

DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**EXAMINING THE EFFICIENCY OF
REGRESSION ESTIMATORS IN MODIFIED
RANKED SET SAMPLING METHODS**

by
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September, 2019

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EXAMINING THE EFFICIENCY OF REGRESSION ESTIMATORS IN MODIFIED RANKED SET SAMPLING METHODS

**A Thesis Submitted to the
Graduate School of Natural And Applied Sciences of Dokuz Eylül University
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**by
Eda DAVASLIOĞLU**

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M.Sc THESIS EXAMINATION RESULT FORM

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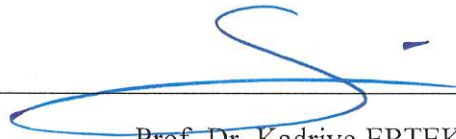
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Eda DAVASLIOĞLU

EXAMINING THE EFFICIENCY OF REGRESSION ESTIMATORS IN MODIFIED RANKED SET SAMPLING METHODS

ABSTRACT

Ranked Set Sampling (RSS) has been a popular sampling method in recent years. RSS is used when visual ranking can be done easily while the variable of interest is difficult or expensive to measure. In the last years, different modifications of the RSS which are Pair RSS, Extreme RSS, Median RSS, Double RSS, LRSS and Truncation Based RSS methods have been used in wide applications and suggested by many researchers. The aim of this study is to estimate the regression estimators and to compare relative efficiencies of mean square errors of the regression models, population mean and regression coefficients for Simple Random Sampling (SRS), RSS and the different modified RSS methods. The Monte Carlo simulation study is performed via R Project with 10,000 repetitions. The performances of the estimators are compared based on bias, mean square error and relative efficiency for different levels of correlation coefficient, set and cycle sizes under Bivariate Normal Distribution (BVN) with different parameters for normal and outlier cases. The results indicate that the regression estimators under the modified RSS methods performs better than the regression estimators under SRS.

Keywords: Ranked set sampling, modified ranked set sampling methods, relative efficiency

MODİFİYE SIRALI KÜME ÖRNEKLEMESİ YÖNTEMLERİNDE REGRESYON KESTİRİCİLERİNİN ETKİNLİKLERİNİN İNCELENMESİ

ÖZ

Sıralı Küme Örneklemesi (SKÖ), son yıllarda popüler bir örneklem yöntemi olmuştur. SKÖ, ilgilenilen değişkenin ölçümü zor veya maliyetliken, görsel sıralamanın kolay bir şekilde yapılabilirdiği durumlarda kullanılmaktadır. Son yıllarda, Eş Sıralı Küme Örneklemesi, Uç Sıralı Küme Örneklemesi, Medyan Sıralı Küme Örneklemesi, Çift Sıralı Küme Örneklemesi, L Sıralı Küme Örneklemesi ve Kesme Tabanlı Sıralı Küme Örneklemesi gibi Sıralı Küme Örneklemesi yönteminin farklı modifikasyonları birçok araştırmacı tarafından önerilmiş ve bu yöntemler geniş çaplı uygulamalarda kullanılmıştır. Bu çalışmanın amacı, Basit Rastgele Örneklem (BRÖ), SKÖ ve farklı modifiye sıralı küme örneklemesi yöntemlerinin regresyon kestiricilerini tahmin etmek ve bu yöntemlerin regresyon modellerine ait hata kareler ortalamaları, kitle ortalaması ve regresyon katsayılarının göreceli etkinliklerini karşılaştırmaktır. R Programlama dili aracılığıyla 10,000 tekrarlı Monte Carlo simülasyon çalışması gerçekleştirilmiştir. Kestiricilerin performansları; yanlılık, hata kareler ortalaması ve göreceli etkinliklerine göre farklı korelasyon katsayıları, küme ve döngü genişlikleri ile farklı parametrelere sahip iki değişkenli normal dağılım altında, aykırı değer içeren ve içermeyen durumlar için karşılaştırılmıştır. Bulgular, modifiye sıralı küme örneklemesi yöntemleri altındaki regresyon kestiricilerinin, BRÖ altındaki regresyon kestiricilerine göre daha iyi performans sergilediğini ortaya koymaktadır.

Anahtar kelimeler: Sıralı küme örneklemesi, modifiye sıralı küme örneklemesi yöntemleri, göreceli etkinlik

CONTENTS

	Page
M.Sc THESIS EXAMINATION RESULT FORM.....	ii
ACKNOWLEDGEMENTS	iii
ABSTRACT	iv
ÖZ.....	v
LIST OF TABLES.....	viii
 CHAPTER ONE – INTRODUCTION.....	 1
 CHAPTER TWO – SAMPLING METHODS AND REGRESSION ESTIMATORS	 4
2.1 Ranked Set Sampling.....	4
2.1.1 Mean Estimators of RSS.....	5
2.1.2 Regression Estimators of RSS.....	6
2.2 Modified Ranked Set Sampling Methods	7
2.2.1 Pair Ranked Set Sampling	7
2.2.2 Extreme Ranked Set Sampling.....	8
2.2.3 Median Ranked Set Sampling	9
2.2.4 Double Ranked Set Sampling	11
2.2.5 L Ranked Set Sampling	12
2.2.6 Truncation Based Ranked Set Sampling	13
2.3 General Form of Regression Estimators of Modified RSS Methods	15
 CHAPTER THREE – SIMULATION STUDY.....	 16
3.1 Normal Case	16
3.2 Outlier Case	24
 CHAPTER FOUR – CONCLUSIONS.....	 32

REFERENCES.....	34
------------------------	-----------

APPENDICES.....	36
------------------------	-----------



LIST OF TABLES

	Page
Table 3.1 RE_1 values of the estimators under normal case	18
Table 3.2 RE_2 and RE_3 values for the mean estimator under normal case	19
Table 3.3 RE_2 and RE_3 values for $\hat{\beta}_0$ under normal case	20
Table 3.4 RE_2 and RE_3 values for $\hat{\beta}_1$ under normal case	21
Table 3.5 RE_2 and RE_3 values for MSE of regression model under normal case ...	22
Table 3.6 Bias values of the estimators $(\bar{y}, \hat{\beta}_0, \hat{\beta}_1)$ for sampling methods under normal case.....	23
Table 3.7 RE_1 values of the estimators under 20% outlier case	25
Table 3.8 RE_2 and RE_3 values for the mean estimator under 20% outlier case	26
Table 3.9 RE_2 and RE_3 values for $\hat{\beta}_0$ under 20% outlier case.....	27
Table 3.10 RE_2 and RE_3 values for $\hat{\beta}_1$ under 20% outlier case.....	28
Table 3.11 RE_2 and RE_3 values for MSE of regression model under 20% outlier case	29
Table 3.12 Bias values of the estimators $(\bar{y}, \hat{\beta}_0, \hat{\beta}_1)$ for sampling methods under 20% outlier case	30

CHAPTER ONE

INTRODUCTION

Sample is a subset of population that represents the characteristics of the population. Sampling is the process of selecting the sample for estimating the population characteristics. Simple random sampling (SRS) method is the most common sampling method. Each individual is chosen randomly and has the same probability to be chosen in the sampling. The sample selected with SRS does not exactly reflect the population when the population of interest is not distributed homogeneously.

Ranked set sampling (RSS) method is an alternative sampling method. In RSS, units can be easily ranked, provided that the actual measurement of sample unit is time-consuming, difficult or expensive. The RSS is suggested by McIntyre (1952) for estimating the population mean of pasture yields in Australia. Halls & Dell (1966) first introduced the name of RSS and used the method for estimation process of the forage yields. Takahasi & Wakimoto (1968) developed the mathematical background of the method and established a very important statistical foundation for the theory of RSS. They proved that the mean estimator of RSS is an unbiased estimator of the population mean with smaller variance when compared to the mean estimator of SRS in case the sample size is equal.

Dell & Clutter (1972) showed that the mean estimator of RSS is an unbiased estimator of the population mean, whether or not there are errors in ranking and it is more efficient than the mean estimator of SRS. Stokes (1977) considered the case when the variable of interest, Y , is difficult to measure or order, however there is a concomitant variable, X , which is correlated with Y and can be used to judge the order of the variable Y .

In the last thirty years, many researchers suggested different modifications of the RSS. Firstly, Samawi et al. (1996) introduced extreme ranked set sampling (ERSS) method to estimate the population mean. They showed that ERSS gives an unbiased

estimator of the population mean in case of symmetric populations and the mean estimator of ERSS is more efficient than the mean estimator of SRS, under the same number of quantified units.

Muttlak (1996) suggested pair ranked set sampling (PRSS) method for estimating the population mean instead of RSS. He showed that the mean estimator of PRSS is better than the mean estimator of SRS. Additionally, the number of sample units are almost half in the PRSS.

Muttlak (1997) proposed median ranked set sampling (MRSS) method to reduce the loss of efficiency in the RSS due to the errors in ranking as well as its effect on the increase in the efficiency of the estimator of the population mean. He suggested that the estimator of the population mean is unbiased for symmetric distributions and biased otherwise.

Al-Saleh & Al-Kadiri (2000) introduced double ranked set sampling (DRSS) method to estimate the population mean. They showed that the estimator of the population mean of DRSS is more efficient than the estimators of the population mean of SRS and RSS methods.

Al-Nasser (2007) suggested a robust ranked set sampling (LRSS) method for estimating the population mean. The proposed method is a generalization for many types of RSS. He found that the mean estimator of LRSS is unbiased if the underlying distribution is symmetric and it is more efficient than the mean estimators of SRS and RSS methods.

Al-Omari & Raqab (2013) proposed truncation based ranked set sampling (TBRSS) method for estimating the population mean and median. They proved that the mean estimator of TBRSS is unbiased and it is more efficient than the mean estimator of SRS when the underlying distribution is symmetric or asymmetric where the coefficient, k is large enough (except exponential distribution).

RSS and modified RSS methods are used to find the regression-type estimators by many researchers. Yu & Lam (1997) proposed the regression estimator of the population mean for the variable of interest Y in RSS. They found that when X and Y jointly follow a Bivariate Normal Distribution (BVN), the regression estimator of RSS is more efficient than the mean estimators of RSS and SRS methods in some cases.

Muttlak (2001) used ERSS and MRSS methods to estimate the regression estimators of the population mean instead of the regression estimator of the population mean of RSS which is suggested by Yu & Lam (1997). It turned out that when the variable of interest and the concomitant variable jointly followed by a BVN, the regression estimator of ERSS dominates all the other estimators considered.

Al-Nasser & Radaideh (2008) used LRSS to estimate a simple linear regression model. They showed that the regression estimator of LRSS is more efficient than the estimators of SRS, RSS and ERSS methods.

The aim of this study is to estimate the regression estimators and to compare the relative efficiencies of mean square errors of the regression models, population mean and regression coefficients for different modified ranked set sampling methods. This study is arranged as follows. In Chapter 2, the motivations of RSS and modified RSS methods (Pair RSS, Extreme RSS, Median RSS, Double RSS, LRSS and Truncation Based RSS) are discussed and details of these methods are described. In Chapter 3, performances of the estimators are investigated for normal and outlier cases with the simulation study carried out under BVN with different levels of correlation coefficients, set and cycle sizes. Finally, Chapter 4 provides concluding remarks.

CHAPTER TWO

SAMPLING METHODS AND REGRESSION ESTIMATORS

2.1 Ranked Set Sampling

Ranked set sampling (RSS) method was suggested by McIntyre (1952) as a process to improve the precision of the sample mean as an estimator of a population mean, when the variable of interest is difficult, time-consuming or expensive to measure. RSS method can be summarized as follows:

Step 1: Select m random samples, each of size m from the population.

Step 2: Rank the units within each sample with respect to the variable of interest by visual inspection or any inexpensive method.

Step 3: Select the i th unit from the i th sample, where $i=1,2,\dots,m$.

Step 1 through 3 are called a cycle. The cycle can be repeated r times to get $N=mr$ units.

Assume that (X,Y) have a BVN. Let $X_{a(b)c}$ and $Y_{a[b]c}$ denote b th the smallest unit of concomitant variable, X , and the corresponding value of response variable, Y , obtained from the a th sample in the c th cycle. The square brackets in the subscripts are used to indicate the possibility of ranking error. With the condition that the ranking process is perfect, the square brackets are replaced with round ones. The RSS can be illustrated as below:

Cycle 1

$$\begin{bmatrix} \boxed{(X_{1(1)1}, Y_{1[1]1})} & (X_{1(2)1}, Y_{1[2]1}) & \cdots & (X_{1(m)1}, Y_{1[m]1}) \\ (X_{2(1)1}, Y_{2[1]1}) & \boxed{(X_{2(2)1}, Y_{2[2]1})} & \cdots & (X_{2(m)1}, Y_{2[m]1}) \\ \vdots & \vdots & \ddots & \vdots \\ (X_{m(1)1}, Y_{m[1]1}) & (X_{m(2)1}, Y_{m[2]1}) & \cdots & \boxed{(X_{m(m)1}, Y_{m[m]1})} \end{bmatrix}$$

Cycle 2

$$\begin{bmatrix} \boxed{(X_{1(1)2}, Y_{1[1]2})} & (X_{1(2)2}, Y_{1[2]2}) & \cdots & (X_{1(m)2}, Y_{1[m]2}) \\ (X_{2(1)2}, Y_{2[1]2}) & \boxed{(X_{2(2)2}, Y_{2[2]2})} & \cdots & (X_{2(m)2}, Y_{2[m]2}) \\ \vdots & \vdots & \ddots & \vdots \\ (X_{m(1)2}, Y_{m[1]2}) & (X_{m(2)2}, Y_{m[2]2}) & \cdots & \boxed{(X_{m(m)2}, Y_{m[m]2})} \end{bmatrix}$$

$\vdots$

Cycle r

$$\begin{bmatrix} \boxed{(X_{1(1)r}, Y_{1[1]r})} & (X_{1(2)r}, Y_{1[2]r}) & \cdots & (X_{1(m)r}, Y_{1[m]r}) \\ (X_{2(1)r}, Y_{2[1]r}) & \boxed{(X_{2(2)r}, Y_{2[2]r})} & \cdots & (X_{2(m)r}, Y_{2[m]r}) \\ \vdots & \vdots & \ddots & \vdots \\ (X_{m(1)r}, Y_{m[1]r}) & (X_{m(2)r}, Y_{m[2]r}) & \cdots & \boxed{(X_{m(m)r}, Y_{m[m]r})} \end{bmatrix}$$

2.1.1 Mean Estimators of RSS

Let $\{(X_{i(i)j}, Y_{i[i]j}) : i = 1, \dots, m; j = 1, \dots, r\}$ be a ranked set sample with the size $N = mr$. The mean estimators of RSS are given

$$\bar{x}_{RSS} = \frac{1}{mr} \sum_{j=1}^r \sum_{i=1}^m X_{i(i)j} \quad (2.1)$$

$$\bar{y}_{RSS} = \frac{1}{mr} \sum_{j=1}^r \sum_{i=1}^m Y_{i[i]j} \quad (2.2)$$

For more details, see Takahasi & Wakimoto (1968).

2.1.2 Regression Estimators of RSS

The mean estimators of RSS are utilized by many researchers. Recently, regression estimators have also been studied. Yu & Lam (1997) were amongst the pioneers of those suggesting the regression estimator of the population mean of the variable of interest Y using the RSS. They had followed the ideas of Stokes that the ranking is performed by means of a concomitant variable, X , which can be measured easily and is positively correlated with the variable of interest, Y , that is expensive or difficult to measure (see details Stokes (1977)). Assume that (X, Y) have a BVN and the regression of Y on X is linear. The regression model is given in Equation 2.3.

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \quad i = 1, 2, \dots, n \quad (2.3)$$

where X and ε are independent and ε has mean 0 and variance $\sigma_y^2(1 - \rho^2)$.

Let $X_{(i)j}$ and $Y_{[i]j}$ be, respectively, the i th smallest X and the corresponding value of Y obtained from the i th sample in the j th cycle. Then we have, from Equation 2.3

$$Y_{[i]j} = \beta_0 + \beta_1 X_{[i]j} + \varepsilon_{[i]j}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, r \quad (2.4)$$

When the population mean μ_x of X is known, the regression estimator of the population mean of RSS and the regression coefficients of RSS $\hat{\beta}_1$ and $\hat{\beta}_0$ are given as below respectively:

$$\bar{y}_{RSSreg} = \bar{y}_{RSS} + \hat{\beta}_1(\mu_x - \bar{x}_{RSS}) \quad (2.5)$$

$$\hat{\beta}_1 = \frac{\sum_{j=1}^r \sum_{i=1}^m (X_{[i]j} - \bar{x}_{RSS})(Y_{[i]j} - \bar{y}_{RSS})}{\sum_{j=1}^r \sum_{i=1}^m (X_{[i]j} - \bar{x}_{RSS})^2} \quad (2.6)$$

$$\hat{\beta}_0 = \bar{y}_{RSS} - \hat{\beta}_1 \bar{x}_{RSS} \quad (2.7)$$

2.2 Modified Ranked Set Sampling Methods

In the past three decades, many researchers have suggested different modifications of the RSS and run through widespread applications. In this section, modified RSS methods; Pair RSS, Extreme RSS, Median RSS, Double RSS, LRSS and Truncation Based RSS are introduced and the mean estimators of them are stated.

2.2.1 Pair Ranked Set Sampling

The pair ranked set sampling (PRSS) method was suggested by Muttlak (1997) for estimating the population mean in place of RSS. The advantage of the PRSS is that it decreases the number of sample units by almost half regarding other modified RSS methods. The PRSS can be summarized as follows:

Step 1: If m is an even number, select $k=m/2$ random samples, each of size m from the population. If m is an odd number, select $k=(m+1)/2$ random samples each of size m .

Step 2: Rank the units within each sample with respect to a variable of interest by visual inspection or any inexpensive method.

Step 3: Select i th and $(m+1-i)$ th units from the k th sample where $i = 1, 2, \dots, m$.

Step 1 through 3 are called a cycle. The cycle can be repeated r times to get $N=mr$ units.

The PRSS for $r=1$, $m=4$ and $m=5$ can be illustrated as below:

$$\begin{bmatrix} (X_{1(1)1}, Y_{1[1]1}) & (X_{1(2)1}, Y_{1[2]1}) & (X_{1(3)1}, Y_{1[3]1}) & (X_{1(4)1}, Y_{1[4]1}) \\ (X_{2(1)1}, Y_{2[1]1}) & (X_{2(2)1}, Y_{2[2]1}) & (X_{2(3)1}, Y_{2[3]1}) & (X_{2(4)1}, Y_{2[4]1}) \end{bmatrix}$$

$$\begin{bmatrix} (X_{1(1)1}, Y_{1[1]1}) & (X_{1(2)1}, Y_{1[2]1}) & (X_{1(3)1}, Y_{1[3]1}) & (X_{1(4)1}, Y_{1[4]1}) & (X_{1(5)1}, Y_{1[5]1}) \\ (X_{2(1)1}, Y_{2[1]1}) & (X_{2(2)1}, Y_{2[2]1}) & (X_{2(3)1}, Y_{2[3]1}) & (X_{2(4)1}, Y_{2[4]1}) & (X_{2(5)1}, Y_{2[5]1}) \\ (X_{3(1)1}, Y_{3[1]1}) & (X_{3(2)1}, Y_{3[2]1}) & (X_{3(3)1}, Y_{3[3]1}) & (X_{3(4)1}, Y_{3[4]1}) & (X_{3(5)1}, Y_{3[5]1}) \end{bmatrix}$$

The mean estimators of PRSS are given for even and odd set sizes in Equations 2.8-2.11, respectively.

$$\bar{x}_{PRSS}^E = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{m/2} X_{i(i)j} + \sum_{j=1}^r \sum_{i=1}^{m/2} X_{i(m+1-i)j} \right] \quad (2.8)$$

$$\bar{y}_{PRSS}^E = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{m/2} Y_{i[i]j} + \sum_{j=1}^r \sum_{i=1}^{m/2} Y_{i[m+1-i]j} \right] \quad (2.9)$$

$$\bar{x}_{PRSS}^O = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{(m+1)/2} X_{i(i)j} + \sum_{j=1}^r \sum_{i=1}^{(m-1)/2} X_{i(m+1-i)j} \right] \quad (2.10)$$

$$\bar{y}_{PRSS}^O = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{(m+1)/2} Y_{i[i]j} + \sum_{j=1}^r \sum_{i=1}^{(m-1)/2} Y_{i[m+1-i]j} \right] \quad (2.11)$$

2.2.2 Extreme Ranked Set Sampling

Extreme ranked set sampling (ERSS) method was put forward by Samawi et al. (1996) for estimating the population mean. The writers discerned that the ERSS provides an unbiased estimator of the population mean contingent upon symmetric populations. The ERSS can be summarized as follows:

Step 1: Select m random samples, each of size m from the population.

Step 2: Rank the units within each sample with respect to a variable of interest by visual inspection or any inexpensive method.

Step 3: For actual measurement, if m is an even number, select the smallest unit from $m/2$ samples and from the other $m/2$ samples select the largest unit. If m is an odd number, select the smallest unit from $(m-1)/2$ samples, from the other $(m-1)/2$ select the largest unit and from the remaining sample select the median of the sample.

Step 1 through 3 are called a cycle. The cycle can be repeated r times to get $N=mr$ units.

The ERSS for $r=1, m=4$ and $m=5$ can be illustrated as below:

$$\begin{bmatrix} \boxed{(X_{1(1)1}, Y_{1[1]1})} & (X_{1(2)1}, Y_{1[2]1}) & (X_{1(3)1}, Y_{1[3]1}) & (X_{1(4)1}, Y_{1[4]1}) \\ \boxed{(X_{2(1)1}, Y_{2[1]1})} & (X_{2(2)1}, Y_{2[2]1}) & (X_{2(3)1}, Y_{2[3]1}) & (X_{2(4)1}, Y_{2[4]1}) \\ (X_{3(1)1}, Y_{3[1]1}) & (X_{3(2)1}, Y_{3[2]1}) & (X_{3(3)1}, Y_{3[3]1}) & \boxed{(X_{3(4)1}, Y_{3[4]1})} \\ (X_{4(1)1}, Y_{4[1]1}) & (X_{4(2)1}, Y_{4[2]1}) & (X_{4(3)1}, Y_{4[3]1}) & \boxed{(X_{4(4)1}, Y_{4[4]1})} \end{bmatrix}$$

$$\begin{bmatrix} \boxed{(X_{1(1)1}, Y_{1[1]1})} & (X_{1(2)1}, Y_{1[2]1}) & (X_{1(3)1}, Y_{1[3]1}) & (X_{1(4)1}, Y_{1[4]1}) & (X_{1(5)1}, Y_{1[5]1}) \\ \boxed{(X_{2(1)1}, Y_{2[1]1})} & (X_{2(2)1}, Y_{2[2]1}) & (X_{2(3)1}, Y_{2[3]1}) & (X_{2(4)1}, Y_{2[4]1}) & (X_{2(5)1}, Y_{2[5]1}) \\ (X_{3(1)1}, Y_{3[1]1}) & (X_{3(2)1}, Y_{3[2]1}) & (X_{3(3)1}, Y_{3[3]1}) & (X_{3(4)1}, Y_{3[4]1}) & \boxed{(X_{3(5)1}, Y_{3[5]1})} \\ (X_{4(1)1}, Y_{4[1]1}) & (X_{4(2)1}, Y_{4[2]1}) & (X_{4(3)1}, Y_{4[3]1}) & (X_{4(4)1}, Y_{4[4]1}) & \boxed{(X_{4(5)1}, Y_{4[5]1})} \\ (X_{5(1)1}, Y_{5[1]1}) & (X_{5(2)1}, Y_{5[2]1}) & \boxed{(X_{5(3)1}, Y_{5[3]1})} & (X_{5(4)1}, Y_{5[4]1}) & (X_{5(5)1}, Y_{5[5]1}) \end{bmatrix}$$

The mean estimators of ERSS are given for even and odd set sizes in Equations 2.12-2.15, respectively.

$$\bar{x}_{ERSS}^E = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{m/2} X_{i(1)j} + \sum_{j=1}^r \sum_{i=(m/2)+1}^m X_{i(m)j} \right] \quad (2.12)$$

$$\bar{y}_{ERSS}^E = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{m/2} Y_{i[1]j} + \sum_{j=1}^r \sum_{i=(m/2)+1}^m Y_{i[m]j} \right] \quad (2.13)$$

$$\bar{x}_{ERSS}^O = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{(m-1)/2} X_{i(1)j} + \sum_{j=1}^r \sum_{i=((m-1)/2)+1}^{m-1} X_{i(m)j} + \sum_{j=1}^r X_{m((m+1)/2)j} \right] \quad (2.14)$$

$$\bar{y}_{ERSS}^O = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{(m-1)/2} Y_{i[1]j} + \sum_{j=1}^r \sum_{i=((m-1)/2)+1}^{m-1} Y_{i[m]j} + \sum_{j=1}^r Y_{m[(m+1)/2]j} \right] \quad (2.15)$$

2.2.3 Median Ranked Set Sampling

Muttlak (1997) suggested median ranked set sampling (MRSS) method for estimating the population mean. MRSS is suggested to reduce ranking error and to increase the efficiency of the estimator under perfect ranking assumption. The MRSS can be summarized as follows:

Step 1: Select m random samples, each of size m from the population.

Step 2: Rank the units within each sample with respect to a variable of interest by visual inspection or any inexpensive method.

Step 3: For actual measurement, if m is an even number, select the $(m/2)$ th smallest unit from the first $m/2$ samples and select the $((m+2)/2)$ th largest unit from the second $m/2$ samples. If m is an odd number, select the $((m+1)/2)$ th smallest unit (the median of the sample) from each sample.

Step 1 through 3 are called a cycle. The cycle can be repeated r times to get $N=mr$ units.

The MRSS for $r=1$, $m=4$ and $m=5$ can be illustrated as below:

$$\begin{bmatrix} (X_{1(1)1}, Y_{1[1]1}) & \boxed{(X_{1(2)1}, Y_{1[2]1})} & (X_{1(3)1}, Y_{1[3]1}) & (X_{1(4)1}, Y_{1[4]1}) \\ (X_{2(1)1}, Y_{2[1]1}) & \boxed{(X_{2(2)1}, Y_{2[2]1})} & (X_{2(3)1}, Y_{2[3]1}) & (X_{2(4)1}, Y_{2[4]1}) \\ (X_{3(1)1}, Y_{3[1]1}) & (X_{3(2)1}, Y_{3[2]1}) & \boxed{(X_{3(3)1}, Y_{3[3]1})} & (X_{3(4)1}, Y_{3[4]1}) \\ (X_{4(1)1}, Y_{4[1]1}) & (X_{4(2)1}, Y_{4[2]1}) & \boxed{(X_{4(3)1}, Y_{4[3]1})} & (X_{4(4)1}, Y_{4[4]1}) \end{bmatrix}$$

$$\begin{bmatrix} (X_{1(1)1}, Y_{1[1]1}) & (X_{1(2)1}, Y_{1[2]1}) & \boxed{(X_{1(3)1}, Y_{1[3]1})} & (X_{1(4)1}, Y_{1[4]1}) & (X_{1(5)1}, Y_{1[5]1}) \\ (X_{2(1)1}, Y_{2[1]1}) & (X_{2(2)1}, Y_{2[2]1}) & \boxed{(X_{2(3)1}, Y_{2[3]1})} & (X_{2(4)1}, Y_{2[4]1}) & (X_{2(5)1}, Y_{2[5]1}) \\ (X_{3(1)1}, Y_{3[1]1}) & (X_{3(2)1}, Y_{3[2]1}) & \boxed{(X_{3(3)1}, Y_{3[3]1})} & (X_{3(4)1}, Y_{3[4]1}) & (X_{3(5)1}, Y_{3[5]1}) \\ (X_{4(1)1}, Y_{4[1]1}) & (X_{4(2)1}, Y_{4[2]1}) & \boxed{(X_{4(3)1}, Y_{4[3]1})} & (X_{4(4)1}, Y_{4[4]1}) & (X_{4(5)1}, Y_{4[5]1}) \\ (X_{5(1)1}, Y_{5[1]1}) & (X_{5(2)1}, Y_{5[2]1}) & \boxed{(X_{5(3)1}, Y_{5[3]1})} & (X_{5(4)1}, Y_{5[4]1}) & (X_{5(5)1}, Y_{5[5]1}) \end{bmatrix}$$

The mean estimators of MRSS are given for even and odd set sizes in Equations 2.16-2.19, respectively.

$$\bar{x}_{MRSS}^E = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{m/2} X_{i(m/2)j} + \sum_{j=1}^r \sum_{i=(m/2)+1}^m X_{i((m+2)/2)j} \right] \quad (2.16)$$

$$\bar{y}_{MRSS}^E = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{m/2} Y_{i[m/2]j} + \sum_{j=1}^r \sum_{i=(m/2)+1}^m Y_{i[(m+2)/2]j} \right] \quad (2.17)$$

$$\bar{x}_{MRSS}^O = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^m X_{i((m+1)/2)j} \right] \quad (2.18)$$

$$\bar{y}_{MRSS}^O = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^m Y_{i[(m+1)/2]j} \right] \quad (2.19)$$

2.2.4 Double Ranked Set Sampling

Al-Saleh & Al-Kadiri (2000) introduced double ranked set sampling (DRSS) method to estimate the population mean. A gain in efficiency is obtained using DRSS without increasing the set size m . Hence, DRSS provides more representative samples than SRS and RSS. The DRSS can be summarized as follows:

Step 1: Select m^3 units from the target population and divide them randomly into m subsets each of size m^2 .

Step 2: The procedure of ranked set sampling (RSS) is applied to m subsets separately.

Step 3: On each of the subsets obtain the m sets of ranked set samples of each size m .

Step 4: The RSS procedure is reapplied. The unit with rank i in the i th set is selected for actual measurement where $i=1,2,\dots,m$.

Step 1 through 4 are called a cycle. The cycle can be repeated r times to get $N=mr$ units.

Let $(X_{i(i)j})^k$ be the i th ranked unit (given in subscript parenthesis) in the i th set, j th cycle for the k th subset ($i=1,2,\dots,m$, $j=1,2,\dots,r$ and $k=1,2,\dots,m$). An example of the DRSS method for $r=1$ and $m=3$ is illustrated below. First 27 units are selected, then these units are divided into 3 subsets each of size 9.

$$\begin{array}{ccc} \begin{bmatrix} (X_{1(1)1})^1 & (X_{1(2)1})^1 & (X_{1(3)1})^1 \\ (X_{2(1)1})^1 & (X_{2(2)1})^1 & (X_{2(3)1})^1 \\ (X_{3(1)1})^1 & (X_{3(2)1})^1 & (X_{3(3)1})^1 \end{bmatrix} & \begin{bmatrix} (X_{1(1)1})^2 & (X_{1(2)1})^2 & (X_{1(3)1})^2 \\ (X_{2(1)1})^2 & (X_{2(2)1})^2 & (X_{2(3)1})^2 \\ (X_{3(1)1})^2 & (X_{3(2)1})^2 & (X_{3(3)1})^2 \end{bmatrix} & \begin{bmatrix} (X_{1(1)1})^3 & (X_{1(2)1})^3 & (X_{1(3)1})^3 \\ (X_{2(1)1})^3 & (X_{2(2)1})^3 & (X_{2(3)1})^3 \\ (X_{3(1)1})^3 & (X_{3(2)1})^3 & (X_{3(3)1})^3 \end{bmatrix} \\ \Downarrow & \Downarrow & \Downarrow \\ S_{11} = \{(X_{1(1)1})^1, (X_{2(2)1})^1, (X_{3(3)1})^1\} & S_{21} = \{(X_{1(1)1})^2, (X_{2(2)1})^2, (X_{3(3)1})^2\} & S_{31} = \{(X_{1(1)1})^3, (X_{2(2)1})^3, (X_{3(3)1})^3\} \end{array}$$

Let s_{kj} be the k th subset for the j th cycle and $W_{(1)1}=\min(s_{11})$, $W_{(2)1}=\text{med}(s_{21})$ and $W_{(3)1}=\max(s_{31})$. For $m=3$ the DRSS units are $\{(W_{(1)1}, Y_{[1]1}), (W_{(2)1}, Y_{[2]1}), (W_{(3)1}, Y_{[3]1})\}$. The mean estimators of DRSS are given below:

$$\bar{w}_{DRSS} = \frac{1}{mr} \sum_{j=1}^r \sum_{i=1}^m W_{(i)j} \quad (2.20)$$

$$\bar{y}_{DRSS} = \frac{1}{mr} \sum_{j=1}^r \sum_{i=1}^m Y_{[i]j} \quad (2.21)$$

2.2.5 L Ranked Set Sampling

Al-Nasser (2007) suggested a robust ranked set sampling (LRSS) method to estimate the population mean. The proposed method is a generalization for many types of ranked set sampling such as RSS, Median RSS and Percentile RSS methods which had been introduced in the literature to estimate the population mean. The LRSS method can be summarized as follows:

- Step 1: Select m random samples, each of size m from the population.
- Step 2: Rank the units within each sample with respect to the variable of interest by visual inspection or any inexpensive method.
- Step 3: Select the LRSS coefficient, $k=[mp]$, where $0 \leq p < 0.5$ and $[x]$ is the largest integer value less than or equal to x .
- Step 4: For each of the first $k+1$ ranked sample, select the unit with rank $k+1$, for each of the last $k+1$ ranked sample, select the unit with rank $m-k$.
- Step 5: Select the unit with rank i in the i th ranked sample for $i=k+2, \dots, m-k-1$.

Step 1 through 5 are called a cycle. The cycle can be repeated r times to get $N=mr$ units.

The LRSS for $r=1$, $m=5$ and $k=1,2$ can be illustrated as below:

$$\begin{array}{c}
 \mathbf{m=5, k=1} \\
 \left[\begin{array}{ccccc}
 (X_{1(1)1}, Y_{1[1]1}) & \boxed{(X_{1(2)1}, Y_{1[2]1})} & (X_{1(3)1}, Y_{1[3]1}) & (X_{1(4)1}, Y_{1[4]1}) & (X_{1(5)1}, Y_{1[5]1}) \\
 (X_{2(1)1}, Y_{2[1]1}) & \boxed{(X_{2(2)1}, Y_{2[2]1})} & (X_{2(3)1}, Y_{2[3]1}) & (X_{2(4)1}, Y_{2[4]1}) & (X_{2(5)1}, Y_{2[5]1}) \\
 (X_{3(1)1}, Y_{3[1]1}) & (X_{3(2)1}, Y_{3[2]1}) & \boxed{(X_{3(3)1}, Y_{3[3]1})} & (X_{3(4)1}, Y_{3[4]1}) & (X_{3(5)1}, Y_{3[5]1}) \\
 (X_{4(1)1}, Y_{4[1]1}) & (X_{4(2)1}, Y_{4[2]1}) & (X_{4(3)1}, Y_{4[3]1}) & \boxed{(X_{4(4)1}, Y_{4[4]1})} & (X_{4(5)1}, Y_{4[5]1}) \\
 (X_{5(1)1}, Y_{5[1]1}) & (X_{5(2)1}, Y_{5[2]1}) & (X_{5(3)1}, Y_{5[3]1}) & \boxed{(X_{5(4)1}, Y_{5[4]1})} & (X_{5(5)1}, Y_{5[5]1})
 \end{array} \right]
 \end{array}$$

$$\begin{array}{c}
 \mathbf{m=5, k=2} \\
 \left[\begin{array}{ccccc}
 (X_{1(1)1}, Y_{1[1]1}) & (X_{1(2)1}, Y_{1[2]1}) & \boxed{(X_{1(3)1}, Y_{1[3]1})} & (X_{1(4)1}, Y_{1[4]1}) & (X_{1(5)1}, Y_{1[5]1}) \\
 (X_{2(1)1}, Y_{2[1]1}) & (X_{2(2)1}, Y_{2[2]1}) & \boxed{(X_{2(3)1}, Y_{2[3]1})} & (X_{2(4)1}, Y_{2[4]1}) & (X_{2(5)1}, Y_{2[5]1}) \\
 (X_{3(1)1}, Y_{3[1]1}) & (X_{3(2)1}, Y_{3[2]1}) & \boxed{(X_{3(3)1}, Y_{3[3]1})} & (X_{3(4)1}, Y_{3[4]1}) & (X_{3(5)1}, Y_{3[5]1}) \\
 (X_{4(1)1}, Y_{4[1]1}) & (X_{4(2)1}, Y_{4[2]1}) & \boxed{(X_{4(3)1}, Y_{4[3]1})} & (X_{4(4)1}, Y_{4[4]1}) & (X_{4(5)1}, Y_{4[5]1}) \\
 (X_{5(1)1}, Y_{5[1]1}) & (X_{5(2)1}, Y_{5[2]1}) & \boxed{(X_{5(3)1}, Y_{5[3]1})} & (X_{5(4)1}, Y_{5[4]1}) & (X_{5(5)1}, Y_{5[5]1})
 \end{array} \right]
 \end{array}$$

The mean estimators of LRSS are given in Equations 2.22 and 2.23.

$$\bar{x}_{LRSS} = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{k+1} X_{i(k+1)j} + \sum_{j=1}^r \sum_{i=k+2}^{m-k-1} X_{i(i)j} + \sum_{j=1}^r \sum_{i=m-k}^m X_{i(m-k)j} \right] \quad (2.22)$$

$$\bar{y}_{LRSS} = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^{k+1} Y_{i[k+1]j} + \sum_{j=1}^r \sum_{i=k+2}^{m-k-1} Y_{i[i]j} + \sum_{j=1}^r \sum_{i=m-k}^m Y_{i[m-k]j} \right] \quad (2.23)$$

Note that, if $m \geq 2$ and $k=0$, the LRSS is the same with classical RSS. If $m \geq 3$ and k is the integer part of $((m-1)/2)$, the LRSS is the same with MRSS.

2.2.6 Truncation Based Ranked Set Sampling

Al-Omari & Raqab (2013) proposed truncation based ranked set sampling (TBRSS) method for estimating the population mean and median. The TBRSS can be summarized as follows:

Step 1: Select m random samples, each of size m from the population.

Step 2: Rank the units within each sample with respect to the variable of interest by visual inspection or any inexpensive method.

Step 3: Select a coefficient, $k=[mp]$, where $0 \leq p < 0.5$ and $[x]$ is the integer part of x .

Step 4: From the first k samples, select the smallest ranked unit and from the last k samples select the largest ranked unit. From the remaining $m-2k$ samples, select the i th ranked unit of the i th sample, for $k+1 \leq i \leq m-k$.

Step 1 through 4 are called a cycle. The cycle can be repeated r times to get $N=mr$ units.

The TBRSS for $r=1$, $m=5$ and $k=0,1,2$ can be illustrated as below:

$m=5, k=0,1$				
$(X_{1(1)1}, Y_{1[1]1})$	$(X_{1(2)1}, Y_{1[2]1})$	$(X_{1(3)1}, Y_{1[3]1})$	$(X_{1(4)1}, Y_{1[4]1})$	$(X_{1(5)1}, Y_{1[5]1})$
$(X_{2(1)1}, Y_{2[1]1})$	$(X_{2(2)1}, Y_{2[2]1})$	$(X_{2(3)1}, Y_{2[3]1})$	$(X_{2(4)1}, Y_{2[4]1})$	$(X_{2(5)1}, Y_{2[5]1})$
$(X_{3(1)1}, Y_{3[1]1})$	$(X_{3(2)1}, Y_{3[2]1})$	$(X_{3(3)1}, Y_{3[3]1})$	$(X_{3(4)1}, Y_{3[4]1})$	$(X_{3(5)1}, Y_{3[5]1})$
$(X_{4(1)1}, Y_{4[1]1})$	$(X_{4(2)1}, Y_{4[2]1})$	$(X_{4(3)1}, Y_{4[3]1})$	$(X_{4(4)1}, Y_{4[4]1})$	$(X_{4(5)1}, Y_{4[5]1})$
$(X_{5(1)1}, Y_{5[1]1})$	$(X_{5(2)1}, Y_{5[2]1})$	$(X_{5(3)1}, Y_{5[3]1})$	$(X_{5(4)1}, Y_{5[4]1})$	$(X_{5(5)1}, Y_{5[5]1})$

$m=5, k=2$				
$(X_{1(1)1}, Y_{1[1]1})$	$(X_{1(2)1}, Y_{1[2]1})$	$(X_{1(3)1}, Y_{1[3]1})$	$(X_{1(4)1}, Y_{1[4]1})$	$(X_{1(5)1}, Y_{1[5]1})$
$(X_{2(1)1}, Y_{2[1]1})$	$(X_{2(2)1}, Y_{2[2]1})$	$(X_{2(3)1}, Y_{2[3]1})$	$(X_{2(4)1}, Y_{2[4]1})$	$(X_{2(5)1}, Y_{2[5]1})$
$(X_{3(1)1}, Y_{3[1]1})$	$(X_{3(2)1}, Y_{3[2]1})$	$(X_{3(3)1}, Y_{3[3]1})$	$(X_{3(4)1}, Y_{3[4]1})$	$(X_{3(5)1}, Y_{3[5]1})$
$(X_{4(1)1}, Y_{4[1]1})$	$(X_{4(2)1}, Y_{4[2]1})$	$(X_{4(3)1}, Y_{4[3]1})$	$(X_{4(4)1}, Y_{4[4]1})$	$(X_{4(5)1}, Y_{4[5]1})$
$(X_{5(1)1}, Y_{5[1]1})$	$(X_{5(2)1}, Y_{5[2]1})$	$(X_{5(3)1}, Y_{5[3]1})$	$(X_{5(4)1}, Y_{5[4]1})$	$(X_{5(5)1}, Y_{5[5]1})$

The mean estimators of the TBRSS method are given in Equations 2.24 and 2.25.

$$\bar{x}_{TBRSS} = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^k X_{i(1)j} + \sum_{j=1}^r \sum_{i=k+1}^{m-k} X_{i(i)j} + \sum_{j=1}^r \sum_{i=m-k+1}^m X_{i(m)j} \right] \quad (2.24)$$

$$\bar{y}_{TBRSS} = \frac{1}{mr} \left[\sum_{j=1}^r \sum_{i=1}^k Y_{i[1]j} + \sum_{j=1}^r \sum_{i=k+1}^{m-k} Y_{i[i]j} + \sum_{j=1}^r \sum_{i=m-k+1}^m Y_{i[m]j} \right] \quad (2.25)$$

Note that, if $m \geq 2$ and k equals 0 or 1, the TBRSS is the same with classical RSS. If $m=2k$, the TBRSS is the same with ERSS.

2.3 General Form of Regression Estimators of Modified RSS Methods

Since 1952, mean and regression estimators of RSS have been investigated by many scientists and these estimators have continued to be used in modified ranked set sampling methods. At the outset, Yu & Lam (1997) suggested the regression estimator of the population mean of the variable of interest Y using the RSS. The details are given for Ranked set sampling in Section 2.1.2.

In this section, a general formulation of regression estimators is employed for modified RSS methods based on Equation 2.3. The modified RSS method used is shown by $Mod.RSS$ and (X_ℓ, Y_ℓ) is the ℓ th unit for created sample with $Mod.RSS$. The general form of the regression model is given below:

$$Y_\ell = \beta_0 + \beta_1 X_\ell + \varepsilon_\ell, \quad \ell = 1, 2, \dots, n \quad (2.26)$$

The general form of regression estimators of modified RSS methods is

$$\bar{y}_{Mod.RSSreg} = \bar{y}_{Mod.RSS} + \hat{\beta}_1(\mu_x - \bar{x}_{Mod.RSS}) \quad (2.27)$$

$$\hat{\beta}_1 = \frac{\sum_{\ell=1}^n (X_\ell - \bar{x}_{Mod.RSS})(Y_\ell - \bar{y}_{Mod.RSS})}{\sum_{\ell=1}^n (X_\ell - \bar{x}_{Mod.RSS})^2} \quad (2.28)$$

$$\hat{\beta}_0 = \bar{y}_{Mod.RSS} - \hat{\beta}_1 \bar{x}_{Mod.RSS} \quad (2.29)$$

where $\bar{x}_{Mod.RSS}$ and $\bar{y}_{Mod.RSS}$ are the mean estimators of X and Y of the created sample, respectively.

CHAPTER THREE

SIMULATION STUDY

In this chapter, the simulation studies for normal and outlier cases are presented. The mean estimator, regression estimators and mean square error of regression model are taken into consideration. The performances of the estimators are compared based on different sampling methods such as SRS, RSS, PRSS, ERSS, MRSS, DRSS, LRSS₁ ($k=1$), LRSS₂ ($k=2$), TBRSS₁ ($k=1$) and TBRSS₂ ($k=2$) methods. In order to compare the methods, the data sets from $(X,Y) \sim \text{BVN}(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$ with different levels of correlation coefficient (ρ), set (m) and cycle sizes (r).

3.1 Normal Case

The simulation study is designed with R Project concluding the processes given below:

1. Generate 5,000 observations from $(X,Y) \sim \text{BVN}(0,0,1,1,\rho)$ and $(X,Y) \sim \text{BVN}(3,7,1,4,\rho)$ where $\rho=0.50, 0.75, 0.95$.
2. The performances of the estimators are investigated for the cycle size $r=2, 3, 4, 5, 6$ and the set size $m=4, 5, 6$.
3. Run the simulation $R=10,000$ times for SRS, RSS, PRSS, ERSS, MRSS, DRSS, LRSS₁, LRSS₂, TBRSS₁ and TBRSS₂.
4. For $\hat{\theta}_j$ is the estimated value of j th number of repetitions, θ is the population parameter. The formulations of bias, the mean square error and the relative efficiency where Mod.RSS represents the each of modified RSS methods are given below:

$$\text{Bias}(\hat{\theta}_j, \theta) = \frac{1}{R}(\hat{\theta}_j - \theta) \quad (3.1)$$

$$\text{MSE}(\hat{\theta}_j, \theta) = \frac{1}{R} \sum_{j=1}^R (\hat{\theta}_j - \theta)^2 \quad (3.2)$$

$$RE_1 = RE(\hat{\theta}_{SRS}, \hat{\theta}_{RSS}) = \frac{MSE(\hat{\theta}_{SRS})}{MSE(\hat{\theta}_{RSS})} \quad (3.3)$$

$$RE_2 = RE(\hat{\theta}_{SRS}, \hat{\theta}_{Mod.RSS}) = \frac{MSE(\hat{\theta}_{SRS})}{MSE(\hat{\theta}_{Mod.RSS})} \quad (3.4)$$

$$RE_3 = RE(\hat{\theta}_{RSS}, \hat{\theta}_{Mod.RSS}) = \frac{MSE(\hat{\theta}_{RSS})}{MSE(\hat{\theta}_{Mod.RSS})} \quad (3.5)$$

The RE_1 values which are greater than 1 indicate that the estimator of RSS method is more efficient than the estimator of SRS method. The RE_2 values that are greater than 1 indicate that the estimator of related sampling method is more efficient than the estimator of SRS method. The RE_3 values that are greater than 1 indicate that the estimator of related sampling method is more efficient than the estimator of RSS method.

The Monte Carlo simulation results of relative efficiencies and bias values of the estimators for $(X,Y) \sim BVN(0,0,1,1,\rho)$ are given in Table 3.1-Table 3.6. The simulation results of $(X,Y) \sim BVN(3,7,1,4,\rho)$ are given in Appendix.

Table 3.1 RE_1 values of the estimators under normal case

ρ	r	m	$\bar{y}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\sigma}^2$
0.50	2	4	1.119	1.079	1.200	0.985
		5	1.224	1.113	1.159	1.003
		6	1.226	1.086	1.170	0.993
	3	4	1.207	1.105	1.145	1.007
		5	1.186	1.045	1.058	0.996
		6	1.256	1.064	1.082	0.997
	4	4	1.189	1.033	1.060	1.001
		5	1.141	0.978	1.048	1.002
		6	1.188	0.993	1.043	1.007
	5	4	1.192	1.036	1.069	0.997
		5	1.171	1.023	1.081	1.006
		6	1.207	1.023	1.050	1.000
	6	4	1.160	1.007	1.066	1.005
		5	1.192	1.042	1.065	0.999
		6	1.227	1.027	1.015	1.002
0.75	2	4	1.435	1.110	1.210	0.988
		5	1.585	1.091	1.180	0.996
		6	1.683	1.135	1.147	0.994
	3	4	1.537	1.113	1.086	1.009
		5	1.574	1.064	1.069	0.993
		6	1.674	1.048	1.111	0.999
	4	4	1.449	1.042	1.113	1.007
		5	1.532	1.034	1.034	0.990
		6	1.657	1.056	1.062	0.995
	5	4	1.459	0.981	1.037	1.003
		5	1.541	1.029	1.055	0.997
		6	1.636	1.030	1.047	0.994
	6	4	1.510	1.048	1.050	0.995
		5	1.590	1.029	1.025	0.993
		6	1.657	1.016	1.067	0.999
0.95	2	4	2.028	1.102	1.194	0.993
		5	2.386	1.079	1.147	1.001
		6	2.658	1.133	1.123	0.997
	3	4	2.117	1.082	1.075	1.011
		5	2.355	1.051	1.077	0.996
		6	2.637	1.049	1.102	1.005
	4	4	2.035	1.058	1.113	1.009
		5	2.277	1.038	1.030	0.993
		6	2.681	1.049	1.040	1.000
	5	4	2.051	1.012	1.058	1.003
		5	2.315	1.016	1.053	1.000
		6	2.594	1.007	1.049	0.993
	6	4	2.068	1.022	1.003	1.004
		5	2.351	1.020	1.030	1.006
		6	2.708	1.043	1.019	0.998

According to Table 3.1, each of the relative efficiency values are greater than 1 for $\bar{y}$ and $\hat{\beta}_1$. These values indicate that the estimators of RSS method are more efficient than the estimators of SRS method. Additionally, the relative efficiencies for the estimators $\hat{\beta}_0$ and the mean square error of the regression model are almost greater than 1.

Table 3.2 RE_2 and RE_3 values for the mean estimator under normal case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.086	1.152	1.157	1.191	1.146	*	1.170	1.113	0.971	1.030	1.034	1.065	1.024	*	1.046	0.995
		5	1.173	1.186	1.209	1.240	1.246	1.240	1.188	1.173	0.958	0.969	0.988	1.013	1.018	1.013	0.971	0.958
		6	1.241	1.209	1.268	1.307	1.255	1.266	1.277	1.216	1.012	0.986	1.034	1.066	1.023	1.032	1.041	0.992
	3	4	1.151	1.176	1.232	1.250	1.191	*	1.171	1.173	0.953	0.974	1.020	1.035	0.986	*	0.970	0.971
		5	1.167	1.167	1.195	1.252	1.221	1.212	1.196	1.182	0.984	0.984	1.008	1.056	1.030	1.022	1.009	0.997
		6	1.206	1.213	1.274	1.280	1.291	1.235	1.244	1.227	0.960	0.966	1.014	1.020	1.028	0.983	0.991	0.977
	4	4	1.147	1.201	1.208	1.284	1.235	*	1.183	1.178	0.964	1.010	1.016	1.080	1.038	*	0.995	0.991
		5	1.135	1.131	1.171	1.241	1.171	1.198	1.164	1.158	0.994	0.990	1.026	1.087	1.026	1.049	1.020	1.015
		6	1.179	1.204	1.223	1.244	1.208	1.234	1.232	1.209	0.992	1.013	1.030	1.047	1.017	1.038	1.037	1.017
	5	4	1.157	1.138	1.200	1.223	1.214	*	1.178	1.183	0.970	0.954	1.006	1.026	1.018	*	0.988	0.993
		5	1.133	1.143	1.166	1.214	1.170	1.213	1.198	1.142	0.967	0.976	0.995	1.036	0.999	1.036	1.023	0.975
		6	1.149	1.180	1.243	1.258	1.254	1.208	1.207	1.184	0.952	0.978	1.030	1.042	1.039	1.001	0.999	0.981
	6	4	1.142	1.142	1.191	1.205	1.191	*	1.186	1.152	0.985	0.985	1.027	1.039	1.027	*	1.023	0.993
		5	1.195	1.169	1.206	1.290	1.234	1.228	1.166	1.173	1.003	0.981	1.011	1.082	1.035	1.030	0.978	0.984
		6	1.194	1.156	1.238	1.281	1.246	1.226	1.230	1.216	0.973	0.942	1.009	1.044	1.016	0.999	1.003	0.991
0.75	2	4	1.266	1.401	1.493	1.674	1.515	*	1.481	1.364	0.882	0.976	1.040	1.167	1.055	*	1.032	0.950
		5	1.434	1.518	1.701	1.777	1.666	1.683	1.555	1.492	0.905	0.958	1.073	1.122	1.051	1.062	0.981	0.941
		6	1.543	1.526	1.782	1.912	1.736	1.779	1.705	1.595	0.917	0.907	1.059	1.136	1.032	1.057	1.013	0.948
	3	4	1.334	1.415	1.577	1.703	1.581	*	1.476	1.428	0.868	0.921	1.026	1.108	1.029	*	0.960	0.929
		5	1.472	1.501	1.648	1.796	1.619	1.629	1.578	1.499	0.935	0.954	1.047	1.142	1.029	1.035	1.003	0.953
		6	1.522	1.536	1.765	1.939	1.805	1.784	1.662	1.617	0.909	0.918	1.054	1.158	1.079	1.066	0.993	0.966
	4	4	1.267	1.365	1.550	1.659	1.534	*	1.447	1.380	0.874	0.941	1.069	1.145	1.058	*	0.999	0.952
		5	1.439	1.502	1.688	1.794	1.671	1.659	1.542	1.493	0.939	0.980	1.102	1.171	1.091	1.083	1.006	0.974
		6	1.502	1.513	1.739	1.875	1.688	1.721	1.623	1.597	0.906	0.913	1.049	1.131	1.019	1.038	0.979	0.963
	5	4	1.307	1.411	1.604	1.717	1.581	*	1.478	1.393	0.896	0.967	1.099	1.177	1.084	*	1.014	0.955
		5	1.448	1.438	1.674	1.756	1.623	1.647	1.549	1.473	0.940	0.933	1.086	1.140	1.053	1.069	1.005	0.956
		6	1.471	1.480	1.774	1.826	1.704	1.701	1.649	1.550	0.899	0.905	1.084	1.116	1.042	1.040	1.008	0.947
	6	4	1.284	1.384	1.602	1.690	1.589	*	1.487	1.405	0.850	0.916	1.061	1.119	1.052	*	0.985	0.930
		5	1.467	1.450	1.743	1.760	1.650	1.713	1.558	1.488	0.923	0.912	1.096	1.107	1.038	1.077	0.980	0.936
		6	1.486	1.467	1.842	1.870	1.747	1.818	1.664	1.554	0.897	0.885	1.112	1.128	1.055	1.097	1.004	0.938
0.95	2	4	1.522	1.804	2.307	2.866	2.247	*	2.033	1.787	0.751	0.889	1.138	1.413	1.108	*	1.002	0.881
		5	2.003	2.170	2.863	3.397	2.715	2.856	2.392	2.087	0.839	0.909	1.200	1.424	1.138	1.197	1.002	0.875
		6	2.109	2.126	3.180	3.865	3.037	3.170	2.721	2.366	0.793	0.800	1.196	1.454	1.143	1.192	1.024	0.890
	3	4	1.597	1.867	2.357	2.826	2.389	*	2.036	1.893	0.755	0.882	1.114	1.335	1.129	*	0.962	0.894
		5	1.984	2.082	2.773	3.340	2.583	2.707	2.336	2.130	0.843	0.884	1.178	1.418	1.097	1.150	0.992	0.904
		6	2.120	2.120	3.189	3.881	3.107	3.182	2.680	2.406	0.804	0.804	1.209	1.472	1.178	1.207	1.016	0.912
	4	4	1.516	1.765	2.317	2.718	2.260	*	2.012	1.801	0.745	0.867	1.139	1.336	1.111	*	0.989	0.885
		5	1.993	2.105	2.811	3.371	2.720	2.751	2.324	2.079	0.875	0.925	1.235	1.481	1.195	1.208	1.021	0.913
		6	2.142	2.116	3.116	3.831	2.943	3.083	2.636	2.426	0.799	0.789	1.162	1.429	1.098	1.150	0.983	0.905
	5	4	1.601	1.870	2.413	2.828	2.341	*	2.039	1.844	0.781	0.912	1.177	1.379	1.142	*	0.994	0.899
		5	1.965	2.033	2.778	3.222	2.573	2.712	2.273	2.082	0.849	0.878	1.200	1.392	1.111	1.171	0.982	0.899
		6	2.080	2.093	3.144	3.802	2.955	3.076	2.672	2.391	0.802	0.807	1.212	1.466	1.139	1.186	1.030	0.922
	6	4	1.601	1.878	2.462	2.841	2.373	*	2.101	1.848	0.774	0.908	1.191	1.374	1.147	*	1.016	0.894
		5	1.931	2.040	2.819	3.272	2.692	2.812	2.329	2.124	0.821	0.868	1.199	1.392	1.145	1.196	0.991	0.903
		6	2.103	2.088	3.123	3.774	2.914	3.149	2.643	2.353	0.777	0.771	1.153	1.394	1.076	1.163	0.976	0.869

* k can be up to 1 when $m=4$

Table 3.2 presents the relative efficiencies of the mean estimators based on SRS, RSS, PRSS, ERSS, MRSS, DRSS, LRSS₁, LRSS₂, TBRSS₁ and TBRSS₂ methods. The mean estimators of all the modified RSS methods are more efficient than the mean estimator of SRS method for all cases. The efficiency values increase when the set size or the correlation level increases. In addition to this, the mean estimators of MRSS and DRSS methods are approximately three times better than the mean estimator of SRS method when $\rho=0.95$. The mean estimator of DRSS method is always more efficient than the mean estimator of RSS method. Furthermore, the relative efficiencies of the mean estimators of MRSS, LRSS₁ and LRSS₂ methods are mostly greater than 1 when compared to the mean estimator of RSS method. Moreover, the relative efficiency values decrease when k increases for TBRSS method.

Table 3.3 RE_2 and RE_3 values for $\hat{\beta}_0$ under normal case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.062	1.167	1.007	1.125	1.002	*	1.117	1.107	0.985	1.082	0.934	1.043	0.929	*	1.036	1.026
		5	1.098	1.123	0.996	1.111	1.062	1.027	1.086	1.139	0.986	1.009	0.895	0.998	0.954	0.923	0.976	1.023
		6	1.125	1.126	1.034	1.127	1.083	1.045	1.127	1.119	1.036	1.037	0.952	1.038	0.997	0.962	1.037	1.030
	3	4	1.101	1.116	1.047	1.125	1.037	*	1.077	1.103	0.997	1.010	0.948	1.018	0.939	*	0.975	0.998
		5	1.046	1.056	0.972	1.067	1.044	0.989	1.051	1.070	1.001	1.010	0.930	1.021	0.999	0.946	1.006	1.024
		6	1.055	1.080	1.037	1.056	1.060	0.993	1.062	1.063	0.992	1.014	0.974	0.992	0.996	0.933	0.998	0.999
	4	4	1.045	1.076	1.021	1.082	1.034	*	1.031	1.073	1.011	1.041	0.987	1.047	1.001	*	0.998	1.038
		5	0.999	0.987	0.959	1.024	0.958	0.959	1.008	1.005	1.021	1.009	0.980	1.048	0.979	0.981	1.031	1.028
		6	1.023	1.043	0.979	1.009	0.995	0.984	1.023	1.031	1.030	1.051	0.986	1.016	1.002	0.991	1.030	1.039
	5	4	1.037	1.029	0.992	1.044	1.021	*	1.045	1.054	1.000	0.993	0.957	1.007	0.985	*	1.008	1.017
		5	1.013	1.009	0.967	1.013	1.005	0.992	1.034	1.007	0.991	0.987	0.946	0.990	0.983	0.970	1.011	0.985
		6	1.015	1.042	1.007	1.033	1.038	0.991	1.022	1.024	0.992	1.019	0.984	1.009	1.014	0.969	0.999	1.001
	6	4	1.020	1.013	0.997	1.010	0.991	*	1.018	1.018	1.013	1.006	0.990	1.003	0.984	*	1.011	1.011
		5	1.049	1.046	0.997	1.081	1.037	1.020	1.017	1.033	1.007	1.003	0.957	1.037	0.995	0.979	0.976	0.991
		6	1.033	0.999	1.005	1.034	1.017	0.987	1.027	1.022	1.006	0.972	0.978	1.006	0.989	0.960	0.999	0.995
0.75	2	4	1.071	1.165	0.998	1.140	1.022	*	1.128	1.130	0.964	1.049	0.899	1.027	0.921	*	1.016	1.018
		5	1.050	1.134	1.013	1.110	1.051	1.006	1.073	1.144	0.962	1.039	0.928	1.017	0.963	0.922	0.984	1.048
		6	1.108	1.119	1.053	1.137	1.069	1.045	1.120	1.119	0.976	0.986	0.927	1.002	0.942	0.921	0.987	0.986
	3	4	1.098	1.086	1.021	1.126	1.042	*	1.080	1.091	0.986	0.976	0.917	1.011	0.936	*	0.970	0.980
		5	1.045	1.077	0.973	1.071	1.048	0.979	1.041	1.058	0.981	1.012	0.914	1.006	0.985	0.920	0.978	0.994
		6	1.046	1.071	0.996	1.075	1.068	1.009	1.053	1.078	0.997	1.021	0.950	1.026	1.018	0.963	1.005	1.028
	4	4	1.022	1.029	1.024	1.051	1.020	*	1.038	1.049	0.981	0.988	0.983	1.009	0.979	*	0.996	1.007
		5	1.036	1.065	1.001	1.054	1.029	1.005	1.061	1.040	1.003	1.031	0.968	1.020	0.996	0.972	1.026	1.006
		6	1.023	1.062	1.023	1.044	1.031	1.013	1.043	1.052	0.969	1.006	0.968	0.989	0.977	0.959	0.988	0.997
	5	4	1.027	1.006	1.016	1.046	1.001	*	1.040	1.022	1.047	1.025	1.036	1.066	1.021	*	1.060	1.042
		5	1.031	1.019	1.001	1.041	1.032	1.003	1.024	1.035	1.002	0.990	0.973	1.012	1.003	0.975	0.995	1.006
		6	1.007	1.007	1.013	0.992	0.980	0.962	1.012	1.018	0.978	0.977	0.983	0.963	0.951	0.934	0.983	0.988
	6	4	1.025	1.054	1.019	1.024	1.023	*	1.038	1.049	0.978	1.006	0.973	0.977	0.977	*	0.991	1.001
		5	1.003	0.989	1.027	1.004	0.972	0.993	0.999	0.998	0.975	0.961	0.998	0.976	0.945	0.965	0.971	0.970
		6	1.017	1.016	1.028	1.008	1.039	1.001	0.995	1.000	1.000	1.000	1.012	0.992	1.022	0.985	0.979	0.984
0.95	2	4	1.059	1.138	1.008	1.117	1.013	*	1.104	1.134	0.961	1.032	0.914	1.013	0.919	*	1.001	1.029
		5	1.069	1.123	0.975	1.126	1.048	1.038	1.076	1.119	0.990	1.041	0.903	1.044	0.971	0.962	0.997	1.037
		6	1.086	1.096	1.076	1.124	1.066	1.022	1.110	1.112	0.959	0.968	0.950	0.992	0.941	0.902	0.980	0.982
	3	4	1.066	1.096	1.048	1.141	1.030	*	1.081	1.093	0.986	1.013	0.969	1.055	0.952	*	0.999	1.010
		5	1.062	1.066	0.992	1.063	1.026	0.980	1.032	1.055	1.011	1.015	0.945	1.011	0.976	0.932	0.982	1.004
		6	1.049	1.066	0.989	1.069	1.034	1.011	1.048	1.058	1.000	1.016	0.942	1.019	0.986	0.963	0.999	1.008
	4	4	1.025	1.038	1.015	1.064	1.009	*	1.045	1.058	0.969	0.982	0.960	1.006	0.954	*	0.988	1.000
		5	1.029	1.077	0.979	1.058	1.043	1.003	1.063	1.034	0.992	1.038	0.943	1.020	1.005	0.967	1.024	0.997
		6	1.058	1.056	1.034	1.055	1.034	1.045	1.042	1.086	1.008	1.006	0.986	1.006	0.985	0.996	0.993	1.035
	5	4	1.023	1.017	0.989	1.037	0.986	*	1.052	1.022	1.011	1.004	0.977	1.024	0.974	*	1.039	1.009
		5	1.032	1.022	0.992	1.018	1.012	1.006	1.015	1.046	1.015	1.006	0.976	1.001	0.996	0.990	0.998	1.030
		6	1.018	0.988	0.992	1.029	1.016	1.004	1.027	1.029	1.011	0.980	0.985	1.021	1.008	0.997	1.019	1.021
	6	4	1.007	1.042	0.993	1.052	0.984	*	1.021	1.047	0.986	1.020	0.972	1.030	0.963	*	1.000	1.025
		5	1.019	1.014	0.965	1.032	0.980	0.971	1.027	1.000	0.998	0.994	0.946	1.011	0.961	0.952	1.007	0.980
		6	1.041	1.012	0.985	1.018	1.011	0.990	1.014	1.028	0.998	0.971	0.945	0.976	0.970	0.949	0.973	0.986

* k can be up to 1 when $m=4$

Table 3.3 shows the relative efficiencies of the regression estimator $\hat{\beta}_0$. According to the regression estimator $\hat{\beta}_0$, the RE_2 and RE_3 values are mostly greater than 1 in all the modified RSS methods. This means that the estimators of the modified RSS methods are more efficient than the regression estimators of SRS and RSS methods.

Table 3.4 RE_2 and RE_3 values for $\hat{\beta}_1$ under normal case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.191	2.041	0.469	1.286	0.462	*	1.176	1.996	0.992	1.700	0.391	1.071	0.385	*	0.980	1.663
		5	1.189	1.794	0.276	1.221	0.522	0.276	1.160	1.773	1.026	1.547	0.238	1.053	0.450	0.238	1.000	1.529
		6	1.158	2.457	0.299	1.146	0.603	0.296	1.170	1.687	0.990	2.100	0.256	0.979	0.515	0.253	1.000	1.442
	3	4	1.101	1.792	0.469	1.157	0.475	*	1.108	1.798	0.962	1.565	0.409	1.010	0.415	*	0.968	1.570
		5	1.101	1.696	0.289	1.162	0.533	0.294	1.100	1.656	1.042	1.604	0.273	1.099	0.504	0.278	1.040	1.566
		6	1.095	2.271	0.294	1.120	0.608	0.301	1.113	1.630	1.012	2.100	0.272	1.035	0.562	0.279	1.029	1.507
	4	4	1.050	1.696	0.451	1.054	0.451	*	1.043	1.664	0.990	1.600	0.426	0.994	0.425	*	0.983	1.569
		5	1.046	1.569	0.291	1.074	0.523	0.279	1.062	1.559	0.998	1.496	0.277	1.024	0.499	0.266	1.013	1.487
		6	1.050	2.147	0.289	1.073	0.590	0.289	1.061	1.523	1.006	2.058	0.277	1.028	0.565	0.277	1.017	1.460
	5	4	1.052	1.667	0.452	1.074	0.456	*	1.058	1.633	0.984	1.558	0.422	1.004	0.427	*	0.990	1.527
		5	1.077	1.598	0.288	1.089	0.532	0.291	1.085	1.593	0.996	1.479	0.267	1.007	0.492	0.269	1.004	1.474
		6	1.020	2.219	0.288	1.072	0.574	0.290	1.070	1.478	0.971	2.113	0.274	1.020	0.547	0.276	1.019	1.407
	6	4	1.041	1.670	0.462	1.070	0.467	*	1.067	1.686	0.976	1.566	0.433	1.003	0.438	*	1.001	1.581
		5	1.036	1.570	0.293	1.047	0.517	0.288	1.074	1.588	0.973	1.474	0.275	0.983	0.486	0.270	1.008	1.490
		6	1.022	2.039	0.286	1.012	0.574	0.284	1.041	1.506	1.007	2.008	0.281	0.996	0.565	0.280	1.025	1.483
0.75	2	4	1.191	2.039	0.467	1.289	0.490	*	1.184	1.988	0.984	1.684	0.386	1.065	0.405	*	0.979	1.642
		5	1.169	1.785	0.286	1.244	0.526	0.278	1.166	1.815	0.991	1.513	0.242	1.055	0.446	0.236	0.988	1.538
		6	1.136	2.444	0.300	1.187	0.618	0.295	1.154	1.697	0.991	2.132	0.261	1.035	0.539	0.257	1.007	1.480
	3	4	1.107	1.809	0.457	1.138	0.464	*	1.102	1.778	1.020	1.666	0.421	1.048	0.427	*	1.015	1.637
		5	1.086	1.651	0.295	1.144	0.536	0.292	1.091	1.641	1.016	1.545	0.276	1.070	0.501	0.273	1.021	1.535
		6	1.094	2.301	0.295	1.160	0.618	0.303	1.139	1.664	0.984	2.070	0.266	1.044	0.556	0.272	1.025	1.498
	4	4	1.083	1.737	0.463	1.143	0.459	*	1.086	1.720	0.973	1.561	0.416	1.027	0.413	*	0.976	1.546
		5	1.052	1.571	0.284	1.082	0.513	0.286	1.039	1.595	1.017	1.519	0.274	1.046	0.496	0.277	1.005	1.542
		6	1.023	2.106	0.287	1.081	0.597	0.289	1.034	1.499	0.963	1.983	0.270	1.018	0.563	0.272	0.974	1.412
	5	4	1.062	1.696	0.451	1.072	0.454	*	1.059	1.689	1.024	1.636	0.435	1.034	0.438	*	1.022	1.629
		5	1.057	1.604	0.285	1.075	0.518	0.305	1.032	1.599	1.002	1.521	0.271	1.019	0.491	0.289	0.978	1.516
		6	1.053	2.158	0.295	1.047	0.581	0.292	1.046	1.499	1.006	2.061	0.282	1.000	0.555	0.279	0.999	1.432
	6	4	1.024	1.711	0.449	1.069	0.443	*	1.062	1.747	0.975	1.630	0.427	1.018	0.422	*	1.011	1.663
		5	1.053	1.632	0.281	1.077	0.507	0.286	1.055	1.613	1.027	1.592	0.274	1.051	0.495	0.279	1.030	1.573
		6	1.054	2.205	0.276	1.061	0.573	0.279	1.064	1.556	0.987	2.066	0.259	0.994	0.537	0.261	0.997	1.458
0.95	2	4	1.171	2.003	0.465	1.240	0.485	*	1.164	1.928	0.981	1.678	0.390	1.038	0.406	*	0.975	1.615
		5	1.153	1.752	0.285	1.211	0.525	0.293	1.132	1.777	1.005	1.527	0.248	1.056	0.458	0.255	0.987	1.549
		6	1.154	2.459	0.309	1.168	0.608	0.303	1.160	1.673	1.027	2.190	0.275	1.040	0.541	0.269	1.033	1.489
	3	4	1.087	1.813	0.471	1.126	0.479	*	1.098	1.780	1.011	1.687	0.438	1.048	0.446	*	1.022	1.656
		5	1.075	1.615	0.295	1.098	0.535	0.286	1.067	1.648	0.999	1.500	0.274	1.020	0.497	0.265	0.992	1.531
		6	1.086	2.296	0.313	1.134	0.598	0.310	1.114	1.643	0.985	2.083	0.284	1.029	0.543	0.281	1.011	1.491
	4	4	1.099	1.726	0.481	1.117	0.476	*	1.079	1.747	0.987	1.551	0.432	1.004	0.428	*	0.970	1.569
		5	1.063	1.591	0.288	1.049	0.513	0.286	1.033	1.584	1.032	1.545	0.280	1.019	0.498	0.278	1.003	1.538
		6	1.049	2.223	0.295	1.051	0.581	0.299	1.037	1.531	1.008	2.136	0.284	1.010	0.558	0.288	0.996	1.472
	5	4	1.052	1.681	0.461	1.066	0.453	*	1.044	1.688	0.994	1.589	0.435	1.008	0.428	*	0.986	1.596
		5	1.067	1.606	0.292	1.058	0.529	0.297	1.081	1.618	1.014	1.525	0.278	1.005	0.503	0.282	1.027	1.537
		6	1.026	2.140	0.296	1.045	0.578	0.288	1.028	1.507	0.978	2.041	0.282	0.996	0.552	0.274	0.980	1.437
	6	4	1.012	1.648	0.431	1.033	0.419	*	1.012	1.607	1.009	1.643	0.430	1.029	0.418	*	1.009	1.602
		5	1.033	1.608	0.279	1.074	0.510	0.282	1.065	1.582	1.004	1.561	0.271	1.043	0.496	0.274	1.034	1.536
		6	1.039	2.187	0.278	1.060	0.572	0.280	1.043	1.509	1.020	2.146	0.273	1.040	0.561	0.275	1.024	1.480

* k can be up to 1 when $m=4$

Table 3.4 indicates that the regression estimators $\hat{\beta}_1$ of all the modified RSS methods are more efficient than the regression estimator of SRS method except MRSS, LRSS₁ and LRSS₂ methods. Additionally, the regression estimators $\hat{\beta}_1$ of ERSS and TBRSS₂ methods are always better than the regression estimator of RSS method. The relative efficiency values increase when k increases for TBRSS method. Moreover, when $m=6$, the regression estimator $\hat{\beta}_1$ of ERSS method is approximately two times better than the regression estimators $\hat{\beta}_1$ of SRS and RSS methods.

Table 3.5 RE_2 and RE_3 values for MSE of regression model under normal case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	0.997	1.001	0.989	0.998	0.991	*	0.994	0.995	1.012	1.016	1.004	1.013	1.006	*	1.009	1.010
		5	1.001	1.007	0.999	1.003	0.999	1.001	1.007	1.011	0.998	1.003	0.996	1.000	0.995	0.998	1.004	1.008
		6	0.986	0.994	0.988	0.995	0.991	0.988	0.984	0.993	0.993	1.001	0.994	1.002	0.998	0.995	0.990	1.000
	3	4	1.014	1.014	0.996	1.008	1.003	*	1.004	1.007	1.007	1.007	0.989	1.001	0.996	*	0.996	1.000
		5	0.996	0.999	0.987	0.996	0.995	0.987	0.991	0.999	1.000	1.003	0.991	1.000	0.999	0.991	0.995	1.003
		6	1.005	1.011	0.999	1.004	1.002	1.000	1.001	1.004	1.007	1.014	1.001	1.007	1.005	1.003	1.003	1.007
	4	4	0.992	1.003	0.998	0.996	0.993	*	0.997	1.003	0.991	1.002	0.998	0.995	0.992	*	0.997	1.003
		5	1.005	1.005	0.995	1.002	0.997	0.995	1.002	1.000	1.003	1.003	0.993	0.999	0.995	0.993	1.000	0.997
		6	1.004	1.010	0.999	1.004	1.002	1.002	1.001	1.000	0.997	1.003	0.992	0.997	0.995	0.995	0.994	0.993
	5	4	1.002	1.004	0.999	0.996	0.991	*	0.995	1.001	1.005	1.007	1.002	0.999	0.994	*	0.997	1.003
		5	1.002	1.010	0.998	1.005	0.999	0.999	1.001	1.005	0.996	1.004	0.992	0.999	0.992	0.993	0.995	0.999
		6	1.001	1.002	0.997	1.002	0.996	1.002	1.002	1.001	1.000	1.002	0.997	1.002	0.996	1.002	1.001	1.001
	6	4	1.006	1.012	1.000	1.004	0.997	*	1.008	1.008	1.001	1.007	0.995	0.999	0.992	*	1.003	1.003
		5	0.998	1.009	0.997	1.004	1.001	0.996	1.006	1.002	1.000	1.010	0.999	1.006	1.002	0.997	1.008	1.003
		6	1.003	1.008	0.994	0.999	0.993	0.990	0.998	0.995	1.001	1.006	0.992	0.997	0.991	0.988	0.995	0.993
0.75	2	4	0.999	0.991	0.994	0.992	0.997	*	0.992	0.987	1.011	1.003	1.006	1.004	1.009	*	1.004	0.998
		5	1.005	1.003	1.002	1.006	0.994	1.008	1.007	1.008	1.010	1.007	1.007	1.011	0.999	1.012	1.012	1.012
		6	0.995	0.994	1.005	0.991	0.994	0.994	0.987	0.993	1.002	1.001	1.012	0.997	1.000	1.000	0.993	0.999
	3	4	1.012	1.009	1.006	1.012	1.011	*	1.009	1.004	1.003	1.000	0.997	1.003	1.002	*	1.000	0.995
		5	0.994	0.997	0.991	1.000	1.003	0.996	0.989	0.997	1.001	1.004	0.998	1.007	1.010	1.003	0.995	1.004
		6	1.006	1.003	1.008	1.003	1.006	1.003	1.003	1.001	1.006	1.004	1.008	1.004	1.006	1.003	1.003	1.002
	4	4	1.007	1.009	1.002	1.006	1.006	*	1.004	1.008	1.000	1.001	0.995	0.999	0.998	*	0.997	1.000
		5	0.998	0.997	0.990	0.995	0.997	0.992	0.991	0.995	1.008	1.008	1.000	1.005	1.007	1.002	1.001	1.006
		6	0.999	1.001	0.998	1.001	1.004	1.001	0.999	1.002	1.005	1.006	1.003	1.006	1.010	1.006	1.005	1.007
	5	4	1.006	1.008	1.004	1.009	1.004	*	1.004	1.002	1.003	1.005	1.001	1.006	1.001	*	1.001	0.999
		5	1.001	1.002	1.002	1.002	1.004	1.002	1.005	1.002	1.004	1.005	1.004	1.005	1.007	1.005	1.008	1.004
		6	0.995	0.996	0.998	0.992	0.993	1.000	0.994	0.995	1.001	1.002	1.004	0.998	0.999	1.006	1.000	1.001
	6	4	0.997	0.984	1.003	0.992	1.004	*	1.001	0.998	1.003	0.989	1.008	0.998	1.009	*	1.007	1.003
		5	0.995	0.993	1.006	0.993	0.998	1.007	0.999	0.989	1.002	1.000	1.013	1.000	1.005	1.014	1.006	0.996
		6	0.997	0.987	1.007	1.002	0.998	1.010	0.999	0.996	0.998	0.987	1.008	1.002	0.999	1.011	1.000	0.996
0.95	2	4	0.996	0.994	0.998	1.000	0.994	*	0.995	0.985	1.003	1.000	1.005	1.007	1.001	*	1.001	0.992
		5	0.997	0.998	1.002	1.004	1.002	1.001	1.001	1.004	0.996	0.997	1.002	1.003	1.001	1.000	1.000	1.003
		6	0.997	0.984	1.004	0.999	0.997	0.995	0.996	0.988	1.001	0.987	1.008	1.002	1.001	0.998	0.999	0.992
	3	4	1.008	1.002	1.011	1.009	1.009	*	1.008	1.002	0.997	0.991	1.000	0.998	0.998	*	0.997	0.990
		5	0.993	0.995	0.998	0.999	1.000	0.995	0.995	0.997	0.996	0.998	1.002	1.002	1.003	0.998	0.999	1.000
		6	1.005	0.996	1.010	1.005	1.008	1.008	1.001	0.999	0.999	0.991	1.004	1.000	1.002	1.002	0.996	0.994
	4	4	1.007	1.009	1.011	1.003	1.009	*	1.007	1.006	0.998	1.000	1.002	0.994	1.000	*	0.998	0.997
		5	0.997	0.994	0.998	0.999	0.999	0.994	0.993	0.999	1.003	1.001	1.004	1.006	1.005	1.000	1.000	1.005
		6	1.002	0.997	1.006	1.000	1.007	1.003	0.999	1.002	1.002	0.997	1.006	1.000	1.007	1.003	0.999	1.002
	5	4	1.002	1.005	1.004	1.010	1.008	*	1.006	1.000	0.999	1.002	1.001	1.006	1.004	*	1.002	0.997
		5	0.999	1.002	1.002	1.001	1.004	1.005	1.002	1.002	0.999	1.001	1.002	1.000	1.004	1.005	1.002	1.001
		6	0.996	0.991	0.999	0.994	0.994	1.003	0.995	0.994	1.003	0.998	1.006	1.002	1.001	1.010	1.003	1.001
	6	4	0.995	1.003	0.989	1.001	0.991	*	0.995	1.003	0.991	0.999	0.986	0.998	0.987	*	0.991	0.999
		5	1.004	1.005	0.996	0.997	0.995	0.991	1.005	1.008	0.998	0.999	0.990	0.991	0.989	0.985	0.999	1.002
		6	0.998	1.012	0.989	0.998	0.996	0.990	0.998	0.998	1.000	1.013	0.991	1.000	0.998	0.991	1.000	1.000

* k can be up to 1 when $m=4$

According to Table 3.5 the relative efficiency values are very close to 1. It can be assumed that the MSE of the regression model of the modified sampling methods are equivalent to the MSE of the regression model of SRS and RSS methods.

From Table 3.6, it can be inferred that, the bias values of the estimators of SRS, RSS and all the modified RSS methods are approximately 0. The bias values of the mean estimator of MRSS method are greater than the bias values of the mean estimator of the other sampling methods. However, the bias values decrease when the values of the correlation coefficient, the cycle size and the set size increase in this method.

Table 3.6 Bias values of the estimators ($\bar{y}$, $\hat{\beta}_0$, $\hat{\beta}_1$) for sampling methods under normal case

ρ		r		$Bias(\bar{y})$												$Bias(\hat{\beta}_0)$												$Bias(\hat{\beta}_1)$																	
				SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂		
0.50	4	2	5	4	0.005	0.005	0.007	-0.002	0.103	-0.004	0.008	*	0.001	-0.001	0.002	0.006	0.006	0.003	-0.002	-0.002	0.004	*	0.003	0.005	0.004	-0.002	0.002	0.002	0.003	-0.015	-0.003	-0.013	*	0.005	-0.001	0.000	-0.007	0.003	0.003	-0.033	0.001	-0.004	-0.023	-0.001	0.001
		2	5	4	-0.006	0.003	0.001	-0.008	0.080	-0.001	0.001	0.004	-0.002	-0.001	-0.005	0.005	0.002	-0.004	-0.004	-0.001	-0.001	-0.002	-0.001	-0.002	-0.001	0.000	-0.007	0.003	0.003	-0.033	0.001	-0.004	-0.023	*	-0.001	0.000	-0.007	0.003	0.003	-0.033	0.001	-0.004	-0.023	-0.001	0.001
		3	5	4	0.003	-0.002	0.002	-0.004	0.067	0.003	0.003	*	0.000	-0.009	0.003	-0.002	0.003	-0.005	-0.001	0.003	0.005	-0.003	-0.001	-0.005	0.005	0.005	-0.005	0.005	-0.030	-0.002	0.000	-0.018	-0.001	0.006	0.000	-0.007	0.003	0.003	-0.030	0.001	-0.004	-0.018	-0.001	0.006	
		3	5	4	-0.001	-0.003	-0.002	-0.001	0.054	0.003	0.007	-0.001	-0.002	-0.007	0.000	-0.002	0.000	-0.002	0.003	0.003	-0.007	-0.001	-0.003	-0.001	-0.003	0.000	-0.002	0.000	-0.001	0.002	0.000	-0.002	0.000	-0.001	0.000	-0.002	0.000	0.000	-0.016	0.000	0.000	0.000	0.000	0.000	
		3	5	4	0.002	-0.002	0.001	-0.009	0.043	0.000	0.004	0.004	-0.002	-0.003	0.001	-0.002	0.000	-0.001	-0.003	0.000	0.001	-0.001	-0.001	-0.002	-0.001	0.000	0.002	0.003	-0.001	0.006	-0.028	0.001	-0.005	-0.020	0.001	0.000	0.000	-0.020	0.001	0.000	0.000	0.000	0.000		
		3	5	4	0.003	0.001	0.004	-0.003	0.051	0.001	0.004	*	-0.001	-0.005	0.003	0.003	0.004	0.001	0.001	-0.001	-0.001	*	-0.002	-0.001	-0.002	0.001	0.000	0.007	0.001	0.001	-0.009	-0.004	-0.013	*	-0.003	0.000	0.000	-0.009	0.004	0.000	-0.013	*	-0.003	0.000	0.000
0.75	4	2	5	4	-0.001	0.001	-0.001	-0.004	0.041	0.001	0.004	0.000	0.002	-0.005	0.000	0.000	-0.004	-0.004	-0.001	0.002	-0.006	0.003	-0.002	0.001	-0.003	0.003	-0.003	0.003	-0.020	0.000	-0.004	-0.024	0.001	0.000	0.000	-0.003	0.000	-0.004	-0.024	0.001	0.003	0.003	0.003		
		2	5	4	0.004	-0.001	-0.003	-0.006	0.040	-0.001	0.003	*	0.001	-0.005	0.002	0.000	0.000	-0.001	0.003	0.000	-0.002	-0.003	0.001	-0.002	0.000	0.002	0.000	0.002	0.001	-0.010	0.000	-0.007	*	0.001	0.003	0.000	-0.007	0.003	0.003	-0.002	-0.001	0.000	0.000	0.000	
		3	5	4	-0.002	0.000	0.001	-0.010	0.026	-0.001	0.002	0.004	-0.002	-0.008	0.001	-0.003	0.001	-0.003	-0.004	-0.001	-0.001	-0.001	-0.002	-0.006	0.002	0.002	0.002	0.003	-0.019	0.000	-0.001	-0.014	-0.001	0.003	0.000	-0.001	0.000	-0.014	-0.001	0.001	0.001	0.001	0.001		
		3	5	4	0.000	0.002	-0.001	-0.005	0.034	0.001	0.002	*	-0.003	-0.001	0.000	0.001	0.000	-0.001	0.002	0.000	-0.003	*	-0.003	0.002	0.000	0.003	0.002	0.000	0.003	-0.011	0.001	-0.008	*	0.001	0.004	0.000	-0.008	0.000	0.000	0.000	0.000	0.000			
		3	5	4	-0.001	0.003	-0.004	-0.003	0.027	0.000	0.005	0.004	0.004	-0.004	-0.001	0.003	-0.002	0.001	-0.004	0.000	0.002	-0.002	0.005	0.000	0.003	0.001	0.001	0.001	-0.020	0.001	-0.002	-0.016	0.001	0.000	0.000	-0.016	0.001	0.000	0.000	0.000	0.000				
		3	5	4	-0.002	-0.001	0.001	-0.007	0.022	0.002	0.003	0.003	-0.001	-0.005	0.001	0.000	0.001	0.000	0.000	0.002	0.000	-0.003	-0.001	-0.001	-0.001	0.000	0.002	-0.001	0.004	-0.001	-0.022	-0.003	0.000	-0.023	0.003	0.000	-0.023	0.003	0.000	0.000	0.000	0.000	0.000	0.000	
0.95	4	2	5	4	0.006	0.005	0.006	-0.004	0.080	-0.002	0.008	*	0.000	-0.001	-0.001	0.004	0.004	0.000	-0.003	-0.001	0.001	*	0.002	0.006	0.005	-0.003	0.000	0.004	-0.001	-0.001	-0.002	*	0.004	0.000	0.000	-0.007	0.003	0.003	-0.002	-0.001	0.000	0.000	0.000	0.000	
		2	5	4	-0.006	0.003	-0.003	-0.005	0.058	0.001	-0.002	0.004	-0.003	0.001	-0.006	-0.002	0.000	-0.003	-0.003	-0.002	-0.002	-0.004	-0.001	-0.001	0.005	0.001	-0.003	0.003	-0.002	0.000	-0.004	-0.004	-0.001	-0.001	-0.001	-0.001	0.000	-0.002	0.000	-0.006	0.000	0.000	0.000	0.000	0.000
		3	5	4	0.003	0.000	-0.002	-0.002	0.052	0.002	0.004	*	0.001	-0.009	0.002	-0.001	0.000	0.000	-0.004	0.001	-0.001	-0.001	-0.001	-0.001	-0.001	0.000	-0.002	0.000	0.000	-0.004	0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
		3	5	4	0.000	-0.005	-0.002	-0.002	0.039	0.001	0.009	0.001	-0.003	-0.005	-0.001	-0.003	-0.001	0.002	0.000	0.001	-0.006	-0.002	-0.002	0.000	0.000	-0.002	0.000	0.004	-0.002	0.000	-0.003	-0.005	-0.002	0.001	0.003	0.000	-0.002	0.001	0.000	-0.008	0.000	0.000	0.000	0.000	0.000
		3	5	4	0.003	0.001	0.001	-0.006	0.031	-0.002	0.002	0.002	-0.002	-0.003	0.000	0.000	0.000	0.004	-0.004	-0.002	-0.003	-0.005	-0.002	0.001	0.003	0.000	0.000	0.001	0.000	0.004	-0.004	0.001	-0.004	-0.004	0.001	0.000	0.000	-0.008	0.000	0.000	0.000	0.000	0.000		
		3	5	4	0.002	0.003	0.001	-0.001	0.038	-0.001	0.000	*	-0.001	-0.002	0.001	0.000	-0.001	0.000	0.005	-0.004	0.000	-0.003	*	-0.001	0.003	0.000	0.000	0.000	0.000	0.000	-0.003	-0.001	-0.002	-0.006	*	0.001	0.002	0.000	-0.006	0.000	0.000	0.000	0.000	0.000	0.000
1.00	4	2	5	4	-0.001	-0.003	0.000	0.000	0.029	0.000	0.003	0.005	0.001	-0.002	0.003	0.002	-0.001	-0.002	0.000	-0.002	-0.002	0.000	0.001	0.000	0.001	0.000	0.002	0.001	-0.009	0.002	-0.003	-0.006	0.001	-0.001	0.000	0.000	0.002	0.001	-0.009	0.002	-0.003	-0.006	0.001	-0.001	
		2	5	4	0.001	-0.003	-0.003	-0.004	0.023	-0.001	0.001	0.006	-0.002	-0.004	0.002	-0.002	-0.003	0.005	-0.006	-0.002	-0.003	-0.002	0.000	0.001	0.000	-0.002	0.000	0.001	-0.015	0.001	-0.003	-0.008	0.002	0.002	0.000	-0.002	0.000	0.000	-0.005	0.003	-0.006	0.000	0.000	0.000	
		3	5	4	0.002	-0.001	-0.004	-0.001	0.031	0.000	0.000	*	0.001	-0.005	0.001	-0.001	-0.001	0.000	-0.002	-0.001	-0.007	*	-0.001	0.000	0.002	0.000	0.000	-0.001	0.000	0.000	-0.005	0.003	-0.006	*	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
		3	5	4	-0.005	0.003	0.001	-0.003	0.023	-0.001	0.006	0.003	0.001	-0.005	-0.003	0.003	0.003	0.001	0.001	-0.003	-0.001	-0.003	0.000	0.002	0.000	0.004	0.001	0.001	0.001	-0.004	0.000	0.000	-0.011	0.000	0.000	0.000	-0.011	0.000	0.000	0.000	0.000	0.000			
		3	5	4	0.001	0.001	0.000	-0.004	0.018	0.000	0.006	0.002	-0.002	-0.005	0.002	0.001	0.002	0.006	-0.006	0.000	0.001	-0.006	-0.002	-0.001	0.000	0.001	-0.001	0.001	0.002	-0.005	0.001	-0.003	-0.008	0.002	0.002	0.000	-0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
		3	5	4	-0.001	-0.003	0.002	0.003	0.027	0.000	-0.006	*	0.000	0.002	-0.002	-0.001	-0.002	0.000	-0.001	0.001	-0.001	0.001	-0.001	0.000	0.000	0.001	0.002	0.000	0.003	0.003	0.004	0.001	-0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

3.2 Outlier Case

The simulation study is designed with R Project concluding the processes given below:

1. Generate 5,000 observations from $(X,Y) \sim (1-\phi)*\text{BVN}(0,0,1,1,\rho) + \phi*\text{BVN}(0,0,1,25,\rho)$ where outlier percents are $\phi=0.10, 0.20, 0.30$ and correlation levels are $\rho=0.50, 0.75$ and 0.95 .
2. The performances of the estimators are investigated for the cycle size $r=2, 3, 4, 5, 6$ and the set size $m=4, 5, 6$.
3. Run the simulation $R=10,000$ times for SRS, RSS, PRSS, ERSS, MRSS, DRSS, LRSS₁, LRSS₂, TBRSS₁ and TBRSS₂.
4. The bias, the mean square error and the relative efficiency formulations are given in Equations 3.1-3.5.

The simulation results for $\phi=0.20$ are summarized in Table 3.7-Table 3.12. The simulation results for $\phi=0.10$ and $\phi=0.30$ give similar results to $\phi=0.20$ and these results are given in Appendix.

Table 3.7 RE_1 values of the estimators under 20% outlier case

ρ	r	m	$\bar{y}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\sigma}^2$
0.50	2	4	1.078	1.075	1.069	0.968
		5	1.134	1.087	1.108	0.973
		6	1.118	1.058	1.113	1.002
	3	4	1.097	1.061	1.070	1.008
		5	1.074	1.022	1.071	1.021
		6	1.141	1.083	1.041	1.001
	4	4	1.081	1.037	1.034	1.020
		5	1.130	1.063	1.049	0.997
		6	1.109	1.019	1.061	0.995
	5	4	1.071	1.012	1.044	0.986
		5	1.079	0.995	1.044	1.001
		6	1.099	1.013	1.027	0.998
	6	4	1.103	1.043	1.034	1.000
		5	1.095	1.012	1.032	1.010
		6	1.103	1.002	1.001	0.990
0.75	2	4	1.238	1.087	1.078	0.983
		5	1.292	1.057	1.083	0.995
		6	1.271	1.029	1.073	1.002
	3	4	1.211	1.031	1.076	1.003
		5	1.248	1.039	1.062	1.022
		6	1.290	1.050	1.041	1.006
	4	4	1.246	1.046	1.027	1.030
		5	1.254	1.024	1.033	0.992
		6	1.249	0.997	1.024	0.989
	5	4	1.218	1.018	1.014	0.988
		5	1.247	1.013	1.034	1.008
		6	1.272	1.005	1.001	1.000
	6	4	1.238	1.036	1.011	0.996
		5	1.250	1.019	1.035	1.008
		6	1.258	0.993	0.982	0.996
0.95	2	4	1.392	1.004	1.034	0.996
		5	1.521	1.026	1.071	1.027
		6	1.542	0.990	1.021	1.017
	3	4	1.382	0.993	1.032	1.014
		5	1.482	1.019	1.052	1.027
		6	1.542	1.007	1.035	1.018
	4	4	1.416	1.001	1.015	1.028
		5	1.475	0.996	0.991	0.996
		6	1.463	0.987	1.003	0.987
	5	4	1.407	0.999	1.019	0.989
		5	1.475	1.015	1.025	1.012
		6	1.537	1.003	0.973	1.002
	6	4	1.379	0.981	0.992	0.993
		5	1.490	1.001	1.049	1.006
		6	1.511	0.985	0.998	0.995

According to Table 3.7, all the relative efficiency values of $\bar{y}$ are greater than 1. These values indicate that the mean estimator of RSS method is more efficient than the mean estimator of SRS method. The relative efficiency values increase as the correlation coefficient increases when the set and the cycle size are constant. Additionally, the relative efficiencies for estimators $\hat{\beta}_0$, $\hat{\beta}_1$ and the mean square error of the regression model are almost greater than 1.

Table 3.8 RE_2 and RE_3 values for the mean estimator under 20% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.067	0.961	1.169	1.079	1.182	*	1.080	0.964	0.990	0.892	1.085	1.001	1.096	*	1.002	0.894
		5	1.088	1.051	1.291	1.140	1.228	1.312	1.093	1.028	0.960	0.927	1.138	1.005	1.082	1.157	0.964	0.906
		6	1.089	0.943	1.302	1.112	1.222	1.309	1.152	1.037	0.974	0.844	1.165	0.994	1.093	1.171	1.030	0.927
	3	4	1.079	0.986	1.224	1.113	1.267	*	1.107	1.001	0.984	0.899	1.116	1.015	1.155	*	1.009	0.913
		5	1.097	1.000	1.295	1.116	1.203	1.292	1.128	0.990	1.021	0.931	1.206	1.040	1.120	1.204	1.050	0.922
		6	1.131	0.958	1.294	1.195	1.239	1.317	1.108	1.032	0.991	0.839	1.134	1.047	1.085	1.154	0.971	0.904
	4	4	1.030	0.963	1.224	1.125	1.239	*	1.086	0.959	0.954	0.891	1.133	1.041	1.147	*	1.005	0.888
		5	1.084	1.008	1.283	1.178	1.241	1.342	1.135	1.021	0.960	0.892	1.136	1.043	1.099	1.188	1.005	0.904
		6	1.076	0.901	1.270	1.100	1.199	1.280	1.092	0.988	0.970	0.812	1.145	0.991	1.080	1.154	0.985	0.891
	5	4	1.036	0.985	1.232	1.120	1.244	*	1.102	0.963	0.967	0.920	1.150	1.045	1.161	*	1.029	0.899
		5	1.087	0.989	1.302	1.137	1.232	1.249	1.100	0.995	1.008	0.917	1.207	1.054	1.142	1.158	1.020	0.922
		6	1.116	0.944	1.279	1.165	1.212	1.291	1.129	1.028	1.015	0.859	1.163	1.060	1.103	1.174	1.027	0.935
	6	4	1.066	0.980	1.219	1.108	1.262	*	1.089	0.997	0.966	0.889	1.105	1.005	1.144	*	0.987	0.904
		5	1.081	0.979	1.272	1.116	1.230	1.264	1.108	0.966	0.987	0.894	1.162	1.019	1.123	1.155	1.012	0.883
		6	1.099	0.880	1.269	1.153	1.204	1.234	1.102	1.002	0.997	0.798	1.151	1.046	1.092	1.119	1.000	0.909
0.75	2	4	1.147	0.990	1.531	1.278	1.499	*	1.195	0.986	0.927	0.800	1.237	1.032	1.211	*	0.965	0.797
		5	1.208	1.088	1.708	1.345	1.594	1.718	1.273	1.061	0.935	0.843	1.322	1.041	1.234	1.330	0.985	0.821
		6	1.235	0.908	1.762	1.356	1.589	1.763	1.317	1.092	0.972	0.714	1.387	1.067	1.250	1.387	1.037	0.859
	3	4	1.183	1.023	1.556	1.286	1.582	*	1.239	1.058	0.976	0.845	1.285	1.062	1.306	*	1.023	0.873
		5	1.188	1.038	1.690	1.336	1.554	1.681	1.242	1.015	0.952	0.832	1.354	1.070	1.245	1.346	0.994	0.813
		6	1.263	0.923	1.787	1.382	1.581	1.733	1.305	1.116	0.979	0.715	1.385	1.071	1.226	1.344	1.011	0.865
	4	4	1.114	0.972	1.534	1.249	1.570	*	1.207	0.990	0.894	0.780	1.231	1.002	1.260	*	0.968	0.794
		5	1.233	1.070	1.723	1.319	1.604	1.769	1.268	1.063	0.984	0.853	1.375	1.052	1.279	1.411	1.011	0.848
		6	1.194	0.881	1.667	1.328	1.559	1.717	1.227	1.057	0.956	0.705	1.335	1.063	1.248	1.374	0.982	0.846
	5	4	1.116	0.994	1.551	1.281	1.579	*	1.220	0.992	0.916	0.817	1.273	1.052	1.297	*	1.002	0.814
		5	1.180	1.015	1.684	1.333	1.527	1.646	1.247	1.023	0.946	0.814	1.351	1.069	1.225	1.320	1.000	0.820
		6	1.258	0.922	1.750	1.347	1.581	1.695	1.285	1.093	0.989	0.725	1.376	1.059	1.243	1.332	1.010	0.859
	6	4	1.151	0.997	1.535	1.280	1.573	*	1.219	1.019	0.930	0.806	1.240	1.034	1.271	*	0.984	0.823
		5	1.217	1.006	1.676	1.282	1.589	1.671	1.222	1.003	0.974	0.805	1.341	1.025	1.271	1.337	0.977	0.802
		6	1.238	0.882	1.680	1.319	1.516	1.657	1.247	1.076	0.984	0.701	1.335	1.049	1.205	1.317	0.991	0.855
0.95	2	4	1.278	1.016	2.171	1.520	2.130	*	1.382	1.010	0.918	0.730	1.560	1.092	1.531	*	0.993	0.726
		5	1.414	1.165	2.797	1.683	2.276	2.728	1.522	1.109	0.930	0.766	1.839	1.107	1.497	1.794	1.001	0.729
		6	1.416	0.909	2.966	1.755	2.223	2.957	1.577	1.158	0.918	0.590	1.923	1.138	1.441	1.917	1.023	0.751
	3	4	1.290	1.056	2.228	1.549	2.200	*	1.416	1.099	0.933	0.764	1.612	1.121	1.592	*	1.024	0.795
		5	1.354	1.086	2.750	1.643	2.201	2.802	1.468	1.079	0.914	0.733	1.855	1.109	1.485	1.891	0.991	0.728
		6	1.457	0.899	2.980	1.747	2.269	2.927	1.509	1.180	0.945	0.583	1.932	1.133	1.471	1.898	0.978	0.765
	4	4	1.210	1.011	2.207	1.506	2.150	*	1.389	1.027	0.854	0.714	1.558	1.063	1.518	*	0.981	0.725
		5	1.439	1.112	2.729	1.633	2.272	2.730	1.512	1.146	0.975	0.754	1.850	1.107	1.540	1.851	1.025	0.777
		6	1.338	0.871	2.855	1.676	2.212	2.850	1.467	1.139	0.914	0.595	1.951	1.145	1.512	1.948	1.002	0.778
	5	4	1.242	1.039	2.141	1.560	2.211	*	1.422	1.028	0.883	0.739	1.522	1.109	1.572	*	1.011	0.731
		5	1.346	1.060	2.737	1.628	2.165	2.611	1.471	1.112	0.912	0.718	1.855	1.104	1.468	1.770	0.997	0.754
		6	1.438	0.908	2.949	1.722	2.250	2.930	1.540	1.185	0.936	0.591	1.919	1.120	1.464	1.907	1.002	0.771
	6	4	1.251	1.044	2.161	1.549	2.213	*	1.387	1.057	0.907	0.757	1.567	1.124	1.605	*	1.006	0.766
		5	1.412	1.076	2.840	1.639	2.209	2.811	1.489	1.061	0.948	0.722	1.906	1.100	1.483	1.886	0.999	0.712
		6	1.399	0.878	2.866	1.690	2.207	2.796	1.486	1.155	0.926	0.581	1.897	1.118	1.461	1.850	0.984	0.764

* k can be up to 1 when $m=4$

Table 3.8 presents the relative efficiencies of the mean estimators between all the modified RSS methods and SRS or RSS methods. The mean estimators of all the modified RSS methods are more efficient than the mean estimator of SRS method except ERSS and TBRSS₁ methods. The relative efficiency values increase when the correlation values increase. The relative efficiency values between DRSS and RSS methods are almost greater 1. Additionally, the mean estimators of MRSS, LRSS₁ and LRSS₂ methods are always more efficient than the mean estimator of RSS method. The mean estimators of MRSS, LRSS₁ and LRSS₂ methods are approximately one and a half times better than the mean estimators of RSS method when $\rho=0.95$. In TBRSS method, the relative efficiency values decrease when k increases.

Table 3.9 RE_2 and RE_3 values for $\hat{\beta}_0$ under 20% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.051	1.005	1.073	1.088	1.084	*	1.078	1.022	0.978	0.936	0.999	1.013	1.009	*	1.003	0.951
		5	1.058	1.043	1.085	1.090	1.128	1.123	1.047	1.021	0.974	0.960	0.999	1.003	1.038	1.033	0.964	0.940
		6	1.020	0.933	1.125	1.046	1.112	1.142	1.099	1.002	0.964	0.882	1.064	0.989	1.051	1.080	1.039	0.947
	3	4	1.025	0.994	1.097	1.064	1.123	*	1.065	0.995	0.966	0.936	1.033	1.002	1.058	*	1.004	0.938
		5	1.046	0.964	1.143	1.045	1.080	1.156	1.063	0.959	1.023	0.943	1.118	1.022	1.057	1.131	1.040	0.939
		6	1.082	0.941	1.159	1.120	1.144	1.168	1.051	0.983	0.999	0.869	1.070	1.035	1.057	1.079	0.971	0.908
	4	4	1.010	0.957	1.127	1.082	1.120	*	1.053	0.945	0.974	0.923	1.086	1.043	1.079	*	1.015	0.911
		5	1.016	0.973	1.131	1.085	1.114	1.187	1.066	0.978	0.956	0.915	1.064	1.021	1.049	1.117	1.003	0.921
		6	1.009	0.865	1.105	1.001	1.087	1.126	1.005	0.935	0.990	0.850	1.085	0.982	1.067	1.106	0.987	0.918
	5	4	0.988	0.945	1.115	1.044	1.102	*	1.039	0.919	0.976	0.934	1.102	1.031	1.089	*	1.027	0.908
		5	1.017	0.930	1.137	1.026	1.102	1.104	1.022	0.924	1.022	0.935	1.142	1.031	1.108	1.110	1.027	0.928
		6	1.042	0.901	1.137	1.065	1.098	1.152	1.043	0.982	1.029	0.889	1.122	1.052	1.084	1.138	1.029	0.970
	6	4	1.026	0.949	1.105	1.033	1.125	*	1.038	0.956	0.983	0.910	1.059	0.990	1.079	*	0.995	0.917
		5	1.003	0.915	1.121	1.019	1.109	1.115	1.026	0.913	0.991	0.905	1.108	1.007	1.096	1.102	1.014	0.903
		6	1.017	0.836	1.110	1.033	1.085	1.095	1.023	0.931	1.015	0.834	1.108	1.031	1.082	1.092	1.021	0.929
0.75	2	4	1.017	0.915	1.197	1.086	1.210	*	1.059	0.929	0.936	0.842	1.101	0.999	1.113	*	0.974	0.855
		5	1.045	0.912	1.248	1.065	1.200	1.190	1.041	0.898	0.989	0.863	1.181	1.007	1.135	1.126	0.985	0.850
		6	1.034	0.759	1.269	1.053	1.212	1.289	1.075	0.892	1.005	0.738	1.234	1.024	1.179	1.253	1.045	0.867
	3	4	1.003	0.893	1.222	1.045	1.232	*	1.043	0.914	0.973	0.866	1.185	1.013	1.194	*	1.011	0.886
		5	1.007	0.876	1.282	1.056	1.216	1.290	1.025	0.850	0.969	0.843	1.234	1.016	1.170	1.241	0.987	0.818
		6	1.049	0.775	1.355	1.069	1.243	1.329	1.063	0.907	0.999	0.738	1.290	1.018	1.184	1.266	1.013	0.864
	4	4	0.997	0.843	1.250	1.016	1.251	*	1.038	0.857	0.953	0.806	1.194	0.971	1.195	*	0.992	0.819
		5	1.039	0.889	1.307	1.026	1.250	1.359	1.029	0.883	1.015	0.868	1.276	1.002	1.221	1.327	1.004	0.863
		6	0.999	0.723	1.254	1.010	1.220	1.296	0.978	0.863	1.002	0.725	1.258	1.013	1.224	1.301	0.982	0.866
	5	4	0.988	0.835	1.239	1.019	1.237	*	1.000	0.843	0.970	0.820	1.217	1.001	1.215	*	0.982	0.828
		5	0.972	0.834	1.260	1.021	1.197	1.254	1.012	0.824	0.960	0.823	1.244	1.008	1.182	1.238	1.000	0.813
		6	1.041	0.760	1.347	1.029	1.226	1.305	1.024	0.896	1.036	0.756	1.340	1.024	1.220	1.299	1.019	0.892
	6	4	1.013	0.851	1.253	1.021	1.249	*	1.024	0.854	0.978	0.822	1.210	0.985	1.205	*	0.988	0.824
		5	1.013	0.827	1.292	1.000	1.249	1.295	0.987	0.834	0.995	0.812	1.268	0.981	1.226	1.271	0.969	0.818
		6	1.006	0.723	1.273	0.999	1.190	1.281	1.003	0.870	1.013	0.729	1.282	1.007	1.198	1.290	1.010	0.876
0.95	2	4	0.980	0.700	1.640	0.990	1.704	*	0.994	0.701	0.976	0.697	1.633	0.986	1.697	*	0.990	0.698
		5	1.004	0.730	2.029	1.011	1.574	1.996	1.007	0.712	0.979	0.712	1.978	0.986	1.535	1.946	0.982	0.694
		6	0.968	0.558	2.110	1.006	1.496	2.120	1.028	0.730	0.978	0.564	2.133	1.017	1.512	2.142	1.039	0.737
	3	4	0.968	0.700	1.711	1.003	1.692	*	0.987	0.715	0.975	0.705	1.723	1.010	1.704	*	0.994	0.720
		5	0.986	0.707	2.164	1.003	1.607	2.226	1.008	0.705	0.967	0.693	2.123	0.984	1.576	2.183	0.989	0.691
		6	1.021	0.558	2.230	1.023	1.549	2.227	1.010	0.739	1.013	0.554	2.214	1.016	1.538	2.211	1.003	0.733
	4	4	0.965	0.690	1.718	1.003	1.752	*	1.005	0.695	0.965	0.690	1.717	1.002	1.751	*	1.005	0.695
		5	1.056	0.717	2.135	1.021	1.651	2.176	1.014	0.736	1.060	0.720	2.144	1.025	1.658	2.184	1.018	0.738
		6	0.980	0.548	2.199	1.000	1.526	2.156	0.967	0.750	0.994	0.555	2.228	1.013	1.547	2.185	0.980	0.760
	5	4	0.991	0.698	1.743	0.997	1.697	*	0.993	0.695	0.992	0.699	1.746	0.999	1.700	*	0.994	0.696
		5	0.968	0.690	2.173	0.978	1.612	2.119	0.999	0.717	0.953	0.679	2.140	0.963	1.589	2.088	0.984	0.706
		6	1.021	0.573	2.241	1.029	1.557	2.251	1.018	0.763	1.018	0.571	2.235	1.026	1.552	2.244	1.015	0.760
	6	4	1.002	0.705	1.732	1.003	1.743	*	1.019	0.711	1.022	0.718	1.765	1.023	1.777	*	1.039	0.724
		5	1.010	0.695	2.229	1.012	1.600	2.259	1.002	0.697	1.008	0.694	2.227	1.011	1.598	2.257	1.001	0.696
		6	0.974	0.545	2.168	1.014	1.511	2.180	0.974	0.733	0.989	0.553	2.200	1.029	1.534	2.212	0.989	0.744

*k can be up to 1 when $m=4$

Table 3.9 shows the relative efficiencies of the regression estimator $\hat{\beta}_0$. The regression estimator $\hat{\beta}_0$ of PRSS, MRSS, DRSS and TBRSS₁ methods are mostly more efficient than the regression estimators of SRS and RSS methods. On the other hand, the regression estimator $\hat{\beta}_0$ of MRSS, LRSS₁ and LRSS₂ methods are always better than the regression estimator $\hat{\beta}_0$ of SRS method. The regression estimator $\hat{\beta}_0$ of MRSS and LRSS₂ methods are approximately two times better than the regression estimator $\hat{\beta}_0$ of SRS and RSS methods when $\rho=0.95$.

Table 3.10 RE_2 and RE_3 values for $\hat{\beta}_1$ under 20% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.097	1.662	0.498	1.152	0.497	*	1.110	1.711	1.026	1.555	0.466	1.077	0.465	*	1.039	1.601
		5	1.129	1.640	0.348	1.191	0.621	0.355	1.086	1.628	1.019	1.481	0.314	1.075	0.561	0.321	0.980	1.469
		6	1.122	2.086	0.373	1.117	0.692	0.376	1.156	1.554	1.008	1.874	0.335	1.003	0.621	0.337	1.038	1.396
	3	4	1.083	1.623	0.545	1.129	0.573	*	1.071	1.662	1.012	1.517	0.509	1.055	0.536	*	1.001	1.554
		5	1.050	1.484	0.356	1.058	0.624	0.363	1.041	1.488	0.981	1.387	0.332	0.988	0.583	0.339	0.972	1.390
		6	1.055	1.982	0.369	1.093	0.688	0.376	1.063	1.414	1.013	1.904	0.354	1.049	0.661	0.362	1.021	1.358
	4	4	1.011	1.491	0.544	1.041	0.555	*	1.057	1.552	0.977	1.441	0.526	1.007	0.537	*	1.022	1.501
		5	1.081	1.430	0.369	1.050	0.622	0.388	1.080	1.507	1.030	1.362	0.351	1.001	0.592	0.370	1.029	1.436
		6	1.075	1.934	0.379	1.046	0.700	0.364	1.042	1.439	1.014	1.824	0.357	0.986	0.660	0.344	0.983	1.357
	5	4	1.057	1.540	0.557	1.062	0.562	*	1.032	1.571	1.013	1.476	0.533	1.018	0.539	*	0.989	1.505
		5	1.025	1.431	0.373	1.029	0.637	0.376	1.027	1.424	0.982	1.371	0.357	0.985	0.610	0.360	0.984	1.364
		6	1.023	1.871	0.372	1.017	0.674	0.397	1.001	1.361	0.996	1.822	0.362	0.991	0.657	0.387	0.975	1.326
	6	4	1.003	1.514	0.555	1.049	0.543	*	1.025	1.484	0.970	1.464	0.537	1.014	0.525	*	0.992	1.435
		5	1.042	1.399	0.380	1.037	0.610	0.378	1.058	1.416	1.010	1.355	0.368	1.005	0.591	0.366	1.025	1.372
		6	1.008	1.817	0.371	1.008	0.684	0.379	1.002	1.334	1.007	1.815	0.371	1.008	0.683	0.378	1.001	1.333
0.75	2	4	1.127	1.527	0.626	1.127	0.630	*	1.090	1.525	1.045	1.417	0.580	1.045	0.585	*	1.011	1.415
		5	1.094	1.447	0.460	1.124	0.764	0.466	1.046	1.423	1.011	1.336	0.425	1.038	0.706	0.431	0.966	1.315
		6	1.084	1.824	0.503	1.069	0.814	0.496	1.098	1.379	1.010	1.700	0.469	0.996	0.759	0.462	1.024	1.285
	3	4	1.072	1.503	0.679	1.063	0.676	*	1.041	1.488	0.996	1.397	0.631	0.988	0.628	*	0.968	1.383
		5	1.031	1.405	0.495	1.072	0.804	0.522	1.048	1.398	0.971	1.323	0.466	1.010	0.757	0.492	0.987	1.316
		6	1.034	1.826	0.518	1.075	0.842	0.532	1.063	1.312	0.993	1.754	0.497	1.032	0.808	0.511	1.020	1.260
	4	4	0.986	1.393	0.699	1.030	0.703	*	1.044	1.441	0.960	1.356	0.680	1.003	0.684	*	1.016	1.403
		5	1.048	1.307	0.508	1.027	0.754	0.526	1.041	1.355	1.014	1.265	0.491	0.994	0.730	0.509	1.007	1.311
		6	1.034	1.746	0.526	1.038	0.840	0.518	1.020	1.295	1.009	1.706	0.514	1.014	0.821	0.506	0.996	1.264
	5	4	1.013	1.406	0.692	1.012	0.693	*	1.002	1.455	0.999	1.387	0.682	0.998	0.683	*	0.988	1.435
		5	0.994	1.357	0.521	1.020	0.765	0.516	1.008	1.314	0.961	1.312	0.504	0.986	0.740	0.499	0.975	1.271
		6	0.991	1.696	0.509	1.001	0.798	0.532	0.999	1.251	0.990	1.695	0.508	1.000	0.798	0.531	0.998	1.250
	6	4	0.998	1.434	0.687	1.056	0.669	*	1.005	1.388	0.987	1.418	0.680	1.044	0.662	*	0.994	1.373
		5	1.056	1.334	0.560	1.042	0.771	0.542	1.059	1.344	1.021	1.289	0.541	1.007	0.745	0.524	1.024	1.299
		6	1.019	1.695	0.524	1.009	0.839	0.544	1.013	1.285	1.037	1.725	0.533	1.027	0.854	0.554	1.032	1.308
0.95	2	4	1.059	1.358	0.861	1.077	0.880	*	1.062	1.327	1.025	1.314	0.833	1.042	0.852	*	1.028	1.284
		5	1.043	1.272	0.730	1.058	0.932	0.720	1.011	1.251	0.974	1.188	0.682	0.988	0.870	0.673	0.944	1.168
		6	1.035	1.585	0.787	1.058	1.012	0.777	1.040	1.215	1.014	1.553	0.771	1.037	0.991	0.761	1.020	1.191
	3	4	1.046	1.368	0.881	1.052	0.919	*	1.021	1.334	1.014	1.326	0.853	1.019	0.890	*	0.989	1.292
		5	1.003	1.302	0.807	1.049	1.012	0.794	1.030	1.302	0.954	1.238	0.768	0.998	0.962	0.755	0.979	1.238
		6	1.002	1.670	0.832	1.049	1.052	0.797	1.043	1.210	0.969	1.614	0.804	1.013	1.017	0.770	1.008	1.170
	4	4	0.978	1.327	0.896	0.985	0.895	*	1.001	1.333	0.963	1.307	0.883	0.970	0.881	*	0.986	1.313
		5	1.020	1.214	0.788	1.003	0.934	0.783	1.004	1.243	1.029	1.225	0.796	1.012	0.942	0.790	1.013	1.255
		6	1.009	1.572	0.784	1.005	1.057	0.803	1.032	1.190	1.005	1.567	0.781	1.002	1.053	0.800	1.028	1.186
	5	4	1.008	1.347	0.904	1.015	0.903	*	1.004	1.324	0.989	1.322	0.887	0.996	0.886	*	0.985	1.299
		5	0.998	1.288	0.812	1.026	1.015	0.821	1.027	1.235	0.973	1.256	0.792	1.001	0.989	0.801	1.001	1.205
		6	0.959	1.543	0.788	0.966	1.021	0.795	0.980	1.171	0.985	1.586	0.810	0.993	1.050	0.817	1.008	1.203
	6	4	0.985	1.337	0.922	1.040	0.898	*	0.998	1.290	0.994	1.349	0.930	1.049	0.906	*	1.006	1.301
		5	1.055	1.301	0.854	1.057	1.026	0.886	1.047	1.295	1.006	1.240	0.814	1.007	0.977	0.845	0.998	1.234
		6	1.029	1.598	0.825	1.022	1.038	0.842	1.019	1.212	1.031	1.601	0.826	1.024	1.040	0.844	1.021	1.215

* k can be up to 1 when $m=4$

Table 3.10 shows the relative efficiencies of the regression estimator $\hat{\beta}_1$. The regression estimator $\hat{\beta}_1$ of PRSS and DRSS methods are almost more efficient than the regression estimators $\hat{\beta}_1$ of SRS and RSS methods. On the other hand, the regression estimators $\hat{\beta}_1$ of ERSS and TBRSS₂ methods are always better than the regression estimators $\hat{\beta}_1$ of SRS and RSS methods. In TBRSS method, the relative efficiency values increase when k increases.

Table 3.11 RE_2 and RE_3 values for MSE of regression model under 20% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	0.988	0.894	1.078	0.964	1.052	*	0.983	0.905	1.020	0.923	1.113	0.995	1.086	*	1.015	0.935
		5	0.977	0.926	1.128	0.991	1.071	1.111	0.998	0.917	1.004	0.952	1.160	1.019	1.101	1.142	1.026	0.942
		6	1.003	0.863	1.126	0.991	1.070	1.123	1.010	0.946	1.001	0.862	1.124	0.990	1.069	1.121	1.008	0.945
	3	4	0.998	0.919	1.102	1.012	1.104	*	1.005	0.935	0.990	0.912	1.093	1.004	1.095	*	0.997	0.927
		5	1.014	0.925	1.134	0.993	1.090	1.143	1.018	0.918	0.993	0.906	1.110	0.972	1.067	1.119	0.997	0.899
		6	1.018	0.859	1.137	1.011	1.050	1.130	1.003	0.934	1.017	0.859	1.136	1.010	1.049	1.129	1.002	0.933
	4	4	0.998	0.927	1.120	1.008	1.119	*	1.008	0.924	0.978	0.908	1.097	0.987	1.096	*	0.987	0.906
		5	1.002	0.917	1.108	0.988	1.089	1.153	0.996	0.928	1.005	0.920	1.111	0.991	1.092	1.157	0.999	0.931
		6	1.000	0.842	1.133	0.996	1.066	1.130	1.001	0.922	1.005	0.847	1.139	1.001	1.071	1.136	1.006	0.926
	5	4	0.987	0.907	1.098	0.988	1.088	*	0.971	0.906	1.000	0.919	1.114	1.002	1.103	*	0.985	0.919
		5	0.984	0.914	1.129	0.997	1.081	1.118	0.996	0.913	0.984	0.913	1.128	0.997	1.080	1.117	0.995	0.912
		6	1.013	0.857	1.126	1.007	1.078	1.140	1.004	0.932	1.016	0.858	1.129	1.009	1.080	1.142	1.006	0.934
	6	4	0.996	0.915	1.090	0.994	1.102	*	1.012	0.913	0.997	0.916	1.091	0.994	1.102	*	1.012	0.913
		5	1.002	0.925	1.151	1.012	1.089	1.149	1.009	0.930	0.992	0.916	1.139	1.002	1.079	1.138	0.999	0.921
		6	0.998	0.840	1.115	0.995	1.072	1.129	0.998	0.923	1.009	0.848	1.126	1.005	1.083	1.141	1.008	0.932
0.75	2	4	1.000	0.810	1.172	0.982	1.176	*	0.995	0.813	1.018	0.824	1.192	0.999	1.197	*	1.012	0.827
		5	0.999	0.841	1.282	1.019	1.204	1.254	1.005	0.835	1.004	0.846	1.289	1.024	1.210	1.261	1.011	0.839
		6	1.010	0.717	1.294	0.997	1.172	1.298	1.019	0.865	1.009	0.716	1.292	0.996	1.171	1.296	1.018	0.863
	3	4	0.997	0.836	1.220	1.018	1.236	*	1.004	0.849	0.994	0.834	1.217	1.016	1.233	*	1.001	0.847
		5	0.998	0.843	1.304	1.000	1.229	1.296	1.010	0.835	0.977	0.825	1.276	0.979	1.203	1.268	0.989	0.817
		6	1.017	0.724	1.306	1.019	1.165	1.297	1.002	0.854	1.011	0.720	1.299	1.013	1.158	1.289	0.997	0.849
	4	4	0.994	0.834	1.241	1.011	1.240	*	1.003	0.837	0.965	0.810	1.205	0.982	1.205	*	0.974	0.813
		5	1.009	0.839	1.283	1.003	1.213	1.313	1.004	0.838	1.016	0.845	1.292	1.010	1.222	1.323	1.011	0.845
		6	0.989	0.715	1.307	0.988	1.193	1.310	0.988	0.847	1.001	0.723	1.322	1.000	1.207	1.325	0.999	0.856
	5	4	0.989	0.823	1.227	0.998	1.233	*	0.983	0.826	1.001	0.833	1.242	1.011	1.248	*	0.995	0.836
		5	0.989	0.830	1.291	0.999	1.224	1.286	1.000	0.827	0.980	0.823	1.281	0.991	1.214	1.276	0.992	0.821
		6	1.011	0.723	1.320	1.007	1.198	1.315	1.005	0.859	1.011	0.723	1.319	1.006	1.198	1.314	1.005	0.858
	6	4	0.997	0.831	1.224	0.996	1.232	*	0.999	0.829	1.001	0.834	1.229	1.001	1.237	*	1.004	0.833
		5	1.012	0.845	1.333	1.011	1.223	1.334	1.009	0.847	1.004	0.838	1.323	1.003	1.214	1.324	1.001	0.840
		6	0.997	0.712	1.322	0.994	1.180	1.307	0.998	0.848	1.001	0.714	1.327	0.998	1.185	1.312	1.001	0.851
0.95	2	4	1.022	0.661	1.683	0.996	1.670	*	1.009	0.653	1.026	0.663	1.690	1.000	1.676	*	1.013	0.655
		5	1.014	0.700	2.115	1.020	1.599	2.119	1.015	0.698	0.988	0.682	2.060	0.994	1.557	2.064	0.989	0.679
		6	1.023	0.532	2.141	1.022	1.488	2.136	1.032	0.737	1.006	0.523	2.106	1.005	1.464	2.101	1.015	0.725
	3	4	1.007	0.681	1.706	1.037	1.732	*	1.017	0.691	0.994	0.672	1.683	1.023	1.708	*	1.004	0.681
		5	0.998	0.707	2.186	1.006	1.634	2.184	1.008	0.702	0.971	0.689	2.128	0.980	1.591	2.126	0.981	0.683
		6	1.022	0.539	2.200	1.014	1.498	2.174	1.006	0.734	1.004	0.529	2.162	0.996	1.472	2.136	0.988	0.721
	4	4	1.006	0.688	1.754	1.014	1.758	*	1.011	0.692	0.979	0.669	1.706	0.986	1.710	*	0.984	0.673
		5	1.017	0.707	2.155	1.008	1.623	2.181	1.000	0.714	1.021	0.710	2.164	1.012	1.629	2.190	1.004	0.716
		6	0.990	0.535	2.192	1.003	1.518	2.209	0.999	0.732	1.003	0.542	2.221	1.016	1.538	2.239	1.012	0.741
	5	4	0.993	0.684	1.726	0.995	1.709	*	0.980	0.679	1.004	0.692	1.746	1.006	1.729	*	0.991	0.687
		5	0.999	0.701	2.154	1.005	1.638	2.152	1.001	0.707	0.987	0.693	2.128	0.994	1.619	2.126	0.989	0.699
		6	1.021	0.544	2.214	1.002	1.540	2.220	1.010	0.738	1.019	0.543	2.209	1.000	1.537	2.215	1.008	0.736
	6	4	0.996	0.687	1.723	0.997	1.742	*	1.002	0.689	1.004	0.692	1.735	1.005	1.754	*	1.009	0.694
		5	1.014	0.711	2.219	1.019	1.626	2.258	1.009	0.715	1.008	0.707	2.206	1.013	1.617	2.245	1.003	0.710
		6	0.997	0.539	2.203	1.001	1.516	2.200	1.002	0.728	1.001	0.541	2.213	1.006	1.523	2.211	1.007	0.732

* k can be up to 1 when $m=4$

According to Table 3.11 the relative efficiency values are very close to 1. It can be assumed that the MSE of the regression model of PRSS, DRSS and TBRSS₁ methods have equivalent to the MSE of the regression model of SRS and RSS methods. Additionally, the MSE of the regression model of MRSS, LRSS₁ and LRSS₂ methods are always smaller than the MSE of the regression model of SRS and RSS methods. The MSE of the regression model of MRSS and LRSS₂ methods are approximately two times better than the MSE of the regression model of SRS and RSS methods when $\rho=0.95$.

Table 3.12 Bias values of the estimators $(\bar{y}, \hat{\beta}_0, \hat{\beta}_1)$ for sampling methods under 20% outlier case

ρ	r	m	$Bias(\bar{y})$												$Bias(\hat{\beta}_0)$												$Bias(\hat{\beta}_1)$											
			SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂						
0.50	4	4	0.012	0.005	0.004	0.020	-0.012	-0.007	-0.009	*	-0.005	0.029	-0.004	-0.001	-0.003	0.006	-0.001	-0.014	0.012	*	-0.008	0.020	0.040	0.029	0.028	-0.011	0.078	0.022	0.075	*	0.020	-0.005						
		5	-0.013	-0.008	0.002	0.014	-0.020	0.009	-0.023	-0.012	0.005	0.020	-0.014	-0.008	0.000	0.006	-0.009	0.008	-0.018	0.002	0.001	0.010	0.021	0.026	0.011	0.001	0.106	0.020	0.066	0.091	0.007	0.002						
		6	-0.001	0.004	0.009	0.039	-0.016	0.000	-0.022	-0.015	0.007	0.014	-0.013	0.000	-0.001	0.025	-0.002	-0.003	-0.022	0.000	0.008	0.009	0.016	0.012	0.022	-0.017	0.097	0.007	0.072	0.088	0.026	-0.003						
		3	-0.002	-0.004	-0.009	0.027	-0.013	0.001	-0.008	*	-0.002	0.022	-0.006	-0.012	-0.003	0.014	-0.008	-0.001	0.000	*	-0.005	0.013	0.017	0.027	0.024	-0.012	0.090	0.027	0.089	*	0.007	0.004						
		5	-0.003	-0.008	-0.003	0.021	-0.018	0.002	-0.019	-0.018	-0.003	0.026	-0.008	-0.012	-0.003	0.015	-0.008	0.000	-0.013	-0.009	-0.003	0.019	0.017	0.026	0.007	-0.016	0.092	0.101	0.067	0.103	0.019	-0.013						
		6	0.003	-0.006	0.006	0.041	-0.020	-0.002	-0.018	-0.011	0.002	0.024	-0.004	-0.008	0.006	0.030	-0.010	-0.005	-0.014	0.000	0.000	0.017	0.006	0.012	0.016	-0.023	0.108	0.000	0.064	0.096	0.006	-0.024						
0.75	4	4	-0.004	-0.003	-0.004	0.016	-0.016	-0.004	-0.020	*	-0.001	0.022	-0.008	-0.006	-0.009	0.005	-0.012	-0.005	-0.012	*	-0.003	0.010	0.002	0.024	0.013	-0.010	0.082	0.022	0.099	*	0.018	-0.011						
		5	-0.001	-0.001	0.009	0.022	-0.015	0.005	-0.026	-0.017	0.003	0.022	-0.003	-0.002	0.004	0.015	-0.004	0.002	-0.019	-0.007	0.002	0.014	0.007	0.006	0.017	-0.026	0.072	-0.001	0.065	0.112	0.011	-0.012						
		6	-0.002	-0.001	0.006	0.039	-0.007	-0.002	-0.017	-0.016	0.001	0.017	-0.001	-0.002	0.004	0.029	0.002	-0.004	-0.013	-0.006	-0.002	0.013	0.010	0.008	0.006	-0.028	0.106	0.009	0.069	0.088	0.015	-0.010						
		4	-0.010	0.009	-0.006	0.014	-0.014	-0.003	-0.015	*	0.004	0.023	-0.011	0.007	-0.007	0.007	-0.007	-0.008	-0.007	*	0.004	0.015	0.021	0.012	0.013	-0.011	0.078	0.004	0.070	*	-0.003	-0.016						
		5	-0.004	-0.001	-0.006	0.020	-0.011	-0.002	-0.023	-0.017	-0.006	0.020	-0.004	-0.001	-0.004	0.017	-0.002	-0.004	-0.019	-0.008	-0.006	0.015	0.013	0.015	0.008	-0.014	0.098	0.005	0.068	0.102	0.011	-0.016						
		6	0.002	0.001	0.005	0.038	-0.014	0.002	-0.025	-0.019	0.003	0.019	0.000	0.001	0.002	0.029	-0.006	0.000	-0.021	-0.010	0.001	0.012	0.010	0.008	0.003	-0.028	0.078	0.008	0.064	0.093	0.004	-0.023						
0.95	4	4	0.001	-0.003	-0.001	0.012	-0.024	-0.007	-0.017	*	0.000	0.009	0.000	-0.006	-0.004	0.005	-0.016	-0.007	-0.011	*	-0.001	0.003	0.005	0.008	0.012	-0.012	0.086	0.007	0.079	*	0.011	-0.015						
		5	0.000	-0.002	-0.001	0.018	-0.016	0.003	-0.022	-0.015	-0.002	0.024	-0.001	-0.003	-0.001	0.013	-0.005	-0.001	-0.017	-0.005	-0.004	0.018	0.010	0.011	0.006	-0.018	0.105	0.003	0.064	0.102	0.007	-0.018						
		6	0.008	-0.005	-0.001	0.040	-0.007	0.002	-0.023	-0.013	0.006	0.016	-0.006	-0.006	0.000	0.030	-0.001	0.002	-0.019	-0.004	-0.005	0.011	0.005	0.011	0.009	-0.033	0.077	0.006	0.071	0.103	0.003	-0.020						
		4	0.013	0.001	0.007	0.031	-0.030	-0.008	-0.025	*	-0.006	0.035	-0.006	-0.005	-0.001	0.019	-0.026	-0.013	-0.017	*	-0.012	0.025	0.042	0.024	0.040	-0.006	0.065	0.017	0.079	*	0.014	-0.007						
		5	-0.013	-0.004	0.002	0.016	-0.037	0.015	-0.025	-0.042	0.002	0.013	-0.013	-0.004	-0.001	0.008	-0.033	0.012	-0.023	-0.035	-0.001	0.002	0.023	0.023	0.029	0.013	-0.001	0.119	0.027	0.088	0.094	0.006	-0.001					
		6	-0.001	-0.007	0.005	0.045	-0.036	-0.004	-0.024	-0.032	0.004	0.017	-0.012	-0.007	-0.001	0.030	-0.029	-0.007	-0.021	-0.025	0.005	0.012	0.017	0.015	0.019	-0.022	0.112	0.002	0.060	0.089	0.030	-0.006						
1.00	4	4	-0.002	-0.008	-0.010	0.030	-0.022	0.006	-0.035	*	0.001	0.022	-0.006	-0.015	-0.013	0.017	-0.023	0.003	-0.032	*	-0.003	0.012	0.018	0.026	0.025	-0.006	0.084	0.024	0.072	*	0.009	0.010						
		5	-0.003	-0.003	-0.001	0.024	-0.036	0.003	-0.025	-0.046	-0.001	0.030	-0.008	-0.007	-0.001	0.017	-0.031	0.001	-0.021	-0.040	-0.001	0.021	0.014	0.021	0.002	-0.016	0.089	0.010	0.071	0.102	0.018	-0.014						
		6	0.004	-0.007	0.001	0.051	-0.031	0.003	-0.020	-0.031	0.001	0.019	-0.006	-0.007	0.000	0.037	-0.023	0.000	-0.017	-0.022	0.000	0.010	0.005	0.008	0.012	-0.021	0.103	0.011	0.059	0.084	0.004	-0.019						
		5	-0.004	-0.004	0.002	0.029	-0.025	0.002	-0.022	*	-0.001	0.027	-0.008	-0.006	-0.001	0.018	-0.020	0.003	-0.015	*	-0.003	0.013	0.003	0.027	0.002	-0.009	0.085	0.015	0.093	*	0.019	-0.008						
		6	-0.001	-0.003	0.005	0.025	-0.037	0.008	-0.025	-0.034	0.002	0.025	-0.002	-0.005	-0.001	0.017	-0.029	0.004	-0.018	-0.028	0.002	0.016	0.009	0.004	0.011	-0.025	0.071	0.006	0.061	0.101	0.010	-0.014						
		6	-0.002	-0.005	0.006	0.053	-0.035	-0.001	-0.019	-0.037	-0.002	0.022	-0.010	0.004	-0.005	0.005	0.039	-0.028	-0.001	-0.015	-0.029	-0.005	0.017	0.012	0.007	0.004	-0.026	0.102	0.003	0.073	0.102	0.006	-0.012					
1.00	4	4	-0.010	0.007	-0.002	0.030	-0.027	0.003	-0.024	*	-0.001	0.033	-0.010	0.004	-0.004	0.024	-0.022	-0.001	-0.018	*	-0.002	0.023	0.022	0.006	0.006	-0.011	0.074	0.006	0.077	*	-0.005	-0.011						
		5	-0.004	0.002	-0.004	0.022	-0.033	-0.005	-0.022	-0.037	-0.007	0.022	-0.003	0.002	-0.002	0.020	-0.028	-0.006	-0.016	-0.031	-0.006	0.015	0.011	0.015	0.009	-0.013	0.083	0.008	0.069	0.097	0.008	-0.019						
		6	0.002	0.002	0.007	0.049	-0.040	-0.002	-0.022	-0.037	0.000	0.020	0.000	0.002	0.003	0.037	-0.031	-0.003	-0.016	-0.028	-0.002	0.011	0.009	0.007	0.008	-0.027	0.088	0.009	0.064	0.103	0.004	-0.021						
		4	0.000	-0.002	-0.003	0.021	-0.028	-0.001	-0.019	*	-0.001	0.018	-0.001	-0.005	-0.007	0.014	-0.020	0.000	-0.013	*	-0.002	0.013	0.007	0.002	0.012	-0.011	0.075	0.007	0.074	*	0.011	-0.016						
		5	0.000	-0.004	0.001	0.023	-0.035	0.003	-0.021	-0.034	0.000	0.028	-0.001	-0.004	0.001	0.017	-0.024	-0.001	-0.016	-0.025	-0.003	0.020	0.005	0.007	0.005	-0.019	0.099	0.008	0.065	0.102	0.009	-0.021						
		6	0.008	-0.003	-0.001	0.048	-0.032	-0.001	-0.023	-0.038	0.007	0.013	0.003	-0.004	0.000	0.036	-0.027	0.000	-0.017	-0.029	0.006	0.008	0.007	0.002	0.007	-0.033	0.096	0.011	0.064	0.100	0.009	-0.019						
1.00	4	4	0.014	0.006	0.007	0.022	-0.024	-0.001	-0.020	*	-0.008	0.030	-0.004	-0.001	0.000	0.009	-0.016	-0.004	-0.009	*	-0.014	0.016	0.034	0.008	0.027	0.008	0.019	0.011	0.026	*	0.009	-0.002						
		5	-0.013	-0.002	0.000	0.019	-0.023	0.003	-0.026	-0.028	-0.001	0.009	-0.009	-0.002	-0.003	0.015	-0.014	-0.001	-0.018	-0.016	-0.001	-0.001	0.014	0.032	0.008	0.002	-0.003	0.015	0.029	-0.006	0.000	-0.007						
		6	0.000	-0.011	0.006	0.037	-0.029	-0.007	-0.021	-0.030	0.001	0.016	-0.008	-0.009	0.000	0.024	-0.018	-0.009	-0.017	-0.019	0.001	0.011	0.011	0.006	0.011	-0.001	0.014	0.009	0.018	0.004	0.022	0.002						
		3	-0.002	-0.002	-0.012	0.032	-0.019	0.002	-0.025	*	0.006	0.017	-0.006	-0.007	-0.011	0.019	-0.018	-0.002	-0.019	*	0.000	0.009	0.010	0.009	0.013	0.004	0.010	0.018	0.022	*	0.002	0.014						
		5	-0.002	-0.003	-0.005	0.024	-0.025	0.007	-0.026	-0.027	0.000	0.027	-0.008	-0.009	-0.005	0.018	-0.015	0.003	-0.021	-0.017	-0.001	0.016	0.008	0.018	-0.006	-0.001	0.023	0.000	0.031	0.015	0.006	-0.004						
		6	0.004	-0.004	0.005	0.047	-0.022	0.000	-0.021	-0.025	0.004	0.022	-0.005	-0.003	0.003	0.033	-0.011	-0.003	-0.019	-0.014	0.003																	

According to Table 3.12, the bias values of the estimators of SRS, RSS and all the modified RSS methods are very close to 0. Additionally, the bias values of the mean estimator of ERSS, MRSS, LRSS₁, LRSS₂ and TBRSS₂ methods are greater than the bias values of the mean estimators of the other sampling methods.



CHAPTER FOUR

CONCLUSIONS

Ranked set sampling method can be used where the exact measurement is expensive, time-consuming or difficult, yet ranking observations are easy and expensive in ecological, medical and environmental studies. This study concerns the mean square error of the regression model, the regression estimator for the population mean and the regression coefficients, $\hat{\beta}_0$ and $\hat{\beta}_1$. In more details, these estimators are obtained for SRS, RSS, PRSS, ERSS, MRSS, DRSS, LRSS₁, LRSS₂, TBRSS₁ and TBRSS₂ methods. By the help of the Monte Carlo simulation using R project, the performances of the estimators are evaluated in terms of the bias and the relative efficiency for different correlation coefficients, the set and the cycle sizes under BVN with different parameters for the normal and the outlier cases.

Based on the results of simulation for the normal case, it can be summarized that the mean estimators of RSS and all the modified RSS methods are more efficient than the mean estimator of SRS method for all cases. The mean estimator of DRSS method is better than the mean estimators of the other modified RSS methods when compared to the mean estimator of the RSS method. The relative efficiency values increase when the set size or the correlation level increases for the mean estimator. The regression estimator $\hat{\beta}_1$ of ERSS and TBRSS₂ methods are better than the regression estimator $\hat{\beta}_1$ of SRS and RSS methods. In addition to all this, the relative efficiency values of $\bar{y}$ in TBRSS method decrease when k increases. The relative efficiency values of $\hat{\beta}_1$ in LRSS method decrease and the relative efficiency values of $\hat{\beta}_1$ in TBRSS method increase when k increases.

Based on the results of simulation for the outlier case, it can be summarized that the mean estimators of RSS and all the modified RSS methods are more efficient than the mean estimator of SRS method except ERSS and TBRSS₂ methods. In all cases, the estimators of MRSS, LRSS₁ and LRSS₂ methods are better than the estimators of SRS and RSS methods except the regression estimator $\hat{\beta}_1$. The regression estimator $\hat{\beta}_1$ of ERSS and TBRSS₂ methods are better than the regression estimators $\hat{\beta}_1$ of SRS

and RSS methods. In addition to all this, the relative efficiency values of LRSS method decrease and the relative efficiency values of TBRSS method increase when k increases for $\bar{y}$, $\hat{\beta}_0$ and MSE of the regression model. The relative efficiency values of the regression estimator $\hat{\beta}_1$ of LRSS method decrease and the relative efficiency values of the regression estimator $\hat{\beta}_1$ of TBRSS method increase when k increases.

As a result, RSS and all the modified RSS methods can be utilized for estimating the population mean instead of SRS method when the population has no outlier. MRSS, LRSS₁ and LRSS₂ methods may be employed for estimating the population mean, $\hat{\beta}_0$ and the MSE of the regression model when the population possesses an outlier. In both normal and outlier cases, ERSS and TBRSS₂ methods might be used for estimating the regression estimator $\hat{\beta}_1$.

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APPENDICES

Table A.1 RE_1 values of the estimators under $(X,Y)\sim\text{BVN}(3,7,1,4,\rho)$

ρ	r	m	$\bar{y}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\sigma}^2$
0.50	2	4	1.163	1.167	1.170	0.995
		5	1.192	1.149	1.169	0.988
		6	1.183	1.150	1.156	1.010
	3	4	1.170	1.121	1.118	0.997
		5	1.226	1.099	1.092	0.998
		6	1.213	1.089	1.092	0.995
	4	4	1.200	1.049	1.057	1.002
		5	1.250	1.075	1.070	1.003
		6	1.217	1.073	1.078	1.001
	5	4	1.205	1.028	1.029	1.001
		5	1.189	1.012	1.020	1.001
		6	1.229	1.048	1.040	0.997
	6	4	1.205	1.057	1.041	0.995
		5	1.188	1.034	1.025	0.996
		6	1.209	1.004	1.006	0.999
0.75	2	4	1.454	1.149	1.143	0.997
		5	1.558	1.131	1.155	0.993
		6	1.611	1.160	1.151	1.011
	3	4	1.476	1.122	1.119	1.007
		5	1.580	1.078	1.078	1.002
		6	1.635	1.101	1.087	0.997
	4	4	1.514	1.060	1.066	0.996
		5	1.588	1.092	1.089	1.000
		6	1.630	1.061	1.063	0.997
	5	4	1.512	1.062	1.062	1.005
		5	1.541	1.037	1.054	1.004
		6	1.644	1.050	1.037	1.002
	6	4	1.503	1.023	1.014	0.995
		5	1.608	1.044	1.045	0.998
		6	1.633	1.002	1.007	1.002
0.95	2	4	2.018	1.120	1.119	0.989
		5	2.344	1.151	1.159	0.990
		6	2.595	1.110	1.119	1.002
	3	4	2.047	1.108	1.103	1.007
		5	2.347	1.106	1.122	1.000
		6	2.685	1.102	1.084	0.997
	4	4	2.106	1.072	1.075	0.991
		5	2.324	1.079	1.082	0.997
		6	2.588	1.051	1.061	1.000
	5	4	2.117	1.064	1.065	1.005
		5	2.358	1.054	1.061	0.998
		6	2.662	1.032	1.022	1.005
	6	4	2.109	1.031	1.020	0.995
		5	2.401	1.035	1.037	1.000
		6	2.607	1.021	1.026	1.000

Table A.2 RE_2 and RE_3 values for the mean estimator under $(X,Y) \sim \text{BVN}(3,7,1,4,\rho)$

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.111	1.195	1.172	1.211	1.226	*	1.177	1.152	0.955	1.027	1.008	1.041	1.054	*	1.013	0.991
		5	1.151	1.184	1.217	1.243	1.219	1.206	1.195	1.161	0.966	0.994	1.021	1.043	1.023	1.012	1.003	0.974
		6	1.122	1.178	1.198	1.242	1.189	1.185	1.191	1.158	0.948	0.995	1.012	1.050	1.005	1.002	1.007	0.979
	3	4	1.109	1.143	1.187	1.216	1.167	*	1.139	1.141	0.948	0.977	1.015	1.040	0.997	*	0.973	0.976
		5	1.155	1.179	1.191	1.208	1.223	1.206	1.180	1.196	0.942	0.962	0.972	0.986	0.997	0.984	0.963	0.976
		6	1.171	1.171	1.230	1.233	1.237	1.242	1.220	1.227	0.966	0.965	1.014	1.016	1.020	1.023	1.005	1.012
	4	4	1.148	1.188	1.212	1.240	1.204	*	1.185	1.160	0.957	0.990	1.010	1.033	1.004	*	0.988	0.967
		5	1.196	1.214	1.228	1.285	1.244	1.201	1.193	1.176	0.957	0.971	0.982	1.028	0.995	0.961	0.955	0.941
		6	1.195	1.187	1.249	1.293	1.222	1.230	1.217	1.213	0.982	0.975	1.026	1.062	1.004	1.011	1.000	0.997
	5	4	1.178	1.191	1.261	1.248	1.232	*	1.192	1.182	0.978	0.989	1.047	1.036	1.022	*	0.990	0.981
		5	1.158	1.127	1.195	1.244	1.225	1.205	1.181	1.167	0.974	0.948	1.004	1.046	1.030	1.013	0.993	0.981
		6	1.169	1.183	1.232	1.294	1.222	1.247	1.220	1.238	0.952	0.963	1.003	1.053	0.995	1.015	0.993	1.008
	6	4	1.131	1.190	1.189	1.229	1.193	*	1.181	1.209	0.938	0.988	0.987	1.020	0.990	*	0.980	1.003
		5	1.136	1.157	1.202	1.242	1.199	1.236	1.178	1.167	0.956	0.974	1.012	1.045	1.009	1.041	0.991	0.982
		6	1.135	1.162	1.213	1.239	1.190	1.192	1.187	1.190	0.939	0.962	1.004	1.025	0.985	0.987	0.982	0.984
0.75	2	4	1.271	1.448	1.541	1.677	1.588	*	1.508	1.397	0.874	0.996	1.059	1.153	1.092	*	1.037	0.960
		5	1.431	1.529	1.676	1.787	1.625	1.706	1.551	1.472	0.919	0.981	1.076	1.147	1.043	1.095	0.996	0.945
		6	1.417	1.466	1.715	1.811	1.672	1.714	1.595	1.511	0.879	0.910	1.065	1.124	1.038	1.064	0.990	0.938
	3	4	1.301	1.406	1.617	1.661	1.552	*	1.457	1.398	0.882	0.953	1.096	1.125	1.051	*	0.987	0.947
		5	1.479	1.476	1.661	1.761	1.689	1.708	1.552	1.465	0.936	0.934	1.052	1.114	1.069	1.081	0.982	0.927
		6	1.481	1.469	1.751	1.888	1.678	1.732	1.631	1.575	0.906	0.899	1.071	1.155	1.026	1.059	0.997	0.963
	4	4	1.326	1.434	1.607	1.692	1.595	*	1.497	1.440	0.876	0.947	1.062	1.118	1.054	*	0.989	0.951
		5	1.486	1.535	1.693	1.818	1.663	1.682	1.558	1.498	0.936	0.966	1.066	1.144	1.047	1.059	0.981	0.943
		6	1.494	1.485	1.741	1.914	1.717	1.760	1.650	1.603	0.916	0.911	1.068	1.174	1.053	1.080	1.012	0.983
	5	4	1.349	1.423	1.613	1.716	1.600	*	1.549	1.454	0.892	0.941	1.067	1.135	1.059	*	1.024	0.962
		5	1.408	1.463	1.674	1.768	1.629	1.659	1.526	1.477	0.914	0.949	1.086	1.147	1.057	1.076	0.990	0.958
		6	1.461	1.493	1.818	1.905	1.733	1.788	1.618	1.604	0.889	0.909	1.106	1.159	1.054	1.088	0.984	0.976
	6	4	1.324	1.404	1.581	1.685	1.570	*	1.504	1.454	0.881	0.935	1.052	1.121	1.044	*	1.001	0.968
		5	1.404	1.480	1.661	1.766	1.669	1.663	1.544	1.476	0.873	0.920	1.033	1.098	1.038	1.034	0.960	0.918
		6	1.439	1.480	1.725	1.869	1.673	1.758	1.592	1.557	0.881	0.906	1.056	1.145	1.024	1.076	0.975	0.953
0.95	2	4	1.539	1.896	2.307	2.816	2.353	*	2.108	1.826	0.763	0.940	1.143	1.396	1.166	*	1.045	0.905
		5	1.958	2.134	2.791	3.325	2.691	2.773	2.348	2.079	0.835	0.910	1.191	1.419	1.148	1.183	1.002	0.887
		6	1.960	2.110	3.033	3.768	2.935	3.042	2.567	2.292	0.755	0.813	1.169	1.452	1.131	1.172	0.989	0.883
	3	4	1.565	1.833	2.402	2.815	2.344	*	2.049	1.821	0.765	0.895	1.174	1.376	1.145	*	1.001	0.890
		5	1.991	2.076	2.750	3.275	2.666	2.796	2.346	2.112	0.849	0.885	1.172	1.396	1.136	1.192	1.000	0.900
		6	2.107	2.097	3.145	3.718	2.949	3.081	2.570	2.356	0.785	0.781	1.171	1.384	1.098	1.147	0.957	0.877
	4	4	1.613	1.870	2.451	2.880	2.399	*	2.068	1.889	0.766	0.888	1.164	1.367	1.139	*	0.982	0.897
		5	2.034	2.143	2.825	3.347	2.683	2.816	2.340	2.079	0.875	0.922	1.216	1.440	1.154	1.212	1.007	0.895
		6	2.081	2.091	3.092	3.823	3.003	3.146	2.604	2.396	0.804	0.808	1.195	1.477	1.160	1.215	1.006	0.926
	5	4	1.663	1.898	2.408	2.918	2.450	*	2.126	1.888	0.786	0.897	1.138	1.379	1.157	*	1.004	0.892
		5	1.943	2.068	2.751	3.377	2.631	2.777	2.351	2.125	0.824	0.877	1.167	1.432	1.116	1.178	0.997	0.901
		6	2.040	2.094	3.095	3.858	2.986	3.175	2.541	2.381	0.766	0.787	1.163	1.449	1.122	1.193	0.954	0.895
	6	4	1.612	1.887	2.349	2.840	2.340	*	2.073	1.940	0.764	0.895	1.114	1.347	1.110	*	0.983	0.920
		5	1.923	2.110	2.765	3.231	2.678	2.794	2.346	2.071	0.801	0.879	1.152	1.346	1.115	1.164	0.977	0.863
		6	1.978	2.085	3.131	3.755	2.864	3.111	2.667	2.355	0.759	0.800	1.201	1.440	1.098	1.193	1.023	0.903

* k can be up to 1 when $m=4$

Table A.3 RE_2 and RE_3 values for $\hat{\beta}_0$ under $(X,Y)\sim\text{BVN}(3,7,1,4,\rho)$

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.160	1.913	0.487	1.258	0.486	*	1.209	1.893	0.993	1.639	0.417	1.078	0.417	*	1.035	1.622
		5	1.191	1.762	0.310	1.231	0.557	0.310	1.156	1.715	1.037	1.534	0.270	1.071	0.485	0.270	1.006	1.493
		6	1.132	2.231	0.302	1.219	0.613	0.314	1.125	1.670	0.985	1.940	0.262	1.060	0.533	0.273	0.979	1.453
	3	4	1.111	1.689	0.482	1.145	0.496	*	1.130	1.725	0.991	1.507	0.430	1.021	0.442	*	1.008	1.539
		5	1.075	1.612	0.308	1.065	0.541	0.299	1.081	1.616	0.978	1.467	0.280	0.969	0.492	0.272	0.983	1.470
		6	1.098	2.115	0.308	1.123	0.609	0.316	1.101	1.586	1.008	1.942	0.283	1.031	0.559	0.290	1.011	1.456
	4	4	1.026	1.595	0.471	1.088	0.475	*	1.051	1.652	0.978	1.520	0.449	1.037	0.453	*	1.001	1.575
		5	1.110	1.592	0.311	1.130	0.540	0.310	1.105	1.554	1.033	1.480	0.289	1.051	0.502	0.288	1.028	1.446
		6	1.045	1.976	0.304	1.084	0.612	0.304	1.061	1.492	0.974	1.841	0.284	1.010	0.571	0.283	0.988	1.390
	5	4	1.016	1.580	0.475	1.078	0.462	*	1.026	1.621	0.989	1.537	0.462	1.049	0.450	*	0.999	1.577
		5	0.990	1.474	0.295	1.032	0.514	0.294	1.034	1.490	0.978	1.456	0.291	1.020	0.507	0.290	1.022	1.472
		6	1.101	1.910	0.300	1.063	0.564	0.299	1.016	1.436	0.971	1.823	0.287	1.014	0.538	0.285	0.969	1.371
	6	4	1.056	1.538	0.464	1.060	0.470	*	1.045	1.537	1.000	1.456	0.440	1.003	0.445	*	0.989	1.455
		5	1.036	1.488	0.311	1.040	0.521	0.307	1.057	1.511	1.002	1.438	0.300	1.005	0.504	0.296	1.022	1.461
		6	1.013	1.891	0.296	1.029	0.572	0.293	1.002	1.419	1.009	1.884	0.294	1.025	0.570	0.292	0.999	1.414
0.75	2	4	1.171	1.864	0.496	1.261	0.499	*	1.176	1.846	1.019	1.623	0.432	1.098	0.434	*	1.024	1.607
		5	1.150	1.696	0.309	1.192	0.568	0.312	1.134	1.651	1.017	1.499	0.273	1.054	0.502	0.276	1.003	1.459
		6	1.139	2.128	0.318	1.194	0.640	0.318	1.115	1.624	0.982	1.835	0.274	1.030	0.552	0.274	0.961	1.400
	3	4	1.116	1.684	0.496	1.145	0.502	*	1.132	1.724	0.995	1.501	0.442	1.020	0.447	*	1.009	1.536
		5	1.094	1.560	0.312	1.114	0.562	0.312	1.093	1.533	1.015	1.447	0.289	1.033	0.522	0.290	1.013	1.422
		6	1.101	2.034	0.332	1.095	0.651	0.332	1.087	1.548	0.999	1.847	0.302	0.994	0.591	0.301	0.987	1.405
	4	4	1.070	1.566	0.479	1.083	0.486	*	1.047	1.630	1.009	1.477	0.452	1.021	0.458	*	0.987	1.537
		5	1.065	1.525	0.321	1.067	0.556	0.322	1.102	1.483	0.976	1.398	0.295	0.977	0.509	0.295	1.009	1.358
		6	1.058	1.878	0.333	1.061	0.627	0.323	1.069	1.454	0.998	1.770	0.314	1.000	0.591	0.304	1.008	1.370
	5	4	1.054	1.575	0.517	1.100	0.500	*	1.037	1.592	0.992	1.483	0.486	1.035	0.471	*	0.977	1.499
		5	1.007	1.507	0.313	1.013	0.553	0.318	1.075	1.426	0.971	1.453	0.302	0.977	0.533	0.307	1.036	1.375
		6	1.026	1.847	0.325	1.057	0.621	0.322	1.061	1.393	0.978	1.759	0.310	1.007	0.591	0.307	1.010	1.327
	6	4	1.038	1.487	0.489	1.051	0.499	*	1.049	1.517	1.014	1.453	0.478	1.027	0.487	*	1.026	1.482
		5	1.060	1.497	0.330	1.055	0.560	0.331	1.077	1.476	1.015	1.434	0.316	1.011	0.537	0.317	1.031	1.413
		6	0.989	1.857	0.320	1.026	0.598	0.309	0.993	1.353	0.987	1.853	0.319	1.024	0.597	0.309	0.991	1.350
0.95	2	4	1.138	1.808	0.480	1.222	0.502	*	1.193	1.857	1.016	1.615	0.429	1.091	0.448	*	1.066	1.658
		5	1.150	1.701	0.309	1.186	0.574	0.320	1.120	1.702	0.999	1.478	0.269	1.030	0.499	0.278	0.973	1.479
		6	1.118	2.091	0.312	1.158	0.625	0.321	1.083	1.607	1.007	1.883	0.281	1.043	0.563	0.289	0.975	1.447
	3	4	1.057	1.676	0.490	1.130	0.491	*	1.076	1.623	0.954	1.513	0.442	1.020	0.443	*	0.972	1.466
		5	1.093	1.579	0.314	1.118	0.587	0.327	1.087	1.599	0.988	1.427	0.283	1.010	0.531	0.296	0.983	1.445
		6	1.079	2.042	0.319	1.104	0.633	0.331	1.097	1.512	0.979	1.852	0.289	1.001	0.574	0.300	0.995	1.372
	4	4	1.081	1.577	0.501	1.087	0.483	*	1.063	1.624	1.009	1.471	0.467	1.014	0.451	*	0.992	1.515
		5	1.049	1.535	0.324	1.096	0.564	0.335	1.091	1.515	0.973	1.424	0.300	1.016	0.523	0.311	1.012	1.405
		6	1.061	1.899	0.312	1.061	0.619	0.328	1.051	1.420	1.009	1.807	0.297	1.010	0.589	0.313	1.000	1.351
	5	4	1.058	1.654	0.507	1.055	0.507	*	1.073	1.605	0.994	1.555	0.477	0.992	0.477	*	1.009	1.509
		5	1.034	1.491	0.312	1.062	0.560	0.306	1.063	1.443	0.981	1.414	0.296	1.007	0.531	0.290	1.008	1.368
		6	1.038	1.885	0.308	1.080	0.612	0.313	1.039	1.393	1.006	1.827	0.299	1.047	0.594	0.303	1.007	1.350
	6	4	1.003	1.560	0.477	1.037	0.507	*	1.062	1.542	0.973	1.514	0.462	1.006	0.492	*	1.030	1.496
		5	1.059	1.523	0.322	1.062	0.564	0.315	1.024	1.479	1.023	1.472	0.311	1.027	0.545	0.305	0.990	1.429
		6	1.036	1.876	0.312	1.036	0.595	0.316	1.044	1.441	1.014	1.837	0.306	1.014	0.582	0.310	1.022	1.411

*k can be up to 1 when $m=4$

Table A.4 RE_2 and RE_3 values for $\hat{\beta}_1$ under $(X,Y)\sim\text{BVN}(3,7,1,4,\rho)$

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.171	2.047	0.465	1.273	0.469	*	1.219	2.018	1.001	1.750	0.397	1.089	0.401	*	1.042	1.725
		5	1.207	1.884	0.292	1.259	0.534	0.292	1.181	1.838	1.032	1.611	0.250	1.076	0.457	0.250	1.009	1.571
		6	1.140	2.513	0.281	1.215	0.586	0.291	1.135	1.746	0.987	2.174	0.243	1.051	0.507	0.252	0.982	1.511
	3	4	1.101	1.782	0.453	1.132	0.462	*	1.117	1.812	0.985	1.594	0.406	1.013	0.413	*	1.000	1.621
		5	1.084	1.689	0.285	1.066	0.510	0.276	1.083	1.690	0.992	1.546	0.261	0.975	0.467	0.252	0.991	1.547
		6	1.091	2.345	0.283	1.140	0.574	0.291	1.098	1.657	0.999	2.146	0.259	1.043	0.525	0.267	1.005	1.517
	4	4	1.028	1.693	0.446	1.086	0.446	*	1.061	1.755	0.973	1.602	0.422	1.028	0.422	*	1.005	1.661
		5	1.113	1.657	0.287	1.128	0.508	0.288	1.118	1.646	1.041	1.549	0.269	1.054	0.475	0.269	1.045	1.538
		6	1.064	2.195	0.280	1.095	0.589	0.283	1.080	1.586	0.987	2.036	0.260	1.016	0.546	0.263	1.002	1.471
	5	4	1.014	1.717	0.447	1.072	0.432	*	1.015	1.706	0.986	1.668	0.434	1.042	0.420	*	0.986	1.657
		5	1.008	1.576	0.274	1.050	0.492	0.274	1.047	1.577	0.989	1.546	0.269	1.030	0.482	0.269	1.027	1.547
		6	1.013	2.130	0.275	1.046	0.537	0.275	1.010	1.476	0.974	2.048	0.265	1.006	0.516	0.265	0.972	1.419
	6	4	1.039	1.620	0.432	1.041	0.439	*	1.035	1.619	0.998	1.557	0.415	1.000	0.421	*	0.995	1.556
		5	1.034	1.563	0.284	1.026	0.491	0.280	1.049	1.579	1.008	1.525	0.277	1.001	0.479	0.273	1.024	1.541
		6	1.029	2.089	0.271	1.036	0.543	0.270	0.995	1.476	1.022	2.076	0.269	1.030	0.539	0.268	0.989	1.467
0.75	2	4	1.171	1.996	0.469	1.250	0.476	*	1.173	1.932	1.025	1.747	0.411	1.094	0.416	*	1.026	1.690
		5	1.157	1.796	0.295	1.219	0.548	0.297	1.154	1.759	1.001	1.555	0.256	1.055	0.475	0.257	0.999	1.523
		6	1.128	2.325	0.294	1.183	0.607	0.294	1.124	1.677	0.980	2.020	0.255	1.028	0.528	0.256	0.976	1.457
	3	4	1.104	1.757	0.472	1.132	0.475	*	1.131	1.801	0.987	1.571	0.422	1.012	0.425	*	1.011	1.610
		5	1.083	1.612	0.292	1.104	0.535	0.292	1.080	1.601	1.004	1.496	0.271	1.024	0.496	0.271	1.002	1.485
		6	1.080	2.189	0.306	1.098	0.619	0.307	1.073	1.598	0.994	2.015	0.282	1.011	0.569	0.282	0.987	1.470
	4	4	1.073	1.655	0.456	1.072	0.463	*	1.051	1.703	1.007	1.553	0.427	1.006	0.434	*	0.986	1.598
		5	1.067	1.590	0.302	1.071	0.532	0.303	1.096	1.558	0.979	1.460	0.277	0.983	0.489	0.278	1.006	1.430
		6	1.067	2.102	0.312	1.068	0.599	0.302	1.057	1.510	1.004	1.978	0.294	1.005	0.564	0.284	0.994	1.421
	5	4	1.057	1.688	0.494	1.104	0.478	*	1.038	1.687	0.996	1.589	0.465	1.040	0.450	*	0.978	1.589
		5	1.033	1.593	0.296	1.035	0.539	0.303	1.088	1.514	0.980	1.512	0.280	0.982	0.512	0.287	1.032	1.436
		6	1.024	2.015	0.298	1.036	0.593	0.297	1.045	1.428	0.988	1.943	0.287	0.999	0.572	0.287	1.008	1.377
	6	4	1.012	1.564	0.455	1.036	0.466	*	1.037	1.574	0.999	1.543	0.449	1.022	0.459	*	1.023	1.552
		5	1.063	1.582	0.307	1.054	0.534	0.310	1.072	1.544	1.018	1.514	0.294	1.009	0.512	0.297	1.027	1.478
		6	0.981	2.038	0.298	1.024	0.571	0.289	0.988	1.403	0.974	2.023	0.296	1.017	0.567	0.287	0.981	1.393
0.95	2	4	1.141	1.902	0.456	1.223	0.478	*	1.186	1.899	1.020	1.700	0.408	1.093	0.427	*	1.060	1.698
		5	1.156	1.788	0.295	1.210	0.555	0.305	1.134	1.804	0.997	1.543	0.254	1.044	0.479	0.263	0.979	1.557
		6	1.117	2.329	0.295	1.163	0.601	0.304	1.095	1.678	0.998	2.081	0.264	1.039	0.537	0.272	0.978	1.500
	3	4	1.052	1.749	0.464	1.133	0.468	*	1.073	1.676	0.954	1.586	0.421	1.028	0.425	*	0.973	1.520
		5	1.088	1.666	0.298	1.122	0.566	0.310	1.095	1.674	0.969	1.485	0.265	1.000	0.504	0.277	0.976	1.492
		6	1.070	2.212	0.297	1.108	0.598	0.309	1.086	1.555	0.987	2.040	0.274	1.023	0.551	0.285	1.002	1.434
	4	4	1.081	1.646	0.474	1.069	0.463	*	1.055	1.690	1.006	1.531	0.441	0.995	0.431	*	0.981	1.572
		5	1.066	1.610	0.306	1.106	0.543	0.319	1.102	1.590	0.986	1.489	0.283	1.022	0.502	0.295	1.018	1.470
		6	1.065	2.121	0.296	1.060	0.603	0.309	1.069	1.501	1.004	1.999	0.279	0.999	0.568	0.292	1.008	1.415
	5	4	1.059	1.743	0.487	1.055	0.486	*	1.075	1.669	0.995	1.636	0.457	0.991	0.457	*	1.009	1.567
		5	1.050	1.583	0.296	1.064	0.541	0.290	1.059	1.530	0.989	1.492	0.279	1.002	0.509	0.273	0.998	1.442
		6	1.029	2.060	0.287	1.071	0.584	0.290	1.022	1.443	1.008	2.016	0.281	1.048	0.572	0.284	1.001	1.412
	6	4	0.997	1.618	0.452	1.034	0.482	*	1.053	1.598	0.978	1.587	0.443	1.014	0.472	*	1.033	1.567
		5	1.059	1.590	0.303	1.053	0.541	0.298	1.025	1.542	1.022	1.534	0.293	1.016	0.522	0.287	0.989	1.488
		6	1.034	2.038	0.292	1.038	0.572	0.297	1.036	1.499	1.008	1.986	0.285	1.011	0.558	0.289	1.009	1.461

* k can be up to 1 when $m=4$

Table A.5 RE_2 and RE_3 values of MSE of the regression model under $(X,Y)\sim\text{BVN}(3,7,1,4,\rho)$

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.000	1.004	0.999	0.996	0.997	*	0.997	0.997	1.005	1.009	1.004	1.001	1.002	*	1.002	1.002
		5	0.996	1.007	1.002	0.998	1.005	1.003	1.006	1.006	1.008	1.019	1.014	1.010	1.017	1.015	1.019	1.018
		6	1.009	1.001	1.006	1.007	1.006	1.010	1.009	1.007	0.999	0.991	0.996	0.997	0.996	1.000	0.999	0.998
	3	4	1.003	1.014	1.000	1.000	1.005	*	1.001	1.000	1.006	1.017	1.003	1.003	1.008	*	1.004	1.003
		5	0.989	1.002	1.005	0.997	0.997	1.005	1.000	0.992	0.991	1.004	1.007	1.000	0.999	1.007	1.002	0.994
		6	0.992	0.998	0.999	1.000	0.997	1.006	0.999	1.000	0.997	1.003	1.004	1.006	1.002	1.011	1.004	1.005
	4	4	1.001	0.997	0.997	1.002	1.002	*	0.997	1.000	0.999	0.995	0.995	1.000	1.000	*	0.994	0.998
		5	1.007	1.005	1.007	1.000	1.003	1.011	1.003	1.001	1.004	1.002	1.004	0.997	1.000	1.008	0.999	0.998
		6	0.997	1.001	1.002	1.005	1.002	1.000	0.997	0.995	0.996	1.000	1.001	1.004	1.001	0.999	0.996	0.994
	5	4	1.002	0.999	1.006	1.002	1.013	*	1.007	1.000	1.000	0.998	1.005	1.001	1.011	*	1.005	0.998
		5	0.994	0.997	0.999	1.001	0.997	0.997	1.001	0.997	0.993	0.996	0.998	1.000	0.996	0.996	1.000	0.996
		6	1.002	0.999	1.003	0.995	0.997	1.003	1.002	1.001	1.005	1.002	1.005	0.997	0.999	1.006	1.004	1.003
	6	4	0.998	0.993	0.996	0.995	0.996	*	0.999	0.995	1.003	0.998	1.001	1.000	1.001	*	1.004	1.000
		5	1.002	0.995	1.002	0.995	1.005	1.000	0.998	0.996	1.005	0.998	1.005	0.998	1.008	1.003	1.002	0.999
		6	0.999	1.003	1.007	1.001	1.002	1.007	0.998	1.001	1.000	1.004	1.008	1.002	1.003	1.008	0.999	1.002
0.75	2	4	0.991	0.986	1.003	0.998	1.013	*	1.007	0.981	0.994	0.989	1.006	1.001	1.016	*	1.010	0.984
		5	0.996	0.988	1.004	0.999	1.010	1.005	0.993	0.994	1.003	0.995	1.012	1.006	1.018	1.012	1.000	1.002
		6	1.009	0.976	1.020	1.010	1.018	1.015	1.011	0.992	0.998	0.965	1.008	0.998	1.007	1.004	1.000	0.980
	3	4	1.002	0.997	1.011	1.002	1.013	*	1.004	0.989	0.995	0.990	1.004	0.996	1.006	*	0.997	0.982
		5	0.998	0.995	1.011	1.003	1.004	1.010	1.005	0.990	0.996	0.993	1.010	1.001	1.002	1.008	1.003	0.988
		6	0.996	0.981	1.010	1.003	1.005	1.006	1.004	0.984	1.000	0.985	1.013	1.007	1.009	1.010	1.007	0.988
	4	4	1.000	0.988	1.010	1.007	1.002	*	0.998	0.987	1.003	0.992	1.014	1.010	1.006	*	1.002	0.990
		5	0.998	0.992	1.009	0.997	1.015	1.015	1.002	0.990	0.998	0.992	1.010	0.997	1.015	1.015	1.002	0.990
		6	0.998	0.976	1.013	0.999	1.011	1.012	0.995	0.989	1.001	0.979	1.016	1.002	1.014	1.015	0.998	0.992
	5	4	1.006	0.990	1.014	1.002	1.012	*	1.006	0.987	1.002	0.986	1.010	0.998	1.008	*	1.001	0.982
		5	0.994	0.987	1.012	0.997	1.008	1.005	1.002	0.986	0.991	0.983	1.008	0.993	1.005	1.002	0.998	0.982
		6	1.001	0.978	1.005	0.997	1.012	1.012	1.002	0.989	0.999	0.976	1.003	0.995	1.011	1.011	1.000	0.987
	6	4	0.995	0.983	1.007	1.001	1.002	*	0.997	0.984	0.999	0.988	1.012	1.005	1.006	*	1.002	0.988
		5	1.002	0.985	1.012	1.001	1.012	1.008	0.997	0.988	1.005	0.988	1.014	1.003	1.015	1.010	1.000	0.990
		6	0.999	0.982	1.012	1.002	1.011	1.009	0.999	0.988	0.997	0.980	1.009	0.999	1.009	1.006	0.997	0.985
0.95	2	4	0.998	0.978	1.006	1.004	1.005	*	1.001	0.981	1.009	0.989	1.017	1.015	1.016	*	1.011	0.992
		5	0.993	0.980	0.999	0.991	1.007	1.009	0.996	0.985	1.004	0.990	1.009	1.002	1.018	1.020	1.006	0.996
		6	1.001	0.974	1.014	1.009	1.013	1.015	0.997	0.987	0.999	0.972	1.012	1.007	1.011	1.013	0.995	0.985
	3	4	1.010	0.991	1.016	1.004	1.026	*	1.002	0.985	1.003	0.983	1.008	0.996	1.018	*	0.995	0.978
		5	1.001	0.991	1.013	1.000	1.008	1.022	1.000	0.987	1.001	0.991	1.013	1.000	1.008	1.022	1.000	0.987
		6	0.997	0.981	1.016	1.007	1.013	1.022	1.003	0.993	1.000	0.984	1.019	1.010	1.016	1.025	1.006	0.996
	4	4	1.004	0.986	1.006	0.994	1.006	*	0.995	0.982	1.013	0.995	1.015	1.003	1.015	*	1.004	0.991
		5	0.997	0.989	1.014	0.996	1.012	1.015	0.999	0.984	1.000	0.992	1.017	0.999	1.015	1.018	1.002	0.987
		6	1.002	0.971	1.016	1.003	1.012	1.015	1.000	0.993	1.002	0.971	1.016	1.003	1.012	1.015	1.000	0.993
	5	4	1.013	0.988	1.014	1.010	1.018	*	1.002	0.990	1.008	0.983	1.009	1.005	1.013	*	0.997	0.986
		5	0.992	0.986	1.013	0.997	1.007	1.010	0.994	0.980	0.994	0.989	1.016	0.999	1.010	1.012	0.997	0.983
		6	1.002	0.982	1.023	1.005	1.015	1.019	1.003	0.995	0.997	0.977	1.018	1.000	1.010	1.014	0.999	0.990
	6	4	0.996	0.981	1.012	0.993	1.007	*	1.002	0.983	1.002	0.986	1.017	0.999	1.012	*	1.007	0.988
		5	1.000	0.988	1.014	1.000	1.015	1.017	0.999	0.988	1.000	0.989	1.015	1.000	1.015	1.018	0.999	0.988
		6	0.998	0.976	1.018	0.998	1.007	1.013	0.997	0.987	0.998	0.976	1.018	0.998	1.007	1.013	0.997	0.987

* k can be up to 1 when $m=4$

Table A.6 Bias values of the estimators under $(X, Y) \sim \text{BVN}(3, 7, 1, 4, \rho)$

ρ		τ		$Bias(\hat{\gamma})$												$Bias(\hat{\beta}_0)$												$Bias(\hat{\beta}_1)$											
				SRS	RRS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RRS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RRS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂						
0.50	4	-0.011	-0.002	0.006	0.014	-0.007	-0.010	-0.001	*	-0.003	0.017	0.040	0.014	0.057	0.009	0.058	0.037	0.123	*	0.036	0.003	-0.017	-0.006	-0.018	0.004	-0.025	-0.016	-0.042	*	-0.014	0.006								
	5	0.004	0.001	0.002	0.008	-0.015	-0.013	0.004	-0.013	0.006	0.012	0.026	0.014	0.035	-0.024	0.112	0.002	0.095	0.146	0.019	0.009	-0.009	-0.005	-0.010	0.011	-0.045	-0.004	-0.032	-0.057	-0.005	0.001								
	6	-0.008	0.002	-0.010	0.006	-0.007	0.006	0.010	-0.006	-0.007	0.002	0.025	0.037	0.027	-0.006	0.187	0.024	0.129	0.121	0.019	-0.009	-0.010	-0.012	-0.010	0.006	-0.067	-0.006	-0.041	-0.045	-0.009	0.004								
	4	-0.011	-0.008	0.007	0.007	-0.003	-0.007	-0.001	*	-0.008	0.009	0.009	0.021	-0.002	0.003	0.131	0.011	0.078	*	0.011	-0.007	-0.004	-0.010	0.002	0.002	-0.046	-0.006	-0.028	*	-0.006	0.006								
	5	-0.003	0.002	0.000	0.008	-0.010	0.005	-0.001	-0.007	-0.004	-0.001	0.000	0.040	-0.001	-0.002	0.126	0.034	0.107	0.162	0.162	0.005	-0.001	-0.014	0.000	0.003	-0.047	-0.010	-0.036	-0.059	-0.006	0.002								
	6	-0.008	0.000	0.001	0.013	-0.014	-0.004	0.002	-0.014	-0.003	0.001	-0.007	0.003	-0.003	-0.003	0.176	0.001	0.069	0.094	0.094	0.012	-0.027	0.000	-0.001	0.006	-0.065	-0.001	-0.023	-0.037	-0.005	0.009								
0.75	4	0.001	0.001	0.001	0.008	0.007	0.006	-0.005	*	0.006	0.009	0.026	0.008	0.032	0.006	0.074	0.033	0.107	*	0.040	-0.013	-0.008	-0.002	-0.010	0.002	-0.025	-0.009	-0.040	*	-0.012	0.009								
	5	0.003	0.003	0.005	0.008	-0.013	-0.001	-0.001	-0.011	0.002	0.010	0.024	0.008	0.007	-0.005	0.124	0.001	0.089	0.108	0.108	0.027	-0.007	-0.008	-0.002	-0.002	0.004	0.000	-0.031	-0.041	-0.009	0.006								
	6	0.000	0.007	-0.002	0.008	-0.013	0.004	0.001	-0.016	0.000	0.005	0.004	0.009	0.011	-0.009	0.130	0.025	0.038	0.138	0.138	0.015	-0.008	-0.003	-0.001	0.007	-0.049	-0.007	-0.014	-0.053	-0.005	0.004								
	4	-0.003	0.002	0.000	0.003	-0.005	0.008	0.004	*	-0.002	0.010	0.005	0.007	0.017	-0.030	0.070	0.010	0.098	*	0.031	-0.002	-0.003	-0.002	-0.006	0.012	-0.026	-0.001	-0.033	*	-0.010	0.004								
	5	0.001	0.011	-0.002	0.000	-0.011	0.006	0.004	-0.010	-0.001	-0.002	0.012	0.037	-0.006	-0.021	0.160	0.000	0.085	0.126	0.126	0.007	-0.017	-0.004	-0.010	0.002	0.008	-0.059	0.002	-0.028	-0.047	-0.002	0.005							
	6	-0.003	0.000	0.009	-0.013	0.000	0.000	-0.006	-0.012	0.000	0.000	-0.005	0.008	-0.016	-0.017	0.091	-0.011	0.059	0.151	0.151	0.014	-0.016	0.002	-0.002	0.006	0.010	-0.036	0.003	-0.022	-0.056	-0.004	0.005							
0.95	4	-0.003	0.000	-0.006	0.005	-0.004	0.001	-0.011	*	-0.002	0.012	0.007	0.008	0.009	-0.021	0.071	0.013	0.091	*	0.021	-0.009	-0.002	-0.002	-0.005	0.009	-0.026	-0.004	-0.034	*	-0.007	0.007								
	5	0.009	0.000	-0.002	0.003	-0.004	-0.005	-0.016	0.000	0.003	-0.007	-0.001	-0.005	-0.032	0.135	0.014	0.090	0.114	0.114	-0.009	-0.021	0.004	0.000	0.001	0.012	-0.048	-0.006	-0.033	-0.045	0.003	0.009								
	6	0.003	-0.001	0.002	0.011	-0.015	-0.001	0.002	-0.008	-0.002	0.000	0.026	0.007	0.011	0.003	0.158	0.000	0.060	0.113	0.113	0.011	-0.024	-0.007	-0.002	-0.003	0.004	-0.059	0.000	-0.020	-0.042	-0.005	0.009							
	4	-0.011	0.003	0.000	0.012	-0.001	-0.006	0.000	*	-0.010	0.012	0.034	0.023	0.033	0.010	0.043	0.026	0.061	*	0.001	-0.002	-0.013	-0.007	-0.011	0.001	-0.017	-0.011	-0.020	*	-0.004	0.004								
	5	0.004	-0.001	-0.002	0.005	0.006	-0.007	0.004	0.002	0.005	0.005	0.027	0.012	0.004	-0.037	0.043	0.002	0.048	0.042	0.042	0.012	-0.012	-0.009	-0.005	-0.002	0.013	-0.013	-0.002	-0.016	-0.015	-0.004	0.004							
	6	-0.008	-0.003	-0.007	-0.002	0.000	0.001	0.002	0.002	-0.006	0.004	0.008	-0.009	0.016	-0.014	0.054	0.003	0.059	0.071	0.071	-0.007	-0.017	-0.004	0.003	-0.004	0.006	-0.018	-0.001	-0.021	-0.023	0.001	0.007							
0.95	4	-0.011	-0.001	0.003	0.004	0.000	0.002	0.002	*	-0.005	0.013	0.000	0.007	0.012	-0.005	0.038	0.028	0.055	*	0.006	-0.001	0.000	-0.003	-0.004	0.004	-0.013	-0.008	-0.018	*	-0.004	0.004								
	5	-0.003	-0.001	0.003	0.012	-0.003	0.000	0.001	0.004	-0.005	-0.002	-0.005	0.006	0.009	-0.004	0.014	0.011	0.031	0.075	0.075	0.000	0.003	0.001	-0.003	-0.002	0.005	-0.006	-0.004	-0.010	-0.024	-0.002	0.000							
	6	-0.008	0.004	0.001	0.009	0.001	0.004	-0.002	0.001	-0.002	0.000	-0.009	-0.009	0.010	-0.011	0.031	0.022	0.037	0.053	0.053	0.010	-0.009	0.002	0.004	-0.003	0.006	-0.010	-0.006	-0.014	-0.018	-0.004	0.003							
	4	0.001	-0.005	-0.003	0.001	0.004	0.003	0.002	*	0.002	0.004	0.014	0.002	0.006	-0.004	0.022	-0.006	0.060	*	0.020	-0.007	-0.004	-0.001	-0.002	0.002	-0.008	0.003	-0.020	*	-0.006	0.005								
	5	0.003	0.002	0.003	0.005	-0.005	0.000	0.003	-0.004	-0.001	0.005	0.012	-0.010	0.009	-0.008	0.067	-0.002	0.043	0.012	0.012	0.000	-0.011	-0.004	0.004	-0.003	0.003	-0.024	0.001	-0.014	-0.005	-0.001	0.005							
	6	0.000	0.004	0.002	0.004	-0.009	0.001	-0.005	-0.006	-0.004	0.003	-0.003	0.006	0.009	-0.016	0.030	0.008	0.038	0.043	0.043	0.003	-0.006	0.000	-0.002	-0.002	0.007	-0.012	-0.003	-0.016	-0.016	-0.002	0.003							
0.95	4	-0.003	0.001	-0.001	0.000	0.001	0.004	-0.002	*	0.000	0.004	0.007	0.009	0.014	-0.011	0.057	0.011	0.064	*	0.016	-0.009	-0.003	-0.003	-0.005	0.004	-0.019	-0.003	-0.023	*	-0.005	0.003								
	5	0.001	0.008	-0.003	-0.004	-0.002	0.003	-0.001	-0.004	0.001	0.001	0.012	0.014	-0.004	-0.016	0.065	0.008	0.053	0.056	0.056	-0.002	-0.014	-0.005	-0.004	0.001	0.006	-0.023	-0.002	-0.018	-0.020	0.001	0.005							
	6	-0.003	-0.003	0.002	0.000	0.003	0.000	0.001	-0.005	0.006	0.000	-0.010	-0.005	-0.020	-0.016	0.058	-0.001	0.059	0.019	0.019	0.009	-0.006	0.004	0.001	0.007	0.006	-0.019	0.000	-0.020	-0.008	0.000	0.002							
	4	-0.003	-0.002	0.001	0.001	-0.005	0.001	-0.004	*	-0.002	0.007	0.000	0.004	-0.002	-0.015	0.065	-0.001	0.046	*	0.013	-0.004	0.001	-0.001	0.001	0.005	-0.024	0.001	-0.016	*	-0.005	0.003								
	5	0.009	0.000	0.001	0.002	-0.001	-0.005	-0.006	-0.007	-0.001	0.005	-0.008	0.005	-0.004	-0.018	0.048	0.010	0.056	0.011	0.011	0.003	-0.013	0.004	-0.002	0.001	0.007	-0.017	-0.005	-0.021	-0.006	-0.001	0.006							
	6	0.003	0.000	-0.001	0.007	-0.005	-0.001	0.000	-0.002	-0.001	0.006	0.018	-0.002	-0.002	0.002	0.033	0.010	0.045	0.056	0.056	0.000	-0.009	-0.005	0.001	0.000	0.002	-0.013	-0.003	-0.016	-0.019	-0.001	0.005							
0.95	4	-0.012	0.005	0.000	0.004	0.005	-0.001	0.005	*	-0.005	0.005	-0.002	-0.004	0.000	-0.012	0.028	-0.006	0.025	*	-0.025	-0.007	0.001	0.002	0.000	0.005	-0.010	0.002	-0.008	*	0.008	0.003								
	5	0.004	-0.001	0.001	0.004	0.006	-0.006	0.006	-0.001	0.006	0.005	0.008	0.003	0.003	0.003	-0.020	-0.004	0.019	0.038	0.038	0.007	-0.002	-0.002	0.000	0.007	-0.015	0.001	-0.006	-0.014	-0.002	0.001								
	6	-0.008	-0.004	-0.006	-0.010	-0.002	0.002	0.002	0.002	-0.001	-0.002	-0.001	-0.002	0.001	-0.010	0.022	-0.004	0.016	0.028	0.028	-0.016	-0.008	0.000	0.000	0.001	0.002	-0.008	0.002	-0.010	-0.005	0.002	0.001							
	4	-0.011	-0.002	0.002	0.001	0.003	-0.003	0.003	*	-0.005	0.007	-0.004	-0.006	-0.002	-0.007	0.014	0.002	0.032	*	-0.010	-0.008	0.002	0.002	0.001	0.003	-0.004	-0.001	-0.011	*	0.003	0.003								
	5	-0.003	-0.002	0.003	0.006	0.003.																																	

Table A.7 RE_1 values of the estimators under 10% outlier case

ρ	r	m	$\bar{y}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\sigma}^2$
0.50	2	4	1.090	1.048	1.135	1.016
		5	1.043	1.018	1.117	0.981
		6	1.132	1.065	1.140	1.011
	3	4	1.126	1.027	1.066	0.997
		5	1.113	1.048	1.075	0.985
		6	1.110	1.010	1.060	0.997
	4	4	1.072	1.005	1.078	0.985
		5	1.075	1.004	1.072	0.975
		6	1.090	0.989	0.977	0.998
	5	4	1.068	1.036	1.036	1.005
		5	1.104	1.018	1.084	1.009
		6	1.115	1.000	0.992	0.993
	6	4	1.091	1.001	0.993	1.002
		5	1.060	0.963	1.043	0.994
		6	1.115	1.019	0.995	0.999
0.75	2	4	1.218	1.040	1.147	1.008
		5	1.216	1.017	1.069	0.993
		6	1.283	1.026	1.091	0.980
	3	4	1.251	0.995	1.062	1.008
		5	1.274	1.030	1.042	0.991
		6	1.362	1.047	1.105	1.020
	4	4	1.223	0.998	1.073	0.992
		5	1.250	1.006	1.042	0.989
		6	1.270	0.983	0.985	0.992
	5	4	1.207	1.034	1.010	0.998
		5	1.270	1.008	1.075	1.007
		6	1.286	0.989	1.013	1.010
	6	4	1.217	0.988	0.999	0.984
		5	1.260	0.972	1.012	0.990
		6	1.314	1.036	0.978	0.998
0.95	2	4	1.408	1.006	1.108	1.029
		5	1.463	0.994	1.050	0.989
		6	1.562	0.985	1.051	0.979
	3	4	1.437	0.976	1.013	0.996
		5	1.524	1.015	1.032	1.008
		6	1.600	1.005	1.053	1.004
	4	4	1.455	0.984	1.047	1.006
		5	1.475	0.973	1.017	0.989
		6	1.511	0.975	1.011	0.982
	5	4	1.393	0.993	0.980	1.001
		5	1.530	1.027	1.037	1.002
		6	1.563	0.980	1.032	1.009
	6	4	1.409	0.967	1.031	0.985
		5	1.497	0.959	0.986	0.986
		6	1.607	1.037	0.971	1.000

Table A.8 RE_2 and RE_3 values for the mean estimator under 10% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.012	0.984	1.243	1.073	1.208	*	1.087	0.984	0.928	0.903	1.141	0.984	1.109	*	0.998	0.903
		5	1.058	1.018	1.258	1.139	1.179	1.261	1.071	1.013	1.015	0.977	1.206	1.092	1.130	1.210	1.027	0.972
		6	1.110	1.000	1.283	1.178	1.235	1.326	1.124	1.085	0.980	0.883	1.133	1.040	1.091	1.171	0.993	0.958
	3	4	1.080	0.990	1.211	1.119	1.223	*	1.074	1.039	0.959	0.879	1.075	0.994	1.086	*	0.954	0.922
		5	1.075	1.067	1.306	1.155	1.181	1.262	1.079	0.994	0.966	0.959	1.174	1.038	1.061	1.134	0.969	0.893
		6	1.100	0.958	1.298	1.122	1.182	1.292	1.118	1.068	0.991	0.862	1.169	1.010	1.064	1.163	1.007	0.962
	4	4	1.071	0.993	1.196	1.082	1.188	*	1.132	1.014	0.999	0.926	1.116	1.009	1.108	*	1.056	0.946
		5	1.063	1.024	1.304	1.118	1.169	1.279	1.072	1.054	0.990	0.953	1.213	1.041	1.088	1.191	0.998	0.981
		6	1.074	0.937	1.262	1.119	1.142	1.266	1.099	1.051	0.985	0.860	1.158	1.027	1.047	1.162	1.008	0.964
	5	4	1.043	1.013	1.198	1.124	1.208	*	1.110	0.976	0.977	0.948	1.122	1.052	1.131	*	1.039	0.913
		5	1.117	0.996	1.270	1.150	1.144	1.273	1.139	0.994	1.012	0.903	1.151	1.042	1.037	1.154	1.032	0.901
		6	1.075	0.974	1.264	1.155	1.194	1.309	1.135	1.076	0.964	0.874	1.133	1.035	1.071	1.174	1.018	0.965
	6	4	1.076	1.011	1.197	1.127	1.211	*	1.090	1.026	0.987	0.927	1.098	1.034	1.110	*	0.999	0.941
		5	1.055	1.010	1.252	1.124	1.164	1.236	1.125	1.027	0.996	0.953	1.181	1.060	1.098	1.166	1.061	0.968
		6	1.114	0.967	1.347	1.165	1.193	1.337	1.130	1.087	0.999	0.867	1.208	1.045	1.070	1.199	1.013	0.975
0.75	2	4	1.105	1.034	1.537	1.210	1.491	*	1.243	1.014	0.907	0.849	1.263	0.994	1.224	*	1.021	0.833
		5	1.181	1.062	1.736	1.332	1.522	1.696	1.252	1.050	0.971	0.873	1.427	1.095	1.252	1.394	1.030	0.864
		6	1.256	0.983	1.780	1.451	1.567	1.834	1.285	1.143	0.979	0.766	1.387	1.131	1.222	1.430	1.008	0.891
	3	4	1.190	1.029	1.530	1.303	1.573	*	1.221	1.065	0.951	0.823	1.223	1.042	1.257	*	0.976	0.852
		5	1.197	1.081	1.711	1.326	1.507	1.730	1.238	1.060	0.940	0.849	1.344	1.041	1.184	1.358	0.972	0.832
		6	1.199	0.925	1.807	1.327	1.465	1.776	1.262	1.118	0.881	0.679	1.326	0.974	1.075	1.303	0.927	0.821
	4	4	1.164	1.011	1.537	1.308	1.532	*	1.252	1.046	0.951	0.827	1.256	1.069	1.252	*	1.023	0.855
		5	1.189	1.085	1.710	1.307	1.530	1.712	1.243	1.108	0.951	0.868	1.368	1.046	1.224	1.370	0.995	0.886
		6	1.215	0.923	1.728	1.353	1.466	1.735	1.238	1.112	0.956	0.726	1.360	1.065	1.154	1.366	0.974	0.875
	5	4	1.138	1.041	1.530	1.294	1.519	*	1.231	1.003	0.943	0.863	1.268	1.072	1.259	*	1.020	0.831
		5	1.208	1.060	1.689	1.333	1.493	1.721	1.280	1.060	0.952	0.835	1.330	1.050	1.176	1.355	1.009	0.835
		6	1.221	0.956	1.742	1.363	1.507	1.779	1.296	1.137	0.949	0.744	1.355	1.060	1.172	1.383	1.008	0.884
	6	4	1.170	1.020	1.517	1.319	1.505	*	1.253	1.071	0.961	0.838	1.247	1.084	1.237	*	1.029	0.880
		5	1.196	1.060	1.671	1.322	1.501	1.697	1.283	1.069	0.950	0.842	1.327	1.050	1.192	1.348	1.019	0.849
		6	1.274	0.932	1.842	1.418	1.540	1.777	1.314	1.141	0.970	0.709	1.402	1.079	1.171	1.352	1.000	0.868
0.95	2	4	1.210	1.082	2.072	1.526	2.136	*	1.424	1.059	0.859	0.768	1.471	1.084	1.516	*	1.011	0.752
		5	1.351	1.166	2.672	1.702	2.118	2.600	1.450	1.121	0.924	0.797	1.827	1.163	1.448	1.777	0.991	0.767
		6	1.473	0.983	2.904	1.793	2.187	2.880	1.548	1.213	0.943	0.630	1.859	1.148	1.400	1.844	0.991	0.777
	3	4	1.307	1.102	2.157	1.647	2.205	*	1.415	1.114	0.910	0.767	1.501	1.146	1.535	*	0.985	0.776
		5	1.370	1.173	2.635	1.666	2.117	2.662	1.468	1.156	0.899	0.770	1.729	1.093	1.389	1.747	0.964	0.759
		6	1.403	0.940	2.734	1.679	2.078	2.743	1.561	1.225	0.876	0.588	1.708	1.049	1.299	1.714	0.976	0.765
	4	4	1.266	1.059	2.110	1.610	2.115	*	1.482	1.096	0.870	0.728	1.450	1.106	1.453	*	1.018	0.753
		5	1.380	1.168	2.616	1.669	2.132	2.520	1.497	1.183	0.935	0.792	1.773	1.132	1.446	1.709	1.015	0.802
		6	1.405	0.911	2.742	1.761	2.102	2.737	1.519	1.219	0.930	0.603	1.815	1.165	1.391	1.812	1.005	0.807
	5	4	1.253	1.089	2.040	1.574	2.065	*	1.396	1.043	0.899	0.781	1.464	1.130	1.482	*	1.002	0.749
		5	1.374	1.130	2.537	1.658	2.066	2.647	1.517	1.152	0.898	0.739	1.658	1.083	1.350	1.730	0.991	0.753
		6	1.450	0.964	2.780	1.709	2.129	2.791	1.631	1.253	0.928	0.617	1.778	1.094	1.362	1.786	1.044	0.802
	6	4	1.299	1.056	2.118	1.629	2.084	*	1.452	1.107	0.922	0.749	1.503	1.156	1.479	*	1.030	0.786
		5	1.385	1.132	2.560	1.652	2.110	2.576	1.545	1.146	0.926	0.756	1.710	1.104	1.409	1.721	1.032	0.765
		6	1.475	0.932	2.895	1.793	2.167	2.768	1.583	1.273	0.918	0.580	1.802	1.116	1.348	1.722	0.985	0.792

* k can be up to 1 when $m=4$

Table A.9 RE_2 and RE_3 values for $\hat{\beta}_0$ under 10% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.019	0.980	1.147	1.050	1.149	*	1.043	0.967	0.972	0.935	1.095	1.003	1.097	*	0.996	0.923
		5	1.041	1.003	1.124	1.100	1.136	1.172	1.056	0.992	1.022	0.985	1.104	1.080	1.115	1.151	1.037	0.974
		6	1.070	0.954	1.186	1.098	1.172	1.191	1.079	1.044	1.004	0.895	1.113	1.031	1.100	1.118	1.013	0.980
	3	4	1.031	0.927	1.105	1.042	1.093	*	1.007	0.983	1.003	0.902	1.076	1.014	1.064	*	0.980	0.957
		5	1.025	0.994	1.171	1.061	1.090	1.127	1.015	0.941	0.978	0.949	1.117	1.013	1.040	1.075	0.968	0.898
		6	1.026	0.881	1.170	1.031	1.088	1.145	1.039	1.001	1.016	0.872	1.158	1.021	1.077	1.134	1.028	0.991
	4	4	1.026	0.927	1.122	0.995	1.095	*	1.066	0.946	1.020	0.922	1.116	0.990	1.089	*	1.060	0.941
		5	0.985	0.941	1.159	1.026	1.076	1.149	0.990	0.977	0.981	0.938	1.154	1.022	1.072	1.145	0.986	0.973
		6	0.969	0.849	1.123	1.003	1.020	1.141	0.998	0.953	0.980	0.858	1.135	1.014	1.032	1.154	1.008	0.963
	5	4	1.007	0.979	1.118	1.055	1.162	*	1.069	0.944	0.972	0.945	1.079	1.018	1.121	*	1.032	0.911
		5	1.042	0.922	1.150	1.039	1.051	1.126	1.052	0.916	1.024	0.906	1.130	1.021	1.033	1.106	1.033	0.900
		6	0.995	0.883	1.131	1.025	1.070	1.173	1.040	0.975	0.995	0.883	1.131	1.025	1.070	1.173	1.040	0.975
	6	4	1.014	0.926	1.091	1.008	1.091	*	0.992	0.945	1.013	0.924	1.090	1.007	1.089	*	0.991	0.944
		5	0.973	0.914	1.124	0.988	1.055	1.105	1.014	0.927	1.010	0.949	1.167	1.026	1.095	1.147	1.053	0.962
		6	1.029	0.879	1.212	1.035	1.080	1.200	1.023	0.990	1.010	0.863	1.189	1.016	1.060	1.177	1.004	0.971
0.75	2	4	0.999	0.889	1.232	1.000	1.227	*	1.040	0.878	0.960	0.855	1.184	0.961	1.180	*	1.000	0.844
		5	1.024	0.907	1.280	1.074	1.250	1.284	1.065	0.894	1.006	0.892	1.258	1.055	1.229	1.262	1.047	0.878
		6	1.050	0.778	1.338	1.103	1.249	1.382	1.034	0.920	1.024	0.759	1.304	1.075	1.218	1.347	1.008	0.897
	3	4	1.013	0.828	1.199	1.009	1.217	*	0.981	0.862	1.018	0.832	1.205	1.014	1.223	*	0.986	0.866
		5	1.022	0.857	1.294	1.022	1.183	1.318	1.003	0.861	0.993	0.832	1.257	0.992	1.149	1.280	0.974	0.837
		6	0.973	0.708	1.355	1.004	1.148	1.335	1.006	0.884	0.929	0.676	1.294	0.958	1.096	1.274	0.961	0.844
	4	4	1.010	0.822	1.245	1.015	1.213	*	1.025	0.841	1.012	0.824	1.248	1.017	1.216	*	1.027	0.843
		5	0.988	0.850	1.312	1.014	1.202	1.285	1.003	0.879	0.982	0.844	1.304	1.007	1.194	1.276	0.996	0.873
		6	0.973	0.703	1.328	1.002	1.146	1.299	0.968	0.861	0.991	0.715	1.352	1.019	1.167	1.322	0.986	0.877
	5	4	0.993	0.884	1.246	1.040	1.269	*	1.044	0.853	0.960	0.856	1.205	1.006	1.227	*	1.010	0.825
		5	0.995	0.844	1.336	1.002	1.209	1.314	1.022	0.856	0.987	0.837	1.325	0.994	1.199	1.303	1.014	0.849
		6	1.004	0.733	1.321	0.989	1.164	1.347	1.002	0.871	1.015	0.741	1.336	1.000	1.177	1.362	1.013	0.881
	6	4	0.989	0.814	1.209	1.006	1.174	*	1.006	0.857	1.001	0.823	1.223	1.018	1.187	*	1.018	0.868
		5	0.985	0.818	1.295	0.973	1.183	1.306	0.992	0.825	1.013	0.842	1.332	1.001	1.217	1.343	1.021	0.849
		6	1.045	0.722	1.395	1.035	1.193	1.349	1.011	0.893	1.009	0.697	1.347	0.999	1.152	1.302	0.976	0.861
0.95	2	4	0.966	0.714	1.627	0.985	1.664	*	1.004	0.708	0.960	0.710	1.618	0.979	1.654	*	0.998	0.703
		5	0.981	0.764	1.994	1.038	1.539	1.938	0.988	0.730	0.987	0.769	2.007	1.044	1.549	1.950	0.995	0.735
		6	1.024	0.566	2.077	1.005	1.444	2.029	0.994	0.734	1.040	0.575	2.109	1.020	1.466	2.061	1.009	0.745
	3	4	0.987	0.705	1.626	1.017	1.646	*	0.956	0.705	1.012	0.722	1.667	1.042	1.687	*	0.980	0.723
		5	0.983	0.720	1.990	0.989	1.520	2.000	0.984	0.729	0.968	0.709	1.961	0.974	1.497	1.971	0.969	0.718
		6	0.971	0.542	2.013	0.979	1.452	1.972	1.021	0.750	0.966	0.539	2.004	0.974	1.445	1.963	1.016	0.747
	4	4	0.974	0.675	1.639	0.996	1.599	*	1.025	0.693	0.990	0.686	1.666	1.012	1.626	*	1.042	0.705
		5	0.975	0.720	1.962	0.985	1.499	1.981	0.984	0.730	1.002	0.740	2.016	1.012	1.541	2.035	1.011	0.751
		6	0.966	0.529	1.995	1.021	1.455	1.963	1.005	0.740	0.991	0.542	2.046	1.047	1.493	2.014	1.031	0.759
	5	4	0.971	0.720	1.639	1.010	1.635	*	1.010	0.705	0.978	0.725	1.651	1.017	1.647	*	1.017	0.710
		5	0.988	0.709	1.982	0.990	1.507	1.991	1.001	0.734	0.961	0.690	1.929	0.964	1.467	1.938	0.975	0.715
		6	1.005	0.557	1.999	0.957	1.426	1.962	1.012	0.747	1.026	0.568	2.040	0.977	1.456	2.003	1.033	0.762
	6	4	0.988	0.664	1.650	0.989	1.577	*	0.975	0.693	1.021	0.686	1.706	1.023	1.630	*	1.008	0.717
		5	0.991	0.690	1.987	0.942	1.510	1.900	0.978	0.692	1.033	0.719	2.071	0.982	1.574	1.980	1.019	0.721
		6	1.037	0.554	2.073	1.021	1.461	2.009	1.013	0.775	1.000	0.534	1.999	0.985	1.409	1.938	0.977	0.747

* k can be up to 1 when $m=4$

Table A.10 RE_2 and RE_3 values for $\hat{\beta}_1$ under 10% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.104	1.826	0.507	1.131	0.533	*	1.157	1.853	0.972	1.609	0.447	0.996	0.470	*	1.020	1.633
		5	1.126	1.645	0.342	1.168	0.591	0.353	1.099	1.589	1.008	1.473	0.306	1.045	0.529	0.316	0.983	1.422
		6	1.167	2.391	0.357	1.231	0.696	0.371	1.127	1.691	1.023	2.097	0.313	1.080	0.611	0.326	0.989	1.483
	3	4	1.074	1.624	0.503	1.079	0.477	*	1.088	1.705	1.008	1.523	0.472	1.012	0.447	*	1.021	1.600
		5	1.066	1.635	0.354	1.150	0.557	0.330	1.077	1.612	0.992	1.522	0.329	1.070	0.519	0.307	1.002	1.500
		6	1.060	2.076	0.344	1.097	0.601	0.344	1.070	1.495	1.000	1.959	0.324	1.035	0.567	0.324	1.010	1.410
	4	4	1.060	1.645	0.500	1.052	0.513	*	1.118	1.698	0.983	1.525	0.464	0.976	0.476	*	1.037	1.575
		5	1.058	1.596	0.355	1.075	0.559	0.341	1.069	1.584	0.986	1.489	0.331	1.002	0.521	0.318	0.997	1.477
		6	1.002	2.003	0.339	1.034	0.572	0.316	1.009	1.428	1.026	2.050	0.346	1.058	0.585	0.323	1.032	1.461
	5	4	1.028	1.645	0.499	1.083	0.515	*	1.083	1.594	0.993	1.588	0.481	1.045	0.497	*	1.045	1.539
		5	1.025	1.505	0.356	1.091	0.544	0.352	1.062	1.566	0.945	1.388	0.329	1.007	0.502	0.325	0.980	1.445
		6	1.021	2.020	0.338	1.019	0.583	0.333	1.023	1.434	1.030	2.037	0.341	1.027	0.588	0.336	1.032	1.446
	6	4	1.031	1.537	0.483	1.007	0.487	*	0.992	1.543	1.038	1.548	0.487	1.014	0.490	*	0.999	1.555
		5	1.019	1.482	0.336	1.046	0.567	0.347	1.059	1.479	0.977	1.421	0.322	1.003	0.544	0.333	1.015	1.417
		6	0.984	1.875	0.321	0.975	0.577	0.331	0.990	1.396	0.989	1.884	0.322	0.979	0.580	0.333	0.995	1.402
0.75	2	4	1.062	1.650	0.629	1.108	0.656	*	1.137	1.658	0.926	1.439	0.548	0.967	0.573	*	0.992	1.446
		5	1.045	1.464	0.478	1.114	0.715	0.457	1.050	1.375	0.978	1.369	0.447	1.042	0.669	0.428	0.982	1.286
		6	1.145	2.076	0.498	1.148	0.837	0.518	1.107	1.459	1.050	1.903	0.457	1.053	0.767	0.475	1.015	1.338
	3	4	1.078	1.533	0.638	1.073	0.618	*	1.122	1.529	1.015	1.443	0.601	1.010	0.582	*	1.057	1.440
		5	1.024	1.439	0.470	1.064	0.717	0.481	1.020	1.458	0.983	1.382	0.451	1.022	0.688	0.462	0.979	1.400
		6	1.037	1.803	0.512	1.052	0.762	0.499	1.019	1.379	0.938	1.632	0.463	0.952	0.689	0.452	0.922	1.248
	4	4	1.040	1.503	0.642	1.051	0.663	*	1.113	1.554	0.970	1.402	0.598	0.980	0.618	*	1.037	1.449
		5	1.021	1.442	0.480	1.030	0.681	0.496	1.077	1.467	0.980	1.384	0.461	0.988	0.653	0.476	1.034	1.408
		6	0.991	1.813	0.506	1.039	0.727	0.484	1.008	1.340	1.007	1.841	0.514	1.055	0.739	0.492	1.024	1.360
	5	4	1.020	1.550	0.630	1.082	0.648	*	1.030	1.447	1.009	1.535	0.623	1.071	0.642	*	1.020	1.433
		5	1.012	1.388	0.512	1.028	0.709	0.506	1.030	1.433	0.942	1.291	0.476	0.956	0.660	0.471	0.958	1.333
		6	1.019	1.862	0.498	1.024	0.716	0.474	1.009	1.372	1.006	1.839	0.492	1.011	0.707	0.468	0.996	1.355
	6	4	1.005	1.438	0.621	1.013	0.616	*	1.000	1.447	1.006	1.440	0.622	1.014	0.616	*	1.001	1.448
		5	0.998	1.377	0.495	0.997	0.691	0.493	1.058	1.384	0.986	1.361	0.489	0.985	0.683	0.487	1.045	1.367
		6	0.942	1.742	0.486	0.983	0.718	0.477	0.993	1.328	0.963	1.781	0.497	1.005	0.734	0.487	1.015	1.358
0.95	2	4	1.044	1.437	0.867	1.077	0.861	*	1.104	1.440	0.942	1.297	0.782	0.971	0.777	*	0.996	1.300
		5	0.991	1.300	0.742	1.057	0.892	0.690	1.004	1.240	0.944	1.238	0.707	1.007	0.849	0.657	0.956	1.181
		6	1.098	1.753	0.776	1.086	1.016	0.752	1.053	1.303	1.045	1.669	0.739	1.033	0.967	0.716	1.002	1.240
	3	4	1.034	1.382	0.866	1.041	0.854	*	1.112	1.383	1.021	1.364	0.855	1.028	0.843	*	1.098	1.365
		5	1.002	1.325	0.717	1.030	0.937	0.747	1.010	1.331	0.971	1.285	0.695	0.999	0.908	0.724	0.979	1.290
		6	1.024	1.625	0.767	1.038	0.957	0.779	1.009	1.256	0.972	1.543	0.729	0.986	0.909	0.740	0.958	1.192
	4	4	1.007	1.368	0.846	1.019	0.868	*	1.089	1.416	0.962	1.307	0.809	0.974	0.830	*	1.041	1.353
		5	1.003	1.312	0.762	1.008	0.846	0.728	1.045	1.352	0.987	1.290	0.749	0.992	0.832	0.716	1.027	1.329
		6	1.009	1.681	0.805	1.082	0.944	0.795	1.044	1.297	0.998	1.663	0.796	1.070	0.934	0.786	1.033	1.283
	5	4	1.008	1.410	0.846	1.047	0.882	*	1.018	1.359	1.028	1.439	0.863	1.068	0.900	*	1.038	1.386
		5	1.003	1.292	0.765	1.016	0.911	0.745	1.009	1.314	0.967	1.246	0.738	0.980	0.879	0.719	0.974	1.268
		6	1.007	1.710	0.779	1.031	0.947	0.756	1.045	1.274	0.976	1.657	0.755	1.000	0.917	0.733	1.013	1.235
	6	4	1.019	1.374	0.884	1.037	0.846	*	1.012	1.380	0.988	1.332	0.857	1.006	0.820	*	0.981	1.338
		5	0.972	1.298	0.757	0.972	0.849	0.745	1.029	1.259	0.986	1.316	0.767	0.985	0.861	0.755	1.043	1.277
		6	0.958	1.603	0.751	0.960	0.906	0.755	0.995	1.244	0.987	1.651	0.774	0.988	0.933	0.777	1.024	1.281

* k can be up to 1 when $m=4$

Table A.11 RE_2 and RE_3 values for MSE of regression model under 10% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	0.980	0.929	1.133	1.000	1.124	*	1.026	0.925	0.965	0.915	1.115	0.985	1.107	*	1.010	0.911
		5	0.974	0.911	1.146	0.982	1.088	1.149	0.973	0.915	0.993	0.929	1.168	1.001	1.108	1.171	0.992	0.933
		6	1.016	0.869	1.148	1.024	1.074	1.188	0.997	0.944	1.005	0.859	1.135	1.013	1.062	1.174	0.986	0.933
	3	4	1.015	0.913	1.086	1.010	1.108	*	0.986	0.912	1.017	0.916	1.089	1.013	1.111	*	0.989	0.915
		5	0.993	0.915	1.173	0.997	1.058	1.162	0.979	0.904	1.008	0.929	1.192	1.013	1.074	1.180	0.994	0.918
		6	1.011	0.846	1.172	1.008	1.074	1.176	0.990	0.944	1.014	0.848	1.176	1.011	1.077	1.179	0.992	0.947
	4	4	1.016	0.926	1.113	0.989	1.111	*	1.028	0.908	1.031	0.940	1.130	1.005	1.128	*	1.043	0.922
		5	0.981	0.920	1.193	0.989	1.076	1.167	0.987	0.935	1.007	0.944	1.224	1.015	1.104	1.197	1.012	0.959
		6	0.994	0.841	1.169	1.004	1.052	1.140	0.995	0.952	0.996	0.843	1.172	1.006	1.054	1.142	0.997	0.955
	5	4	0.995	0.917	1.101	0.998	1.124	*	0.990	0.902	0.990	0.912	1.095	0.993	1.118	*	0.985	0.897
		5	1.011	0.915	1.166	1.009	1.069	1.172	1.014	0.933	1.002	0.907	1.156	1.000	1.060	1.162	1.006	0.925
		6	1.005	0.868	1.188	1.006	1.059	1.178	1.001	0.957	1.012	0.874	1.196	1.013	1.067	1.186	1.008	0.964
	6	4	0.995	0.901	1.107	0.998	1.093	*	1.000	0.911	0.993	0.899	1.105	0.996	1.091	*	0.997	0.909
		5	0.991	0.918	1.166	0.997	1.084	1.157	1.008	0.932	0.997	0.924	1.173	1.004	1.090	1.164	1.015	0.938
		6	0.999	0.843	1.175	0.999	1.056	1.171	0.992	0.937	1.000	0.844	1.176	1.000	1.057	1.172	0.994	0.939
0.75	2	4	0.980	0.853	1.238	1.004	1.237	*	1.029	0.831	0.972	0.846	1.228	0.996	1.227	*	1.021	0.824
		5	0.981	0.824	1.294	1.002	1.211	1.309	0.996	0.832	0.988	0.830	1.303	1.010	1.220	1.318	1.003	0.838
		6	1.010	0.730	1.293	1.007	1.164	1.376	1.002	0.859	1.031	0.745	1.320	1.028	1.188	1.404	1.022	0.877
	3	4	1.013	0.827	1.202	1.010	1.227	*	0.997	0.826	1.005	0.821	1.193	1.002	1.217	*	0.989	0.820
		5	0.986	0.829	1.321	0.988	1.183	1.329	0.975	0.821	0.994	0.836	1.333	0.997	1.194	1.341	0.984	0.828
		6	0.997	0.709	1.343	0.979	1.166	1.334	0.984	0.861	0.978	0.695	1.317	0.960	1.143	1.308	0.965	0.844
	4	4	1.011	0.829	1.244	0.993	1.244	*	1.022	0.825	1.018	0.835	1.254	1.001	1.253	*	1.029	0.832
		5	0.989	0.838	1.347	0.992	1.225	1.323	0.990	0.854	1.000	0.847	1.361	1.002	1.238	1.338	1.001	0.863
		6	0.986	0.706	1.349	0.995	1.176	1.345	0.990	0.857	0.993	0.712	1.359	1.002	1.185	1.355	0.998	0.863
	5	4	0.997	0.835	1.247	1.001	1.253	*	1.000	0.821	0.998	0.836	1.249	1.002	1.255	*	1.001	0.822
		5	0.996	0.834	1.343	1.005	1.221	1.358	1.004	0.855	0.990	0.828	1.334	0.998	1.213	1.349	0.997	0.849
		6	1.002	0.733	1.366	1.000	1.182	1.357	1.011	0.864	0.992	0.726	1.352	0.990	1.170	1.343	1.001	0.856
	6	4	1.002	0.818	1.243	1.000	1.236	*	0.996	0.825	1.017	0.831	1.263	1.015	1.256	*	1.012	0.838
		5	0.991	0.841	1.330	0.997	1.208	1.346	1.012	0.846	1.001	0.850	1.344	1.008	1.221	1.361	1.023	0.855
		6	1.002	0.713	1.344	1.002	1.180	1.347	0.986	0.857	1.003	0.714	1.346	1.004	1.182	1.349	0.987	0.859
0.95	2	4	0.979	0.690	1.660	1.001	1.671	*	1.029	0.675	0.951	0.670	1.613	0.973	1.623	*	1.000	0.655
		5	0.992	0.702	2.038	0.986	1.540	2.015	0.982	0.699	1.003	0.710	2.062	0.997	1.558	2.039	0.994	0.708
		6	1.023	0.551	2.100	1.006	1.462	2.100	0.989	0.741	1.044	0.562	2.144	1.028	1.493	2.144	1.010	0.757
	3	4	1.023	0.693	1.646	1.022	1.668	*	0.987	0.682	1.027	0.696	1.653	1.026	1.675	*	0.991	0.685
		5	0.984	0.710	2.065	0.982	1.546	2.090	0.974	0.700	0.977	0.705	2.050	0.975	1.535	2.075	0.967	0.695
		6	0.996	0.541	2.071	0.988	1.450	2.082	0.986	0.743	0.992	0.539	2.063	0.984	1.445	2.074	0.982	0.740
	4	4	1.014	0.696	1.685	1.000	1.687	*	1.023	0.694	1.008	0.692	1.675	0.994	1.677	*	1.017	0.690
		5	0.999	0.718	2.130	0.996	1.555	2.073	1.003	0.733	1.010	0.726	2.154	1.008	1.573	2.097	1.014	0.742
		6	0.982	0.540	2.074	1.004	1.458	2.095	0.997	0.736	1.000	0.550	2.112	1.022	1.485	2.133	1.015	0.750
	5	4	0.998	0.698	1.678	0.998	1.688	*	1.002	0.689	0.997	0.697	1.676	0.997	1.687	*	1.001	0.689
		5	1.000	0.711	2.114	1.006	1.562	2.132	1.007	0.727	0.998	0.709	2.109	1.004	1.558	2.128	1.005	0.725
		6	1.013	0.562	2.135	0.997	1.469	2.129	1.022	0.748	1.004	0.557	2.116	0.988	1.456	2.110	1.013	0.741
	6	4	1.003	0.690	1.694	0.996	1.675	*	0.991	0.689	1.019	0.701	1.720	1.012	1.701	*	1.006	0.700
		5	0.994	0.721	2.111	1.010	1.566	2.113	1.022	0.723	1.008	0.731	2.140	1.024	1.587	2.142	1.036	0.733
		6	0.996	0.555	2.147	0.999	1.483	2.130	0.992	0.747	0.996	0.555	2.147	0.999	1.483	2.131	0.992	0.747

* k can be up to 1 when $m=4$

Table A.12 Bias values of the estimators $(\hat{y}, \hat{\beta}_0, \hat{\beta}_1)$ for sampling methods under 10% outlier case

			$Bias(\hat{y})$										$Bias(\hat{\beta}_0)$										$Bias(\hat{\beta}_1)$											
ρ	r	m	SRS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂			
0.50	4	4	0.005	-0.006	-0.001	-0.003	-0.006	0.005	0.001	*	-0.009	0.004	-0.002	-0.009	-0.006	-0.002	-0.021	-0.001	0.000	0.000	*	-0.011	0.003	-0.022	-0.023	-0.013	0.000	-0.058	-0.032	-0.019	*	-0.009	-0.002	
		5	-0.001	0.003	0.001	0.005	-0.001	0.005	-0.010	-0.002	-0.004	0.019	-0.002	-0.001	0.001	0.005	-0.026	0.003	-0.012	-0.008	-0.006	0.019	-0.002	-0.009	-0.008	0.004	-0.046	-0.010	-0.045	-0.037	-0.011	-0.009		
		6	0.012	0.000	0.002	0.004	0.007	0.000	0.002	-0.007	-0.004	0.014	0.008	-0.003	0.000	0.009	-0.031	-0.001	-0.003	-0.010	-0.005	0.012	-0.010	-0.011	0.004	0.016	-0.063	0.003	-0.034	-0.044	-0.005	0.005		
		3	5	-0.006	0.004	0.003	-0.001	-0.002	-0.006	-0.011	-0.002	0.010	0.010	-0.005	0.002	0.000	0.003	-0.026	-0.008	-0.015	-0.008	0.008	0.014	0.003	-0.005	0.002	0.012	-0.037	-0.001	-0.029	-0.044	-0.013	0.005	
		6	-0.009	0.000	0.002	0.007	-0.010	-0.002	-0.008	-0.002	0.006	0.006	-0.006	-0.002	0.002	0.013	-0.017	-0.004	-0.011	-0.007	0.004	0.007	-0.003	0.002	0.002	0.009	-0.034	-0.004	-0.034	-0.052	-0.010	0.008		
		5	4	0.003	0.008	-0.003	0.011	-0.004	0.013	-0.003	*	0.001	0.000	0.002	0.005	-0.005	0.013	-0.024	0.011	-0.009	*	0.000	0.000	-0.008	-0.009	-0.004	0.004	-0.034	-0.006	-0.035	*	0.006	0.009	
0.75	4	5	-0.007	0.008	-0.001	0.004	-0.008	0.005	-0.007	0.003	0.005	-0.008	0.007	-0.002	0.004	-0.019	0.004	-0.009	-0.011	0.004	0.008	0.001	-0.003	-0.006	-0.009	-0.035	-0.001	-0.047	-0.043	-0.007	0.015	0.015		
		6	0.005	0.004	0.000	0.014	-0.011	0.004	-0.006	-0.007	-0.001	0.005	0.001	0.003	0.001	0.017	-0.016	0.003	-0.007	-0.012	-0.002	0.006	0.006	-0.004	-0.007	0.003	0.011	-0.031	0.004	-0.034	-0.051	-0.004	0.014	
		4	-0.001	0.000	-0.001	0.001	-0.001	0.004	-0.003	*	-0.004	0.005	-0.004	-0.002	-0.001	0.005	-0.027	0.003	-0.004	*	-0.004	0.004	-0.005	-0.008	-0.002	0.007	-0.051	-0.007	-0.032	*	-0.003	0.006		
		5	0.004	0.001	-0.004	0.007	-0.008	0.002	-0.002	-0.002	-0.007	0.006	0.000	0.000	-0.005	0.009	-0.019	0.001	-0.005	-0.007	-0.005	0.006	0.007	0.000	0.004	0.006	-0.004	0.006	-0.033	0.002	-0.034	-0.044	0.007	0.015
		6	-0.007	-0.001	-0.001	0.014	-0.005	-0.002	-0.010	-0.007	0.006	0.002	-0.007	-0.002	-0.002	0.017	-0.022	-0.002	-0.013	-0.012	0.004	0.005	0.000	0.000	-0.004	0.002	0.017	-0.043	0.001	-0.031	-0.047	0.001	0.012	
		5	4	0.006	0.000	-0.004	-0.001	-0.003	-0.005	-0.007	*	0.003	0.005	0.003	-0.001	-0.006	0.001	-0.025	-0.006	-0.011	*	0.001	0.006	0.001	0.000	0.002	0.006	-0.041	-0.005	-0.039	*	0.000	0.008	
0.95	4	5	0.002	-0.008	0.001	0.004	-0.010	0.001	-0.005	-0.008	-0.002	0.000	-0.001	-0.009	-0.001	0.008	-0.018	0.001	-0.009	-0.013	-0.002	0.001	-0.006	-0.003	-0.002	0.007	-0.043	-0.004	-0.021	-0.043	0.007	0.014		
		6	0.000	0.001	-0.003	0.010	-0.008	-0.001	-0.008	-0.001	-0.002	0.010	-0.001	-0.001	-0.004	0.015	-0.020	-0.001	-0.010	-0.006	0.000	0.011	-0.003	0.002	0.000	0.015	-0.042	-0.002	-0.033	-0.039	-0.002	0.008		
		4	0.004	-0.006	0.000	0.011	-0.016	0.006	-0.013	-0.016	-0.017	-0.007	0.023	-0.007	-0.006	0.000	0.017	-0.012	-0.002	-0.021	-0.023	-0.013	0.019	-0.002	-0.006	-0.009	0.070	-0.008	0.021	0.062	-0.007	-0.022		
		5	-0.001	0.000	0.006	0.020	-0.012	0.003	-0.016	-0.017	-0.007	0.023	-0.007	-0.006	0.000	0.017	-0.012	-0.002	-0.021	-0.023	-0.013	0.019	-0.002	-0.006	-0.009	0.070	-0.008	0.021	0.062	-0.007	-0.022			
		6	0.012	0.006	-0.002	0.030	-0.003	-0.006	-0.015	-0.019	-0.001	0.021	0.002	0.001	-0.005	0.037	-0.018	-0.007	-0.023	-0.024	-0.006	0.018	-0.006	0.018	-0.006	-0.014	0.006	-0.001	0.053	-0.002	0.014	0.062	-0.006	-0.007
		5	4	0.010	-0.006	-0.002	0.012	-0.021	0.003	-0.012	*	0.000	0.014	-0.005	-0.008	-0.009	0.015	-0.003	-0.002	-0.021	*	-0.004	0.015	0.001	-0.002	-0.003	-0.008	0.009	-0.001	0.015	*	-0.001	-0.005	
0.95	4	5	-0.006	0.006	-0.003	0.012	-0.012	-0.002	-0.018	-0.015	0.007	0.018	-0.007	0.001	-0.008	0.016	-0.011	-0.005	-0.024	-0.021	0.003	0.022	0.004	-0.004	-0.003	-0.001	0.059	-0.004	0.022	0.051	-0.009	-0.007		
		6	-0.009	-0.002	0.001	0.035	-0.021	-0.005	-0.025	-0.012	0.007	0.020	-0.008	-0.004	-0.001	0.044	-0.003	-0.009	-0.030	-0.020	0.001	0.020	-0.001	0.009	-0.002	-0.012	0.065	-0.006	-0.001	0.054	-0.008	-0.004		
		4	0.002	0.005	-0.006	0.024	-0.019	0.008	-0.014	*	-0.002	0.009	-0.003	-0.001	-0.009	0.024	-0.006	0.003	-0.024	*	-0.004	0.009	-0.007	0.000	-0.003	-0.010	0.011	-0.009	0.019	*	0.007	0.007		
		5	-0.007	0.008	-0.003	0.019	-0.014	0.006	-0.020	-0.015	-0.004	0.013	-0.010	0.005	-0.006	0.019	-0.007	0.004	-0.026	-0.022	-0.004	0.017	-0.001	-0.005	-0.007	-0.006	0.046	-0.001	0.001	0.050	-0.004	0.000		
		6	0.005	0.003	-0.004	0.040	-0.017	0.004	-0.015	-0.016	0.002	0.015	-0.002	0.001	-0.004	0.045	-0.005	0.002	-0.020	-0.024	0.000	0.018	-0.002	-0.007	-0.003	-0.009	0.060	0.003	0.002	0.047	0.001	0.001		
		5	4	-0.001	-0.002	-0.004	0.011	-0.016	0.002	-0.020	*	-0.004	0.014	-0.006	-0.005	-0.006	0.017	-0.008	-0.001	-0.023	*	-0.006	0.014	-0.004	-0.004	-0.005	-0.004	0.013	-0.007	0.014	*	-0.006	-0.008	
0.95	4	5	0.005	0.000	-0.002	0.017	-0.016	-0.001	-0.013	-0.014	-0.001	0.017	-0.003	-0.002	-0.003	0.021	-0.006	-0.002	-0.019	-0.020	-0.002	0.018	0.005	0.001	-0.005	-0.009	0.048	0.004	0.010	0.047	0.002	0.002		
		6	-0.007	0.003	0.003	0.037	-0.016	0.003	-0.020	-0.018	0.001	0.017	-0.008	0.001	0.002	0.040	-0.006	0.002	-0.024	-0.024	-0.002	0.022	0.000	0.000	0.000	0.048	-0.004	-0.006	0.054	0.002	-0.002			
		4	0.006	-0.001	-0.003	0.017	-0.011	-0.007	-0.022	*	0.009	0.016	0.000	-0.003	-0.007	0.020	-0.011	-0.008	-0.026	*	0.006	0.018	0.003	0.002	0.001	-0.009	0.013	-0.005	0.012	*	-0.005	-0.004		
		5	0.002	-0.006	0.002	0.014	-0.021	0.003	-0.017	-0.022	-0.008	0.011	-0.002	-0.008	-0.001	0.018	-0.003	0.003	-0.022	-0.028	-0.007	0.014	-0.004	-0.004	-0.006	-0.002	-0.005	0.044	-0.005	0.012	0.044	0.007	0.000	
		6	0.000	-0.003	0.002	0.033	-0.018	0.000	-0.020	-0.014	-0.002	0.018	-0.003	-0.003	-0.001	0.042	-0.005	0.000	-0.024	-0.023	-0.003	0.020	-0.001	0.000	-0.002	-0.007	0.043	-0.004	0.002	0.047	-0.005	-0.003		
		5	4	0.004	-0.006	0.002	0.021	-0.024	0.011	-0.018	*	-0.009	0.025	-0.019	-0.015	-0.008	0.025	0.008	0.002	-0.031	*	-0.014	0.021	-0.015	-0.010	-0.018	-0.008	-0.010	-0.026	-0.008	*	-0.005	-0.010	
0.95	4	5	-0.002	-0.005	0.003	0.023	-0.021	-0.004	-0.017	-0.021	-0.006	0.023	-0.010	-0.012	-0.004	0.019	0.006	-0.010	-0.026	-0.035	-0.012	0.018	-0.003	-0.007	-0.015	-0.004	0.009	-0.009	-0.007	-0.011	-0.014	-0.015		
		6	0.012	0.006	-0.007	0.036	-0.019	-0.003	-0.021	-0.025	0.003	0.023	-0.005	-0.002	-0.010	0.045	0.006	-0.004	-0.032	-0.034	-0.004	0.020	-0.010	-0.017	0.004	0.005	0.002	-0.008	-0.006	-0.001	-0.011	-0.002		
		4	0.010	-0.004	0.001	0.018	-0.028	0.001	-0.017	*	0.001	0.020	-0.009	-0.007	-0.008	0.022	0.006	-0.006	-0.031	*	-0.003	0.019	-0.005	-0.014	-0.005	-0.004	-0.011	-0.003	-0.011	*	-0.007	-0.003		
		5	-0.006	0.002	0.001	0.020	-0.022	-0.002	-0.027	-0.020	0.005	0.023	-0.009	-0.006	-0.005	0.023	0.006	-0.002	-0.035	-0.031	-0.001	0.028	-0.001	-0.001	-0.008	0.004	-0.006	-0.002	-0.001	-0.012	-0.004			
		6	-0.009	0.000	0.000	0.044	-0.022	-0.002	-0.025	-0.020	0.001	0.021	-0.010	-0.004	-0.004	0.055	0.005	-0.006	-0.039	-0.031	-0.003	0.020	-0.003	-0.001	-0.005	-0.003	-0.004	-0.009	-0.013	0.000	-0.007	0.000		

Table A.13 RE_1 values of the estimators under 30% outlier case

ρ	r	m	$\bar{y}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\sigma}^2$
0.50	2	4	1.091	1.142	1.167	1.019
		5	1.143	1.111	1.195	1.008
		6	1.101	1.065	1.125	0.988
	3	4	1.121	1.083	1.067	1.002
		5	1.084	1.051	1.056	0.995
		6	1.095	1.030	1.023	0.994
	4	4	1.078	1.028	1.105	0.987
		5	1.085	1.020	1.001	0.990
		6	1.095	1.016	1.044	1.003
	5	4	1.106	1.065	1.081	1.000
		5	1.085	1.015	1.046	0.998
		6	1.127	1.070	1.014	0.992
	6	4	1.086	1.024	1.042	0.992
		5	1.064	1.015	1.047	1.005
		6	1.118	1.048	1.018	0.998
0.75	2	4	1.230	1.089	1.160	1.026
		5	1.299	1.061	1.090	1.014
		6	1.298	1.033	1.094	0.993
	3	4	1.285	1.045	1.058	1.003
		5	1.240	1.024	1.035	0.989
		6	1.313	1.031	1.011	0.995
	4	4	1.213	0.997	1.061	0.992
		5	1.209	0.986	0.968	0.988
		6	1.297	1.031	1.029	1.014
	5	4	1.272	1.078	1.042	1.002
		5	1.261	1.015	1.008	0.999
		6	1.267	1.027	0.986	0.991
	6	4	1.219	1.002	1.045	0.996
		5	1.229	1.017	1.034	1.007
		6	1.301	1.027	0.998	0.998
0.95	2	4	1.428	1.010	1.071	1.042
		5	1.547	1.015	1.047	1.024
		6	1.517	0.951	1.045	1.002
	3	4	1.481	1.005	1.002	1.008
		5	1.473	0.983	1.023	0.996
		6	1.555	0.977	1.013	1.001
	4	4	1.413	0.958	1.018	0.999
		5	1.450	0.971	0.988	1.000
		6	1.566	1.019	1.047	1.009
	5	4	1.490	1.056	1.030	1.007
		5	1.484	1.022	1.006	1.000
		6	1.551	1.031	0.976	0.989
	6	4	1.402	0.962	1.023	0.997
		5	1.462	0.993	0.999	1.000
		6	1.535	0.996	0.997	0.999

Table A.14 RE_2 and RE_3 values for the mean estimator under 30% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.093	1.115	1.151	1.139	1.133	*	1.125	1.058	1.001	1.022	1.055	1.044	1.039	*	1.031	0.969
		5	1.124	1.090	1.211	1.177	1.171	1.186	1.160	1.081	0.983	0.953	1.059	1.029	1.024	1.037	1.015	0.945
		6	1.115	1.049	1.162	1.152	1.145	1.185	1.112	1.108	1.013	0.953	1.056	1.047	1.040	1.077	1.011	1.007
	3	4	1.078	1.089	1.173	1.142	1.142	*	1.125	1.102	0.961	0.971	1.046	1.019	1.019	*	1.004	0.983
		5	1.048	1.033	1.116	1.094	1.138	1.104	1.106	1.061	0.967	0.953	1.030	1.010	1.051	1.019	1.021	0.979
		6	1.138	1.072	1.168	1.149	1.182	1.201	1.146	1.077	1.039	0.979	1.066	1.049	1.080	1.097	1.046	0.984
	4	4	1.031	1.028	1.115	1.100	1.099	*	1.061	1.029	0.956	0.954	1.035	1.020	1.019	*	0.985	0.954
		5	1.074	1.030	1.134	1.093	1.097	1.126	1.063	1.056	0.990	0.949	1.045	1.007	1.011	1.038	0.980	0.973
		6	1.101	1.030	1.176	1.120	1.169	1.139	1.108	1.082	1.005	0.941	1.074	1.022	1.067	1.040	1.011	0.988
	5	4	1.102	1.072	1.175	1.131	1.136	*	1.117	1.061	0.997	0.969	1.063	1.023	1.027	*	1.010	0.959
		5	1.079	1.063	1.152	1.124	1.092	1.127	1.078	1.065	0.994	0.980	1.061	1.036	1.007	1.038	0.993	0.981
		6	1.096	1.040	1.117	1.106	1.113	1.155	1.080	1.072	0.972	0.923	0.991	0.981	0.988	1.025	0.958	0.951
	6	4	1.019	1.050	1.143	1.135	1.137	*	1.079	1.035	0.938	0.967	1.053	1.045	1.047	*	0.994	0.953
		5	1.060	1.059	1.155	1.129	1.108	1.119	1.113	1.044	0.996	0.995	1.086	1.062	1.042	1.052	1.046	0.982
		6	1.064	1.034	1.141	1.134	1.087	1.137	1.108	1.074	0.951	0.925	1.021	1.015	0.972	1.017	0.991	0.961
0.75	2	4	1.167	1.073	1.549	1.304	1.550	*	1.263	1.047	0.949	0.872	1.260	1.060	1.260	*	1.027	0.851
		5	1.231	1.121	1.741	1.414	1.556	1.715	1.318	1.113	0.947	0.862	1.339	1.088	1.198	1.319	1.014	0.856
		6	1.281	0.967	1.691	1.370	1.495	1.715	1.296	1.131	0.986	0.745	1.302	1.055	1.151	1.321	0.998	0.871
	3	4	1.168	1.065	1.590	1.368	1.531	*	1.286	1.089	0.909	0.829	1.237	1.065	1.192	*	1.001	0.848
		5	1.199	1.062	1.634	1.258	1.496	1.635	1.272	1.068	0.966	0.856	1.317	1.014	1.206	1.318	1.026	0.861
		6	1.263	0.963	1.746	1.412	1.568	1.762	1.307	1.136	0.962	0.733	1.330	1.075	1.194	1.341	0.995	0.865
	4	4	1.121	1.019	1.504	1.275	1.447	*	1.211	1.013	0.924	0.840	1.239	1.051	1.192	*	0.998	0.835
		5	1.203	1.039	1.592	1.338	1.498	1.647	1.234	1.071	0.994	0.859	1.317	1.107	1.239	1.362	1.020	0.886
		6	1.222	0.949	1.711	1.376	1.481	1.714	1.271	1.109	0.943	0.732	1.320	1.061	1.142	1.322	0.980	0.855
	5	4	1.203	1.046	1.505	1.318	1.534	*	1.279	1.053	0.945	0.822	1.183	1.036	1.205	*	1.005	0.828
		5	1.227	1.080	1.707	1.352	1.468	1.627	1.235	1.090	0.973	0.857	1.354	1.072	1.164	1.290	0.979	0.864
		6	1.243	0.930	1.722	1.345	1.536	1.708	1.278	1.114	0.981	0.734	1.359	1.061	1.212	1.347	1.008	0.879
	6	4	1.110	1.017	1.521	1.323	1.480	*	1.206	1.030	0.911	0.834	1.247	1.085	1.214	*	0.990	0.845
		5	1.204	1.084	1.659	1.341	1.478	1.686	1.274	1.087	0.980	0.883	1.350	1.091	1.203	1.372	1.037	0.885
		6	1.202	0.941	1.688	1.363	1.477	1.697	1.283	1.125	0.924	0.724	1.297	1.047	1.135	1.304	0.986	0.865
0.95	2	4	1.289	1.066	2.272	1.583	2.301	*	1.451	1.058	0.902	0.747	1.591	1.108	1.611	*	1.016	0.741
		5	1.441	1.130	3.004	1.804	2.319	3.006	1.516	1.179	0.931	0.730	1.941	1.166	1.499	1.943	0.980	0.762
		6	1.468	0.918	3.150	1.774	2.308	3.175	1.624	1.207	0.968	0.605	2.077	1.170	1.522	2.093	1.071	0.796
	3	4	1.295	1.066	2.329	1.681	2.270	*	1.503	1.106	0.875	0.720	1.573	1.135	1.533	*	1.015	0.747
		5	1.399	1.100	2.876	1.620	2.180	2.868	1.447	1.083	0.950	0.747	1.953	1.100	1.480	1.947	0.983	0.736
		6	1.485	0.925	3.113	1.784	2.331	3.234	1.554	1.225	0.955	0.595	2.002	1.147	1.499	2.080	0.999	0.787
	4	4	1.239	1.030	2.240	1.562	2.188	*	1.419	1.018	0.877	0.729	1.585	1.105	1.548	*	1.004	0.720
		5	1.378	1.062	2.825	1.678	2.222	2.823	1.459	1.091	0.950	0.732	1.947	1.157	1.532	1.947	1.006	0.752
		6	1.420	0.927	3.107	1.737	2.241	3.089	1.526	1.178	0.907	0.592	1.985	1.110	1.431	1.973	0.975	0.752
	5	4	1.333	1.056	2.287	1.653	2.255	*	1.471	1.054	0.894	0.709	1.535	1.109	1.513	*	0.987	0.708
		5	1.393	1.138	2.927	1.733	2.186	2.899	1.493	1.110	0.938	0.767	1.973	1.168	1.473	1.954	1.006	0.748
		6	1.415	0.902	2.978	1.705	2.300	3.064	1.510	1.190	0.912	0.582	1.920	1.099	1.483	1.975	0.974	0.767
	6	4	1.230	1.018	2.242	1.622	2.226	*	1.376	1.040	0.878	0.726	1.600	1.157	1.589	*	0.982	0.742
		5	1.355	1.115	2.803	1.664	2.199	2.898	1.494	1.100	0.927	0.763	1.917	1.138	1.504	1.982	1.022	0.753
		6	1.398	0.903	3.023	1.725	2.188	2.938	1.574	1.177	0.910	0.588	1.969	1.123	1.425	1.913	1.025	0.767

* k can be up to 1 when $m=4$

Table A.15 RE_2 and RE_3 values for $\hat{\beta}_0$ under 30% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.161	1.194	1.118	1.172	1.102	*	1.171	1.133	1.017	1.046	0.979	1.027	0.965	*	1.026	0.992
		5	1.099	1.081	1.078	1.148	1.091	1.071	1.140	1.073	0.989	0.974	0.971	1.034	0.982	0.964	1.027	0.966
		6	1.060	1.029	1.027	1.101	1.072	1.069	1.081	1.073	0.996	0.966	0.964	1.034	1.006	1.003	1.015	1.007
	3	4	1.036	1.068	1.062	1.084	1.069	*	1.095	1.072	0.956	0.985	0.980	1.001	0.987	*	1.010	0.990
		5	1.010	1.004	1.010	1.036	1.060	1.005	1.048	1.041	0.961	0.955	0.961	0.986	1.008	0.956	0.997	0.991
		6	1.059	1.007	1.036	1.050	1.084	1.069	1.074	1.009	1.029	0.977	1.006	1.020	1.052	1.037	1.042	0.980
	4	4	1.001	1.007	1.055	1.048	1.020	*	1.029	0.993	0.973	0.979	1.026	1.019	0.992	*	1.000	0.965
		5	1.016	0.967	1.028	1.009	1.017	1.012	0.995	1.001	0.996	0.948	1.008	0.989	0.997	0.993	0.976	0.982
		6	1.032	0.957	1.064	1.019	1.071	1.023	1.023	1.019	1.015	0.941	1.047	1.003	1.053	1.006	1.007	1.003
	5	4	1.059	1.054	1.093	1.070	1.059	*	1.073	1.021	0.994	0.989	1.026	1.004	0.994	*	1.007	0.959
		5	1.016	0.991	1.056	1.039	1.010	1.028	1.014	1.012	1.002	0.976	1.040	1.024	0.996	1.013	0.999	0.998
		6	1.039	0.970	1.024	1.031	1.042	1.069	1.018	1.014	0.971	0.907	0.957	0.963	0.973	0.998	0.951	0.948
	6	4	0.983	0.985	1.050	1.047	1.045	*	1.017	0.991	0.960	0.962	1.025	1.022	1.021	*	0.993	0.968
		5	1.013	0.998	1.052	1.037	1.040	1.032	1.050	0.998	0.998	0.983	1.037	1.022	1.025	1.017	1.035	0.983
		6	1.018	0.967	1.047	1.036	1.010	1.038	1.035	1.014	0.972	0.923	0.999	0.988	0.964	0.991	0.988	0.968
0.75	2	4	1.078	0.948	1.280	1.103	1.265	*	1.114	0.947	0.990	0.871	1.176	1.013	1.162	*	1.023	0.869
		5	1.035	0.922	1.310	1.089	1.257	1.355	1.096	0.905	0.976	0.869	1.235	1.027	1.185	1.277	1.033	0.853
		6	1.032	0.765	1.286	1.040	1.159	1.330	1.034	0.898	0.999	0.741	1.245	1.007	1.122	1.287	1.001	0.869
	3	4	1.010	0.885	1.284	1.070	1.259	*	1.084	0.892	0.967	0.847	1.228	1.024	1.204	*	1.037	0.854
		5	1.003	0.876	1.293	0.974	1.182	1.316	1.027	0.877	0.980	0.855	1.262	0.951	1.153	1.284	1.003	0.856
		6	1.028	0.734	1.346	1.045	1.219	1.376	1.038	0.876	0.997	0.712	1.305	1.014	1.182	1.334	1.006	0.850
	4	4	0.992	0.854	1.240	1.021	1.198	*	1.013	0.825	0.995	0.856	1.243	1.024	1.201	*	1.016	0.827
		5	1.007	0.833	1.298	1.029	1.219	1.292	1.002	0.861	1.021	0.845	1.317	1.044	1.236	1.310	1.016	0.873
		6	1.022	0.742	1.329	1.034	1.171	1.340	0.998	0.890	0.992	0.720	1.289	1.003	1.136	1.300	0.968	0.863
	5	4	1.032	0.883	1.256	1.061	1.273	*	1.058	0.855	0.958	0.819	1.166	0.985	1.181	*	0.982	0.793
		5	1.020	0.861	1.382	1.040	1.207	1.342	1.015	0.877	1.005	0.848	1.361	1.024	1.189	1.321	0.999	0.864
		6	1.041	0.722	1.362	1.040	1.238	1.370	1.042	0.897	1.014	0.704	1.327	1.013	1.206	1.335	1.015	0.874
	6	4	0.980	0.822	1.231	1.028	1.234	*	0.993	0.847	0.978	0.820	1.228	1.025	1.231	*	0.991	0.845
		5	1.026	0.870	1.346	1.027	1.224	1.351	1.039	0.889	1.009	0.856	1.323	1.010	1.204	1.328	1.021	0.874
		6	1.012	0.731	1.351	1.025	1.200	1.361	1.039	0.911	0.986	0.712	1.316	0.999	1.169	1.326	1.012	0.887
0.95	2	4	0.987	0.703	1.816	0.999	1.832	*	1.044	0.714	0.977	0.696	1.797	0.989	1.814	*	1.034	0.707
		5	1.000	0.699	2.320	1.022	1.694	2.332	1.021	0.713	0.985	0.688	2.285	1.007	1.668	2.297	1.006	0.702
		6	0.967	0.529	2.295	0.990	1.565	2.310	1.015	0.706	1.017	0.556	2.413	1.041	1.646	2.429	1.067	0.743
	3	4	0.986	0.703	1.845	1.054	1.869	*	1.037	0.710	0.982	0.700	1.836	1.049	1.861	*	1.032	0.706
		5	0.983	0.692	2.293	0.975	1.568	2.303	0.959	0.693	1.000	0.703	2.332	0.992	1.595	2.342	0.976	0.705
		6	1.004	0.526	2.299	1.005	1.558	2.370	0.987	0.714	1.028	0.539	2.352	1.028	1.594	2.425	1.010	0.731
	4	4	0.990	0.675	1.769	0.980	1.737	*	0.975	0.646	1.033	0.704	1.846	1.023	1.812	*	1.017	0.674
		5	0.999	0.684	2.314	1.026	1.665	2.337	0.969	0.702	1.028	0.704	2.383	1.056	1.714	2.406	0.998	0.722
		6	1.009	0.564	2.393	1.000	1.579	2.368	0.985	0.749	0.991	0.554	2.349	0.981	1.550	2.325	0.967	0.735
	5	4	1.012	0.711	1.854	1.048	1.819	*	1.040	0.689	0.958	0.673	1.756	0.993	1.723	*	0.985	0.652
		5	0.994	0.718	2.398	1.054	1.652	2.399	1.014	0.707	0.973	0.703	2.346	1.031	1.617	2.347	0.992	0.692
		6	1.004	0.541	2.334	1.001	1.630	2.402	0.991	0.745	0.974	0.524	2.262	0.971	1.580	2.329	0.961	0.723
	6	4	0.975	0.648	1.782	0.985	1.778	*	0.957	0.682	1.014	0.674	1.852	1.023	1.848	*	0.994	0.709
		5	0.978	0.703	2.398	1.007	1.644	2.408	0.998	0.706	0.985	0.708	2.416	1.014	1.656	2.426	1.006	0.711
		6	1.005	0.542	2.362	0.995	1.548	2.324	1.039	0.749	1.009	0.544	2.371	0.999	1.554	2.333	1.044	0.752

* k can be up to 1 when $m=4$

Table A.16 RE_2 and RE_3 values for $\hat{\beta}_1$ under 30% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.221	1.948	0.524	1.277	0.530	*	1.204	2.024	1.047	1.669	0.449	1.095	0.454	*	1.032	1.734
		5	1.174	1.755	0.312	1.209	0.566	0.319	1.163	1.744	0.983	1.469	0.261	1.012	0.474	0.267	0.974	1.460
		6	1.121	2.340	0.313	1.171	0.636	0.308	1.150	1.661	0.996	2.081	0.278	1.042	0.565	0.274	1.022	1.477
	3	4	1.104	1.656	0.486	1.146	0.504	*	1.114	1.684	1.035	1.552	0.455	1.074	0.472	*	1.044	1.578
		5	1.112	1.613	0.316	1.113	0.572	0.310	1.075	1.590	1.053	1.527	0.299	1.054	0.542	0.294	1.018	1.506
		6	1.002	2.058	0.313	1.033	0.586	0.307	1.054	1.455	0.980	2.011	0.306	1.010	0.573	0.301	1.030	1.422
	4	4	1.067	1.726	0.507	1.115	0.474	*	1.113	1.672	0.965	1.562	0.459	1.009	0.429	*	1.007	1.513
		5	1.019	1.513	0.312	1.038	0.539	0.311	1.029	1.520	1.018	1.511	0.311	1.037	0.539	0.311	1.028	1.519
		6	0.989	2.020	0.308	1.054	0.598	0.313	1.041	1.438	0.948	1.935	0.296	1.010	0.573	0.300	0.997	1.378
	5	4	1.057	1.647	0.496	1.110	0.480	*	1.067	1.597	0.978	1.523	0.459	1.027	0.444	*	0.987	1.477
		5	1.060	1.534	0.324	1.077	0.551	0.314	1.032	1.555	1.013	1.466	0.310	1.029	0.527	0.300	0.987	1.486
		6	1.007	1.977	0.308	1.038	0.598	0.306	1.046	1.461	0.993	1.950	0.304	1.024	0.590	0.302	1.032	1.440
	6	4	1.015	1.586	0.495	1.039	0.514	*	1.029	1.598	0.974	1.522	0.475	0.997	0.493	*	0.987	1.534
		5	1.029	1.485	0.318	1.059	0.569	0.326	1.043	1.510	0.983	1.418	0.303	1.011	0.544	0.311	0.996	1.442
		6	1.012	1.984	0.316	1.031	0.585	0.312	1.032	1.430	0.994	1.949	0.311	1.012	0.574	0.306	1.013	1.405
0.75	2	4	1.188	1.671	0.673	1.205	0.691	*	1.182	1.711	1.024	1.441	0.580	1.039	0.596	*	1.019	1.476
		5	1.081	1.540	0.446	1.144	0.697	0.456	1.127	1.492	0.992	1.413	0.409	1.050	0.639	0.419	1.034	1.369
		6	1.088	1.952	0.460	1.114	0.771	0.465	1.110	1.436	0.994	1.784	0.420	1.019	0.705	0.425	1.014	1.312
	3	4	1.058	1.488	0.628	1.071	0.619	*	1.076	1.481	1.000	1.406	0.593	1.012	0.585	*	1.017	1.399
		5	1.071	1.439	0.457	1.069	0.678	0.475	1.045	1.426	1.034	1.390	0.442	1.033	0.656	0.459	1.010	1.378
		6	1.005	1.778	0.462	1.027	0.728	0.484	1.008	1.318	0.994	1.759	0.457	1.016	0.721	0.479	0.998	1.305
	4	4	1.047	1.543	0.645	1.075	0.652	*	1.072	1.492	0.987	1.455	0.608	1.014	0.615	*	1.010	1.406
		5	1.010	1.391	0.463	1.038	0.675	0.466	1.021	1.405	1.043	1.437	0.478	1.072	0.697	0.482	1.054	1.451
		6	1.003	1.797	0.477	1.027	0.746	0.483	1.022	1.348	0.974	1.746	0.464	0.998	0.725	0.469	0.993	1.310
	5	4	1.031	1.448	0.625	1.063	0.624	*	1.026	1.426	0.989	1.390	0.600	1.020	0.599	*	0.984	1.368
		5	1.059	1.416	0.487	1.033	0.673	0.488	1.020	1.445	1.051	1.405	0.483	1.025	0.668	0.484	1.012	1.434
		6	0.985	1.747	0.484	1.007	0.723	0.464	1.031	1.324	0.999	1.773	0.491	1.021	0.733	0.471	1.046	1.343
	6	4	1.012	1.471	0.639	1.039	0.638	*	1.021	1.500	0.969	1.408	0.611	0.994	0.611	*	0.977	1.435
		5	1.013	1.336	0.487	1.029	0.688	0.483	1.015	1.393	0.980	1.292	0.471	0.995	0.665	0.467	0.982	1.348
		6	1.003	1.804	0.479	1.006	0.722	0.470	1.019	1.317	1.005	1.808	0.480	1.008	0.724	0.471	1.021	1.320
0.95	2	4	1.050	1.357	0.893	1.076	0.937	*	1.084	1.391	0.980	1.266	0.834	1.004	0.874	*	1.012	1.298
		5	1.035	1.317	0.801	1.108	0.974	0.824	1.087	1.303	0.989	1.258	0.765	1.059	0.930	0.787	1.038	1.245
		6	1.029	1.602	0.811	1.064	1.003	0.804	1.055	1.249	0.984	1.532	0.776	1.018	0.960	0.769	1.010	1.195
	3	4	1.019	1.312	0.892	1.034	0.863	*	1.020	1.317	1.017	1.310	0.891	1.032	0.862	*	1.018	1.315
		5	1.043	1.281	0.798	1.027	0.941	0.812	1.015	1.294	1.019	1.252	0.780	1.004	0.920	0.793	0.992	1.265
		6	1.002	1.569	0.827	1.033	1.002	0.846	1.002	1.195	0.990	1.549	0.817	1.020	0.990	0.836	0.990	1.180
	4	4	1.027	1.362	0.915	1.042	0.920	*	1.050	1.333	1.009	1.339	0.899	1.024	0.905	*	1.032	1.310
		5	1.007	1.277	0.800	1.011	0.946	0.788	0.992	1.289	1.019	1.293	0.810	1.023	0.958	0.798	1.004	1.305
		6	1.006	1.630	0.841	1.054	1.003	0.815	1.030	1.239	0.960	1.556	0.803	1.007	0.958	0.778	0.984	1.183
	5	4	0.988	1.295	0.883	1.020	0.875	*	1.016	1.311	0.960	1.258	0.858	0.990	0.850	*	0.986	1.273
		5	1.017	1.294	0.832	1.021	0.940	0.822	1.008	1.322	1.010	1.285	0.827	1.014	0.934	0.817	1.002	1.313
		6	0.978	1.571	0.800	0.987	0.973	0.810	1.027	1.218	1.002	1.611	0.820	1.012	0.997	0.830	1.053	1.249
	6	4	1.010	1.343	0.894	1.018	0.927	*	1.012	1.379	0.987	1.313	0.873	0.995	0.906	*	0.989	1.348
		5	0.999	1.227	0.813	1.007	0.970	0.796	1.002	1.290	1.000	1.228	0.814	1.008	0.971	0.797	1.004	1.292
		6	0.993	1.607	0.817	1.006	0.982	0.812	1.007	1.210	0.996	1.612	0.819	1.009	0.985	0.815	1.011	1.214

* k can be up to 1 when $m=4$

Table A.17 RE_2 and RE_3 values for MSE of regression model under 30% outlier case

ρ	r	m	RE_2								RE_3							
			PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂
0.50	2	4	1.010	0.997	1.040	1.011	1.049	*	1.033	0.992	0.991	0.978	1.020	0.991	1.029	*	1.013	0.973
		5	0.998	0.975	1.040	1.016	1.028	1.029	1.028	0.973	0.990	0.968	1.032	1.009	1.021	1.022	1.020	0.965
		6	1.000	0.945	1.030	1.006	1.020	1.032	0.990	0.978	1.012	0.957	1.043	1.019	1.033	1.045	1.002	0.990
	3	4	1.012	0.977	1.053	1.008	1.049	*	1.030	0.991	1.010	0.976	1.051	1.006	1.047	*	1.028	0.989
		5	0.995	0.972	1.032	0.989	1.027	1.030	0.991	0.977	1.000	0.977	1.036	0.994	1.032	1.035	0.996	0.982
		6	1.001	0.951	1.039	1.003	1.016	1.043	1.005	0.979	1.007	0.957	1.045	1.009	1.022	1.049	1.011	0.985
	4	4	0.997	0.975	1.043	0.995	1.018	*	1.005	0.955	1.010	0.988	1.056	1.008	1.031	*	1.018	0.968
		5	0.996	0.970	1.029	0.993	1.019	1.023	0.989	0.974	1.006	0.979	1.039	1.003	1.029	1.033	0.998	0.983
		6	1.003	0.944	1.037	1.005	1.029	1.042	1.015	0.979	1.000	0.941	1.033	1.002	1.026	1.038	1.012	0.976
	5	4	0.996	0.971	1.036	1.008	1.024	*	1.006	0.970	0.997	0.971	1.036	1.009	1.024	*	1.007	0.970
		5	1.012	0.975	1.044	1.006	1.025	1.045	0.988	0.969	1.013	0.977	1.046	1.008	1.026	1.047	0.990	0.971
		6	1.001	0.940	1.030	0.988	1.021	1.029	0.992	0.974	1.009	0.948	1.038	0.996	1.029	1.037	1.000	0.982
	6	4	0.983	0.962	1.027	0.988	1.026	*	0.996	0.965	0.991	0.970	1.035	0.996	1.034	*	1.004	0.973
		5	0.999	0.968	1.043	1.003	1.031	1.032	0.996	0.979	0.994	0.963	1.038	0.998	1.026	1.027	0.991	0.974
		6	0.996	0.942	1.025	1.003	1.016	1.036	1.000	0.976	0.998	0.944	1.027	1.005	1.018	1.038	1.002	0.978
0.75	2	4	1.018	0.837	1.261	1.021	1.247	*	1.042	0.828	0.993	0.817	1.229	0.995	1.216	*	1.016	0.807
		5	1.007	0.850	1.328	1.010	1.198	1.310	1.029	0.850	0.993	0.839	1.310	0.997	1.181	1.292	1.015	0.838
		6	0.997	0.716	1.327	0.997	1.162	1.321	0.990	0.863	1.004	0.722	1.337	1.004	1.170	1.331	0.997	0.869
	3	4	1.023	0.836	1.261	1.020	1.248	*	1.029	0.838	1.019	0.833	1.256	1.017	1.244	*	1.026	0.835
		5	1.010	0.848	1.325	0.995	1.198	1.330	0.995	0.842	1.021	0.857	1.339	1.006	1.212	1.345	1.006	0.851
		6	0.999	0.728	1.336	1.008	1.165	1.357	1.010	0.864	1.004	0.732	1.343	1.013	1.172	1.365	1.015	0.868
	4	4	1.003	0.830	1.235	1.001	1.236	*	1.006	0.814	1.011	0.838	1.246	1.010	1.246	*	1.015	0.821
		5	0.988	0.837	1.328	0.998	1.192	1.337	1.001	0.843	1.000	0.846	1.344	1.010	1.206	1.352	1.012	0.853
		6	1.002	0.726	1.353	1.008	1.179	1.357	1.006	0.868	0.988	0.716	1.334	0.994	1.162	1.338	0.991	0.856
	5	4	1.002	0.828	1.240	1.010	1.243	*	1.010	0.832	0.999	0.826	1.238	1.008	1.241	*	1.008	0.830
		5	1.013	0.848	1.336	1.010	1.194	1.342	0.992	0.840	1.014	0.848	1.337	1.011	1.194	1.342	0.993	0.841
		6	0.990	0.722	1.341	0.995	1.170	1.325	1.001	0.863	0.999	0.728	1.353	1.004	1.180	1.336	1.010	0.871
	6	4	0.987	0.816	1.233	0.990	1.223	*	0.993	0.824	0.991	0.819	1.238	0.994	1.228	*	0.997	0.828
		5	1.002	0.845	1.355	1.006	1.216	1.339	1.002	0.850	0.994	0.839	1.346	0.999	1.207	1.329	0.994	0.844
		6	0.999	0.726	1.336	1.000	1.165	1.341	0.998	0.859	1.002	0.728	1.339	1.002	1.168	1.344	1.000	0.861
0.95	2	4	1.042	0.671	1.816	1.039	1.831	*	1.039	0.657	1.000	0.643	1.742	0.997	1.757	*	0.997	0.630
		5	1.009	0.691	2.344	1.022	1.640	2.325	1.033	0.695	0.985	0.675	2.289	0.999	1.602	2.271	1.009	0.679
		6	1.008	0.513	2.351	1.011	1.543	2.347	1.015	0.721	1.006	0.512	2.347	1.009	1.540	2.343	1.013	0.720
	3	4	1.024	0.670	1.849	1.034	1.836	*	1.034	0.672	1.015	0.664	1.834	1.026	1.821	*	1.026	0.667
		5	1.006	0.694	2.314	0.991	1.652	2.342	0.996	0.687	1.010	0.696	2.323	0.995	1.658	2.350	0.999	0.690
		6	1.011	0.521	2.373	1.016	1.559	2.388	1.002	0.722	1.010	0.521	2.371	1.015	1.558	2.386	1.001	0.722
	4	4	1.003	0.669	1.809	1.007	1.816	*	0.994	0.663	1.004	0.670	1.811	1.008	1.819	*	0.996	0.664
		5	0.992	0.690	2.345	1.002	1.651	2.353	1.002	0.691	0.992	0.690	2.346	1.003	1.652	2.354	1.002	0.691
		6	1.011	0.524	2.377	1.010	1.561	2.380	1.006	0.727	1.003	0.520	2.356	1.001	1.548	2.360	0.998	0.720
	5	4	1.006	0.672	1.822	1.011	1.820	*	1.007	0.671	0.999	0.667	1.810	1.004	1.807	*	1.000	0.666
		5	1.003	0.696	2.373	1.005	1.667	2.369	0.996	0.695	1.003	0.696	2.374	1.005	1.668	2.370	0.996	0.695
		6	0.991	0.525	2.348	0.993	1.548	2.367	0.991	0.722	1.002	0.531	2.373	1.004	1.565	2.392	1.001	0.730
	6	4	0.990	0.665	1.807	1.000	1.814	*	0.994	0.667	0.993	0.667	1.811	1.003	1.819	*	0.997	0.669
		5	1.004	0.697	2.376	1.005	1.681	2.387	1.007	0.698	1.004	0.697	2.376	1.005	1.681	2.388	1.007	0.698
		6	1.002	0.530	2.373	1.001	1.555	2.371	0.997	0.722	1.002	0.530	2.375	1.002	1.556	2.373	0.998	0.723

* k can be up to 1 when $m=4$

Table A.18 Bias values of the estimators $(\bar{y}, \hat{\beta}_0, \hat{\beta}_1)$ for sampling methods under 30% outlier case

			$Bias(\bar{y})$										$Bias(\hat{\beta}_0)$										$Bias(\hat{\beta}_1)$										
ρ	r	m	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	SRS	RSS	PRSS	ERSS	MRSS	DRSS	LRSS ₁	LRSS ₂	TBRSS ₁	TBRSS ₂	
0.50	4	4	0.008	-0.004	0.005	0.018	-0.014	-0.014	0.013	*	0.005	0.007	-0.005	-0.009	-0.008	0.017	-0.018	-0.013	0.004	*	0.008	0.009	-0.034	-0.019	-0.006	-0.006	-0.058	-0.030	-0.067	*	-0.011	-0.005	
		5	0.019	0.004	-0.006	0.023	-0.028	-0.020	-0.015	-0.017	-0.012	0.014	0.009	0.003	-0.009	0.022	-0.041	-0.020	-0.026	-0.023	-0.015	0.011	-0.011	-0.008	-0.009	-0.080	-0.010	-0.068	-0.105	-0.007	0.000		
		6	-0.009	0.001	0.001	0.034	-0.016	-0.004	-0.015	-0.013	0.002	0.014	-0.020	-0.003	-0.002	0.036	-0.023	-0.006	-0.017	-0.018	0.001	0.011	-0.020	-0.011	-0.006	0.026	-0.089	-0.010	-0.046	-0.096	-0.006	0.000	
		3	5	-0.001	-0.008	0.004	0.014	-0.019	-0.008	-0.015	-0.010	0.002	0.021	-0.001	-0.008	0.001	0.013	-0.025	-0.011	-0.021	-0.013	-0.003	0.018	0.013	-0.010	0.006	0.003	-0.083	-0.010	-0.048	-0.087	-0.004	0.014
		6	-0.005	-0.006	-0.001	0.032	-0.022	0.006	-0.025	-0.030	0.004	0.009	-0.011	-0.007	-0.004	0.034	-0.029	0.005	-0.027	-0.036	0.004	0.008	-0.017	-0.017	-0.009	0.016	-0.065	-0.008	-0.045	-0.081	-0.007	0.008	
		5	4	0.002	-0.007	0.000	0.016	-0.022	0.009	-0.018	*	0.001	0.019	-0.008	-0.007	0.001	0.013	-0.026	0.008	-0.020	*	-0.002	0.021	0.001	-0.005	-0.008	0.009	-0.054	-0.004	-0.050	*	-0.004	0.002
0.75	4	5	0.009	-0.006	0.001	0.008	-0.022	0.002	-0.017	-0.018	-0.002	0.017	0.005	-0.006	-0.003	0.010	-0.028	0.001	-0.024	-0.024	0.000	0.018	0.001	0.000	0.001	0.006	-0.081	-0.010	-0.047	-0.077	-0.001	0.015	
		6	-0.002	0.010	-0.006	0.031	-0.015	0.006	-0.007	-0.014	0.002	0.017	-0.006	0.008	-0.008	0.033	-0.020	0.007	-0.012	-0.020	0.002	0.017	0.007	-0.008	-0.007	-0.096	-0.007	-0.044	-0.073	0.002	0.008		
		5	4	0.000	-0.005	0.001	0.018	-0.011	0.015	-0.012	*	-0.010	0.021	-0.005	-0.006	0.000	0.019	-0.014	0.011	-0.018	*	-0.012	0.023	-0.009	0.003	-0.005	0.009	-0.058	0.003	-0.057	*	0.006	0.003
		6	-0.003	0.007	0.002	0.017	-0.012	-0.009	-0.018	-0.012	-0.007	0.014	-0.004	0.005	0.003	0.017	-0.021	-0.009	-0.020	-0.019	-0.009	0.018	-0.002	-0.014	0.005	0.014	-0.072	0.006	-0.038	-0.085	0.000	0.004	
		5	6	0.001	0.003	0.002	0.033	-0.010	0.003	-0.021	-0.015	0.009	0.021	0.000	0.003	0.001	0.036	-0.016	0.003	-0.021	-0.018	0.006	0.020	0.002	-0.007	0.005	0.012	-0.081	-0.008	-0.050	-0.069	-0.005	0.006
		6	5	0.003	-0.009	-0.001	0.017	-0.018	0.002	-0.005	*	-0.010	0.006	0.001	-0.010	-0.002	0.017	-0.021	0.002	-0.009	*	-0.009	0.010	-0.009	0.003	-0.015	0.005	-0.041	-0.004	-0.024	*	-0.008	0.009
0.95	4	5	0.005	0.006	-0.001	0.007	-0.014	0.009	-0.011	-0.017	-0.003	0.012	0.000	0.007	-0.003	0.007	-0.018	0.008	-0.011	-0.023	-0.004	0.011	-0.008	0.003	-0.009	0.006	-0.078	-0.006	-0.039	-0.075	0.000	0.010	
		6	0.006	-0.005	0.004	0.029	-0.016	-0.004	-0.010	-0.020	0.008	0.006	0.003	-0.005	0.003	0.032	-0.020	-0.003	-0.009	-0.025	0.006	0.008	-0.002	0.004	-0.001	0.019	-0.089	0.003	-0.045	-0.070	-0.005	0.011	
		5	4	0.009	0.012	-0.002	0.018	-0.015	0.000	-0.014	*	0.020	0.015	-0.009	0.005	-0.020	0.017	-0.021	0.001	-0.031	*	0.017	0.022	-0.049	-0.014	-0.008	-0.018	-0.012	-0.034	-0.006	*	0.003	-0.009
		6	0.020	0.001	-0.007	0.030	-0.036	-0.016	-0.014	-0.021	-0.013	0.016	0.000	0.000	-0.010	0.027	-0.036	-0.012	-0.025	-0.027	-0.012	0.012	-0.007	-0.006	-0.012	-0.007	-0.054	-0.005	-0.023	-0.028	-0.001	-0.003	
		5	6	-0.008	-0.003	-0.010	0.049	-0.032	-0.003	-0.026	-0.030	0.004	0.023	-0.020	-0.006	-0.011	0.049	-0.035	-0.003	-0.029	-0.032	0.004	0.019	-0.017	-0.004	-0.005	0.014	-0.021	-0.006	-0.027	-0.032	-0.006	-0.006
		6	4	-0.010	-0.006	-0.007	0.026	-0.018	0.003	-0.021	*	0.004	0.022	-0.017	-0.014	-0.008	0.024	-0.025	0.000	-0.021	*	0.003	0.019	-0.026	-0.019	-0.005	-0.006	-0.009	-0.003	-0.022	*	-0.003	0.000
6	5	4	-0.002	-0.001	0.011	0.023	-0.034	0.003	-0.023	-0.023	0.003	0.028	-0.002	-0.001	0.006	0.021	-0.035	-0.001	-0.028	-0.029	-0.001	0.024	0.007	-0.012	0.013	0.006	-0.030	-0.013	-0.035	-0.012	-0.009	0.007	
		5	6	-0.005	0.005	0.002	0.046	-0.028	-0.004	-0.029	-0.026	0.007	0.015	-0.008	0.004	-0.002	0.048	-0.032	0.006	-0.030	-0.027	0.008	0.014	-0.016	-0.014	0.012	0.008	-0.045	-0.009	-0.036	-0.009	-0.003	0.005
		4	0.002	0.000	-0.002	0.018	-0.022	0.004	-0.033	*	0.005	0.030	-0.010	-0.001	0.001	0.014	-0.027	0.003	-0.030	*	0.001	0.032	-0.003	-0.014	0.004	-0.020	-0.005	-0.024	*	-0.012	-0.008		
		5	0.009	-0.001	0.006	0.016	-0.028	0.000	-0.011	-0.028	0.003	0.021	0.005	-0.001	0.002	0.020	-0.035	0.000	-0.019	-0.030	0.006	0.022	0.006	-0.003	-0.003	0.000	-0.033	0.003	-0.034	-0.027	0.003	0.012	
		6	-0.002	0.008	-0.003	0.045	-0.024	0.005	-0.016	-0.029	0.000	0.021	-0.008	0.004	-0.004	0.046	-0.027	0.005	-0.022	-0.035	0.001	0.021	0.004	-0.002	-0.007	0.002	-0.044	-0.009	-0.029	-0.023	-0.002	0.005	
		6	4	0.000	-0.002	-0.001	0.021	-0.022	0.007	-0.022	*	-0.011	0.026	-0.006	-0.004	0.001	0.020	-0.024	0.001	-0.028	*	-0.012	0.026	-0.009	0.000	-0.010	-0.001	-0.021	0.004	-0.023	*	0.003	-0.001
6	6	5	-0.003	0.001	0.004	0.023	-0.024	-0.008	-0.022	-0.027	0.001	0.025	-0.005	-0.002	0.005	0.021	-0.030	-0.007	-0.023	-0.032	-0.001	0.031	-0.004	-0.017	0.011	0.004	-0.026	0.006	-0.048	-0.024	-0.008	-0.001	
		6	0.001	0.006	0.002	0.052	-0.026	0.002	-0.029	-0.025	0.008	0.027	-0.001	0.006	0.000	0.054	-0.032	0.002	-0.028	-0.028	0.005	0.023	0.001	-0.013	-0.009	0.006	-0.028	-0.003	-0.040	-0.040	0.002	0.006	
		4	0.003	-0.008	-0.005	0.025	-0.027	-0.005	-0.022	*	-0.007	0.024	-0.001	-0.008	-0.005	0.025	-0.029	-0.004	-0.024	*	-0.005	0.027	-0.007	0.003	-0.016	-0.001	-0.023	-0.002	-0.022	*	-0.010	0.004	
		5	0.005	-0.001	-0.003	0.013	-0.030	0.003	-0.023	-0.026	-0.001	0.018	-0.001	0.000	-0.005	0.014	-0.031	0.000	-0.025	-0.030	-0.002	0.018	-0.012	0.001	-0.005	0.002	-0.022	-0.006	-0.026	-0.037	-0.003	0.009	
		6	0.006	-0.008	-0.001	0.043	-0.034	-0.003	-0.025	-0.029	-0.002	0.016	0.003	-0.008	-0.002	0.045	-0.036	-0.002	-0.025	-0.032	-0.004	0.019	-0.001	-0.003	0.003	0.012	-0.034	0.003	-0.026	-0.023	-0.004	0.004	
		6	4	0.009	0.016	0.000	0.014	-0.002	0.000	-0.006	*	0.017	0.009	-0.006	0.005	-0.018	0.014	-0.008	0.004	-0.019	*	0.010	0.016	-0.027	0.002	0.004	-0.011	0.047	-0.014	0.068	*	0.017	-0.010
6	5	5	0.020	-0.001	-0.008	0.030	-0.014	-0.019	-0.019	-0.021	-0.008	0.008	-0.006	-0.003	-0.008	0.024	-0.012	-0.011	-0.026	-0.020	-0.006	0.003	0.004	0.012	-0.002	-0.013	0.063	0.012	0.024	0.061	0.016	0.001	
		6	-0.008	-0.007	-0.002	0.033	-0.020	0.003	-0.019	-0.021	0.006	0.019	-0.015	-0.008	0.001	0.028	-0.018	0.001	-0.021	-0.020	0.006	0.014	0.000	-0.002	0.001	-0.002	0.067	0.008	0.032	0.060	0.014	-0.009	
		4	-0.009	-0.006	-0.006	0.017	-0.014	-0.001	-0.021	*	0.004	0.019	-0.013	-0.015	-0.008	0.014	-0.018	-0.007	-0.020	*	0.002	0.017	-0.009	-0.009	0.002	-0.019	0.043	0.013	0.035	*	0.005	-0.008	
		5	6	-0.002	-0.006	0.008	0.020	-0.020	0.004	-0.017	-0.015	0.001	0.018	0.005	-0.006	0.002	0.016	-0.018	0.000	-0.021	-0.018	-0.002	0.011	0.013	-0.009	0.013	0.001	0.039	-0.006	0.020	0.061	-0.002	-0.005
		6	4	0.005	-0.001	-0.002	0.037	-0.018	-0.003	-0.022	-0.014	0.002	0.018	-0.005	-0.003	-0.007	0.038	-0.019	-0.004	-0.019	-0.015	0.004	0.016	-0.004	-0.004	0.000	-0.011	0.047	0.004	0.028	0.073	-0.003	-0.006
		6	5	0.003	-0.006	-0.005	0.018	-0.013	-0.003	-0.032	*</																						