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NODE LOCALIZATION IN WSN BASED ON RANGE MEASUREMENT AND REFERENCE LOCATIONS

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Master's Thesis

Supervisor

Asst. Prof. Dr. Oğuz KARAN

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The thesis NODE LOCALIZATION IN WSN BASED ON RANGE MEASUREMENT AND REFERENCE LOCATIONS prepared by JUMAN MOHAMMED YAHYA AL-ANI and submitted on 27/04/2023 has been **accepted unanimously** for the degree of Master of Science in Information Technology.

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I hereby declare that this thesis/dissertation meets all format and submission requirements of a …………………………… (Master's/Ph.D.) thesis/dissertation.

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Juman Mohammed Yahya Al-ANI

Signature

DEDICATION

To My Beloved Family (My Dad, Mom, and Sisters), and My Teachers In The Past, Present, And Future. To Asst.Prof.Dr.Oguz Karan, To My Friends And To All Those Who Truly Believed In Me And Never Let Me Down Throughout The Way.

ABSTRACT

NODE LOCALIZATION IN WSN BASED ON RANGE MEASUREMENT AND REFERENCE LOCATIONS

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This research presents a decentralized method for addressing the difficulties associated with the n-dimensional static sensor network node localization. Consider a network in which n+1 nodes are anchored in place, but the remaining nodes are fully free-floating without any connections to other nodes. By applying barycentric coordinates obtained from range measurements, the non-convex node localization problem may be reduced in the noiseless situation into a conventional linear system of equations. This is accomplished by the use of barycentric coordinates. When independent zero mean Gaussian noise is introduced to range measurements, all barycentric coordinates become dependent random variables without a known standard distribution, and the distributions of these random variables may not be identical to one another. To solve the problem of noisy range-only localization, we provide a method that is based on online optimization techniques and a distributed online gradient descent algorithm.

Keywords: Barycentric Coordinates, Cayley-Menger Determinants, Localization, Wireless Sensor Networks, Angle of Arrival.

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ABBREVIATIONS

WSN	:	Wireless Sensor Network
MEMS	:	Micro-Electro-Mechanical Systems
DSN	:	Distributed Sensor Networks
OCARI	:	Open Communication Protocol For Ad Hoc Reliable Industrial Instrumentation
GPS	:	Global Positioning System
TOF	:	Global Positioning System
TDOA	:	Time Difference of Arrival
RSSI	:	Received Signal Strength Indicator
AOA	:	Angle of Arrival
UWB	:	Ultra-Wide Band
CPE	:	Convex Position Estimation
APIT	:	Approximate Point-In-Triangulation
DV-hop	:	Distance Vector-Hop
IoT	:	Internet Of Things
VANETS	:	Vehicular Ad Hoc Networks
HAN	:	Home Area Network
MBOTS	:	Mobile Robot
RS	:	Robotic Sensor
DV	:	Distance Vector
APIT	:	Approximate Point In Triangulation

GC	:	Gradient Centralization
DOGB	:	Distributed Online Gradient Descent

1. INTRODUCTION

The development of wireless communication technology has made it possible to produce sensors that are multifunctional, affordable, and efficient in their use of energy. These sensors are also able to communicate data over shorter distances and use fewer resources. In a wireless sensor network, the individual battery-powered sensors, which are referred to as "sensor nodes," collaborate to conduct calculations and share data with one another.

The omnipresent monitoring and control of buildings, public spaces, and the environment are now made possible by the widespread deployment of intelligent sensor networks. In addition, there is a range of applications for them that can bring cutting-edge technology to the areas of surveillance and the military. The integration of sensors into machines, buildings, and environments, in conjunction with the effective conveyance of the data that is gleaned from those sensors, has the potential to provide enormous benefits to businesses. Recent developments in wireless communication have made it possible to create multi-purpose sensors that are both tiny and able to communicate over very short distances. These sensors are also affordable, power-efficient, and able to serve several purposes. Sensor nodes are the building blocks of wireless sensor networks. These sensor nodes are very small, battery-powered devices that can communicate and share data with other nodes in the network as well as execute calculations based on the signals they receive.

Large-scale deployments of intelligent sensor networks now make it possible to control and monitor virtually every aspect of large ecosystems, as well as individual buildings and urban regions. In addition to this, they may be implemented in a wide variety of settings, such as the creation of cutting-edge technology for defense and surveillance. The integration of sensors into machines, buildings, and the environment around them, in conjunction with effective data exchange, has the potential to provide enormous benefits to enterprises.

By deploying a sensor network, which is comprised of sensing, processing, and communication components, an administrator can monitor, instrument, and react to occurrences and phenomena that occur inside a certain environment. Over the course of a wireless link, several gateways, also known as "base stations," in a wireless sensor network (WSN) trade data with a large number of sensors. Following the completion of the data collection process, the node will either directly or indirectly relay the information to the base station in a compressed format. When they reach the base station, the bits that have been transferred are put to productive use.

1.1 INTRODUCTION

Because of recent developments in wireless communication and MEMS (Micro-Electro-Mechanical Systems), wireless sensor networks are currently a very popular subject of discussion in academic circles [1]. The early investigations in this topic were motivated by defense applications like the "Distributed Sensor Networks (DSN) program" [2] and "Smart Dust" [3]. Researchers at Berkeley University devised the "PicoRadio project" [4] to establish a low-power and low-cost ubiquitous sensor network that may be deployed in civilian areas. They hoped that this network would be useful. Since then, other solutions for sensor networks have been proposed, such as the one called "OCARI," which was created by researchers in France [5]. Recent research has focused on the potential applications of wireless sensor networks in the field of civilian applications. Some of these applications include hospital surveillance [6], smart home [7] [8], environmental monitoring [9] [10], and object tracking [11] [12]. These are just a few examples.

A network of sensor nodes that can communicate with one another wirelessly is referred to as a "wireless sensor network," and it is this type of network that is described by the phrase. As can be seen in Figure 1.1, each individual sensor node is made up of a variety of different parts and pieces. Sensor nodes are generally very tiny electrical devices that get their power from localized battery packs. It possesses sensors that can detect several aspects of the environment, including temperature, sound, vibration, pressure, humidity, and motion, among other things. A wireless transceiver is added in order to permit a bidirectional connection with additional sensors.

One of the characteristics of a sensor node is its small size. Other characteristics include low power consumption, constrained computing power, near proximity communications, and limited memory storage.

When paired with cutting-edge networking protocols, sensor nodes may produce a broad variety of wireless networks that improve the quality of life for people in a variety of different ways. For example, sensor nodes can be employed in hospitals to monitor patients' vital signs and other important data. With the help of these sensor nodes, it is possible to monitor and send information regarding a patient's heart rate and blood oxygen level [13]. With the assistance of the wireless network that is created by these nodes, as shown in Figure 1.2, medical professionals are able to monitor their patients at any time and from any location using a computer or a smartphone.

Wireless sensor nodes, in contrast to more traditional wireless devices such as mobile phones, require just a bidirectional connection with other nodes in their near proximity. Instead, of relying on a network that has already been set up, each sensor node makes its own contribution to the overall infrastructure. Through the use of this architecture for ad hoc networking, numerous nodes are able to connect with one another in a mesh-like fashion. Due to the dynamic flexibility of the mesh design, it is possible to compensate for failed nodes while also allowing for the inclusion of more nodes.

Several applications need users to be aware of the location of each sensor node inside the network. This is because consumers frequently desire not only information about what occurs, but also the location of events of interest or the location of the objective itself. During a disaster relief operation that makes use of WSN, for example, to identify survivors in a building that has fallen, sensors must provide monitoring information together with their position [13] [6]. In the event of an emergency requiring hospital supervision, knowing the patient's location can assist medical staff in getting to the appropriate location as soon as feasible. On the other hand, the accessibility of the position characteristics of sensor nodes is crucial to the success of many network management tasks. This is especially true for the process of routing, which has seen the presentation of a number of geographical algorithms [14]. Topology control [15] adjusts the network structure in order to decrease power consumption [16]. Geolocation data is used in this process.

There have been many different methodologies put forward in the hopes of locating specific nodes inside WSNs. There are three distinct interpretations of this statement. Techniques may be broken down into two primary groups: range-based and range-free. These groups are differentiated from one another based on whether or not they need accurate ranging.

Before range-based systems can function properly, the range information between the essential devices—whether it be the distance or the angle—must be accurately recorded.

Methods such as trilateration and triangulation could be utilized by [17] in order to pinpoint the needed position. Techniques such as Received Signal Strength Indicator, Time of Flight, and Angle of Arrival are the most common types of range finders. A number of different localization systems have seen rapid development in recent years. The Global Positioning System (GPS) is probably the most well-known example of a range-based method that makes use of TOF or TDOA [18]. TDOA stands for "Time Difference of Arrival." GPS devices, on the other hand, are notoriously power-hungry and lose most of their usefulness once you enter a building. GSM is another system that employs the RSSI and AOA methodologies (Global System for Mobile Communications). UWB (Ultra-Wide Band) technology, which is used in today's cutting-edge systems and gives exceptionally precise time-of-flight measurements, is at the forefront of technological advancement. In general, range-based methodologies suffer from two significant shortcomings. To begin, range data is famously prone to be affected by noise, environmental variations, and multipath fading. Second, additional range devices are often necessary, which typically increases both the total cost and the amount of energy utilized.

The range-based technique considers the physical spacing or orientation of nodes, whereas the range-free strategy places more emphasis on the connections that exist between the nodes. In this system, nodes that are aware of their placements are referred to as anchors, while all other nodes are referred to as regular nodes. Anchors are distinguished from regular nodes by the fact that they are aware of their positions. Anchors always remain in the same location, in contrast to normal nodes, which might move around. After collecting information on its connections and anchors, a typical node will first work out its own location for the network. In this scenario, the hop counts between nodes have the potential to act as a proxy for the connection information associated with a node, which is symbolized by the letter N. The degree to which this node N is connected to other nodes is used as a proxy for determining how close it is to those other nodes. For example, the nodes that are within the transmission range of node N are referred to as their neighbors since they are just one hop away. Since the range-free technique does not require any ranging information, it is suitable for usage in wireless sensor networks that have a limited budget. Range-free schemes also have the advantage of being robust, which means that the connection data between nodes in the network is less likely to become damaged as a result of events that are external to the network. Consequently, the range-free system is the focus of the research at this time.

Throughout the years, several range-free approaches like Centroid, CPE (Convex Position Estimation), Approximate Point-In-Triangulation (APIT), DV-hop (Distance Vector-hop), and many more have been presented. On the other hand, the methods that are now accessible do not have a precision that is enough. As a result, there is more work that has to be done to improve the accuracy of localization.

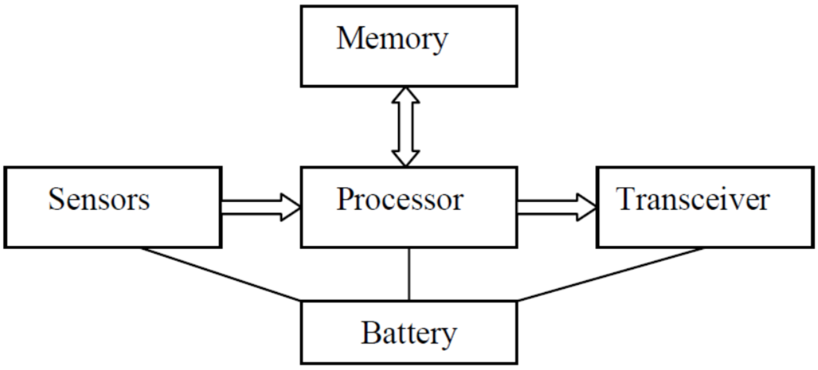

Figure 1.1: Sensor Node Structure.

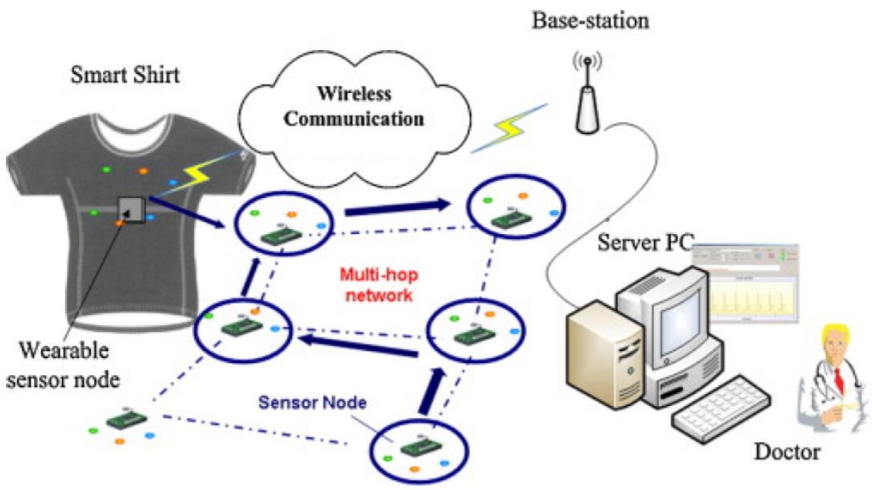

Figure 1.2: Hospital Monitoring WSN.

1.2 BACKGROUND

The general public has shown a substantial amount of interest in expansive and varied wireless network architectures. When talking about "heterogeneous wireless networks," it is referring to networks in which no two devices are the same and where the software and hardware platforms on which the devices operate are vastly different from one another. Some of the applications of these networks that are brought up in conversation the most frequently include the Internet of Things (IoT), VANETs (Vehicular ad hoc networks), HAN (Home Area Networks), mBots (Mobile Robot), and RS (Robotic Sensor).

The technologically advanced home of today serves as an excellent illustration of the Internet of Things (IoT) due to the fact that it contains numerous pieces of electrical and household appliances manufactured by a variety of companies that are capable of communicating with one another and/or with user interfaces, which are typically developed as apps for mobile smartphones [19], [20]. The popular Nest thermostat is an early example of this sort of technology in action. It was originally supposed to function as a centralized hub for controlling the temperature inside of a home, replete with the capability to learn and integrate over WiFi with other devices. Later on, the same firm also began selling carbon monoxide and smoke detectors, in addition to a collection of interconnected cameras. It is now touted as being interoperable with up to 3,000 different devices (including Google's own voice interface, "Hello Google") for hands-free operation, and Alphabet Inc., the parent company of Google, controls the rights to the trademark as well as the underlying technology. Amazon.com, Inc., another industry leader, also creates collaborations with other firms in order to make its "Alexa" speech interface compatible with a broad variety of devices on the same WiFi and Bluetooth networks as those used by those other businesses.

A growing trend in the market for home automation is the introduction of personal healthcare gadgets and items for everyday use, such as electric toothbrushes and Bipap machines, that connect with smartphones to provide user-friendly management interfaces and historical data. Other examples of this type of product include fitness trackers and blood pressure monitors. Taking this concept to its logical conclusion, it is possible that in the not-too-distant future, autonomous transportable robots may be used to aid with the medical care of the local people [21], [22]. They predict that these robots will become commonplace in homes where care is required for elderly people and infants, particularly for tasks that are considered a chore by their families or in which the family members are not trained to perform or are unable to perform, such as the cleaning of human waste, the application of intravenous medicines, or physically lifting impaired people. This is especially true for tasks that are considered a chore by their families.

The architectures of communication between cars as well as the architectures of communication between vehicles and permanent infrastructure, such as roads, are the fundamental building elements of VANETs. The development of these protocols will allow for the provision of new vehicle features and road services, such as the enhancement of the functionality and dependability of autonomous vehicles, the management of traffic, and the exchange of information regarding the existence of potential risks to vehicles. Despite the fact that several automobile manufacturers, technological companies, and academic organizations are constantly exploring and implementing such technologies, there are presently no communication and sensor network protocols and standards that are generally acknowledged. Researchers are putting forth a lot of effort to get self-driving cars ready for public usage, despite the fact that autonomous vehicles are not currently popular in the academic community. The end goal is for autonomous vehicles to be able to self-coordinate and plan their operations in a cooperative manner with other vehicles and with the road infrastructure. This would allow for things like local and global traffic management and control, the propagation of road alerts throughout the network so that vehicles can plan their actions in advance, and vehicle platooning in formations ranging in size from small (road-wide) to large. This would be the ultimate goal (city-wide). Due to the increased redundancy of sensors, network connectivity, and system intelligence, there will be less and less human involvement with the system. As a result, car accidents that result in injured or dead passengers or pedestrians will be extremely rare in the near future, possibly within a few decades [23].

The prospect is that automobiles of the future will be mobile robots and you might disregard the potential that robots could be utilized in the automation of your house, but you can't ignore the tremendous uprising of drones that is occurring right now. Both large and small businesses are investigating the ways in which they may use drone technology in their operations to reap the potential benefits it offers. Amazon.com, Inc. is yet another well-known corporation that is investigating the possibility of utilizing drones for package delivery in heavily populated regions. Another important field where drones are being researched and put to use is the one involving law enforcement, search and rescue operations, and the military forces. Exploration, infiltration, target search and track, target pacification, package, item, and ordnance delivery are some of the many possible applications for drones. Drones may also be employed in a number of other scenarios. The general public and certain politicians are concerned about its utilization since, when paired with cameras and other recording devices, it may lead to intrusions of privacy. Additionally, it can lead to accidents, damage to property, and criminal activity.

Even if there are many goods and pieces of software that are currently available for purchase and put to use, the great majority of them are still in the preliminary stages of research and development. The attention is concentrated on one facet of the many different kinds of research and development that are relevant to us, namely the requirement of comprehending the particular applications of their findings. It's possible that those who use the devices and those who maintain them, in addition to the equipment themselves, have an interest in fulfilling this criterion.

1.3 PROBLEM STATEMENT

The ability of a system to locate itself in its surroundings can affect both its behavior of the system and its utility of the system. It is possible that sensors and robots will not be able to carry out the duties for which they were designed if they were placed in an environment that is foreign to them, just as people would face the same challenges. One example that is easier to understand is that of a network of air sensors, the purpose of which is to collect meteorological data for general use. If their positions are not known, the data that is gathered by these sensors is almost completely meaningless. It is cause for serious concern to be aware that a storm is approaching but to have no notion where the storm is or where it is heading. It is important to note that in this scenario, all sensors will be put in permanent places that are equipped with GPS. This will make it possible to utilize a single measurement for an endless period. But what about networks where anything like this never happens? For example, a wireless network that is contained within a building or that has mobile nodes would fall under this category.

1.4 OBJECTIVES

The precise localization of network nodes, particularly fixed nodes that are able to independently detect their position in space, is the primary focus of the efforts at this time. Because they are so widely used in residential, commercial, and industrial contexts, static networks are where to start the investigation into the many potential solutions to the problem of localization. On the other hand, it may be able to make a significant contribution to the theory of static network localization by developing a method that is applicable in every possible spatial dimension and lays fewer restrictions on the network that is being used. One of the arguments that might be made against relying exclusively on range measurements is the fact that utilizing both range measurements and bearing measurements can yield more comprehensive information regarding inter-node placement. On the other hand, one may argue that the restriction to range measurements alone is really a benefit because it reduces the number of sensors that are required. This is not to suggest that we are not interested in investigating the possibility of including bearing measurements into the method in the near future; it is possible that doing so will enable us to build a strategy that is more efficient.

1.5 PROPOSED METHOD

The proposed method is able to localize nodes in a static network provided that the actual and noiseless locations of d + 1 nodes of the network in d dimensional Euclidean space (these nodes are referred to as anchor nodes), the range measurements between all neighboring nodes, and the requirement that each node must have at least d + 1 neighbor are all provided. It is possible that certain nodes that meet these characteristics are not localizable if their neighbors are unable to be localized. Because of this, it is feasible for a single untraceable node to affect a significant portion of a network. As a consequence of this, localizable nodes are responsible for the formation of a network that contains anchor nodes and in which every node has a minimum of d + 1 neighbors. Despite these criteria, the relative location of nodes to anchors is not bound, as is the case with other techniques. This is crucial because it indicates that the technique may be used to localize networks with a greater range of nodes, independent of where the anchors are situated. This opens up a lot of possibilities for future applications. A straightforward illustration is provided in three-dimensional space (3-D), where other approaches would call for two sets of four anchors, of which no more than three can be shared between the sets, and two iterations of the algorithm in order to locate nodes in a rectangular prism-shaped environment (a common room with rectangular floor plan).

1.6 THESIS OUTLINE

In the first chapter, there is a discussion of a potential examination of the problem and its history.

In Chapter 2, we will discuss the history of research into node localization solutions, and then in the next chapter, we will proceed to a more fundamental knowledge of the area.

In Chapter 3, we will go more deeply into the methodology that was used in the study so that you may have a comprehensive understanding of the significance of the findings.

The primary objective of this chapter is to conduct an investigation into the experimental implementations of many models for asymmetric data streams and to draw comparisons between those implementations.

The conclusion, as well as a summary of the findings and some recommendations for more study, will be presented in Chapter 5.

2. LITERATURE REVIEW

2.1 RELATED WORK

In order for a heterogeneous wireless network to perform as its designers, managers, and end users have envisioned it, the network must fulfill a number of predetermined conditions. Nodes in such networks have the flexibility to be either immobile or mobile, and they may be outfitted with whatever sensors are necessary for decentralized or centralized operation, depending on the needs of the network. It is probable that the way in which they do business will be impacted not just by their location but also by the neighborhood that surrounds them. This necessitates that every node in a network is conscious of its location in both space and time in relation to some sort of established reference system. In harsh operational conditions, where GPS access is denied, the surroundings are featureless, and the sensed information is lower-dimensional than the sensor states of interest (for example, using bearing-only or range-only measurements for three-dimensional (3-D) position and orientation estimation), solving relative localization robustly is particularly difficult but essential. In these conditions, the surroundings are featureless. It is frequently required to know the locations of a small subset of the nodes in a frame of reference, which are referred to as anchor nodes. This is so that an accurate identification of the frame of reference can be made.

All of the other nodes are referred to as "unknowns" throughout this work because they are not clearly addressed elsewhere in the text. In order to find a solution to the problem of wireless localization in dimensions ranging from two to three and even higher generic dimensions, researchers have examined a wide variety of measurement methodologies and forms of noise distributions. Some methods [24] rely on both range measurements and bearing measurements, whereas others, such as bearing-only [25] and range-only [26], rely on just one of these observations. In most cases, algorithms start off in a centralized fashion [27], which means that all of the network information is sent to a "central" node that is responsible for the more intensive computational needs to localize all of the network nodes. If this is not the case, the algorithm is considered distributed [28], which means that each node continues to be responsible for its own localization based on shared local information.

The advancement of research into localization over the course of many years has resulted in the creation of a variety of diverse methodologies. Triangulation, often known as trilateration, is one of the oldest and most extensively used techniques. It may be found in [29], [30]. The latter is summed up as the iterative solution of a non-linear system of equations that translates range measurements between nodes to Euclidean distance estimations for each unknown node. This system of equations maps range measurements to estimate the distance between each unknown node. In the following example for two dimensions, the set of equations is demonstrated that must be satisfied to determine the location of node l with regard to nodes I and J. (equation 2.1).

$$\begin{cases} \|x_l - x_i\|_2 = z_{li} \\ \|x_l - x_j\|_2 = z_{lj} \\ \|x_l - x_k\|_2 = z_{lk} \end{cases} \tag{2.1}$$

Trilateration approaches are iterative, as was previously stated, and the focus of each iteration is on the localization of a single unknown node utilizing either known anchor nodes or unknown nodes that have been identified before. On the other hand, there are methods in which all of the data is gathered, and all of the nodes are localized at the same time, either in a centralized or distributed fashion. There are several references to the Kalman filter [31], Multilateration [32], MDS MultiDimensional Scaling [33], and barycentric coordinates [34]. The Multidimensional Scaling and Barycentric Approaches are two of the ones that are brought to your attention here. The MDS serves as the benchmark against which to judge the newly suggested approach, which is based on barycentric coordinates. Minimizing the following function as part of an optimization problem is the conventional approach to dealing with MDS (equation 2.2).

$$Stress(x1; \dots; xn) = \left(\frac{\sum_{i,j} \left(Z_{ij} - \|x_l - x_j\|_2 \right)^2}{\left(Z_{ij} \right)^2} \right)^{1/2} \tag{2.2}$$

MDS methods have an advantage in that they can be solved by manipulating algebraic equations in matrix form. This makes the solution to the minimization problem equivalent to the eigenvalue and eigenvector decomposition of a matrix, which is a problem for which there are optimized algorithms. Despite the absence of a convex or linear function to optimize, MDS methods have this advantage. Their solution has a number of flaws, one of which is that it is heavily dependent on the accuracy of the initial assumptions. It is possible that it will be necessary to carry out the operation a number of times before a good answer can be discovered. This is due to the potential expenses that are introduced to solutions with stress values ranging from minor to large by the use of random initialization.

The computation of barycentric coordinates has been designed with the intention of concealing any non-linearities that may exist in the underlying issue. After one has identified the barycentric coordinates of each node in the network, the process of solving the noiseless system is as straightforward as the process of solving a linear system. Khan et al. were the ones who first presented this methodology [35]. It is intriguing to observe how the system divides itself among the numerous network nodes on its own. Each node is able to calculate its own barycentric coordinates utilizing only the data that is accessible locally, and this phenomenon is fascinating to observe.

When noise is added to the range measurements, this technique becomes more difficult to answer since the computation of all barycentric coordinates requires the solution of an enormously non-linear set of equations. In addition, the addition of even uncomplicated Gaussian noise to the range measurements makes it difficult to arrive at a closed-form solution for the barycentric coordinates.

In order to solve this last problem, Khan et al. [36] came up with a solution that was based on averages.

There have been other proposals made for different iterations of the original barycentric algorithm [35], [36]. To be more specific, Khan et al. [35] provide us with a broad range of research that expands this framework in static and mobile networks as well as other applications where unknown nodes are securely contained within the convex hull of anchors. Additionally, Diao et al. [26] presented a method that operates in two dimensions and enables anchors to be positioned freely among unknown nodes. Unfortunately, their technique for overcoming the localization issue relied on a geometrical identification of all criteria necessary to reliably compute barycentric coordinates' signs in dimensions beyond two. This was a must in order for their strategy to work. Han et al. [27] suggested a hybrid technique that blends barycentric coordinates with MDS in order to solve a similar issue that arises in three dimensions.

2.2 IMPORTANCE OF LOCALIZATION

It is of the highest significance for some applications, such as those that entail monitoring forests and/or fields, to position sensor nodes as precisely as possible. A sophisticated localization algorithm is able to locate the specific geographic position of devices or nodes by gathering data from sensor nodes.

In addition, location-based routing protocols can save a significant amount of power since they eliminate the need for route finding and improve the efficiency with which services are deployed. This results in a more optimum distribution of services. The difficulty of locating unidentified sensor nodes is the crux of the localization problem that arises in wireless sensor networks. Limitations are required to be used to an important degree in many different scenarios, including reconnaissance while an attack is in progress, military supplies for target identification, activity checks, and many others. When considering the aftermath of a disaster, in which homes are destroyed and people are injured, killed, or go missing somewhere in the rubble, every effort must be made to determine the people and locations that were impacted by the event. This must be done regardless of the cost. To achieve the objective of identifying the people or items, there are a variety of approaches available, ranging from the time-honored practice of human spotting by emergency medical workers to the most cutting-edge technical breakthroughs. The events of late Ascos offer us a valuable lesson: if it is necessary to access more modern technology, it is possible to save the lives of individuals who were exploited in a far timelier manner. Wearable gadgets, such as Zigbee 802.15 and analogous devices from the medical aid men or halfway controlled devices, may be used to out all persons and precious goods in this scenario, which would enable them to build up a remote hoc system like the one depicted in Figure 2.1.

In the future, each sensor node will be able to calculate its own coordinates as well as those of the region around it by employing a position estimation. At that moment, the emergency medical workers will have successfully reached the location that houses the person or thing that was the focus of their mission (s). In addition, the population needs aid in evaluating the surrounding region and planning an escape plan in order to get out of this predicament.

The problem of localization gives rise to two important challenges that need to be addressed. To begin, there is the challenge of creating an appropriate direction structure that is appropriate. The second challenge, which is more difficult to assess, consists of determining the actual distance that separates each sensor (the ranging problem). It is possible to find a workable solution to any of these issues by implementing any one of the approaches that are detailed in the paragraphs that follow.

In this section, the concept of localization will be discussed which plays an important part in a number of different scenarios when a dispersed network of sensors is put into place. In order to ascertain the locations of these nodes, the procedure of localization must first be carried out.

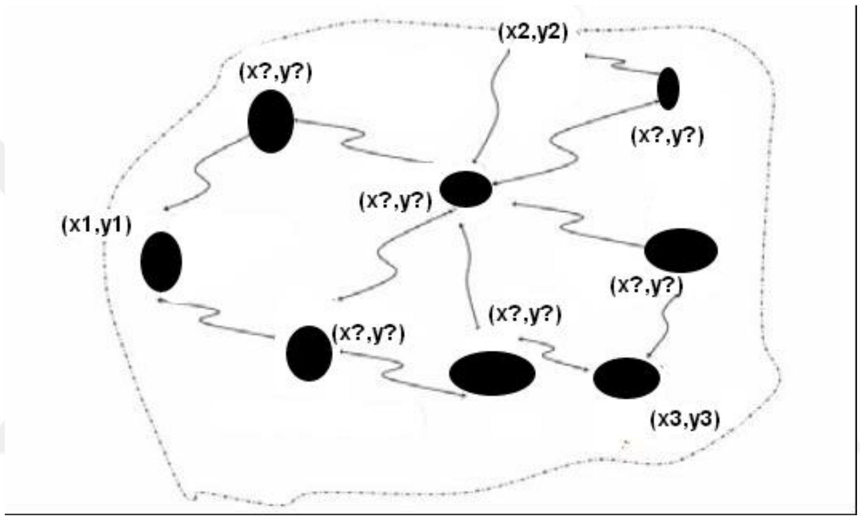

Figure 2.1: Requirement of Localization.

2.3 CLASSIFICATION OF LOCALIZATION

Any of the following techniques can be used for localization:

a. Centralized v/s distributed.

b. Anchor-free v/s Anchor based.

c. Range-free v/s Range based.

2.3.1 Centralized v/s Distributed

It is essential to determine whether the required calculations should be performed generally, by the participants, based on some mainly accessible estimations or whether all estimations should be accounted for to a central station that finds the location of nodes in the network and appropriates them back to the participants. If the required calculations should be performed generally, by the participants, based on some mainly accessible estimations, then it is essential to determine whether this should be done. Scalability and efficiency are the two aspects of a system that need to be considered first and foremost.

The execution of centralized algorithms on a single computer is the primary objective of these types of algorithms. Each sensor node engages in two-way communication with its immediate surroundings in order to generate distance estimates. These distance estimates are then transmitted to a base station, where they are analyzed in order to identify the precise location of each sensor node. Centralized algorithms are able to avoid the issues that are brought on by computational bottlenecks at the nodes because they are ready to bear the expense of rerouting data back to the central station, which is a part of the communication cost. Due to the system's overwhelming concentration on centers in close proximity to the base station, this trade-off will become less practicable as the system generates more. In addition, there is a possibility that the data will be delayed while being transmitted to the control center, which means that centralized operations won't be able to fulfill the requirements of a diverse range of inquiries (e.g., mobile nodes).

Distributed algorithms, on the other hand, are developed to function properly inside a system in which data is handled locally, at each sensor node. Every node is responsible for independently determining its position using the data it has about its surrounding nodes. Due to the fact that the number of neighbors is often not very high (between ten and twenty), the number of connections is typically a couple of requests orders of magnitude less, which results in a considerable decrease in the need for calculating. The distributed computing approach is tolerant of failure at individual nodes and distributes the cost of correspondence equitably across all of the sensor nodes. In spite of this, information loss is a regular problem that arises during the implementation of distributed methods; as a result, the results that may be achieved are often less reliable. For distributed localization, it is possible to utilize either a range-based strategy or a range-free approach.

2.3.2 Anchor Free v/s Anchor Based

Anchor Nodes, which are also frequently referred to as Beacon Nodes or Seeds, are required in order to localize a system inside a global coordinate system. This is accomplished by the utilization of Beacon Nodes. The signals that different nodes get from anchor nodes allow them to make an educated guess as to their own location. Anchor nodes are standard sensor nodes that have been pre-programmed with their own global position coordinates. Either this data is hard-coded, or it is received by other means, such as a receiver for a Global Positioning System (GPS). The same is true for the anchor-free nodes, often known as the nodes, that do not have a GPS receiver [37]. In this step, nodes determine their location within the network by speculating intelligently about the distances that separate them from other nodes. Treating itself as the system's origin and selecting two non-collinear, one-jump neighbors as the axes are the steps that are taken by each sensor in the network in order to generate its own coordinate system for its immediate surroundings. Following that, the coordinates of the closest neighbors are dealt with in the right manner. The very last thing that has to be done before a global coordinate system can be formulated is to determine the location of each node in the network. It is necessary to have at least three non-collinear signals in order to confine the hubs in both of the measurements. If three-dimensional directions are to be supplied, it is necessary to have access to a minimum of four beacons that are not coplanar with one another.

Discovering the coordinates of typical nodes in a more straightforward manner.

The application of GPS technology comes with a number of significant limitations.

a. GPS signals are either unable to penetrate thick vegetation or are unable to pass through it.

b. The high-power needs of the GPS component substantially reduce the amount of time that a sensor node's battery may remain functional.

2.3.3 Range Based Localization

Estimates of the distance traveled and the angle of approach is substituted for precise measurements in range-based localization. Among the most important techniques applied in range-based localization are the received signal strength indication (RSSI), angle of arrival (AOA), time difference of arrival (TDOA), and time of arrival (TOA) (TOA).

The Received Signal Strength Indicator (RSSI) technology [38], which is also a technique that is range-based, may be beneficial to hardware-limited systems.

The signal intensity at a node is used as a surrogate for the node's distance from the signal source when the RSSI scale is used. The signal is interrupted by noise and clutter, as well as the type of the antenna that is being utilized, which results in a substantial lack of precision in the localization process. The node's distance from the signal source is accessible by determining the strength of the signal it receives when it arrives at the node. The greater the distance, the weaker the signal will be when it ultimately reaches the destination node. In theory, the signal's strength will diminish at a rate that is proportional to the inverse of the square of the distance as it travels further away.

The angle of approach, or AoA, refers to the direction in which the signal traveled from the transmitter to the receiver. The angle at which the signals are received is utilized by AoA in conjunction with basic geometric calculations in order to arrive at a conclusion regarding the distance that exists between the transmitter and the receiver. The attractiveness of the AoA measurement can be attributed, in part, to the ease with which it can be calculated (triangulation). The multi-path refractions are the primary cause of this method's most significant shortcoming, which is an erroneous estimation of the directions being traversed.

2.3.4 Range Free Localization

In range-free localization methods, it is not assumed that absolute point-to-point distance measurements are easily accessible between sensor nodes. This is because there is no need for these values. Because of this, an approximation of the position of each node may be computed by making use of either the data on the radio connection or the sensing capabilities of each sensor. These methods might benefit from the addition of a few reference nodes. Several examples of range-free techniques include the distance vector (DV) hop, hop terrain, the centroid system, the APIT, and the gradient algorithm. Radio communication is essential to range-free technology because it allows for the transmission of location data between sensor nodes. A solution to the problem of range-free localization may be obtained using any one of the following methods by making use of just the data information that is contained in messages sent from the reference nodes to all of the other sensor nodes:

Methods Using a Centroid as the Basis: Finding the geographic center of all of the beacon signals a sensor node has received allows for the location of that sensor node to be identified. Anchors have the potential to significantly cut down on location mistakes if they are carefully positioned, however, this cannot be accomplished with spontaneous placements of the anchors. It is essential to have a large number of seeds located in close proximity to one another for the centroid techniques [38] to be successful. This will allow each node to pick up on many signals. It is possible to improve the method's precision in terms of its position by employing a weighted approach.

DV With the aid of the hop-counting approach, it is possible to isolate networks that have a limited number of anchors. The purpose of using hop-counting systems is to disseminate location data throughout a network [39]. Counting the number of hops between two nodes in a DV network allows us to determine the distance between those nodes [40]. In order for a DV hop to be valid, there must be a minimum of three reference nodes that broadcast their coordinates and hop count. Every node in the network shares the data with the nodes that are immediately adjacent to it. If a message that contains data about reference nodes is received by an adjacent node, then the hop count will rise by one each time this occurs. Therefore, it is possible to determine the distance between the anchor node and the node whose location is unknown. It is possible to calculate the route that takes the least amount of time between each of the anchor nodes and the unknown nodes. The average hop distance may be calculated by taking the distance between two nodes and dividing it by the total number of hops. The triangulation technique is used to approximate the position of unknown nodes by calculating the shortest distance between three or more anchor nodes. This is done by counting the number of hops between each pair of nodes.

A Procedure Using APIT The Approximate Point in Triangulation (APIT) [41] architecture utilizes satellite positioning systems (GPS) or radio transmitters as the primary sources of location data for the anchor nodes. When a node's location is unknown, the network will calculate the node's approximate position by utilizing the location information from the overlapping triangles that are located nearby. Unknown nodes are responsible for storing the anchor IDs, signal strengths, and positions of anchor nodes in APIT. These unknown nodes get this information from anchor nodes in the form of beacon messages. The area is subdivided into many sections by overlapping circles. After selecting three anchor nodes as the starting point, the unknown nodes determine whether or not the three anchor nodes form a triangle. This kind of investigation is referred to as a "point-in-triangulation" test, which is also the meaning of the abbreviation PIT. This approach is carried out on a continual basis until a trustworthy means of finding unknown nodes is identified. In conclusion, the location of the mystery node may be deduced by placing it at the intersection of all the triangles in the network.

2.4 BARRIERS ENCOUNTERED IN LOCALIZATION

It is difficult to generate a node distribution in which every region is covered by at least one node due to the random placement of sensor nodes, which makes this a desirable outcome.

Due to differences in the amount of power consumed by individual sensor nodes, sensing, and communication is rendered ineffective in some locations.

There is a possibility that wireless sensor networks will be installed in a random manner in locations that are difficult to access, such as hot spots of violence, inhospitable regions, and other similar places. There will be a great number of networks in places where natural obstacles, such as mountains or buildings, already exist.

A significant paucity of resources is causing problems for many sensor networks. In most cases, the power supply for sensor nodes consists of batteries. As the node interacts, analyzes and detects movement, its lifetime will progressively get shorter. As a direct consequence of this, they need to reduce the amount of money that they spend on power, gear, and rollout.

2.5 MEASUREMENT TECHNIQUES

Calculated distances between a node and its neighbors are one of the primary factors that algorithms use while attempting to determine the precise position of the node in question. The following are examples of well-known methods of measurement that are put to use:

Trilateration It is possible to arrive at an estimate of the location of a single sensor node that is unknown by comparing the distances between that unknown sensor node and three anchor sensors that are known. The process in question is called trilateration. It is possible to consider a sensor's location to be unique if at least three anchor nodes in a two-dimensional space are linked to the sensor's unknown position. By finding the point at which these three circles meet with one another, the coordinates of the sensor that cannot be localized are obtainable, as shown in Figure 2.2. It is possible that due to inaccuracies, the position at which the three circles meet does not represent a genuine single point.

Triangulation When determining the position of sensor nodes, triangulation is a straightforward approach that makes use of the AoA. This method is utilized anytime the angle of the sender node is evaluated in place of the distance between the nodes. The angular position of each anchor node in relation to the unlocalized sensor node in some reference frame is what is utilized to calculate the coordinates of the unlocalized sensor node using the laws of trigonometry. As can be seen in Figure 2.3, this scenario calls for the measurement of two separate distances.

c: Using multi-alteration, it is possible to ascertain the location of a sensor node even if it is too far away from the necessary three anchor sensors for the process of localization to be successful. The location of a hypothetical sensor node can be determined using multi-alteration localization algorithms [13]. These approaches work by reducing the sum of squared distances that separate the node from each anchor point. In order to obtain the coordinates (x, y) of node S, the subsequent function [10] can be utilized, with Figure 2.4 serving as an example.

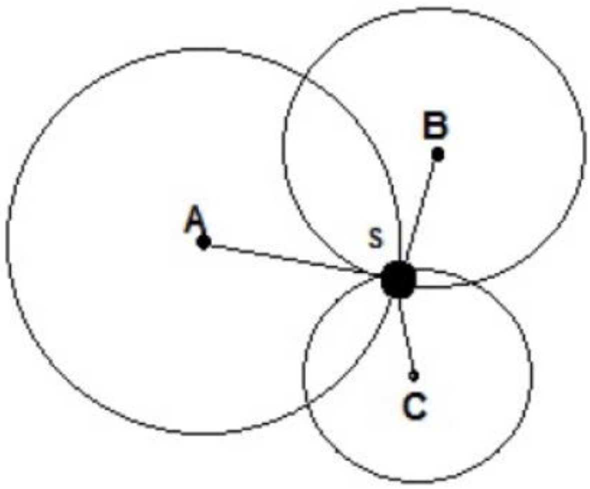

Figure 2.2: Trilateration.

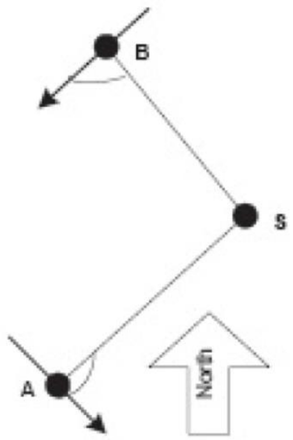

Figure 2.3: Triangulation.

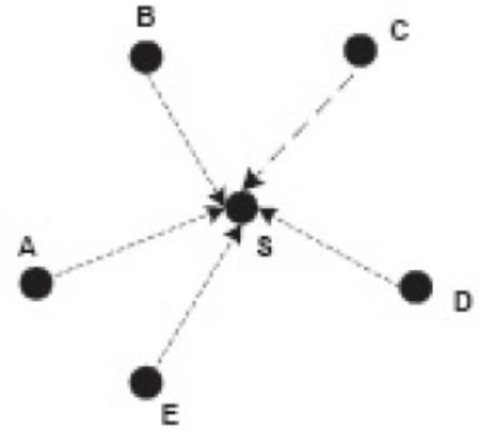

Figure 2.4: Multilateration.

2.6 DESIGN ISSUES

The following paragraphs will provide an explanation of the conceptual challenges that must be taken into account while working on wireless sensor networks.

2.6.1 Computation Power

The central processing unit is responsible for the majority of the power consumption. Processing data requires a far lower amount of energy than moving data between nodes in a network. As a consequence of this, a location-discovery algorithm ought to make use of contextual knowledge concerning the region immediately around the target site in order to save as much energy as possible.

Before beginning to work on the successful localization of sensor nodes, it is essential to first have an understanding of the multiple design challenges that are presented by wireless sensor networks.

2.6.2 Mobility

If some of the nodes are mobile, they will be able to move their position after the network has been deployed in order to improve wireless connectivity. This will result in a network that is more durable and better able to adapt to changing conditions. WSNs are categorized as either entirely mobile or partially mobile according to the number of nodes that are moving around at any one time.

One of the most important differences between the two is that in the first scenario, only part of the nodes are moving, but in the second scenario, all of the nodes are moving. The wind, earthquakes, and landslides are just a few of the natural occurrences that are capable of creating movement in objects. In point of fact, the nature of the node's mobility decides whether the node has active or passive mobility.

2.6.3 Sensing Coverage

It is possible, in light of the distinctions between sensing and radio coverage, to draw the conclusion that the latter is inherently more extensive. Because of this, sensing coverage is a trait that is quite significant from a functional point of view. There are three ways to classify the level of protection provided by a network: sparse, in which case only a portion of the investment zone is guarded; dense, in which case the entirety of the infrastructure is protected; and redundant, in which case a large number of sensor nodes accept the same data from the same location.

2.7 GENERAL PROBLEM DEFINITION

A static sensor network may be represented in d-dimensional Euclidean space as a weighted directed graph G with a vertex set V. This graph can have any number of vertices. This bigger set is used to create anchor nodes with known places, which are referred to as set A, and nodes with unknown locations, which are set U. As a convenient shortcut for describing its position, Cartesian coordinates are used. Therefore, the set of all node sites is denoted by the letter X, the set of anchor node locations is denoted by the letter XA, and the set of unknown node locations is denoted by the letter XU.

A model of such a network is shown in a three-dimensional format in Figure 2.5. In this particular instance, each of the sixty-four vertices is situated on a lattice that has a lattice constant value of one. The greatest distance that may be traveled between nodes for the purposes of measurements and transmissions is two units. Figure 2.5 also includes a representation of the degree1 of each of the vertices that make up this example network.

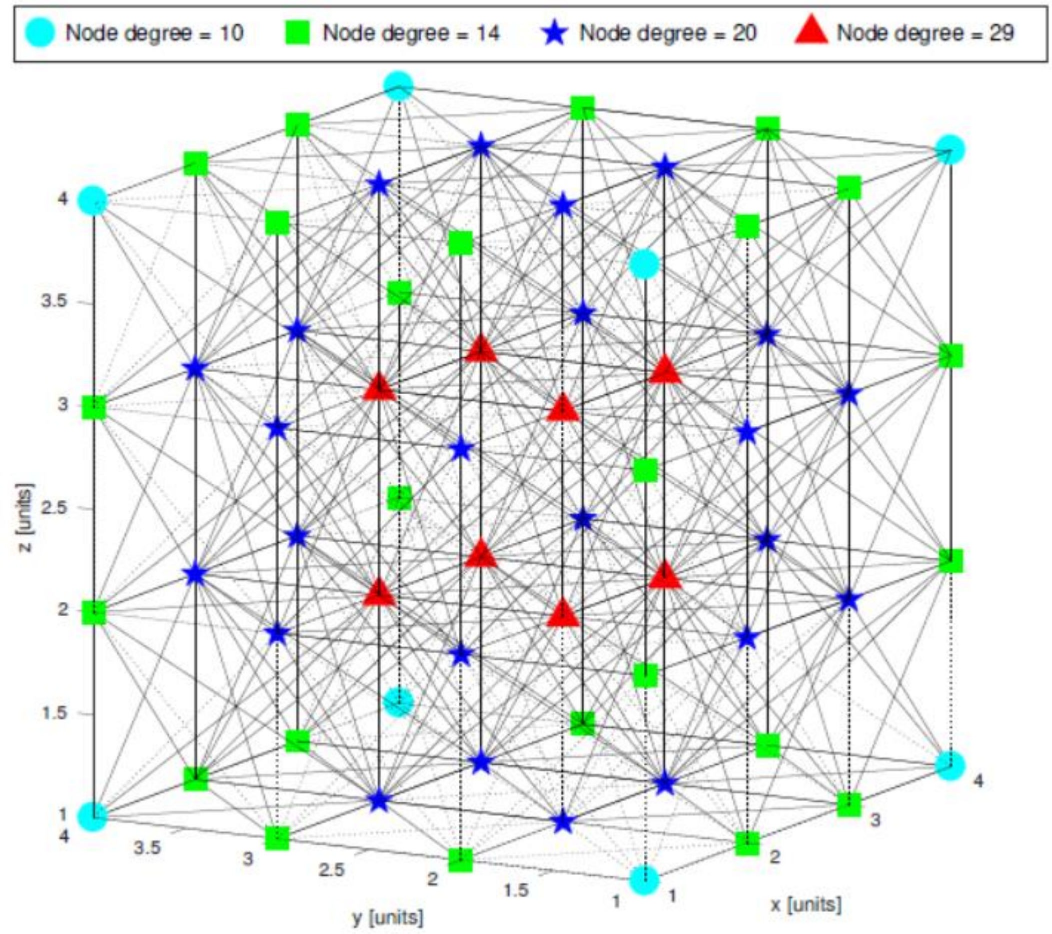

Figure 2.5: Wireless Node Network Example.

The assumption is that at any single moment in time t is used, each given pair of nodes may collect range measurements that are capable of being arranged into a set of square matrices. As a result of the fact that all pairings are independent and independent variables, there is no known distribution for any of the conceivable outcomes. The resulting graph takes on a certain form because the quantities being monitored are independent random variables. In contrast to this, a formulation of an undirected graph is produced when there is no background noise. The localization problem may be simplified into a more general form using the principles presented in the previous paragraphs, as shown in Figure 2.6.

Problem. *Given the anchor node locations* $\mathcal{X}_A \subset \mathcal{X}$ *of a static sensor network and noisy range measurements* $\mathcal{Z}$, *estimate the locations of all unknown nodes* $\mathcal{X}_{\mathcal{U}} \subset \mathcal{X}$.

Figure 2.6: Problem Statement.

Formal definitions of Cayley-Menger bi-determinants will be presented next as well as barycentric coordinates. Next, how to use the first to derive the second for any topology with any node placement will be shown by demonstrating how to use the first to generate the second. After this, the section will explain how the same approach may be used to solve the distributed noisy case of the generic localization issue using barycentric coordinates, and then a way for solving the centralized noiseless version of the problem using barycentric coordinates will be presented. The most common symbols and their definitions are included in a convenient chart that may be seen in Figure 2.7.

Symbol	Meaning
i, j, k	Generic indices
l	Index referencing the l^{th} node
t	Index referencing time iteration
d	Number of dimensions
n	Number of nodes $n := \|\mathcal{V}\|$
a	Number of anchor nodes $a := \|\mathcal{A}\|$
u	Number of unknown nodes $u := \|\mathcal{U}\|$
r_l	Maximum sense range of node l
$\delta_{ij}(t)$	Random noise at edge (i, j) at time t
$\mathbb{K}_n$	Group of complete graphs with n vertices
$\mathcal{P}$, $\mathcal{Q}$	Generic sets
$\mathcal{V}$	Set of vertices or nodes
$\mathcal{E}$	Set of edges
$\mathcal{Z}$	Set of range measurement matrices
$\mathcal{A}$	Subset of anchor nodes
$\mathcal{U}$	Subset of unknown nodes
$\mathcal{N}^{(l)}$	Set of neighbors of node l
$\mathcal{I}^{(l)}$	Family of sets of $d+1$ indexes given by all possible combinations of $d+1$ neighbors of node l which form a subgraph in $\mathbb{K}_{d+1}$
$\mathcal{X}^{(l)}$	Family of sets of Cartesian coordinates of neighbors of l given by $\mathcal{I}^{(l)}$
$\mathcal{X}^{(l)}_i$	i^{th} set of family $\mathcal{X}^{(l)}$
$\hat{\mathcal{X}}^{(l)}_{i_j}$	i^{th} set of family $\mathcal{X}^{(l)}$ with j^{th} element modified
$\mathbf{p}, \mathbf{q}$	Generic real column vectors with some specified dimension
$\mathbf{1}$	Vector of ones with correct dimension
$\mathbf{x_l}$, $\mathbf{x}^{(l)}$	Location of node l in $\mathbb{R}^d$
$[\mathbf{x_l}]_i$, $\mathbf{x}^{(l)}_i$	Element at i^{th}-row of vector $\mathbf{x}^{(l)}$
P, Q	Generic matrices with some specified dimension
X	Location matrix in $\mathbb{R}^{d\times n}$
Z_t	Range measurement matrix at time t
L_l, $L^{(l)}$	Matrix of barycentric coordinates of node l
$[L_l]_{ij}$, $L^{(l)}_{ij}$	Element at i^{th}-row and j^{th}-column of matrix $L^{(l)}$
$\mathbf{L}^{(l)}$	Generalized barycentric matrix (GBC) of node l
$\mathbf{L}^{(l)}_{:\mathcal{A}}$	The columns of GBC of l that are indexed by $\mathcal{A}$
$\mathbf{L}^{(l)}_{:\mathcal{U}}$	The columns of GBC of l that are indexed by $\mathbf{U}$

Figure 2.7: Symbols.

2.8 CAYLEY-MENGER DETERMINANT

Cayley-Menger determinants, [41], provide a link between the Euclidean distances of points in space and the signed volume of the simplex that is created when such points are used. This link can be found further down in this section, in both Figures 2.8 and 2.9, as well as in Definition 1 and Theorem 1 of the first section.

But why is the Cayley-Menger determinant so important when it comes to finding a solution to the problem that is now being faced? As a response, one could calculate the experimental signed volume squared, which will be used to calculate barycentric coordinates in the subsequent section, by merely converting the computed Euclidean distances of points in P to real range measurements. This will allow one to determine how to determine barycentric coordinates in the subsequent section.

Definition 1. *Let the tuple of* $d+1$ *points,* $\mathcal{P} = (\mathrm{p}_0, \ldots, \mathrm{p}_d)$, *be defined by its Cartesian coordinates, such that* $\mathrm{p}_i \in \mathbb{R}^d$ *for* $0 \leq i \leq d$. *Then, the Cayley-Menger determinant of* $\mathcal{P}$ *is defined as follows:*

$$D(\mathcal{P}) = D(\mathrm{p}_0, \ldots, \mathrm{p}_d) = 2\left(-\frac{1}{2}\right)^{d+1} \begin{vmatrix} 0 & 1 & 1 & \ldots & 1 \\ 1 & 0 & d(\mathrm{p}_0, \mathrm{p}_1)^2 & \ldots & d(\mathrm{p}_0, \mathrm{p}_d)^2 \\ 1 & d(\mathrm{p}_1, \mathrm{p}_0)^2 & 0 & \ldots & d(\mathrm{p}_1, \mathrm{p}_d)^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & d(\mathrm{p}_d, \mathrm{p}_0)^2 & d(\mathrm{p}_d, \mathrm{p}_1)^2 & \ldots & 0 \end{vmatrix}.$$

Figure 2.8: Cayley-Menger Definition.

Theorem 1. *The Cayley-Menger determinant of a tuple of* $d+1$ *points in* $\mathbb{R}^d$, $\mathcal{P} = (\mathrm{p}_0, \ldots, \mathrm{p}_d)$, *is related to the square of its signed volume as in*

$$D(\mathcal{P}) = (d!)^2 \, Vol(\mathcal{P})^2.$$

Figure 2.9: Cayley-Menger Theorem.

2.9 APPLYING BARYCENTRIC COORDINATES

The definition of barycentric coordinates often involves affine frames, affine spaces, and points as the fundamental building components. An affine space, denoted by the letter P, is created by the points, the vector space, and the function all working together. An affine frame is a set of points in an affine space with origin p0... such that vectors f... are linearly independent. This serves as a basis for the embedded vector space. When a field K similar to R is utilized, the definition of barycentric coordinates appears as shown in Figure 2.10. According to the implication made by Definition 2, it is necessary to have knowledge of the locations of d + 1 nodes within an affine frame within a d-dimensional space in order to determine the location of a node in the network that is unknown. This indicates that in order to provide an appropriate affine frame for the underlying affine space, each network has to contain a minimum of d + 1 anchor nodes. If a tuple of points in space is provided, then one can apply Definition 2 and Theorem 1 to get the absolute value of any barycentric coordinate. This is illustrated in Figure 2.11 below.

Definition 2 ([66, Prop.3.6.2 Modified]). *Let* $\{\mathrm{p_i}\}_{i=0,1,\ldots,d}$ *be a frame for an affine space* $\mathbf{P}$. *For any (point)* $\mathrm{p} \in \mathbf{P}$ *there exist* $\lambda_i \in \mathbf{K}$, $0 \leq i \leq d$, *such that* $\sum_i \lambda_i = 1$ *and* $\mathrm{p} = \sum_i \lambda_i \mathrm{p_i}$. *The scalars* λ_i *are uniquely defined by this property and are called barycentric coordinates of* p *in the frame* $\{\mathrm{p_i}\}_{i=0,1,\ldots,d}$.

Figure 2.10: Barycentric Coordinates Definition 1.

Definition 3. *Let the tuple* $\mathcal{P} = (\mathrm{p_0}, \cdots, \mathrm{p_d})$ *be a frame for an affine space* $\mathbf{P}$. *Then, given a point* $\mathrm{p} \in \mathbf{P}$, *the* ***absolute*** *value of its barycentric coordinates in relation to the tuple* $\mathcal{P}$ *is:*

$$|\lambda_i| = \sqrt{\frac{D(\mathrm{p_0}, \ldots, \mathrm{p_{i-1}}, \mathrm{p}, \mathrm{p_{i+1}}, \ldots, \mathrm{p_d})}{D(\mathrm{p_0}, \ldots, \mathrm{p_d})}}$$

Figure 2.11: Barycentric Coordinates Definition 2.

3. METHODOLOGY

3.1 PROPOSED APPROACH

When compared to the previous research, the method stands out due to the following reasons: first, a novel will be provided and formal way to compute barycentric coordinates for any point in closed form; second, the formulation of barycentric coordinates will be used to extend the case where one allows arbitrary placement of anchors among unknowns to d-dimensions; and third, both a noiseless centralized algorithm as well as a noisy distributed algorithm is provided that can solve the proposed problem. All of these reasons make the method unique in the next paragraph, the Cayley-Menger bi-determinants will be defined in more detail.

3.1.1 Cayley-Menger Bi-Determinants

The concept of Cayley-Menger bi-determinants is introduced, which are described in [36] but do not have a formal proof, and which operate on two different sets of points in a space of d dimensions. Two different sets of points serve as inputs for Cayley-Menger bi-determinants, which then provide a relation between the product of the volumes of each set and the Euclidean distances between the points in the sets as their output (Figure 3.1).

The formal statement of the relationship between the Cayley-Menger bi-determinant of a set of tuples P and Q in d-dimensional Euclidean space and the signed volumes of these sets is provided by Theorem 2 (Figure 3.2), which may be found below.

In order to correctly compute a determinant, it is essential to bear in mind that determinants come in alternating forms and that the order in which the determinant's elements are structured is extremely important. This becomes a much more important consideration for bi-determinants given that changing the sequence in which the points are presented might affect the signal. While it is important to keep the sequence of the Cayley-Menger determinants, the magnitude of the barycentric coordinates is not as significant.

In order to obtain results equivalent to [37], but more succinctly, Theorem 2 is used to derive the barycentric coordinates of points within and outside the convex hull of its $d + 1$ neighbors. This will allow us to reach comparable conclusions in a shorter amount of time.

Definition *Let two tuples of $d+1$ points, $\mathcal{P} = (\mathbf{p_0}, \ldots, \mathbf{p_d})$ and $\mathcal{Q} = (\mathbf{q_0}, \ldots, \mathbf{q_d})$, be defined by their Cartesian coordinates, such that $\mathbf{p_i}$ and $\mathbf{q_i} \in \mathbb{R}^d$ for $0 \leq i \leq d$. Then, the Cayley-Menger bi-determinant of $\mathcal{P}$ and $\mathcal{Q}$ is defined as follows:*

$$D(\mathcal{P};\mathcal{Q}) = D(\mathbf{p_0}, \ldots, \mathbf{p_d}; \mathbf{q_0}, \ldots, \mathbf{q_d}) =$$

$$2\left(-\frac{1}{2}\right)^{d+1} \begin{vmatrix} 0 & 1 & 1 & \ldots & 1 \\ 1 & d(\mathbf{p_0},\mathbf{q_0})^2 & d(\mathbf{p_0},\mathbf{q_1})^2 & \ldots & d(\mathbf{p_0},\mathbf{q_d})^2 \\ 1 & d(\mathbf{p_1},\mathbf{q_0})^2 & d(\mathbf{p_1},\mathbf{q_1})^2 & \ldots & d(\mathbf{p_1},\mathbf{q_d})^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & d(\mathbf{p_d},\mathbf{q_0})^2 & d(\mathbf{p_d},\mathbf{q_1})^2 & \ldots & d(\mathbf{p_d},\mathbf{q_d})^2 \end{vmatrix}.$$

Figure 3.1: Cayley-Menger Bi-Determinants Definition.

Theorem 2. *The Cayley-Menger bi-determinant of two tuples of $d+1$ points in $\mathbb{R}^d$, $\mathcal{P} = (\mathbf{p_0}, \ldots, \mathbf{p_d})$ and $\mathcal{Q} = (\mathbf{q_0}, \ldots, \mathbf{q_d})$, is related to the products of the signed volumes of each independent tuple, by*

$$D(\mathcal{P};\mathcal{Q}) = (d!)\,\mathit{Vol}(\mathcal{P}) \cdot (d!)\,\mathit{Vol}(\mathcal{Q}).$$

Figure 3.2: Cayley-Menger Bi-Determinants Theorem.

3.1.2 Computing Barycentric Coordinates

Our work on the computation of barycentric coordinates is now finished, and it has been done so in line with the third theorem (Figure 3.3). Note that the sign of a point's barycentric coordinates reveals its relative placement to the points that create the affine frame used in the computation. This is important information to keep in mind. In an affine frame, the only way for a point to have positive coordinates is for it to fall wholly within the convex hull; otherwise, the point will have at least one value that is either zero or negative. Negative signs are not allowed for any non-zero barycentric coordinates at any time. In addition, the order in which the individual signals are presented is determined by the partial ordering of d+1 hyperplanes generated on d-1 dimensions that are superimposed over d-dimensional space.

If r each node is considered to be a point in an affine space, then allowing that point to reside outside the convex hull formed by its neighbors is equivalent to giving that point barycentric coordinates that are either zero or negative.

As can be seen in Figure 3.4, there are conceivable places in three-dimensional space where the signs of the barycentric coordinates are either strictly positive or strictly negative. These locations can be thought of as points. In order to highlight the geometric qualities of the convex hull that surrounds the set of d+1 points that compose the affine frame, all of the edges of the hull have been displayed in black. Figure 3.5 illustrates a few of the locations that might be taken into account while thinking about precisely zero coordinates. Some of these locations include: Bear in mind that some of the portions are hidden by others, while others are either flat or even pointlike, which means that are unable to exhibit all of them.

Theorem 3. *Let the tuple $\mathcal{P} = (\mathrm{p_0}, \cdots, \mathrm{p_d})$ be a frame for an affine space $\mathbf{P}$. Then, any point $\mathrm{p} \in \mathbf{P}$ can be localized in relation to the tuple $\mathcal{P}$ by its barycentric coordinates $\boldsymbol{\lambda}$ as follows:*

$$\lambda_i = \frac{D(\mathrm{p_0}, \ldots, \mathrm{p_d}; \mathrm{p_0}, \ldots, \mathrm{p_{i-1}}, \mathrm{p}, \mathrm{p_{i+1}}, \ldots, \mathrm{p_d})}{D(\mathrm{p_0}, \ldots, \mathrm{p_d})}$$

Proof. From Definition 2, the following relations hold:

$$\mathrm{p} = \sum_i \lambda_i \mathrm{p_i}, \quad \sum_i \lambda_i = 1 \quad \Leftrightarrow \quad \begin{bmatrix} P \\ \mathbb{1}^T \end{bmatrix} \lambda = \begin{bmatrix} \mathrm{p} \\ 1 \end{bmatrix}.$$

Figure 3.3: Computing Barycentric Coordinates Theorem.

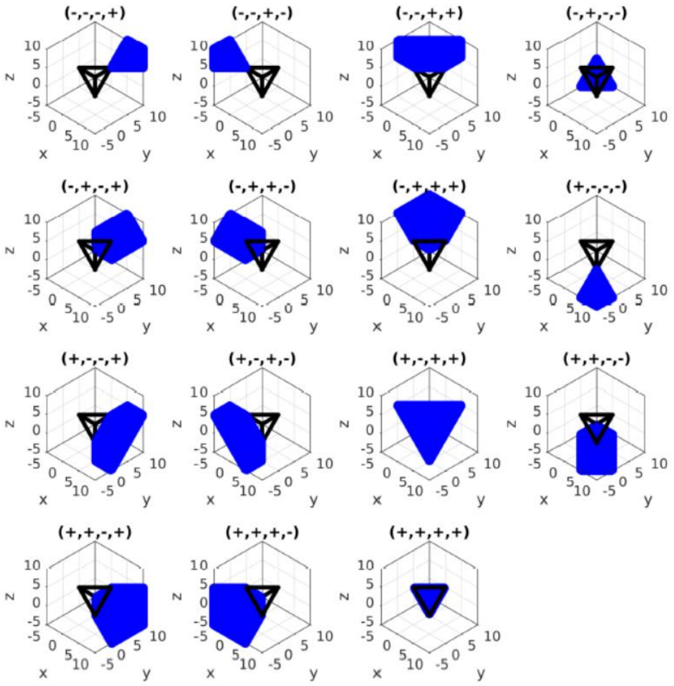

Figure 3.4: Regions of Different Barycentric Coordinate.

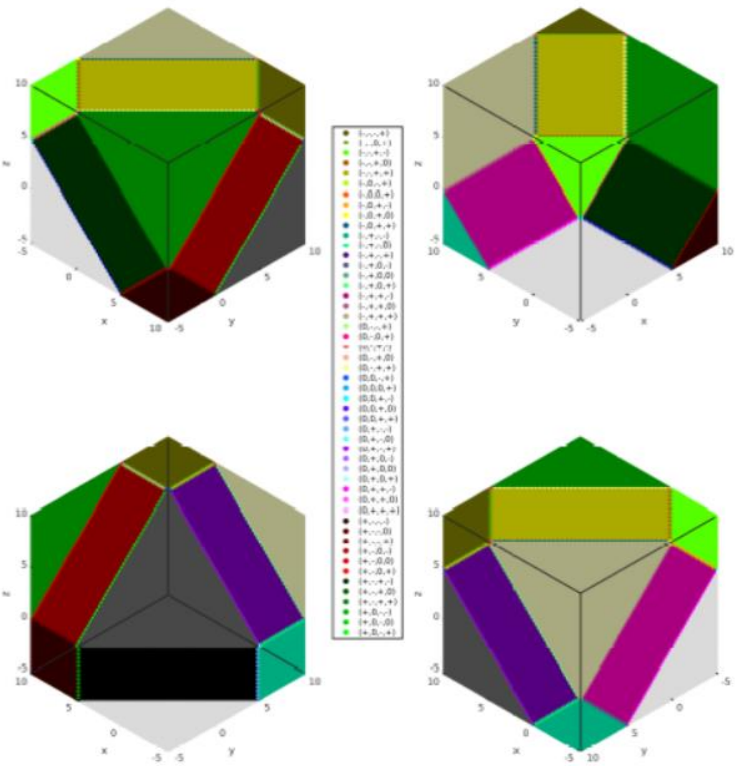

Figure 3.5: Regions of Different Barycentric Coordinate Signs with Zero Elements.

3.1.3 Computing Barycentric Coordinates for Every Node

The problem of localization can be solved with a minimum of effort if every unknown node has a direct link to every anchor, and if the anchors form an affine frame. After that, the barycentric coordinate of each node may be determined by measuring the distance between each node and the anchors.

Hence, Cartesian coordinates are implemented at each node by making use of the previous results. Because a single unknown node is not directly linked to all anchors, it is essential to have a combination of its neighbors that construct an affine frame and are themselves localizable and dependent on neighbors. With the use of this method, the traditional trilateration methodology is turned on its head. It is necessary to find the first group of neighbors that are connected to the anchors, followed by the second set, and so on. Obtaining all of the necessary barycentric coordinates for each unknown node in such a way that the system of equations has a unique solution, that is, all nodes become localizable simultaneously, is an example of a solution that is comparable to those presented in [26], [35]. This is not a straightforward problem to solve since the number of barycentric coordinates for a node is determined by multiplying its neighbor count by d plus one. As a result, the number of options may rapidly increase in networks that are both extremely dense and extremely vast.

In order to solve this problem, Diao et al. [26] suggested computing a generalized barycentric coordinate, which they defined as the median of a node's potential barycentric coordinates. This was done so that the problem could be solved. Next, a different formulation that takes use of all barycentric coordinates is presented for your consideration.

Theorem 4. *For each node l and each set $\mathcal{I}_i^{(l)}$, if $D(\mathcal{X}_i^{(l)}) \neq 0, \forall i$, then all possible barycentric coordinates of node l, with respect to its neighbors identified by the set $\mathcal{I}_i^{(l)}$, can be arranged in matrix form $L^{(l)} \in \mathbb{R}^{|\mathcal{I}^{(l)}| \times n}$*

$$L_{ik}^{(l)} = \begin{cases} \dfrac{D\left(\mathcal{X}_i^{(l)}; \widehat{\mathcal{X}}_{i_j}^{(l)}\right)}{D\left(\mathcal{X}_i^{(l)}\right)}, & \text{if } k = \mathcal{I}_{ij}^{(l)} \\ 0, & \text{otherwise} \end{cases} .$$

Figure 3.6: A Noiseless Environment Using a Centralized Algorithm.

3.2 A NOISY DISTRIBUTED SOLUTION

The previously suggested centralized noiseless method has the potential for two significant enhancements: (1) allowing the system to accept zero mean additive random noise to each range measurement, and (2) changing the algorithm's calculation from centralized to distributed. Both of these modifications have the potential to significantly improve the accuracy of the method. In the first case, the notion of doubling up is applied to averaging by first averaging the range measurement inputs and then averaging the resulting barycentric coordinates. This helps us get more accurate results. It makes sense to apply averages to noisy range measurements with zero mean additive random noise because, in accordance with the Law of Large Numbers, a sufficient number of samples will cause the average to converge to the noiseless value. This will make it possible to determine whether or not the noise is additive. On the other hand, conventional probability distributions cannot be applied to barycentric coordinates since their calculations are based on the determinants of squared averaged noisy range values. This makes it impossible to apply conventional probability distributions to barycentric coordinates.

The second approach for averaging is utilized in order to bring the value gaps between the successively obtained barycentric coordinates to a more manageable level. These changes might be the result of the non-linear nature of the barycentric coordinates computation, or they could be the result of numerical inaccuracy brought about by values that are close to 0 in the denominator. In addition to this, it is essential to check for these almost volume-less simplifications and to impose restrictions on how they can be used. Simulations in MATLAB are carried out in order to evaluate the performance of this double-averaging method. These simulations take into consideration not only the centralized case solution but also a derived gradient descent approach that makes use of data that has been averaged over time. The second adjustment, which entails dispersing the procedure, is based on earlier research that Shahrampour and Jadbabaie [42] conducted on an optimization strategy for networks with cost functions that change over the course of time. Under this context, the Bregman Divergence and a Distributed Consensus Algorithm served as the inspiration for the development of this approach. It is feasible to obtain gradually better location estimations over the course of time when the technique grows into a modified gradient descent approach with convex cost functions. This becomes achievable when the technology improves over time. This is a result of the fact that the Bregman Divergence is frequently computed utilizing the Euclidean distance.

3.2.1 Averaging Processes

The problem of relative localization, when measurements are only approximate, is investigated here. The primary challenge is that the measurement noise distribution is not always known, and even if an assumption were made about the class of the noise (for example, common options include Gaussian, Laplace, Rayleigh, or Rice), the posterior distribution would not be a standard or stable distribution after being propagated through the nonlinear barycentric coordinate function in Theorem 4. This is the primary challenge. To get started, two different ways are provided for averaging in order to lessen the influence of the noise and make it possible to apply Theorem 5 (Figure 3.7) in a noisy environment.

In order to overcome this challenge, an implementation of the technique was done that was suggested by Khan et al. [34]. They reason that by averaging the iid measurements over a period of time, they will eventually arrive at the noiseless range measurement d(xi;xj), which can then be used to calculate the matrix L. This is because they believe that this will lead to a more accurate estimate of the distance between the two points (l). To be more explicit, L(l; t) is determined not by utilizing the raw data at each time step t but rather by using the averaged range readings in the following table. These readings may be found in the following:

$$\left|\overline{Z}_t\right|_{ij} = \frac{t-1}{t}\left|\overline{Z}_{t-1}\right|_{ij} + \frac{1}{t}\left|\overline{Z}_t\right|_{ij}, \left|\overline{Z}_0\right|_{ij} = 0 \tag{3.1}$$

Theorem 5. *The localization problem proposed can be solved through the solution of the linear system given by* (3.32). *Its centralized solution for the unknown nodes is:*

$$X_{\mathcal{U}} = -\left(\sum_{l=1}^{n} \mathbb{L}_{:\mathcal{U}}^{(l)T} \mathbb{L}_{:\mathcal{U}}^{(l)}\right)^{-1} \left(\sum_{l=1}^{n} \mathbb{L}_{:\mathcal{U}}^{(l)T} \mathbb{L}_{:\mathcal{A}}^{(l)}\right) X_{\mathcal{A}},$$

Figure 3.7: Theorem 5.

After sufficient time has elapsed, L (l; t) will be close to the matrix that results from the authentic node barycentric coordinates, then the issue of noisy relative localization may once again be tackled with the help of Theorem 5. Even when data are averaged, it is possible for an incorrect L value to be determined if there is significant instantaneous noise at time t or numerical inaccuracy from near zero volumes (l; t). In order to eliminate the oscillations that were brought on by the most recent measurement, a second step of averaging is carried over all of the components of matrix L. (l; t). This method for computing time-averaged blocks of L(l; t) is broken down into the following parts to provide further clarity and context for each step.

$$\begin{aligned} \overline{\mathbb{L}}_{:\mathcal{A}}^{(l,t)} &= \frac{t-1}{t}\overline{\mathbb{L}}_{:\mathcal{A}}^{(l,t-1)} + \frac{1}{t}\mathbb{L}_{:\mathcal{A}}^{(l,t)} \\ \overline{\mathbb{L}}_{:\mathcal{U}}^{(l,t)} &= \frac{t-1}{t}\overline{\mathbb{L}}_{:\mathcal{U}}^{(l,t-1)} + \frac{1}{t}\mathbb{L}_{:\mathcal{U}}^{(l,t)} \end{aligned} \tag{3.2}$$

3.2.2 Centralized Gradient Descent

Gradient Centralization (GC), a simple and straightforward strategy for improving DNNs, zero-means the gradient vectors in order to make them more focused on certain values. It makes it possible to train DNNs more quickly and can improve their overall generalization performance.

3.3 DISTRIBUTED ONLINE GRADIENT DESCENT

The approach that was presented (DOGB) can be used for the online categorization that takes place via networks as a potential solution to the problem of dispersed data streams. When there is just one agent in the network, the DOGB that was recommended can be reduced to the OGB approach that was provided in Beygelzimer et al. [43]. DOGB uses the distributed online gradient descent strategy that was developed by Zhang et al. [44] in situations where the boosting model has just one learner that is not performing up to expectations (Figure 3.8).

Input: For agent k, the local classifier consists of N weak learners and a learning rate η_t. We set $f_0^k = 0$.
Output: Local classifier f_t^k.
for $t = 1, 2, \dots$ **do**
    Receive example (x_t^k, y_t^k);
    for $i = 1, \dots, N$ **do**
        Compute the residual $r_{i,t}^k$ and the gradient $\nabla_{\lambda_{i,t}^k} \mathcal{F}_t^k(r_{i,t}^k, h_{i,t}^k)$;
        Update the parameters $\lambda_{i,t+1}^k$ based on (6);
        Update the weight $\alpha_{i,t+1}^k = \frac{1}{t}$;
        Update the learner $f_{i,t+1}^k$.
    end for
    return Local classifier: $f_t^k = \frac{1}{N} \sum_{i=1}^{N} h_{i,t}^k$.
end for

Figure 3.8: Distributed Online Gradient Boosting.

4. EXPERIMENT AND RESULTS

The employment of simulations is presented and discussed in this chapter as a potential method that may be utilized to demonstrate the proposed solutions, represent them, and make it easier to compare them to one another. MATLAB, which is made by MathWorks, was utilized for all of the codings of the simulations. In this section, the primary focus is on comparing and contrasting an existing MDS (Multidimensional Scaling) method with the proposed centralized GBC (Generalized Barycentric Coordinate) and centralized/distributed GBM (Generalized Barycentric Matrix) algorithms. MDS stands for "multidimensional scaling."

The centralized GBC is implemented in a manner that is quite similar to that which was provided by Diao et al. in [33], with the difference that Cayley-Menger bi-determinants are utilized in the calculation of barycentric coordinates.

MATLAB's MDS implementation, which is supposed to be more efficient, is employed despite the fact that certain one-of-a-kind algorithms are implemented using core MATLAB features, which may lead to a less-than-optimal implementation. The implementation makes use of the default version of DILOC, which may be inefficient in terms of both the amount of time it takes to complete an algorithm and the amount of algorithmic complexity it requires.

To get things started, the simulation results for centralized GBC and GBM algorithms are provided, comparing the former with DILOC and MATLAB's MDS and the latter with MATLAB's MDS. After that, simulations are shown later on demonstrating RMSE values over time for a number of different localized networks that were created using the distributed GBM approach. Comparing DILOC to other approaches once it was demonstrated in [32] that DILOC's location estimations converge to the truth for nodes that are located inside the convex hull of the d + 1 anchors. Because of this, the number of simulated nodes that are able to fulfill the requirements of the constraint is often insufficient for truly random networks. The MDS approach, on the other hand, is more analogous to the proposed algorithms since it can find nodes regardless of where they are placed in relation to anchors. This makes it more versatile.

4.1 NOISELESS CENTRALIZED ALGORITHMS

As was mentioned before, the midscale function found in MATLAB's MDS may be utilized to analyze and contrast all centralized methods. It is started out in a haphazard fashion, has a maximum of one thousand repeats, and uses a stress criterion that is squared and normalized by the sum of the powers of dissimilarities. The maximum number of repetitions is one thousand. In addition, the user has the ability to determine the maximum allowed a number of random re-initializations that the program can do in order to locate solutions that are free of stress. To illustrate that each random beginning of MDS may produce a unique localization estimate, the number of repetitions is set to 1 in this experiment. The goal was to show that this is possible. There will be a wide range of stress levels across all of these different options. The ideal solution is achieved when there are no stresses; but, as the load increases, the estimations become less exact. As a consequence of this, MDS is frequently iterated until the estimate that causes the least amount of stress is discovered.

Yet, centralized algorithms will either be successful or unsuccessful when it comes to giving a solution. The response is dependent on the constituents that make up the network. Each non-localized node is required to have d+1 neighbors, who collectively make up a whole subgraph.

4.2 GENERALIZED BARYCENTRIC COORDINATES

The strategy recommends constructing a random network for the subsequent comparisons by employing a 6x6x6 unit-spaced regular lattice into which zero-mean, unit-variance Gaussian noise has been independently introduced to each node coordinate. This approach is based on the idea that random networks are more representative of real-world networks. The maximum range criteria for all of the nodes have been set to ri = 3 units. In place of a network graph for the sake of this explanation, figure 4.1 shows a histogram of the degrees of each node. In the end, anchor nodes were chosen by hand in order to strike a balance between nodes located inside and nodes located outside of their convex hull. The results of the GBC approach, which were displayed in Figure 4.1, show that out of a total of 216 nodes in the example, 215 were correctly localized. The unlocalized nodes and anchors are represented by black circles and squares, whereas the positions of the estimated node positions are shown in red dots. The unlocalized nodes are shown in black circles. Due to the fact that it is the lone black star, it serves as a representation of the true coordinates of the decentralized node.

As can be seen in Figure 4.2b, the default method utilized by DILOC correctly pinpoints all of the nodes that are included inside the convex hull of the anchor nodes. Nevertheless, in order to do this, it is important to use measurement ranges that were greater than the maximum of three units that it is necessary to intend use at six of the fifteen unknown nodes inside the convex hull. Some nodes won't be localized if the radius of DILOC is set to 3 since they won't have enough neighbors to compute their barycentric coordinates. This occurs when the radius of DILOC is set to 3. This conclusion fulfills all of the prerequisites and expectations outlined in the proposed application and theory of DILOC in an impeccable manner.

On the other hand, the MDS approach in Matlab involves running a simulation one thousand times using only one copy. A histogram illustrating the stresses that MDS was subjected to may be found in Figure 4.3. This histogram was created in a mechanical manner by employing the square-root binning approach. This graphic provides an overview of the anticipated range of network stressors based on MDS measurements.

The top and bottom node coordinates that were determined from all one thousand MATLAB MDS simulations are displayed in Figure 4.3-b. By employing [68], it is possible to construct error ellipsoids that were centered on the mean node location estimations. The eigenvalues and eigenvectors of the covariance matrix are used to determine the size of these ellipsoids as well as the direction they point in. For the entirety of the calculating procedure, a confidence interval of 90% is applied to each of their volumes. The data imply that this technique may produce results that are below average, and they illustrate the significant contrast between estimates that are ideal and estimates that are poor.

This highlights the need of adopting a large number of replications in the research process.In order to make a comparison, one thousand simulations of MATLAB's MDS implementation are conducted, in addition to one hundred simulations of the technique itself. The MDS simulations were run with a single replication throughout the whole process. The first step of the GBC method, which consists of building candidate families of neighbor sets, requires a total of 140.13 seconds on average, with a standard variation of 2.71 seconds. The second part of the problem takes an average of 49.15 seconds to complete, with a standard deviation of 1.32 seconds. This part involves constructing and solving the linear system. The time required to run MATLAB's MDS implementation with a single replica was, on average, 2.80 seconds, with a standard variation of 0.71 seconds. Even though the run lengths are 68 times longer than MDS's single replication rounds, it is likely that one will need to compute a few rounds of MDS in order to create proper minimum stresses. This is seen in Figure 4.3-a. This behavior suggests that the algorithm run lengths are not as harsh as indicated, given that MDS can solve the problem in a single iteration. Considering this, the problem can be solved in a single iteration.

In Figure 4.2, the black star denotes a node that could not be localized using any of the methods that were open to researchers at the time. Not only does it not lie within the convex hull of the anchor, but it also has less than d + 1 neighbors that are located within the limitations of the maximum measurement that is practically possible. This indicates that it goes against not just the assumptions and restrictions of DILOC but also the procedures that are advocated. As a consequence of this, there are many who believe that it is preferable to state that "it can't be localized" rather than provide the erroneous location.

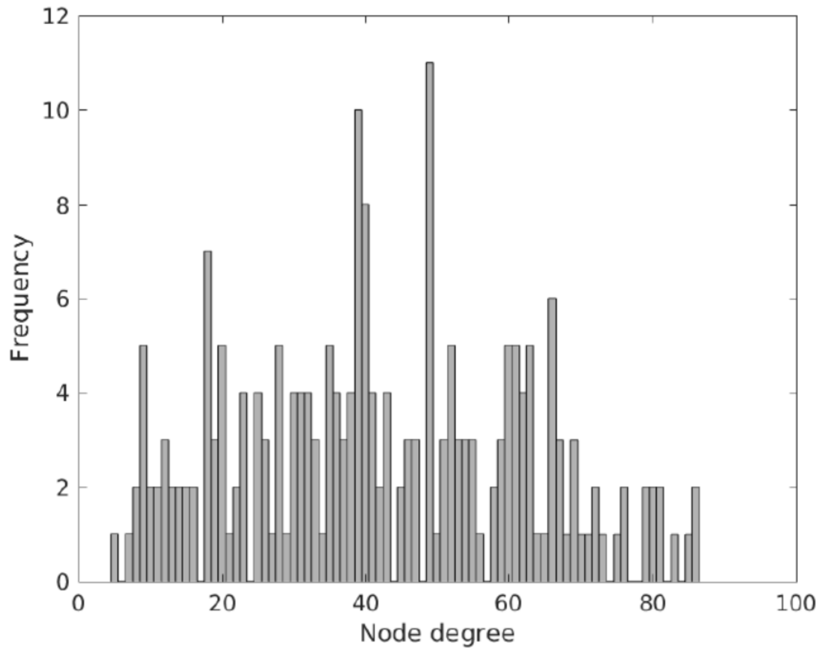

Figure 4.1: Histogram of Node Degrees as an Illustration of the GBC Method.

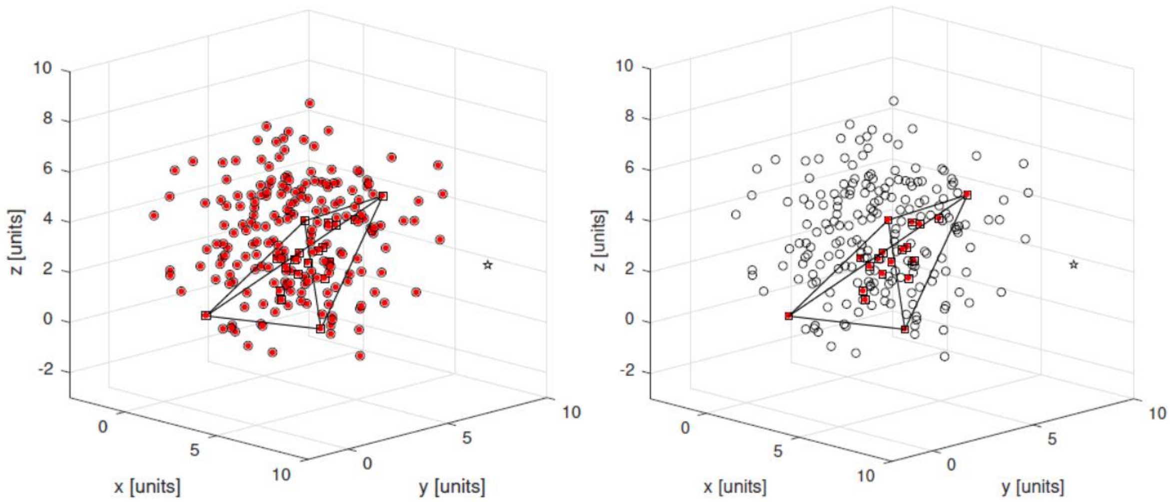

Figure 4.2: Location Estimates Comparison: a) Generalized Barycentric Coordinate Algorithm's Location Estimates, b) DILOC's Location Estimates.

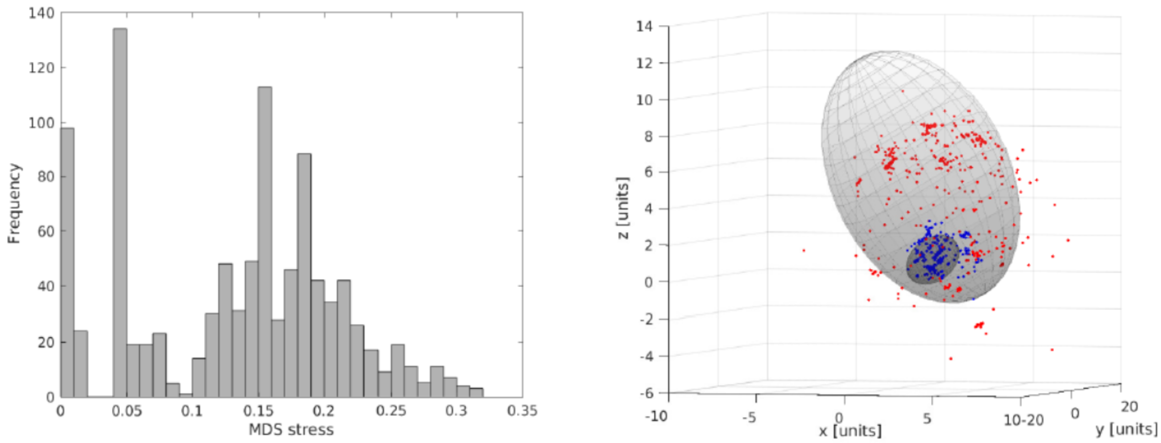

Figure 4.3: Features of MATLAB's Multidimensional Scaling (MDS) Method.

4.3 NOISY DISTRIBUTED ALGORITHM

In the next part, the results of the suggested distributed GBM method will be shown. As a result of this, 32 randomly formed networks are produced, each of which consists of 4 randomly chosen anchors and 10 nodes whose identities are unknown. These vertices are evenly distributed around a cube with sides that are 10 units long, while the cube itself is a unit long. In light of the need of maintaining the accurate localization of each unknown node, all networks make use of a range radius that is equal to 9 units. All of the simulations were carried out with a constant step size because there does not appear to be a way for us to include dynamic step sizes in the distributed algorithms.

Simulations of the implementation of the method that was suggested are provided below for the networks that are illustrated in Figure 4.4.

All range measurements include an extra random variable that has a Gaussian distribution, a mean of zero, and a standard deviation that is equal to one-tenth of the actual distance that exists between the two nodes. As a natural consequence, nodes located further apart will have range measurements that are less precise when employing this approach.

Figure 4.5 depicts the relative standard error (RMSE) of the position estimations for each node in the network. These estimations are used to determine the positions of the nodes. In Figure 4.6, only three of the 32 networks that were taken into consideration are used to illustrate instances of some of the potential behaviors. A red network is used to illustrate an example of a network that successfully converged to the desired solution, while a black network is used to illustrate an example of a network that did not successfully converge within the simulated number of iterations (network 21). It's possible that the estimates of the algorithmic solutions for this network may ultimately converge; however, it might take a little bit more time than was first predicted. The dashed blue curve represents the progression of events that take place during an oscillation (network 20).

Figure 4.7 illustrates the solution over time, which is sometimes referred to as trajectories, for this particular network so that the solutions for this particular network may be shown more clearly. The network was successful in locating the correct response (network 1).

Simulations of the recommended algorithms, whether centralized or decentralized, provide results that are comparable to one another and fall within the margin of error. Even if it is possible that it will go through some oscillations along the way, the distributed GBM approach will continue to repeat itself until it obtains the result that is necessary. It's possible that the distributed strategy might be sped up with a few minor adjustments, such as a variable optimum step size and a reduction in the number of neighbor groups chosen for each node. Figure 4.8 shows the compatibility between network 1 current and true location.

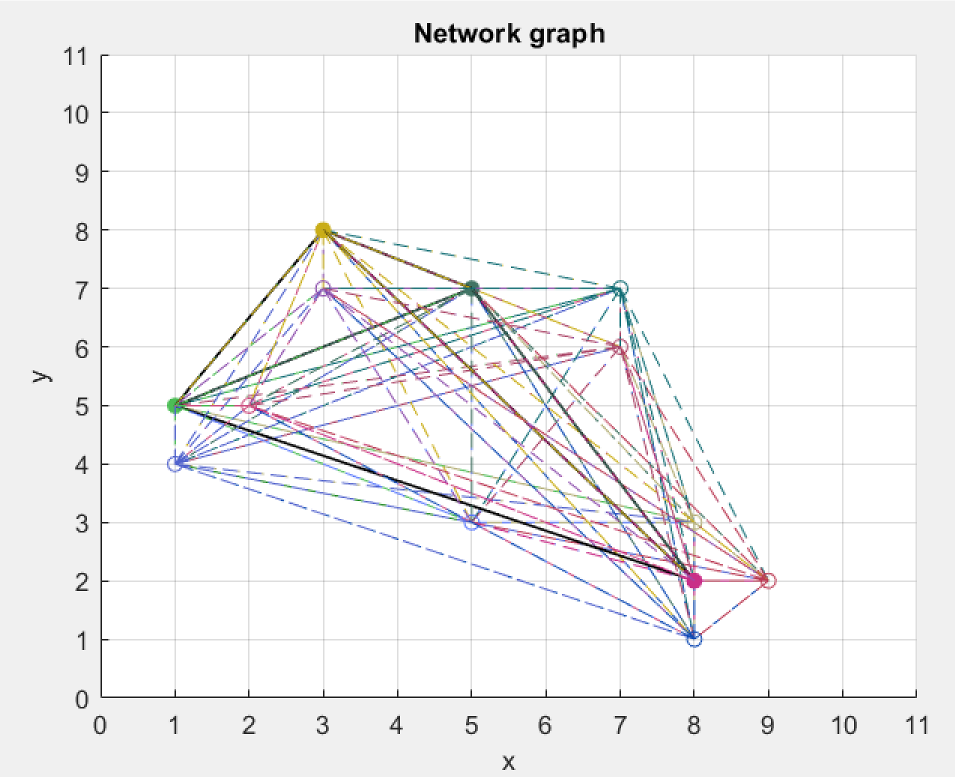

Figure 4.4: Network Graph Before Processing.

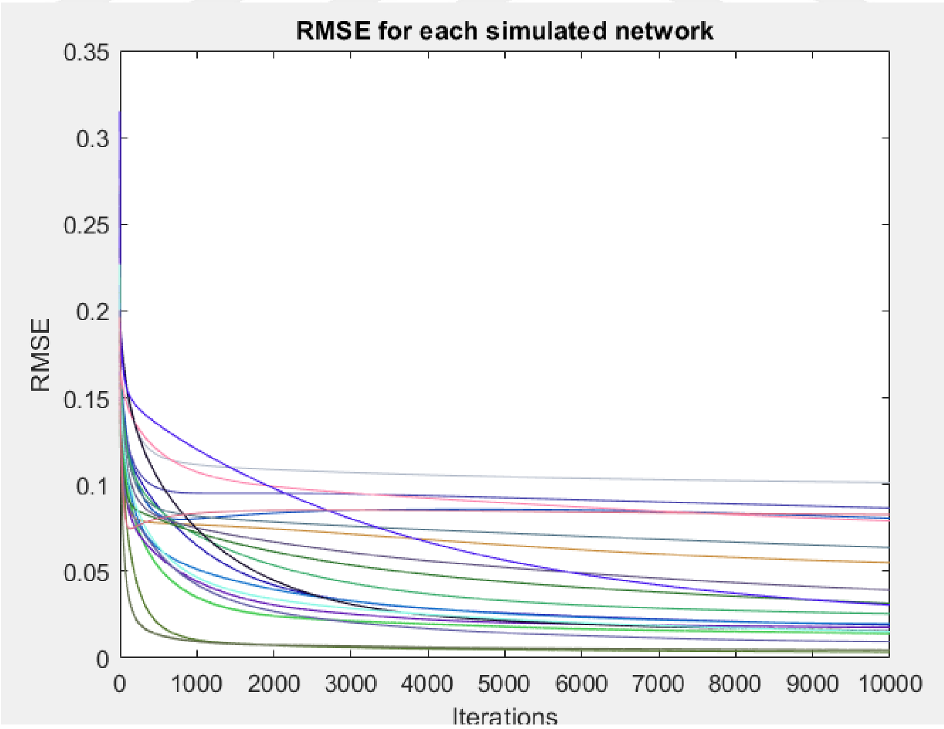

Figure 4.5: RMSE Values for Networks, Different Colors Indicate Different Networks.

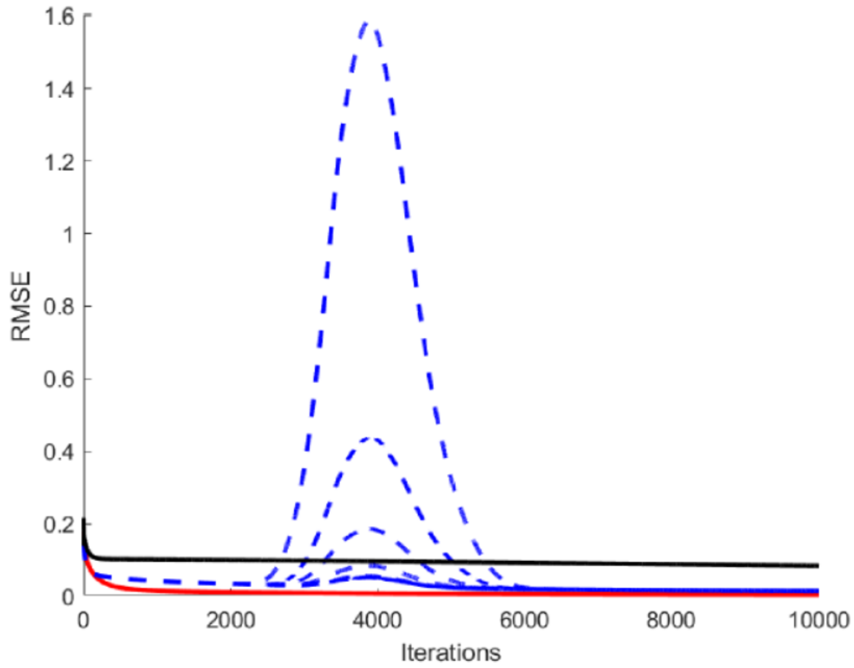

Figure 4.6: RMSE Values for Networks 1 (Red Continuous Line), 20 (Blue Dashed Line), and 21 (Black Continuous Line).

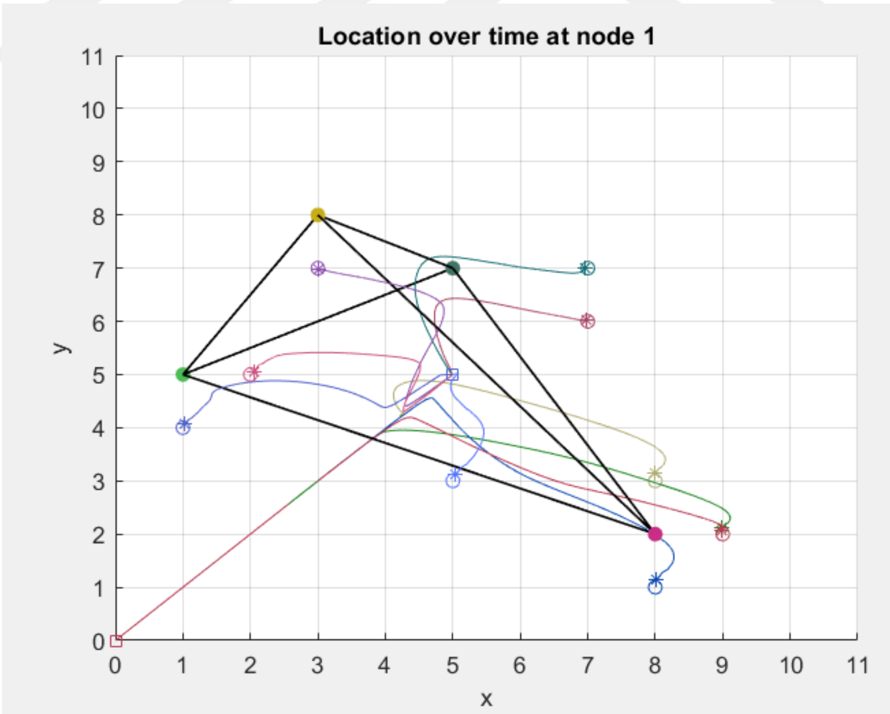

Figure 4.7: Solution Trajectories Over Iterations.

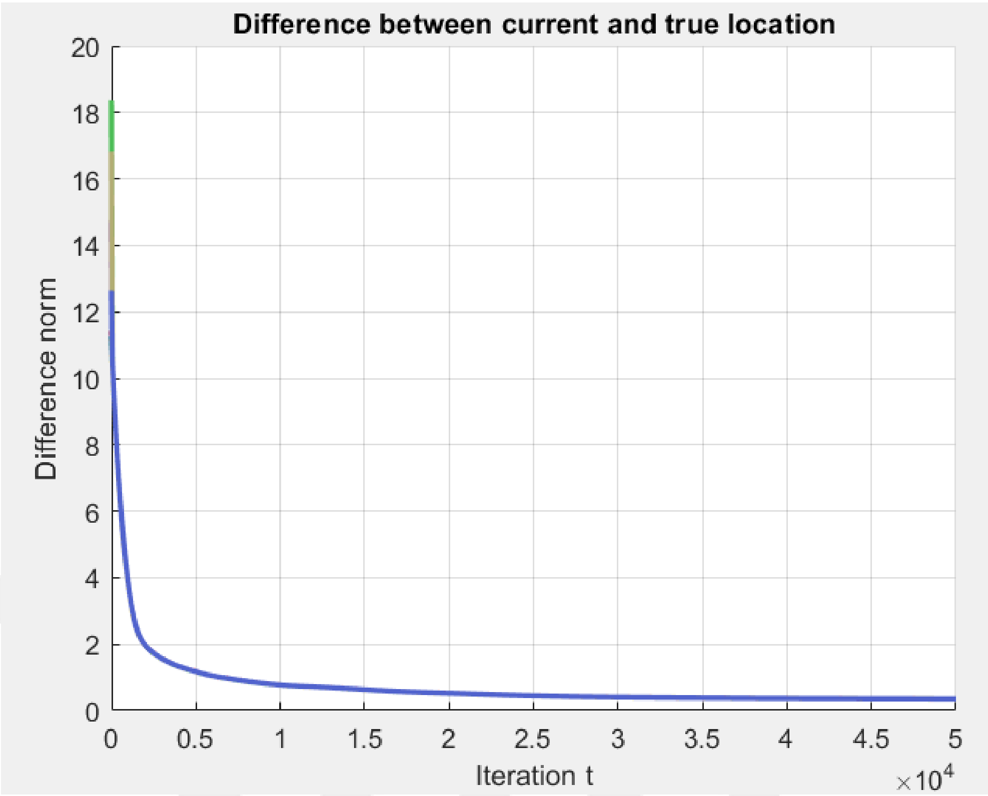

Figure 4.8: Difference Between Current and True Location for Network 1.

5. CONCLUSION

The topic of range-only localization in wireless sensor networks has been the focus of this study from the very beginning, and the purpose of this endeavor has been to provide a thorough approach and physical experimental setup for resolving the problem. In this dissertation, the aims and priorities were outlined, and successful in accomplishing a significant portion of what set out to do. For the purpose of providing a closed-form formula, Cayley-Menger bi-determinants are applied. This allows one to derive barycentric coordinates in arbitrary d-dimensional node topologies. These bi-determinants are a version of the Cayley-Menger determinants, which are used more frequently to handle localization concerns utilizing barycentric networks. Nevertheless, the bi-determinants are not as popular as the Cayley-Menger determinants. The use of bi-determinants substantially simplified an existing theory for computing barycentric coordinates on arbitrary node configurations, and these results were also expanded to apply to an unconstrained d-dimensional scenario.

Depending on the architecture of the network, this method will either produce the correct answer or demonstrate that it cannot produce the correct answer. This strict binary behavior is viewed favorably because it eliminates the possibility of being misled by inaccurate estimates. Additionally, if a solution is not available, the network can be investigated to rule out any unknown nodes that are not localizable according to the theory. Examples of these nodes include those that do not have an adequate number of neighbors or are part of a sub-graph of the network that can be partitioned by cutting at most d + 1 edges.

On the basis of the currently available theoretical framework, a number of strategies have been suggested for coping with noisy range measurements. The centralized method would do a search for its output during each cycle, but the amount of noise it encountered would vary from cycle to cycle.

The numerical results of the simulations conducted on the network were made accessible. Both the centralized and the distributed algorithms made advantage of the first noisy range measurements that were taken by the neighbors to begin compiling their estimations.

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