A POOLING-BASED STOCHASTIC ORDER-PICKING SYSTEM CONSIDERING PRIORITY QUEUES

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A POOLING-BASED STOCHASTIC ORDER-PICKING SYSTEM CONSIDERING PRIORITY QUEUES

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ABSTRACT

A POOLING-BASED STOCHASTIC ORDER-PICKING SYSTEM CONSIDERING PRIORITY QUEUES

E-commerce is a phenomenon that shapes the shopping needs of today's world and enables every product to reach our homes with a single click, shaping the life of the consumption society of the 21st century. E-commerce is a market whose users are increasing every year. Especially in the post-covid period, people far from e-commerce applications have also started to use this system.

In this thesis, the bottleneck created by online orders and the increasing workload in online order retailers on special days are focused on. Markov models are a widely used method in modeling the preparation and waiting process of such orders. Future orders can be determined with Markov chains and the waiting times in the queue can be found as exact solutions without the need for simulation. Classifying orders and prioritizing certain order classes can avoid bottlenecks. Priority orders are generally expected to include more valuable product lines, as retailers want to deliver more valuable items sooner to satisfy relevant customers sooner. In this study, an order fulfillment model using the priority model is envisaged and two models are considered with priority orders. In the first model, we assume different service rates for different priority classes. Thus, we aim to specify the conditions where the priority order model decreases expected waiting time and increase the customer satisfaction. In the second model, same service rates for different priority classes are assumed but order cancellations are also considered by introducing a threshold waiting time level for every customer's waiting sensitivity. As waiting time sensitivities, deterministic and exponential values are considered. Moreover, the proportion of serviced customers as well as the expected waiting time is regarded as another performance measure of the system because order cancellations are possible. Lastly, numerical results are obtained to show our statements.

ÖZET

ÖNCELİKLİ KUYRUKLARI DİKKATE ALAN HAVUZ TEMELLİ STOKASTİK SİPARİŞ TOPLAMA SİSTEMİ

E-ticaret, günümüz dünyasının alışveriş ihtiyaçlarını şekillendiren ve her ürünün tek tıkla evimize ulaşmasını sağlayan, 21. yüzyılın tüketim toplumunun hayatını şekillendiren bir olgudur. E-ticaret her yıl kullanıcısı artan bir pazardır. Özellikle covid sonrası dönemde e-ticaret uygulamalarından uzak kişiler de bu sistemi kullanmaya başladı.

Bu tezde online siparişlerin, depolardaki artan iş yükünün özel günlerde oluşturduğu darboğaza odaklanılıyor. Markov modelleri, bu tür siparişlerin hazırlanma ve bekleme süreçlerinin modellenmesinde yaygın olarak kullanılan bir yöntemdir. Gelecekteki siparişler Markov zincirleri ile belirlenebilir ve kuyruktaki bekleme süreleri simülasyona ihtiyaç duymadan kesin çözümler olarak bulunabilir. siparişlerin sınıflandırılması ve bazı emir sınıflarına öncelik verilmesi darboğazları önleyebilir. Perakendeciler sipariş tutarı yüksek müşterileri daha önce memnun etmek için daha değerli ürünleri daha erken teslim etmek istediklerinden, öncelikli siparişlerin genellikle daha değerli ürün gruplarını içermesi beklenir. Bu çalışmada, önceliklendirme modelinin kullanıldığı bir sipariş tamamlama modeli öngörülmektedir ve önceliklendirmenin olduğu iki model ele alınmıştır. İlk modelde, farklı öncelik sınıfları için farklı hizmet oranları varsayıyoruz. Böylece öncelik sıralaması modelinin beklenen bekleme süresini azalttığı ve müşteri memnuniyetini arttırdığı durumları belirlemeyi amaçlıyoruz. İkinci modelde, farklı öncelik sınıfları için aynı hizmet oranları varsayılır, ancak sipariş iptalleri de her müşterinin bekleme hassasiyeti için bir eşik bekleme süresi seviyesi getirilerek dikkate alınır. Bekleme süresi hassasiyetleri olarak deterministik ve üstel değerler dikkate alınmıştır. Ayrıca, hizmet verilen müşterilerin oranı ve beklenen bekleme süresi, sipariş iptallerinin mümkün olması nedeniyle sistemin bir başka performans ölçüsü olarak kabul edilmektedir. Son olarak iddialarımızı göstermek için sayısal sonuçlar elde edilmiştir.

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LIST OF SYMBOLS/ABBREVIATIONS

λ	The arrival rate for whole orders
λ_1	The arrival rate for priority orders
λ_2	The arrival rate for non-priority orders
μ	The service rate for whole orders
μ_1	The service rate for priority orders
μ_2	The service rate for non-priority orders
ρ	The utilization factor which is / μ
$E[w]$	The expected waiting time for the system 1
$E[w_1]$	The expected waiting time for priority orders
$E[w_2]$	The expected waiting time for non-priority orders
W_1	The average waiting time for priority orders
W_2	The average waiting time for non-priority orders
p	The proportion of priority orders in the system
q	The proportion of non-priority orders in the system
λ_{eff}	The effective arrival rate of customers
S	The maximum waiting time a customer can tolerate
γ	The customers' waiting time tolerance level S has an exponential distribution with a rate
Cs	Customer Satisfaction

1. INTRODUCTION

Online shopping platforms have grown quickly over the past few years. With such platforms, customers make their shopping online, the service providers prepare the orders of customers, and then the orders are delivered to the residence of the customers. The increase in Internet usage and Covid-19 pandemic, online shopping platforms have become more popular than ever. However, a growing number of customers using online shopping platforms causes customers to wait for a long time to receive their orders resulting in customer dissatisfaction.

Especially on certain days like Valentine's Day, Black Friday and Cyber Monday, the boom in online shopping causes service providers to encounter excessive demand and creates bottleneck in warehouses where orders are prepared. Moreover, many retailers make their sale plans on Black Friday after thanksgiving to meet their high profit targets before closing the financial year [1]. Therefore, service providers on online platforms must come up with alternative solutions to deal with these highly volatile customer fluctuations because the boom season in online shopping result in excessive waiting time for customers. However, excessive waiting time for customers is a big problem for the service providers because customer waiting time is one of the most important measure that determines customer loyalty for online shopping platforms.

The growth of online shopping platforms creates a wide range of research areas. When the process structure of the order preparation process through online platforms are considered, queuing models are frequently used to understand the customer satisfaction by computing their waiting times [2].

In another area of research, the effect of priority classes on waiting time of queuing models is studied. Charging for priority class is an efficient and prevalent strategy to manage congestion and revenue in many service systems [3]. Offering products which are different only in their delivery times is also a popular strategy in the systems where heterogeneous time-sensitive customers exist [4]. Therefore, much of queuing theory is devoted to analyzing priority queues, where customers are labeled and served in accordance with a priority scheme [5]. Moreover, many queuing systems with multiple classes in the literature are modeled by assuming Markovian processes [6,7].

These research areas are the motivation for our study. In this thesis, we study the expected waiting time of customers in service systems that can be regarded as a measure of customer satisfaction. We consider an M/M/1 service system where there are priority classes. Our model is different from the literature in that the market is homogeneous and the customers are not differentiated out of the system. For the sake of simplicity, we divide orders in two groups: on the one hand there are standard orders, these orders include multiple items at reasonable prices; on the other hand, there are prioritized orders with single item having considerably greater price. We suppose that it is possible to decrease overall expected waiting time by giving priority to some orders.

In the first part of the thesis, we consider a model where order cancellation is not possible. In this model, we assume different service rates for different order groups and compare the average waiting time of customers in a priority system with different service rates and a standard system with a single service rate. We investigate the relationship between service rates to obtain a better service system with smaller expected waiting time in a priority service system.

We also consider an alternative model where the service rate does not change with order classes. Therefore, we assume a constant service rate for both standard system and priority system. However, in this system, balking is possible in that some customers can cancel their orders and leave the system without shopping. Studying balking altogether with priority decisions brings an advantage that is without balking only a system with workload less than one can be studied due to the stability requirement. Because we analyze an online shopping system during boom seasons, it makes much sense to assume a workload greater than 1. Moreover, we assume that a customer cancels his order whenever his waiting time is greater than the time he can tolerate. Therefore, a waiting time sensitivity value is assumed for each customer.

We compute numerical results for both of our two proposed models under different parameter values that indicate us how grouping orders and giving priority affect customers' expected waiting time and the proportion of customers who can take the service. Then, we determine the conditions where grouping orders reduces expected waiting time and makes the customers better off.

This thesis is structured as follows. In Chapter 2, we review the literature on Markov chains and queuing theory, priority queuing models and the relationship between waiting time and customer satisfaction. In Chapter 3, we present our first priority model with two priority classes. We assume that the service rate for different classes is also different. Then, we give a second model in which balking is possible in Chapter 4. However, the same service rate for different classes are assumed in this model. Finally, conclusions are drawn in Chapter 5 and directions for future work are offered in Chapter 6.

2. LITERATURE REVIEW

E-commerce is a milestone for commerce. Customers can purchase items in the comfort of their own homes or workplaces through the internet. It is easy to cancel the transactions, and a wide variety of products are available at a lower price. E-commerce is defined as "buying goods and services without any effort and selling products with minimum effort and using the internet." [5–8]. The most critical part of e-commerce is customer satisfaction. The goods and services are available at places in traditional commerce. Customers can see and touch the product to check the quality of the goods. The quality of e-commerce is to meet customer needs in the most effective way through the internet [9,10].

E-commerce is more popular than onsite sales because of the wide variety of products, ease of search, and competitive prices for the same goods and services, and so on [1,11]. It is common because people hurry to reach goods, and the time is insufficient for face-to-face shopping [12]. People use e-commerce after work-hour (from 5.00 p.m. to 9.00 p.m.). Many markets and services closed during that period, so it raises the number of e-commerce users [12,13]. Considering the impact of COVID-19 pandemic crisis, new shoppers have adapted themselves to e-commerce very quickly. Ordinary food supplies can order online, especially after a pandemic, as predicted by some authors [6,14]. In addition, sales on Black Friday continue to increase every year, thanks to the internet.

This study examines how the big increase in sales volume especially on special days like Black Friday, and Valentine’s Day affects the workload for the preparation of online orders. In addition, this thesis, we examine the literature in 3 main parts.

2.1. Markov Chain and Queue Theory

2.2. Priority Queuing Models

2.3. The Relationship Between Waiting Time and Customer Satisfaction

2.1. MARKOV CHAIN AND QUEUING THEORY

Markov models are quite important in the literature since they allow exact solutions for complex stochastic problems. One implementation area of Markov Chain models is game theory. Game theory was started by John Nash. Majumder and Groeneveld use game theory to formulate the Markov chain [15]. Today, Markov models also come up with more innovative ideas. For instance, there are studies that include the product lifecycle [16]. One of these studies includes the life cycle of paper in the paper industry and finds its life cycle to be inferior to that of a quality product. A similar study was conducted in 2015 for plastic products as well.

Markov models are used in service systems where customer waiting time modelling is important. In this case, the decision-making process of customers can also be modelled by using Markov chains. In [17], customers decided to stay or leave the line based on the information they receive. This work is also important because it assesses information flow in queue systems. The system examines no information, partial information, and full information cases. The customer's knowledge of the whole process increases their decision to leave the system. This situation negatively affects customer satisfaction. The queue relaxes in the short term, but it becomes longer again with the entry of new customers.

In a 2-server queue model, customer behavior with knowledge of the queue is examined [18]. In this study, customers chose the shorter of the 2 servers to shorten the sojourn time. Using a Markovian model, it can statistically predict which server the customers will choose. In this model, when we look at the length of the queue, the shorter queue means a shorter sojourn time.

The probabilities of the order in which certain events occur are the focus of Markov Models. In the study [19], a supplier interruption model with a Bayesian network was created by the author. The Markov models show graphically and explain the transitions between the current states. Markov chains are used in more than one place. For example, there are researchers who have used it for inventory problems [20–24].

Optimizing the queue reduces the waiting time. That also increases labor productivity [25]. In [25], a full-information system based on Naor's assumptions was investigated by the author. In [26], researchers studied systems for which there is no information. In [27], Poisson arrivals were considered by the author, and we also assume Poisson arrivals in our model. In [28], a 2-client-class with fixed times was predicted. There is no fixed time in our study.

2.2. PRIORITY QUEUING MODELS

The classification of customers is more frequent than ever due to the increasing competition in marketing. For example, the payment line are opened for orders including fewer than five products at the supermarket. Another example can be seen in banks through card scanning. Banks prioritize their own customers over others, so you benefit from shorter wait times. In another example, if the customer is in the VIP category, the service stops for other customers, and the operation focuses on the VIP customer in luxury shopping stores. This is called priority in the literature. In sequential models, a priority model can increase the waiting time for certain products. This reduces the satisfaction of some customers and the average decreases. There are M/M/C studies for priority models [29]. For priority studies, M/M/1 models can obtain values more easily, while M/M/C structures can be used for non- priority studies. It is more convenient because equal service rates are anticipated between the servers.

Kamoun tried this assumption for the model in tandem. Due to this approach, departures from the system should result in a realistic queuing system [30]. This study also addresses the negative impact of the change in low-priority traffic. The study of priority and non-priority actions is also evident in the studies of Kargahi and Movaghar [8]. The Earliest-Deadline-First Policy is used in this thesis. The values which obtain are between deterministic and exponential results. The assumption of values as deterministic and exponential in our thesis is relevant to this study. The authors also use cancel operations. In this study, a definite analytical solution was not presented, but the importance of the policy problem was revealed. Our thesis envisages the classical first-come, first-served (FCFS) situation and also waiting time tolerance.

When the related study examined that thesis, that thesis used the "first in, first out policy". In addition, two types of classification were used. One of them was high priority, and the other was low priority. It is similar to our study both in terms of policy and classification [5]. The results were obtained using Little's Law by calculating the average wait divided by the average arrival. In this thesis, it reduces to a M/M/1 system, as a study on the queue length of high-priority and low-priority customers reveals. In our thesis, the queue length are not included in the analysis due to the waiting time tolerance value. Thanks to the Cancel operation, the queue length can stay under control. In addition, there are priority systems that use M/M/1 [10]. Le Nyl used explicit formulas in his study. Also, his work included

FIFO policy. With the Maple function, the first or second type of priority situation was examined. Recursive equation values were calculated. Owing to this study, we see that keeping the classification scale at two types would be healthier for our thesis. There are other studies that used explicit recursive formulas [13]. This approach is used for both priority and non- priority. The matrix is in line with Neuts Theory. In this study used for both discreate and continues time processes.

In addition, penalty costs were assumed by Liberopoulosa, Tsikisa and Delikouras [31]. They made this assumption for back orders. Back Order refers to completing the unallocated number of products that cannot be completed in the warehouse. Backorders prioritize their status as they wait for a long time. As a result, they proposed a Schwartz's model penalty coefficient for the backorder. Similar to our thesis, two classes predict, and the authors used discrete-time priority [11]. In this study, delay was predicted by the author, but there was no cancel order process as in our model. This study theoretically shows the effect of service time on performance in a priority system with numerical examples.

A warehouse includes complex activities, and optimizing the warehouse focus can decrease the operation time in the warehouse. Nowadays, diminishing time is not enough for operations. Leadership skills are also crucial for handling a high volume of orders. Optimizing the process is not just about shortening warehouse processing times. It is necessary to look at the system as a whole and synchronize it [32]. An improvement study in the paper gives the expected performance from the system. Mathematical optimization creates disadvantages in terms of time and complexity [33]. Queuing for the warehouse process is an expected situation. A queue can appear as a pattern wherever you expect it. The first study on the queuing model was made by Agner Krarup Erlang, and it was about telephone exchanges [17,33]. When today's conditions are considered, the system produces a more realistic working structure. To improve the optimization of the warehouse, inventory management models, queueing theory, and methods of graph theory [32].

Analyzing impatient customers is as crucial as optimizing the warehouse for operation. Shukla, Khan, and Shivhare examined the queue model on online shopping sites in their study in 2019 [34]. It was a model that was used to accelerate the flow of customers in the system by using the Lining Hypothesis. This study once again demonstrated the difficulties of retaining customers in the system. Working with particularly impatient customer groups requires a more straightforward and effective approach. For instance, giving information to the users in the warehouse reinforces the perception of value in the eyes of the customer. The simplest example of this is the cargo stage, and starting the time from the warehouse process will help keeping the curiosity of the customer under control.

2.3. THE RELATIONSHIP BETWEEN WAITING TIME AND CUSTOMER SATISFACTION

Waiting time and customer satisfaction interrelate with each other considerably. As the increase in waiting time reduces satisfaction, the customer may not want to work with the same seller again, and the customer can cancel the orders [12,17,35]. The term "delay means the time endured to obtain a good or service". The waiting time described in the previous section relates to the delay. When waiting time increases, it triggers a delay. That causes a decrease in customer satisfaction.

Service time is important for both delays and waiting times. The service time only changes with the creation of new servers. New servers create new queues, which decrease the waiting time and the delay. Creating new servers affects the revenue of the process. The waiting time can vary with the number of services. However, the cost must be considered in that case. In [36], who studied income, argued that the necessary attention was not given to this issue. Especially, return operation decreases the net asset value.

When we look at the difference between generations, the y-generation (millennials) is more patient than the z-generation. With Z-generation, the sensitivity of the waiting time increases. This situation starts to decrease customer satisfaction faster. For example, a package damaged in the warehouse can be returned even if the product is not broken, reducing customer satisfaction [12]. It should not be forgotten that waiting time likewise decreases satisfaction. Customers do not like long waiting times. This is how it is in general, no matter what product or service it is. Analyses are a way to reduce waiting times and increase customer satisfaction. Every manager wants to see waiting times near zero at their facility. However, the cost is very high. Therefore, the ideal is to determine an optimum level and approach to these values [37]. Waiting in service lines is disliked by the clients [38,39].

The business-to-business, B2B, systems has become the main element of e-commerce [2]. Yang and Ding designed a survey to measure customer satisfaction with their thesis. In this study, they identified five main elements. These ware usability, information quality, system quality, security, privacy, and visual design. Based on this study, there may be changes in the waiting time tolerance situation in line with the knowledge level of the customers. Incomplete or complete information status may determine the cancellation or continuation of the order.

According to the dissonance theory, knowing the waiting time ahead of time minimizes a person's unhappiness while they are waiting [40]. For example, if we inform the customer about the process, we can preserve customer satisfaction [40,41]. For example, process flow animations on e-commerce sites or notifications for cargo help protect customer satisfaction and foster a sense of trust. In this way, the customer wants to work with the same vendors and companies over and over again.

Waiting is an operation that has consequences. It creates problems in terms of both time and cost [39]. The waiting time may also cause complaints by the customer. In the study, they worked on the barcode system for customer losses [42]. In the model they are interested in the company uses barcodes, but it mostly uses them for commercial reasons, not picking or shipment operations. In the system, a system that is used barcodes and controls the process with barcode readers are used by the staff.

In other studies, the subject of customer loss was covered with data mining [43]. Customer groups and select algorithm issues affect customer loss. In this study, individualization of services was suggested by author. In terms of warehouse management, maintaining the correct stock amount also affect the system. Thus, stocks more than needed cause unnecessary expenses and waste of space. In addition, the area that cannot be used due to excessive stock destroys the opportunity for another product and causes customer losses [44].

Since service quality increases satisfaction, the relationship between waiting time and service quality is investigated [9]. There are resources available to support these ideas. In particular, [45] emphasized that customer satisfaction, waiting time and service quality are critical. They found that time is a more crucial driver of customer happiness than service quality, based on their research with bank clients. Another crucial component of customer happiness is the expected waiting time.

Waiting time is an average value. However, since this time varies for each customer, a random waiting period is determined and simulated for each customer in today's studies. This is also true in this study. In this way, the real world can simulate Random waiting time generation is also included in this article. This randomness and the idea of simulating conditions close to those in the real world has come from the study described in the [46].

At the beginning of the waiting time section, we said, "Waiting time and customer satisfaction interrelate.". Due to the advertising activities used today and the curiosity in the market, the opposite of this situation can occur. This is a kind of reverse psychology. Preexisting knowledge, consumption intentions, and the degree to which quality is difficult to objectively judge all act as mediators of this effect [47].

There are cases where customers are willing to wait. It may apply to technological products. A sequel may also apply to an expected movie or game [9]. Since waiting will increase the expectation, if the satisfaction level is not satisfied, the customer's trust will be lost, and customers will not shop from the same store in the future. However, this approach has an adverse effect on customer satisfaction, as the customer knows what he is getting and what he expects in the warehouse. Therefore, that approach is not correct for the warehouse.

Some researchers focus on the delay information since delay information affects customer satisfaction with the system. The study of [27] considers three levels of delay information. These were: no information, partial information, and full information (precise waiting time). The first research on the partial information system was published by the author [48]. In addition, the cost is determined by the total duration of the sojourn time, not simply the delay [49].

The delay issue can be handled mathematically, but since customers are involved, it is difficult. This indicates that we need to understand its impact on human behavior as well. In [50], investigated the uncertain situation of the customer's behavior. The customers' reactions to delays were studied empirically. In [51], proved that the delay affects the customer evaluation survey. Customers' knowledge of the process increases satisfaction. For example, notifications about cargo status keep customer satisfaction constant. Customers want to work with companies that give them value. The information provided to the customer will increase customer satisfaction. In [39,52] were studied in this area.

3. CUSTOMER WAITING TIME IN M/M/1 SYSTEM WITH DIFFERENT SERVICE RATES

In this section, we present M/M/1 service system in which orders are not differentiated out of the system, but we put them in one of the different priority classes based on both the value and the number of the items they include. In order to keep our system simple, we suppose that there are two types of priority classes called priority and non-priority. Priority orders include orders with single item having considerably greater price and have priority over non-priority orders with multiple items at reasonable prices. Here non-priority means that when non-priority order is processed and another priority order is taken, then the non-priority order is not interrupted, and the server provider proceeds with non-priority order. In this thesis, we consider two different models to check if classifying orders and giving priority to some orders can increase customers' satisfaction. In the first model, we assume that all orders are processed, and order cancellations are not possible. However, the service rates for different order classes are different. We assume that the service rate for priority orders is greater than the service rate for the non-priority orders because they include single item while non-priority orders include multiple items.

3.1. MODEL ASSUMPTIONS

- A single server to prepare orders for customers is assumed to simplify our model
- There are two priority classes for orders based on their value
- All orders arrive with Poisson distribution rate Λ and the service time has an exponential distribution with rate μ
- We are interested in orders waiting time in the system before they are started to be prepared, denoted by W
- Order cancellation is not possible

3.2. MODEL FORMULATION

In this model, a service system is considered which processes all orders, so order cancellation is not possible. In order to guarantee that the queue length does not explode, we suppose the following condition is satisfied.

$$\frac{\Lambda}{\mu} < 1 \tag{3.1}$$

We suggest comparing the expected waiting time of customers in two different systems. The first one is the standard system where orders are not classified. The second one is the system where orders are classified into two groups as Priority orders and Non- Priority orders. The comparison of the expected waiting time of customers in two different systems enables us to compare the customer satisfaction level of customers since an increase in waiting time decreases customer satisfaction in all systems.

Since the first system is a standard M/M/1 system, the expected waiting time of customers in the first system becomes equation (3.2).

$$E[W] = \frac{\Lambda}{\mu(\mu - \Lambda)} \tag{3.2}$$

In the second system, orders are classified into two groups as Priority orders and Non-Priority orders and Priority orders have priority over Non- Priority orders. Let the proportion of Priority orders be p and the proportion of Non- Priority orders be q. Then;

$$p + q = 1 \tag{3.3}$$

Decomposition property of Poisson distribution implies that both Priority and Non Priority orders arrive with Poisson distribution rate $\lambda 1$ and $\lambda 2$ where;

$$\lambda_1 = \Lambda p \tag{3.4}$$

$$\lambda_2 = \Lambda q \tag{3.5}$$

Because Priority orders include single item, we assume a greater service rate for Priority orders. Let $\mu 1$ be the service rate for Priority orders and $\mu 1 > \mu$. Moreover, we assume a smaller service rate for Non- Priority orders than μ. Let $\mu 2$ be the service rate for Non-

Priority orders and $\mu 2 < \mu$. Then, the Markov diagram for system 2 becomes as given in Figure 3.1.

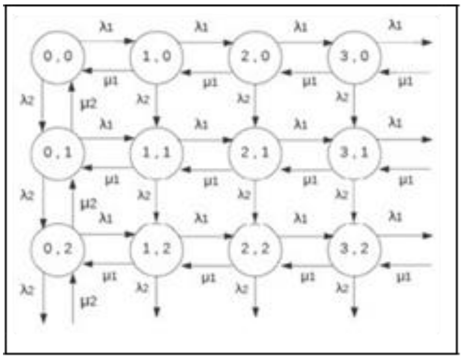

Figure 3. 1 Markov diagram of priority order system

The expected waiting time of customers with Priority orders and Non- Priority orders of course become different from each other. Moreover, it is clear that the expected waiting time of customers with Priority orders and Non- Priority orders depends on both service rates and their proportions.

Proposition 1: The expected waiting time of customers with Priority orders becomes

$$E[W_1] = \frac{\Lambda/\mu_0}{(\mu_0 - \lambda_1)} \tag{3.6}$$

and the expected waiting time of customers with Non- Priority orders becomes.

$$E[W_2] = \frac{\Lambda/\mu_0}{(1 - \Lambda/\mu_0)(\mu_0 - \lambda_1)} \tag{3.7}$$

where;

$$\mu_0 = \frac{\lambda_1/\mu_1}{\lambda_1/\mu_1 + \lambda_2/\mu_2}\mu_1 + \frac{\lambda_2/\mu_2}{\lambda_1/\mu_1 + \lambda_2/\mu_2}\mu_2 \tag{3.8}$$

Proof: The priority system in Figure 3.1 is a quasi-birth and death process and it can be transformed into a birth and death process if $\mu 1 = \mu 2$ because we are only interested in computation of expected waiting time in the system. $\mu 1$ is the departure rate of Priority orders while $\mu 2$ is the departure rate of Non- Priority orders. Because in the long run, the proportion of Priority orders in the system is $\rho 1/(\rho 1 + \rho 2)$ where $\rho 1$ is $\lambda 1/\mu 1$ and $\rho 2$ is $\lambda 2/\mu 2$. The average departure rate of orders, which can be denoted by $\mu 0$, becomes

$$\mu_0 = \frac{\lambda_1/\mu_1}{\lambda_1/\mu_1 + \lambda_2/\mu_2}\mu_1 + \frac{\lambda_2/\mu_2}{\lambda_1/\mu_1 + \lambda_2/\mu_2}\mu_2 \tag{3.9}$$

Then, it is possible to compute the expected waiting time of Priority orders in the queue as

$$E[W_1] = \frac{\Lambda/\mu_0}{(\mu_0 - \lambda_1)} \tag{3.10}$$

and the expected waiting time Priority orders in the queue as,

$$E[W_2] = \frac{\Lambda/\mu_0}{(1 - \Lambda/\mu_0)(\mu_0 - \lambda_1)} \tag{3.11}$$

The simulation values show a consistent structure with the theoretical values.

Proposition 2: If $\frac{\Lambda\mu_1\mu_2}{\mu(\lambda_1\mu_2+\lambda_2\mu_1)} > 1$ the expected waiting time of customers in the priority system is smaller than the expected waiting time of the customers in the non- priority system.

Proof: In order to find the critical μ_0, it is needed to make the expected waiting times of customers in both priority and non-priority system equal to each other that is $E\,[W] = pE\,[W_1] + qE\,[W_2]$.Because,

$$pE[W_1] + qE[W_2] = p\frac{\Lambda/\mu_0}{(\mu_0 - \lambda_1)} + q\frac{\Lambda/\mu_0}{(1 - \Lambda/\mu_0)(\mu_0 - \lambda_1)} = \frac{\Lambda/\mu_0}{(\mu_0 - \Lambda)} \tag{3.12}$$

As long as μ_0 is greater than μ, the expected waiting time in priority system becomes smaller than the expected waiting time in non-priority system. Moreover,

$$\mu_0 = \frac{\lambda_1/\mu_1}{\lambda_1/\mu_1 + \lambda_2/\mu_2}\mu_1 + \frac{\lambda_2/\mu_2}{\lambda_1/\mu_1 + \lambda_2/\mu_2}\mu_2 = \frac{\Lambda\mu_1\mu_2}{(\lambda_1\mu_2 + \lambda_2\mu_1)} \tag{3.13}$$

Therefore, if $\frac{\mu_0}{\mu}>1$, priority system becomes better off. Thus, the critical ratio is $\frac{\Lambda\mu_1\mu_2}{\mu(\lambda_1\mu_2+\lambda_2\mu_1)}$.

3.3. NUMERICAL RESULTS

In order to obtain some results to check Proposition 1, we first obtain expected waiting time of both Priority orders and Non-priority orders and are shown in “Table 3.1”.

Table 3. 1. Comparison of theoretical and simulation results

Λ = 5						Theoretical Results		Simulation Results	
(p, q)	λ_1	λ_2	μ_1	μ_2	μ_0	E[W_1]	E[W_2]	W_1	W_2
(0.25 , 0.75)	1.25	3.75			5.45	0.218	2.616	0.213	2.637
(0.5 , 0.5)	2.5	2.5	7.5	5	6	0.237	1.424	0.238	1.428
(0.75 , 0.25)	3.75	1.25			6.67	0.257	1.028	0.257	1.028
(0.25 , 0.75)	1.25	3.75			5.71	0.196	1.568	0.196	1.565
(0.5 , 0.5)	2.5	2.5	10	5	6.67	0.18	0.72	0.181	0.717
(0.75 , 0.25)	3.75	1.25			8	0.147	0.392	0.148	0.391
(0.25 , 0.75)	1.25	3.75			6.15	0.166	0.884	0.165	0.874
(0.5 , 0.5)	2.5	2.5	20	5	8	0.114	0.303	0.115	0.302
(0.75 , 0.25)	3.75	1.25			11.42	0.057	0.101	0.057	0.100

“Table3.1.” compares theoretical results and simulation results. In that table, the Λ value is five; this is the whole order quantity to come into our system. The p value indicates the number of priority orders received by our system; the calculation of the p value is explained in the model formulation section. The q value shows the number of non-priority orders that have come into our system. Calculated orders show up in the system as λ_1 and λ_2. In that case, we have three different possibilities. These are as follows: λ_1 and λ_2 can be equal; λ_1 can be greater than λ_2; or λ_1 can be less than λ_2. In our assumption, priority order is crucial, so they pick immediately. That’s why μ_1 is greater than μ_2 in our assumption. The μ_0 is calculated, which is already a defined value in the table, and formulation is explained in the model formulation section. When we compare theoretical results with simulation results, both results show the same value. This demonstrates that our simulation generates appropriate values for our next assumptions.

Table 3. 2. Comparison of non-priority and priority results

Λ=5				Non-Priority	Priority		
(p, q)	**(λ_1 , λ_2)**	μ	μ_0	E[W]	E[W_1]	E[W_2]	E[W]
(0.25,0.75)	(1.25 , 3.75)	6	5.45	0.833	0.218	0.633	2.039
(0.5,0.5)	(2.5 , 2.5)	6	6	0.833	0.238	0.417	0.833
(0.75,0.25)	(3.75 , 1.25)	6	6.67	0.833	0.257	0.281	0.449

(0.25,0.75)	(1.25 , 3.75)	6.67	5.71	0.449	0.196	0.512	1.233
(0.5,0.5)	(2.5 , 2.5)	6.67	6.67	0.449	0.180	0.281	0.449
(0.75,0.25)	(3.75 , 1.25)	6.67	8	0.449	0.147	0.156	0.208
(0.25,0.75)	(1.25 , 3.75)	8	6.15	0.208	0.166	0.378	0.707
(0.5,0.5)	(2.5 , 2.5)	8	8	0.208	0.114	0.156	0.208
(0.75,0.25)	(3.75 , 1.25)	8	11.42	0.208	0.057	0.059	0.068

Table3.2 is the result of the second proposition. The Λ value is five; this is the whole order quantity to come into our system. The values of λ_1 and λ_2 are calculated using the p and q ratios. The same assumption is made when using Table 3.1. Table3.2 compares the expected waiting time of non-priority values with the expected waiting time of priority values. In that operation, we focus on the calculation of the expected waiting time. Therefore, we don't use two different mu values in the system; we just have one mu value for both priority and non-priority systems, but μ_0 is calculated, which is already a defined value in the table.

In the data obtained, the critical ratio value of the first line is 1.10. In this case, non-priority gives a better result because equality cannot be achieved. When we look at the second line, the critical ratio value is 1. Therefore, the expected waiting time values of both systems are equal. Looking at the third row, the critical ratio is 0.899. In this case, the priority system has a better expected waiting time. The study gives the same result when being tried with different μ values. Other values are also met with the same result.

To define Customer Satisfaction, let us denote by Cs, we use the derivative of the customer satisfaction formula obtained by Davis and Heineke in their 1997 study [53]. In this way, a concave formula obtains that decreases with time instead of a linear formula. The derivative of the customer satisfaction, Cs is below.

$$Cs = 4.25 - 0.027 * W - 0.018 * W^2 \tag{3.14}$$

Table 3. 3. Customer satisfaction non-priority and priority systems

(p, q)	Λ=5 (λ_1 , λ_2)	μ	μ_0	CS $_{\text{Non-Priority}}$	CS$_1$	CS$_2$	CS $_{\text{Priority}}$
(0.25,0.75)	(1.25 , 3.75)	6	5.45	4.215	4.243	4.226	4.120
(0.5,0.5)	(2.5 , 2.5)	6	6	4.215	4.243	4.236	4.215
(0.75,0.25)	(3.75 , 1.25)	6	6.67	4.215	4.242	4.241	4.234
(0.25,0.75)	(1.25 , 3.75)	6.7	5.71	4.234	4.244	4.231	4.189

(0.5,0.5)	(2.5 , 2.5)	6.7	6.67	4.234	4.245	4.241	4.234
(0.75,0.25)	(3.75 , 1.25)	6.7	8	4.234	4.246	4.245	4.244
(0.25,0.75)	(1.25 , 3.75)	8	6.15	4.244	4.245	4.237	4.222
(0.5,0.5)	(2.5 , 2.5)	8	8	4.244	4.247	4.245	4.244
(0.75,0.25)	(3.75 , 1.25)	8	11.4	4.244	4.248	4.248	4.248

Based on the explanations in the comparison section, the priority system is successful when the critical ratio in the second proportion is less than one. Within the scope of this information, when the non-priority and priority systems are analyzed, the most successful result is obtained when 75% of the incoming orders are priority orders and the μ value is 8. All orders can receive service from the system.

4. CUSTOMER WAITING TIME IN M/M/1 SYSTEM WITH WAITING TIME SENSITIVITIES

As in Section 3, we again consider an M/M/1 service system in which orders are not differentiated out of the system, and it is possible to give priority to some orders based on the number of items in the order and the total value of the order. Thus, two types of priority classes as priority and non-priority are supposed and the service rate for both priority orders and non-priority orders are identical. Order cancellations are possible. Therefore, we assume that customers have waiting time sensitivities and they can cancel their orders if they have to wait greater than they can tolerate. Thus, customer losses are observed in this model.

4.1. MODEL ASSUMPTIONS

- A single server to prepare orders for customers is assumed to simplify our model
- There are two priority classes for orders based on their value
- All orders arrive with Poisson distribution rate Λ and the service time has an exponential distribution with rate μ
- We are interested in orders waiting time in the system before they are started to be prepared, denoted by W
- Order cancellation is possible if customers have to wait more than they can tolerate
- The maximum waiting time a customer can tolerate is assumed to be constant and exponential random variable.

4.2. MODEL FORMULATION

In this model, we consider a service system in which customers can cancel their orders if they have to wait more than they can tolerate. Therefore, each customer has a waiting time limit beyond which they do not take the service and an order is lost. All orders are assumed to arrive with Poisson distribution rate Λ and the service time has an exponential distribution with rate μ. We assume that the service rate does not change even if orders are classified in priority groups. Furthermore, we do not need to suppose the following condition.

$$\frac{\Lambda}{\mu} = \rho < 1 \tag{4.1}$$

since some orders are cancelled due to long waiting times.

Let S be the maximum waiting time a customer can tolerate, so a customer can take the order if S is greater than or equal to W that is customers' waiting time. Since some orders can be cancelled due to long waiting times, our effective arrival rate is expected to be smaller than Λ. Let λeff be the effective arrival rate of customers, then

$$\lambda eff = \Lambda P(W \leq S) \tag{4.2}$$

4.3. NUMERICAL RESULTS

In this thesis, we assume that S can be both deterministic and exponentially distributed random variable and numerical results are obtained for both cases.

4.3.1. Deterministic Waiting Time Tolerance Level

In this part of our thesis, we assume that customers are homogeneous, and their waiting time tolerance level, S can only take constant values. Moreover, for some specific values of S, we obtain the expected waiting time of customers and the proportion of customers who complete their order without cancellations.

In our first numerical results, the S value is taken as 0.5, and 1 hour. The ρ values are set to 0.5, 0.8, 1, 1.6, 2, 2.5 and 3 where the service rate, μ is always 25. The expected waiting times for both systems are given in the table.

In this section, we have some performance measurements. The expected waiting time of priority orders is supposed to be smaller than that of non-priority orders. In addition, the expected waiting time of priority orders is supposed to be smaller than the non-priority system. In this case, priority orders get service faster than others. The expected waiting time value of the priority system must be equal to or less than the non-priority system. When this condition is achieved, the priority system is proven to be acceptable.

When the waiting time tolerance is added to the system, a cancel order will start in the system. The proportion of customers is more important when we start to use waiting time tolerance in the system. Deterministic customer cancellation predicts that customers will cancel the order when it reaches the tolerance value, but customers are independent, so it is unrealistic to have a fixed period of cancellation. It would be more correct to think of such a cancellation as a warehouse policy. In this assumption, when the orders reach the tolerance value, the warehouse operation is cancelled because it cannot complete them.

The proportion of customers for priority orders must be equal to or higher than the proportion of the non-priority system. The proportion of customers for non-priority orders may be smaller than the proportion of customers for non-priority systems because priority systems put an emphasis on covering priority systems.

Excepted waiting time and proportion of customers are the performance criteria of the simulation for us, and it is expected that the expected waiting time value in the system for priority orders is low and the proportion of customers value is high for priority orders. When the customer satisfaction formula is examined by using the expected waiting time and proportion of customer values for the non-priority and priority systems, the priority system gives more successful results in cases where the Rho value is higher than 0.8.

In the table, we observe the following results.

- Expected waiting times of customer in both non-priority and priority models are equal to each other for both S values when $\rho \leqslant 0.8$.

- The expected waiting time of customers in priority system is smaller than the expected waiting time of customers in non-priority system when $0.8 \leqslant \rho \leqslant 3$ for S values considered. under a new title.

- The proportion of the serviced customers increases with increasing S value but there is no significant difference in the proportion of the serviced customers in the non-priority and priority system.

Table 4. 1. Proportion of serviced customers and their expected waiting time with deterministic waiting time tolerance

Inputs (μ = 25)				Non-Priority		Priority					
Λ	λ_1	λ_2	S	P(W <= S)	E[W]	P(W_1 <= S)	P(W_2 <= S)	E[W_1]	E[W_2]	P(W <= S)	E[W]
13	6.25	6.25	0.5	99.9%	0.043	100.0%	99.3%	0.028	0.056	99.7%	0.042
	3.25	9.75		99.9%	0.043	100.0%	99.8%	0.024	0.049	99.8%	0.042
	9.75	3.25		99.9%	0.043	99.9%	98.1%	0.034	0.067	99.5%	0.042
	6.25	6.25	1	100.0%	0.043	100.0%	99.9%	0.028	0.058	99.9%	0.043
	3.25	9.75		100.0%	0.043	100.0%	100.0%	0.024	0.050	100.0%	0.043
	9.75	3.25		100.0%	0.043	100.0%	99.8%	0.034	0.070	99.9%	0.043
20	10	10	0.5	98.6%	0.127	99.9%	93.1%	0.052	0.166	96.6%	0.109
	5	15		98.6%	0.128	100.0%	96.9%	0.039	0.142	97.7%	0.116
	15	5		98.6%	0.127	99.7%	84.9%	0.075	0.239	96.1%	0.116
	10	10	1	99.9%	0.156	100.0%	98.2%	0.053	0.221	99.1%	0.137
	5	15		99.9%	0.156	100.0%	99.5%	0.040	0.184	99.7%	0.148
	15	5		99.9%	0.155	100.0%	93.5%	0.079	0.270	98.4%	0.127
25	12.5	12.5	0.5	93.1%	0.250	99.9%	80.4%	0.072	0.262	90.2%	0.167
	6.25	18.75		93.1%	0.250	99.9%	88.9%	0.049	0.252	91.7%	0.201
	18.75	6.25		93.1%	0.250	98.9%	62.1%	0.126	0.300	89.8%	0.170
	12.5	12.5	1	96.3%	0.500	100.0%	87.8%	0.075	0.510	93.9%	0.293
	6.25	18.75		96.3%	0.500	100.0%	93.8%	0.051	0.501	95.4%	0.388
	18.75	6.25		96.3%	0.501	99.9%	70.6%	0.148	0.568	92.6%	0.253
40	20	20	0.5	62.5%	0.458	98.2%	26.4%	0.162	0.467	62.3%	0.314
	10	30		62.5%	0.458	99.9%	49.9%	0.067	0.456	62.5%	0.359
	30	10		62.5%	0.458	82.1%	3.3%	0.372	0.493	62.5%	0.402
	20	20	1	62.5%	0.958	99.8%	25.1%	0.194	0.967	62.5%	0.580
	10	30		62.5%	0.958	100.0%	49.9%	0.067	0.956	62.5%	0.733
	30	10		62.5%	0.958	83.2%	0.3%	0.839	0.999	62.5%	0.879
50	25	25	0.5	50.0%	0.480	92.6%	7.3%	0.268	0.494	49.9%	0.381
	12.5	37.5		49.9%	0.480	99.9%	33.3%	0.079	0.482	49.9%	0.382
	37.5	12.5		49.9%	0.480	66.6%	0.1%	0.447	0.500	50.0%	0.460
	25	25	1	50.0%	0.980	96.1%	3.8%	0.519	0.997	50.0%	0.758
	12.5	37.5		49.9%	0.980	100.0%	33.3%	0.080	0.982	49.9%	0.757
	37.5	12.5		49.9%	0.980	66.6%	0.0%	0.947	1.000	59.9%	0.960
63	31.5	31.5	0.5	39.7%	0.490	78.7%	0.6%	0.395	0.500	39.7%	0.447
	15.75	47.25		39.7%	0.490	99.6%	19.6%	0.105	0.493	39.7%	0.396
	47.25	15.75		39.7%	0.490	99.6%	19.6%	0.105	0.493	39.7%	0.202
	31.5	31.5	1	39.7%	0.990	79.3%	0.1%	0.879	1.000	39.7%	0.940
	15.75	47.25		39.7%	0.990	100.0%	19.5%	0.108	0.993	39.7%	0.772
	47.25	15.75		39.7%	0.990	52.9%	0.1%	0.976	1.000	31.8%	0.982
75	37.5	37.5	0.5	33.4%	0.493	66.5%	0.1%	0.447	0.500	33.3%	0.474
	18.75	56.25		33.3%	0.493	98.8%	11.4%	0.141	0.497	33.3%	0.408
	56.25	18.75		33.3%	0.493	44.4%	0.1%	0.486	0.500	33.3%	0.368
	37.5	37.5	1	33.3%	0.993	66.6%	0.1%	0.947	1.000	53.8%	0.973
	18.75	56.25		33.3%	0.993	99.9%	11.1%	0.158	0.997	33.3%	0.787
	56.25	18.75		33.3%	0.993	46.6%	0.1%	0.158	1.000	33.3%	0.369

The results given in the table above are summarized in the following figure according to the workload in the system that is five different ρ values.

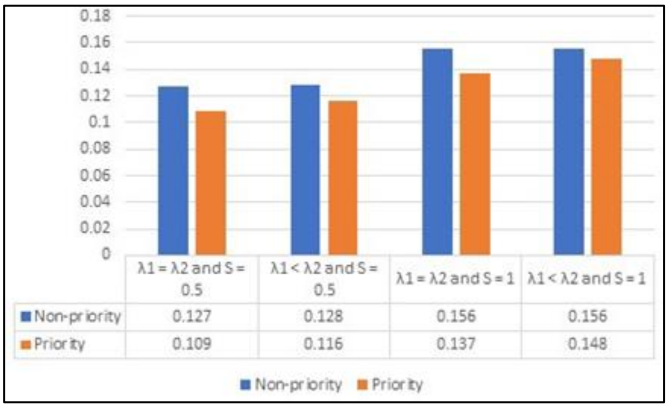

Figure 4. 1. Expected waiting time with different deterministic waiting time tolerances (ρ=0.8)

The figure above indicates that the expected waiting time values in the priority system are always smaller than the expected waiting time values in the non-priority system under all conditions where ρ is 0.8.

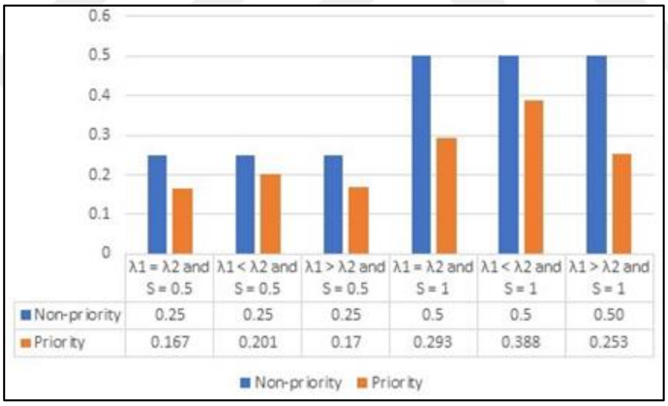

Figure 4. 2. Expected waiting time with different deterministic waiting time tolerances (ρ=1)

The figure above also indicates that the expected waiting time values in the priority system are always smaller than the expected waiting time values in the non-priority system under all conditions where ρ is 1. Moreover, the difference between the expected waiting time values of two systems is greater since ρ is bigger. Thus, we state that as the workload in the system increases the performance of priority system in customers waiting time becomes much better off than the performance of non-priority system.

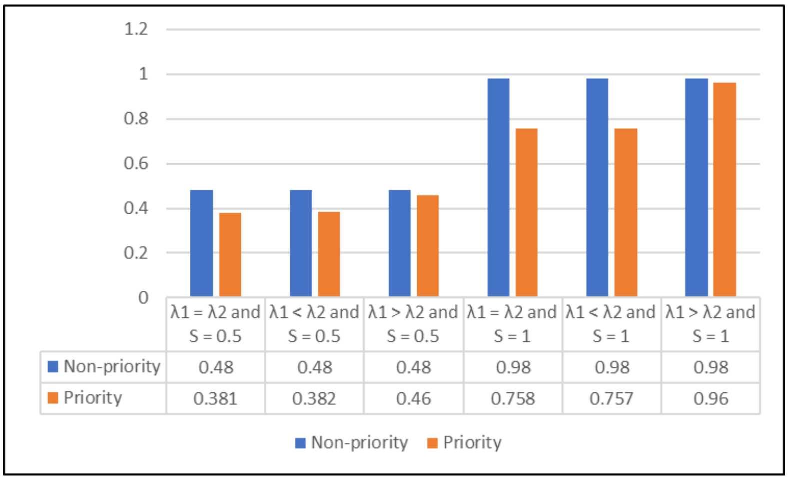

Figure 4. 3. Expected waiting time with different deterministic waiting time tolerances (ρ=2)

The figure above shows that the expected waiting time difference between non-priority and priority systems becomes insignificant for λ_1, and λ_2 values. Therefore, we cannot benefit from a priority system if we include a great number of orders in the priority class.

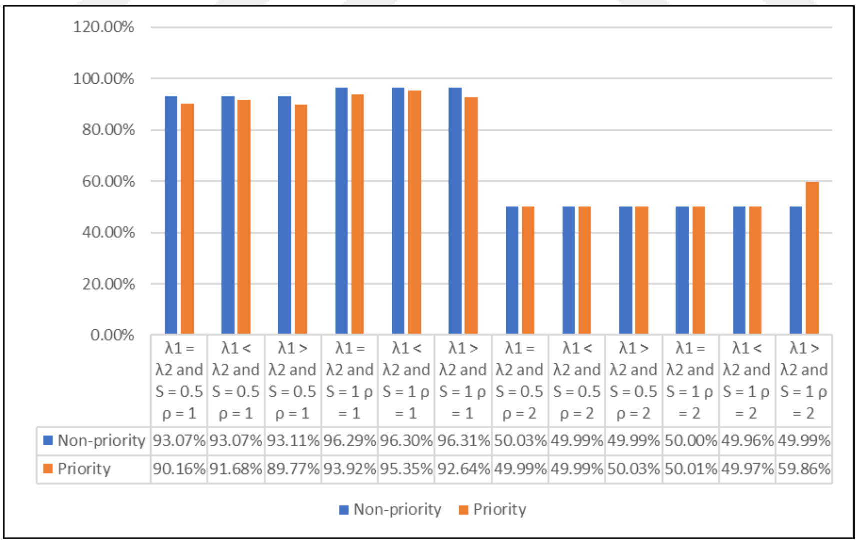

Figure 4. 4. The proportion of the serviced customers

The figure indicates that there no significant difference in the proportion of the serviced customers in two systems. Therefore, we can claim that a priority system with two classes of orders can increase the customers' satisfaction level by reducing their waiting times without decreasing the number of potential customers.

Table 4. 2. Customer satisfaction calculation with deterministic waiting time tolerance

Inputs (μ = 25)				Non-Priority		Priority						
Λ	λ_1	λ_2	S	$CS_{Non\ Priority}$	E[W]	CS_1	CS_2	$E[W_1]$	$E[W_2]$	$CS_{Priority}$	E[W]	Ratio
20	10	10	0.5	4.246	0.127	4.249	4.245	0.052	0.166	4.247	0.109	0.013
	5	15	0.5	4.246	0.128	4.249	4.246	0.039	0.142	4.247	0.116	0.009
	15	5	0.5	4.246	0.127	4.248	4.243	0.075	0.239	4.247	0.116	0.008
	10	10	1	4.245	0.156	4.249	4.243	0.053	0.221	4.246	0.137	0.014
	5	15	1	4.245	0.156	4.249	4.244	0.04	0.184	4.246	0.148	0.006
	15	5	1	4.245	0.155	4.248	4.241	0.079	0.27	4.246	0.127	0.021
25	12.5	12.5	0.5	4.242	0.25	4.248	4.242	0.072	0.262	4.245	0.167	0.068
	6.25	18.8	0.5	4.242	0.25	4.249	4.242	0.049	0.252	4.244	0.201	0.041
	18.8	6.25	0.5	4.242	0.25	4.246	4.240	0.126	0.3	4.245	0.17	0.065
	12.5	12.5	1	4.232	0.5	4.248	4.232	0.075	0.51	4.241	0.293	0.202
	6.25	18.8	1	4.232	0.5	4.249	4.232	0.051	0.501	4.237	0.388	0.114
	18.8	6.25	1	4.232	0.501	4.246	4.229	0.148	0.568	4.242	0.253	0.238
40	20	20	0.5	4.234	0.458	4.245	4.233	0.162	0.467	4.240	0.314	0.139
	10	30	0.5	4.234	0.458	4.248	4.234	0.067	0.456	4.238	0.359	0.098
	30	10	0.5	4.234	0.458	4.237	4.232	0.372	0.493	4.236	0.402	0.056
	20	20	1	4.208	0.958	4.244	4.207	0.194	0.967	4.228	0.58	0.491
	10	30	1	4.208	0.958	4.248	4.208	0.067	0.956	4.221	0.733	0.307
	30	10	1	4.208	0.958	4.215	4.205	0.839	0.999	4.212	0.879	0.113
50	25	25	0.5	4.233	0.48	4.241	4.232	0.268	0.494	4.237	0.381	0.099
	12.5	37.5	0.5	4.233	0.48	4.248	4.233	0.079	0.482	4.237	0.382	0.098
	37.5	12.5	0.5	4.233	0.48	4.234	4.232	0.447	0.5	4.234	0.46	0.021
	25	25	1	4.206	0.98	4.231	4.205	0.519	0.997	4.219	0.758	0.308
	12.5	37.5	1	4.206	0.98	4.248	4.206	0.08	0.982	4.219	0.757	0.309
	37.5	12.5	1	4.206	0.98	4.208	4.205	0.947	1	4.207	0.96	0.029
63	31.5	31.5	0.5	4.232	0.49	4.237	4.232	0.395	0.5	4.234	0.447	0.045
	15.8	47.3	0.5	4.232	0.49	4.247	4.232	0.105	0.493	4.236	0.396	0.095
	47.3	15.8	0.5	4.232	0.49	4.247	4.232	0.105	0.493	4.244	0.202	0.268
	31.5	31.5	1	4.206	0.99	4.212	4.205	0.879	1	4.209	0.94	0.073
	15.8	47.3	1	4.206	0.99	4.247	4.205	0.108	0.993	4.218	0.772	0.304
	47.3	15.8	1	4.206	0.99	4.207	4.205	0.976	1	4.206	0.982	0.012
75	37.5	37.5	0.5	4.232	0.493	4.234	4.232	0.447	0.5	4.233	0.474	0.020
	18.8	56.3	0.5	4.232	0.493	4.246	4.232	0.141	0.497	4.236	0.408	0.087
	56.3	18.8	0.5	4.232	0.493	4.233	4.232	0.486	0.5	4.238	0.368	0.126
	37.5	37.5	1	4.205	0.993	4.208	4.205	0.947	1	4.207	0.973	0.030
	18.8	56.3	1	4.205	0.993	4.245	4.205	0.158	0.997	4.218	0.787	0.289
	56.3	18.8	1	4.205	0.993	4.245	4.205	0.158	1	4.238	0.369	0.764

The results obtained by using the customer satisfaction formula given in Section 3.3. yield more successful results for the priority system in this section. Looking at the Non-priority and Priority systems, the ratio between the systems is calculated to find the performance. When the ratios are examined in Table 4.2, the improvement is 76% for the case where the Λ is 75, and 75% of the total orders are priority orders. However, this value is unrealistic when only 33.34% of total customers are served in this example.

Examining the system under the assumption that at least half of the customers who come to the system will be served may enable the system to operate under a more realistic approach. For this reason, when the Λ is 50 and the priority orders are 25% of the total orders, the priority system achieved a higher result as customer satisfaction at a rate of 30.9% as it can be seen in Table 4.2.

Under the assumption that at least 50% of the customers will receive service, the value that provides the optimum value is the situation where the Λ is 40 and the number of orders is evenly distributed. In this case, the difference between the formula values of the Priority system and the non-priority system is 49.1%. The optimum value for the system is that 62.49% of the customers receive service from the system and the expected waiting time value is 0.58, while customer satisfaction is 4.228.

4.3.2. Exponential Waiting Time Tolerance Level

We also assume that customers' waiting time tolerance level S has an exponential distribution with rate γ and we compute the expected waiting time of customers and the proportion of customers who complete their order without cancellations for different values of γ. The same table uses for exponential waiting time. In this way, we expect to obtain more realistic values. In this way, customers in the system are allowed to cancel their orders on their own, and a more realistic structure is created.

The assumptions remain valid from the previous section. The expected waiting time value is lower for priority orders in the priority system. The expected waiting time of non-priority orders are higher than non-priority system. This is because the system tends to process priority orders, and non-priority orders are cancelled instead of being completed. Especially when the Rho value increases, the bottleneck effect will be seen show in the system, and the proportion of customer value will decrease.

We have important questions like "how many customers are lost in the warehouse process, and will the operation remain at healthy levels?". In both calculations (deterministic and exponential), when the rho value approaches 2, the system starts to move away from the operationally healthy point.

When the waiting time tolerance value generates exponential distribution, the expected waiting time is equal to each other. For that reason, we should check W1 and W2 values to see the effect of the priority system.

Table 4. 3. Proportion of serviced customers and their expected waiting time with exponential waiting time tolerance

Inputs (μ = 25)				Non-Priority		Priority					
Λ	λ_1	λ_2	γ	P(W <= S)	E[W]	P(W_1 <= S)	P(W_2 <= S	E[W_1]	E[W_2]	P(W <= S)	E[W]
13	6.25	6.25	2	94.13%	0.029	95.47%	92.75%	0.023	0.036	94.11%	0.029
	3.25	9.75		94.13%	0.029	95.96%	93.53%	0.020	0.032	94.14%	0.029
	9.75	3.25		94.13%	0.029	94.86%	91.92%	0.026	0.040	94.12%	0.029
13	6.25	6.25	1	96.57%	0.034	97.52%	95.63%	0.025	0.044	96.58%	0.034
	3.25	9.75		96.57%	0.034	97.80%	96.16%	0.022	0.039	96.57%	0.034
	9.75	3.25		96.57%	0.034	97.09%	94.97%	0.029	0.050	96.56%	0.034
20	10	10	2	88.37%	0.058	92.51%	84.25%	0.037	0.079	88.38%	0.058
	5	15		88.37%	0.058	93.79%	86.56%	0.031	0.067	88.37%	0.058
	15	5		88.37%	0.058	90.75%	81.27%	0.046	0.094	88.38%	0.058
20	10	10	1	92.22%	0.078	95.68%	88.75%	0.043	0.112	92.21%	0.078
	5	15		92.22%	0.078	96.56%	90.77%	0.034	0.092	92.22%	0.078
	15	5		92.22%	0.078	94.34%	85.87%	0.057	0.142	92.22%	0.078
25	12.5	12.5	2	82.72%	0.086	90.27%	75.24%	0.049	0.124	82.76%	0.086
	6.25	18.75		82.74%	0.086	92.42%	79.55%	0.038	0.102	82.77%	0.086
	18.75	6.25		82.74%	0.086	87.15%	69.44%	0.064	0.153	82.72%	0.086
25	12.5	12.5	1	86.87%	0.131	94.21%	79.59%	0.058	0.204	86.90%	0.131
	6.25	18.75		86.86%	0.131	95.72%	83.94%	0.043	0.161	86.88%	0.131
	18.75	6.25		86.86%	0.131	91.56%	72.76%	0.084	0.272	86.87%	0.131
40	20	20	2	61.62%	0.192	83.81%	39.51%	0.081	0.302	61.66%	0.192
	10	30		61.60%	0.192	89.54%	52.33%	0.052	0.238	61.63%	0.192
	30	10		61.60%	0.192	74.02%	24.34%	0.130	0.378	61.60%	0.192
40	20	20	1	62.37%	0.377	89.47%	35.20%	0.105	0.648	62.33%	0.377
	10	30		62.36%	0.376	94.14%	51.78%	0.058	0.482	62.37%	0.376
	30	10		62.36%	0.376	78.34%	14.61%	0.217	0.854	62.40%	0.376
50	25	25	2	49.91%	0.250	79.17%	20.74%	0.104	0.396	49.96%	0.250
	12.5	37.5		49.96%	0.250	88.26%	37.17%	0.059	0.314	49.94%	0.250
	37.5	12.5		49.91%	0.250	64.30%	6.89%	0.179	0.465	49.94%	0.250
50	25	25	1	49.99%	0.500	84.90%	15.08%	0.151	0.849	49.98%	0.500
	12.5	37.5		49.99%	0.500	93.35%	35.56%	0.067	0.645	50.01%	0.500
	37.5	12.5		50.03%	0.500	66.18%	1.52%	0.338	0.985	50.02%	0.500
63	31.5	31.5	2	39.70%	0.302	71.85%	7.48%	0.141	0.463	39.67%	0.302
	15.75	47.25		39.68%	0.301	86.49%	24.12%	0.068	0.379	39.70%	0.301

	47.25	15.75		39.68%	0.301	52.67%	0.75%	0.237	0.496	39.69%	0.301
63	31.5	31.5	1	39.68%	0.603	75.92%	3.41%	0.241	0.966	39.66%	0.603
	15.75	47.25		39.67%	0.603	91.99%	22.24%	0.080	0.778	39.68%	0.603
	47.25	15.75		39.67%	0.603	52.91%	0.02%	0.471	1.000	39.76%	0.603
75	37.5	37.5	2	33.33%	0.333	64.24%	2.43%	0.179	0.488	33.34%	0.333
	18.75	56.25		33.32%	0.333	84.49%	16.29%	0.078	0.418	33.34%	0.333
	56.25	18.75		33.32%	0.333	44.24%	0.01%	0.277	0.500	33.34%	0.333
75	37.5	37.5	1	33.33%	0.667	66.18%	0.50%	0.338	0.995	33.34%	0.667
	18.75	56.25		33.34%	0.666	90.28%	14.35%	0.097	0.856	33.33%	0.666
	56.25	18.75		33.34%	0.666	66.18%	0.01%	0.555	1.000	33.33%	0.666

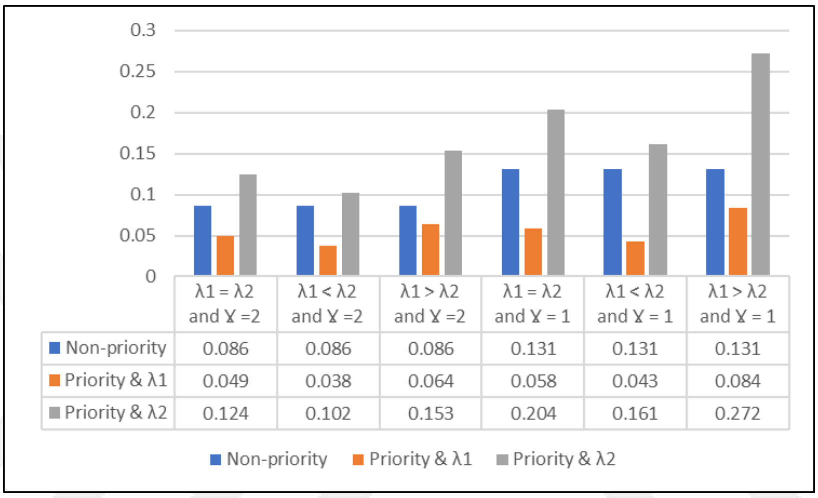

Figure 4. 5. Expected waiting time with different exponential waiting time tolerances (ρ=1)

The expected waiting time for priority orders is better than for non-priority orders in the priority system. Because priority orders have priority, the expected waiting time for priority orders is smaller than for non-priority orders.

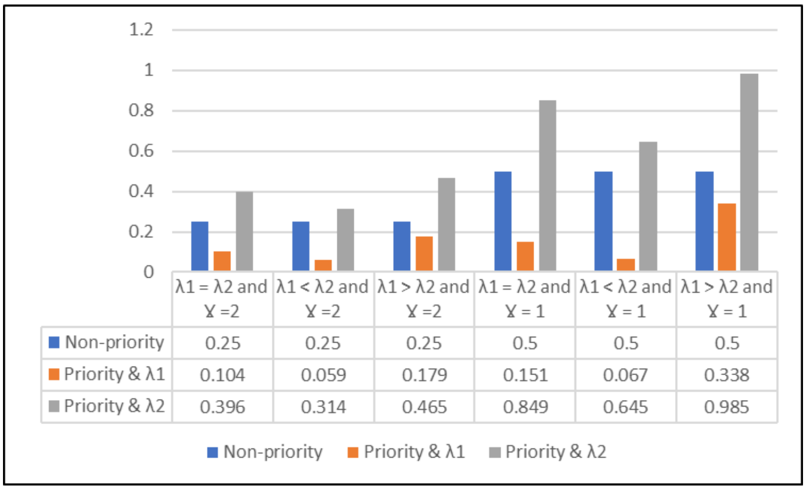

Figure 4. 6. Expected waiting time with different exponential waiting time tolerances (ρ=2)

The result in Figure 4.6. are consistent with the result in Figure 4.3. The expected waiting times increases with decreasing Ɣ as expected.

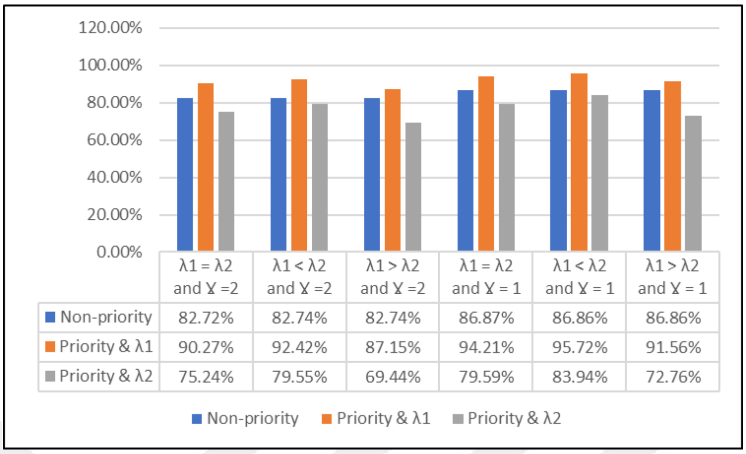

Figure 4. 7. The proportion of the serviced customers (ρ = 1)

The proportion of the serviced customers is close in both non-priority and priority systems. The proportion of the serviced customers with priority orders in the priority system is higher than in overall serviced customers in non-priority. Thus, the satisfaction rate for priority orders is also better than non-priority orders.

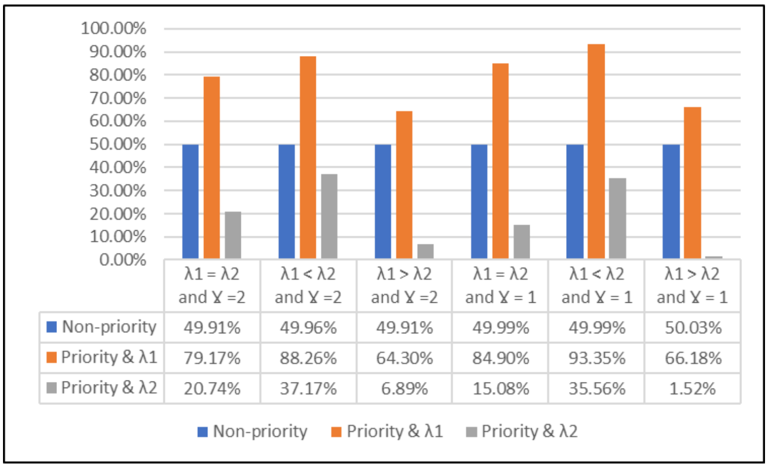

Figure 4. 8. The proportion of the serviced customers (ρ = 2)

As ρ increases, the proportion of the serviced customers for priority orders increases, in which case the proportion of the serviced customers for non-priority will decrease. The results indicate that there is a threshold value for ρ. Here threshold value is 2. Whenever ρ is smaller than 2, priority system shows a better performance in proportion of the serviced customers than non-priority system. Moreover, if ρ exceeds 2, non-priority system gets better results.

5. CONCLUSION

In the scope of this thesis, we investigate expected waiting time for online shopping during the special time period of the year like Black Friday. We study an order queuing system modeled by Markov chains with Poisson arrivals and exponential service times.

In our first model, both the theoretical and the simulation results for the expected waiting time of a non-priority and a priority system are compared for different parameter settings. Results obtained from both methods are consistent showing that the expected waiting time can be computed numerically without implementing simulation. Moreover, we obtain the expected waiting time of both non priority and priority system, so a threshold value for the change in service rates for different order classes are stated to make a priority system better than a non-priority system.

In our second model, we again compare a non-priority and a priority system where the service rates are the same, but order cancellations are possible. The expected waiting time values obtained in the second model specify the cases where a priority system gives better results. The data considered in the model is a preliminary stage for the case of order cancellation. For the ρ values greater than 0.8, a priority system shows a better performance in the expected waiting time, so customer satisfaction. Thus, a priority order model can be used on special days. As ρ increases, the priority model becomes more successful in the results for the expected waiting time.

Moreover, order cancelations are also supposed in the system depending on certain waiting time tolerances at different ρ values. Here, we assume both a deterministic waiting time tolerance and an exponential waiting time tolerance. Then, it is shown that the success of the priority model increases with the increase in the ρ value, but after a critical ρ value, a non-priority system becomes better. In addition, we calculated customer satisfaction in both non-priority and priority systems, and we found when the system works efficiently. However, since we do not expect the number of customers to triple, a priority system is still suggested during special days.

6. RECOMMENDATIONS

For future studies, our customer satisfaction formula can be improved by a formula using the expected waiting times. In addition to the cancellation process, incoming orders can be directed to the nearest retailer. The three retailer orders closest to each other can be shared and returned according to the transaction density.

As a result of this study, increasing number of order classes will increase the accuracy rate in the model. Separating the products into two groups did not have an adequate effect on that model. A price-based approach has adopted for the separation of orders. In this case, it facilitates the prioritization process. A distinction to be made according to the aisle proximity within the service system can also have positive effects on the waiting time. These two concepts can be evaluated together. A formula for customer satisfaction can be developed and satisfaction measured over time. Break-even points can determine systems whose orders can cancel. In this way, it is possible to work on models that aim for minimum waiting time and maximum customer satisfaction.

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APPENDIX A: PRIORITY AND NON-PRIORITY OPERATION WAITING TIME GENERATION

In this section, we present the R code to generate waiting time.

We keep the total number of first type orders coming to the system as NQ1. Likewise, NQ2 second type orders appear. LQ1 and LQ2 keeps the number of orders leaving the first type and second type system. The S value in the code shows the current status of the system. It can take three values. These are; 0, 1 and 2. 0 indicates that there is no order acting on the server. 1 indicates that the first type of order is in service in the system. 2 is used for orders of the second type. The code generally keeps the exponential and random arrival times of first and second type orders as A1 and A2. Since the values are vectors, they hold different values for each order. Likewise, service times in the system are calculated exponentially. Therefore, D1 and D2 values keep the departure time values. Since it is used as a vector, the departure time of each order is different from each other. W1 and W2 values hold the waiting time values. It is kept separately for each order and averaged for the relevant order type at the end of the transaction. i1 and i2 values are used as arrival time. i1 holds the first order of type first the first time. Then, when A1 used, i1 holds the second order of the first type and so on. i2 does the same for A2. The j1 and j2 values are also used for orders leaving the system, as they are for i1 and i2. The nextevent value is used to keep the time of the next transaction that will come to the system. The value of know is used to keep the current time of the system.

In this system, LQ1, LQ2 values will come as zero since there is no execution status of orders. Likewise, when using the R code, the last two values will give the cancellation rate of the first type orders. Since these are not LQ1 and LQ2 values, they will zero. When the last W function uses to call the processes in the system, the system will generate the system value with the corresponding reputation and times. Here, by changing $\lambda1$ and $\lambda2$ values and $\mu1$ and $\mu2$ values, the loads coming to the system and their outputs can adjust. By changing the P value in the last part, the system can be used as priority and non- priority, and values can obtain.

```
mm1<-function(n=100,T=8,lambda1=10,lambda2=10,mu1=25, mu2=25, p=1){
tnow<-0
NQ1<-0
NQ2<-0
S<-0
A1<-rep(0,n)
A2<-rep(0,n)
D1<-rep(0,n)
D2<-rep(0,n)
w1<-rep(0,n)
w2<-rep(0,n)
i1<-1
i2<-1
j1<-0
j2<-0
A1[i1]<-rexp(1,lambda1)
A2[i2]<-rexp(1,lambda2)
nextevent <- min(A1[i1],A2[i2])
l<-1
while(tnow<T){
l<-l+1
if(nextevent==A1[i1]){
if(S==0){
tnow<-nextevent
S<-1
i1<-i1+1
A1[i1]<-tnow+rexp(1,lambda1)
j1<-j1+1
w1[j1]<-0
D1[j1]<-tnow+rexp(1,mu1)
nextevent<-min(A1[i1],A2[i2],D1[j1])
}
else{
tnow<-nextevent
NQ1<-NQ1+1
i1<-i1+1
A1[i1]<-tnow+rexp(1,lambda1)
if(S==1){nextevent<-min(A1[i1],A2[i2],D1[j1])}
else{nextevent<-min(A1[i1],A2[i2],D2[j2])}
}}
else if(nextevent==A2[i2]){
if(S==0){
tnow<-nextevent
S<-2
```

```
i2<-i2+1
A2[i2]<-tnow+rexp(1,lambda2)
j2<-j2+1
w2[j2]<-0
D2[j2]<-tnow+rexp(1,mu2)
nextevent<-min(A1[i1],A2[i2],D2[j2])
}
else{
tnow<-nextevent
NQ2<-NQ2+1
i2<-i2+1
A2[i2]<-tnow+rexp(1,lambda2)

if(S==1){nextevent<-min(A1[i1],A2[i2],D1[j1])}
else{nextevent<-min(A1[i1],A2[i2],D2[j2])}
}}

else{
if(p==1){
if(NQ1>0){
tnow<-nextevent
S<-1
j1<-j1+1
w1[j1]<-tnow-A1[j1]
D1[j1]<-tnow+rexp(1,mu1)
NQ1<-NQ1-1
nextevent<-min(A1[i1],A2[i2],D1[j1])
}
else if(NQ1==0 & NQ2>0){
tnow<-nextevent
S<-2
j2<-j2+1
w2[j2]<-tnow-A2[j2]
D2[j2]<-tnow+rexp(1,mu2)
NQ2<-NQ2-1
nextevent<-min(A1[i1],A2[i2],D2[j2])
}
else{
tnow<-nextevent
nextevent<-min(A1[i1],A2[i2])
S=0
}}
else{
if(NQ1+NQ2>0){
tnow<-nextevent
if(NQ1!=0 & A1[j1+1]<A2[j2+1]){
```

```
S<-1
j1<-j1+1
w1[j1]<-tnow-A1[j1]
D1[j1]<-tnow+rexp(1,mu1)
NQ1<-NQ1-1
nextevent<-min(A1[i1],A2[i2],D1[j1])
}
else{
S<-2
j2<-j2+1
w2[j2]<-tnow-A2[j2]
D2[j2]<-tnow+rexp(1,mu2)
NQ2<-NQ2-1
nextevent<-min(A1[i1],A2[i2],D2[j2])
}
}
else{
S<-0
tnow<-nextevent
nextevent<-min(A1[i1],A2[i2])
}}
}
}
#j<-c(j1,j2)
w<-cbind(w1,w2)
#res<-rbind(j,w)
res<-matrix(0,n+1,2)
res[1,]<-c(j1,j2)
res[2:(n+1),]<-w
res
}
mm1(n=100,T=8,lambda1=10,lambda2=10, mu1=25, mu2=25, p=0)
```

APPENDIX B: PRIORITY AND NON-PRIORITY OPERATION WAITING TIME GENERATION WITH DETERMINISTIC ACCEPTABLE WAITING TIME

In this system, unlike Appendix A, the situation in which there is a constant period of exit from the system examine. Since the values in the system are given as hours, 30 minutes of an hour is 0.5. Waiting time value holds the acceptable waiting time value. In the system, the values in the five, ten and fifteen-minute wait states examine, respectively.

By taking these values as constant, W1, W2 values and expected waiting time values take from the system. The value of the number of customers leaving the system with J1 and J2 is kept. LQ1 and LQ2 values and the number of customers who left the system without receiving service were kept. Priority Loss and Non-priority Loss values are LQ1 / J1 values, and they keep the values of the unserved order, that is, the customer, in the system.

```
mm1<-function(n=101,T=8,lambda1=10,lambda2=10, mu1=25, mu2=25, p=1){
 waitingtime<- 1
 tnow<-0
 NQ1<-0
 NQ2<-0
 LQ1<-0 #Number of Leave i1 order
 LQ2<-0 #Number of Leave i2 order
 S<-0
 A1<-rep(0,n)
 A2<-rep(0,n)
 D1<-rep(0,n)
 D2<-rep(0,n)
 w1<-rep(0,n)
 w2<-rep(0,n)
 l1<-rep(0,n) #leave 1
 l2<-rep(0,n) #leave 2
 i1<-1
 i2<-1
 j1<-0
 j2<-0
 k1<-0 #first order leave
 k2<-0 #second order leave
 A1[i1]<-rexp(1,lambda1)
 A2[i2]<-rexp(1,lambda2)
 nextevent <- min(A1[i1],A2[i2])
```

```
l<-1
while(tnow<T){
 l<-l+1
 if(nextevent==A1[i1]){
 if(S==0){
 tnow<-nextevent
 S<-1
 i1<-i1+1
 A1[i1]<-tnow+rexp(1,lambda1)
 j1<-j1+1
 w1[j1]<-0
 D1[j1]<-tnow+rexp(1,mu1)
 nextevent<-min(A1[i1],A2[i2],D1[j1])
}
else{
 tnow<-nextevent
 NQ1<-NQ1+1
 i1<-i1+1
 A1[i1]<-tnow+rexp(1,lambda1)
 if(S==1){nextevent<-min(A1[i1],A2[i2],D1[j1])}
 else{nextevent<-min(A1[i1],A2[i2],D2[j2])}
}
}
else if(nextevent==A2[i2]){
if(S==0){
 tnow<-nextevent
 S<-2
 i2<-i2+1
 A2[i2]<-tnow+rexp(1,lambda2)
 j2<-j2+1
 w2[j2]<-0
 D2[j2]<-tnow+rexp(1,mu2)
 nextevent<-min(A1[i1],A2[i2],D2[j2])
}
else{
 tnow<-nextevent
 NQ2<-NQ2+1
 i2<-i2+1
 A2[i2]<-tnow+rexp(1,lambda2)
 if(S==1){nextevent<-min(A1[i1],A2[i2],D1[j1])}
 else{nextevent<-min(A1[i1],A2[i2],D2[j2])}
}}

else{#Departure
if(p==1){
if(NQ1>0){
```

```
 tnow<-nextevent
 j1<-j1+1
 w1[j1]<-tnow-A1[j1]
 if (w1[j1] <= waitingtime) {
  S<-1
  D1[j1]<-tnow+rexp(1,mu1)
  NQ1<-NQ1-1
  nextevent<-min(A1[i1],A2[i2],D1[j1])
 }else{
  S<-0
  NQ1<-NQ1-1
  LQ1 <- LQ1 +1
  nextevent<-min(A1[i1],A2[i2],tnow)
  w1[j1]<- waitingtime
 }
}
else if(NQ1==0 & NQ2>0){
 tnow<-nextevent
 j2<-j2+1
 w2[j2]<-tnow-A2[j2]
 if (  w2[j2] <= waitingtime) {
  S<-2
  D2[j2]<-tnow+rexp(1,mu2)
  NQ2<-NQ2-1
  nextevent<-min(A1[i1],A2[i2],D2[j2])
 }else{
  S<-0
  NQ2<-NQ2-1
  LQ2 <- LQ2 +1
  nextevent<-min(A1[i1],A2[i2],tnow)
  w2[j2]<- waitingtime
 }
}
else{
 tnow<-nextevent
 nextevent<-min(A1[i1],A2[i2])
 S=0
}}
else{
if(NQ1+NQ2>0){
 tnow<-nextevent
if(NQ1!=0 & A1[j1+1]<A2[j2+1]){
 j1<-j1+1
 w1[j1]<-tnow-A1[j1]
 if (  w1[j1] <= waitingtime) {
  S<-1
  D1[j1]<-tnow+rexp(1,mu1)
  NQ1<-NQ1-1
```

```
  nextevent<-min(A1[i1],A2[i2],D1[j1])
 }else {
  s<-0
  NQ1<-NQ1-1
  LQ1 <- LQ1 +1
  nextevent<-min(A1[i1],A2[i2],tnow)
  w1[j1]<- waitingtime
 }
}
else{
 j2<-j2+1
 w2[j2]<-tnow-A2[j2]
 if (  w2[j2] <= waitingtime) {
  S<-2
  D2[j2]<-tnow+rexp(1,mu2)
  NQ2<-NQ2-1
  nextevent<-min(A1[i1],A2[i2],D2[j2])
 }else{
  S<-0
  NQ2<-NQ2-1
  LQ2 <- LQ2 +1
  nextevent<-min(A1[i1],A2[i2],tnow)
  w2[j2]<- waitingtime
 }
}
}
else{
 S<-0
 tnow<-nextevent
 nextevent<-min(A1[i1],A2[i2])
}}
}
}
 #j<-c(j1,j2)
 w<-cbind(w1,w2,j1,j2,LQ1,LQ2)
 #res<-rbind(j,w)
 res<-matrix(0,n+1,6)
 res[1,]<-c(j1,j2, 1, 1, 1, 1)
 res[2:(n+1),]<-w
 res
}
mm1(n=101,T=8,lambda1=10,lambda2=10, mu1=25, mu2=25, p=1)
```

APPENDIX C: PRIORITY AND NON-PRIORITY OPERATION WAITING TIME GENERATION WITH EXPONENTIAL ACCEPTABLE WAITING TIME

In this system, the values are brought to a more realistic structure by introducing different acceptable waiting times for each order, that is, for each customer, exponentially, of the system developed in addition to Appendix B. Instead of the same expected waiting time for each order with constant value, a separate acceptable waiting time for each order define exponentially with the Gamma value.

```
mm1<-function(n=101,T=8,lambda1=10,lambda2=10, mu1=25, mu2=25, p=1){
  gamma <- 2
  tnow<-0
  NQ1<-0
  NQ2<-0
  LQ1<-0 #Number of Leave i1 order
  LQ2<-0 #Number of Leave i2 order
  S<-0
  A1<-rep(0,n)
  A2<-rep(0,n)
  D1<-rep(0,n)
  D2<-rep(0,n)
  w1<-rep(0,n)
  w2<-rep(0,n)
  l1<-rep(0,n) #leave 1
  l2<-rep(0,n) #leave 2
  i1<-1
  i2<-1
  j1<-0
  j2<-0
  k1<-0 #first order leave
  k2<-0 #second order leave
  A1[i1]<-rexp(1,lambda1)
  A2[i2]<-rexp(1,lambda2)
  nextevent <- min(A1[i1],A2[i2])
l<-1
while(tnow<T){
  l<-l+1
  if(nextevent==A1[i1]){
  if(S==0){
  tnow<-nextevent
```

```
S<-1
 i1<-i1+1
 A1[i1]<-tnow+rexp(1,lambda1)
 j1<-j1+1
 w1[j1]<-0
 D1[j1]<-tnow+rexp(1,mu1)
 nextevent<-min(A1[i1],A2[i2],D1[j1])
}
else{
 tnow<-nextevent
 NQ1<-NQ1+1
 i1<-i1+1
 A1[i1]<-tnow+rexp(1,lambda1)
 if(S==1){nextevent<-min(A1[i1],A2[i2],D1[j1])}
 else{nextevent<-min(A1[i1],A2[i2],D2[j2])}
}
}
else if(nextevent==A2[i2]){
if(S==0){
 tnow<-nextevent
 S<-2
 i2<-i2+1
 A2[i2]<-tnow+rexp(1,lambda2)
 j2<-j2+1
 w2[j2]<-0
 D2[j2]<-tnow+rexp(1,mu2)
 nextevent<-min(A1[i1],A2[i2],D2[j2])
}
else{
 tnow<-nextevent
 NQ2<-NQ2+1
 i2<-i2+1
 A2[i2]<-tnow+rexp(1,lambda2)
 if(S==1){nextevent<-min(A1[i1],A2[i2],D1[j1])}
 else{nextevent<-min(A1[i1],A2[i2],D2[j2])}
}}
else{#Departure
if(p==1){
if(NQ1>0){
 tnow<-nextevent
 j1<-j1+1
 w1[j1]<-tnow-A1[j1]
 waitingtime <- rexp(1,gamma)
 if (w1[j1] <= waitingtime) {
  S<-1
  D1[j1]<-tnow+rexp(1,mu1)
```

```
 NQ1<-NQ1-1
   nextevent<-min(A1[i1],A2[i2],D1[j1])
  }else{
   S<-0
   NQ1<-NQ1-1
   LQ1 <- LQ1 +1
   nextevent<-min(A1[i1],A2[i2],tnow)
   w1[j1]<- waitingtime
  }
 }
else if(NQ1==0 & NQ2>0){
  tnow<-nextevent
  j2<-j2+1
  w2[j2]<-tnow-A2[j2]
  waitingtime <- rexp(1,gamma)
  if (  w2[j2] <= waitingtime) {
   S<-2
   D2[j2]<-tnow+rexp(1,mu2)
   NQ2<-NQ2-1
   nextevent<-min(A1[i1],A2[i2],D2[j2])
  }else{
   S<-0
   NQ2<-NQ2-1
   LQ2 <- LQ2 +1
   nextevent<-min(A1[i1],A2[i2],tnow)
   w2[j2]<- waitingtime
  }
 }
else{
  tnow<-nextevent
  nextevent<-min(A1[i1],A2[i2])
  S=0
}}
else{
if(NQ1+NQ2>0){
  tnow<-nextevent
if(NQ1!=0 & A1[j1+1]<A2[j2+1]){
  j1<-j1+1
  w1[j1]<-tnow-A1[j1]
  waitingtime <- rexp(1,gamma)
  if (  w1[j1] <= waitingtime) {
   S<-1
   D1[j1]<-tnow+rexp(1,mu1)
   NQ1<-NQ1-1
   nextevent<-min(A1[i1],A2[i2],D1[j1])
  }else {
   s<-0
   NQ1<-NQ1-1
```

```
    LQ1 <- LQ1 +1
    nextevent<-min(A1[i1],A2[i2],tnow)
    w1[j1]<- waitingtime
   }
  }
  else{
   j2<-j2+1
   w2[j2]<-tnow-A2[j2]
   waitingtime <- rexp(1,gamma)
   if (  w2[j2] <= waitingtime) {
    S<-2
    D2[j2]<-tnow+rexp(1,mu2)
    NQ2<-NQ2-1
    nextevent<-min(A1[i1],A2[i2],D2[j2])
   }else{
    S<-0
    NQ2<-NQ2-1
    LQ2 <- LQ2 +1
    nextevent<-min(A1[i1],A2[i2],tnow)
    w2[j2]<- waitingtime
   }
  }
  }
  else{
   S<-0
   tnow<-nextevent
   nextevent<-min(A1[i1],A2[i2])
  }}
  }
  }
   #j<-c(j1,j2)
   w<-cbind(w1,w2,j1,j2,LQ1,LQ2)
   #res<-rbind(j,w)
   res<-matrix(0,n+1,6)
   res[1,]<-c(j1,j2, 1, 1, 1, 1)
   res[2:(n+1),]<-w
   res
  }
  mm1(n=101,T=8,lambda1=10,lambda2=10, mu1=25, mu2=25, p=1)
```