

ELECTROMAGNETIC IMAGING OF THREE-DIMENSIONAL CONDUCTING OBJECTS USING THE NEWTON MINIMIZATION APPROACH

A THESIS

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By

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August 2013

I certify that I have read this thesis and that in my opinion it is fully adequate,
in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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M.S. in Electrical and Electronics Engineering

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The main goal of shape reconstruction is to retrieve the location and shape of an unknown target. This approach is used in a wide range of areas, from detecting cancer tumors to finding buried objects. Various methods can be applied to detect objects in different applications. One of the important challenges in many of these methods is to solve the non-linearity and non-uniqueness of the solutions.

Inverse scattering is one of the most efficient ways to retrieve shapes and locations of targets. By illuminating the objects with electromagnetic waves and collecting the scattering fields using appropriate methods, we try to obtain the shape of unknown object. To achieve this goal, we start with an initial guess of the unknown object, then by comparing the scattered far-field patterns of the guess and the real object, we evolve that object and update it iteratively such that we decrease the difference between the patterns and finally achieve the shape of the unknown object.

In this thesis, we model the object by one of its parameters, such as the location of the nodes on the surface of the object, or by the conductivity, permittivity, and permeability of the discretized space in which the object is placed. Then, the model parameters are updated iteratively by minimizing the mismatch between the measured data of the target and the collected data from the modeled object. Using surface nodes to model a three-dimensional object is a good choice because we decrease the number of unknowns.

Keywords: Inverse scattering, shape reconstruction, iterative solution, newton minimization approach.

ÖZET

NEWTON ENKÜÇÜLTME YAKLAŞIMI KULLANARAK ÜÇ BOYUTLU İLETKEN CİSİMLERİN ELEKTROMANYETİK GÖRÜNTÜLENMESİ

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Elektromanyetik görüntülemenin amacı, bilinmeyen bir cismin yerini ve şeklini bulmaktır. Bu yaklaşım, kanser tümörlerini saptamaktan, gömülü cisimlerin bulunmasına kadar çeşitli alanlarda kullanılmaktadır. Çeşitli uygulamalarda cisimlerin şeklinin belirlenmesi için farklı yöntemler kullanılabilir. Bu yöntemlerdeki başlıca zorluklar, tek bir çözümün olmaması ve problemin doğrusal olmamasıdır.

Cisimlerin yerinin ve şeklinin belirlenmesi için kullanılan yollardan en etkili ters saçılımıdır. Cisim elektromanyetik dalgalarla aydınlatılır, saçılan dalgalar toplanır ve saçılan dalgalar kullanılarak cismin şekli tahmin edilmeye çalışılır. Bunun için, ilk olarak cisim için tahmini bir şekil atanır, sonra cismin ve tahmini şeklin uzak-alan saçılımları karşılaştırılır, tahmini şekil cismin saçılımına daha yakın saçılım verecek şekilde güncellenir ve iteratif bir şekilde tahmini şekil cismin şekline yaklaştırılır.

Bu tezde, cisim içinde bulunduğu ayrıklaştırılmış uzayın herhangi bir özelliğini (yüzey noktaları, iletkenlik, geçirgenlik vb.) kullanarak modellenmiştir. Daha sonra, modellenen cismin seçilen özelliği iteratif bir şekilde değiştirilerek, ölçüm yapılan cisimden toplanan veriyle yaklaşık veriler alınması sağlanmıştır. Üç boyutlu bir cismi yüzey noktalarıyla modellemek bilinmeyen sayısını düşürdüğü için uygun bir yöntemdir.

Anahtar sözcükler: Elektromanyetik görüntüleme, elektromanyetik ters problemler, Newton enküçültme yaklaşımı.

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Contents

1	Introduction	1
1.1	Motivations	3
1.2	Historical Background	4
1.3	Contributions	4
1.4	Simulation Environment and Computational Resources	5
2	Cost Function and Its Role in the Minimization Process	8
2.1	Cost Function	9
2.2	Taylor-Series Expansion of the Cost Function	11
3	Minimization of the Cost Function	15
3.1	The Newton Minimization Approach	16
3.1.1	The Possible Solutions of the Newton Minimization Approach	17
3.1.2	The Gradient Vector of the Cost Function	17
3.1.3	The Hessian Matrix of the Cost Function	19
3.2	The Steepest-Descent Method	20

3.3	Numerical Calculation of the Jacobian Matrix	22
4	Numerical Results	26
4.1	Reconstruction of a Conducting Ellipsoid	27
4.2	Comparison of Two Unknown Representations in the Reconstruc- tion Process	30
4.3	Reconstruction of a Conducting Sphere	34
4.4	Reconstruction of the Objects with Complex Geometries	37
4.4.1	A Star-Shaped Target	39
4.4.2	A Concave Target	43
4.5	Reconstruction of a Conducting Cube	43
4.6	Reconstruction of a Target with a Shifted Initial Guess	48
4.7	Reconstruction of Dielectric Objects	49
4.8	Comments on the Numerical Experiments	52
5	Conclusion	57

List of Figures

1.1	The configuration of the inverse scattering problem for PEC object.	2
1.2	Flow chart of the inverse electromagnetic scattering technique. . .	2
1.3	Flow chart of the shape reconstruction problems.	3
1.4	Surface models of the closed triangular meshed geometries using NX8 software. Triangular meshed modelling of (a) cube, (b) concave object, (c) sphere, and (d) ellipsoid.	7
2.1	The mismatch between the measurements of two spheres with 20 mm and 30 mm diameters with respect to different numbers of receiving antennas is shown by (a) the cost function and (b) the normalized cost function.	12
2.2	The mismatch between the measurements of a sphere with a 20 mm diameter and a star-shaped object with an average radius of 13 mm with respect to different numbers of receiving antennas is shown by (a) the cost function and (b) the normalized cost function.	13
3.1	Derivative of the scattered electric field on the x - y plane with respect to the x component of a node on a sphere with a 50 mm radius illuminated with a plane wave from ($\theta = 90^\circ, \phi = 0^\circ$) direction in 3 GHz: (a) derivative of the theta component's magnitude and (b) derivative of the phi component's magnitude.	24

3.2	Derivative of the scattered electric field on the x - y plane with respect to the x component of a node on a sphere with a 50 mm radius illuminated with a plane wave from $(\theta = 45^\circ, \phi = -120^\circ)$ direction in 3 GHz: (a) derivative of the theta component's magnitude and (b) derivative of the phi component's magnitude. . . .	25
4.1	Some of the reconstructed targets with complex geometries: (a) an ellipsoid, (b) an egg-shaped object, (c) a star-shaped object, and (d) a concave object.	28
4.2	Cost function for the reconstruction of an ellipsoid in 21 iterations.	31
4.3	Step vector of the shape reconstruction problem of an ellipsoid at 10 GHz, which is separated into three parts to update the (a) x components, (b) y components, and (c) z components of evolving object nodes in the first iteration.	32
4.4	Reconstruction of an ellipsoid at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 21st iteration.	33
4.5	Reconstruction of an egg-shaped target at 10 GHz for the r representation of unknowns, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the fourth iteration, and (d) in the 11th iteration.	35
4.6	Reconstruction of an egg-shaped target at 10 GHz for the xyz representation of unknowns, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the fourth iteration, and (d) in the 11th iteration.	36

4.7	Step vector of the shape reconstruction problem of an egg-shaped target at 10 GHz, which updates the r components of evolving object nodes in first iteration (the unknowns are r components of the nodes).	37
4.8	Step vector of the shape reconstruction problem of an egg-shaped target at 10 GHz, which is separated into three parts to update the (a) x components, (b) y components, and (c) z components of evolving object nodes in the first iteration (the unknowns are xyz components of the nodes).	38
4.9	Cost function for the reconstruction of the egg-shaped target for two representations of unknowns in 11 iterations.	39
4.10	Cost function for the reconstruction of a conducting sphere at 15 GHz using two different initial guesses: (a) a sphere with a 30 mm diameter and (b) a sphere with a 20 mm diameter.	40
4.11	Reconstruction of a sphere at 15 GHz with a sphere initial guess of a 30 mm diameter, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 18th iteration.	41
4.12	Reconstruction of a sphere at 15 GHz with a sphere initial guess of a 20 mm diameter, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the third iteration, (c) in the sixth iteration, and (d) in the 18th iteration.	42
4.13	Cost function for the reconstruction of the star-shaped target in 25 iterations.	43

4.14	Reconstruction of a star-shaped object at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 25th iteration.	44
4.15	Reconstruction of a concave object at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 25th iteration.	45
4.16	Cost function for the reconstruction of the concave target in 25 iterations.	46
4.17	Reconstruction of a cube with an edge length of 26 mm at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 25th iteration.	47
4.18	Cost function for the reconstruction of the cube in 25 iterations. .	48
4.19	Cost function for the reconstruction of two sphere targets with (a) 5 mm and (b) 10 mm shifts with respect to the initial guess. .	50
4.20	Reconstruction of two sphere targets with 5 mm and 10 mm shifts with respect to the initial guess, where the transparent object is the target and the red object is the evolving object. Reconstruction of the target with a 5 mm shift, (a) in the first iteration (initial guess), (b) in the second iteration, and (c) in the 23rd iteration. Reconstruction of the target with a 10 mm shift, (d) in the first iteration (initial guess), (e) in the second iteration, and (f) in the 30th iteration.	51

4.21	Reconstruction of a dielectric star-shaped object with the relative permittivity of 4.0 at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the third iteration, (c) in the fifth iteration, and (d) in the 28th iteration.	53
4.22	Cost function for the reconstruction of the dielectric star-shaped target in 28 iterations.	54
4.23	Distribution of the (a) transmitting antennas in six directions and (b) receiving antennas in 26 directions around the target.	55

List of Tables

4.1	List of illumination and scattering directions.	56
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This thesis is dedicated to my parents
for their love and endless support.

Chapter 1

Introduction

In this thesis, we investigate reconstructing the shape and location of three-dimensional perfect electric conductor (PEC) objects. We also present a new approach to solving the inverse scattering problem of three-dimensional PEC targets.

Over the last few years, inverse scattering has been the subject of a wide range of electromagnetic research studies [1–18]. In electromagnetic inverse scattering problems, the main goal is to acquire knowledge about the electromagnetic properties of an unknown object, such as its conductivity, permittivity, and permeability or about its geometrical parameters, such as shape and location.

In general, to obtain the above knowledge about a scattering object, this object must be illuminated by incident waves, and the scattered fields from the object must be measured and collected as the data set of the unknown object. Figure 1.1 shows the configuration of the inverse scattering problem for PEC objects. Because we can not measured the object's desired unknown parameters and we only collect the scattered fields, we need to use an appropriate technique to transform the collected data into useful information about the target's parameters. This technique is shown as the third block of the general flowchart of inverse problems in Fig. 1.2.

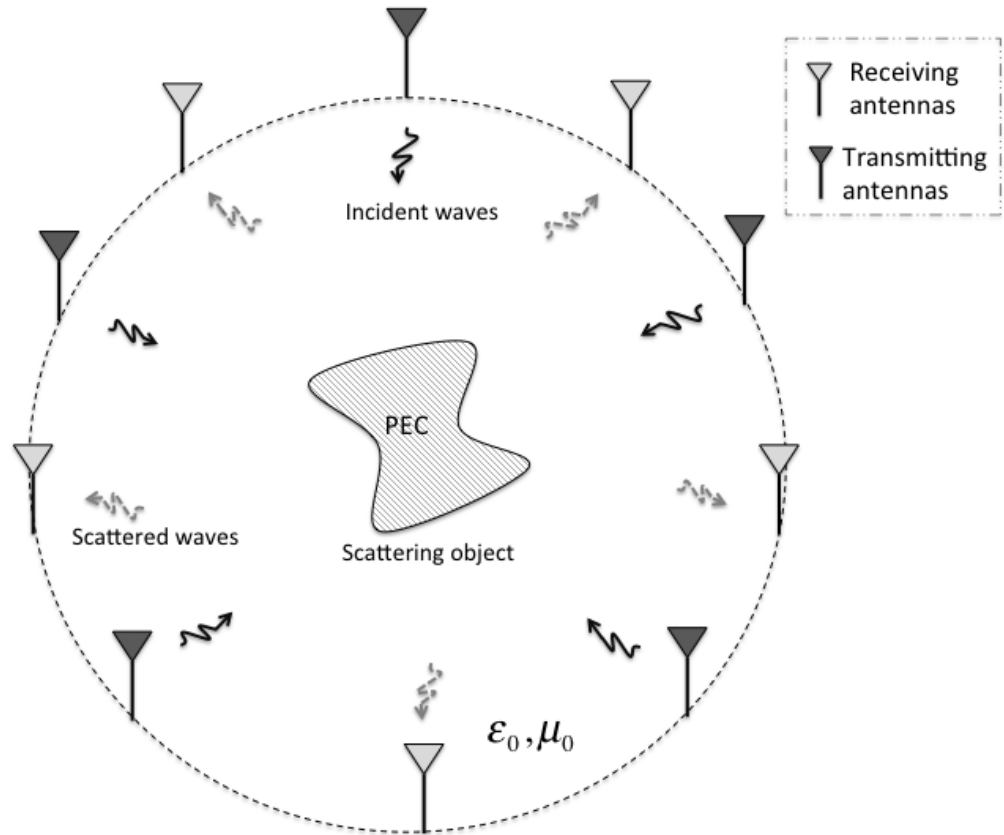


Figure 1.1: The configuration of the inverse scattering problem for PEC object.

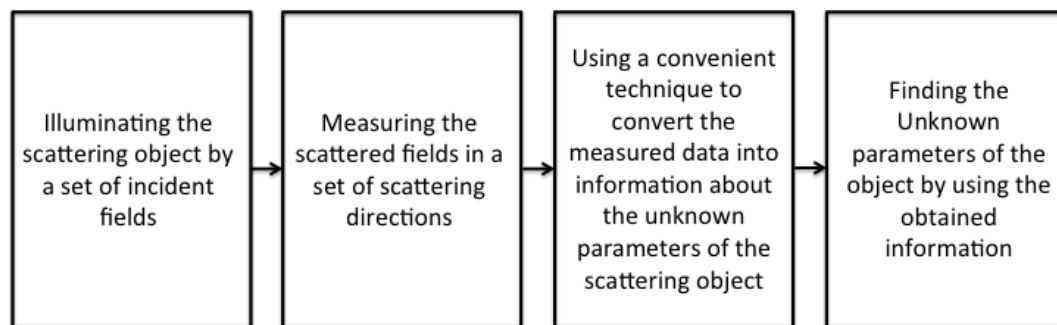


Figure 1.2: Flow chart of the inverse electromagnetic scattering technique.

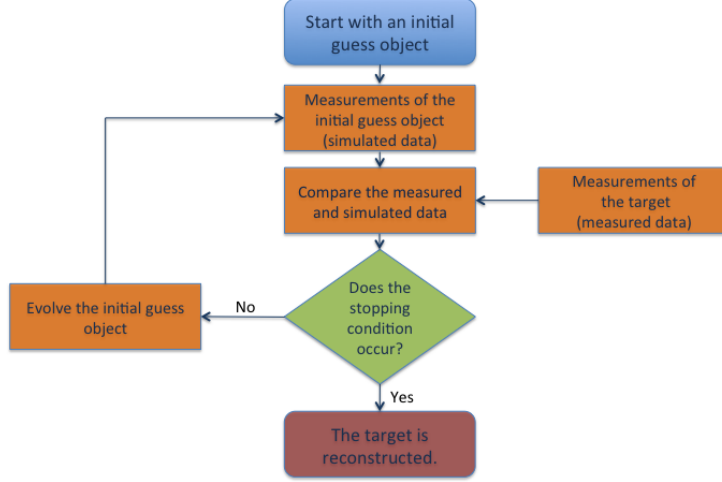


Figure 1.3: Flow chart of the shape reconstruction problems.

1.1 Motivations

Applications of inverse electromagnetic scattering emerge in many areas. For instance, because electromagnetic waves can deeply penetrate the ground, inverse electromagnetic scattering is widely used in geophysics, such as in detecting the shape, location, and electromagnetic properties of valuable buried objects [2–4, 9, 10]. In other applications, inverse electromagnetic scattering can determine where to drill for petroleum and natural gas. Inverse electromagnetic scattering applications are also widely used in biomedical imaging studies, such as detecting small tumor regions on different organs or identifying leukemia [11–14].

Strong shape reconstruction methods will improve the performance of inverse scattering problems. As explained above, the applications of these problems vary greatly. In some cases, the target objects are very big, such as petroleum resources, and in other cases, the target objects may be of microscopic size. The properties of the target objects may also be different. All these diversities should be considered when implementing shape reconstruction algorithms.

1.2 Historical Background

In the literature, different methods are used to solve inverse scattering problems. In earlier works, these methods were generally developed for two-dimensional cases. The Born iterative method is one of the the most popular methods for solving the inverse scattering problems that updates the permittivity and conductivity profiles of a computational domain to reconstruct their distribution in a specified region [1]. In consecutive works, the distorted Born iterative method (DBIM) is implemented to accelerate convergence. The DBIM is similar to the Born iterative method, but the Green's function is updated at each iteration in the DBIM [2–4]. These methods reconstruct the electromagnetic property profile of the computational domain; however, in related works, an initial object is modeled by a suitable method, and then the target is reconstructed by evolving the object iteratively. The level set method is one of the most well-known methods for inverse scattering problems. In such works, the most important part in the shape reconstruction formulation is to find an appropriate speed function to evolve the level sets in the proper direction of retrieving the targets [17, 18].

A modeling vector is introduced in [7] that contains information about the shape and location of the target, and it also models the electromagnetic properties of the computational domain. The Newton minimization approach is applied to update this modeling vector iteratively such that the shape, location, and other target properties can be reconstructed. Some of the numerical results for different cases are illustrated in [6–8].

1.3 Contributions

The main objective of this thesis is to formulate a new technique for reconstructing the shape of three-dimensional PEC objects using the Newton minimization approach. In Chapter 2, the cost function is defined as the fundamental parameter of the shape reconstruction technique and the inverse scattering problem turns into the optimization problem of the cost function. The vector of unknowns is

defined as a vector that should be updated iteratively, in such a way that the cost is reduced at each iteration. The Taylor-series expansion of the cost function is presented to show its decreasing direction.

In Chapter 3, the Newton minimization method and the steepest-descent method are introduced, with their formulations for updating the vector of unknowns derived from the Taylor-series expansion of the cost function. In this work, the formulations are derived for complex-valued measurements; therefore, we are no longer restricted to real-valued measurements. At the end of the chapter, a numerical method for calculating the Jacobian matrix of the residual vector is described in detail.

In Chapter 4, the numerical results of the shape reconstructions of different targets are presented, and the method's strengths and weaknesses will be discussed. In the final chapter, a conclusion about the proposed shape reconstruction method is presented and future works are discussed.

1.4 Simulation Environment and Computational Resources

We used NX Unigraphics 8 (NX8), which is a computer-aided design software, to model the geometries. This program has the capability of meshing the geometries; therefore, we use it to provide triangular meshed geometries. Figure 1.4 shows some triangular meshed geometries designed from the NX8 program.

As described earlier, in inverse scattering problems the scattering object should be illuminated from different directions and the scattered fields should be collected; thus, a forward solver is required to provide a full collection of the scattered field data. In this work, we use forward solvers of the Bilkent University Computational Electromagnetics Research Center (BiLCEM). The solvers are implemented in the Fortran programming language. Surface integral equations are

used in these solvers to formulate scattering and radiation problems. These integral equations are discretized with the method of moments, which yields a dense matrix equation. The matrix equation is solved iteratively by the forward solvers, where the required matrix-vector multiplications are accelerated by the multilevel fast multipole algorithm [19,20]. It is important to notice that the geometry file exported by the NX8 program is compatible with the input file of the forward solver.

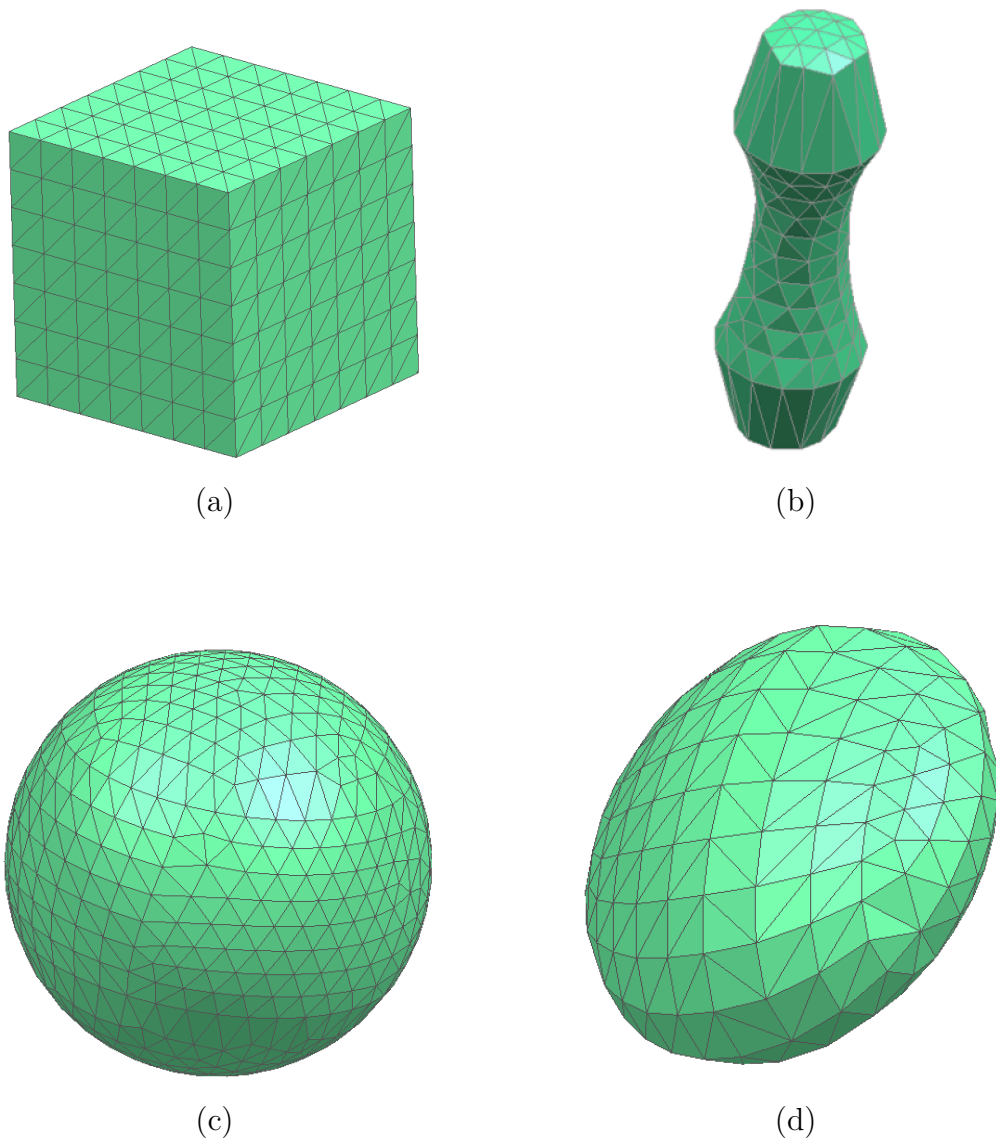


Figure 1.4: Surface models of the closed triangular meshed geometries using NX8 software. Triangular meshed modelling of (a) cube, (b) concave object, (c) sphere, and (d) ellipsoid.

Chapter 2

Cost Function and Its Role in the Minimization Process

The general idea of solving the shape reconstruction problem is to begin with an initial guess object and evolve this object in an appropriate direction to reduce the difference between the evolving object and the target. The general flowchart of the shape reconstruction problems is shown in Fig. 1.3. To achieve this goal, we need to define a criterion to determine the difference between the evolving object and the target object. In this chapter, we will introduce the cost function as a parameter to show this difference. In addition, we will describe how the cost function is related to the complex-valued measurements of the object. Finally, we will introduce the Taylor-series expansion of the cost function to choose a suitable direction to decrease the cost function and reconstruct the target by evolving the initial guess object.

2.1 Cost Function

It is necessary to define a criterion to determine the difference between the target object and the evolving object, the latter of which should ultimately match the target. A suitable criterion is the total mismatch of the measurements between the target and the evolving object. The residual vector is defined as the mismatches of measurements between the target and evolving object, and can be shown as

$$\mathbf{e}(\mathbf{x}) = \mathbf{S}(\mathbf{x}) - \mathbf{m} = \begin{bmatrix} e_1(\mathbf{x}) \\ \vdots \\ e_j(\mathbf{x}) \\ \vdots \\ e_M(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} S_1(\mathbf{x}) - m_1 \\ \vdots \\ S_j(\mathbf{x}) - m_j \\ \vdots \\ S_M(\mathbf{x}) - m_M \end{bmatrix}, \quad (2.1)$$

where $e_j(\mathbf{x}) = S_j(\mathbf{x}) - m_j$ is the mismatch between the j th measured data from the evolving object $S_j(\mathbf{x})$ and the j th measured data from target m_j , and M is the number of measurements. For the sake of generality, we assume that the measurement values are complex-valued numbers, and by considering this assumption, the formulation of the minimization problem is obtained (described in later sections); however, in some works, the residual vector, and consequently, the formulations, are obtained for the real-valued measurements. Note that the presented formulations in this thesis for the complex-valued measurements are valid for the real-valued measurements.

In Eq. (2.1), $\mathbf{x}$ is the vector of unknowns that consists of the elements that we want to update in each iteration. As a result, the elements of the vector of unknowns should be chosen relevant to the situation. For example, the unknowns can be chosen as the conductivity of a specific volume's pixels for the situation of detecting underground petroleum sources. In addition, for cases with a high contrast between the dielectric properties of the target and the environment, the unknowns can be chosen as the dielectric properties of the computational domain.

In this thesis, the material properties are *a priori* information and we will assume that they are constant; thus, we will not update their values during the iterations. We therefore choose the location of the object nodes, which model

the object's surface, as the inverse problem's unknowns. In Chapter 4, where we show the numerical experiments, we give a complete explanation about two different representations of the node locations in the vector of unknowns.

After introducing the residual vector, we can define the summation of the magnitude of the measurement mismatches as the cost function, that is the above-mentioned criterion. Hence, we define the cost function as

$$C(\mathbf{x}) = \sum_{i=1}^M \|e_i(\mathbf{x})\|^2 + \alpha R(\mathbf{x}). \quad (2.2)$$

We did not include the noise in this expression because we have neglected its effect in our measurements. In addition, an extra term of $R(\mathbf{x})$ is added to the cost function in Eq. (2.2) as a regularization term. The regularization term is usually used to prevent quick changes of the vector of unknowns that models the evolving object. As a consequence, the variations of the unknowns will be controlled in each iteration and the object will evolve smoothly. This term can be chosen in different ways according to the problem. A common choice for the regularization term is to define it as $(\mathbf{x} - \mathbf{x}_p)$, where $\mathbf{x}_p$ is the previous iteration's unknown values, so by choosing such regularization terms, we can control the rate of the unknown's changes with respect to previous iterations. α is the regularization parameter and determines the relative importance of the two terms of the cost function. In this thesis, we do not use the regularization term; therefore, we say that $\alpha = 0$. Thus, we can rewrite the cost function in Eq. (2.2) as

$$C(\mathbf{x}) = \sum_{i=1}^M \|e_i(\mathbf{x})\|^2 = \sum_{i=1}^M \|S_i(\mathbf{x}) - m_i\|^2. \quad (2.3)$$

In some cases, it is better to use the normalized cost function, which can be written as

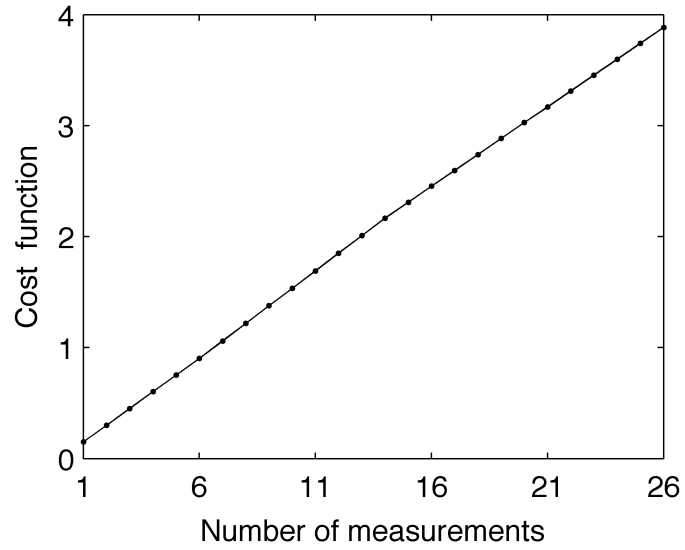
$$C_n(\mathbf{x}) = \frac{\sum_{i=1}^M \|S_i(\mathbf{x}) - m_i\|^2}{\sum_{i=1}^M \|m_i(\mathbf{x})\|^2}. \quad (2.4)$$

Obviously, by increasing the number of measurements, the cost function will be increased. However, the normalized cost function may not necessarily increase.

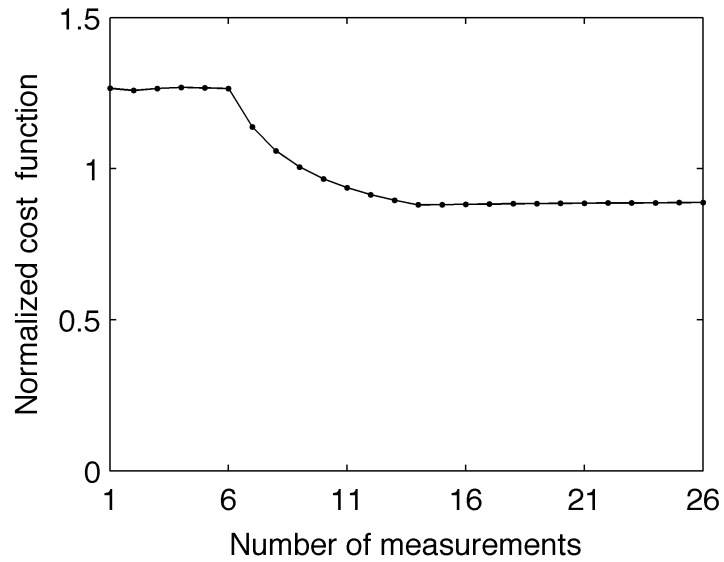
For example, consider the case where the target is a sphere with 30 mm diameter and the initial guess is another sphere with 20 mm diameter. Both objects are centered at the origin and they are illuminated by 12 incident fields from six directions with theta and phi polarizations. The measurements are the scattered electric fields, which are measured in the far-field region. We will increase the number of measurements to determine its effects on the cost function. The incident and scattering directions are shown in Table 4.1. Figure 2.1(a) shows that by increasing the number of measurements, the cost function increases. In Fig. 2.1(b), we can see that after adding 15-th measurement direction to the normalized cost function measurements, adding new measurement directions will not change the normalized cost function significantly; therefore, it can be concluded that increasing the number of measurements will not give us any further information about the target. As a second example, we substitute the big sphere with a star-shaped object with an average radius of 13 mm. In Fig. 2.2(a) we can see that the cost function increases by adding new measurements; however, the rate of increase is not similar to the first example because the star-shaped object is not as symmetrical as a sphere. Figure 2.2(b) shows that we have considerable changes in the normalized cost function until the twentieth measurement for the cost function. As a result, by choosing 26 scattering directions for the numerical experiments, we can guarantee that we have collected enough samples from the scattered fields. As evident in Fig. 4.23, the transmitting and receiving antennas are uniformly distributed around the object.

2.2 Taylor-Series Expansion of the Cost Function

In the previous section, we defined a parameter to show the difference between the target and the evolving object at each iteration; therefore, to retrieve the object properly, we should minimize the cost function. As a result, we should update the vector of unknowns at each iteration such that the new set of unknowns reduces the cost function.

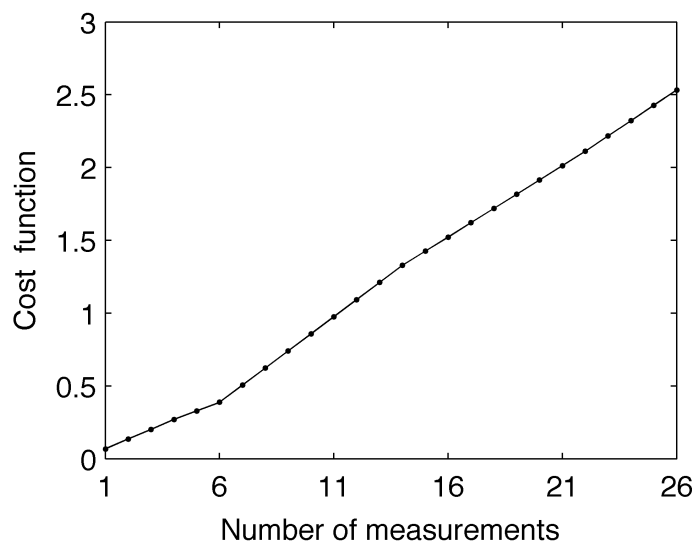


(a)

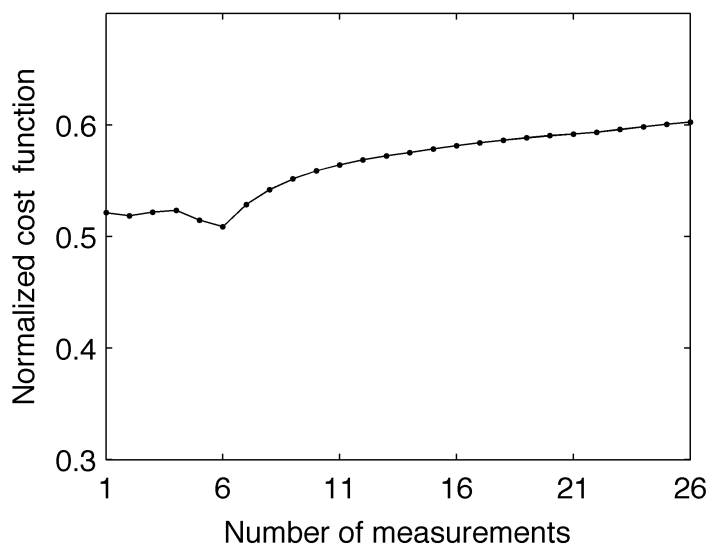


(b)

Figure 2.1: The mismatch between the measurements of two spheres with 20 mm and 30 mm diameters with respect to different numbers of receiving antennas is shown by (a) the cost function and (b) the normalized cost function.



(a)



(b)

Figure 2.2: The mismatch between the measurements of a sphere with a 20 mm diameter and a star-shaped object with an average radius of 13 mm with respect to different numbers of receiving antennas is shown by (a) the cost function and (b) the normalized cost function.

In similar works, different methods are used to decrease the cost function with respect time or iterations [5, 17]. For example, the derivative of the cost function with respect to time is provided, and then by using the fact that the derivative of the cost function should be negative to have a decreasing cost function, a shape reconstruction method is presented. In this thesis, we will use the Taylor-series expansion of the cost function to update the vector of unknowns at each iteration in such a way that the values of the cost function decrease continuously. In Eq. (2.5), we have the expansion of the cost function around the unknown vector of the k -th iteration, $\mathbf{x}_k$, for an update vector of $\mathbf{p}_k$ that can be written as

$$C(\mathbf{x}_k + \mathbf{p}_k) = C(\mathbf{x}_k) + \nabla C(\mathbf{x}_k)^T \cdot \mathbf{p}_k + \frac{1}{2} \mathbf{p}_k^T \cdot \nabla \nabla C(\mathbf{x}_k) \cdot \mathbf{p}_k + \dots \quad (2.5)$$

As a conclusion for this chapter, we define the cost function in Eq. (2.2) and choose it as the main parameter of the minimization problem, and also discuss the validation of this choice. Then, the Taylor-series expansion of the cost function is presented. In the next chapter, by starting from Eq. (2.5), possible solutions for the minimization problem are obtained, and possible modifications are introduced. Finally, an alternative solution that can be applicable to our work is presented.

Chapter 3

Minimization of the Cost Function

In this chapter, we will formulate a method for the shape reconstruction problem. To retrieve the shape and location of an unknown object iteratively, we need to update the vector of unknowns at each iteration in such a way that the value of the cost function in the next iteration is diminished. As indicated above, the obtained solutions are not necessarily unique. Therefore, some applicable modifications are introduced for more-reliable solutions. In addition, the steepest-descent method is introduced as an appropriate method for our inverse problem. Finally, the numerical method used to calculate the Jacobian matrix of the vector of unknowns is fully described.

3.1 The Newton Minimization Approach

In Chapter 2, the Taylor-series expansion of the cost function was shown in Eq. (2.5). Since this expansion is an infinite series, we use Newton's method in optimization problems that uses the function's quadratic form; in other words, we will only use the first three terms of the expansion of the cost function, which can be written as

$$C(\mathbf{x}_k + \mathbf{p}_k) \approx C(\mathbf{x}_k) + \mathbf{g}^T(\mathbf{x}_k) \cdot \mathbf{p}_k + \frac{1}{2} \mathbf{p}_k^T \cdot \overline{\mathbf{G}}(\mathbf{x}_k) \cdot \mathbf{p}_k, \quad (3.1)$$

where $\mathbf{p}_k$ is the step vector that will update the unknown vector in the k th iteration. The vector $\mathbf{g}(\mathbf{x}_k) = \nabla C(\mathbf{x}_k)$ is the gradient of the cost function and the matrix $\overline{\mathbf{G}}(\mathbf{x}_k) = \nabla \nabla C(\mathbf{x}_k)$ is the Hessian matrix of the cost function. It can be shown that the error of ignoring other terms of the Taylor-series expansion of the cost function is in the order of $\mathcal{O}(\|\mathbf{p}_k\|^2)$ [21]. Therefore, as long as we have small step vectors, the approximation used in Eq. (3.1) is acceptable.

In Eq. (3.1), the left-hand side of the equation is the updated cost function in the $(k+1)$ th iteration. The first term of the right-hand side is the cost function of the k th iteration, and the next two terms can be considered as an update of the cost function in the k th iteration. By considering the fact that the cost function should be decreased iteratively, the step vector must be chosen in such a way that we achieve the minimum value of the last two terms in Eq. (3.1). In other words, we need to find the minimum of

$$U(\mathbf{p}_k) = \mathbf{g}^T(\mathbf{x}_k) \cdot \mathbf{p}_k + \frac{1}{2} \mathbf{p}_k^T \cdot \overline{\mathbf{G}}(\mathbf{x}_k) \cdot \mathbf{p}_k. \quad (3.2)$$

To find the critical points of the function $U(\mathbf{p}_k)$, we should find the points that make the gradient of $U(\mathbf{p}_k)$ zero. In other words, the solution of

$$\nabla U(\mathbf{p}_o) = \mathbf{g}(\mathbf{x}_o) + \overline{\mathbf{G}}(\mathbf{x}_o) \cdot \mathbf{p}_o = 0, \quad (3.3)$$

which is a linear algebraic equation, provides the desired critical points as

$$\overline{\mathbf{G}}(\mathbf{x}_o) \cdot \mathbf{p}_o = -\mathbf{g}(\mathbf{x}_o). \quad (3.4)$$

3.1.1 The Possible Solutions of the Newton Minimization Approach

The solution of the matrix equation shown in Eq. (3.4) is dependent on the Hessian matrix $\overline{\mathbf{G}}(\mathbf{x})$ and the gradient vector $\mathbf{g}(\mathbf{x})$. Assume that the Hessian matrix of the cost function is singular. In this case, if we can express the gradient vector $\mathbf{g}(\mathbf{x})$ as a linear combination of the columns of the Hessian matrix, we will have infinite solutions as critical points for the function $U(\mathbf{p})$; otherwise, there will be no solutions, and consequently, no critical points for the function $U(\mathbf{p})$.

For the second case, where we deal with a non-singular Hessian matrix, the definiteness of the Hessian matrix determines the solution of Eq. (3.4). If $\overline{\mathbf{G}}(\mathbf{x})$ is a positive definite matrix, it is guaranteed that Eq. (3.4) has a unique solution, and the obtained solution is the minimum of the function. Otherwise, for an indefinite $\overline{\mathbf{G}}(\mathbf{x})$ we will have a unique solution; however, this solution is not necessarily a minimum. One possible solution is to replace the Hessian matrix $\overline{\mathbf{G}}(\mathbf{x})$ with a related positive definite matrix $\overline{\mathbf{K}}(\mathbf{x})$ constructed from the Hessian matrix elements. In the next two sections, we derive the expressions of the gradient vector $\mathbf{g}(\mathbf{x})$ and the Hessian matrix $\overline{\mathbf{G}}(\mathbf{x})$, which are required for the minimization problem.

3.1.2 The Gradient Vector of the Cost Function

To find the gradient vector of the cost function, we expand this function in terms of the complex residuals of the measurements:

$$C(\mathbf{x}_k) = \mathbf{e}^H(\mathbf{x}_k) \cdot \mathbf{e}(\mathbf{x}_k) = e_1^* e_1 + \dots + e_M^* e_M, \quad (3.5)$$

where the superscript H signifies the complex conjugate and transpose of the vector. Using Eq. (3.5), the gradient vector of the cost function $\mathbf{g}(\mathbf{x}_k)$ can be written as

$$\mathbf{g}(\mathbf{x}_k) = \nabla C(\mathbf{x}_k) = \left[\frac{\partial C}{\partial x_1} \quad \dots \quad \frac{\partial C}{\partial x_i} \quad \dots \quad \frac{\partial C}{\partial x_N} \right]^T. \quad (3.6)$$

By substituting Eq. (3.5) in Eq. (3.6), we obtain

$$\begin{aligned}
\Rightarrow \mathbf{g}(\mathbf{x}_k) &= \begin{bmatrix} \frac{\partial}{\partial x_1} e_1^* e_1 + \dots + \frac{\partial}{\partial x_1} e_M^* e_M \\ \vdots \\ \frac{\partial}{\partial x_i} e_1^* e_1 + \dots + \frac{\partial}{\partial x_i} e_M^* e_M \\ \vdots \\ \frac{\partial}{\partial x_N} e_1^* e_1 + \dots + \frac{\partial}{\partial x_N} e_M^* e_M \end{bmatrix} \\
&= \begin{bmatrix} \left(\left(\frac{\partial}{\partial x_1} e_1^* \right) e_1 + e_1^* \left(\frac{\partial}{\partial x_1} e_1 \right) \right) + \dots + \left(\left(\frac{\partial}{\partial x_1} e_M^* \right) e_M + e_M^* \left(\frac{\partial}{\partial x_1} e_M \right) \right) \\ \vdots \\ \left(\left(\frac{\partial}{\partial x_i} e_1^* \right) e_1 + e_1^* \left(\frac{\partial}{\partial x_i} e_1 \right) \right) + \dots + \left(\left(\frac{\partial}{\partial x_i} e_M^* \right) e_M + e_M^* \left(\frac{\partial}{\partial x_i} e_M \right) \right) \\ \vdots \\ \left(\left(\frac{\partial}{\partial x_N} e_1^* \right) e_1 + e_1^* \left(\frac{\partial}{\partial x_N} e_1 \right) \right) + \dots + \left(\left(\frac{\partial}{\partial x_N} e_M^* \right) e_M + e_M^* \left(\frac{\partial}{\partial x_N} e_M \right) \right) \end{bmatrix}. \tag{3.7}
\end{aligned}$$

Each element of the above vector is the summation of complex-valued terms and their conjugates; therefore, we can write the gradient vector as

$$\begin{aligned}
\Rightarrow \mathbf{g}(\mathbf{x}_k) &= \begin{bmatrix} 2 \operatorname{Re}\left\{ \left(\frac{\partial}{\partial x_1} e_1^* \right) e_1 \right\} + \dots + \operatorname{Re}\left\{ \left(\frac{\partial}{\partial x_1} e_M^* \right) e_M \right\} \\ \vdots \\ \operatorname{Re}\left\{ \left(\frac{\partial}{\partial x_i} e_1^* \right) e_1 \right\} + \dots + \operatorname{Re}\left\{ \left(\frac{\partial}{\partial x_i} e_M^* \right) e_M \right\} \\ \vdots \\ \operatorname{Re}\left\{ \left(\frac{\partial}{\partial x_N} e_1^* \right) e_1 \right\} + \dots + \operatorname{Re}\left\{ \left(\frac{\partial}{\partial x_N} e_M^* \right) e_M \right\} \end{bmatrix} \\
&= \begin{bmatrix} 2 \operatorname{Re} \left\{ \left(\frac{\partial}{\partial x_1} \mathbf{e}^H(\mathbf{x}) \right) \cdot \mathbf{e}(\mathbf{x}) \right\} \\ \vdots \\ 2 \operatorname{Re} \left\{ \left(\frac{\partial}{\partial x_i} \mathbf{e}^H(\mathbf{x}) \right) \cdot \mathbf{e}(\mathbf{x}) \right\} \\ \vdots \\ 2 \operatorname{Re} \left\{ \left(\frac{\partial}{\partial x_N} \mathbf{e}^H(\mathbf{x}) \right) \cdot \mathbf{e}(\mathbf{x}) \right\} \end{bmatrix}, \tag{3.8}
\end{aligned}$$

and finally, the gradient vector can be written as

$$\mathbf{g}(\mathbf{x}_k) = \nabla C(\mathbf{x}_k) = \operatorname{Re}\{\bar{\mathbf{J}}^H(\mathbf{x}_k) \cdot \mathbf{e}(\mathbf{x})\}, \tag{3.9}$$

where $\bar{\mathbf{J}}(\mathbf{x})$ is $M \times N$ Jacobian matrix of the residual vector, and can be written as

$$\bar{\mathbf{J}}(\mathbf{x}) = \begin{bmatrix} \frac{\partial}{\partial x_1} e_1 & \cdots & \frac{\partial}{\partial x_j} e_1 & \cdots & \frac{\partial}{\partial x_N} e_1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial}{\partial x_1} e_i & \cdots & \frac{\partial}{\partial x_j} e_i & \cdots & \frac{\partial}{\partial x_N} e_i \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial}{\partial x_1} e_M & \cdots & \frac{\partial}{\partial x_j} e_M & \cdots & \frac{\partial}{\partial x_N} e_M \end{bmatrix} \quad (3.10)$$

3.1.3 The Hessian Matrix of the Cost Function

In this section, we take the gradient of $\mathbf{g}(\mathbf{x})$ to find the Hessian matrix of the cost function. To achieve this goal, we put the gradient vector beside the vector (3.9) and by expanding the corresponding dyadic of these two vectors, we will obtain $\bar{\mathbf{G}}(\mathbf{x})$ in the following sequence:

$$\begin{aligned} \bar{\mathbf{G}}_k(\mathbf{x}) = \nabla \nabla C(\mathbf{x}_k) &= \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \vdots \\ \frac{\partial}{\partial x_i} \\ \vdots \\ \frac{\partial}{\partial x_N} \end{bmatrix} \begin{bmatrix} \text{Re}\{(\frac{\partial}{\partial x_1} \mathbf{e}^*(\mathbf{x})) \cdot \mathbf{e}(\mathbf{x})\} \\ \vdots \\ \text{Re}\{(\frac{\partial}{\partial x_i} \mathbf{e}^*(\mathbf{x})) \cdot \mathbf{e}(\mathbf{x})\} \\ \vdots \\ \text{Re}\{(\frac{\partial}{\partial x_N} \mathbf{e}^*(\mathbf{x})) \cdot \mathbf{e}(\mathbf{x})\} \end{bmatrix} \\ &= \text{Re} \begin{bmatrix} \vdots \\ \cdots \left(\frac{\partial}{\partial x_i} ((\frac{\partial}{\partial x_j} \mathbf{e}_1^*) e_1) + \cdots + \frac{\partial}{\partial x_i} ((\frac{\partial}{\partial x_j} \mathbf{e}_M^*) e_M) \right) \cdots \\ \vdots \end{bmatrix}. \end{aligned} \quad (3.11)$$

By expanding each element of the Hessian matrix, we can rewrite it as

$$\bar{\mathbf{G}}_k(\mathbf{x}) = \text{Re} \begin{bmatrix} \vdots \\ \cdots \left(\left((\frac{\partial^2}{\partial x_i \partial x_j} \mathbf{e}_1^*) e_1 + (\frac{\partial}{\partial x_j} \mathbf{e}_1^*) (\frac{\partial}{\partial x_i} e_1) \right) + \cdots + \left((\frac{\partial^2}{\partial x_i \partial x_j} \mathbf{e}_M^*) e_M + (\frac{\partial}{\partial x_j} \mathbf{e}_M^*) (\frac{\partial}{\partial x_i} e_M) \right) \right) \cdots \\ \vdots \end{bmatrix}. \quad (3.12)$$

Each element of the Hessian matrix consists of two terms, so the matrix can be separated into two matrices and written as

$$\begin{aligned} \overline{\mathbf{G}}_k(\mathbf{x}) = & \text{Re} \begin{bmatrix} & & \vdots & & \\ & & & & \\ \dots & \left(\left(\frac{\partial}{\partial x_j} e_1^* \right) \left(\frac{\partial}{\partial x_i} e_1 \right) + \dots + \left(\frac{\partial}{\partial x_j} e_M^* \right) \left(\frac{\partial}{\partial x_i} e_M \right) \right) & \dots & \\ & & \vdots & & \end{bmatrix} \\ & + \text{Re} \begin{bmatrix} & & \vdots & & \\ & & & & \\ \dots & \left(\left(\frac{\partial^2}{\partial x_i \partial x_j} e_1^* \right) e_1 + \dots + \left(\frac{\partial^2}{\partial x_i \partial x_j} e_M^* \right) e_M \right) & \dots & \\ & & \vdots & & \end{bmatrix}. \end{aligned} \quad (3.13)$$

Finally, we can write the compact form of the Hessian matrix as

$$\overline{\mathbf{G}}(\mathbf{x}_k) = \nabla \nabla C(\mathbf{x}_k) = \text{Re}\{\overline{\mathbf{J}}^H(\mathbf{x}) \cdot \overline{\mathbf{J}}(\mathbf{x}) + \overline{\mathbf{Q}}(\mathbf{x})\}, \quad (3.14)$$

where $\overline{\mathbf{Q}}(\mathbf{x}) = \sum_{m=1}^M e_m(\mathbf{x}) \overline{\mathbf{F}}_m^H$ and $\overline{\mathbf{F}}_m = \nabla \nabla e_m(\mathbf{x})$. In this work, the $\overline{\mathbf{Q}}(\mathbf{x})$ matrix, which contains second order derivatives of the measurement mismatches, is neglected in calculating the Hessian matrix.

3.2 The Steepest-Descent Method

We explained earlier that it is not always possible to have a unique solution for the matrix equation of Eq. (3.4), which can guarantee decreasing the cost function in each iteration. In similar works, some possible modifications are presented to change the conditioning and definiteness of the Hessian matrix. After all possible modifications, we may still be unable to obtain the step vector from Eq. (3.4) because the most realistic problems deal with a large number of unknowns and a huge number of measurements, thus it may not be applicable to find the inverse matrix of $\overline{\mathbf{G}}(\mathbf{x})$ at each iteration.

As an alternative solution, we apply the steepest-descent method, which simply chooses the step vector of each iteration in the opposite direction of the

gradient of the cost function:

$$\mathbf{p}_k = -\gamma_k \nabla C(\mathbf{x}_k) = -\gamma_k \mathbf{g}_k. \quad (3.15)$$

By substituting Eq. (3.15) in Eq. (3.1), we can rewrite it as

$$C(\mathbf{x}_k + \mathbf{p}_k) \approx C(\mathbf{x}_k) - \gamma_k |\mathbf{g}_k|^2 + \frac{1}{2} \gamma_k^2 \mathbf{g}_k^T \cdot \overline{\mathbf{G}}(\mathbf{x}_k) \cdot \mathbf{g}_k. \quad (3.16)$$

Because our goal in the shape reconstruction problem is to decrease the cost function, we should minimize the last two terms of Eq. (3.16). Thus, γ_k should be chosen as:

$$\gamma_k = -\frac{|\mathbf{g}_k|^2}{\mathbf{g}_k^T \cdot \overline{\mathbf{G}}(\mathbf{x}_k) \cdot \mathbf{g}_k}, \quad (3.17)$$

so the step vector will be

$$\mathbf{p}_k = -\frac{|\mathbf{g}_k|^2}{\mathbf{g}_k^T \cdot \overline{\mathbf{G}}(\mathbf{x}_k) \cdot \mathbf{g}_k} \mathbf{g}_k. \quad (3.18)$$

In addition, by substituting Eq. (3.17) in Eq. (3.16), we can see that the cost function at each iteration will be decreased in the form of

$$C(\mathbf{x}_k + \mathbf{p}_k) - C(\mathbf{x}_k) = -\frac{1}{2} \frac{|\mathbf{g}_k|^4}{\mathbf{g}_k^T \cdot \overline{\mathbf{G}}(\mathbf{x}_k) \cdot \mathbf{g}_k}. \quad (3.19)$$

As expressed in Eq. (3.9) and Eq. (3.14), to obtain the gradient vector $\mathbf{g}$ and the Hessian matrix $\overline{\mathbf{G}}$, the Jacobian matrix of the residual vector is required. To calculate the Jacobian matrix elements, different methods can be used.

As we can see in Eq. (3.10), the Jacobian matrix consists of the derivatives of the measurement mismatches with respect to the unknowns. For some kinds of measurements and unknowns, mathematical expressions of these derivatives are available. In these situations, the Jacobian matrix filling will be straightforward. However, it is not always possible to find an expression for these derivatives. For instance, in our case, where we have the node locations of the object as the unknowns and the scattered electric fields as the measurements of the shape reconstruction problem, mathematical expressions for the Jacobian matrix elements are not available. In these situations, one solution for calculating the Jacobian matrix elements is to use the chain rule for each derivative and convert it into two derivatives with available mathematical expressions.

3.3 Numerical Calculation of the Jacobian Matrix

In this work, we use a numerical method to calculate the Jacobian matrix elements. To compute the j th column elements of the matrix, the corresponding unknown of that column will be perturbed slightly, and the small variation in all measurement mismatches will be recorded. Finally, by using the first-order derivative approximation, each element of the column will be calculated. For instance, if the mismatch in the i th measurement is e_i and after a small perturbation of δx of the j th unknown, this mismatch changes to e'_i , the element of the i th row and the j th column of the Jacobian matrix will be calculated as

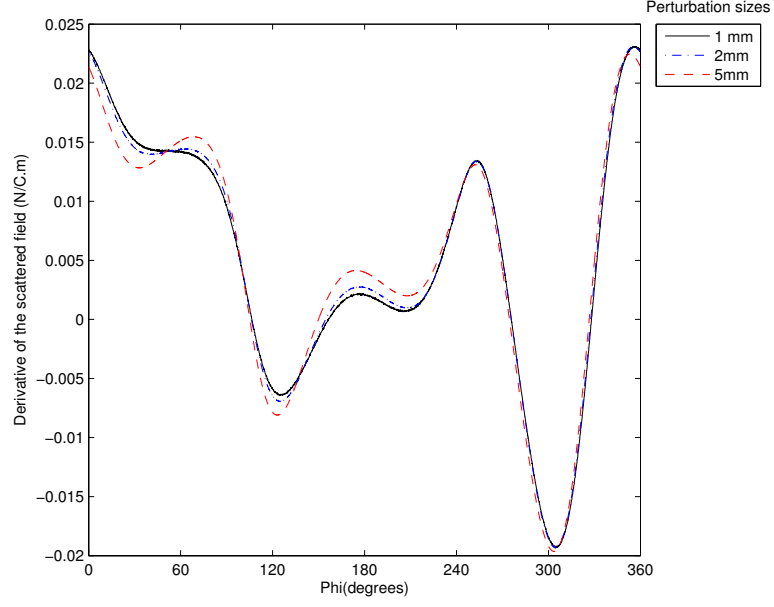
$$J_{ij} \approx \frac{e'_i - e_i}{(x_j + \delta x) - (x_j)}. \quad (3.20)$$

In choosing the perturbation size, we should consider reasonable upper and lower limits. One of the parameters that can determine the upper limit of the perturbation size is the mesh size of the object. Clearly, perturbations that are larger than the mesh size will change the topology of the object. In addition, numerical calculation of the derivatives imposes an upper limit for the perturbation size. Obviously, large perturbation sizes lead to less accurate numerical results. On the other hand, the lower limit of choosing the perturbation size depends on the computational accuracy of the forward solver, which computes the scattered electric fields.

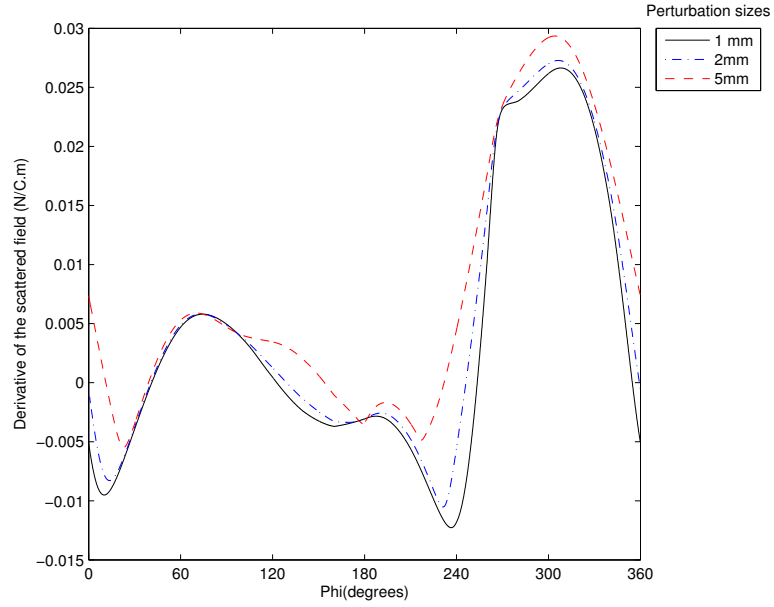
In Fig. 3.1, the numerically computed derivative of the magnitude of the theta and phi components of the scattered electric field with respect to the X component of a node on the surface of a sphere is presented. The sphere is located at the origin with a 50 mm radius and the tested node is located on $(\theta = 41^\circ, \phi = -120^\circ)$ direction of the sphere. The object is illuminated by a plane wave in $(\theta = 90^\circ, \phi = 0^\circ)$ direction. In this experiment, the frequency is 3 GHz. In addition, the derivatives are calculated for perturbation sizes of 1 mm, 2 mm, and 5 mm. Numerical experiments show that for perturbations smaller

than one-fiftieth of the wavelength, the calculated derivatives are accurate and acceptable.

In the second example, we repeat the same experiment, for another illumination direction of $(\theta = 45^\circ, \phi = -120^\circ)$. The important point in this test is that the incident field is in the direction of the perturbed node; thus, the nodes perturbation has much more effect on the scattered field. As we can see in Fig. 3.2, because of the importance of the perturbed node's location in the scattered electric field pattern, for the largest perturbation size, we fail to compute the derivatives correctly. It is important to mention that by decreasing the perturbation size, the computed derivatives converge to a set of values. Therefore, these converged values are assumed to be the correct results. The numerical results show that one-hundredth of the wavelength is an appropriate choice for the perturbation size. In the numerical experiments of Chapter 4, we used this size for calculating the Jacobian matrix elements.

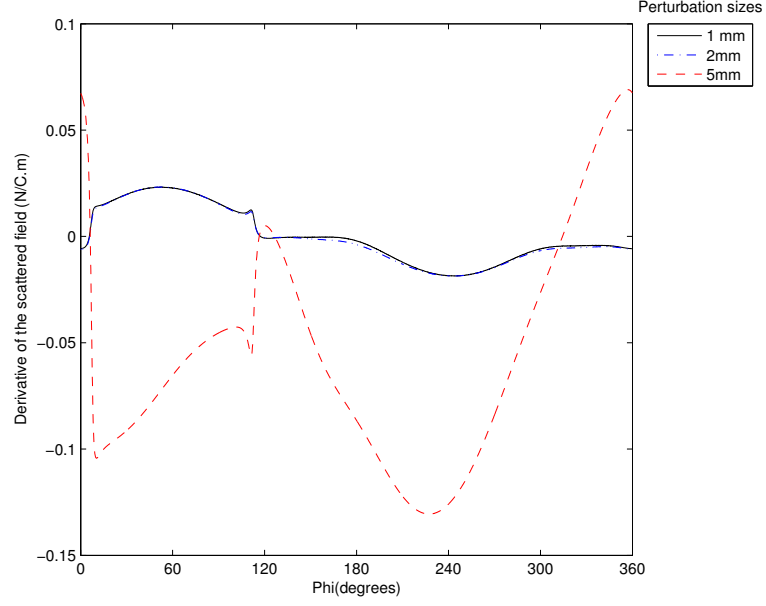


(a)

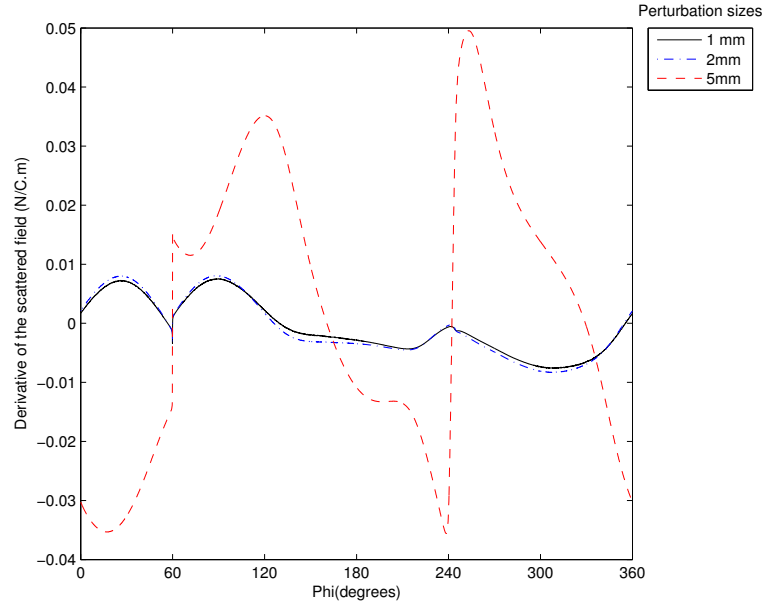


(b)

Figure 3.1: Derivative of the scattered electric field on the x - y plane with respect to the x component of a node on a sphere with a 50 mm radius illuminated with a plane wave from $(\theta = 90^\circ, \phi = 0^\circ)$ direction in 3 GHz: (a) derivative of the theta component's magnitude and (b) derivative of the phi component's magnitude.



(a)



(b)

Figure 3.2: Derivative of the scattered electric field on the x - y plane with respect to the x component of a node on a sphere with a 50 mm radius illuminated with a plane wave from $(\theta = 45^\circ, \phi = -120^\circ)$ direction in 3 GHz: (a) derivative of the theta component's magnitude and (b) derivative of the phi component's magnitude.

Chapter 4

Numerical Results

In this chapter, a set of numerical results will be presented to demonstrate the advantages and disadvantages of the inverse scattering method introduced in this thesis. Therefore, some of the important microwave imaging cases will be presented, and the numerical results of these cases will be analyzed.

In the numerical experiments of the inverse scattering problems, plenty of parameters, such as frequency, number of measurements, and number of unknowns, have significant effects on the results. To realize these effects, we give a brief explanation of the optimization algorithm that we used to obtain the results, and then present the numerical results.

The Inverse Problem Algorithm

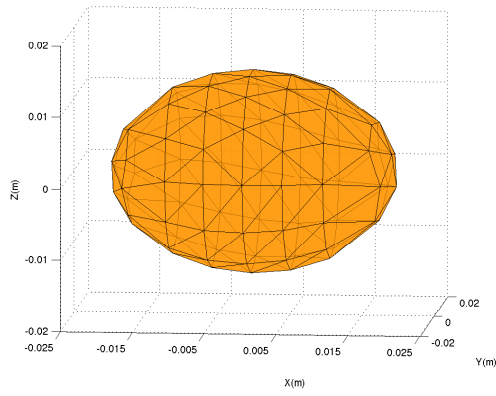
The algorithm of the shape reconstruction of three-dimensional PEC objects using the Newton minimization approach is summarized as follows:

- 1) Choose a suitable initial guess of the target. Provide the triangular mesh representation of the initial guess and the list of node locations, on the surface of the object. Due to the symmetric the shape of sphere in three-dimensional problems, different sizes of spheres are used as the initial guess.
- 2) Do the measurements of the target and collect the results as measured data.
- 3) Do the measurements of the evolving object, which is the initial guess in the first iteration, and collect the results as the simulated data.
- 4) Calculate the Jacobian matrix (described in previous chapter).
- 5) Use the steepest-descent method and update the modeling parameters of the object, which in this thesis are the node locations of the evolving object.
- 6) Calculate the cost function and reconstruct the object by the updated values of the modeling parameters.
- 7) Go to Step 3 and repeat till convergence of the cost function occurs.

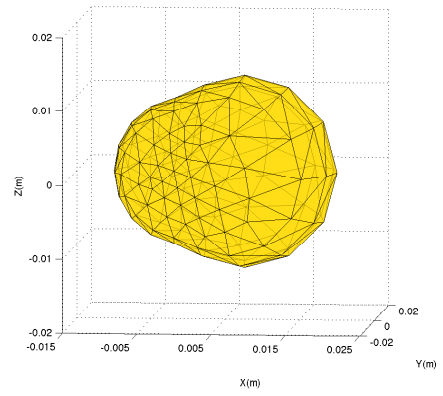
The algorithm is implemented in the shell script, which contains Fortran programs for numerical calculations.

4.1 Reconstruction of a Conducting Ellipsoid

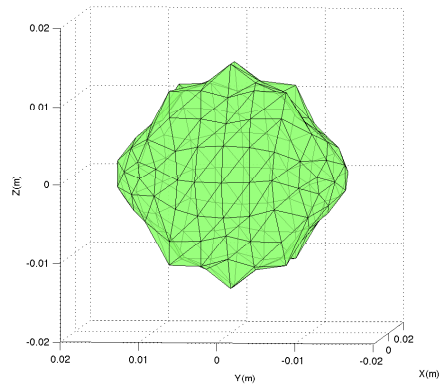
In the first example, the reconstruction of a conducting ellipsoid shown in Fig. 4.1(a) is investigated. The ellipsoid is located at the origin and its diameters along x , y , and z axes are 40 mm, 30 mm, and 30 mm respectively. The initial guess is a sphere with diameter of a 20 mm. The sphere is located at the origin



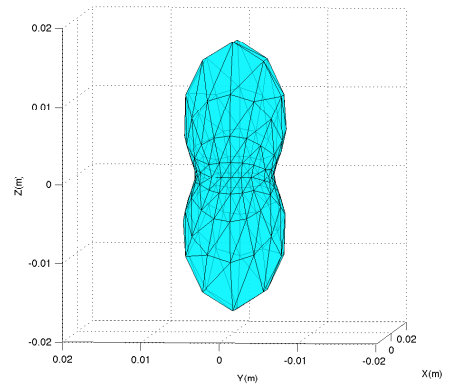
(a)



(b)



(c)



(d)

Figure 4.1: Some of the reconstructed targets with complex geometries: (a) an ellipsoid, (b) an egg-shaped object, (c) a star-shaped object, and (d) a concave object.

and its triangular meshed presentation has 156 nodes. The initial guess and the target are shown in Fig. 4.4(a). Twelve incident fields from six directions with both theta and phi polarizations are employed to illuminate the object. In addition, the measurements are obtained from 26 scattering directions. Because we measured the theta and phi components of the scattered electric field separately, for each scattering direction, two measurements have been done. Furthermore, it is important to know that the measurement of the scattered field in the i th direction corresponding to the j th incident field is not the same as the measurement of the scattered field corresponding to the k th incident field. As a result, the number of measurements is equal to $M = N_{inc} \times (2 \times N_{sca})$, where N_{inc} is the number of incident fields and N_{sca} is the number of scattering directions, which, for this example, is $M = 12 \times (2 \times 26) = 624$. The list of transmitting and receiving antennas' directions are given in Table 4.1 and their distribution around the target is shown in Fig. 4.23. The frequency of 10 GHz is employed for this case.

Figure 4.4 shows how the evolving object retrieves the shape of ellipsoid in 21 iterations. In the first three iterations, the general shape of target is retrieved, and the details of the target's shape are reconstructed in the next iterations. As we can see in Fig. 4.2, the cost function drops rapidly, to about 7.8% of its initial value in the first five iterations, and finally in 21th iteration, the cost function is reduced to 4.3% of its initial value.

In this example, unknowns are the x , y , and z components of the nodes, which are located on the surface of the object. These unknowns are placed in the vector

of unknowns in the form of

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \\ y_1 \\ \vdots \\ y_N \\ z_1 \\ \vdots \\ z_N \end{bmatrix}, \quad (4.1)$$

where x_i , y_i , and z_i are the components of the i th node's location, and N is the number of nodes; thus, the number of unknowns for this case is $N' = 3 \times N = 468$. In Fig. 4.3, we can see the step vector on the surface of the evolving object that updates the nodes' x , y , and z components in the first iteration. For this representation of unknowns, the duration of the ellipsoid reconstruction was approximately 15 hours.

In the next example, this representation of unknowns will be compared to a compressed form of unknowns representation.

4.2 Comparison of Two Unknown Representations in the Reconstruction Process

In the second example, we analyze the results of reconstructing an egg-shaped target shown in Fig. 4.1(b). The left half of the target is a hemisphere. For the right half, the nodes of a hemisphere are perturbed in the radial direction. The perturbation size of each node is proportional to the x component of the node. In this example, the frequency of 10 GHz is employed and the initial guess is a sphere with diameter of a 20 mm and 156 nodes. Twelve incident fields are used to illuminate the target and 26 scattering directions are used to measure the theta and phi components of the scattered electric fields. For this experiment,

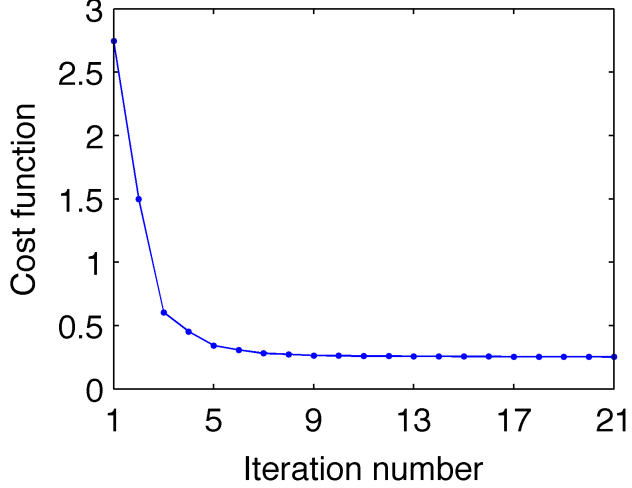


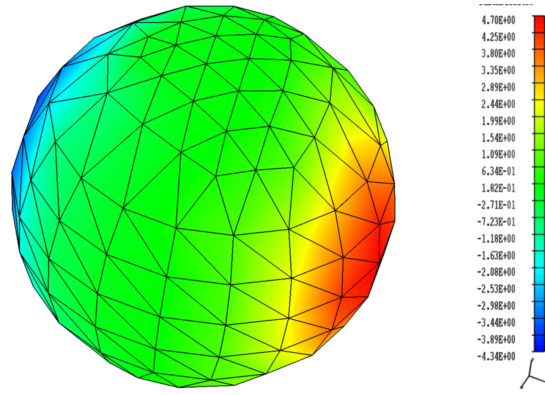
Figure 4.2: Cost function for the reconstruction of an ellipsoid in 21 iterations.

we present the results of the shape reconstruction of the target for two different representations of unknowns, and discuss their strengths and weaknesses. In one case, the unknowns are the x , y , and z components of the nodes in the form of (4.1), and in the second case, the unknowns are the r component of the nodes. In this case, the vector of unknowns can be written as

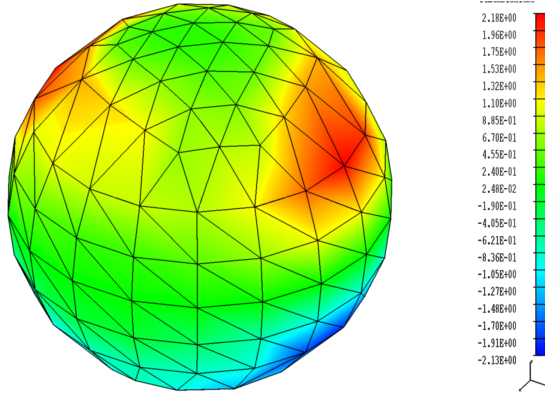
$$\mathbf{x} = \begin{bmatrix} r_1 \\ \vdots \\ r_i \\ \vdots \\ r_N \end{bmatrix}, \quad (4.2)$$

where r_i is the r component of the i th node and N is the number of nodes.

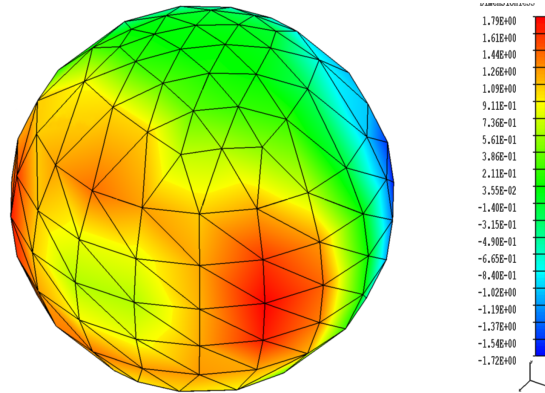
In the first representation of the unknowns, the number of unknowns is equal to $3N$. However, for the second representation, the number of unknowns drops to N . As a result, the duration of each iteration drops to about 33% of its initial value. Note that the calculation of the Jacobian matrix that should be done at each iteration, is time consuming because we run the forward solver for each column of the Jacobian matrix. As a consequence, because the Jacobian



(a)

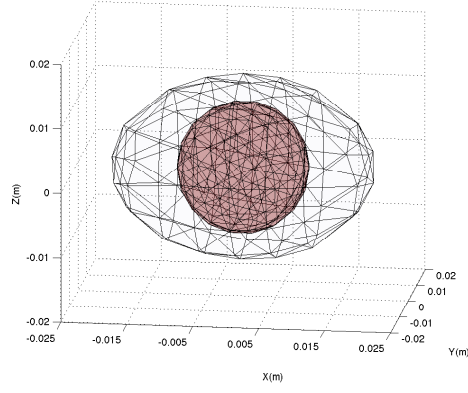


(b)

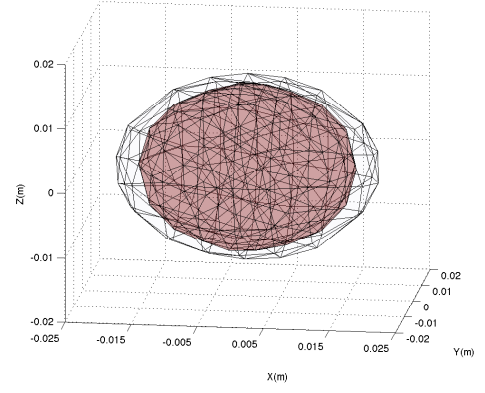


(c)

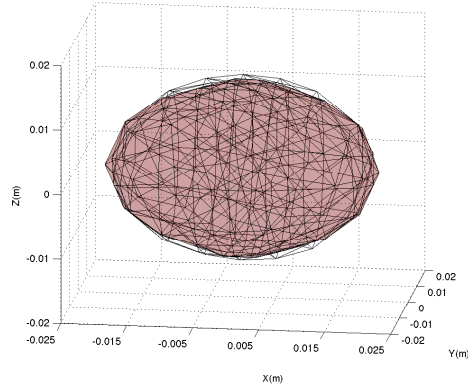
Figure 4.3: Step vector of the shape reconstruction problem of an ellipsoid at 10 GHz, which is separated into three parts to update the (a) x components, (b) y components, and (c) z components of evolving object nodes in the first iteration.



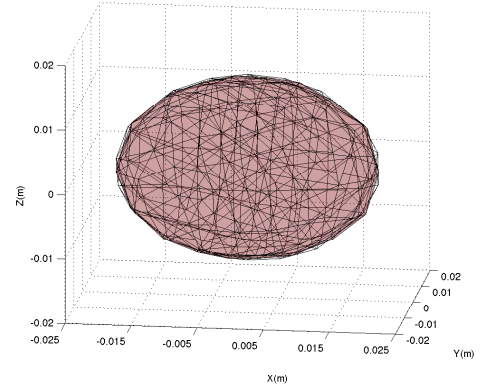
(a)



(b)



(c)



(d)

Figure 4.4: Reconstruction of an ellipsoid at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 21st iteration.

matrix calculation basically depends on the number of unknowns (i.e., equal to the number of columns), any reduction in the number of unknowns will decrease the duration of each iteration significantly.

However, the representation of unknowns in the form of (4.2) reduces the time duration of each iteration considerably, we put a restriction on the nodes to only move in the radial direction. In other words, the other two components of the nodes (θ, ϕ) will be fixed to their initial value. As a result, with this unknown representation, we will not be able to retrieve all given targets.

In Fig. 4.9, however, we can see that we used a much fewer number of unknowns in one case compared to the other one, but their cost functions are almost the same (decreased to about 2% of their first value in the first iteration). As we can see in Fig. 4.5 and Fig. 4.6, these two cases have a close evolution pattern in retrieving the target; thus, it can be concluded that the unknowns in both cases are updated similarly. We can see in Fig. 4.7 and Fig. 4.8, which show the step vectors of both cases in the first iteration of the optimization problem, that the update of the unknowns in both cases are close to each other. Note that update of the unknowns by the step vectors are not the same for the two cases. For instance, the x , y , and z components of the nodes should be updated by the update values shown on the surface of the object in Figs. 4.8(a), 4.8(b), and 4.8(c), respectively.

4.3 Reconstruction of a Conducting Sphere

In this example, the shape reconstruction of a conducting sphere with a 40 mm diameter is presented. The operating frequency is 15 GHz. We illuminated the sphere with 12 plane waves from six directions and we have done 52 measurements for each illumination. The illumination and scattering directions can be obtained in the Table 4.1. The unknowns are xyz components of the nodes, so the vector of unknowns is in the form of (4.1).

In the first case, the initial guess is a sphere with a 30 mm diameter. As we can

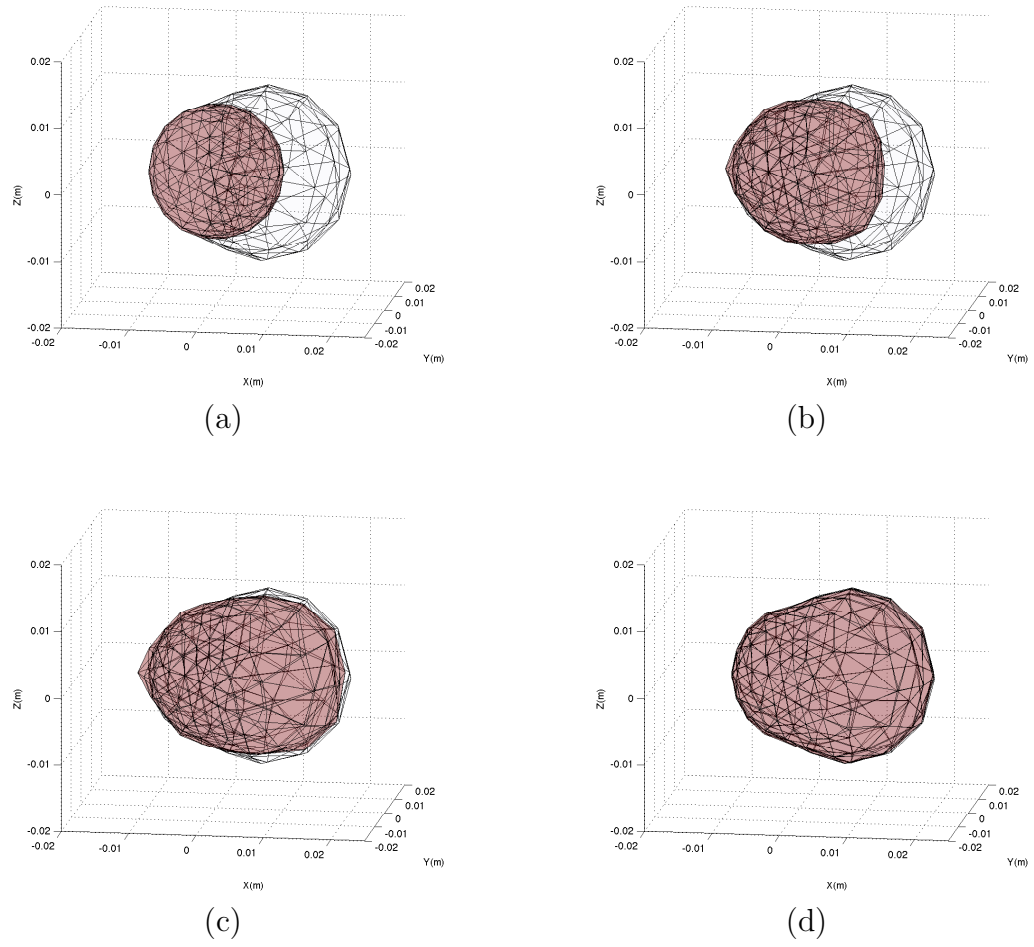


Figure 4.5: Reconstruction of an egg-shaped target at 10 GHz for the r representation of unknowns, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the fourth iteration, and (d) in the 11th iteration.

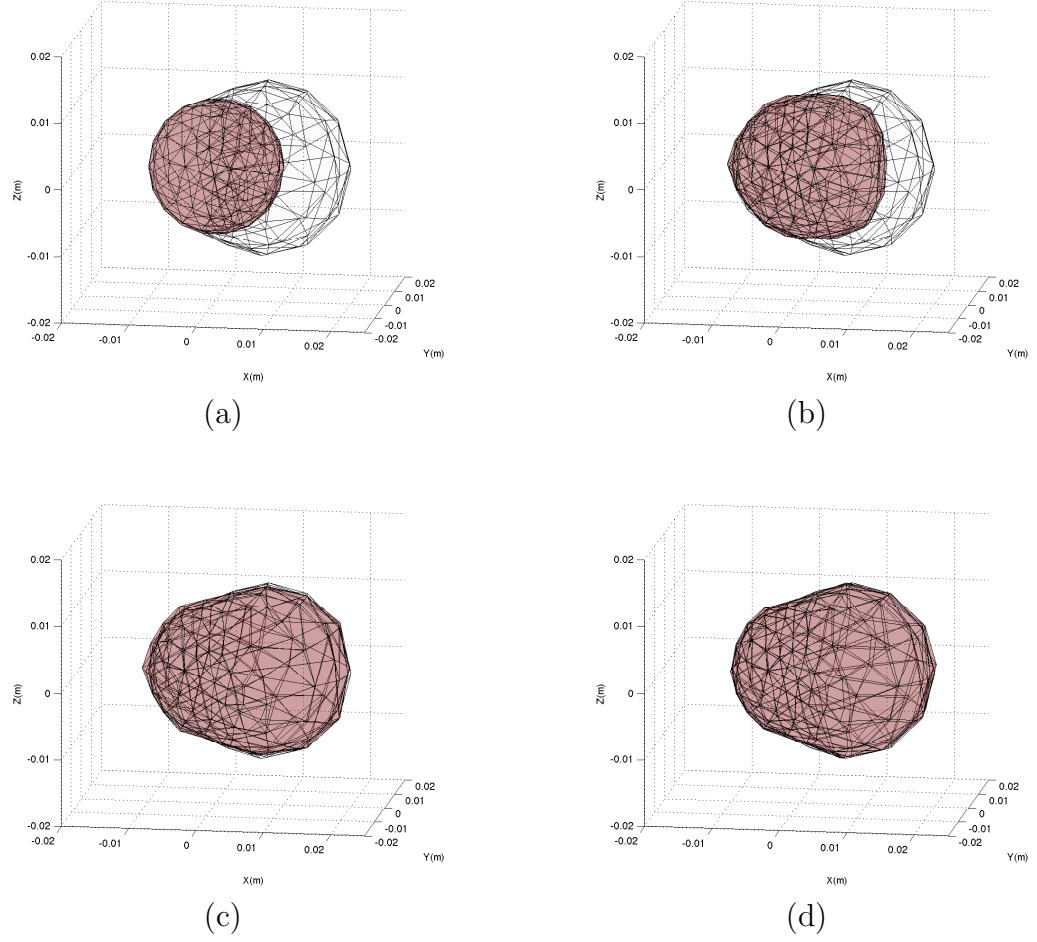


Figure 4.6: Reconstruction of an egg-shaped target at 10 GHz for the xyz representation of unknowns, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the fourth iteration, and (d) in the 11th iteration.

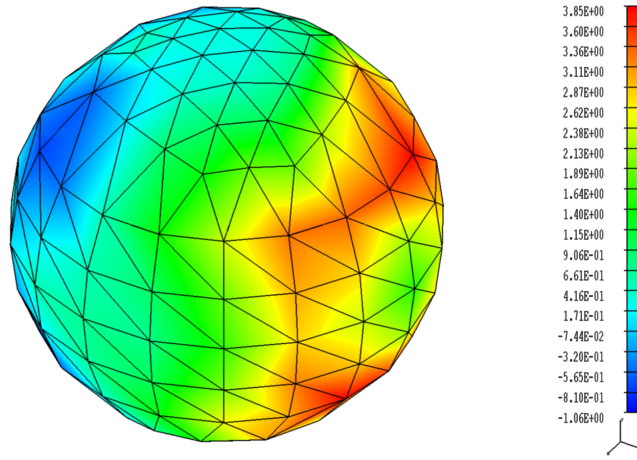


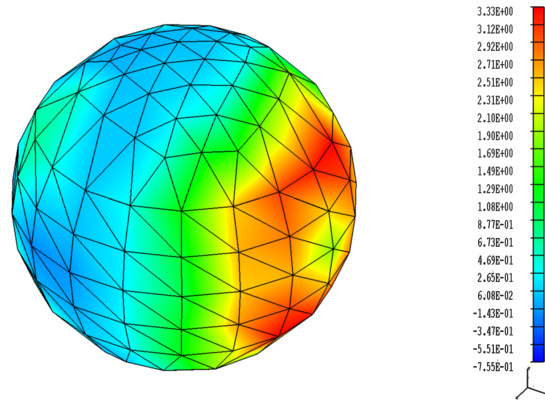
Figure 4.7: Step vector of the shape reconstruction problem of an egg-shaped target at 10 GHz, which updates the r components of evolving object nodes in first iteration (the unknowns are r components of the nodes).

see in Fig. 4.10(a), the cost function is reduced to 1.8% of its initial value after 18 iterations. In the second case, the initial guess is a sphere with a 20 mm diameter, which is sufficiently far from the target (one wavelength difference between the target and the initial guess). Figure 4.10(b) shows that the cost function is reduced to 2.1% of its initial value after 18 iterations. The reconstruction of the conducting sphere is shown in Fig. 4.11 and Fig. 4.12 for two different initial guesses.

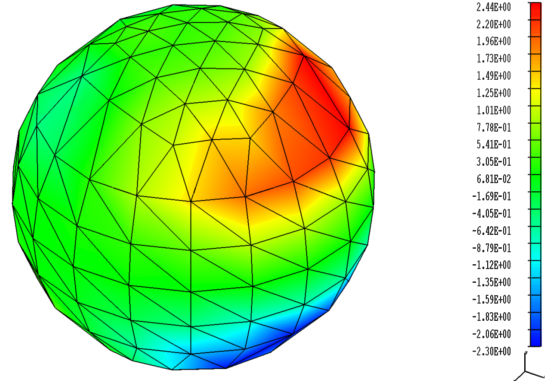
4.4 Reconstruction of the Objects with Complex Geometries

In this section, the numerical results of the shape reconstruction of two targets with more-complex geometries will be presented. For both cases, the initial guess is chosen to be a sphere with a 10 mm radius and 156 nodes. The unknowns are in the form of (4.1); thus, the number of unknowns is 468 (3×156) for both cases.

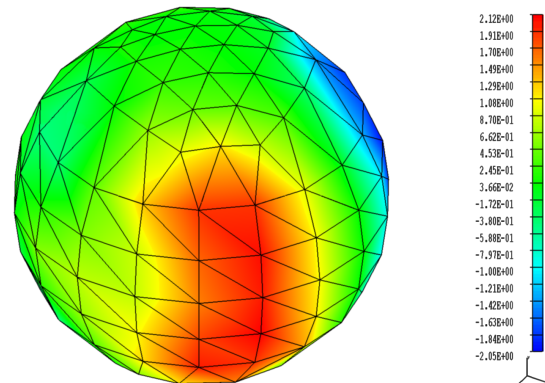
We used 12 incident fields with theta and phi polarizations in six illumination



(a)



(b)



(c)

Figure 4.8: Step vector of the shape reconstruction problem of an egg-shaped target at 10 GHz, which is separated into three parts to update the (a) x components, (b) y components, and (c) z components of evolving object nodes in the first iteration (the unknowns are xyz components of the nodes).

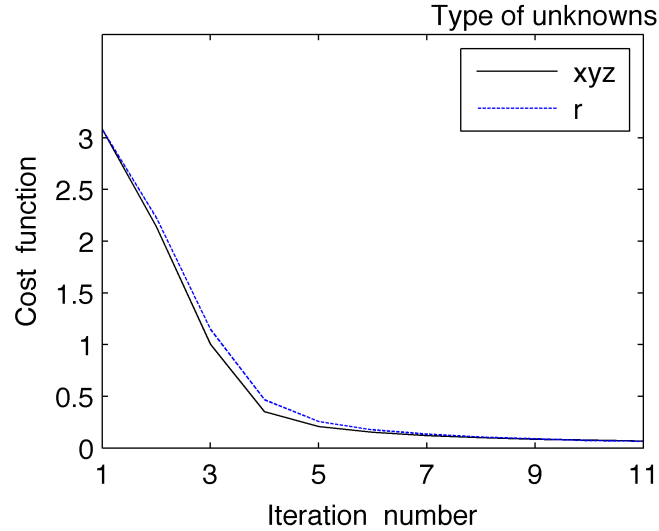
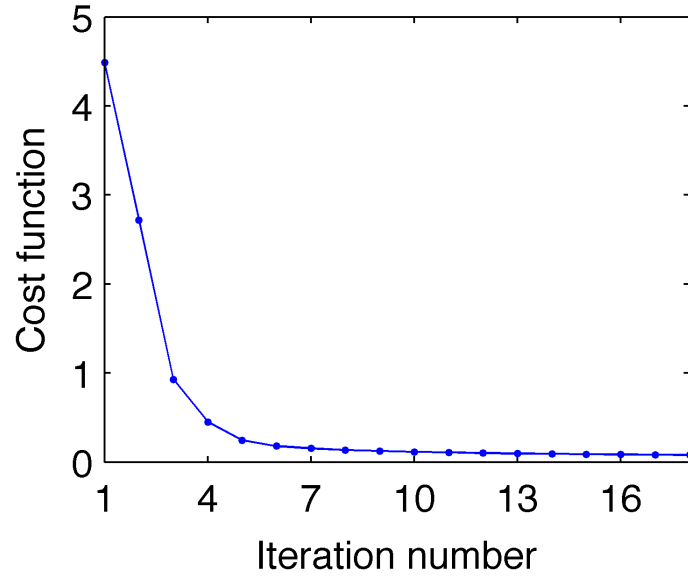


Figure 4.9: Cost function for the reconstruction of the egg-shaped target for two representations of unknowns in 11 iterations.

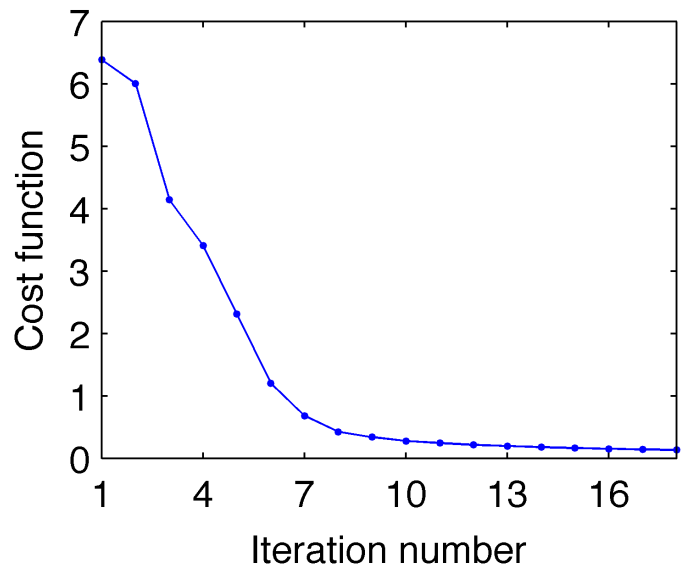
directions. In addition, the measurements have been done from 26 scattering directions. At each direction, we measured the theta and phi components of the scattered electric fields separately. Therefore, we have done 52 measurements for each illumination. The operating frequency is 10 GHz.

4.4.1 A Star-Shaped Target

In this case, the target is a star-shaped object shown in Fig. 4.1(c) and it is centered at the origin. The radius of the target changes between 11 mm and 15 mm. As shown in Fig. 4.13, the cost is dropped to 3.1% of its initial value after 25 iterations. Figure 4.14(b) shows that the general shape of the target is retrieved in the third iteration, and then, the shape of the target is completely reconstructed in the 25th iteration.

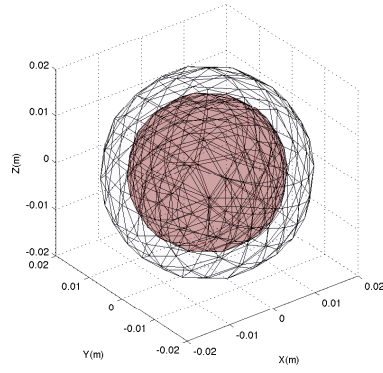


(a)

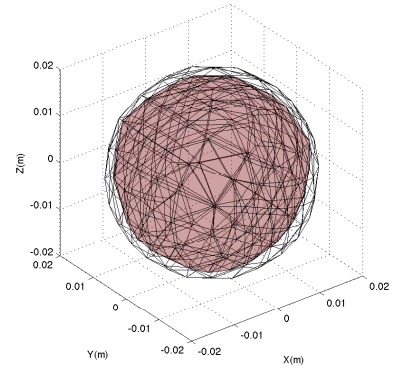


(b)

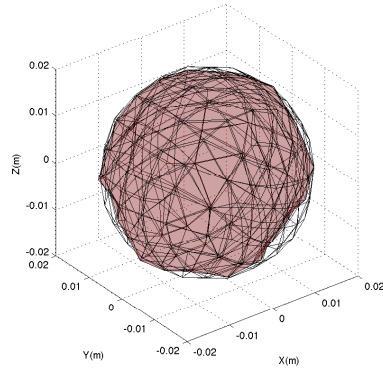
Figure 4.10: Cost function for the reconstruction of a conducting sphere at 15 GHz using two different initial guesses: (a) a sphere with a 30 mm diameter and (b) a sphere with a 20 mm diameter.



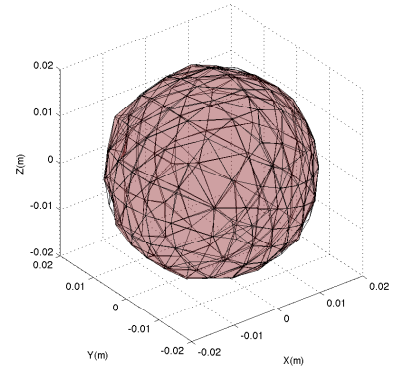
(a)



(b)

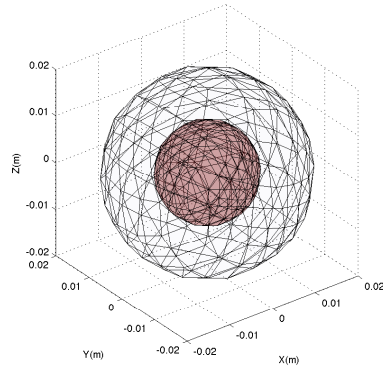


(c)

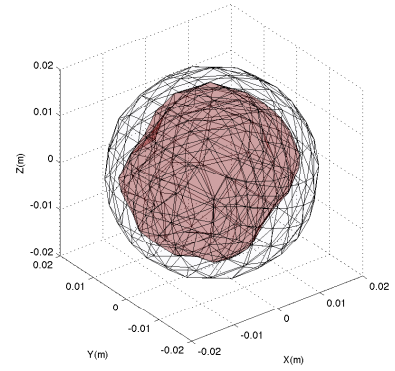


(d)

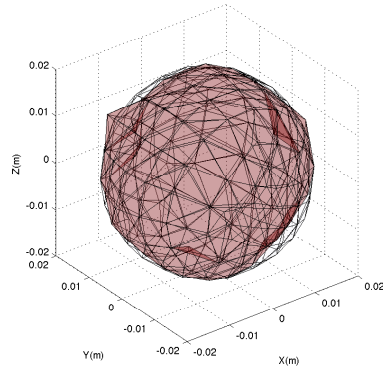
Figure 4.11: Reconstruction of a sphere at 15 GHz with a sphere initial guess of a 30 mm diameter, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 18th iteration.



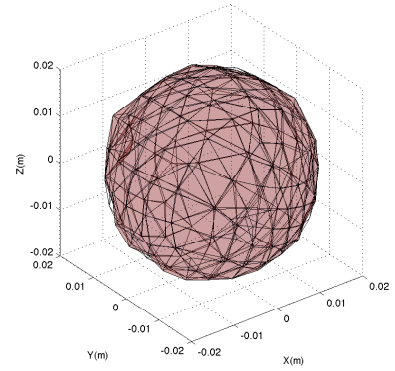
(a)



(b)



(c)



(d)

Figure 4.12: Reconstruction of a sphere at 15 GHz with a sphere initial guess of a 20 mm diameter, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the third iteration, (c) in the sixth iteration, and (d) in the 18th iteration.

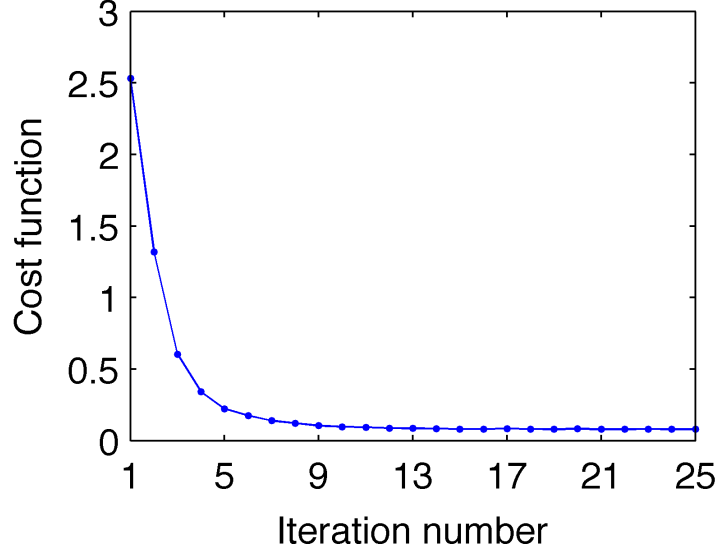


Figure 4.13: Cost function for the reconstruction of the star-shaped target in 25 iterations.

4.4.2 A Concave Target

The second target is a concave object shown in Fig. 4.1(d), which has 35 mm height. The reconstruction of this target is quite complicated because of the fact that some of the nodes on the surface of the initial guess object should get closer to the origin; while, the other nodes should move away from the origin. As evident in Fig. 4.16, the cost function is decreased to 1.2%, which shows that the shape of the target is fully retrieved in the 25th iteration. The shape reconstruction of the concave target is demonstrated in Fig. 4.15 for different iterations.

4.5 Reconstruction of a Conducting Cube

Due to the roughness of the cube's surface, the shape reconstruction of cubes is considered as a notable experiment case. Therefore, in this example, the shape reconstruction of a cube with an edge length of 26 mm is demonstrated. The

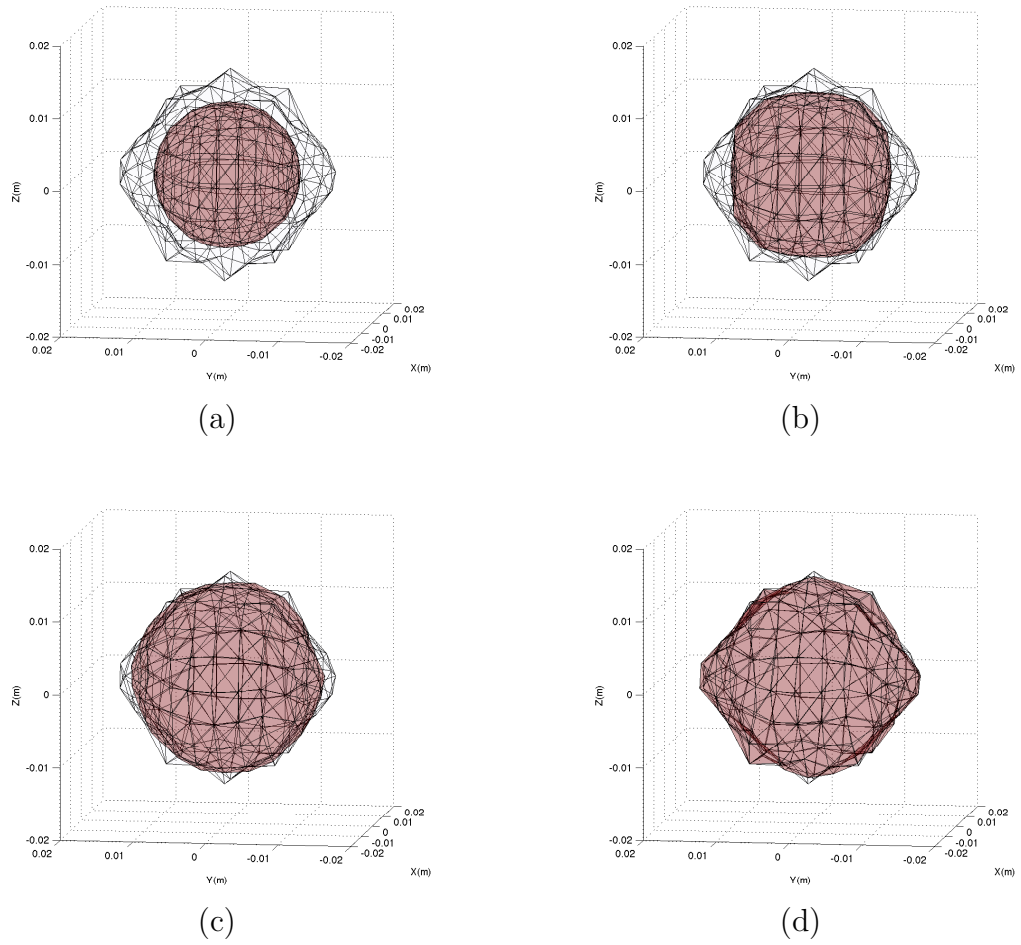
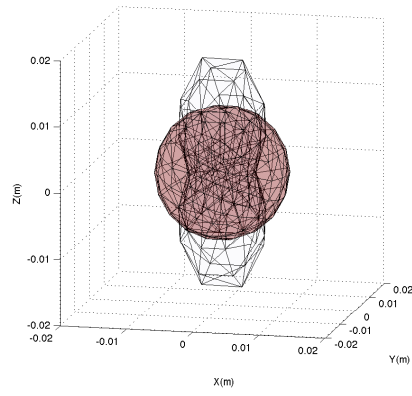
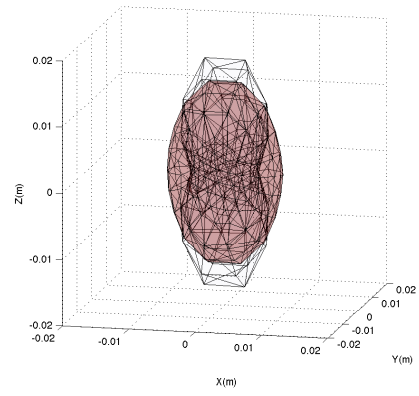


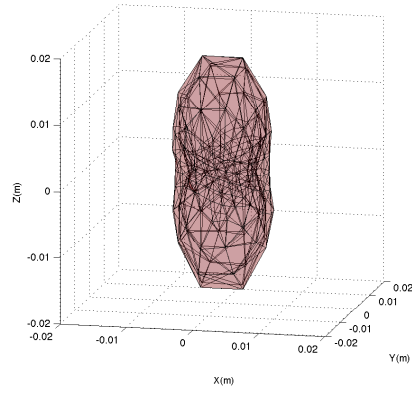
Figure 4.14: Reconstruction of a star-shaped object at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 25th iteration.



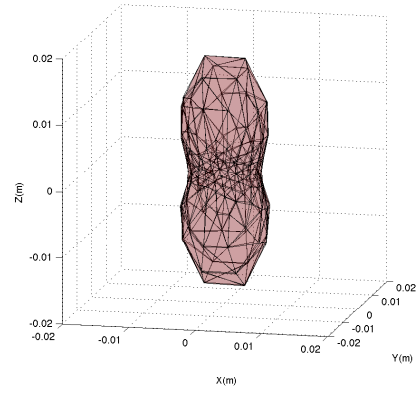
(a)



(b)



(c)



(d)

Figure 4.15: Reconstruction of a concave object at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 25th iteration.

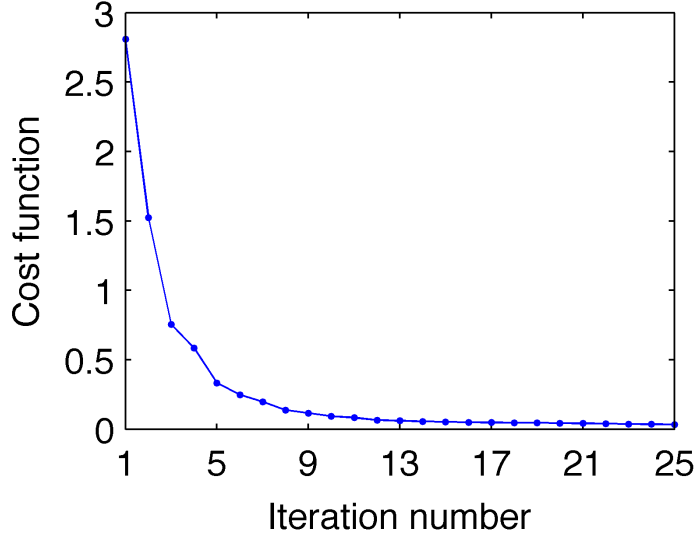
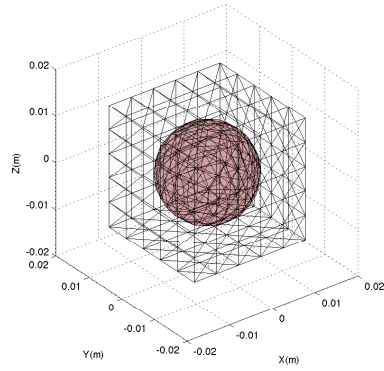
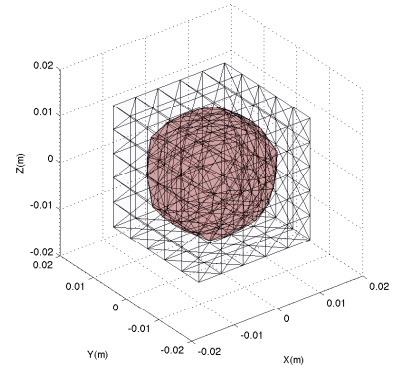


Figure 4.16: Cost function for the reconstruction of the concave target in 25 iterations.

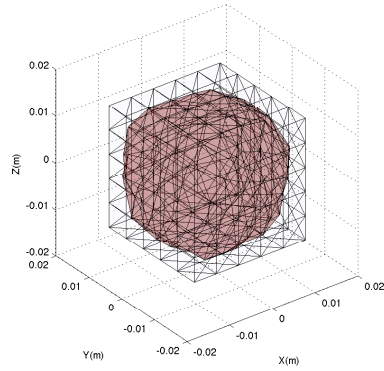
initial guess is a sphere with a radius of 10 mm and 156 nodes. We used 12 incident fields and we have done 52 measurements for each illumination. The illumination and scattering directions can be obtained in the Table 4.1. The cost function is reduced to 6.5% of its initial value, which is an acceptable number for this case, because we reconstructed a rough target with a smooth sphere. As we can see in Fig. 4.18, the general shape of the cube is obtained in the third iteration, and finally, the cube is roughly retrieved in the 25th iteration. To obtain better results for this case, more complicated modeling methods should be implemented.



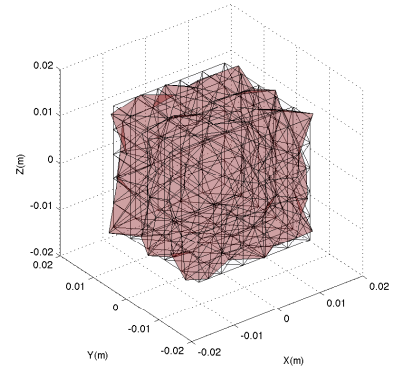
(a)



(b)



(c)



(d)

Figure 4.17: Reconstruction of a cube with an edge length of 26 mm at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the second iteration, (c) in the third iteration, and (d) in the 25th iteration.

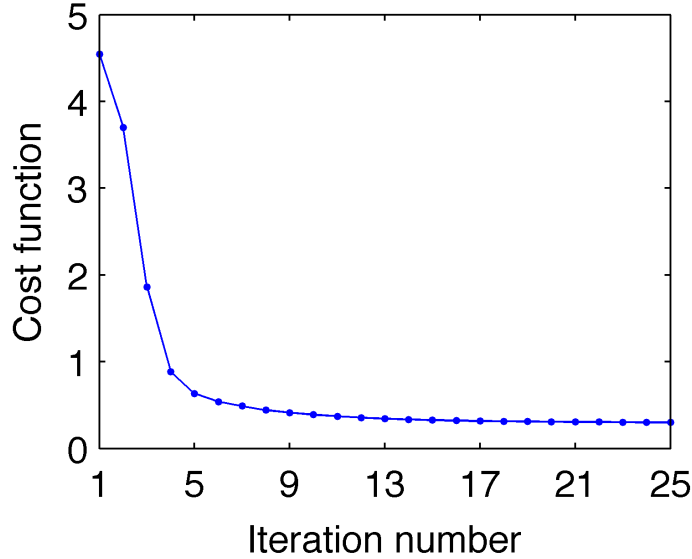


Figure 4.18: Cost function for the reconstruction of the cube in 25 iterations.

4.6 Reconstruction of a Target with a Shifted Initial Guess

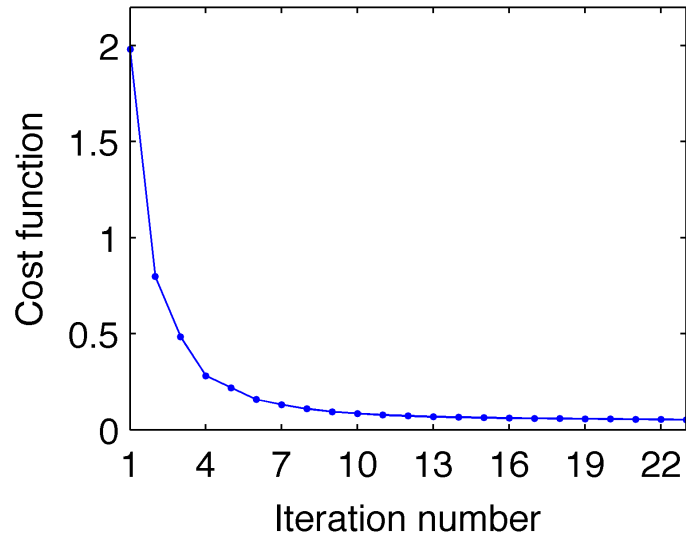
In all previous examples, the initial guess and the target were chosen to be concentric objects. However, in this example, we investigate the cases where these objects are non-concentric. The results of two different cases will be presented to discuss the performance of our shape reconstruction method. In both cases, the initial guess is a sphere with a 20 mm diameter, centered at the origin. The targets are spheres with 20 mm diameter in both cases; in the first case, the sphere is centered at (5mm, 0, 0), and in the second case, the sphere is centered at (10mm, 0, 0). Similar to the previous examples, 12 incident fields and 52 measurements have been done for both theta and phi polarizations in the directions shown in Table 4.1. The frequency is $f = 10$ GHz. Figure 4.19 shows the cost function of both cases. For the case with a 5 mm shift between the initial guess and the target, the cost function is decreased sufficiently, to 2.6% of its initial value; however, for the other case, with a 10 mm shift, the cost function is decreased

to 10.1% of its initial value, which shows that we could not retrieve the shape of target completely. As evident from Fig. 4.20, the evolving object reconstructed the target completely in the case with a 5 mm shift; however, for the second case, the evolving object reconstructed the general shape of the target, but it could not construct the whole shape of the target in detail. One explanation for the shape reconstruction method's failure may be that the initial guess object is reached at a local minimum. Possible solutions for such problems are discussed in the last chapter.

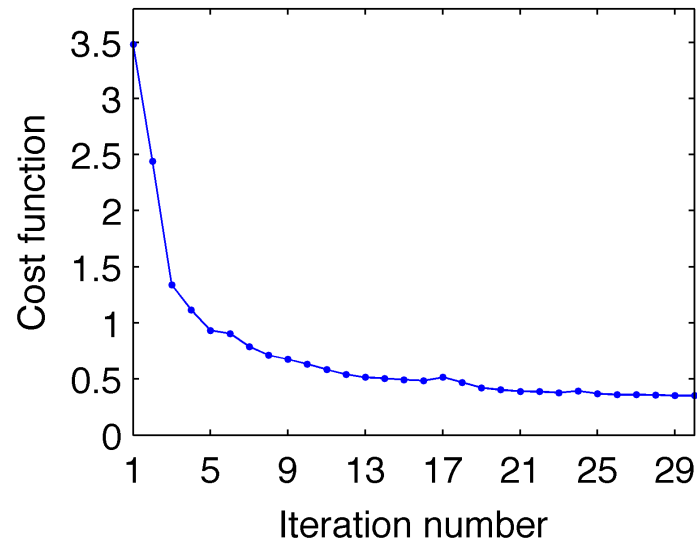
4.7 Reconstruction of Dielectric Objects

Although we presented a solution for the inverse scattering problem of the three-dimensional perfectly conducting objects, a similar solution can be used for the inverse scattering problem of the dielectric targets. One essential step in this solution is to find the scattered fields from the surface of the dielectric object. A forward solver that satisfies the boundary condition for dielectric media is used. One of the commonly used methods to formulate the scattering problems is surface integral equation (SIE). As a need for the numerical solution of SIE, the method-of-moment discretization of electric and magnetic current combined field integral equation (one of the possible choices of available SIEs for dielectric problems) yields a matrix equation. The multilevel fast multipole algorithm is used to accelerate the solution for this problem [22–24]. After providing an appropriate dielectric forward solver, a similar numerical method can be applied to calculate the Jacobian matrix. Then, by computing the gradient vector and the Hessian matrix, we can obtain the step vector to update the unknowns iteratively.

As a numerical example for the shape reconstruction of the dielectric objects, we will show the results of retrieving a star-shaped target shown in Fig. 4.1(c). The radius of the target changes between 11 mm and 15 mm. The relative permittivity and permeability of the target are assumed to be *a priori* information. In this example, the relative permittivity and permeability of the target are 4.0 and 1.0 respectively. The target is located in the free space. The initial guess is

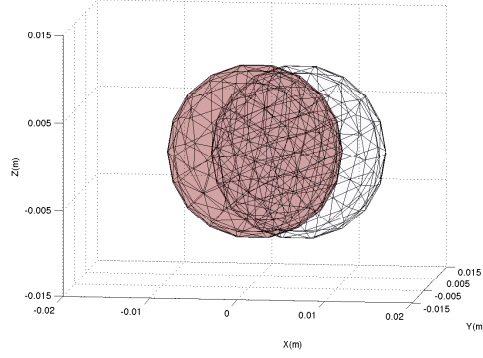


(a)

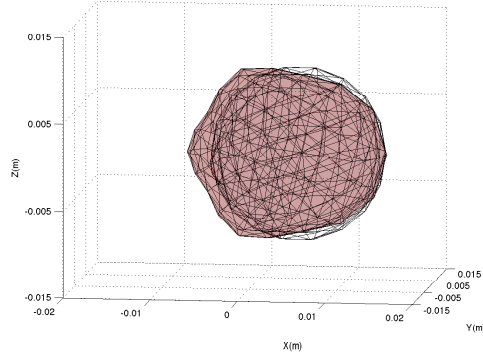


(b)

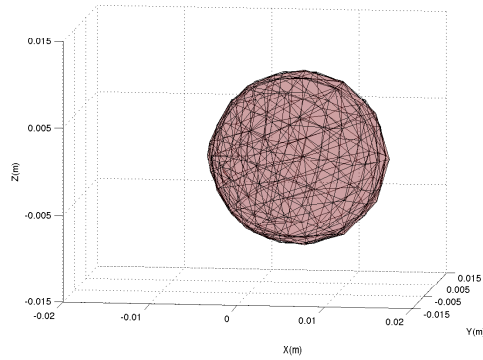
Figure 4.19: Cost function for the reconstruction of two sphere targets with (a) 5 mm and (b) 10 mm shifts with respect to the initial guess.



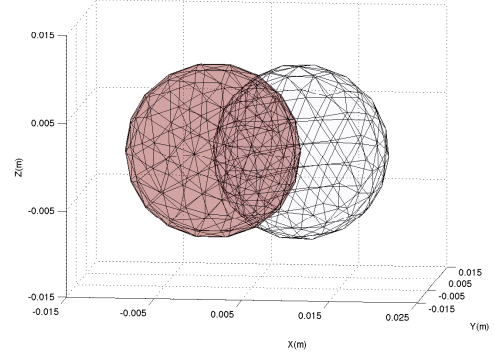
(a)



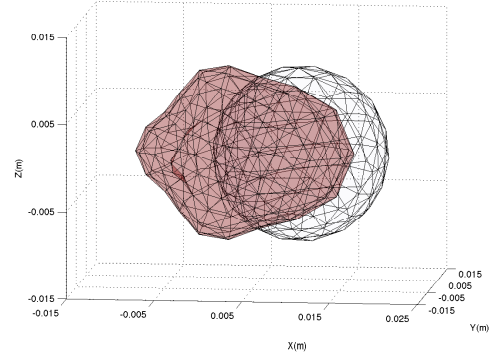
(b)



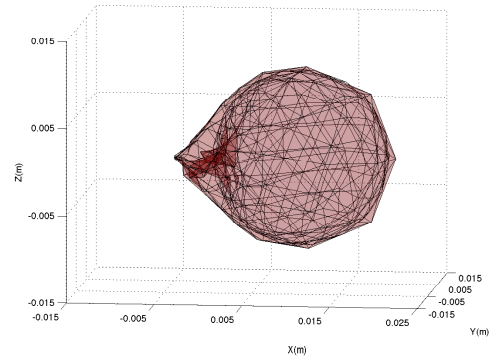
(c)



(d)



(e)



(f)

Figure 4.20: Reconstruction of two sphere targets with 5 mm and 10 mm shifts with respect to the initial guess, where the transparent object is the target and the red object is the evolving object. Reconstruction of the target with a 5 mm shift, (a) in the first iteration (initial guess), (b) in the second iteration, and (c) in the 23rd iteration. Reconstruction of the target with a 10 mm shift, (d) in the first iteration (initial guess), (e) in the second iteration, and (f) in the 30th iteration.

a sphere with a 10 mm radius and 156 nodes. The unknowns are in the form of (4.2), so we have 156 unknowns in this example. 26 scattering directions and 6 illumination directions of the Table 4.1 for both theta and phi polarizations are used in this example. As evident in Fig. 4.22, the cost function is decreased significantly in the first five iterations, and it is dropped to 2.4% of its initial value in the 28th iteration. The general shape of the object is retrieved in the first five iterations. The retrieved object in the 28th iteration is shown in Fig. 4.21(d).

4.8 Comments on the Numerical Experiments

In order to complete this chapter, we will present some possible explanations about the experimental results:

- In the experimental results, the receiving antennas are located in the same distance from the scattering objects. For the cases that we are not able to put the receiving antennas in equal distances from the target, we can use a weighting matrix in the cost function to adjust the impact of each antenna in the inverse scattering problem.
- We use 26 scattering directions to have a complete and uniform set of measurements. As it was explained in a numerical example in Chapter 2, if we reduce the number of receiving antennas, we may lose an important part of the scattering fields data. The direction of the receiving antennas can be chosen in different ways. One may locate them uniformly in the θ - ϕ domain. As evident in Fig. 4.23, we choose the location of the receiving antennas such that they are uniformly distributed around the target.
- Incident fields are illuminated from $\pm x$, $\pm y$, and $\pm z$ directions. Therefore, we will illuminate the targets from six main directions, uniformly. Note that by increasing the number of illuminations, we will obtain more information about the scattering object, but the duration of calculating the Jacobian matrix will be increased significantly.

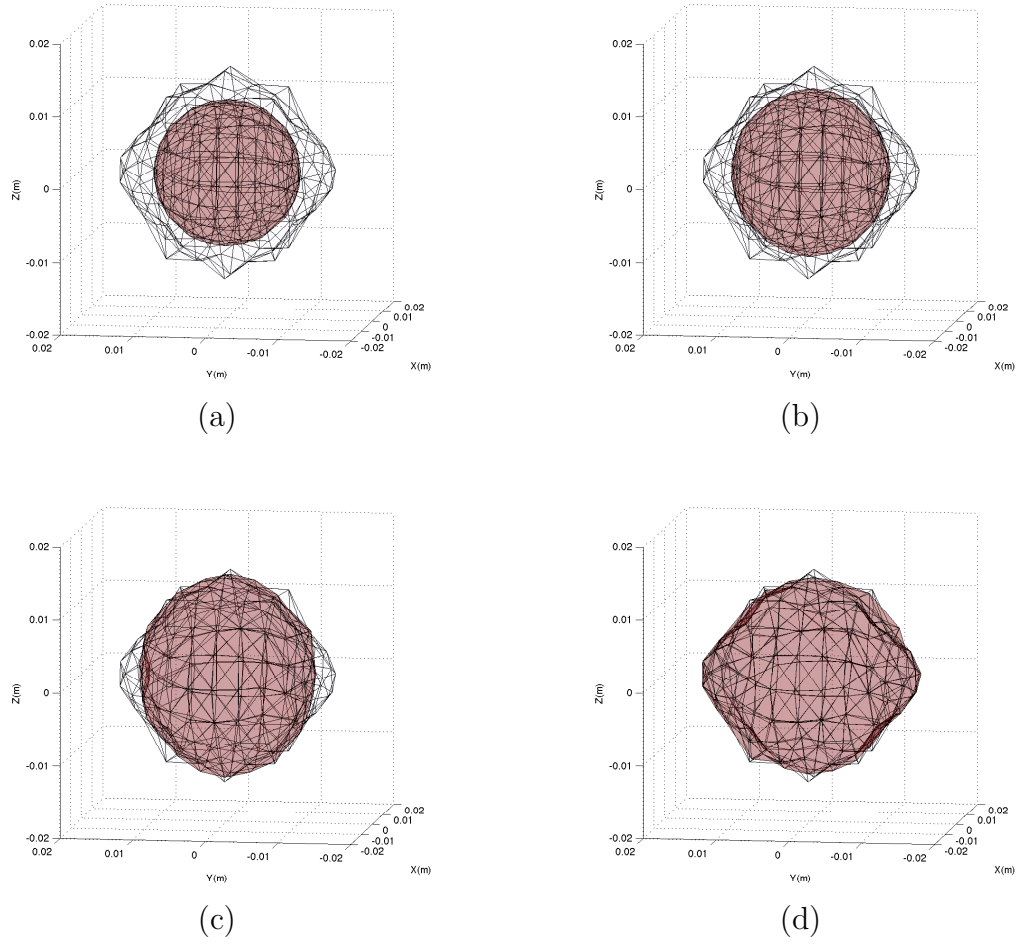


Figure 4.21: Reconstruction of a dielectric star-shaped object with the relative permittivity of 4.0 at 10 GHz, where the transparent object is the target and the red object is the evolving object, (a) in the first iteration (initial guess), (b) in the third iteration, (c) in the fifth iteration, and (d) in the 28th iteration.

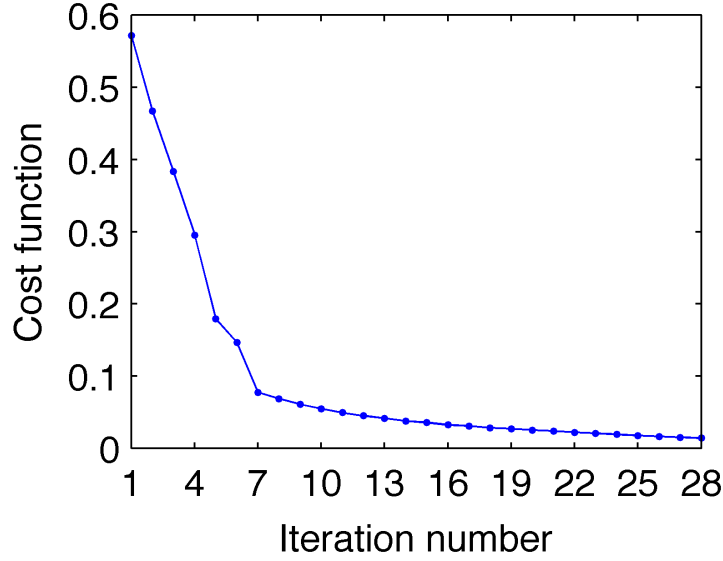
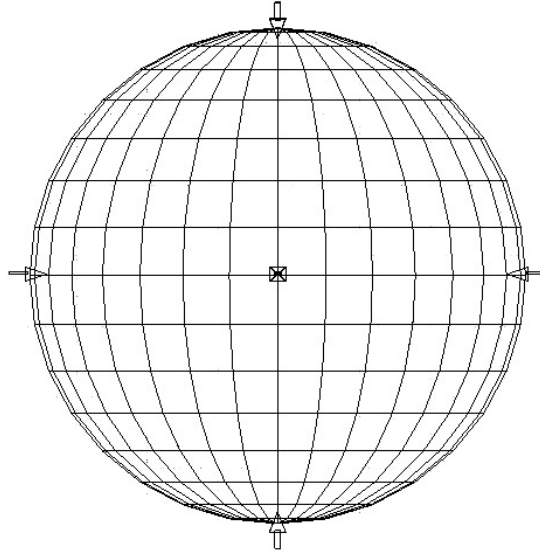
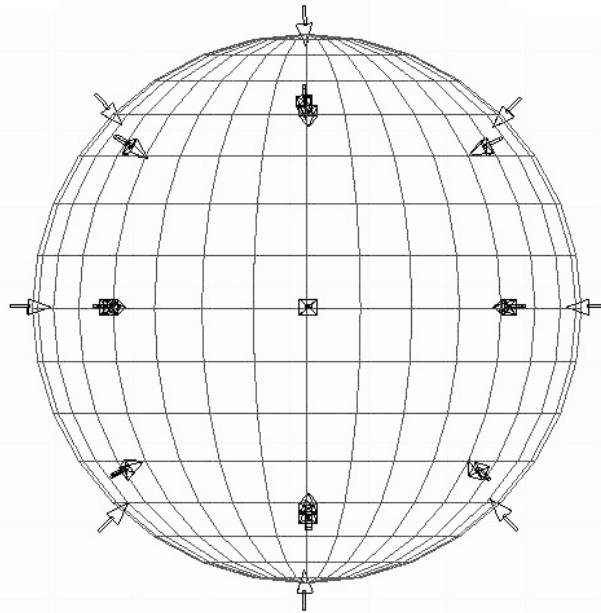


Figure 4.22: Cost function for the reconstruction of the dielectric star-shaped target in 28 iterations.

- All calculations are done on a workstation, which has two quad-core Intel Xeon X5355 processors with a clock rate of 2.66 GHz and 32 GB memory. The calculation associated with Jacobian matrix of an evolving object with 156 nodes, in the form of (4.2), requires the solution of 156 forward problems. The solution time of each forward problem is 4.3 seconds for 12 different incident plane waves. Therefore, the duration of each iteration is about $156 \times 4.3 = 670.8$ seconds. The other representation of unknowns, in the form of (4.1), takes about 2012 seconds rather than 670.83 seconds.



(a)



(b)

Figure 4.23: Distribution of the (a) transmitting antennas in six directions and (b) receiving antennas in 26 directions around the target.

Table 4.1: List of illumination and scattering directions.

Illumination directions (θ , ϕ) (degrees)	Scattering directions (θ , ϕ) (degrees)
(90.0 , 0.0)	(90.0 , 0.0)
(90.0 , 180.0)	(90.0 , 180.0)
(90.0 , 90.0)	(90.0 , 90.0)
(90.0 , 270.0)	(90.0 , 270.0)
(0.0 , 0.0)	(0.0 , 0.0)
(180.0 , 180.0)	(180.0 , 180.0)
	(54.7 , 45.0)
	(125.3 , 225.0)
	(54.7 , 135.0)
	(125.3 , 315.0)
	(54.7 , 225.0)
	(125.3 , 45.0)
	(54.7 , 315.0)
	(125.3 , 135.0)
	(45.0 , 0.0)
	(135.0 , 180.0)
	(45.0 , 90.0)
	(135.0 , 270.0)
	(45.0 , 180.0)
	(135.0 , 0.0)
	(45.0 , 270.0)
	(135.0 , 90.0)
	(90.0 , 45.0)
	(90.0 , 225.0)
	(90.0 , 135.0)
	(90.0 , 315.0)

Chapter 5

Conclusion

In this thesis, we study the inverse scattering problem of three-dimensional PEC objects. A general framework for the shape reconstruction of perfectly conducting objects is developed and suitable modelings of the evolving objects are introduced.

In Chapter 2, the cost function was defined as the main parameter of our minimization problem and the general form of the cost function was introduced. The influence of two basic terms of the cost function in the inverse scattering problem is discussed. The prominent role of the measurements in the performance of the cost function in the inverse scattering problem was reviewed, and finally, the Taylor-series expansion of the cost function was introduced to show how the cost function changes by updating the vector of unknowns.

In Chapter 3, we used the Newton minimization approach to find the solution for the shape reconstruction problem. To update the unknowns iteratively, two different methods were developed, and their strong and weak aspects were discussed. In addition, we obtained the expression of the Hessian matrix and the gradient vector of the cost function in terms of the residual vector and the Jacobian matrix of the residual vector. In the end, a numerical technique for calculating the Jacobian matrix was presented and the validity of this technique was fully discussed.

In Chapter 4, the shape reconstruction of different conducting and dielectric objects was presented. The numerical results of the shape reconstruction for the cases that the initial guess and target are concentric objects were highly satisfactory. In these cases, the general shape of the target was reconstructed after only three iterations and the cost function decreased significantly, which is a remarkable achievement compared to the similar works in the literature. However, it was shown that we may not be able to fully reconstruct a target with an initial guess located far from the target because the error of modeling the cost function by a quadratic function, which is in the order of $\mathcal{O}(\| \mathbf{p}_k \|^2)$, will be large in such cases. In Chapter 4, we also discussed the duration of the reconstruction process, which is relatively large. Therefore, a compact representation of the unknowns was introduced to decrease the duration of each iteration, and consequently, the total duration of the shape reconstruction of the target. Finally, a shape reconstruction method for the dielectric objects was briefly introduced, and also the corresponding numerical results were presented to analyze the performance of our shape reconstruction method for the dielectric targets.

Future works may include possible efforts to reduce the duration of the numerical calculation of the Jacobian matrix. As indicated earlier, it takes a long time to calculate the elements of the Jacobian matrix because a large number of forward problems must be solved to calculate the Jacobian matrix at each iteration; thus, we should find an appropriate way to decrease the number of forward problems or a solution to reduce the duration of the forward problem. In addition, we can replace our numerical method for calculating the Jacobian matrix with any other efficient method that calculates it in less time. Furthermore, some improvements can be made to the modeling of the cost function because the quadratic modeling may not be always sufficient to reconstruct the target's shape.

In order to simulate the real-life inverse scattering problems, we should put the receiving antennas in the near-field region, and also we should use available incident waves in real-life cases. Therefore, we should change the forward-solver

program in such a way that we can illuminate the targets with the desired incident fields and measure the scattered electric fields in the near-field region. We may need to model the receiving and transmitting antennas, which are located in the near-field region, to include their possible effects in the measurements. Additionally, instead of using plane waves for illuminating the target, we can use other types of excitation sources, such as delta-gap sources and dipoles. Finally, we should include the effect of noise in our measurements to be able to simulate the real-life problems.

In future works, we should improve the shape reconstruction method of the dielectric objects to reconstruct the dielectric properties of the targets, such as their permittivity and conductivity. Additionally, we should develop an improved version of the presented method to detect multiple disconnected targets. For this goal, more complex techniques for modeling the evolving objects should be incorporated. In addition, we should improve our shape reconstruction method to be able to retrieve the shape and location of the targets with different sizes in a wide frequency range.

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