

A NUMERICAL SOLUTION FOR NANOSECOND SINGLE CRYSTAL UPCONVERSION OPTICAL PARAMETRIC OSCILLATORS

A THESIS

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By

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June, 2005

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ABSTRACT

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A model of single-crystal upconversion optical parametric oscillators (OPO's) is a valuable tool for design and optimization. In these devices, a single nonlinear crystal is used both for optical parametric amplification (OPA) and sum-frequency/second-harmonic generation (SFG/SHG). This thesis presents a computational model for single-crystal upconversion OPO's with pump pulse durations on the order of few nanoseconds. When the duration of the pump pulse is longer than the round trip time of the cavity used in the OPO, the transient nature of the problem must be taken into consideration. Besides the evolutions of the interacting light beams in time, the beam profiles and effects due to diffraction are incorporated into the model. Besides the model, incorporating nonlinear interaction and diffraction inside the crystal, solutions for different cavity components are given in detail. Computational results for a class-D sum-frequency OPO are presented, which are in good agreement with experimental measurements.

Keywords: Second order nonlinear interaction, non-uniform beam profile, optical parametric oscillator, sum-frequency generation, second-harmonic generation, numeric model.

ÖZET

NANOSANIYE TEK-KRİSTALLİ YUKARI-DÖNÜŞÜM OPTİK PARAMETRİK OSİLATÖRLER İÇİN SAYISAL ÖRNEKLEME

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Tek kristalli yukarı dönüşüm optik parametrik osilatörleri (OPO) için bir örnekleme değerli bir araçtır. Bu cihazlarda, tek bir doğrusal olmayan kristal hem optik parametrik yükseltme hem de toplam-frekans/ikinci-harmonik için kullanılmaktadır. Bu tez, giriş pompa ışını titreşim uzunluğu birkaç nanosaniye-den uzun tek kristalli yukarı dönüşüm OPO'ları için bir sayısal örnekleme sunmaktadır. Giriş pompa ışını titreşim uzunluğu OPO kovuğu dolanım zamanından uzun olduğunda, problemin süreksiz doğası hesaba katılmalıdır. Etkileşen ışınların zaman içindeki değişiminin dışında ışın şekli ve dağılım etkileri hesaba katılmıştır. Örnekleme kullanılarak D sınıfı bir toplam-frekans OPO'su için çözümler ve sonuçlar verilmiş, deneysel ölçümlerle uygunluğu gösterilmiştir.

Anahtar sözcükler: Doğrusal olmayan ikinci derece etkileşim, düzgün olmayan ışın şekli, optik parametrik osilatör, toplam-frekans üretimi, ikinci harmonik üretimi, sayısal örnekleme.

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Chapter 1

Introduction

Lasers find many applications in entertainment, telecommunications, manufacturing, medical instrumentation and treatment. Different applications place different requirements on the lasers' output color, beam shape and power.

The profile and power of a laser beam depends on the cavity topology and pumping level of the design, whereas the output wavelength is determined by the gain medium. The wavelength of a laser beam is determined by the difference between the two radiating energy levels of the gain medium used. For a given wavelength (color) an appropriate material may not exist, or may be hard to obtain. Even though a gain medium with an appropriate energy gap (between radiating energy levels) for a specific wavelength exists, the material may not meet the requirements of the laser to be designed. Designing a laser for a specific wavelength may not be possible. It is clear that an alternative method for generating beams with desired wavelengths is needed.

Using second-order nonlinear optical processes, in nonlinear media, it is possible to use a laser beam to generate or amplify other beams with longer wavelengths. Moreover, two laser beams can be used to generate a beam with a wavelength shorter than both of the waves. Generation and amplification of a beam, in the expense of a beam with a different wavelength, is the conversion of wavelength of light. The ability to convert the wavelength of a laser beam has

great technological importance. Conversion of beam wavelengths of easily available, commonly used, well established lasers to desired wavelengths has many advantages.

Nonlinear optical processes originate from the nonlinear response of a material to an optical excitation [1]. In any medium, a polarization density field is generated due to the excitation, which in turn induces an electric field. The nonlinear coefficients coupling the electric field to the nonlinear polarization density inside a medium are small, so the nonlinearities get effective only at high intensities. To obtain these high intensity fields, either continuous wave lasers with high powers or lasers with short pulse durations and high intensities are used.

In this thesis, we will be focusing on second-order nonlinear interactions. Second-order nonlinear interactions lead to three types of processes:

- **Sum frequency generation (SFG)**: is the generation of a wave whose frequency is the sum of the two input frequencies.
- **Difference frequency generation (DFG)**: is the generation of a wave with a frequency equal to the difference of the frequencies of two input beams.
- **Second harmonic generation (SHG)**: is a special case of SFG in which the two input frequencies are the same. The process is also called frequency doubling.

When the DFG process is used to amplify a wave at the expense of a higher frequency (lower wavelength) wave, this amplification is called optical parametric amplification (OPA). When this amplifier is placed inside a cavity, resonant at the frequency of the amplified wave (forming an optical parametric oscillator), as a gain medium and the unsaturated gain is higher than the cavity losses, oscillation starts. The seed for the oscillation is generally the noise photons present in the cavity due to parametric fluorescence [27] .

Using an optical parametric oscillator (OPO), it is only possible to convert the wavelengths of source lasers to longer wavelengths. If the desired wavelength is

shorter than the source wavelength, some form of upconversion must be performed either using SFG or SHG. There is more than one way of combining an OPO with an upconversion process:

- A frequency doubler can be placed at the output of an OPO [2].
- A frequency doubler can be placed at the input of an OPO [3],[4].
- A frequency doubler can be placed inside the cavity of an OPO [2].
- SFG of the pump and signal waves inside the cavity may be achieved [5].
- SFG of the pump and signal waves at the output of the cavity may be achieved.
- The same nonlinear crystal can be used for both OPO and SFG/SHG [6], [7].

By placing the second nonlinear process inside the OPO cavity, it is possible to take advantage of the high intensity fields inside the cavity, increasing conversion efficiency.

A model of nonlinear optical processes is a valuable tool for design and optimization. For a model to produce results in good agreement with real world processes, it should incorporate the critical factors affecting them.

The main purpose of this thesis is to present a numerical model for single-crystal upconversion OPO's with source beam pulse durations on the order of ten nanoseconds. In this time regime, when the input pulse is first incident on the OPO cavity, noise photons are amplified inside the cavity and oscillation starts. During this oscillation, the pump pulse intensity and amplified wave intensities change in time due to the pulse nature of the source wave. Since the amount of interaction is dependent on the intensities of the waves involved, the evolution of their intensities in time is critical.

Another critical factor affecting the conversion process is the transverse spatial profiles of the interacting beams. It is known [8] that the spatial profiles of the

interacting beams affect the nonlinear process, so they must be incorporated into the model. Since most laser beams do not have perfectly gaussian or flat-top beam profiles, the model should incorporate the interaction of arbitrary beam profiles.

Besides the temporal and spatial profiles of interacting beams, the cavity topology, nonlinear crystal and cavity components are critical factors effecting the conversion process. Even though the input beam profile and pulse duration are critical parameters for conversion, they may not be modifiable. By incorporating the modifiable design parameters into the model, like the cavity topology and components, optimization of the design is possible.

In this thesis, a brief introduction to second-order nonlinear optical processes, followed by a discussion of single-crystal upconversion OPO's is given. Our model for two simultaneous nonlinear optical processes, incorporating both the transverse spatial and temporal profiles of the beams is discussed in detail. In Chapter 4, the computational results obtained from the model is compared with experimental results.

Chapter 2

Second-Order Nonlinear Processes

In this chapter, the basic wave theory of second-order nonlinear interactions will be presented. The discussion will begin with an introduction to second-order nonlinear wave theory, followed by the derivation of the differential equations governing the propagation of waves inside a nonlinear crystal. The three processes, difference frequency generation, sum-frequency generation and second harmonic generation will be explained in detail. Finally, the theory of two simultaneously phase matched second-order nonlinear optical processes will be given.

2.1 The Driven Wave Equation

In a nonlinear medium, an optical excitation generates a nonlinear polarization density field which in turn induces an electric field. The propagation through the crystal and the nonlinear processes are governed by the driven wave equation.

Maxwell's equations,

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \quad (2.1)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}, \quad (2.2)$$

$$\nabla \cdot \mathbf{D} = 0, \quad (2.3)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2.4)$$

where

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \quad (2.5)$$

$$\mathbf{B} = \mu_0 \mathbf{H}, \quad (2.6)$$

for nonmagnetic media with no free charges and currents, is our starting point.

The polarization density $\mathbf{P}$ is dependent on the medium properties and the electric field $\mathbf{E}$ applied to the material. For any medium, the polarization density $\mathbf{P}$ can be expressed as a sum of powers of $\mathbf{E}$ as

$$\mathbf{P} = \epsilon_0 [\chi^{(1)} \cdot \mathbf{E} + \mathbf{E} \cdot \chi^{(2)} \cdot \mathbf{E} + \mathbf{E} \cdot (\mathbf{E} \cdot \chi^{(3)} \cdot \mathbf{E}) + \dots], \quad (2.7)$$

$$\mathbf{P} = \mathbf{P}^{(1)} + \mathbf{P}^{(2)} + \mathbf{P}^{(3)} + \dots, \quad (2.8)$$

where $\chi^{(n)}$ represents the n^{th} order susceptibility tensor of the medium. For linear media the polarization term simplifies into

$$\mathbf{P} = \epsilon_0 \chi^{(1)} \cdot \mathbf{E} \quad (2.9)$$

which, for isotropic media, leads [9] to the familiar

$$\mathbf{D} = \epsilon \mathbf{E}. \quad (2.10)$$

The coefficients concerning the second-order nonlinear processes ($\chi^{(2)}$) are small, which results in nonlinear processes only occurring at high intensities. To be able to obtain strong nonlinear effects, materials with high nonlinear coefficients are used. In this thesis only the second-order interactions between the polarization density $\mathbf{P}$ and electric field will be analyzed.

When Equation (2.2) is substituted into Equation (2.1) and the nonlinear polarization density field is assumed to be quadratic, then the driven wave equation can be written as

$$-\nabla^2 \mathbf{E} + \frac{n^2}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial \mathbf{P}^{(2)}}{\partial t^2}, \quad (2.11)$$

where n is the refractive index of the medium. The fields of our concern can be assumed to be localized TEM waves [10] with linear polarizations, so the wave equation, hence the waves can be written in scalar form. Notice that the driving term in the driven wave equation is the second-order nonlinear polarization density field.

2.2 Coupled Mode Equations

The complex electric field E_i ,

$$E_i(x, y, z, t) = A_i e^{-j(\omega_i t - k_i z)} + A_i^* e^{j(\omega_i t - k_i z)}, \quad (2.12)$$

is defined as the monochromatic spectral component of the electric field at frequency ω_i . If only three monochromatic waves exist in the medium with frequencies ω_1 , ω_2 and ω_3 , then the second-order polarization density,

$$P^{(2)} = (E_1 + E_2 + E_3) \chi^{(2)} (E_1 + E_2 + E_3), \quad (2.13)$$

leads to

$$P^{(2)} = \chi^{(2)} \left(E_1^2 + E_2^2 + E_3^2 + 2E_1 E_2 + 2E_1 E_3 + 2E_2 E_3 \right). \quad (2.14)$$

When $P^{(2)}$ is written as a function of frequency, $P^{(2)}(\omega)$ is nonzero for only 13 frequencies. The polarization density field expressions at these 13 frequencies are:

$$P^{(2)}(0) = A_1 A_1^* + A_2 A_2^* + A_3 A_3^*, \quad (2.15)$$

$$P^{(2)}(2\omega_1) = A_1^2 \cdot e^{-j2(\omega_1 t - k_1 z)} + \text{complex conjugate}(c.c.), \quad (2.16)$$

$$P^{(2)}(2\omega_2) = A_2^2 \cdot e^{-j2(\omega_2 t - k_2 z)} + c.c., \quad (2.17)$$

$$P^{(2)}(2\omega_3) = A_3^2 \cdot e^{-j2(\omega_3 t - k_3 z)} + c.c., \quad (2.18)$$

$$P^{(2)}(\omega_1 + \omega_2) = 2A_1 A_2 \cdot e^{-j((\omega_1 + \omega_2)t - (k_1 + k_2)z)} + c.c., \quad (2.19)$$

$$P^{(2)}(\omega_1 + \omega_3) = 2A_1A_3 \cdot e^{-j((\omega_1+\omega_3)t-(k_1+k_3)z)} + c.c., \quad (2.20)$$

$$P^{(2)}(\omega_2 + \omega_3) = 2A_2A_3 \cdot e^{-j((\omega_2+\omega_3)t-(k_2+k_3)z)} + c.c., \quad (2.21)$$

$$P^{(2)}(\omega_1 - \omega_2) = 2A_1A_2^* \cdot e^{-j((\omega_1-\omega_2)t-(k_1-k_2)z)} + c.c., \quad (2.22)$$

$$P^{(2)}(\omega_1 - \omega_3) = 2A_1A_3^* \cdot e^{-j((\omega_1-\omega_3)t-(k_1-k_3)z)} + c.c., \quad (2.23)$$

$$P^{(2)}(\omega_2 - \omega_3) = 2A_2A_3^* \cdot e^{-j((\omega_2-\omega_3)t-(k_2-k_3)z)} + c.c., \quad (2.24)$$

$$P^{(2)}(\omega_2 - \omega_1) = [P^{(2)}(\omega_1 - \omega_2)]^*, \quad (2.25)$$

$$P^{(2)}(\omega_3 - \omega_1) = [P^{(2)}(\omega_1 - \omega_3)]^*, \quad (2.26)$$

$$P^{(2)}(\omega_3 - \omega_2) = [P^{(2)}(\omega_2 - \omega_3)]^*. \quad (2.27)$$

When the polarization density field expressions are substituted into the driven wave equation [Equation (2.11)] and expressed as a function of frequency

$$-\nabla^2 E(\omega) + \frac{n^2}{c^2} \frac{\partial^2 E(\omega)}{\partial t^2} = -\mu_0 \frac{\partial P^{(2)}(\omega)}{\partial t^2} = \mu_0 \chi^{(2)} \omega^2 P^{(2)}(\omega) \quad (2.28)$$

is obtained.

If the frequencies of the three waves satisfy the condition $\omega_3 = \omega_1 + \omega_2$ and the field expressions are inserted into the driven wave equation, assuming non-uniform transverse spatial profiles, for the wave with frequency ω_3 we obtain

$$\frac{\partial A_3(x, y)}{\partial z} = \frac{j}{2k_3} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] A_3(x, y) + j \frac{\omega_3 d_e}{n_3 c} A_1(x, y) A_2(x, y) e^{j(k_1+k_2-k_3)z}, \quad (2.29)$$

where $\Delta k = k_3 - k_2 - k_1$ is the phase mismatch. Similarly, the differential equations for the waves with frequency ω_1 and ω_2 can be written as

$$\frac{\partial A_1(x, y)}{\partial z} = \frac{j}{2k_1} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] A_1(x, y) + j \frac{\omega_1 d_e}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j\Delta k z} \quad (2.30)$$

and

$$\frac{\partial A_2(x, y)}{\partial z} = \frac{j}{2k_2} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] A_2(x, y) + j \frac{\omega_2 d_e}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j\Delta k z}, \quad (2.31)$$

respectively. The three differential equations [Equations (2.29)-(2.31)] governing the interaction between the three waves are called the coupled mode equations. These equations are the basis of the numerical model which is presented in Chapter 3.

The first summation term in the right hand side of the coupled mode equations [Equations (2.29)-(2.31)] represents diffraction due to the propagation of the wave inside the medium. The second summation term represents the nonlinear interaction dependent on the other two field intensities. For ease of use, Equation (2.29) can be rewritten in the form

$$\frac{\partial A_3}{\partial z} = D_3(x, y) + N_3(x, y), \quad (2.32)$$

where D and N represent the diffraction and nonlinear interaction terms respectively.

The nonlinear interaction terms for the case $\omega_3 = \omega_1 + \omega_2$ can be written as

$$N_1(x, y) = j \frac{\omega_1 d_e}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j\Delta k z}, \quad (2.33)$$

$$N_2(x, y) = j \frac{\omega_2 d_e}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j\Delta k z} \quad (2.34)$$

and

$$N_3(x, y) = j \frac{\omega_3 d_e}{n_3 c} A_1(x, y) A_2(x, y) e^{j\Delta k z}. \quad (2.35)$$

2.3 Phase Matching

The phase matching ($\Delta k = k_3 - k_2 - k_1 = 0$) of the three waves is the key requirement for second-order nonlinear processes to occur [1]. From a quantum mechanical point of view, this condition can also be viewed as the condition for conservation of momentum of photons in the process. In materials with normal dispersion the refractive index of the medium increases with increasing frequency. To simultaneously satisfy the conservation of energy ($\omega_3 = \omega_1 + \omega_2$) and phase matching conditions, the interacting fields must be subject to different refractive indices. One common way of achieving this condition is by using the natural birefringence of nonlinear crystals.

Another way of achieving the phase matching condition is by using periodical domain reversals of the nonlinear crystal, satisfying a quasi phase matched (QPM) condition. Using QPM, it is possible to phase match any process within the transparency range of the crystal [11].

2.4 Birefringent Phase Matching

In a medium with normal dispersion, the refractive index increases with increasing frequency. Due to this increase in refractive index, the phase matching condition,

$$n_3\omega_3 = n_1\omega_1 + n_2\omega_2, \quad (2.36)$$

cannot be satisfied while also satisfying the conservation of energy ($\omega_3 = \omega_1 + \omega_2$) condition. To satisfy both equations, natural birefringence of anisotropic crystals can be used.

Birefringence, also called double refraction, occurs when a material has two different refractive indices for two orthogonal polarization directions [12]. These two orthogonal axes are called the fast and slow axes (fast having the smaller refractive index). In anisotropic crystals, for a given propagation direction, two orthogonal eigenmodes with different refractive indices are present. The refractive indices for orthogonal polarizations depend on the propagation direction, which can be changed to satisfy the phase matching condition. By finding an appropriate propagation direction for a given set of frequencies birefringent phase matching (BPM) can be achieved. It should be noted that it may not be possible to find a crystal and propagation direction for a given set of frequencies.

For the phase matching condition to be satisfied, the wave with the higher frequency should be incident on the crystal polarized along the fast axis. With the highest frequency wave's polarization set, three possibilities exist for the polarizations of the two remaining waves, for the phase matching condition to be satisfied. Possible types of BPM can be seen in Table 2.1 [13].

Type	w_1	w_2	w_3
I	slow	slow	fast
II	fast	slow	fast
III	slow	fast	fast

Table 2.1: Possible BPM types for phase matching to be possible in a birefringent crystal. $w_3 = w_1 + w_2$.

2.5 Quasi-Phase Matching

In QPM, the nonlinear crystal is periodically poled to obtain periodic domain reversals. Using domain reversals, it is possible to compensate the phase mismatch, making Δk effectively zero. The main disadvantage of using QPM is the higher cost and increased complexity of the design. The advantage of QPM systems is the availability of a phase matching condition for any three (energy conserving) waves, within the transparency range of the crystal [11]. In this thesis, only the numerical model for birefringent phase matched systems will be discussed.

2.6 Optical Parametric Amplification and Oscillation

Optical parametric amplification (OPA) is the term used for the amplification of a weak wave (at frequency ω_2), in the expense of a higher frequency intense wave (at frequency ω_3) using second-order nonlinear optical processes. The third wave (at frequency ω_1) is generated in the amplification process. The phase of the generated wave matches the phase mismatch between the pump and signal waves [13]. The generation of the wave with frequency ω_1 is also called difference frequency generation (DFG). The waves interacting in the process are called the pump (ω_3), signal (ω_2) and idler (ω_1) waves.

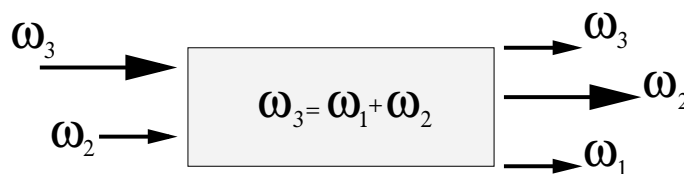


Figure 2.1: Optical parametric amplification.

When an OPA is placed inside a cavity, resonating only the signal (singly-resonant) or both the signal and idler waves (doubly-resonant), an optical parametric oscillator (OPO) is formed. The basic operation principles of an OPO is similar to a laser's. The difference between a laser and an OPO is the nature of

the optical gain in the gain medium. In a laser, the gain is achieved by stimulated emission whereas in an OPO it is achieved by parametric amplification.

In singly resonant OPO's, as in a laser, all mirrors of the cavity except the output coupler is highly reflective to the signal wave frequency. For the cavity to be singly resonant, the mirror reflectivities should be small for the idler wave, for the purpose of minimizing the feedback due to the generated idler wave on the generation process. If an idler wave is present at the input of the crystal, the nonlinear process becomes sensitive to the phase of the idler, disrupting the stability of oscillation.

In doubly resonant OPO's, both the signal and idler waves are resonated inside the cavity. Since idler feedback is existent in these types of OPO's, the phase of the three waves must match for oscillation to be stable inside the cavity. The main disadvantage of using doubly resonant cavities is the cavity complexity. To sustain the resonance of both of the waves inside the cavity generally piezoelectric or motorized actuators are used. However using a doubly resonant cavity increases oscillation threshold of the OPO, which makes them suitable when high intensity pump waves are not available, or high nonlinear interactions are required.

There are two types of OPO's depending on the source of the signal wave at the beginning of oscillation. In seeded OPO's, a weak external laser (seed) at frequency ω_2 is present. This weak wave is amplified inside the cavity and if the unsaturated gain is higher than the cavity losses, oscillation starts. In seedless OPO's, no external seed laser exists, so the amplification of the signal wave starts from noise photons. Using an external seed laser lowers the threshold of oscillation significantly, moreover decreases the spectral bandwidth of the generated signal wave [14], [15].

2.7 Sum-Frequency Generation

When a phase matched crystal (for $\omega_3 = \omega_1 + \omega_2$) is used to generate a high frequency wave (ω_3), in the expense of two waves with lower frequencies (ω_1 and

ω_2), SFG is achieved. As in the OPA case, if the sum-frequency wave is lacking at the input of the crystal, it will be generated in the process. For SFG, if a wave with frequency ω_3 exists at the input of the cavity, its phase should match the phase mismatch between the two input waves (ω_1 and ω_2).

Since the nonlinear crystal is phase matched for the same set of frequencies, it can also be used for DFG. The difference between the DFG and SFG cases is the intensity of the fields at the input of the crystal (initial boundary conditions of the coupled mode equations).

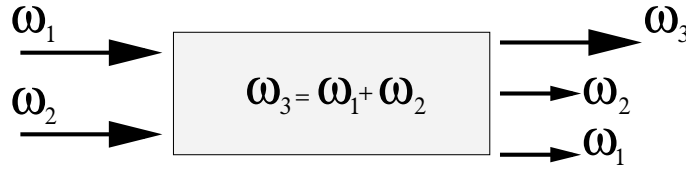


Figure 2.2: Sum-frequency generation.

2.8 Second Harmonic Generation

In second harmonic generation, the incident wave at ω_1 (called the fundamental) generates a polarization density field at $2\omega_1$ leading to the generation of a wave at $\omega_3 = 2\omega_1$. As in the OPA and SFG case, the phase of the incident second-harmonic ($2\omega_1$) wave with respect to the fundamental wave is important for the nonlinear process.

In SHG, if phase matching is achieved with type-II or type-III BPM, the fundamental wave must have components at both of the two orthogonal eigenmodes of the nonlinear crystal. In these cases of operation, SHG is a special case of SFG with $\omega_1 = \omega_2$. The nonlinear interaction terms are the same as of SFG [Equations (2.33)-(2.35)] with $\omega_1 = \omega_2$.

However, if type-I BPM is used, the fundamental wave is required only at the slow axis of the crystal. For a SHG process, with type-I BPM, the number of interacting waves reduces to two. For a type-I BPM SHG the nonlinear interaction

terms are given as [13]

$$N_1(x, y) = j \frac{w_1 d_e}{n_1 c} A_3(x, y) A_1^*(x, y) e^{j \Delta k z} \quad (2.37)$$

and

$$N_3(x, y) = j \frac{1}{2} \frac{w_3 d_e}{n_3 c} A_1(x, y) A_1(x, y) e^{j \Delta k z}, \quad (2.38)$$

where N_1 and N_3 are the fundamental and second harmonic waves respectively.

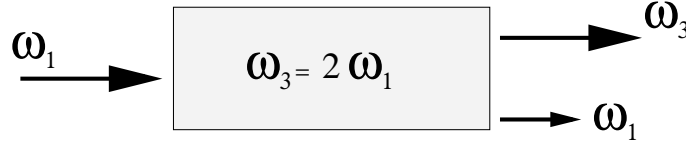


Figure 2.3: Second harmonic generation.

2.9 Single Crystal Up-Conversion

Using an OPO, with a single pump input, it is not possible to obtain an output wave which has a higher frequency than the input. To be able to obtain a higher frequency output from a single crystal, the crystal can be set so that a second process (SFG or SHG) is also phase matched for the same propagation direction. By simultaneously phase matching the upconversion process, the high intensity intra-cavity fields can be used, increasing conversion efficiency. Since the signal wave is not required at the output of the cavity, the output coupler can be highly reflective to the signal wave, increasing the intra-cavity signal intensity significantly.

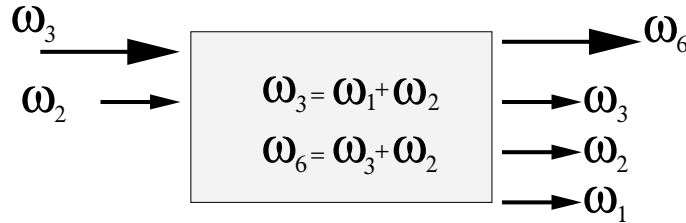


Figure 2.4: An example OPA-SFG system.

2.10 Phase-Matched OPA and Sum-Frequency Generation

When OPA and SFG are simultaneously phase matched for the same propagation direction inside a nonlinear crystal, single crystal up-conversion can be achieved. There are many combinations which may conform the phase matching condition, some of which require the polarization rotation of some of the fields. All of the different combinations of BPM for OPO-SFG fall into four distinct classes [13] defined as in Table 2.2. The polarization rotations needed for different classes of operation can be seen in Table 2.3.

	OPO Type I	OPO Type II	OPO Type III
SFG Type I	C	C	B
SFG Type II	B	B	C
SFG Type III	A	A	D

Table 2.2: Class definitions of possible OPO-SFG

When two processes occur simultaneously inside the crystal, two sets of nonlinear interaction terms combine with each other to form the terms for the complete process. Simultaneous phase matching of the processes inside the crystal does not effect the diffraction terms.

Class	Polarization rotation
A	none
B	both signal and pump
C	pump
D	signal

Table 2.3: Polarization rotation requirements for different classes of operation of OPO-SFG's.

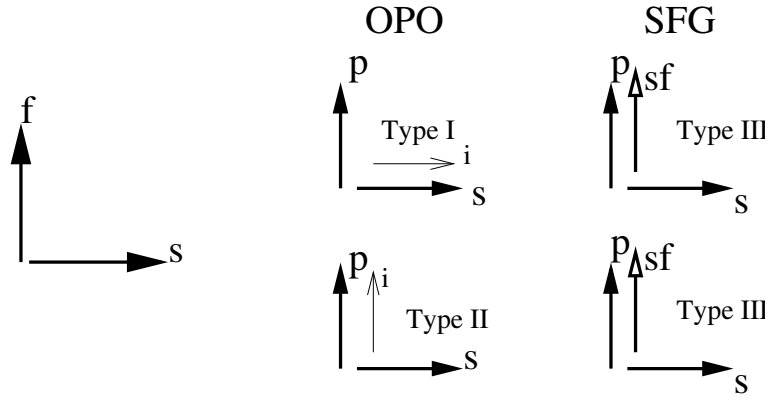


Figure 2.5: Class-A OPO-SFG BPM configurations.

2.10.1 Class-A OPO-SFG

In class-A OPO-SFG's the coupling between the two simultaneous nonlinear processes is established by the signal and the pump fields. Notice that none of the waves are rotated inside or outside the cavity. The interaction terms, the combination of nonlinear interaction terms for the OPO and SFG processes, are

$$N_1(x, y) = j \frac{\omega_1 d_{e1}}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j\Delta k_1 z}, \quad (2.39)$$

$$N_2(x, y) = j \frac{\omega_2 d_{e1}}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j\Delta k_1 z} + j \frac{\omega_2 d_{e2}}{n_2 c} A_6(x, y) A_3^*(x, y) e^{j\Delta k_2 z}, \quad (2.40)$$

$$N_3(x, y) = j \frac{\omega_3 d_{e1}}{n_3 c} A_1(x, y) A_2(x, y) e^{j\Delta k_1 z} + j \frac{\omega_3 d_{e2}}{n_3 c} A_6(x, y) A_2^*(x, y) e^{j\Delta k_2 z}, \quad (2.41)$$

$$N_6(x, y) = j \frac{\omega_6 d_{e2}}{n_6 c} A_2(x, y) A_3(x, y) e^{j\Delta k_2 z}, \quad (2.42)$$

where N_6 is the nonlinear interaction expression for the sum-frequency wave. Notice that the phase mismatch and effective nonlinear coefficients for the two processes are different.

2.10.2 Class-B OPO-SFG

In class-B OPO-SFG's, the two processes occurring in the single crystal are not coupled to each other, hence is not any different than an OPO with two nonlinear crystals phase matched for different processes. The equations governing the

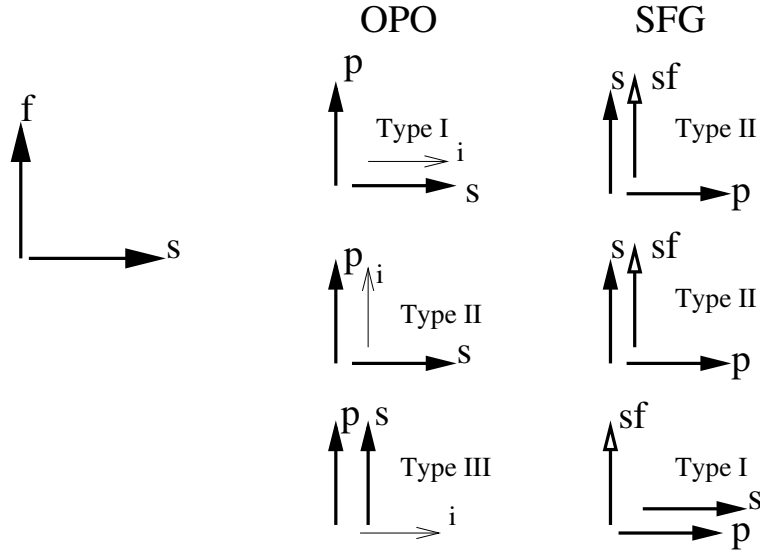


Figure 2.6: Class-B OPO-SFG BPM configurations.

processes are two uncoupled sets of triple equations. The first three being the equations of the OPO, and the latter three of the SFG. In this type of OPO-SFG's intra-cavity signal and extra-cavity pump polarization rotation is needed. The nonlinear interaction terms for class-B OPO-SFG are

$$N_1(x, y) = j \frac{\omega_1 d_{e1}}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j \Delta k_1 z}, \quad (2.43)$$

$$N_2(x, y) = j \frac{\omega_2 d_{e1}}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j \Delta k_1 z}, \quad (2.44)$$

$$N_3(x, y) = j \frac{\omega_3 d_{e1}}{n_3 c} A_1(x, y) A_2(x, y) e^{j \Delta k_1 z}, \quad (2.45)$$

$$N_4(x, y) = j \frac{\omega_4 d_{e2}}{n_4 c} A_6(x, y) A_5^*(x, y) e^{j \Delta k_2 z}, \quad (2.46)$$

$$N_5(x, y) = j \frac{\omega_5 d_{e2}}{n_5 c} A_6(x, y) A_4^*(x, y) e^{j \Delta k_2 z}, \quad (2.47)$$

$$N_6(x, y) = j \frac{\omega_6 d_{e2}}{n_6 c} A_4(x, y) A_5(x, y) e^{j \Delta k_2 z}, \quad (2.48)$$

where N_4 , N_5 , and N_6 are the nonlinear interaction expressions for the rotated signal, rotated pump and sum-frequency waves respectively.

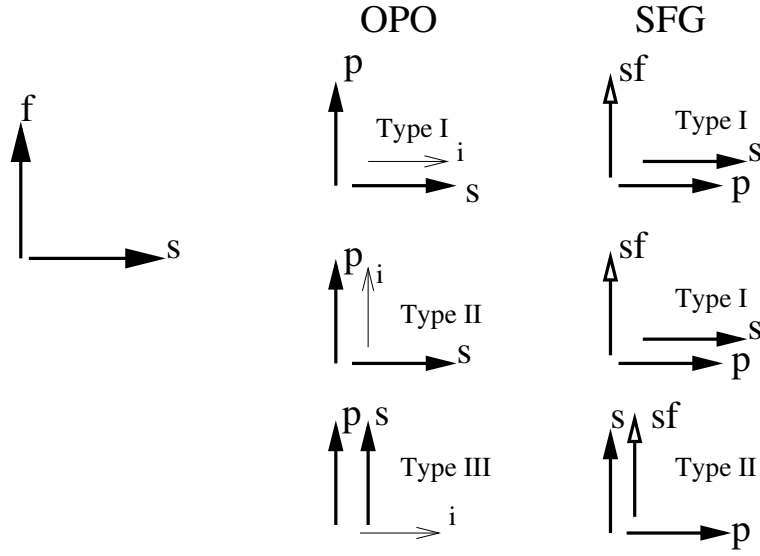


Figure 2.7: Class-C OPO-SFG BPM configurations.

2.10.3 Class-C OPO-SFG

In class-C OPO-SFG's, the rotated pump and the signal combines to generate the sum-frequency. In this class, only the extra-cavity polarization rotation of the pump is needed. The nonlinear interaction terms associated with this class are

$$N_1(x, y) = j \frac{\omega_1 d_{e1}}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j \Delta k_1 z}, \quad (2.49)$$

$$N_2(x, y) = j \frac{\omega_2 d_{e1}}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j \Delta k_1 z} + j \frac{\omega_2 d_{e2}}{n_2 c} A_6(x, y) A_5^*(x, y) e^{j \Delta k_2 z}, \quad (2.50)$$

$$N_3(x, y) = j \frac{\omega_3 d_{e1}}{n_3 c} A_1(x, y) A_2(x, y) e^{j \Delta k_1 z}, \quad (2.51)$$

$$N_5(x, y) = j \frac{\omega_5 d_{e2}}{n_5 c} A_6(x, y) A_2^*(x, y) e^{j \Delta k_2 z}, \quad (2.52)$$

$$N_6(x, y) = j \frac{\omega_6 d_{e2}}{n_6 c} A_2(x, y) A_5(x, y) e^{j \Delta k_2 z}, \quad (2.53)$$

where N_5 and N_6 are the nonlinear interaction expressions for the rotated pump and sum-frequency (SF) waves respectively. In class-B OPO-SFG's the coupling between the processes is through the signal wave.

2.10.4 Class-D OPO-SFG

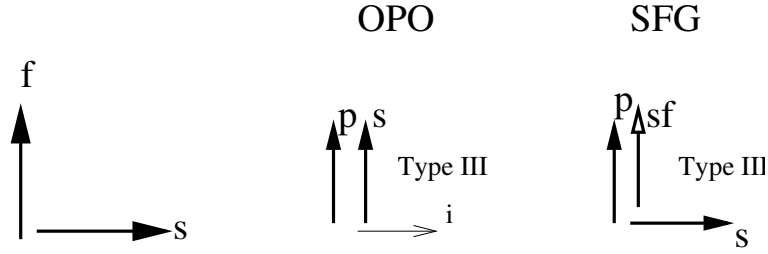


Figure 2.8: Class-D OPO-SFG BPM configuration.

In class-D OPO-SFG's the signal is rotated inside the cavity to combine with the non-rotated pump wave to generate the sum-frequency component. In Chapter 4, the numerical calculations of a class-D OPO-SFG will be compared with experimental results. The corresponding nonlinear interaction terms for a class-D OPO-SFG are

$$N_1(x, y) = j \frac{\omega_1 d_{e1}}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j \Delta k_1 z}, \quad (2.54)$$

$$N_2(x, y) = j \frac{\omega_2 d_{e1}}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j \Delta k_1 z}, \quad (2.55)$$

$$N_3(x, y) = j \frac{\omega_3 d_{e1}}{n_3 c} A_1(x, y) A_2(x, y) e^{j \Delta k_1 z} + j \frac{\omega_3 d_{e2}}{n_3 c} A_6(x, y) A_4^*(x, y) e^{j \Delta k_2 z}, \quad (2.56)$$

$$N_4(x, y) = j \frac{\omega_4 d_{e2}}{n_4 c} A_6(x, y) A_3^*(x, y) e^{j \Delta k_2 z}, \quad (2.57)$$

$$N_6(x, y) = j \frac{\omega_6 d_{e2}}{n_6 c} A_3(x, y) A_4(x, y) e^{j \Delta k_2 z}, \quad (2.58)$$

where N_4 and N_6 are the nonlinear interaction expressions for the rotated signal and sum-frequency waves respectively. It can be seen from the interaction expressions that the coupling of the two processes are through the pump wave.

2.11 Phase Matched OPO and Second Harmonic Generation

When the second phase matched process is SHG, an OPO-SHG is formed. In Table 2.4 all possible types of BPM are presented. Different combinations of BPM for the OPO and SHG lead to six different possibilities. When the combinations are classified, according to the polarization rotation requirements, three different classes emerge [13].

	OPO Type I	OPO Type II	OPO Type III
SHG Type I	A	A	B
SHG Type II	C	C	C

Table 2.4: Possible classes of BPM for simultaneously phase matched OPO and SHG. Classes B and C need polarization rotation for the signal wave.

2.11.1 Class-A OPO-SHG

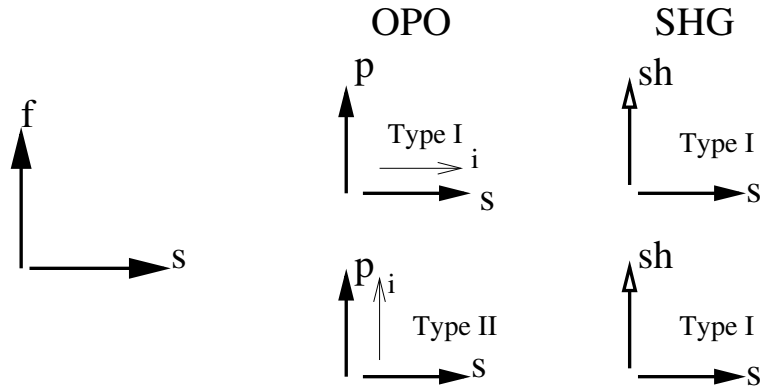


Figure 2.9: Class-A OPO-SHG BPM configurations.

Since in class-A OPO-SHG's the SHG process is a type-I process, and since the signal wave output of the OPA coincides with the slow axis of the crystal, no polarization rotation is needed. Coupling of the two processes inside the crystal is achieved by the signal wave. When the nonlinear interaction expressions for OPO and SHG are combined, due to the coupling, four equations can be found which define the process. These equations are

$$N_1(x, y) = j \frac{\omega_1 d_{e1}}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j \Delta k_1 z}, \quad (2.59)$$

$$N_2(x, y) = j \frac{\omega_2 d_{e1}}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j \Delta k_1 z} + j \frac{\omega_2 d_{e2}}{n_2 c} A_6(x, y) A_2^*(x, y) e^{j \Delta k_2 z}, \quad (2.60)$$

$$N_3(x, y) = j \frac{\omega_3 d_{e1}}{n_3 c} A_1(x, y) A_2(x, y) e^{j \Delta k_1 z}, \quad (2.61)$$

$$N_6(x, y) = j \frac{1}{2} \frac{\omega_6 d_{e2}}{n_6 c} A_2(x, y) A_2(x, y) e^{j \Delta k_2 z}, \quad (2.62)$$

where N_6 is the nonlinear interaction term for the second harmonic wave.

2.11.2 Class-B OPO-SHG

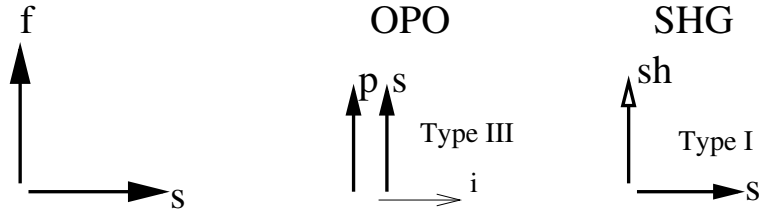


Figure 2.10: Class-B OPO-SHG BPM configurations.

Even though the SHG process is a type-I process, in class-B OPO-SHG's intra-cavity signal rotation is needed. In class-B OPO-SHG's there is no coupling between the OPO and SHG processes, so the five nonlinear interaction terms are the combination of the three for the OPO and two for the SHG process. The resultant nonlinear interaction terms are

$$N_1(x, y) = j \frac{\omega_1 d_{e1}}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j \Delta k_1 z}, \quad (2.63)$$

$$N_2(x, y) = j \frac{\omega_2 d_{e1}}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j \Delta k_1 z}, \quad (2.64)$$

$$N_3(x, y) = j \frac{\omega_3 d_{e1}}{n_3 c} A_1(x, y) A_2(x, y) e^{j \Delta k_1 z}, \quad (2.65)$$

$$N_5(x, y) = j \frac{\omega_5 d_{e2}}{n_5 c} A_6(x, y) A_5^*(x, y) e^{j \Delta k_2 z}, \quad (2.66)$$

$$N_6(x, y) = j \frac{1}{2} \frac{\omega_6 d_{e2}}{n_6 c} A_5(x, y) A_5(x, y) e^{j \Delta k_2 z}, \quad (2.67)$$

where N_5 and N_6 are the nonlinear interaction terms for the rotated signal and second harmonic waves respectively.

2.11.3 Class-C OPO-SHG

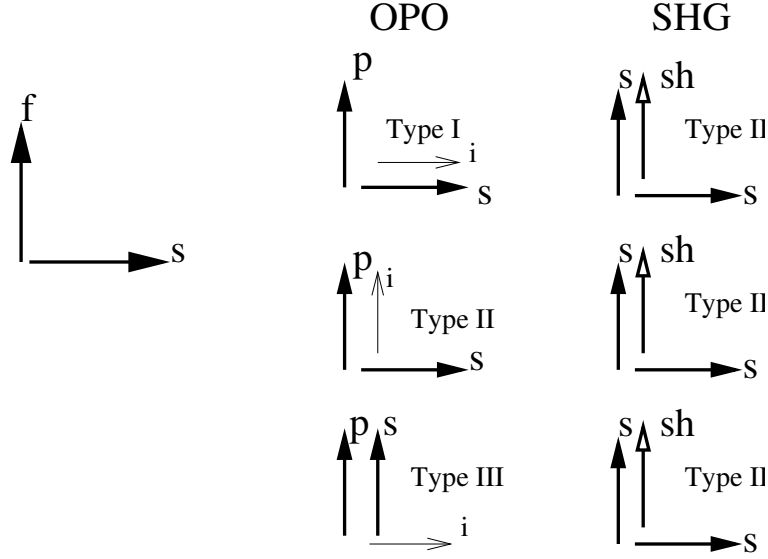


Figure 2.11: Class-C OPO-SHG BPM configurations.

In class-C OPO-SHG's, the SHG process is a type-II process, hence requiring the signal wave at both orthogonal polarizations. Intra-cavity polarization rotation is needed for the signal wave. In class-C type OPO-SHG's, the processes are coupled through one of the orthogonal signal waves. The equations governing the processes are the combination of the OPO and SHG processes, coupled through one of the signal waves.

The nonlinear interaction expressions can be written as

$$N_1(x, y) = j \frac{\omega_1 d_{e1}}{n_1 c} A_3(x, y) A_2^*(x, y) e^{j \Delta k_1 z}, \quad (2.68)$$

$$N_2(x, y) = j \frac{\omega_2 d_{e1}}{n_2 c} A_3(x, y) A_1^*(x, y) e^{j \Delta k_1 z} + j \frac{\omega_2 d_{e2}}{n_2 c} A_6(x, y) A_5^*(x, y) e^{j \Delta k_2 z}, \quad (2.69)$$

$$N_3(x, y) = j \frac{\omega_3 d_{e1}}{n_3 c} A_1(x, y) A_2(x, y) e^{j \Delta k_1 z}, \quad (2.70)$$

$$N_5(x, y) = j \frac{\omega_5 d_{e2}}{n_5 c} A_6(x, y) A_2^*(x, y) e^{j \Delta k_2 z}, \quad (2.71)$$

$$N_6(x, y) = j \frac{\omega_6 d_{e2}}{n_6 c} A_2(x, y) A_5(x, y) e^{j \Delta k_2 z}, \quad (2.72)$$

where N_5 and N_6 are the nonlinear interaction terms for the rotated signal and second harmonic waves.

Chapter 3

Solution for Second Order Processes

In this chapter, a numerical model for two simultaneously phase matched nonlinear optical processes is presented. In addition to the numerical model for the nonlinear interaction and diffraction of the waves inside the nonlinear crystal, solutions for diffraction due to air propagation and cavity components are given in detail. Finally, the calculations for some specific analysis parameters are given.

3.1 Motivation

An accurate model of real world practical OPO's is a valuable tool for design and optimization. As presented in Chapter 2, there are many classes of operation for single crystal upconversion processes. Constructing a cavity for each possible class of operation to determine the one with the maximum conversion efficiency is not practical. Moreover, for a given class of operation, it may not be possible to experiment with the design parameters for optimization. Using a numerical model, the optimum class of operation and design parameters can be calculated. For model to be accurate, the problem should be well defined and its parameters accurate.

One critical factor affecting the operation of an OPO is the duration of the pump pulse. OPO's have been reported for a great span of pump pulse durations. Nonlinear conversion in continuous wave OPO's have been reported [16], [17]. On the other end, OPO's working in the femtosecond regime has been reported [18] [19].

In continuous wave conversion systems, the main shortcoming is the availability of high intensity sources. Since the intensities which can be obtained from continuous wave sources are limited, the conversion efficiencies associated with the process are generally low. However, in short pulsed systems, very high intensity light wave pulses can be generated. The high intensity of these lasers, combined with highly nonlinear media can lead to effective nonlinear interactions. The main disadvantage in using very short pulses is the low amount of energy contained in one pulse of light. Since the amount of energy fed into the system is low, the output converted pulse energy is low, even with good conversion efficiencies. To increase the amount of energy converted while maintaining an acceptable efficiency, pulses with durations on the order of tens of nanoseconds can be used.

For the computational modeling of continuous wave OPO's, a steady-state model can be used. For very short pulses, methods dependent on finite-difference time domain (FDTD) and finite-difference frequency domain (FDFD) have been reported [20], [21]. In the nanosecond pulse regime, there are different computational methods for the calculation of nonlinear interactions [22]. Using a time-slice approach, Smith et. al. have designed a numerical model and reported results with good agreement to experimental results for an OPO [23], [14], [24].

In this thesis, a numerical model for single-crystal upconversion OPO's with pump pulse durations on the order of tens of nanoseconds is presented. In this time regime, the pump pulse duration is longer than the round trip time of the OPO cavity. When the pump pulse is incident on the cavity, the noise photons are amplified and oscillation starts. During oscillation, due to the pulse nature of the pump, the pump and intra-cavity signal intensities does not remain constant. To accurately model the operation of OPO's working in this time regime, a transient analysis is needed.

The beam transverse spatial profiles of the input and generated waves is also a critical factor in the conversion process. Pawel et. al. have conducted a study on the effect of the beam spatial profile on the nonlinear interaction [8]. If spatial filtering is not applied, the output beam profile of a laser is generally not gaussian or flat-top. To accurately model the nonlinear processes, the arbitrary spatial profiles of the pump and other interacting beams must be taken into account. In the model presented in following sections the beam profile of the waves can be given arbitrarily. When arbitrary beam profiles are used, due to diffraction, the beams diverge and spurious reflections occur from computational boundaries. The requirement on the arbitrary beam profiles is low divergence, so that the profiles remain inside computational boundaries.

3.2 Discretization in Time and Space Domain

To be able to represent real electric and polarization fields in a computer, their discretization both in the space and time domain is needed.

The optical electric field inside the cavity is discretized in time and space. The electric field is defined as

$$E_i[n, m, k] = E_i(n\Delta x, m\Delta y, k\Delta t), \quad (3.1)$$

where Δx , Δy are the grid feature sizes for the x and y directions, and Δt is the round trip time of the cavity. Here the x and y directions are in the transverse plane, ie. the plane orthogonal to the wave propagation direction. The index i represents the monochromatic wave with frequency ω_i .

For each frequency involved in the computations, only one transverse spatial profile is stored. The spatial beam profiles are stored for a square region generally 4.7 mm wide. The field inside the square region is mapped onto a grid with feature sizes Δx and Δy . The accuracy of the model depends on the number of grid points taken and the frequency of variations of the beam profile. For the calculations given in the following chapters, grid sizes were generally set to either 32x32 or 64x64.

The electric field is discretized in time with Δt equal to the round trip time of the cavity, assuming that the input and output of the cavity are changing slowly with respect to the round-trip time of the cavity. For short cavities (few centimeters) and input pulses in the order of tens of nanoseconds, this assumption is generally valid.

The discretization resolution of the transverse spatial beam profiles can be increased easily in the expense of computational time. However, in our current model, it is not possible to change the temporal resolution of the calculations. This restriction makes the numerical model suitable for systems which the input is slowly varying with respect to the round trip time of the cavity, and short cavities.

3.3 Propagation Through the Cavity

At the beginning of each time step, the input pump pulse spatial profile is multiplied with the according coefficient (which corresponds to a value in the time profile). The input pump fields are then summed up with intra-cavity fields. The fields are then propagated through the cavity components to find the output spatial profile and intensities of the waves.

In the first time step, the electric field for every frequency is assumed to be non-existent. The fields inside the cavity are assumed to be very low power noise waves. There are many studies on parametric fluorescence which try to model the generation of fields from noise [25], [26]. In this thesis, the fields which remain zero are set to be very low power beams with large beam radii. The power representing the noise fields is set to be four orders of magnitude smaller than the one photon/mode noise level used by Byer and Harris [27]. The power level representing the noise photons inside the cavity was obtained by decreasing the noise power level until the solutions converged to a solution.

For each time step, the spatial profiles of the waves (pump, signal, idler, rotated signal, rotated pump, sum-frequency, second-harmonic) are propagated

through the cavity components. After the spatial profiles propagate through the cavity once, the fields inside the cavity are summed up with the input fields. These spatial profiles are recorded for later use. In Fig 3.8, the flowchart of computation for a specific cavity topology (Figure 3.3) can be seen.

In the following sections, the solutions for different types of cavity components will be described in detail.

3.3.1 Propagation Through Air

All propagation through air is simulated by Fourier transforming the spatial profiles, multiplying by the transfer function of free space and taking the inverse Fourier transform [12]. The numerical steps which the field transverse spatial profiles go through can be written as

$$A_j(x, y) \xrightarrow{\mathcal{F}} a_j(\nu_x, \nu_y), \quad (3.2)$$

$$a'_j(\nu_x, \nu_y) = a_j(\nu_x, \nu_y) \cdot \exp \left[-i2\pi \left(\frac{1}{\lambda^2} - \nu_x^2 - \nu_y^2 \right)^{\frac{1}{2}} d \right], \quad (3.3)$$

$$a'_j(\nu_x, \nu_y) \xrightarrow{\mathcal{F}^{-1}} A'_j(x, y), \quad (3.4)$$

where $A'_j(x, y)$ is the spatial profile of the propagated wave. Since there are no absorbing boundary conditions, the spatial beam profile should be small at the edges of the computation domain. If the beam profile exceeds the computation domain, spurious reflections will occur. Absorbing boundary conditions may be placed at the edge of the computation domain to decrease the computation time, trading off with complexity.

3.3.2 Reflection From and Transmission Through Mirrors

Using the mirror reflection, transmission and absorption coefficients, the transmitted and reflected spatial profiles and fields are calculated. Moreover, in the input mirror, the fields entering the cavity and the fields inside the cavity are summed. The transmitted and reflected fields can be written as

$$A_{i(transmitted)}(x, y) = \sqrt{T(\omega_i)} \cdot A_{i(incident)}(x, y) \quad (3.5)$$

$$A_{i(reflected)}(x, y) = \sqrt{R(\omega_i)} \cdot A_{i(incident)}(x, y) \quad (3.6)$$

where $T(\omega_i)$ and $R(\omega_i)$ are the transmittance and reflectance of the mirror for the frequency ω_i .

3.3.3 Polarization Rotation

In the numerical model, and the differential equations, orthogonal polarizations of a wave are considered as separate waves with the same wavelength λ . Polarization rotation of a wave is accomplished by coupling these separate waves depending on the rotation angle.

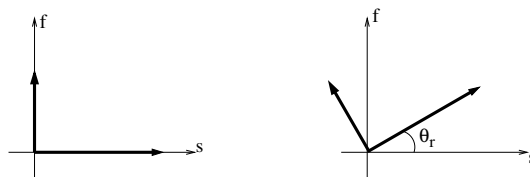


Figure 3.1: Polarization rotation by α_r degrees.

The coupling between the two orthogonal waves is calculated by

$$\begin{bmatrix} A'_f(x, y) \\ A'_s(x, y) \end{bmatrix} = \begin{bmatrix} \cos(\alpha_r) & \sin(\alpha_r) \\ -\sin(\alpha_r) & \cos(\alpha_r) \end{bmatrix} \begin{bmatrix} A_f(x, y) \\ A_s(x, y) \end{bmatrix}, \quad (3.7)$$

where A'_f and A'_s are the fields A_f and A_s , rotated by α_r degrees.

3.3.4 Propagation Through the Nonlinear Crystal

The differential equations governing the nonlinear interaction and diffraction inside the crystal are given in Chapter 2. Using the fields at the input of the crystal the differential equations are integrated using Runge-Kutta Cash-Carp method, updating the polarization fields at every integration step [23].

The differential equations governing the diffraction and second order nonlinear processes can be generalized as

$$\frac{\partial A_i(x, y)}{\partial z} = \frac{j}{2k_i} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] A_i(x, y) + N_i(x, y) \quad (3.8)$$

where N_i is the nonlinear interaction expression, dependent on the polarization density. Substituting the Fourier representation of the electric and polarization fields,

$$A_i(x, y) = \int a_i(f_x, f_y) e^{j2\pi(f_x x + f_y y)} df_x df_y \quad (3.9)$$

and

$$N_i(x, y) = \int n_i(f_x, f_y) e^{j2\pi(f_x x + f_y y)} df_x df_y \quad (3.10)$$

into Equation (3.8),

$$\frac{\partial a_i(f_x, f_y)}{\partial z} = -j \left[\frac{2\pi^2}{k_i} (f_x^2 + f_y^2) \right] a_i(f_x, f_y) + n_i(f_x, f_y) \quad (3.11)$$

is obtained. Equation (3.11) is the differential equation which will be used to calculate the change in the field as it propagates through the crystal.

At the input of the crystal, using the electrical fields, the nonlinear interaction terms $N_i(x, y)$'s are calculated, using the appropriate expression sets defined in Chapter 2. The nonlinear interaction terms and electric field amplitudes are then Fourier transformed to obtain $n_i(f_x, f_y)$'s and $a_i(f_x, f_y)$'s.

Integrating ordinary differential equations using numerical techniques is a well studied subject. Let's assume that we have a differential equation in the form

$$\frac{d}{dx} y(x) = f(x, y).$$

If we are given an initial condition ($y_{initial} = y(x_{initial})$), it is possible to find the value of our function at $x = x_{initial} + \Delta x$ using the Euler method formula

$$y(x_{initial} + \Delta x) = y_{initial} + f(x_{initial}, y_{initial}) \Delta x. \quad (3.12)$$

However, the Euler method is not computationally efficient, nor stable enough for scientific calculations [28]. In our computations Cash-Karp Runge-Kutta method (an improved version of the Euler method) is used to propagate the profiles inside the crystal.

After the fields are propagated a small step in the crystal, by the Cash-Karp Runge-Kutta method [28], $a_i(f_x, f_y)$'s are inverse Fourier transformed to obtain the electric fields to calculate the new $N_i(x, y)$'s. This process is iterated until the end of the crystal is reached. A flow chart of the computations done inside the crystal can be seen in Figure 3.2.

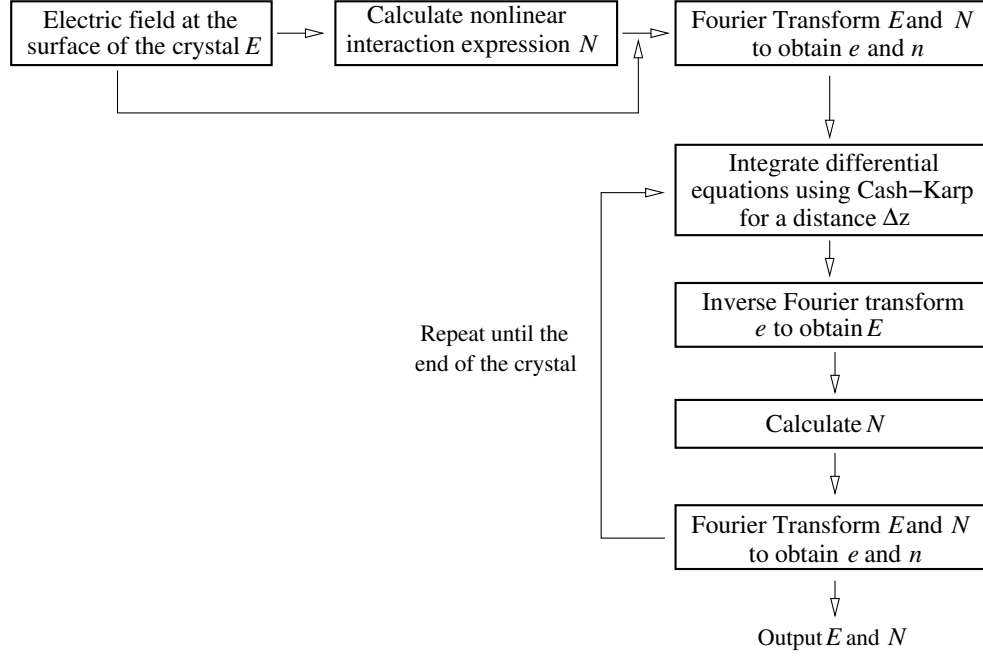


Figure 3.2: Flow chart of processing inside the crystal.

3.4 Results

All computational results given in the following chapters will be results for the singly resonant cavity topology in Figure 3.3. The thin plates designated as WP1 and WP2 are the wave-plates used for polarization rotation of the pump and signal waves respectively. The mirrors are designated with the letter M. The input mirror M1 is transparent to the pump wave but highly reflective to the signal. The mirrors M2 and M3 are only highly reflective to the signal wave.

In the following sections, computational results of a simultaneously phase

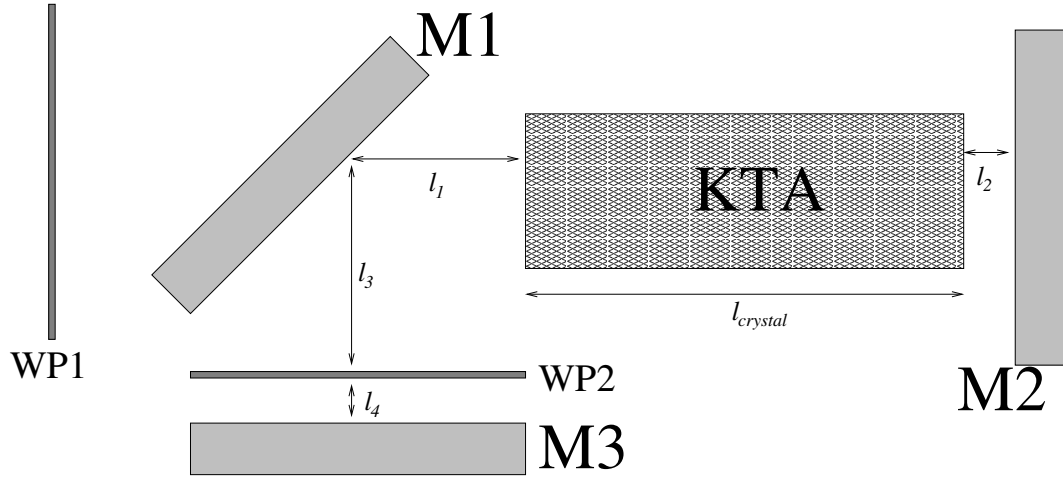


Figure 3.3: Cavity topology.

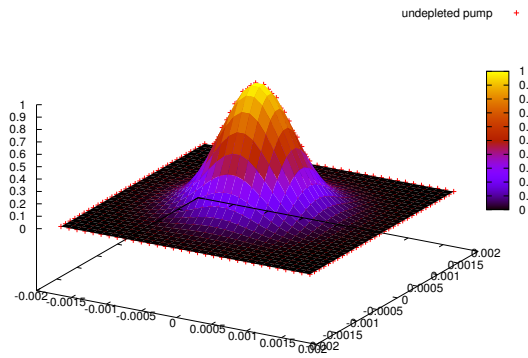


Figure 3.4: Input gaussian beam profile, with beam radius 1.8 mm.

matched class-D OPO-SFG will be given and compared with computational results. In the experiment performed, the nonlinear crystal is a KTA crystal, so we will be performing the calculations for the KTA crystal. However, any other nonlinear crystal which exhibits second order nonlinearity can be used. More detailed information about the cavity components can be seen in Table 4.1. The beam profile used for the pump pulse is a spherically symmetric Gaussian beam with radius 1.8 mm, as seen in Figure 3.4.

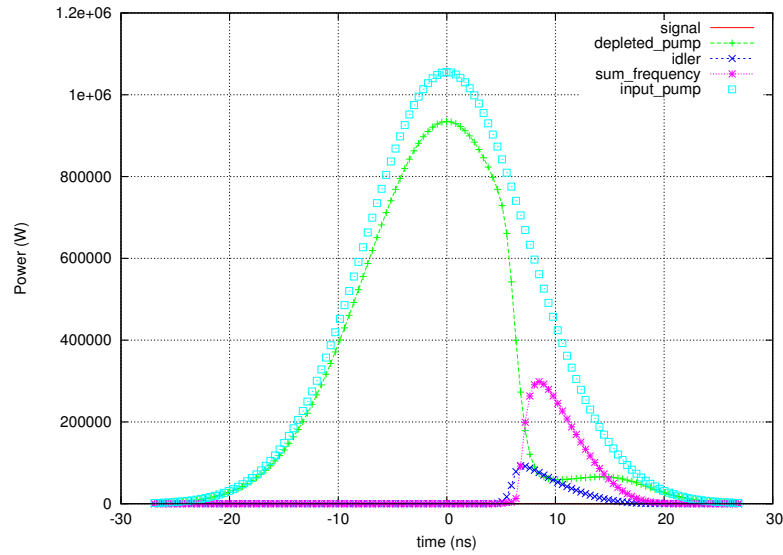


Figure 3.5: Power time profiles of the input pump, depleted pump, idler and sum-frequency. $E_{pump} = 20$ mJ, polarization rotation angle = 30 degrees.

3.4.1 Time Profiles

To be able to gain intuition about the processes occurring inside the cavity, the evolution of the fields inside the cavity is invaluable to us. By recording the intensity values for each round trip time it is possible to obtain the time profiles for the different fields. The power associated with a spatial profile for a given time step can be calculated as

$$P_i[k] = \sum_{x=1}^N \sum_{y=1}^N \Delta x \Delta y \frac{|A_i[x, y, k]|^2}{2\eta}, \quad (3.13)$$

where k is the time step index. The total energy of a pulse, using the power definition in Equation (3.13) can be written as

$$Energy_i = \sum_{k=0}^K \Delta t \cdot P_i[k], \quad (3.14)$$

where K is the total number of time steps computed. The time power profiles of different waves for a single pulse can be seen in Figure 3.5.

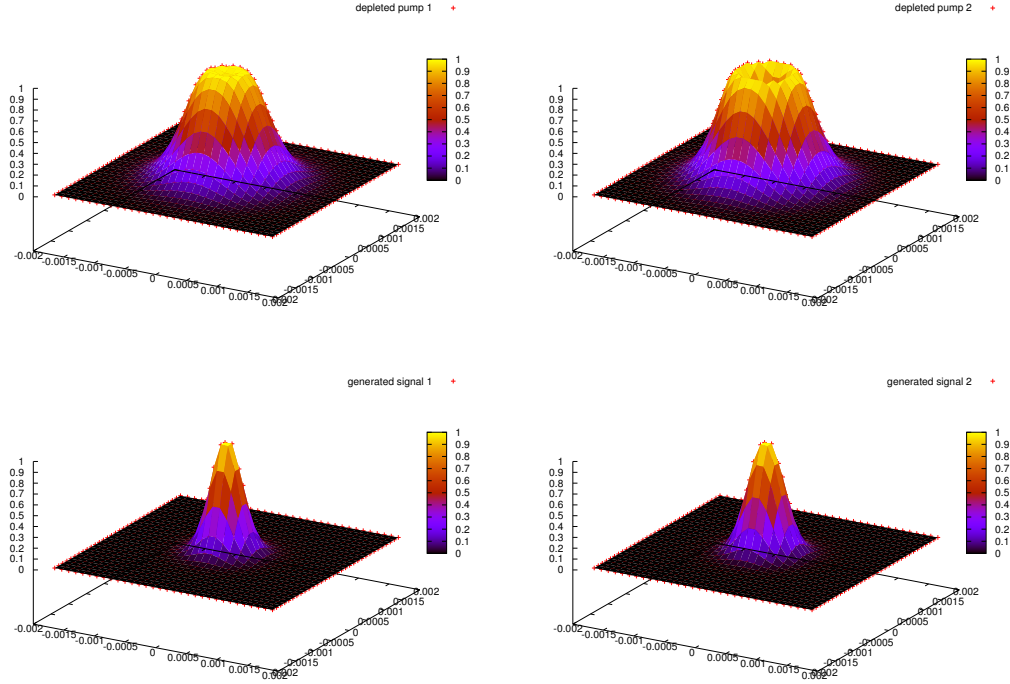


Figure 3.6: The depleted pump and generated signal wave spatial profiles for two time instances. $E_{in} = 20$ mJ, polarization rotation = 30 degrees.

3.4.2 Spatial Profiles

The conversion efficiency and beam shape of the generated signal is dependent on the input pump spatial profile, so the spatial profile results is also of great importance. Besides the field total intensities, spatial intensities of all the different waves are recorded. In Figure 3.6 the output spatial profiles of the pump and sum-frequency waves for two time steps where the conversion can be clearly seen are given.

The input pump beam profiles used in these calculations are gaussian beams, however any beam profile with low divergence can be used.

3.4.3 Maximum Efficiency

The conversion efficiency calculated as

$$\eta = \frac{Energy_{sum_frequency}}{Energy_{pump}} \quad (3.15)$$

is dependent on the amount of intra-cavity polarization rotation. By sweeping the rotation angle, it is possible to find the maximum efficiency and the optimum angle for a given input pulse energy. The conversion efficiency vs polarization rotation degree can be seen in Figure 3.7.

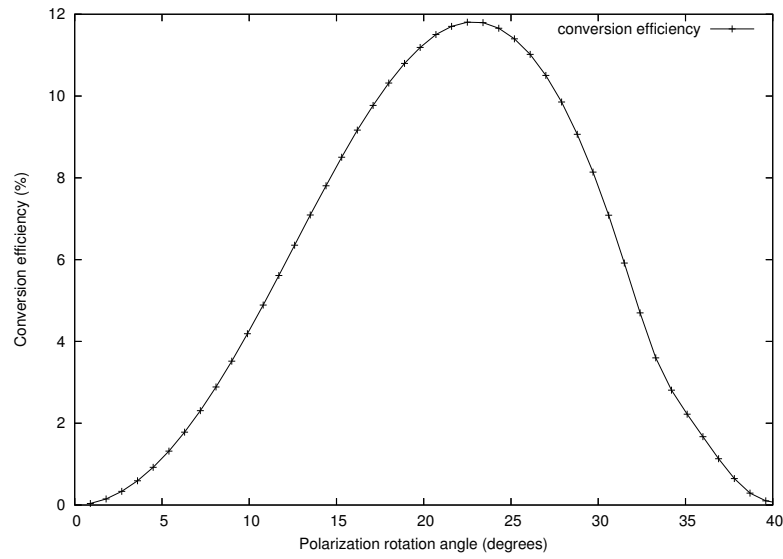


Figure 3.7: Conversion efficiency vs. polarization rotation angle. $E_{in} = 20$ mJ with gaussian beam profile (beam radius 1.8 mm).

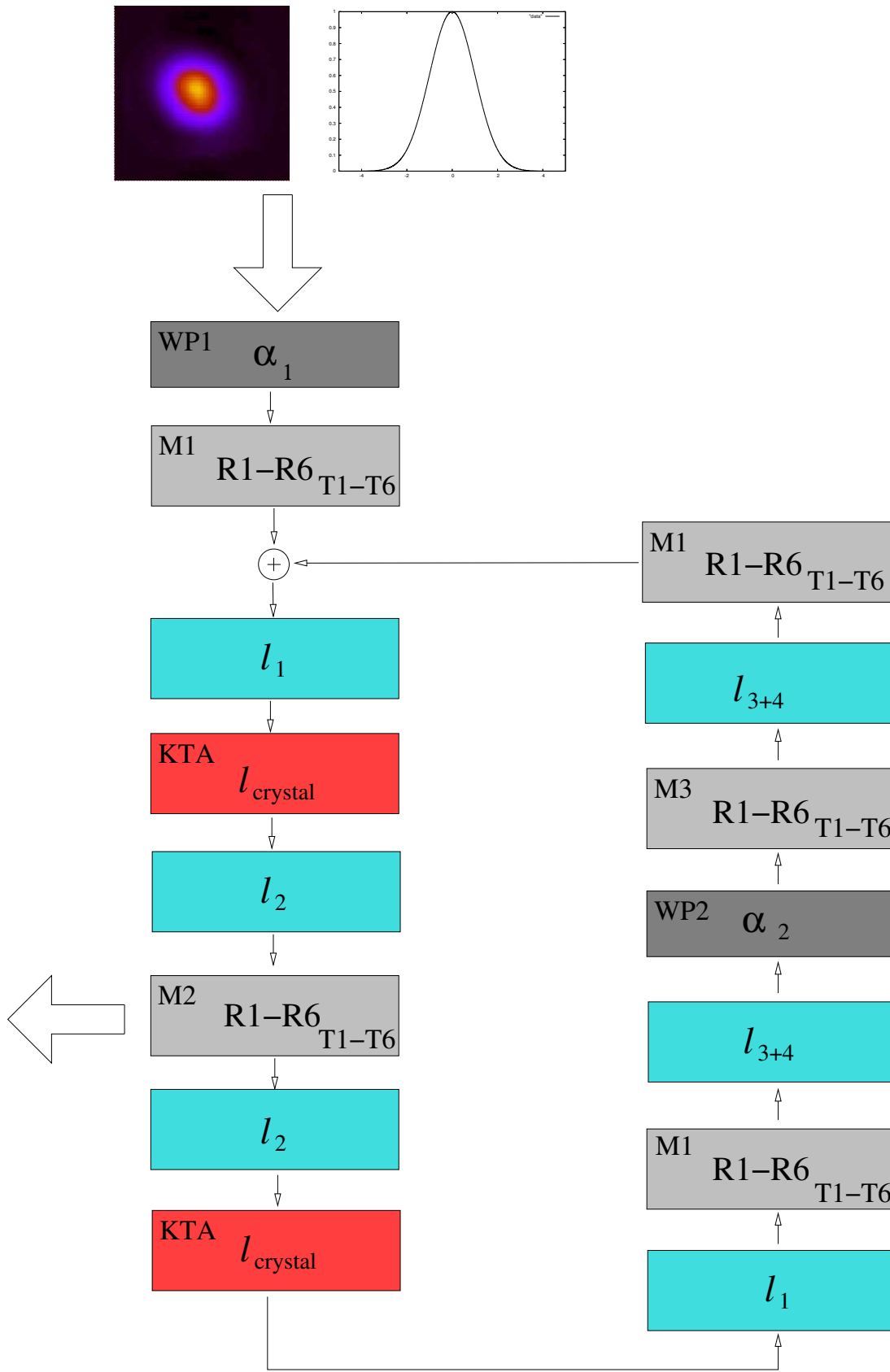


Figure 3.8: Flow chart of processing through the cavity (topology in Figure 3.3).

Chapter 4

Comparison With Experimental Measurements

To reassure the validity of our calculations, a comparison of numerical results with the measurements from an actual experiment is imperative. In this chapter we will focus on the comparison of measurements obtained from an experiment performed by Gürkan Figen in the Photonics Research Laboratory in Bilkent University with numerical computations.

Measurements will be given and compared with calculations for two different experiments. In the first experiment, the pump pulse passes through the crystal only once, which is defined as single pass operation. In the second experiment, the pump pulse is reflected back into the cavity to increase the conversion efficiency. This mode of operation is defined as double pass operation, regarding the multiple incidences of the pump pulse on the nonlinear crystal.

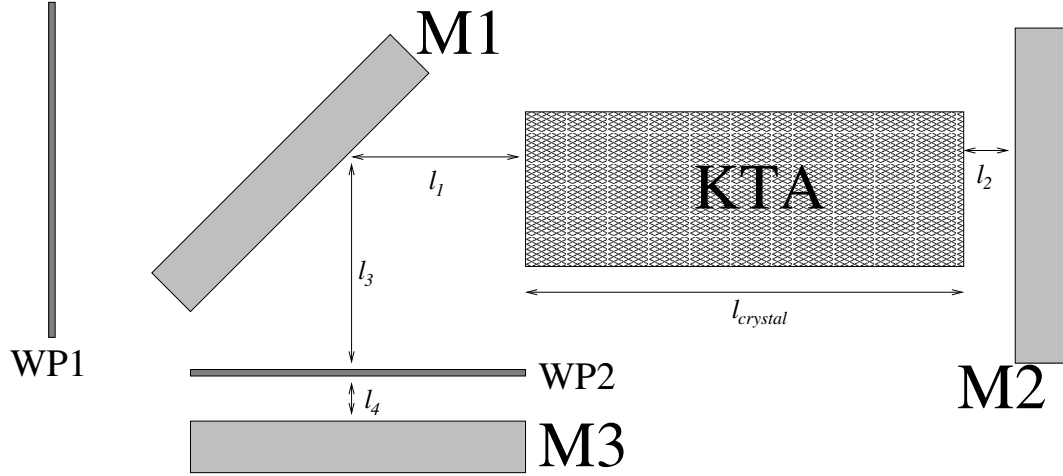


Figure 4.1: Cavity topology.

4.1 Single Pass Operation

4.1.1 The Experiment

The cavity in which the crystal was placed was constructed according to the cavity topology given in Figure 4.1. The crystal is placed such that the OPO and SFG processes are phase matched as a class-D OPO-SFG. Since no extra-cavity polarization rotation of the pump wave is needed in class-D OPO-SFG's, the polarization rotation due to WP1 is set to be zero.

The crystal used in the experiment is a 20 mm long KTA crystal. KTA is commonly used because of its high damage threshold and nonlinear coefficient d_e . The surfaces of the cavity are anti-reflection coated for the signal and pump waves, with a reflection of 0.06% for the signal wave. The coating is not effective at the sum-frequency wavelength λ_6 , which leads to a reflection around 7.6% for the sum-frequency wave.

The three mirrors M1, M2 and M3 are highly reflective to the signal wave at λ_2 . The reflectivity of the mirrors to the pump idler and sum-frequency waves are low. It is assumed that no absorption is present in the output coupler. The

output coupler reflectivity (and the reflectivity of the other cavity mirrors) for the signal wave can be seen in Table 4.1.

The polarization rotation device WP2 used is a single-order quarter wave plate for the signal wavelength λ_2 . Due to the cavity topology, the wave passes through the waveplate twice, hence the polarization rotation angle is twice the angle of rotation of the waveplate. The waveplate is assumed to introduce a total 1% loss to the waves inside the crystal. The useless loss of the cavity is set to be 2.5% for the signal wave, which conforms to experimental measurements.

Detailed information about cavity components is tabulated in Table 4.1. The parameter N represents the size of the computational grid, which is generally 64 (for low error and moderate computation time).

4.1.2 Time Profiles

In this section, the temporal power measurements of the signal, depleted pump and sum-frequency waves will be compared to numerical computations. In Figures 4.2-4.5 the undepleted pump, depleted pump, intra-cavity signal and sum-frequency temporal power comparisons are given.

4.1.3 Spatial Profiles

An important factor in the conversion process is the transverse spatial profile of the input and generated waves. In Figure 4.6 the input spatial profile of the pump pulse can be seen. It is assumed that the pump spatial profile shape does not change for the duration of the pulse. The spatial profiles are captured using a Cohu 6410 CCD camera, which captures the intensity in a square of 4.7 mm by 4.7 mm.

The spatial profile of the pump pulse is critical in finding the depleted pump and sum-frequency spatial profiles. The capability of being able to calculate the spatial profile for every time step is invaluable in giving an intuition about the

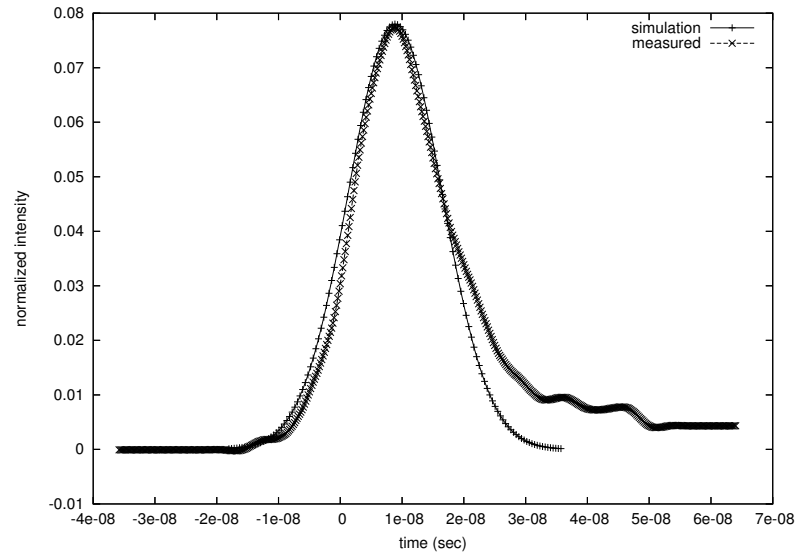


Figure 4.2: Undepleted pump time profile comparison. $E_{in} = 30.7$ mJ with the optimum polarization rotation angle.

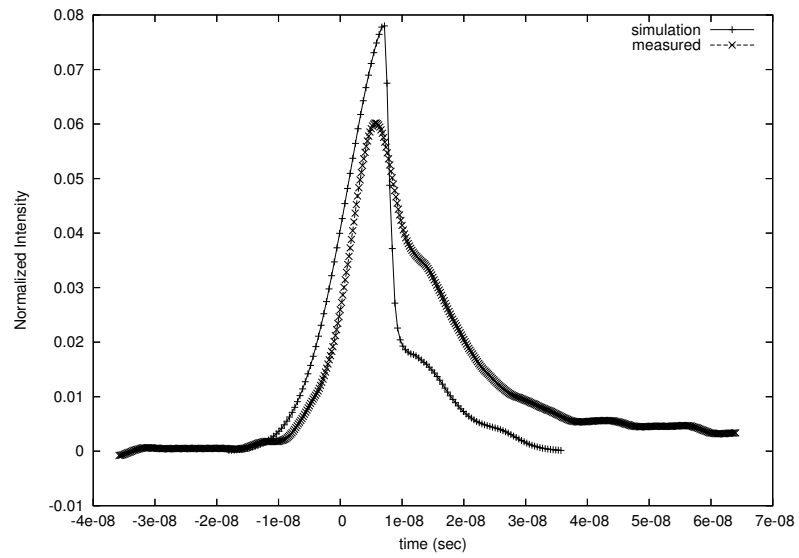


Figure 4.3: Depleted pump time profile comparison. $E_{in} = 30.7$ mJ with the optimum polarization rotation angle.

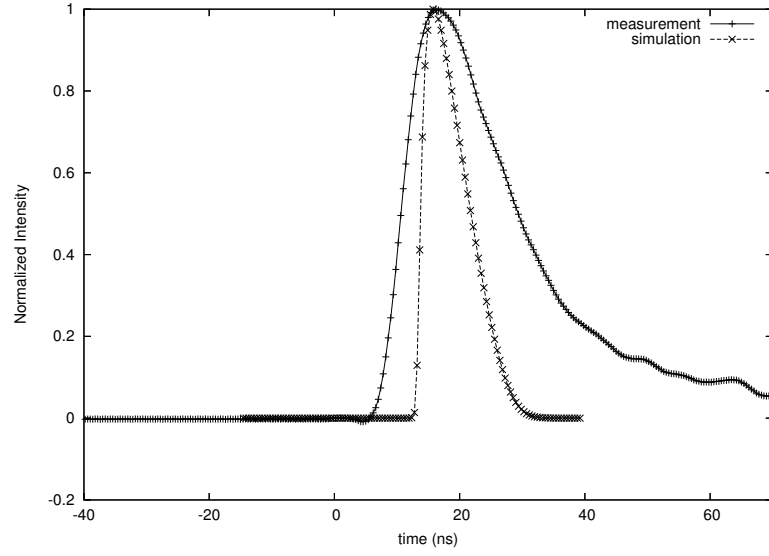


Figure 4.4: Signal wave time profile. $E_{in} = 30.7$ mJ with the optimum polarization rotation angle.

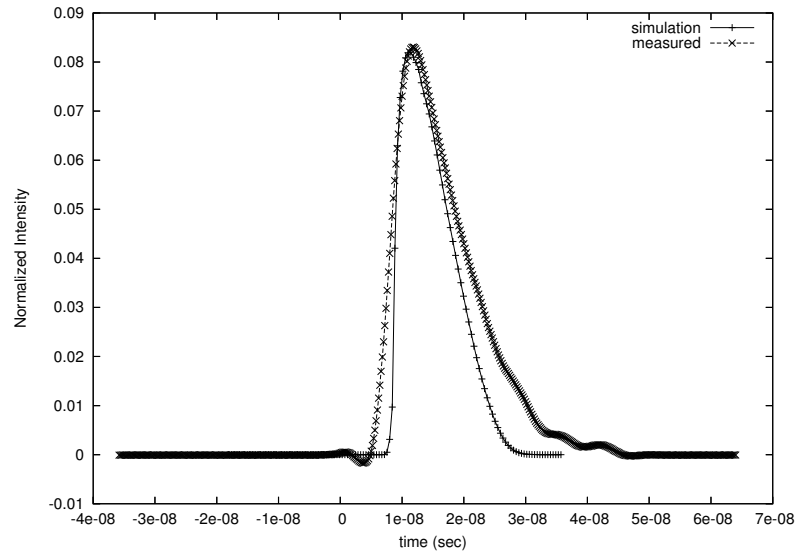


Figure 4.5: Sum-frequency wave time profile. $E_{in} = 30.7$ mJ with the optimum polarization rotation angle.

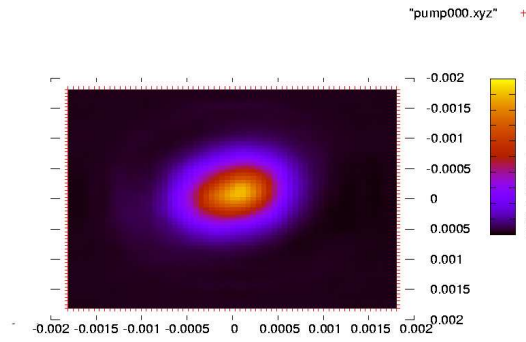


Figure 4.6: Input pump pulse spatial profile.

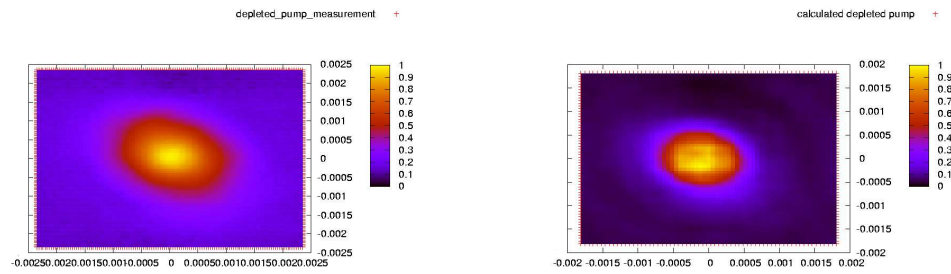


Figure 4.7: Measured and computed depleted pump transverse spatial profiles.

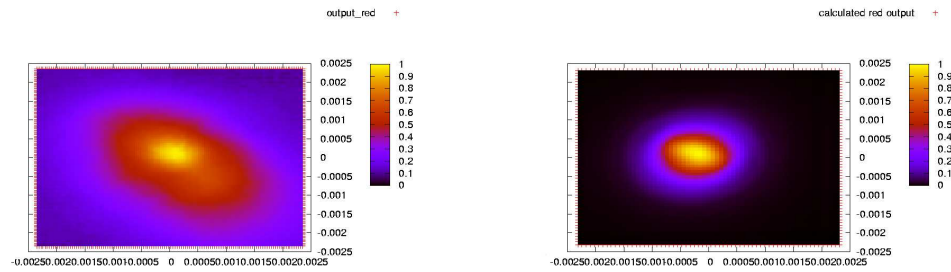


Figure 4.8: Measured and computed sum-frequency output transverse spatial profiles.

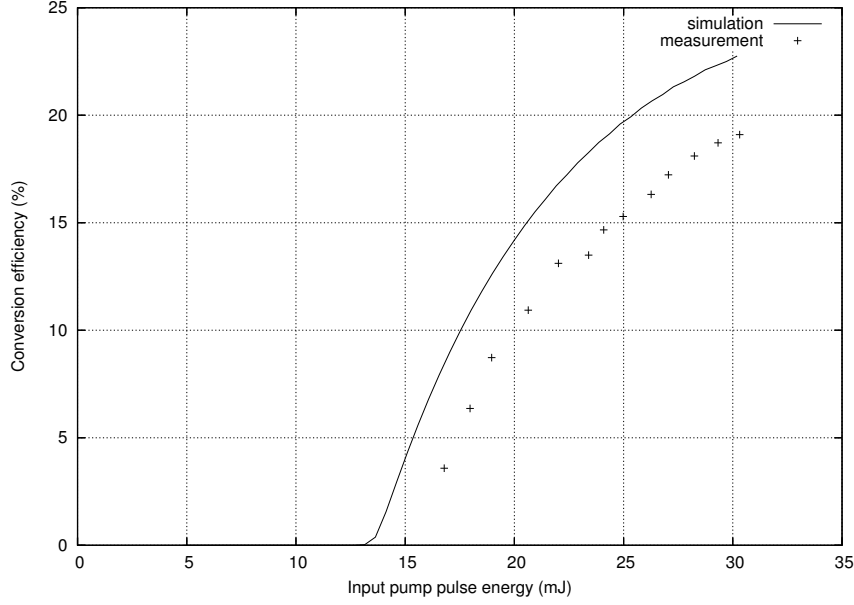


Figure 4.9: Conversion efficiency as a function of the input pump pulse energy. The polarization rotation angle is kept constant at 30.56 degrees (The optimum angle for $E_{in} = 30.7$ mJ).

evolution of the process. The depleted pump and sum-frequency wave profiles (measured and calculated) can be seen in Figures 4.7 and 4.8.

4.1.4 Conversion Efficiency vs. Input Power

The conversion efficiency defined as

$$\eta = \frac{E_{sum-frequency}}{E_{pump}}, \quad (4.1)$$

depends on the input pump pulse energy incident on the OPO cavity, where $E_{sum-frequency}$ and E_{pump} are the energies of the output sum-frequency and input waves respectively. In Figure 4.9 the calculated and measured conversion efficiencies with respect to the input pump pulse energy can be seen. The polarization rotation is kept constant (at which conversion is maximum for $E_{pump} = 30.1$ mJ) for every input.

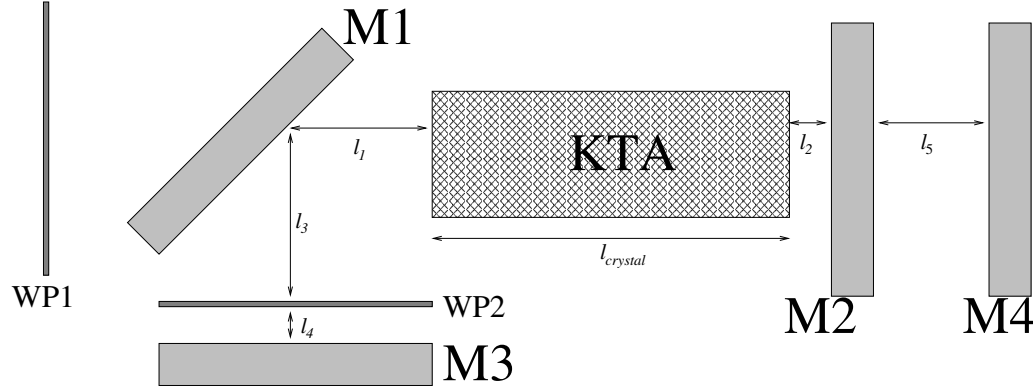


Figure 4.10: Cavity topology for double pass operation.

4.2 Double Pass Operation

In this section, calculations regarding the double pass system will be presented. In this cavity topology, the pump pulse is reflected back into the cavity using a YAG mirror, for a second pass through the cavity in the opposite direction. The YAG mirror used to reflect the pump wave back into the cavity has a high reflectivity to the pump wave, and a high transmission to the sum-frequency wave. The cavity topology for the double pass of operation can be seen in Figure 4.10. Detailed information on the YAG mirror can be seen in Table 4.2.

The main purpose of dual pass operation is to utilize the unused pump at the output of the cavity to increase conversion. For the opposite direction of propagation inside the cavity, only the OPA process is phase matched.

4.2.1 Temporal Profiles

The undepleted and depleted time power profiles can be seen in Figures 4.11-4.14. The input pump pulse energy is set to be $E_{in} = 20.7mJ$, with a polarization rotation angle of 29.86 (maximum convergence). Due to experimental limitations, it was not possible to measure the depleted pump profiles. However, the computational results for the depleted pump wave can be seen in 4.12.

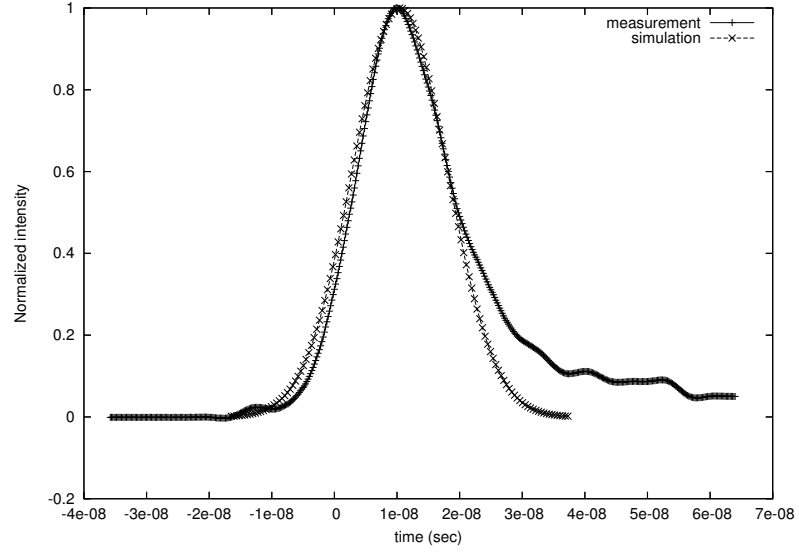


Figure 4.11: Undepleted pump time profile comparison.

4.2.2 Transverse Spatial Profiles

Due to the mirror at the output of the system, reflecting the depleted pump back into the crystal, it was not possible to measure the transverse spatial profile of the depleted pump. In Figure 4.15 the measured and calculated transverse spatial profiles for the sum-frequency wave can be seen.

4.2.3 Conversion Efficiency vs. Angle

For the double pass operation, the conversion efficiency vs polarization rotation angle can be seen in Figure 4.16 with comparison to the experimental results. It should be noted that the conversion efficiencies are higher than that of the single pass operation ones.

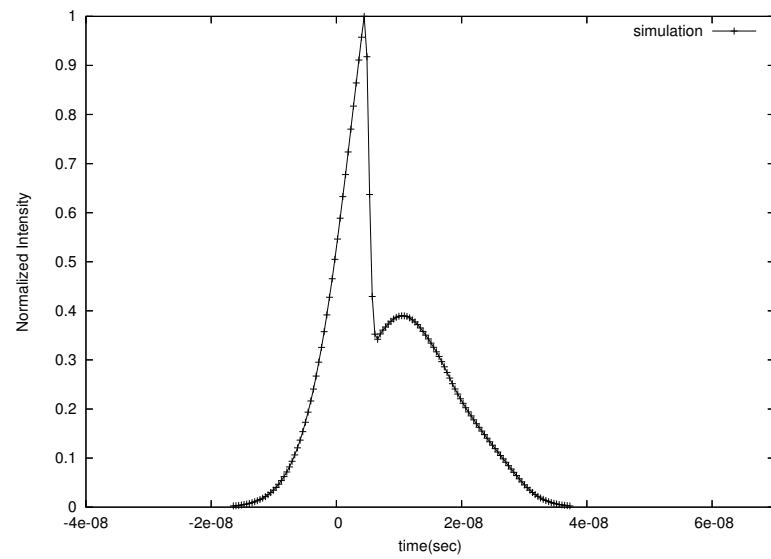


Figure 4.12: Depleted pump time profile.

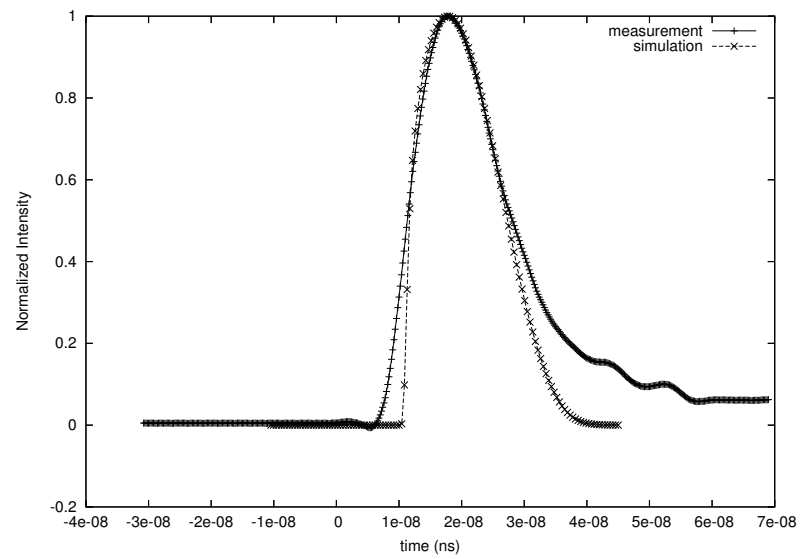


Figure 4.13: Signal wave time profile comparison.

Property	Value	
λ_1	3519.7 nm	Wavelength of the idler wave
λ_1	1064 nm	Wavelength of the signal wave
λ_1	1525 nm	Wavelength of the pump wave
λ_1	626.7 nm	Wavelength of the sum-frequency wave
n_1	1.785741	Refractive index of the crystal to the idler wave
n_1	1.775587	Refractive index of the crystal to the signal wave
n_1	1.8091567	Refractive index of the crystal to the pump wave
n_5	1.854626	Refractive index of the crystal to the rotated signal wave
n_6	1.813968	Refractive index of the crystal to the sum-frequency wave
$R_1(1525nm)$	0.99501	Reflectivity of M1 to 1525nm
$\alpha_1(1064nm)$	0.02	Absorption due to transmission through M1
d_{e_OPO}	2.9 pm/V	Effective nonlinear coefficient for the OPO process
d_{e_SFG}	3.1 pm/V	Effective nonlinear coefficient for the SFG process
$R_2(1064nm)$	0.03	Reflectivity of M2 to 1064nm
$R_2(1525nm)$	0.9982123	
$R_2(3550nm)$	0.01	
$R_2(628nm)$	0.03	
$R_3(1525nm)$	0.997729	Reflectivity of M3 to 1525nm
l_1	1 mm	
l_{cavity}	20 mm	
l_2	5 mm	
l_3	17 mm	
l_4	5 mm	
Δt	420 ps	Round trip time of the cavity
ΔX	4.7 mm	Computation domain length in X direction
ΔY	4.7 mm	Computation domain length in Y direction
Δx	$\frac{\Delta X}{N}$	Grid feature size in X direction
Δy	$\frac{\Delta Y}{N}$	Grid feature size in Y direction
t_p	17.8 ns	
Δk_1	0	Phase mismatch for the OPO process
Δk_2	0	Phase mismatch for the SFG process

Table 4.1: Variables used in the model.

Property	Value	
l_5	12 mm	Distance of the YAG mirror to the output coupler
$R_{double_pass}(1064nm)$	0.99	Reflectivity of YAG mirror to 1064nm
$T_{double_pass}(628nm)$	0.97	Transmission of mirror to 628nm

Table 4.2: Properties of the YAG mirror reflecting the depleted pump back into the cavity.

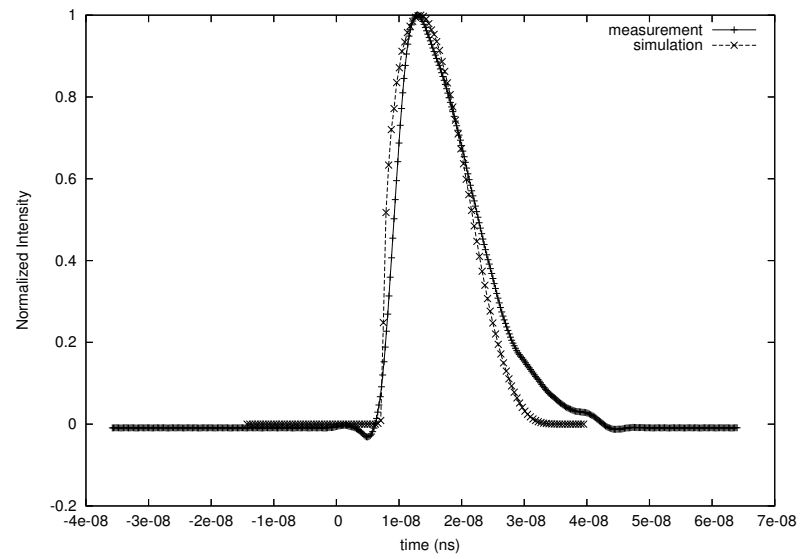


Figure 4.14: Sum-frequency wave time profile comparison.

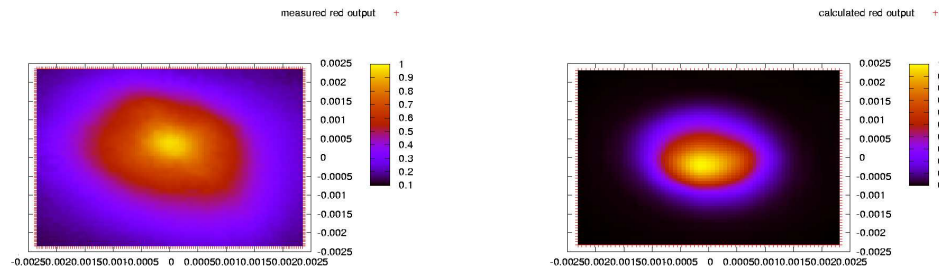


Figure 4.15: Measured and calculated transverse spatial profiles for the sum-frequency wave.

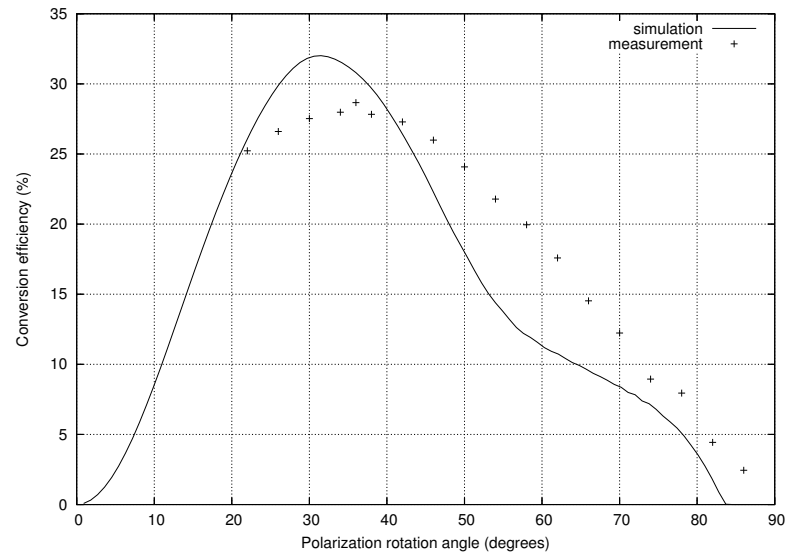


Figure 4.16: Conversion efficiency vs polarization rotation angle for $E_{in} = 20.7\text{mJ}$.

Chapter 5

Conclusion

Simultaneous phase matching of two second order nonlinear processes is a method for upconverting the frequency of a laser beam. It gives us the ability to convert the frequency of light from readily available powerful sources to desired frequencies.

The operation of a single crystal up-conversion system involves many parameters such as the cavity topology, nonlinear crystal properties, mirror reflectivities and transverse spatial profiles. It is of great importance to be able to analyze the effects of changing these parameters using a numerical model which incorporates all of them.

In this thesis, a numerical model, an analysis tool for computationally calculating simultaneously phase matched OPO-SFG/SHG processes was presented. The numeric model results were compared to experimental measurements, and were found to be in good agreement with each other.

In the numerical solutions presented in this thesis, the fields were discretized with a time step equal to the round trip time of the cavity. By setting the time step equal to the round trip time, we assume that the input and output of the cavity are changing slowly with respect to the cavity round trip time. Due to this assumption, the solution is applicable to devices with long input pulses and

short cavities. For the solution to be accurate for devices with longer cavities or shorter pulses, it should be able to incorporate a smaller time step size (Δt).

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