

Survey of Modeling and Reduction Techniques for Low Frequency Noise in MOSFET Circuits

by

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ABSTRACT

Low frequency noise is a great concern especially for analog and RF electronic circuits. It affects circuits that operate at high frequencies due to the frequency translation and up-conversion of noise via mixers and oscillators. This thesis is a comprehensive literature survey on low frequency noise, encompassing the review of literature on physical modeling of basic low frequency noise generating mechanisms in semiconductors, compact modeling of noise in MOSFETs, and circuit design techniques that were proposed to reduce the adverse effects of low frequency noise. The noise models proposed in the literature are reviewed from the perspective of generality and validity especially for time-varying bias conditions.

ÖZETÇE

Düşük frekans gürültüsü özellikle analog ve radyo frekansı devreleri için kritik önem arz etmektedir. Bu gürültü yüksek frekanslarda çalışan devreleri frekans çevrimi sebebiyle ve gürültünün karıştırıcı ve işaret üreteçleriyle yüksek frekanslara taşınması yüzünden etkilemektedir. Bu tez düşük frekans gürültüsü hakkında kapsamlı bir literatür araştırmasıdır. Yarı iletkenlerde düşük frekans gürültüsü üreten mekanizmaların fiziksel modellenmesi, MOS transistörlerde gürültünün kompakt modellenmesi ve düşük frekans gürültüsünün ters etkilerini düşürmek için geliştirilen devre tasarım teknikleri incelenen literatür konularını oluşturmaktadır. Literatürdeki gürültü modelleri özellikle zamanla değişen besleme koşullarına genellenebilirlikleri ve bu koşullarda geçerlilikleri açılarından incelenmiştir.

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NOMENCLATURE

α	Parameter for polynomial fit in correlated model
α_H	Hooge's parameter
α_t	Tunneling constant
$\Delta\mu$	Mobility fluctuation
ΔN	Number fluctuation
γ	Frequency exponent parameter of 1/f noise
λ	Used for trap rates
μ	Carrier mobility
$\overline{\Delta N^2}$	Variance of number fluctuations
σ	Standard deviation
τ_0	Time constant of traps at low state
τ_1	Time constant of traps at high state
τ_c	Capture time constant of traps
τ_e	Emission time constant of traps
$c(t)$	Autocovariance function

C_N	Capture rate
C_d	Depletion capacitance
C_{it}	Interface capacitance
C_{ox}	Oxide capacitance
E_C	Conduction band energy
E_f	Fermi energy
E_N	Emission rate
E_V	Valence band energy
f	Frequency
f_i	Corner frequency
$g(V, E)$	A parameter for Enz's noise calculation methodology
g_m	Transconductance
$H_0(f)$	Transfer function
I_d	Drain current
k	Boltzman's constant
L	MOS transistor length
N_t	Density of total traps

n_t	Density of total filled traps
N_{tr}	Number of traps
P_f	Fermi function
p_t	Density of total empty traps
Q	Charge
q	Unit charge
Q_{acc}	Accumulation charge
Q_{inv}	Inversion charge
R	Reimbold's parameter
S_N	PSD of number fluctuations
T	Period
T_K	Temperature in Kelvin
V_{bs}	Bulk-to-source voltage
V_{ds}	Drain-to-source voltage
V_d	Drain voltage
V_{fb}	Flat-band voltage
V_{gs}	Gate-to-source voltage

V_g	Gate voltage
V_{os}	Offset voltage
V_t	Threshold voltage
W	MOS transistor width
ACF	Auto-covariance Function
BJT	Bipolar Junction Transistor
BSIM	Berkeley Short Channel IGFET Model
CDS	Correlated Double Sampling
CMOS	Complementary MOS transistor
EKV	Enz-Krummenacher-Vittoz
FET	Field Effect Transistor
FSB	Forward Substrate Bias
GBW	Gain Bandwidth
GR	Generation - Recombination
IC	Integrated Circuit
LEM	Langevin Equation Model
LF	Low Frequency

LSE Large Signal Excitation

LTI Linear Time Invariant

MIT Massachusetts Institute of Technology

MOS Metal Oxide Semiconductor

NMOS n-channel MOS transistor

P Power

PMOS p-channel MOS transistor

PSD Power Spectral Density

PSP Acronym for a surface potential based MOS model

RF Radio Frequency

RTS Random Telegraph Signal

SAM Stochastic Automaton Model

SNR Signal-to-noise ratio

SPICE Simulation Program with Integrated Circuit Emphasis

SRH Shockley-Read-Hall

TAT Trap Assisted Tunneling

VCO Voltage Controlled Oscillator

Chapter 1

INTRODUCTION

Noise is a phenomenon that exists almost in every branch of engineering and natural sciences. In electrical engineering, noise is defined as spontaneous fluctuations in electrical quantities such as voltage and current. There are different noise sources which effect performance of electrical devices. We can classify noise sources as external and internal ones. Examples for externally generated noise types can be interference noise from power lines or other coupled noise types between different devices such as crosstalk or impulse noises. Apart from these externally generated ones there are also internal noise sources. External and internal classification is about the origin of the noise and dealing with the former one is much easier than the latter one. Shielding the circuits from outer environment, grounding in a better way or routing digital/power lines in integrated circuits in a careful way can be effective solutions when dealing with external noise sources. However, in case of internally generated noise it is not such an easy task to eliminate noise or to get rid of its effects.

Internal noise can also be divided into two categories which are operational noise and inherently generated noise. Operational noise is the result of circuit operation such as intermodulation noise in communication circuits. Although it is not as easy as dealing with external noise sources, operational noise is also avoidable in certain conditions. Generally it is required to have strong circuit design skills to deal with these type of noises. Inherently generated noises are the second type of internal noises which are directly related to the material properties. Thermal noise, shot noise and

low frequency (LF) noise are all inherently generated noises.

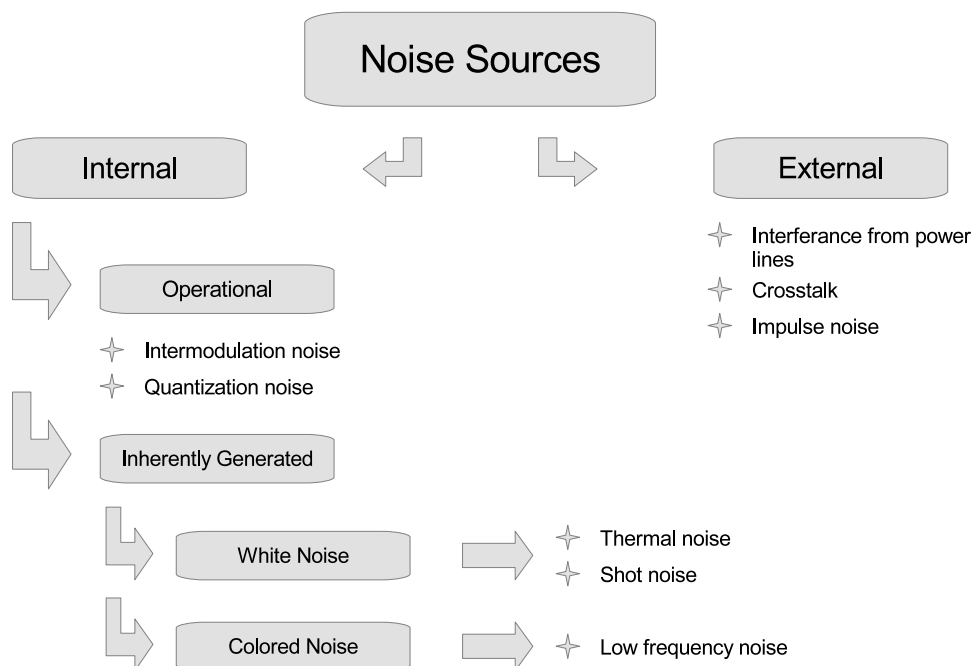


Figure 1.1: Classification of noise sources in electronic devices

Thermal and shot noises are well-understood white noise processes and these are directly related to the semiconductor material in use. Although it is not possible to eliminate thermal and shot noises, effects of them can be reduced by using some techniques since they are frequency independent. The term *white noise* is often used for the description of frequency independent noise types. Power spectral density (PSD) of a white noise is assumed to be flat across whole frequency band. However, mathematically it is not possible for a noise process to have a constant PSD across all the frequency spectra since in that case the resulting total noise power will be infinity. Indeed there is no exact white noise in nature. White noise definition is used

for practical purposes. It is valid in the frequency interval of interest. For example, PSD of thermal noise is flat up to roughly 100 THz [10].

Reducing the effect of white noise is possible. Averaging and oversampling techniques are used for this purpose. Performance criterion in noise reduction is the signal to noise ratio (SNR) which is simply the ratio of signal power to noise power.

$$SNR = \frac{P_{signal}}{P_{noise}} \quad (1.1)$$

If there exists only white noise in a circuit, it is possible to improve SNR by just averaging. Averaging means measuring the signal and noise and taking their average over a specified bandwidth. After the averaging, noise will reduce whereas original signal's power remains same. With number of X measurements, SNR will improve with a factor of $\sqrt{X}$. Another technique is the oversampling method which is used especially in data converters to improve SNR in which SNR improvement is obtained by sampling the signal beyond the nyquist rate. Although it is possible to reduce the effect of white noise with averaging and oversampling techniques, noise itself is not actually reduced in both cases. However, reducing the effect of noise may also be as important as reducing the noise itself in electronic circuit design.

If the noise is not white, it is named as colored noise. Low frequency (LF) noise is one of the colored noises and sometimes it is called as *pink noise* or *flicker noise*. Another name of low frequency noise is *1/f noise* due to its frequency dependent characteristic which linearly decreases as frequency increases. In Fig. 1.2, a frequency spectrum example for electrical devices that shows different noise sources is given.

LF noise is becoming to be a more and more important issue for IC design both for analog and digital applications. Transistors are scaling down in a continuous manner which results in noisier devices. In these sub-micron devices LF noise dominates especially in digital circuits [3], [4]. Analog circuits are also suffering from LF noise

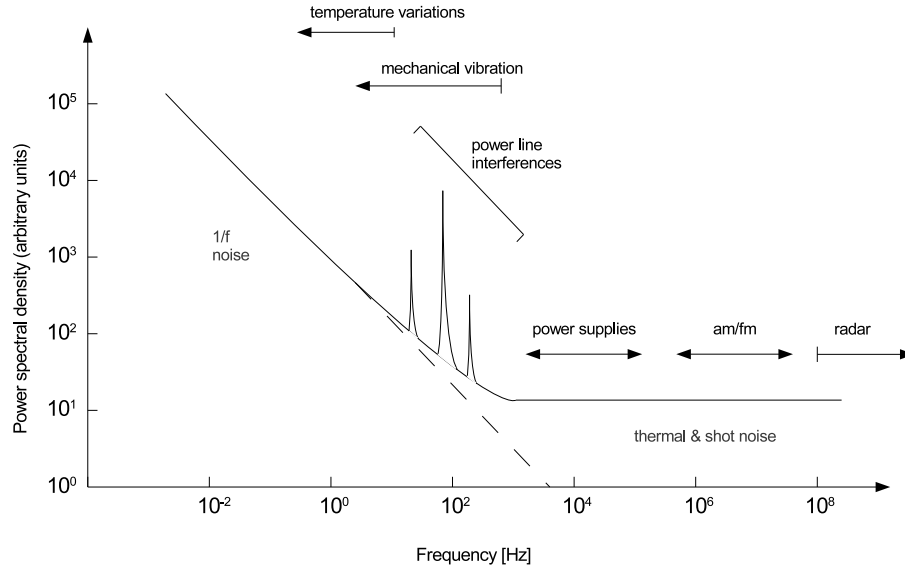


Figure 1.2: Noise spectrum example for electrical devices

in high precision and low frequency applications. Active pixel sensor circuits (APS) are good examples that suffer from LF noise since in digital cameras noise is one of the most important design parameter [109], [58]. Since low frequency noise is upconverted to the higher frequencies via mixers in RF circuits, these circuits have also been subject of LF noise studies [38].

There are significant challenges for engineers in LF noise modeling and simulation. First of all, underlying physics of LF noise in transistors have not yet been well-understood. In the advanced transistor models noise depends on time-varying quantities such as voltage across devices [20]. In that case non-stationarity is also an important concern which move the researches. All these processes require knowledge of advanced mathematics and physics with high computational skills. Moreover, reducing the effects of LF noise is not as easy as in the white noise case. Averaging

and oversampling methods do not reduce the effects of LF noise. Although numerous models have been developed, there is no universal model that truly captures LF noise behavior. LF noise is still a subject of active research in MOS transistor applications.

1.1 Scope of the Thesis

This thesis is a comprehensive literature survey in the subject of low frequency noise in MOS transistors. From classical works dating back to 1950s to the novel models of last few years, we have given a review of most prominent physical and circuit simulation oriented analytical LF noise models for MOS transistors. Finally, LF noise reduction techniques are given in the last chapter. Our original comments usually are left to the discussion sections. In other sections, we have tried to discriminate our own comments from the others by explicitly referring to an author or an existing work.

The motivations of this work are:

- i. First of all there is a naming convention problem in literature among researchers. Historically, low frequency (LF) noise is used closely with *flicker noise*, *1/f noise*, *generation recombination noise*, *burst noise*, *popcorn noise*, *random telegraph noise* terms. Although some of these terms are exactly used for the same phenomenon, there are differences between the others. In this thesis, we have clarified this problem by analyzing LF noise models in a comparative way. A chronological method is used, focusing on effects of technological advances through the years and their impacts on MOS transistor modeling.
- ii. There is no universally accepted LF noise model for MOS transistors. There have been long debates about even in the physical origin of the LF noise. These debates are still going on. We have first summarized theoretical models and then we have discussed if they were verified experimentally. After each model

a critical discussion part was provided where both weaknesses and strengths of these models are given.

- iii. Apart from on going debates on physical origin of the LF noise, existing models use very different approaches. We grouped them in two as physical models and circuit simulation oriented compact analytical models. Implementing physical models into circuit simulators is a challenging task and requires both analog circuit designer perspective and strong mathematical skills. In this thesis we have summarized some state of the art analytical models that were designed for circuit simulators.

Thesis plan is given as follows:

Chapter 1 is the introduction chapter. Scope of the thesis and a physical background section were given. Then comes a section which covers basic operation principles of MOS transistors. This introduction chapter was prepared to help reader in order to be familiar with the subject before digging into complex physical models.

After the introduction chapter there is Chapter 2 in which we have given an introduction to noise types on semiconductors. Fundamental noise sources were divided into two as broadband noise and LF noise. Challenging nature of LF noise was explained briefly in this section. LF noise types were introduced and their differences and relations to each other were examined.

Chapter 3 focuses on physical modeling of LF noise. Theories that try to explain physical origin of LF noise were summarized under three main headings: number fluctuation, mobility fluctuation and correlated models.

Chapter 4 is about compact modeling of LF noise. Flicker noise models of BSIM and SPICE were reviewed. A compact noise model which is not implemented yet in any circuit simulator was also reviewed. Their relations to classical physical models that were given in Chapter 3 were discussed.

In Chapter 5, we focused on RTS noise modeling. We reviewed RTS noise models in different bias cases including transient and constant gate biasing. Advanced nonstationary models that capture LF noise behavior in time-varying conditions were briefly reviewed.

Chapter 6 is about LF noise reduction methods. Forward substrate biasing, large signal excitation, correlated double sampling and chopper stabilization methods were reviewed. LF noise modeling in these scenarios were discussed in the light of advanced RTS models that were given in Chapter 5.

1.2 Physical Background

1.2.1 Fermi Energy ¹

Fermi-Dirac statistics gives us insight for the distribution of particles in certain conditions. In particular, when finding the probability of an electron to be in a certain energy level we use Fermi-Dirac statistics [49]. *Fermi function* which is derived using Fermi-Dirac statistics gives us that probability. $P_f(E)$ definition is given in [49]:

$$P_f(E) = \frac{1}{1 + e^{(E-E_F)/kT_K}} \quad (1.2)$$

In eq. 1.2, E_F is the fermi energy and E is the energy level that is occupied with probability $P_f(E)$. k is Boltzmann's constant and T_K is temperature in Kelvin degree. Noting that if E is above a few kT_K than E_F , eq. 1.2 is approximated as [49]:

$$P_f(E) \approx e^{-(E-E_F)/kT_K} \quad (1.3)$$

According to Eq. 1.3, $P_f(E)$ decreases exponentially at high E levels. In contrast, if E is below a few kT_K than E_F , eq. 1.2 can be approximated as follows:

¹This part is mostly summarized from [49]

$$P_f(E) \approx 1 - e^{-(E_F - E)/kT_K} \quad (1.4)$$

In that case $P_f(E)$ approaches to one for lower E levels. Fermi function is plotted

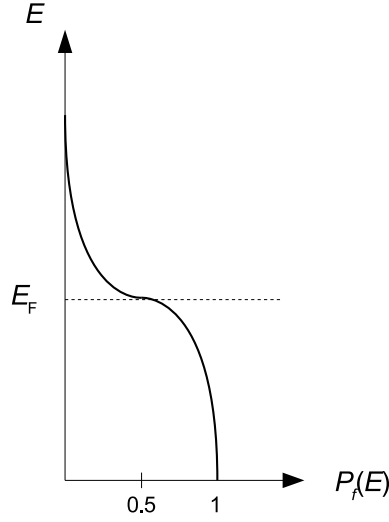


Figure 1.3: Fermi function [49]

in Fig. 1.3. Understanding behaviour of fermi function is important for dealing with RTS noise in MOS transistors. $P_f(E)$ is determined by E and E_F difference. It can be seen that at very high or very low energy levels, electrons tend to keep their existing state, e.g. corresponding energy level is either fully occupied by that electron or just empty. However, if the energy level is close to the fermi level, probability of occupation is 0.5 which means that electron can be at that state or not with equal probabilities. So *switching behavior* is likely in that energy levels which forms the basis of trapping-detrapping process in semiconductors.

1.2.2 Si-SiO₂ Interface ²

Since 1/f noise is seen in very different areas of science and engineering such as in semiconductors, rate of traffic flow, average seasonal temperature and economic data, some researchers searched the roots of 1/f noise in mathematics [59]. Milotti has explained this effort as the trial of finding a universal all-encompassing explanation of 1/f behavior in these different systems [77]. Simoen stated that in order to have a 1/f spectrum, strategically distributed lorentzian waveforms that come from individual RTSs is adequate [101]. 1/f noise studies in electrical engineering mainly focused on mechanisms that should provide suitable time constants that forms the bases of these RTS waveforms. Trapping-detrapping mechanism in MOS transistors became popular explanation of 1/f noise among researchers since resulting time constants will lead to 1/f spectrum in frequency domain as also verified by Kirton in [60] with extensive measurements [101]. Researchers attributed this trapping behavior to the interfacial defects in the Si-SiO₂ interface [126]. Formation of these defects were extensively studied and number of theories have been developed. In experimental studies, it is found that there were different type of defects [17]. Some of them resides in Si-SiO₂ interface and the others resides in the oxide. Not only their locations but also electrical and physical properties of these defects were different. Some of them communicates with MOS channel via trapping-detrapping mechanism whereas the others do not [17], [31]. In this section, we will review these works on Si-SiO₂ interface.

In order to overcome confusion and misunderstandings about defects, a committee was established by IEEE to recommend a terminology. Deal and the other members of committee published a paper that offers a terminology [17]. Four types of traps were defined as shown in Fig. 1.4 [17]:

According to terminology [17], fixed oxide charges are positive charges that are not

²This part heavily is based on the works given in [17], [112], [31]

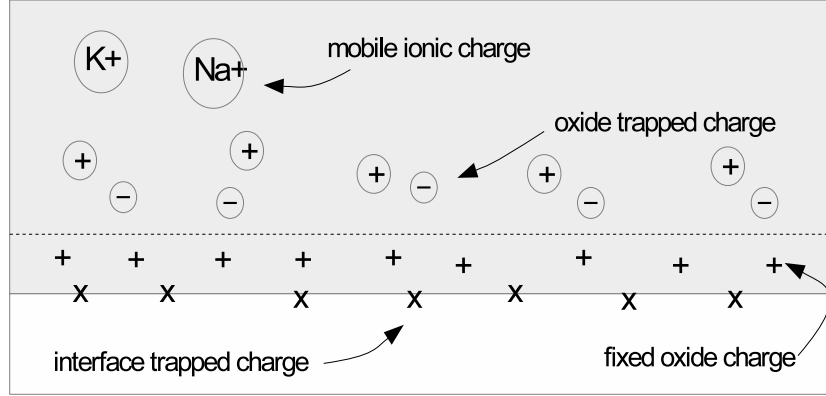


Figure 1.4: Four types of traps [17]

mobile. Structural defects in the oxide are the main reason of these charges. Fixed oxide charges are not deep in oxide, they are close to the interface. Mobile ionic charges are generally results of ionic impurities such as Li^+ , Na^+ , K^+ . In some cases heavy metals and also some negative ions can also contribute mobile ionic charges [17]. Oxide trapped charges can be either positive or negative depending on the type of the trapped carrier [17].

Fletwood underlined the fact that these four defects differ from each other not only with their physically different structures but also with their electrical properties [31]. According to him, some of the traps are in communication with channel via trapping-detrapping process. For example, interface trapped charges strongly communicates with underlying channel. Fixed oxide charges and oxide trapped charges can also be in electrical communication with channel, however, it is said that their interaction with channel is slower than the interface traps [31]. Therefore trapping-detrapping mechanisms in the oxide traps and interface traps produce different time constants which consequently changes their contribution to $1/f$ noise in terms of frequency

range. In [112] it is shown that oxide traps contribute to lower frequency component of $1/f$ noise whereas interface traps contributes to the high frequency components of $1/f$ noise. Due to their different contributions to $1/f$ spectrum interface traps was called as fast states and other oxide traps were called as slow states [112].

Kirton stated that traps that contribute to $1/f$ noise in n-channel devices are mostly acceptors whereas in p-channel devices they are donors [60]. According to [111] acceptor type traps are neutral when they are above fermi level and they are negative when they are below fermi level whereas donors are positive if they are above fermi level, and neutral if they are below the fermi level.

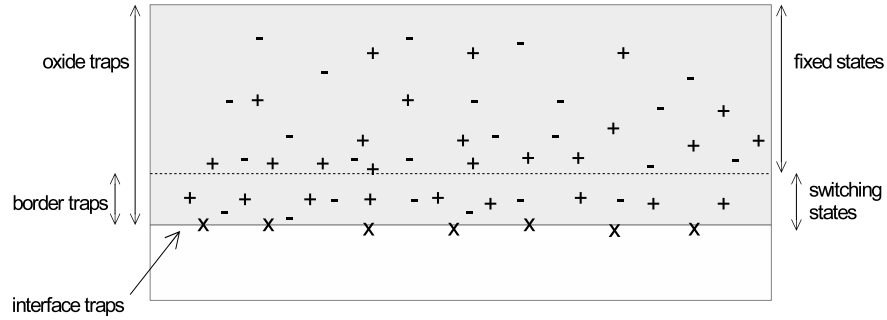


Figure 1.5: "Trap" and "state" discrimination [31]

Fletwood presented a nice picture that clarifies *state* and *trap* terms where they were used interchangeably by previous authors [31]. According to him there were two classifications of defects in the Si – SiO₂ interface in the literature. One classification was about physical structures of defects and the second classification was about electrical properties of defects. Some of the oxide traps were assumed to be not in electrical communication with the channel if their time constants is far beyond the available measurement times [31]. This brought an ambiguity about the classifications of defects according to Fletwood. He defined *border traps* in which

electrical and physical properties of defects are distinguished from each other [31]. In Fig. 1.5, border traps are shown. Note that they are the oxide traps which are in electrical communication with channel. Border trap definition distinguish switching and non-switching states in which they may have the same physical structure.

Traps not only effect dynamic behavior of transistors via trapping - detrapping processes but they also change the static characteristics of transistor. For example it is reported that oxide and interface traps cause threshold voltage and flatband voltage shifts after irradiation and annealing steps of MOS manufacturing process [29]. It is shown that oxide and interface traps cause voltage shifts in different polarities [29]. In high and low frequency $C - V$ measurements, it is seen that oxide and interface traps shifts the $C - V$ curves of MOS transistors [98]. In conclusion, it should be kept in mind that effects of oxide and interface traps are not limited to $1/f$ noise but also they change static parameters of MOS transistors such as threshold, flatband voltages and also $C - V$ curves.

1.3 Introduction to MOSFET³

Although first patent of the basic principle of field effect transistor (FET) date back to 1925, actual MOSFET device appeared in 1948 after a long development period and became practical for electronic design in 1958 [9]. Early transistors were in bipolar technology. Although bipolar junction transistors (BJT) and FETs have different physical structures, for principles of operation they are not so different. Both transistors can be simply modelled as controlled current sources [99]. In MOS transistors, gate voltage controls drain-source current whereas in BJTs base current controls the collector-emitter current. In [9] it is reported that two main advantages of MOS technology over BJTs were thought to be: first, the fewer processing steps

³This part is mostly summarized from [49] and [117]

which results low cost for IC design and the second is the scalability of MOS transistors without degrading performance which consequently results speed improvement and less power consumption.

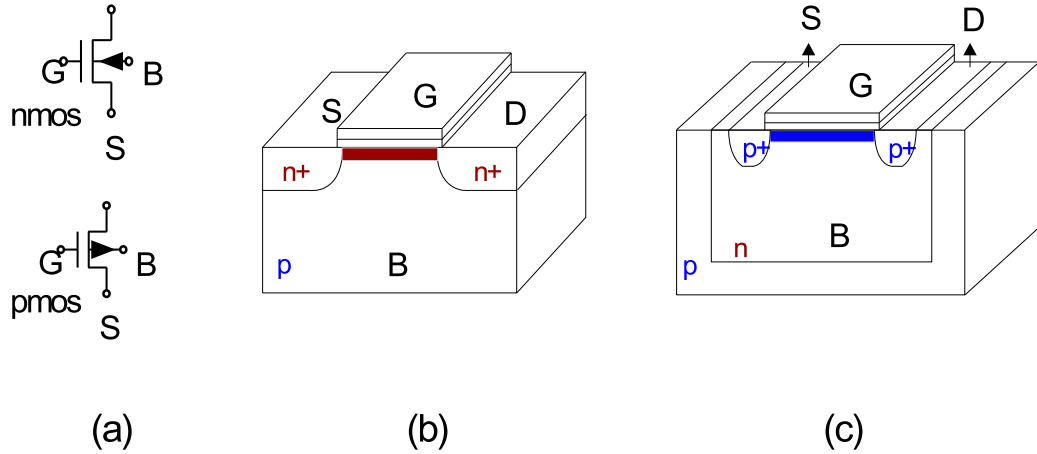


Figure 1.6: NMOS and PMOS transistors [99]

MOSFET is a four terminal device with two versions. In Fig. 1.6 symbolic representation of PMOS and NMOS versions and their physical structures are given. Gate is the first terminal where the control voltage is applied. Drain and source terminals are the other terminals that are connected to the outer environment. Finally, bulk is the reference terminal which is always at highest or lowest potential of the circuit depending on the type of MOS transistor. Similar to BJTs, MOS transistors operate complementary and therefore they are also called as CMOS (complementary MOS) transistors. As can be seen in Fig. 1.6b, NMOS transistor is formed on a p-type silicon wafer. There are two n-well regions that are excavated during fabrication process. One of the wells is for source terminal and the other one is for drain terminal. Channel is created between these two wells. So the charge carriers are electrons in NMOS transistors since the channel is formed by inverting the p-type silicon to n-type by

applying a positive voltage to the gate terminal. Complement of the NMOS transistor is the p-type MOS transistor (PMOS). As can be seen in Fig. 1.6c, PMOS is built on an n-type silicon well which is excavated in a p-type silicon wafer. Between two p-wells, channel is formed where the charge carriers are now holes instead of electrons.

According to Chenming Hu, a good starting point to MOS transistor analysis is thinking it as a parallel plate capacitor as it was given in Fig. 1.7 [49]. MOS transistors differ from MOS capacitor given in Fig. 1.7 only by having two more $n+$ regions. Therefore it is very useful to start from a simple capacitor metaphor before digging into more complex analyses. Poly-silicon is preferred as the gate material in

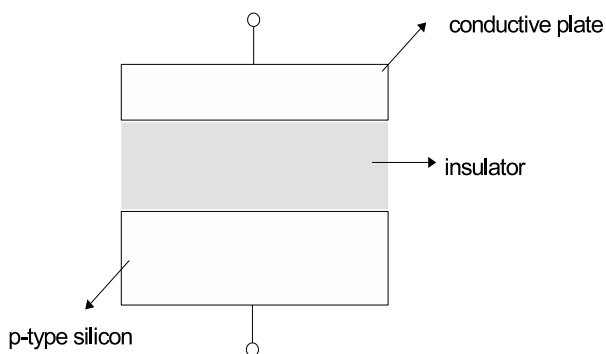


Figure 1.7: MOS transistor as a capacitor [49]

modern MOS transistors. Conductive plate is therefore poly-silicon for this so-called capacitor. However, the bottom plate which is the substrate of MOS transistor is intrinsic silicon. Since the gate and substrate materials are not same, there exists a built-in voltage between these two plates.

Band diagrams are very useful tools to understand the effect of the built-in voltage in MOS transistors. They are also very instructive for comparing different operation regions of MOS transistors. In Fig. 1.8, band diagram of an NMOS transistor when

there is no bias voltage is applied to gate terminal is given. Lowest bound of con-

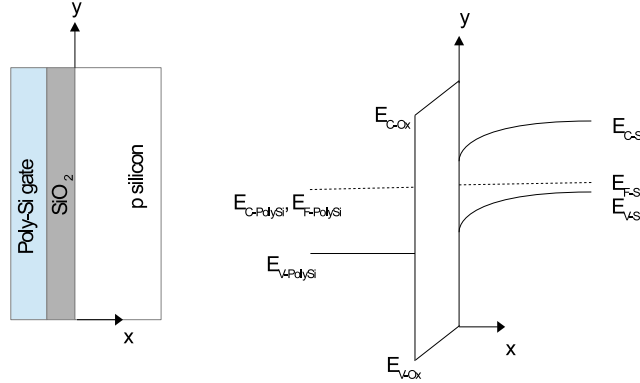


Figure 1.8: Band Structure [49]

duction band energy is E_C and upper bound of valence band energy is E_V . Fermi energy level is denoted as E_F . As can be seen in Fig. 1.8, E_C , E_F and E_V of gate poly-silicon, thin gate oxide and silicon substrate are all in different levels. This is the result of built-in voltage. It is assumed that voltage change in the oxide (SiO_2) is linear. In the substrate the situation is different, voltage change is exponential [49]. Change in the energy levels follows the same argument with only polarity difference. Note that there is significant potential difference between conduction band energy in oxide (E_{C-Ox}) and conduction band energy in silicon body (E_{C-Si}), which is approximately 3.1 eV at $Si - SiO_2$ interface. This potential difference is *electron energy barrier* which prohibits electrons to pass through the oxide [49]. There is also similar potential barrier for holes since E_{V-Ox} is approximately 4.8 eV less than E_{V-Si} [49]. Due to these potential barriers carriers normally cannot pass through the oxide. However, as we will see later, there are *traps* in the $Si - SiO_2$ interface and deep in oxide that can capture and release electrons. This capture and release process is thought as one of the main reasons of LF noise in MOS transistors [60].

There are three different region of operations: accumulation, depletion and inversion. In order to differentiate these regions people two critical levels for gate voltage

are defined. *Threshold voltage* stands for the depletion-inversion boundary, whereas *flat-band voltage* is related to accumulation-depletion boundary [117]. As given in Fig. 1.8 band diagram is not flat if the gate voltage is zero. E_C of substrate bends upward towards the surface in that case. In order to have flat energy band structure, negative voltage should be applied to gate which is called as flat-band voltage [117].

1.3.1 MOSFET Operation Regions

If the gate voltage is more negative than the flat-band voltage, transistor is in the accumulation region. Band diagram is given in Fig. 1.9b where in the poly side energy bands are pushed upward since the gate voltage is decreased which means more negative potential levels are represented with E_C and E_V in the poly side [49]. As a result E_C and E_V bend up towards the surface in the substrate side to satisfy continuity. Since E_V is closer to the E_F , gate electrode attract holes to the surface. This is called as *accumulation* and it is illustrated in Fig. 1.9a [49]. One important

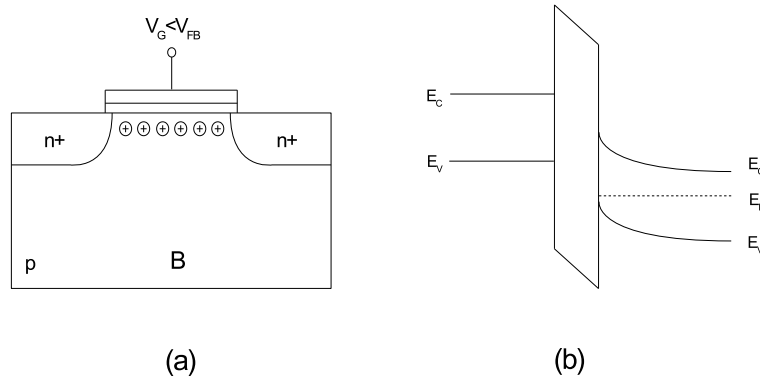


Figure 1.9: Accumulation Region [49]

property of MOS transistor in this region is the capacitive behavior. Accumulation charge is given as [49]:

$$Q_{acc} = -C_{ox}(V_g - V_{fb}) \quad (1.5)$$

where C_{ox} is oxide capacitance and V_{fb} is the flat-band voltage. If V_g is slightly more positive than V_{fb} , band diagram will be different than the one in accumulation region. In this case energy bands in the Si body side will be higher than the ones in the gate side as given in Fig. 1.10b. Since neither E_C nor E_V is closer to the E_F , gate does not attract any charge carriers [117]. So the channel is *depleted* from carriers in this *depletion* region but some electrons are attracted to the surface side of the channel since the gate voltage is positive as can be seen in Fig. 1.10a [117].

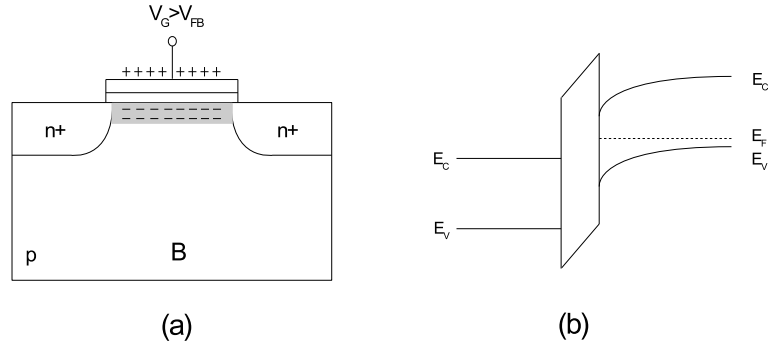


Figure 1.10: Depletion Region [49]

If the gate voltage is increased more and more, at some point surface electron concentration will be equal to the bulk doping concentration. In this case channel is said to be inverted from p-type to n-type in which electrons are now majority carriers. This critical voltage level is called as *threshold voltage* and beyond this voltage transistor operates in *inversion* region [117]. Physical structure and band diagram of transistor in inversion region is given in Fig. 1.11. Threshold voltage definition can be easily formulated using the band diagram. At threshold value following equality should be satisfied [49]:

$$(E_C - E_F)_{surface} = (E_F - E_V)_{bulk} \quad (1.6)$$

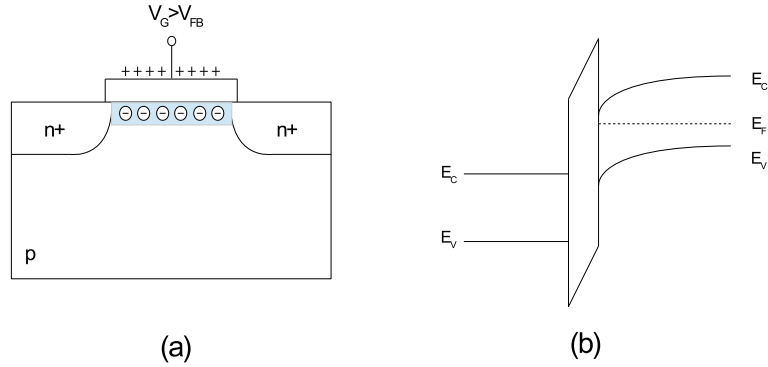


Figure 1.11: Inversion Region [49]

In inversion region, inversion charge Q_{inv} presents and similar to Q_{acc} it can be formulated as $Q_{inv} = -C_{ox}(V_g - V_t)$. It will be instructive to compare NMOS and PMOS band structures in inversion regions. They are given in Fig. 1.12, respectively. Note that band structures of the transistors clearly demonstrates that why in n-channel devices acceptor type of traps are active whereas in p-channel devices donor type of traps are active. The reason is explained in [111] as acceptor traps are closer to E_F in NMOS transistors and donor traps are closer to E_F in PMOS transistors.

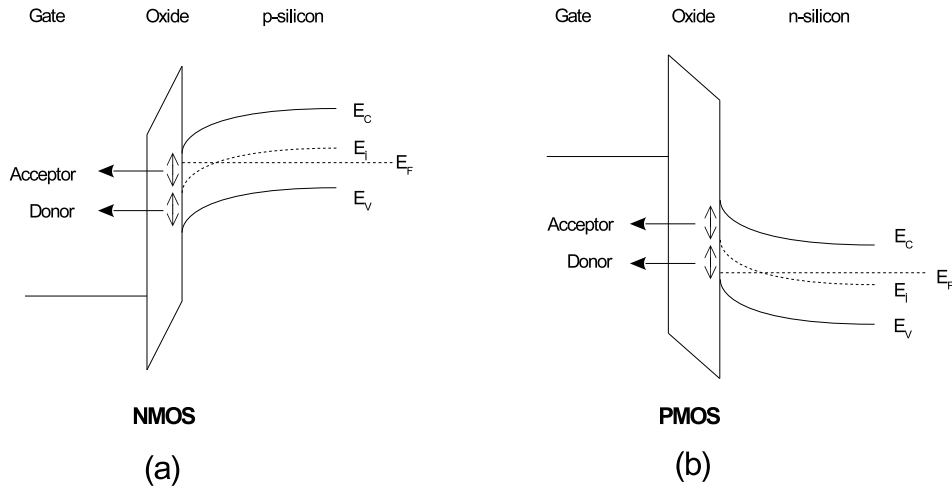


Figure 1.12: NMOS and PMOS in inversion regions [49]

Chapter 2

FUNDAMENTAL NOISE SOURCES IN SEMICONDUCTORS

History of the noise studies in semiconductors goes back to the early years of the 20th century [130]. In 1905, Einstein's paper on Brownian motion was published [24]. Einstein's statement was that the Brownian motion of free carriers can lead to fluctuations on the potential difference across a resistor. Einstein's explanation is a milestone that accelerates noise studies. First noise theory that was developed on the basis of Einstein's statement is the thermal noise theory. In this section, a brief introduction to noise in semiconductors will be given. In the first section white noise sources such as thermal and shot noise will be given under *broadband noise* topic. Second section is about *low frequency noise* which includes $1/f$ noise, generation-recombination (GR) noise and random telegraph signal (RTS) noise.

2.1 *Broadband Noise*

Noise power spectrum of an electronic device includes the effects of all noise sources within that device. Generally noise spectrum can be divided into two regions as given in Fig. 2.1 where LF noise dominates in the first region and broadband noise dominates in the second region. Broadband noise consist of thermal and shot noises. Its PSD is flat across the spectrum. LF noise is the *excess* noise on top of broadband noise. LF noise is inversely proportional to frequency and it dominates the power spectrum at lower frequencies.

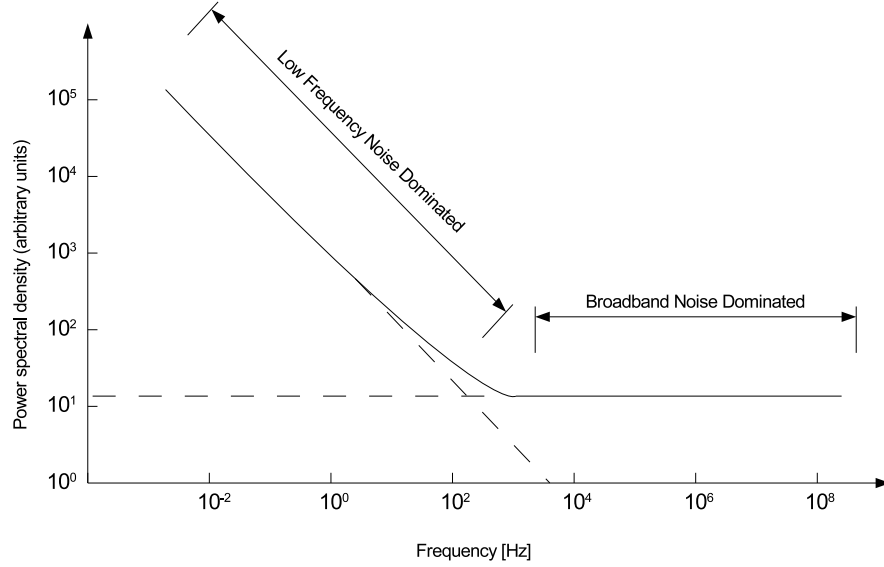


Figure 2.1: Noise spectrum is divided into two parts

2.1.1 Thermal Noise

Thermal noise is related to the random motion of charge carriers as Einstein stated. The effect of the random motion on electrical potential was observed by Johnson [56]. This is one of the well-understood electrical noise mechanisms. Charge carriers in semiconductors have thermal velocities due to their kinetic energies that are independent of whether an electrical field is applied or not. Hu stated that in an imperfect crystal structure, charge carriers follow random paths due to the scattering off mechanisms that forces carriers to change their directions [49]. This random thermal motion is called thermal noise. It is directly proportional to the absolute temperature (T_K) and has many frequency components. Output voltage when thermal noise is

measured from a resistor and the resulting PSD is given as:

$$v_n^2 = 4KT_K R \Delta f \quad S_{v_n} = 4KTR \quad (2.1)$$

PSD of thermal noise is independent of frequency and it is a *white noise*.

2.1.2 Shot Noise

Current is carried by electrons which are discrete in nature. So the current flow is not smooth and there exists fluctuations in the number of carriers that pass from a reference point in semiconductor. Shot noise is about this fluctuations that results from discrete nature of current [84]. In semiconductors, shot noise is observed when carriers pass from *pn* junctions [20]. Mean square value and PSD of current are:

$$i_n^2 = 2qI\Delta f \quad S_I(f) = 2qI \quad (2.2)$$

where q is the electron charge and I is average direct current flow in semiconductor and Δf is the bandwidth. Similar to thermal noise, shot noise is also a white noise process. Once shot noise and thermal noise are together introduced in the same conductor, it is not possible to distinguish them from each other and therefore these two contributors to noise spectrum are together called as broadband noise.

2.2 Low Frequency (LF) Noise

At lower frequencies electronic devices exhibit higher noise and this excess noise is called as low frequency noise. The terms *low frequency noise* and *1/f noise* sometimes are used interchangeably. It is a subject of debate since there are different type of noise mechanisms that contribute to LF noise. To clarify this issue, we have summed all noise sources that contribute to the frequency dependent part of the power spectrum

into LF noise. Subject of this thesis is LF noise which consists of $1/f$ noise, generation-recombination noise and burst noise.

Low frequency noise can be seen in various electronic devices and components. It is reported that MOS transistors, BJTs, metal-insulator-metal (MIM) junctions and high-electron-mobility transistors (HEMT), i.e. GaAs and GaN, are subjected to LF noise [57], [18], [66]. This thesis is focused on LF noise in MOS transistors. This section provides a brief introduction to noise sources that contribute LF noise.

2.2.1 $1/f$ noise¹

Noise is not a sufficient term to describe $1/f$ behavior alone. It is rather a phenomenon that can be seen in very different areas [59]. Press drew attention to this property of $1/f$ behavior in different systems in his famous paper. He stated that in undersea ocean current velocities, meteorological data, loudness fluctuation spectra for radio stations and sunspot number data $1/f$ spectrum can be seen [85].

Generally spectrum is not exactly $1/f$. Frequency dependency varies as $1/f^\gamma$ with a γ parameter that ranges between 0.5 and 1.5 [77]. Due to the presence of $1/f$ behavior in so different systems, some researchers prefer to describe this behavior as a random process that have $1/f$ like spectrum [59]. Although first explanations of $1/f$ behavior go back to the Johnson, first mathematical formulation is given by Schottky in which he used the concept of relaxation processes [77]. According to him, electron releasing from traps contributes current and consequently gives rise to $1/f$ noise. Here, we will summarize the work of Schottky as given in [77].

Trapping sites release electrons according to a relaxation process that is formulated with $N(t) = N_0 e^{-\lambda t}$ for $t \geq 0$. If same relaxation process is extended such as $N(t, t_k) = N_0 e^{-\lambda(t-t_k)}$ for $t \geq t_k$ and $N(t, t_k) = 0$ for $t \leq t_k$. Fourier transform of these processes

¹This part is mostly summarized from [77]

can be obtained as [77]:

$$F(\omega) = \int_{-\infty}^{+\infty} \sum_k N(t, t_k) e^{-j\omega t} dt = \frac{N_0}{\lambda + j\omega} \sum_k e^{j\omega t_k} \quad (2.3)$$

Based on eq. 2.3, PSD is defined in the following form in [77]:

$$S(\omega) = \frac{C}{\lambda^2 + \omega^2} \quad (2.4)$$

where C is a constant that depends on the relaxation process parameters. Note that this formulation assumes that all relaxation processes have the same rate λ . Assuming uniform λ distribution between λ_1 and λ_2 for different relaxation processes, PSD definition is updated to [77]:

$$S(\omega) = \frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} \frac{C}{\lambda^2 + \omega^2} d\lambda = \frac{C}{\omega(\lambda_2 - \lambda_1)} \left(\arctan \frac{\lambda_2}{\omega} - \arctan \frac{\lambda_1}{\omega} \right) \quad (2.5)$$

PSDs that are given in eq. 2.4 and eq. 2.5 are plotted in Fig. 2.2. Note that in distributed λ case, spectrum is divided into three regions. For $\omega \ll \lambda_1$ spectrum is flat, for $\omega \gg \lambda_2$ spectrum decreases with square of f and in between there is a $1/f$ region. This is the basic mathematical formulation of $1/f$ noise. Some $1/f$ noise models, i.e. number fluctuation model and correlated models were developed upon this picture. In Chapter 3 these models will be analyzed in detail.

2.2.2 Generation - Recombination (GR) Noise ²

GR noise is one of the fundamental noise sources in semiconductors. This noise is attributed to random emission and capture of charge carriers by the defect centers in the depletion region of the semiconductors [132]. It is reported that first appearance

²This section is mainly summarized from [86] and [125]

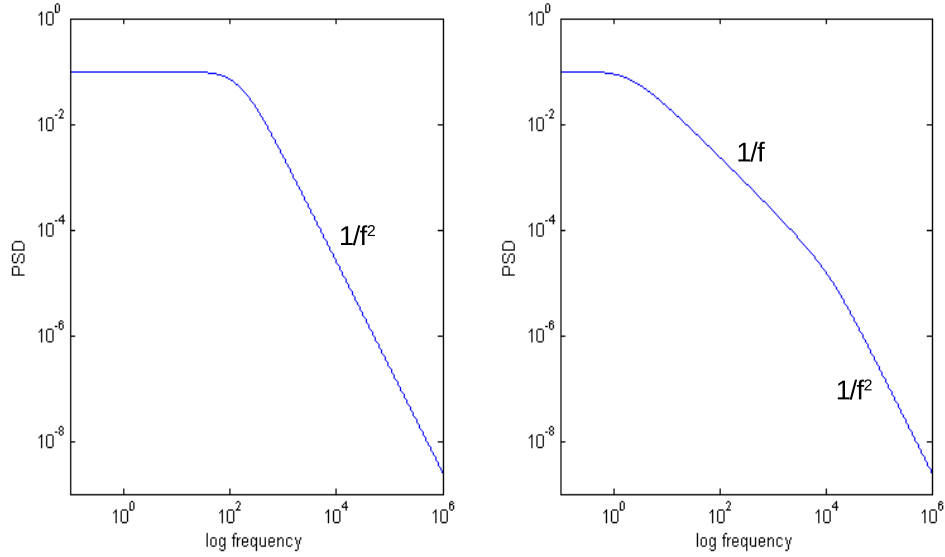


Figure 2.2: $1/f$ spectrum based on eq. 2.4 (left) and eq. 2.5 (right)

of GR noise in literature dates back to 1951 [78]. When some number of carriers are captured by a trap this results a PSD in the form [125]:

$$\frac{S_N(f)}{N^2} = \frac{\overline{\Delta N^2}}{N^2} \frac{4\tau}{1 + (2\pi f)^2 \tau^2} \quad (2.6)$$

This looks like the basic assumption of number fluctuation models that we presented briefly in previous section. Indeed, GR noise is closely related to RTS noise. This will be more clear after we review RTS noise in the next section.

Haartman listed four type of processes that can be collected under GR noise in [125] as: (i) A free electron and a free hole can recombine. (ii) A free electron and a free hole can be generated. (iii) A free electron or hole can be captured by an empty defect center. (iv) A free electron or hole can be released by a full defect center. In [86] it is stated that first two GR mechanisms are two-stated where electrons and holes move between valence and conduction bands. This is called *band to band* type GR mechanism. Last two mechanisms are three stated in which there is a mid-band

level between valence and conduction bands. This mid-level corresponds to a trap site in general. This three stated GR structure is called as *trap assisted* GR noise [86]. In Fig. 2.3 trap assisted GR mechanism for an electron is pictorially shown.

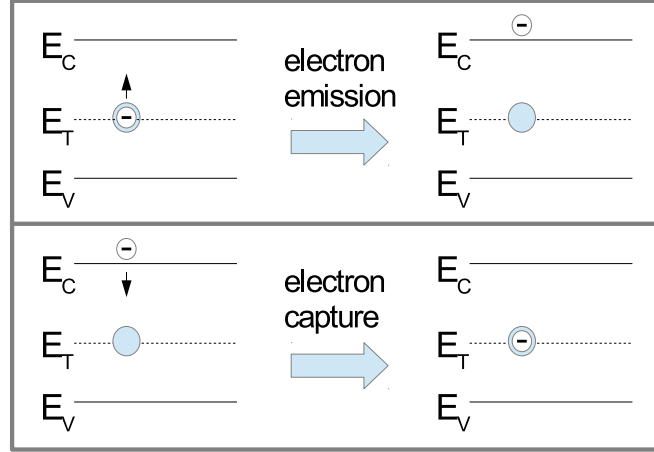


Figure 2.3: Capture and emission process of an electron by a trap center [86]

Trap assisted GR noise is modeled with Shockley-Read-Hall (SRH) statistics. Emission and capture rates for electrons are defined in SRH theory as [86]:

$$E_n = e_n n_t \quad C_n = \sigma_n \langle v \rangle_{th} p_t n = \alpha_n p_t n \quad (2.7)$$

where e_n is emission rate coefficient for electrons, n_t is the density of total filled traps, σ_n is the capture cross-section, $\langle v \rangle_{th}$ is thermal velocity and α_n is effective volume element that is swept out. n corresponds to density of electrons in the conduction band. Similar expressions for holes, i.e. C_p for capture rate and E_p for emission rate can be written just replacing indicator n with p in corresponding definitions.

Rate of electrons leaving from conduction band and holes from valence band can be defined simply as the difference of capture and emission rates, respectively [86]:

$$-\frac{dn}{dt} = \alpha_n p_t n - e_n n_t \quad -\frac{dp}{dt} = \alpha_p n_t p - e_p p_t \quad (2.8)$$

The net rate of increase in total filled traps n_t is defined in [86] as:

$$\frac{dn_t}{dt} = -\frac{dn}{dt} + \frac{dp}{dt} \quad (2.9)$$

Total density of traps is given as $N_t = n_t + p_t$. Putting the definitions given in eq. 2.8 into eq. 2.9, net rate of increase in total filled traps is calculated in [86] as:

$$\frac{dn_t}{dt} = (c_n + e_p)N_t - (c_n + e_p + c_p + e_n)n_t \quad (2.10)$$

As stated in [86], eq. 2.10 is the kinetic equation that completely captures GR process in which one can obtain carrier densities in each of three bands (conduction, valence and trap) at a given time t . In order to calculate emission rates from a trap, fermi function is used that is defined in eq. 1.2. In thermal equilibrium capture and emission rates for electrons are equal i.e. $E_n = C_n$. Same holds for holes in which $E_p = C_p$. However, if there is a generation of an electron and a hole, this balance of capture and emission rates is no longer valid. This case is also discussed in [86], further detailed information about GR kinetics can be found there.

GR noise studies are now replaced with RTS noise studies. GR noise was studied not only in MOS transistors but also in GaN transistors as can be seen in the works [92], [91]. However, RTS noise studies focused on MOS transistors. RTS noise is a special form of GR noise in which there are very few traps that contribute to the capture and release process. As devices scaled down to sub-micron levels it became possible to observe drain current fluctuations in the time domain. It also became possible to study single traps in small devices such as MOS transistors [87], [61].

2.2.3 Burst - Popcorn - Random Telegraph Signal (RTS) Noise

Burst noise is also called as popcorn noise and random telegraph signal (RTS) noise as indicated in the heading of this section. We will prefer to use the name “RTS noise” in this thesis. RTS noise can be seen as step transitions between two or more levels in time domain voltage and current waveforms [60]. Although this noise was discovered in early semiconductors it is more pronounced after some popular operational amplifiers such as 709 that is designed by Bob Widlar in 1965 [2].

Similar to GR noise, RTS noise results from trapping-detrapping of charge carriers in MOS transistor channels by the trap centers in the gate oxide. In [101] it is stated that trap centers can be either electrically neutral or charged in their empty states. Trapping occurs both electrically and physically. In Fig. 2.3, electrical structure of trapping has been shown. Physical structure of trapping and detrapping process in MOS transistor is illustrated in Fig. 2.4. It is critical to understand properties of trap centers. Detailed study of them was given in Section 1.2.2

In Fig. 2.4, a typical RTS noise waveform is given. RTS noise is modeled with

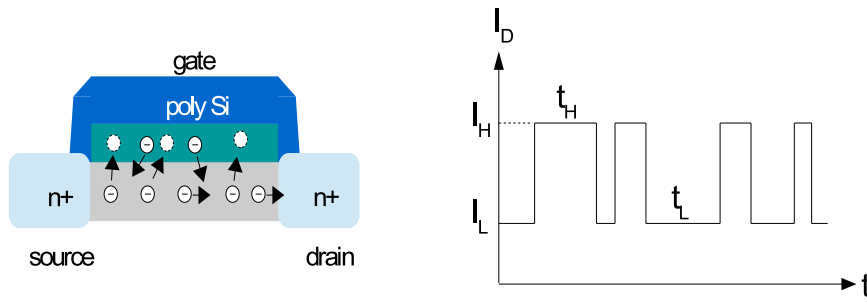


Figure 2.4: Trapping-detrapping process (*left*). RTS noise waveform example (*right*)

two levels and poisson distributed switching events [125]. However, RTS noise does not always have to be two leveled. In some studies, it is shown that RTS signals may have more than two states [42]. For simplicity we are assuming RTS noise waveform

has only two discrete levels. PSD of such an RTS signal is given in [125] as:

$$S_{I(f)} = \frac{4\overline{\Delta I^2}}{(\tau_L + \tau_H)[(1/\tau_L + 1/\tau_H)^2 + (2\pi f)^2]} \quad (2.11)$$

in which τ_L and τ_H are the mean low and high times of RTS, respectively. Note that if mean low and high times are equal, i.e. $\tau = \tau_L = \tau_H$, eq. 2.11 simplifies to eq. 2.6.

After Kirton's seminal paper [60] it is widely accepted that origin of LF noise is the individual RTSs that result from single trap activities. Kirton stated that the sum of only a few RTS noise waveforms can lead to a $1/f$ spectrum which is observed in measurements [60]. In Fig. 2.5, this statement is illustrated. However, in contrast

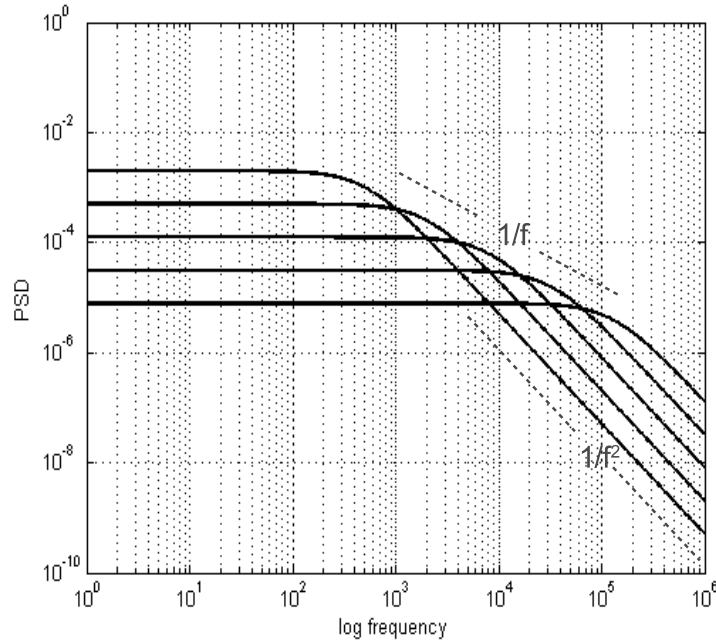


Figure 2.5: RTS noise gives rise to LF noise [125]

to this widely accepted opinion that the RTS noise is the dominant factor of LF noise, Vandamme argued that origins of LF noise and the RTS noise are different and the

observed Lorentzian like spectrum is the sum of these two different contributors [122].

RTS noise is defined with its three parameters: ΔI_d , τ_H , τ_L . τ_H and τ_L can be calculated from SRH theory. Difference between SRH analysis of GR noise and RTS noise is that only one trap is active in RTS noise. This results minor differences in formulations. For example, capture and emission rates are rewritten in [65] as:

$$E_n = \frac{P_f}{\tau_e} \quad C_n = \frac{1 - P_f}{\tau_c} \quad (2.12)$$

where τ_e and τ_c are mean emission and capture times. In [60], it is written that:

$$\frac{\tau_c}{\tau_e} = \exp\left(\frac{E_T - E_F}{kT_K}\right) \quad (2.13)$$

So far, we have given the simplest formulation of RTS noise. There are more advanced analytical RTS noise models. They will be reviewed in detail in Chapter 5.

2.2.4 Conclusion

Relationship between RTS noise, 1/f noise and GR noise has been subject of an endless debate. In number fluctuation models, 1/f noise was considered to be the sum of individual lorentzian waveforms as illustrated in Fig. 2.5. Supporters of mobility fluctuation model criticized this assumption. In his review paper, Hooge stated that 1/f and RTS noises have different origins [123]. Hooge did not limit the debate with MOS devices. Since MOS devices are surface devices, RTS noise may dominate LF noise in MOS transistors where other contributions to 1/f noise including bulk effects may be very small and can be neglected. Similar argument to ours was given by Jones where he argues that if there is a universal physical phenomenon that contribute to the 1/f noise, it is so small and the extrinsic noise that comes from the defects almost always dominates [57]. This debate is still going on.

Chapter 3

PHYSICAL LOW FREQUENCY NOISE MODELS

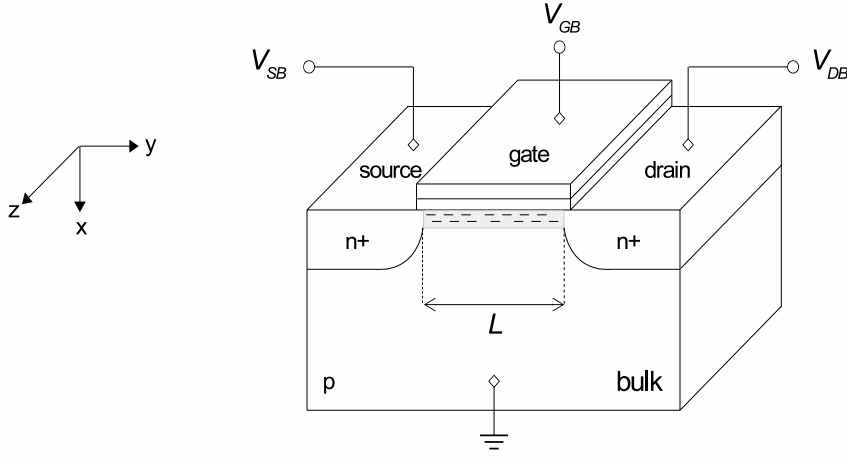


Figure 3.1: NMOS transistor cross section [99]

In Fig. 3.1, an NMOS transistor cross section is given. Current density for this transistor can be calculated as the sum of drift and diffusion components. Following expression can be obtained for current density as given in [97]:

$$J_n = -qn\mu_n \frac{dV_c}{dy} \quad (3.1)$$

q denotes unit charge in this expression, n is the electron concentration, μ_n is the electron mobility coefficient and $\frac{dV_c}{dy}$ is the derivative of the channel potential in lateral direction. Drain current expression can be found by integrating J_n in x and z directions. In literature, general assumption is that if there are fluctuations in the

current, it is either the result of fluctuations in the n or in the μ_n or in both of them as eq. 3.1 indicates. This is also the basis of classification of the LF noise models.

We can divide LF noise studies into two periods. First one is the classical age where $1/f$ noise studies focused on physical LF noise modeling. In the second period, analytical and circuit simulation oriented models became popular. In this chapter, only physical LF noise models will be reviewed.

Physical origin of LF noise has been subject of endless debates since 1950s. Two school of thought have emerged where one of them attributed the origin of LF noise to the fluctuations in the number of carriers in semiconductors. The other school tried to explain LF noise as the result of fluctuations in the mobility of carriers. First school considered the LF noise as a surface effect where the latter school explained LF noise as a bulk effect in semiconductors. Number fluctuation (ΔN) model has a solid physical explanation for LF noise as we will see in the next section. However, mobility fluctuation ($\Delta\mu$) model is based on empirical formulations and experimental facts which will also be shown in corresponding review section.

ΔN model is originally developed by McWorther as a surface effect in metal films [74]. According to McWorther's model, physical origin of the LF noise is the trapping-detrapping of charge carriers into the trap centers in semiconductors via tunneling mechanism. Christensson applied McWorther's model to MOS transistors in [16]. Other ΔN models such as the ones [88] and [36] were developed on top of it.

$\Delta\mu$ model was based on Hooge's empirical formula. Simoen called this model as "purely empirical" [101]. Hooge developed his model after the experimental measurements, offering a formulation for noise current PSD that represents these observations [44]. Vandamme applied Hooge's model to MOS transistors in [119], [120]. Measurement results showed that LF noise of PMOS transistors can better be explained with Hooge's $\Delta\mu$ model whereas NMOS transistors exhibit a noise characteristic that is more consistent with McWorther's model as given in [14] and [121].

In order to develop a universal LF noise model, some researchers put forward the idea that both mobility and number fluctuations have effect on LF noise in MOS transistors. Early models counted these effects in an uncorrelated manner (examples are Surya [103] and Mikoshiba's models [76]). Later, Jayaraman from MIT [55] and Hung from Berkeley [53] developed unified models, respectively. Both models included mobility fluctuations in conjunction with the number fluctuations.

3.1 Number Fluctuation (ΔN) Model

3.1.1 McWorther's Model ¹

The number fluctuation model was first proposed by McWorther. He stated that the origin of LF noise in semiconductors is the trapping-detrapping of charge carriers into the trap centers in the oxide. McWorther assumed that this trapping-detrapping process is carried out by a tunneling mechanism [74]. PSD of the number fluctuation in MOS transistors was derived in sec.2.2.2 as a GR process with the eq. 2.6. Using the relationship $S_Q = q^2 S_N$ and eq. 2.6 PSD of number fluctuations is rewritten as:

$$S_Q(f) = \frac{q^2}{W^2 L^2} \overline{\Delta N^2} \frac{4\tau}{1 + (2\pi f)^2 \tau^2} \quad (3.2)$$

In order to calculate variance $\overline{\Delta N^2}$ Fermi-Dirac statistics is used. As given in [125], variance $\overline{\Delta N^2}$ is calculated as:

$$\overline{\Delta N^2} = (1 - P_f(E))^2 P_f(E) + (0 - P_f(E))^2 (1 - P_f(E)) = P_f(E)(1 - P_f(E)) \quad (3.3)$$

¹This part is mostly summarized from [125] and [16]

Eq. 3.2 is for a single trap. In order to obtain total PSD, integration is carried out in three dimensions x, y, z and also in energy E as given in [125]:

$$S_Q(f) = \frac{q^2}{W^2 L^2} \int_{E_V}^{E_C} \int_0^W \int_0^L \int_0^{t_{ox}} 4N_t P_f(E)(1 - P_f(E)) \frac{\tau}{1 + (2\pi f)^2 \tau^2} dx dy dz dE \quad (3.4)$$

This is the master integral of ΔN models. In order to evaluate it, some assumptions are used. These assumptions are summarized from [101] and [16] as follows:

- i. Product $P_f(E)(1 - P_f(E))$ equals to $-kT_K dP_f(E)/dE$. This product can be simplified since it shows peaked behavior near quasi-fermi level [101].
- ii. Traps are distributed uniformly in oxide i.e. uniform in $y - z$ plane if x is given as the distance from $Si - SiO_2$ interface.
- iii. Carriers that are tunneling to traps have the same energy (E).
- iv. Tunneling time constants of the carriers are given with τ which varies with x according to the relationship $\tau = t_0 e^{-\alpha_t x}$ where α_t is the tunneling parameter.

Integral in eq. 3.4 is evaluated using these assumptions. In [125], PSD is given as:

$$S_{V_{fb}}(f) = \frac{S_{Id}(f)}{g_m^2} = \frac{q^2 k T_K N_t}{W L C_{ox}^2 \alpha_t f} \quad (3.5)$$

Eq. 3.5 shows that $1/f$ noise reduces as device area increases. Input referred noise is inversely proportional to the square of oxide capacitance C_{ox} . Finally this model indicates that $1/f$ noise is bias voltage independent since N_t is bias independent provided that N_t spreads uniformly in the oxide. Effects of non-uniformity in the space and energy distribution of traps were studied in [16] and [104]. In that studies it is reported that even in non-uniform distribution case, noise PSD can be approximated by eq. 3.5 with a minor change in term f , that is given in non-uniform case as f^γ .

McWorther's model was only valid for low V_{DS} values in which transistor operates in the linear region. In [54] and [88], comprehensive models that cover all operation regions from subthreshold to saturation were developed. In Reimbold's model, relationship between local fluctuations in the number of occupied traps δN and resulting local fluctuations in the channel carriers δN_t is reconsidered. A coupling parameter R is defined using charge conservation principle as [88]:

$$R = \left| \frac{\delta Q_n}{\delta Q_t} \right| = \frac{-\beta Q_n}{C_{ox} + C_d + C_{it} - \beta Q_n} \quad (3.6)$$

Q_t and Q_n is trapped charge and channel charge, respectively. C_{ox} , C_d and C_{it} are oxide, depletion and interface charges. β is given as q/kT_K [88]. If R is used in McWorther's model final expression for drain current noise PSD is obtained as:

$$\frac{S_{Id}}{I_D^2} = \frac{q^4}{kT_K W L \alpha_t} \frac{1}{f^\gamma} \frac{N_t}{(C_{ox} + C_d + C_{it} - \beta Q_n)^2} \quad (3.7)$$

Reimbold's model covers all operating regions as shown in [88]:

- i. In weak inversion: $|\beta Q_n| \ll Q_{ox} + Q_d + Q_{it}$.

$$\frac{S_{Id}}{I_D^2} = \frac{q^4}{kT_K W L \alpha_t} \frac{1}{f^\gamma} \frac{N_t}{(C_{ox} + C_d + C_{it})^2} \quad (3.8)$$

- ii. In strong inversion: $|\beta Q_n| \gg Q_{ox} + Q_d + Q_{it}$.

$$\frac{S_{Id}}{I_D^2} = \frac{q^4}{kT_K W L \alpha_t} \frac{1}{f^\gamma} \frac{N_t}{(\beta Q_n)^2} \quad (3.9)$$

3.1.2 Alternatives to McWorther's model ²

So far, we have given the basis of ΔN models. McWorther's model is based on quantum-tunneling mechanism. Tunneling mechanism is used in many ΔN models (see [16] and [88] for examples). However, it is not the unique physical mechanism that leads to $1/f$ spectrum which is the summation of individual Lorentzian spectra. In [101], alternative models were discussed in which they are based on different physical processes to obtain suitable time constants that gives $1/f$ like PSDs. These models were illustrated in Fig. 3.2. Simoen stated that Fig. 3.2a is the McWorther's model in which traps reside only in oxide and the tunneling takes place between these oxide traps and free carriers in the channel [101]. Sah's model which is given in Fig. 3.2b added interface traps into this physical picture. These new interface traps are used to explain charge tunneling between fast surface states and deeper oxide states which is thought to be a SRH or a thermal process [33] [101]. Another explanation is given in

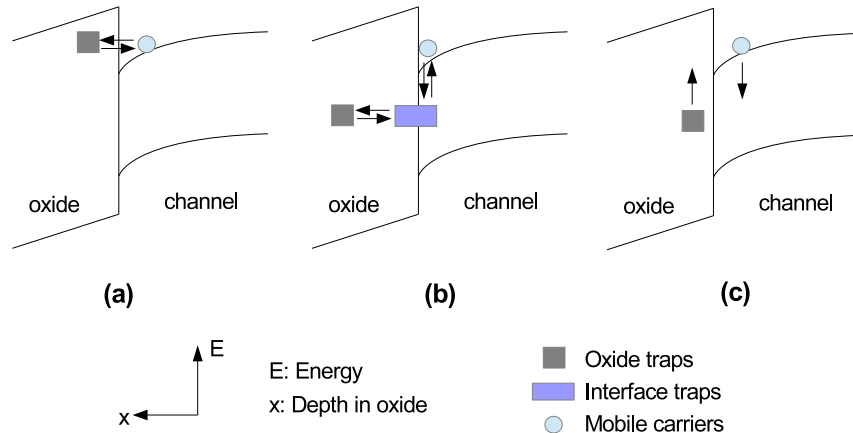


Figure 3.2: ΔN models [101]

²This part is mostly summarized from [101]

Figure 2.1.c which is based on a thermal process [101]. It is reported that this model assumes that carriers interact with oxide traps via phonon-assisted transitions. As in given in [101], the time constants of this thermally activated transitions are:

$$\tau_{th} = t_0 e^{E/kT_K} \quad (3.10)$$

Simoen stated in [101] that this formulation has been used to obtain 1/f like spectrum in some studies including [104] and [105]. It is also important that all these models that are given in Fig. 3.2 root back to the earliest 1/f noise model of Schottky that was given in Section 2.2.1 which was based distribution of time constants. In [57], it is concluded that distribution of time constants were mainly explained by two physical processes in later models where the first one is quantum-mechanical tunneling and the latter one is thermally activated trapping model.

3.1.3 Discussion

First critics to number fluctuation (ΔN) models was about their bias independent formulations. Measurements show that both n-channel and p-channel devices show bias dependency on 1/f noise magnitude [62], [76], [12]. The exponent γ is also reported to be a bias dependent parameter [104], [13]. Among ΔN models, only Reimbold's model includes bias dependency. (Q_n in the denominator of eq.3.9 brought bias dependency to current PSD). However, even Reimbold's model cannot explain different bias dependencies between PMOS and NMOS drain current noise PSDs.

Studies show that measurement results from n-channel transistors are consistent with ΔN models [88], [30], [14]. Another important factor that makes ΔN model popular was that strong correlation was seen between 1/f noise and the trap densities in numerous studies including [93], [48], [5] and [33]. However, unlike the n-channel ones, p-channel devices show strong bias dependency (as shown in [104], [105], [30]

and [14]) that cannot be explained with ΔN models.

Finally, it is important to emphasize that n-channel devices that are consistent with ΔN models are relatively long channel devices. In advanced MOS technologies in which device dimensions shrinks to sub-micron levels, short channel effects dominate device performances. ΔN models cannot model short channel effects.

3.2 Bulk Mobility Fluctuation ($\Delta\mu$) Model

Mobility fluctuation model is based on an empirical formulation which was developed by Hooge [44]. Hooge stated that $\Delta\mu$ model is based on experimental facts [46]. He argued that 1/f noise model of homogeneous semiconductors can be represented with the following empirical expression:

$$\frac{S_I}{I^2} = \frac{\alpha_H}{Nf} \quad (3.11)$$

In that formulation, I represents the total current flowing in homogeneous semiconductor and S_I corresponds to current noise PSD. α_H is Hooge's parameter which is an empirical constant. α_H varies between 10^{-6} and 10^{-4} [44]. The $\Delta\mu$ theory is explained with scattering mechanisms that result in fluctuations in carrier mobility and these effects can be summed according to Matthiessen's rule [45]:

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}} \quad (3.12)$$

$\mu_{lattice}$ is the lattice scattering and $\mu_{impurity}$ is the impurity scattering. $\mu_{impurity}$ is neglected in Hooge's formulation. Hooge defined the following relationship [47]:

$$\alpha_H = \alpha_{lattice} \left(\frac{\mu}{\mu_{lattice}} \right)^2 \quad (3.13)$$

	Linear	Saturation
S_{Id}	$K_m \left(\frac{q\mu I_d V_{ds}}{L^2 f} \right)$	$K_m \left(\frac{q\mu I_d (V_{gs} - V_t)}{L^2 f} \right)$
S_{Vg}	$K_m \left(\frac{q(V_{gs} - V_T)}{C_{ox} W L f} \right)$	$K_m \left(\frac{q(V_{gs} - V_T)}{2C_{ox} W L f} \right)$

Table 3.1: PSDs in mobility fluctuation model

Eq. 3.11 and Eq. 3.13 together constitute $\Delta\mu$ model. Hooge's formulation is applied to MOS transistors in later works such as [62] and [116]. Obtained results were summarized in Table 3.1 in which K_m is given as $\alpha_H / (\mu / \mu_{lattice})^2$.

3.2.1 Discussion

In $\Delta\mu$ model, S_{Vg} increases linearly with overdrive voltage ($V_{gs} - V_t$) in both saturation and linear region of operations. In [14] and [36], it is shown that PMOS devices follows this rule. In ΔN model, S_{Vg} was independent from overdrive voltage (see eq. 3.5). Another result is that input referred noise inversely proportional to C_{ox} in both linear and saturation regions. In McWorther's model, C_{ox} dependency was $1/(C_{ox}^2)$ which is not linear as in $\Delta\mu$ model. Hooge stated that $\Delta\mu$ model considers LF noise as a bulk effect whereas ΔN model considers LF noise as a surface effect [45].

Although $\Delta\mu$ model was studied extensively, it is not as popular as ΔN based models especially for analog designers since popular compact models like SPICE and BSIM are based on ΔN or correlated models. However, it should be noted that Hooge's parameter can be useful in various areas, for example it can be used as a diagnosis tool for material quality. For example, Simoen stated that α_H parameter is accepted as a material parameter [101].

3.3 Correlated ΔN - $\Delta\mu$ Models

As discussed in previous sections, measurement results in literature show that flicker noise of p-channel devices can be better explained with mobility fluctuation model whereas n-channel devices are more consistent with the number fluctuation model. None of these models can explain flicker noise in both n-channel and p-channel devices correctly. So the lack of a universal model has motivated researchers to develop some new unified models that incorporates mobility fluctuations $\Delta\mu$ and number fluctuations ΔN which have different physical origins.

Early models incorporated $\Delta\mu$ and ΔN effects in an uncorrelated manner as discussed in Section 3. These uncorrelated models are not given here. Jayaraman's model and Hung's unified model are correlated models. These two models are very similar in which they share almost the same formulations. However, motivations behind these two works were very different. Jayaraman offers a comprehensive model that captures $1/f$ noise in MOS transistors in almost every operation condition [55]. Hung's model is more circuit design oriented in which they tried to develop a practical model for analog applications [52]. Since these two models are similar, we will review only Hung's model in this thesis.

3.3.1 Hung's Unified Model ³

In 1988, Hung showed that mobility fluctuations which result from coulomb scattering is also important in addition to the carrier number fluctuations [51]. Based on these previous works, Hung developed a unified model which captures both mobility fluctuation and number fluctuation effects [53], [52]. Proposed model is based on the

³This part is mostly summarized from [53] and [52]

following equation:

$$\frac{\delta I_d}{I_d} = - \left(\frac{1}{\Delta N} \frac{\delta \Delta N}{\delta \Delta N_t} \pm \frac{1}{\mu_{eff}} \frac{\delta \mu_{eff}}{\delta \Delta N_t} \right) \delta \Delta N_t \quad (3.14)$$

N_t corresponds to the occupied traps per unit area. ΔN and ΔN_t are the change in total number of channel carriers and occupied traps, respectively. In order to evaluate eq. 3.14 $\delta \Delta N / \delta \Delta N_t$ and $\delta \mu_{eff} / \delta \Delta N_t$ should be known. The first term is called as coupling coefficient. It is taken as one [53]. The second term is related to the carrier mobility and oxide charge density relation. Hung used Matthiessen's rule that was given in eq. 3.12 and assumed that scattering from the oxide charges is related to the change in number of occupied traps (N_t) via a coupling coefficient α [53]. Finally second term of eq. 3.14 is expressed as:

$$\frac{\delta \mu_{eff}}{\delta \Delta N_t} = - \frac{\alpha \mu_{eff}^2}{W \Delta x} \quad (3.15)$$

Eq. 3.14 is evaluated using $\delta \Delta N / \delta \Delta N_t$ and $\delta \mu_{eff} / \delta \Delta N_t$ definitions and the following drain current PSD is obtained in [53]:

$$S_{\Delta I_d}(x, f) = \left[\frac{I_d}{\Delta N} (1 \pm \alpha \mu_{eff} N) \right]^2 S_{\Delta N_t}(x, f) \quad (3.16)$$

Calculation of coupling coefficient α is discussed in Hung's work. It is reported that although there are physical models for calculating α such as given in [94], they do not match with the measurement results [53]. As a practical solution, extraction of α from the RTS measurement results is offered. In that time RTS measurement results were available for shorter channel devices and therefore this kind of empirical solution with a polynomial fit for α is offered [53]. The expression $S_{\Delta N_t}(x, f)$ is already given in eq. 3.4 that was developed in McWorther's model. One can use the relationship $S_Q = q^2 S_{\Delta N_t} / (W^2 L^2)$ to switch from one to the other. After the evaluation of $S_{\Delta N_t}$

and integrating $S_{\Delta I_d}$ over channel drain current PSD is calculated as [53]:

$$S_{Id}(f) = \frac{kT_K I_d^2}{\alpha_t f W L} \left(\frac{1}{N} + \alpha \mu_{eff} \right)^2 N_t(E_f) \quad (3.17)$$

Note that this formulation is valid only for low V_{ds} values. As can be seen in eq. 3.17, Hung's model captures mobility effect via the term $\alpha \mu_{eff}$.

So far, a basic structure of Hung's model is given. In [52], extension of this model is given such that it covers all operating regions from weak to strong inversion. Two modifications were done to this model. First modification is about coupling coefficient $\delta \Delta N / \delta \Delta N_t$ of eq. 3.14. It is approximately equal to one in strong inversion but this assumption is not valid in weak inversion as Reimbold showed in [88]. $\delta \Delta N / \delta \Delta N_t$ is updated as R in which R is the Reimbold's parameter (see eq. 3.6). Second modification is about the channel charge definitions at source and drain ends of the transistor. Hung showed that channel charge is given with the following formulation [52]:

$$qN(x) = C_{ox}[V_g - V_{th}(x) - V(x)] \quad (3.18)$$

in which $N(x)$ is carrier density and $V(x)$ is the channel potential at point x starting from the source end of the transistor. Channel charge is linearized by Hung as [52]:

$$qN(x) = C_{ox}[V_g - V_{th0} - aV(x)] \quad (3.19)$$

with the help of these modifications Hung rewrite the master equation for linear operating region as follows [52]:

$$S_{Id}(f) = \frac{kT_K q^2 I_d \mu_{eff}}{a \alpha_t f L^2 C_{ox}} \int_{N_L}^{N_0} N_t^*(E_f) \frac{R^2}{N} dN \quad (3.20)$$

In this equation N_0 and N_L are given as carrier densities at points $x = 0$ and $x = L$,

respectively. $x = 0$ corresponds to the source end of the channel. In [52] channel charges at $x = 0$ and $x = L$ are given as:

$$qN_0 = C_{ox}(V_g - V_{th0}) \quad qN_L = C_{ox}(V_g - V_{th0} - aV_d) \quad (3.21)$$

$N_t^*(E_f)$ in eq. 3.20 is defined as the equivalent oxide trap density in [52] and it is approximated with a second order polynomial [52]:

$$N_t^*(E_f) = A + BN + CN^2 \quad (3.22)$$

Eq. 3.20 is evaluated with given channel charge definitions and full expression for noise current PSD is obtained for linear operating region. Similar master equations are also written for subthreshold and saturation regions in [52]. In all operating regions channel charges are also defined with the linearization approach that is shown in equations 3.18 and 3.19 (see Section III in [52] for details). Hung's model that is given here is a circuit simulation oriented model which is developed for BSIM models.

3.3.2 Discussion

In Hung's model, LF noise is modeled for all operating regions. Some important effects that are not considered in number and mobility fluctuation models were taken into account. For example channel length modulation effect in saturation is modeled and also a smoothing function between weak and strong inversion regions is derived [52]. In their works, extensive measurement results were performed and superiority of their model to previous ΔN and $\Delta\mu$ models were shown [52], [53]. This model is also implemented in BSIM compact transistor models.

There is a critical discussion paper from Vandamme on Hung's unified model [118]. In this paper, assumption of Hung's unified model that mobility (which is

limited to Coulomb scattering mechanism) do not depend inversion carrier density is criticized. In [118], Vandamme stated that there are experimental results in which coulomb scattering limited mobility is proportional to the square-root of inversion carrier density as shown in the works [64], [106] and [79].

3.4 Conclusion

This section compared physical low frequency noise models. Although there are various models, none of them is fully capable of modeling LF noise in both n-channel and p-channel MOS transistors. Hung's model was the most successful among them and it is used in circuit simulators [52]. SPICE, BSIM and PSP models still uses modified versions of Hung's unified model. Compact modeling history of LF noise will be reviewed in the next chapter. In this contemporary era, existing compact models no more satisfy the needs of circuit design community.

One of the main reasons that physical LF noise models were not capable of capturing all LF noise behavior was the limited information that noise power spectrum gives. Wel stated that ensemble averaged power spectrum contains less information than the time domain noise waveforms as phase information is lost [126]. All the models that were reviewed in this section based on modeling LF noise with a stationary PSD term, i.e. $S_{Id}(f)$ or $S_{Vg}(f)$. As devices scaled down to submicron levels it became possible to obtain significant time domain waveforms that clearly show switching events due to the trapping-detrapping processes [60]. This progress provided valuable data for better understanding LF noise and also initiated circuit simulation oriented analytical noise modeling studies.

Chapter 4

COMPACT MODELING OF LOW FREQUENCY NOISE

A compact model is simply the circuit model that is developed for simulation softwares for observing circuit behavior in a reasonable accuracy. For easy implementation to simulation softwares, compact models have to be as simple as possible. Main challenging task for a compact model is to satisfy both simplicity and accuracy in which most of the time these two requirements conflict with each other [40]. Rapid developments in technology further complicates the objectives of compact modeling since each progress brings new challenges along.

MOS transistors are the most challenging circuit elements for compact modeling. In advanced sub-micron MOS models short channel effects dominate. Compared to long channel ones device-to-device variability in performance is also higher which makes the exact behavior of a single transistor unpredictable. Therefore statistical analysis should be incorporated into compact models which is a rising demand to have more accurate predictions of device behavior. As transistors scale down, supply and threshold voltages are also reduced. High performance requirements in low voltage design pushed researchers to novel techniques such as operating transistors in weak inversion. For compact modeling, this means robust and single piece models should be developed to meet industrial requirements. In addition to all these problems, there is emerging difficulty in obtaining reasonable computational efficiency. Although computational capacities of simulation softwares are growing, electronic circuits now have more transistors which consequently boosts computational costs.

Compact modeling requires an interdisciplinary work in which device physicists

and electronic design engineers collaborate together. Lack of this kind of interdisciplinary work gave rise to wrong models in compact modeling history. This brought financial losses to companies where chips did not work after production. Woltjer noted that in order to meet industrial requirements compact model developers at least should take into account feedbacks of circuit designers [129]. McAndrew stated that developers should be aware of intended application areas of the device and they should have pre-information including bias ranges and scale of the system that device will be used in [73]. A good compact model should satisfy following criteria¹:

- i. First of all, it should give accurate electrical behavior of the transistor. I-V characteristic, charge conversation, small signal parameters should be correctly modeled. These small signal parameters should be continuous with respect to the terminal voltages [113]. Derivatives of small signal parameters should also be well defined since they are very important for electronic circuit simulation.
- ii. The model should be valid for all reasonable bias points and it should cover temperature range of interest. Geometrical dimensions should be well defined and user should be able to use all width and length combinations safely [113]. An important issue with operating point coverage of the model is the regional modeling. A single piece model is better than regional models especially in terms of accuracy in the transition regions such as boundary of linear and saturation operation regimes [73].
- iii. The model should have as few parameters as possible. Physical parameters should be preferred to empirical parameters. Empirical models based on polynomial equations are one of the main reasons of numerical problems such as asymptotic incorrectness [73]. An efficient parameter extraction method should

¹This list is summarized from [113], [73] and [129]

be provided to users [113]. If the model is based on physical parameters, extraction method will not get complicated in further development process. In empirical models it may be needed to start from scratch after the model is updated which is cumbersome for developers.

- iv. Different analysis methods should be supported. Time and frequency domain analyses including small signal AC, large signal harmonic balance and periodic steady state should be supported [129].
- v. The model should be free of numerical errors. A challenging metric for advanced models is C_∞ continuity property for small signal parameters in which the functions that represent small signal parameters are differentiable for all degrees of differentiation [73]. C_∞ continuity is preferable for a good compact model since critical accuracy errors due to differentiation is avoided.

We have given the general industrial requirements for a good compact MOS transistor model. As for compact modeling of LF noise, requirements are further tightened. For example statistical analysis is required for modeling of trap behavior. Another example is device-to-device variability of RTS noise in sub-micron transistors which consequently lead us to the fact that statistical modeling for physical parameters is also needed. Non-stationarity of RTS noise is another issue which makes the LF noise analysis even more complicated.

4.1 Existing Compact Models of MOS Transistors

SPICE (Simulation Program with Integrated Circuit Emphasis) is an open source circuit simulator. It is originally developed at Electronics Research Laboratory in University of California Berkeley in 1960s. SPICE has different versions that different MOS transistor models was implemented in. Early SPICE versions have primitive

MOS transistor models such as Level 1 and Level 2. As expressed in [1], SPICE Level 3 is rather more important model than previous two in which more empirical and less analytical modeling is performed. This brought slightly less accuracy but better convergence performance in which computational costs are also reduced [1].

BSIM models are more advanced models than the SPICE level 2 & 3. BSIM1 which corresponds to SPICE Level 13 was developed for shorter channel devices [1]. However, this model was too empirical which extensively used polynomial equations [1]. As MOS transistors have been developed further, BSIM models have also been updated. BSIM2 (Level 39), BSIM3v2 (Level 47) and BSIM3v3 (Level 49) were some of the later models. BSIM3v3 has been very popular and also it has been accepted as first industry standard model [100], [6]. BSIM4 is developed in 2000s and it is widely used with the novel technologies down to 28 nm as stated in [6]. BSIM6 is the latest model of BSIM line. Main motivation for BSIM6 development is said to be bringing Gummel symmetry property which is important especially in RF circuit design [6].

Except BSIM6, BSIM models directly relates drain current and terminal charges to the node voltages of transistors. In [34], it is stated that this brings inaccuracy especially for transition regions such as weak to strong inversion region. Researchers developed alternative models to BSIM to overcome these problems. There are two alternatives to conventional modeling approach that BSIM models use [15]. Instead of directly relating drain current and terminal charges to the node voltages, they use inversion charge based or surface potential based approaches that indirectly relates drain current and terminal charges to the node voltages of transistors [34].

Surface potential based approach is used in PSP model which is developed by NXP Semiconductors and Arizona State University. This model is said to be able to capture major physical effects of new generation transistors without too much model complexity [39]. For example, PSP model can calculate higher order derivatives of currents and charges which is very important issue for distortion analyses [102].

Another model is EKV model which is developed by Swiss researchers in EPFL. EKV model is charge-sheet based model in which inversion charge calculation is used as the core of the model [39]. It is stated that this model is one of the most physical models that allows direct calculation of device characteristics [39]. The model is intended to be used in low voltage and low current applications [26]. EKV 2.6 model uses very few model parameters and Enz stated that EKV model is also suitable for hand calculations [25]. In BSIM6 model development, EKV and BSIM groups have teamed up to develop an industry standard open source circuit simulator. Core of the BSIM6 model has been updated after this collaboration. The new model is very different from the earlier models such as BSIM4 and BSIM3v3 [7].

4.2 Low Frequency Noise in Compact Models

Earliest models such as SPICE 3 used simple LF noise models that can be easily transformed to number fluctuation or mobility fluctuation models. Hung's unified flicker noise is implemented in BSIM4. BSIM4 was very popular model and it is widely used in circuit simulators [6]. Latest version of BSIM and PSP still use modified versions of Hung's unified model. In addition to these models we will review the models of Li et. al [68] that is not implemented any compact model yet.

4.2.1 Flicker Noise in SPICE Level 3

One of the earliest models is SPICE Level 3 in which there are three options for flicker noise.² In SPICE Level 3, NLEV is defined as a parameter that permits user to switch between these three flicker noise models. In Table 4.1 these models were given.

KF and AF are two SPICE parameters where the former one is flicker noise coefficient and the latter one is flicker noise exponent. As can be seen in Table 4.1, NLEV

²In SPICE terminology, *flicker noise* is used instead of *low frequency (LF) noise*. We followed their terminology and preferred to use *flicker noise* in this section which is nothing but LF noise.

SPICE Parameter	Formulation
NLEV=0	$I_{dn}^2 = \frac{KF * I_d^{AF}}{C_{ox} * L_{eff}^2 * f}$
NLEV=1	$I_{dn}^2 = \frac{KF * I_d^{AF}}{C_{ox} * W_{eff} * L_{eff} * f}$
NLEV=2,3	$I_{dn}^2 = \frac{KF * g_m^2}{C_{ox} * W_{eff} * L_{eff} * f^{AF}}$

Table 4.1: SPICE Level 3 Flicker Noise Models

0 model could not take into account even basic geometrical effects on transistors. Another important point is that flicker noise formulation of NLEV 2 & 3 models is different than NLEV 0 and NLEV 1 in which noise current PSD is proportional to the square of transconductance g_m instead of drain source current I_d in NLEV 2 & 3.

It is possible to obtain ΔN and $\Delta\mu$ models from SPICE Level 3 equations. By choosing NLEV 2 equation and selecting $AF = 1$, ΔN model is obtained. Similarly, $\Delta\mu$ model can be extracted from by selecting NLEV 0 equation with $AF = 2$.

4.2.2 Flicker Noise in BSIM Models

Hung's correlated LF noise model that was reviewed in Section 3.3.1 was implemented in BSIM 3v3. In BSIM4 and BSIM6, there are some improvements on this model. Improvements in BSIM4 are inclusion of bulk charge effect, improved effective channel length description and smoother transition region characteristic as stated in [83]. In the final version (BSIM6.1.0) there are important updates especially in channel charge definitions. In BSIM4v4.8.0 channel charge $qN(x)$ value at drain end that is given in eq. 3.21 has changed with a minor modification as follows [83]:

$$qN(L) = C_{ox}(V_g - V_{th0})\left(1 - \frac{A_{bulk}V_{ds}}{V_g - V_{th0} + 2\phi_t}\right) \quad (4.1)$$

$qN(0)$ (channel charge at source end) definition did not change, only $qN(L)$ expression has been updated. However, in BSIM6.1.0 there are major changes in charge definitions. BSIM and EKV groups have collaborated in BSIM6 development and inversion charge definition that was given in the model of Sallese et al. has been implemented in BSIM6. EKV model had also a special version of this inversion charge definition [95]. Therefore, structure of channel charge definitions have completely changed in BSIM6. Detailed information of updated channel charge definitions can be found in [7] and [95]. We will give only updated expressions for $qN(0)$ and $qN(L)$:

$$qN(0) = \frac{2n_q C_{ox} V_t q_s}{q} \quad qN(L) = \frac{2n_q C_{ox} V_t q_d}{q} \quad (4.2)$$

q_s and q_d are source side and drain side charges, respectively. n_q is the inversion charge linearization factor that is defined in [95]. Despite these important changes in channel charge definitions, physical basis of LF noise formulation remains same in BSIM6. It is still based on the framework of Hung's unified model [7].

Similar to Spice Level 3, transition to $\Delta\mu$ or ΔN models is possible in BSIM models. In a review paper this transition is investigated [124]. Their work provides a guide for parameter extraction of BSIM models. In BSIM models specific parameters of LF noise are $NOIA$, $NOIB$, $NOIC$. These parameters correspond to coefficients of the polynomial that is given in eq. 3.22. BSIM formulation can be related to classical physical models via appropriate selection of $NOIA$, $NOIB$, $NOIC$. However, this transformation is plausible for only linear region of operation as shown in [124].

4.2.3 A Compact Channel Noise Model³

In 2006, Roy and Enz have offered a new methodology for noise modeling in MOS transistors [89]. According to Roy, problem of noise modeling in MOS transistors can

³This part is mostly summarized from [89] and [68]

be grouped into two categories. One problem is modeling the local noise sources and their statistical properties. Second problem is relating the local noise sources to the terminal currents and voltages. Let's consider eq. 3.16 which was given as:

$$S_{\Delta Id}(x, f) = \left[\frac{I_d}{\Delta N} (1 \pm \alpha \mu_{eff} N) \right]^2 S_{\Delta N_t}(x, f) \quad (4.3)$$

Roy assumes that first problem is modeling $S_{\Delta N_t}(x, f)$ which represents the spectral density of a local noise source. Latter problem is relating $S_{\Delta N_t}(x, f)$ to $S_{\Delta Id}(x, f)$. Roy's methodology offers a solution to the second problem where a general expression was derived that relates all local noise sources to the device terminals [89].

Roy stated that there have been three popular noise calculation methodologies which are Klaassen-Prins approach, impedance field method and equivalent circuit method [89]. According to Roy although these methods works well for long channel devices, nonlinearity effects emerge for short channel devices and they cannot be handled with these conventional noise calculation methodologies. Roy's work starts with the analysis of three methodologies in which they incorporated the short channel effects to each of them. Finally, they ended up with same drain and gate noise current expressions. This approach constitutes the basis of their generalized methodology.

Li et al. have developed a novel compact channel noise model using the noise calculation methodology of Roy et al. [68]. Their model includes both thermal noise and flicker noise. We will be interested in only flicker noise part and the methodology that it is based on. Roy's formulation was based on the following core equation:

$$S_{Id} = \frac{1}{I_{ds} L_c^2} \int_0^{V_{ds}} \frac{g_c^2(V, E)}{g(V, E)} S_{\delta i_n^2} dV \quad (4.4)$$

where $g(V, E)$, $g_c(V, E)$ and L_c are parameters that are dependent on the device that is subjected to noise calculation methodology. $S_{\delta i_n^2}$ is defined as the local current fluc-

tuations [68]. Li calculated $g(V, E)$, $g_c(V, E)$ and L_c using MOS transistor parameters. In order to calculate $g(V, E)$, Li rewrite I_{ds} expression as [68]:

$$I_{ds}(x) = g(V, E)E = W\mu(x)Q_{inv}(x)E \quad (4.5)$$

Note that $g(V, E)$ not only depends on channel potential V but also its first order derivative since $E = -dV/dx$ holds. This point is important since the dependence of current to the electrical field means inclusion of field dependent mobility effect for the drain current [89]. This effect is more pronounced in the velocity saturation according to Roy. This is one of the key contributions of Roy's formulation to noise calculation studies [89]. Q_{inv} expression was taken directly from BSIM4 model and following expression is obtained for $g(V, E)$ [68]:

$$g(V, E) = W \frac{\mu_{eff}}{1 + E/E_c} C_{ox} V_{gs} \left(1 - \frac{A_{bulk} V}{V_{gs} + 2\phi_t}\right) \quad (4.6)$$

A_{bulk} is the bulk effect parameter that is defined in BSIM model and ϕ_t is the thermal voltage. Electrical field definition is also obtained using Q_{inv} expression of BSIM4 model and also the equations 4.6 and 4.5 [68]. Derivation of $g_c(V, E)$ and L_c are given in [68] in detail. We just give the results here:

$$g_c(V, E) = g(V, E)(1 + E/E_c) \quad L_c = L_{elec} + V_{ds}/E_c \quad (4.7)$$

Enz's formulation of the drain current PSD was given in eq. 4.4. In order to obtain full expression, $S_{\delta i_n^2}$ should be calculated. Li follows the work of Hung's unified model with some improvements for modeling $S_{\delta i_n^2}$. Critical discussion of Vandamme in [118] to Hung's model was taken into account by Li [68] (see Section 3.3.2). Li included dependence of inversion carrier density to mobility definition and updated the scattering parameter expression [68]. In his model, scattering parameter is defined

in two parts. Above the threshold, α was given as [68]:

$$\alpha = \frac{\sqrt{q}/\sqrt{Q_{inv}(x)}}{5.9 \times 10^6 [m/Vs]} \quad (4.8)$$

and below the threshold:

$$\alpha = 3 \times 10^{-15} [Vs] \quad (4.9)$$

$S_{\delta i_n^2}$ is obtained in [68] as follows:

$$S_{\delta i_n^2} = \left[I_d \left(\frac{q}{Q_{inv}(x) + nC_{ox}\phi_t} + \alpha\mu_{eff} \right) \right]^2 \frac{N_{ot}}{Wf} \quad (4.10)$$

Finally, integral in eq. 4.4 is evaluated in [68] using $g(V, E)$, $g_c(V, E)$, L_c and $S_{\delta i_n^2}$ definitions:

$$S_{Id} = \frac{I_d N_{ot} \mu_{eff}}{f L_c^2} \int_0^{V_{ds}} \left[\frac{Q_{inv}^2(x)}{Q_{inv}(x) - (I_d/(\mu_{eff} W E_c))} \left(\frac{q}{Q_{inv}(x) + nC_{ox}\phi_t} + \alpha\mu_{eff} \right)^2 \right] dV \quad (4.11)$$

Note that according to the operation region, integral in eq. 4.11 will take two different forms due to the definition of scattering coefficient α which is given in eq. 4.8 and eq. 4.9. More explicit expression can be found in [68].

4.2.4 Discussion

Performance of SPICE Level 3 and BSIM3v3 models have been reviewed in some studies with measurements of both short and long channel devices [131], [134]. In that studies devices that are used in measurements are from $0.6\mu m$ to $5\mu m$ length. Although these transistors are not at sub-micron levels they showed short channel effects during the measurements. This allows comparison of compact models in terms of performances about capturing short channel device behavior. NMOS devices were measured from subthreshold to saturation region [131] and PMOS devices were mea-

sured only in saturation region of operation [134]. It was shown that in short channel NMOS devices input referred noise increases with increasing V_{gs} . The reason is reported to be the velocity saturation which results g_m to vary with V_{gs} . In long channel NMOS devices g_m remains constant as V_{gs} increases (see Fig. 4 in [131]). In subthreshold region input referred noise increases with increasing V_{gs} for both long and short channel devices (see Fig. 10 in [131]). This behavior is best captured by BSIM3v3 model for short channel devices and by SPICE Level 3 NLEV 2&3 model for long channel devices (see Fig. 11 and Fig. 12 in [131]). One problem is SPICE Level 3 NLEV 2&3 model does not contain any subthreshold model and BSIM3v3 model does not support flicker noise modeling of long channel devices such as $5\mu m$ length transistors. Modeling of such longer transistors may not be need by industry today, however it is still a problem if we want an ideal compact model (see the criteria that we have given in the beginning of Chapter 4).

Measurement of PMOS devices gave interesting results. It is reported that for long channel devices flicker noise is due to the mobility fluctuations and for short channel devices it is due to the number fluctuations [134]. This discrimination is made by comparing the overdrive voltage ($V_{gs} - V_{th}$) dependence of input referred voltage noise. As we have given in Section 4.2.1 mobility fluctuation theory can be selected by choosing $NLEV = 0, AF = 2$ in SPICE Level 3. Long channel PMOS transistors are best modeled by this configuration. For short channel devices BSIM3v3 model was able to capture the drain current noise behavior but not that good (see Fig. 6 in [134]). Especially it fails in capturing the input referred noise behavior (see Fig. 5 in [134]). In conclusion, BSIM3v3 is not very good in capturing flicker noise behavior of short channel PMOS transistors.

PSP model and BSIM models both use Hung's unified flicker noise model in their flicker noise formulations. Therefore we will not give flicker noise modeling in PSP here separately. There is a manual where the translation of BSIM parameters to PSP

parameters have been explained [28]. In [28], it is stated that although parameter translation is possible, it is not an exact one-to-one translation. We believe that the reason of this fact is the result of different frameworks of these models. PSP model is surface potential based model and it has a hierarchical structure. There are two distinct parameter sets in the PSP formulation as global level parameters and local level parameters [102]. Due to these basic differences exact translation of flicker noise parameters from BSIM to PSP may not be possible.

In Section 4.2.3 of this chapter we have given a compact model that is an extension of Hung's unified model. Although it is not implemented in simulation softwares it is worth to consider its capabilities. Advantages of this model as we noticed are:

- i. It includes field dependent mobility effect. Note that this was one of the main critics that was used against Hung's unified model by Vandamme [118].
- ii. It uses the generalized noise calculation methodology of Enz that was given in [89] which relates the local noise fluctuations to the drain and source terminals of transistor in a more physical way than the conventional methods.
- iii. It includes more short channel effects compared to Hung's unified model.

However, we believe that this model still have some drawbacks:

- i. It is a two part regional model, not a single piece one. This may cause some problems in compact models as we discussed in the beginning of Chapter 4.
- ii. We could not see its implementation to any circuit simulator. In order to have an industry standard compact model, one should test its compact model within a simulation framework.

- iii. It is for only frequency domain simulations and tries to model LF noise with a PSD term. Note that the trend is now time domain noise simulations that allow better capturing of noise correlations.

4.3 Conclusion

Compact models develop in a continuous manner. BSIM6 is the most complex and the newest version of BSIM line. It includes lots of features including Gummel symmetry, short channel effects and also more physical charge definitions [7]. PSP model is also a nice model which has surface potential based equations and provides smoother transition between operation regions. Similar to BSIM6, flicker noise model of PSP is also based on Hung's unified model.

Time domain simulations are available now as the result of observation possibility of time domain waveforms of RTS noise in short channel transistors. However, RTS and GR noise models have not been implemented in circuit simulators yet. PSP model and BSIM models (after BSIM4) includes trap assisted tunneling (TAT) effect. However, they do not consider this effect in terms of noise but they use it in leakage junction current calculation. In PSP, in addition to trap assisted tunneling, band to band tunneling current and also shockley-read-hall (SRH) parameters were defined [102]. However, correlation between trap assisted tunneling and LF noise has not been studied so far. The same physical mechanism which is the trapping-detrapping process should be the reason to study this correlation. In the next section we will review some compact models of RTS noise that were developed to fill the gap in the compact noise model development for time domain noise simulations.

Chapter 5

COMPACT MODELING OF RTS NOISE

Before time domain simulations are available that allow observations of single switching events, only power spectrum data were available to researchers. Earlier LF noise models were founded on this limited data in which *complete* characterization of LF noise as a stochastic process was not possible. After time domain simulations that provide new useful data, characterization studies of RTS noise were accelerated [60]. Another reason of the interest shift to RTS noise modeling was that RTS noise is believed to be one of the main contributors to LF noise [60].

In classical LF noise models that were given in Chapter 3 and Chapter 4, standard *AC analysis* was used. In [20] it is stated that this AC analysis is based on small-signal assumptions and linear time invariant (LTI) system theory. Transistors are nonlinear devices in terms of their I-V relationships. Nonlinear devices like transistors are linearized using small-signal models in standard AC analysis. Nonlinear behavior of the devices are approximated using linear equations that are evaluated around a stable DC bias point. Motivation of linearizing nonlinear models is that solving a system of linear equations is far easier than solving a system of nonlinear equations. These small-signal models are assumed to be valid as long as DC bias point does not vary too much. Demir stated that although linear system theory that is based on small-signal analysis works well with lots of basic circuits it may not suitable for some other circuits including mixers and oscillators [20]. In RF circuits operating points vary significantly and therefore small-signal assumptions do not hold anymore.

Time invariance assumption is also not valid for all circuits. Low frequency models

that are given up to this section assumed that noise sources can be treated as stationary processes in which means and the variances of these processes do not change over time. This assumption allowed researchers to model the systems as time invariant. However, as Keshner indicated in 1982, LF noise should be modeled as a nonstationary process due to the long time correlations between noise samples [59]. Unlike the thermal and shot noise, LF noise cannot be treated as a stationary process. Therefore LTI system based analysis techniques are not suitable for modeling LF noise correctly.

Inefficiency of stationary RTS noise models was seen clearly in circuits where the transistors are subjected to large signal excitation, i.e. having transient bias applied to their gate instead of a constant bias. Such circuits especially popular in RF electronics. Tian stated that traditional LF noise models overestimated LF noise in switched bias circuits [109]. Difference between measured and simulated noise powers were reported to be more than 10 dB [65], [109]. Such a major error gave rise to a big demand from the circuit design community for accurate modeling of RTS noise. In this chapter we will review several models that were developed in the last decades.

In the first section we will review a statistical model that is one of the first attempts of modeling effects of statistical variations of transistor parameters to LF noise performance. This model is also a nice transition model example between classical LF noise models and RTS noise models. In the second section, well-known stationary RTS model is given since basic nonstationary models were indeed the extensions of this model. In the third section, cyclostationary RTS noise that results from the large signal excitation of a transistor is discussed with two different models. Kolhatkar's work provide simple models for trap parameters which is a good introduction to nonstationary modeling of RTS noise. Tian's model with the clarified version by Mahmutoglu has analytical expressions for nonstationary RTS noise PSD. However, both models given in this sections were based on an idealized trap model. In the last section we review Mahmutoglu's advanced nonstationary models that are based on

physically realistic trap definitions. Efficient non Monte Carlo techniques were used for the simulation of these models that reduces simulation costs in comparison to Monte Carlo based simulation techniques.

5.1 Wirth's Statistical Model ¹

Wirth et al. developed a statistical LF noise model in 2005 where they primarily focused on effects of statistical parameters on LF noise of CMOS devices [128]. It is a nice transition example between compact modeling of RTS noise and classical modeling approaches that are given in Section 4. In this section, we will try to elucidate their efforts that mirrors the trend in compact modeling of LF noise in the last decade.

Their motivation for a statistical model is understandable. As MOS transistors get smaller, device to device variability of characteristic parameters of transistors increases. Even in transistors that are produced in same wafer with same geometries, different LF noise performances were measured (see Fig. 1 in [128]). So the first question was simple: “Which parameters are the main sources of these statistical variations?”. Based on this question a new simple and useful perspective on LF noise modeling that include statistical characterization of critical parameters is developed by Wirth et al.

The formulation of LF noise in their study is divided into two categories. For large area devices they developed an LF noise model that is similar to classical LF noise models that we reviewed in previous sections. However, their models for small area devices is different than the classical LF noise modeling approaches. We will review basic properties of this model.

They differentiated small area devices from large area devices. They used a dis-

¹This part is mostly summarized from [128]

crete summation method instead of integration to calculate the PSD of LF noise for small area devices [128]:

$$S(f) = \sum_{i=1}^{N_{tr}} A_i^2 \frac{1}{f_i} \frac{1}{1 + (f/f_i)^2} \quad (5.1)$$

In this formulation contribution of individual traps are summed. N_{tr} is the number of traps and f_i is the corner frequency of each trap which is given as $f_i = 1/(2\pi\tau(E, x))$ where τ is the trap time constant. A_i is defined as the Fourier transformed amplitude of gate voltage spectral density for trap i [128]. Wirth defined N_{tr} , f_i and A_i parameters as the sources of statistical variations of LF noise in transistors. For the standard deviation of LF noise, each of these parameters analyzed independently in [128]. Wirth defined the standard deviation of noise PSD as:

$$\sigma_{S(f)} = \sqrt{\langle S_f^2 \rangle - \langle S_f \rangle^2} \quad (5.2)$$

$\langle S_f \rangle$ is formulated in [128] as:

$$\langle S_f \rangle = \left\langle \left\langle \left\langle \sum_{i=1}^{N_{tr}} A_i^2 \frac{1}{f_i} \frac{1}{1 + (f/f_i)^2} \right\rangle_{A_i} \right\rangle_{f_i} \right\rangle_{N_{tr}} \quad (5.3)$$

Similarly, $\langle S_f^2 \rangle$ is formulated as:

$$\langle S_f^2 \rangle = \left\langle \left\langle \left\langle \sum_{i=1}^{N_{tr}} \frac{A_i^4}{f_i^2} \frac{1}{(1 + (f/f_i)^2)^2} + \sum_{i \neq j}^{N_{tr}} \frac{A_i^2 A_j^2}{f_i f_j} \frac{1}{1 + (f/f_i)^2} \frac{1}{1 + (f/f_j)^2} \right\rangle_{A_i} \right\rangle_{f_i} \right\rangle_{N_{tr}} \quad (5.4)$$

These two expressions are evaluated over N_{tr} and f_i assuming suitable N_{tr} and f_i distributions (see Section A in [128] for details). Normalized variance of PSD is

obtained in [128] as follows:

$$\frac{\sigma_{S(f)}}{\langle S_f \rangle} = \frac{\sqrt{2}}{\pi \sqrt{N_{dec} W L}} \sqrt{\frac{\langle A^4 \rangle}{\langle A^2 \rangle^2}} \quad (5.5)$$

As can be seen in eq. 5.5, variations of fluctuation amplitude has strong effect on standard deviation. However, this formulation of standard deviation does not say very much about physical sources of fluctuation amplitudes and its bias voltage dependency as Wirth stated. He stated that in order to have a deep understanding about variations of A the actual physical sources should have been examined. Wirth borrowed the definition of A by Wel et al. which is given as [126] [128]:

$$A_i^2 = \delta I_d^2 \frac{\beta}{(1 + \beta)^2} \quad (5.6)$$

in which β is given as τ_c/τ_e . So, Wel et al. defined A_i with the amplitude of the current fluctuation δI_d and the ratio of capture and emission time constants τ_c and τ_e . Standard deviation of A^2 is formulated by Wirth using the definition of A^2 that is given in eq. 5.6 and the definition of drain current I_d as [128]:

$$(\sigma_{A^2})^2 = \left(\frac{\partial A^2}{\partial (\delta \mu / \mu)} \right)^2 \sigma_{\delta \mu / \mu}^2 + \left(\frac{\partial A^2}{\partial (\delta N_C / N_C)} \right)^2 \sigma_{\delta N_C / N_C}^2 + \left(\frac{\partial A^2}{\partial \beta} \right)^2 \sigma_{\beta}^2 \quad (5.7)$$

Terms $\sigma_{\delta \mu / \mu}$, $\sigma_{\delta N_C / N_C}$ and σ_{β} should be calculated for the evaluation of eq. 5.7. Here, we will not give all derivation steps of these parameters. We will give only the results and their meaning, since in this work we give emphasis to the methodology that Wirth's statistical model is based on rather than the actual results that they have obtained. Detailed information can be found in [128]. Wirth calculated the normalized spectral density that comes from mobility fluctuations and from local

current density fluctuations as:

$$\frac{\sigma(\delta\mu/\mu)}{\langle\delta\mu/\mu\rangle} = \frac{1}{\sqrt{5}} = k_d \quad \frac{\sigma(\delta N_C/N_C)^2}{\langle\delta N_C/N_C\rangle^2} \approx \frac{V_d^5}{V_{g,eff}^5} \quad (5.8)$$

The last term in eq. 5.7 could not have been modeled since the bias dependencies of τ_c and τ_e are not known as Wirth et al. stated. They assumed in their study the contribution is constant (k_β) with a first order approximation.

The methodology that they have followed is important. They underlined the problem in device to device variability in short channel devices and tried to find the sources of statistical variations. Secondly they emphasize the bias dependency of τ_c and τ_e voltages which is the main issue that the advanced cyclostationary and nonstationary analyses try to deal with. Therefore it can be counted as a transition model between classical stationary LF noise models and RTS noise based nonstationary models.

5.2 Stationary RTS Model ²

Before nonstationary models, stationary RTS model should be understood carefully. In Section 2.2.3, RTS noise was introduced with the emphasis on its relation to GR noise. In this section, we will review the basic stationary RTS model in detail.

Machlup has derived the spectrum of two-level RTS signal as given already in eq. 2.11 in Section 2.2.3. Starting point of Machlup is the derivation of autocorrelation function of the RTS. Referring to Fig. 2.4, if the lower value is represented with x_0 and the higher value of the RTS with x_1 , and the amplitude of the RTS is defined as ΔI , $x_1 = \Delta I$ holds. Assuming the initial value of RTS is zero i.e. $x_0 = 0$, following probabilities can be defined [60]:

$$p_1 = \frac{\tau_1}{\tau_1 + \tau_0} \quad p_0 = \frac{\tau_0}{\tau_1 + \tau_0} \quad (5.9)$$

²This part is summarized from [60]

Here p_1 is the probability that RTS is in state 1 and p_0 is the probability that RTS is in state 0. Autocorrelation function $c(t)$ is defined as [60]:

$$c(t) = \langle x(s)x(s+t) \rangle = \sum_i \sum_j x_i x_j \times P[x(s) = x_i] \times P[x(s+t) = x_j | x(s) = x_i] \quad (5.10)$$

As given in [60], $c(t)$ is rewritten using probability definitions as:

$$c(t) = (\Delta I)^2 \times P[x(s) = \Delta I] \times p_{11}(t) = (\Delta I)^2 \times p_1 \times p_{11}(t) \quad (5.11)$$

$p_{11}(t)$ is the probability of *even* number of transitions given that starting state is 1. In order to calculate it, p_{10} is defined (probability of *odd* number of transitions given that starting state is 1) providing that $p_{10}(t) + p_{11}(t) = 1$. $p_{11}(t + dt)$ is given as [60]:

$$p_{11}(t + dt) = p_{10}(t) \frac{dt}{\tau_0} + p_{11}(t) \left(1 - \frac{dt}{\tau_1} \right) \quad (5.12)$$

This formulation is nothing but the summation of two mutually exclusive events. In order to have even number of transitions after a time interval dt , at time t there have to be odd number of events *and* one switching event (0 to 1, denoted by dt/τ_0) *or* at time t even number of events *and* no switching event in time interval dt . According to Kirton, one assumption here is that dt is small enough and probability of more than one transition in dt is zero [60]. Using above definitions differential equation for $p_{11}(t)$ is obtained in [60]:

$$\frac{dp_{11}(t)}{dt} + p_{11}(t) \left(\frac{1}{\tau_0} + \frac{1}{\tau_1} \right) = \frac{1}{\tau_0} \quad (5.13)$$

After solving this equation, $p_{11}(t)$ is inserted in eq. 5.11 and the Fourier transform is

applied to $c(t)$. PSD that is given already in eq. 2.11 is obtained as the result [60]:

$$S_{I(f)} = \frac{\overline{4\Delta I^2}}{(\tau_0 + \tau_1)[(1/\tau_0 + 1/\tau_1)^2 + (2\pi f)^2]} \quad (5.14)$$

One may refer to Fig. 2.5 for the spectral density drawings for different time constants based on eq. 5.14. One important result of Machlup's formulation is about the total average power of RTS. Kolhatkar integrated eq. 5.14 over all frequency band and obtained following result for total average power of RTS [65]:

$$P_{total} = \frac{\Delta I^2}{(\tau_1 + \tau_0)(1/\tau_1 + 1/\tau_0)} \quad (5.15)$$

If P_{total} is plotted with respect to ratio of time constants (τ_1/τ_0) , Fig. 5.1 is obtained [65]. Note that maximum value of P_{total} is reached for equal time constants, i.e. $\tau_1 = \tau_0$. Here we should comment that this result is very important for further RTS noise analysis. Remember the issue where Wirth's statistical model could not overcome: "modeling bias dependence of time constants". In here, we end up with similar point to them in which it is clear that RTS power is very much effected by the variations in the ratio of time constants. Being the most critical source of variation in time constants, modeling of bias voltage dependency is crucial for RTS noise analysis. It is also important in large signal excitation (LSE) modeling of RTS noise.

5.3 Transient Bias: Cyclostationary RTS Noise

Cyclostationary processes are defined as the statistical processes in which their characteristics change periodically with time [35]. So far, we have dealt with stationary noise processes that their statistical characteristics do not change over time. Necessary mathematical background for statistical noise analysis in electronic circuit will not given here. However, reader can find useful information on random variables

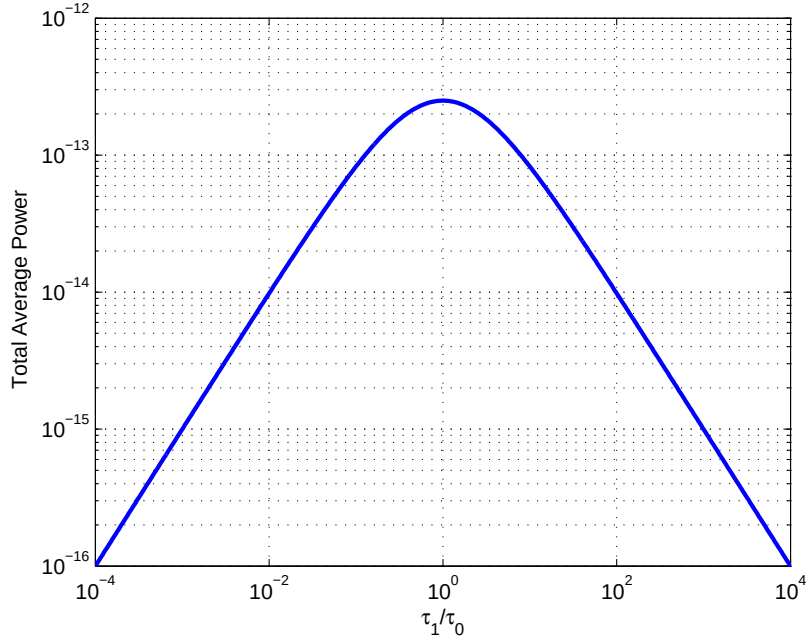


Figure 5.1: Total average power with respect to ratio of time constants [65]

and stochastic processes in [81], on cyclostationary processes in [35] and also a brief mathematical background information for noise analysis in the second chapter of [20].

In Fig. 5.2, a transient bias scheme for a MOS transistor is given in comparison to the constant bias case. Note that in transient bias case, gate-source voltage V_{gs} of the MOS transistor is switching between *on* and *off* states periodically. Therefore transistor do not operate in steady-state condition anymore. Since it is also periodic, resulting drain current noise will be cyclostationary. Since bias points vary, event rates will also be time dependent. In [70] it is reported that bias dependency of time constants is the main source of nonstationarity of RTS noise. Therefore switching bias case is important for nonstationary RTS noise analysis.

In this section we will review two models. In the first one, Kolhatkar and Wel modeled the cyclostationary behavior of RTS noise. Part of this review based on their doctoral theses. Their model can be counted as an extension of stationary RTS

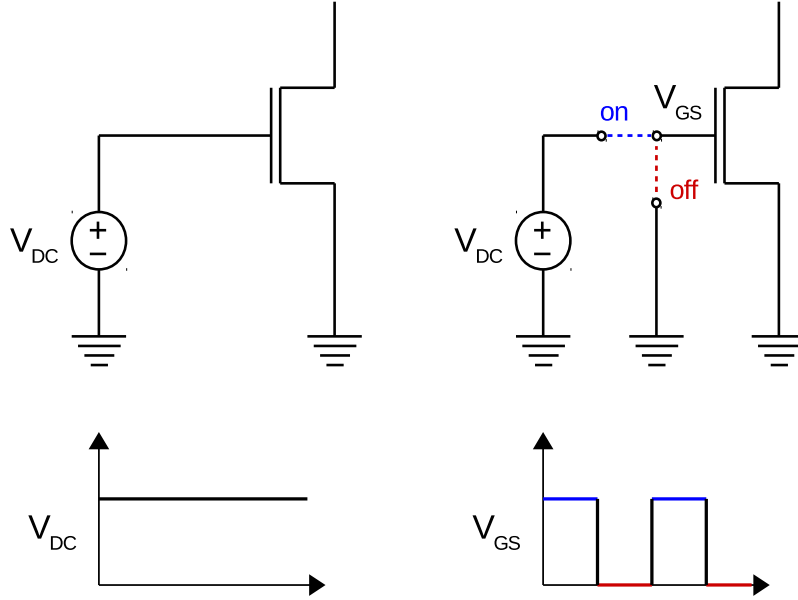


Figure 5.2: Constant vs transient bias [65]

model that is reviewed already in Section 5.2 [65] [126]. In the second section, we will review Tian and Gamal's model. There are some oversights in this model that consequently give incorrect expressions. Their model then revisited and clarified by Mahmutoglu and Demir [109], [70]. Both Kolhatkar's and Tian's models analyze LF noise in transient bias case. Therefore we will review them consecutively.

5.3.1 Kolhatkar's Cyclostationary RTS Model ³

Probability of a trap being occupied with a carrier was given in eq. 1.2. Kolhatkar rewrote trap occupation probability in [65] as:

$$P_f = \frac{\tau_e}{\tau_e + \tau_c} \quad (5.16)$$

³This part is mostly summarized from [65] and [126]

Note that τ_1 is replaced with τ_e and τ_0 is replaced with τ_c . This trap occupation probability P_f is valid only in steady state conditions. More general expression of P_f can be found from the net capture rate of the traps. Capture and emission rates were defined already in eq. 2.12. The net capture rate is given by Kolhatkar as [65]:

$$\frac{dP_f(E, t)}{dt} = C_n(E, t) - E_n(E, t) \quad (5.17)$$

Solution of this differential equation is given in [126] as:

$$P_f(E, t) = \frac{\tau_e}{\tau_e + \tau_c} + K \times \exp\left(-\left(\frac{1}{\tau_e} + \frac{1}{\tau_c}\right)t\right) \quad (5.18)$$

K is the integration constant and its value depends on the initial conditions. This expression is the general expression for trap occupancy that includes both steady state and non steady state cases as stated in [126]. In order to obtain eq. 5.16, taking limit in eq. 5.18, i.e. $t \rightarrow \infty$ is sufficient. $P_f(t)$ is plotted in Fig. 5.3 as given in [65]. Kolhatkar stated that switching frequency of transient bias is much higher than the RTS corner frequency in this analysis [65]. Bias dependency of time constants is studied in Kolhatkar's work. In [65] it is reported that when V_{gs} voltage of transistor is increased conduction band energy bends towards to fermi energy level in which electron concentration gets higher. τ_c decreases as a result of this fact since it is defined as the "mean time before electron capture". unlike bias dependency of τ_c , bias dependency of τ_e is more complex. It is reported that interface traps and oxide traps behave differently in case of varying bias. Bias dependency of time constants were summarized in Table 5.1 [65]. Bias dependency difference of interface and oxide traps was discussed in [65]. As given in Fig. 3.2, different models were developed for trapping-detrapping process. In [65] it is stated that "capture and tunneling" scheme that was illustrated in Fig. 3.2b can better explain bias dependencies of interface and

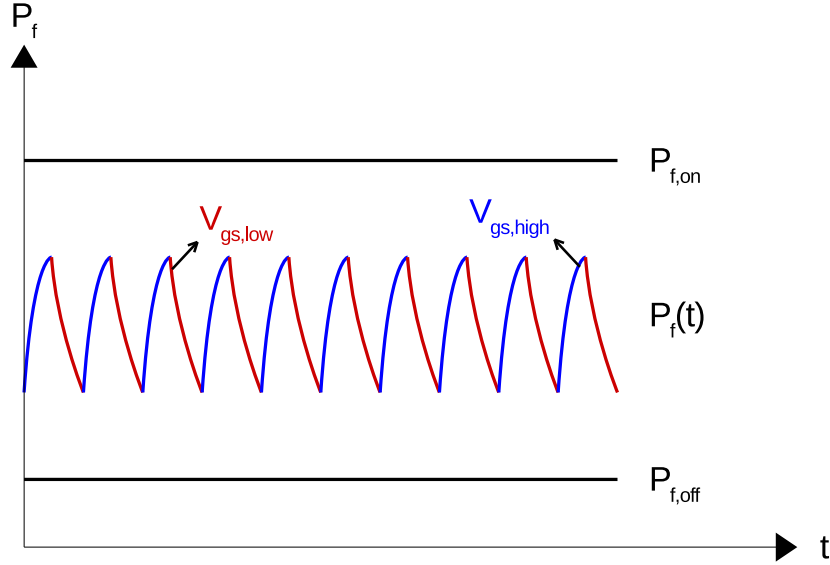


Figure 5.3: Trap occupancy function in constant and transient bias cases [65]

$V_{gs} \uparrow$	Interface Traps	Oxide Traps
τ_c	decreases	decreases
τ_e	constant	increases

Table 5.1: Change of time constants as V_{gs} is increased

oxide traps.

Trap occupancy function change instantaneously in case of a transient bias as shown in Fig. 5.3. $P_{f,on}$ and $P_{f,off}$ corresponds two steady state cases, respectively. Kolhatkar stated that if a constant high voltage is applied to the gate of the transistor, transistor will be in *on* state and corresponding trap occupancy function will be $P_{f,on}$. Similarly, if a constant voltage that is below the threshold level is applied to the gate terminal, device will be in *off* state [65]. From the definition of trap

occupancy function that is given in eq. 5.16 it is obvious that $P_{f,off}$ will be well below than $P_{f,on}$. Between these two steady state levels, there is time dependent trap occupancy function $P_f(t)$. In Fig. 5.3, increasing blue parts corresponds to *on* states and decreasing red parts corresponds to *off* states of transistor.

In [65] it is stated that since switching frequency of gate bias voltage is higher than the corner frequency of RTS, transient trap occupancy function cannot reach steady state values as can also be seen in Fig. 5.3. Since the behavior is cyclostationary, Kolhatkar tried to stationarize cyclostationary time constants by averaging over one switching period of the transient gate bias [65]. There are four different time constants which are $\tau_{e,on}$, $\tau_{c,on}$, $\tau_{e,off}$ and $\tau_{c,off}$. Transition rates based on these time constants are given in [65] as:

$$C_{n,on} = \frac{1 - P_{f,avg}}{\tau_{c,on}} \quad E_{n,on} = \frac{P_{f,avg}}{\tau_{e,on}} \quad (5.19)$$

$$C_{n,off} = \frac{1 - P_{f,avg}}{\tau_{c,off}} \quad E_{n,off} = \frac{P_{f,avg}}{\tau_{e,off}} \quad (5.20)$$

$P_{f,avg}$ is the average occupancy value of transient $P_f(t)$ [65]. If duty cycle is 0.5, it is simply the arithmetic mean of $P_{f,on}$ and $P_{f,off}$. Averaging over one period gives [65]:

$$C_{n,cyclo} = \frac{1}{T} \int_0^T C_n(t) dt \quad E_{n,cyclo} = \frac{1}{T} \int_0^T E_n(t) dt \quad (5.21)$$

After the integration, time constants of stationarized process is obtained in [65] as:

$$\tau_{c,cyclo} = \frac{1 - P_{f,avg}}{C_{n,cyclo}} \quad \tau_{e,cyclo} = \frac{P_{f,avg}}{E_{n,cyclo}} \quad (5.22)$$

In [65], analytical results were compared with the measurement results and good matching between them was observed. For example, Fig. 4.11 and Fig. 4.12 in [65] verified the behavior of trap occupancy function that is given in Fig. 5.3 here. Bias

dependencies of time constants that are summarized in Table 5.1 was verified with measurement results that can be found in Fig. 4.8 and Fig. 4.9 in [65]. One important measurement result is also given in Fig. 4.18 of [65] in which PSD of both constant bias and transient bias cases were given. This result is important and we will use it in the following section. As the next step of Kolhatkar's work, Wel implemented their model in a simulator [126]. In the simulator, bias dependencies were modeled as:

$$\tau_{c,off} = \tau_{c,on} \times m_c \qquad \tau_{e,off} = \tau_{e,on}/m_e \qquad (5.23)$$

τ_c approaches to infinity in the *off* state therefore m_c is very large as stated in [126]. These steady state time constants were used in [126] to calculate the effective time constants of cyclostationary RTS as:

$$\tau_{c,cyclo} = \frac{2m_c}{m_c + 1} \tau_{c,on} \qquad \tau_{e,cyclo} = \frac{2}{m_e + 1} \tau_{e,on} \qquad (5.24)$$

According to Wel, since m_c approaches to infinity, bias dependence of τ_c is not too significant in the limit case as time constant changes at most with a factor of two [126]. However, in transient bias case, τ_e can differ too much from the steady state value. Therefore bias dependency of τ_e is more important in terms of noise performance of transistors in large signal excitation [126]. Wel also reported that one important result is that unlike the steady state, maximum total power of RTS is not reached when the time constants are equal in the transient bias case. Fig. 8 of [115] illustrates this “shift” of the point that maximum total average power is reached.

In this part we have reviewed Kolhatkar's and Wel's works. Their works are based on the most basic nonstationary scheme of a transistor operation: a periodic transient bias applied to the gate terminal of the MOS transistor that is abruptly changing between two states. Although these basic assumptions are unrealistic, their

works provide good information about bias dependency of RTS parameters which is very important for nonstationary RTS noise modeling. They do not only contribute with their RTS parameter modeling approaches to LF noise literature but also with their inclusive measurement results that were given in [65] and [126].

5.3.2 Tian's Nonstationary Model ⁴

Methodology that is used in Tian's model is first deriving auto-covariance function of RTS noise and then obtaining PSD by taking Fourier transform of the auto-covariance function. Same setup is used that is given in Fig. 5.2. A square wave transient bias is applied to the gate terminal of the MOS transistor and a single trap is assumed as the source of trapping-detrapping process [109]. Before modeling under transient bias, nonstationary extension of RTS noise in the constant bias case is derived by Tian. Trapping-detrapping process is modeled with trapped electron number $N(t)$ which is mathematically equivalent to RTS with rates λ_c and λ_e . Note that there is a notation difference between Tian's and Kolhatkar's models. Tian's model is based on capture and emission rates rather than time constants. Capture rate is given as $\lambda_c = 1/\tau_c$ and emission rate is given as $\lambda_e = 1/\tau_e$. In constant bias case, it is assumed that λ_c and λ_e are equal and they are represented with a single parameter λ . Auto-covariance of $N(t)$ in steady state and corresponding PSD are given by Tian as [109]:

$$c_\lambda(\tau) = \frac{1}{4}e^{-2\lambda\tau} \quad S_\lambda(f) = \frac{1}{4} \frac{\lambda}{\lambda^2 + (\pi f)^2} \quad (5.25)$$

In the switching bias case, auto-covariance function is given with the following expression in [70]:

$$c_\lambda(t, \tau) = E[N(t - \tau/2)N(t + \tau/2)] - E[N(t - \tau/2)]E[N(t + \tau/2)] \quad (5.26)$$

⁴This part is summarized from [109] and [70]

According to Mahmutoglu, in order to have correlation between $N(t)$ and $N(t + \tau)$, t and $t + \tau$ have to be in same *on* state. If they are in different cycles, no correlation is observed. With careful calculations of state probabilities and using the property that $N(t)$ is a Bernoulli variable Mahmutoglu calculated $c_\lambda(t, \tau)$ as [70]:

$$c_\lambda(t, \tau) = \frac{1}{4} \left(e^{-2\lambda|\tau|} - e^{-4\lambda t} \right), \quad \left| \frac{\tau}{2} \right| < t < \frac{T}{2} - \left| \frac{\tau}{2} \right| \quad (5.27)$$

Note that Mahmutoglu checked the negative values of τ that Tian did not included in his paper (see their slightly different expression for this part in eq. 9 of [109]). $c_\lambda(t, \tau)$ is zero outside of the region that is given in eq. 5.27 [70]. This expression includes dependency on t . However, in eq. 5.25 auto-covariance function (ACF) was independent from t since $N(t)$ was stationary in constant bias case. As rate λ varies with transient bias $N(t)$, resulting auto-covariance function is now time dependent [70] [109].

In order to calculate PSD, Fourier transform of ACF should be calculated. However, ACF have to be stationary to perform the Fourier transform. Mahmutoglu stated that it is possible to calculate time-averaged PSD by calculating time-average of ACF. This is achieved by taking time average [70]. Tian et al.'s analysis is plagued by an oversight at this point. In [70], limits of integration is carefully determined in the averaging step. Corrected expression of time-averaged ACF is given in [70] as:

$$c_\lambda^s(\tau) = \frac{1}{T} \int_{|\tau/2|}^{T/2-|\tau/2|} c_\lambda(t, \tau) dt = \frac{1}{4T} \left[\left(\frac{T}{2} - \frac{1}{4\lambda} - |\tau| \right) e^{-2\lambda|\tau|} + \frac{e^{-2\lambda T}}{4\lambda} e^{2\lambda|\tau|} \right] \quad (5.28)$$

with τ is restricted to $|\tau| \leq \frac{T}{2}$. Outside of this region $c_\lambda^s(\tau) = 0$. Since stationary auto-covariance is obtained, Fourier transform is taken by Mahmutoglu and the following

expression for PSD is obtained in [70]:

$$S_{\lambda}(w) = \frac{1}{2T(4\lambda^2 + \omega^2)} \left(2\lambda T - e^{-2\lambda T} - \frac{12\lambda^2 - \omega^2}{4\lambda^2 + \omega^2} + \frac{8\lambda e^{-\lambda T}}{4\lambda^2 + \omega^2} \left[2\lambda \cos\left(\frac{T\omega}{2}\right) - \omega \sin\left(\frac{T\omega}{2}\right) \right] \right) \quad (5.29)$$

In [70], this expression is divided into two part:

$$S_{\lambda}(w) = \frac{S_{\lambda}^L(w)}{2} - S_{\lambda}^+(w) \quad (5.30)$$

in which $S_{\lambda}^L(w)$ is the PSD in the constant bias case which was calculated in eq. 5.25:

$$S_{\lambda}^L(w) = \frac{2\lambda}{4\lambda^2 + (\omega)^2} \quad (5.31)$$

and $S_{\lambda}^+(w)$ is the noise reduction term as the result of transient biasing [70]:

$$S_{\lambda}^+(w) = \frac{1}{2T(4\lambda^2 + \omega^2)} \left(e^{-2\lambda T} - \frac{12\lambda^2 - \omega^2}{4\lambda^2 + \omega^2} + \frac{8\lambda e^{-\lambda T}}{4\lambda^2 + \omega^2} \left[2\lambda \cos\left(\frac{T\omega}{2}\right) - \omega \sin\left(\frac{T\omega}{2}\right) \right] \right) \quad (5.32)$$

In Fig. 5.4, comparison of stationary and switched bias power spectrum densities for a single trap was given. λ was taken as $10^9 s^{-1}$ and T is $10^{-11} s$. Note that blue curve is the power spectral density under constant bias based on eq. 5.25. Black curve that is 3dB below the blue curve is the switched bias case under stationary assumptions. Tian stated that one of the biggest motivation of modeling RTS in switched bias is this wrong result that traditional LF noise models had given [109]. As can be seen from Fig. 5.4, compared to corrected waveform (red curve), noise reduction due to the transient bias is mispredicted under stationary assumptions. In Chapter 6, we will discuss this in detail under the topic of noise reduction techniques. Another difference is about the corner frequencies of RTS. In the corrected PSD, corner frequency is exactly at switching frequency of transient bias, i.e. at $1/T$. Mahmutoglu reminded that this plot is valid for the cases where the switching frequency of transient bias

is much higher than the trap rates [70]. In this case $\lambda T = 10^{-2} < 1$ where in other cases such as $\lambda T \approx 1$ and $\lambda T > 1$ this observation is not valid [70]. However, given analytical expressions are valid for all cases as also stated by Mahmutoglu.

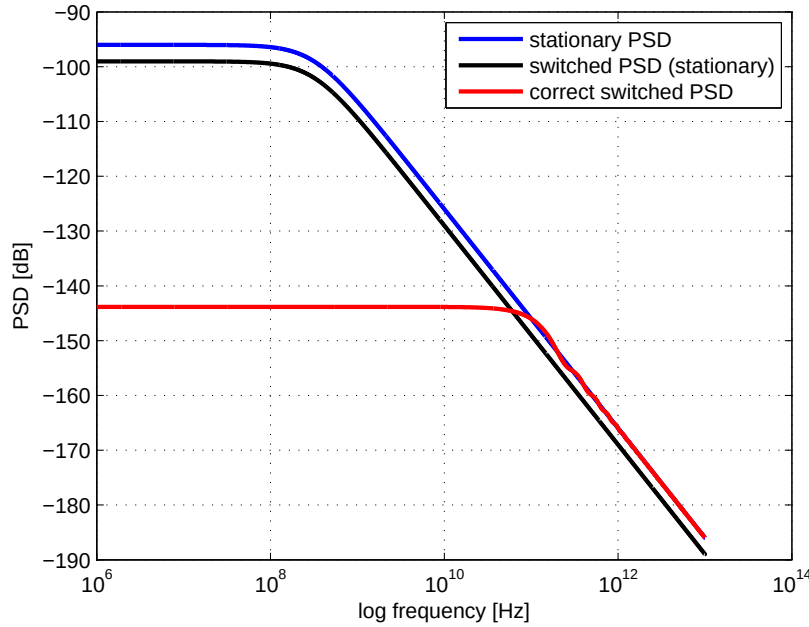


Figure 5.4: Noise spectrum of a single trap: stationary and switched [70]

The whitening effect of switching bias is explained by Mahmutoglu with the disappearance of long time correlations of trap states. After the corner frequency ($\approx 1/T$) traps are correlated with each other in stationary case. However, in the case of switching bias, due to the reset mechanism after each period, correlations disappear for traps that are separated with longer than one period. This effects the spectrum such that PSD will be flat like a white noise spectrum before the corner frequency [70].

Fig. 5.4 shows the noise PSD as a result of single trap activity in both constant and transient bias cases. In order to have a spectrum that include multitude of traps, a distribution function can be used. In Fig. 4 of [70] such a distribution function is

used to obtain a $1/f$ spectrum that distributes λ values in a given range (λ_L, λ_H) :

$$S_{1/f}(\omega) = \int_{\lambda_L}^{\lambda_H} g(\lambda) S_{\lambda}(\omega) d\lambda \quad (5.33)$$

5.3.3 Discussion

Both Tian's and Wel's models were based on idealized traps that switch abruptly between two states. According to Mahmutoglu this idealized model cannot truly capture the real behavior of trap states in physical systems [70].

One problematic aspect of Tian's model is that it assumes trap to be empty in the *off* state. This is discussed in [70] by Mahmutoglu. After their realistic trap models [71] they concluded that in the *off* state trap do not change its state with high probability, i.e. if it is full, it will not release the carrier or if it is empty it will not capture a carrier. He noted that this observation also violates Tian's zero correlation assumption between trap states that are separated more than a period T .

In [70] it is reminded that corner frequency of RTS in switching bias case is not exactly at $1/T$ as idealized trap model indicates. It is showed that realistic trap model assumes a corner frequency at a lower frequency than the switching frequency of transient bias [70]. Figure 4.8 in [126] is a nice plot that gives an idea about this overestimation of idealized trap model. Figure 8 of [70] and Figure 4.8 of [126] can be checked together to see the probable locations of corner frequency in switching bias case. It is said that corner frequency is determined by "emission rate in the *off* state", i.e., λ_{off} [70]. Therefore the corner frequency depends on the circuit variables.

5.4 Advanced Nonstationary Models

Simulations are inseparable parts of electronic circuit design. A novel transistor noise model should be easily implemented to existing compact transistor models in order

to be successful. It should also not overload the computational costs. These demands require noise models to be developed in the context of simulation framework.

In [20], it is noted that LTI system based analysis methods such as standard AC analysis are not suitable for nonlinear circuits that are used extensively in RF circuit design. Different nonstationary noise models were developed for the analysis of these nonlinear circuits and these models triggered the development of specialized simulation techniques as reported in [71]. For example, noise analysis techniques for phase noise in oscillator circuits are developed in [21], for mixer circuits in [50] and effective simulation methods were developed for nonlinear circuits in [108], [20], [90], [80]. Tian's and Kolhatkar's models that we reviewed were the first attempts to nonstationary RTS noise modeling. However, as we discussed in Section 5.3.3 these models rely on simplistic trap definitions which are not realistic [70].

Mahmutoglu and Demir developed two physical nonstationary RTS noise models [71], [69]. First one is Stochastic Automaton Model (SAM) which is based on traps that are modeled using discrete Markov chain random processes and the second one is the Langevin Equation Model (LEM) that uses trap definition based on stochastic differential equations. Mahmutoglu et. al stated that both models are developed by inspiring from simulation and modeling ion channel noise in nerve cells in biology [71]. They noted that traps in transistors and ion channels in nerve cells behave in a similar manner. Both of them have two states that change stochastically in a voltage dependent manner [71]. There are numerous studies about modeling behavior of ion channels including [127], [43], [41], [32]. SAM and LEM models for traps were developed by inspiring from this existing work as stated in [71].

5.4.1 Stochastic Automaton Model (SAM) ⁵

As in Section 5.3.2, $N(t)$ is used for trap occupancy function with two states. $N(t)$ is modeled with a Markov chain. Unlike analysis of Tian in Section 5.3.2, capture and release rates λ_c and λ_e do not have to be equal in SAM model [71]. $P(n, t)$ is defined as the “probability that $N(t)$ is equal to n ” where n is either 1 or 0. It gives the probability of trap state to be n . In order to capture the evolution of $N(t)$, three events that are H_0, H_c, H_e is defined in SAM model as follows [71]:

$$P(H_0) = P(n, t) \quad P(H_c) = P(n - 1, t) \quad P(H_e) = P(n + 1, t) \quad (5.34)$$

$P(A)$ is given as $P(n, t + dt)$, following conditional probabilities are defined in [71]:

$$P(A|H_c) = \lambda_c(t)dt \quad P(A|H_e) = \lambda_e(t)dt \quad (5.35)$$

$$P(A|H_0) = 1 - [(1 - n)\lambda_c(t)dt + n\lambda_e(t)dt] \quad (5.36)$$

dt is assumed to be small enough. In [71] it is stated that $P(A|H_c)$ means “probability of trap to be in state n at time $t + dt$ given that trap is in state $n - 1$ at time t ”. That requires one capture event in time interval dt which is equal to $\lambda_c(t)dt$ [71]. Similarly, $P(A|H_0)$ requires that no event occur in time interval dt . Derivation of $P(A|H_0)$ expression is based on the fact that trap cannot capture electron if it is full (in state 1) and cannot emit an electron if it is empty (in state 0) as stated in [71]. $P(A)$ is calculated using total probability law in the following form [71]:

$$P(A) = P(A|H_0)P(H_0) + P(A|H_c)P(H_c) + P(A|H_e)P(H_e) \quad (5.37)$$

⁵This part is summarized from [71] and [69]

Using definitions given above, Mahmutoglu obtained $P(n, t+dt)$. Then by rearranging the terms and taking limit $dt \rightarrow 0$, *Chapman-Kolmogorov* equation is obtained [71]:

$$\frac{P(n, t)}{dt} = -[(1-n)\lambda_c(t) + n\lambda_e(t)]P(n, t) + \lambda_c(t)P(n-1, t) + \lambda_e(t)P(n+1, t) \quad (5.38)$$

This is the core equation of SAM model. In [71] it is noted that this equation completely describe stochastic nature of the Markov chain model of the traps [71]. Since n takes only two discrete values, eq. 5.38 is evaluated to two equations by Mahmutoglu:

$$\frac{P(0, t)}{dt} = -\lambda_c(t)P(0, t) + \lambda_e(t)P(1, t) \quad (5.39)$$

$$\frac{P(1, t)}{dt} = -\lambda_e(t)P(1, t) + \lambda_c(t)P(0, t) \quad (5.40)$$

5.4.2 Langevin Equation Model (LEM) ⁶

Electronic circuits are modeled using ordinary differential equations, or partial differential equations or finite state machines [20]. Demir stated that if the noise is modeled as a stochastic process, differential equations that mathematically describe the system should also include stochastic terms [20]. Langevin model is developed using eq. 5.38 as the basis of the Fokker-Planck equation that is used in stochastic differential equation (SDE) representation. One of the main differences between SAM and LEM models is that LEM is a continuous state model of traps whereas SAM is a discrete state model as stated in [71]. In LEM formulation, it is assumed that trap can be fractionally occupied that is it may take values other than 0 and 1. In [71] continuous time and continuous space stochastic process version of trap state definition $N(t)$ is defined as N_t . Mahmutoglu showed that N_t can be defined with an Ito stochastic differential equation [69]. After using the own calculus of SDE

⁶This part is summarized from [71] and [69]

representation they obtained the following governing equation for LEM model:

$$dN_t = [(1 - N_t)\lambda_c(t) - N_t\lambda_e(t)]dt + \sqrt{(1 - N_t)\lambda_c(t) + N_t\lambda_e(t)}dW_t \quad (5.41)$$

where W_t corresponds to Wiener process. This core equation of Langevin model captures the time evolution of the trap states as stated in [69]. Mahmutoglu noted that Taylor approximation is used in one of the derivation steps of LEM model from the SAM model. He added that SAM model is the most accurate one [69]. However, it is also noted in [69] that LEM model is adequate for the first and second order probabilistic characterization of trap states (including means and variances). Demir stated that up to second order characterization of a noise process is most of the time adequate for electronic circuit design and can be preferred to full characterization that includes finding all finite-dimensional distributions [20].

So far we have given the core analytical expressions for SAM and LEM models. Definitions of capture and emission rates of traps are also important. Similar to previous RTS models such as Kolhatkar and Tian's, Mahmutoglu defined trap rates using SRH statistics [69]. Bilateral relationship between transistor currents/voltages and the trap parameters were noted to be significant issue [69]. These two group of parameters effect each other. Bias dependency of trap parameters were already shown in Wel and Kolhatkar's model. Mahmutoglu stated that capturing and releasing a carrier lead to a change in flat-band voltage which is the opposite effect [71]:

$$\Delta V_{fb} = \frac{q}{C_{ox}W_{eff}L_{eff}} \quad (5.42)$$

In SAM and LEM models τ_c is defined as $1/(c\bar{v}\bar{\sigma})$ and relationship between τ_c and τ_e is taken as given in eq. 2.13 [71]. Trap definitions were implemented in PSP model for simulation purposes. Parameters that are required for calculating time constants are

taken from PSP model during the simulations. Mapping trap parameters to circuit parameters is not strictly defined in this work. Mahmutoglu noted that it is possible to implement other mapping methodologies to the simulation framework [71].

5.4.3 Discussion

There is a significant methodology difference between developments of SAM and LEM models. In the Stochastic Automaton Model, evolution of trap states are modeled with the Markov chains. Noise is defined in an implicit manner in this methodology [69]. Trapping-detrapping mechanism causes shifts in some of the circuit variables, e.g. drain current, however, this shift in the circuit variables also effects the trap parameters. Mahmutoglu stated that inseparability of noise dynamics and circuit dynamics limits the available solution techniques [69]. Some specialized Monte Carlo techniques were used to perform simulations of SAM model. *Poisson thinning* and *adaptive uniformization* are example of these techniques. However these Monte Carlo based techniques are computationally costly as stated in [71].

Although LEM model can be used with Monte Carlo based simulation techniques, it is not restricted to them due to its stochastic differential equation baseline [69]. Traditional Monte Carlo based techniques were successful especially for nonlinear circuits since they do not need small signal assumptions. However, since they use ensemble averages over lots of lengthy simulations, they are cumbersome in terms of simulation costs. In [107] it is noted that after development of matrix-implicit iterative methods, simulation costs were reduced considerably. These methods were already adapted to popular time and frequency domain simulation techniques such as shooting-newton and harmonic balance [107]. There are also non Monte Carlo techniques that are developed for frequency domain simulations [90] [69] and also for time domain simulations [20] in which noise is treated as an additive component to the circuit. A small noise expansion is used which is based on *perturbation* method

in the analysis of systems that have nonlinear stochastic dynamics in these methods [20]. Demir stated that Modified Nodal Analysis scheme can also be used in these techniques with the stochastic noise terms added [20]. Advantage of LEM model is the significant simulation cost reduction when it is used with non Monte Carlo techniques as shown in [69]. Since it is out of the scope of this thesis noise analysis techniques were not given here. Detailed reviews can be found in [20], [75], [107].

Realistic trap models of SAM and LEM were shown to be superior to idealized trap models of Tian's and Kolhatkar's works in [70]. Reappearance of $1/f$ noise in the power spectrum was seen in various measurements [114], [8]. At low frequencies power spectral density bends up in which excess $1/f$ noise appears at very low frequencies. Mahmutoglu stated that this behavior cannot be modeled with idealistic trap model. However, Mahmutoglu's realistic trap model can capture this effect as shown in Fig. 10 in [70]. In [70] it is also shown that unlike the other models, realistic trap model can correctly capture the noise reduction behavior at all frequencies in the switching bias case (see Figure 9 in [70]).

5.5 Conclusion

In this chapter we have reviewed nonstationary RTS noise models that can handle time varying bias conditions in nonlinear circuits. We have started with a review of a simplistic statistical model that is a nice transition model between LF noise models and RTS noise models. This model also clarifies the need of nonstationary models that truly capture the kinetics of traps. In the following section we have given well-known stationary model that is the basis of basic nonstationary models.

Abruptly switching bias with idealized trap definitions is the most simple case for studying time-varying bias effects on RTS noise. In Section 5.3, this case is studied with two major works. Kolhatkar and Wel's model provide valuable measurement

data and also good information on bias dependency of trap time constants [65], [126]. Although Tian's model [109] includes oversights, it provides a good starting point for nonstationary RTS noise modeling. This model was revisited and clarified by Mahmutoglu and Demir [70]. Corrected model is reviewed in this section.

In the last section, two models for nonstationary RTS noise were reviewed. First model is Stochastic Automaton Model that is based on discrete space-continuous time trap definitions whereas second model is Langevin Equation Model that is based on continuous space-continuous time trap definitions. These models were based on realistic trap models instead of idealized trap models that Kolhatkar and Tian's original works rely on. SAM model truly captures trap behaviors under varying bias conditions with its realistic trap model whereas LEM model is able to capture the first and second order probabilistic characteristics of RTS noise. LEM model have rigorous mathematical foundations that enables non Monte Carlo techniques which results significant computational cost savings [69]. One of the most important properties of SAM and LEM models is that they can be easily incorporated to device models such as PSP and also to circuit simulators [69]. Enabling implementation to novel noise analysis algorithms in circuit simulators is an important property for a compact model as we discussed in the beginning of Chapter 4.

Chapter 6

LOW FREQUENCY NOISE REDUCTION TECHNIQUES

Low frequency noise models were reviewed in Chapter 3, 4 and 5. In this chapter, most prominent noise reduction techniques for LF noise will be discussed from a circuit designer point of view. These techniques are correlated double sampling (CDS), chopper stabilization, forward substrate bias (FSB) and large signal excitation (LSE). Although there may be other ways to deal with LF noise, these are the most popular ones that are well understood and also validated with measurements. One common nice property of these techniques is that a circuit designer can benefit from them in design phase of the circuit. Parameters that are adjustable for the circuit designer are very limited. These are restricted to transistor dimensions W and L and bias voltages. Two of the techniques that we are listed above based on using specialized biasing schemes. For example, in LSE switching bias is used for transistors instead of a constant bias. In FSB technique, substrate is biased independently instead of short-circuiting to the source node. CDS and chopper stabilization did not based on biasing manipulations. CDS is based on sampling theory whereas chopper stabilization is based on the modulation theory.

Schmid noted that there are three significant properties of LF noise in which each of these properties had been used by researchers to develop a proper noise reduction technique [96]. He listed them as follows:

- i. Keshner indicated that unlike white noise, LF noise has memory that the present values of process are not independent from the past values of the process [59].

This means that auto-covariance functions of the LF noise process cannot reach

the steady state in a short time and therefore long time correlations exists. According to Schmid, this rather interesting property of LF noise can be useful for noise reduction. For example, if one can reset the noise process, long time memory contributions to LF noise can be suppressed. Large signal excitation method successfully uses this property of LF noise.

- ii. Since LF noise process inherently includes strong correlation between samples that are not too far apart, it is possible to eliminate noise by subtracting strongly correlated noise samples from each other and therefore minimizing the noise effect on the original signal [96]. This technique is similar to the offset cancellation technique that is used in operational amplifier circuits. CDS is based on this technique and it is used for both offset and noise cancellation.
- iii. LF noise dominates in the lower frequency band of the spectrum. One important technique which is called as *chopping* used for upconverting the signal to higher frequencies in order to separate it from the LF noise. In the next phase, signal is down-converted to its original frequency without effecting from LF noise. This modulation of signal is said to be useful if bandwidth of the signal is separated enough from the chopping frequency [96].

Forward substrate biasing is another way of reducing LF noise. It is reported that forward biasing body-to-source junction, i.e. connecting source node to a positive voltage improves noise performance of the MOS transistors [82], [19], [133], [110]. In this chapter we will review these four techniques in three sections. CDS and chopper stabilization will be discussed under the same topic.

6.1 Large Signal Excitation (LSE)

LSE technique was already discussed in Section 5.3. Applying a switching bias to the gate terminals of transistors were shown to be effective in terms of RTS noise performance of devices. In this section we will focus one of the most important benefits of this technique, namely, *intrinsic* noise reduction. The term *intrinsic* is important here since LSE provides a noise reduction that is not about the effect of noise on circuit performance but it is about the reduction of noise itself as stated in [63]. In CDS and chopper stabilization techniques, only the effect of noise is reduced.

First observers of the noise reduction due to the switching bias is Bloom and Nemirovsky. They showed that cycling between inversion and accumulation operation regions reduces the LF noise of transistors [11]. They keep the on voltage $V_{gs,on}$ constant such that transistor will be in strong inversion and of voltage $V_{gs,off}$ as variable. They concluded that towards the accumulation region noise reduction will increase and after some point it stays constant (see Fig. 3 in [11]). They have observed a reduction by a factor of 3 for single NMOS transistors at 1 kHz (see Fig. 2 in [11]). Later, LSE effect was related to RTS noise by another group in which they noticed the change in RTS time constants during the LSE [22].

First detailed analysis of LSE effect on circuits was performed by Klumperink [63]. They have performed phase noise analysis of a three stage ring oscillator using switched biased NMOS and PMOS transistors [38]. They have also performed phase noise analysis of a six stage coupled sawtooth oscillator [37]. They have obtained approximately 8 dB noise reduction in $1/f^3$ phase noise of these oscillators [37], [63].

Physical mechanism that is behind noise reduction by LSE is explained in detail in Section 5.3. Main idea is the reset mechanism in each switching period that eliminates long term correlations of RTS noise process [70]. In addition to this noise reduction, LSE also provides reduction in power consumption [63]. Klumperink noted that in

LSE there should be additional switches for bias network as can be seen in Fig. 5.2. These switches consume some power and also may cause nonidealities, e.g. glitches to the circuit due to the charge injection. However, during the *off* state, power consumption is reduced considerably. Moreover, overall noise is also reduced by an inherent mechanism during cycling from accumulation and strong inversion regions [63]. As long as net noise reduction is good enough as compared to the effect of other nonidealities, LSE seems to be preferable in circuit design.

6.2 Forward Substrate Bias (FSB)

MOS transistor is a four terminal device as given in Fig. 1.6. Most of the time, bulk terminal which represents the substrate of the transistor is connected to the highest potential of the circuit for PMOS devices and to the lowest potential of the circuit for NMOS devices. In [19] it is reported that forward and reverse biasing are also used in certain circuit applications in which reverse biasing for low current applications and forward biasing for low voltage applications were given as examples. In several studies, it has been observed that forward biasing substrate can reduce LF noise [82], [19], [133], [110]. In this section we will review effects of FSB on LF noise.

In [82] LF noise performances of both NMOS and PMOS transistors under FSB were investigated. They used 0.25μ transistors. $|V_{ds}|$ was kept constant as 1V, overdrive voltage V_{gt} was varied between -0.2V and 0.3 and body to source voltage V_{bs} between -2.5V and 0.5V. Therefore transistors operated either in subthreshold or in saturation regions. Since V_t is effected from the change in V_{bs} , V_{gs} was chosen accordingly for keeping drain current constant under varying V_{bs} [82]. Noise current spectral density S_{Id} was measured at 1 kHz. In subthreshold region, S_{Id} is reduced approximately 10 dB for NMOS devices in forward bias case ($V_{bs} = 0.5$) with respect to the normal case where $V_{bs} = 0$. In this study reverse biasing was also applied with

varying V_{bs} between -2.5V and 0V. In this range no change in noise was seen. Similar observation was also seen for PMOS devices. When devices are in saturation no noise reduction was observed even in FSB [82]. Park has offered two possible explanation of noise reduction due to the FSB. One explanation of Park is the decrease in depletion capacitance in subthreshold region under FSB. Other reasoning is the increased average distance between the carriers in the channel and Si – SiO₂ interface [82].

In [72], measurements were performed with NMOS and PMOS devices with device lengths $l = 0.2\mu$, $l = 0.8\mu$, $l = 10\mu$. Results were consistent with the ones that Park has obtained. Measurements showed a considerable decrease in noise under forward bias but noise is less effected in reverse bias (see Fig. 10 and Fig. 11 in [72]). They stated that noise is reduced approximately %50 in both NMOS and PMOS devices in weak inversion. They also noted that changing V_{bs} did not effect noise performance of devices in strong inversion conditions. They have explained results with McWorther's number fluctuation theory using following relationship between inversion charge fluctuation δQ_{inv} and trapped oxide charge fluctuation δQ_{ox} [72]:

$$\delta Q_{inv} = \frac{-C_{inv}}{C_{ox} + C_d + C_{it} - C_{inv}} \delta Q_{ox} \quad (6.1)$$

Marin stated that in the weak inversion $C_{ox} + C_d + C_{it} > C_{inv}$ holds and therefore δQ_{inv} and δQ_{ox} relationship become bias dependent due to the substrate bias dependencies of capacitances in term $C_{ox} + C_d + C_{it}$, however, in strong inversion since C_{inv} dominates the denominator of eq. 6.1, δQ_{inv} become fully correlated with δQ_{ox} , i.e. $\delta Q_{inv} \approx \delta Q_{ox}$ in which V_{bs} dependency vanishes [72].

Effects of forward substrate bias on circuit performances was studied in [82] using substrate biased transistors in simple differential amplifiers and in [110] using a VCO circuit. Differential amplifier circuits are given in Fig. 6.1. PMOS version is given in Fig. 6.1.a that has PMOS transistors in input terminal. NMOS version is shown in

	PMOS-SUB	PMOS-STR	NMOS-SUB	NMOS-STR
$V_{bs} = 0$	+9.6dB	+12.5dB	+21.7dB	+26.5dB
$V_{bs} = 0.5$	REF	+12.6dB	+12.2dB	+26.5dB

Table 6.1: S_{V_g} @100Hz normalized to version PMOS-SUB [82]

Fig. 6.1.b. Details about transistor sizes can be found in [82]. Both amplifier versions (PMOS and NMOS) were operated in weak and strong inversions and also forward biased. V_{bs} was varied from -2V to 0.5V for NMOS version and -0.5V to 2V for PMOS version while supply voltage is set to 2.5V. Gate to source voltages of M5, M1 and M2 were adjusted so as to keep tail current $40\mu A$. Four variations of amplifiers were defined. PMOS-SUB is the amplifier that is the PMOS version with subthreshold operation region whereas PMOS-STR is in strong inversion. The others are NMOS-SUB and NMOS-STR which are NMOS versions. Input referred noise power spectral densities (S_{V_g}) of these amplifiers were measured. S_{V_g} values at 100 Hz were given in Table 6.1. As can be seen in Table 6.1, in weak inversion FSB is very effective.

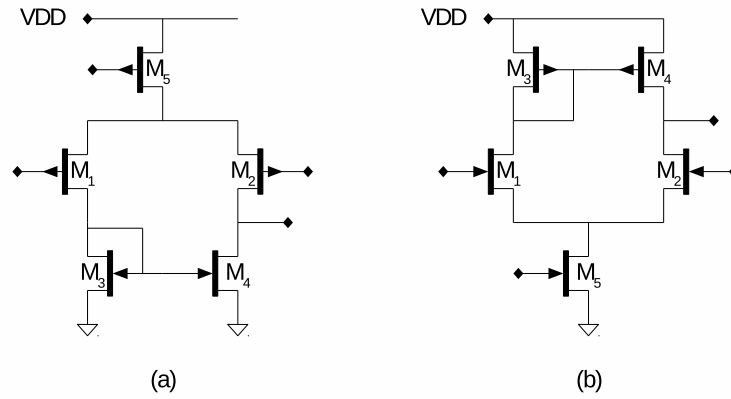


Figure 6.1: Differential amplifier circuits (a) PMOS version (b) NMOS version [82]

PMOS-SUB version had 9.6dB noise reduction when subjected to forward bias and

NMOS-SUB version had 9.5dB noise reduction. However, devices were not effected much from FSB in strong inversion [82]. PMOS-STR and NMOS-STR versions had almost same noise levels in $V_{bs} = 0$ and $V_{bs} = 0.5$ values. Complete PSD plots can be seen in Fig. 6.8 in [82] and S_{Vg} vs V_{bs} graph in Fig. 6.9 in [82]. They reported that apart from LF noise reduction, there are also some other effects of forward biasing on differential amplifiers. For example, they stated that there is a trade-off between gain-bandwidth product (GBW) and LF noise performance. In PMOS-SUB case, GBW is less than approximately 22 dB than PMOS-STR version (see Table 6.2 in [82]). They pointed capacitive loading of differential amplifier, in which GBW can be improved with a capacitive load in PMOS-SUB version. Huge GBW difference in weak inversion with respect to strong inversion can be reduced to acceptable limits by loading [82] (see Fig. 6.10 in [82] for S_{Vg} in loaded cases).

One nice study of forward bias is given in [110] in which effects of FSB and LSE techniques were studied together. They performed measurements in six different cases for 0.1μ PMOS transistors. After single transistor measurements they also used their biasing schemes in a VCO design for phase noise measurements. In Fig. 6.2, different biasing schemes were given [110]. In that figure, CB corresponds to constant gate bias whereas SB corresponds to switching bias. CZSB and CFSB are two cases for substrate biasing in which CZSB stands for “constant zero substrate bias” and CFSB for “constant forward substrate bias” [110]. Fig. 6.2e and Fig. 6.2f are the last two cases for substrate biasing in which it is biased as in-phase with the switching gate bias in former one and as out of phase with the switching gate bias in the latter one, respectively. In this work, measurements were performed with PMOS devices operating in strong inversion. First four measurements of Fig. 6.2 was performed and PSD of normalized noise current was plotted (see Fig. 5 in [110]). They showed that under constant bias (CB), FSB has negligible effect on noise performance. However, forward biasing provides considerable noise reduction in switching bias (SB) case. They

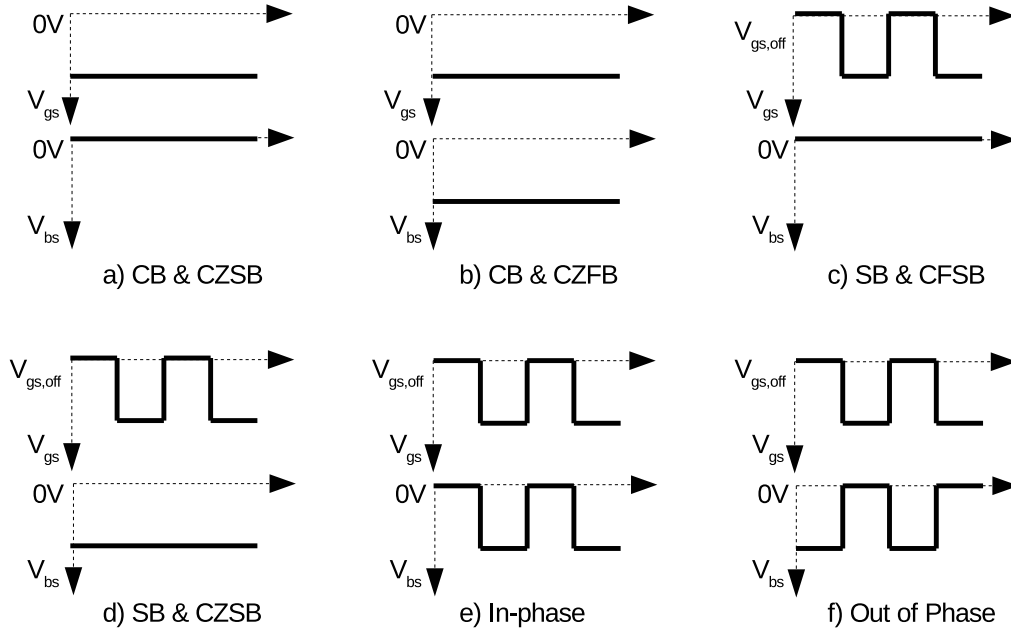


Figure 6.2: Biasing schemes as given in [110]

also showed that reverse biasing did not effect noise performance in neither CB nor SB conditions (see Fig. 6 in [110]). Tiebout's measurement results in this study are consistent with the measurements in [82] in which Park had shown that FSB is not effective in saturation operation region. However, Park only did measurements under CB. Tiebout in [110] showed that by applying SB and FSB together noise can be reduced considerably. In order to understand the underlying mechanism of this noise reduction they used in-phase and out of phase switching to gate bias as given in Fig. 6.2e and Fig. 6.2f. They noticed that noise reduction is seen only in out of phase case, i.e. when FSB is applied in the *off* state of the transistor (see Fig. 7 in [110]).

Tiebout concluded that noise reduction is the result of inducing accumulation to the oxide interface in the *off* state [110]. They have shown that forcing transistor to the deep accumulation region by adjusting $V_{gs,off}$, amount of noise reduction that is obtained by applying both FSB and LSE can also be obtained by applying only

LSE with forcing device to deep accumulation in *off* state (see Fig. 9 in [110]). In conclusion they showed that noise reduction can be obtained either using LSE (switching bias) and forward biasing techniques together or using only LSE but with adjusting $V_{gs,off}$ such that device is in deep accumulation region in the *off* state.

6.3 Correlated Double Sampling (CDS) and Chopper Stabilization ¹

Chopper stabilization and CDS are two well known techniques that are used by analog design engineers since 1980s. These techniques are mainly used for offset cancellation and noise reduction in circuits. In [27] it is reported that these techniques are used in various analog circuits including data converter stages, integrators and also comparators. Enz noted that CDS technique is based on sampling theory whereas chopper stabilization technique is based on the modulation theory [27]. In this section we will focus only on LF noise reduction effect of these techniques.

According to Schmid, fundamental mechanism of CDS technique is taking a noise sample at a time without the fundamental signal and taking another sample at a different time with the fundamental signal and then subtracting these two samples to obtain a noise free signal [96]. Enz stated that this subtraction can be performed at input or output stages of the system, or even in an intermediate stage [27]. According to Enz, critical point here is that two noise samples should be correlated to each other for cancellation. Therefore since LF noise is a correlated process it can be reduced by this technique whereas broadband noise cannot be eliminated. Moreover broadband noise is even increased with CDS technique since its uncorrelated nature [27]. Another critical issue with CDS technique is reported to be the aliasing problem that comes with the sampling process [27]. In Fig. 6.3, a simple application of CDS technique on a comparator circuit is given [96]. This circuit operates with two phases. In ϕ_1 phase

¹This part is mostly summarized from [96] and [27]

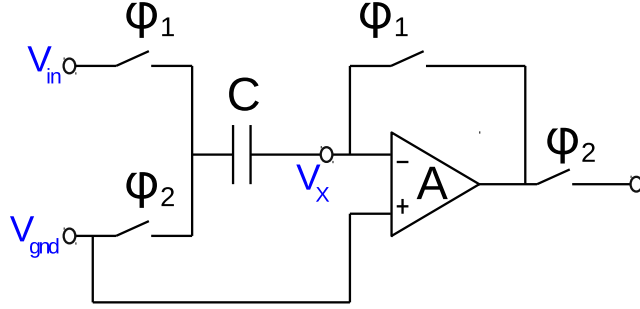


Figure 6.3: Comparator structure for CDS [96]

capacitor C is charged to voltage $V_{in} - V_{os1}$ where V_{os1} is the offset voltage of the amplifier. In ϕ_2 phase, input is grounded and only new offset voltage V_{os2} remained in node X . In this phase the initial value of capacitor and V_{os2} is compared by the comparator as given in [96]:

$$V_{os2} - (V_{os1} - V_{in}) \quad (6.2)$$

This comparison step includes subtraction of two offset voltages. During this subtraction noise components are also subtracted if they are correlated [96]. According to Schmid, charge injections due to the switches are the main issue in CDS operation especially for the offset cancellation [96].

Detailed mathematical analysis of CDS technique is given in [27]. Enz stated that CDS is based on auto-zeroing principle and auto-zeroing process basically provides a high pass filtering to the circuit [27]. However, he also stated that although the baseband noise is suppressed by auto-zeroing technique, due to the sampling process a foldover component imposed at the output. Enz showed that output noise PSD of a circuit that auto-zeroing is applied can be divided into two components:

$$S_{AZ}(f) = |H_0(f)|^2 S_N(f) + S_{fold}(f) \quad (6.3)$$

where $|H_0(f)|^2 S_N(f)$ is the baseband term which is reduced by auto-zeroing and $S_{fold}(f)$ is the foldover term due to the aliasing. Enz showed that baseband term is reduced by auto-zeroing both for broadband and LF noise. However, even foldover component resulting from the aliasing of LF noise is not critical so much, foldover component resulting from the aliasing of broadband noise dominates the total output noise in practical circuits as Enz stated in [27] (see Fig. 5, Fig. 6 and Fig. 7 in [27]).

In [67], CDS effect on RTS noise is studied. Experiments showed that LF noise is reduced considerably in CMOS imager circuits by applying CDS technique [67]. However, they noted that amount of noise reduction strongly depends on trap parameters such as corner frequencies of traps. They obtained different noise reductions for transistors from different processes (see Fig. 8 in [67]).

Chopper stabilization technique is illustrated in Fig. 6.4. Main idea in this technique is giving signal to the amplifier with different polarities at different phases [96]. $m(t)$ in Fig. 6.4, is the modulation signal which is a square wave that takes values -1 and +1 consecutively. Schmid stated that the signal is modulated to a chopping frequency before entering amplifier (so offset and noise is not added yet) and then at the output of amplifier it is demodulated back to the original band while offset and noise is modulated to the chopping frequency at the same time [96]. Therefore the output after chopping processes became offset free and noise free. Enz stated

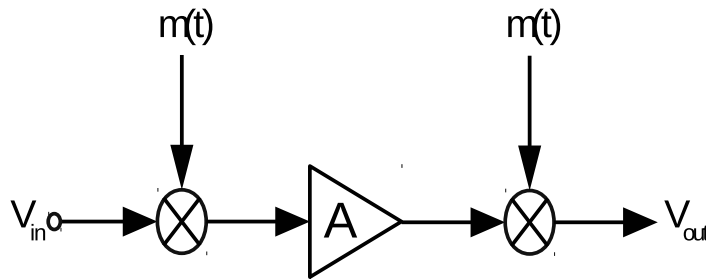


Figure 6.4: Chopper stabilization technique [96]

that chopping upconverts the signal to the odd harmonics of the chopping frequency. However, due to the finite bandwidth of the amplifier, some unwanted spurs emerge at even harmonics of chopper frequency [27]. He stated that these spectral components should be filtered out at the output. Another issue that Enz noted is the phase shift that comes from the delay in the amplification phase [27]. This delay is said to be critical since it may cause DC gain degradation down to zero. Enz reminded that phase shift between modulators should be adjusted according to the phase shift that amplifier introduce [27].

A measurement result was shown in [27] (Fig. 15) in which an amplifier is subjected to chopper stabilization. Power spectrum is plotted before and after the technique is applied. It is shown that low frequency noise dominated part of the spectrum is flattened after chopper stabilization technique [27].

6.4 Conclusion

In this section we have reviewed the most popular LF noise reduction techniques. Basic theories of these techniques were explained briefly and application examples of these techniques were given from existing literature.

Forward biasing technique can be applied to both continuous-time and sampled data circuits. LSE and CDS techniques are more suitable for sampled data circuits and chopper stabilization technique is more appropriate for continuous time circuits. Enz compared CDS and chopper stabilization techniques and stated that CDS suppresses LF noise by introducing high pass filtering whereas chopper stabilization reduces LF noise by upconverting it to a different frequency band [27]. He also added that unlike CDS, chopper technique did not introduce a foldover component at the output due to an aliasing since sampling is not used [27].

Some of the techniques that were introduced in this section can be applied together

at same circuits. In Section 6.2, application of forward substrate bias together with LSE technique is discussed with the help of Tiebout's work [110]. One nice study is [115] in which LSE and CDS techniques are applied together to a CMOS imager circuit. Wel et al. showed that CDS should be applied carefully along with LSE since LF noise may become even worse otherwise [115]. They stated that two samples that were taken during CDS should have same *bias histories* in order to have a benign effect on LF noise performance when applied along with LSE [125].

Low frequency noise reduction techniques that were given in that chapter were empirically evaluated techniques. Apart from LSE, they do not provide a solid physical explanation of the low frequency noise reduction especially for time-varying conditions. Note that both Enz [27] and Tiebout [110] treated noise as a stationary process in their analyses. However, as we know the works from Tian and Mahmutoglu [109] [70] things get complicated even in a very simple circuit in time-varying bias conditions. For example, in [109] and [70] it is shown that even in a single transistor common source amplifier circuitry, noise analysis that is based on stationary assumptions gives wrong PSD expression. CDS and chopper stabilization techniques are applied to very complex circuits that include lots of transistors. LF noise analysis of these circuits in time-varying conditions is plausible only in a carefully treated simulations with accurate physical device models. Nonstationary noise models that were given in Chapter 5 such as SAM and LEM models may be used in LF noise analysis of these complicated circuits since they were developed considering novel noise analysis algorithms for effective noise simulations. Work of Tiebout in which effects of LSE and FSB techniques are applied together to MOS transistors can also be revisited in order to study time-varying bias effects by using non-stationary models that were developed in Chapter 5. It may provide better understanding of noise reduction in these circuits.

Appendix 1

SIMPLE RTS NOISE SIMULATOR IN MATLAB[®]

Simple script for generating RTS noise waveforms in both stationary and non-stationary cases is given in this appendix section. This simple simulator is written in MATLAB[®] software. This script can generate RTS noise waveforms and estimates power spectral densities for both constant bias and switching bias cases.

This code indeed performs simulations for the setup that was given in Tian's and Mahmutoglu's works which were reviewed in Sec. 5.3.2 in detail. Thus we have used the idealized trap model that their studies were based on. First we have generated RTS waveform using *exprnd* and *cumsum* functions of MATLAB[®]. After generating switching times of RTS signal, we have generated RTS waveform by switching the signal at these switching time points. In this code we have benefited the method that is used in Example 17.6.1 of [23]. Switching rates of emission and capture processes are taken to be equal and they were represented with parameter λ (taken as 10^8 Hz). RTS is assumed to be abruptly switching between *on* and *off* states and takes the values 0 or 1. An example RTS noise waveform that was generated with this script is given in Fig. A.1. This RTS noise corresponds to the stationary case where the bias voltages of transistor do not change over time.

For the non-stationary case switching bias is assumed (see Sec. 5.3.2 for details). In that case, noise process is reset at the beginning of each *off* times and started in the beginning of each *on* times. Main reason of this resetting mechanism is that emission rates became to high and the capture rates became very low in the *off* times. In this code we set RTS signal to 0 during *off* times. T is taken as 10^{-10} Hz

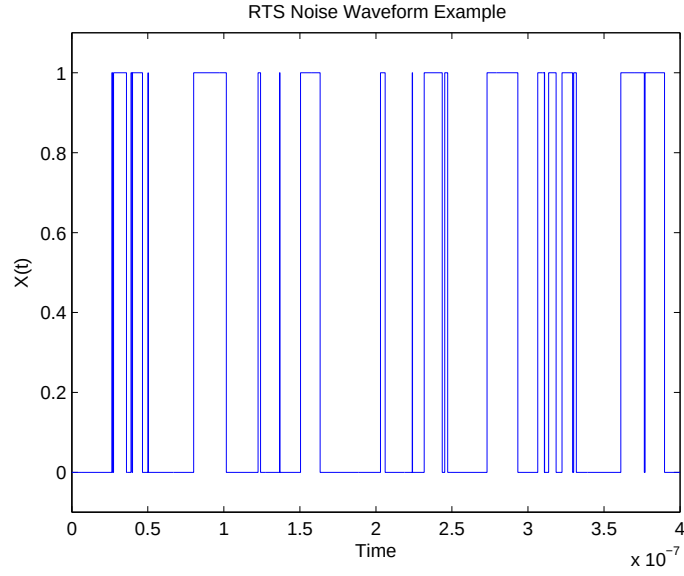


Figure A.1: RTS noise waveform example (stationary case)

in order to satisfy the condition that $\lambda T \ll 1$ as Mahmutoglu stated in [70]. RTS noise waveform example for this non-stationary case is given in Fig. A.2

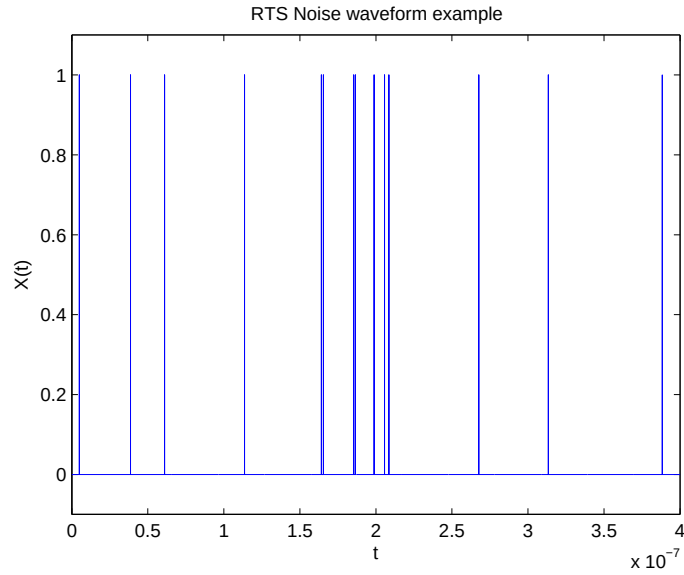


Figure A.2: RTS noise waveform example (non-stationary case)

Spectrum estimation is performed with *pwelch* function of MATLAB®. Simulations were performed in a Monte-Carlo manner in which lots of RTS waveforms were generated for the two cases and averaging was performed over PSDs. PSDs of RTS noise in both stationary and non-stationary cases were given in Fig. A.3. As can be seen, corner frequency is shifted to $1/T$ from λ/π . Noise reduction and whitening effect of noise can also be seen in switching bias case. Analytical results of Mahmutoglu was already given in Fig. 5.4. Simulation results here are consistent with the results of Mahmutoglu. Simulation scripts are given at the end of this section.

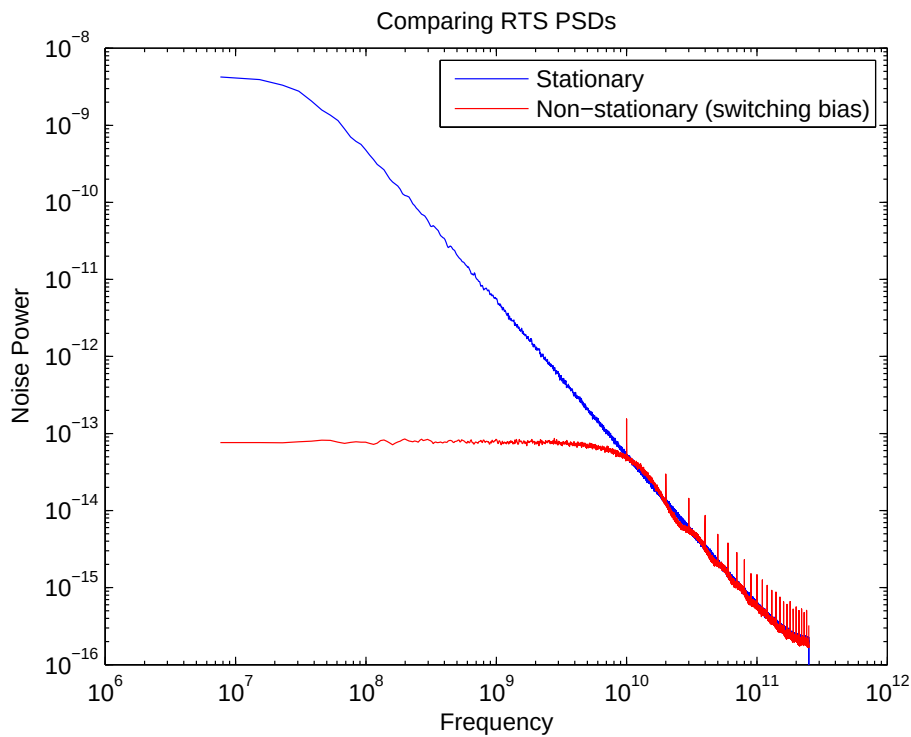


Figure A.3: RTS noise waveform example

Code-1 (stationary case):

```
%clear all, clc, close all
```

```
% Constant bias case
% Stationary RTS noise analysis

hold off

% number of switches in realization (approximation)
N = 500;

% switching rate (emission and capture rates are equal)
lambda = 1e8;

X = [];

% number of generated RTS waveforms:
nRTS = 100;

% generate exponentially distributed random variables:
E = exprnd(1/lambda, [nRTS, N]);

for u = 1:nRTS
    % compute switching times:
    V(u, :) = cumsum(E(u, :));

    % define time array:
    tmax = 4e-7;
    tres = 2e-12;
    t = [0:tres:tmax];

    % initialize X(t):
    cnt = 1; amp = 0; sgn = -1;

    % generating RTS, X(t):
    for k = 1:length(t)
        if (t(k) >= V(u, cnt))
            cnt = cnt + 1;
```

```

        sgn = -sgn; % switch sign
        amp = amp + sgn;

    end

    X(u,k) = amp;

end

% subtract mean:
X_zMean(u,:)=(X(u,:))-mean(X(u,:))*ones(size(X(u,:)));

% calculate individual PSDs:
[Px,Fx] = pwelch(X_zMean(u,:),[],[],[],1/tres);

% store PSDs:
P2(u,:)=Px;
F2(u,:)=Fx;

end

% average over PSDs
P_sta=sum(P2,1)/(nRTS);
F_sta=sum(F2,1)/nRTS;

figure(1)
%plot an example RTS waveform
plot(t,X(1,:))
title('RTS Noise Waveform Example');
xlabel('Time');
ylabel('X(t)');
axis([0 tmax -0.1 1.1])

figure(2)
loglog(F_sta,P_sta,'b-')
```

Code-2 (non-stationary case):

```
clear all, clc, close all

% Two stated switching bias case
% non-stationary RTS noise analysis

% number of switches (approximation)
N = 500;

% switching rates in the "on" state (lambda=lambda_e=lambda_c)
lambda = 1e8;

% period of switching bias
T = 1e-10;

X = [];

% number of generated RTS waveforms
nRTS = 100;

for u = 1:nRTS

    %define time array:
    tmax = 4e-7;
    tres = 2e-12;
    t = [0:tres:tmax];
    tsize = length(t);    % size of total time array
    tratio = T/tres;      % how many points in one period
    Tnum = ceil(tmax/T);  % number of switching periods

    %finding "on" times:
    tp-on = linspace(1,tsize,Tnum+1);
```

```

%off times:
tp_off = floor((tratio/2))+linspace(1,tsize,Tnum+1);

% separate "on times" and "off times"

for k = 1:Tnum %on times

    amp = 0; sgn = -1; %initialize X(t)
    cnt_on = 1;
    % generate N uniformly distributed random variables:
    E_on = exprnd(1/lambda,[nRTS,N]);
    % compute switching times:
    V_on(u,:) = cumsum(E_on(u,:)) + t(tp_on(k));

    for j = tp_on(k):1:tp_off(k)

        if (t(j) >= V_on(u,cnt_on))
            cnt_on = cnt_on + 1;
            sgn = -sgn; %switch sign
            amp = amp + sgn;
        end
        X(u,j) = amp;
    end
end

for k = 1:Tnum %off times
    for j = tp_off(k):1:tp_on(k+1)
        X(u,j) = 0;
    end
end

% subtract mean:
X_zm(u,:)=(X(u,:))-mean(X(u,:))*ones(size(X(u,:)));

% calculate individual PSDs:

```

```
[Px,Fx] = pwelch(X_zm(u,:),[],[],[],1/tres);  
  
% store PSDs:  
  
P(u,:)=Px;  
F(u,:)=Fx;  
  
end  
  
% average over PSDs  
  
P=sum(P,1)/nRTS;  
F=sum(F,1)/nRTS;  
  
figure(1)  
  
%plot an example RTS waveform  
plot(t,X(1,:))  
title('RTS Noise waveform example');  
xlabel('t');  
ylabel('X(t)');  
axis([0 tmax -0.1 1.1])  
  
figure(2)  
loglog(F,P,'r-')
```

BIBLIOGRAPHY

- [1] Online hspice manual. http://www.ece.uci.edu/docs/hspice/hspice_2001_2-168.html. Accessed: 2015-05-02.
- [2] Operational amplifier noise prediction. Technical report, Intersil, 1996.
- [3] Karthik Aadithya, Sriramkumar Venogopalan, Alper Demir, and Jaijeet Roychowdhury. Mustard: A coupled, stochastic/deterministic, discrete/continuous technique for predicting the impact of random telegraph noise on srams and drams. In *Proceedings of the 48th Design Automation Conference*, pages 292–297. ACM, 2011.
- [4] Karthik V Aadithya, Alper Demir, Sriramkumar Venugopalan, and Jaijeet Roychowdhury. Samurai: An accurate method for modelling and simulating non-stationary random telegraph noise in srams. In *Design, Automation & Test in Europe Conference & Exhibition (DATE), 2011*, pages 1–6. IEEE, 2011.
- [5] Gerald Abowitz, Emil Arnold, and EA Leventhal. Surface states and 1/f noise in mos transistors. *Electron Devices, IEEE Transactions on*, 14(11):775–777, 1967.
- [6] Harshit Agarwal, Sarad Venugopalan, Maria-Anna Chalkiadaki, Navid Paydavosi, Juan Pablo Duarte, Shantanu Agnihotri, Chandresh Yadav, Pragya Kushwaha, Yogesh Singh Chauhan, CC Enz, et al. Recent enhancements in bsim6 bulk mosfet model. In *Simulation of Semiconductor Processes and Devices (SISPAD), 2013 International Conference on*, pages 53–56. IEEE, 2013.

- [7] Duarte J. P. Chauhan Y.S. Niknejad A. M. Hu C. Agarwal H., Khandelwal S. *BSIM6.1.0 MOSFET Compact Model - Technical Manual*. University of California, Berkeley, 2014.
- [8] A. Arnaud and M.R. Miguez. Very low frequency cyclostationary $1/f$ noise in mos transistors. In *Noise and Fluctuations (ICNF), 2013 22nd International Conference on*, pages 1–4, June 2013.
- [9] Robert G Arns. The other transistor: early history of the metal-oxide semiconductor field-effect transistor. *Engineering Science & Education Journal*, 7(5):233–240, 1998.
- [10] Razavi Behzad. Design of analog cmos integrated circuits. *International Edition*, 2001.
- [11] I Bloom and Y Nemirovsky. $1/f$ noise reduction of metal-oxide-semiconductor transistors by cycling from inversion to accumulation. *Applied physics letters*, 58(15):1664–1666, 1991.
- [12] Zeynep Celik and TY Hsiang. Study of $1/f$ noise in n-mosfet's: linear region. *Electron Devices, IEEE Transactions on*, 32(12):2797–2802, 1985.
- [13] Z Celik-Butler and TY Hsiang. Spectral dependence of $1/f$ noise on gate bias in n -mosfets. *Solid-state electronics*, 30(4):419–423, 1987.
- [14] Jimmin Chang, AA Abidi, and CR Viswanathan. Flicker noise in cmos transistors from subthreshold to strong inversion at various temperatures. *Electron Devices, IEEE Transactions on*, 41(11):1965–1971, 1994.
- [15] Yogesh Singh Chauhan, Sarad Venugopalan, Maria-Anna Chalkiadaki, Muhammed Ahosan Ul Karim, Harshit Agarwal, Sourabh Khandelwal, Navid

- Paydavosi, Juan Pablo Duarte, Christian C Enz, Ali M Niknejad, et al. Bsim6: Analog and rf compact model for bulk mosfet. *Electron Devices, IEEE Transactions on*, 61(2):234–244, 2014.
- [16] S Christensson, I Lundström, and C Svensson. Low frequency noise in mos transistors theory. *Solid-State Electronics*, 11(9):797–812, 1968.
- [17] Bruce E Deal. Standardized terminology for oxide charges associated with thermally oxidized silicon. *Journal of the Electrochemical Society*, 127(4):979–981, 1980.
- [18] M Jamal Deen, John Ilowski, and Ping Yang. Low frequency noise in polysilicon-emitter bipolar junction transistors. *Journal of applied physics*, 77(12):6278–6288, 1995.
- [19] M Jamal Deen and Ognian Marinov. Effect of forward and reverse substrate biasing on low-frequency noise in silicon pmosfets. *Electron Devices, IEEE Transactions on*, 49(3):409–413, 2002.
- [20] A. Demir and A. Sangiovanni-Vincentelli. *Analysis and simulation of noise in nonlinear electronic circuits and systems*. The Springer International Series in Engineering and Computer Science Series. Kluwer Academic Publishers, 1998.
- [21] Alper Demir, Amit Mehrotra, and Jaijeet Roychowdhury. Phase noise in oscillators: a unifying theory and numerical methods for characterization. *Circuits and Systems I: Fundamental Theory and Applications, IEEE Transactions on*, 47(5):655–674, 2000.
- [22] B Dierickx and E Simoen. The decrease of random telegraph signalnoise in

- metal-oxide-semiconductor field-effect transistors when cycled from inversion to accumulation. *Journal of Applied Physics*, 71(4):2028–2029, 1992.
- [23] Dean G Duffy. *Advanced engineering mathematics with MATLAB*. CRC Press, 2010.
- [24] Albert Einstein. Eine neue bestimmung der moleküldimensionen. *Annalen der Physik*, 324(2):289–306, 1906.
- [25] Christian C Enz. A short story of the ekv mos transistor model. *Solid-State Circuits Society Newsletter, IEEE*, 13(3):24–30, 2008.
- [26] Christian C Enz, François Krummenacher, and Eric A Vittoz. An analytical mos transistor model valid in all regions of operation and dedicated to low-voltage and low-current applications. *Analog integrated circuits and signal processing*, 8(1):83–114, 1995.
- [27] Christian C Enz and Gabor C Temes. Circuit techniques for reducing the effects of op-amp imperfections: autozeroing, correlated double sampling, and chopper stabilization. *Proceedings of the IEEE*, 84(11):1584–1614, 1996.
- [28] Scholten et al. *From BSIM3/4 to PSP - Translation of flicker noise and junction parameters*. Philips Research Europe, 2006.
- [29] Daniel M Fleetwood. Total ionizing dose effects in mos and low-dose-rate-sensitive linear-bipolar devices. *Nuclear Science, IEEE Transactions on*, 60(3):1706–1730, 2013.
- [30] Daniel M Fleetwood, Timothy L Meisenheimer, and John H Scofield. 1/f noise and radiation effects in mos devices. *Electron Devices, IEEE Transactions on*, 41(11):1953–1964, 1994.

- [31] DM Fleetwood, PS Winokur, RA Reber Jr, TL Meisenheimer, JR Schwank, MR Shaneyfelt, and LC Riewe. Effects of oxide traps, interface traps, and border traps on metal-oxide-semiconductor devices. *Journal of applied physics*, 73(10):5058–5074, 1993.
- [32] Ronald F Fox and Yan-nan Lu. Emergent collective behavior in large numbers of globally coupled independently stochastic ion channels. *Physical Review E*, 49(4):3421, 1994.
- [33] Horng-Sen Fu and Chih-Tang Sah. Theory and experiments on surface 1/f noise. *Electron Devices, IEEE Transactions on*, 19(2):273–285, 1972.
- [34] C Galup-Montoro, M Schneider, VC Pahim, and R Rios. Comparison of surface-potential-based and charge-based mosfet core models. *Proc. WCM 2005*, pages 13–18, 2005.
- [35] William A Gardner, Antonio Napolitano, and Luigi Paura. Cyclostationarity: Half a century of research. *Signal processing*, 86(4):639–697, 2006.
- [36] G Ghibaudo, O Roux, Ch Nguyen-Duc, F Balestra, and J Brini. Improved analysis of low frequency noise in field-effect mos transistors. *physica status solidi (a)*, 124(2):571–581, 1991.
- [37] Sander Laurentius Johannes Gierkink. *Control linearity and jitter of relaxation oscillators*. Universiteit Twente, 1999.
- [38] Sander LJ Gierkink, Eric AM Klumperink, Arnoud P van der Wel, Gian Hoogzaad, EAJM van Tuijl, and Bram Nauta. Intrinsic 1/f device noise reduction and its effect on phase noise in cmos ring oscillators. *Solid-State Circuits, IEEE Journal of*, 34(7):1022–1025, 1999.

- [39] Gennady Gildenblat, Xin Li, Weimin Wu, Hailing Wang, Amit Jha, Ronald Van Langevelde, Geert DJ Smit, Andries J Scholten, and Dirk Klaassen. Psp: An advanced surface-potential-based mosfet model for circuit simulation. *Electron Devices, IEEE Transactions on*, 53(9):1979–1993, 2006.
- [40] Gennady Sh Gildenblat. *Compact modeling: principles, techniques and applications*. Springer Science & Business Media, 2010.
- [41] Daniel T Gillespie. Stochastic simulation of chemical kinetics. *Annu. Rev. Phys. Chem.*, 58:35–55, 2007.
- [42] Tibor Grasser. Stochastic charge trapping in oxides: From random telegraph noise to bias temperature instabilities. *Microelectronics Reliability*, 52(1):39–70, 2012.
- [43] Desmond J Higham. Modeling and simulating chemical reactions. *SIAM review*, 50(2):347–368, 2008.
- [44] FN Hooge. 1/f noise. *Physica B+ C*, 83(1):14–23, 1976.
- [45] FN Hooge. 1/f noise sources. *Electron Devices, IEEE Transactions on*, 41(11):1926–1935, 1994.
- [46] FN Hooge, TGM Kleinpenning, and LKJ Vandamme. Experimental studies on 1/f noise. *Reports on progress in Physics*, 44(5):479, 1981.
- [47] FN Hooge and LKJ Vandamme. Lattice scattering causes 1/f noise. *Physics Letters A*, 66(4):315–316, 1978.
- [48] ST Hsu, DJ Fitzgerald, and AS Grove. Surface-state related 1/f noise in p-n junctions and mos transistors. *Applied Physics Letters*, 12(9):287–289, 1968.

-
- [49] Chenming Hu. *Modern semiconductor devices for integrated circuits*, volume 1. Prentice Hall Upper Saddle River, NJ, 2010.
- [50] Christopher D Hull and Robert G Meyer. A systematic approach to the analysis of noise in mixers. *IEEE transactions on circuits and systems. 1, Fundamental theory and applications*, 40(12):909–919, 1993.
- [51] KK Hung, PK Ko, C Hu, and YC Cheng. Flicker noise characteristics of advanced mos technologies. pages 34–37, 1988.
- [52] Kwok K Hung, Ping K Ko, Chenming Hu, and Yiu C Cheng. A physics-based mosfet noise model for circuit simulators. *Electron Devices, IEEE Transactions on*, 37(5):1323–1333, 1990.
- [53] Kwok K Hung, Ping K Ko, Chenming Hu, and Yiu C Cheng. A unified model for the flicker noise in metal-oxide-semiconductor field-effect transistors. *Electron Devices, IEEE Transactions on*, 37(3):654–665, 1990.
- [54] C Jakobson, I Bloom, and Y Nemirovsky. 1/f noise in cmos transistors for analog applications from subthreshold to saturation. *Solid-State Electronics*, 42(10):1807–1817, 1998.
- [55] Raj Jayaraman and Charles G Sodini. A 1/f noise technique to extract the oxide trap density near the conduction band edge of silicon. *Electron Devices, IEEE Transactions on*, 36(9):1773–1782, 1989.
- [56] John Bertrand Johnson. Thermal agitation of electricity in conductors. *Physical Review*, 32(1):97, 1928.
- [57] Brian K Jones. Low-frequency noise spectroscopy. *Electron Devices, IEEE Transactions on*, 41(11):2188–2197, 1994.

- [58] Nobuhiro Kawai and Shoji Kawahito. Noise analysis of high-gain, low-noise column readout circuits for cmos image sensors. *Electron Devices, IEEE Transactions on*, 51(2):185–194, 2004.
- [59] Marvin S Keshner. 1/f noise. *Proceedings of the IEEE*, 70(3):212–218, 1982.
- [60] MJ Kirton and MJ Uren. Noise in solid-state microstructures: A new perspective on individual defects, interface states and low-frequency (1/f) noise. *Advances in Physics*, 38(4):367–468, 1989.
- [61] MJ Kirton, MJ Uren, and S Collins. Individual interface states and their implications for low-frequency noise in mosfets. *Applied Surface Science*, 30(1):148–152, 1987.
- [62] F.M. Klaassen. Characterization of low 1/f noise in mos transistors. *Electron Devices, IEEE Transactions on*, 18(10):887–891, Oct 1971.
- [63] Eric AM Klumperink, Sander LJ Gierkink, Arnoud P van der Wel, and Bram Nauta. Reducing mosfet 1/f noise and power consumption by switched biasing. *Solid-State Circuits, IEEE Journal of*, 35(7):994–1001, 2000.
- [64] Junji Koga, S Takagi, and A Toriumi. A comprehensive study of mosfet electron mobility in both weak and strong inversion regimes. In *Electron Devices Meeting, 1994. IEDM'94. Technical Digest., International*, pages 475–478. IEEE, 1994.
- [65] Jay Sudhir Kolhatkar. *Steady-state and Cyclo-stationary RTS noise in MOS-FETs*. Unversity of Twente, 2005.

- [66] DV Kuksenkov, H Temkin, R Gaska, and JW Yang. Low-frequency noise in algan/gan heterostructure field effect transistors. *Electron Device Letters, IEEE*, 19(7):222–224, 1998.
- [67] C Leyris, JC Vildeuil, F Roy, Fabiola Martinez, M Valenza, and A Hoffmann. Response of correlated double sampling cmos imager circuit to random telegraph signal noise. In *Devices, Circuits and Systems, Proceedings of the 6th International Caribbean Conference on*, pages 109–114. IEEE, 2006.
- [68] Zhiyuan Li, Jianguo Ma, Yizheng Ye, and Mingyan Yu. Compact channel noise models for deep-submicron mosfets, 2009.
- [69] A.G. Mahmutoglu and A. Demir. Modeling and analysis of nonstationary low-frequency noise in circuit simulators: Enabling non monte carlo techniques. In *Computer-Aided Design (ICCAD), 2014 IEEE/ACM International Conference on*, pages 309–315, Nov 2014.
- [70] A.G. Mahmutoglu and A. Demir. Analysis of low-frequency noise in switched mosfet circuits: Revisited and clarified. *Circuits and Systems I: Regular Papers, IEEE Transactions on*, 62(4):929–937, April 2015.
- [71] A.G. Mahmutoglu, A. Demir, and J. Roychowdhury. Modeling and analysis of (nonstationary) low frequency noise in nano devices: A synergistic approach based on stochastic chemical kinetics. In *Computer-Aided Design (ICCAD), 2013 IEEE/ACM International Conference on*, pages 500–507, Nov 2013.
- [72] Mario Marin, MJ Deen, M de Murcia, P Llinares, and JC Vildeuil. Effects of body biasing on the low frequency noise of mosfets from a 130 nm cmos technology. *IEE Proceedings-Circuits, Devices and Systems*, 151(2):95–101, 2004.

- [73] Colin C McAndrew. Practical modeling for circuit simulation. *Solid-State Circuits, IEEE Journal of*, 33(3):439–448, 1998.
- [74] AL McWhorter. $1/f$ noise and germanium surface properties. *Semiconductor surface physics*, pages 207–228, 1957.
- [75] Amit Mehrotra. *Simulation and modelling techniques for noise in radio frequency integrated circuits*. PhD thesis, UNIVERSITY of CALIFORNIA at BERKELEY, 1999.
- [76] Hiroaki Mikoshiba. $1/f$ noise in n-channel silicon-gate mos transistors. *Electron Devices, IEEE Transactions on*, 29(6):965–970, 1982.
- [77] Edoardo Milotti. $1/f$ noise: a pedagogical review. *arXiv preprint physics/0204033*, 2002.
- [78] Varani L. Mitin V., Reggiani L. Generation-recombination noise in semiconductors. In Balandin A., editor, *Noise and Fluctuations Control in Electronic Devices*, pages 11–30. APS, 2002.
- [79] Aon Mujtaba, S-I Takagi, and Robert Dutton. Accurate modeling of coulombic scattering, and its impact on scaled mosfets. In *VLSI Technology, 1995. Digest of Technical Papers. 1995 Symposium on*, pages 99–100. IEEE, 1995.
- [80] Makiko Okumura, Hiroshi Tanimoto, Tetsuro Itakura, and Tsutomu Sugawara. Numerical noise analysis for nonlinear circuits with a periodic large signal excitation including cyclostationary noise sources. *Circuits and Systems I: Fundamental Theory and Applications, IEEE Transactions on*, 40(9):581–590, 1993.
- [81] Athanasios Papoulis and S Unnikrishna Pillai. *Probability, random variables, and stochastic processes*. Tata McGraw-Hill Education, 2002.

-
- [82] Namkyu Park. *1/f noise of MOS field effect transistors for analog circuit applications*. PhD thesis, University of Florida, 2001.
- [83] Lu D. D. Yang W. Dunga M. V. Xi X. He J. Liu W. Kanyu Cao M. Jin X. Ou J. J. Chan M. Niknejad A. M. Hu C. Paydavosi N., Morshed T. H. *BSIM 4v4.8.0 MOSFET Model - User's Manual*. University of California, Berkeley, 2013.
- [84] Raoul Pettai. *Noise in receiving systems*. Wiley, 1984.
- [85] William H Press. Flicker noises in astronomy and elsewhere. *Comments on Astrophysics*, 7:103–119, 1978.
- [86] Umar Saeed Qurashi. *STUDY OF DEFECT CHARACTERISTICS IN SOME TECHNOLOGICALLY IMPORTANT COMPOUND SEMICONDUCTORS*. PhD thesis, Quaid-i-Azam University, Islamabad, 1996.
- [87] KS Ralls, WJ Skocpol, LD Jackel, RE Howard, LA Fetter, RW Epworth, and DM Tennant. Discrete resistance switching in submicrometer silicon inversion layers: Individual interface traps and low-frequency (1/f) noise. *Physical review letters*, 52:228–231, 1984.
- [88] G Reimbold. Modified 1/f trapping noise theory and experiments in mos transistors biased from weak to strong inversion - influence of interface states. *Electron Devices, IEEE Transactions on*, 31(9):1190–1198, 1984.
- [89] Ananda S Roy, Christian C Enz, and J-M Sallese. Noise modeling methodologies in the presence of mobility degradation and their equivalence. *Electron Devices, IEEE Transactions on*, 53(2):348–355, 2006.

- [90] Jaijeet Roychowdhury, David Long, and Peter Feldmann. Cyclostationary noise analysis of large rf circuits with multitone excitations. *Solid-State Circuits, IEEE Journal of*, 33(3):324–336, 1998.
- [91] Sergey L Rumyantsev, Nezih Pala, Michael S Shur, Michael E Levinshtein, Remis Gaska, M Asif Khan, and Grigory Simin. Generation-recombination noise in gan-based devices. *International Journal of High Speed Electronics and Systems*, 14(01):175–195, 2004.
- [92] Serguei L Rumyantsev, Nezih Pala, Michael S Shur, Elena Borovitskaya, Alexander P Dmitriev, Michael E Levinshtein, Remis Gaska, M Asif Khan, Jinwei Yang, Xuhong Hu, et al. Generation-recombination noise in gan/algan heterostructure field effect transistors. *Electron Devices, IEEE Transactions on*, 48(3):530–534, 2001.
- [93] Chih-Tang Sah and Frank H Hielscher. Evidence of the surface origin of the 1 f noise. *Physical Review Letters*, 17(18):956, 1966.
- [94] CT Sah, TH Ning, and LL Tschopp. The scattering of electrons by surface oxide charges and by lattice vibrations at the silicon-silicon dioxide interface. *Surface Science*, 32(3):561–575, 1972.
- [95] Jean-Michel Sallese, Matthias Bucher, François Krummenacher, and Pierre Fazan. Inversion charge linearization in mosfet modeling and rigorous derivation of the ekv compact model. *Solid-State Electronics*, 47(4):677–683, 2003.
- [96] Hanspeter Schmid. Offset, flicker noise, and ways to deal with them. *Power*, 10:3, 2008.

-
- [97] Márcio Cherem Schneider and Carlos Galup-Montoro. *CMOS analog design using all-region MOSFET modeling*. Cambridge University Press, 2010.
- [98] Dieter K Schroder. *Semiconductor material and device characterization*. John Wiley & Sons, 2006.
- [99] Adel S Sedra and Kenneth Carless Smith. *Microelectronic circuits*, volume 1. Oxford university press, 1998.
- [100] Michael Shur and Tor A Fjeldly. *Silicon and beyond: Advanced device models and circuit simulators*, volume 15. World Scientific, 2000.
- [101] Eddy Simoen and Cor Claeys. On the flicker noise in submicron silicon mosfets. *Solid-State Electronics*, 43(5):865–882, 1999.
- [102] Scholten A.J. Klaassen D.B.M. van der Toorn R. Smit, G.D.J. *PSP 103.3*. NXP Semiconductors, 12 2013. Model Summary.
- [103] C Surya and TY Hsiang. Surface mobility fluctuations in metal-oxide-semiconductor field-effect transistors. *Physical review B*, 35(12):6343, 1987.
- [104] Charles Surya and Thomas Y Hsiang. Theory and experiment on the $1/f^{\gamma}$ noise in p-channel metal-oxide-semiconductor field-effect transistors at low drain bias. *Physical Review B*, 33(7):4898, 1986.
- [105] Charles Surya and Thomas Y Hsiang. A thermal activation model for $1/f$ noise in si-mosfets. *Solid-state electronics*, 31(5):959–964, 1988.
- [106] Shin-ichi Takagi, Akira Toriumi, Masao Iwase, and Hiroyuki Tango. On the universality of inversion layer mobility in si mosfet’s: Part i-effects of substrate

- impurity concentration. *Electron Devices, IEEE Transactions on*, 41(12):2357–2362, 1994.
- [107] R Telichevesky, K Kundert, I Elfadel, and J White. Fast simulation algorithms for rf circuits. In *Custom Integrated Circuits Conference, 1996., Proceedings of the IEEE 1996*, pages 437–444. IEEE, 1996.
- [108] Ricardo Telichevesky, Ken Kundert, and Jacob White. Efficient ac and noise analysis of two-tone rf circuits. In *Design Automation Conference Proceedings 1996, 33rd*, pages 292–297. IEEE, 1996.
- [109] Hui Tian and Abbas El Gamal. Analysis of 1/f noise in switched mosfet circuits. *Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on*, 48(2):151–157, 2001.
- [110] Marc Tiebout, Nicola Zanolla, Peter Baumgartner, Claudio Fiegna, et al. Noise reduction in cmos circuits through switched gate and forward substrate bias. *Solid-State Circuits, IEEE Journal of*, 44(7):1959–1967, 2009.
- [111] Ming-Horn Tsai. *Random Telegraph Signal and 1/f Noise in MOS Transistors*. PhD thesis.
- [112] MingHorn Tsai and T.-P. Ma. Effect of radiation-induced interface traps on 1/f noise in mosfet’s. *Nuclear Science, IEEE Transactions on*, 39(6):2178–2185, Dec 1992.
- [113] Yannis Tsividis and Ken Suyama. Mosfet modeling for analog circuit cad: problems and prospects. *Solid-State Circuits, IEEE Journal of*, 29(3):210–216, 1994.

-
- [114] AP Van der Wel, EAM Klumperink, SLJ Gierink, RF Wassenaar, and H Wallinga. Mosfet 1/f noise measurement under switched bias conditions. *Electron Device Letters, IEEE*, 21(1):43–46, 2000.
- [115] Arnoud P Van Der Wel, Eric AM Klumperink, Jay S Kolhatkar, Eric Hoekstra, Martijn F Snoeij, Cora Salm, Hans Wallinga, and Bram Nauta. Low-frequency noise phenomena in switched mosfets. *Solid-State Circuits, IEEE Journal of*, 42(3):540–550, 2007.
- [116] Adv Van der Ziel. Flicker noise in electronic devices. *Advances in electronics and electron physics*, 49:225–297, 1979.
- [117] Bart Van Zeghbroeck. Principles of semiconductor devices. *Colorado University*, 2004.
- [118] Ewout P Vandamme and Lode KJ Vandamme. Critical discussion on unified 1/f noise models for mosfets. *Electron Devices, IEEE Transactions on*, 47(11):2146–2152, 2000.
- [119] LKJ Vandamme. Model for 1/f; noise in mos transistors biased in the linear region. *Solid-State Electronics*, 23(4):317–323, 1980.
- [120] LKJ Vandamme and HMM De Werd. 1/f; noise model for mosfets biased in nonohmic region. *Solid-State Electronics*, 23(4):325–329, 1980.
- [121] LKJ Vandamme, Xiaosong Li, and Dominique Rigaud. 1/f noise in mos devices, mobility or number fluctuations? *Electron Devices, IEEE Transactions on*, 41(11):1936–1945, 1994.
- [122] Lode KJ Vandamme and FN Hooge. What do we certainly know about 1/f noise in mosfets? *IEEE transactions on electron devices*, 55(11):3070–3085, 2008.

- [123] Lode KJ Vandamme and FN Hooge. What do we certainly know about 1/f noise in mosfets. *IEEE transactions on electron devices*, 55(11):3070–3085, 2008.
- [124] JC Vildeuil, M Valenza, and D Rigaud. Extraction of the bsim3 1/f noise parameters in cmos transistors. *Microelectronics journal*, 30(2):199–205, 1999.
- [125] Martin Von Haartman and Mikael Östling. *Low-frequency noise in advanced MOS devices*. Springer Science & Business Media, 2007.
- [126] Arnoud Pieter Wel. *Mosfet 1/f noise under large signal excitation*. University of Twente, 2005.
- [127] John A White, Jay T Rubinstein, and Alan R Kay. Channel noise in neurons. *Trends in neurosciences*, 23(3):131–137, 2000.
- [128] Gilson Wirth, Jeongwook Koh, Roberto Da Silva, Roland Thewes, Ralf Bredel, et al. Modeling of statistical low-frequency noise of deep-submicrometer mosfets. *Electron Devices, IEEE Transactions on*, 52(7):1576–1588, 2005.
- [129] Reinout Woltjer, Luuk Tiemeijer, and Dick Klaassen. An industrial view on compact modeling. *Solid-State Electronics*, 51(11):1572–1580, 2007.
- [130] Hei Wong. Low-frequency noise study in electron devices: review and update. *Microelectronics Reliability*, 43(4):585–599, 2003.
- [131] Dingming Xie, Mengzhang Cheng, and Leonard Forbes. Spice models for flicker noise in n-mosfets from subthreshold to strong inversion. *Computer-Aided Design of Integrated Circuits and Systems, IEEE Transactions on*, 19(11):1293–1303, 2000.

-
- [132] Leopaldo D Yau and RL-Y Sah. Theory and experiments of low-frequency generation-recombination noise in mos transistors. *Electron Devices, IEEE Transactions on*, 16(2):170–177, 1969.
- [133] Nicola Zanolla, Domagoj Šiprak, Marc Tiebout, Peter Baumgartner, Enrico Sangiorgi, and Claudio Fiegna. Reduction of rts noise in small-area mosfets under switched bias conditions and forward substrate bias. *Electron Devices, IEEE Transactions on*, 57(5):1119–1128, 2010.
- [134] Junlin Zhou, Mengzhang Cheng, and Leonard Forbes. Spice models for flicker noise in p-mosfets in the saturation region. *Computer-Aided Design of Integrated Circuits and Systems, IEEE Transactions on*, 20(6):763–767, 2001.