ÇUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES

PhD THESIS

A Novel Approach To The Solution Of Diffraction Problem For The Strip

Zeynep BAZ

Electrical and Electronics Engineering Department

June, 2024

ÇUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES

PhD THESIS APPROVAL

A Novel Approach To The Solution Of Diffraction Problem For The Strip

Zeynep BAZ

Electrical and Electronics EngineeringDepartment

This Doctorate Thesis was evaluated by the following Jury Members on 10/06/2024 and was approved by unanimity.

Jury	: Prof. Dr. Turgut İKİZ	(Advisor)	…………
	: Prof. Dr. Ahmet TEKE		…………
	: Prof. Dr. Bekir ÖZÇELIK		…………
	: Prof. Dr. Eldar VELİEV		…………
	: Prof. Dr. Vasil TABATADZE		…………

This Thesis was written in the Department of Electrical and Electronics Engineering, Institute of Natural and Applied Sciences.
ThesisNumber:

Prof.Dr. Sadık DİNÇER
Director
Institute of Natural and Applied Sciences

CONTENTS

CUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES

PhD THESIS

A Novel Approach To The Solution Of Diffraction Problem For The Strip

Zeynep BAZ

Advisor: Prof. Dr. Turgut İKİZ

Department of Electrical and Electronics Engineering

ABSTRACT

The development of numerical methods for solving scattering problems has been dependent advancement in computer technology. Unlike analytical methods, numerical approaches, relying on matrix inversion, are often perceived as less intricate, although their effectiveness is limited by computer capacity. Both analytical and numerical methods come with drawbacks and constraints. Notably, when dealing with high-frequency sources or large electromagnetic structures, extensive discretization is necessary, posing challenges in achieving desired accuracy. Analytical methods, meanwhile, are typically applicable to a finite range of geometries. Hence, hybrid methods that amalgamate the strengths of both analytical and numerical approaches have emerged to tackle these issues.

In our thesis, we utilize a hybrid method inspired by Veliev's prior research to explore an efficient alternative for solving a specific class of integral equations. This novel approach is anticipated to streamline calculations, expedite processing, and extend applicability across various materials, surpassing the capabilities of existing methods in the field. Our research focuses on resolving the two-dimensional thin strip diffraction problem using this innovative approach, which combines analytical and numerical techniques. Furthermore, we delve into a detailed examination of the current distribution on the two-dimensional thin strip under 0 and 1 fractional order limits, leveraging simulations to derive insights. The resultant near-field and far-field distributions hold promise for application in antenna design or devices aimed at directing electromagnetic waves.

Keywords: Fractional boundary condition, electromagnetic diffraction, fractional strip, integral equation, fractional calculus

ÇUKUROVA ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ

DOKTORA TEZİ

Şeritten Saçılma Probleminin Çözümüne Yeni Bir Yaklaşım

Zeynep BAZ

Danışman: Prof. Dr. Turgut İKİZ

Elektrik Elektronik Mühendisliği Anabilim Dalı

ÖZ

Saçılma problemlerinin çözümü için sayısal tekniklerin gelişimi her zaman bilgisayar teknolojisindeki ilerlemeye paralel olmuştur. Analiz için matris ters çevirme prosedürü nedeniyle sayısal yöntemler, bilgisayar kapasitesi çözülebilecek problemin boyutunu sınırladığından analitik yöntemlere göre daha az karmaşık kabul edilebilir. Analitik ve sayısal yöntemlerin çözümlerinin yapılmasında bazı sınırlamalar ve dezavantajlarıvardır. Frekans kaynakları ve elektriksel olarak büyük nesneler, istenilen doğruluğun sağlanması açısından özellikle saçıcının elektriksel boyutu göz önünde bulundurularak geniş ölçüde ayrıklaştırma gerektirir.Bu durum sayısal olarak problemin çözümünün uygulanabilirliğini sınırlamaktadır. Analitik metotlar genellikle belirli bir sayıda geometri için uygulanır. Bu yüzden, problemlerin çözümünde analitik ve sayısal yaklaşımların avantajlarını bir araya getiren hibrit yöntemler geliştirilmiştir.

Tezde Veliev'in daha önceki çalışmalarında da sunulduğu gibi hibrit yöntem kullanılmıştır. Bir integral denklem sınıfını çözmek için etkili ve alternatif yeni bir yöntem araştırılmaktadır. Yeni yöntemin, mevcut yöntemlere kıyasla daha basit hesaplamalar, daha hızlı işlem ve çeşitli malzemelere daha geniş bir uygulanabilirlik sağlaması beklenmektedir. Tez çalışması, analitik ve sayısal yöntemler bir araya getirerek, iki boyutlu ince şerit kırınım problemine yeni bir yaklaşımla odaklanmayı amaçlamaktadır. Ayrıca çalışmamda iki boyutlu ince bir şerit üzerindeki akım dağılımı 0 ve 1 kesirli derece limitleri durumunda detaylı olarak incelenmiştir. Sonuçlar simülasyon yoluyla elde edildi. Yakın alan ve uzak alan dağılımlarında şerit üzerindeki dağılımların, elektromanyetik dalgaları yönlendirmek için kullanılan anten veya cihazların tasarımında kullanıma uygun olabileceği düşünülmektedir.

Anahtar Kelimeler: Kesirli Sınır Koşulu, Elektromanyetik Kırınım, Kesirli Şerit, Integral Denklem, Kesirli Hesap

GENİŞLETİLMİŞ ÖZET

Saçılma problemlerinin çözümü için sayısal tekniklerdeki gelişme her zaman bilgisayar teknolojisindeki gelişmelere paralel olmuştur. Sayısal yöntemler analitik yöntemlere göre daha az karmaşık kabul edilse de, analiz için kullanılan matris ters çevirme prosedürü nedeniyle bilgisayar kapasitesi, ele alınabilecek problemin boyutunu kısıtlamaktadır. Genel olarak maksimum boyutu birkaç dalga boyuna sahip olan engeller için sayısal yöntemler doğru çözümler sağlayabilir. Integral denklemler genellikle sayısal yöntemlerle çözülse de, bazı analitik yöntemler kullanılarak bir takım cebirsel denklemlere de dönüştürülebilir. Elde edilen matris denklemi daha sonra standart matris ters çevirme algoritmaları ile çözülebilir. Bu matris denkleminin çözümü için gereken süre, elde edilen matrisin boyutuyla orantılıdır. Bu nedenle özellikle büyük cisimler için Radar Kesiti (RCS) tahmini için gereken süre çok büyük olabilir, bu nedenle matris boyutunun mümkün olduğu kadar küçük tutulması gerekir. Oldukça fazla sayıda güçlü analitik teknik olmasına rağmen, sayısal tekniklerin temel avantajı, bunların isteğe bağlı şekildeki bir dağıtıcıya uygulanabilmesi ve genellikle yalnızca dağıtıcının boyutuyla sınırlı olmasıdır. Ancak bu sınırlama pratik bir sorundur. Teorik olarak saçılma problemini tanımlayan bir dizi doğrusal denklem oluşturulabilir ancak elde edilen küme çözülemeyecek kadar büyük olabilir. Bilgisayar teknolojisindeki gelişmeler sayesinde birçok elektromanyetik problemin istenilen doğrulukta çözümü mümkün olmaktadır. Karmaşık şekilli cisimler söz konusu olduğunda problemin zorluğu artar.

Analitik ve sayısal yöntemlerin avantaj ve dezavanytajları vardır. Bunları özetlemek gerekir ise;

- Analitik yöntemlerde; avantaj olarak, matematiksel ifadeler aracılığıyla sonuçlar üreten bu yöntemler doğru ve hızlıdır, problemdeki fiziksel mekanizmanın daha derin bir şekilde anlaşılmasına olanak tanır. Genellikle geniş bir parametre yelpazesi üzerinde genel çözümler sunarlar. Dezavantaj olarak, karmaşık geometriler ve sınır koşulları için çözümler elde etmek zor olabilir ve bunlar doğrusal olmayan problemlere uygulanamazlar.
- Sayısal yöntemlerde, avantaj olarak, karmaşık geometriler ve sınır koşullarıyla başa çıkmak için kullanılırlar. Gelişmiş bilgisayar yazılımları ve algoritmalar sayesinde doğrusal olmayan ve karmaşık problemleri çözmede etkilidirler. Dezavantajları ise, hesaplamak çok zaman almaktadır ve maliyeti yüksektir ayrıca sayısal hataları oldukça fazda olmasından dolayı özelleştirme gereksinimlerne ihtiyaç duyulmaktadır.

Tez çalışmamda özette de belirtildiği gibi hibrit yöntemler kullanılarak analitik ve sayısal yöntemlerin avantajları birleştirilerek analitik ve sayısal yöntemlerin bir arada kullanılmasıyla bu zorluklar aşılabilir.

Hibrit yöntemler, esneklik, daha hızlı çözümler, hassasiyet ve kararlılık gibi özellikler sunmaktadır. Özellikle de karmaşık geometri ve sınır koşulları içeren problemler için, şerit problemleri gibi karmaşık mühendislik problemlerini çözmede önemli avantajlar sağlarlar. Tez çalışmasında, daha önce Veliev tarafından yapılan çalışmalarda hibrit (melez) yöntem kullanılmıştır ve çözümün önceden belirlenmiş herhangi bir doğruluğu içerdiği alternatif bir yöntem geliştirilmiştir (Veliev et.al., 2011). Elde edilmek istenen sonuca en yakın ve kesin bir doğrulukta hesaplamanın yapılması gerekir. Elektriksel boyut, belirli bir problem için sayısal çözümün uygulanabilirliğini sınırlar çünkü yüksek frekanslı kaynaklar ve büyük elektriksel nesneler, daha fazla ayrıklaştırmayı gerektirir. Bu da hesaplama gücü ihtiyacını artırır. Analitik yöntemlerin karmaşık olmayan ancak belirli geometrileriler için uygulanabilirliği vardır. Bu sebeplerden dolayı ise sayısal ve analitik yöntemlerin avantajlarının birlikte ele alınarak bahsedilen zorlukları aşmak amacıyla hibrit yöntemler geliştirilmiştir. Analitik yöntemler genellikle yüksek frekans rejiminde kapalı form çözümler sağlayabilirken, melez yöntemler daha geniş frekans aralıklarında alan ifadelerini hesaplayabilirler. Bir integral denklem sınıfını çözmek için etkili ve alternatif yeni bir yöntem araştırılmıştır. Saçılan alan, problemin çözümünü oluşturmak için bir dizi avantaj sunan karşılık gelen yüzey akım yoğunluğunun Fourier dönüşümü kullanılarak temsil edilmiştir. Elektromanyetik dalgaların geometrik ve fiziksel süreksizliklerden saçılması, elektromanyetik dalga teorisinin en önemli alanlarından biridir. Basit geometrisi nedeniyle şerit konusu birçok bilim insanı tarafından çalışmalarda araştırılmıştır. Yaklaşımlarımda şerit halinde birçok teknik önerilmiş ve ciddi çözüme kavuşturulmuştur (Butler, 1985). Saçılma problemlerinin çözümü için sayısal tekniklerin gelişmesi ve aynı zamanda bilgisayar teknolojisinin gelişmesiyle birlikte bu gelişmeler birçok elektromanyetik problemin gerekli doğruluk derecesinde çözülmesini mümkün kılmıştır (Uchida et.al., 1989). Araştırmanın amacı özette de belirtildiği gibi soruna alternatif ve yeni bir yaklaşım geliştirmek ve genelleştirmektir. Önerilen yöntemin, literatürde sunulmuş olan diğer yöntemlere göre oldukça basit hesaplamalar, çok hızlı işleme ve çeşitli malzemelere daha geniş uygulanabilirlik sunması beklenmiştir. Bu çalışmada ele alınan problemlerden biri, Şekil 3.2'de gösterildiği gibi, iki boyutlu bir şerit (strip) üzerinde düzlemsel H-polarize dalganın kırınımıdır. Çeşitli geometrilerdeki yüzeylerin saçılma özelliklerini belirlemek için, Neumann ve Dirichlet sınır koşullarına karşılık gelen integral sınır koşulları kullanılmaktadır. Bu integral sınır koşulları, Dirichlet ve Neumann sınır koşullarının bir tür genelleştirmesidir. Yani, belirli bir geometriye sahip bir yüzeydeki saçılma problemlerini çözmek için bu tür sınır koşulları yaygın olarak kullanılır ve hem Dirichlet hem de Neumann sınır koşullarının özelliklerini kapsarlar. Bazı örtünük ifadeler yalnızca yüksek frekanslı durumlar için elde edilebilirlerken, hibrit yöntemlerin kullanılmasıyla oldukça geniş frekans aralıklarında alan ifadelerini (elektrik alanın veya manyetik alanın) hesaplayabilir ve ayrıca ince şerit problemlerinde rezonansları araştırmak için kullanılabilirler. Tez çalışmasında ele alınan problemi çözmek için, kırınım problemlerini çözmek amacıyla ortogonal polinomlar yöntemi kullanılmıştır.

Öncelikle Şekil 3.2' de gösterilen saçılma alanı, bir integral formülüyle tanımlanmıştır ve bu formülü elde etmek için Green ve Fourier analizleri kullanılmıştır. Daha sonra problem analitik bir yaklaşım kullanılarak çözülmüştür. Genel çözüm için integral denklemi, geometri, şeritlerdeki akım yoğunluğu ve diğer koşullara göre belirlenererek, seçilen özel ortogonal fonksiyonlar setiyle ifade edilerek ve bu fonksiyonların katsayıları belirlenerek bulunmuştur. Burada diğer koşullar olarak:

- ✓ Problemin çözümü için gerekli olan koşullar (sınır koşulları, başlangıç koşulları vb.) belirlenir.
- ✓ Problemin doğasına ve koşullarına uygun olan özel ortogonal fonksiyonlar seçilir.
- ✓ Seçilen özel ortogonal fonksiyonlar, bir genişletilmiş fonksiyon seti oluşturur. Bu fonksiyon seti, genel çözümü ifade etmek için kullanılır.
- ✓ Problemin verilen koşulları altında, seçilen özel ortogonal fonksiyonların lineer bir kombinasyonu olarak genel çözüm ifade edilir. Bu genel çözüm, problemdeki tüm değişkenleri ve koşulları içerecek şekilde belirlenir.

Tezde ayrıca, Şekil 3.3' de gösterildiği gibi, iki boyutlu doğrusal ince bir şerit üzerinde silindirik bir dalga kaynağı kullanılarak şerit üzerindeki yakın alan dağılımı için akım yoğunlukları, uzak alan deseninde (radyasyon deseni, Radiation pattern (RP)) kesirli derecenin limit sınırları olan 0 ve 1 olması durumu detaylı olarak incelenmiştir. Literatürdeki önceki çalışmalarda, 0 ile 1 arasındaki sınır koşulları için mükemmel elektriksel iletken (Perfect Electric Conduction (PEC)) veya mükemmel manyetik iletken (Perfect Magnetic Conduction (PMC)) şeritlerden veya empedans şeritlerinden saçılma problemleri üzerine odaklanılmıştır (Veliev et.al., 2018a; Veliyev et.al., 2018b). Kesirli türev yöntemi, sınır koşullarını genişleterek ve kesirli türev yaklaşımıyla mevcut problemleri en genel şekilde çözmeyi mümkün kılar. Kesirli hesaplamalar, elektromanyetik teoride yaklaşık 30 yıldır aktif bir şekilde kullanılmaktadır ve genellikle metamalzemeler ve hafızalı malzemeler için uygulanır. Elektromanyetik alanında, bu yöntem birçok araştırmacı tarafından tercih edilmektedir. Elektromanyetik teoriye kesirli yaklaşımın uygulanmasına dair ilk çalışmalar 1990'larda Nader Engheta tarafından gerçekleştirilmiştir. N. Engheta, elektromanyetik alanın sürekli ara aşamaları olduğunu ve bu kavramı 90'lı yıllarda "elektromanyetikte fraksiyonelleşme" olarak tanıtmıştır (Engheta, 1996; Engheta, 1997; Engheta, 2000). Saçılma problemleri üzerine yapılan çeşitli araştırmalarda, tek şerit saçılma durumunda, yaklaşıklıklarla analitik ifadeler elde edilmiş ve bu ifadeler daha önce yapılmış çalışmalarla karşılaştırılmıştır. Ayrıca, bu çalışmada, mükemmel elektriksel iletken bir şerit için çizgisel kaynaktan saçılan alanın yüzey akımları, Fiziksel Optik ve Momentler Yöntemi ile modellenmiş ve bu tezde önerilen yeni yöntemle karşılaştırılmıştır. Elde edilen sonuçlar, Momentler Yöntemi ve önerilen yeni yöntemin, Fiziksel Optik yöntemine kıyasla daha iyi sonuçlar verdiğini göstermektedir. Ayrıca, literatürde önerilen yöntemle henüz çalışılmamış birçok farklı geometrinin bulunduğu belirtilmiştir. Kesirli (integral) sınır koşulu, Dirichlet ve Neumann sınır koşulları arasında bir ara sınır koşuluna karşılık gelmiştir ki, farklı geometrik özelliklere sahip olan yüzeyler üzerindeki saçılma özelliklerini tanımlamada kullanılmıştır. Tez çalışmasında, önerilen yeni yaklaşımla sınır koşulları ile saçılma özellikleri incelenmiş ve yeni bir malzeme özelliği (PMC, PEC veya bunların arası özellikler) tanımlanmıştır. Yukarıda da belirtildiği gibi, kesirli sınır koşulu şöyle özetlenebilir:

Bir yüzey üzerinde,toplam elektrik alanın yüzey normali yönündeki teğetsel bileşeninin kesirli türevi sıfırdır. Kesirli mertebe 0 olduğunda, bu koşul Dirichlet Sınır Koşuluna denk gelirken, kesirli mertebe 1 olduğunda, Neumann Sınır Koşuluna eşit olur. Bu sınır koşulu, sırasıyla mükemmel elektrik iletken ve mükemmel manyetik iletken gibi farklı malzemelerin yüzeylerine karşılık gelir. Bu çalışmada, kesirli derecenin 0 ve 1 olduğu limit değerleri için şerit üzerindeki alan dağılımları, antenlerin ve elektromanyetik dalgaların yönlendirilmesinde kullanılan cihazların tasarımında potansiyel olarak yararlı olacaktır. Bu tür yapılar, anten sentezi, dalga kılavuzları ve rezonatör problemleri gibi uygulamalarda kullanılabilir. Çalışmanın önemli bir bulgusu ise, kesirli derecenin 0 olduğu durumlarda (α=0), yüzeyin mükemmel elektrik iletken özelliklerine yakın olduğu, kesirli derecenin 1 olduğu durumlarda ise (α=1) mükemmel manyetik iletken özelliklere daha yakın olduğudur.

ACKNOWLEDGEMENTS

I would like to extend my heartfelt gratitude to my esteemed advisor, Prof. Dr. Turgut İKİZ, from the Department of Electrical and Electronics Engineering at Çukurova University, for his continuous support and assistance throughout the thesis and research process. I am deeply thankful to him for helping me at every stage until the completion of my thesis. During my doctoral studies, I consider myself fortunate to have had the opportunity to meet and work with the distinguished academician in the fields of Mathematical Physics and Electrical Engineering, Prof. Dr. Eldar VELIEV, from the National University of 'Kharkiv Polytechnic Institute'. I would like to express my immense gratitude to him for his invaluable assistance and efforts. He not only dedicated considerable time to my thesis but also guided me in approaching problems and deriving correct solutions. His constant motivation and positive outlook in all circumstances encouraged me to view situations positively. Additionally, I would like to thank the esteemed professors who served on my thesis jury and accepted to examine this work: Prof. Dr. Bekir ÖZÇELİK from the Department of Physics at Çukurova University, Prof. Dr. Vasil TABATADZE from the Informatics Institute at Istanbul Technical University, and Prof. Dr. Ahmet TEKE from the Department of Electrical and Electronics Engineering at Çukurova University, for their valuable comments and suggestions.

Lastly, I want to express my heartfelt gratitude to my dear and highly esteemed parents, who have supported me in every aspect of my life and have always believed in me. Words cannot fully express my gratitude to my parents for their unwavering support. I am proud and happy to be their daughter. I also want to thank my dear siblings, Fatma and H. Veli, for their constant moral support, motivation, and encouragement at every moment of my life. I am truly grateful to have them in my life.

To My Family,

LIST OF FIGURES

SYMBOLS AND ABBREVIATIONS

BSCS	: Backscattering Cross Section
DIE	: Dual Integral Equations
FBC	: Fractional Boundary Condition
FDTD	: Finite Difference Time Domain Method
FEM	: Finite Element Method
FMM	: Fast Multipole Method
FO	: Fractional Order
GRIN	: Graded-index
IE	: Integral Equation
MAS	: Method of Auxiliary Sources
MLFMA	: Multi-Level Fast Multipole Algorithm
MoM	: Method of Moments
PC	: Polygonal Cylinder
PEC	: Perfect Electric Conductor
PMC	: Perfect Magnetic Conductor
PO	: Physical Optics
PWTD	: Plane Wave Time-Domain
RCS	: Radar Cross Section
RP	: Radiation Pattern
SCS	: Scattered Cross Section
ISLAE	: Inhomogeneous System of Linear Algebraic Equations
SLAE	: System of Linear Algebraic Equations
TRCS	: Total Radar Cross Section

1. INTRODUCTION

The advancement of numerical techniques for solving scattering problems has consistently progressed alongside developments in computer technology. Even though numerical techniques are regarded as simpler compared to analytical methods, the computational capabilities of computers limits the extend of the problem that can be tackled due to the matrix inversion procedure used for analysis. Typically, numerical techniques offer precise solutions for obstacles with dimensions that are a few wavelengths at most. While integral equations are commonly resolved using numerical methods, they can also be transformed into algebraic equations and tackled using specific analytical techniques.The resulting matrix equation is solved with standard matrix inversion algorithms.The time needed to solve the matrix equation increases in proportion to the size of the resulting matrix.

That’s why, particularly for sizable objects, the time required for Radar Cross Section (RCS) estimation can be very large, so the matrix size needs to be kept as small as possible. Although there are quite a number of the primary benefit of numerical techniques is their ability to be employed with distributors of any shape, unlike powerful analytical techniques, which typically have limitations based on distributor size.Nevertheless, this constraint poses a practical challenge. In theory, it's possible to formulate a set of linear equations representing the scattering issue, yet the resultant set might be excessively large for not solve.Thanks to advancements in the realm of computer technology, it is possible to solve many electromagnetic problems with the desired accuracy. When intricate shapes are involved, the complexity of the problem escalates. However, these challenges can be surmounted by employing a blend of analytical and numerical approaches. With the proposed alternative method, it was ensured that the solution reached a predefined level of accuracy (Veliev et al.,2011).

Representing the scattered field through the Fourier transform of the related surface current density provides several advantages in resolving the problem. The scattering of electromagnetic waves from geometric and physical inconsistencies have been standed as one of the pivotal domains in electromagnetic wave theory.Because of its straightforward geometry, the subject of ribbon has been researched by many scientists. Many techniques have been proposed and seriously solved in my approaches (Butler, 1985). These developments, together with the advancements of numerical techniques for tackling scattering quandaries have been evolved in tandem with advancements in computer technology (Uchida et al.,1989), these advancements make it feasible to tackle numerous electromagnetic problems with the necessary degree of precision.The aim of this thesis is to tackle the two-dimensional thin strip diffraction problem utilizing a novel method. The goal of the investigative, as mentioned in the overview. The objective is to devise and generalize a fresh approach to the problem, offering simpler calculations, quicker processing, and wider applicability across various materials compared to existing methods in literature. As shown Figure 3.2 and Figure

3.3. our work, has been focused on the diffraction of plane waves and cylindrical waves. The integral boundary condition corresponding to the Dirichlet boundary condition is employed to characterize the scattering characteristics of surfaces with diverse geometries.In this study, the integral boundary condition, which is a combined form of Dirichlet and Neumann boundary conditions, was used. This limiting system has been used, referred to as the hybrid method. The hybrid method provides the advantages of analytical and numerical methods. by combining. This approach offers a more flexible and effective way to solve complex problems. Meantime certain implicit expressions are only attainable for the high-frequency range, hybrid methods can compute field expressions across broader frequency ranges, enabling the exploration of resonances in thin strip problems. In addressing the problem outlined in this study, the method of orthogonal polynomials was employed for solving diffraction problems.In solving the problem, the scattered field is initially defined as an integral, obtained using Green's and Fourier analyses. Subsequently, an analytical approach is employed to solve the problem. For the general solution, the integral equation is expressed as a sum of special orthogonal functions, taking into account the current density on the strips, geometry, and all other relevant conditions.

In the thesis work, a thorough analysis of the current distribution on the two-dimensional thin strip is conducted in the scenario of fractional order limits of 0 and 1. Solutions to these problems for perfect conductors and magnetic conductors are available in the literature. In previous studies in the literature, there are studies on scattering problems from strips that exhibit perfect electrical or magnetic conductivity, or impedance stripsunder specified conditions. As implied by its name, the fractional derivative method enables the generalization of boundary conditions and offers a comprehensive solution to the current problem through the fractional derivative approach. This fractional approximation has been employed in electromagnetics for three decades and is commonly utilized by numerous scientists in the field, particularly for metamaterials and memory materials.

In the 1990’s, Nader Engheta pioneered the application of fractional techniques in electromagnetic theory, introducing the concept of "fractionalization in electromagnetics" to elucidate the continual intermediate states between canonical electromagnetic field states. Subsequent to Engheta's work, numerous studies have investigated scattering problems, particularly in single-lane scenarios. Analytical expressions were derived, with certain approximations, for single-lane scattering, and comparisons were drawn with prior research. Limit values were obtained analytically by approximating the linear source's placement in the far field and examining the scattered field accordingly. Additionally, surface currents induced by the scattered field from the linear source on a perfectly electrically conductive single strip were modeled using the Physical Optics and Moment Method, with comparisons made against the method proposed in this thesis. Results indicated that both the Moment Method and the thesis method outperformed Physical Optics. The thesis further elucidates intermediate boundary conditions between canonical cases using the fractional derivative approach, detailing field distribution, radiation patternand current distribution. However, numerous geometries in the literature remain unexplored with the proposed method.

The fractional boundary condition, also referred to as the integral boundary condition, serves as an intermediary description between Dirichlet and Neumann boundary conditions. It provides a framework for characterizing the scattering behavior of surfaces with diverse geometries. By investigating scattering properties using newly proposed boundary conditions, a novel material property is delineated, distinguishing between perfect electrical conductor (PEC), perfect magnetic conductor (PMC), or states in between. Essentially, the fractional boundary condition extends beyond the specifics of Dirichlet and Neumann boundary conditions. It stipulates that the fractional derivative of the tangential component of the total electric field in the surface normal direction must be zero at the scatterer's surface. When the fractional order is zero, this aligns with the Dirichlet Boundary Condition, whereas a fractional order of one corresponds to the Neumann Boundary Condition.

The intermediate boundary condition responsed to materials that case between perfectly electrically conducting (PEC) and perfectly magnetically conducting (PMC) surfaces. In our research, we will utilize the hybrid method as outlined in Veliev's previous work. There are several rationales for employing the hybrid method. Both analytical and numerical methods come with drawbacks and limitations. The desired level of accuracy and the size of the scatterer in electrical terms impose limitations on the suitability of numerical solutions for a given problem. Higher frequency sources and larger electrical objects require finer discretization, leading to increased computational demands. Conversely, analytical methods are typically confined to a finite set of geometries. Therefore, hybrid methods have been developed to combine the beneficial aspects of both analytical and numerical approaches. While analytical methods may only provide closed-form expressions for the high-frequency regime, hybrid methods can expand field expression calculations to a wider range of frequencies. This capability enables the exploration of resonances in double-strip problems using hybrid methods. In this thesis, the orthogonal polynomials method was utilized for addressing diffraction problems.

We can outline the primary approach to solving the diffraction problem as follows: Initially, the scattered field is formulated as an integral, utilizingto derive this integral expression. The total area is then adjusted to satisfy the fractional boundary condition, leading to the formulation of the integral equation. For fractional orders of 0 and 1, analytical solutions are obtained through specific methodologies. In the general solution, the current density on the strips is represented as a combination of special orthogonal functions, taking into account the geometry and boundary conditions. In these scenarios, the current distribution is expanded as a series of Gegenbauer polynomials with unknown constant coefficients associated with the geometry. This manipulation enables the transformation of the integral equation into a system of linear algebraic equations with unknown coefficients, which is achieved using orthogonality relations. The determination of these coefficients is accomplished by solving the resulting system of linear equations through inversion.

In thesis study, suggests that varying the fractional order between 0 and 1 can lead to distributions on the strip suitable for application in designing antennas or devices for directing electromagnetic waves, particularly in antenna synthesis, waveguides, and resonator problems. Another significant finding of the study is that when the fractional degree is zero ($\alpha = 0$), the surface exhibits characteristics akin to a perfectly electrically conductive surface, whereas when the fractional degree is one ($\alpha = 1$), it resembles a perfectly magnetic conductive surface.

2. PRELIMINARY WORK

There is a lot of interest in the modern literature in the application of fractional operators to obtain new solutions with new properties of the problems under consideration. Over the last decade, have been characterized by the development of new approaches (methods) and mathematical models for describing physical phenomena in electrodynamics. For electromagnetic diffraction by some canonical methods in the 1950's objects such as half planes, wedges, strips and cylinders were intensively studied and analytical solutions were developed. All studies on electromagnetic diffraction in the first half of the last century consist of excellently conductive structures such as half planes, wedges and cylinders (Vinod, 1946; Senior, 1952; Senior T., 1979).

On the other hand, the first study for diffraction. It was made by Senior in 1952 for the diffraction of a flat wave E-polarized by a metallic semi-plane with finite conductance. Notwithstanding significant attention focused on diffraction with superb conductive structures, not many notable achievements have been made when finite conductivity has been introduced. At the present time studies have shown that assuming appropriate boundary conditions, the diffraction problem in a metal sheet has definitely same solution. A set of Wiener-Hopf integral equations corresponding to each of the two essential polarizations are derived to determine the electric and 'magnetic' currents existing in the plate. One of these equations is solved precisely, and the solutions of the other three are derived accordingly, taking symmetry into account. Using the generalized steepest descent method then helps identify diffracted areas. A comparison was made between theoretical and experimental forms of the scattered field by briefly refining the circularly polarized incident wave case (Senior, 1952; Senior T., 1979).

Through the last fifteen years N. Engheta for solve a large class of problems in the field of fractional operators developed the method of fractional operatorsFractional operators in electromagnetics have been studied by many researchers. Following that, the fractional area wave propagation in different electromagnetic scenarios such as chiral media waveguides, reflection, and scattering problems was examined. New fractional boundary conditions (FBC) on plane boundaries were introduced. FBC are considered as an intermediate case between perfect electric conductor and perfect magnetic conductor. Fractional binary solutions obtained with the fractional curl operator and the sources that depends on them are discussed (Engheta, 1996; Engheta ,1997; Engheta, 2000).

Electromagnetic theory is based on mathematical modeling, as in the basic science of physics and in all branches of sciences such as engineering. If we explain the interest in electromagnetic scattering; under predefined conditions, the problem itself is modeled using mathematics and tools. The solution part needs to be done so that its solution has been studied by many research for many years with partial differential equations and integral equations and modeled using these equations. The solution can be obtained by analytical, numerical-analytical, and numerical methods. In the analytical methods, the exact solution of the problem can be obtained whereas the number of the problem which can be solved by analytical methods is very limited since the analytical approach can be applied for only canonical geometries (Hasanov, 2004; Hasanov and Polat, 2000).

Computing for electrically large objects is stil challenging today. In such cases approximate analytical techniques are used for specific problems to overcome difficulties. In this study, a technique is proposed. The first differential or integral equation is decomposed numerically, then the equation is tried to be solved for the given conditions to find the field distribution over the required region in the time domain or frequency domain. Known techniques and methods could be listed as follows: Method of Moments (MoM), Finite Element Method (FEM), Finite Difference Time Domain Method (FDTD), and Method of Auxiliary Sources (MAS) (Sadiku, 2018; Gibson, 2014; Anastassiu, 2003).

These techniques require enormous computational power. Researchers are developing new approaches for reducing computational costs. These are fast multipole method (FMM) (frequency domain), the multi-level fast multipole algorithm (MLFMA) (frequency domain), the plane wave time-domain (PWTD) method (time-domain) (Sadiku, 2018; Gibson, 2014; Anastassiu, 2003).

Due to developments in microwave communication systems and circuits, high frquency electromagnetic waves have become an area of interest for scientists since the 20^{th} century. Electrically small obstacles, edges and discontinuties can't be ignored. Therefore, a general strategy to address these issues involves breaking down the problem into fundamental components and addressing them separately. Subsequently, the main problem can be viewed as a composite of canonical problems. This approach enables the problem to be formulated by asymptotically solving the diffracted waves in the high-frequency regime without sacrificing the physical characteristics of the obstacle and space. In essence, the problem boils down to this: the amalgamation of canonical problems typically constitutes a boundary value problem for the Helmholtz equation with mixed boundary conditions. This problem has been handled by using different techniques such as Wiener Hopf, Orthogonal Polynomials in the field distribution having the boundary, edge and radiation condition, the geometric theory of the diffraction, the physical theory of the diffraction, Maliuzhinets etc. (Copson, 1946; Senior, 1952; Senior, 1979; Miles, 1949; Maliuzhinets, 1958).

In this research, the solution of the plane wave diffraction problem with two axisymmetric strips have been examined in different dimensions. Fractional boundary conditions are required for each strip on the surface. Various scenario regarding the size, configurations and fractional grades of the strip have been considered andnumerical results were obtained. Near electric field distribution, Total Radar Section Frequency properties and Poynting vector distribution around these strips were calculated. The solution for fractional order 0.5 was found analytically (Tabatadze et al., 2019).

After obtaining the solution of half-plane, wedge and cylinder structures with different polarization and finite conductivity, many researchers have carried out studies on more complex structures such as apertures, slits or gratings. To solve these complex geometries, some approximations and modifications to the solution of the half plane are used. The edge-to-edge interaction in diffraction has been ignored by the researchers in the solutions. Later, many geometries such as strip, slot, half plane, wedge, truncated waveguide, stepped strips with different boundary situations such as Dirichlet, Neumann and Mixed Boundary Conditions have been obtained by different types of modified Wiener-Hopf techniques (Lawrie and Abrahams, 2007; Idemen, 2000; Kobayashi, 1993).

Discretization is main key in numerical calculations. The process of solving valid differential or integral equations by applying the necessary conditions have followed after the discretization process. The solution has made using a system of linear algebraic equations, which generally requires computational power to solve the equations. In the Method of Moments, the surface currents induced on the object are expanded as the sum of the basis function obtained by applying the boundary condition and the corresponding integral equation. The integral equation includes unknown coefficients for current densities. To solve the integral equation, transformed in a system of linear algebraic equations. After inversion, parameters corresponding to the current density at the surface are obtained. Post-processing for field distributions and other physical properties have been then performed on the solution (Gibson, 2014).

Both analytical and numerical methods have some disadvantages and limitations. In particular, the desired accuracy and electrical size of the scatterer have placed limits on the application of the numerical solution to a given problem, as higher frequency sources and electrically large objects require a greater number of discretizations. The existence of limits necessitated the need for computing power. On the other hand, analytical methods can generally be applied to some finite number of geometries. Therefore, hybrid methods have been developed to combine the advantageous aspects of both analytical and numerical approaches. While some closed expressions can only be obtained for the high frequency regime in analytical methods, Hybrid methods can calculate field expressions for wider frequency regimes. Hybrid methods, in some cases, have expressed electrically small, resonant and electrically large regions by utilizing the tools of analytical and numerical methods (Avşar and İkiz, 2016; Avşar, 2016; Ikiz and Avşar, 2017; Ikiz et al., 2001).

The basic approach to the solution of wave scattering by perfectly conducting polygonal cylinders (PC) is formulated in terms of integral equations of the first kind, the problems of which are satisfied by the unknown functions associated with the current induced on the cylinder surface.In general, integral equations are obtained by applying potential theory with Dirichlet or Neumann boundary conditions on the scatterer surface. Numerical techniques, such as the Galerkin moment method, can often be used to reduce these integral equations to a system of linear algebraic equations (SLAE) (Mei and Bladel, 1963; Abdelmessih and Sinclar, 1967; Shafai, 1971).

For example, results based on the method of moments for diffraction with PC structures have presented (Aidala and Bolle, 1968).

To improve the accuracy of the solution to a certain degree, some approaches have been proposed that explicitly take into account the Meixner edge state, where the current distribution and far-field behavior on PC surfaces are investigated (Abdelmessih and Sinclar, 1967; Shafai, 1971; Hunter, 1972; Hunter, 1973).

PC has been considered as a combination of basic elements such as flat strips. Then the original scattering problem involving the computer is equivalent to scattering by N key elements. Secondly, the field from each direction of the cylinder is represented using the Fourier transform of the corresponding surface current density, which offers a number of advantages for establishing the solution of the problem. Application of the boundary conditions to the employed model then leads to a coupled system of dual integral equations (DIE) with the kernel in trigonometric functions, has satisfied by the unknown Fourier transform. To obtain the desired solution, unlike other methods that rely on the semi-inversion procedure for equation operators and the method of moments, a hybrid technique has been used for special cases and for exact solutions (Mittra et al., 1979; Sologub 1976; Fihmanas and Fridberg, 1978).

In this study, the hybrid method was used as presented in Veliev's previous studies. In the analyses, the current distribution is expanded as the sum of Gegenbauer polynomials whose constant coefficients related to the geometry are unknown. The boundary condition considered in the approaches used in his study is different from the known boundary conditions. Here the boundary condition is called fractional boundary condition. The fractional boundary condition is a generalization of the Dirichlet and Neumann boundary conditions. In this case, the fractional derivative of the tangential component of the total electric field in the direction of the surface normal is zero at the surface of the scatterer. When the fractional degree was zero, this corresponded to the Dirichlet Boundary Condition, while when the fractional degree was one, it meanted that the boundary condition was equaled to the Neumann Boundary Condition. In the middle, where there is a boundary condition, it has corresponded to different materials between perfect electrical conductivity and perfect magnetic conductivity surfaces (Veliev, et al., 2008; Veliev, et al., 2011; Veliev and Engheta, 2003; Veliyev et al., 2018; Veliyev et al., 2020; Tabatadze et al., 2020; Karaçuha et al., 2019; Tarasov, V. E. 2019).

Signal processing, modeling and control are one of the areas that are subject to intensive studies in the world. As pioneering studies examining the application of fractional derivatives in recent decades (Oustaloup, 1991); covering a wide range of topics from a frequency response perspective the application of fractional calculus in automatic control is also mentioned in the book (Podlubny, 1999) and studies have shown that it is also important in this field. We can also see that another book introduces fractional derivatives related to electromagnetic theory in 1920 and then another book we can see applications of fractional derivatives in elasticity problems (Gemant, 1936).

Nowadays it is accepted and proven that there are many advantages of using Fractional Calculus (FC). Increasing number of articles and special issues in journals.In recent years, the field of fractional calculus has shown evidence of attracting a lot of attention from researchers in various fields such as mathematics, physics, chemistry, engineering and even finance and social sciences.

Why fractional calculus is important is a matter of research. Until recently, fractional calculus was considered a mathematical theory whose application was rather esoteric. However, the last (few) decades have attracted a lot of attention due to a significant explosion in research activities on the application of fractional calculus to a wide variety of fields.Fractional calculus was used in various scientific fields, from the physics of diffusion and advection phenomena to control systems, finance, economics and technology. In fact, applications related to fractional calculus today (Machado, 2011);

1) partial control of engineering systems,
2) development of Variation Calculation and Optimal Control to fractional dynamic systems,
3) analytical and numerical tools and techniques,
4) fundamental research of mechanical, electrical and thermal structural relationships and other properties of various materials, engineering materials such as viscoelastic polymers, foams, gels and animal tissues and their engineering and scientific applications,
5) fundamental understanding of wave and diffusion phenomena, including applications in plasma physics (such as diffusion in Tokamak),
6) bioengineering and biomedical applications,
7) thermal modeling of engineering systems such as brakes and machine tools,
8) image and signal processing has emerged in the fields of their measurements and verifications

In this study, inspired by the concept of fraction and using it solutions have been developed for the non-homogeneous Helmholtz equation. Dirac delta function sourceswith he presented an efficient intermediate solution between scalar Green functions of integer-dimensional Helmholtz operators.These solutions, which we refer to as “fractional” solutions, are due to the source distributions enunciable as fractional integrals of integer-dimensional Dirac delta functions. These solutions (according to variables) along a particular direction in the remote region scalar Green's function with k^2 and lower bound zero the lower integer dimension was written as fractional integration.This is also for remote area at points on a certain direction, the magnitude of these functionsGreen's decline as a function of distance with a rate that is also an "intermediate" rate between two corresponding decline rates. They have also represented as functions of neighboring integer-dimensional states. For source allocation intermediate between one dimensional and two dimensional Dirac delta functionsin remote area. It has been shown that it consists of cylindrical and plane wave parts (Engheta, 1996).

This special article has reviewed some of the roles and applications of fractional calculus in electromagnetics, which have recently been introduced and have been very intensively researched. Although these cases are limited and inherently interesting properties of fractional derivatives and integrals and their possible utility in electromagnetic theory have been achieved in studies. Since fractional derivatives/integrals are effectively an intermediate case between traditional integer order differentiation integration, it is thought that the use of these fractional operators in electromagnetics may provide interesting, new, "intermediate" cases in electromagnetics. The study applied the concept of fractional derivatives/integrals to a variety of special electromagnetic problems and obtained promising results and ideas that show that these mathematical operators can be interesting and useful tools in electromagnetic theory. Cases such as fractional multipolarity, fractional solutions of Helmholtz equations, and fractional image methods are examined here. Additionally, some other cases such as the breakdown of the curl operator and its electromagnetic applications will also be examined by the researcher later (Engheta, 1997).

In this study, two approaches to the definition of fractional derivatives are explained. The accuracy of the analysis method for solving the fractional degree problem was investigated. Some improvements have also been made to prove the existence and uniqueness of the solution in fractional differential equations. The improvement of the fractional derivative operator has been made by associating it with two or three variable extended Appell hypergeometric functions and three variable Lauricella hypergeometric functions (Samko et al., 1993).

The concept of ordinary fractional order Fourier transforms has been investigated. Fourier transform is a 1^{st} order transformation. Integral representation of thistransform was used to create a table of fractional order Fourier transforms. generalized operational analysis, ordinary transformation was developed in parallel with the known calculation. Emerging in quantum mechanics a practical technique is provided for solving certain classes of ordinary and partial differential equations from classical quadratic Hamiltonians.The solution method is first of all it is demonstrated by applying free and forced quantum mechanical harmonic oscillators. The corresponding Green's functions were obtained in closed form.The new technique was later extended to three-dimensional problemsquantum mechanical motion of electrons in a constant magnetic field wereapplied to the expression (Namias, 1980).

In this study, the ordinary Fourier transform was determined as p = 1 degree. For example, when the fractional Fourier transform of order p = 1/2 is applied twice in succession, an ordinary Fourier transform is performed. They converted the fractional Fourier transform into optics based on the fact that a Fourier transform was performed on a segment of graded index (GRIN) fiber of appropriate length. Cutting this part of the GRIN fiber into shorter pieces have corresponded to dividing the ordinary Fourier transform into fractional transforms (Özaktaş and Mendlovic, 1993).

The issue of fractional Fourier transforms has been approached in two different ways. Firstly, image rotation has drew attention to the algorithmic isomorphism between rotation and fractional Fourier transform of the Wigner distribution function. Secondly, two optical devices capable of performing fractional Fourier transform have been proposed (Lohmann, 1993).

Differential forms of various degrees are intertwined with multiple integrals. It has become clear that these create a situation that challenges the laws of physics. However, some of its structures (external algebra, external differential operators, and others) are not widely known or used. In this study, we have been focused on the relevance of "external calculation" to electromagnetics. It has been shown that it is quite natural to relate differential forms to electromagnetic quantities. The basic relationships between quantities shown in flow diagrams have been used a single "d" operator (external differential) instead of the familiar curl, grad and div operators of vector calculus.Their covariance properties (their behavior under variation of abilities) are discussed. These formulas in space-time have a conspicuously compendious and graceful expression. Furthermore, they do not change under any change in coordinates covering both space and time. The physical dimensions of electromagnetic forms are such that only two units (coulomb and weber, or e and g) are needed. A few examples of non-mathematical applications have been made, mostly to familiarize researchers with the aspect of equations written this way that are discussed.The transition from differential to integral formulas has been carried out neatly via Stokes' theorem (briefly expressed in forms). When it was come to integrations over moving fields, the concepts of flow and Lie derivative come into play. The relationship between the topology of a region and the existence of valid potentials in that region; the magnetic field due to a constant electric current and the vector potential of the B-field due to the Dirac magnetic monopole has been illustrated with two examples (Deschamps,1981).

In this study the fractional curl operator and its potential applications in electromagnetic problems are discussed. As distinguishing features have been considered radiation from sheet and line current distributions. Applying fractionalized curl operator have been obtained new "fractional" fields and corresponding "fractional" currents, and have been analyzed their physical meanings. In this study "Fractional" field has been considered as "intermediate" one between the original and dual fields. It has been treated as an extension of conventional duality principle. Expressions for the "fractional" fields and relation between "fractional" currents and the original currents have been presented (Ivakhnychenko and Veliev, 2004).

In this study, has been devoted to the description of new boundaries using the fractional field, which has been constructedby applying the fractional curl operator. Fractional field has been permitted to describe the solution of problem of reflection fromspecific boundaries of “impedance type”, which generalize canonical boundaries (perfect electric conductor (PEC),perfect magnetic conductor (PMC), isotropic impedance boundary). Fractional field approach has been obtained boundaries with new features. The fractional curl operator has been allowed to obtain intermediate solutions in electromagnetic problems.In reflection problems for boundaries of the impedance type, the fractional solution is a useful tool for defining solutions of anisotropic and bi-anisotropic boundaries of special kids. New frontiers generalizing both PEC and PMC also isotropic impedance limits, it has been used as a tool in boundaries versus fractional area. Fractional operators have been used to obtain electromagnetic intermediate states (Ahmedov et al., 2006).

During the recent years, we have been focused too much interested in exploring the roles and potential applications of fractional calculus in electromagnetic theory, and in finding physical implications and possible utilities of these operators in certain electromagnetic problems. Fractional calculus has been an area of mathematics that addresses generalization of the mathematical operations of differentiation and integration to arbitrary, general, non-integer orders that can be fractional or even complex One of the definitions of fractional integrals is the one known as the Riemann-Liouville integral. It is written as the idea of non-integer-order differentiation and integration in mathematics, which dates back to as early as the late part of the seventeenth century, has been a subject of interest for many mathematicians in pure and applied mathematics over the years (Engetha, 1996a; Engetha, 1996b, Engetha, 1995;Engetha, 1994).

In this study, fractional derivatives and fractional integrals have interesting mathematical properties that may be utilized in treating certain mathematical problems. Researchers in studies the concept of fractional deriva- tiveshntegrals in several specific electromagnetic problems, and have obtained promising results and ideas that demonstrate that these mathematical operators can be interesting and useful tools in electromagnetic theory. In this study, firstly has been reviewed of the general principles, definitions, and several features of fractional derivative integrals, and then has been reviewed some of ideas and findings in exploring potential applications of fractional calculus in some electromagnetic problems (McBride and Roach, 1985; Kiryakova, 1994; Davis, 1936; Love, 1971; Erdtlyi and Sneddon, 1962; Kalisch, 1967; Osler, 1970; Shilov, 1964; Duran and Kalla, 1992; Rubin, 1994; Abel, 1881;Lutzen, 1984).

In this study, have been investigated several substantial questions arising in the diffraction by circular surfaces with the fractional boundary condition, which is the generalization of Dirichlet and Neumann boundary conditions. In the study, analysis was made on electromagnetic E-polarized plane wave diffraction by a slotted circular cylinder with fractional boundary condition.For the first time, the fractional boundary condition regarding circular geometries have been employed in the literature.In this study, the solution of the electromagnetic diffraction problem for a circular arcfractional boundary condition between 0 and 1 different electromagnetic has been demonstrated different behaviors for the arc. Additively, the resonance peaks have been higher for certain values.

It has also been noticed that there is a transition between field modes. PEC and PMC fields have been validated when changing the fractional order from 0 to 1.The resonance characteristics for different boundary conditions, angle of incidence, and aperture sizes have been analyzed. .

The study has been explored the solution to the comparison between perfect electric conductor (PEC) and perfect magnetic conductor (PMC) cases, focusing on a specific fractional order and impedance. It observes new resonances when the surface deviates from perfect electric or magnetic conducting surfaces. In other words, for fractional orders between 0 and 1, resonant phenomena are observed and analyzed. Additionally, the findings are compared with analytical results derived from zeros of the Bessel function and its derivative for PEC and PMC cases, respectively. This study marks the first investigation into circular geometries with fractional boundary conditions. As such, it has introduced a novel approach to expressing Green's function and computing the fractional derivative of Bessel and Hankel functions. It is propose a concise formulation for the fractional derivative of the Hankel function (Karaçuha et al., 2023).

3. MATERIAL AND METHOD

All the characteristics of the material used, the place where the sample was taken, the way it was taken, the changes it underwent, and its status from the beginning to the end of the research should be explained. As a method, the methods applied during the research should be reported, and any changes made in the method should be explained. However, if a special and new method is used, it should be written in detail.

This study aims to address the two-dimensional thin strip diffraction problem with a new method. The research objective is to develop and generalize an alternative and new approach to the problem, as stated in the summary. The proposed method is expected to offer simpler calculations, faster processing, and wider applicability to various materials when compared to the existing methods in the literature. The problem considered in this study is the diffraction of a plane H-polarized wave, as shown in Figure 3.2. To describe the scattering properties of surfaces in various geometries, we utilize the integral boundary condition, which corresponds to a boundary condition between the Dirichlet and Neumann boundary conditions. The integral boundary condition is a generalization of the Dirichlet and Neumann boundary conditions. In our study, we employ the hybrid method as presented in previous works. The hybrid method combines the advantages of both analytical and numerical approaches to develop methods. While some implicit expressions can only be obtained for the high-frequency regime, hybrid methods can calculate field expressions in wider frequency regimes, allowing the investigation of resonances for thin strip problems with hybrid methods. To solve the problem presented in this study, we use the method of orthogonal polynomials to tackle the diffraction problems. Firstly, in Figure 3.2, we define the scattered area as an integral and employ Green and Fourier analysis to obtain this integral. The problem is then solved using an analytical approach. For the general solution, the integral equation is expressed as the sum of the special orthogonal functions, taking into account the current density on the strips, geometry, and other relevant conditions.

The fractional derivative method, as its name suggests, allows us to generalize the boundary conditions and solve the current problem in the most general way using the fractional derivative approach. The fractional boundary condition can be briefly summarized as follows: the fractional derivative of the tangential component of the total electric field in the surface normal direction is zero at the surface of the scatterer. When the fractional order is zero, this corresponds to the Dirichlet Boundary Condition, while when the fractional order is equal to one, it means the boundary condition is equal to the Neumann Boundary Condition. In our study, the hybrid method will be used as presented in Veliev's previous works. There are many reasons for using the hybrid method. Both analytical and numerical methods have some disadvantages and some limitations. The desired accuracy and the electrical size of the scatterer impose a limit on the applicability of the numerical solution for a given problem, since higher frequency source and electrically large objects require a greater number of discretizations. Analytical methods can generally be applied to some finite number of geometries. Therefore, hybrid methods have been developed to combine the advantageous aspects of both analytical and numerical approaches.

In this thesis, the orthogonal polynomials method was used to solve two-dimensional thin strip diffraction problems. The main approach to solving the diffraction problem is; First, the scattered field is defined as an integral. Green's Theorem and Fourier analysis are used to obtain this integral. The total area is then forced to satisfy the fractional boundary condition. Then the integral equation is obtained. In the case of fractional order 0 and 1, the problem is solved analytically with some approaches, in Figure 3.3. For the general solution, to solve the integral equation, the current density on the strips is expressed as the sum of special orthogonal functions, taking into account the geometry and edge condition. In the problems here, the current distribution is expanded as a sum of Gegenbauer polynomials with unknown constant coefficients related to the geometry. This approach allows transforming the integral equation into a system of linear algebraic equations with unknown constant coefficients using orthogonality relations. Finding the coefficients is done by inverting the resulting system of linear equations.

3.1. Formulatıon of The Problems

3.1.1. Fractional Calculus

Before starting the problem definition in the thesis, it is necessary to define the fractional derivative and fractional derivative of the exponential function. Why do we need fractional derivatives and non-integer order integrals? It is necessary to ask this question. Fractional calculus, the theory of fractional order integrals and derivatives, describes a wide variety of different types of operators of non-integer degree (Rothwell, 2018; Engheta, 1996; Engheta, 1998; Engheta, 1999; Tabatadze et al., 2016; Veliev et al., 2018; Ivakhnychenko and Veliev, 2004). Fractional calculus stems from the inquiry of whether the concept of derivative, which pertains to integer orders, can be extended in a manner that remains valid when the order is not an integer. This question was first by L'Hopital on September 30, 1695. On this date, *L'Hopital*corresponded with Leibniz incuring about $D^n x/Dx^n$, his notation of the n^{th} derivative of a linear function $f(x) = x$. *L'Hopital* asked with great curiosity what the result would be if $n = \frac{1}{2}$. Leibniz remarked that this would constitute "an apparent paradox from which useful conclusions will one day be drawn" (Tabatadze, 2019).

Following this distinctive and unparalleled discourse, the field of fractional analysis garnered the interest of other esteemed mathematicians, with many of them making direct or indirect contributions to its advancement. By the end of the 19th century, a theory of fractional calculus in a more comprehensive form began to emerge, largely attributable to Liouville (Liouville, 1832; Liouville, 1834; Liouville, 1835;Liouville, 1937), Riemann (Riemann, 1876), Grünwald (Geünwald, 1867), Letnikov (Letnikov, 1868a; Letnikov, 1868b; Letnikov, 1872; Letnikov, 1874) and Marchaud (Ferrari, 2018; Rogosin, 2018).

In 1819, Lacroix became the first mathematician to publish an article on the subject on the topic fractional derivatives (Lacroix, 1819). The equation starts like this:

If $y = x^m$ is where m is a positive integer,

$$\frac{d^n y}{dx^n} = \frac{m!}{(m-n)!} x^{m-n}, m \geq n \tag{3.1}$$

$$\frac{d^n y}{dx^n} = \frac{\Gamma(m+1)}{(m-n+1)} x^{m-n} \tag{3.2}$$

If where $m = 1$ and $n = \frac{1}{2}$, as formule obtained,

$$\frac{d^{\frac{1}{2}} y}{dx^{\frac{1}{2}}} = \frac{2\sqrt{x}}{\sqrt{\pi}} \tag{3.3}$$

Nevertheless, the initial application of fractional operations was not attributed to Lacroix but to Abel in 1823. Abel utilized fractional calculus to address an integral equation stemming from the problem of the tautochrone shape, where the objective was to find a curve such that the time taken for an object to descend along it under uniform gravity is invariant regardless of the object's initial position (Lacroix, 1819; Abel, 1823).

Over the years, numerous mathematicians, employing their distinct notations and methodologies, have devised diverse definitions that align with the concept of non-integer order integrals or derivatives. It's quite noteworthy and intriguing that when a fractional derivative is examined according to the Riemann-Liouville definition, it yields the same outcome as the equation (3.3) previously derived by Lacroix. Given that most other formulations of fractional calculus are largely variations of the Riemann-Liouville approach, this version will predominantly be referenced in addressing the problems outlined in the thesis (Lacroix, 1819).

At present, the most commonly used fractional integrals and derivatives in applications are the left- and right-sided Riemann–Liouville integrals (Podlubny, 1999). (Fractional integrals are as described in equations (3.6) and (3.7)).

$$D_{a,x}^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \frac{d}{dx} \int_{a}^{x} \frac{f(t)}{(x-t)^{1-\alpha}}, \alpha > 0 \tag{3.4}$$

$$D_{b,x}^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \frac{d}{dx} \int_{x}^{b} \frac{f(t)}{(t-x)^{1-\alpha}}, \alpha > 0 \tag{3.5}$$

The Riemann–Liouville derivatives on the left and right sides,

$$D_{a,x}^{\alpha} f(x) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dx^m} \int_{\mathrm{a}}^{x} \frac{f(t)}{(x-t)^{\alpha+1-m}},\ m-1 \le \alpha < m \in Z^{+} \tag{3.6}$$

$$D_{x,b}^{\alpha} f(x) = \frac{(-1)^m}{\Gamma(m-\alpha)} \frac{d^m}{dx^m} \int_{x}^{b} \frac{f(t)}{(t-x)^{\alpha+1-m}},\ m-1 \le \alpha < m \in Z^{+} \tag{3.7}$$

The left fractional derivative over the closed interval [a, x] and similarly the right fractional derivative over the closed interval [x, b] are expressed. The left Riemann-Liouville fractional derivative is considered as expected, and it also exists for $0 < \alpha \le 1$.

Thus, for $a \to -\infty$, $f(x)$ as the approaches to zero in Equation (3.6):

$$D_x^{\alpha} f(x) \overset{\text{def}}{=} \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_{-\infty}^{x} \frac{f(t)}{(x-t)^{\alpha}} dt \tag{3.8}$$

Here, $\Gamma(1-\alpha)$ represents the Gamma function, and D_x^{α} is the fractional derivative operator indicating differentiation with respect to x of order α, ranges from $(0,1)$ inclusive of the edges.

To solve the diffraction problem, the boundary value problem needs to be solved where the boundary condition is the fraction boundary condition (Ivakhnvchenko and Veliev, 2004; Ivakhnvchenko et al.,2008). The fractional boundary condition, which corresponding the fractional derivative of the total tangential field component with respect to the direction of the surface normal, is the generalization of the Dirichlet and Neumann boundary conditions. In that case, it is crucial to derive the fractional derivative of the exponential function, e^{-ikx} where $''i''$ is equal to $\sqrt{-1}$ (imaginary unit) and k is a real number. Then the expression is found when $f(x) = e^{-ikx}$.

First, it is required to apply the change of variable as $x - t = s$. Note that, in the integrand of (3.8), t is a dummy variable.

$$D_x^{\alpha} f(x) = \frac{d^{\alpha} f(x)}{dx^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_{0}^{\infty} \frac{f(x-\mathrm{s})}{s^{\alpha}} ds \tag{3.9}$$

In equation (3.9), e^{-ikx} is written instead of $f(x)$;

$$\frac{d^{\alpha} e^{-ikx}}{dx^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_{0}^{\infty} \frac{e^{-ik(x-\mathrm{s})}}{s^{\alpha}} ds = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} e^{-ikx} \int_{0}^{\infty} \frac{e^{iks}}{s^{\alpha}} ds$$

$$= \frac{1}{\Gamma(1-\alpha)} (-ik) e^{-ikx} \int_{0}^{\infty} \frac{e^{iks}}{s^{\alpha}} ds \tag{3.10}$$

If $\int_0^\infty \frac{e^{iks}}{s^\alpha} ds$ expression is replaced by T for short in the equation. After defining T, the change of variable$\left(s = \frac{iy}{k}\right)$ is required to make a resemblance between function T and Gamma function. Equation then, *T;*

$$T = \left(\frac{1}{-ik}\right)^{-\alpha+1} \int_0^{-i\infty} e^{-y} y^{-\alpha} dy \tag{3.11}$$

If the definition of the Gamma Function (Γ) is expressed,

$$\Gamma(x) = \int_0^\infty y^{x-1} e^{-y} dy \approx \Gamma(1-\alpha) = \int_0^\infty e^{-y} y^{-\alpha} dy. \tag{3.12}$$

As we know from Complex Integral, by choosing a closed loop and if there is no singularity in it, the closed loop integral will be zero in the complex plane according to the Cauchy Principle (Brown and Churchill, 2009; Harrington, 1993). We can see the integral path in Figure 3.1.

Under the condition that $0 < \alpha < 1$, there is a singularity at zero. Here path 2 represents the path of the Gamma function by definition with the argument (1-α). It should be noted that according to our approach in the equations (integral T) the reverse direction of path 1 is required.

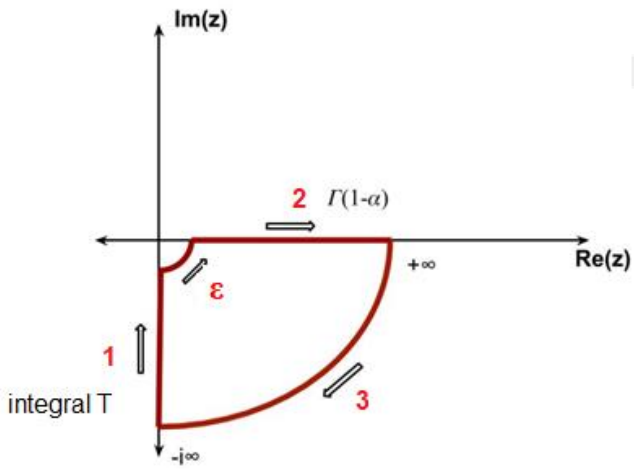


Figure 3.1. Integration Path

From Cauchy's Principle (Brown and Churchill, 2009; Harrington, 1993),

$$\oint e^{-z} z^{-\alpha} dz = \int_1 e^{-z} z^{-\alpha} dz + \int_2 e^{-z} z^{-\alpha} dz + \int_3 e^{-z} z^{-\alpha} dz + \int_\epsilon e^{-z} z^{-\alpha} dz = 0 \tag{3.13}$$

From Jordan Theorems (Brown and Churchill, 2009; Harrington, 1993):

$$\lim_{z \to 0} z f(z) = 0, \quad then \int_\epsilon f(z) dz = 0$$

$$\lim_{z\to\infty} zf(z) = 0\,, \quad then \int_3 f(z)dz = 0 \tag{3.14}$$

By Jordan Theorems,

$\int_\epsilon e^{-z}z^{-\alpha}dz$and $\int_3 e^{-z}z^{-\alpha}dz$ go to zero.

Then,

$$\int_1 e^{-z}z^{-\alpha}dz = -\int_2 e^{-z}z^{-\alpha}dz \tag{3.15}$$

Note that,

$\int_2 e^{-z}z^{-\alpha}dz = \Gamma(1-\alpha)$ and $\int_1 e^{-z}z^{-\alpha}dz = -T$

This yields that,

$$\begin{aligned}\frac{d^\alpha e^{-ikx}}{dx^\alpha} &= \frac{1}{\Gamma(1-\alpha)}(-ik)e^{-ikx}\left(\frac{1}{-ik}\right)^{-\alpha+1}\int_0^{-i\infty} e^{-y}y^{-\alpha}dy \\ &= \frac{1}{\Gamma(1-\alpha)}(-ik)^\alpha e^{-ikx}\Gamma(1-\alpha) \\ &= (-ik)^\alpha e^{-ikx}\end{aligned} \tag{3.16}$$

As seen in equation (3.16), the fractional derivative of the exponential function is obtained. By using this property, the fractional derivative of an exponential function can be easily obtained without going through the formal definition given in equation (3.8).

3.2. Two-Dimensional Thin Strip Diffraction Problem Solution and Boundary Conditions

This study aims to handled the two-dimensional thin strip diffraction problem using a novel method. The research objective, as stated in the summary, is to develop and generalize an alternative and innovative approach to the problem. The proposed method is expected to provide simpler calculations, faster processing, and broader applicability to various materials compared to the existing methods in the literature. The problem considered in this thesis is the diffraction of a plane H-polarized wave, as illustrated in Figure 3.2. To depict the scattering characteristics of surfaces in diverse geometries, we employ the integral boundary condition, which represents a boundary condition intermediate between the Dirichlet and Neumann boundary conditions. The integral boundary condition is defined as a generalized form both the Dirichlet and Neumann boundary conditions. In our investigation, we utilize the hybrid method, as outlined in prior research. This approach different methods the benefits of both analytical and numerical techniques, while some implicit expressions can only be obtained for the high-frequency regime, hybrid methods can calculate field expressions in wider frequency regimes, allowing the investigation of resonances for thin strip problems with hybrid methods. To solve the problem presented in this study, we use the method of orthogonal polynomials to tackle the diffraction problems.

Firstly, in Figure 3.2, we define the scattered area as an integral and employ Green and Fourier analysis to obtain this integral. The problem is then solved using an analytical approach. For the general solution, the integral equation is expressed as the sum of the special orthogonal functions, taking into account the current density on the strips, geometry, and other relevant conditions (Baz et al., 2024)

The diffraction problem of the Plane H-polarized wave shown in in Figure 3.2, will be discussed and analytical and numerical solutions will be made.

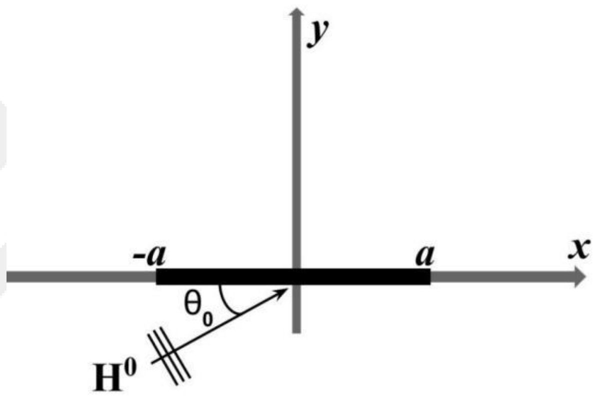


Figure 3.2. Geometry of the planar H-polarized diffraction problem (Baz et al., 2024)

$$H_z^0 = e^{ik\left(\alpha_0 x + \sqrt{1-\alpha_0^2} y\right)} \tag{3.17}$$

On a flat strip (see Figure3.2). Here, $\alpha_0 = cos\theta_0$, and θ_0 is the angle of incidence. Total magnetic field can be expressed as

$$H_z = H_z^0 + H^s \tag{3.18}$$

Where in equation 3.18 the function H^s is represented as a scattered wave, (Hönl et al., 1987; Ivakhnychenko et al., 2008; Prudnikov et al.,2002; Otsuki, 1976),

$$H_z(\eta', \xi') = \frac{i}{2}\int_{-1}^{1} \mu(\eta) \frac{\partial y}{\partial \xi'} H_0^{(1)}\left(\epsilon_j \sqrt{(\eta' - \eta^0)^2 + \xi'^2}\right) d\eta^0 \tag{3.19}$$

In equation (3.19), $\eta = \frac{x}{a}$, $\xi = \frac{y}{a}$ are entered as dimensionless coordinates, then the Hankel function of the first kind is entered into the definition of $H_0^{(1)}$ and it is a zero-order function.$H_z$function in the total magnetic field equation describes the surface current density function that complies with the condition at (-1,1) integral boundaries.

$$\mu_E^{(j)}(\eta) \sim (1-\eta^2)^{\frac{1}{2}},\ \eta \to \pm 1 \tag{3.20}$$

For the H_z scattered field in the equations, we can write the corresponding expression in the Fourier domain of the current density function as stated in equation 3.21 given below,

$$H_z^s(\eta', \xi') = -\frac{\epsilon}{2\pi}\frac{|\xi'|}{\xi'}\int_{-\infty}^{\infty} h(\alpha) e^{i\epsilon[\alpha\eta' + \sqrt{1-\alpha^2}|\xi'|]} d\alpha \tag{3.21}$$

$$\mu(\eta_j) = \frac{\epsilon}{2\pi}\int_{-\infty}^{\infty} h(\alpha) e^{i\epsilon\alpha\eta} d\alpha \tag{3.22}$$

After performing the Fourier transformation, we define the serial expansion of the current density function using Gegenbauer orthogonal polynomials, as seen in equation (3.23) $\{C_n^{\nu}\}_{n=0}^{\infty}$ (Hönl et al., 1987).

$$\mu(\eta) = (1-\eta^2)^{\nu} \sum_{n=0}^{\infty} x_n C_n^{\nu+\frac{1}{2}}(\eta),\ \frac{1}{2} \le \nu < 1 \tag{3.23}$$

where, x_n is unknown coefficient. When $\nu = \frac{1}{2}$, $C_n^{\nu+\frac{1}{2}}(\eta)\Big|_{\nu=\frac{1}{2}} = U_n(\eta)$ Note that $U_n(\eta)$ is Chebyshev polynomials, a polynomial of the second kind, are defined.

The current density function shown in equation (3.23) is obtained again after Fourier transformation is performed.

$$h(\alpha) = \frac{\epsilon_j}{2\pi}\int_{-1}^{1} \mu(\eta) e^{-i\epsilon\alpha\eta} d\eta$$

$$h(\alpha) = \frac{2\pi}{\Gamma\left(\nu+\frac{1}{2}\right)} \sum_{n=0}^{\infty} (-i)^n x_n \beta_n^{\left(\nu+\frac{1}{2}\right)} \frac{J_{n+\nu+\frac{1}{2}}(\epsilon\alpha)}{(2\epsilon\alpha)^{\nu+\frac{1}{2}}}\Bigg|_{\nu=\frac{1}{2}} = 2\pi \sum_{n=0}^{\infty} (-i)^n x_n (n+1) \frac{J_{n+1}(\epsilon\alpha)}{(2\epsilon\alpha)}$$

$$\beta_n^{(\nu+\frac{1}{2})} = \frac{\Gamma(n+2\nu+1)}{\Gamma(n+1)}\bigg|_{\nu=\frac{1}{2}} = n+1 \tag{3.24}$$

After Fourier and Gegenbauer polynomials are defined, Neumann boundary conditions are defined on the straight strip.

$$\left[\frac{\partial}{\partial \vec{n}}\left(H_z^0 + H_z^p\right)\right]_L = 0 \tag{3.25}$$

To determine the unknown x_{2k}coefficients, a system of infinite algebraic equations is obtained and defined in equation (3.26).,

$$\begin{cases} (-1)^k x_{2k} - \sum_{n=0}^{\infty}(-1)^n(2n+1)x_{2n}d_{2k2n} = -2i\frac{\sqrt{1-\alpha_0^2}}{\alpha_0^2}J_{2k+1}(\epsilon_1\alpha_0) \\ (-1)^k x_{2k+1} - \sum_{n=0}^{\infty}(-1)^n(2n+2)x_{2n+1}d_{2k+12n+1} = 2\frac{\sqrt{1-\alpha_0^2}}{\alpha_0^2}J_{2k+2}(\epsilon_1\alpha_0) \end{cases} \tag{3.26}$$

indicates the d_{kn}matrix elements specified in equation (3.26) and,

$$d_{kn}^{(1)} = 2\int_0^{\infty}\gamma(\alpha)J_{k+1}(\epsilon\alpha)J_{n+1}(\epsilon\alpha)\frac{d\alpha}{\alpha} \tag{3.27}$$

Where $\gamma(\alpha) = 1 + \frac{i}{|\alpha|}\sqrt{1-\alpha^2}$

After all definitions and transformations are made, the values of the unknowns given in Equation (3.26) are placed the equation after being defined in the Fourier function. the integral equations given in Equation (3.28) are obtained (Prudnikov et al., 2002).

$$h_1(\pm\alpha) = \frac{1}{2}[h_1^+(\alpha) \pm h_1^-(\alpha)]$$

$$h_1^+(\alpha) = 2\pi\sum_{n=0}^{\infty}(-1)^n(2n+1)x_{2n}^{(1)}\frac{J_{2n+1}(\epsilon_1\alpha)}{\epsilon_1\alpha}$$

$$h_1^-(\alpha) = -2\pi i\sum_{n=0}^{\infty}(-1)^n(2n+2)x_{2n+1}^{(1)}\frac{J_{2n+2}(\epsilon_1\alpha)}{\epsilon_1\alpha} \tag{3.28}$$

After the Fourier transformation is performed, the integral equations given in equation (3.29) are obtained.

$$h_1^+(\alpha) = -i\frac{4\pi}{\epsilon_1}\sqrt{1-\alpha_0^2}K^+(\alpha,\alpha_0) + 2\int_0^\infty \beta\gamma(\beta)K^+(\alpha,\beta)h_1^+(\beta)d\beta$$

$$h_1^-(\alpha) = -\frac{4\pi}{\epsilon_1}\sqrt{1-\alpha_0^2}K^-(\alpha,\alpha_0) + 2\int_0^\infty \beta\gamma(\beta)K^-(\alpha,\beta)h_1^-(\beta)d\beta \tag{3.29}$$

The kernels of the equations $h_1^+(\alpha)$ and $h_1^-(\alpha)$ have the form of a bilinear expansion through the Bessel functions $J_n(x)$ (Baz et al., 2024).

$$K^+(\alpha,\beta) = \frac{1}{\alpha\beta}\sum_{K=0}^{\infty}(2K+1)J_{2K+1}(\epsilon\alpha)J_{2K+1}(\epsilon\beta)$$

(3.30)

$$K^-(\alpha,\beta) = \frac{1}{\alpha\beta}\sum_{K=0}^{\infty}(2K+2)J_{2K+2}(\epsilon\alpha)J_{2K+2}(\epsilon\beta)$$

These bilinear expansions of Bessel functions in the form of infinite series can be examined in detail in the reference we have mentioned (Otsuki, 1976). In the last case, expressions that are simple enough for the kernels have been now obtained.

$$K^+(\alpha,\beta) = \frac{\epsilon}{2(\alpha^2-\beta^2)}[\alpha J_1(\epsilon\alpha)J_0(\epsilon\beta) - \beta J_0(\epsilon\alpha)J_1(\epsilon\beta)]$$

(3.31)

$$K^+(\alpha,\alpha) = \frac{\epsilon^2}{4}[J_0^2(\epsilon\alpha) + J_1^2(\epsilon\alpha)]$$

$$K^-(\alpha,\beta) = \frac{\epsilon}{2(\alpha^2-\beta^2)}[\beta J_1(\epsilon\alpha)J_0(\epsilon\beta) - \alpha J_0(\epsilon\alpha)J_1(\epsilon\beta)]$$

(3.32)

$$K^-(\alpha,\alpha) = -\frac{\epsilon}{2\alpha}J_0(\epsilon\alpha)J_1(\epsilon\alpha) + K^+(\alpha,\alpha)$$

The equations given in Equation (3.28) are the new integral equations with simple kernels that we mentioned in the last case. After obtaining this equation, the following equations are obtained respectively for the radiation pattern (RP) and cross section of the total scattering (TCS) expressed with unknown coefficients.

$$\Phi(\psi) = -\frac{\pi\sin(\psi)}{2\epsilon\cos(\psi)}\sum_{n=0}^{\infty}(-i)^n(n+1)x_n^{(1)}J_{n+1}(\epsilon\cos(\psi)) \tag{3.33}$$

$$\frac{\sigma_s^H}{4a} = \frac{\pi}{\epsilon^2}\frac{sin\theta_0}{cos\theta_0}\sum_{n=0}^{\infty}(n+1)J_{n+1}(\epsilon\cos\theta_0)Re\left\{(-i)^n x_n^{(1)}\right\} \quad (3.34)$$

3.3. Two Dimensional Strip Diffraction with Fractional Boundary Conditions

The final investigation in the thesis is to analyzed the current distribution for a single strip with cylindrical wave excitation, specifically for the cases where the fractional order (FO) values are α=0 and α=1. Previously, diffraction by a strip with fractional boundary condition in the range [0,1] have been studied (Veliev et al., 2018a; Veliyev et al., 2018b; Veliyev et al., 2020; Karaçuha et al.,2019). The solution approach is the same. Therefore, a brief explanation is given in this section. Heres, near electric field distribution, far field pattern are investigated. The problem is two-dimensional. The strip with 2a and is located on the plane $y = 0$. The strip has an infinite extension along the z-axis and infinitesimal height along the y-axis.

When the cylindrical source on the thin strip surface is used to solve the problem, the current distribution, near electric field and far field electric field are investigated. In the problem, It has a geometry given in Figure Figure 3.3., at the point (x_o,y_o) using a cylindrical wave source with a width of 2a in the y = 0 plane on an infinite two-dimensional plane strip along the z axis.

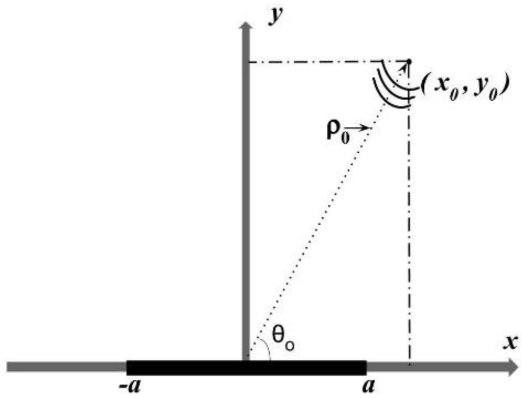


Figure 3.3. The geometry of the problem

The time dependency $e^{-i\omega t}$ and through the problem, it will be ignored. Unlike the problems previously examined on this subject, the incident wave in this problem is a cylindrical wave.Therefore, also the incidence wave should be expressed in terms of the exponential function to take the fractional derivative more easily for the further procedure (Veliev et al., 2018a; Veliyev et al., 2018b; Veliyev et al., 2020; Karaçuha et al.,2019).

The incidence electric field $\vec{E}_z^i$ mathematically can be represented as equation (3.35) under the condition of $e^{-i\omega t}$ time dependency (Veliev et al., 2018a; Veliyev et al., 2018b; Veliyev et al., 2020; Karaçuha et al., 2019; Balanis, 2016).

$$\vec{E}_z^i(x,y) = -\vec{J}_e \frac{\eta_0 k}{4} H_0^{(1)}\left(k\sqrt{(x-x_o)^2+(y-y_o)^2}\right) \quad (3.35)$$

Where, $H_0^{(1)}$ is the Hankel function of the first kind and zero-order, η_0 is the impedance of free space, and k is the wavenumber. The total electric field is defined as a superposition of the incidence and the scattered electric fields as given in equation (3.36).

$$\vec{E}_z = \vec{E}_z^i + \vec{E}_z^s \quad (3.36)$$

After obtaining the total electric field equation, there are two main steps to be achieved. The first is to express the scattered field mathematically and then, to apply the fractional boundary condition to obtain an equation to solve with mathematical techniques. As demonstrated earlier, the scattered electric field can be represented as the convolution of the Green's function with the induced current density on the strip in equation (3.37) (Veliev et al., 2018a; Veliyev et al., 2018b; Veliyev et al., 2020; Karaçuha et al.,2019; Karaçuha et al., 2020).

$$E_z^s(x,y) = \int_{-\infty}^{\infty} f^{1-\alpha}(x') G^{\alpha}(x-x',y)dx' \quad (3.37)$$

Where, $f^{1-\alpha}(x')$ is entitled the fractional current density which exists only on the strip and $G^{\alpha}(x)$ is the fractional Green's function which has the following form (Veliev et al., 2018a; Veliyev et al., 2018b; Veliyev et al., 2020; Karaçuha et al.,2019). In solving problems, the main motivation is to express everything in terms of the exponential functions. Therefore, the spectral representation of the Hankel function will be utilized to facilitate the application of the fractional boundary condition.

$$G^{\alpha}(x-x',y) = -\frac{i}{4}\mathfrak{D}_{ky}^{\alpha} H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right) \quad (3.38)$$

where,

$$H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right) = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{ik((x-x')\alpha+|y|\sqrt{1-\alpha^2})} \frac{d\alpha}{\sqrt{1-\alpha^2}} \quad (3.39)$$

Note that, $Im\{\sqrt{1-\alpha^2} > 0\}$ is assumed. Im is the imaginary operator which gives the imaginary part of the function of a function. By usingequation (3.38) in equation (3.39) and taking into account the Fourier transform, the scattered field is obtained as follows.

$$E_z^s(x,y) = -i\frac{e^{i\frac{\pi}{2}\alpha}}{4\pi}\int_{-\infty}^{\infty} F^{1-\alpha}(\tau)e^{ik\left[\tau x + y\sqrt{1-\tau^2}\right]}(1-\tau^2)^{\frac{\alpha-1}{2}}d\tau \quad (3.40)$$

where,

$$F^{1-\alpha}(\tau) = \int_{-1}^{1}\tilde{f}^{1-\alpha}(\xi)e^{-i\varepsilon\tau\xi}d\xi, \tilde{f}^{1-\alpha}(\xi) = af^{1-\alpha}(a\xi)$$

$$\varepsilon = ka,\ \ \xi = \frac{x}{a},\quad \tilde{f}^{1-\alpha}(\xi) = \frac{\varepsilon}{2\pi}\int_{-\infty}^{\infty}F^{1-\alpha}(\tau)e^{i\varepsilon\tau\xi}d\tau \quad (3.41)$$

Here, $F^{1-\alpha}(\tau)$ is the Fourier transform of $\tilde{f}^{1-\alpha}(\xi)$ which is the normalized current density between $x \in [-1,1]$ on the strip. As mentioned above, the second step is to apply the fractional boundary condition after having the mathematical expression for each component of the total electric fields. In equation (3.42), the fractional boundary condition is given (Veliev et al., 2018; Veliyev et al., 2018; Veliyev et al., 2020; Karaçuha et al., 2019).

$$\mathfrak{D}_y^{\alpha}E_z(x,y) = 0,\ y \to \pm 0 \quad (3.42)$$

We will use the $\mathfrak{D}_y^{\alpha}$ symbol in the analytical solution of the problem under consideration. What we actually mean here is the fractional derivative on the semi-axis. $\mathfrak{D}_y^{\alpha} \equiv -\infty\ \mathfrak{D}_y^{\alpha}$

The incident field $\vec{E}$ polarized plane electromagnetic wave. The infinite thin strip of width 2a is located in the *y* = 0 plane and at infinity along the *z* axis. The incidence field $\overrightarrow{E^i} = \vec{z}E_z^i(x,y)$ is defined with following the expression,

$$E_z^i(x,y) = e^{-ik(xcos\theta + ysin\theta)} \quad (3.43)$$

where, θ is the incidence angle, $k = \frac{2\pi}{\lambda}$ and k wave number. The time dependency is given as $e^{-i\omega t}$ and through the problem, it will be ignored. The scattered field, $\vec{E} = \vec{z}E_z(x,y)$ is the sum of the incident and scattered fields;

$$E_z(x,y) = E_z^i(x,y) + E_z^s(x,y) \quad (3.44)$$

The problem solution, the function $E_z(x,y)$, must must provide the following conditions:

<u>*Condition 1*</u>: The Helmholtz equation in whole space, noninclusive the strip surface;

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)E_z=0$$

Condition 2: The Somerfield radiation condition in the infinity for the scattered field (Colton,1987; Veliev and Shestopalov, 1988);

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial E_z^s}{\partial r}-iE_z^s\right)=0\ ,\qquad r=\sqrt{x^2+y^2}$$

Condition 3: The Meixner condition in the ends of the strip;

y=0, x→±a (Hönl, 1987)

Condition 4: The FBC on the surface of the strip:

$$\mathfrak{D}_{ky}^{\alpha}E_z(x,y)|_{y\to\pm0}=\ \mathfrak{D}_{ky}^{\alpha}\left[E_z^i(x,y)+E_z^s(x,y)\right]|_{y\to\pm0}=0,\qquad (-a<x<a)$$

For convenience the derivative is taken with due to the dimensionless value *ky*. According the results of the first chapter, let's represent the function $E_z^s(x,y)$with the fractional Green's function,

$$E_z^s(x,y)=\int_{-\infty}^{\infty}f^{1-\alpha}(x')G^{\alpha}(x-x',y)dx' \tag{3.45}$$

Where, $f^{1-\alpha}(x)$represents the unknown function, referred to as the fractional potential density.

In the two-dimensional scenario, the fractional Green's function G^{α} in the equation for $E_z^s(x,y)$ is represented by the fractional derivative of the classical Green's function.

$$G^{\alpha}(x-x',y)=-\frac{i}{4}\mathfrak{D}_{ky}^{\alpha}H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right) \tag{3.46}$$

Where, Hankel's function of the first kind $H_0^{(1)}$, and the function $f^{1-\alpha}(x)$, which expresses the potential density in the integral equation (3.45), is associated with the discontinuity of the fractional derivative of the function $E_z(x,y)$when it crosses the boundary at *y*=0.

$H_0^{(1)}$ function of the first kind is a mathematical function that arises in the context of solving certain differential equations, particularly those with circular symmetry. It's often used in problems involving wave propagation or scattering, especially in cylindrical coordinates.

$$f^{1-\alpha}(x) = \mathfrak{D}_{ky}^{1-\alpha} E_z(x,y)|_{y=+0} - E_z(x,y)|_{y=-0}\ , \qquad -\mathrm{a} < x < a \tag{3.47}$$

$f^{1-\alpha}(x)$can be expressed through integration and plays a role in determining the behavior of the electrical and magnetic currents on the surface.

If the equation is written in new form for scattered field $E_z^s(x,y)$, on the surface we examine, $(y = 0, -\mathrm{a} < x < a)$;

$$E_z(x,y) = \frac{1}{4\pi}\int_{-a}^{a} \mathfrak{D}_{ky}^{-\alpha}\,\mathrm{G}(x-x',y)\nabla\ \mathfrak{D}_{ky}^{\alpha} E_z(x',y)\,dx'$$
$$-\frac{1}{4\pi}\int_{-a}^{a} \mathfrak{D}_{ky}^{\alpha} E_z(x',y)\,\nabla\,\mathfrak{D}_{ky}^{-\alpha}\,\mathrm{G}(x-x',y)\,dx' \tag{3.48}$$

The right side of the second integral of the equation (3.48) isequal to zero, where the function $f^{1-\alpha}(x)$is determined by the discontinuity of the fractional derivative of the fractional boundary condition E_z.

The equation given in equation (3.45), gives the known representations of the potentials from the first and second layers, which used in solving the diffraction problem it gives the known representations of potentials with Dirichlet and Neumann BC respectively (Ivakhnychenko and Veliev, 2004; Veliev et al., 2011):

$$U(x,y) = -\frac{\mathrm{i}}{4}\int_{-\mathrm{a}}^{\mathrm{a}} f^{1-\alpha}(x')\,H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right), \qquad \alpha = 0 \tag{3.49}$$

The potential density function has physical meaning in the equation; defines the density of electric currents on the surface.

$$f^{1-\alpha}(x) = \frac{\partial E_z(x,y)}{\partial y}|_{y=+0} - \frac{\partial E_z(x,y)}{\partial y}|_{y=-0}\ , \quad \alpha = 0 \tag{3.50}$$

If we represent the $E_z(x,y)$function in equation (3.45) as fractional boundary conditions FBC in equation (3.42), if the equation is rewritten;

$$\lim_{y\to 0}\ \mathfrak{D}_{ky}^{\alpha}\int_{-\infty}^{\infty} f^{1-\alpha}(x')G^{\alpha}(x-x',y)dx' = -\lim_{y\to 0}\ \mathfrak{D}_{ky}^{\alpha}\ E_z^{i}(x,y) \tag{3.51}$$

On the left side of the equation, $f^{1-\alpha}(x)$, represents the unknown function, while the knowns are on the right side of the equation. It is assumed that the function f is zero outside the interval $[-a, a]$ .According to this situation Fourier transform,

$$\tilde{f}^{1-\alpha}(\xi) = \frac{ka}{2\pi} \int_{-\infty}^{\infty} F^{1-\alpha}(\beta) e^{ika\xi\beta} d\beta$$

$$\tilde{f}^{1-\alpha}(\xi) \equiv a f^{1-\alpha}(a\xi) \tag{3.52}$$

$$F^{1-\alpha}(\beta) = \int_{-1}^{1} \tilde{f}^{1-\alpha}(\xi) e^{-ika\xi\beta} d\xi$$

In equation (3.52), dimensionless variable $\xi = \frac{x}{a} \epsilon [-1, 1]$.

When the expression of the Fractional Green function is used to define the Fourier transform $F^{1-\alpha}(\beta)$ for the scattered field;

$$E_z^s(x, y) = -i \frac{e^{\pm i\pi\alpha/2}}{4\pi} \int_{-\infty}^{\infty} F^{1-\alpha}(\beta) e^{ik[\beta x + |y|\sqrt{1-\beta^2}]} (1 - \beta^2)^{\alpha - 1/2} d\beta \tag{3.53}$$

According to the equation (3.51),unknown functions $F^{1-\alpha}(\beta)$ reduce to the following equations;

$$\int_{-\infty}^{\infty} F^{1-\alpha}(\beta) e^{ika\xi\beta} (1 - \beta^2)^{\alpha - 1/2} d\beta = -4\pi e^{\frac{i\pi}{2(1-a)}} |\sin\theta|^{\alpha} e^{-ika\xi\cos\theta}, \quad \xi \epsilon [-1, 1]$$

$$(3.54)$$

$$\int_{-\infty}^{\infty} F^{1-\alpha}(\beta) e^{ika\xi\beta} d\beta = 0, |\xi| > 1$$

In equation (3.54), α=0, gives the coupled integral equations (CIE) for The diffraction problem involving an *E*-polarized plane wave on the perfect electric conductor surface (PEC) infinitely thin strip, and for α=1, the surface integral equation (SIE) describes the diffraction problem by the perfect magnetic conductor surface (PMC) strip. The SIE equation (3.54) is more general and includes SIE, which is explained as a special case.

The examination in this thesis concentrates on two specific cases: when the fractional order is 0 and when it is 1.When α=0 and α=1 are written one by one in the fractional part of equation, (3.55).

$$E_z^s(x,y) = \begin{cases} \frac{1}{4\pi}\int_{-\infty}^{\infty} F^1(\beta) e^{ik\left[\beta x + |y|\sqrt{1-\beta^2}\right]}(1-\beta^2)^{-1/2} d\beta & ,\alpha = 0 \\ \\ \frac{e^{\pm i\pi/2}}{4\pi}\int_{-\infty}^{\infty} F^0(\beta) e^{ik\left[\beta x + |y|\sqrt{1-\beta^2}\right]}(1-\beta^2)^{1/2} d\beta & ,\alpha = 1 \end{cases} \quad (3.55)$$

Now, in solving the problem, on the strip of dimension 2a in the $y = 0$ plane given in Figure 3.3. The problem of diffraction of the H-polarized electromagnetic wave by the strip with FBC in the incident field plane in the geometry of the problem $\overrightarrow{H^i}(0,0,H_z^i)$ has been examined. The wave falls from the side y>0 and takes the following form;

$$\mathrm{H}_z^i(x,y) = e^{-ik(x\cos\theta + y\sin\theta)} \quad (3.56)$$

where, θ represents the angle of incidence, $k = \frac{2\pi}{\lambda}$ denotes the wave number. The time dependency is expressed as $e^{-i\omega t}$. The scattered field, $\vec{H}^s = \vec{z}\mathrm{H}_z^s(x,y)$, when the total field $\vec{H} = \vec{z}H_z(x,y)$ is the sum of scattered and incident waves and is denoted by,

$$H_z(x,y) = H_z^i(x,y) + H_z^s(x,y) \quad (3.57)$$

The components of the electric field $\vec{E}(E_x, E_y, 0)$is defined form the Maxwell's equation

$$\vec{E} = -\frac{\eta_0}{k}\nabla \vec{H} \quad (3.58)$$

Where, η_0 is the wave resistance of the free space.

$$E_x = -\frac{\eta_0}{k}\frac{\partial H_z}{\partial y}, \qquad E_x = \frac{\eta_0}{ik}\frac{\partial H_z}{\partial x} \quad (3.59)$$

The solution to the problem we will construct with the next conditions:

Condition 1: $H_z(x,y)$satisfies the Helmholtz equation everywhere outside the strip

Condition 2: $H_z^s(x,y)$satisfies the Somerfield's radiation condition in the infinity

Condition 3: finiteness of energy in any finite volume of space to the situation called Meixner's edge conditing (y=0, x→± a) (Hönl, 1987);

Condition 4: $H_z(x,y)$ satisfies the FBC on the strip surface which have the next form

$$H_z(x,y)|_{y=\pm 0} = [H_z^i(x,y) + H_z^s(x,y)]|_{y=\pm 0} = 0, \quad x \in (-a,a) \quad (3.60)$$

Analogically to the E polarization case, let's represent the unknown function $H_z^s(x,y)$ based on the fractional Green's function as the surface integral,

$$H_z^s(x,y) \equiv \int_{-a}^{a} f^{\alpha}(x')G^{1-\alpha}(x-x',y)dx' \tag{3.61}$$

The function $f^{\alpha}(x)$ represents the fractional potential density, which is characterized by the discontinuity of the fractional derivative of the function $H_z(x,y)$ at the boundary y=0;

$$f^{\alpha}(x) = H_z(x,y)|_{\mathrm{y=+0}} - H_z(x,y)|_{\mathrm{y=-0}}\ , x\epsilon\,(-a,a) \tag{3.62}$$

Requiring from $f^{\alpha}(x)$ the FBC in equation (3.60), we will get the equation;

$$\lim_{y\to 0}\frac{\partial\mathfrak{D}}{\partial y}\int_{-a}^{a} f^{\alpha}(x')\frac{\partial}{\partial y}H_0^1(x-x',y)dx' = -\lim_{y\to 0}H_z^0(x,y) \quad , \alpha = 0 \tag{3.63}$$

The main difference between fractional integro-differantial equation (FIDE) (3.63) and equation (3.51) is that the order of the fractional derivative α changes with the $1-\alpha$ and the right part of the equation changes. The FIDE is reduced to the system of the coupled integral equation (CIE) with respect to the Fourier transform;

$$F^{\alpha}(\beta) = \int_{-1}^{1}\tilde{f}^{\alpha}(\xi)e^{-ika\xi\beta}d\xi\ , \qquad \left(\tilde{f}^{\alpha}(\xi) \equiv\ af^{\alpha}(a\xi)\right) \text{ as} \tag{3.64}$$

$$\begin{cases}\int_{-\infty}^{\infty}\mathrm{F}^{\alpha}(\beta)\mathrm{e}^{\mathrm{ika\xi\beta}}(1-\beta^2)^{1/2-\alpha}\mathrm{d}\beta = -4\pi\mathrm{i}^{\alpha}|\sin\theta|^{\frac{1-\alpha}{2}}\mathrm{e}^{-\mathrm{ika\xi}\cos\theta}, & \xi\epsilon\,[-1,1] \\ \quad\int_{-\infty}^{\infty}\mathrm{F}^{\alpha}(\beta)\mathrm{e}^{\mathrm{ika\xi\beta}}\mathrm{d}\beta = 0, & |\xi| > 1\end{cases} \tag{3.65}$$

The problem of diffraction of the H-polarized wave by the infinitely thin planar PE (Perfect Electric) strip at α= 0 is reduced to the CIE equation (3.65).

For the scenario where the fractional order value α=0, the solution to the diffraction problem can be obtained analytically,

$$\tilde{f}^{\,0}(\xi) = -4\pi e^{-ika\xi\cos\theta}\sin^{1/2}\theta\, e^{i\varepsilon\xi\cos\theta + i\pi/2} \tag{3.66}$$

$$F^0(\beta) = -4\,sin^{1/2}\,\theta\, e^{i\pi/2}\frac{\sin ka(\beta-cos\theta_0)}{(\beta-cos\theta_0)} \tag{3.67}$$

$$\tilde{f}^{\alpha}(\xi) = (1-\xi^2)^{1/2-\alpha}\sum_{n=0}^{\infty} f_n^{\alpha}C_n^{1-\alpha}(\xi) \tag{3.68}$$

Where f_n^α is the unknown coefficients. In case of α= 0 Gegenbauer polynomials, that is $C_n^\alpha(\xi)$, in the equation, it behaves like a Chebishev polynomials. For the Fourier transform F^α(β)can be obtained the next representation

$$F^\alpha(\beta) = \frac{2\pi}{\Gamma(1-\alpha)} \sum_{n=0}^{\infty} (-i)^n f_n^\alpha \mathrm{q}_n^{1-\alpha} \frac{\mathcal{J}_{n+1-\alpha}(ka\beta)}{(2\mathrm{ka}\beta)^{1-\alpha}} \tag{3.69}$$

Where, $\mathrm{q}_n^{1-\alpha} = \frac{\Gamma(n+2(1-\alpha))}{\Gamma(n+1)}$and $\mathcal{J}_{n+1-\alpha}(ka\beta)$ stands for the Bessel functions. Analogically to the E-polarization, for f_n^α unknown coefficients determination we can get the integral linear algebraic equations (ISLAE) in the next form,

$$\sum_{n=0}^{\infty} (-i)^n \mathrm{q}_n^{1-\alpha} f_n^\alpha C_{kn}^{\alpha,H} = \gamma_k^{\alpha,H}, \qquad k = 0,1,2 \ldots. \tag{3.70}$$

Where matrix elements solution of the equations in case of α= 0 given equations,

$$C_{kn}^H = \int_{-\infty}^{\infty} \mathcal{J}_{n+1}(ka\beta) \mathcal{J}_{k+1}(ka\beta)(1-\beta^2)^{\frac{1}{2}} \frac{d\beta}{\beta^2} \tag{3.71}$$

And

$$\gamma_k^H = -2\Gamma(2ka) \frac{\sin\theta\, \mathcal{J}_{k+1}(ka\cos\theta)}{\cos\theta} \tag{3.72}$$

This is the SLAE problem, with the help of which, based on the reduction α method, the coefficients f_n^α can be found with any predetermined accuracy. According to the coefficients f_n^α found, the potential density function and the Fourier transform $F^\alpha(\beta)$ are determined by formulas equations (3.68) and (3.69), respectively. We present expressions for determining the electrodynamic characteristics of a scattered field. To determine the RP we write down the value of the field $H_z^s(x,y)$ in the far zone $kr \to \infty$. For a scattered field $H_z^s(x,y)$ on the basis of the integral representation for the Green function, we can obtain the expression,

$$H_z^s(r,\varphi) = \frac{i}{4\pi} (\pm i)^{1-\alpha} \int_{-\infty}^{\infty} F^\alpha(\cos\beta) e^{ikr\cos(\varphi\pm\beta)} \sin^{1-\alpha}\beta\, d\beta \tag{3.73}$$

The far-field expression of the scattered field can be derived by considering $kr \to \infty$. In this case, the steepest descent method is employed to obtain the expression for the far-field $H_z^s(r,\varphi)$as outlined equations below (Veliev et al., 2018a; Veliyev et al., 2018b; Veliyev et al., 2020; Karaçuha et al.,2019; Balanis, 2016). (This method can be viewed in the Appendix B). In here, $r = \sqrt{x^2 + y^2}$ and φ is the angle between the $\hat{e}_r$ unit vector along in r and $\hat{e}_x$.

$$H_z^s(r,\varphi) = A(kr)\Phi^\alpha(\varphi), \qquad kr \rightarrow \infty \tag{3.74}$$

where,

$$A(kr) = \sqrt{\frac{2}{\pi kr}} e^{ikr - i\pi/4}$$

$$\Phi^0(\varphi) = -\frac{i}{4} F^0(cos\varphi)sin\varphi \tag{3.75}$$

In this context, $A(kr)$and $\Phi^\alpha(\varphi)$represent the radial and angular components of the scattered electric field in the far zone, respectively. The primary focus of the scattered field is the angular variation of the field. Therefore, $\Phi^\alpha(\varphi)$ is called the scattered radiation pattern. For $H_z^s(r,\varphi)$, the sign (+) corresponds to the angle values φ ϵ [0,π], and the sign (-) to the values φ ϵ [π,2π] (Karaçuha et al.,2019; Balanis, 2016).

In equation $\Phi^\alpha(\varphi)$, the RP is determined through the Fourier image. The function can be defined uncomplicatedly in the physical optics approximation.

The coupled integral equation (CIE) it has an apparent solution. The physical optics (PO) approach is a technique used to address problems involving the scattering or reflection of electromagnetic waves. This method is applicable when the dimensions of the wave are much larger in scale. Specifically, it is applied when the wave is at a considerable distance relative to its wavelength ($ka \gg 1$) and the function $\Phi^\alpha(\varphi)$,

$$\lim_{ka\rightarrow\infty} \frac{\sin ka(\tau-\beta)}{(\tau-\beta)} = \pi\delta(\tau - \beta) \tag{3.76}$$

$$F^\alpha(\beta) = -4i^\alpha \frac{\sin^{1-\alpha}\theta}{(1-\beta^2)^{1-2\alpha/2}} \frac{\sin ka(\beta - cos\theta)}{(\beta - cos\theta)}, \qquad ka \gg 1, \tag{3.77}$$

$$\Phi^\alpha(\varphi) \approx \ sin\theta \left(\frac{\sin\varphi}{\sin\theta}\right) \frac{\sin ka(cos\varphi - cos\theta)}{(cos\varphi - cos\theta)}, \qquad ka \gg 1, \tag{3.78}$$

For the particular values of the parameter α RP $\Phi^\alpha(\varphi)$, rewrite as follows

$$\Phi^\alpha(\varphi) \approx (\sin\theta) \frac{\sin ka(cos\varphi - cos\theta)}{(cos\varphi - cos\theta)}, \qquad \alpha = 0 \tag{3.79}$$

$$\Phi^{\alpha}(\varphi) \approx (\sin\varphi)\frac{\sin ka(\cos\varphi-\cos\theta)}{(\cos\varphi-\cos\theta)}, \qquad \alpha = 1 \tag{3.80}$$

Surface currents are defined by scattering cross section (SCS), $\frac{\sigma_{2D}}{\lambda}$ and back scattering cross section (BSCS), σ_{2D}^{mono}.(Equations in these surface currents can be viewed in the Appendix A). The scattering widths can be found through the RP $\Phi^{\alpha}(\varphi)$from the corresponding definitions for scattering cross section (SCS) and back scattering cross section (BSCS). Surface currents are defined not as a jump in field components as they pass along the edge of a band, but as currents flowing along the surface of a conductor due to changes in the surrounding environment or discontinuities in the conductor itself.

$$J_z^{\alpha(m)} = E_x(x,+0) - E_x(x,-0), \tag{3.81}$$

$$J_x^{\alpha(e)} = H_z(x,+0) - H_z(x,-0), \tag{3.82}$$

For the H – polarization the electric current has only x component, $J^{\alpha(e)} = \boldsymbol{x}J_x^{\alpha(e)}$, when the magnetic current has only z component $J^{\alpha(m)} = \boldsymbol{z}J_z^{\alpha(m)}$. The physical currents can be expressed with the density $\tilde{f}^{\alpha}(\xi)$ as the integrals,

$$J_x^{\alpha(e)}(x) = -2i\cos\left(\frac{\pi\alpha}{2}\right)\frac{i}{4\pi}\int_{-\infty}^{\infty} F^{\alpha}(\beta)e^{ikax}(1-\beta^2)^{-\alpha/2}d\beta \tag{3.83}$$

$$J_z^{\alpha(m)}(x) = -2\sin\left(\frac{\pi\alpha}{2}\right)\frac{i}{4\pi}\int_{-\infty}^{\infty} F^{\alpha}(\beta)e^{ikax}(1-\beta^2)^{-\alpha/2+1/2}d\beta \tag{3.84}$$

At the fractional order values for $\alpha = 0$, there only electric current is left $J^{\alpha(e)} = 0$ and for $\alpha = 1$, there only magnetic current is left $J^{\alpha(m)} = 0$.

$$J_x^{\alpha(e)}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F^{0}(\beta)e^{ikax}d\beta\ , \qquad \alpha = 0 \tag{3.85}$$

$$J_z^{\alpha(m)}(x) = -2\sin\left(\frac{\pi}{2}\right)\frac{i}{4\pi}\int_{-\infty}^{\infty} F^{1}(\beta)e^{ikax}d\beta, \qquad \alpha = 1 \tag{3.86}$$

4. RESULTS AND DISCUSSIONS

4.1. Results of Numerical Simulation

After analytical solutions are made for the problems discussed in this section, numerical results are presented. Here, the near-field distribution, the far-field radiation pattern and the current distribution, at the given parameters. First, an efficient and new method for solving a class of integral equations have been studied, secondly, line source diffraction by single strip for the fractional-order 0 and 1 cases are given. For calculations, the MatLab simulation tool is employed whereas, for the illustrations, both LAE Service and MatLab Simulation Tools are utilized.

Expressing the current density on the obstacles using the Gegenbauer polynomials with the weighting factor $(1-\xi^2)^{\alpha-\frac{1}{2}}$ leads to having fastly converging series to the actual values of the current density found by analytical or numerical methods for the same problem because the edge condition is satisfied with that factor by default (Rotwell, 2018; Tabatadze et al., 2019; Meixner, 1972; Herman and Volakis, 1987). In this study, the summations are calculated up to $\varepsilon+5$ values. For higher accuracy demand, this value can be increased. For more than 95% accuracy, the predefined value $\varepsilon+5$ is enough. Note that ε is equal to ka which inherently, gives the information about the relative dimension of the scattered with respect to the wavelength of the incidence wave (i.e. electrical length) (Ivakhnvchenko et al., 2004).

In the thesis, the hybrid method is employed for the diffraction problem. This method is called also, the numerical-analytical method which stems from the combination of the analytical and numerical methods. The first steps of the method start with analytical manipulations. The field components are expressed as analytical forms as summations or integrals regarding the geometry. Then, by the boundary conditions, the problem is reduced to a boundary value problem. After that, the initial operator equation is converted to an infinite system of a linear algebraic equation which is truncated for the numerical calculation as expected by introducing the orthogonal polynomials concerning the geometry. Here, the important point is that obtained SLAE utilizing analytical manipulations is much better than full numerical approaches for computing due to the requirement of less size of the inverted matrix and highly converting trends in the summations or integrals standing for the field components (Ivakhnvchenko et al., 2004; Avşar, 2016; Veliyev et al., 2020).

4.2. Two Dimensional Thin Strip Diffraction Problem Solution and Boundary Conditions

In Figure 4.1, the issue of diffraction of a H- polarized plane wave is addressed.The method we propose, unlike the classical method, have used a new integral equation solution to calculate the near-field distribution, far-field radiation pattern and current density, of the H-polarized wave sent planarly on a strip at different frequencies and where the angle of incidence of the plane wave varies, depending on the scattered field. According to the scattered field, how the current density of the plane wave changes at different frequency values depending on the angle of incidence, and what behavior and propagation it shows in the near and far fields has been examined. Also, current distribution were obtained by calculating the radiation pattern (RP) and cross section of the total scattering (TCS) expressed with unknown coefficients.

The integral calculation of the flux density at each point very close to the strip, that is, according to its coordinates, was calculated one by one withMatLab Simulation Tool.

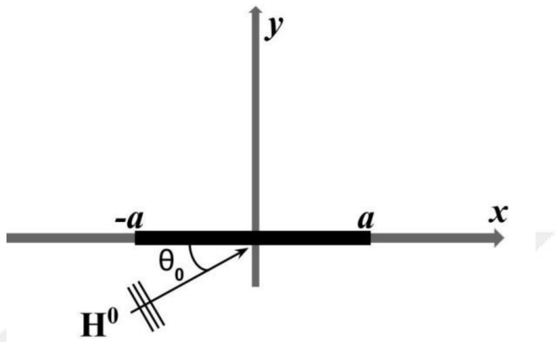


Figure 4.1. The geometry of the problem

The near-field distribution refers to the spatial distribution of electromagnetic fields in close proximity to scattering object. Unlike the far-field distribution, which describes the radiation pattern at distances much larger than the dimension of the object, the near-field distribution characterizes the electromagnetic fields in close proximity to the object. In this region, which typically extends up to a few wavelengths away from the object, the electromagnetic fields exhibit complex behavior with significant variations in amplitude, phase, and polarization. The near-field distribution is influenced by the geometry, material properties, and excitation of the object, as well as any surrounding structures or boundaries.

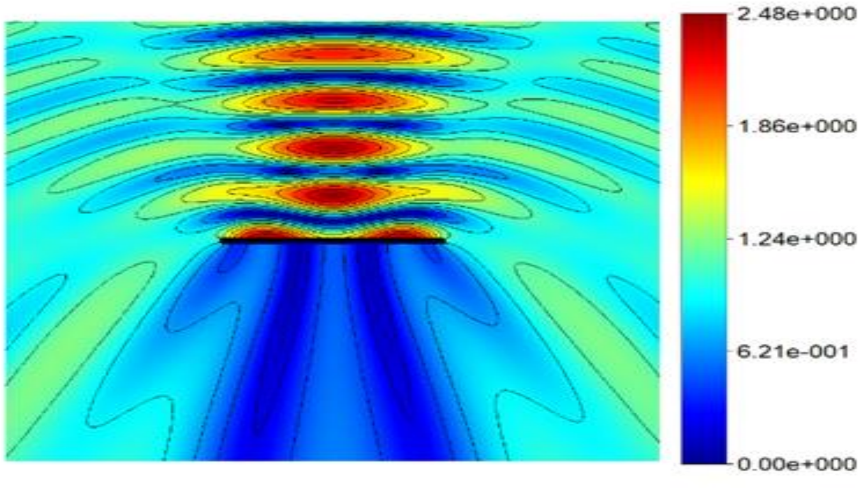
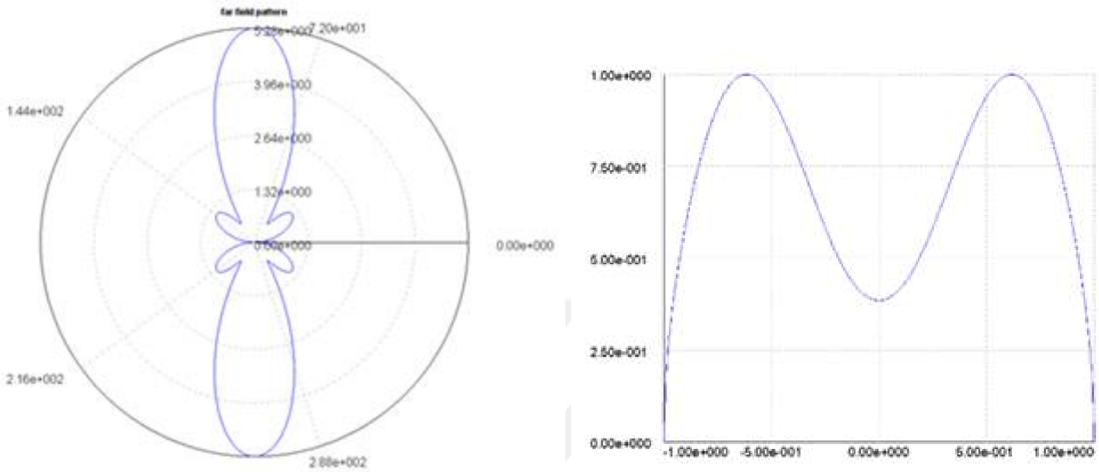


Figure 4.2.Near field distribution $\varepsilon = ka = 5, \theta_0 = -\frac{\pi}{2}$

In Figure 4.2., In the near field region, when,$\varepsilon = ka = 5, \theta_0 = -\frac{\pi}{2}$at the given wavelength from the strip. The propagation and distribution of the electromagnetic wave in a wide radiation pattern in thenear field has been increasingly changed. In strip, Neumann and Dirichlet Boundary conditions are provided for the PMC (Tabatadze, 2019).



a) b)

Figure 4.3. a)Far Field Pattern$\varepsilon = ka = 5, \theta_0 = -\frac{\pi}{2}$,b) Current density $\varepsilon = ka = 5, \theta_0 = -\frac{\pi}{2}$ (Baz et al., 2024)

In Figure 4.3.a and Figure 4.3.b, these parameters define the dimensional and geometric properties of the scattering phenomenon.The radiation model (RP) is called the far field model,this pattern illustrates how electromagnetic energy propagates to distant locations and how it concentrates in certain directions. It clearly defines the distribution and characteristic features of the electromagnetic field as we move away from the radiating or scattering object. As observed in the far-field pattern, the predominant portion of the scattered field is emitted in an upward direction, accompanied by a minor back lobe. The comment on Figure 4.3.a may be; the radiation pattern is the far-field pattern that corresponds to the scattered field. The scattered field is directed through symmetrical direction mainly with small back lobes (Tabatadze et al., 2020). In Figure 4.3.b, in the electromagnetic field, the current density has been distributed symmetrically.

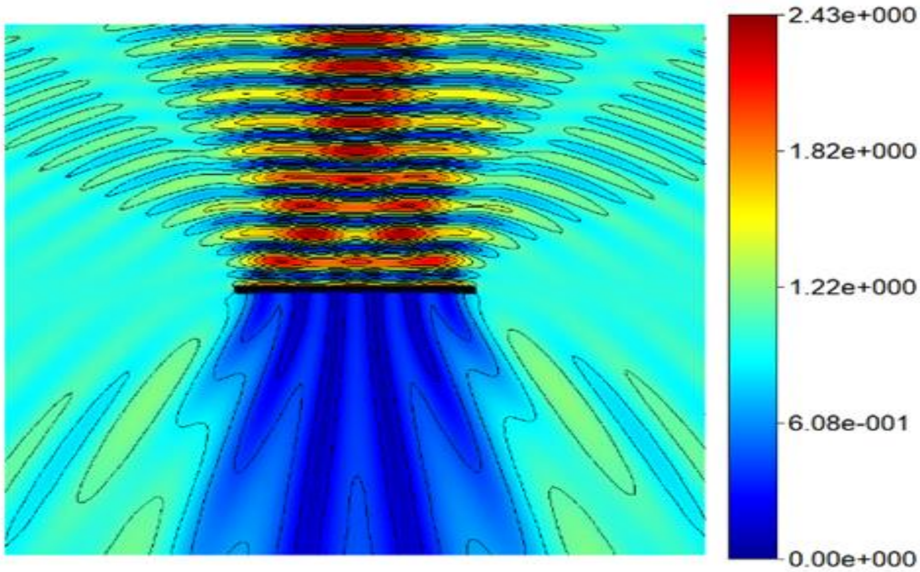
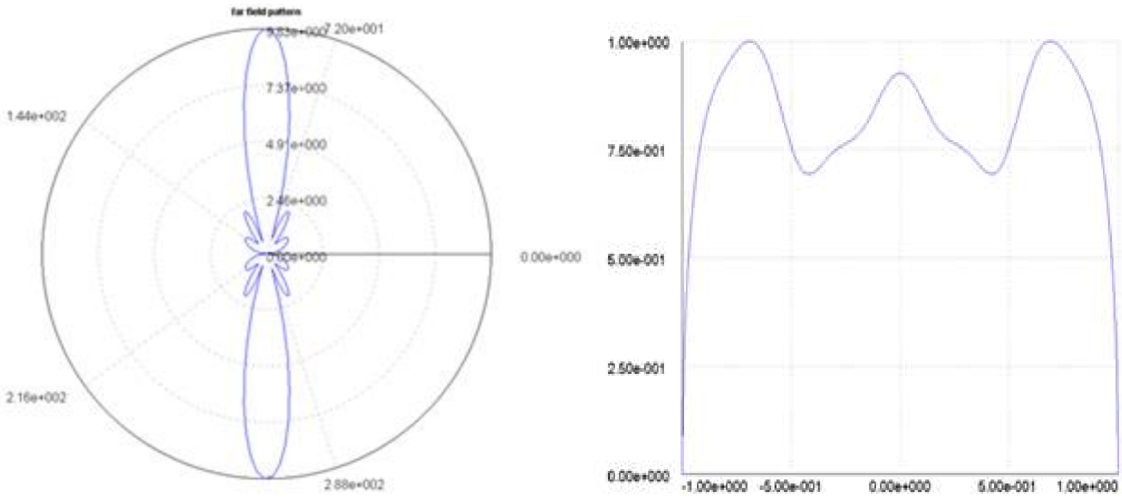


Figure 4.4.Near field distribution$\varepsilon = ka = 10, \theta_0 = -\frac{\pi}{2}$

In Figure 4.4., In the near field region, in a plane wave frequency and angle of incidence of the plane wave respectively,$\varepsilon = ka = 10, \theta_0 = -\frac{\pi}{2}$. at the given wavelength from the strip. Having $\varepsilon = ka = 10$ means a larger one, unlike the scenario in Figure 4.2., it indicates that the value of ε and therefore the wavelength is larger than the characteristic length scale. This indicates that the characteristic dimensions of the scattering event are larger and therefore the high wavenumber approximation may be more accurate. Neumann and Dirichlet Boundary conditions are satisfied for the PMC case on the strip (Tabatadze, 2019).



a)b)

Figure 4.5.a) Far field pattern $\varepsilon = ka = 10, \theta_0 = -\frac{\pi}{2}$, b) Current density $\varepsilon = ka = 10, \theta_0 = -\frac{\pi}{2}$

In Figure 4.5.a, it is seen from the radiation pattern, the majority of scattered field is radiated upward with a small back lobe. As the, ε value increases, the radiation pattern becomes narrower. The scattered field is directed through symmetrical direction mainly with small back lobes

(Tabatadze et al., 2020). In Figure 4.3.b, in the electromagnetic field, the current density has been distributed symmetrically.

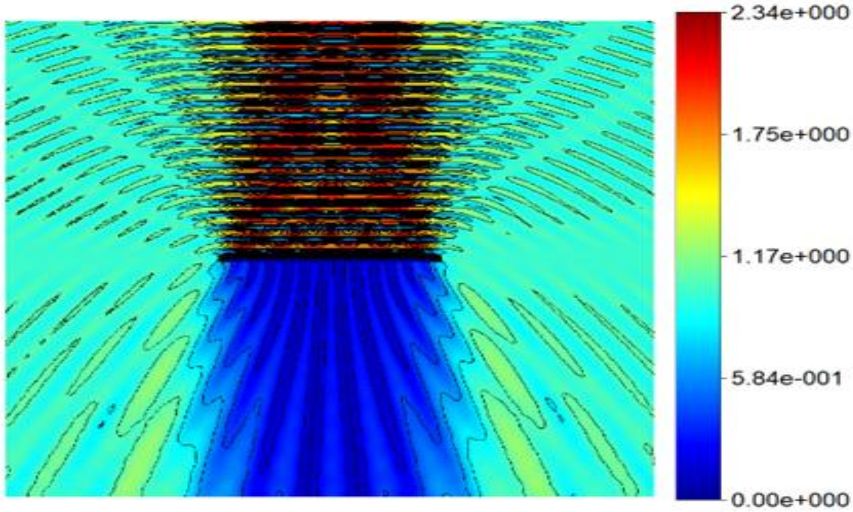
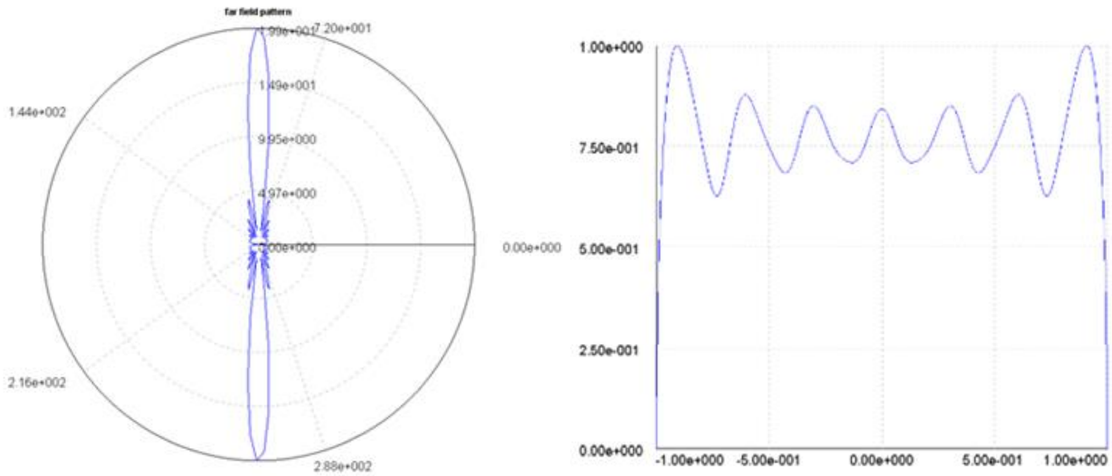


Figure 4.6.Near field distribution$\varepsilon = ka = 20, \theta_0 = -\frac{\pi}{2}$

In Figure 4.6., having $\varepsilon = ka = 20$, unlike ε =5 and ε =10in all other scenarios, it indicates that the value of ε and therefore the wavelength is the biggest than the characteristic length scale. This indicates that the characteristic dimensions the high wavenumber approximation may be more accurate. Neumann and Dirichlet Boundary conditions are provided for the PMC case on the strip (Tabatadze, 2019).



a) b)

Figure 4.7. a) Far field pattern $\varepsilon = ka = 20, \theta_0 = -\frac{\pi}{2}$,b) Current density $\varepsilon = ka = 20, \theta_0 = -\frac{\pi}{2}$ (Baz et al., 2024)

As seen from the far-field pattern in Figure 4.7.a, most of the scattered field spreads upward with a small posterior lobe. As the value of ε increases, the radiation pattern becomes very narrow and the radiation is very strong in this region. The scattered field is mainly directed in a symmetrical direction by small posterior lobes (Tabatadze et al., 2020). In Figure 4.7.b, the current density in the electromagnetic field showed a symmetrical and strong distribution.

From Figure 4.8 to Figure 4.13, the near field and far field distribution and current density are $\varepsilon = ka = 5$ and $\theta_0 = -\frac{3\pi}{4}$, respectively. For this situation, the scattering properties of the surface and the radiation characteristics depending on the angle were analyzed. In this analysis, the source for the incoming wave is positioned at a certain angle and the scattered field changes with this given angle are found. Neumann and Dirichlet boundary conditions are provided for the Perfect Magnetic Conductor (PMC) scenario on the strip (Tabatadze, 2019).

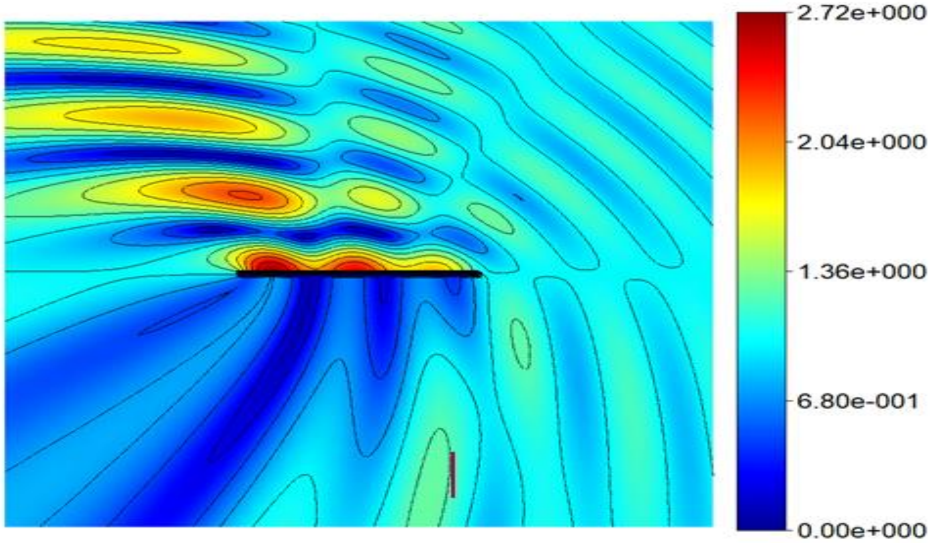
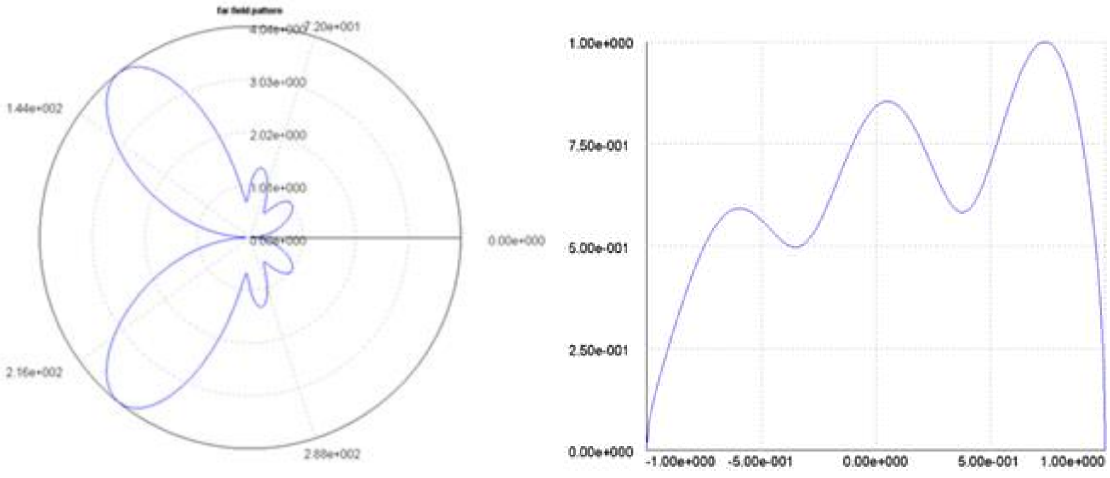


Figure 4.8. a) Near field distribution $\varepsilon = ka = 5, \theta_0 = -\frac{3\pi}{4}$

In Figure 4.8., when ε=ka=5, θ= -3π/4 in the near-field distribution, it is seen that the radiation scattered by the electromagnetic wave at the edge of the strip in the near-field distribution is towards the lateral directions, and it has been obtained that this scattering changes in a small range.



a) b)

Figure 4.9. a)Far field pattern $\varepsilon = ka = 5, \theta_0 = -\frac{3\pi}{4}$, b) Current density $\varepsilon = ka = 5, \theta_0 = -\frac{3\pi}{4}$

When the far field pattern and the distribution of current density in the scattering field in Figures 4.9.a and 4.9.b are examined, most of the scattered field is spread angularly with a small back lobe, creating a wide radiation pattern laterally. The scattered field is mainly distributed in a symmetrical direction by the small back lobes (Tabatadze et al., 2020). In Figure 4.9.b, the current density in the electromagnetic field showed a non-symmetrical distribution.

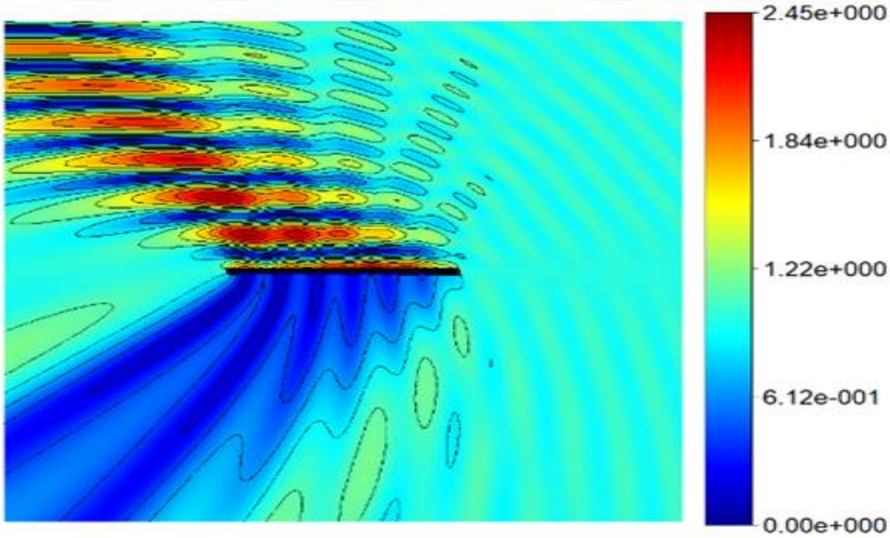
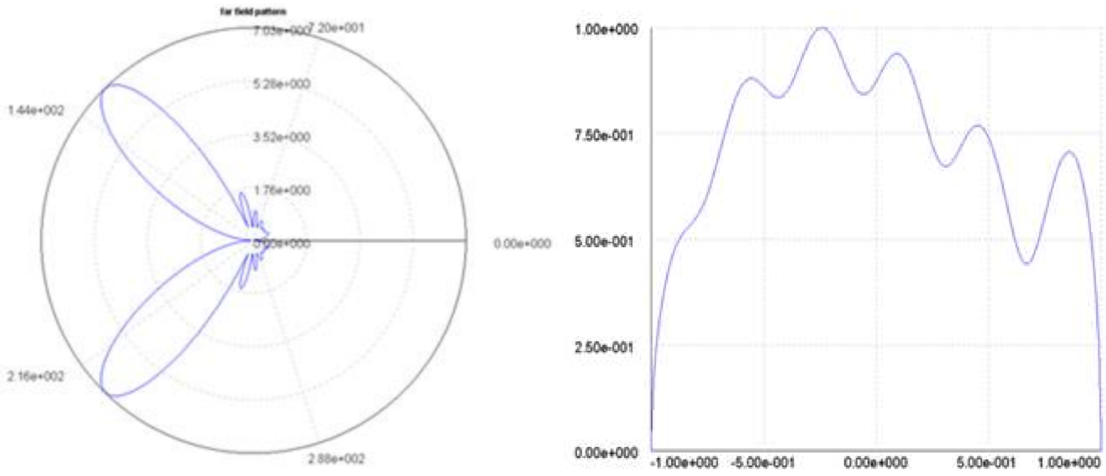


Figure 4.10. a) Near field distribution $\varepsilon = ka = 10, \theta_0 = -\frac{3\pi}{4}$

In Figure 4.10., In the near field distribution, when the frequency and angle of incidence of the plane wave are respectively,$\varepsilon = ka = 10,\ \theta_0 = -\frac{3\pi}{4}$ at the given wavelength from the strip. It shows that the radiation scattered by the electromagnetic wave in the near field in the region at the edge of the strip spreads and changes towards a wide range in the lateral directions compared to the distribution in Figure 4.8.



a) b)

Figure 4.11. a)Far field pattern $\varepsilon = ka = 10, \theta_0 = -\frac{3\pi}{4}$,b)Current density $\varepsilon = ka = 10, \theta_0 = -\frac{3\pi}{4}$

When the far field pattern and current density distribution in the scattering area in Figures 4.11.a and 4.11.b are examined, a large part of the scattered area spreads angularly with a small back lobe, creating a rather narrow radiation pattern laterally. The scattered field is distributed in a symmetrical direction mainly by the small posterior lobes (Tabatadze et al., 2020). In Figure 4.11.b, the current density in the electromagnetic field shows a strong non-symmetrical distribution.

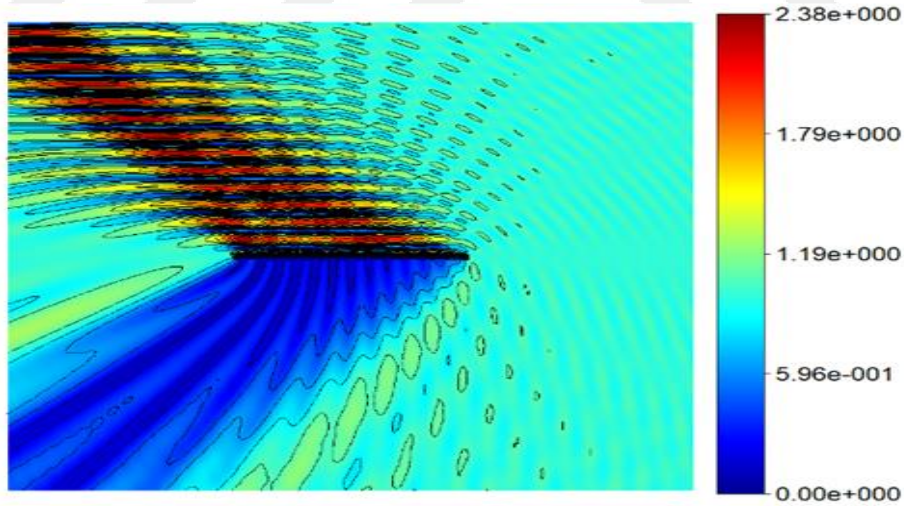
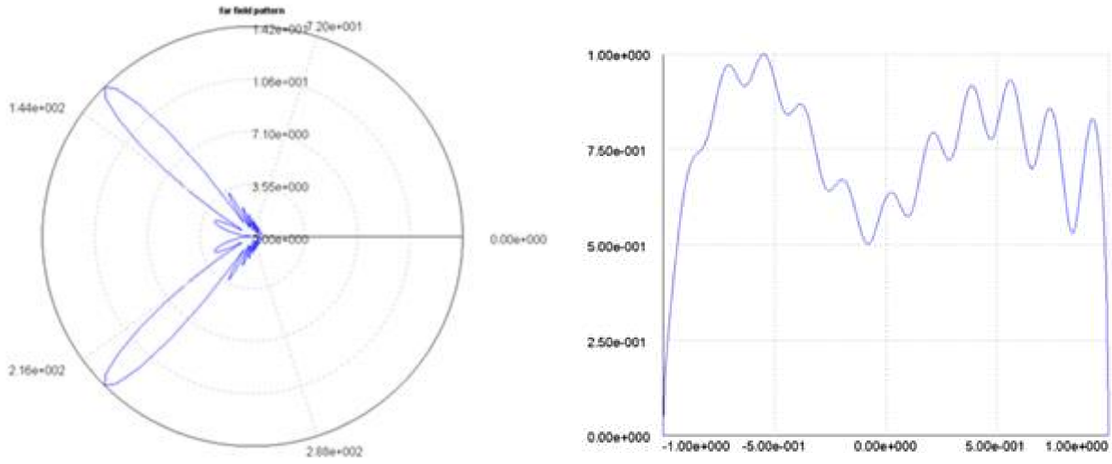


Figure 4.12. a) Near field distribution $\varepsilon = ka = 20, \theta_0 = -\frac{3\pi}{4}$

In Figure 4.12., In the near field distribution, where angle of incidence of the plane wave and the frequency,$\varepsilon = ka = 20$, $\theta_0 = -\frac{3\pi}{4}$ at the given wavelength from the strip. Compared to the frequency change in Figure 4.8 and Figure 4.10, the radiation scattered by the electromagnetic wave in the near field in the region at the edge of the strip has changed by spreading strongly over a wide range in lateral directions from the strip.



a) b)

Figure 4.13. a)Far field pattern $\varepsilon = ka = 20, \theta_0 = -\frac{3\pi}{4}$,b)Current density $\varepsilon = ka = 20, \theta_0 = -\frac{3\pi}{4}$

In Figure 4.13.a, when the ε value reaches the largest value, the radiation pattern becomes much narrower and the radiation becomes very strong in this region. The scattered field is mainly directed in a symmetrical direction by the small posterior lobes (Tabatadze et al., 2020). In Figure 4.9.b, the current density in the electromagnetic field shows the non-symmetrical and strongest distribution.

4.3. Two Dimensional Strip Diffraction with Fractional Boundary Conditions

In this section, the research will examine the numerical and detailed analyzes of the two dimensional strip diffraction problem for the only fractional order 0 and 1 case. Previously, fractional derivatives were utilized for addressing the dilemma involving two-dimensional diffraction of a plane wave. Research has been carried out using the diffraction approach based on plane and cylindrical waves on a strip (Veliev et al., 2018; Veliyev et al., 2018; Karaçuha et al., 2019). In these previous studies, only numerical and detailed analyzes were studied for fractional order 0.5 cases and between values of the interval [0,1], the general solution was investigated numerically. (Veliev et al., 2018a; Veliyev et al.,2018b; Karaçuha et al., 2019). However, no study or analysis has been done for the fractional degree case, 0 and 1.

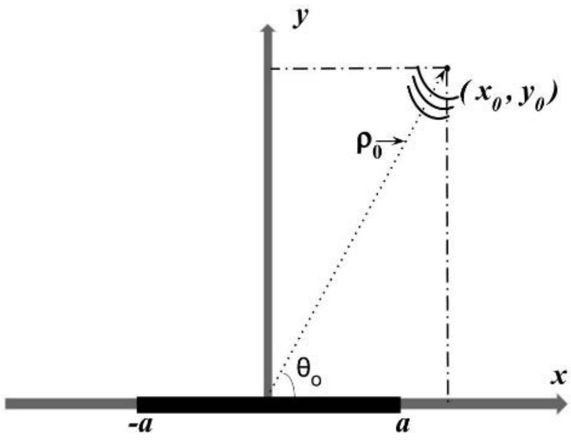


Figure 4.14. The geometry of the problem

The significance of the scenarios where the fractional order equals 0 and 1 lies in the ability to derive analytical expressions for both the plane wave case and the cylindrical wave as an incident wave. This includes obtaining closed-form expressions for the induced current and its Fourier transform. In the analytical expression to be obtained for the Fourier transform, the amount of current can be found with some approaches (Veliyev et al., 2018; Tabatadze et al., 2020; Karaçuha et al.,2019).Here, current densities, near and far field distributions, and radiation model are examined. For numerical analysis, $B = -J_e \frac{\eta_0 k}{4\pi}$ given in equation (3.35) was used. This term was assumed to be 1 in our solutions as a result of our numerical analysis. In Figures 4.15 and 4.16, the near-field distributions of current densities was obtained for on a metal strip when the fractional derivative are 0 and 1.

While one type of current density was expressed the problem, other methods require two different current densities to solve the same problem (Ivakhnychenko et al.,2008).In Figures 4.17and 4.18, the far-field distributions of current densities, when the flux density is viewed from the edge points of the strip, our approach to solving this physical problem has been controlled with current densities, thanks to the hybrid method we have used. In the hybrid method we have used, it is sufficient to obtain quite accurate results by using the terms a half-width of the strip and k wave number to obtain a reliable and accurate approximation in the total expression of the fractional current density.

Scattering distributions were examined for cases where $\nu = 0,\ r_{inc} = (0,1),\ k = 3, a = 1$ and $\nu = 1,\ r_{inc} = (0,1),\ k = 3, a = 1$in the near field distribution and far field distribution.

Where, ν is the fractional order, r_{inc} indicates the observation point of the scattering event. This expression describes the position in a two-dimensional plane. The coordinates on the *x* and *y* axis are respectively 0 and 1, i.e., it shows that it corresponds to (0,1) coordinates. These coordinates do not specify whether the observation point is above or below the object, but simply describe its location in a two-dimensional plane.$k$is the wavevector represents the spatial frequency of the wave, $a$can represent the length, width, or characteristic size of the scattering strip.

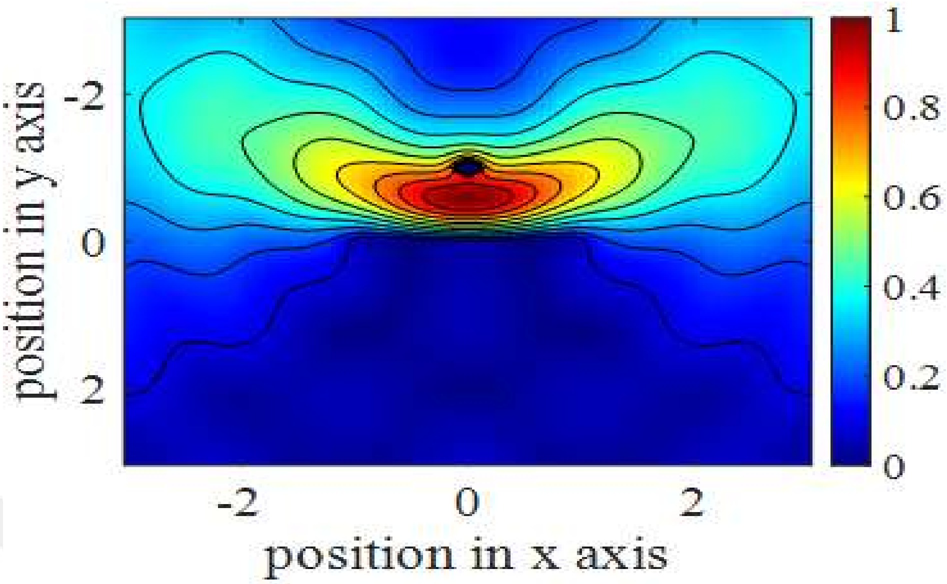


Figure 4.15.Near field distributionfor$\nu = 0,\ r_{inc} = (0,1),\ k = 3, a = 1$

In Figure 4.15., current densities for the fractional order $\nu = 0$ the value of the near field distribution flux density the wave number is taken as 3, the length of the strip being constant (The value of $''a''$ is fixed), and for *E*-polarized case was obtained.

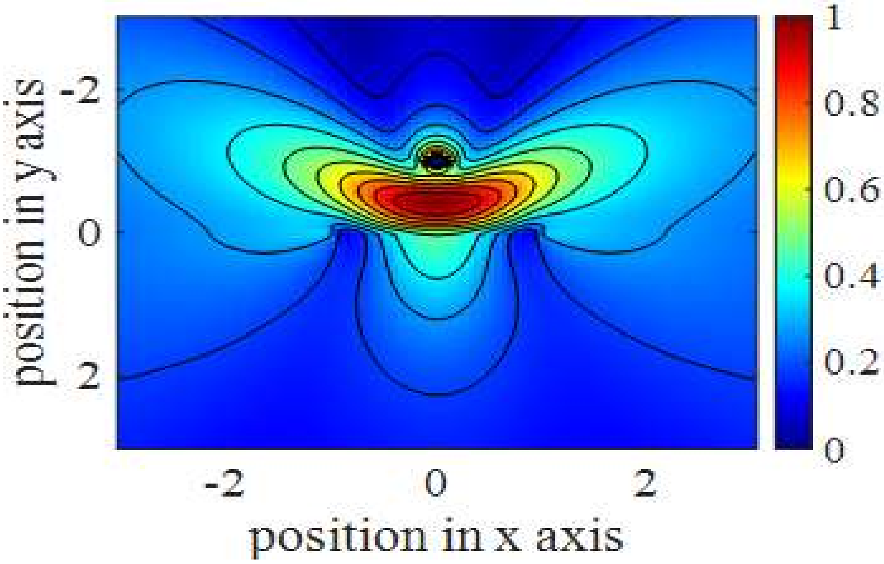


Figure 4.16. Near field distributionfor$\nu = 1,\ r_{inc} = (0,1),\ k = 3, a = 1$

In Figure 4.16., current densities for the fractional order $\nu = 1$ the value of the near field distribution flux density the wave number is taken as 3, the length of the strip being constant (The value of $''a''$ is fixed), and for *H*-polarized case was obtained.

Antenna patterns (azimuth and elevation plane patterns) are often represented using plots in polar coordinates. This enables viewers to readily visualize the radiation characteristics of the antenna in all directions, simulating the antenna's orientation or mounting position (Kraus and Marhefka, 2002).

The Far Field pattern, it is also called the radiation pattern. In this region, various antenna parameters are taken into account, including antenna directivity and radiation pattern, among others (Balanis, 1997). In Figure 4.17 and Figure 4.18, a radiation pattern (RP) view is obtained for the *E*-polarized and *H*-polarized cases, respectively, with lobes labeled in each drawing type.

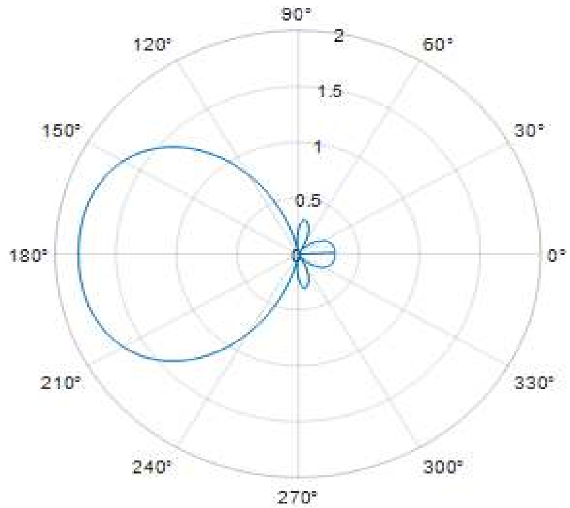


Figure 4.17.Far Field pattern for $v = 0,\ r_{inc} = (0,1),\ k = 3, a = 1$

An antenna radiation pattern is obtained in Figure 4.17, where the main lobe points at 180◦ and the side lobes have smaller gains. The axis of the main lobe is defined as the viewing angle of the antenna and extends through the peak gain, which is the maximum gain in all directions. Beamwidth refers to the angle in the directions on either side of the viewing angle. Ideally, directional antennas are assumed to have a constant value of gain in the main lobe and zero outside the main lobe.

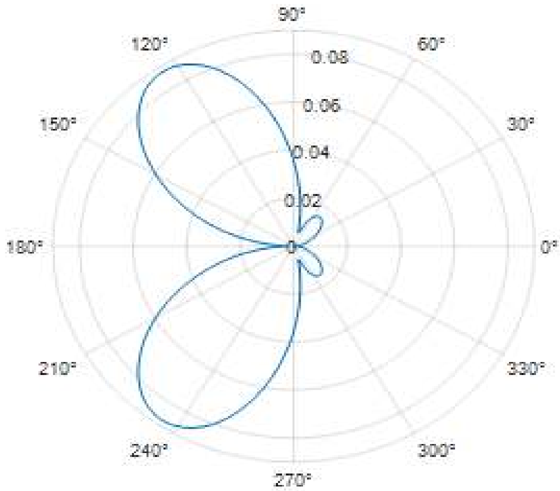


Figure 4.18. Far Field pattern for$\nu = 1,\ r_{inc} = (0,1),\ k = 3, a = 1$

As it can be seen from Figure 4.18, the boundary conditions for the magnetic field (H-polarized) are satisfied. On strip, the Neumann boundary condition and Dirichlet boundary condition is satisfied and the field's derivative becomes zero. The reason why field distribution is symmetrical is due to having one strips with fraction orders. For the fractional order ν=1, a strip with perfect magneticconductivity (PMC) is obtained (Engheta, 1998). In the antenna radiation model given in the figure, it is seen that the main lobe points to 120° and the side lobes have smaller gain values.

5. CONCLUSIONS

In this section, information, values and their interpretation indicating the extent to which the goals stated in the introduction of the thesis have been achieved are regularly given. Using the hybrid method, which is a new approach method, two different electromagnetic scattering problems were solved analytically and examined by numerical analysis. In this thesis, the original theoretical background for the solution of the diffraction problem with strips is given. The electromagnetic scattering problem on a planethin strip was analyzed using the hybrid method to compare the Method of Moments (MoM) and other methodsFinite Element Method (FEM), Finite Difference Time Domain (FDTD), Boundary Element Method (BEM), Spherical Harmonics etc.

The newly proposed boundary conditions characterize a new material property (Perfect Electric Conductor (PEC) and Perfect Magnetic Conductor (PMC)). Furthermore, Dirichlet and Neumann boundary conditions have been employed to describe the scattering in the integral boundary condition solution. The problems we deal with in the thesis are two-dimensional and are expressed in terms of the integral of the scattered wavelength, starting from the solution of the total magnetic field. The integral used in our solution here was obtained by Fourier transforming the current density on the strip. To obtain the fractional integral equation, the Fourier transform of the Hankel function is performed and boundary conditions are used. After transforming into this form, the Fourier transform of the current density on the strip must be determined. For this, the integral equation must be solved., the current density is explained as a sum of orthogonal polynomials. In our work, these polynomials are selected to be Gegenbauer polynomials because the problems solved in the thesis involve strip geometries that are finite in the xy-plane, making Gegenbauer polynomials defined in a finite region. To address the boundary condition of the current density and guarantee the swift convergence of the polynomials to the true value of the current density on the strip, weighting is incorporated into the expression of the Gegenbauer polynomials. Subsequently, orthogonality (inner product) is applied to uniquely identify the unknown coefficients in the expression of the current density. After solving the system of linear algebraic equations, the coefficients are determined. Finally, the Fourier transform of the current density is derived. This allows for the determination of the electric field distribution with any desired accuracy. The advantages of the fractional derivative approach lie in its consistency in the numerical part after the theoretical part for any fractional order. In this approach, the numerical part remains the same regardless of the specific fractional order, with only the order itself being variable. Therefore, it allows for solving integral equations or combined integral equation sets for any fractional order. In essence, the fractional derivative approach generalizes boundary conditions and simplifies the computational burden. Unlike other methods, where obtaining two different current densities is typically required to solve a similar problem, in this new method, expressing the current density is sufficient. Various scenarios, such as different operating frequencies, strip lengths, fractional orders, will be examined both theoretically and numerically. Subsequently, solutions obtained with the hybrid method presented in this thesis has been compared with other methods (e.g., Moments Method and Physical Optics). In this hybrid method, the expression $\sum_{n=0}^{N} x_n C_{n,m}$ represents, unknown coefficients x_n, that determine the elements of the matrix $C_{n,m}$.

In solving the problem, N→ ∞ (taken to approach infinity). $N = [ka = \varepsilon] + 5$ (half-width of the strip is a and k is frequency). It is reliable and accurate for $0 < ka < 15$.The inclusion of the $(ka + 5)$, terms in the total expression of the fractional current density yields sufficiently accurate results. For larger kavalues (where $ka > 15$), increasing the terms in the total expression of the fractional current density is necessary to obtain highly accurate results.For larger (ka)values, the method provides approximate results. In numerical analysis, the choice of *N* is crucial to achieve a solution with minimal error and to solve the problem easily. In the first part of the thesis, a plane wave H-polarized was onto a two dimensional strip, and an integral calculation was performed using a new approach.A new integral equation has been proposed, and in solving the problem, the series expressions have been transformed into Bessel functions through Fourier transformation. In other words, the analytical solution has been converted into an integral solution to find the current densities. The near-field distribution of an H-polarized plane wave on a two dimensional strip has been investigated, and it has been shown that as the frequency values increase with respect to the angle (at different frequency values at a fixed angle), the characteristic sizes suggest that a high wave number approximation may be more accurate. An infinite analytical solution is obtained for the far-field distribution (Radiation Field (RP)) of an plane H-polarized wave incidence on a two dimensional strip. As the value of ε increases, the radiation pattern narrows, and radiation in this region becomes stronger. Such variations play a crucial role in antenna applications; for instance, they can significantly impact the gain of the antenna (which increases with ε). Additionally, the electromagnetic field's current density exhibits a strong distribution. For the PMC (perfect magnetic conductor) condition on the strip, both Neumann and Dirichlet boundary conditions are satisfied (Tabatadze, 2019). This ensures complete reflection suppression on the strip surface, providing the desired performance in various applications.In the final part of the thesis, a wave is sent from a cylindrical source onto a two-dimensional strip, and only the fractional order 0 and 1 cases are examined. The significance of the fractional order being equal to 0 and 1 lies in obtaining an analytical expression for the plane wave case and also in obtaining the incident wave, the closed form of the induced current, and the Fourier transform. Some approaches can determine the amount of current in the analytical expression to be obtained for the Fourier transform. Near-field distribution, far-field distribution, and current densities have been analyzed for the fractional orders α=0 and α=1 at the observation point with coordinates (0,1) on a two-dimensional plane.In the near field distribution, an interesting solution has been achieved in the case of α = 1, unlike the fractional order

$\alpha = 0$, as they physically affect each other. The results indicate that the direction of radiation modeling is highly correlated with the choice of fractional order. Such a structure can be utilized in antenna synthesis, waveguides, and resonator problems. In the radiation field distribution, for the fractional order $\nu=0$, a strip with perfect electrical conductivity (PEC) is obtained. For the fractional order $\nu=1$, a strip with perfect magnetic conductivity (PMC) is obtained. In the problems we have addressed within the scope of our thesis study, near field distribution, far field radiation model and current distribution parameters have been obtained numerically in the examination of electromagnetic scattering by using different sources.Numerical calculations were made based on theory and the results are presented. Additionally, unknown expressions were obtained from the coefficients of the radiation model and the cross-sectional area of the total scattering, and the analytical numerical results of the problem were analyzed according to the given algorithm. With this new hybrid approach, results have been obtained in a shorter time and with greater accuracy. In future studies, research can be done on surfaces with different lengths and different geometry, diffraction of multiple randomly positioned strips with different boundary conditions and widths.

REFERENCES

Abel, N.H., 1881. Solution de quelques problèmes à l'aide d'intégrales définies: "Œuvrescomplètesde Niels Henrik Abel. Nouvelle edition", L. Sylow and S. Lie, Grøndahl & Søn, Christiania, , Ch. II, 11–27.

Ahmedov, T.M., Ivakhnychenko, M.V, Veliev, E. I., 2006. New Generalized Electromagnetic Boundaries: Fractional Operators Approach, 11^{th} Int. Conf. on Mathematical Methods in Electromagnetic Theory.

Baz, Z., Helvacı, M., İkiz, T., Veliev, E.I., 2024. Integral Equations For The Problem Of Wave Equations For The Problem Of Wave Diffractıon On A Flat Strip: ALTERNATIVE REPRESENTATION, TWMS J. App. and Eng. Math. 14(1), 1-10.

Butler, C. M., 1985. General Solution of the Narrow Strip Integral Equations. IEEE Trans. on Ant. and Proc., 33(10), 1085-1090.

Copson, E. T., 1946. An integral-equation method of solving plane diffraction problems, Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, 186 (1004), 100–118.

Colton D., 1987. Kress R. Methods of integral equations in the theory of scattering, 9, 311.

Davis, H. T., 1936. The Theory of Linear Operators, Bloomington, Indiana, The Principia Press, 64-77 and 276-292.

Engheta, N., 1996. Use of fractional integration to propose some ``fractional'' solutions for the scalar helmholtz equation, Progress In Electromagnetics Research, 12 107–132.

Engheta, N., 1998. Fractional curl operator in electromagnetics, Microwave and Optical Technology Letters, 17 (2), 86–91.

Engheta, N., 1999. Phase and amplitude of fractional-order intermediate wave, Microwave and optical technology letters, 21 (5), 338–343.

Engheta, N., 2000. Fractionalization methods and their applications to radiation and scattering problems, Proceedings of MMET*00, Vol. 1, 34–40.

Engheta, N., 1996.Use of fractional integration to propose some``fractional''solutions for the scalar helmholtz equation, Progress In Electromagnetics Research, 12 107–132.

Engheta, N., 1998. Fractional curl operator in electromagnetics, Microwave and Optical Technology Letters, 17 (2), 86–91.

Engheta, N., 1999. Phase and amplitude of fractional-order intermediate wave, Microwave and optical technology letters, 21 (5), 338–343.

Engheta, N. 1997, On the role of fractional calculus in electromagnetic theory, IEEE Antennas and Propagation Magazine, 39, 35-46.

Engheta, N., 1996. On Fractional Calculus and Fractional Multipoles in Electromagnetism, IEEE Transactions on Antennas and Propagation, AP-44, 554-566.

Engheta, N., 1996. Electrostatic ‘Fractional’ Image Methods for Per- fectly Conducting Wedges and Cones,” IEEE Transactions on Antennas and Propagation, AP-44, 1565- 1574.

Engheta, N., 1995. A Note on Fractional Calculus and the Image Method for Dielectric Spheres,Journal of Electromagnetic Waves andApplications, 1179-1188.

Engheta, N., 1996. Use of Fractional Integration to Propose Some ‘Fractional’ Solutions for the Scalar Helmholtz Equation, in Jin A. Kong (ed.), Progress in Electromagnetics Research (PIER) Monograph Series, Cambridge, MA, EMW Pub., 12, 107-132.

Engheta, N., 1994. Fractional Differintegrals and Electrostatic Fields and Potentials near Sharp Conducting Edges,” presented in “Ecole d’ETE Intemationale sur Geometrie Fractale et Hyperbolique, Derivation Fractionnaire et Fractal, Applications dans les Sciences de 1’Ingenieur et en Economie,” Bordeaux, France, organized by Alain Le MChaute from Institut SupCrieur des Mate- riaux du Mans (ISMANS), Le Mans, France, and A. Oustaloup from Equipe CRONE, Laboratoire d’Automatique et de Produc- tique, ENSERB UniversitC Bordeaux I, Bordeaux, France.

Erdtlyi, A., Sneddon, I. N., 1962. Fractional Integration and Dual Integral Equations, Canadian Journal of Mathematics, XIV, 685-693.

Grünwald, A.K. Uber “begrenzte” Derivationen und deren Anwendung. Z. Angew. Math. und Phys. 1867, 12, 441–480.

Hussain, A., Naqvi, Q. A., 2006. Fractional curl operator in chiral medium and fractional non-symmetric transmission line, Progress In Electromagnetic Research, Vol. 59, 199–213.

Hussain, A., Ishfaq, S., Naqvi, Q. A., 2006. Fractional curl operator and fractional waveguides, Progress In Electromagnetics Research, PIER 63, 319–335.

Hönl, H., Maue, A. W., Westpfahl, K., 1987. Theorie der beugung In Kristalloptik, Beugung/Crystal Optics, Diffraction, 218-573.

Ishimam, A., 1991. Electromagnetic Wave Propagation, Radiation, and Scattering, Prcntice-Hall, Englewood.

Ivakhnychenko, M. V., Veliev, E. I., and Ahmedov, T. M., 2008. Scattering properties of the strip with fractional boundary conditions and comparison with the impedance strip,PIERC (Progress in Electromagnetics Research C), 02, 189-205.

Ivakhnychenko, M. V., E. I. Veliev, and T. V. Ahmedov, 2007. Fractional operators approach in electromagnetic wave reflection problems, Journal of Electromagnetic Waves and Applications, 21 (18), 1787–1802.

Ivakhnvchenko, M.V., Veliev, E.I., 2004. Fractıonal Curl Operator in Radıation Problems, 10th Int. Conf. On Mathematical Methods in Electromagnetic Theory Sept. 14-17.

Ivakhnychenko M.V., Veliev E.I., 2004. Fractional curl operator in radiation problems. 10th IEEE International Conference on Mathematical Methods in Electromagnetic Theory; Ukraine; 231-233.

Kalisch, G. K., 1967. On Fractional Integrals of Pure Imaginary Order in L, Proceedings of the American Mathematical Society,18, 136-139.

Karaçuha, K., Veliyev, E. I., Tabatadze, V., and Karaçuha, E., 2019. Application of the method of fractional derivatives to the solution of the problem of plane wave diffraction by two axisymmetric strips of different sizes. Proceedings of 2019 URSI International Symposium on Electromagnetic Theory (EMTS), San Diego, USA, August.

Karaçuha, K., Veliyev, E. I., Tabatadze, V., and Karaçuha, E., 2019. Analysis of Current Distributions and Radar Cross Sections of Line Source Scattering from Impedance Strip by Fractional Derivative Method, Advanced Electromagnetics, 8 (2), 108–113.

Karaçuha ,K., Tabatadze V., Alperen, Ö.P., 2023. Electromagnetic Diffraction by a Slotted Cylinder with the Fractional Boundary Condition, Progress in Electromagnetics Research C, 128, 61-71.

Kiryakova, V., 1994. Generalized Fractional Calculus and Its Appli- cations, (Pitman Research Notes in Mathematics Series 301), Essex, UK, Longman Scientific & Technical, Longman Group.

Lacroix, S.F., 1819. Traité du Calcul Différentiel et du Calcul Intégral. 2nd Edition, Mme, Paris, Ve Courcier, Tome Troisiéme, 409-410.

Lakhtakia, A., 2001. A representation theorem involving fractional derivatives for linear homogeneous chiral media, Microwave Opt. Tech. Lett., Vol. 28, 385–386.

Letnikov, A.V., 1868. Theory of differentiation with an arbitrary index (Russian). Mat. Sb., 3, 1–66.

Letnikov, A.V., 1868. On historical development of differentiation theory with an arbitrary index (Russian). Mat. Sb., 3, 85–112.

Letnikov, A.V.,1872. On explanation of the main propositions of differentiation theory with an arbitrary index (Russian). Mat. Sb., 6, 413–445.

Letnikov,A.V., 1874. Investigations on the theory of integrals the form x a (x − u)p−1 f(u)du (Russian). Mat. Sb., 7, 5–205.

Liouville, J., 1832. Mémories sur quelques questions de géométrie et de mécanique, et sur un nouveau genre de calcul pour résoudre ces questions. J. l'Ecole Roy. Polytéchn., 13, 1–69.

Liouville, J., 1832. Mémories sur le calcul des différentielles à indices quelconques. J. l'Ecole Roy. Polytéchn., 13, 71–162.

Liouville, J., 1832. Mémories sur l'integration de l'équation différentielles indices quelconques. J. l'Ecole Roy. Polytéchn., 13, 163–186.

Liouville, J., 1834. Mémorie sur le théoréme des fonctions complémentaries. J. fur Reine und Ungew. Math. ,11, 1–19.

Liouville, J., 1834. Mémorie sur une formule d'analys. J. fur Reine und Ungew. Math., 12, 273–287.

Liouville, J., 1835. Mémorie sur l'usage que l'on peut faire de la formule de Fourier, dans le calcul des differentielles à indices quelconques. J. fur Reine und Ungew. Math.,13, 219–232.

Liouville, J., 1837. Mémorie sur l'intégration des équetions différentilles à indices fractionnaries. J. l'Ecole Roy. Polytéchn., 15, 58–84.

Lohmann, A. W., 1993. Image Rotation, Wigner Rotation, and the Fractional Fourier Transform, J. Opt. Soc. Amer. A, 10, 2181-2186.

Love, E. R., 1971. Fractional Derivatives of Imaginary Order, Jour-the London Mathematical Society, 3, Part 2, Second Series, 241-259.

McBride, A. C., Roach, G. F., 1985. Fractional Calculus, (Research Notes in Mathematics 138), Boston, Pitman Advanced Publishing Program.

Namias, V., 1979. The Fractional Order Fourier Transform and its Application to Quantum Mechanics Department of Physics, Purdue University Calumet, Hammond, Indiana, J. Inst. Maths Applies 25, 241-265,

Namias, V., 1980. The Fractional Order Fourier Transform and Its Application to Quantum Mechanics, J. Inst. Math. Appl., 25, 241-265.

Naqvi, Q. A., Rizvi, A. A., 2000. Fractional dual solutions and corresponding sources, Progress In Electromagnetics Research, PIER 25, 223–238.

Oldham, K. B., Spanier, J., 1974. The Fractional Calculus, New York, Academic Press.

Otsuki, T., 1976. Reexamination of diffraction problem of a slit by a method of Fourier-orthogonal functions transformation, J. Phys. Soc. Jpn.,, 41(6), 2046-2051.

Özaktaş, H. M., Mendlovic, D., 1993. Fractional Order Fourier Transforms and Their Optical Applications, J. Opt. Soc. Am. A, 10(12), 2522-2531.

Prudnikov, A. P., Brychkov, I. A., and Marichev, O. I., 2002. Special functions,Taylor and rancis, 750.

Potapov, A.A., 2008. Fractals, scaling and fractional operators as the basis of new methods for processing information and designing fractal radio systems // Technologies and Designing in Electronic Equipment, № 5, 3–19.

Podlubny, I., 1999. Fractional Differential Equations; Academic Press: San Diego, CA, USA.

Riemann, B., 1953. Versuch einer allgemeinen Auffasung der Inte- gration und Differentiation, in H. Weber (ed.), The Collected Works of Bernhard Riemann, (second edition), New York, Dover, 19, 353-366.

Samko, S. G., Kilbas, A. A., Marchichev, 0. I., 1993. Fractional Integrals and Derivatives, Theory and Applications, Langhorne, Pennsylvania, Gordon and Breach Science Publishers,(Originally published in Russian by Minsk, Nauka i Tekhnika, 1987).

Senior, T. B. A., 1952. Diffraction by a semi-infinite metallic sheet, Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, 213 (1115), 436–458.

Senior, T., 1979. Backscattering from resistive strips, IEEE Transactions on Antennas and Propagation, 27 (6), 808–813.

Tabatadze, V., Karaçuha, K., Veliyev, E.I., 2019. The Fractional Derivative Approach for Diffraction Problems: Plane Wave Diffraction by Two Strips with Fractional Boundary Conditions, Progress In Electromagnetics Research C, 95, 251–264.

Tabatadze, V., Karaçuha, K., and Veliyev, E. I., 2020. The solution of the plane wave diffraction problem by two strips with different fractional boundary conditions, Journal of Electromagnetic Waves and Applications, 34 (7), 881–893.

Tabatadze, V., Karaçuha, K., Veliyev, E. I., and Karaçuha, E., 2020. The Diffraction by Two Half-Planes and Wedge with the Fractional Boundary Condition, Progress In Electromagnetics Research, 91 1–10.

Tarasov, V. E., 2019. On history of mathematical economics: Application of fractional calculus, Mathematics, 7 (6), 509.

Uchida, K., Noda, T., Matsunaga,T., 1989. Electromagnetic wave scattering by a conducting strip Spectral domain analysis, Electronics and Communications in Jpn., 72(11), 717-722.

Veliev, E. I., Ivakhnychenko, M. V., 2004. Fractional curl operator in radiation problems, Proceedings of MMET*04, Dniepropetrovsk, 231–233.

Veliev, E. I., Ivakhnychenko, M. V., 2006. Elementary fractional dipoles,” Proceedings of MMET*06, Kharkiv, 485–487.

Veliev, E. I., Ahmedov, T. M., Ivakhnychenko, M. V., 2006. New generalized electromagnetic boundaries — Fractional operators approach, Proceedings of MMET*06, 434–437.

Veliev, E. I., Karaçuha, K., and Karaçuha, E., 2018. Scattering of a cylindrical wave from an impedance strip by using the method of fractional derivatives, Proceedings of 2018 XXIII[rd] International Seminar/Workshop on Direct and Inverse Problems of Electromagnetic and Acoustic Wave Theory (DIPED), Tbilisi, Georgia.

Veliev, E. I., Ivakhnychenko, M. V, and Ahmedov, T. M., 2008. Fractional boundary conditions in plane waves diffraction on a strip, Progress in electromagnetics research, (79) 443–462.

Veliev, E., Ahmedov, T., and Ivakhnychenko, M., 2011. Fractional operators approach and fractional boundary conditions, Electromagnetic Waves, 117.

Veliev, E. I. and Engheta, N., 2003. Generalization of Green’s theorem with fractional differintegration. Proceedings of 2003 IEEE AP-S International Symposium & USNC/URSI National Radio Science Meeting, USA.

Veliyev, E. I., Karaçuha, K., Karaçuha, E., and Dur, O., 2018. The use of the fractional derivatives approach to solve the problem of diffraction of a cylindrical wave on an impedance strip, Progress In Electromagnetics Research, 77 19–25.

Veliyev, E. I., Tabatadze, V., Karaçuha, K., and Karaçuha, E., 2020. The diffraction by the half-plane with the fractional boundary condition, Progress In Electromagnetics Research, 88 101–110.

Veliev E.I., Shestopalov V.P., 1988. On a general method for solving pair integral equations, DAN SSSR, , 300 (4), 827-832.

Veliev E.I., Ahmedov T.M., Ivakhnychenko M.V., 2011. Fractional operators approach and fractional boundary conditions. In: Zhurbenko V (editor). Electromagnetic Waves. Rijeka, Croatia: IntechOpen.

CURRICULUM VITAE

Zeynep BAZ received her B.Sc. degrees from the Physics and Electrical and Electronics Engineering Department of Çukurova University in Adana. She received her M.Sc. and Ph.D. degrees in the Physics Department at Çukurova University in Adana. Between 2018 and 2021, She worked at the TUBITAK Space Technologies Research Institute in Ankara. Since 2021, She has been working at the TUBITAK Defense Industries Research and Development Institute in Ankara.

Publications On The Thesis

- Baz, Z., Helvacı, M., İkiz, T., Veliev, E.I., 2024. Integral Equations For The Problem Of Wave Equations For The Problem Of Wave Diffractıon On A Flat Strip: ALTERNATIVE REPRESENTATION, TWMS J. App. and Eng. Math. 14(1), 1-10.

APPENDICES

APPENDIX A: The Properties of Surface Scattering

To compare the numerical characteristics of the scattered field in diffraction problem solutions, have been examined the properties of scattering such as SCS (Scattered Cross Section) and BSCS (Backscattering Cross Section).

When$kr \to \infty$, $E_z^s(x,y)$ is expressed differently using the stationary phase method.

$$E_z^s(r,\varphi) \approx A(kr)\Phi^{\alpha}(\varphi) \tag{A.1}$$

$$A(kr) = \sqrt{\frac{2}{\pi kr}}e^{ikr-i\pi/4} \tag{A.2}$$

$$\Phi^{\alpha}(\varphi) = -\frac{i}{4}(\pm i)^{\alpha}F^{1-\alpha}(\cos\varphi)sin^{\alpha}\varphi \tag{A.3}$$

The function $\Phi(\alpha,\varphi)$ determines the FFP (Far-Field Pattern) of the scattered field and is expressed using the coefficients f_n^{α}, which are found from the solution of the ISLAE (Inhomogeneous System of Linear Algebraic Equations).

$$\Phi^{\alpha}(\varphi) = -\frac{\pi i(\pm i)^{\alpha}}{2\Gamma(a+1)}(tan)^{\alpha}\varphi\sum_{n=0}^{\infty}(-i)^n f_n^{\alpha}\frac{\Gamma(n+2\alpha)}{\Gamma(n+1)}\frac{J_{n+\alpha}(ka\cos\varphi)}{(2ka)^{\alpha}} \tag{A.4}$$

Scattering cross section (SCS) $\frac{\sigma_{2D}}{\lambda}$ can be expressed with the FFP (Far-Field Pattern) (φ),

$$\frac{\sigma_{2D}}{\lambda}(\varphi) = \frac{2}{\pi}|\Phi(\varphi)|^2 \tag{A.5}$$

Back scattering cross section (BSCS), denoted as σ_{2D}^{mono}, it signifies the value of the scattering cross section (given by equation A.5) when the observation angle(φ) is equal to the incident angle (θ) (Potapov,2008).

$$\sigma_{2D}^{mono} = \frac{\sigma_{2D}}{\lambda}(\theta) = \frac{2}{\pi}|\Phi(\theta)|^2 \tag{A.6}$$

Within the Physical Optics (PO) approximation, we have the following expression $ka >> 1$:

$$\frac{\sigma_{2D}}{\lambda} = \frac{2}{\pi}\sin^2\varphi\left(\frac{\sin\theta}{\sin\varphi}\right)^{2\alpha}\left\{\frac{\sin ka(\cos\varphi+\cos\theta)}{\cos\varphi+\cos\theta}\right\}^2 \tag{A.7}$$

$$\sigma_{2D}^{mono} = \frac{2}{\pi}\sin^2\theta\left\{\frac{\sin ka(2\cos\theta)}{2\cos\theta}\right\}^2 \tag{A.8}$$

Surface currents represent the discontinuity of the field component as it passes through the strip boundary and are denoted by $J_z^{\alpha(e)}$ and $J_z^{\alpha(m)}$.

APPENDIX B: The Steepest Descent Method

This appendix describes the steepest descent method. Equations and formulas are taken from the reference book (Balanis, 2016). The steepest descent method is an iterative optimization algorithm used to minimize or maximize a function. It is particularly useful for finding the local minimum or maximum of a function. The basic idea behind the steepest descent method is to iteratively move in the direction of the negative gradient of the function at each iteration. This means that at each step, the algorithm takes a small step in the direction that will decrease the value of the function the most. The steepest descent method may converge slowly, especially if the function has narrow valleys or plateaus. However, it is simple to implement and can be effective for many optimization problems. Various modifications, such as the use of line search techniques or preconditioning, can be applied to improve its convergence rate and efficiency.
For large values of β, the value of the following integral can be found approximately.

$$I(\beta) = \int_C F(z)e^{\beta f(z)}dz \tag{B.1}$$

where $f(z)$ is an analytical function and the path of integration C is on the complex plane (Balanis, 2016). $I(\beta)$ can be found as follows if there exists one saddle point.

$$I(\beta) \cong \sqrt{\frac{2\pi}{-\beta f''(z_s)}}F(z_s)e^{\beta f(z_s)} \tag{B.2}$$

where $z_s = x_s + iy_s$ and $\left.\frac{df}{dz}\right|_{z=z_s} = f'(z = z_s) = 0$.