most common approach for MFWP is proposed by Cooper in [3] as an iterative heuristic. Location and allocation components are iteratively visited. After initialization of randomly generated m facilities, demand points are assigned to the closest one, then, m single facility location problems are solved individually using the well known iterative procedure of Weiszfeld [4] for determining the location of each. Heuristic is alternating between these steps until no further improvement is possible and no further change occurs at least in the location of one facility. This procedure can be repeated either for a predetermined times or during the limited time range. Sites of facilities are recalculated and location and allocation steps are repeated until no further improvement is possible according to a convergence criterion. İyigün and Israel in [27] proposed a generalization of Weiszfeld’s method for multi-facility location problem. Demand points are assigned to the facilities probabilistically. As the method functions iteratively, the assignment probabilities, the facility locations and all individual clusters are refreshed every iteration. Love and Juel in [28], proposed neighborhood change for the set of points near to the current solution by exchanging a determined number of assignments from their facilities to new one. 2-exchange neighborhood is observed to be appropriate to escape the local optimum traps of one exchange method.

A constructive heuristic is proposed by Moreno et al. in [29] where initially facilities are generated and number of facilities are gradually decreased until its lower bound is reached so that in each drop, increase in objective value is minimized. Computational results could be obtained for instances up to ($m = 10$) and ($n = 900$). An opposite constructive approaches on “drop” and “add” moves are proposed by Brimberg et al. in [30] suggesting to increase the number of facilities gradually while attempting to maximize the decrease in objective value. Hansen et al. in [31] proposed an heuristic approach to MFWP problem by first solving p -median problem

exactly. Afterwards, individual single facility location problems are solved where the facility sites are mapped to the continuous counterpart.

Another neighborhood exchange heuristic is proposed by Brimberg et al. in [30] based on relocation of facilities instead of reallocation of customers. Simply, this approach is useful for locating the facilities at unexplored regions. Neighborhood exchange is constructed as one exchange type representing every single move of a facility. Cooper's well known heuristic is then applied to conduct a local search with the initial configurations formed by the points selected among one exchange neighborhood. A constructive heuristic based on a furthest distance rule is developed by Gamal and Salhi in [32] where facilities are avoided to be inserted to the locations close to the other facilities. Osman and Christofides in [6] used the similar constructive heuristic to generate initial solutions required so that a location of a facility is chosen in a way that the product of distances from the newly selected facility to the other facilities is maximized. This enable to have good spread of initial medians over the plane. Gamal and Salhi solve discrete p -median problem and use the solutions of it in their second heuristic.

Brimberg et al. in [33] developed a solution method using multi-stage heuristic for the MFWP with fixed opening costs. They emphasized the fact that a major deficiency of the classical model containing only transportation costs, is that the number of facilities to be located are given a priori. However, some other constraints such as investment limitation, operational infeasibility, government regulations, physical infeasibility can form an upper bound on this number. Thereby, a major cost component of fixed opening costs are taken in to consideration and best trade-off is left to decision maker. In the first stage, discrete counterpart of the same problem is to be solved to determine the number of facilities required. In the second stage, given the initial form of clusters obtained by the first stage; Weiszfeld's and Cooper's well known algorithms and refinements are iteratively performed for location and allocation, respectively. In the last stage, because the number of facilities m found to

be optimal is not necessarily best possible for continuous problem, a local search can be added optionally and first two stages can be repeated for different values of m . Brimberg and Salhi in [34] introduced a new problem with zone dependent fixed costs considering the fact that cost of land may vary in accordance with the government regulation. Number of facilities m may or may not be specified. An algorithm is proposed to solve single facility location problem with zone dependent fixed cost and this algorithm is incorporated in another heuristic to solve multi-facility case. Brimberg et al. proposed a local search heuristic in [35] for solving continuous location problems. The reasoning behind the idea is the strong relation between continuous problem and its discrete counterpart. Basically, local search is carried out in the continuous problem to obtain a local optimum. Then, heuristic switches to the discretized model to obtain an improved solution. This iterative procedure alternates between continuous and discrete spaces according to the corresponding formulation until no further improvement is possible. Fernandes et al. proposed a global optimization algorithm in [36], to minimize the sum of distances from demand points to facilities where all distances are limited by an amount of predetermined threshold. That is, distance limited service constraints are employed representing the case that service provided by any facility become insensitive after a predetermined threshold.

Gokbayrak and Kocaman proposed multi stage heuristic algorithm in [2] to solve distance limited uncapacitated continuous location-allocation problem. They augment the set of candidate locations by solving planar set covering problem and use the augmented set to solve the discrete counterpart of the original problem. We study on the extension of this problem as the facilities are assumed to be capacitated in our problem. In addition to augment the set of candidate locations with the points yielded by planar set covering problem solution, we augment the set of candidate locations by employing new equiangular circular points around the demand points. We modify the Weiszfeld's algorithm by a different projection method. After directly performing the Weiszfeld's algorithm, we adjust the location of the point

converged by the algorithm in order to obey the predetermined D_{max} constraint; whereas they apply the modifications in every iteration of Weiszfeld's algorithm. We modify the Cooper's algorithm as well. Because the facilities are assumed to be capacitated, Cooper's algorithm is performed by using priority measures based on a regret function in our problem.

Papers discussed in this section so far are all studied uncapacitated Multi Facility Weber Problem. The extension of this problem is Capacitated Multi Facility Weber Problem (CMFWP) where m facilities are required to be located on planar space, to serve n customers with known demand and spatial positions to minimize the sum of Euclidean distances without violating the determined capacity of the facilities. Because MFWP is uncapacitated, each customer can be assigned to the closest facility. However, for CMFWP, this could not be possible for each customer, then a priority assessment may be required. Literature on the capacitated problem is not that extensive; even deemed to be scarce, in contrast with uncapacitated problem and seems to need more technical advance.

Brimberg et al. proposed a dynamic programming formulation to the capacitated problem on a real line in [37], minimizing the total service and fixed cost of practical considerations (zoning regulations) to locate p facilities. Chaim et al. proposed heuristic approach yielding reasonable lower bounds for the same problem in [38], without restricting the facilities to the candidate set of points, permit to be located anywhere on the line for hierarchical distribution networks. Even if facilities are subject to be located along the line, fixed customers are assumed to be located off the line.

Brimberg et al. suggested a course of actions in [1] for the CMFWP such that solving the uncapacitated problem to be initial configuration for the capacitated problem. However; because the best case for the capacitated problem does not necessarily originate from the best solution of the uncapacitated problem, generating

several uncapacitated solutions to be the initial solution of capacitated problem is another way suggested. That is, capacitated problem is required to be solved several times as well. Even few initial configurations would suffice to have reasonable solutions for the capacitated problem. Nevertheless, some selecting schemes could be applied to differentiate the configuration of initial solutions.

A perturbation based heuristic is developed by Zainuddin and Salhi [39] to assign the so called borderline customers to the most appropriate cluster. As suggested by Brimberg et al. in [1], initial solution configurations are preferred to be chosen among different uncapacitated solutions. This comprehension is implemented by firstly initiating the facilities in accordance with furthest distance rule and iteratively applying Cooper's alternate method afterwards. The borderline customers whose distances to their closest and second closest facilities are more or less the same, are taken into consideration to form better configuration of clusters. Basically, these type of customers are temporarily removed from the customer set giving the others priority to be assigned first on the assumption that they have almost equal costly alternatives. After allocation phase is completed in this manner, points removed are re-inserted and assigned to the best possible alternative. Location phase is performed via the well known iterative procedure of Weiszfeld [4]. A neighborhood reduction is put forward to get rid of the computational burden of some instances.

Series of constructive heuristics are developed by Luis et al. in [40] for solving CMFWP by generating restricted regions and forbidding them to locate new facility if they are close to the existing ones. The firstly proposed restricted regions which are represented by circles having fixed radius r . A dynamic restriction scheme is also proposed in accordance with the spatial distribution of customers, where r could adaptively be changed considering the density/scarcity of customers over the plane. Another heuristic they proposed is the partition of the plane in to fixed number of cells, each represented by its center of mass to discretize the continuous plane. Region-rejection based approach is also inserted to this configuration to form a new

heuristic.

A guided reactive algorithm is developed by Luis et al. in [41] to construct restricted candidate list (RCL) for the CMFWP using adaptive learning. Greedy randomized adaptive search procedure (GRASP) is put forward to clarify the restricted regions where new facilities are infeasible to be physically located as locations could be on lakes, mountains etc. The regions where previously located facilities are deemed to be so close in distance are also incorporated in to the restricted candidate list so that site generation in these regions are avoided.

Aras et. al proposed a series of 3 heuristics [42] for solving CMFWP, each based on one of the three well known metaheuristic approaches: simulated annealing (SA), genetic algorithm (GA) and threshold acceptance (TA) algorithm. Whereas SA and TA are analogous in terms of the accepting better solutions with no doubt, acceptance criterion for the worse solutions differs. That is, in TA, inferior solutions are accepted only if the difference between current and new solution is below some threshold value θ . Thereby, better solutions are always accepted as well, while worse solutions can only be accepted if it differs from current solution at most by an amount of θ . For TA, one variable exchange and two variable exchange neighborhood structures are implemented as well. Contrary to TA and SA, GA does not search the neighborhood of a single solution but conducts search among the population of solutions. The implemented algorithm incorporate the fundamental characteristics of a common GA such as selection, crossover and mutation, one variable and two variable exchange structures are implemented as well. All algorithms are put forward to solve a few variants of CMFWP; RCMFWP, ECMFWP, SECMFWP and LpCMFWP where “R”, “E”, “SE”, “Lp” representing the norms of rectilinear, Euclidean, squared-Euclidean and l_p distances, respectively. SA heuristic performed with two variable exchange is observed to outperform the other heuristics and variants.

A hybrid algorithm is proposed in [43], associating genetic algorithm (GA) and

PSO (Particle Swarm Optimization) for solving capacitated continuous location problem with service level constraints and fixed opening costs, where the number of facilities is unknown. Chromosome is varying with respect to the number of facilities to be located, that is why the used GA is called variable length GA. Special mutation and crossover operators are developed and used to attain the required number of facilities. Some inferior solutions are also inserted to the population so as to use possible better seeds of them. Alternate transportation location heuristic and PSO is combined to determine the location of the facilities of each individual cluster.

Aras et al. in [44] proposed new heuristic methods for solving CMFWP with euclidean, squared euclidean and l_p distances. Heuristics are based on the approximation of mixed integer linear programming formulation. Lagrangean, p -median and cellular heuristics are developed and compared with the benchmark instances present in the literature. Proposed p -median heuristic is observed to attain better results in all instances. Cellular heuristic yields reasonable results in shorter CPU times than the exact methods. Regardless of the distance function and problem instance, p -median heuristic is found to have the highest accuracy. The reasoning behind this observation is that the optimal locations generally appear in the very close vicinity of demand points. Even employing grids over the plane as the set of candidate locations is observed to be unable to compete with p -median heuristic.

Akyuz et al. in [45], proposed two types of branch and bound algorithm for the l_r distance for CMFWP where $1 \leq r \leq \infty$. New branching selection and lower bounding methods are presented for allocation-space-based-branch and bound algorithm. Location space is partitioned and new lower bounding and upper bounding methods are proposed in the newly introduced location-based-branch and bound algorithm. Computational experiments indicates that the proposed algorithms outperform the existing ones.

Ajdad et al. in [46] proposed an iterative two stage heuristic algorithm for variant

of CMFWP with single source constraints. In Single Source Capacitated Multi-Facility Weber Problem (SSCMFWP), demand of all customers have to be fulfilled by a single facility. This variant of CMWP; interestingly, suffers from the scarcity of interest and there is very limited number of works conducted upon that problem. Basically, in addition to the priority value of $\theta_c = d(c, f_2) - d(c, f_1)$, firstly introduced by Mulvey and Beck [14] as a regret function, a new priority value $\theta_c = \sum_{i=1}^m d(c, f_i) - d(c, f_1)$ is suggested for the method called parallel assignment where $d(c, f_i)$ represents the Euclidean distance between point c and i^{th} closest facility to it. Weiszfeld's procedure is used for the location phase as well. The Cooper's alternating location allocation algorithm is iteratively carried out until a termination criterion is satisfied. Termination criteria can either be fulfilled by predetermined number of iterations or predefined number of iterations without improvement. Alternatively, N different initial solutions could be tried to refresh the algorithm that would hopefully prevent from local traps. A simulated annealing algorithm is also implemented for comparison purposes and two phase heuristic is found to be superior in comparison with SA.

The problem we propose in this thesis differs from all the problems introduced so far. Basically, we introduce capacitated location and allocation problem in a site generating environment with distance limited service constraints and fixed facility opening costs. In deference to the SSCMFWP; the variant of CMFWP with single source constraints, fixed opening cost of facilities are concurrently taken into consideration with connection costs, recognizing the need for cost effective planning of distribution systems. That is, number of facilities required to serve all demand points are not known a priori. On top of that, in addition to the upper bound of service amount that a facility can provide, we also employ a constraint on the facilities to make them serve only up to a limited distance. This limitation interprets the physical realities in our model.

2.3 Other Extensions of Capacitated Continuous Location Allocation Problems

Different extensions of capacitated site selecting problems discussed in the literature so far are presented in this section with the brief summary of solution methodologies.

Negreiros and Palhano in [47] proposed two variants of CCP. p -CCCP is the one where the center of a cluster is not necessarily a demand point. That is, each center is located at *geometric center (centroid)* of that cluster and this position is calculated with respect to the coordinates of the individuals assigned to that cluster. That is why this variant of CCP is called Capacitated Centred Clustering Problem (CCCP). CCCP is a generalized form of CPMP as well and could be defined as clustering set of n items into p disjoint groups in a way that total dissimilarity between each item and the center of the cluster that it is assigned to is minimized considering that total weight of each cluster may not exceed the determined capacity limit. The main difference between CCCP and CPMP is that center of a cluster in CCCP is located at the *geometric center (centroid)* of the points that belong to the cluster, whereas in CPMP, clusters are centered by the median of the points. Accordingly, spatial position of the centers depends on the points that form the cluster; that is, candidate location of a center is not known in advance. Second variant is *Generic-CCCP* in which fixed opening costs of clusters are inserted to the objective function; thereby, in addition to the total dissimilarity of the clusters, the number of clusters to cover all individuals is subject to the minimization as well. The proposed heuristic method is composed of two phases. In the first phase, constrained variants of “K-means” method are used with log-polynomial (q -tree) geometric tree search and in the second phase VNS is performed. The instances are chosen among garbage collection zones, food distribution areas and frequently used in CCP, CPMP literature.

Clustering Search (CS) algorithm is developed by Chaves and Lorena in [48] for

the CCCP known also as Capacitated Continuous Clustering Problem. Mainly, CS algorithm determines the promising sites in the search space and performs local search only in these sites.

Akyuz et al. developed beam search algorithm in [49] for the single and multi commodity capacitated multi facility Weber problem (MCCMFWP), which is an extension of CMFWP and known to be non-convex optimization problem. Transportation costs depend on the commodity type. Branch and bound algorithms and beam search heuristics are developed for solving both single and multi commodity CMFWP. In another study on MCCMFWP [50], the same authors developed discrete approximation heuristics for solving the MCCMFWP; using lagrangean relaxations, using block norms and using customer locations for determining the candidate location of facilities.

Singhtaun and Charnsethikul proposed and compared exact methods in [51] for solving Single Source Capacitated Multi Facility Weber Problem in rectilinear distance. Decomposition algorithm is the first proposed method. Branch and bound algorithm is also used as a classical exact algorithm by considering the p -median problem and transforming the original formulation to corresponding binary integer formulation. This formulation is composed by making use of common observations for the optimal facility locations which says the facility locations are generally on customer locations or very close to them. Aras et al. in [52] has also studied on rectilinear distance capacitated multi-facility Weber problem and proposed two heuristic based algorithms: p -median and cellular heuristics by again exploiting the same observations for the optimal facility locations. In the cellular heuristic, the selection of the promising sites are decided by partitioning the rectangular area including all customers into the cells. Basically, cells are evaluated in terms of their potential of containing the optimal facility location. Cells are represented by the centroids of the customer locations which resides in that cell.

Durmaz et al. proposed a discrete approximation heuristic in [53] for the probabilistic extension of CMFWP (PCMFWP) with probabilistic customer locations assuming that the customers are randomly distributed with respect to a bivariate probability distribution. They formulate discrete approximation of the problem as a mixed integer linear model where candidate set of location points is subject to a discrete domain. They presented three heuristics and compared the performance of their heuristics with probabilistic alternate transportation location heuristic (PATL) which is the adapted version of Cooper's alternating transportation location heuristic (ATL) [54] originally developed for solving the deterministic version of the problem. The new heuristics are based on the evaluation of the expected distances between demand points and facilities. They proposed approximation methods to enable the heuristics to be used for different type of distance functions and probability distributions. Altinel et al. [55] proposed a location allocation heuristic for the same problem. Probabilistic version of an alternate location allocation local search heuristic is proposed by tailoring the ideas originally used in the deterministic problem.

Chapter 3

Problem Definition and Formulation

We propose a variant of CMFWP with single source constraints where required amount of demand by each customer are to be served by single facility only. Single Source Capacitated Multifacility Weber Problem (SSCMFWP), interestingly suffers from the lack of interest despite its real life applications generally in energy distribution [56], [2]. Thereby, we even extend the SSCMFWP by incorporating fixed opening costs and distance limited service constraints. A comprehensive review on the literature we conduct reveals that the problem proposed in this thesis is newly introduced.

We consider a rectangular greenfield region of $L \times W$ dimensions where N customer points are spread over. Demand Point i is located at (a_i, b_i) and requires demand by an amount of w_i . A multi-facility continuous location and allocation problem is proposed with service and distance limited facilities. That is, there is an upper bound not only on the amount of service that a single facility could provide

(capacitated facilities) but there is also limitation on the service distance for each facility associated with their spatial coverage. Capacitated and service distance limited facilities are to be opened on continuous plane $\mathbb{R}^2$ to fulfill the known demands of customers, given their spatial locations, in a way that sum of connection and fixed opening costs is minimized. Distance limitation constraints are incorporated to mathematical model for realizing some physical facts such as energy loss, voltage drop, pressure drop and etc. Both the electrical voltage and the water pressure drop is directly proportional with distance [2]. Distance limitation is associated to prevent over drops and represented by a circular region with a predefined radius D_{max} , which can be determined adaptively with a series of basic and electrical engineering calculations [57]. Thereby, facilities become disabled outside the mentioned circular region and demand points located there could not be served. This limitation can be expressed by the straightforward electrical engineering calculations and linear operations.

Example: We consider a consumption of 50kWh per month with a load factor 20%. Maximum allowable voltage drop percent is 2.5% for line voltage of 220V.

Load factor is defined by $F_{LD} = \frac{AverageLoad}{PeakLoad} = \frac{AverageLoad \times T}{PeakLoad \times T}$, in [57] where T denotes the period. Then, $F_{LD} = \frac{UnitsServed}{PeakLoad \times T} \implies F_{LD} = \frac{50kWh}{PeakLoad \times 30 \times 24} = \frac{1}{5}$, which yields $PeakLoad = 347W$. Because $P = I \times V$, this peak power consumption triggers the maximum current demand of $I = P/V = 347/220 = 1.58A$.

Allowable voltage drop VD can be calculated as $VD = 220 \times \frac{2.5}{100} = 5.5V$.

$VD = K \cdot I \cdot L \cdot r$, as given in [58], where K is VD per unit length and equals 2 for single phase distribution, L is line length and r is unit resistance per length.

For a squirrel type cable having unit resistance $r_{sqr} = 1.37\Omega/km$, we come up with;
 $5.5 = 2 \times 1.58 \times L \times \frac{1.37}{10^3} \implies L = 1270.44m$.

For a mole type cable having unit resistance $r_{ml} = 2.7\Omega/km$, we come up with;
 $5.5 = 2 \times 1.58 \times L \times \frac{2.7}{10^3} \implies L = 644.63m$. Obtained L value would basically employ an upper bound on the cable length that does not lead an over drop.

Distribution is subject to the star topology in which the customers are individually connected to the facility they are assigned to. That is, central node is the server and other nodes are clients directly connected to the server. This network topology are used when the central node performs data distribution, the network controls the data transit or the network requires electrical power sources [59], [60]. Star topologies may also be used in wireless routers, ethernet/cabled structures. Hence, connections are formed between each customer and the corresponding facility.

In contrast to the classical continuous location and allocation problems, number of facilities to be opened is not assumed to be known. That is, the number of facilities m to be opened is not known a priori and the best trade-off between connection costs and fixed opening costs satisfying the required demand of customers is left to the decision maker. In this context, we need to determine the number of facilities m , in addition to the location and allocation configuration (assignment of customers to the facilities) of each.

“Capacitated Single Source Multifacility Continuous Location Allocation Problem with Distance Limited Service Constraints for Spatial Planning of Decentralized Distribution Systems” defines a single level resource distribution network in star configuration where all distances are assumed to be in Euclidean (l_2) norm. The problem is mathematically formulated as a mixed integer quadratically constraint model. Parameters, decision variables and mathematical formulation are provided for the problem denoted by (DLim-CapCLAP) as follows:

Parameters

N	: Set of demand points
$L \times W$	: Rectangular greenfield region
(a_i, b_i)	: Spatial position of i^{th} customer
D_{max}	: Maximum coverage distance
w_i	: Demand of i^{th} customer
F	: Unit cost of a facility
C	: Capacity of a facility
Cnn_{cost}	: Unit cost of connection

Decision Variables

z_{ij}	: Assignment of i^{th} customer to j^{th} facility
u_j	: Siting/Opening a facility on j^{th} location
(x_j, y_j)	: Spatial position of j^{th} facility
δ_i	: Euclidean distance between i^{th} customer and its corresponding facility
d_{ij}	: Euclidean distance between i^{th} customer and j^{th} facility

$$z_{ij} = \begin{cases} 1, & \text{if customer } i \text{ is served by facility } j, \\ 0, & \text{otherwise.} \end{cases}$$

$$u_j = \begin{cases} 1, & \text{if facility } j \text{ is open,} \\ 0, & \text{otherwise.} \end{cases}$$

Mathematical Model: (DLim-CapCLAP)

$$\min \sum_{i=1}^N \delta_i Cnn_{cost} + \sum_{j=1}^N u_j F \quad (3.1)$$

$$\text{s.t. } \sum_{j=1}^N z_{ij} = 1 \quad \forall i \in \{1, 2, \dots, N\}, \quad (3.2)$$

$$z_{ij} \leq u_j \quad \forall i, j \in \{1, 2, \dots, N\}, \quad (3.3)$$

$$\sum_{i=1}^N z_{ij} w_i \leq u_j C \quad \forall j \in \{1, 2, \dots, N\}, \quad (3.4)$$

$$\delta_i \leq D_{max} \quad \forall i \in \{1, 2, \dots, N\}, \quad (3.5)$$

$$\delta_i \geq \sqrt{(L^2 + W^2)}(z_{ij} - 1) + d_{ij} \quad \forall i, j \in \{1, 2, \dots, N\}, \quad (3.6)$$

$$d_{ij}^x = a_i - x_j \quad \forall i, j \in \{1, 2, \dots, N\}, \quad (3.7)$$

$$d_{ij}^y = b_i - y_j \quad \forall i, j \in \{1, 2, \dots, N\}, \quad (3.8)$$

$$d_{ij}^2 \geq (d_{ij}^x)^2 + (d_{ij}^y)^2 \quad \forall i, j \in \{1, 2, \dots, N\}, \quad (3.9)$$

$$x_j, y_j \in R \quad \forall j \in \{1, 2, \dots, N\}, \quad (3.10)$$

$$u_j \in \{0, 1\} \quad \forall j \in \{1, 2, \dots, N\}, \quad (3.11)$$

$$z_{ij} \in \{0, 1\} \quad \forall i, j \in \{1, 2, \dots, N\}, \quad (3.12)$$

$$d_{ij}^x, d_{ij}^y \geq 0 \quad \forall i, j \in \{1, 2, \dots, N\}, \quad (3.13)$$

$$d_{ij} \geq 0 \quad \forall i, j \in \{1, 2, \dots, N\}, \quad (3.14)$$

$$\delta_i \geq 0 \quad \forall i \in \{1, 2, \dots, N\} \quad (3.15)$$

We minimize the total cost of the system (3.1) consisting of two main components: connection costs between the demand points and customers and fixed opening costs. Single-source constraint set (3.2) enables to assign each customer to exactly one facility. We ensure with (3.3) that a customer can be assigned to a site only if a facility is opened there. Total amount of demand assigned to a facility can not exceed the capacity of the facility, by means of (3.4). The distance between a customer and its

corresponding facility is bounded by an upper bound D_{max} by (3.5). According to the dimensions of the rectangular region defined for the problem, lower bounds are presented for all Euclidean distances in (3.6). It guarantees that δ_i is equal to the distance d_{ij} if demand point i is assigned to facility j and it is redundant constraint otherwise. In (3.7) and (3.8), differences between customers and candidate location points in continuous plane, $\mathbb{R}^2$, are defined in terms of x and y coordinates, respectively. Considering the differences defined in (3.7) and (3.8), the set of quadratic constraints are defined in (3.9) to represent Euclidean distances between the customers and the facilities. The decision variables of the problem are represented in (3.10)-(3.12). Non-negativity constraints for Euclidean distances are provided in (3.13)-(3.15).

Chapter 4

Solution Methodology

The problem we proposed is reduced to be “CMFWP with single source constraints (SSCMFWP)”, when the number of facilities is known and there is no distance limitation. Even the discrete counterpart of SSCMFWP, CPMP, is known to be NP-Hard [61]. Similarly, MFWP and CMFWP (even all customers are located along a line) are also known to be NP-hard as shown by Sherali and Nordai in [62]. As the problem is quadratically constrained NP-hard problem, even small instances having a cardinality of 10, could not be exactly solved. Therefore, we propose a multistage heuristic framework to solve the continuous location and allocation problem making use of the strong relation between continuous problem and its discrete counterpart. In order to solve continuous location and allocation problem, we propose a solution approach that exploit the ideas of Hansen et al. [63]. Basically, the idea lies behind their famous p -median heuristic is the use of common observations for the optimal facility locations which are generally on demand points or very close to them. In many studies discussed so far in the literature of continuous problems such as [44],[51],[52],[35], the optimal facility locations are observed to be on the demand points or very near vicinity of them. On the other side, defining the set of candidate locations as it only

consists of the demand points would restrict the feasible region of the problem and downgrade the solution quality accordingly.

In order to form a new set of candidate locations while still keeping demand points in the set, we propose to insert a reasonable number of new promising candidate points. That is, we still use the mentioned relation between continuous problem and its discrete counterpart by augmenting the domain of the discrete problem. We propose to employ new points attained by solving Planar Set Covering Problem (PSCP) as Gökbayrak and Kocaman have done in [2].

We solve the (PSCP) by exploiting the ideas of Church [64]. He simply suggested to obtain an optimum solution of PSCP by solving SCP in the presence of circle intersection points. These points are obtained by the intersections of the circles, centered on (a_i, b_i) with a radius of D_{max} . The union of these intersection points and set of demand points, denoted by *CIPS*, constitutes the domain of SCP to be solved. Basically, the optimum solution of this problem (SCP) generates the minimum number of points that can cover all of the demand points. Namely, the solution of the PSCP yields the points to be inserted to the set of candidate locations to find a solution for the continuous problem by solving its discrete counter part.

Alternatively, we propose to select equally spaced (equiangular) additional points over each circle centered on (a_i, b_i) with a radius of D_{max} to augment the set of candidate locations. Besides, for comparison purposes, we partition the rectangular plane in to square grids as Aras et al. proposed in their cellular heuristics [52],[44] to augment the set of candidate locations, in the computational experiments.

We adaptively use the ideas of famous heuristic of Hansen et al. [63]. Making use of the observations on optimal facility locations discussed before, they firstly solved the p -median problem where set of candidate locations are subject to demand points. In the second step, individual Weber problems; aiming to find a point on

the continuous plane to minimize the sum of distances from this point to n different end points, is solved for each cluster. As they solved the discrete counterpart of MFWP which is p -median problem, in their p -median heuristic; likewise, we solve the discrete counter part of DLim-CapCLAP either with or without newly employed candidate locations. In the next stage, we implement the Cooper's well known alternate heuristic that iteratively switches between location and allocation steps to find better allocation configuration of customers to the facilities. Since the facilities are assumed to be capacitated and some of the demand points could not be assigned to their nearest facility, allocation steps of Cooper are adapted with respect to the priority value of each demand point. The flow chart representing the stages of the heuristic framework is provided in 4.1.

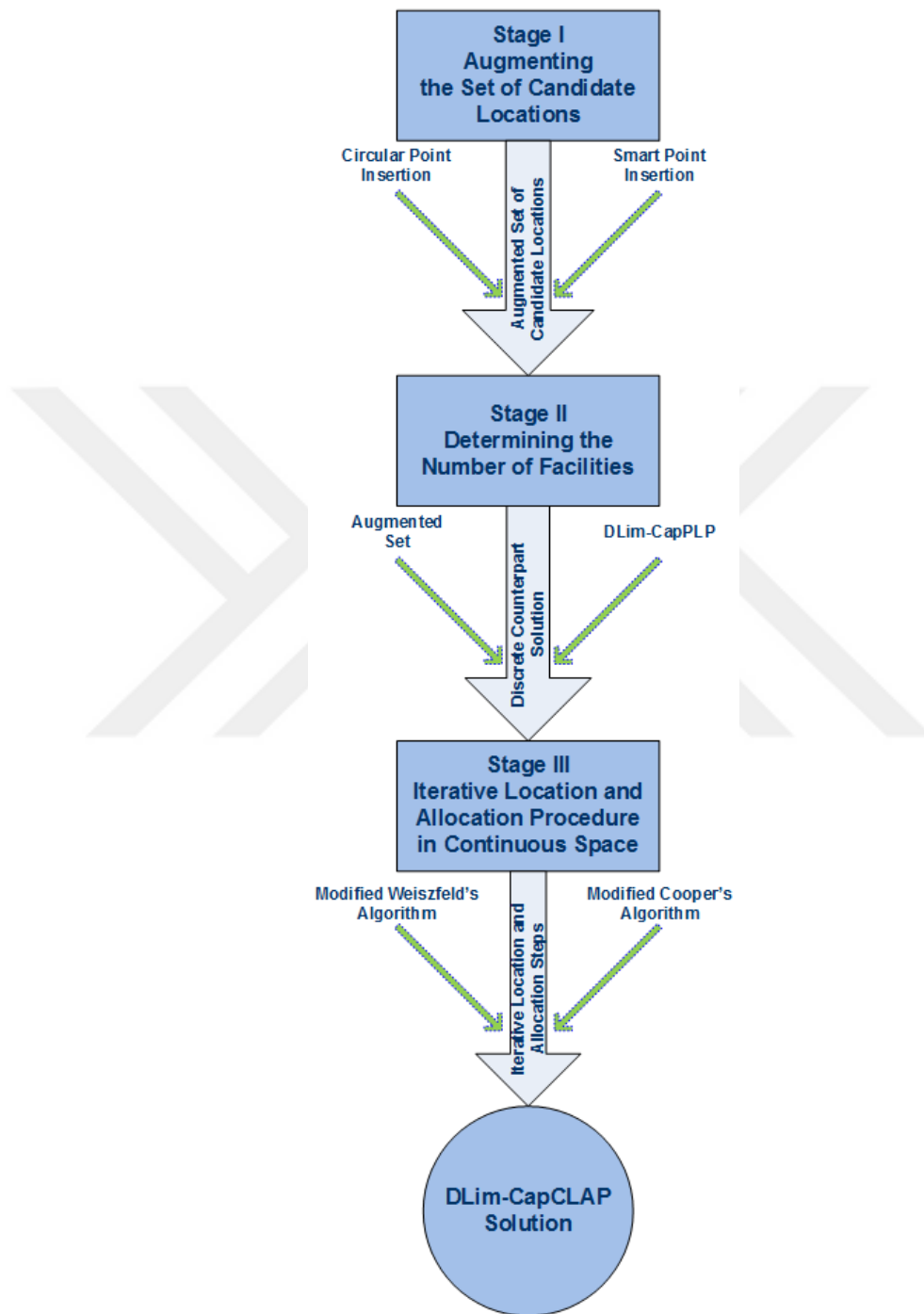


Figure 4.1: Flow Chart of Multistage Heuristic Framework

4.1 Stage I: Augmenting the Set of Candidate Locations

In this stage, we augment the set of candidate locations which initially consists of demand points. We propose two different augmentation ways to insert new candidate points to the domain. Smart points produced by the solution of PSCP and equiangular circular points selected over the circles centered around each demand point, are used to augment the set of candidate points separately.

4.1.1 Smart Point Insertion

PSCP is solved to acquire smart points as the promising candidate points in this method. For this purpose, all intersection points of the circles centered around each demand location (a_i, b_i) with the radius D_{max} , are determined. The union of intersection points set and the set of demand points denoted together by $CIPS$, forms the domain of the SCP to be solved. Note that if the circle centered at the demand point i does not intersect with any other circle, then the demand point itself is certainly determined to be a facility. The reasoning behind this idea is the fact that this demand point could not be spatially reached by any $CIPS$ other than itself, under the defined distance coverage.

In order to graphically represent the mentioned arguments, an example with cardinality of 50 is selected among the frequently used instances of Osman and Christofides [6] in location literature. Demand points and corresponding intersection points are shown in Figure 4.2 and Figure 4.3, respectively. Note that, because the circle around the point located at northeastward of the plane does not intersect any other circle, this point will be chosen as a facility by PSCP solution shown in Figure 4.5. The maximum spatial coverage that could be provided by the set of all $CIPS$ is indicated

by the circles drawn in Figure 4.4. The circles drawn by cyan in Figure 4.4, shows the coverage region of the CIPS being concurrently the demand points. Because they could not be covered by any point other than themselves, these demand points will be determined as facility by both PSCP and discrete counterpart solutions as shown in 4.5 and 4.8, respectively.

All the demand points form a set to be served by the facilities. The spatial coverage area of all *CIPS* is shown in Figure 4.4. We are to find the minimum possible subset of this area that can cover all the demand points. Hence, the set covering problem we propose in that stage yields the minimum number of facilities required to cover all the demand points. We define a coverage indicator α_{ic} , $\forall i \in N$ and $\forall c \in CIPS$, representing the coverage state of i^{th} demand point by c^{th} *CIPS*. Our proposed model for SCP to determine the set of minimum number of facilities for spatially covering all the demand points formulated as follows;

Parameters

CIPS : $N \cup$ intersection points set

d_{ic} : Euclidean distance between i^{th} demand point and c^{th} *CIPS*

α_{ic} : Coverage indicator of i^{th} demand point by c^{th} *CIPS*

$$\alpha_{ic} = \begin{cases} 1, & d_{ic} \leq D_{max} \\ 0, & \text{otherwise.} \end{cases}$$

Decision Variables

v_c : Siting/Opening a facility on c^{th} point in *CIPS*

$$v_c = \begin{cases} 1, & \text{if a facility is open on } c^{th} \text{ point in } CIPS, \\ 0, & \text{otherwise.} \end{cases}$$

Mathematical Model

$$\min \sum_{c=1}^{|CIPS|} v_c \quad (4.1)$$

$$\text{s.t. } \sum_{c=1}^{|CIPS|} \alpha_{ic} v_c \geq 1 \quad \forall i \in \{1, 2, \dots, N\}, \quad (4.2)$$

$$v_c \in \{0, 1\} \quad \forall c \in CIPS \quad (4.3)$$

The number of facilities that can cover all the demand points is minimized by (4.1). We ensure with (4.2) that each demand point is covered by at least one of the points in $CIPS$. In other words, it guarantees that each demand point is in the D_{max} range of at least one point in $CIPS$. The objective value of the optimal solution of PSCP gives the minimum number of facilities required to cover all demand points. Note that, total number of facilities $\sum_{c=1}^{|CIPS|} v_c$ opened also includes the facilities chosen among demand points. Therefore, this is an upper bound on the number of new points to augment the set of candidate locations. In fact, number of opened facilities chosen among intersection points $\sum_{c \in CIPS \setminus N} v_c$, yields us how many points will be inserted to the set of demand points to form the augmented set of candidate locations. Consequently, the difference between the total number of facilities opened and total number of facilities opened on the demand points actually yields us the number of points to be inserted. In other words, union of demand points and points yielded as facilities by PSCP solution forms the augmented set of candidate locations. The PSCP solution of the example instance is provided in Figure 4.5.

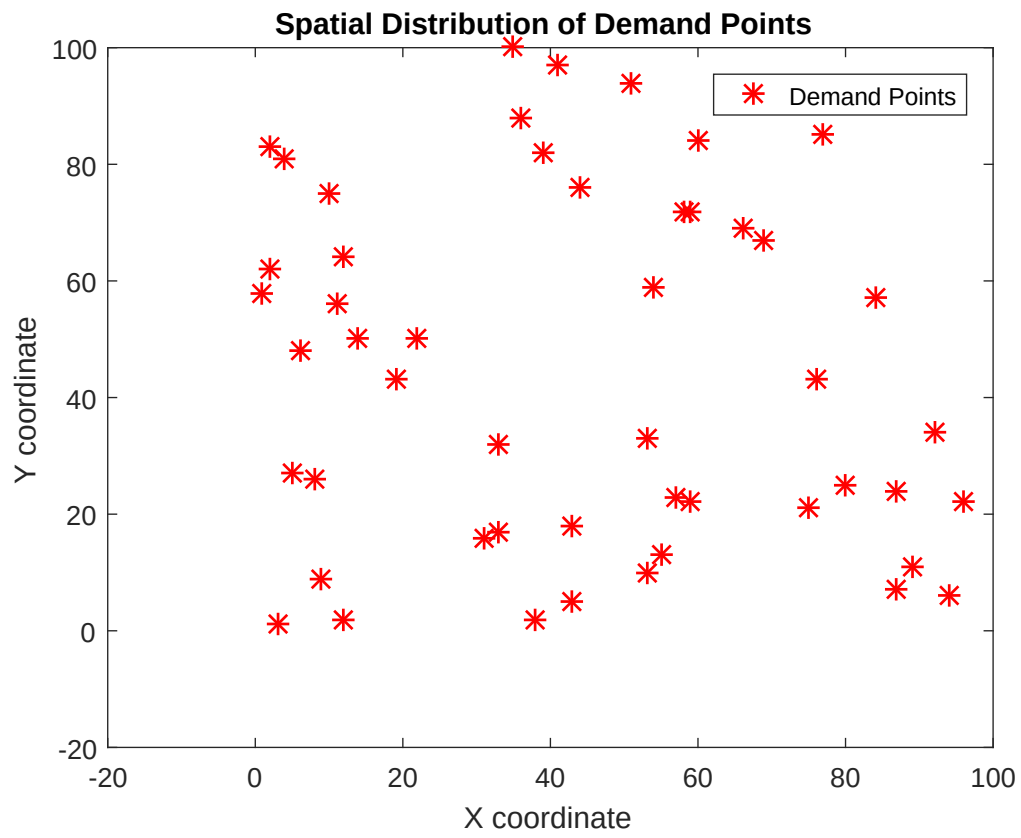


Figure 4.2: Demand Points

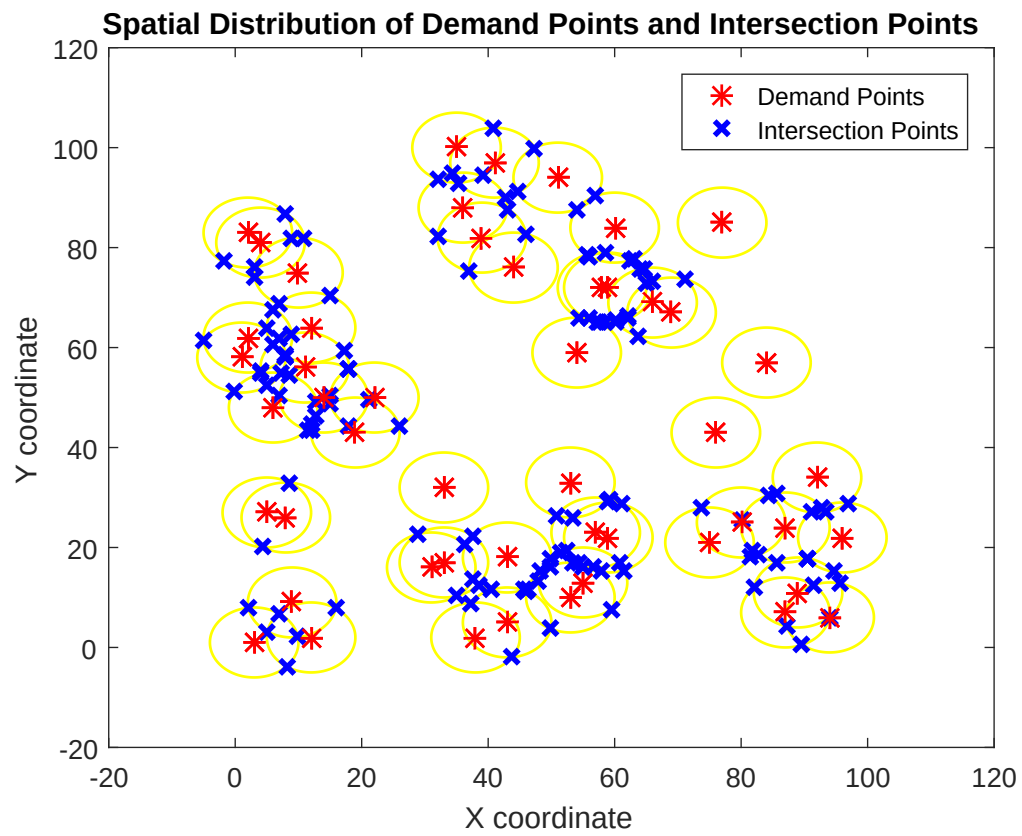


Figure 4.3: Circle Intersection Points

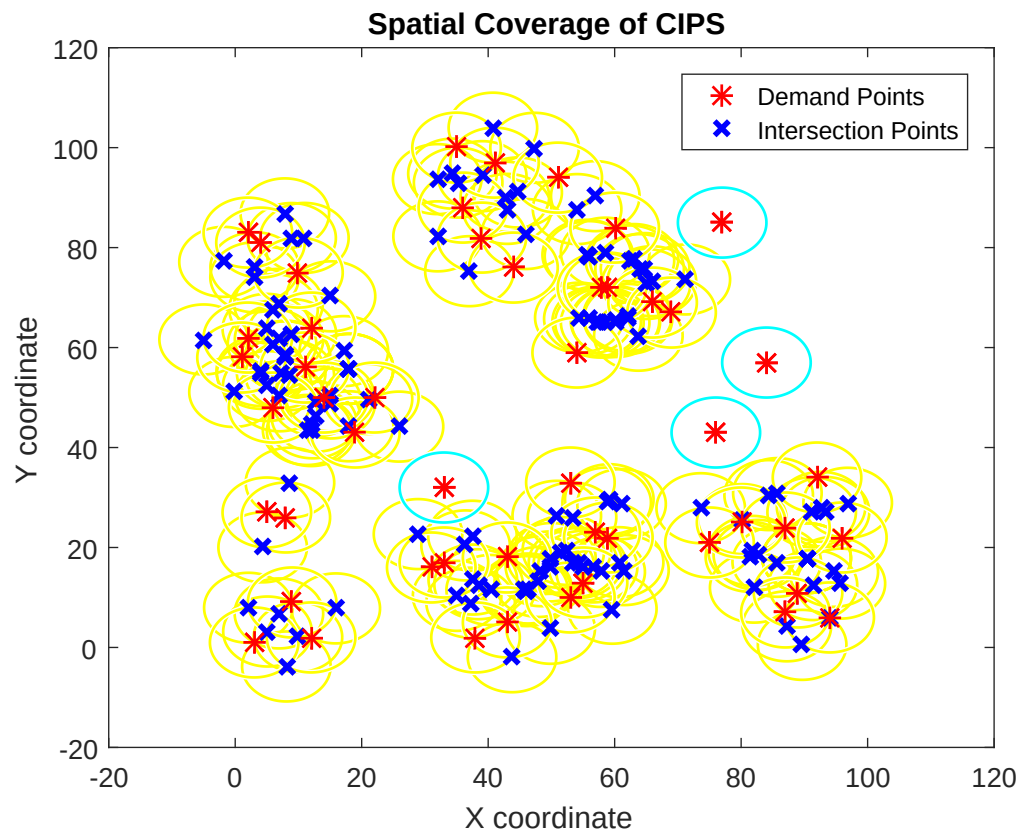


Figure 4.4: Spatial Coverage of CIPS

4.1.2 Circular Point Insertion

As a variant of insertion methodology mentioned in the previous section, we propose to augment the set of candidate locations by employing equiangular points on the circles centered at each demand point with radius D_{max} . For the problem instances that we perform computational experiments, we employ 8 equiangular points being 45° spaced, for all demand points. This number and density is determined empirically with respect to the tradeoff between the improvement in solution quality and computational burden due to employing more number of points.

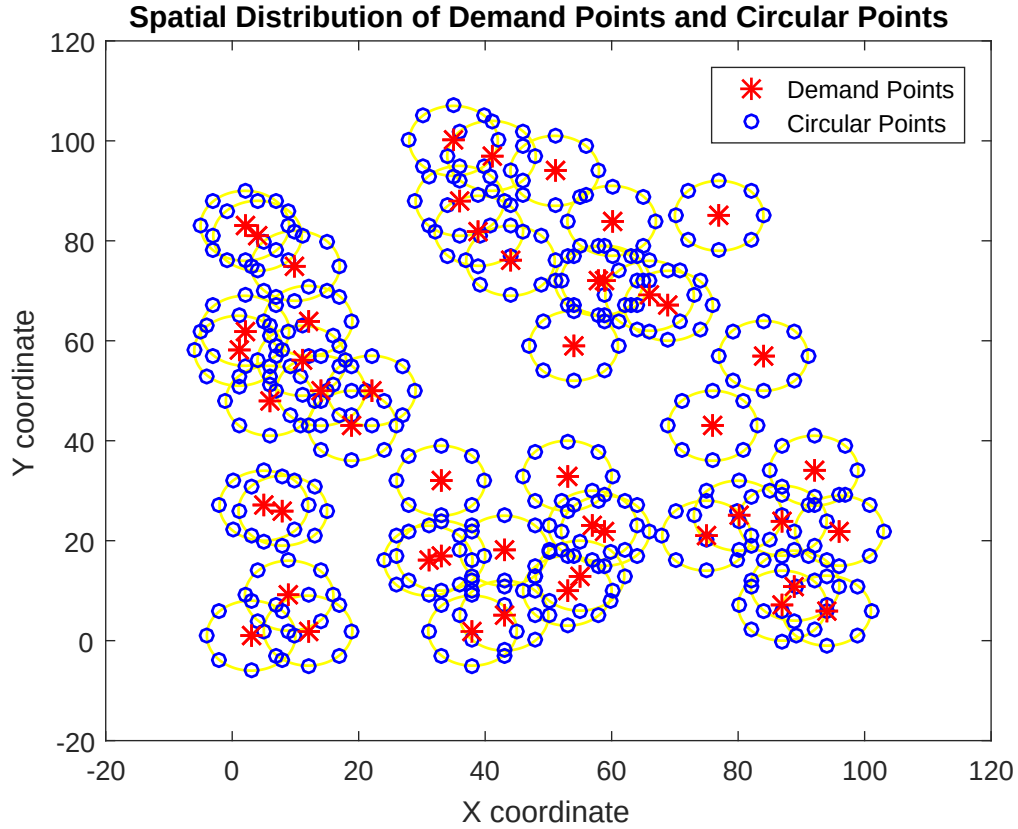


Figure 4.6: Circular Points

This amount can be deemed as the cut off density of equiangular circular points in accordance with the mentioned tradeoff. That is, the insertion of new points more than 8, does not provide considerable improvement that is worthy of computational burden. The graphical representation of the circular point insertion is provided in Figure 4.6.

4.2 Stage II: Determining the Number of Facilities

In this stage, we determine the number of facilities, initial locations of facilities and initial allocation of demand points to them. However, rather than using the ideas of well known p -median heuristic of Hansen et al. [63] directly, as Brimberg et al. did in [33], we solve the discrete counterpart of the DLim-CapCLAP with an augmented set of candidate locations. A good initial estimate we need for the solution of DLim-CapCLAP is not yielded by the original discrete counterpart having domain that consists only of demand points. Therefore, the augmented set of candidate points is preferred while solving discrete counterpart to have better estimate. Nevertheless; for comparison purposes, we also use the original domain of the discrete problem while solving DLim-CapCLAP in our computational experiments. Thus, the set of candidate locations denoted by $M = N \cup \{c \in CIPS | v_c = 1\}$ is used. The mathematical formulation is provided for the discrete counter part of DLim-CapCLAP, denoted by DLim-CapPLP (distance limited capacitated plant location problem) as follows:

Parameters

- N : Set of demand points
 M : Augmented set of candidate locations
 D_{max} : Maximum coverage distance
 w_i : Demand of i^{th} customer
 F : Unit cost of a facility
 C : Capacity of a facility
 Cnn_{cost} : Unit cost of connection

Decision Variables

- z_{ij} : Assignment of i^{th} customer to j^{th} facility
 u_j : Siting/Opening a facility on j^{th} location
 d_{ij} : Euclidean distance between i^{th} customer and j^{th} facility

$$z_{ij} = \begin{cases} 1, & \text{if customer } i \text{ is served by facility } j, \\ 0, & \text{otherwise.} \end{cases}$$

$$u_j = \begin{cases} 1, & \text{if facility } j \text{ is open,} \\ 0, & \text{otherwise.} \end{cases}$$

Mathematical Model: : (DLim-CapPLP)

$$\min \sum_{i=1}^N \sum_{j=1}^M z_{ij} d_{ij} Cnn_{cost} + \sum_{j=1}^M u_j F \quad (4.4)$$

$$\text{s.t. } \sum_{j=1}^M z_{ij} = 1 \quad \forall i \in \{1, 2, \dots, N\}, \quad (4.5)$$

$$z_{ij} \leq u_j \quad \forall i \in \{1, 2, \dots, N\}, \forall j \in \{1, 2, \dots, M\}, \quad (4.6)$$

$$\sum_{i=1}^N z_{ij} w_i \leq u_j C \quad \forall j \in \{1, 2, \dots, M\}, \quad (4.7)$$

$$z_{ij} d_{ij} \leq D_{max} \quad \forall i \in \{1, 2, \dots, N\}, \forall j \in \{1, 2, \dots, M\}, \quad (4.8)$$

$$u_j \in \{0, 1\} \quad \forall j \in \{1, 2, \dots, M\}, \quad (4.9)$$

$$z_{ij} \in \{0, 1\} \quad \forall i \in \{1, 2, \dots, N\}, \forall j \in \{1, 2, \dots, M\}, \quad (4.10)$$

$$d_{ij} \geq 0 \quad \forall i \in \{1, 2, \dots, N\}, \forall j \in \{1, 2, \dots, M\} \quad (4.11)$$

We minimize the total distribution cost (4.4) consisting of two main components, connection and fixed facility opening costs. Single-source constraint set (4.5) enables to assign each customer to exactly one facility. We ensure with (4.6) that a customer can be assigned to a site only if a facility is opened there. Total amount of demand required by the customers assigned to a facility can not exceed the predetermined capacity of a facility can serve, by means of (4.7). The distance between a customer and the facility that serves to the customer is bounded by an upper bound D_{max} by (4.8). The decision variables for allocation and location are represented in (4.9)-(4.10) respectively. The Euclidean distances between the customers and the facilities are defined to be non-negative in (4.11). Note that, the set of quadratic constraints disappear as we switch to the discrete space and d_{ij} becomes a parameter so is known a priori whereas it was a decision variable in continuous counterpart.

In our running example with $|N| = 50$, the cardinality of augmented set of candidate locations was found to be $|M| = 66$. Actually, solving PSCP yielded the minimum number of facilities required to spatially cover the set of demand points was $\sum_{c=1}^{|CIPS|} v_c = 22$. However, the number of facilities opened on the intersection points is $\sum_{c \in CIPS \setminus N} v_c = 16$, which means 6 facilities are chosen among the set of demand points N . That is why, the number of newly employed points 16 and total number of points in the augmented set of candidate locations is $|M| = 66$ but not 72. In other words, minimum number of facilities required to cover all the demand points; $\sum_{c=1}^{|CIPS|} v_c$, is the upper bound for the number of newly employed points as some of them are the demand points.

All parameters of the running example is provided as follows: $|N| = 50$, $|M| = 66$, $D_{max} = 7$, $w_i = 1 \quad \forall i \in N$, $F = 3000$, $C = 10$, $Cnn_{cost} = 10$. DLim-CapPLP solutions without and with additional smart points are provided in Figure 4.7 and Figure 4.8, respectively. Note that, even if we add reasonable number of additional points (16) to the set of candidate location points, a significant cost reduction from 108596.28 to 71278.28 is observed in the objective value, that is by an amount of %34.36. The reason lie behind this reduction is because the required number of facilities yielded by DLim-CapPLP is decreased from 36 to 23, so 13 facilities are dropped while still ensuring the capacitated and distance limited coverage of the facilities. Note also that 11 over 23 facilities are selected to be opened among the candidates yielded by PSCP solution.

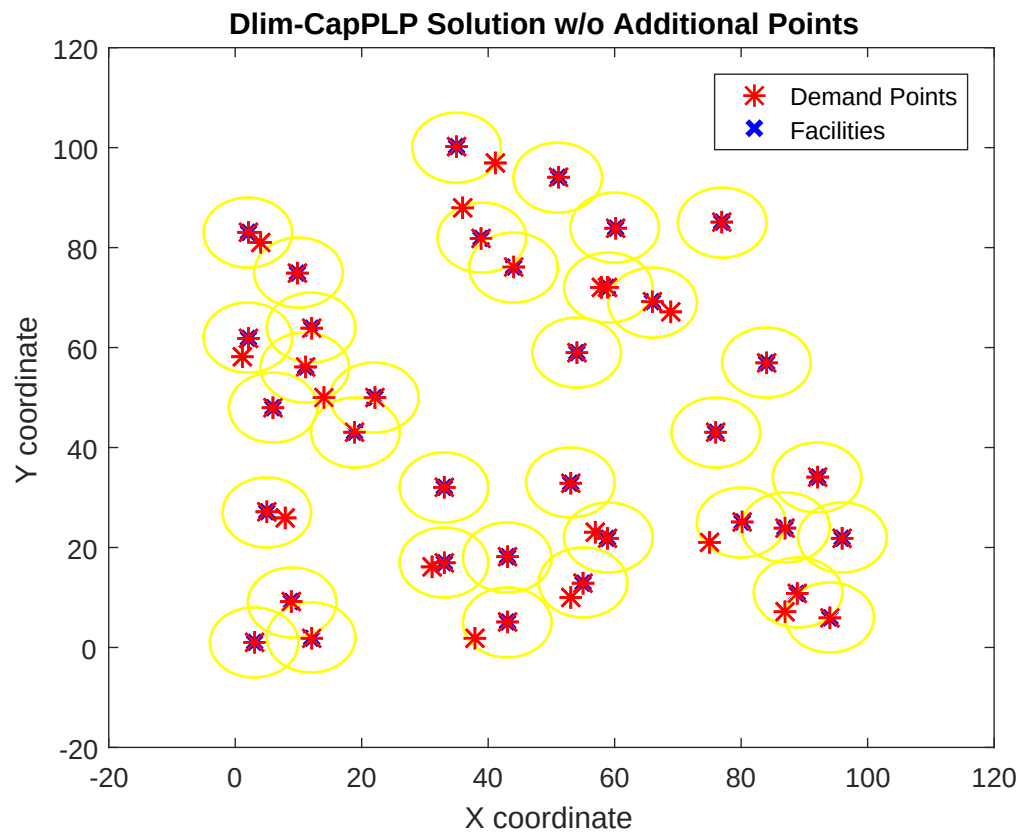


Figure 4.7: DLim-CapPLP Solution without Smart Points

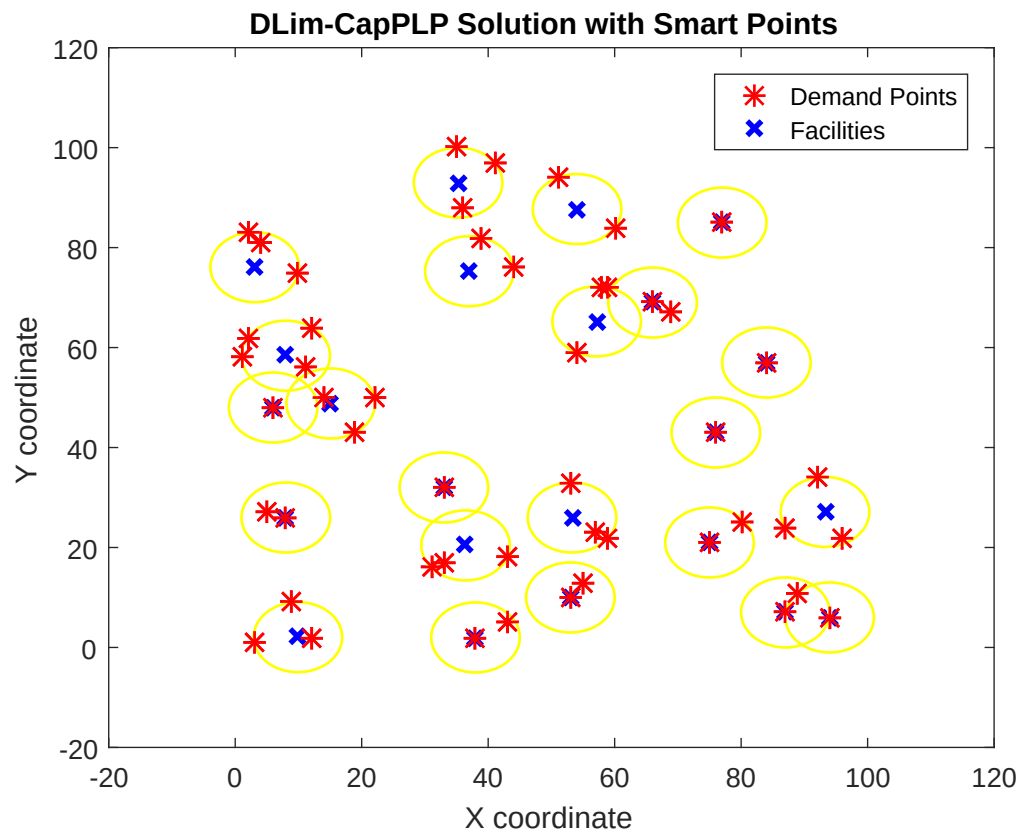


Figure 4.8: DLim-CapPLP Solution with Smart Points

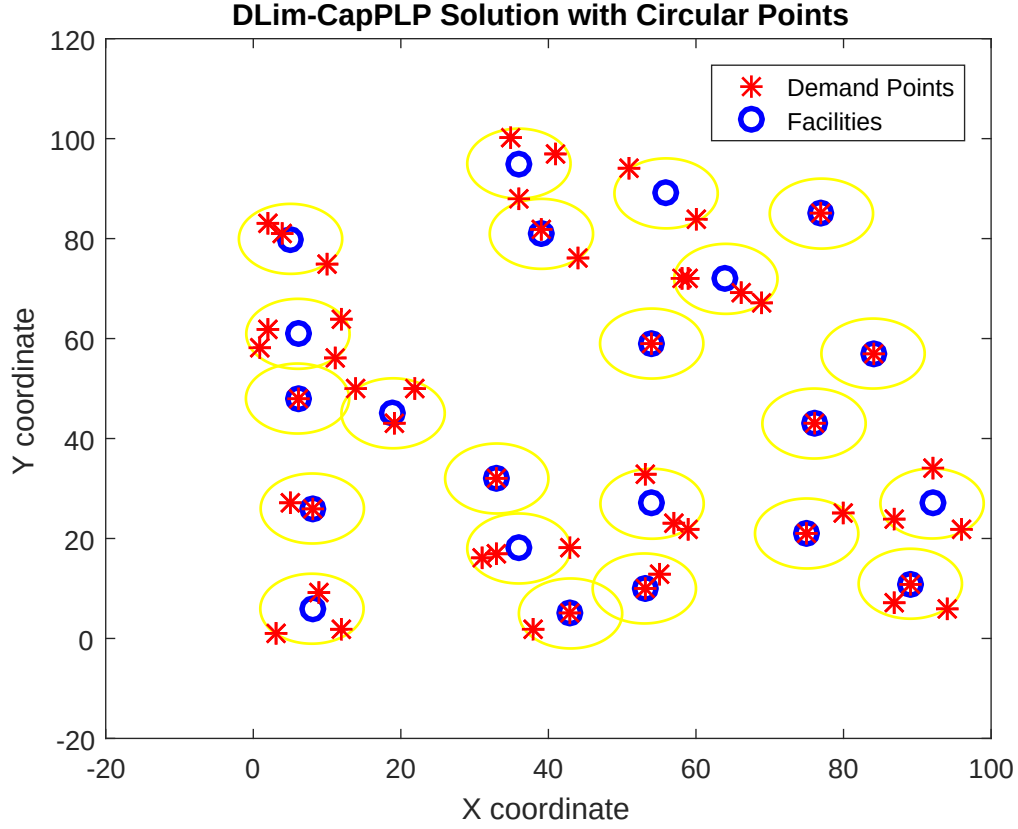


Figure 4.9: DLim-CapPLP Solution with Circular Points

DLim-CapPLP solution with additional circular points mentioned in Section 4.1.2 is also provided in Figure 4.9. Note that, this time we insert 400 additional points to the set of candidate location points as we add 45° spaced equiangular points for each demand point. Significant cost reduction is observed once again in the objective value in comparison with the discrete solution without additional points. The solution is improved even further, with objective value 68112.87, that is by an amount of %37.27. The required number of facilities to cover the demand points is decreased from 36 to 22 this time where as it was 23 with smart points. That is, 14 facilities are dropped while still ensuring the capacitated and distance limited coverage of the facilities. The computational experiments has shown that these two

point insertion methods compete in most instances and even yield very approximate results. However in large instances, computational burden and significant increase in solution times is observed with circular point insertion as much more points are employed than the smart points as candidate points. Note that 11 over 22 facilities are selected to be opened among the circular points.

4.3 Stage III: Iterative Location and Allocation Procedure in Continuous Space

In the last stage, we implement the Cooper's well known alternate heuristic that iteratively switches between location and allocation steps to find better allocation configuration of customers to the facilities. Considering that facilities are assumed to be capacitated and some of the demand points could not be assigned to their nearest facility, allocation steps of Cooper are adapted with respect to the priority value of each demand point. As a priority measure, we use the regret value as introduced by Mulvey et al. in [14]. The regret value is determined for each demand point as the difference between the distances to the first and second nearest feasible facility that it can be assigned without violating the capacity constraint. After each complete allocation step, to determine the new locations of the facilities, Weiszfeld's iterative convergence algorithm [4], [5] is applied with some adaptations to conform the distance limitation constraints. That is, to estimate the geometric median (point minimizing the sum of the distances between the demand points and the facility that serves to them) of each cluster, Weiszfeld's iterative algorithm is modified subject to the distance limitations. Location and allocation steps of Cooper's heuristic is revisited until no further improvement is possible and no further change occurs at least in the location of one facility.

4.3.1 Weiszfeld's Algorithm and Modified Geometric Median

Once the allocation configuration of demand points to the facilities are determined, the best possible location of the facility is to be determined. That is, when the clusters are formed, the best place for the facility should be determined in accordance with the spatial positions of all points belonging that cluster. This requirement typically defines the Weber Problem where the aim is to find a point on a continuous plane to minimize the sum of distances to n different end points from this point. Each demand/end point could have different weight/cost factors to be multiplied with corresponding distance and the objective function could be the sum of weighted distances of each demand point accordingly. When the associated weights of each end point are assumed to be the same, Weber Problem reduces to find the *geometric median* of the set of points.

Although it is straightforward to conceptually understand the *geometric median*, no mathematical formulation exists to exactly calculate it. In fact, it is proven that no exact algorithm or formulation could be driven [65]. Instead, approximation and convergence algorithms are used to find reasonable estimates. Indeed, approximation algorithms with iterative procedures yield strong estimates easily.

The reliability behind this type of approximation procedure arises from the convexity of the sum of distances to the sample points. Because the sum of convex distance functions is also convex, procedure usually does not stick around a local optima. Even though the algorithm is able to converge for most of the starting points, it can converge to a local optima only if an estimate in an iteration overlaps with one of the points forming the cluster. We basically start with a different initial point if such an overlap exists.

One of the most famous iterative procedures is Weiszfeld's approximation algorithm [4]. The iterative equations of Weiszfeld are provided in (4.12). $X_j^{(k)}$ and $Y_j^{(k)}$

represents the x and y coordinates of j^{th} facility in k^{th} iteration. $p_i = (a_i, b_i)$ is the spatial position of demand point i and $P_j^{(k)}$ is the position of the facility in the k^{th} iteration. So, $P_j^{(k)} = (X_j^{(k)}, Y_j^{(k)})$. The number of points forming the cluster is represented by n and the corresponding weights are w_i . The distance expressions appeared in the denominators as $d(P_j^{(k-1)}, p_i)$ states the Euclidean distance between the i^{th} demand point and position of the facility in previous $(k - 1)$ iteration.

$$X_j^{(k)} = \frac{\sum_{i=1}^n w_i a_i / d(P_j^{(k-1)}, p_i)}{\sum_{i=1}^{n_j} w_i / d(P_j^{(k-1)}, p_i)}, \quad Y_j^{(k)} = \frac{\sum_{i=1}^n w_i b_i / d(P_j^{(k-1)}, p_i)}{\sum_{i=1}^{n_j} w_i / d(P_j^{(k-1)}, p_i)} \quad (4.12)$$

When weights are assumed to be all the same, the approximation above in (4.12) defines the *geometric median* and reduces to (4.13). The graphical representation of geometric median of several clusters is shown in Figure 4.10.

$$GeomX^{(k)} = \frac{\sum_{i=1}^n a_i / d(P_j^{(k-1)}, p_i)}{\sum_{i=1}^{n_j} 1 / d(P_j^{(k-1)}, p_i)}, \quad GeomY^{(k)} = \frac{\sum_{i=1}^n b_i / d(P_j^{(k-1)}, p_i)}{\sum_{i=1}^{n_j} 1 / d(P_j^{(k-1)}, p_i)} \quad (4.13)$$

Although the assignment of the demand points to the facilities are carried out subject to the distance constraints in the previous stage, implementing Weiszfeld's algorithm directly, may lead to violations on D_{max} limitation. That is, even if the solution of DLim-PLP ensures the distances from demand points to facilities does not exceed D_{max} , Weiszfeld's equations does not guarantee any limitation on distance values. Thereby, while solving each individual Weber problem to find the best facility location for each cluster, we consider the problem below as proposed by Gökbayrak and Kocaman in [2] to model the distance limited Weber problem.

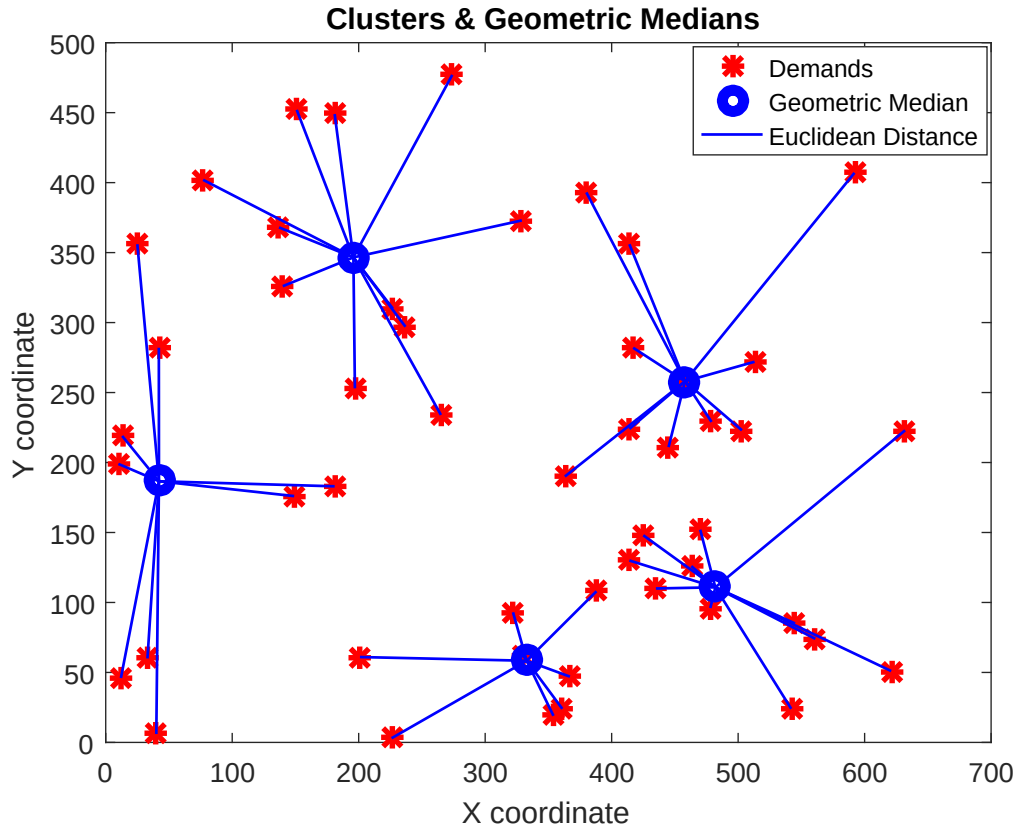


Figure 4.10: Clusters and Geometric Medians

Decision Variables

(x_p, y_p) :Spatial position of p^{th} facility

d_{ip} :Euclidean distance between i^{th} customer and p^{th} facility

Mathematical Model

$$\min \sum_{i \in C_p} d_{ip} \quad (4.14)$$

$$\text{s.t. } d_{ip} \leq D_{max} \quad \forall i \in C_p, \quad (4.15)$$

$$d_{ip}^x = a_i - x_p \quad \forall i \in C_p, \quad (4.16)$$

$$d_{ip}^y = b_i - y_p \quad \forall i \in C_p, \quad (4.17)$$

$$d_{ip}^2 \geq (d_{ip}^x)^2 + (d_{ip}^y)^2 \quad \forall i \in C_p, \quad (4.18)$$

$$d_{ip}^x, d_{ip}^y \geq 0 \quad \forall i \in C_p, \quad (4.19)$$

$$d_{ip} \geq 0 \quad \forall i \in C_p, \quad (4.20)$$

$$d_{ip}^x, d_{ip}^y \in R \quad \forall i \in C_p, \quad (4.21)$$

$$x_p, y_p \in R \quad (4.22)$$

We minimize the total distance of demand points to the facility (modified geometric median) in (4.14). We ensure with (4.15) that the distances from the demand points to the facility are bounded by an upper bound D_{max} . In (4.16) and (4.17), differences between demand points and the facility in continuous plane; $\mathbb{R}^2$, are defined in terms of x and y coordinates, respectively. Considering the differences defined in (4.16) and (4.17), the set of quadratic constraints are defined in (4.18) to represent Euclidean distances between the demand points and the facility. The Euclidean distances between the demand points and the facility of the cluster are defined to be non-negative in (4.19) and (4.20). The decision variables of the problem are represented in (4.21),(4.22) subject to the domains of each. C_p represents the cluster where p is the modified geometric median point minimizing the sum of distances. Note that, this problem is actually Weber Problem if the distance limitation constraints 4.15 are not introduced. Since we could not solve this problem exactly for large instances, we exploit the ideas of Weiszfeld with straightforward modification.

Algorithm 1: Modified Geometric Median

Data: $i, p, C_p, a_i, b_i, x_p, y_p, d_{ip}, D_{max}$

Result: Modified Geometric Median P

```
1 Find geometric median  $p$  as in 4.13;
2  $count \leftarrow 0$ ;
3  $P \leftarrow p$ ;
4  $d_{iP} \leftarrow d_{ip}$ ;
5 while  $\max_i(d_{iP}) > D_{max}$  do
6   Find farthest customer  $\{I^{far} \in C_p | d_{I^{far}P} = \max_{i \in C_p}(d_{iP})\}$ ;
7   Find the point  $P'$  on line segment  $\overline{I^{far}P}$  st.  $d(I^{far}, P') = D_{max}$ ;
8   Set  $P \leftarrow P'$ ;
9   Recalculate  $d_{iP} \quad \forall i \in C_p$  with (4.16), (4.17), (4.18);
10   $count \leftarrow count + 1$ ;
11  if  $count > 2 \times |C_p|$  &  $\max_{i \in C_p}(d_{iP}) > D_{max}$  then
12     $P \leftarrow p$ ;
13    break;
14  else
15     $P \leftarrow P$ ;
```

In Algorithm 1, we adaptively implement the iterative procedure of Weiszfeld with some modifications. Firstly, we determine the geometric median of the cluster C_p with respect to the equations defined in (4.13). After determining geometric median P which is a point minimizing the sum of distances to the demand points in the cluster, we find the farthest demand point to it $\{I^{far} \in C_p | d_{I^{far}P} = \max_{i \in C_p}(d_{iP})\}$. If the distance between the farthest demand point and P exceeds D_{max} , then we basically, slide the point P through demand point I^{far} , along the line segment $\overline{I^{far}P}$ till the distance between I^{far} and P becomes D_{max} ; and we call the new point P' . That is, we update P and carry it to P' in order to obey the D_{max} limitation. We recalculate d_{iP} in accordance with its update position and find the farthest point to

its new position. If the distance to the farthest point exceeds D_{max} , we apply the same projection and update its position once more. We repeat these straightforward steps until no further points lie outside the circle centered at P with radius D_{max} . Because we start with feasible clusters generated by the DLim-CapPLP solution, the projection we propose could find feasible solutions for all problem instances. Namely, the clusters formed by the discrete counterpart solution has already complied with the distance limitation constraints, our modification algorithm did not get stuck in the infinite loop. The projection we propose is represented geometrically by Figure 4.11.

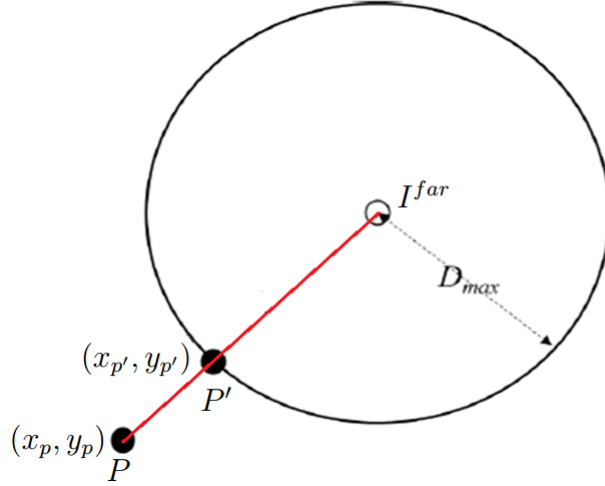


Figure 4.11: Modification for Geometric Median

4.3.2 Cooper's ALA Algorithm and Regret Priority

Once the allocation of the facilities are determined; for locations decisions of facilities, we implement the modified version of Weiszfeld's algorithm. The location steps are all followed by allocation steps. That is, we implement Cooper's well known Alternating Location Allocation (ALA) heuristic that switches between location and allocation steps to find better allocation form of customers to the facilities. Considering that the facilities are assumed to be capacitated in service they could supply, some demand points could not be allocated the nearest facility to them. That is why, the allocation steps of Cooper are modified in accordance with the priority value (ties broken arbitrarily) of each demand point. As a priority measure, regret value is used. The regret value for demand point i ; $R_i = d(i, P_{2nd}) - d(i, P_{1st})$, is defined for each demand point as the absolute difference in distance to the first (P_{1st}) and second (P_{2nd}) nearest feasible facility that it can be assigned without violating the capacity constraint. That is, the customers having higher regret values are more privileged in being assigned to their closest facility. The reasoning behind this approach is that assigning the customers having lower priority first, may possibly lead to the costly burden due to inappropriate formation of clusters. After allocation of each customer is completed, medians are recalculated so that allocation step is repeated for the newly calculated medians. In short, location and allocation steps of Cooper's heuristic is revisited until no further improvement is possible and no further change occurs in the locations of facility set.

Algorithm 2: Cooper's ALA Heuristic with Priority Measures

Data: $i, j, N, M, P, p, C_p, a_i, b_i, X_P, Y_P, d_{iP}, D_{max}$

Result: Re-Allocation $Z = (z_{ij})$ and Re-Location (P) of Facilities

```
1 OldSumDist  $\leftarrow$  BigNumber;
2 SumDist  $\leftarrow \sum_{i=1}^N \sum_{j=1}^M z_{ij} d_{ij}$ ;
3 OldMedians  $\leftarrow$  zeros(size(P));
4 NewMedians  $\leftarrow$  P;
5 while
    abs(sum(sum(OldMedians - NewMedians))) & SumDist < OldSumDist
do
6   Set OldMedians  $\leftarrow$  NewMedians;
7   Set OldSumDist  $\leftarrow$  SumDist;
8   Calculate  $R_i = d(i, P_{2nd}) - d(i, P_{1st}) \forall i$  ;
9   Fixing  $P = (P_1, P_2, P_3, \dots, P_M)$  assign each customer to the nearest
    possible facility in decreasing order of  $R_i$ , obtain new allocations as
     $Z = (z_{ij})$ ;
10  Set SumDist  $\leftarrow \sum_{i=1}^N \sum_{j=1}^M z_{ij} d_{ij}$  ;
11  Fixing Z, find modified geometric median  $P_j$  of each cluster individually
    as in Algorithm 1;
12  Set NewMedians  $\leftarrow$  P ;
```

In Algorithm 2, starting with an initial solution, we alternate between location and allocation steps of Cooper iteratively to locally search for a better solution. Initial solution can either be the solution of DLim-CapPLP (the discrete counter part of our problem) or the solution where we additionally implement the Algorithm 1 for all clusters yielded by DLim-CapPLP. In the former case we consider the medians as the DLim-CapPLP solution determines directly, whereas in the latter case we basically find the modified geometric median of each cluster and submit these locations to Algorithm 2 as medians. Demand points are assigned to the nearest median possible,

in the decreasing order of their regret values R_i . After the allocation is completed and clusters are reformed based upon this approach, medians are recalculated again in accordance with Algorithm 1. This procedure revisited as long as location of at least one facility changes and decrease in total distance proceeds.

To indicate the affect of the allocation approach with respect to the regret values, we consider an instance that is provided in the Figure 4.12 showing the location and allocation configuration of demand points to the facilities without D_{max} limitation. The solution indicated graphically in Figure 4.12 is obtained by implementing Cooper's alternate heuristic with multi-start random initial median set. That is, Algorithm 2 is executed predetermined number of times for different random initials. We observe that solving the discrete counterpart of this problem and mapping the medians of each cluster to the geometric median individually, yields completely the same solution even in longer time period. In other words, Cooper's heuristic with multi start random initials yields as good solutions as p -median heuristic (CPMP solution followed by individual Weiszfeld's heuristic for each cluster) in shorter times. Note that, one of the points served by the facility on upper left corner is not assigned to the nearest facility. That is because it is sorted at the end of the regret priority list. This instance is the first of the (50x5) problems of Osman and Christofides in [6] widely used in location literature.

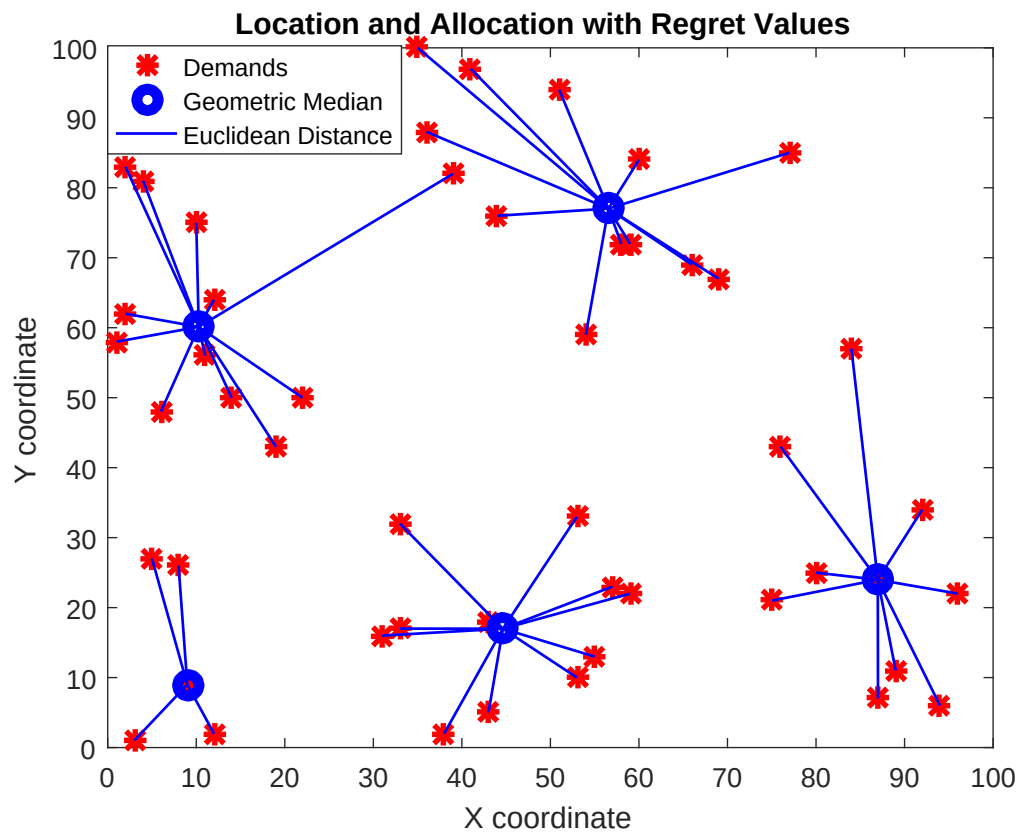


Figure 4.12: Location and Allocation with Regret Values

Chapter 5

Computational Results

In this chapter, we perform computational experiments on five different data sets. TA100 is the 100 customer instance being the largest of the instances generated by Negreiros and Palhano in [47]. This instance has been widely used in CPMP and CCP literature. SJC4 is the 402 customer instance being the largest of the instances generated by Lorena et al. [12]. This instance is also one of the most common instances used in CPMP, CCP, CCCP literature. Instances denoted with “U654” and “U1060” are the widely used and most common instances for MFWP and its other extensions. These 654 and 1060 customer problems are obtained from the TSP library [66]. These common data sets are also used by Luis et al. in [41] for CMFWP in the presence of fixed costs, Brimberg et al. in [33] for MFWP with constant opening costs, Zainuddin and Salhi in [39] for CMFWP, Luis et al. in [40] for CMFWP, Ajdad et al. in [46] for CMFWP with single source constraints, Gokbayrak and Kocaman in [2] for distance limited continuous location allocation problem and in many other related problems. The last one is a real life instance containing the data of 993 customer spread over a greenfield in Bonsaaso, which is located in the Amansie-West District of the Ashanti Region of Ghana.

In all instances the demand of the each customer w_i is assumed to be 1 considering the fact that all households consumes approximately same amount of resource. Indeed; TA100, U654 and U1060 data sets have already been generated with this setting. So, we don't alter anything on these data sets. However, in SJC4, enormously diversified and relatively big demand values are generated. Because these demand values are incoherent for the problem we propose, we assume all demands to be the same; $w_i = 1$, for this data set as well.

For the computational experiments, opening cost of a facility F , takes values $\{5000, 8000, 12000\}$ in compliance with the capacity values $C \{25, 50, 100\}$ respectively, for each data set. The capacity values are chosen among different type of many others, being some of the common sizes in stand alone systems like a diesel generator. Corresponding cost values are determined by observing the differentiating prices of public products. As is seen, economies of scale applies on the cost of serving unit amount of source. Cnn_{cost} is chosen in a similar manner, by observing the prices of public products such as single phase low voltage cable. Cnn_{cost} representing the unit cost of connection is set to 10 for all instances, as Kocaman et al. did in [67] for the unit price of low voltage cable. Thereby, it does not appear in the following tables.

D_{max} values are defined in accordance with the spatial distribution of demand points over the plane. That is, for each data set 6 different D_{max} values are chosen individually as a D_{max} value defined for one data set could be redundant for another one. Thereby, we perform a straightforward distance analysis for each data set. Basically, $N \times N$ distance matrix is obtained for each data set and minimum distance from each point to other than itself is obtained. After minimum distances (for each demand point) are found, the average and the largest of them are taken into consideration for determining the D_{max} values. More importantly, the number of customers that stays out of D_{max} range is also observed to determine the appropriate D_{max} values. Namely, number of customers out of range could be the same, for very different D_{max} values. Choosing such D_{max} values yields similar results and could

not contribute much to the analysis of the problem. Even if the largest minimum distance of the data set is a very big number (only one point is located far away), other minimum distances may stand in a narrow range (points can intensify in specific region). Hence, we consider also the average of minimum distances while selecting D_{max} values.

By considering the average minimum distance and largest minimum distances of the data set, we also observe the number of points that can not be reached (far more than D_{max} away) by any demand point other than itself for several D_{max} values. The set of such points can be expressed as:

$N_{out} = \{i \in N \mid \min_{k \in N \setminus \{i\}} d_{ik} > D_{max}\}$, where N represents the set of demand points. Consequently, 6 different values are determined in the decreasing order of number of customers ($|N_{out}|$) out of corresponding D_{max} range. That is, the minimum D_{max} is chosen as a number close to the average minimum distance. Then, increasing D_{max} values are determined observing the reduction in the number of customers out of range ($|N_{out}|$). The problem instances are provided in Tables 5.1 and 5.2.

As a result for 5 data sets, we consider 3 facility opening costs (3 capacity types accordingly) and 6 D_{max} distances; hence, work totally on 90 instances. Our computational experiments are performed on 2.59 GHz Intel Core i7-6500U CPU with 12GB RAM. The convergence threshold in finding geometric median mentioned in Section 4.3.1 is regarded as 0.001. The optimization models are solved by CPLEX 12.6.3 via MATLAB R2016a.

Table 5.1: Problem Instances

Problem Set 1					
Data Set	Distance Analysis		Cost	Cap	Dmax Nout
TA100	Average Min Distance	29.94	5K	25	30 41
					40 25
					45 18
					50 11
					60 6
					300 0
	Largest Min Distance	111.02	8K	50	30 41
					40 25
					45 18
					50 11
					60 6
					300 0
	Largest Min Distance	111.02	12K	100	30 41
					40 25
					45 18
					50 11
					60 6
					300 0

Problem Set 2					
Data Set	Distance Analysis		Cost	Cap	Dmax Nout
SJC4	Average Min Distance	81.39	5K	25	80 173
					90 131
					100 88
					120 26
					150 6
					500 0
	Largest Min Distance	216.3	8K	50	80 173
					90 131
					100 88
					120 26
					150 6
					500 0
	Largest Min Distance	216.3	12K	100	80 173
					90 131
					100 88
					120 26
					150 6
					500 0

Problem Set 3					
Data Set	Distance Analysis		Cost	Cap	Dmax Nout
U654	Average Min Distance	30.91	5K	25	30 109
					40 76
					50 53
					100 21
					150 13
					1000 0
	Largest Min Distance	600.37	8K	50	30 109
					40 76
					50 53
					100 21
					150 13
					1000 0
	Largest Min Distance	600.37	12K	100	30 109
					40 76
					50 53
					100 21
					150 13
					1000 0

Problem Set 4					
Data Set	Distance Analysis		Cost	Cap	Dmax Nout
U1060	Average Min Distance	147.47	5K	25	150 433
					200 240
					250 107
					300 40
					400 13
					2000 0
	Largest Min Distance	1839.6	8K	50	150 433
					200 240
					250 107
					300 40
					400 13
					2000 0
	Largest Min Distance	1839.6	12K	100	150 433
					200 240
					250 107
					300 40
					400 13
					2000 0

Table 5.2: Bonaasoo Instances

Problem Set 5					
Data Set	Distance Analysis		Cost	Cap	Dmax Nout
Bonaasoo	Average Min Distance	23.86	5K	25	20 295
					25 149
					30 84
					40 39
					50 26
					2000 0
			8K	50	20 295
					25 149
					30 84
					40 26
					50 39
					2000 0
	Largest Min Distance	1590.6	12K	100	20 295
					25 149
					30 84
					40 39
					50 26
					2000 0

We present the results for each instance with and without augmenting the candidate set of demand points. For each instance, model solution-only, model solution followed by modified Weiszfeld's algorithm as in Section 4.3.1 and model solution followed by modified Cooper's algorithm with regret priority as in Section 4.3.2 are presented in Tables 5.6, 5.7, 5.8, 5.11. These methods are denoted by S1, S2 and S3 in the tables, respectively. For each solution, objective value (cost), time (in seconds), number of facilities are provided. In addition to the augmentation methodologies discussed in Section 4.1.1 and Section 4.1.2, for comparison purposes, we also execute our algorithm by two other augmentation ways for 2 data sets (TA100 and SJC4). Firstly, we partition the plane to square grids and augment the set of candidate demand points with the centroid of these grids; secondly, we select random points over the continuous plane and employ them to observe the effect of smart and

circular point insertion.

5.1 PSCP Results

In this section we provide the results of the PSCP. In Section 4.1.1, we propose smart point insertion methodology by solving PSCP for a given D_{max} value. Then, the output of this problem is used as input in the following stage to solve the problem with augmented set of candidate demand points. PSCP solutions are reported in Table 5.3, where the first and second column represent data sets and corresponding D_{max} values. In the third column, the number of intersection points is presented excluding the demand points, for each D_{max} value. Note that intersection points together with the set of demand points constitutes the *CIPS* set and forms the domain of PSCP. Hence, the set to augment the candidate set of locations are chosen among the *CIPS*. The minimum required number of facilities to cover the data set under distance limitation is presented in the fourth column. Corresponding time values are provided in the last column.

Table 5.3: PSCP Solutions

Data Set	Dmax	CIPS-N	Solution	Time
TA100	30	349	44	0.6
	40	560	34	0.6
	45	702	27	0.6
	50	848	25	0.7
	60	1178	20	0.7
	300	9776	2	1
SJC4	80	2112	146	0.8
	90	2646	126	0.9
	100	3284	109	0.9
	120	4668	82	1
	150	7084	63	1.8
	500	54040	10	5.3
U654	30	10635	159	1.5
	40	15416	114	1.8
	50	18824	97	2
	100	34868	57	3.5
	150	44583	44	4.3
	1000	126800	7	15
U1060	150	5438	419	1.4
	200	8118	299	1.8
	250	11210	239	2
	300	14758	188	2.4
	400	22968	128	4.3
	2000	345786	12	430
Bonsaaso	20	4981	356	1.4
	25	7724	275	1.6
	30	10771	225	1.9
	40	17933	159	2.8
	50	26222	121	3.4
	2000	278538	6	73

5.2 Effect of Smart Point Augmentation

In this section we present the results of the solutions where the candidate set of demand points to solve DLim-CapPLP is augmented by an amount of $\sum_{c=1}^{|CIPS|} v_c$. The v_c variables are determined by solving PSCP as we discuss in Section 4.1.1 . We present in Table 5.5 and Table 5.4 the cost reduction by percentage in reference to the corresponding solutions where candidate set of demand points are not augmented at all.

Briefly, the first set of columns represents the problem instances. The second set of columns indicates the cost improvements by means of the additional points. S1, S2, S3 represent the cost differences by percentage for the model (DLim-CapPLP) solution, model solution followed by the modified Weiszfeld's algorithm and model solution followed by the modified Cooper's algorithm with regret values, respectively.

The solution results and mentioned comparison are presented in Tables 5.6, 5.7 and 5.8 for each data set and problem instance. In the tables, details of the problem instances are given in the first set of columns. Objective values (cost), the number of required facilities p and corresponding time values of the solutions for non-augmented set of candidate points are presented in the second set of columns. In the third set of columns; same information of the solutions for the augmented set of candidate points are provided. The last set of columns represents again the related cost improvements by percentage. Note that, cost reductions are simply obtained by calculating the difference between the costs of former and latter method divided by the cost of former method. Indeed, Tables 5.5 and 5.4 are the summary of Tables 5.6, 5.7 and 5.8.

Considerable reduction in cost is observed in almost all instances with smart point augmentation in the set of candidate location points . The improvements are reached up to nearly 40% for SJC4 instances. For TA100 and Bonsaaso instances,

improvements are obtained up to 36.29% and 35.2%, respectively. Similar drastic reduction in cost values yields improvements up to 27.72% and 28.82% in U654 and U1060 instances as well. Tables 5.4 and 5.5 summarizes all these improvements by percentage. When the number of facilities p is taken in to consideration for both cases in the Tables 5.6, 5.7 and 5.8, a significant decrease is observed in almost all instances. This decrease conduces to further reduction in cost (so increase improvement), as the cost of a facility increases. Consequently, employing relatively small number of new smart points, cause substantial decrease in cost without any computational burden and significant increase in solution times. In most of the instances, solution times do not exceed 10 seconds. Despite the outstanding performance of smart point insertion, solution times change only in the order of few seconds. Actually, this insignificant increase in time is mostly because of solving additional problem PSCP as discussed in Section 4.1.1 . In fact, even solving DLim-CapPLP with that small number of additional points does not affect the mentioned increase in time.

Table 5.4: Comparison of Smart Point Augmentation on Bonaaso

Problem Instance				% Cost Improvement with Smart Point Augmentation		
Data Set	F	C	Dmax	S1	S2	S3
Bonaaso	5K	25	20	32.99	33.42	33.45
			25	30.41	30.89	30.90
			30	25.76	26.25	26.31
			40	18.28	18.87	19.02
			50	12.21	12.61	12.71
			2000	-	-	-
		50	20	34.28	34.55	34.57
			25	32.30	32.60	32.61
			30	28.08	28.40	28.44
			40	21.54	21.93	22.03
			50	15.93	16.22	16.29
			2000	-	-	-
	12K	100	20	35.01	35.19	35.20
			25	33.37	33.58	33.58
			30	29.43	29.64	29.67
			40	23.51	23.79	23.85
			50	18.33	18.54	18.59
			2000	-	-	-

Table 5.5: Comparison of Smart Point Augmentation on TA100, SJC4, U654, U1060

Problem Instance				% Cost Improvement with Smart Point Augmentation			Problem Instance				% Cost Improvement with Smart Point Augmentation		
Data Set	F	C	Dmax	S1	S2	S3	Data Set	F	C	Dmax	S1	S2	S3
TA100	5K	25	30	27.01	27.53	27.53	SJC4	5K	25	80	30.94	32.29	32.29
			40	29.00	30.12	30.16				90	29.05	30.49	30.64
			45	30.96	31.42	31.43				100	26.14	27.85	27.91
			50	25.12	26.49	26.52				120	15.54	16.79	16.78
			60	22.01	22.89	22.87				150	5.05	6.06	5.94
			300	-	-	-				500	0.00	0.03	0.03
	8K	50	30	28.58	28.91	28.91		8K	50	80	36.20	37.06	37.07
			40	31.50	32.21	32.24				90	35.34	36.27	36.37
			45	34.21	34.50	34.51				100	33.26	34.38	34.42
			50	28.92	29.82	29.84				120	23.45	24.33	24.33
			60	26.61	27.20	27.18				150	11.87	12.88	12.80
			300	-	-	-				500	-	-	-
	12K	100	30	29.46	29.68	29.68		12K	100	80	39.22	39.81	39.82
			40	32.92	33.40	33.42				90	39.03	39.66	39.72
			45	36.08	36.29	36.29				100	37.52	38.30	38.32
			50	31.16	31.76	31.78				120	28.59	29.22	29.22
			60	29.38	29.79	29.78				150	17.29	18.04	18.10
			300	-	-	-				500	-	-	-
Problem Instance				% Cost Improvement with Smart Point Augmentation			Problem Instance				% Cost Improvement with Smart Point Augmentation		
Data Set	F	C	Dmax	S1	S2	S3	Data Set	F	C	Dmax	S1	S2	S3
U654	5K	25	30	20.71	21.03	20.98	U1060	5K	25	150	14.96	18.24	18.37
			40	22.78	23.27	23.25				200	8.43	11.79	11.83
			50	13.93	14.64	14.69				250	1.94	2.84	2.84
			100	4.27	5.02	5.13				300	0.17	0.26	0.27
			150	0.61	0.81	0.81				400	0.01	- 0.04	- 0.04
			1000	-	-	-				2000	-	-	-
	8K	50	30	22.36	22.54	22.51		8K	50	150	22.66	24.85	24.94
			40	25.72	26.05	26.04				200	18.31	21.30	21.41
			50	17.09	17.57	17.61				250	8.29	10.52	10.74
			100	6.98	7.43	7.45				300	2.12	3.05	3.04
			150	1.66	2.04	1.87				400	0.18	0.38	0.42
			1000	-	- 0.02	-				2000	-	-	0.00
	12K	100	30	23.32	23.45	23.43		12K	100	150	27.25	28.74	28.80
			40	27.48	27.72	27.70				200	25.17	27.29	27.37
			50	19.07	19.41	19.43				250	14.90	16.90	17.05
			100	8.91	9.23	9.28				300	6.06	7.35	7.55
			150	2.58	2.88	2.75				400	1.19	1.63	1.55
			1000	-	-	-				2000	-	-	-

Table 5.6: Effect of Smart Point Augmentation on TA100 and SJC4

Problem Instance				w/o Additional Points							Smart Points							%Cost Improvement		
				S1		S2		S3			S1		S2		S3					
Data Set	F	C	Dmax	p	Cost	Time	Cost	Time	Cost	Time	p	Cost	Time	Cost	Time	Cost	Time	S1	S2	S3
TA100	5K	25	30	64	327319	0.1	327166	0.1	327166	0.2	44	238912	0.7	237092	0.7	237092	0.8	27.01	27.53	27.53
			40	53	276675	0.1	276415	0.1	276415	0.2	34	196443	0.7	193171	0.7	193039	0.8	29.00	30.12	30.16
			45	45	240757	0.1	240609	0.1	240548	0.2	27	166213	0.7	165001	0.8	164938	0.8	30.96	31.42	31.43
			50	39	214413	0.1	214156	0.2	214096	0.2	25	160558	0.7	157418	0.7	157309	0.8	25.12	26.49	26.52
			60	31	179516	0.1	179025	0.2	178965	0.2	20	140009	0.7	138040	0.8	138040	0.8	22.01	22.89	22.87
			300	9	104833	0.3	104173	0.3	103981	0.4	9	104833	1.2	104173	1.2	103981	1.3	-	-	-
	8K	50	30	64	519319	0.1	519166	0.1	519166	0.2	44	370912	0.7	369092	0.8	369092	0.8	28.58	28.91	28.91
			40	53	435675	0.1	435415	0.2	435415	0.2	34	298443	0.7	295171	0.8	295039	0.8	31.50	32.21	32.24
			45	45	375757	0.1	375547	0.2	375547	0.2	27	247213	0.7	246002	0.8	245938	0.8	34.21	34.50	34.51
			50	39	331413	0.1	331156	0.2	331096	0.2	25	235558	0.7	232418	0.8	232309	0.8	28.92	29.82	29.84
			60	31	272516	0.1	272025	0.2	271965	0.2	20	200009	0.7	198040	0.8	198040	0.8	26.61	27.20	27.18
			300	7	126520	0.2	125382	0.3	125119	0.3	7	126520	1.2	125382	1.2	125119	1.3	-	-	-
SJC4	12K	100	30	64	775319	0.1	775167	0.2	775167	0.2	44	546912	0.7	545092	0.8	545092	0.8	29.46	29.68	29.68
			40	53	647674	0.1	647415	0.2	647415	0.2	34	434443	0.7	431171	0.8	431039	0.8	32.92	33.40	33.42
			45	45	555757	0.1	555609	0.2	555548	0.2	27	355213	0.7	354002	0.8	353938	0.8	36.08	36.29	36.29
			50	39	487413	0.1	487156	0.2	487096	0.2	25	335558	0.7	332418	0.8	332309	0.8	31.16	31.76	31.78
			60	31	396516	0.1	396025	0.2	395965	0.2	20	280009	0.7	278040	0.8	278040	0.8	29.38	29.79	29.78
			300	5	149249	0.3	148755	0.3	148755	0.3	5	149249	1.2	148755	1.3	148755	1.3	-	-	-
	5K	25	80	268	1426427	0.5	1425968	0.6	1425968	0.8	146	985060	1.5	965589	1.6	965589	1.9	30.94	32.29	32.29
			90	237	1299130	0.5	1298678	0.6	1298678	0.8	126	921736	1.5	902698	1.6	900775	1.9	29.05	30.49	30.64
			100	205	1172703	0.5	1171996	0.6	1171996	0.8	109	866195	1.4	845612	1.5	844886	1.7	26.14	27.85	27.91
			120	139	925881	0.5	923176	0.6	922829	0.8	82	781956	1.5	768147	1.6	768023	1.8	15.54	16.79	16.78
			150	95	791900	0.7	788998	0.8	788049	1	70	751902	2.3	741209	2.4	741209	2.5	5.05	6.06	5.94
			500	53	705524	1.3	702317	1.4	702055	1.6	53	705519	5.8	702106	5.9	701845	6	0.00	0.03	0.03
SJC4	8K	50	80	268	2230427	0.5	2229968	0.6	2229968	0.8	146	1423060	1.5	1403589	1.6	1403234	1.9	36.20	37.06	37.07
			90	237	2010130	0.5	2009678	0.6	2009678	0.8	126	1299736	1.5	1280698	1.6	1278775	1.9	35.34	36.27	36.37
			100	205	1787703	0.5	1786996	0.6	1786996	0.8	109	1193195	1.5	1172612	1.6	1171886	1.9	33.26	34.38	34.42
			120	139	1342881	0.5	1340176	0.6	1339829	0.8	82	1027956	1.5	1014095	1.6	1013817	1.9	23.45	24.33	24.33
			150	95	1076900	1.6	1073998	1.7	1073049	1.9	64	949073	2.3	935659	2.4	935659	2.5	11.87	12.88	12.80
			500	35	839879	1.3	838200	1.4	837702	1.5	35	839879	5.9	838200	6	837702	6.1	-	-	-
	12K	100	80	268	3302427	0.6	3301968	0.7	3301968	0.9	146	2007060	1.5	1987589	1.6	1987234	1.9	39.22	39.81	39.82
			90	237	2958295	0.5	2957696	0.6	2957696	0.6	126	1803736	1.5	1784698	1.6	1782775	1.9	39.03	39.66	39.72
			100	205	2607703	0.5	2606996	0.6	2606996	0.8	109	1629195	1.5	1608612	1.6	1607886	1.9	37.52	38.30	38.32
			120	139	1898881	0.5	1896176	0.6	1895829	0.8	82	1355956	1.5	1342095	1.6	1341817	1.9	28.59	29.22	29.22
			150	95	1456900	1.2	1453998	1.3	1453049	1.5	64	1205074	2.3	1191659	2.4	1190079	2.5	17.29	18.04	18.10
			500	30	971074	1.6	968967	1.7	968967	1.7	30	971074	6.2	968967	6.3	968967	6.3	-	-	-

Table 5.7: Effect of Smart Point Augmentation on U654 and U1060

Problem Instance				w/o Additional Points							Smart Points							%Cost Improvement			
				S1		S2		S3			S1		S2		S3						
Data Set	F	C	Dmax	p	Cost	Time	Cost	Time	Cost	Time	p	Cost	Time	Cost	Time	Cost	Time	S1	S2	S3	
U654	5K	25	30	213	1160853	7	1159677	14	1158695	26	159	920409	3	915846	3.2	915625	5	20.71	21.03	20.98	
			40	166	949483	2	947617	2	946768	3	114	733231	3.62	727073	4	726640	5	22.78	23.27	23.25	
			50	127	789033	3	785563	3	785563	3	97	679113	4	670575	4	670185	4.3	13.93	14.64	14.69	
			100	76	600209	1.8	596826	1.8	595861	2.1	66	574568	5.2	566855	5.2	565305	5.5	4.27	5.02	5.13	
			150	64	561653	2.2	558459	2.2	557494	2.5	62	558247	6.3	553918	6.3	552953	6.6	0.61	0.81	0.81	
			1000	57	546950	4.2	543188	4.3	542223	4.5	57	546950	19.2	543188	19.3	542223	19.5	-	-	-	
		8K	50	30	213	1799853	7.2	1798097	7.4	1797067	8.2	159	1397409	3.3	1392814	3.4	1392593	3.9	22.36	22.54	22.51
		40		166	1447483	2.1	1445698	2.2	1444808	2.8	114	1075231	3.6	1069073	3.6	1068639	4	25.72	26.05	26.04	
		50		127	1170033	4.4	1166564	4.5	1166564	4.8	97	970113	3.9	961575	4	961184	4.2	17.09	17.57	17.61	
		100		68	819381	2.2	811498	2.2	810781	2.5	58	762171	6.3	751200	6.4	750364	6.6	6.98	7.43	7.45	
		150		51	732135	2.4	728364	2.5	727120	2.7	48	719993	7.6	713509	7.7	713509	7.9	1.66	2.04	1.87	
		1000		34	678334	4	673445	4.1	672379	4.2	34	678334	19	673557	19.1	672379	19.2	-	- 0.02	-	
	12K	100	30	213	2651852	6.9	2650238	7	2649402	7.9	159	2033409	3.5	2028814	3.6	2028593	4.2	23.32	23.45	23.43	
			40	166	2111483	1.8	2109935	1.8	2108729	2.7	114	1531231	3.7	1525073	3.8	1524640	4.1	27.48	27.72	27.70	
			50	127	1678033	2.4	1674564	2.5	1674564	2.8	97	1358113	3.8	1349575	3.8	1349184	4.1	19.07	19.41	19.43	
			100	68	1091381	2	1082989	2.1	1082989	2.3	58	994171	6	983030	6	982434	6.1	8.91	9.23	9.28	
			150	51	936135	2.3	932521	2.3	931120	2.6	48	911992	7.3	905659	7.4	905509	7.7	2.58	2.88	2.75	
			1000	32	808712	4.1	803447	4.2	802543	4.3	32	808712	19.1	803447	19.2	802543	19.3	-	-	-	
		5K	25	150	466	3719466	3.1	3715771	3.2	3714632	6.4	423	3163034	6.6	3038052	6.8	3032202	7.4	14.96	18.24	18.37
				200	512	3217631	3.4	3209071	3.5	3209071	5.3	334	2946322	6.8	2830847	7	2829304	8.4	8.43	11.79	11.83
				250	378	2880564	3.4	2863843	3.6	2862439	5.3	327	2824538	6.6	2782538	6.8	2781135	8.3	1.94	2.84	2.84
				300	291	2723008	3.4	2709762	3.6	2709762	4.5	286	2718414	6.6	2702663	6.8	2702318	8.1	0.17	0.26	0.27
				400	257	2676531	3.2	2664825	3.3	2664825	4.2	256	2676302	8.5	2665982	8.6	2665982	9.3	0.01	- 0.04	- 0.04
				2000	257	2672050	7.3	2661101	7.4	2661101	8	247	2672050	438	2661101	438	2661101	439	-	-	-
8K	50	150	666	5717466	3	5713771	3.2	5712632	6.3	419	4421880	7.5	4293951	7.6	4288101	9.5	22.66	24.85	24.94		
		200	510	4748621	3.1	4740105	3.2	4739755	5.5	301	3879136	6.5	3730265	6.7	3724987	8.1	18.31	21.30	21.41		
		250	374	4007821	3.3	3991323	3.4	3988609	5.2	265	3675682	6.6	3571593	6.8	3560162	8.7	8.29	10.52	10.74		
		300	273	3564483	3.2	3545256	3.4	3543535	4.6	235	3488831	6.3	3437274	6.5	3435662	7.5	2.12	3.05	3.04		
		400	208	3355321	3.6	3337515	3.8	3334300	4.7	202	3349382	8.5	3324963	8.7	3320165	9.6	0.18	0.38	0.42		
		2000	182	3303099	7.6	3290877	7.7	3289961	8.6	182	3303099	438	3290877	438	3289882	439	-	-	-		
	12K	100	150	666	8381466	3	8377771	3.1	8376632	6.3	419	6097880	6.6	5969951	6.8	5964101	8.7	27.25	28.74	28.80	
			200	510	6788621	3.4	6780105	3.5	6779755	5.8	300	5079839	6.4	4929734	6.5	4924457	8	25.17	27.29	27.37	
			250	374	5503821	3.5	5487323	3.6	5484609	5.4	246	4683990	6.3	4560165	6.5	4549456	7.9	14.90	16.90	17.05	
			300	270	4651355	3.3	4630559	3.4	4624718	4.8	208	4369299	6.4	4290067	6.5	4275714	8.2	6.06	7.35	7.55	
			400	188	4140024	3.2	4112946	3.3	4107640	4.6	173	4090554	7.8	4045935	7.9	4043812	9.1	1.19	1.63	1.55	
			2000	136	3929079	7	3915332	7.1	3915017	7.8	136	3929079	438	3915332	438	3915017	439	-	-	-	

Table 5.8: Effect of Smart Point Augmentation on Bonsaaso

Problem Instance				w/o Additional Points							Smart Points							%Cost Improvement		
				S1		S2		S3			S1		S2		S3					
Data Set	F	C	Dmax	p	Cost	Time	Cost	Time	Cost	Time	p	Cost	Time	Cost	Time	Cost	Time	S1	S2	S3
Bonsaaso	5K	25	20	562	2874348	2.8	2873689	3	2873616	5.2	357	1925996	5.5	1913301	5.6	1912452	7.1	32.99	33.42	33.45
			25	430	2246992	2.8	2245594	2.9	2245594	4.2	277	1563646	5.2	1551964	5.3	1551810	6.3	30.41	30.89	30.90
			30	338	1819373	2.8	1817776	2.9	1817738	4.3	229	1350784	5.5	1340547	5.6	1339444	6.4	25.76	26.25	26.31
			40	226	1319390	2.9	1316186	3	1315988	4	163	1078219	6.4	1067789	6.5	1065708	7.6	18.28	18.87	19.02
			50	162	1064390	6.1	1058827	6.2	1057439	7.2	123	934436	6.9	925278	7	923070	7.9	12.21	12.61	12.71
			2000	65	762815	16.5	760509	16.6	760018	16.9	65	762815	95.5	760509	95.6	760018	95.9	-	-	-
	8K	50	20	562	4560348	2.6	4559689	2.7	4559616	5.3	357	2996996	5.5	2984301	5.6	2983452	7.1	34.28	34.55	34.57
			25	430	3536992	2.5	3535594	2.6	3535594	3.8	277	2394646	5.1	2382964	5.2	2382810	6.2	32.30	32.60	32.61
			30	338	2833373	2.6	2831776	2.7	2831738	4	229	2037784	5.6	2027547	5.7	2026444	6.5	28.08	28.40	28.44
			40	226	1997390	2.8	1994186	2.9	1993988	3.8	163	1567219	6.3	1556789	6.4	1554708	7.4	21.54	21.93	22.03
			50	162	1550390	6.7	1544827	6.8	1543439	7.8	123	1303436	6.9	1294278	7.1	1292070	7.9	15.93	16.22	16.29
			2000	51	926861	9.6	923074	9.7	922239	10	51	926861	88.6	923074	88.7	922239	89	-	-	-
	12K	100	20	562	6808348	2.9	6807689	3	6807616	5.2	357	4424996	5.6	4412301	5.7	4411452	7.2	35.01	35.19	35.20
			25	430	5256992	2.6	5255594	2.7	5255594	3.9	277	3502646	5.3	3490964	5.4	3490810	6.4	33.37	33.58	33.58
			30	338	4185373	2.6	4183776	2.7	4183738	4	229	2953784	5.8	2943547	5.9	2942444	6.9	29.43	29.64	29.67
			40	226	2901390	3.3	2898186	3.4	2897988	4.3	163	2219219	6.8	2208789	6.9	2206708	8.1	23.51	23.79	23.85
			50	162	2198390	10.3	2192827	10.4	2191439	11.6	123	1795436	7.2	1786278	7.3	1784070	8.2	18.33	18.54	18.59
			2000	40	1108754	12	1104490	12.1	1104074	12.4	40	1108754	91	1104490	91.1	1104074	91.4	-	-	-

5.3 Effect of Circular Point Augmentation

In this section we present the results of the solutions where the candidate set of demand points to solve DLim-CapPLP is augmented by an amount of $8 \times N$ with circular points. These new circular points are employed as 8 equiangular points on the circles centered at each demand point with radius D_{max} . That is, we employ 8 equiangular points being 45° spaced, for each demand point as mentioned in Section 4.1.2. Because this time we insert much more points compared to smart point insertion, we receive out of memory warnings for some of the instances in big data sets. Hence, we report our computational results for this second augmentation method on two data sets: TA100 and SJC4.

We present in Tables 5.9 and 5.10, the cost reduction by percentage in reference to the corresponding solutions where candidate set of demand points are not augmented at all. Similar to the previous tables in Section 5.2, the first set of columns represent the problem instances. The second set of columns shows the cost improvements by means of the additional circular points. S1, S2, S3 represent the cost differences by percentage for the model (DLim-CapPLP) solution, model solution followed by modified Weiszfeld's algorithm and model solution followed by modified Cooper's algorithm, respectively. The solution results and mentioned comparison are also presented in table (5.11), for these two data sets. In those tables, details of the problem instances, objective values (cost), number of required facilities (p) and corresponding time values of the solutions for both non-augmented and augmented set of candidate points are presented in the same order as previously mentioned. Last set of columns represents again the related cost improvements by percentage. Note that, cost reductions are obtained by calculating the difference between the costs of former and latter method divided by the cost of former method. Indeed, Tables 5.9 and 5.10 are the summary of Table 5.11.

Substantial reduction in cost is observed in almost all instances when circular points are added to the candidate set in all three methods. For SJC4 instances, improvements are obtained up to 39.09%. Similar reduction in cost values yields improvements up to 33.5% in TA100 instances. Tables 5.9 and 5.10 summarizes all these improvements by percentage. Note that, significant decrease is observed in p

Table 5.9: Comparison of Circular Point Augmentation on TA100

Problem Instance				% Cost Improvement with Circular Point Augmentation		
Data Set	F	C	Dmax	S1	S2	S3
TA100	5K	25	30	26.15	26.25	26.25
			40	30.10	30.34	30.34
			45	27.52	27.98	27.96
			50	26.24	26.71	26.74
			60	23.56	23.76	23.73
			300	0.58	0.56	0.43
	8K	50	30	27.45	27.52	27.52
			40	32.20	32.35	32.35
			45	30.41	30.69	30.69
			50	29.65	29.95	29.97
			60	27.63	27.77	27.75
			300	0.59	0.20	- 0.01
	12K	100	30	28.19	28.24	28.24
			40	33.39	33.50	33.50
			45	32.08	32.28	32.27
			50	31.65	31.86	31.87
			60	30.08	30.18	30.17
			300	0.22	- 0.02	- 0.02

value for most instances. New circular points also cause substantial decrease in the cost. However, these improvements could be reached with circular point insertion as long as at least 8 equiangular points are employed for each demand point. Because the amount of newly introduced points are very big in comparison with the previous method, memory problems and computation burden arise. We observe that both methods yields similar quality solutions by means of both objective value and time for TA100 instances. However, as the problem instance enlarges, time exponentially increases. Although it is not time efficient method in comparison with the smart

point insertion method, it competes in all instances and outperforms the former method in some instances defined with those data sets.

Table 5.10: Comparison of Circular Point Insertion on SJC4

Problem Instance				% Cost Improvement with Circular Point Augmentation		
Data Set	F	C	Dmax	S1	S2	S3
SJC4	5K	25	80	31.67	32.11	32.11
			90	29.31	29.75	29.77
			100	27.23	27.75	27.75
			120	15.25	15.68	16.06
			150	7.33	7.59	7.53
			500	0.40	0.19	0.23
	8K	50	80	36.26	36.54	36.54
			90	34.46	34.75	34.76
			100	33.13	33.48	33.48
			120	21.65	21.99	22.01
			150	12.99	13.20	13.29
			500	0.28	0.21	0.16
	12K	100	80	38.90	39.09	39.09
			90	37.49	37.68	37.68
			100	36.67	36.91	36.91
			120	25.85	26.09	26.10
			150	17.41	17.66	17.67
			500	0.16	0.10	0.12

We present the similar tables for comparing the method with smart point insertion method for 2 data sets: TA100 and SJC4. Similarly, the summary indicating the method differences by percentage is provided first in Table 5.12. Negative cost differences indicate the outperformance of smart point insertion, whereas positive differences show that circular point insertion yields better solutions. In Table 5.13, solutions are provided in detailed and better solutions are marked in bold. The format of both tables are similar to the previously presented tables as they contain same outputs of the resulted solutions.

Table 5.11: Effect of Circular Point Augmentation on TA100 and SJC4

Problem Instance				w/o Additional Points							Circular Points							%Cost Improvement		
				S1		S2		S3			S1		S2		S3					
Data Set	F	C	Dmax	p	Cost	Time	Cost	Time	Cost	Time	p	Cost	Time	Cost	Time	Cost	Time	S1	S2	S3
TA100	5K	25	30	64	327319	0.1	327166	0.1	327166	0.2	45	241741	0.2	241287	0.3	241287	0.3	26.15	26.25	26.25
			40	53	276675	0.1	276415	0.1	276415	0.2	34	193388	0.3	192561	0.3	192561	0.4	30.10	30.34	30.34
			45	45	240757	0.1	240609	0.1	240548	0.2	29	174490	0.4	173284	0.5	173284	0.5	27.52	27.98	27.96
			50	39	214413	0.1	214156	0.2	214096	0.2	25	158156	0.3	156960	0.4	156851	0.4	26.24	26.71	26.74
			60	31	179516	0.1	179025	0.2	178965	0.2	20	137228	0.5	136490	0.6	136490	0.6	23.56	23.76	23.73
			300	9	104833	0.3	104173	0.3	103981	0.4	8	104225	1.3	103586	1.3	103533	1.4	0.58	0.56	0.43
	8K	50	30	64	519319	0.1	519166	0.1	519166	0.2	45	376741	0.3	376287	0.4	376287	0.4	27.45	27.52	27.52
			40	53	435675	0.1	435415	0.2	435415	0.2	34	295388	0.3	294561	0.4	294561	0.4	32.20	32.35	32.35
			45	45	375757	0.1	375547	0.2	375547	0.2	29	261490	0.4	260284	0.5	260284	0.5	30.41	30.69	30.69
			50	39	331413	0.1	331156	0.2	331096	0.2	25	233156	0.3	231960	0.4	231851	0.4	29.65	29.95	29.97
			60	31	272516	0.1	272025	0.2	271965	0.2	20	197228	0.5	196490	0.6	196490	0.6	27.63	27.77	27.75
			300	7	126520	0.2	125382	0.3	125119	0.3	7	125775	1.4	125127	1.5	125127	1.5	0.59	0.20	- 0.01
SJC4	12K	100	30	64	775319	0.1	775167	0.2	775167	0.2	45	556741	0.3	556287	0.4	556287	0.4	28.19	28.24	28.24
			40	53	647674	0.1	647415	0.2	647415	0.2	34	431388	0.3	430561	0.4	430561	0.4	33.39	33.50	33.50
			45	45	555757	0.1	555609	0.2	555548	0.2	29	377490	0.4	376284	0.5	376284	0.5	32.08	32.28	32.27
			50	39	487413	0.1	487156	0.2	487096	0.2	25	333156	0.3	331960	0.4	331851	0.4	31.65	31.86	31.87
			60	31	396516	0.1	396025	0.2	395965	0.2	20	277228	0.6	276490	0.7	276490	0.7	30.08	30.18	30.17
			300	5	149249	0.3	148755	0.3	148755	0.3	5	148914	1.6	148781	1.7	148781	1.7	0.22	- 0.02	- 0.02
	5K	25	80	268	1426427	0.5	1425968	0.6	1425968	0.8	149	974620	3.6	968099	3.7	968099	3.8	31.67	32.11	32.11
			90	237	1299130	0.5	1298678	0.6	1298678	0.8	133	918365	6.7	912321	6.8	912126	7	29.31	29.75	29.77
			100	205	1172703	0.5	1171996	0.6	1171996	0.8	114	853399	4.1	846752	4.2	846752	4.3	27.23	27.75	27.75
			120	139	925881	0.5	923176	0.6	922829	0.8	90	784656	8.4	778432	8.5	774659	8.7	15.25	15.68	16.06
			150	95	791900	0.7	788998	0.8	788049	1	70	733832	7.8	729141	7.9	728741	8.1	7.33	7.59	7.53
			500	53	705524	1.3	702317	1.4	702055	1.6	52	702701	8.9	700981	9	700442	9.1	0.40	0.19	0.23
SJC4	8K	50	80	268	2230427	0.5	2229968	0.6	2229968	0.8	149	1421620	3.7	1415099	3.8	1415099	4	36.26	36.54	36.54
			90	237	2010130	0.5	2009678	0.6	2009678	0.8	133	1317345	17.3	1311325	17.4	1311131	17.6	34.46	34.75	34.76
			100	205	1787703	0.5	1786996	0.6	1786996	0.8	114	1195399	14.3	1188752	14.4	1188752	14.6	33.13	33.48	33.48
			120	139	1342881	0.5	1340176	0.6	1339829	0.8	89	1052092	9.5	1045458	9.6	1044946	9.8	21.65	21.99	22.01
			150	95	1076900	1.6	1073998	1.7	1073049	1.9	67	937039	63	932192	63.1	930482	63.4	12.99	13.20	13.29
			500	35	839879	1.3	838200	1.4	837702	1.5	36	837518	11	836398	11.1	836398	11.1	0.28	0.21	0.16
	12K	100	80	268	3302427	0.6	3301968	0.7	3301968	0.9	149	2017620	3.9	2011099	4	2011099	4.2	38.90	39.09	39.09
			90	237	2958295	0.5	2957696	0.6	2957696	0.6	133	1849345	22.5	1843325	22.6	1843131	22.8	37.49	37.68	37.68
			100	205	2607703	0.5	2606996	0.6	2606996	0.8	119	1651399	53.3	1644752	53.4	1644752	53.5	36.67	36.91	36.91
			120	139	1898881	0.5	1896176	0.6	1895829	0.8	89	1408092	73.4	1401458	73.5	1400946	73.7	25.85	26.09	26.10
			150	95	1456900	1.2	1453998	1.3	1453049	1.5	66	1203203	533	1197274	533	1196357	534	17.41	17.66	17.67
			500	30	971074	1.6	968967	1.7	968967	1.7	30	969504	25	968.040	25.1	967850	25.2	0.16	0.10	0.12

Table 5.12: Comparison of Smart and Circular Point Augmentation on TA100 and SJC4

Problem Instance				% Cost Diff		
Data Set	F	C	Dmax	S1	S2	S3
TA100	5K	25	30	- 1.18	- 1.77	- 1.77
			40	1.56	0.32	0.25
			45	- 4.98	- 5.02	- 5.06
			50	1.50	0.29	0.29
			60	1.99	1.12	1.12
			300	0.58	0.56	0.43
	8K	50	30	- 1.57	- 1.95	- 1.95
			40	1.02	0.21	0.16
			45	- 5.78	- 5.81	- 5.83
			50	1.02	0.20	0.20
			60	1.39	0.78	0.78
			300	0.59	0.20	- 0.01
	12K	100	30	- 1.80	- 2.05	- 2.05
			40	0.70	0.14	0.11
			45	- 6.27	- 6.29	- 6.31
			50	0.72	0.14	0.14
			60	0.99	0.56	0.56
			300	0.22	- 0.02	- 0.02
SJC4	5K	25	80	1.06	- 0.26	- 0.26
			90	0.37	- 1.07	- 1.26
			100	1.48	- 0.13	- 0.22
			120	- 0.35	- 1.34	- 0.86
			150	2.40	1.63	1.68
			500	0.40	0.16	0.20
	8K	50	80	0.10	- 0.82	- 0.85
			90	- 1.35	- 2.39	- 2.53
			100	- 0.18	- 1.38	- 1.44
			120	- 2.35	- 3.09	- 3.07
			150	1.27	0.37	0.55
			500	0.28	0.21	0.16
	12K	100	80	- 0.53	- 1.18	- 1.20
			90	- 2.53	- 3.28	- 3.39
			100	- 1.36	- 2.25	- 2.29
			120	- 3.84	- 4.42	- 4.41
			150	0.16	- 0.47	- 0.53
			500	0.16	0.10	0.12

Table 5.13: Detailed Comparison of Smart and Circular Point Augmentation on TA100 and SJC4

Problem Instance				Smart Points							Circular Points							%Cost Diff				
				S1		S2		S3			S1		S2		S3							
Data Set	F	C	Dmax	p	Cost	Time	Cost	Time	Cost	Time	p	Cost	Time	Cost	Time	Cost	Time	S1	S2	S3		
6L	TA100	5K	25	30	44	238912	0.7	237092	0.7	237092	0.8	45	241741	0.2	241287	0.3	241287	0.3	- 1.18	- 1.77	- 1.77	
				40	34	196443	0.7	193171	0.7	193039	0.8	34	193388	0.3	192561	0.3	192561	0.4	1.56	0.32	0.25	
				45	27	166213	0.7	165001	0.8	164938	0.8	29	174490	0.4	173284	0.5	173284	0.5	- 4.98	- 5.02	- 5.06	
				50	25	160558	0.7	157418	0.7	157309	0.8	25	158156	0.3	156960	0.4	156851	0.4	1.50	0.29	0.29	
				60	20	140009	0.7	138040	0.8	138040	0.8	20	137228	0.5	136490	0.6	136490	0.6	1.99	1.12	1.12	
				300	9	104833	1.2	104173	1.2	103981	1.3	8	104225	1.3	103586	1.3	103533	1.4	0.58	0.56	0.43	
		8K	50	30	44	370912	0.7	369092	0.8	369092	0.8	45	376741	0.3	376287	0.4	376287	0.4	- 1.57	- 1.95	- 1.95	
				40	34	298443	0.7	295171	0.8	295039	0.8	34	295388	0.3	294561	0.4	294561	0.4	1.02	0.21	0.16	
				45	27	247213	0.7	246002	0.8	245938	0.8	29	261490	0.4	260284	0.5	260284	0.5	- 5.78	- 5.81	- 5.83	
				50	25	235558	0.7	232418	0.8	232309	0.8	25	233156	0.3	231960	0.4	231851	0.4	1.02	0.20	0.20	
				60	20	200009	0.7	198040	0.8	198040	0.8	20	197228	0.5	196490	0.6	196490	0.6	1.39	0.78	0.78	
				300	7	126520	1.2	125382	1.2	125119	1.3	7	125775	1.4	125127	1.5	125127	1.5	0.59	0.20	- 0.01	
	SJC4	12K	100	30	44	546912	0.7	545092	0.8	545092	0.8	45	556741	0.3	556287	0.4	556287	0.4	- 1.80	- 2.05	- 2.05	
				40	34	434443	0.7	431171	0.8	431039	0.8	34	431388	0.3	430561	0.4	430561	0.4	0.70	0.14	0.11	
				45	27	355213	0.7	354002	0.8	353938	0.8	29	377490	0.4	376284	0.5	376284	0.5	- 6.27	- 6.29	- 6.31	
				50	25	335558	0.7	332418	0.8	332309	0.8	25	333156	0.3	331960	0.4	331851	0.4	0.72	0.14	0.14	
				60	20	280009	0.7	278040	0.8	278040	0.8	20	277228	0.6	276490	0.7	276490	0.7	0.99	0.56	0.56	
				300	5	149249	1.2	148755	1.3	148755	1.3	5	148914	1.6	148781	1.7	148781	1.7	0.22	- 0.02	- 0.02	
		5K	25	80	146	985060	1.5	965589	1.6	965589	1.9	149	974620	3.6	968099	3.7	968099	3.8	1.06	- 0.26	- 0.26	
				90	126	921736	1.5	902698	1.6	900775	1.9	133	918365	6.7	912321	6.8	912126	7	0.37	- 1.07	- 1.26	
				100	109	866195	1.4	845612	1.5	844886	1.7	114	853399	4.1	846752	4.2	846752	4.3	1.48	- 0.13	- 0.22	
				120	82	781956	1.5	768147	1.6	768023	1.8	90	784656	8.4	778432	8.5	774659	8.7	- 0.35	- 1.34	- 0.86	
				150	70	751902	2.3	741209	2.4	741209	2.5	70	733832	7.8	729141	7.9	728741	8.1	2.40	1.63	1.68	
				500	53	705519	5.8	702106	5.9	701845	6	52	702701	8.9	700981	9	700442	9.1	0.40	0.16	0.20	
		8K	50	80	146	1423060	1.5	1403589	1.6	1403234	1.9	149	1421620	3.7	1415099	3.8	1415099	4	0.10	- 0.82	- 0.85	
				90	126	1299736	1.5	1280698	1.6	1278775	1.9	133	1317345	17.3	1311325	17.4	1311131	17.6	- 1.35	- 2.39	- 2.53	
				100	109	1193195	1.5	1172612	1.6	1171886	1.9	114	1195399	14.3	1188752	14.4	1188752	14.6	- 0.18	- 1.38	- 1.44	
				120	82	1027956	1.5	1014095	1.6	1013817	1.9	89	1052092	9.5	1045458	9.6	1044946	9.8	- 2.35	- 3.09	- 3.07	
				150	64	949073	2.3	935659	2.4	935659	2.5	67	937039	63	932192	63.1	930482	63.4	1.27	0.37	0.55	
				500	35	839879	5.9	838200	6	837702	6.1	36	837518	11	836398	11.1	836398	11.1	0.28	0.21	0.16	
	12K			100	80	146	2007060	1.5	1987589	1.6	1987234	1.9	149	2017620	3.9	2011099	4	2011099	4.2	- 0.53	- 1.18	- 1.20
					90	126	1803736	1.5	1784698	1.6	1782775	1.9	133	1849345	22.5	1843325	22.6	1843131	22.8	- 2.53	- 3.28	- 3.39
					100	109	1629195	1.5	1608612	1.6	1607886	1.9	119	1651399	53.3	1644752	53.4	1644752	53.5	- 1.36	- 2.25	- 2.29
					120	82	1355956	1.5	1342095	1.6	1341817	1.9	89	1408092	73.4	1401458	73.5	1400946	73.7	- 3.84	- 4.42	- 4.41
					150	64	1205074	2.3	1191659	2.4	1190079	2.5	66	1203203	533	1197274	533	1196357	534	0.16	- 0.47	- 0.53
					500	30	971074	6.2	968967	6.3	968967	6.3	30	969504	25	968040	25.1	967850	25.2	0.16	0.10	0.12

We generally observe that the solutions with less number of facilities outperform the other for the problem instances defined on these two data sets. Less number of facility solutions are generally yielded by smart points insertion method. In 21 over 36 instances, smart point insertion could find less number of facilities where 19 of them are better than circular point insertion in terms of objective value. Interestingly, better solutions could be attained by circular point insertion even if more required number of facilities determined in two instances. On the other hand, only in 2 instances, less number of facilities could be found by circular point insertion. They both could able to find the same number of facilities in 13 instances. In 11 of these 13 instances, the difference between the objectives of 2 methods does not exceed 1%. Relatively big percentages are observed in between the solutions where number of facilities are found to be different. 19 out of 36 solutions are found to be better by smart point insertion, where 15 better solutions could be found by circular point insertion. In 2 instances very near (0.01% diff) solutions could be obtained by both methods. Even though 21 better solutions could be found by circular point insertion for S1, this score downgrades to 16 and 15 in S2 and S3, respectively. The differences of the instances in which smart point insertion get the better, are generally greater than the differences of the instances circular point insertion could do. All better solutions are given in bold in Tables 5.12 and 5.13.

For TA100 instances, circular point insertion method can be performed in shorter computational times as smart point insertion method additionally need to solve the PSCP. On the other hand, as problem instances enlarge; so in SJC4 instances, DLim-CapPLP can be solved with circular point insertion in relatively longer times. Additionally, as D_{max} constraint relaxes, even more computational time is required to solve the problem as the additional number of inserted points is much more than the inserted smart points. Smart point insertion method is observed to take shorter and stationary durations as the number of additional points is relatively small and the PSCP solution can be obtained for large instances without any computational burden as well.

5.4 Effect of Grid Points and Random Points Augmentation

Both augmentation methodologies discussed so far crucially affect the solution quality in all instances. In order to experiment the effect of augmentation scheme, we perform two other augmentation approaches to see whether the ways we previously proposed are wise enough or not. To that end, we enlarge the domain comparably and generate new set of candidate points. Hence, we solve DLim-CapPLP model subject to these domains and run our algorithm for two data sets: TA100 and SJC4 once again.

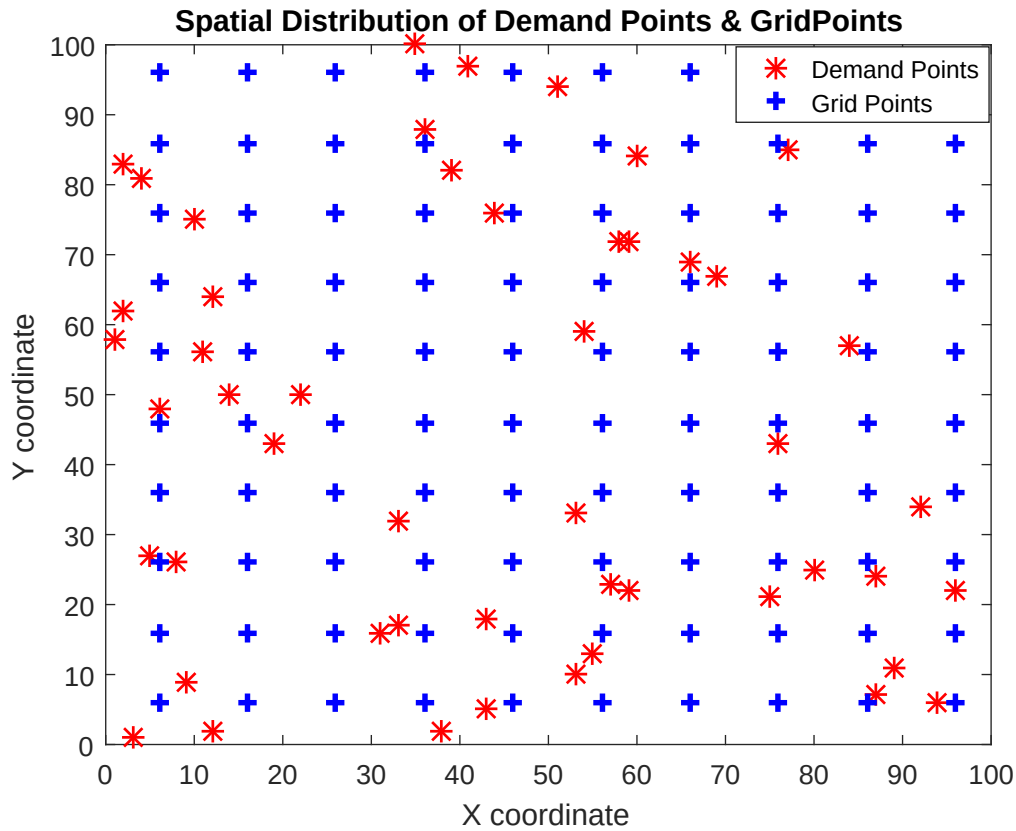


Figure 5.1: Grid Points

We simply partition the rectangular planes in to square grids so as to employ sufficient number of new candidates. The sizes of the squares are determined so that the desired number of new points could be produced. Simply, small sized grids yields more points. For TA100 instances, 17.5 sized square grids produces 1008 new points and for SJC4 instances 49.5 sized square grids produces 3640 new points. Basically, $\max_{i \in N} a_i$, $\min_{i \in N} a_i$, $\max_{i \in N} b_i$, $\min_{i \in N} b_i$, are taken in to consideration to draw the exact frame of the rectangular area. Note that, upper bounds, $\max_{i \in N} a_i$ and $\max_{i \in N} b_i$, could be smoothly ceiled to the nearest integer multiple of the determined grid size. Once the extreme points are set in this manner, new points inserted equally sized, being the first and the last (in a row) located far half size away the frame edge. The graphical representation of this type of augmentation scheme is provided in Figure 5.1.

As another benchmark tool, we try the augmentation scheme of employing random points. We consider the number of grid points to determine the cardinality of the random points. Hence for TA100 and SCJ4 instances, we select again 1008 and 3640 randomly generated points to augment the set of candidate points, respectively. Definitely, results of the solutions where domain of the problem formed by such way, would change depending on the quality of random selections. That is why for each instance, we make 10 random selections and report the best solution obtained among all.

In Table 5.14, S3 solutions of each instance are reported. That is, D-LimPLP is solved with corresponding augmented set of candidate points, then Cooper's and Weiszfeld's modified algorithms are performed as previously discussed. Hence, table summarizes the comparisons for the complete algorithm with the mentioned benchmark approaches. First set of columns represents problem instance as usual. Second set of columns contains the objective and time values of the solutions obtained without augmenting the set of candidate points at all. Hence, its denoted as baseline. Following 4 set of columns present objective value (cost), time, additional number

of newly employed points and difference in cost by percentage; for smart point insertion, circular point insertion, grid points insertion and random point insertion, respectively. Percentage differences are obtained by the decrease in cost with respect to the baseline cost divided by the baseline cost. The best solution for each instance is marked as bold.

Although the number of new points employed by the smart point insertion is a lot smaller than the others, this method outperforms the other methods in most of the instances defined for these two data sets. Basically, it only competes with circular point insertion as we discussed before. Although the benchmark solutions dominate the baseline solutions as expected, they could not dominate the solutions of smart point and circular point insertion in any instance. Despite the substantial augmentation of grid points and random points insertion, explicit effect of smart point augmentation could not be reached. Random points also augment the set of candidate points by multiple times more than smart points do. However, this does not cause a remarkable improvement in objective value. Time values are in the order of few seconds with smart point augmentation, whereas it increases multiple times with grid and random points, especially as the problem instance enlarges. Grid point insertion method seems to yield better solutions than the random point insertion method do in most instances. Nevertheless, solutions with random points could be deemed as reasonable in reference to it, when the randomness of the augmentation is taken into consideration. Even in few instances random points insertion outperforms grid solution. In fact, most of the instances that D_{max} is relaxed, random points insertion yields better solutions. As problem instance enlarges, time values for few instances increase for both augmentation methods as they employ the same number of new points. Note that, as the D_{max} value is adequately relaxed, all methods yield very close solutions and the power of smart and circular point insertion disappears.

In Table 5.15, we simply report the required number of facilities and cost differences by percent in comparison with the baseline cost. Hence, we summarize the

highlights of Table 5.14 with Table 5.15. To explain the significant differences in the objective value, we provide p values with cost differences together. The main reason in the substantial cost improvements is observed to be resulted by the decrease in number of facilities. When we specifically focused on the solutions that an alternative method yields relatively better solution, we see that the number of facilities are less than the other alternative method yields. Similarly, worse solutions produced by these alternative methods determine the much more number of facilities to be opened. Clearly, smart point insertion method outperforms the others in both solution quality and computational time for the instances generated in these data sets. Even though circular point insertion method competes with it, for large instances the computational burden of this method complicates the problem.

Another point to consider is that even in the case where D_{max} constraint is redundant, determined number of facilities are much more than the required amount to fulfill the total demand. As capacity of facility increases, a decrease in the number of facilities is observed. However, this decrease could not trigger a reduction in overall cost because increase in distribution costs clearly dominates the decrease in fixed opening costs. That is why, the model does not determine less number of facilities even in the instances where D_{max} is relaxed. Therefore, in bigger capacity instances total costs are much more than the smaller capacity instances despite the economies of scale of a facility.

Table 5.14: Comparison of the Methods with Grid Points and Random Points Augmentation

Problem Instance				Baseline		Smart Points				Circular Points				Grids				Random Points			
				S3		S3				S3				S3				S3			
Data Set	F	C	Dmax	Cost	Time	Cost	Time	Added	% Diff	Cost	Time	Added	% Diff	Cost	Time	Added	% Diff	Cost	Time	Added	% Diff
TA100	5K	25	30	327166	0.2	237092	0.8	44	27.53	241287	0.3	800	26.25	263940	0.7	1008	19.33	264624	0.5	1008	19.12
			40	276415	0.2	193039	0.8	34	30.16	192561	0.4	800	30.34	201608	0.6	1008	27.06	210335	0.5	1008	23.91
			45	240548	0.2	164938	0.8	27	31.43	173284	0.5	800	27.96	180568	0.6	1008	24.93	180488	0.7	1008	24.97
			50	214096	0.2	157309	0.8	25	26.52	156851	0.4	800	26.74	163405	0.6	1008	23.68	168157	0.7	1008	21.46
			60	178965	0.2	138040	0.8	20	22.87	136490	0.6	800	23.73	146612	1.3	1008	18.08	147369	0.8	1008	17.65
			300	103981	0.4	103981	1.3	2	-	103533	1.4	800	0.43	103584	2.7	1008	0.38	103514	2.6	1008	0.45
		50	30	519166	0.2	369092	0.8	44	28.91	376287	0.4	800	27.52	413940	0.6	1008	20.27	414624	0.5	1008	20.14
			40	435415	0.2	295039	0.8	34	32.24	294561	0.4	800	32.35	309608	0.8	1008	28.89	324335	0.5	1008	25.51
			45	375547	0.2	245938	0.8	27	34.51	260284	0.5	800	30.69	273568	0.5	1008	27.15	273488	0.6	1008	27.18
			50	331096	0.2	232309	0.8	25	29.84	231851	0.4	800	29.97	244405	0.6	1008	26.18	252157	0.6	1008	23.84
			60	271965	0.2	198040	0.8	20	27.18	196490	0.6	800	27.75	212612	0.7	1008	21.82	216369	4.5	1008	20.44
			300	125119	0.3	125119	1.3	2	-	125127	1.5	800	- 0.01	125127	3.2	1008	- 0.01	125120	3.1	1008	- 0.00
	12K	100	30	775167	0.2	545092	0.8	44	29.68	556287	0.4	800	28.24	613940	0.5	1008	20.80	614624	0.5	1008	20.71
			40	647415	0.2	431039	0.8	34	33.42	430561	0.4	800	33.50	453608	0.6	1008	29.94	476335	0.5	1008	26.43
			45	555548	0.2	353938	0.8	27	36.29	376284	0.5	800	32.27	397568	0.6	1008	28.44	397488	0.7	1008	28.45
			50	487096	0.2	332309	0.8	25	31.78	331851	0.4	800	31.87	352405	0.6	1008	27.65	364157	0.6	1008	25.24
			60	395965	0.2	278040	0.8	20	29.78	276490	0.7	800	30.17	300612	1.3	1008	24.08	308369	5.5	1008	22.12
			300	148755	0.3	148755	1.3	2	-	148781	1.7	800	- 0.02	148824	3.4	1008	- 0.05	148797	3.4	1008	- 0.03
		25	80	1425968	0.8	965589	1.9	146	32.29	968099	3.8	3216	32.11	1063101	6.9	3640	25.45	1074430	7	3640	24.65
			90	1298678	0.8	900775	1.9	126	30.64	912126	7	3216	29.77	973224	6.2	3640	25.06	984533	5.5	3640	24.19
			100	1171996	0.8	844886	1.7	109	27.91	846752	4.3	3216	27.75	892922	6.8	3640	23.81	905887	4.8	3640	22.71
			120	922829	0.8	768023	1.8	82	16.78	774659	8.7	3216	16.06	795041	6.1	3640	13.85	810881	11.4	3640	12.13
			150	788049	1	741209	2.5	63	5.94	728741	8.1	3216	7.53	733739	7.5	3640	6.89	738522	17.5	3640	6.28
			500	702055	1.6	701845	6	10	0.03	700442	9.1	3216	0.23	700428	12.6	3640	0.23	700322	10.7	3640	0.25
SJC4	5K	25	80	2229968	0.8	1403234	1.9	146	37.07	1415099	4	3216	36.54	1588295	7.4	3640	28.77	1605430	6.5	3640	28.01
			90	2009678	0.8	1278775	1.9	126	36.37	1311131	17.6	3216	34.76	1426242	8	3640	29.03	1443533	5.7	3640	28.17
			100	1786996	0.8	1171886	1.9	109	34.42	1188752	14.6	3216	33.48	1273922	11.5	3640	28.71	1298887	7.6	3640	27.31
			120	1339829	0.8	1013817	1.9	82	24.33	1044946	9.8	3216	22.01	1084766	9.1	3640	19.04	1113014	42	3640	16.93
			150	1073049	1.9	935659	2.5	63	12.80	930482	63.4	3216	13.29	946584	67.6	3640	11.79	958112	106	3640	10.71
			500	837702	1.5	837702	6.1	10	-	836398	11.1	3216	0.16	836346	13.8	3640	0.16	836203	12.6	3640	0.18
		50	80	3301968	0.9	1987234	1.9	146	39.82	2011099	4.2	3216	39.09	2288295	7.6	3640	30.70	2313430	6.3	3640	29.94
			90	2957696	0.6	1782775	1.9	126	39.72	1843131	22.8	3216	37.68	2030242	8.2	3640	31.36	2055533	5.6	3640	30.50
			100	2606996	0.8	1607886	1.9	109	38.32	1644752	53.5	3216	36.91	1781922	18.5	3640	31.65	1822887	6.1	3640	30.08
			120	1895829	0.8	1341817	1.9	82	29.22	1400946	73.7	3216	26.10	1468766	14.9	3640	22.53	1517014	66.4	3640	19.98
			150	1453049	1.5	1190079	2.5	63	18.10	1196357	534	3216	17.67	1227051	228	3640	15.55	1247470	139	3640	14.15
			500	968967	1.7	968967	6.3	10	-	967850	25.2	3216	0.12	967688	22.9	3640	0.13	967755	20.4	3640	0.13

Table 5.15: Comparison Summary and p Values

Problem Instance				Baseline	Smart Points		Circular Points		Grid Points		Random Points	
Data Set	F	C	Dmax	p	p	% Diff	p	% Diff	p	% Diff	p	% Diff
TA100	5K	25	30	64	44	27.53	45	26.25	50	19.33	50	19.12
			40	53	34	30.16	34	30.34	36	27.06	38	23.91
			45	45	27	31.43	29	27.96	31	24.93	31	24.97
			50	39	25	26.52	25	26.74	27	23.68	28	21.46
			60	31	20	22.87	20	23.73	22	18.08	23	17.65
			300	9	9	-	8	0.43	8	0.38	8	0.45
		50	30	64	44	28.91	45	27.52	50	20.27	50	20.14
			40	53	34	32.24	34	32.35	36	28.89	38	25.51
			45	45	27	34.51	29	30.69	31	27.15	31	27.18
			50	39	25	29.84	25	29.97	27	26.18	28	23.84
			60	31	20	27.18	20	27.75	22	21.82	23	20.44
			300	7	7	-	7	- 0.01	7	- 0.01	7	- 0.00
	12K	100	30	64	44	29.68	45	28.24	50	20.80	50	20.71
			40	53	34	33.42	34	33.50	36	29.94	38	26.43
			45	45	27	36.29	29	32.27	31	28.44	31	28.45
			50	39	25	31.78	25	31.87	27	27.65	28	25.24
			60	31	20	29.78	20	30.17	22	24.08	23	22.12
			300	5	5	-	5	- 0.02	5	- 0.05	5	- 0.03
		25	80	268	146	32.29	149	32.11	176	25.45	177	24.65
			90	237	126	30.64	133	29.77	151	25.06	153	24.19
			100	205	109	27.91	114	27.75	127	23.81	131	22.71
			120	139	82	16.78	90	16.06	97	13.85	103	12.13
			150	95	70	5.94	70	7.53	73	6.89	75	6.28
			500	53	53	0.03	52	0.23	52	0.23	52	0.25
SJC4	5K	25	80	268	146	37.07	149	36.54	175	28.77	177	28.01
			90	237	126	36.37	133	34.76	151	29.03	153	28.17
			100	205	109	34.42	114	33.48	127	28.71	131	27.31
			120	139	82	24.33	89	22.01	96	19.04	101	16.93
			150	95	64	12.80	67	13.29	71	11.79	73	10.71
			500	35	35	-	36	0.16	35	0.16	35	0.18
		50	80	268	146	39.82	149	39.09	175	30.70	177	29.94
			90	237	126	39.72	133	37.68	151	31.36	153	30.50
			100	205	109	38.32	119	36.91	127	31.65	131	30.08
			120	139	82	29.22	89	26.10	96	22.53	101	19.98
			150	95	64	18.10	66	17.67	70	15.55	72	14.15
			500	30	30	-	30	0.12	30	0.13	30	0.13
	12K	100	80	268	146	39.82	149	39.09	175	30.70	177	29.94
			90	237	126	39.72	133	37.68	151	31.36	153	30.50
			100	205	109	38.32	119	36.91	127	31.65	131	30.08
			120	139	82	29.22	89	26.10	96	22.53	101	19.98
			150	95	64	18.10	66	17.67	70	15.55	72	14.15
			500	30	30	-	30	0.12	30	0.13	30	0.13

5.5 Effect of Weisfeld's and Cooper's Algorithms

Another point to consider is the transition among the stages. After solving DLim-CapPLP with or without augmented set of candidate points, modified versions of

Weiszfeld's and Cooper's algorithms are followed as discussed in Section 4.3. The corresponding solutions of these stages are represented by S1, S2 and S3. The contributions and the effects of each stage transition within the separate methods is provided in this section. The results of the comparisons between the stages are also presented for each separate method; without additional points, with smart point insertion and with circular point insertion. For this purpose, we provide the differences among the stages, by percentage, for the separate methods.

Basically, S1-S2 represent the effect of modified Weiszfeld's algorithm, as it finds the location of the minimizing point for each cluster. In fact, this tool maps the solution of D-LimPLP to the continuous space. S1-S3 represents the effect of Cooper's algorithm alternating between location and allocation steps. In this stage, allocation steps are iteratively revisited, after each location step is performed and new locations of clusters are determined. Namely, it represents the effect of reallocation of demand points. Note that, Weiszfeld's algorithm is inherently repeated in Cooper's algorithm multiple times. Hence, S2-S3 difference indicates the effect of iterative allocation steps and effect of local changes in facility locations, as in S2 Weiszfeld's algorithm is performed only once.

In the following tables, as usual, first set of columns represents the problem instances. Second set consisting of three columns presents the differences between the stages within a specific method. In Table 5.16 differences are provided for the solutions without augmented set of candidate points. S1-S2 differences up to %0.77 are observed, representing the effect of modified Weiszfeld's algorithm. S1-S3 differences up to %0.88 are observed, representing the effect of both Weiszfeld's and Cooper's algorithm as Cooper's algorithm inherently uses the Weiszfeld's algorithm. Generally, very minor differences occurred in between S2 and S3 shown as S2-S3. That is because the Cooper's algorithm can not iterate that much. In other words, algorithm can not conduct an effective local search and converges instantly because no further change can occur in the locations of the facilities. The reasoning lied

behind this observation is the fact that allocation decisions are properly made by D-LimPLP. Starting with a single high quality solution leads to a quick convergence of this algorithm. The solution times for S3 are also indicates the fast convergence of the algorithm. Furthermore, allocation with respect to regret function may sometimes deteriorate the power of it as some distance values exceed the predetermined D_{max} during the execution of algorithm. For instance, when distance to the second nearest facility is $d(i, P_{2nd}) = D_{max} + \epsilon$ and distance to the first nearest facility is $d(i, P_{1st}) = D_{max} - \epsilon$, regret value is calculated as 2ϵ and demand point is put in order in priority list, although it is not reasonable to assign it to a facility far more than D_{max} away from it.

In Table 5.17 stage differences are presented for the solutions with smart point insertion. S1-S2 differences up to %3.95 are observed, representing the effect of Weiszfeld's algorithm. S1-S3 differences up to %4.14 are observed, representing the effect of both Weiszfeld's and Cooper's algorithm. At most %0.33 difference is observed in S2-S3 due to the mentioned reasons. The solution times of S2 and S3 are also close. Table 5.18 provides the differences between the stages on Bonsaaso for both non-augmented and augmented sets. Table 5.19 presents the similar results on TA100 and SJC4 instances with circular point insertion. At most %0.48 difference is observed in S2-S3 indicating the fact that Cooper's algorithm could not provide further remarkable improvement once Weiszfeld's algorithm is performed.

Table 5.16: Comparison of Stages w/o Additional Points on TA100, SJC4, U654, U1060

Problem Instance				% Diff Between Stages w/o Additional Points			Problem Instance				% Diff Between Stages w/o Additional Points		
Data Set	F	C	Dmax	S1-S2	S1-S3	S2-S3	Data Set	F	C	Dmax	S1-S2	S1-S3	S2-S3
TA100	5K	25	30	0.05	0.05	-	SJC4	5K	25	80	0.03	0.03	-
			40	0.09	0.09	-				90	0.03	0.03	-
			45	0.06	0.09	0.03				100	0.06	0.06	-
			50	0.12	0.15	0.03				120	0.29	0.33	0.04
			60	0.27	0.31	0.03				150	0.37	0.49	0.12
			300	0.63	0.81	0.18				500	0.45	0.49	0.04
	8K	50	30	0.03	0.03	-		8K	50	80	0.02	0.02	-
			40	0.06	0.06	-				90	0.02	0.02	-
			45	0.06	0.06	-				100	0.04	0.04	-
			50	0.08	0.10	0.02				120	0.20	0.23	0.03
			60	0.18	0.20	0.02				150	0.27	0.36	0.09
			300	0.90	1.11	0.21				500	0.20	0.26	0.06
	12K	100	30	0.02	0.02	-		12K	100	80	0.01	0.01	-
			40	0.04	0.04	-				90	0.02	0.02	-
			45	0.03	0.04	0.01				100	0.03	0.03	-
			50	0.05	0.07	0.01				120	0.14	0.16	0.02
			60	0.12	0.14	0.02				150	0.20	0.26	0.07
			300	0.33	0.33	-				500	0.22	0.22	-
U654	5K	25	30	0.10	0.19	0.08	U1060	5K	25	150	0.10	0.13	0.03
			40	0.20	0.29	0.09				200	0.27	0.27	-
			50	0.44	0.44	-				250	0.58	0.63	0.05
			100	0.56	0.72	0.16				300	0.49	0.49	-
			150	0.57	0.74	0.17				400	0.44	0.44	-
			1000	0.69	0.86	0.18				2000	0.41	0.41	-
	8K	50	30	0.10	0.15	0.06		8K	50	150	0.06	0.08	0.02
			40	0.12	0.18	0.06				200	0.18	0.19	0.01
			50	0.30	0.30	-				250	0.41	0.48	0.07
			100	0.96	1.05	0.09				300	0.54	0.59	0.05
			150	0.52	0.68	0.17				400	0.53	0.63	0.10
			1000	0.72	0.88	0.16				2000	0.37	0.40	0.03
	12K	100	30	0.06	0.09	0.03		12K	100	150	0.04	0.06	0.01
			40	0.07	0.13	0.06				200	0.13	0.13	0.01
			50	0.21	0.21	-				250	0.30	0.35	0.05
			100	0.77	0.77	-				300	0.45	0.57	0.13
			150	0.39	0.54	0.15				400	0.65	0.78	0.13
			1000	0.65	0.76	0.11				2000	0.35	0.36	0.01

Table 5.17: Comparison of Stages with Smart Points on TA100, SJC4, U654, U1060

Problem Instance				% Diff Between Stages Smart Points			Problem Instance				% Diff Between Stages Smart Points		
Data Set	F	C	Dmax	S1-S2	S1-S3	S2-S3	Data Set	F	C	Dmax	S1-S2	S1-S3	S2-S3
TA100	5K	25	30	0.76	0.76	-	SJC4	5K	25	80	1.98	1.98	-
			40	1.67	1.73	0.07				90	2.07	2.27	0.21
			45	0.73	0.77	0.04				100	2.38	2.46	0.09
			50	1.96	2.02	0.07				120	1.77	1.78	0.02
			60	1.41	1.41	-				150	1.42	1.42	-
			300	0.63	0.81	0.18				500	0.48	0.52	0.04
			30	0.49	0.49	-				80	1.37	1.39	0.03
			40	1.10	1.14	0.04				90	1.46	1.61	0.15
			45	0.49	0.52	0.03				100	1.73	1.79	0.06
			50	1.33	1.38	0.05				120	1.35	1.38	0.03
			60	0.98	0.98	-				150	1.41	1.41	-
			300	0.90	1.11	0.21				500	0.20	0.26	0.06
	8K	50	30	0.33	0.33	-		8K	50	80	0.97	0.99	0.02
			40	0.75	0.78	0.03				90	1.06	1.16	0.11
			45	0.34	0.36	0.02				100	1.26	1.31	0.05
			50	0.94	0.97	0.03				120	1.02	1.04	0.02
			60	0.70	0.70	-				150	1.11	1.24	0.13
			300	0.33	0.33	-				500	0.22	0.22	-
	12K	100	30	0.33	0.33	-		12K	100	80	0.97	0.99	0.02
			40	0.75	0.78	0.03				90	1.06	1.16	0.11
			45	0.34	0.36	0.02				100	1.26	1.31	0.05
			50	0.94	0.97	0.03				120	1.02	1.04	0.02
			60	0.70	0.70	-				150	1.11	1.24	0.13
			300	0.33	0.33	-				500	0.22	0.22	-
U654	5K	25	30	0.50	0.52	0.02	U1060	5K	25	150	3.95	4.14	0.19
			40	0.84	0.90	0.06				200	3.92	3.97	0.05
			50	1.26	1.31	0.06				250	1.49	1.54	0.05
			100	1.34	1.61	0.27				300	0.58	0.59	0.01
			150	0.78	0.95	0.17				400	0.39	0.39	-
			1000	0.69	0.86	0.18				2000	0.41	0.41	-
			30	0.33	0.34	0.02				150	2.89	3.03	0.14
			40	0.57	0.61	0.04				200	3.84	3.97	0.14
			50	0.88	0.92	0.04				250	2.83	3.14	0.32
			100	1.44	1.55	0.11				300	1.48	1.52	0.05
			150	0.90	0.90	-				400	0.73	0.87	0.14
			1000	0.70	0.88	0.17				2000	0.37	0.40	0.03
	8K	50	30	0.23	0.24	0.01		8K	50	150	2.10	2.19	0.10
			40	0.40	0.43	0.03				200	2.95	3.06	0.11
			50	0.63	0.66	0.03				250	2.64	2.87	0.23
			100	1.12	1.18	0.06				300	1.81	2.14	0.33
			150	0.69	0.71	0.02				400	1.09	1.14	0.05
			1000	0.65	0.76	0.11				2000	0.35	0.36	0.01
	12K	100	30	0.23	0.24	0.01		12K	100	150	2.10	2.19	0.10
			40	0.40	0.43	0.03				200	2.95	3.06	0.11
			50	0.63	0.66	0.03				250	2.64	2.87	0.23
			100	1.12	1.18	0.06				300	1.81	2.14	0.33
			150	0.69	0.71	0.02				400	1.09	1.14	0.05
			1000	0.65	0.76	0.11				2000	0.35	0.36	0.01

Table 5.18: Comparison of Stages on Bonsaaso

Problem Instance				% Diff Between Stages w/o Additional Points			% Diff Between Stages Smart Points		
Data Set	F	C	Dmax	S1-S2	S1-S3	S2-S3	S1-S2	S1-S3	S2-S3
Bonsaaso	5K	25	20	0.02	0.03	0.00	0.66	0.70	0.04
			25	0.06	0.06	-	0.75	0.76	0.01
			30	0.09	0.09	0.00	0.76	0.84	0.08
			40	0.24	0.26	0.02	0.97	1.16	0.19
			50	0.52	0.65	0.13	0.98	1.22	0.24
			2000	0.30	0.37	0.06	0.30	0.37	0.06
	8K	50	20	0.01	0.02	0.00	0.42	0.45	0.03
			25	0.04	0.04	-	0.49	0.49	0.01
			30	0.06	0.06	0.00	0.50	0.56	0.05
			40	0.16	0.17	0.01	0.67	0.80	0.13
			50	0.36	0.45	0.09	0.70	0.87	0.17
			2000	0.41	0.50	0.09	0.41	0.50	0.09
	12K	100	20	0.01	0.01	0.00	0.29	0.31	0.02
			25	0.03	0.03	-	0.33	0.34	0.00
			30	0.04	0.04	0.00	0.35	0.38	0.04
			40	0.11	0.12	0.01	0.47	0.56	0.09
			50	0.25	0.32	0.06	0.51	0.63	0.12
			2000	0.38	0.42	0.04	0.38	0.42	0.04

Table 5.19: Comparison of Stages with Circular Points on TA100 and SJC4

Problem Instance				% Diff Between Stages Circular Points			Problem Instance				% Diff Between Stages Circular Points		
Data Set	F	C	Dmax	S1-S2	S1-S3	S2-S3	Data Set	F	C	Dmax	S1-S2	S1-S3	S2-S3
TA100	5K	25	30	0.19	0.19	-	SJC4	5K	25	80	0.67	0.67	-
			40	0.43	0.43	-				90	0.66	0.68	0.02
			45	0.69	0.69	-				100	0.78	0.78	-
			50	0.76	0.83	0.07				120	0.79	1.27	0.48
			60	0.54	0.54	-				150	0.64	0.69	0.05
			300	0.61	0.66	0.05				500	0.24	0.32	0.08
	8K	50	30	0.12	0.12	-		8K	50	80	0.46	0.46	-
			40	0.28	0.28	-				90	0.46	0.47	0.01
			45	0.46	0.46	-				100	0.56	0.56	-
			50	0.51	0.56	0.05				120	0.63	0.68	0.05
			60	0.37	0.37	-				150	0.52	0.70	0.18
			300	0.52	0.52	-				500	0.13	0.13	-
	12K	100	30	0.08	0.08	-		12K	100	80	0.32	0.32	-
			40	0.19	0.19	-				90	0.33	0.34	0.01
			45	0.32	0.32	-				100	0.40	0.40	-
			50	0.36	0.39	0.03				120	0.47	0.51	0.04
			60	0.27	0.27	-				150	0.49	0.57	0.08
			300	0.09	0.09	-				500	0.15	0.17	0.02

Chapter 6

Conclusion

In this thesis, we studied on a continuous site-generating location and allocation problem that enables the distribution of resources from capacitated facilities to the end points (customers) for the spatial planning of stand-alone resource distribution systems. In addition to the distribution costs, a major cost component of fixed opening costs of facilities are inserted to the objective function of our problem to make better cost effective assessment. Hence, in addition to the location and allocation configuration of facilities, number of facilities to be opened is unknown and also subject to the decision maker. Additional insights are combined with the ordinary characteristics of common site generating problems to obtain more realistic estimates for contributing the greenfield development.

To consider the mentioned trade-off and to make a cost effective network design, problem is mathematically modeled subject to the specified constraints. We proposed a MIQCP model and a heuristic approach to solve the problem under the mentioned setting. Our heuristic approach is based on the idea that the solution of the discrete counter part of the continuous location problems yields reasonable estimates. This

initial stage of our approach is performed with enlarged domain to determine the number of facilities required to cover the set of demand points. Weiszfeld's and Cooper's well known heuristics are adopted in the following stages and mapped the problem in to the continuous space. Because there are limitations on both the service that a facility can supply and the distance it can reach to, Cooper's and Weiszfeld's heuristics are modified to protect the feasibility with priority measures on assignment decisions and projections on location decisions, respectively.

Augmented set of candidate locations forming the domain of the discrete counterpart problem is obtained by insertion of smart points and circular points which are determined in accordance with the spatial distribution of demand points. After solving the discrete counterpart of the problem with augmented domain, Cooper's well known heuristic that alternates between allocation and location steps is followed to find better allocation configuration of customers to the facilities. Considering the fact that facilities are assumed to be capacitated and some of the demand points could not be assigned to their nearest facility, allocation steps are performed with respect to the priority value of each demand point. After each allocation step, to determine the new locations of the facilities, Weiszfeld's iterative convergence algorithm is applied with some modifications. These iterative heuristics enabled the algorithm to make fine tuning on continuous space while still ensuring the capacity and distance constraints.

Wise augmentation of the set of candidate points with newly introduced very small number of points, resulted substantial improvements in objective values. In deed, this improvements are observed in the presence of D_{max} constraints. That is, as the D_{max} constraint is relaxed, the effect of the method deteriorates. The reasoning behind this fact is the drastic decrease in the required number of facilities when D_{max} is present as a tight constraint. However, when the constraint is relaxed adequately, newly employed candidate points becomes obsolete as all baseline points (demand points) can serve anywhere in the plane. In deed, the experimental results indicates

this fact for all data sets.

The substantial reduction in cost are all observed to be a result of the decrease in number of facilities. This fact shows the importance of such problem setting. Hence, determining the number of facilities rather than using it as a parameter end up with better cost estimates.

Another extension of this problem could be proposed by considering different type of capacitated facilities with different fixed opening costs. As economies of scale applies, unit cost of serving a demand is more costly with smaller capacity facilities. That is why, one may want to choose a large capacity facility. Hence, choosing large capacity facilities would definitely decrease the required number of facilities. This would; on the other hand, increase the total length of the lines in the grid serving to the demand points. Therefore, a serious trade-off occurs among these decisions. If this trade-off could be properly evaluated, better cost estimates could be obtained for the greenfield development.

Centralized extension of the similar capacitated and distance limited settings would also be a serious deal where stand-alone distribution centers become intermediate facilities being connected to a central source in a tree configuration. Tree formation decisions would further complicate the problem and require wise approaches as well. Even the cable type or connection infrastructure would be chosen in accordance with the demand it would carry. That is, appropriate cable/connection sizing could also be performed to further decrease the distribution costs.

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