MULTI-TRIP PERIODIC VEHICLE ROUTING PROBLEM WITH TIME WINDOWS, SPLIT DELIVERY FOR SIMULTANEOUS PICK-UP AND DELIVERY

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To My Dear Family

ABSTRACT

Vehicle routing and delivery planning problems are one of the main problems that logistics systems face with. Given pick-up and demand quantities for each customer, available working hour, shift times, total distance between two customers, cost parameters and a variety of transportation constraints, planners search for an effective delivery and, in some cases, pick-up schedule in order to satisfy customers' needs. Delivery planners identify vehicles to be used, start time of service, the shortest vehicle routes within the determined time window, shifts and number of a tour of a specific vehicle in a period to increase vehicle utilization and also to meet these requirements at minimum cost. In this paper, in order to solve such a vehicle routing problem, we propose three mixed integer programming models which reduce the logistic costs and number of vehicle traffic by combining shipments from different customers. The first model includes a heterogeneous fleet of vehicles that defines service to customers with time windows, simultaneously pick-up and/or delivery requests, and shift planning of these vehicles. The second model tackles the first problem with split pick-up and delivery. Besides these two models, the third detailed model deals with vehicles performing multiple tours in a given period. These models have been implemented and tested with Solomon data set up to 25 and 50 customers to verify the models and make comparisons between their performances. In order to solve large problems, we propose a decomposition based heuristics which separates periods by considering time windows and setting an objective that minimizes the number of used vehicles in each period, firstly. Then, an updated mathematical model does not contain shift parameter but counts multi-trip, customer time windows, split delivery, simultaneous pick-up and delivery.

The results show that inefficient transportation between two nodes is avoided, numbers of vehicles are decreased, the utilization rate of each vehicle is increased and vehicle traffic is reduced at each of the customer areas.

Keywords: Vehicle Routing Problems, Multi-Trip, Vehicle Routing Problems with Time Window, Vehicle Routing Problems with Simultaneous Pick-up and Delivery, Mixed Integer Programming, Decomposition

ÖZETÇE

Araç rotalama ve teslimat problemleri, lojistik sistemlerinin karşılaştığı ana problemlerden biridir. Her müşteri için verilen talep ve teslim alma miktarları, uygun çalışma saati, vardiya süreleri, iki nokta arasındaki toplam mesafe, maliyet parametreleri ve çeşitli ulaşım kısıtlamaları göz önüne alındığında, planlamacılar müşterilerin ihtiyaçlarını yerine getirmek için etkili bir teslimat ve bazı durumlarda toplama programı arar. Teslimat planlayıcıları araç kullanımını artırmak ve bu gereksinimleri minimum maliyetle karşılamak için kullanılacak araçları, servis başlama zamanını, belirlenen zaman aralığındaki en kısa araç rotalarını, vardiyaları ve spesifik bir aracın bir periyottaki tur sayısını tanımlarlar. Böyle bir araç rotalama problemini çözmek amacıyla bu yazıda, farklı müşterilerden gelen gönderileri birleştirerek, lojistik maliyetleri ve araç trafiğinin sayısını azaltmak için üç tane karışık tamsayılı programlama modelleri önerdik. İlk model, müşterilere zaman pencereli, eşzamanlı teslim alma ve/veya teslimat talepleri ve bu araçların vardiya planlaması ile hizmet tanımlayan heterojen bir araç filosunu içerir. İkinci model, ilk sorunu ayrı toplama ve teslimatla birlikte ele alır. Bu iki modelin yanı sıra, üçüncü ayrıntılı model, verilen bir periyotta birden çok tur yapan araçlarla ilgilidir. Modelleri doğrulamak ve modellerin performansları arasında bir karşılaştırma yapmak için bu modeller 25 ve 50 müşteriye kadar Solomon veri seti ile uygulanmış ve test edilmiştir. Büyük problemleri çözmek için ilk olarak, zaman pencerelerini göz önünde bulundurarak periyotlara bölen ve her periyotta kullanılan araç sayısını minimize etmeyi amaçlayan ayrıştırma sezgisel yöntemini ileri sürüyoruz.Güncellenmiş matematiksel model vardiya parametresi içermez, ancak çoklu seyahat, müşteri zaman pencereleri, bölünmüş teslimat, eşzamanlı teslim alma ve teslim etmeyi ele almaktadır.

Sonuçlar, müşteri alanlarının her birinde iki düğüm arasında verimsiz ulaşımın önlendiğini, araç sayısının azaldığını, her aracın kullanım oranının arttığını ve araç trafiğinin azaldığını göstermektedir.

Anahtar kelimeler: Araç Rotalama Problemi, Çok Seferli, Zaman Kısıtlı Araç Rotalama Problemi, Eş zamanlı Topla-Dağıt Araç Rotalama Problemi, Karışık Doğrusal programlama, Ayrıştırma

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CHAPTER I

INTRODUCTION

In recent years, researchers try to develop decision-making models to control the logistics processes, effectively utilize vehicle transport network and decrease total cost. It is crucial that vehicle movements and usages in the network are effectively planned while minimizing the costs. Planning for these transportation activities is a real application of Vehicle Routing Problem (VRP) which has already been thoroughly investigated, studied and describe by many operational researchers(see, for example, the surveys of Bodin et al. (1983), Laporte and Nobert (1987), Goel and Maini (2017)). VRP takes part in the literature as the "vehicle scheduling"(Land and Doig (1960)), "vehicle dispatching"(Bracken and McCormick (1968), Little et al. (1963)) or in short as the "delivery"problem(Mao (1969)) .It is used in very different aspects of daily life: waste collection(Dotoli and Epicoco (2017)), distribution of goods(Osvald and Stirn (2008)), transport of handicapped persons(Feillet et al. (2014)), route planning of school buses(Li and Fu (2002)), etc.

VRP is considered as a main problem in the fields of distribution, transportation and logistics. When suppliers deliver their goods to customer by their own vehicles, logistics cost and inefficiency increases by partially filled vehicles which causes increasing number of partially filled vehicles, traffic jam in the gates of customer and excessive waiting times. This is not preferred since customers do not want to face with delays causing extra costs in the operation.

VRP is categorized as NP-hard meaning that the needed solution time increases significantly with size of customers and vehicles. VRP is the basis of many logistics systems which contains a distribution center or a depot to keep consumer goods and provide all the shipments to the customers. Customer demands are satisfied by vehicles whose starting points and terminating points are at the depot. VRP can also consist of time windows of customers, scheduling of vehicle fleet and driver, amount of pick-up and delivery goods in the planning horizon. In this case, the idea is to determine the projection of fleet and driver and routing planning of the vehicles. One of the main objective is to schedule a set of vehicle routes to overcome overall demands by visiting each customer at least once and to minimize the total travelling cost by taking a variety of operational constraints into consideration.

There are different variants of the VRP, depending on the restrictions that are imposed. These restrictions(i.e. capacity, time) form the types of classical VRP.The classical VRP, also called the Capacitated VRP (CVRP), mainly aims to determine a set of routes for a fleet of vehicles so that all the requirements of a predetermined number of customers are met at the same time, minimize the transportation costs which is determined by the total distance traveled and the fixed costs for the use of the vehicles. Besides these, for instance, customers that are planned to be served can be sorted. Vehicles having capacity and total trip time restrictions may be directed to a set of customers with known demands. Customer demands may be satisfied within predefined time windows at minimum cost Kumar and Panneerselvam (2012).

The Vehicle Routing Problem with Time Windows (VRPTW) contains customers which have to be served within a certain time period in such a way that vehicles serve these customers in their time windows. If a vehicle arrives before the customer starts working, it may wait due to the availability of customer capacity. To overcome this problem, appointments in specific time periods can be created by the customer. The VRPTW focuses on issues in which a set of routes for a fleet of vehicles based at one or several depots must be determined for some geographically dispersed cities or customers as stated in Dabia et al. (2011).

VRP with pickup and delivery (VRPPD) elaborates the VRP by concerning goods transmitted from a depot to customers and also the other way around, but there is no transportation of goods between customers as summarized in Wassan and Nagy (2014). VRPSPD which was firstly introduced by Min (1989), comes from pick-up and delivery of books in a public library system consisting of a main library and 22 local libraries. It is a matter of determining the optimal vehicle routes to distribute customer demand which consists of pick-up and delivery service with capacity constraints. If customers get goods from the depot, they are known as linehauls or deliveries. On the other side, customers who send goods back to the depot are called as backhauls or pick-ups. There is a possibility that a customer may send and also receive goods which is defined as combined demands. On the contrary, if all customers are either send or receive goods are known as single demands. When VRPPD has a constraint that vehicles make a single stop to both pick-up and deliver goods, VRP with Simultaneous Pickup and Delivery (VRPSPD) problem shows up.

The classical vehicle routing problem (VRP) supposes that the demand of a customer can be covered by visiting the customer once. However, in practice, there may be some customers having demand quantities more than vehicle capacities. In this case, customer demands can be split and vehicles can serve more than once to get split deliveries. In other words, demand of the customer can be covered by dividing deliveries if the sizes of the customer orders are more than the capacity of a vehicle. This methodology is called as Split Delivery Vehicle Routing Problem (SDVRP), a variant of the Capacitated Vehicle Routing Problem (CVRP), in which the number of available vehicles is considered as unlimited. Each customer can be visited at least once, that is in contrast to the classical VRP, by a fleet of heterogeneous vehicles. On the other hand, the sum of the split delivery quantities in each tour cannot exceed the capacity of the vehicles (Archetti et al. (2006)).The interesting point on the SDVRP is about the fact that cost savings can be obtained by performing split deliveries(Archetti and Speranza (2007)).

Classical VRP types seek for planning the routes of the vehicles in the VRP system for a single working period (a day or a shift). Therefore, VRPTW is not in line with the some real situations, where there are continuous operations to route vehicles to satisfy demand. Generally, routing and scheduling in a day period consists of shifts. In such cases, VRP with shift is desired that all vehicles has to turn back to the depot before the end of each shift (Ren et al. (2010)).

Another variant of capacitated vehicle routing is the multi-trip vehicle routing problem (MTVRP) where vehicles may be dispatched several times in a scheduling period. A vehicle that returns from a short route may be used once again or even a number of times until it reaches the maximum allowed driving time. This is a frequently used method in the transportation networks having vehicles limited capacity and short travel durations. In other words, when the planning period lasts long with respect to the route duration, some of vehicles in the system can be a candidate to perform multiple routes in the period (Mingozzi et al. (2013)). Vehicle capacities in each route still lower than the maximum allowed vehicle capacity.The main objective of the MTVRP is to identify a set of routes and assign each route to a vehicle in such a way that the total travel time is minimized (Cattaruzza et al. (2014)).

VRP is referred to as NP-hard. Lenstra and Rinnooy Kan (1979) state that as problem complexity increases exponentially with the size of the problem, it is generally considered that exact solutions to these problems cannot be found easily. Only for small scale problems an acceptable solution can be found within an acceptable time period. Operational researchers work hard on heuristic procedures, which are designed for giving an approximate solution more quickly when classic methods are too slow or fail to find any exact result, to achieve the best possible solution. Investigating the successful applications for solving various vehicle routing problems such as VRP with multiple use of vehicles (Karaoğlan (2015)), VRP with time windows and multiple use of vehicles (Koç and KARAOĞLAN (2012)), multi-trip with time windows(Nguyen et al. (2014)), we develop three mixed-integer programming (MILP) formulations for the mentioned problems in this paper and to solve model in efficient time, we develop decomposition based heuristic that gives solution in big data set. Heuristics decomposes considering customer time interval and available vehicles.

In this paper, we aim to find a solution to the more realistic problem that we focus on the vehicle routing problem which consists of vehicles performing Multi-Trip Periodic Routing with Time Windows, Split Delivery for Simultaneous Pick-up and Delivery (MT-PVRPTW-SD-SPD). When a vehicle is decided to be used in a shift, it visits at least one customer and may execute more than one tour in the shift. Despite many publications dealing with the vehicle routing problem, there is no explicit algorithm and study on MT-PVRPTW-SD-SPD according to the best of our knowledge. In order to lighten this area, we propose three mathematical models for the Multi-Trip Periodic VRP with Time Windows, Split Delivery for Simultaneous Pickup and Delivery (MT-PVRPTW-SD-SPD). In order to solve this big size of problem, we suggest a decomposition based heuristic that give good solutions in a given time.

The rest of this paper is organized as follows. Section 2 reviews the literature on vehicle routing problems. Section 3 and 4 describe the problem, its objective and used methodology respectively. Section 5 presents the results of a computational analysis of our model and comparison results. Finally, Section 6 concludes results with guidance for future research areas.

CHAPTER II

LITERATURE REVIEW

The literature review section addresses and provides comprehensive summary of the previous studies which created VRP with multi-trip, time windows, split delivery, shifts that is called periodic VRP in the literature and simultaneous pick-up and delivery. The purpose of this section is to provide foundation of knowledge on related studies which throw some light on our problem by discussing various kinds of solution methods and approaches.

As mentioned in VRP review article of Braekers et al. (2016a), VRP was firstly introduced by Dantzig and Ramser (1959). They define a model discussing the shortest route for delivering gasoline to several gas stations using homogeneous trucks. This study is called as “The truck dispatching problem” in the literature and prompt Clarke and Wright (1964) to structure this problem as a linear optimization problem which defines how to supply geographically dispersed customers around the main depot as in Figure 1 by using a fleet of trucks having different capacities.

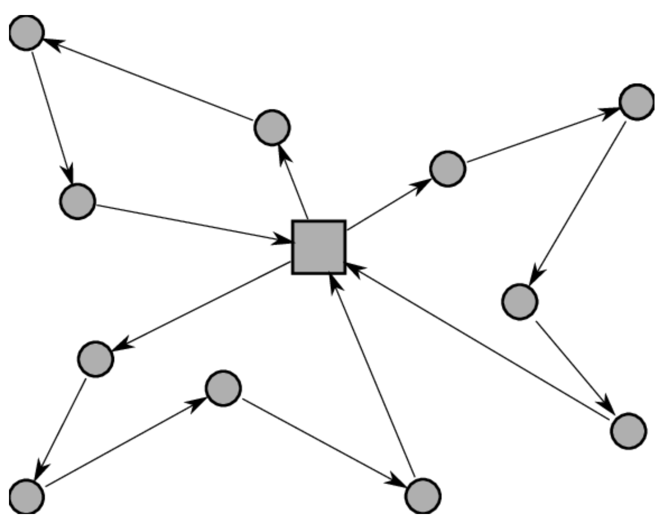

Figure 1: Illustration of VRP

The studies are mentioned most often and became known as the "Vehicle Routing Problem" (VRP). Over the years, a variety of researches were carried and VRP branched out for different cases.

In some sectors, transportation costs have a significant impact on added value to goods. Thus, according to Zmazek et al. (2005), the more we use developed programming models for those fields, the more savings we obtain. Generally, a lot of side restrictions arises in the real world VRPs. We focus some of the constraints identified in the literature and used below items in our study which is also illustrated in Figure 2:

- The vehicles have different size and thus different costs (Heterogeneous VRP)
- Each vehicle starts its trip from the depot, serves to the customer and it returns back to the depot when its shift or capacity is finished (Periodic VRP)
- The vehicles are with a limited capacity (Capacitated VRP)
- The customers receive their demands within their time windows (VRP with time windows)
- The customers may also send back some goods to the depot (VRP with Pick-up and Delivery)
- The customers may be served by different vehicles (Split Delivery VRP)
- Each vehicle can perform multi trip for each working day (Multi Trip VRP) and also, may have multiple tours in a shift

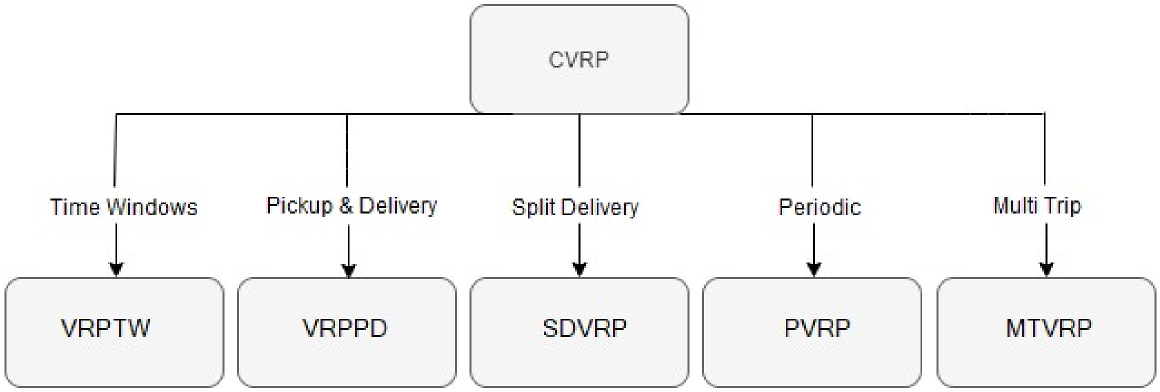

Figure 2: Classification of VRP in our study

Mixed Fleet VRP or also called as Heterogeneous Fleet VRP, which is firstly evaluated in a structured way in Golden et al. (1984), includes a fleet of vehicles having different capacities and costs Baldacci et al. (2008). Another study carried out by Gheysens et al. (1984) helps to solve the composition and size of a fleet of vehicles and creates an associated set of routes for different types of vehicles. They consider a two-stage heuristic in such a way that vehicle mix is stated using the lower bound methodology of Golden et al. (1984) which yields good vehicle mixes; then VRP solved by using that initial fleet.

Many researches on VRP give an output as Capacitated VRP(CVRP), whose fundamental constraint is the capacity of vehicle, and VRP with Time Windows (VRPTW), which is the extension of CVRP as described in Feliu et al. (2007). In VRPTW, customer demands are covered within a defined time interval of customers and schedule of vehicle trips are determined. Customer imposed time restrictions prevent starting delivery to customer before the earliest time and service cannot begin after prescribed latest time as in study of Bräysy and Gendreau (2005). Namely, it is not allowed to arrive after the end of time window. Moreover, departure time of each vehicle from the depot is adjusted and so, unnecessary idle times are managed.

The exact methods for the VRPTW can be categorized as lagrange relaxation based methods, column generation and dynamic programming (El-Sherbeny (2010)).

Lagrange relaxation-based methods reveals different approaches such as variable splitting followed by lagrange relaxation, K-tree approach followed by lagrange relaxation and shortest path with side constraints approach followed by lagrange relaxation. The variable splitting defined by Fisher et al. (1997) divides variables into different copies and applies Lagrangian relaxation which results in a number of independent subproblems of the problem. The K-tree approach of Fisher et al. (1997) is the improved type of 1-tree approach of Held and Karp (1970, 1971) which solves traveling salesman problem with vehicles having capacity constraints. It is supposed that each tour serves minimum of two customers. K-tree problem is created by a Lagrangian relaxation of the flow conversation constraints as well as the capacity and time window constraints. Violated constraints are handled by generating new constraints. Kohl and Madsen (1997) relaxes constraint set for requests of customers. Then, optimal Lagrangian multiplier of main problem is found and the subproblem yields results for a shortest path problem with time windows and capacity constarints.

The VRP with time windows can be solved by column generation, which is a generalization of Dantzig-Wolfe decomposition, in order to find optimal solution. Using Dantzig-Wolfe decomposition(Dantzig and Wolfe (1961)), VRPTW is divided into two such as Set Partitioning problem as master problem and Elementary Shortest Path problem with Capacity and Time Windows Constraints as pricing problem. The master problem retains the objective function, the constraint which is visiting of each customer exactly once, and the binary requirements on the flow variables. The subproblem includes a modified objective function.

The dynamic programming for VRPTW was firstly presented by Kolen et al. (1987) who extended dynamic programming coupled with state space relaxation (Christofides et al. (1981)) to determine lower bound on all feasible solution at each branch-and-bound node. The branch-and-bound algorithm of Kolen et al. (1987) defines following three sets: a set *F(a)* of fixed tours starting and ending at depot contains node a of the search tree, a set *P(a)* of partial tours starting at depot and a set *C(a)* of customers that are not allowed to be next on *P(a)*. *F(a)* and *C(a)* are firstly empty sets, yet *P(a)* contains the depot. The branching is initialized by selecting a customer i. This customer is not in *F(a)* or *P(a)* and the customer is allowed. A selection heuristic is used to perform selection. The main idea is to relax the constraint that each customer which is not in a route should be visited exactly once.

An exact integer solution for VRPTW can be introduced by making branch and cut decisions on the binary decision variables (Cordeau et al. (2002)). At each branching node, decomposition process including Lagrangian relaxation or column generation is applied repeatedly. It is possible to have binary or integer decisions on arc flow variables. On the other side, binary decisions on path flow variables are introduced. Lastly, sub-tour elimination, 2-path cuts, multiple path cuts and parallelism are explained. Cordeau et al. (2002) also defines integer decisions on time variables to make decision of branching and cutting.

Many applications of VRP contain pick-up and delivery services between the depot and customer location. This type of VRP is referred as VRP with pickup and delivery (VRPPD). Mosheiov (1998) presents two heuristics of tour partitioning. First heuristic is grounded on destroying a traveling salesman tour passing through all customer locations and it creates disjoint segments that are serviced by a different vehicle. This type of heuristic is referred to iterated tour partitioning (ITP) which was firstly defined by Haimovich and Rinnooy Kan (1985) and developed further by Altinkemer and Gavish (1990) who has studied Q Iterated Optimal Tour Partition (QIOTP) and best optimal tour partition (BOTP) algorithms. Second heuristic, Route first-cluster second method, which was introduced by Beasley (1983) considers to generate a big tour starting from depot, visiting all customers and terminating at the depot. Next, customers in the tour are split into clusters to obtain feasible routes.

VRPPD has another branch considering pick-up and delivery actions at the same time by the same vehicle and all delivered goods are arose from the depot and all pick-up goods are sent back to the depot (Belgin et al. (2018), Desaulniers et al. (2002) and Montané and Galvão (2002)). This study is called as Simultaneous pick-up and delivery and it is the one kind of the VRP operations. SPD, which includes customers having simultaneously pick-up and delivery request, aims to return and reuse the products. Therefore, vehicle planning is needed to reverse the flow. The reverse logistics of the goods has to be managed, since it generates additional revenue by capturing the value again as in Gajpal and Abad (2009).

Another relaxation of the classical capacitated vehicle routing problem is Split Delivery VRP (SDVRP) which was formally defined by Dror and Trudeau (1989) with the incentive that allowing split deliveries can be concluded in significant transportation cost savings. The problem is presented to be NP-hard (Dror and Trudeau (1990)), and it is not easier to overcome when the amounts to be delivered to each customer by each vehicle is also unknown as described by Ozbaygin et al. (2018). In SDVRP, the demand of a customer can be split and satisfied by different vehicles which relaxes the VRP constraint that each customer is visited exacty once. The main purpose is to find a set of most economical delivery routes for a vehicle fleet that starts and ends at the depot, the demand of every customer is fully covered, and the total demand dedicated to any route is less than or equal to the vehicle capacity. Dror and Trudeau (1989) comes up with local search algorithm to solve this problem and verified that the solution gives better results than that of VRP when customer demands are larger. Furthermore, Xie (2006) presenta tabu search algorithm (TSA) and a genetic algorithm (GA) by concluding that the performance of TSA in terms of solution optimality and time consumption is better than GA.

In order to find multi-period vehicle routes, period vehicle routing problems (PVRP) are modeled (Baldacci et al. (2011)). There are different kinds of PVRP definitions in the literature. One of them explains that in PVRP, time horizon of several days are introduced and assigning proper combinations for customer deliveries is considered. PVRP tries to find the delivery route planning for a vehicle fleet in the planning period. Customers are expected to be visited once or several times on a defined time horizon by this fleet. Each of the vehicles starts its trip from the depot, provides service to a set of customers. Then, the vehicle turns back to the depot before its work period (shift) is terminated. The objective is to meet required service of all customers assigned to each day and minimize the overall distribution cost. The heuristic algorithms mentioned in Christofides and Beasley (1984) show routing costs by changing the vehicle routing problem for each day of the period by a median problem and a traveling salesman problem. Moreover, in the study of Campbell and Savelsbergh (2004), driver work duration is considered under the constraint of 16-hour shift time limit, 10-hour drive time limit and a limit of 70 total hours in any 8 ensuing days. For example, If a driver works five days with 14-hour in each day, this driver is not allowed to work in the next three days due to mentioned restrictions. In order to overcome shift time limit, Campbell and Savelsbergh (2004) focus on an insertion heuristic method. In our problem, we discuss another version of PVRP explained more in Angelelli and Speranza (2002). This PVRP contains set of customers to be visited at least once in a given planning horizon by a fleet of vehicles. Each vehicle starts its trip from the depot, provide service to the customer and it returns back to the depot when its shift or capacity is finished.

First attempt to address Multi Trip Vehicle Routing Problem is done by Fleischmann (1990). He tackles a problem with a heterogeneous fleet of vehicles and a single-depot VRP. Vehicles perform several tours within their time windows. This is applicable if customer demands are high and vehicle usage time to supply is short; so a vehicle needs to execute multiple tours a day. Based upon this study, a new heuristic using the Rochat-Taillard principle is proposed by Taillard et al. (1996).

The proposal consists of generation of large set of good vehicle routes compatible with VRP constraints, selection of subset of these routes and combination of selected routes with proper working days.

When split delivery and multi depot vehicle routing problems are taken into account together, the multi-depot split delivery vehicle routing problem(MDSDVRP) is constructed as in Gulczynski et al. (2011). A heuristic solution is applied in two phases. Firstly, a customer is assigned to a depot which is the closest depot to the customer among all depots as defined in Golden et al. (1975). If a customer is in between two depots, then the cheapest depot is chosen. Next, unassigned customers are directed to the cheapest depots. After all assignments are completed, initial solution of MDSDVRP is obtained without splits a heuristic method Gulczynski (2010). Then, a mixed integer program (MILP) deals with the split deliveries including inter-depot split deliveries.

Finally, we checked whether our MT-PVRPTW-SD-SPD study has been discussed earlier. Its subtopics such as multi-trip VRP, periodic VRP, time windows, split delivery, simultaneous pick-up and delivery are discussed separately. A comprehensive review of the studies on this subject can be found in the following articles: Kumar and Panneerselvam (2012), Goel and Maini (2017), Braekers et al. (2016b), Şen and Bülbül (2008). After going through related articles, we concluded that all these subtopics are not included and evaluated in one unique article in the literature. Therefore, we did not encounter with an article which studies our topic fully.

CHAPTER III

PROBLEM DESCRIPTION

Let G(N,A) is a network graph where N is set of all nodes and A is set of arcs A= {(i, j): i=0, 1,. . . ..,N, j=1, 2,. . . ..,N} between the nodes. $N = 0$ is the depot and N_c represents the set of customers that need to be served. Distances between these arcs are stated as $Dist_{ij}$ where $\forall i, j \in N | i \neq j$. Let V be the set of heterogeneous vehicles such that each vehicle has a finite capacity Cap_v and D_j specifies the demand of customer $j \in Nc$. We assume that the number of vehicles is limited and each has average speed Spd_v. Our problem contains working periods which is called as shift throughout the study. There are some customers whose time window overlaps in two different working period, so we impose shift term for the problem. Vehicles should complete their route and come back to depot in the time interval of same shift [LS_s, US_s] which corresponds to start and end time of each shift s .Let a_j and b_j be respectively the opening and closing time of node j which depend on availability of customers to be served, namely time window of customer. Vehicle transports the demand of each node j and/or gets back pick-up quantity P_j. While giving and/or receiving the goods, the vehicle spends time which is called as service time of vehicle $SrvT_v$. Each route is traversed by a vehicle with a fixed and finite capacity and heterogeneous vehicles, so they have different costs per kilometer. Before leaving the depot, the type of vehicle is determined and the total demand delivered in each route should not exceed the capacity of the vehicle. The service available time of each customer is known, so service at a customer point is only allowed to start within its time interval. When the vehicle arrives before the shift time of the customer, it waits until the customer is ready to begin service and then, delivers the customer demand within associated time window. When the customer has pick-up demand, this is also taken by the same vehicle while satisfying the demand. The shift time of depot is also known and therefore, the vehicle should complete its tour within this predefined shift time. To be more precise, each vehicle returns back to the depot when its shift or capacity is exceeded after serving a set of customers to be visited at least once in a given planning horizon.

Firstly, we introduce PVRPTW-SPD to find solution for problems including vehicles, time windows, shifts. These vehicles should serve demands and take back returns simultaneously. PVRPTW-SD-SPD is derived from PVRPTW-SPD by allowing when the customer demand is more than the vehicle capacity and split delivery is inevitable. Demand of each customer represented by D_j and it can be covered by several vehicles each having split pick-up quantity P_j and delivery quantities of each customer can be received by different vehicles. The PVRPTW-SD-SPD consists of finding a set of least-cost feasible routes for all shifts such that the demands of all customer are met.

The required number of vehicles can be decreased by forcing a particular vehicle to visit several nodes during the shift. In other words, a vehicle, which is decided to be used visits at least one customer. Regarding to time, distance and capacity limitation constraints, vehicle performs multi-trip with the same shift s. This is called MT-PVRPTW-SD-SPD. Also, due to the restrictions of driving duration, the number of tours are restricted by R_{max} which is the maximum number of tours in the shift.

CHAPTER IV

OBJECTIVE AND METHODOLOGY

In order to overcome the problem defined above, we develop three MILP models as

i)Capacitated Periodic Vehicle Routing Problem with Time Windows for Simultaneous Pick-up and Delivery,

ii)Capacitated Periodic Vehicle Routing Problem with Time Windows, Split Delivery for Simultaneous Pick-up and Delivery and

iii)Multi-Trip Periodic VRP with Time Windows, Split Delivery for Simultaneous Pick-up and Delivery

which is constructed with the help of two predetermined decision support systems. For model i and ii, our goal is to reduce the total logistics costs by minimizing the numbers of vehicles used in shifts, finding shortest route to decrease utilization time of vehicles. For model iii, we aim to minimize total fixed costs, traveling expenses and the number of used vehicles in the system.

The problem is to define number of vehicles and their route with minimum total cost under these assumptions:

- Each customer should be visited at least once during its time window,
- Each vehicle must turn back to depot before the end of the shift,
- Each route begins and ends in the depot so that the known demand of all customers are satisfied,
- Each vehicle has a limited capacity,
- Total demand along each tour should not exceed the capacity of vehicle

Here, we present the parameters, decision variables and mathematical models for the problem described above.

4.1 Capacitated Periodic VRP with Time Windows for Simultaneous Pick-up and Delivery

First problem consists of customers having definite demand and pick-up quantities that are taken back by vehicles simultaneously. Customers share their time windows and there may be some customers whose time window overlaps in two working periods of vehicles. In this case, we use "shift" term to indicate vehicles with limited capacity that visit customers at least once. These vehicles work in different periods within a given time horizon. They provide service to customers with predefined time windows and when their work shift or capacity is finished, they return to the depot. In order to overcome this PVRPTW-SPD problem, a mixed-integer programming (MILP) formulation is developed. The parameters and decision variables of this problem are given in below tables.

Table 1: Parameters of PVRPTW-SPD

Parameter	Description
N	Set of nodes, i={0, 1,. . . ..,N}
Nc	Set of customer nodes, j={1, 2,. . . ..,N}
V	Set of vehicle, v={1, 2,. . . ..,V}
S	Set of shifts, s={1, 2,. . . ..,S}
D_j	Demand or delivery quantity of node j ($\forall j \in Nc$)
P_j	Pick-up quantity from node j ($\forall j \in Nc$)
$[LS_s, US_s]$	Start and finish time of shift s ($\forall s \in S$)
$Dist_{ij}$	Distance between node i and node j ($\forall i, j \in N \| i \neq j$)
$[a_i, b_i]$	Time window of node i ($\forall i \in N$)
Spd_v	Average speed of vehicle v ($\forall v \in V$)
$SrvT_v$	Service time of loading or unloading per goods for vehicle v ($\forall v \in V$)

Cap_v	Capacity of vehicle v in goods ($\forall v \in V$)
CFs_v	Fix cost of vehicle ($\forall v \in V$)
Ckm_v	Cost per kilometer of vehicle v ($\forall v \in V$)
M	A large number

Table 2: Decision Variables of PVRPTW-SPD

Decision Variable	Definition
$x_{ijvs} \in \{0,1\}$	1 iff vehicle v travels from node i to j in shift s ($\forall i,j \in N \| i \neq j, \forall v \in V, \forall s \in S$)
$w_{vs} \in \{0,1\}$	1 iff vehicle v is used in shift s ($\forall v \in V, \forall s \in S$)
$y_{jvs} \in \{0,1\}$	1 iff vehicle v visit node j in shift s($\forall j \in Nc, \forall v \in V, \forall s \in S$)
$st_{ivs} \in \mathbb{R}$	Service start time of vehicle v at node i in shift s($\forall i \in N, \forall v \in V, \forall s \in S$)
$l_{ivs} \in \mathbb{R}$	Load of vehicle v after leaving node i in shift s ($\forall i \in N, \forall v \in V, \forall s \in S$)
$tt_{vs} \in \mathbb{R}$	Travel time of vehicle v in shift s($\forall v \in V, \forall s \in S$)
$u_{ivs} \in \{0,1\}$	Additional variable for sub tour eliminations ($\forall i \in Nc, \forall v \in V, \forall s \in S$)

Objective Function

The objective function (1) is to minimize number of used vehicle, total distance traveled by these vehicles for all shifts.

$$\textbf{Min} \sum_{i,j \in N} \sum_{v \in V} \sum_{s \in S} x_{ijvs} \cdot Dist_{ij} \cdot Ckm_v + \sum_{s \in S} \sum_{v \in V} w_{vs} \cdot CFs_v \tag{1}$$

s.t.

Constraint (2) expresses that demand of all customers are satisfied.

$$\sum_{s \in S} \sum_{i \in N | i \neq j} \sum_{v \in V} x_{ijvs} = 1 \qquad \forall j \in Nc \tag{2}$$

Constraint (3) and (4) ensures that each vehicle should start and end its route in depot.

$$\sum_{j \in Nc} x_{0jvs} = w_{vs} \qquad \forall v \in V, \forall s \in S \tag{3}$$

$$\sum_{j \in Nc} x_{j0vs} = w_{vs} \qquad \forall v \in V, \forall s \in S \tag{4}$$

Constraint (5) and (6) guarantee that, if a vehicle visits one node it must leave that node.

$$\sum_{j \in N} x_{jivs} = y_{ivs} \qquad \forall i \in Nc, \forall v \in V, \forall s \in S \tag{5}$$

$$\sum_{j \in N} x_{ijvs} = y_{ivs} \qquad \forall i \in Nc, \forall v \in V, \forall s \in S \tag{6}$$

Constraints (7) and (8) eliminate sub tours.

$$u_{ivs} - u_{jvs} + N \cdot x_{ijvs} \leq N - 1 \qquad \forall i, j \in Nc, \forall v \in V, \forall s \in S \tag{7}$$

$$u_{ivs} \geq 0 \qquad \forall i \in Nc, \forall v \in V, \forall s \in S \tag{8}$$

Constraint (9) determines the load of vehicles upon leaving the depot.

$$l_{0vs} = \sum_{i \in N} \sum_{j \in Nc | i \neq j} D_j \cdot x_{ijvs} \qquad \forall v \in V, \forall s \in S \tag{9}$$

Constraints (10) imposes consistency of loads of vehicles.

$$l_{jvs} \geq l_{ivs} + P_j - D_j - M \cdot (1 - x_{ijvs}) \qquad \forall i \in N, \forall j \in Nc | i \neq j, \forall v \in V, \forall s \in S \tag{10}$$

Constraint (11) makes sure that the loads of vehicles do not exceed vehicle capacity during each route and (12) provides that vehicle's load does not exceed vehicle capacity when it leaves depot.

$$l_{jvs} \leq Cap_v \cdot y_{jvs} \qquad \forall j \in Nc, \forall v \in V, \forall s \in S \tag{11}$$

$$l_{0vs} \leq Cap_v \cdot w_{vs} \qquad \forall v \in V, \forall s \in S \tag{12}$$

Constraints (13) and (14) impose consistency of the visiting times of nodes.

$$LS_s \cdot w_{vs} \leq st_{0vs} \qquad \forall v \in V, \forall s \in S \tag{13}$$

$$\begin{aligned} st_{jvs} \geq st_{ivs} + (Dist_{ij}/Spd_v) + SrvT_v \cdot (D_i + P_i) - M \cdot (1 - x_{ijvs}) \\ \forall i \in N, \forall j \in Nc | i \neq j, \forall v \in V, \forall s \in S \end{aligned} \tag{14}$$

Constraint (15) determines total travel time of each vehicle.

$$\begin{aligned} tt_{vs} \geq st_{jvs} + (Dist_{j0}/Spd_v) + SrvT_v \cdot (D_j + P_j) - M \cdot (1 - x_{j0vs}) - LS_S \\ \forall j \in Nc, \forall v \in V, \forall s \in S \end{aligned} \tag{15}$$

Constraint (16) ensure that the arrival times of nodes are within time windows.

$$a_j \cdot y_{jvs} \leq st_{jvs} \leq b_j \cdot y_{jvs} \qquad \forall j \in Nc, \forall v \in V, \forall s \in S \tag{16}$$

Constraints (17) and (18) ensure that each tour starts and ends at the same shift

$$LS_s \cdot y_{jvs} \leq st_{jvs} \leq US_s \cdot y_{jvs} \qquad \forall j \in Nc, \forall v \in V, \forall s \in S \tag{17}$$

$$tt_{vs} \leq (US_s - LS_s) \cdot w_{vs} \qquad \forall v \in V, \forall s \in S \tag{18}$$

4.2 Capacitated Periodic VRP with Time Windows, Split Delivery for Simultaneous Pick-up and Delivery

PVRPTW-SD-SPD is a branch of PVRPTW-SPD. It includes an additional decision variable to solve a problem in cases where the customer demand exceeds the vehicle capacity and the vehicle cannot carry all customer loads. Separate delivery is unavoidable in this case and customer burdens are split. Thus, the vehicle can get these divided loads in different services to that customer. All parameters and decisions are listed below and then, mathematical model is given.

Table 3: Parameters of PVRPTW-SD-SPD

Parameter	Description
N	Set of nodes, i={0, 1,. . . ..,N}
Nc	Set of customer nodes, j={1, 2,. . . ..,N}
V	Set of vehicle, v={1, 2,. . . ..,V}
S	Set of shifts, s={1, 2,. . . ..,S}
D_j	Demand or delivery quantity of node j ($\forall j \in Nc$)
P_j	Pick-up quantity from node j ($\forall j \in Nc$)
$[LS_s, US_s]$	Start and finish time of shift s ($\forall s \in S$)
$Dist_{ij}$	Distance between node i and node j ($\forall i, j \in N \| i \neq j$)
$[a_i, b_i]$	Time window of node i ($\forall i \in N$)
Spd_v	Average speed of vehicle v ($\forall v \in V$)
$SrvT_v$	Service time of loading or unloading per goods for vehicle v ($\forall v \in V$)
Cap_v	Capacity of vehicle v in goods ($\forall v \in V$)
CFs_v	Fix cost of vehicle ($\forall v \in V$)
Ckm_v	Cost per kilometer of vehicle v ($\forall v \in V$)
M	A large number

Table 4: Decision Variables of PVRPTW-SD-SPD

Decision Variable	Definition
$q_{jvs} \in \mathbb{R}$	The split load of vehicle v to be delivered to node j in shift s$(\forall j \in Nc, \forall v \in V, \forall s \in S)$
$z_{jvs} \in \mathbb{R}$	The split load of vehicle v received from node j in shift s$(\forall j \in Nc, \forall v \in V, \forall s \in S)$
$x_{ijvs} \in \{0,1\}$	1 iff vehicle v travels from node i to j in shift s $(\forall i, j \in N \mid i \neq j, \forall v \in V, \forall s \in S)$
$w_{vs} \in \{0,1\}$	1 iff vehicle v is used in shift s $(\forall v \in V, \forall s \in S)$
$y_{jvs} \in \{0,1\}$	1 iff vehicle v visit node j in shift s$(\forall j \in Nc, \forall v \in V, \forall s \in S)$
$st_{ivs} \in \mathbb{R}$	Service start time of vehicle v at node i in shift s$(\forall i \in N, \forall v \in V, \forall s \in S)$
$l_{ivs} \in \mathbb{R}$	Load of vehicle v after leaving node i in shift s $(\forall i \in N, \forall v \in V, \forall s \in S)$
$tt_{vs} \in \mathbb{R}$	Travel time of vehicle v in shift s$(\forall v \in V, \forall s \in S)$
$u_{ivs} \in \{0,1\}$	Additional variable for sub tour eliminations $(\forall i \in Nc, \forall v \in V, \forall s \in S)$

Objective Function

The objective function (19) is to minimize number of hired vehicles, total distance traveled by vehicles for all shifts.

$$\textbf{Min} \sum_{i,j \in N} \sum_{v \in V} \sum_{s \in S} x_{ijvs} \cdot Dist_{ij} \cdot Ckm_v + \sum_{s \in S} \sum_{v \in V} w_{vs} \cdot CFs_v \tag{19}$$

s.t.

Constraint (20) guarantees that demands of each customer must be satisfied.

$$D_j = \sum_{s \in S} \sum_{v \in V} q_{jvs} \qquad \forall j \in Nc \tag{20}$$

Constraint (21) ensures that returns of each customer must be received.

$$P_j = \sum_{s \in S} \sum_{v \in V} z_{jvs} \qquad \forall j \in Nc \tag{21}$$

Constraint (22) guarantees that each vehicle can supply goods of customer if it goes to that customer.

$$q_{jvs} \leq \sum_{i \in N} M \cdot x_{ijvs} \qquad \forall j \in Nc, \forall v \in V, \forall s \in S \tag{22}$$

Constraint (23) shows that each vehicle can take returns of customer if it goes to that customer.

$$z_{jvs} \leq \sum_{i \in N} M \cdot x_{ijvs} \qquad \forall j \in Nc, \forall v \in V, \forall s \in S \tag{23}$$

Constraint (24) expresses that each customer must be visited at least once.

$$\sum_{s \in S} \sum_{i \in N | i \neq j} \sum_{v \in V} x_{ijvs} \geq 1 \qquad \forall j \in Nc \tag{24}$$

Constraints (25) and (26) guarantee that each vehicle should starts and ends its route in depot.

$$\sum_{j \in Nc} x_{0jvs} = w_{vs} \qquad \forall v \in V, \forall s \in S \tag{25}$$

$$\sum_{j \in Nc} x_{j0vs} = w_{vs} \qquad \forall v \in V, \forall s \in S \tag{26}$$

Constraints (27) and (28) guarantee that if vehicle visits a node, it must leave that node.

$$\sum_{j \in N} x_{jivs} = y_{ivs} \qquad \forall i \in Nc, \forall v \in V, \forall s \in S \tag{27}$$

$$\sum_{j \in N} x_{ijvs} = y_{ivs} \qquad \forall i \in Nc, \forall v \in V, \forall s \in S \tag{28}$$

Constraints (29) and (30) avoid sub tours.

$$u_{ivs} - u_{jvs} + N \cdot x_{ijvs} \leq N - 1 \qquad \forall i, j \in Nc, \forall v \in V, \forall s \in S \tag{29}$$

$$u_{ivs} \geq 0 \qquad \forall i \in Nc, \forall v \in V, \forall s \in S \tag{30}$$

Constraints (31) imposes consistency of loads of vehicles.

$$l_{jvs} \geq l_{ivs} - q_{jvs} + z_{jvs} - M \cdot (1 - x_{ijvs}) \qquad \forall i \in N, \forall j \in Nc | i \neq j, \forall v \in V, \forall s \in S \quad (31)$$

Constraint (32) ensures that the vehicle capacities are not exceeded.

$$l_{ivs} \leq Cap_v \cdot w_{vs} \qquad \forall i \in N, \forall v \in V, \forall s \in S \quad (32)$$

Constraint (33) provides that vehicle load at depot satisfies total split delivery quantity of customers.

$$l_{0vs} \geq \sum_{j \in Nc} q_{jvs} \qquad \forall v \in V, \forall s \in S \quad (33)$$

Constraint (34) evaluates the load of vehicle at node i.

$$\sum_{j \in Nc} q_{jvs} - q_{ivs} + z_{ivs} \leq Cap_v \cdot w_{vs} \qquad \forall i \in Nc, v \in V, \forall s \in S \quad (34)$$

Constraint (35) and (36) ensure that the start time of service at customer.

$$LS_s \cdot w_{vs} \leq st_{0vs} \leq US_s \cdot w_{vs} \qquad \forall v \in V, \forall s \in S \quad (35)$$

$$\begin{aligned} st_{jvs} \geq st_{ivs} + (Dist_{ij}/Spd_v) + SrvT_v \cdot (q_{jvs} + z_{jvs}) - M \cdot (1 - x_{ijvs}) \\ \forall i \in N, \forall j \in Nc | i \neq j, \forall v \in V, \forall s \in S \end{aligned} \quad (36)$$

Constraint (37) determines total travel time of each vehicle.

$$\begin{aligned} tt_{vs} \geq st_{jvs} + (Dist_{j0}/Spd_v) + SrvT_v \cdot (q_{jvs} + z_{jvs}) - M \cdot (1 - x_{j0vs}) - LS_S \\ \forall j \in Nc, \forall v \in V, \forall s \in S \end{aligned} \quad (37)$$

Constraint (38) ensures that the arrival times of nodes are within time windows.

$$a_j \cdot y_{jvs} \leq st_{jvs} \leq b_j \cdot y_{jvs} \qquad \forall j \in Nc, \forall v \in V, \forall s \in S \quad (38)$$

Constraint (39) ensures that each tour starts and ends at the same shift.

$$LS_s \cdot y_{jvs} \leq st_{jvs} \leq US_s \cdot y_{jvs} \qquad \forall j \in Nc, \forall v \in V, \forall s \in S \tag{39}$$

Constraint (40) ensure that total travel time of each vehicle cannot exceed the shift duration.

$$tt_{vs} \leq (US_s - LS_s) \cdot w_{vs} \qquad \forall v \in V, \forall s \in S \tag{40}$$

4.3 Multi-Trip Periodic VRP with Time Windows, Split Delivery for Simultaneous Pick-up and Delivery

We obtain a multi-trip periodic vehicle routing problem with time windows, split delivery for simultaneous pick-up and delivery (MT-PVRPTW-SD-SPD) by inserting multi-trip capability to PVRPTW-SD-SPD. The problem we studied in this study is an extension of the fundamental VRP. It consists of logistics system where the fleet of finite capacity vehicles is based, a supply point (or depot) where customer loads are available during particular time windows, loads picked-up at customers may be brought during one of their time windows, and customers waiting for their loads to be delivered or picked-up, or both, during their time windows. Also, we consider that picked-up goods cannot be used to supply delivery requirements. Each customer is visited at least once in its time window by a vehicle whose shift time is allowing to provide service to that customer and before the end of each shift, all vehicles have to return to the depot. The route planning is to be performed for a certain shift length is that each route having visit for one or several facilities (hence the "multi-trip" characterization) during their respective time windows brings in or takes away time-dependent customer loads. Therefore, the system provides an increase in logistics efficiency and a decrease in logistics cost.

Firstly, we clarify parameters and decision variables for MT-PVRPTW-SD-SPD. Next, we give mathematical formulation for our main goal to find number of vehicles with their route by minimizing overall logistic cost according to problem constraints defined below.

Table 5: Parameters of MT-PVRPTW-SD-SPD

Parameter	Description
N	Set of nodes, i={0, 1,. . . ..,N}
Nc	Set of customer nodes, j={1, 2,. . . ..,N}
V	Set of vehicle, v={1, 2,. . . ..,V}
S	Set of shifts, s={1, 2,. . . ..,S}
R	The number of tours in a shift, i={1, 2,. . . ..,R_{max}}
R_{max}	Maximum tour number
D_j	Demand or delivery quantity of node j ($\forall j \in Nc$)
P_j	Pick-up quantity from node j ($\forall j \in Nc$)
$[LS_s, US_s]$	Start and finish time of shift s ($\forall s \in S$)
$Dist_{ij}$	Distance between node i and node j ($\forall i, j \in N \vert i \neq j$)
$[a_i, b_i]$	Time window of node i ($\forall i \in N$)
Spd_v	Average speed of vehicle v ($\forall v \in V$)
$SrvT_v$	Service time of loading or unloading per goods for vehicle v ($\forall v \in V$)
Cap_v	Capacity of vehicle v in goods ($\forall v \in V$)
$CFix_{vr}$	Fixed cost of vehicle v in tour r ($\forall v \in V$,$\forall r \in R$)
CFs_{vs}	Fixed cost of vehicle v in shift s ($\forall v \in V$,$\forall s \in S$)
Ckm_v	Cost per kilometer of vehicle v ($\forall v \in V$)
M	A large number

Table 6: Decision Variables of MT-PVRPTW-SD-SPD

Decision Variable	Definition
$x_{rijvs} \in \{0, 1\}$	1 iff vehicle v travels from node i to j in shift s in tour r ($\forall r \in R, \forall i, j \in N \vert i \neq j, \forall v \in V, \forall s \in S$)
$q_{rjvs} \in \mathbb{R}$	Number of goods supplied to node j by vehicle v in shift s in tour r ($\forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S$)

$z_{rjvs} \in \mathbb{R}$	Number of returned goods at node j loaded to vehicle v in shift s in tour r ($\forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S$)
$w_{rvs} \in \{0,1\}$	1 iff vehicle v is used in shift s in tour r ($\forall r \in R, \forall v \in V, \forall s \in S$)
$y_{rjvs} \in \{0,1\}$	1 iff vehicle v visit node j in shift s in tour r ($\forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S$)
$st_{rivs} \in \mathbb{R}$	Service start time of vehicle v at node i in shift s in tour r($\forall r \in R, \forall i \in N, \forall v \in V, \forall s \in S$)
$l_{rivs} \in \mathbb{R}$	Load of vehicle v after leaving node i in shift s in tour r ($\forall r \in R, \forall i \in N, \forall v \in V, \forall s \in S$)
$tt_{rvs} \in \mathbb{R}$	Travel time of vehicle v in shift s in tour r ($\forall r \in R, \forall v \in V, \forall s \in S$)
$u_{rivs} \in \{0,1\}$	Additional variable for sub tour eliminations ($\forall r \in R, \forall i \in Nc, \forall v \in V, \forall s \in S$)

Objective Function

The objective function (41) is to minimize the number of used vehicles by encouraging them to perform multi-trips in their working shifts, total distance traveled by vehicles for all shifts.

$$\begin{aligned}\textbf{Min} \sum_{r \in R} \sum_{i,j \in N} \sum_{v \in V} \sum_{s \in S} x_{rijvs} \cdot Dist_{ij} \cdot Ckm_v + \sum_{r \in R} \sum_{j \in Nc} \sum_{v \in V} \sum_{s \in S} x_{r0jvs} \cdot CFs_{vs} \\ - \sum_{r \in 2..Rmax} \sum_{v \in V} \sum_{s \in S} w_{rvs} \cdot CFix_{vr}\end{aligned} \tag{41}$$

subject to

If vehicle goes from node i to customer j, constraint (42) assures that the vehicle can supply split delivery of the customer.

$$q_{rjvs} \leq \sum_{i \in N} M \cdot x_{rijvs} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S \tag{42}$$

If vehicle goes from node i to customer j, constraint (43) guarantees that the vehicle can get returns of the customer as split pick-up.

$$z_{rjvs} \leq \sum_{i \in N} M \cdot x_{rijvs} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S \tag{43}$$

Constraint (44) guarantees that demands of each customer must be satisfied.

$$D_j = \sum_{r \in R} \sum_{s \in S} \sum_{v \in V} q_{rjvs} \qquad \forall j \in Nc \tag{44}$$

Constraint (45) guarantees that pick-up requests of each customer must be satisfied.

$$P_j = \sum_{r \in R} \sum_{s \in S} \sum_{v \in V} z_{rjvs} \qquad \forall j \in Nc \tag{45}$$

Constraint (46) expresses that each customer must be visited at least once.

$$\sum_{r \in R} \sum_{i \in N | i \neq j} \sum_{v \in V} \sum_{s \in S} x_{rijvs} \geq 1 \qquad \forall j \in Nc \tag{46}$$

Constraint (47) expresses that vehicle can start consecutive tour after completing previous tour in a shift.

$$\sum_{j \in Nc} x_{(r-1)0jvs} \geq \sum_{j \in Nc} x_{r0jvs} \qquad \forall rin\{2, .., Rmax\}, v \in V, s \in S \tag{47}$$

Constraints (48) and (49) guarantee that each vehicle should start and end its route in depot

$$\sum_{j \in Nc} x_{r0jvs} = w_{rvs} \qquad \forall r \in R, \forall v \in V, \forall s \in S \tag{48}$$

$$\sum_{j \in Nc} x_{rj0vs} = w_{rvs} \qquad \forall r \in R, \forall v \in V, \forall s \in S \tag{49}$$

Constraints (50) and (51) guarantee that if vehicle visits a node, it must leave that node.

$$\sum_{j \in N} x_{rjivs} = y_{rivs} \qquad \forall r \in R, \forall i \in Nc, \forall v \in V, \forall s \in S \tag{50}$$

$$\sum_{j \in N} x_{rijvs} = y_{rivs} \qquad \forall r \in R, \forall i \in Nc, \forall v \in V, \forall s \in S \tag{51}$$

Constraints (52) and (53) avoid sub tours.

$$u_{rivs} - u_{rjvs} + N \cdot x_{rijvs} \leq N - 1 \qquad \forall r \in R, \forall i, j \in Nc, \forall v \in V, \forall s \in S \tag{52}$$

$$u_{rivs} \geq 0 \qquad \forall r \in R, \forall i \in Nc, \forall v \in V, \forall s \in S \tag{53}$$

Constraint (54) determines the initial quantity when vehicle leave depot.

$$l_{r0vs} \geq \sum_{j \in Nc} q_{rjvs} \qquad \forall r \in R, \forall v \in V, \forall s \in S \tag{54}$$

Constraint (55) states the vehicle load at customer j cannot exceed vehicle capacity.

$$l_{rjvs} \leq Cap_v \cdot y_{rjvs} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S \tag{55}$$

Constraint (56) indicates that the load quantity in the vehicle before leaving from customer. In other words, the picked-up quantities plus the demand quantities still to be delivered presents the load l_{rjvs} for each customer j.

$$\begin{gathered} l_{rjvs} \geq l_{rivs} - q_{rjvs} + z_{rjvs} - M \cdot (1 - x_{rijvs}) \\ \forall r \in R, \forall i \in N, \forall j \in Nc | i \neq j, \forall v \in V, \forall s \in S \end{gathered} \tag{56}$$

Constraint (57) ensures that the vehicle load at ith node is less than the capacity of used vehicle.

$$\sum_{j \in Nc} q_{rjvs} - q_{rivs} + z_{rivs} \leq Cap_v \cdot w_{rvs} \qquad \forall r \in R, \forall i \in Nc, \forall v \in V, \forall s \in S \tag{57}$$

Constraint (58) ensures that the start time of service at depot is in the shift time interval of shift s.

$$LS_s \cdot w_{rvs} \leq st_{r0vs} \leq US_s \cdot w_{rvs} \qquad \forall r \in R, \forall v \in V, \forall s \in S \tag{58}$$

Constraint (59) ensures that start time of service at jth node is greater than the sum of start time at ith node, time for travelling between node i and j, and spent time for delivering and taking the returns from node j.

$$\begin{aligned} st_{rjvs} \geq st_{rivs} + (Dist_{ij}/Spd_v) + SrvT_v \cdot (q_{rjvs} + z_{rjvs}) - M \cdot (1 - x_{rijvs}) \\ \forall r \in R, \forall i \in N, \forall j \in Nc | i \neq j, \forall v \in V, \forall s \in S \end{aligned} \tag{59}$$

Constraint (60) determines start time for the next tour.

$$\begin{aligned} st_{r0vs} \geq st_{(r-1)ivs} + (Dist_{j0}/Spd_v) + SrvT_v \cdot (q_{rjvs} + z_{rjvs}) - M \cdot (1 - x_{rj0vs}) \\ \forall r \in \{2, .., Rmax\}, \forall i \in N, \forall j \in Nc | i \neq 0, \forall v \in V, \forall s \in S \end{aligned} \tag{60}$$

Constraint (61) states that the start time of service should be within time window.

$$a_j \cdot y_{rjvs} \leq st_{rjvs} \leq b_j \cdot y_{rjvs} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S \tag{61}$$

Constraint (62) ensures that each tour starts and ends at the same shift.

$$LS_s \cdot y_{rjvs} \leq st_{rjvs} \leq US_s \cdot y_{rjvs} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S \tag{62}$$

Constraint (63) determines total travel time of each vehicle.

$$\begin{aligned} tt_{rvs} \geq st_{rjvs} + (Dist_{j0}/Spd_v) + SrvT_v \cdot (q_{rjvs} + z_{rjvs}) - M \cdot (1 - x_{rj0vs}) - st_{r0vs} \\ \forall r \in R, \forall j \in Nc, \forall v \in V, \forall s \in S \end{aligned} \tag{63}$$

Constraint (64) ensures that the sum of total travel time for each rank cannot exceed the shift time for every vehicle.

$$\sum_{r \in R} tt_{rvs} \leq (US_s - LS_s) \qquad \forall v \in V, \forall s \in S \tag{64}$$

4.4 Decomposition Based Heuristics

When a problem is hard to solve or solution takes too much of time, it is common to seek for ways to decompose the original problem into smaller sub-problem sets which are often easier to solve. By this way, it is expected that applying decomposition to the problem instances speeds up the process. It is also concluded in the study of Bun (2008) that CPLEX is able to yield solution for more problem instances after applying decomposition method and this brings a significant enhancement to the computation duration needed for CPLEX to give solution to the problem instances.

In our problem, we have a list of customers whose time windows coincide with vehicle shifts. In order to decrease the complexity of this problem, we present a decomposition approach. This approach offers a mathematical model which creates three sub-problems and yields three separate shift sets. Each set contains customers having close time windows so that these customers are treated in the same shift.

Hereinbelow, we describe how we applied the decomposition method to our problem. We show our model with defined parameters and decision variables in below tables.

4.4.1 Clustering Heuristics

We develop clustering heuristics in such a way that it offers a solution by defining which customers to be served in which shift and the number of vehicles to be used in order to cover customer desire by minimizing the overall cost. When a customer time window overlaps in two shift time window, model chooses best shift for that customer. It does not assign a customer more than one shift. It also decides how many vehicles should be used to serve all customers in the shift and does not put unnecessary vehicles in order to avoid extra costs.

Table 7: Parameters of Decomposition Model

Parameter	Description
Nc	Set of customer nodes, j={1, 2,. . . ..,N}
V	Set of vehicle, v={1, 2,. . . ..,V}
S	Set of shifts, s={1, 2,. . . ..,S}
D_j	Demand or delivery quantity of node j ($\forall j \in Nc$)
$[LS_s, US_s]$	Start and finish time of shift s ($\forall s \in S$)
$[a_j, b_j]$	Time window of node j ($\forall j \in Nc$)
Cap_v	Capacity of vehicle v in goods ($\forall v \in V$)
CEx_v	Cost of using unnecessary/extra vehicle v ($\forall v \in V$)
M	A large number

Table 8: Decision Variables of Decomposition Model

Decision Variable	Definition
$p_{js} \in \{0, 1\}$	1 iff customer j assigned to shift s ($\forall j \in Nc, \forall s \in S$)
$r_{vs} \in \{0, 1\}$	1 iff vehicle v assigned to shift s ($\forall v \in V, \forall s \in S$)
$sel_s \in \{0, 1\}$	1 iff s is selected shift ($\forall s \in S$)
$cd_s \in \mathbb{R}$	Real demand of shift s($\forall s \in S$)

Objective Function

The objective function (65) is to minimize the number of used vehicles for each shift.

$$\textbf{Min} \sum_{v \in V} \sum_{s \in S} r_{vs} \cdot CEx_v \tag{65}$$

subject to

Constraint (66) makes sure that each customer is assigned to a shift.

$$\sum_{j \in Nc | LS_s \leq a_j < US_s \ \vee \ LS_s < b_j \leq US_s} p_{js} = 1 \qquad \forall j \in Nc \tag{66}$$

Constraint (67) shows the real demand of each shift.

$$cd_s = \sum_{j \in Nc | LS_s \leq a_j < US_s \ \vee \ LS_s < b_j \leq US_s} D_j \cdot p_{js} \qquad \forall s \in S \tag{67}$$

Constraint (68) ensures that shift demand shall be covered by minimum number of vehicles.

$$\sum_{v \in V} Cap_v \cdot r_{vs} \geq cd_s \qquad \forall s \in S \tag{68}$$

If customer is assigned to the vehicle at shift s, then constraint (69) guarantees that the vehicle is located at that shift.

$$\sum_{s \in S} r_{vs} \leq 1 \qquad \forall v \in V \tag{69}$$

If real demand of shift s is greater than zero, constraint (70) ensures that shift is created.

$$cd_s \leq M \cdot sel_s \qquad \forall s \in S \tag{70}$$

4.4.2 MT-PVRPTW-SD-SPD After Decomposition

After obtaining three different shift sets, we introduce parameters and decision variables of MT-PVRPTW-SD-SPD for each shift. We no longer use shift parameter and solve below reconstructed mathematical model for each shift. Next, we sum three solutions derived by each shift to get overall cost.

Table 9: Parameters of MT-PVRPTW-SD-SPD After Decomposition

Parameter	Description
N	Set of nodes, i={0, 1,. . . ..,N}

Nc	Set of customer nodes, j={1, 2,. . . ..,N}
V	Set of vehicle, v={1, 2,. . . ..,V}
R	The number of tours in a shift, i={1, 2,. . . ..,R_{max}}
R_{max}	Maximum tour number
D_j	Demand or delivery quantity of node j ($\forall j \in Nc$)
P_j	Pick-up quantity from node j ($\forall j \in Nc$)
$[LS_s, US_s]$	Start and finish time of shift s ($\forall s \in S$)
$Dist_{ij}$	Distance between node i and node j ($\forall i, j \in N \mid i \neq j$)
$[a_j, b_j]$	Time window of node j ($\forall j \in Nc$)
Spd_v	Average speed of vehicle v ($\forall v \in V$)
$SrvT_v$	Service time of loading or unloading per goods for vehicle v ($\forall v \in V$)
Cap_v	Capacity of vehicle v in goods ($\forall v \in V$)
$CFix_{vr}$	Fixed cost of vehicle v in tour r ($\forall v \in V$,$\forall r \in R$)
CFs_v	Fixed cost of vehicle v in shift s ($\forall v \in V$)
Ckm_v	Cost per kilometer of vehicle v ($\forall v \in V$)
M	A large number

Table 10: Decision Variables of MT-PVRPTW-SD-SPD After Decomposition

Decision Variable	Description
$x_{rijv} \in \{0, 1\}$	1 iff vehicle v travels from node i to j in shift s in tour r ($\forall r \in R, \forall i, j \in N \mid i \neq j, \forall v \in V$)
$q_{rjv} \in \mathbb{R}$	Number of goods supplied to node j by vehicle v in shift s in tour r ($\forall r \in R, \forall j \in Nc, \forall v \in V$)
$z_{rjv} \in \mathbb{R}$	Number of returned goods at node j loaded to vehicle v in shift s in tour r ($\forall r \in R, \forall j \in Nc, \forall v \in V$)
$w_{rv} \in \{0, 1\}$	1 iff vehicle v is used in shift s in tour r ($\forall r \in R, \forall v \in V$)
$y_{rjv} \in \{0, 1\}$	1 iff vehicle v visit node j in shift s in tour r ($\forall r \in R, \forall j \in Nc, \forall v \in V$)
$st_{riv} \in \mathbb{R}$	Service start time of vehicle v at node i in shift s in tour r($\forall r \in R, \forall i \in N, \forall v \in V$)

$l_{riv} \in \mathbb{R}$	Load of vehicle v after leaving node i in shift s in tour r ($\forall r \in R, \forall i \in N, \forall v \in V$)
$tt_{rv} \in \mathbb{R}$	Travel time of vehicle v in shift s in tour r ($\forall r \in R, \forall v \in V$)
$u_{riv} \in \{0, 1\}$	Additional variable for sub tour eliminations ($\forall r \in R, \forall i \in Nc, \forall v \in V$)

Objective Function

The objective function (71) is to minimize the number of used vehicles by promoting them to serve customers by multi-trips in their working shifts and to minimize total distance traveled by vehicles during shift.

$$\begin{aligned} \textbf{Min} \sum_{r \in R} \sum_{i,j \in N} \sum_{v \in V} x_{rijv} \cdot Dist_{ij} \cdot Ckm_v + \sum_{r \in R} \sum_{j \in Nc} \sum_{v \in V} x_{r0jv} \cdot CFs_v \\ - \sum_{r \in 2..Rmax} \sum_{v \in V} w_{rv} \cdot CFix_{vr} \end{aligned} \tag{71}$$

subject to

If vehicle goes from node i to customer j, constraint (72) assures that the vehicle can supply split delivery of the customer.

$$q_{rjv} \leq \sum_{i \in N} M \cdot x_{rijv} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V \tag{72}$$

If vehicle goes from node i to customer j, constraint (73) guarantees that the vehicle can get returns of the customer as split pick-up.

$$z_{rjv} \leq \sum_{i \in N} M \cdot x_{rijv} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V \tag{73}$$

Constraint (74) guarantees that demands of each customer must be satisfied.

$$D_j = \sum_{r \in R} \sum_{v \in V} q_{rjv} \qquad \forall j \in Nc \tag{74}$$

Constraint (75) guarantees that pick-up requests of each customer must be satisfied.

$$P_j = \sum_{r \in R} \sum_{v \in V} z_{rjv} \qquad \forall j \in Nc \tag{75}$$

Constraint (76) expresses that each customer must be visited at least once.

$$\sum_{r \in R} \sum_{i \in N | i \neq j} \sum_{v \in V} x_{rijv} \geq 1 \qquad \forall j \in Nc \tag{76}$$

Constraint (77) expresses that vehicle can start consecutive tour after completing previous tour in a shift.

$$\sum_{j \in Nc} x_{(r-1)0jv} \geq \sum_{j \in Nc} x_{r0jv} \qquad \forall rin\{2, .., Rmax\}, v \in V \tag{77}$$

Constraints (78) and (79) guarantee that each vehicle should start and end its route in depot

$$\sum_{j \in Nc} x_{r0jv} = w_{rv} \qquad \forall r \in R, \forall v \in V \tag{78}$$

$$\sum_{j \in Nc} x_{rj0v} = w_{rv} \qquad \forall r \in R, \forall v \in V \tag{79}$$

Constraints (80) and (81) guarantee that if vehicle visits a node, it must leave that node.

$$\sum_{j \in N} x_{rjiv} = y_{riv} \qquad \forall r \in R, \forall i \in Nc, \forall v \in V \tag{80}$$

$$\sum_{j \in N} x_{rijv} = y_{riv} \qquad \forall r \in R, \forall i \in Nc, \forall v \in V \tag{81}$$

Constraints (82) and (83) avoid sub tours.

$$u_{riv} - u_{rjv} + N \cdot x_{rijv} \leq N - 1 \qquad \forall r \in R, \forall i, j \in Nc, \forall v \in V \tag{82}$$

$$u_{riv} \geq 0 \qquad \forall r \in R, \forall i \in Nc, \forall v \in V \tag{83}$$

Constraint (84) determines the initial quantity when vehicle leave depot.

$$l_{r0v} \geq \sum_{j \in Nc} q_{rjv} \qquad \forall r \in R, \forall v \in V \tag{84}$$

Constraint (85) states the vehicle load at customer j cannot exceed vehicle capacity.

$$l_{rjv} \leq Cap_v \cdot y_{rjv} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V \tag{85}$$

Constraint (86) indicates that the load quantity in the vehicle before leaving from customer. In other words, the picked-up quantities plus the demand quantities still to be delivered presents the load l_{rjvs} for each customer j.

$$\begin{gathered} l_{rjv} \geq l_{riv} - q_{rjv} + z_{rjv} - M \cdot (1 - x_{rijv}) \\ \forall r \in R, \forall i \in N, \forall j \in Nc | i \neq j, \forall v \in V \end{gathered} \tag{86}$$

Constraint (87) ensures that the vehicle load at ith node is less than the capacity of used vehicle.

$$\sum_{j \in Nc} q_{rjv} - q_{riv} + z_{riv} \leq Cap_v \cdot w_{rv} \qquad \forall r \in R, \forall i \in Nc, \forall v \in V \tag{87}$$

Constraint (88) ensures that the start time of service at depot is in the shift time interval of shift s.

$$LS_s \cdot w_{rv} \leq st_{r0v} \leq US_s \cdot w_{rv} \qquad \forall r \in R, \forall v \in V \tag{88}$$

Constraint (89) ensures that start time of service at jth node is greater than the sum of start time at ith node, time for travelling between node i and j, and spent time for delivering and taking the returns from node j.

$$\begin{aligned} st_{rjv} \geq st_{riv} + (Dist_{ij}/Spd_v) + SrvT_v \cdot (q_{rjv} + z_{rjv}) - M \cdot (1 - x_{rijv}) \\ \forall r \in R, \forall i \in N, \forall j \in Nc | i \neq j, \forall v \in V \end{aligned} \tag{89}$$

Constraint (90) determines start time for the next tour.

$$st_{r0v} \geq st_{(r-1)iv} + (Dist_{j0}/Spd_v) + SrvT_v \cdot (q_{rjv} + z_{rjv}) - M \cdot (1 - x_{rj0v})$$
$$\forall r \in \{2, .., Rmax\}, \forall i \in N, \forall j \in Nc | i \neq 0, \forall v \in V \tag{90}$$

Constraint (91) states that the start time of service should be within time window.

$$a_j \cdot y_{rjv} \leq st_{rjv} \leq b_j \cdot y_{rjv} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V \tag{91}$$

Constraint (92) ensures that each tour starts and ends at the same shift.

$$LS_s \cdot y_{rjv} \leq st_{rjv} \leq US_s \cdot y_{rjv} \qquad \forall r \in R, \forall j \in Nc, \forall v \in V \tag{92}$$

Constraint (93) determines total travel time of each vehicle.

$$tt_{rv} \geq st_{rjv} + (Dist_{j0}/Spd_v) + SrvT_v \cdot (q_{rjv} + z_{rjv}) - M \cdot (1 - x_{rj0v}) - st_{r0v}$$
$$\forall r \in R, \forall j \in Nc, \forall v \in V \tag{93}$$

Constraint (94) ensures that the sum of total travel time for each rank cannot exceed the shift time for every vehicle.

$$\sum_{r \in R} tt_{rv} \leq (US_s - LS_s) \qquad \forall v \in V \tag{94}$$

CHAPTER V

COMPUTATIONAL STUDY AND RESULTS

In this section, the computational studies on the Solomon data, which are available at http://www.vrp-rep.org/references/item/solomon-1987.html and widely used as a benchmark since 1987, are performed to interpret the results of proposed approaches described in previous section. We used solution quality and computational performances as comparison parameters. The results of the decomposition method become the inputs for our vehicle routing problem. The proposed mathematical model and heuristic methods are solved using CPLEX 12.10 with the time limit of 3600 seconds (1 hour). All of the results are obtained from a Windows 10 operating system with Intel®Core ™i5-430M 2.5 GHz processor and 6 GB RAM.

5.1 Data

To conduct experimental analysis, we use the values provided in the Solomon data set, which are elaborated in Appendix A and acquired by Solomon (1987). We modify these benchmark values to generate appropriate instances for our model. There are six groups of problems: R1, R2, C1, C2, RC1 and RC2 as explained in Rieck and Zimmermann (2013).

- The position of customers is generated randomly with a uniform distribution in groups R1 and R2.
- In groups C1 and C2, the customers are placed in clusters.
- In groups RC1 and RC2, some of the customers are placed randomly and some are placed in clusters, so the RC groups are the mix of clustered and randomly created customer locations.

- Groups R1, C1 and RC1 have a tight time windows, short service time, homogeneous fleet of vehicles having 200 units of capacity so that they allow only some customers in each route.

- Groups R2, C2 and RC2 have longer planning horizon, longer service time, and they can supply more customers than first group (R1, C1, RC1) for each route because they have higher capacity fleet of vehicles varying from 100 to 1000 units.

- In each group, location of the customers, their demands and the service time for each node do not change.

In addition to the customer location, service time, customer demand, time window of each customer, service time, maximum number of available vehicles and vehicle capacities defined by Solomon (1987), the parameters to be used in our problem are determined as follows. Our problem consisting of 25 vehicles was generated with 5 different types of vehicles moving at an average speed of 40, 45, 50, 60, 70 km/h. The customers and depot are located at different coordinates on 100x100 grid. The data P_j for the pick-up quantity is generated randomly in the range of $\pm 50\,\%$ of the demand quantities given in Solomon instances, so we obtain customers having pick-up quantity greater and less than their demand quantity. Overall working time is divided into three time periods such as morning, afternoon and night as shifts and so, there occur 3 working shifts as in real industries. Vehicles are allowed to perform maximum 3 trips in their working shifts. Shift duration is generated by applying decomposition approach to Solomon's data. Fixed costs for morning, afternoon and night shifts are 1500TL, 2000TL and 2500TL respectively. Moreover, fixed costs of vehicles in the tours are considered as 500TL and traveling cost per kilometer differs from 0.5TL to 0.95TL as per vehicle.

5.2 *Results of Exact Models*

In this section we evaluate the results of three exact models and observe the reaction of them for different Solomon data sets. We use Solomon data sets called as C101, C201, R101, R201, RC101 and RC201. This means that we discuss clustered, randomly located and the mix of clustered and randomly located customers. Each set contains 25 customers located in different places and vehicle capacities differ from one set to another one.

5.2.1 Results of Model 1 - PVRPTW-SPD

We carry out tests for PVRPTW-SPD model and obtained results are shown in Table 11. We observe that model does not propose a solution when the customers are not randomly. It better works for data sets having clustered customers.

We execute tests in below table where we work on clustered, randomly scattered, mix of clustered and randomly scattered customers existing on the coordinate plane. We use vehicles available in Solomon data having capacities of 200, 700 and 1000. The number of constraints, integer variables and binary variables are the same for all test sets and less than of Model 2 and Model 3 since the complexity of the problem is lower. We obtain the best integer and best-bound values of the solutions that are called upper and lower bound respectively. We define gap as the absolute difference between upper and lower bounds divided by absolute value of upper bound. Our solution time limit for all instances is 3600 seconds. We call each data set starting with Mx where x is the model number. Then we specify what kind of Solomon data set we used (i.e. R1, R2, C1, C2, RC1, or RC2), and the last part indicates how many customers are in the set.

Table 11: Test Results of PVRPTW-SPD Model

Set Name	Vehicle Capacity	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M1_C101_25	200	154225	3825	52650	3200.96	3200.96	0%	102.22
M1_C201_25	700	154225	3825	52650	2922.20	2922.20	0%	1047.05
M1_R101_25	200	154225	3825	52650	3354.99	2472.29	26.31%	3600
M1_R201_25	1000	154225	3825	52650	2298.61	1802.75	21.57%	3600
M1_RC101_25	200	154225	3825	52650	-	1510.31	-	3600
M1_RC201_25	1000	154225	3825	52650	-	1635.76	-	3600

5.2.2 Results of Model 2 - PVRPTW-SD-SPD

We use six different Solomon data sets as inputs for PVRPTW-SD-SPD model and get results as specified in Table 12.

Table 12: Test Results of PVRPTW-SD-SPD Model

Set Name	Vehicle Capacity	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M2_C101_25	200	302250	7575	52650	3722.89	3722.89	0%	1250.44
M2_C201_25	700	302250	7575	52650	2694.72	2694.72	0%	1784.20
M2_R101_25	200	302250	7575	52650	3930.81	2585.11	34.23%	3600

M2_R201_25	1000	302250	7575	52650	2298.61	1802.75	21.57%	3600
M2_RC101_25	200	302250	7575	52650	4478.30	1701.79	62%	3600
M2_RC201_25	1000	302250	7575	52650	1841.10	1841.10	0%	2089.84

5.2.3 Results of Model 3 - MT-PVRPTW-SD-SPD

This model has been firstly tested with different data having different customer numbers using C101 data set. We observed that as the number of customers increases in the model, percentage of gap in a given time limit increases too. It is explicitly seen that model barely finds solution when there are more than 15 customers in the input. The results of test executions are given in Table 13. As it is seen from the table, when the number of customers increases, the number of constraints, binary and integer variables increases and it turns out that it is hard to solve such a problem.

Table 13: Test Results of MT-PVRPTW-SD-SPD with Different Number of Customers

Number of Customers	Total Demand of Customers	Total Pick-up quantity of Customers	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
5	70	62	9420	945	1890	11093.16	11093.16	0%	1.26
10	150	146	62790	3690	11880	11100.93	11100.93	0%	8.42
15	260	269	198360	8235	36720	14676.64	12703.50	13.44%	3600
20	360	372	454380	14580	83160	14710.99	13202.23	10.26%	3600

25	460	578	869100	22725	157950	14724.50	10775.76	26.82%	3600

Next, we check how this mathematical model responds to different type of data sets as shown in Table 14. Model can count best bound but cannot yield solution for most of the data sets. Therefore, we propose a new approach since it is obviously understood that we need to develop a method to find solution for such complex instances.

Table 14: Test Results of MT-PVRPTW-SD-SPD Model

Set Name	Vehicle Capacity	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_C101_25	200	869100	22725	157950	14724.50	10775.76	26.82%	3600
M3_C201_25	700	869100	22725	157950	-	6241.63	-	3600
M3_R101_25	200	869100	22725	157950	-	6396.46	-	3600
M3_R201_25	1000	869100	22725	157950	-	6261.44	-	3600
M3_RC101_25	200	869100	22725	157950	-	4920.36	-	3600
M3_RC201_25	1000	869100	22725	157950	-	6212.55	-	3600

5.2.4 Results of Heuristic Models

We propose a decomposition based heuristic model. This method takes initial data and tries to create three shift sets by evaluating customer location, demand and time window. At the end it gives the number of customers and vehicles in each set by taking customer pick-up and demand quantities into consideration.

With the purpose of finding solution for data having more than 15 customers, we propose a decomposition approach to have less number of customers in input data set by dividing initial big data set having 25 customers into three sets and then process these cluster sets in updated model as shown in Figure 3. Thanks to this approach, our model deals with three small data sets and it can give solution within an acceptable duration. It does not assign unnecessary or extra vehicles than required in order to be for the benefit of cost minimization objective function.

We show decomposition results of each data set in Table 15. M3 stands for Model 3 and next to it we refer to Solomon data type (R1.., R2.., C1.., C2.., RC1.., RC2..) we used. We call final result of decomposition model as "_Total".

The results of heuristic model for given 25 customers with different Solomon data is detailed more in Appendix B and results for given 50 customers are available in Appendix C.

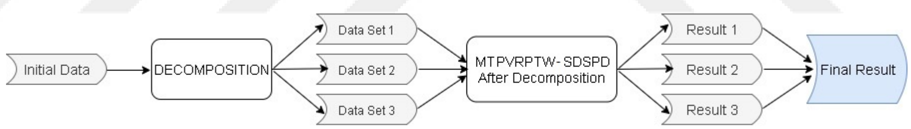

Figure 3: Decomposition and Updated Model Processing

Table 15: Test Results of MT-PVRPTW-SD-SPD Model After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_C101_Total	9481	618	1788	12215.55	12215.55	0%	8

M3_C201_Total	4726	309	882	6503.53	6503.53	0%	1.5
M3_R101_Total	4648	1278	876	8416.65	8416.65	0%	1.91
M3_R201_Total	4794	309	894	6443.26	6443.26	0%	20.05
M3_RC101_Total	4614	309	870	6624.51	6624.51	0%	31.65
M3_RC201_Total	4614	309	870	6622.63	6622.63	0%	3.39

We analyze mathematical model results in Table 14. It shows that as the location of customers is scattered, mathematical model cannot provide best integer value within 3600 seconds. It can only show lower bound result and cannot offer a solution. On the other hand, when we apply heuristic approach, we find solution for all data sets within a very short time compared to 3600 seconds, so this leads to zero gap as indicated in Table 15. We also notice that data set type has impact on the solution time. When customers are close to each other so they are clustered in an area, model easily gives solution. However, if customers are randomly located, model spend more time to yield result.

5.3 Comparison of Model 3 and Model 3 After Decomposition

In this section we deal with MT-PVRPTW-SD-SPD from shift aspect. Namely, we check route solution of each shift in C101 solution of Model 3. Next, we take the results of each shift of Model 3 after decomposition and then, we compare route solutions between the two models.

In the first shift of Model 3, two vehicles are assigned and these vehicles serve customers that are located in the Cartesian coordinates (x,y) as depicted in Figure 4.

Second shift of Model 3 uses 2 vehicles and these vehicles visits 9 different customers to cover their demands as in Figure 5.

Model 3 stops by 3 different customers by using 2 vehicles in the third shift of working day as shown in Figure 6. Vehicle 2 visits the same customer which is served by first vehicle as well since the first vehicle does not have enough time to pick-up all quantities of that customer.

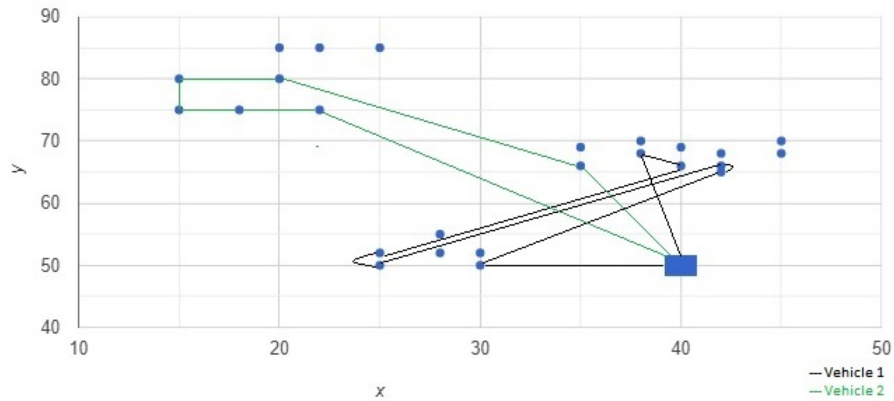

Figure 4: Illustration of Shift 1 of Model 3

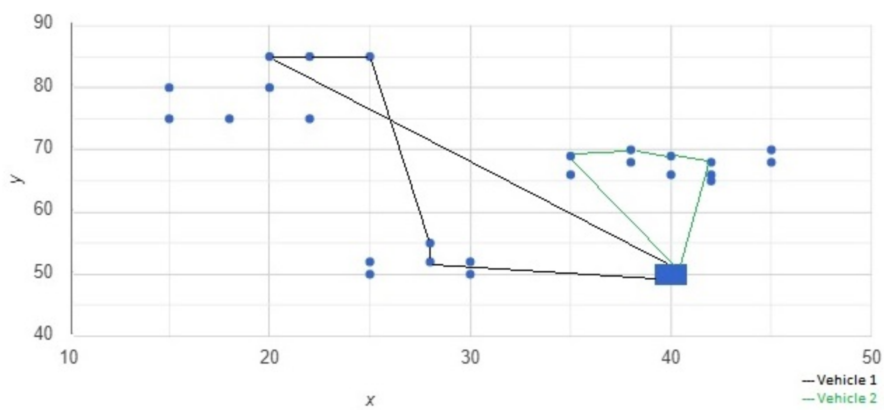

Figure 5: Illustration of Shift 2 of Model 3

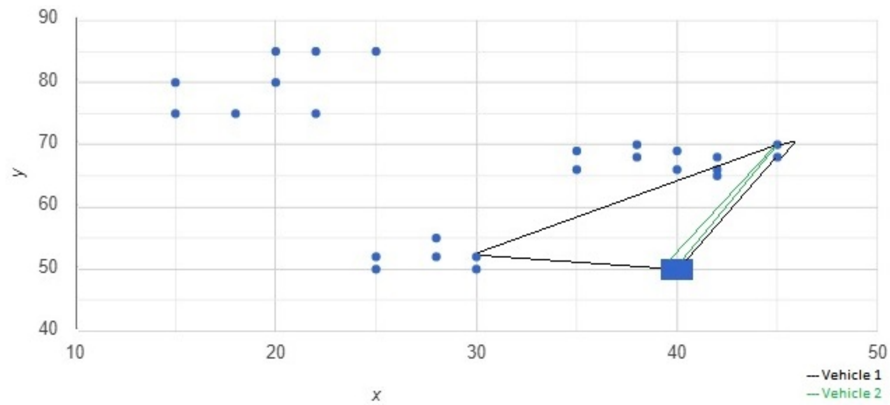

Figure 6: Illustration of Shift 3 of Model 3

As an alternative way to find solution for big data, we define decomposition based heuristic which creates three shifts. Figures 7, 8, 9 point out an advantage coming from the proposed heuristic which is using only one vehicle to serve all customers in the defined shift by performing multi-trips.

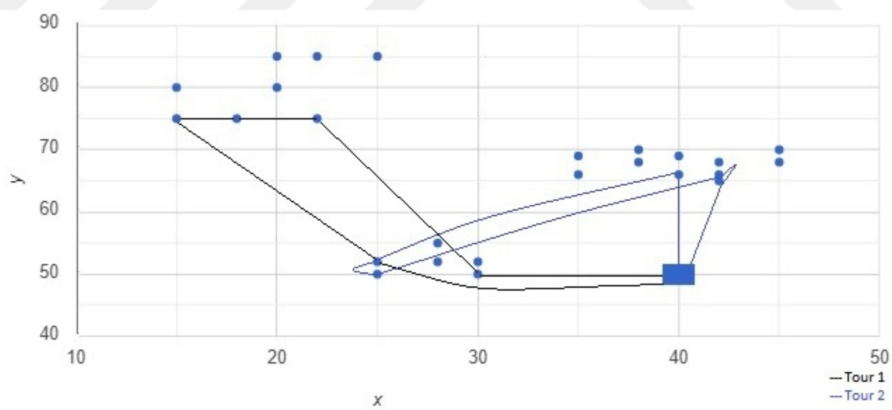

Figure 7: Illustration of Shift 1 of Model 3 After Decomposition

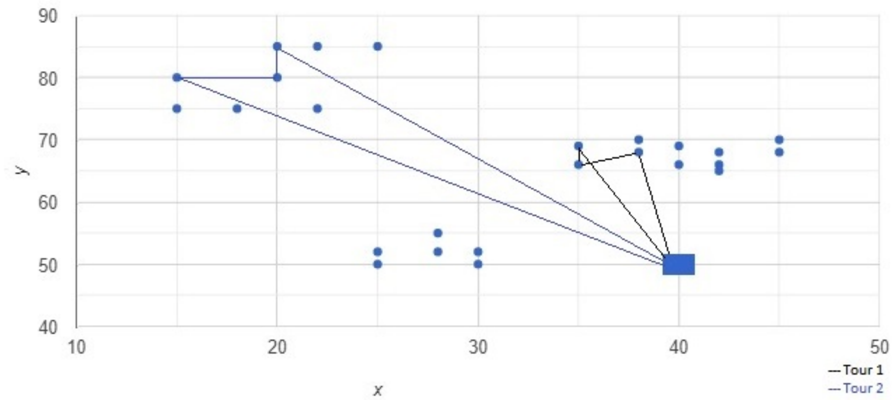

Figure 8: Illustration of Shift 2 of Model 3 After Decomposition

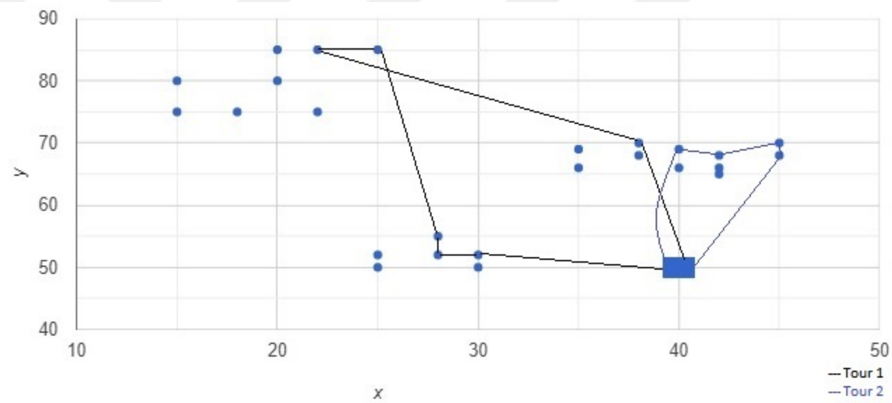

Figure 9: Illustration of Shift 3 of Model 3 After Decomposition

CHAPTER VI

CONCLUSION AND FURTHER RESEARCH

In this paper, we consider a different types of the typical vehicle routing problem. We offer three mixed integer linear programming formulations. First model, called as Capacitated Periodic VRP with Time Windows for Simultaneous Pick-up and Delivery (PVRPTW-SPD), gives a solution and searches for minimum logistics cost considering shortest path for vehicles, available times of customers and shift times of vehicles. Secondly, Capacitated Periodic VRP with Time Windows, Split Delivery for Simultaneous Pick-up and Delivery (PVRPTW-SD-SPD), adds split delivery characteristic to meet pick-up and demand quantities of suppliers by using different vehicles. As a third step, we develop Multi-Trip Periodic VRP with Time Windows, Split Delivery for Simultaneous Pick-up and Delivery (or MT-PVRPTW-SD-SPD). We describe a problem in which capacity limited vehicles are allowed to perform at least one tour in a shift in order to meet foreknown customer demands and receive back pick-up quantities. Our problem combines pick-ups and deliveries with split deliveries, so this makes it more challenging. Vehicles start to serve customers before and after the earliest and latest time window bounds, respectively. In order to solve large instance, we develop decomposition based heuristic which aims to create three different shift sets under defined constraints in the model. This model has the ability to assign customer to the proper shift set when time window of that customer overlaps with the time duration of two shifts. We take the whole data and generate three clusters taking customer time windows into consideration by using our heuristic based model. We eliminate shift parameter thanks to this classification and deal with multi trip, split pick-up delivery, time windows, capacity constraints. Then we process each shift data and find VRP solution which basically defines the order of customers to be served.

Furthermore, we also design the test instances derived from the literature of vehicle routing problem. We carry out experimental comparisons over these data sets. We observe that there are different advantages of these three models such as PVRPTW-SPD can solve problems quickly, which can save time when data become more complex. We figure out that it can give results for clustered data easily compared to random and random-clustered mix. On the other hand, PVRPTW-SD-SPD can yield a more optimal objective result to all three kinds (C, R, RC) of data but spends more time to give a solution. The third model, MT-PVRPTW-SD-SPD, find it difficult to solve since the total constraints tackled by this model are almost the triple of the second model. In order to decrease the number of constraints and the iterations, we eliminate shift constraint by a new approach called decomposition heuristic. This approach yields three shift sets and we process them separately to find the optimal solution. The advantage of this approach is that it can get a solution for all kinds of data even though the third model cannot find within the same solution time. The computational results validate that the proposed model solves the given data in a better way and shows results more quickly under complicated constraints. Namely, outputs show that our model provides good quality solutions to the defined problem, while making reasonable computational efforts. Also, most instances are solved close to the optimum point and vehicle utilization decimated.

The proposed model approach can be used in companies having three working shifts and need to deliver and receive back products from different points which have different types of quantities to be handled by simultaneous split pick-up and delivery. Additionally, this paper can be extended in several aspects. Our model can be modified to take into account more realistic aspects of the MT-PVRPTW-SD-SPD with multiple depots, such as combining our model with the problem of optimizing transport flows between depots nearest to the closest customer set and finding the optimal set of vehicle routes. It would also be worthwhile to investigate some heuristic methods on the performance of the suggested model. For example, Large Neighborhood Search (LNS) heuristic, which is firstly suggested by Shaw (1998). LNS method tries to develop an initial solution by performing both destroy and fix neighborhood operators. Namely, LNS contains a bunch of removal and insertion processed. If the method finds a new solution is better than the best solution currently, the current solution is replaced with this new current solution. Also, this is used as an input in the next step to find another better solution. More improved version of this heuristic is Adaptive Large Neighborhood Search (ALNS) which firstly defined byPisinger and Ropke (2007) to solve various vehicle routing problems. Unlike LNS which uses one large neighborhood, the ALNS applies many removal and insertion processes to a given initial solution. The neighborhood of a given set of feasible routes is checked by taking out some requests and put them back to the solution. The decision of which operator to use for removal and insertion processes is made as per past and current performances of the operators. For this purpose, each operator takes a probability of being chosen. If the new solution complies with the acceptance criteria (e.g., simulated annealing, tabu search) of the defined ALNS heuristic, it is considered as a solution.

All in all, our model can propose solutions to the problem introduced in previous sections within a reasonable time and quality even if previous models cannot find a result. It can also open new doors to the researches conducted on popular heuristics such as LNS and ALNS recently and be a new method for complex instances.

APPENDIX A

INPUT DATA

Table 16: C101 Test Data

CUST NO.	XCOORD.	YCOORD.	DEMAND	PICKUP	READY TIME	DUE DATE	SERVICE TIME
0	40	50	0	0	0	1236	0
1	45	68	10	13	912	967	90
2	45	70	30	34	825	870	90
3	42	66	10	11	65	146	90
4	42	68	10	15	727	782	90
5	42	65	10	11	15	67	90
6	40	69	20	19	621	702	90
7	40	66	20	27	170	225	90
8	38	68	20	29	255	324	90
9	38	70	10	13	534	605	90
10	35	66	10	13	357	410	90
11	35	69	10	10	448	505	90

12	25	85	20	27	652	721	90
13	22	75	30	42	30	92	90
14	22	85	10	14	567	620	90
15	20	80	40	54	384	429	90
16	20	85	40	59	475	528	90
17	18	75	20	21	99	148	90
18	15	75	20	30	179	254	90
19	15	80	10	14	278	345	90
20	30	50	10	5	10	73	90
21	30	52	20	28	914	965	90
22	28	52	20	14	812	883	90
23	28	55	10	10	732	777	90
24	25	50	10	10	65	144	90
25	25	52	40	55	169	224	90

Table 17: C201 Test Data

CUST NO.	XCOORD.	YCOORD.	DEMAND	PICKUP	READY TIME	DUE DATE	SERVICE TIME
0	40	50	0	0	0	3390	0

1	52	75	10	13	311	471	90
2	45	70	30	29	213	373	90
3	62	69	10	11	1167	1327	90
4	60	66	10	8	1261	1421	90
5	42	65	10	8	25	185	90
6	16	42	20	29	497	657	90
7	58	70	20	13	1073	1233	90
8	34	60	20	16	2887	3047	90
9	28	70	10	11	2601	2761	90
10	35	66	10	6	2791	2951	90
11	35	69	10	7	2698	2858	90
12	25	85	20	23	2119	2279	90
13	22	75	30	23	2405	2565	90
14	22	85	10	11	2026	2186	90
15	20	80	40	50	2216	2376	90
16	20	85	40	36	1934	2094	90
17	18	75	20	23	2311	2471	90
18	15	75	20	15	1742	1902	90
19	15	80	10	14	1837	1997	90
20	30	50	10	6	10	170	90

21	30	56	20	15	2983	3143	90
22	28	52	20	18	22	182	90
23	14	66	10	10	1643	1803	90
24	25	50	10	13	116	276	90
25	22	66	40	38	2504	2664	90

Table 18: R101 Test Data

CUST NO.	XCOORD.	YCOORD.	DEMAND	PICKUP	READY TIME	DUE DATE	SERVICE TIME
0	35	35	0	0	0	230	0
1	41	49	10	15	161	171	10
2	35	17	7	8	50	60	10
3	55	45	13	9	116	126	10
4	55	20	19	24	149	159	10
5	15	30	26	37	34	44	10
6	25	30	3	3	99	109	10
7	20	50	5	3	81	91	10
8	10	43	9	12	95	105	10
9	55	60	16	22	97	107	10

10	30	60	16	24	124	134	10
11	20	65	12	14	67	77	10
12	50	35	19	16	63	73	10
13	30	25	23	24	159	169	10
14	15	10	20	22	32	42	10
15	30	5	8	5	61	71	10
16	10	20	19	26	75	85	10
17	5	30	2	3	157	167	10
18	20	40	12	6	87	97	10
19	15	60	17	11	76	86	10
20	45	65	9	12	126	136	10
21	45	20	11	7	62	72	10
22	45	10	18	22	97	107	10
23	55	5	29	42	68	78	10
24	65	35	3	3	153	163	10
25	65	20	6	5	172	182	10

Table 19: R201 Test Data

CUST NO.	XCOORD.	YCOORD.	DEMAND	PICKUP	READY TIME	DUE DATE	SERVICE TIME
0	35	35	0	0	0	1000	0
1	41	49	10	13	707	848	10
2	35	17	7	9	143	282	10
3	55	45	13	7	527	584	10
4	55	20	19	11	678	801	10
5	15	30	26	30	34	209	10
6	25	30	3	4	415	514	10
7	20	50	5	5	331	410	10
8	10	43	9	13	404	481	10
9	55	60	16	12	400	497	10
10	30	60	16	21	577	632	10
11	20	65	12	16	206	325	10
12	50	35	19	19	228	345	10
13	30	25	23	25	690	827	10
14	15	10	20	23	32	243	10
15	30	5	8	5	175	300	10
16	10	20	19	13	272	373	10

17	5	30	2	3	733	870	10
18	20	40	12	15	377	434	10
19	15	60	17	15	269	378	10
20	45	65	9	8	581	666	10
21	45	20	11	16	214	331	10
22	45	10	18	15	409	494	10
23	55	5	29	38	206	325	10
24	65	35	3	2	704	847	10
25	65	20	6	8	817	956	10

Table 20: RC101 Test Data

CUST NO.	XCOORD.	YCOORD.	DEMAND	PICKUP	READY TIME	DUE DATE	SERVICE TIME
0	40	50	0	0	0	240	0
1	25	85	20	23	145	175	10
2	22	75	30	19	50	80	10
3	22	85	10	8	109	139	10
4	20	80	40	37	141	171	10
5	20	85	20	10	41	71	10

6	18	75	20	24	95	125	10
7	15	75	20	10	79	109	10
8	15	80	10	14	91	121	10
9	10	35	20	25	91	121	10
10	10	40	30	24	119	149	10
11	8	40	40	58	59	89	10
12	8	45	20	28	64	94	10
13	5	35	10	6	142	172	10
14	5	45	10	11	35	65	10
15	2	40	20	25	58	88	10
16	0	40	20	10	72	102	10
17	0	45	20	18	149	179	10
18	44	5	20	14	87	117	10
19	42	10	40	24	72	102	10
20	42	15	10	6	122	152	10
21	40	5	10	11	67	97	10
22	40	15	40	53	92	122	10
23	38	5	30	37	65	95	10
24	38	15	10	7	148	178	10
25	35	5	20	12	154	184	10

Table 21: RC201 Test Data

CUST NO.	XCOORD.	YCOORD.	DEMAND	PICKUP	READY TIME	DUE DATE	SERVICE TIME
0	40	50	0	0	0	960	0
1	25	85	20	16	673	793	10
2	22	75	30	43	152	272	10
3	22	85	10	5	471	591	10
4	20	80	40	53	644	764	10
5	20	85	20	26	73	193	10
6	18	75	20	17	388	508	10
7	15	75	20	30	300	420	10
8	15	80	10	6	367	487	10
9	10	35	20	24	371	491	10
10	10	40	30	21	519	639	10
11	8	40	40	59	195	315	10
12	8	45	20	11	223	343	10
13	5	35	10	11	653	773	10
14	5	45	10	15	35	155	10
15	2	40	20	26	174	294	10

16	0	40	20	24	255	375	10
17	0	45	20	19	703	823	10
18	44	5	20	22	335	455	10
19	42	10	40	46	254	374	10
20	42	15	10	6	537	657	10
21	40	5	10	14	215	335	10
22	40	15	40	59	375	495	10
23	38	5	30	36	201	321	10
24	38	15	10	12	681	801	10
25	35	5	20	18	784	904	10

APPENDIX B

DETAILED DECOMPOSITION RESULTS FOR 25 CUSTOMERS

In this section, we elaborate more on decomposition result of 25 customers. After decomposition, we obtain three different shift sets. Firstly, we define the number of customers and vehicles for each shift set. The first row of the next table includes mathematical model result for the same data with 25 customers. In the same table, we also present results for three shift sets separately. Then, we get final result of decomposition model and put a name it as "_Total". Shift set naming keeps the information of Solomon data type (R1.., R2.., C1.., C2.., RC1.., RC2..), number of customers assigned and shift number (S1, S2, S3) respectively.

Table 22: Decomposition Result of C101

Set Name	Number of Customers in Set	Number of Vehicles in Set
C101_9_S1	9	1
C101_6_S2	6	1
C101_10_S3	10	1

Table 23: C101 Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_C101_25	869100	22725	157950	14724.50	10775.76	26.82%	3600
M3_C101_Total	9481	618	1788	12215.55	12215.55	0%	8
M3_C101_9_S1	3489	222	660	3080.73	3080.73	0%	5
M3_C101_6_S2	1742	150	336	4066.87	4066.87	0%	1
M3_C101_10_S3	4250	246	792	5067.96	5067.96	0%	2

Table 24: Decomposition Result of C201

Set Name	Number of Customers in Set	Number of Vehicles in Set
C201_10_S1	10	1
C201_7_S2	7	1
C201_8_S3	8	1

Table 25: C201 Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)

M3_C201_25	869100	22725	157950	-	6241.63	-	3600
C201_Total	4726	309	882	6503.53	6503.53	0%	1.5
C201_10_S1	2140	123	396	1712.07	1712.07	0%	0.5
C201_7_S2	1144	87	216	2251.21	2251.21	0%	0.5
C201_8_S3	1442	99	270	2540.25	2540.25	0%	0.5

Table 26: Decomposition Result of R101

Set Name	Number of Customers in Set	Number of Vehicles in Set
R101_7_S1	7	1
R101_9_S2	9	1
R101_9_S3	9	1

Table 27: R101 Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_R101_25	869100	22725	157950	-	6396.46	-	3600
R101_Total	4648	1278	876	8416.65	8416.65	0%	1.91
R101_7_S1	1132	87	216	1600.93	1600.93	0%	0.06
R101_9_S2	1758	111	330	4158.35	4158.35	0%	1.40
R101_9_S3	1758	111	330	2657.37	2657.37	0%	0.45

Table 28: Decomposition Result of R201

Set Name	Number of Customers in Set	Number of Vehicles in Set
R201_10_S1	10	1
R201_6_S2	6	1
R201_9_S3	9	1

Table 29: R201 Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_R201_25	869100	22725	157950	-	6261.44	-	3600
R201_Total	4794	309	894	6443.26	6443.26	0%	20.05
R201_10_S1	2140	123	396	1677.00	1677.00	0%	18.27
R201_6_S2	880	75	168	2097.18	2097.18	0%	0.39
R201_9_S3	1774	111	330	2669.09	2669.09	0%	1.39

Table 30: Decomposition Result of RC101

Set Name	Number of Customers in Set	Number of Vehicles in Set
RC101_8_S1	8	1
RC101_8_S2	8	1
RC101_9_S3	9	1

Table 31: RC101 Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_RC101_25	869100	22725	157950	-	4920.36	-	3600
RC101_Total	4614	309	870	6624.51	6624.51	0%	31.65
RC101_8_S1	1428	99	270	1618.30	1618.30	0%	0.7
RC101_8_S2	1428	99	270	2248.98	2248.98	0%	10.75
RC101_9_S3	1758	111	330	2757.24	2757.24	0%	20.2

Table 32: Decomposition Result of RC201

Set Name	Number of Customers in Set	Number of Vehicles in Set
RC201_8_S1	8	1
RC201_8_S2	8	1
RC201_9_S3	9	1

Table 33: RC201 Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_RC201_25	869100	22725	157950	-	6212.55	-	3600
RC201_Total	4614	309	870	6622.63	6622.63	0%	3.39
RC201_8_S1	1428	99	270	1633.15	1633.15	0%	0.86
RC201_8_S2	1428	99	270	2262.24	2262.24	0%	1.47
RC201_9_S3	1758	111	330	2757.24	2757.24	0%	1.06

APPENDIX C

RESULTS FOR 50 CUSTOMERS

Table 34: Decomposition Result of C101 with 50 customers

Set Name	Number of Customers in Set	Number of Vehicles in Set
C101_15_S1	15	2
C101_15_S2	15	2
C101_20_S3	20	2

Table 35: C101 with 50 Customers - Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_C101_50	3331050	45225	596700	-	6170.71	-	3600
M3_C101_Total	32922	1218	6036	12439.12	12439.12	0%	223
M3_C101_15_S1	8859	366	1632	3117.97	3117.97	0%	180
M3_C101_15_S2	8859	366	1632	4161.58	4161.58	0%	10
M3_C101_20_S3	15204	486	2772	5159.57	5159.57	0%	33

Table 36: Decomposition Result of C201 with 50 customers

Set Name	Number of Customers in Set	Number of Vehicles in Set
C201_16_S1	16	1
C201_17_S2	17	1
C201_17_S3	17	1

Table 37: C201 with 50 Customers - Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_C201_50	3331050	45225	596700	-	-	-	3600
M3_C201_Total	16264	609	2970	6876.275	6876.275	0%	60
M3_C201_16_S1	5020	195	918	1678.00	1678.00	0%	4
M3_C201_17_S2	5622	207	1026	2432.45	2432.45	0%	36
M3_C201_17_S3	5622	207	1026	2765.83	2765.83	0%	20

Table 38: Decomposition Result of R101 with 50 customers

Set Name	Number of Customers in Set	Number of Vehicles in Set
R101_16_S1	16	2
R101_15_S2	15	2
R101_19_S3	19	2

Table 39: R101 with 50 Customers - Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_R101_50	3331050	45225	596700	-	-	-	3600
M3_R101_Total	32838	1218	5988	12629.08	12629.08	0%	448
M3_R101_16_S1	10052	390	1836	3185.81	3185.81	0%	165
M3_R101_15_S2	8915	366	1632	4195.17	4195.17	0%	25
M3_R101_19_S3	13871	462	2520	5248.10	5248.10	0%	258

Table 40: Decomposition Result of R201 with 50 customers

Set Name	Number of Customers in Set	Number of Vehicles in Set
R201_16_S1	16	1
R201_16_S2	16	1
R201_18_S3	18	1

Table 41: R201 with 50 Customers - Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_R201_50	3331050	45225	596700	-	-	-	-
M3_R201_Total	16298	609	2976	7027.821	7027.821	0%	408
M3_R201_16_S1	5020	195	918	1782.12	1782.12	0%	210
M3_R201_16_S2	5020	195	918	2383.94	2383.94	0%	38
M3_R201_18_S3	6258	219	1140	2861.76	2861.76	0%	160

Table 42: Decomposition Result of RC101 with 50 customers

Set Name	Number of Customers in Set	Number of Vehicles in Set
RC101_17_S1	17	2
RC101_16_S2	16	2
RC101_17_S3	17	2

Table 43: RC101 with 50 Customers - Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)

M3_RC101_50	3331050	45225	596700	-	-	-	-
M3_RC101_Total	32566	1218	5940	12615.84	12562.83	1.48%	7637
M3_RC101_17_S1	11257	414	2052	3201.52	3172.76	0.90%	3600
M3_RC101_16_S2	10052	390	1836	4212.37	4188.12	0.58%	3600
M3_RC101_17_S3	11257	414	2052	5201.95	5201.95	0%	437

Table 44: Decomposition Result of RC201 with 50 customers

Set Name	Number of Customers in Set	Number of Vehicles in Set
RC201_18_S1	18	1
RC201_15_S2	15	1
RC201_17_S3	17	1

Table 45: RC201 with 50 Customers - Test Results After Decomposition

Set Name	Number of constraints	Number of integer variables	Number of binary variables	Best Integer	Best Bound - Objective	Gap	Solution Time-Time Limit (s)
M3_RC201_50	3331050	45225	596700	-	-	-	-
M3_RC201_Total	16426	609	2982	8766.035	7270.426	45.38%	7718
M3_RC201_18_S1	6292	219	1140	3296.08	1800.47	45.38%	3600
M3_RC201_15_S2	4480	183	816	2455.57	2455.57	0%	3016
M3_RC201_17_S3	5654	207	1026	3014.39	3014.39	0%	1102

Bibliography

Altinkemer, K. and B. Gavish

1990. Heuristics for delivery problems with constant error guarantees. *Transportation Science*, 24(4):294–297.

Angelelli, E. and M. G. Speranza

2002. The periodic vehicle routing problem with intermediate facilities. *European journal of Operational research*, 137(2):233–247.

Archetti, C. and M. G. Speranza

2007. An overview on the split delivery vehicle routing problem. In *Operations Research Proceedings 2006*, Pp. 123–127. Springer.

Archetti, C., M. G. Speranza, and A. Hertz

2006. A tabu search algorithm for the split delivery vehicle routing problem. *Transportation science*, 40(1):64–73.

Baldacci, R., E. Bartolini, A. Mingozzi, and A. Valletta

2011. An exact algorithm for the period routing problem. *Operations research*, 59(1):228–241.

Baldacci, R., M. Battarra, and D. Vigo

2008. Routing a heterogeneous fleet of vehicles. In *The Vehicle Routing Problem: Latest Advances and New Challenges*, volume 43 of *1*, Pp. 3–27. Springer.

Beasley, J. E.

1983. Route first—cluster second methods for vehicle routing. *Omega*, 11(4):403–408.

Belgin, O., I. Karaoglan, and F. Altiparmak

2018. Two-echelon vehicle routing problem with simultaneous pickup and delivery: Mathematical model and heuristic approach. *Computers & Industrial Engineering*, 115:1 – 16.

Bodin, L., B. Golden, A. Assad, and M. Ball

1983. Routing and scheduling of vehicles and crews - the state of the art. volume 10, Pp. 63–212. Computers and Opertions Research.

Bracken, J. and G. P. McCormick

1968. Selected applications of nonlinear programming. Technical report, Research Analysis Corp Mclean va.

Braekers, K., K. Ramaekers, and I. Van Nieuwenhuyse

2016a. The vehicle routing problem: State of the art classification and review. *Computers & Industrial Engineering*, 99:300 – 313.

Braekers, K., K. Ramaekers, and I. Van Nieuwenhuyse

2016b. The vehicle routing problem: State of the art classification and review. *Computers & Industrial Engineering*, 99:300–313.

Bräysy, O. and M. Gendreau

2005. Vehicle routing problem with time windows, part i: Route construction and local search algorithms. *Transportation science*, 39(1):104–118.

Bun, A.

2008. Comparing vehicle routing approaches with artificial intelligence approaches on solving instances of the logistics planning problem domain.

Campbell, A. M. and M. Savelsbergh

2004. Efficient insertion heuristics for vehicle routing and scheduling problems. *Transportation science*, 38(3):369–378.

Cattaruzza, D., N. Absi, D. Feillet, and T. Vidal

2014. A memetic algorithm for the multi trip vehicle routing problem. *European Journal of Operational Research*, 236(3):833–848.

Christofides, N. and J. E. Beasley

1984. The period routing problem. *Networks*, 14(2):237–256.

Christofides, N., A. Mingozzi, and P. Toth

1981. State-space relaxation procedures for the computation of bounds to routing problems. *Networks*, 11(2):145–164.

Cordeau, J.-F., M. Gendreau, G. Laporte, J.-Y. Potvin, and F. Semet

2002. A guide to vehicle routing heuristics. *Journal of the Operational Research society*, 53(5):512–522.

Dabia, S., S. Røpke, T. Van Woensel, and T. De Kok

2011. Branch and cut and price for the time dependent vehicle routing problem with time windows. In *Agra, Agostinho and Doostmohammadi, Mahdi (2011) A Polyhedral Study of Mixed 0-1 Set. In: Proceedings of the 7th ALIO/EURO Workshop. ALIO-EURO 2011, Porto, pp. 57-59.*, P. 141.

Dantzig, G. and J. Ramser

1959. The truck dispatching problem. *Management Science*, 6(1):80 – 91.

Dantzig, G. B. and P. Wolfe

1961. The decomposition algorithm for linear programming. notes on linear programming and extensions. part 57. Technical report, RAND CORP SANTA MONICA CA.

Desaulniers, G., J. Desrosiers, A. Erdmann, M. Solomon, and F. Soumis

2002. *VRP with Pickup and Delivery*, Pp. 225–242.

Dotoli, M. and N. Epicoco

2017. A vehicle routing technique for hazardous waste collection. *IFAC-PapersOnLine*, 50(1):9694 – 9699. 20th IFAC World Congress.

Dror, M. and P. Trudeau

1989. Savings by split delivery routing. *Transportation Science*, 23(2):141–145.

Dror, M. and P. Trudeau

1990. Split delivery routing. *Naval Research Logistics (NRL)*, 37(3):383–402.

El-Sherbeny, N. A.

2010. Vehicle routing with time windows: An overview of exact, heuristic and metaheuristic methods. *Journal of King Saud University-Science*, 22(3):123–131.

Feillet, D., T. Garaix, F. Lehuédé, O. Péton, and D. Quadri

2014. A new consistent vehicle routing problem for the transportation of people with disabilities. *Networks*, 63.

Feliu, J. G., G. Perboli, R. Tadei, and D. Vigo

2007. The two-echelon capacitated vehicle routing problem. In *Proceedings of the 22nd European Conference on Operational Research, Prague*, Pp. 1–40.

Fisher, M. L., K. O. Jörnsten, and O. B. Madsen

1997. Vehicle routing with time windows: Two optimization algorithms. *Operations research*, 45(3):488–492.

Fleischmann, B.

1990. The vehicle routing problem with multiple use of vehicles.

Gajpal, Y. and P. Abad

2009. An ant colony system (acs) for vehicle routing problem with simultaneous delivery and pickup. *Computers & Operations Research*, 36(12):3215 – 3223. New developments on hub location.

Gheysens, F., B. Golden, and A. Assad

1984. A comparison of techniques for solving the fleet size and mix vehicle routing problem. *Operations-Research-Spektrum: Quantitative Approaches in Management*, 6(4):207.

Goel, R. and R. Maini

2017. Vehicle routing problem and its solution methodologies: a survey. *International Journal of Logistics Systems and Management*, 28(4):419–435.

Golden, B., A. Assad, L. Levy, and F. Gheysens

1984. The fleet size and mix vehicle routing problem. *Computers & Operations Research*, 11(1):49–66.

Golden, B. L., T. L. Magnanti, and H. Q. Nguyen

1975. Implementing vehicle routing algorithms. Technical report, MASSACHUSETTS INST OF TECH CAMBRIDGE OPERATIONS RESEARCH CENTER.

Gulczynski, D.

2010. Integer programming-based heuristics for vehicle routing problems.

Gulczynski, D., B. Golden, and E. Wasil

2011. The multi-depot split delivery vehicle routing problem: An integer programming-based heuristic, new test problems, and computational results. *Computers & Industrial Engineering*, 61(3):794 – 804.

Haimovich, M. and A. H. Rinnooy Kan

1985. Bounds and heuristics for capacitated routing problems. *Mathematics of operations Research*, 10(4):527–542.

Held, M. and R. M. Karp

1970. The traveling-salesman problem and minimum spanning trees. *Operations Research*, 18(6):1138–1162.

Held, M. and R. M. Karp

1971. The traveling-salesman problem and minimum spanning trees: Part ii. *Mathematical programming*, 1(1):6–25.

Karaoğlan, I.

2015. A branch-and-cut algorithm for the vehicle routing problem with multiple use of vehicles. *Int'l J. of Lean Thinking*, 6(1).

Koç, Ç. and Ġ. KARAOĞLAN

2012. Çok kullanimli ve zaman pencereli araç rotalama problemi için bir matematiksel model. *Journal of the Faculty of Engineering & Architecture of Gazi University*, 27(3).

Kohl, N. and O. B. Madsen

1997. An optimization algorithm for the vehicle routing problem with time windows based on lagrangian relaxation. *Operations research*, 45(3):395–406.

Kolen, A. W., A. Rinnooy Kan, and H. W. Trienekens

1987. Vehicle routing with time windows. *Operations Research*, 35(2):266–273.

Kumar, S. N. and R. Panneerselvam

2012. A survey on the vehicle routing problem and its variants. *Intelligent Information Management*, 4(03):66.

Land, A. and A. Doig

1960. An automatic method of solving discrete programming problems. *Econometrica*, 28(3):497–520.

Laporte, G. and Y. Nobert

1987. Exact algorithms for the vehicle routing problem. In *North-Holland Mathematics Studies*, volume 132, Pp. 147–184. Elsevier.

Lenstra, J. K. and A. H. G. Rinnooy Kan

1979. Complexity of vehicle routing and scheduling problems. *Econometric Institute Archives*.

Li, L. and Z. Fu

2002. The school bus routing problem: a case study. *Journal of the Operational Research Society*, 53(5):552–558.

Little, J. D., K. G. Murty, D. W. Sweeney, and C. Karel

1963. An algorithm for the traveling salesman problem. *Operations research*, 11(6):972–989.

Mao, J. C.

1969. *Quantitative analysis of financial decisions*. Prentice Hall.

Min, H.

1989. The multiple vehicle routing problem with simultaneous delivery and pick-up points. *Transportation Research Part A: General*, 23(5):377–386.

Mingozzi, A., R. Roberti, and P. Toth

2013. An exact algorithm for the multitrip vehicle routing problem. *INFORMS Journal on Computing*, 25(2):193–207.

Montané, F. A. T. and R. D. Galvão

2002. Vehicle routing problems with simultaneous pick-up and delivery service. *Opsearch*, 39(1):19–33.

Mosheiov, G.

1998. Vehicle routing with pick-up and delivery: tour-partitioning heuristics. *Computers & Industrial Engineering*, 34(3):669–684.

Nguyen, P. K., T. G. Crainic, and M. Toulouse

2014. *Multi-zone multi-trip pickup and delivery problem with time windows and synchronization*. CIRRELT.

Osvald, A. and L. Z. Stirn

2008. A vehicle routing algorithm for the distribution of fresh vegetables and similar perishable food. *Journal of food engineering*, 85(2):285–295.

Ozbaygin, G., O. Karasan, and H. Yaman

2018. New exact solution approaches for the split delivery vehicle routing problem. *EURO Journal on Computational Optimization*, 6(1):85–115.

Pisinger, D. and S. Ropke

2007. A general heuristic for vehicle routing problems. *Computers & operations research*, 34(8):2403–2435.

Ren, Y., M. Dessouky, and F. Ordóñez

2010. The multi-shift vehicle routing problem with overtime. *Computers & Operations Research*, 37(11):1987–1998.

Rieck, J. and J. Zimmermann

2013. Exact solutions to the symmetric and asymmetric vehicle routing problem with simultaneous delivery and pick-up. *Business Research*, 6(1):77–92.

Şen, A. and K. Bülbül

2008. A survey on multi trip vehicle routing problem.

Shaw, P.

1998. Using constraint programming and local search methods to solve vehicle routing problems. In *International conference on principles and practice of constraint programming*, Pp. 417–431. Springer.

Solomon, M. M.

1987. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations research*, 35(2):254–265.

Taillard, E. D., G. Laporte, and M. Gendreau

1996. Vehicle routeing with multiple use of vehicles. *The Journal of the Operational Research Society*, 47(8):1065–1070.

Wassan, N. A. and G. Nagy

2014. Vehicle routing problem with deliveries and pickups: modelling issues and meta-heuristics solution approaches. *International Journal of Transportation*, 2(1):95–110.

Xie, Y.

2006. The study of logistics vehicle routing problem with split demand. *Tongji University.*

Zmazek, B., A. Taranenko, and M. Smid

2005. Capacitated vrp with time windows and multiple trips within working day. In *27th International Conference on Information Technology Interfaces, 2005.*, Pp. 104–109.