

**Numerical Modeling of Dispersive and Nonlinear Pulse
Distortions in a Cr²⁺:ZnSe Regenerative Amplifier**

by

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**A Thesis Submitted to the
Graduate School of Engineering
in Partial Fulfillment of the Requirements for
the Degree of**

**Master of Science
in
Optoelectronics and Photonics Engineering**

Koc University

August 2013

Koc University
Graduate School of Sciences and Engineering

This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by the final
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ABSTRACT

$\text{Cr}^{2+}:\text{ZnSe}$ is an attractive gain medium for ultra-short pulse generation and amplification in the mid IR spectrum at room temperature. It shows a high and broadband stimulated emission cross section coupled with a quantum yield of almost unity. A drawback is the exceptionally high nonlinear refractive index of $1.7 \cdot 10^{-18} \text{ m}^2/\text{W}$, which makes it susceptible to self-phase modulation effects. In amplifiers the pulse shaping effects should be calculated to make an ideal recompression possible. In the thesis work presented here, we demonstrate a numerical approach to determine both the pulse power and the dispersive broadening of a pulse, while it is propagating in a regenerative amplifier. The Frantz-Nodvik equations were extended by accounting for distortion effects from self-phase modulation and media dispersion for a Gaussian pulse. The result is a numerical code that determines the ideal extraction time and the ideal compressor dispersion for a $\text{Cr}^{2+}:\text{ZnSe}$ regenerative amplifier. The code has been successfully used with experimental results to estimate the stimulated emission cross section of the amplifier gain medium and the amount of chirp in the pulse. The best-fit value of the stimulated emission cross section for the $\text{Cr}^{2+}:\text{ZnSe}$ medium was determined to be $2.055 \cdot 10^{-24} \text{ m}^2$.

ÖZET

$\text{Cr}^{2+}:\text{ZnSe}$ kristali, oda sıcaklığında ve orta kızılaltı bölgesinde kısa darbe üretiminde ve darbe yükselteçlerinde kullanılan, birçok elverişli fiziksel özelliğe sahip bir katıhal kazanç ortamıdır. Bu kristal, geniş bir dalgaboyu aralığında kuvvetli uyarılı ışıma ara kesitine ve oda sıcaklığında neredeyse birim kuantum verimliliğine sahiptir. Ancak, $\text{Cr}:\text{ZnSe}$ kristalinin yüksek değere sahip doğrusal olmayan kırınım katsayısı ($n_2=1.7 \times 10^{-18} \text{ m}^2/\text{W}$), kazanç ortamından geçmekte olan optik darbelerde faz bozulumuna sebep olmakta ve bu durum, bazı uygulamalar için dezavantaj yaratabilmektedir. Lazer darbe yükselteçlerinin tasarımında, darbeleri bozulmasına neden olan etkilerin hesaplanması, en uygun sıkıştırma yöntemini belirleyebilmek adına oldukça önemlidir. Bu çalışmada, sayısal modelleme yöntemleri kullanılarak, üretilen darbelerin rejeneratif yükselteç içerisinde ilerlerken sahip olduğu güç ve dispersiyona bağlı genişlemeleri incelenmiştir. Frantz-Nodvik denklemleri, faz modülasyonu ve darbe profil biçimini bozan etkiler dahil edilerek genelleştirilmiş ve Gauss profiline sahip bir darbe için sayısal hesaplamalarda kullanılmıştır. Sonuç olarak, $\text{Cr}^{2+}:\text{ZnSe}$ rejeneratif yükselteçleri için, darbe sıkıştırıcısının sahip olması gereken ideal dispersiyon miktarını ve darbenin yükselteçten çıkartılması için gerekli en uygun zamanı hesaplayan sayısal bir program geliştirilmiştir. Deneysel veriler kullanılarak, darbe yükseltecinin uyarılı ışıma kesiti ve darbenin sahip olduğu artı dispersiyon miktarı bu çalışmada geliştirilen program ile başarılı bir şekilde hesaplanmıştır. Sayısal program kullanılarak yapılan hesaplamalarda, $\text{Cr}^{2+}:\text{ZnSe}$ kazanç ortamı için en uygun uyarılı ışıma arakesit değeri $2.06 \times 10^{-24} \text{ m}^2$ olarak belirlenmiştir.

ACKNOWLEDGEMENTS

Koç University gave me the chance to work and graduate in the field of Optoelectronics and Photonics Engineering, while supporting me with a generous scholarship. I am grateful for being able to work in the Laser Research Laboratory (LRL) and being supervised by Prof. Dr. Alphan Sennaroğlu throughout the past two years. I thank my advisor for his guidance and for assigning me this interesting research topic as it considerably extended my theoretical understanding of pulse propagation and pulse shaping.

I thank all of my group members for their support and expertise. Especially, I would like to thank Ersen Beyatlı, Natali Çizmeciyan and Dr. Hüseyin Çankaya for performing the experiments on the regenerative $\text{Cr}^{2+}:\text{ZnSe}$ amplifier. Their work was essential as it provided both the motivation and the necessary experimental results for the work presented in my thesis. Also, I would like to thank Asst. Prof. Dr. Şükrü Ekin Kocabaş and Asst. Prof. Dr. Uğur Ünal for finding the time to serve in my thesis committee and giving valuable comments.

Valerie Taşiran who was my lecturer in the academic writing course, helped me immensely by polishing up my writing style and proofreading my thesis. My colleague and friend Solmaz Naghizadeh also spent much time discussing and giving many helpful comments on how to present and discuss data with a good academic style.

I would also like to warmly thank my family in Germany for their support, love and patience while I was studying a thousand kilometers away in Istanbul. It is them, to whom I dedicate this thesis.

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Abbreviations

SVE **S**lowly **V**arying **E**nvelope
GDD **G**roup **D**elay **D**ispersion
TOD **T**hird **O**rders **D**ispersion
CPA **C**hirped **P**ulse **A**mplification
SPM **S**elf-**P**hase **M**odulation

HR **H**igh **R**eflektor
M **M**irror
OC **O**utput **C**oupler

PR **P**Rism
GR **G**Rating

PC **P**ockels **C**ell
FR **F**araday **R**otator
PBS **P**olarizing **B**eam**S**plitter
OPO **O**ptical **P**arametric **O**scillator
TFP **T**hin-**F**ilm **P**olarizer

KTP **KTiOPO₄** (potassium titanyl phosphate)

$\lambda/2$ Half Waveplate

$\lambda/4$ Quarter Waveplate

Physical Constants

speed of light	c	$=$	$2.997\,924\,58 \times 10^8 \text{ m/s}$
Planck's constant	h	$=$	$6.626\,069\,57 \times 10^{-34} \text{ Js}$
reduced Planck constant	$\hbar$	$=$	$1.054\,571\,73 \times 10^{-34} \text{ Js}$
permittivity of free space	ϵ_0	$=$	$8.854\,187\,82 \times 10^{-12} \text{ Fm}^{-1}$

Symbols

E	electric field
B	magnetic field
D	electric displacement field
P	polarization density
χ	susceptibility
ϵ_0	dielectric constant of vacuum
n	refractive index
t	time
$a_{(t)}$	pulse envelope
$\varphi_{(t)}$	phase-shift
$\Delta\omega_{(t)}$	dispersion induced frequency shift
λ	wavelength
Ω	angular frequency
Ω_0	central angular frequency
$\tilde{a}_{(\Omega)}$	pulse spectrum
$\phi_{(\Omega)}$	spectral phase

ϕ''	group delay dispersion
ϕ'''	third order dispersion
n_2	non-linear refractive index
φ_{max}	maximum phase-shift (induced by TOD)
W	energy
F	photon flux
I	intensity
N	upper state population
σ	absorption cross section
J	fluence
f	focal length
d	grating period

Chapter 1

Introduction

Pulsed laser sources have always been of special interest to scientists as they allowed for a higher energy density than continuous lasers. When the time-bandwidth product of such a pulse approaches the boundaries set by Heisenberg's uncertainty relation coupled with a large spectral width, we refer to them as ultra short pulses. Laser sources, capable of generating these pulses have enabled many new research applications in various disciplines. In bio-medical imaging an ultra fast laser source can be used for two photon microscopy, to obtain three-dimensional images of living tissue[4]. This technique has both higher penetration depth and lower background noise than confocal microscopy. In material processing the high peak intensity of these lasers makes it possible to cut through material before a thermal equilibrium can be reached. The preferred gain media for these kind of lasers are solid state bulks or fibers. The most important property of such a material is a broad fluorescence spectrum. The necessary width in frequency domain will result in a short wave package in time domain. For the work presented in this thesis, a $\text{Cr}^{2+}:\text{ZnSe}$ gain medium was used. This medium is a II-VI compound semiconductor, doped with the transition metal Cr^{2+} [5]. This gain medium shows a high quantum efficiency due to its lack of excited state absorption. It can be pumped at around 1800 nm, which is suitable for Tm- and Er- based Lasers. The emission ranges from 2 to 3 μm in the mid infrared spectrum, which translates to a minimum pulse length of just

under 20 fs in mode-locked operation. $Cr^{2+}:ZnSe$ is relatively cheap in production and well-suited for applications like higher harmonic generation, spectroscopy or the pumping of optical parametric oscillators.

Mode-locked laser cavities are not able to produce arbitrarily high output powers. High energy density will be an issue when the pump power is increased or cavity length is extended. Due to thermal lensing and non-linear dispersion the laser becomes unstable. Non-linear effects like self-phase modulation can even lead to a splitting of the pulse, which is known as double pulsing. The laser power has to stay below a threshold, so that a stable operation is ensured. One solution to the instability issue is having a dispersed pulse traveling in the cavity. As the pulseduration is longer, the gain medium is exposed to a decreased energy flux. However a dispersed pulse does not contain the full emission spectrum which limits the compressibility. To overcome these shortcomings it makes sense to split the laser pulse generation into two stages. A seed resonator that generates low-energy, clean, transform-limited pulses and an amplification stage. If, however, the pulse were injected directly into an amplifier, the same issues that are present in a high energy ultra short laser, would be present in the amplifier as well. The energy flux has to be reduced before the pulse enters the amplification stage. This can be achieved by stretching the pulse with grating stretcher. The pulse maintains its full spectrum and can be re-compressed after the amplification step is complete. This is the concept of chirped pulse amplification.

Amplification is an actively controlled process and the ideal duration has to be determined for which the pulse should stay in the cavity. If it gets extracted from the amplifier too soon, it will not fully utilize the given energy. However, If it overstays, it will start to decay due to absorption. The build up and decay of a trapped pulse

can be modeled with the Frantz-Nodvik equations[6]. This method has also been successfully applied to zig-zag beam geometries[7][8] and diverging laser beams[9]. The Frantz-Nodvik equations have also been modified for application in broad gain spectra[10]. Using this modification it is possible to simulate the effects of gain narrowing in the amplification process. It has also been shown that a modified version of the Frantz-Nodvik equations can be used to model the interaction of a short pulse with solid state saturable absorbers[11]. However, dispersion from the optical components in the amplifier stage has a strong impact on the amplification process and need to be considered. Especially so, considering the high nonlinear refractive index of $n_2 = 170 \times 10^{-20} \frac{m^2}{W}$ [2] for $Cr^{2+}:ZnSe$, compared to only $3 \times 10^{-20} \frac{m^2}{W}$ in $Ti:Saph$. After pulse extraction it is crucial to determine the net chirp that was imposed on the pulse. Otherwise we can not re-compress the pulse to its bandwidth-limited duration. The pulse shaping mechanisms are partly of linear and partly of non-linear nature. The non-linear contribution comes from the self-phase modulation. This effect is dependent on intensity and spectral phase of the propagating pulse. It has been shown that the chirp from self-phase modulation can be calculated from energy and original chirp of the pulse[12]. However, spectral phase and intensity are interconnected. The Frantz-Nodvik equations have to be extended in order to determine both the pulse energy and chirp. In this work I developed a program that calculates both the ideal amplifier extraction times and the accumulated dispersion from linear and non-linear effects. The program was further used to indirectly determine the stimulated emission cross-section of the amplifier gain medium. Pulse energy and shape are calculated iteratively for each round-trip in the amplifier cavity. This is an approximation for Gaussian pulses and a non wavelength-dependent gain spectrum. These approximations allow us to use compact mathematical expressions for both the intensity and dispersion. This has

proven to be a valid approximation as it agrees with the autocorrelation results of the seed laser(see figure 3.8). The obtained results are sufficient to provide a guideline on both the pockels cell timings and the compression stage dimensions. However, if further adjustment is needed, it can be done from output power measurements or autocorrelation results.

Chapter 2

Theory of Pulses and Dispersion

2.1 Analysis of Pulse Propagation with Kerr Non-linearity

To understand dispersion, the basic equations that govern the propagation of a Gaussian pulse have to be looked at. This derivation can be found in the book *Fundamentals of Photonics* [13] in more detail. We start from Faraday's law of induction:

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (2.1)$$

Now we act with $\nabla \times$ from the left and use $D = \epsilon_0 E + P$.

$$\begin{aligned} \nabla \times (\nabla \times E) &= \nabla(\nabla \cdot E) - \nabla^2 E \\ \nabla \times (\nabla \times E) &= \mu_0 \frac{\partial^2 D}{\partial t^2} \end{aligned}$$

We obtained the wave equation for homogeneous, isotropic medium. Here P is the polarization density.

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2} \quad (2.2)$$

Now we can add the non-linear polarization contribution $P = \epsilon_0 \chi E + P_{NL} = P_0 + P_{NL}$

$$\left[\nabla^2 - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \right] E = \mu_0 \frac{\partial^2}{\partial t^2} (P_L + P_{NL}) \quad (2.3)$$

The above expression has to be transformed into Ω domain. Further we introduce the propagation parameters $\beta(\Omega)$ and $\beta(0) = \beta(\Omega_0) = \frac{\Omega_0}{c}$, where Ω_0 is the central frequency.

$$[\nabla^2 + \beta(\Omega)^2] \tilde{E} = \mu_0 \Omega^2 \tilde{P}_{NL} \quad (2.4)$$

This is the non-linear Helmholtz equation.

We now introduce the frequency in relation to the central frequency $\Delta\Omega = \Omega - \Omega_0$. Further we split $\tilde{E}$ and $\tilde{P}$ into an envelope and an oscillating term.

$$\begin{aligned} \tilde{E}(z, \Delta\Omega) &= \tilde{a}(z, \Delta\Omega) \exp(-i\beta_0 z) \\ \tilde{P}_{NL}(z, \Delta\Omega) &= \tilde{a}_{NL}(z, \Delta\Omega) \exp(-i\beta_0 z) \end{aligned}$$

We can bring (2.4) into following form.

$$\left[\frac{\partial^2}{\partial z^2} + \beta(\Omega)^2 \right] \tilde{a}(z, \Delta\Omega) \exp(-i\beta_0 z) = -\mu_0 \Omega^2 \tilde{a}_{NL}(z, \Delta\Omega) \exp(-i\beta_0 z) \quad (2.5)$$

Now we need a number of assumptions to proceed. We assume Ω to be close to the central frequency, so $\Omega^2 \approx \Omega_0^2$. We also apply the slowly varying envelope(SVE) approximation:

$$\left(\frac{d^2}{dz^2} \right) [\tilde{a} \exp(-i\beta_0 z)] \approx \left[-i2\beta_0 \frac{d\tilde{a}}{dz} - \beta_0^2 \tilde{a} \right] \exp(-i\beta_0 z) \quad (2.6)$$

Then we obtain:

$$\left[-i2\beta_0 \frac{d\tilde{a}}{dz} \right] \tilde{a} + [\beta^2(\Omega_0 + \Delta\Omega) - \beta_0^2] \tilde{a} = -\mu_0 \Omega_0^2 \tilde{a}_{NL} \quad (2.7)$$

Now we assume weak dispersion around the center of the pulse.

$$\beta^2(\Omega_0 + \Delta\Omega) - \beta_0^2 \approx 2\beta_0 [\beta(\Omega_0 + \Delta\Omega) - \beta_0] \quad (2.8)$$

Further we estimate the term $\beta(\Omega_0 + \Delta\Omega)$ to its third Taylor expansion:

$$\beta(\Omega_0 + \Delta\Omega) \approx \beta_0 + \beta' \Delta\Omega + \frac{1}{2} \beta'' \Delta\Omega^2 \quad (2.9)$$

We obtain:

$$-i2\beta_0 \frac{d\tilde{a}}{dz} + 2\beta_0(\Delta\Omega\beta' + \frac{1}{2}\Delta\Omega^2\beta'')\tilde{a} = -\mu_0\Omega_0^2\tilde{a}_{NL} \quad (2.10)$$

Now we assume a plane wave optical pulse and write the resulting non-linear polarization density:

$$\begin{aligned} E &= \text{Re} \{ a(z, t) \exp[-i(\Omega_0 t - \beta_0 z)] \} \\ P_{NL} &= \text{Re} \{ a_{NL}(z, t) \exp[-i(\Omega_0 t - \beta_0 z)] \} \end{aligned}$$

We know the non-linear polarizability with $P_{NL} = 4\chi^{(3)}E^3$ so the envelope function will be $a_{NL} = 3\chi^{(3)}|a|^2 a$. Knowing this, we can now transform (2.10) into time domain and obtain the non-linear SVE wave equation:

$$\frac{1}{2} \left(\frac{\phi''}{l} \right) \frac{\partial^2 a}{\partial t^2} + \rho |a|^2 a + i \left(\frac{\partial}{\partial z} + \frac{1}{u_g} \frac{\partial}{\partial t} \right) a = 0 \quad (2.11)$$

where $u_g = \frac{1}{\beta'}$ is the group velocity and $\frac{\phi''}{l} = \beta''$ the dispersion coefficient. Also, we defined $\rho = \frac{\pi n_2}{\lambda_0 \eta}$ with η being the electromagnetic impedance.

The above equation can be rewritten in the form of the non-linear Schrodinger equation. Here we take t_0 as the pulse length and $z_0 = \frac{t_0^2}{2\beta''}$ as the dispersion length. For normalizing the wave function we further need the envelope peak amplitude with: $a_0 = \sqrt{(-\beta''/\rho)}/t_0$.

Now we bring the variables into their dimensionless form by making use of following normalizations:

$$Z = \frac{z}{2z_0}, \quad \Psi = \frac{a}{a_0} \quad (2.12)$$

Shifting the time into retarded frame of reference, we obtain:

$$T = \frac{t - \frac{z}{v}}{t_0} \quad (2.13)$$

We arrive at the standard form of the non-linear Schrodinger equation:

$$\frac{1}{2} \frac{\partial^2 \Psi}{\partial T^2} + |\Psi|^2 \Psi + i \frac{\partial \Psi}{\partial Z} = 0 \quad (2.14)$$

The simplest solution to this is the fundamental soliton. We start by assuming a space-time separable function $\Psi(Z, T) = \psi(T) \exp[ix(Z)]$. With $\psi(Z, T)$ and $x(Z)$ being real functions. Using separation of variables we can put (2.14) in following form:

$$\begin{aligned} x(Z)' &= \vartheta \\ \psi(T)'' &= 2(\vartheta - \psi^2)\psi \end{aligned}$$

with ϑ being a constant.

This can be solved by direct integration with the boundary conditions $\psi = \psi' = 0$ at $|t| \rightarrow \infty$ and $\psi' = 0$ at $t = 0$ (center of the incident pulse). We obtain:

$$\Psi(Z, T) = \text{sech}(T) \exp\left(i \frac{Z}{2}\right) \quad (2.15)$$

Using the normalizations from 2.12, we arrive at:

$$a(z, t) = a_0 \operatorname{sech} \left(\frac{t - \frac{z}{v}}{t_0} \right) \exp \left(i \frac{z}{4z_0} \right) \quad (2.16)$$

2.2 Medium Dispersion

For a bulk of refractive index n_λ the GDD can easily be defined as the second order derivative of the spectral phase or wave number

$$\phi'' = \frac{\partial^2 \phi}{\partial \Omega^2} = l \frac{d^2 k}{d\Omega^2} \quad (2.17)$$

It can also be written in terms of the wavelength-dependent refractive index:

$$\frac{d^2 k}{d\Omega^2} = \frac{2}{c} \frac{dn}{d\Omega} + \frac{\Omega}{c} \frac{d^2 n}{d\Omega^2} = \left(\frac{\lambda}{2\pi c} \right) \frac{1}{c} \left(\lambda^2 \frac{d^2 n}{d\lambda^2} \right) \quad (2.18)$$

so ϕ'' can be written as:

$$\phi'' = l \frac{\lambda^3}{2\pi c^2} \frac{d^2 n}{d\lambda^2} \quad (2.19)$$

with l being the bulk length. $n(\lambda)$ can be obtained from the appropriate Sellmeier equation. Now we will to obtain the dispersion introduced phase shift for a Gaussian pulse:

$$E(z, t) = \frac{1}{2} a(z, t) e^{i\varphi(t)} \quad (2.20)$$

The Gaussian envelope $a(z, t)$ has the form:

$$a(z, t) = \exp \left[-1.385 \left(\frac{t - \frac{z}{c}}{t_{\frac{1}{2}}} \right)^2 \right] \quad (2.21)$$

The phase shift at position z can be expressed as:

$$\varphi(z, t) = \frac{2\alpha^2 t^2 k'' z}{1 + 4\alpha^2 k''^2 z^2} - \frac{1}{2} \tan^{-1}(2\alpha k'' z) \quad (2.22)$$

where $\alpha = \frac{2\ln(2)}{\Delta t^2}$.

Then we can write the dispersion induced frequency shift $\Delta\omega(t) = \frac{\partial\varphi(z,t)}{\partial t}$, considering $k''z = \phi''$.

$$\Delta\omega(t) = \frac{4\alpha^2 t k'' z}{1 + 4\alpha^2 k''^2 z^2} = \frac{16\phi'' \ln(2)^2}{t_{\frac{1}{2}}^4 + (4\phi'' \ln(2))^2} t \quad (2.23)$$

for an input gaussian pulse with an electric field of:

$$E(z, t) = \frac{1}{2} a(z, t) e^{i\varphi_0} e^{i\omega_0 t + \Delta\omega(t)} \quad (2.24)$$

Where $t_{\frac{1}{2}}$ stands for the full width half max duration of the pulse. The field has an envelope $a(z, t)$ with a respective carrier envelope phase of φ_0 . Further $\omega_0 (= \Omega_0)$ represents the central frequency. The full derivation of (2.23) can be found in the book *Photonics and Laser Engineering*[14].

2.3 Self-Phase Modulation from Third Order Non-Linearities

Third order non-linearities will make the refractive index vary with the beam power[14]. This variation will make the bulk act like a grin lens as the refractive index is higher in the center of the pulse, than at the edge. When the spatial beam profile causes a focusing behavior it is known as self focusing. This section however, is on similar effect, which occurs in time domain. As the pulse center has a higher intensity than the leading or trailing part of the pulse, it propagates with lower speed. The pulse is thereby deformed and shifted in phase, which is referred to as self-phase modulation(SPM). Starting with the equation for SPM:

$$\frac{\partial a}{\partial z} = -i\gamma a(z, t)P(z, t) \quad (2.25)$$

with $\gamma = \frac{2\pi}{\lambda} \frac{n_2}{A_{eff}}$ and $a(z, t)$ being the envelope function of the incoming pulse.

Now we assume that $P(z, t)$ remains constant over a short distance.

$$a(z, t) \simeq a(0, t)e^{-i\gamma P(0, t)z} \quad (2.26)$$

and with a non-linear phase shift of

$$\varphi_{NL}(z, t) = -\gamma P(0, t)z \quad (2.27)$$

Now for a Gaussian input pulse of envelope $a(t) = a_0 e^{-\alpha t^2}$ the phase shift can be expressed as:

$$\varphi_{NL}(z, t) = -\varphi_{max} e^{-2\alpha t^2} \quad (2.28)$$

with $\varphi_{max} = \gamma z P_0$.

From this we can derive the frequency shift $\Delta\omega(t)$, which can then be related to (2.23) :

$$\Delta\omega(t) = \frac{\partial\varphi}{\partial t} = \frac{\partial P(0, t)}{\partial t} \quad (2.29)$$

$$\Delta\omega(t) = 4 \ln(2) \frac{\varphi_{max}}{t_{\frac{1}{2}}} e^{-2\alpha t^2} \frac{t}{t_{\frac{1}{2}}} \quad (2.30)$$

Now to find a GDD equivalent expression for the self phase modulation induced dispersion, we set equations (2.23) = (2.30). We further assume $t_{\frac{1}{2}}$ to be large, compared to $4\phi'' \ln(2)$. Also near the pulse center we take $e^{-2\alpha t^2} \simeq 1$. This is a rough approximation but it leads to a compact expression:

$$\frac{16 \ln(2)^2}{t_{\frac{1}{2}}^2} \phi'' = 4 \gamma z P_0 \ln(2) \quad (2.31)$$

$$\phi'' = \frac{t_{\frac{1}{2}}^2 \varphi_{max}}{4 \ln(2)} \quad (2.32)$$

2.4 Self-Phase Modulation from Third Order Non-Linearities for Heavily Chirped Input Pulses

Formula (2.32) fails as soon as we depart from a soliton-like operation regime. In case of a regenerative amplifier, we have to deal with a heavily chirped input pulse. Before the pulse reaches the amplifier, it passes a stretcher which increases the pulse length by around a 100 fold. However, it is still possible to find an analytic expression for the acquired chirp through non-linearities. We can add dispersion terms to the field in frequency-domain and we can add the non-linear refractive index in time-domain[12]. Using a gaussian pulse envelope(2.21), the field of the pulse can be written as:

$$a_1(t) = a_0 \exp(-\alpha t^2) \quad (2.33)$$

note that $\alpha = \frac{2 \ln(2)}{t_1^2}$ for any Gaussian function. We first bring the above equation in frequency domain in order to add a chirp term.

$$\tilde{a}_1(\Omega) = \frac{a_0}{\sqrt{2\alpha}} \exp \left[-\frac{\Omega^2}{4\alpha} \right] \quad (2.34)$$

Now we add a GDD of Φ'' which is normalized by $\Phi'' = 2\alpha\phi''$

$$\tilde{a}_2(\Omega) = \frac{a_0}{\sqrt{2\alpha}} \exp \left[-\frac{\Omega^2}{4\alpha} (1 + i\Phi'') \right] \quad (2.35)$$

We have to bring $\tilde{a}_{(\Omega)2}$ into time domain, in order to add the non-linear refractive index.

$$a_2(t) = \frac{a_0}{\sqrt{1 + i\Phi''}} \exp \left[-\frac{\alpha t^2}{1 + i\Phi''} \right] \quad (2.36)$$

When applying the normalization constant α , we see that the time-dependent phase-shift is equivalent to $\Delta\omega_{(t)}$ as derived in equation 2.23. In the presence of GDD but without higher order chirp we can assume the Intensity as:

$$I_2(t) = I_0 \exp \left[-\frac{2\alpha t^2}{1 + \Phi''} \right] \approx I_0 \left[1 - \frac{2\alpha t^2}{1 + \Phi''} \right] \quad (2.37)$$

from this we can derive the non-linear refractive index as:

$$n(t) = n + n_2 I_0 - n_2 I_0 \frac{2\alpha t^2}{1 + \Phi''} \quad (2.38)$$

Now we apply $\varphi_{max} = \gamma z P_0$ as introduced in (2.28). The field envelope in time domain can then be written as:

$$a_3(t) = \frac{a_0}{\sqrt{1 + i\Phi''}} \exp \left[-\frac{\alpha t^2}{1 + \Phi''} \{1 - i(2\varphi_{max} + \Phi'')\} \right] \quad (2.39)$$

To extract the net GDD form SPM and original pulse chirp, we need to bring this expression back to frequency domain.

$$\tilde{a}_3(\Omega) = \frac{a_0}{\sqrt{2\alpha}} \sqrt{\frac{1 - i\Phi''}{1 - i(2\varphi_{max} + \Phi'')}} \exp \left[-\frac{\Omega^2 (1 + \Phi'')}{4\alpha \{1 - i(2\varphi_{max} + \Phi'')\}} \right] \quad (2.40)$$

In order to extract the net GDD (Φ''_{out}) from the above equation, it needs to be rearranged according to (2.35). Once in the form of (2.35) the GDD can be determined from the imaginary part of the complex exponential.

$$\Phi''_{out} = \frac{(1 + \Phi''^2)(2\varphi_{max} + \Phi'')}{1 + (2\varphi_{max} + \Phi'')^2} \quad (2.41)$$

Assuming an unchirped input pulse ($\Phi'' = 0$), we see that this expression is analogous to (2.32), which we derived for a soliton-like pulse. From the real part of the exponent of (2.35) we can extract the spectral width.

$$\Delta\Omega = \frac{4 \ln(2)}{t_{\frac{1}{2}}} \sqrt{\frac{1 + (2\varphi_{max} + \Phi'')^2}{1 + \Phi''^2}} \quad (2.42)$$

The third order dispersion(TOD) can be calculated, using a similar approach[12]. The TOD is normalized by $\Phi''' = 2\alpha\Omega_0\phi'''$ with Ω_0 being the central angular frequency.

$$\Phi'''_{out} = \frac{(1 - 3\Phi''^2) \{1 + (2\varphi_{max} + \Phi'')^4\} + 2\Phi''(1 + \Phi''^2)(2\varphi_{max} + \Phi'') \{3 - (2\varphi_{max} + \Phi'')^2\}}{\{1 + (2\varphi_{max} + \Phi'')^2\}^3} \Phi''' \quad (2.43)$$

Note that the TOD contribution from SPM goes to 0, when the input TOD= 0. This means that third order contribution can be neglected in the case of a CPA system, because there is no significant source of TOD other than the self-phase modulation.

2.5 Frantz-Nodvik Equation

First we will derive the Frantz-Nodvik equation which provides the approximation of pulse energy, which in turn will be needed for the calculation of non-linear effects. The full derivation can be found in the book *Principles of lasers*[1]. This is a shortened version.

We start by taking a look at the rate of change in photon energy of a pulse that travels a path dz in the gain medium.

using $\sigma = \frac{W}{F}$ and $F = \frac{I}{h\nu}$ we obtain:

$$\frac{\partial \rho}{\partial t_{gain}} = WNh\nu = \sigma NI \quad (2.44)$$

With F being the photon flux, I the intensity, N the upper state population, W the absorption probability and σ the absorption cross section.

The difference in incoming and outgoing laser power, traveling a length dz in the gain medium, can be written as:

$$\frac{\partial \rho}{\partial t_{shape}} dz = I_{t,z} - I_{t,z+dz} = -\frac{\partial I}{\partial z} dz \quad (2.45)$$

We can combine the above expressions while using $\frac{\partial \rho}{\partial t} = \frac{\partial I}{c \partial t}$:

$$\frac{1}{c} \frac{\partial I}{\partial t} + \frac{\partial I}{\partial z} = \sigma NI \quad (2.46)$$

Which is a time-dependent transport equation. One should note that we neglected any kind of amplifier losses. To further describe the population inversion we need another assumption. We take the pulse duration to be small compared to the upper state lifetime. By this we can neglect both spontaneous emission and the pumping while the pulse is travelling in the amplifier. So we can write:

$$\frac{\partial N}{\partial t} = -WN = -\frac{NI}{J_{sat}} \quad (2.47)$$

with $J_{sat} = \frac{h\nu}{\sigma}$ being the saturation fluence, which can be solved for:

$$N_{\infty} = N_0 e^{-\frac{J}{J_{sat}}} \quad (2.48)$$

With N_0 being the original excited state population. Now we are going back to (2.47) and integrate over time:

$$\int_{-\infty}^{\infty} \frac{NI}{J_{sat}} dt = N_0 - N_{\infty} = N_0 \left(1 - e^{-\frac{J}{J_{sat}}}\right) \quad (2.49)$$

The same integration can be performed on the transport equation (2.46), while using the above result and the fact that $I_{-\infty,z} = I_{\infty,z} = 0$

$$\frac{dJ}{dz} = g_0 J_{sat} \left(1 - e^{-\frac{J}{J_{sat}}}\right) \quad (2.50)$$

with $g_0 = \sigma N_0 = \frac{J_{stored}}{J_{sat}}$ being the small signal gain. We arrive at the final expression which is called the Frantz-Nodvik equation.

$$J_{out} = J_{sat} \ln \left[1 + e^{\eta g_0} \left(e^{\frac{J_{in}}{J_{sat}}} - 1 \right) \right] \quad (2.51)$$

Here we further introduced the factor η to use the conventions of the paper[15]. We define $\eta = 1 - \eta_{ESA}$, with η_{ESA} being the fraction of gain which was lost due to excited state absorption.

Chapter 3

Experimental Setup

3.1 Chirped Pulse Amplification

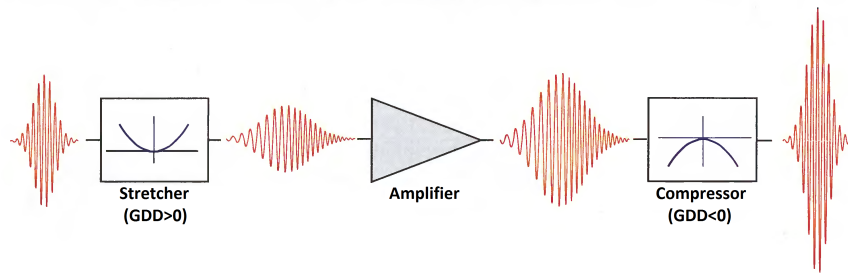


FIGURE 3.1: principle of chirped pulse amplification

First it is necessary to point out the stages, that make up a chirped pulse amplification system. The Stretcher imposes a positive chirp on the seeded pulse, to make it ready for the amplification. The amplifier is the centerpiece of this setup, as it increases the pulse intensity. In the compressor stage a negative GDD is imposed to counteract the effects of the stretcher and other dispersive effects in the amplifier.

3.2 Gain medium

For both the seed laser and the amplifier a $\text{Cr}^{2+}:\text{ZnSe}$ gain medium was used. This medium is a II-VI compound semiconductor, doped with the transition metal Cr^{2+} [5]. It has good absorption at around 1800 nm and an emission ranging from 2 to 3 μm . As is common for transition metal doped solid state media, there is a strong interaction between the electronic states and lattice vibration. This vibronic interaction causes a broadening of the fluorescence spectrum and thereby enables ultra short pulse generation. Operation at room temperature is exceptionally stable, as $\text{Cr}:\text{ZnSe}$ is among the highest gain solid state media with vibronic broadening [2]. The medium further shows a low amount of parasitic processes. Its low non-radiative decay rate leads to a quantum yield of around unity. The transitions from the upper energy level are spin forbidden and thereby excited state absorption is not an issue. In contrary to other broad band solid state media, $\text{Cr}:\text{ZnSe}$ can be pumped directly from laser diodes due to its low threshold for long wave transitions [2].

Gain narrowing can be an issue in regenerative amplification systems. This causes the spectrum to narrow which leads to a crippled re-compressibility. Ideally the stimulated emission cross section should be uniform over the whole pulse spectrum. In figure 3.2 it can be seen that the emission cross-section completely covers the experimental emission spectrum at modelocked operation. This leads us to assume, that gain narrowing is not an issue and can be neglected in the simulations.

Like other Cr^{2+} -doped laser crystals $\text{Cr}:\text{ZnSe}$ is susceptible to self-phase modulation due to its high third order nonlinearity of $n_2 = 120 \times 10^{-20} \frac{\text{m}^2}{\text{W}}$ [16], compared to only $3 \times 10^{-20} \frac{\text{m}^2}{\text{W}}$ in $\text{Ti}:\text{Saph}$ [2]. This shows that nonlinear pulse shaping mechanisms are

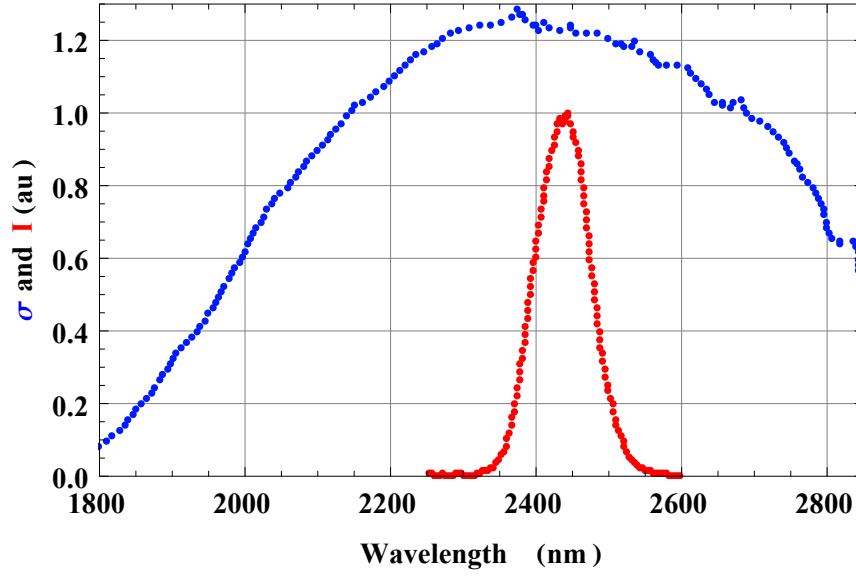


FIGURE 3.2: emission cross-section[2] and experimental emission spectrum of $\text{Cr}^{2+}:\text{ZnSe}$ at modelocked operation as a function of wavelength

not negligible inside the amplification stage. These effects have been considered in the Simulation section.

3.3 Pulse Shaping with Gratings

For the amplification a strong pulse stretching is needed (in the order of 10^3 fold). Inside the resonator prisms were used for dispersion control due to their low insertion losses. In a stretcher setup losses are not as important as in the resonator and the stronger angular dispersion of gratings is needed. The dispersive behavior of this system can be calculated starting by determining the optical path length as a function of wavelength[17].

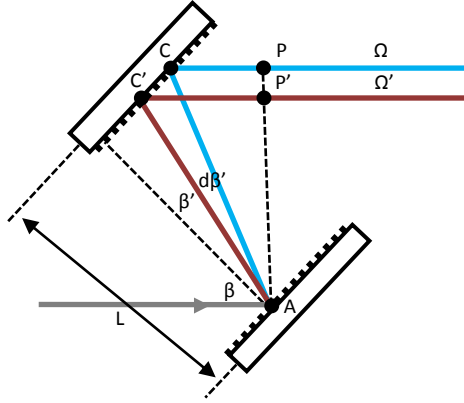


FIGURE 3.3: dispersion from a grating pair

At first the path length of a monochromatic light beam has to be determined, assuming light of frequency Ω between the parallel gratings.

$$\overline{ACP} = \frac{L}{\cos \beta'} [1 + \cos(\beta' + \beta)] \quad (3.1)$$

Light enters with an incident angle of β and is refracted into an angle β' . The gratings have an equal grating period of d and a normal separation of L . We assume the grating period to be small enough to produce only a single refractive order. Now the refraction angle β' can be related to the frequency Ω of the light beam, with the basic grating equation:

$$\sin \beta' - \sin \beta = \frac{2\pi c}{\Omega d} \quad (3.2)$$

Further the path length has to be adjusted for a different frequency Ω' . The original phase factor is given by $\phi = \frac{\Omega}{c} \overline{ACP}$. When β' changes and the point C moves a

distance of one grating period, an additional phase of 2π is added to ϕ . The phase ϕ for the path $\overline{ACP}$ is given by:

$$\phi = \frac{\Omega}{c} \overline{ACP} - 2\pi \frac{L}{d} \tan(\beta') \quad (3.3)$$

Using the above relation, the group delay can be obtained by differentiation of the phase:

$$\begin{aligned} \frac{d\phi}{d\Omega} = & + \frac{L}{c} \frac{1 + \cos(\beta + \beta')}{\cos \beta'} \\ & + \frac{\Omega L}{c \cos^2 \beta'} \{ \sin \beta' [1 + \cos(\beta + \beta')] - \cos \beta' \sin(\beta + \beta') \} \frac{d\beta'}{d\Omega} \\ & - \frac{2\pi}{d} \frac{L}{\cos^2 \beta'} \frac{d\beta'}{d\Omega} \end{aligned}$$

Using (3.2) $\frac{d\beta'}{d\Omega}$ can be solved. The second and third term vanish and we arrive at the compact form:

$$\frac{d\phi}{d\Omega} = \frac{L}{c} \frac{1 + \cos(\beta + \beta')}{\cos \beta'} \quad (3.4)$$

Differentiating a second time will yield the group delay dispersion:

$$\frac{d^2\phi}{d\Omega^2} = \frac{L}{c} \frac{1}{\cos^2 \beta'} \{ \sin \beta' [1 + \cos(\beta + \beta')] - \cos \beta' \sin(\beta + \beta') \} \frac{d\beta}{d\Omega} \quad (3.5)$$

Using (3.2) yields:

$$\phi'' = -\frac{4\pi Lc}{\Omega^3 d^2 \cos^3 \beta'} \quad (3.6)$$

The above equation can be evaluated at the central frequency $\Omega_0 = \frac{2\pi c}{\lambda_c}$.

$$\phi'' = -\frac{\lambda^3 L}{2\pi c^2 d^2} \frac{1}{\cos^3 \beta'} \quad (3.7)$$

The above can also be expressed as a function of the incident beam angle β , the grating period d and the wavelength λ . Applying equation 3.2 yields:

$$\phi'' = -\frac{\lambda^3 L}{2\pi c^2 d^2} \left[1 - \left(\frac{\lambda}{d} - \sin \beta \right)^2 \right]^{-\frac{3}{2}} \quad (3.8)$$

The use of focusing elements allows us to increase the grating separation regardless of mirror and grating size. A telescope setup between the gratings also inverts the picture so a *positive* GDD can be produced from parallel gratings. For calculating the modified dispersive behavior the analogue value of L has to be determined[1].

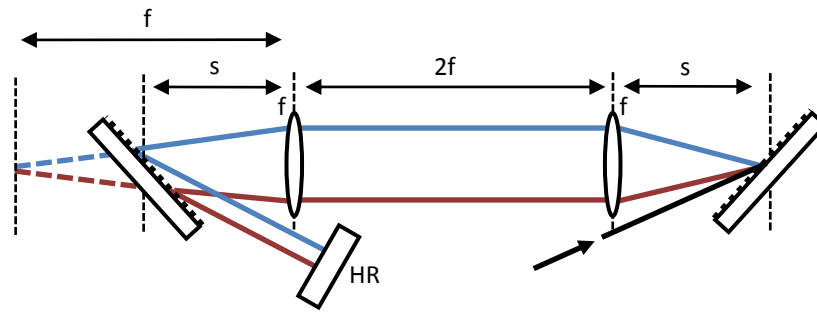


FIGURE 3.4: pulse stretcher setup with telescope between gratings

The GDD of this setup can be tuned by varying the distance between grating and lens ($0 \leq s \leq f$). The effective separation length of the gratings then becomes $L = -2(f - s)$.

We can put the new effective length into the formula for GDD of a parallel grating pair(3.8) to yield the modified equation. However, in this configuration(see figure 3.4) there are 4 bounces on the gratings, as the mirror HR sends the light back onto the grating. Thus the GDD in the equation below has an added factor of 2.

$$\phi'' = \frac{2\lambda^3(f - s)}{\pi c^2 d^2} \left[1 - \left(\frac{\lambda}{d} - \sin \beta \right)^2 \right]^{-\frac{3}{2}} \quad (3.9)$$

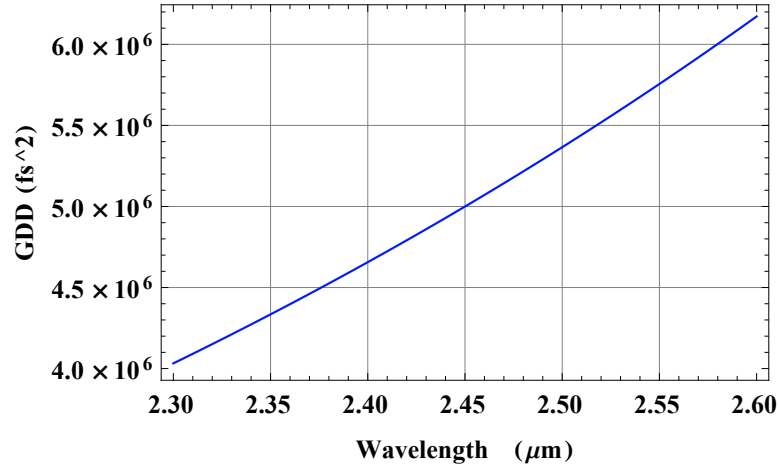


FIGURE 3.5: group delay dispersion of the pulse stretcher(figure 3.9) as a function of central wavelength from equation 3.9 with $f = 1 \text{ m}$, $s = 0.5 \text{ m}$, $d = 3 \times 10^5 \frac{1}{\text{m}}$ and $\beta = 32^\circ$

3.4 Numerical Calculation of Pulse Shaping

Deriving the dispersive behavior of an optical setup can be difficult with geometries more complex than those described in the previous section. The analytic approach does not consider dispersion from lens aberrations. A raytracing software can be applied to any setup geometry and will provide reliable results. It can also be used to check the validity of an analytic expression. Even spatial beam shaping effects can be simulated by using a multitude of spatially displaced probe rays. However, sampling only the central beam is sufficient for determining the dispersive behavior. After reconstructing the whole dispersive setup (in our case the pulse stretcher setup as shown in figure 3.9), the raytracer calculates the optical pathlength $P(\omega)$ for a range of input wavelengths.

The whole setup can be regarded as a dispersive bulk medium, as in equation 2.19. The optical path length $P(\omega)$ can be used in the same way as a refractive index n times a length l to obtain the GDD.

$$\phi'' = \frac{\lambda^3}{2\pi c^2} \frac{d^2 P(\omega)}{d\lambda^2} \quad (3.10)$$

This approach also works with higher order chirp components like third order dispersion. However, in a grating stretcher setup these contributions were found to be negligible.

3.5 Seed Laser

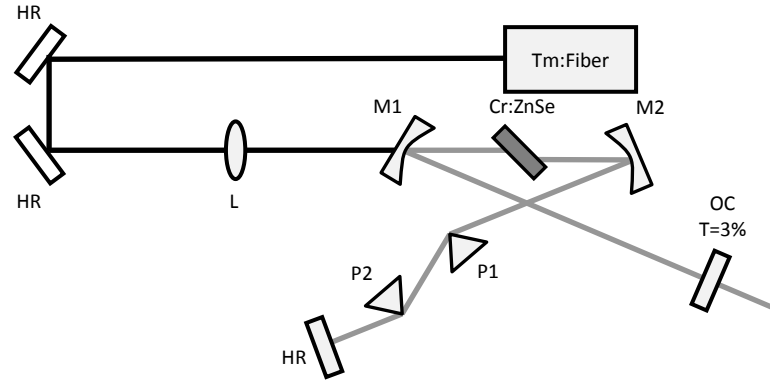


FIGURE 3.6: $\text{Cr}^{2+}:\text{ZnSe}$ seed laser setup

The seed laser is a Kerr lens modelocked (KLM) cross cavity resonator (see figure 3.6). A Tm:Fiber laser generates a pump power up to $3W$ at around $1.8\mu\text{m}$. At this wavelength the gain medium shows good small-signal absorption of 94%. In CW operation the setup showed a slope efficiency of 6.7% for a 1% output coupler and 14.5% for a 6% coupler. This corresponds to a maximum output power of $158mW$ and $370mW$ respectively. A CaF_2 prism pair is used for dispersion control (P1 and P2 in figure 3.6), they provide a GDD of $-1700fs^2$ for minimizing the pulse length. Other configurations for dispersion control were also investigated [16]. Inserting a YAG crystal bulk into the beam is another option for managing the dispersion.

With this configuration (see figure 3.6) it was possible to generate a spectrum of around $76nm$ in width. The autocorrelation showed a width of $143fs$ which corresponds to an approximate pulse duration of $92fs$.

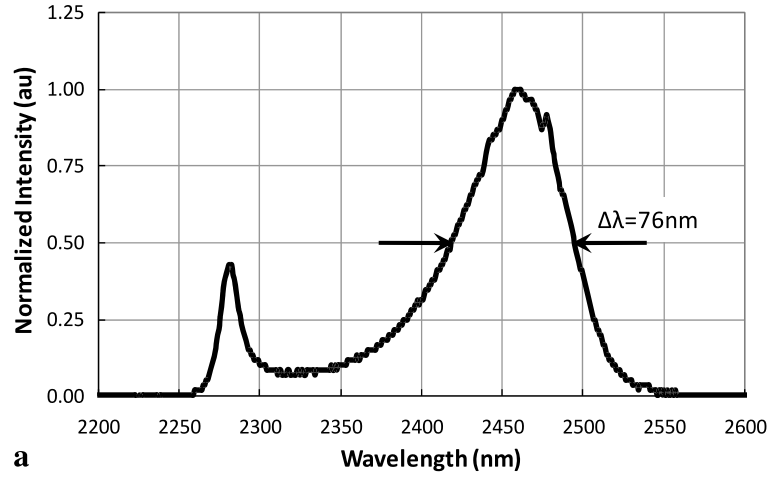


FIGURE 3.7: measured spectrum of the $\text{Cr}^{2+}:\text{ZnSe}$ seed laser using a 6% output coupler and a CaF_2 prism pair[16]

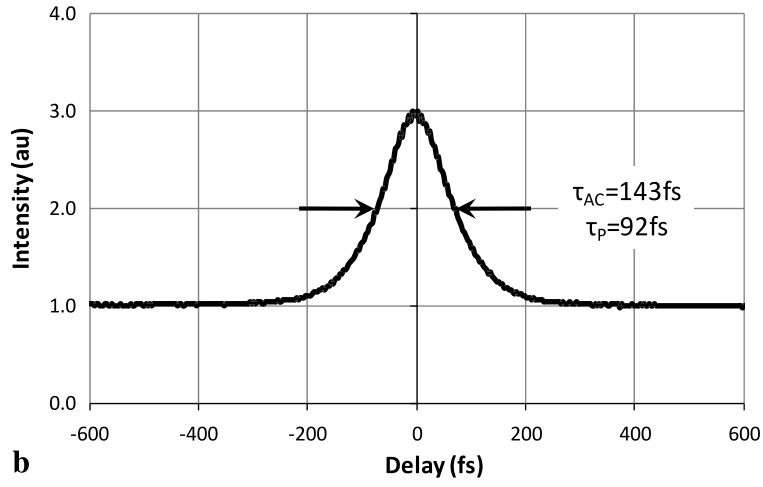


FIGURE 3.8: measured autocorrelation of the $\text{Cr}^{2+}:\text{ZnSe}$ seed laser using a 6% output coupler and a CaF_2 prism pair[16]

3.6 Pulse Stretcher

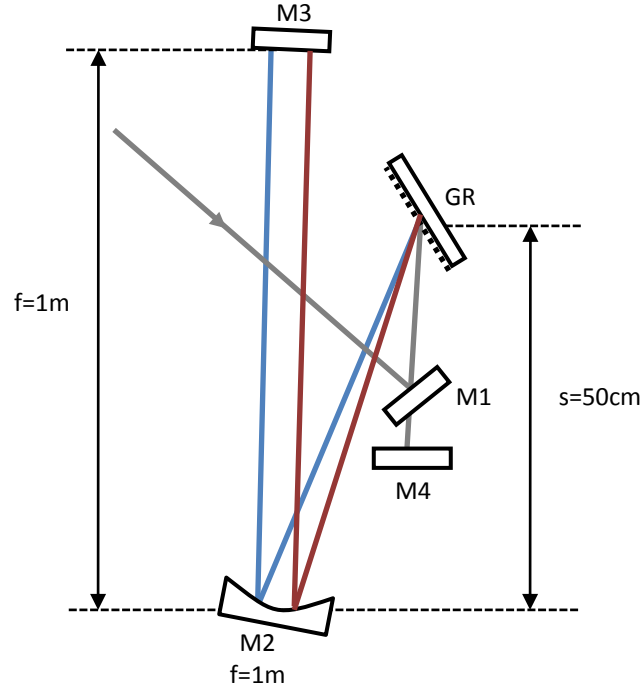


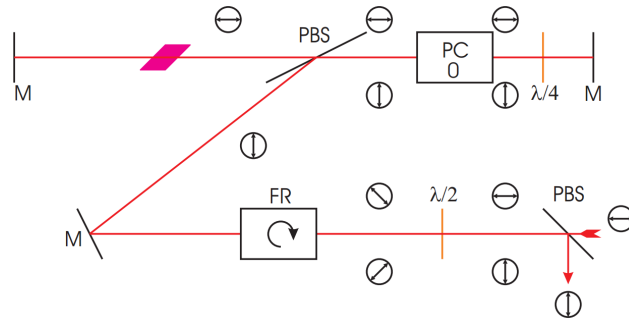
FIGURE 3.9: pulse stretcher setup with $f = 1\text{ m}$, $s = 0.5\text{ m}$, $d = 3 \times 10^5 \frac{1}{\text{m}}$ and grating incident angle $\beta = 32^\circ$

The pulse stretcher (see figure 3.9) increases the pulse length to about 1000 times its original duration in one round trip. The incoming pulse first hits M1, then diffracts on the grating 'GR'. The grating constant is $d = 3 \times 10^5 \frac{1}{\text{m}}$, which is sufficient to create just a single diffraction order at the given wavelength of $2.45\text{ }\mu\text{m}$. This setup is a folded version of the geometry shown in figure 3.4 with equivalent parameters s and f . The curved mirror M2 allows for positive dispersion with only one grating. This mirror also limits the necessary aperture for M3. A net GDD of $5 \times 10^6 fs^2$ has

been calculated(3.9) for this setup. The pulse enters the stretcher upon bouncing on M1. The first refraction on the grating GR creates angular dispersion. Upon the second bounce on the grating the beam is collimated again. The beam is then reflected on M4 and bounces off the grating surface twice. After the pulse hits M1 for the second time, it leaves the stretcher setup. Upon exiting the setup angular and spatial dispersion are completely canceled. However, each spectral component propagated on a different path within the stretcher setup. For the setup seen in figure 3.9 small wavelengths leave the setup slightly earlier than longer wavelengths. The result is a group delay dispersion within the pulse.

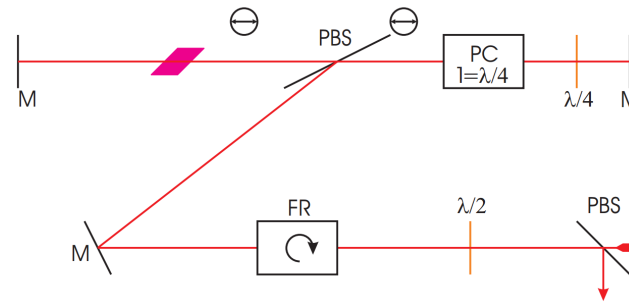
3.7 Amplifier

The number of passes through the gain medium of the amplifier needs to be controlled. Having too few or too many round trips will reduce the effectiveness of the amplification. A pockels cell is used as a fast shutter to control the number of passes. This electro-optical modulator can turn the polarization of transmitted light by 90° . The switching time is in the range of nanoseconds which is shorter than the time the light takes for one pass.



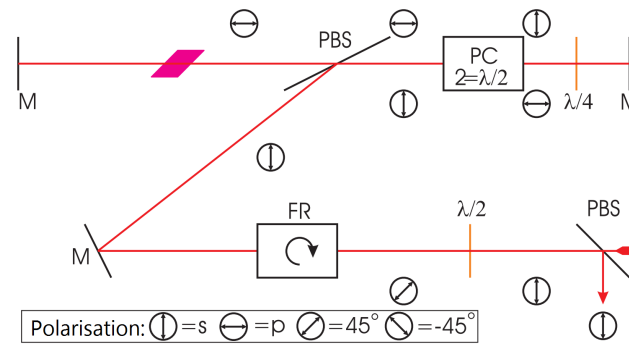
(a)

(a) insertion stage



(b)

(b) trapping stage



(c)

(c) release stage

FIGURE 3.10: principle of amplification stage in three steps[3]

There are three distinct phases in the operation of the amplification stage(see figure 3.10).

- (a) While the pulse is inserted the Pockels cell 'PC' is in its non-polarizing state. The pulse enters in a p-polarized state. The $\frac{\lambda}{2}$ plate is positioned for a 45° polarization change. At the Faraday rotator 'FR', another 45° is added for a total polarization rotation of 90° . The incoming light is in the proper angle for total reflection on the polarizing beam splitter 'PBS'. After passing the $\frac{\lambda}{4}$ plate twice, the pulse can pass the PBS and enter the gain medium. Note that the combination of a $\frac{\lambda}{2}$ plate and a Faraday rotator also cancels back reflection into the pulse stretcher and seed laser stage.
- (b) At this phase the pockels cell is set from the insertion to the trapping state. Here 'PC' is set to $\frac{\lambda}{4}$ which causes a net rotation of 180° rotation, including the $\frac{\lambda}{4}$ plate. The pulse will stay in a p-polarized state and accumulate energy in the cavity.
- (c) After a sufficient number of round trips, 'PC' is switched into $\frac{\lambda}{2}$ state, which initiates the extraction of the pulse. The total polarization rotation of the Pockels cell and the $\frac{\lambda}{4}$ plate now creates s-polarized light at the beam splitter and thus enabling the pulse to exit the cavity upon reflection. After passing the Faraday rotator and the $\frac{\lambda}{2}$ plate, the amplified pulse can be separated from the seed pulse at the outer beam splitter. The extracted pulse is now directed to the compression stage.

The pump source in our setup(see figure 3.12) is a Q-switched Nd:YAG laser, producing $1.064\mu m$ pulses at a rate of $1kHz$. An optical parametric oscillator(OPO)

was used for converting the wavelength to $1.570\mu\text{m}$ for matching the absorption band of the gain medium.

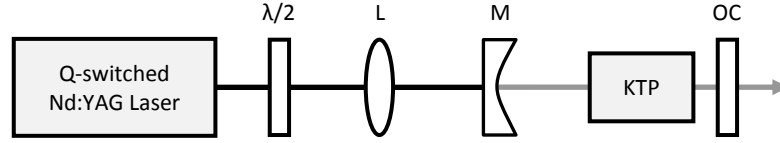


FIGURE 3.11: optical parametric oscillator setup(OPO)

Inside the OPO a nonlinear crystal(KTP) splits the pump wavelength into a signal and an idler component. Depending on the angle of the KTP, there is a different phase matching condition and thereby different wavelengths in the signal and idler components. This is, how the OPO can be tuned for ideal match to the gain medium.

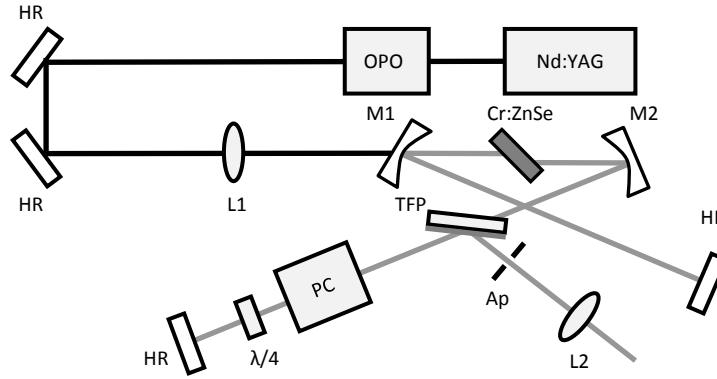


FIGURE 3.12: regenerative $\text{Cr}^{2+}:\text{ZnSe}$ amplifier setup

Note that the interval between two pump pulses is much greater than the whole amplification process. We can thereby assume no pumping while a pulse is circling in the amplifier, which greatly simplifies our calculations.

3.8 Compressor

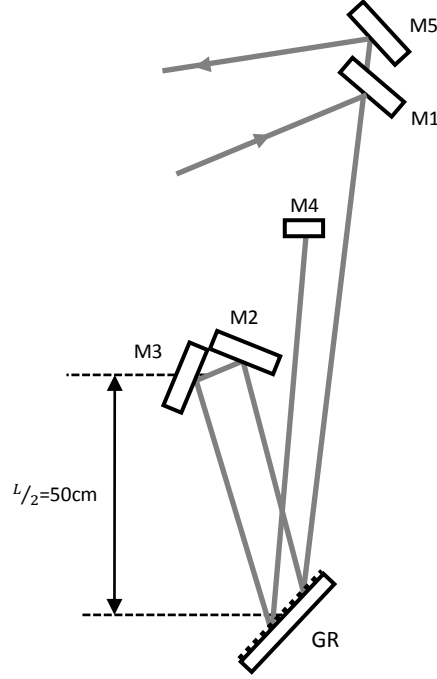


FIGURE 3.13: pulse compressor setup with $L = 1\text{ m}$, $d = 3 \times 10^5 \frac{1}{\text{m}}$ and grating incident angle $\beta = 32^\circ$

Like the pulse stretcher, this setup (see figure 3.13) is also a four-bounce grating setup. Note that the equivalent grating separation here is $L = 100\text{ cm}$ whereas for the stretcher setup it was $L = -2(f - s) = -100\text{ cm}$. Given the same grating period of $3 \times 10^5 \frac{1}{\text{m}}$ and same incident angle, the compressor will perfectly reverse the dispersion of the stretcher. However, when passing the amplification stage additional GDD was imposed on the pulse. A modification of the compressor setup is needed for ideal recompression. The mirror pair (M2 and M3 in figure 3.13) can be placed on a translation stage, which makes the net GDD tune-able.

Upon the fourth bounce on the grating 'GR' the spectral components synchronize and recombine. Therefore the pulse is in its most compressed state. In high power amplifiers this is a very critical point, as the grating is exposed to the highest energy flux occurring in all of the CPA system. Gratings always absorb several percent of the incoming light as they are metal reflectors. This can easily lead to damage and is a major limiting factor of the CPA system in general. Big curved gratings are often used to counteract this destructive effect.

Chapter 4

Simulation

4.1 Calculation of Stretcher Dispersion

In the raytracer software Oslo we modeled the pulse stretcher and extracted the optical path length for the relevant wavelength range from 2.3 to 2.6 μm . In Mathematica we can numerically obtain the group delay dispersion from this data(3.10).

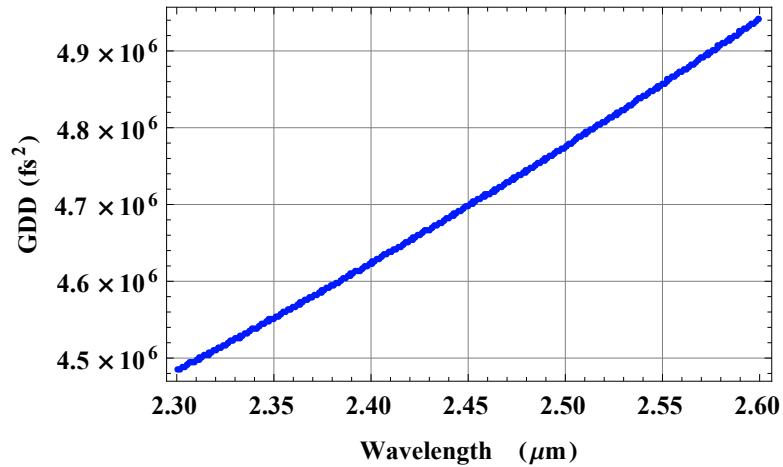


FIGURE 4.1: group delay dispersion of the pulse stretcher setup(see figure 3.9) as a function of central wavelength from raytracing data with setup parameters $f = 1 \text{ m}$, $s = 0.5 \text{ m}$, $d = 3 \times 10^5 \frac{1}{\text{m}}$ and $\beta = 32^\circ$

With the results, shown in figure 4.1 the numerical approach through raytracing is verified as it perfectly agrees with the results shown in figure 3.5. The duration of a pulse, after acquiring a certain GDD can be obtained from equation 2.36. Using $\alpha = \frac{2\ln(2)}{t_{\frac{1}{2}}^2}$ we can solve for the pulse duration $t_{\frac{1}{2}}$.

$$t_2 = t_1 \sqrt{1 + \left(4 \ln 2 \frac{\phi''}{t_1^2}\right)^2} \quad (4.1)$$

where t_1 is the input pulse duration and t_2 the duration after passing the stretcher. This result is shown in the figure below.

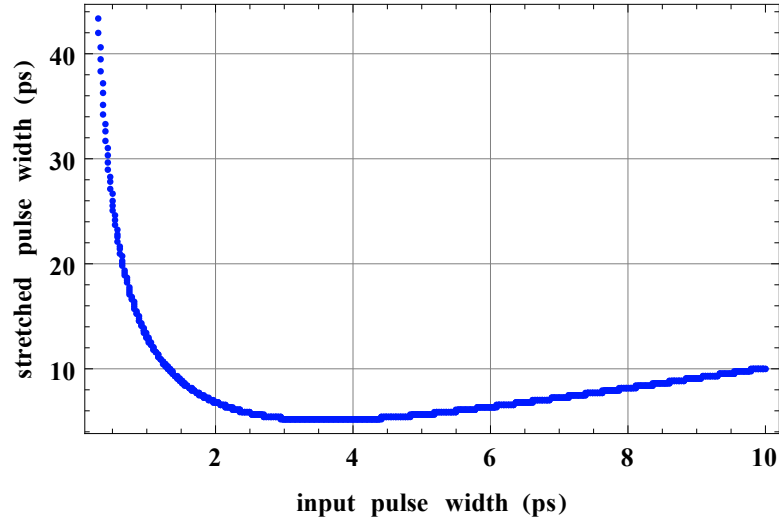


FIGURE 4.2: output pulse length after exposure to the given group delay dispersion for a GDD of $\phi'' = 4.7 \times 10^6 fs^2$

For the central wavelength of $2.45 \mu m$ we obtained a GDD of $\phi'' = 4.7 \times 10^6 fs^2$ which is very close to $\phi'' = 5 \times 10^6 fs^2$ which was obtained from the analytic description(3.8). The raytracing approach takes into account all linear optical effects like spherical aberration and should thereby be more exact then the result from

the analytic description. Assuming an input pulse duration of 100fs, the stretched pulse duration is 130ps for the analytic result and 139ps for the raytracing result respectively.

4.2 Investigation of Self-Phase Modulation Effects

Now we would like to visualize the effects of self-phase modulation on dispersed pulses(2.41). This can be compare with the approximate solution(2.32) which is not dependent on the instantaneous frequency of the incoming pulse. We now assume a pulse of an initial duration of $t_{\frac{1}{2}}$. This pulse first travels through a stretcher and acquires a chirp of initial GDD and is stretched to a duration of $t'_{\frac{1}{2}}$. Afterwards the pulse passes a nonlinear medium and is modulated by a maximum phase shift φ_{max} thus changes the duration to $t''_{\frac{1}{2}}$.

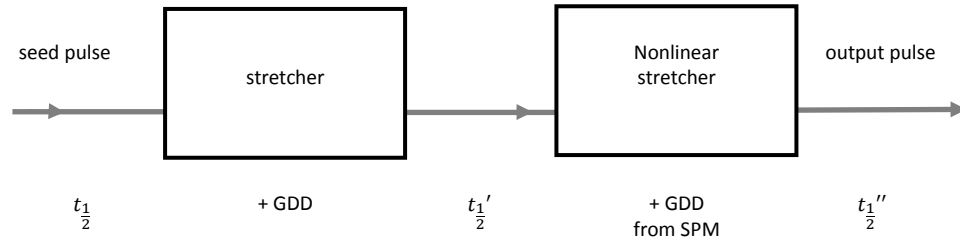


FIGURE 4.3: schematic of dispersion stages

We can keep either $t_{\frac{1}{2}}$ or $t'_{\frac{1}{2}}$ constant, they can be related using formula 4.1 and inserting the GDD from the pulse stretcher. Figure 4.4 shows the newly acquired GDD from self-phase modulation using both approaches from equation 2.32 and 2.41 for constant $t_{\frac{1}{2}}$.

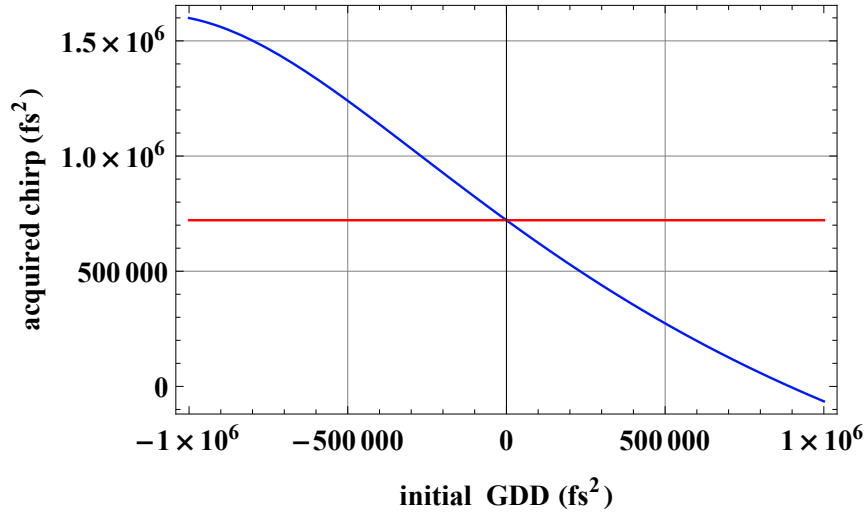


FIGURE 4.4: dispersion from self-phase modulation for initial pulselength of $t_{\frac{1}{2}} = 2ps$ and a maximum phaseshift of $\varphi_{max} = 0.5$, calculated using equation 2.32 (red) and 2.41 (blue)

We can also keep the pulse length after passing the dispersive medium $t'_{\frac{1}{2}}$ constant (see figure 4.5). We can then see the difference between a transform-limited or a dispersed pulse of equal duration. The initial GDD strongly influences the effects of self-phase modulation.

However one should note that the $t_{\frac{1}{2}}$ from equation 2.41 is the transform limited pulse duration before the dispersion from the dispersive bulk. Assuming a Gaussian pulse and a time-bandwidth product of 0.44, which is typical for a Gaussian, we can estimate the minimal bandwidth of a pulse with $\Delta\Omega \approx \frac{0.44}{t_{\frac{1}{2}}}$. This bandwidth corresponds to the values of initial GDD seen in figure 4.5. The minimal bandwidth is displayed in figure 4.6.

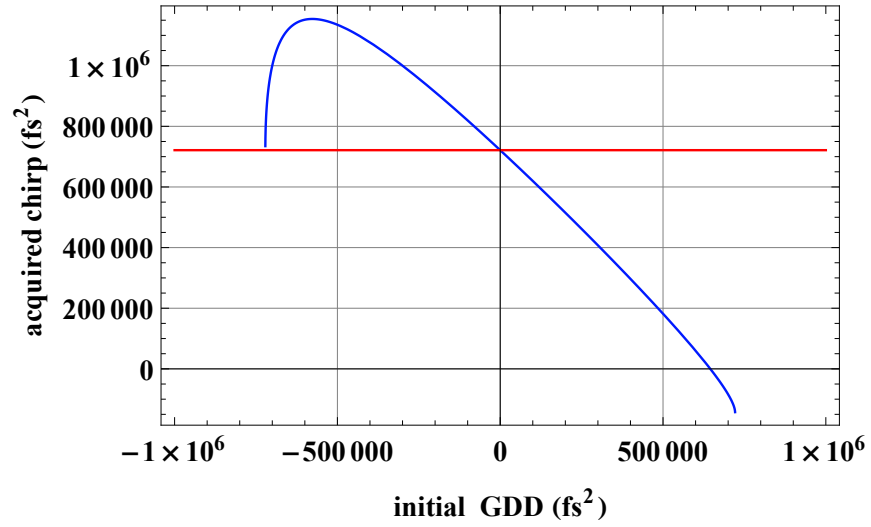


FIGURE 4.5: dispersion from self-phase modulation for dispersed pulse length of $t'_{\frac{1}{2}} = 2ps$ and a maximum phaseshift of $\varphi_{max} = 0.5$, calculated using equation [2.32](#)(red) and [2.41](#)(blue)

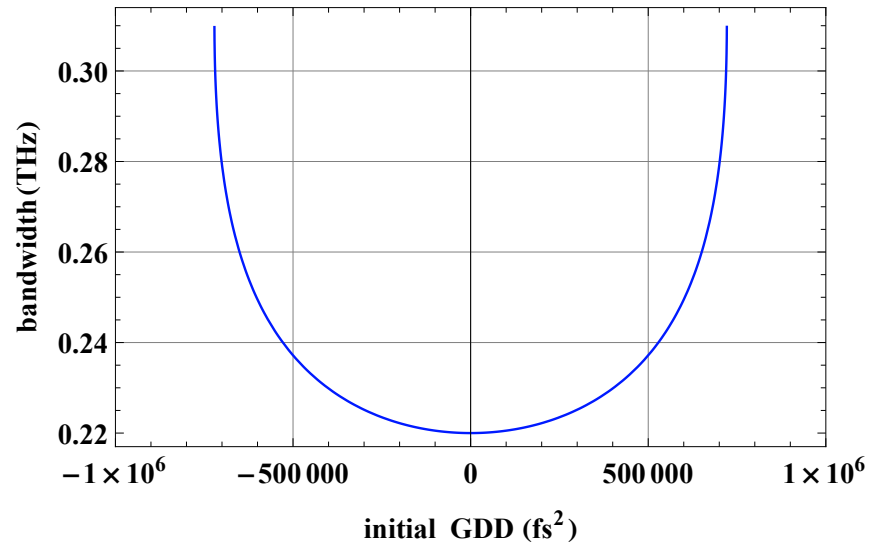


FIGURE 4.6: minimal bandwidth necessary to produce a dispersed pulse of length $t_{\frac{1}{2}} = 2ps$ and a time-bandwidth product of 0.44 at the given amount of chirp

4.3 Simulation of Pulse Evolution

The heart of this work was to put together a code which describes the evolution of a pulse inside of a regenerative amplifier. This was based on the approximation of the pulse maintaining a Gaussian envelope shape throughout the whole amplification process. The first steps are preliminary calculations which are shown by A, B & C in figure 4.7 and are described in further detail below. Afterwards the pulse energy and shape are iteratively calculated, using the methods described in section 2.4 & 2.5. A flow chart displays all relevant steps of the computation (figure 4.7). On the right side (grey boxes) we see the user parameters on which the calculations will be based. Most of these values are used in the preliminary calculations which will be described below in further detail.

(A) In this module we process the input parameters which are related to the Frantz-Nodvik equation (2.51). The absorbed energy E_{abs} can be calculated from the pump energy and the absorbed fraction, according to the Beer-Lambert law [14]. For the effective beam area A_{eff} , first the beam waist at the focal point was measured (w_{rms}). From this we obtained the average beam area over the whole gain medium volume, using $A_{eff} = \frac{1}{2}n_{ZnSe}\pi w_{rms}^2$. From normalizing the energy with the beam area and taking into account the quantum defect between pump and lasing wavelength ($Q = \frac{\lambda_P}{\lambda_L}$), we obtain the stored fluence $J_{sto} = \frac{E_{abs}}{A} \frac{\lambda_P}{\lambda_L}$. We further need the saturation fluence (J_{sat}). For this the stimulated-emission cross section (σ) is needed, this is a material constant that can be determined with various methods [18]. We then obtain $J_{sat} = \frac{hc}{\lambda_L \sigma}$.

Finally we get the small signal gain which is used for the first iteration of the Frantz-Nodvik equation $g_0 = \frac{J_{sto}}{J_{sat}}$.

- (B) For calculating the effects of self-phase modulation we need to find the Kerr nonlinearity coefficient as defined in [14]. It can be obtained from the nonlinear refractive index (n_2) and the effective beam area (A_{eff}), with $\gamma = \frac{2\pi}{\lambda_L} \frac{n_2}{A_{eff}}$. Also the normalization (α) as used in (2.33): $\alpha = \frac{2\ln(2)}{t_{\frac{1}{2}}^2}$. This is constant, as $t_{\frac{1}{2}}$ is the duration of the transform limited pulse, as measured directly from the seed laser. The nonlinearity parameters are only calculated for the gain medium, as the beam is focused here and thus generating an energy density which is high enough to be considered for nonlinear effects.
- (C) For the chirp we need to use the Sellmeier equations ($n(\lambda)$) for all involved solids (see the setup in figure 3.12). However inside the Pockels cell there is a birefringent medium (LiNbO₃) which means that the refractive index is a function of polarization. The polarization of light inside the cell is $\theta = 45^\circ$ between ordinary and extraordinary axis. This is true for the switched-on state, where the cell acts like a half wave plate and thus traps the pulse inside the cavity. The effective refractive index can be found from $\frac{1}{n_{eff}^2} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}$ as derived in [14], with n_o & n_e being the ordinary and extraordinary refractive indices respectively. Dispersion also arises from other optical components. Thus the refractive index functions of the quarter-wave plate (CaF₂), the gain host-medium (ZnSe) are also needed. To obtain the GDD these refractive index functions $n(\lambda)$ are used in $\phi'' = \frac{\partial^2 \phi}{\partial \Omega^2} = l \frac{d^2 k}{d\Omega^2}$ as shown in equation 2.19.

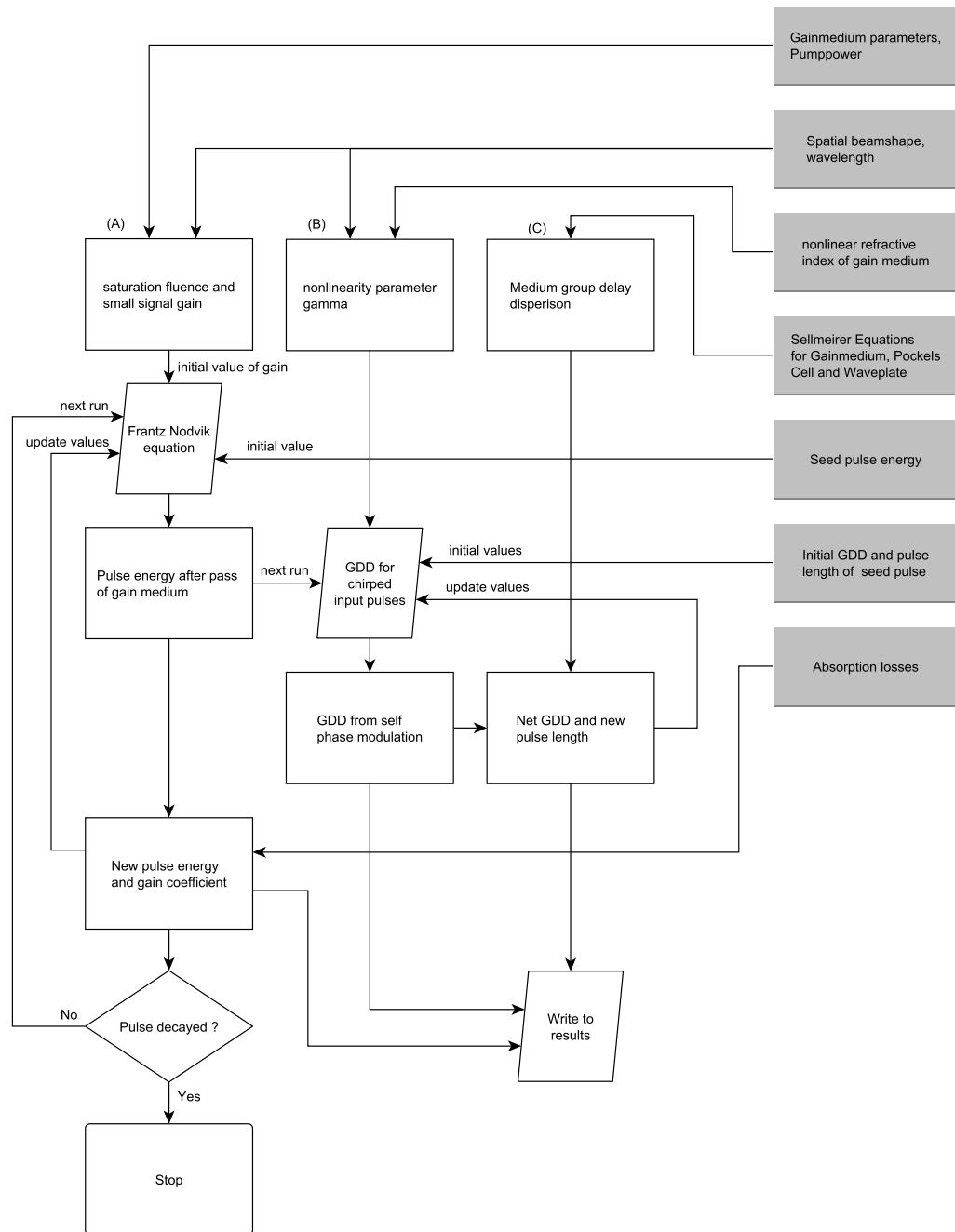


FIGURE 4.7: flowchart of the main program with user input parameters in grey boxes on the right

Once the preliminary processing is done, the Frantz-Nodvik equation(2.51) is executed for the first iteration. In this run J_{in} is the fluence of the seed pulse and the small signal gain(g_0) is the initial value from box (A). After obtaining the pulse energy the calculation of nonlinear effects can begin. We first calculate the peak power from the peak energy of a Gaussian pulse and the pulse duration with $P_{peak} = 0.94 \frac{J_{out} A_{eff}}{t_{\frac{1}{2}}}$. From this value, the nonlinearity parameter(γ) and the gain medium length, we get $\varphi_{max} = \gamma P_{peak} L_{gain}$. This can then be inserted to equation 2.41 to obtain the GDD from self-phase modulation. After adding the GDD from box (C) we can sum up the net chirp and estimate the new pulse length, using(4.1). The new chirp and duration will give the new parameters for the next iteration of the nonlinear chirp calculation(2.41). Taking into account absorption losses which occur at every pass, we get J_{in} for the next iteration from J_{out} of the current iteration. The gained fluence ($J_{out} - J_{in}$) is subtracted from the stored fluence(J_{sto}) to obtain the new small signal gain(g_0) for the next iteration.

This loop can be run until the pulse is decayed below a certain threshold. After every loop iteration, the pulse energy, duration and chirp is written to a result table. Following graphs were obtained for a pump power of $50mW$, a repetition rate of $1kHz$ and a stimulated emission cross section of $\sigma = 3.38 \times 10^{-23} m^2$ [19]. Further we used a nonlinear refractive index $n_2 = 120 \times 10^{-20} \frac{m^2}{W}$ which was determined in our own laboratory[16]. A passive cavity loss of 14 % per pass was assumed. From the knife edge method we obtained the beam waist at the center of the gain medium. The result was $w_{rms} = 35.5 \times 10^{-6} m$ which translates to a effective beam area of $A_{eff} = 4.83 \times 10^{-9} m^2$. The input pulse has an energy of $58 pJ$. The first graph(4.8) shows the pulse energy for every pass of the gain medium. In figure 4.8 we can see that the pulse should be extracted when it peaks around the seventh pass. After this point the resonator losses outweigh the gain and cause the pulse to slowly decay.

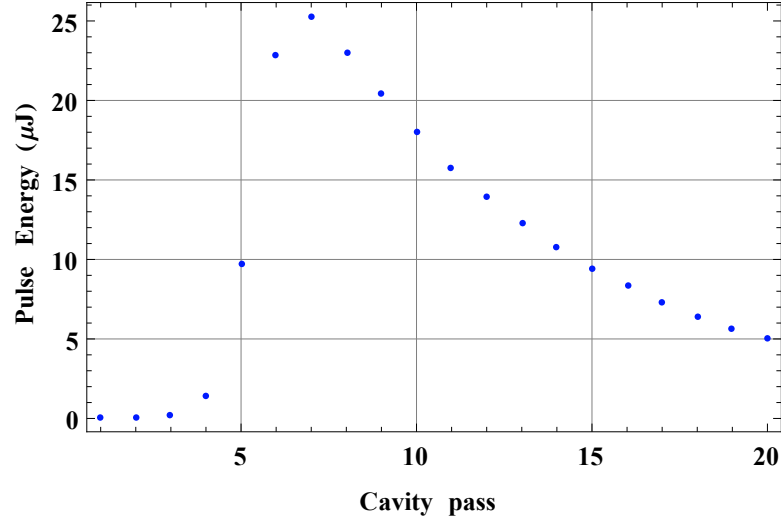


FIGURE 4.8: pulse energy as a function of cavity pass, for a pump power of $50mW$, repetition rate of $1kHz$, transform limited seed pulse length of $t_{\frac{1}{2}} = 100fs$ and an initial GDD of $\phi'' = 5 \times 10^6 fs^2$ and $\sigma = 3.38 \times 10^{-23} m^2$

Figure 4.9 shows the GDD from self-phase modulation. Here we used a transform limited seed pulse length of $t_{\frac{1}{2}} = 100fs$ and an initial GDD from stretcher of $\phi'' = 5 \times 10^6 fs^2$. We can see that the pulse energy up to pass 5, is not sufficient to cause relevant nonlinearities. Also the curve flattens out in the end, as the pulse slowly decays.

The medium GDD is simply a constant, that adds up in every pass. Its contribution is comparable to the self-phase modulation(see figure 4.10).

Summing up the GDD of both contributions, we can calculate the evolution of pulse duration in figure 4.11.

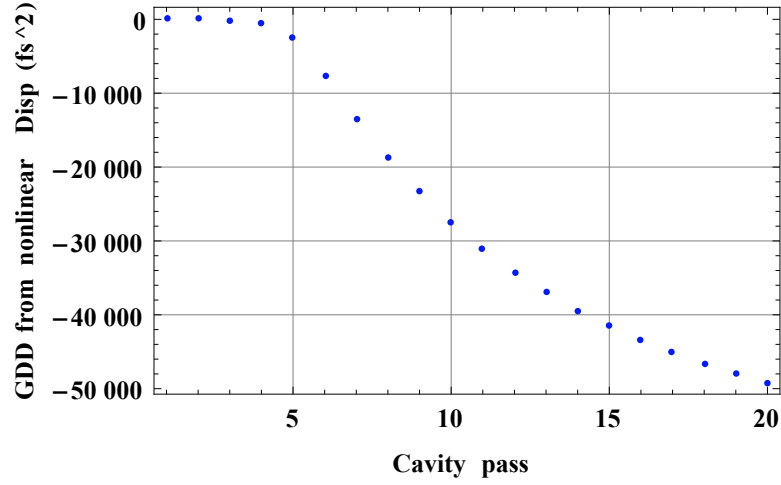


FIGURE 4.9: GDD from self-phase modulation as a function of cavity pass, for a pump power of $50mW$, repetition rate of $1kHz$, transform limited seed pulse length of $t_{\frac{1}{2}} = 100fs$ and an initial GDD of $\phi'' = 5 \times 10^6 fs^2$ and $\sigma = 3.38 \times 10^{-23} m^2$

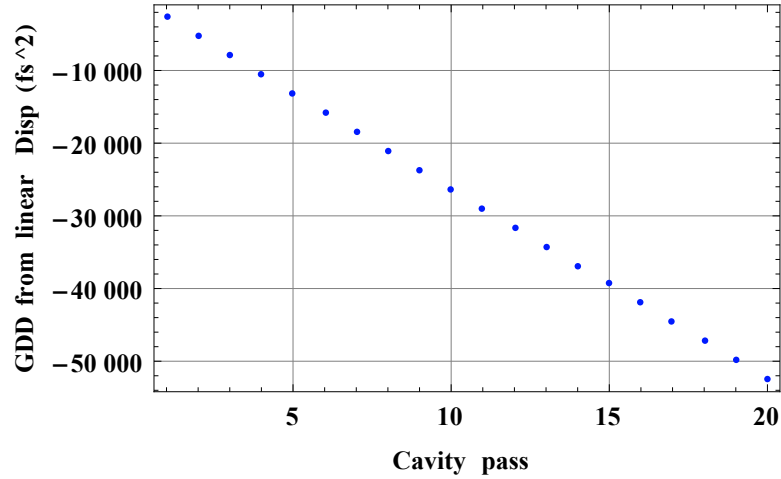


FIGURE 4.10: media GDD as a function of cavity passes for a pump power of $50mW$, repetition rate of $1kHz$, transform limited seed pulse length of $t_{\frac{1}{2}} = 100fs$ and an initial GDD of $\phi'' = 5 \times 10^6 fs^2$ and $\sigma = 3.38 \times 10^{-23} m^2$

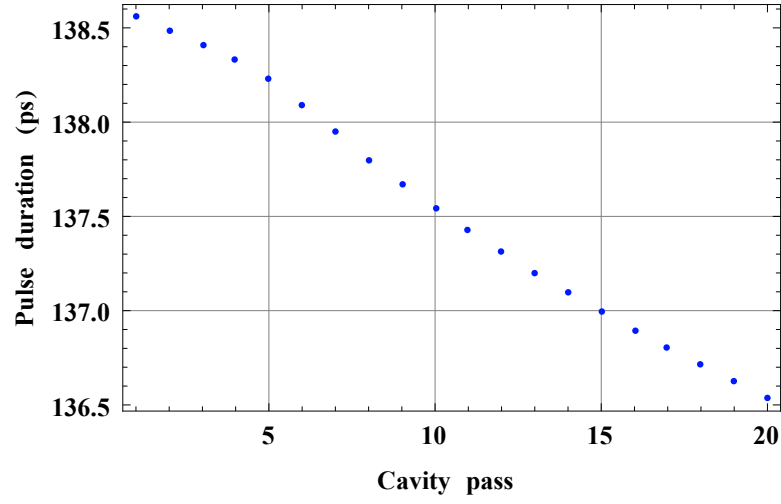


FIGURE 4.11: pulse duration as a function of cavity pass, for a pump power of $50mW$, repetition rate of $1kHz$, transform limited seed pulse length of $t_{\frac{1}{2}} = 100fs$ and an initial GDD of $\phi'' = 5 \times 10^6 fs^2$ and $\sigma = 3.38 \times 10^{-23} m^2$

Determining stimulated emission cross-section σ is vital for a reliable simulation of the amplifier. However, measuring this parameter with high precision is not a trivial task[18]. Results can be obtained indirectly from the reciprocity method or the Füchtbauer-Ladenburg equation[18]. Measuring the gain is a direct approach for determining the emission cross-section. With a pump-probe technique the small-signal gain can be observed which is connected to σ . In a lasing setup, the emission cross-section can also be obtained from the output power[20]. However, an error of 20% is common in these measurements which translates to a strong impact on the Frantz-Nodvik equation(2.51). In this work we used experimental results and the Frantz Nodvik equation to determine the emission cross-section. The results are shown in the final chapter. An example for the impact of σ on the output energy and build up time is given in figures 4.12 and 4.12.

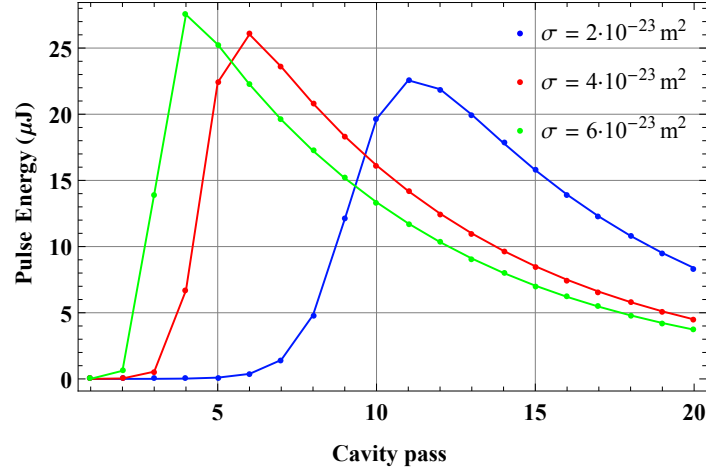


FIGURE 4.12: evolution of pulse energy in the amplifier for three values of stimulated emission cross-section, using a pump power of 50mW , repetition rate of 1kHz , transform limited seed pulse length of $t_{\frac{1}{2}} = 100\text{fs}$ and an initial GDD of $\phi'' = 5 \times 10^6 \text{fs}^2$ and $\sigma = 3.38 \times 10^{-23} \text{m}^2$

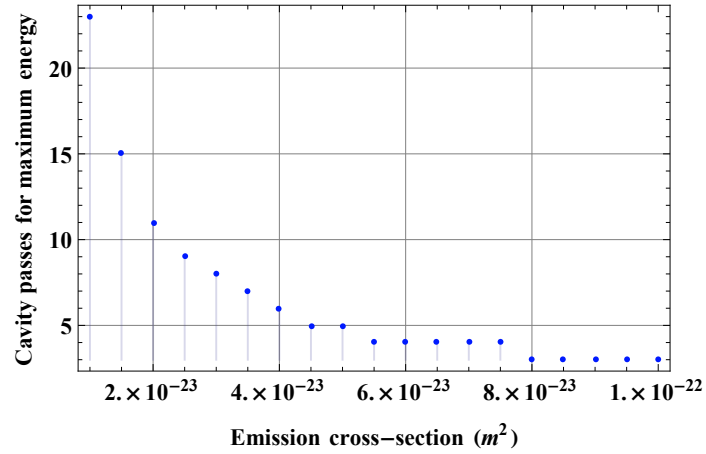


FIGURE 4.13: number of amplifier passes for highest output power as a function of emission cross-section for a pump power of 50mW , repetition rate of 1kHz , transform limited seed pulse length of $t_{\frac{1}{2}} = 100\text{fs}$ and an initial GDD of $\phi'' = 5 \times 10^6 \text{fs}^2$ and $\sigma = 3.38 \times 10^{-23} \text{m}^2$

Chapter 5

Results and Discussion

5.1 Determination of Emission Cross-Section and Absorption Coefficient

The emission cross-section and passive amplifier losses are hard to measure directly. Thus the value for the emission cross-section and passive amplifier losses were determined by applying the measured output power for a pump power of 325 mW , as shown in figure [5.1](#), along with the measured trapping time to the code. We kept all other parameters as given in the previous chapter. For a pump power of 325 mW an ideal trapping time of 79 ns was measured experimentally. The amplifier cavity had a length of 73.5 cm which translates to a roundtrip time of 4.904 ns . Thus for a pump power of 325 mW , 32 roundtrips were needed for ideal amplification. Using both output energy and trapping time for a pump power of 325 mW , we found the emission cross-section $\sigma = 2.055 \times 10^{-24}\text{ m}^2$ and the passive losses to be 41.5%. Detailed simulation results can be found in tables [B.1](#), [B.2](#), [B.3](#) and [B.4](#) in appendix B.

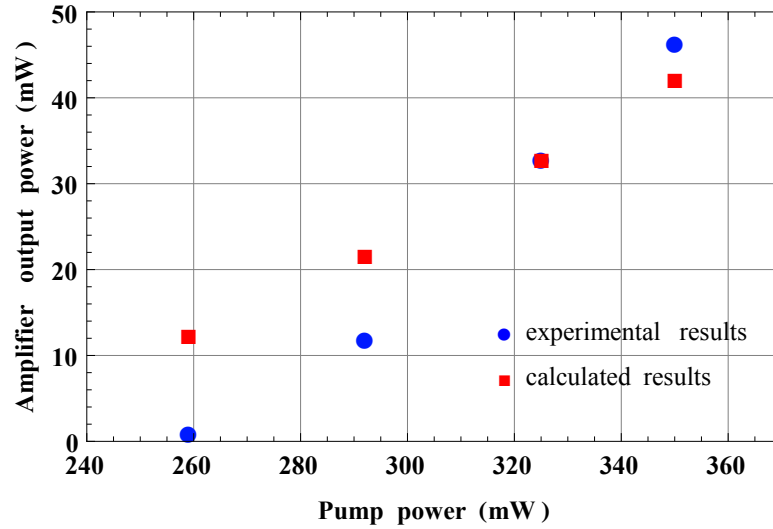


FIGURE 5.1: Output power of the amplifier as a function of pump power from experimental results and simulation, using $\sigma = 2.055 \times 10^{-24} m^2$ and absorption of 41.5% per pass, a repetition rate of $1 kHz$, transform limited seed pulse length of $t_{\frac{1}{2}} = 100 fs$ and an initial GDD from the stretcher of $5 \times 10^6 fs^2$

The found stimulated emission cross-section of $\sigma = 2.055 \times 10^{-24} m^2$ was about one order of magnitude lower than we expected. Literature values of σ for $Cr^{2+}:ZnSe$ range from $1.6 \times 10^{-23} m^2$ [2] to $13 \times 10^{-23} m^2$ [21]. This difference can arise from the fact that the stimulated emission cross-section can vary between crystals due to different quality of the host medium crystal. The passive losses of 41.5% per pass were higher than we had expected. In the non-lasing setup (see figure 3.12) a passive loss from absorption and reflection of 14% was estimated by summing up the measured loss at lasing wavelength for each optical component separately. However, in complex optical setups slight misalignments can easily lead to strong losses.

The simulation results for power efficiency are compared to the measured values in figure 5.1. In the experimental results the output power shows a cutoff at 260 *mw* of pump power. This might point to a strong absorption coefficient. One should also note that the ideal trapping time for this pump power was calculated to be 129.85 *ns* which surpasses the maximum trapping time of the Pockels cell. The Pockels cell can only remain in the $\frac{\lambda}{4}$ mode for a maximum of 100 *ns* which is equivalent to 41 cavity passes. Taking into account this premature pulse extraction at the 41th pass, an output power of 2.6 *mW* was calculated (see table B.1). This agrees well with the experimental value of 1 *mW* and explains the cutoff in figure 5.1 for a pump power of 259 *mW*.

5.2 Results from Simulation of Dispersion

Looking at the case of 325 *mW* pump power and the other parameters as described in section 4.3, a net GDD of $\phi'' = -1.2433 \times 10^5 fs^2$ has been calculated (see table B.3). This consists of a contribution from SPM of $-0.4026 \times 10^5 fs^2$ and a contribution from material dispersion of $-0.8406 \times 10^5 fs^2$. It is obvious that this is not a negligible amount of dispersion. Consideration of nonlinear pulse shaping effects is necessary in a $Cr^{2+}:ZnSe$ based amplifier, even after increasing the pulse duration by around 1000 times in the stretcher. If the compressor was not adjusted to the newly acquired chirp from the amplifier, a compressed pulse duration of 3448 *fs* would be the result (using equation 4.1). This pulse duration is far from the original seed pulse duration of 100 *fs*. From equation 3.8 we calculated a displacement of $\Delta L = 2.49 cm$ in the pulse compressor setup (see figure 3.13) for perfect compression.

Appendix A

Mathematica Code

(* General and power relevant parameters *)

```

c = 299792458 ;(* speed of light, m/s *)

h = 6.626068 * 10^-34 // N; (* Planck's constant, Js *)

λL = 2.38 * 10^-6 ;(* lasing wavelength, m *)

λP = 1.570 * 10^-6 ;(* pump wavelength, m *)

PumpPower = 0.325;(* W *)

σ = 0.2055 * 10^-23;
(* emission cross section of the gain medium @ lasing wavelength, m^2*)

frep = 1 * 10^3; (* repetition rate in Hz*)

nZnSe =  $\sqrt{9.01536 + \frac{0.24482}{(\lambda L * 10^6)^2 - 0.29934^2} + \frac{7229.931303}{(\lambda L * 10^6)^2 - 48.38^2}}$ ;

wRMS = 35.5 * 10^-6;
Area = 0.5 * nZnSe * π * (wRMS)^2; // N (*effective laser mode area, m^2*)

η = 1; (* ηesa(excited state absorption)=0 *)

LL = 6.6 * 10^-3 // N; (*Length of the crystal, m*)

Eabs = 0.92 * PumpPower / frep // N; (* Stored pump energy J*)

Jsto = (Eabs / Area) * (λP / λL) // N (* stored energy density J/m^2 *)

EE = Jsto * Area; (* Energy normalization *)

Jsat = (h * c) / (σ * λL); (*saturation fluence of the amplifying medium*)

g0 = Jsto / Jsat; (* small signal gain *)

Gain[1] = g0; (*setting the small signal gain for the first iteration*)

Loss = 0.415; (*passive losses of the amplifier setup per pass*)

Jin[1] = 0.058 * 10^-9 * (1 - Loss) / Area; (*inout pulse fluence*)

```

(* nonlinear Dispersion relevant parameters *)

```
tFWHM = 100;(*duration in femtoseconds*)
```

```
n2znse = 1.2 * 10^-18;
```

```
 $\gamma = (2 * \text{Pi} * n2znse) / (\lambda L * \text{Area});$ 
```

```
a = 4 * Log[2] / tFWHM^2;
```

```
 $\Phi_{\text{max}}[0] = 0;$ 
```

(* linear Dispersion relevant parameters *)

(*caution: in this section the standart length unit is millimeter,
wavelengths in micrometer. the Disp2slabBrewster
additionally computes dispersion for brewster angle incidence*)

```
LLiNbO3 = 20;
```

```
LZnSe = LL * 10^3;
```

```
LCaF2 = 5;
```

(*Sellmeier Equations*)

(*this is the material of the pockels cell*)

```
neLiNbO3[ $\lambda$ _] :=  $\sqrt{(1 + (2.9804 * \lambda^2) / (\lambda^2 - 0.02047) +$   

 $(0.5981 * \lambda^2) / (\lambda^2 - 0.0666) + (8.9543 * \lambda^2) / (\lambda^2 - 416.08))};$ 
```

```
noLiNbO3[ $\lambda$ _] :=  $\sqrt{(1 + (2.6734 * \lambda^2) / (\lambda^2 - 0.01764) +$   

 $(1.2290 * \lambda^2) / (\lambda^2 - 0.05914) + (12.614 * \lambda^2) / (\lambda^2 - 474.60))};$ 
```

```
n45LiNbO3[ $\lambda$ _] :=
```

```
 $\sqrt{(1 / ((\text{Cos}[\text{Pi} / 4])^2 / \text{neLiNbO3}[\lambda]^2 + (\text{Cos}[\text{Pi} / 4])^2 / \text{noLiNbO3}[\lambda]^2))};$   

(* for polarization rotation of pi/2, the light should be incident  

with pi/4 with respect to the ordinary axis of the chrystal,  

as the driver voltage will keep the pockels cell in  

"half wave plate mode" until pulse is extracted*)
```

```
ZnSe[ $\lambda$ _] :=  $\sqrt{(9.01536 + 0.24482 / (\lambda^2 - 0.29934^2) + 7229.931303 / (\lambda^2 - 48.38^2))};$ 
```

```
nCaF2[ $\lambda$ _] :=  $\sqrt{(1 + 0.437387571 * \lambda^2 / (\lambda^2 - 0.00173799328) +$   

 $0.449211397 * \lambda^2 / (\lambda^2 - 0.00782718648) + 0.152068715 * \lambda^2 / (\lambda^2 - 0.0124086125) +$   

 $13.0020420 * \lambda^2 / (\lambda^2 - 4039.76541))};$ 
```

```

(*Second order dispersion of a slab of length L in fs2,
Brewster angle correction *)
Disp2SlabBrewster[λ_, L_, n_] :=
  ((λ3 * L) / (2 * π * c2 * Sin[ArcTan[n[λ]]])) * n'[λ] * 1021;
(*Second order dispersion of a slab of length L in fs2*)
Disp2Slab[λ_, L_, n_] := ((λ3 * L) / (2 * π * c2)) * n'[λ] * 1021;

GDDLsum = Disp2SlabBrewster[λL * 106, LLiNbO3, n45LiNbO3] +
  Disp2SlabBrewster[λL * 106, LZnSe, ZnSe] + Disp2Slab[λL * 106, LCaF2, nCaF2];
(*linear dispersion for one pass through pockels cell,
gain medium and beam splitter *)

(* GDDLsum is the GDD from media dispersion for ONE pass.*)

GDDL[0] = 0; (*GDDL is the net GDD from media dispersion*)

GDDN[0] = 0; (*GDDN is the net GDD from nonlinear dispersion*)
GDDS[0] = 4.7 * 106; (*GDDS is the overall net
GDD. start value comes from the GDD of the pulse stretcher*)

```

**(* loop starts here, will calculate pulse energy,
linear and nonlinear dispersion*)**

**(* in the loop everything is calculated per pass,
not per roundtrip ! (a roundtrip is equal to 2 passes)*)**

```
For [ii = 1, ii < 51, ii++,
```

(* General and power *)

```

Jout[ii] = Jsat * Log[1 + Exp[η * Gain[ii]] * (Exp[Jin[ii] / Jsat] - 1)];
(*the Frantz Nodvik equation*)

```

```

Gain[ii + 1] = Gain[ii] - ((Jout[ii] - Jin[ii]) / Jsat);
(*determining the new small signal gain from the extracted energy*)

```

```
Jin[ii + 1] = Jout[ii] * (1 - Loss);
```

(* nonlinear Dispersion *)

```

Ppeak[ii] = 0.94 * Jout[ii] * Area / (TPULSE[ii - 1] * 10-15);
(*calculating the peak power of the pulse*)
Φmax[ii] = γ * LL * Ppeak[ii];
(*calculating the maximum SPM induced phase shift of the pulse*)

```

```

GDDNone[ii] = (((1 + (a * GDDS[ii - 1])^2) * (2 *  $\Phi$ max[ii] + a * GDDS[ii - 1])) /
  (1 + (2 *  $\Phi$ max[ii] + a * GDDS[ii - 1])^2)) / a - GDDS[ii - 1];
(*GDDNone is the GDD from nonlinear dispersion for one
pass of the amplifier*)

(* linear Dispersion *)

GDDL[ii] = GDDL[ii - 1] + GDDLsum; (*updating the value of GDDL*)

(* Pulse duration from net GDD *)
GDDN[ii] = GDDN[ii - 1] + GDDNone[ii];
(*updating the value of GDDN*)
GDDS[ii] = GDDS[ii - 1] + GDDNone[ii] + GDDLsum; (*updating the value of GDDS*)

TPULSE[ii] = tFWHM * Sqrt[1 + (GDDS[ii] * a)^2];
(*determining the current pulse duration*)

(*end of the loop*)
]

(* By using obtained values from the for loop,
this part creates plotable vectors *)

ENERGY = Table[Jout[i], {i, 50}] * Area * 10^6;
PMAx = Table[ $\Phi$ max[i], {i, 50}];
GDDL = Table[GDDL[i], {i, 50}];
GDDN = Table[GDDN[i], {i, 50}];
TPULSE = Table[TPULSE[i], {i, 50}] * 10^-3;
TIME = Table[i, {i, 50}];
GAIN = Table[Gain[i], {i, 50}];

```

Appendix B

Result Tables from Mathematica Code

Pass	Pulse Energy (μJ)	Phimax	GDDnonlin (fs^2)	GDDlin (fs^2)	netGDD (fs^2)	Duration (ps)	Gain
1	0.0000755582	2.35748×10^{-6}	-0.0170056	-2627.02	-2627.04	130.239	0.800603
2	0.0000984317	3.07288×10^{-6}	-0.0391717	-5254.04	-5254.08	130.166	0.800603
3	0.00012823	4.00536×10^{-6}	-0.0680642	-7881.07	-7881.13	130.093	0.800603
4	0.000167048	5.2208×10^{-6}	-0.105724	-10508.1	-10508.2	130.02	0.800603
5	0.000217617	6.80508×10^{-6}	-0.154813	-13135.1	-13135.3	129.948	0.800602
6	0.000283496	8.87011×10^{-6}	-0.218797	-15762.1	-15762.4	129.875	0.800601
7	0.000369316	0.0000115618	-0.302197	-18389.2	-18389.5	129.802	0.800601
8	0.000481116	0.0000150702	-0.410906	-21016.2	-21016.6	129.729	0.8006
9	0.000626759	0.0000196433	-0.552602	-23643.2	-23643.8	129.656	0.800598
10	0.00081649	0.000025604	-0.737296	-26270.2	-26271.	129.583	0.800597
11	0.00106365	0.0000333735	-0.978035	-28897.2	-28898.2	129.51	0.800594
12	0.00138563	0.0000435004	-1.29182	-31524.3	-31525.6	129.438	0.800591
13	0.00180506	0.0000567	-1.70083	-34151.3	-34153.	129.365	0.800587
14	0.00235145	0.0000739044	-2.23393	-36778.3	-36780.5	129.292	0.800582
15	0.0030632	0.0000963284	-2.9288	-39405.3	-39408.3	129.219	0.800576
16	0.00399034	0.000125555	-3.83448	-42032.4	-42036.2	129.146	0.800567
17	0.00519805	0.000163647	-5.01495	-44659.4	-44664.4	129.073	0.800556
18	0.00677115	0.000213293	-6.55353	-47286.4	-47292.9	129.	0.800541
19	0.00882014	0.000277994	-8.55883	-49913.4	-49922.	128.928	0.800522
20	0.0114888	0.000362311	-11.1723	-52540.4	-52551.6	128.855	0.800498
21	0.0149644	0.000472184	-14.5784	-55167.5	-55182.	128.782	0.800465
22	0.0194905	0.000615347	-19.0172	-57794.5	-57813.5	128.709	0.800423
23	0.025384	0.000801867	-24.8014	-60421.5	-60446.3	128.636	0.800369
24	0.0330568	0.00104484	-32.3383	-63048.5	-63080.9	128.563	0.800297
25	0.0430442	0.00136129	-42.1579	-65675.6	-65717.7	128.49	0.800205
26	0.0560413	0.00177334	-54.9498	-68302.6	-68357.5	128.416	0.800084
27	0.0729497	0.00230969	-71.6106	-70929.6	-71001.2	128.343	0.799927
28	0.0949372	0.00300756	-93.3055	-73556.6	-73649.9	128.27	0.799722
29	0.123514	0.0039151	-121.547	-76183.6	-76305.2	128.196	0.799456
30	0.160629	0.00509448	-158.295	-78810.7	-78969.	128.122	0.79911
31	0.208788	0.00662571	-206.089	-81437.7	-81643.8	128.048	0.79866
32	0.271204	0.00861141	-268.206	-84064.7	-84332.9	127.974	0.798075
33	0.351972	0.0111825	-348.87	-86691.7	-87040.6	127.898	0.797316
34	0.456278	0.0145049	-453.498	-89318.7	-89772.2	127.823	0.796331
35	0.590629	0.018787	-589.013	-91945.8	-92534.8	127.746	0.795056
36	0.7631	0.0242876	-764.204	-94572.8	-95337.	127.668	0.793407
37	0.983538	0.0313227	-990.138	-97199.8	-98190.	127.589	0.79128
38	1.2637	0.04027	-1280.61	-99826.8	-101107.	127.508	0.788544
39	1.61721	0.0515679	-1652.56	-102454.	-104106.	127.425	0.785038
40	2.05914	0.0657024	-2126.45	-105081.	-107207.	127.339	0.780567
41	2.6051	0.0831788	-2726.38	-107708.	-110434.	127.25	0.774897
42	3.2695	0.104466	-3479.82	-110335.	-113815.	127.156	0.767764
43	4.06282	0.12991	-4416.73	-112962.	-117379.	127.057	0.758873
44	4.98775	0.159609	-5567.77	-115589.	-121157.	126.953	0.747922
45	6.03452	0.193265	-6961.45	-118216.	-125177.	126.841	0.734623
46	7.17623	0.230032	-8620.18	-120843.	-129463.	126.722	0.718748
47	8.36569	0.268411	-10555.5	-123470.	-134026.	126.596	0.700178
48	9.53579	0.306259	-12763.6	-126097.	-138861.	126.462	0.67895
49	10.605	0.34096	-15221.8	-128724.	-143946.	126.321	0.655308
50	11.4883	0.369771	-17887.6	-131351.	-149239.	126.174	0.629705
51	12.1113	0.390276	-20701.1	-133978.	-154679.	126.023	0.60279
52	12.4239	0.400828	-23590.6	-136605.	-160196.	125.87	0.575333
53	12.4089	0.400832	-26480.2	-139232.	-165712.	125.717	0.548141
54	12.0835	0.390795	-29297.4	-141859.	-171157.	125.566	0.521956
55	11.4929	0.37214	-31980.2	-144486.	-176466.	125.419	0.497384
56	10.6999	0.346869	-34481.	-147113.	-181594.	125.277	0.474851
57	9.7735	0.317198	-36767.9	-149740.	-186508.	125.141	0.454597
58	8.77919	0.285238	-38824.5	-152367.	-191192.	125.011	0.436698
59	7.77251	0.252793	-40647.3	-154994.	-195642.	124.887	0.421104
60	6.79623	0.221259	-42242.8	-157621.	-199864.	124.77	0.407674

TABLE B.1: beam parameters per pass from the Mathematica code for a pump power of 259mW, maximum value of pulse energy is shown in figure

Pass	Pulse Energy (μJ)	Phimax	GDDnonlin (fs^2)	GDDlin (fs^2)	netGDD (fs^2)	Duration (ps)	Gain
1	0.0000836725	2.61066×10^{-6}	-0.0188319	-2627.02	-2627.04	130.239	0.902611
2	0.000120708	3.76832×10^{-6}	-0.0460145	-5254.04	-5254.09	130.166	0.902611
3	0.000174137	5.43932×10^{-6}	-0.0852509	-7881.07	-7881.15	130.093	0.90261
4	0.000251215	7.85131×10^{-6}	-0.141886	-10508.1	-10508.2	130.02	0.90261
5	0.000362409	0.0000113328	-0.223635	-13135.1	-13135.3	129.948	0.902609
6	0.00052282	0.0000163582	-0.341634	-15762.1	-15762.5	129.875	0.902608
7	0.000754231	0.0000236119	-0.511958	-18389.2	-18389.7	129.802	0.902606
8	0.00108807	0.0000340821	-0.757808	-21016.2	-21016.9	129.729	0.902604
9	0.00156966	0.0000491948	-1.11267	-23643.2	-23644.3	129.656	0.902601
10	0.00226439	0.0000710084	-1.62489	-26270.2	-26271.8	129.583	0.902596
11	0.00326659	0.000102494	-2.36422	-28897.2	-28899.6	129.51	0.902589
12	0.0047123	0.000147938	-3.43137	-31524.3	-31527.7	129.438	0.902579
13	0.00679772	0.000213527	-4.97164	-34151.3	-34156.3	129.365	0.902565
14	0.00980579	0.000308189	-7.19475	-36778.3	-36785.5	129.292	0.902544
15	0.0141445	0.000444801	-10.4033	-39405.3	-39415.7	129.219	0.902515
16	0.0204017	0.000641936	-15.0339	-42032.4	-42047.4	129.146	0.902472
17	0.0294249	0.000926372	-21.7162	-44659.4	-44681.1	129.073	0.90241
18	0.0424342	0.00133669	-31.3584	-47286.4	-47317.8	129.	0.902321
19	0.0611856	0.00192846	-45.2693	-49913.4	-49958.7	128.927	0.902192
20	0.0882031	0.00278158	-65.334	-52540.4	-52605.8	128.853	0.902007
21	0.127109	0.00401082	-94.2657	-55167.5	-55261.7	128.78	0.90174
22	0.183091	0.00578057	-135.963	-57794.5	-57930.4	128.706	0.901356
23	0.26355	0.00832561	-196.019	-60421.5	-60617.5	128.631	0.900802
24	0.378998	0.0119796	-282.432	-63048.5	-63331.	128.556	0.900005
25	0.544259	0.0172133	-406.596	-65675.6	-66082.1	128.48	0.89886
26	0.780023	0.0246845	-584.65	-68302.6	-68887.2	128.402	0.897217
27	1.11473	0.035298	-839.257	-70929.6	-71768.9	128.322	0.894866
28	1.58661	0.0502713	-1201.86	-73556.6	-74758.5	128.239	0.891512
29	2.24532	0.0711884	-1715.32	-76183.6	-77899.	128.152	0.886753
30	3.15207	0.100005	-2436.59	-78810.7	-81247.2	128.059	0.880044
31	4.37597	0.138936	-3438.58	-81437.7	-84876.3	127.958	0.870679
32	5.98357	0.190126	-4809.64	-84064.7	-88874.3	127.848	0.857783
33	8.01775	0.254983	-6648.22	-86691.7	-93339.9	127.724	0.840345
34	10.4651	0.333138	-9050.05	-89318.7	-98368.8	127.584	0.817336
35	13.2184	0.421242	-12086.7	-91945.8	-104032.	127.427	0.787923
36	16.0522	0.512179	-15778.3	-94572.8	-110351.	127.252	0.751779
37	18.6393	0.595545	-20070.2	-97199.8	-117270.	127.06	0.709405
38	20.6217	0.659882	-24825.3	-99826.8	-124652.	126.856	0.662297
39	21.7164	0.696032	-29840.6	-102454.	-132294.	126.644	0.612801
40	21.803	0.699978	-34884.3	-105081.	-139965.	126.431	0.563636
41	20.9478	0.673652	-39738.5	-107708.	-147446.	126.224	0.517292
42	19.3582	0.623557	-44232.	-110335.	-154567.	126.026	0.475561
43	17.304	0.558262	-48255.5	-112962.	-161217.	125.842	0.439379
44	15.0451	0.486094	-51759.2	-115589.	-167348.	125.672	0.408923
45	12.7881	0.413731	-54741.7	-118216.	-172958.	125.516	0.383852
46	10.6731	0.345733	-57234.2	-120843.	-178077.	125.374	0.363547
47	8.77908	0.284702	-59287.	-123470.	-182757.	125.245	0.347288
48	7.13831	0.231733	-60958.	-126097.	-187055.	125.125	0.334375
49	5.7516	0.186893	-62305.7	-128724.	-191030.	125.015	0.324175
50	4.60123	0.149645	-63384.9	-131351.	-194736.	124.912	0.316149
51	3.66035	0.119143	-64244.2	-133978.	-198222.	124.816	0.309851
52	2.8991	0.0944374	-64925.3	-136605.	-201530.	124.724	0.304918
53	2.28828	0.0745951	-65463.3	-139232.	-204695.	124.636	0.301058
54	1.80132	0.0587621	-65887.2	-141859.	-207746.	124.552	0.298041
55	1.41501	0.0461915	-66220.3	-144486.	-210707.	124.47	0.295684
56	1.10974	0.0362499	-66481.8	-147113.	-213595.	124.39	0.293844
57	0.869207	0.0284112	-66686.8	-149740.	-216427.	124.311	0.292408
58	0.680133	0.0222451	-66847.2	-152367.	-219214.	124.234	0.291288
59	0.531773	0.0174035	-66972.7	-154994.	-221967.	124.157	0.290413
60	0.415524	0.0136074	-67070.9	-157621.	-224692.	124.082	0.289731

TABLE B.2: beam parameters per pass from the Mathematica code for a pump power of 292mW, maximum value of pulse energy is shown in figure

Pass	Pulse Energy (μJ)	Phimax	GDDnonlin (fs^2)	GDDlin (fs^2)	netGDD (fs^2)	Duration (ps)	Gain
1	0.0000926582	2.89102×10^{-6}	-0.0208543	-2627.02	-2627.04	130.239	1.00462
2	0.000148027	4.62115×10^{-6}	-0.0541888	-5254.04	-5254.1	130.166	1.00462
3	0.00023648	7.38666×10^{-6}	-0.107472	-7881.07	-7881.17	130.093	1.00462
4	0.00037779	0.0000118072	-0.192643	-10508.1	-10508.3	130.02	1.00462
5	0.000603538	0.0000188731	-0.328784	-13135.1	-13135.4	129.948	1.00462
6	0.000964179	0.0000301676	-0.546397	-15762.1	-15762.7	129.875	1.00461
7	0.00154031	0.0000482209	-0.894237	-18389.2	-18390.	129.802	1.00461
8	0.0024607	0.0000770776	-1.45023	-21016.2	-21017.6	129.729	1.00461
9	0.00393099	0.000123202	-2.33894	-23643.2	-23645.5	129.656	1.0046
10	0.00627971	0.000196923	-3.75944	-26270.2	-26274.	129.583	1.00458
11	0.0100315	0.000314751	-6.02989	-28897.2	-28903.3	129.51	1.00456
12	0.016024	0.000503059	-9.65868	-31524.3	-31533.9	129.437	1.00453
13	0.0255947	0.000803973	-15.4581	-34151.3	-34166.7	129.364	1.00448
14	0.0408773	0.00128475	-24.7256	-36778.3	-36803.	129.291	1.0044
15	0.0652739	0.00205268	-39.5325	-39405.3	-39444.9	129.218	1.00427
16	0.104202	0.00327873	-63.1833	-42032.4	-42095.5	129.145	1.00406
17	0.166275	0.00523482	-100.944	-44659.4	-44760.3	129.071	1.00372
18	0.265139	0.00835212	-161.191	-47286.4	-47447.6	128.996	1.00318
19	0.422317	0.0133111	-257.208	-49913.4	-50170.6	128.921	1.00233
20	0.67149	0.0211772	-409.964	-52540.4	-52950.4	128.844	1.00097
21	1.0647	0.0335981	-652.31	-55167.5	-55819.8	128.764	0.998805
22	1.68074	0.053071	-1035.1	-57794.5	-58829.6	128.681	0.995382
23	2.63497	0.0832557	-1635.59	-60421.5	-62057.1	128.591	0.989994
24	4.087	0.129225	-2567.56	-63048.5	-65616.1	128.492	0.981581
25	6.23672	0.197347	-3990.67	-65675.6	-69666.2	128.38	0.968616
26	9.28958	0.294204	-6111.93	-68302.6	-74414.5	128.249	0.949028
27	13.3656	0.423728	-9166.47	-70929.6	-80096.1	128.091	0.920295
28	18.3446	0.582293	-13363.	-73556.6	-86919.6	127.902	0.879898
29	23.7111	0.753747	-18793.7	-76183.6	-94977.4	127.678	0.826287
30	28.557	0.909381	-25344.2	-78810.7	-104155.	127.424	0.760177
31	31.8716	1.01696	-32668.3	-81437.7	-114106.	127.148	0.685375
32	32.995	1.05509	-40266.5	-84064.7	-124331.	126.865	0.60813
33	31.8961	1.02223	-47628.5	-86691.7	-134320.	126.588	0.53504
34	29.0829	0.934107	-54356.7	-89318.7	-143675.	126.328	0.470894
35	25.2886	0.813909	-60220.3	-91945.8	-152166.	126.093	0.417802
36	21.1859	0.683137	-65142.7	-94572.8	-159716.	125.883	0.375653
37	17.2492	0.557124	-69158.	-97199.8	-166358.	125.699	0.343096
38	13.7439	0.444556	-72362.5	-99826.8	-172189.	125.538	0.318365
39	10.7746	0.348961	-74878.3	-102454.	-177332.	125.395	0.299758
40	8.34484	0.270576	-76829.2	-105081.	-181910.	125.268	0.285831
41	6.40472	0.207879	-78328.3	-107708.	-186036.	125.154	0.275432
42	4.88253	0.158618	-79472.2	-110335.	-189807.	125.049	0.267675
43	3.70338	0.120412	-80340.6	-112962.	-193303.	124.952	0.26189
44	2.79845	0.0910592	-80997.3	-115589.	-196586.	124.861	0.257575
45	2.10871	0.0686657	-81492.6	-118216.	-199709.	124.775	0.254356
46	1.58564	0.0516689	-81865.3	-120843.	-202708.	124.691	0.251954
47	1.19045	0.0388175	-82145.3	-123470.	-205615.	124.611	0.250161
48	0.892716	0.0291279	-82355.4	-126097.	-208452.	124.532	0.248822
49	0.66886	0.0218376	-82512.9	-128724.	-211237.	124.455	0.247822
50	0.50081	0.0163611	-82630.9	-131351.	-213982.	124.379	0.247076
51	0.3748	0.0122519	-82719.3	-133978.	-216697.	124.304	0.246518
52	0.280394	0.0091714	-82785.5	-136605.	-219391.	124.229	0.246101
53	0.20971	0.00686352	-82835.	-139232.	-222067.	124.155	0.24579
54	0.156813	0.00513533	-82872.	-141859.	-224731.	124.081	0.245557
55	0.11724	0.0038417	-82899.7	-144486.	-227386.	124.007	0.245383
56	0.0876443	0.00287361	-82920.4	-147113.	-230034.	123.934	0.245253
57	0.0655139	0.00214929	-82936.	-149740.	-232676.	123.861	0.245156
58	0.0489684	0.00160744	-82947.5	-152367.	-235315.	123.787	0.245083
59	0.0365998	0.00120213	-82956.2	-154994.	-237951.	123.714	0.245029
60	0.0273542	0.000898992	-82962.7	-157621.	-240584.	123.641	0.244989

TABLE B.3: beam parameters per pass from the Mathematica code for a pump power of 325mW, maximum value of pulse energy is shown in figure

Pass	Pulse Energy (μJ)	Phimax	GDDnonlin (fs^2)	GDDlin (fs^2)	netGDD (fs^2)	Duration (ps)	Gain
1	0.000100103	3.12329×10^{-6}	-0.0225298	-2627.02	-2627.04	130.239	1.0819
2	0.000172768	5.39353×10^{-6}	-0.0614358	-5254.04	-5254.11	130.166	1.0819
3	0.000298181	9.31393×10^{-6}	-0.128622	-7881.07	-7881.19	130.093	1.0819
4	0.000514631	0.0000160839	-0.244642	-10508.1	-10508.3	130.02	1.08189
5	0.000888201	0.0000277748	-0.444995	-13135.1	-13135.6	129.948	1.08189
6	0.00153294	0.0000479631	-0.790975	-15762.1	-15762.9	129.875	1.08189
7	0.00264567	0.000082825	-1.38843	-18389.2	-18390.5	129.802	1.08188
8	0.00456605	0.000143025	-2.42013	-21016.2	-21018.6	129.729	1.08188
9	0.00788019	0.000246974	-4.20167	-23643.2	-23647.4	129.656	1.08186
10	0.0135993	0.000426457	-7.2779	-26270.2	-26277.5	129.583	1.08183
11	0.0234677	0.00073633	-12.5894	-28897.2	-28909.8	129.51	1.08179
12	0.0404927	0.00127123	-21.7593	-31524.3	-31546.	129.437	1.08171
13	0.0698558	0.00219429	-37.5877	-34151.3	-34188.9	129.364	1.08157
14	0.120473	0.00378641	-64.9007	-36778.3	-36843.2	129.29	1.08134
15	0.207652	0.00653013	-112.005	-39405.3	-39517.3	129.216	1.08093
16	0.357578	0.0112514	-193.165	-42032.4	-42225.5	129.141	1.08023
17	0.614745	0.0193545	-332.774	-44659.4	-44992.1	129.064	1.07903
18	1.05391	0.0332007	-572.253	-47286.4	-47858.6	128.985	1.07697
19	1.79818	0.0566822	-981.092	-49913.4	-50894.5	128.901	1.07343
20	3.04336	0.0959953	-1673.45	-52540.4	-54213.9	128.809	1.06741
21	5.08175	0.160406	-2830.24	-55167.5	-57997.7	128.704	1.05727
22	8.30062	0.262223	-4721.01	-57794.5	-62515.5	128.578	1.04045
23	13.0966	0.414136	-7706.44	-60421.5	-68127.9	128.423	1.01332
24	19.6237	0.621283	-12183.7	-63048.5	-75232.2	128.226	0.971342
25	27.3809	0.868206	-18438.	-65675.6	-84113.6	127.98	0.910414
26	34.9484	1.11029	-26433.2	-68302.6	-94735.8	127.685	0.829424
27	40.3916	1.28618	-35692.4	-70929.6	-106622.	127.356	0.733003
28	42.3083	1.3507	-45415.	-73556.6	-118972.	127.013	0.631406
29	40.5947	1.29948	-54769.6	-76183.6	-130953.	126.681	0.536265
30	36.249	1.16342	-63146.5	-78810.7	-141957.	126.376	0.455564
31	30.6201	0.985129	-70241.6	-81437.7	-151679.	126.106	0.391891
32	24.8175	0.800151	-76006.2	-84064.7	-160071.	125.874	0.34394
33	19.5166	0.630405	-80549.	-86691.7	-167241.	125.675	0.308771
34	15.0167	0.485822	-84050.8	-89318.7	-173370.	125.505	0.283313
35	11.3742	0.368478	-86707.2	-91945.8	-178653.	125.358	0.264979
36	8.51829	0.27628	-88699.3	-94572.8	-183272.	125.23	0.25179
37	6.32763	0.205439	-90180.7	-97199.8	-187381.	125.116	0.242294
38	4.67275	0.151848	-91275.8	-99826.8	-191103.	125.013	0.235446
39	3.43601	0.11175	-92081.8	-102454.	-194536.	124.918	0.2305
40	2.51882	0.0819829	-92673.1	-105081.	-197754.	124.829	0.226922
41	1.84234	0.0600076	-93105.9	-107708.	-200814.	124.744	0.224331
42	1.34536	0.0438502	-93422.2	-110335.	-203757.	124.662	0.222452
43	0.981293	0.0320047	-93653.1	-112962.	-206615.	124.583	0.221089
44	0.715133	0.0233388	-93821.4	-115589.	-209410.	124.506	0.2201
45	0.520841	0.0170086	-93944.1	-118216.	-212160.	124.429	0.219382
46	0.379165	0.0123896	-94033.5	-120843.	-214876.	124.354	0.21886
47	0.275936	0.00902194	-94098.5	-123470.	-217569.	124.279	0.21848
48	0.200764	0.00656808	-94145.9	-126097.	-220243.	124.205	0.218205
49	0.146046	0.00478079	-94180.4	-128724.	-222904.	124.131	0.218004
50	0.106227	0.00347941	-94205.5	-131351.	-225557.	124.058	0.217858
51	0.0772581	0.00253204	-94223.8	-133978.	-228202.	123.985	0.217753
52	0.0561854	0.0018425	-94237.1	-136605.	-230842.	123.911	0.217676
53	0.0408584	0.00134067	-94246.7	-139232.	-233479.	123.838	0.21762
54	0.0297115	0.000975483	-94253.8	-141859.	-236113.	123.765	0.217579
55	0.0216051	0.000709754	-94258.9	-144486.	-238745.	123.692	0.217549
56	0.0157101	0.000516402	-94262.6	-147113.	-241376.	123.619	0.217528
57	0.0114234	0.000375718	-94265.3	-149740.	-244006.	123.546	0.217512
58	0.00830635	0.000273358	-94267.3	-152367.	-246635.	123.474	0.217501
59	0.00603978	0.000198883	-94268.7	-154994.	-249263.	123.401	0.217493
60	0.00439166	0.000144698	-94269.8	-157621.	-251891.	123.328	0.217487

TABLE B.4: beam parameters per pass from the Mathematica code for a pump power of 350mW, maximum value of pulse energy is shown in figure

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