

MANAGING WORD-OF-MOUTH WITH CAPACITY ALLOCATION AND
ADVERTISEMENT

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ASYA DİPKAYA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

SEPTEMBER 2021

Approval of the thesis:

**MANAGING WORD-OF-MOUTH WITH CAPACITY ALLOCATION AND
ADVERTISEMENT**

submitted by **ASYA DİPKAYA** in partial fulfillment of the requirements for the degree of **Master of Science in Industrial Engineering Department, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Esra Karasakal
Head of Department, **Industrial Engineering**

Assist. Prof. Dr. Sakine Batun
Supervisor, **Industrial Engineering, METU**

Assist. Prof. Dr. Bahar Çavdar
Co-supervisor, **ETID, Texas A&M University**

Examining Committee Members:

Assoc. Prof. Dr. Seçil Savaşeneril Tüfekci
Industrial Engineering, METU

Assist. Prof. Dr. Sakine Batun
Industrial Engineering, METU

Assist. Prof. Dr. Bahar Çavdar
ETID, Texas A&M University

Assist. Prof. Dr. Özgen Karaer
Industrial Engineering, METU

Assist. Prof. Dr. Banu Yüksel Özkaya
Industrial Engineering, Hacettepe University

Date:



I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Surname: Asya Dipkaya

Signature :

ABSTRACT

MANAGING WORD-OF-MOUTH WITH CAPACITY ALLOCATION AND ADVERTISEMENT

Dipkaya, Asya

M.S., Department of Industrial Engineering

Supervisor: Assist. Prof. Dr. Sakine Batun

Co-Supervisor: Assist. Prof. Dr. Bahar Çavdar

September 2021, 117 pages

We study the advertising and capacity decisions of a service firm under endogenous demand and multiple time periods. Consumers are exposed to two types of information: experiences of other consumers regarding fill rate of the service through WoM communication and the information provided by the firm through advertisements. As a result, consumer perception towards the firm keeps getting updated, which impacts demand. We examine fixed versus variable capacity cases. In each case, we study how the firm's decision changes when it is aware of the endogenous demand structure and when it is not.

Keywords: WoM, advertising, capacity, service, fill rate, endogenous demand, consumer perception, brand reputation

ÖZ

MÜŞTERİLER ARASI BİLGİ PAYLAŞIMININ KAPASİTE VE REKLAM KARARLARI İLE YÖNETİMİ

Dipkaya, Asya

Yüksek Lisans, Endüstri Mühendisliği Bölümü

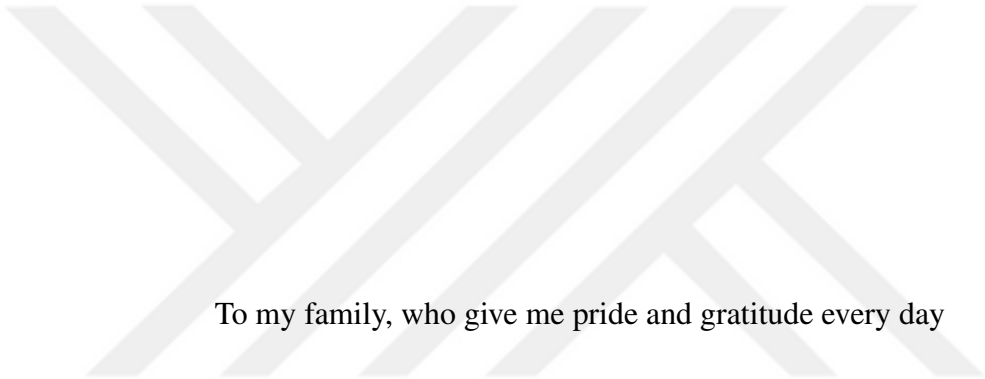
Tez Yöneticisi: Dr. Öğr. Üyesi. Sakine Batun

Ortak Tez Yöneticisi: Dr. Öğr. Üyesi. Bahar Çavdar

Eylül 2021 , 117 sayfa

Servis sektöründe, talebin iç dinamiklere bağlı olduğu, uzun periyotlu bir problem durumunda, firmaların kapasite ve tanıtım kararları çalışılmaktadır. Tüketiciler WoM davranışı ile firmanın sipariş karşılama oranına yönelik bilgiyi birbiri ile paylaşmakta olup, aynı zamanda firmanın tanıtım aktiviteleri ile yaydığı bilgiyi de edinmektedir. Bu sayede, müşteri algısı sürekli güncellenmekte ve talep de buna bağlı olarak değişmektedir. Sabit ve değişken kapasite senaryoları incelenmekte olup, bu iki senaryo, firmanın içsel talebin farkında olduğu ve olmadığı durumlar için çalışılmaktadır.

Anahtar Kelimeler: WoM, tanıtım, kapasite, servis, sipariş karşılama oranı, içsel talep, tüketici algısı, marka itibarı



To my family, who give me pride and gratitude every day

ACKNOWLEDGMENTS

This study would not be possible without my advisors Assist. Prof. Dr. Sakine Batun and Assist. Prof. Dr. Bahar avdar, who have guided me through every step of the process with upmost wisdom, understanding and support.

I would like to express my gratitude for the jury members, Assoc. Prof. Dr. Seil Savařaneril Tfekci, Assist. Prof. Dr. zgen Karaer and Assist. Prof. Dr. Banu Yksel zkaya for their insightful and highly beneficial feedback.

I will be eternally grateful to my beloved sister Melis Dipkaya, mother Nursen Dipkaya and father Melih Dipkaya for being exactly the way they are and for countless other things that it is pointless to even try to express.

Finally, I would like to thank all my friends, who were always there for me.

TABLE OF CONTENTS

ABSTRACT	v
ÖZ	vi
ACKNOWLEDGMENTS	viii
TABLE OF CONTENTS	ix
LIST OF TABLES	xii
LIST OF FIGURES	xv
CHAPTERS	
1 INTRODUCTION	1
2 LITERATURE REVIEW	5
3 PROBLEM DEFINITION	11
4 VARIABLE CAPACITY PROBLEM	17
4.1 Complete-Information Firm	18
4.1.1 Analysis of Alternative Optima	24
4.2 Incomplete-Information Firm	29
4.3 Computational Results on Value of Information (VoI)	37
4.3.1 Impact of Starting Reputation	40
4.3.2 Impact of the Advertisement Resistance	41
4.3.3 Impact of Weight of WoM	42

4.3.4	Impact of Reservation Utility	44
4.3.5	Impact of Reputation's Monetary Value	45
4.3.6	Impact of the Length of the Planning Horizon	46
4.3.7	Impact of the Advertisement Levels	47
4.3.8	Impact of the Service Price	49
4.3.9	Impact of Advertisement Cost	50
4.3.10	Impact of Capacity Cost	51
4.3.11	Impact of the Market Size	52
4.4	Summary Of The Variable Capacity Problem	53
5	CONSTANT CAPACITY PROBLEM	57
5.1	Classification of the Firm's Demand Realization Ability	58
5.2	Complete-Information Firm	62
5.2.1	Special Cases	63
5.3	Incomplete-Information Firm	67
5.4	Computational Results on Value of Information (VoI)	71
5.4.1	Impact of Weight of WoM	74
5.4.2	Impact of Advertisement Resistance	76
5.4.3	Impact of Reputation's Monetary Value	79
5.4.4	Impact of Service Price	81
5.4.5	Impact of Starting Reputation	83
5.4.6	Impact of Reservation Utility	86
5.4.7	Impact of Service Capacity	88
5.4.8	Impact of Advertisement Cost	90

5.4.9	Impact of Advertisement Levels	93
5.4.10	Impact of the Length of the Planning Horizon	97
5.4.11	Impact of Market Size	99
5.5	Summary Of The Constant Capacity Problem	101
6	CONCLUSION	109
	REFERENCES	115



LIST OF TABLES

TABLES

Table 4.1	Parameter distributions of variable capacity problem computations.	37
Table 4.2	Advertising policies of complete and incomplete-information firms under variable capacity with respect to starting reputation.	40
Table 4.3	Advertising policies of complete and incomplete-information firms under variable capacity with respect to advertisement resistance.	42
Table 4.4	Advertising policies of complete and incomplete-information firms under variable capacity with respect to weight of WoM.	43
Table 4.5	Advertising policies of complete and incomplete-information firms under variable capacity with respect to reservation utility.	44
Table 4.6	Advertising policies of complete and incomplete-information firms under variable capacity with respect to monetary value of reputation.	46
Table 4.7	Advertising policies of complete and incomplete-information firms under variable capacity with respect to the length of the planning horizon.	47
Table 4.8	Advertising policies of complete and incomplete-information firms under variable capacity with respect to high advertising level.	48
Table 4.9	Advertising policies of complete and incomplete-information firms under variable capacity with respect to low advertising level.	49
Table 4.10	Advertising policies of complete and incomplete-information firms under variable capacity with respect to service price.	49

Table 4.11 Advertising policies of complete and incomplete-information firms under variable capacity with respect to advertising cost.	50
Table 4.12 Advertising policies of complete and incomplete-information firms under variable capacity with respect to capacity cost.	52
Table 4.13 Advertising policies of complete and incomplete-information firms under variable capacity with respect to market size.	52
Table 5.1 Parameter distributions of constant capacity problem computations. .	72
Table 5.2 Advertising policies of complete and incomplete-information firms under constant capacity with respect to weight of WoM.	75
Table 5.3 Advertising policies of complete and incomplete-information firms under constant capacity with respect to advertisement resistance.	78
Table 5.4 Advertising policies of complete and incomplete-information firms under constant capacity with respect to monetary value of reputation. . . .	80
Table 5.5 Advertising policies of complete and incomplete-information firms under constant capacity with respect to service price.	82
Table 5.6 Advertising policies of complete and incomplete-information firms under constant capacity with respect to starting reputation 1.	84
Table 5.7 Advertising policies of complete and incomplete-information firms under constant capacity with respect to starting reputation 2.	85
Table 5.8 Advertising policies of complete and incomplete-information firms under constant capacity with respect to reservation utility.	87
Table 5.9 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the service capacity.	89
Table 5.10 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the advertisement cost 1.	91

Table 5.11 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the advertisement cost 2.	92
Table 5.12 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the high advertisement level. . . .	94
Table 5.13 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the low advertisement level.	96
Table 5.14 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the length of the problem horizon. .	98
Table 5.15 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the market size.	100

LIST OF FIGURES

FIGURES

Figure 3.1	Order of Events	14
Figure 4.1	Behavior of $\Pi(j)$ When $R_1 > \frac{u+p-L^\alpha}{1-L^\alpha}$	28
Figure 4.2	Behavior of $\Pi(j)$ When $R_1 \leq \frac{u+p-L^\alpha}{1-L^\alpha}$	28
Figure 4.3	Behavior of $\Pi(j)$ When $R_1 \leq \frac{u+p-L^\alpha}{1-L^\alpha}$	29
Figure 4.4	Behavior of $\Pi(j)$ When $R_1 \leq \frac{u+p-L^\alpha}{1-L^\alpha}$	29
Figure 4.5	R_i Series Of Variable Capacity Firms	36
Figure 4.6	Cumulative π_i Series Of Variable Capacity Firms	36
Figure 4.7	Reputation vs Fill Rate Under Variable Capacity	54
Figure 5.1	Cumulative Advertising Cost Of Constant Capacity Firms	70
Figure 5.2	Cumulative Revenue Of Constant Capacity Firms	70
Figure 5.3	Cumulative Profit Of Constant Capacity Firms	71
Figure 5.4	Average % VoI versus average correlation between the two firms' advertising sequences, with respect to Ω	73
Figure 5.5	Reputation vs Fill Rate Under Constant Capacity When $\Omega \subset \{C, E, F\}$	102
Figure 5.6	Reputation vs Fill Rate Under Constant Capacity Under Proposition 6	103

Figure 5.7	Reputation vs Fill Rate Under Constant Capacity With Long Planning Horizon	104
------------	---	-----

Figure 5.8	Reputation vs Fill Rate Under Constant Capacity With Short Planning Horizon	104
------------	--	-----



CHAPTER 1

INTRODUCTION

Easy access to information has created many opportunities for consumers and service providers. Increasing communication channels and the vast amount of information available has been changing consumers' purchase habits, as well as creating new means for the companies to better understand consumer perception.

Studies have shown that WoM (Word-of-Mouth) communication is highly effective in shaping consumers' opinions, since it typically occurs between acquaintances, and consumers tend to rely on the transmitted information [1]. Online reviews, ratings and social media constitute a less traditional concept that has been drawing attention lately. This sort of communication is named *eWoM* (electronic Word-of-Mouth). One advantage eWoM has over traditional WoM, from consumers' point of view, is that information sharer sends the message directly to other consumers, rather than having it transmitted from person to person. Due to this *broadcasting* of information, dissemination is faster and the message does not get distorted on the way to the recipients [1]. In this study, we will use *WoM* to refer to the joint effect of traditional WoM and eWoM.

In the online marketplaces, consumers do not make the purchase by visiting the supplier and have to collect all the needed information, including evidence regarding service quality and firm's credibility, online. WoM plays an important role as a decision support mechanism for consumers, by helping them have a better understanding of the service quality. There are different approaches to define service quality, depending on the type of the service. In this study, we assume one type of service is provided, where speed is of essence (e.g. online grocery shopping applications, call centers). Therefore, in this context service quality corresponds to the fill rate of the

company, and information regarding the fill rate is shared through WoM.

Studies regarding consumer behavior and purchase decisions suggest that each consumer has some expectation from a product/service, that gets altered as the service level changes [2] [3]. Adopting this observation, we consider consumers to have prior beliefs or first impressions regarding the fill rate, which are updated as consumers receive new information on the fill rate.

Having underlined the significance of WoM, as it is not realistic to consider firms as passive agents, watching WoM manipulate consumer perception without doing anything, the actions taken by firms should be accounted for as well. Firms often place attempts to influence consumers' purchasing decisions, such as promotions and advertisements [4]. The intensity of such efforts determines how much, or through how many different channels, consumers are exposed to the new information provided by the firm. Therefore, we consider the amount of effort put into such activities as a factor affecting consumer perception, besides WoM.

Studies often suggest that companies should take WoM into consideration when constructing marketing plans [5]. We take this result one step further by suggesting that companies should consider WoM while making operational decisions as well, in our case: service capacity allocation. How much capacity a company maintains is a determinant of whether consumers will be served on time, or simply the fill rate. As a result, it creates new information for the consumers to spread through WoM. To illustrate, we expect firms that cannot maintain enough capacity to fail in fulfilling demand and suffer negative WoM while firms that can maintain enough capacity to enjoy positive WoM.

Taking the introduced factors into account, we model consumer utility. We state that consumers' perception of the service, determines how much utility they believe they will gain if they purchase the service and if it is worth it. Then, we derive a comprehensive demand model based on consumer utility. We consider endogenous demand, since consumers' perception is affected by capacity decisions (with the resulting fill rate information) and advertising efforts. In the upcoming chapters, we introduce a definition, namely *reputation*, which is used to keep track of how consumers feel about the service at the moment (perception of the service or fill rate expectation).

We use this term to incorporate the effects of advertisement and WoM on demand.

To summarize, we consider a service environment, where consumers' collective perception towards company's service quality (or fill rate expectation), determines the demand for the service. Company's actions to impact demand are making advertisements and setting the service quality to manipulate WoM. In this context, since quality is related to service availability, setting the service quality is equivalent to determining how much service capacity to maintain. In the meanwhile, consumers are active players sharing their experiences with the service through WoM, and changing the perception towards the service. We aim to observe how the company's advertising policies and operational decisions should be under this endogenous demand structure. We focus on strategies to balance the traditional operational decisions with the management of customer perception through WoM. The research questions we aim to study are as follows:

- What kind of advertisement strategy should a firm adopt?
- When should the firm advertise more aggressively?
- How do the parameters affect firm's strategy?
- How much capacity should the firm maintain?
- Can the firm exploit WoM?
- Should the firm rely on WoM to capture a greater portion of the market?
- Does WoM motivate the firm to provide better service quality?

The remainder of this thesis is organized as follows. In the next chapter, a literature review on WoM communication, consumer behavior, demand functions and operational decision making under endogenous demand, is presented. In Chapter 3, we provide a problem definition and introduce the necessary notations. Chapters 4 and 5 then cover the problem formulation and structural results, as well as computational experiments. Finally, Chapter 6 closes with concluding remarks and suggestions for future studies.



CHAPTER 2

LITERATURE REVIEW

Literature review conducted for this study consists mainly of WoM, consumer behavior, characteristics of the demand function and operational decision making under endogenous demand.

There are various different definitions for WoM [1]. Some studies define it as unidirectional information transmission between two individuals when the receiver believes that the sender has no commercial concern [5] while some define it as bidirectional communication between customers, where the senders are without commercial concern [6]. In our study, we have WoM occurring between multiple agents that have no commercial concern, which accommodates eWoM.

When it comes to eWoM, it is discussed that such online mechanisms function as a trust-builder and motivate collaboration among strangers that share information about companies, products or services [7]. Comparing eWoM with traditional WoM, eWoM's great accessibility and the faster diffusion of information stand out [1]. With motivated consumers to share their experiences, the amount of available information grows rapidly in online marketplaces. Moreover, customers tend to regard the information they gather from such platforms more than what the companies promise through promotion activities [8]. It is clear that eWoM is largely affecting consumer decisions, and therefore demand. As a result, it is no surprise that companies' may view this as a downside, since their influence on demand is lower than it used to be [9]. There is an increasing number of studies emphasizing that incorporating the information exchange between customers and the impacts of it in operational decision making is important [10]. However, to our knowledge [11] is the first study that integrates the WoM into an operational problem. They study a consolidated ship-

ment policy problem considering the information exchange between different classes of customers, and present results on how to manage WoM communication through operational decisions.

In order to understand consumer behavior better and model it correctly, we study the literature regarding consumer behavior and the relationship between consumers' experiences, expectations, beliefs and demand. Later, we combine these findings with WoM behavior to construct our model. To begin with, Shapiro (1982) suggests that each customer has some expectation from the service, which determines whether she will decide to purchase; implying that demand for the service is based on customers' expectations [3]. Additionally, it is discussed that customers' expectations depend on several factors, including brand reputation, price and advertising [4] [12] [13] [14]. Studies show that changes in the service level have a significant effect on how customers rate service quality while customers' behavior is affected by their prior attitude as well [2]. These results lead us to defining a measure to reflect customers' current perception of the company as well as their expectations from the service. On top of that, we consider consumers that exchange information regarding service quality by WoM, get exposed to advertisement and update their perception. Then, we model demand based on this perception measure.

In Adelman's (2013) study on how to allocate a limited capacity among a set of customers, it is argued that customers remember company's past performance in fulfilling demand, and their purchase decisions are affected by it [15]. In our study, we consider a time-sensitive service environment, where perception towards service quality is largely determined by company's performance in fulfilling demand in a timely manner. Therefore, we accommodate Adelman's observation in our demand model, with a consumer mass that share information regarding firm's fill rate performance. This time-sensitive service environment underlines the importance of service capacity in fulfilling demand (e.g. number of couriers of an online grocery shopping firm). Therefore, we incorporate firm's operational decisions that affect the fill rate. As firm's capacity decision will determine the fill rate and consumers will share that information to update demand, we consider endogenous demand. An example of operational decision making under endogenous demand is provided in a study carried out by Mandal et al. (2018), where the classical newsvendor problem is extended by

introducing behavioral biases [16]. They also consider endogenous demand which depends on how probable the consumers think getting served is. In their problem, this probability depends on the inventory level while corresponding to service capacity level in our case. They show that this probability is equal to firm's fill rate. They highlight the importance of decision making by accounting for endogenous demand, and show that such demand models motivate firms to keep the fill rate high. A similar finding is presented in the study carried out by Urban (2005) regarding inventory-dependent demand models [17]. Urban classifies such demand models in two types, first of which covers the models where demand is affected by the initial inventory while the second covers demand models that incorporate instantaneous inventory levels. Our model falls under the second category.

There are extensive empirical studies on the relationship between fill rate and demand, which report similar results. Craig et al. (2016) examine the relationship between service level and demand for a well-known functional apparel product supplier and show that inventory level of the supplier has a statistically significant impact on demand [18]. In parallel, Heim and Sinha (2001) establish that service availability and timeliness are strongly associated with customer retention [19] in their study related with electronic food retailers. Similar results on customers looking for substitutes or not making the purchase at all in case of stockouts are shown for different product groups as well [20] [21].

When considering a service environment, where fill rate is essential to maintain demand, new questions arise regarding how service capacity is managed. One method in use is *shared services* (insourcing), which are organizations established by companies to carry out similar business functions for different centers/units of the same company [22]. Another method is outsourcing the service capacity entirely while Fuhrman (1999) suggests *co-sourcing* as a better alternative, which is the practice of meeting some of the demand using internal resources and outsourcing the rest [23]. These methods, if applicable, allow firms to operate with different service capacity levels throughout a planning horizon [24]. Note that the studies we have mentioned so far regarding the service capacity management are mostly devoted to call center optimization, which is a befitting service environment example where purchase decisions of customers depend on whether they are able to get served in a timely manner,

or not [25]. Meanwhile, Dorsch and Häckel (2014) provide a study on managing service capacity by different outsourcing options [26]. They consider a service firm outsourcing capacity by different contract types: (i) outsourcing a fixed capacity from a vendor, (ii) outsourcing elastic capacity and (iii) outsourcing surplus capacity. The first two options rely on long-term contracts while the third can be executed in short notice. Outsourcing surplus capacity is often observed via *Business Process as a Service (BPaaS)*, in which business processes are outsourced through cloud computing services on demand. When applicable, this practice can provide firms with great agility to meet volatile demand.

Next, we direct our efforts into studying different types of demand functions in a service environment. Huang et al. (2013) present a comprehensive review of different demand function types used in the literature, in terms of different aspects companies make use of to compete in the market [27]. They suggest that the factors that influence consumers' purchasing decisions are companies' tools in trying to capture greater portions of the market. These factors include price, rebate, lead-time, space (area occupied on shelves), quality and advertisement. Aiming to represent perceived quality's effect on consumers' purchasing decisions and capturing consumer behavior better; we use a demand model based on consumer utility as a function of service quality. In the preferred demand model, Moorthy (1988) and Tirole (1988) suggest that the utility obtained by a single consumer when the service is purchased can be modeled as follows [28] [29].

$$u(p) = \theta y - p, \quad (2.1)$$

where y denotes product/service quality, p denotes price, u denotes utility as a function of price and θ is how much utility the consumer gains for unit quality if she receives the product/service. θ is used to define different customer types. A customer with low θ has lower quality valuation; therefore, gains less utility from unit quality. Consumers with positive net surplus make the purchase and others do not. When θ is uniformly distributed between 0 and 1, and market size is a , demand function is derived as follows.

$$d(p, a) = a \left(1 - F \left(\frac{p}{y} \right) \right), \quad (2.2)$$

where F is the c.d.f. of θ .

We extend this model by accounting for advertisement and WoM, which is explained in the upcoming chapters. Note that when advertisement's effect on demand is considered, most of the demand models use increasing and concave demand functions, to account for "advertising saturation effect" [27].

We aim to describe a service environment that accounts for the above described factors by an endogenous demand structure. In a nutshell, we consider a set of consumers that have a first impression about the service, get new information from two sources (advertisements and WoM communication on the past fill rate performance of the firm), generate demand based on utility and observe the new fill rate. In the meanwhile, firm is able to make capacity and advertisement decisions to affect consumer expectation.

In the current literature, operational decision making under endogenous demand is mainly observed for product suppliers, rather than service providers. In our model, we capture consumer behavior in a service environment by a rigorous demand mechanism, using multiple time periods to observe the turn of events and long-term policies, which is our main contribution to the literature. We have included few aggregations, which introduces high complexity, considering the multi-period problem horizon.



CHAPTER 3

PROBLEM DEFINITION

In this study, we consider a system where a company provides one type of service/product, and customers share information about the company's performance through WoM (e.g., online reviews). Customers make their purchase decisions considering the knowledge gained by this information exchange. We consider a time-sensitive service; therefore, company's ability to fulfill demand on time, or fill rate, is the main determinant of service quality, as in shipping services. Accordingly, we consider an endogenous demand structure, and our focus is on understanding the impact of this structure on operational decision making in a multi-period problem. In each time period $i \in \{1 \dots, N\}$, the company observes the customer demand D_i and fulfills the demand as the capacity in the current period (S_i) allows. For each unit of service provided, company receives revenue. We assume that the cost of maintaining one unit of capacity is c to the company.

To model the demand, we consider a customer population of size Λ . Each customer has a *taste* for the service provided by the company, denoted by θ . We assume that $\theta \sim U(0, 1)$. Customers are faced with the choice of requesting a service from the company or looking for substitutes. When they are making this decision, their belief about the company's performance in fulfilling demand, or fill rate, plays an important role. If the company has been successful in fulfilling demand in the past, customers find it more appealing. The information on whether their service request will be fulfilled by the company or not is not available to customers. However, they can infer the fill rate of the company based on the available information. One source of information is WoM communication. As customers use the service provided by the company, they share their experiences through communication channels. In addition

to this, company can also take an action to alter the perception of the fill rate. These actions can include advertising, promotions, etc. Therefore, customers are exposed to two sources of information: historical fill rate and promises made by company regarding the future service. The former is available through WoM, the latter depends on the advertisement/promotion efforts of the company. Observing these two sources of information, customers develop an idea about the fill rate in case they request the service. We name this fill rate expectation as *reputation* of the company. We denote the reputation of the company in the beginning of period i by R_i . R'_i , or the post-advertisement reputation, denotes the reputation of the company after the advertisement effort is made in period i . We assume that advertisement's effect on reputation is immediate. Since the reputation of the company is based on the fill rate, it takes a value in $[0, 1]$. We denote the advertisement effort in period i by A_i , and $A_i \in [0, 1]$ for all i . We model the post-advertisement reputation in period i as follows:

$$R'_i = R_i + (1 - R_i)A_i^\alpha \quad (3.1)$$

The parameter $\alpha > 0$ in equation (3.1) is a reflection of customers' reaction to the observed advertisement. Higher values of α implies that customers are more resistant to advertisement and more prone to basing their decisions on the firm's performance; therefore we refer to α as *advertisement resistance* of the the customers. This structure requires companies with higher reputation to place more advertisement efforts to increase their reputation compared to companies with lower reputation, since advertisement's effect diminishes at higher reputation levels [27].

A customer observing the post-advertisement reputation R'_i in period i determines whether she will request the service from the company or go somewhere else. Customers' collective decision determines the demand the company faces in period i and fill rate F_i , and hence affects the future reputation through WoM. When customers are faced with multiple options, customer behavior is generally modeled by utility-based demand functions [27]. We extend a utility model suggested, and derive the demand function as described below [28] [29].

To compute the overall demand drawn by the population Λ , we calculate the utility obtained by a customer if she makes the purchase and derive the overall demand.

If customers do not prefer to request their service from the company, they receive the reservation utility u . A customer with a taste, or *reputation valuation*, θ for the service, receives the following utility, at period i , if she request the service from the company.

$$m \theta R'_i - (u + p) \quad (3.2)$$

In the above expression, m is the coefficient to convert reputation to monetary terms, and p is the service price. By purchasing the service, the customer receives the utility of $m \theta R'_i$ and sacrifices a utility of $u + p$. If the difference is positive, it is worth for the customer to purchase the service. So, customers create a demand from this company if their taste parameter θ satisfies the following condition:

$$\theta > \frac{u + p}{m R'_i} \quad (3.3)$$

Then, the fraction of customers, who create demand at stage i is $1 - F(\frac{u+p}{m R'_i})$, where F is the c.d.f. of θ . Since θ is uniformly distributed between 0 and 1, this fraction can be written as $1 - \min\{1, \frac{u+p}{m R'_i}\}$.

Accordingly, demand of period i can be expressed as $D_i = \max\{0, \Lambda(1 - \frac{u+p}{m R'_i})\}$. We will represent the outcome of the function $\Lambda(1 - \frac{u+p}{m R'_i})$ by γ_i , in order to express the cases where it is negative, leading to zero demand.

Having observed the demand, the company operates throughout period i , and the fill rate F_i is realized as $\frac{\min\{D_i, S_i\}}{D_i}$. F_i reflects the actual experience of the customers. Customers share this experience through WoM affecting the reputation of the company in the future. We model impact of the fill rate in the reputation through a linear function as follows:

$$R_{i+1} = \begin{cases} (1 - w)R'_i + wF_i, & \text{if } \gamma_i > 0 \\ R'_i, & \text{otherwise} \end{cases} \quad (3.4)$$

where w is the weight assigned to the information on the most recent fill rate gained by WoM and $0 \leq w \leq 1$. $w = 1$ implies that customers only use the most recent information to determine the perception of the company, while $w = 0$ implies that customers are not affected by the WoM. The second condition in Equation (3.4) assumes that if there is no demand generated in the current period, there is no additional information on the fill rate, and the reputation of the company remains the

same. Here, we assume that reputation is altered by Equation 3.4 at the end of each time period.

Note that reputation level increases in advertisement while fill-rate's effect on reputation can be in both directions. Fill-rate increases reputation if $F_i > R'_i$, and a decreases it if $F_i < R'_i$. This is interpreted as consumers expecting firm's performance to be aligned with its reputation. If the firm performs better than expected, reputation goes up; if it performs worse, reputation goes down; and if it performs just as consumers expect, reputation does not get updated.

In period i , the company collects the profit π_i :

$$\pi_i = p \min\{S_i, D_i\} - c S_i - c_A A_i, \quad (3.5)$$

where c_A is the cost of unit advertisement. We assume that price of one unit of service is greater than its capacity cost, that is $p > c$ so that operating is meaningful for the company. The company's objective is to maximize the total profit Π over the planning horizon, $\Pi = \sum_{i=1}^N \pi_i$. Capacity selection and advertisement efforts are the main decisions made by the company in each period.

Note that this problem is different than the classical operational problems due to the endogenous demand structure. To illustrate the endogenous demand, the order of events that shape demand for period i is represented in Figure 3.1, where the first three events take place instantaneously at the beginning of each period while the last two occur instantaneously at the end.

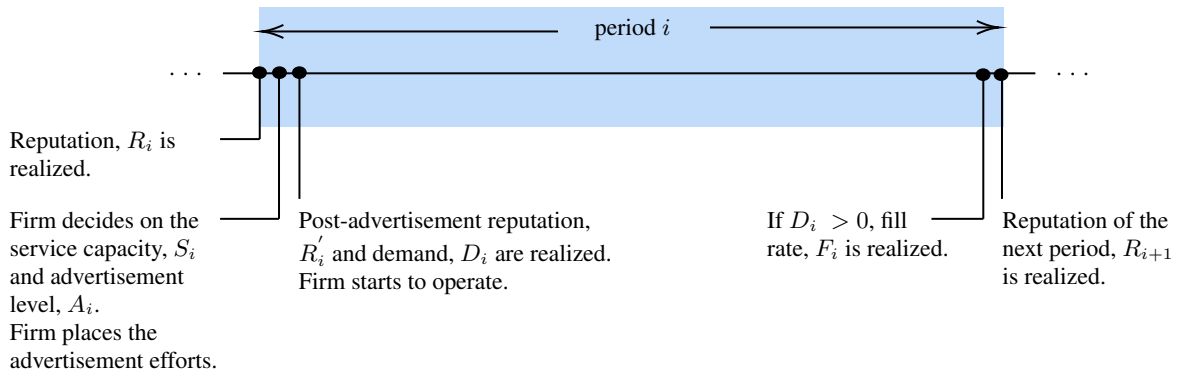


Figure 3.1: Order of Events

To summarize, as the events given above occur, variables in the system are determined in the following order.

- R_i is realized as a result of the previous period.
- S_i and A_i are determined by the firm, simultaneously and immediately. Firm puts the effort to advertise.
- Post-advertisement reputation is determined as $R'_i = R_i + (1 - R_i)A_i$, and demand is determined as $D_i = \max \left\{ 0, \Lambda \left(1 - \frac{u+p}{mR'_i} \right) \right\}$. Firm starts performing to fulfill demand.
- At the end of the period, if $D_i \neq 0$, fill rate is realized as $F_i = \frac{\min\{S_i, D_i\}}{D_i}$.
- Next period begins with:

$$R_{i+1} = \begin{cases} (1-w)R'_i + wF_i, & \text{if } \gamma_i > 0 \\ R'_i, & \text{otherwise} \end{cases} \quad (3.6)$$

We study this problem in two cases. In the first case, the company determines the capacity for each period i ; in the second case the capacity is predetermined and constant over all periods. The first case addresses firms that can easily change service capacity (e.g. by making short-term contracts with employees or being able to rent work force through some agency) while the latter is for firms that cannot change capacity so easily (e.g. other constraints due to infrastructure or facility). We would like to observe how firm would make use of WoM or be affected by it, as well as what kind of pattern reputation follows throughout the problem horizon in each case.

Advertisement decision can be handled in different ways. In this study, we consider that company has two options: high level and low level advertisements denoted by H and L respectively. Therefore, in an N -period problem, the firm has to decide on an advertisement sequence such as $A_1 = H, A_2 = L, A_3 = L, \dots, A_N = L$, or shortly $A = \{H, L, L, \dots, L\}$.

When the company is making operational decisions, it also needs to consider the impact of these on the reputation and future demand as a result. For example, ignoring

the WoM effect will hurt the reputation and demand in the future if the company attracts more customers than they can serve.

In the following sections, we analyse this problem under different settings. We will first address the case where the company can adjust the service capacity in each period. In the second setting, we will consider a fixed-capacity problem. The former is associated with the cases where increasing/decreasing the capacity is not costly. This could be the case if the company can make short-term contracts. In both settings, we analyze the problem from different perspectives; we consider a firm that is not aware of the WoM impact and the endogenous demand structure, and a firm who has complete information on the demand realization and can take the impact of operational decisions on demand into consideration.



CHAPTER 4

VARIABLE CAPACITY PROBLEM

In this chapter, we present and analyze the problem under variable capacity. The firm can adjust the capacity in each period to avoid lost sales or capacity surplus, depending on its strategy. In addition to the capacity decision, the firm also chooses between the high (H) and low (L) advertisement levels in each period.

We consider two types of service providers:

- (i) the service provider is aware of the WoM communication and considers its impact on the endogenous demand structure when making operational decisions (complete-information firm),
- (ii) the service provider is not aware of the WoM communication and its impact on customer demand (incomplete-information firm).

Under the first case, the firm can make better decisions in terms of capacity and advertisement levels since it can incorporate the impact of WoM on the demand in the decision making process. On the other hand, in the second scenario, the firm's decisions are limited because it cannot capture the impact of WoM communication.

We study these scenarios and identify the relationship between the two types of service providers' decisions. Then, we present computational results to understand the impact of problem parameters on the value of the having full information on endogenous demand structure.

4.1 Complete-Information Firm

In this section, we examine the optimal capacity and advertisement decisions of the complete-information firm under variable capacity. The firm tries to maximize the total profit throughout an N -period planning horizon by determining A_i and S_i for each period i . Having full information on the market dynamics, at any period i , the firm is able to make decisions for all future periods. The problem can be expressed as the following dynamic program, where the stages correspond to time periods, the state is expressed by the current reputation R_i , the actions are advertisement level A_i and service capacity S_i . The profit throughout periods $i, i + 1, \dots, N$ is represented by $P_i(R_i)$.

$$P_i(R_i) = \max_{A_i, S_i} \{ \pi_i + P_{i+1}(R_{i+1}) \}$$

$$P_N(R_N) = \max_{A_N, S_N} \{ \pi_N \}$$

$$\text{where } R_{i+1} = \begin{cases} (1-w)(R_i + (1-R_i)A_i^\alpha) + w \frac{\min\{D_i, S_i\}}{D_i}, & \text{if } D_i > 0 \\ (R_i + (1-R_i)A_i^\alpha), & \text{if } D_i = 0 \end{cases}$$

$$\pi_i = p \min\{D_i, S_i\} - cS_i - c_A A_i,$$

$$D_i = \max\{0, \gamma_i\},$$

$$\gamma_i = \Lambda \left(1 - \frac{u+p}{m(R_i + (1-R_i)A_i^\alpha)} \right) \text{ and}$$

$$A_i \in \{H, L\}, \quad S_i \geq 0.$$

The objective is to maximize the total profit over the planning horizon, $P_1(R_1)$. In the remainder of this thesis, we also use Π to represent the total profit, that is $\Pi = P_1(R_1)$. The problem above can be handled by backward recursion. However, the structure of the demand function and the continuous state space for the reputation result in a highly complicated formulation to derive a closed form solution. Therefore, we direct our efforts into developing some structural results to help us to solve this problem more efficiently and understand the behavior of the optimal solution.

Our first result is regarding the optimal capacity allocation.

Proposition 1. *In the variable-capacity problem, in each period, the optimal capacity is equal to the demand to be realized.*

Proof. We want to show that the optimal capacity decision $S_i^* = D_i^*$ for all i , where S_i^* represents the optimal capacity level for period i and D_i^* denotes the corresponding demand to be realized under the optimal policy.

We will first prove that in the optimal solution, in any period i , picking a capacity level greater than demand level never occurs.

If we have $S_i = D_i$, then we obtain the following profit in period i :

$$\pi_i = (p - c) D_i - c_A A_i. \quad (4.1)$$

If we choose a capacity S_i for period i such that $S_i > D_i$, the profit in the current period would be

$$\pi_i = p D_i - c S_i - c_A A_i. \quad (4.2)$$

The profit obtained in Equation 4.1 is greater than of Equation 4.2 by $c (S_i - D_i)$. Therefore, choosing a capacity level greater than the demand generates less profit for period i compared to having capacity equal to demand.

Both capacity choices result in the same fill rate (i.e., $F_i = 1$) and hence result in the same reputation in the next period. Then, choosing $S_i > D_i$ for some i cannot help generate more demand in the following periods, as well. As a result, maintaining more capacity than demand for any i , results in lower Π than having $S_i = D_i$ for all i . Thus, a capacity choice greater than demand cannot be an optimal decision in any period i .

Now, we will show that a capacity choice $S_i < D_i$ is suboptimal for all i , given $p > c$.

Since S_i^* cannot be greater than the demand, we can rewrite the profit function as

follows:

$$\begin{aligned}
\pi_i &= (p - c) S_i - c_A A_i \\
s.t. \quad & \\
0 &\leq S_i \leq D_i
\end{aligned} \tag{4.3}$$

Note that π_i is increasing in S_i , and in D_i as a result. Choosing a capacity S_i that is less than D_i in a period i results in loss of profit opportunity in the current period. It also decreases the company's reputation through the fill rate in the following periods leading to smaller demand. Therefore, for any set of advertisement decisions, having $S_i < D_i$ for some i generates a smaller profit than having $S_i = D_i$ for all i , which concludes the second part of the proof. $\square$

Proposition 1 simplifies the capacity decision in the problem to $S_i^* = D_i^*$ for all i . Therefore, the problem reduces to finding the optimal advertisement policy. Since the firm has to choose between discrete advertisement levels at each period, the number of solutions is finite. An obvious alternative to backward recursion, which comes to mind with this finding is complete enumeration. However, the number of iterations required grows exponentially with the number of periods N . The problem complexity is $O(2^N)$. Having said that, let us pause the discussion on solution method again, and continue with the structural analysis and the resulting implications.

An important implication of Proposition 1 is that we have $F_i = 1$ for all periods. Therefore, as a result of WoM communication, the reputation of the company increases throughout the horizon. Hence, regarding the relationship between the reputation and demand between consecutive periods, we can write the following:

$$\begin{aligned}
R_{i+1}^* &\geq R_i^* \\
D_{i+1}^* &\geq D_i^* \\
\gamma_{i+1}^* &\geq \gamma_i^*
\end{aligned} \tag{4.4}$$

Accordingly, we have the following corollary.

Corollary 1. *In the variable-capacity problem, total profit is a non-decreasing function of the starting reputation, R_1 .*

Corollary 1 results from the fact that we have $R_i^* \geq R_j^*$ for all $i > j$; higher reputation values lead to higher demand and greater profit as a result.

Our next result is regarding the optimal advertisement decision.

Proposition 2. *In the variable-capacity problem under complete-information on WoM communication, if $A_i^* = L$, then $A_j^* = L$ for all $j > i$.*

Proof. To prove the proposition, we will show that $A_{i+1} = H$ cannot be optimal when $A_i = L$.

Let us consider the following advertisement sequence for an N-period problem.

$$\begin{aligned} V &= \{V_1, V_2, \dots, V_N\}, \text{ where each } V_i \text{ is either } H \text{ or } L, \\ \text{and } V_k &= L, V_{k+1} = H, \text{ for some } 1 \leq k \leq N-1. \end{aligned} \quad (4.5)$$

Then, using V , let us define a second sequence Q such that:

$$\begin{aligned} Q_i &= V_i, \text{ for all } i \in \{1, 2, \dots, k-1, k+2, \dots, N\}, \text{ and} \\ Q_k &= H, Q_{k+1} = L. \end{aligned} \quad (4.6)$$

We denote the parameters such as reputation, post-advertisement reputation, service capacity, fill rate, demand and profit at period i , using the sequence information (e.g. firm's reputation at sequence V and period i is represented by R_i^V).

By Proposition 1, the variable capacity profit function is $\pi_i = (p - c) D_i - c_A A_i$. Since, in the above sequences, all decisions are common before the k^{th} period, we have the following.

$$\pi_1^Q + \pi_2^Q + \dots + \pi_{k-1}^Q = \pi_1^V + \pi_2^V + \dots + \pi_{k-1}^V \quad (4.7)$$

The reputation level at the beginning of period k is the same for both sequences, that is $R_k^V = R_k^Q$. The sequence Q has high level advertisement at period k while sequence V has low. Therefore, we have $R_k'^Q > R_k'^V$, indicating $D_k^Q \geq D_k^V$.

Then, we have either one of the following cases regarding period $k+1$:

- (i) If $D_k^Q = 0$, we have $D_k^V = 0$ as well. As a result, new information on fill rate is not created, and WoM does not happen. Then, we have the following expression

for post-advertisement reputation of period $k + 1$, which is symmetric around H and L allowing R_{k+1}^Q and R_{k+1}^V to be equal.

$$\begin{aligned} R_{k+1}^Q &= R_{k+1}^V = R_k^V + H - HR_k^V - LR_k^V - HL + HLR_k^V + L \\ &= H(1 - R_k^V) + L(1 - R_k^V) - HL(1 - R_k^V) + R_k^V \end{aligned} \quad (4.8)$$

In that case, we have $D_k^Q + D_{k+1}^Q = D_k^V + D_{k+1}^V$.

- (ii) If $D_k^Q > 0$ while $D_k^V = 0$, we have the following expressions for post-advertisement reputation of period $k + 1$; thus, we have $R_{k+1}^Q > R_{k+1}^V$.

$$\begin{aligned} R_{k+1}^Q &= R_k^V(1 - w) + w + H(1 - R_k^V - w + wR_k^V) \\ &\quad + L(1 - R_k^V - w + wR_k^V) + HL(-1 + R_k^V + w - wR_k^V) \end{aligned} \quad (4.9)$$

$$R_{k+1}^V = H(1 - R_k^V) + L(1 - R_k^V) - HL(1 - R_k^V) + R_k^V$$

In that case, we have $D_k^Q + D_{k+1}^Q > D_k^V + D_{k+1}^V$.

- (iii) If $D_k^Q > 0$ while $D_k^V > 0$, we have the following expression for post-advertisement reputation of period $k + 1$, which is symmetric around H and L allowing R_{k+1}^Q and R_{k+1}^V to be equal.

$$\begin{aligned} R_{k+1}^Q &= R_{k+1}^V \\ &= R_k^V + H - HR_k^V - wR_k^V - wH + wHR_k^V + w - LR_k^V - HL + HLR_k^V + \\ &\quad wR_k^VL + wHL - wHLR_k^V - wL + L \\ &= R_k^V(1 - w) + w + H(1 - R_k^V - w + wR_k^V) + L(1 - R_k^V - w + wR_k^V) + \\ &\quad HL(-1 + R_k^V + w - wR_k^V) \end{aligned} \quad (4.10)$$

In that case, we have $D_k^Q + D_{k+1}^Q = D_k^V + D_{k+1}^V$.

Since $D_k^Q + D_{k+1}^Q \geq D_k^V + D_{k+1}^V$, firm realizes more revenue in sequence Q .

We already know that cost of advertisement at k and $k + 1$ is $c_A(H + L)$ for both sequences. Then, we obtain:

$$\pi_k^Q + \pi_{k+1}^Q \geq \pi_k^V + \pi_{k+1}^V \quad (4.11)$$

Since $R_{k+1}^Q \geq R_{k+1}^V$, we have that $R_{k+2}^Q \geq R_{k+2}^V$. Then, by Corollary 1, we have:

$$\pi_{k+2}^Q + \pi_{k+3}^Q + \dots \pi_N^Q \geq \pi_{k+2}^V + \pi_{k+3}^V + \dots \pi_N^V \quad (4.12)$$

Finally, we conclude that the overall profit of the sequence Q is an upper bound for V 's. $\square$

As a result of Proposition 2, under variable capacity, in an optimal advertising sequence, firm never makes high level advertisement after making low level advertisement for the first time. Therefore, an optimal advertising sequence should consist of a number of high level advertisements in the earlier periods and continue with low level advertisements after a point. To illustrate, an optimal advertising sequence could look like: $\{H, H, \dots H, L, L, \dots L\}$. Thus the problem becomes when to make the first low level advertisement, which reduces the required number of iterations from 2^N to $N+1$ (+1 is for the advertisement sequence consisting of only high level advertisements), implying a complexity of $O(N)$. Consequently, we are able to solve the problem in polynomial time.

We call the first period we observe a low level advertisement, *the switching point*. For example, in a problem with $N = 10$ and $\{H, H, H, L, L, L, L, L, L, L\}$ as the advertising sequence, the switching point would be 4.

In Proposition 1, we have seen that it is always more profitable for the firm to fully meet the demand and exploit positive WoM. When Proposition 2 is considered, it is seen that firm prefers to boost consumer expectations through advertisement when the returns of advertisement are high, that is at the earlier periods when R_i is the lowest. Then, in the later periods, the firm does not invest in advertising efforts as much, since advertisement's marginal return on reputation is lower at high R_i . Moreover, extra effort put into advertisement at any period i will contribute to R_j for every $j > i$. These propositions together imply that under variable capacity, the optimal strategy suggests aggressive growth. Firm builds up expectations and then builds them up even further by fully meeting them.

In the upcoming section, we examine whether a more low-key strategy could be an alternative to a more aggressive one, yielding the optimal Π . In a more moderate strategy, firm makes less high level advertisements, and focuses on cutting costs rather

than aggressive reputation growth. We question whether two advertising sequences as follows can yield the same overall profit, where $j \neq k$, or simply whether the optimal switching point is unique.

- $A^1 = \{A_1 = H, \dots, A_{j-1} = H, A_j = L, A_{j+1} = L, \dots, A_N = L\}$
- $A^2 = \{A_1 = H, \dots, A_{k-1} = H, A_k = L, A_{k+1} = L, \dots, A_N = L\}$

4.1.1 Analysis of Alternative Optima

In this section, we question whether there are alternative optimal switching points.

Let us denote the total profit of N periods, with switching point j , by $\Pi(j)$. We want to observe whether $\Pi(j)$ has only one local optima, therefore is concave in j , or not.

Now, let us define the following functions such that $\Pi(j) = M(j) - C(j)$, where j is the switching point:

- $\mu_i(j)$ represents the revenue and capacity cost of period i , when switching point is at period j . Then, $\mu_i(j) = (p - c)D_i$ or $\mu_i(j) = (p - c)S_i$ by Proposition 1. Note that $A_i = H$ if $i < j$ and $A_i = L$ if $i \geq j$.
- $M(j)$ represents the total revenue and total capacity cost of N periods, that is $M(j) = \sum_{i=1}^N \mu_i(j)$.
- Finally, $C(j)$ represents the total advertisement expenditure of N periods, that is $C(j) = \sum_{i=1}^N c_A A_i$.

It is clear that,

$$C(j) = (j - 1)H + (N - j + 1)L \quad (4.13)$$

$$= -H + NL + L + j(H - L) \quad (4.14)$$

Therefore, $C(j)$ is a linear, increasing function of j .

To study the other component of total profit, which is $M(j)$, we will consider the difference between having the switching point at period j versus $j + 1$, and see how $M(j + 1) - M(j)$ changes as j increases.

Comparing $M(j)$ and $M(j+1)$, it is seen that periods before j are equivalent. Therefore, $\sum_{i=1}^{j-1} \mu_i(j+1) = \sum_{i=1}^{j-1} \mu_i(j)$. Thus, investigating the following is sufficient:

- $\mu_j(j)$ versus $\mu_j(j+1)$
- $\sum_{i=j+1}^N \mu_i(j)$ versus $\sum_{i=j+1}^N \mu_i(j+1)$

Firstly, we present the comparison between $\mu_j(j)$ and $\mu_j(j+1)$ in the following result.

Lemma 2. *In the variable capacity problem under complete information, if $D_j \neq 0$ regardless of the advertisement level at period j ; the difference between $\mu_j(j)$ and $\mu_j(j+1)$, diminishes as R_j increases.*

Proof. $\mu_j(j)$ represents the revenue and capacity cost of period j when switching point is at j ; therefore we have $A_j = L$, and $\mu_j(j)$ is:

$$\mu_j(j) = (p - c)\Lambda \left(1 - \frac{u + p}{m(R_j + (1 - R_j)L^\alpha)} \right) \quad (4.15)$$

In the meanwhile, $\mu_j(j+1)$ represents the revenue and capacity cost of period j when switching point is at $j+1$, therefore we have $A_j = H$, and $\mu_j(j+1)$ is:

$$\mu_j(j+1) = (p - c)\Lambda \left(1 - \frac{u + p}{m(R_j + (1 - R_j)H^\alpha)} \right) \quad (4.16)$$

Note that, in both cases, $A_i = H$ for all $i < j$; hence, R_j is the same.

The interval, in which demand can be generated regardless of the advertisement decision is derived as follows.

$$\begin{aligned} D_i(L) &> 0 \\ \gamma_i(L) &> 0 \\ R_i &> \frac{\frac{u+p}{m} - L^\alpha}{1 - L^\alpha} \end{aligned} \quad (4.17)$$

Then, we need to show that whenever $R_j > \frac{\frac{u+p}{m} - L^\alpha}{1 - L^\alpha}$, the following is a non-increasing function of R_j :

$$(p - c)\Lambda \left(1 - \frac{u + p}{m(R_j + (1 - R_j)H^\alpha)} \right) - \left(1 - \frac{u + p}{m(R_j + (1 - R_j)L^\alpha)} \right) \quad (4.18)$$

This expression simplifies as:

$$\frac{(p-c)\Lambda(u+p)(H^\alpha - L^\alpha)}{m} \frac{1 - R_j}{(H^\alpha + (1 - H^\alpha)R_j)(L^\alpha + (1 - L^\alpha)R_j)} \quad (4.19)$$

The term $\frac{(p-c)\Lambda(u+p)(H^\alpha - L^\alpha)}{m}$ is a positive constant. $1 - R_j$ is a decreasing function of R_j and while $(H^\alpha + (1 - H^\alpha)R_j)(L^\alpha + (1 - L^\alpha)R_j)$ is an increasing function of R_j . Then, the expression given by Equation 4.19 becomes a decreasing function of R_j , which concludes the proof. $\square$

Since, R_i increases with i , Lemma 2 implies that the difference between $\mu_j(j)$ and $\mu_j(j+1)$ diminishes as j increases.

Now, let us consider $\sum_{i=j+1}^N \mu_i(j)$ and $\sum_{i=j+1}^N \mu_i(j+1)$, and let R_i^j denote the reputation level of period i when switching point is at j .

Clearly, R_{j+1}^{j+1} is larger than R_{j+1}^j , since it gets exposed to one more high level advertisement instead of low. Then by Corollary 1, we have $R_i^{j+1} \geq R_i^j$ for all $i > j$. Thus, we have $\sum_{i=j+1}^N \mu_i(j) \leq \sum_{i=j+1}^N \mu_i(j+1)$. We want to observe how the difference between those terms changes with j .

We have the following result regarding this comparison.

Lemma 3. *In the variable capacity problem under complete information, if two advertisement sequences with switching points at j and $j+1$ both have $D_j \neq 0$, the difference between $\sum_{i=j+1}^N \mu_i(j)$ and $\sum_{i=j+1}^N \mu_i(j+1)$ diminishes as j gets larger.*

Proof. Given that $R_j^j > \frac{u+p-L^\alpha}{1-L^\alpha}$ holds, after the switching point j , we have a reputation series defined by the following relationship, where $i \geq j$, since all $A_i = L$ and WoM takes effect.

$$R_{i+1} = (1-w)(R_i + (1-R_i)L^\alpha) + w \quad (4.20)$$

The above equation defines a series of R_i that is concave in i .

We need to compare the revenue and capacity costs of two such reputation series that start with different R_{j+1} 's:

- $R_{j+1}^j, R_{j+2}^j, R_{j+3}^j, \dots, R_N^j$, where $R_{j+1}^j = (1-w)(R_j + (1-R_j)L^\alpha) + w$.

- $R_{j+1}^{j+1}, R_{j+2}^{j+1}, R_{j+3}^{j+1}, \dots, R_N^{j+1}$, where $R_{j+1}^{j+1} = (1-w)(R_j + (1-R_j)H^\alpha) + w$.

Due to diminishing returns of advertisement, the difference between R_{j+1}^{j+1} and R_{j+1}^j will be smaller at high R_j , or large j .

Then, as γ_i is a concave function of R_i , $\sum_{i=j+1}^N D_i^{j+1} - D_i^j$ will be smaller at large j as well.

As a result, $\sum_{i=j+1}^N \mu_i(j+1) - \mu_i(j) = (p-c) \sum_{i=j+1}^N D_i^{j+1} - D_i^j$ diminishes with j , which concludes the proof. □

Proposition 3. *Under variable capacity and complete information, the optimal solution is unique if $R_1 > \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$.*

Proof. By Lemmas 2 and 3, we have that $\mu_j(j)$ and $\sum_{i=j+1}^N \mu_i(j)$, are both concave in j , given $R_i > \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$ for all i . We have already declared that $\sum_{i=1}^{j-1} \mu_i(j) = \sum_{i=1}^{j-1} \mu_i(j+1)$.

Since $R_i \leq R_{i+1}$ for all i , we can say that $M(j)$ is concave in j if $R_1 > \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$.

We have that $C(j)$ is a linear function of j . Therefore, $\Pi(j)$ is concave in j whenever $R_1 > \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$. As a result, the optimal switching point is unique when $R_1 > \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$ holds. □

From Proposition 3, we infer that if a firm is perceived well enough to generate the smallest amount of demand, there is a unique number of optimal high level advertisements to make. Making any number of high level advertisements that is less than the optimal, hoping to cut cost and depend on WoM to increase reputation, will never yield as much profit to the firm. Similarly, making more high level advertisements than the optimal level will yield high costs and lower profits. All in all, acting any more aggressively or modestly than optimal can never result in the same profit when firm is able to realize demand starting from the first period.

However, if $R_1 > \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$ does not hold, the behavior of $\Pi(j)$ cannot be generalized, due to $M(j)$'s behavior.

For one thing, especially when N is small, we can have $M(j) = 0$ for small j 's, since abandoning high level advertisement too early can mean $D_i = 0$ for all i . Moreover, suppose for some switching point s , the first time demand is realized is at some $k > s$ while for $s + 1$ demand is observed at an earlier period than k , say $k - \epsilon \geq s + 1$. As a result, we get $M(s) = (p - c) \sum_{i=k}^N D_i^s$ while $M(s + 1) = (p - c) \sum_{i=k-\epsilon}^N D_i^{s+1}$. Notice that $M(j)$ is steeper at $s + 1$, since there are more periods with $D_i > 0$ contributing to the revenue. Moreover, the first period demand is realized introduces WoM to the system, boosting reputation, and $M(j)$ even further. $M(j)$ contains such slope changes until some $\bar{j}$, such that $R_{\bar{j}}^{\bar{j}} > \frac{\frac{u+p}{m} - L^\alpha}{1 - L^\alpha}$. $M(j)$ is continuous for all j 's such that $j \geq \bar{j}$.

Below, we present $\Pi(j)$, $M(j)$ and $C(j)$ under different parameter sets. Optimal switching points, j^* , and the points after which $M(j)$ is continuous, $\bar{j}$, are marked in the figures. It can be observed that in Figure 4.1, where $R_1 > \frac{\frac{u+p}{m} - L^\alpha}{1 - L^\alpha}$, $\Pi(j)$ is continuous and concave in j , leading to single optimal solution. Therefore in this figure, there is no $\bar{j}$. Meanwhile, Figures 4.2, 4.3 and 4.4 represent $\Pi(j)$'s that are not continuous before $\bar{j}$. Notice that in Figure 4.2 and 4.3, $\bar{j}$ occurs before j^* whereas in Figure 4.4, j^* comes first. Also, in Figures 4.2 and 4.4, $\Pi(j)$ decreases in j before $\bar{j}$ while in Figure 4.2, it increases in j before $\bar{j}$.

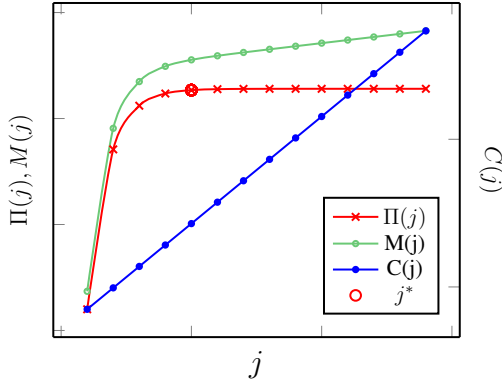


Figure 4.1: Behavior of $\Pi(j)$ When $R_1 > \frac{\frac{u+p}{m} - L^\alpha}{1 - L^\alpha}$

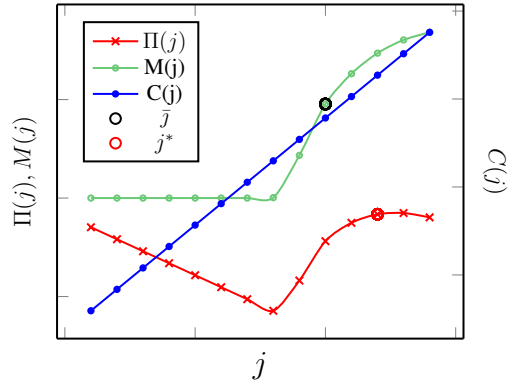


Figure 4.2: Behavior of $\Pi(j)$ When $R_1 \leq \frac{\frac{u+p}{m} - L^\alpha}{1 - L^\alpha}$

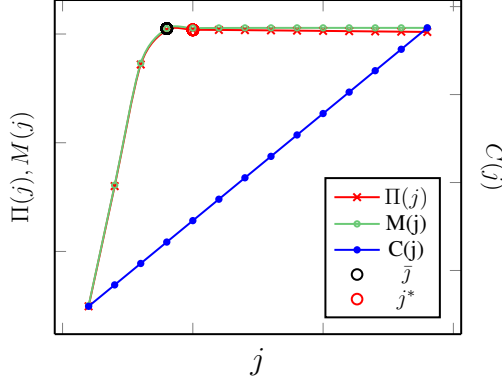


Figure 4.3: Behavior of $\Pi(j)$ When $R_1 \leq \frac{u+p-m}{1-L^\alpha}$

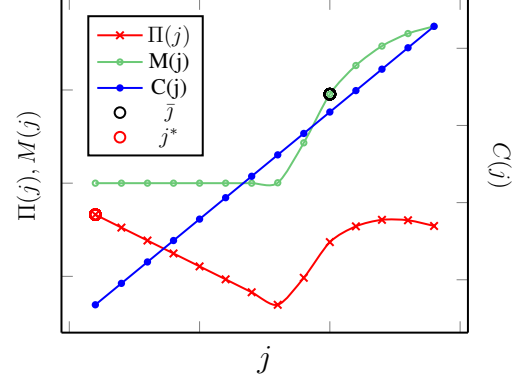


Figure 4.4: Behavior of $\Pi(j)$ When $R_1 \leq \frac{u+p-m}{1-L^\alpha}$

These imply that when firm has so negative recognition that it cannot realize demand without performing a series of advertising activities, it is possible that there are different optimal strategies. One such strategy is to put modest advertising efforts and wait for WoM to repair the reputation. The other strategy is to invest a lot of resources in advertisement activities and fix the reputation as soon as possible.

In this section we consider a firm that can capture the impact of the current performance on the company reputation and demand. In the following section, we consider the same problem under incomplete information where the firm is not aware of the feedback mechanism between the performance and demand.

4.2 Incomplete-Information Firm

In this section, we consider a firm that can observe its reputation at the beginning of each period and expect advertisement's effect on demand. Meanwhile, customer perception evolves according to the underlying WoM communication and utility-based demand structure. However, the firm is not aware of this mechanism, and assumes that there is some exogeneity in demand. Therefore, the firm observes its reputation at the beginning of each period, and determines the capacity and advertisement levels, to maximize the profit of the current period. This implies single-period profit maximization and the firm's problem can be expressed as follows.

$$\max_{A_i, S_i} \pi_i = p \min\{D_i, S_i\} - cS_i - c_A A_i \quad (4.21)$$

$$\text{where } D_i = \max\{0, \gamma_i\} \quad (4.22)$$

$$\gamma_i = \Lambda \left(1 - \frac{u+p}{m(R_i + (1-R_i)A_i^\alpha)} \right) \quad (4.23)$$

$$A_i \in \{H, L\}, S_i \geq 0 \quad (4.24)$$

Notice that while complete-information firm observes R_1 and plans the optimal advertisement and capacity policies of all periods, accounting for WoM; incomplete-information firm observes R_i at the beginning of each period, and makes the optimal advertisement and capacity decisions of each period individually.

Since the structure of this problem is different, we do not presume that there will be a single switching point policy for the advertisement choice, i.e., $A_i = L$ for all periods $i \geq j$ and $A_i = H$ for all $i < j$, as in complete-information firm case.

Proposition 1 applies here, since having $S_i = D_i$ maximizes π_i . Therefore, fill rate will either not exist or be equal to 1, implying $R_{i+1}^* \geq R_i^*$ for any i . As a result, we have $\pi_i = (p - c) D_i - c_A A_i$.

Then, the advertisement decision of period i is as follows.

- When $(p - c)\Lambda - c_A(H - L) \leq 0$, it is trivial that we have $A_i = L$; since, this condition means that advertisement is so costly that capturing the entire market would not help compensate the difference between high and low level advertising costs.
- When $(p - c)\Lambda - c_A(H - L) > 0$:
 - If no demand can be realized, meaning we have $R_i \leq \frac{\frac{u+p}{m} - H^\alpha}{1 - H^\alpha}$, then $A_i = L$.
 - If demand can only be realized under high advertisement, meaning we have $\frac{\frac{u+p}{m} - H^\alpha}{1 - H^\alpha} < R_i \leq \frac{\frac{u+p}{m} - L^\alpha}{1 - L^\alpha}$;
 - * if $R_i > \frac{\frac{\Lambda(p-c)(u+p)}{m(\Lambda(p-c)-c_A(H-L))} - H^\alpha}{1 - H^\alpha}$, then $A_i = H$;

* otherwise $A_i = L$.

– If demand can be generated under low level advertisement, meaning we have $R_i > \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$;

* if $\frac{1}{R_i+L^\alpha-R_iL^\alpha} - \frac{1}{R_i+H^\alpha-R_iH^\alpha} > \frac{mc_A(H-L)}{\Lambda(p-c)(u+p)}$, then $A_i = H$;

* otherwise $A_i = L$.

Since $\frac{1}{R_i+L^\alpha-R_iL^\alpha} - \frac{1}{R_i+H^\alpha-R_iH^\alpha}$ gets smaller as reputation increases, the firm will always makes low level advertisement once reputation exceeds a threshold.

Let us denote the expressions used in the above decision mechanism as follows for simplicity:

- $\iota = \frac{\frac{u+p}{m} - H^\alpha}{1-H^\alpha}$
- $\rho = \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$
- $\kappa = \frac{\frac{\Lambda(p-c)(u+p)}{m(\Lambda(p-c)-c_A(H-L))} - H^\alpha}{1-H^\alpha}$
- $\tau = \Lambda(p-c) - c_A(H-L)$
- ν st. $\frac{1}{\nu+L^\alpha-\nu L^\alpha} - \frac{1}{\nu+H^\alpha-\nu H^\alpha} = \frac{mc_A(H-L)}{\Lambda(p-c)(u+p)}$

Then the optimal decision for any period i can be summarized as below.

$$A_i = \begin{cases} H, & \text{if } \tau > 0 \text{ and } \iota < \rho \text{ and } R_i \in (\min\{\max\{\iota, \kappa\}, \rho\}, \max\{\rho, \nu\}) \\ L, & \text{otherwise} \end{cases} \quad (4.25)$$

Note that since we have $R_{i+1} \geq R_i$,

- (i) firm will make low level advertisement when $R_i \leq \min\{\max\{\iota, \kappa\}, \rho\}$,
- (ii) as reputation increases between consecutive periods, at some period, we will have $\min\{\max\{\iota, \kappa\}, \rho\} < R_i < \max\{\rho, \nu\}$, and firm will start making high level advertisements,

- (iii) finally, reputation will exceed $\max\{\rho, \nu\}$, and firm will continue with low level advertisements until the end of the last period.

Thus, firm will adopt an advertisement pattern such as:

$$A = \{L, L, L, H, H, H, L, L\} \quad (4.26)$$

We say that such solutions have *two switching points*. In the above example, where $N = 8$, the first switching point is at $j = 4$ and the second is at $j = 7$. Depending on the starting reputation, R_1 , firm may adopt a solution with one or two switching points. While the structure of the optimal advertisement policies are different under complete and incomplete-information, there is a relation between the optimal advertisement strategies of the two problems. The amount of high level advertisements in the incomplete-information problem creates a bound for the complete-information problem. In the following part, we present our structural results showing this relationship, where we use the following notation:

- $H_{R_1}^C$: the optimal number of high level advertisements in complete-information case when the starting reputation is R_1
- $H_{R_1}^I$: the optimal number of high level advertisements in incomplete-information case when the starting reputation is R_1

Proposition 4. *In the variable capacity problem with discrete advertisement, incomplete information firm never makes more high level advertisements than its complete information equivalent.*

Proof. Firstly, we will show that $H_{R_1}^C$ is (i) a non-decreasing function of N and (ii) a non-increasing function of R_1 . These results regarding the complete-information firm will be used in the upcoming parts of the proof of Proposition 4.

- (i) When the optimal switching point of an N period problem is j^* , it produces more Π than any later switching point. Thus, $\Pi(j^*) \geq \Pi(j^* + 1)$, and we have the following.

$$(p - c) \left(\sum_{i=1}^N D_i^{j^*+1} - D_i^{j^*} \right) \leq c_A(H - L) \quad (4.27)$$

As N increases, the left-hand side of Equation 4.27 increases as well. When N increases enough so that the above inequality does not hold, the optimal switching point becomes greater than j^* , which proves that $H_{R_1}^C$ is a non-decreasing function of N .

- (ii) Let us again consider an N -period problem with j^* as the optimal switching point. j^* produces more Π than any earlier switching point. Thus, $\Pi(j^*) \geq \Pi(j^* - 1)$, and we have the following.

$$(p - c) \left(\sum_{i=1}^N D_i^{j^*} - D_i^{j^*-1} \right) > c_A(H - L) \quad (4.28)$$

As R_1 goes up, R_i for all i go up as well, and the firm will not be able to increase reputation with advertisement as much, due to diminishing returns of advertisement. Left-hand side of the above inequality will decrease, and the optimal switching point will be less than j^* , which proves that $H_{R_1}^C$ is a non-increasing function of R_1 .

Next, we show that Proposition 4 holds under every condition:

1. We first prove that Proposition 4 holds when $\tau \leq 0$ or $\iota \geq \rho$:

$\iota \geq \rho$ implies that $\frac{u+p}{m} \geq 1$. Since $0 < R'_i < 1$, $\gamma_i = \Lambda \left(1 - \frac{u+p}{mR'_i} \right)$ becomes negative, which means it is not possible to realize demand at all. Then, making high level advertisement becomes meaningless, and the incomplete-information firm makes low level advertisement in all periods, which is optimal for complete-information firm as well. A similar situation occurs when we have $\tau \leq 0$; since it means that advertisement is so costly that making high level advertisement can never yield higher π_i than low level advertisement. In both these special cases, firms focus on cutting costs, and none of them makes high level advertisement. Therefore, $H_{R_1}^C = H_{R_1}^I = 0$, and Proposition 4 holds.

2. Next, we will prove that Proposition 4 holds when $\tau > 0$ and $\iota < \rho$:

2.1. Suppose, $R_1 \in [\max\{\rho, \nu\}, 1]$.

It is established that $R_i \leq R_{i+1}$ for all i . Therefore, when the sequence starts with $R_1 \in [\max\{\rho, \nu\}, 1]$, we say that $R_i \in [\max\{\rho, \nu\}, 1]$ for

all i . Then, the incomplete-information firm's advertisement decision is the same for every period, which is $A_i = L$.

As a result, $H_{R_1}^C \geq H_{R_1}^I = 0$, and Proposition 4 holds.

2.2. Suppose, $R_1 \in (\min \{\max\{\iota, \kappa\}, \rho\}, \max\{\rho, \nu\})$.

When we have $R_1 \in (\min \{\max\{\iota, \kappa\}, \rho\}, \max\{\rho, \nu\})$; the incomplete-information firm makes high level advertisement. Reputation only increases during the problem horizon; therefore, after a while, the firm reaches a point where $R_i \in [\max\{\rho, \nu\}, 1]$. Then, it continues with low level advertisements in the remaining periods.

These imply that the incomplete-information firm adopts a *single switching point solution*.

Every time the incomplete-information firm decides on making high level advertisement at a period i , the complete-information firm does so as well, since it increases both π_i and $\sum_{k=i+1}^N \pi_k$ (by Corollary 1, since R_{i+1} is higher after making high level advertisement).

When the incomplete-information firm switches to low level advertisement at period j , we have $\pi_j(j) \geq \pi_j(j+1)$, where $\pi_i(j)$ is the profit of period i when switching point is at j . However, since R_{j+1} will be greater if switching point is greater than j , we have that $\sum_{k=j+1}^N \pi_k(j+1) > \sum_{k=j+1}^N \pi_k(j)$. Then, if the following holds, the complete-information firm makes high level advertisement at j .

$$\sum_{k=j+1}^N \pi_k(j+1) - \sum_{k=j+1}^N \pi_k(j) > \pi_j(j) - \pi_j(j+1) \quad (4.29)$$

Thus, we have $H_{R_1}^C \geq H_{R_1}^I$, and Proposition 4 holds.

2.3. Finally, suppose, $R_1 \in (0, \min \{\max\{\iota, \kappa\}, \rho\}]$.

We consider the case where, we have significantly small reputation level at the first period. Thus, we should observe a pattern with *two switching points*.

Let us call the first switching point s , as in $A_s = H$ and $A_i = L$, for all $i < s$.

At period s , we have $R_s \in (\min \{\max\{\iota, \kappa\}, \rho\}, \max\{\rho, \nu\})$. We can consider the periods after s as an $N - s + 1$ period problem with R_s as the

starting reputation. Then, this new problem has a single switching point, and by result 2.2 of this proof, we have $H_{R_s}^I \leq H_{R_s}^C$.

The actual complete-information case has $N > N - s + 1$ periods and starts with $R_1 < R_s$. Then, as $H_{R_1}^C$ is a non-decreasing function of N and a non-increasing function of R_1 , we have $H_{R_1}^C \geq H_{R_s}^C$. Finally, we obtain $H_{R_1}^C \geq H_{R_s}^C \geq H_{R_s}^I = H_{R_1}^I$, and Proposition 4 holds.

We have shown that Proposition 4 holds in all possible cases, which concludes the proof.

□

Proposition 4, implies that in the complete-information scenario, the firm may prefer high advertisement costs as an investment on the reputation, and be rewarded in the upcoming periods. However, the incomplete-information firm makes to low level advertisement if the returns on advertisement investment will not be realized immediately. We call this the *myopic behavior* of the incomplete-information firm.

Using Proposition 4, we can reduce the number of iterations needed to find the optimal advertising sequence of the complete-information firm even further from $N + 1$ to $N - H_{R_1}^I + 1$. On top of that, it is possible to reduce the number of iterations even further by Proposition 3, since it indicates that $\Pi(j)$ is concave in switching point j after $\bar{j}$.

We have observed that company can make more informed decisions under complete-information on WoM communication and can interfere in customer demand. As a result, complete-information firm is able to gain reputation faster and gain higher profits. We name the difference between two firms' profits *Value of Information (VoI)*. In order to visualize this concept, we present Figure 4.5 on how reputation evolves for both firms, and Figure 4.6, on the growth of cumulative profit throughout N periods. Both figures represent the optimal solutions of the two firms with the parameter set: $R_1 = 0.2983$, $\alpha = 0.9762$, $H = 0.3749$, $L = 0.1400$, $m = 23.8867$, $u = 2.6973$, $p = 18.6337$, $w = 0.4150$, $\Lambda = 99,293$, $c = 1.4156$, $c_A = 382.5346$, $N = 20$.

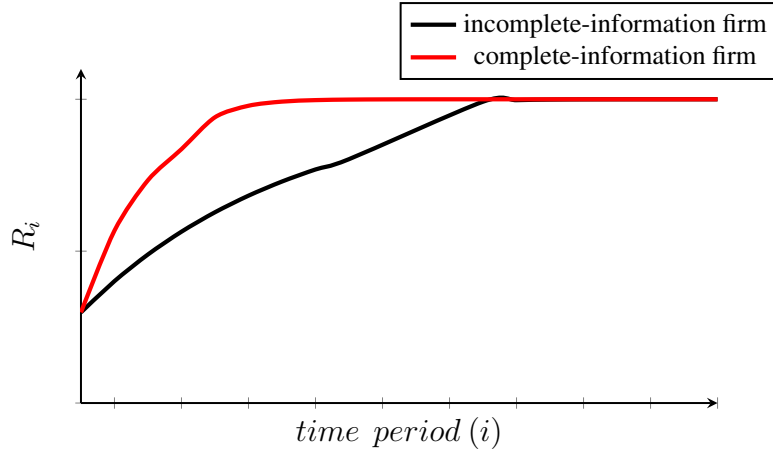


Figure 4.5: R_i Series Of Variable Capacity Firms

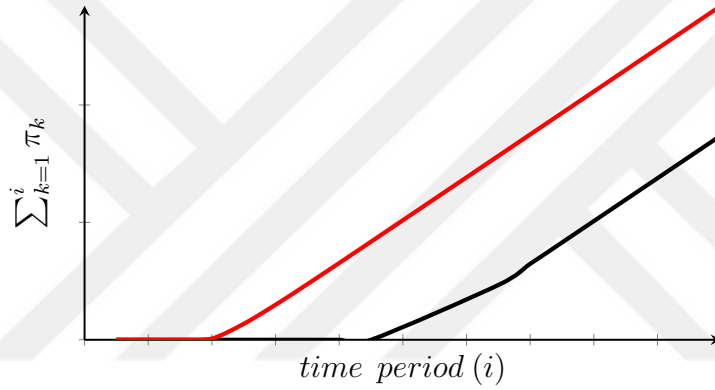


Figure 4.6: Cumulative π_i Series Of Variable Capacity Firms

Notice that reputation curves of both firms approach 1, and if N is large enough, reputations converge to 1. Correspondingly, once the gap between the two reputation curves become insignificantly small, the rate by which profit curves' gap widens becomes stable. As a result, in Figure 4.6, we observe that after a number of periods, the two cumulative profit curves become close to parallel. VoI corresponds to the difference between the two lines at period N of Figure 4.6. Therefore, when reputation curves increase fast and converge to 1, late periods do not contribute to VoI much.

The difference between two firms is greater if incomplete-information firm is not inclined to advertise and grow reputation as fast as its complete-information equivalent. Depending on the parameters, VoI may be larger, which indicates how advantageous complete-information firm is. Having said that, access to reliable information on customer behavior (e.g., sensitivity to advertisement, reliance on past information) does

not come at no cost. Therefore, it is important to understand the factors affecting the VoI. In the following section, we present our computational results and discussion to understand the factors affecting the VoI.

4.3 Computational Results on Value of Information (VoI)

We performed computational experiments to observe the impact of problem parameters on the VoI. We generated the test instances and solved them with enumeration-based methods, which are significantly improved by the structural analysis of the optimal solution, and can be solved in polynomial time. We coded the instance generation mechanism and the solution method using Python Programming Language version 3.8.5, where the test environment is Intel(R) Core(TM) i7-8700 CPU @ 3.20 GHz, 16GB RAM Windows 10 PC.

To test the impact of each parameter, we use 200 parameter sets randomly generated by the distributions from Table 4.1, and alter the parameter of concern 20 times while keeping the others constant.

Table 4.1: Parameter distributions of variable capacity problem computations.

parameter	distribution
R_1	$U(0, 1)$
w	$U(0, 1)$
L	$U(0, 1)$
H	$U(L, 1)$
N	$U(1, 50)$
c_A	$TN(500, 500, 0)$
α	$U(0, 1)$
m	$TN(15, 10, 0)$
Λ	$TN(10^5, 10^3, 0)$
u	$TN(0.5, 3, 0)$
c	$TN(2, 5, 0)$
p	$TN(c, 10, 0)$

We consider 12 parameters, which are the starting reputation (R_1), low advertisement level (L), high advertisement level (H), advertisement resistance (α), weight of WoM (w), market size (Λ), capacity cost (c), reputation's monetary value (m), reservation utility (u), price (p), cost of advertisement (c_A) and number of periods (N). In total, we generated 48,000 test instances.

Test instances are solved for both types of firms. For each test instance, the complete-information firm's optimal solution is obtained using enumeration while incomplete-information firm's optimal policy is obtained by going through problem horizon once and recording the decision at each time period based on the current reputation (by Equation 4.25). For both firms, every time a period is visited, the impacts of the advertisement decision and the resulting WoM are reflected in the system, and the parameters get updated. Therefore, both firms are subjected to the same market mechanisms in each test instance.

In a test instance, if the resulting advertising sequences of complete and incomplete-information firms of an observation are similar, we interpret this as low VoI, since incomplete information firm is able to approximate complete-information firm's optimal decisions without the information.

Before presenting each parameter's impact on VoI, let us provide some core observations that are used to discuss the experiment results.

Firstly, in the parameter sets where reputation increases fast in the earlier periods of the problem horizon, the gap between two firms' reputation curves, which is presented in Figure 4.5) becomes narrower. This is reflected in Π , and incomplete-information firm's total profit approaches complete-information's. Thus, while studying the experiment results, the parameters that boost reputation in the earlier periods will be said to decrease the VoI.

Secondly, as discussed earlier, in the optimal advertising sequences of two types of firms, we may encounter the following switching points:

- (i) complete-information firm switching from high to low level advertisement
- (ii) incomplete-information firm switching from low to high level advertisement

(iii) incomplete-information firm switching from high to low level advertisement

Note that, switching action in (ii) is only observed when starting reputation, R_1 , is below $\min\{\max\{\iota, \kappa\}, \rho\}$. If switching action described in (ii) does not exist, the incomplete-information firm can have a single switching action described in (iii).

Let us consider a setting where all three switching points are observed. That is, complete-information firm changes advertisement choice once, and incomplete-information firm changes the advertisement choice twice. If we had a higher starting reputation in the same setting; reputation would converge faster. This would reduce the need for advertisement, and complete-information firm would switch earlier. In the meanwhile, by Equation 4.25, switching points (ii) and (iii) would occur earlier as well. Suppose we repeat the experiment a number of times, each time having a higher R_1 than the last. In that case, all three switching points would move backwards between experiments, and get squeezed to the earlier periods. In other words, similarity between the two types of service suppliers' decisions increases.

Before diving into the experiment results, let us note a third observation. As advertisement levels are discrete, in each period firm has to choose between two options. Therefore, parameters' effect on VoI may not be continuous. To illustrate, consider we have some weight of WoM, w , for which both firms switch once. If we conduct another experiment with the same parameter set and the weight of WoM as $w + \epsilon$ where $1 \gg \epsilon > 0$, it is not guaranteed that both switching actions' locations will be different than the first experiment. We may observe one of the switching points changing its location while the other remains stable. This creates fluctuations in VoI; however, there is a clear trend between each parameter and the corresponding VoI. We make our interpretations based on this relationship.

Keeping these observations in mind, we examine each parameter in terms of their impact on VoI. We base our comparison on the locations of the switching points and how they change between experiments with respect to the parameter of concern.

For the sake of brevity, we present a subset of our computational results to provide important observations regarding the impact of parameters, in Tables 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12 and 4.13, where switching points are marked

for readability. Each table examines a parameter by gradually altering its value while keeping the other parameters fixed, between experiments. We report the optimal advertisement sequences of the two firms, along with the % VoI, that is calculated by:

$$\frac{\Pi_{complete\ info.}^* - \Pi_{incomplete\ info.}^*}{\Pi_{complete\ info.}^*} \quad (4.30)$$

We start by discussing the relationship between the starting reputation and VoI.

4.3.1 Impact of Starting Reputation

Regarding the impact of R_1 on VoI, we present a sample from the computations in Table 4.2, where each experiment uses the same parameter set except R_1 .

Table 4.2: Advertising policies of complete and incomplete-information firms under variable capacity with respect to starting reputation.

Test Parameter: R_1

$m : 15.87$	$L : 0.10$	$H : 0.28$	$\alpha : 0.62$	$w : 0.21$
$N : 19$	$c : 8.83$	$u : 6.13$	$p : 9.67$	$c_A : 806.28$
$\Lambda : 204, 744$				
R_1	Complete-Information Firm	Incomplete-Information Firm	% VoI	
0.207	H, H, H, H, H, H, H, H, H, H, L , ... L	L, ... L, L, L, L, L, L, L, L, L, L, L, L	135%	
0.251	H, H, H, H, H, H, H, H, H, H, L , ... L	L, ... L, L, L, L, L, L, L, L, L, L, L, H	134%	
0.426	H, H, H, H, H, H, H, H, H, L , L, ... L	L, ... L, L, L, L, L, L, L, L, L, L, H, L	122%	
0.561	H, H, H, H, H, H, H, H, L , L, ... L	L, ... L, L, L, L, L, L, L, L, L, H, L , L	110%	
0.658	H, H, H, H, H, H, H, L , L, ... L	L, ... L, L, L, L, L, L, L, H, L , L, L	98%	
0.718	H, H, H, H, H, H, L , L, L, ... L	L, ... L, L, L, L, L, L, L, H, L , L, L	94%	
0.753	H, H, H, H, H, L , L, L, ... L	L, ... L, L, L, L, L, L, H, L , L, L, L	86%	
0.851	H, H, H, H, H, L , L, L, L, ... L	L, ... L, L, L, L, L, H, L , L, L, L, L	66%	
0.925	H, H, H, H, L , L, L, L, L, ... L	L, ... L, L, L, H, L , L, L, L, L, L, L	49%	
0.948	H, H, H, L , L, L, L, L, L, ... L	L, ... L, H, L , L, L, L, L, L, L, L, L	35%	
0.998	H, L , L, L, L, L, L, L, L, L, ... L	L, ... L, L, L, L, L, L, L, L, L, L, L	1%	

We observe that as the starting reputation, R_1 , increases, the need for advertisement decreases, and the complete-information firm makes fewer high level advertisements. Meanwhile, the contribution of high level advertisement is more crucial for the incomplete-information firm. Therefore, the switching point (ii) (i.e., from L

to H) occurs earlier in the planning horizon.

As a result, VoI decreases. As Table 4.2 reveals, % VoI differs significantly for different starting reputations.

We infer that information on WoM communication is more influential for firms that have newly entered the market or the firms that are trying to improve their reputation after a brand damage.

4.3.2 Impact of the Advertisement Resistance

A sample from the computations regarding the advertisement resistance, α , is given in Table 4.3.

As α increases, consumers become less and less effected by advertisement; therefore, it gets harder for the firms to increase reputation. Therefore, complete-information firm makes more frequent high level advertisements to fight against the increasing resistance, as long as it is worth it. In Table 4.3, the incomplete-information firm switches once under the optimal advertising policy when α is low. Then, as α goes up, firm first starts to increase the number of high level advertisements, following its complete-information equivalent. However, as α increases further, the firm adopts a two-switching-point sequence, and both switching points move to later periods. Therefore, VoI increases in α , until it becomes meaningless to invest on advertisement and both firms choose to make low level advertisement in all periods. High α can be interpreted as consumers being less prone to updating their opinions; therefore, capacity decisions made by the firm in the past become more important, highlighting the importance on WoM awareness. Table 4.3 underlines the strong relationship between advertisement resistance and VoI, as % VoI changes significantly with α .

Table 4.3: Advertising policies of complete and incomplete-information firms under variable capacity with respect to advertisement resistance.

Test Parameter: α

$R_1 : 0.20$	$L : 0.02$	$H : 0.04$	$m : 18.98$	$w : 0.81$
$N : 17$	$c : 1.75$	$u : 0.70$	$p : 13.78$	$c_A : 700.29$
$\Lambda : 75, 871$				

α	Complete-Information Firm	Incomplete-Information Firm	% VoI
0.03	H, H, L , L, L, L, L, L, L, L, L, L, L, L, L	H, H, L , L, L, L, L, L, L, L, L, L, L, L, L	0%
0.09	H, H, H, L , L, L, L, L, L, L, L, L, L, L, L, L	H, H, H, L , L, L, L, L, L, L, L, L, L, L, L, L	0%
0.11	H, H, H, H, L , L, L, L, L, L, L, L, L, L, L, L	L, H , H, H, L , L, L, L, L, L, L, L, L, L, L, L	0%
0.21	H, H, H, H, H, L , L, L, L, L, L, L, L, L, L, L	L, H , H, H, H, L , L, L, L, L, L, L, L, L, L, L	1%
0.22	H, H, H, H, H, L , L, L, L, L, L, L, L, L, L, L	L, L, H , H, H, L , L, L, L, L, L, L, L, L, L, L	4%
0.26	H, H, H, H, H, H, L , L, L, L, L, L, L, L, L, L	L, L, H , H, H, H, L , L, L, L, L, L, L, L, L, L	1%
0.30	H, H, H, H, H, H, L , L, L, L, L, L, L, L, L, L	L, L, L, H , H, H, H, L , L, L, L, L, L, L, L, L	6%
0.36	H, H, H, H, H, H, H, L , L, L, L, L, L, L, L, L	L, L, L, L, H , H, H, H, L , L, L, L, L, L, L, L	7%
0.40	H, H, H, H, H, H, H, L , L, L, L, L, L, L, L, L	L, L, L, L, H , H, H, H, L , L, L, L, L, L, L, L	3%
0.41	H, H, H, H, H, H, H, L , L, L, L, L, L, L, L, L	L, L, L, L, L, H , H, H, H, L , L, L, L, L, L, L	9%
0.45	H, H, H, H, H, H, H, L , L, L, L, L, L, L, L, L	L, L, L, L, L, L, H , H, H, H, L , L, L, L, L, L	15%
0.47	H, H, H, H, H, H, H, H, L , L, L, L, L, L, L, L	L, L, L, L, L, L, H , H, H, H, L , L, L, L, L, L	10%
0.53	H, H, H, H, H, H, H, H, L , L, L, L, L, L, L, L	L, L, L, L, L, L, L, H , H, H, H, L , L, L, L, L	21%
0.56	H, H, H, H, H, H, H, H, L , L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, H , H, H, H, L , L, L, L	24%
0.57	H, H, H, H, H, H, H, H, L , L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, H , H, H, H, L , L, L	33%
0.59	H, H, H, H, H, H, H, H, H, L , L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, H , H, H, H, L , L	38%
0.62	H, H, H, H, H, H, H, H, H, L , L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, H , H, H, H, L	45%
0.66	H, H, H, H, H, H, H, H, H, H, L , L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, H , H, H	69%
0.70	H, H, H, H, H, H, H, H, H, H, H, L , L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L, L	100%
0.76	H, H, H, H, H, H, H, H, H, H, H, H, H, H, H	L, L, L, L, L, L, L, L, L, L, L, L, L, L, L	100%

4.3.3 Impact of Weight of WoM

Another factor we examine is the weight the customers assign to the information gained by WoM communication (w). We present a subset of related computational results in Table 4.4.

Table 4.4: Advertising policies of complete and incomplete-information firms under variable capacity with respect to weight of WoM.

Test Parameter: w

$R_1 : 0.454$	$L : 0.002$	$H : 0.512$	$\alpha : 0.54$	$m : 21.064$
$N : 21$	$c : 10.884$	$u : 5.721$	$p : 13.366$	$c_A : 525.303$
$\Lambda : 98, 536$				

w	Complete-Information Firm	Incomplete-Information Firm	% VoI
0.06	H, H, H, H, H, H, L , L, ... L	L, L, ... L, H , H, H, H	86%
0.16	H, H, H, H, H, H, L , L, L, ... L	L, L, ... L, H , H, H, H	86%
0.21	H, H, H, H, H, H, L , L, L, ... L	L, L, ... L, H , H, H, H	86%
0.26	H, H, H, H, H, L , L, L, L, ... L	L, L, ... L, H , H, H, H	86%
0.36	H, H, H, H, H, L , L, L, L, ... L	L, L, ... L, H , H, H, H	86%
0.41	H, H, H, H, H, L , L, L, L, ... L	L, L, ... L, H , H, H, L	86%
0.46	H, H, H, H, L , L, L, L, L, ... L	L, L, ... L, H , H, H, L	85%
0.56	H, H, H, H, L , L, L, L, L, ... L	L, L, ... L, H , H, H, L	85%
0.66	H, H, H, H, L , L, L, L, L, ... L	L, L, ... L, H , H, H, L	85%
0.81	H, H, H, L , L, L, L, L, L, ... L	L, L, ... L, H , H, L , L	85%

When w is high, consumers are easily impressed by the latest fill rate. Therefore, reputation increases fast for both complete and incomplete-information firms since fill rate is always 1 once demand is realized for the first time. One way to read this situation is that the information shared through WoM is not permanent, as it quickly gets overridden by the next fill rate. Therefore, firm's past actions lose importance, and VoI decreases in w . This is a counter intuitive result, since it indicates that as the strength of WoM communication increases, being aware of its effect on demand loses importance. This result is highly dependent on the fact that firm can adjust capacity in each period, and the direction of WoM effect is fixed (always increasing R_i).

On the other hand, when w is small, its effect on reputation is negligible, so it is expected to have low VoI at low w as well.

These two mechanisms function in opposing directions, canceling each other out. Finally, as the computations reveal, under variable capacity, VoI changes negligibly with w .

4.3.4 Impact of Reservation Utility

We now consider the impact of the reservation utility, u , and present a sample from the computational results in Table 4.5.

Table 4.5: Advertising policies of complete and incomplete-information firms under variable capacity with respect to reservation utility.

Test Parameter: u

$R_1 : 0.454$	$L : 0.02$	$H : 0.512$	$\alpha : 0.896$	$w : 0.775$
$N : 21$	$c : 10.884$	$m : 21.064$	$p : 13.366$	$c_A : 525.303$
$\Lambda : 98, 536$				
u	Complete-Information Firm	Incomplete-Information Firm	% VoI	
2.45	H, H, H, L , L, L, ... L	H, H, H, L , L, L, L, L, L, L, L, L, L, L, L, L, ...	0%	
2.60	H, H, H, H, L , L, L, ... L	L, L, L, L, L, L, H , H, H, L , L, L, L, L, L, L, L, ...	29%	
2.75	H, H, H, H, L , L, L, ... L	L, L, L, L, L, L, L, L, L, L, L, L, L, L, H , H, H, L , ...	70%	
2.90	H, H, H, H, L , L, L, ... L	L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, ...	100%	
3.10	H, H, H, H, L , L, L, ... L	L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, ...	100%	
5.10	H, H, H, H, L , L, L, ... L	L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, ...	100%	
7.10	H, H, H, H, H, L , L, ... L	L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, ...	100%	
8.10	L, L, L, L, L, L, L, ... L	L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, L, ...	0%	

As u increases, it becomes more difficult to attract customers. Therefore, complete-information firm's single switching point (i) moves to later periods in the horizon. However, if u becomes too high, it becomes very difficult and costly to generate demand, and switching point (i) jumps back to 1. On the other hand, for the incomplete-information firm, generating demand in the earlier periods becomes more difficult; therefore, both switching points (ii) and (iii) move to future periods. Mainly, u increases VoI up to a point, where it does not make sense to invest on advertisement regardless of the type of the firm.

In this particular example, at the last experiment in Table 4.5, complete-information firm does not operate, since total demand becomes zero when $A_i = L$ and making $A_i = H$ for some i simply is not worth it as u is too high. In the earlier experiments where $u = 2.90$, $u = 3.10$, $u = 5.10$ and $u = 7.10$; we have $\sum_{i=1}^N D_i = 0$ when $A_i = L$ for all i as well. Therefore, the incomplete-information firms decides not

operate at much lower values of u than the complete-information firm. This indicates that when u is high, it is difficult to generate demand but it can be done by early high level promotions. This may be costly at the beginning but may have a return later. However, the incomplete-information firm is not able to predict this, since it is approaching the problem myopically.

4.3.5 Impact of Reputation's Monetary Value

We present the computational results regarding the monetary value of the company's reputation in Table 4.6.

We observe that when m is very low, it becomes difficult to generate demand, and making high level advertisement is not worthwhile. Therefore, both firms adopt all-low advertisement policies. When m becomes large enough, the switching point of the complete-information firm jumps from 1 to a latter period. As m increases further, for the same reputation levels, complete-information firm is able to seize a greater portion of the market, and the need for advertisement decreases. As a result, switching point moves backwards, until m becomes so high that high level advertisement is not necessary at all.

In this particular example, having $A_i = L$ corresponds to realizing no demand at all. We observe that in the experiments where m is low, the incomplete-information firm gets stuck in the no-demand region as a result of its myopic decisions, since the problem horizon is not long enough for it to reach $R_i \geq \frac{\frac{u+p}{m} - L^\alpha}{1-L^\alpha}$ by making only low level advertisements. As m increases, the firm adopts a two switching point solution and starts to realize demand. Finally, when m increases further, the firm becomes able to generate demand in the early periods, so it adopts a pattern with single switching action. The switching point (iii) starts to move backwards until it reaches 1. To sum up, when consumers do not value reputation much, VoI is low, since firms focus on cutting costs rather than attracting demand. When the valuation increases enough, VoI drastically increases, and starts gradually decreasing. Information is more valuable when it is neither easy, nor impossible to generate demand.

Table 4.6: Advertising policies of complete and incomplete-information firms under variable capacity with respect to monetary value of reputation.

Test Parameter: m

$R_1 : 0.454$	$L : 0.002$	$H : 0.512$	$\alpha : 0.896$	$w : 0.775$
$N : 21$	$c : 10.884$	$u : 3.1$	$p : 13.366$	$c_A : 525.303$
$\Lambda : 98,536$				
m	Complete-Information Firm	Incomplete-Information Firm	% VoI	
15.94	L, L, L, L, L, L, L, L, L, L, L, L, ...	L, L, L, L, L, L, L, L, L, L, L, L, ...	0%	
16.68	H, H, H, H, H, H, L , L, L, L, L, L, L, ...	L, L, L, L, L, L, L, L, L, L, L, L, ...	100%	
21.32	H, H, H, H, L , L, L, L, L, L, L, L, L, ...	L, L, L, L, L, L, L, L, L, L, L, L, ...	100%	
21.32	H, H, H, H, L , L, L, L, L, L, L, L, L, ...	L, L, L, L, L, L, L, L, L, L, L, L, ...	100%	
21.62	H, H, H, H, L , L, L, L, L, L, L, L, L, ...	L, L, L, L, L, L, L, L, L, L, H , H, H, L , ...	49%	
21.68	H, H, H, H, L , L, L, L, L, L, L, L, L, ...	L, L, L, L, L, L, L, L, L, L, H , H, H, L , L, ...	39%	
21.69	H, H, H, H, L , L, L, L, L, L, L, L, L, ...	L, L, L, L, L, L, L, L, L, L, H , H, H, L , L, L, ...	34%	
21.78	H, H, H, H, L , L, L, L, L, L, L, L, L, ...	L, L, L, L, L, L, L, L, L, L, H , H, H, L , L, L, ...	19%	
21.86	H, H, H, L , L, L, L, L, L, L, L, L, L, ...	L, H , H, H, L , L, L, L, L, L, L, L, L, ...	5%	
21.87	H, H, H, L , L, L, L, L, L, L, L, L, L, ...	H, H, H, L , L, L, L, L, L, L, L, L, L, ...	0%	

4.3.6 Impact of the Length of the Planning Horizon

When number of periods' effect is examined, it is seen that complete-information firm is inclined to increase advertisement in longer problem horizons, since there is simply more time to collect the returns; therefore, switching point (i) moves forward. On the other hand, incomplete-information firm is not able to develop long-term advertising strategies. Therefore, its switching points do not move as N goes up or down.

A sample from the computations is given in Table 4.7. When $N = 3$, the complete-information firm realizes demand in the last period by placing high advertisement effort in all periods. The incomplete-information firm, however, fails to predict that and does not realize demand at all; therefore, it is not meaningful for it to operate. This suggests that when the market is more challenging than average, as in demand realization requires high efforts, the incomplete-information firm may not enter the market while complete-information firm does.

As the planning horizon gets longer, we observe two factors effecting VoI in opposite directions. Firstly, the advertising sequences differentiate more as the problem horizon gets longer, which increases VoI. However, the second factor playing out is that the incomplete-information firm has more time to increase reputation in longer horizons. Due to the increasing reputation, the π_i 's obtained in the later periods are closer to complete-information firm's. Therefore, as N increases, the advantage gained by the complete-information firm in the earlier periods become less significant, reducing % VoI. The second factor is more dominant than the first; therefore, the overall effect of N is a decrease in VoI.

Table 4.7: Advertising policies of complete and incomplete-information firms under variable capacity with respect to the length of the planning horizon.

Test Parameter: N				
$m : 12.10$	$L : 0.04$	$H : 0.08$	$\alpha : 0.30$	$R_1 : 0.40$
$w : 0.22$	$c : 4.68$	$u : 0.24$	$p : 10.56$	$c_A : 1,304.38$
$\Lambda : 292,128$				
N	Complete-Information Firm	Incomplete-Information Firm	% VoI	
3	H, H, H	L, L, L	100%	
5	H, H, H, H, H	L, L, L, H , H	44%	
8	H, H, H, H, H, H, H, H	L, L, L, H , H, H, H, H	19%	
10	H, H, H, H, H, H, H, H, H, L	L, L, L, H , H, H, H, H, H, H	13%	
11	H, H, H, H, H, H, H, H, H, H, L	L, L, L, H , H, H, H, H, H, H, L	12%	
26	H, H, H, H, H, H, H, H, H, H, L , L, ... L	L, L, L, H , H, H, H, H, H, H, L , L, ... L	4%	
48	H, H, H, H, H, H, H, H, H, H, L , L, ... L	L, L, L, H , H, H, H, H, H, H, L , L, ... L	2%	

4.3.7 Impact of the Advertisement Levels

Our next discussion is regarding the impact of the advertisement levels (H and L) on VoI, starting with H .

There are two ways H affects VoI, that function in the opposite directions. Firstly, when H is small (or close to L), we simply have two similar advertisement options in each period; therefore, regardless of how different the resulting advertising sequences may look, we expect small VoI. However, the second factor suggests that when H increases, reputation converges faster, and the complete-information firm makes fewer

high advertisement decisions. Since the higher levels of advertisement make it easier to attract demand, for the incomplete-information firm, switching point (ii) occurs earlier. As a result, reputation increases faster, and the switching point (iii) moves backward as well. This suggests that VoI decreases with H . As a result of this contradiction, we do not observe a significant change in % VoI for different values of H . Samples from the computations are given in Table 4.8.

Table 4.8: Advertising policies of complete and incomplete-information firms under variable capacity with respect to high advertising level.

Test Parameter: H				
$R_1 : 0.01$	$m : 19.80$	$L : 0.29$	$\alpha : 0.95$	$w : 0.81$
$N : 19$	$c : 1.35$	$u : 2.83$	$p : 8.66$	$c_A : 993.80$
$\Lambda : 98, 256$				
H	Complete-Information Firm	Incomplete-Information Firm	% VoI	
0.30	H, H, H, H, H, L , ... L	L, L, H , H, H, L , ... L	0%	
0.31	H, H, H, H, H, L , ... L	L, L, H , H, H, L , ... L	0%	
0.32	H, H, H, H, H, L , ... L	L, L, H , H, H, L , ... L	1%	
0.33	H, H, H, H, H, L , ... L	L, L, H , H, H, L , ... L	1%	
0.34	H, H, H, H, L , ... L	L, L, H , H, H, L , ... L	4%	
0.35	H, H, H, H, L , ... L	L, L, H , H, H, L , ... L	4%	
0.55	H, H, H, H, L , ... L	L, H , H, H, L , L, ... L	2%	
0.59	H, H, H, L , L, ... L	H, H, H, L , L, L, ... L	0%	
0.73	H, H, H, L , L, ... L	H, H, L , L, L, L, ... L	0%	
0.77	H, H, L , L, L, ... L	H, H, L , L, L, L, ... L	0%	

Examining the impact of low advertisement level, we present Table 4.9 with the samples from the computations.

When the low advertisement level is increased, the need for high level advertisement decreases; therefore, switching points (i) and (iii) move backwards. Moreover, low level advertisement increases reputation faster, pulling switching point (ii) backwards as well. Additionally, the increase in L makes the advertisement costs of the two choices closer. As a result, VoI decreases in L .

Table 4.9: Advertising policies of complete and incomplete-information firms under variable capacity with respect to low advertising level.

Test Parameter: L

$R_1 : 0.23$	$m : 29.37$	$H : 0.38$	$\alpha : 0.44$	$w : 0.30$
$N : 35$	$c : 11.49$	$u : 2.34$	$p : 20.21$	$c_A : 1,466.87$
$\Lambda : 120,553$				

L	Complete-Information Firm	Incomplete-Information Firm	% VoI
0.0002	H, H, H, H, H, H, L , ... L	L, L, L, L, L, L, L, H , H, H, H, H, L , ... L	20%
0.0019	H, H, H, H, H, H, L , ... L	L, L, L, H , H, H, H, H, L , L, L, L, L, ... L	8%
0.0831	H, H, H, H, H, H, L , ... L	L, H , H, H, H, H, L , L, L, L, L, L, ... L	1%
0.0878	H, H, H, H, H, L , L, ... L	L, H , H, H, H, H, L , L, L, L, L, L, ... L	1%
0.3746	H, H, H, H, H, L , L, ... L	L, H , H, H, H, L , L, L, L, L, L, L, ... L	0%

4.3.8 Impact of the Service Price

To represent the impact of service price on VoI, a sample from the computations is given in Table 4.10.

Table 4.10: Advertising policies of complete and incomplete-information firms under variable capacity with respect to service price.

Test Parameter: p

$R_1 : 0.43$	$L : 0.10$	$H : 0.37$	$\alpha : 0.62$	$w : 0.21$
$N : 8$	$c : 3.42$	$m : 10.88$	$u : 0.36$	$c_A : 603.12$
$\Lambda : 98,721$				

p	Complete-Information Firm	Incomplete-Information Firm	% VoI
3.64	H, H, H, L , L, L, L, L	H, H, H, L , L, L, L, L	0%
4.13	H, H, H, H, H, L , L, L	H, H, H, H, L , L, L, L	0%
5.37	H, H, H, H, H, H, L , L	H, H, H, H, H, L , L, L	0%
7.28	H, H, H, H, H, H, L , L	H, H, H, H, H, H, L , L	0%
7.75	H, H, H, H, H, H, H, L	L, H , H, H, H, H, H, L	7%
8.44	H, H, H, H, H, H, H, L	L, L, H , H, H, H, H, H	20%
9.73	H, H, H, H, H, H, H, L	L, L, L, L, L, H , H, H	65%
10.29	H, H, H, H, H, H, H, H	L, L, L, L, L, L, L, L	101%

When price is gradually increased, consumer utility decreases, and it becomes harder to attract demand. Complete-information firm's switching point (i) starts to move forward. Incomplete-information firm's switching point (ii) moves forward, since it gets harder to generate enough demand to make profit. Consequently, switching point (iii) moves forward as well. As a result, VoI increases with price.

As the example in Table 4.10 reveals, when $p = 10.29$, the incomplete-information firm fails to realize demand. Due to the myopic behavior, the firm cannot foresee the future benefits of paying for high level advertisement to build up reputation while complete-information firm can plan to collect the future profits.

4.3.9 Impact of Advertisement Cost

Regarding the impact of advertisement cost on VoI, we present a sample from the computations in Table 4.11.

Table 4.11: Advertising policies of complete and incomplete-information firms under variable capacity with respect to advertising cost.

Test Parameter: c_A				
$R_1 : 0.08$	$m : 23.332$	$H : 0.665$	$L : 0.311$	$w : 0.15$
$N : 10$	$c : 7.275$	$u : 0.783$	$p : 17.90$	$\alpha : 0.832$
$\Lambda : 180,498$				
c_A	Complete-Information Firm	Incomplete-Information Firm	% VoI	
38.88	H, H, H, H, H, H, H, H, L , L	L, H , H, H, H, H, H, H, L	7%	
175.43	H, H, H, H, H, H, H, L , L, L	L, H , H, H, H, H, H, L , L	7%	
1,150.99	H, H, H, H, H, H, L , L, L, L	L, H , H, H, H, H, L , L, L, L	7%	
1,365.27	H, H, H, H, H, H, L , L, L, L	L, H , H, H, H, H, L , L, L, L	7%	
10,000.00	H, H, H, H, H, L , L, L, L, L	L, H , H, H, H, L , L, L, L, L	8%	
350,000.00	H, H, L , L, L, L, L, L, L	L, L, H , L , L, L, L, L, L	20%	
500,000.00	H, H, L , L, L, L, L, L, L	L, L, H , L , L, L, L, L, L	25%	

When cost of advertising is gradually increased, complete-information firm's switching point (i) moves backward to cut costs.

The incomplete-information firm aims to maximize single period profit. As c_A increases, making low level advertisement becomes more profitable than its alternative. When reputation is low in the earlier periods, the firm cannot realize enough demand to make up for picking H . Meanwhile, in the late periods, reputation is so high that diminishing returns of advertisement takes effect, and picking L becomes more profitable. As a result, the switching point (ii) moves forward while (iii) moves backward.

Notice that unlike most of the other parameters' effects on the switching actions, increase in c_A does not move all switching points in the same direction.

The most significant effect on VoI is caused by switching point (ii), especially at low starting reputation. The gap between two firms' π_i 's is the largest in the earlier periods, and having the first switching action of incomplete-information firm late amplifies this difference. Having switching points (i) and (iii) in the earlier periods acts in the opposite direction in terms of impacting VoI. Since the movement of switching point (ii) is the most influential one, in the computations, we observe that VoI increases in c_A .

4.3.10 Impact of Capacity Cost

In the variable-capacity problem, the company chooses a capacity level that matches the demand. As c increases, the complete-information firm makes fewer high level advertisements to avoid higher demand levels as it becomes more costly to serve. Therefore, switching point (i) moves to earlier periods. In a similar manner, switching point (iii) of the incomplete-information firm moves backward as well. These factors act to decrease VoI. However, the firms start to pay more cost for unit demand fulfilled; therefore, switching point (ii) moves forward in the planning horizon, acting to increase VoI.

When all factors play out, VoI goes up. A sample from the computations is given in Table 4.12.

Table 4.12: Advertising policies of complete and incomplete-information firms under variable capacity with respect to capacity cost.

Test Parameter: c				
$R_1 : 0.001$	$L : 0.031$	$H : 0.512$	$\alpha : 0.257$	$w : 0.156$
$N : 10$	$c_A : 525.303$	$u : 5.721$	$p : 13.366$	$m : 21.064$
$\Lambda : 98, 536$				
c	Complete-Information Firm	Incomplete-Information Firm	% VoI	
7.284	H, H, H, H, L , L, L, L, L, L	L, H , H, H, H, L , L, L, L, L	10%	
8.362	H, H, H, H, L , L, L, L, L, L	L, H , H, H, H, L , L, L, L, L	10%	
8.583	H, H, H, H, L , L, L, L, L, L	L, H , H, H, H, L , L, L, L, L	10%	
9.625	H, H, H, H, L , L, L, L, L, L	L, L, H , H, H, L , L, L, L, L	16%	
13.000	H, H, H, L , L, L, L, L, L, L	L, L, H , H, L , L, L, L, L, L	16%	

4.3.11 Impact of the Market Size

A sample from the computations is given in Table 4.13.

Table 4.13: Advertising policies of complete and incomplete-information firms under variable capacity with respect to market size.

Test Parameter: Λ				
$R_1 : 0.001$	$L : 0.02$	$H : 0.753$	$\alpha : 0.246$	$w : 0.168$
$N : 15$	$c : 0.377$	$u : 0.998$	$p : 4.603$	$c_A : 612.02$
$m : 5.903$				
Λ	Complete-Information Firm	Incomplete-Information Firm	% VoI	
4, 613	H, H, L , L, L, L, ... L	L, L, H,L , L, L, ... L	11%	
21, 153	H, H, L , L, L, L, ... L	L, H , H L , L, L, ... L	5%	
51, 361	H, H, H, L , L, L, ... L	L, H , H L , L, L, ... L	6%	
74, 172	H, H, H, L , L, L, ... L	L, H , H L , L, L, ... L	6%	
111, 851	H, H, H, L , L, L, ... L	L, H , H, H L , L, ... L	5%	
149, 530	H, H, H, L , L, L, ... L	L, H , H, H L , L, ... L	5%	

As market size increases, complete-information firm is motivated to make more frequent high level advertisements; therefore switching point (i) moves forward. For the

same reason, the incomplete-information firm's switching point (ii) moves backward and (iii) moves forward. Again, the movement of switching point (ii) affect VoI in the opposite manner compared with the other two. VoI mainly decreases with the market size, since switching point (ii) creates a more significant impact. However, when market size is sufficiently high, the incomplete-information firm only makes one switching action from H to L . In that case, the effect of market size on VoI becomes insignificant. Mainly, VoI is the highest for moderate market size.

Generally, a changing condition can convince the complete-information firm easier to place more effort in advertisement; however, the incomplete-information firm has to see the benefit of the investment immediately, otherwise it is not convinced. When the changing condition motivates firms to cut cost and advertise less; the complete-information firm reduces the advertising efforts only if it is convinced that there is no gain it is missing out in the future. Meanwhile, the incomplete-information firm immediately gives up on placing high advertising effort if it is more profitable in the current period. Correspondingly, when a change in a problem parameter moves the switching points of both firms forward, switching point (i) moves faster than the others. On the other hand, it moves slower when moving backwards. We deduce that complete-information firm is more inclined to increase/preserve revenue, since it can expect the extra future gains to collect for the high advertisement efforts put in the early periods. In the meanwhile incomplete-information firm gives priority to cutting costs, since it cannot estimate the future benefits of early advertising.

4.4 Summary Of The Variable Capacity Problem

In the variable capacity problem, we have examined firm's advertisement and service capacity decisions under complete and incomplete information regarding WoM communication and its impacts. We have shown that under variable capacity, both complete and incomplete-information firms choose service capacity levels equal to demand, and exploit positive WoM to increase demand in the upcoming periods, by Proposition 1. An important implication is that since firm always fulfills demand, WoM will always be positive, and firm's perception will only improve in time. We reflect this situation in Figure 4.7 below, which is constructed with the same param-

eter set as Figures 4.6 and 4.5. As the figure reveals, until the first time demand is realized, reputation grows with the advertisement efforts, and after that, fill rate starts to contribute to reputation as well.

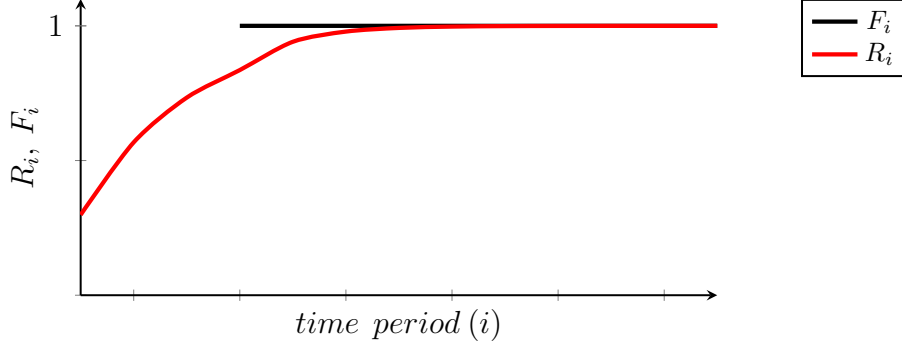


Figure 4.7: Reputation vs Fill Rate Under Variable Capacity

This continuous improvement in reputation implies that consumers keep expecting more from the service, and the firm will keep investing more in capacity. Yet, as reputation approaches 1, demand and, hence the service capacity will converge to $\Lambda \left(1 - \frac{u+p}{m}\right)$, so firm's additional capacity acquisitions between consecutive periods will diminish in time.

When it comes to the optimal advertisement policy, due to the complexity of our demand model, a closed form solution cannot be obtained. However, structural results on the optimal policy have strong implications, as well as reducing the problem complexity from $O(2^N)$ to $O(N)$. One of the most significant results we obtained is that complete-information firm places high effort in advertising activities at the earlier time periods and continues with a reduced effort, by Proposition 2. It is more profitable for the firm to make use of advertisement to increase demand when reputation is low, which corresponds to earlier time periods. We call the period in which the firm makes low level advertisement for the first time as the *switching point*. Notice that switching point characterizes the optimal policy on its own.

Then, we have shown that for a firm that can generate positive demand, there is a unique optimal advertising policy, or unique optimal switching point, by Proposition 3. For firms that cannot generate demand immediately and have to use gradual advertisement to build on their reputation, there may be alternative optimal advertising

policies. These are the firms that are just entering the market or that have suffered brand damage.

Propositions 1 and 2 together imply that it is optimal for the firm to boost the expectations of the customers by advertisements and by maintaining enough service capacity, and to meet the expectations. Firm raises the bar by these actions for itself; therefore, it gets harder to provide highest quality service but the firm does so anyway. From these results, we infer that WoM motivates firms to provide high service quality.

In the incomplete-information scenario, the firm observes its reputation at the beginning of each period and decides on S_i and A_i . The firm is not aware of WoM; therefore, how reputation changes from R'_i to R_{i+1} is believed to be exogenous. As a result, firm makes myopic decisions to maximize π_i , instead of Π . Advertisement's contribution to reputation is never lost, since reputation never drops. Therefore, putting more effort in advertisement is considered investing in reputation to increase future demand. The incomplete-information firm is unable to account for this; therefore makes fewer high level advertisements than its complete-information equivalent, as presented by Proposition 4.

Finally, we present the computational results to compare the two types of firms in terms of VoI. Note that extreme values of the parameters lead to zero VoI, since decisions become trivial. Assuming no extreme conditions, we observed that VoI is the highest when starting reputation (R_1), weight of WoM (w), monetary value of reputation (m), number of periods (N), both advertisement levels (H and L) are low; while advertisement resistance (α), reservation utility (u), price (p), advertisement cost (c_A) and capacity cost (c) are high; and market size (Λ) is medium.



CHAPTER 5

CONSTANT CAPACITY PROBLEM

In this chapter, we examine the system under a fixed service capacity that is predetermined and given to us as S . In this new setting, the firm only makes the advertisement decisions, choosing between the high (H) and low (L) advertisement levels for each period.

Constant capacity problem introduces new challenges into the advertisement decision. Since the firm cannot change service capacity, advertisement is the only tool to manage demand. However, since the advertisement decisions are discrete, the firm may increase the reputation more than desired with high level reputation (or not as much with low level advertisement) considering the capacity. If the firm attracts more demand than the capacity, the fill rate will be low and will lead to a decrease in the reputation through WoM. Therefore, there may be fluctuations in the reputation, and the single switching point solution is often not optimal for the constant capacity firm.

We again consider two types of firms in terms of the availability of information on WoM communication and its impacts: (i) complete-information firm, and (ii) incomplete-information firm. In this section, we first examine the solution structure for these scenarios. Then, we present computational results to understand the impact of problem parameters on the value of having full information on endogenous demand structure.

To ensure that we address the problem in a meaningful setting, we make the following assumptions:

- (i) Capacity is less than the market size ($S < \Lambda$).

- (ii) We have $\frac{u+p}{m} < 1$; otherwise, regardless of the company reputation, no demand can be generated.

Our analysis is based on the conditions of the firm at the beginning of each period. Before going into the discussion, we introduce these conditions.

5.1 Classification of the Firm's Demand Realization Ability

At the beginning of each time period, the firm makes an advertisement decision, affecting the demand generated in that period. We provide a classification for the status of the firm at the beginning of a period, in terms of its demand generating ability. We define a set of *states* by comparing the amount of demand the firm can generate under high and low level advertisements with capacity S and with 0.

Under the assumptions $\frac{u+p}{m} < 1$ and $\Lambda > S$; at the beginning of any period i , the firm can be in one of the following states:

State A:

Demand will be greater than or equal to capacity regardless of the advertisement level, that is $D_i \geq S$ when $A_i = L$ or $A_i = H$.

State B:

Demand will be greater than or equal to capacity if high level advertisement is picked, and will be less than capacity (but positive) otherwise, that is $D_i \geq S$ when $A_i = H$, and $S > D_i > 0$ when $A_i = L$.

State C:

Demand will less than capacity (but positive) regardless of the advertisement level, that is $S > D_i > 0$ when $A_i = H$ and $A_i = L$.

State D:

Demand will be greater than or equal to capacity if high level advertisement is picked, and will be zero otherwise, that is $D_i \geq S$ when $A_i = H$, and $D_i = 0$ when $A_i = L$.

State E:

Demand will be less than capacity (but positive) if high level advertisement is picked, and zero otherwise, that is $S > D_i > 0$ when $A_i = H$, and $D_i = 0$ when $A_i = L$.

State F:

Demand will be zero regardless of the advertisement level, that is $D_i = 0$ when $A_i = H$ and $A_i = L$.

The states defined above help us understand firm's position at any time period, and provide simplicity in our expressions. Notice that for each period i , firm's state is a direct result of R_i . Therefore, each state defines an interval for reputation by the conditions given above, and occurs when R_i falls into that interval. In order to determine these intervals, let us introduce the following thresholds for reputation.

- We denote the minimum reputation level required to have $D_i \geq S$ ($\gamma_i \geq S$), given $A_i = L$, by X_1 calculated as follows:

$$X_1 = \frac{\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} - L^\alpha}{1 - L^\alpha}. \quad (5.1)$$

- We denote the minimum reputation level required to have $D_i \geq S$ ($\gamma_i \geq S$), given $A_i = H$, by X_2 calculated as follows:

$$X_2 = \frac{\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} - H^\alpha}{1 - H^\alpha}. \quad (5.2)$$

- We denote the minimum reputation level required to have $\gamma_i \geq 0$, given $A_i = L$, by Y_1 calculated as follows:

$$Y_1 = \frac{\frac{u+p}{m} - L^\alpha}{1 - L^\alpha}. \quad (5.3)$$

- We denote the minimum reputation level required to have $\gamma_i \geq 0$, given $A_i = H$, by Y_2 calculated as follows:

$$Y_2 = \frac{\frac{u+p}{m} - H^\alpha}{1 - H^\alpha}. \quad (5.4)$$

By the definitions of these thresholds, it is clear that $X_1 > X_2$, $Y_1 > Y_2$, $X_1 > Y_1$ and $X_2 > Y_2$. However, the relationship between Y_1 and X_2 depends on the parameters.

If we have $Y_1 > X_2$, occurrences of the states A, B, C, D, E and F, on a reputation axis, can be shown below. For instance, if $X_2 < R_i < Y_1$ at some period i , the firm is at state D.



If $Y_1 < X_2$, states A, B, C, D, E and F occur by the following reputation axis.



Let us denote the set of states that can be observed with a given parameter set by Ω . Regarding Ω , we have the following results.

(i) If $Y_1 > X_2$, $A \in \Omega$.

Proof. We have assumed that $\frac{u+p}{m} < 1$; therefore, by definition, we have $Y_1 < 1$ and $Y_2 < 1$. Then, when $Y_1 > X_2$, we say $1 > X_2$ as well, which indicates that $\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} < 1$. Therefore, $X_1 < 1$ must hold, meaning that there must be some $1 > \epsilon > 0$ such that $X_1 + \epsilon < 1$, in other words, state A is always observable. $\square$

(ii) If $Y_1 < X_2$; and if $\{E, F\} \cap \Omega \neq \emptyset$, then $C \in \Omega$.

Proof. We have $\frac{u+p}{m} < 1$; therefore, $Y_2 < Y_1 < 1$. $\{E, F\} \cap \Omega \neq \emptyset$ implies that reputation can take smaller values than Y_1 ; therefore, $Y_1 > 0$. Then, there must be some $1 > \epsilon > 0$ such that $0 < Y_1 + \epsilon < 1$, allowing state C to occur. We conclude that Ω cannot include E and/or F while excluding C . $\square$

(iii) If $Y_1 < X_2$; and if $B \in \Omega$, then $A \in \Omega$.

Proof. If $B \in \Omega$, we should have $1 > X_2$, meaning $\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} < 1$. This indicates that $X_1 < 1$. Then, there must be some $1 > \epsilon > 0$ such that $X_1 + \epsilon < 1$, implying that $A \in \Omega$. $\square$

Given a parameter set, cardinality of Ω can be reduced by ruling out some of the states if the corresponding reputation intervals of those states do not intersect with $[0, 1]$. Using the results above, Ω of a given parameter set can be one of the following.

- If $Y_1 > X_2$:

- $\Omega = \{A, B, D, E, F\}$, if $H^\alpha < \frac{u+p}{m}$ and $\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} < 1$.
- $\Omega = \{A, B, D, E\}$, if $H^\alpha \geq \frac{u+p}{m} > L^\alpha$ and $H^\alpha < \frac{u+p}{m} \frac{\Lambda}{\Lambda-S} < 1$.
- $\Omega = \{A, B, D\}$, if $H^\alpha \geq \frac{u+p}{m} > L^\alpha$ and $\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} \leq H^\alpha$.
- $\Omega = \{A, B\}$, if $L^\alpha \geq \frac{u+p}{m}$ and $L^\alpha < \frac{u+p}{m} \frac{\Lambda}{\Lambda-S} \leq H^\alpha$.
- $\Omega = \{A\}$, if $L^\alpha \geq \frac{u+p}{m} \frac{\Lambda}{\Lambda-S}$.

- If $Y_1 < X_2$:

- $\Omega = \{A, B, C, E, F\}$, if $H^\alpha < \frac{u+p}{m}$ and $\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} < 1$.
- $\Omega = \{A, B, C, E\}$, if $H^\alpha \geq \frac{u+p}{m} > L^\alpha$ and $H^\alpha < \frac{u+p}{m} \frac{\Lambda}{\Lambda-S} < 1$.
- $\Omega = \{A, B, C\}$, if $L^\alpha \geq \frac{u+p}{m}$ and $H^\alpha < \frac{u+p}{m} \frac{\Lambda}{\Lambda-S} < 1$.
- $\Omega = \{A, B\}$, if $L^\alpha \geq \frac{u+p}{m}$ and $L^\alpha < \frac{u+p}{m} \frac{\Lambda}{\Lambda-S} \leq H^\alpha$.
- $\Omega = \{A\}$, if $L^\alpha \geq \frac{u+p}{m} \frac{\Lambda}{\Lambda-S}$.
- $\Omega = \{C, E, F\}$, if $H^\alpha < \frac{u+p}{m}$ and $\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} \geq 1$.
- $\Omega = \{C, E\}$, if $H^\alpha \geq \frac{u+p}{m} > L^\alpha$ and $\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} \geq 1$.
- $\Omega = \{C\}$, if $L^\alpha \geq \frac{u+p}{m}$ and $\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} \geq 1$.

Note that states with higher reputation values, such as A, yield lower fill rates than the others. At the end of a period i with state C, E or F, we have $R_{i+1} > R_i$. Same holds for states B and D if $A_i = L$. Then we can make interpretations such as: state $i + 1$ cannot be E or F when state i is C; since states E and F occur at lower reputation levels than C, and F_i is always 1 at state C.

When we are at state A or when $A_i = H$ at states B and D; the relationship between R_i and R_{i+1} may differ depending on advertisement choice and the exact value of fill rate.

Keeping these in mind, let us continue the constant capacity scenario with the complete-information firm.

5.2 Complete-Information Firm

Under the complete-information scenario, the firm tries to maximize profit considering the endogenous demand, through the advertising decision. The problem can be modeled as the following dynamic program, where stages correspond to time periods, the state corresponds to R_i , and the action is A_i .

$$\begin{aligned}
P_i(R_i) &= \max_{A_i} \{ \pi_i + P_{i+1}(R_{i+1}) \} \\
P_N(R_N) &= \max_{A_N} \{ \pi_N \} \\
\text{where } R_{i+1} &= \begin{cases} (1-w)(R_i + (1-R_i)A_i^\alpha) + w \frac{\min\{D_i, S\}}{D_i}, & \text{if } \gamma_i > 0 \\ (R_i + (1-R_i)A_i^\alpha), & \text{otherwise} \end{cases} \\
\pi_i &= p \min\{D_i, S\} - cS - c_A A_i, \\
D_i &= \max\{0, \gamma_i\}, \\
\gamma_i &= \Lambda \left(1 - \frac{u+p}{m(R_i + (1-R_i)A_i^\alpha)} \right) \text{ and} \\
A_i &\in \{H, L\}.
\end{aligned}$$

To imagine the system better, consider a parameter set such that $\Omega = \{A, B, C, E, F\}$. Suppose that in the first period, we have some R_1 that can generate demand regardless of the advertisement level, and the firm has enough capacity to fulfill it (e.g. state C). Then, firm will face positive WoM impact, and its reputation will increase. In the upcoming periods, unless demand exceeds capacity ($D_i > S$), reputation will keep increasing, and so is demand. Since firm has to pick among the discrete advertisement levels, it can only slow down the increase in reputation, rather than avoiding it. Therefore, at some period i , we will eventually have $D_i > S$ (e.g. state A) and $F_i < 1$. If fill rate drops enough, the firm will have $R_i > R_{i+1}$. This decrease in reputation will continue until fill rate is high enough. Then it would start increasing again. This behavior is observed as oscillations in R_i . Firm's advertisement decisions affect the shape and period of these oscillations, which sometimes appear in repetitive patterns. On the other hand, in other parameter sets that have high enough capacity (e.g. $\Omega = \{C, E, F\}$), R_i^* 's construct a monotone non-decreasing series. There are different behaviors of R_i^* 's as well, depending on Ω and the parameters.

Due to different characteristics of different parameter sets and the instability of reputation, firm's optimal advertisement strategy often cannot be expressed in a structured manner like the single switching point of the variable capacity firm. Therefore, in an N -period problem, we need to enumerate 2^N solutions to find the optimal advertising sequence, which consumes significant computational effort. We analyze firm's advertisement decision to identify the conditions that reduce problem's complexity. In the next section, we present our findings regarding such special cases.

5.2.1 Special Cases

We identify the special cases, under which some solutions dominate others.

We start by introducing Proposition 5.

Proposition 5. *Under constant capacity and complete-information on WoM communication, if demand is always lower than the service capacity; if $A_i^* = L$, then $A_j^* = L$ for all $j > i$.*

Proof. We have $R_{i+1} = R'_i$ when $D_i = 0$, and $R_{i+1} = wF_i + (1-w)R'_i$ when $D_i > 0$. Therefore, for simplicity, we can use the following function to set fill rate, and stick with the formula $R_{i+1} = wF_i + (1-w)R'_i$ to determine R_{i+1} .

$$F_i = \begin{cases} R'_i, & D_i = 0 \\ \frac{\min\{S, D_i\}}{D_i}, & D_i > 0 \end{cases} \quad (5.5)$$

Since $D_i \leq S$ always holds, we have that $F_i = R'_i$ or $F_i = 1$ for all i . As a result, $R_{i+1} \geq R_i$, which implies that $F_i = R'_i$ is true up to the first period demand is observed and from that point and on, we have $F_i = 1$.

Suppose we have an advertisement sequence $V = \{V_1, V_2, \dots, V_N\}$, where $V_k = L$ and $V_{k+1} = H$, for some k , in an N -period problem. Then, using V , let us define a second sequence Q such that $Q_i = V_i$, for all $i \in \{1, 2, \dots, k-1, k+2, \dots, N\}$ while $Q_k = H$, $Q_{k+1} = L$.

We denote the parameters such as reputation, post-advertisement reputation, fill rate, demand and profit at period i , using the sequence information (e.g. firm's reputation

at sequence V and period i is represented by R_i^V).

In both sequences, the number of times high and low level advertisements are made and the total capacity costs are the same. Therefore, our sole concern in comparing the sequences is the total revenue, which is expressed as $\sum_{i=1}^N p D_i$, since demand cannot exceed capacity. As p is constant as well, we can reduce the comparison to the sum of demands, that is $\sum_{i=1}^N D_i^V$ versus $\sum_{i=1}^N D_i^Q$.

Up to period k , the two sequences are equivalent, always making the same decisions. Thus, $\sum_{i=1}^{k-1} D_i^Q = \sum_{i=1}^{k-1} D_i^V$.

At period k , we have that $Q_k = H$ while $V_k = L$; therefore, $R_k^V \leq R_k^Q$ and consequently $F_k^V \leq F_k^Q$ and $D_k^V \leq D_k^Q$.

At period $k + 1$, we have the following post-advertisement reputation for sequence V , where $F_k^V = 1$ if $D_k > 0$ and $F_k^V = R_k + (1 - R_k)H^\alpha$ otherwise:

$$R_{k+1}^V = H^\alpha + (1 - H^\alpha)(wF_k^V + (1 - w)(L^\alpha + (1 - L^\alpha)R_k)), \quad (5.6)$$

and the following post-advertisement reputation for sequence Q , where $F_k^Q = 1$ if $D_k > 0$ and $F_k^Q = R_k + (1 - R_k)L^\alpha$ otherwise:

$$R_{k+1}^Q = L^\alpha + (1 - L^\alpha)(wF_k^Q + (1 - w)(H^\alpha + (1 - H^\alpha)R_k)). \quad (5.7)$$

We have $R_{k+1}^V < R_{k+1}^Q$ if $F_k^V = R_k + (1 - R_k)H^\alpha$ while $F_k^Q = 1$, and otherwise $R_{k+1}^V = R_{k+1}^Q$. Then, $F_{k+1}^V \leq F_{k+1}^Q$ and $D_{k+1}^V \leq D_{k+1}^Q$.

Since $R_{k+1}^V \leq R_{k+1}^Q$ and $F_{k+1}^V \leq F_{k+1}^Q$, we have $R_{k+2}^V \leq R_{k+2}^Q$. Therefore, for all remaining periods we can say that $D_i^Q \geq D_i^V$, where $i > k + 1$.

Finally, we conclude that $\sum_{i=1}^N D_i^Q \geq \sum_{i=1}^N D_i^V$, leading to $\Pi^Q \geq \Pi^V$. $\square$

Proposition, 5 implies that when the firm can guarantee high service quality, it is more profitable to capture greater portions of the market as fast as possible, by boosting reputation.

By Proposition, 5, whenever the following holds, we can say that our problem has a single switching point from H to L , as in the variable capacity problem, where the left-hand-side represents the required post-advertisement reputation level to have

demand equal to capacity.

$$\frac{u+p}{m} \frac{\Lambda}{\Lambda-S} \geq 1 \quad (5.8)$$

When this condition holds, we say that demand can never reach or exceed capacity, and we have $\Omega \subset \{C, E, F\}$.

This allows us to decrease complexity from $O(2^N)$ to $O(N)$.

Another result we present is regarding the situations where realizing high demand is easier. By Lemma 4, whenever we have $\Omega \subset \{A, B, D\}$, we can calculate an upper bound for the number of high level advertisements in the optimal solution.

Lemma 4. *Under constant capacity, for any $R_i \in (0, 1)$, if we have $D_i \geq S$ for $A_i = H$, the optimal number of high level advertisements cannot be greater than the one obtained by the following heuristic.*

- pick L when state is A
- pick H when state is not A

Proof. Let us denote the number of high level advertisements this heuristic results in by h^* .

Following the given heuristic, firm obtains the maximum possible revenue, that is pSN . Making more high level advertisements would not increase the total revenue but it would increase the total cost. Therefore, the above heuristic provides a better solution than any advertising sequence with more high level advertisements than h^* . $\square$

By Lemma 4, we infer that when firm is able to seize enough demand to utilize service capacity just by a single advertisement action at each period, making high level advertisement only when demand needs a boost to exceed service capacity provides a lower bound to the optimal amount of profit. Having complete information on the endogenous demand structure, firm can be able to come up with less costly advertising policies to keep demand high.

Lemma 4 reduces the number of iterations required from 2^N (or $\sum_{k=0}^N \binom{N}{k}$) to the following.

$$\sum_{k=0}^{h^*} \binom{N}{k} \quad (5.9)$$

For our final special case, let us consider the conditions under which the firm drops to a lower-reputation state from state A (R_i drops below X_1). Firstly, to observe this situation, we need to have a parameter set such that $A \in \Omega$ and $\Omega \neq \{A\}$. Therefore, we need to have $0 < X_1 < 1$, which translates as follows.

$$\begin{aligned} \frac{u+p}{m} \frac{\Lambda}{\Lambda-S} &< 1 \\ \frac{u+p}{m} \frac{\Lambda}{\Lambda-S} &> L^\alpha \end{aligned} \quad (5.10)$$

Dropping from state A means, next period begins with a reputation level less than X_1 , as in $R_{i+1} < X_1$, which is expressed as follows.

$$w \frac{S}{\Lambda \left(1 - \frac{u+p}{mR'_i}\right)} + (1-w)R'_i < X_1 \quad (5.11)$$

By working on this expression, we obtain the following.

$$0 > (1-w)R'^2_i + \left(\frac{wS}{\Lambda} - X_1 - (1-w)\frac{u+p}{m} \right) R'_i + X_1 \frac{u+p}{m} \quad (5.12)$$

Using the quadratic formula, we obtain the roots for post-advertisement reputation that satisfy Equation 5.12:

$$r'_{1,2} = \frac{-\left(\frac{wS}{\Lambda} - X_1 - (1-w)\frac{u+p}{m}\right) \pm \sqrt{\left(\frac{wS}{\Lambda} - X_1 - (1-w)\frac{u+p}{m}\right)^2 - 4(1-w)X_1 \frac{u+p}{m}}}{2(1-w)} \quad (5.13)$$

Within the interval defined by the roots, the firm drops from state A, while outside the interval, it remains at state A. Note that these roots are for the post-advertisement reputation. The corresponding R_i roots can be obtained as follows.

$$r_{1,2} = \frac{\left(\frac{-\left(\frac{wS}{\Lambda} - X_1 - (1-w)\frac{u+p}{m}\right) \pm \sqrt{\left(\frac{wS}{\Lambda} - X_1 - (1-w)\frac{u+p}{m}\right)^2 - 4(1-w)X_1 \frac{u+p}{m}}}{2(1-w)} \right) - A_i^\alpha}{1 - A_i^\alpha} \quad (5.14)$$

Proposition 6. *Under constant capacity, given that firm is at state A in some period i , if $[X_1, 1)$ and the interval defined by the Equation 5.14 with $A_i = L$, do not have an intersection, firm makes low level advertisement in all remaining periods.*

Proof. Proposition 6 results from the fact that state A brings the maximum possible revenue (pS) regardless of the advertisement level, while low level advertisement has the lowest advertisement cost.

As a result, when the given intervals do not overlap, firm is able to stay at state A for the lowest possible cost forever. Then, it always makes low level advertisement once state A is observed for the first time. $\square$

Proposition 6 is applicable when Equations 5.10 hold, and the roots of Equation 5.14 are complex, negative or define an interval that simply does not coincide with $[X_1, 1]$. At low w , the roots of Equation 5.14 define a large interval but they are negative. At slightly higher w , the roots are positive but the interval between the roots is narrower, so it does not intersect $[X_1, 1]$. At larger w , roots are complex numbers. Therefore, up to this point, the proposition is applicable. Finally, when w increases further, roots become positive real numbers, and the interval between them starts to get wider, making the proposition not applicable. The same behavior is observed for α and $\frac{u+p}{m}$, while L behaves in the exact opposite manner. To sum up, when the Equations 5.10 hold, and w , α and $\frac{u+p}{m}$ are low while L is high; it is easy for the firm to keep demand high once it is raised above a certain level. Consumers' tend to stick with their existing opinions rather than relying on WoM. This may occur for services, for which WoM communication has not become a habit yet. Alternatively, highly needed or beneficial products and services may invite such situations, as well as the ones without substitutes.

In this section, we have identified different market conditions that help us understand the problem better and reduce computational complexity, which concludes the complete-information scenario under constant capacity.

5.3 Incomplete-Information Firm

Under the incomplete-information scenario, the firm tries to maximize profit by choosing the advertisement level in each period i based on the observed reputation R_i . Having fewer actions than the variable capacity problem and less information than

the complete-information firm, this is the most restrictive scenario we examine. This problem the company solves in each period is as follows.

$$\max \pi_i = p \min\{D_i, S\} - cS - c_A A_i \quad \text{where} \quad (5.15)$$

$$D_i = \max\{0, \gamma_i\} \quad (5.16)$$

$$\gamma_i = \Lambda \left(1 - \frac{u + p}{m(R_i + (1 - R_i)A_i^\alpha)} \right) \quad (5.17)$$

$$A_i \in \{H, L\} \quad (5.18)$$

Advertisement decision is made after observing R_i . Solving the above problem for each possible state (A, B, C, D, E, F), we obtain the following rules:

- if firm is at state A or F, $A_i = L$ maximizes π_i .
- if firm is at state B, we have $A_i = H$ if the following holds, and $A_i = L$ otherwise.

$$R_i < \frac{\frac{p\Lambda}{p\Lambda - pS + c_A(H-L)} - L^\alpha}{1 - L^\alpha} \quad (5.19)$$

- if firm is at state C, we have $A_i = H$ if the following holds, and $A_i = L$ otherwise.

$$\frac{1}{R_i + (1 - R_i)L^\alpha} - \frac{1}{R_i + (1 - R_i)H^\alpha} > \frac{mc_A(H - L)}{p\Lambda(u + p)} \quad (5.20)$$

- if firm is at state D, we have $A_i = H$ if the following holds, and $A_i = L$ otherwise.

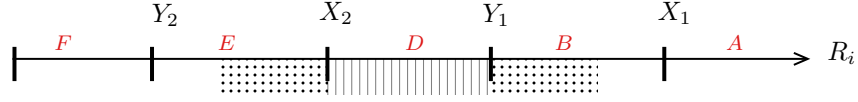
$$pS > c_A(H - L) \quad (5.21)$$

Notice that firm's decision does not vary depending on R_i at case D.

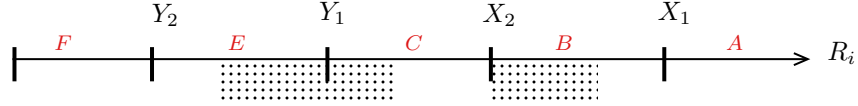
- if firm is at case E, we have $A_i = H$ if the following holds, and $A_i = L$ otherwise.

$$R_i > \frac{\frac{u+p}{m} \frac{p\Lambda}{p\Lambda - c_A(H-L)} - H^\alpha}{1 - H^\alpha} \quad (5.22)$$

Then, if $Y_1 > X_2$; firm's decision can be expressed via the following figure. The dotted areas represent when firm picks H as the advertisement level; unscanned areas represent when firm picks L ; and at the vertically striped area, firm picks H if $pS > c_A(H - L)$ holds and picks L otherwise.



Similarly, if $Y_1 < X_2$; firm's decision can be expressed via the following figure. The dotted areas represent when firm picks H as the advertisement level, and unscanned areas represent when firm picks L .



Let us denote the expressions used in the decision as follows for simplicity:

- $\zeta = \frac{\frac{u+p}{m} \frac{p\Lambda}{p\Lambda - c_A(H-L)} - H^\alpha}{1 - H^\alpha}$
- $\omega = \frac{\frac{p\Lambda}{p\Lambda - pS + c_A(H-L)} - L^\alpha}{1 - L^\alpha}$
- $\phi = pS - c_A(H - L)$
- $\beta, \text{ st. } \frac{1}{\beta + (1-\beta)L^\alpha} - \frac{1}{\beta + (1-\beta)H^\alpha} = \frac{mc_A(H-L)}{p\Lambda(u+p)}$

Then the optimal decision for any period i can be summarized as below.

$$A_i = \begin{cases} R_i \in (\zeta, \min\{X_2, \beta\}) \text{ or} \\ H, \text{ if } R_i \in (\max\{Y_1, X_2\}, \omega) \text{ or} \\ \phi > 0 \text{ and } X_2 < R_i < Y_1 \\ L, \text{ otherwise} \end{cases} \quad (5.23)$$

Notice that incomplete-information firm's optimal advertisement decision is more complex than its variable capacity equivalent. In the variable capacity scenario, the firm picks high level advertisement only if R_i falls within the interval defined in Equation 4.25. Under constant capacity, however, by Equation 5.23, we define a more complex decision mechanism. This results from the fact that under constant capacity, we can observe $S > D > 0$ and $D > S$, which is not the case in the variable capacity scenario. Moreover, unlike the variable capacity scenario, we can observe instances, where incomplete-information firm makes more frequent high level

advertisements than the complete-information firm. This highlights the importance of having complete information on endogenous demand structure; since, by having complete-information, firms can make more profit even if they place less effort in advertising.

For illustration, we present an example by Figures 5.1, 5.2 and 5.3, under parameters: $R_1 = 0.304$, $\alpha = 0.316$, $H = 0.445$, $L = 0.069$, $m = 12.297$, $u = 0.915$, $p = 9.309$, $w = 0.816$, $\Lambda = 99,196$, $c = 2.21$, $c_A = 1,117.097$, $N = 22$, $S = 440$. The figures below represent the cumulative advertisement costs, cumulative revenues and the resulting cumulative profits of the two firms, when the respective optimal advertisement policies are carried out. In this example, it can be seen that even though incomplete-information firm puts more effort in advertising, it results in a smaller revenue than the complete-information equivalent due to attracting demand that cannot be met and resulting in a low fill rate.

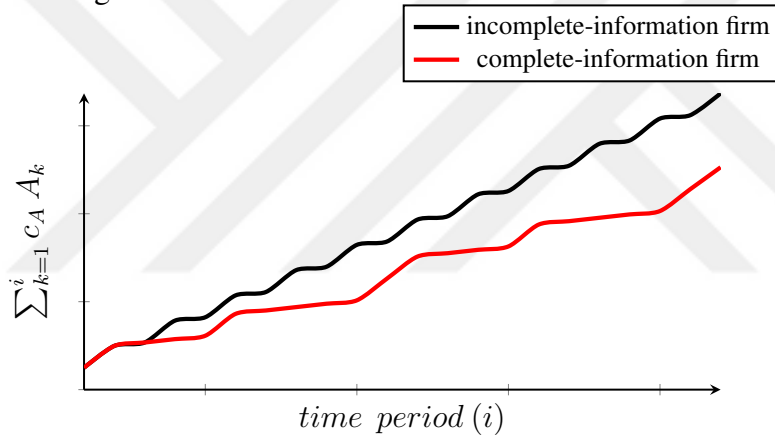


Figure 5.1: Cumulative Advertising Cost Of Constant Capacity Firms

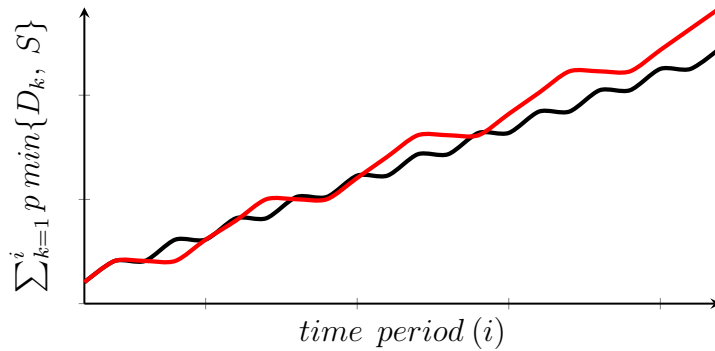


Figure 5.2: Cumulative Revenue Of Constant Capacity Firms

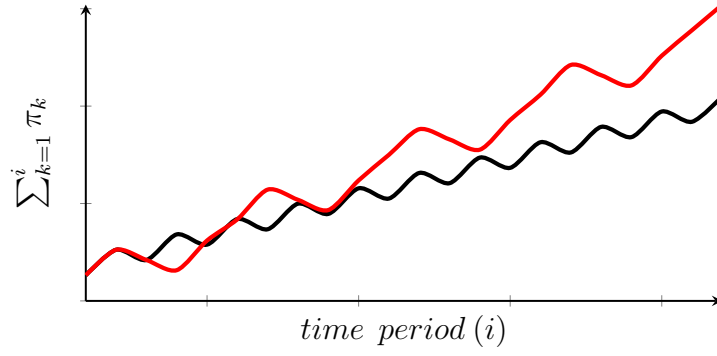


Figure 5.3: Cumulative Profit Of Constant Capacity Firms

To better understand the how VoI changes in different settings, we perform an extensive computational study. In the following section, we present and discuss these results.

5.4 Computational Results on Value of Information (VoI)

In this section, we analyze the VoI under constant-capacity. Our experimental setting was generated in a similar fashion as before. We generated the test instances and solved the problem for each of them with enumeration-based methods, which are improved by the introduced special cases. As most of the problem instances require non-polynomial solving times, we conducted our experiments using Docker Software version 3.3.1. The coding language used is Python Programming Language version 3.8.5.

To test the impact of each parameter on VoI, we consider 200 parameter sets randomly generated based on the distributions presented in 5.1, and alter the parameter of concern 8 times while keeping the others constant. The 12 parameters tested are the starting reputation (R_1), low advertisement level (L), high advertisement level (H), advertisement resistance (α), weight of WoM (w), market size (Λ), service capacity (S), reputation's monetary value (m), reservation utility (u), price (p), cost of advertisement (c_A) and number of periods (N). Since the capacity and unit capacity cost are the same in both problems, we do not consider their effects. In total, 19,200 instances are considered. Similar to the earlier computations in Chapter 4, both types

of firms are subjected to the same market mechanisms in each time period, as a result of their past actions.

We present representative cases along with their interpretations.

parameter	distribution
R_1	$U(0, 1)$
w	$U(0, 1)$
L	$U(0, 1)$
H	$U(L, 1)$
N	$U(5, 25)$
c_A	$TN(500, 500, 0)$
α	$U(0, 1)$
m	$TN(15, 10, 0)$
Λ	$TN(10^5, 10^3, 0)$
u	$TN(0.5, 3, 0)$
c	$TN(2, 5, 0)$
p	$TN(c, 10, 0)$
S	$TN(5^5, 10^3, 0)$

Table 5.1: Parameter distributions of constant capacity problem computations.

Before we discuss the effects of each parameter on VoI, notice that under constant capacity, the decisions of the two types of service providers are more complex than they are under variable capacity. Under variable capacity, complete-information firm adopts a single switching point solution while incomplete-information firm adopts either a single or a two switching point solution. Therefore, the relationship between each parameter and VoI can be inferred by observing how the locations of the optimal switching points change with respect to the parameter of concern. However, in the constant capacity problem, we observe several points, where we have $A_i \neq A_{i+1}$. To identify the similarity between the optimal advertising sequences of complete and incomplete-information firms, we measure the correlation, r , between the two resulting advertising sequences, and have not observed any significant relationship between

r and Π . However, when we introduced Ω into the analysis, we have observed the following.

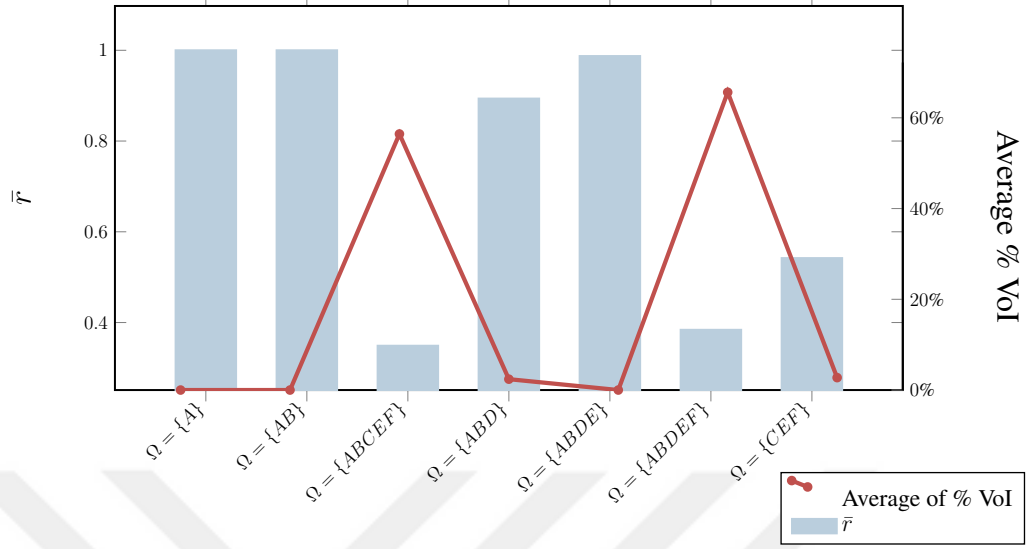


Figure 5.4: Average % VoI versus average correlation between the two firms' advertising sequences, with respect to Ω

Figure 5.4 shows how similar the two firms' optimal advertising sequences are on the average by $\bar{r}$, and the ratio of VoI to the complete-information firm's optimal profit (% VoI), with respect to firms' demand generating ability (Ω). Note that the Ω types that are rarely observed, $\{A, B, C\}$ (0.04% of the experiments), $\{A, B, C, E\}$ (0.02% of the experiments), $\{A, B, C, E, F\}$ (0.04% of the experiments), $\{C\}$ (0.01% of the experiments) and $\{C, E\}$ (0.01% of the experiments), are not presented.

The figure reveals that VoI is the most significant, when Ω has the highest cardinality, as in $\Omega = \{A, B, C, E, F\}$ and $\Omega = \{A, B, D, E, F\}$. In the meanwhile, the difference between two firms' optimal advertising sequences are the most dissimilar as well. This implies that when firm's demand generating ability can vary greatly throughout the problem horizon, more complex advertisement sequences are needed, which cannot be estimated easily by the incomplete-information firm. To construct such complex advertisement policies, firm has to be aware of the factors that affect demand, and make long-term advertising plans.

Finally, let us note that when $\Omega \in \{C, E, F\}$ for all experiments in a sample, the

parameters' effects on VoI is the same as they are for the variable capacity scenario.

We first discuss the effect of the weight of WoM on VoI.

5.4.1 Impact of Weight of WoM

We present a sample of experiments regarding the weight of WoM communication in determining consumers' perceptions in Table 5.2, in which we have 8 experiments with the same parameter set, and between experiments we gradually increase w . We report the resulting optimal advertising sequences of the two firms, Ω and % VoI.

Weight of WoM, w , does not affect X_1 , X_2 , Y_1 and Y_2 ; therefore, Ω . Yet, its effect is still significant. We observe that VoI increases in w , when $\Omega \notin \{C, E, F\}$. This is due to the fact that with these Ω types, we can have $F_i < 1$, $F_i = 1$ and the case where fill rate is not applicable due to zero demand. Therefore, we can observe WoM's effect on reputation in both directions, by positive WoM and negative WoM. In addition to this direction variety, when w is high, fluctuations in reputation are stronger and more frequent, requiring serious planning for the advertisement efforts. Note that this result is the opposite of what we have observed under variable capacity. This difference is due to having both positive and negative WoM effect under constant capacity while having only positive WoM under variable capacity.

Meanwhile, when $\Omega \subset \{C, E, F\}$, which is a rarely observed situation, w 's effect on VoI is the same as it was under variable capacity scenario.

In some instances, we observe that when w is high, firm makes more high level advertisements to recover the effects of negative WoM while in other instances, firm reduces the number of high level advertisements. The second behavior sometimes results from the fact that advertisement's effect lasts so short that it is not worth it. Also, sometimes firm tries to make use of WoM as a tool for advertising.

5.4.2 Impact of Advertisement Resistance

Next, we present a sample from the experiments regarding the impact of α on VoI in Table 5.3.

It is observed that when α is significantly low, X_1 is low as well; therefore, we have $\Omega = \{A\}$. In that case, advertisement decision is trivial, and we have $A_i = L$ for all i for both complete and incomplete-information firms. As a result, VoI becomes zero. This implies that when consumers have significantly low resistance towards advertisement, no matter how negative WoM is, firm can always recover its reputation even by putting low effort in advertising.

As α increases, cardinality of Ω goes up. We begin to observe $\Omega \subset \{A, B, D\}$. This increases the complexity of the optimal advertising policy, however, it is still possible to pull demand above capacity level by a single period's advertisement effort. Therefore, VoI slightly increases in some instances while it is mostly still zero. Notice that in the computations, where Proposition 6 holds; the incomplete-information firm is more likely to approach complete-information policy, which is the case at the first row of Table 5.3. In that particular example, as firms are at state A in the first period and it is possible to stay there with low effort, we observe $A_i = L$ for all i .

Rarely, we observe $\Omega = \{A, B, C\}$, $\Omega = \{A, B, C, E\}$ and $\Omega = \{A, B, D, E\}$ when α is further increased; however those cases are too rare to comment on.

Finally, when α becomes sufficiently high, we observe $\Omega = \{A, B, C, E, F\}$ and $\Omega = \{A, B, D, E, F\}$, which introduces more complexity and increases VoI. Typically, as consumers are more resistant towards advertisement, WoM gains importance. One strategy the complete-information firm chooses is to boost demand by increasing advertisement, and row against negative WoM while another strategy is to keep a low profile in terms of advertising and exploit WoM. Notice that at the last three experiments in Table 5.3, the incomplete-information firm loses money; therefore, it would not operate at those regions.

Some problem instances start with $\Omega = \{C\}$, and evolve to $\Omega = \{C, E\}$ and $\Omega = \{C, E, F\}$, as α increases. Complete-information firm sticks with a single

switching point solution by Proposition 5 while incomplete-information firm starts with single switching point solutions and then adopts two switching point solutions. Three switching points move forward with α . In such cases, VoI increases with α , in a similar manner with variable capacity scenario.



Table 5.3 Advertising policies of complete and incomplete-information firms under constant capacity with respect to advertisement resistance.

Test Parameter: α

$R_1 : 0.12$	$L : 0.06$	$H : 0.80$	$m : 19.08$	
$w : 0.40$	$S : 1, 110$	$N : 17$	$c : 10.46$	
$u : 1.06$	$p : 17.40$	$c_A : 685.02$	$\Lambda : 100, 030$	
α	Complete-Information Firm	Incomplete-Information Firm	Ω	% Vol
0.030	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D\}$	0%
0.038	H, H, H, H, H, H, H, H, H, H, H, H, H, H	H, H, H, H, H, H, H, H, H, H, H, H, H, H	$\Omega = \{A, B, D\}$	0%
0.080	H, H, H, H, H, H, H, H, H, H, H, H, H, H	H, H, H, H, H, H, H, H, H, H, H, H, H, H	$\Omega = \{A, B, D\}$	0%
0.108	H, L, H, H, H, H, H, H, H, H, H, H, H, H	H, L, H, H, H, H, H, H, H, H, H, H, H, H	$\Omega = \{A, B, D, E\}$	0%
0.243	H, H, H, H, H, H, H, H, H, H, H, H, H, H	L, H, L, H, H, H, H, H, H, H, H, H, H, H	$\Omega = \{A, B, D, E, F\}$	2%
0.328	H, H, H, H, H, H, H, H, H, H, H, H, H, H	L, L, H, L, H, H, H, H, H, H, H, H, H, H	$\Omega = \{A, B, D, E, F\}$	20%
0.645	H, H, H, H, H, L, H, L, H, L, H, L, H, L	L, L, L, L, L, L, L, H, L, H, L, H, L, L	$\Omega = \{A, B, D, E, F\}$	203%
0.710	H, H, H, H, L, H, L, L, H, L, H, L, H, L	L, L, L, L, L, L, L, L, L, H, L, H, L, L	$\Omega = \{A, B, D, E, F\}$	301%
0.965	H, L, L, H, L, H, L, L, L, L, H, L, H, H, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	1,345%

5.4.3 Impact of Reputation's Monetary Value

A sample from the computations is given in Table 5.4.

When m is small, the utility a consumer thinks she will receive from unit reputation is low. Consumers need very high reputation levels in order to be convinced. Therefore, firm either cannot realize demand at all or it is able to capture a small portion of the market.

In the region where at least one firm operates, we have $\Omega = \{C, E\}$, $\Omega = \{C, E, F\}$ or $\Omega = \{C\}$. Complete-information firm adopts a single switching point advertising sequence while incomplete-information firm adopts either a single or a two-switching point solution. As m increases, all switching points start to slide backwards, since the necessity for advertisement drops. This reduces the VoI, just as it is the case for the variable capacity scenario.

When m becomes large enough, we have $\Omega = \{A, B, C, E\}$, $\Omega = \{A, B, C, E, F\}$, $\Omega = \{A, B, C\}$, $\Omega = \{A, B, D, E\}$ or $\Omega = \{A, B, D, E, F\}$. VoI makes a peak when such Ω 's are observed for the first time, due to the increased complexity, which can be observed at Table 5.4, where $m = 17.54$. As we keep increasing m , attracting high demand for lower reputations become possible, and the complete-information firm becomes less advantageous. Consequently, VoI goes down.

For larger m , the observed Ω types are $\Omega = \{A, B, C\}$, $\Omega = \{A, B, D\}$ and $\Omega = \{A, B\}$. Complexity is reduced, and VoI keeps going down. Finally, we get $\Omega = \{A\}$, where VoI is zero.

The conditions defined in Proposition 6 hold at large m . Notice that these conditions hold when $m = 50$ at Table 5.4, where the state at period i is state A; therefore, we observe $A_i = L$ for all i , for both firms. The proposition applies in the experiments with higher m as well.

5.4.4 Impact of Service Price

The impact of service price on VoI is given by Table 5.5.

As price is increased between computations, Ω changes from $\Omega = \{A\}$ to having higher cardinality as in $\Omega = \{A, B, D\}$. At significantly low price, firms do not operate as the revenue will be insignificant anyway. When price is large enough, they start operating, and as we keep increasing p , % VoI exhibits insignificant fluctuations for a while.

Increased price makes it harder for the firms to realize demand; therefore, states A, B and D start to occur less frequently. We have $\Omega = \{A, B, D, E, F\}$, and more elaborate advertisement policies are required, which distinguish complete and incomplete-information firms. Therefore, as price increases % VoI goes up.

When price is increased enough, it becomes impossible to get demand exceed the service capacity, no matter the advertisement policy. Therefore, we begin to observe $\Omega \subset \{C, E, F\}$. Then, the complete-information firm adopts a single switching point solution as suggested by Proposition 5. The switching point moves forward as price keeps increasing, since more effort is required to realize demand. Besides, it becomes more worthwhile to invest on advertisement since the revenue obtained from the same amount of demand realized is higher due to the increased price. Meanwhile, in the last few examples provided by Table 5.5, the incomplete-information firm fails to predict that demand can be realized in the future if high effort is placed in advertisement early in the planning horizon. As a result of this myopic behavior, it gets stuck in the no demand region, where it would not operate even though the potential revenue it could collect increases (the complete-information firm collects the highest revenue at $p = 29.96$). This implies that under constant capacity, when price is high, the incomplete-information firm may decide on staying out of the market and miss the opportunity to collect the profits the complete-information firm gains. Finally, when price is too high, the complete-information firm gives up on operating as demand is too low and it cannot generate profit (the last two rows of Table 5.5).

5.4.5 Impact of Starting Reputation

To represent the impact of starting reputation, R_1 , we provide Tables 5.6 and 5.7 with samples from computations.

R_1 does not affect Ω . When $\Omega \subset \{C, E, F\}$, the starting reputation decreases VoI, as it is the case under variable capacity. When $\Omega \not\subset \{C, E, F\}$, we observe that R_1 has no regular effect on VoI, and its effect highly depends on other parameters. The highest VoI's are recorded when $R_1 < \zeta$, since the incomplete-information firm does not realize demand in the early periods (see Equation 5.23). When w is high, R_i makes notable fluctuations, which reduces the importance of R_1 . On the other hand, when the planning horizon N is short, R_1 becomes important, since the firm can employ an advertising policy that is able to keep R_i high until the N^{th} period, by fighting negative effects of WoM through advertisement. Table 5.6 represents a parameter set with long planning horizon, where R_1 does not have a significant effect on VoI while Table 5.7 represents a parameter set with lower N , where R_1 gains importance. It can be seen from Table 5.7 that when R_1 is low, VoI is high.

Table 5.7 Advertising policies of complete and incomplete-information firms under constant capacity with respect to starting reputation 2.

Test Parameter: R_1

$p : 4.663$		$L : 0.033$	$H : 0.125$	$\alpha : 0.418$
$w : 0.629$		$S : 548$	$N : 13$	$c : 0.584$
$u : 5.202$		$m : 10.354$	$c_A : 743.084$	$\Lambda : 100, 177$
R_1	Complete-Information Firm	Incomplete-Information Firm	Ω	% VoI
0.010	H, H, H, H, H, L, L, L, L, L	L, L, L, L, L, L, L, L, L, H, L, L	$\Omega = \{A, B, D, E, F\}$	187%
0.554	H, H, H, L, L, L, H, H, H, L, H, H	L, L, L, L, L, L, H, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	159%
0.832	H, H, H, H, H, L, H, H, H, H, H, H	L, L, L, H, L, L, L, L, L, L, L, H	$\Omega = \{A, B, D, E, F\}$	78%
0.837	H, H, L, L, H, H, H, L, L, H, H, H, L	L, L, L, H, H, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	73%
0.898	H, L, L, H, H, H, H, H, H, H, H, H	L, H, L, L, L, L, L, L, L, L, H, L, L	$\Omega = \{A, B, D, E, F\}$	56%
0.934	H, H, H, L, L, H, H, H, L, L, L, L	H, L, L, L, L, L, H, L, H, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	54%
0.970	L, H, H, H, L, L, H, H, H, H, L, L	L, L, L, L, L, L, L, L, H, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	41%

5.4.6 Impact of Reservation Utility

A sample from the computations regarding the impact of reservation utility is given in Table 5.8.

We observe that when reservation utility, u , is low; it is easy to convince consumers to purchase the service. Therefore, firms can boost demand by single advertisement actions, as in $\Omega \subset \{A, B, D\}$. As a result, it is easier for the incomplete-information firm to approximate complete-information firm's advertising policy, and VoI is low.

When we increase u , cardinality of Ω increases, making the advertisement decisions more complex. To gain the optimal amount of profit, firm has to make more elaborate advertisement policies. VoI increases with u . Finally, when u becomes sufficiently high, realizing a demand level that matches or exceeds capacity becomes impossible, which means $\Omega \subset \{C, E, F\}$. This reduces the problem complexity significantly. Complete-information firm picks a single switching point solution while the incomplete-information firm picks a single or two switching point solution. The resemblance between the two advertising sequences peaks, and VoI drops significantly.

Notice that in the experiment with $u = 3.40$, shown in Table 5.8, the incomplete-information firm loses money while complete-information firm is able to generate profit. What's more, in this particular example, the complete-information firm generates more revenue with one less high level advertisement compared with its incomplete-information equivalent. This underlines the importance of timing in placing efforts in promotion.

Table 5.8 Advertising policies of complete and incomplete-information firms under constant capacity with respect to reservation utility.

Test Parameter: u

$p : 9.75$	$L : 0.60$	$H : 0.75$	$\alpha : 0.58$	
$w : 0.63$	$S : 386$	$N : 16$	$c : 2.90$	
$R_1 : 0.86$	$m : 13.23$	$c_A : 990.69$	$\Lambda : 101, 330$	
u	Complete-Information Firm	Incomplete-Information Firm	Ω	% Vol
0.777	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D\}$	0%
1.072	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D\}$	0%
1.112	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D\}$	0%
1.589	L, H, H, H, H, H, H, H, H, H, H, H, H, H, H	L, H, H, H, H, H, H, H, H, H, H, H, H, H, H	$\Omega = \{A, B, D, E, F\}$	0%
3.333	H, L, H, L, H, L, H, L, L, L, L, L, H, H, L	L, H, L, H, L, H, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	78%
3.400	H, L, L, L, H, L, H, L, L, L, L, H, L, L, L	L, L, H, H, L, L, L, L, L, L, L, L, L, L, H	$\Omega = \{A, B, C, E, F\}$	105%
3.425	H, H, L, L, L, L, L, L, L, L, L, L, L, L	L, L, H, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{C, E, F\}$	6%
3.450	H, H, H, L, L, L, L, L, L, L, L, L, L, L	L, L, H, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{C, E, F\}$	11%

5.4.7 Impact of Service Capacity

A sample from the computations is given in Table 5.9.

As service capacity increases, it becomes easier to meet demand, therefore the region of reputation, where we have state C, widens. Ω goes from $\Omega \subset \{A, B, D\}$ to $\{A, B, D, E, F\}$ to $\{A, B, C, E, F\}$ and finally to $\Omega \subset \{C, E, F\}$. % VoI makes peaks when Ω becomes $\{A, B, D, E, F\}$ or $\{A, B, C, E, F\}$ for the first time. When we finally have $\Omega \subset \{C, E, F\}$, the complete-information firm adopts the single switching point solution while the incomplete-information firm switches once or twice. As a result, VoI decreases.

When we increase service capacity, simply having more playground increases the revenue of the complete-information firm. The incomplete-information firm's revenue mostly increases as well. However, in some instances we observe that incomplete-information makes myopic decisions in the early periods, reducing the size of its own playground for the future periods and ends up making less revenue than it did with lower S values. In the experiments included in Table 5.9, $S = 2,206$, $S = 2,702$ and $S = 4,089$ correspond to such falls, and the result is having peaks in % VoI. Apart from such peaks, the general trend is a decline in VoI as S increases.

Table 5.9 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the service capacity.

Test Parameter: S									
$p : 7.035$	$L : 0.435$			$H : 0.501$	$\alpha : 0.939$				
$w : 0.777$	$u : 4.103$			$N : 16$	$c : 2.779$				
$R_1 : 0.799$	$m : 13.529$			$c_A : 1, 091.878$	$\Lambda : 99, 386$				
S	Complete-Information Firm				Incomplete-Information Firm				% Vol
500	H, H, H, L, L, H, H, L, L, H, H, H, L, L	H, H, H, L, L, H, H, L, L, H, H, H, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$				212%
1,000	H, H, H, H, L, L, H, H, L, L, H, H, H, L, H	H, H, H, L, L, H, H, L, L, L, L, H, L, H	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$				153%
1,718	L, H, L, H, L, L, L, H, L, L, H, H, L, L, L	L, H, L, L, L, L, H, H, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, H, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, D, E, F\}$				47%
1,945	L, H, L, L, H, H, L, L, H, H, L, L, H, H, H	L, H, L, L, L, L, H, H, L, L, H, H, L, L, H	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, H, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, D, E, F\}$				27%
1,950	L, H, L, L, H, H, L, L, H, H, L, L, L, L, L	L, H, L, L, L, L, H, H, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, H, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, D, E, F\}$				24%
2,206	L, H, L, L, L, L, H, H, L, L, L, L, H, H, H	L, H, L, L, L, L, H, H, L, L, L, L, H, H, H	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, H, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, D, E, F\}$				62%
2,702	H, H, L, L, L, L, L, L, L, L, L, L, L, L, L	H, H, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, H, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				82%
3,164	H, H, L, L, L, L, H, H, L, L, L, L, H, H, L, L	H, H, L, L, L, L, H, H, L, L, L, L, H, H, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, H, L, L	$\Omega = \{A, B, C, E, F\}$				37%
4,089	H, L, L, L, L, L, L, L, H, H, L, L, L, L, L	H, L, L, L, L, L, L, L, H, H, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				54%
5,231	L, H, L, L, L, L, H, H, L, L, H, H, H, L, L	L, H, L, L, L, L, H, H, L, L, H, H, H, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, H, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				15%
5,500	L, L, L, H, H, H, H, L, L, L, L, L, H, H, L	L, L, L, H, H, H, H, L, L, L, L, L, H, H, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				44%
7,000	H, H, H, H, L, L, L, L, L, L, L, L, H, L, L	H, H, H, L, L, L, L, L, L, L, L, L, H, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				31%
8,000	H, H, L, H, H, H, L, L, H, H, H, L, H, H, L	H, H, L, L, L, L, H, H, H, L, H, H, H, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	H, L, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				45%
9,000	H, H, L, H, H, L, H, H, L, L, H, L, H, L, L	H, H, L, L, L, L, H, H, L, L, H, L, H, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	H, L, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				28%
10,000	H, L, L, L, H, H, L, L, H, L, H, L, H, L, L	H, L, L, L, L, L, H, L, H, L, H, L, H, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	H, L, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				0%
15,000	H, L, L, L, H, L, L, L, H, L, H, L, H, L, L	H, L, L, L, L, L, H, L, H, L, H, L, H, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	H, L, L, L, L, L, L, L, L, L, L, L, H, L	$\Omega = \{A, B, C, E, F\}$				0%
20,000	H, H, H, L, L, L, L, L, L, L, L, L, L, L, L	H, H, H, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	H, H, H, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{C, E, F\}$				0%
50,000	H, H, H, L, L, L, L, L, L, L, L, L, L, L, L	H, H, H, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	H, H, H, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{C, E, F\}$				0%
90,000	H, H, H, L, L, L, L, L, L, L, L, L, L, L, L	H, H, H, L, L, L, L, L, L, L, L, L, L, L, L	L, L, L, L, L, L, L, L, L, L, L, L, L, L	H, H, H, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{C, E, F\}$				0%

5.4.8 Impact of Advertisement Cost

Regarding the advertisement cost, computation samples are given in Tables 5.10 and 5.11.

Cost of advertisement does not change Ω ; since it does not affect X_1 , X_2 , Y_1 or Y_2 . Its effect on VoI varies depending on other parameters. To begin with, when $\Omega \subset \{C, E, F\}$, it increases VoI as in the variable capacity scenario. For other Ω types, as c_A increases, for a while the firms do not change their advertising sequences. Then, the profits decrease, and % VoI increases due to this scale change. However, when cost of advertisement becomes significantly high, both firms start to put less effort in advertising, which reduces % VoI. All in all, % VoI first follows a slow increase, and then decreases with cost of advertisement. Table 5.10 represents the case where c_A increases VoI while Table 5.11 represents the latter. Note that in the first two experiments in Table 5.11, the incomplete-information firm loses money, and therefore would not operate there.

Table 5.10 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the advertisement cost 1.

Test Parameter: c_A

$p : 8.36$	$L : 0.260$	$H : 0.458$	$\alpha : 0.987$	
$w : 0.802$	$u : 0.452$	$N : 9$	$c : 0.532$	
$R_1 : 0.742$	$m : 12.805$	$S : 249$	$A : 101, 115$	
c_A	Complete-Information Firm	Incomplete-Information Firm	Ω	% Vol
109.581	L, H , H , H , H , H , H	L, L, L, H , L, L, H , L, L	$\Omega = \{A, B, D, E, F\}$	46%
238.395	L, H , H , H , H , H , H	L, L, L, H , L, L, H , L, L	$\Omega = \{A, B, D, E, F\}$	47%
281.280	L, H , H , H , H , H , H	L, L, L, H , L, L, H , L, L	$\Omega = \{A, B, D, E, F\}$	47%
429.005	L, H , H , H , H , H , H	L, L, L, H , L, L, H , L, L	$\Omega = \{A, B, D, E, F\}$	49%
548.802	L, H , H , H , H , H , H	L, L, L, H , L, L, H , L, L	$\Omega = \{A, B, D, E, F\}$	50%
891.379	L, H , H , H , H , H , H	L, L, L, H , L, L, H , L, L	$\Omega = \{A, B, D, E, F\}$	54%
1,081.764	L, H , H , H , H , H , H	L, L, L, H , L, L, H , L, L	$\Omega = \{A, B, D, E, F\}$	58%
1,091.337	L, H , H , H , H , H , H	L, L, L, H , L, L, H , L, L	$\Omega = \{A, B, D, E, F\}$	58%

Table 5.11 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the advertisement cost 2.

Test Parameter: c_A

$p : 5.668$	$L : 0.124$	$H : 0.277$	$\alpha : 0.545$
$w : 0.748$	$u : 2.981$	$N : 19$	$c : 4.767$
$R_1 : 0.722$	$m : 14.697$	$S : 119$	$\Lambda : 100, 105$
c_A	Complete-Information Firm	Incomplete-Information Firm	Ω
394.722	L, H, H, H, H, H, H, H, H, H, H, H, H, H	L, H, H, H , L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$
590.958	L, H, H, H, H, H, H, H, H, H, H, H, H, H	L, H, H, H , L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$
675.332	L, H, H, H, H, H, H, H, H, H, H, H, H, H	L, H, H, H , L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$
702.048	L, H, H, H, H, H, H, H, H, H, H, H, H, H	L, H, H, H , L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$
889.825	L, H, L, H, H, L, L, L, H, H, H, H, H, H, L	L, H, H, H , L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$
1,286.772	L, H, L, H, H, L, L, L, H, H, H, H, H, H, L	L, H, H, H , L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$
1,346.009	L, H, L, H, H, L, L, L, H, H, H, H, H, H, L	L, H, H, H , L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$

5.4.9 Impact of Advertisement Levels

Samples from computations are presented for high and low level advertisements in Tables 5.12 and 5.13, respectively.

We first discuss the high level advertisement's impact.

As H increases, X_2 and Y_2 decrease. Therefore, the region occupied by states E and F shrinks. The parameter sets yielding to $\Omega \subset \{C, E, F\}$ evolve to $\Omega = \{C, E\}$, and Ω does not change any further. This indicates that firms can always realize demand by having $A_i = H$. In such Ω 's, as in the variable capacity problem, H does not have a significant effect on VoI.

Other parameter sets change Ω from $\{A, B, C, E, F\}$ to $\{A, B, D, E, F\}$ and then to $\{A, B, D\}$. Alternatively, if we have $\Omega \subset \{A, B\}$, Ω does not change with H . In other words, as H goes up, the states in which the firm can utilize its entire capacity by a single advertisement action, occur more frequently (states A, B and D).

If $\Omega \not\subset \{C, E, F\}$, when H is close to L , trivially, VoI is low, since there is simply very little difference between the two options the firms face. Otherwise, as H increases, if we have high service price p and low advertisement cost c_A , both firms increase their investments on advertisement, which decreases VoI. If the service price is not high and the advertisement cost is not low however, both firms reduce their efforts, and VoI decreases again. These conditions make the problem relatively easy, which is the reason behind the decrease in VoI. The decision becomes more complex when neither of these conditions hold. Generally, VoI is high when H is moderate.

Finally, let us note that the first two rows of Table 5.12 represent cases where the incomplete-information firm does not realize the profit opportunities in the upcoming periods and makes successive low level advertisements in the early periods, yielding no positive profit at the end of the planning horizon. Therefore, it would not operate under such conditions.

Table 5.12 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the high advertisement level.

Test Parameter: H

$p : 9.233$	$L : 0.034$	$c_A : 250.08$	$\alpha : 0.871$	
$w : 0.627$	$u : 3.574$	$N : 24$	$c : 3.551$	
$R_1 : 0.128$	$m : 22.197$	$S : 92$	$\Lambda : 100,498$	
H	Complete-Information Firm	Incomplete-Information Firm	Ω	% Vol
0.312	H, H, H, H, L, L, H, H, H, H, H, H, H, H, L	L, L, L, L, L, H, L, L, L, H, L, L, L, L, L, H, L	$\Omega = \{A, B, D, E, F\}$	175%
0.351	H, L, H, H, L, L, H, L, H, H, L, H, H, H, L	L, L, L, L, H, H, L, L, H, L, L, H, L, L, H, L, H	$\Omega = \{A, B, D, E, F\}$	313%
0.386	H, H, L, L, L, H, L, H, H, L, L, H, H, H, H, H	L, L, L, H, H, L, H, L, L, H, L, L, H, L, H, L, L	$\Omega = \{A, B, D, E, F\}$	92%
0.430	H, H, H, H, H, H, H, H, H, H, H, H, H, H, H	L, L, H, H, H, H, H, H, H, H, H, H, H, H, H, H, H	$\Omega = \{A, B, D, E, F\}$	7%
0.567	H, H, H, H, H, H, H, H, H, H, H, H, H, H, H	H, H, H, H, H, H, H, H, H, H, H, H, H, H, H, H, H	$\Omega = \{A, B, D\}$	0%

We continue our discussion on advertisement levels' impact by low level advertisement. As L is increased, X_1 and Y_1 decrease. If the parameter set of concern yields $\Omega \subset \{C, E, F\}$, Ω 's content does not change with L . For such parameter sets, VoI decreases in L in the same manner with the variable capacity problem.

When $\Omega \subset \{A, B, D\}$, as L is increased, it reaches $\Omega = \{A\}$, and does not change any further, which again indicates low complexity. Additionally, for such parameter sets, as we increase L , Proposition 6 is more likely to hold. The decision becomes easier, and VoI decreases in L .

Finally, when we have $\Omega = \{A, B, D, E, F\}$ and gradually increase L , it evolves to $\Omega = \{A, B, C, E, F\}$. In that case, VoI follows a decreasing trend as L increases; however, its decrease is not always monotone.

5.4.10 Impact of the Length of the Planning Horizon

A sample from computations is presented by Table 5.14.

Length of the planning horizon does not have a regular effect on VoI, unless we have $\Omega \subset \{C, E, F\}$ in which case N reduces VoI as in the variable capacity problem.

When $\Omega \not\subset \{C, E, F\}$, however, we observe that VoI makes small fluctuations as we increase N . After a point where N is sufficiently large, as we keep increasing it, the existing advertisement patterns of both firms start to repeat themselves. This means that the profits of the two firms increase in an almost linear fashion, keeping the % VoI stable.

Let us mark that when $N = 1$, the sole purpose is maximizing that period's profit with no concerns about the resulting reputation. In fact, the complete and incomplete-information firms' problems are the same. When N gets larger, to be able to obtain the maximum profit in all periods, maintaining the reputation becomes a necessity. As a result, even if the firm's optimal advertising sequence consists of repeating patterns, often at the last period(s), the pattern is broken. As a result, in shorter planning horizons, we do not observe repeating patterns.

Table 5.14 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the length of the problem horizon.

Test Parameter: N

$p : 8.360$	$H : 0.458$	$L : 0.260$	$\alpha : 0.986$		
$w : 0.802$	$u : 0.452$	$c_A : 350.910$	$c : 0.532$		
$R_1 : 0.742$	$m : 12.805$	$S : 249$	$\Lambda : 101, 115$		
N	Complete-Information Firm	Incomplete-Information Firm	Ω	% Vol	
7	L, H , H , H , H	L, L, L, H , L, L, H	$\Omega = \{A, B, C, E, F\}$	28%	
13	L, H , H , H , H , H , H , H , H	L, L, L, H , L, L, H , L, L, H , L, H	$\Omega = \{A, B, C, E, F\}$	33%	
15	L, H , H , H , H , H , H , H , H , H	L, L, L, H , L, L, H , L, L, H , L, L	$\Omega = \{A, B, C, E, F\}$	45%	
19	L, H , H , H , H , H , H , H , H , H , H , H	L, L, L, H , L, L, H , L, L, H , L, L, H , L, L, H	$\Omega = \{A, B, C, E, F\}$	35%	

5.4.11 Impact of Market Size

A sample from computations is presented by Table 5.15.

In Table 5.15, we observe that the incomplete-information firm is not aware of the fact that it can make profit in the future periods if it invests in advertisement, and makes only low level advertisements for all market sizes. It gets stuck in the no demand region, and hence it simply would not operate under these conditions. Meanwhile the complete-information firm realizes the opportunities implied by the larger market and increases its profit. As a result, VoI increases. Other than that, increase in market size amplifies revenue for both firms. As a result, when the market is significantly large, the advertisement cost becomes negligible, and % VoI decreases. We conclude that VoI is the largest when market size is moderate.

Table 5.15 Advertising policies of complete and incomplete-information firms under constant capacity with respect to the market size.

Test Parameter: Λ

$p : 7.035$	$H : 0.501$	$L : 0.435$	$\alpha : 0.939$	
$w : 0.777$	$u : 4.103$	$N : 16$	$c : 2.779$	
$R_1 : 0.799$	$m : 13.529$	$S : 794$	$c_A : 1,091.878$	
Λ	Complete-Information Firm	Incomplete-Information Firm	Ω	% Vol
97,838	L, H , H , H , H , L, L, H , L, H , L, L	L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	141%
97,889	H , H , H , L, L, H , L, H , L, H , L, H , H	L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	153%
99,180	H , H , H , L, L, H , H , H , H , L, H , L or H , H , H , L, L, H , L, H , H , H , L or H , H , H , L, L, H , L, H , H , H , H	L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	154%
100,154	H , H , L, L, H , H , L, H , L, H , L, L	L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	163%
100,612	H , H , L, L, H , H , L, H , L, H , L, L	L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	161%
100,661	H , H , L, L, H , H , L, H , L, H , L, L	L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, D, E, F\}$	161%
101,437	H , H , L, H , H , L, H , L, H , L, L	L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, C, E, F\}$	158%
101,790	H , H , L, H , H , L, H , L, H , L, H , L	L, L, L, L, L, L, L, L, L, L, L, L, L	$\Omega = \{A, B, C, E, F\}$	156%

5.5 Summary Of The Constant Capacity Problem

In the constant capacity problem, we have studied firm's advertisement decisions under complete and incomplete-information on the endogenous demand structure. The constant capacity problem is more challenging than the variable capacity problem. We approach the problem by classifying the position of the firm in terms of realizing demand. We define states to represent firm's demand realization ability under different advertisement levels, and identify which states can occur for a given parameter set. Note that the state of the firm in period i is a direct result of R_i . Consequently, each state defines an interval for R_i .

We call the set that contains the observable states for the parameter set at hand Ω . Despite having 6 possible states in total, namely A, B, C, D, E, F, we have seen that not all subsets of $\{A, B, C, D, E, F\}$ are observable. By inspecting the interval limits for the reputation that define the states, we reduce the number of possible Ω sets from $2^N - 1$ to 11.

These studies shed light on some special cases for the complete-information firm that reduce problem complexity significantly. We have first shown that when $\Omega \subset \{C, E, F\}$, meaning that firm is always able to fulfill demand, the problem becomes equivalent to the variable capacity problem, and the complete-information firm adopts a single switching point solution. In that case, all results obtained in the variable capacity problem hold. One such result is that fill rate is always 1 after the first period some demand is generated. This implies that reputation approaches to its highest value as time passes, due to the impact of advertisement and positive WoM. The profits obtained increase throughout the problem horizon, and the increase between consecutive π_i^* 's diminish as the increase in reputation slows down. An example from the computations is shown below to represent the relationship between fill rate and reputation (Figure 5.5), where the parameter set is: $R_1 = 0.679$, $L = 0.518$, $H = 0.570$, $\alpha = 0.810$, $w = 0.128$, $\Lambda = 101,421$, $c = 3.355$, $m = 7.422$, $u = 2.137$, $p = 5.278$, $c_A = 307.560$, $N = 20$, $S = 448$, and the optimal switching point is at period 8.

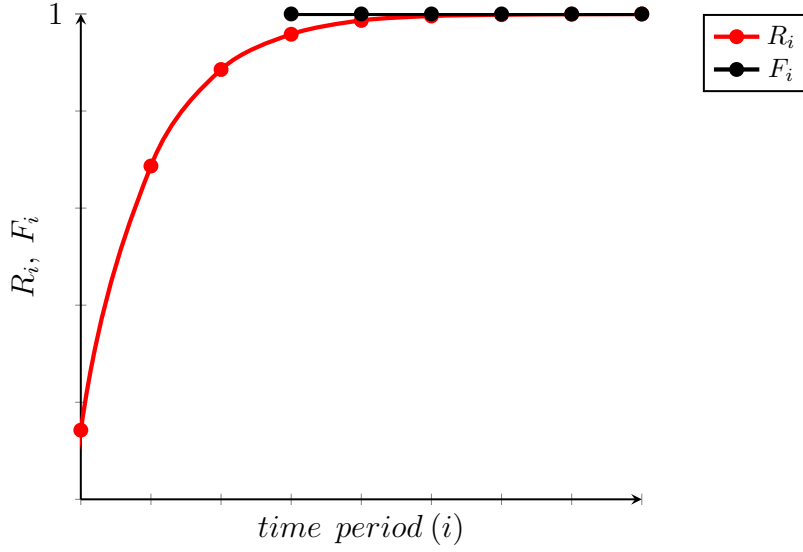


Figure 5.5: Reputation vs Fill Rate Under Constant Capacity When $\Omega \subset \{C, E, F\}$

Next, we have shown an upper bound for the optimal number of high level advertisements by Lemma 4 when $\Omega \subset \{A, B, D\}$, meaning that firm is able to utilize its entire capacity any time by a single advertisement action. A heuristic to obtain the maximum possible revenue is presented, by which we obtain a lower bound for the optimal amount of profit.

Another special case is introduced by Proposition 6, by which we reduce the size of the solution space significantly. The proposition is useful when the parameters dictate that firm can always utilize the service capacity following a low level advertisement action once state A is reached. In that case making low level advertisements at the remaining periods dominates other solutions. An example from the computations, where this proposition applies, is given below to represent the relationship between fill rate and reputation (Figure 5.6), where the parameter set is: $R_1 = 0.053$, $L = 0.132$, $H = 0.273$, $\alpha = 0.464$, $w = 0.372$, $\Lambda = 99,096$, $c = 0.435$, $m = 20.729$, $u = 1.657$, $p = 9.773$, $c_A = 890.325$, $N = 17$, $S = 1,050$. Notice that firm is able to stay above X_1 for all future time periods once reputation exceeds this threshold. This result is meaningful when A is an element of Ω but not the only element of it.

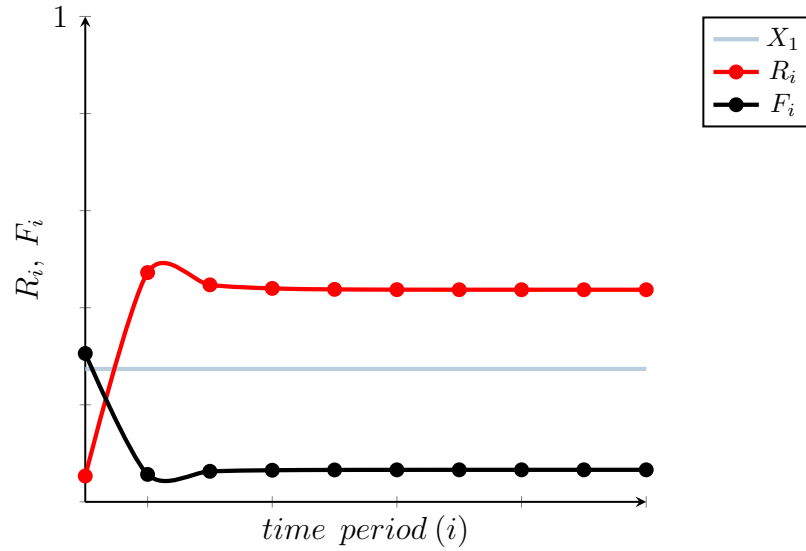


Figure 5.6: Reputation vs Fill Rate Under Constant Capacity Under Proposition 6

In the settings where no special case applies, we observe different behaviors of R_i . When the planning horizon is long and Ω has high cardinality, we observe fluctuating R_i . These fluctuations are stronger when WoM is more influential. One such example is shown in Figure 5.7 for the parameters: $R_1 = 0.722$, $L = 0.124$, $H = 0.277$, $\alpha = 0.545$, $w = 0.748$, $\Lambda = 100, 105$, $c = 0.767$, $m = 14.697$, $u = 2.981$, $p = 5.668$, $c_A = 1, 061.993$, $N = 19$, $S = 3, 285$. Y_2 and X_2 are far below the region R_i is at, therefore are not shown in the figure for the sake of simplicity. The optimal advertising sequence of this instance is $L, H, L, H, H, L, H, H, \dots, L, H, H, L, L$. Notice that the optimal advertisement policy consists of a repeating pattern, and so are R_i and F_i . As a result, firm's state follows a pattern as well. The states we observe are: $A, D, A, A, D, A, \dots, A, D, A, A$. The firm breaks this pattern at the end of the problem horizon. The implication is that as more periods are left behind, making profit in the current period becomes more important than investing in reputation as there are simply less periods ahead to use the built up reputation. The aim at the last period is to maximize the current period's profit.

Another point we would like to draw attention to is the relationship between fill rate and reputation. Figure 5.7 underlines how fill rate pulls reputation closer to itself. The result is observed in the upcoming period. The strength of the fluctuations is greater for fill rate, as $w < 1$. If we had $w = 1$, we would observe two identical curves that follow each other with 1 period of lag.

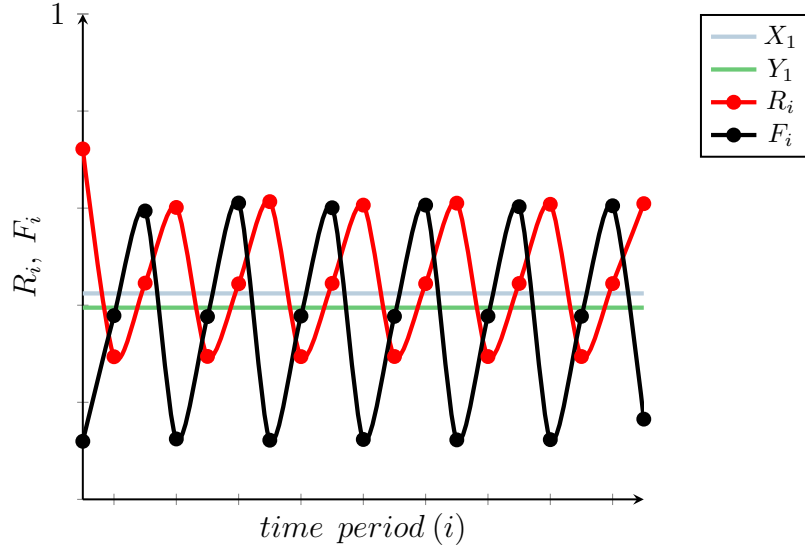


Figure 5.7: Reputation vs Fill Rate Under Constant Capacity With Long Planning Horizon

A final example regarding how R_i^* evolves over time is presented by Figure 5.8, where parameters are: $R_1 = 0.0002$, $L = 0.036$, $H = 0.151$, $\alpha = 0.344$, $w = 0.187$, $\Lambda = 101,305$, $c = 1.694$, $m = 27.962$, $u = 1.130$, $p = 23.202$, $c_A = 484.781$, $N = 6$, $S = 1,308$.

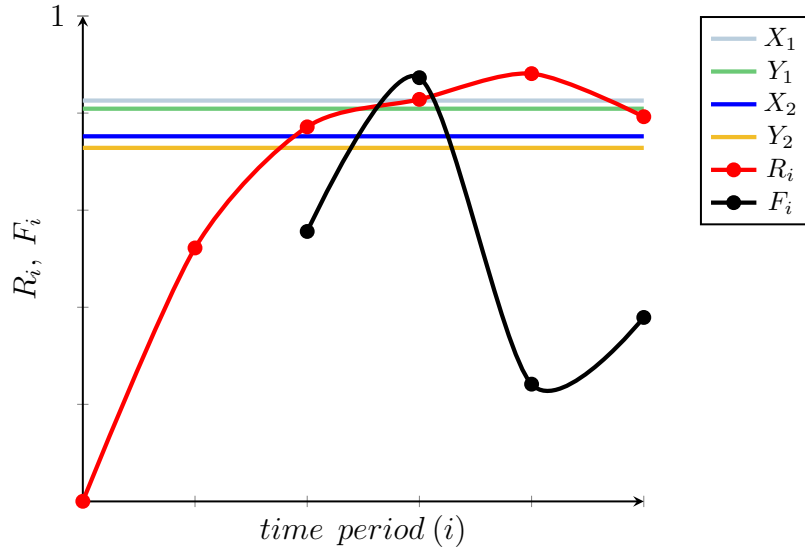


Figure 5.8: Reputation vs Fill Rate Under Constant Capacity With Short Planning Horizon

In this example, the optimal advertising sequence is H, H, H, L, L, H , and the states observed from period 1 through 6 are F, F, D, A, A, D . Notice that, since the problem horizon is short, we do not observe a repeating pattern in R_i . Firm's optimal strategy is to increase reputation in the early periods to collect maximum revenue in the last 4 periods. Weight of WoM communication is low in this example, therefore fill rate cannot pull reputation closer to itself strongly, it just slows down the increase in reputation.

In the next section, we discussed the optimal advertisement decision of the incomplete-information firm under constant capacity. For each state, we have identified the optimal decision, and summarized the results in Equation 5.23. When this decision mechanism is compared with the one for the incomplete-information firm of the variable capacity problem, which is given by Equation 4.25, it is seen that constant capacity decision is more complex.

Finally, the computational results are presented regarding VoI. For the constant capacity problem, analysis of parameters' effects on VoI is not as straightforward as variable capacity problem; since, the optimal advertisement policies of the two types of firms often consist of fluctuating advertisement levels. However, when the % VoI is measured for each Ω type, it is seen that when Ω has the highest cardinality, the average % VoI is significantly high while the average correlation between the advertising sequences of the two firms is significantly low. The Ω types that result in the highest VoI are $\Omega = \{A, B, C, E, F\}$ and $\Omega = \{A, B, D, E, F\}$. For such parameter sets, demand can take values from zero to above S . This indicates that there are many values fill rate can take throughout the planning horizon, creating the necessity for more elaborate advertisement policies. These are hard to construct for the myopic incomplete-information firm, resulting in high VoI. Additionally, when $\Omega \subset \{C, E, F\}$, as the problem becomes similar to the variable capacity problem, parameters' effects on VoI are the same as they were for the variable capacity VoI analysis. Considering all these, we evaluate VoI with respect to the Ω type.

The computations reveal that parameters' effects on VoI may differ between variable and constant capacity problems. We have the following remarks.

To begin with, we had seen that weight of WoM reduces VoI rather insignificantly

under variable capacity whereas its impact is crucial under constant capacity. Since WoM can be both positive and negative, high w makes it harder for the firm to manage demand, and VoI increases with w . This is interpreted as when it is not guaranteed that firms can keep up with the expectations of the consumers, unlike the variable capacity problem, it needs to pay more attention to WoM communication that takes place.

Advertisement resistance increases VoI in both problems; however, its effect is amplified under constant capacity since it changes Ω .

As we increase the reservation utility, in the variable capacity problem, VoI goes up. Under constant capacity, u first increases VoI by increasing the cardinality of Ω , then we observe $\Omega \subset \{C, E, F\}$, where VoI makes a sharp fall and finally, keeps increasing slower than before. This indicates that as it becomes harder to attract customers, the firm needs to put more effort and employ more complex advertisement policies. However, when it becomes impossible to utilize the service capacity, the firm starts to adopt more straightforward advertisement policies and VoI drops. After that point, the firm simply keeps increasing the advertisement efforts, which increases VoI but not as much as before, as it is easier for the incomplete-information firm to match the newly applied advertisement policies.

Similarly, the monetary value of reputation decreases VoI under variable capacity while under constant capacity, it first decreases VoI, then firm's demand generating ability changes (cardinality of Ω increases), sky rocketing VoI, and finally, it keeps decreasing from that point.

Service price increases VoI in both problems.

As the planning horizon becomes longer, VoI decreases under variable capacity. Under constant capacity, however, N does not affect VoI on its own. The length of the planning horizon is rather important in examining other parameters' effects on VoI.

When the impact of R_1 is examined under constant capacity, it is observed that the highest differences between complete and incomplete-information firms' profits are recorded when R_1 is very low and the planning horizon is short. When the planning horizon becomes longer, R_i fluctuates, and the point where it started loses importance.

This effect is similar to the variable capacity problem.

Studying the impacts of the high and low advertisement levels, we observe that L reduces VoI for both variable and constant capacity problems. At variable capacity, H does not have a significant effect on VoI. Meanwhile, under constant capacity, when H is too high or too low, the decision becomes easier; therefore, VoI is the highest when H is moderate.

Under constant capacity, when c_A is increased, the firms make less high level advertisements which makes their advertisement policies more similar. On the other hand, under variable capacity, the incomplete-information firm postpones the advertisement efforts which increases the profit gap between the two firms. Therefore, c_A reduces VoI at the constant capacity problem while increasing it at the variable capacity problem.

Increase in the market size creates opportunities for both types of firms, and the complete-information firm is able to realize such opportunities faster. When the incomplete-information firm fails to seize the opportunity, VoI goes up, while in the instances where both firms can seize the opportunity, VoI goes down due to scale. In general, its effect is not very significant, and VoI is the highest when market is medium size. This is the case for both constant and variable capacity problems.

Finally, service capacity mainly decreases VoI; however, as discussed earlier, it causes peaks in VoI due to the short sighted greedy decisions made by the incomplete-information firm, which damage reputation. The firm can only realize that reputation is beyond repair or very costly to repair at later periods.



CHAPTER 6

CONCLUSION

In this study, we consider a firm that serves a set of customers who are sensitive to the past performance of the company. Customers use the information they have regarding the potential fill rate to determine whether to request the service or not. This information can be obtained from multiple sources. One of them is the Word-of-Mouth communication which reflects the service experiences of other customers. The other one is the advertisements made by the company as a reflection of future promises. Using these two sources of information, customers develop a belief on company's reputation, which determines the demand. In this setting, we consider the company's problem to determine the advertisement level and capacity decisions. We study this problem from two aspects: (i) the company has full information on WoM communication and its impacts, (ii) company does not know the impact of WoM on reputation, assumes the reputation is mostly external and infer it at the beginning of each period.

While there are several studies stating that WoM should be considered as a marketing method; the studies on how a firm can benefit from making operational and marketing decisions at once while taking WoM into account are scarce. Understanding the mechanisms involved in WoM communication is significant for firms. As eWoM is getting more influential on consumer decisions, firms have the opportunity of observing the communication flow between consumers easier, through online review platforms, etc. and learn how WoM acts as a determinant of demand.

To model the the impact of WoM, we introduce an endogenous demand structure in a service environment. The typical example of the type of service we consider is online grocery delivery services, where fulfilling customers' demand in a timely

manner is the main criterion in the purchasing decisions of the customers. In our setting, consumers' perception towards the service is expressed by *reputation*, which is the fill rate they presume. Reputation is updated by advertisements and consumers sharing their experiences with each other regarding the fill rate through WoM, and the demand is realized based on the reputation. We study the problem under fixed and variable capacity scenarios, in a problem horizon with multiple time periods. In each setting, we examine complete and incomplete-information firms. Finally, we carry out computational experiments, analyze their results and provide a discussion for each scenario.

Firstly, the variable capacity problem is studied. It is observed that service capacity should always match demand, which means that the fill rate is always 1. Therefore, only positive effect of WoM is observed under this setting. For the complete-information firm, it is optimal to increase demand as fast as possible in the early periods, which then stabilizes. To achieve this, the firm adopts a rather aggressive advertisement policy, placing successive high level advertisements for a number of time periods starting with the first one, and then switching to placing less effort for the rest of the periods. These indicate that firm raises the bar in terms of service quality in each time period and keeps reaching it by providing high-quality service. In other words, WoM motivates firms to provide high-quality service.

The single switching point structure reduces the problem complexity significantly. Meanwhile, the incomplete-information firm makes separate advertisement decisions at each time period. When these successive decisions are considered, it is seen that the firm does not place high efforts in advertising for a number of periods until higher demand can be realized to make it worthwhile. Then the firm places high advertisement efforts, and finally when demand stabilizes at a high enough level, it goes back to making low level advertisements. It is seen that the incomplete-information firm can never place more effort in advertisement than its complete-information equivalent. This is interpreted as complete-information firm not only makes high level advertisements to generate more profit at the current time period, but it also makes high level advertisements as an investment on reputation to benefit from in the future periods. Maximizing only the current period's profit, the incomplete-information firm makes more myopic decisions. Due to this difference, the complete-information firm uses its

advantage over the incomplete-information firm by realizing more revenue. If the two types of firms can achieve high levels of demand fast, they end up with close profits, and if they do not, having information on WoM mechanism gains importance.

Under variable capacity, we observe that, by considering the effects of WoM on demand, firms can increase their brand reputation faster. Not accounting for the positive WoM, however, can cause firms to overlook this opportunity. In that case, if capturing demand is not easy, they overestimate how challenging a market is, and may even give up on operating.

For the case where the firm can adjust service capacity, having complete information on WoM is vital when (i) the initial perception of the consumers regarding the service is not very positive (low R_1), (ii) consumers' opinions are weakly affected by the recent WoM (low w), (iii) the amount of utility hinted by firm's current perception is low (low m), (iv) planning horizon is short (low N), (v) advertisement levels are low (low H and low L), (vi) consumers are not easily impressed by advertisements (high α), (vii) the amount of utility consumers think they will gain from substitutes is high (high u), (viii) market size is medium (medium Λ), (ix) service price is high (high p), (x) advertisement cost is high (high c_A), and (xi) the unit cost of service capacity is high (high c).

When the fixed capacity scenario is studied, it is seen that not being able to manipulate WoM by determining service capacity introduces new challenges to the firm. Reputation of the firm fluctuates between time periods, requiring complicated advertisement policies to maximize profit. Mainly, when the amount of demand firm can realize for different reputation levels varies greatly, the problem becomes more complex. Under this setting, we identify some special cases where solving the problem requires less computational effort owing to the significantly reduced problem complexity.

We have seen that if the service capacity is constant, information on endogenous demand becomes vital. When firms do not take WoM into account, they do not anticipate how much demand can grow due to positive WoM, and eventually suffer negative reviews. Having said that, if the firm has comprehensive understanding of the WoM effect, it can act anticipating a dose of negative WoM to keep demand at desired levels. For such manoeuvres, firms need to identify elaborate policies, which

allow them to keep a low profile when needed and advertise more aggressively at other times. A strong observation on this matter shows that not incorporating WoM effect in the decision making process can yield higher promotion expenditures and yet, less revenue than the optimal policy. Note that in the variable capacity problem, the complete-information firm beats the incomplete-information firm by making more revenue. However, under constant capacity, the complete-information firm can collect more revenue, has less expenditure or does both at the same time compared with its incomplete-information equivalent. On top of that, the incomplete-information firm may end up reducing reputation beyond repair after a few bad decisions, reducing the size of its own playground for the future periods.

For the case where the firm must operate with a fixed service capacity, having complete information on WoM is vital when (i) consumers' opinions are strongly affected by the recent WoM (high w), (ii) consumers are not easily impressed by advertisements (high α), (iii) service price is high (high p), (iv) the firm can either put moderate effort or very low effort in advertisement (medium H and low L), and improve its perception according to the amount of effort placed, (v) the advertisement cost is low (low c_A), and (vi) the market size is medium (medium Λ). The other factors have counter intuitive implications: (vii) the amount of utility hinted by firm's current perception (m) decreases the importance of this information up to a point, then creates a peak, and continues to decrease it, while (viii) the amount of utility consumers think they will gain from substitutes (u) has the exact opposite effect, and (ix) when service capacity is low or moderate (low or medium S), the value of information on WoM is high. Finally, (x) the initial perception of the consumers regarding the service (R_1) becomes important only if the (xi) planning horizon (N) is short, in which case having complete information on endogenous demand becomes important when consumers think poorly of the firm at the beginning.

Ultimately, the lack of information on WoM can result in loss of opportunities and unmet demand, especially if the firm does not have the means to increase the service capacity.

An interesting future study could be to observe how WoM reduces the influence of unrealistic messages given by advertisement and adjusts how demand is distributed in

the market based on consumer utility. It is possible that WoM helps markets become more rational by clearing the information pollution and making the decision making mechanism of the consumer mass healthier. In addition, introducing competition and price decision to the model can be considered, as well as modifying the demand model to represent different consumer or product types. Having reputation increase slower at low levels can reflect the situations where consumers require more convincing if they think poorly of the firm or if they have little information about the firm. Products or services, for which customer retention is difficult, or resentful consumer populations can be expressed by such models. Additionally, considering time value of money and examining its impact on the optimal policies of the firms could reveal interesting outcomes. Besides these, the nature of the incomplete-information can be diversified. The firm may not be aware of the exact relationship between reputation and demand or it may have failed to estimate the parameters acting in the system, leading to inaccurate demand expectations at firm's side. These extensions may introduce new computational complexities to the already analytically challenging problem. Finally, we would like to mark that due to the self-feeding nature of the endogenous demand structure, approaching the problem with the inclusion of other disciplines such as control theory could be an interesting research topic for the future.



REFERENCES

- [1] N. Huete-Alcocer, “A literature review of word of mouth and electronic word of mouth: Implications for consumer behavior,” *Frontiers in psychology*, vol. 8, p. 1256, 2017.
- [2] R. N. Bolton and J. H. Drew, “A longitudinal analysis of the impact of service changes on customer attitudes,” *Journal of marketing*, vol. 55, no. 1, pp. 1–9, 1991.
- [3] C. Shapiro, “Consumer information, product quality, and seller reputation,” *The Bell Journal of Economics*, pp. 20–35, 1982.
- [4] V. A. Zeithaml, “Consumer perceptions of price, quality, and value: a means-end model and synthesis of evidence,” *Journal of marketing*, vol. 52, no. 3, pp. 2–22, 1988.
- [5] T. Daugherty and E. Hoffman, “ewom and the importance of capturing consumer attention within social media,” *Journal of Marketing Communications*, vol. 20, no. 1-2, pp. 82–102, 2014.
- [6] S. W. Litvin, R. E. Goldsmith, and B. Pan, “Electronic word-of-mouth in hospitality and tourism management,” *Tourism management*, vol. 29, no. 3, pp. 458–468, 2008.
- [7] C. Dellarocas, “The digitization of word of mouth: Promise and challenges of online feedback mechanisms,” *Management science*, vol. 49, no. 10, pp. 1407–1424, 2003.
- [8] J. Nieto, R. M. Hernández-Maestro, and P. A. Muñoz-Gallego, “Marketing decisions, customer reviews, and business performance: The use of the toprural website by spanish rural lodging establishments,” *Tourism Management*, vol. 45, pp. 115–123, 2014.

- [9] F. X. Yang, “Effects of restaurant satisfaction and knowledge sharing motivation on ewom intentions: the moderating role of technology acceptance factors,” *Journal of Hospitality & Tourism Research*, vol. 41, no. 1, pp. 93–127, 2017.
- [10] D. Mayzlin, “Promotional chat on the internet,” *Marketing science*, vol. 25, no. 2, pp. 155–163, 2006.
- [11] B. Çavdar and N. Erkip, “Word-of-mouth on action: Analysis of operational decisions when customers are resentful,” *Working paper*, 2021.
- [12] M. D. Johnson, E. W. Anderson, and C. Fornell, “Rational and adaptive performance expectations in a customer satisfaction framework,” *Journal of consumer research*, vol. 21, no. 4, pp. 695–707, 1995.
- [13] W. B. Dodds, K. B. Monroe, and D. Grewal, “Effects of price, brand, and store information on buyers’ product evaluations,” *Journal of marketing research*, vol. 28, no. 3, pp. 307–319, 1991.
- [14] W. Boulding, A. Kalra, R. Staelin, and V. A. Zeithaml, “A dynamic process model of service quality: from expectations to behavioral intentions,” *Journal of marketing research*, vol. 30, no. 1, pp. 7–27, 1993.
- [15] D. Adelman and A. J. Mersereau, “Dynamic capacity allocation to customers who remember past service,” *Management Science*, vol. 59, no. 3, pp. 592–612, 2013.
- [16] P. Mandal, R. Kaul, and T. Jain, “Stocking and pricing decisions under endogenous demand and reference point effects,” *European Journal of Operational Research*, vol. 264, no. 1, pp. 181–199, 2018.
- [17] T. L. Urban, “Inventory models with inventory-level-dependent demand: A comprehensive review and unifying theory,” *European Journal of Operational Research*, vol. 162, no. 3, pp. 792–804, 2005.
- [18] N. Craig, N. DeHoratius, and A. Raman, “The impact of supplier inventory service level on retailer demand,” *Manufacturing & Service Operations Management*, vol. 18, no. 4, pp. 461–474, 2016.

- [19] G. R. Heim and K. K. Sinha, "Operational drivers of customer loyalty in electronic retailing: An empirical analysis of electronic food retailers," *Manufacturing & Service Operations Management*, vol. 3, no. 3, pp. 264–271, 2001.
- [20] G. J. Fitzsimons, "Consumer response to stockouts," *Journal of consumer research*, vol. 27, no. 2, pp. 249–266, 2000.
- [21] A. C. McKinnon, D. Mendes, and M. Nababteh, "In-store logistics: an analysis of on-shelf availability and stockout responses for three product groups," *International Journal of Logistics Research and Applications*, vol. 10, no. 3, pp. 251–268, 2007.
- [22] O. Z. Aksin and A. Masini, "Effective strategies for internal outsourcing and offshoring of business services: An empirical investigation," *Journal of Operations Management*, vol. 26, no. 2, pp. 239–256, 2008.
- [23] F. Fuhrman, "Co-sourcing: A winning alternative to outsourcing call center operations," *Call Center Solutions*, vol. 17, pp. 100–109, 1999.
- [24] O. Z. Akşin, F. De Véricourt, and F. Karaesmen, "Call center outsourcing contract analysis and choice," *Management Science*, vol. 54, no. 2, pp. 354–368, 2008.
- [25] O. Garnett, A. Mandelbaum, and M. Reiman, "Designing a call center with impatient customers," *Manufacturing & Service Operations Management*, vol. 4, no. 3, pp. 208–227, 2002.
- [26] C. Dorsch and B. Häckel, "Combining models of capacity supply to handle volatile demand: The economic impact of surplus capacity in cloud service environments," *Decision Support Systems*, vol. 58, pp. 3–14, 2014.
- [27] J. Huang, M. Leng, and M. Parlar, "Demand functions in decision modeling: A comprehensive survey and research directions," *Decision Sciences*, vol. 44, no. 3, pp. 557–609, 2013.
- [28] K. S. Moorthy, "Product and price competition in a duopoly," *Marketing science*, vol. 7, no. 2, pp. 141–168, 1988.
- [29] J. Tirole and T. Jean, *The theory of industrial organization*. MIT press, 1988.