

Novel Techniques for Electromechanical Modeling of Straight/Curved Panels Integrated with Multiple Piezoelectric Patches

by

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Panel Integrated with Multiple Piezoelectric Patches**

**Çoklu Piezoelektrik Yamalar ile Entegre Düz / Kavisli
Panellerin Elektromekanik Modellemesi için Yeni Teknikler**

Koç University

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Abstract

Two-dimensional thin plates are widely used in many applications. This thesis will investigate vibration of these thin plates integrated with piezoelectric patches. Analyzing the dynamics of such electromechanical systems requires precise modeling tools that properly consider the coupling between the piezoelectric elements and the host structure. At first, a methodology and a formulation for separately shunted piezoelectric patches for achieving higher performance on vibration attenuation is developed. The developed model includes mass and stiffness contribution of the piezoelectric patches as well as the electromechanical coupling effect. It was shown that separately shunted piezoelectric patches are more effective compared to the connected ones for a wide range of frequencies. The results are validated with experimental studies to demonstrate the effect of the separately shunted circuits. In addition, a superior electromechanical model for predicting the dynamics of thin and/or moderately thick plates with surface-integrated piezo-patches is developed using spectral Tchebychev (ST) technique. This method has been shown to have the exponential convergence and superior stability characteristics, enabling prediction of the dynamics of mechanical structures accurately and efficiently. The boundary value problem is derived through an energy based approach using Mindlin plate theory. The electromechanical frequency response functions (FRFs) including the vibration response and electrical output of the system under a transverse point force excitation are obtained using the ST model. The case studies demonstrate that the computational efficiency of the developed method is significantly higher than that of FE simulations. Finally, an electromechanical model for predicting the dynamics of curved panels with multiple surface-integrated piezo-patches is developed. The boundary value problem governing the electro-elastic dynamic behavior of a (doubly-) curved panel and piezo-patch structure is derived following the first order shear deformation (FSDT) theory. Spectral Tchebychev approach is used to numerically solve the system dynamics and obtain voltage and mechanical frequency response functions (FRFs). Furthermore, the effect of mode jumping phenomenon is investigated to achieve higher voltage values in the application of energy harvesting.

For future studies, the novel models developed in this thesis can be used for straight/curved electro-mechanical systems for optimizing the harvested energy and improving the vibration control performance.

ÖZETÇE

İki boyutlu ince plakalar birçok uygulamada yaygın olarak kullanılmaktadır. Bu çalışmada, piezoelektrik yamalar ile entegre edilmiş ince plakaların titreşimi incelenecektir. Bu tür elektromekanik sistemlerin dinamiklerini analiz etmek, piezoelektrik elemanlar ile panel arasındaki bağlantıyı doğru bir şekilde dikkate alan hassas modelleme araçlarını gerektirir. İlk olarak, titreşim azaltmada daha yüksek performans elde etmek için ayrı ayrı olarak bağlanmış piezoelektrik yamalar için bir metodoloji ve bir formülasyon geliştirilmiştir. Geliştirilen model, piezoelektrik yamaların kütle ve sertlik katsayısının yanı sıra elektromekanik bağlantı etkisini de içerir. Ayrı ayrı bağlanmış piezoelektrik yamaların, geniş bir frekans aralığı için toptan bağlı olanlara göre daha etkili olduğu gösterilmiştir. Sonuçlar, ayrı ayrı bağlanmış devrelerin etkisini göstermek için deneysel çalışmalarla doğrulanmıştır. Ek olarak, spektral Tchebychev (ST) tekniği kullanılarak yüzeye entegre piezo-yamalar ile ince ve / veya orta derecede kalın plakaların dinamiklerini tahmin etmek için daha etkili bir elektromekanik model geliştirilmiştir. Bu yöntemin, mekanik yapıların dinamiklerinin doğru ve verimli bir şekilde tahmin edilmesini sağlayan üstün kararlılık özelliklerine sahip olduğu gösterilmiştir. Sınır değeri problemi, Mindlin plaka teorisi kullanılarak enerji temelli bir yaklaşımla elde edilir. Nokta kuvvet uyarımı altında sistemin titreşim tepkisini ve elektriksel çıktısını içeren elektromekanik frekans tepki fonksiyonları (FRF'ler), ST modeli kullanılarak elde edilir. Vaka çalışmaları, geliştirilen yöntemin hesaplama verimliliğinin, FE simülasyonlarından önemli ölçüde daha yüksek olduğunu göstermektedir. Son olarak, birden çok sayıda piezo yamalar ile entegre edilmiş kavisli panellerin dinamiklerini tahmin etmek için elektromekanik bir model geliştirilmiştir. Çift kavisli bir panel ve piezo-yama yapısının elektro-elastik dinamik davranışını yöneten sınır değeri problemi, birinci dereceden kayma deformasyonu (FSDT) teorisini takiben türetilmiştir. Spektral Tchebychev yaklaşımı, sistem dinamiklerini sayısal olarak çözmek ve voltaj/mechanik yer değiştirme frekans yanıt fonksiyonlarını (FRF'ler) elde etmek için kullanılır. Ayrıca, enerji hasadı uygulamasında daha yüksek voltaj değerleri elde etmek için mod atlama olgusunun etkisi araştırılmıştır.

Gelecekteki çalışmalarda, bu tezde geliştirilen yeni modeller, hasat edilen enerjiyi optimize etmek ve titreşim kontrol performansını iyileştirmek için düz / kavisli elektro-mekanik sistemler için kullanılabilir.

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1 Introduction

Piezoelectric materials have been widely used in many applications such as vibration control (Casagrande, Gardonio, and Zillett 2017; Bailey and Hubbard 1985), energy harvesting (Gozum et al. 2018; Ugur Aridogan, Basdogan, and Erturk 2014) and shunt damping (Han, Neubauer, and Wallaschek 2013; Motlagh, Aghakhani, and Basdogan 2018b) over the past few decades to convert mechanical (electrical) energy to electrical (mechanical) energy. Because piezoelectric transducers are easier to manufacture and present higher power density (Alper Erturk and Inman 2011a), they are preferred compared to other types of transducers such as electromagnetic (Yang et al. 2009) and electrostatic (Torres and Rincon-Mora 2009). Many studies in the literature investigate the modeling approaches for electromechanical structures consisting of a host (which can be a thin beam or a panel structure) integrated with piezoelectric patches or layers. Although lumped parameter models (Roundy and Wright 2004) are used previously, distributed parameter models are required to accurately predict the vibration behavior and voltage or mechanical frequency response functions (FRFs). For instance, Erturk *et al.* (A Erturk and Inman 2009) introduced an experimentally validated distributed parameter model to predict the electromechanical response of uni-morph and bimorph beams. Wang and Quek (Wang and Quek 2000) showed free vibration responses of an isotropic beam with an integrated piezoelectric layer. In the later studies, the analyses performed for beams are extended to panel type structures. Reddy (Reddy 1999) developed a finite element based model using both the classical plate theory (CPT) and first/third-order shear deformation theory (FSDT) to control the dynamic response of laminated panels including piezoelectric layers. In a recent study, Aridogan *et al.* (Ugur Aridogan, Basdogan, and Erturk 2016b) predicted electromechanical response by neglecting the mass and inertia contribution of single/multiple piezoelectric patches; note that this assumption may lead to errors when piezo patch sizes are large. Then, Bayik et al. (Bayik et al. 2016) based on the modeling approach described in (Ugur Aridogan, Basdogan, and Erturk 2016b), developed a multi-mode equivalent circuit model of a piezo-patch integrated to a thin panel coupled with a standard AC–DC conversion circuit to investigate energy harvesting. Although a large body of literature has been devoted for modeling and analysis of electromechanical structures, the majority of these studies are focused on the analysis of straight panels. In other words, there are still limitations on

the number of studies investigating the piezoelectric ceramic embedded smart structures on shells and curved panels. Kapuria (Kapuria, Dumir, and Sengupta 1997) expands a three-dimensional axisymmetric solution to investigate simply-supported shells and a model of smart shells. Then, Wang (Wang and Liew 2003) proposed a general solution for wave propagation in a piezoelectric coupled cylindrical shell based on the Cooper's study (Naghdi and Cooper 1956) using power and Fourier series expansion method for bending analysis of the laminated piezoelectric structure. Recently, Zhou (Zhou, Zhu, and Liu 2019) analyzed the dynamics of the piezoelectric laminated shells using a state-space technique for the bending problem of laminated piezoelectric cylindrical shells. There are other studies (Nath and Kapuria 2012; Donoso, Bellido, and Chacón 2010; Alibeigloo and Shakeri 2009) which used zigzag theory, the third-order theory, B-spline approximation, and Fourier series to study the bending behavior of piezoelectric shells under mechanical, thermal and electrical excitation; however, in those studies, the analysis is based on the structures which have a full surface covered by piezoelectric layers. Therefore, there is still a need to develop a more generalized model that can predict the electromechanical coupling of curved structures integrated with patches that cover the surface partially. Another common approach to model electromechanical structures is the use finite element (FE) approach (De Marqui Junior, Erturk, and Inman 2009; Rochus and Geuzaine 2012; Aten, Jensen, and Howell 2012); but most of the studies used 3D models which are computationally inappropriate for the discretization of the structure such as thin-layer electromechanical actuators and sensors. To improve the computational efficiency, equivalent single layer (ESL) (E. Carrera, Brischetto, and Cinefra 2010) and layer-wise (LW) (D'Ottavio et al. 2006) approaches (which is based on FE to predict behavior of 2D models) are developed. For instance, Cinefra et al. (Cinefra, Valvano, and Carrera 2015) presents a model based on LW to predict natural frequencies of electromechanical systems. Recently, Carrera and Brischetto (Erasmus Carrera and Valvano 2017) used FE models based on LW and ESL based for free vibration analysis and electromechanical coupling investigation of composite plates and shells embedded with piezoelectric layers. Further, for predicting global/local dynamics of piezoelectric patches, Carrera (E. Carrera, Valvano, and Kulikov 2018b; 2018a) developed a FE modeling approach which can use simultaneous multi-models for 2D elements with node-dependent kinematics to predict electromechanical behavior of composite panels with integrated piezoelectric layers. Later, Kulikov et al. (Kulikov, Plotnikova, and Carrera 2018) extends this work to develop a hybrid-mixed four-node

piezoelectric solid-shell element. These methods can predict layered composites accurately and efficiently but still there exists a need for developing a model which can predict both natural frequency and dynamic behavior of partially integrated piezoelectric patches on panels accurately and efficiently. For instance, Gozum et al. (Gozum et al. 2018) introduced an analytical modeling approach based on Rayleigh-Ritz solution including the mass and stiffness contribution of piezoelectric patches. They followed the classical plate theory (CPT) assumptions and used Heaviside function to include the mass and stiffness of piezoelectric patches. However, such modeling approaches cannot accurately estimate the dynamics of thin and thick plates with relatively large piezoelectric patch sizes. For that reason, their modeling approach has been improved by using Mindlin and Spectral Tchebychev method in chapter 3 and it was further improved by using first shear deformation theory (FSDT) approach to include the modeling of curved structures.

Overall, in Chap. 2, by using classical plate theory (CLPT) a new approach is proposed to obtain broadband frequency shunt damping. The Raleigh Ritz model developed in (Gozum et al. 2018) is extended to cover the electro-mechanical equations for separately shunted circuits and also neutral axis shift is included in the electro-mechanical model to accommodate when a single patch is used on one side of the host plate. The results of the extended electro-mechanical model are validated by FEM results and experiments are conducted to demonstrate the accuracy of the proposed technique for the separately shunted circuits. In this study, Equivalent Radiated Power (ERP) metric is used to investigate the shunt damping performance. The shunt damping performance is investigated for different thickness values, area ratios and piezoelectric patch numbers. In addition, an optimization study is performed to determine the optimized design parameters of the shunt circuit (i.e. Resistor values and number of patches) to minimize the vibration amplitudes at the first three modes of the structure.

To achieve higher accuracy and reducing computational cost, Chap. 3 presents a general electromechanical modeling of plates with structurally integrated piezo-patches using a new series based on the spectral numerical approach which employs Tchebychev polynomials to discretize the boundary value problem (BVP). Due to the exponential convergence and superior stability characteristics (Gottlieb, Orszag, and Foundation (U.S.) 1977), the developed solution technique enables predicting the dynamics of mechanical structures accurately and efficiently. Another important advantage of the presented spectral Tchebychev approach is that the differentiation and integral operations

are performed exactly through the derived differentiation and inner-product matrices which increases the computational efficiency of the modeling process. In this approach, the system matrices for the host plate and the piezo-patch(es) are first derived using Hamilton's principle. Then, a novel coupling algorithm is developed to include the two-way electromechanical coupling effect between the host plate and piezo-patches and also to obtain the mass and stiffness matrices of the overall coupled system. The compatibility equations for the electromechanical coupling and the boundary condition equations are incorporated into the boundary value problem through projection matrices approach. This projection matrices approach also eliminates the need to use different basis/trial functions for each problem with different boundary conditions (Bediz 2018; Yagci et al. 2009) following the previously developed ST approaches in literatures, the proposed ST framework in this study is able to predict not only the mechanical behavior but also the electrical response of the two-dimensional electromechanical structures accurately and efficiently. To validate the accuracy and demonstrate the computational efficiency of the presented technique, several case studies including one-sided and two-sided piezo-patches are presented. The modal analysis results as well as the displacement and voltage frequency response functions are compared to those obtained by a commercial finite-element (FE) software and the existing Rayleigh-Ritz method from the literature. Spectral Tchebychev approach enables predicting the dynamics of mechanical structures accurately and efficiently because of the exponential convergence and superior stability characteristics (Gottlieb, Orszag, and Foundation (U.S.) 1977; Serhat, Bediz, and Basdogan 2020).

Finally, in Chap. 4, a general solution approach is presented that is based on first order shear deformation theory (FSDT) to predict the electromechanical response of curved piezoelectric patches on a (doubly-) curved host structure. The integral boundary value problem (IBVP) is solved numerically using the spectral Tchebychev (ST) technique. To include the mass and inertia contribution of the multiple piezo-patches, a novel structural coupling algorithm is proposed. Furthermore, a detailed analysis is carried out to benefit from multiple patches configuration to increase energy harvesting performance. To validate the results of the presented solution approach, a commercial finite-element (FE) software is used.

Chap.5 will conclude the thesis and propose some paths for further studies.

Chapter 2

2 Multimode piezoelectric shunt damping of thin plates with arrays of separately shunted patches

2.1 Introduction

Mechanical structures are exposed to vibrations caused by operational or environmental sources, which can be undesirable. Over the decades, many passive and semi-passive systems have been proposed for reducing these vibrations (Alper Erturk and Inman 2011b; Bailey and Hubbard 1985). Piezoelectric structures have been widely used in a range of applications including vibration control (Bailey and Hubbard 1985; Gripp and Rade 2018), energy harvesting (Bayik et al. 2016; Aghakhani et al. 2019; Aghakhani and Basdogan 2017), structural health monitoring (Tinoco et al. 2016; Giurgiutiu 2007). Over the past few decades, among the transducers that convert the mechanical (electrical) energy to electrical (mechanical) energy, piezoelectric transducers are mostly preferred to the electromagnetic (Beeby et al. 2007; Yang et al. 2009), and electrostatic (Beeby et al. 2007; Torres and Rincon-Mora 2009), ones due to their high power density and ease of manufacturing at different size scales (Powering, n.d.). The most common use of piezoelectric materials in the form of patches/layers is by integrating them to the surfaces of flexible beam/plate-like structures, and then utilize them in bending motion for generating an electrical signal and vice versa (applying a voltage to generate a bending deformation). Piezoelectric patches have been used for shunt damping applications that focus on designing simple electrical circuits that efficiently reduce the structural vibrations.

In this chapter, a new approach is proposed in this study to obtain broadband frequency shunt damping. The Raleigh Ritz model developed in (Gozum et al. 2018) is extended to cover the electro-mechanical equations for separately shunted circuits and also neutral axis shift is included in the electro-mechanical model to accommodate when a single patch is used on one side of the host plate. The results of the extended electro-mechanical model are validated by FEM results and experiments are conducted to demonstrate the accuracy of the proposed technique of the separately shunted circuits. In this study, Equivalent Radiated Power (ERP) metric is used to investigate the shunt damping performance. At each step, the ERP is calculated while the R value is varied

between short circuit ($R=100\ \Omega$) to open circuit ($R=1\ \text{M}\Omega$). Then the optimum resistance R_{opt} value is obtained when the ERP is minimum. The same procedure is repeated for different thickness values, area ratios and piezoelectric patch numbers. In addition, an optimization study is performed to determine the optimized design parameters of the shunt circuit (i.e. Resistor values and number of patches) to minimize the vibration amplitudes at the first three modes of the structure. It is shown that the distribution of the optimized resistor values of the shunt circuits follows a pattern very similar to the strain mode shape of the targeted mode and also improves the shunt damping performance which is a significant observation of the present study.

2.2 Analytical model of a thin plate with multiple piezoelectric patches

In this section, a brief description of the model of a thin plate with multiple piezoelectric patches is given based on the Kirchhoff plate theory (Gozum et al. 2018; Yoon, Youn, and Kim 2016a). Figure 2.1, presents the host plate with all four edges clamped (CCCC) boundary conditions and the structurally integrated piezoelectric patches in separated and connected configurations, respectively. As it can be seen from the figure, in the separated configuration, each patch is connected independently to a shunted electric circuit and the negative end is grounded on the plate, whereas in the connected configuration, all the positive ends of the patches are connected to the positive end of each shunted electric circuit and the negative ends are grounded on the plate.

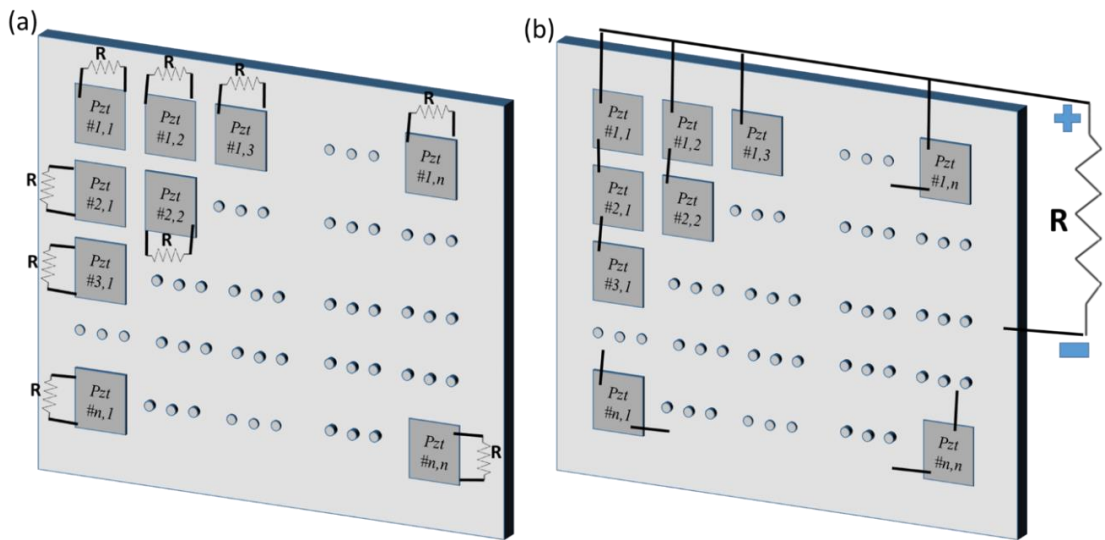


Figure 2.1. (a) separated configuration of the piezoelectric patches and the host plate, (b) connected configuration of the piezoelectric patches and the host plate

2.3 Constitutive equations of the thin plate with piezoelectric patches based on Kirchhoff plate theory

Since the piezoelectric patches are typically manufactured as a thin plate, piezoelectric patch skin can be modeled as a two-dimensional Kirchhoff plate. According to Kirchhoff plate theory (Yoon, Youn, and Kim 2016a), the deflection of the middle surface is assumed to be small compared to the thickness of the skin. Since the normal stress σ_{zz} in the thickness direction is much smaller compared to the in-plane stresses, it can be ignored under the assumption of the thin plate theory. The material of the structural layer is assumed to be isotropic. Therefore, based on the Kirchhoff plate theory, the constitutive equations for the structural layer are reduced to Eq. (2.1) where Y_s is the Young's modulus and ν_s is the Poisson's ratio of the structural layer. The in-plane stress and strain components for the piezoelectric patches are denoted by σ_{ij} and $\gamma_{ij}, \epsilon_{ij}$ respectively, and the constitutive equations of a piezoelectric patch are expressed in a reduced form as Eq. (2.2) and (1.2) (Yoon, Youn, and Kim 2016a):

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{pmatrix} = \frac{Y_s}{1 - \nu_s^2} \begin{bmatrix} 1 & \nu_s & 0 \\ \nu_s & 1 & 0 \\ 0 & 0 & (1 - \nu_s)/2 \end{bmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{pmatrix} \quad (2.1)$$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ D_3 \end{pmatrix} = \begin{bmatrix} \bar{c}_{11} & \bar{c}_{12} & 0 & -\bar{e}_{31} \\ \bar{c}_{12} & \bar{c}_{11} & 0 & -\bar{e}_{31} \\ 0 & 0 & \bar{c}_{66} & 0 \\ \bar{e}_{31} & \bar{e}_{31} & 0 & \epsilon_{33}^s \end{bmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \\ E_{33} \end{pmatrix} \quad (2.2)$$

Where $\bar{c}_{ij}$ are reduced elastic moduli of the piezoelectric patches, $\bar{e}_{ij}$ and ϵ_{33}^s are piezoelectric constant and dielectric permittivity, respectively (Yoon, Youn, and Kim 2016a). Hamilton's principle is used to determine the equation of motion Eq. (2.3) :

$$\delta \oint_{t_1}^{t_1} (KE - PE + W_p) dt = 0 \quad (2.3)$$

Where kinetic energy, potential energy, and applied external work is indicated as KE , PE , and W_p respectively (Marion 2013). And the indicator function $P(x, y)$ is defined to identify the areas where k piezoelectric patches are attached to the surface of the structural layer by Eq. (2.4) :

$$P(x, y) = \sum_{i=1}^k [H(x - x_{i,1}) - H(x - x_{i,2})] \times [H(y - y_{i,1}) - H(y - y_{i,2})] \quad (2.4)$$

Where x_1, x_2, y_1 , and y_2 are the vertices of the area along x- and y-axes, respectively, and H denotes the Heaviside unit step function. The equivalent mass per unit area can be derived as Eq. (2.5) :

$$m(x, y) = \rho_s h_s + \rho_p h_p P(x, y) \quad (2.5)$$

Here ρ_s, ρ_p , are densities and h_s and h_p are the thicknesses of the host plate and the piezoelectric patch, respectively. The stress distribution in the areas covered by the piezoelectric patch is not the same as that in the uncovered areas. Thus, the neutral surface is shifted from the middle surface of the structural layer towards the piezoelectric patch. Based on the stress equilibrium, the location of the neutral surface can be determined as Eq. (2.6) :

$$z_0 = \frac{\bar{c}_{11} h_p (h_s + h_p)}{2 \left\{ \frac{Y_s h_s}{(1 - v_s^2)} + \bar{c}_{11} h_p \right\}} \quad (2.6)$$

Then, the equation of motion of the plate and piezoelectric patches can be derived as Eq. (2.7) in which w is displacement of the system:

$$\begin{aligned} m(x, y) \ddot{w} + \left[\{D^s + P(x, y) D^{sp}\} \left\{ \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2v_s \left(\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right) \right\} \right. \\ \left. + 2(1 - v_s) \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \\ + P(x, y) D_{11}^p \left\{ \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right\} \\ + 2P(x, y) \left\{ D_{12}^p \left(\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right) + 2D_{66}^p \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} \\ + P(x, y) \bar{e}_{31} v(t) \left(\frac{h_s + h_p}{2} - z_0 \right) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ = f(t) \delta(x - x_0) \delta(y - y_0) \end{aligned} \quad (2.7)$$

where $D_{11}^p, D_{12}^p, D_{66}^p, D^s$ and D^{sp} are as follows

$$\begin{aligned}
D_{11}^p &= \bar{c}_{11} \left(\frac{h_p^3}{3} + \frac{h_s^2 h_p}{4} + \frac{h_s h_p^2}{2} - z_0(h_p h_s + h_p^2) + z_0^2 h_p \right) \\
D_{12}^p &= \bar{c}_{12} \left(\frac{h_p^3}{3} + \frac{h_s^2 h_p}{4} + \frac{h_s h_p^2}{2} - z_0(h_p h_s + h_p^2) + z_0^2 h_p \right) \\
D_{66}^p &= \bar{c}_{66} \left(\frac{h_p^3}{3} + \frac{h_s^2 h_p}{4} + \frac{h_s h_p^2}{2} - z_0(h_p h_s + h_p^2) + z_0^2 h_p \right)
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
D^s &= \frac{Y_s h_s^3}{12(1 - \nu_s^2)} \\
D^{sp} &= \frac{Y_s}{1 - \nu_s^2} \left(\frac{h_s^3}{12} + z_0^2 h_p \right)
\end{aligned} \tag{2.9}$$

2.3.1 Modal analysis using the Rayleigh-Ritz method

The natural frequencies and corresponding mode shapes are calculated for the fully clamped (CCCC) boundary conditions. Based on the modal expansion, the relative displacement of piezoelectric patch skin is approximated by a linear combination of the assumed modes $U_{ij}W_{ij}(x, y)$ as Eq. (2.10), where $\mu_{ij}(t)$ are the generalized modal coordinates, N is the total number of vibration modes in y and R is the total number of vibration modes x coordinates. Assumed modes are indicated by $U_{ij}W_{ij}(x, y)$, where $W_{ij}(x, y)$ are the trial functions satisfying the boundary conditions, and U_{ij} are the corresponding coefficients.

$$w(x, y, t) = \sum_{i=1}^N \sum_{j=1}^N U_{ij} W_{ij}(x, y) \mu_{ij}(t) \tag{2.10}$$

The eigenvalue equation can be written as Eq. (2.11) for more information, the reader can be referred to (Yoon, Youn, and Kim 2016a):

$$[K_{rn,kl} - \omega_{rn}^2 M_{rn,kl}][U_{rn}] = \{0\} \tag{2.11}$$

Here, assumed mode shape coefficients U_{ij} 's are the eigenvectors and natural frequencies ω_{ij} 's are the square root of the eigenvalues of Eq. (2.11).

2.3.2 Electromechanical coupling between patches and host structure

The electric current can be calculated using Gauss's law (i.e., Gauss's flux theorem) with respect to the electric displacement component D_3 . This value is the same

as the scalar product of the electric displacement field D and the outward normal unit vector n (A Erturk and Inman 2009) as Eq. (2.12) in which $v(t)$, R_l and S_p are voltage over the piezoelectric patch, resistor attached to each patch and area of piezoelectric patch, respectively.

$$\frac{v(t)}{R_l} = \frac{d}{dt} \int_{S_p} (D \cdot n) dS_p = \frac{d}{dt} \int_{S_p} D_3 dS_p \quad (2.12)$$

By substituting D_3 from Eq. (2.2) in Eq. (2.12) and substituting strain Eq. (2.2), Eq. (2.12) can be written as Eq. (2.13).

$$\frac{v(t)}{R_l} = -\bar{e}_{31} \left(\frac{h_p + h_s}{2} - z_0 \right) \frac{\partial}{\partial t} \int_{S_p} \nabla^2 w(x, y, t) dS_p - \frac{dv(t)}{dt} \int_{S_p} \frac{\varepsilon_{33}^s}{h_p} dS_p \quad (2.13)$$

The last term on the right-hand side of Eq. (2.13) can be considered as the capacitance C_p^k of the k^{th} piezoelectric patch which can be defined as Eq. (2.14) :

$$C_p^k = \int_{S_p} \frac{\varepsilon_{33}^s}{h_p} dS_p \quad (2.14)$$

The Kirchhoff laws can be applied to the electrical circuits to obtain Eq. (2.15), by comparing Eq. (2.13) and Eq. (2.15), the current passing through k^{th} piezoelectric patch is defined as Eq. (2.16) (A Erturk and Inman 2009).

$$C_p^k \left(\frac{dv(t)}{dt} \right) + \frac{v(t)}{Z_l} = i_k(t) \quad (2.15)$$

$$i_k(t) = -\bar{e}_{31} \left(\frac{h_p + h_s}{2} - z_0 \right) \frac{\partial}{\partial t} \int_{S_p} \nabla^2 w(x, y, t) dS_p \quad (2.16)$$

Then, the current output in modal coordinates can be obtained by substituting Eq. (2.10) into Eq. (2.16), and written as :

$$i_k(t) = - \sum_{n=1}^N \sum_{r=1}^R \frac{d\eta_{rn}(t)}{dt} \tilde{\theta}_{rn}^k \quad (2.17)$$

Where $\tilde{\theta}_{rn}^k$ can be defined as Eq. (2.17):

$$\tilde{\theta}_{rn}^k = -\bar{e}_{31} \left(\frac{h_p + h_s}{2} - z_0 \right) \int_{S_p} U_{rn} \nabla^2 W_{rn}(x, y) dS_p \quad (2.18)$$

2.3.3 Steady-state response

The steady-state equations for piezo patches on a plate can be divided into two categories, connected and separated configurations. The connected patches were previously presented in literature (Bayik et al. 2016; Aghakhani et al. 2019; Ugur Aridogan, Basdogan, and Erturk 2016a). In this section, first, connected configuration equations are summarized then the separated configuration equations will be derived. For each of these cases, the current through each of piezoelectric patch can be represented using Eq. (2.17) (Ugur Aridogan, Basdogan, and Erturk 2016a). If the transverse force excitation on the host plate is assumed to be harmonic of the form $f(t) = F_0 e^{j\omega t}$ (where F_0 is the force amplitude and ω is the excitation frequency), the linear system assumption implies that the mechanical and electrical responses can be represented as harmonic functions: $\eta_{rn}(t) = H_{rn} e^{j\omega t}$ and $v_k(t) = V_k e^{j\omega t}$, respectively.

2.3.3.1 Connected patches

In the connected configuration, positive nodes of all of the patches are connected to each other whereas all of their negative nodes are considered grounded ($v = 0$) as in Figure 2.1. For this configuration, the circuit equation can be written as Eq. (2.19) using Kirchhoff's current law, where Z_l is impedance of load attached to each piezoelectric patch (Ugur Aridogan, Basdogan, and Erturk 2016a):

$$\sum_{k=1}^n C_p^k \left(\frac{dv(t)}{dt} \right) + \frac{v(t)}{Z_l} = \sum_{k=1}^n i_k(t) \quad (2.19)$$

A complete methodology description of connected patches is presented in (Ugur Aridogan, Basdogan, and Erturk 2016a), in which the displacement of the system $w(x, y, t)$ is given by:

$$w(x, y, t) = \sum_{r=1}^R \sum_{n=1}^N U_{rn} W_{rn}(x, y) \left\{ \frac{F_0 U_{rn} W_{rn}(x_0, y_0) + V_k \sum_{k=1}^K \tilde{\theta}_{rn}^k}{\omega_{rn}^2 - \omega^2 + 2j\xi_{rn}\omega_{rn}\omega} \right\} e^{j\omega t} \quad (2.20)$$

2.3.3.2 Separated patches

In the separated configuration, each patch is connected to an electric circuit independently as in Figure 2.1-(a), by applying Kirchhoff's current law for each patch, the circuit equation can be written as Eq. (2.21):

$$C_p^k \left(\frac{dv_k(t)}{dt} \right) + \frac{v_k(t)}{(Z_l)_k} = i_k(t), (k = 1, 2, \dots, \text{Number of patches}) \quad (2.21)$$

The independent differential equations of motion in modal coordinates was developed previously in (A Erturk and Inman 2009), (Yoon, Youn, and Kim 2016a) as Eq. (2.22) where the voltage $v_k(t)$ will be different on each patch:

$$\frac{d^2 \eta_{rn}(t)}{dt^2} + 2\omega_{rn} \xi_{rn} \frac{d\eta_{rn}(t)}{dt} + \omega_{rn}^2 \mu_{rn}(t) - \sum_{k=1}^K \theta_{rn}^k v_k(t) = f_{rn}(t) \quad (2.22)$$

Where $f_{rn}(t)$ (Yoon, Youn, and Kim 2016a) is :

$$f_{rn}(t) = f(t) U_{rn} W_{rn}(x_0, y_0) \quad (2.23)$$

From Eq. (2.17) and Eq. (2.21), the relationship between W_{rn} and $v_k(t)$ can be written as Eq. (2.24):

$$\begin{aligned} \left[\frac{1}{(Z_l)_k} + j\omega C_p^k \right] V_k(t) + j\omega \sum_{n=1}^N \sum_{r=1}^N \frac{F_0 U_{ij} W_{ij}(x_0, y_0) \tilde{\theta}_{ij}^k}{\omega_{ij}^2 - \omega^2 + (2j)\xi_{ij}\omega_{ij}\omega} \\ + j\omega \sum_{i=1}^N \sum_{j=1}^N \tilde{\theta}_{ij}^k \frac{\sum_{k=1}^K \tilde{\theta}_{ij}^k v_k(t)}{\omega_{ij}^2 - \omega^2 + (2j)\xi_{ij}\omega_{ij}\omega} = 0 \end{aligned} \quad (2.24)$$

Then $V_k(t)$ are the only unknowns in this equation, and they can be derived as:

$$\begin{bmatrix} V_1 \\ \vdots \\ V_k \\ \vdots \\ V_n \end{bmatrix} = A^{-1} \begin{bmatrix} b_1 \\ \vdots \\ b_k \\ \vdots \\ b_n \end{bmatrix} \quad (2.25)$$

Where A is defined as Eq. (2.26) :

$$A = \begin{bmatrix} \left\{ \frac{1}{(Z_l)_1} + j\omega C_p^1 + a'_{11} \right\} & \cdots & a'_{1k} & \cdots & a' \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a'_{k1} & \ddots & \left\{ \frac{1}{(Z_l)_k} + j\omega C_p^k + a'_{kk} \right\} & \ddots & a' \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a'_{n1} & \cdots & a'_{nk} & \cdots & \left\{ \frac{1}{(Z_l)_n} + j\omega \right\} \end{bmatrix} \quad (2.26)$$

where a and b 's are defined as

$$a'_{ls} = j\omega \sum_{j=1}^N \sum_{i=1}^N \frac{\tilde{\theta}_{ij}^l \tilde{\theta}_{ij}^s}{\omega_{ij}^2 - \omega^2 + 2j\xi_{ij}\omega_{ij}\omega}, (i, j = 1, \dots, N), (l, s = 1, 2, \dots, \text{number of patches}) \quad (2.27)$$

$$b_k = j\omega \sum_{i=1}^N \sum_{j=1}^N \frac{F_0 U_{ij} W_{ij}(x_0, y_0) \tilde{\theta}_{ij}^k}{\omega_{rn}^2 - \omega^2 + 2j\xi_{rn}\omega_{rn}\omega}, (k = 1, 2, \dots, \text{number of patches}) \quad (2.28)$$

By finding $V = [v_1 \ \cdots \ v_k \ \cdots \ v_n]^T$ from (2.25), and inserting in (2.22), the relative displacement of the plate can be defined as Eq. (2.29):

$$w(x, y, t) = \sum_{i=1}^N \sum_{j=1}^N U_{ij} W_{ij}(x, y) \left\{ \frac{F_0 U_{ij} W_{ij}(x_0, y_0) + \sum_{k=1}^K \theta_{ij} A^{-1} [b_1 \ \cdots \ b_k \ \cdots]}{\omega_{ij}^2 - \omega^2 + 2j\xi_{ij}\omega_{ij}\omega} \right\} \quad (2.29)$$

2.3.4 Vibration Metric to evaluate the Shunt Damping Performance

In this study, Equivalent Radiated Power (ERP) is used as the vibration metric to evaluate the shunt damping performance of the system. ERP is mostly used for vibrating panels which includes information about maximum possible acoustic radiation at the excitation frequencies and it is used to demonstrate the amount of reduction on a structure when some sort of vibration control technique is performed (Fritze, Marburg, and Hardtke 2009; Serhat and Basdogan 2018). The ERP can be defined as Eq. (2.30) and normalized one as Eq. (1.31) :

$$\text{ERP}(t) = \frac{1}{2} \rho_f c_f \int_S \dot{w}(x, y, t)^2 dS \quad (2.30)$$

$$\overline{\text{ERP}}(t) = \frac{\frac{1}{2} \rho_f c_f \int_S \dot{\omega}_{rel}(x, y, t)^2 dS}{\frac{1}{2} \rho_f c_f \int_S dS} \quad (2.31)$$

Once the normalized ERP ($\overline{\text{ERP}}$) is calculated, percent reduction compared to the open circuit (OC) condition is calculated using Eq. (1.32). The results are summarized in Section 5.

$$\begin{aligned} & (\text{Percentage reduction via shunt damping})_i \\ &= 100 \times \left| \frac{\max(\overline{\text{ERP}}_{\text{minimom}})_i - \max(\overline{\text{ERP}}_{\text{OC}})_i}{\max(\overline{\text{ERP}}_{\text{OC}})_i} \right| \end{aligned} \quad (2.32)$$

2.4 Finite element model

Commercial finite element analysis (FEA) software ANSYS is used for validating the modal solutions obtained using the analytical model. The plate is meshed with 20-node structural solid elements (SOLID 186). The piezoelectric patches are meshed with 20-node coupled field solid elements (SOLID 226). The host structure and the piezoelectric patches are bonded by generating constraint equations at the interface (Bayik et al. 2016). The external resistive load connected to piezoelectric patches is modeled with a piezoelectric circuit element (CIRCU 94). Natural frequencies and mode shapes are obtained by modal analysis.

2.5 Experimental setup

The experimental setup used for comparing the shunt damping performance of the separated and connected patch configurations is presented in Figure 2.2. A fully clamped aluminum plate is used as the host plate. Three electrode piezoelectric patches (T105-A4E-602 by Piezo Systems, Inc.) are employed for shunt damping by attaching them on the plate. The geometric and material properties of the aluminum plate and the piezoelectric patches are given in Table 2.1. The three piezoelectric patch harvesters (labeled as PZT-I, PZT-II and PZT-III) having identical polarities are perfectly bonded on the surface of the plate. The bottom (negative) and top (positive) surfaces of the piezoelectric patches are covered by thin vacuum sputtered nickel electrodes of a negligible thickness (by the manufacturer) and the bonding areas on the aluminum plate are electrically insulated with 3M Scotch 1601 spray. A modal shaker is used to excite

the plate. The attachment point of the shaker's stinger rod can be seen in Figure 2.2. The applied force is monitored by placing a force transducer (PCB 208C01) between the shaker's armature and the plate surface. A laser Doppler vibrometer (LDV—Polytec PDV 100) is used to measure the transverse velocity of the plate at the target point shown in Figure 2.2. The signals obtained from the force transducer and the laser vibrometer are sent to the data acquisition system for time-domain and frequency response analyses.

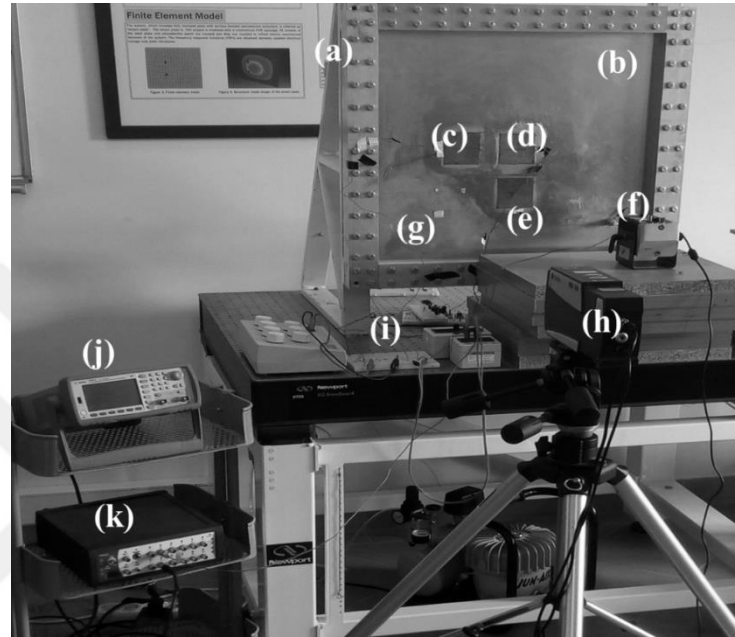


Figure 2.2. Experimental setup, (a) clamping frame, (b) aluminum plate, (c) PZT-I, (d) PZT -II, (e) PZT -III, (f) modal shaker, (g) measurement point, (h) laser vibrometer, (i) electrical circuit, (j) signal generator and (k) signal analyzer.

Table 2.1. Geometric and material properties of piezo-patches and the host plate

Properties	Plate	Piezoelectric (PZT-5A)
Length (mm)	540	72.4
Width (mm)	580	72.4
Thickness (mm)	1.9	0.267
Young's modulus (GPa)	70	69
Mass density (kg/m ³)	2700	7800
Piezoelectric constant (C/m ²)	-	-190
Permittivity constant (nF/m)	-	9.57
Damping ratio	0.01	0.01

2.6 Results and discussions

In this section, Experimental validation, Modal analysis comparison with Ansys, and Electromechanical frequency response functions (FRF) results for connected and separated configurations will be presented.

2.6.1 Experimental validations

In order to validate the analytical model presented in the previous section, the FRFs of the system are obtained using the experimental set-up in Section 5. The velocity FRFs are compared for the separated and connected configurations in Figure 2.2. For both cases, optimum R (R_{opt}) is obtained by the analytical model by sweeping the resistor values (R) from short circuit ($R_{sc} = 100\Omega$) to open circuit ($R_{oc} = 1\text{ M}\Omega$), then picking the best R that achieves the maximum shunt damping performance. Once the R_{opt} is obtained from the model, it is also implemented in the experimental setup and then compared with the OC (open circuit) configuration. Figures 3a and 3b compare the results for connected and separated configurations, respectively. Please note that, although the R_{opt} values are obtained for the first mode, reductions in the second and third modes can be also observed. The velocity FRFs in the second column of Figure 2.4, demonstrate the zoomed section for the 2nd and 3rd modes to observe the reductions more clearly.

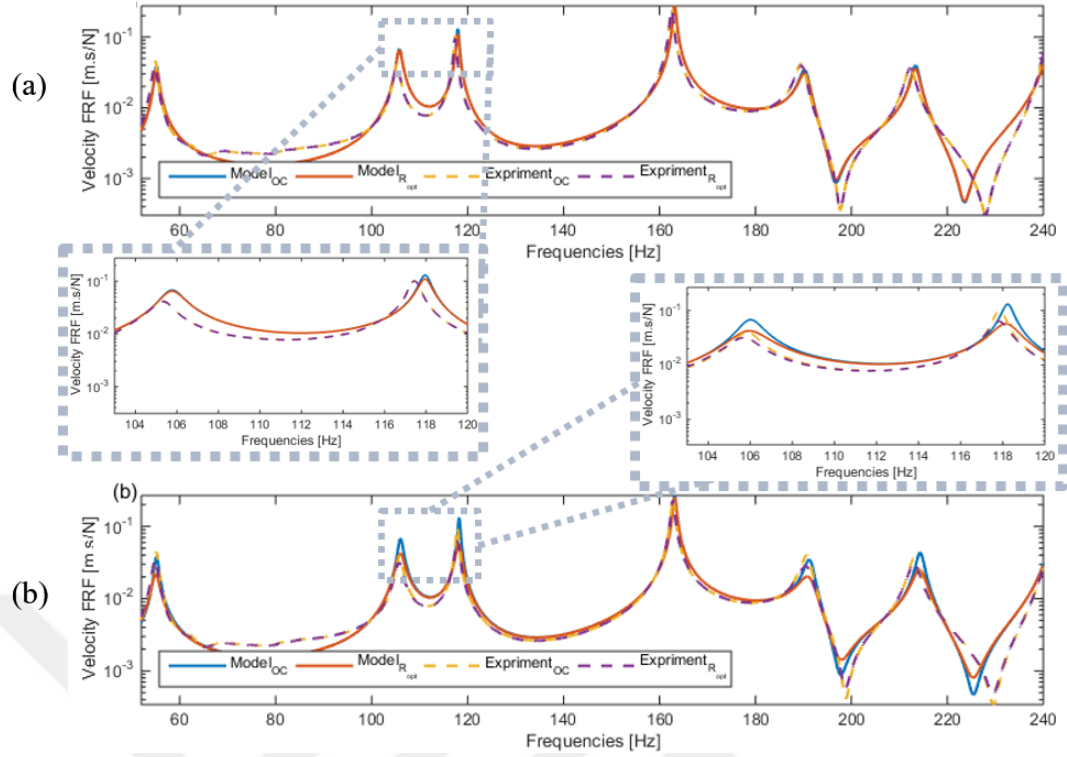


Figure 2.3. Comparison of the analytical model and experimental results (a) connected configuration, (b) separated configuration. The zoomed area shows the reductions in the 2nd and 3rd modes.

It can be observed from the figure that the analytical model accurately predicts the velocity FRFs near resonance and anti-resonance frequencies up to 240 Hz. The velocity percent reductions for each mode are listed in Table 2.2. As can be seen from the listed results, separated configuration shunt damping performance results are significantly better than the connected configuration.

Table 2.2. Velocity percent reduction comparison between the experimental and analytical model

Mode #	Connected patches		Separated patches	
	Model	Experimental	Model	Experimental
1	16.2	15.3	43.8	41.2
2	1.65	1.54	36.4	35.1
3	5.5	5.2	44.1	43.2

2.6.2 Comparison of the analytical and finite-element models

Validation of the analytical model via FEM for a wide range of piezoelectric patch size/thickness is presented in Figure 2.4 for a single patch. The area ratio is changed from $\frac{A_p}{A_s} = .005$ up to $\frac{A_p}{A_s} = .75$ (whereas A_p and A_s are the piezoelectric patch and the

host structure area, respectively). The thickness ratio is also changed from $\frac{h_p}{h_s} = .1$ up to $\frac{h_p}{h_s} = 1$ (whereas h_p and h_s are thickness of the piezoelectric patch and the host structure, respectively.). Figure 2.4 shows the comparison of the natural frequencies of the first 6 modes when size and thickness ratios are varied. The first and second column show the Ansys and analytical model results, respectively. The third column shows the percent difference between those results and it can be observed that for all the first 6 modes of the structure, the difference between the analytical and Ansys model remains within 1%. It shows that the analytical model accurately predicts the natural frequencies of the system when the patch size and thickness are varied.

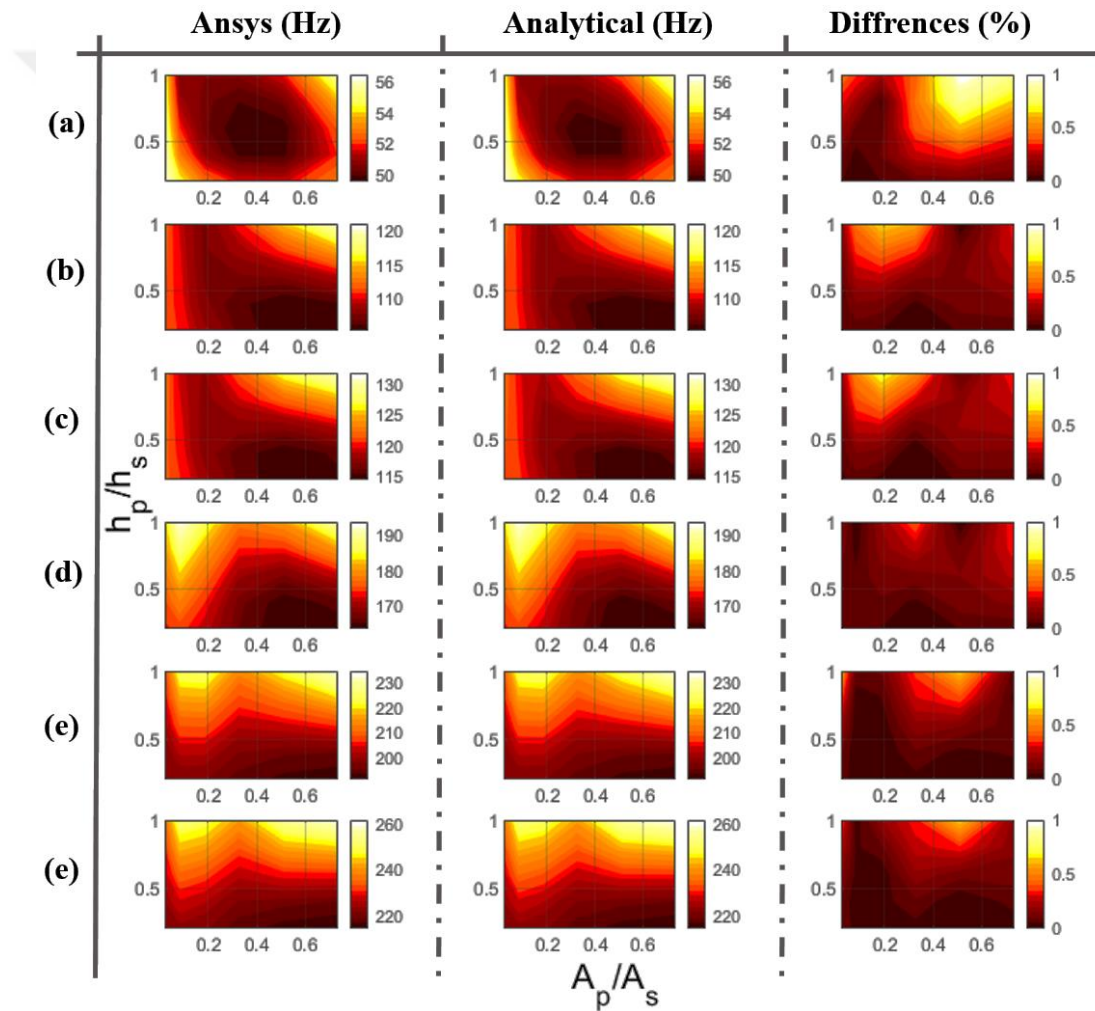


Figure 2.4. Comparison of the natural frequencies of the first 6 mode shapes. The first and second column shows the Ansys and analytical model results, respectively. The third column shows the percent difference between those results.

2.6.3 One patch configuration: changing thickness/area of the patch

This section includes investigation of the shunt damping performance via one patch which is located at the center of the plate. Figure 2.5 represents a demonstration of the cases where the piezoelectric patch size is varied. The thickness ratio of the patch to the plate $\frac{h_p}{h_s}$ varies from 0.1 to 0.9 and the area coverage ratio $\frac{A_p}{A_s}$ varies from 0.005 up to 0.75.

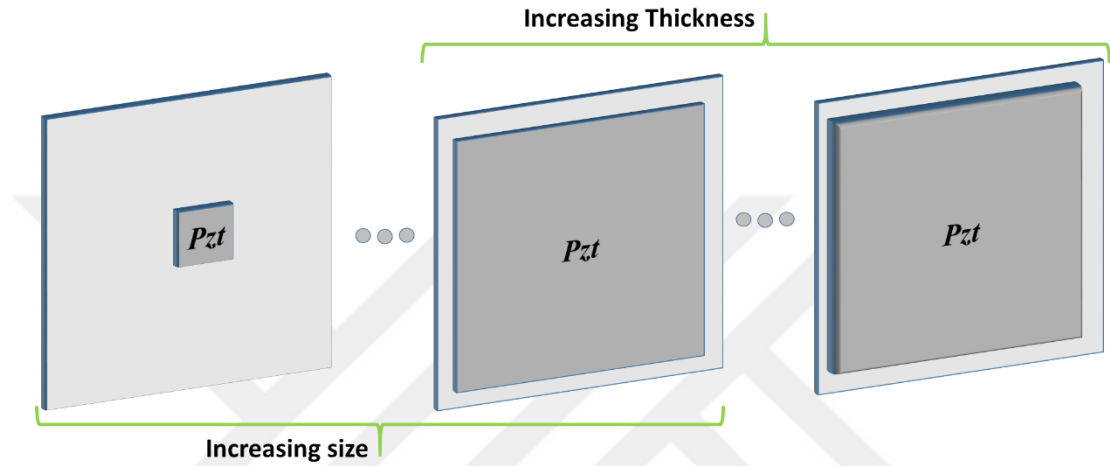


Figure 2.5. Demonstration of increasing size and thickness of one patch configuration

Figure 2.6. shows the reduction percentage of the $\overline{ERP}$ on the first 6 modes of the system when the $\frac{h_p}{h_s}$ and $\frac{A_p}{A_s}$ values are varied. At each step, the $\overline{ERP}$ is calculated while the R value in Eq. (2.32) is varied between short circuit ($R=100\Omega$) to open circuit ($R=1\text{ M}\Omega$). Then the optimum resistance R_{opt} value is obtained when the $\overline{ERP}$ is minimum. The same procedure is repeated at different thickness and area ratios and then the percent reduction is calculated using Eq. (1.32).

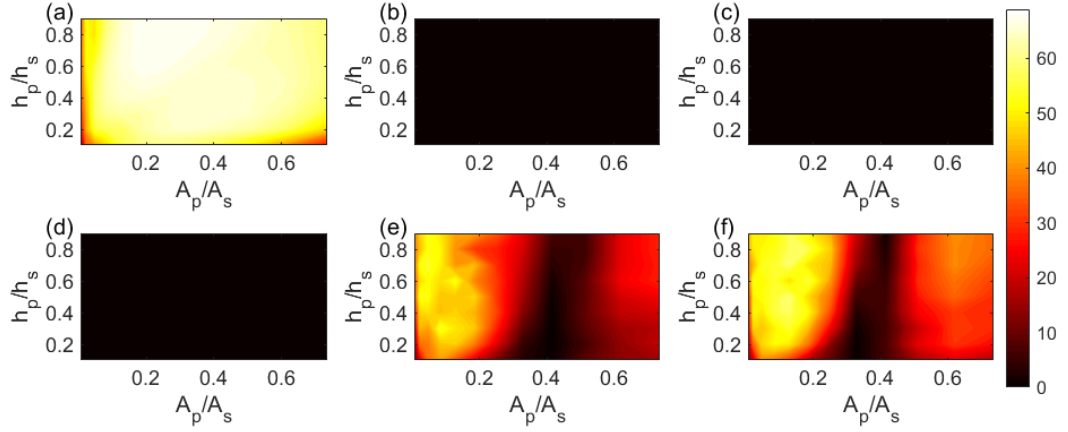


Figure 2.6. $\overline{\text{ERP}}$ Percent reduction on the first 6 modes by one patch with R shunted circuit, (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5 and (f) mode 6; R values are the same for each case.

Figure 2.6 shows that, due to the patch location and the strain mode shapes, $\overline{\text{ERP}}$ reduction in modes 2, 3 and 4 cannot be achieved. There is a black gap near $\frac{A_p}{A_s} = 0.36$ at modes 1, 5 and 6 which indicates that increasing the piezoelectric patch size has some negative-effect which could be linked to the cancelation of the piezoelectric effect due to the symmetry of the strain mode shapes. The maximum percent reduction is 69% and only achievable for the first mode.

2.6.4 Multiple patch configurations: changing thickness/number of the patches

This section investigates the $\overline{\text{ERP}}$ percent reduction for separated and connected configurations, Figure 2.7 demonstrates how the number and thickness of the patches are varied. The location of the patches is adjusted while increasing the number of the patches in a symmetric pattern with respect to the middle axis of the plate. The patches are chosen with sizes $40 \times 40 \text{ mm}^2$ and the number of patches are increased from 1 to 144. When 144 patches are positioned on the plate, the area ratio of total patches to the area of the plate will be equal to 0.75 the same as the biggest patch in the previous section. The thickness ratio of the patches is also varied from 0.1 to 0.9, such that the results are comparable with the previous section.

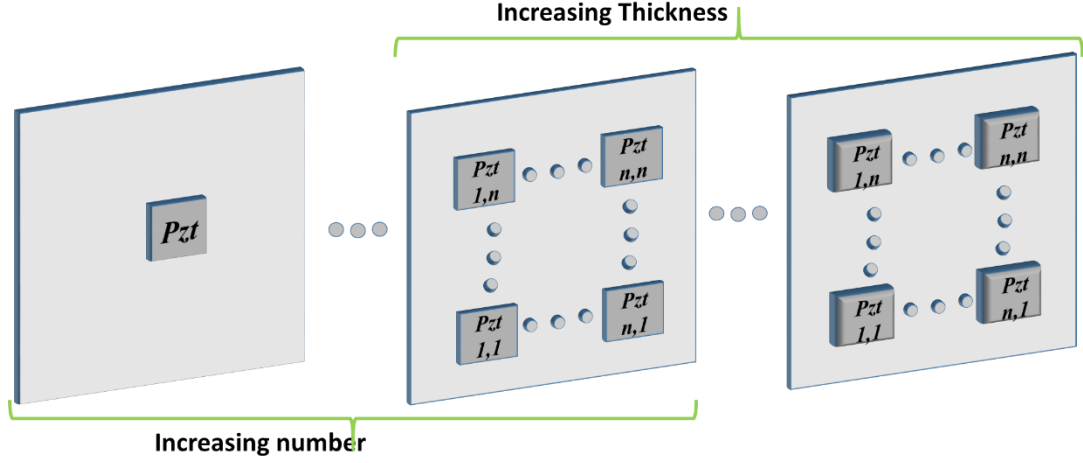


Figure 2.7. Demonstration of increasing number/ thickness of the multiple patches

The $\overline{\text{ERP}}$ percent reduction for the connected and separated configurations are presented in Figure 2.8. At each step, the $\overline{\text{ERP}}$ is calculated while the R value in Eq. (2.32) is varied between short circuit ($R=100\Omega$) to open circuit ($R=1 \text{ M}\Omega$). Then the optimum resistance R_{opt} value is obtained when the $\overline{\text{ERP}}$ is minimum. The same procedure is repeated for different patch thicknesses. Figure 2.8 shows that the shunt circuit in connected configuration is ineffective for modes 2, 3, and 4 as in the case of single patch configuration in the previous section. This result indicates that the connected patches work as one patch and making the Eq. (2.16) ineffective. When patch locations are symmetric relative to the middle axis of the plate, the percent reduction gets worse where the strain mode shape is also symmetric relative to the middle axis of the plate. It is observed that the maximum percent reduction achieved in the connected configuration is also similar to one patch configuration which is 69%. After adding 81 patches on the plate, a black gap occurs in modes 5 and 6, which can be explained due to the fact that the patches cover the symmetric strain region of the modes 5 and 6, and their effect is canceled and adding more patches do not help in reducing the vibration amplitudes. Figure 2.8-(ii) shows much better performance when the patches are separately shunted, compared to the connected configuration. The percent reduction can be up to 79 % for all of the modes. It can be also observed that 49 patches would be sufficient to achieve this performance without a need for further increasing the number of patches.

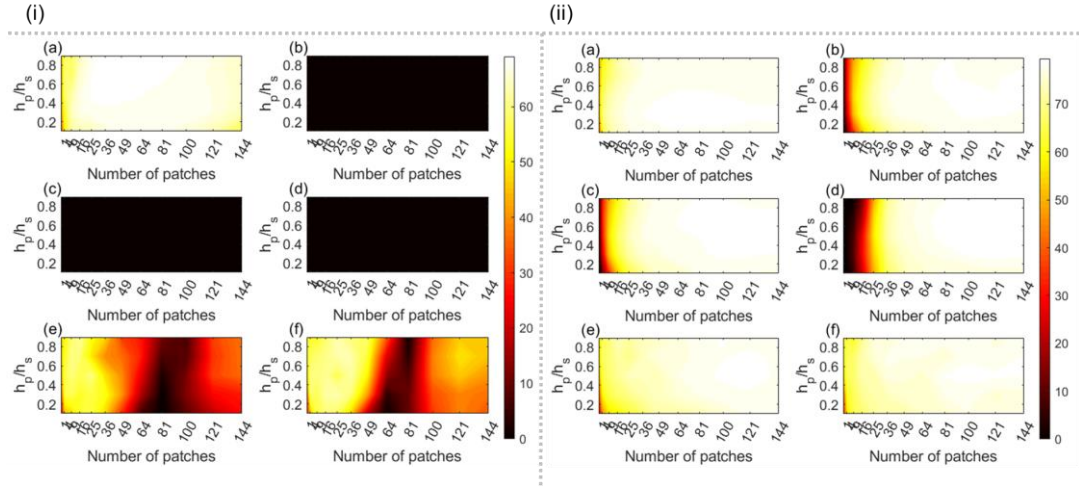


Figure 2.8. (i) percent reduction by multiple patches in connected configuration, (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5 and (f) mode 6, R values are the same for each case on all modes; (ii) percent reduction by multiple patches in separated configuration, (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5 and (f) mode 6; R values are the same for each case on all modes.

2.6.5 Multiple patches covering the same effective area: changing thickness and number of the patches

This section investigates the influence of the effective area of the patches and their sizes on shunt damping performance. Figure 2.9 demonstrates the cases used in this section. Effective area is varied from $\frac{A_p}{A_s} = 0.01$ to $\frac{A_p}{A_s} = 0.75$ while the numbers of patches are increased. Then connected and separated configurations are compared.

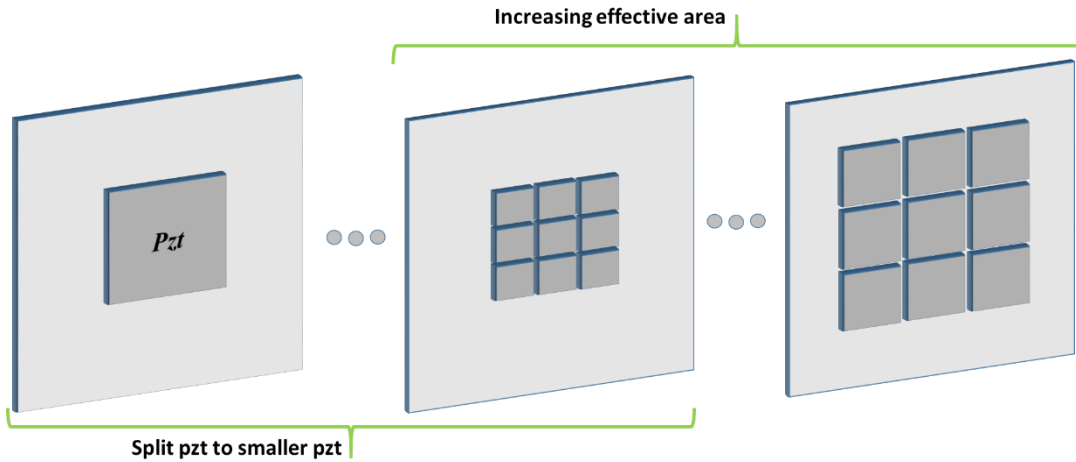


Figure 2.9. Demonstration of splitting effective area to multiple patches

An effective area is the part of the host structure which is covered by all of the patches. For simplicity, numbers of the patches are increased from 1 to 36, all symmetric relative to the center of the plate. At each step the $\overline{\text{ERP}}$ is calculated while the R value in

Eq. (2.32) is varied between short circuit ($R=100 \Omega$) to open circuit ($R=1 \text{ M}\Omega$). Then the optimum resistance R_{opt} value is obtained when the $\overline{\text{ERP}}$ is minimum. The same procedure is repeated at a different number of patches and effective area ratios. Figure 2.10 represents the percent reduction for the connected and separated configurations, respectively. Figure 2.10-(i) shows the percent reduction for connected configuration when the effective area and number of patches are changed. The ineffectiveness of the patches at modes 2, 3 and 4 can be observed when the effective area is near $\frac{A_p}{A_s} = 0.36$. The black gaps can be also observed on modes 5 and 6, which is again due to the symmetric nature of these modes and the cancelation effect. Shunt damping performance doesn't improve as the number of patches increase, because connected patches behave as a single patch when the effective area is kept constant. The maximum percent reduction is achieved only in the first mode which is 69 %. Figure 2.10-(ii) shows the percent reduction on the separated configuration. Unlike the connected configuration, as the number of patches increases the percent reduction gets improved, so for the case of 1 patch, the results are the same as Figure 2.6 and it gets improved as the number of patches increases. For case $\frac{A_p}{A_s} = 0.36$, when the patches are separately shunted, the black region in Figure 2.10-(i) reduces and it can be only seen in the region where the numbers of patches are low. The maximum percent reduction is 79% same as the separated configuration in the previous section.

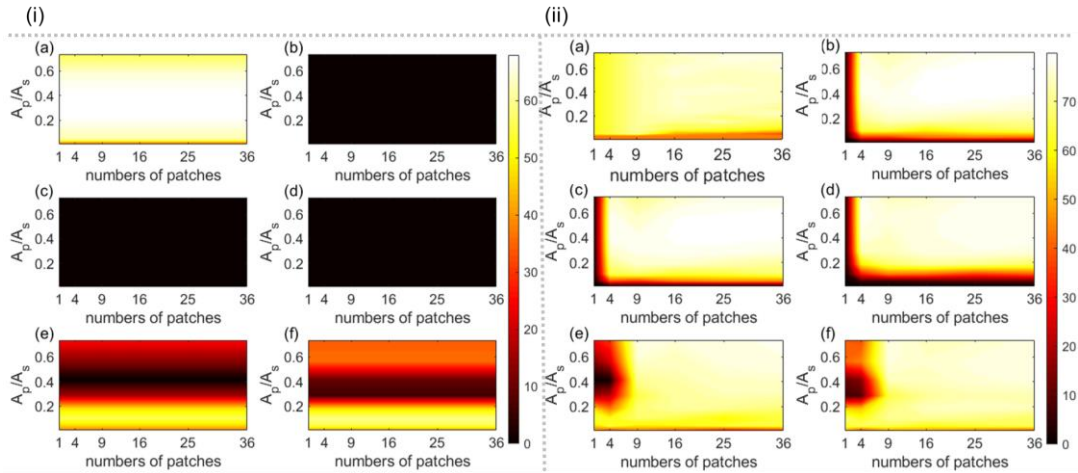


Figure 2.10. (i) Percent reduction by multiple patches in connected configuration with the same effective area, for the first 6 modes, (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5 and (f) mode 6; R values are the same for each case on all modes.; (ii) percent reduction by multiple patches in separated configuration with the same effective area, for the first 6 modes, (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4, (e) mode 5 and (f) mode 6; R values is the same for each case on all modes.

For a better comparison between cases of separated patches configurations, the $\overline{ERP}$ is investigated for the first 6 modes of the system and 1 patch and 36 patch results are compared in Figure 2.11. At each step, the $\overline{ERP}$ is calculated while the R value in Eq. (2.32) is varied between short circuit ($R=100\Omega$) to open circuit ($R=1\text{ M}\Omega$). Then the optimum resistance R_{opt} value is obtained when the $\overline{ERP}$ is minimum over the whole frequency range. The same procedure is repeated for 1 patch and 36 patches. As the numbers of patches are increased, the amplitude of the optimum case is reduced over the full range of frequencies.

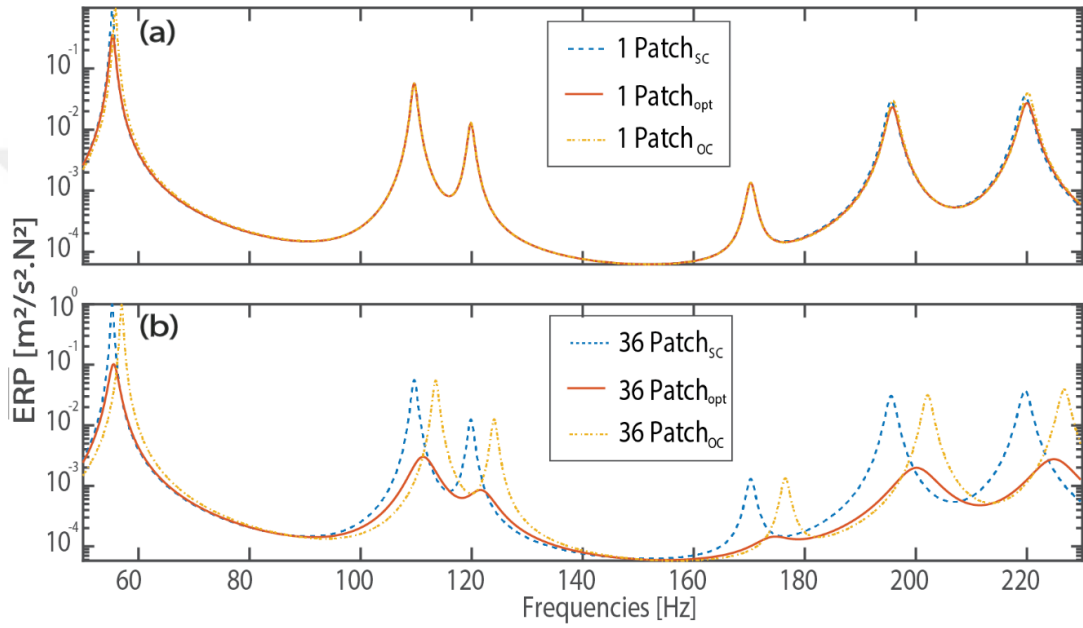


Figure 2.11. Comparison between cases of (a) 1 patch vs (b) 36 patches, $\overline{ERP}$, dashed blue lines are for the case of SC, solid red lines are for the optimum case and dashed orange ones are for the case of OC.

2.6.6 Optimization of R values for separated patches

Previously, the variation of the size, thickness and number of patches are investigated in both separated and connected configurations. There is further improvement, which can be achieved by optimizing the R values, where each patch has different values of R. This study will be conducted only on the separated configuration since further improvement is targeted by the optimization study. The plate dimensions remained the same as the previous cases but for a large patch (area ratio of $\frac{A_p}{A_s} = 0.75$). This ratio is chosen because this patch can cover almost 90% of the plate and the effective area can be divided into 4, 9, 16 and 25 small patches. Excitation force is located in the middle of the plate and the $\overline{ERP}$ is investigated. Initially, the optimization study is

conducted only for the first mode and the repeated for the first three modes of the structure. There are two major results which are presented in Figure 2.12; optimum values of R and optimized values of R. The optimum values are obtained using the same procedure as in the previous section where R is swept from SC to OC and same R values are used for all of the patches. The optimized value will be the case when the patches have different R values and each one of them is swept from SC to OC. In a case when there are m patches and n variation of R values, the overall cases are in the order of m^n cases where, the computation cost is increasing as the number of patches increases. It can be shown that as the number of patches increases, the amplitude can be reduced further with the optimized R values. For the first mode, the optimized R values are near to the optimum values but as the number of patches increases, the distribution of the R values follows the strain mode shape of the first mode. The amplitude reduction can be also improved by adding more patches to the system. Figure 2.12 includes the comparison between the cases of optimized and optimum values. Table 2.3 compares the percent reduction when the numbers of patches are increased for the optimum and optimized values of R. It is observed that when 25 patches are used, 3.3% improvement can be achieved when the R values are optimized for the first mode.

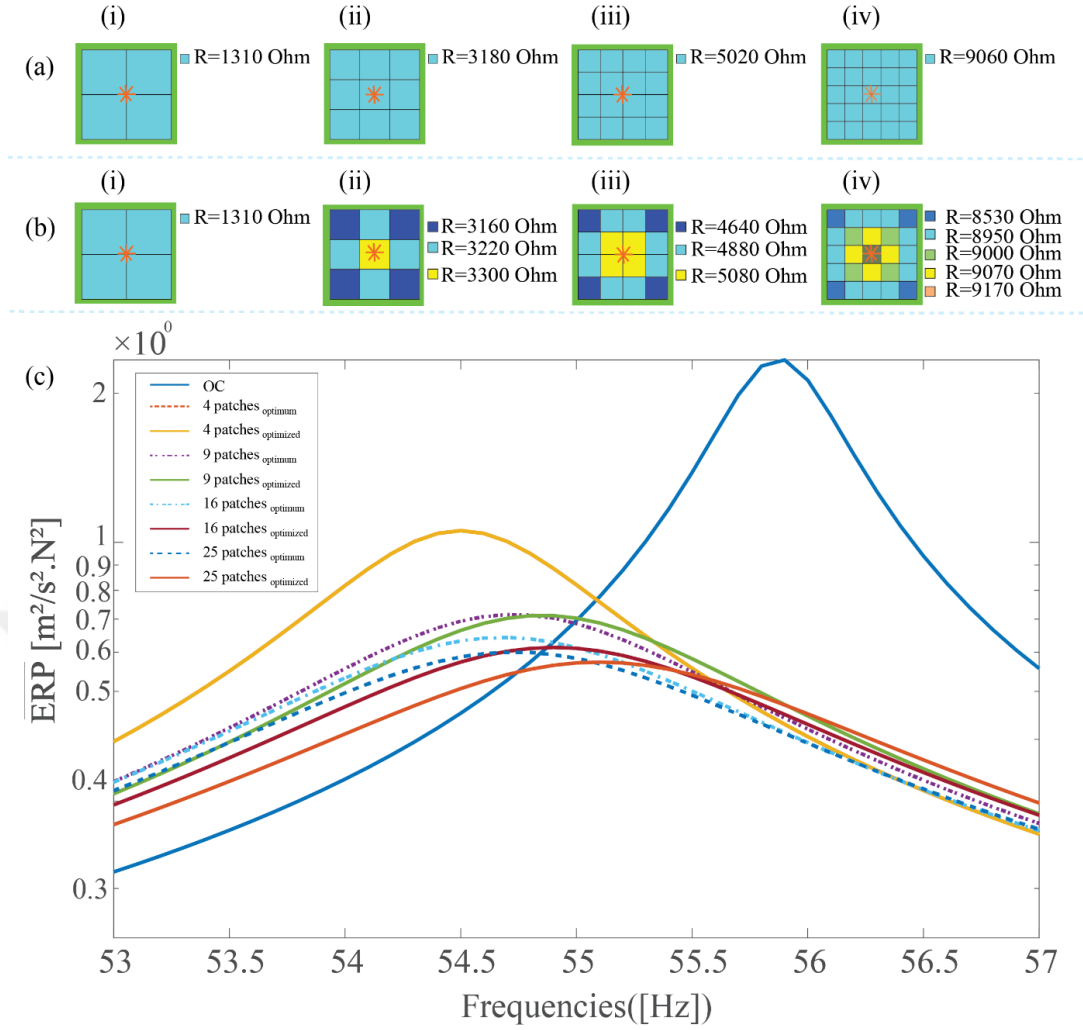


Figure 2.12. R values distribution for cases (a) Optimum R values: (i) 4 patches, (ii) 9 patches, (iii) 16 patches, (iv) 25 patches; (b) Optimized R values: (i) 4 patches, (ii) 9 patches, (iii) 16 patches, (iv) 25 patches; (c) $\overline{ERP}$ for part (a) and (b). * represent the force location.

Table 2.3. Reduction of $\overline{ERP}$ amplitude

Number of patches	Percent Reduction for the case of optimum values	Percent Reduction for the case of optimized values	Improvement (%)
4	64.79	64.79	0.00
9	68.99	69.55	0.57
16	72.08	73.74	1.66
25	72.17	75.47	3.30

Further investigation is done on modes 2 and 3 by optimizing the R values for each patch. Table 2.4 lists the percent improvements for the case of 25 patches for each mode. Figure 13 shows the similarities of the mode shapes and the R distribution pattern for the first three modes of the structure. There is also a significant improvement when the $\overline{ERP}$ reduction is also considered (see Table 2.4). It is observed that near the edges, R values gets the lowest values.

Table 2.4. Reduction of $\overline{ERP}$ amplitude 5*5 patches case.

Modes	Optimum case	Optimized case	Differences (%)
2	66.98	76.74	9.76
3	61.80	73.47	11.67

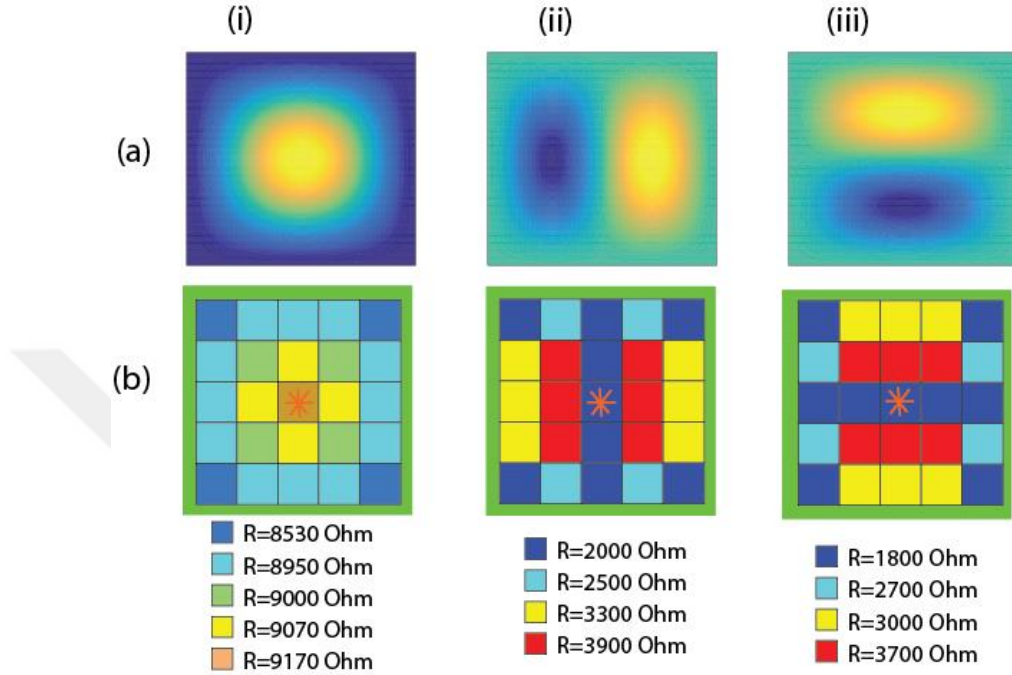


Figure 2.13. Comparison cases of (a) mode shapes for the case of $\frac{A_p}{A_s} = 0.75$: (i) first mode, (ii) second mode, (iii) third mode; (b) resistor values for optimized case: (i) first mode, (ii) second mode, (iii) third mode.

2.7 Conclusion

This chapter presents a methodology and a formulation for separately shunted piezoelectric patches for achieving higher performance on vibration attenuation. The Rayleigh-Ritz method was used for performing the modal analysis of the electro-mechanical system. The developed model includes mass and stiffness contribution of the piezoelectric patches as well as the electromechanical coupling effect. The vibration performance of the separately shunted piezoelectric patches were compared with the connected configurations. An experimental setup was also built to validate the performance of the separately shunted piezoelectric patches. Finite element simulations were performed in ANSYS and compared with the analytical model results for verification of the presented methodology. A thorough investigation was performed by changing the area, thickness and number of the patches to improve the shunt damping performance and demonstrate the effect of those parameters. It was shown that separately

shunted piezoelectric patches were more effective compared to connected patches and also more effective when broadband vibration attenuation was considered. Further improvements were also observed when the number of patches were increased while keeping the same effective area. Finally, an optimization study was performed to investigate the effect of the distribution of the resistor values for the first three modes of the structure. It was shown that the distribution of the optimized resistor values followed a pattern very similar to the strain mode shapes of the targeted modes and at the same time provided an improvement of $\sim 12\%$ in the $(\overline{ERP})$ metric. Future studies can be conducted by adding more electrical circuit elements (i.e. Inductance) to the system and their design parameters can be optimized using a similar procedure. However, due to the number of variables, the case studies would increase significantly and make the computational cost very high. For that reason, various optimization methods and modeling approaches can be explored to reduce the computational cost in future studies.

Chapter 3

3 A general electromechanical model for plates with integrated piezo-patches using spectral-Tchebychev method

3.1 Introduction

Piezoelectric structures have been widely used in a range of applications including vibration control (Bailey and Hubbard 1985; Moheimani and Fleming 2006), energy harvesting (Alper Erturk and Inman 2011a; Elvin and Erturk 2013; Aghakhani and Basdogan 2017), structural health monitoring (Giurgiutiu 2007; Zou, Tong, and Steven 2000; Tinoco et al. 2016), and also surface haptics (Poupyrev, Okabe, and Maruyama 2004; Emgin et al. 2019) over the past few decades. Among the transducers that convert the mechanical (electrical) energy to electrical (mechanical) energy, piezoelectric transducers are mostly preferred to the electromagnetic (Yang et al. 2009; Beeby et al. 2007) and electrostatic (Torres and Rincon-Mora 2009; Beeby, Tudor, and White 2006) ones due to their high power density and ease of manufacturing at different size scales (Powering, n.d.; “Estimation of Electric Charge Output for Piezoelectric Energy Harvesting - Sodano - 2004 - Strain - Wiley Online Library” n.d.). The most common use of piezoelectric materials in the form of patches/layers is by integrating them to the surfaces of flexible beam/plate-like structures. Consequently, the bending motion could be exploited for generating electrical signal and vice versa, applying an electrical field to generate a bending deformation. There are established analytical and numerical methods for modeling the piezoelectric patches/layers bonded on thin beam- and plate-like structures.

Much of the theoretical studies on dynamics of two-dimensional structures with surface bonded piezo-patches have considered the host plate modal solution and only accounted the induced moments at the edges of the patches for prediction of the system electromechanical response. However, such modeling approaches cannot accurately estimate the dynamics of thin and thick plates with relatively large piezoelectric patch sizes. To this end, this paper presents a general electromechanical modeling of plates with structurally integrated piezo-patches using a new series based on the spectral numerical approach which employs Tchebychev polynomials to discretize the boundary value problem (BVP). Due to the exponential convergence and superior stability

characteristics (Gottlieb, Orszag, and Foundation (U.S.) 1977), the developed solution technique enables predicting the dynamics of mechanical structures accurately and efficiently. Another important advantage of the presented spectral Tchebychev approach is that the differentiation and integral operations are performed exactly through the derived differentiation and inner-product matrices which increases the computational efficiency of the modeling process. In this approach, the system matrices for the host plate and the piezo-patch(es) are first derived using Hamilton's principle. Then, a novel coupling algorithm is developed to include the two-way electromechanical coupling effect between the host plate and piezo-patches and also to obtain the mass and stiffness matrices of the overall coupled system. The compatibility equations for the electromechanical coupling and the boundary condition equations are incorporated into the boundary value problem through projection matrices approach. This projection matrices approach also eliminates the need to use different basis/trial functions for each problem with different boundary conditions (Bediz 2018; Yagci et al. 2009). Therefore, the major contributions of this study are **(i)** the integration of the electromechanical equations to the classical Spectral-Tchebychev method for the first time, and **(ii)** the versatile coupling algorithm that enables accurate capturing of the dynamics of plate-piezo configurations. Consequently, following the previously developed ST approaches by the authors, the proposed ST framework in this study is able to predict not only the mechanical behavior but also the electrical response of the two-dimensional electromechanical structures accurately and efficiently. To validate the accuracy and demonstrate the computational efficiency of the presented technique, several case studies including one-sided and two-sided piezo-patches are presented. The modal analysis results as well as the displacement and voltage frequency response functions are compared to those obtained by a commercial finite-element (FE) software and the existing Rayleigh-Ritz method from the literature.

3.2 Modal analysis of a thin/thick plate with piezoelectric patches

The schematic of a rectangular plate with an integrated with one-sided and two-sided piezo patches are depicted in Figure 3.1. Here, h represents the thickness of the structures, a and b show the width and length of the structures, respectively. Here and hereafter, the subscripts 's' and 'p' are used to represent the host plate and the piezo patch, respectively. Since, plate modeling approaches/assumptions are followed, the

main difference in Figure 3.1 (a) and (b) is due to the shifting of the neutral surface in the case of one-sided surface-bonded piezo-patch towards the piezoelectric patch.

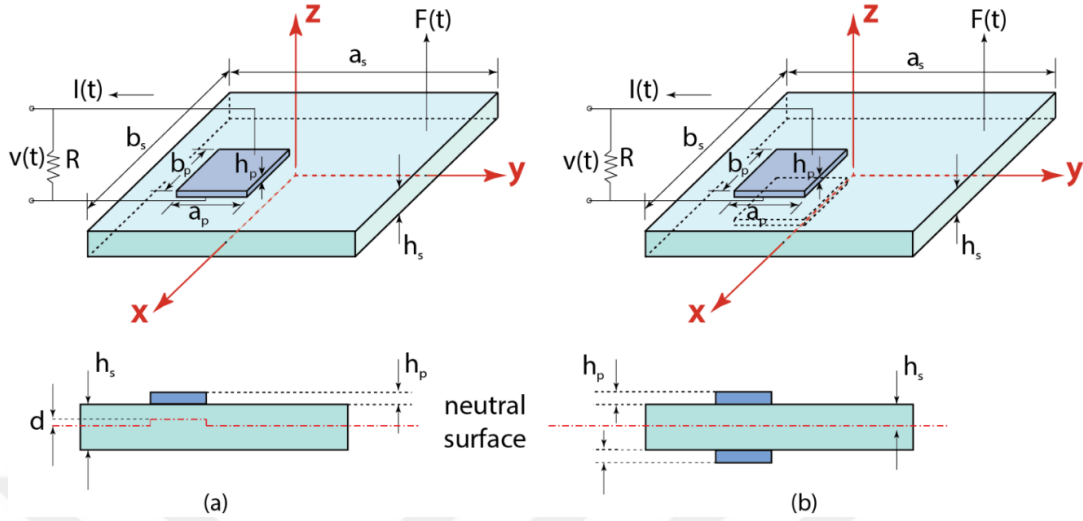


Figure 3.1. Geometry of a rectangular (host) plate with (a) one-sided and (b) two-sided piezo patches configurations. Bottom figures show the front views, and the dashed red lines represent the neutral surface corresponding to each different configuration.

3.2.1 Determining the location of the neutral surface

The neutral surface of a plate with isotropic material properties is basically the mid-plane of the structure. For the case of two-sided surface-bonded piezo-patches, the overall system is symmetric; thus the neutral surface remains as the mid-plane of the host plate (*i.e.*, $\delta = 0$). However, in the case of one-sided piezo patch configuration, the symmetricity of the structure is disturbed. Therefore, the neutral plane shifts to the piezo-patch side for the in-plane region where the patch is attached. The location of the neutral surface can be determined using stress-equilibrium (Yoon, Youn, and Kim 2016b)

$$\delta = \frac{\bar{c}_{11} h_p (h_s + h_p)}{2(Y_s h_s / (1 - \nu_s^2) + \bar{c}_{11} h_p)} \quad (3.1)$$

where δ represents the location of the neutral surface with respect to the mid-plane of the host plate. Here, Y_s and ν_s are the Young's Modulus and Poisson's ratio of the host plate, respectively; and $\bar{c}_{11}$ is the reduced elastic modulus of the piezoelectric patch. Note that Eq. (2.1) will lead to discontinuities at the edges of the piezo-patch. To achieve a continuous transition of the neutral surface, we used piecewise cubic interpolation.

3.2.2 Spectral-Tchebychev approach

In this study, spectral-Tchebychev technique is applied for the first time for predicting the electroelastic behavior of structures. Therefore, before proceeding with the derivation of the boundary value problem, the basic properties of the spectral-Tchebychev approach will be briefly summarized in this section. However, the reader is referred to (Yagci et al. 2009; Filiz et al. 2012; 2014) for the details of the derivations.

Tchebychev polynomials are recursive and orthogonal polynomials, that exhibit exponential convergence behavior (Gottlieb, Orszag, and Foundation (U.S.) 1977; Yagci et al. 2009). A function can be expressed using a finite summation of Tchebychev polynomials as

$$y(x) = \sum_{k=0}^N a_k T_k(x) = \sum_{k=0}^N a_k \cos(k \cos^{-1}(x)) \quad (3.2)$$

where a_k 's are the coefficients of expansion and T_k 's are the Tchebychev polynomials. For numerical calculations, the functions need to be sampled at certain spatial increments. There exists a one-to-one mapping between the expansion coefficients and the sampling points. Therefore the coefficients of the expansion can be related to the sampled function's values as $f = \Gamma_B a$ or $a = \Gamma_F f$ where f and a are the vectors containing the sampled function's values and the coefficients of expansion, respectively; Γ_B and Γ_F are the backward and forward transformation matrices, respectively (Yagci et al. 2009).

In modeling two-dimensional problems such as membranes, plates, and shells, the functions need to be expressed using a double Tchebychev expansion (Bediz 2018). Furthermore, the sampling will lead to a second rank tensor whose elements are

$$f_{kl} = f(x(k), y(l)) , \quad (3.3)$$

where, $k = 1, \dots, N_x$ and $l = 1, \dots, N_y$ are the indices of the tensor; N_x and N_y are the number of polynomials used along the x and y directions. To transform the second rank tensor to a vector, a mapping algorithm is used as follows

$$f_c = f_{kl} , \quad c = (k - 1) N_y + l . \quad (3.4)$$

Next, it is required to calculate the spatial derivatives of the deflection vector and perform integral operations to calculate the mass and stiffness matrices of the system. Similar to Eq. (2.2), the derivation of a function can be expressed as Tchebychev

polynomials expansion where the coefficients of expansion is b . It is possible to write a relation between the coefficients of the original function and the derivative of the original function such as $b = Da$ through a derivative matrix (D). Therefore, using the forward and backward matrices, and the derivative matrix, it is possible to obtain the n^{th} spatial derivative $(\partial^{(n)}f/\partial q_i^{(n)})$ of a function as $f^{(n)} = Q_{q_i}^n f$, where $Q_{q_i}^n$ is the n^{th} spatial derivative matrix (Bediz 2018; Yagci et al. 2009; Filiz et al. 2012). Here, the subscript q_i denotes the spatial variable; for instance for a two dimensional plate problem $q_i = x, y$.

Similarly, to perform the area integral operations, the inner product matrix approach is followed. According to this approach, the inner product of two functions f and g can be expressed as

$$\int_A f(x, y) g(x, y) dx dy = f^T \mathbb{V} g, \quad (3.5)$$

where $\mathbb{V}$ is the inner product matrix. However, if the integral operation includes a weighting function, $r(x, y)$ as in the case of one-sided piezo-patch configuration where the neutral plane is changing within the defined domain of the host plate, the weighted inner product matrix, $\mathbb{V}^r$, need to be calculated following the derivations in (Bediz and Aksoy 2018).

Note that the method can be applied with any type of sampling algorithm. However, the calculation of the backward, forward, derivative and inner product matrices becomes simpler if the domain of the problem is sampled using Gauss-Lobatto sampling scheme (Pasquetti and Rapetti 2004).

3.2.3 Derivation of the boundary value problem

The boundary value problem (BVP) of a plate with a surface bonded piezoelectric energy harvester can be derived using the Hamilton's equation, using the terms of kinetic energy (E_k), potential (strain) energy (E_p), electrical energy (W_{ie}), and work done by non-conservative forces (W_{nc}). For a Mindlin plate, the deformations u , v , and w along x , y , and z directions, respectively, can be expressed as

$$u = z\phi_y(x, y) \quad (3.6)$$

$$v = z\phi_x(x, y) \quad (3.7)$$

$$w = w(x, y) \quad (3.8)$$

where $\phi_x(x, y)$ and $\phi_y(x, y)$ denote the rotations of the plate around x and y axes, respectively; z denotes the distance of a point from the neutral-plane of the plate.

The kinetic energy of the assembled system can be written as a summation of the kinetic energies of the host plate, E_{k_s} , and the piezo-patch, E_{k_p} , as follows

$$\begin{aligned} E_k &= E_{k_s} + E_{k_p} \\ &= \frac{1}{2} \int_{V_s} \rho_s [\dot{u}_s^T \dot{u}_s + \dot{v}_s^T \dot{v}_s + \dot{w}_s^T \dot{w}_s] dV_s + \frac{1}{2} \int_{V_p} \rho_p [\dot{u}_p^T \dot{u}_p \\ &\quad + \dot{v}_p^T \dot{v}_p + \dot{w}_p^T \dot{w}_p] dV_p . \end{aligned} \quad (3.9)$$

Here, ρ is the density, and V is the volume of the corresponding structure. Inserting Eqs. (2.6)-(2.8) into Eq. (2.9), the kinetic energy for the host plate can be expressed as

$$\begin{aligned} E_{k_s} &= \frac{1}{2} \int_{A_s} \rho_s \int_{z_l}^{z_u} [z^2 (\frac{\partial \phi_y}{\partial t})_s^2 + z^2 (\frac{\partial \phi_x}{\partial t})_s^2 + (\frac{\partial w}{\partial t})_s^2] dz dA_s \\ &= \frac{1}{2} \int_{A_s} \rho_s \dot{q}_s^T \Lambda_{q_s} \dot{q}_s dA_s \end{aligned} \quad (3.10)$$

Here, $q_s = \{\Phi_{y_s}; \Phi_{x_s}; w_s\}$ represents the deflection vector, z_l and z_u are the limits of integral operation along the z direction, and Λ_{q_s} is the weighting function. For instance, for a two-sided configuration, the limits of integral operation become $z_l = -h_s/2$ and $z_u = h_s/2$. Thus, the kinetic energy term can be expressed using $\Lambda_{q_s} = \text{diag}[\Lambda_{\phi_{y_s}}; \Lambda_{\phi_{x_s}}; \Lambda_{w_s}]$ where $\Lambda_{\phi_{y_s}}$, $\Lambda_{\phi_{x_s}}$, and Λ_{w_s} matrices are diagonal matrices whose

elements are $h^3/12$, $h^3/12$, and h , respectively. However, in the case of one-sided piezo-patch configuration, the limits of the integral operation performed along the z axis will be $z_l = -(h_s/2 + \delta)$ and $z_u = (h_s/2 - \delta)$ over the domain that the piezo-patch is attached. Here, δ represent the shift amount in the neutral plane that is calculated using Eq. (2.1).

Similarly, the kinetic energy of the piezo-patch(es) can be found as

$$\begin{aligned} E_{k_p} &= \frac{1}{2} \int_{A_p} \rho_p \int_{z_l}^{z_u} [z^2 \left(\frac{\partial \phi_y}{\partial t}\right)_p^2 + z^2 \left(\frac{\partial \phi_x}{\partial t}\right)_p^2 + \left(\frac{\partial w}{\partial t}\right)_p^2] dz dA_p \\ &= \frac{1}{2} \int_{A_p} \rho_p \dot{q}_p^T \Lambda_{q_p} \dot{q}_p dA_p \end{aligned} \quad (3.11)$$

For one-sided piezo-patch configuration, these limits are $z_l = (h_s/2 - \delta)$ and $z_u = h_p + (h_s/2 - \delta)$. On the other hand, for the two-sided (symmetric) piezo-patch configuration, the kinetic energy needs to be calculated for two piezo-patch elements. Since, the kinetic energies of these two piezo-patches are identical to each other, the kinetic energy of one of the elements can be found and multiplied by two. In this case, to calculate the kinetic energy of one of the elements, the limits of the integral operation along the z direction can be taken as $z_l = h_s/2$ and $z_u = h_s/2 + h_p$.

Next, the strain energy of the system can be obtained as a summation of the strain energies of the host plate and the piezo-patch(es). Accordingly, the strain energy of the host plate can be obtained from the elasticity equations as

$$E_{p_s} = \frac{1}{2} \int_{V_s} S_s^T T_s dV_s. \quad (3.12)$$

Here, S is the strain vector and T is the stress vector. The stresses and the strains are related to each other through the constitutive matrix, $T = C S$ and the strains and the displacements are related to each other using the differential operator matrix, $S = B q$. Note that the differential operator matrix includes the spatial variable, z ; thus it is possible to rewrite B as

$$B = \underbrace{\begin{bmatrix} z & 0 & 0 & 0 & 0 \\ 0 & z & 0 & 0 & 0 \\ 0 & 0 & z & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{Z_I} \underbrace{\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & 1 & \frac{\partial}{\partial y} \\ 1 & 0 & \frac{\partial}{\partial x} \end{bmatrix}}_{B'} . \quad (3.13)$$

Inserting the constitutive and differential operator matrices into Eq. (2.12), and performing the integral operation along the thickness direction similar to the kinetic energy part, the potential energy for the host plate can be derived as

$$\begin{aligned} E_{p_s} &= \frac{1}{2} \int_{A_s} q_s^T B_{s'}^T \left\{ \int_{z_l}^{z_u} Z_I^T C_s Z_I dz \right\} B_{s'} q_s dA_s \\ &= \frac{1}{2} \int_{A_s} q_s^T B_{s'}^T C_{h_s} B_{s'} q_s dA_s , \end{aligned} \quad (3.14)$$

where

$$C_{h_s} = \begin{bmatrix} \frac{h^3}{12} \lambda & \frac{h^3}{12} \nu \lambda & 0 & 0 & 0 \\ \frac{h^3}{12} \nu \lambda & \frac{h^3}{12} \lambda & 0 & 0 & 0 \\ 0 & 0 & \frac{h^3}{12} \mu & 0 & 0 \\ 0 & 0 & 0 & h k^2 \mu & 0 \\ 0 & 0 & 0 & 0 & h k^2 \mu \end{bmatrix} . \quad (3.15)$$

for the case in which the mid-plane of the plate is the neutral plane. For the one-sided piezo attachment case, the elements of the C_{h_s} matrix can be obtained using the integration limits as described in the formulation of kinetic energies part. However, note that since the neutral plane is shifted only at the location of the piezo-patch, the value of C_h matrix is not constant throughout the domain of the problem. Therefore, it should be treated as a weighting function and the integral operation can be performed by calculating the weighted inner product matrix, $\mathbb{W}^{C_h}$.

The strain energy of the piezoelectric patch can be derived using the electroelastic constitutive relations as described in here:

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_4 \\ T_5 \\ T_6 \\ D_3 \end{Bmatrix} = \begin{bmatrix} \bar{c}_{11}^E & \bar{c}_{12}^E & 0 & 0 & 0 & -\bar{e}_{31} \\ \bar{c}_{12}^E & \bar{c}_{11}^E & 0 & 0 & 0 & -\bar{e}_{31} \\ 0 & 0 & \bar{c}_{55}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{c}_{55}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{c}_{66}^E & 0 \\ \bar{e}_{31} & \bar{e}_{31} & 0 & 0 & 0 & \bar{\epsilon}_{33}^S \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_4 \\ S_5 \\ S_6 \\ E_3 \end{Bmatrix}. \quad (3.16)$$

where c_{ij} 's are the reduced elastic moduli, $\bar{e}_{ij}$ is the piezoelectric constant (where $e = \{\bar{e}_{31}; \bar{e}_{31}; 0; 0; 0\}$ is the vector of piezoelectric constants), $\bar{\epsilon}_{33}^S$ is the dielectric permittivity, D_3 represents the electric displacements, and E_3 is the electric field vector whose elements' magnitude can be defined in terms of the electric potential difference as $E_3 = -v(t)/h_p$.

Thus, potential energy for the piezoelectric patch(es) can be derived as

$$E_{p_p} = \underbrace{\frac{1}{2} \int_{V_p} S_p^T T_p dV_p}_{E_{p_{p1}}} - \underbrace{\frac{1}{2} \int_{V_p} S_p^T e^T E dV_p}_{E_{p_{p2}}}. \quad (3.17)$$

The first term of the above integral is similar to the host plate's potential energy; thereby can be written similarly as in Eq. (2.14). The second term of Eq. (2.17) includes the coupling term between the structural and the electric part of the problem. Using the electroelastic relations, and performing the integral operation along the thickness direction, the coupling term becomes

$$E_{p_{p2}} = \mathbb{C} \int_{A_p} \left[\frac{\partial}{\partial x} \phi_{y_p} \bar{e}_{31} v(t) + \frac{\partial}{\partial y} \phi_{x_p} \bar{e}_{31} v(t) \right] dA_p \quad (3.18)$$

where $\mathbb{C}$ equals to $\frac{h_s+h_p-2\delta}{4}$ or $\frac{h_s+h_p}{2}$ for one-sided or two-sided configuration, respectively.

The electrical energy term in the Hamilton's equation becomes

$$W_{ie} = \frac{1}{2} \int_{A_p} E^T D dA_p = \frac{1}{2} \left[\int_{A_p} E^T e S dA_p + \int_{A_p} E^T \bar{\epsilon}_{33}^S E dA_p \right]. \quad (3.19)$$

Here, the first term is the same as the $E_{p_{p2}}$ term; and the second term represents capacitance of the piezo that can be simplified to $C_P = \bar{\epsilon}_{33}^S (a_p b_p) / h_p$ (A Erturk and Inman 2009).

Finally, the work done by non-conservative (external) forces can be expressed as

$$W_{nc} = \int_A f^T q dA \quad (3.20)$$

where $f = \{f_{\phi_y}; f_{\phi_x}; f_w\}$ represents the forces acting in the domain.

Therefore, using the terms representing kinetic energy, potential energy, electrical energy, and the work done by non-conservative forces into the Hamilton's equation, the system matrices for the host plate and the piezo-patch(es) for the two-sided (symmetric) configuration can be obtained as

$$M_s = \frac{\rho_s h_s^3}{12} \mathbb{I}_{\phi_{ys}}^T \mathbb{V}_s \mathbb{I}_{\phi_{ys}} + \frac{\rho_s h_s^3}{12} \mathbb{I}_{\phi_{xs}}^T \mathbb{V}_s \mathbb{I}_{\phi_{xs}} + \rho_s h_s \mathbb{I}_{w_s}^T \mathbb{V}_s \mathbb{I}_{w_s}, \quad (3.21)$$

$$M_p = 2 \rho_p \left\{ \left[\frac{h_p^3}{3} + \frac{h_s^2 h_p}{4} + \frac{h_s h_p}{2} \right] (\mathbb{I}_{\phi_{yp}}^T \mathbb{V}_p \mathbb{I}_{\phi_{yp}} + \mathbb{I}_{\phi_{xp}}^T \mathbb{V}_p \mathbb{I}_{\phi_{xp}}) + h_p \mathbb{I}_{w_p}^T \mathbb{V}_p \mathbb{I}_{w_p} \right\}, \quad (3.22)$$

$$K_s = B_s^T \mathbb{V}_s C_{h_s} B_s, \quad (3.23)$$

$$K_p = 2 \{ B_p^T \mathbb{V}_p C_{h_p} B_p \} \quad (3.24)$$

$$\boldsymbol{\theta} = (h_s + h_p) (\boldsymbol{\varepsilon}_{xxp}^T \mathbb{V}_p \bar{e}_{31} + \boldsymbol{\varepsilon}_{yy_p}^T \mathbb{V}_p \bar{e}_{31}). \quad (3.25)$$

Here $\boldsymbol{\theta}$ is the electro-mechanical coupling term. Similarly, the system matrices for the one-sided configuration are

$$M_s = \rho_s \mathbb{I}_{\phi_{ys}}^T \mathbb{V}_s^{\Lambda_{qs}} \mathbb{I}_{\phi_{ys}} + \rho_s \mathbb{I}_{\phi_{xs}}^T \mathbb{V}_s^{\Lambda_{qs}} \mathbb{I}_{\phi_{xs}} + \rho_s \mathbb{I}_{w_s}^T \mathbb{V}_s^{\Lambda_{qs}} \mathbb{I}_{w_s}, \quad (3.26)$$

$$M_p = \rho_p \left\{ \frac{1}{3} \left[\left(\frac{h_s}{2} - \delta + h_p \right)^3 - \left(\frac{h_s}{2} - \delta \right)^3 \right] (\mathbb{I}_{\phi_{yp}}^T \mathbb{V}_p \mathbb{I}_{\phi_{yp}} + \mathbb{I}_{\phi_{xp}}^T \mathbb{V}_p \mathbb{I}_{\phi_{xp}}) + h_p \mathbb{I}_{w_p}^T \mathbb{V}_p \mathbb{I}_{w_p} \right\}, \quad (3.27)$$

$$K_s = B_s^T \mathbb{V}_s^{C_{h_s}} B_s, \quad (3.28)$$

$$K_p = B_p^T \mathbb{V}_p C_{h_p} B_p \quad (3.29)$$

$$\boldsymbol{\theta} = \frac{h_s + h_p - 2\delta}{2} (\boldsymbol{\varepsilon}_{xxp}^T \mathbb{V}_p \bar{e}_{31} + \boldsymbol{\varepsilon}_{yy_p}^T \mathbb{V}_p \bar{e}_{31}). \quad (3.30)$$

Note that mechanical damping matrix is not included here. However it can introduced to the system in the form of proportional (Rayleigh) or structural damping while predicting the structural and voltage frequency response functions (FRFs).

3.2.4 Structurally coupling the dynamics of host plate and the piezo-patch(es) and imposing the boundary conditions

To obtain the complete model for the system, a component mode synthesis technique is used to combine the dynamics of the host plate and the piezo patch(es). To address this, compatibility equations are written at the connection boundaries at which the degrees of freedom of the coupled structures should be equal to each other. However, note that for numerical efficiency, Gauss-Lobatto sampling is used to discretize the problem domain. Therefore the sampling points for the piezo patches and the host plate may not be matching as shown in Figure 3.2.

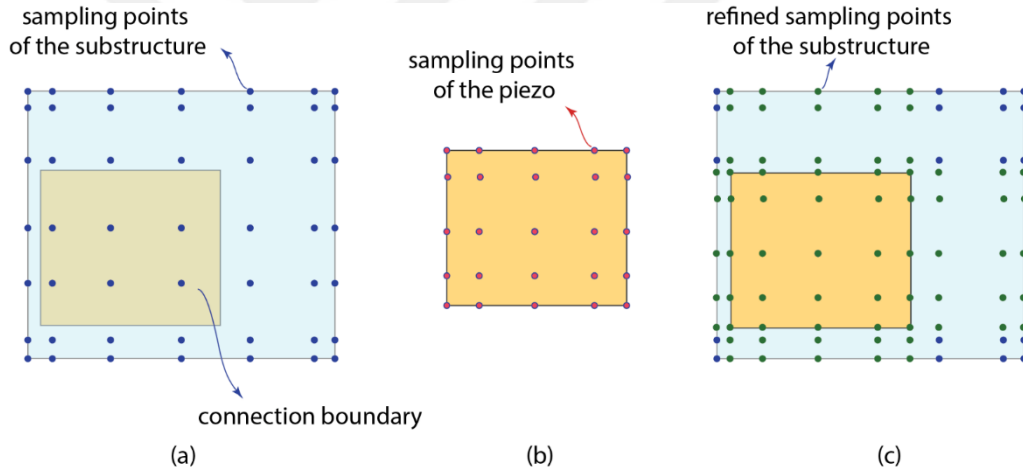


Figure 3.2. Component mode synthesis approach for host plate and piezo-patch: (a) Gauss-Lobatto sampling points of host plate, (b) Gauss-Lobatto sampling points of piezo patches, and (c) refined sampling points of the host plate (dots show the sampling points).

First, the sampling of the host plate is modified/refined to include the sampling nodes of the piezo patch(es). Then the refined deflections are related to the original deflection variables using the forward and backward transformation matrices of the host plate

$$q_s^* = \Gamma_{B_s}^* a_s = \Gamma_{B_s}^* \Gamma_{F_s} q_s. \quad (3.31)$$

Here, Γ_{F_s} represents $(N_x N_y)_s \times (N_x N_y)_s$ global forward transformation matrix for the host plate, and $\Gamma_{B_s}^*$ represent $(N_x N_y)_s^* \times (N_x N_y)_s$ refined global backward

transformation matrix whose elements are evaluated at the refined points. Thus, using the refined sampling points, compatibility equations can be expressed simply by equating the degrees of freedom at the connection boundaries of the coupled sections:

$$\boldsymbol{\beta}_{s-p}^* q_T^* = [A_1 \quad A_2] \begin{Bmatrix} q_s^* \\ q_p \end{Bmatrix} = 0, \quad (3.32)$$

where $\boldsymbol{\beta}_{s-p}^*$ is the matrix containing the compatibility equations between the refined sampling points of the host plate and the sampling points of the piezo-patch(es). Using Eq. (2.31), we can obtain the compatibility equations between the original sampling points of the host plate and the piezo-patches, $\boldsymbol{\beta}_{s-p}$, as

$$\boldsymbol{\beta}_{s-p} q_T = [(A_1 \quad \Gamma_{B_s}^* \Gamma_{F_s}) \quad A_2] \begin{Bmatrix} q_s \\ q_p \end{Bmatrix} = 0. \quad (3.33)$$

Note that there is definitely no limitation regarding the position of the piezo-patch on the host plate. In the presented coupling algorithm, depending on the location of the piezo-patch and the original sampling of the host plate, a refined sampling distribution is formed following Eq. (2.31). Finally, the corresponding compatibility equations are calculated using Eq. (2.33).

The integral boundary value problem approach eliminates the necessity to impose natural boundary conditions. Yet, the essential boundary conditions need to be incorporated to the formulations. Thus, boundary equations are written in a matrix form as $\boldsymbol{\beta} q_T = 0$ can be obtained. Here, $q_T = \{q_s; q_p\}$ represents the deflection vector of the host plate and the piezo-patch(es).

Hence, to impose the essential boundary conditions and compatibility equations, projection matrices approach can be followed as described in detail in (Yagci et al. 2009) as $q_T = Pq_d + Rq_b$, where, P and R matrices represent the projection matrices obtained through the singular value decomposition of the $\boldsymbol{\beta}$ matrix, q_d is the deflection vector within the domain, and q_b is the deflection vector on the boundary. Thus, the global system matrices can be derived as

$$\mathbb{M} = P^T \begin{bmatrix} M_s & 0 \\ 0 & M_p \end{bmatrix} P, \quad \mathbb{K} = P^T \begin{bmatrix} K_s & 0 \\ 0 & K_p \end{bmatrix} P, \quad \mathbb{f} = P^T \begin{Bmatrix} f_s \\ \vdots \\ f_p \end{Bmatrix}. \quad (3.34)$$

3.2.5 Electromechanical version of component mode synthesis

After obtaining the stiffness and mass matrices, forcing vector, and the electromechanical coupling vector, the coupled electromechanical differential equations can be written as follows:

$$\begin{bmatrix} M_s & 0 \\ 0 & M_p \end{bmatrix} \begin{Bmatrix} \ddot{q}_s \\ \ddot{q}_p \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_p \end{bmatrix} \begin{Bmatrix} q_s \\ q_p \end{Bmatrix} - \begin{Bmatrix} 0 \\ \boldsymbol{\theta} \end{Bmatrix} v(t) = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix} \quad (3.35)$$

$$\boldsymbol{\theta}^T \dot{q}_p + C_p \dot{v}(t) + v(t)Y = 0. \quad (3.36)$$

Here, C_p is the capacitance of the piezo and $Y = 1/R$ where R is the load resistance. Using the projection matrices, and premultiplying both sides of the Eq. (2.35) by P^T yields

$$\mathbb{M} \ddot{q}_d + \mathbb{K} q_d - P^T \begin{Bmatrix} 0 \\ \boldsymbol{\theta} \end{Bmatrix} v(t) = \mathbf{f} \quad (3.37)$$

Similarly, the electrical circuit equation (2.36) needs to be written in terms of the q_d vector. Using Eqs. (32)-(33), the electrical circuit equation becomes,

$$\boldsymbol{\theta}^T \underbrace{\left[A_2^{-1} (A_1 \Gamma_{B_s}^* \Gamma_{F_s}) \right]}_B P \dot{q}_d + C_p \dot{v}(t) + v(t)Y = 0 \quad (3.38)$$

3.2.6 Steady-state response of the system

To obtain the electromechanical steady-state responses of the whole system under harmonic excitation, let $\mathbf{f} = \mathbf{F}e^{j\omega t}$ which is based on the linear system assumption. This leads the generalized coordinates to be of harmonic forms $v(t) = \mathbf{V}e^{j\omega t}$ and $\mathbf{q}_d = \mathbf{Q}_d e^{j\omega t}$. Next, substituting the steady-state forms of the force, displacement and voltage variables in Eqs. (35) and (36), the complex valued amplitudes of $\mathbf{V}$ and $\mathbf{Q}_d$ are obtained as

$$\mathbf{Q}_d = \{ -\mathbb{M} \omega^2 + \mathbb{K} + j\omega \mathbb{C} - P^T \left(\frac{j\omega}{j\omega C_p + Y(\omega)} \right) \begin{Bmatrix} 0 \\ \boldsymbol{\theta} \end{Bmatrix} \boldsymbol{\theta}^T B P \}^{-1} P^T \mathbf{F} \quad (3.39)$$

$$\mathbf{V} = \frac{-j\omega}{j\omega C_p + Y(\omega)} \boldsymbol{\theta}^T B P \mathbf{Q}_d \quad (3.40)$$

3.3 Numerical studies

In this section, first we present the modal analysis solution of a plate structure with surface-bonded piezo-patch(es) using the developed spectral-Tchebychev (ST) solution technique and compare the results with the finite-element analysis and the Rayleigh-Ritz (RR) solution (Yoon, Youn, and Kim 2016b; Gozum et al. 2018) from the literature. Next, the electromechanical frequency response functions (FRFs) of the system for three cases studies, representing the thin and moderately thick host plates, are validated by comparing them against the RR solution and FE modeling. For the sake of simplicity, the numerical studies presented will focus on the fully cantilevered plates under point-force excitation.

3.3.1 Modal analysis of plate integrated with single/paired piezo-patch

In this subsection, the modal analysis results of representative thin and moderately thick host plates with a surface-bonded single/paired piezo-patch(es) are obtained using the two-dimensional spectral-Tchebychev (2D-ST) approach and validated by the finite-element simulations. The material and geometric properties of the host plate and piezoelectric patch used for the case studies throughout the paper are summarized in Table 3.1.

Table 3.1 Material and geometric properties of the structure and the piezoelectric patch(es)

Property	Aluminum plate	Piezoceramic patch (PZT-5A)
Young's modulus [GPa]	70	69
Shear Correction factor	$\pi/\sqrt{12}$	$\pi/\sqrt{12}$
Mass density [kg/m^3]	2700	7800
Piezoelectric constant [C/m^2]	—	-190
Permittivity constant [nF/m]	—	9.57
Length (a_s or a_p) [m]	0.5	0.2 - 0.4
Width (b_s or b_p) [m]	0.5	0.2 - 0.4
Height (h_s or h_p) [mm]	1.0 - 5.0 - 20.0	0.2 - 1.0 - 2.0

Prior to the comparison of the results, a convergence analysis is performed to determine the sufficient number of polynomial numbers to be used along x and y direction for the host plate and the piezo-patch(es) in the 2D-ST approach. In this analysis, the polynomial numbers along each direction is increased incrementally, and the calculated natural frequencies are compared to a reference case using

$$LC_{N_x, N_y}^i = \log\left(\frac{|\omega_{N_x, N_y}^i - \omega_r^i|}{\omega_r^i}\right). \quad (3.41)$$

Here, the subscript (N_x, N_y) represent the values of the polynomial numbers along x and y directions to predict the natural frequencies, the superscript represent the i^{th} natural frequency, and LC is the logarithmic convergence value. In this study, the convergence is assessed based on the average LC values for the first eight modes. For instance, an LC value of -3 corresponds to an average difference in natural frequencies less than 0.1 %.

Figure 3.3 shows an example convergence plot based on the assembled system. Note that since square plates are investigated in this study, the polynomial numbers in both directions for the host plate and the piezo-patches are kept to be equal to each other. The natural frequencies for the reference case is calculated by selecting $N_{x_s} = N_{y_s} = 60$ and $N_{x_p} = N_{y_p} = 30$. The contours in Figure 3.3 represent the logarithmic convergence values calculated by taking the logarithm of the relative error with respect to the reference case using Eq. (2.41).

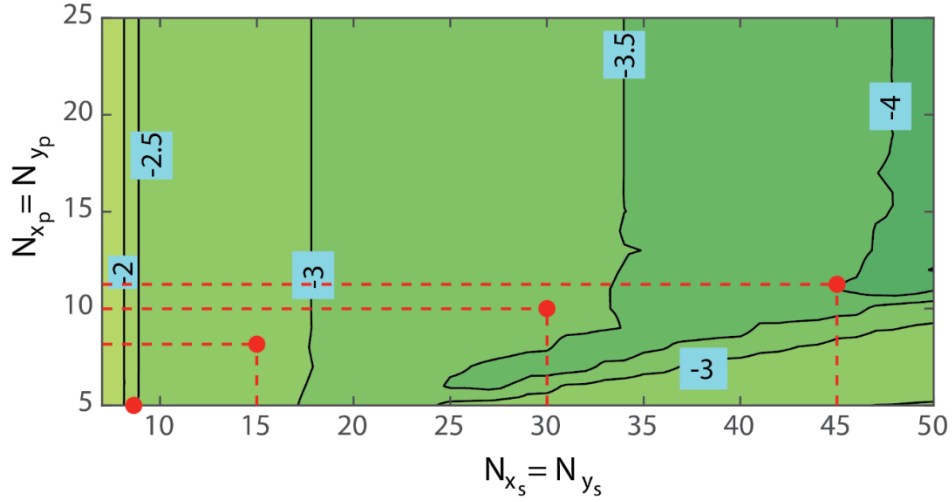


Figure 3.3. An example logarithmic convergence (LC) map representing the average LC values of first eight vibration modes for a square plate with a surface-bonded piezo-patch pair having the geometric properties of $h_s = 20$ mm, $h_p = 2$ mm, $l_s (= a_s = b_s) = 500$ mm, $l_p (= a_p = b_p) = 200$ mm. The piezo-patch pair is located at the center of the host plate. The red markers indicate examples of polynomial number pairs for the host plate and piezo-patches to achieve different LC values.

To exemplify further, four points are selected in Figure 3.3 corresponding to different LC values and the corresponding natural frequencies are listed in Table 3.2. For instance, selecting $N_{x_s} = N_{y_s} = 9$ and $N_{x_p} = N_{y_p} = 5$ results in the average of the differences in the first eight natural frequencies as 0.26 % which corresponds to a LC value of -2.7; or selecting $N_{x_s} = N_{y_s} = 15$ and $N_{x_p} = N_{y_p} = 8$ results in the average of the differences in the first eight natural frequencies as 0.14 which corresponds to a LC value of -2.9.

Table 3.2. Convergence analysis regarding natural frequencies (in [Hz]) of a square plate with a surface-bonded (at the center of the host plate) piezo-patch pair having the geometric properties of $h_s = 20$ mm, $h_p = 2$ mm, $l_s (= a_s = b_s) = 500$ mm, $l_p (= a_p = b_p) = 200$ mm.

Modes	N _s -N _p 9-5		N _s -N _p 15-8		N _s -N _p 30-10		N _s -N _p 45-12		N _s -N _p 60-30	
	Freq. (Hz)	Diff. (%)	Freq. (Hz)	Diff. (%)	Freq. (Hz)	Diff. (%)	Freq. (Hz)	Diff. (%)	Freq. (Hz)	
1	646.85	0.22	646.14	0.11	645.67	0.04	645.51	0.01	645.44	
2	1366.95	0.49	1363.74	0.25	1361.42	0.08	1360.59	0.02	1360.29	
3	1366.95	0.49	1363.74	0.25	1361.42	0.08	1360.59	0.02	1360.29	
4	2040.14	0.27	2037.4	0.14	2035.5	0.05	2034.82	0.01	2034.57	
5	2453.21	0.07	2452.75	0.05	2451.75	0.01	2451.48	0	2451.41	
6	2467.79	0.19	2465.93	0.12	2463.88	0.03	2463.27	0.01	2463.08	
7	3071.02	0.15	3068.82	0.08	3067.2	0.03	3066.61	0.01	3066.4	
8	3071.02	0.15	3068.82	0.08	3067.2	0.03	3066.61	0.01	3066.4	

As a result, using this convergence analysis, the sufficient number of polynomials can be selected based on the LC value. For instance, the sufficient number of polynomials can be selected to be $N_{x_s} = N_{y_s} = 20$ and $N_{x_p} = N_{y_p} = 9$ or $N_{x_s} = N_{y_s} = 45$ and $N_{x_p} = N_{y_p} = 12$ to achieve a convergence level of -3 or -4, respectively, for the case having the geometric properties of $h_s = 20$ mm, $h_p = 2$ mm, $l_s (= a_s = b_s) = 500$ mm, $l_p (= a_p = b_p) = 200$ mm.

Next, to validate the accuracy/precision and demonstrate the computational efficiency and capabilities of the presented solution technique, several case studies are presented in the following subsections.

3.3.1.1 Centered single/paired piezo-patches

The predicted natural frequencies for the single and paired piezo-patch(es) cases are compared to those found using a finite element (FE) solver (ANSYS[®] v18.1) and the Rayleigh-Ritz (RR) method. A similar convergence analysis is performed for the FE simulations and the required element numbers are found for each case study. For FE simulations, the host plate and piezo-patch were meshed using 20-node structural solid elements (SOLID186) and 20-node coupled solid elements (SOLID226), respectively. As an example, for the case of one-sided piezo-patch with $l_p/l_s = 0.8$ in Table 3.5, a swept meshing was performed using 4(1) and 125(200) number of slicing for the

thickness and length of the plate(piezo-patch). The CEINTF command was used to fully bond the piezo-patch and the host structure nodes at their shared surface area. For simulating the resistive load, a piezoelectric circuit element CIRC94 was used. Furthermore, the top and bottom surfaces of the piezo-patch were forced to have equal electric potential, ensuring a single voltage output on highly conductive electrodes.

Table 3.3, Table 3.4 and Table 3.5 show the calculated first eight natural frequencies of surface bonded single and piezo-patch(es) for different host plate and piezo thickness values (given in Table 3.1), respectively. For each case, the errors between the presented solution approach and FE results are less than 0.7%; and the average error value is calculated to be 0.3%. However, the errors of the RR method increases significantly by increasing the host plate thickness; for instance the average error is around 0.37% for $h_s = 1$ mm case, and it is increased to 6.3% for $h_s = 20$ mm case.

Table 3.3. The comparison of natural frequencies (in [Hz]) of a square thin plate with a surface-bonded single piezo-patch or a piezo-patch pair, using the ST solution, Rayleigh-Ritz method (RR) and FE simulations. The geometric properties of the host plate and the piezo-patch are $l_s = 500$ mm, $h_s = 1$ mm, $h_p = 0.2$ mm.

Modes	one-sided piezo-patch						two-sided piezo-patches					
	$l_p/l_s = 0.4$			$l_p/l_s = 0.8$			$l_p/l_s = 0.4$			$l_p/l_s = 0.8$		
	ST	RR	FEM	ST	RR	FEM	ST	RR	FEM	ST	RR	FEM
1	33.0	33.0	33.2	32.7	32.5	32.8	31.7	31.7	31.7	31.6	31.6	31.6
2	70.6	70.6	70.7	68.7	68.5	68.8	69.5	69.5	69.2	68.7	68.6	68.7
3	70.6	70.6	70.7	68.7	68.5	68.8	69.5	69.5	69.2	68.7	68.6	68.7
4	107.0	106.9	107.1	102.1	101.8	102.3	108.1	108.0	107.6	102.6	102.5	102.5
5	130.1	129.9	130.6	126.2	125.8	126.3	132.5	132.5	132.5	128.2	128.2	128.4
6	130.6	130.5	131.0	126.9	126.6	127.2	133.0	132.8	133.1	129.2	129.1	129.3
7	164.4	164.3	164.7	158.1	157.6	158.2	167.9	167.7	167.5	160.2	160.2	160.1
8	164.4	164.3	164.7	158.1	157.6	158.2	167.9	167.7	167.5	160.2	160.2	160.1

Table 3.4. The comparison of natural frequencies (in [Hz]) of a square thin plate with a surface-bonded single piezo-patch or a piezo-patch pair, using the ST solution, Rayleigh-Ritz method (RR), and FE simulations. The geometric properties of the host plate and the piezo-patch are $l_s = 500$ mm, $h_s = 5$ mm, $h_p = 1$ mm.

Modes	one-sided piezo-patch						two-sided piezo-patches					
	$l_p/l_s = 0.4$			$l_p/l_s = 0.8$			$l_p/l_s = 0.4$			$l_p/l_s = 0.8$		
	ST	RR	FEM	ST	RR	FEM	ST	RR	FEM	ST	RR	FEM
1	164.7	164.7	165.5	162.5	162.5	163.5	158.3	158.4	158.1	157.9	157.9	157.7
2	352.1	352.8	352.2	342.1	342.4	343.0	346.2	347.4	345.5	342.4	343.0	342.4
3	352.1	352.8	352.2	342.1	342.4	343.0	346.2	347.4	345.5	342.4	343.0	342.4
4	533.0	534.3	533.3	507.8	509.1	509.4	538.3	540.0	535.8	510.7	512.5	510.0
5	647.9	649.3	650.5	627.1	629.1	628.9	659.8	661.7	659.2	637.8	640.8	638.2
6	650.4	652.1	651.9	631.0	632.8	633.1	662.2	663.8	662.8	642.5	645.5	642.9
7	818.3	821.1	819.3	784.4	788.0	786.6	835.0	838.1	833.1	795.9	800.9	794.7
8	818.3	821.1	819.3	784.4	788.0	786.6	835.0	838.1	833.1	795.9	800.9	794.7

Table 3.5. The comparison of natural frequencies (in [Hz]) of a square thin plate with a surface-bonded single piezo-patch or a piezo-patch pair, using the ST solution, Rayleigh-Ritz method (RR), and FE simulations. The geometric properties of the host plate and the piezo-patch are $l_s = 500$ mm, $h_s = 20$ mm, $h_p = 2$ mm.

Modes	one-sided piezo-patch						two-sided piezo-patches					
	$l_p/l_s = 0.4$			$l_p/l_s = 0.8$			$l_p/l_s = 0.4$			$l_p/l_s = 0.8$		
	ST	RR	FEM	ST	RR	FEM	ST	RR	FEM	ST	RR	FEM
1	665.7	677.9	668.7	657.3	670.5	659.6	645.5	658.0	647.1	634.6	648.9	635.2
2	1376.8	1424.0	1380.7	1339.6	1389.3	1345.5	1360.6	1410.2	1361.6	1311.2	1366.5	1316.4
3	1376.8	1424.0	1380.8	1339.6	1389.3	1345.6	1360.6	1410.2	1361.7	1311.2	1366.5	1316.7
4	2032.0	2128.8	2039.3	1954.5	2057.8	1964.9	2034.9	2135.6	2038.2	1916.5	2031.5	1925.4
5	2449.5	2584.6	2463.4	2376.9	2523.5	2392.0	2451.5	2594.8	2463.8	2344.2	2509.9	2360.4
6	2457.0	2593.6	2468.1	2392.8	2537.0	2408.0	2463.4	2606.7	2471.3	2361.8	2524.8	2377.7
7	3051.4	3257.7	3066.4	2942.6	3163.5	2961.8	3066.7	3282.2	3078.0	2896.8	3144.1	2915.5
8	3051.4	3257.7	3066.4	2942.6	3163.5	2962.2	3066.7	3282.2	3078.1	2896.8	3144.1	2916.2

From Table 3.3, Table 3.4 and Table 3.5, it can be concluded that the results obtained by the presented solution technique are in excellent agreement with those found using FEM. However, the crucial point here is the computational efficiency. To assess the computational duration, the total duration of the modal analysis simulation to find the

first eight eigenvalues and eigenvectors is measured based on the time spend by the central processing unit (CPU)¹. Table 3.6 demonstrates the required DOFs to achieve the preset convergence criteria and the corresponding simulation durations measured using presented ST, Rayleigh-Ritz and FE approaches for the one-sided piezo-patch case having the geometries of $l_s = 500$ mm, $l_p = 400$ mm, $h_s = 20$ mm, $h_p = 2$ mm. On average, the presented technique decreases the computation duration of the modal analysis simulation at least one-to-two orders of magnitude. The main reason for this increase in computational efficiency is due to the fact that the same level of accuracy/precision can be achieved using less DOFs in the presented ST approach.

Table 3.6. The simulation duration and the corresponding DOF information comparison for the one-sided piezo-patch case having the geometries of $l_s = 500$ mm, $l_p = 400$ mm, $h_s = 20$ mm, $h_p = 2$ mm studied in Table 3.5.

	Poly. Num. (or # of DOFs)	Δt [s]	Poly. Num. (or # of DOFs)	Δt [s]	Poly. Num. (or # of DOFs)	Δt [s]
ST	$N_s - N_p = 15 - 9$ (# of DOFs = 507)	1.5	$N_s - N_p = 30 - 15$ (# of DOFs = 2352)	14.5	$N_s - N_p = 45 - 21$ (# of DOFs = 5547)	78.1
RR	$N_y = N_x = 10$ (# of DOFs = 100)	24.6	$N_y = N_x = 15$ (# of DOFs = 225)	53.3	$N_y = N_x = 20$ (# of DOFs = 400)	94.0
FEM	# of DOFs = 21768	53	# of DOFs = 164922	139.0	# of DOFs = 1012159	1084.0

In addition to the natural frequency comparison, the level of similarity in predicted mode shapes are investigated through modal assurance criteria (MAC). To perform the MAC analyses, the modal vectors obtained through two different solutions are required. First, the position and associated deflection at each sampling point in 2D-ST approach is obtained. Second, the deflection information corresponding to the sampling points in 2D-ST method is found in FE simulations. Following this procedure, the MAC values are calculated for the first eight modes for each case. The MAC plots for the double sided configuration are plotted in Figure 3.5. As observed, the MAC values set forth the similarity between the mode shapes calculated using two different approaches. The average MAC value is calculated to be 0.96 and the minimum MAC value is obtained as 0.72 for the symmetric modes corresponding to the case study in

¹ To assess the computational efficiency, both simulations are performed on the same workstation (Dell Precision Tower 7810).

which the geometric properties selected to be $l_s = 500$ mm, $l_p = 400$ mm, $h_s = 20$ mm, and $h_p = 2$ mm.

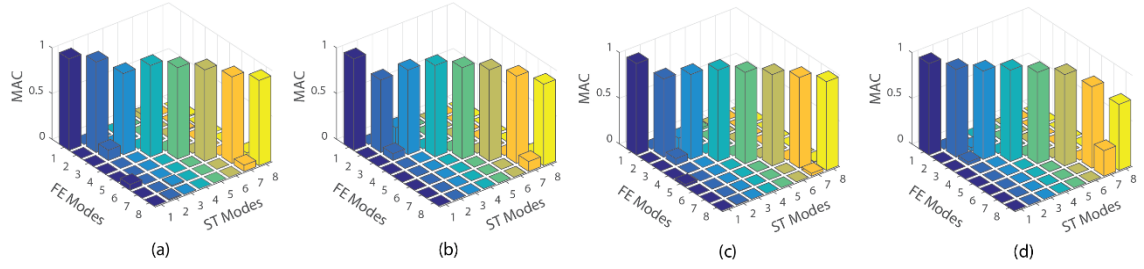


Figure 3.4 Example MAC plots for the double sided piezo-patch configuration cases whose geometric properties are (a) $l_s = 500$ mm, $l_p = 200$ mm, $h_s = 5$ mm, and $h_p = 1$ mm, (b) $l_s = 500$ mm, $l_p = 400$ mm, $h_s = 5$ mm, and $h_p = 1$ mm, (c) $l_s = 500$ mm, $l_p = 200$ mm, $h_s = 20$ mm, and $h_p = 2$ mm, and (d) $l_s = 500$ mm, $l_p = 400$ mm, $h_s = 20$ mm, and $h_p = 2$ mm.

3.3.1.2 Off-centered single piezo-patch

To demonstrate the capability of the presented modeling approach in terms of the location of the piezo-patch, two case studies are performed where the dynamics of a host plate and a surface-bonded piezo patch located at an arbitrary position (as shown in Figure 3.2) is investigated. The center of the piezo-patch is placed at $(x, y) = (-100, -50)$ mm and $(x, y) = (-20, -100)$ mm based on the coordinate frame located at the center of the host plate. In both case studies, the host plate and the piezo patch have the geometric properties of $h_s = 5$ mm, $h_p = 1$ mm, $l_s (= a_s = b_s) = 500$ mm, $l_p (= a_p = b_p) = 200$ mm.

Table 3.7 shows the calculated first eight natural frequencies using the presented ST technique and the FE approach. As expected, independent of the piezo-patch location, the results are in excellent agreement; the maximum difference in natural frequencies is calculated to be 0.56 %.

Table 3.7. The comparison of natural frequencies (in [Hz]) of a square thin plate with an off-centered surface-bonded single piezo-patch using the ST solution and FE simulations. The geometric properties of the host plate and the piezo-patch are $l_s = 500$ mm, $l_p = 200$ mm, $h_s = 5$ mm, $h_p = 1$ mm.

Modes	off-centered one-sided piezo-patch			
	$(x, y) = (-100, -50)$ mm		$(x, y) = (-20, -100)$ mm	
	ST	FEM	ST	FEM
1	169.5	169.5	168.8	168.8
2	350.2	351.8	351.5	353.2
3	360.8	360.9	359.1	358.2
4	529	529.4	529.8	529.6
5	636.5	637.2	634.5	636
6	649.7	648.2	654.8	652.1
7	803.1	802.9	806.8	806.1
8	810.1	810.5	814.5	814.5

3.3.2 Electromechanical FRFs

To validate the electromechanical response of the system, the displacement and voltage FRFs are calculated using the Eqs. (40) and (39) derived through the ST formulation. A transverse point force is applied on the host plate at 0.2 m away in x and y axes from the center of structure, and the corresponding voltage output (from the terminals of the piezo-patch) and displacement output (at symmetric position from the forcing location with respect to the plate center) are calculated. The forcing and response locations used in the FRF simulations are shown in Figure 3.5.

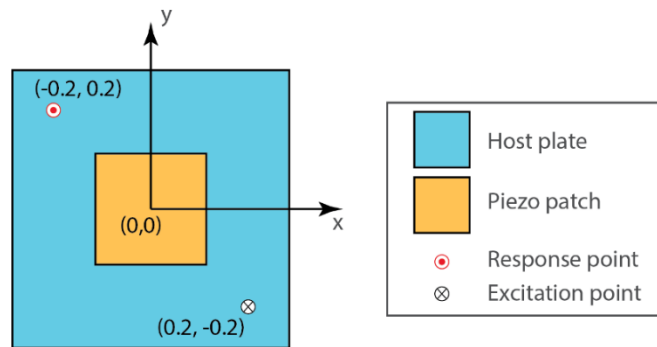


Figure 3.5. Forcing and response locations used in the FRF simulations.

For simplicity, the case studies are performed for the host plates with a single piezo-patch configuration. The electromechanical FRFs calculated by the presented ST approach, and the results are compared against the FEM and RR method from the

literature. In each case, the structural damping coefficient is set to $\eta = 0.01$. Figure 3.6, Figure 3.7 and Figure 3.8 show the displacement and voltage FRFs for all case studies investigated in Section 3.1.1: (i) $h_s = 1$ mm, $l_s = 500$ mm, $h_p = 0.2$ mm, $l_p/l_s = 0.4$, (ii) $h_s = 5$ mm, $l_s = 500$ mm, $h_p = 1$ mm, $l_p/l_s = 0.4$, and (iii) $h_s = 20$ mm, $l_s = 500$ mm, $h_p = 2$ mm, $l_p/l_s = 0.8$.

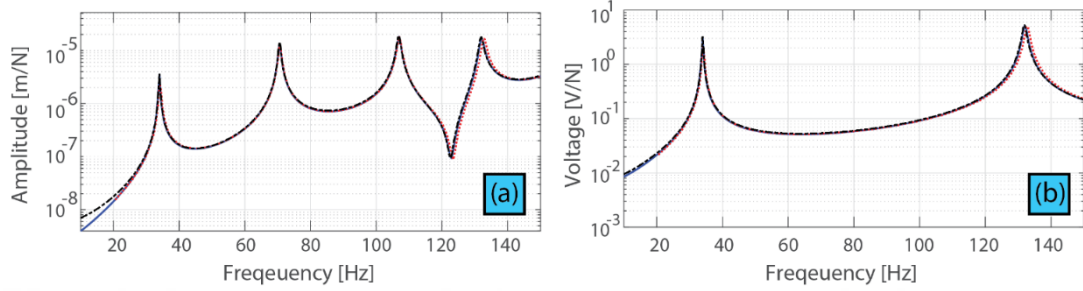


Figure 3.6. Comparison of the open-circuit (a) displacement FRFs and (b) voltage FRF against Rayleigh-Ritz and ANSYS simulation results for the single piezo-patch case where $h_s = 1$ mm, $l_s = 500$ mm, $h_p = 0.2$ mm, $l_p/l_s = 0.4$ (note that excitation and response locations are given in Figure 3.5). The solid blue, dashed black, and dashed red lines represent the Mindlin plate (2D-ST), Kirchhoff plate (RR), and 3D-FEM (ANSYS), respectively.

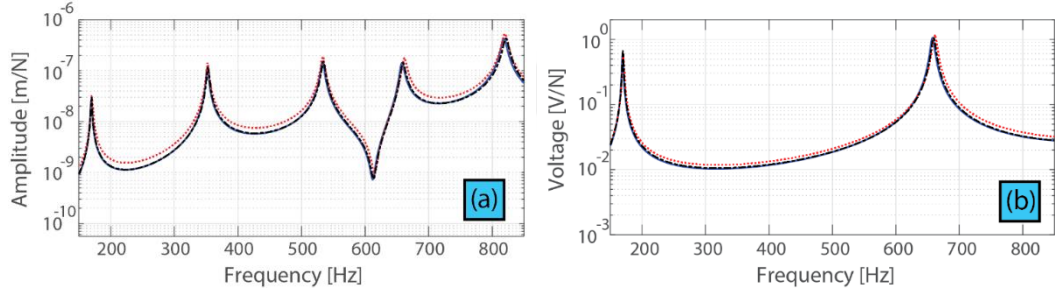


Figure 3.7. Comparison of the open-circuit (a) displacement FRFs and (b) voltage FRF against Rayleigh-Ritz and ANSYS simulation results for the single piezo-patch case where $h_s = 5$ mm, $l_s = 500$ mm, $h_p = 1$ mm, $l_p/l_s = 0.4$ (note that excitation and response locations are given in Figure 3.5). The solid blue, dashed black, and dashed red lines represent the Mindlin plate (2D-ST), Kirchhoff plate (RR), and 3D-FEM (ANSYS), respectively.

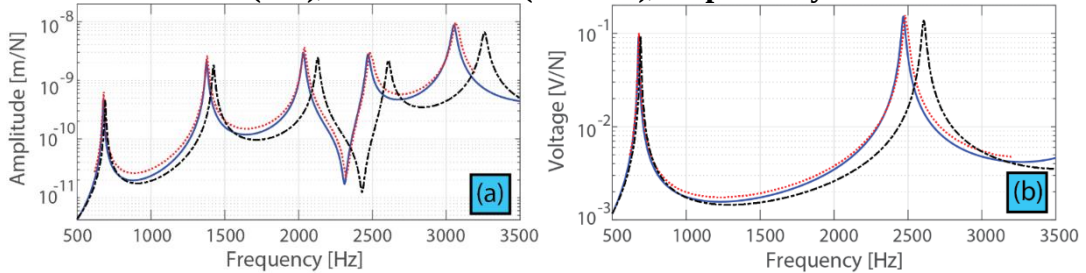


Figure 3.8. Comparison of the open-circuit (a) displacement FRFs and (b) voltage FRF against ANSYS simulation results for the single piezo-patch case where $h_s = 20$ mm, $l_s = 500$ mm, $h_p = 2$ mm, $l_p/l_s = 0.4$ (note that excitation and response locations are given in Figure 3.5). The solid blue, dashed black, and dashed red lines represent the Mindlin plate (2D-ST), Kirchhoff plate (RR), and 3D-FEM (ANSYS), respectively.

In each case study, it is clear that the ST model can accurately predict the mechanical vibration and electrical responses of the system. On the other hand, for the first and second case studies, the RR method enables accurate prediction of the electromechanical FRFs. However, for the moderately thick host plate (see Figure 3.8), the deviation between the calculated FRFs significantly increases. This could be due to the Kirchhoff plate theory assumption employed in the Rayleigh-Ritz method which neglects the shear deformation terms in the governing equations of motion. Also, it is worth noting that for calculation of FRFs using RR method in all three case studies, 20×20 number of expansion modes (400 modes in total) were used to achieve converged results.

The computational efficiency of the harmonic simulation is also analyzed based on the CPU time. To obtain the structural and voltage FRFs presented in Figure 3.6, Figure 3.7 and Figure 3.8, the CPU time of the ST simulations is measured to be three orders of magnitudes lower than the FE simulations. The main reason for this significant decrease is the fact that the spectral Tchebychev solution can achieve the same level of precision with orders of magnitude less degrees of freedom compared to FEM analysis. Therefore, we can state that the accuracy of the developed model allows optimization of the electromechanical responses of smart plate-like structures with lower computational cost than that required by FE simulations.

3.4 Conclusion

In this work, a general electromechanical modeling framework for thin and moderately thick plates with structurally integrated piezo-patches was developed using spectral Tchebychev (ST) method. This method has been shown to have the exponential convergence and superior stability characteristics, enabling prediction of the dynamics of mechanical structures accurately and efficiently. The boundary value problem is derived through an energy based approach using Mindlin plate theory. Having incorporated any type of boundary condition into a boundary value problem via projection matrices approach, the need to use different basis/trial functions for different boundary conditions is eliminated. Furthermore, in this study a novel approach was developed to couple the individually modeled host plate and the piezo-patches and thus to construct the governing equations of motions as a boundary value problem. The electromechanical coupling terms, that behave as compatibility equations, were imposed

to the system matrices similar to boundary conditions. Therefore, the overall system's mass and stiffness components can be used in the eigenvalue and eigenvector analyses as well as the vibration and electrical frequency response calculations; where this consideration has been typically ignored in the other studies focusing on active/passive vibration control and energy harvesting areas.

To validate the accuracy and demonstrate the efficiency of the presented framework, several case studies were presented and the results of modal analysis and frequency responses of the system were compared to those obtained by a commercial finite-element (FE) software and the Rayleigh-Ritz method. It was shown that the RR method is accurate for the thin plate case, but its accuracy for prediction of the electromechanical response fails for the thicker plate configurations. Therefore, the developed ST model in this study provides a more general and reliable solution for predicting the electroelastic dynamics of smart plate structures. On the other hand, the results obtained using the presented ST approach are in excellent agreement with FE results. Moreover, the computational cost of the developed method was shown to be significantly lower than that of FEM simulations. Therefore, the presented approach can be employed as an efficient and accurate framework for modeling the electroelastic dynamics of two dimensional plates with structurally integrated piezo-patches in a variety of vibration control and energy harvesting applications.

Chapter 4

4 A spectral Tchebychev solution for electromechanical analysis of thin curved panels with multiple integrated piezo-patches

4.1 Introduction

Piezoelectric materials have been widely used in many applications such as vibration control (Casagrande, Gardonio, and Zilletti 2017; Bailey and Hubbard 1985), energy harvesting (Gozum et al. 2018; Ugur Aridogan, Basdogan, and Erturk 2014) and shunt damping (Han, Neubauer, and Wallaschek 2013; Motlagh, Aghakhani, and Basdogan 2018b) over the past few decades to convert mechanical (electrical) energy to electrical (mechanical) energy. Because piezoelectric transducers are easier to manufacture and present higher power density (Alper Erturk and Inman 2011a), they are preferred compared to other types of transducers such as electromagnetic (Yang et al. 2009) and electrostatic (Torres and Rincon-Mora 2009). Many studies in the literature investigate the modeling approaches for electromechanical structures consisting of a host (which can be a thin beam or panel structure) integrated with piezoelectric patches or layers. Although lumped parameter models (Roundy and Wright 2004) are used previously, distributed parameter models are required to accurately predict the vibration behavior and voltage or mechanical frequency response functions (FRFs). For instance, Erturk *et al.* (A Erturk and Inman 2009) introduced a distributed parameter model to predict the electromechanical response of uni-morph and bimorph beams and the results are validated experimentally. In the later studies, the analyses performed for beams are extended to panel type structures. Reddy (Reddy 1999) developed a finite element based model using both the classical plate theory (CPT) and first/third-order shear deformation theory (FSDT) to control the dynamic response of laminated panels including piezoelectric layers. Although a large body of literature has been devoted to modeling and analysis of electromechanical structures, the majority of these studies are focused on the analysis of straight panels. In other words, there are still limitations on the number of studies investigating the piezoelectric ceramic embedded smart structures on shells and curved panels. Kapuria (Kapuria, Dumir, and Sengupta 1997) expands a three-dimensional axisymmetric solution to investigate simply-supported shells and a model of smart shells. Then, Wang (Wang and Liew 2003) proposed a general solution for wave

propagation in a piezoelectric coupled cylindrical shell based on the Cooper's study (Naghdi and Cooper 1956) using power and Fourier series expansion method for bending analysis of the laminated piezoelectric. In those studies, the analysis is based on the structures which have a full surface covered by piezoelectric layers. Therefore, there is still a need to develop a more generalized model that can predict the electromechanical coupling of curved structures integrated with patches that cover the surface partially.

In this chapter, a general solution approach is presented that is based on first order shear deformation theory (FSDT) to predict the electromechanical response of curved piezoelectric patches on a (doubly-) curved host structure. The integral boundary value problem (IBVP) is solved numerically using the spectral Tchebychev (ST) technique. To include the mass and inertia contribution of the multiple piezo-patches, a novel structural coupling algorithm is proposed. Furthermore, a detailed analysis is carried out to benefit from multiple patches configuration to increase energy harvesting performance. To validate the results of the presented solution approach, a commercial finite-element (FE) software is used.

4.1.1 Modal analysis of panels with multiple piezoelectric patches

The schematic of a doubly-curved panel (having a thickness of h_s) integrated with piezoelectric patches (having a thickness of h_p) is depicted in Figure 4.1 for parallel connection configuration in which all patches are interconnected and connected to the same load. Panel dimensions and curvatures are expressed in terms of non-dimensional parameters: a/b , a/r_1 and b/r_2 , where, a and b represent the length and width of the panel, and r_1 and r_2 are curvature amounts around x and y directions, respectively. The piezo patches can be also connected independently where each patch is connected to a separated electrical circuit.

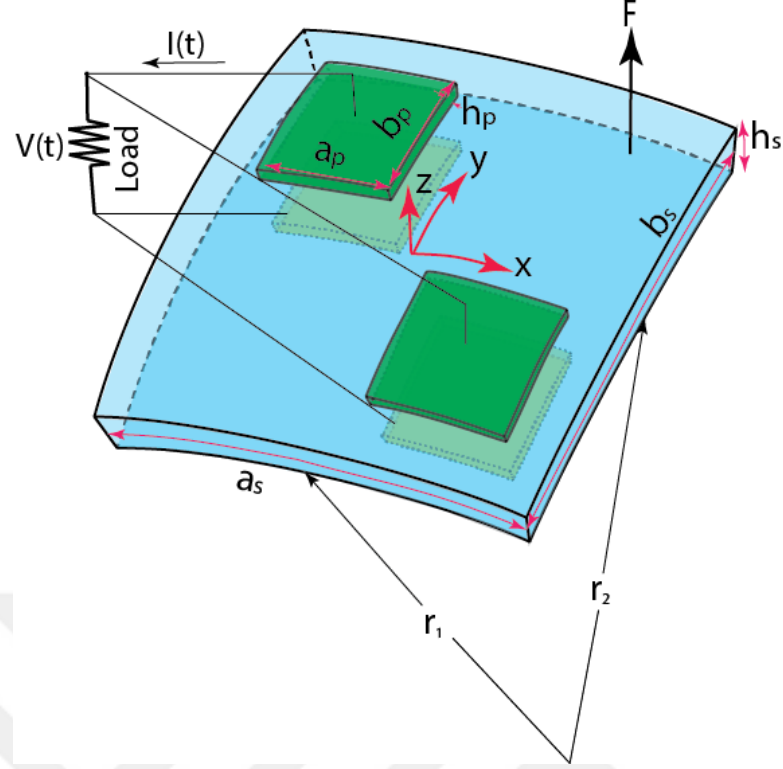


Figure 4.1. Schematic of a doubly-curved panel with multiple piezoelectric patches with parallel connection.

To obtain the governing BVP for the panel dynamics including the effects of piezoelectric patches, the kinematic relations based on the FSDT are used. To accurately and efficiently solve the BVP, the two-dimensional ST approach is applied. The following section describes the derivation of the boundary value problem and the spectral Tchebychev approach to numerically solving the dynamic behavior.

4.1.2 Derivation of boundary value problem

Based on the FSDT assumption, the displacement of any point on the curved panel can be expressed as:

$$\begin{aligned}
 u(x, y, z, t) &= u_o(x, y, z, t) + z\phi_x(x, y, z, t) \\
 v(x, y, z, t) &= v_o(x, y, z, t) + z\phi_y(x, y, z, t) \\
 w(x, y, z, t) &= w_o(x, y, z, t)
 \end{aligned} \tag{4.1}$$

where subscript 'o' represent the deformations on the neutral surface (mid-plane of the curved panel), ϕ_x and ϕ_y are the rotations of the panel around y and x -axes, respectively, and z denotes the distance of a point from the neutral surface. Therefore, the relationship

between the strains ε_{ij} and the panel deformations, $\mathbf{q} = \{u_0; v_0; w_0; \phi_x; \phi_y\}$, can be written as:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{1}{r_1} & z \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} & \frac{1}{r_2} & 0 & z \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & z & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \\ 0 & -\frac{1}{r_2} & \frac{\partial}{\partial y} & 0 & 1 \\ -\frac{1}{r_1} & 0 & \frac{\partial}{\partial x} & 1 & 0 \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \phi_x \\ \phi_y \end{Bmatrix} = \mathbf{B} \mathbf{q} \quad (4.2)$$

The BVP governing the dynamics of a doubly-curved electromechanical structure is derived using Hamilton's principle:

$$\int_{t_1}^{t_2} [\delta(\mathbf{T} - \mathbf{U} + \mathbf{W}_{ie}) + \delta \mathbf{W}_{nc}] dt = 0 \quad (4.3)$$

Here, T is kinetic energy, U is potential energy, W_{ie} is electrical energy and W_{nc} is non-conservative work. The kinetic energy of the electromechanical system can be derived as a summation of kinetic energies of host (panel) structure, T_s , and piezoelectric patches, T_p , as:

$$\begin{aligned} \mathbf{T} &= \mathbf{T}_s + \sum_{k=1}^n (\mathbf{T}_p)_k = \\ &\frac{1}{2} \int_{V_s} \rho_s \left[\left(\frac{\partial u_0}{\partial t} \right)_s^2 + \left(\frac{\partial v_0}{\partial t} \right)_s^2 + \left(\frac{\partial w_0}{\partial t} \right)_s^2 + z^2 \left(\frac{\partial \phi_x}{\partial t} \right)_s^2 + z^2 \left(\frac{\partial \phi_y}{\partial t} \right)_s^2 \right] dV_s \\ &+ \sum_{k=1}^n \left(\frac{1}{2} \int_{V_p} \rho_p \left[\left(\frac{\partial u_0}{\partial t} \right)_p^2 + \left(\frac{\partial v_0}{\partial t} \right)_p^2 + \left(\frac{\partial w_0}{\partial t} \right)_p^2 + z^2 \left(\frac{\partial \phi_x}{\partial t} \right)_p^2 \right. \right. \\ &\left. \left. + z^2 \left(\frac{\partial \phi_y}{\partial t} \right)_p^2 \right] dV_p \right) \end{aligned} \quad (4.4)$$

where subscripts 's' and 'p' denote the host structure and piezoelectric patches, respectively, n represent the number of piezoelectric patches, subscript k represents the k^{th} piezo-patch, ρ is the density, and V is the volume of corresponding structure. Thus, the kinetic energy can be simplified as:

$$\begin{aligned}
\mathbf{T} = & \frac{1}{2} \int_{V_s} \rho_s \dot{\mathbf{q}}_s^T \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & z^2 & 0 \\ 0 & 0 & 0 & 0 & z^2 \end{bmatrix}}_{\Lambda} \dot{\mathbf{q}}_s dV_s \\
& + \sum_{k=1}^n \left(\frac{1}{2} \int_{V_{p_k}} \rho_{p_k} \dot{\mathbf{q}}_{p_k}^T \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & z^2 & 0 \\ 0 & 0 & 0 & 0 & z^2 \end{bmatrix}}_{\Lambda} \dot{\mathbf{q}}_{p_k} dV_{p_k} \right) \quad (4.5)
\end{aligned}$$

Note that the volume integrals in the kinetic energy equation can be transformed into area integrals by performing the integral operation along the z direction, analytically. The integral limits along the z direction are $(z_{l_s}, z_{u_s}) = (-h_s/2, h_s/2)$ and $(z_{u_p}, z_{l_p}) = (h_s/2 + h_p, h_s/2)$ for the host structure and the piezoelectric patches, respectively.

The potential energy of the system can be derived as a summation of potential energies of the host structure and the piezoelectric patches. Using the stress-strain ($\sigma = C\epsilon$) and strain-deformation ($\epsilon = Bq$) relations, the strain energy of the host structure can be written as

$$\mathbf{U}_s = \frac{1}{2} \int \epsilon_s^T \sigma_s dV_s = \frac{1}{2} \int \mathbf{q}_s^T \mathbf{B}^T \mathbf{C}_s \mathbf{B} \mathbf{q}_s dV_s \quad (4.6)$$

where the constitutive matrix, $\mathbf{C}_s$, is expressed in terms of lame parameters (λ and μ) and Poisson's ratio (ν) as follows

$$\mathbf{C}_s = \begin{bmatrix} \frac{h_s^3}{12} \lambda & \frac{h_s^3}{12} \nu \lambda & 0 & 0 & 0 \\ \frac{h_s^3}{12} \nu \lambda & \frac{h_s^3}{12} \lambda & 0 & 0 & 0 \\ 0 & 0 & \frac{h_s^3}{12} \mu & 0 & 0 \\ 0 & 0 & 0 & h_s k^2 \mu & 0 \\ 0 & 0 & 0 & 0 & h_s k^2 \mu \end{bmatrix} \quad (4.7)$$

In the case of piezoelectric materials, the constitutive equations are defined using the electro-elastic relations as:

$$\begin{bmatrix} \sigma \\ D \end{bmatrix} = \begin{bmatrix} \mathbf{C}_p^E & -\mathbf{e}^T \\ \mathbf{e} & \epsilon^S \end{bmatrix} \begin{bmatrix} \epsilon \\ E \end{bmatrix}, \quad E = -\frac{v(t)}{h_p} \quad (4.8)$$

where $(c_{ij})'_p$ s are reduced elastic moduli for piezoelectric patches, $e = \{\bar{e}_{31}; \bar{e}_{31}; 0; 0; 0\}$ is the vector of piezoelectric constants, ε^S is dielectric permittivity, D is the piezoelectric displacement, and E is the electric field vector. Then, the potential energy of the piezoelectric patches can be defined as:

$$\begin{aligned} U_p &= \sum_{k=1}^N U_{p_{k_1}} - \sum_{k=1}^N U_{p_{k_2}} = \sum_{k=1}^N \frac{1}{2} \int \varepsilon_{p_k}^T \sigma_{p_k} dV_{p_k} - \sum_{k=1}^N \frac{1}{2} \int \varepsilon_{p_k}^T e^T E dV_{p_k} \\ &= \sum_{k=1}^N \frac{1}{2} \int \mathbf{q}_{p_k}^T \mathbf{B}^T \mathbf{C}_{p_k} \mathbf{B} \mathbf{q}_{p_k} dV_{p_k} - \sum_{k=1}^N \frac{1}{2} \int \varepsilon_{p_k}^T e^T E dV_{p_k} \end{aligned} \quad (4.9)$$

The electrical energy can be expressed as:

$$\begin{aligned} W_{ie} &= \sum_{k=1}^N \frac{1}{2} \int E^T D dV_{p_k} = \sum_{k=1}^N (W_{ie_{k_1}} + W_{ie_{k_2}}) \\ &= \frac{1}{2} \left(\sum_{k=1}^N \int E^T e \varepsilon_{p_k} dV_{p_k} + \sum_{k=1}^N \int E^T \varepsilon^S E dV_{p_k} \right) \end{aligned} \quad (4.10)$$

Here, the first term is exactly the same as the second term in Eq. (3.9) and can be expressed explicitly as follows,

$$W_{ie_{k_1}} = U_{p_{k_2}} = \int \left\{ \int_{z_l}^{z_u} \left[\frac{\left[z \frac{\partial \phi_x}{\partial x} \right] e_{31} v(t)}{2h_{p_k}} + \frac{\left[z \frac{\partial \phi_y}{\partial y} \right] e_{32} v(t)}{2h_{p_k}} \right] dz \right\} dA_{p_k} \quad (4.11)$$

The second term in Eq. (3.10) can be simplified as $C_{p_k} = \varepsilon^S (a_{p_k} b_{p_k}) / h_{p_k}$. Finally, the non-conservative work term due to the external forces ($\mathbf{f} = \{f_{u_0}; f_{v_0}; f_{w_0}; f_{\phi_x}; f_{\phi_y}\}$) can be written as:

$$W_{nc} = \frac{1}{2} \int \mathbf{f}^T \mathbf{q} dV \quad (4.12)$$

4.1.3 Spectral-Tchebychev (ST) approach

The basis of the presented solution approach is described in detail in the authors' previous publications (Aghakhani et al. 2019; Filiz et al. 2012). Therefore, the basic properties of the 2D-ST method will be briefly summarized here. First of all, the problem domain is discretized using the Gauss-Lobatto sampling algorithm, and then double-

expansion of Tchebychev polynomials are used for each degree of freedom in the deflection vector:

$$\mathbf{q}_i = \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} a_{q_i} \mathcal{T}_{k-1}(x) \mathcal{T}_{l-1}(y), \quad (4.13)$$

$$i = (m-1)N_y + n$$

where $\mathcal{T}$'s are the recursive and orthogonal Tchebychev polynomials of the first kind, a 's are coefficients of expansion, and N_x and N_y are the numbers of polynomials used along x and y directions, respectively. After the discretization process, the deflection vector for FSDT can be expressed as:

$$\begin{aligned} u_o &= [\mathbb{I} \quad \mathbb{O} \quad \mathbb{O} \quad \mathbb{O} \quad \mathbb{O}] \mathbf{q} = \mathbb{I}_u \mathbf{q}, \\ v_o &= [\mathbb{O} \quad \mathbb{I} \quad \mathbb{O} \quad \mathbb{O} \quad \mathbb{O}] \mathbf{q} = \mathbb{I}_v \mathbf{q}, \\ w_o &= [\mathbb{O} \quad \mathbb{O} \quad \mathbb{I} \quad \mathbb{O} \quad \mathbb{O}] \mathbf{q} = \mathbb{I}_w \mathbf{q}, \\ \Phi_x &= [\mathbb{O} \quad \mathbb{O} \quad \mathbb{O} \quad \mathbb{I} \quad \mathbb{O}] \mathbf{q} = \mathbb{I}_{\Phi_x} \mathbf{q}, \\ \Phi_y &= [\mathbb{O} \quad \mathbb{O} \quad \mathbb{O} \quad \mathbb{O} \quad \mathbb{I}] \mathbf{q} = \mathbb{I}_{\Phi_y} \mathbf{q} \end{aligned} \quad (4.14)$$

where $\mathbb{O}$ and $\mathbb{I}$ are $(N_x N_x \times N_y N_y)$ zero and identity matrices, respectively. To perform the derivative and integral operations exactly and efficiently, we develop special matrices in ST technique. To derive those matrices, we use extended backward ($\mathbf{\Gamma}_B$) and forward ($\mathbf{\Gamma}_F$) transformation matrices (the details of the derivation of these matrices are given in (Bediz 2018; Bediz and Aksoy 2018)) that relate the function's values at the sampling points to the expansion coefficients as:

$$\mathbf{q} = \mathbf{\Gamma}_B \mathbf{a} \quad \text{or} \quad \mathbf{a} = \mathbf{\Gamma}_F \mathbf{q} \quad (4.15)$$

To perform the derivative operation, we first use double-expansion of Tchebychev polynomials to express the derivative of the function as $\partial \mathbf{q} / \partial \mathbf{q}_t = \mathbf{\Gamma}_B \mathbf{b}$ where $\mathbf{q}_t$ represents x or y variables. Then, a relationship between the coefficients of expansions can be written as $\mathbf{b} = D^{q_t} \mathbf{a}$ where D^{q_t} is the differentiation matrix along q_t . Finally, following the derivations performed in (Bediz 2018; Gottlieb, Orszag, and Foundation (U.S.) 1977), the n^{th} spatial derivative, $\partial^{(n)} \mathbf{q} / \partial \mathbf{q}_t$ can be calculated using $\mathbf{q}^{(n)} = \mathbf{Q}_{q_t}^{(n)} \mathbf{q}$, where $\mathbf{Q}_{q_t}^{(n)}$ is the derivative matrix in Tchebychev domain (obtained using extended forward and backward transformation matrices). Since the material properties (ρ and $\mathbf{C}_{ij}$'s) are constant within the material, the integral operations include the deformation

functions as the only variables. According to the inner product matrix approach, integral of multiplication of two functions can be calculated as follows:

$$\int_A f(x, y)g(x, y)dxdy = \mathbf{f}^T \boldsymbol{\xi} \mathbf{g} \quad (4.16)$$

where $\mathbf{f}$ and $\mathbf{g}$ are the unknown functions (deformation vector, q , in our case) and $\boldsymbol{\xi}$ is the inner product matrix. For the details of the derivation of the inner product matrix, the reader is referred to (Bediz 2018; Yagci et al. 2009).

Therefore, inserting the energy and work terms into Hamilton's equation, the mass and stiffness matrices, and the electromechanical coupling vector of electromechanical structure can be obtained as follows:

$$\mathbf{M}_s = \rho_s \left(h_s (\mathbb{I}_{u_s}^T \boldsymbol{\xi}_s \mathbb{I}_{u_s} + \mathbb{I}_{v_s}^T \boldsymbol{\xi}_s \mathbb{I}_{v_s} + \mathbb{I}_{w_s}^T \boldsymbol{\xi}_s \mathbb{I}_{w_s}) + \frac{h_s^3}{12} (\mathbb{I}_{\phi_{xs}}^T \boldsymbol{\xi}_s \mathbb{I}_{\phi_{xs}} + \mathbb{I}_{\phi_{ys}}^T \boldsymbol{\xi}_s \mathbb{I}_{\phi_{ys}}) \right) \quad (4.17)$$

$$\mathbf{M}_{pk} = \left\{ \left[\frac{h_p^3}{3} + \frac{h_p^2 h_p}{4} + \frac{h_s h_p}{2} \right] (\mathbb{I}_{\phi_{xp}}^T \boldsymbol{\xi}_p \mathbb{I}_{\phi_{xp}} + \mathbb{I}_{\phi_{yp}}^T \boldsymbol{\xi}_p \mathbb{I}_{\phi_{yp}}) + 2\rho_p \left(h_p (\mathbb{I}_{u_p}^T \boldsymbol{\xi}_p \mathbb{I}_{u_p} + \mathbb{I}_{v_p}^T \boldsymbol{\xi}_p \mathbb{I}_{v_p} + \mathbb{I}_{w_p}^T \boldsymbol{\xi}_p \mathbb{I}_{w_p}) \right) \right\}_k \quad (4.18)$$

$$\mathbf{K}_s = \mathbf{B}_s^T \boldsymbol{\xi}_s \mathbf{C}_{h_s} \mathbf{B}_s \quad (4.19)$$

$$\mathbf{K}_{pk} = (\mathbf{B}_p^T \boldsymbol{\xi}_p \mathbf{C}_{h_p} \mathbf{B}_p)_k \quad (4.20)$$

$$\boldsymbol{\theta}_k = \left[\left(\frac{h_p + h_s}{2} \right) (\varepsilon_{xxp}^T \boldsymbol{\xi}_p e_{31} + \varepsilon_{yy_p}^T \boldsymbol{\xi}_p e_{31}) \right]_k \quad (4.21)$$

Note that the damping can be included in calculating the structural or voltage FRF as proportional or structural damping.

4.1.4 Structural coupling of piezoelectric patches to the host structure

To predict the dynamic behavior of the electromechanical system, the component mode synthesis approach is used. In this approach, compatibility equations are written at the connection boundaries between the degrees of freedom of the host structure and

piezoelectric patches. Note that, the system matrices for the host structure and the piezo patches are obtained using Gauss-Lobatto sampling to improve the numerical efficiency. However, since the sampling of each part is performed separately, the sampling points corresponding to the host structure and the piezo patches may not coincide (see Figure 4.2). Therefore, to express the compatibility equations, the host structure is resampled to include the sampling points of the piezo patches (see Figure 4.2c). Using the forward and backward transformation matrices, it is possible to relate the original sampling points (q_s) to the refined sampling points of the host structure (q_s^*):

$$\mathbf{q}_s^* = \Gamma_{B_s}^* \mathbf{a}_s = \Gamma_{B_s}^* \Gamma_{F_s} \mathbf{q}_s \quad (4.22)$$

where $\Gamma_{B_s}^*$ is the $(N_x N_y)_s^* \times (N_x N_y)_s$ refined global backward transformation matrix whose elements are evaluated at the refined points. After refining sampling of host structure, the new sampling points of the host structure coincide with the sampling points of the piezo patches. Therefore, we can easily equate the degrees of freedom at the connection boundaries to obtain the compatibility equations as

$$\beta_{s-p} \mathbf{q}_T = \begin{bmatrix} \mathbf{A}_{s_1} & \mathbf{A}_{p_1} & 0 & \dots & \dots \\ \mathbf{A}_{s_2} & 0 & \mathbf{A}_{p_2} & 0 & \dots \\ \dots & \dots & \dots & \ddots & 0 \\ \mathbf{A}_{s_n} & 0 & \dots & 0 & \mathbf{A}_{p_n} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{q}_{p_1} \\ \dots \\ \mathbf{q}_{p_n} \end{bmatrix} = 0, \quad (4.23)$$

where $\mathbf{A}_{s_i} = \mathbf{A}_{s_i}^* \Gamma_{B_s}^* \Gamma_{F_s}$

Here, β_{s-p} is the compatibility matrix, $\mathbf{q}_T$ is the deflection vector for the complete system, and $\mathbf{A}$'s are the matrices that includes the coupling equations. Note that there is no limitation on the position of piezoelectric patches on the host structure. As seen from Eqs. (22) and (23), the developed algorithm can be applied for coupling any number of piezoelectric patches to the host structure. Furthermore, the coupling can also be done using a reduced number of sampling points of the piezoelectric patches to increase the computational efficiency.

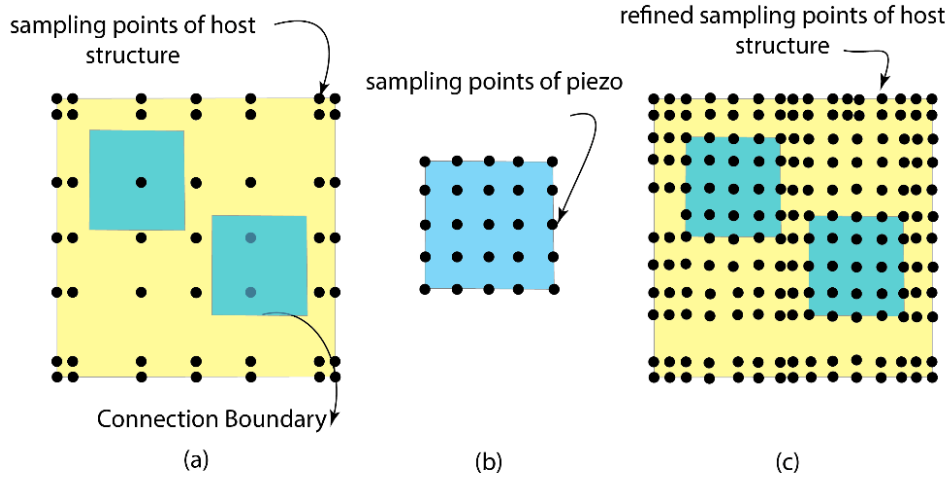


Figure 4.2. Component mode synthesis of structure, (a) Gauss-Lobatto sampling of the host structure, (b) Gauss-Lobatto sampling of the piezoelectric patch, (c) new refined sampling of the host structure

4.1.5 Applying boundary conditions

The boundary conditions can be expressed in the matrix form as $\beta_{BC} \mathbf{q}_T = \mathbf{0}$. Note that since an integral BVP approach is used in this study, only the essential boundary conditions should be included in the β matrix. Since the coupling and the boundary condition equations are in the same form, it is possible to combine them as,

$$\begin{bmatrix} \beta_{s-p} \\ \beta_{BC} \end{bmatrix} \mathbf{q}_T = \beta \mathbf{q}_T = \mathbf{0} \quad (4.24)$$

To impose the effect of boundary conditions and coupling equations on the derived system matrices, the projection matrices approach is used. Following this technique, it is possible to obtain the projection matrix $\mathbf{P}$ based on the basis recombination approach using the singular value decomposition of the β matrix. Based on this approach, $N_T \times (N_T - M)$ $\mathbf{P}$ matrix can be obtained where N_T is the total number of degrees of freedom, and M is the number of equations in the β matrix. This projection matrix basically defines a coordinate transformation as $\mathbf{q}_T = \mathbf{P} \mathbf{q}_d$ (for more detail reader can refer to (Filiz et al. 2012; Yagci et al. 2009; Filiz et al. 2014)).

4.1.6 Electromechanical component mode synthesis, Steady-state response

The differential equation governing the electromechanical behavior can be defined using the derived system matrices and electromechanical coupling vector as:

$$\begin{aligned}
& \begin{bmatrix} \mathbf{M}_s & 0 & \dots & \dots \\ 0 & \mathbf{M}_{p_1} & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ \dots & \dots & 0 & \mathbf{M}_{p_n} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_s \\ \ddot{\mathbf{q}}_{p_1} \\ \vdots \\ \ddot{\mathbf{q}}_{p_n} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_s & 0 & \dots & \dots \\ 0 & \mathbf{K}_{p_1} & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ \dots & \dots & 0 & \mathbf{K}_{p_n} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_s \\ \mathbf{q}_{p_1} \\ \vdots \\ \mathbf{q}_{p_n} \end{Bmatrix} \\
& - \begin{bmatrix} 0 & \dots & \dots & 0 \\ \boldsymbol{\theta}_1 & 0 & \dots & 0 \\ 0 & \boldsymbol{\theta}_2 & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ 0 & \dots & 0 & \boldsymbol{\theta}_n \end{bmatrix} \begin{Bmatrix} \mathbf{v}_{p_1} \\ \vdots \\ \mathbf{v}_{p_n} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_s \\ \mathbf{f}_{p_1} \\ \vdots \\ \mathbf{f}_{p_n} \end{Bmatrix} \quad (4.25)
\end{aligned}$$

where, $\mathbf{v}_{p_i}$ represent the voltage vector of each individual piezo patch. Then, using the projection matrices approach (*i.e.* inserting $\mathbf{q}_T = \mathbf{P}\mathbf{q}_d$), premultiplying each term by $\mathbf{P}^T$ and assuming a harmonic excitation as $\mathbf{f} = \bar{\mathbf{F}}e^{-i\omega t}$ that leads the deflection and voltage vector to be in a harmonic form as $\mathbf{q}_d = \bar{\mathbf{Q}}_d e^{-i\omega t}$ and $\mathbf{v}_p = \bar{\mathbf{V}}_d e^{-i\omega t}$ based on the linear system assumption, top equation can be rewritten as

$$\begin{aligned}
& \{-\omega^2 \mathbb{M} + \mathbb{K}\} \bar{\mathbf{Q}}_d - \mathbf{P}^T \begin{bmatrix} 0 \\ \hat{\boldsymbol{\theta}}_{ind} \end{bmatrix} \bar{\mathbf{V}}_p = \mathbf{P}^T \bar{\mathbf{F}}, \text{ where } \mathbb{M} = \mathbf{P}^T \mathbf{M} \mathbf{P}, \\
& \mathbb{K} = \mathbf{P}^T \mathbf{K} \mathbf{P}, \hat{\boldsymbol{\theta}}_{ind} = \begin{bmatrix} \boldsymbol{\theta}_1 & 0 & \dots & 0 \\ 0 & \boldsymbol{\theta}_2 & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ 0 & \dots & 0 & \boldsymbol{\theta}_n \end{bmatrix} \quad (4.26)
\end{aligned}$$

For the electrical circuit differential equation, two cases will be considered in this paper: **(i)** independent and **(ii)** parallel connection. In the case of independent connection, the voltage of each individual piezoelectric patch will be coupled to the deflection vector as follows:

$$\boldsymbol{\theta}_i^T \dot{\mathbf{q}}_{p_i} + c_{p_i} \dot{\mathbf{v}}_{p_i} + Y_{p_i} \mathbf{v}_{p_i} = 0 \quad (4.27)$$

and in the case of parallel connection, the voltage is the same for the piezo patches connected in parallel and coupled to the deflection vector as:

$$\sum_{i=1}^N \boldsymbol{\theta}_i^T \dot{\mathbf{q}}_{p_i} + \dot{\mathbf{v}} \sum_{i=1}^N c_{p_i} + Y \mathbf{v} = 0 \quad (4.28)$$

Here, c_{p_i} is piezoelectric patch capacitance and Y_{p_i} represents the impedance. By extending Eq. (3.27) **Error! Reference source not found.** for the multiple patch case, the electrical circuit differential equation can be written in matrix form as:

$$\begin{aligned}
& \begin{bmatrix} \theta_1^T & 0 & \dots & \dots \\ 0 & \theta_2^T & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ \dots & \dots & 0 & \theta_n^T \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}_{p_1} \\ \dot{\mathbf{q}}_{p_2} \\ \dots \\ \dot{\mathbf{q}}_{p_n} \end{Bmatrix} + \begin{bmatrix} c_{p_1} & 0 & \dots & \dots \\ 0 & c_{p_2} & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ \dots & \dots & 0 & c_{p_n} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{v}}_{p_1} \\ \dot{\mathbf{v}}_{p_2} \\ \dots \\ \dot{\mathbf{v}}_{p_n} \end{Bmatrix} \\
& + \begin{bmatrix} \mathbf{Y}_{p_1} & 0 & \dots & \dots \\ 0 & \mathbf{Y}_{p_2} & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ \dots & \dots & 0 & \mathbf{Y}_{p_n} \end{bmatrix} \begin{Bmatrix} \mathbf{v}_{p_1} \\ \mathbf{v}_{p_2} \\ \dots \\ \mathbf{v}_{p_n} \end{Bmatrix} = 0
\end{aligned} \tag{4.29}$$

Note that we need to relate voltage to the same deflection vector, $\mathbf{q}_d$, used in the coupled electromechanical differential equation. Using the compatibility equations defined in Eq. (3.23), it is possible to relate the piezo-patches' and host structure's deflection vectors as

$$\underbrace{\begin{bmatrix} \mathbf{A}_{s_1} \\ \mathbf{A}_{s_2} \\ \vdots \\ \mathbf{A}_{s_n} \end{bmatrix}}_{\mathbf{A}_s} \mathbf{q}_s + \underbrace{\begin{bmatrix} \mathbf{A}_{p_1} & 0 & \dots & \dots \\ 0 & \mathbf{A}_{p_2} & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ \dots & \dots & 0 & \mathbf{A}_{p_n} \end{bmatrix}}_{\mathbf{A}_p} \underbrace{\begin{Bmatrix} \mathbf{q}_{p_1} \\ \mathbf{q}_{p_2} \\ \dots \\ \mathbf{q}_{p_n} \end{Bmatrix}}_{\mathbf{q}_p} = \mathbf{A}_s \mathbf{q}_s + \mathbf{A}_p \mathbf{q}_p = 0 \tag{4.30}$$

$$\mathbf{q}_p = -\mathbf{A}_p^{-1} \mathbf{A}_s \mathbf{q}_s \tag{4.31}$$

Thus, the deflection vector can be related to $\mathbf{q}_d$ as

$$\mathbf{q}_p = \begin{bmatrix} -\mathbf{A}_p^{-1} \mathbf{A}_s & 0 \end{bmatrix} \mathbf{P} \mathbf{q}_d \tag{4.32}$$

Inserting Eq. (3.32) into Eq. (3.29), the displacement and voltage variables can be obtained as

$$\begin{aligned}
\bar{\mathbf{Q}}_d = & \left\{ -\omega^2 \mathbb{M} + \mathbb{K} \right. \\
& \left. - \mathbf{P}^T \begin{bmatrix} 0 \\ \hat{\boldsymbol{\theta}}_{ind} \end{bmatrix} \left(\frac{j\omega}{j\omega C_p + Y_p} \right) \hat{\boldsymbol{\theta}}_{ind}^T \begin{bmatrix} \mathbf{A}_p^{-1} \mathbf{A}_s & 0 \end{bmatrix} \mathbf{P} \right\}^{-1} \mathbf{P}^T \bar{\mathbf{F}}
\end{aligned} \tag{4.33}$$

$$\bar{\mathbf{V}}_p = \frac{-j\omega}{j\omega C_p + Y_p} (\hat{\boldsymbol{\theta}}^T)_{ind} \begin{bmatrix} \mathbf{A}_p^{-1} \mathbf{A}_s & 0 \end{bmatrix} \mathbf{P} \mathbf{Q}_d \tag{4.34}$$

where

$$\mathbf{C}_p = \begin{bmatrix} c_{p_1} & 0 & \dots & \dots \\ 0 & c_{p_2} & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ \dots & \dots & 0 & c_{p_n} \end{bmatrix} \quad \text{and} \quad \mathbf{Y}_p = \begin{bmatrix} Y_{p_1} & 0 & \dots & \dots \\ 0 & Y_{p_2} & \dots & \dots \\ \dots & \dots & \ddots & 0 \\ \dots & \dots & 0 & Y_{p_n} \end{bmatrix} \quad (4.35)$$

Following the same procedure in the case of parallel connection, and keeping the electromechanical coupled equations as Eq. (3.33), the voltage and deflection vectors are defined as:

$$\{-\omega^2 \mathbb{M} + \mathbb{K}\} \bar{\mathbf{Q}}_d - \mathbf{P}^T \begin{bmatrix} 0 \\ \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \\ \dots \\ \boldsymbol{\theta}_n \end{bmatrix} \mathbf{v} = \mathbf{P}^T [\bar{\mathbf{F}}] \quad (4.36)$$

$$\begin{aligned} \bar{\mathbf{Q}}_d &= \left\{ -\omega^2 \mathbb{M} + \mathbb{K} \right. \\ &\quad \left. - \mathbf{P}^T \begin{bmatrix} 0 \\ \hat{\boldsymbol{\theta}}_{ind} \end{bmatrix} \left(\frac{j\omega}{j\omega \sum_{i=1}^N c_{p_i} + Y} \right) \mathbf{P}^T \begin{bmatrix} 0 \\ \boldsymbol{\theta}_1 \\ \dots \\ \boldsymbol{\theta}_n \end{bmatrix} [\boldsymbol{\theta}_1^T \quad \dots \quad \boldsymbol{\theta}_n^T] [\mathbf{A}_p^{-1} \mathbf{A}_s \quad 0] \mathbf{P} \right\}^{-1} \mathbf{I} \end{aligned} \quad (4.37)$$

$$\bar{\mathbf{V}}_p = \frac{-j\omega}{j\omega \sum_{i=1}^N c_{p_i} + Y} [\boldsymbol{\theta}_1^T \quad \boldsymbol{\theta}_2^T \quad \dots \quad \boldsymbol{\theta}_n^T] [\mathbf{A}_p^{-1} \mathbf{A}_s \quad 0] \mathbf{P} \mathbf{Q}_d \quad (4.38)$$

4.2 Results and discussion

In this section, we demonstrated the accuracy and performance of the presented solution approach considering fully-clamped straight ($a/r_1 = 0$, $b/r_2 = 0$) and (doubly-) curved ($a/r_1 = \pi/40$, $b/r_2 = \pi/40$) host structure for two cases: **(i)** single patch located at the center having a size of $a_p = 200$ mm, $b_p = 200$ mm, **(ii)** two patches located at $[x,y]=[-100$ mm, -100 mm] and $[x,y]=[100$ mm, 100 mm] having a size of $a_p = 150$ mm, $b_p = 150$ mm. Figure 4.3 depicts the top view of the host panel and piezo patch(es) geometries of the investigated cases. Material and geometry properties of the investigated host structure and piezoelectric patches are given in Table 4.1.

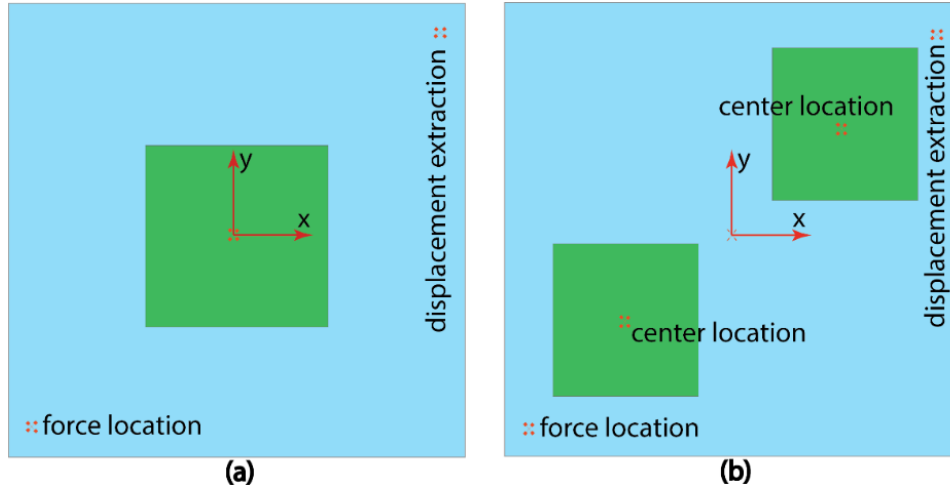


Figure 4.3. Top view schematic of (a) one patch (case-a) and (b) two patches (case-b) configuration cases. Blue and green squares represent the host structure and the piezoelectric patches, respectively.

In the first part, the validations are performed comparing the results (natural frequencies, structural and electromechanical FRFs) to those obtained using a commercial finite element modeling (FEM) software (COMSOL v5.5). In the case of FRF comparisons, a force (with an amplitude of 1N) is applied along the z direction at $x = -220$ mm and $y = -220$ mm, and the corresponding voltage output (from the terminals of the piezo-patch) and the displacement output (measured at $x = 220$ mm and $y = 220$ mm) are calculated. To present the effect of electrical connection between the piezo patches, analyses are performed in parallel configurations (the results for the independent configuration are given in Appendix. A). In the second part, the mode jump effect for curved panels is investigated to increase the harvested energy.

Table 4.1 Material and geometry properties of host panel and piezoelectric patches

Property	Aluminum (host) panel	Piezoelectric patches (PZT-5A)
Young Modulus [GPa]	70	69
Shear correction factor	$\pi/\sqrt{12}$	$\pi/\sqrt{12}$
Mass density [kg/m ³]	2700	7800
Piezoelectric constant [C/m ²]	-	-190
Permittivity constant [nF/m]	-	9.57
Length (a_s and a_p) [m]	0.5	0.15 - 0.4
Width (b_s and b_p) [m]	0.5	0.15 - 0.4
Height (h_s and h_p) [mm]	1.0-4.0	0.2-1.0

4.2.1 Convergence analysis

In the presented modeling approach, the accuracy of the solution depends on the number of polynomials used in modeling the host plate and the piezo patches. Therefore, prior to validation, the convergence analysis of the presented ST modeling approach is performed to determine the required polynomial numbers along the x and y directions for both the host panel and piezoelectric patches. Since the geometries of the aluminum panel and the piezoelectric patches are selected to be square (however note that there is no geometry limitation of the presented solution technique), the polynomial numbers along each direction is restricted to be equal to each other ($N_x = N_y$). The case which is used in the convergence analysis is case-b as shown in Figure 4.3. The geometry of panel and piezo patches are $a_s = b_s = 500$ mm, $h_s = 4$ mm, $a_p = b_p = 150$ mm, and $h_p = 1$ mm.

In the convergence analysis, the polynomial numbers are increased incrementally, and the results are compared to a reference case (in the reference case, the polynomial numbers are chosen as $N_x=N_y= 40$ for piezo patches and $N_x=N_y=70$ for the panel) in terms of the logarithm of the relative difference as,

$$C^i = \log \left(\frac{|\omega_{N_x, N_y}^i - \omega_r^i|}{\omega_r^i} \right) \quad (4.39)$$

Here, ω_{N_x, N_y}^i and ω_r^i are the i^{th} natural frequency corresponding to the selected polynomial number and the reference case. This analysis can be done either for a specific natural frequency or for a set of natural frequencies by averaging the differences for the investigated natural frequencies. The convergence behavior, that is calculated considering the first ten natural frequencies, is plotted in Figure 4.4 where the contour lines show the level of convergence, C values in Eq. (3.39). To better understand the precision, some of the cases are chosen and compared numerically with the reference case in Table 4.2. Based on this convergence study, the polynomial numbers are determined to be $N_{x_s} = 25$, $N_{x_p} = 11$ to satisfy the convergence of 0.1 %.

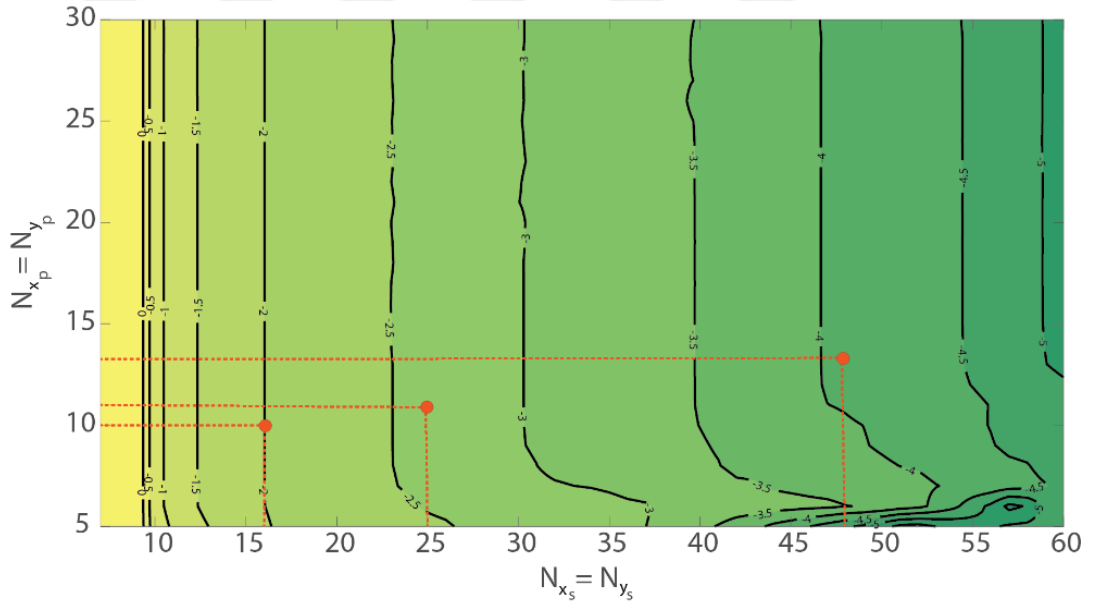


Figure 4.4. An example of a convergence plot considering the first ten natural frequencies for two patch configuration. The dimensions are: $a_s = b_s = 500$ mm, $h_s = 4$ mm, $a_p = b_p = 150$ mm, and $h_p = 1$ mm.

Table 4.2. Convergence studies of first ten natural frequencies for one patch case on the panel with $a_p = 100$ mm, $a_s = 500$ mm, $h_s = 4$ mm, $h_p = 1$ mm.

Modes	$N_s-N_p = 10-16$		$N_s-N_p = 11-25$		$N_s-N_p = 13-47$		$N_s-N_p = 40-70$
	Frequency (Hz)	Diff. (%)	Frequency (Hz)	Diff. (%)	Frequency (Hz)	Diff. (%)	Frequency (Hz)
1	69.139	0.121	69.100	0.063	69.069	0.019	69.056
2	140.155	0.213	140.026	0.121	139.91	0.038	139.857
3	146.91	0.043	146.887	0.027	146.856	0.006	146.847
4	211.828	0.12	211.730	0.073	211.617	0.02	211.575
5	259.692	0.188	259.467	0.102	259.257	0.02	259.204
6	262.12	0.182	261.899	0.098	261.695	0.019	261.644
7	325.819	0.129	325.633	0.072	325.446	0.015	325.398
8	328.074	0.143	327.876	0.083	327.656	0.016	327.604
9	413.85	0.141	413.577	0.075	413.34	0.018	413.266
10	423.939	0.081	423.728	0.032	423.62	0.006	423.594

4.2.2 Model validation

The validation study is composed of two parts: **(i)** natural frequency prediction and **(ii)** structural/voltage FRF comparison. To validate the results of the presented solution approach, a commercial finite element software (COMSOL v5.5) is used. Note that a similar convergence analysis similar to the convergence analysis of the presented modeling approach is performed for FE models to determine the required number of elements. In modeling the curved structures in FE, a curvilinear coordinate is assigned along the curvature of the model as in the presented modeling approach. The important point here is that with the shell elements, it won't be possible to define the top and bottom electrodes; therefore, 3D (hexahedral) elements are used in FE simulations. Regarding the nature of the investigated problem where the piezo patch is partially covering the host plate surface, 3D finite element modeling is the only option that yields accurate prediction of the electromechanical response. After construction the 3D model, the piezoelectric effect is applied to piezoelectric patches, and subsequently by using electrostatic module the top surfaces of patches are used as an output terminal and bottom surfaces are used as the ground (the top and bottom surfaces of the piezo-patch were forced to have equal electric potential, ensuring a single voltage output on highly conductive electrodes). To impose the boundary conditions, the edges of plate chosen as fixed. To compare the results with the presented 2D modeling approach, an extra element along the thickness direction is assigned to make sure that there is an internal node on the neutral plane to capture the force and displacement locations similar to presented

modeling approach. Based on the convergence analysis, in one patch and two patches configurations are modeled with 5546 and 6362 elements, respectively. To determine the natural frequencies and frequency response functions, the modal and harmonic modules of COMSOL are used, respectively.

In the first part, a modal analysis study is performed and the predicted natural frequencies for one piezo patch and two piezo patches cases on straight and curved host panels are listed in Table 4.3. As shown, the predicted natural frequencies using ST and FE analyses are in excellent agreement; the maximum and average percentage differences are calculated as 0.73% and 0.33%, respectively.

Table 4.3. Natural frequency (in Hz) comparison between the ST model and FEM model for one piezo patch and two piezo patches cases on straight and curved host panels.

Mode	one patch						two patches					
	Straight			Curved			Straight			Curved		
	FEM	ST	Diff. (%)	FEM	ST	Diff. (%)	FEM	ST	Diff. (%)	FEM	ST	Diff. (%)
1	125.31	125.83	-0.42	188.69	189.09	-0.21	133.63	133.77	-0.10	209.68	210.02	-0.16
2	275.07	277.14	-0.75	302.06	303.44	-0.46	261.03	262.35	-0.51	282.86	284.03	-0.41
3	275.07	277.14	-0.75	302.14	303.44	-0.43	304.33	303.58	0.25	335.26	334.58	0.20
4	436.95	437.56	-0.14	459.22	458.78	0.10	412.97	414.12	-0.28	431.30	432.20	-0.21
5	537.84	537.53	0.06	555.11	553.47	0.30	512.75	514.65	-0.37	527.32	528.99	-0.32
6	540.75	539.00	0.32	563.54	560.38	0.56	521.05	521.97	-0.18	539.63	540.20	-0.10
7	678.50	677.16	0.20	694.00	690.76	0.47	632.02	635.91	-0.62	643.40	646.85	-0.53
8	678.51	677.16	0.20	694.27	690.76	0.51	638.19	641.10	-0.46	651.19	653.74	-0.39
9	865.63	867.14	-0.17	877.45	876.09	0.15	831.42	835.53	-0.49	841.10	844.63	-0.42
10	865.64	867.14	-0.17	878.02	876.09	0.22	836.58	839.91	-0.40	846.45	849.08	-0.31

Next, to validate the electromechanical response of the system, displacement and voltage FRFs are calculated using Eqs. (34) - (38), and the results are compared to those obtained using FE simulations. Four different case studies are investigated: **(i-ii)** one patch on a straight/curved host panel, **(iii-iv)** two patches on a straight/curved host structure with parallel connection (the geometries, configurations, and excitation and response locations are shown in Figure 4.3 and Table 4.1). Figure 4.5 shows the structural and voltage FRF comparisons for cases of **(i-ii)** in the open circuit ($R=10^9 \Omega$) condition. As shown, the results obtained using the ST and FE methods are perfectly matching. Note that, the piezo is centered on the host panel for the one-patch configuration, however, there is no limitation in terms of the location or the size of the piezo patch in the presented solution technique as described in the model derivation part.

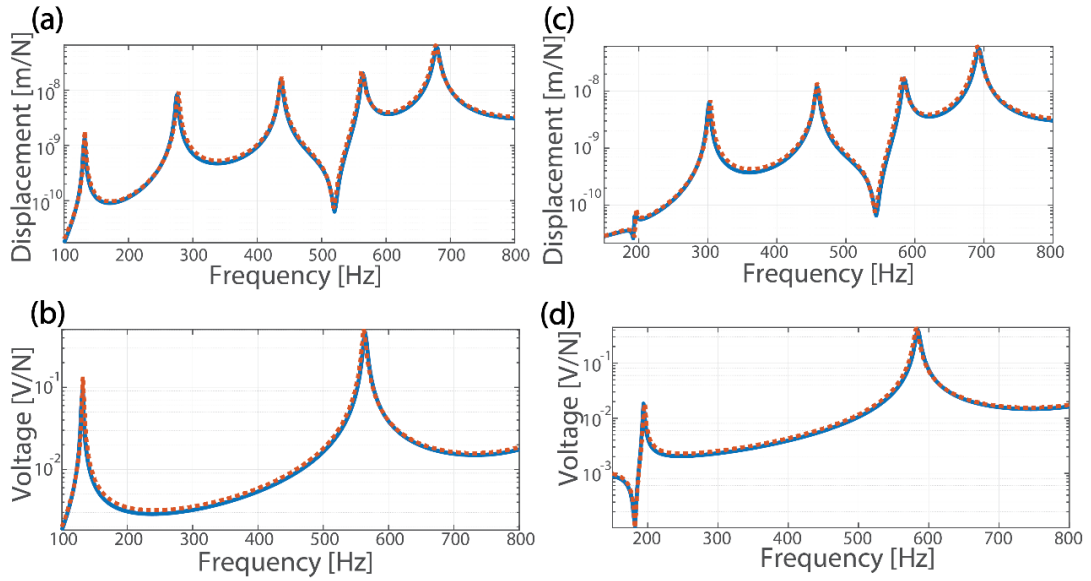


Figure 4.5. (a,c) Structural and (b,d) open circuit voltage FRFs for (a, b) straight and (c, d) curved panel with (centered) piezo patch ($a_s = 500$ mm, $b_s = 500$ mm, $h_s = 4$ mm, $a_p = 200$ mm, $b_p = 200$ mm, $h_p = 1$ mm) The solid blue and dashed red lines represent the results calculated using FE and presented ST solution methods.

The results in the case of multiple patches are shown in Figure 4.6 for parallel configuration. Similar to the one-patch case, the results for multiple patch configurations (Figure 4.3) are in close agreement with FE results. The comparison for independent configuration can be seen in Appendix A.

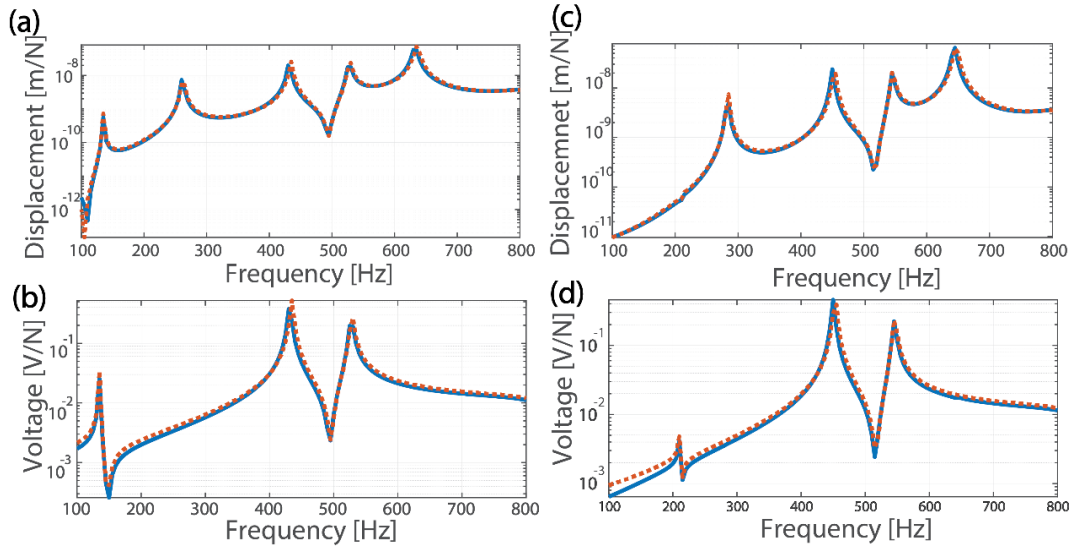


Figure 4.6. (a,c) Structural and (b,d) open circuit voltage FRFs for (a,b) straight and (c,d) curved panel with two piezo patches connected in parallel, ($a_s = 500$ mm, $b_s = 500$ mm, $h_s = 4$ mm, $a_p = 150$ mm, $b_p = 150$ mm, $h_p = 1$ mm). The solid blue and dashed red lines represent the results calculated using FE and ST solution methods.

4.3 Mode jumping phenomenon and advantages for energy harvesting

In this section, we investigated the benefits of using multiple patches compared to single patch configuration, especially for (doubly-) curved panels. As the curvature increases, mode jumping/shifting occurs (Serhat et al. under review). For instance, Figure 4.7 shows the effect of curvature on the first mode shape (*i.e.* the mode shape corresponding to the fundamental frequency) of the panel structure having a size of $a_s = 500$ mm, $b_s = 500$ mm, $h_s = 1$ mm under fully clamped (CCCC) boundary condition. It can be seen that the mode jumping occurs even with a small curvature; for instance, the first mode shape for $a/r_1 = b/r_2 = 0.1$ case, that is a repeated mode which is similar to the second mode shape of the straight panel. Note that, as the curvature increases (*e.g.* $a/r_1 = b/r_2 = 0.5$), even higher deformation modes corresponding to the straight panel becomes the first mode shape. This phenomenon can be used as an advantage to increase the harvested energy when multiple patches are used instead of a single patch configuration as shown in Figure 4.8. The harvested energy can be even improved significantly using the proper configurations of the patch pole connections.

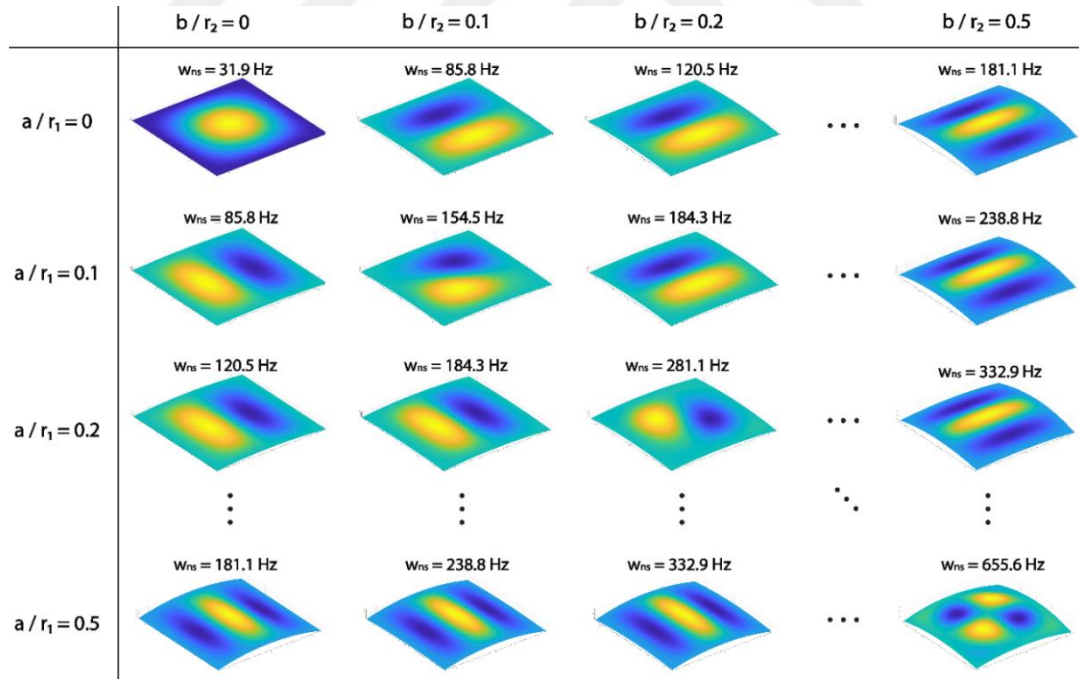


Figure 4.7. Fundamental frequency mode shape jumping effect due the curvature for a panel of with dimensions of $a_s = 500$ mm, $b_s = 500$ mm, and $h_s = 1$ mm.

To better visualize the benefits of multiple patches for energy harvesting purposes (using the mode jumping phenomenon), voltage FRFs are calculated for the piezo/panel geometries and configurations shown in Figure 4.8. Note that the excitation force is in

the form of a harmonic excitation as $f = \bar{F}e^{-i\omega t}$ in which $\bar{F}$ assumed to have 1 N amplitude. Thus, by sweeping the excitation frequency (ω), the voltage FRF can be obtained by dividing the voltage output by the force input amplitude. In this model, the curvature is set to be $a/r_1 = b/r_2 = 0.1$, and the area covered by patches is set to be: **(i)** $a_p = 400$ mm, $b_p = 400$ mm, $h_p = 0.2$ mm for single patch and located on the center of the panel and **(ii)** $a_p = 194$ mm, $b_p = 194$ mm, $h_p = 0.2$ mm and the centers of each piezoelectric patch are located at $[x, y] = [-100$ mm, -100 mm], $[x, y] = [-100$ mm, 100 mm], $[x, y] = [100$ mm, -100 mm] and $[x, y] = [100$ mm, 100 mm].

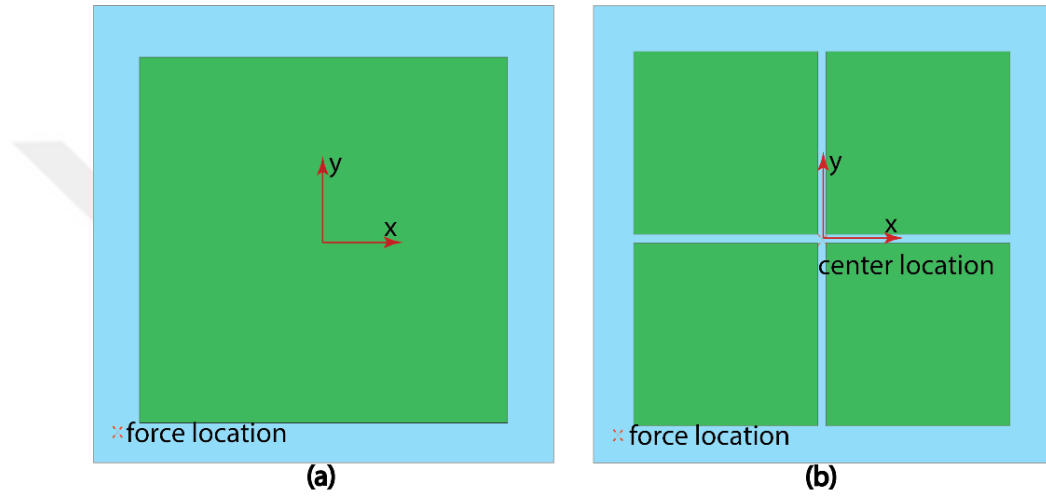


Figure 4.8. Configurations of (a) one patch and (b) four patches on the doubly-curved host structure. Green and blue squares represent the piezoelectric patches and host panel, respectively.

For the one patch configuration, there are no other alternatives for achieving high voltage. However, in four patch configuration, for each set of patches, there are two possible options: **(a)** Positive configuration in which the outer surfaces of the patches are connected to the positive end of the resistor and inner surfaces of the patches are connected to the negative end of the resistor; **(b)** Negative configuration in which the outer surfaces of the patches are connected to the negative end of the resistor and inner surfaces of the patches are connected to the positive end of the resistor. The positive and negative configurations are demonstrated in Figure 4.9 in detail. For constructing the four patches configuration, there are several options and here just four of them are considered as shown in Figure 4.9. Consequently, all of the patches are connected in parallel to the load and the positive and negative configurations mean that the interconnections of the patches are different.

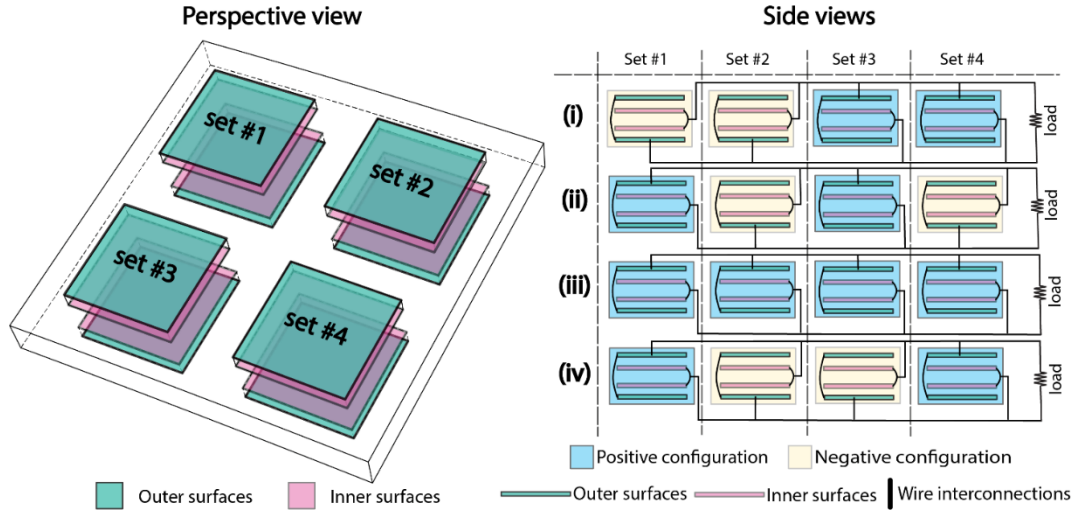


Figure 4.9. Demonstration of the four patches case that leads to the positive and negative configurations.

The first six mode shapes of the coupled electromechanical system are plotted in Figure 4.10(a). The connection configuration of the piezo-patches is determined based on the deformation behavior. In this process, we label the patches which are located on the positive peaks of the mode shapes as a positive pole with ‘+’ sign and on the negative peaks with ‘-’ sign. Considering the first six mode shapes, four unique connection configurations are chosen to increase the harvested voltage within the interested frequency range. Then, these four unique configurations can be obtained by connecting the piezo patches placed on the locations that have the same sign. Thus, these unique configurations can be labeled as (i), (ii), (iii) and (iv) (see Figure 4.9 for the configuration options) to achieve the best possible performances in the frequency ranges where the first, second, third, and fourth mode shapes of the structure are the dominant deformation mechanism, respectively. Figure 4.10(b) shows the comparison of voltage FRFs for one piezo patch case and four piezo patches cases corresponding to the four unique connection configurations. Several important observations can be inferred based on the FRF simulations. First of all, in the first and second modes (repeated modes), half of the panel deforms upward, the other half deforms downward; thereby either in one piezo patch case or configurations (iii) and (iv), the total deformation experienced by the piezo patch is canceled out because of symmetry and this leads to a low voltage output. Secondly, for the configurations (i) and (ii), higher voltage output can be observed around the first natural frequency (154.46 Hz) due to the fact that the poles of the patches are configured such that the cancellation effect is minimized. As a result, the connection

configuration of piezo-patches can be designed to maximize the voltage output around a specific frequency range of interest.

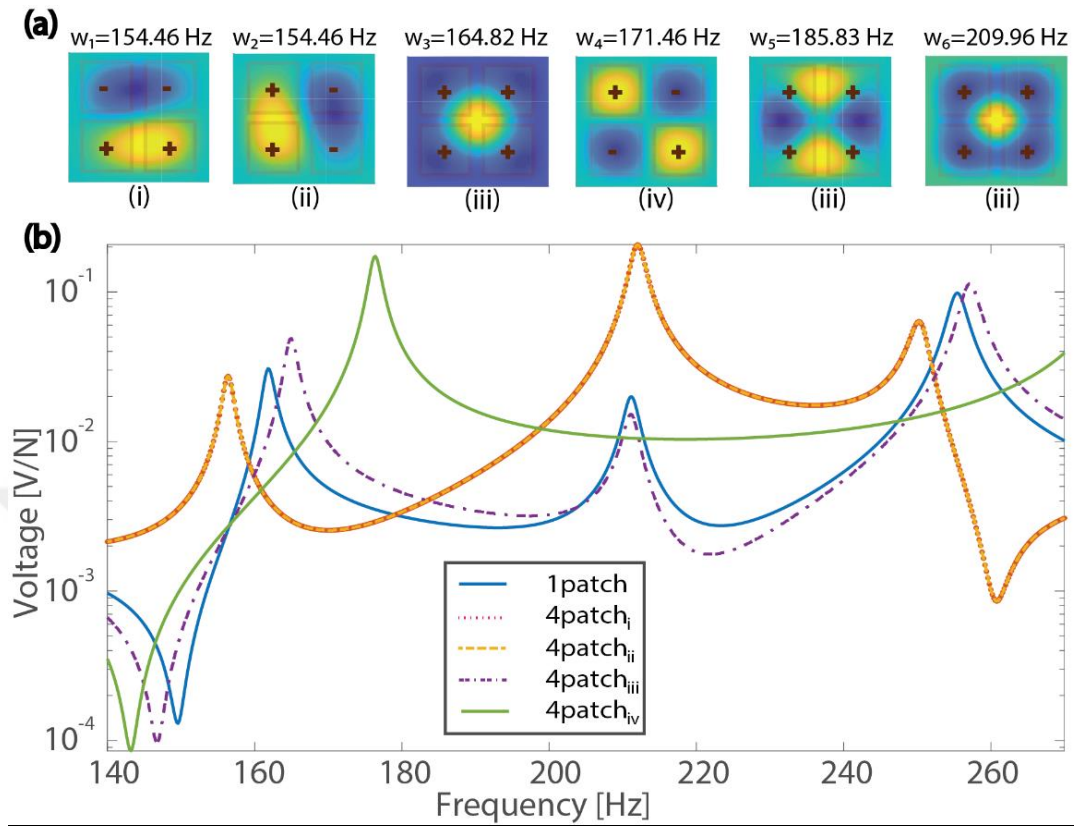


Figure 4.10. Voltage FRFs and mode shapes for the case of one patch versus four patches: (a) mode shapes; (b) voltage FRFs. The solid blue line represents the one patch case, dashed red, dashed orange, dashed purple, and solid green lines represent the cases (i)-(iv), respectively.

Furthermore, it is shown that the voltage output in the entire frequency range can be significantly increased using multiple piezo patches as demonstrated in Figure 4.11 if a switching circuitry that enables to change the configuration of piezo patches automatically based on the excitation frequency is used. It shows due to the new configuration of four patches, voltage output near the first mode increased more than 8 folds compared to one patch configuration and also the overall voltage output over the first ten modes of the systems can also be increased more than 2 folds compared to one patch configuration.

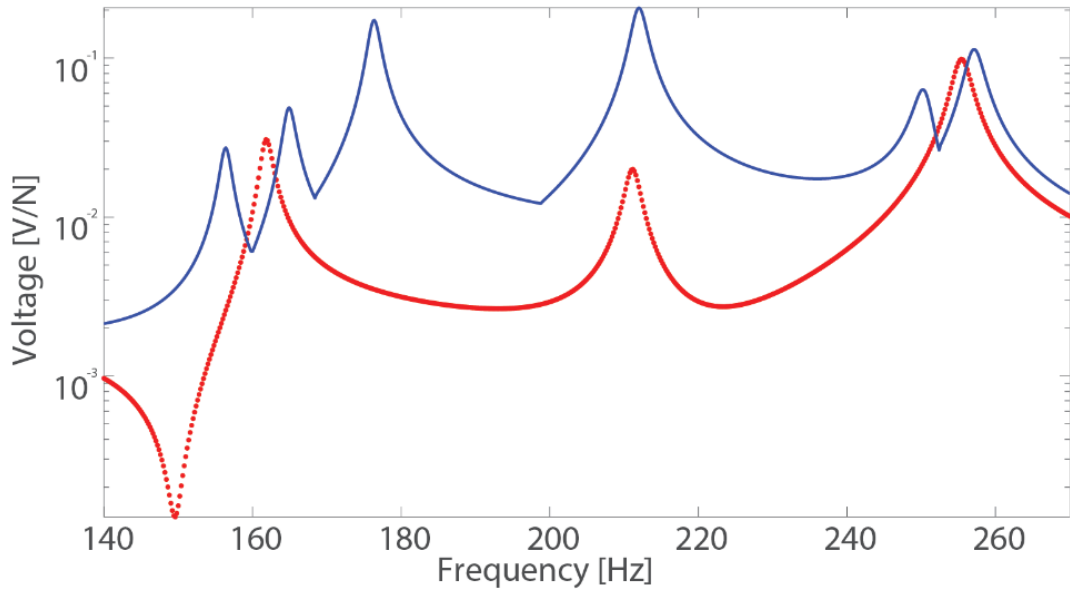


Figure 4.11. Voltage FRF comparison of multiple patches with a switching circuitry and one patch cases. Solid blue and dashed red lines represent voltage FRF corresponding to the multiple piezo patches and one patch case, respectively.

4.4 Conclusion

In this study, a novel accurate and efficient modeling framework to predict the dynamic behavior of (doubly-) curved panels integrated with multiple piezo patches is presented. The BVP that governs the electro-mechanical behavior is derived based on FSDT. In order to couple the piezoelectric patches and the host structure, a novel coupling algorithm based on component mode synthesis is developed. The modal analysis (to predict natural frequencies and mode shapes) and harmonic analysis (to obtain voltage and displacement FRFs) are solved numerically using a two-dimensional spectral-Tchebychev (ST) approach. To demonstrate the accuracy and performance of the presented solution technique, several case studies including straight and curved panels integrated with multiple piezo patches are investigated. The predicted results are compared to those obtained using a commercial FE software (COMSOL v5.5). It is shown that the results are in excellent agreement both for the predicted natural frequencies and voltage/displacement FRFs.

To demonstrate the effect of the mode jumping phenomenon in curved panels, we performed several case studies. Due to this phenomenon, the deformation behavior (mode shape) corresponding to the fundamental natural frequency changes depending on the curvature amount. This change may lead to a decrease in harvested energy compared to straight panels if the piezo-patch configuration is not selected properly since the

deformations may cancel out the harvested energy, especially in the case of single patch cases. Therefore, it is shown that multiple piezo patches connected in parallel configuration considering the deformation behavior around the frequency range of interest enable to improve the voltage output of the system at the frequency range of interest. Furthermore, the effect of the possibility of changing the connection configuration electronically on harvested energy is investigated; and it is shown that the harvested energy can be increased significantly in the entire frequency range of interest.

In future studies, the modeling approach can be extended to model the electro-mechanical behavior including the varying voltage distributions through the thickness of the piezo-patches using a LW approach. In such studies, the modeling approach can also be extended to accurately model the thicker host plates and piezo-patches.



Chapter 5

5 Conclusion and future work

This thesis consists of three chapters in which, firstly a new methodology and a formulation for separately shunted piezoelectric patches for achieving higher performance on vibration attenuation is developed. The Rayleigh-Ritz method was used for performing the modal analysis of the electro-mechanical system. The developed model includes mass and stiffness contribution of the piezoelectric patches as well as the electromechanical coupling effect. The shunt damping performance of the separately shunted piezoelectric patches were compared with the connected configurations. An experimental setup was also built to validate the performance of the separately shunted piezoelectric patches. Finite element simulations were performed in ANSYS and compared with the analytical model results for verification of the presented methodology. A thorough investigation was performed by changing the area, thickness and number of the patches to improve the shunt damping performance and demonstrate the effect of those parameters. It was shown that separately shunted piezoelectric patches were more effective compared to connected patches and also more effective when broadband vibration attenuation was considered. Further improvements were also observed when the number of patches were increased while keeping the same effective area. Finally, an optimization study was performed to investigate the effect of the distribution of the resistor values for the first three modes of the structure. It was shown that the distribution of the optimized resistor values followed a pattern very similar to the strain mode shapes of the targeted modes and at the same time provided an improvement of $\sim 12\%$ in the $(\overline{ERP})$ metric. Future studies can be conducted by adding more electrical circuit elements (i.e. Inductance) to the system and their design parameters can be optimized using a similar procedure.

In addition, a general electromechanical modeling framework for thin and moderately thick plates with structurally integrated piezo-patches was developed using spectral Tchebychev (ST) method. This method has been shown to have the exponential convergence and superior stability characteristics, enabling prediction of the dynamics of mechanical structures accurately and efficiently. The boundary value problem is derived through an energy based approach using Mindlin plate theory. Having incorporated any type of boundary condition into a boundary value problem via

projection matrices approach, the need to use different basis/trial functions for different boundary conditions is eliminated. Furthermore, in this study a novel approach was developed to couple the individually modeled host plate and the piezo-patches and thus to construct the governing equations of motions as a boundary value problem. The electromechanical coupling terms, that behave as compatibility equations, were imposed to the system matrices similar to boundary conditions. Therefore, the overall system's mass and stiffness components can be used in the eigenvalue and eigenvector analyses as well as the vibration and electrical frequency response calculations; where this consideration has been typically ignored in the other studies focusing on active/passive vibration control and energy harvesting areas. To validate the accuracy and demonstrate the efficiency of the presented framework, several case studies were presented and the results of modal analysis and frequency responses of the system were compared to those obtained by a commercial finite-element (FE) software and the Rayleigh-Ritz method. It was shown that the RR method is accurate for the thin plate case, but its accuracy for prediction of the electromechanical response fails for the thicker plate configurations. Therefore, the developed ST model in this study provides a more general and reliable solution for predicting the electroelastic dynamics of smart plate structures. On the other hand, the results obtained using the presented ST approach are in excellent agreement with FE results. Moreover, the computational cost of the developed method was shown to be significantly lower than that of FEM simulations. Therefore, the presented approach can be employed as an efficient and accurate framework for modeling the electroelastic dynamics of two dimensional plates with structurally integrated piezo-patches in a variety of vibration control and energy harvesting applications.

Finally, a novel accurate and efficient modeling framework to predict the dynamic behavior of (doubly-) curved panels integrated with multiple piezo patches is presented. The Boundary value problem (BVP) that governs the electro-mechanical behavior is derived based on First shear deformation theory (FSDT). To couple the piezoelectric patches and the host structure, a novel coupling algorithm based on component mode synthesis is developed. The modal analysis (to predict natural frequencies and mode shapes) and harmonic analysis (to obtain voltage and displacement FRFs) are solved numerically using a two-dimensional spectral-Tchebychev (ST) approach. To demonstrate the accuracy and performance of the presented solution technique, several case studies including straight and curved panels integrated with multiple piezo patches are investigated. The predicted results are compared to those

obtained using a commercial FE software (COMSOL v5.5). It is shown that the results are in excellent agreement both for the predicted natural frequencies and voltage/displacement FRFs. To demonstrate the effect of the mode jumping phenomenon in curved panels, we performed several case studies. Due to this phenomenon, the deformation behavior (mode shape) corresponding to the fundamental natural frequency changes depending on the curvature amount. This change may lead to a decrease in harvested energy compared to straight panels if the piezo-patch configuration is not selected properly since the deformations may cancel out the harvested energy, especially in the case of single patch cases. Therefore, it is shown that multiple piezo patches connected in parallel configuration considering the deformation behavior around the frequency range of interest enable to improve the voltage output of the system at the frequency range of interest. Furthermore, the effect of the possibility of changing the connection configuration electronically on harvested energy is investigated; and it is shown that the harvested energy can be increased significantly in the entire frequency range of interest.

5.1 Future work

Due to the number of variables, the case studies would increase significantly and make the computational cost very high. For that reason, various optimization methods and modeling approaches can be explored to reduce the computational cost in future studies to extend work of chapter 2. Additionally, workload of chapter 3 can also be extended to accurately model the thicker host plates and piezo-patches. And later, in future studies, the modeling approach can be extended to model the electro-mechanical behavior including the varying voltage distributions through the thickness of the piezo-patches using a LW approach.

In addition, host structures made of composite materials can be also studied to improve the energy produced by the electromechanical system. Especially composites produced by functionally graded materials can be a good candidate for exploring these new techniques for vibration control/energy harvesting of electromechanical systems.

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Appendix A

This section demonstrates the comparison between FE and ST results for the case of two piezo-patches and independent configuration. As shown in Figure ., excellent agreement is observed between the ST and FE results.

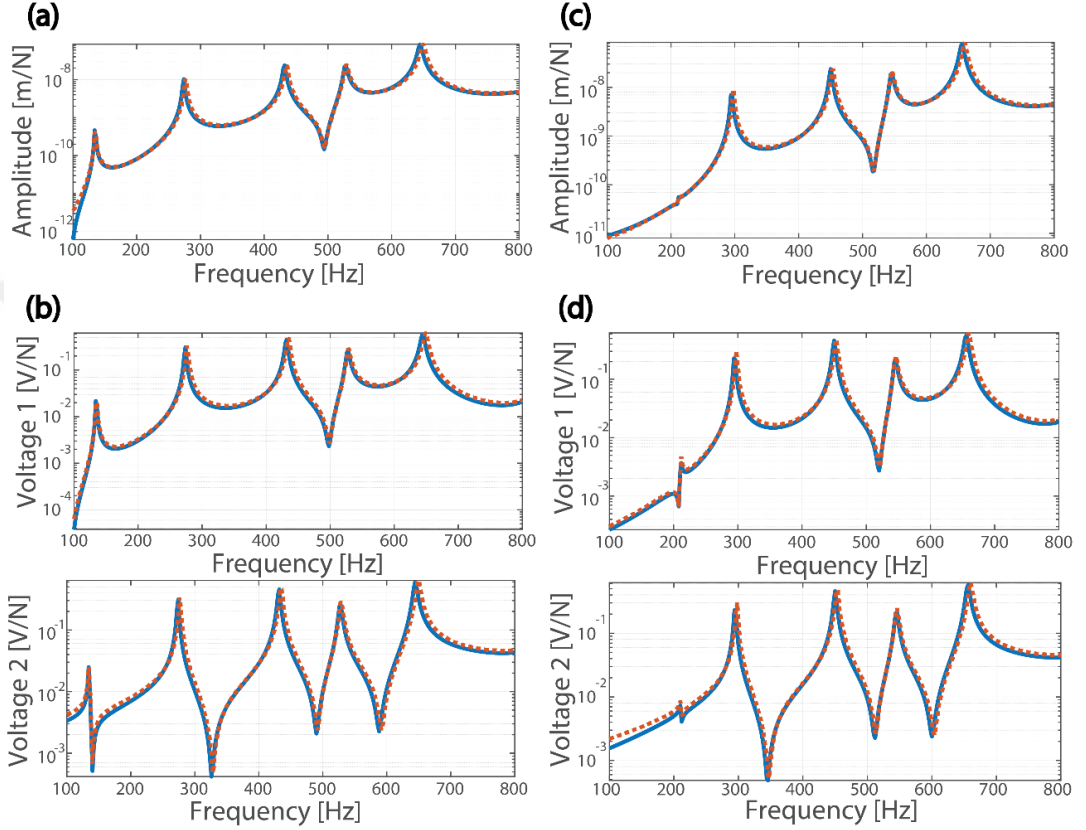


Figure A.1. (a,c) Structural and (b,d) open circuit voltage FRFs for (a,b) straight and (c,d) curved panel with independently connected two piezo patches, having the geometry of $a_p = 150$ mm, $b_p = 150$ mm, $h_p = 1$ mm, $a_s = 500$ mm, $b_s = 500$ mm, $h_s = 4$ mm. The solid blue and dashed red lines represent the results calculated using FE and presented ST solution methods.