

**APPLICATION OF MODEL PREDICTIVE CONTROL FOR
OPTIMIZATION OF CENTRALIZED AND DECENTRALIZED SUPPLY
CHAIN SYSTEMS**

by

Esen Mestan

736748

**A Thesis Submitted to the
Graduate School of Engineering
in Partial Fulfillment of the Requirements for
the Degree of
Master of Science
in**

Industrial Engineering

Koç University

October, 2003

**T.C. YÖKSEK ÖĞRETİM KURULU
DOKÜMANTASYON MERKEZİ**

Koç University
Graduate School of Sciences and Engineering

This is to certify that I have examined this copy of a master's thesis by

Esen Mestan

and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by the final
examining committee have been made

Committee Members:



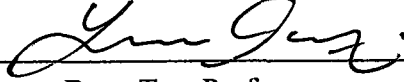
Yaman Arkun, Prof. (Advisor)



Metin Türkay, Ph. D. (Advisor)



Süleyman Özekici, Prof.



Barış Tan, Prof.



Fikri Karaesmen, Ph. D.

Date: _____

YÜKSEK LİSANS VE DOKTORA
TEZLERİNİN KONTROLÜ
VE YERLEŞTİRİLMESİ
BÖLÜMÜ

To my grandmother



ABSTRACT

This thesis studies Model Predictive Control (MPC) as an optimization tool for supply chain systems and compares its results on multi-product centralized and decentralized systems. The main challenge in supply chain optimization studies is to reduce the negative effects of rapid changes in demand. Previous work on application of MPC techniques onto supply chain systems aimed to keep inventory or customer satisfaction levels at a certain value, rather than optimizing the supply chain as a whole. In this study overall profit of the supply chain systems are optimized through MPC and compared with open-loop optimization results. Different MPC optimization horizon lengths are studied and MPC performance on centralized supply chains with different production size, initial inventory values and unpredictable demand change are observed. Semi-decentralized and decentralized supply chain systems are optimized with different MPC horizon lengths and their performance are studied. Performances of the decentralized configurations are compared with that of the centralized configuration while each configuration observes the same demand pattern and implements MPC with the same horizon length. It is observed that for both centralized and decentralized systems as the MPC horizon length increases, higher profits are obtained, yet a trade off must be made between high profits and low solution times. In the decentralized cases, the lack of information causes high bullwhip effect in the supply chains, increasing the costs and decreasing the profits.

ACKNOWLEDGEMENTS

I would like to thank Prof. Yaman Arkun and Asst. Prof. Metin Türkay, who have been my advisors throughout my graduate study, for all their guidance and support. I would like to express my sincere gratitude to Prof. Yaman Arkun, for even spending his leisure time during his official holiday, in reading and correcting my thesis. I would also like to especially thank to Metin Türkay for making my attendance to INFORMS San Jose Conference possible and letting me to gain important experiences and knowledge which were very helpful in completing my thesis. I would like to thank Prof. Mehmet Çamurdan from my undergraduate university, for introducing process control to me. I would also like to thank Prof. Uğur Akman from Boğaziçi University, for introducing what chemical engineering is all about as well as teaching linear programming and optimization methods and helping me with my senior year project.

I am also very thankful to Başak Candemir for motivating me to work on my thesis, all the way from Norway, in her unique ways. I thank to my parents, my grandmother and my sister for helping, motivating and directing me not only through my graduate study but in my entire life.

Finally, I would like to thank two special people, Mehmet Tuğrul Tekin, for all his help, support, and friendship in especially the last four difficult months of my study, and Arda Gezdur, for everything in the last seven years of my life.

TABLE OF CONTENTS

List of Tables	ix
List of Figures	x
Nomenclature	xix
Chapter 1: Introduction	1
1.1 Motivation	1
1.2 Related Work	2
1.3 Contributions of This Work	4
1.4 Thesis Outline	5
Chapter 2: Model Predictive Control of a Single Product Supply Chain	6
2.1 Modeling of a Single Product Continuous Supply Chain	6
2.2 Optimization Formulation	9
2.3 Model Predictive Control.	10
2.4 Illustrative Example	11
2.4.1 Open-Loop Optimization.	14
2.4.2 MPC Optimization.	19
2.4.3 Single Product Discrete Supply Chain.	23
2.4.4 Open Loop vs. MPC Optimization of the Discrete System.	24
Chapter 3: Model Predictive Control of Multiple Product Hybrid Supply Chains	38
3.1 Modeling of a Three-Product Hybrid Supply Chain with Batch Production-Illustrative Example	38
3.1.1 Manufacturing Plant (Batch Operation).	40

3.2	Optimization Formulation	42
3.3	Open Loop Optimization of the Hybrid System.	44
3.4	Model Predictive Control of the Hybrid System	72
3.4.1	Optimization Horizon 20.	72
3.4.2	Optimization Horizon 15 & Optimization Horizon 25.	99
3.5	Modeling of a Three-Product Hybrid Supply Chain with Variable Production Size.	110
3.5.1	Open-Loop Optimization	110
3.5.2	Model Predictive Control	118
3.6	Modeling of a Three-Product Hybrid Supply Chain with Reduced Initial Inventory.	142
3.6.1	Open-Loop Optimization.	142
3.6.2	Model Predictive Control.	150
3.7	Modeling of a Three-Product Hybrid Supply Chain with Unpredictable Demand Change	173

**Chapter 4: Model Predictive Control of
Multiple Product Decentralized Supply Chains 178**

4.1	Modeling of a Three-Product Decentralized & Semi-decentralized Supply Chain	178
4.2	Model Predictive Control Formulation.	182
4.2.1	Decentralized Supply Chain Optimization Models.	182
4.2.2	Semi-Decentralized Supply Chain Optimization Models.	186
4.3	Model Predictive Control of Decentralized & Semi-decentralized Supply Chain.	188
4.3.1	Optimization Horizon 15	188
4.3.2	Optimization Horizon 25	208
4.3.3	Optimization Horizon 15 vs. Horizon 25	223
4.4	Centralized vs. Semi-decentralized vs. Decentralized Supply Chains . . .	256

Chapter 5: Conclusions	261
5.1 Directions for Future Work	262
Bibliography	264
Appendix A	267
Appendix B	270



LIST OF TABLES

2.1	CPU times, Objective values and Sizes of different Optimization Approaches	25
3.1	Sales prices, costs and production amounts for the three products.	46
3.2	Initial inventory and material transfer levels.	48
3.3	MPC Horizon Lengths, Objective Function values and CPU times.	109
3.4	MPC Horizon Lengths, Obj Fct values and CPU times for cont. production. .	141



LIST OF FIGURES

2.1	A Typical Single Product Supply Chain.	7
2.2	Single Product Supply Chain with Centralized Optimizer	9
2.3	The schematic representation of basic concept of MPC	11
2.4	Single product demand profile	14
2.5	Open-loop optimization inventory level results.	15
2.6	Open-loop optimization material transfer level results.	16
2.7	Manufacturing Plant Production Amount.	16
2.8	Warehouse node optimal orders.	17
2.9	Dist Center node optimal orders.	17
2.10	Retailer node optimal orders.	18
2.11	Retailer node material transfer level.	19
2.12	Distribution center node material transfer level.	20
2.13	Warehouse node material transfer level.	20
2.14	Optimal production rates	21
2.15	Retailer node inventory level	21
2.16	Distribution center inventory level.	22
2.17	Warehouse node inventory level	22
2.18	Retailer Node Mat. Transfer (OLO vs. MPC).	26
2.19	Retailer Node Mat Transfer (Update interval 4 vs. 8).	26
2.20	Retailer Node Mat Transfer (Fixed horizon 35 vs. 15).	27
2.21	DC Node Mat. Transfer (OLO vs. MPC).	27
2.22	WH Node Mat. Transfer (OLO vs. MPC).	28
2.23	Retailer Node Inventory (OLO vs. MPC).	28
2.24	Retailer Node Inventory (update interval 4 vs. 8).	29
2.25	Retailer Node Inventory (Fixed horizon 35 vs. 15).	29
2.26	DC Node Inventory (OLO vs. MPC).	30
2.27	WH Node Inventory (OLO vs. MPC).	30
2.28	Retailer Node Orders (OLO vs. MPC).	31
2.29	Retailer Node Orders (Update interval 4 vs. 8).	31
2.30	Retailer Node Orders (Fixed horizon 35 vs. 15).	32
2.31	.DC Node Orders (OLO vs. MPC).	32
2.32	WH Node Orders (OLO vs. MPC)	33
2.33	Production Node (OLO vs. MPC).	33
2.34	Production Node (update interval 4 vs. 8).	34
2.35	Production Node (Fixed horizon 35 vs. 15).	34
2.36	Material transferred from retailer vs. actual customer demand.	35
3.1	A typical multiple product Supply Chain.	38
3.2	Demand profiles for the three product supply chain.	45
3.3	Optimal production schedule for the plant.	47
3.4	Retailer node orders for product A.	49
3.5	Retailer node inventory level for product A.	49
3.6	Retailer node material transfer level for product A.	50
3.7	Instantaneous objective function for product A.	50
3.8	Cumulative objective function for product A.	51

3.9	Material transferred to customer vs. actual customer demand for product A. . .	52
3.10	Penalty cost paid for product A.	52
3.11	Revenue gained from product A	53
3.12	Distribution center order levels for product A.	54
3.13	Distribution center material transfer level for product A.	54
3.14	Warehouse order levels for product A.	55
3.15	Warehouse material transfer level for product A.	55
3.16	Distribution center inventory levels for product A.	56
3.17	Warehouse inventory levels for product A.	56
3.18	Retailer order levels for product B.	57
3.19	Distribution center order levels for product B.	58
3.20	Warehouse order levels for product B.	58
3.21	Retailer material transfer level for product B.	59
3.22	Distribution center material transfer level for product B.	59
3.23	Warehouse material transfer level for product B.	60
3.24	Material transferred to customer vs. actual customer demand for product B. . .	60
3.25	Penalty cost paid for product B.	61
3.26	Revenue gained from product B.	61
3.27	Retailer inventory level for product B.	62
3.28	Distribution center inventory levels for product B.	62
3.29	Warehouse inventory levels for product B.	63
3.30	Instantaneous objective function for product B.	63
3.31	Cumulative objective function for product B.	64
3.32	Retailer order levels for product C.	65
3.33	Distribution center order levels for product C.	65
3.34	Warehouse order levels for product C.	66
3.35	Retailer material transfer level for product C.	66
3.36	Distribution center material transfer level for product C.	67
3.37	Warehouse material transfer level for product C.	67
3.38	Material transferred to customer vs. actual customer demand for product C. . .	68
3.39	Penalty cost paid for product C.	68
3.40	Revenue gained from product C.	69
3.41	Retailer inventory level for product C.	69
3.42	Distribution center inventory levels for product C.	70
3.43	Warehouse inventory levels for product C.	70
3.44	Instantaneous objective function for product C.	71
3.45	Cumulative objective function for product C.	71
3.46	Optimal production schedule for the plant (MPC vs. open loop)	73
3.47	Retailer inventory level for product A (MPC vs. open loop)	74
3.48	Retailer material transfer level for product A (MPC vs. open loop)	74
3.49	Retailer order levels for product A (MPC vs. open loop)	75
3.50	Distribution center inventory level for product A (MPC vs. open loop)	75
3.51	Distribution center material transfer level for product A (MPC vs. open loop) .	76
3.52	Distribution center order levels for product A (MPC vs. open loop)	76
3.53	Warehouse inventory level for product A (MPC vs. open loop)	77
3.54	Warehouse material transfer level for product A (MPC vs. open loop)	77

3.55	Warehouse order levels for product A (MPC vs. open loop)	78
3.56	Material transferred to customer vs. actual cost, demand for product A (MPC) .	79
3.57	Penalty cost paid for product A (MPC vs. open loop)	79
3.58	Total inventory costs paid for product A (MPC vs. open loop)	80
3.59	Revenue gained from product A (MPC vs. open loop)	81
3.60	Material transfer costs for product A(MPC vs. open loop)	82
3.61	Instantaneous objective function for product A(MPC vs. open loop)	82
3.62	Cumulative objective function for product A(MPC vs. open loop)	83
3.63	Retailer inventory level for product B (MPC vs. open loop)	84
3.64	Retailer material transfer level for product B (MPC vs. open loop)	84
3.65	Retailer order levels for product B (MPC vs. open loop)	85
3.66	Distribution center inventory level for product B (MPC vs. open loop)	85
3.67	Distribution center material transfer level for product B (MPC vs. open loop) .	86
3.68	Distribution center order levels for product B (MPC vs. open loop)	86
3.69	Warehouse inventory level for product B (MPC vs. open loop)	87
3.70	Warehouse material transfer level for product B (MPC vs. open loop)	87
3.71	Warehouse order levels for product B (MPC vs. open loop)	88
3.72	Material transferred to cust. vs. actual customer demand for product B (MPC) .	88
3.73	Penalty cost paid for product B (MPC vs. open loop)	89
3.74	Total inventory costs paid for product B (MPC vs. open loop)	89
3.75	Revenue gained from product B (MPC vs. open loop)	90
3.76	Instantaneous objective function for product B(MPC vs. open loop)	90
3.77	Cumulative objective function for product B (MPC vs. open loop)	91
3.78	Retailer inventory level for product C (MPC vs. open loop)	91
3.79	Retailer material transfer level for product C (MPC vs. open loop)	92
3.80	Retailer order levels for product C (MPC vs. open loop)	92
3.81	Distribution center inventory level for product C (MPC vs. open loop)	93
3.82	Distribution center material transfer level for product C (MPC vs. open loop) .	93
3.83	Distribution center order levels for product C (MPC vs. open loop)	94
3.84	Warehouse inventory level for product C (MPC vs. open loop)	94
3.85	Warehouse material transfer level for product C (MPC vs. open loop)	95
3.86	Warehouse order levels for product C (MPC vs. open loop)	95
3.87	Material transferred to customer vs. actual cost, demand for product C (MPC) .	96
3.88	Penalty cost paid for product C (MPC vs. open loop)	96
3.89	Total inventory costs paid for product C (MPC vs. open loop)	97
3.90	Revenue gained from product C (MPC vs. open loop)	97
3.91	Instantaneous objective function for product C(MPC vs. open loop)	98
3.92	Cumulative objective function for product C(MPC vs. open loop)	98
3.93	Optimal production schedule for the plant (Different MPC horizons)	99
3.94	Retailer node material transfer level for product A (Different MPC horizons) .	100
3.95	Retailer node inventory level for product A (Different MPC horizons)	101
3.96	Retailer node order levels for product A (Different MPC horizons)	101
3.97	Distribution center inventory level for product A (Different MPC horizons) . .	102
3.98	Distribution center mat. transfer level for product A (Different MPC horizons) .	103
3.99	Distribution center order levels for product A (Different MPC horizons)	103
3.100	Warehouse inventory level for product A (Different MPC horizons)	104

3.101	Warehouse material transfer level for product A (Different MPC horizons) . . .	104
3.102	Warehouse order levels for product A (Different MPC horizons)	105
3.103	Mat. transfrd to cust. vs. actual cust. demand for prod A (Diff MPC horizons)	105
3.104	Penalty cost paid for product A (Different MPC horizons)	106
3.105	Total inventory costs paid for product A (Different MPC horizons)	106
3.106	Revenue gained from product A (Different MPC horizons)	107
3.107	Instantaneous objective function for product A(Different MPC horizons)	107
3.108	Cumulative objective function for product A(Different MPC horizons)	108
3.109	Retailer inventory level for product C (Different MPC horizons)	109
3.110	Retailer node inventory level for product A (olo vs. batch olo)	111
3.111	Retailer node material transfer level for product A (olo vs. batch olo).	112
3.112	Customer dissatisfaction cost paid for product A (olo vs. batch olo)	112
3.113	Production amounts for product A (olo vs. batch olo).	113
3.114	Retailer node inventory level for product B (olo vs. batch olo).	113
3.115	Retailer node material transfer level for product B (olo vs. batch olo).	114
3.116	Customer dissatisfaction cost paid for product B (olo vs. batch olo).	115
3.117	Production amounts for product B (olo vs. batch olo).	116
3.118	Retailer node inventory level for product C (olo vs. batch olo).	117
3.119	Retailer node material transfer level for product C (olo vs. batch olo).	117
3.120	Retailer inventory level for product A (MPC vs. open loop).	118
3.121	Optimal production schedule of the plant for product A (MPC vs. open loop)	119
3.122	Material transferred to customer vs. actual cost. demand for product A	120
3.123	Penalty cost paid for product A (MPC vs. open loop)	120
3.124	Retailer order levels for product A (MPC vs. open loop).	121
3.125	Distribution center material transfer level for product A (MPC vs. open loop). .	122
3.126	Distribution center inventory level for product A (MPC vs. open loop).	122
3.127	Distribution center order levels for product A (MPC vs. open loop).	123
3.128	Warehouse inventory level for product A (MPC vs. open loop)	123
3.129	Warehouse material transfer level for product A (MPC vs. open loop)	124
3.130	Warehouse order levels for product A (MPC vs. open loop)	124
3.131	Cumulative objective function for product A(MPC vs. open loop)	125
3.132	Instantaneous objective function for product A(MPC vs. open loop)	125
3.133	Optimal production schedule of the plant for product B (MPC vs. open loop). .	127
3.134	Retailer material transfer level for product B (MPC vs. open loop).	127
3.135	Retailer inventory level for product B (MPC vs. open loop)	128
3.136	Retailer order levels for product B (MPC vs. open loop).	128
3.137	Distribution center inventory level for product B (MPC vs. open loop).	129
3.138	Distribution center material transfer level for product B (MPC vs. open loop) .	129
3.139	Distribution center order levels for product B (MPC vs. open loop)	130
3.140	Warehouse inventory level for product B (MPC vs. open loop)	130
3.141	Warehouse material transfer level for product B (MPC vs. open loop)	131
3.142	Warehouse order levels for product B (MPC vs. open loop).	131
3.143	Material transferred to customer vs. actual cost. demand for product B (MPC).	132
3.144	Penalty cost paid for product B (MPC vs. open loop).	132
3.145	Instantaneous objective function for product B(MPC vs. open loop)	133

3.146	Cumulative objective function for product B (MPC vs. open loop).	133
3.147	Retailer inventory level for product C (MPC vs. open loop).	134
3.148	Retailer material transfer level for product C (MPC vs. open loop).	134
3.149	Retailer order levels for product C (MPC vs. open loop).	135
3.150	Distribution center inventory level for product C (MPC vs. open loop).	135
3.151	Distribution center material transfer level for product C (MPC vs. open loop).	136
3.152	Distribution center order levels for product C (MPC vs. open loop).	136
3.153	Warehouse inventory level for product C (MPC vs. open loop).	137
3.154	Warehouse material transfer level for product C (MPC vs. open loop).	137
3.155	Warehouse order levels for product C (MPC vs. open loop).	138
3.156	Material transferred to customer vs. actual cost. demand for product C (MPC).	138
3.157	Penalty cost paid for product C (MPC vs. open loop).	139
3.158	Instantaneous objective function for product C(MPC vs. open loop).	139
3.159	Cumulative objective function for product C(MPC vs. open loop).	140
3.160	Optimal production schedule of the plant for product C (MPC vs. open loop).	140
3.161	Retailer node mat transfer level for product A (inv 100 OLO vs. inv 50 OLO).	142
3.162	Penalty cost paid for product A (inv 100 OLO vs. inv 50 OLO).	143
3.163	Retailer node inventory level for product A (inv 100 OLO vs. inv 50 OLO).	143
3.164	Production amounts for product A (inv 100 OLO vs. inv 50 OLO).	144
3.165	Retailer node inventory level for product B (inv 100 OLO vs. inv 50 OLO).	145
3.166	Retailer node mat transfer level for product B (inv 100 OLO vs. inv 50 OLO).	146
3.167	Penalty cost paid for product B (inv 100 OLO vs. inv 50 OLO).	146
3.168	Production amounts for product B (inv 100 OLO vs. inv 50 OLO).	147
3.169	Retailer node inventory level for product C (inv 100 OLO vs. inv 50 OLO).	148
3.170	Retailer node mat transfer level for product C (inv 100 OLO vs. inv 50 OLO).	149
3.171	Penalty cost paid for product C (inv 100 OLO vs. inv 50 OLO).	149
3.172	Retailer inventory level for product A (MPC vs. open loop).	151
3.173	Material transferred to customer vs. actual cost. demand for product A	152
3.174	Penalty cost paid for product A (MPC vs. open loop).	152
3.175	Optimal production schedule of the plant for product A (MPC vs. open loop).	153
3.176	Retailer order levels for product A (MPC vs. open loop).	154
3.177	Distribution center inventory level for product A (MPC vs. open loop).	154
3.178	Distribution center material transfer level for product A (MPC vs. open loop).	155
3.179	Distribution center order levels for product A (MPC vs. open loop).	155
3.180	Warehouse inventory level for product A (MPC vs. open loop).	156
3.181	Warehouse material transfer level for product A (MPC vs. open loop).	156
3.182	Warehouse order levels for product A (MPC vs. open loop).	157
3.183	Instantaneous objective function for product A(MPC vs. open loop).	157
3.184	Cumulative objective function for product A(MPC vs. open loop).	158
3.185	Retailer inventory level for product B (MPC vs. open loop).	158
3.186	Optimal production schedule of the plant for product B (MPC vs. open loop).	160
3.187	Retailer material transfer level for product B (MPC vs. open loop).	160
3.188	Retailer order levels for product B (MPC vs. open loop).	161
3.189	Distribution center inventory level for product B (MPC vs. open loop).	161

3.190	Distribution center material transfer level for product B (MPC vs. open loop). .	162
3.191	Distribution center order levels for product B (MPC vs. open loop).	162
3.192	Warehouse inventory level for product B (MPC vs. open loop).	163
3.193	Warehouse material transfer level for product B (MPC vs. open loop).	163
3.194	Warehouse order levels for product B (MPC vs. open loop).	164
3.195	Penalty cost paid for product B (MPC vs. open loop).	164
3.196	Instantaneous objective function for product B(MPC vs. open loop).	165
3.197	Cumulative objective function for product B (MPC vs. open loop).	165
3.198	Retailer inventory level for product C (MPC vs. open loop).	166
3.199	Penalty cost paid for product C (MPC vs. open loop).	167
3.200	Mat. transferred to cust. vs. actual cust. demand for product C (MPC vs. OLO)	167
3.201	Optimal production schedule of the plant for product C (MPC vs. open loop). .	168
3.202	Retailer order levels for product C (MPC vs. open loop).	168
3.203	Distribution center inventory level for product C (MPC vs. open loop).	169
3.204	Distribution center material transfer level for product C (MPC vs. open loop)	169
3.205	Distribution center order levels for product C (MPC vs. open loop).	170
3.206	Warehouse inventory level for product C (MPC vs. open loop).	170
3.207	Warehouse material transfer level for product C (MPC vs. open loop).	171
3.208	Warehouse order levels for product C (MPC vs. open loop).	171
3.209	Instantaneous objective function for product C(MPC vs. open loop).	172
3.210	Cumulative objective function for product C(MPC vs. open loop).	172
3.211	New Demand Profiles	173
3.212	Actual demand vs. MPC interpretation of Demand for product A.	174
3.213	Inventory changes for the sinusoidal change in demand (MPC vs. OLO).	175
3.214	Production schedule for the sinusoidal change in demand (MPC vs. OLO). . .	175
3.215	Mat transferred to cust for the sinusoidal change in demand (MPC vs. OLO) .	176
4.1	Decentralized Configuration.	179
4.2	Semi-Decentralized Configuration.	180
4.3	Demand profiles for the three product supply chain.	188
4.4	Retailer node inventory level for product A.	189
4.5	Retailer node inventory level for product B.	190
4.6	Retailer node inventory level for product C.	190
4.7	Retailer node material transfer level for product A.	191
4.8	Retailer node material transfer level for product B.	191
4.9	Retailer node material transfer level for product C.	192
4.10	Material demanded from Plant and Warehouse of Product A.	193
4.11	Material demanded from Plant and Warehouse of Product B.	193
4.12	Material demanded from Plant and Warehouse of Product C.	194
4.13	Distribution center node inventory level for product A.	195
4.14	Distribution center node inventory level for product B.	196
4.15	Distribution center node inventory level for product C.	196
4.16	Distribution center node material transfer level for product A.	197
4.17	Distribution center node material transfer level for product B.	197

4.18	Distribution center node material transfer level for product C	198
4.19	Plant and Warehouse node inventory level for product A	198
4.20	Plant and Warehouse node inventory level for product B.	199
4.21	Plant and Warehouse node inventory level for product C.	199
4.22	Plant and Warehouse node material transfer level for product A.	200
4.23	Plant and Warehouse node inventory level for product B.	200
4.24	Plant and Warehouse node inventory level for product C	201
4.25	Production of A.	202
4.26	Production of B.	202
4.27	Production of C.	203
4.28	Instantaneous System Profit.	204
4.29	Demand vs. Material Transfer at Retailer for Product A.	205
4.30	Demand vs. Material Transfer at Retailer for Product B.	206
4.31	Demand vs. Material Transfer at Retailer for Product C.	206
4.32	Retailer node inventory level for product A.	208
4.33	Retailer node inventory level for product B.	209
4.34	Retailer node inventory level for product C.	209
4.35	Retailer node material transfer level for product A.	210
4.36	Retailer node material transfer level for product B.	210
4.37	Retailer node material transfer level for product C.	211
4.38	Distribution center node inventory level for product A.	211
4.39	Distribution center node inventory level for product B.	212
4.40	Distribution center node inventory level for product C.	212
4.41	Distribution center node material transfer level for product A.	213
4.42	Distribution center node material transfer level for product B.	213
4.43	Distribution center node material transfer level for product C.	214
4.44	Material demanded from Plant and Warehouse of Product A.	214
4.45	Material demanded from Plant and Warehouse of Product B.	215
4.46	Material demanded from Plant and Warehouse of Product C.	215
4.47	Plant and warehouse node inventory level for product A.	216
4.48	Plant and warehouse node inventory level for product B.	216
4.49	Plant and warehouse node inventory level for product C	217
4.50	Plant and warehouse node material transfer level for product A.	217
4.51	Plant and warehouse node material transfer level for product B.	218
4.52	Plant and warehouse node material transfer level for product C.	218
4.53	Production of A.	219
4.54	Production of B.	219
4.55	Instantaneous System Profit.	220
4.56	Cumulative System Profit.	221
4.57	Demand vs. Material Transfer at Retailer for Product A.	221
4.58	Demand vs. Material Transfer at Retailer for Product B.	222
4.59	Demand vs. Material Transfer at Retailer for Product C.	222
4.60	Retailer node inventory level for product A.	223
4.61	Retailer node inventory level for product B.	224
4.62	Retailer node inventory level for product C.	224
4.63	Retailer node material transfer level for product A.	225

4.64	Retailer node material transfer level for product B.	225
4.65	Retailer node material transfer level for product C.	226
4.66	Distribution center node inventory level for product A.	226
4.67	Distribution center node inventory level for product B.	227
4.68	Distribution center node inventory level for product C.	227
4.69	Distribution center node material transfer level for product A.	228
4.70	Distribution center node material transfer level for product B.	228
4.71	Distribution center node material transfer level for product C.	229
4.72	Plant and warehouse node inventory level for product A.	229
4.73	Plant and warehouse node inventory level for product B.	230
4.74	Plant and warehouse node inventory level for product C.	230
4.75	Plant and Warehouse node material transfer level for product A.	231
4.76	Plant and Warehouse node material transfer level for product B.	231
4.77	Plant and Warehouse node material transfer level for product C.	232
4.78	Material demanded from Plant and Warehouse of Product A.	233
4.79	Material demanded from Plant and Warehouse of Product B.	233
4.80	Material demanded from Plant and Warehouse of Product C.	234
4.81	Production of A.	235
4.82	Production of B.	235
4.83	Production of C.	236
4.84	Instantaneous System Profit.	236
4.85	Cumulative System Profit.	237
4.86	Demand vs. Material Transfer at Retailer for Product A.	237
4.87	Demand vs. Material Transfer at Retailer for Product B.	238
4.88	Demand vs. Material Transfer at Retailer for Product C.	238
4.89	Retailer node inventory level for product A.	239
4.90	Retailer node inventory level for product B.	240
4.91	Retailer node inventory level for product C.	240
4.92	Retailer node material transfer level for product A.	241
4.93	Retailer node material transfer level for product B.	241
4.94	Retailer node material transfer level for product C.	242
4.95	Distribution center node inventory level for product A.	242
4.96	Distribution center node inventory level for product B.	243
4.97	Distribution center node inventory level for product C.	243
4.98	Distribution center node material transfer level for product A.	244
4.99	Distribution center node material transfer level for product B.	244
4.100	Distribution center node material transfer level for product C.	245
4.101	Material demanded from Plant and Warehouse of Product A.	245
4.102	Material demanded from Plant and Warehouse of Product B.	246
4.103	Material demanded from Plant and Warehouse of Product C.	246
4.104	Plant and warehouse node inventory level for product A.	247
4.105	Plant and warehouse node inventory level for product B.	247
4.106	Plant and warehouse node inventory level for product C.	248
4.107	Plant and warehouse node material transfer level for product A.	248
4.108	Plant and warehouse node material transfer level for product A.	249
4.109	Plant and warehouse node material transfer level for product C.	249

4.110	Production of A.	250
4.111	Production of B.	250
4.112	Production of C.	251
4.113	Instantaneous System Profit.	251
4.114	Cumulative System Profit.	252
4.115	Demand vs. Material Transfer at Retailer for Product A.	252
4.116	Demand vs. Material Transfer at Retailer for Product B.	253
4.117	Demand vs. Material Transfer at Retailer for Product C.	253
4.118	Order levels between retailer and distribution center for product A.	254
4.119	Order levels between retailer and distribution center for product B.	254
4.120	Order levels between retailer and distribution center for product C.	255
4.121	Retailer node inventory level for product A.	256
4.122	Production of A.	256
4.123	Production of B.	257
4.124	Production of C.	257
4.125	Customer satisfaction of A.	258
4.126	Customer satisfaction of B.	258
4.127	Customer satisfaction of C.	259
4.128	Instantaneous System Profit.	259

NOMENCLATURE

APIOBCS	Automatic Pipeline, Inventory and Order Based Production Control System
$CP_m(k)$	amount of product m produced at plant at time k
CPU	Central Processing Unit
CPU Time	Time dedicated to optimization by computer
CS	Customer satisfaction penalty per unit of material in the single product case
CSD_m	Customer satisfaction cost per unit of product m at distribution center
CS_i	Customer satisfaction penalty for product i
CSR_m	Customer satisfaction cost per unit of product m at retailer
CSW_m	Customer satisfaction cost per unit of product m at plant and warehouse
$\hat{d}(i/j)$	estimate of $d(i)$ based on measurements available at time j
$d(k)$	demand at time unit k
DC	Distribution Center Node
d_{ck}	demand of end customer from node k in the single product case
$DD_m(k)$	Demand of product m observed by distribution center at time k
Dec	Fully decentralized configuration of supply chain
Dec15	Decentralized configuration with MPC horizon length 15 for each node in the supply chain
Dec25	Decentralized configuration with MPC horizon length 25 for each node in the supply chain
DHL15	Decentralized configuration with MPC horizon length 15 for each node in the supply chain
DHL25	Decentralized configuration with MPC horizon length 25 for each node in the supply chain
d_j	demand for product j
$d_m(k)$	measured value of the demand at time unit k
$DR_m(k)$	Demand of product m observed by retailer at time k
$DW_m(k)$	Demand of product m observed by plant and warehouse at time k
FC_j	Fixed production cost for product j
f_m	Fixed production cost for product m
η_j	Fraction of total orders that arrive to node i from immediate downstream node in the single product case, equals 1 in this work

η_{jm}	Fraction of total orders regarding product m from immediate downstream node that arrive to node j regarding product m, equals 1 in this work
I^*	Safety inventory
I_0	Initial inventory level
IC	inventory cost per unit of product per period at all nodes in the single product case
ICD_m	Inventory cost per unit of product m per period at distribution center
IC_i	Inventory cost for product i per period
IC_{jm}	Inventory cost per unit of product m at node j per period
ICR_m	Inventory cost per unit of product m per period at retailer
ICW_m	Inventory cost per unit of product m per period at plant and warehouse
$ID_m(k)$	Inventory level of product m at distribution center at time k
ID_{m1}	Initial inventory level of product m at distribution center
I_{ij}	Inventory at node i for product j
I_j	Inventory at node j (for single product)
$I_j[t]$ or $I_j(t)$	Inventory level at node j at time t in the single product case
$I_j^{\text{lowerbound}}$	Lower limit on inventory in the single product case
I_{jm}	Inventory of product m at node j
$I_{jm}(k)$	Inventory of product m at node k at time k
$I_{jm}^{\text{lowerbound}}$	Lower limit on inventory for product m
$I_{jm}^{\text{upperbound}}$	Upper limit on inventory for product m
$I_j^{\text{upperbound}}$	Upper limit on inventory in the single product case
$IR_m(k)$	Inventory level of product m at retailer at time k
IR_{m1}	Initial inventory level of product m at retailer
$IW_m(k)$	Inventory level of product m at plant and warehouse at time k

IW_m1	Initial inventory level of product m at plant and warehouse
K_f	optimal value of Kalman filter
$M1_m(k)$	total excess units of product m delivered to downstream node at time k
$M2_m(k)$	total shortage units of product m delivered to downstream node at time k
MILP	Mixed Integer Linear Programming
MLD	Mixed Logic Dynamical
MPC	Model Predictive Control
MPC15	MPC with horizon length of 15 weeks
MPC20	MPC with horizon length of 20 weeks
MT_j	Minimum time to deliver material from node j to immediate downstream node
MT_{jm}	Minimum time to deliver product m from node j to immediate downstream node
μ	money units
$OD_m(k)$	Order level of product m at distribution center at time k
o_{jk}	Order placed in node j by downstream node k
$o_{jk}[t]$ or $o_{jk}(t)$	Order placed in j by downstream node k at time t in the single product case
o_{jkm}	order placed to node j , for product m , from its lower node k
o_k	optimal order level for node k decided by central optimizer in the single product case
OLO	Open Loop Optimization
OLO INV100	OLO with initial inventory level of 100 at all nodes
OLO INV50	OLO with initial inventory level of 50 at all nodes
$OR_m(k)$	Order level of product m at retailer at time k
$OW_m(k)$	Order level of product m at plant and warehouse at time k

P	Length of optimization horizon in MPC
PC	production cost per unit of product in the single product case
pc_{mt}	Production cost for product m at time t
PDD_m	Purchase cost per unit of product m by distribution center
PDR_m	Purchase cost per unit of product m by retailer
PDW_m	Purchase cost per unit of product m by plant and warehouse
PID	Proportional-Integral-Derivative
P_j	Binary production variable for product j
$P_m(k)$	production variable for product m , which is 1 if the plant produces product j at time t and 0 otherwise
$p_m(t)$	production variable for product m , which is 1 if the plant produces product j at time t and 0 otherwise
PR	price per unit of product at retailer in the single product case
PrD_m	Price per unit of product m at distribution center
PR_i	Selling price of product i per unit of material
PrR_m	Price per unit of product m at retailer
PrW_m	Price per unit of product m at plant and warehouse
R	Retailer Node
r	total shortage units delivered to end customer in the single product case
r_{mt}	given batch size of product m at time t
s	total excess units delivered to end customer in the single product case
S_0	Initial order accumulation level
$SDHL15$	Semi-decentralized configuration with MPC horizon length 15 for each node in the supply chain
$SDHL25$	Semi-decentralized configuration with MPC horizon length 25 for each node in the supply chain
$SD_m(k)$	Accumulated order level of product m at distribution center at time t
SD_{m1}	Initial accumulated order level of product m at distribution center

sec	seconds (time)
SemDec	Semi-decentralized configuration of supply chain
SemDec15	Semi-decentralized configuration with MPC horizon length 15 for each node in the supply chain
SemDec25	Semi-decentralized configuration with MPC horizon length 25 for each node in the supply chain
S_j	Order accumulation in node j
$S_j[t]$ or $S_j(t)$	Accumulated order level at node j at time t in the single product case
S_{jm}	order accumulation of node j for product m
s_{mt}	variable production size of product m at time t
$SR_m(k)$	Accumulated order level of product m at retailer at time t
SR_{m1}	Initial accumulated order level of product m at retailer
ST_j	Standard time to deliver an order from node j to downstream node
ST_{jm}	Standard time to deliver an order of product m from node j to immediate downstream node
$SW_m(k)$	Accumulated order level of product m at plant and warehouse at time t
SW_{m1}	Initial accumulated order level of product m at plant and warehouse
TCD_m	Material transfer cost per unit of product m at distribution center
TC_i	Material transfer cost for product i per unit of material
TC_{jm}	Material transfer cost per unit of product m at node j
TCR_m	Material transfer cost per unit of product m at retailer
TCW_m	Material transfer cost per unit of product m at plant and warehouse
UC	Upper limit on the size of the production

UI	Upper bound on inventory level
UO	Upper bound on order level
Uy	Upper bound on material transfer level
VC_j	Variable production cost for product j per unit of material
v_m	Variable production cost for product m
VMI	Vendor Managed Inventory
$w(k)$	white noise with known intensity
WH	Warehouse Node
$yD_m(k)$	Material transfer level of product m from distribution center to retailer at time k
y_j	material transfer from node j when there is only one product and material transfer only allowed to the immediate downstream node
y_{jim}	the material transfer, for product m , from the upper node i to node j
y_{jk}	Material transfer from node j to downstream node k
$y_{jk}[t]$ or $y_{jk}(t)$	material transfer level from node j to downstream node k at time t in the single product case
$y_j^{\text{lowerbound}}$	Lower limit on material transfer in the single product case
$y_{jm}(k)$	Material transfer of product m from node j to its immediate downstream node at time k
$y_{jm}^{\text{lowerbound}}$	Lower limit on material transfer for product m
$y_{jm}^{\text{upperbound}}$	Upper limit on material transfer for product m
$y_j^{\text{upperbound}}$	Upper limit on material transfer in the single product case
$y_{pm}(k)$	the amount of product m produced at plant at time k
$yR_m(k)$	Material transfer level of product m from retailer to end customer at time k
$yR_m(k)$	Material transfer of product m from retailer to end customers at time k
$yW_m(k)$	Material transfer level of product m from plant and warehouse to distribution center at time k

CHAPTER 1

INTRODUCTION

1.1 Motivation

The definition of supply chain management, quoted by Lambert and Cooper [1], as developed in the Global Supply Chain Forum is as follows:

“Supply chain management is the integration of key business processes from end user through original raw material suppliers that provides products, services, and information that adds value for customers and other stakeholders. One of the most important transformations of modern business management is that individual businesses do not compete as independent entities, but rather as supply chains. Instead of brand versus brand or store versus store, it is supply chain versus supply chain. “

As the world changes into a highly integrated global market, customer demands change rapidly, making planning considerably difficult due to changing operating conditions of the units in a process system. The operation of a supply chain, in which the demands are known accurately, is tractable. The challenge is to take over the economical potentials of new markets and to reduce the negative effect of rapid changes in demand. A typical supply chain system consists of customers, retailers, distributors, warehouses, and manufacturing plants that can be represented as nodes in a highly integrated dynamic network. A typical supply chain optimization consists of addressing issues such as determining what product(s) the customer desires, forecasting the demand for the product(s), optimizing the system operation to achieve the desired profit margins, and/or customer satisfaction.

Supply chain systems are traditionally optimized by treating the performance of its individual nodes separately ignoring the dynamic interaction among them, in a decentralized fashion. Such an approach may result in poor performance of the supply chain system and in infeasibilities to meet the customer demand. Consequently, the success of a firm depends on its ability to integrate and coordinate the network of business relationships among supply chain participants [2]. Supply chains can be considered as integrated dynamic systems in centralized fashion and advanced control approaches can be very effective in optimizing the performance of the supply chains

subject to time-varying demand conditions. A control-oriented methodology for supply chain management must also handle the physical constraints that may be present while maintaining certain degree of robustness against uncertainty that is present due to modeling errors.

In this thesis, the aim is to introduce such a control-oriented methodology, Model Predictive Control, for the optimization of both centralized and decentralized hybrid, multi-product supply chain systems.

1.2 Related Work

First efforts to formulate a dynamic approach to study supply chain management problems are seen in the pioneering work of Forrester [3]. In this work, discrete dynamic mass balances and linear/nonlinear delays were used to model and simulate industrial systems. For the next twenty years, Forrester's attempt of dynamic formulation of supply chains remained as the sole example of its kind, until Towill [4], attempted to analyze the dynamics of an inventory and order based production control system. As the capabilities of computerized systems increased in 90's, the interest in integrated dynamic approaches for supply chain management has also increased. Traditionally, the nodes of the supply chains were analyzed and optimized separately, in decentralized fashion. Perea-Lopez et. al. [5] provided a bigger picture of a supply chain, where they integrated manufacturing and distribution channels were integrated simultaneously. This study takes one step forward from the studies performed by Backx et al. [6], where the manufacturing plant was also considered to be an integral part of the supply chain. In this study, some of the dynamic models proposed by Perea-Lopez et al will be used to implement a model predictive controller for supply chain optimization.

Perea-Lopez et al. addressed decentralized control of a multiple product supply chain system and focused on classical control laws such as Proportional-Integral-Derivative (PID). They analyzed the performance under dynamical changes in demand in terms of total cost, customer satisfaction and demand amplification. They extended their study in their following work [7] to propose an MPC strategy for optimization of a centralized supply chain.

Bose and Perkny [8] studied a single product, consumer goods supply chain, in which model predictive control methodology is used to ensure desired customer service level while minimizing average inventory. They mainly focused on the coordination between the production and the demand units and concluded that, the centralized control coordination gives more satisfactory results compared with decentralized coordination, in terms of overall efficiency of the supply

chain. Finally in the work performed by Braun et al [9], it is concluded that a decentralized MPC implementation for inventory management, under simultaneous demand forecast inaccuracies and plant-model mismatch, stabilizes supply chains and allow nodes in supply chain to appropriately buffer the inventory before large changes in demand take place.

Lakhal et al [10] modeled supply chain as a network of activities and the objective is to maximize the value added to gain competitive advantage, where an activity is defined to be a function which uses resources to transform input products into output products. Petrovic et al [11] viewed supply chain as a network consisting of facilities which are capable and responsible of procurement of raw materials, transformation of these into intermediate and end-products, distribution and selling of end products to final buyers. The paper considers a supply chain consisting of facilities in serial connection. The supply chain is assumed to be operating in an uncertain environment and the uncertainties are associated with customer demand, supply deliveries along the supply chain and external supply. Uncertainties are represented by fuzzy sets. The model formulation is directed to determine the order levels for each inventory in the supply chain that provide an acceptable service level for decentralized and partially coordinated control configurations while minimizing total possible cost. In the partial coordination, each inventory in the supply chain is coordinated with the preceding inventory. Petrovic [12] extends this work by allowing uncertainty in lead times.

Kallrath and Timpe [13] proposed a general MILP based on time-indexed formulation including the aspects of production, distribution and marketing. In addition to these, attachment of different time scales to production and distribution, the use of periods with different lengths, the modeling of batch and campaign production are considered. Batch production operates in integer multiples of batches where a batch is the smallest unit to be produced. Successive batches constitute a campaign. Production and marketing is distinguished. Production takes place in plants or production sites whereas marketing takes place in sales points. Because of the accuracy associated with each of these, different time scales are attached to each of these. The model supports several objective functions including maximizing total sales, minimizing costs and maximizing contribution margin.

In Chen and Chu [14] a mathematical model is developed for a market demand possessing the property of linearity. The model is directed towards evaluating the production rate which may affect the inventory level and the optimal sales rate at each time through different pricing strategies so as to achieve the maximum profit for a given planning horizon.

Disney and Towill [15] observes that many companies improve the supply chain performance by sharing demand and inventory information with suppliers and customers, which led to the creation of vendor managed inventory system (VMI). VMI, in short, is a production/distribution and inventory control system where stock positions and demand rates are known across more than one echelon of the supply chain. The paper considers the performance of a production or distribution scheduling algorithm termed Automatic Pipeline, Inventory and Order Based Production Control System (APIOBCS) embedded within a VMI where demand profile is deemed to change significantly over time. Properties of optimal systems are evaluated via various Bullwhip, customer service level and inventory cost metrics.

Arntzen et al [16], developed an MILP model for a multi-product and multi-stage supply chain. The model has taken into account of the interdependence of the fundamental supply chain processes, namely, production, transportation and inventory. Tapiero and Soliman [17] solved multi-product and multi-regional production and inventory control problem subject to uncertain demand problem by utilizing optimal control theory.

Lee and Billington [18] developed a stochastic program which includes the material flow of production, distribution and marketing. Lee et al [19] observed that variability in orders skew the customer demand which leads to a mismatch between supply and demand in the supply chain and investigated the characteristics of the induced bullwhip effect. Towill et al [20] developed simulation models to improve the supply chain performance including bullwhip effect in a multi-player environment. The study tries to identify a decision rule which reduces lead times and coordinates information flow throughout the supply chain..

Newhart et al [21] used spreadsheet based inventory and mathematical models to minimize the cost of operations in the supply chain.

1.3 Contributions of This Work

The contribution of this work can be listed as:

- Comparing the performance MPC control strategy with open loop optimization implemented by a centralized supply chain to observe the differences between forecasted performance, and performance under perfect information.
- Investigating the effects of different MPC optimization horizon lengths implemented on a centralized supply chain

- Investigating the effects of different initial inventory levels on MPC results implemented on a centralized supply chain
- Investigating the effects of a demand pattern change on MPC results implemented by a centralized supply chain
- Inspecting the dynamics of various decentralized supply chain configurations implementing MPC with the same horizon length
- Comparing the performance of the decentralized configurations with that of centralized configuration while each configuration observes the same demand pattern and implements MPC with the same horizon length.

1.4 Thesis Outline

In this thesis we first consider a continuous, single product supply chain system. In the first part of Chapter 2 the dynamic model for a single product supply chain is derived and its performance is dynamically optimized in an open-loop fashion where future demands are assumed to be exactly known. Then centralized model predictive control approach is implemented to this continuous, single product system in the presence of demand uncertainties, and the results of MPC approach is compared with open-loop optimization results.

In chapter 3, a multiple product, hybrid supply chain, which contains continuous retailer, distribution center and warehouse nodes along with a discrete manufacturing plant, is optimized in centralized fashion under different batch and continuous production rules, with various initial inventories and demand profiles. The open loop and MPC results are compared in terms of total profit, inventory and material transfer levels, and customer satisfaction.

In chapter 4, a multi-product hybrid supply chain is evaluated under fully decentralized and semi-decentralized control configurations. In the fully decentralized configuration retailer, distribution center and, plant and warehouse all represent independent entities aiming to maximize their own profits. In the semi-decentralized configuration retailer and distribution center are coordinated by a single optimizer and plant and warehouse represent another independent entity. For both of the configurations each independent entity applies MPC to optimize profit. The results are compared in terms of total profit, inventory and material transfer levels and customer satisfaction.

Finally chapter 5 summarizes the contributions of this work and discusses the directions for future work in this area.

CHAPTER 2

**MODEL PREDICTIVE CONTROL OF A SINGLE PRODUCT
SUPPLY CHAIN**

A typical, simple supply chain system consists of customer, retailer, distribution center, warehouse and manufacturing plant nodes. The number of nodes may vary depending on the diversity of the products or the location constraints. The information sharing between the nodes may also vary. For example, in some supply chain systems customers can directly communicate with distribution centers, warehouses and even with manufacturing plants. A typical single product supply chain is illustrated in Figure 1. In the next section we give the dynamic model of the supply chain.

2.1 Modeling of a Single Product Continuous Supply Chain

Let I and S represent the materials and orders stored, respectively. Inventory levels, I , increase when materials arrive from the supplier nodes and decrease when materials are transferred to the downstream nodes. Order levels, S , increase as replenishment orders arrive from the downstream nodes and decrease as materials are transferred from the supplier nodes.

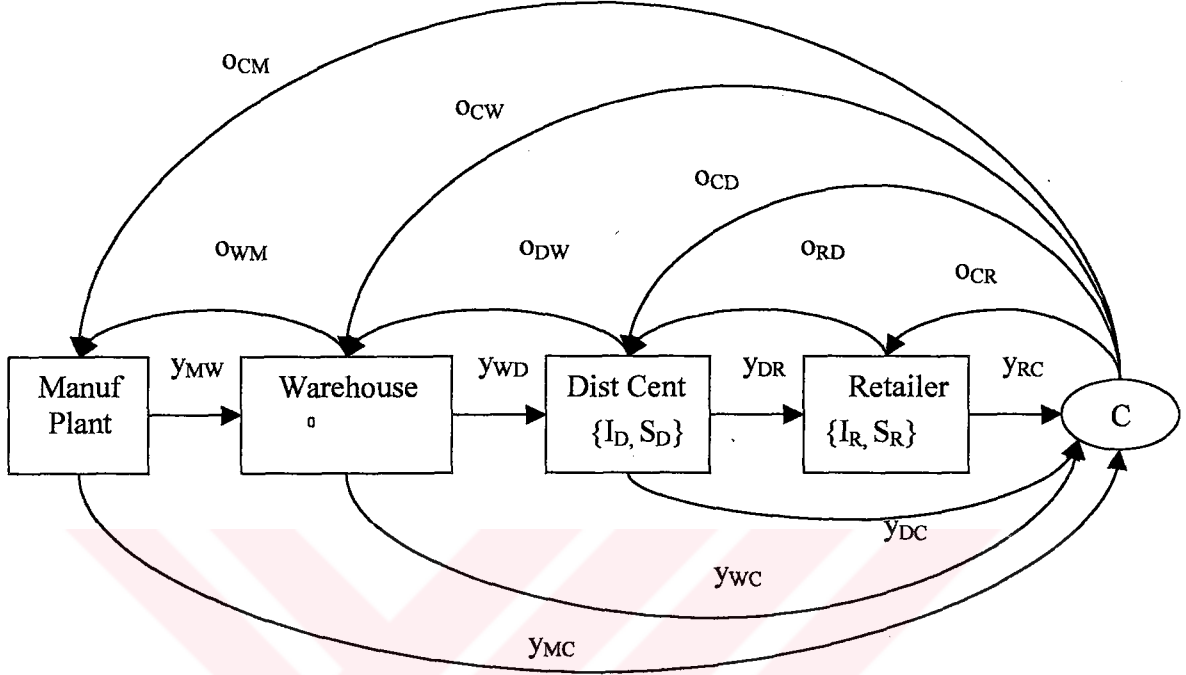


Figure 2.1: A Typical Single Product Supply Chain

The differential inventory balance equations at node j are given as follows:

$$\frac{dI_j}{dt} = \sum_{\substack{\text{from} \\ \text{upstream} \\ \text{nodes}}} y_{j,i} - \sum_{\substack{\text{to} \\ \text{downstream} \\ \text{nodes}}} y_{j,k} \quad (2.1)$$

where y_{ji} is the material transfer from the upper node i to node j and y_{jk} is the material transfer from node j to its lower node k .

The order accumulation at j , i.e., orders received at node j from downstream node k minus material delivered from the node j node k , are given as:

$$\frac{dS_j}{dt} = \sum_{\substack{\text{from} \\ \text{downstream} \\ \text{nodes}}} o_{j,k} - \sum_{\substack{\text{to} \\ \text{downstream} \\ \text{nodes}}} y_{j,k} \quad (2.2)$$

where o_{jk} is the order placed to node j from its lower node k and y_{kj} is the material transferred from node i to its lower node j .

For material transfer we use a function proposed by Grossmann et al. (2001):

$$y_j = \begin{cases} \left(MT_j + \frac{I^*}{I_j \eta_j} \cdot ST_j \right)^{-1} \cdot S_j & S_j > 0 \\ 0 & S_j = 0 \end{cases} \quad (2.3)$$

where;

$$\eta = \frac{S_j}{\sum S_j} \quad (2.4)$$

MT is the minimum time to deliver material from node j to downstream node, I^* safety inventory, ST is the standard time to deliver an order from node j to downstream node.

Physical bounds for inventories, material transfer rates and orders must also be considered:

$$\left[\begin{array}{l} I_j^{upperbound} \geq I_j \geq I_j^{lowerbound} \\ y_j^{upperbound} \geq y_j \geq y_j^{lowerbound} \end{array} \right] \quad (2.5)$$

for all nodes j , with known initial conditions, $I_j(0)$ and $S_j(0)$ at each node.

Such supply chain systems operate as ‘pull systems’ which are driven by orders between their nodes. Any orders placed to the system activate the supply chain and “pulls” material from it. If there are no orders, the system does not respond. When a customer places an order at retailer level, the order is recorded in the backlog of orders in the retailer. If retailer is able to supply the product, it is shipped to the customer and the backlog of the orders is deleted. If the retailer can not supply the necessary material, it places a replenishment order to its supplying node, e.g. to the distribution center. When the product arrives, retailer sends it to the customer and the order is removed from the backlog. When the manufacturing plant receives the orders, it generates a production plan to meet them.

2.2 Optimization Formulation

The type of supply chain systems described above are usually optimized in decentralized fashion (on an individual node basis), resulting in sub-optimality. Bullwhip effect, which causes shortages or surpluses in the customer deliveries, and excess inventories accumulated in the nodes are the main manifestations of these sub-optimality. [22] A centralized optimizer, which sees the system as a whole to decide on inventory, material transfer and production amounts, is therefore a more promising alternative. Here we consider a centralized optimizer as shown in Fig. 2.2.

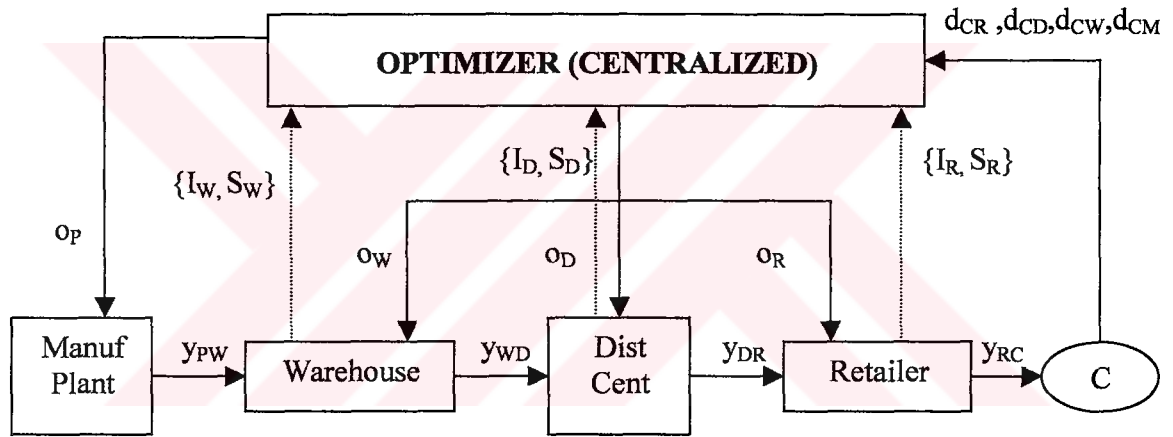


Figure 2: Single Product Supply Chain with Centralized Optimizer

The objective function of the supply chain incorporates both profit and customer satisfaction:

$$J = (PR \cdot y_R) - IC \cdot (I_R + I_D + I_W) - (PC \cdot y_P) - TC \cdot (y_{WD} + y_{DR} + y_{RC}) - CS \cdot (s + r) \quad (2.6)$$

where PR is the price of the product, IC inventory cost, PC production cost, TC transfer cost, CS is the penalty cost for under/over production. The retailer node can transfer more or less than the actual demand to the customer. However, each excess/shortage unit is subject to a penalty cost, CS . The s term is the total excess unit delivered and r is the total shortage unit delivered for each time unit.

The centralized optimizer receives the actual customer demands for the whole optimization horizon and determines the optimal orders (o_w, o_D), optimal order for customer delivery rate (o_R) and the amount to be produced (o_p), at the manufacturing plant by solving the following dynamic optimization problem: .

$$\text{Max}_{o_P(t), o_R(t), o_D(t), o_W(t)} \int_0^{t_f} J(t) dt \quad (2.7)$$

subject to equations (2.1) – (2.6).

If the future demand values over the time horizon $[0, t_f]$ were known without any uncertainty, the above optimization could be solved once and the determined optimal orders implemented over the total horizon $[0, t_f]$ in an open-loop fashion. This strategy is called open-loop optimization (OLO). However, usually future customer demands are not known exactly, and open loop optimization strategy needs to be suitably modified to account for this uncertainty. In this thesis, Model Predictive Control (MPC) is used to accomplish this.

2.3 Model Predictive Control

Many uncertainties are present in a supply chain system, the most important being the uncertainty in demand. Since the future values of demand for a product can seldom be perfectly known. To handle the difficulties caused by these uncertainties Model Predictive Control approach is proposed as an optimization tool in supply chain management.

Model Predictive Control (MPC) is a preferred algorithm for robust, multivariable control in the process industries. MPC utilizes a process model for two central tasks [23] :

1. Explicit prediction of future plant behavior
2. Computation of appropriate corrective control action.

MPC techniques perform an open-loop optimization for a time horizon at each time step on the basis of online, updated estimates of plant states and parameters, as illustrated in figure 3. Since the open-loop optimization is repeated at every sample time after a measurement update, the methodology is also referred as “open loop optimal feedback control”.

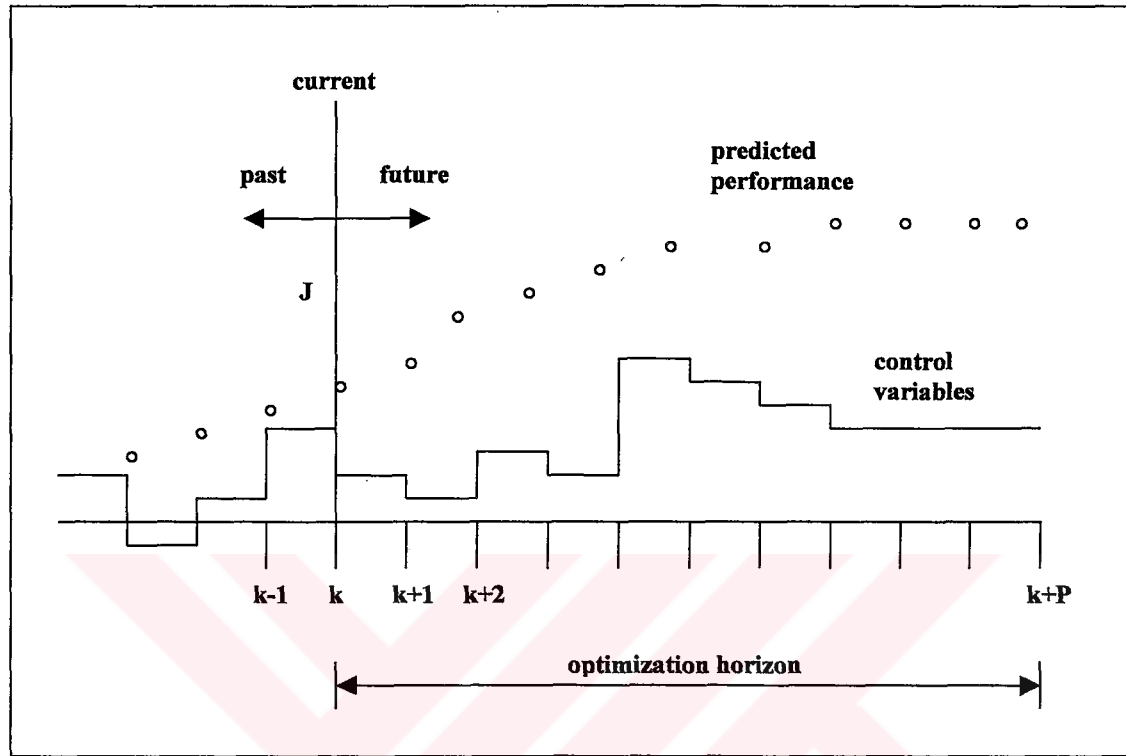


Figure 2.3: The schematic representation of basic concept of MPC

Fig. 2.3 demonstrates an MPC computation performed at k^{th} sampling time (called current time). The length of the optimization horizon is denoted by P (sampling times). MPC computes the future control moves (e.g. orders) so that the predicted plant performance behaves in a certain way. In the current case MPC maximizes the future performance by solving Eqn. 2.7. Once the optimization problem is solved, the first set of control moves at time k is implemented and the process is repeated at next time $k+1$. Whenever new information becomes available via measurements, it is incorporated into the model to update the predictions. MPC is a receding horizon control strategy since a fixed horizon optimization is repeatedly solved after a feedback update.

2.4 Illustrative Example

The system to be studied is the supply chain of a single type of a product. The demand for that product is assumed to be changing monthly. The shipment times are assumed to be very short, which would allow a zero-lead time assumption.

The process model of the sample problem is given in state-space form, where I and S (inventory and order levels) are the state variables and o 's are the controlled (decision) variables.:

$$\dot{x} = f(x) + Bu \quad (2.8)$$

$$y = h(x) \quad (2.9)$$

where

$$x = \begin{bmatrix} I_W \\ I_D \\ I_R \\ S_P \\ S_W \\ S_D \\ S_R \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}; \quad u = \begin{bmatrix} O_P \\ O_W \\ O_D \\ O_R \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}; \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad y = \begin{bmatrix} y_W \\ y_D \\ y_R \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}; \quad (2.10)$$

$$f(x) = \begin{bmatrix} -\left(MT + \frac{I^*}{x_1} * ST\right)^{-1} * x_5 \\ \left(MT + \frac{I^*}{x_1} * ST\right)^{-1} * x_5 - \left(MT + \frac{I^*}{x_2} * ST\right)^{-1} * x_6 \\ \left(MT + \frac{I^*}{x_2} * ST\right)^{-1} * x_6 - \left(MT + \frac{I^*}{x_3} * ST\right)^{-1} * x_7 \\ 0 \\ -\left(MT + \frac{I^*}{x_1} * ST\right)^{-1} * x_5 \\ -\left(MT + \frac{I^*}{x_2} * ST\right)^{-1} * x_6 \\ -\left(MT + \frac{I^*}{x_3} * ST\right)^{-1} * x_7 \end{bmatrix} \quad (2.11)$$

$$h(x) = \begin{bmatrix} \left(MT + \frac{I^*}{x_1} * ST \right)^{-1} * x_5 \\ \left(MT + \frac{I^*}{x_2} * ST \right)^{-1} * x_6 \\ \left(MT + \frac{I^*}{x_3} * ST \right)^{-1} * x_7 \end{bmatrix} \quad (2.12)$$

Where $MT=1$, $ST=1$ and $I^*=100$ for all nodes.

$$700 \geq I_i \geq 0, \quad 100 \geq y_i \geq 0, \quad I_i(0) = 100, \quad S_i(0) = 0$$

Customer demand is modeled as integrated white noise (Brownian-motion-type drifts). The discrete-time demand model is given by:

$$d(k+1) = d(k) + w(k) \quad (2.13)$$

where $w(k)$ is white noise with known intensity.

The optimal demand predictions are obtained by first estimating the current demand using a Kalman filter [24] and computing the future values using the dynamic model. The linear estimator is given by

$$\hat{d}(k/k) = \hat{d}(k/k-1) + K_f(d_m(k) - \hat{d}(k/k-1)) \quad (2.14)$$

$$\hat{d}(k/k-1) = \hat{d}(k-1/k-1) \quad (2.15)$$

where $d_m(k)$ is the measured value of the demand.

$\hat{d}(i/j)$ denotes an estimate of $d(i)$ based on measurements available at time j . K_f is the optimal value of the Kalman filter. When the measurement noise is equal to zero, $K_f=1$ which simplifies the above equations to:

$$\hat{d}(k/k) = d_m(k) \quad \text{and} \quad \hat{d}(k+i/k) = d_m(k) \quad \text{for all } i=1,2 \quad (2.16)$$

Thus the best estimate at time k is the measured value of demand and all future values are equal to this value (i.e., demand is assumed constant). If demand measurement contains any noise, K_f can be computed optimally for given noise level and predictions computed accordingly.

2.4.1 Open-Loop Optimization

To perform the open loop optimization, the demand model is first simulated to generate a profile. In the open loop optimization this demand profile is assumed to be perfectly known. The inventory and material transfer measurements and updates are assumed to be made weekly. One of such demand profile for 52 weeks is given in figure 4.

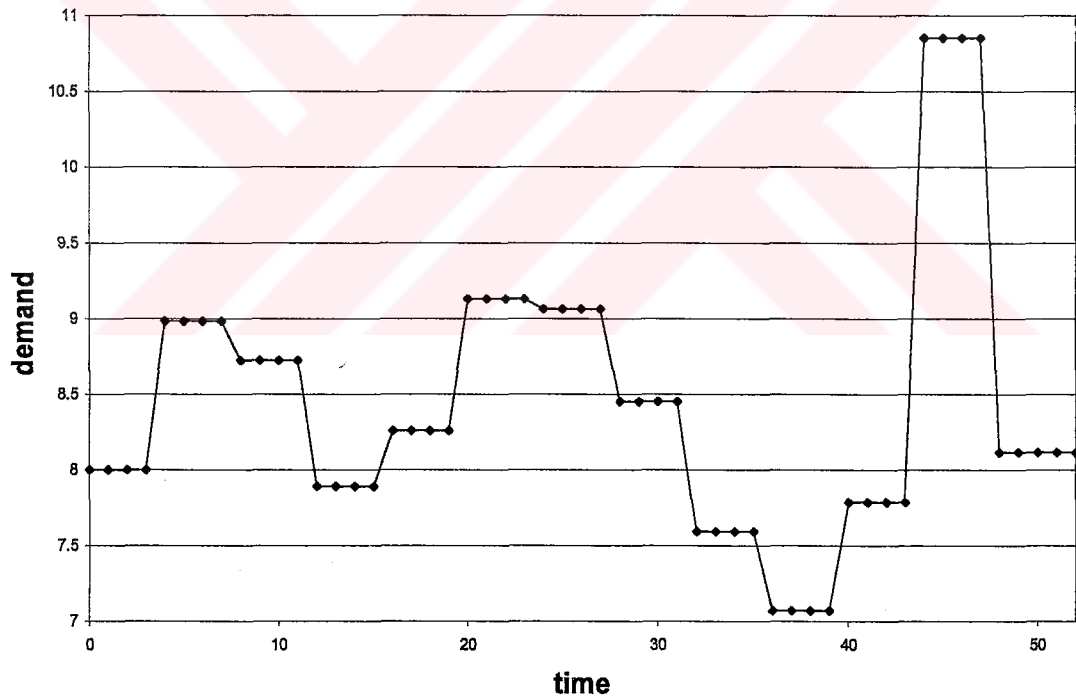


Figure 2.4: Single product demand profile

Inventory costs at each node are 2 monetary units and material transfer costs between nodes are 1 monetary units. The final product is sold to the customer for a price of 50 monetary units. Over and under sales are allowable, however 50 monetary units of penalty cost occurs for each surplus or shortage.

Using the demand profile in figure 2.8, and the state equations 2.8 to 2.12, the system is coded into gPROMS software that is able to handle differential and nonlinear equations to perform dynamic optimization. As a result of the dynamic optimization the maximum profit came out to be 16624 monetary units. The optimal values of the state and control variables are given in the following figures.

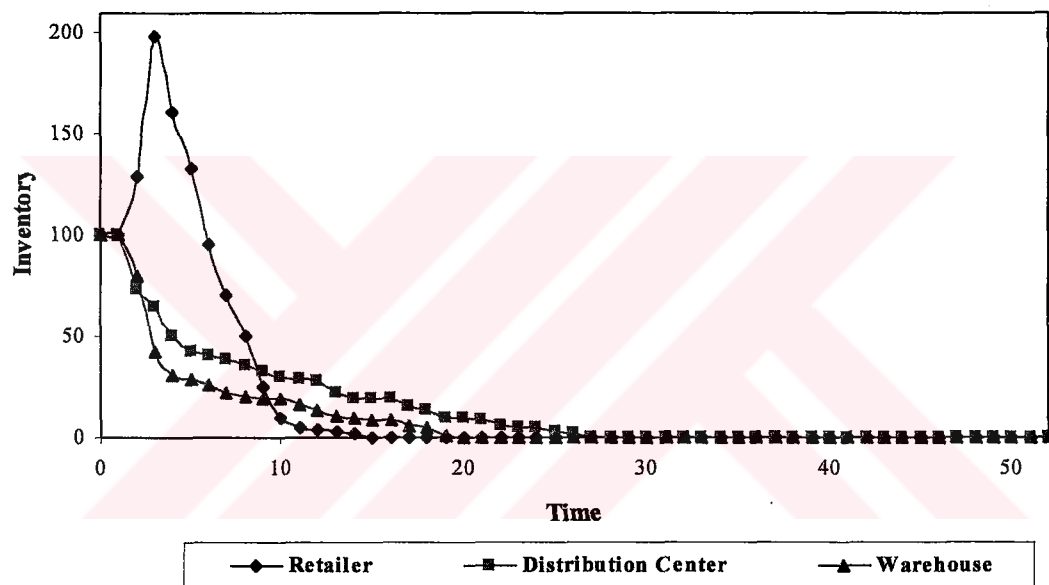


Figure 2.5: Open-loop optimization inventory level results

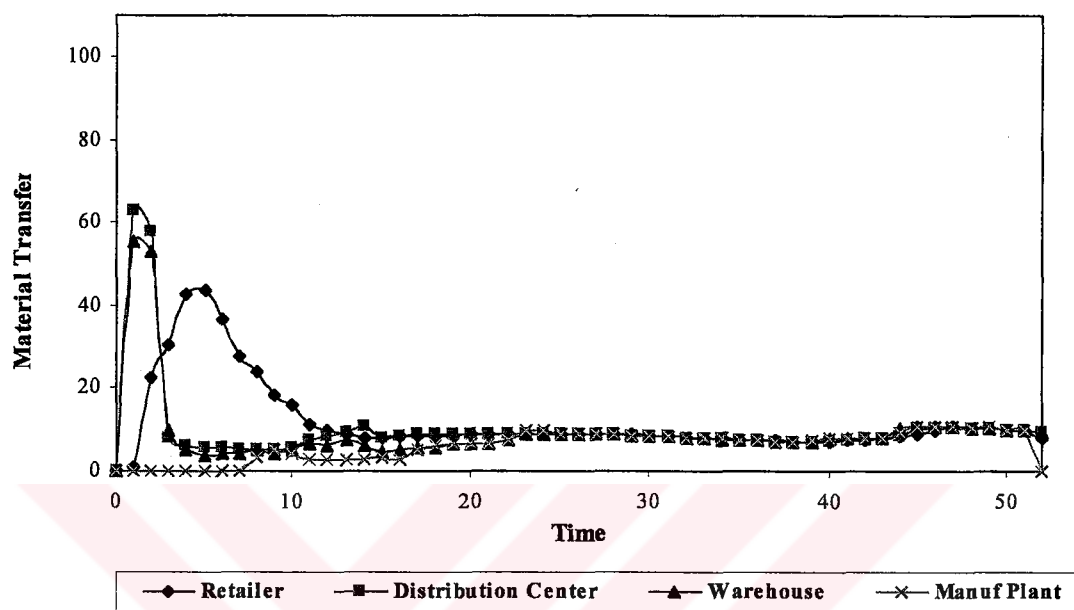


Figure 2.6: Open-loop optimization material transfer level results

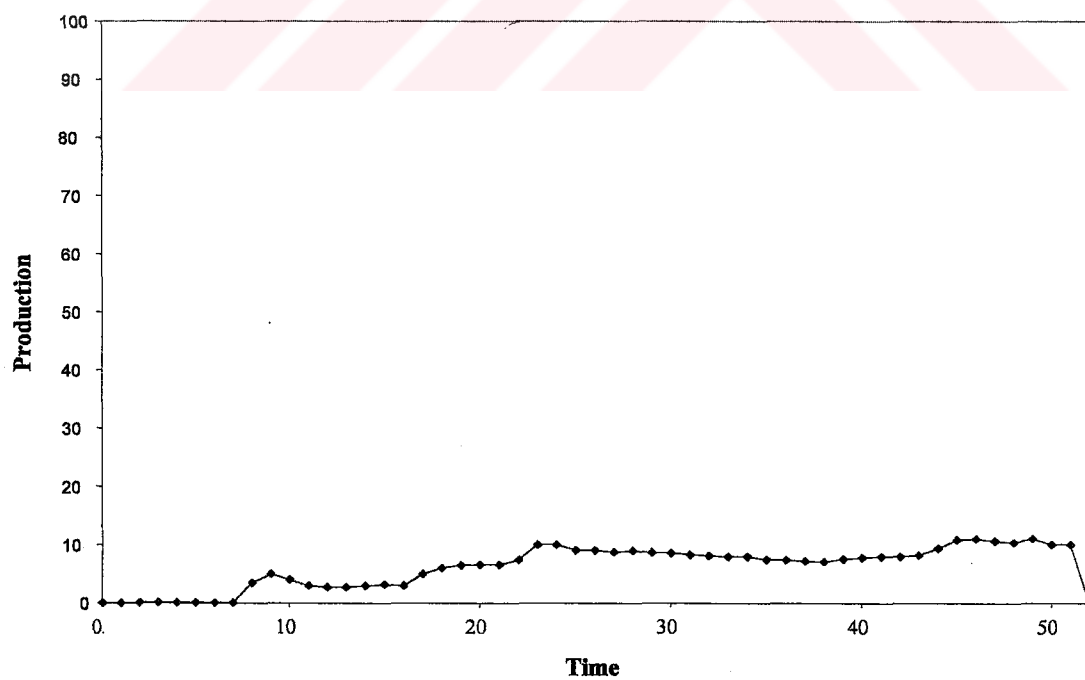


Figure 2.7: Manufacturing Plant Production Amount

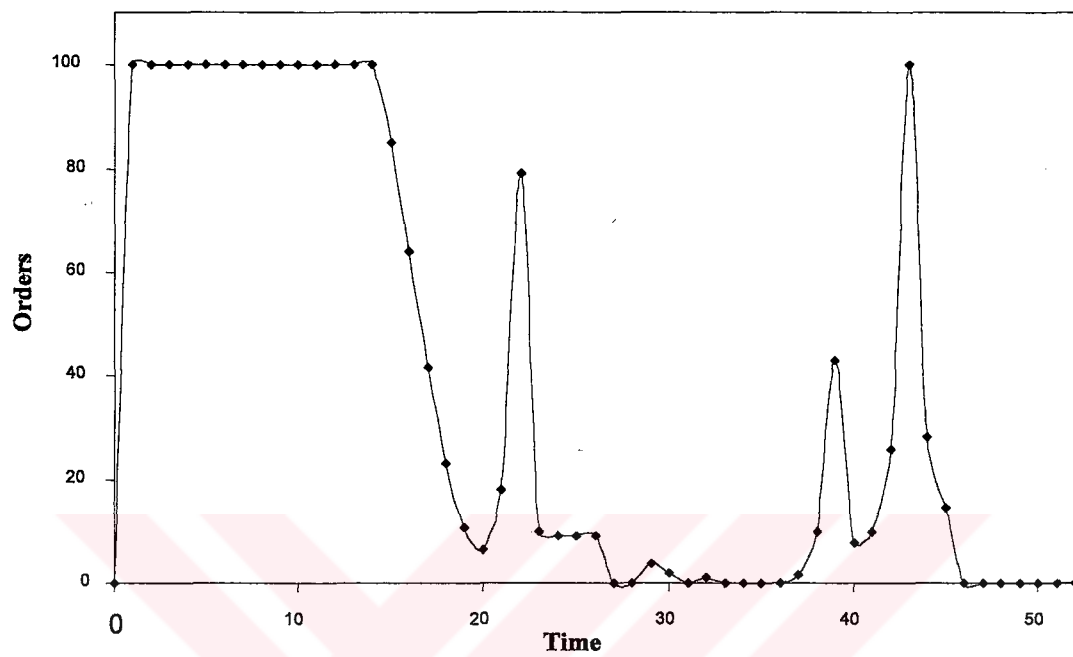


Figure 2.8: Warehouse node optimal orders

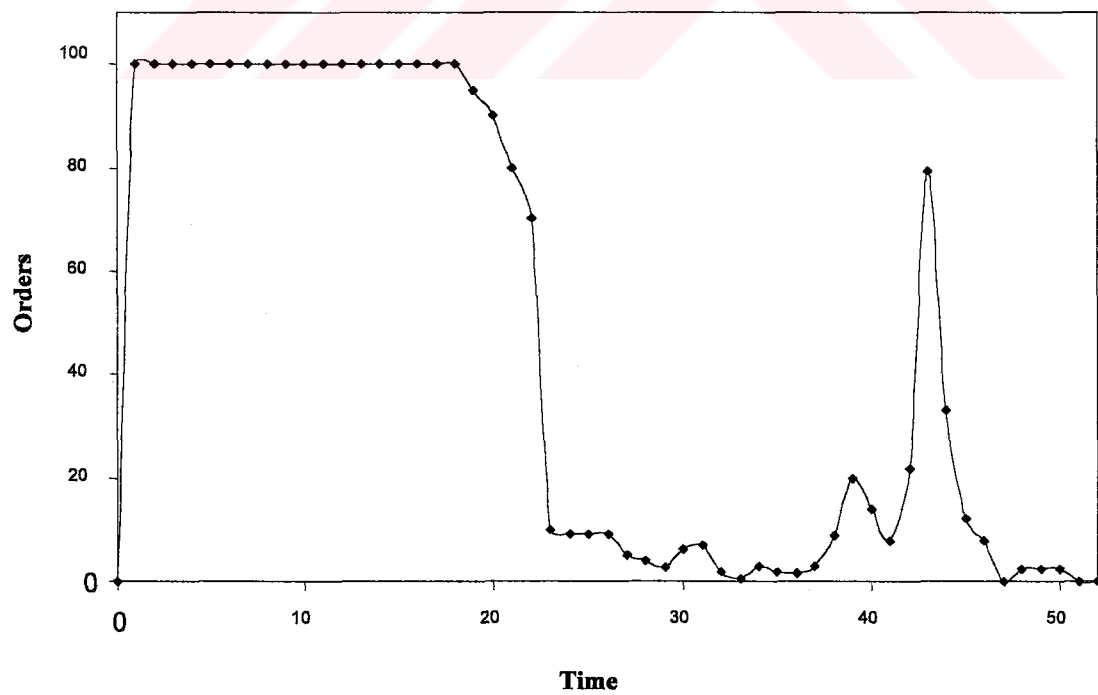


Figure 2.9: Dist Center node optimal orders

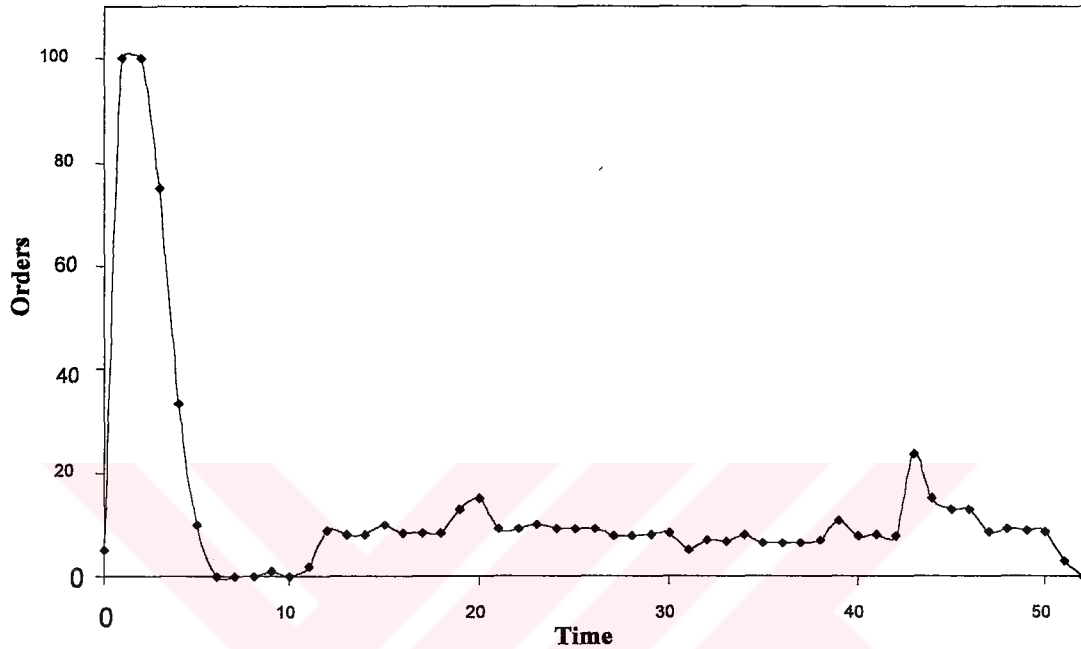


Figure 2.10: Retailer node optimal orders

As a result of open loop optimization (OLO), at the beginning transferring high levels of material to customer regardless of the penalty cost, is more profitable for the supply chain. Material transfer from retailer to customer follows an increasing trend until the end of fifth period. To be able to handle this high transfer rate from retailer to customer, the transfer levels between distribution center-retailer and warehouse-distribution center (Fig. 2.6) also rise. Since the inventory costs are the same at each node, to save time, the system tends to accumulate its inventory in the retailer level. High levels of material transferred from warehouse and distribution center nodes increase the inventory level in the retailer node, while the inventory levels at distribution center and warehouse are depleted (Fig. 2.5). After eight time units, the system chooses to produce due to the decreased inventory levels. Initially the system stores the produced materials in the distribution center and warehouse levels, hence the inventory depletion at these nodes become more gradual. After around ten time units, it depletes its entire inventory in the retailer and warehouse nodes and chooses to deliver from its distribution center node. After

around another 10 time units all inventories in all nodes are depleted and the plant directly transfers all its production without storing inventories. The production rule states that the amount produced is exactly equal to the orders placed from the warehouse and in time the amount produced becomes almost equal to the actual demand (Fig. 2.7).

2.4.2 MPC Optimization

In the MPC application the optimization horizon P is set equal to 5 which corresponds to 20 weeks. At each sampling time (every 4 weeks) new demands are measured; future demands are predicted; inventory and order accumulations (states) are updated, and a new optimization is performed. Between the measurement updates the plant is simulated with the actual demand values and the optimal control values computed with MPC. This procedure is repeated until the final time is reached. MPC optimization gives a profit of 15786 monetary units. Other results are given with the open loop results in the following figures.

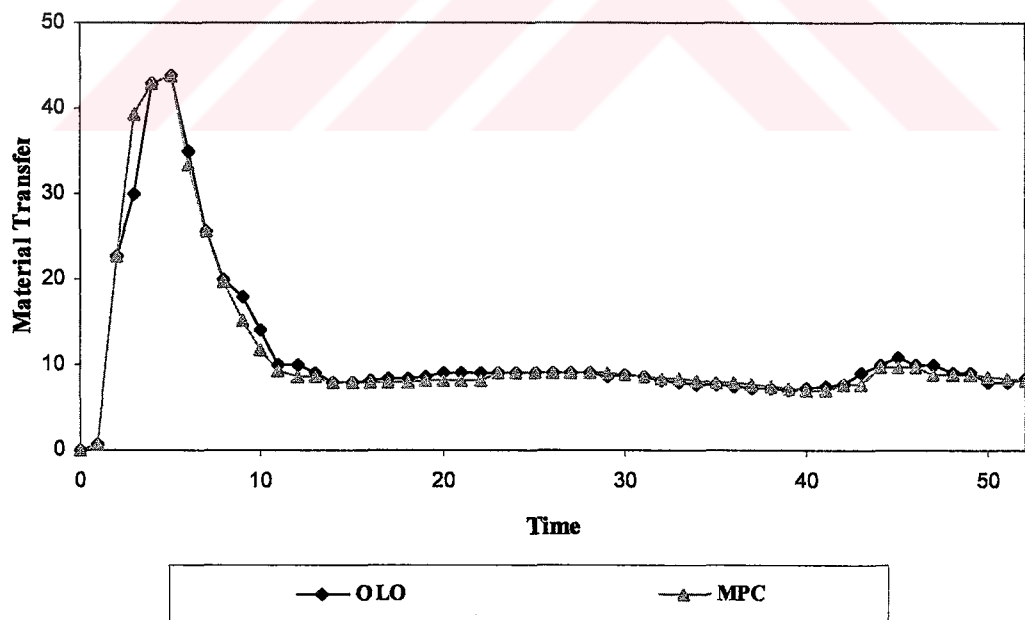


Figure 2.11: Retailer node material transfer level

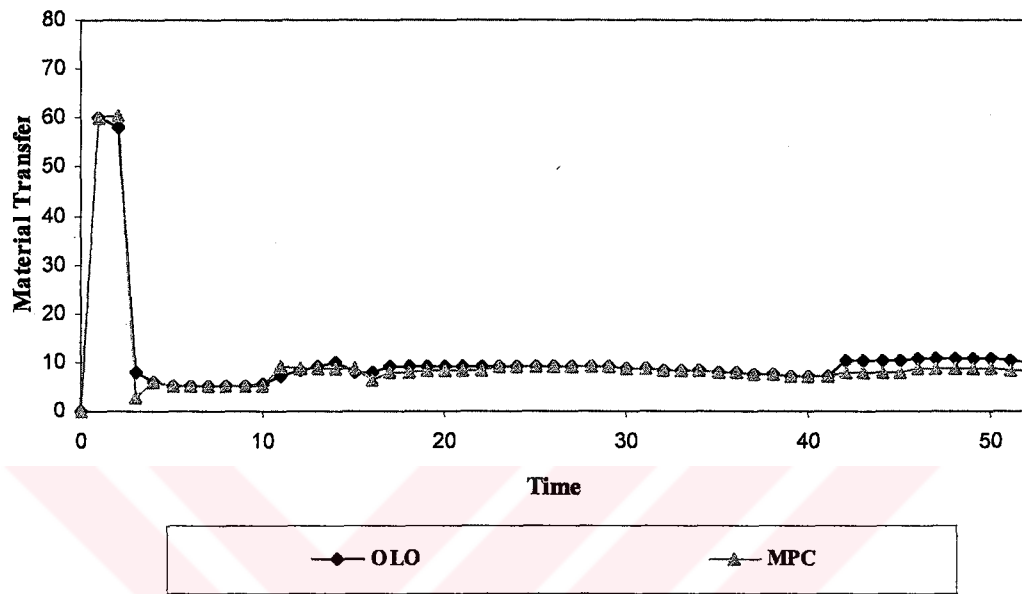


Figure 2.12: Distribution center node material transfer level

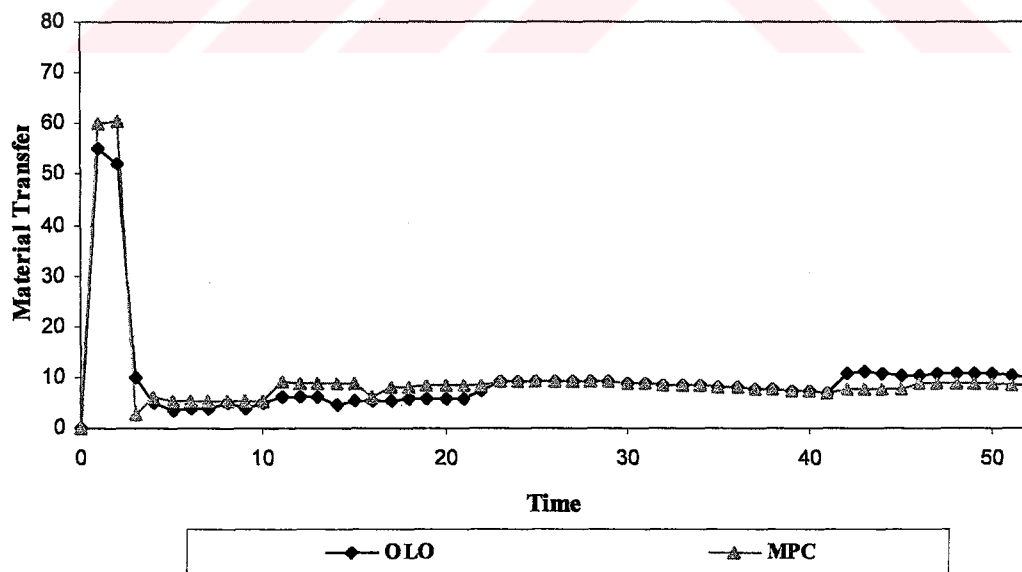


Figure 2.13: Warehouse node material transfer level

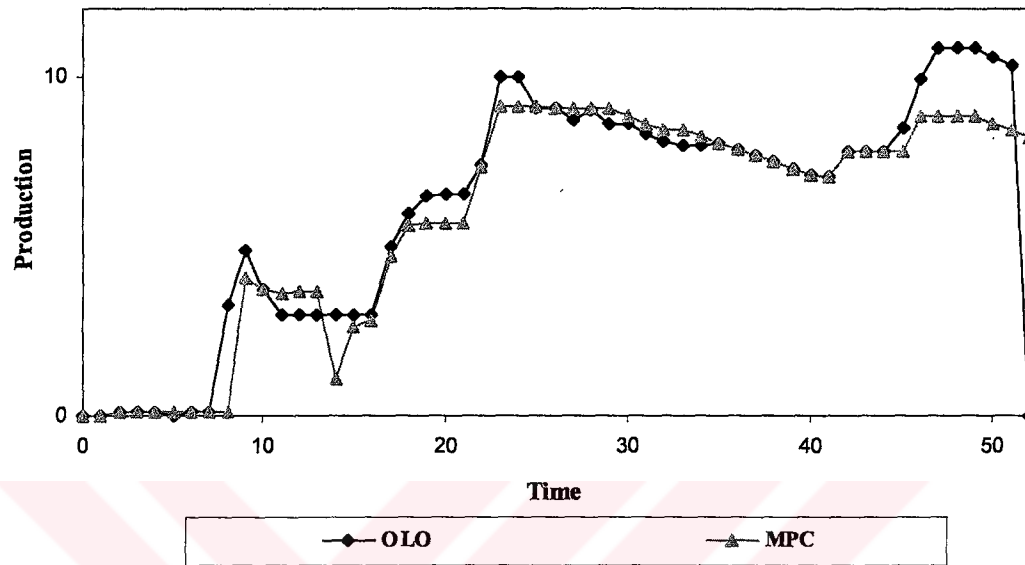


Figure 2.14: Optimal production rates

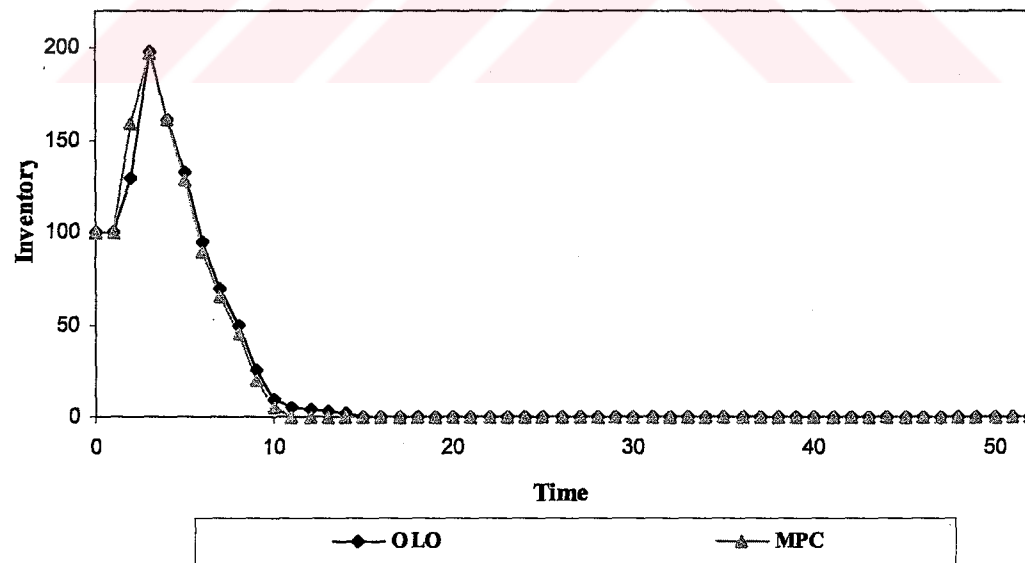


Figure 2.15: Retailer node inventory level

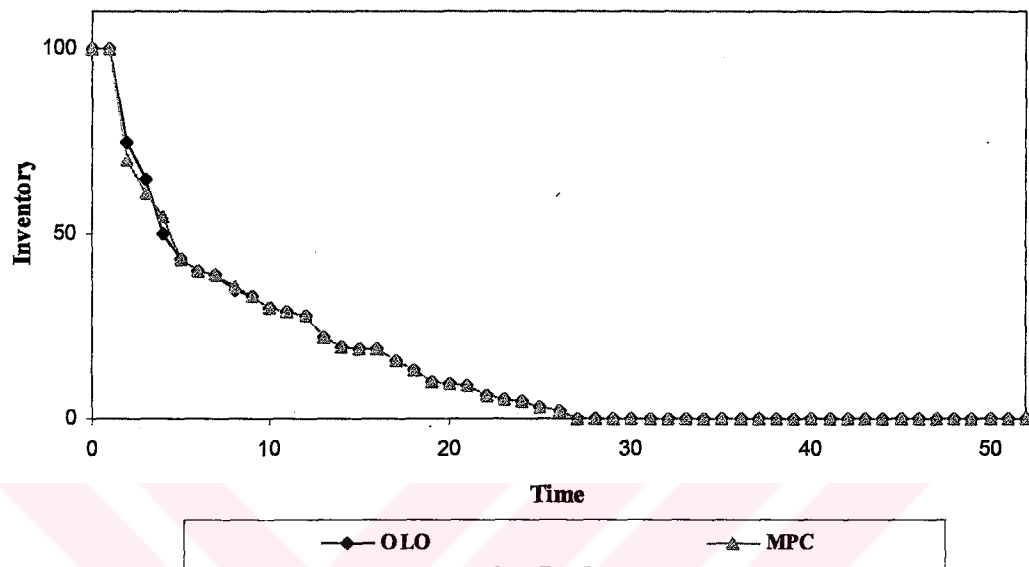


Figure 2.16: Distribution center inventory level

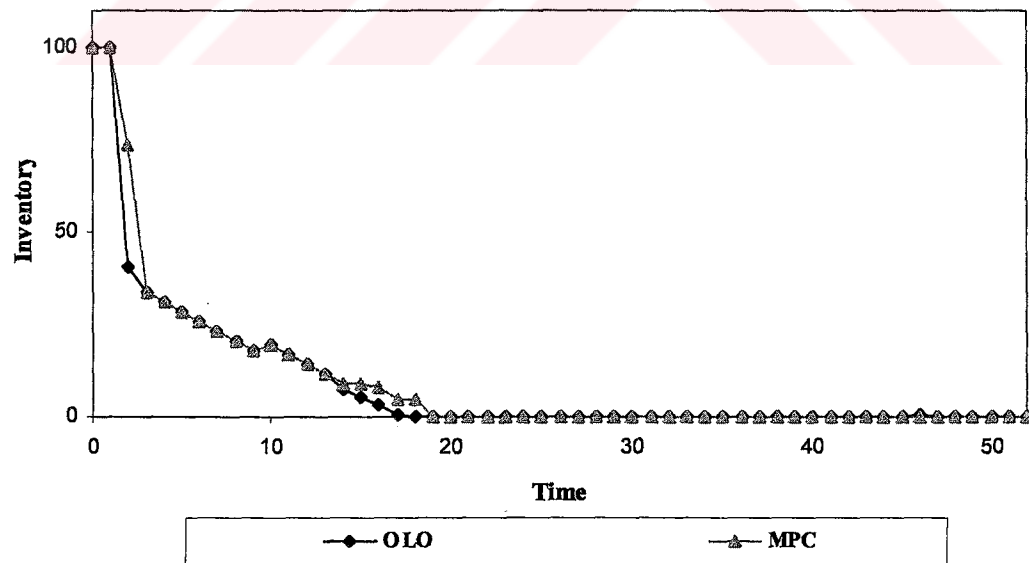


Figure 2.17: Warehouse node inventory level

2.4.3 Single Product Discrete Supply Chain

One of the aims of this thesis is to implement MPC approach in more complex supply chain systems, which contain discrete units as well as typical continuous nodes. To be able to enlarge the scope of the thesis to include these more complex systems as given in the following chapters, the present single product continuous model is discretized.

The differential equations governing the state variables I and S are discretized through difference equations as:

$$\frac{dI_j}{dt} = \sum_{\substack{\text{from} \\ \text{upstream} \\ \text{nodes}}} y_{j,i} - \sum_{\substack{\text{to} \\ \text{downstream} \\ \text{nodes}}} y_{j,k} \quad \Rightarrow \quad I_j[t] = I_j[t-1] + \sum_i y_{ji}[t-1] - \sum_k y_{jk}[t-1] \quad (2.17)$$

$$\frac{dS_j}{dt} = \sum_{\substack{\text{from} \\ \text{downstream} \\ \text{nodes}}} o_{j,k} - \sum_{\substack{\text{to} \\ \text{downstream} \\ \text{nodes}}} y_{j,k} \quad \Rightarrow \quad S_j[t] = S_j[t-1] + \sum_k o_{jk}[t-1] - \sum_k y_{jk}[t-1] \quad (2.18)$$

The linear forms of the non-linear material transfer equations are obtained through regression. The continuous inventory, order accumulation and material transfer function results are obtained from the gPROMS simulation for several different initial inventory values. These data are then regressed to obtain the linear form of the material transfer equations. The statistical analysis of this regression is presented in Appendix A. The highly popular linearization method of Taylor expansion is not used in this case since the convergence of the bilinear function is not guaranteed to be continuous at any point since S_j can make discontinuous jumps. The linearized equations are given as follows:

$$y_R[t] = 0.19 S_R[t] + 0.052 I_R[t] - 5.397 \quad (2.19)$$

$$y_D[t] = 0.031 S_D[t] + 1.1 I_D[t] - 50 \quad (2.20)$$

$$y_W[t] = 0.024 S_W[t] + 0.9 I_W[t] - 30.6 \quad (2.21)$$

The production equation:

$$y_p[t] = u_w[t] \quad (2.22)$$

2.4.4 Open Loop vs MPC Optimization of the Discrete System

Open loop optimization of the discrete system is performed using the same demand profile for the continuous case. The optimal profit comes out to be 16235 monetary units, slightly different from the continuous case, mainly arising from the regression process performed to linearize the material transfer terms.

The open loop optimization of the discrete system is coded into the OPL Studio software which runs CPLEX in the background on a pentium IV 1.7GHz processor, to optimize linear models. MPC optimization is also run, similar to the continuous case, for the fixed horizon approach. To simulate another case, the control update interval, was increased from 4 to 8. This causes some performance loss since demand changes every 4 weeks but measurement update is done less frequently, every 8 weeks. The running times and optimal results for different optimization approaches are given in table 1 and the following figures:

Table 1: CPU times, Objective values and Sizes of different Optimization Approaches

Control update interval	MPC horizon length	CPU time (sec)	Obj. value	No of iterations	No of constraints	No of variables
4	15	10	15156	1118	8008	3913
4	20	13	15758	1534	10608	5148
4	25	17	15999	1807	13208	6383
4	35	21	16122	4147	18408	8853
8	35	18	14014	1938	8496	4086
OLO	52	6.46	16235	660	2151	1059

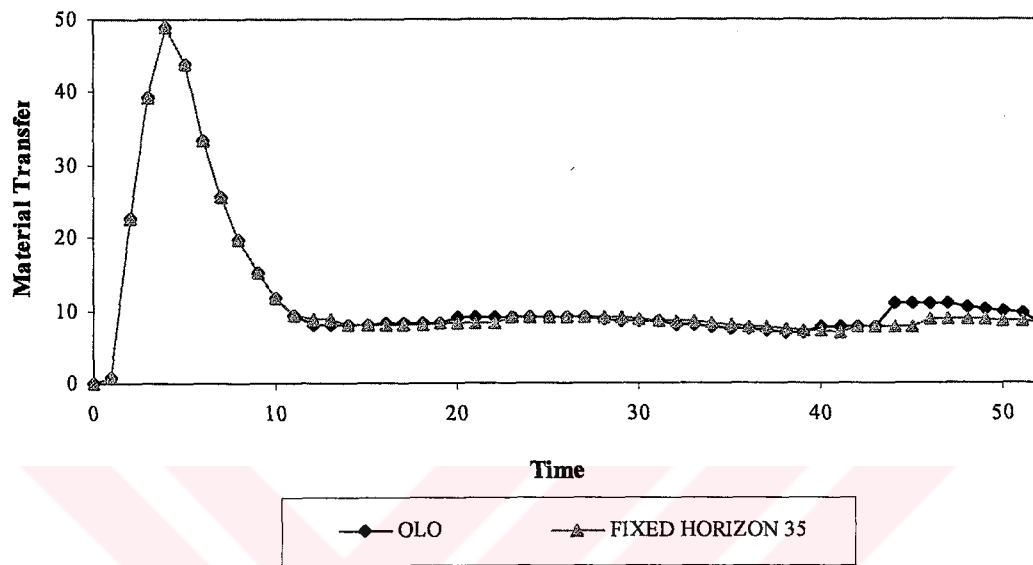


Figure 2.18: Retailer Node Mat. Transfer (OLO vs. MPC)

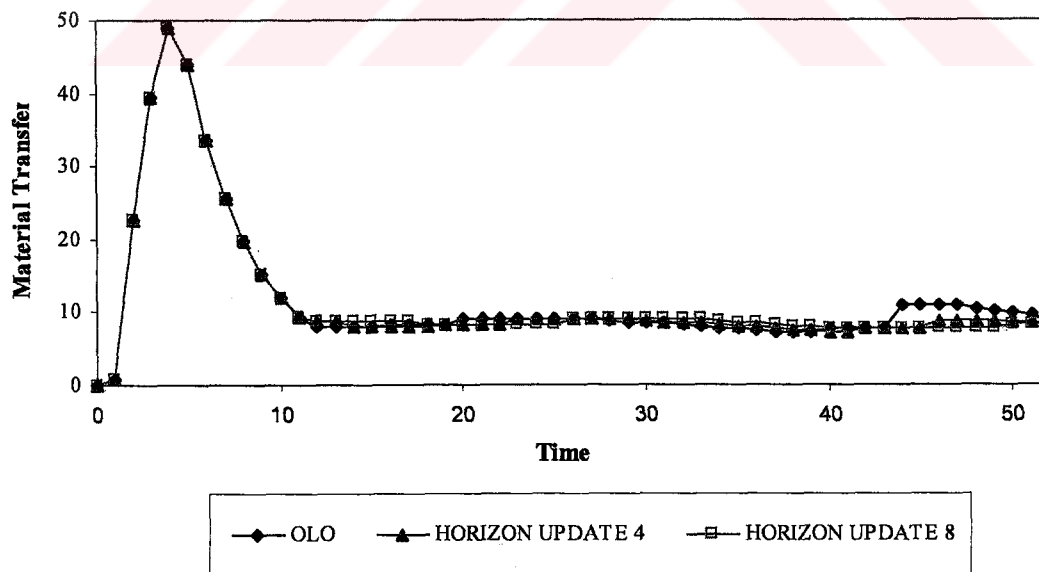


Figure 2.19: Retailer Node Mat Transfer (Update interval 4 vs. 8)

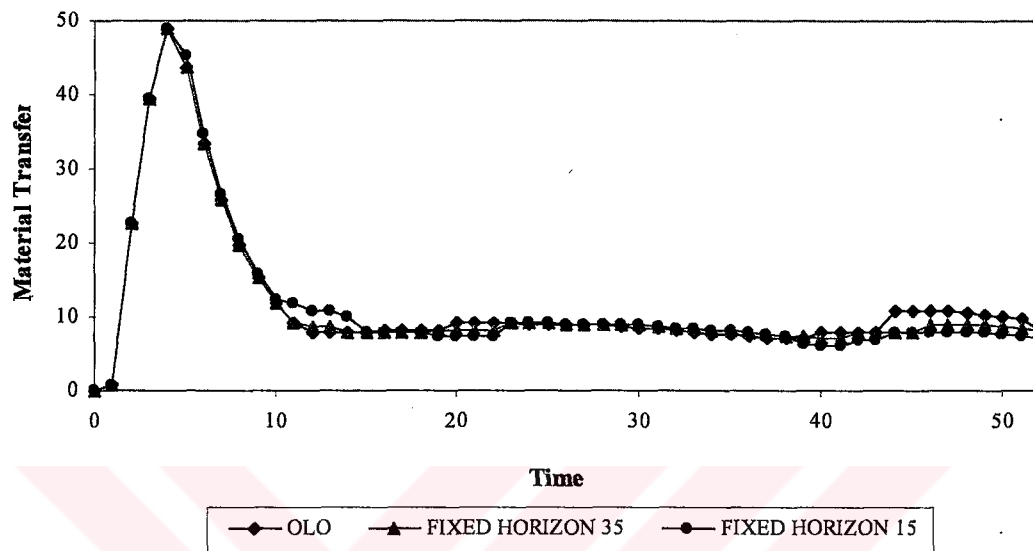


Figure 2.20: Retailer Node Mat Transfer (Fixed horizon 35 vs. 15)

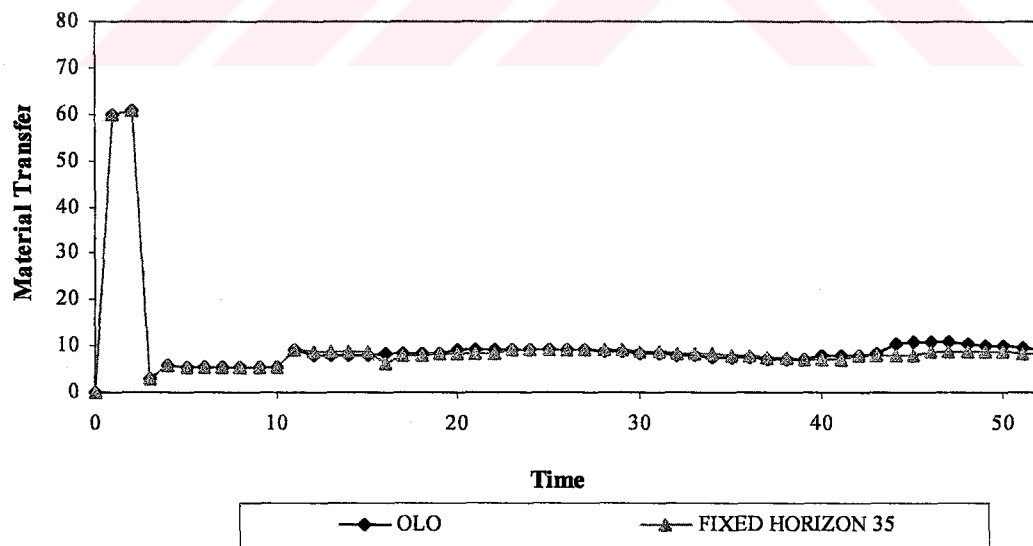


Figure 2.21: DC Node Mat. Transfer (OLO vs. MPC)

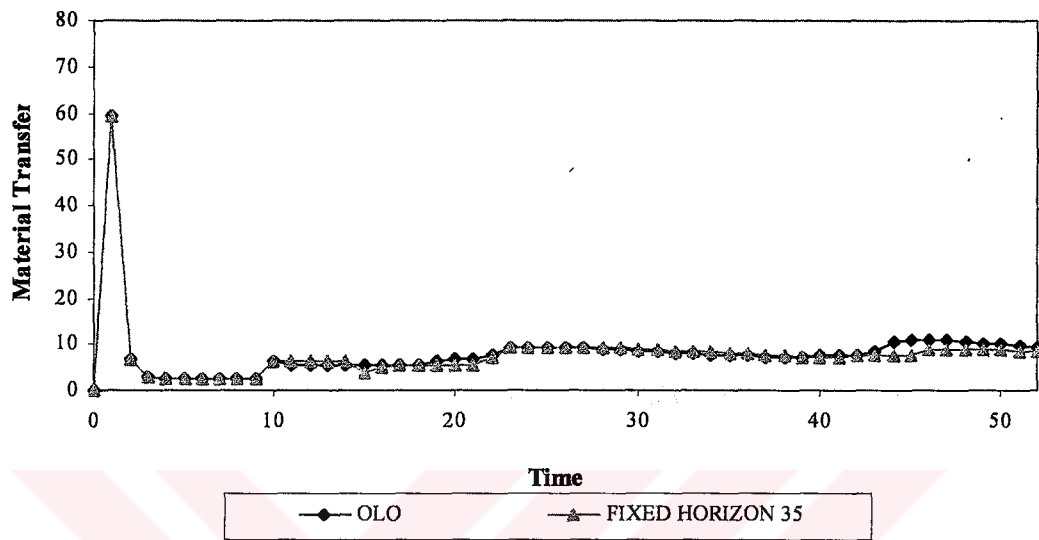


Figure 2.22: WH Node Mat. Transfer (OLO vs. MPC)

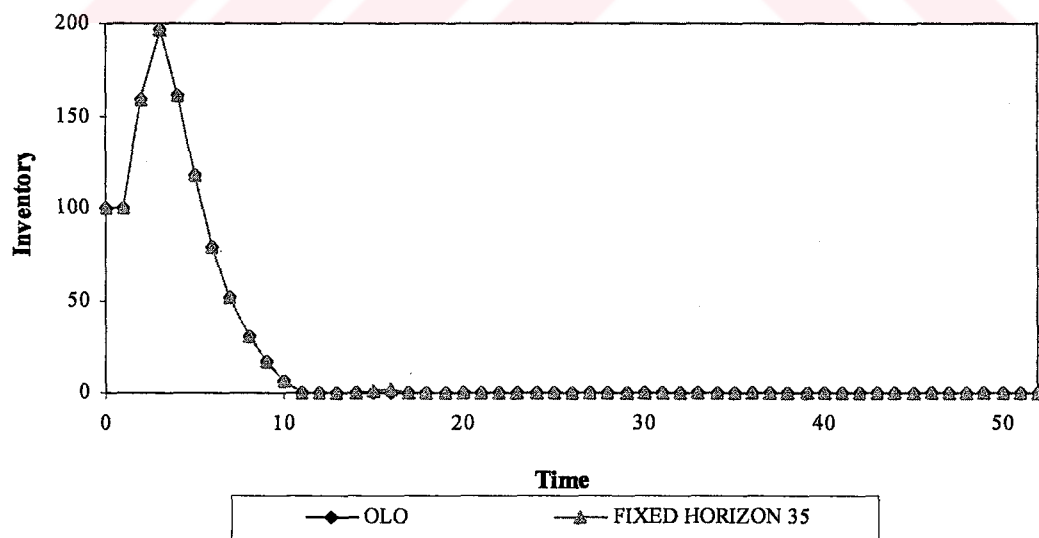


Figure 2.23: Retailer Node Inventory (OLO vs. MPC)

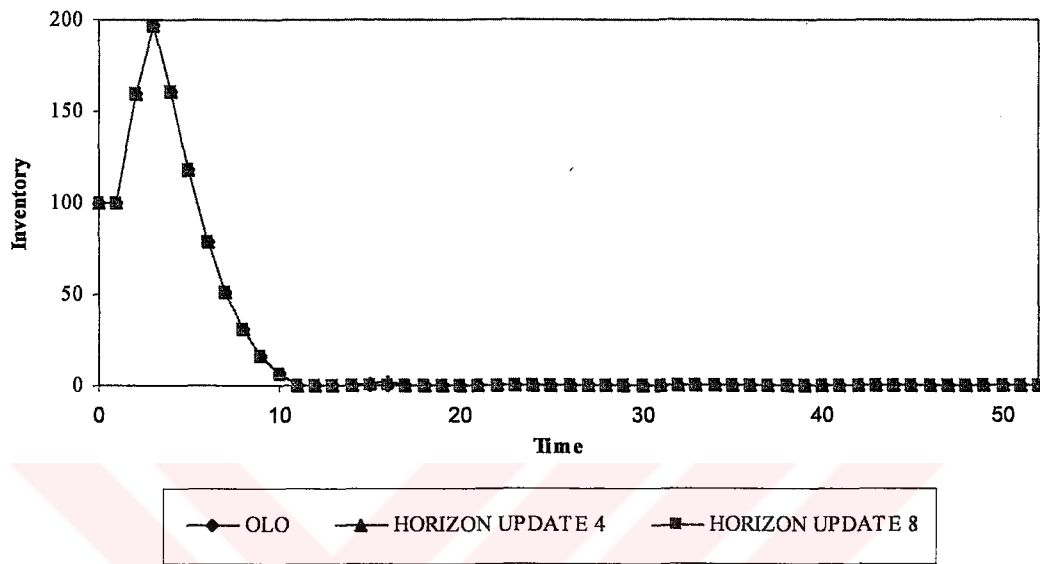


Figure 2.24: Retailer Node Inventory (update interval 4 vs. 8)

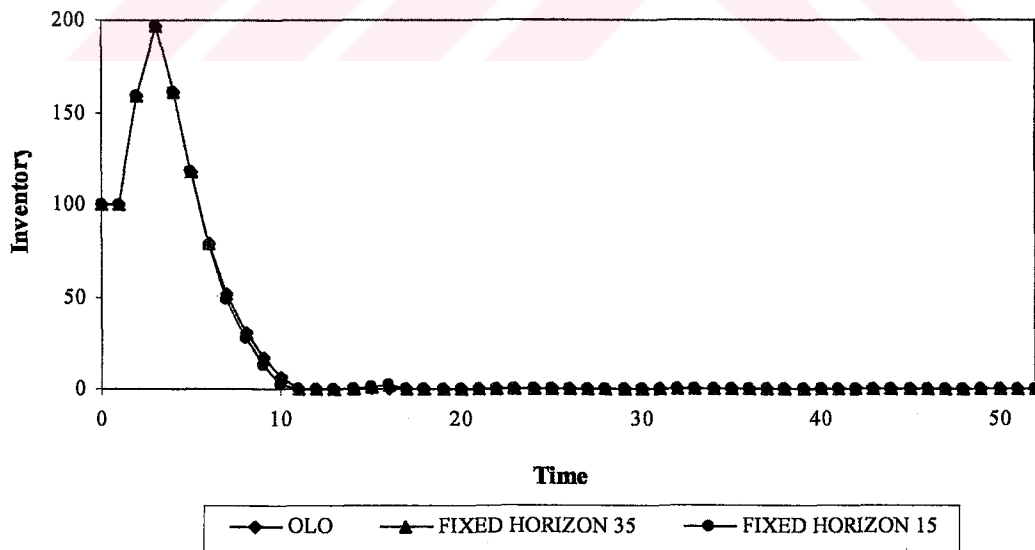


Figure 2.25: Retailer Node Inventory (Fixed horizon 35 vs. 15)

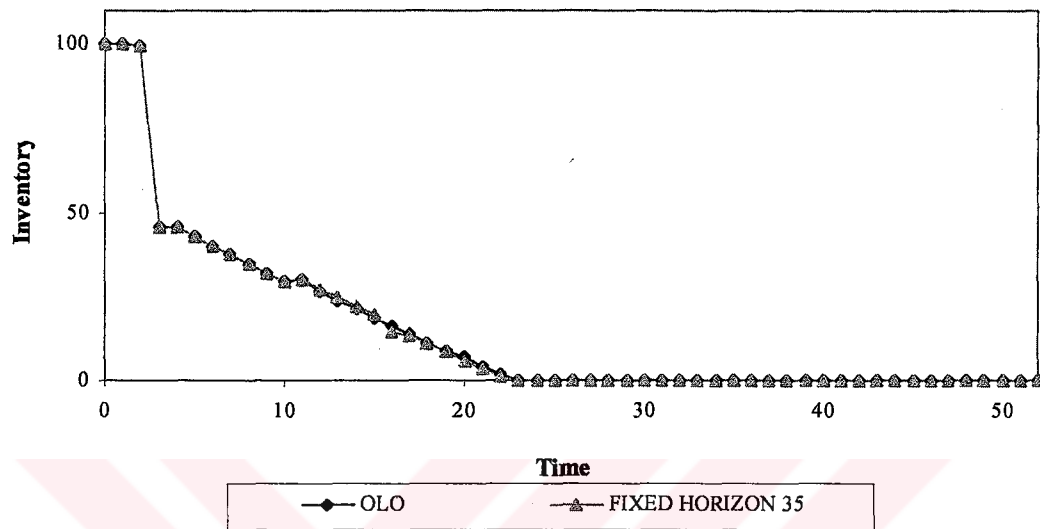


Figure 2.26: DC Node Inventory (OLO vs. MPC)

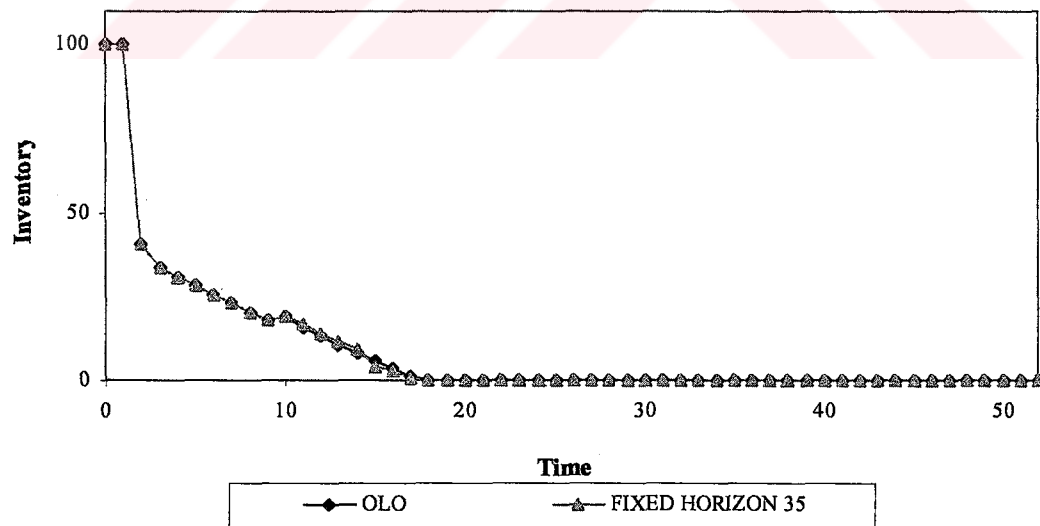


Figure 2.27: WH Node Inventory (OLO vs. MPC)

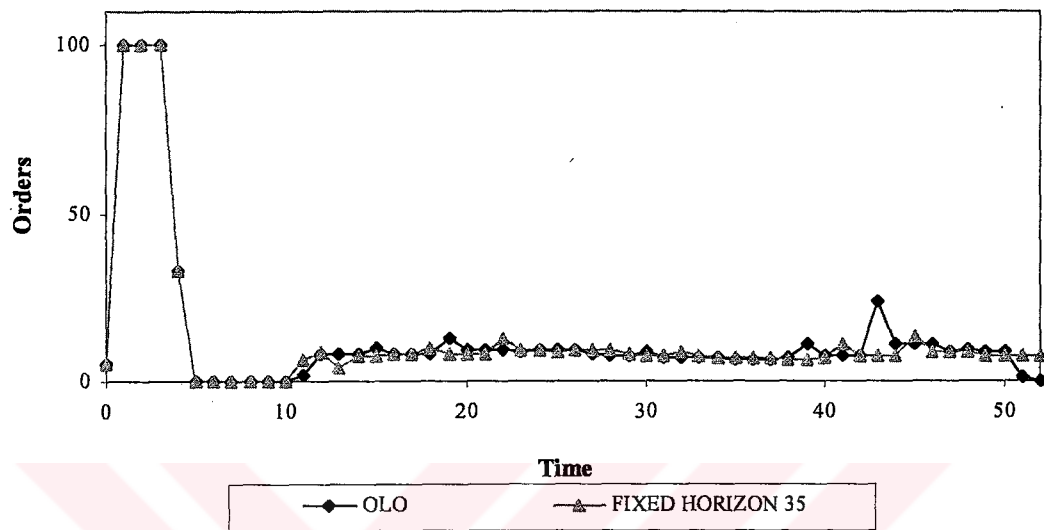


Figure 2.28: Retailer Node Orders (OLO vs. MPC)

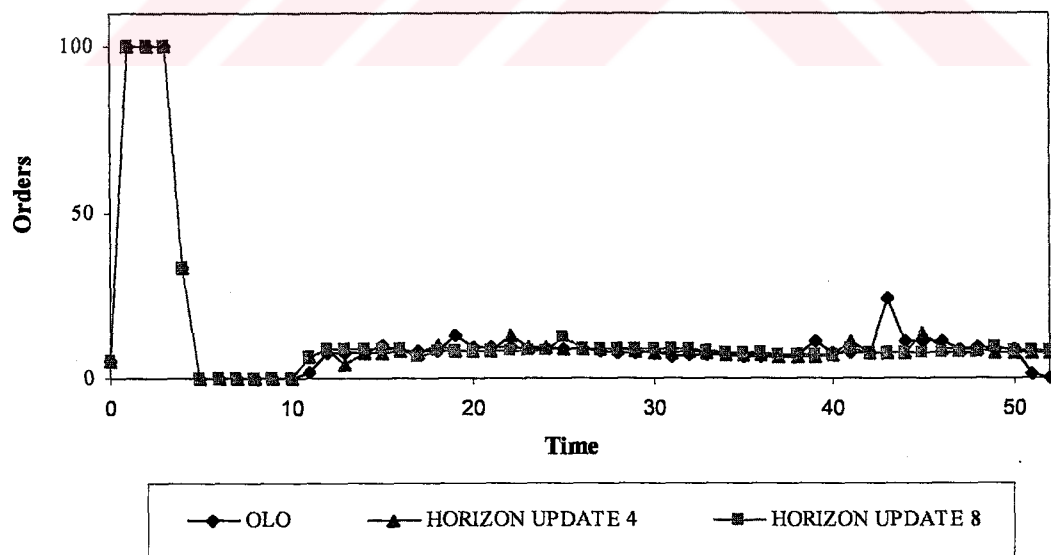


Figure 2.29: Retailer Node Orders (Update interval 4 vs. 8)

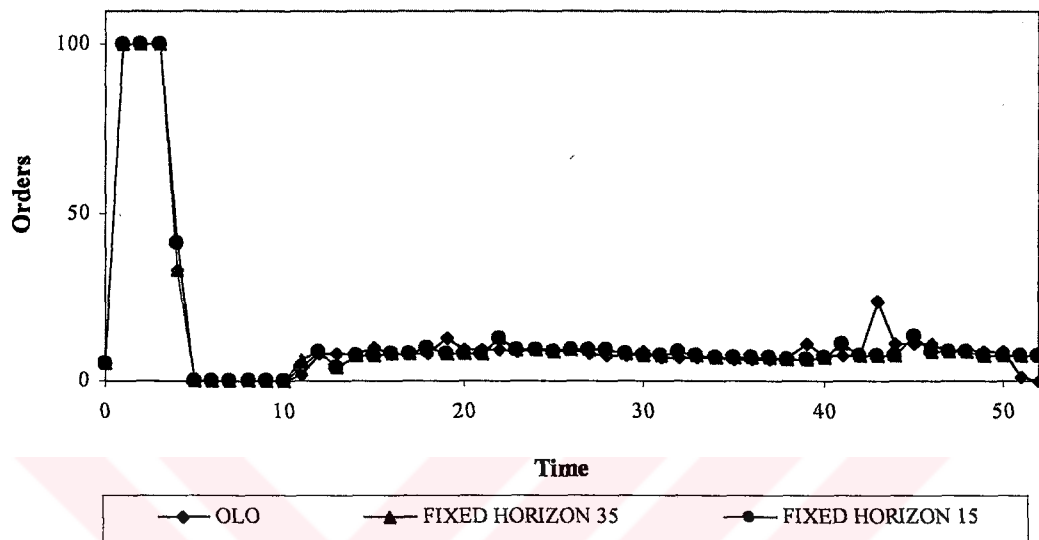


Figure 2.30: Retailer Node Orders (Fixed horizon 35 vs. 15)

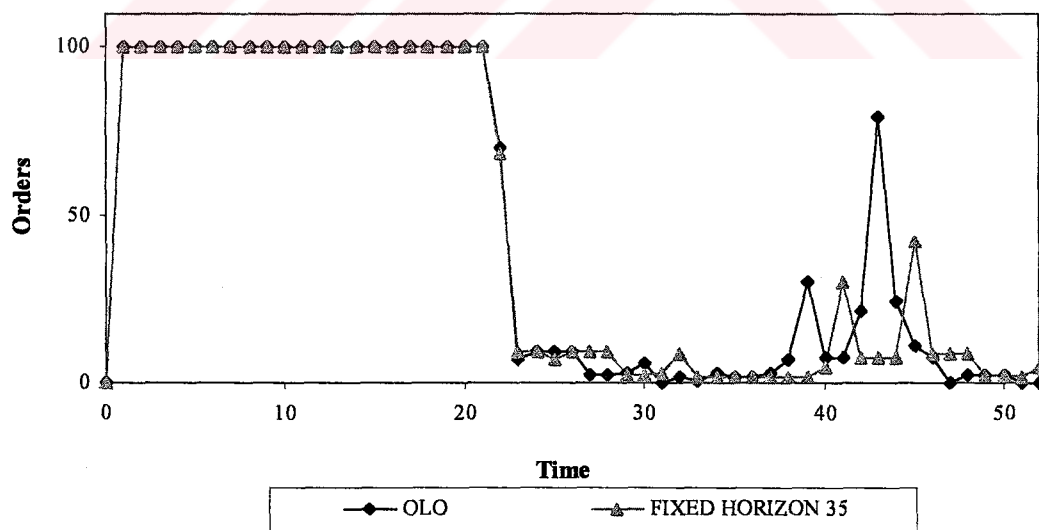


Figure 2.31: DC Node Orders (OLO vs. MPC)

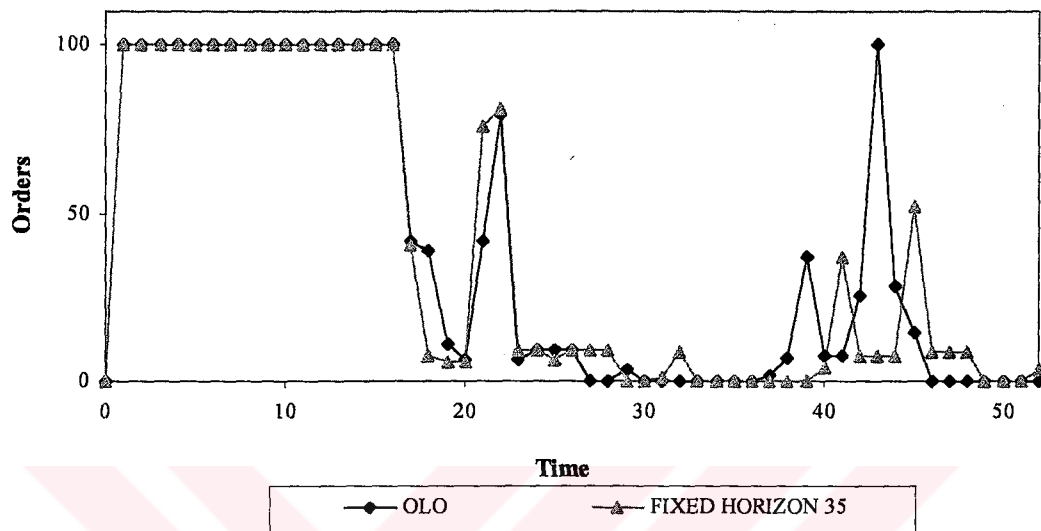


Figure 2.32: WH Node Orders (OLO vs. MPC)

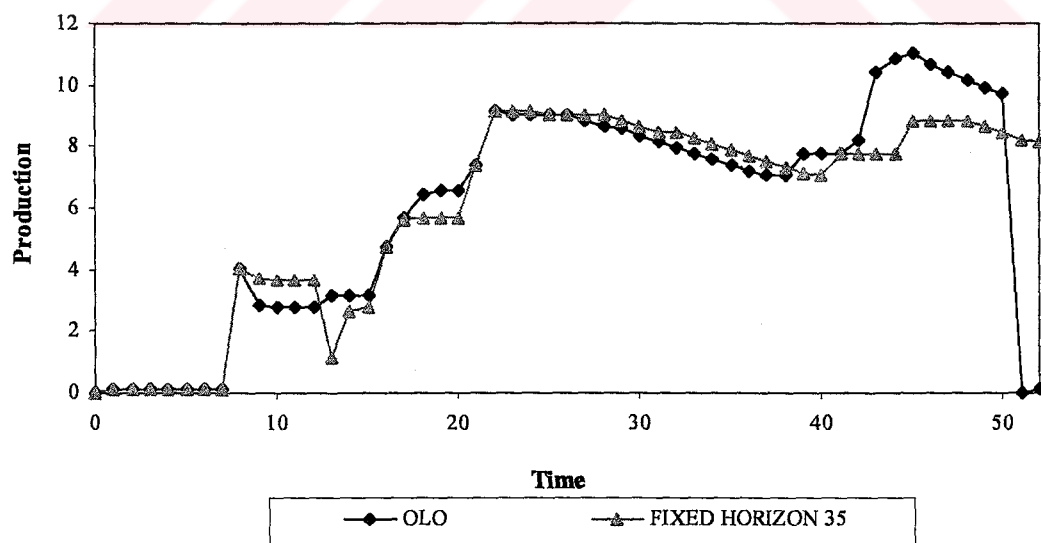


Figure 2.33: Production Node (OLO vs. MPC)

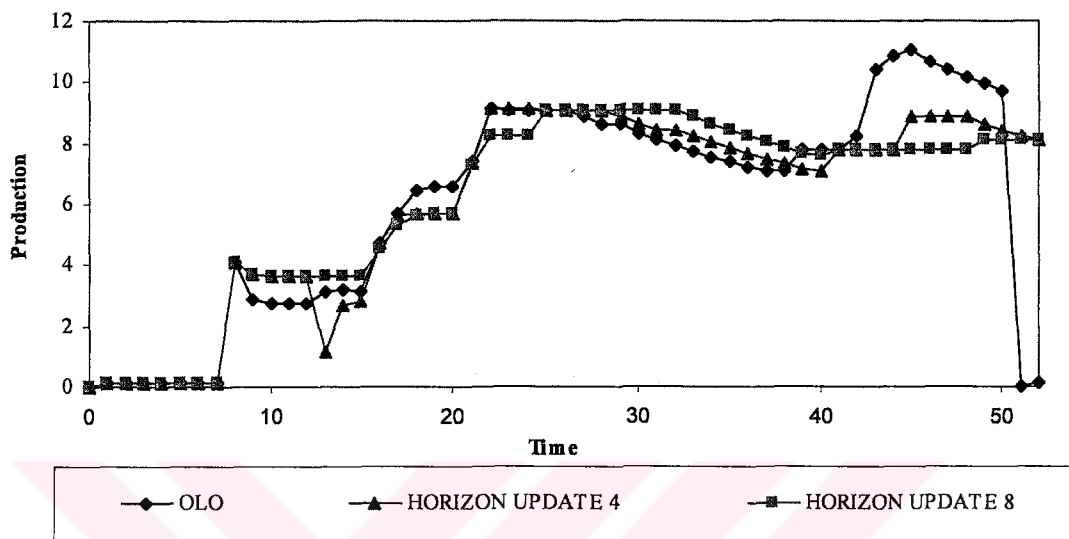


Figure 2.34: Production Node (update interval 4 vs. 8)

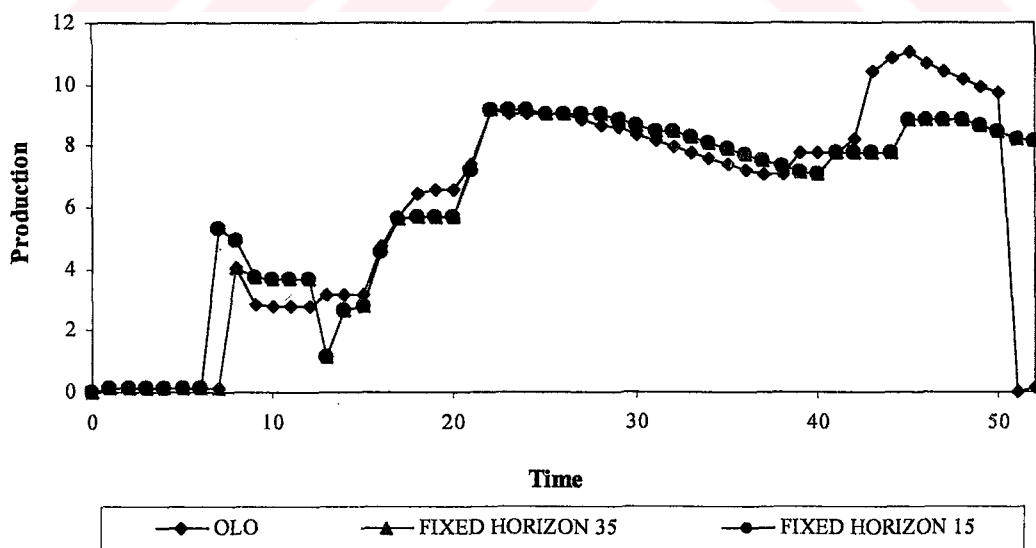


Figure 2.35: Production Node (Fixed horizon 35 vs. 15)

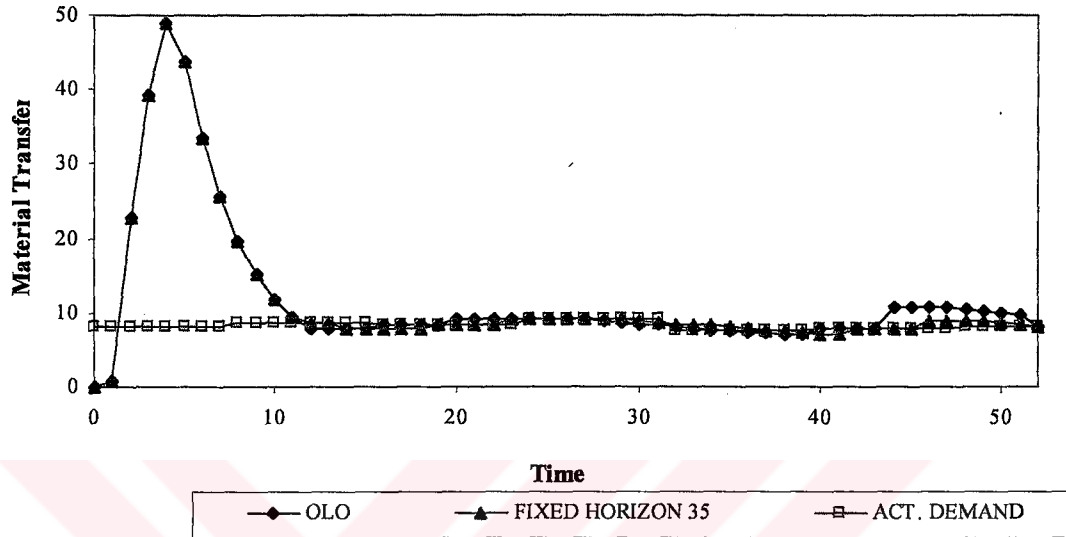


Figure 2.36: Material transferred from retailer vs. actual customer demand

According to the objective function, if the initial inventories are high, the system initially favors the transfer of all material from the inventory in the nodes, without producing new material or without taking customer satisfaction into account, due to high value of the stored product as reflected by the value of y_R in equation 5, and as shown in Fig. 2.18. However, there are upper and lower limits on the inventories. Therefore the system tends to use these inventories such that, as it benefits from the high value of the sold material and it also finds the optimal production schedule over time.

Optimizer makes a tradeoff between the high revenues gained by transferring the entire product to the customer and the high costs incurred by production and customer dissatisfaction. At the beginning, up to around time 4, while the revenue coefficients are very high, it transfers large quantities of material to the customer, then the amount transferred decreases and gradually the amount transferred becomes equal to the actual customer order, as shown in Fig. 2.36. It reduces the inventories in all nodes to zero and directly transfers the amounts produced, since the production and material transfer costs are low relative to the inventory holding costs, as shown in Figs. 2.23, 2.26, 2.27.

Open loop and MPC optimization results do not deviate much from each other at the beginning where initial inventories are sufficient to meet customer demand directly. Initially both approaches aim to decrease their inventories. Bulk deliveries are made at the very beginning to benefit from high revenues, yet in time they become equal to the actual customer demand. The main differences between the open loop and MPC approaches arise when production begins.

For the open loop case, the optimizer sees the actual demand pattern for the whole optimization horizon (52 weeks) at the beginning, and schedules its decision variables accordingly. As shown in Fig. 2.4, demand either gradually increases or decreases, with one exception between time 43 and 44 where it increases from 7.78 to 10.85, the most drastic change in the profile. The open loop optimizer recognizes this change at the beginning, and increases the order amounts between the nodes, around 43rd time unit, to be able to meet the customer high customer demand from the 44th time unit (Figs. 2.28, 2.31, 2.32). Production level also starts to increase and attains its maximum value at 45th time unit.

In the MPC cases, the same pattern of selling out the initial inventory at the beginning and transferring the newly produced product in time, is observed, as in the open loop case. However, in the MPC approaches, the optimizers do not see the entire actual demand pattern, but assumes that it is constant for each optimization period. The fixed horizon MPC, for example, recognizes that change in demand at the beginning of the 44th time unit, but it predicts that the demand will continue at that value for the next 35 time units. Instead of taking immediate precautions against that demand change, it gradually schedules the system s , and results in falling short of meeting the actual customer demand, as shown in Fig. 2.36.

A smaller fixed horizon MPC performs worse at the beginning, compared with the longer one, since it underestimates the actual process, as can be seen in figures 20, 25, 30, 35 and 36. For example at the end of the 8th time unit, fixed-35 horizon optimizes the system until the end of 43rd time unit, while fixed-15 horizon optimizes until the end of 23rd time unit. For the specific problem, the termination time of the process is known. Therefore a longer optimization horizon estimates the actual case more closely. However after the 20th time unit, the longer horizon over estimates the actual case. For example after the 32nd time unit, fixed-35 horizon predicts until the 67th time unit while fixed-15 horizon optimizes until 47th time. On the other hand, processes frequently do not tend to terminate after a specific time, but rather continue in cyclic fashion. In this case, the demand pattern is expected to replicate itself for the next 52 time units. One important advantage of short horizon is that due to its smaller number of variables and

constraints, it gives solution quicker than the long horizon case. (Table 1). For this single product and simple manufacturing policy case, solution time does not create problems, yet as the system gets more complex, time factor can be a decision variable in choosing the most appropriate MPC approach.

In the case where the measurement update interval was taken as 8, different from the demand change interval which is set at 4, MPC performance deteriorates somewhat. For this specific case, since the demand deviates with white noise, the lost demands do not cause extreme problems. However, for more drastic changes in demand, the MPC approach would highly deviate from the optimal. At the 44th time, MPC misses the most intensive change of demand in the demand profile. As a result, it does not increase its production amount and ceases to meet the customer demand (Figs. 2.19, 2.24, 2.29 and 2.34).



CHAPTER 3

MODEL PREDICTIVE CONTROL OF MULTIPLE PRODUCT HYBRID SUPPLY CHAINS

3.1 Modeling of a Three-Product Hybrid Supply Chain with Batch Production-Illustrative Example

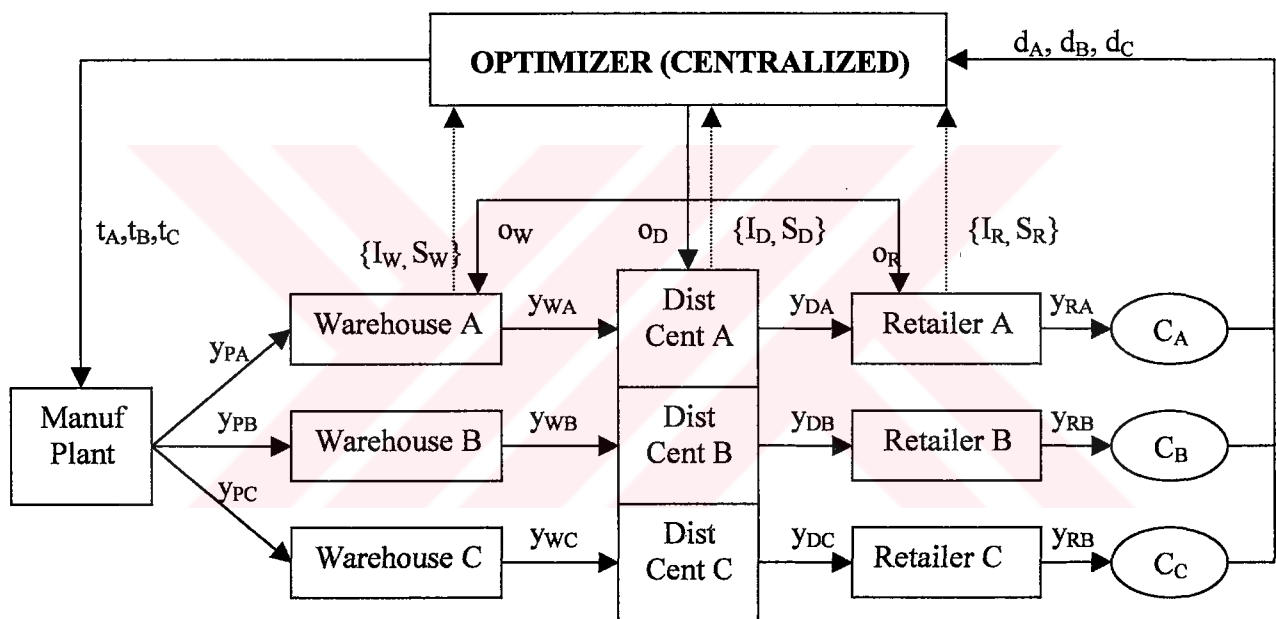


Figure 3.1: A typical multiple product supply chain

For the three product supply chain, shown in Fig. 3.1, each product has its own retailer, distribution center, warehouse nodes, but all products are manufactured in a single manufacturing plant. The inventory balance equation at node j for product m is given as follows:

$$\frac{dI_{jm}}{dt} = \sum_{\substack{\text{from} \\ \text{upstream} \\ \text{nodes}}} y_{jim} - \sum_{\substack{\text{to} \\ \text{downstream} \\ \text{nodes}}} y_{jkm} \quad (3.1)$$

where y_{jim} is the material transfer, for product m , from the upper node i to node j and y_{jkm} is the material transfer for, product m , from node j to its lower node k . The order accumulation of node j for product m is defined as S_{jm} and the equations for S_{jm} are given as:

$$\frac{dS_{jm}}{dt} = \sum_{\substack{\text{from} \\ \text{downstream} \\ \text{nodes}}} o_{jkm} - \sum_{\substack{\text{to} \\ \text{downstream} \\ \text{nodes}}} y_{jkm} \quad (3.2)$$

where o_{jkm} is the order placed to node j , for product m , from its lower node k , and y_{jkm} is the material transferred for product m from j to its lower node k .

The material transfer function proposed for the single product case is also used here, in its three product form:

$$y_{jkm} = \begin{cases} \left(MT_{jm} + \frac{I^*}{I_{jm}\eta_{jm}} \cdot ST_{jm} \right)^{-1} \cdot S_{jm} & S_{jm} > 0 \\ 0 & S_{jm} = 0 \end{cases} \quad (3.3)$$

$$\eta = \frac{S_{jm}}{\sum S_{jm}} \quad (3.4)$$

Physical bounds on the inventories, material transfer rates and control variables are also present:

$$\left[\begin{array}{l} I_{jm}^{upperbound} \geq I_{jm} \geq I_{jm}^{lowerbound} \\ y_{jim}^{upperbound} \geq y_{jim} \geq y_{jim}^{lowerbound} \end{array} \right] \quad (3.5)$$

3.1.1 Manufacturing Plant (Batch Operation):

Manufacturing plant produces three different products, A , B or C , based on the optimal manufacturing schedule generated by the centralized optimizer. Each product has a different production time. Plant can only manufacture one type of product, in a specific production time. Plant is designed to produce a constant batch amount for each product, once the production order for that product is received. The binary variable $p_m(t)$ is the production variable for product m , which is 1 if the plant produces product j at time t and 0 otherwise. In order to model both the continuous/discrete dynamics and switching between different operating conditions, Mixed Logic Dynamical (MLD) systems, and generalized disjunctive programming approach proposed by Raman&Grossmann [25] and Turkay&Grossmann [26] will be used.

The discrete-time formulation of the MLD system allows to develop numerically tractable schemes for solving complex problems such as stability [27], state estimation[28], and control[29]. In particular, MLD models were proven successful for recasting hybrid dynamic optimization problems, such as this particular problem, into mixed-integer linear and quadratic programs solvable via branch and bound techniques.

The first step in the derivation of the MLD form of a hybrid system is to associate with a logical statement S , that can be either true or false, a binary variable [30], such as the $p_m(t)$ production variable in this case, that is “if and only if the statement is true.” Then the combination of elementary statements $S_1, \dots, S_q$ into a compound statement via the Boolean operators AND($\wedge$), OR($\vee$), NOT($\neg$), can be represented as linear inequalities over the corresponding binary variables. Such a combination of elementary statements for that case is given in Eqn. 3.6, below.

$$\left[\begin{array}{c} p_{mt} \\ yp_{mt} = r_{mt} \\ pc_{mt} = (f_m \cdot p_m + v_m \cdot yp_{mt}) \end{array} \right] \vee \left[\begin{array}{c} \neg p_{mt} \\ yp_{mt} = 0 \\ pc_{mt} = 0 \end{array} \right] \quad \forall m, t \quad (3.6)$$

where $\neg$ NOT operator negates the variable p_{mt} (i.e., if p_{mt} is 1, $\neg p_{mt}$ is 0, vice versa) $\vee$ is the OR operator that selects exactly one condition above. Above statement says that either p_{mt} is true, (i.e., production takes place, then the production amount yp_{mt} equals to the batch production amount r_{mt} for that specific product, and the production cost pc_{mt} is the summation of fixed and variable production costs) or p_{mt} is false (i.e. production amount and production costs are zero) for each product at each time t .

The production duration for product A is two time units in this case. This signifies that once the plant starts to produce product A at time t , it cannot produce any other product at time t and at time $t+1$. Likewise, the production duration for product B is three, i.e. when the plant starts to produce B at time t , it can not produce any other product at time t , $t+1$ and $t+2$. However, the production time of C is short compared with the other two products, so that if C is produced at time t , the plant is blocked to produce any other product only on time t . In other words, the system must satisfy the following logical conditions:

$$\Omega(p) = \text{True} \Rightarrow \left\{ \begin{array}{l} p_{A,t} \Rightarrow \neg \left(\bigwedge_{m=A,B,C} p_{m,t+1} \right) \\ p_{B,t} \Rightarrow \neg \left(\bigwedge_{m=A,B,C} p_{m,t+1} \wedge \bigwedge_{m=A,B,C} p_{m,t+2} \right) \\ p_{A,t} \Rightarrow \neg \left(\bigwedge_{m=B,C} p_{m,t} \right) \\ p_{B,t} \Rightarrow \neg \left(\bigwedge_{i=A,C} p_{m,t} \right) \\ p_{C,t} \Rightarrow \neg \left(\bigwedge_{m=A,B} p_{m,t} \right) \end{array} \right. \quad (3.7)$$

The formulation of the convex hull of disjunction in Eqn. 3.6 and logical conditions in Eqn. 3.7:

$$pc = \sum_t \sum_m pc_{mt} \quad (3.8)$$

$$pc_{mt} = (f_m \cdot p_m + v_m \cdot yp_{mt}) \quad \forall m, t \quad (3.9)$$

$$p_{A,t} + p_{A,t+1} + p_{B,t+1} + p_{C,t+1} \leq 1 \quad \forall t \in \{0 \text{ to } 51\} \quad (3.10)$$

$$p_{B,t} + (p_{A,t+1} + p_{B,t+1} + p_{C,t+1}) + (p_{A,t+2} + p_{B,t+2} + p_{C,t+2}) \leq 1 \quad \forall t \in \{0 \text{ to } 50\} \quad (3.11)$$

$$p_{A,t} + p_{B,t} + p_{C,t} \leq 1 \quad \forall t \quad (3.12)$$

3.2 Optimization Formulation

The following profit functional is maximized:

$$J = R_{sales} - C_{inventory} - C_{transfer} - C_{production} - C_{customer\ satisfaction} \quad (3.13)$$

R_{sales} is the revenue gained by selling material from retailer level to customer. The customer buys all that is transferred to him/her regardless of being less or more than the actual demand. In discrete form, for discrete time k , where J is maximized over a specified horizon P :

$$R_{sales} = \sum_{k=1}^P \sum_m^{A,B,C} Pr_m \cdot y_{Rm}(k) \quad (3.14)$$

$C_{inventory}$ is the cost for holding inventory in each node:

$$C_{inventory} = \sum_{k=1}^P \sum_m^{A,B,C} \left[\sum_j^{R,D,W} IC_{jm} \cdot I_{jm}(k) \right] \quad (3.15)$$

$C_{transfer}$ is the cost for transferring material between the nodes and from retailer node to customer:

$$C_{transfer} = \sum_{k=1}^P \sum_m^{A,B,C} \left[\sum_j^{R,D,W,M} TC_{jm} \cdot y_{jm}(k) \right] \quad (3.16)$$

$C_{production}$ is the cost for producing material in the manufacturing plant. It consists of two components, fixed cost (FC) which is incurred each time a product is produced regardless of its production volume, and variable cost (VC) which depends on the amount of production:

$$C_{production} = \sum_{k=1}^P \left[\sum_m^{A,B,C} (FC_m \cdot p_m(k) + VC_m \cdot yp_m(k)) \right] \quad (3.17)$$

$C_{customer\ satisfaction}$ is the penalty cost for selling less or more than the actual demand to the customer. To obtain high customer satisfaction, this penalty cost must be high enough, so that the optimizer would try to decrease its negative effects.

$$C_{customer\ satisfaction} = \sum_{k=1}^P \left[\sum_m^{A,B,C} CS_m \cdot |y_{Rm}(k) - d_m(k)| \right] \quad (3.18)$$

Material transfer functions between the nodes are given in section 2 through Eqn. 2.11.

Redefining material transfer variables, Eqns. 2.11 can be expressed as:

$$\tilde{y}_R = y_R[t] + 5.397 = 0.19 S_R[t] + 0.052 I_R[t] \quad (3.19)$$

$$\tilde{y}_D = y_D[t] + 50 = 0.031 S_D[t] + 1.1 I_D[t] \quad (3.20)$$

$$\tilde{y}_W = y_W[t] + 30.6 = 0.024 S_W[t] + 0.9 I_W[t] \quad (3.21)$$

Thus the overall optimization problem can be written as:

$$\max \quad J \quad (3.22)$$

subject to

$$x(t+1) = Ax(t) + B_1 u(t) + B_2 p(t) \quad (3.23)$$

$$y(t) = Cx(t) + D_1 u(t) + D_2 p(t) \quad (3.24)$$

$$E_1 p(t) + E_2 p(t+1) + E_3 p(t+2) \leq E_4 \quad (3.25)$$

$$\begin{aligned}
 x = \begin{bmatrix} I_{WA} \\ I_{DA} \\ I_{RA} \\ I_{WB} \\ I_{DB} \\ I_{RB} \\ I_{WC} \\ I_{DC} \\ I_{RC} \\ S_{PA} \\ S_{WA} \\ S_{DA} \\ S_{RA} \\ S_{PB} \\ S_{WB} \\ S_{DB} \\ S_{RB} \\ S_{PC} \\ S_{WC} \\ S_{DC} \\ S_{RC} \end{bmatrix} &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \\ x_{16} \\ x_{17} \\ x_{18} \\ x_{19} \\ x_{20} \\ x_{21} \end{bmatrix}; u = \begin{bmatrix} O_{PA} \\ O_{WA} \\ O_{DA} \\ O_{RA} \\ O_{PB} \\ O_{WB} \\ O_{DB} \\ O_{RB} \\ O_{PC} \\ O_{WC} \\ O_{DC} \\ O_{RC} \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \\ u_{11} \\ u_{12} \end{bmatrix}; p = \begin{bmatrix} p_A \\ p_B \\ p_C \end{bmatrix}; y = \begin{bmatrix} \tilde{y}_{PA} \\ \tilde{y}_{WA} \\ \tilde{y}_{DA} \\ \tilde{y}_{RA} \\ \tilde{y}_{PB} \\ \tilde{y}_{WB} \\ \tilde{y}_{DB} \\ \tilde{y}_{RB} \\ \tilde{y}_{PC} \\ \tilde{y}_{WC} \\ \tilde{y}_{DC} \\ \tilde{y}_{RC} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \end{bmatrix} \quad (3.26)
 \end{aligned}$$

The values for A , B_1 , B_2 , C , D_1 , D_2 , E_1 , E_2 , E_3 and E_4 are given in Appendix B.

As explained in Chapter 2, customer demands are modeled as integrated white noise for each of the three products and the discrete-time demand models are as given by Eqn. 2.13. The optimal demand predictions are obtained by estimating the current demand using a Kalman filter and the model to compute the future values as given by Eqn. 2.16.

3.3 Open Loop Optimization of the Hybrid System

To perform the open loop optimization, the demand profiles for the three products for the next year are assumed to be perfectly known and they are shown in Fig. 3.2.

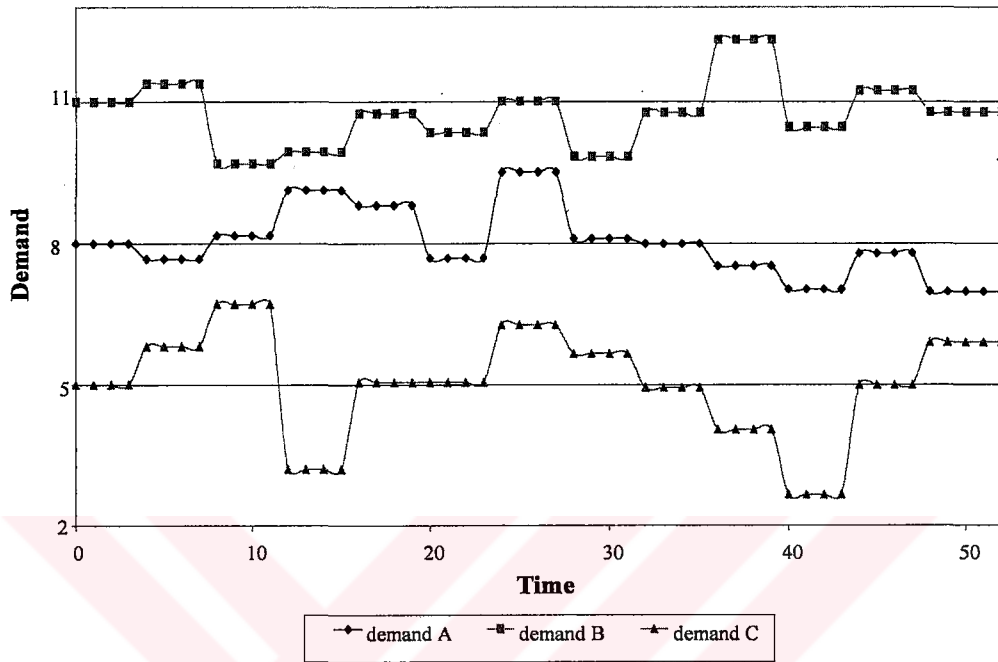


Figure 3.2: Demand profiles for the three product supply chain

Inventory costs are 2 monetary units and material transfer costs are 1 monetary unit. The final product is sold to the customer for a price of 50 monetary units. One of the testing objectives is to see whether C is ever produced when it is not profitable to produce. In order to guarantee that C is not profitable it suffices to choose the price of C less than or equal to its variable production cost, which is 84 monetary units. On the other hand, products A and B are desired to be produced. The variable production cost of A is 24 monetary units and the fixed cost is 12 monetary units. If only one unit of A is produced then, the total production cost of one unit of A is 36. This product is going to incur three times inventory costs, which is 6; and three times material transfer costs, which is 3. Thus, one unit of A incurs a cost of 45 if it stays in each inventory only one time unit. Therefore, setting the price greater than or equal to 45 guarantees that even producing one unit of A is profitable. Bearing in mind that the price of C is chosen to be lower than 84, and for the ease of simple modeling parameters, the prices of all products are taken to be 50, which is a number between 45 and 84. Over and under sales are allowable, however 50 monetary units of penalty cost is charged for each surplus or shortage. If the plant receives an order to produce product A, it produces a constant batch amount of 75 monetary units, while this

batch amount is 100 monetary units for product *B* and 50 monetary units for product *C*. A fixed production cost occurs each time a product is being produced, which is 18 monetary units for product *A*, 6 monetary units for product *B* and 42 monetary units for product *C*. Variable costs also occur which are 24 monetary units/unit *A* produced, 12 monetary units/unit *B* produced, and 84 monetary units/unit *C* produced. All these values are summarized in Table 3.1.

Table 3.1: Sales prices, costs and production amounts for the three products

PRODUCT	SALES PRICE	TRANS. COST	INV. COST	CUST. DISSAT.. COST	VAR. PROD COST	FIXED PROD COST	BATCH PROD. AMOUNTS
A	50	1	2	50	24	18	75
B	50	1	2	50	12	6	100
C	50	1	2	50	84	42	50

Using the demand profiles in Fig. 3.2, and the state Eqn. 3.23, the system is coded into OPL Studio software, which runs CPLEX in the background. Dynamic optimization was performed for a 52 week horizon and optimal total profit is calculated as 12215 monetary units.

If only product *A* is studied, from the form of the objective function, it can be observed that when the initial inventories are high, the system favors to transfer all material without producing or without taking customer satisfaction into account, since the revenue of delivering *A* is very high. However, there are limits on the inventories. Therefore the system tends to use these inventories in such a way that, while it benefits revenue obtained from *A*, and it also finds the optimal production schedule in terms of cost. The customer dissatisfaction also has negative effects on the objective function.

Since the optimizer sees the yearly demand profile at the beginning, it decides on the optimal production times for each product, by taking the production rules in terms of the production durations into account. It makes a tradeoff between the high revenues gained by transferring the entire product to the customer and the high costs incurred by production and customer dissatisfaction. The results are shown in Fig 3.3..

Optimizer decides to produce product *A* on the 22nd, 30th and 40th time units. However production costs are relatively high compared with the other costs. Steep decreases in the objective function can be seen in Fig. 3.7. As a result the system tends to take materials from

inventories until they are depleted (as observed in Figs. 3.5, 3.16 and 3.17) The demand profile gradually increases from 8th to 20th time units. A similar gradual increase is observed between 23rd to 28th time units. However a similar increasing pattern can not be observed around time 40. Therefore although, it can be concluded that the manufacturing plant tends to produce after high increases in demand or when there is non available in inventory, since the manufacturing decisions of *A* also depend on *B* and *C*, their production profiles should also be observed. Manufacturing costs of *A* is very low compared to *C* and somewhat high compared to *B*, hence the system tends to produce *A* more than *C* but less than *B*.

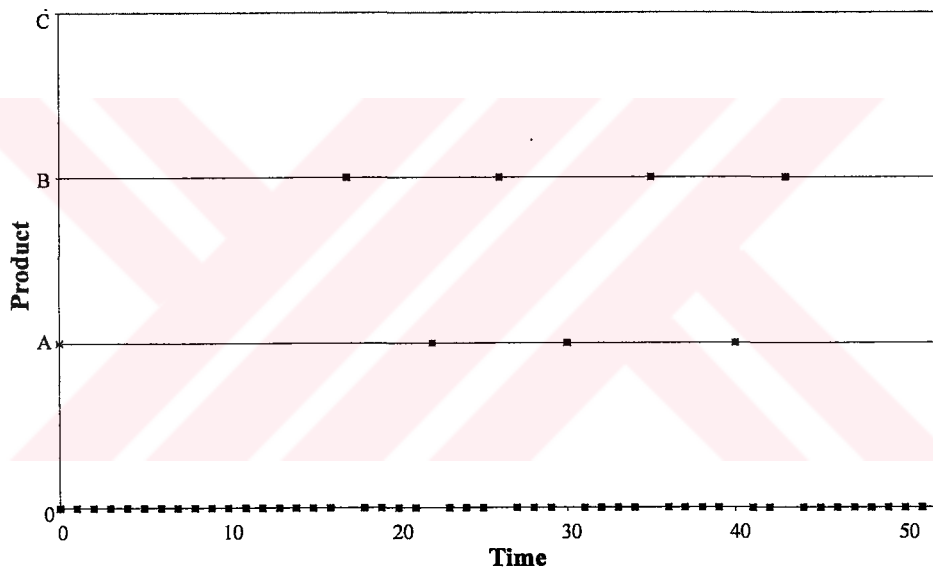


Figure 3.3: Optimal production schedule for the plant

Table 3.2: Initial inventory and material transfer levels

NODE & PRODUCT	INITIAL INVENTORY	INITIAL MAT. TRANSFER	INITIAL ACCUMULATED ORDER VALUES
Retailer A	100	0	0
Dist Center A	100	0	0
Warehouse A	100	0	0
Retailer B	100	0	0
Dist Center B	100	0	0
Warehouse B	100	0	0
Retailer C	100	0	0
Dist Center C	100	0	0
Warehouse C	100	0	0

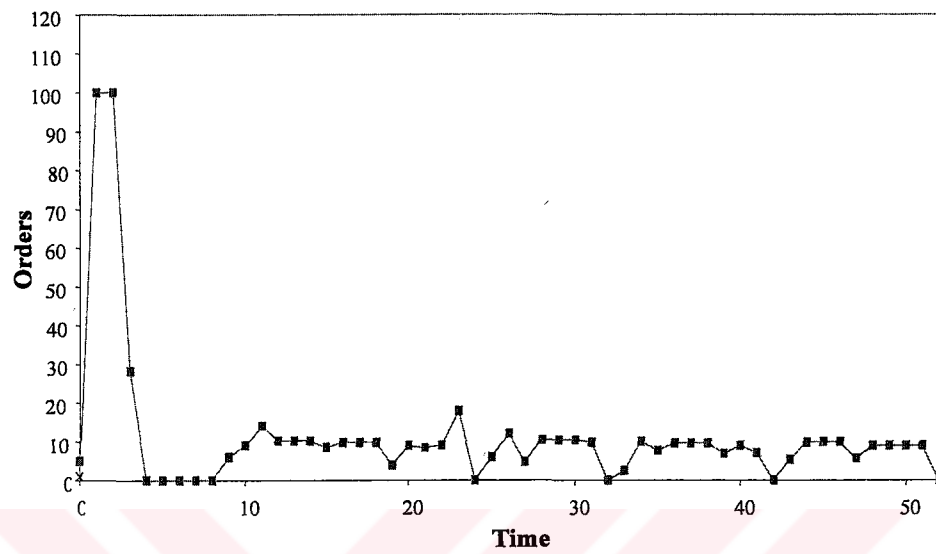


Figure 3.4: Retailer node orders for product A

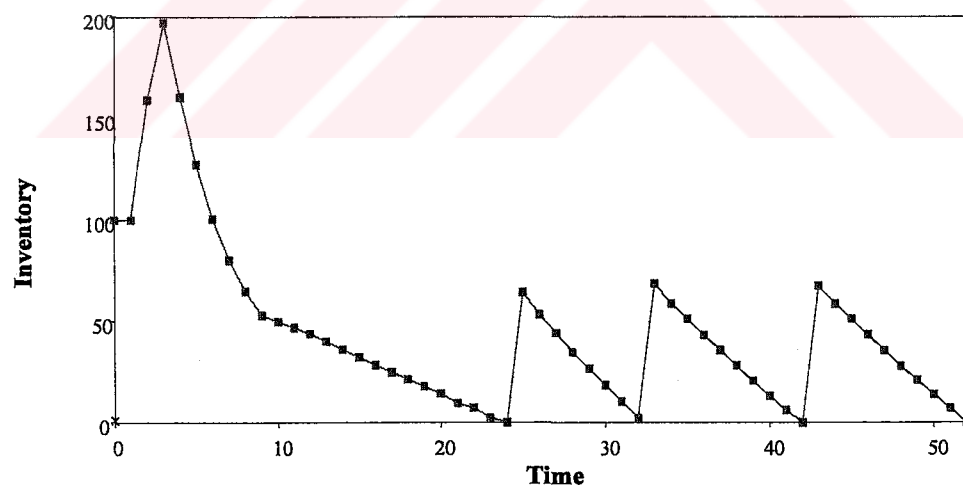


Figure 3.5: Retailer node inventory level for product A

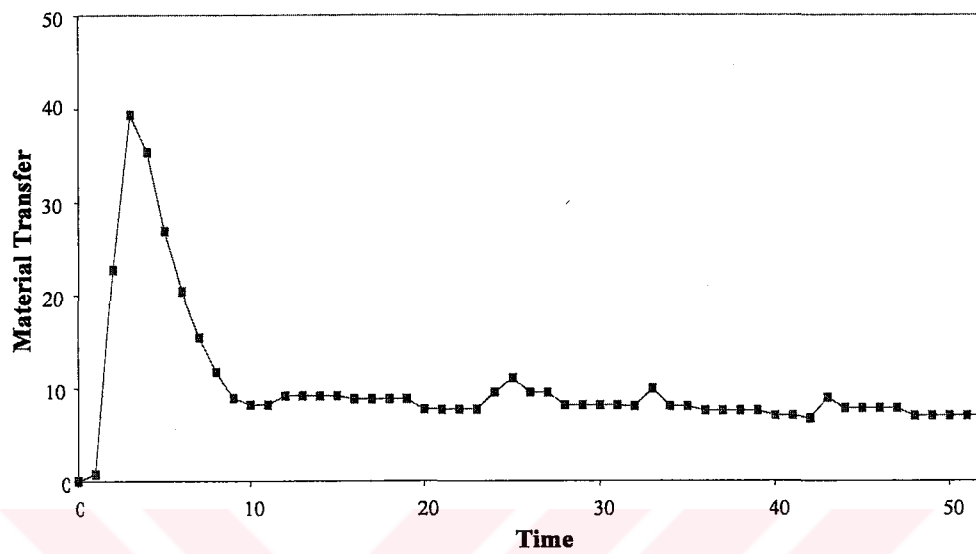


Figure 3.6: Retailer node material transfer level for product A

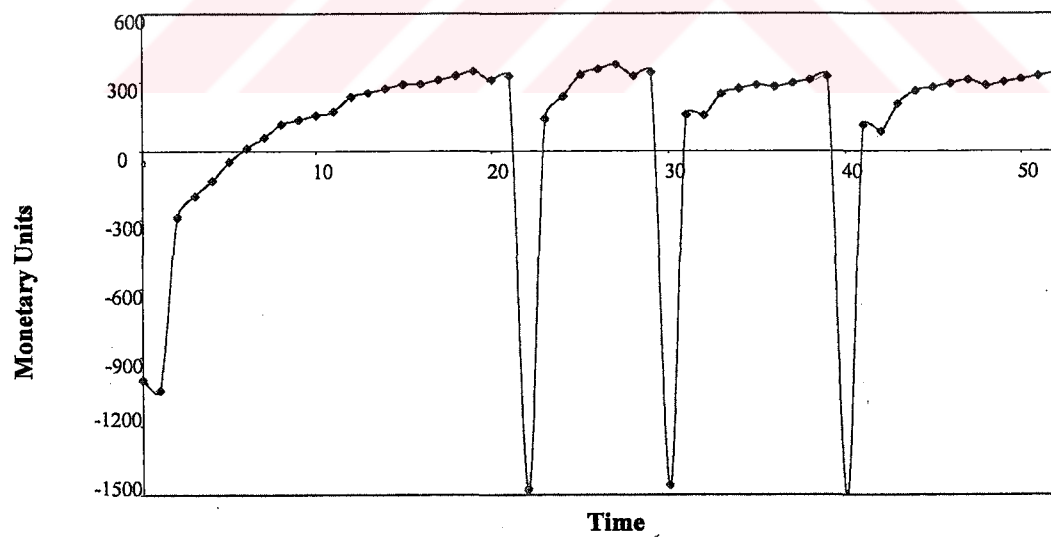


Figure 3.7: Instantaneous objective function for product A

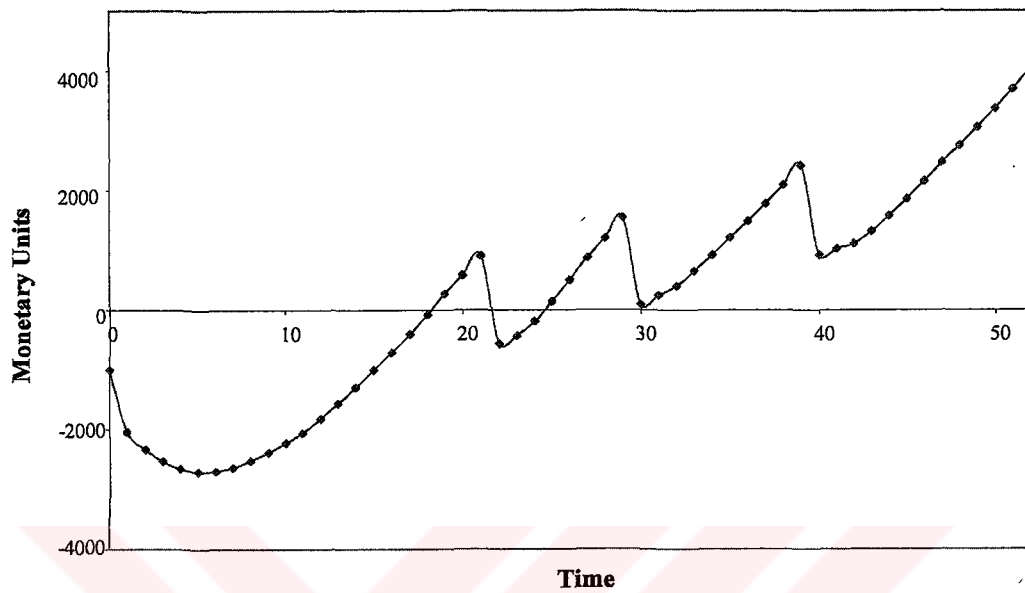


Figure 3.8: Cumulative objective function for product A

As it is seen from the results above, the system initially chooses to benefit from the high revenue of the material transfer at the retailer level and by transferring high quantities of material disregarding the customer satisfaction, since product value is higher than the penalty over production. (Figs. 3.9, 3.10, 3.11) However, delivering large quantities decrease the inventory levels, and this then leads to new production and in time the material transfer amount decreases. Eventually the material transfer becomes approximately equal to the actual customer demand.

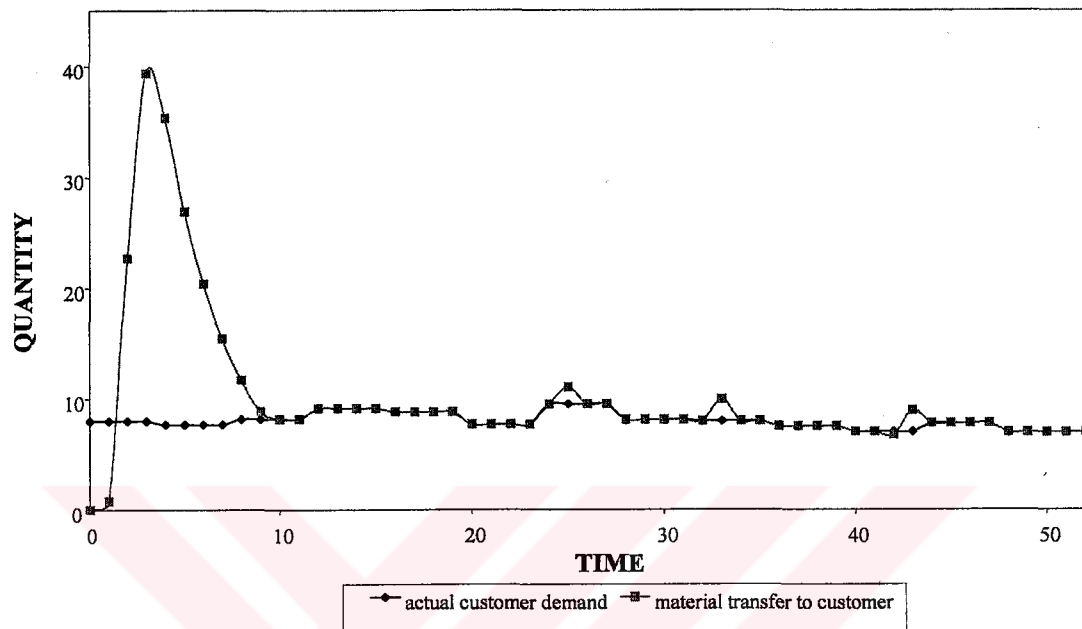


Figure 3.9: Material transferred to customer vs actual customer demand for product A

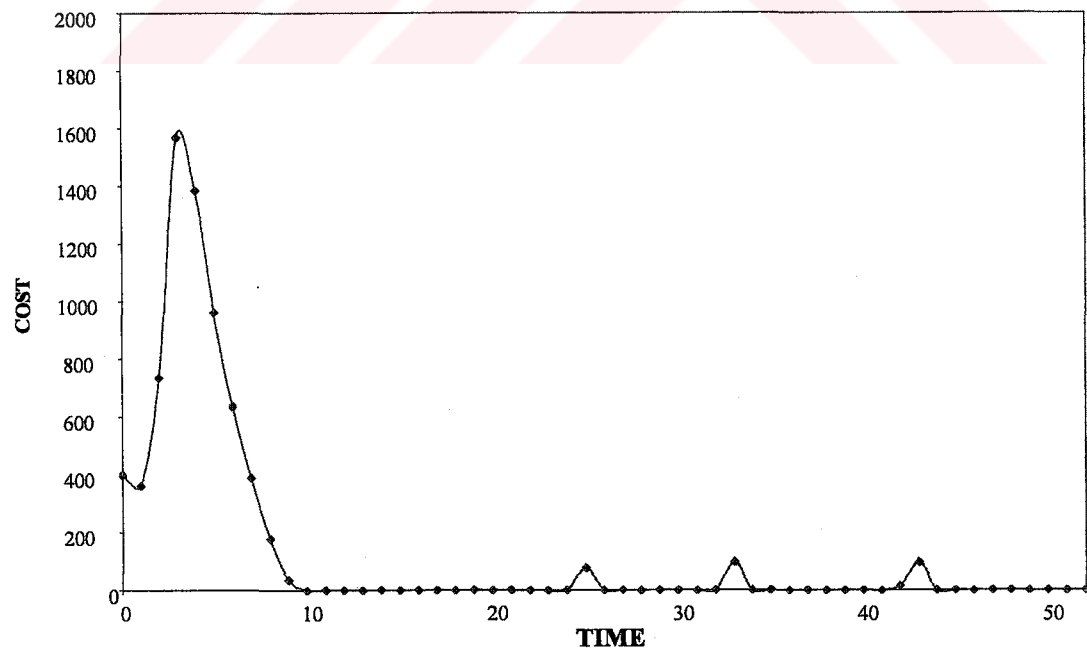


Figure 3.10: Penalty cost paid for product A

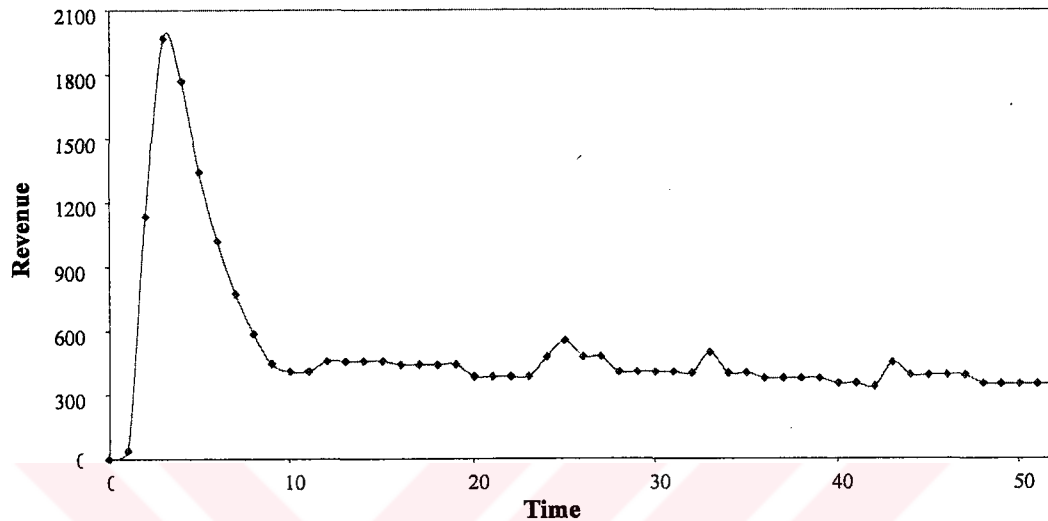


Figure 3.11: Revenue gained from product A

From the two graphs above it can be seen that initially it is more profitable for the system to overlook the customer dissatisfaction, since at the maximum delivery rate at $t=4$, the revenue gained by transferring the material to customer is around 2100mu (Fig. 3.11) while the penalty is around 1600mu. For these high production rates the revenue from the material transfer to the customer exceeds the penalty from the customer dissatisfaction.

Initially the optimizer places orders to the distribution center and warehouse (see o_{RA} , o_{DA}) since initially retailer node does not have enough inventory to meet high optimal customer demand. The retailer inventory levels are increased by transferring material from the warehouse and distribution center nodes. Accordingly the material transfer from these nodes increases (although these transfer rates have negative effects on the objective function, the high revenues from the product A compensate for them.)

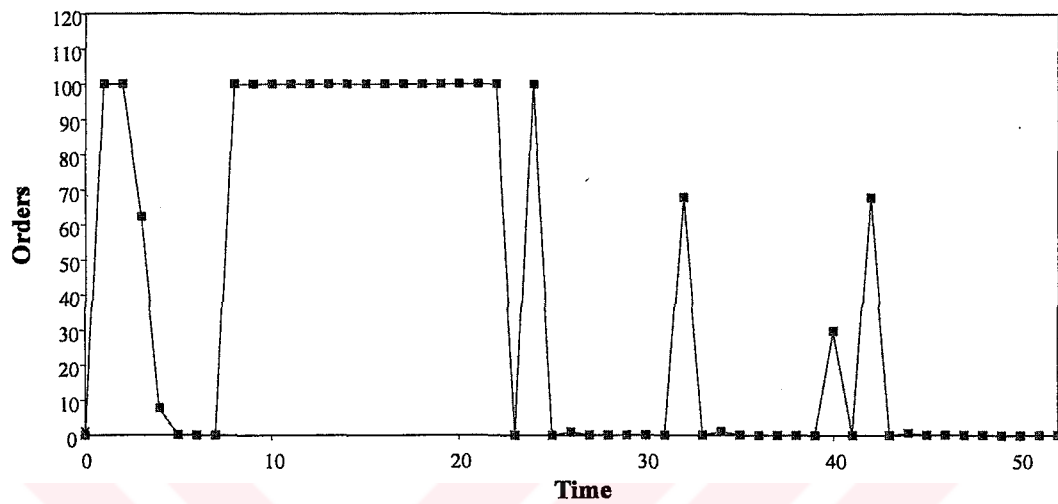


Figure 3.12: Distribution center order levels for product A

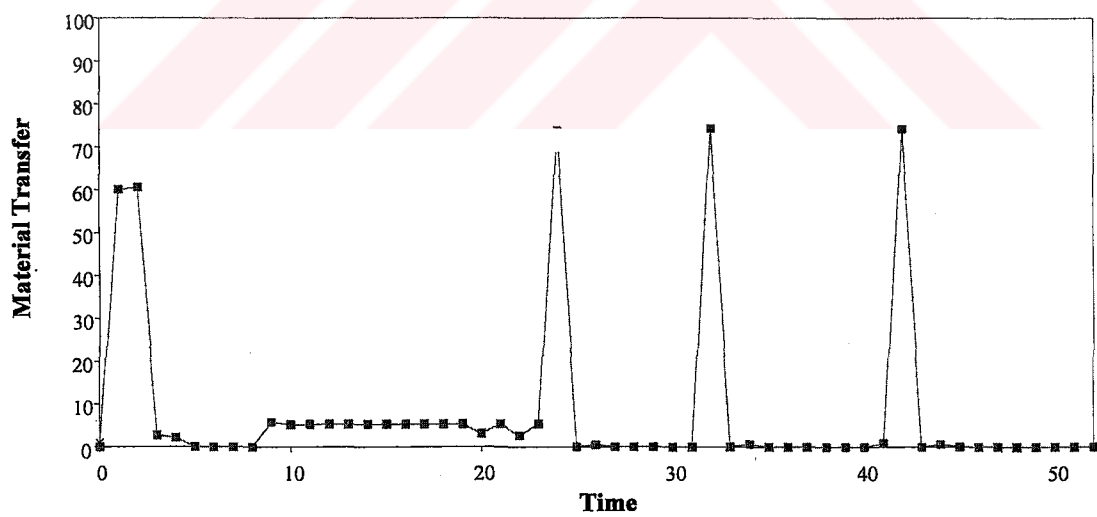


Figure 3.13: Distribution center material transfer level for product A

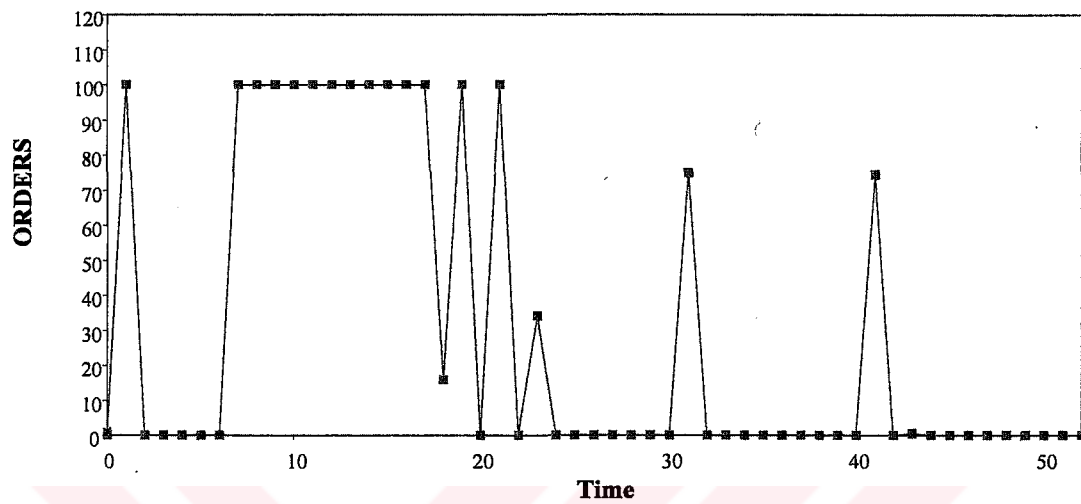


Figure 3.14: Warehouse order levels for product A

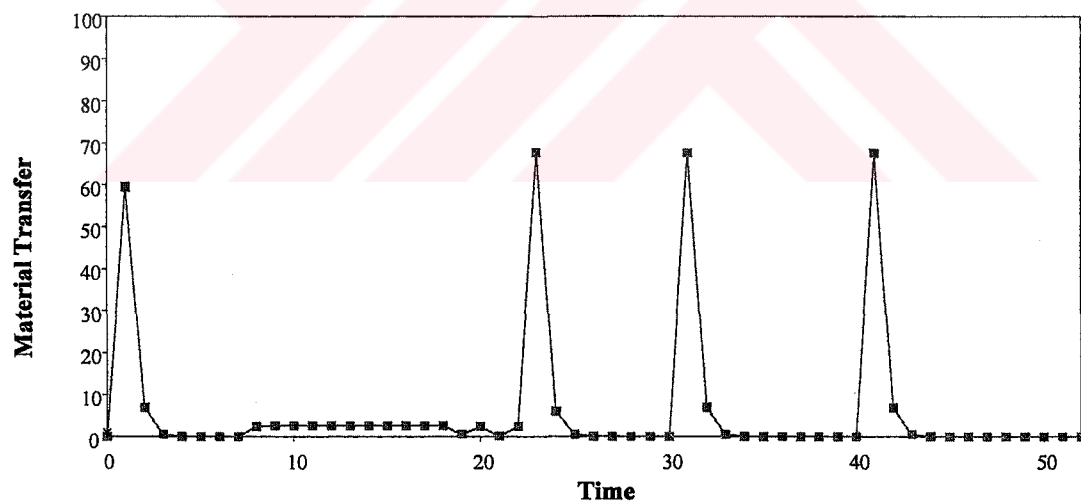


Figure 3.15: Warehouse material transfer level for product A

When the system starts to take the negative effects of production and customer dissatisfaction into account, the material transfer rates from distribution center to retailer and warehouse to distribution center also begin to decrease.

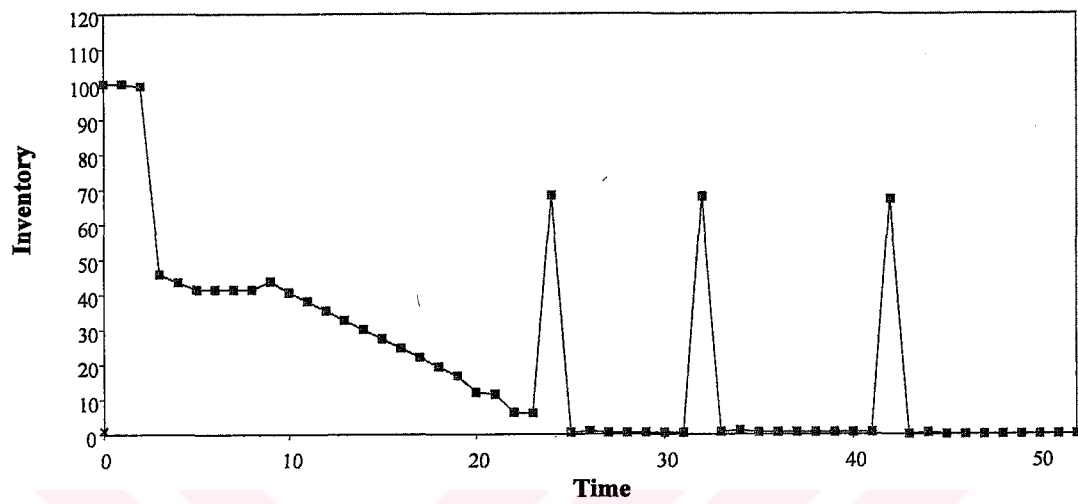


Figure 3.16: Distribution center inventory levels for product A

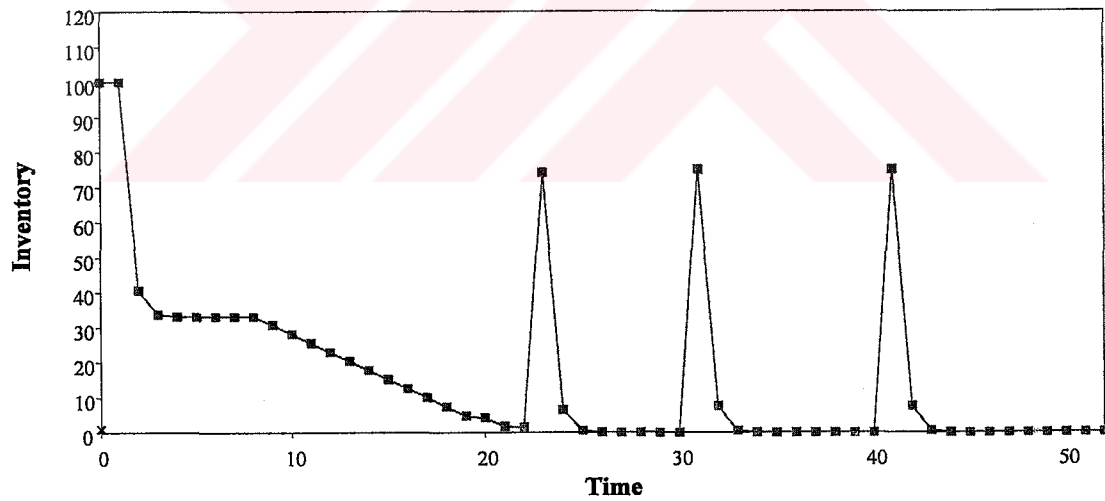


Figure 3.17: Warehouse inventory levels for product A

Similar inventory, material transfer and production trends are also observed for product *B*. However, since the production costs of *B* is the lowest of the three and small compared to the

other elements in the objective function, system tends to produce B more frequently and decreases its inventory levels for B more quickly than for other products.

When we observe the production schedule for B , we see that B is produced at the 16th, 25th, 34th and 42nd time units. There is a gradual increase from the 7th time unit to 15th time unit in demand. Demand also increases from 23rd to 24th time units. However there are gradual decreases in demand from 24th to 40th time units. Furthermore, we see that a production order for B is given in that interval where no increase in demand was observable. It cannot be concluded that a definite production scheme exists for B , like it does not produce after every gradual increase since its production times also depend on A . However, since its production costs are low, the supply chain determines its production schedule mainly based on A and C and it optimizes B based on the remaining allowable times and the demands of B . Also the demand profile of B does not contain many increases, hence the supply chain can not define definite production rules.

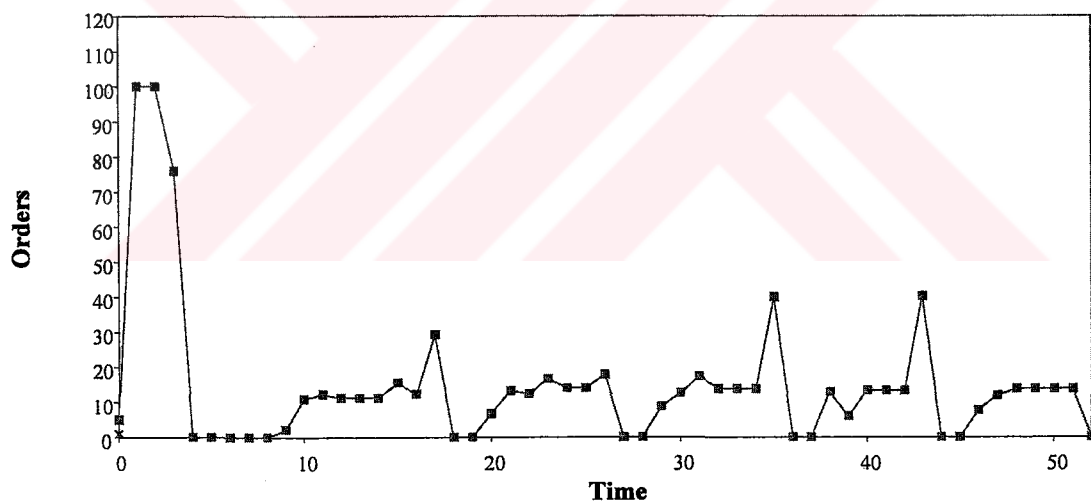


Figure 3.18: Retailer order levels for product B

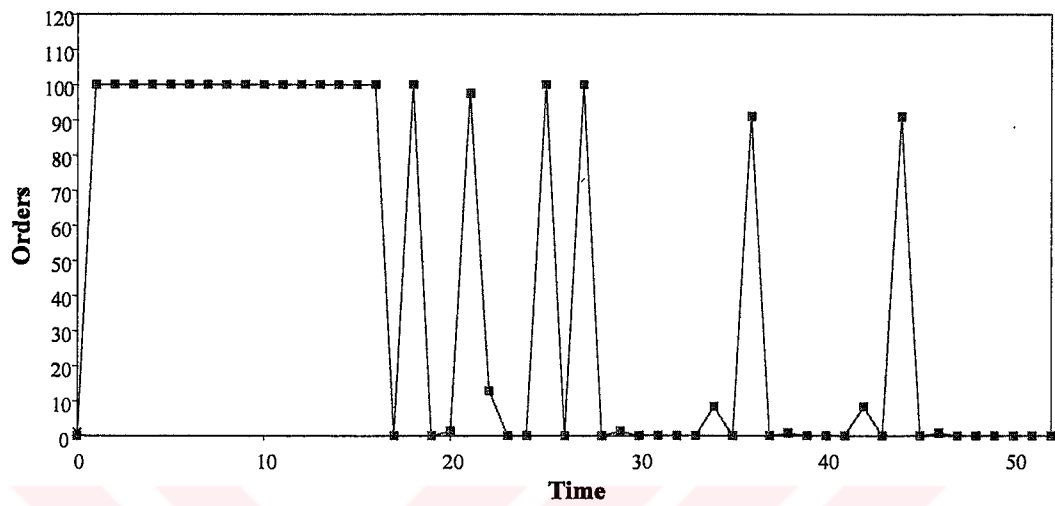


Figure 3.19: Distribution center order levels for product B

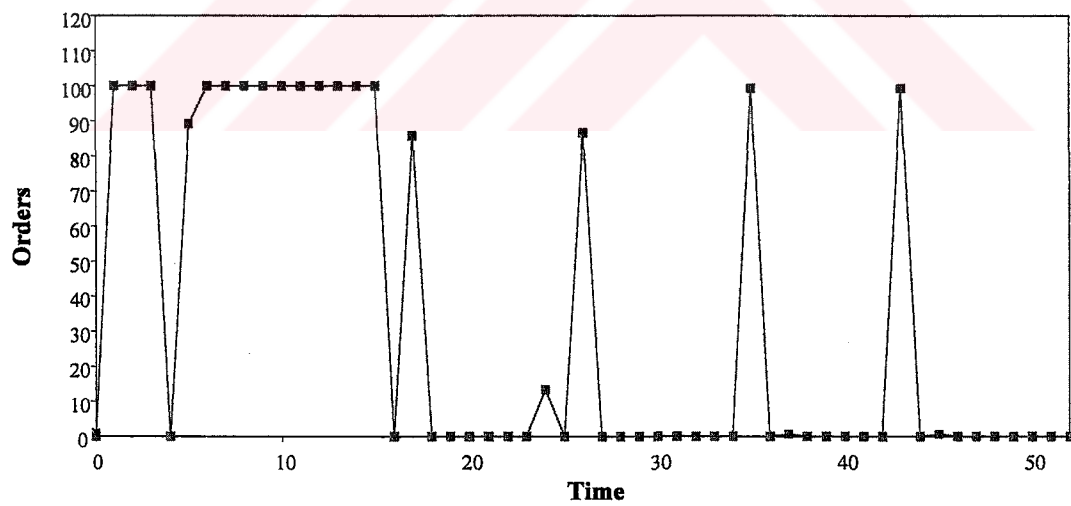


Figure 3.20: Warehouse order levels for product B

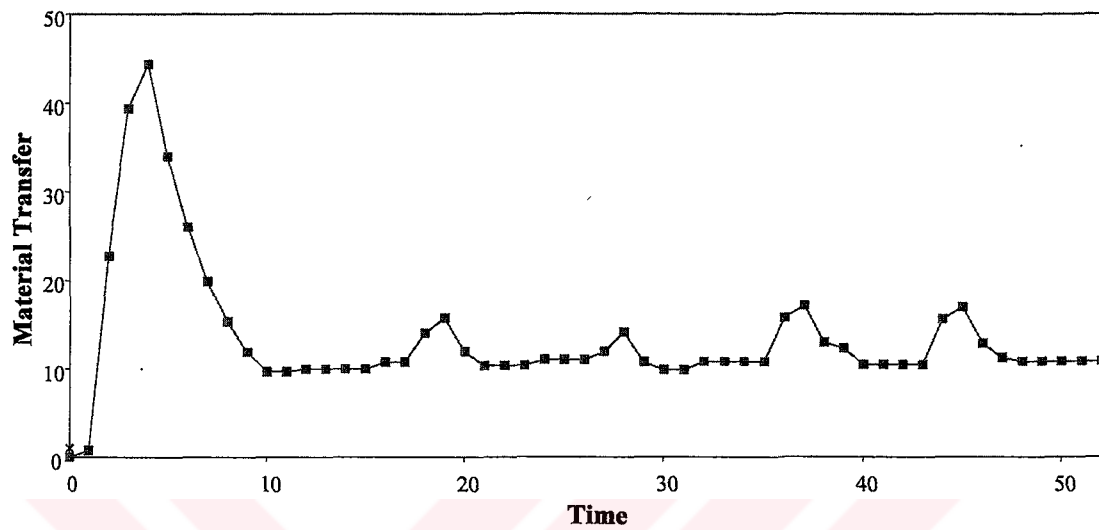


Figure 3.21: Retailer material transfer level for product B

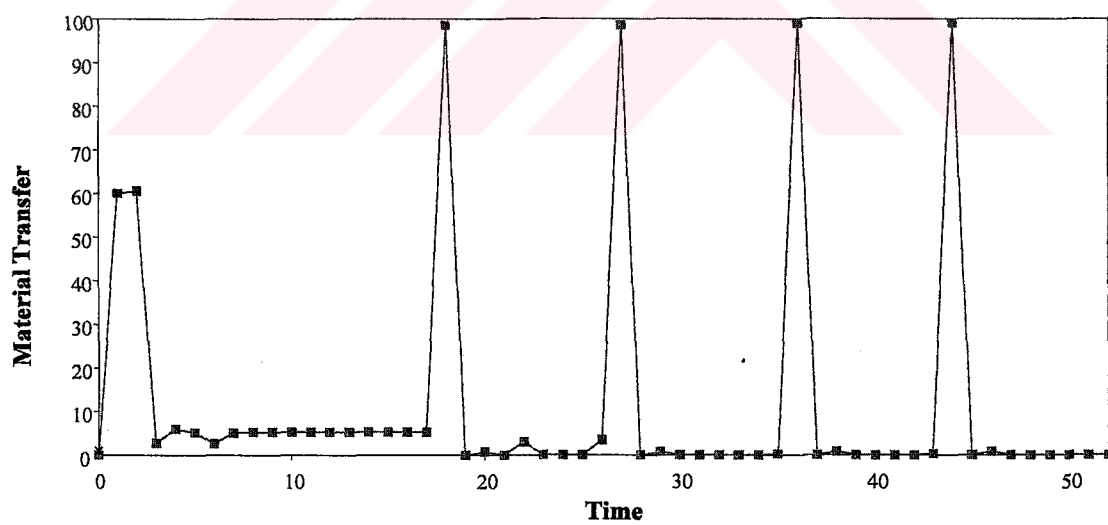


Figure 3.22: Distribution center material transfer level for product B

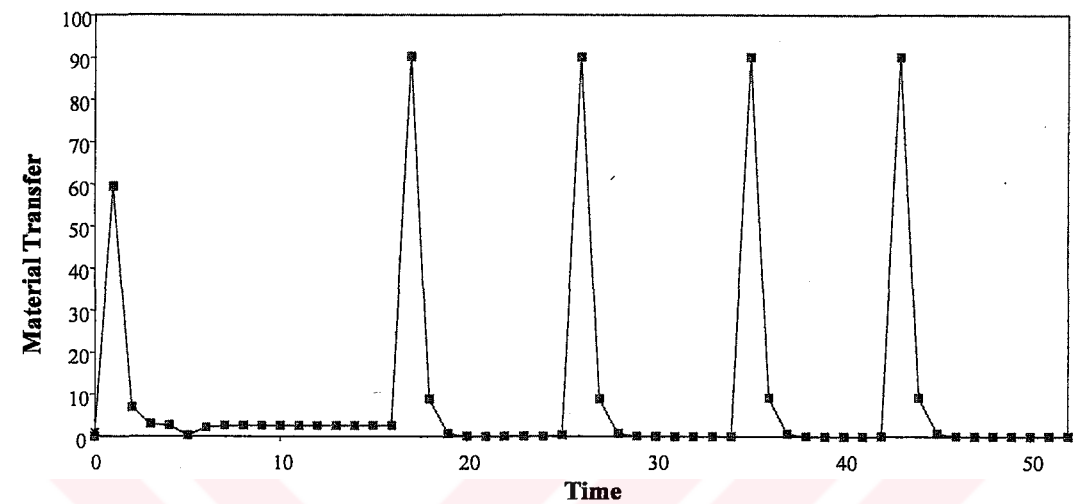


Figure 3.23: Warehouse material transfer level for product B

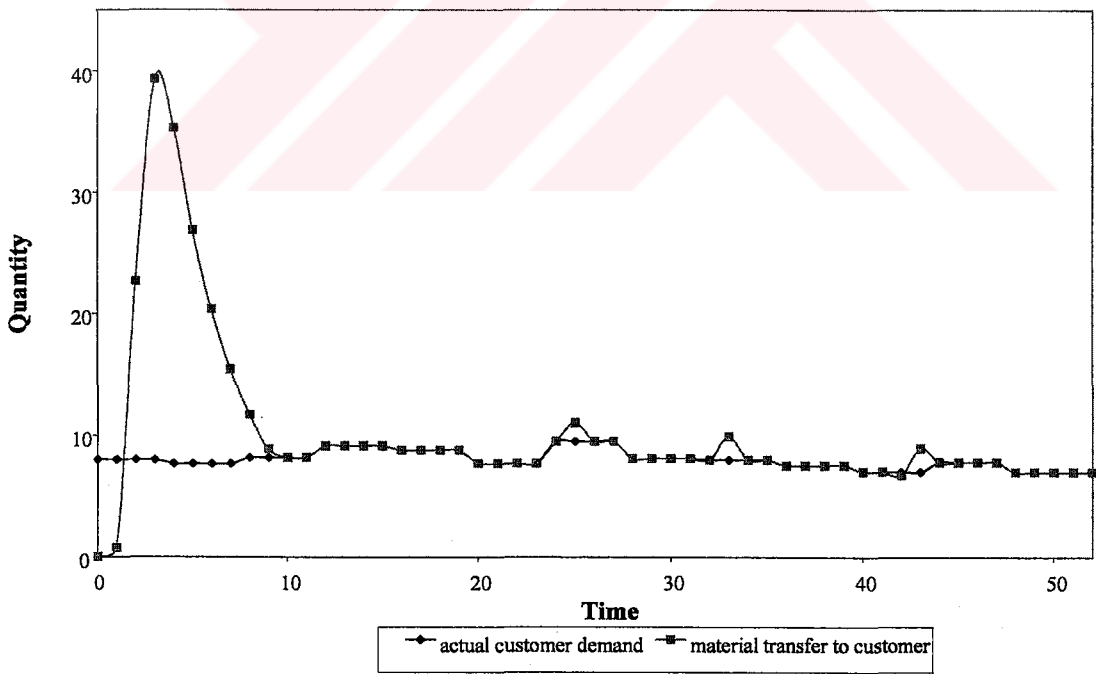


Figure 3.24: Material transferred to customer vs actual customer demand for product B

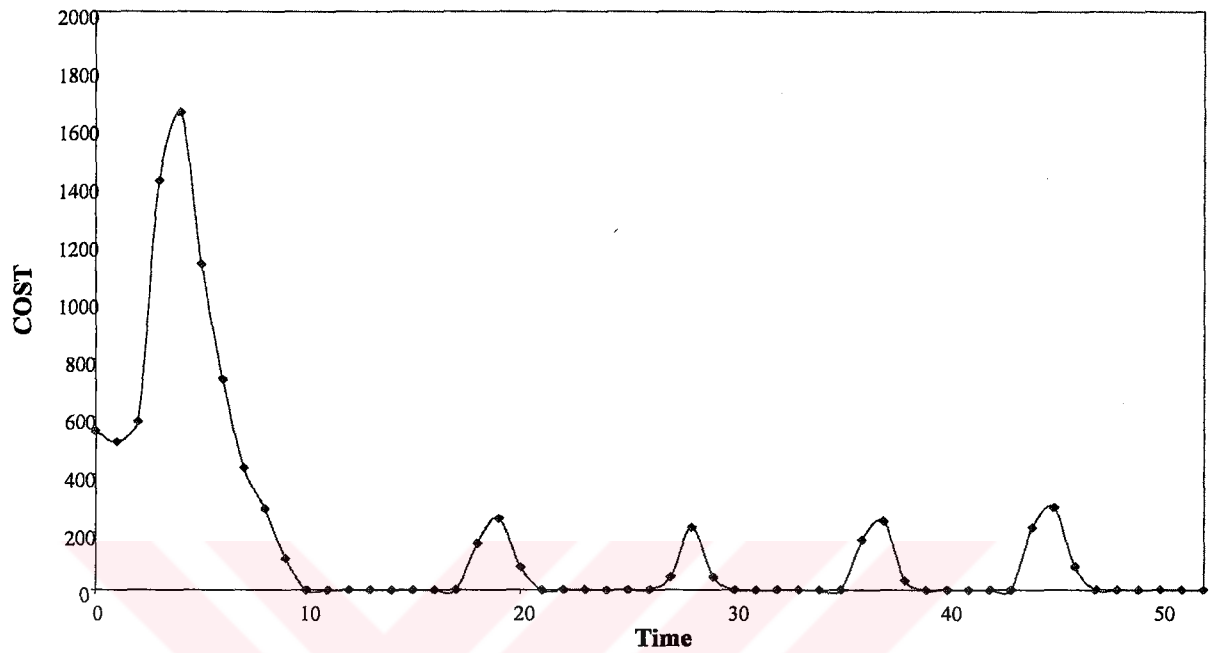


Figure 3.25: Penalty cost paid for product B

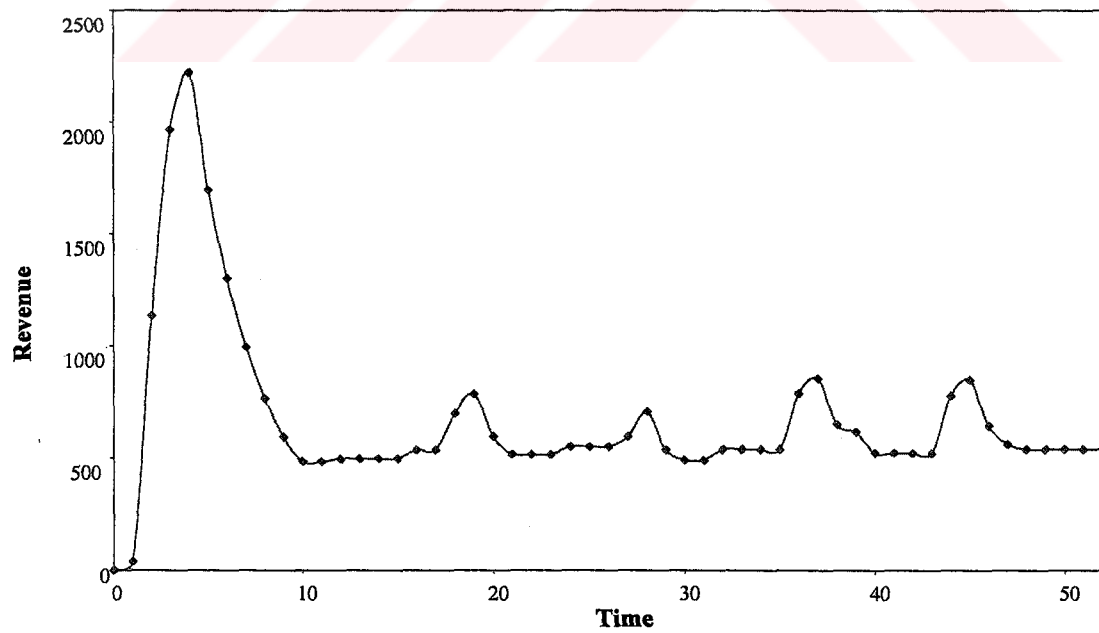


Figure 3.26: Revenue gained from product B

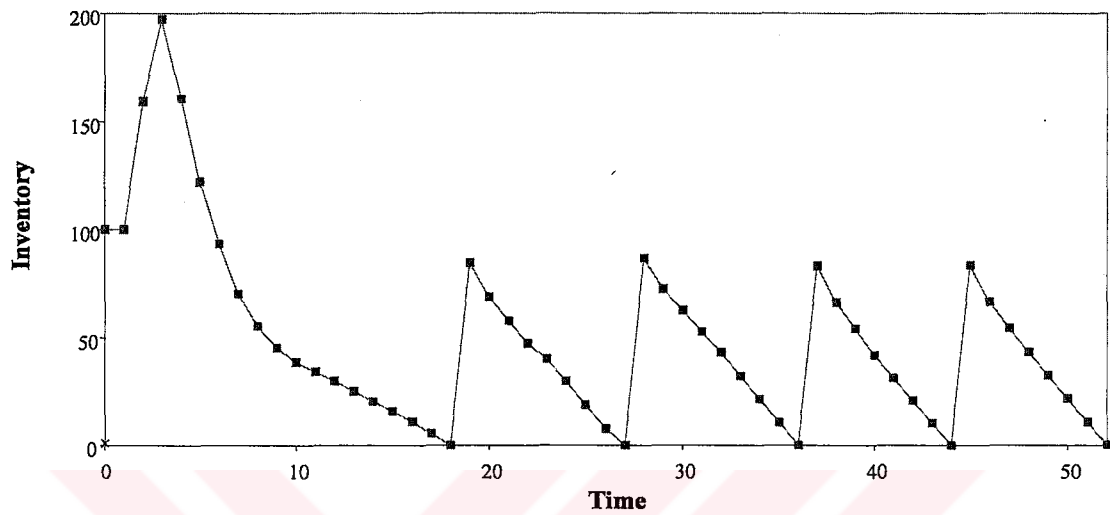


Figure 3.27: Retailer inventory level for product B

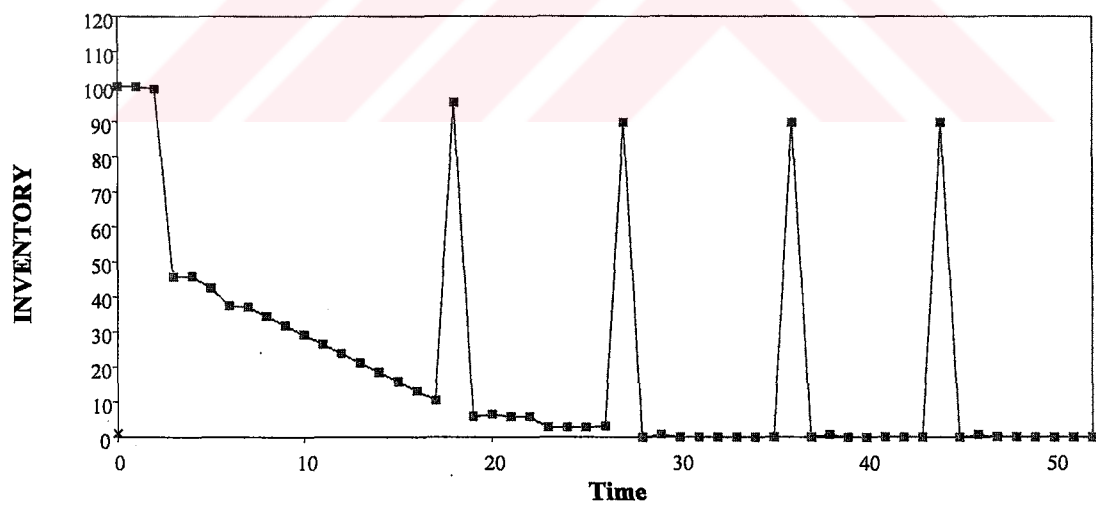


Figure 3.28: Distribution center inventory levels for product B

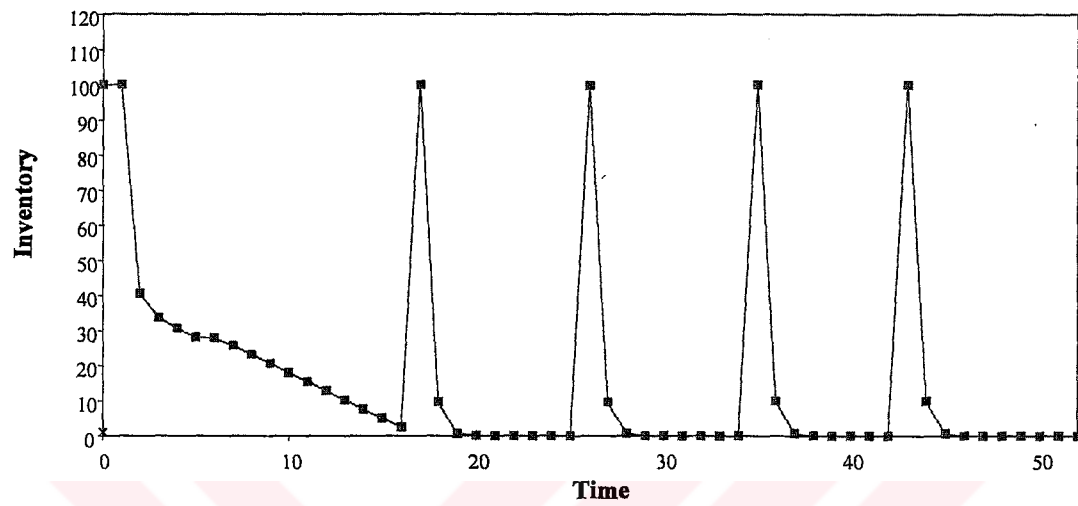


Figure 3.29: Warehouse inventory levels for product B

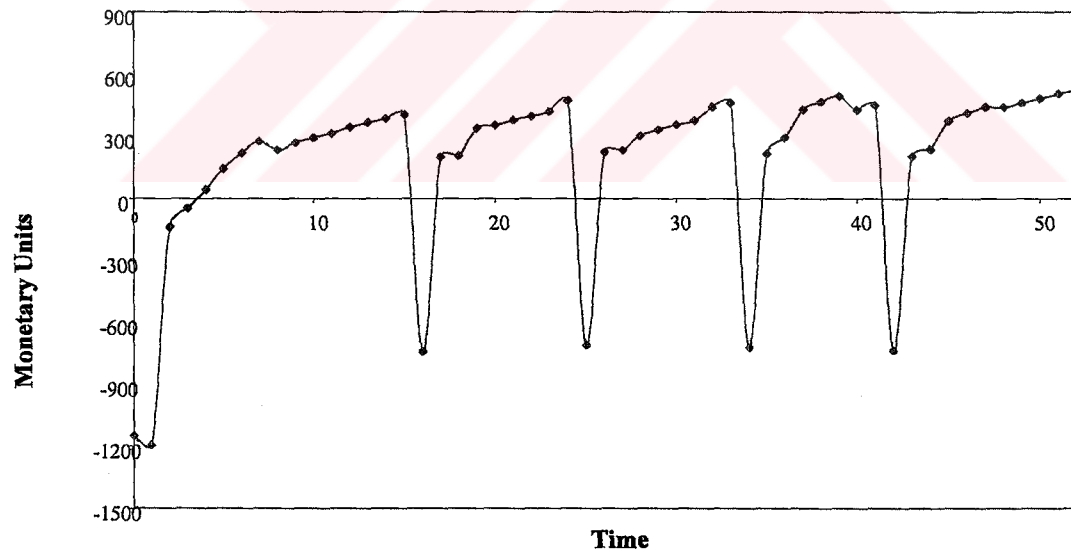


Figure 3.30: Instantaneous objective function for product B

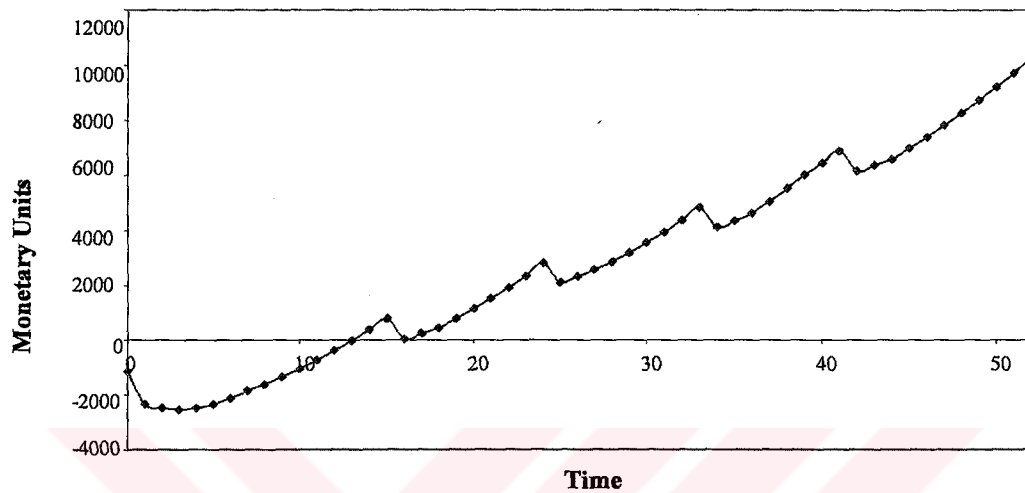


Figure 3.31: Cumulative objective function for product B

For product *C*, the production costs are the highest. The optimizer does not prefer to produce *C*, instead, it keeps the inventory levels very high. Also the demand for *C* is low compared to the other three, therefore the material transfer does not decrease the inventory amounts very much. The system does not decrease the inventory level at the retailer node to zero until the very end. However it reduces the inventory levels in distribution center and warehouse to zero, very late compared to the other three products.

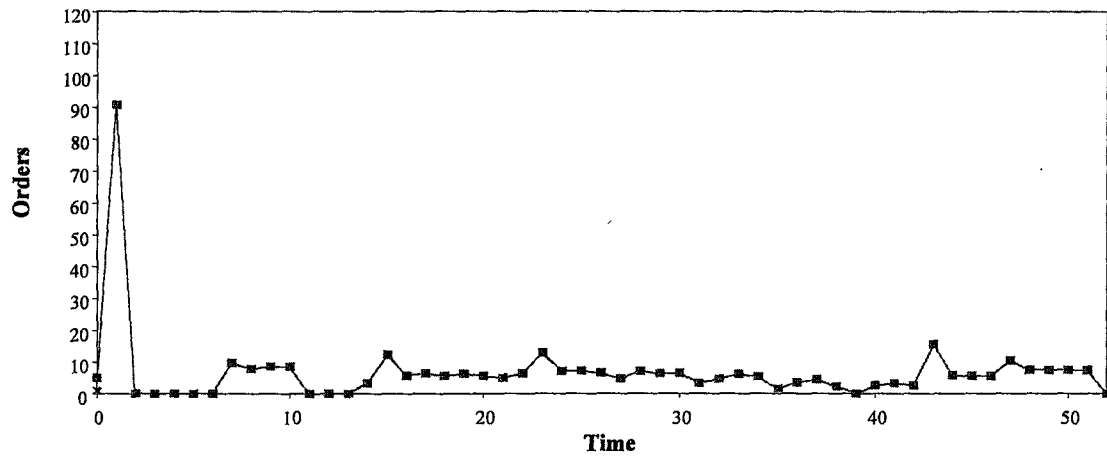


Figure 3.32: Retailer order levels for product C

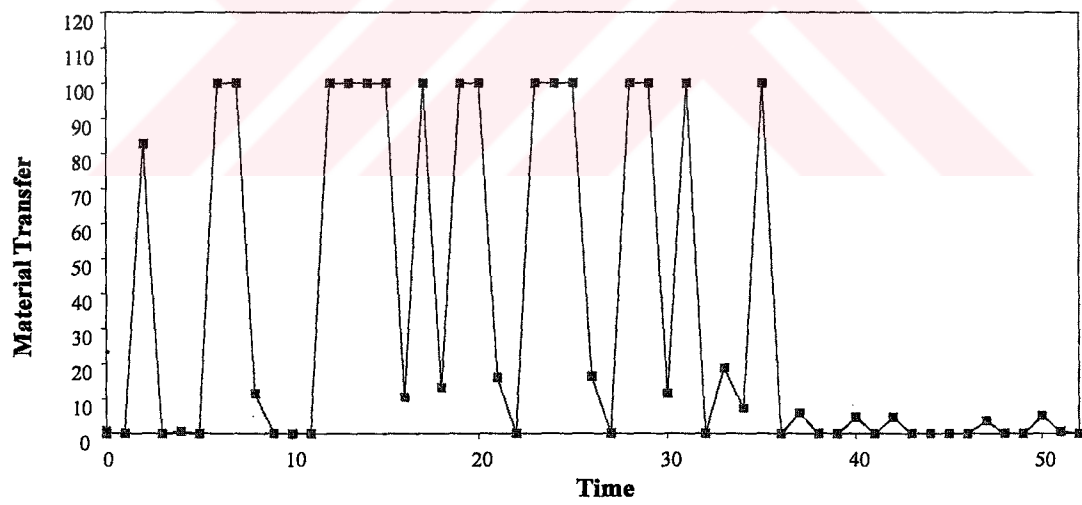


Figure 3.33: Distribution center order levels for product C

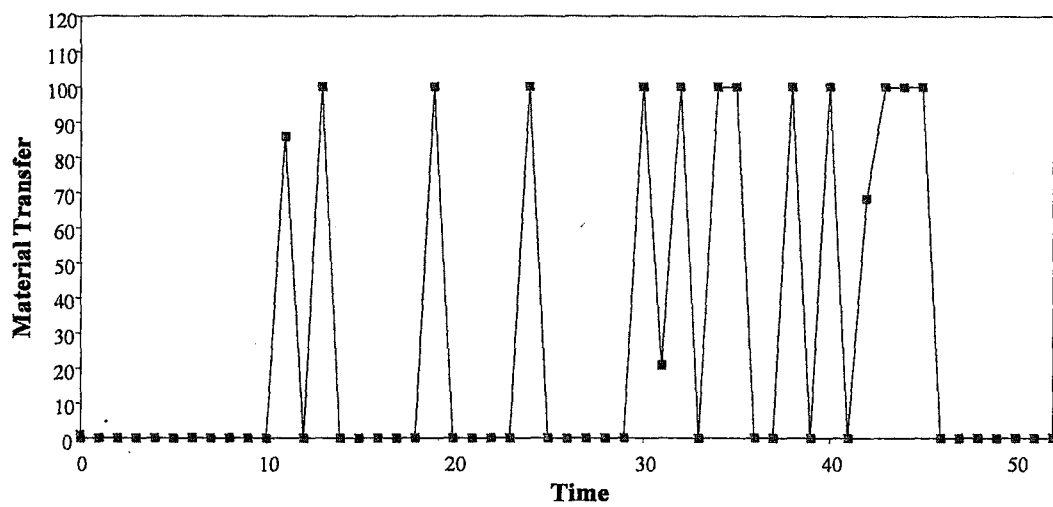


Figure 3.34: Warehouse order levels for product C

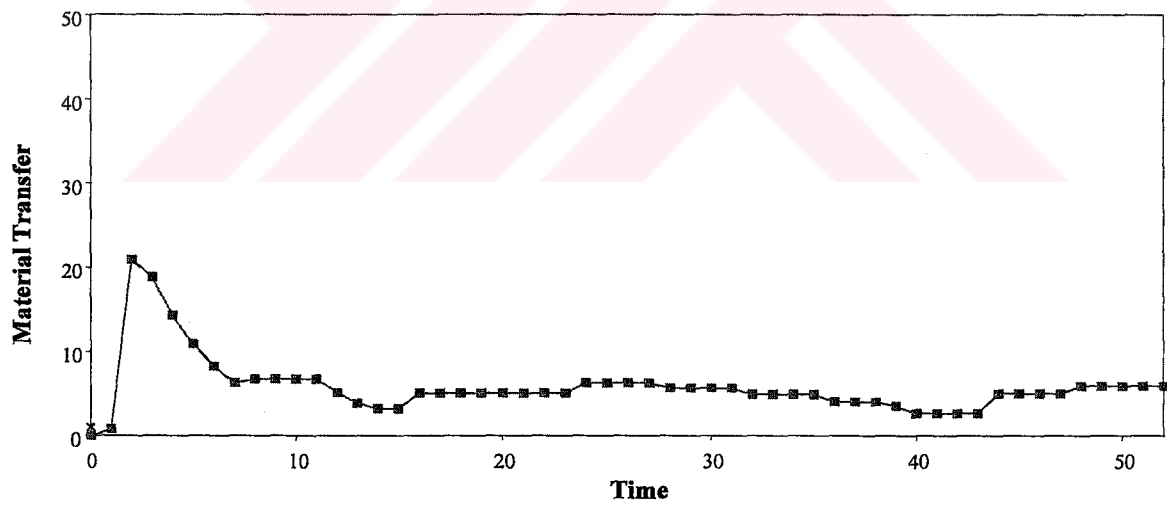


Figure 3.35: Retailer material transfer level for product C

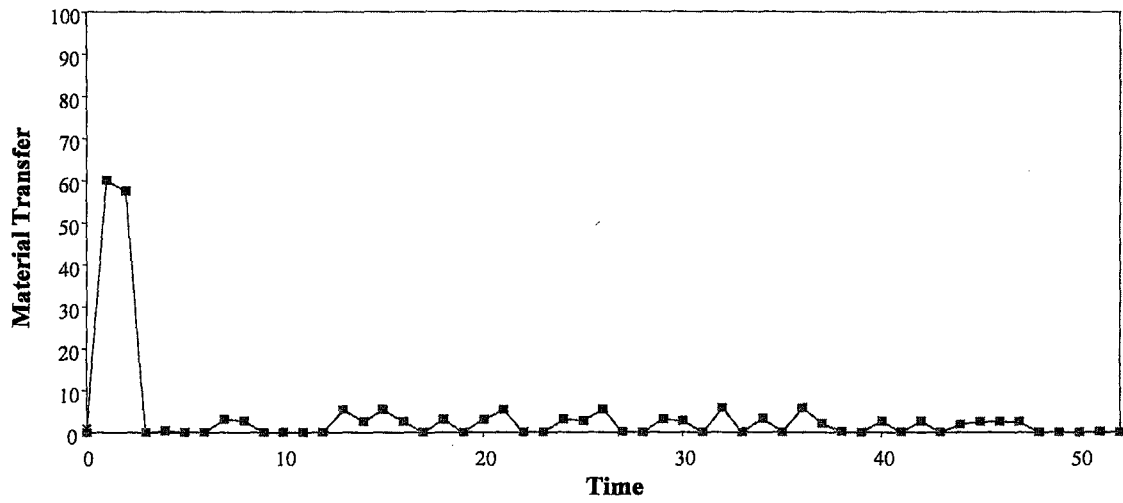


Figure 3.36: Distribution center material transfer level for product C

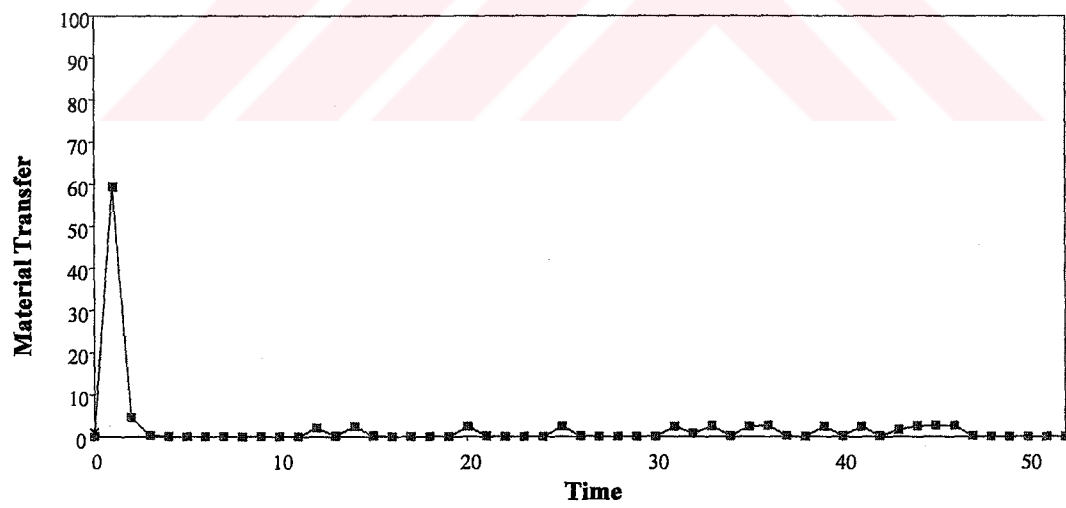


Figure 3.37: Warehouse material transfer level for product C

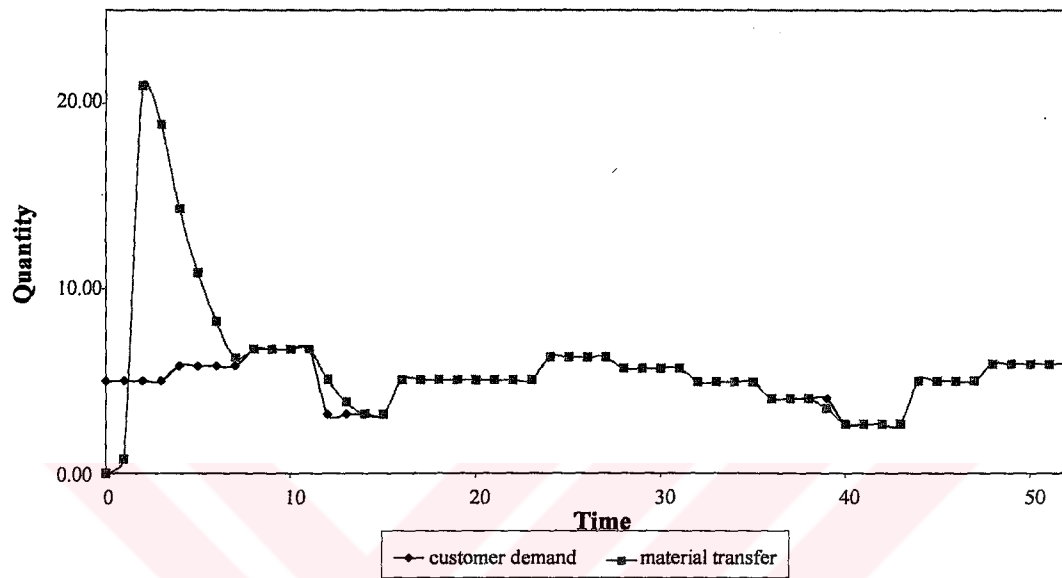


Figure 3.38: Material transferred to customer vs actual customer demand for product C

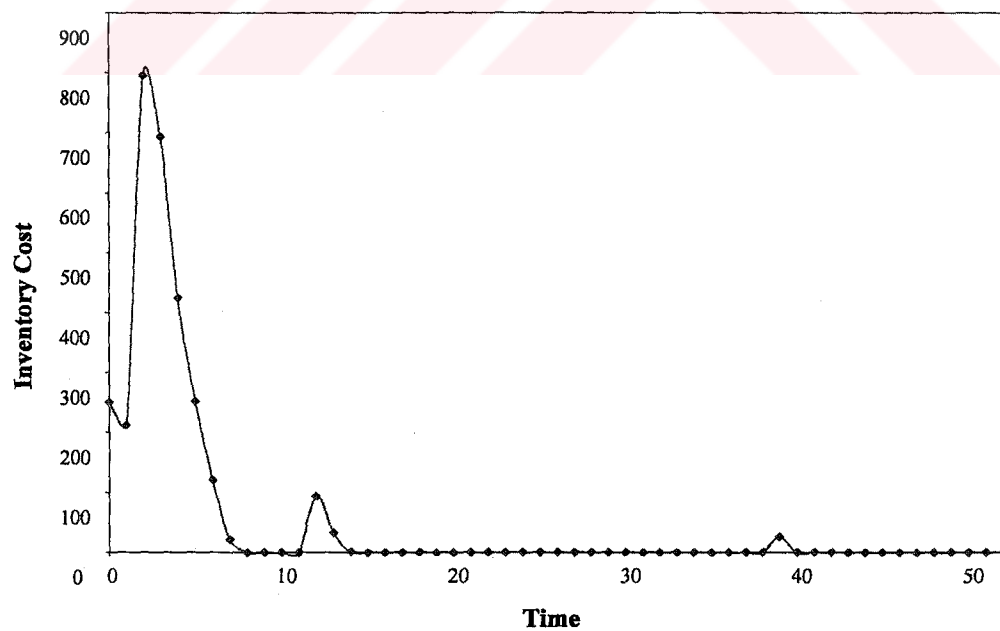


Figure 3.39: Penalty cost paid for product C

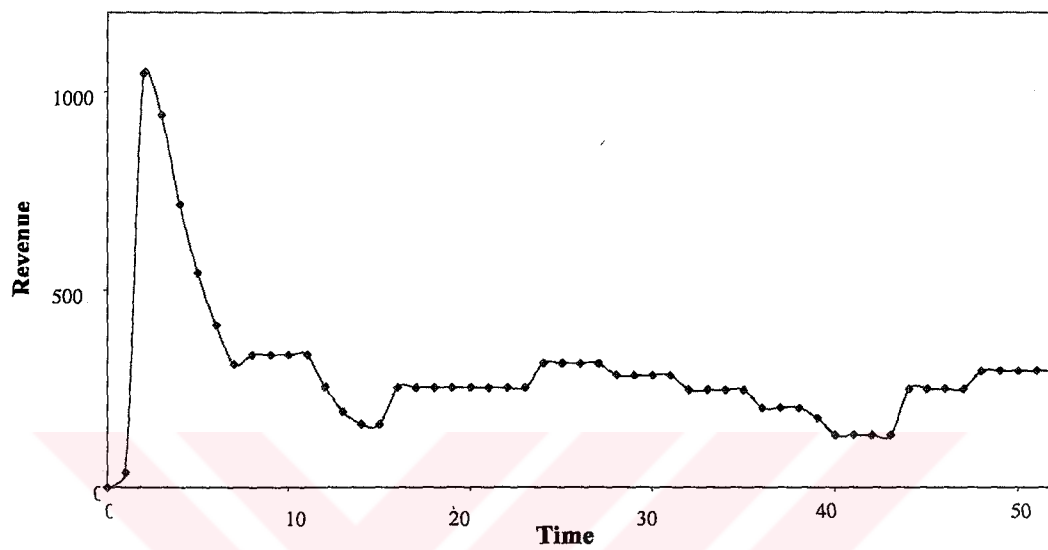


Figure 3.40: Revenue gained from product C

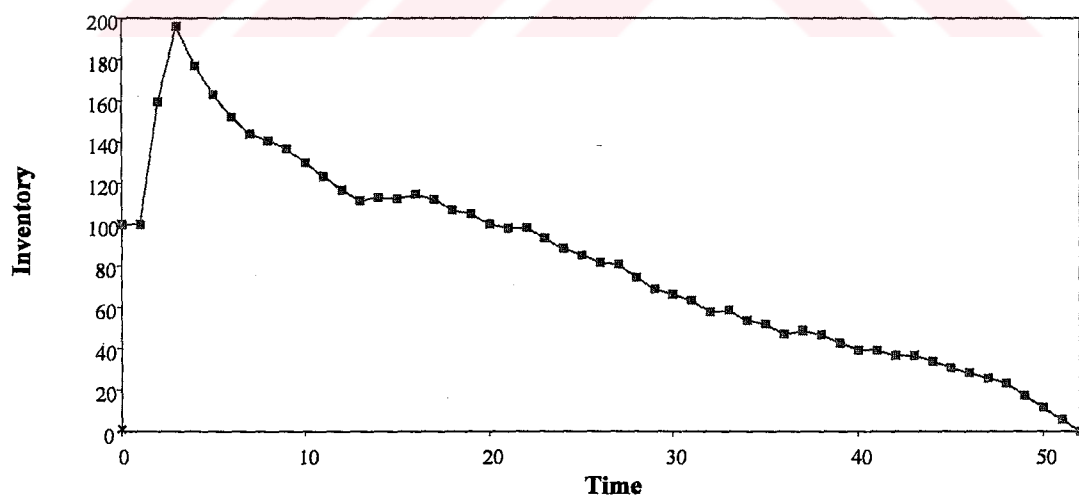


Figure 3.41: Retailer inventory level for product C

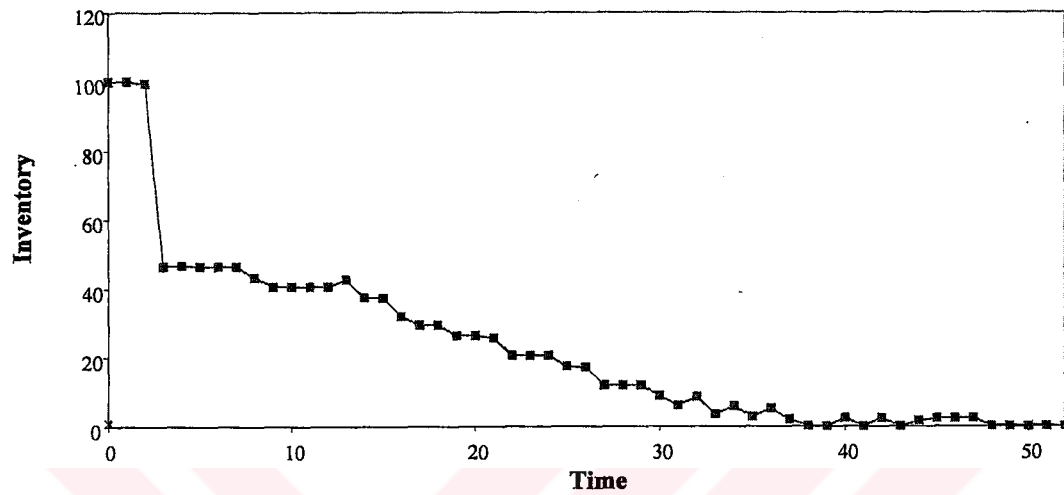


Figure 3.42: Distribution center inventory levels for product C

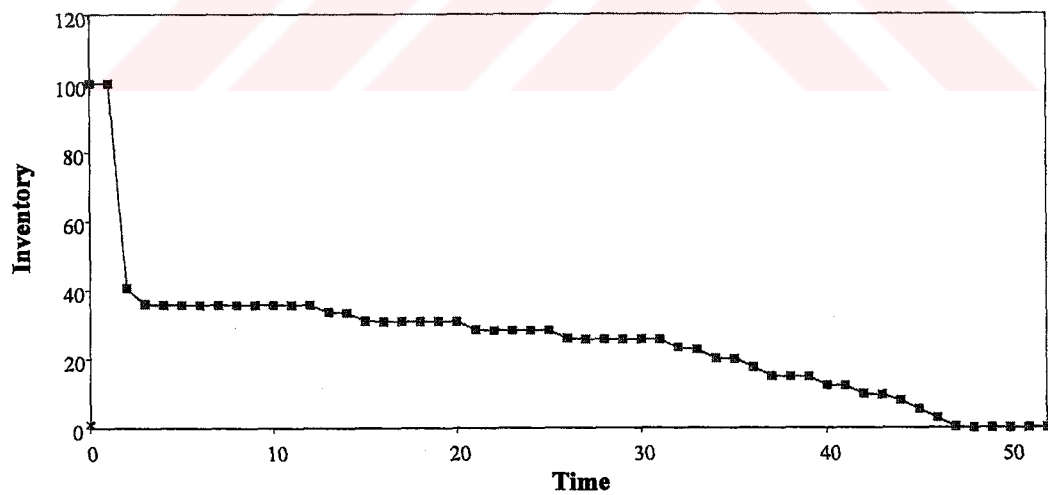


Figure 3.43: Warehouse inventory levels for product C

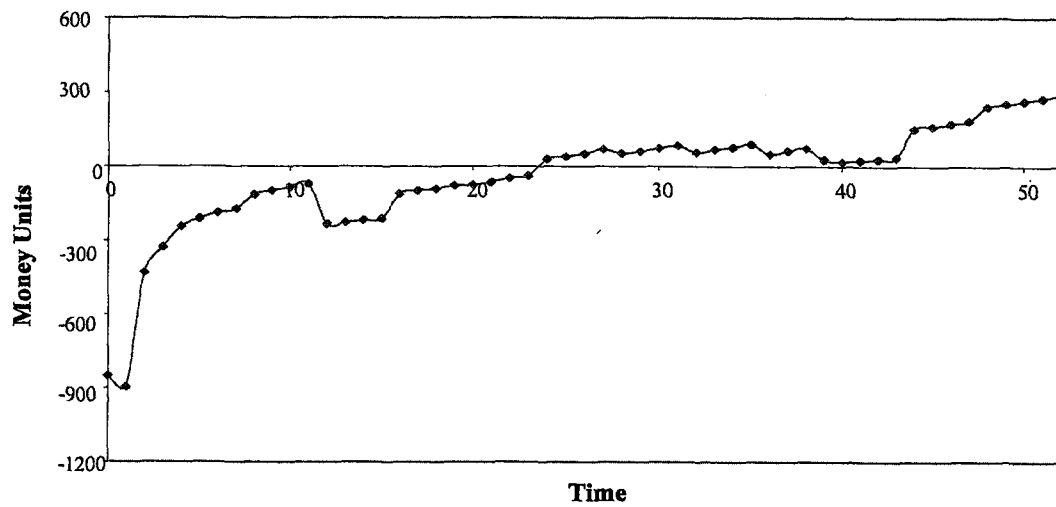


Figure 3.44: Instantaneous objective function for product C

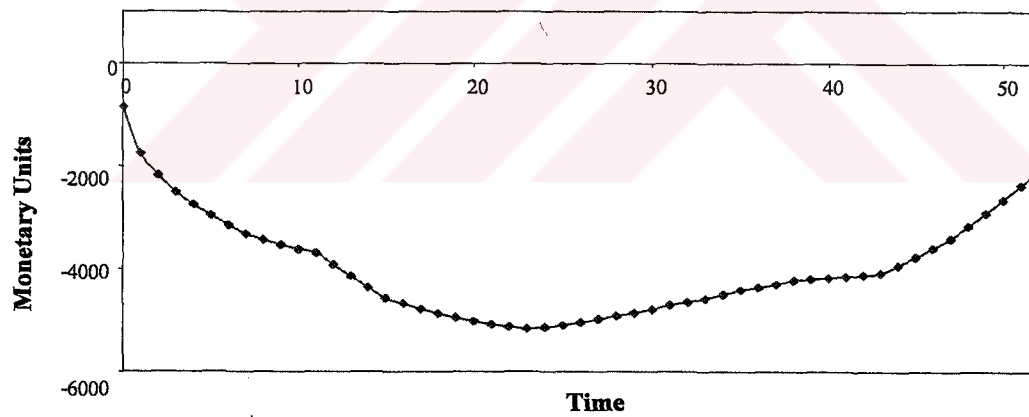


Figure 3.45: Cumulative objective function for product C

3.4 Model Predictive Control of the Hybrid System

In the application of the MPC approach, the initial horizon optimization horizon is taken as 20 time units, and the control interval is taken to be equal to 4. At $t=0$, the only information on the actual demands are that they deviate around the initial value with white noise. Therefore, the best demand model predictions for next 20 time units is made by setting them constant and equal to the initial demand values for each product types.

The supply chain is optimized using these values and optimal decision variables for the predicted horizon are obtained. The system is then simulated using these decision variables and the actual demand values within the control interval. At the end of the first control interval (4 time units), inventory and order states are updated; the new demand profile is again predicted as constant and equal to the measured demand at the 4th time unit and the system is re-optimized for another 20 time units. The same procedure is repeated until the end of the process time. The MPC approach is repeated with different optimization horizon lengths and the results are compared.

3.4.1 Optimization Horizon: 20:

In this case the system is optimized over a shorter horizon compared with the open loop optimization. During each optimization period the optimizer tries to make decisions for that specific period regardless of the future behavior beyond that period. Therefore it takes short-sighted decisions, and does not hesitate to allocate all its resources in earlier periods.

In the open loop optimization case, initially a large delivery to the customer is favored. Since the time horizon is smaller while the initial inventories are the same, compared with the open loop case, initially MPC with horizon 20, tends to deliver more material to the customer compared with the open loop optimization to maximize its initial revenue at the end of the optimization period (Fig. 3.48). Also it reduces its inventory faster compared to the open loop case. For example in the first optimization for product *A*, since it can not see the increase in demand in that 20 time units, it models it as a constant demand. However, since the system reduces its inventories quicker, in that short horizon, that creates problems in the successive steps. Starting from the third optimization step, the inventories become insufficient to meet the customer demand, hence the production orders are given. For example the production orders for product *A* are given at the 14th, 22nd, 30th, 37th and 46th time units as opposed to the 22nd, 30th and 40th of the open loop optimization. They are more in number and the production takes place

earlier. Product *B* is also produced 5 times, one more than the optimal case, and product *C* is produced 2 times, two more than the open loop case. The sudden increases in inventories and material transfer levels after production are explained in the open loop cases. Production of product *A* tends to happen after high increases in demand, as can be seen from the demand profile. This is because, for a high demand, MPC considers the system constant at that high demand and tends to reduce its inventories until the end of the optimization period based on this high demand. However actually the demand is not that large and in fact there is no need to reduce the inventories as much. Moreover at the next optimization these reduced inventories are not sufficient to supply materials to the customer and the need for production arises. As explained before, this arises from the rapid usage of the inventories due to the small horizon length. This effect can also be observed in Fig. 3.47, where the inventories are decreased to 0 quicker than the open loop case.

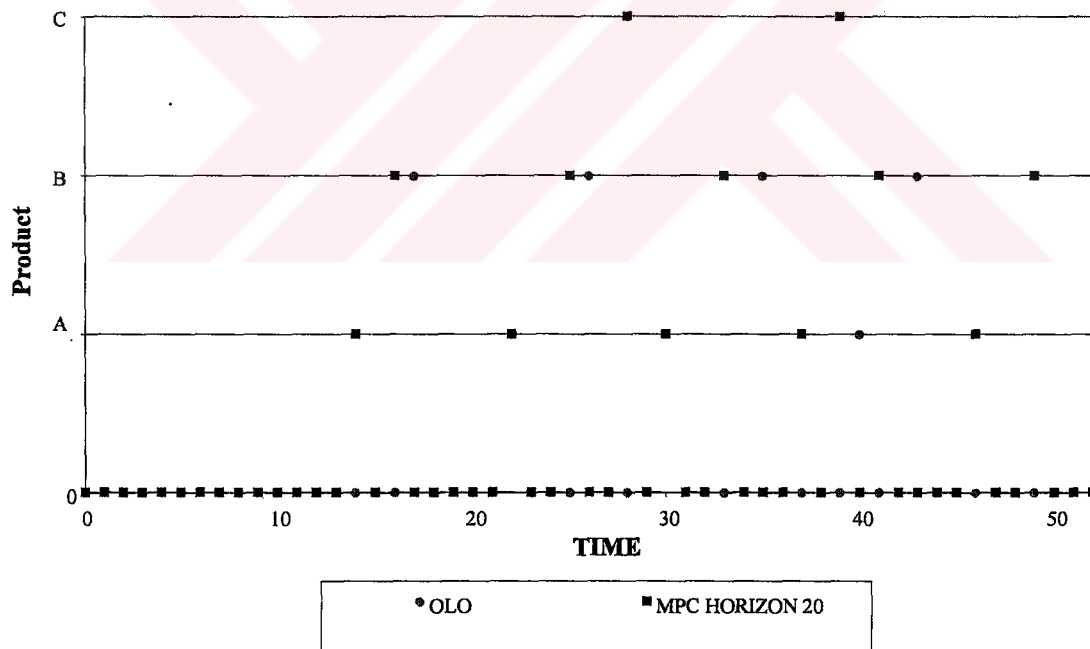


Figure 3.46: Optimal production schedule for the plant (MPC vs open loop)

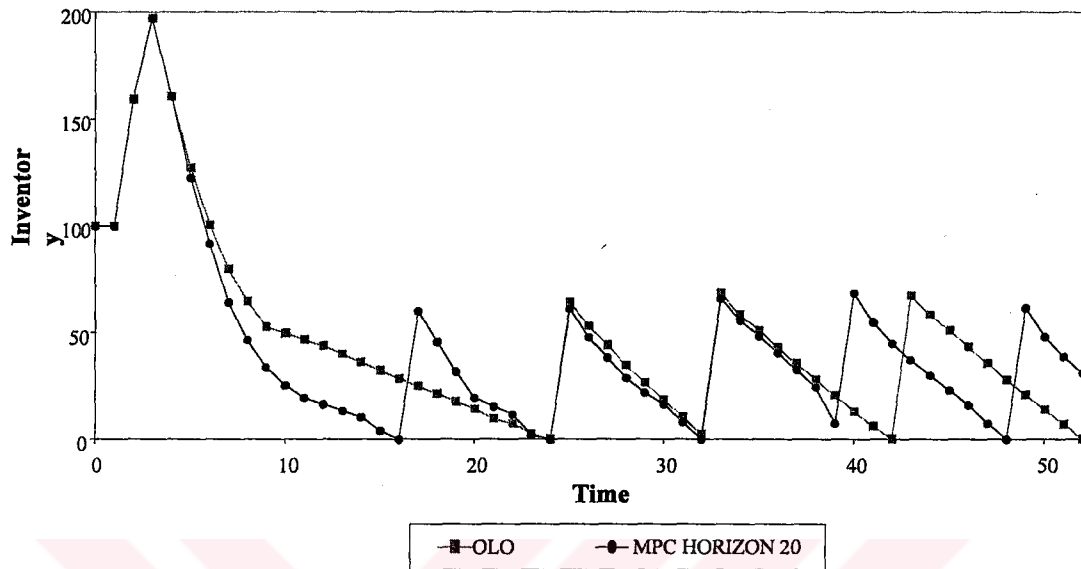


Figure 3.47: Retailer inventory level for product A (MPC vs open loop)

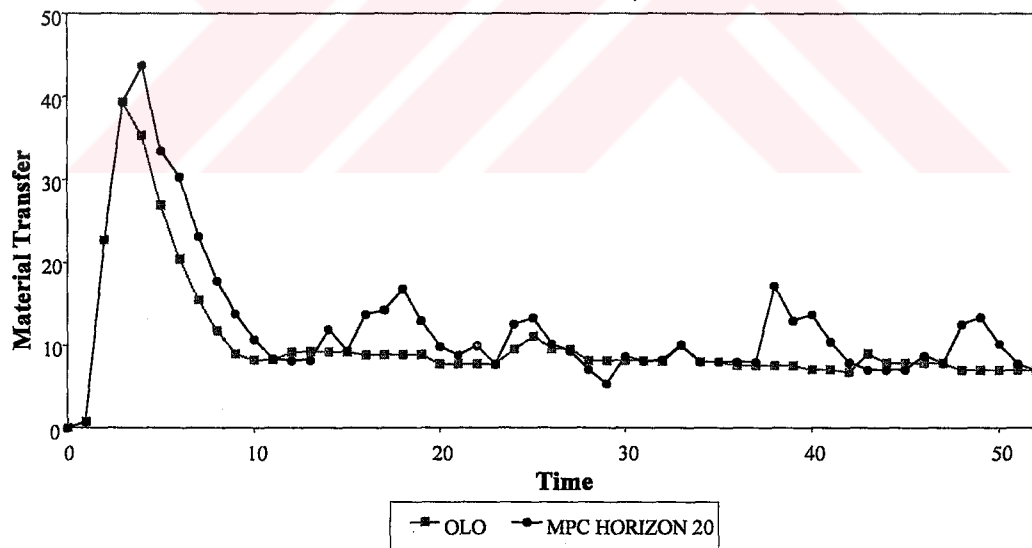


Figure 3.48: Retailer material transfer level for product A (MPC vs open loop)

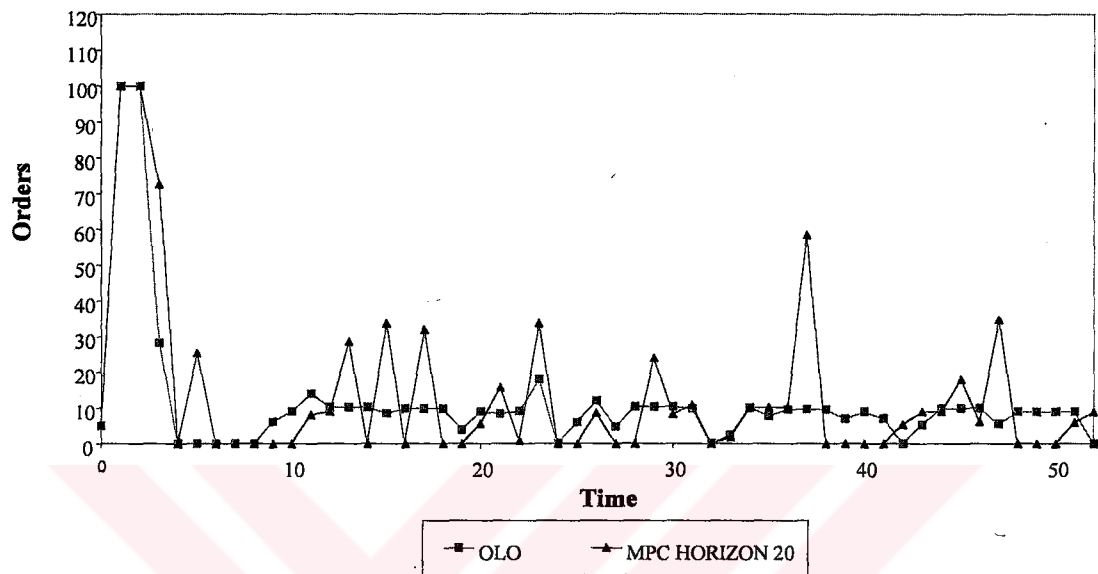


Figure 3.49: Retailer order levels for product A (MPC vs open loop)

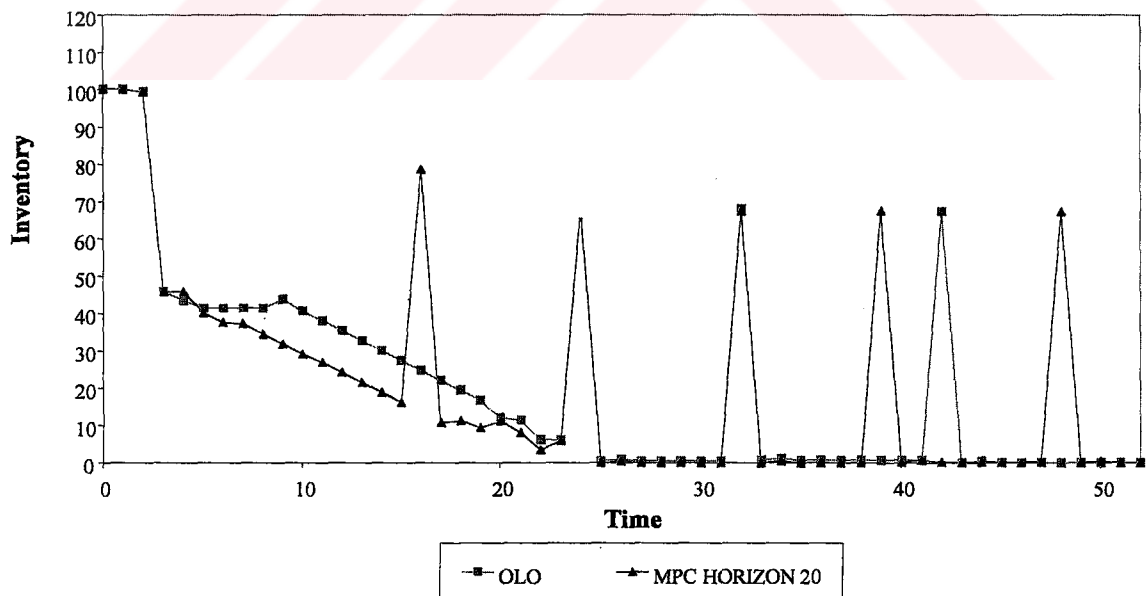


Figure 3.50: Distribution center inventory level for product A (MPC vs open loop)

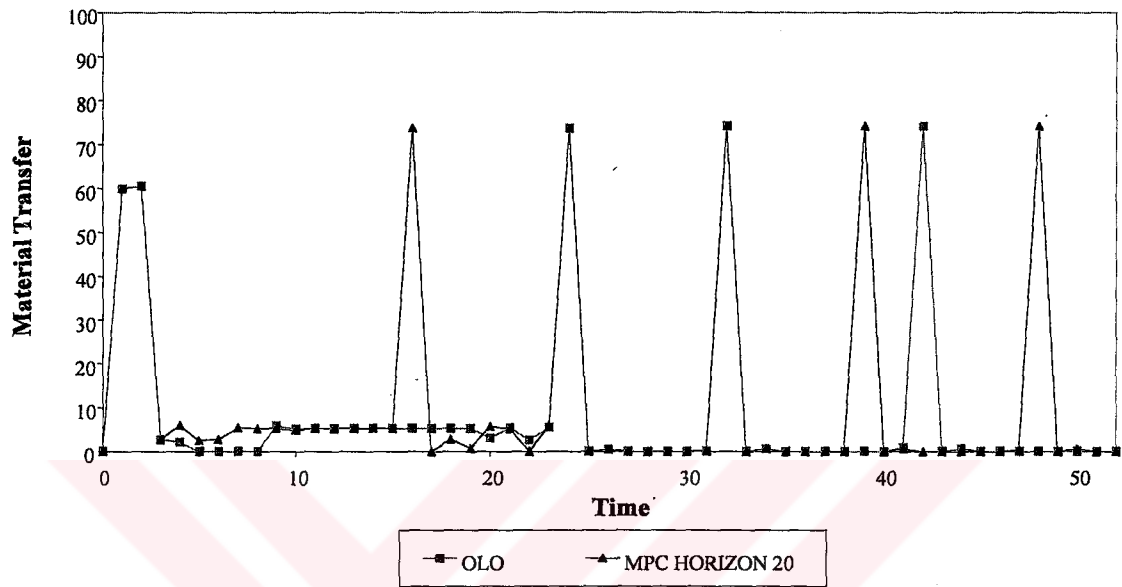


Figure 3.51: Distribution center material transfer level for product A (MPC vs open loop)

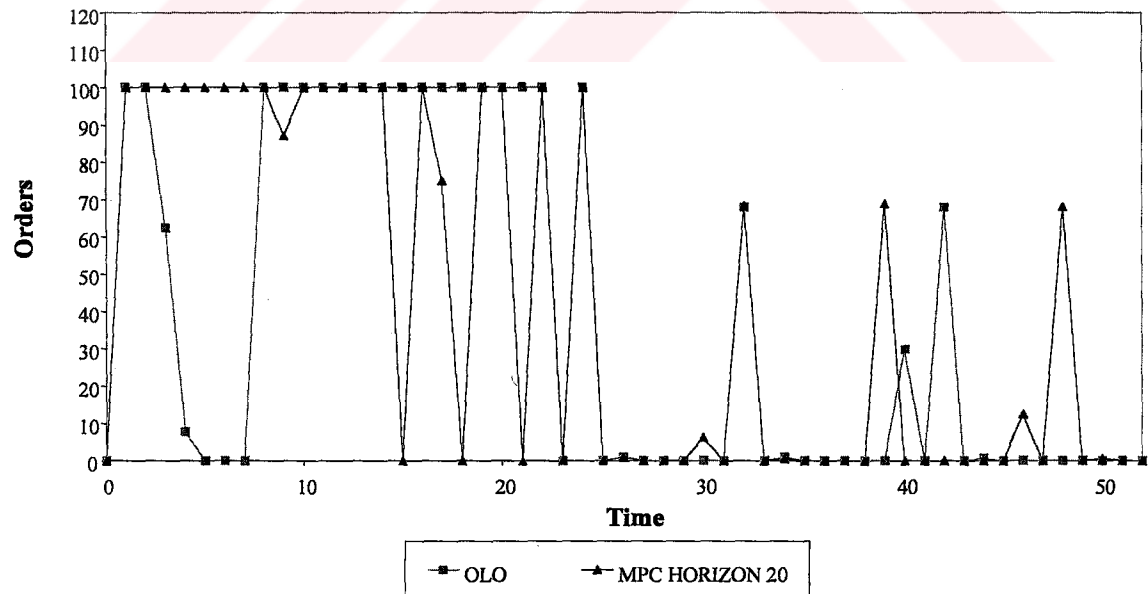


Figure 3.52: Distribution center order levels for product A (MPC vs open loop)

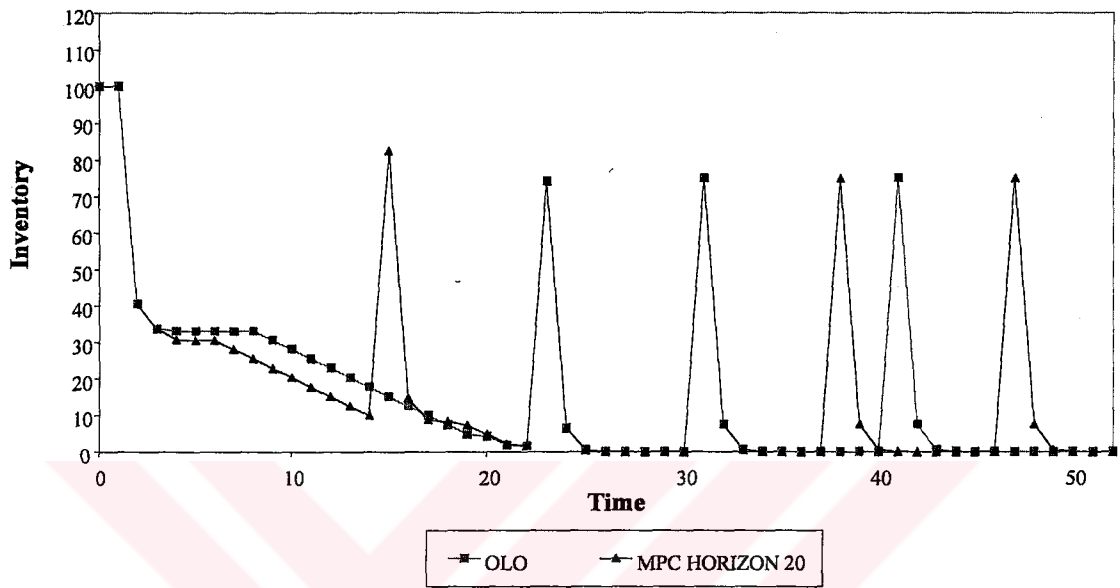


Figure 3.53: Warehouse inventory level for product A (MPC vs open loop)

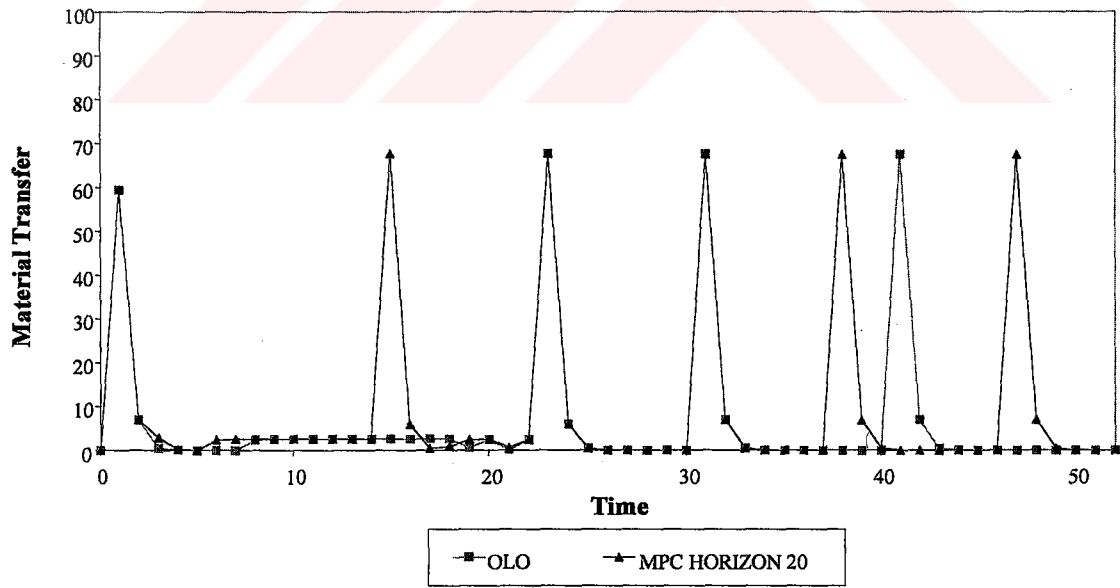


Figure 3.54: Warehouse material transfer level for product A (MPC vs open loop)

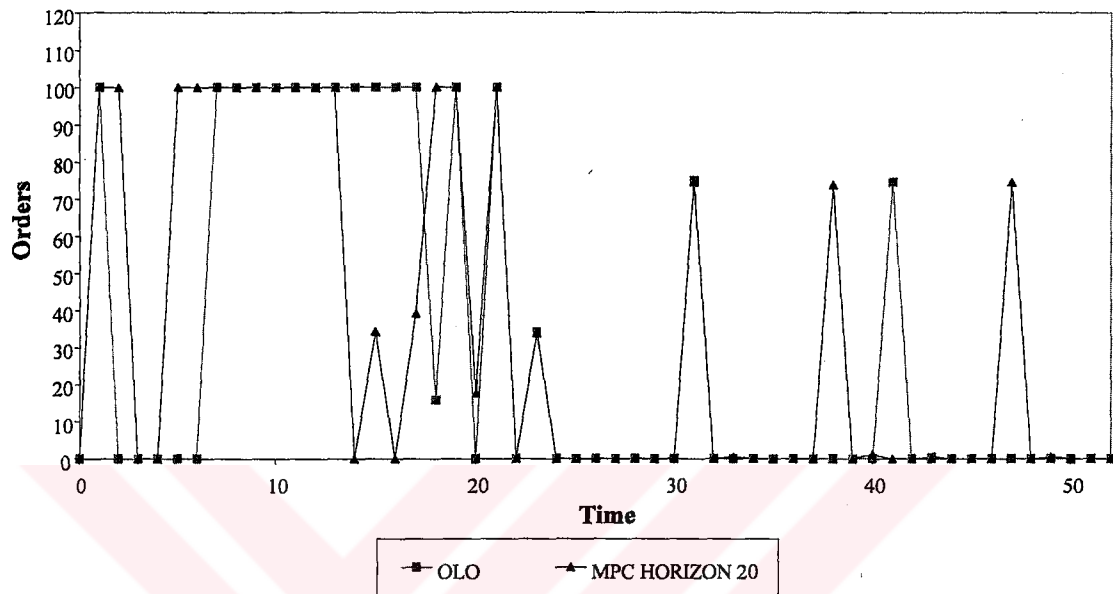


Figure 3.55: Warehouse order levels for product A (MPC vs open loop)

Due to the high production costs, MPC with 20 optimization period, gives a drastically lower objective function value. The effects of other variables of the objective function, compared to the open loop case are given below.

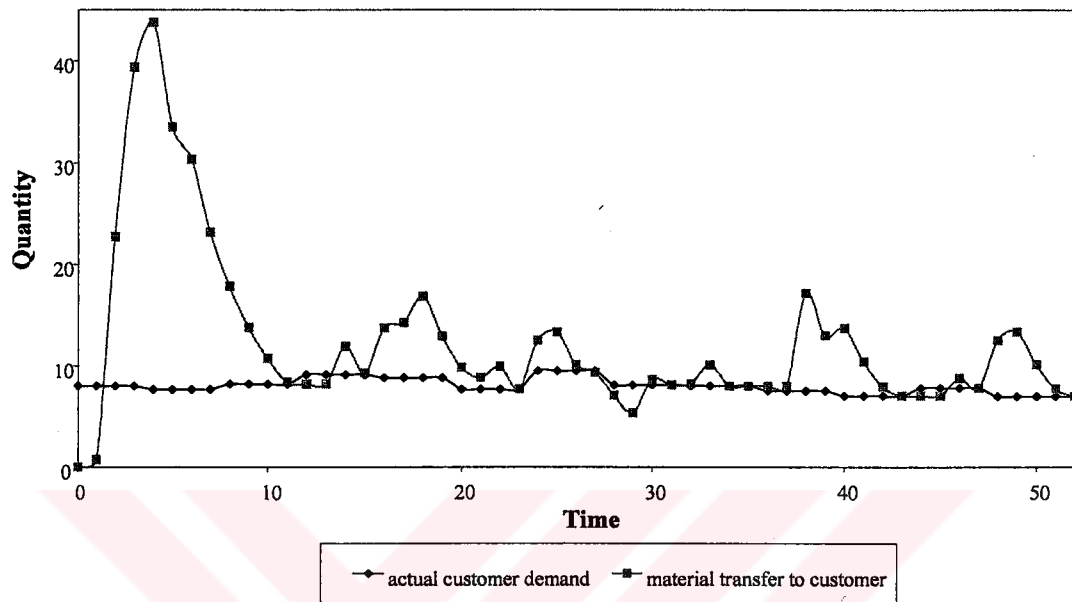


Figure 3.56: Material transferred to customer vs actual cost. demand for product A (MPC)

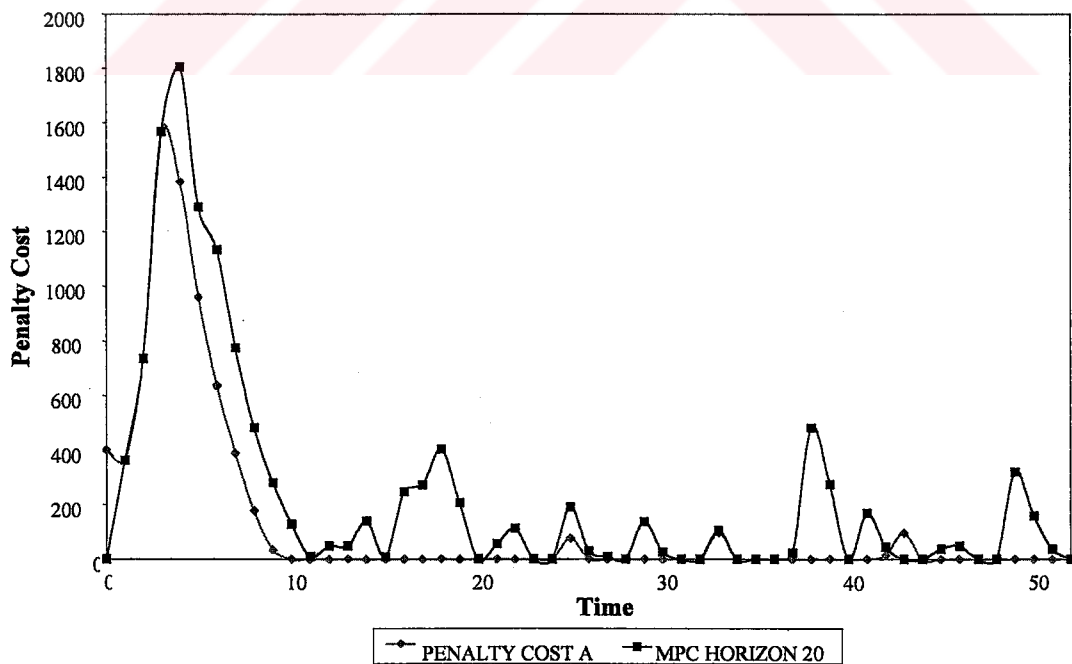


Figure 3.57: Penalty cost paid for product A (MPC vs open loop)

It can be observed from the Figs. 56 and 57 that, the initial aim is to sell as much product to the customer as possible, using the initial inventory levels. The high revenues received for selling compensate for the over production penalty costs, as explained in the open loop case.

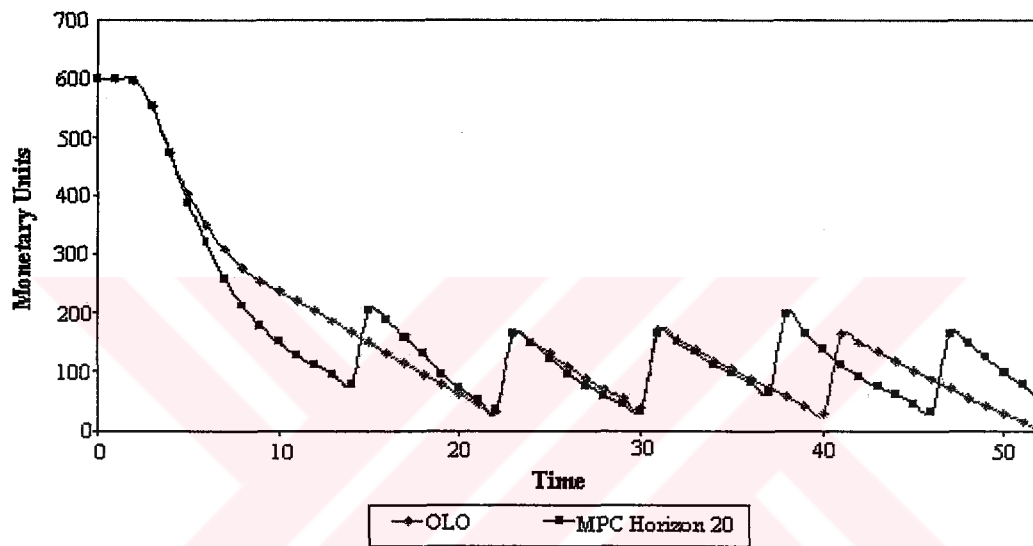


Figure 3.58: Total inventory costs paid for product A (MPC vs open loop)

Inventory costs for the retailer level are also shown above. Overall MPC inventory costs are lower compared with the open loop case, since the MPC tends to transfer all its material rather than accumulating them in the inventories. This high material transfer shows itself in the revenue gained by MPC. As shown in Fig. 3.59, revenue gained by MPC is high compared with open loop case, especially when production takes place, in which most of the produced material is transferred to the customer.

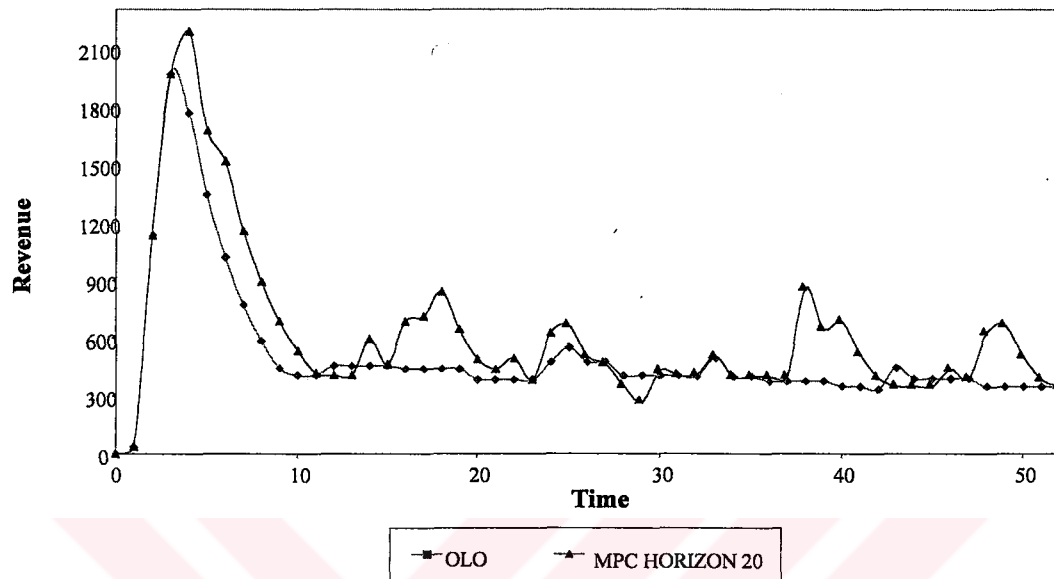


Figure 3.59: Revenue gained from product A (MPC vs open loop)

However, despite the low inventory costs and high revenues obtained by MPC case, customer dissatisfaction is higher in the MPC case arising from high levels of excess product delivered to the customer (Fig. 3.56). All the extra revenue made by MPC is compensated by the customer dissatisfaction penalty in the objective function. Material transfer costs are also high in MPC case due to the high product delivery (Fig. 3.60). Overall the large differences between the cumulative objective functions of the MPC and open loop case arises as a result of more frequent production in the MPC case (Fig. 3.62).

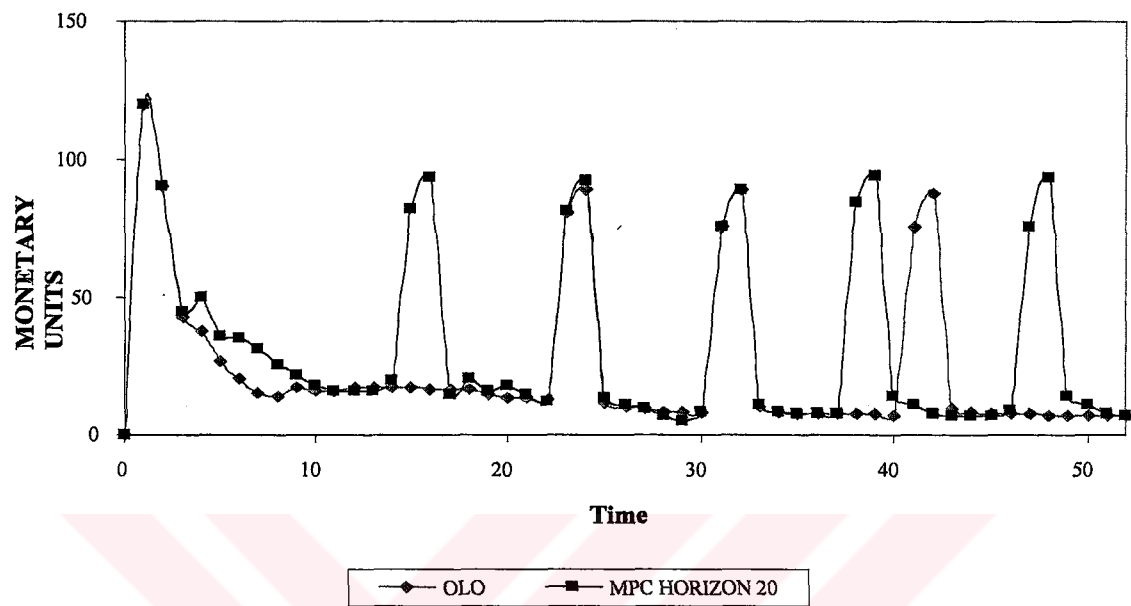


Figure 3.60: Material transfer costs for product A(MPC vs open loop)

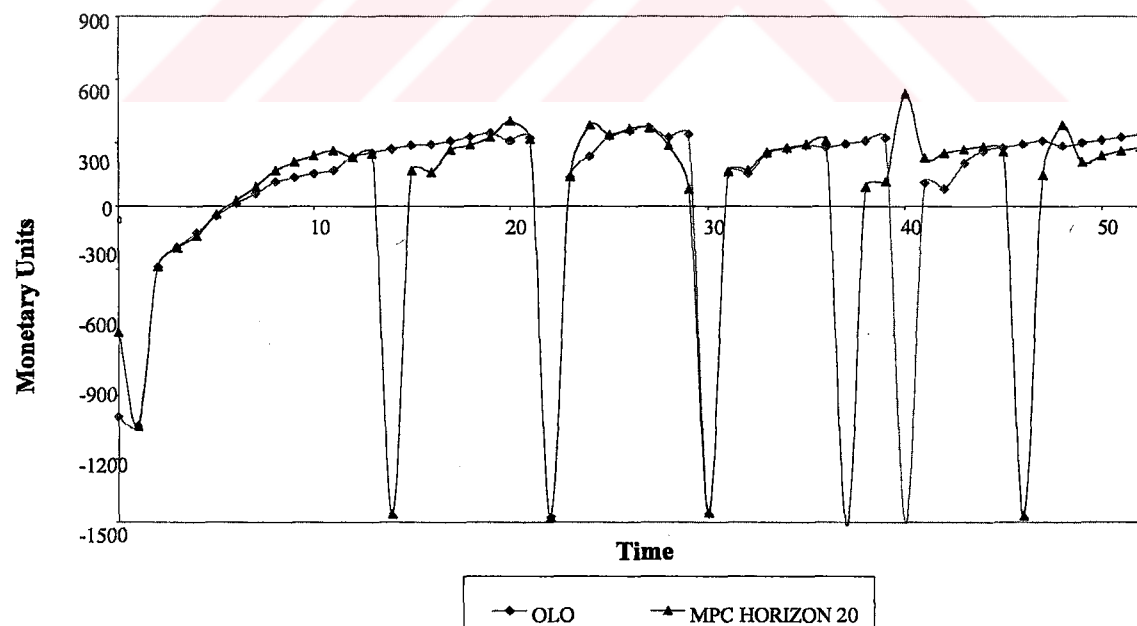


Figure 3.61: Instantaneous objective function for product A(MPC vs open loop)

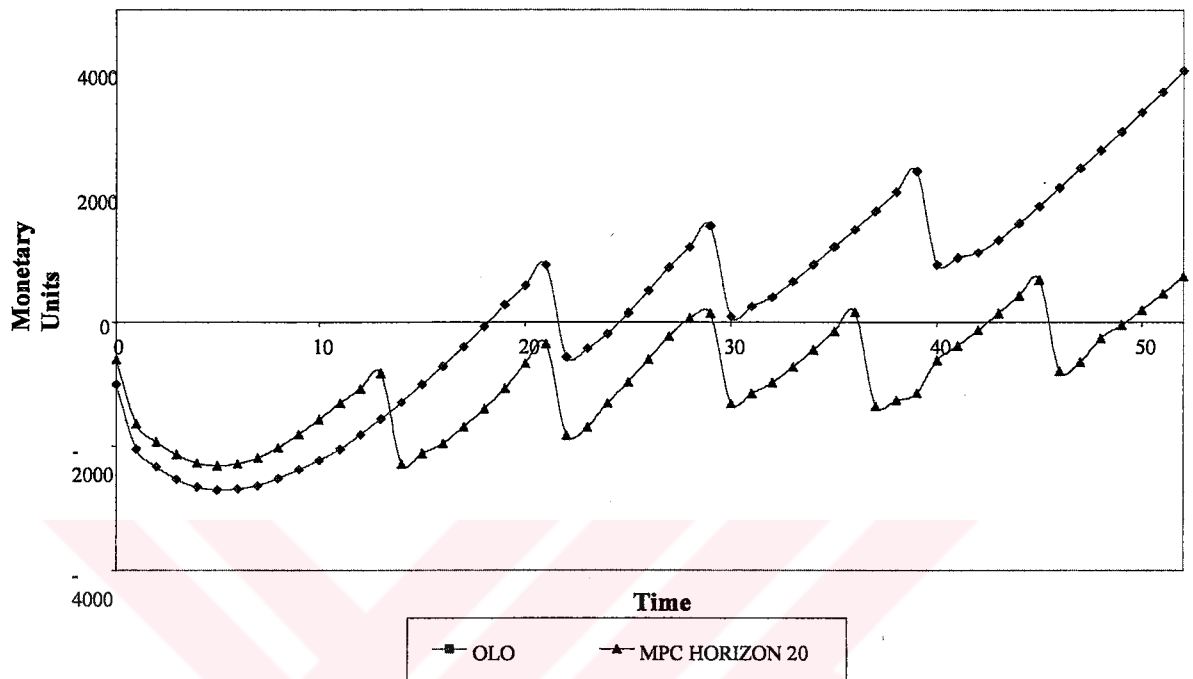


Figure 3.62: Cumulative objective function for product A(MPC vs open loop)

It can be observed that at the beginning due to the high material transfer rates at the retailer level, the objective function of MPC is higher than the open loop case. However due to the production costs the cumulative objective function tends to be lower as shown in Fig. 3.61.

Same patterns are observed for other products B and C as shown in the figures below.

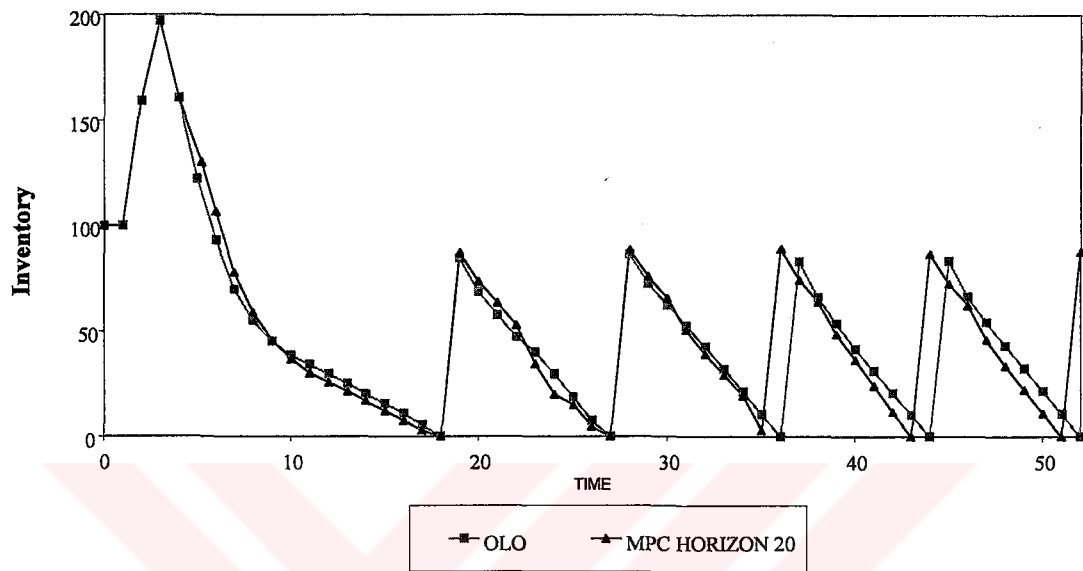


Figure 3.63: Retailer inventory level for product B (MPC vs open loop)

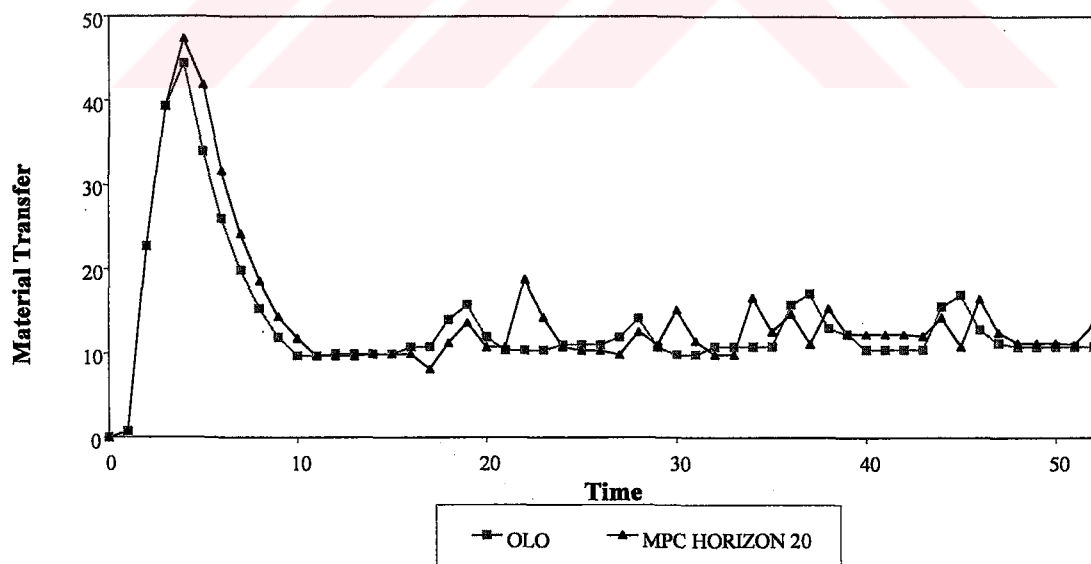


Figure 3.64: Retailer material transfer level for product B (MPC vs open loop)

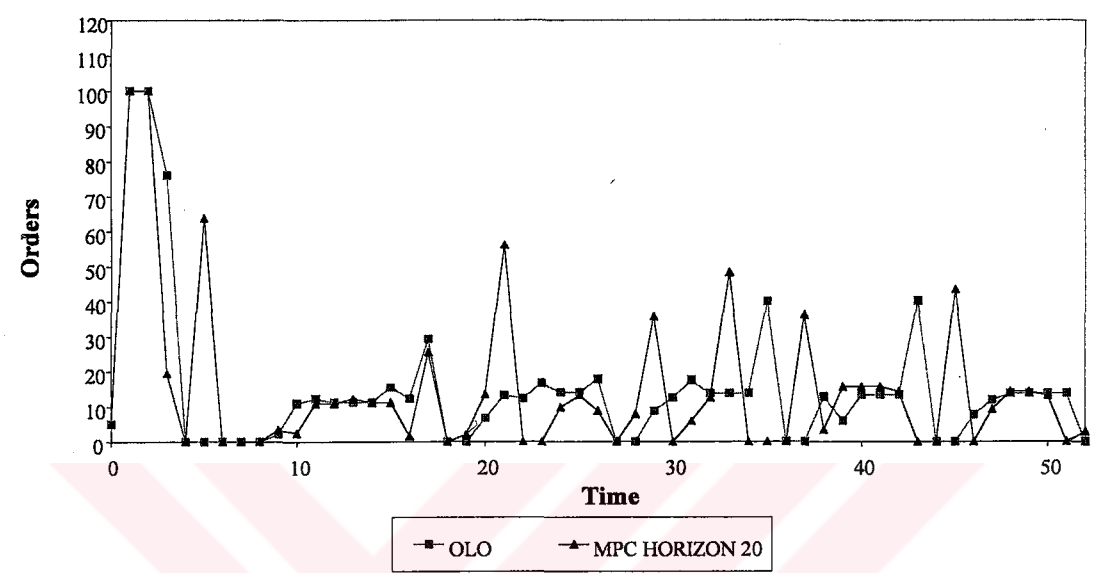


Figure 3.65: Retailer order levels for product B (MPC vs open loop)

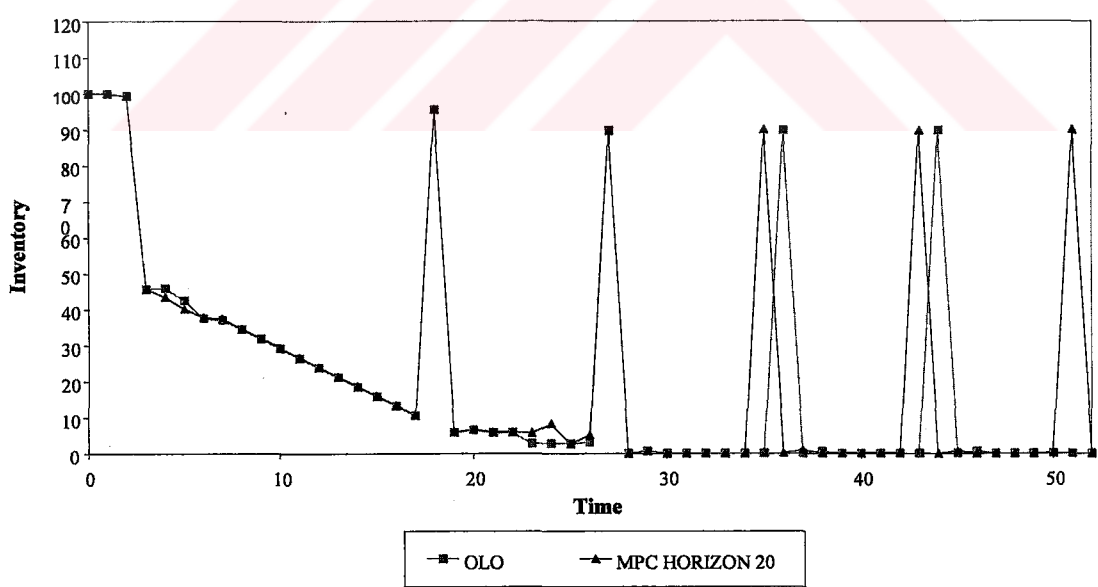


Figure 3.66: Distribution center inventory level for product B (MPC vs open loop)

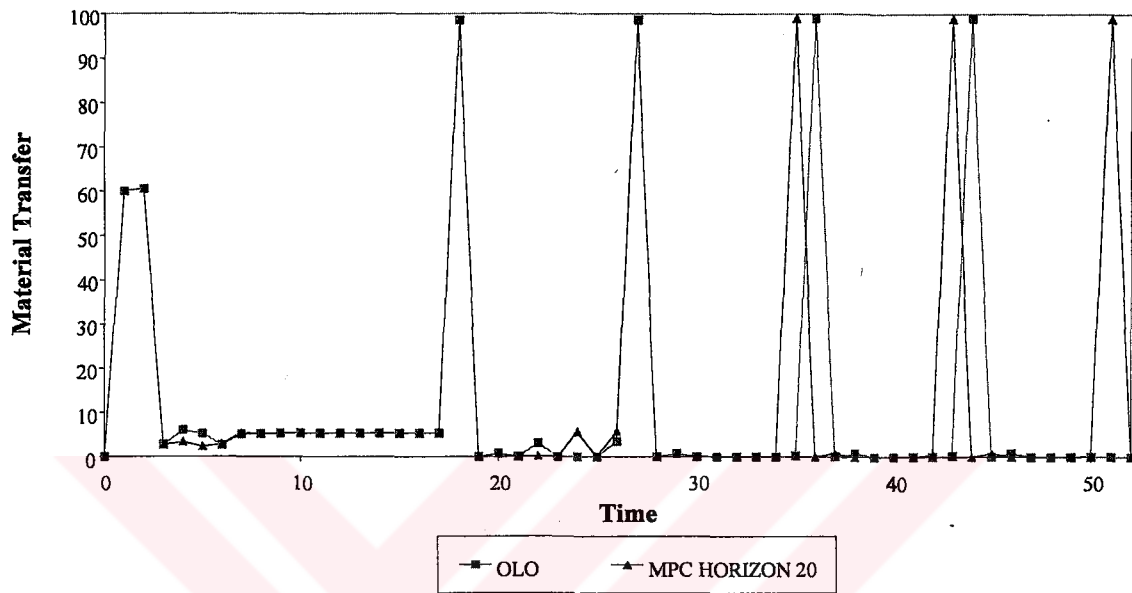


Figure 3.67: Distribution center material transfer level for product B (MPC vs open loop)

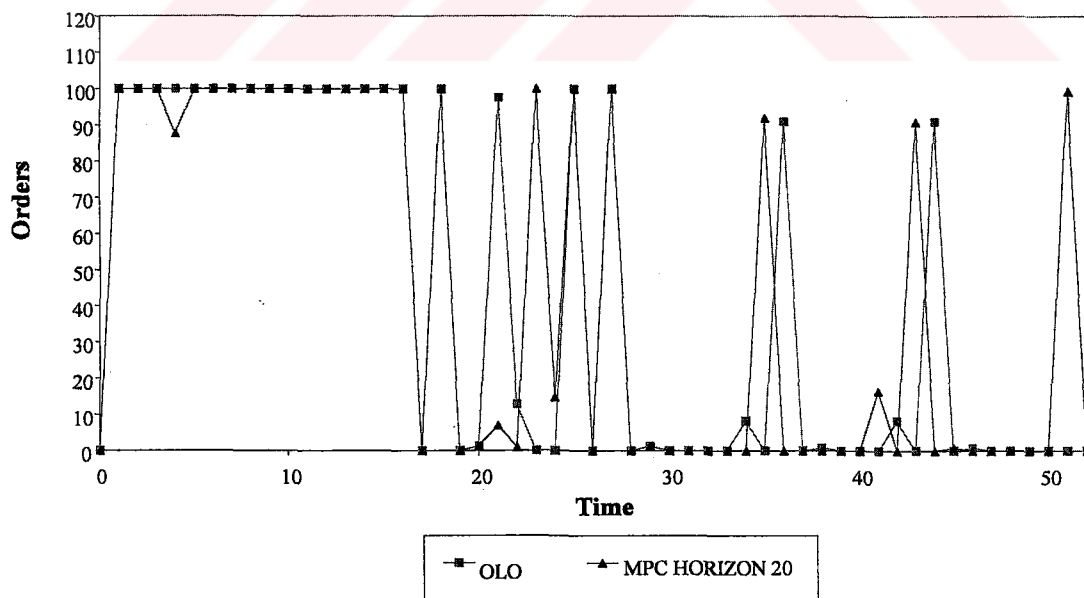


Figure 3.68: Distribution center order levels for product B (MPC vs open loop)

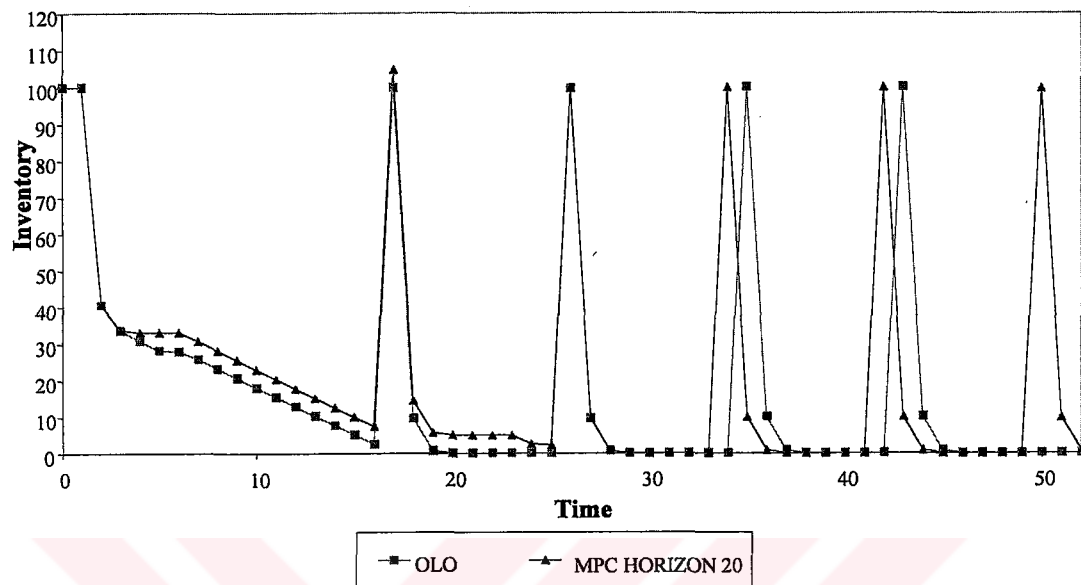


Figure 3.69: Warehouse inventory level for product B (MPC vs open loop)

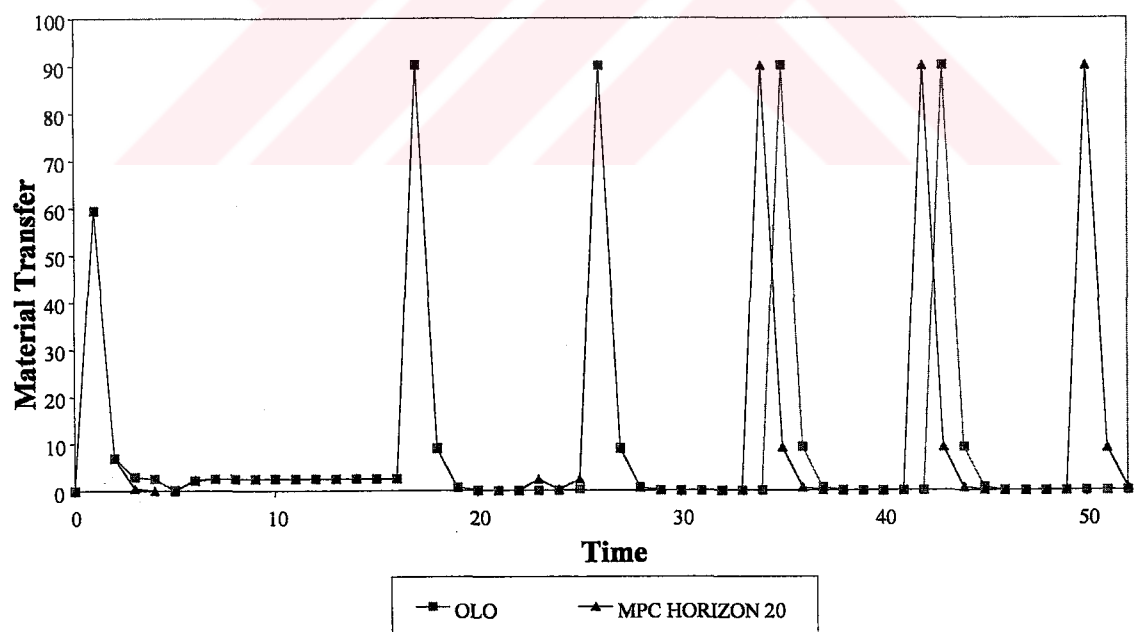


Figure 3.70: Warehouse material transfer level for product B (MPC vs open loop)

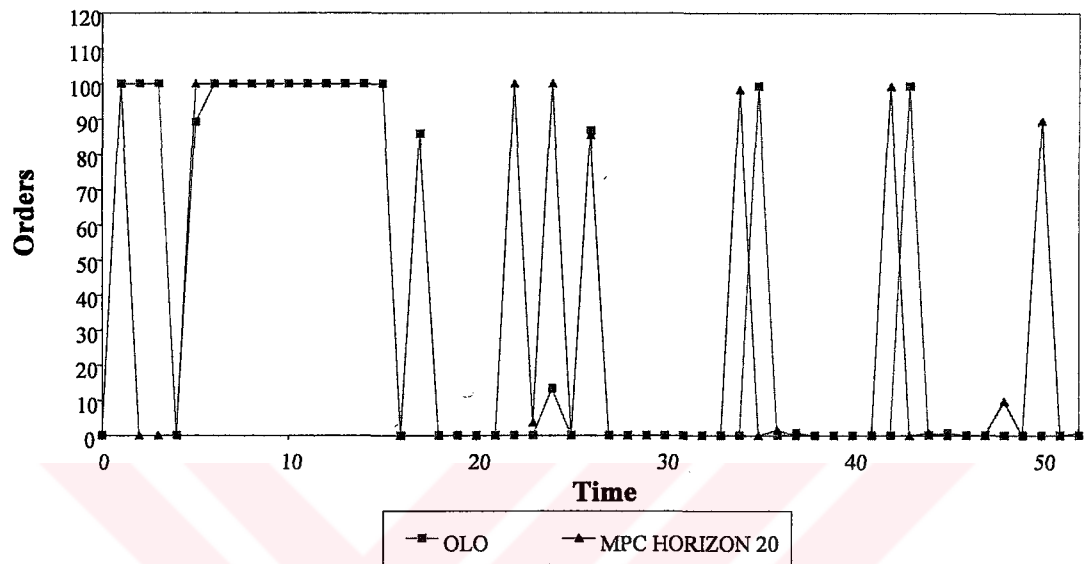


Figure 3.71: Warehouse order levels for product B (MPC vs open loop)

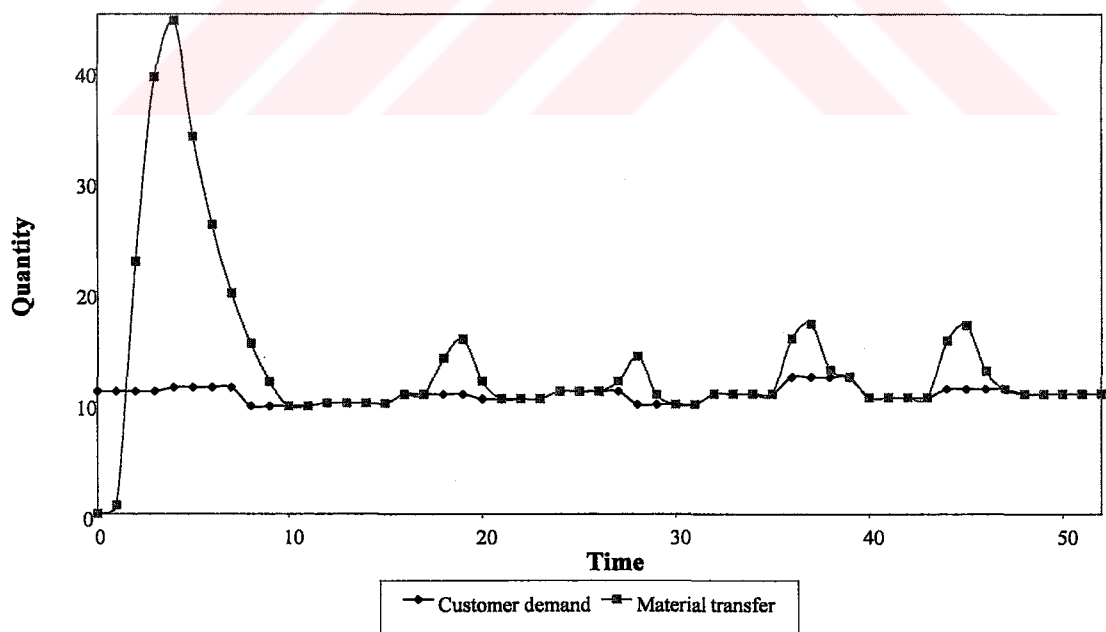


Figure 3.72: Material transferred to customer vs actual customer demand for product B (MPC)

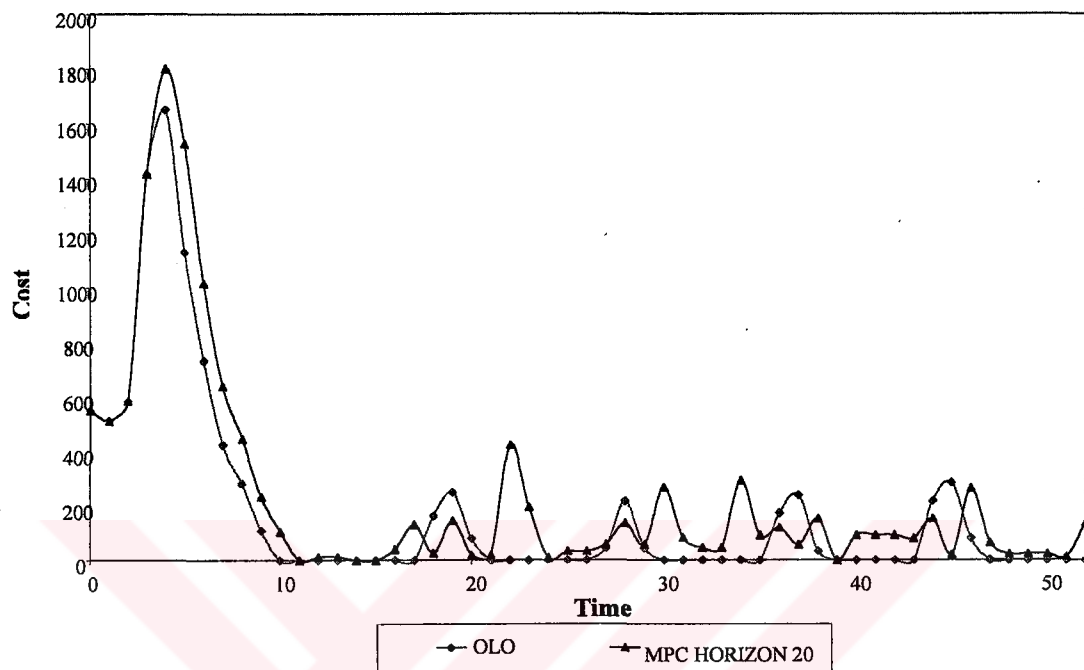


Figure 3.73: Penalty cost paid for product B (MPC vs open loop)

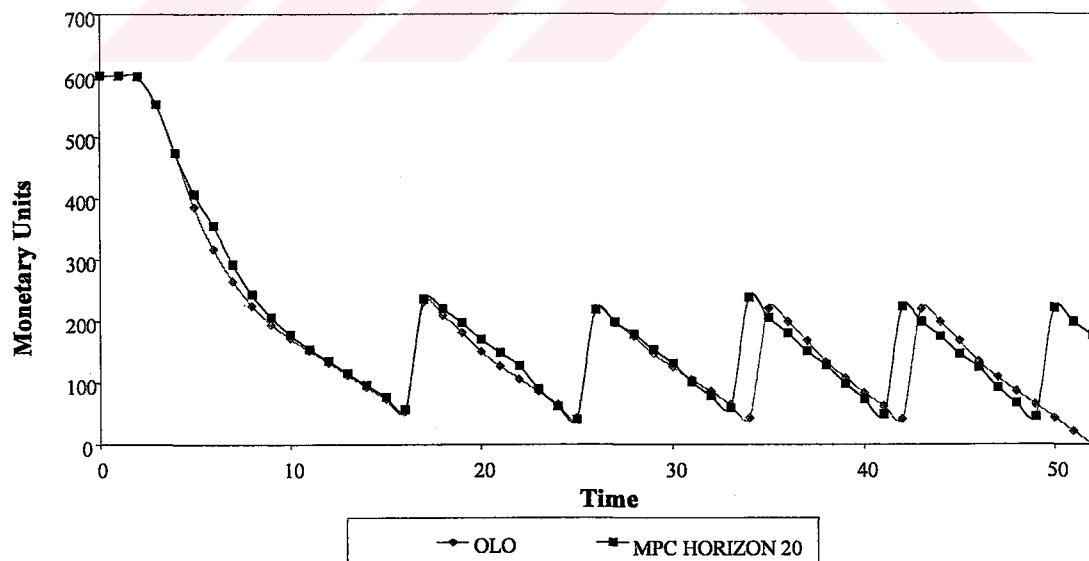


Figure 3.74: Total inventory costs paid for product B (MPC vs open loop)

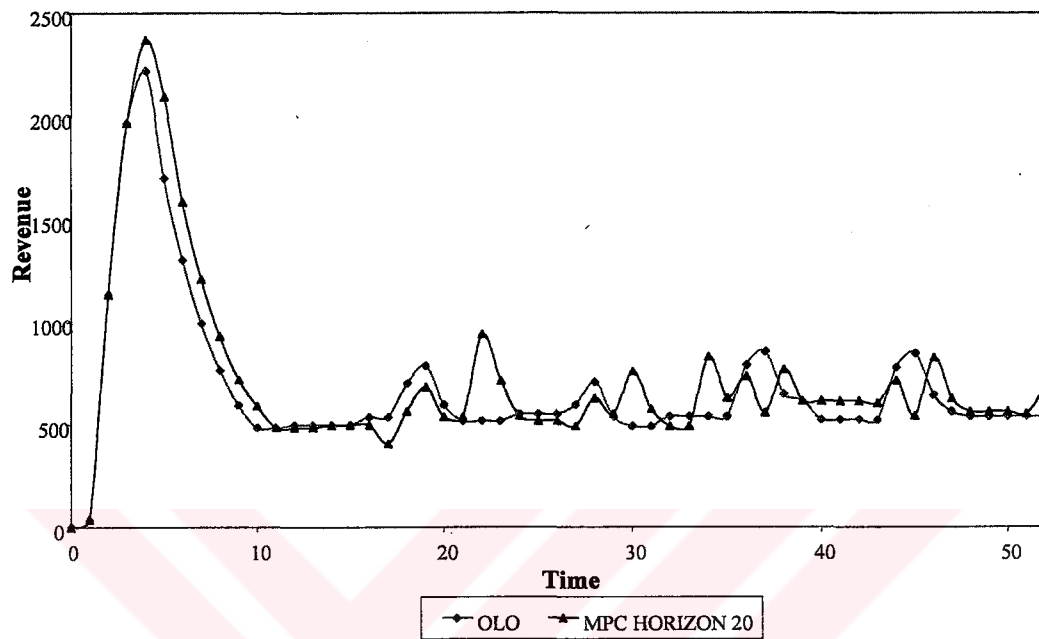


Figure 3.75: Revenue gained from product B (MPC vs open loop)

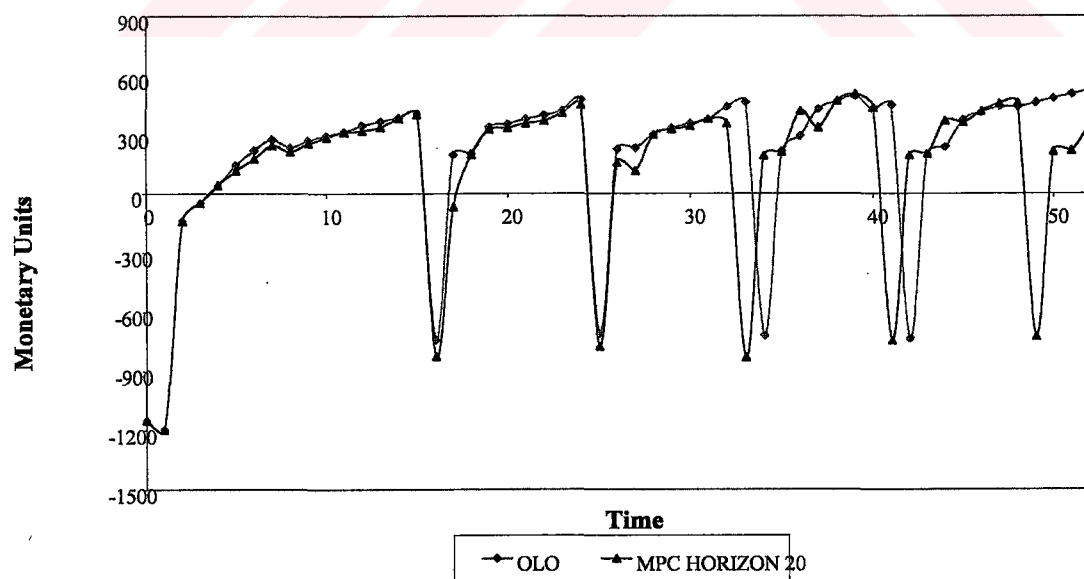


Figure 3.76: Instantaneous objective function for product B (MPC vs open loop)

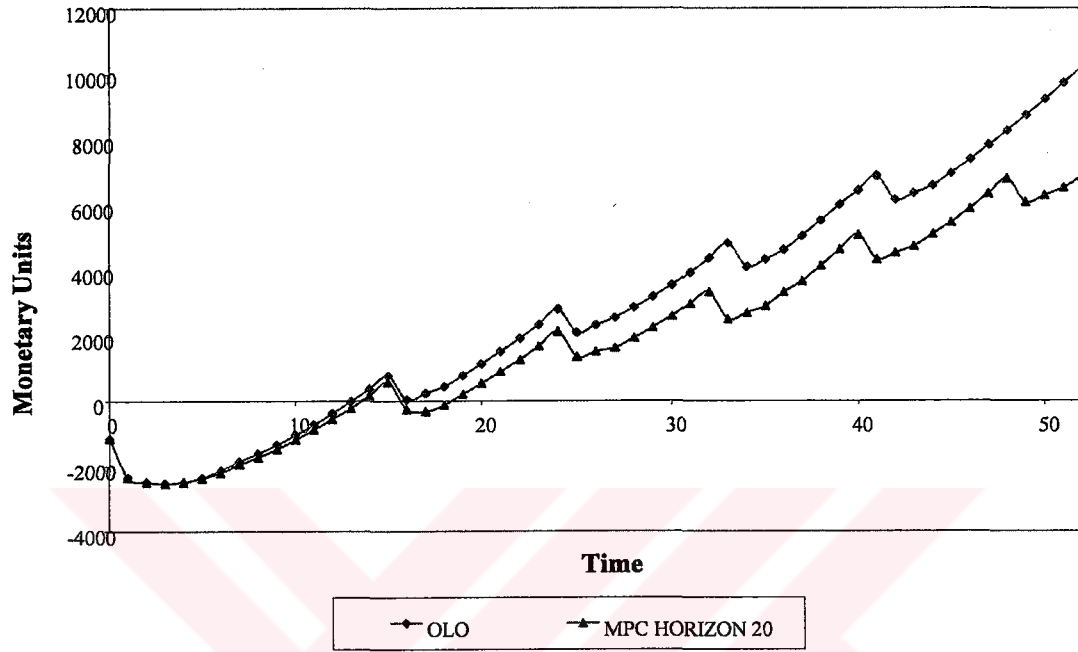


Figure 3.77: Cumulative objective function for product B (MPC vs open loop)

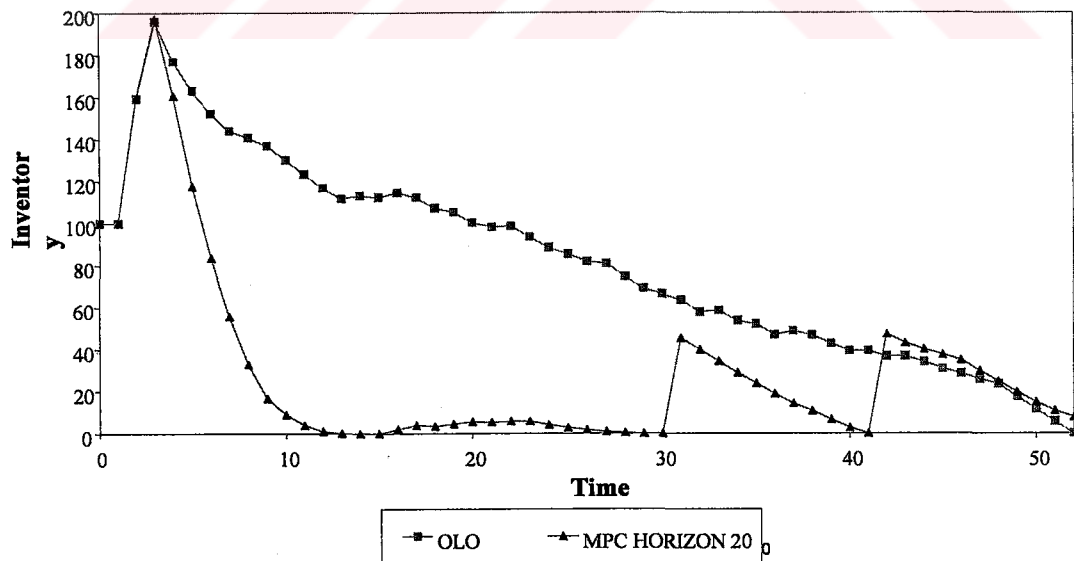


Figure 3.78: Retailer inventory level for product C (MPC vs open loop)

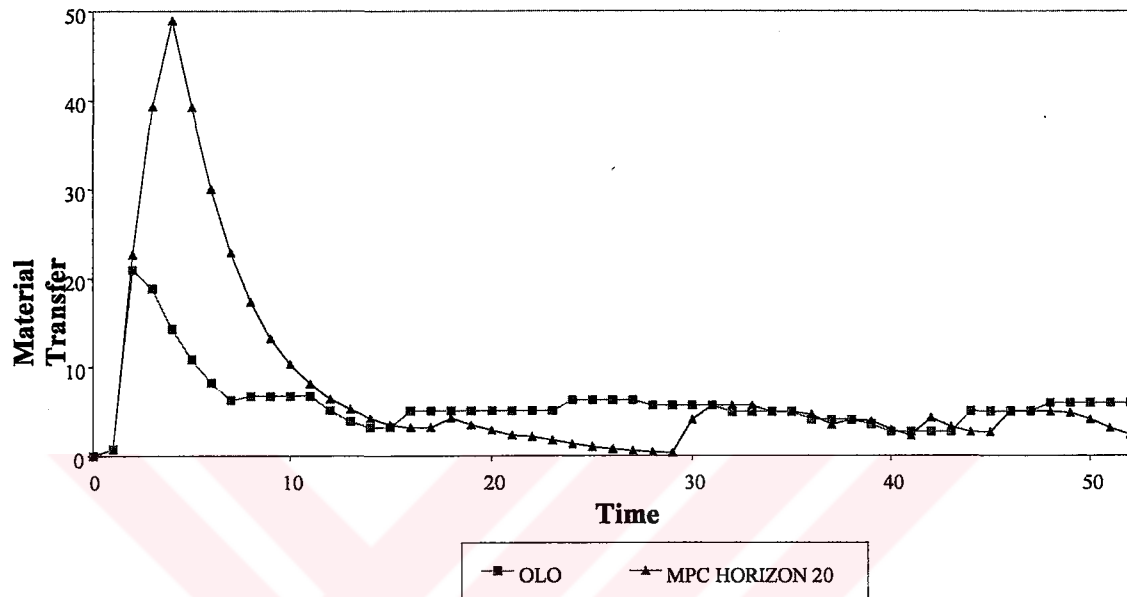


Figure 3.79: Retailer material transfer level for product C (MPC vs open loop)

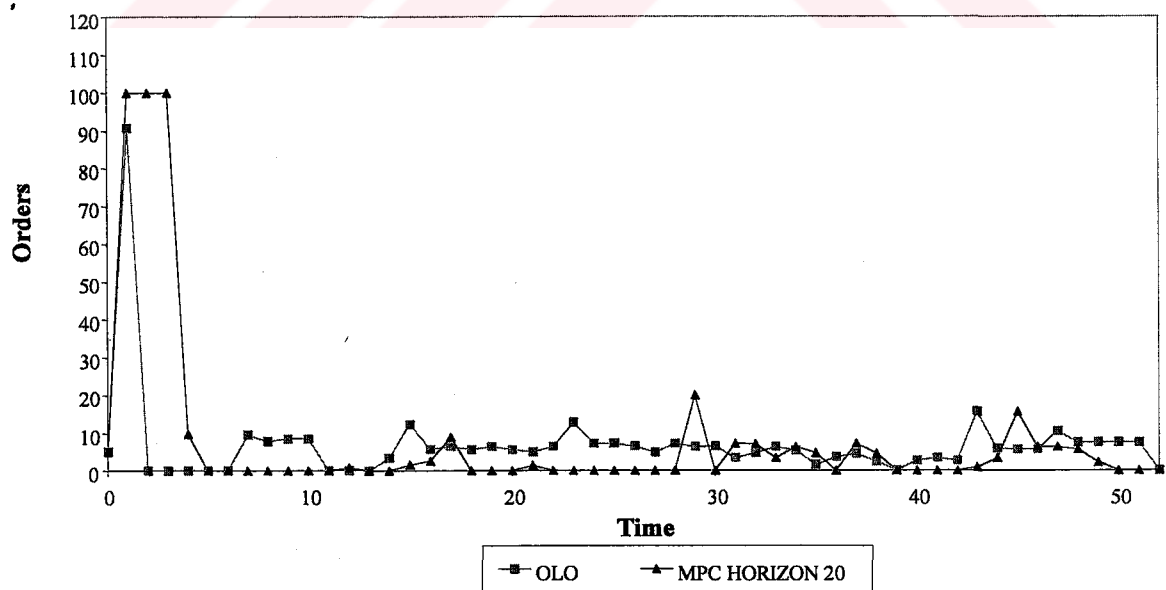


Figure 3.80: Retailer order levels for product C (MPC vs open loop)

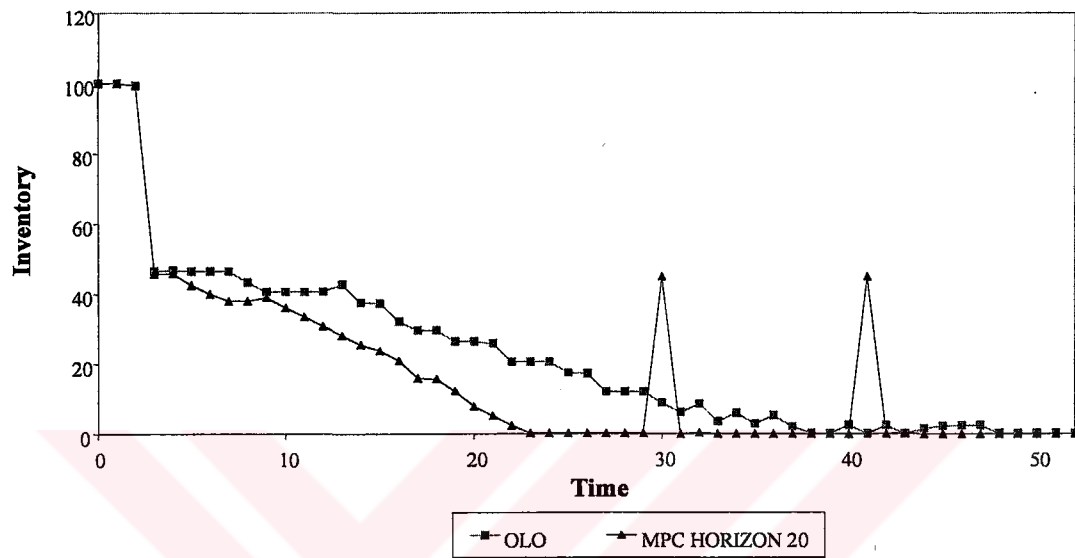


Figure 3.81: Distribution center inventory level for product C (MPC vs open loop)

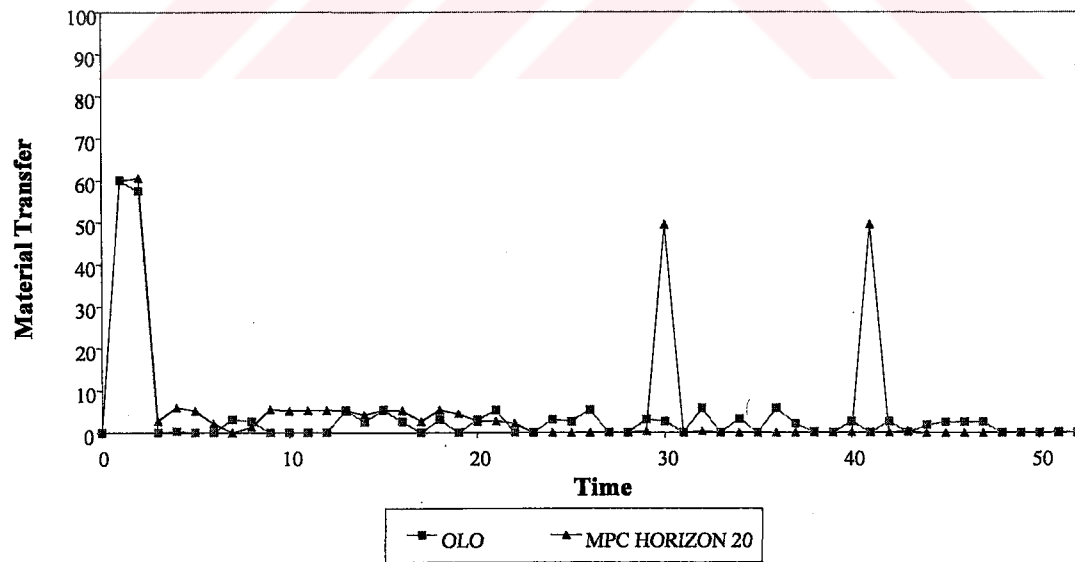


Figure 3.82: Distribution center material transfer level for product C (MPC vs open loop)

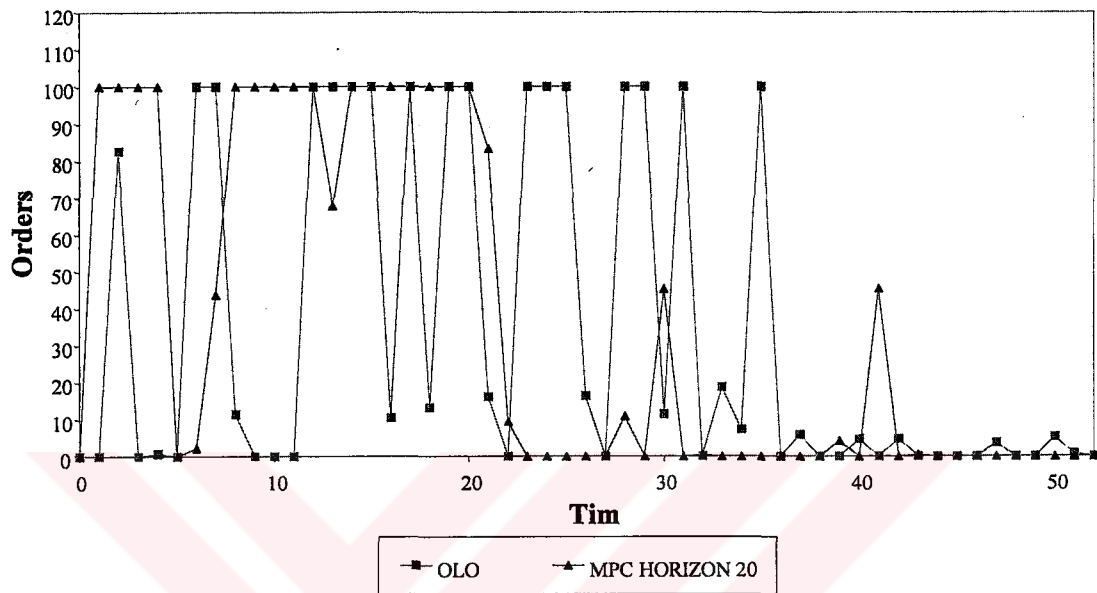


Figure 3.83: Distribution center order levels for product C (MPC vs open loop)

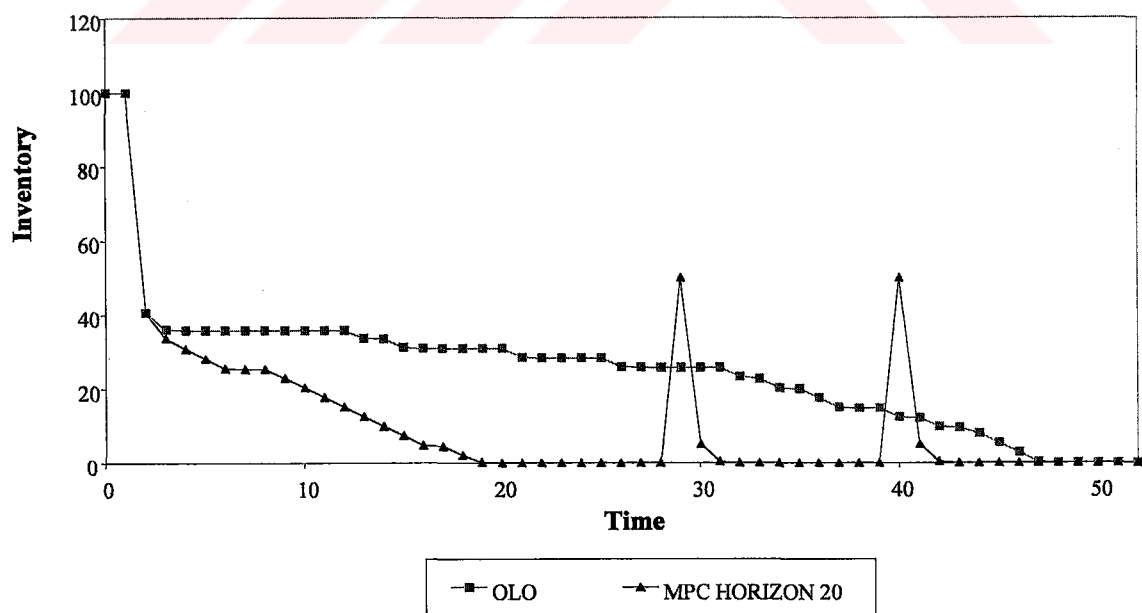


Figure 3.84: Warehouse inventory level for product C (MPC vs open loop)

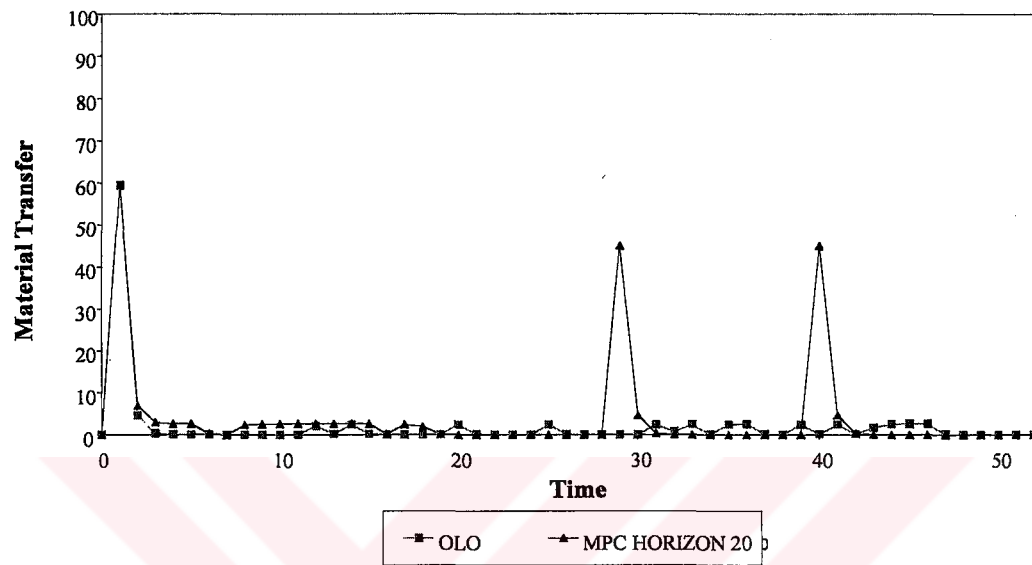


Figure 3.85: Warehouse material transfer level for product C (MPC vs open loop)

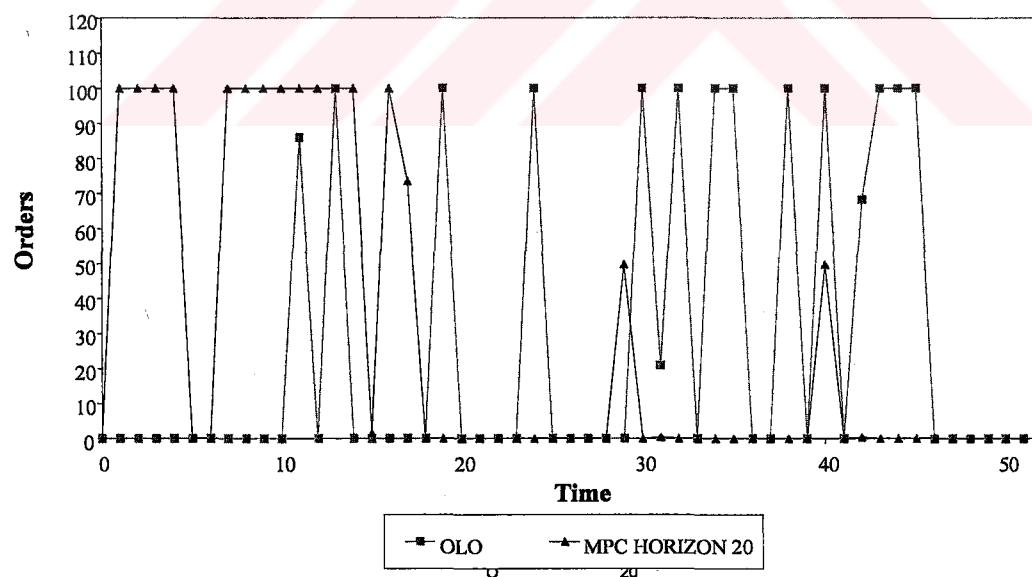


Figure 3.86: Warehouse order levels for product C (MPC vs open loop)

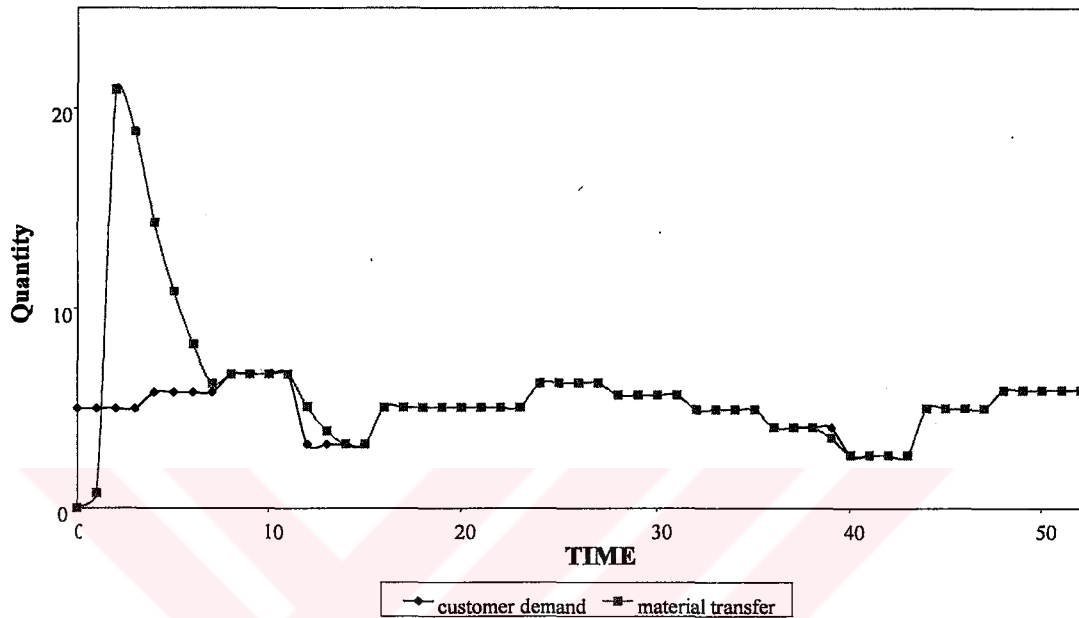


Figure 3.87: Material transferred to customer vs actual cost. demand for product C (MPC)

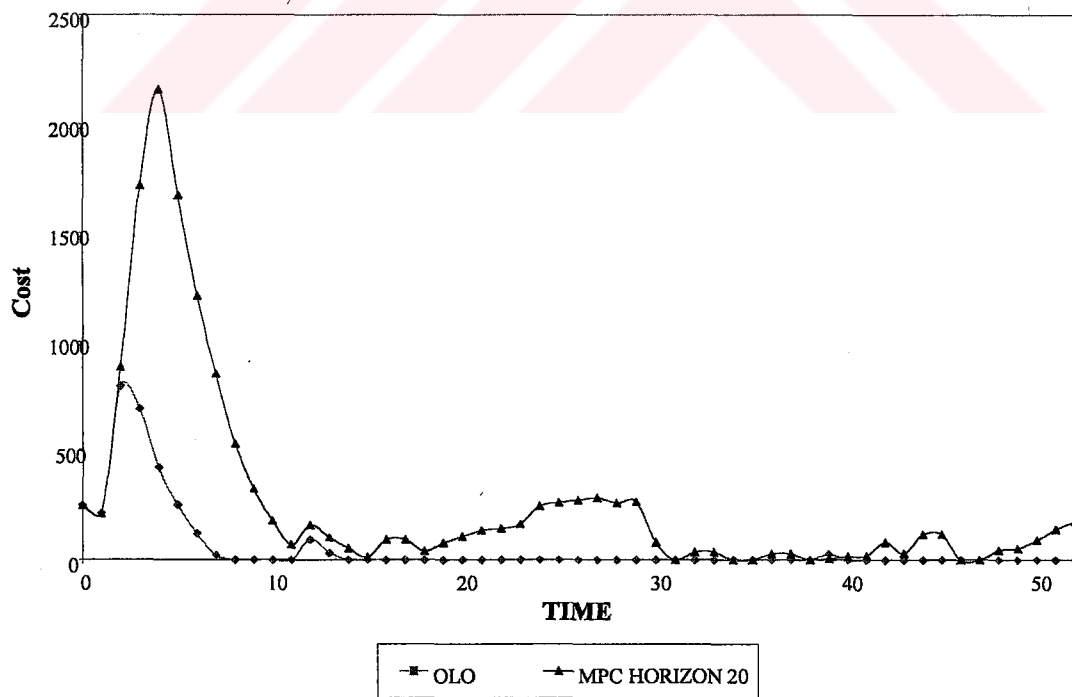


Figure 3.88: Penalty cost paid for product C (MPC vs open loop)

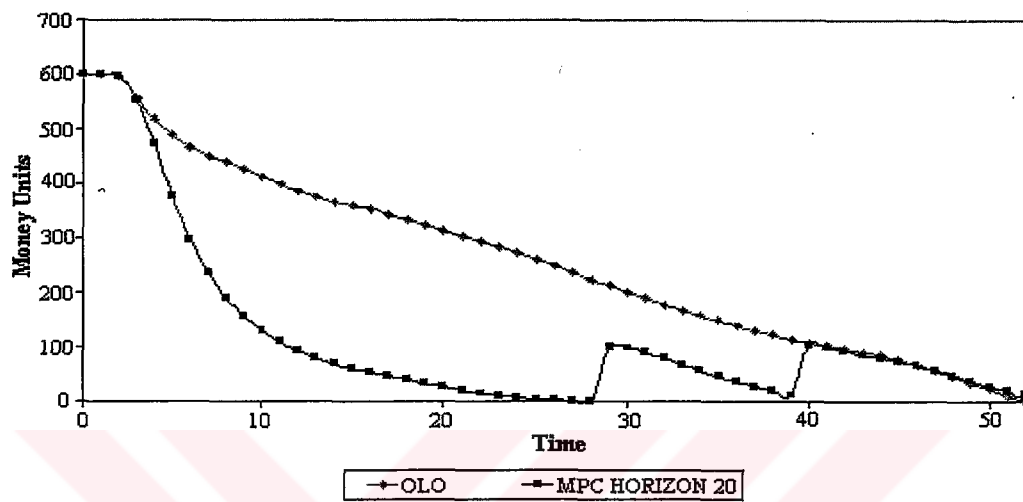


Figure 3.89: Total inventory costs paid for product C (MPC vs open loop)

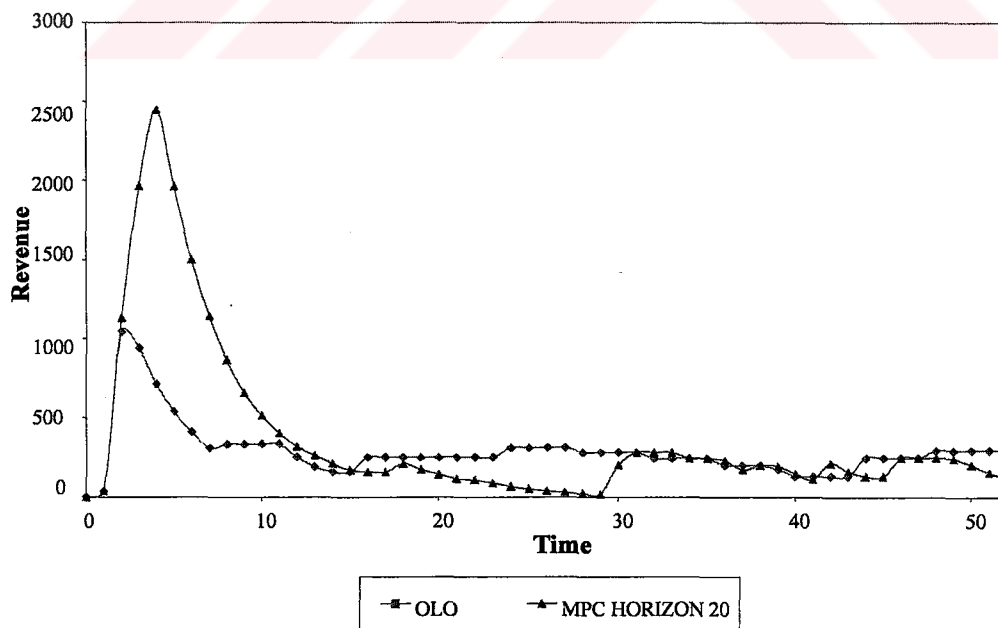


Figure 3.90: Revenue gained from product C (MPC vs open loop)

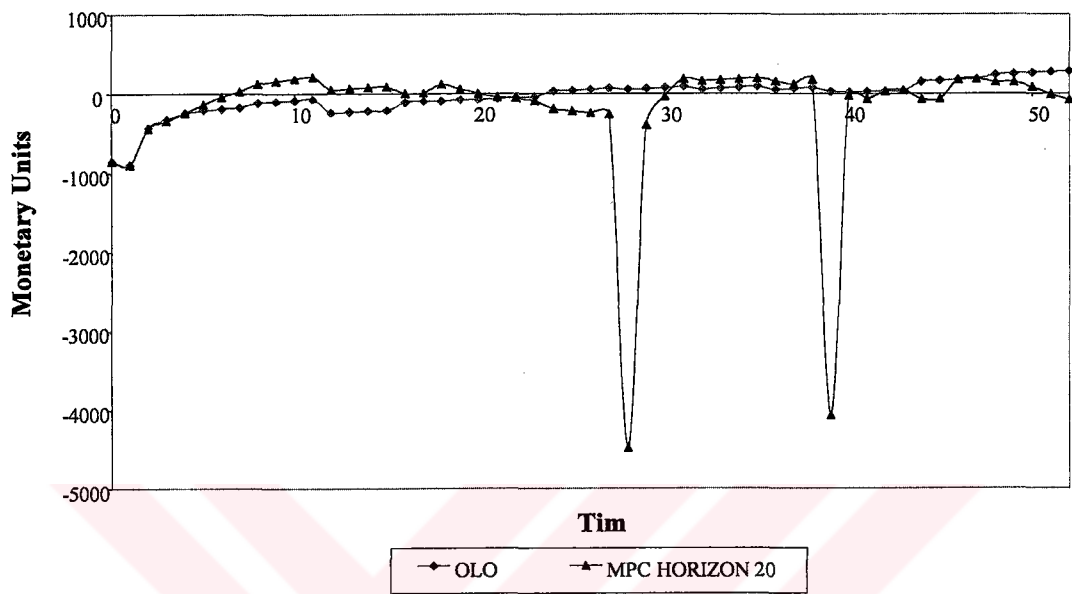


Figure 3.91: Instantaneous objective function for product C(MPC vs open loop)

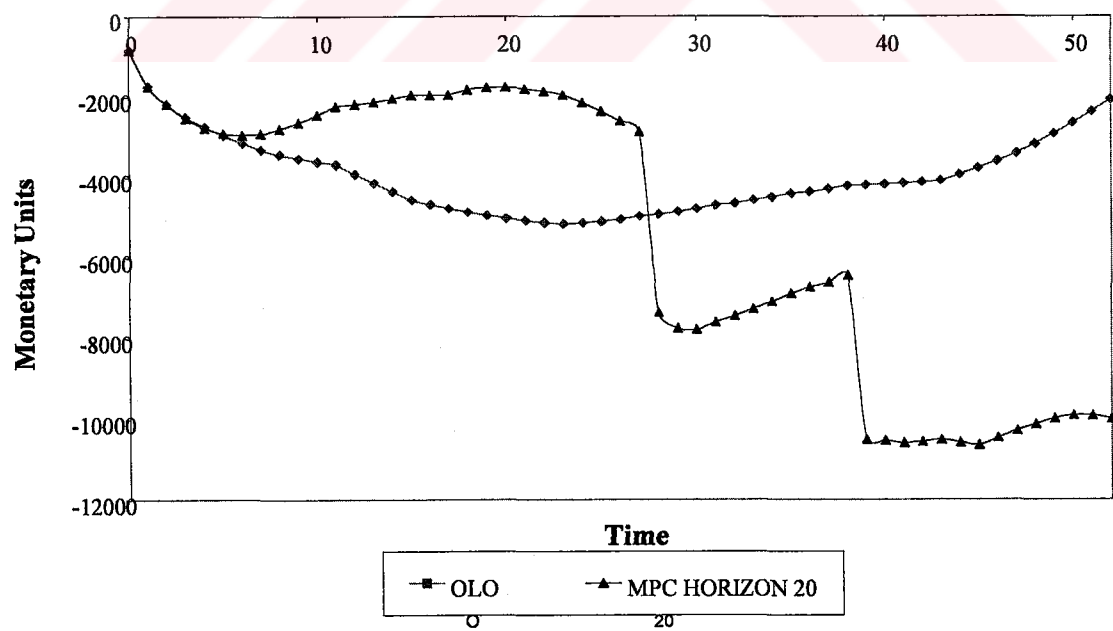


Figure 3.92: Cumulative objective function for product C(MPC vs open loop)

Over all it can be stated that, the deviations in the MPC case largely occurs due to the higher number of production caused by short sighted optimization horizon.

3.4.2 Optimization Horizon: 15 & Optimization Horizon :25:

Once the optimization horizon is decreased further, due to all reasons stated in above case, the system transfers higher material from the retailer level initially, causing higher number of production, resulting in higher inventory costs and higher customer dissatisfaction. However due to the decreased number of decision variables, system operates faster

On the other hand for the longer horizon case, once the optimization horizon is increased, it gets closer to the optimal case, however the demand profile prediction this time gets worse. But as explained before since in the actual demand profile, there are no steep changes with different natures, predicting demand as constant does not give very poor results. Initially due to the longer horizon, it does not reduce its inventories immediately, but also takes production into account. However it still produces higher amounts compared with the open loop case.

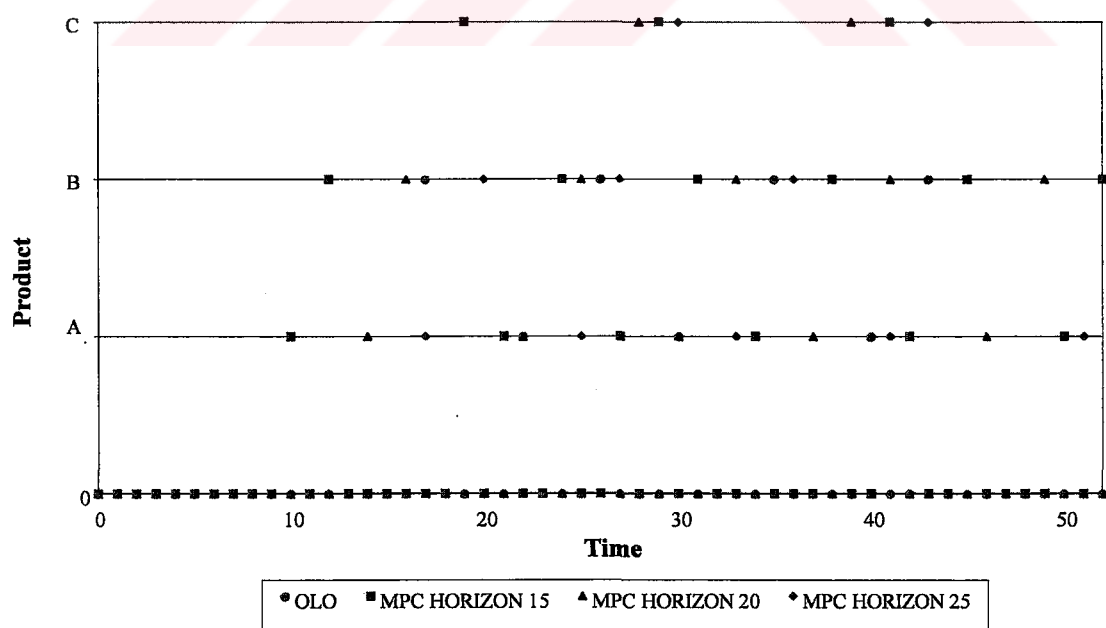


Figure 3.93: Optimal production schedule for the plant (Different MPC horizons)

In the open loop optimization, the optimizer can see the whole actual customer demand and tends to use its inventories first rather than starting to produce. The open loop optimization chooses to produce product A 3 times, B 4 times and chooses not to produce C any time due to its high production costs. However, as the MPC horizon decreases, the MPC optimizer behaves more myopic and tends to decrease its inventory immediately, therefore it needs to produce more in the next optimization sampling times to meet the customer demand. In the MPC with horizon length 15 case, product A is produced 6 times, B 6 times and C 3 times, in MPC with horizon length 20 product A is produced 5 times, B 5 times and C 2 times, in MPC with horizon length 25, A is produced 5 times, B produced 4 times and C is produced 2 times. (Fig. 3.93). However as the horizon length is increased, although the optimizer will act as less myopic, it will never act as the well as open loop optimization case since it still can not see the actual customer demand but approximates it.

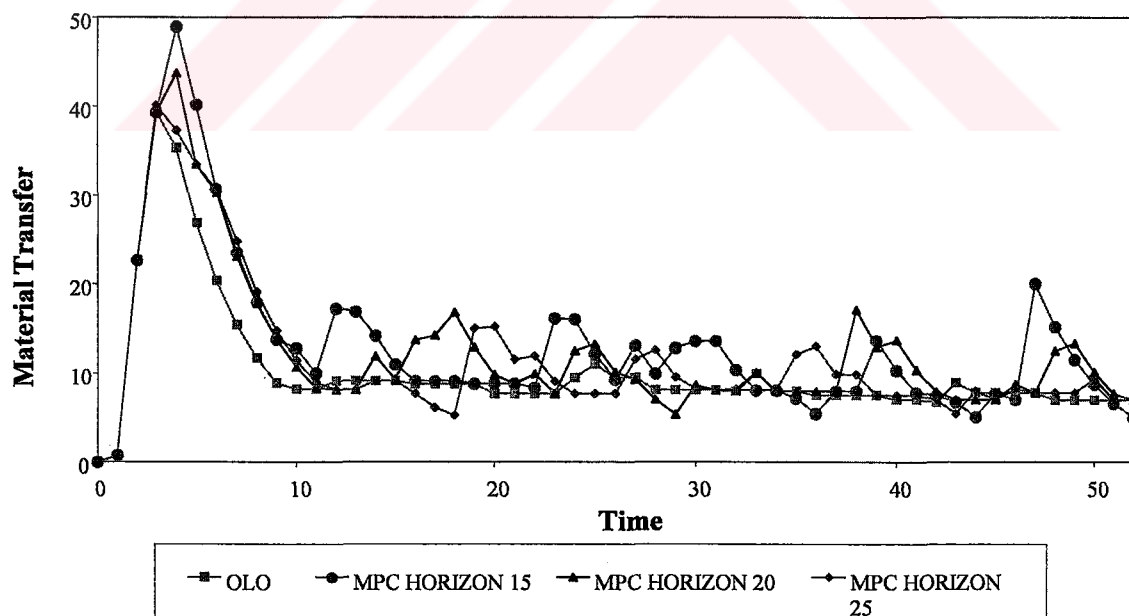


Figure 3.94: Retailer node material transfer level for product A (Different MPC horizons)

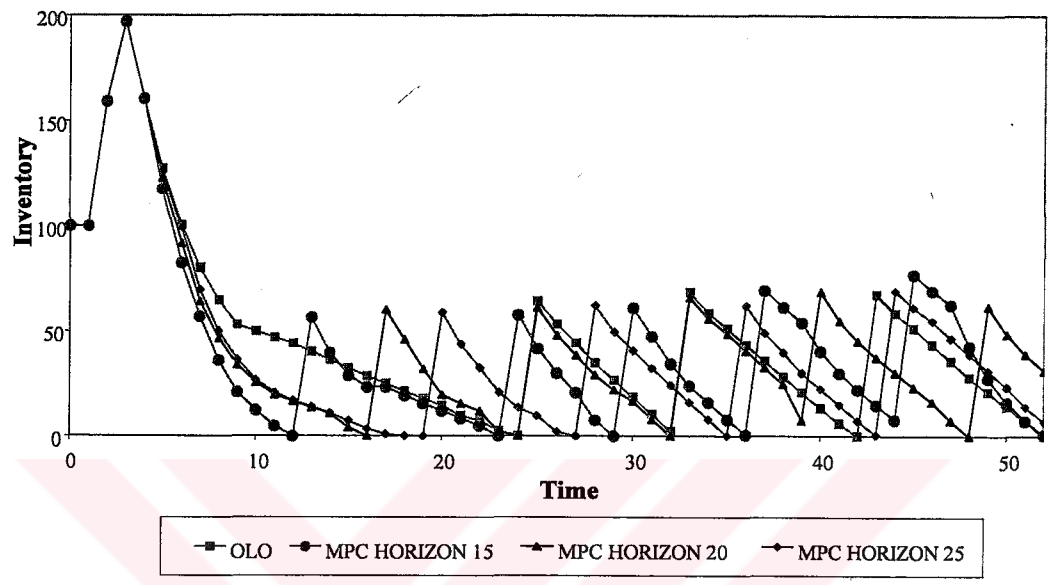


Figure 3.95: Retailer node inventory level for product A (Different MPC horizons)

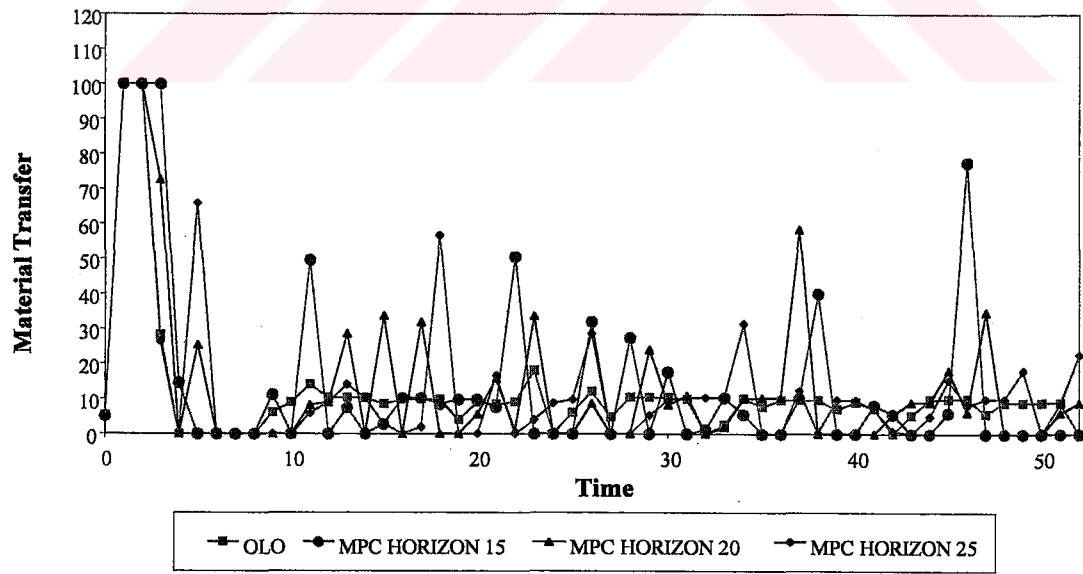


Figure 3.96: Retailer node order levels for product A (Different MPC horizons)

As shown in Fig. 3.94, the initial material transfer is higher in MPC with horizon 15 case due to its myopic nature. Inventory is also depleted faster and order levels are higher to increase the inventory levels in the retailer to meet customer demand, in the MPC with horizon 15 case (Figs. 3.95, 3.96)

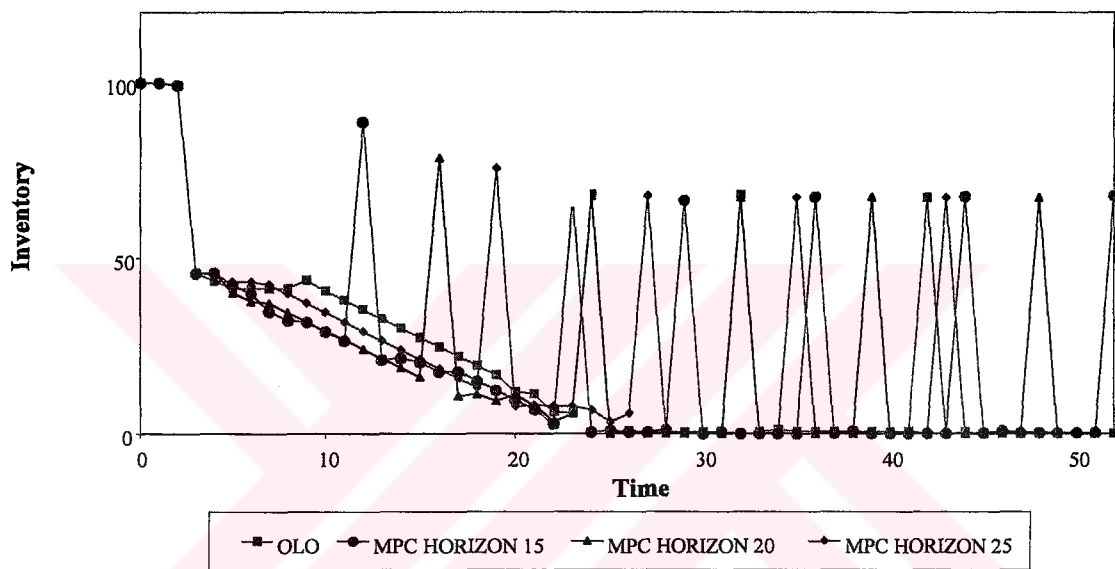


Figure 3.97: Distribution center inventory level for product A (Different MPC horizons)

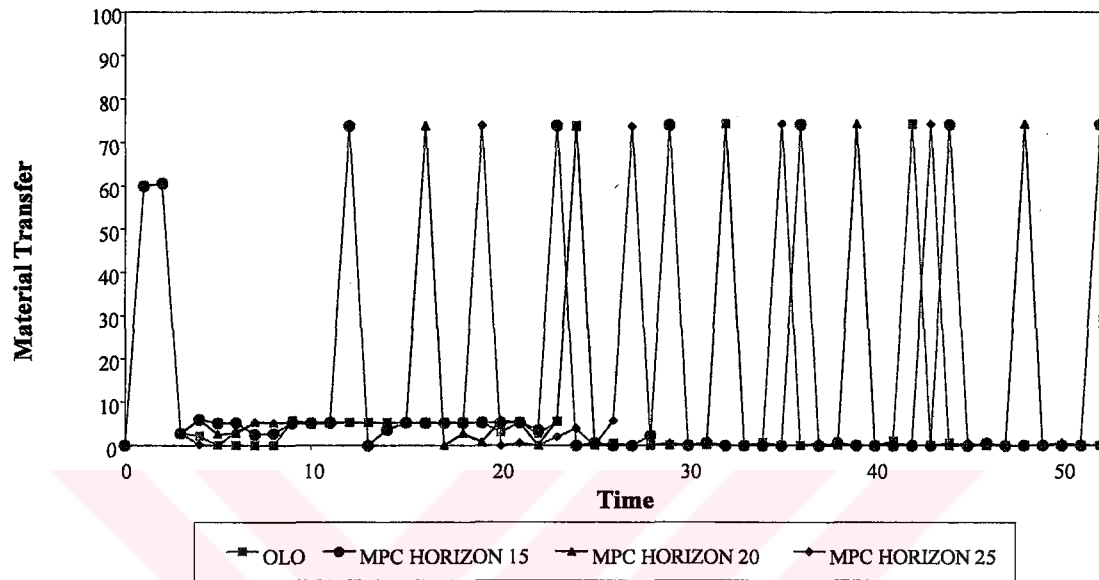


Figure 3.98: Distribution center material transfer level for product A (Different MPC horizons)

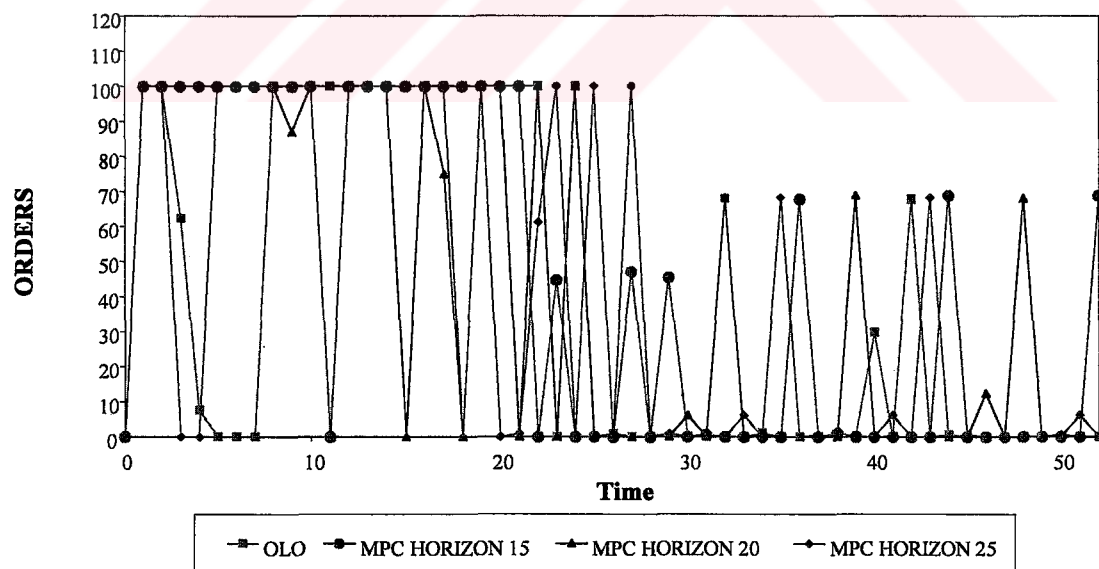


Figure 3.99: Distribution center order levels for product A (Different MPC horizons)

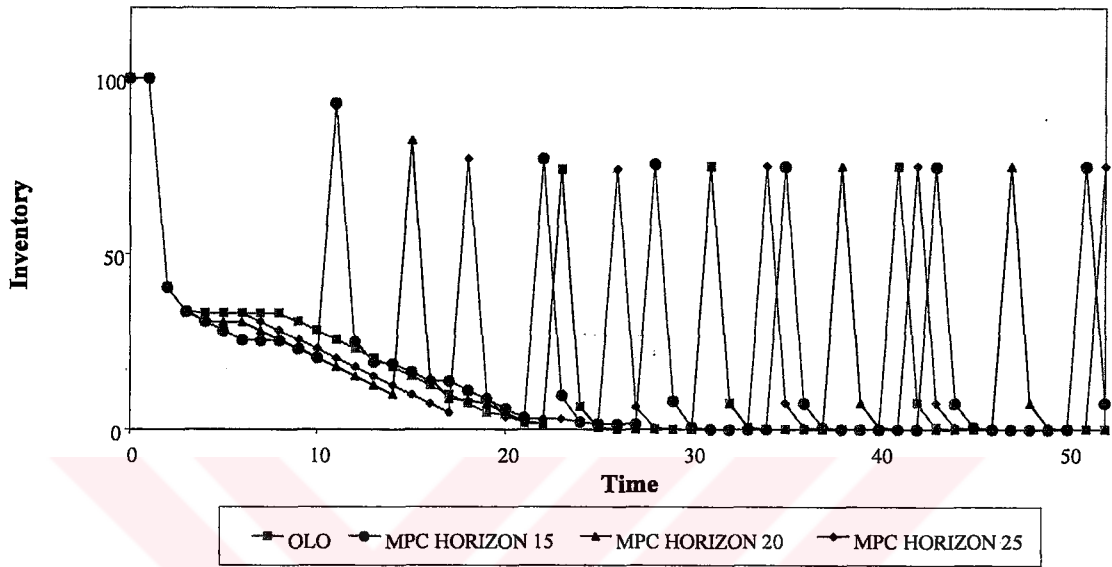


Figure 3.100: Warehouse inventory level for product A (Different MPC horizons)

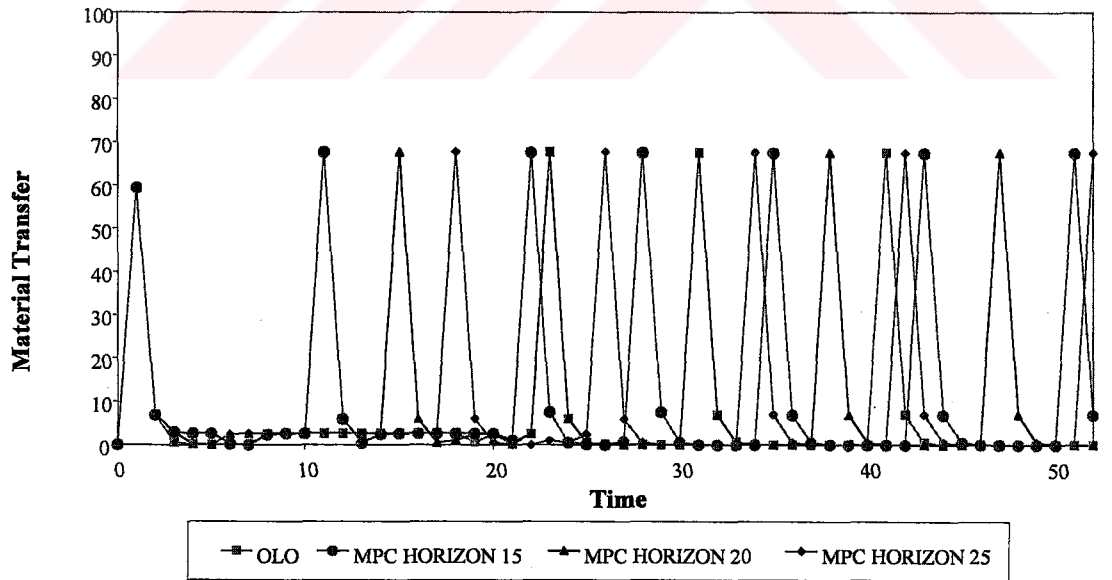


Figure 3.101: Warehouse material transfer level for product A (Different MPC horizons)

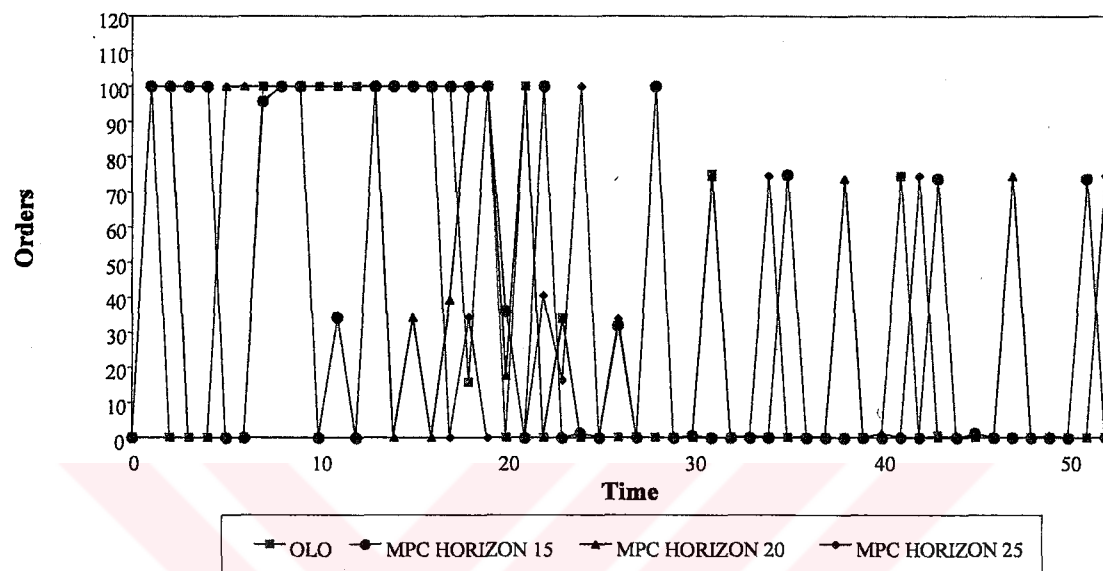


Figure 3.102: Warehouse order levels for product A (Different MPC horizons)

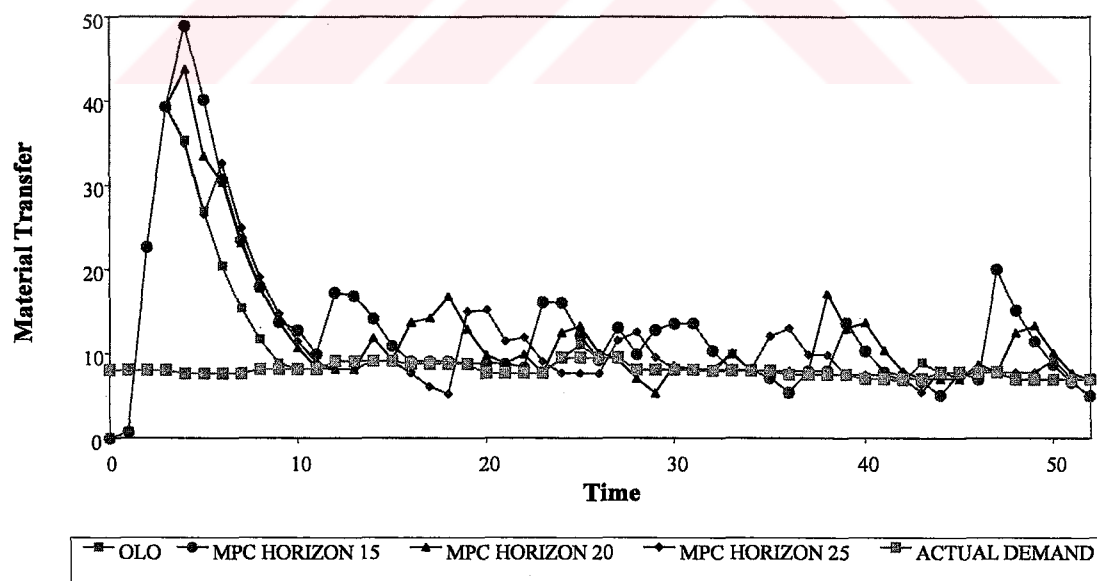


Figure 3.103: Material transferred to customer vs actual customer demand for product A (Different MPC horizons)

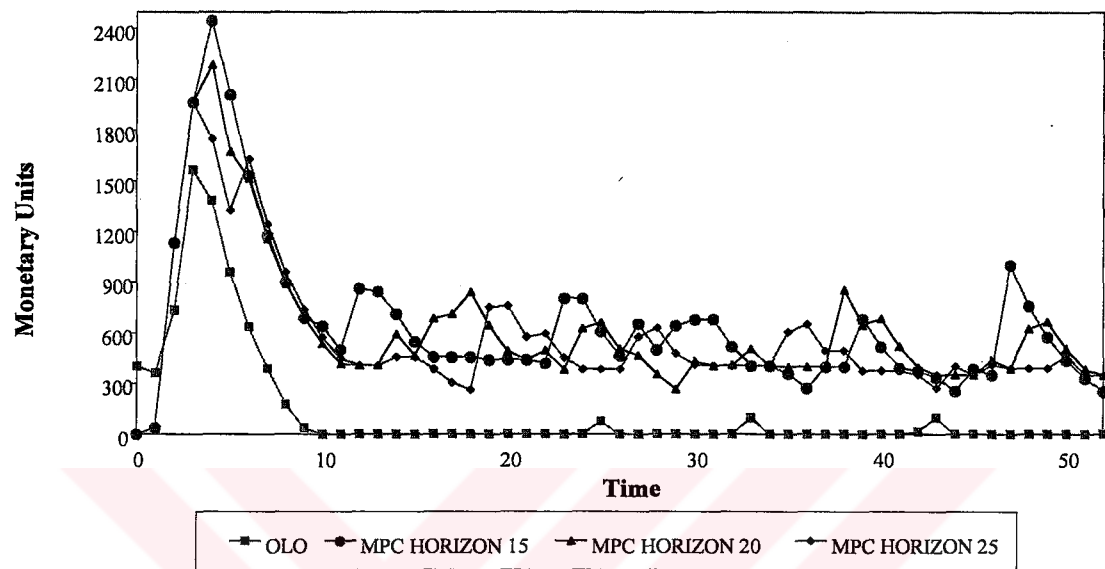


Figure 3.104: Penalty cost paid for product A (Different MPC horizons)

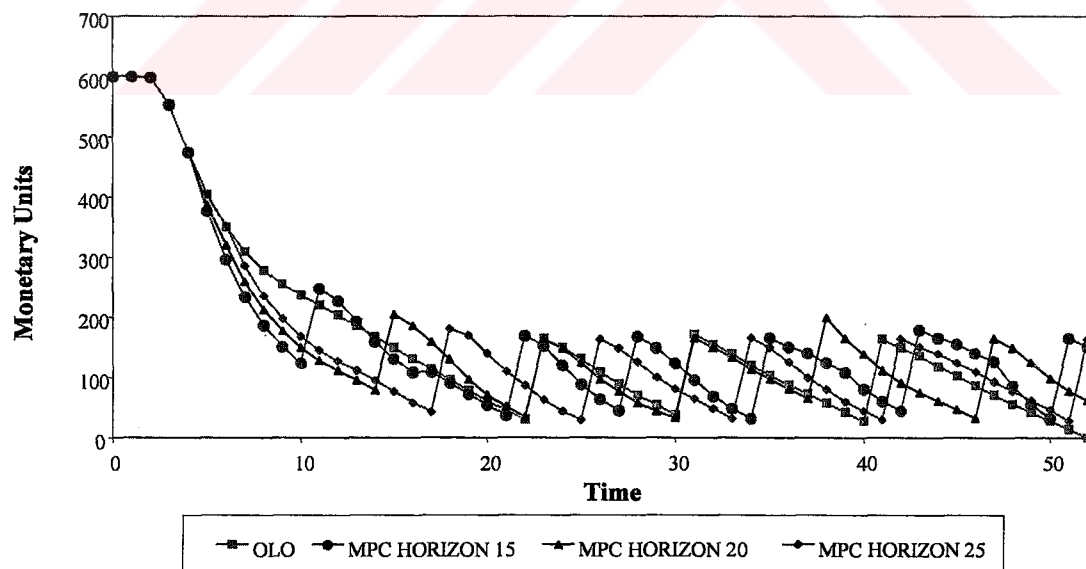


Figure 3.105: Total inventory costs paid for product A (Different MPC horizons)

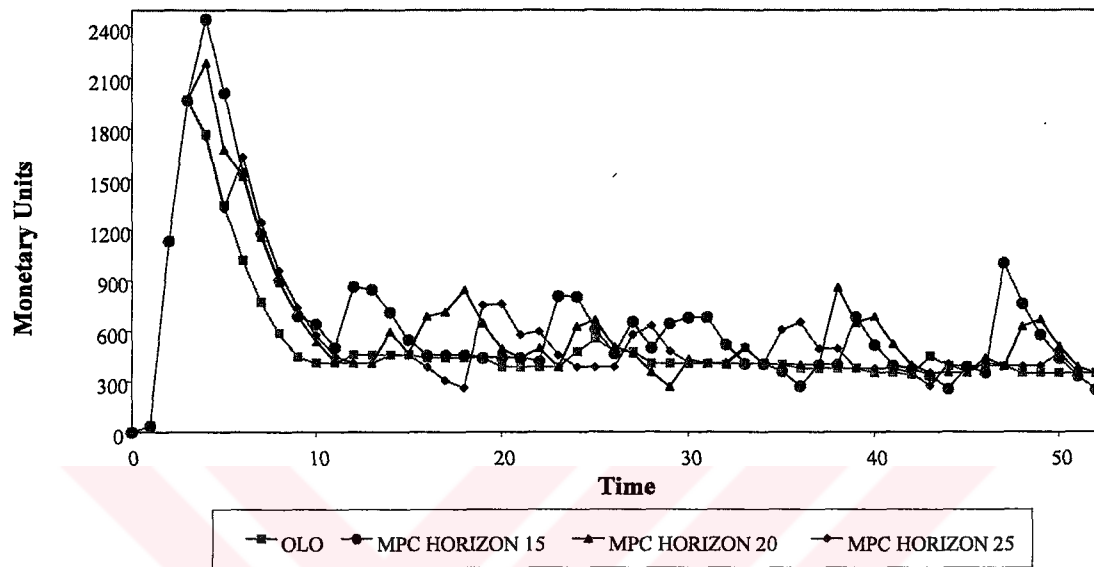


Figure 3.106: Revenue gained from product A (Different MPC horizons)

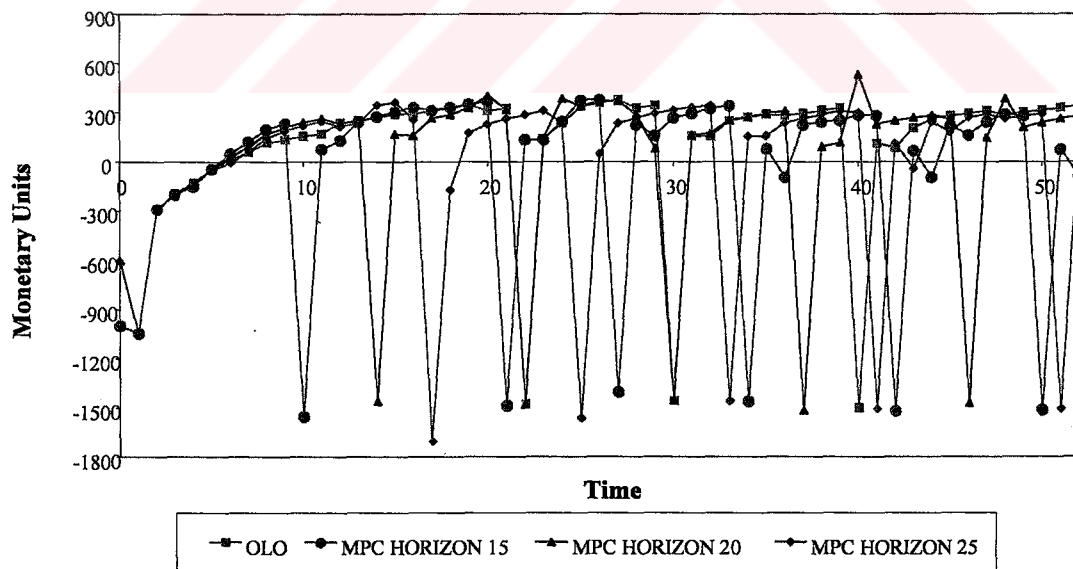


Figure 3.107: Instantaneous objective function for product A (Different MPC horizons)

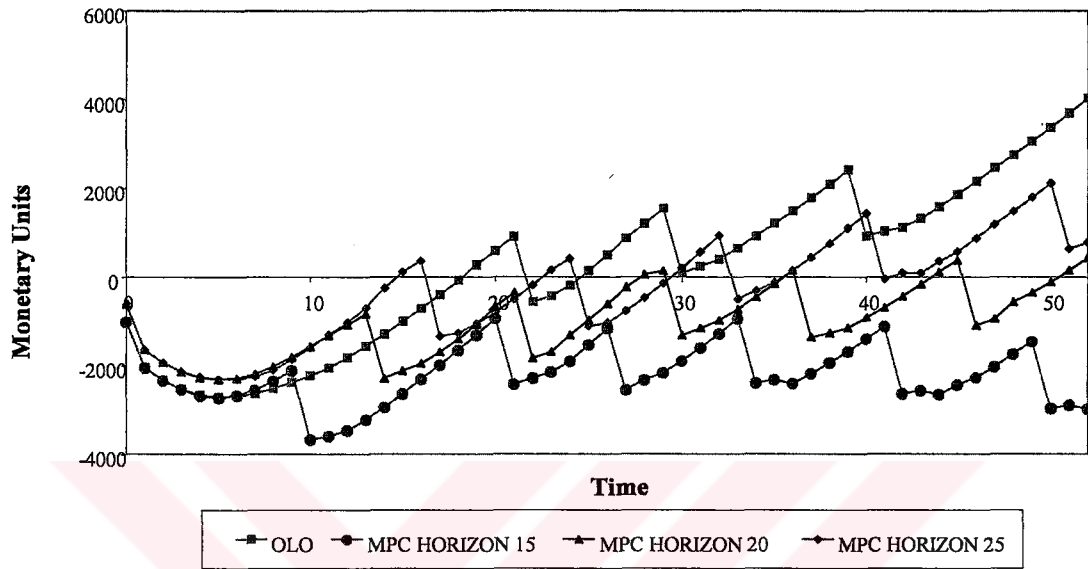


Figure 3.108: Cumulative objective function for product A (Different MPC horizons)

Same trends are also observed for product *B* and product *C*. One of the most interesting results is observed with product *C*. As explained before since open loop optimizer sees a longer horizon it chooses not to produce *C*. However, all MPC cases decrease their inventories without seeing the entire horizon, therefore need to produce *C*.

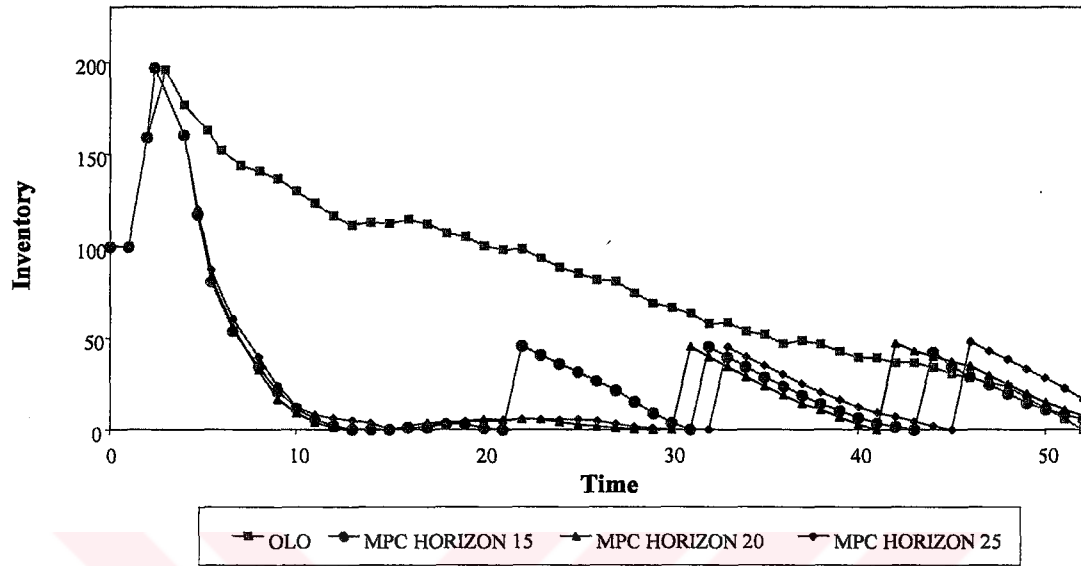


Figure 3.109: Retailer inventory level for product C (Different MPC horizons)

As shown in single product case, as the MPC optimization horizon length increases, the solution time of the optimization problem increases. However when the demand profile does not deviate much from the predicted model, as in this case, longer optimization horizon approaches to the open loop optimization result. A trade off must be made between the improved results and reasonable solution times. MPC horizon lengths, objective function values and CPU times in seconds are given in table 3.3.

Table 3.3: MPC Horizon Lengths, Objective Function values and CPU times

HORIZON LENGTH	OBJECTIVE FCT VALUES	CPU TIMES (sec)	NUMBER OF CONSTRAINTS	NUMBER OF VARIABLES
25	2599	8639	39598	21151
20	-3773	1150	33358	17875
15	-9696	105	23998	12961
52 (OLO)	12215	18035	5203	2863

3.5 Modeling of a Three-Product Hybrid Supply Chain with Variable Production Size

3.5.1 Open loop optimization

As in the batch production case, the manufacturing plant produces three different products, A , B or C , based on the optimal manufacturing schedule generated by the centralized optimizer. However, this time size of the production is also a decision variable. The plant has the perfectly flexible manufacturing capabilities, that there are no lower or upper limits on the production size of the plant. The aim is to compare a perfectly flexible plant with a batch production plant which is more common in real life. As before, each product has a different production time and the plant can only manufacture one type of product, in a specific production time. The binary variable $p_m(t)$ is the production variable for product m , 1 if the plant produces product j at time t and 0 otherwise, and $yp_m(t)$ is the size of production for product m at time t .

The framework of Mixed Logic Dynamic (MLD) systems, and generalized disjunctive programming approach explained in previous section will be used again to model the variable production size plant operations.

$$\begin{bmatrix} p_{mt} \\ yp_{mt} = s_{mt} \\ pc_{mt} = (f_m \cdot p_m + v_m \cdot yp_{mt}) \end{bmatrix} \vee \begin{bmatrix} \neg p_{mt} \\ yp_{mt} = 0 \\ pc_{mt} = 0 \end{bmatrix} \quad \forall m, t \quad (3.24)$$

Above statement is similar to equation 3.6, yet it has the variable production size, s_{mt} , equal to the production amount yp_{mt} .

As before, production duration for product A is two, production duration for B is three and for C one. Therefore the system must satisfy the logical conditions and their equations given in by equations 3.7 to 3.12.

The same objective function given in Eqn. 3.13 is to be maximized with the same demand patterns given in Fig. 3.2.

The system is coded into OPL Studio software. Dynamic optimization was performed for a 52 week horizon and optimal total profit came out to be 15367 monetary units, which is higher than the batch production case, due to the flexibility of the system.

If only product A is studied, as before, at the beginning the system favors to transfer all material without producing or without taking customer satisfaction into account, since the

revenue of delivering A is very high. However, this time the system does not decrease the retailer inventory as fast as the batch production case (Fig. 3.110).

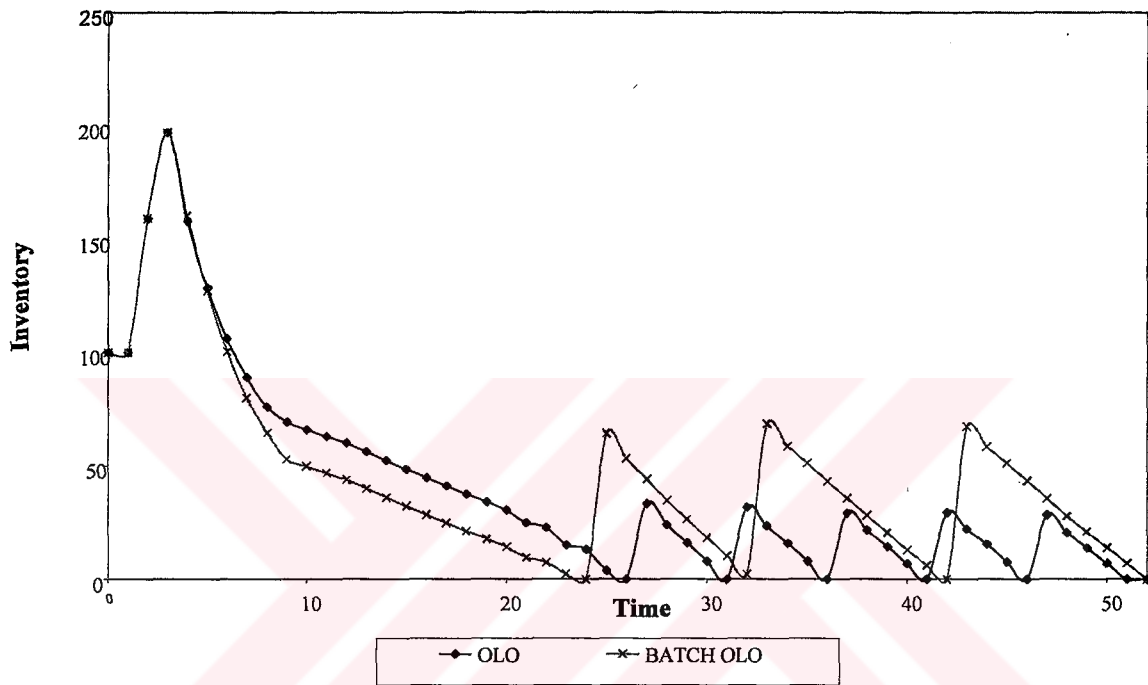


Figure 3.110: Retailer node inventory level for product A (olo vs batch olo)

At the beginning it transfers relatively low amounts compared to the batch case (Fig. 3.111), hence it decreases the customer dissatisfaction cost. (Fig. 3.112). The system produces more frequently than the batch case, however in smaller production sizes. In a way the system benefits from low fixed production costs while avoiding large variable costs. Overall production cost incurred for the batch process is 5454 monetary units for the batch production case, while the production cost for the variable production is 4953 monetary units. Total amount of production in the process horizon is 225 units of product A for batch case and 201 units for variable production (Fig. 3.113). Total inventory cost at retailer level is 4845 for the batch case and 4350 the variable production case which means that although at the beginning the variable production case does not decrease its inventory as fast as the batch case and pays a higher inventory cost, in time it produces fewer product and stores fewer, although more frequently.

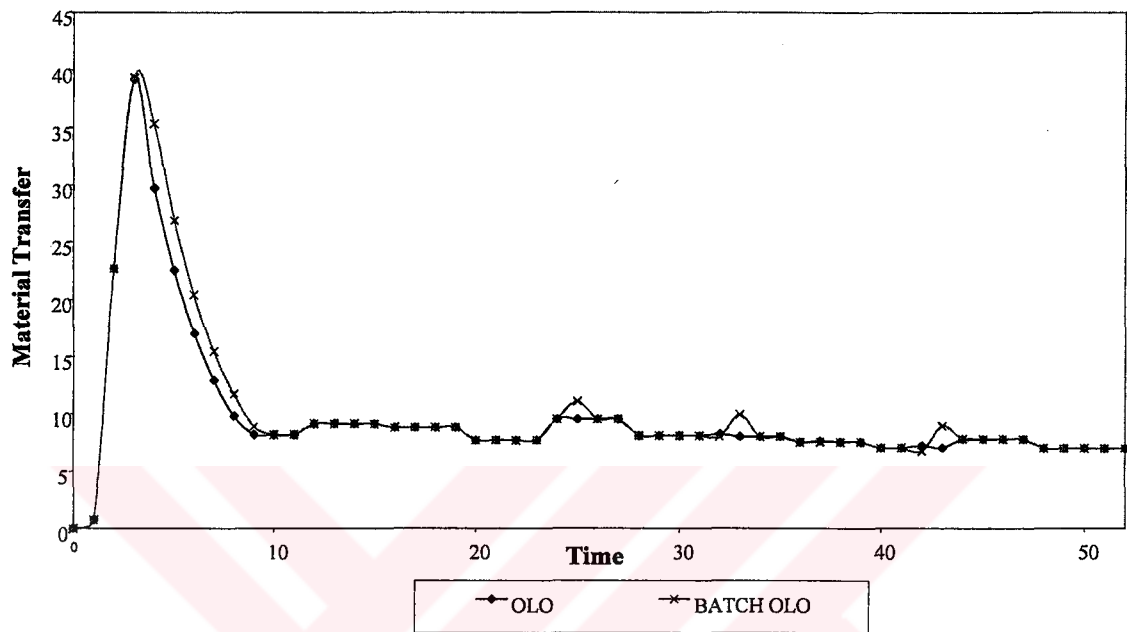


Figure 3.111: Retailer node material transfer level for product A (olo vs batch olo)

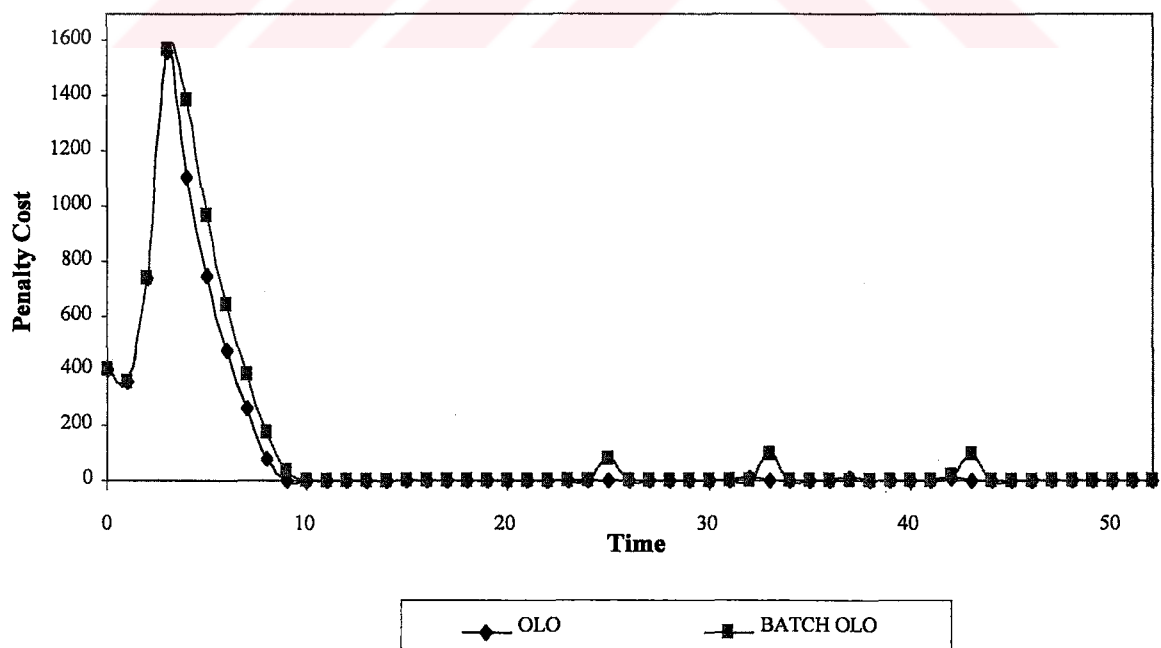


Figure 3.112: Customer dissatisfaction cost paid for product A (olo vs batch olo)



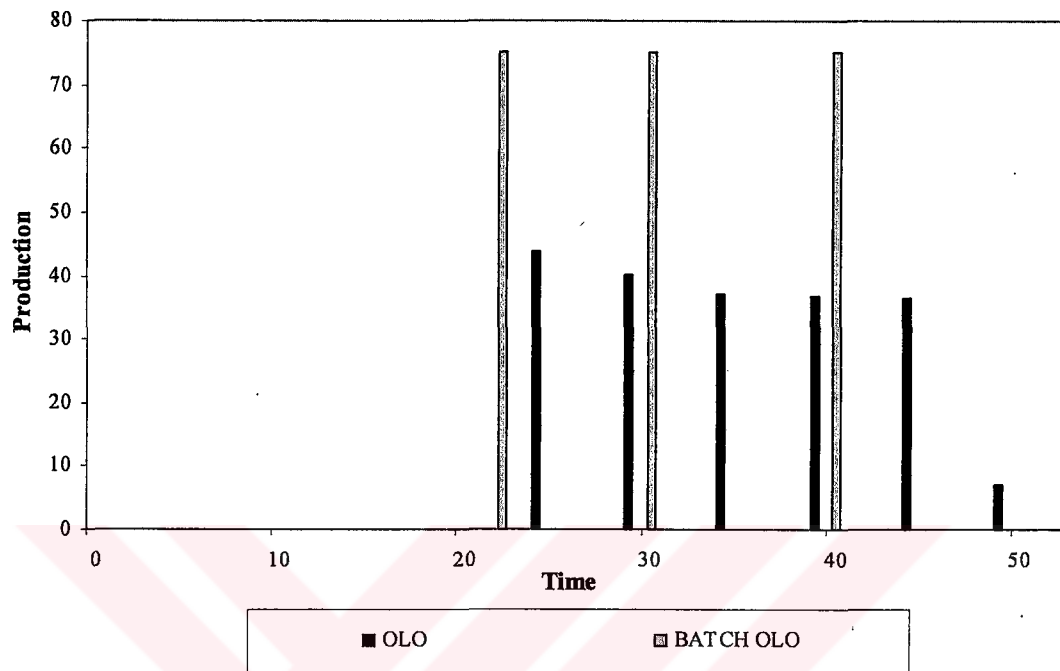


Figure 3.113: Production amounts for product A (olo vs batch olo)

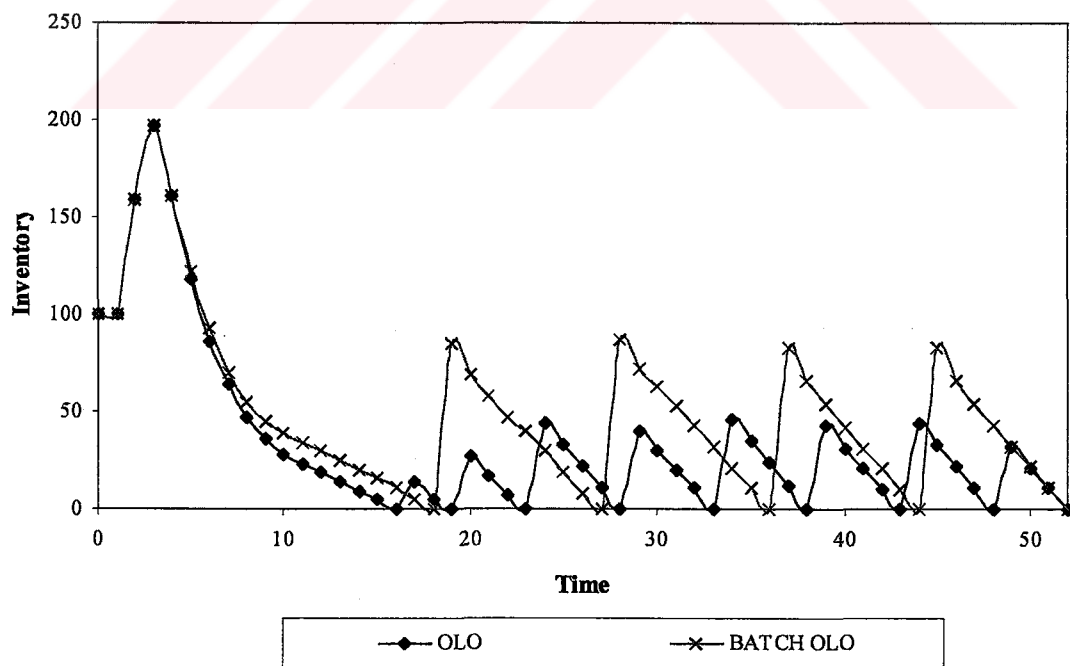


Figure 3.114: Retailer node inventory level for product B (olo vs batch olo)

For product *B* whose production costs are very low compared to products *A* and *C*, (table 2), the system decreases its inventory faster than the batch production case and chooses to meet the customer demand by directly producing.

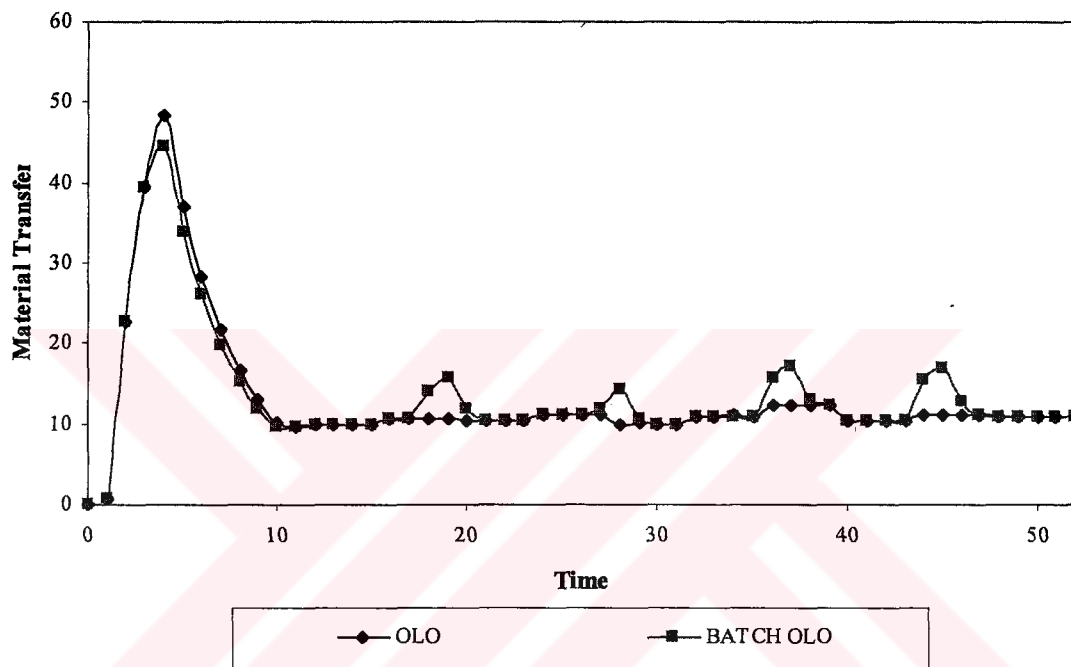


Figure 3.115: Retailer node material transfer level for product B (olo vs batch olo)

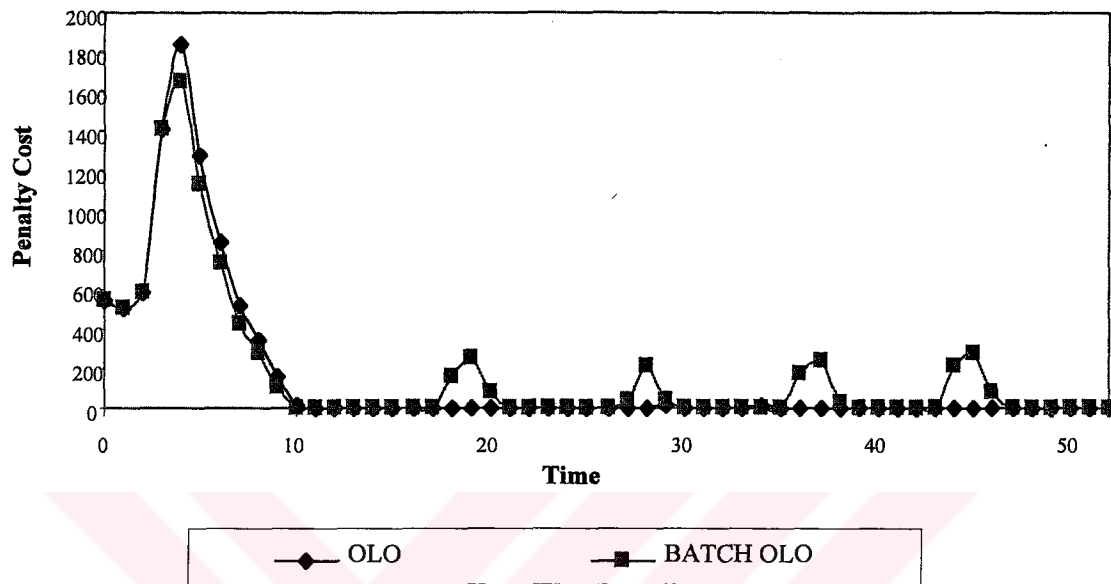


Figure 3.116: Customer dissatisfaction cost paid for product B (olo vs batch olo)

In the beginning it transfers relatively higher amounts compared to the batch case, different from product *A* (Fig. 3.115), hence it increases the customer dissatisfaction cost slightly. (Fig. 3.116). However although the initial customer dissatisfaction is higher in variable production case, the overall penalty paid is 8107 monetary units while it is 9217 monetary units for the batch production case. In other words although the system sells more than what the customer wants in the beginning, in time it stores less in its inventory by producing less and therefore it does not need to transfer high amounts of material to customer to deplete its inventory. As for product *A*, the system produces more frequently than the batch case, however in smaller production sizes. Overall production cost incurred for the batch process is 4824 monetary units for the batch production case, while the production cost for the variable production is 4533 monetary units. Total amount of production in the process horizon is 400 units of product B for batch case and 378 units for variable production (Fig. 3.117).

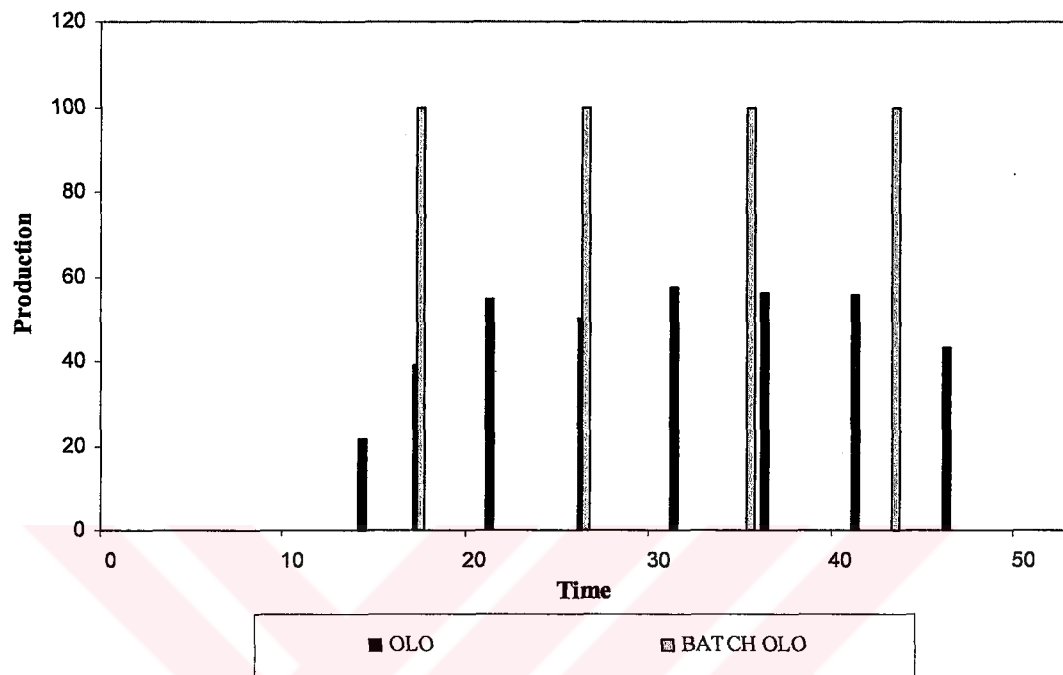


Figure 3.117: Production amounts for product B (olo vs batch olo)

Total inventory cost at retailer level is 5271 for the batch case and 3687 the variable production case. This high difference in the inventory costs are expectable since at the beginning inventory depletion is higher in variable case and moreover total production is less ensuring that even less inventory is held in retailer.

Although the batch production size of 50 is relaxed for product *C* in variable production size case, the system does not choose to produce any *C* because of its high production costs. The optimal inventory and material transfer profiles are therefore the same as the batch case (Figs. 3.118, 3.119).

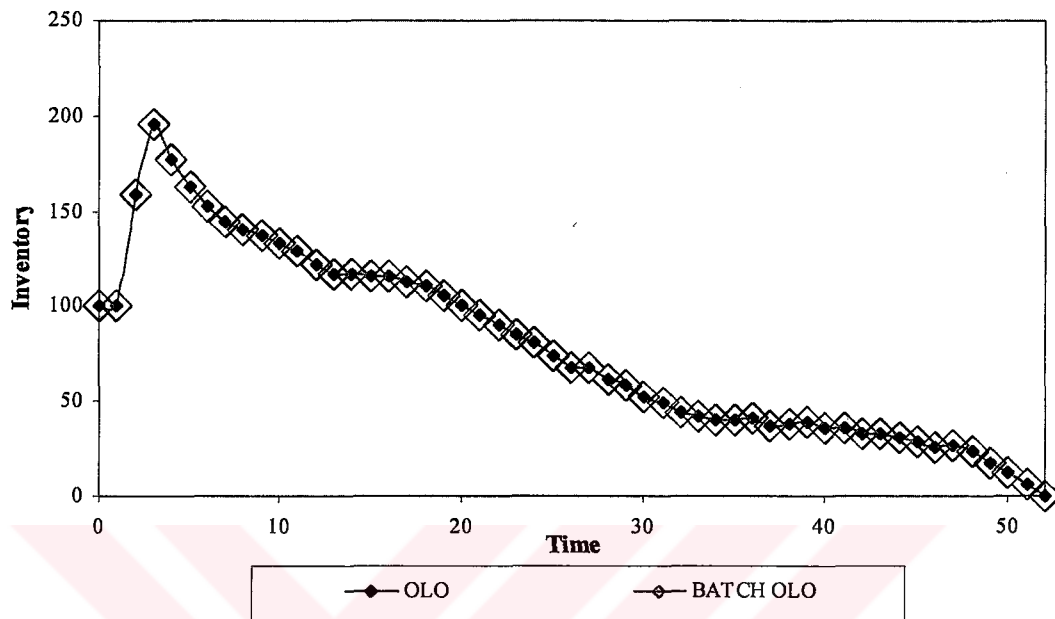


Figure 3.118: Retailer node inventory level for product C (olo vs batch olo)

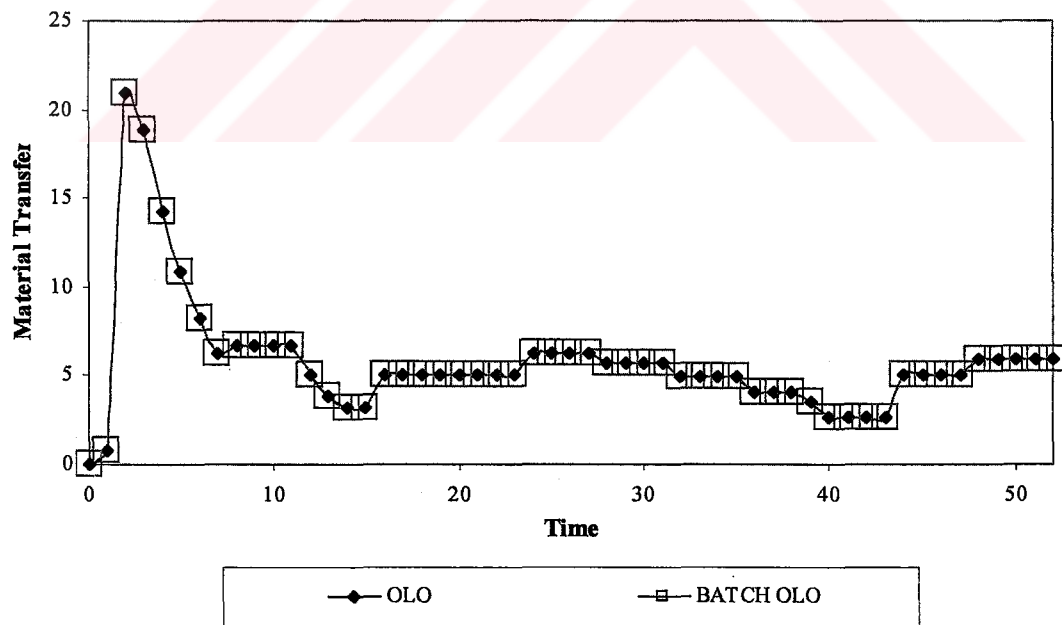


Figure 3.119: Retailer node material transfer level for product C (olo vs batch olo)

3.5.2 Model Predictive Control

MPC with horizon length 15 and 20 are performed for optimization of this particular supply chain system with perfectly flexible manufacturing capabilities to observe the effects of MPC optimization horizon length on the optimal results. MPC application on the supply chain is performed as explained in section 3.4.

As in batch production case, as the horizon length decreases, MPC takes short-sighted decisions and allocates its resources i.e., inventories in earlier periods (Fig. 3.120). As a result to meet the customer demand, production frequency and production amounts increase. (Fig. 3.121)

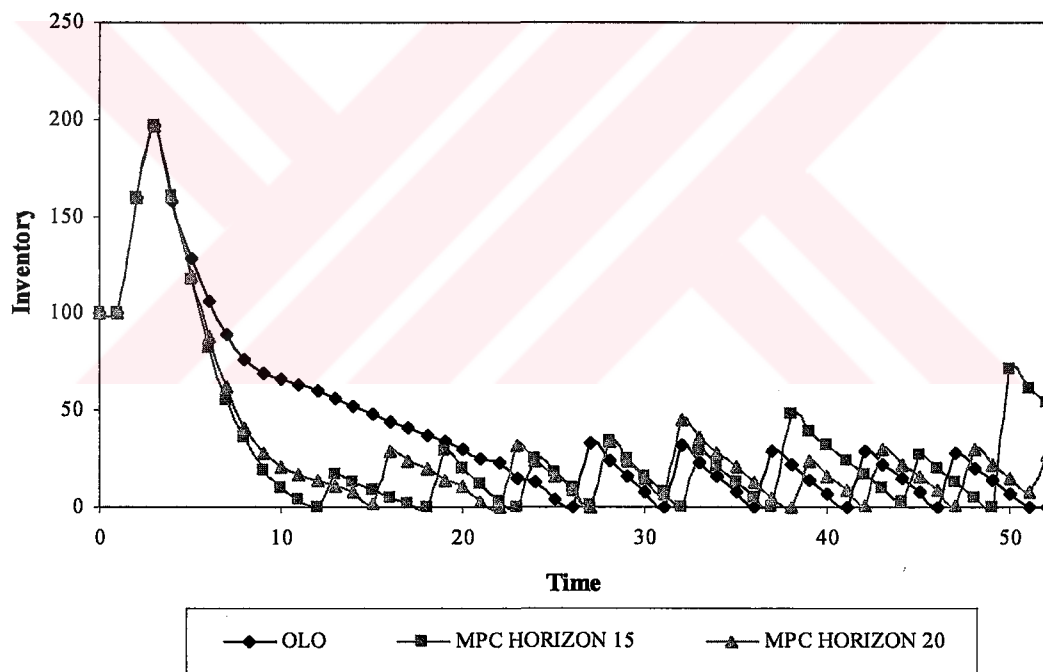


Figure 3.120: Retailer inventory level for product A (MPC vs open loop)

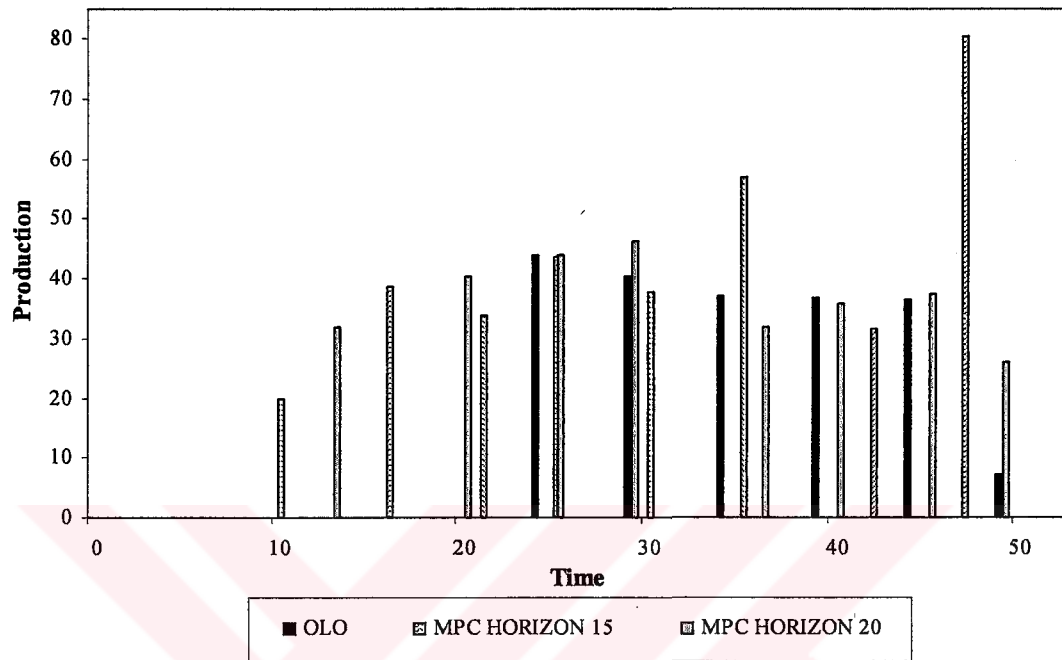


Figure 3.121: Optimal production schedule of the plant for product A (MPC vs open loop)

In the open loop case the plant produces product A six times with a total amount of 201 units. MPC with horizon length 15 produces eight times with total amount 342 units and MPC with horizon length 20 produces eight times with total amount of 294 units. Overall even though MPC with shorter horizon depletes its inventory earlier and attain a lower inventory holding cost and high sales revenues, high customer dissatisfaction penalty decreases the total profit.

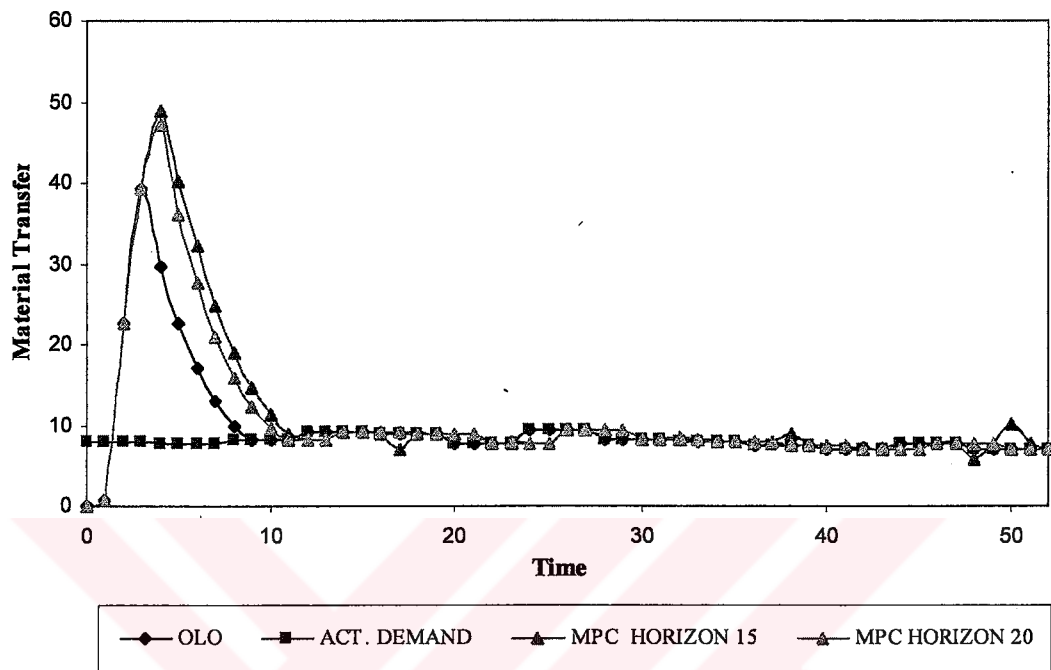


Figure 3.122: Material transferred to customer vs actual cost. demand for product A

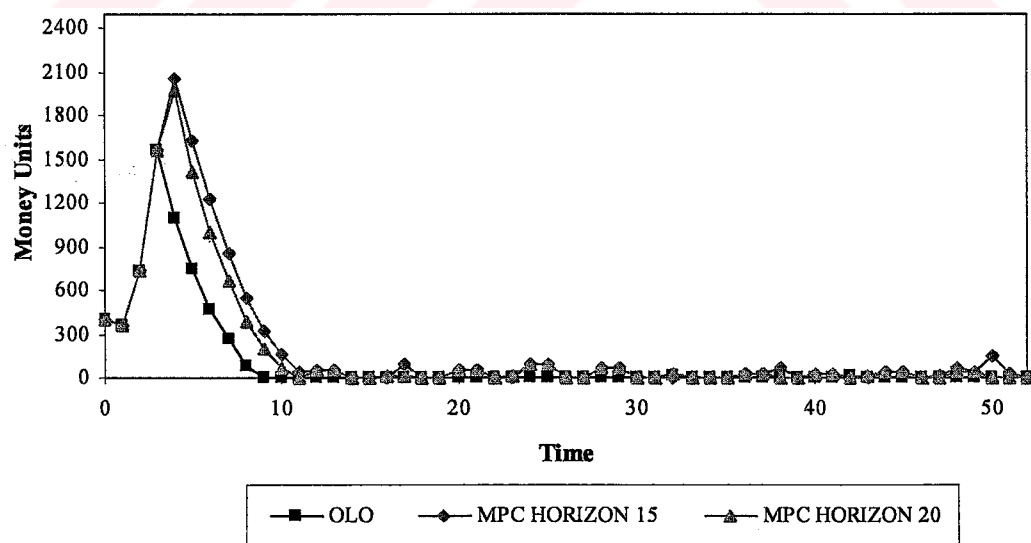


Figure 3.123: Penalty cost paid for product A (MPC vs open loop)

For example even though MPC with horizon length 15 attains the smallest inventory holding cost compared with MPC horizon length 20 and open loop optimization with a value of 7568 monetary units it has the highest customer satisfaction penalty paid with a value of 11091. Where MPC with horizon length has a total inventory holding cost of 7604 monetary units and customer dissatisfaction penalty of 9661 and OLO has a total inventory cost of 8425 monetary units and the smallest penalty of 5742 monetary units. (Figs. 3.122, 3.123)

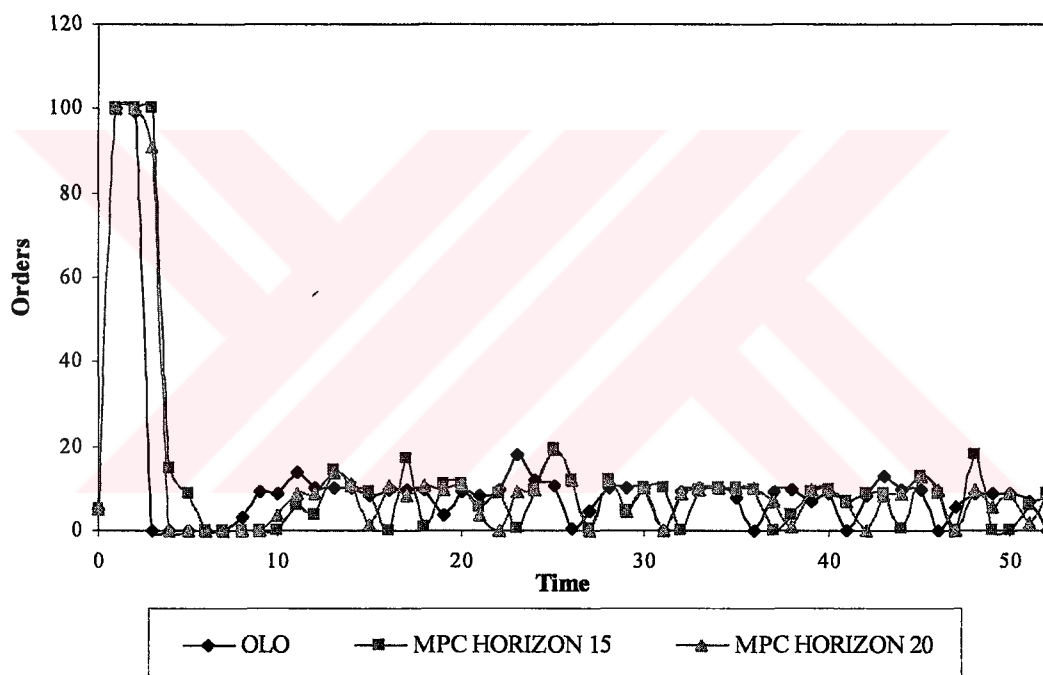


Figure 3.124: Retailer order levels for product A (MPC vs open loop)

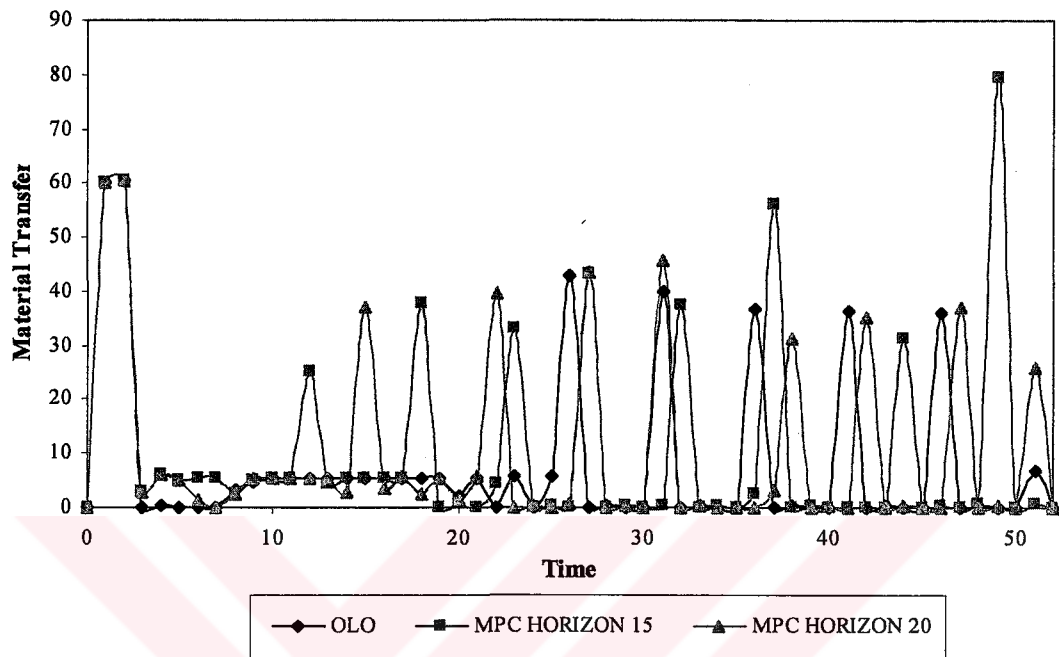


Figure 3.125: Distribution center material transfer level for product A (MPC vs open loop)

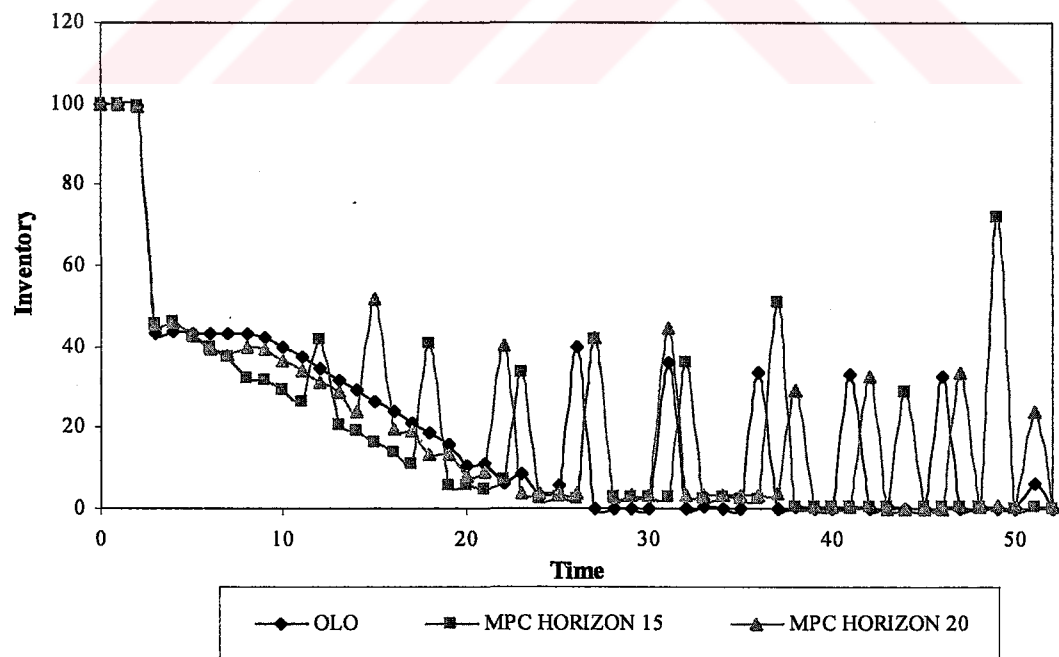


Figure 3.126: Distribution center inventory level for product A (MPC vs open loop)

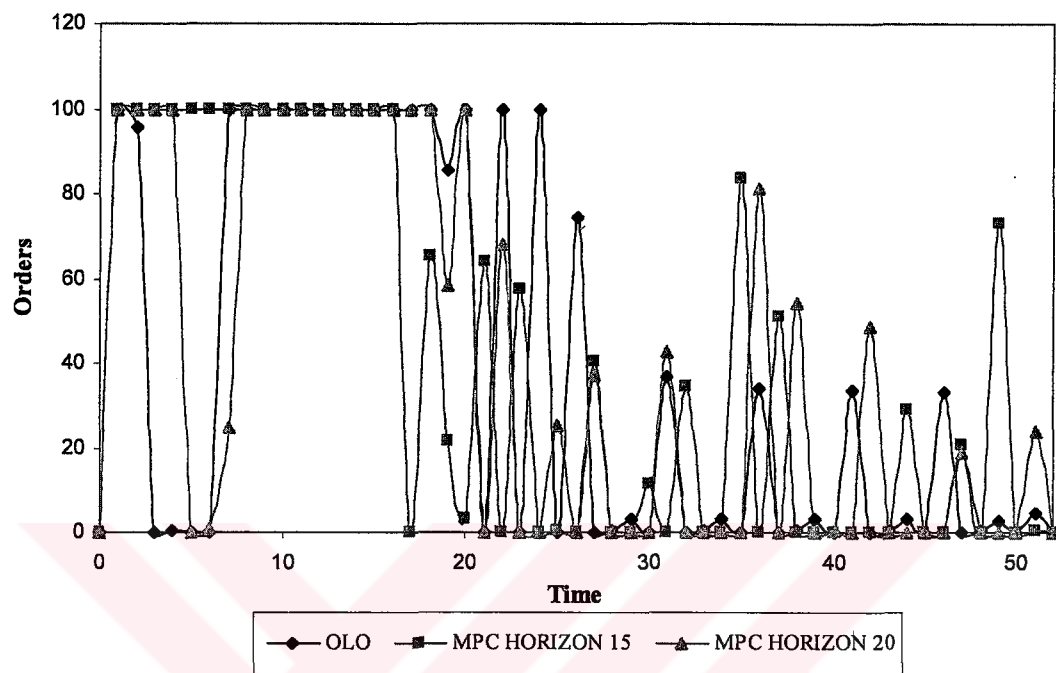


Figure 3.127: Distribution center order levels for product A (MPC vs open loop)

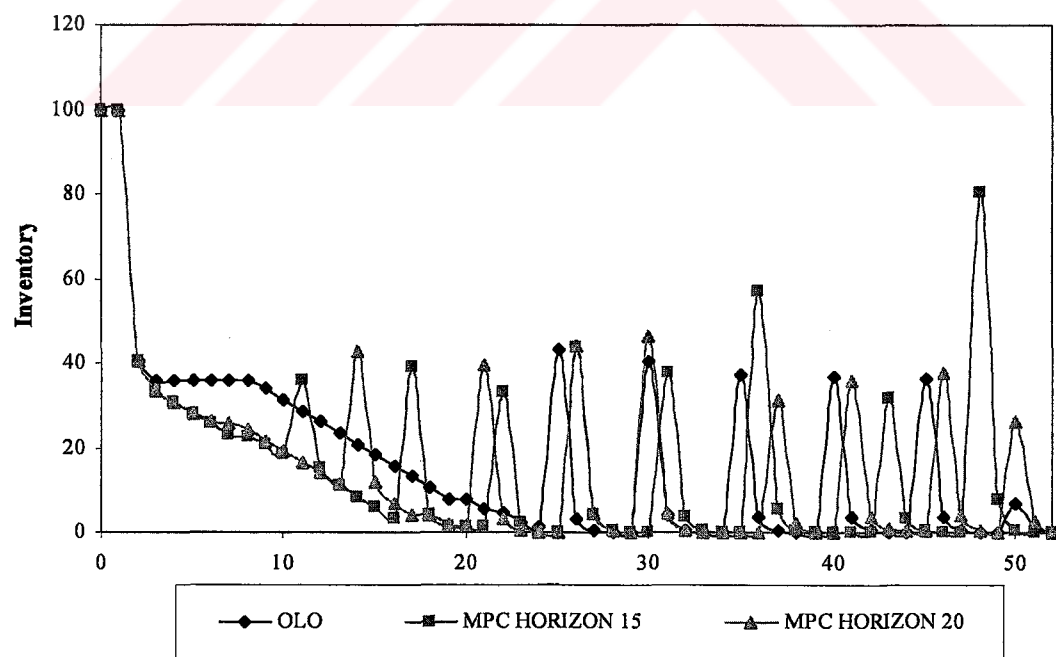


Figure 3.128: Warehouse inventory level for product A (MPC vs open loop)

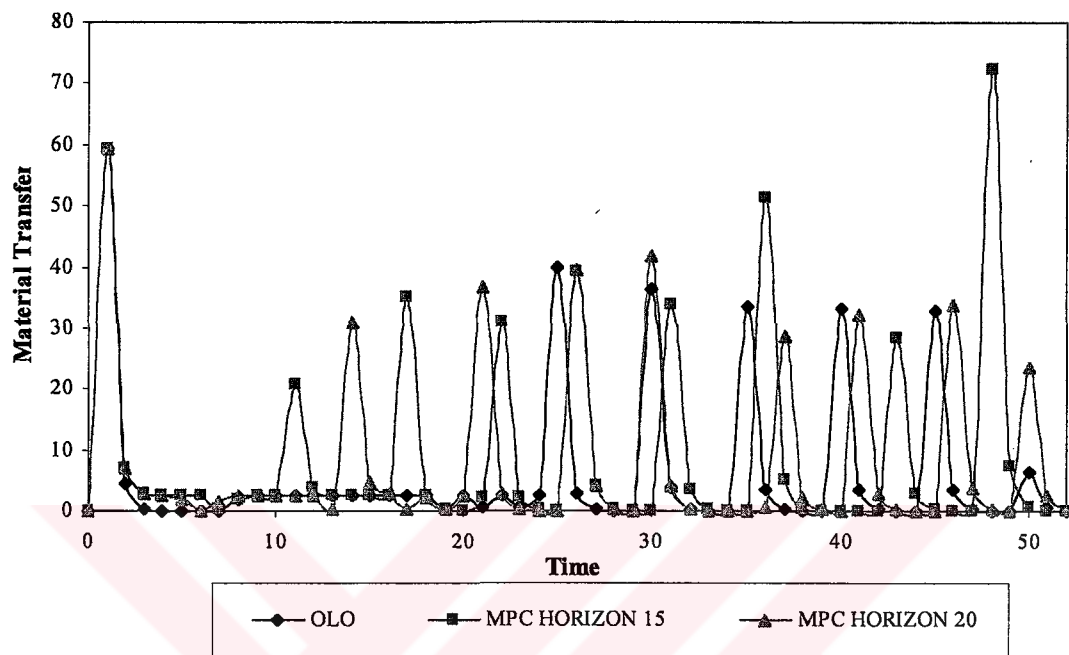


Figure 3.129: Warehouse material transfer level for product A (MPC vs open loop)

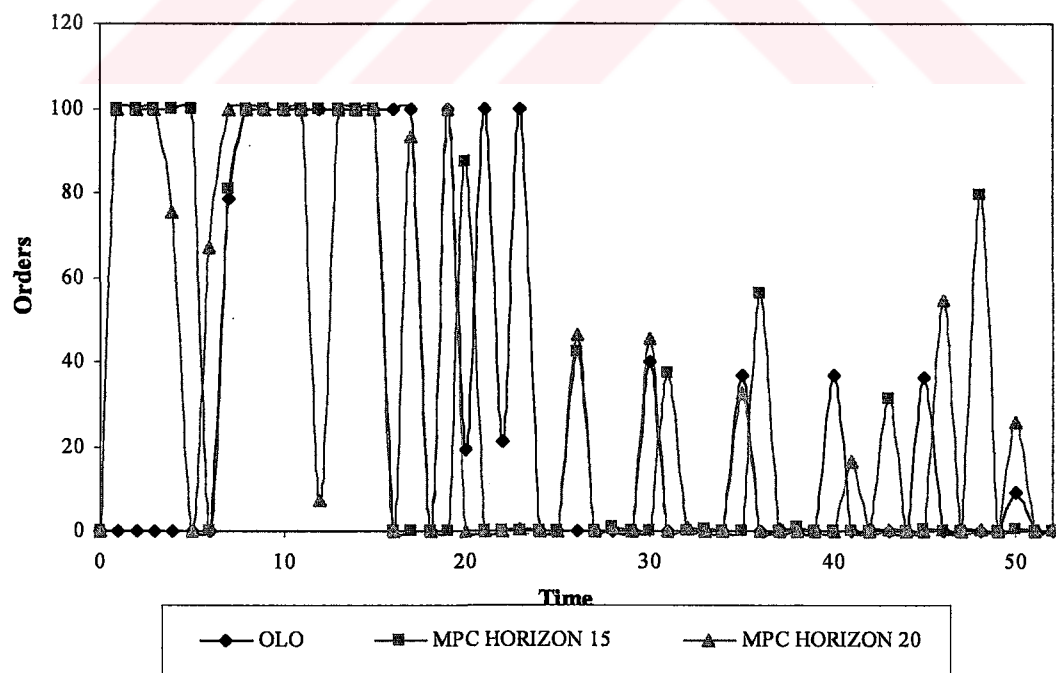


Figure 3.130: Warehouse order levels for product A (MPC vs open loop)

MPC with horizon 15 reduces distribution center and warehouse inventories fastest as well.(Figs. 3.126, Fig. 3.128), by increasing the material transfer rates between these nodes. (Figs. 3.125 , 3.129)

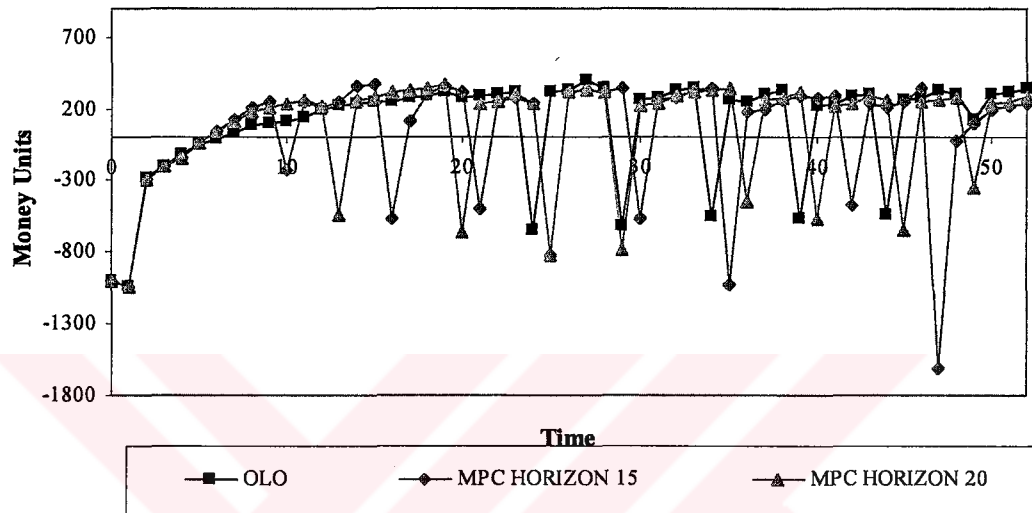


Figure 3.131: Instantaneous objective function for product A(MPC vs open loop)

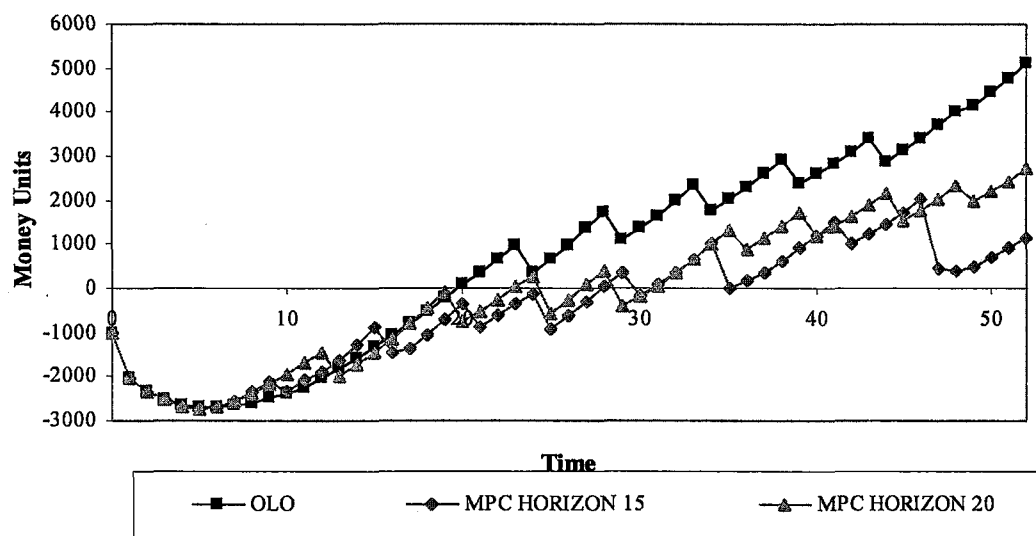


Figure 3.132: Cumulative objective function for product A(MPC vs open loop)

For product *C* the same behaviors are observed while for product *B*, whose production costs are very low compared with the two other products, a different situation arises. From the plots of production schedules of products *A* (Fig. 3.121) and *B* (3.133), it is obvious that in the beginning the inventory (Figs. 3.135, 3.137, 3.140) and the material transfer (Figs. 3.134, 3.138, 3.141) behaviors of product *B* is dictated by the production schedule of product *A*. From the trend seen in the retailer inventory level plot of *B* (figure 3.135), it can be foreseen that the inventory of *B* is going to be depleted at time 16 or 17. In order to avoid this shortage on time, the production of *B* must start at time 14 at latest. However, at time 13 product *A* is produced at the plant, which practically avoids any other production at times 13 and 14. Furthermore, in order to produce product *A* at time 13 and not to deplete inventory of product *B*, the plant must produce *B* at time 11 at latest, which is exactly what is done to minimize induced the inventory holding costs.

Moreover, similar reasons explain the behavior that MPC20 is slower in the depletion of of the inventory of product *B*. In addition to this, in any horizon length of MPC, the production order and the amount to be produced is decided on the expectation that the current demand will remain the same throughout the horizon. However, this expectation could be misleading and if the observed demand turns out to be less than the expectation that is made when the production order is given, then it is possible that at the time of the arrival of the produced material to the node (in this case the node is retailer) the inventory at that node has not been depleted.

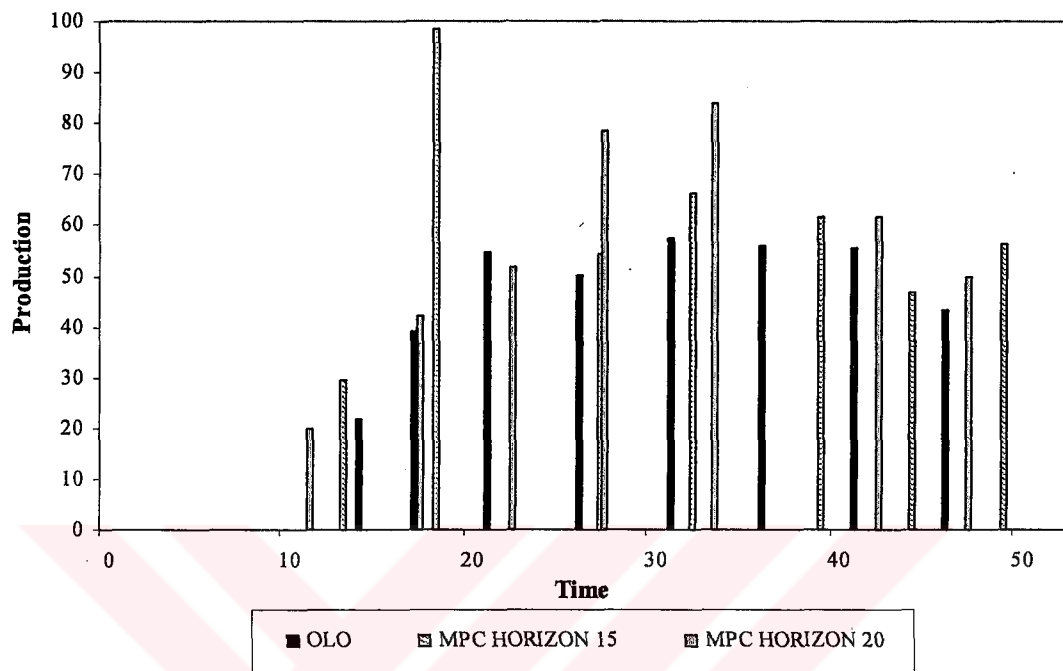


Figure 3.133: Optimal production schedule of the plant for product B (MPC vs open loop)

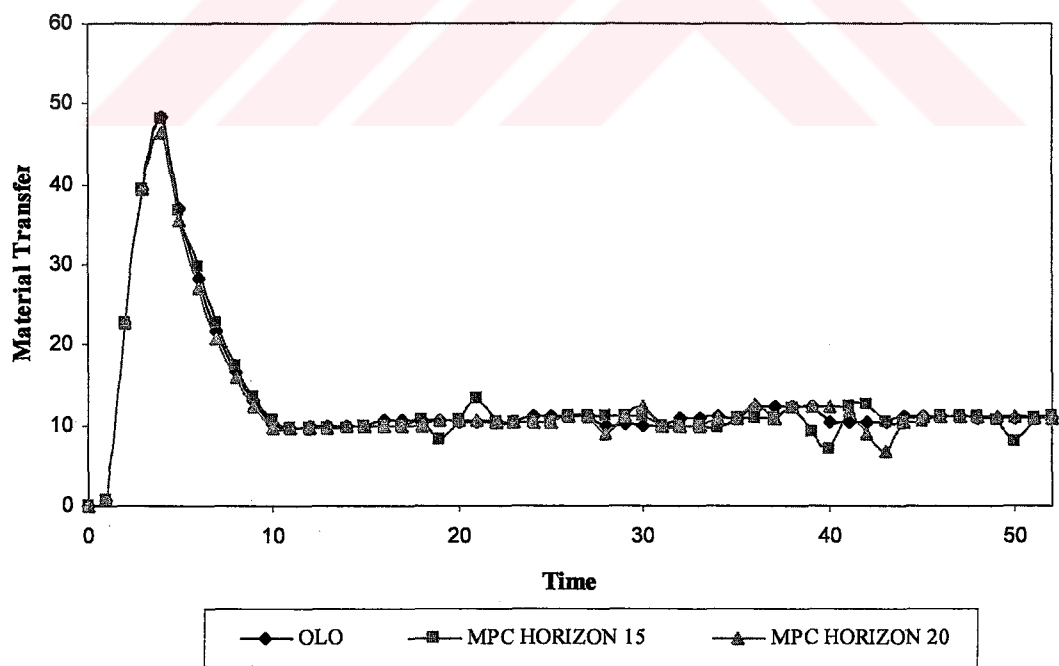


Figure 3.134: Retailer material transfer level for product B (MPC vs open loop)

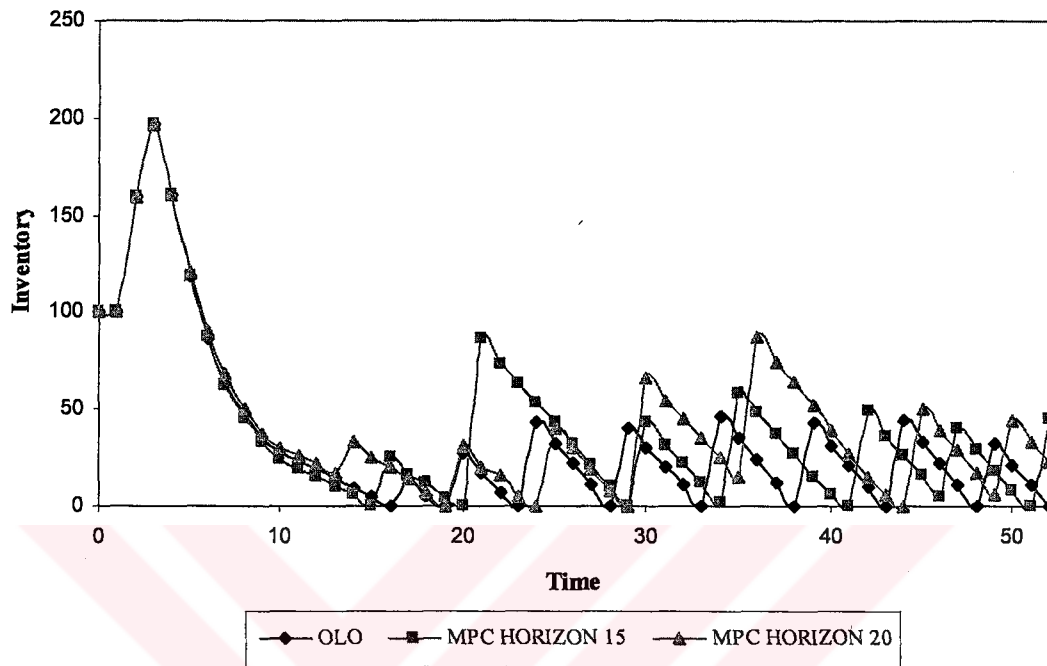


Figure 3.135: Retailer inventory level for product B (MPC vs open loop)

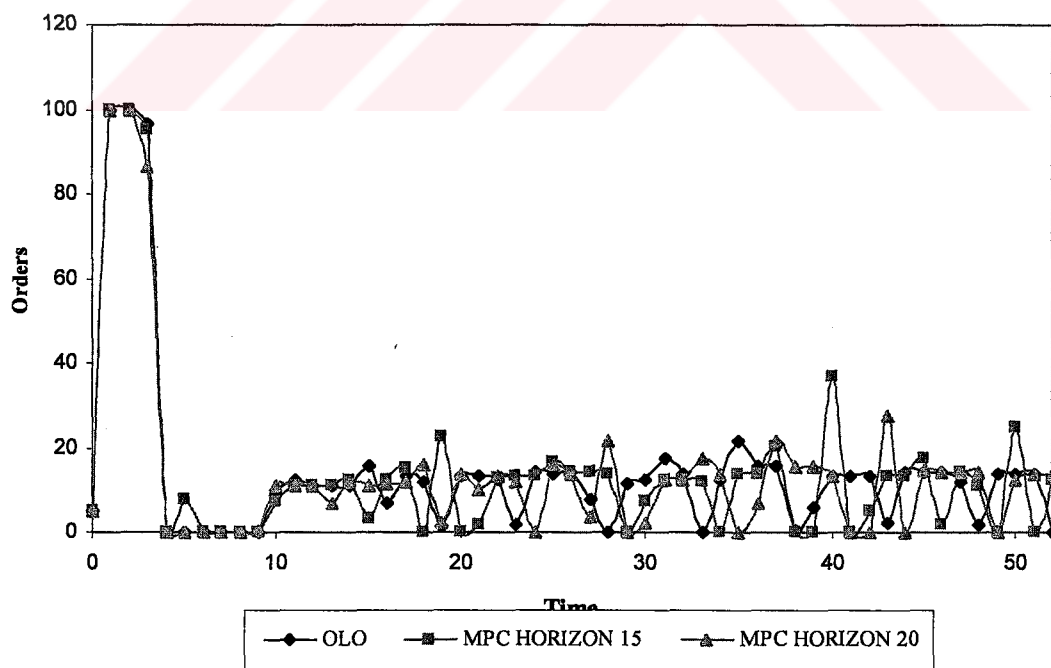


Figure 3.136: Retailer order levels for product B (MPC vs open loop)

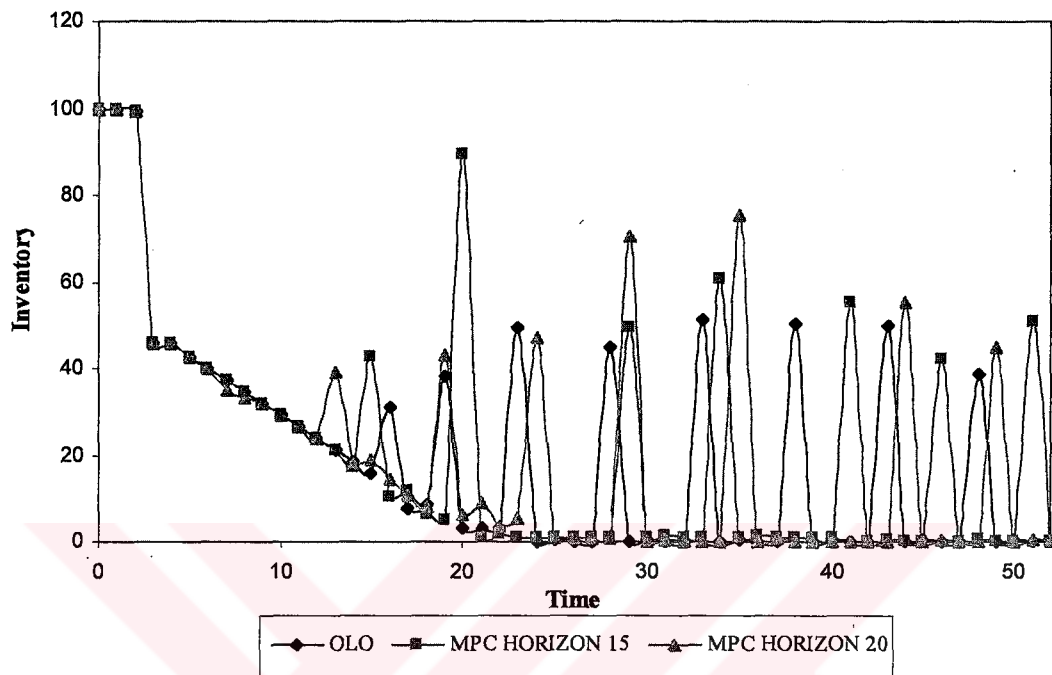


Figure 3.137: Distribution center inventory level for product B (MPC vs open loop)

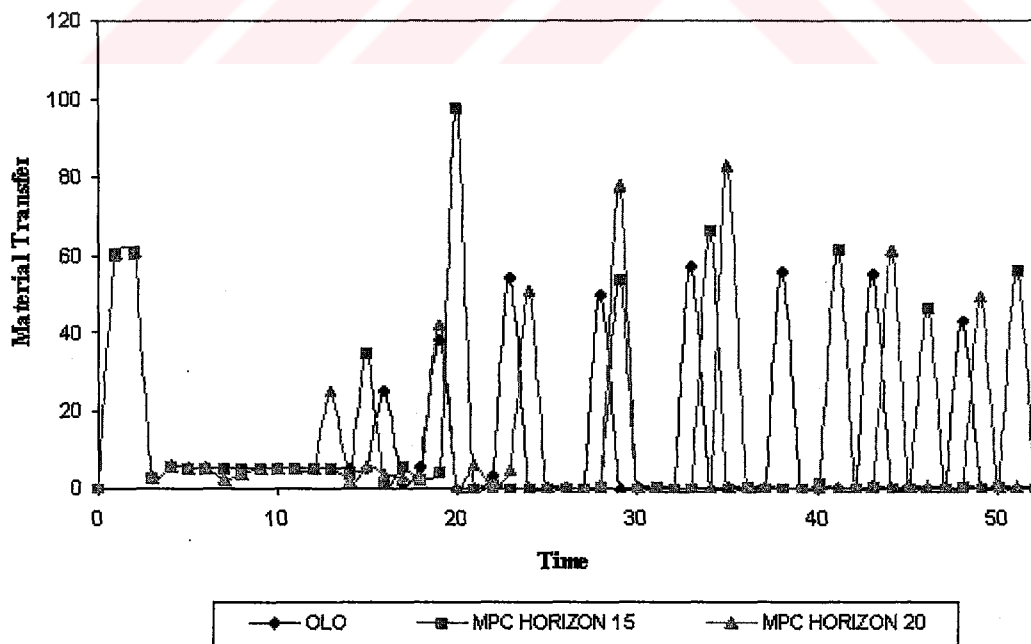


Figure 3.138: Distribution center material transfer level for product B (MPC vs open loop)

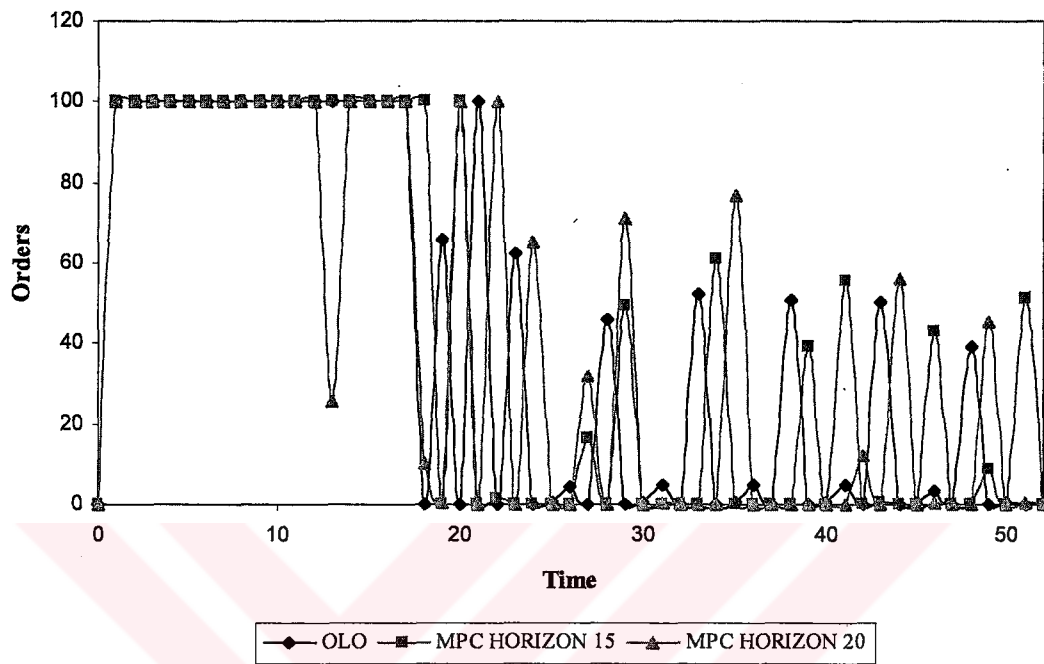


Figure 3.139: Distribution center order levels for product B (MPC vs open loop)

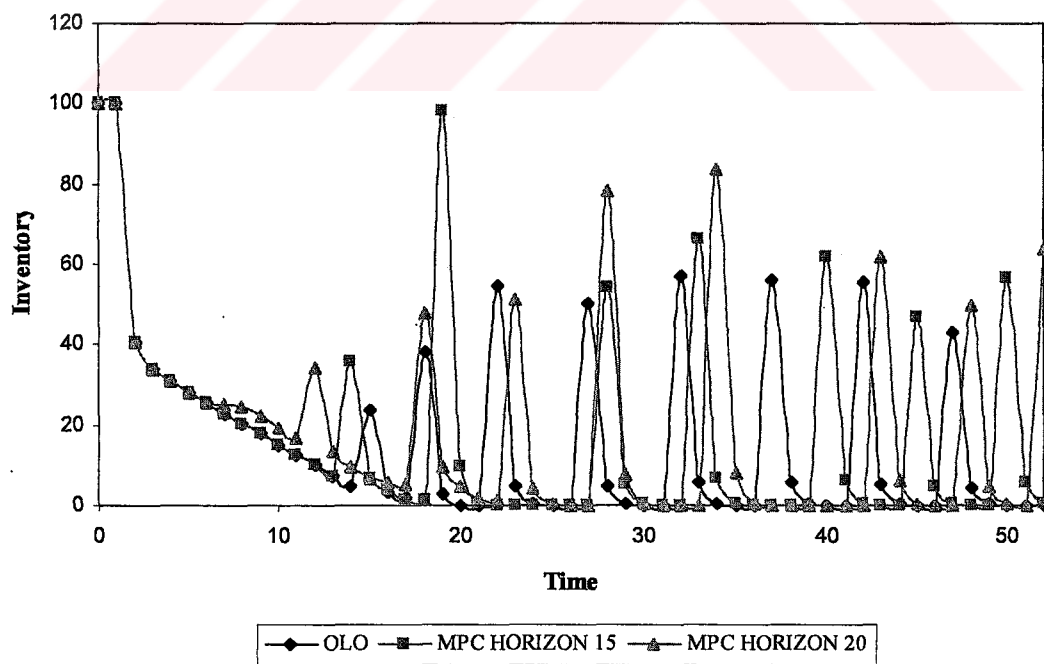


Figure 3.140: Warehouse inventory level for product B (MPC vs open loop)

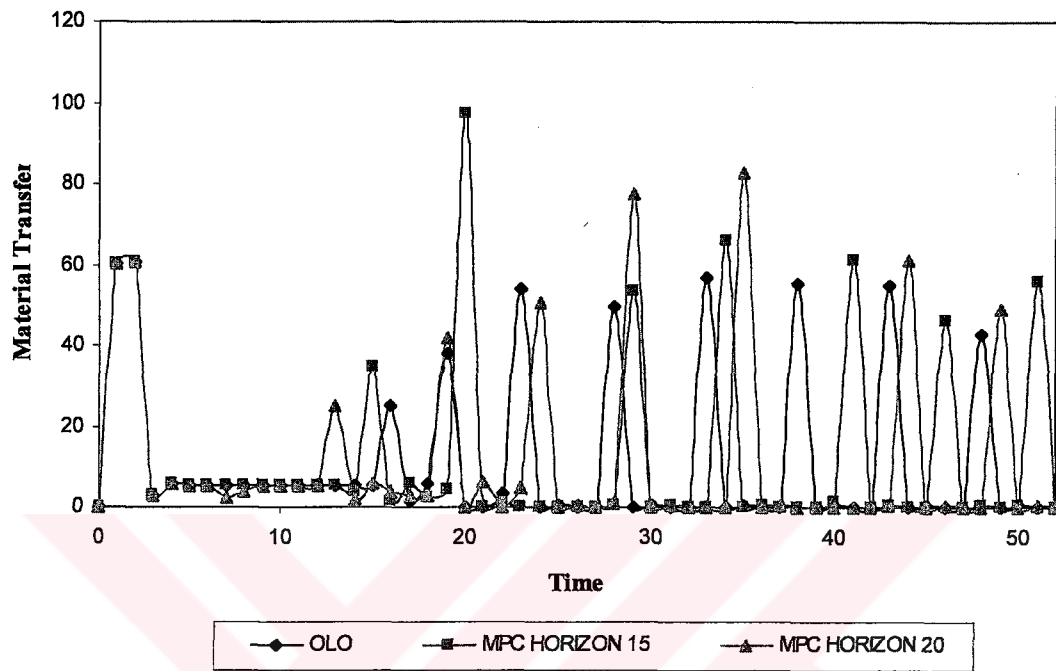


Figure 3.141: Warehouse material transfer level for product B (MPC vs open loop)

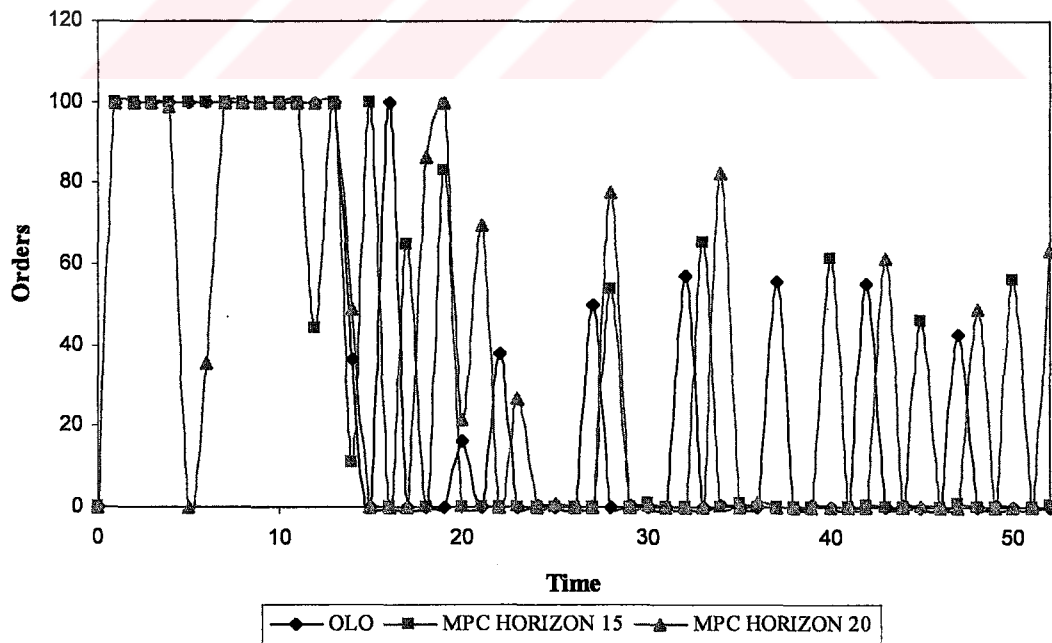


Figure 3.142: Warehouse order levels for product B (MPC vs open loop)

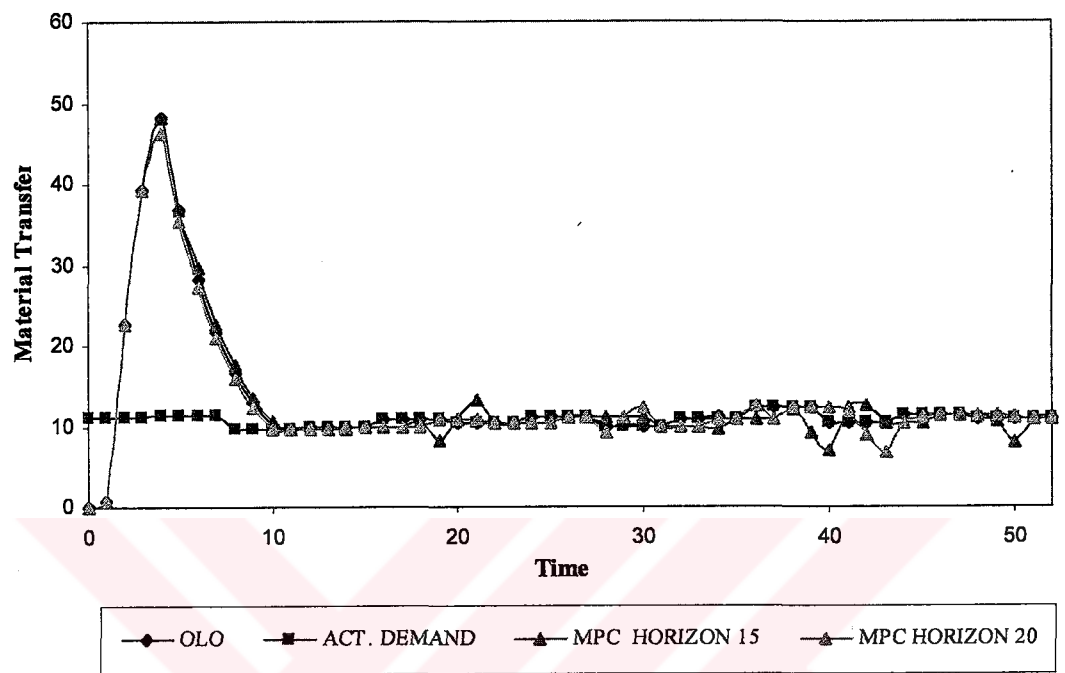


Figure 3.143: Material transferred to customer vs actual cost. demand for product B (MPC)

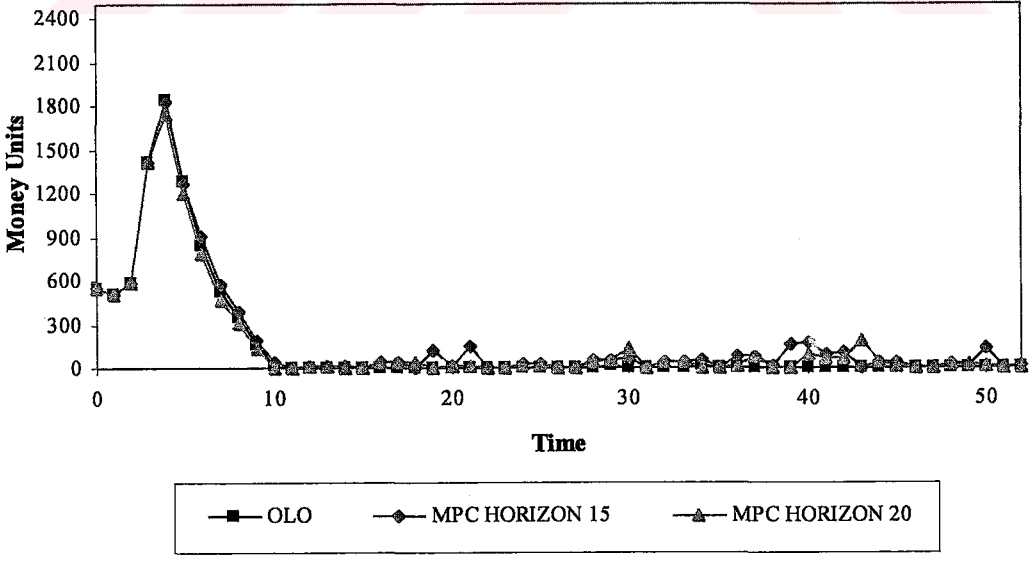


Figure 3.144: Penalty cost paid for product B (MPC vs open loop)

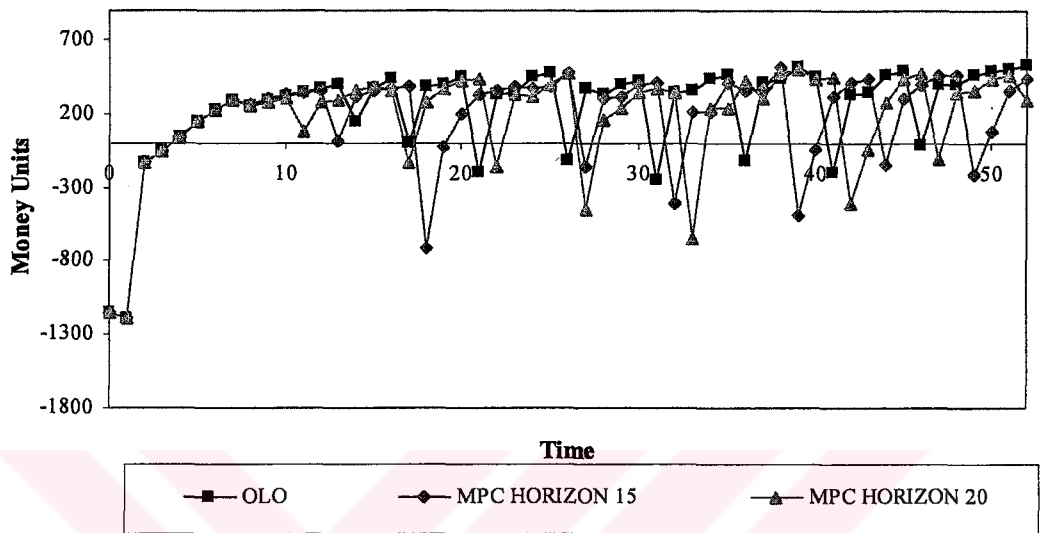


Figure 3.145: Instantaneous objective function for product B(MPC vs open loop)

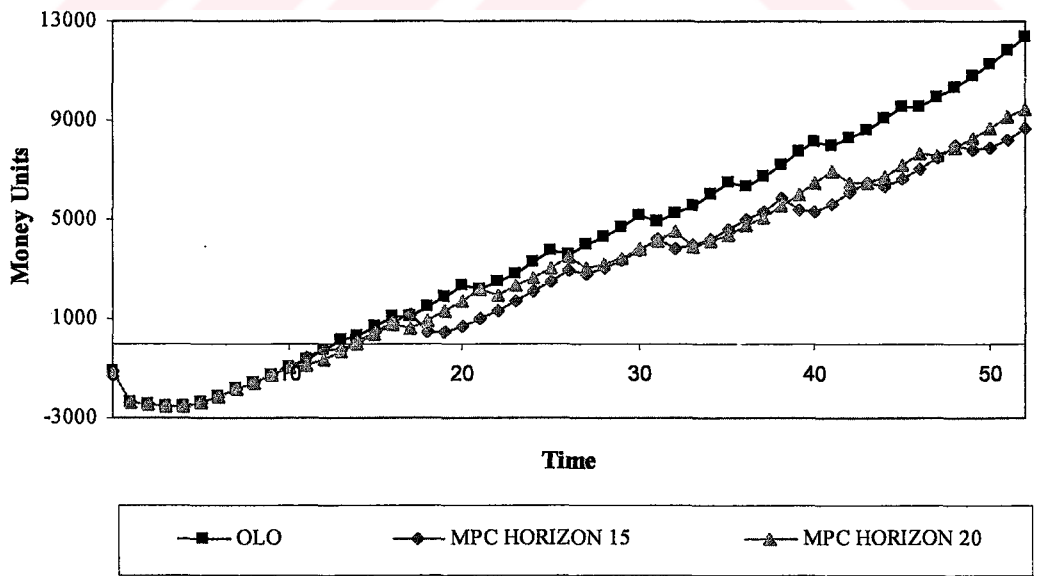


Figure 3.146: Cumulative objective function for product B (MPC vs open loop)

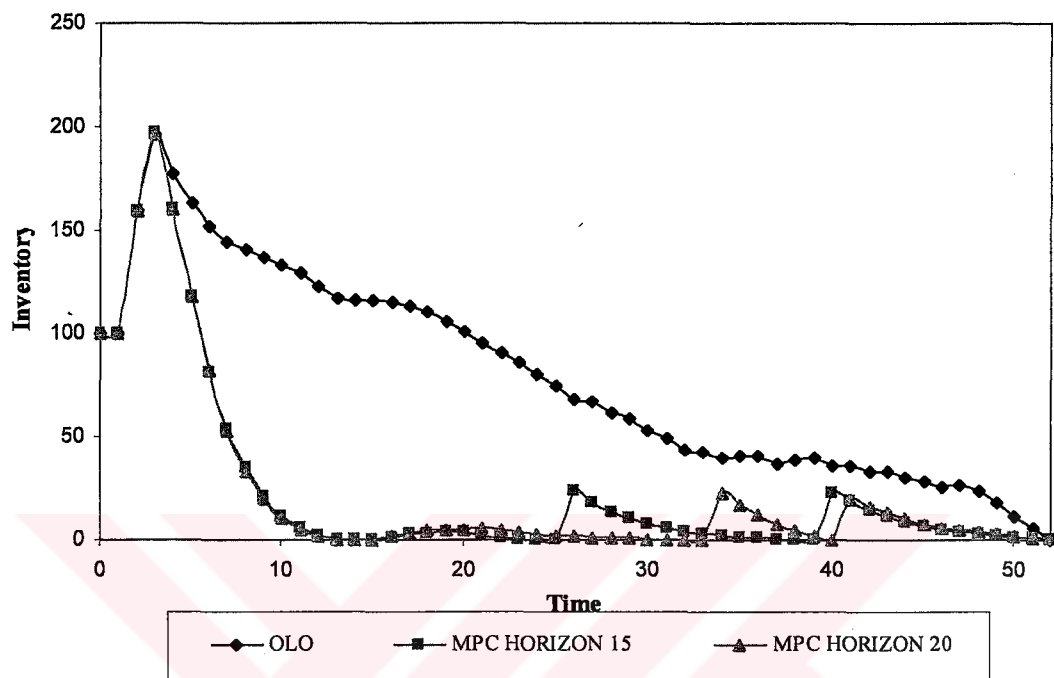


Figure 3.147: Retailer inventory level for product C (MPC vs open loop)

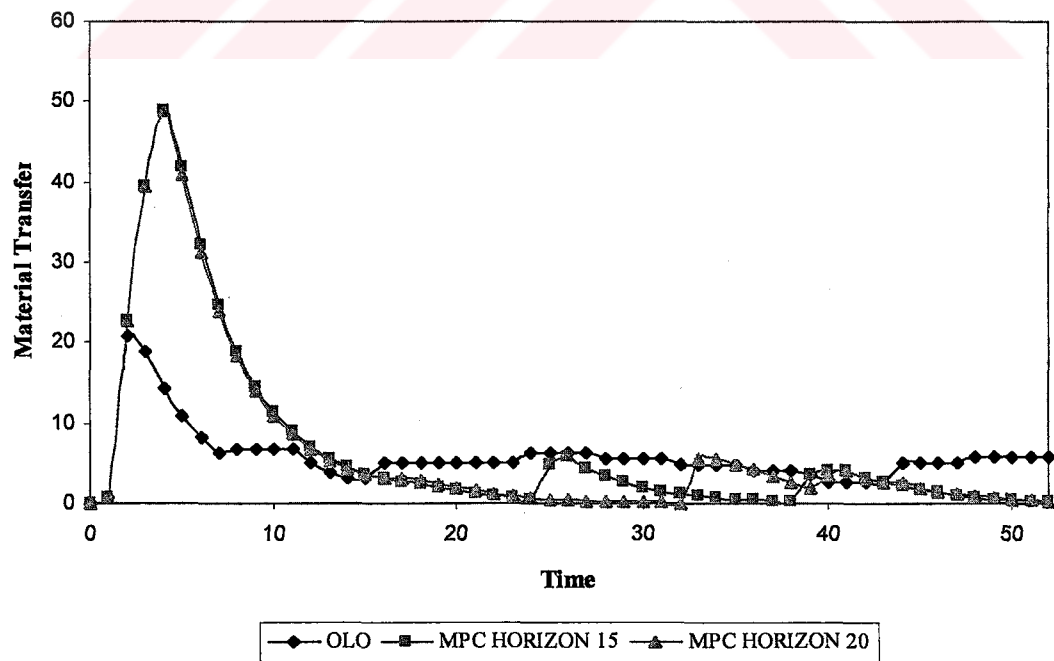


Figure 3.148: Retailer material transfer level for product C (MPC vs open loop)

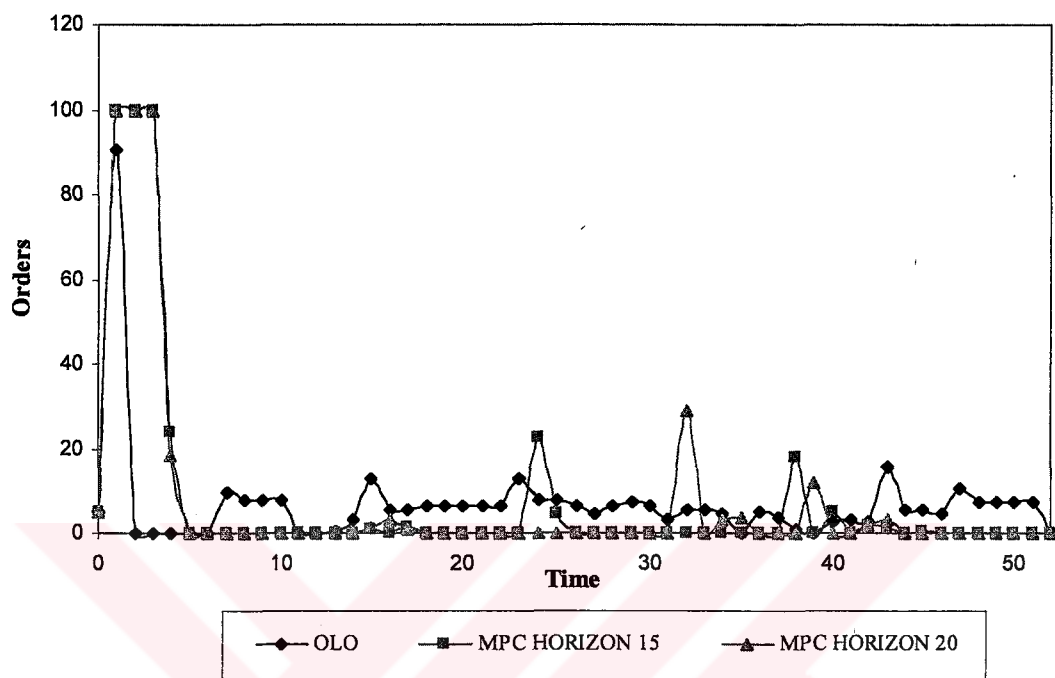


Figure 3.149: Retailer order levels for product C (MPC vs open loop)

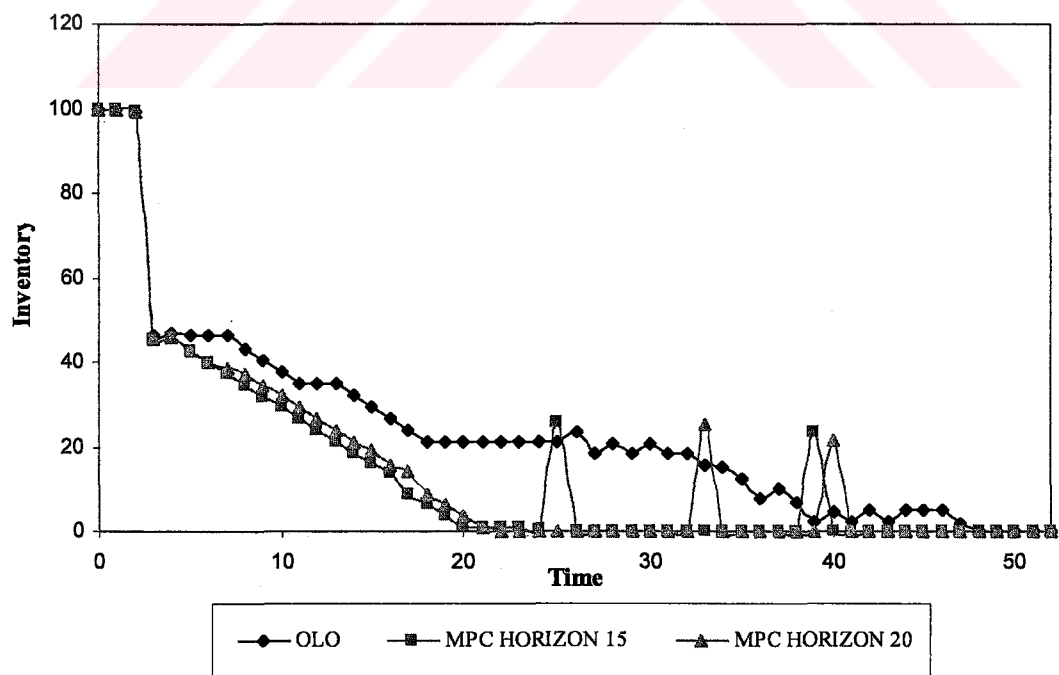


Figure 3.150: Distribution center inventory level for product C (MPC vs open loop)

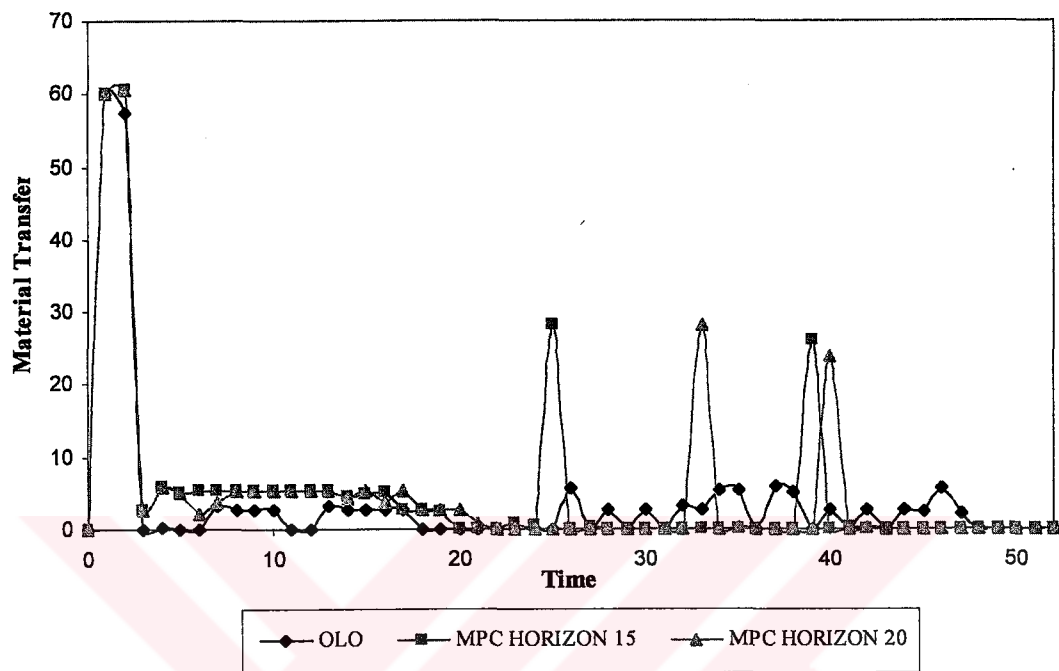


Figure 3.151: Distribution center material transfer level for product C (MPC vs open loop)

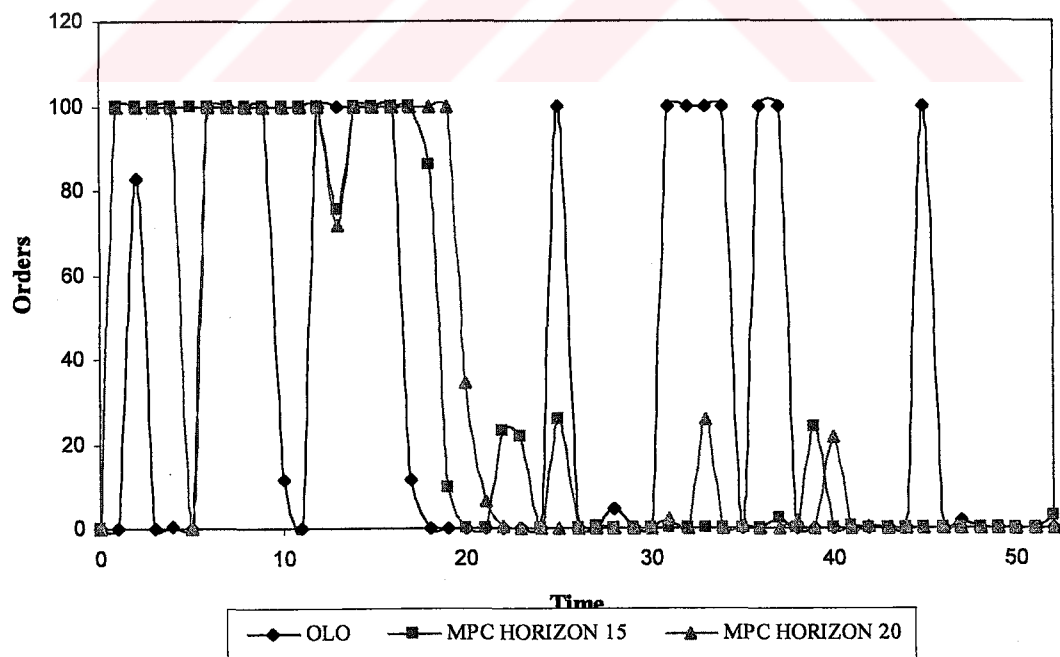


Figure 3.152: Distribution center order levels for product C (MPC vs open loop)

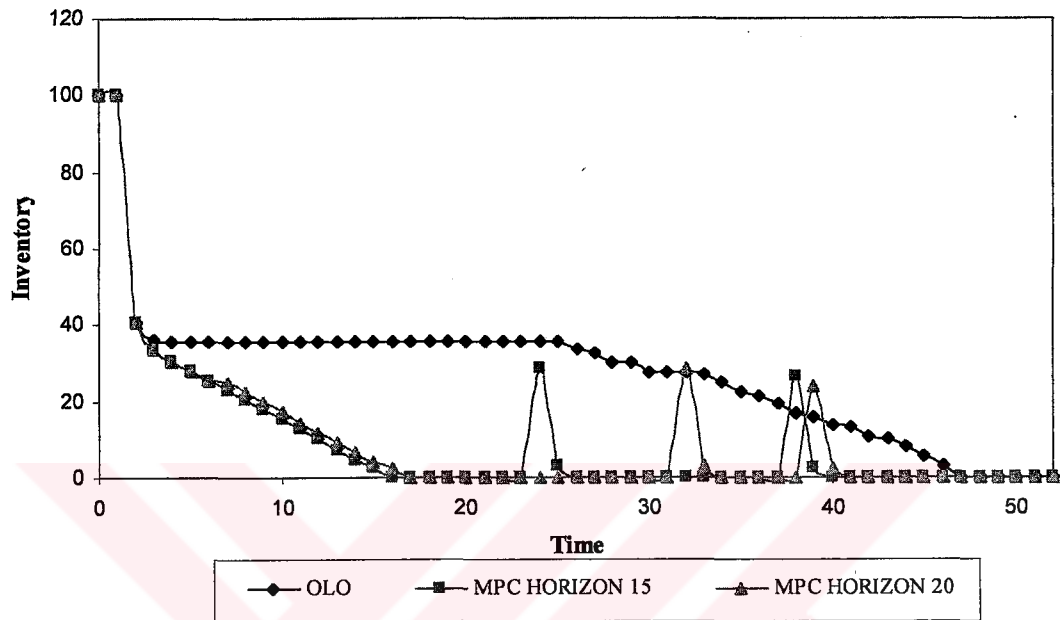


Figure 3.153: Warehouse inventory level for product C (MPC vs open loop)

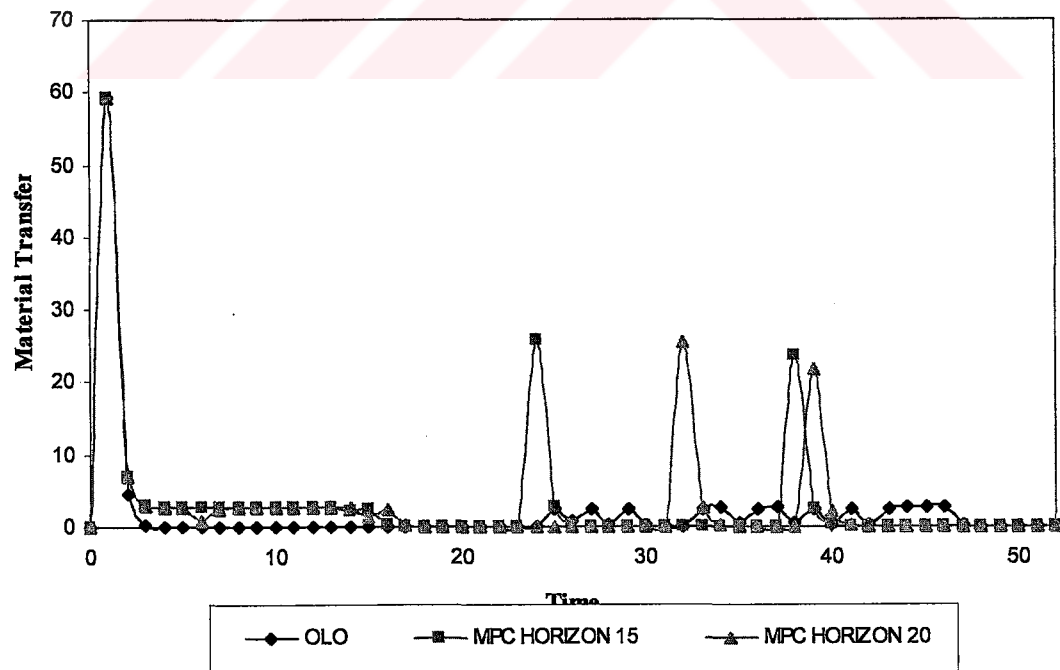


Figure 3.154: Warehouse material transfer level for product C (MPC vs open loop)

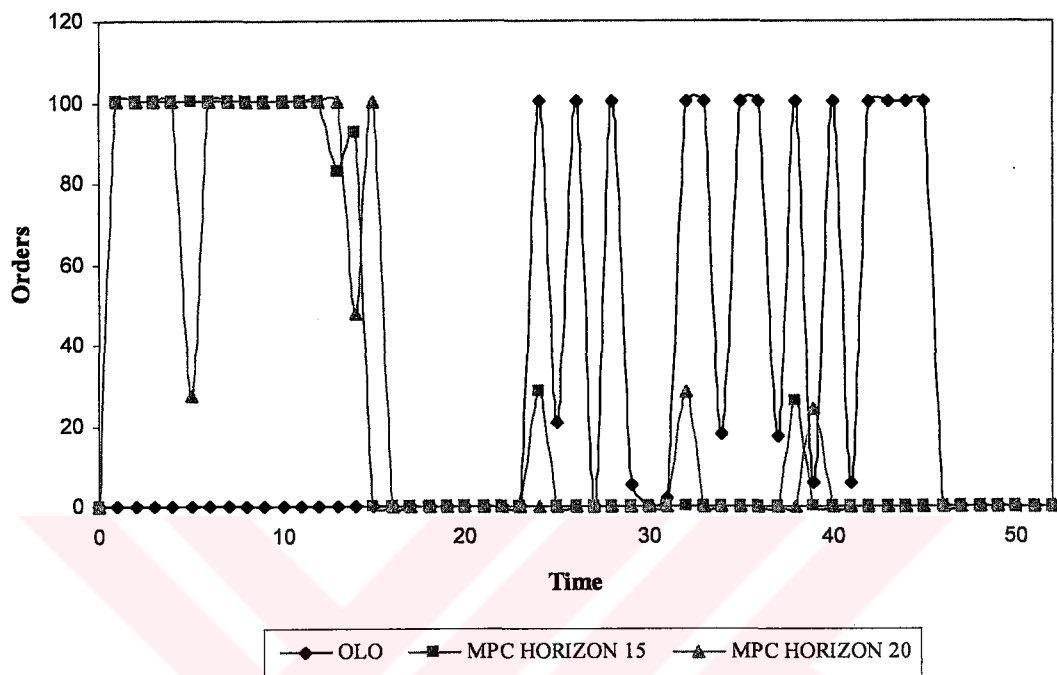


Figure 3.155: Warehouse order levels for product C (MPC vs open loop)

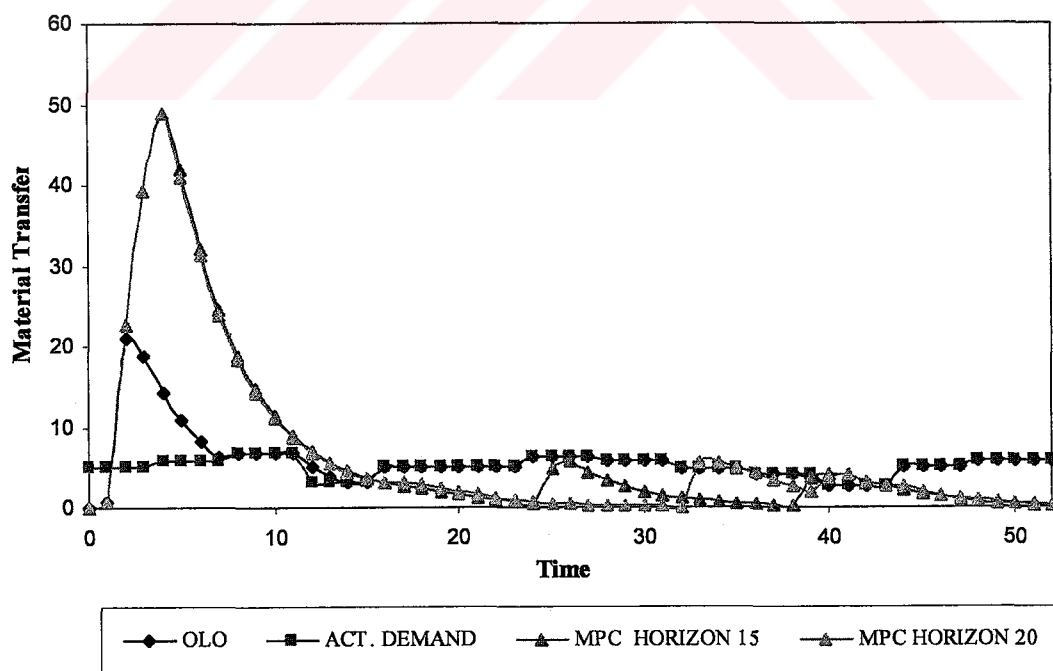


Figure 3.156: Material transferred to customer vs actual cost. demand for product C (MPC)

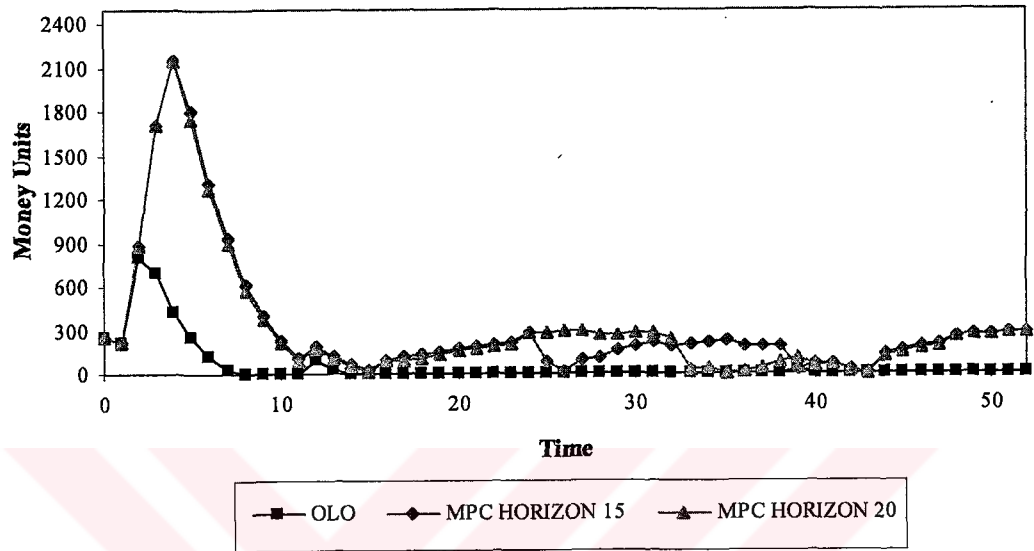


Figure 3.157: Penalty cost paid for product C (MPC vs open loop)

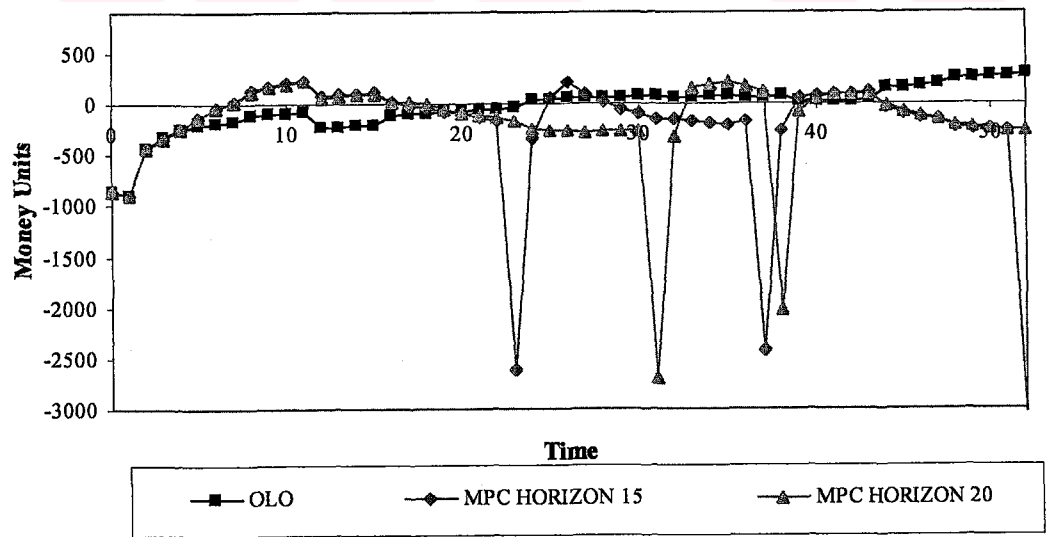


Figure 3.158: Instantaneous objective function for product C(MPC vs open loop)

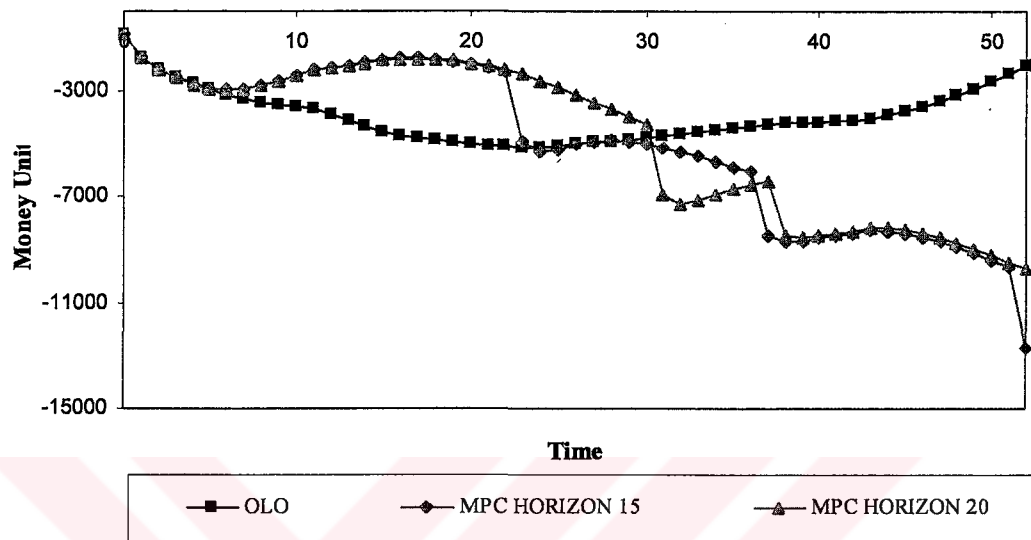


Figure 3.159: Cumulative objective function for product C(MPC vs open loop)

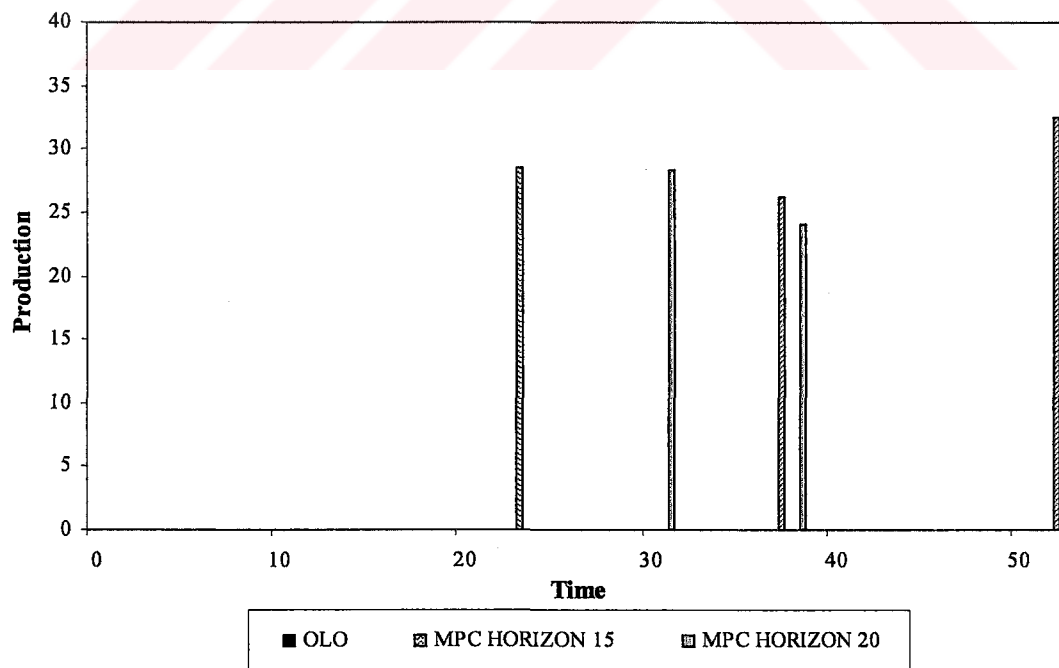


Figure 3.160: Optimal production schedule of the plant for product C (MPC vs open loop)

Table 3.4: MPC Horizon Lengths, Objective Function values and CPU times for continuous production

HORIZON LENGTH	OBJECTIVE FCT VALUES	CPU TIMES (sec)	NUMBER OF CONSTRAINTS	NUMBER OF VARIABLES
20	2457	1874	38821	20177
15	-2900	114	28431	15379
52 (OLO)	15367	24826	5524	3022

3.6 Modeling of a Three-Product Hybrid Supply Chain with Reduced Initial Inventory

3.6.1 Open-Loop Optimization

In the previous cases discussed above initial inventories are taken as 100 units for each product at each node. As a result the system initially tends to reduce its inventories and once the inventories are depleted it supplies material to its customers through production. Once the initial inventories in the nodes are reduced, inventory costs are expected to decrease. Also at earlier stages of the process to reduce its inventories faster, the system tends to supply more material to the customers resulting in higher penalty costs. With reduced initial inventory sizes these high penalties are also expected to be decreased.

Fig. 3.161 shows the material transfer levels from retailer node to customer. As expected, the initial high material transfer rate to decrease the high inventory levels are not observed in the reduced inventory case. As a result the high rates of customer dissatisfaction penalties are not paid as shown in Fig. 3.162.

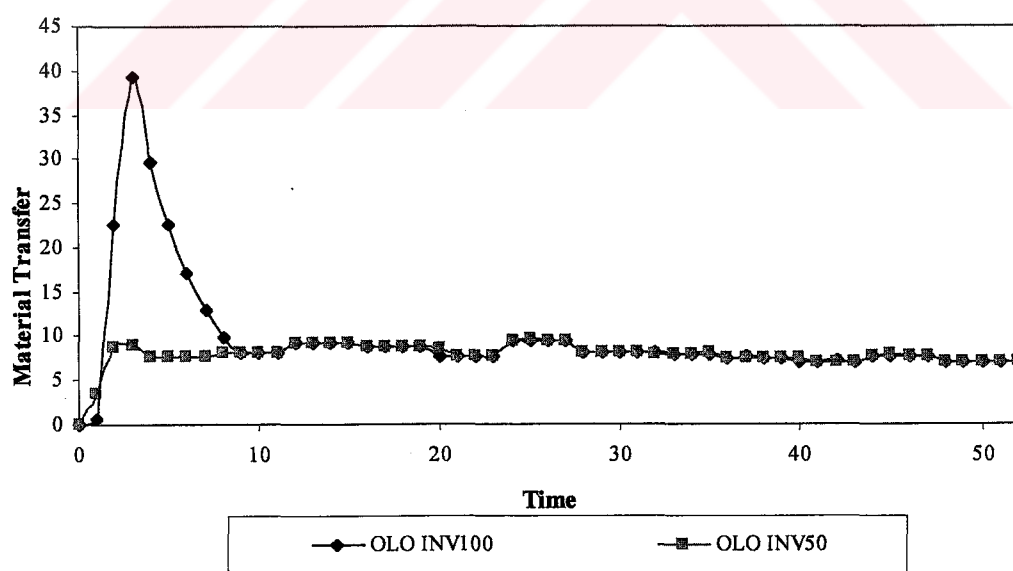


Figure 3.161: Retailer node material transfer level for product A (inv 100 OLO vs inv 50 OLO)

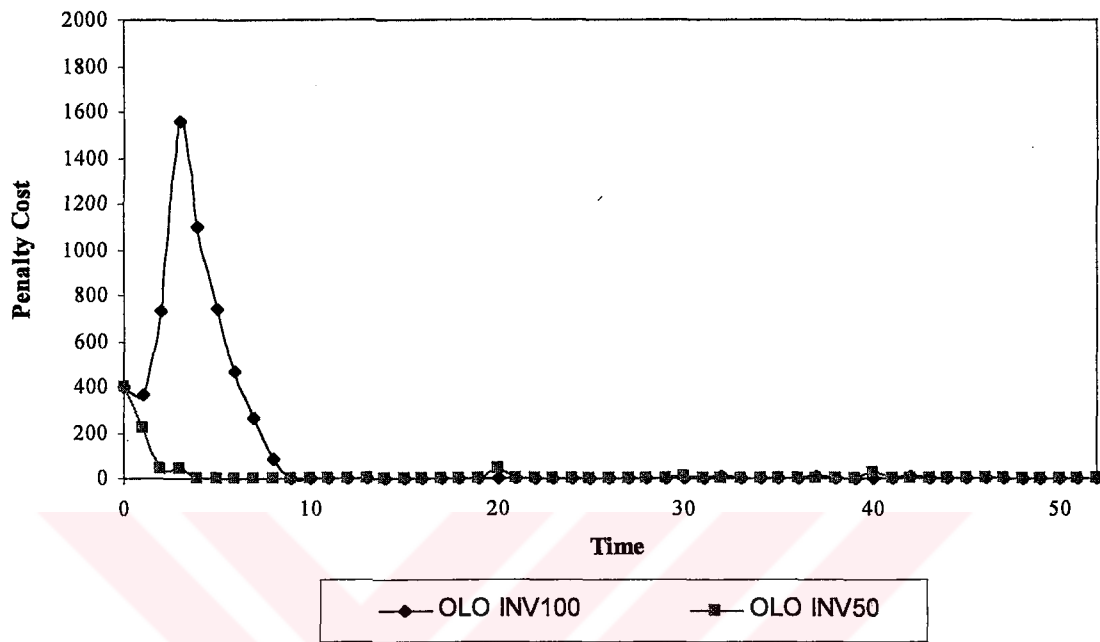


Figure 3.162: Penalty cost paid for product A (inv 100 OLO vs inv 50 OLO)

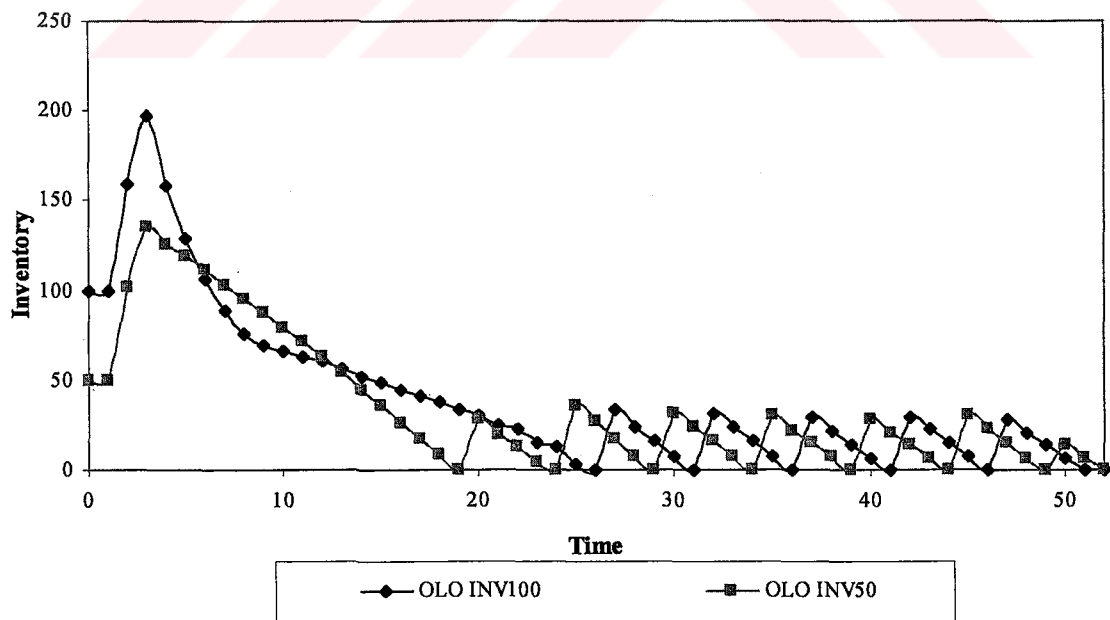


Figure 3.163: Retailer node inventory level for product A (inv 100 OLO vs inv 50 OLO)

As observed in Fig. 3.163, retailer node inventory level also increases as in the large initial inventory case. However this increase is not as high as before and the inventory decrease rate is more gradual compared with the fast inventory depletion rates between the third and ninth time units of the large inventory case. The total inventory cost for the lower initial inventory case is 5332 monetary units and the total penalty paid is 808 monetary units, whereas these values are 8425 monetary units and 5742 monetary units respectively, for the larger inventory case. This low inventory cost and low penalty advantage of the small initial inventory case is however brings along the higher and more frequent production disadvantage.

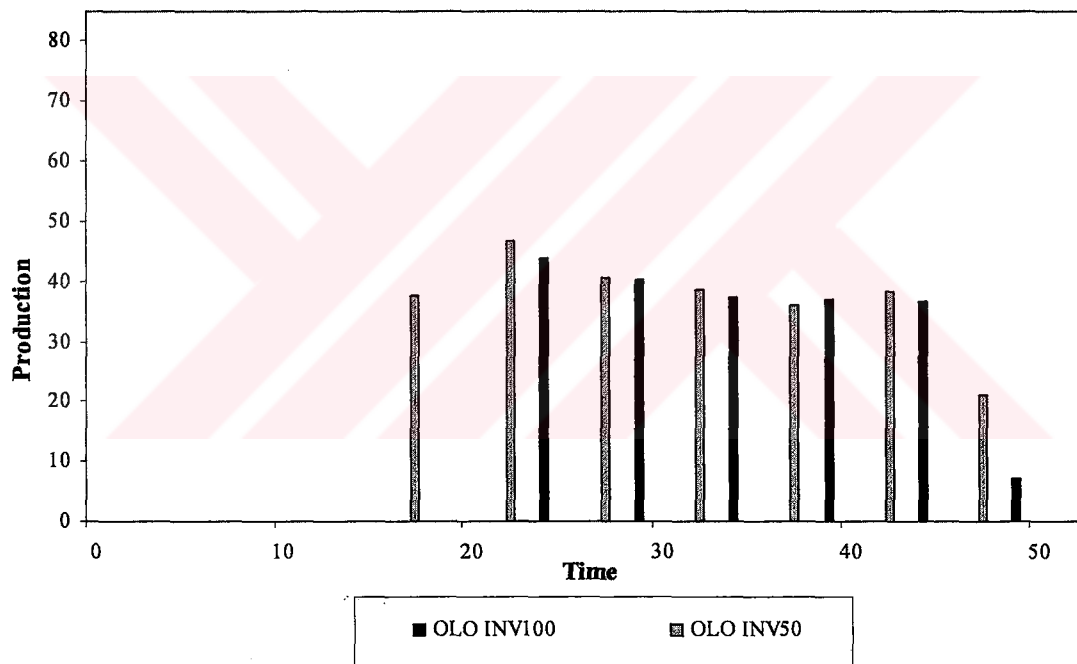


Figure 3.164: Production amounts for product A (inv 100 OLO vs inv 50 OLO)

Total fixed production cost for product A in case of low inventory is 126 monetary units where as total variable production cost is 6207 monetary units which are both higher than large inventory case. Despite these high production costs total profit gained in small initial inventory case is 7224 monetary units for product A which is higher than the large inventory case of 5107 monetary units.

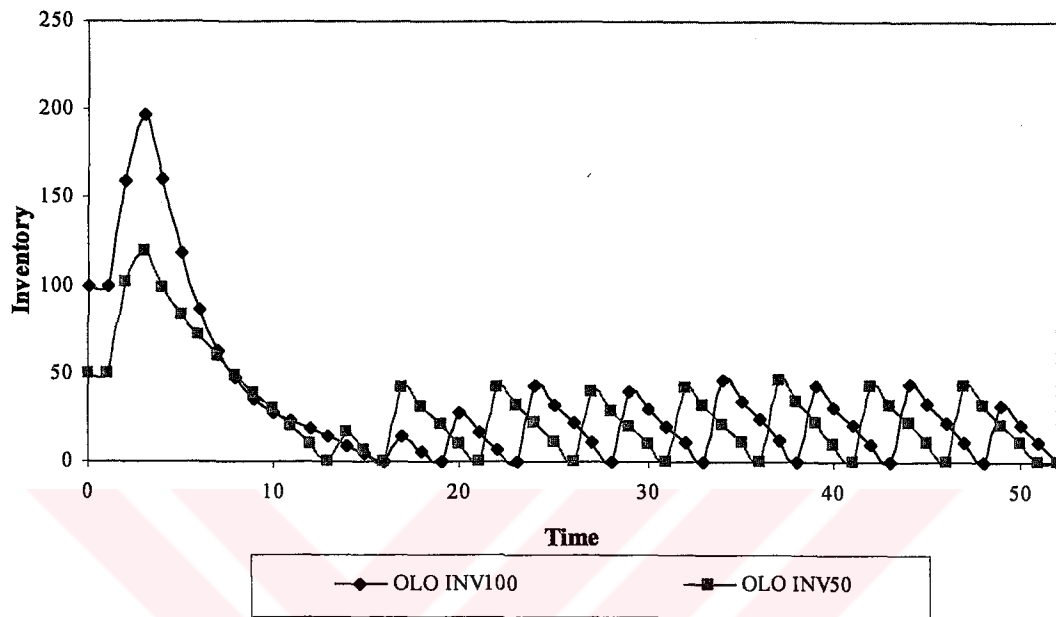


Figure 3.165: Retailer node inventory level for product B (inv 100 OLO vs inv 50 OLO)

As for product *A*, retailer inventory level initially increases for product *B*, to decrease the distribution center and warehouse inventories. However a different behavior than product *A* arises for product *B*. Production duration for *B* is longer than *A* and *C* and production costs are higher. Therefore the system finds it profitable to arrange its production schedule based on *A* and *C* production. Since the production of *B* takes a longer time and disables the production of other products for a longer duration, system decides to produce *B* earlier than *A*. Therefore it tends to reduce its inventory earlier than the other two products, transfers a higher initial amount to the customer (Fig. 3.166), increasing the customer dissatisfaction cost (Fig. 3.167). However, in higher inventory case, inventory amount is enough to handle a production schedule which does not have to arrange production of product *B* according to *A*.

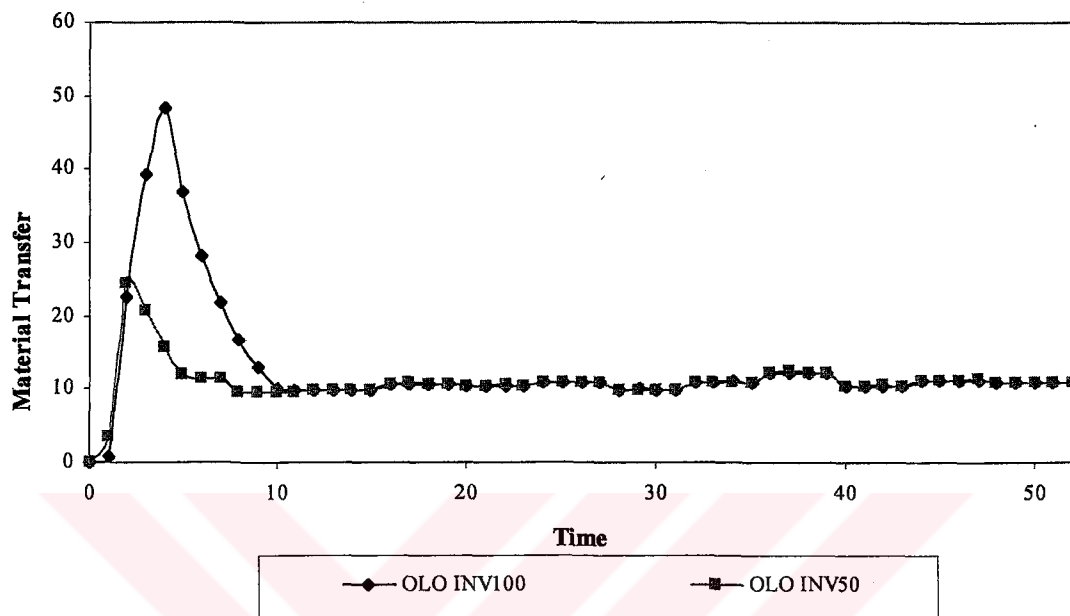


Figure 3.166: Retailer node material transfer level for product B (inv 100 OLO vs inv 50 OLO)

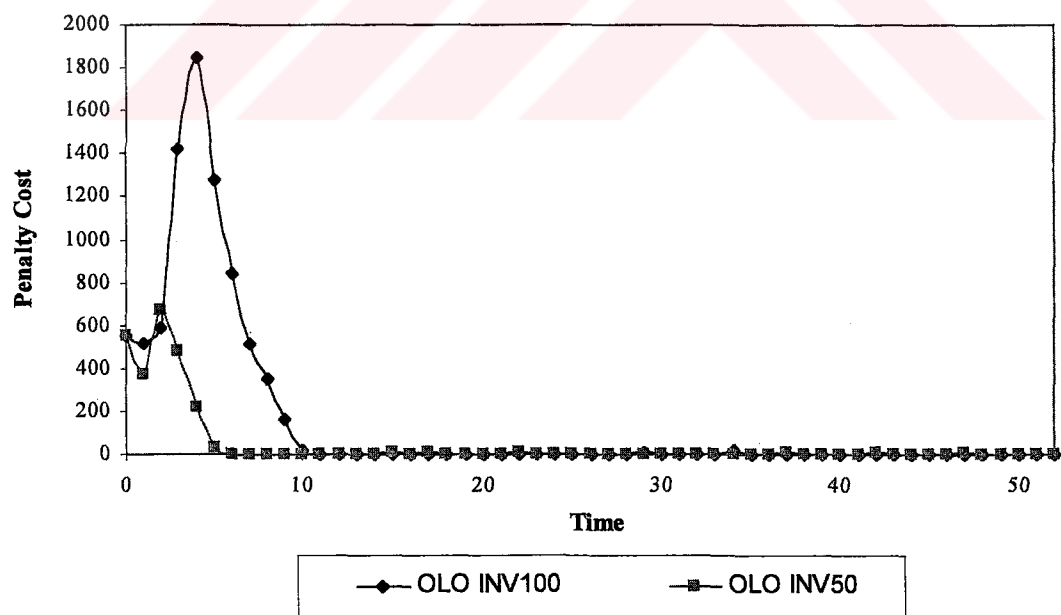


Figure 3.167: Penalty cost paid for product B (inv 100 OLO vs inv 50 OLO)

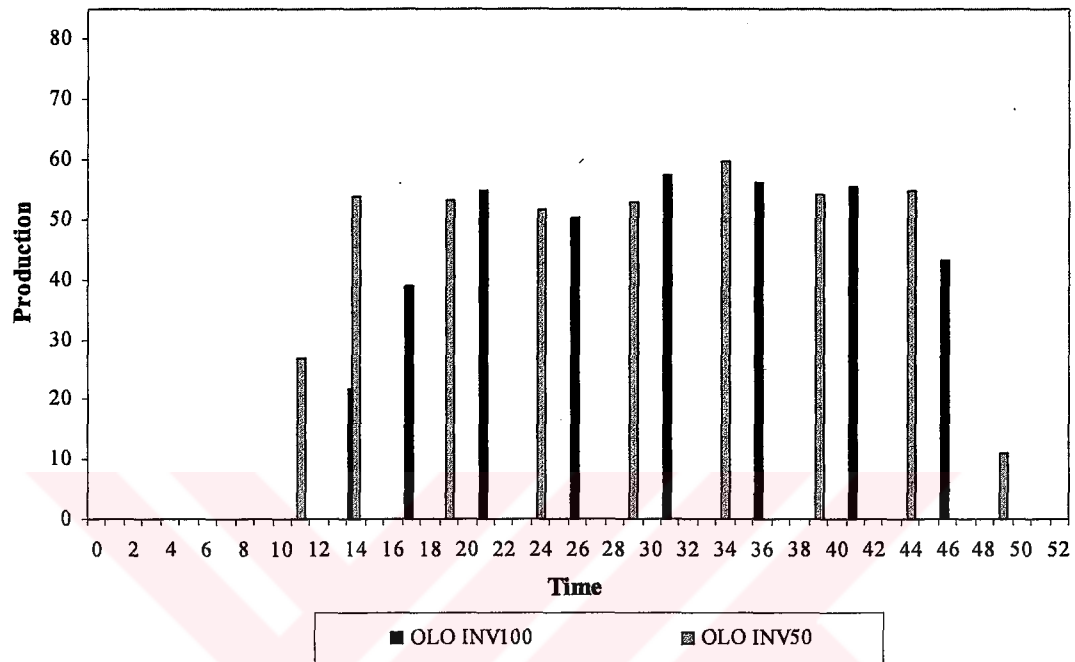


Figure 3.168: Production amounts for product B (inv 100 OLO vs inv 50 OLO)

Despite the higher penalty cost paid at time two and higher production costs, the total profit obtained through *B* is higher in the lower inventory case with a value of 14607 monetary units, than the profit of higher inventory case which is 12338 monetary units.

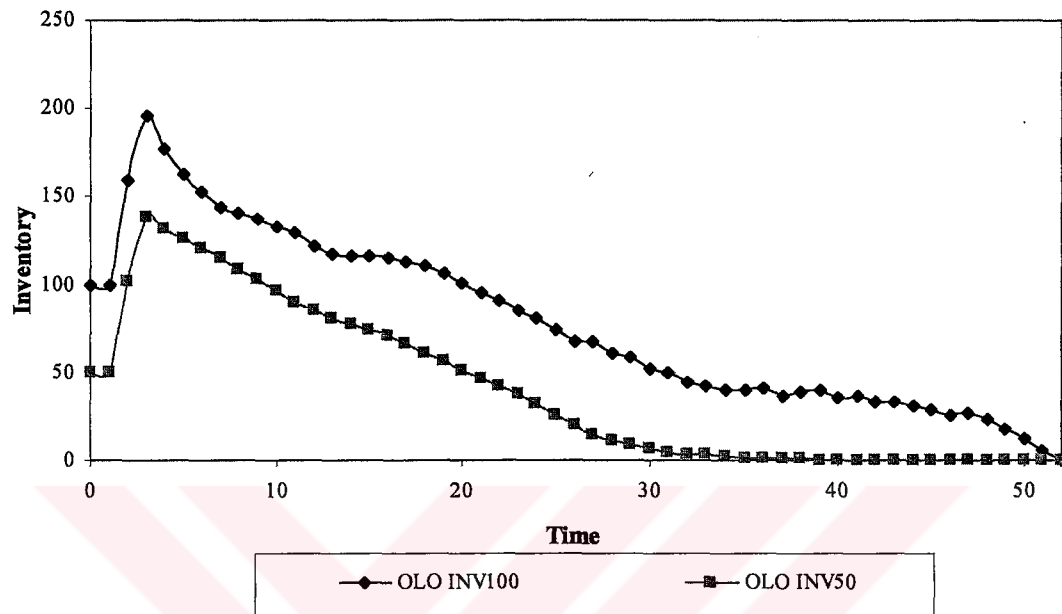


Figure 3.169: Retailer node inventory level for product C (inv 100 OLO vs inv 50 OLO)

The retailer inventory level for product *C* also increases in the beginning as in other products' case, to enable sufficient material transfer to the customer at the beginning (Fig. 3.169, 3.170). However the production cost for product *C* is too high to make its production optimal at any time. The higher inventory case does not deplete its inventories until the very end, whereas the lower inventory case consumed its inventory at 37th time unit, does not make production at any time unit and does not transfer any material to the customer despite the high customer dissatisfaction cost (Fig. 3.171).

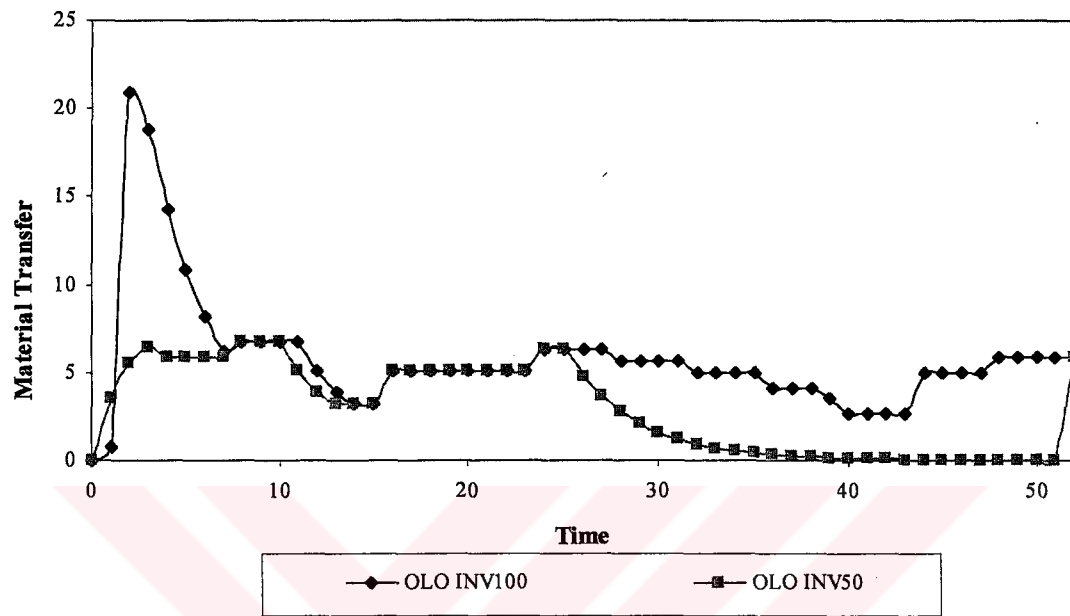


Figure 3.170: Retailer node material transfer level for product C (inv 100 OLO vs inv 50 OLO)

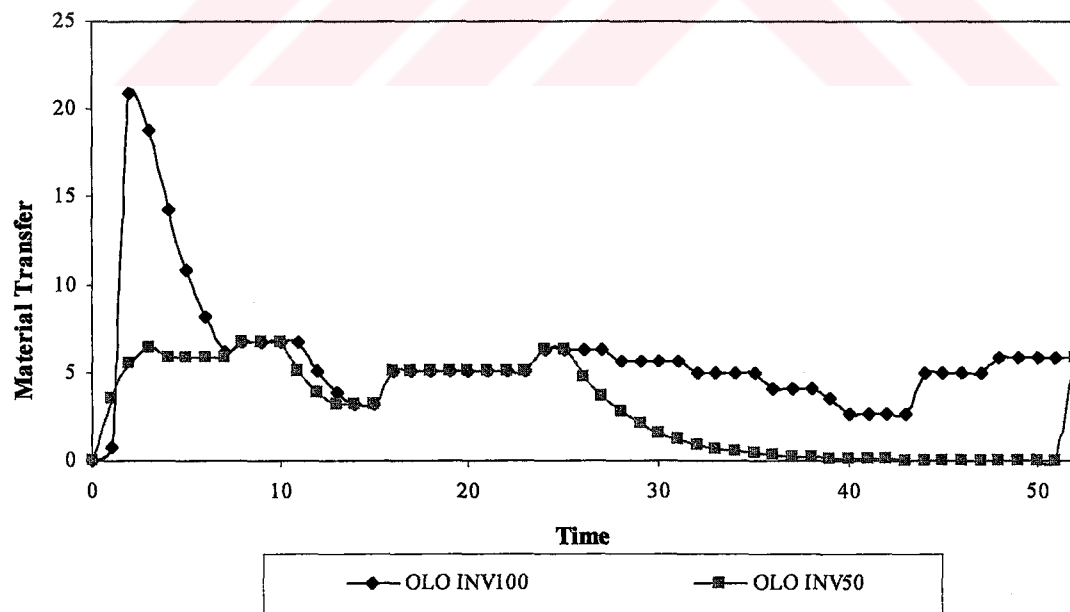


Figure 3.171: Penalty cost paid for product C (inv 100 OLO vs inv 50 OLO)

The total inventory cost paid for product *C* in the lower initial inventory case is 4792 monetary units which is very small compared with the large inventory case's total inventory cost of 13844 monetary units. However, the total revenue gained is 7794 monetary units and penalty cost paid is 5815 in the lower initial inventory case. For the high initial inventory case these values are 15295 monetary units and 2923 monetary units respectively. The overall total profit is therefore higher in the larger initial inventory case compared with the lower inventory case.

However the total profit of the system is higher for the lower inventory case with a value of 18713 monetary units than the higher inventory case with total profit of 15367 monetary units.

3.6.2 Model Predictive Control

As with other supply chain systems studied before, the MPC with horizon length 15 and 20 are performed for optimization of this lower initial inventory system to observe the effects of the MPC optimization horizon length. The MPC application on the supply chain is performed as explained in section 3.4.

MPC again makes short sighted decisions yet, the nature of these short sighted decisions differ now due to the length of the horizon. For product *A*, MPC with horizon length 20, and initial inventory 50, optimally decides to produce at 17th time unit in the first optimization cycle. In other words, even though it has a relatively short horizon it has to produce to meet the customer demand, therefore it depletes its inventory slowly at the beginning, similar to the OLO case, to decrease its production frequency. However, MPC with horizon 15 acts relatively more myopic and since its optimization results does not propose production orders in the first optimization cycle; it diminishes its inventories faster without noticing the necessity of producing in the later stages for the first optimization cycle. Fig. 3.172 shows these behavior differences between the two MPC horizons, in which MPC with horizon 20, spends its inventory slower compared with horizon 15.

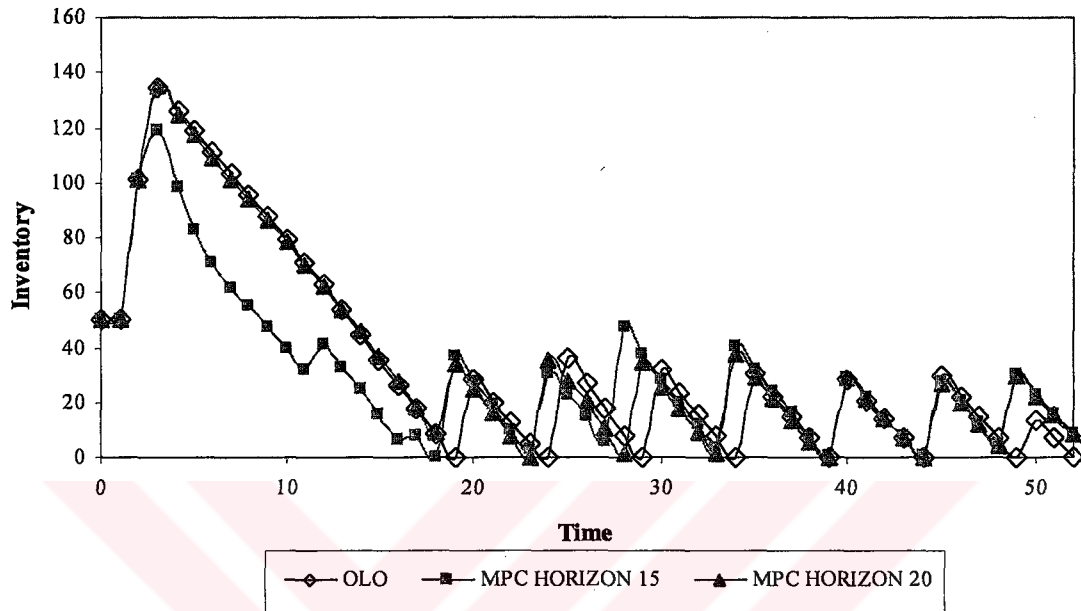


Figure 3.172: Retailer inventory level for product A (MPC vs open loop)

Since MPC with horizon 15 does not see the necessity of production, and aims to diminish its inventories to obtain higher profit, material transfer of MPC with horizon 15 is also higher compared to the similar behavior of OLO and MPC with horizon 20. OLO and MPC with horizon 20, diminish their inventories slower and their material transfers are closer to the actual customer demand to decrease the further negative effects of customer dissatisfaction. (Fig. 3.173). Total customer dissatisfaction cost paid for OLO is 808 monetary units while 1683 monetary units for MPC with horizon 20 and the highest penalty is paid by MPC with horizon 15 with 3726 monetary units. (Fig. 3.174)

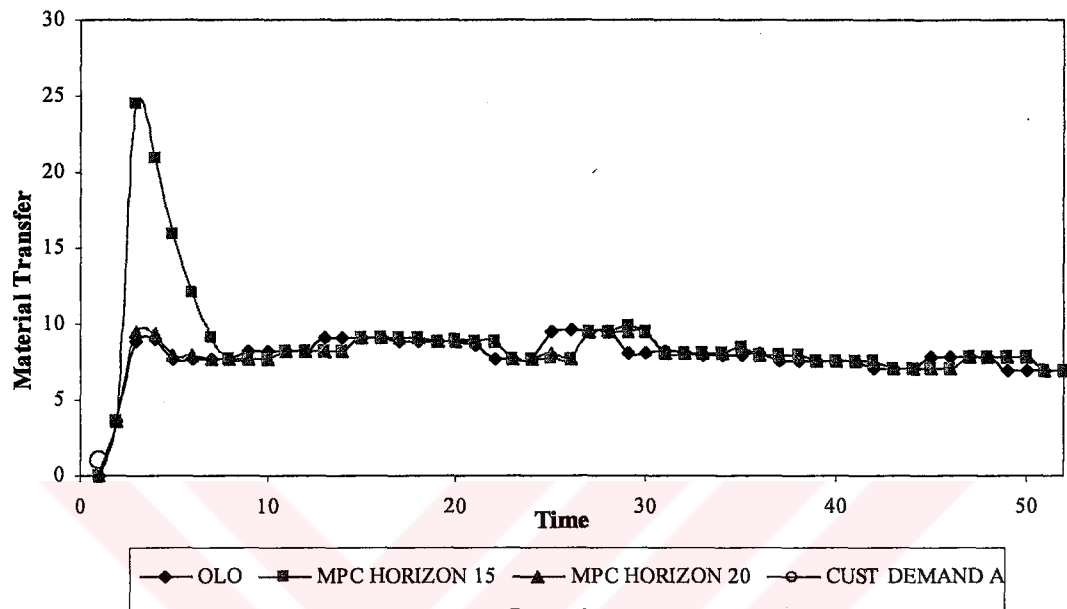


Figure 3.173: Material transferred to customer vs actual cost. demand for product A

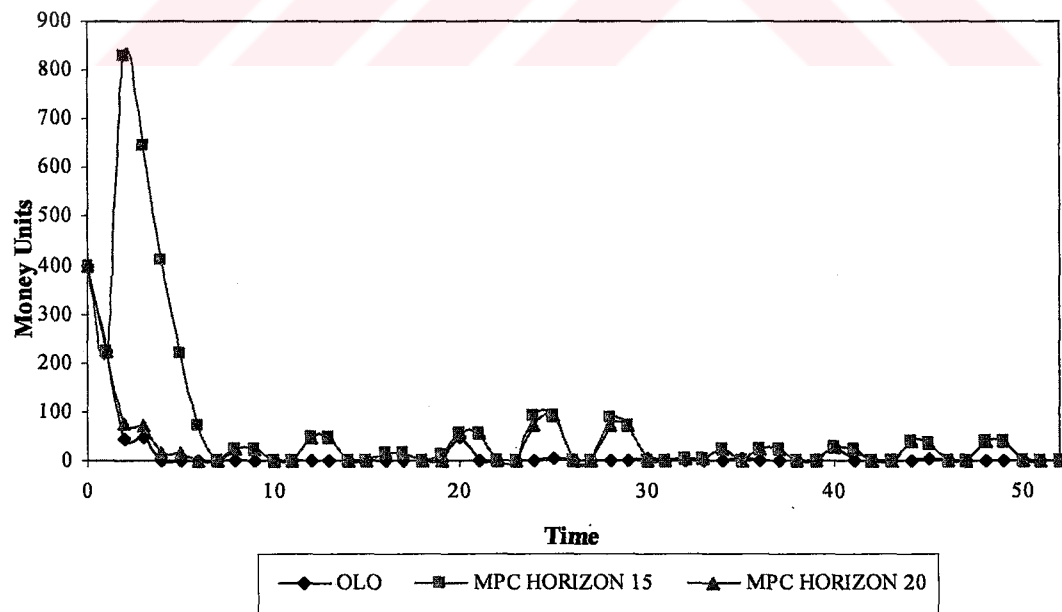


Figure 3.174: Penalty cost paid for product A (MPC vs open loop)

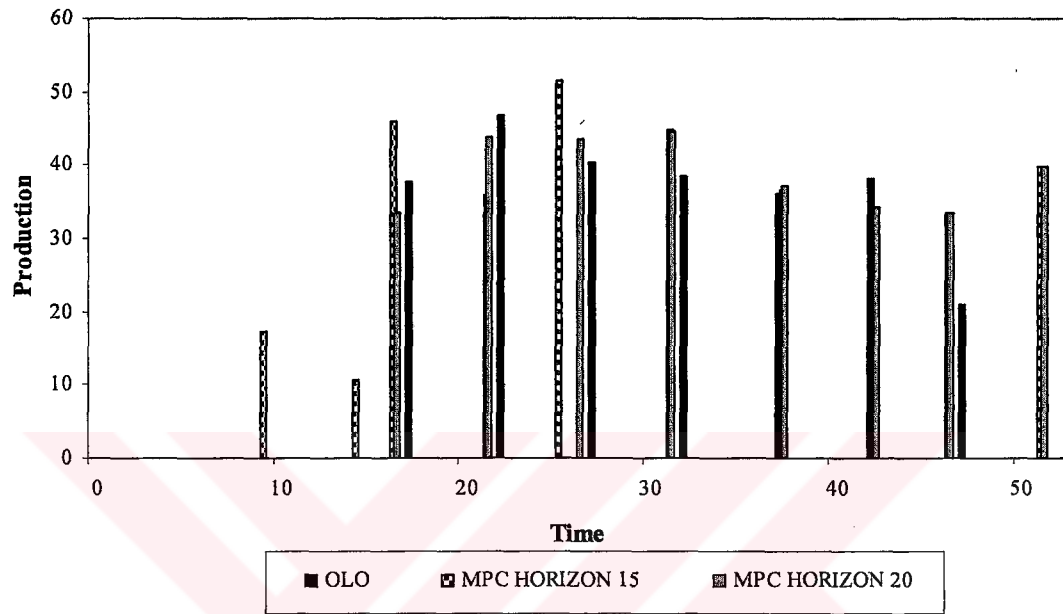


Figure 3.175: Optimal production schedule of the plant for product A (MPC vs open loop)

As in other cases, myopic characteristics of MPC result in more frequent and higher production amounts. MPC with horizon length 20 produces 8 times with a total production size of 310 units, while MPC with horizon length 15 produces 10 times with size 350 units of product A. OLO on the other hand produces 7 times with a total batch size of 259 units. Order levels, inventory levels and material transfer levels at other nodes are given in the following figures.

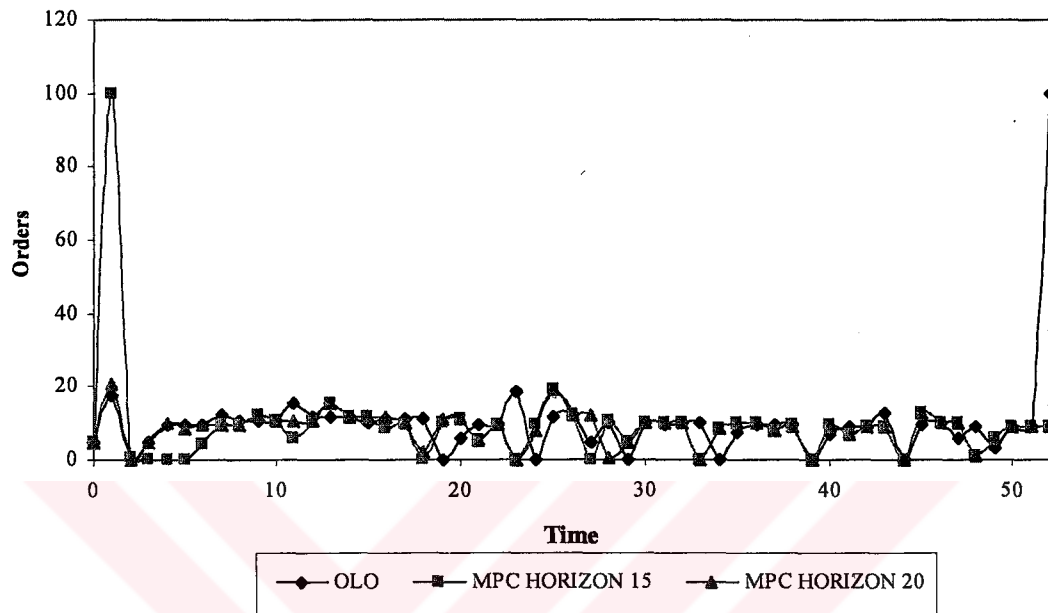


Figure 3.176: Retailer order levels for product A (MPC vs open loop)

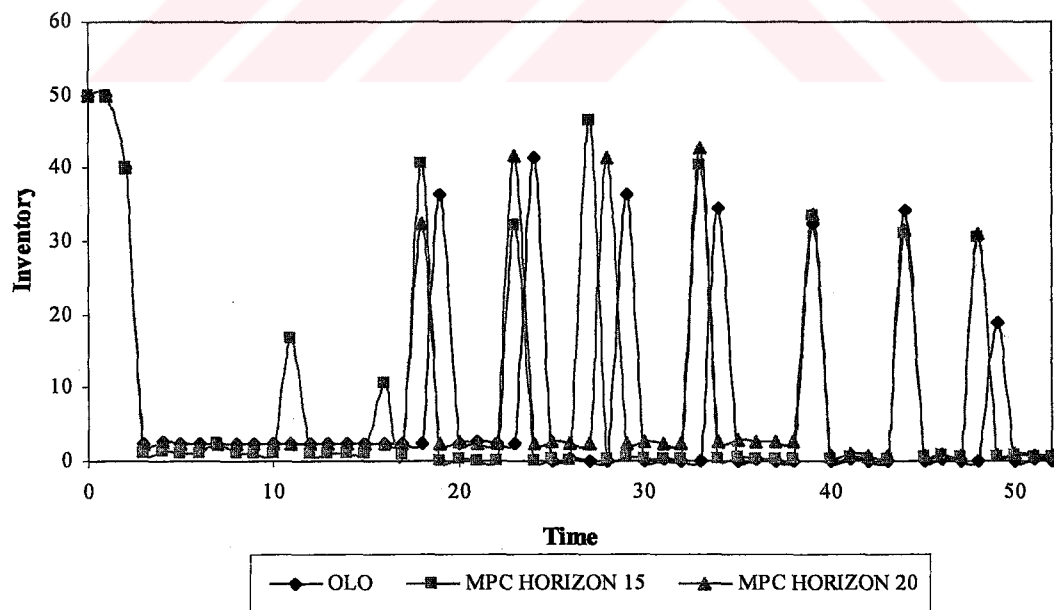


Figure 3.177: Distribution center inventory level for product A (MPC vs open loop)

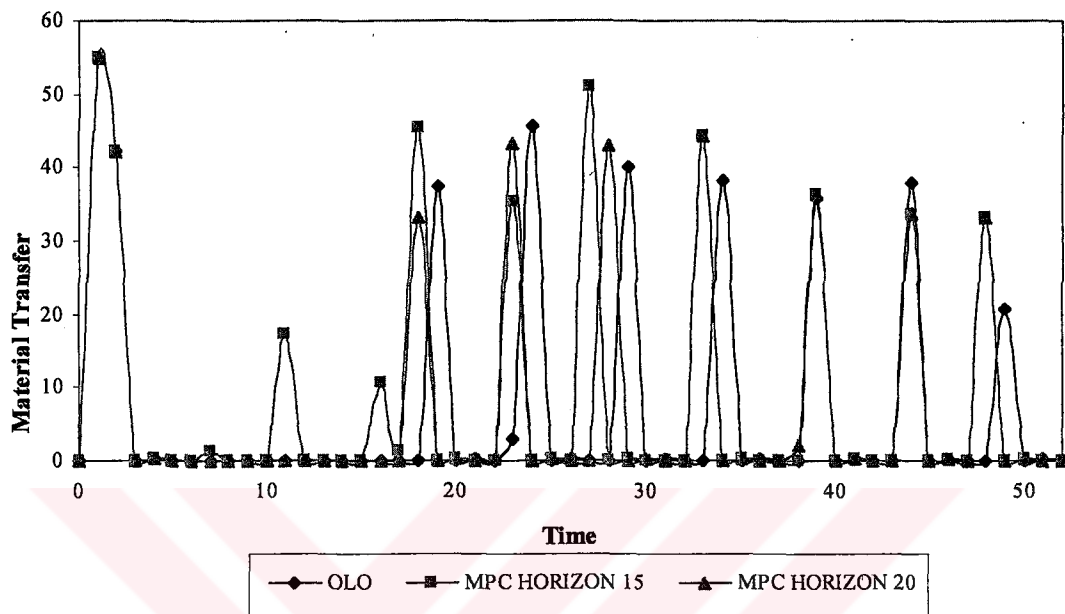


Figure 3.178: Distribution center material transfer level for product A (MPC vs open loop)

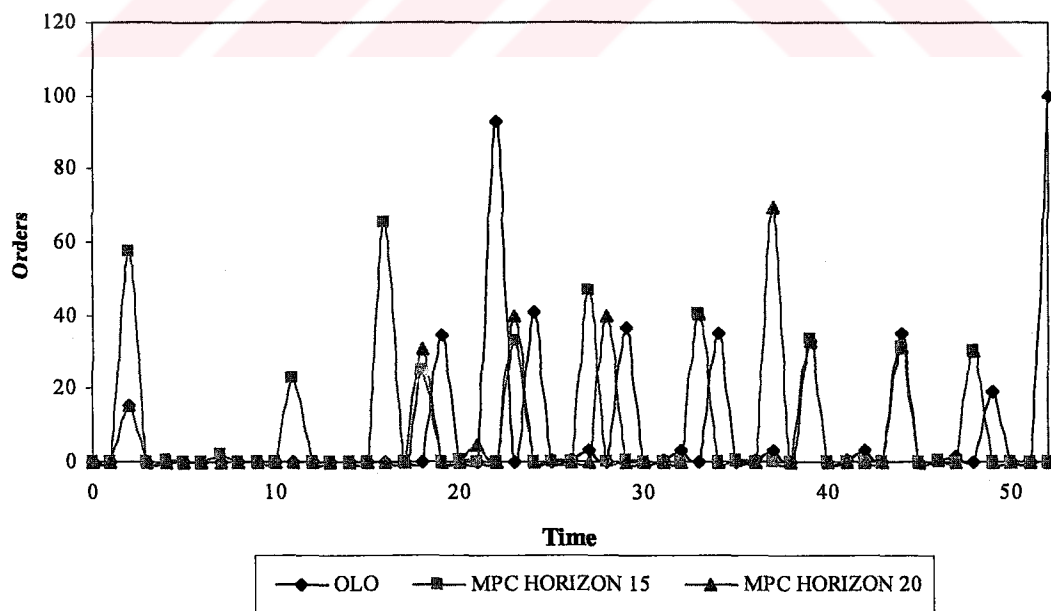


Figure 3.179: Distribution center order levels for product A (MPC vs open loop)

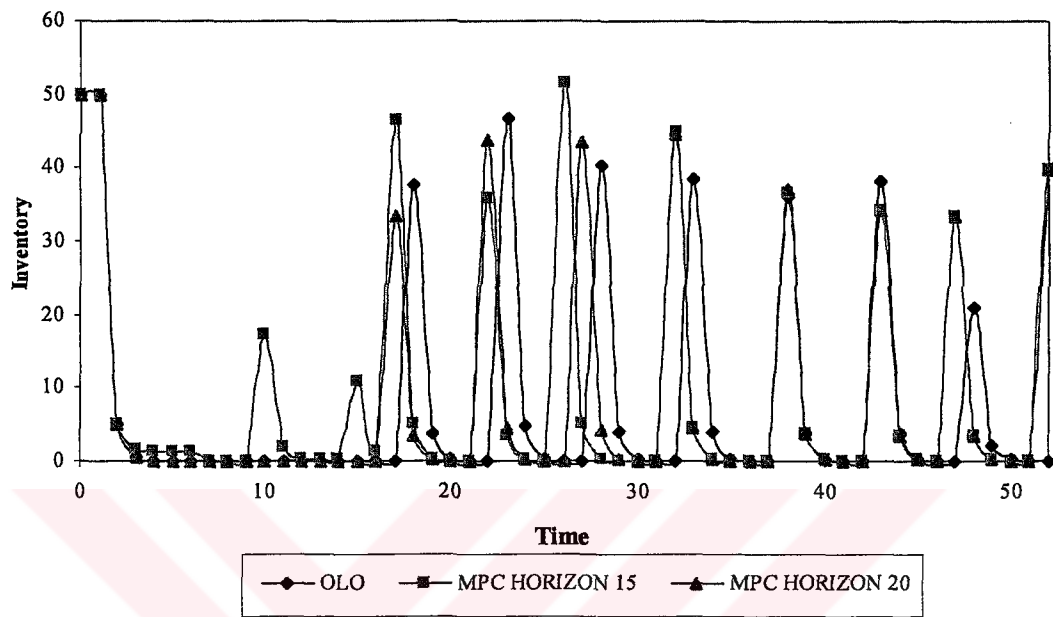


Figure 3.180: Warehouse inventory level for product A (MPC vs open loop)

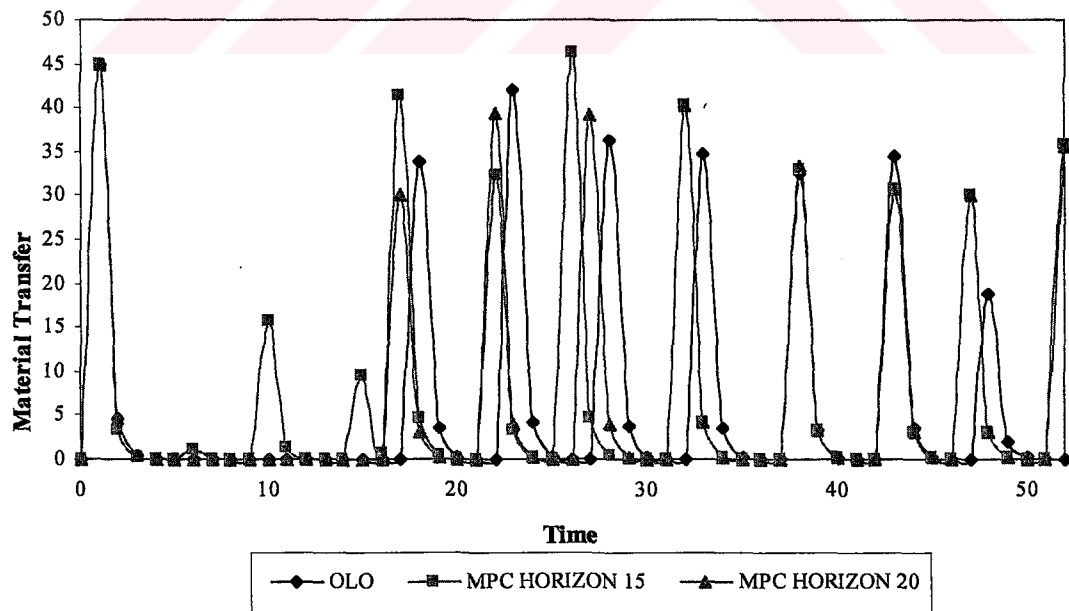


Figure 3.181: Warehouse material transfer level for product A (MPC vs open loop)

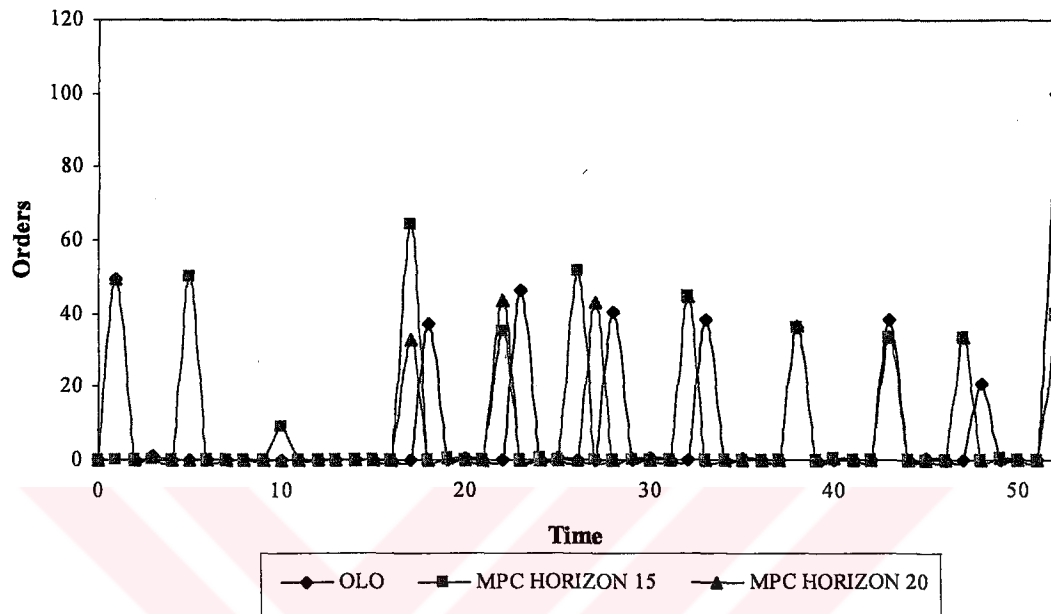


Figure 3.182: Warehouse order levels for product A (MPC vs open loop)

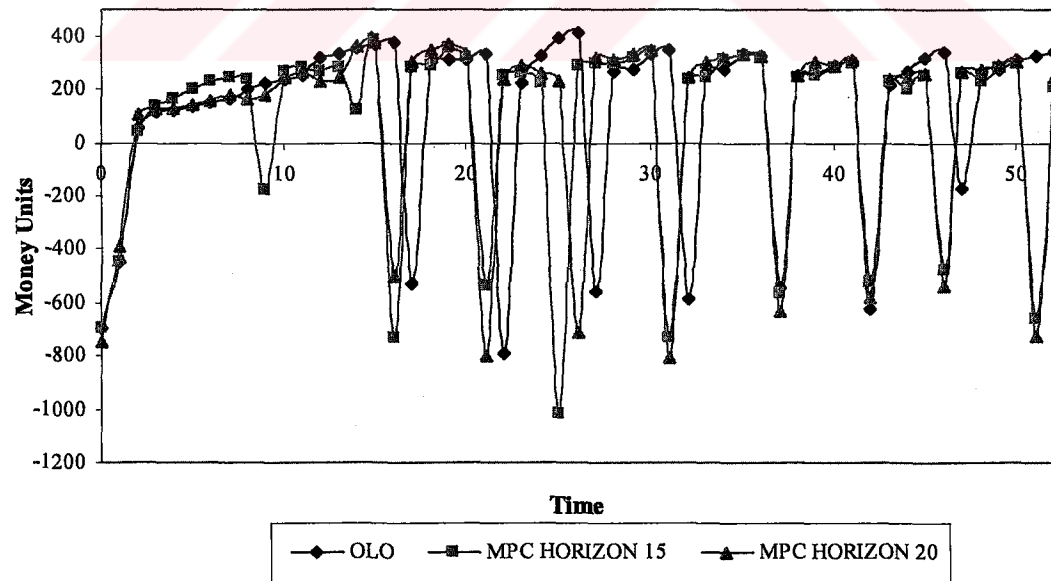


Figure 3.183: Instantaneous objective function for product A (MPC vs open loop)

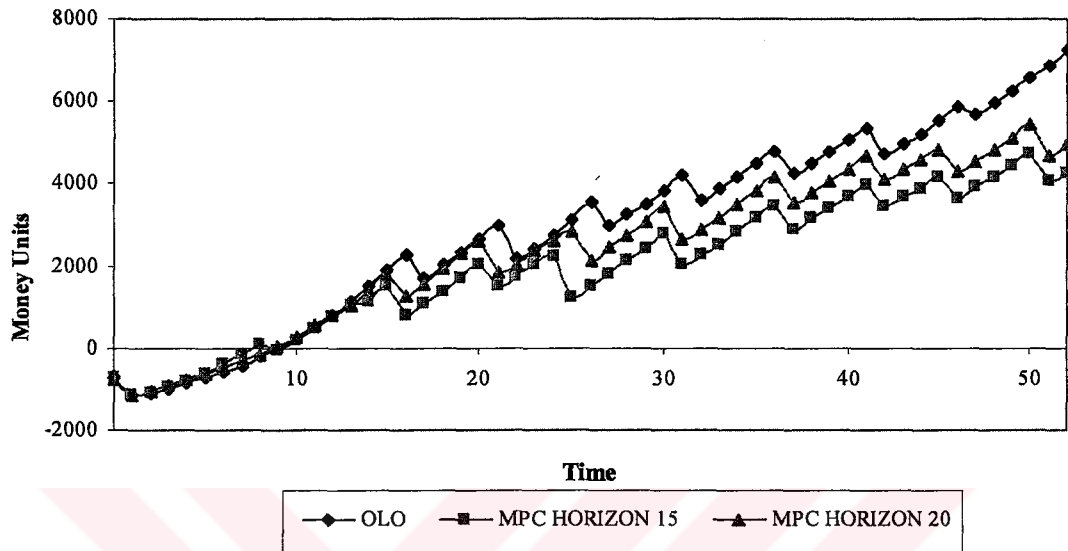


Figure 3.184: Cumulative objective function for product A(MPC vs open loop)

On the other hand, the inventory at retailer for product B does not show the same behavior as product A. (Fig. 3.185).

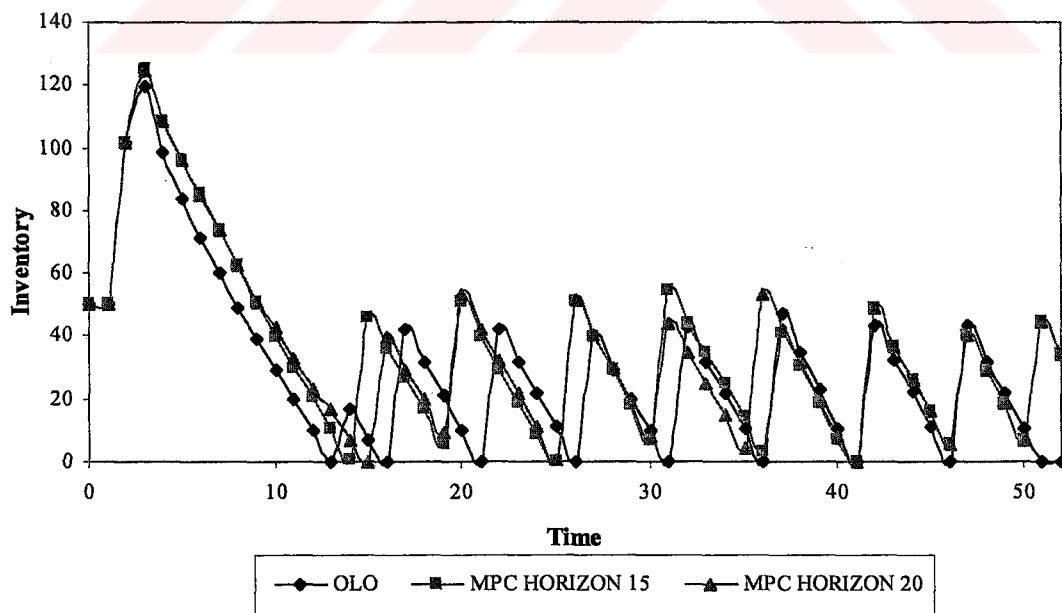


Figure 3.185: Retailer inventory level for product B (MPC vs open loop)

The cause of this behavior can be explained with the production schedule of product *A*. (Fig. 3.186). Looking at the first production time of product *A* for OLO (time=17, Fig. 3.175), and the production times of product *B* thereafter (Fig. 3.175), it is clear that the plant schedules products *A* and *B* successively, i.e. *A* is scheduled at time 17, *B* is scheduled at time 19, *A* is scheduled at time 22, *B* is scheduled at time 24 etc. On the other hand, observing the inventory of product *A* at retailer (Fig. 3.172), it is clear that the inventory of *A* behaves exactly as expected, whereas for product *B* (Fig. 3.185), the OLO depletes initial inventory in a shorter period of time compared to both MPC horizon lengths. From the production schedules of products *A* and *B* and the inventory behavior of product *A*, it is observed that the production schedule and the inventory behavior of product *B* is determined by the production schedule, inventory position of *A* and demand pattern of *A*, giving priority to product *A*. The reason for this is, although both products have the same selling price and inventory, material transfer, and customer dissatisfaction costs, product *A* has higher production costs compared to product *B*. Product *B*'s significantly lower production costs, make it more flexible, therefore production priority is given to product *A*. It is clear that the open loop optimization decides to deplete the inventory of product *B* in order not to carry excess inventory when the production from the plant arrives, which does not imply a myopic depletion policy, but implies a tradeoff in the customer satisfaction costs, inventory carrying costs, material transfer costs and the production costs. In terms of the total profit obtained at the end for product *B*, OLO has a value of 14607 monetary units which is higher than the profits obtained by MPC 15 and MPC 20 with values 12143 and 12273 monetary units respectively.

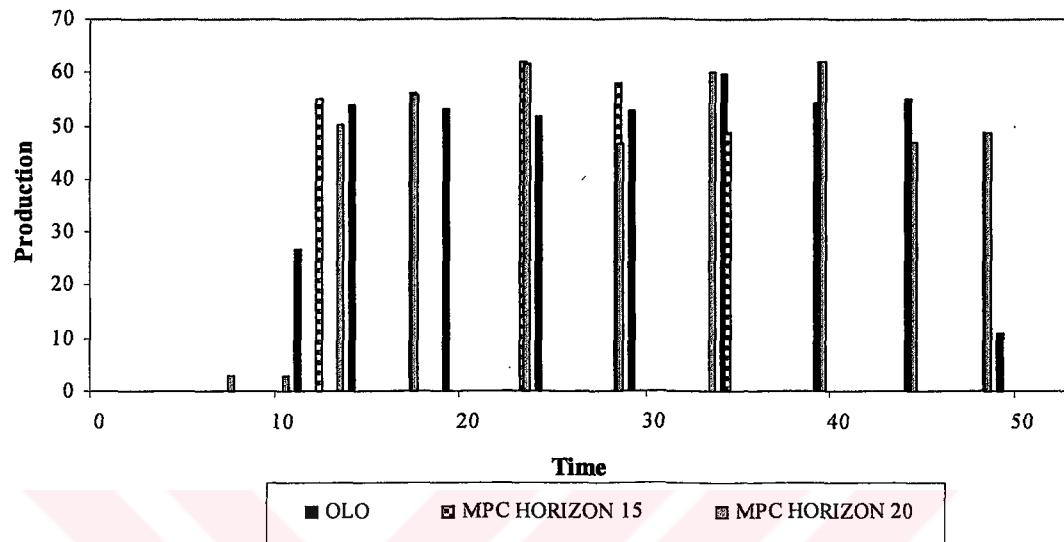


Figure 3.186: Optimal production schedule of the plant for product B (MPC vs open loop)

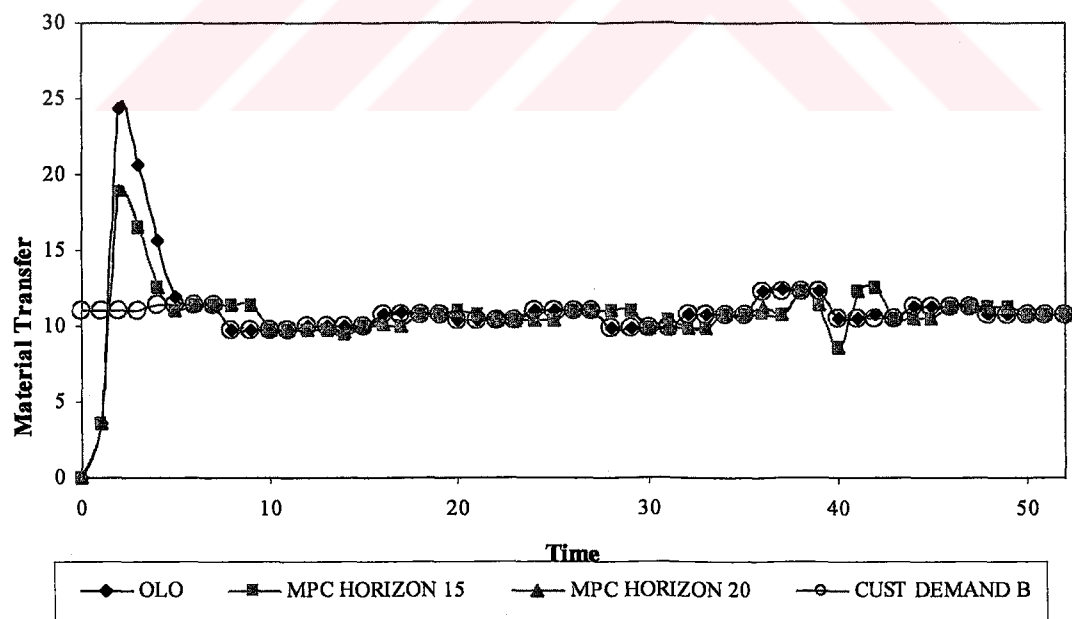


Figure 3.187: Retailer material transfer level for product B (MPC vs open loop)

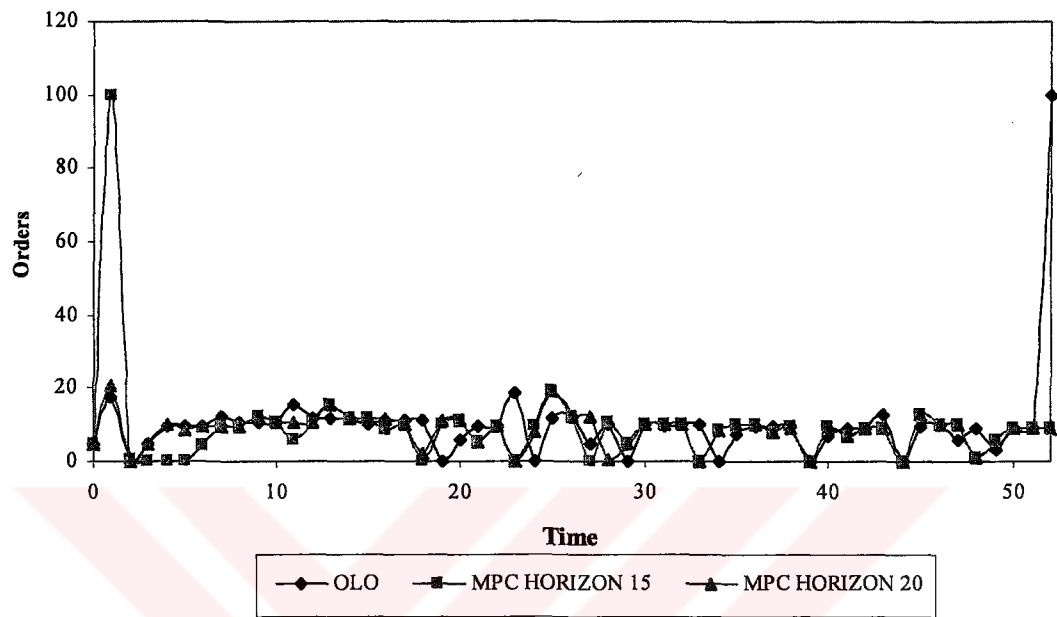


Figure 3.188: Retailer order levels for product B (MPC vs open loop)

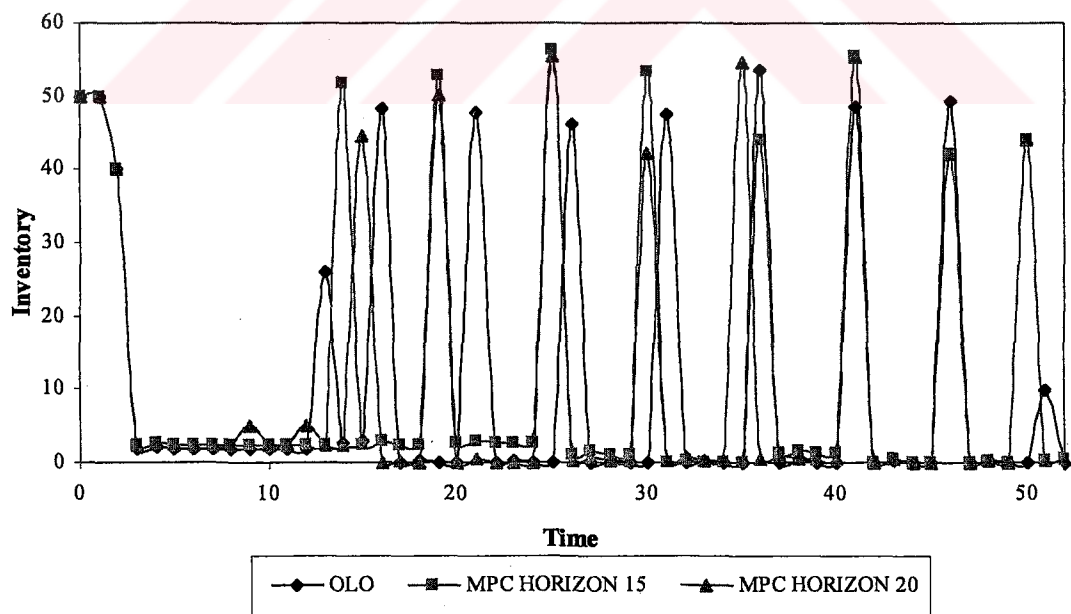


Figure 3.189: Distribution center inventory level for product B (MPC vs open loop)

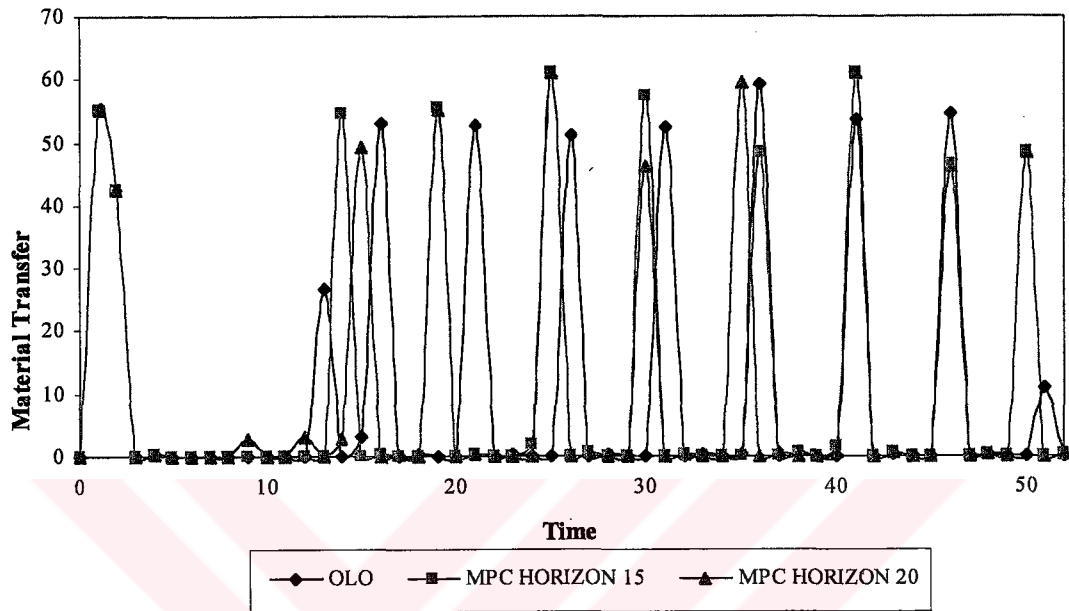


Figure 3.190: Distribution center material transfer level for product B (MPC vs open loop)

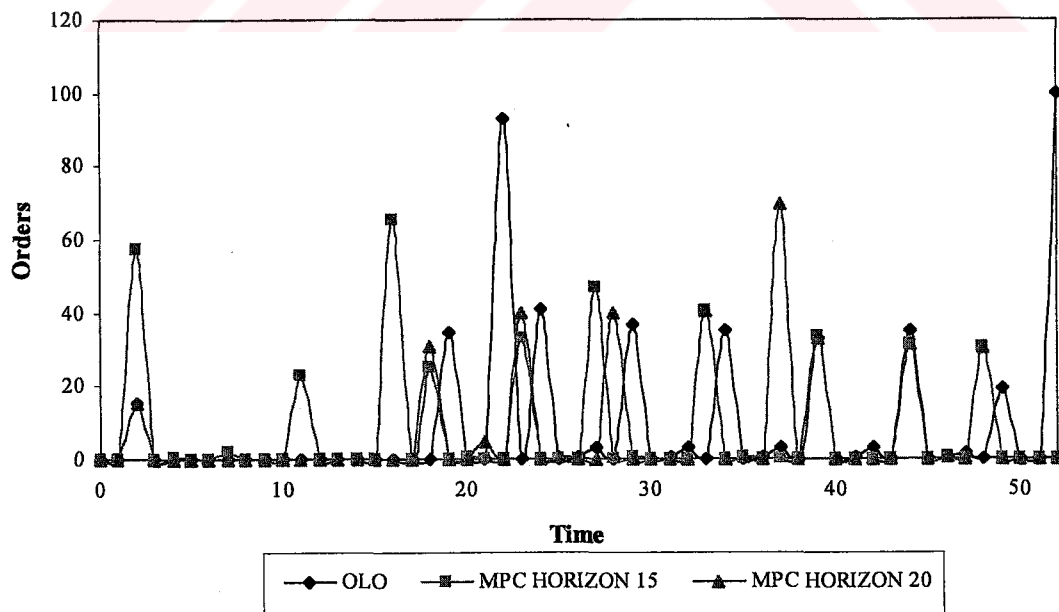


Figure 3.191: Distribution center order levels for product B (MPC vs open loop)

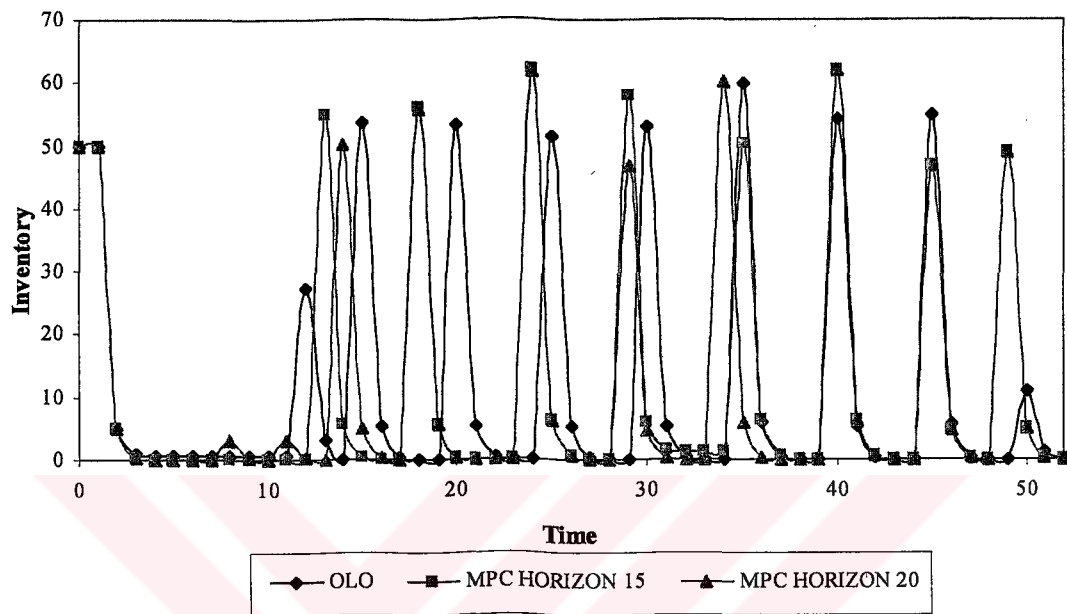


Figure 3.192: Warehouse inventory level for product B (MPC vs open loop)

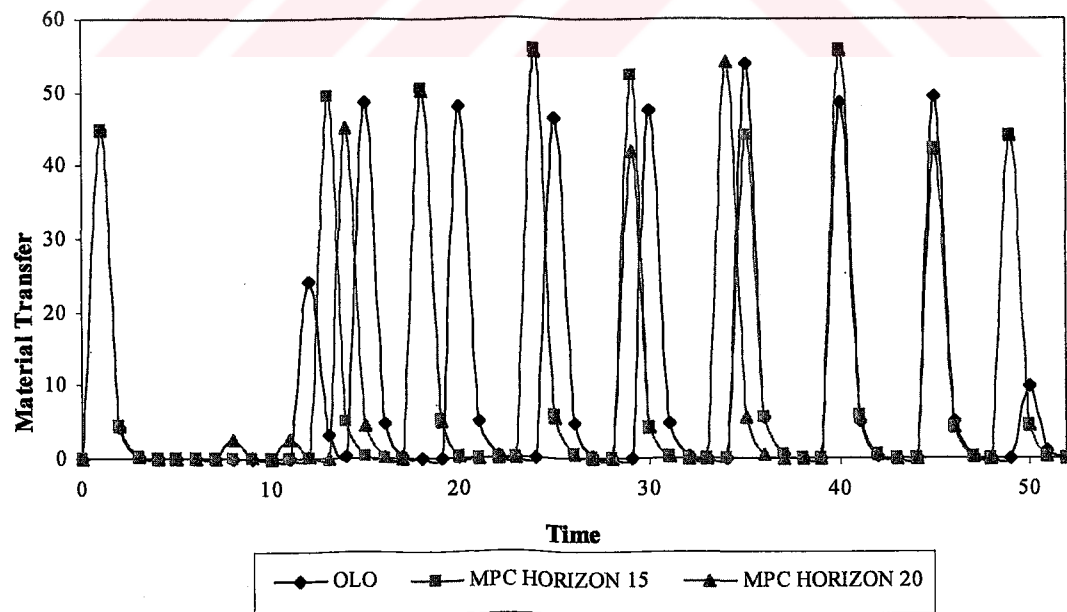


Figure 3.193: Warehouse material transfer level for product B (MPC vs open loop)

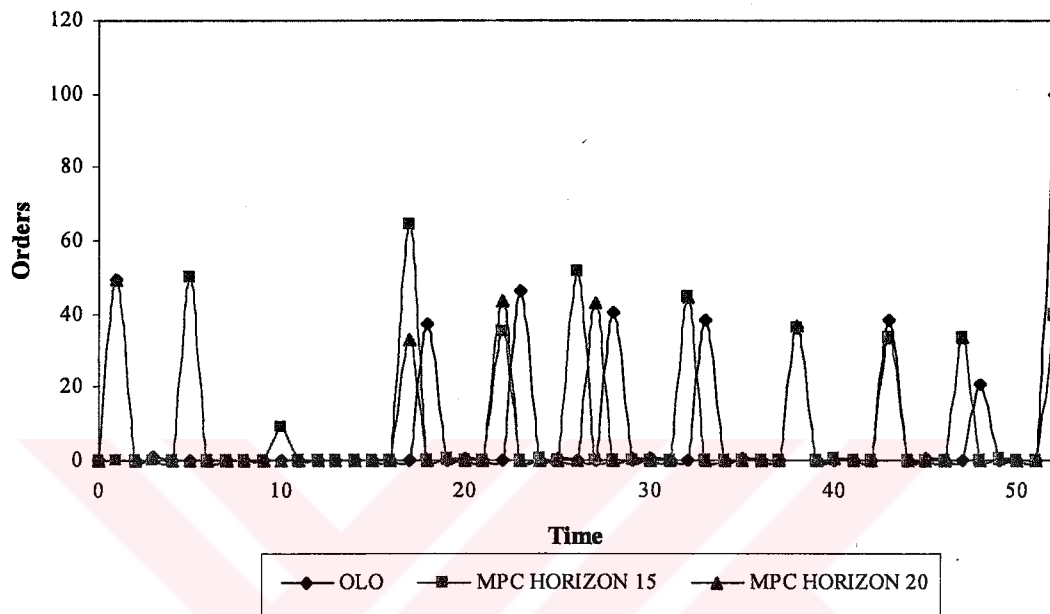


Figure 3.194 Warehouse order levels for product B (MPC vs open loop)

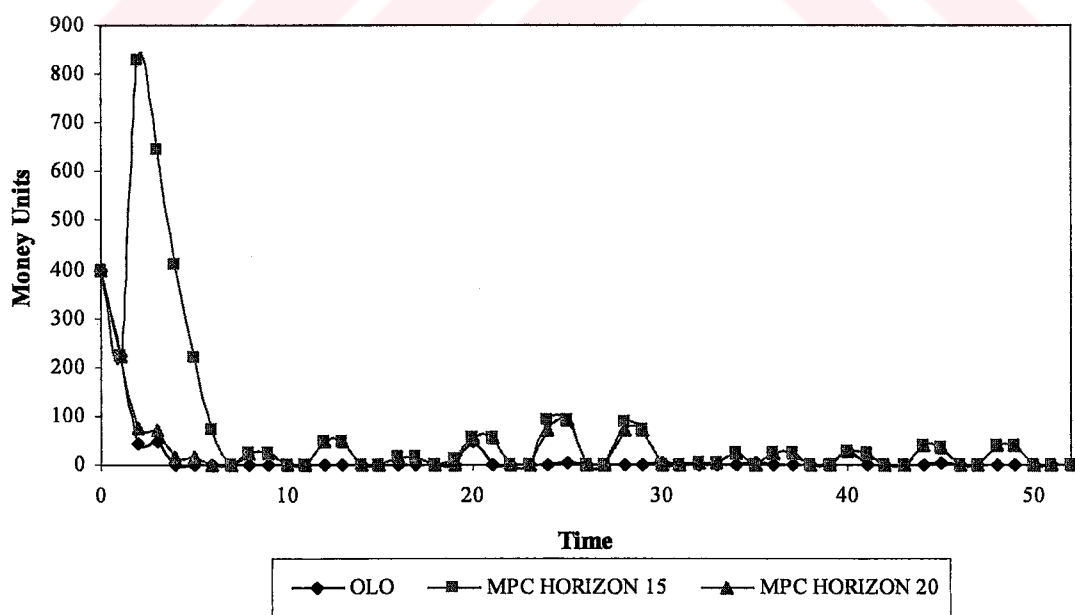


Figure 3.195: Penalty cost paid for product B (MPC vs open loop)

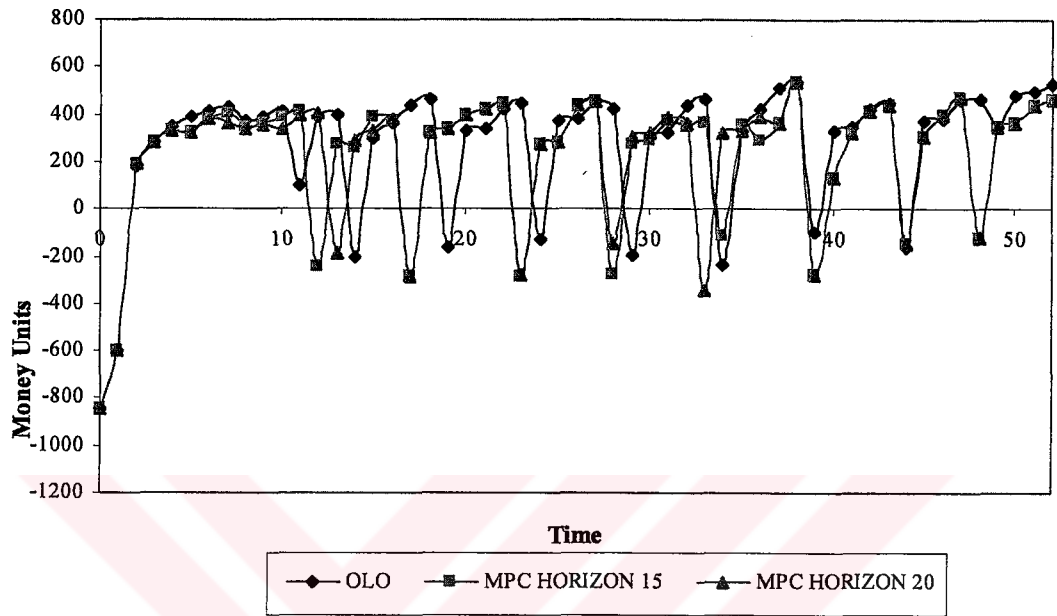


Figure 3.196: Instantaneous objective function for product B(MPC vs open loop)

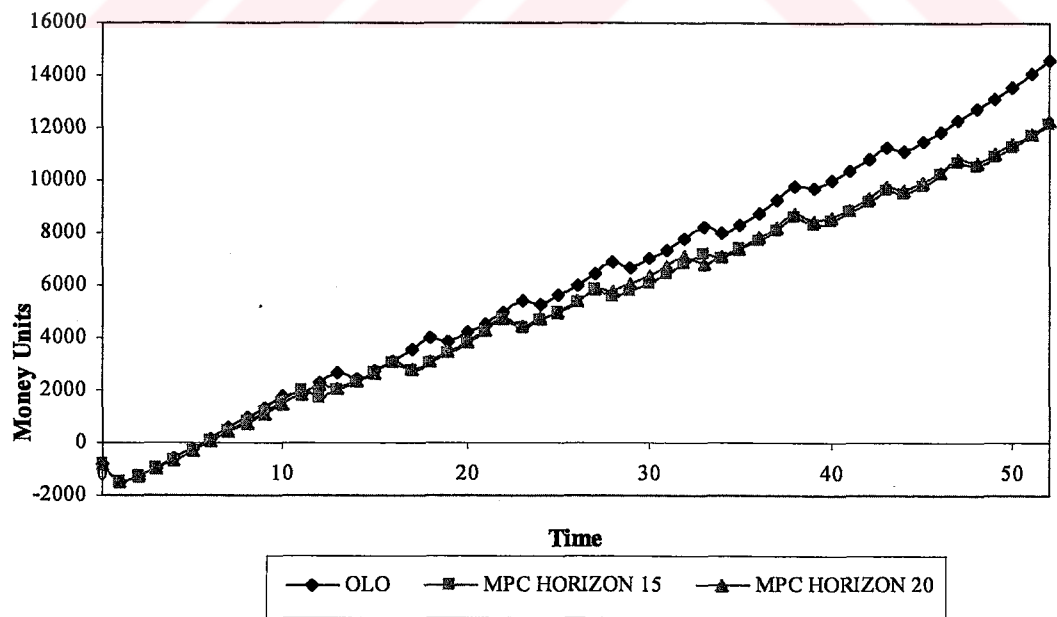


Figure 3.197: Cumulative objective function for product B (MPC vs open loop)

Product *C* follows the same behavior patterns suggested for product *A*, arising from the myopic characteristics of MPC optimization. Inventory levels (Fig. 3.198) are depleted in earlier times compared with the OLO case and in time production of *C* takes place, increasing the total profit drastically due to high production costs. Total objective function value obtained by OLO at the end of the process horizon is -4218 monetary units, which is very high compared with the profits of -11818 and -9789 monetary units for MPC 15 and MPC 20, despite the high customer satisfaction penalties paid. (Fig. 3.199)

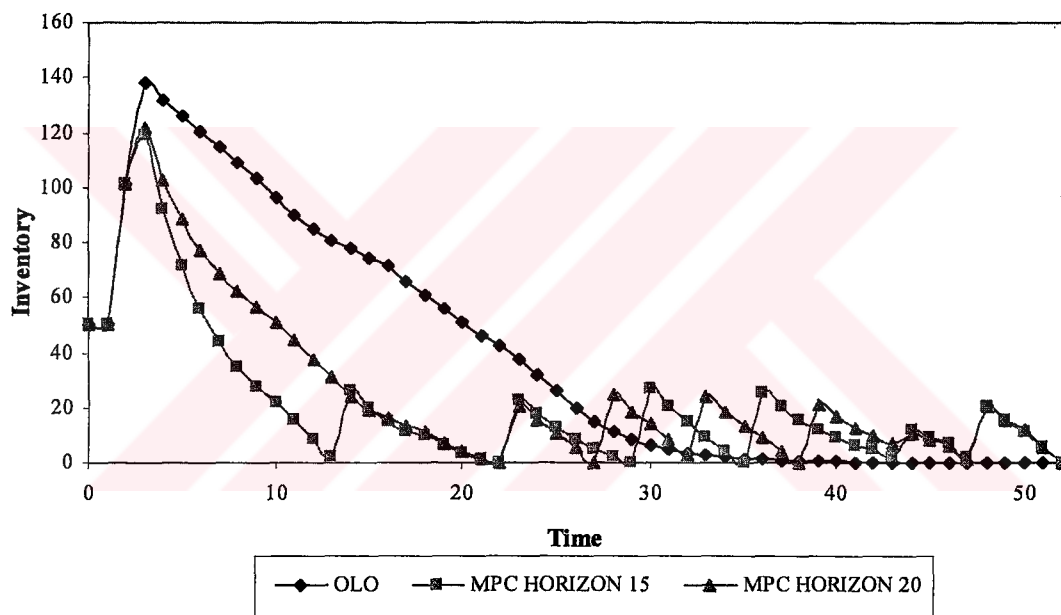


Figure 3.198: Retailer inventory level for product C (MPC vs open loop)

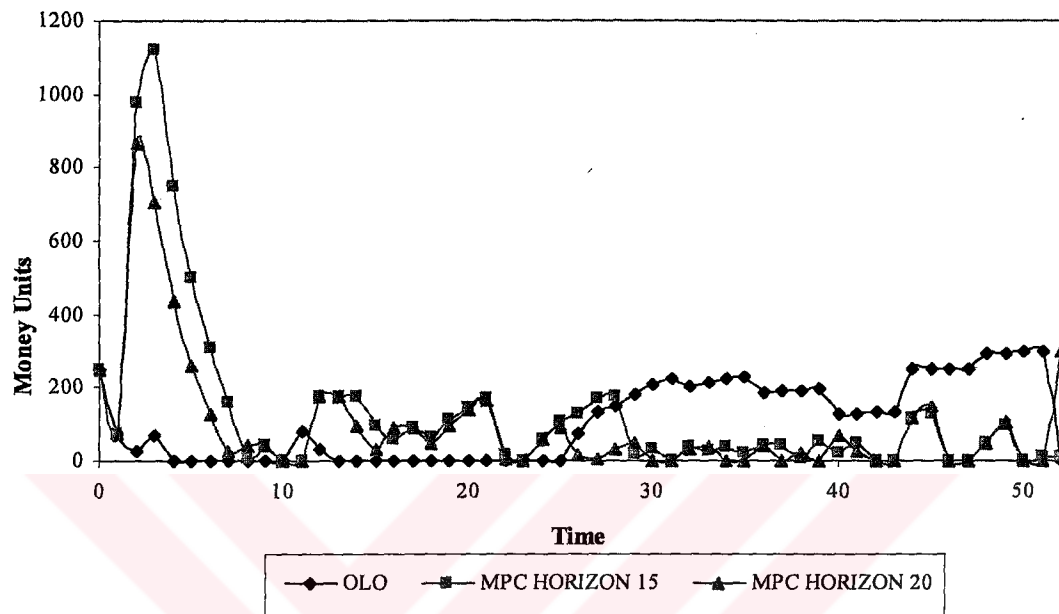


Figure 3.199: Penalty cost paid for product C (MPC vs open loop)

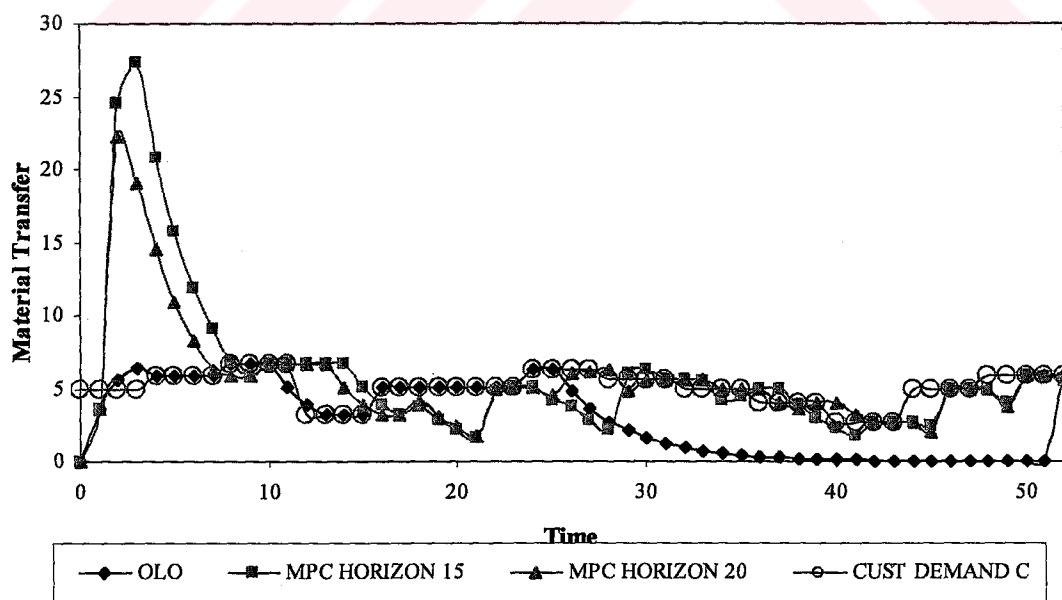


Figure 3.200: Mat. transferred to cust. vs actual cost. demand for product C (MPC vs open loop)

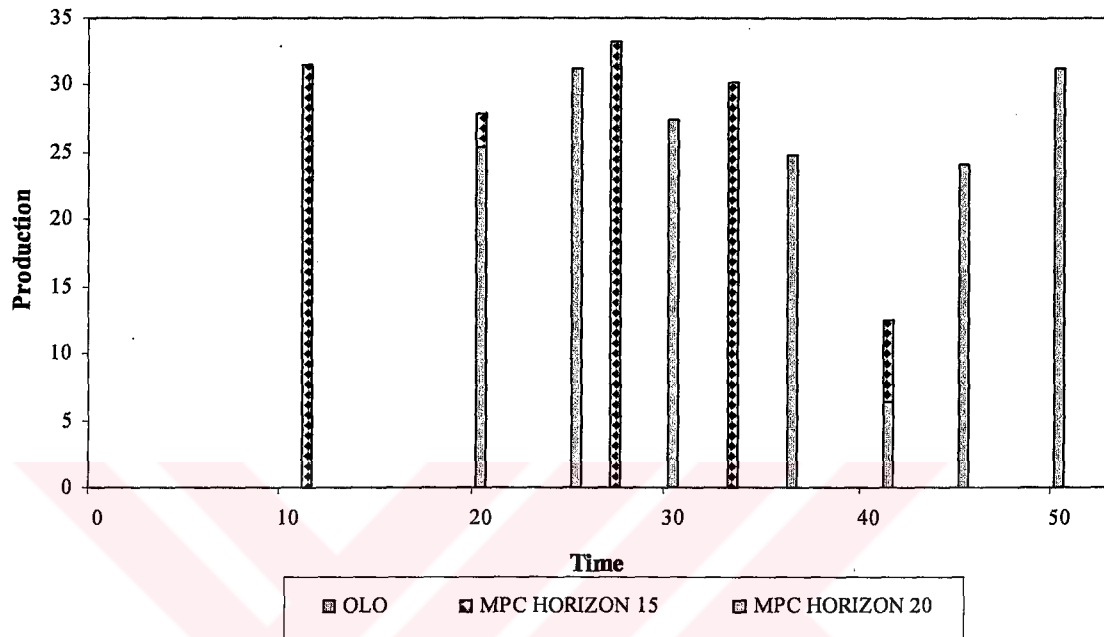


Figure 3.201: Optimal production schedule of the plant for product C (MPC vs open loop)

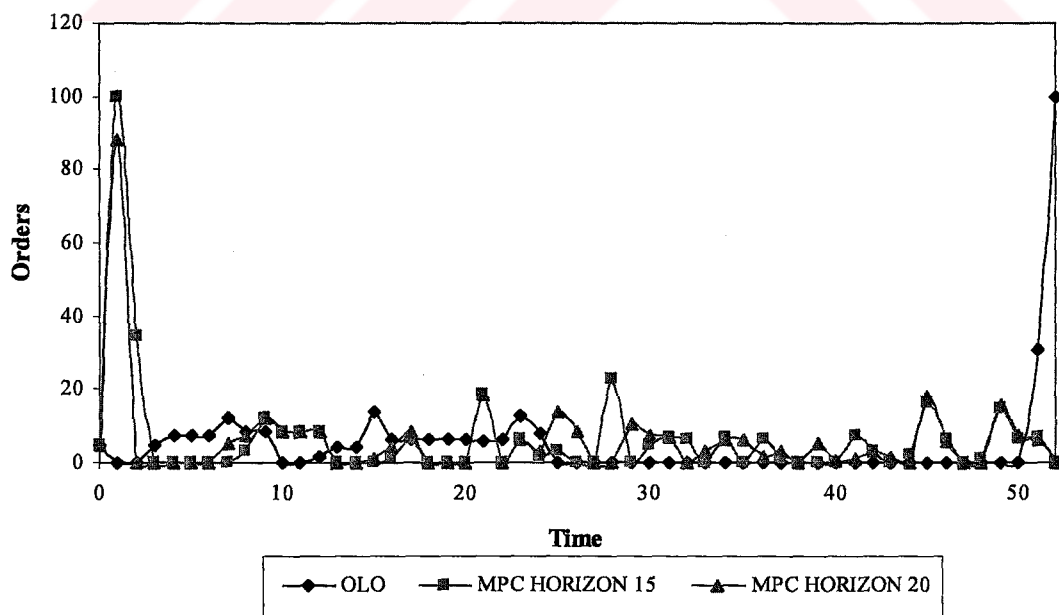


Figure 3.202: Retailer order levels for product C (MPC vs open loop)

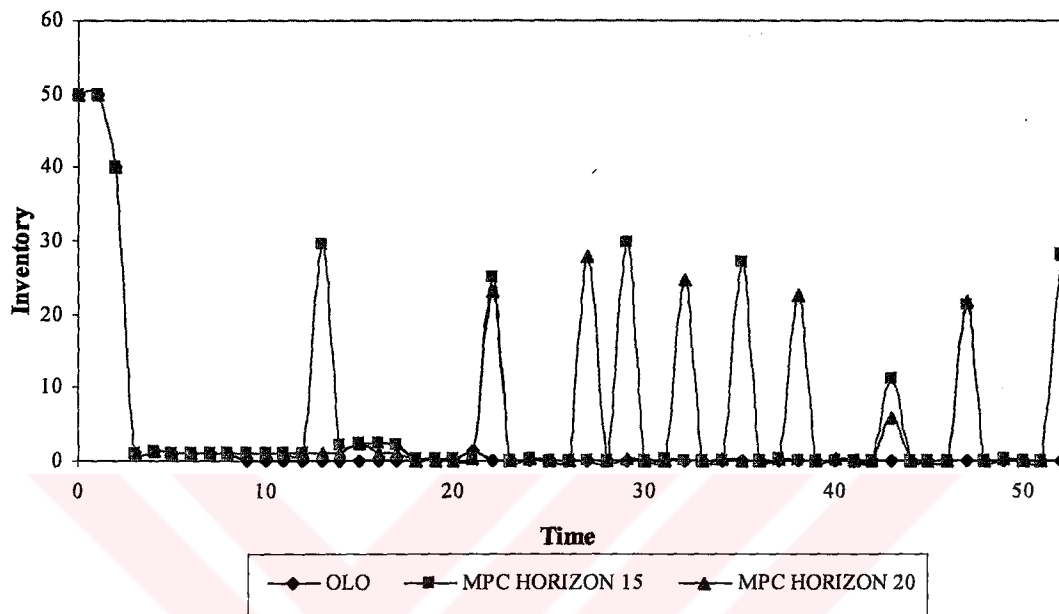


Figure 3.203: Distribution center inventory level for product C (MPC vs open loop)

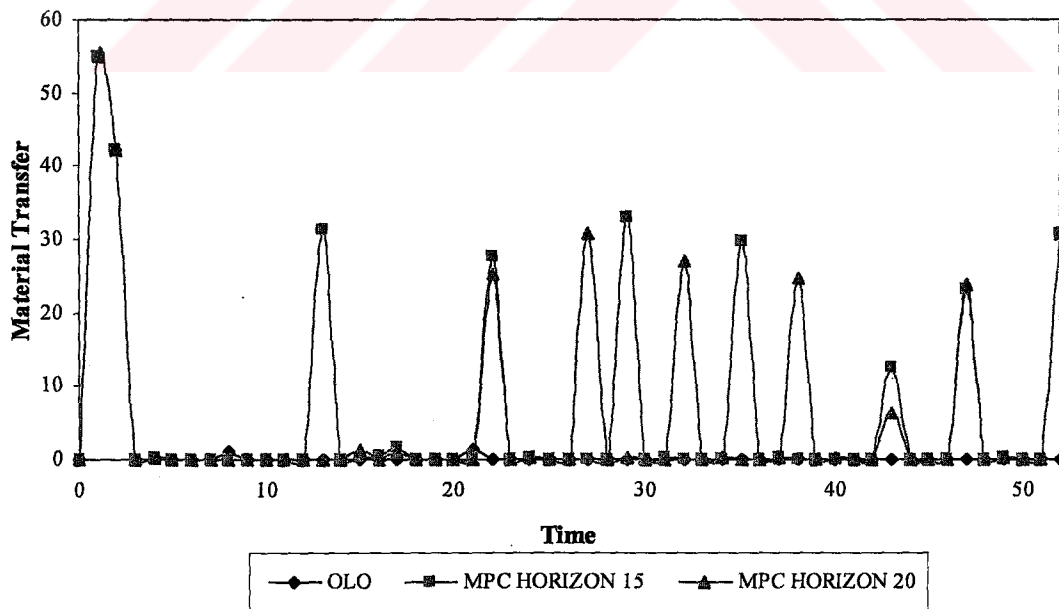


Figure 3.204: Distribution center material transfer level for product C (MPC vs open loop)

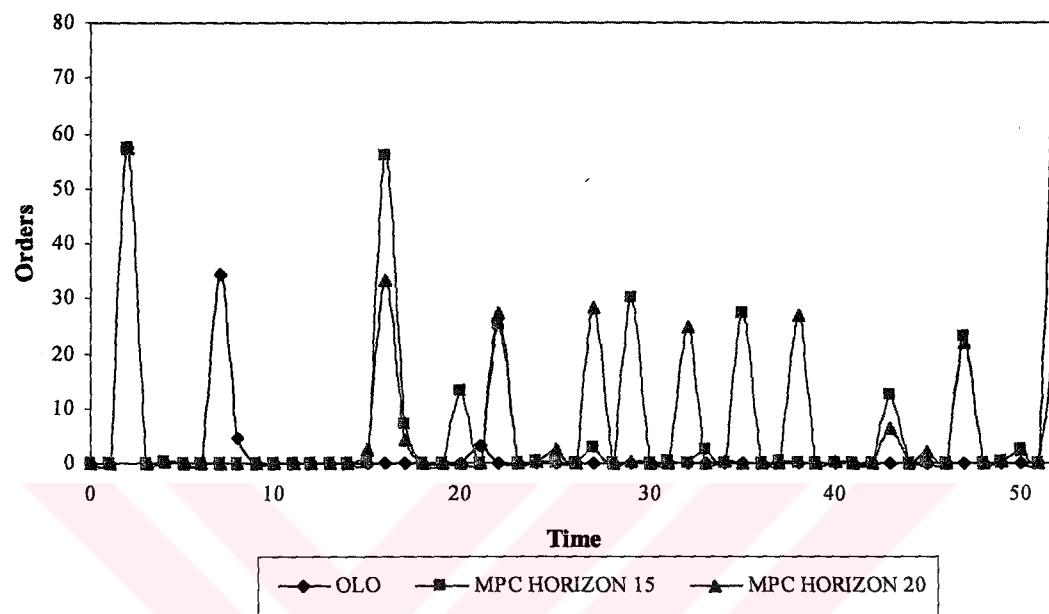


Figure 3.205: Distribution center order levels for product C (MPC vs open loop)

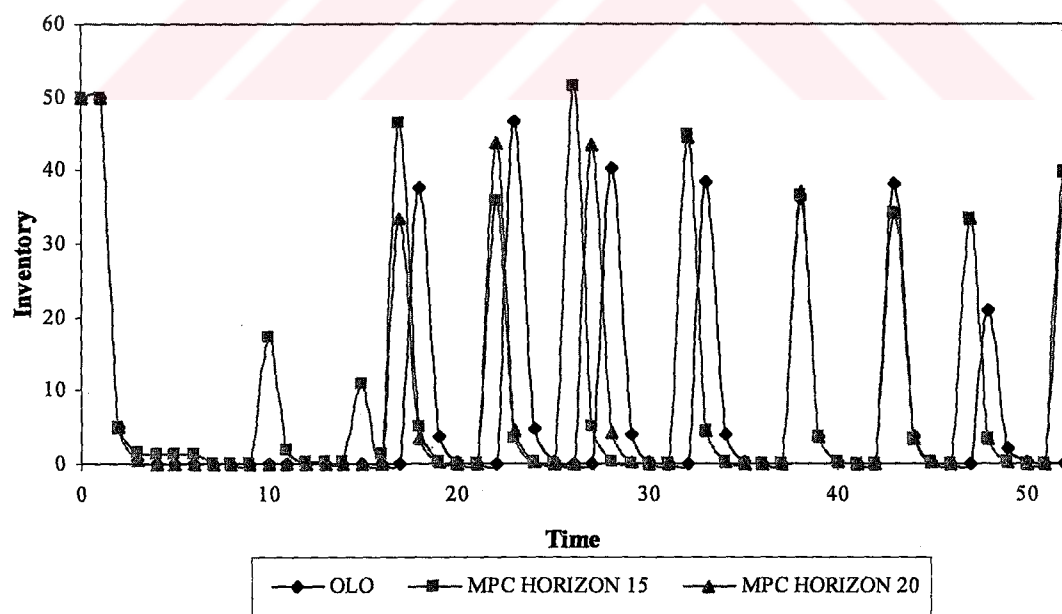


Figure 3.206: Warehouse inventory level for product C (MPC vs open loop)

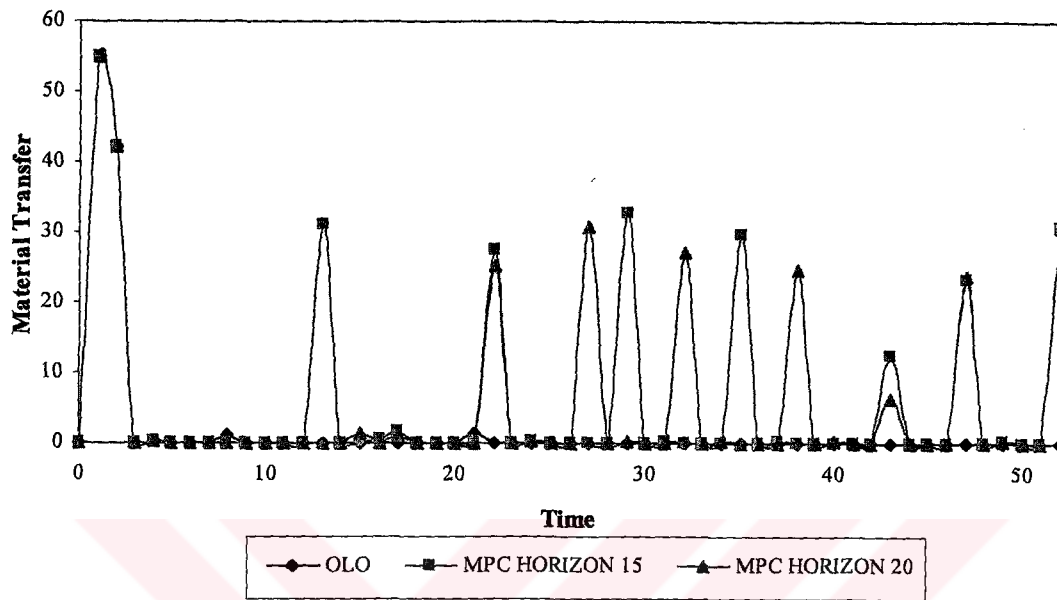


Figure 3.207: Warehouse material transfer level for product C (MPC vs open loop)

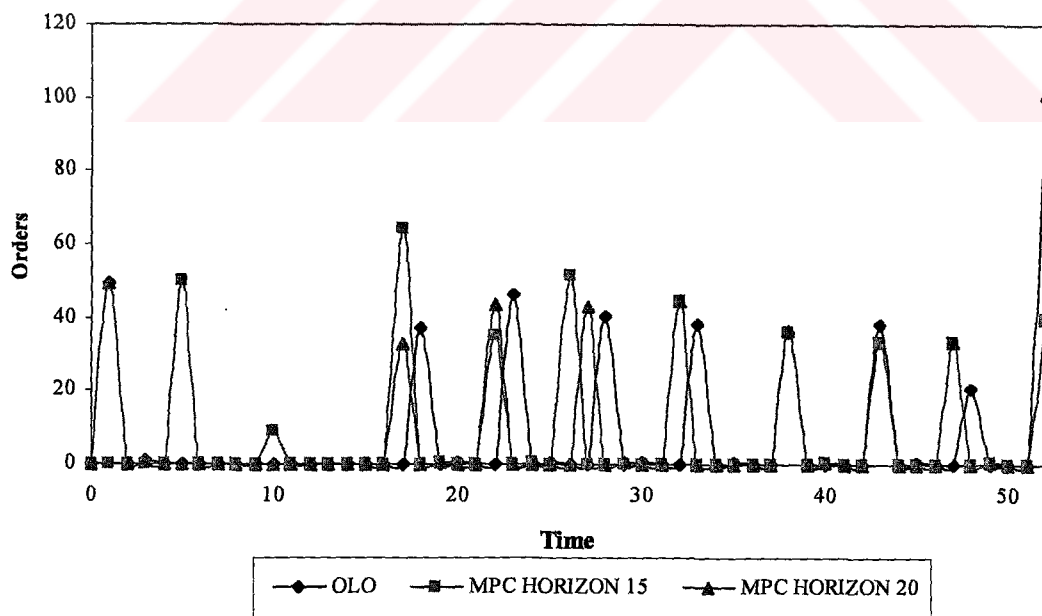


Figure 3.208: Warehouse order levels for product C (MPC vs open loop)

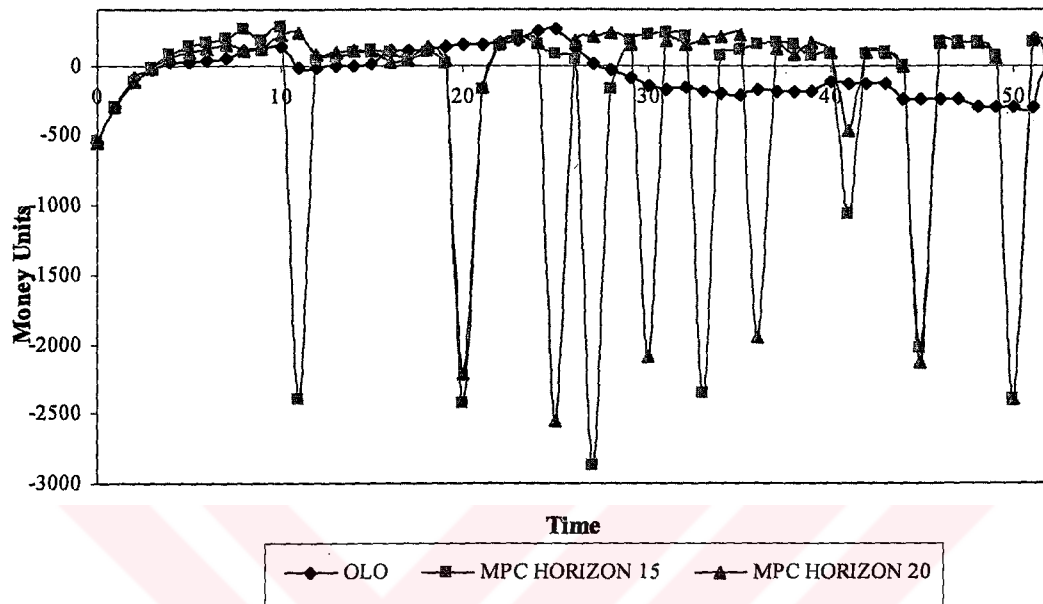


Figure 3.209: Instantaneous objective function for product C(MPC vs open loop)

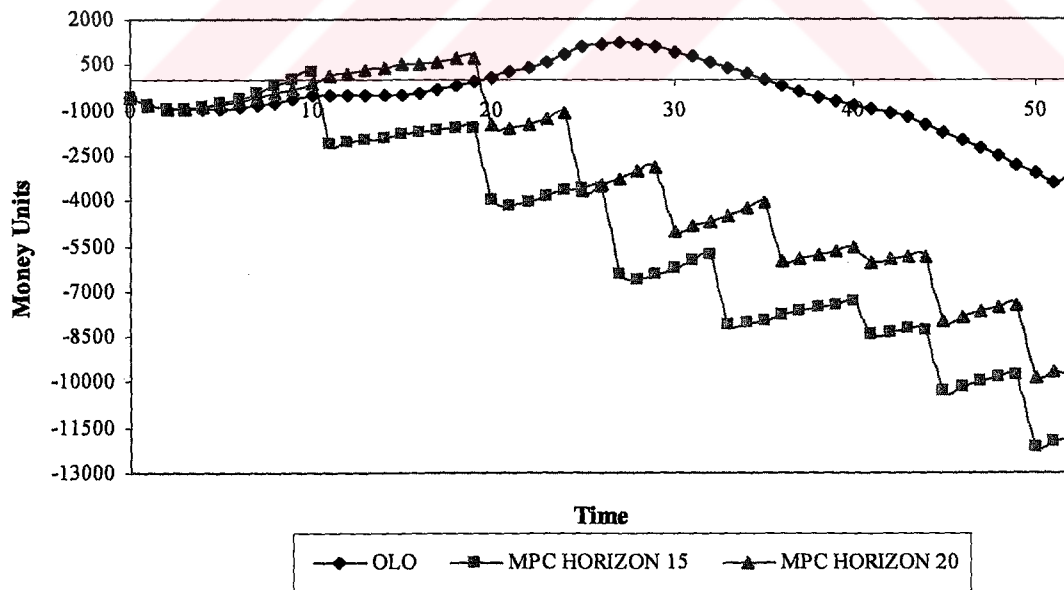


Figure 3.210: Cumulative objective function for product C(MPC vs open loop)

3.7 Modeling of a Three-Product Hybrid Supply Chain with Unpredictable Demand Change

In all the previous cases observed, customer demand was modeled as integrated white noise whose equation was given in equation 10 of section 2. However, in real life, deviations from the modeled demand are possible due to various external factors. In this section MPC performance against unpredictable demand changes will be studied.

In this case demand follows the old white noise model until the 16th time unit, thereafter, demand changes around the same mean following a sinusoidal pattern until the 40th time unit. The demand profiles for the three products under these conditions are given in Fig. 3.211.

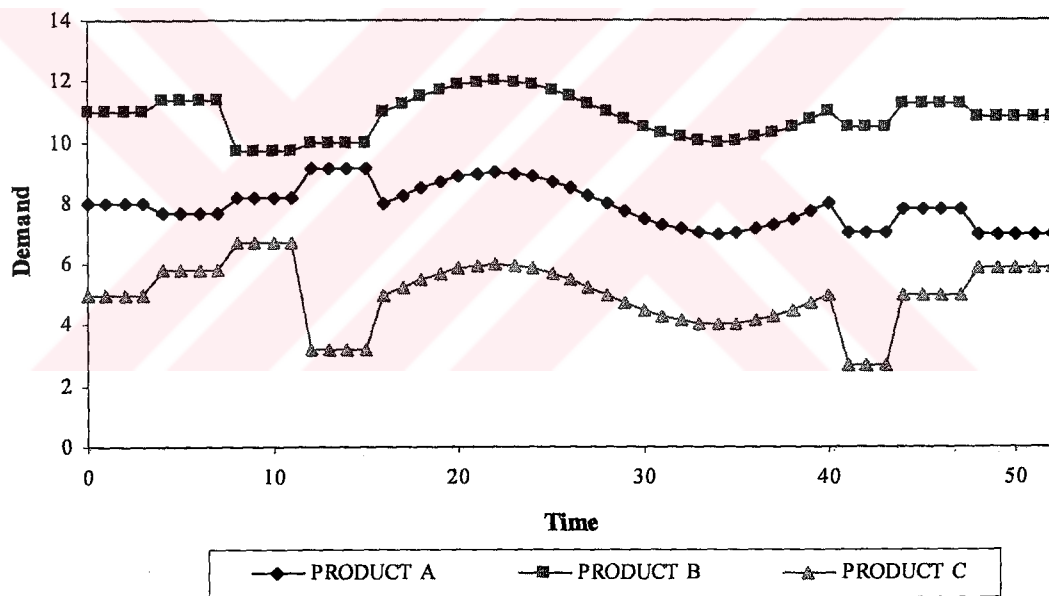


Figure 3.211: New Demand Profiles

OLO sees all the future demand, including the unpredictable changes, on the other hand MPC does not change its prediction scheme and assumes the demand to remain constant for all future values. All demand profile changes take place at the same time for all three products.

For the sinusoidal demand change encountered in this case, MPC predicts the demand as constant for every four time units as shown in Fig. 3.212, for product A.

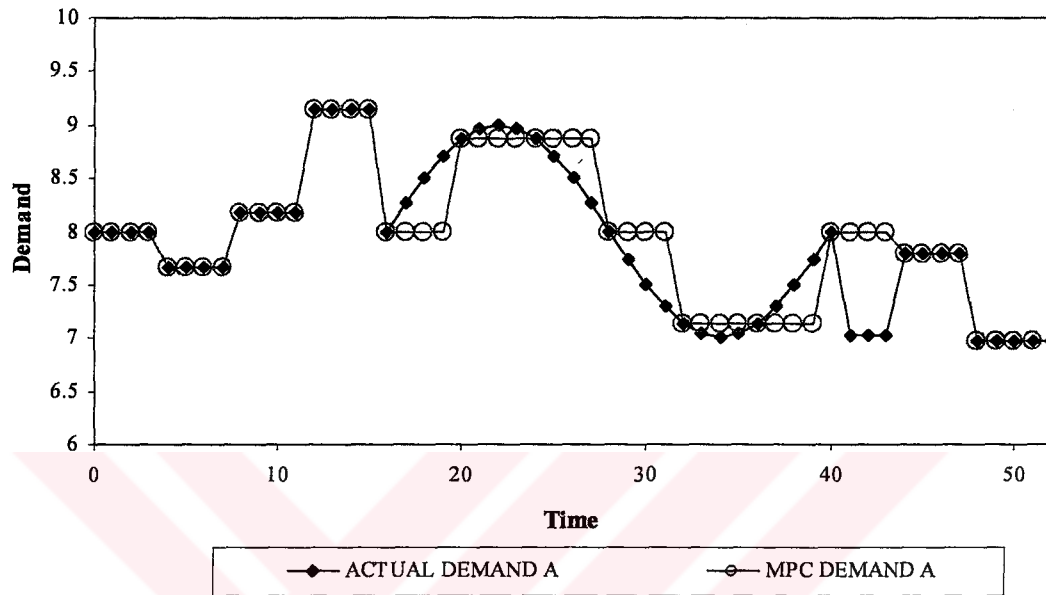


Figure 3.212: Actual demand vs MPC interpretation of Demand for product A

Between 16th - 20th , 20th -24th , and 36th- 40th time units, MPC underestimates the actual customer demand. Between 24th - 28th, 28th - 32nd , 32nd- 36th , and 40th - 44th time units MPC overestimates the actual customer demand. Except between 40th and 44th time units all underestimated amounts are equal to the overestimated amounts.

OLO sees all the demand change in each time unit, hence takes its precautions taking these timely demand changes into account.

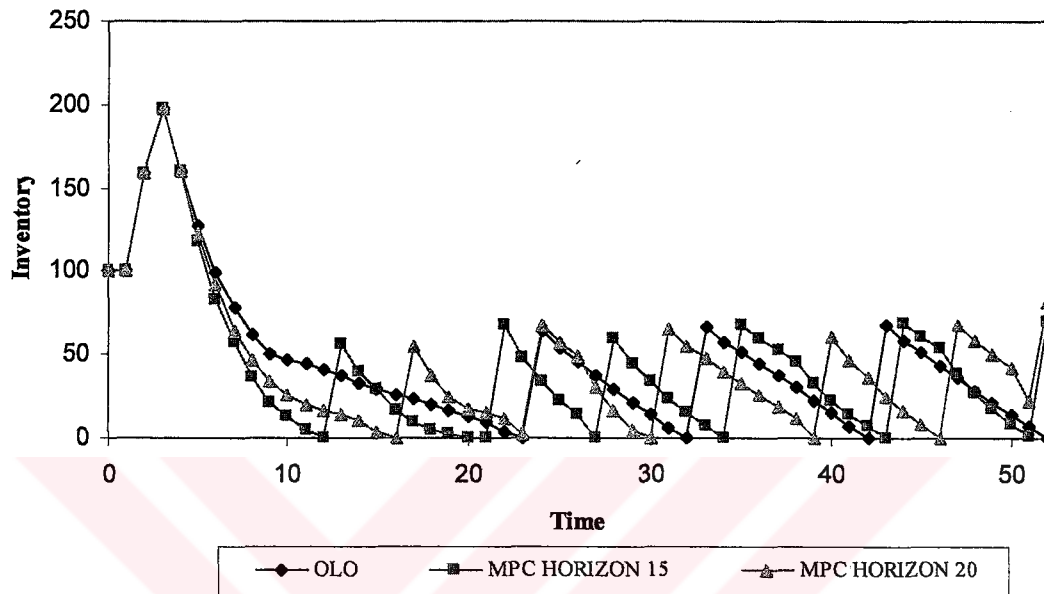


Figure 3.213: Inventory changes for the sinusoidal change in demand (MPC vs OLO)

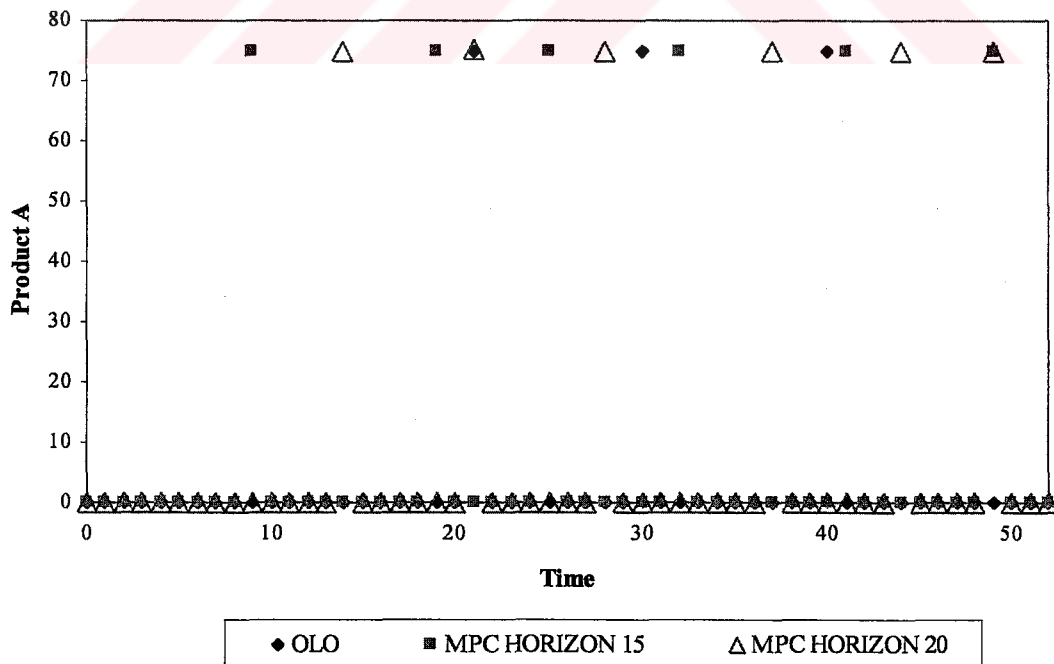


Figure 3.214: Production schedule for the sinusoidal change in demand (MPC vs OLO)

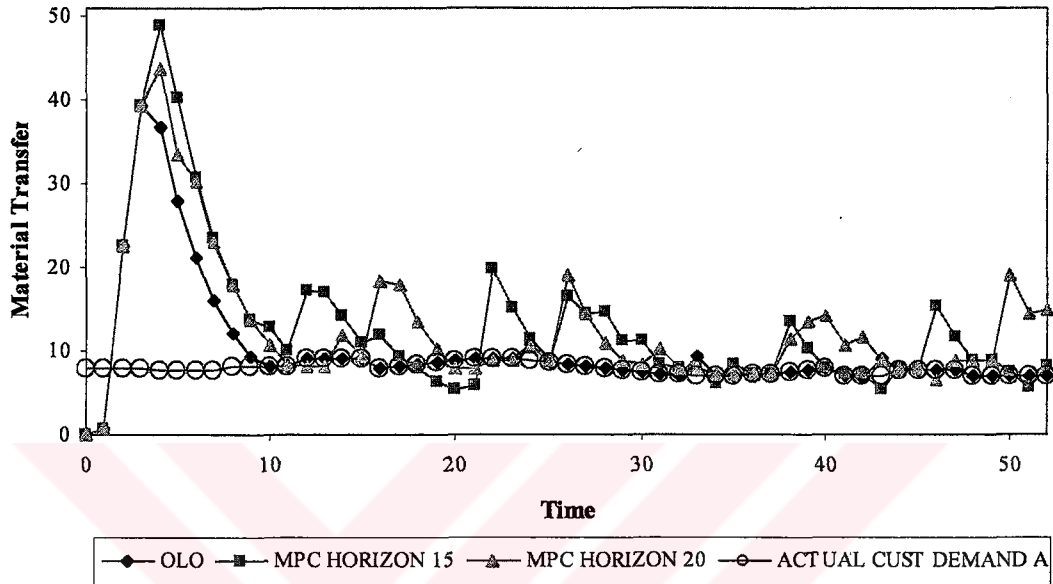


Figure 3.215: Material transferred to customer for the sinusoidal change in demand (MPC vs OLO)

As in earlier cases, MPC sees the whole demand pattern at the 0th time and optimizes its inventory holding policies and production schedules accordingly. It does not reduce its inventory for product *A* until the end of 23rd time unit (Fig. 3.213), and undergoes production for three times. First production with constant batch size takes place on 22nd time unit which is also the highest demand point of the sinusoidal demand pattern. Other production times are 31st and 41st time units which are followed by increased demand values. (Fig. 3.214).

MPC 15, initially reduces inventory much faster compared with the OLO case, due to its myopic behavior. As a result it has to produce at 9th time unit to meet the customer demand. At the 16th time unit, it underestimates the actual sinusoidal customer demand and starts to deplete its inventory much slower until the 20th time unit, by transferring smaller amounts than the actual customer demand (Fig. 3.215). At 19th time unit it produces again and this produced amount is placed to the retailer inventory at 22nd time unit. However, MPC recognizes the increase in demand at the beginning of 20th time unit and it depletes its inventory faster to meet customer satisfaction. Yet a definite production scheme can not be predicted for product *A*, such as *A* is

produced before each increase in demand, since the schedule of B and C are also decision factors in determination of A 's schedule. MPC 20, on the other hand, sees a larger horizon and shows a less myopic behavior compared with the MPC 15 case, as discussed in earlier sections.

Other products also follow similar characteristics resulting with a total profit of 12257 monetary units for OLO, -13109 monetary units for MPC with horizon 15 and -8940 monetary units for MPC with horizon 20. When the demand did not contain any unpredictable changes, such as the results presented in table 4, OLO and MPC results deviated highly, however, in the unpredictable demand change case, these deviations are even higher. In the previous case, MPC used to model the demand change as constant, which was a correct assumption at least for the next four time units. However in this case, during the sinusoidal case, even though the amplitude of change is not very high, modeling it as constant does not give very satisfactory results. Hence OLO and MPC results deviate even higher.

CHAPTER 4

**MODEL PREDICTIVE CONTROL OF MULTIPLE PRODUCT
DECENTRALIZED SUPPLY CHAINS*****4.1 Modeling of a Three-Product Decentralized & Semi-decentralized Supply Chain***

Many of the contemporary supply chain systems consist of a number of decentralized independent nodes, each seeking its own good. In this chapter two configurations of such decentralized chains will be analyzed. One of them is completely decentralized to levels, which will be referred to as decentralized chain that consists of plant and warehouse, distribution center, and retailer as independent units; and the other one is semi-decentralized in the sense that it consists of two units, plant and warehouse, and, distribution center and retailer, and this configuration will be referred to as semi-decentralized chain. It is a partial form of integration between fully decentralization and fully centralization. In this chapter, again, three products will be considered and those will be the same as before. The two configurations are presented in Figs. 4.1 and 4.2.

It is important to note that in these illustrations, all products are managed by a single unit in any node of the supply chain. For example, a single retailer is responsible for all products, although each product has a different inventory and the inventory level of one does not affect the storage capacity of the other.

In the decentralized chain, retailer observes the demand for products and maximizes its profit for the MPC horizon length by forecasting the demand for that horizon. In this process, it creates its own demand values for each period in the MPC horizon length and sends only the current one to distribution center. Distribution center, plant and warehouse repeat the same process except that plant and warehouse does not create demand but instead decides on the production schedule. Each unit assumes that the upper level node will transfer material as much as it demands each time interval. However, the actual amounts transferred to retailer and distribution center nodes are determined by the optimization results of the plant and warehouse node.

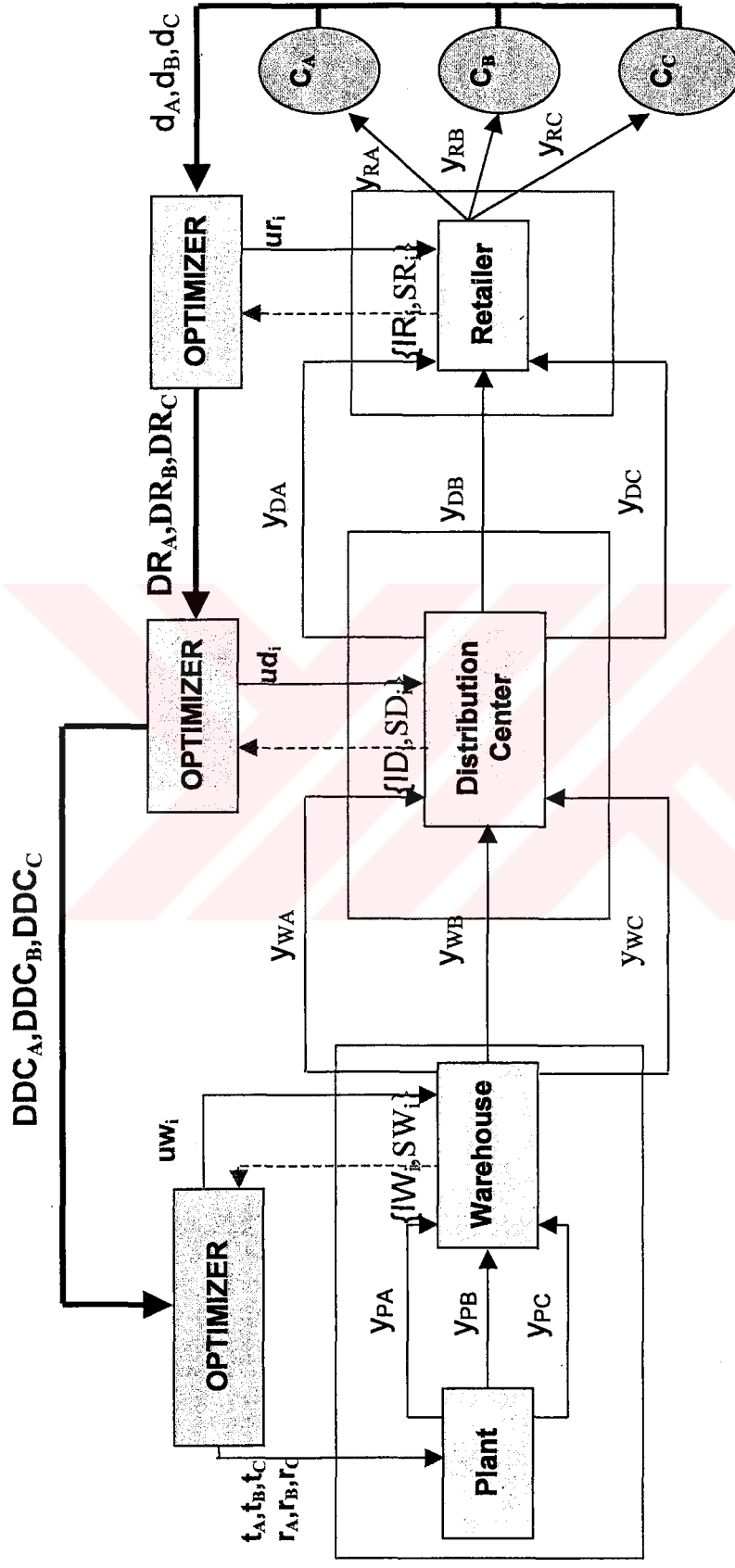


Figure 4.1: Decentralized Configuration

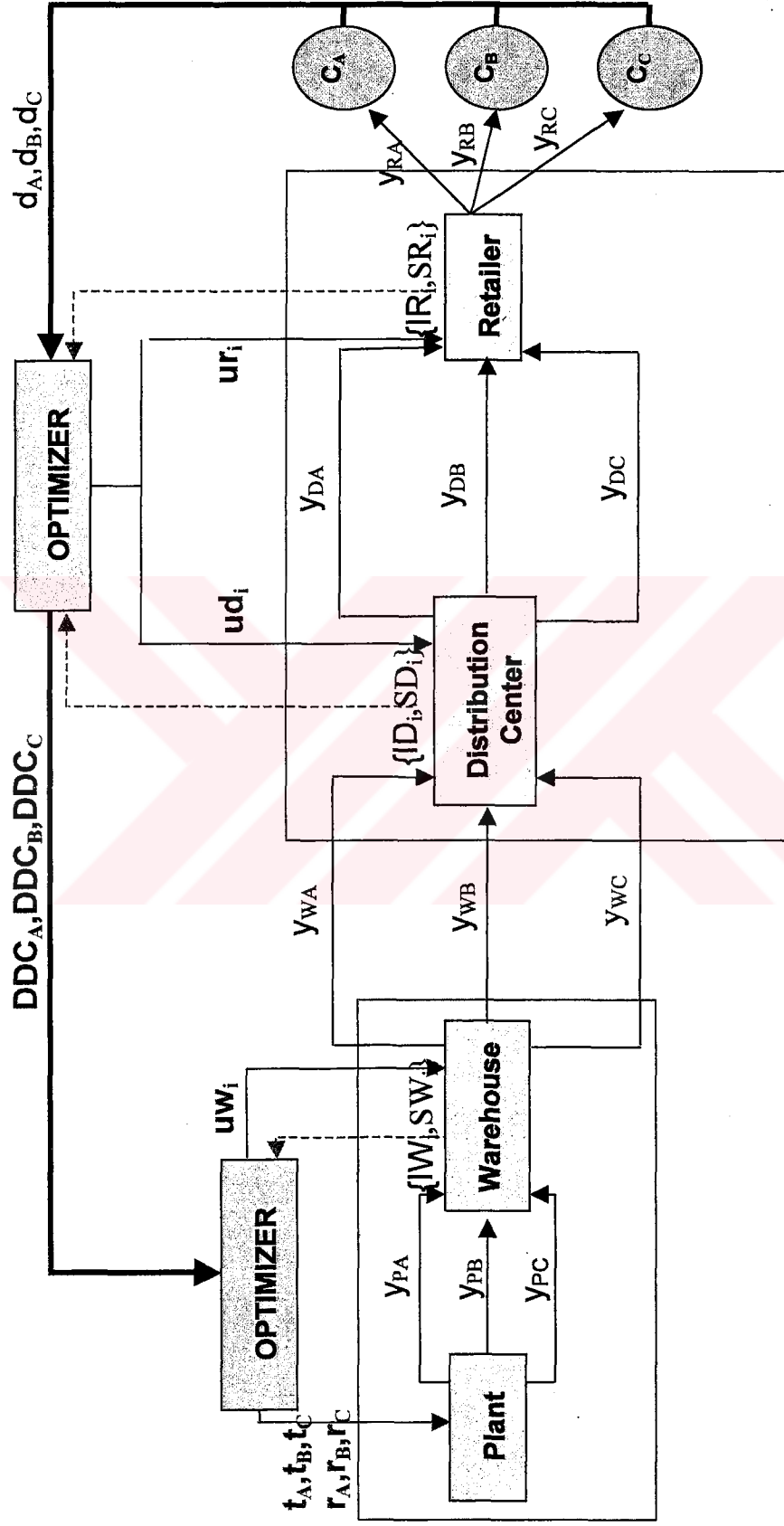


Figure 4.2: Semi-Decentralized Configuration

In the semi-decentralized chain, the retailer and distribution center unit observes the demand for products and maximizes its profit for the MPC horizon length for that horizon. During this optimization process the material transfer requests from the plant and warehouse, the material transfer levels from distribution center to retailer and from retailer to end customers are created. The material transfer requests of the distribution center and retailer complex constitute the demand observed by the plant and warehouse. Plant and warehouse operate in the same manner as in the decentralized case.

All of the physical boundaries as well as the inventory and order balance equations still apply. For the sake of illustration, those equations are re-presented.

The inventory balance equations for the three product case are given as follows:

$$\frac{dI_{j,m}}{dt} = \sum_{\substack{\text{from} \\ \text{upstream} \\ \text{nodes}}} y_{j,i,m} - \sum_{\substack{\text{to} \\ \text{downstream} \\ \text{nodes}}} y_{j,k,m} \quad (4.1)$$

The order accumulation equations between nodes are given as:

$$\frac{dS_{jm}}{dt} = \sum_{\substack{\text{from} \\ \text{downstream} \\ \text{nodes}}} o_{j,k,m} - \sum_{\substack{\text{from} \\ \text{upstream} \\ \text{nodes}}} y_{i,j,m} \quad (4.2)$$

The material transfer function proposed for the single product case is also used here, in its three product form:

$$y_{jm} = \begin{cases} \left(MT_{lim} + \frac{I^*}{I_{jm} \eta_{jm}} \cdot ST_{jm} \right)^{-1} \cdot S_{jm} & S_{jm} > 0 \\ 0 & S_{jm} = 0 \end{cases} \quad (4.3)$$

Physical bounds on the inventories, material transfer rates and control variables are also present as given in the previous chapters.

4.2 Model Predictive Control Formulation

All units in the supply chain are assumed to use MPC to optimize their profits.

4.2.1 Decentralized Supply Chain Optimization Models

Retailer and distribution center independently maximize the following objective function:

$$J = R_{\text{sales}} - C_{\text{inventory}} - C_{\text{transfer}} - C_{\text{purchase}} - C_{\text{customer satisfaction}} \quad (4.4)$$

For retailer, R_{sales} is the revenue gained by selling material to customers. The customer buys all that is transferred to him/her regardless of being less or more than the actual demand. C_{purchase} is the cost of purchasing the material transferred by the distribution center. For distribution center, R_{sales} is the revenue gained by selling material to retailer. Retailer buys all that is transferred regardless of the amount sent. C_{purchase} is the cost of purchasing the material transferred by the plant and warehouse. At each node the sales price decreases by a factor of 10 %, i.e. 50 monetary units, sales price of the material to customer from the retailer node, decreases to 45 monetary units in distribution center as sales price of the material to retailer node. In order to analyze the behavior of distribution center and retailer, the same profit margin is provided to both of them. Storing one unit of product for one time unit and then transferring it to customer cost 3 monetary units. Therefore, the profit margin provided must be greater than or equal to 3 monetary units. Furthermore, the equal-decreasing margin should provide the plant and warehouse high enough selling price so that producing even one unit of A and B remains profitable. Choosing 10% satisfies all of these requirements.

Plant and warehouse maximizes the following objective function:

$$J = R_{\text{sales}} - C_{\text{inventory}} - C_{\text{transfer}} - C_{\text{production}} - C_{\text{customer satisfaction}} \quad (4.5)$$

R_{sales} is the revenue gained by selling material to distribution center. Distribution center buys all that is transferred regardless of the amount. The plant is free to produce any amount material, that is, there is no constant batch size. The production scheduling constraints remain the same as before.

The formulation of these cost and revenue items remains the same except the fact that each independent unit is responsible of its own costs and revenue disregarding those of others. These formulations are given below.

For retailer:

$$R_{sales} = \sum_{k=1}^P \sum_m^{A,B,C} \text{Pr} R_m \cdot y_{Rm}(k) \quad (4.6)$$

where $\text{Pr} R_m$ is the price of product m sold to customer from retailer.

$$C_{inventory} = \sum_{k=1}^P \sum_m^{A,B,C} [ICR_m \cdot I_m(k)] \quad (4.7)$$

where ICR_m is the inventory holding cost for product m at retailer level.

$$C_{transfer} = \sum_{k=1}^P \sum_m^{A,B,C} [TCR_m \cdot y_m(k)] \quad (4.8)$$

where TCR_m is the transfer cost for product m at retailer level to customer. Each upstream unit is responsible for its material transfer cost to the downstream unit.

$$C_{purchase} = \sum_{k=1}^P \sum_m^{A,B,C} [PDR_m \cdot y_m(k)] \quad (4.9)$$

where PDR_m is the material purchase cost of retailer node from distribution center node which is also equal to $\text{Pr} D_m$, sales price of distribution center's material to retailer. As expressed before PDR_m equals to 90 % of $\text{Pr} R_m$.

$$C_{customer\ satisfaction} = \sum_{k=1}^P \left[\sum_m^{A,B,C} CSR_m \cdot |y_{Rm}(k) - d_m(k)| \right] \quad (4.10)$$

where CSR_m is the customer dissatisfaction cost at retailer level arising from over/under delivery to the customer.

Following constraints and initial conditions are used in the retailer optimizer, which assumes that orders placed to the distribution center (DR_m) are equal to the assumed material transfer y_{Rm} :

$$IR_m(k) \leq UI \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.11)$$

$$IR_m(1) = IR1_m \quad \forall m \in \{A, B, C\} \quad (4.12)$$

$$IR_m(k) = IR_m(k-1) - y_{Rm}(k-1) + DR_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.13)$$

$$OR_m(k) \leq UO \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.14)$$

$$SR_m(1) = SR1_m \quad \forall m \in \{A, B, C\} \quad (4.15)$$

$$SR_m(k) = SR_m(k-1) - y_{Rm}(k-1) + OR_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.16)$$

$$y_{Rm}(k) \leq Uy \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.17)$$

$$DR_m(k) \leq Uy \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.18)$$

$$y_{Rm}(k) = 0.052 * IR_m(k) + 0.190 * OR_m(k) - 5.397 \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.19)$$

$$M1_m(k) - M2_m(k) = y_{Rm}(k) - D_m(k) \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.20)$$

For distribution center:

$$R_{sales} = \sum_{k=1}^P \sum_m^{A,B,C} \text{Pr } D_m \cdot y_{Dm}(k) \quad (4.21)$$

where $\text{Pr } D_m$ is the price of product m sold to retailer from distribution center.

$$C_{inventory} = \sum_{k=1}^P \sum_m^{A,B,C} [ICD_m \cdot I_m(k)] \quad (4.22)$$

where ICD_m is the inventory holding cost for product m at distribution center.

$$C_{transfer} = \sum_{k=1}^P \sum_m^{A,B,C} [TCD_m \cdot y_m(k)] \quad (4.23)$$

where TCD_m is the transfer cost for product m at distribution level to retailer.

$$C_{purchase} = \sum_{k=1}^P \sum_m^{A,B,C} [PDD_m \cdot y_m(k)] \quad (4.24)$$

where PDD_m is the material purchase cost of retailer node from distribution center node.

$$C_{customer\ satisfaction} = \sum_{k=1}^P \left[\sum_m^{A,B,C} CSD_m \cdot |y_{Dm}(k) - DR_m(k)| \right] \quad (4.25)$$

where CSD_m is the customer dissatisfaction cost at distribution center level arising from over/under delivery to the retailer.

Following constraints and initial conditions are used in the distribution center optimizer, which assumes that orders placed to the distribution center (DD_m) are equal to the assumed material transfer y_{Dm} :

$$ID_m(k) \leq UI \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.27)$$

$$ID_m(1) = ID1_m \quad \forall m \in \{A, B, C\} \quad (4.27)$$

$$ID_m(k) = ID_m(k-1) - y_{Dm}(k-1) + DD_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.28)$$

$$OD_m(k) \leq UO \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.29)$$

$$SD_m(1) = SD1_m \quad \forall m \in \{A, B, C\} \quad (4.30)$$

$$SD_m(k) = SD_m(k-1) - y_{Dm}(k-1) + OD_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.31)$$

$$y_{Dm}(k) \leq Uy \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.32)$$

$$DD_m(k) \leq Uy \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.33)$$

$$y_{Dm}(t) = 1.1 * ID_m(k) + 0.031 * OD_m(k) - 50 \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.34)$$

$$M1_m(k) - M2_m(k) = y_{Dm}(k) - DR_m(k) \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.35)$$

For plant and warehouse:

$$R_{sales} = \sum_{k=1}^P \sum_m^{A,B,C} \text{Pr } W_m \cdot y_{Wm}(k) \quad (4.36)$$

where $\text{Pr } W_m$ is the price of product m sold to distribution center from warehouse.

$$C_{inventory} = \sum_{k=1}^P \sum_m^{A,B,C} [ICW_m \cdot I_m(k)] \quad (4.37)$$

where ICW_m is the inventory holding cost for product m at warehouse.

$$C_{transfer} = \sum_{k=1}^P \sum_m^{A,B,C} [TCW_m \cdot y_m(k)] \quad (4.38)$$

where TCW_m is the transfer cost for product m at warehouse to distribution center.

$$C_{production} = \sum_{k=1}^P \left[\sum_m^{A,B,C} (FC_m \cdot p_m(k) + VC_m \cdot yp_m(k)) \right] \quad (4.39)$$

$$C_{customer\ satisfaction} = \sum_{k=1}^P \left[\sum_m^{A,B,C} CSW_m \cdot |yW_m(k) - DD_m(k)| \right] \quad (4.40)$$

where CSW_m is the customer dissatisfaction cost at warehouse level arising from over/under delivery to the distribution center.

Following constraints and initial conditions are used in the retailer optimizer, which assumes that orders placed to the distribution center (DW_m) are equal to the assumed material transfer yW_m :

$$IW_m(k) \leq UI \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.41)$$

$$IW_m(1) = IW1_m \quad \forall m \in \{A, B, C\} \quad (4.42)$$

$$IW_m(k) = IW_m(k-1) - yW_m(k-1) + CP_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.43)$$

$$OW_m(k) \leq UO \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.44)$$

$$SW_m(1) = SW1_m \quad \forall m \in \{A, B, C\} \quad (4.45)$$

$$SW_m(k) = SW_m(k-1) - yW_m(k-1) + OW_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.46)$$

$$yW_m(k) \leq Uy \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.47)$$

$$yW_m(k) = 0.9 * IW_m(k) + 0.024 * OW_m(k) - 30.6 \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.48)$$

$$M1_m(k) - M2_m(k) = yW_m(k) - DD_m(k) \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.49)$$

$$CP_m(k) \leq UC \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.50)$$

$$P_A(1) + P_B(1) + P_C(1) \leq 1 \quad (4.51)$$

$$P_A(1) + P_A(2) + P_B(1) + P_B(2) + P_C(2) \leq 1 \quad (4.52)$$

$$P_A(k-1) + P_A(k-2) + P_B(k-2) + P_B(k-1) + P_B(k) + P_C(k) \leq 1 \quad \forall k \in \{3, \dots, P\} \quad (4.53)$$

4.2.2 Semi-Decentralized Chain Optimization Models

Retailer and distribution center together maximize the following objective function:

$$J = R_{\text{sales}} - C_{\text{inventory}} - C_{\text{transfer}} - C_{\text{purchase}} - C_{\text{customer satisfaction}} \quad (4.54)$$

For the retailer, R_{sales} is the revenue gained by selling material to customers. The customer buys all that is transferred to him/her regardless of being less or more than the actual demand. C_{purchase} is the cost of purchasing the material transferred by the plant and warehouse. C_{purchase} is the cost of purchasing the material transferred by the plant and warehouse.

Plant and warehouse maximizes the following objective function:

$$J = R_{\text{sales}} - C_{\text{inventory}} - C_{\text{transfer}} - C_{\text{production}} - C_{\text{customer satisfaction}} \quad (4.55)$$

R_{sales} is the revenue gained by selling material to distribution center. Distribution center and retailer buys all that is transferred regardless of the amount. The plant is free to produce any amount material, that is, there is no constant batch size, and however, the batch size is limited above by a given amount. The production scheduling constraints remain the same as before.

The formulation of these cost and revenue items remains the same except the fact that each independent unit is responsible of its own costs and revenue disregarding those of the other. These formulations are given below.

For retailer & distribution center:

$$R_{\text{sales}} = \sum_{k=1}^P \sum_m^{A,B,C} \text{Pr} R_m \cdot y_{Rm}(k) \quad (4.56)$$

where $\text{Pr} R_m$ is the price of product m sold to customer from retailer.

$$C_{\text{inventory}} = \sum_{k=1}^P \sum_m^{A,B,C} \left[\sum_j^{R,D} IC_{jm} \cdot I_{jm}(k) \right] \quad (4.57)$$

$$C_{\text{transfer}} = \sum_{k=1}^P \sum_m^{A,B,C} \left[\sum_j^{R,D} TC_{jm} \cdot y_{jm}(k) \right] \quad (4.58)$$

$$C_{\text{purchase}} = \sum_{k=1}^P \sum_m^{A,B,C} [PWR_m \cdot y_m(k)] \quad (4.59)$$

where PWR_m is the material purchase cost of distribution center node from warehouse node.

$$C_{\text{customer satisfaction}} = \sum_{k=1}^P \left[\sum_m^{A,B,C} CSR_m \cdot |y_{Rm}(k) - d_m(k)| \right] \quad (4.60)$$

where CSR_m is the customer dissatisfaction cost at retailer level arising from over/under delivery to the customer.

Following constraints and initial conditions are used in the retailer optimizer, which assumes that orders placed to the warehouse node (DD_m) are equal to the assumed material transfer yD_m :

$$IR_m(k) \leq UI \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.61)$$

$$IR_m(1) = IR1_m \quad \forall m \in \{A, B, C\} \quad (4.62)$$

$$IR_m(k) = IR_m(k-1) - yR_m(k-1) + yD_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.63)$$

$$ID_m(k) \leq UI \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.64)$$

$$ID_m(1) = ID1_m \quad \forall m \in \{A, B, C\} \quad (4.65)$$

$$ID_m(k) = ID_m(k-1) - yD_m(k-1) + DD_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.66)$$

$$OR_m(k) \leq UO \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.67)$$

$$SR_m(1) = SR1_m \quad \forall m \in \{A, B, C\} \quad (4.68)$$

$$SR_m(k) = SR_m(k-1) - yR_m(k-1) + OR_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.69)$$

$$OD_m(k) \leq UO \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.70)$$

$$SD_m(1) = SD1_m \quad \forall m \in \{A, B, C\} \quad (4.71)$$

$$SD_m(k) = SD_m(k-1) - yD_m(k-1) + OD_m(k-1) \quad \forall m \in \{A, B, C\}, \forall k \in \{2, \dots, P\} \quad (4.72)$$

$$yR_m(k) \leq Uy \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.73)$$

$$yR_m(k) = 0.052 * IR_m(k) + 0.190 * OR_m(k) - 5.397 \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.74)$$

$$yD_m(k) \leq Uy \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.75)$$

$$DD_m(k) \leq Uy \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.76)$$

$$yD_m(k) = 1.1 * ID_m(k) + 0.031 * OD_m(k) - 50 \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.77)$$

$$M1_m(k) - M2_m(k) = yR_m(k) - D_m(k) \quad \forall m \in \{A, B, C\}, \forall k \in \{1, \dots, P\} \quad (4.78)$$

Plant and warehouse model of the semi-decentralized model remains the same, as given by Eqns. 4.36 to 4.53.

In the application of the MPC approach, optimization horizon is taken to be a constant, and the control interval is taken to be equal to 1. Each unit in the supply chain forecasts the demand for the optimization horizon to be equal to the current demand.

The supply chain is optimized using these values and optimal decision variables for the predicted horizon are obtained. The system is then simulated using these decision variables and the actual demand values within the control interval. At the end of the first control interval (1 time unit), the initial inventory and order values are updated, the new demand profile is again predicted as constant and equal to the demand at the 1st time unit the system is re-optimized for another optimization horizon time units, and the same procedure is repeated until the end of the process time. The MPC approach is repeated with different optimization horizon lengths.

The demand is assumed to be the same as before and is re-illustrated in figure 4.3

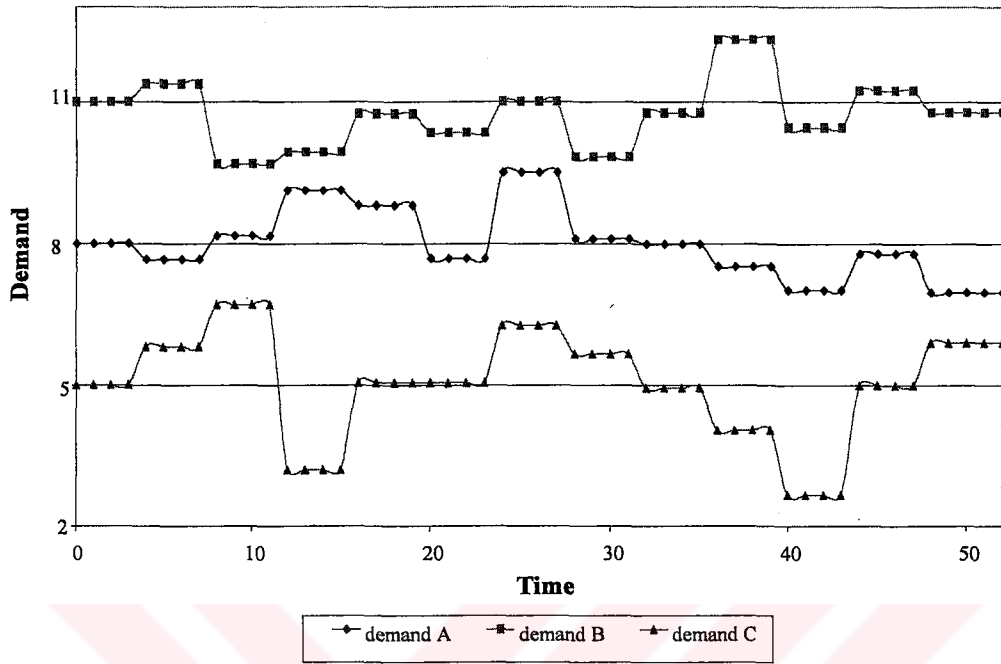


Figure 4.3: Demand profiles for the three product supply chain

Inventory costs are 2 monetary units and material transfer costs are 1 monetary unit. The final product is sold to the customer for a price of 50 monetary units. The selling price at the distribution center is 45 monetary units, whereas selling price is 40 at the warehouse. Over and underselling cost is equal to the selling price at each unit. The production costs remain the same. The material transfer cost between two units is incurred by the seller unit.

4.3 Model Predictive Control of Decentralized and Semi-Decentralized Supply Chain

In the application of the MPC approach, the initial horizon optimization horizon is taken as 15 time units, and the control interval is taken to be equal to 1. The MPC approach is repeated with different optimization horizon lengths. In each case, the system is coded in OPL Studio with CPLEX as MILP solver. The length of the simulation is 52 periods.

4.3.1 Optimization Horizon 15:

Both systems are optimized for a shorter horizon than the actual length of the demand pattern. Therefore, the systems behave as though they are not aware of the future beyond the length of the

optimized horizon, which surely may create a tendency to take myopic decisions, as in centralized case.

Semi-decentralized configuration always manages initial inventories at retailer better than decentralized configuration do in the sense that initial inventory is depleted in a longer horizon of time, which implies a less myopic approach for semi-decentralized configuration. The inventory levels at retailer are plotted in figures 4.4 to 4.6. The material transfer levels out of retailer are plotted in figures 4.7 to 4.9.

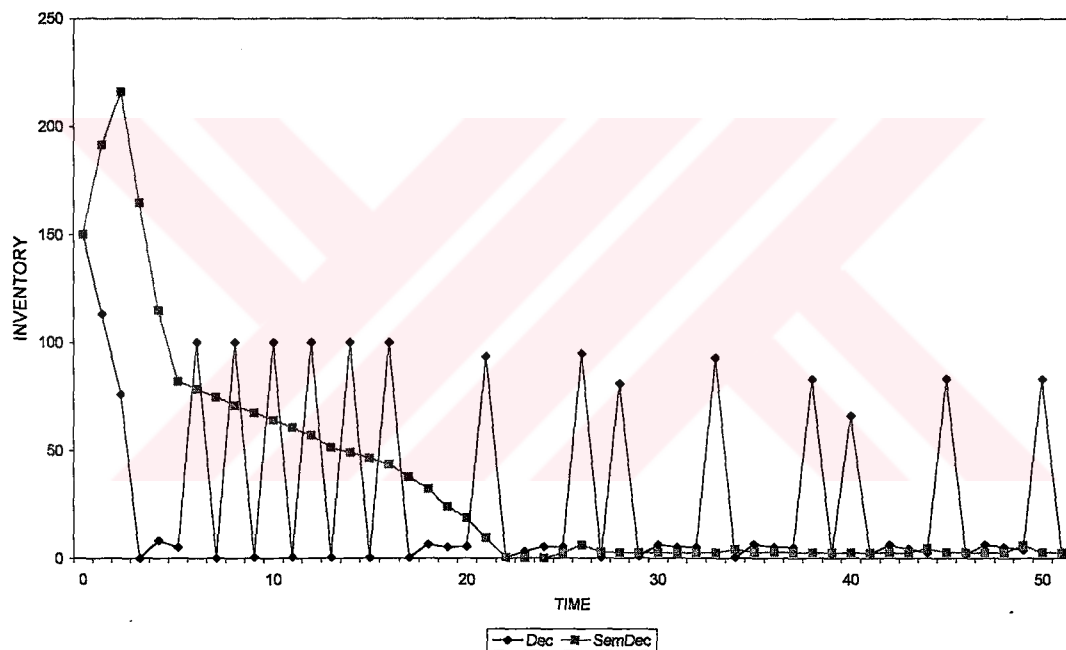


Figure 4.4: Retailer node inventory level for product A

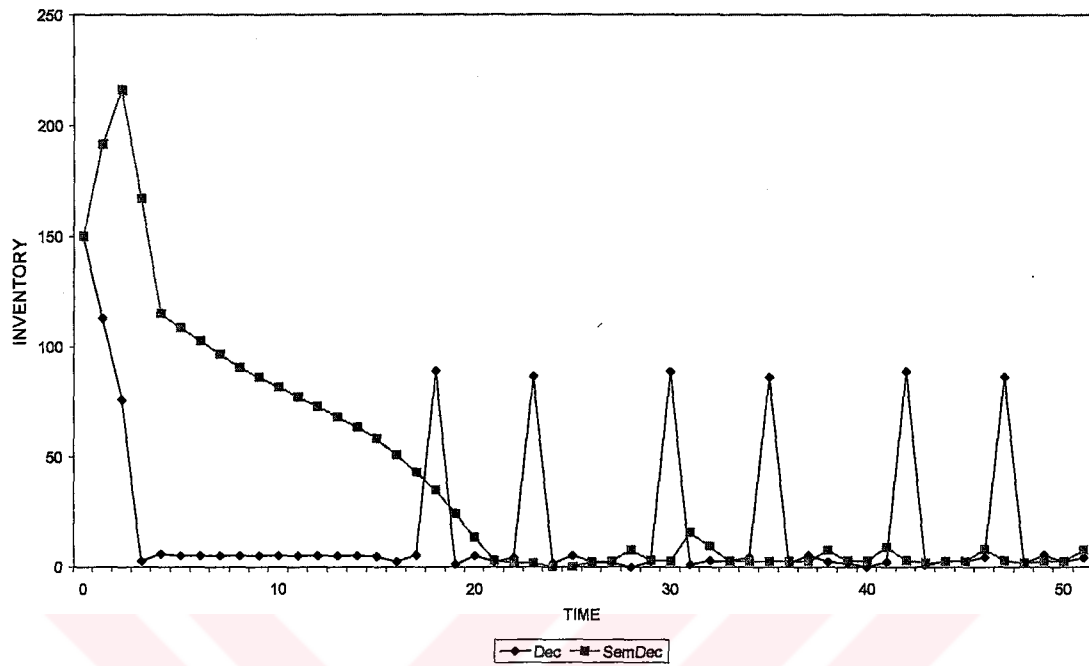


Figure 4.5: Retailer node inventory level for product B

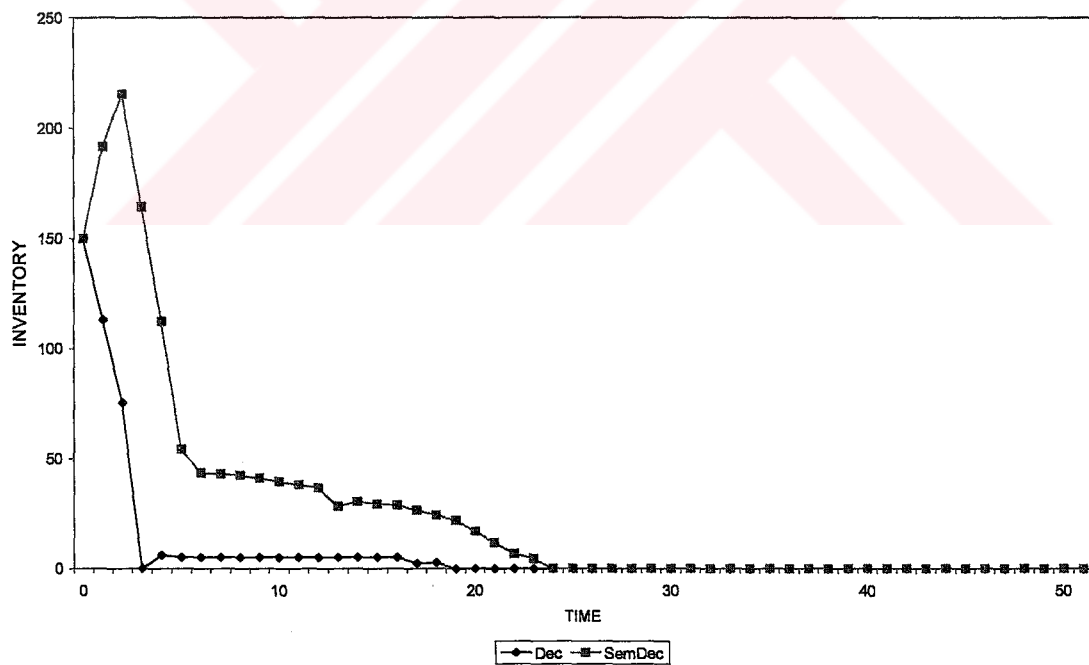


Figure 4.6: Retailer node inventory level for product C

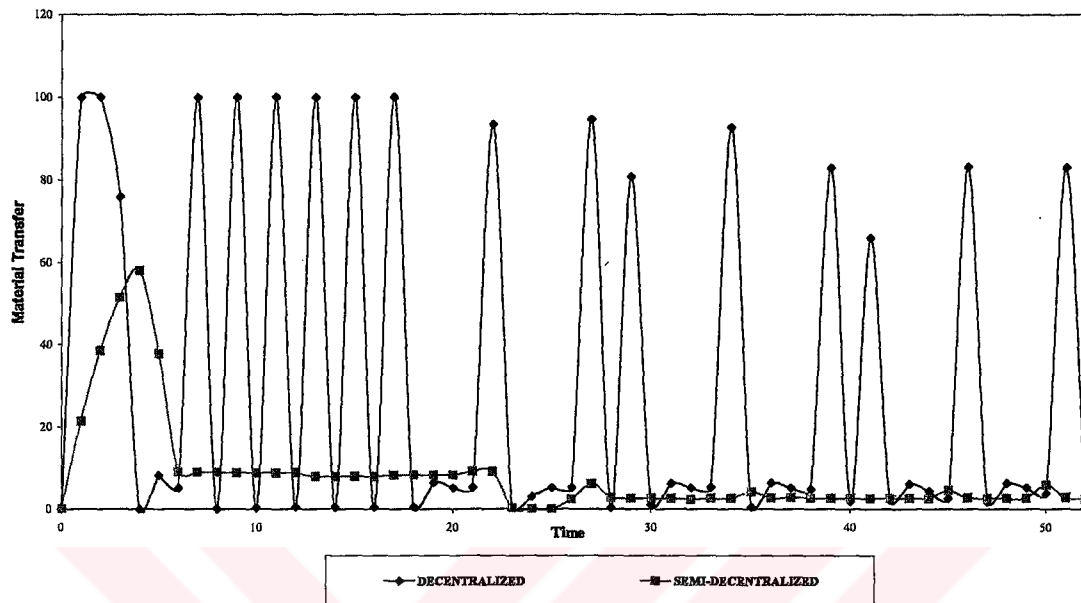


Figure 4.7: Retailer node material transfer level for product A

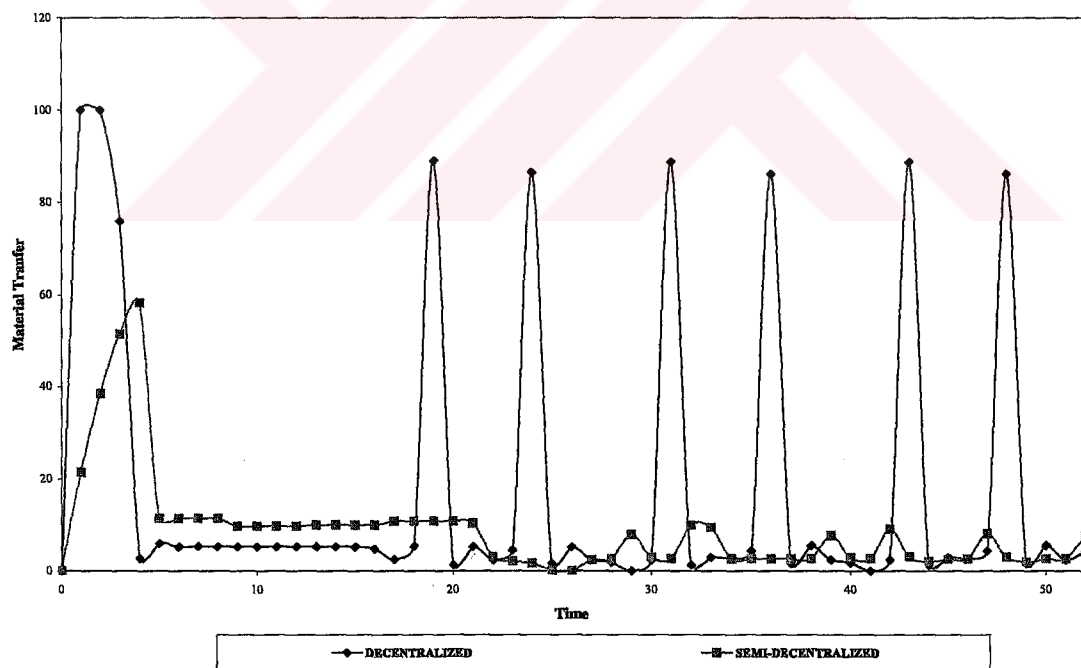


Figure 4.8: Retailer node material transfer level for product B

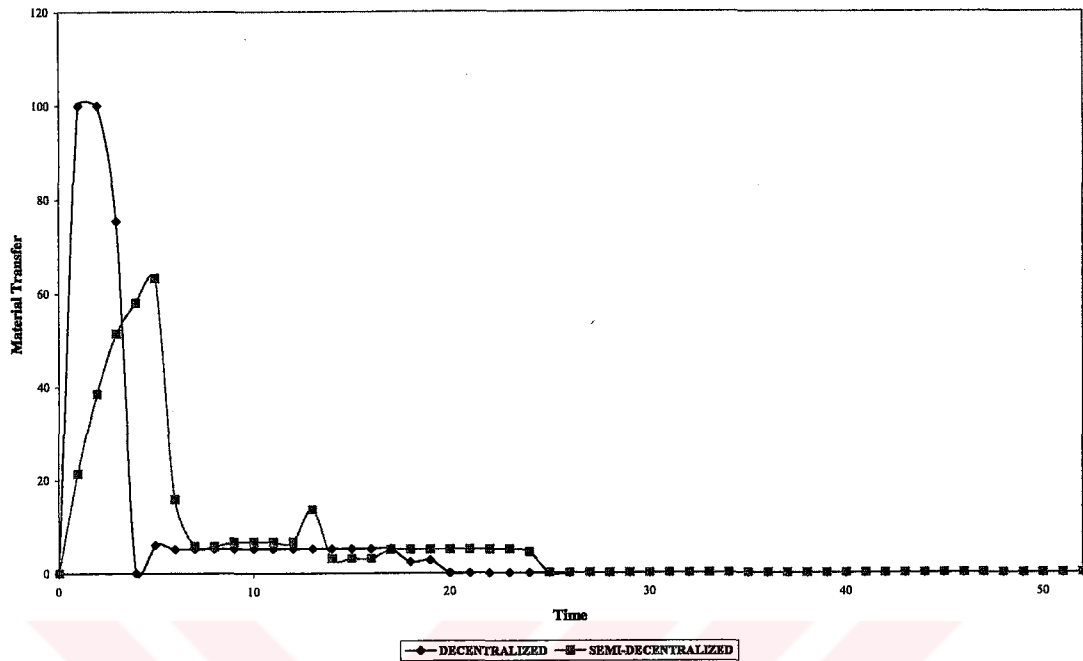


Figure 4.9: Retailer node material transfer level for product C

This result is to be expected, since decentralized configuration optimizes the material transfer out of retailer by just knowing the amount of material in the retailer whereas semi-decentralized configuration optimizes the material transfer level out of retailer by knowing the amount of material in the distribution center and retailer, thus optimizing the inventory level by utilizing more deterministic information. It could be argued that since the distribution center and retailer complex of semi-decentralized system has more initial inventory, this complex might try to transfer more material out of retailer, which is not the case. This implies that in order to explain the lower material transfer level out of the retailer of the distribution center and retailer complex of the semi-decentralized configuration, it must be the case that the material demanded from the plant and warehouse complex must be lower for the semi-decentralized than that of decentralized so that the flow of material into distribution center is lower than that of decentralized configuration which is the case, and this is presented in figures 4.10 to 4.12. Therefore, it is obvious that the distribution center in the semi-decentralized case makes smaller amounts of material demands, which accounts for the stability of the system.

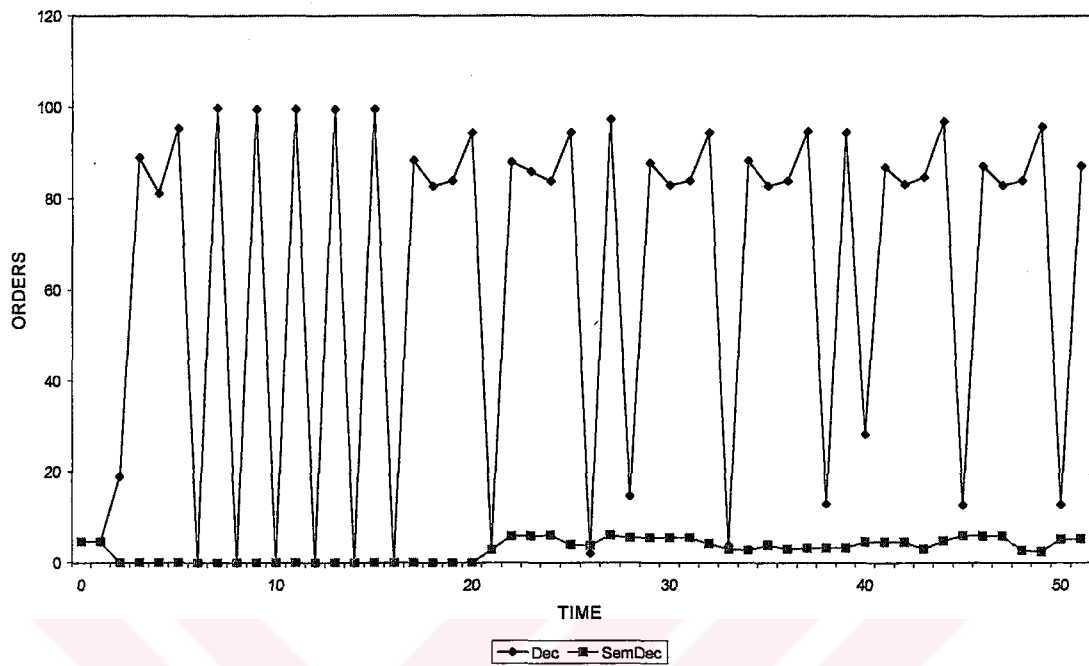


Figure 4.10: Material demanded from Plant and Warehouse of Product A

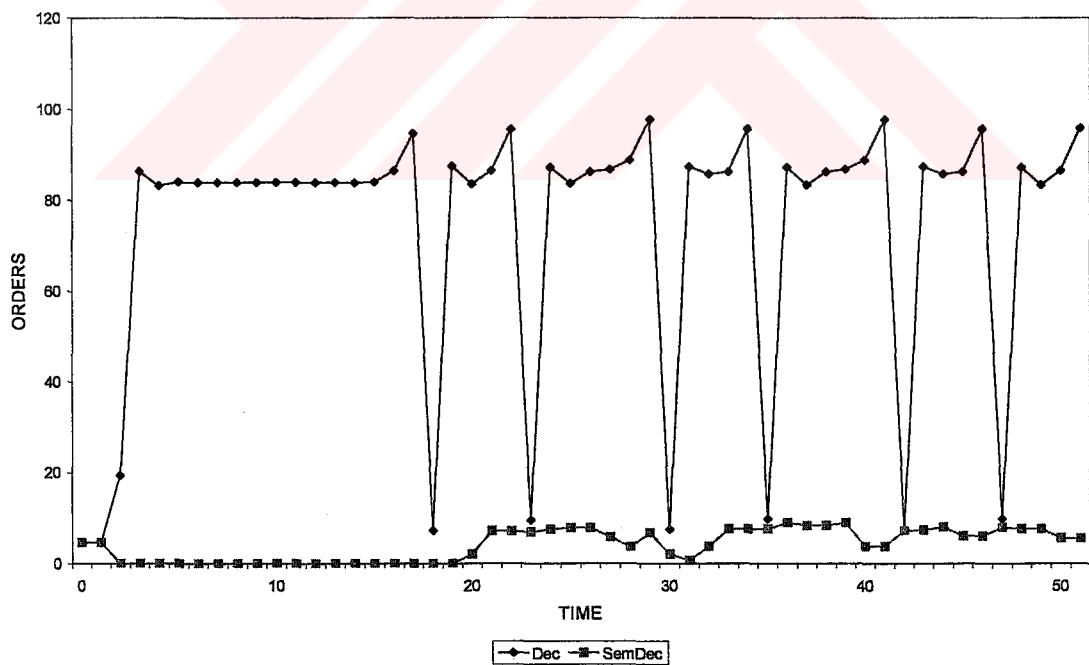


Figure 4.11: Material demanded from Plant and Warehouse of Product B

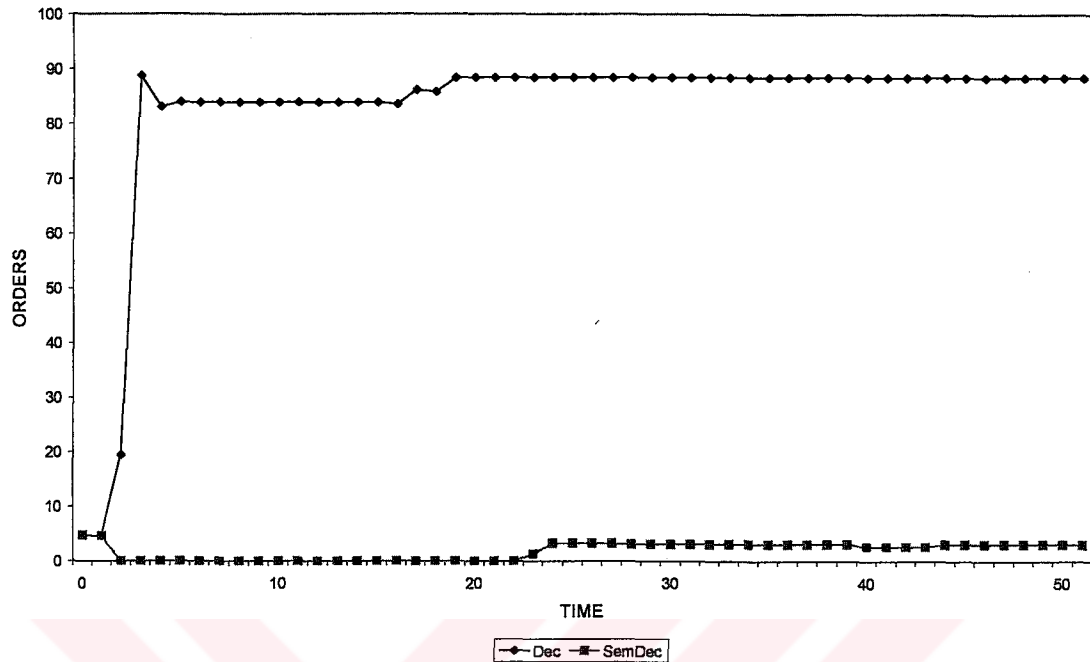


Figure 4.12: Material demanded from Plant and Warehouse of Product C

The order levels of semi-decentralized system from plant and warehouse is nearly always lower than those of decentralized system. This can be resolved in two factors. First, in the decentralized configuration the distribution center has to forecast the demand of the retailer. Whenever the retailer makes a material transfer request, the distribution center assumes that that demand level is going to be maintained in the next optimization horizon time units, which may exaggerate the needs of the retailer as perceived by the distribution center, hence forcing the distribution center to place high order levels to plant and warehouse. In the semi-decentralized configuration the optimizer knows the exact needs of the retailer and distribution center since they are in the same complex, hence avoiding the exaggeration of the needs. Second one is the fact that in the semi-decentralized configuration, the retailer and distribution center complex must pay the entire inventory holding costs and material transfer costs throughout the travel of the material through the complex. On the other hand, in the decentralized configuration, the distribution center does not care about the inventory holding cost and the material transfer cost of the retailer and only optimizes its own profit. And in order to optimize its own profit, disregarding that of the retailer, the distribution center may place high order levels. The observed phenomenon, the exaggeration of demand levels and therefore material transfer levels, is called the *demand amplification problem* or the *bullwhip effect*.

Another interesting result is the fact that other than product *C*, average inventory level at retailer of the semi-decentralized system is lower than that of the decentralized system. Observing the production schedules and amounts of both systems presented in Figs. 4.25 to 4.27, it becomes clear that *C* is never produced and depleting *C* in a longer period of time means managing inventory to satisfy the customer needs for a longer period of time, which is also explained by the more deterministically known inventory approach.

Other than product *C*, from the inventory plots of products *A* and *B*, it is clear that after the effect of initial inventory disappears, the retailer inventory level of semi-decentralized system is lower than that of the decentralized system most of the time.

In addition to these, it can also be observed that other than inventory for product *C*, inventory levels at distribution center is lower for semi-decentralized system than that of the decentralized system. For the case of product *C*, distribution center uses initial inventory for product *C* for a longer period of time since *C* is never produced. The inventory levels and the material transfer levels at distribution center are plotted in Figs. 4.13 to 4.18.

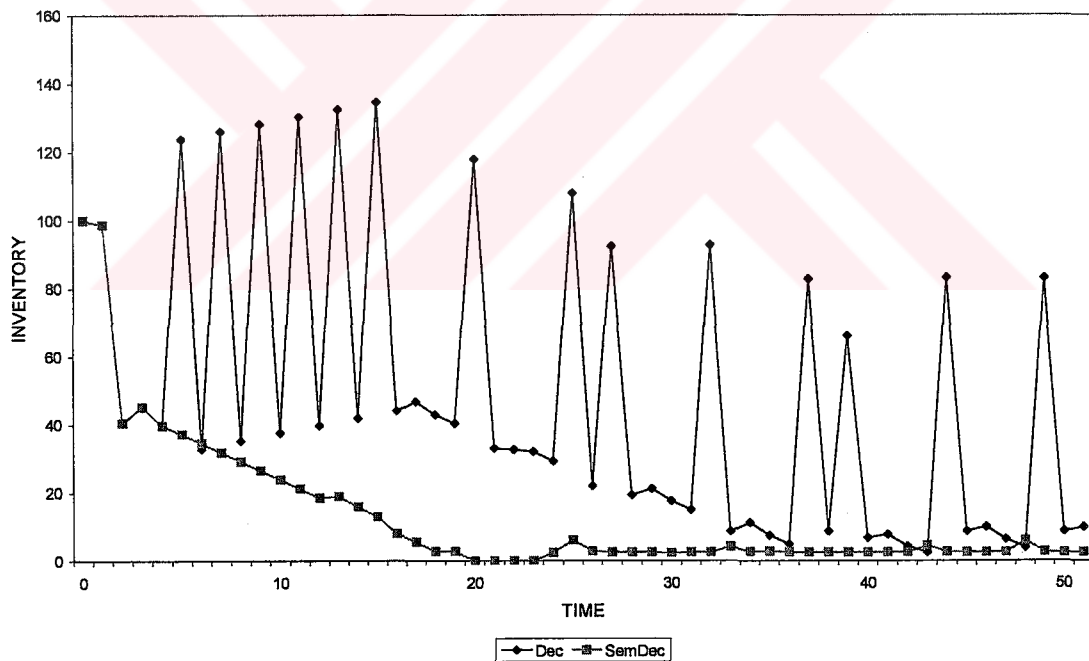


Figure 4.13: Distribution center node inventory level for product A

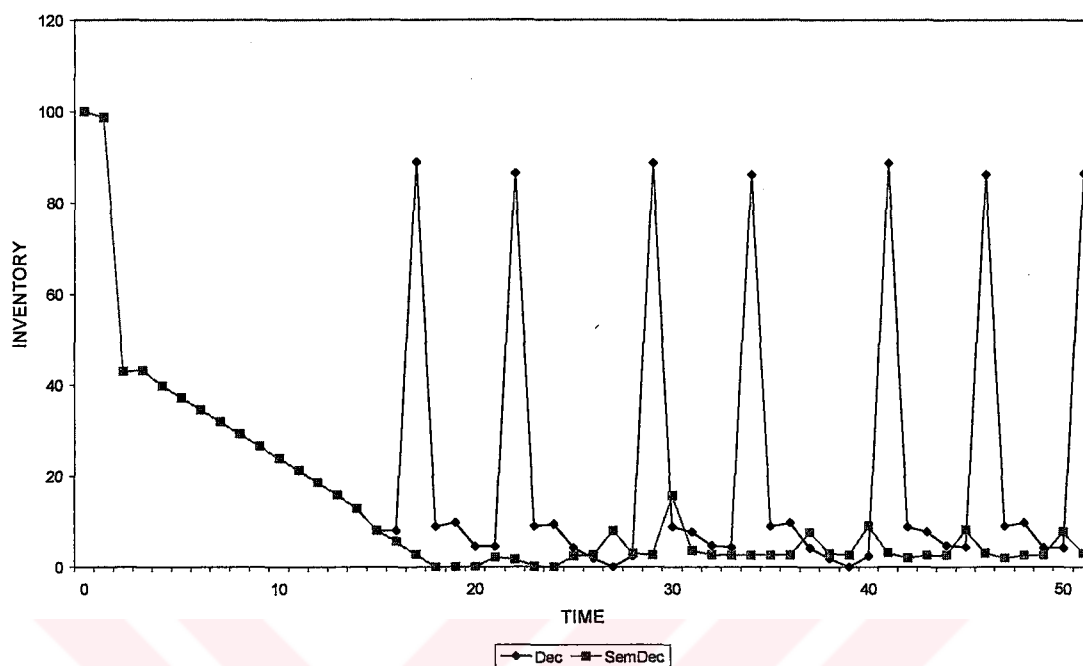


Figure 4.14: Distribution center node inventory level for product B

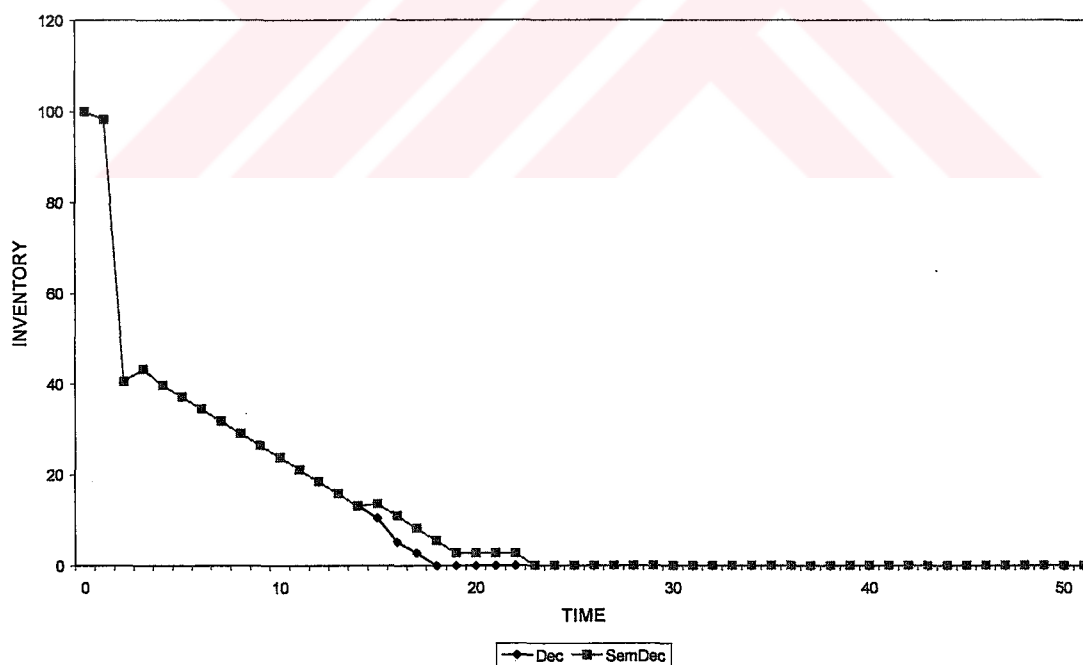


Figure 4.15: Distribution center node inventory level for product C

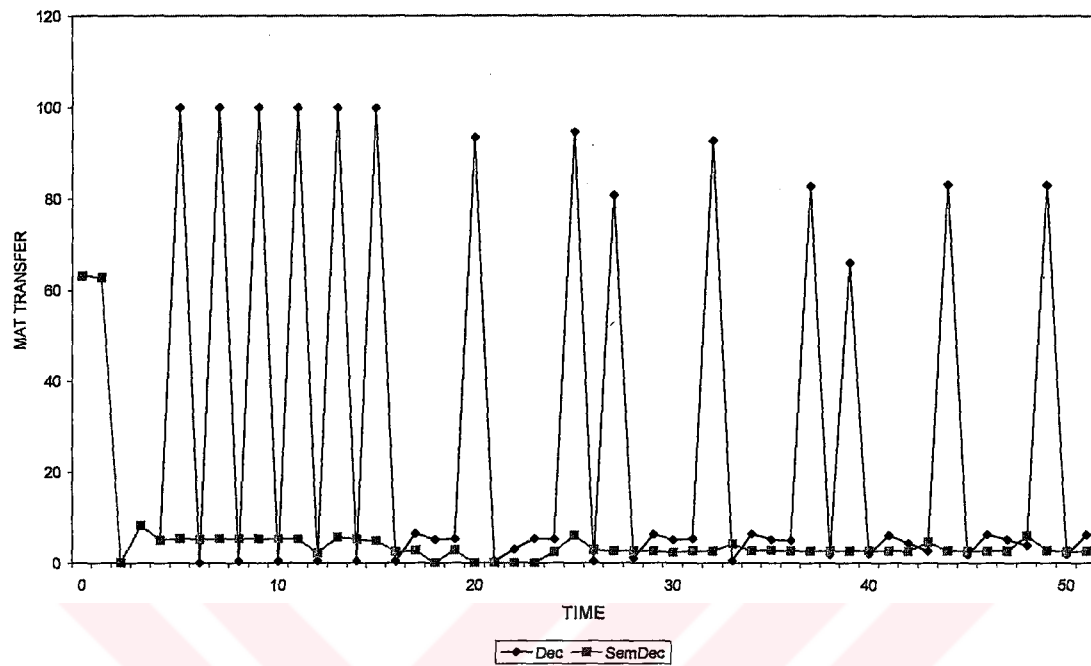


Figure 4.16: Distribution center node material transfer level for product A

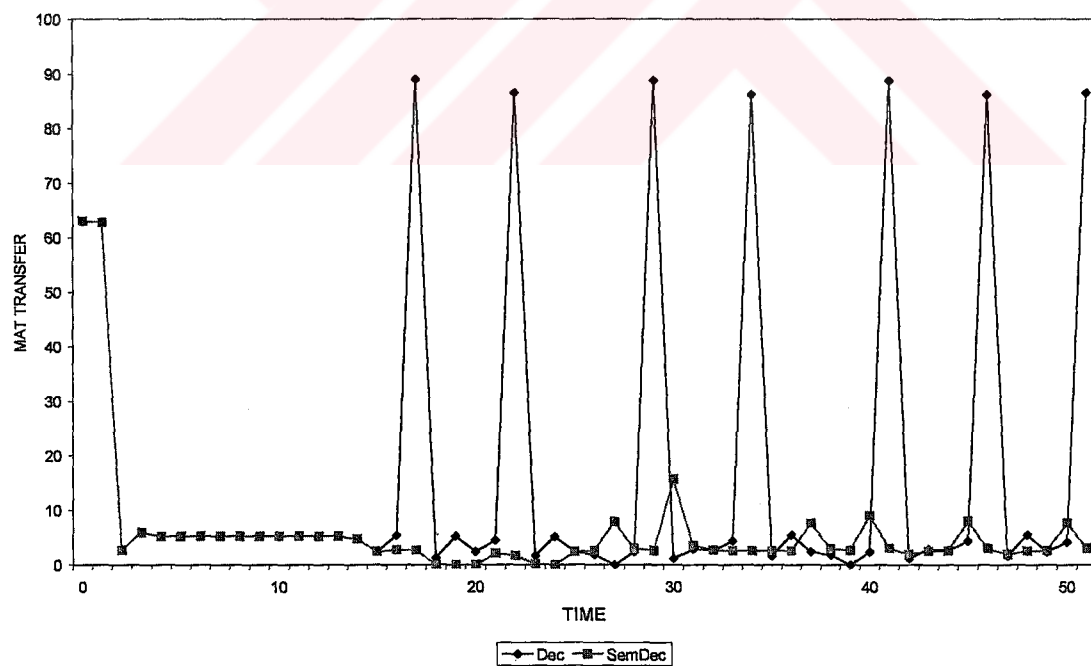


Figure 4.17: Distribution center node material transfer level for product B

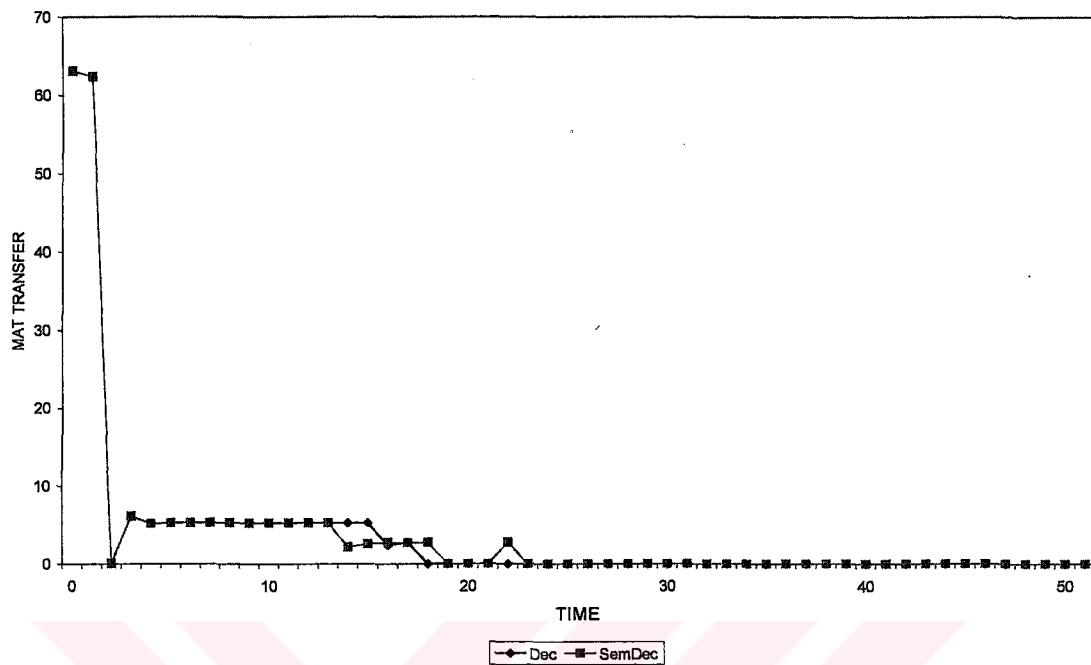


Figure 4.18: Distribution center node material transfer level for product C

The interpretations of the inventory levels and material transfer levels of plant and warehouse is similar to those of distribution center and the plots of these are presented in Figs 4.19 to 4.24.

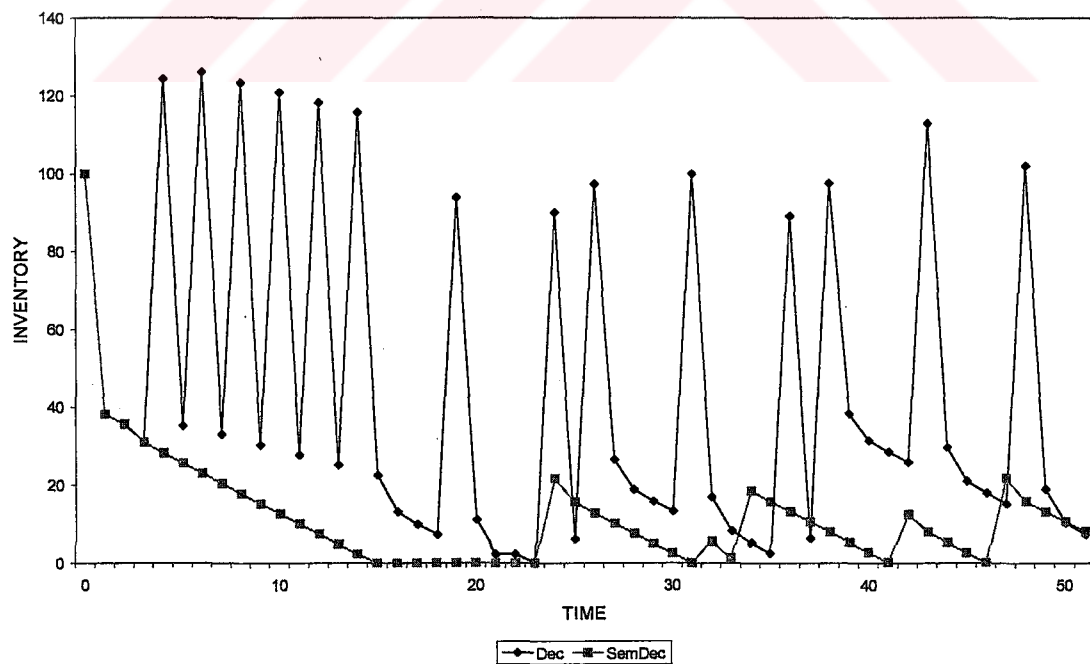


Figure 4.19: Plant and Warehouse node inventory level for product A

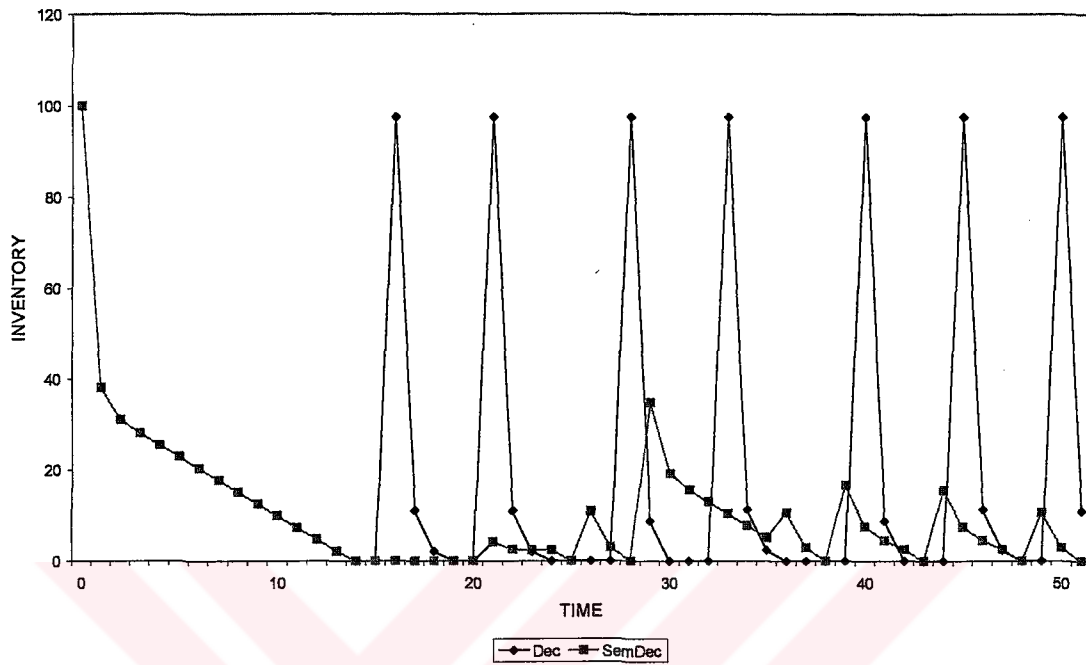


Figure 4.20: Plant and Warehouse node inventory level for product B

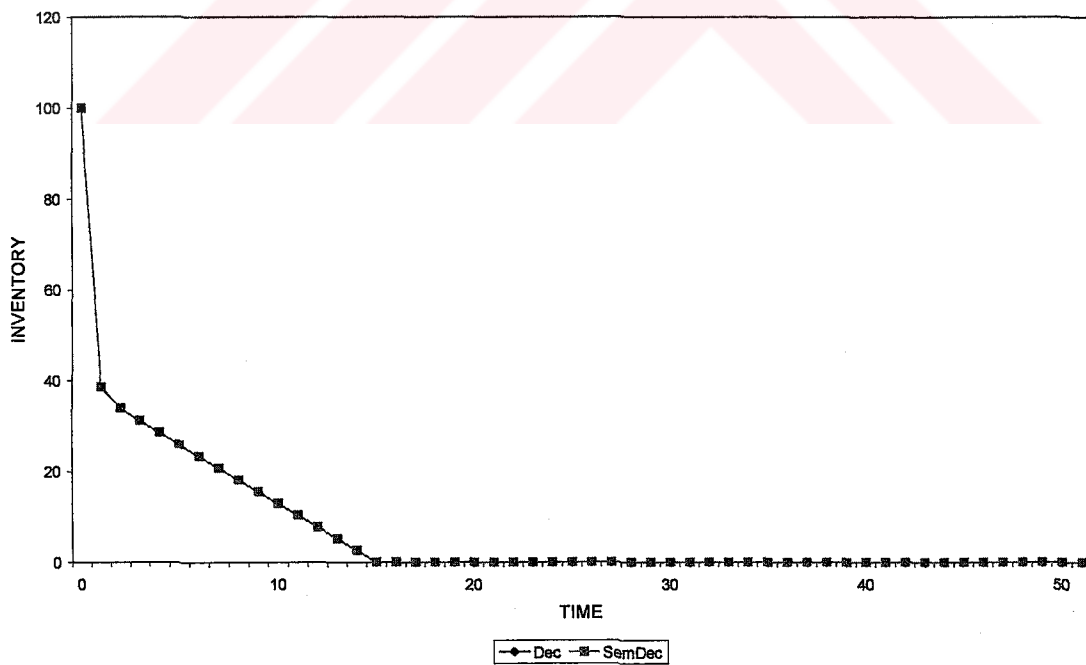


Figure 4.21: Plant and Warehouse node inventory level for product C

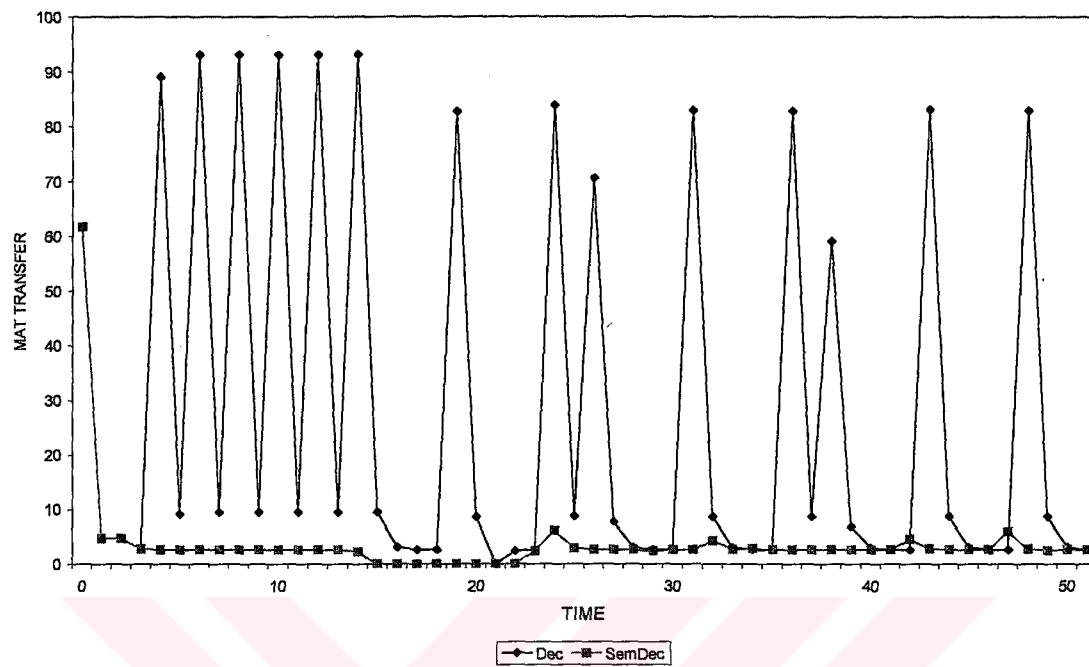


Figure 4.22: Plant and Warehouse node material transfer level for product A

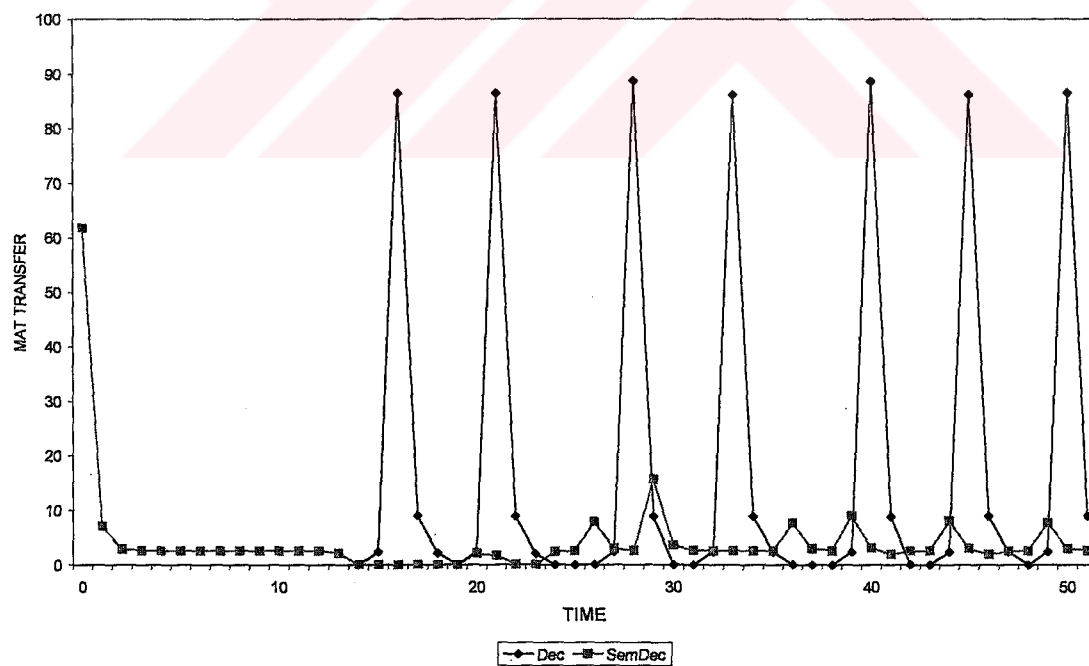


Figure 4.23: Plant and Warehouse node inventory level for product B

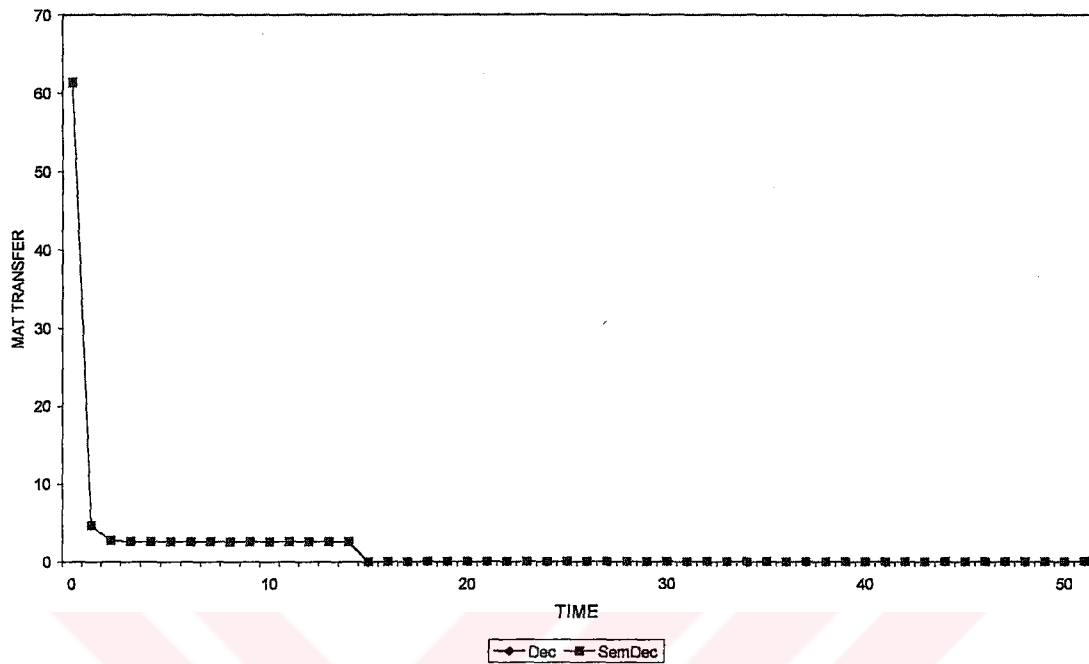


Figure 4.24: Plant and Warehouse node inventory level for product C

In general, semi-decentralized configuration maintains lower inventory levels than decentralized configuration noting the exception for product C, which is never produced. This is due to the fact that the need for forecasting is less and the accuracy of the needs is better known in the semi-decentralized configuration because of the integration of the distribution center and the retailer, as explained before by bullwhip effect. Therefore, semi-decentralized configuration is able to carry fewer inventories because it does not anticipate exaggerated demand levels.

The production schedule and the production amount of products are plotted in Figs. 4.25 to 4.27. From the fixed and variable production costs, it is a clear fact that the system must try to produce as much as possible but in small amounts since variable production cost is more than fixed production cost. From Figs. 4.25 to 4.27, it is clear that semi-decentralized system is better in accomplishing this. The production sizes of the systems are consistent with the material demanded from the plant and warehouse, which is plotted in Figs. 4.10 to 4.12. Product C is never produced in both systems because of it is not profitable due to the production costs. Therefore, the success of the semi-decentralized system in lowering the production costs is caused by the lower demand levels of the distribution center, which is explained by the severity of the bullwhip effect.

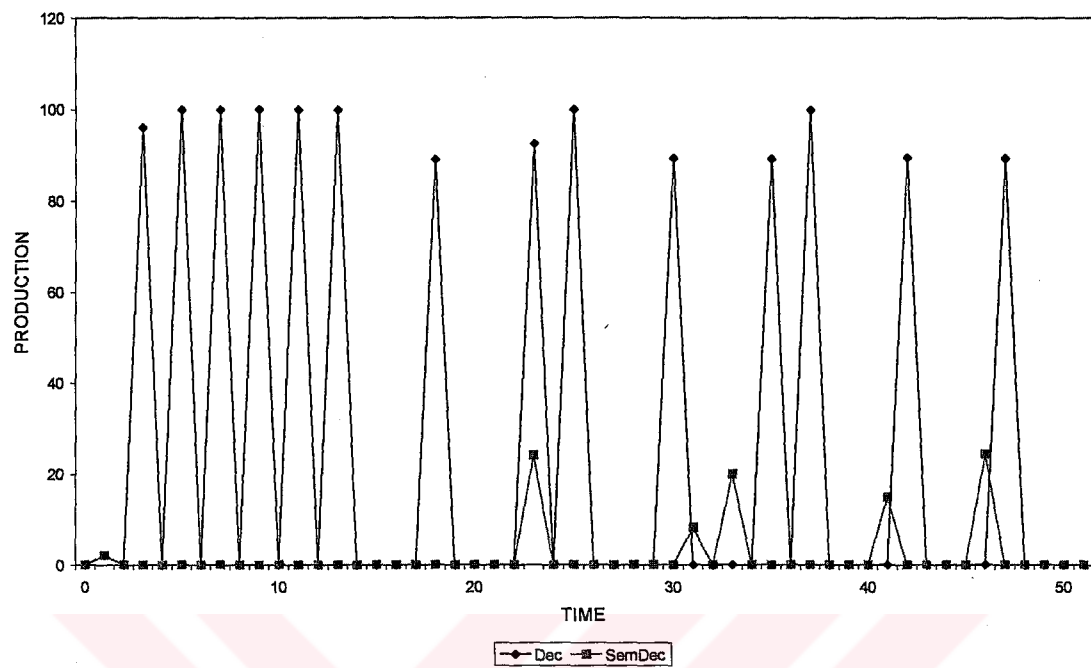


Figure 4.25: Production of A

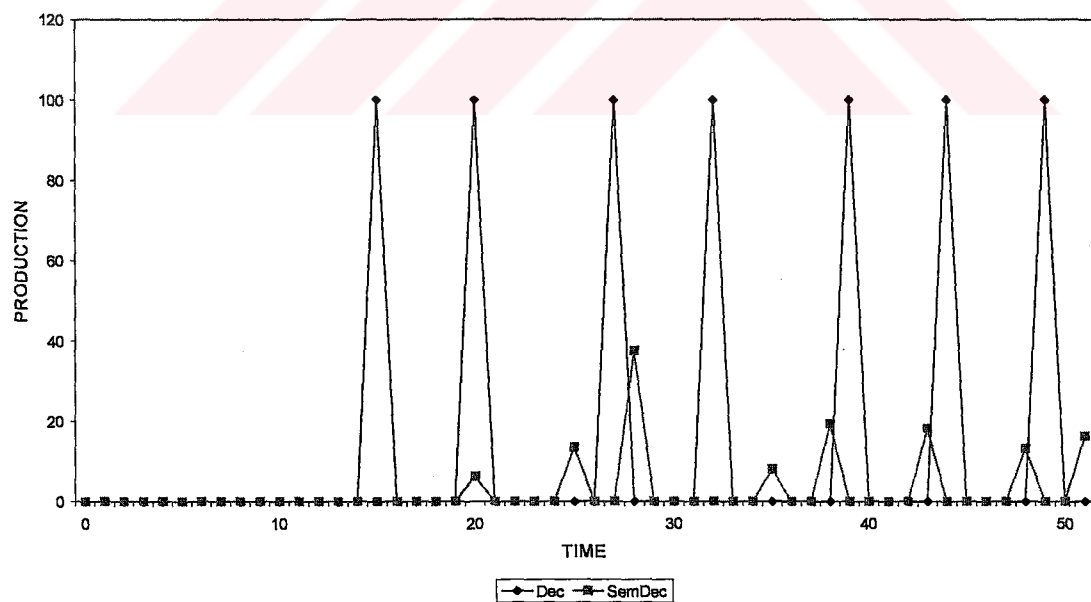


Figure 4.26: Production of B

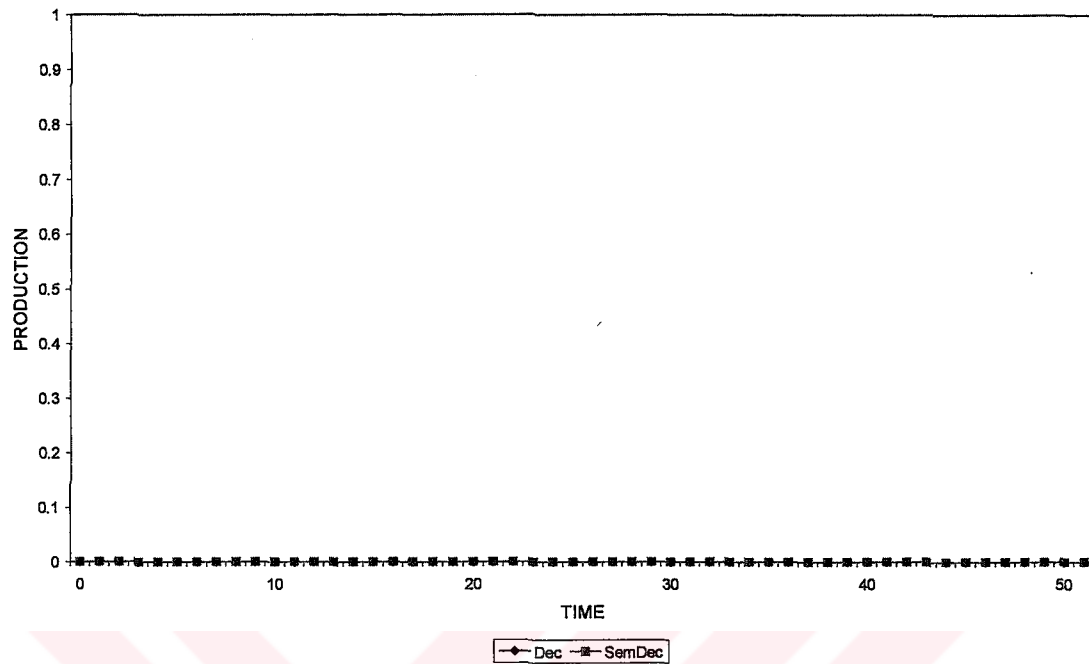


Figure 4.27: Production of C

Cumulative and instantaneous system profits for both configurations are given in figures 4.28 and 4.29. Semi-decentralized system is clearly better than decentralized system in terms of profit, which is caused by smaller and less frequent production, lower inventory levels, and better customer satisfaction levels as will be seen later, all of which are induced by the less severe bullwhip effect, the reasons of which are explained before.

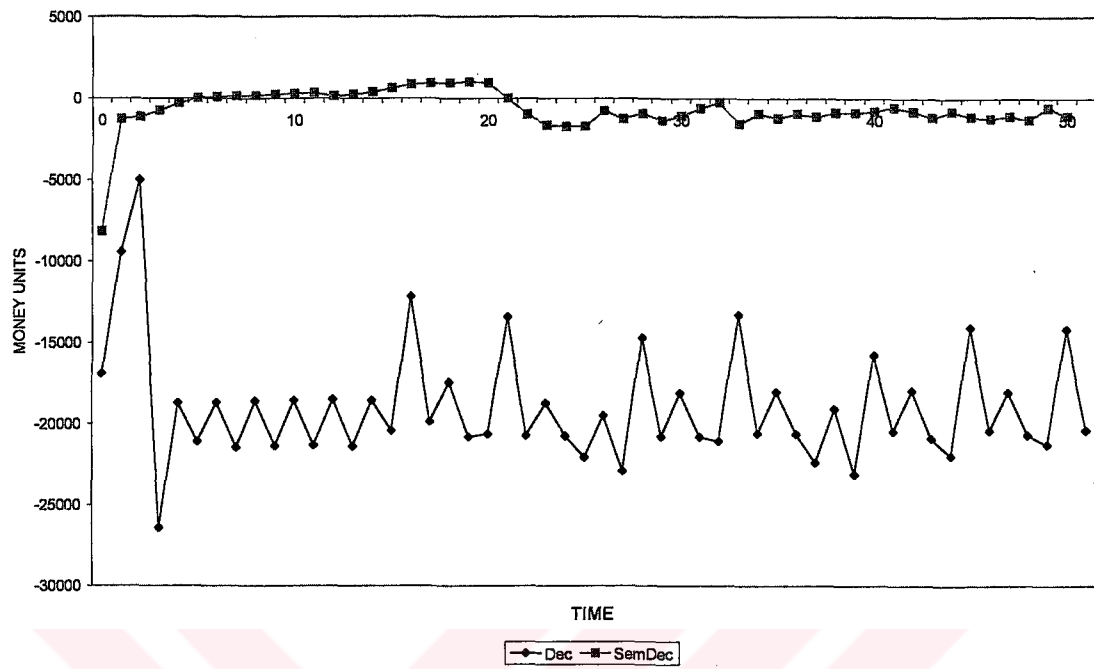


Figure 4:28: Instantaneous System Profit

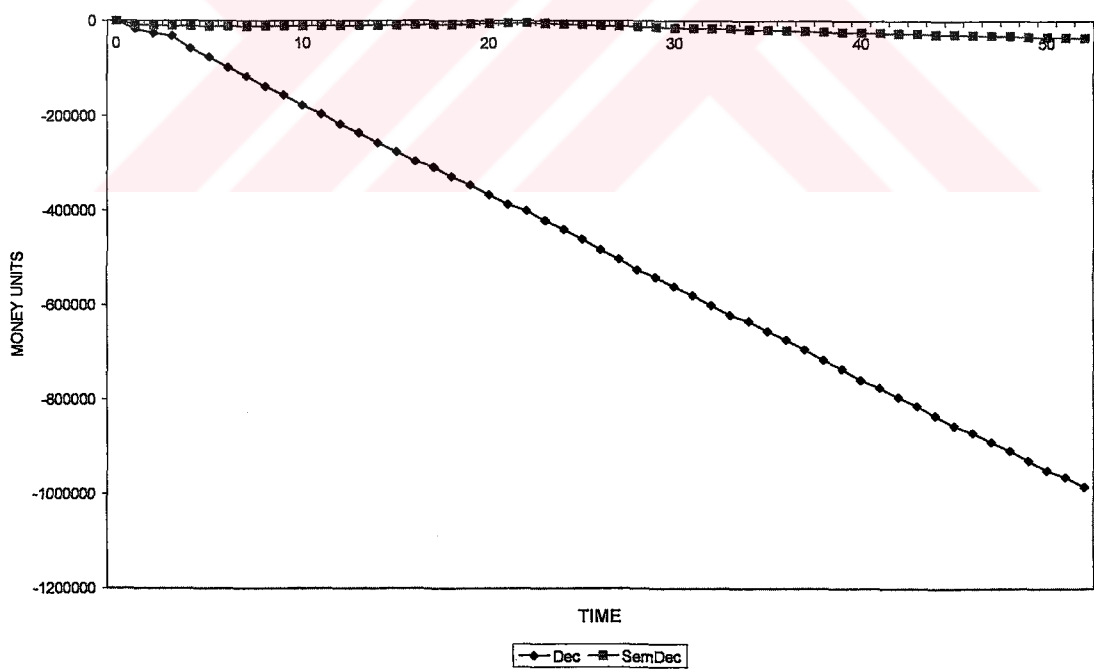


Figure 4:28: Cumulative System Profit

Finally, the end customer satisfaction is better in semi-decentralized configuration than in decentralized configuration and graphs are presented in Figs. 4.29 to 4.31. This is attributable to the reason that distribution center and retailer complex in the semi-decentralized configuration manages inventories and material transfers better, which is the end result of better coordination.

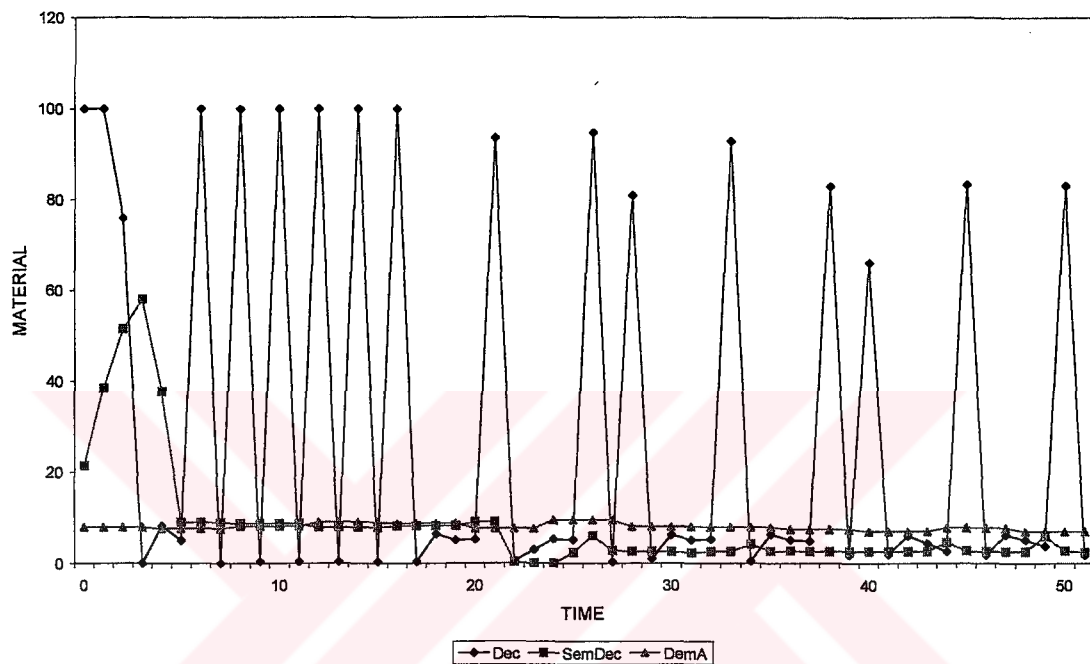


Figure 4:29: Demand vs Material Transfer at Retailer for Product A

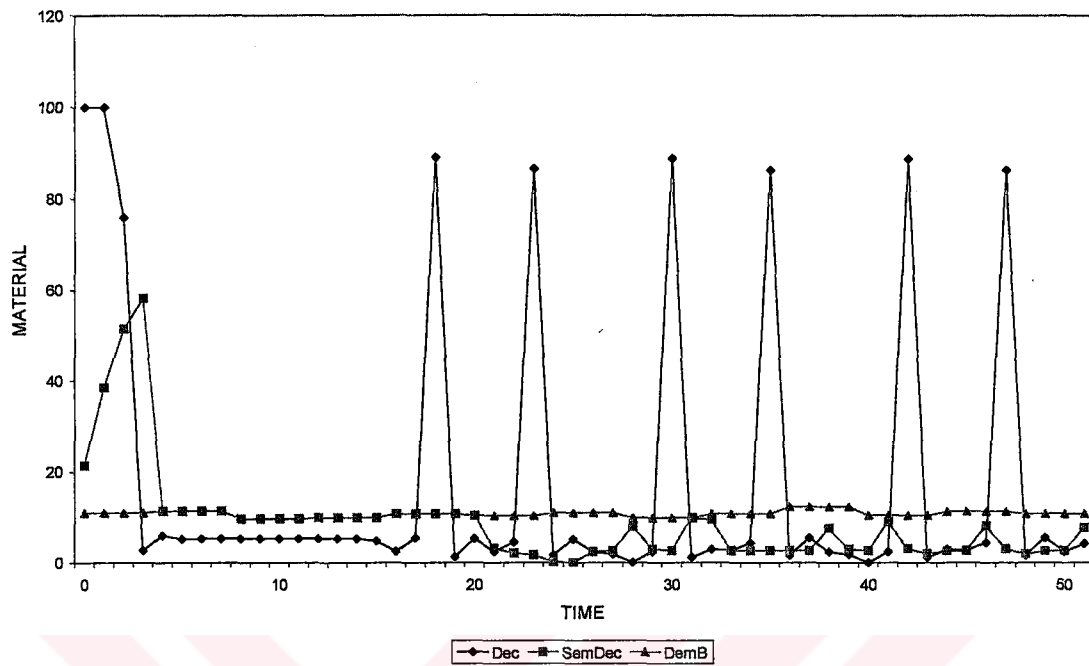


Figure 4:30: Demand vs Material Transfer at Retailer for Product B

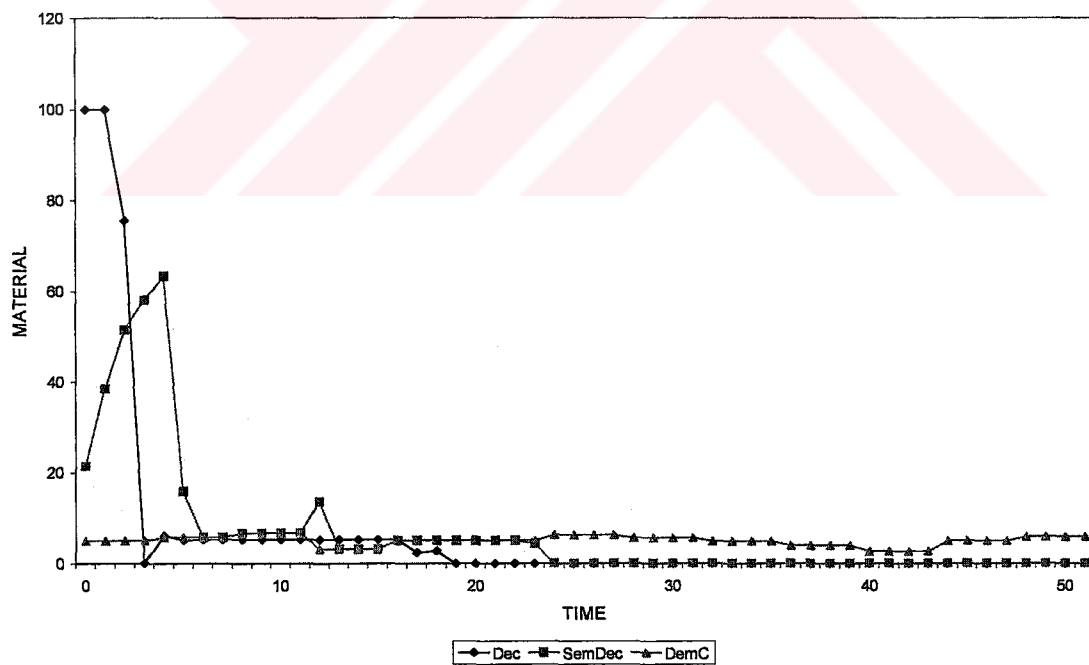


Figure 4:31: Demand vs Material Transfer at Retailer for Product C

In all of the system variables, it is easily observable that the decentralized system has more variability than semi-decentralized system, in the sense that the variables take higher and lower values and the frequency of change is more intense. It must be stated that the bullwhip effect is more severe in decentralized configuration, which can be verified by checking the material transfer request levels from plant and warehouse. The reason for the repetitive occurrence of this phenomenon is the *"inability of the system to learn"*. That is, initially the decentralized configuration observes that the inventory levels are high and tries to transfer this material to the downstream nodes, including the end customers. Because of the bullwhip effect, the plant and warehouse configuration produces a large batch of products, which creates a large amount of inventory in the warehouse. Then, in the next time step, seeing the large amount of inventory in the warehouse and the demand from the distribution center, most of the materials are transferred to the distribution center, just like what happens with the initial inventory. When the material arrives to the distribution center, the distribution center observes the large amount of material in the inventory and the demand from the retailer, it imitates what happened with the initial inventory levels, gets rid of the large inventory, and assumes that the plant and warehouse will deliver as much as wanted whenever desired. Just the same occurs for the retailer, which leads to the large variability in the decentralized configuration. Although the same repetitive occurrence is also present in the semi-decentralized configuration, since the bullwhip effect is not as severe as in the decentralized configuration, the same amount of variability is not observed, despite the fact that both the variability and the repetitive occurrence is present.

The behavioral difference between the two supply chain configurations can be summarized as follows:

- Lower inventory levels for products A and B, and higher inventory level for product C until the depletion of initial inventory for semi-decentralized supply chain
- Lower production sizes as the first priority and less frequent production initiation for the semi-decentralized supply chain, leading to less production cost compared to that of decentralized configuration
- Lower material transfer levels between units while achieving better end customer satisfaction
- Higher total system profit for semi-decentralized system

All of these results increase the profitability of the system and can be attributed to the difference between the induced bullwhip effects in both configurations, which can be summarized as follows:

- Exaggerated forecasts lead to exaggerated material demands in decentralized configuration compared to those of semi-decentralized configuration, since no information is present about the exact needs of the downstream unit and units only optimize their own profit and assume other units will behave as expected.
- Repetitive occurrence of the bullwhip effect, present in both systems, caused by the inability of the system to learn

4.3.2 Optimization Horizon 25:

All of the interpretations performed for horizon length 15 remain valid for this case as well. The graphs are given through Figs. 4.32 to 4.59.

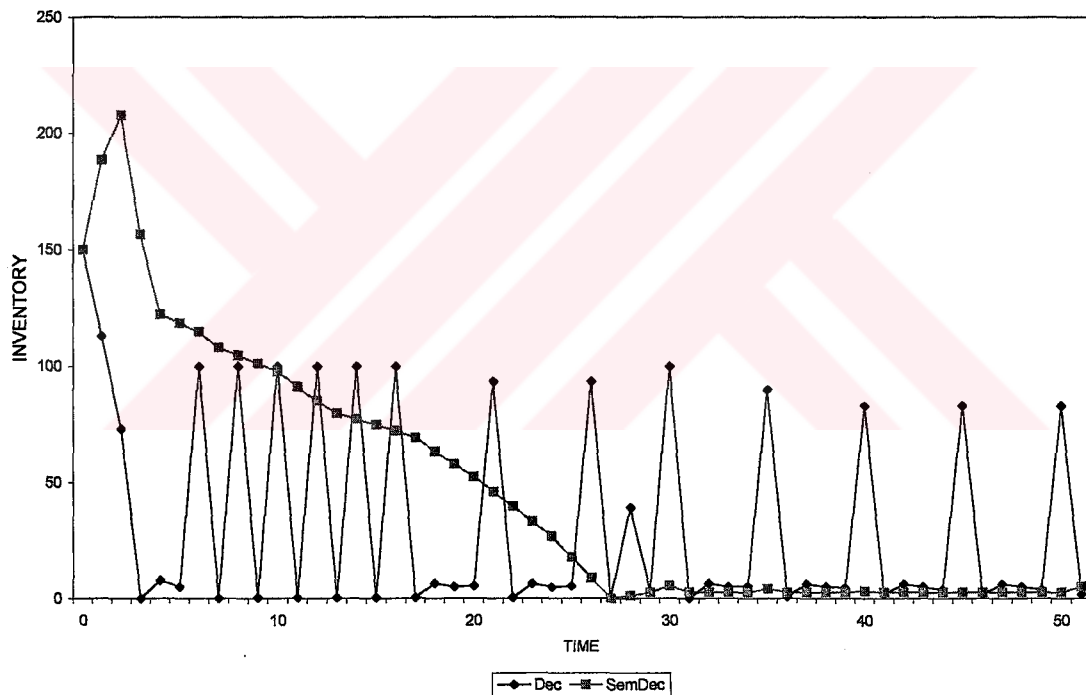


Figure 4.32: Retailer node inventory level for product A

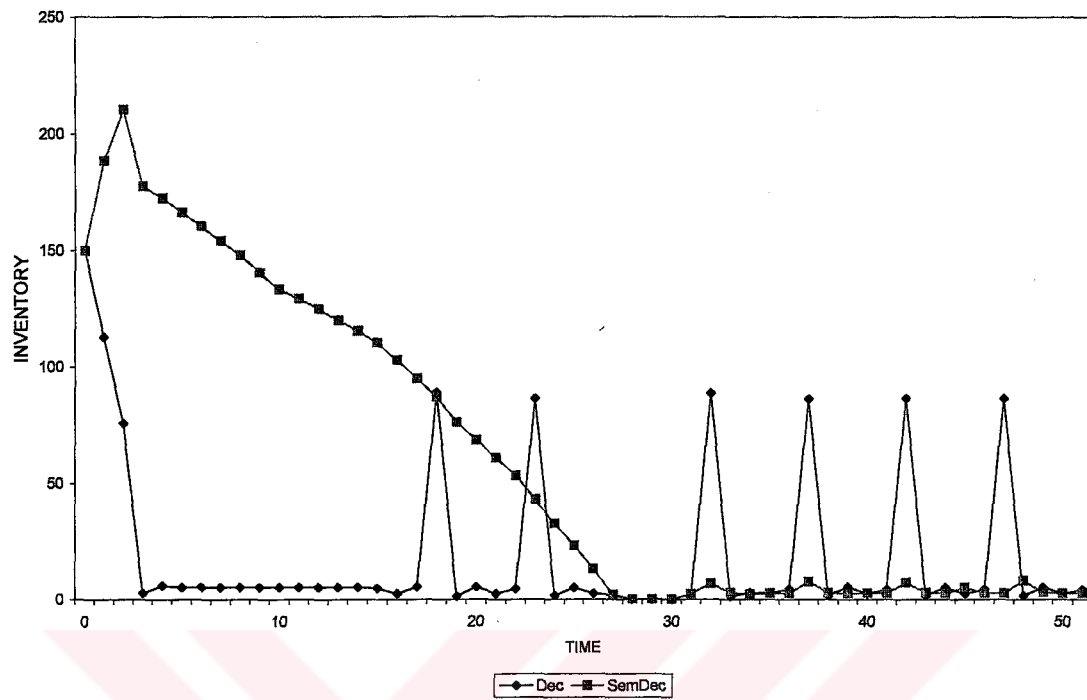


Figure 4.33: Retailer node inventory level for product B

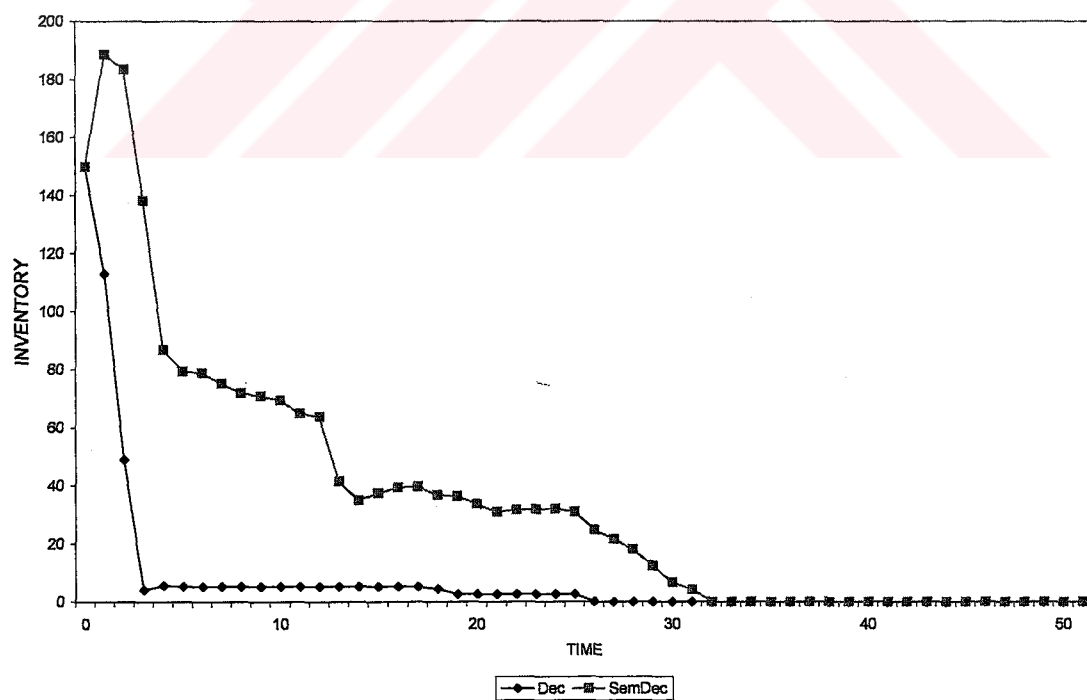


Figure 4.34: Retailer node inventory level for product C

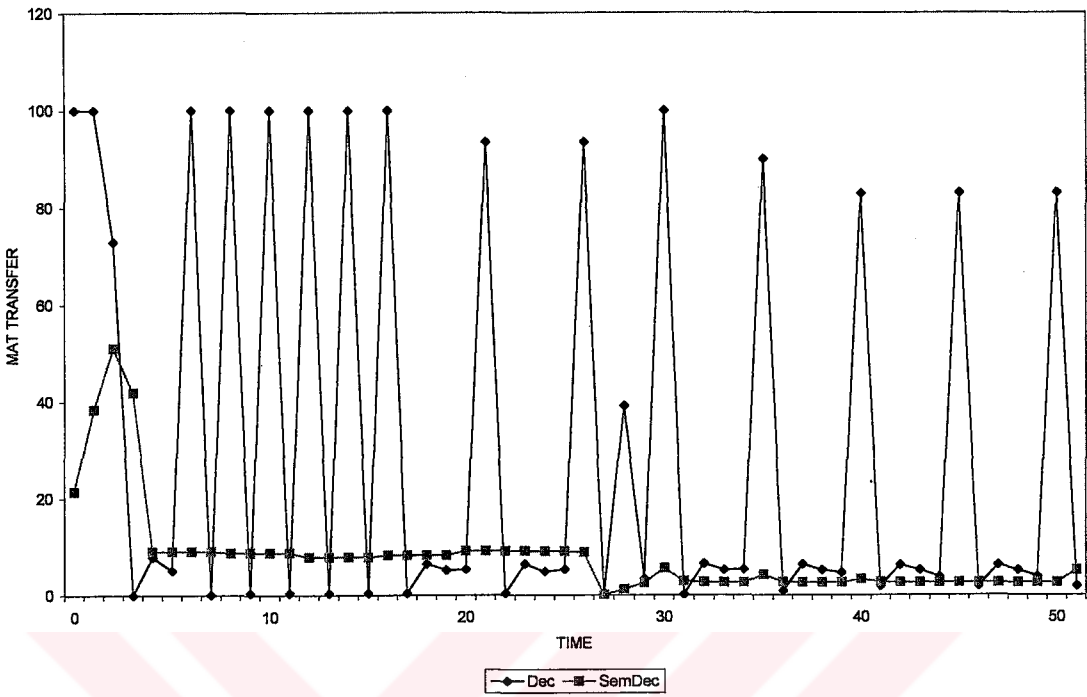


Figure 4.35: Retailer node material transfer level for product A

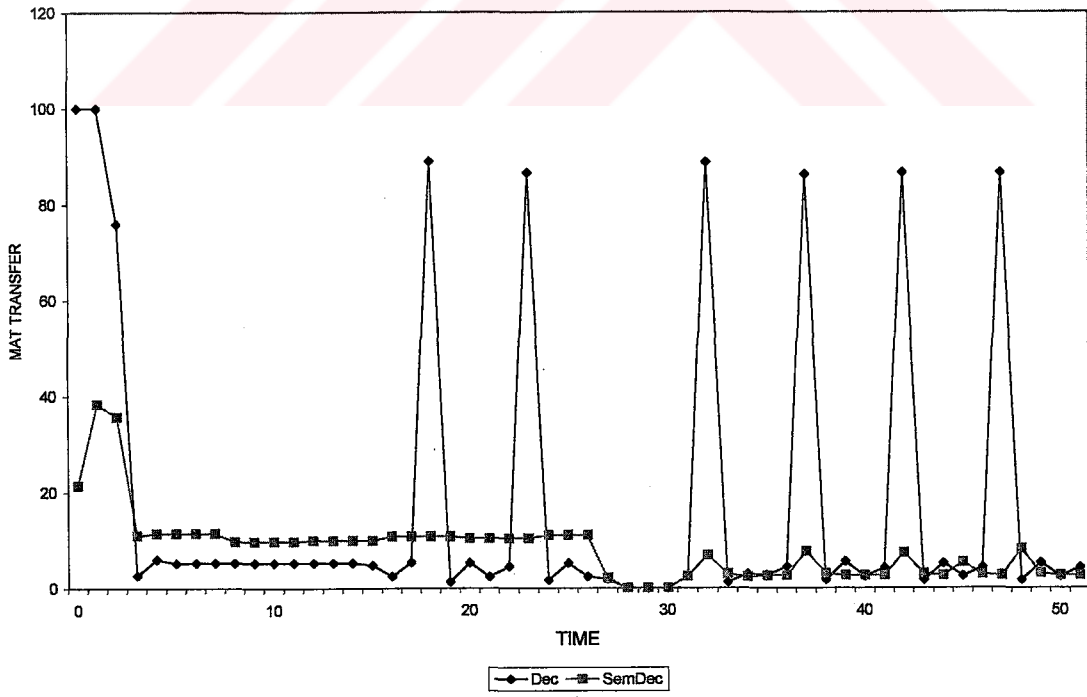


Figure 4.36: Retailer node material transfer level for product B

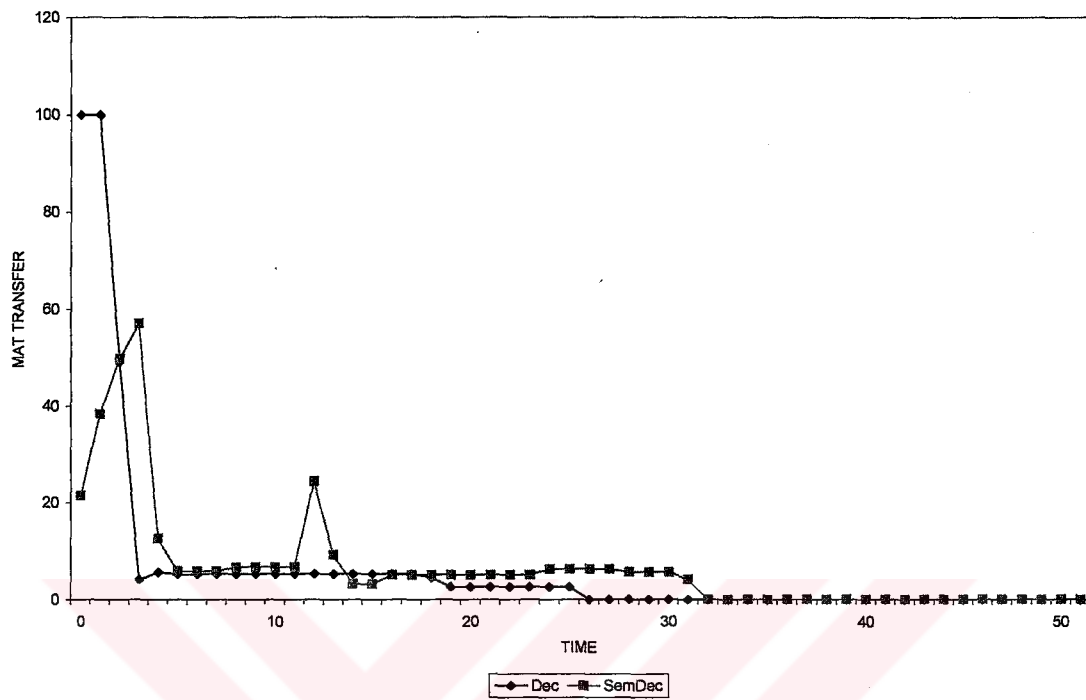


Figure 4.37: Retailer node material transfer level for product C

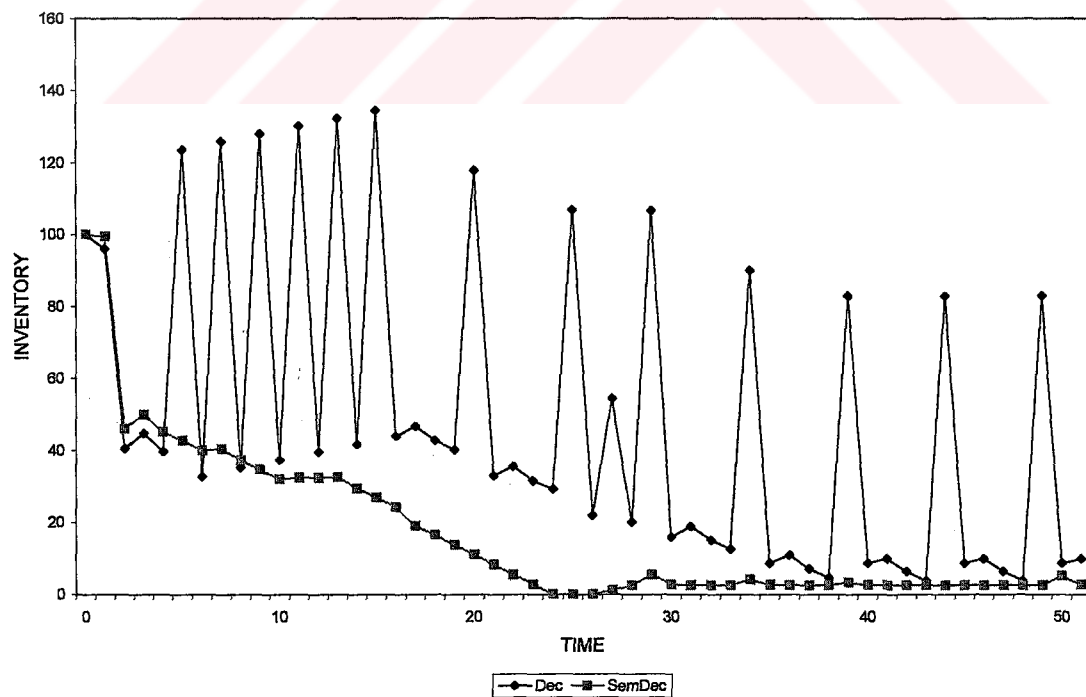


Figure 4.38: Distribution center node inventory level for product A

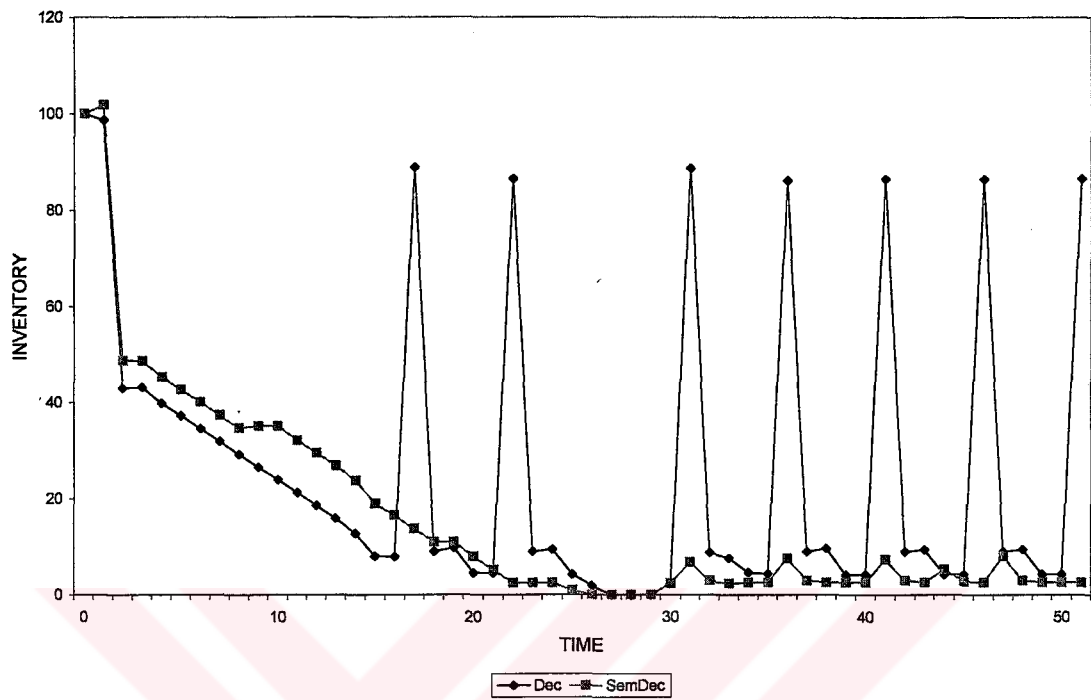


Figure 4.39: Distribution center node inventory level for product B

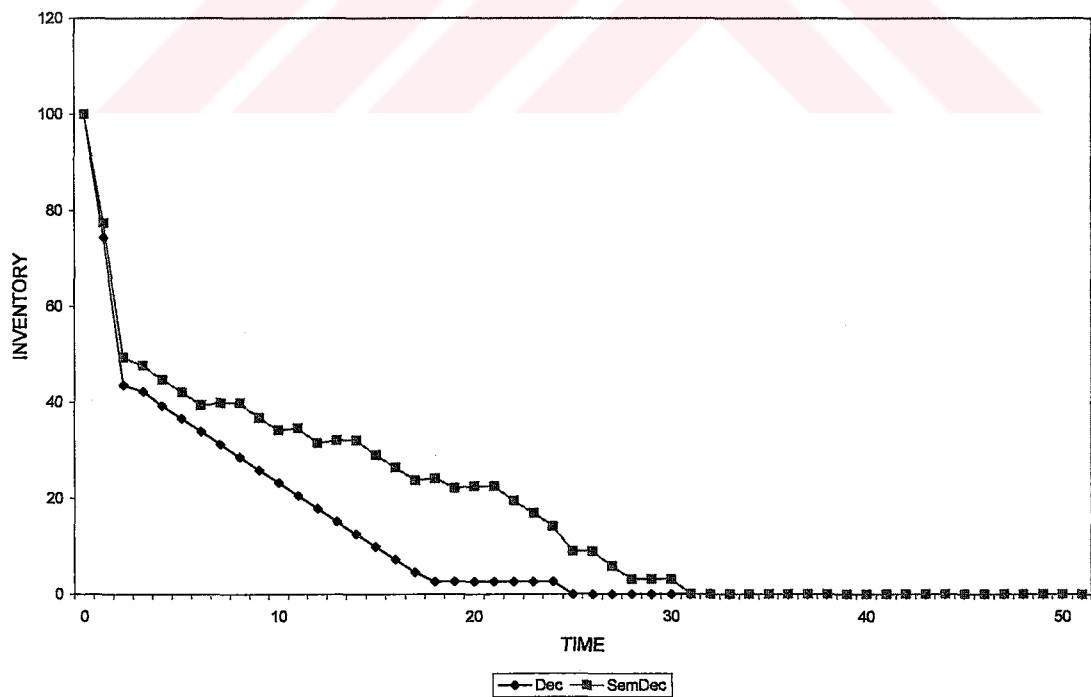


Figure 4.40: Distribution center node inventory level for product C

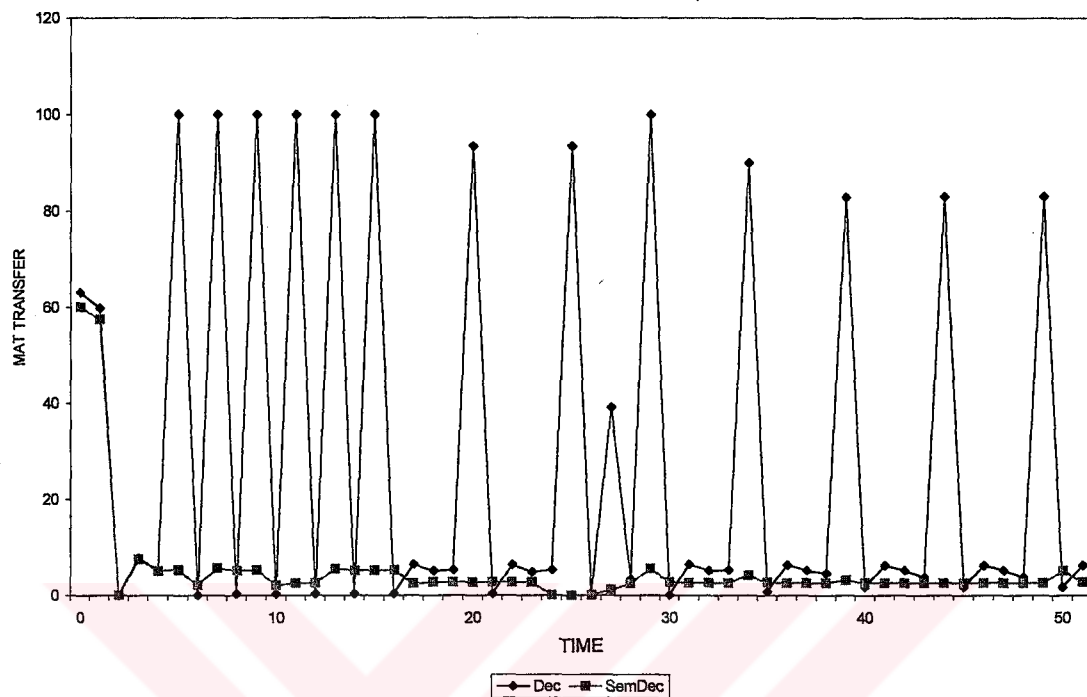


Figure 4.41: Distribution center node material transfer level for product A

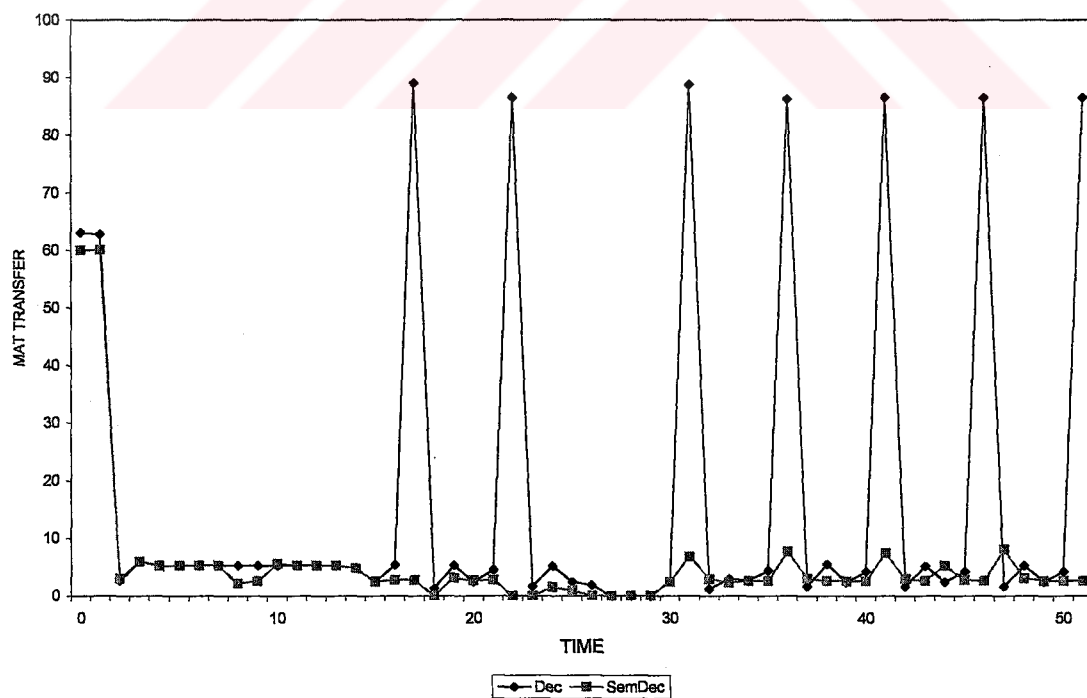


Figure 4.42: Distribution center node material transfer level for product B

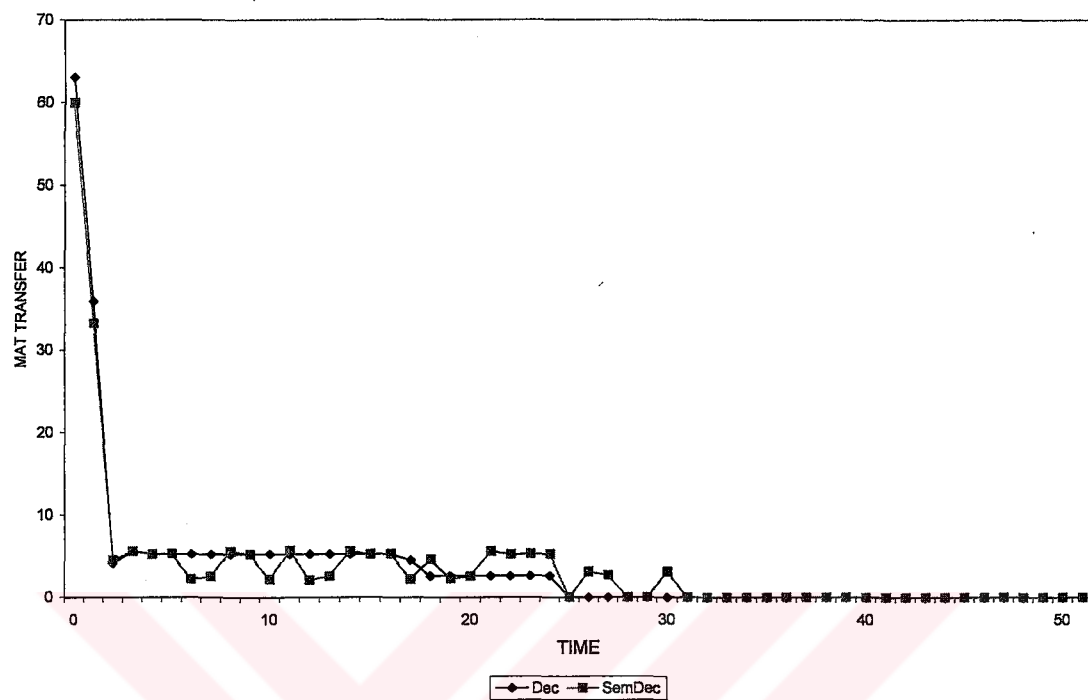


Figure 4.43: Distribution center node material transfer level for product C

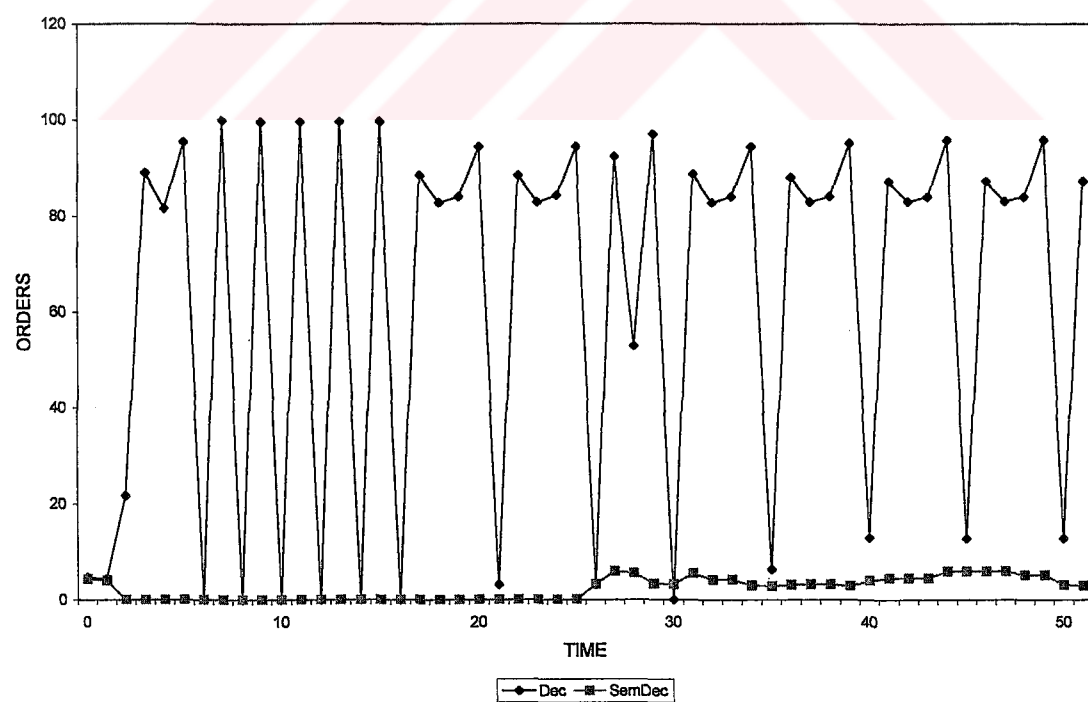


Figure 4.44: Material demanded from Plant and Warehouse of Product A

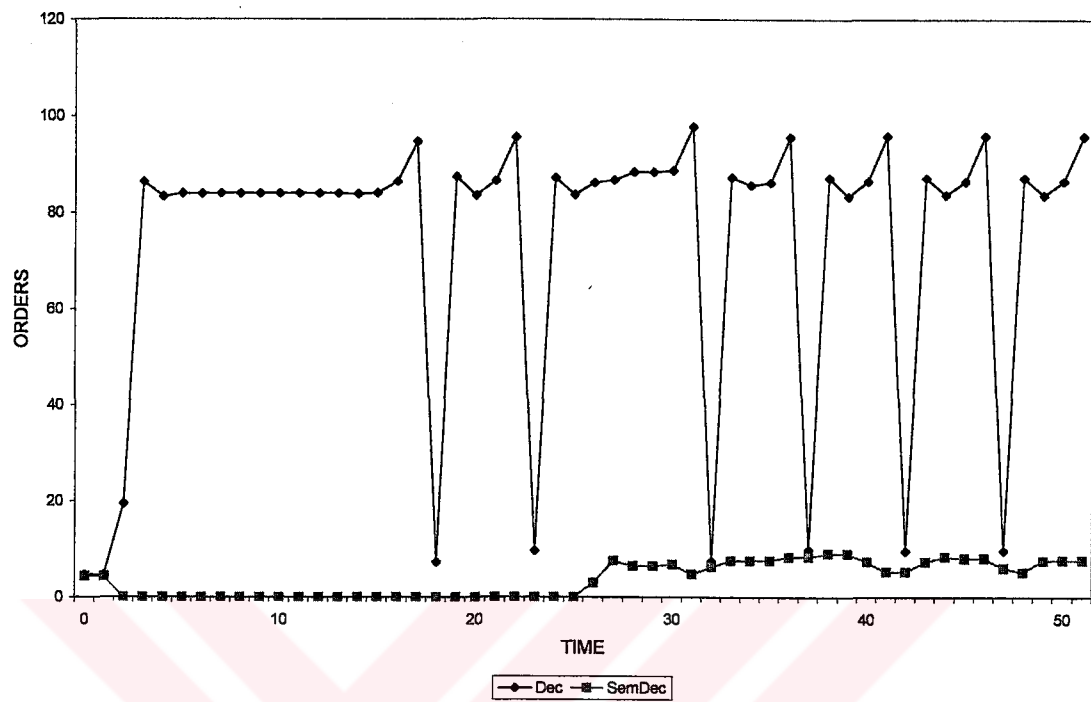


Figure 4.45: Material demanded from Plant and Warehouse of Product B

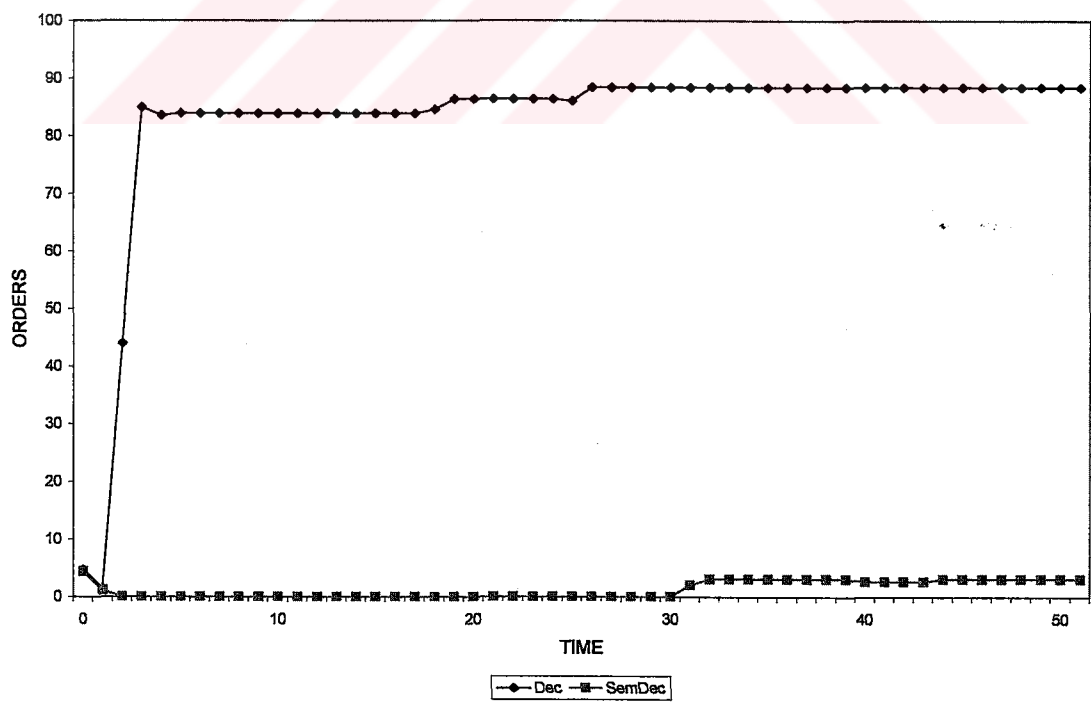


Figure 4.46: Material demanded from Plant and Warehouse of Product C

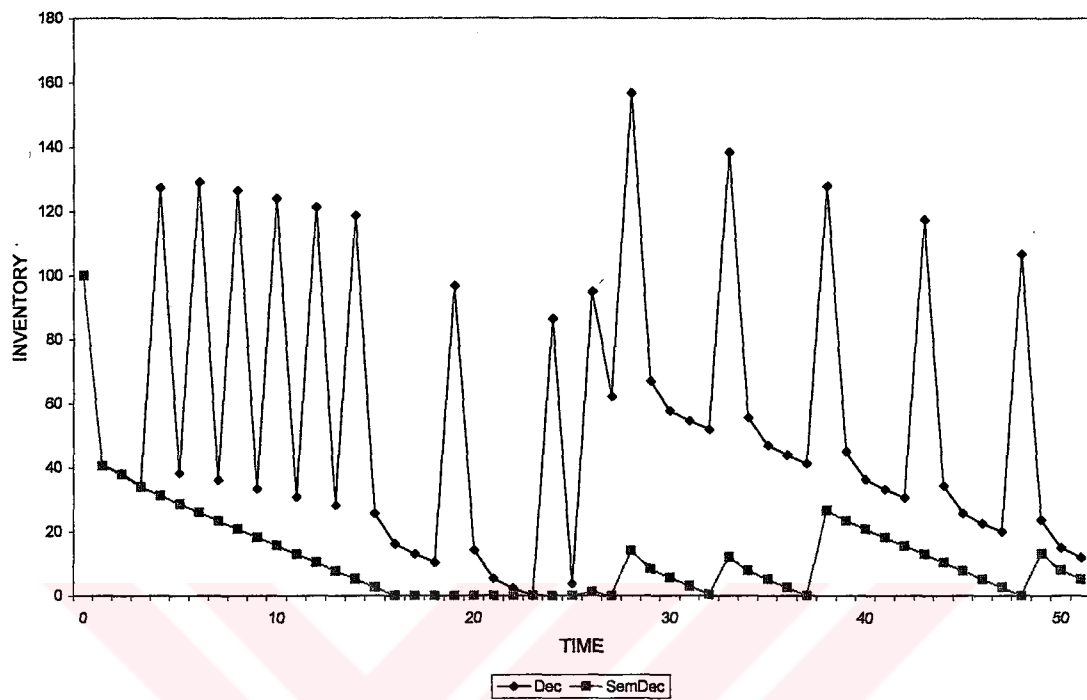


Figure 4.47: Plant and warehouse node inventory level for product A

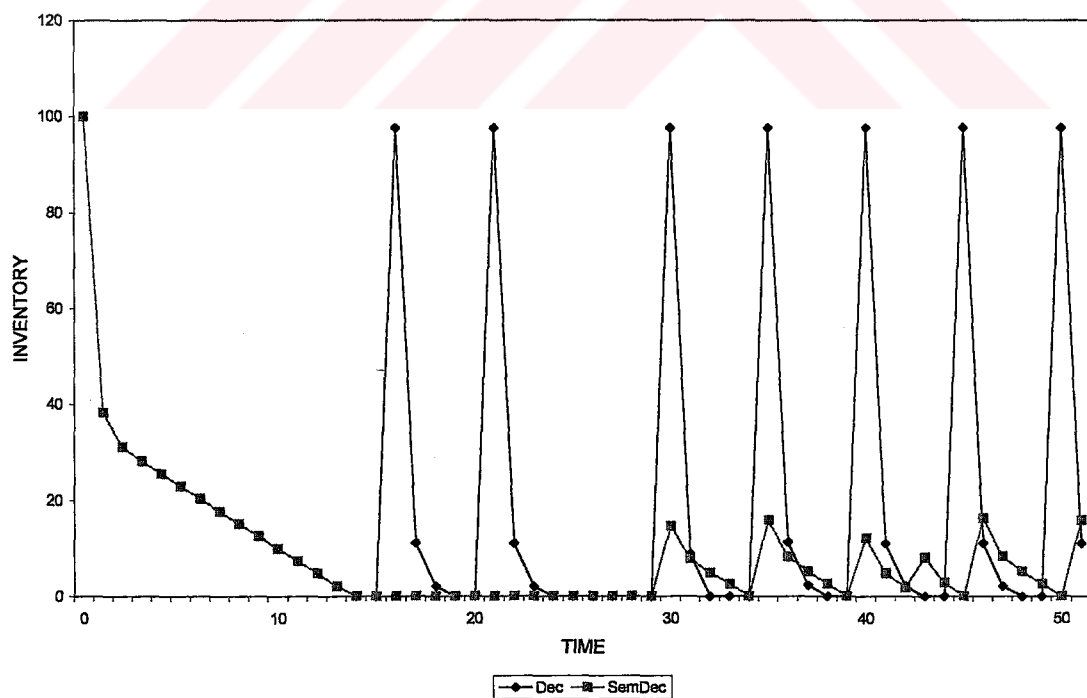


Figure 4.48: Plant and warehouse node inventory level for product B

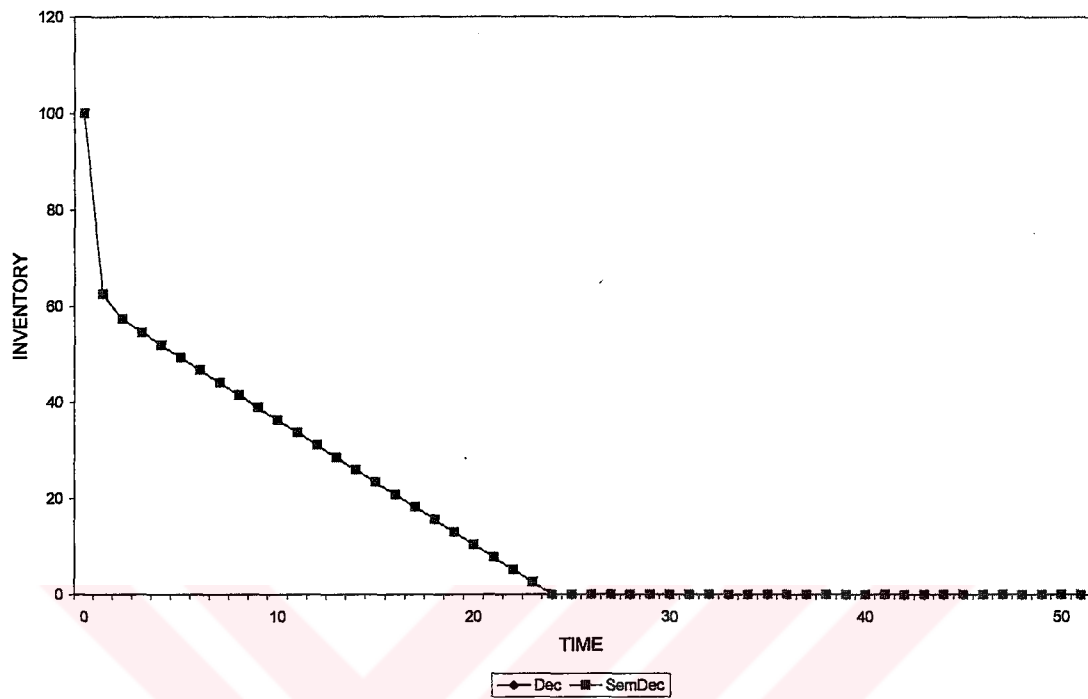


Figure 4.49: Plant and warehouse node inventory level for product C

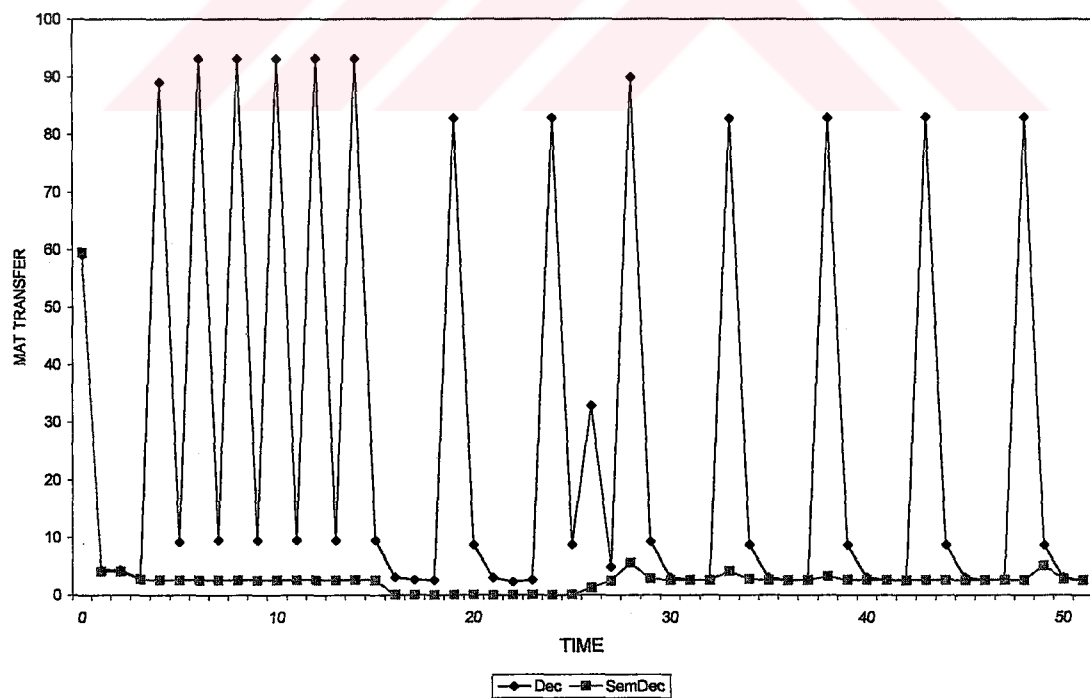


Figure 4.50: Plant and warehouse node material transfer level for product A

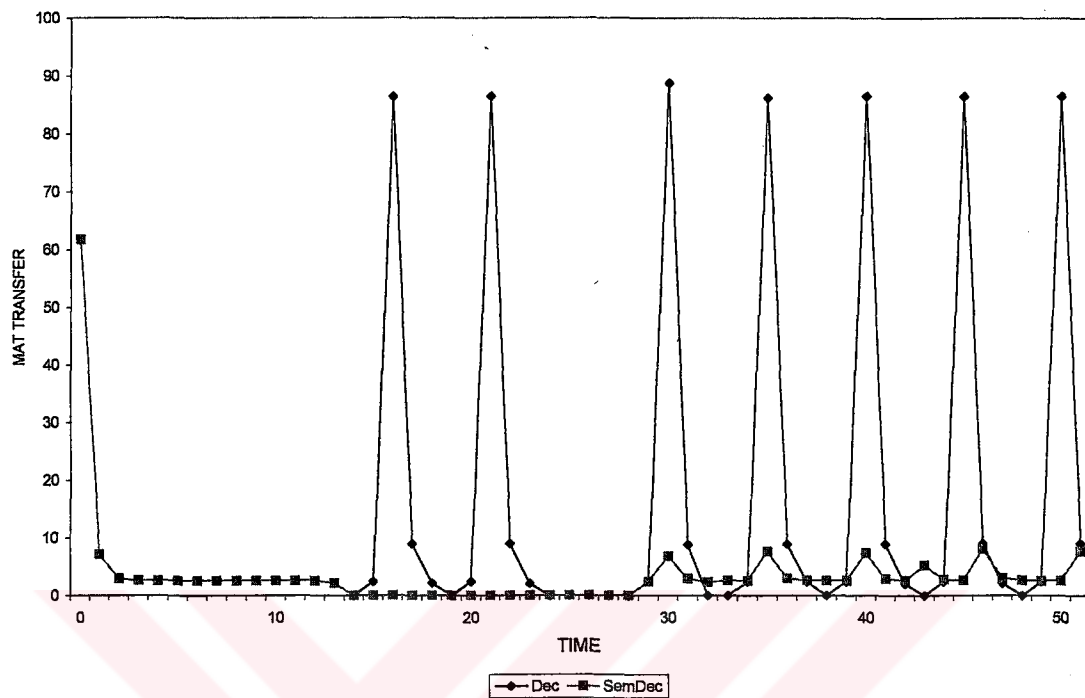


Figure 4.51: Plant and warehouse node material transfer level for product B

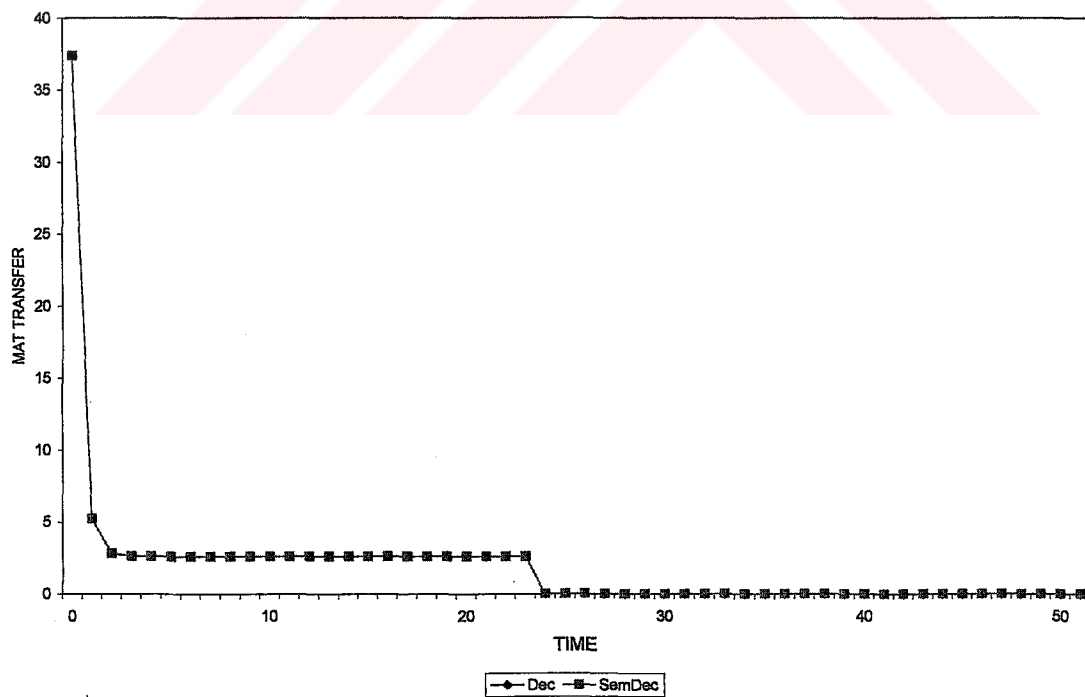


Figure 4.52: Plant and warehouse node material transfer level for product C

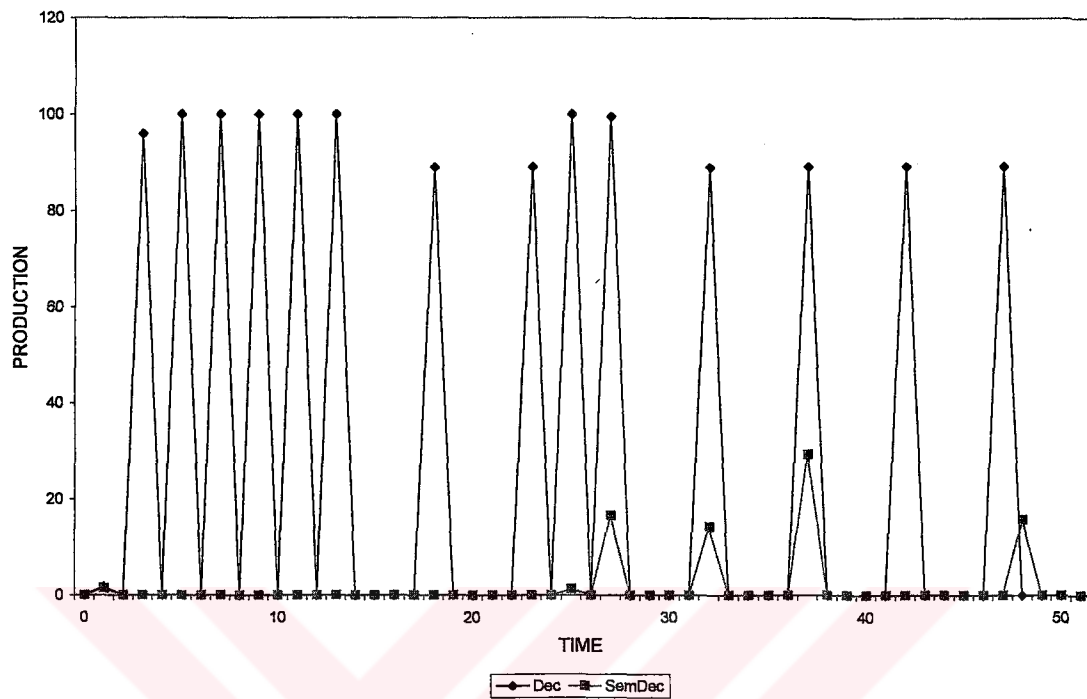


Figure 4.53: Production of A

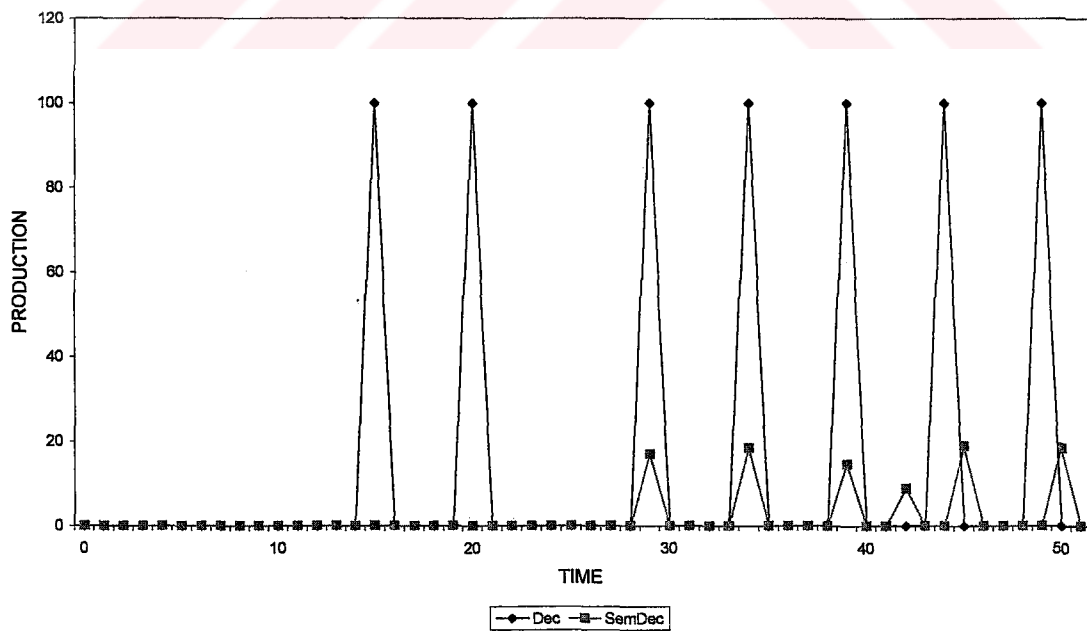


Figure 4.53: Production of B

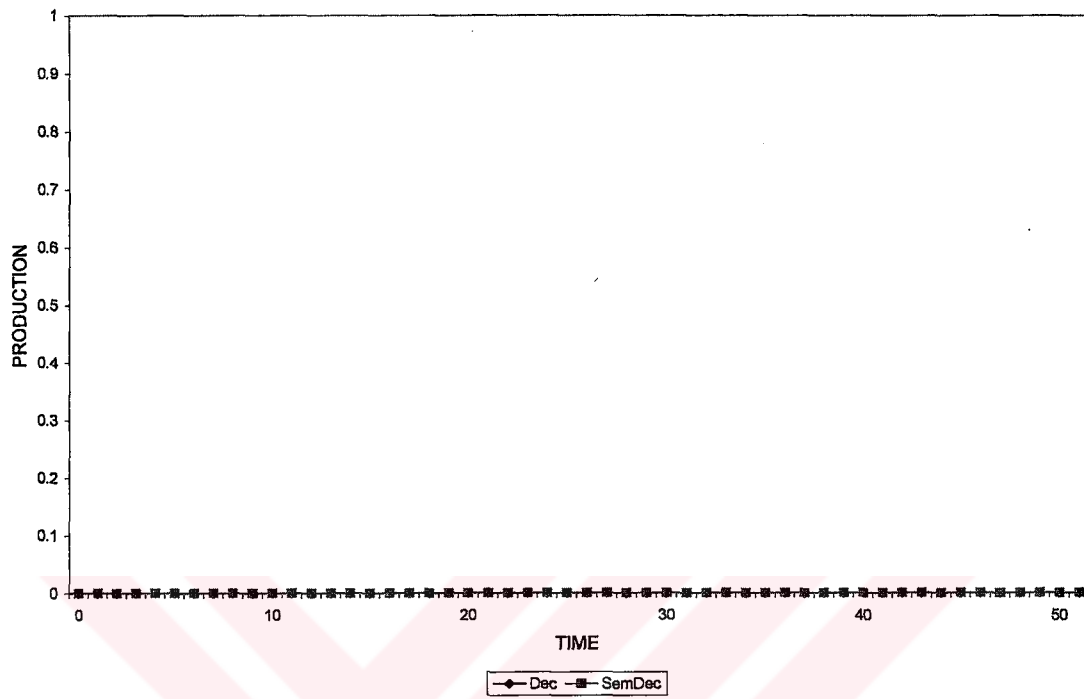


Figure 4.54: Production of C

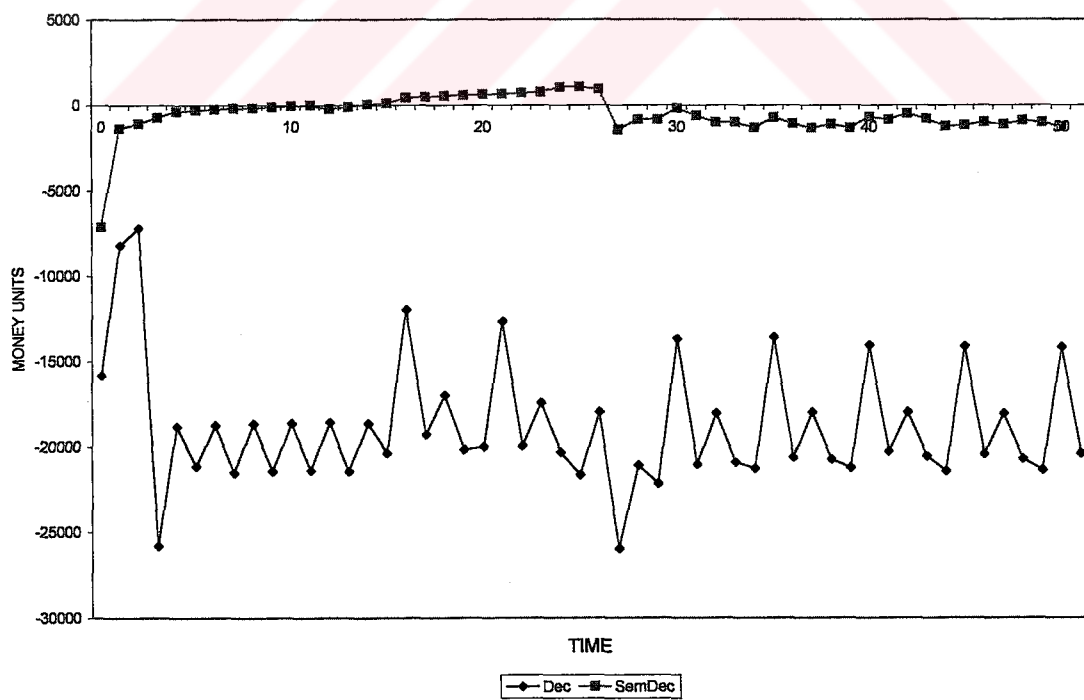


Figure 4.55: Instantaneous System Profit

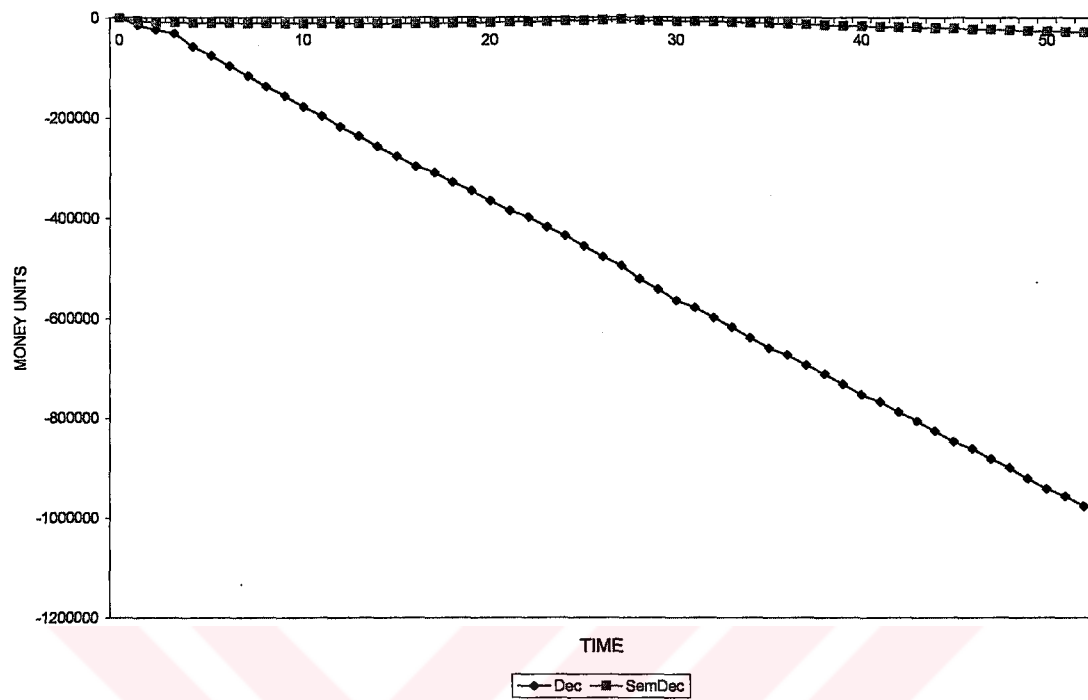


Figure 4:56: Cumulative System Profit

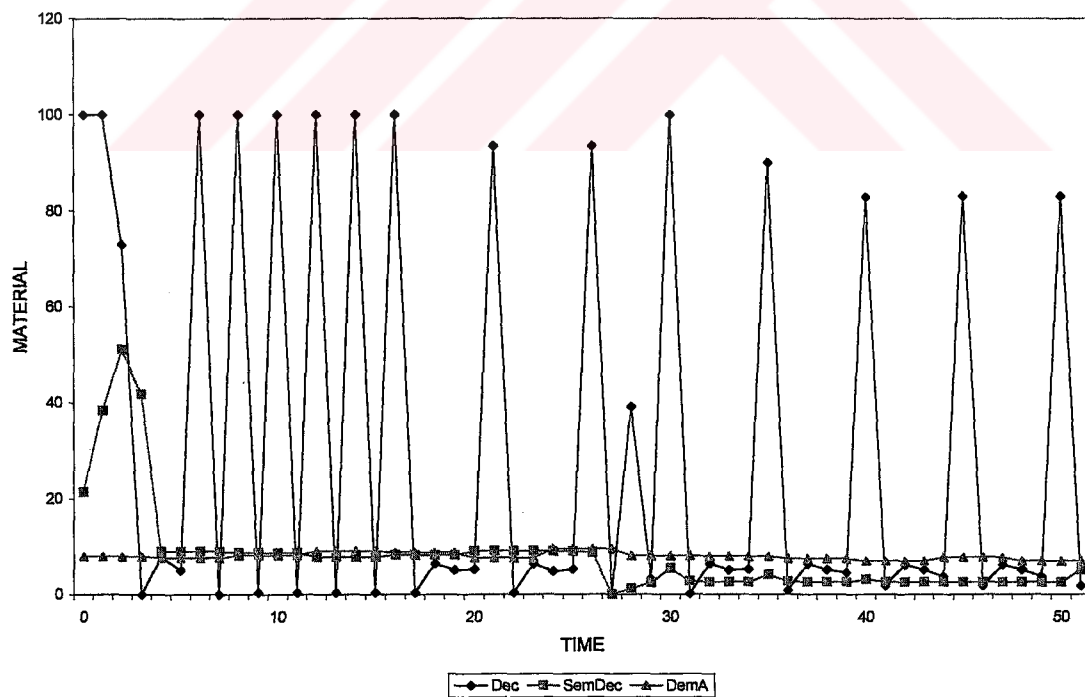


Figure 4:57: Demand vs Material Transfer at Retailer for Product A

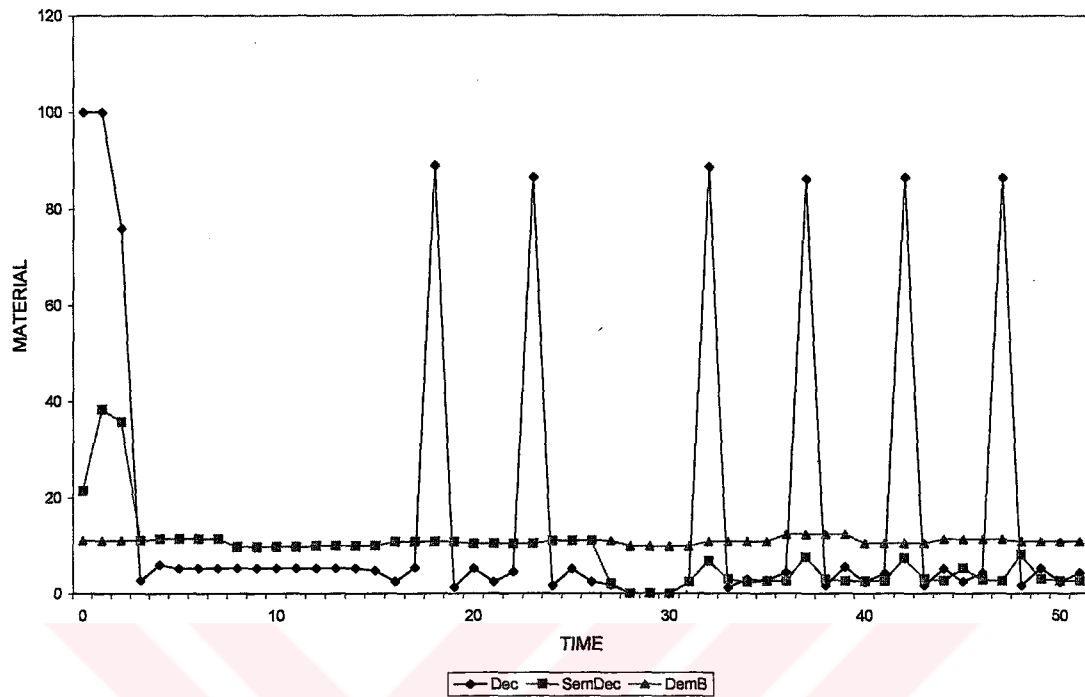


Figure 4:58: Demand vs Material Transfer at Retailer for Product B

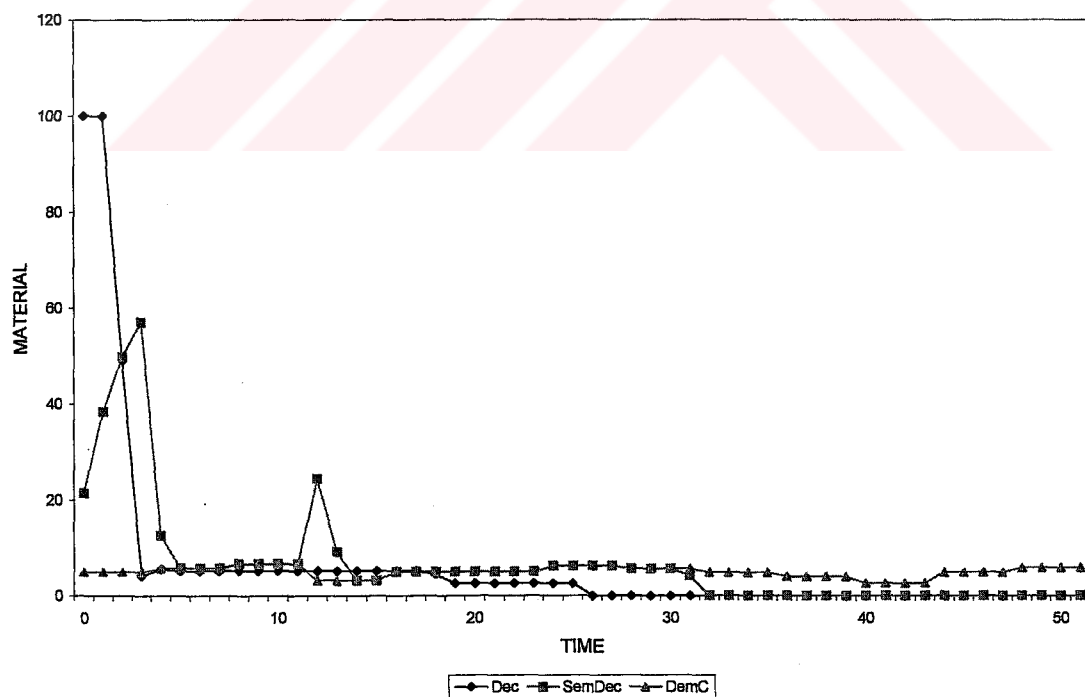


Figure 4:59: Demand vs Material Transfer at Retailer for Product C

4.3.3 Optimization Horizon 15 vs Horizon 25:

To save space, Semi-Decentralized Supply Chain with Horizon Length 15 will be abbreviated as (SDHL15) and Semi-Decentralized Supply Chain with Horizon Length 20 will be abbreviated as (SDHL25)

In terms of inventories, as SDHL25 depletes initial inventory in either equal or a longer period of time for all inventories, which is an expected result since in any optimization step, SDHL15 optimizes inventory on hand in for a longer period of time. This confirms the intuition that SDHL25 should be less myopic than SDHL15. In time, there is very little to distinguish between horizon lengths.

The plots for inventory levels and material transfers are presented in Figs. 4.60 to 4.77.

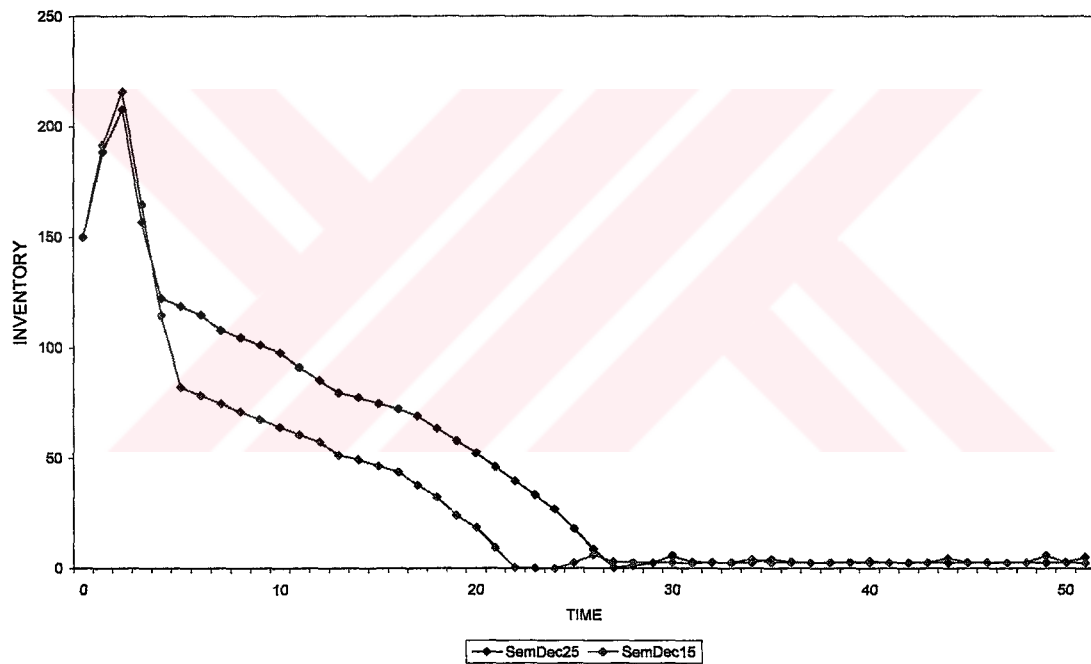


Figure 4.60: Retailer node inventory level for product A

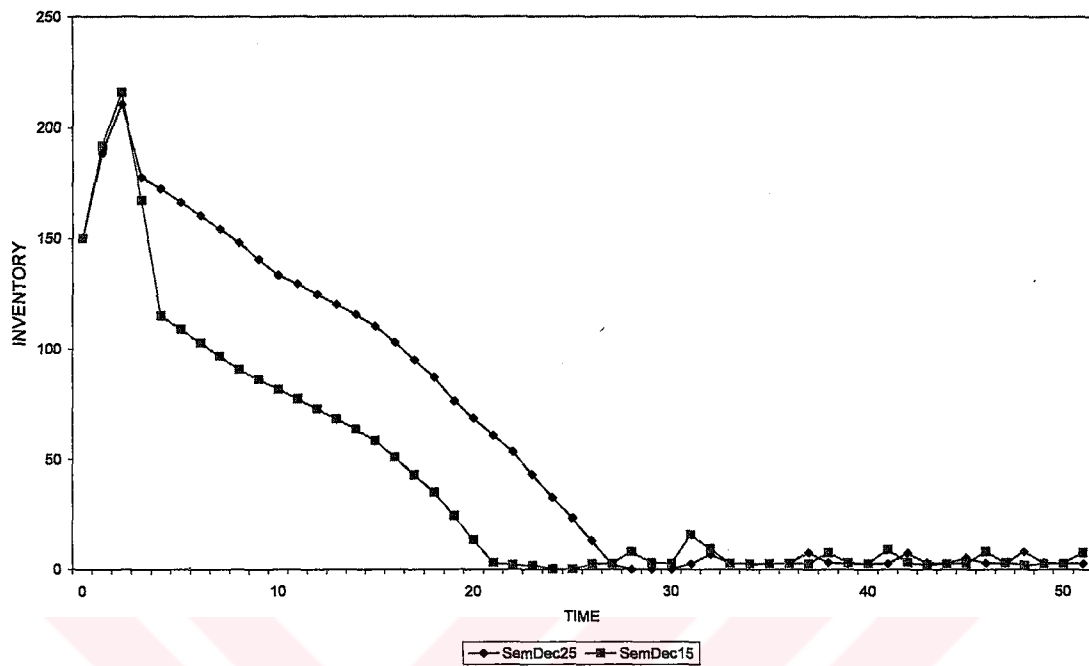


Figure 4.61 Retailer node inventory level for product B

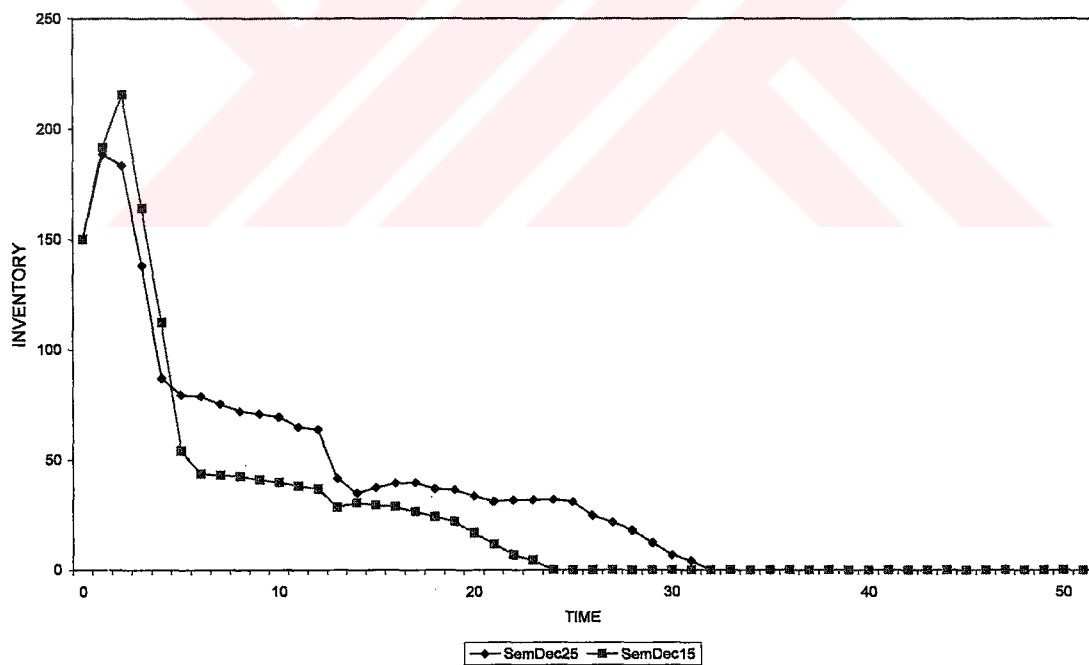


Figure 4.62: Retailer node inventory level for product C

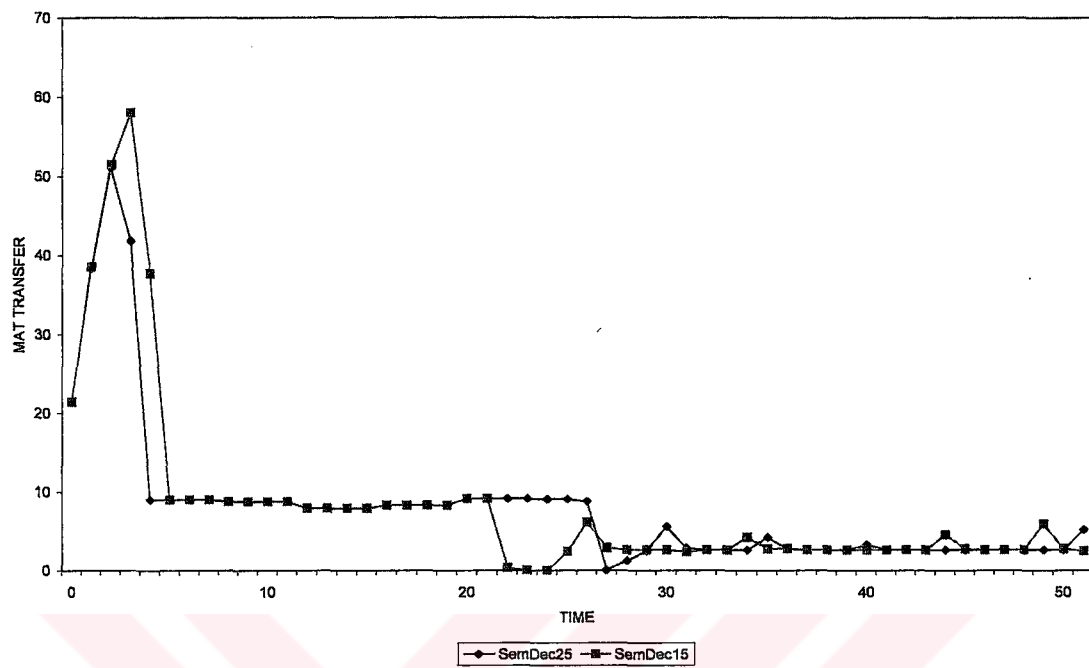


Figure 4.63: Retailer node material transfer level for product A

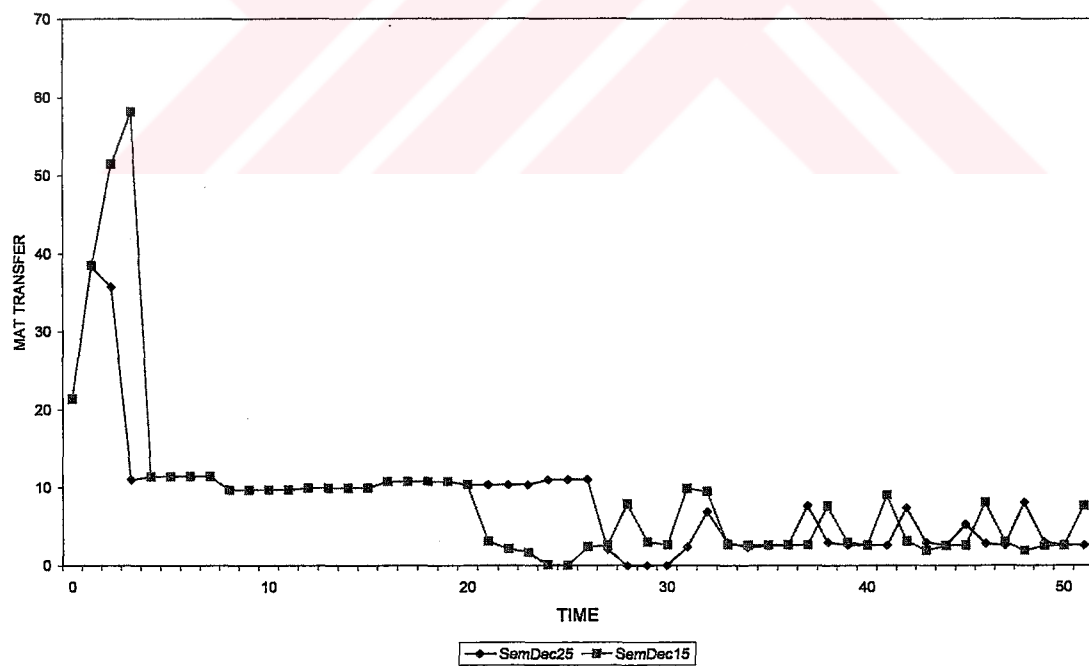


Figure 4.64: Retailer node material transfer level for product B

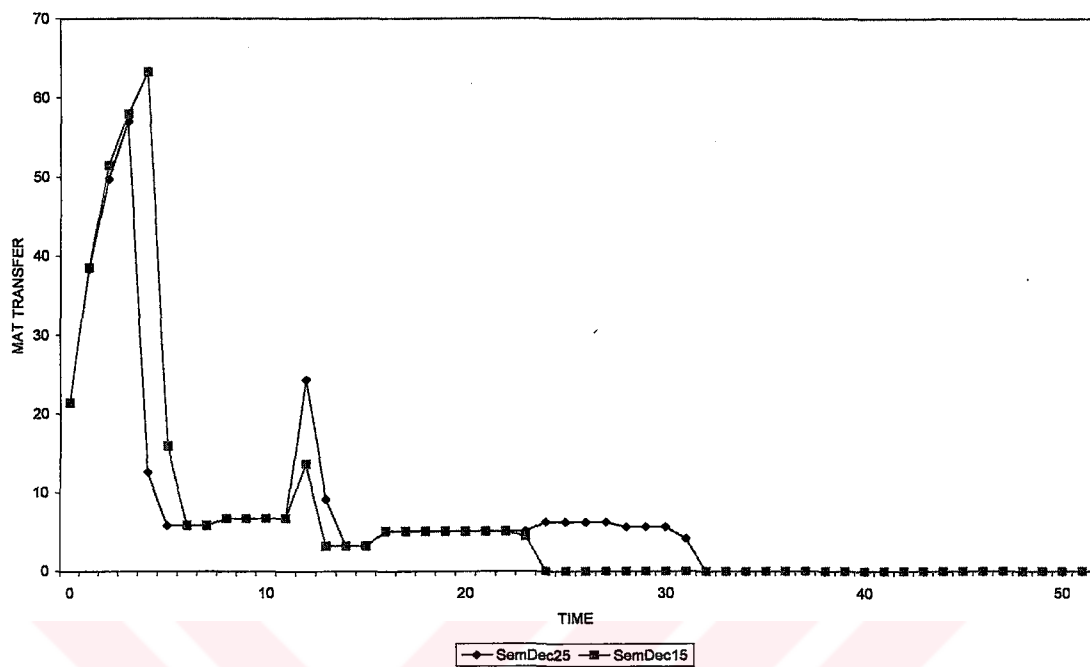


Figure 4.65: Retailer node material transfer level for product C

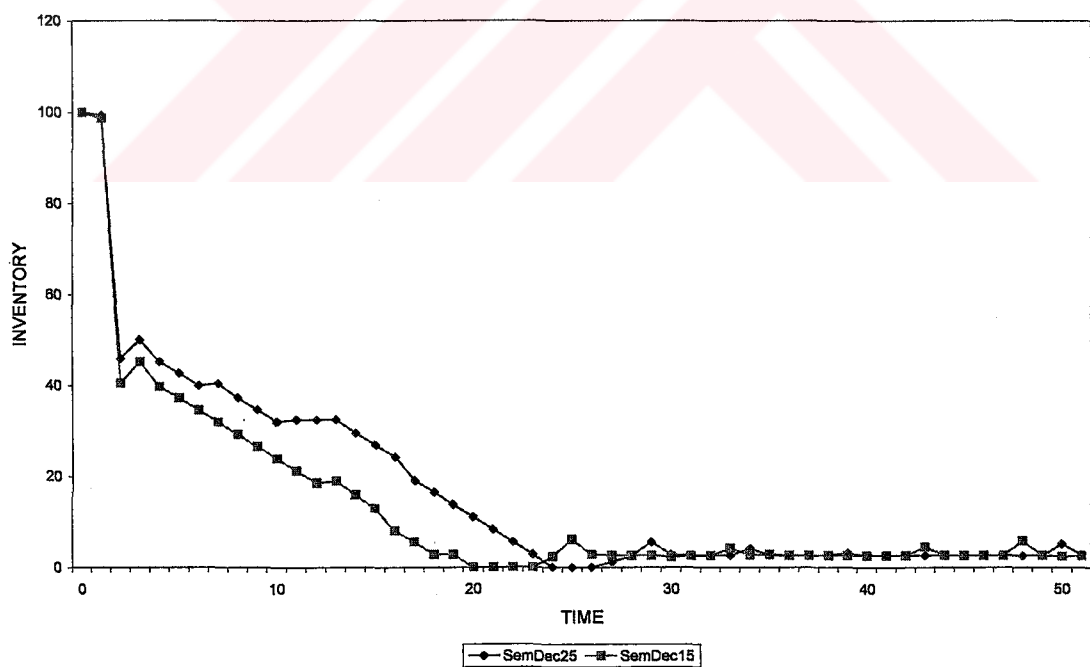


Figure 4.66: Distribution center node inventory level for product A

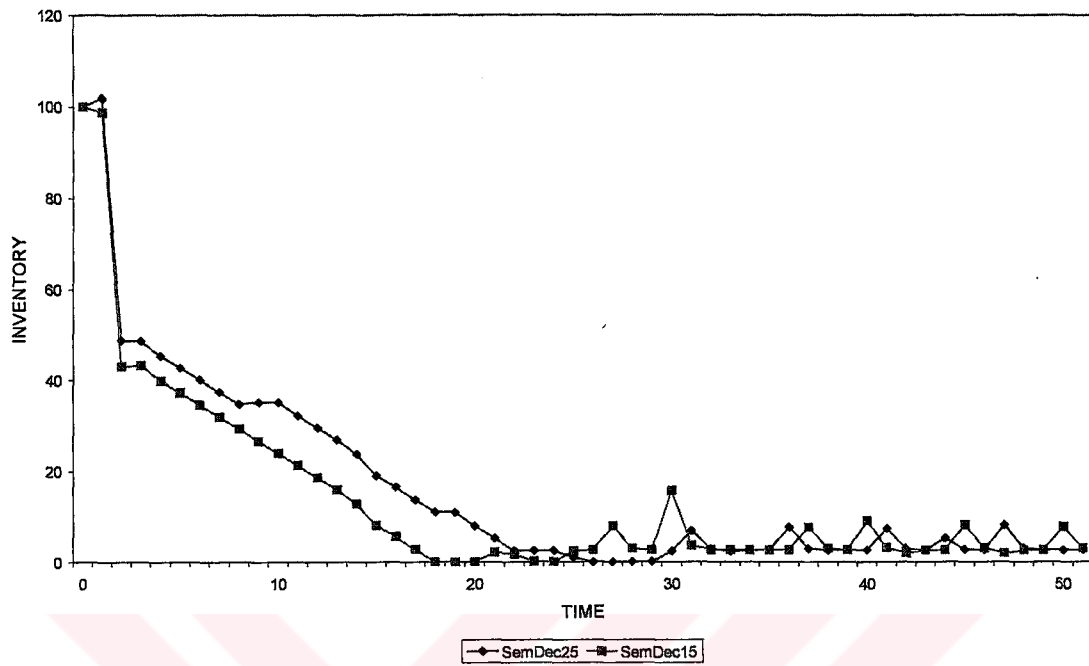


Figure 4.67: Distribution center inventory level for product B

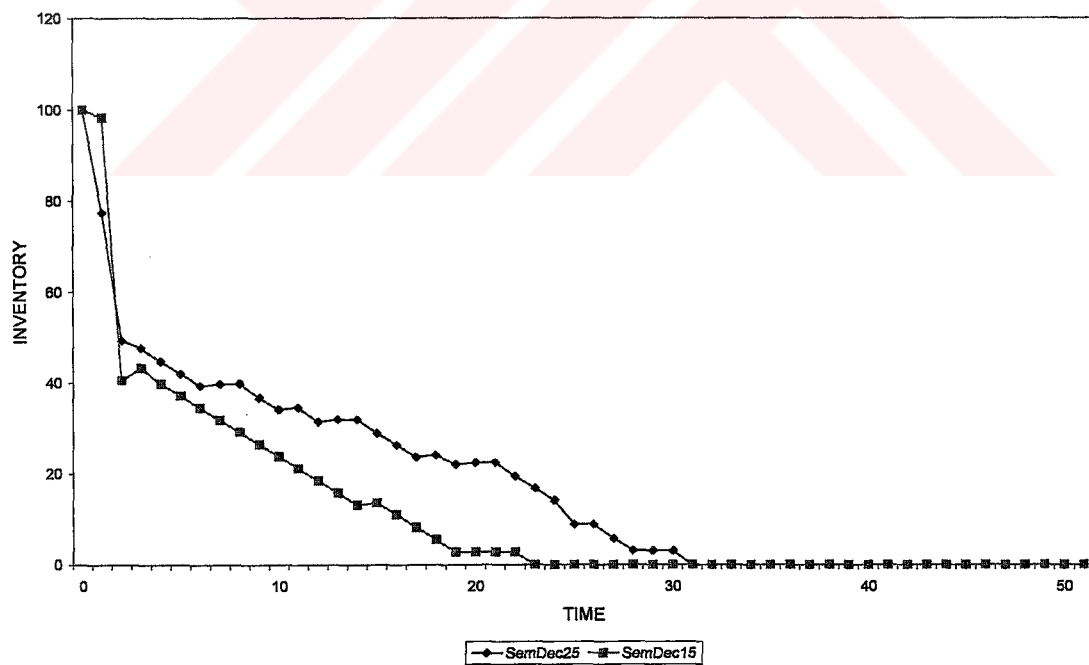


Figure 4.68: Distribution center node inventory level for product C

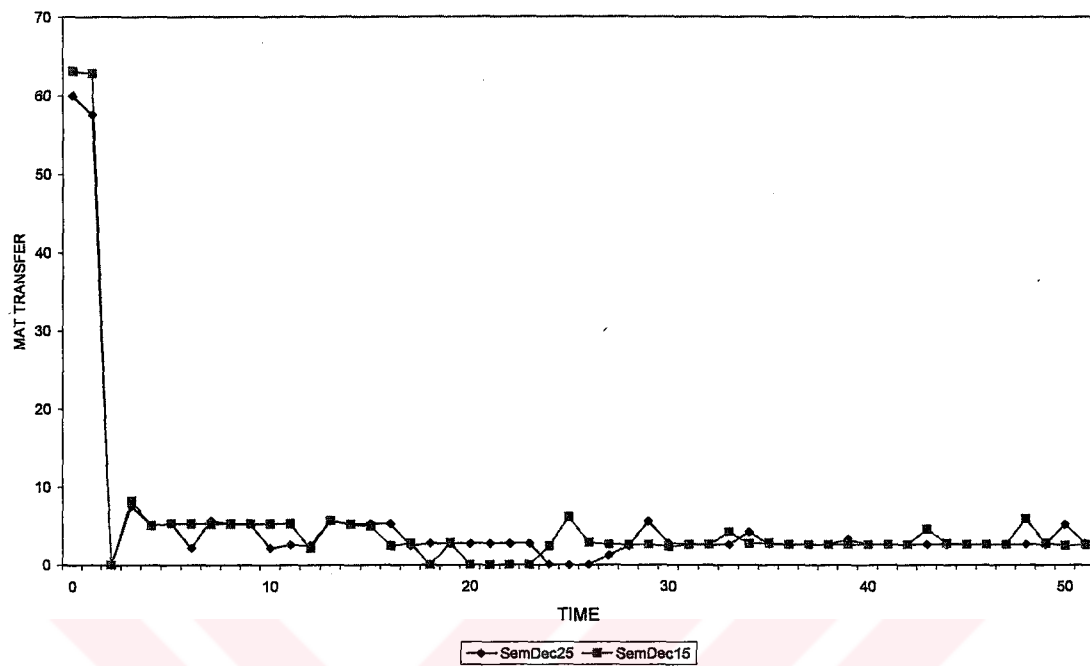


Figure 4.69: Distribution center node material transfer level for product A

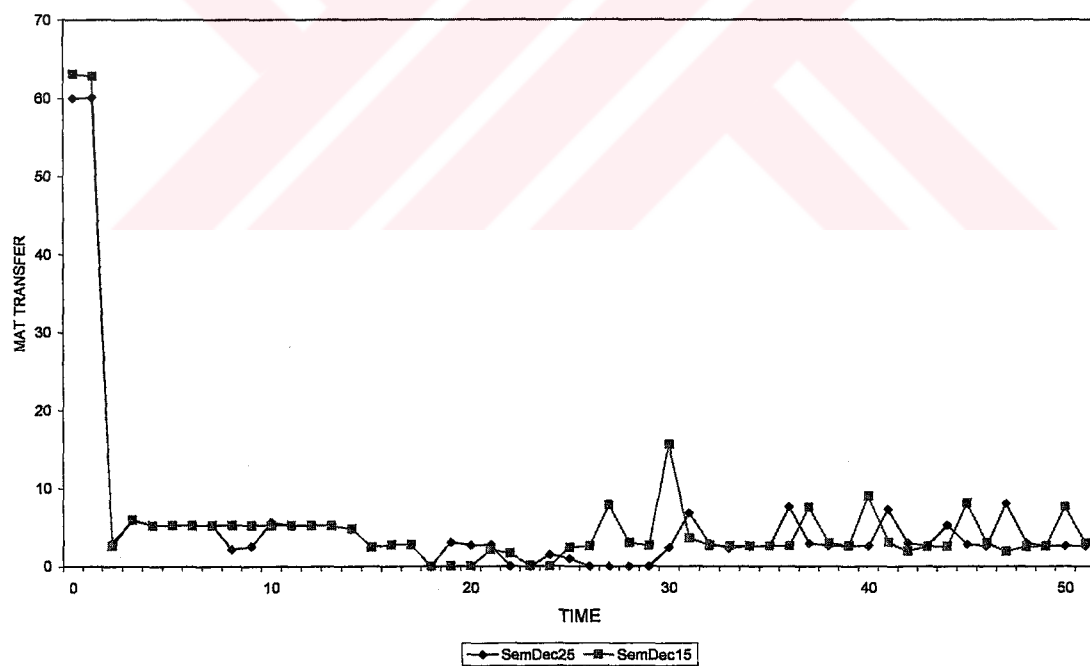


Figure 4.70: Distribution center node material transfer level for product B

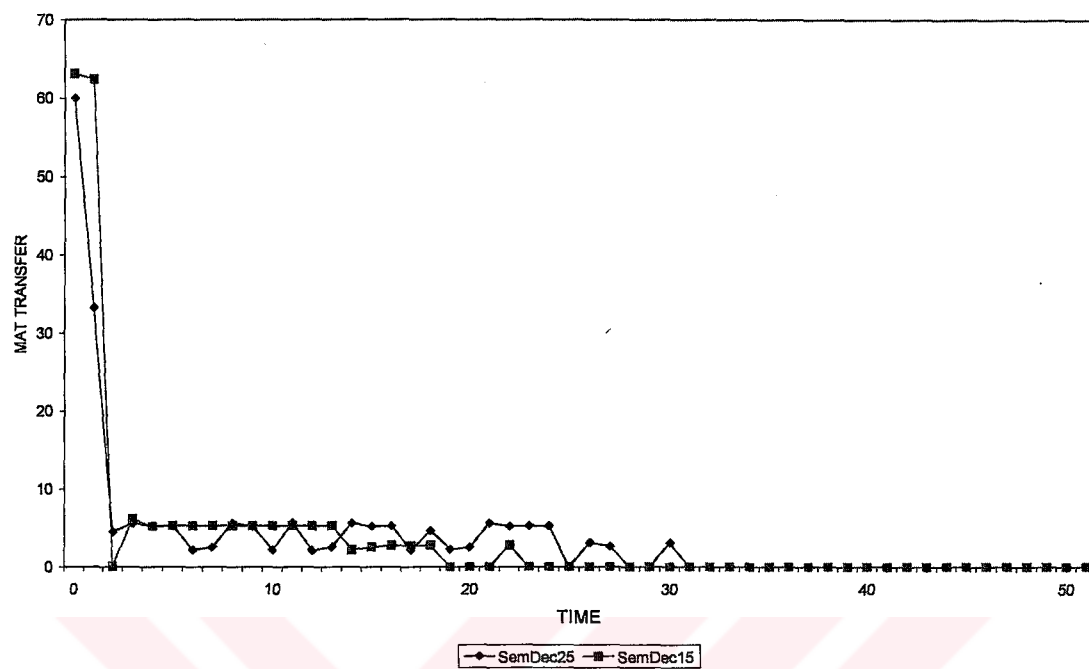


Figure 4.71: Distribution center node material transfer level for product C

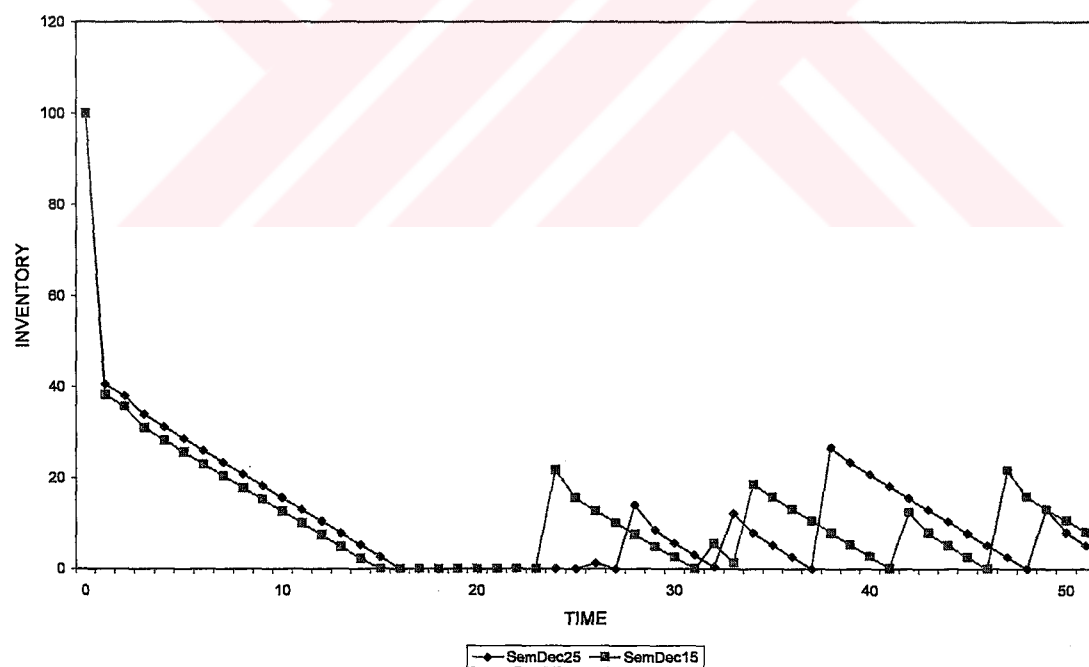


Figure 4.72: Plant and Warehouse node inventory level for product A

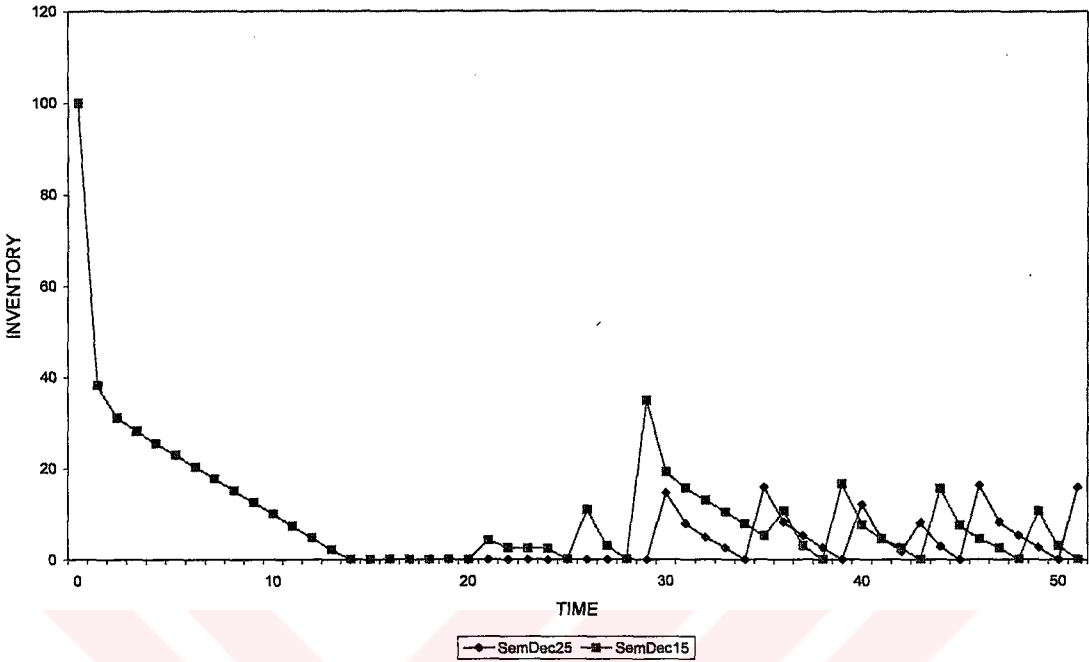


Figure 4.73: Plant and Warehouse node inventory level for product B

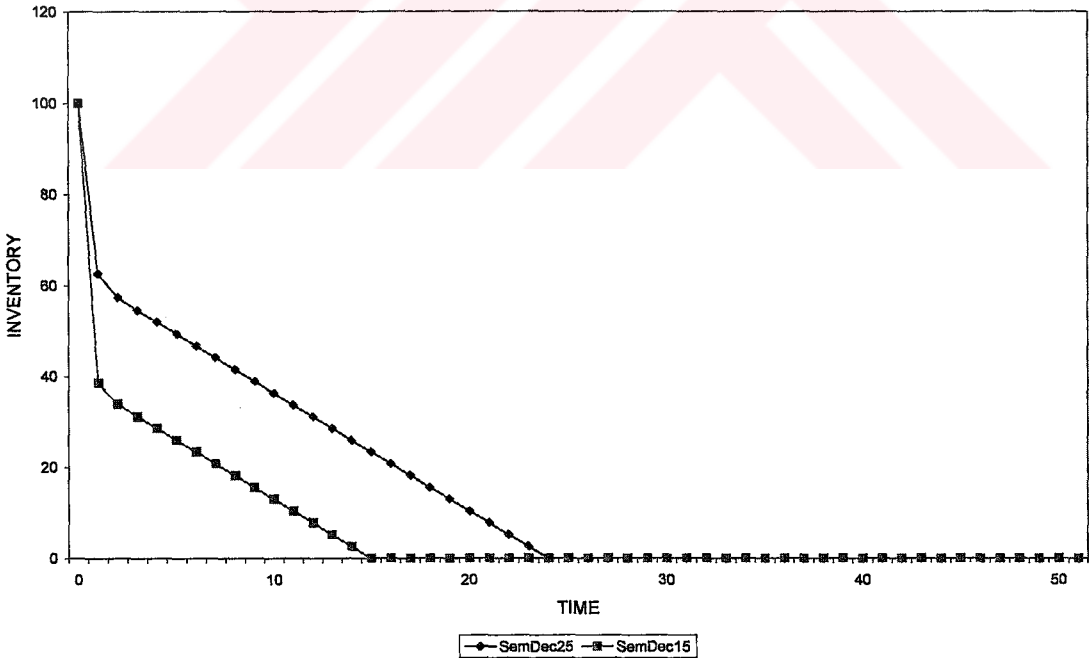


Figure 4.74: Plant and Warehouse node inventory level for product C

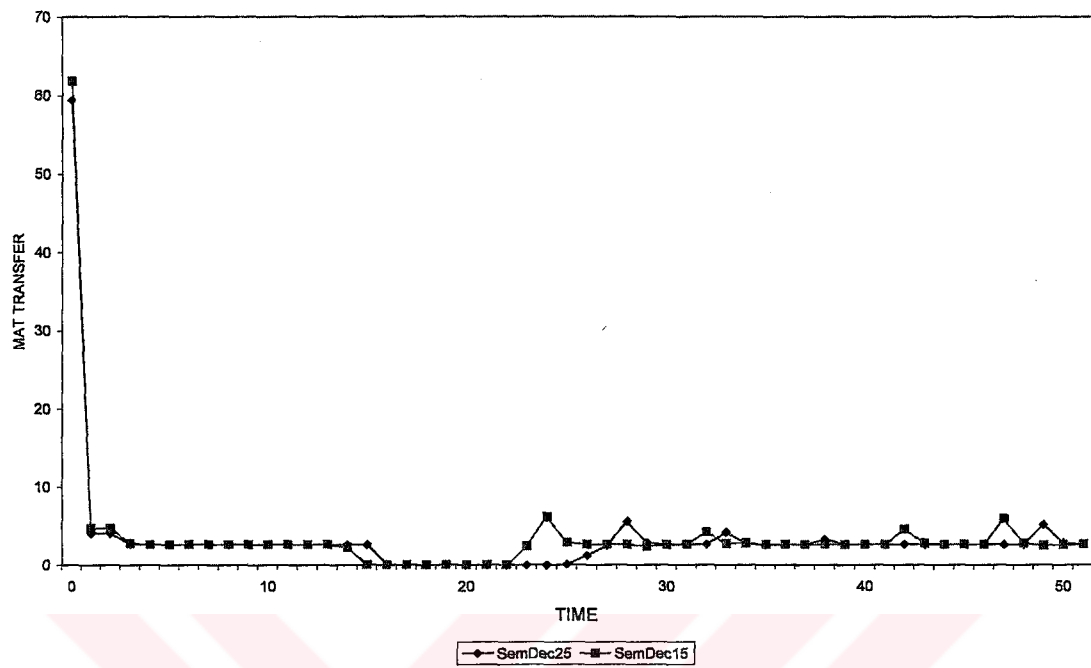


Figure 4.75: :Plant and Warehouse node material transfer level for product A

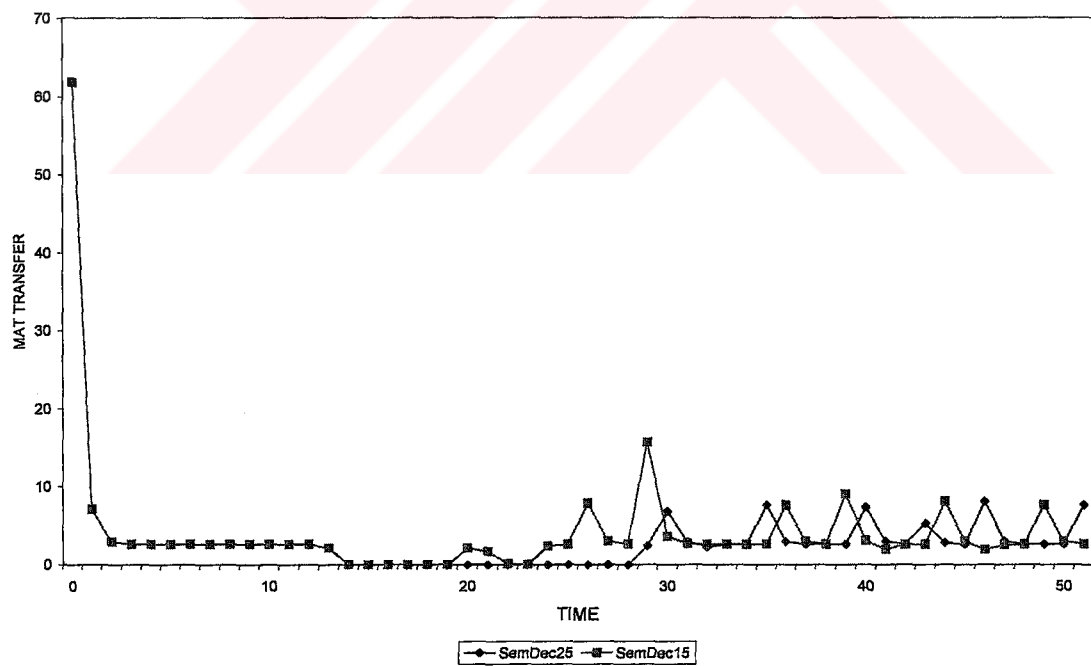


Figure 4.76 Plant and Warehouse node material transfer level for product B

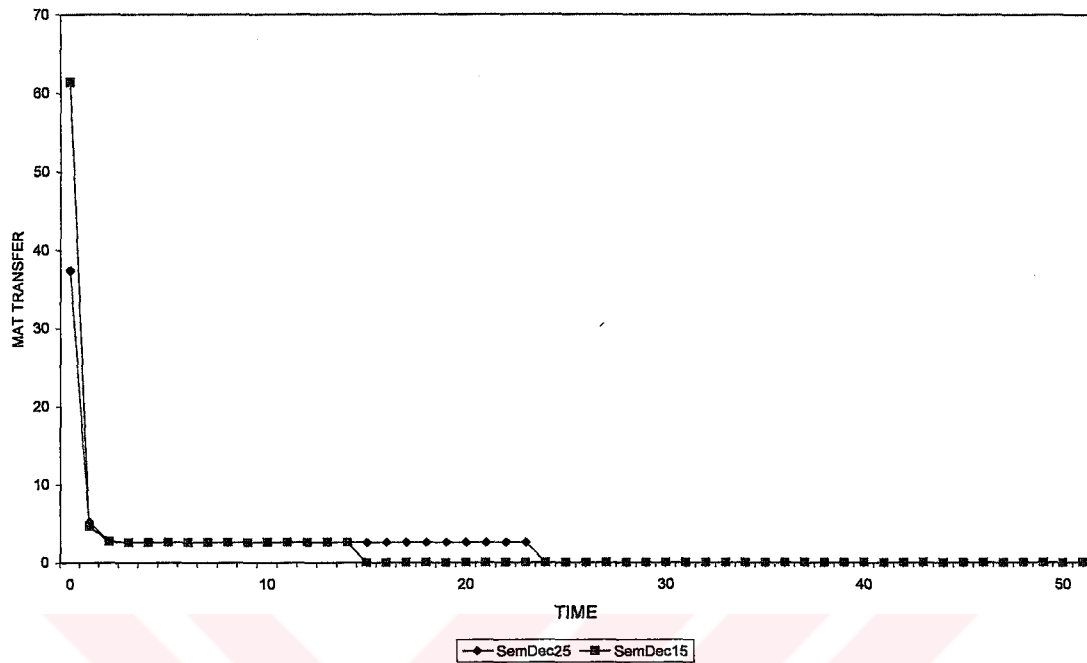


Figure 4.77: Plant and Warehouse node material transfer level for product C

In terms of the material demanded from plant and warehouse, the most important distinction is that after the initial material transfer requests, SDHL25 always makes the next non-zero material request later than SDHL15 do. These non-zero requests coincide with the depletion of the first depletion of the inventory at distribution center, therefore, this lateness is caused by the fact that SDHL25 depletes initial inventory in a longer period of time than SDHL15 do. The plots of material requests from plant and warehouse are presented in figures 4.78 to 4.80.

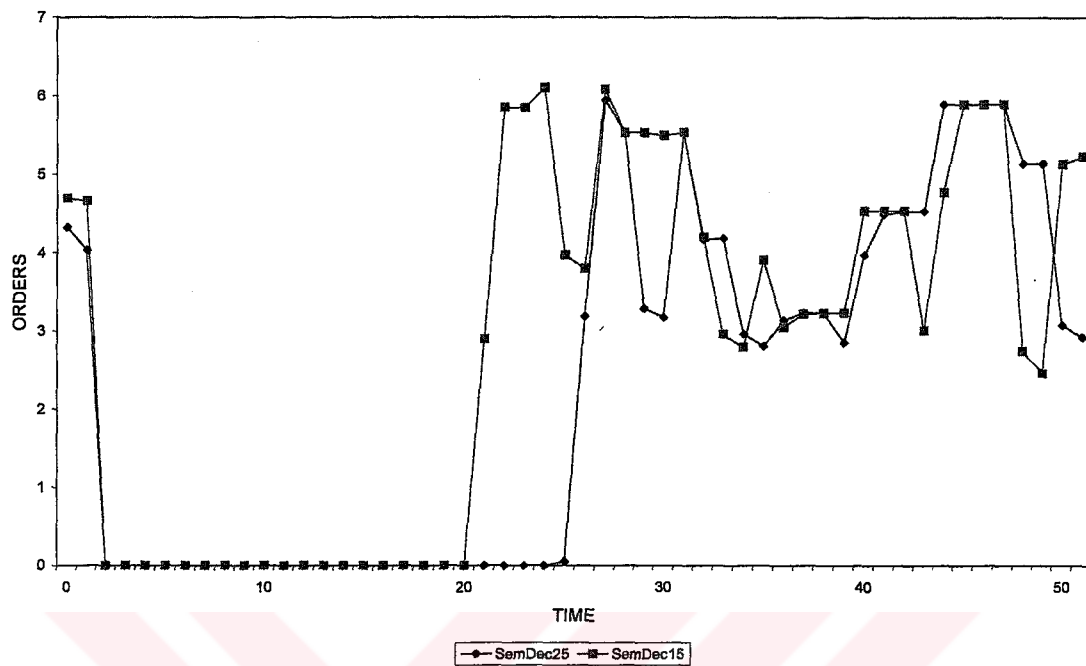


Figure 4.78: Material demanded from Plant and Warehouse of Product A

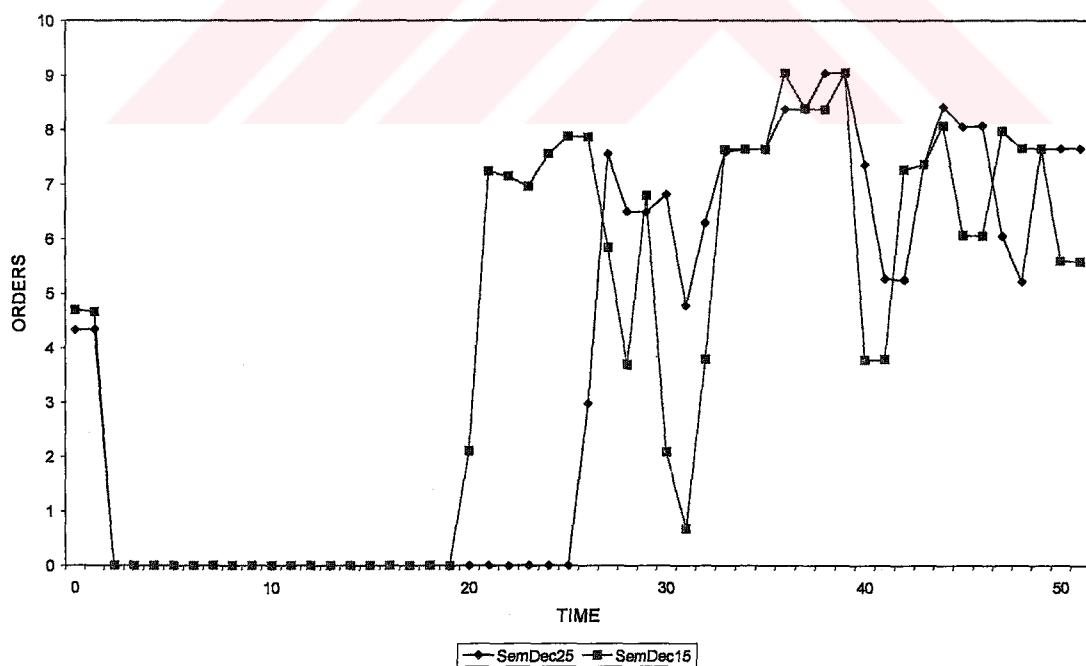


Figure 4.79: Material demanded from Plant and Warehouse of Product B

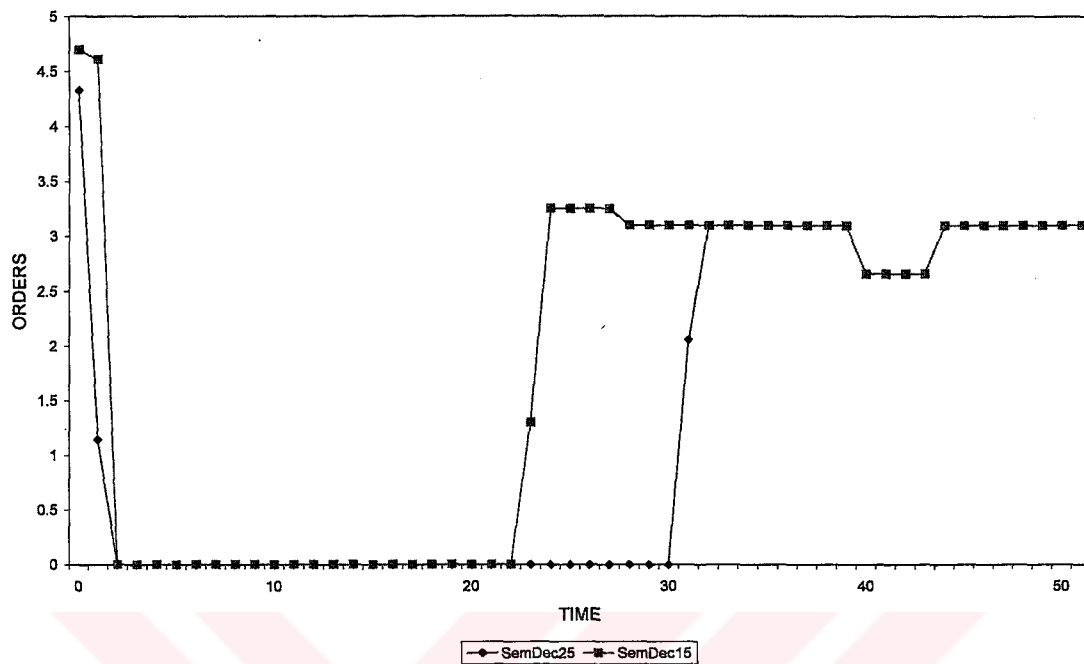


Figure 4.80: Material demanded from Plant and Warehouse of Product C

As discussed before, because of the fact that the variable production cost is very high compared to the fixed production cost, the system should try to reduce the production size whenever possible. SDHL25 does better than SDHL15 in reducing both production size and frequency for product B, and in reducing production size while maintaining and equivalent frequency for product A. Both never produce product C, which is expected because of the high production costs. Production plans of both systems are presented in Figs. 4.81 to 4.83.

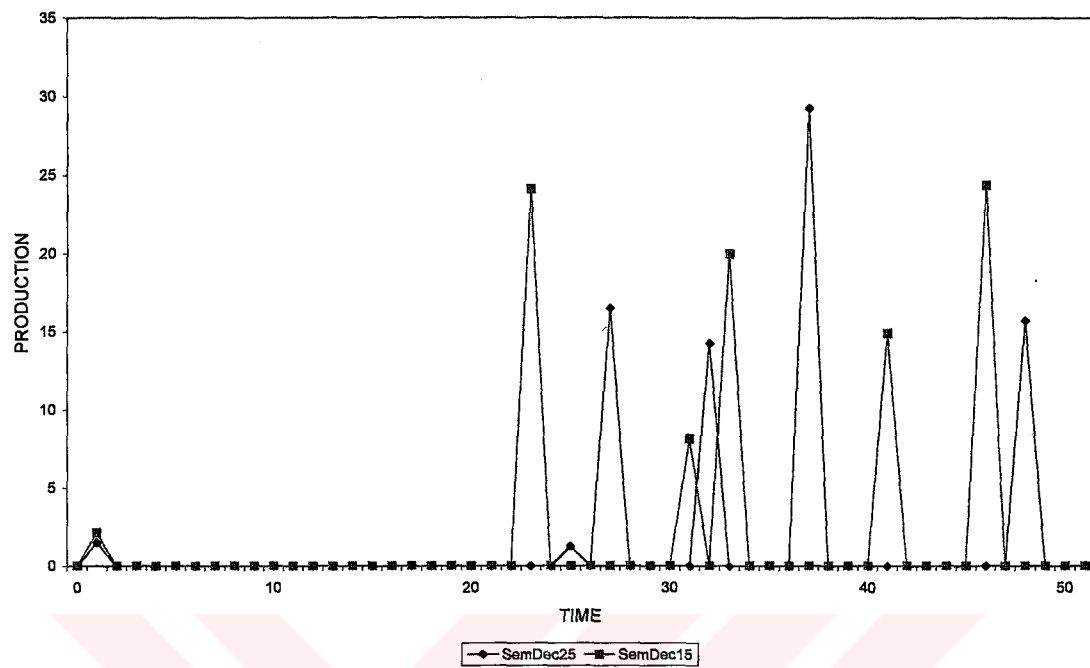


Figure 4.81: Production of A

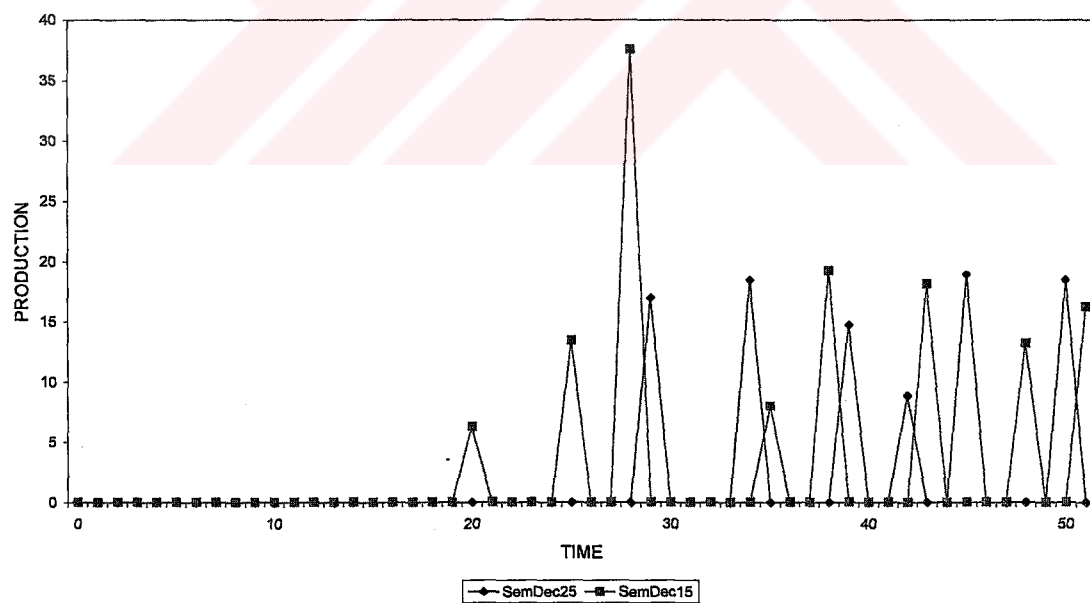


Figure 4.82: Production of B

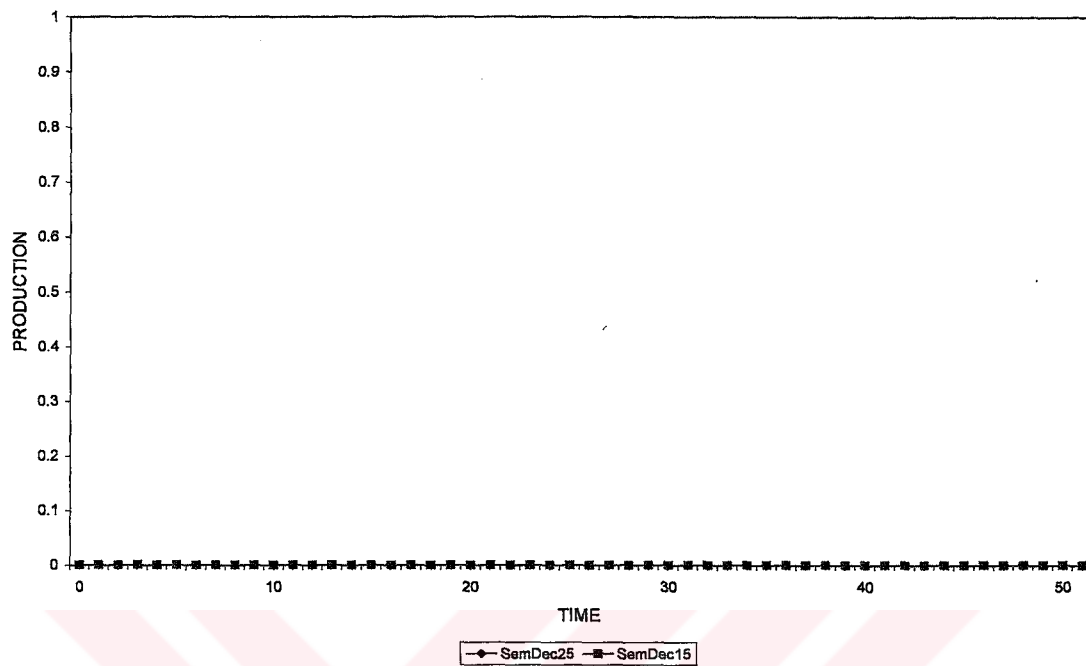


Figure 4.83: Production of C

Instantaneous profit and cumulative profit plots are given in Figs. 4.84 and 4.85. Until the depletion of the initial inventory of SDHL15, SDHL15 performs better than SDHL25. After the depletion of the initial inventory of SDHL25, there is very little to separate the two.

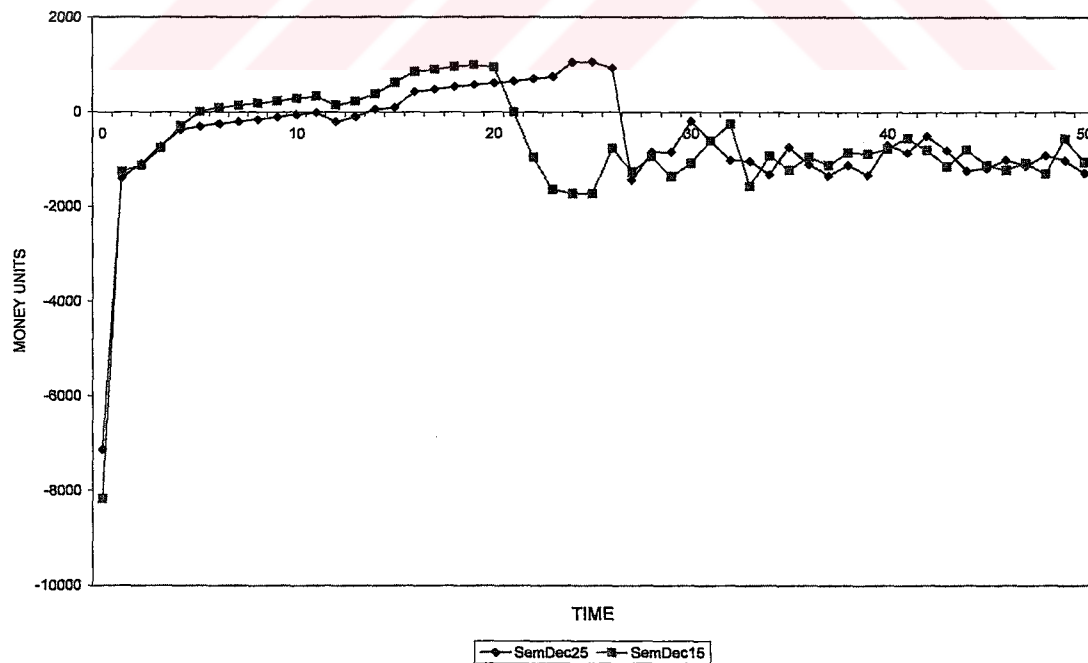


Figure 4.84: Instantaneous System Profit

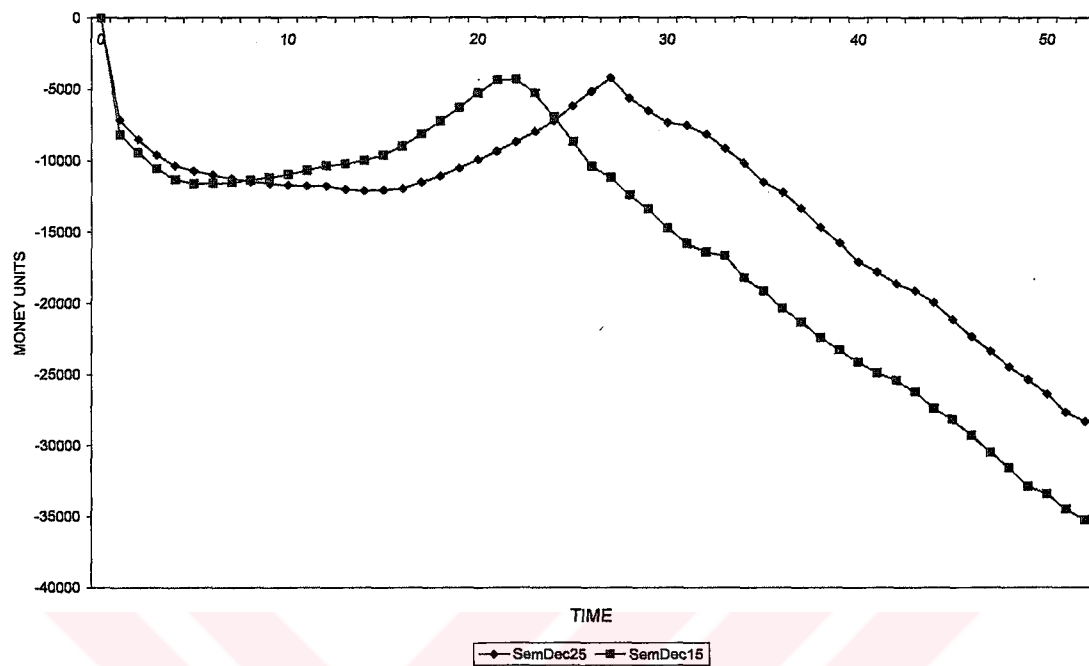


Figure 4.85: Cumulative System Profit

Similar to the situation in profit, SDHL25 is better than SDHL15 in terms of customer satisfaction until the depletion of the initial inventory. After then, there is very little to distinguish between the two. The graphs are given in figures 4.86 to 4.88.

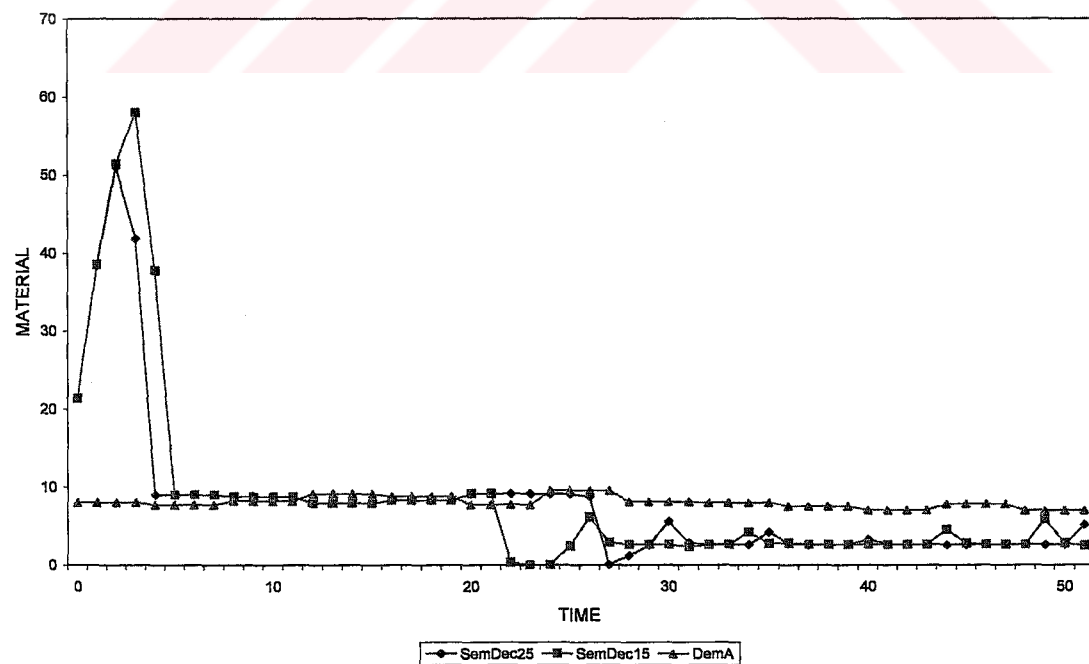


Figure 4.86: Demand vs Material Transfer at Retailer for Product A

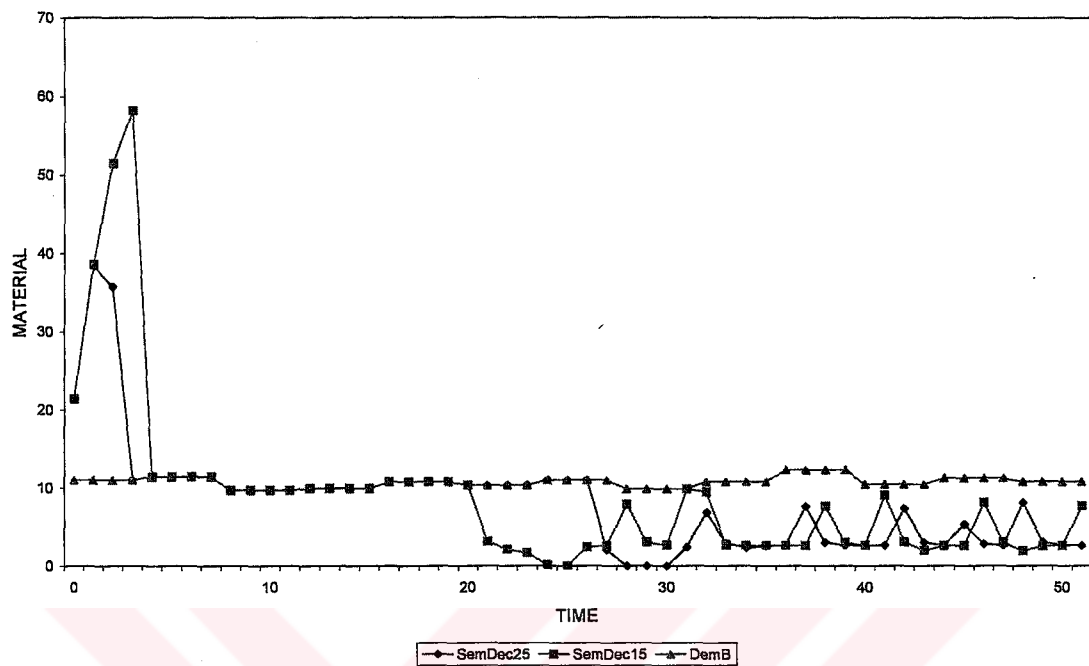


Figure 4.87: Demand vs Material Transfer at Retailer for Product B

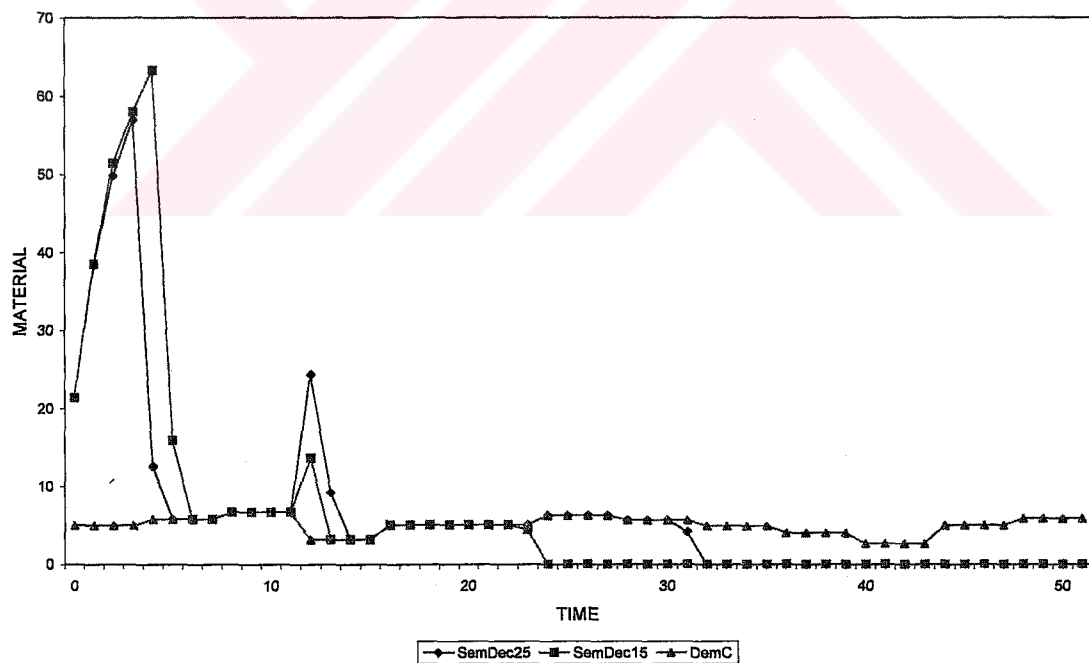


Figure 4.88: Demand vs Material Transfer at Retailer for Product C

Since both SDHL25 and SDHL15 observe the end customer demands, and their forecasts and since the actual demand is consistent with the forecasting procedure, the severity of the bullwhip

effect is very similar in both systems, as can be observed in the material demanded from the plant and warehouse. The only difference is the starting time of the non-zero material demands, which is later for SDHL25. The reason for that is the late depletion of the initial inventory caused by the length of the optimization horizon. Therefore, the bullwhip effect starts later in SDHL25 because of the later depletion of the initial inventory, and the severity of the bullwhip effect is very similar for both of SDHL25 and SDHL15. Because the demand that the forecasts are based is consistent with the forecasting procedure used. Thus, the difference between the overall profits is caused by the depletion policy of the initial inventory levels, however, after the depletion of the initial inventories, both horizon lengths show similar behavior, which is explained by the similar severity of the bullwhip effect in both of SDHL25 and SDHL15.

To save space Decentralized Supply Chain with Horizon Length 15 will be referred as DHL15 and Decentralized Supply Chain with Horizon Length 25 will be referred as DHL25. The only remarkable distinction between these is the product C. DHL15 uses initial inventory of product C in a longer period of time, resulting in a better customer satisfaction for product C. The results are presented in Figs. 4.89 to 4.117.

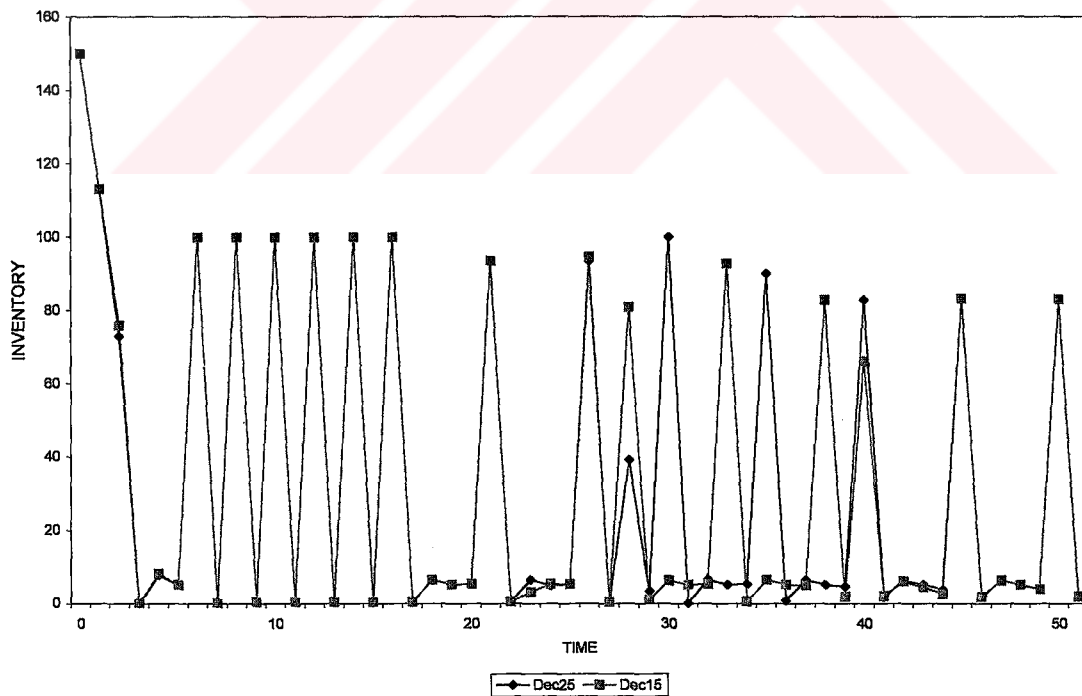


Figure 4.89: Retailer node inventory level for product A

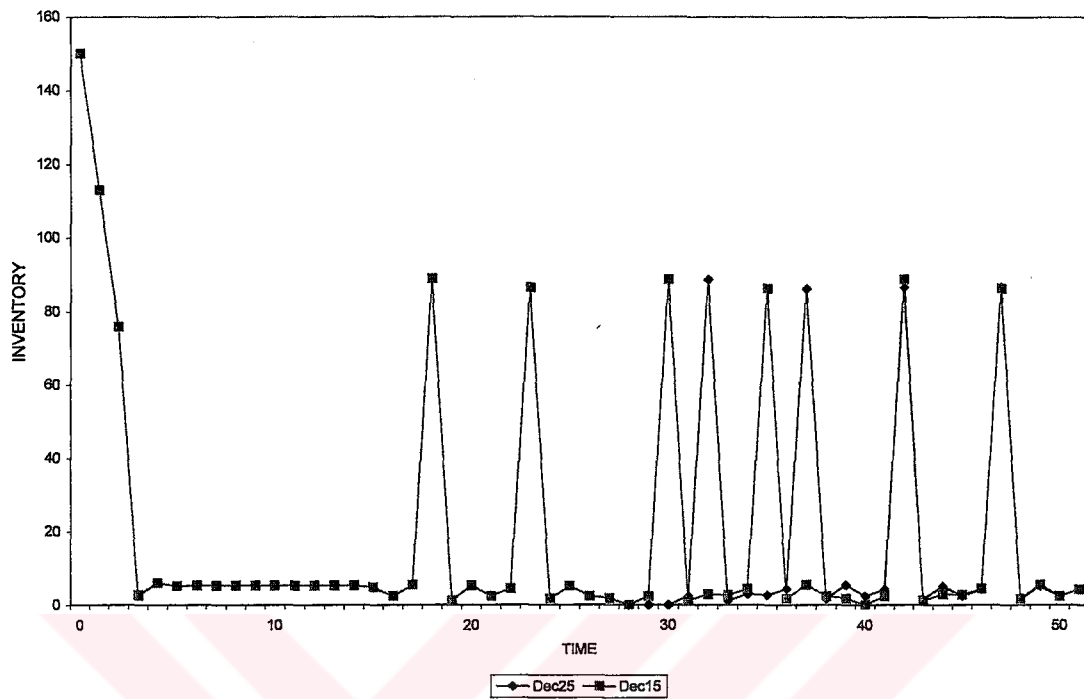


Figure 4.90: Retailer node inventory level for product B

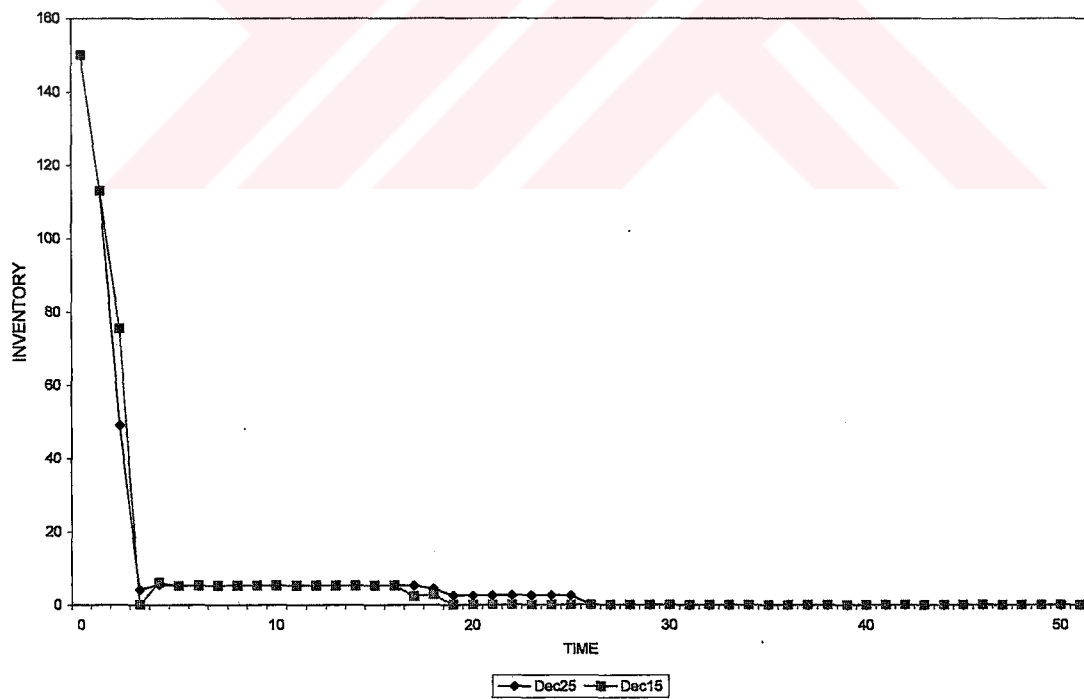


Figure 4.91: Retailer node inventory level for product C

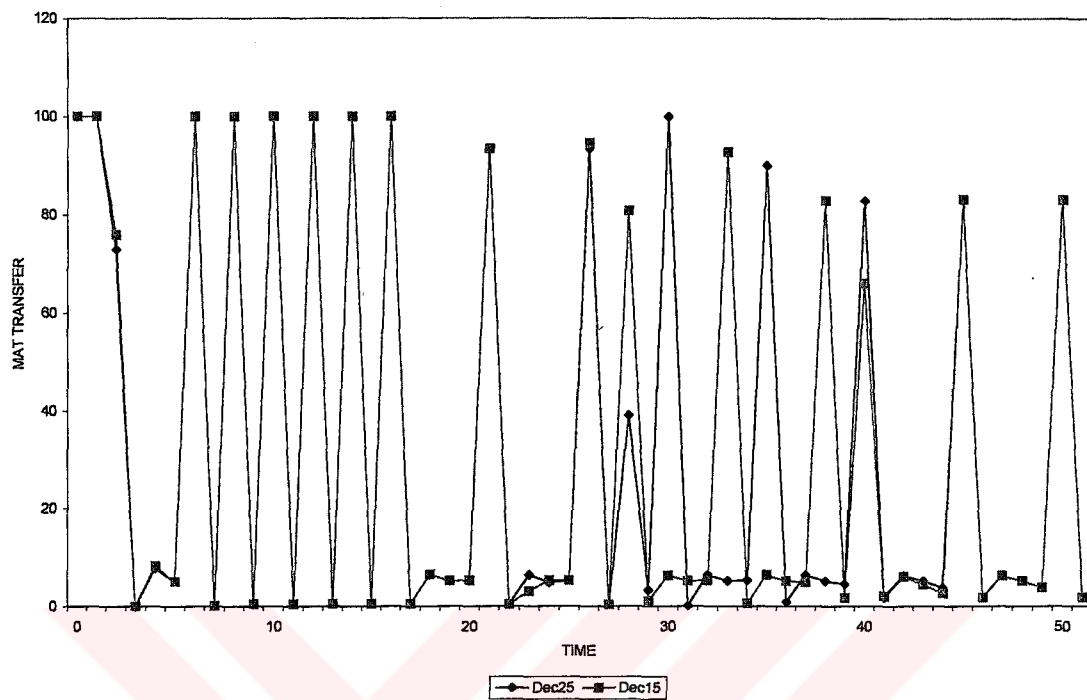


Figure 4.92: Retailer node material transfer level for product A

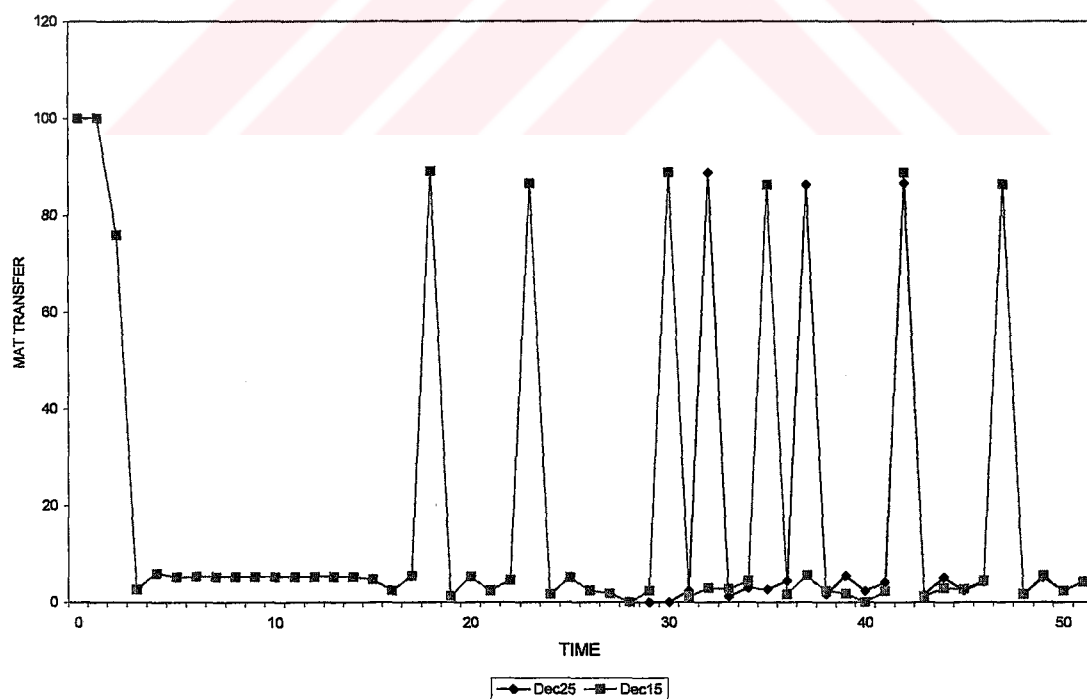


Figure 4.93: Retailer node material transfer level for product B

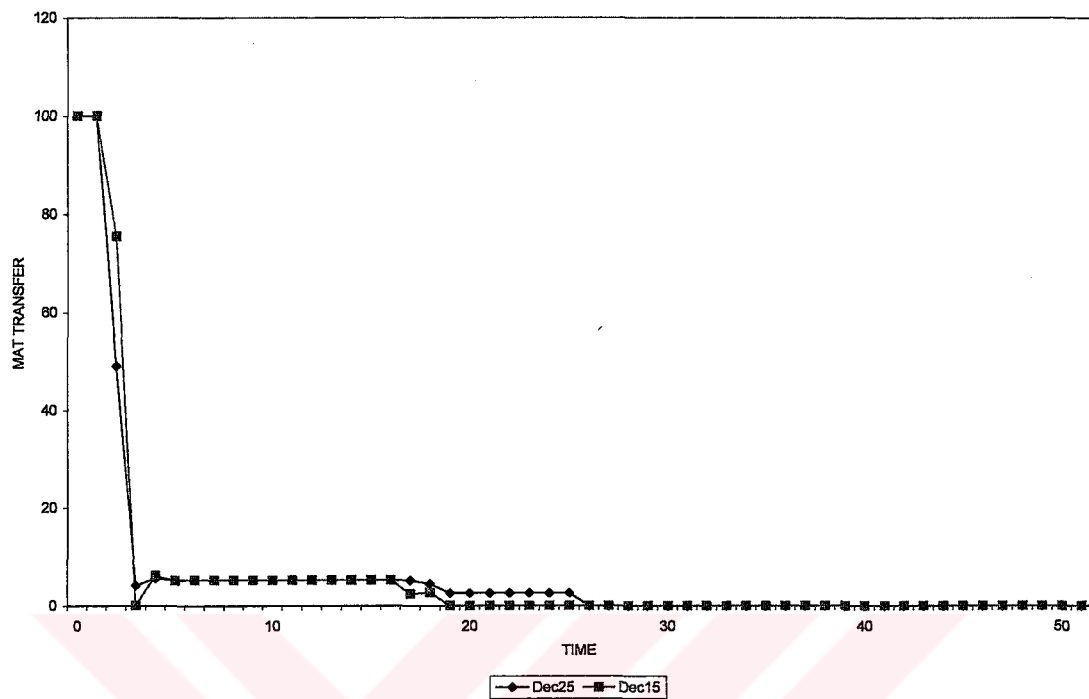


Figure 4.94: Retailer node material transfer level for product C

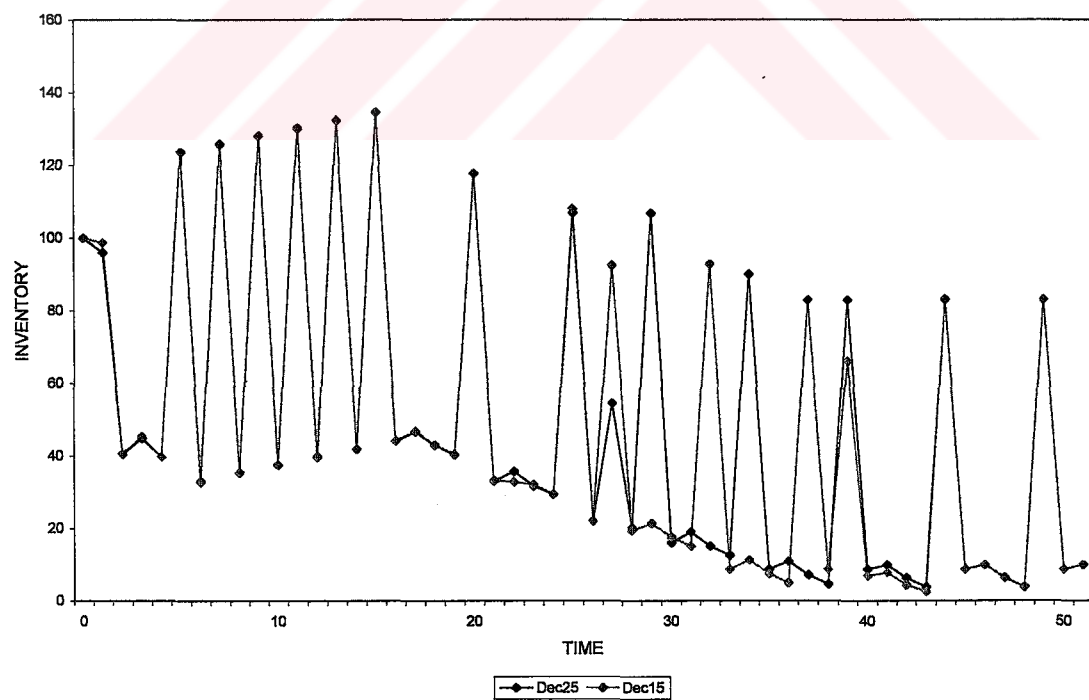


Figure 4.95: Distribution center node inventory level for product A

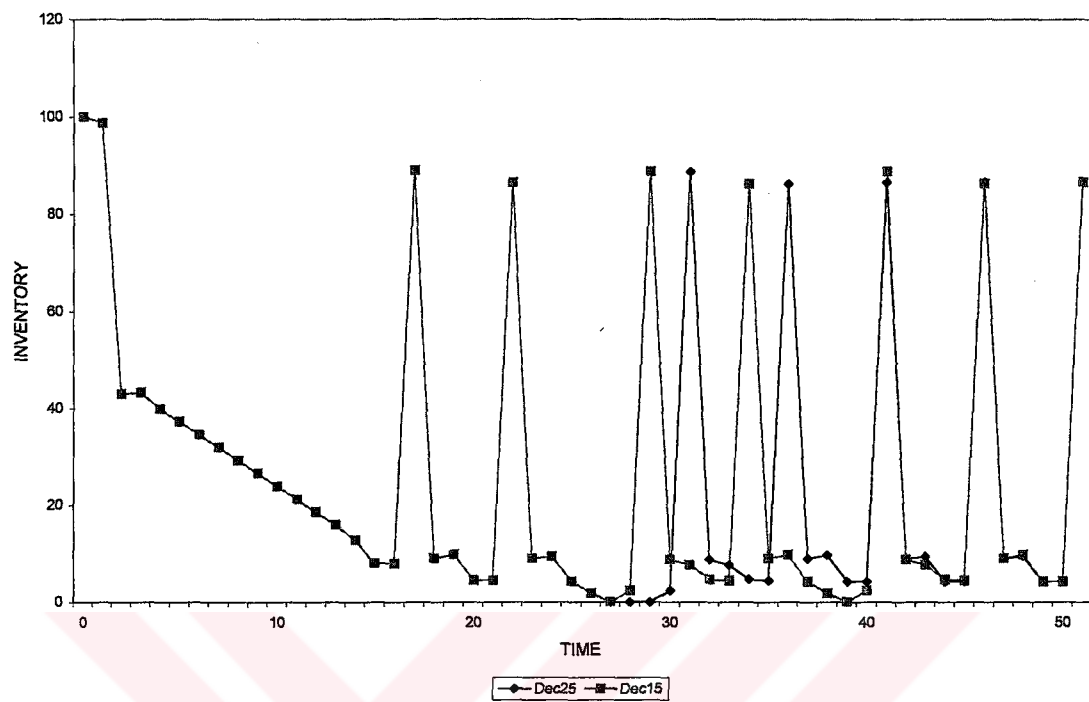


Figure 4.96: Distribution center node inventory level for product B

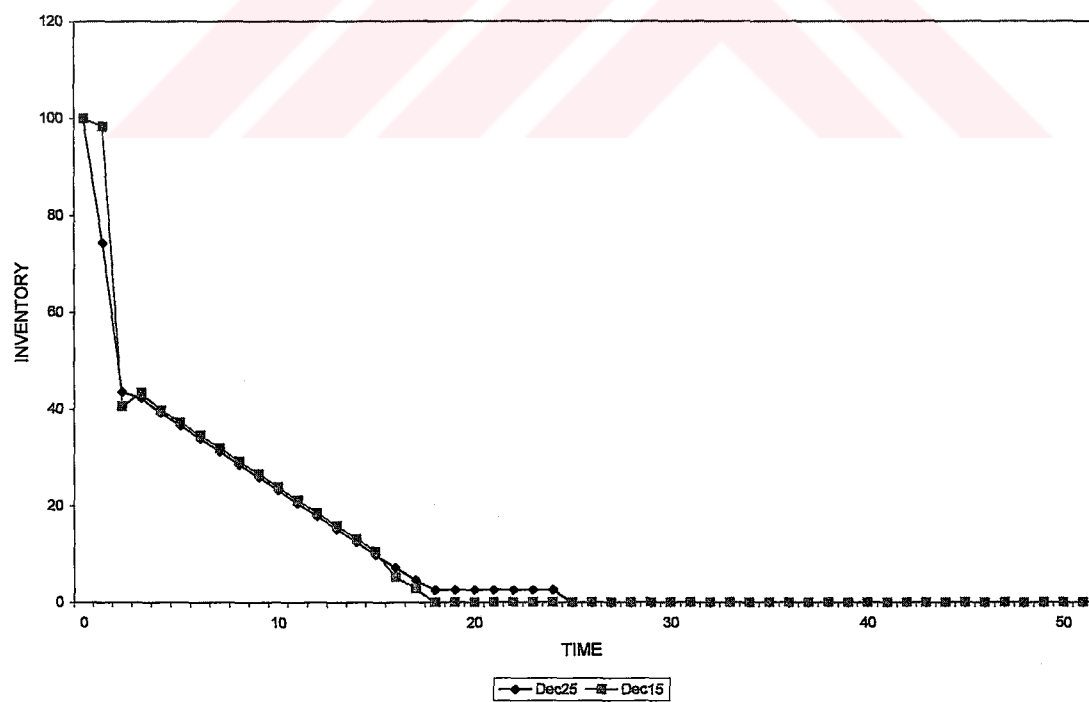


Figure 4.97: Distribution center node inventory level for product C

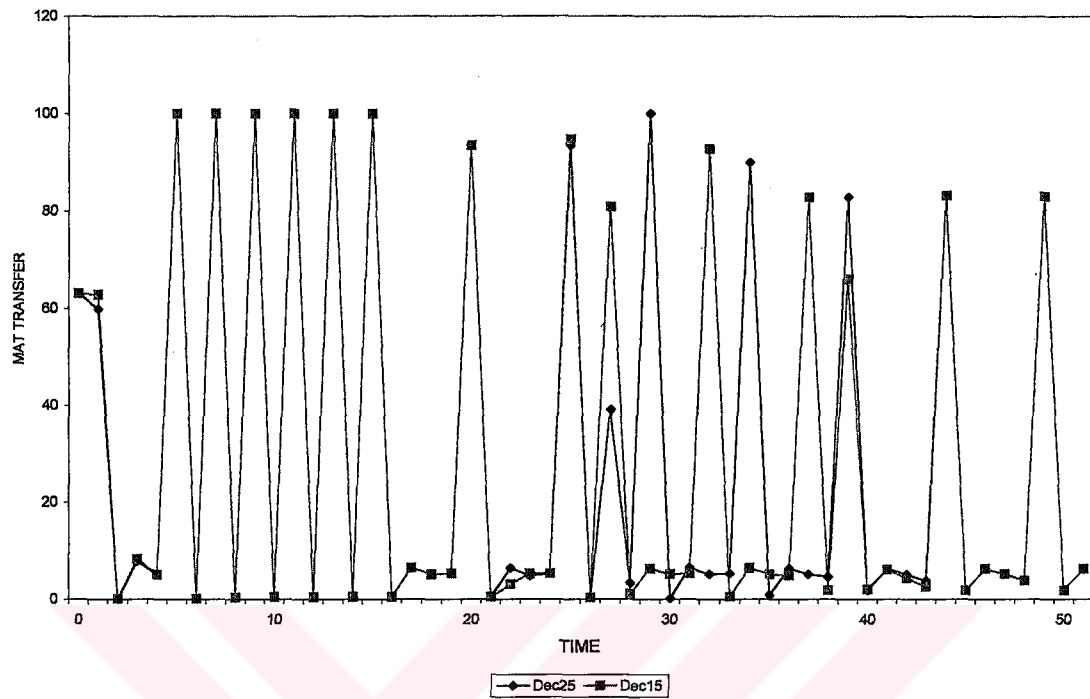


Figure 4.98: Distribution center node material transfer level for product A

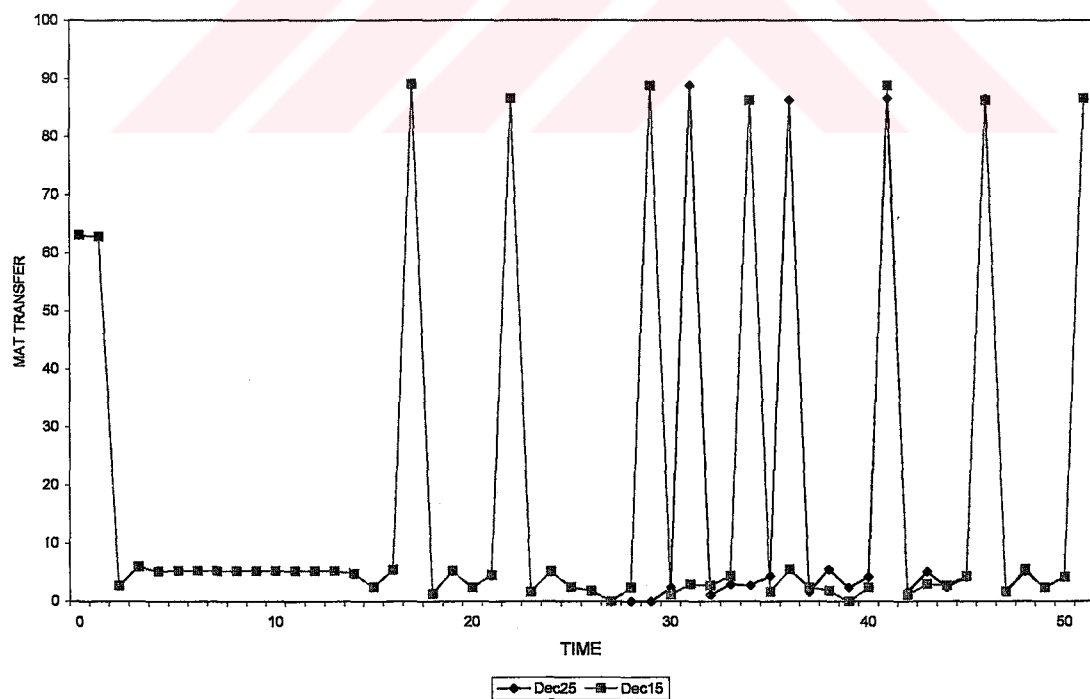


Figure 4.99: Distribution center node material transfer level for product B

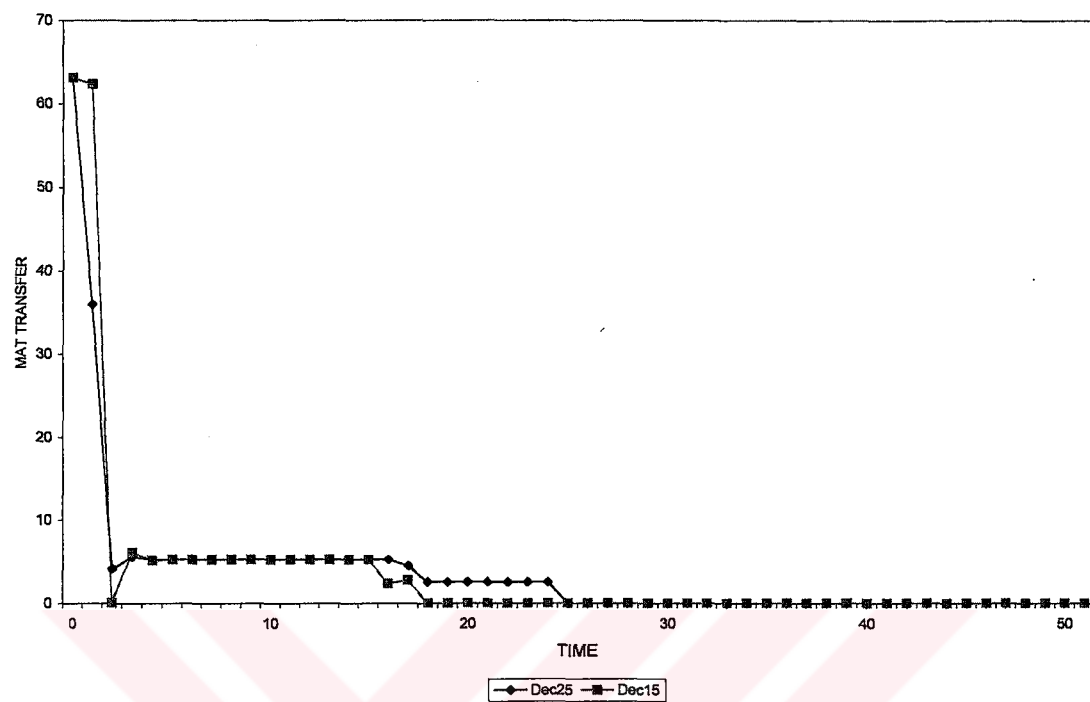


Figure 4.100: Distribution center node material transfer level for product C

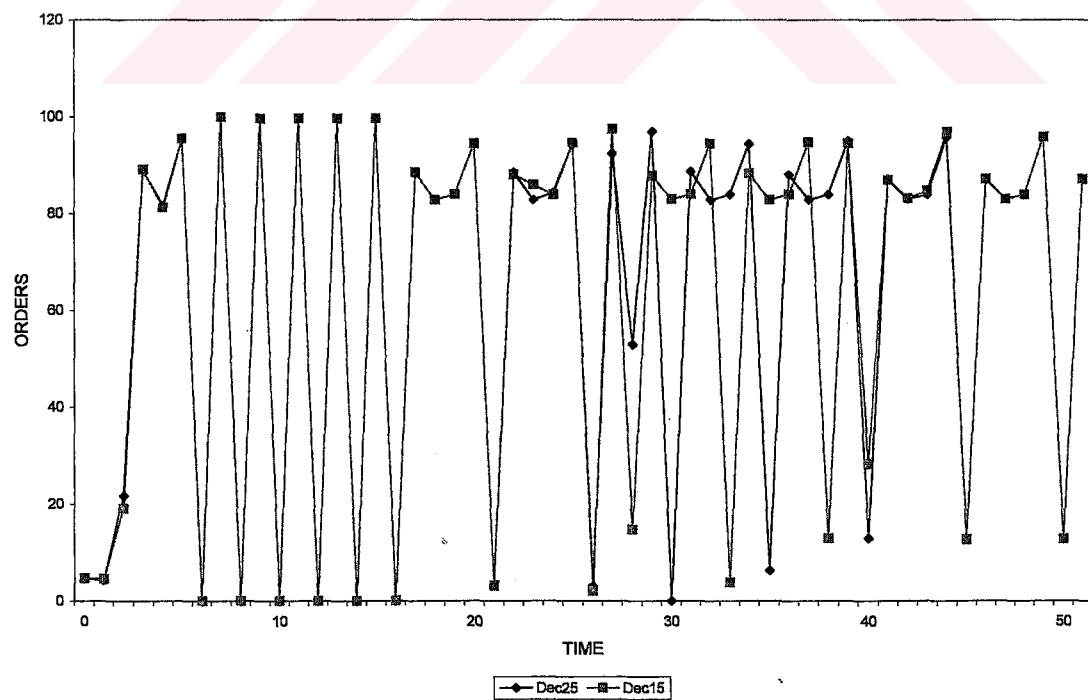


Figure 4.101 Material demanded from plant and warehouse for product A

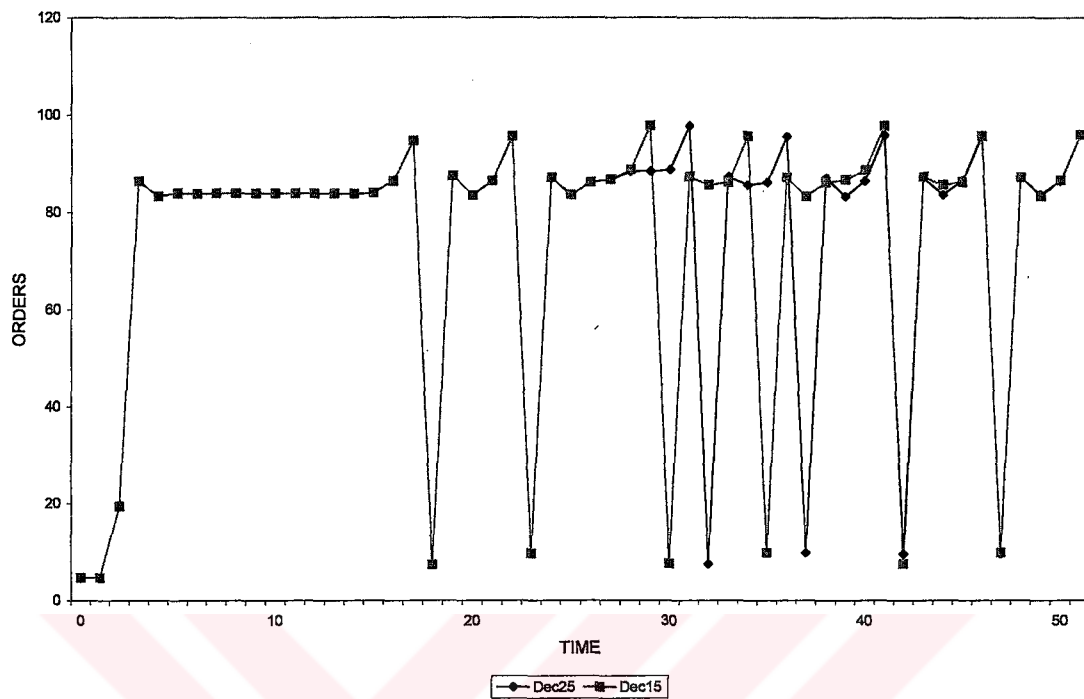


Figure 4.102: Material demanded from plant and warehouse for product B

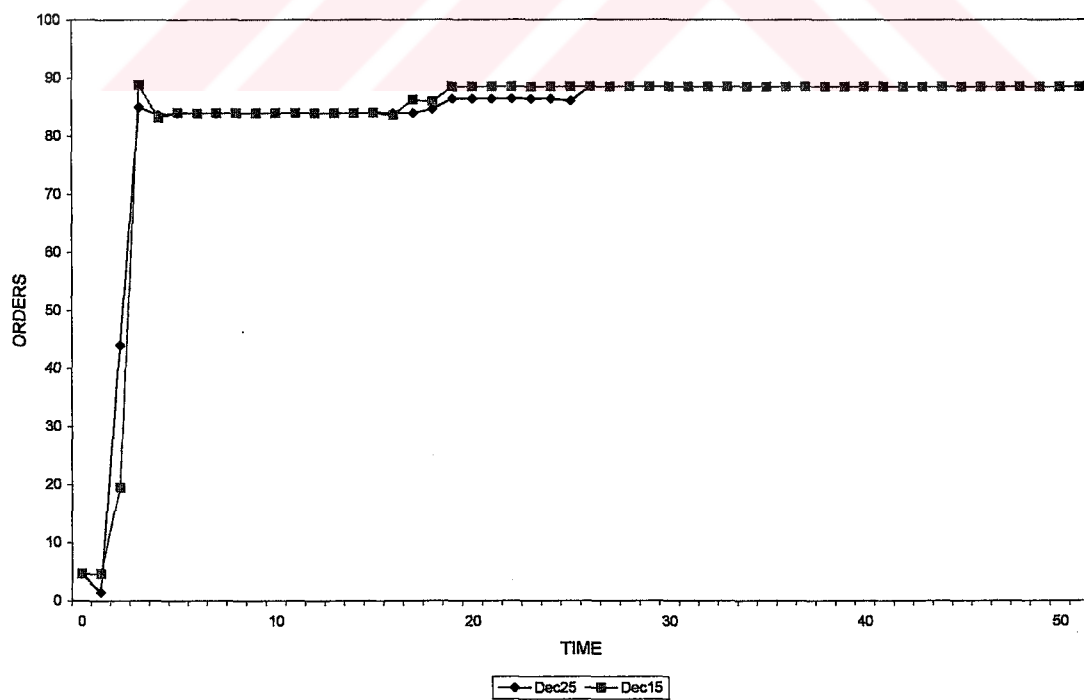


Figure 4.103: Material demanded from plant and warehouse for product C

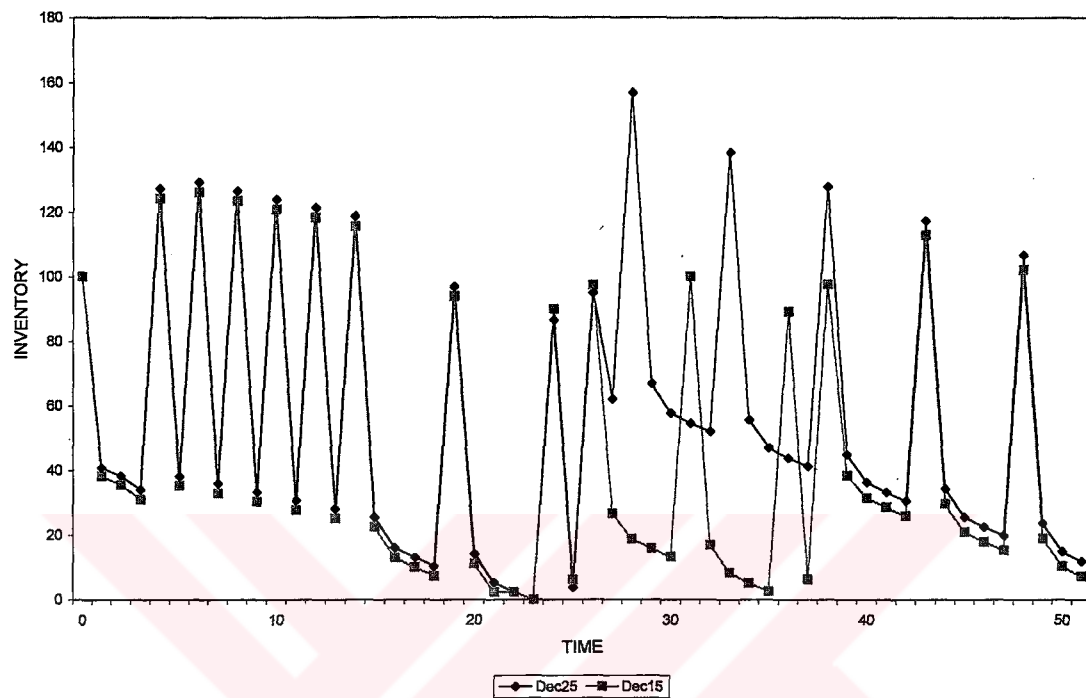


Figure 4.104: Plant and warehouse node inventory level for product A

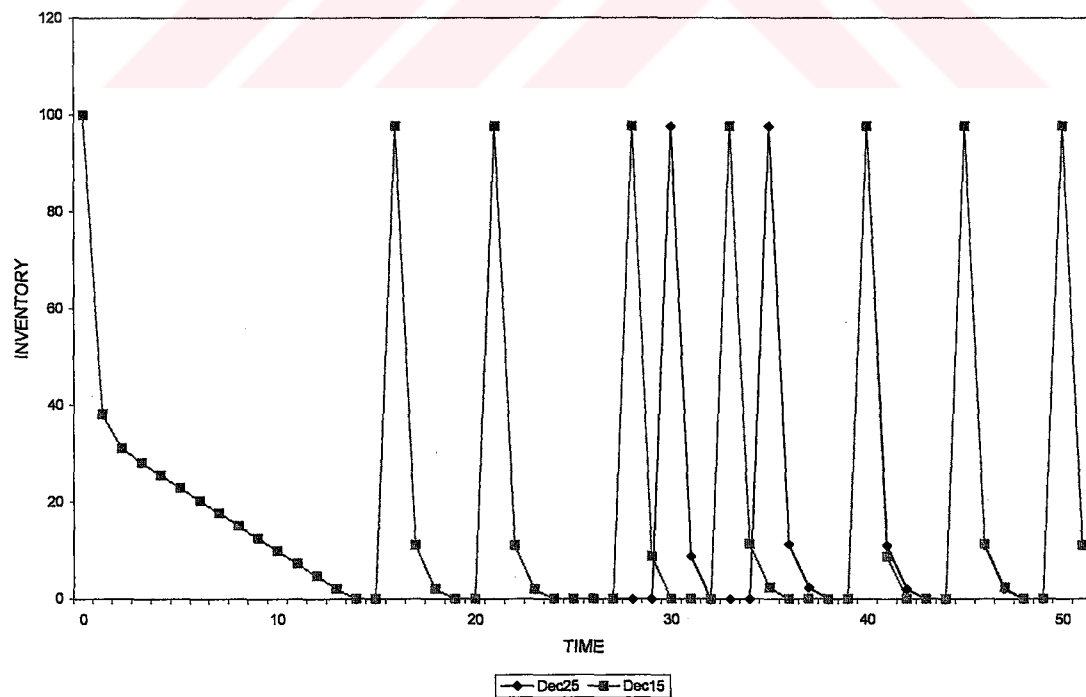


Figure 4.105: Plant and warehouse node inventory level for product B

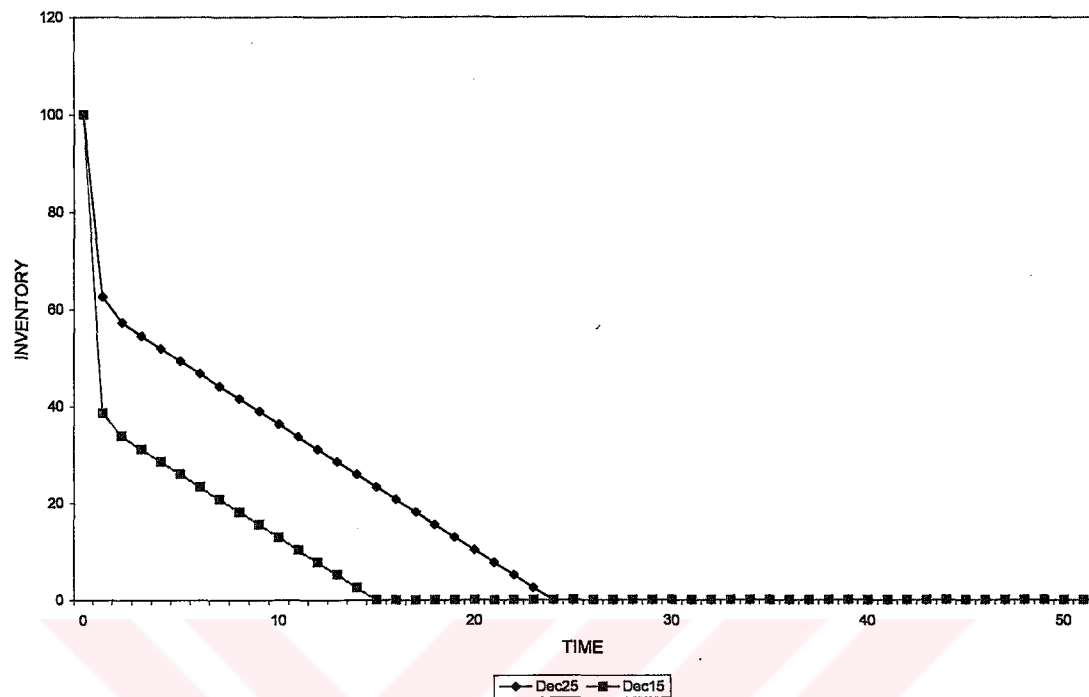


Figure 4.106: Plant and warehouse node inventory level for product C

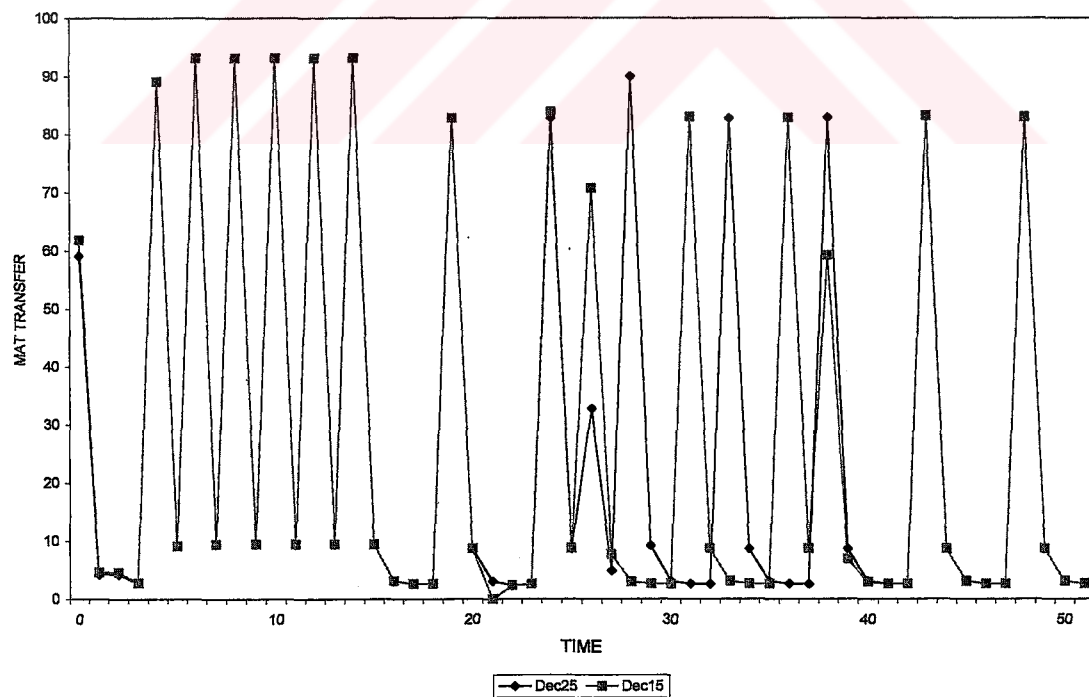


Figure 4.107: Plant and warehouse node material transfer level for product A

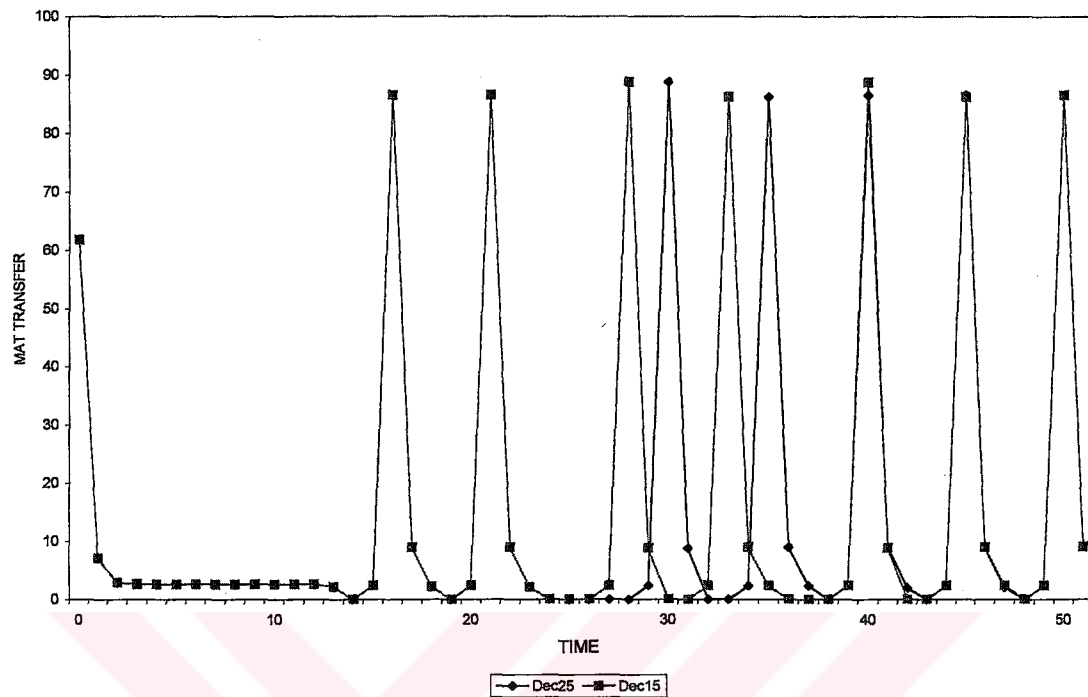


Figure 4.108: Plant and warehouse node material transfer level for product B

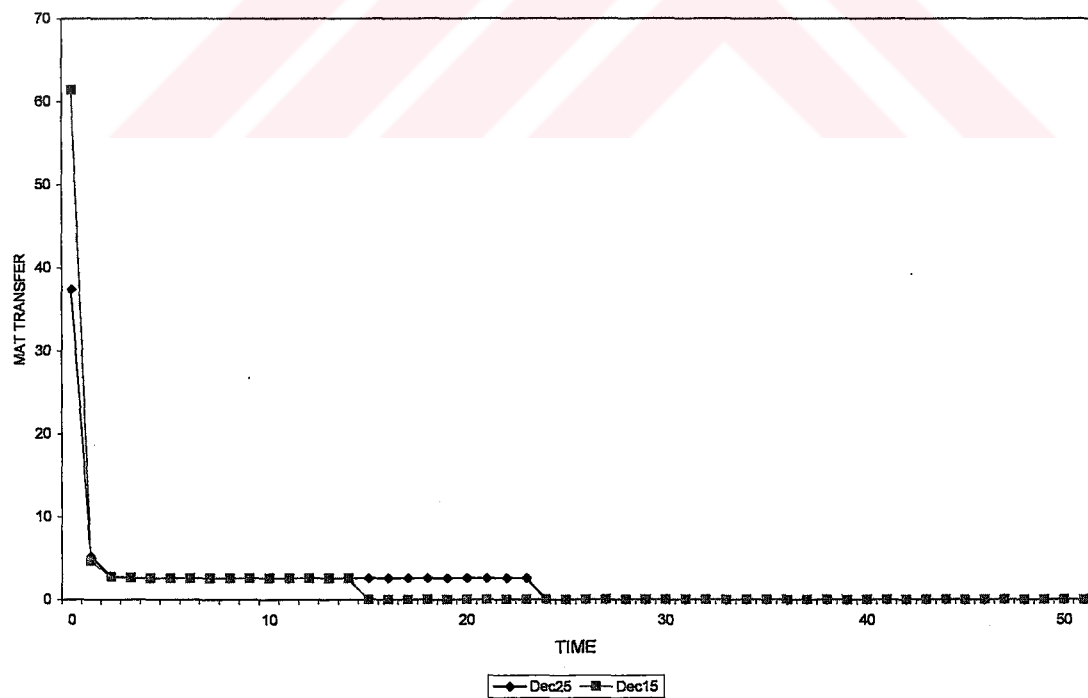


Figure 4.109: Plant and warehouse node material transfer level for product C

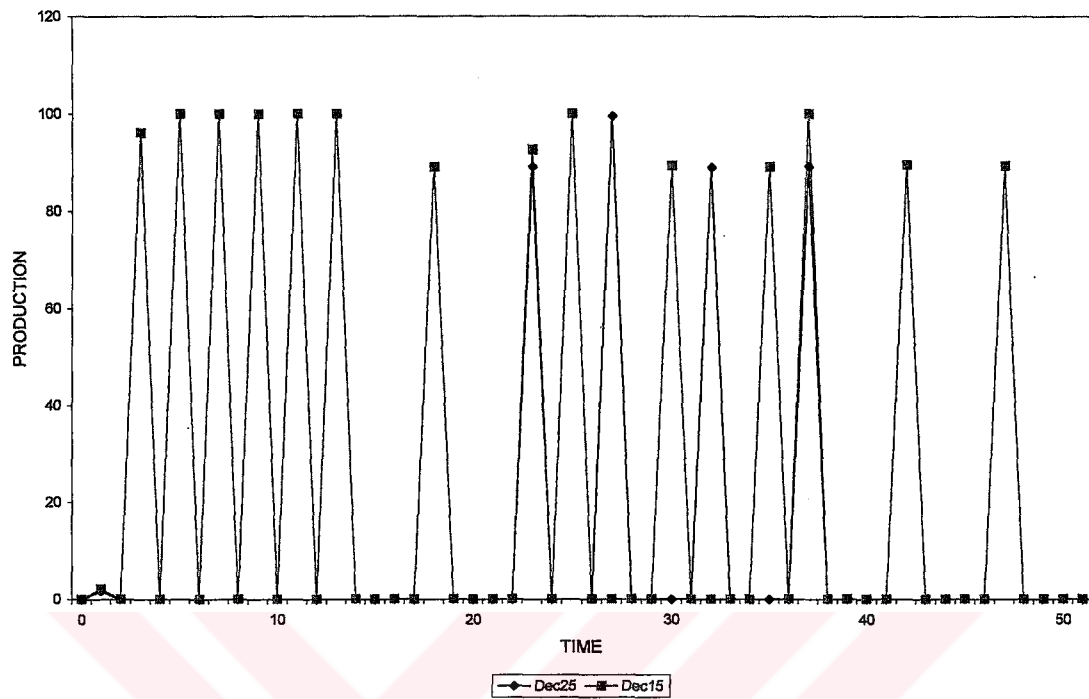


Figure 4.110: Production of A

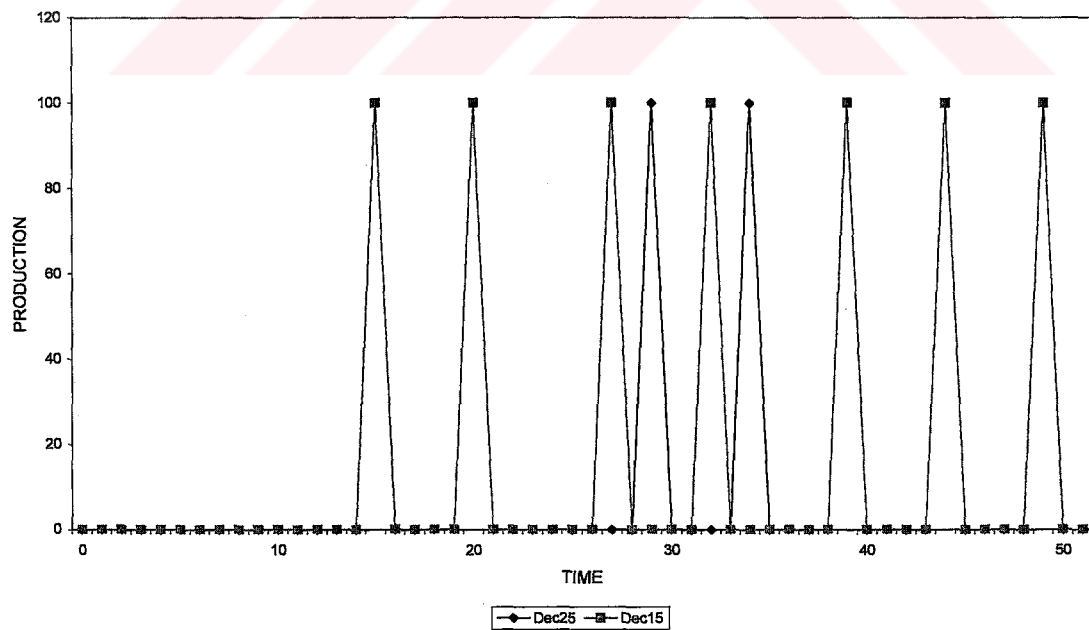


Figure 4.111: Production of B

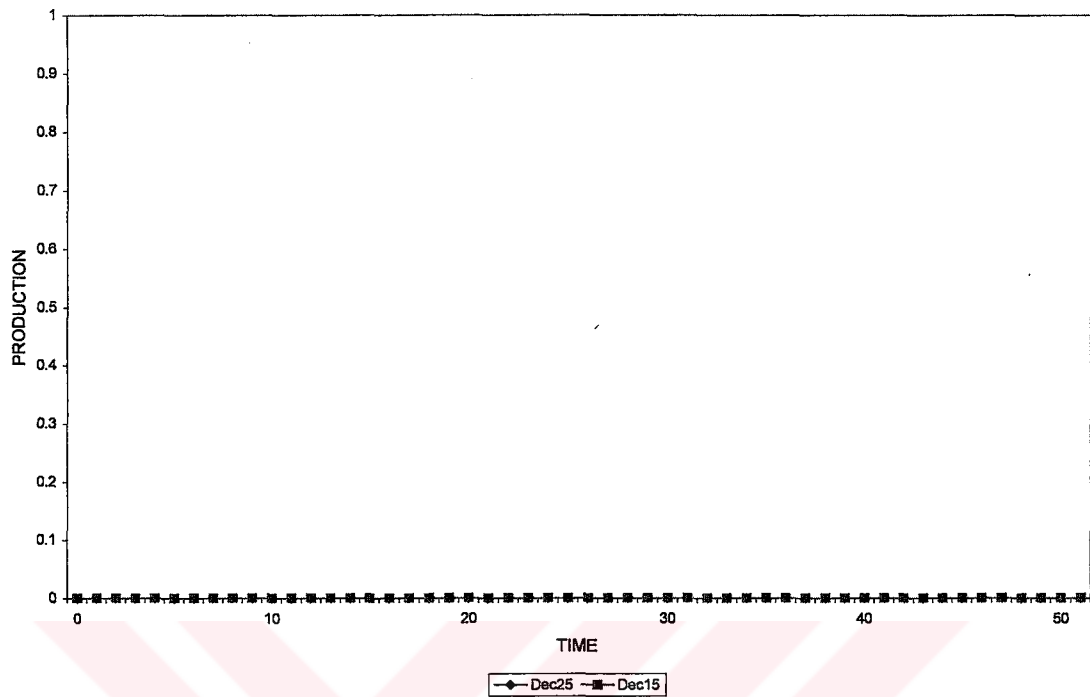


Figure 4.112: Production of C

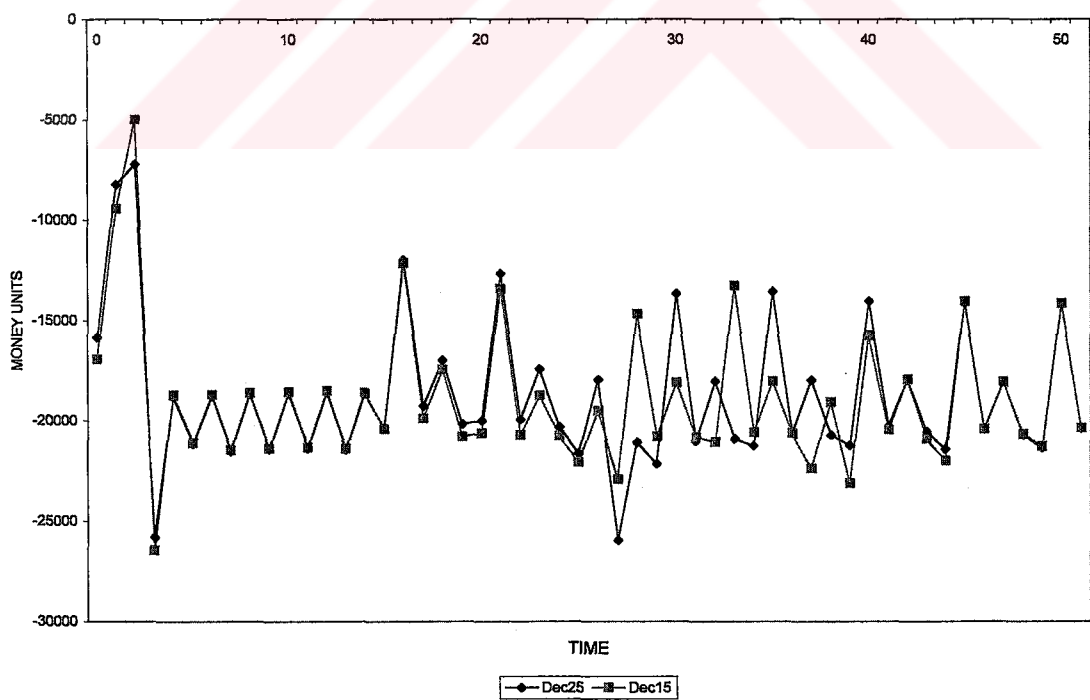


Figure 4.113: Instantaneous System Profit

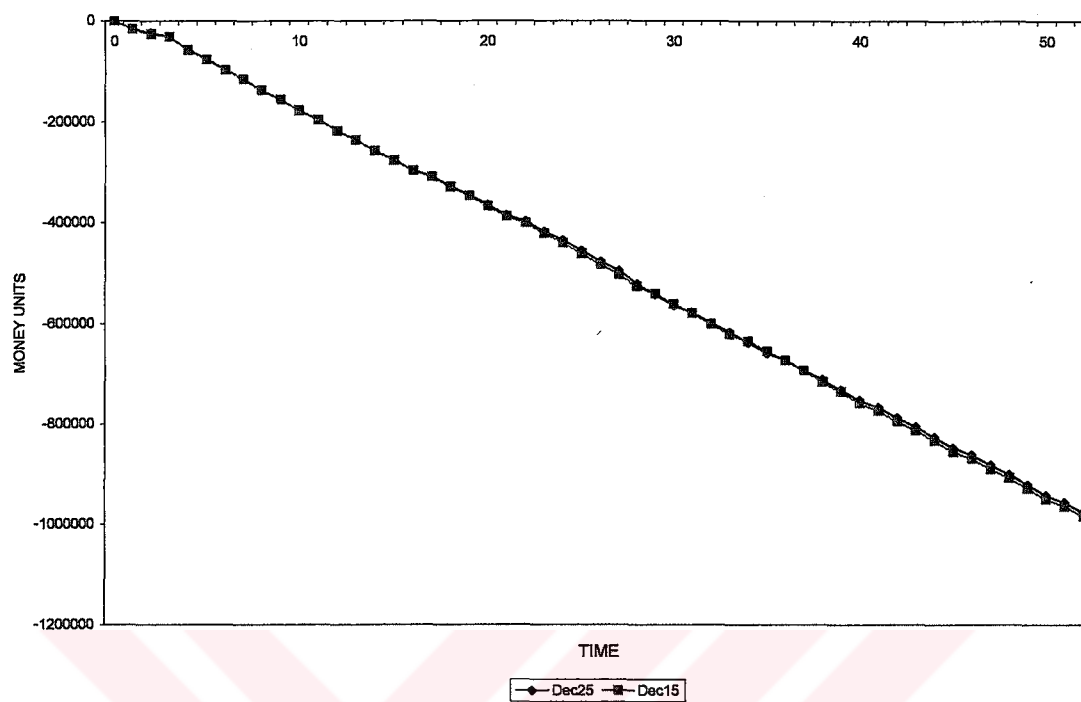


Figure 4.114: Cumulative System Profit

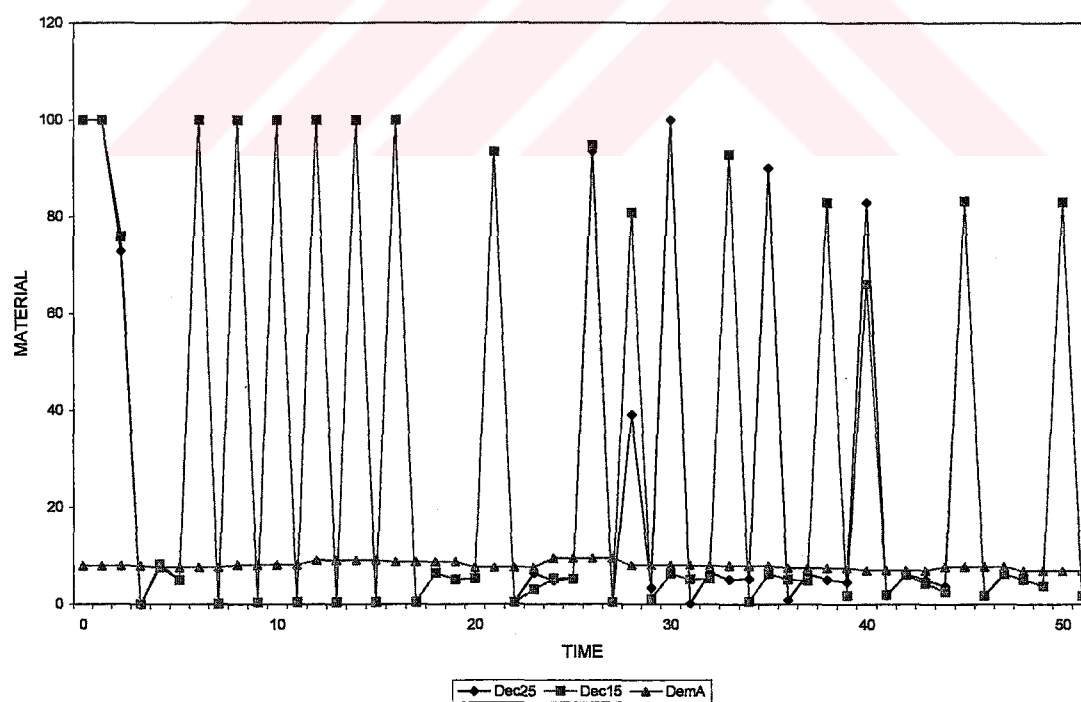


Figure 4.115: Demand vs Material Transfer at Retailer for Product A

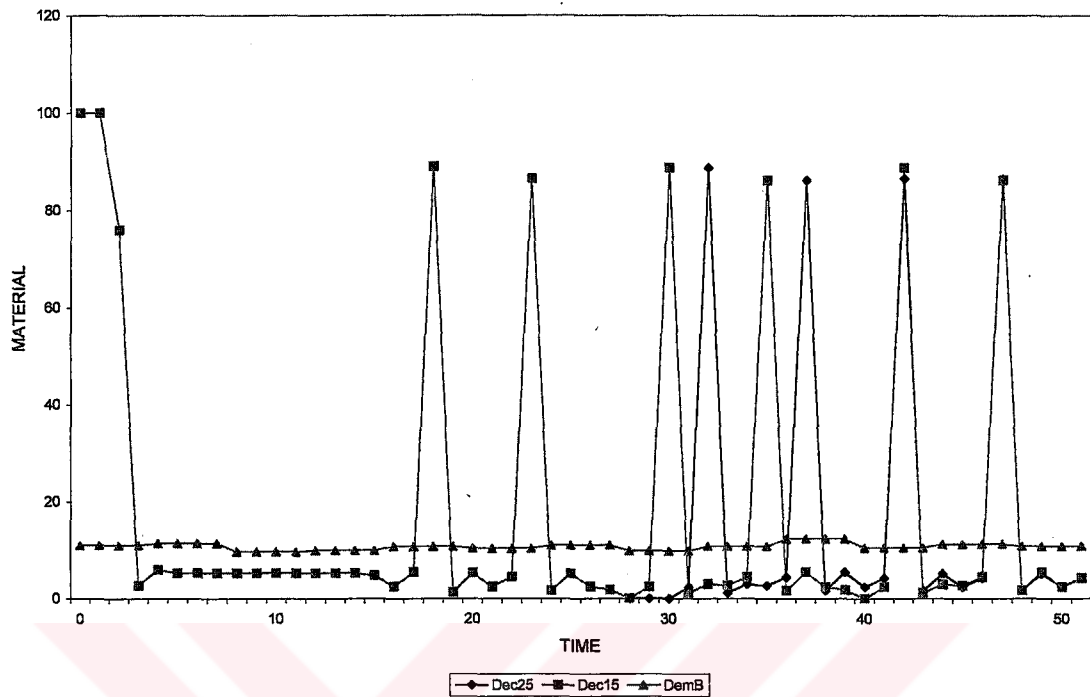


Figure 4:116: Demand vs Material Transfer at Retailer for Product B

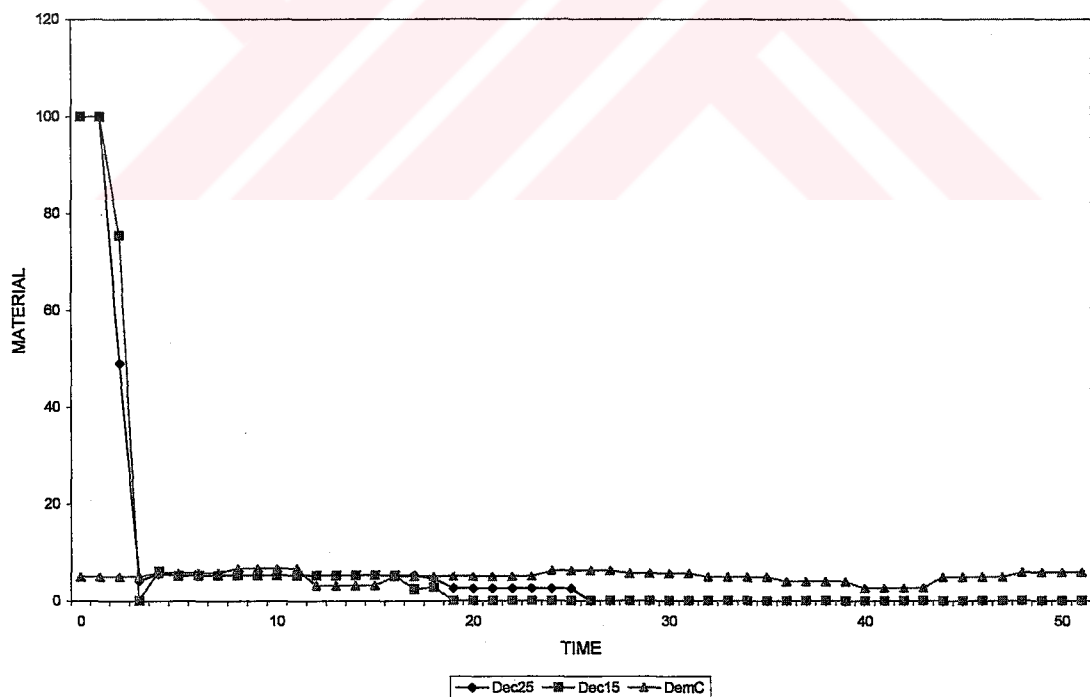


Figure 4:117: Demand vs Material Transfer at Retailer for Product C

The sizes of the bullwhip effects in both horizon lengths are similar as it can be observed from the material demanded from plant and warehouse and from distribution center. (Figs. 4.118, 4.119, 4.120)

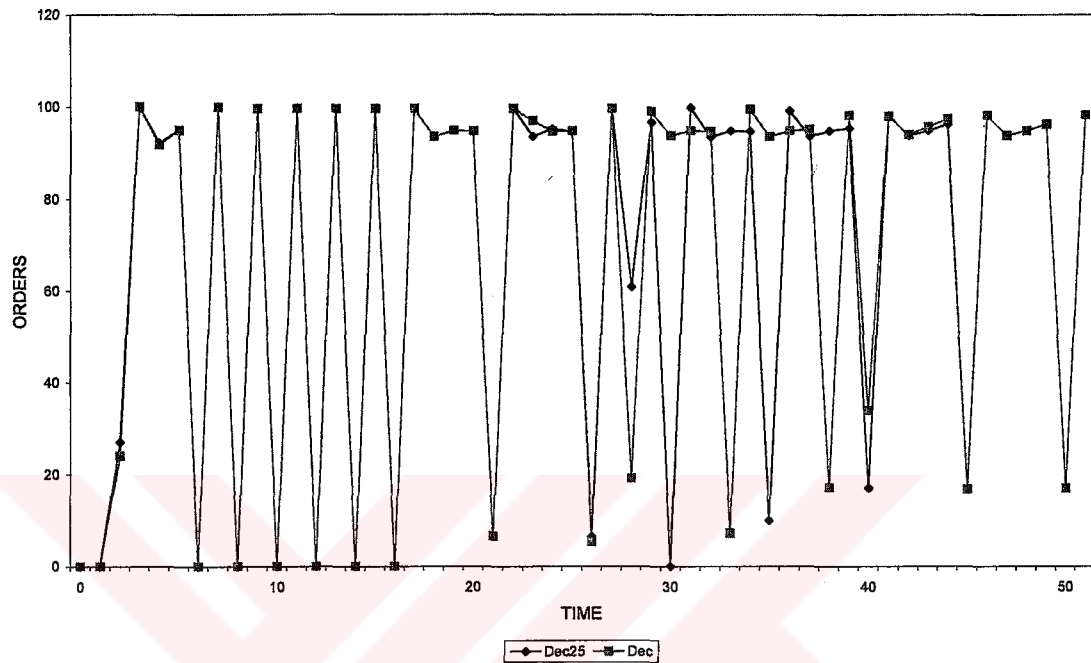


Figure 4.118: Order levels between retailer and distribution center for product A

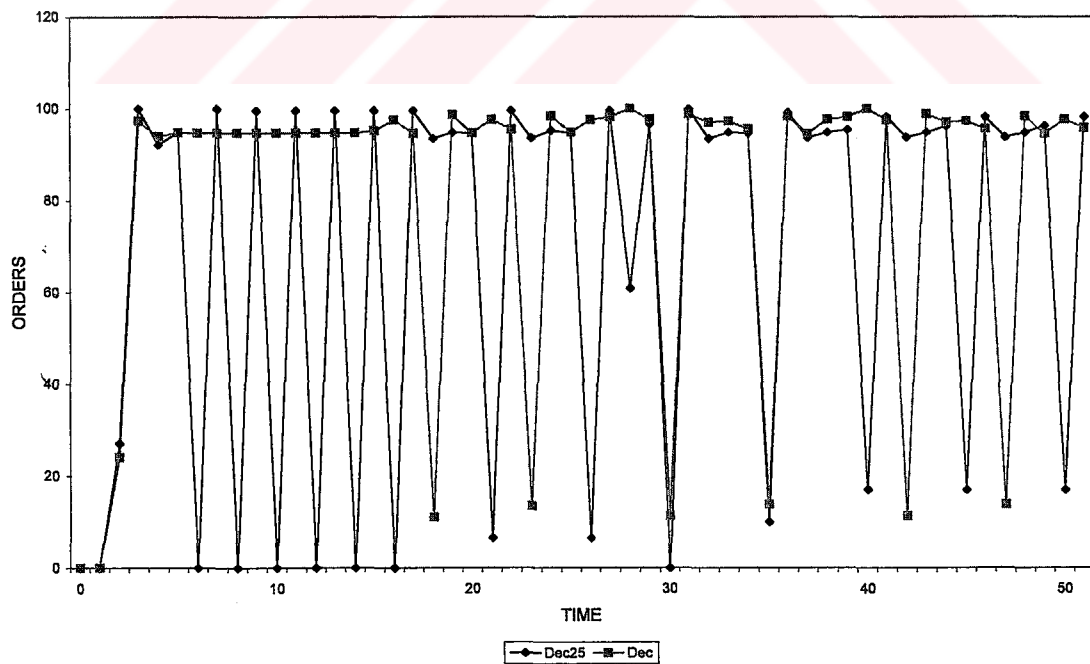


Figure 4.119: Order levels between retailer and distribution center for product B

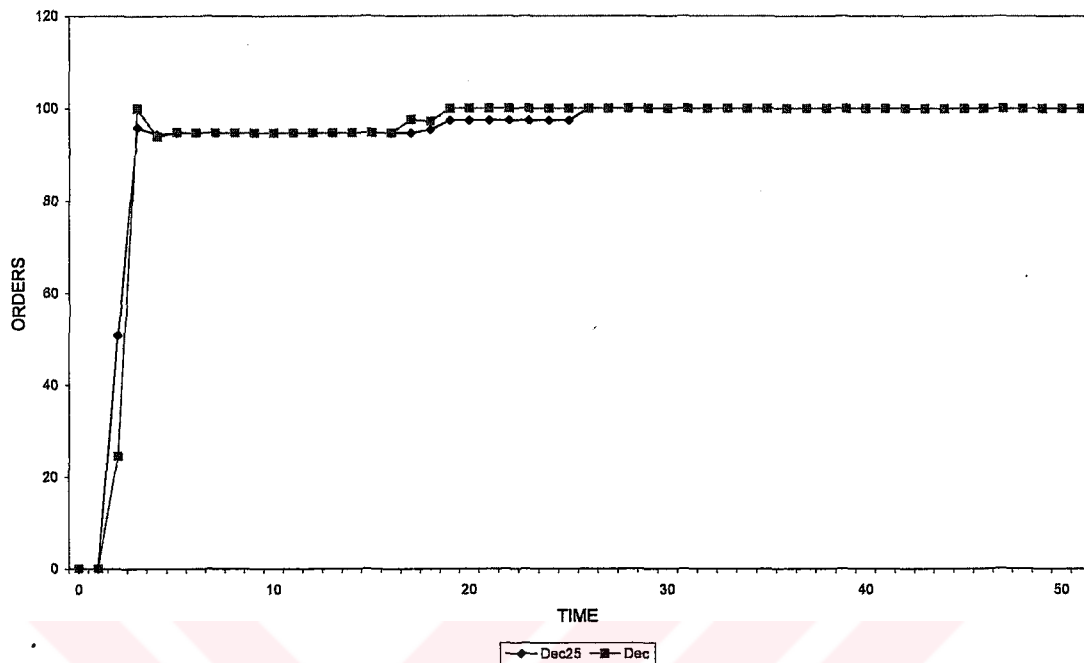


Figure 4:119: Order levels between retailer and distribution center for product C

Furthermore, the amount of material demanded for both horizon lengths is the maximum amount, thus if indeed there is a difference, it is not observable under these conditions. The sizes of the bullwhip effects are similar since both horizon lengths use the same forecasting procedure at all levels and also both of the horizon lengths have their material demand levels at the maximum, therefore if there is any difference, it is not observable. Other than product C, the behavior of the decision variables are nearly the same including the initial inventory behaviors. The reason for this behavior is that in both configurations there is no composite unit which knows the exact state of other units. Hence in both horizon lengths, the units make the same assumptions under the same state conditions and the horizon length does not change the behavior. Because optimal behavior under the same assumptions and state conditions is to deplete the entire inventory. Actually, the behavioral differences between the two horizon lengths only occur when there is a difference between the production schedules of the plants in different horizon lengths. However, observing the production times and production amounts for both horizons, it is seen that the total production amounts and total production times are the same. This implies that the profit of plant and warehouse does not change because of these changes in the production schedule, which is what matters for plant and warehouse. Therefore, it is highly likely that these are alternative optimal solutions for plant and warehouse.

4.4 Decentralized vs Semi-decentralized vs Centralized:

As the integration level increases, performance of the supply chain increases in terms of all performance measures discussed before. Some of the graphs are presented in Figs. 4.121 to 4.128.

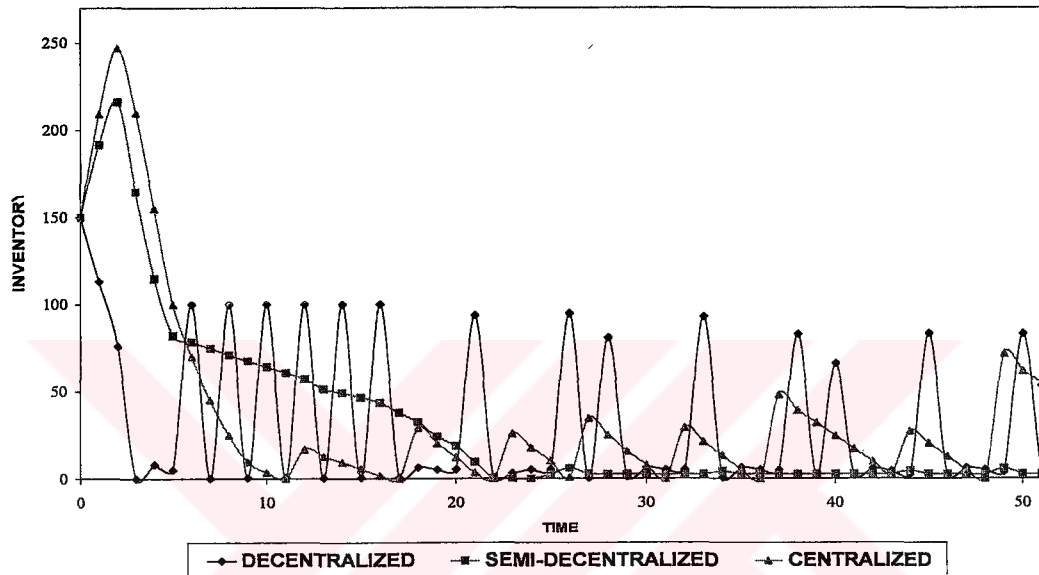


Figure 4.121. Retailer node inventory level for product A

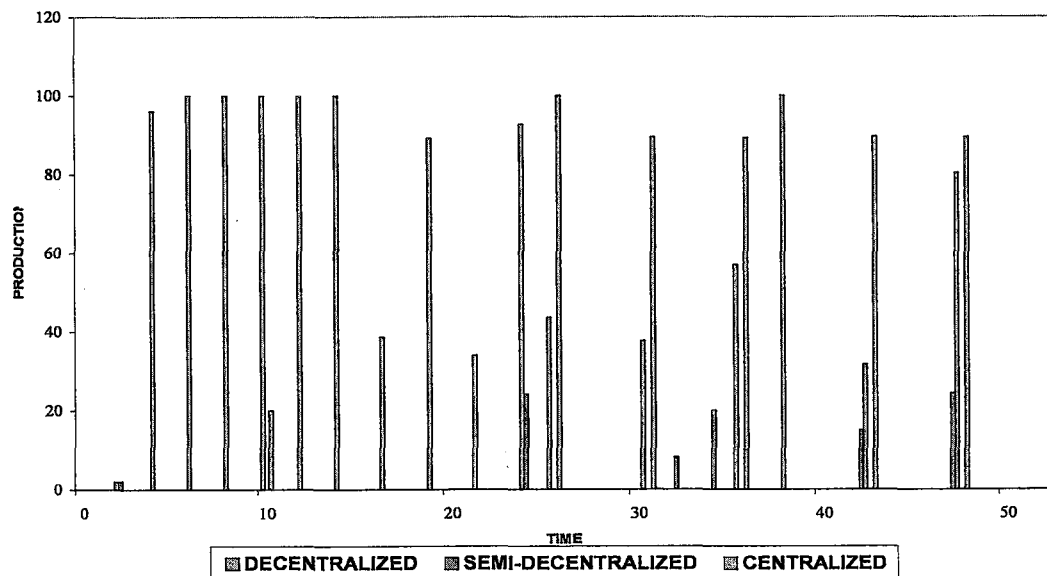


Figure 4.122. Production of A

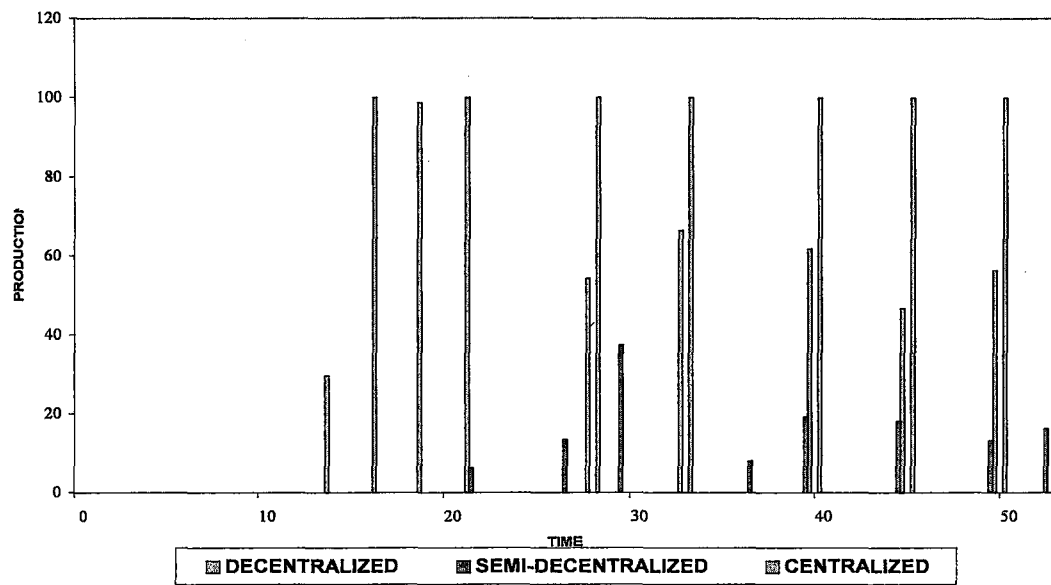


Figure 4.123. Production of B

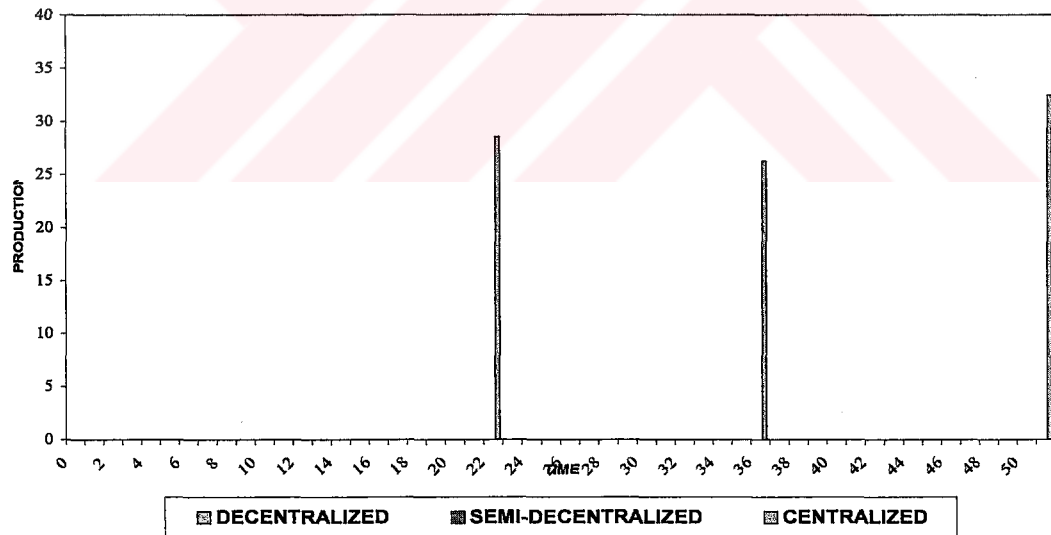
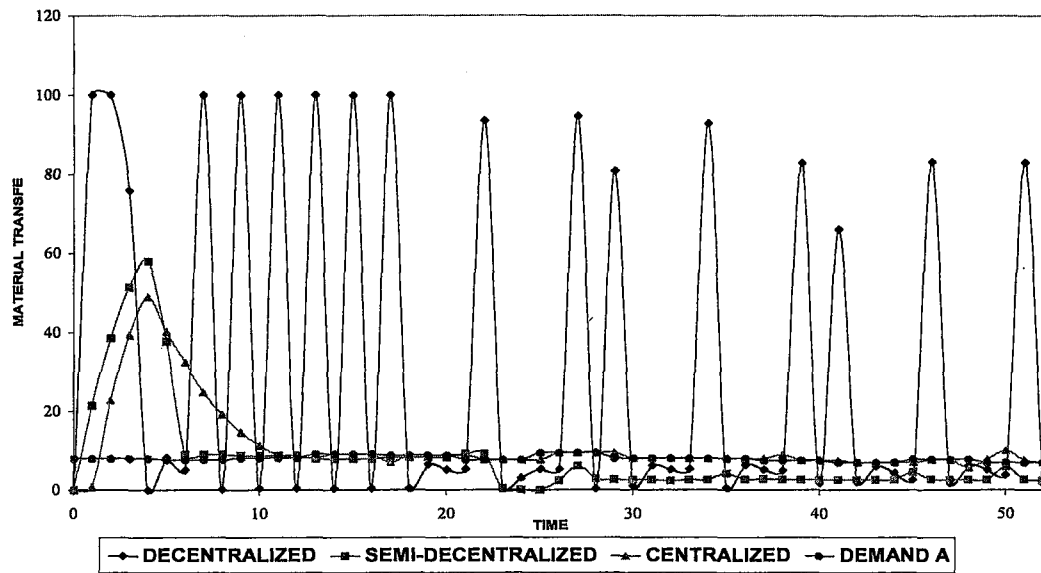


Figure 4.124. Production of C



4.125. Customer satisfaction of A

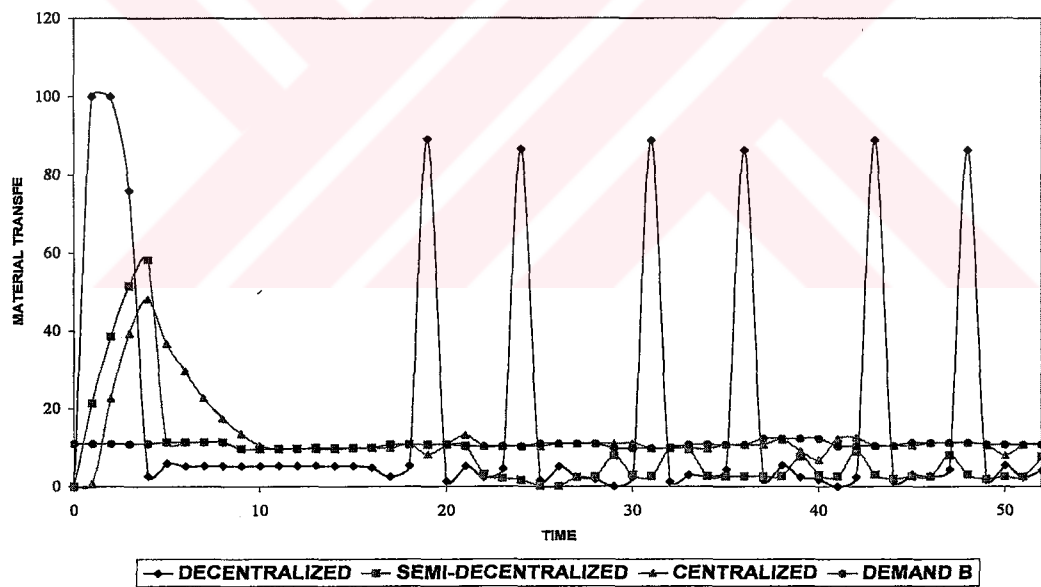


Figure 4.126. Customer satisfaction of B

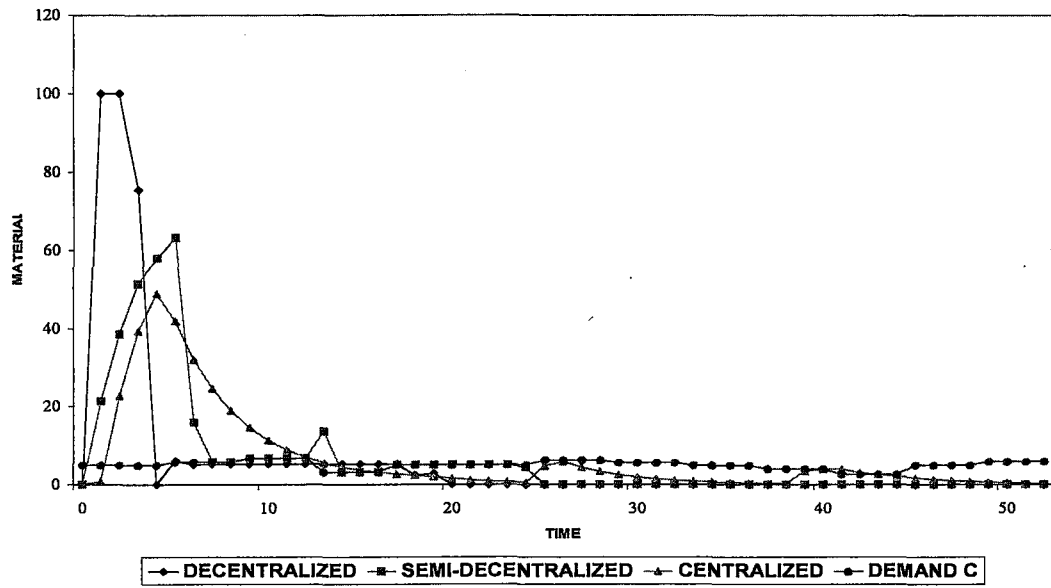


Figure 4.127. Customer satisfaction of C

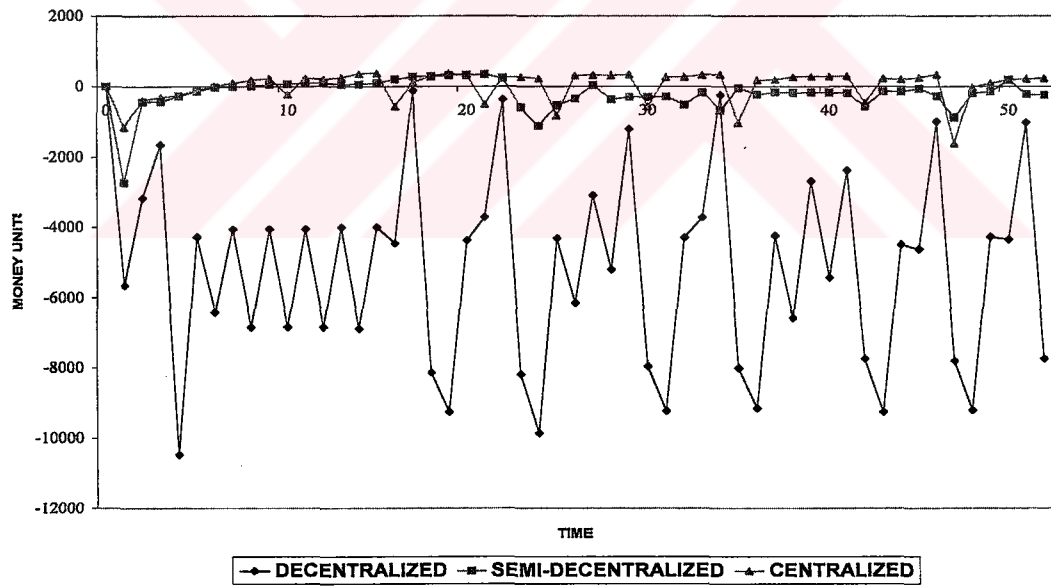


Figure 4.128. Instantaneous profit

Decentralized configurations perform worse than centralized configuration due to lack of coordination, which is induced by basically two main causes.

- Local optimization
- False assumptions that the upstream node will send as much as wanted and downstream node's demand will be the same for the following periods

Local optimization means that all the independent nodes only consider their own good disregarding those of others. This may lead to the creation of overly optimistic demand values since any node optimizing its own profit does not know the state of the upstream node and does not take it into account.

On the other hand, it is clear that the stated false assumptions will lead to a lack of coordination. In other words, if the upstream node does not send as much as wanted, then either over-stocking or under-stocking will occur, which certainly would damage the optimization of the control variables of the following periods henceforth reduce the profit. Furthermore, during each implementation of MPC, each node assumes that the demand provided by the downstream node is going to stay constant for the following periods, which is not necessarily the case. For example, let N be a node and D be its downstream node. Let the demand values created by D be $\{100, 0, 0, 0\}$. D sends 100 to N as its demand. N's forecast for the demand of D is $\{100, 100, 100, 100\}$. Based on this demand pattern, it is clear that N will demand more than necessary amount of materials from its upstream node. It is clear that this will have a significant effect on the optimization of all upstream nodes since this false assumption leads to a "chain reaction".

Centralized control suppresses both of these causes, providing better control levels. The discussion regarding the comparison of semi-decentralized and decentralized controls is given in previous sections.

CHAPTER 5

CONCLUSIONS

Model Predictive Control (MPC) methodology is used to optimize the performance of supply chain systems in this work. The supply chains considered in this study are multi-product systems that exhibit hybrid behavior. The application of MPC for both centralized and decentralized operation models are examined in detail.

The performance of a single product supply chain system is optimized using the MPC methodology in different horizon lengths and measurement update intervals. Then, the performance of this system is compared with the results obtained from open-loop optimization where the demand pattern is perfectly known. In MPC approaches, optimizers do not see the entire actual demand pattern, leading to myopic decisions, deviating from the OLO results which know the entire demand pattern. A smaller fixed horizon MPC performs worse at the beginning compared with the longer one, since it underestimates the actual process. However, in time, the longer horizon overestimates the actual case, deviating from OLO. In the case where the measurement update interval is taken different from the demand change interval, the MPC performance becomes worse.

After observing the effects of MPC optimization on a simple supply chain, a multiple product, hybrid supply chain system, which contains continuous retailer, distribution center and warehouse nodes along with a discrete manufacturing plant, is optimized in centralized fashion under different batch and continuous production rules, with various initial inventories and demand profiles. As the MPC optimization horizon length increased, the results get closer to the OLO values, yet the CPU times increase accordingly, causing a trade-off between optimal results and allowable solution times. When the production strategy is relaxed from batch production size to variable production sizes, the system gives better results. Supply chain systems with different initial inventory levels and demand patterns are also studied in terms of MPC performance. Compared with the higher inventory case studied, the lower initial inventory case gave better results due to the reduced inventory costs. MPC gives satisfactory results when an unpredictable sinusoidal change in demand takes place.

The same multi-product hybrid supply chain is then evaluated under fully decentralized and semi-decentralized control configurations. In the fully decentralized configuration retailer, distribution center and, plant and warehouse are all independent nodes trying to maximize their own profits. In the semi-decentralized configuration, the retailer and distribution center are coordinated by a single optimizer and plant and warehouse represent another independent entity. For both of the cases each independent entity applies MPC to optimize profit. The decentralized, semi-decentralized and centralized supply chain results are compared in terms of total profit, inventory and material transfer levels and customer satisfaction. In the fully decentralized and semi-decentralized models, lack of coordination is the key reason for obtaining worse results compared with centralized model. This lack of coordination causes the bullwhip effect, especially in the fully decentralized model, which damages the optimal performance of the supply chain.

5.1 Directions for Future Work:

- A more realistic supply chain model including
 - more realistic plant models
 - detailed production system modeling
 - detailed cost schemes
 - more realistic distribution network models
 - different locations for each node
 - different lead times for different products
 - different lead times between different nodes
 - different transportation between different nodes for each product
 - different inventory holding costs for different nodes and products
 - different demand locations with different demands and prices
 - stochastic behavior at all levels
- Effects of partnership models on partially or completely decentralized supply chains such as
 - Vendor Managed Inventory

- Send all demand values to the upstream node instead of the current one, reducing the risk of false forecasts
- Sharing of inventory and order levels between nodes



BIBLIOGRAPHY

- [1] D.M. Lambert, and M.C. Cooper, "Issues in supply chain management," *Industrial Marketing Management*, vol. 29, pp. 65-83, 2000.
- [2] D.M. Lambert, M.C. Cooper, and J.D Pagh, "Supply chain management: Implementation issues and research opportunities," *The International Journal of Logistics Management*, vol. 9 (2), pp. 1-19, 1998.
- [3] J. W. Forrester, *Industrial Dynamics*, MIT Press, Cambridge, MA, 1961.
- [4] D.R. Towill, "Dynamic analysis of an inventory and order based production control system," *International Journal of Production Research*, vol. 6, pp. 671-687, 1982.
- [5] E. Perea-Lopez, I.E. Grossmann and B.E. Ydstie, "Dynamic Modeling and decentralized control of supply chains," *Industrial and Engineering Chemistry Research*, vol. 40, pp. 3369-3383, 2001.
- [6] T. Backx, O. Bosgra and W. Marquardt, "Towards international dynamics in supply chain conscious process operations," In *Proceedings of Third International Conference on Foundations of Computer-Aided Process Operations*; J.F. Pekny, G.E. Blau, AIChE: New York, pp. 62-74, 1998.
- [7] E. Perea-Lopez, B.E. Ydstie, and I.E. Grossmann, "A model predictive control strategy for supply chain optimization," *Computers and Chemical Engineering*, vol. 27, pp. 1201-1218, 2003.
- [8] S. Bose, and J.F. Pekny, "A model predictive framework for planning and scheduling problems: a case study of consumer goods supply chain," *Computers and Chemical Engineering*, vol. 24, pp. 329-335, 2000.
- [9] M.W. Braun, D.E. Rivera, W.M. Carlyle and K.G. Kempf, "Model predictive control framework for robust management of multi-product, multi-echelon demand networks," *15th Triennial World Congress*, Barcelona, Spain, 2002.
- [10] S. Lakhal, A. Martel, O. Kettani and M. Oral, "Theory and methodology on the optimization of supply chain networking decisions," *European Journal of Operational Research*, vol. 129, pp. 259-270, 2001.

- [11] D. Petrovic, R. Roy, and R. Petrovic, "Supply chain modeling using fuzzy sets," *International Journal of Production Economics*, vol. 59, pp. 443-453, 1999.
- [12] D. Petrovic, "Simulation of supply chain behavior and performance in an uncertain environment," *International Journal of Production Economics*, vol. 71, pp. 429-438, 2001.
- [13] C. H. Timpe, and J. Kallrath, "Optimal planning in large multi-site production networks," *European Journal of Operational Research*, vol. 126, pp.422-435, 2000.
- [14] M. S. Chen, and M.C. Chu, "The analysis of optimal control model in matching problem between manufacturing and marketing," *European Journal of Operational Research*, vol. 150, pp. 293-303, 2003.
- [15] S. M. Disney, and D. R. Towill, "A procedure for the optimization of the dynamics response of a vendor managed inventory system," *Computers & Industrial Engineering*, vol. 43, pp. 27-58, 2002.
- [16] B.C. Arntzen, G.G. Brown, T.P. Harrison, and L. Trafton, "Global supply chain management at digital equipment corporation," *Interfaces*, Jan-Feb, 1995.
- [17] C.S. Tapiero and M.A. Soliman, "Multi-commodities transportation schedules over time," *Networks*, vol. 2, pp. 311-327, 1972.
- [18] H.L. Lee, and C. Billington "Material management in decentralized supply chains," *Operations Research*, vol. 41 (5), pp. 835-847, 1993.
- [19] H.L. Lee, V. Padmanabhan and S. Whang, "Information distortion in a supply chain: The bullwhip effect," *Management Science*, vol. 43 (4), pp. 546-558, 1997.
- [20] D. R. Towill, M.M. Naim, and J. Wikner "Industrial dynamics simulation models in the design of supply chains," *International Journal of Physical Distribution and Logistics Management*, vol. 22 (5), pp. 3-13, 1992.
- [21] D.D. Newart, K.L. Stott, and F.J. Vasco, "Consolidating product sizes to minimize inventory levels for a multi-stage production and distribution systems," *Journal of the Operational Research Society*, vol. 44 (7), pp. 637-644, 1993.
- [22] H.L. Lee, V. Padmanabhan and S. Whang, "The bullwhip effect in supply chains," *Sloan Management Review*, Spring, 1997.

- [23] B.A. Ogunnaike, and W.H. Ray, *Process Dynamics, Modeling and Control*, Oxford University Press, New York, NY, 1994.
- [24] H. Kwakernaak, and R. Sivan *Linear Optimal Control Systems*, John Wiley & Sons New York, NY, 1972.
- [25] R. Raman, and I.E. Grossmann "Modeling and computational techniques for logic based integer programming," *Computers in Chemical Engineering*, vol. 18, pp. 563-578, 1994.
- [26] M. Türkay, and I.E. Grossmann "Disjunctive programming techniques for the optimization of process systems with discontinuous investment costs-multiple size regions," *Industrial and Engineering Chemistry Research*, vol. 35, pp. 2611-2623, 1996.
- [27] G. Ferrari-Trecate, F.A. Cuzzola, D. Mignone and M. Morari, "Analysis and control with performance of piecewise affine and hybrid systems," *Proc. American Control Conference*, pp. 200 –205, 2001.
- [28] G. Ferrari-Trecate, D. Mignone and M. Morari, "Moving horizon estimation for hybrid systems," *IEEE Transactions on Automatic Control*, vol. 47(10), pp. 1663-1676, 2002.
- [29] A. Bemporad, and M. Morari , "Control of systems integrating logic, dynamics and constraints," *Automatica*, vol. 35 (3), pp. 407 –427, 1999.
- [30] G. Ferrari-Trecate, E. Gallestey, P. Letizia, M. Spedicato, M. Morari, and M. Antoine, "Modeling and control of co-generation power plants: A hybrid system approach," *Hybrid Systems: Computation and Control (HSCC, 2002)*, Stanford, USA, 25-27 March 2002.

APPENDIX A

Statistical Analysis on y_{RA} :

Regression Statistics	
Multiple R	0,978624323
R Square	0,977250538
Adjusted R Square	0,977223714
Standard Error	0,73864604
Observations	208

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	2	40567,96	20283,98	37177,52	3,3335E-263
Residual	205	111,8476	0,545598		
Total	207	40679,8			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95,0%</i>	<i>Upper 95,0%</i>
Intercept	-5,397004512	0,2635942	-21,2525	1,01E-53	-5,397	-5,397	-5,397	-5,397
X Variable 1	0,191047431	0,0009406	203,1147	2,5E-238	0,189193	0,192902	0,189193	0,192902
X Variable 2	0,051533184	0,0009866	52,23454	2E-120	0,049588	0,053478	0,049588	0,053478

Statistical Analysis on y_{DA} :

Regression Statistics	
Multiple R	0,978624323
R Square	0,977250538
Adjusted R Square	0,977223714
Standard Error	0,73864604
Observations	208

ANOVA					
	df	SS	MS	F	Significance F
Regression	2	946597,4	473298,7	558493,3	0
Residual	205	173,7286	0,847456		
Total	207	946771,2			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95,0%	Upper 95,0%
Intercept	-49,56122748	0,3285172	-150,863	4,9E-212	-50,2089	-48,9135	-50,2089	-48,9135
X Variable 1	0,03098645	0,0011723	25,31816	5,06E-65	0,028675	0,033298	0,028675	0,033298
X Variable 2	1,099866085	0,0012296	894,5163	0	1,097442	1,10229	1,097442	1,10229

Statistical Analysis on y_{WA} :

<i>Regression Statistics</i>	
Multiple R	0,979619898
R Square	0,97923994
Adjusted R Square	0,979232525
Standard Error	1,531691701
Observations	208

ANOVA					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	2	632293,2	316146,6	134755,3	0
Residual	205	480,9463	2,346079		
Total	207	632774,1			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95,0%</i>	<i>Upper 95,0%</i>
Intercept	-30,59340951	0,5466016	-55,6373	1,1E-125	-31,6711	-29,5157	-31,6711	-29,5157
X Variable 1	0,02398645	0,0019504	13,07981	7,99E-29	0,020141	0,027832	0,020141	0,027832
X Variable 2	0,899991889	0,0020458	439,0489	8,7E-307	-0,0002	1,800188	-0,0002	1,800188

APPENDIX B

[illegible]

$B_1 =$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$B_2 =$	75	0	0	$B_3 =$	30.6
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0		19.4
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0		-44.603
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	100	0		30.6
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0		19.4
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0		-44.603
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	50		30.6
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0		19.4
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0		-44.603
	1	0	0	0	0	0	0	0	0	0	0	0	0	0		-75	0	0		0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0		30.6
	0	0	1	0	0	0	0	0	0	0	0	0	0	0		0	0	0		50
	0	0	0	1	0	0	0	0	0	0	0	0	0	0		0	0	0		5.392
	0	0	0	0	1	0	0	0	0	0	0	0	0	0		0	-100	0		0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0		0	0	0		30.6
	0	0	0	0	0	0	1	0	0	0	0	0	0	0		0	0	0		50
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	5.392			
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-50	0			
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	30.6			
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	50			
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	5.392			

[illegible]

$$\begin{aligned}
 D_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 D_2 &= \begin{bmatrix} 75 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 50 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
 D_3 &= \begin{bmatrix} 0 \\ -5.397 \\ -50 \\ -30.6 \\ 0 \\ -5.397 \\ -50 \\ -30.6 \\ 0 \\ -5.397 \\ -50 \\ -30.6 \end{bmatrix}
 \end{aligned}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad E_3 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad E_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$