



**PORTFOLIO OPTIMIZATION WITH
OPTION IMPLIED INFORMATION:
AN APPLICATION IN BORSA ISTANBUL**

**Thea ANGURIDZE
(Ph.D Dissertation)
Eskisehir, 2019**

**PORTFOLIO OPTIMIZATION WITH OPTION IMPLIED INFORMATION:
AN APPLICATION IN BORSA ISTANBUL**



Thea ANGURIDZE

Ph.D Dissertation

Department of Business Administration

Supervisor: Asst. Prof. Dr. Özlem SAYILIR

Eskisehir

Anadolu University

Graduate School of Social Sciences

August, 2019

FINAL APPROVAL FOR THESIS

This thesis titled “**Portfolio Optimization with Option Implied Information: an Application in Borsa İstanbul**” has been prepared and submitted by **Thea ANGURIDZE** in partial fulfillment of the requirements in “Anadolu University Directive on Graduate Education and Examination” for the **PhD Department of Business Administration Program in Finance** Department has been examined and approved on 22/08/2019.

Committee Members

Member (Supervisor)	: Asisst.Prof.Dr.Özlem SAYILIR
Member	: Prof.Dr.Bülent AÇMA
Member	: Prof.Dr.Aslı AFŞAR
Member	: Assoc.Prof.Dr. Arzum ERKEN ÇELİK
Member	: Assist.Prof.Dr.Alp POLAT

Signature



22/08/2019
Date

Prof.Dr.Bülent GÜNŞOY
Director
Graduate School of Sciences



ÖZET
OPSİYONLARDAN ELDE EDİLEN BİLGİLERLE PORTFÖY OPTİMİZASYONU:
BORSA İSTANBUL’DA BİR UYGULAMA

Thea ANGURIDZE

İşletme Bölümü,

Anadolu Üniveristesi, Sosyal Bilimler Enstitüsü, Ağustos 2019

Danışman: Dr. Öğretim Üyesi Özlem SAYILIR

Bu çalışmada, opsiyonlardan elde edilen bilgilere dayanan optimal portföylerin, tarihi bilgilere dayanan optimal portföylerden daha başarılı olup olmadığı araştırılmaktadır. Araştırmada, Borsa İstanbul Vadeli İşlem ve Opsiyon Piyasası’nda işlem gören 20 hisse senedine ait veriler kullanılmıştır. Örneklem dönemi, Mart 2017’den Temmuz 2018’e kadardır. Tarihi hisse senedi verileri kullanılarak Ortalama varyans ve minimum varyans portföyleri oluşturulmuş ve opsiyon fiyatları kullanılarak portföy optimizasyon modelleri geliştirilmiştir. Örneklemdeki hisse senetlerinin opsiyonlardan elde edilen volatiliteleri, Black-Scholes opsiyon fiyatlama modeli ile hesaplanmıştır. Hisse senetleri arasındaki opsiyonlardan elde edilen korelasyonların hesaplanmasında, Buss ve Vilkov’un modeli kullanılmıştır. Opsiyonlardan elde edilen ortalama varyans ve minimum varyans portföyleri, kovaryans matrisinde tarihi bilgilerin yerine, opsiyonlardan elde edilen bilgilerin konulması ile oluşturulmuştur. Portföylerin başarılarının değerlendirilmesinde, şu üç kriter kullanılmıştır: portföyün yıllık getirisi, portföyün yıllık volatilitesi ve portföyün Sharpe Rasyosu. Daha sonra, opsiyonlardan elde edilen bilgilerle portföy oluşturma, opsiyonlardan elde edilen bilgileri dikkate almayan portföylerden daha iyi başarı ölçütleri sağlayıp sağlamadığı sınanmıştır. Bulgular, opsiyonlardan elde edilen bilgilere dayanan optimal portföylerin, tarihi bilgilere dayanan optimal portföylerden daha başarılı olduğunu göstermektedir.

Anahtar Sözcükler: Portföy optimizasyonu, Tarihi volatilitesi, Opsiyonlardan elde edilen volatilitesi, Portföy başarıları, Borsa İstanbul

ABSTRACT
PORTFOLIO OPTIMIZATION WITH OPTION IMPLIED INFORMATION:
AN APPLICATION IN BORSA ISTANBUL

Thea ANGURIDZE

Department of Business Administration

Anadolu University, Graduate School of Social Sciences, August 2019

Supervisor: Asst. Prof. Dr. Özlem SAYILIR

This study explores if optimal portfolios based on option-implied information perform better than optimal portfolios based on historical information. We used option prices of 20 stocks, which have been trading in the Futures and Options Market of Borsa İstanbul. The sample period is from March 2017 to July 2018. We developed portfolio optimization models using option prices as well as mean variance and minimum variance portfolios using historical stock price data. We calculated implied volatility of the sample stocks from option prices using Black-Scholes option pricing model. We employed Buss and Vilkov's model for the calculation of implied correlations between stocks. Option implied mean variance and option implied minimum variance portfolios are based on the covariance metrics developed after historical information is replaced by option-implied information. For the evaluation of portfolio performance, we used the following three criteria: annualized portfolio return, annualized portfolio volatility and portfolio Sharpe ratio. Then, we test if creating portfolios with option-implied information can yield better performance measures than portfolios that ignore option-implied information. The findings show that optimal portfolios based on option-implied information perform better than optimal portfolios based on historical information.

Keywords: Portfolio optimization, Historical volatility, Option-implied volatility, Portfolio performance, Borsa İstanbul

ACKNOWLEDGMENTS

Foremost, I would like to express my very sincere gratitude to my PhD thesis supervisor Asst. Prof. Dr. Özlem SAYILIR, for the continuous support to my PhD course, for her motivation, guidance, encouragement and incisive comments. Without her support, meaningful comments and patience on every single stage of my PhD study I could never be able to complete this level. It was great honor to work with her during whole my PhD study. I could not have imagined a better mentor and supervisor than she is.

Besides my supervisor, I am also grateful to the other members of my thesis committee: Prof. Dr. Bülent AÇMA and Prof. Dr. Aslı AFŞAR for their insightful comments, encouragement and helpful suggestions to complete my PhD work.

We would also like to thank the management and employees of Borsa Istanbul for providing us with stock-option data to conduct this study.

My PhD dissertation is sincerely dedicated to my parents and I want to thank them for everything they have done for me, for their tolerance, continuous support and always being next to me, especially, during my PhD study.

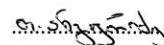
Very special thanks go to William Matthew James Bowler, without his technical help and support the success of this work simply would not be possible. I would never forget his continuing support.

Very special thanks also go to my friends and advisors for standing next to me during one of the most difficult times in my life. Especially, Mr. Sanday AMOS and Mr. Dr. Sharif KOMBO from the Anadolu University's PhD program. I am very grateful to them and appreciate their support, without them the success of this work would not be possible.

22/08/2019

STATEMENT OF COMPLIANCE WITH ETHICAL PRINCIPLES AND RULES

I hereby truthfully declare that this thesis is an original work prepared by me; that I have behaved in accordance with the scientific ethical principles and rules throughout the stages of preparation, data collection, analysis and presentation of my work; that I have cited the sources of all the data and information that could be obtained within the scope of this study, and included these sources in the references section; and that this study has been scanned for plagiarism with “scientific plagiarism detection program” used by Anadolu University, and that “it does not have any plagiarism” whatsoever. I also declare that, if a case contrary to my declaration is detected in my work at any time, I hereby express my consent to all the ethical and legal consequences that are involved.



Thea Anguridze

TABLE OF CONTENTS

TITLE PAGE	i
FINAL APPROVAL FOR THESIS	ii
ÖZET	iii
ABSTRACT.....	iv
ACKNOWLEDGMENTS	v
STATEMENT OF COMPLIANCE WITH ETHICAL PRINCIPLES AND RULES.....	vi
TABLE OF CONTENTS.....	vii
LIST OF TABLES.....	ix
LIST OF FIGURES	x
1 INTRODUCTION.....	1
1.1 Modern Portfolio Theory	3
1.1.1 Mean-variance model	5
1.1.2 Semi-variance model	7
1.2 Capital Asset Pricing Model (CAPM)	9
1.3 Options	11
1.4 Volatility	13
1.5 Variance as a Measure of Risk.....	14
1.6 Covariance of Returns.....	15
1.7 Correlation of Returns.....	16
1.8 Emerging Markets and Borsa Istanbul.....	16
1.8.1 MSCI indices	19
1.8.2 MSCI Turkey index	21
1.8.3 Single stock options contract specification on Borsa Istanbul	23
1.8.4 BIST 30 index options contract specification.....	24
1.9 Research Problem.....	25
1.9.1 Significance of the study.....	25
1.9.2 Goals of the study	26
1.9.3 Limitations of the study	26
1.9.4 Research questions.....	26
1.9.5 Research hypothesis.....	26

2	LITERATURE REVIEW	28
3	DATA AND METHODOLOGY	34
3.1	Data	34
3.2	Portfolio Optimization	36
3.2.1	Minimum variance optimization.....	37
3.2.2	Mean-variance optimization	38
3.2.3	Optimization with option implied information.....	41
3.2.3.1	Implied volatility (“Black-Scholes option pricing model”).....	41
3.2.3.2	Implied correlations	44
3.3	Performance Measurement.....	46
3.3.1	Portfolio return.....	46
3.3.2	Portfolio volatility (Standard deviation)	47
3.3.3	Sharpe ratio	48
3.3.4	T-test	49
4	FINDINGS	50
4.1	Historical and Implied Volatilities	50
4.2	Historical Correlations	60
4.3	Implied Correlation	63
4.4	Stock Performance	67
4.5	Optimized Portfolio Weights	69
4.6	Portfolio Performances.....	72
4.7	Comparison of the Results with Other Similar Research Results.....	76
5	CONCLUSION	78
	REFERENCES	80
	APPENDIX	
	CURRICULUM VITAE	

LIST OF TABLES

Table 1. 1 Breakdown of Domestic and Foreign Investor's Shares Trades in Borsa Istanbul Stock Market.....	17
Table 1. 2 Annual performance of MSCI Turkey, MSCI EM and MSCI AC.....	21
Table 1. 3 The net returns of MSCI Turkey, MSCI EM and MSCI AC.....	22
Table 1. 4 Index risk and return characteristics of MSCI Turkey, MSCI EM and MSCI AC	23
Table 1. 5 MSCI Turkey's top 10 constituents.....	23
Table 3. 1 The list of sample companies.....	36
Table 4. 1 Performance of portfolios (benchmark, minimum variance sample (historical), minimum variance implied, mean variance sample (historical) and mean variance implied).....	73
Table 4. 2 T-test of portfolio performance (Mean Variance Implied versus Mean Variance Sample)	74
Table 4. 3 T-test of portfolio performance (Minimum Variance Implied versus Minimum Variance Sample).....	75
Table 4.4 T-test of portfolio performance (Mean Variance Implied versus Benchmark).....	71
Table 4.5 T-test of portfolio performance (Minimum Variance Implied versus Benchmark).....	72

LIST OF FIGURES

Figure 4. 1 The relationship between implied and historical volatilities and adjusted stock return (AKBNK).	51
Figure 4. 2 The relationship between implied and historical volatilities and adjusted stock return (ARCLK).	51
Figure 4. 3 The relationship between implied and historical volatilities and adjusted stock return (EKGYO).	52
Figure 4. 4 The relationship between implied and historical volatilities and adjusted stock return (EREGL).	52
Figure 4. 5 The relationship between implied and historical volatilities and adjusted stock return (GARAN).	53
Figure 4. 6 The relationship between implied and historical volatilities and adjusted stock return (HALKB).	53
Figure 4. 7 The relationship between implied and historical volatilities and adjusted stock return (ISCTR).	54
Figure 4. 8 The relationship between implied and historical volatilities and adjusted stock return (KCHOL).	54
Figure 4. 9 The relationship between implied and historical volatilities and adjusted stock return (KRDMD).	55
Figure 4. 10 The relationship between implied and historical volatilities and adjusted stock return (PETKM).	55
Figure 4. 11 The relationship between implied and historical volatilities and adjusted stock return (PGSUS).	56
Figure 4. 12 The relationship between implied and historical volatilities and adjusted stock return (SAHOL).	56
Figure 4. 13 The relationship between implied and historical volatilities and adjusted stock return (SISE).	57
Figure 4. 14 The relationship between implied and historical volatilities and adjusted stock return (TCELL).	57

Figure 4. 15 The relationship between implied and historical volatilities adjusted stock return (THYAO).....	58
Figure 4. 16 The relationship between implied and historical volatilities and adjusted stock return (TOASO).	58
Figure 4. 17 The relationship between implied and historical volatilities and adjusted stock return (TTKOM).	59
Figure 4. 18 The relationship between implied and historical volatilities and adjusted stock return (TUPRS).....	59
Figure 4. 19 The relationship between implied and historical volatilities and adjusted stock return (VAKBN).	60
Figure 4. 20 The relationship between implied and historical volatilities and adjusted stock return (YKBNK).	60
Figure 4. 21 The implied correlation matrix, between stock adjusted returns (Heat map) ..	61
Figure 4. 22 Annualized stock return.....	67
Figure 4. 23 Annualized standard deviation	68
Figure 4. 24 Annualized Sharpe ratio	68
Figure 4. 25 Implied mean variance weights allocation	70
Figure 4. 26 Implied minimum variance weights allocation	70
Figure 4. 27 Sample minimum variance weights allocation.....	71
Figure 4. 28 Sample mean variance weights allocation.....	71
Figure 4. 29 Cumulative returns of 5 different portfolios (benchmark, minimum variance sample, minimum variance implied, mean variance sample and mean variance implied)...	72

1 INTRODUCTION

Portfolio optimization is one of the most important topics in the field of investment management. Investment management is a science about making good investment decisions. Investments should match investors' objectives and assets should be allocated properly by balancing risk against return to attain superior performance. Researchers use different empirical methodologies during the asset allocation and portfolio optimization process. When we talk about portfolio optimization, the first theory that comes to mind is Harry Markowitz's Modern Portfolio Theory (MPT), which is also known as "Portfolio Management Theory". In fact, the mean-variance model developed by Markowitz laid the basis of modern portfolio theory. The mathematical problem of portfolio optimization was investigated by Professor Harry Markowitz in 1952 and he was awarded the Nobel Prize in economics in 1990. The Modern Portfolio Theory presumes that investors focus on minimizing risk while obtaining the highest possible return. According to this theory, it is possible for different portfolios to have different levels of return and risk. The fundamental goal of Modern Portfolio Theory (MPT) is to maximize portfolio's expected return for a given amount of risk. This means that an investor can determine how much risk they are willing to take on, and after that they can diversify their portfolios accordingly.

Markowitz' model has had an important impact on the investment community and is still widely used for solving hedging, asset allocation, portfolio construction and other portfolio management problems. Mean variance optimization (MVO) is an easily solvable model, therefore it can be used as an optimizer even, when there are thousands of assets in consideration. In Markowitz's approach, the optimization problem is formulated with respect to two criteria: the reward of a portfolio that should be maximized and the portfolio risk that should be minimized. Investors have always been exceedingly aware of returns. Risk is related to the volatility of future outcomes. The higher is the risk, the higher will be the expected return, as there is a trade-off between return and risk. Total risk of investment is measured by variance or standard deviation.

It is not necessary for an investor to accept the total risk, as he/she can make diversification and some individual investment risk can be avoided by diversification. If the investor decides to put all his funds in a single security, this indicates acceptance of total risk.

Thus, it is not recommended for the investor to put all his/her funds in a single investment, because it exposes the investor to more risk than necessary. Simply, the essence of diversification is: “do not put all your eggs in one basket”. However, not all risk can be avoided by diversification. The risk related to the movements in an economy is defined as market risk and sometimes is considered as non-diversifiable risk (Dobbins et al., 1994).

The main perception of MPT is that risk and return should not be considered alone, they should be estimated by how an investment influences the portfolio’s overall return and risk. Generally, the lower the correlation between securities, the lower is the risk of portfolio. Hence, risk-averse investors tend to select securities, which have low correlations. Markowitz (1959) MPT makes a presumption that investors are risk-averse, which means that investors preferred less risky portfolio rather than a riskier one for a given level of return. This suggests that an investor is willing to take more risk if she or he expects more reward. In the presence of two criteria, there is not a single optimal solution (portfolio), but a set of optimal portfolios, the so-called efficient portfolios, which tradeoff between risk and return (Anagnostopoulos & Mamanis, 2011). Modern Portfolio Theory suggests that it is possible to construct an “efficient frontier” of optimal portfolios that offers maximum return for a given level of risk.

The Turkish stock market like other emerging markets attracts a large amount of funds from developed markets and depend on capital inflows from foreign investors.

The primary aim of this empirical study is to examine the performance of portfolios constructed with historical (back-forwarding) and implied (looking-forward) information and by comparing their performances confirming that portfolios based on looking-forward information overperforms the portfolios based on historical information.

First, we calculated historical and implied volatilities. Historical volatility was computed based on 3 years rolling window method and implied volatility was calculated by Black-Scholes model.

1.1 Modern Portfolio Theory

Markowitz is considered as the father of modern portfolio theory on the basis of his work on portfolio selection in 1952. However, Roy (1952) is the forgotten father of modern portfolio theory, who claims the equal portion of this honor. In this section we summarize their contributions.

Markowitz's (1952) article is about portfolio selection which presented variance of return V and the expected, mean return E of the portfolio. This article assumes that predictions and beliefs about the securities follow the same probability rules that random variables do. According to this assumption, the expected or future return of a portfolio is the average of expected returns of individual securities in the portfolio and also the variance of the portfolio return is a function of the variances of the individual securities as well as the covariances between individual securities and the weights of these securities in the portfolio. Markowitz's paper Markowitz (1952) distinguishes between inefficient and efficient portfolios. Markowitz describes efficient frontier as a set of efficient mean-variance combinations and suggests investors select the set for the desired risk-return combination. He uses geometrical analyses of three and four security examples in order to illustrate properties of efficient sets by assuming non-negative investments. In the 1952 article, Markowitz showed that the set of efficient portfolios is linear and the set of efficient mean-variance combination is parabolic.

Mean variance model is a mathematical structure, which helps investors to maximize expected return with a given level of risk. Modern Portfolio Theory (MPT) is an extension and formalization of diversification in investment, the idea that claims that owning different kind of assets is less risky rather than owning just one type of asset. MPT uses variance of asset prices as a proxy of risk. According to MPT, there is trade-off between return and risk. For years, investment managers and advisers were focused on returns, as risk was not understood. MPT pays attention to risk at least as much as return. Investors can make decisions about the risk they are ready to take, though they cannot make any decisions about the return they will achieve, as it depends on factors beyond their control. Yet, they can predict that the greater the risk, the higher will the future return (Markowitz, 1952)

Roy (1952) also suggested to make portfolio choices on the basis of variance and mean of the portfolio entirely. Concretely, he recommended to choose the portfolio which maximizes portfolio's $(E-d)/\sigma$. Where, d is a devastating return and σ is a standard deviation of the return. Roy's formula is similar to Markowitz's formula and includes the covariances between the returns of securities. The principal difference is that Markowitz's formula requires non-negative investments, while Roy's allows the amount invested in any security to be negative or positive. The second difference is that Markowitz suggests the investors to choose desired portfolios from the efficient frontier, while Roy proposes to choose a specific portfolio.

The main focus of Markowitz (1959) is to explain portfolio theory to the readers who lack advanced mathematics. He explains the concept of mean, mean-variance analysis, variance and covariance and obtains formulas for mean and variance of the portfolio, defines efficiency of mean-variance and represents geometric analysis of efficient sets. He explains analysis of portfolios with large number of securities and focuses on portfolio selection rather than securities. He claims that analysis of the large portfolio which consists of many different kinds of assets has many covariances. The portfolio problem is defined as the choice of the averages and variances of the portfolios composed of different securities (Markowitz, 1959).

Another important issue that Markowitz (1991) deals with is the relationship between securities. The assets should be chosen not only based on their own characteristics, but also on their relations with other entities. However, if securities' returns are not correlated, the risk may be eliminated by diversifying the portfolio. The correlation among returns is not the same for all kind of securities. Generally, it is expected that the return of the securities from the same industries are more correlated than those from different industries. In order to reduce risk, we should avoid a portfolio with securities which all are highly correlated to each other. For investors the most desired is portfolio on the efficient frontier, because it contains portfolios with the highest return for different levels of risk. In MPT, portfolio optimization is based on the mean-variance model, where risk is defined as variance from the efficient frontier. Markowitz's optimization model has disadvantages as well. It has difficulties with computing large quadratic problems.

Markowitz's diversification is the actual type of diversification actively used by portfolio analysis. This kind of diversification is different from naive diversification, which

is commonly used by asset salespeople and in some investment publications. These sources describe diversification as “not putting all eggs in one basket”. Naive diversification ignores the covariance between results and securities in unessential diversification. Markowitz’s diversification includes combination of assets with less rather than perfect positive correlation to reduce risk in the portfolio, without sacrificing any return of the portfolio. Generally, the lower the correlation between securities of the portfolio, the less risky is the portfolio.

“Not only does the E-V hypothesis imply diversification, it implies the "right kind" of diversification for the "right reason." The adequacy of diversification is not thought by investors to depend solely on the number of different securities held. A portfolio with sixty different railway securities, for example, would not be as well diversified as the same size portfolio with some railroad, some public utility, mining, various sort of manufacturing, etc. The reason is that it is generally more likely for firms within the same industry to do poorly at the same time than for firms in dissimilar industries. Similarly, in trying to make variance small it is not enough to invest in many securities. It is necessary to avoid investing in securities with high covariances among each other. We should diversify across industries because firms in different industries, especially industries with different economic characteristics, have lower covariances than firms within an industry” (Markowitz, 1952, “portfolio selection”, pg.89).

1.1.1 Mean-variance model

To achieve an optimal trade-off between return and risk is a challenge for every portfolio manager. The Markowitz model considers the first two moments of the asset return, mean and variance, in order to measure the risk and return of the portfolio. In the financial world, this model is known as Markowitz’s MVO (mean-variance optimization).

The weight of the asset in the portfolio is the proportion of total funds invested in that asset. The risk of a portfolio is estimated as a quadratic function of the weights, which is the variance of the portfolio and the return of portfolio is estimated as linear function in the weights, representing the expected return of portfolio. The trade-off between risk and return is solved by simple quadratic programming (Ceria. S & Sivaramakrishnan. K.K, 2013).

$$\max_w \alpha^T w - \lambda w^T Q w \quad (1.1)$$

Where, α is the expected return, Q is the covariance matrix of returns and $\lambda > 0$ is the risk aversion parameter which presents the investor's preference about how to tradeoff risk and return.

The solution to the QP determines the asset weights in an efficient portfolio – the portfolio with the minimum risk level for a given level of the expected return or (equivalently) the one with the largest expected return for a given level of allowed risk (Francis J.C & Kim.D, 2013).

When λ is smaller, the portfolio risk contribution is small, guiding to higher portfolio risk with higher return of portfolio. On the contrary, large λ is generates less risky portfolios with low return. Solving MVO model is possible using different λ values starting from zero in order to construct portfolio with different return and risk. The set of all those portfolios determine efficient frontier that gives a chance to investors to choose the portfolio from the efficient frontier depending on their risk and return wishes and mandates. MVO method does not return single optimal portfolio, but a family of them lying on the efficient frontier. For a given target of return the portfolio that lies on the efficient frontier gives the least risky portfolio. The same way for a given target of risk, the portfolio lying on the efficient frontier gives the portfolio with highest return (Ceria. S & Sivaramakrishnan. K.K, 2013).

The efficient frontier is the set of optimal portfolios, which offer the greatest expected return for a given level of risk, or the lowest risk for a defined level of future or expected return (Markowitz, 1952). Portfolios below the efficient frontier are sub-optimal as they do not provide enough return for a given level of risk. Portfolios on the right of the efficient frontier are sub-optimal, since they have higher level of risk for a given level of (Frank et al., 2011).

The efficient frontier for MVO model is shown below in figure 1.1. The main goal of portfolio theory is to define the optimal allocation among different assets. Though there are many models to determine the optimal allocation, mean-variance optimization (MVO), which was developed by Harry Markowitz in 1952, is one of the most widely used models in the investment industry. The two key reasons which makes it popular are its simplicity and aesthetic attractiveness. Traditional MVO optimizer creates only one efficient frontier, which allows the users to make comparative efficiency of two or more portfolios easily (Frank et al., 2011).

The aim of MVO is straightforward: helping the user to determine “efficient” portfolios. According to Markowitz, a portfolio is efficient if there is no other portfolio with a higher expected return for a given level of risk or lower risk for a given level of expected return. For MVO model, three parameters are needed: standard deviation, returns and correlations. These estimates create the efficient frontier (Markowitz, 1952). Figure 1.1 shows these combinations of investments with the highest return per unit of risk.

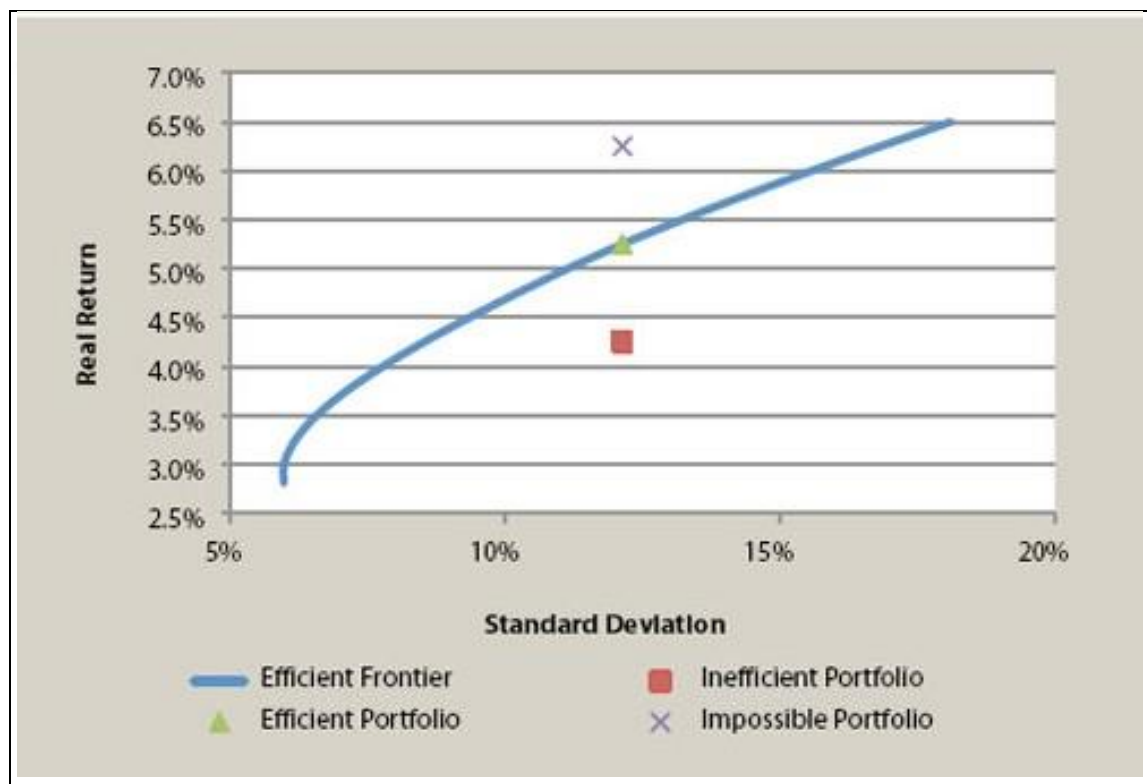


Figure 1. 1 *Efficient Frontier for MVO*

(source:<https://www.onefpa.org/journal/Pages/Incorporating%20Time%20into%20the%20Efficient%20Frontier.aspx>)

1.1.2 Semi-variance model

Markowitz’s mean-variance approach, which leads to optimal the decision of investments, has two important limitations. First, if underlying return data is not normally distributed, the estimation of variance can generate misleading results. Several studies have

shown that investment returns are not normally distributed ((Dennis W. Jansen and Casper G. de Vries, 1991) and (Fama & Roll, 1968). Asset returns are supposed to be asymmetrically distributed. They usually follow a lognormal distribution. If the returns are not normally distributed and investors use standard deviation or variance for measuring risk, they are going to lead to wrong asset allocation decisions. Second, the mean-variance approach ignores the investors' risk aversion. As the variance can only measure the dispersion of the returns around the mean, it cannot be tailored to account for individual investors' risk aversion (Boasson et al. 2011).

In Markowitz's mean-variance approach, risk is measured in terms of the variance of portfolio's expected return. The main assumption of using variance as an appropriate measure of risk is that investors can estimate the probability of negative returns equally against positive returns. According to several researchers, variance measures both downside and upside movements of the asset's return. Hence, it is an improper risk measurement. To construct the efficient frontier with an improper risk measurement may lead to irrational results in portfolio optimization.

Semi-variance is the measurement, which can be used for estimating the investment portfolio's potential downside risk. Downside risk first was modeled by Roy in 1952, which is almost the same time when Markowitz was working on developing the mean-variance theory. Markowitz also realized the weakness of variance as a risk measure. He concluded that the variance measurement and downside risk measurement can give the same results when return distribution is normal. However, when the return distribution is not normal, the downside risk measurement is thought to provide a better solution (Markowitz, 1970). Due to the shortcomings of the mean-variance model, Markowitz, in his article in 1991, developed the semi-variance model, which can make portfolio optimization more effective since it is only concerned with adverse deviations. Markowitz (1991) claimed "semi-variance is a more plausible measure of risk" rather than his mean-variance model, because for investors the risk of loss is a more significant concern than the probability of gain. Hence, semi-variance is a more appropriate risk measure for investors, rather than variance (Markowitz, 1991).

Semi-variance is almost identical to variance, but it considers only observations which fall below the target value or mean of the set of data. Semi-variance is useful in asset analysis or portfolio construction as it provides measurement of downside risk. Semi-variance

considers merely negative fluctuations of asset returns. We can use semi-variance to compute the average loss of a portfolio, as it neutralizes all values that are above investor's target return or above the mean. Semi-variance is calculated as:

$$\sum_{for\ all\ X_i \leq B} (X_i - B)^2 / (n - 1) \quad (1.2)$$

Where: B is the target and n is the number of observations (CFA Institute, 2017).

In short, semi-variance is calculated by measuring the dispersions of all observations which fall below the target value or mean of the dataset. Semi-variance is the average of the squared deviations of values less than the mean. Since investors are more concerned about the downside risk rather than general volatility, measuring risk by semi-variance instead of variance can enable constructing better portfolios (Markowitz, 1959).

1.2 Capital Asset Pricing Model (CAPM)

After Markowitz's two-parameter portfolio analysis model, researchers started investigating what would happen if every investor used Markowitz's model during their investment decisions. As a result of this investigation, the capital asset pricing model (CAPM) was developed. The CAPM is also known as a security market line (SML).

The CAPM describes relationship between an asset's, especially a stock's, future return and systematic risk. Below is the formula for calculating an asset's expected return for a given level of risk (Dobbins et al., 1994).

$$ER_i = R_f + \beta_i(ER_m - R_f) \quad (1.3)$$

Where:

ER_i = Expected return of investment

R_f = Risk-free rate

β_i = Beta of the investment

ER_m = Expected return of market

$(ER_m - R_f) = \text{Market risk premium (Dobbins et al., 1994).}$

The aim of CAPM formula is to assess if the stock is fairly valued, when its time value of money and risk are compared to its future return. Constructing a portfolio using CAPM helps investors to manage their risk. If investors are able to use CAPM for optimizing portfolio's return relative to its risk, then they will choose portfolios on the efficient frontier as shown on the graph below:

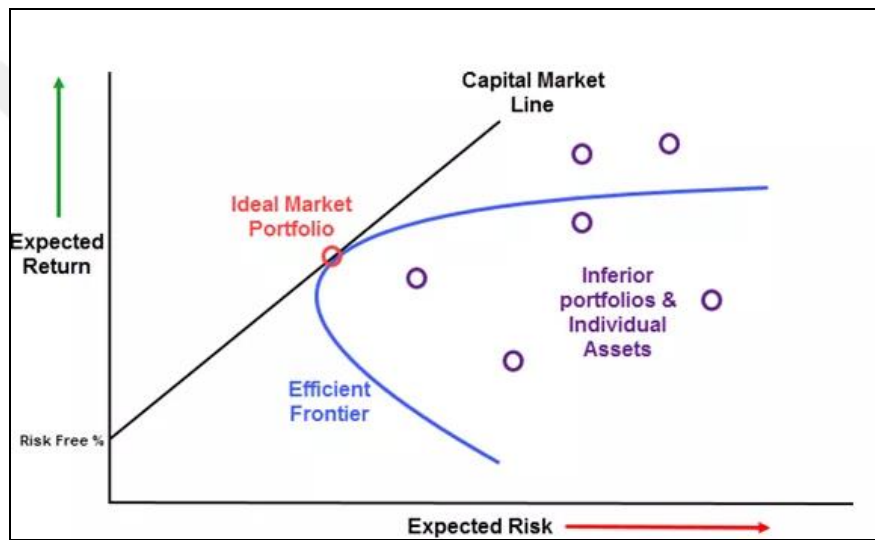


Figure 1. 2 The CAPM and Efficient Frontier (Source: <https://www.investopedia.com/terms/c/capm.asp>)

On the graph, we can see that greater expected return requires greater risk. MPT recommends that starting with risk free rate, the future return of a portfolio increases relatively as risk increases. Any kind of portfolio on the capital market line (CML) is better rather than any portfolio on the right of the line. Efficient frontier and CML illustrates the trade-off between increased risk and increased return. As in real life it is not possible to construct a portfolio on the CML, it is more common for investors to take more risk in order to attain additional return (Markowitz, 1952).

1.3 Options

Many different kinds of options, swaps, forward contracts and other derivatives are regularly traded by fund managers and financial institutions. We can define a derivative as a financial instrument, whose value depends on the underlying asset. For example, a stock option is a derivative with the value dependent on the stock price.

Though the history of options covers over some centuries, it was not till 1973 that exchange-listed, government regulated and standardized options became obtainable (Friedentag, 2009). Call and put options were first introduced in London in 1694. More than two and half centuries later, call and put options became favorite speculative tools of the Wall Street's old time Wolves. When the Put and Call Dealers Association and Brokers established standards and rules which caused a degree of the reputability for call and put options, stock options became commonly used and better understood by investors as a tool of hedging against price changes and gaining possible tax savings (Friedentag, 2009).

Since the importance of derivatives as both a risk-management tools and investment vehicle become broadly familiar, option markets opened all over the world. Options are not traded just only on traditional products, such as, commodities, stocks, interest rate and foreign currencies, but also on new products as well, such as, insurance, inflation, pollution and real estate. Not just number of option markets have increased dramatically, but also the knowledge of investors have become more and more advanced. Currently, many retail customers have the same level of knowledge as a professional trader (Natenberg, 2014).

Options are traded in the over-the-counter market as well as on organized exchanges. However, most trading of stock options takes place on the exchanges. In the USA the major exchanges are the Philadelphia Stock Exchange, the Chicago Board Option Exchange, the American Stock Exchange, the Boston Options Exchange and the International Securities Exchange. Trading of stock options include more than 1000 stocks. One contract gives the holder the right to sell or buy 100 shares at a particular strike price (Hull, 2003).

The largest exchange for trading the stock options in the world is the Chicago Board Options Exchange. The Chicago Board Options Exchange (CBOE) started trading call option contracts on 16 stocks in 1973. Options had been trading before to 1973, though CBOE succeeded in creating an orderly market with elaborated contracts. In 1977 it started trading of put option contracts. Currently, CBOE trades options on over 1000 stocks and many various stock indices. Similar to futures, options have shown to be very popular contracts. Nowadays, many different exchanges all over the world trade options (Hull, 2003).

Despite the complexity, options can be defined by a single word: choice. Except options, all financial contracts are based upon locked-in assurances for the seller and the buyer. Buying the option gives the investor a choice. We should underline that option gives the right the holder to do something and the holders do not have to exercise this right. This characteristic makes options different from other derivatives such as futures and forwards, where the holders have to sell or buy the underlying asset. (Hull, 2003).

While to enter into futures or forward contracts costs nothing, there is cost of obtaining an option. The price the investors pay for options is called “option premium”, When the investor buys an option, she or he buys a price insurance. The investor’s risk is limited. He/she can be protected against certain price movements and can receive monetary compensation. On the other hand, if the investors sell the option, they grant insurance to someone else. If an unfavorable case happens, the option seller compensates the other party and it can be highly expensive (Ward, 2004).

There are two kind of options: a call option and a put option. The call option gives right the holder to buy an underlying asset by a specific date for a specific price. The put option gives right the holder to sell an underlying asset by a specific date for a specific price (Hull, 2003). Options are known by the underlying asset involved. If the underlying asset is one of the particular indexes, such as Standard & Poor (S&P 100), then these options are called the index options. If the underlying product includes common stock, then such kind of options are called equity options. Except equity and index options, options on treasury securities, interest rates, futures and commodities are available (Eisen, 2000).

The price noticed in the contract is known as the strike price or exercise price. The date determined in the contract is known as, maturity or expiration date. American options can be exercised included at any time up to the expiration time, but European options can only be exercised on the expiration date. On exchanges, mostly American options are traded. In the exchange-traded equity options market, within one contract, there is an agreement to sell or buy 100 shares. Analysis of European options are usually easier rather than American ones and some of the characteristics of the American options are often deduced from its corresponding European ones (Hull, 2003).

1.4 Volatility

Investors and options traders are interested in the direction of the market and they are sensitive to speed of the market reactions. In a way, volatility is measured according to market speed. If the market moves slowly, it means that market is low-volatility market. On the other hand, a market, which moves fast, is a high-volatility market (Natenberg, 2014).

The second name of volatility is the standard deviation and the Greek letter sigma (σ) is a traditional symbol of the standard deviation. Similar to interest rates, volatility is also expressed as an annualized number. Annual volatility shows us the possibility of price changes through shorter periods of time. While interest rate is proportional to time, volatility is proportional to the square root of time. In order to calculate the standard deviation, for over more than one-year period we have to multiply the annual volatility by the squared root of time. Where t expresses years (Natenberg, 2014).

$$Volatility_t = Volatility_{annual} \times \sqrt{t} \quad (1.4)$$

Generally, investors and traders calculate volatility by observing price changes with regular intervals. For calculation of volatility many traders assume that there are 256 trading days per year, since the square root of 256 is a whole number, 16. Traders

can make the following presumption: (Natenberg, 2014). In order to approximate daily volatility, traders can divide the annual volatility by 16.

$$Volatility_{daily} = volatility_{annual} \times \sqrt{1/256} = volatility_{annual} \times 1/16 = \frac{volatility_{annual}}{16} \quad (1.5)$$

Among all inputs, which are required for option evaluation, volatility is the most difficult one for investors to understand. At the same time, volatility plays the main role in actual trade decisions. Changes in the investors' presumptions about the volatility may have a dramatic impact on the option's price (Natenberg, 2014).

Volatility is a measure of fluctuations in the stock price. The underlying share's price volatility influences the option premium. The greater the volatility, the higher is the premium (Friedentag, 2009).

Stock price volatility is a measure of how uncertain the investors are about the future movements of the stock price. For the investors who holds the stock, these, two kind, of consequences tend to balance each other. However, this can impact on the owner of call and put differently. The call owner benefits when the price increases, with restricted downside risk in case of price decreases. Likewise, the put option's owner benefits from price decreases, with restricted risk in case of price increases. Therefore, both call and put option's values increase as volatility increases. Generally, typical stock volatility is between 15% and 60% (Hull, 2003).

1.5 Variance as a Measure of Risk

The word risk can be defined as the dispersion of outcomes around the future value. Simply, the word riskier means there is more dispersion around the future outcome. Mathematically, the term standard deviation and variance measure the dispersion around the future return (Fabozzi & Markowitz, 2011).

The variance of the variable is the measure of a variability of the possible outcomes around the expected return. The variance can be calculated with the following formula:

$$var(R_i) = \sum_{n=1}^N p_n [r_n - E(R_i)]^2 \quad (1.6)$$

Standard deviation is the square root of variance.

1.6 Covariance of Returns

Occasionally, one random variable is connected with another random variable. Statistically the measurement of the connection between two variables is called covariance. It represents the direction of the connection. The covariance is positive if variables tend to move in the same direction, and it is negative when they move in the opposite direction.

This statistical perception can be used to analyze the situation when a price movement of one asset is connected with other assets. In this situation, the covariance between the returns of two assets, i and j , symbolized as σ_{ij} and can be calculated as follow (Francis J.C & Kim.D, 2013):

$$\sigma_{ij} = E\{[r_i - E(r_i)][r_j - E(r_j)]\} = \sum_{s=1}^S P_s \{[r_{is} - E(r_i)][r_{js} - E(r_j)]\} \quad (1.7)$$

Where, r_{is} is a rate of i asset when state s happens. Some variable's covariance with itself equals that variable's variance and when $i=j$, then the above equation becomes:

$$\sigma_{ii} = E[r_i - E(r_i)]^2 = \sigma_i^2 \quad (1.8)$$

1.7 Correlation of Returns

Correlation coefficient is another statistical measure between two random variables and is obtained from the covariance. The difference between these two kinds of measures is that the correlation coefficient is standardized by dividing the covariance by the product of standard deviation of two variables. The correlation coefficient between two variables X and Y is computed as follows:

$$P_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \quad (1.9)$$

The correlation coefficient is always less than or equal to 1 and bigger than or equal to -1. $-1 \leq P_{XY} \leq +1$

For constructing a diversified portfolio, the analyst must know the correlation coefficients between all assets under consideration. If p_{ij} is +1, then the returns of assets i and j are perfectly positively correlated, at the same time they move to the same direction. If p_{ij} is 0, then the return of assets i and j are not correlated. If p_{ij} is -1, assets i and j fluctuate conversely and are perfectly negatively correlated. (Francis J.C & Kim.D, 2013)

Covariance can be determined in terms of the standard deviation and correlation.

$$\sigma_{ij} = p_{ij} \sigma_i \sigma_j \quad (1.10)$$

1.8 Emerging Markets and Borsa Istanbul

Investing in emerging markets (EM) has become attractive especially since 2010. According to estimation of International Monetary Fund¹, emerging economies are expected to grow two to three times more than developed economies. An important benefit that emerging markets offer to investors is diversification, as they perform differently from developed markets and have inverse relationship with mature west economies. Morgan

¹ <https://www.forbes.com/pictures/eglg45gdjd/why-invest-in-emerging-markets-2/#46485a2572e0>

Stanley's Emerging Markets Index was launched in 1988² as an index for EM, which consisted of just 10 countries, that was 1% of the world market capitalization. Nowadays, EM consists of 24 countries which is 10% of the world market capitalization. This index is available for a number of market segments, regions and covers about 85% of the free float-adjusted market capitalization for each of the 24 countries: Brazil, Chile, Columbia, Mexico, Peru, Czech Republic, Egypt, Greece, Hungary, Poland, Qatar, Russia, South Africa, Turkey, United Arab Emirates, China, India, Indonesia, Korea, Malaysia, Pakistan, Philippines, Taiwan, Thailand. Among 10 big emerging markets (China, India, Argentina, Poland, Turkey, Brazil, Indonesia, South Africa, Mexico and South Korea). Turkish stock market is one of the biggest emerging markets, with domestic and international investor interest. As Table 1.1 shows, foreign investors' trade share is increasing, which means that Turkish market is becoming more attractive internationally. On Borsa Istanbul 402 companies are traded with TL 795 billion market capitalization which makes Borsa Istanbul the second most liquid trading platform in the world, with 242% of share turnover ratio. Comparing to 2017 the derivatives trading value has increased by 46% and 6% of total consolidated revenues is contributed to the derivatives. In the derivative markets the highest daily trading value was on 10th of August in 2018, with TL 13.3 billion, which broke the record.³

Table 1. 1 *Breakdown of Domestic and Foreign Investor's Shares Trades in Borsa Istanbul Stock Market*

(source: <https://www.borsaistanbul.com/en/data/data/viop-derivatives-market>)

PERIOD	DOMESTIC INVESTORS %	FOREIGN INVESTORS %
2013	75.31	24.69
2014	76.29	23.71
2015	71.57	28.43
2016	72.68	27.32
2017	71.96	28.04
2018	65.21	34.79
2019	66.12	33.88
JANUARY	68.93	31.07
FEBRUARY	63.11	36.89

² MSCI Emerging markets Index <https://www.msci.com/emerging-markets>

³ Borsa Istanbul annual report, <https://www.borsaistanbul.com/docs/default-source/kurumsal-yonetim/borsa-2018-annual-report.pdf?sfvrsn=6>

Istanbul Stock Exchange is the first Turkish organization which is providing trading of bonds, equities, bills, private sector bonds and international securities. ISE was established as autonomous organization in early 1986. Borsa Istanbul (BIST) is the stock exchange organization of Turkey which combines the former Istanbul Stock Exchange (ISE), Derivatives Exchange Istanbul (VOB) and Istanbul Gold Exchange (IAB) under one umbrella. It was established on April 3, 2013 as a consolidated company with capital 423,234,00 TL. BIST started operating on April 5, 2013 with the slogan “worth investing”. The biggest shareholders of BIST is the Turkish Government with 49%. The rest of the shares are divided as follows: IMK with 41%, VOB with 5%, IMKB members with 4%, IMKB brokers with 1% and IAB members 0.3%.

There are four main markets in Borsa Istanbul: debt securities market, equity market and precious metals, diamond market and derivatives market. Turkish derivatives market (hereafter VIOP) is designed for trading options and futures contracts based on capital market instruments and financial or economic indicators and other derivative products electronically. Borsa Istanbul Derivative Market (VIOP) and Turkish Derivatives Exchange (TURKDEX) started trading on 5th of August in 2013. All option and future contracts in Turkey can be traded at the single platform under the VIOP.

BIST Stock indices have been created in order to measure the return and price performances of group of stocks which are traded on Borsa Istanbul. BIST 30, BIST 100, BIST Industrials and BIST Banks Price Indices are once calculated during the session and spread in real-time. Foreign currency and return indices which are calculated and spread once at the end of the session. BIST30 Index includes 30 stocks which are traded on the BIST Main markets and BIST Stars and the real estate investment venture and trust capital investments trusts are traded on the Structured and Collective Products Market.

1.8.1 MSCI indices

MSCI denotes Morgan Stanley Capital International. The MSCI Index is a measure of performance of stock markets in a specific area. Morgan Stanley published the Capital International Indexes in 1968. These indexes were first indexes for markets outside the United States. It took almost 20 years, for the Emerging Markets Index to be published in 1987 and All Country Indexes for emerging markets and developing markets. The exchange-traded funds follow the MSCI indexes. Managed mutual funds try to outperform them by picking better stocks.⁴ Each Index sums up the total value of market capitalization of all stocks. The market caps are computed both in local currency and U.S dollar, which gives the idea how index is doing without exchange rates impact. MSCI has the indexes for different geographic areas and global indexes for stock categories from small to large-cap. MSCI Emerging Market Index and MSCI All Country Index are one of the most popular tracks.

The MSCI Emerging Markets Index represents the performance of large and mid-cap securities in the following 26 developing countries: China, Korea, Brazil, Columbia, Mexico, Peru, Greece, Malaysia, Pakistan, Hungary, Egypt, Argentina, Turkey, Poland, Russia, Philippines, Qatar, South Africa, Saudi Arabia, Thailand, Taiwan, United Arab Emirates, Czech Republic, Chile, Indonesia, India. According to, data of December 2018, it had more than 1100 constituents covering about 85% of free float-adjusted capitalization in each country.⁵

On the chart below we can see the percentage allocation of county in MSCI Emerging Markets Index. The chart is according to March 29, 2019.

⁴ <https://www.thebalance.com/msci-index-what-is-it-and-what-does-it-measure-3305948>

⁵ <https://www.msci.com/documents/1296102/15035999/USLetter-MIS-EM-May2019-cbren.pdf/fb580e1e-d54c-4c68-1314-977bbff69bd7?t=1559125400402>

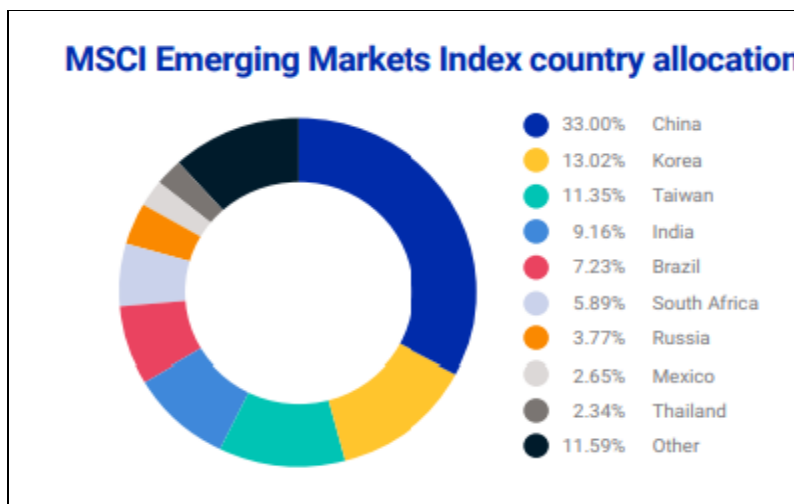


Figure 1. 3 MSCI EM Index country allocation

(source: <https://www.msci.com/documents/1296102/15035999/USLetter-MIS-EM-May2019-cbr-en.pdf/fb580e1e-d54c-4c68-1314-977bbff69bd7?t=1559125400402>)

The MSCI ACWI Index represents the performance of the large and mid-cap stocks of 23 developed countries (Australia, Ireland, Germany, Italy, Norway, Austria, Switzerland, Portugal, Belgium, Denmark, Japan, Spain, the U.K, New Zealand, Finland, France, Hong Kong, Israel, Canada, Netherlands, Singapore, Sweden, and the U.S.) and 26 emerging markets. With 2844 constituents, the index covers about 85% of the global investable equity opportunity set. On the chart below we can see the MSCI ACWI country allocation.

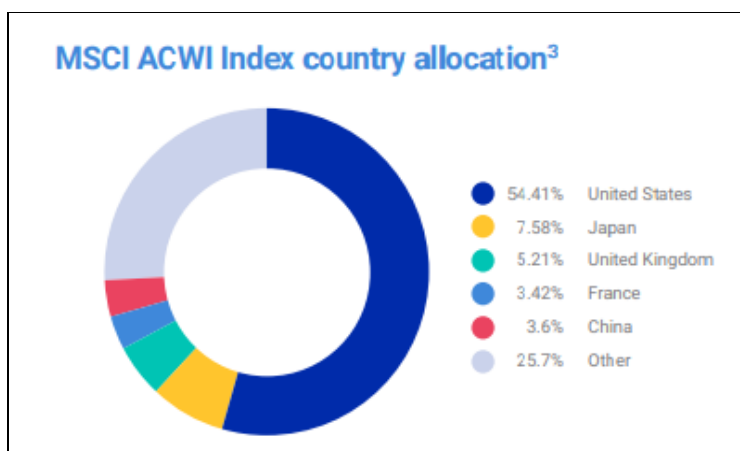


Figure 1. 4 MSCI AC Index country allocation

(source: <https://www.msci.com/documents/1296102/15035999/USLetter-MIS-ACWI-Apr2019-cbr-en.pdf/9de006ef-9cf0-8bd7-62ec-e67430e9155f?t=1559105406131>)

1.8.2 MSCI Turkey index

MSCI Turkey Index is developed to measure the performance of the large and mid-cap segments of the Turkish market. With 16 constituents, the index covers approximately 85% of the equity universe in Turkey, below we can see the performance of MSCI Turkey Index comparable to MSCI EM and MSCI AC. According to annual performance, the lowest performance for MSCI Turkey was in 2008 and 2018 (-60.34 per cent and -38.45 per cent) The highest performances were in 2005 and 2009 (79.84 per cent and 91.35 per cent). These ups and downs are caused by political and economic factors inside and outside of Turkey, which impacts Turkish stock market performance.

Table 1. 2 Annual performance of MSCI Turkey, MSCI EM and MSCI AC

ANNUAL PERFORMANCE (%)			
Year	MSCI Turkey	MSCI Emerging Markets	MSCI ACWI IMI
2018	-38.45	-10.26	-5.54
2017	21.53	20.59	8.87
2016	-5.72	14.51	11.60
2015	-24.10	-5.23	8.96
2014	35.19	11.38	18.24
2013	-29.92	-6.81	18.21
2012	61.69	16.41	14.60
2011	-33.25	-15.70	-4.81
2010	29.20	27.14	22.29
2009	91.35	72.94	32.16
2008	-60.34	-50.92	-39.36
2007	57.02	25.74	0.26
2006	-16.99	18.20	8.19
2005	79.84	54.41	28.54

Source: (<https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23>)

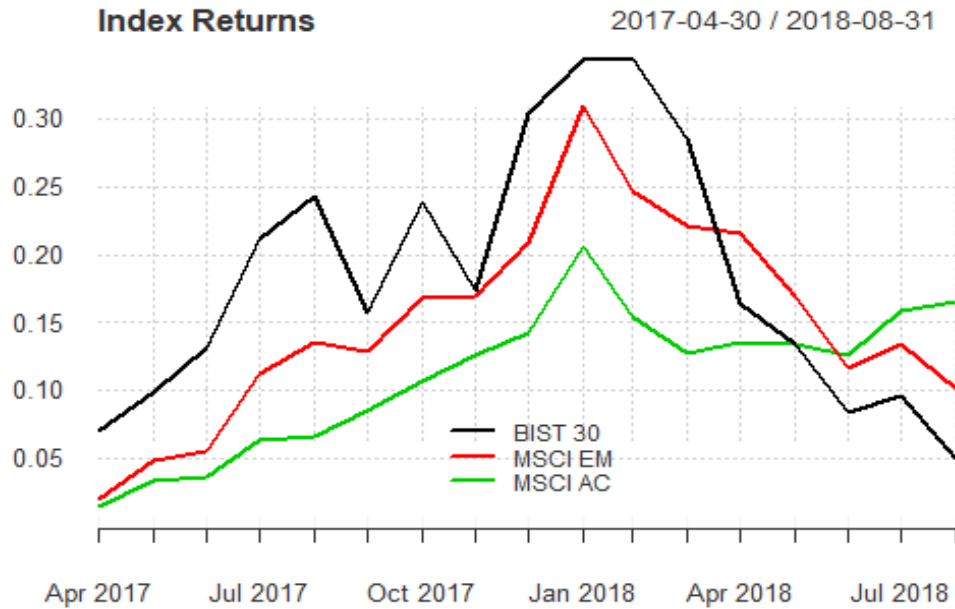


Figure 1. 5 The return indexes of BIST30 and MSCI Indexes (MSCI All Country World and MSCI Emerging Markets).

The chart on the Figure 1.5 above shows performance comparison of the return indexes of BIST30 and MSCI Indexes (MSCI All Country World and MSCI Emerging Markets).

Table 1. 3 The net returns of MSCI Turkey, MSCI EM and MSCI AC

INDEX PERFORMANCE — NET RETURNS (%) (JUL 31, 2019)								
	1 Mo	3 Mo	1 Yr	YTD	ANNUALIZED			
					3 Yr	5 Yr	10 Yr	Since Dec 29, 2000
MSCI Turkey	13.88	19.67	4.53	13.83	-7.83	-8.59	-0.49	1.66
MSCI Emerging Markets	1.03	-2.03	2.80	12.15	8.58	5.66	7.12	7.89
MSCI ACWI IMI	2.59	1.02	7.24	19.54	10.08	10.41	12.09	4.73

(source: <https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23>)

When MSCI EM, MSCI ACWI are compared, we find that MSCI Turkey seems to be the most volatile index with the highest turnover ratio. Moreover, MSCI Turkey index has the lowest Sharpe Ratio.

Table 1. 4 Index risk and return characteristics of MSCI Turkey, MSCI EM and MSCI AC
(source: <https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23>).

INDEX RISK AND RETURN CHARACTERISTICS (JUL 31, 2019)									
	Turnover (%) ¹	ANNUALIZED STD DEV (%) ²			SHARPE RATIO ^{2, 3}			MAXIMUM DRAWDOWN	
		3 Yr	5 Yr	10 Yr	3 Yr	5 Yr	10 Yr	Since Dec 29, 2000 (%)	Period YYYY-MM-DD
MSCI Turkey	13.43	33.12	29.70	29.90	-0.07	-0.14	0.13	0.22	68.07
MSCI Emerging Markets	7.11	11.06	13.47	13.63	0.84	0.50	0.57	0.42	59.79
MSCI ACWI IMI	2.61	10.73	11.90	10.80	0.99	0.92	1.11	0.29	53.48
¹ Last 12 months ² Based on monthly net returns data ³ Based on ICE LIBOR 1M									

Table 1. 5 MSCI Turkey's top 10 constituents
(source: <https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23>)

TOP 10 CONSTITUENTS			
	Float Adj Mkt Cap (EUR Billions)	Index Wt. (%)	Sector
TURKIYE GARANTI BANKASI	3.67	13.14	Financials
AKBANK	3.49	12.47	Financials
BIM BIRLESIK MAGAZALAR	3.22	11.50	Cons Staples
TUPRAS TURKIYE PETROL	2.83	10.12	Energy
TURKCELL ILETISIM HIZMET	2.32	8.29	Comm Srvcs
KOC HOLDING	2.30	8.22	Industrials
EREGLI DEMIR CELIK FABRI	1.68	6.01	Materials
TURKIYE IS BANKASI C	1.59	5.67	Financials
SABANCI HLDG (HACI OMER)	1.46	5.23	Financials
TURK HAVA YOLLARI	1.11	3.97	Industrials
Total	23.67	84.62	

On the table above, we see top 10 constituents among 20 MSCI Turkey constituents (those stocks are in our sample except for BIM Birleşik Mağazalar).

1.8.3 Single stock options contract specification on Borsa Istanbul

With the important merger of Borsa Istanbul and Turkish Derivatives Exchange, all derivative contracts are traded on VOB and VIOP and are integrated on a single platform and all contracts are traded under the one roof of Borsa Istanbul Futures and Options Market. There are following contracts trading:

- Single stock options contracts
- Single stock futures contracts
- BIST 30 options contracts

- BIST 30 futures contracts
- TRYUSD futures contracts
- TRYEUR futures contracts
- EUR/USD cross currency futures contracts
- Gold futures contracts
- USD/Ounce gold futures contracts
- Aegean cotton futures contracts
- Anatolian red wheat futures contracts
- Base-Load electricity futures contracts

There are two kind of option classes, call and put options.

Exercise style of options listed on VIOP is European, which means that the options can only be exercised on the expiry date.

Contract size, there are 100 shares of underlying stock in per contract.

Tick size, prices are offered for the premium value of one underlying asset. TL 0.01 per underlying asset = TL 1.00 per contract (contracts size 100 shares) (<https://www.borsaistanbul.com/en/products-and-markets/products/options/single-stock-options>).

1.8.4 BIST 30 index options contract specification

The name of the underlying asset is BIST 30 Price Index, with options class: call and put. With European exercise style, which can be exercised on the expiration date.

Contract size, as the underlying security is the 1/1000 of the index value. Index options' contract size is 100 underlying securities (<https://www.borsaistanbul.com/en/products-and-markets/products/options/equity-index-options/bist30-index-options-contract-specification>).

1.9 Research Problem

Recent studies on portfolio optimization have suggested that portfolios based on the historical data yield poor performance and that measures derived from option prices such as option-implied volatility risk premium are helpful in predicting stock returns.). (Plyakha, Uppal, & Vilkov, 2012). Historical volatility shows how stock prices fluctuate on a day to day basis over one-year period. Implied volatility does not depend on historical prices of stock, but it forecasts how the marketplace anticipates stock's volatility in the future, based on the changes in the stock options prices (Canina & Figlewski, 1993).

We investigate if implied information in stock options prices can be used in order to improve portfolio's performance. Using option-implied volatility as forward-looking information to create a portfolio strategy, we develop a multi-stage portfolio optimization model using the option prices to simplify market risk. We explore if using option-implied volatility and option-implied correlations can help improve portfolio performance.

In our research, we calculated implied volatility using Black-Scholes option pricing model. BSOPM calculates the theoretical value of a European option using the option's strike price, expected interest rate, time of expiration, expected volatility, expected dividends and current stock price. For comparison, we used the performance of the benchmark portfolio, which is the BIST30 Index. We measured portfolio performance with the following measures: Annualized return, annualized portfolio volatility (standard deviation) and Sharpe ratio.

1.9.1 Significance of the study

The findings may enable local and foreign investors, future investors, researchers and policy makers to utilize forward-looking information implied based on option prices in order to improve the out-of-sample performance of their portfolios.

1.9.2 Goals of the study

The main purpose of this study is to examine whether investors can use forward-looking information to improve the selection of portfolios and improve their out-of-sample performance. We aim to investigate if optimizing portfolios using information in option-prices can yield better performance measures rather than optimizing portfolios ignoring option-implied information. For the evaluation of portfolio performance, we used the following three criteria: annualized portfolio return, annualized portfolio volatility (standard deviation), portfolio Sharpe ratio. Based on our empirical analysis we can demonstrate that prices of stock options contain information that can be useful to improve the out-of-sample performance of portfolios.

1.9.3 Limitations of the study

In this study we use option price data of stock options traded at Borsa Istanbul. Our sample consists of 20 stock options and the sample period is from March 2017 to July 2018.

1.9.4 Research questions

The research aims to answer the following questions:

- Is forward-looking information helpful in portfolio selection process?
- Does option-implied volatility improve portfolio performance?
- Is the performance of portfolio based on option-implied volatility superior to mean-variance or minimum variance portfolio?

1.9.5 Research hypothesis

H_0 : Optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information.

H₁: Optimal portfolios based on option implied information perform better than optimal portfolios based on historical information.

This thesis is organized as follows: Having introduced the topic in chapter 1, chapter 2 presents review of related literature. Chapter 3 presents the data used and the methodology of the study. Chapter 4 presents the empirical findings and chapter 5 presents conclusion and recommendations.



2 LITERATURE REVIEW

Since portfolio selection is an important issue among investors and policy makers, researchers have given a special attention to the subject of portfolio optimization. There are numerous papers about portfolio optimization using different methodologies and techniques. According to many studies, option-implied volatility is a strong forecaster of future volatility in the equity market. Thus, option implied information is used in several studies for portfolio optimization purposes. This chapter presents the summary of relevant literature about portfolio optimization and option implied volatility. Previous studies are sorted out according to similar methodologies and techniques.

H. Markowitz, who is considered as father of Modern Portfolio Theory (MPT) wrote his first paper titled “Portfolio Selection” in 1952. This seminal paper is about portfolio optimization and helps investors to allocate investments between different securities. He introduced the efficient set of portfolios with maximum return with a given level of risk. Mean-variance optimization (MVO) is a quantitative tool, which allows investors to make allocation by considering the trade-off between return and risk. Markowitz (1959) in his book titled “Portfolio Selection: Efficient Diversification of Investments” importantly extended findings from his paper portfolio selection (1952).

James Tobin (1958) in his article titled “Liquidity Preferences as Behavior towards Risk” based on Markowitz work created the “efficient frontier” and “capital market line”. According to Tobin’s model, investors, no matter what their risk tolerance level is, will maintain stock portfolios in the same proportion since they have similar expectations about the future. As a result, Tobin concludes that their investment portfolios will diverge just in their proportion of bonds and stocks.

Later, Harry Markowitz (1959) suggested another measurement of risk, semi-variance of returns. Markowitz (1959) discussed semi-variance and claimed that portfolios based on semi-variance have better performance compared to those based on variance. Moreover, Markowitz (1991) suggested that semi-variance is a more reasonable measure of risk and claimed that as investors worry about underperformance than over-performance, semi-variance is a more suitable measure of risk rather than variance.

Boasson et al. (2011) used mean-semi-variance approach in order to measure the downside risk in optimal portfolio selection. They measured return dispersions below the expected return of the investment return. They used the sample of 7 exchange traded index funds which includes several kinds of securities to test and compare the differences between asset allocations and optimal portfolios. They constructed portfolios using mean-semi-variance approach and traditional mean-variance approach. They showed that semi-variance approach provides desirable benefits. Portfolio optimization under the semi-variance model improved portfolio's expected return, while minimizing its downside risk exposure.

Konno, Waki, & Yuuki (2002) explained the attractiveness and importance of using lower partial risk also called downside risk in portfolio management. The aim of this work is to review the important characteristics of this measurement and to obtain alternative measurements, such as lower semi-absolute deviation, lower semi-variance, conditional value-at risk and below the target risk. They proposed that these risk measurements are useful for controlling the downside risk, when the asset distribution is non-symmetric. Their sample consists of 104 assets and 105 scenarios. According to results MCVaR, M-LSAD and mean-lower partial risk models can control portfolio's downside risk, when the distribution of returns are not symmetric nor normal.

Grootveld & Hallerbach (1999) explained that the popularity of downside risk among the investors has been growing. The paper focuses on the similarities and differences between downside risk and variance measures, from the theoretical and empirical point of view in USA. Empirical results showed the differences between portfolios which are based on variance and semi-variance. According to the findings, downside risk approach tends to favor stocks with the minimum risk point. The differences in portfolio composition with downside risk optimization are significantly large compared to the portfolio optimization with mean-variance approach.

Galsband (2012) investigated the downside risk of international stock returns in 14 major industrialized economies worldwide, including Canada, the US and twelve major EAFE countries between 1975-2010. His findings show that the world's largest equity markets can be rationalized by differences in sensitivities of international asset returns to the downside shocks. Generally, international value stocks are sensitive to market's permanent downside shocks.

Yan, Miao, & Li (2007) utilized Markowitz's semi-variance portfolio selection model. They formulated multi-period semi-variance model. For solving this model, they use genetic algorithm (GA) and particle swarm optimizer (PSO). According to the authors, for the need of investors and reality of financial markets, measuring the risk by variance should be replaced by semi-variance as the findings showed that the method is evidently superior.

Chang, Christoffersen, Jacobs, & Vainberg (2012) found option-implied skewness and volatility as a strong predictor of future beta, and suggest that company beta can be estimated from skewness and option-implied volatility measures from index options and equity. They compare option-implied beta with historical beta and find out that option-implied betas contain information which is not contained in historical beta and option-implied betas have important predictive power for future betas which increase beyond options' maturities. According to the empirical findings of this paper, if the underlying risk-neutral distribution is more negatively skewed, then option-implied betas are higher. They also prove that the stock's option-implied beta is relatively determined by the difference between the skew of the index's risk-neutral distribution and stock's risk-neutral distribution.

Kempf & Korn (2012) developed new estimators of the covariance matrix, which totally relied on forward-looking information. This estimator required implied volatility and implied correlation. Second, the core contribution is that they tested this new method on GMVP, which is based on the covariance matrix of implied estimators and found that it performs much better in the out-of-sample for a sample of 30 stocks from Dow Jones Industrial Average (DJIA). The findings of the empirical study give 2 recommendations to investors. First, strategies which are based on fully-implied estimators outperform the benchmark strategies, and mostly are not beaten by other strategies. Second, if investors want to use historical estimators, they should use the most recent date and shrinkage estimators.

Plyakha et al. (2012) used option-implied information in order to improve the selection of the mean-variance portfolio, with large number of stocks and to identify which option-implied information is the most useful to improve the out-of-sample performance. They measure the performance of portfolios using Sharpe ratio, turnover and volatility. As the benchmark, they use 1/N portfolio. Their findings prove that using option-implied volatility reduces volatility of the portfolio. Also using implied volatility and option implied model-free skewness can achieve higher Sharpe ratio rather than by ignoring option-implied

information. They conclude that stock options prices include information which can be used to improve the performance of the out-of-sample performance.

Kostakis, Panigirtzoglou, & Skiadopoulos (2010) used market option prices (also known as option-implied distribution) for asset allocation problem. They used risky and risk-free assets, implied spread, which was taken from S&P500 futures options and then converted to risk-adjusted ones. They used the stock index implied distribution as an input for calculating the optimal portfolio. According to empirical findings using option-implied information increases obtained risk-adjusted return of investor's which makes meaningfully better off compared with the historical distribution.

Driessen, Maenhout, & Vilkov (2013) developed a model of correlation risk pricing for stock returns. They used two samples, which are, the narrow stock market index DJ30, from 1997 to 2012 and the broad stock market index S&P500 from 1996 to 2012. According to findings, there is a big negative risk premium in 1996-2012, with average option-implied correlation 39.5% for S&P500 and 46% for DJ30, with average realized correlation 32.6% for S&P500 and 35.5% for DJ30. Index options seem expensive in the sense that risk-neutral expected correlation in their prices is considerably higher than average realized correlation, but this high price reflects insurance cost against the unexpected correlation increases in risk and resulting loss in diversification benefits. Another core result is that implied correlation has significant predictive power for future stock market excess returns, particularly, at the 6-month and 1-year horizons.

Bahaludin, Abdullah, & Tolos (2017) used option-implied distribution as an input in asset allocation. The data is divided into two, option prices data and historical prices data. The data consists of stocks, which are listed in Dow Jones Industrial Average (DJIA) index. The sample period is 1/1/2009-12/31/2015. As an alternative of using historical prices, they used option prices to build a portfolio. The performance is measured by Sharpe ratio and standard deviation of portfolio. Findings showed that the portfolio based on risk-world and risk-neutral densities exhibit statistically significant differences. Moreover, the portfolio constructed from the risk-world density performs better rather than from the risk-neutral density.

Rehman & Vilkov (2011) used U.S. exchange-traded individual stock option data and showed that ex ante skewness is positively correlated with future stock returns. They

measured ex ante by using the model free implied skewness (MFIS) of the distribution of risk-neutral return and explained that high MFIS stocks perform better, rather than low MFIS. Using MFIS, it is possible to identify the variation of a value of a firm from its basic value. The most overrated stocks have the most negative ex ante skewness. In contrast to historical skewness estimates, ex ante skewness is related positively to the future returns of the stocks.

Vilkov & Xiao (2013) proposed a forward-looking variable, which directly evaluates the expected stock or market crash size, and condition of realization. With this variable, they made predictions of crash in the cross-section stocks and market index. The main aim of their paper is to estimate a stock specific and market wide tail loss measure (TLM) from the put options, traded at out of money and to show that an ex ante magnitude is positively correlated to the future realized crashes. When there is no crash, higher loss expectations through the immediate decline in the value of asset are related with the higher premium to be gained for taking risk. TLM contains information about the future stock returns over implied correlation (IC) and variance risk premium (VRP) to predict the future returns of market. For getting final results they proceed some steps: first, they use (EVT) extreme value theory, in order to estimate the TLM from perceived out-of-the-money option prices. Then, they conduct time-series test, using the robust regression and show that when standard deviation increases in the TLM, it causes ultimate positive change in the weekly return of market. Next, they exercise optimization of portfolios using option-implied variables and compare it to a non-informative portfolio, and find out that the only variable which gives significantly useful information is implied correlation. On the final stage they study stock-specific TLM and stock returns cross-section and they conclude that TLM is positively correlated with the expected return.

Buss, Schoenleber, & Vilkov (2017) used variance risk premium and implied correlation for predicting market returns, with variance risk premium predicting market return only one quarter ahead. They address two core issues in the paper: first, they build an ex ante covariance (correlation) matrix from the option prices without using any historical information to estimate linear factor model obtained by current option prices, and second they identify which link of implied correlation forecasts the market returns. They found that correlation and variance risk premiums differ according to sectors of economy. That means that stocks in various sectors have heterogeneous exposure to underlying factor. Implied

correlation forecasts systematic diversification risk and option-implied covariance matrix which is based on implied correlation between and within economic sectors and it might be used for extracting statistical factors which better explain dynamics of stock returns.

Davari-ardakani, Aminnayeri, & Seifi (2016) considered multistage portfolio optimization model with NYSE options and stocks, they developed optimization model, which employs options for mitigating market risk. In their methodology, they used dependence structure of different security returns and they considered serial correlations of each security return. They used Black-Scholes model for determining European option's call and put prices. They also used back-testing simulations for comparing multistage models to single-stage ones. According to their results, multistage models show better performance and options can be considered as a core instrument for controlling investment portfolio's market risk.

Vial (2013) analyzed option-implied information in the context of portfolio optimization. Option-implied information is obtained from DJIA and S&P100 index, with sample period from January 1996 to January 2012. The paper focused on the derivation of the option-implied covariance and the stability of forward-looking covariance was increased by Frobenius norm, also known as an, Euclidean norm. Nevertheless, option-implied portfolios performed better than portfolios based on historical information. However, the difference often is insignificant. The results for different estimation periods and strategies are robust assuming that forward-looking information is inherent in exchange-traded options.

Vilkov & Xiao (2013) computed extreme returns using option-implied information. From the observed option prices, they estimated forward-looking tail loss measure (TLM). TLM predicts the probability of market crashes and market returns. In addition to TLM, they also consider the option-implied predictors of asset returns such as model free implied variance, kurtosis and skewness, variance risk premium and implied correlation. They found that TLM is positively correlated to the future market returns, especially in the short run. The other variables, which importantly explain future market returns are implied correlation and variance risk premium. They also find that constructing portfolios using implied correlation gives better results than using historical information.

3 DATA AND METHODOLOGY

This chapter presents the methods employed in the study in order to construct different kind of portfolios using stock options data which is obtained from Borsa Istanbul and includes 20 companies. We compare portfolios performances constructed using historical and implied information.

In R, we used PortfolioAnalytics to calculate the portfolio weights. The method that is used was to generate thousands of portfolios and estimate what their risk-return characteristics are based on the covariance matrix used. Then the weights of the portfolio that minimizes variance and the portfolio that maximizes the mean-variance are used to calculate subsequent performance. The portfolios were constrained to be long-only, such that asset weights were $\geq 0\%$, fully invested, such that the sum of asset weights = 100%, and individual asset weights were capped at 20%. The DerivMkts package was used to estimate the implied volatility of the options. Options were chosen as the call price with the strike closest to the call and the expiration date was selected as the closest month where there was both an option for the underlying asset and the BIST30. The reason for this was that the BIST30 implied volatility was required to estimate the implied correlation and it was consistent to use options that had the same expiration.

Our research utilizes option-implied volatility in order to improve the out-of-sample performance of portfolios. We employ Black-Scholes option pricing model (BSOPM), which is the most widely used model in option pricing. For constructing mean-variance and minimum-variance portfolios we use Markowitz's models. In this part, we define our data and explain different portfolio construction methods and the metrics used in order to compare the benchmark portfolio with the portfolios based on historical and option implied information.

3.1 Data

We utilize stock price data and option price data of 20 stocks, traded in the Futures and Options Market of Borsa Istanbul since March of 2017. The data set includes daily data between March 2017 until July 2018.

From 2013 to 2015 only 10 stock options were traded on Borsa Istanbul. From 2015 onwards number increased gradually and became 20. We decided to employ data starting from 2017, as it includes 20 stocks and until 2017 the trading volumes of stock options were minimal. The risk free (RF) rate was computed from the benchmark (with a maturity of 2 years) government bond data.

We built different portfolios from these 20 stocks with 20% maximum weight restriction of assets in the portfolio. For the benchmark portfolio, we used BIST30 index data, as our sample consists of 20 stock options.

Stock closing prices and stock total returns were obtained from Thomson Reuters Database. The discrete return of an individual investment or portfolio can be calculated as follows (Dobbins et al., 1994):

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} \quad (3.1)$$

Where,

R is a periodical return

P_{t-1} is an initial price of the period

$P_t - P_{t-1}$ is a capital loss or gain

D_t is a dividend in the end of the period

P_t is the last price of the period.

Historical volatilities of returns were computed using 3 years “rolling window method” from the stocks total return. Stock option data was obtained from Borsa Istanbul. Total number of the months of the data are 17 months from March 2017 to July 2018. We decided to work on monthly basis, thus, for calculations we chose the last trading dates of each month.

Data was filtered as follows: we choose the last trade date of each month, then we choose the expiry date, which is at least one month ahead (1 month or 2 months ahead) and we considered the option contract (when both the BIST30 Index Option and the stock option were traded on the same date), which had the closest strike price to the observed stock close price on the expiry date of the option contract.

The data includes 20 stock options and one index option. In total, data consists of 340 stock options and 17 index options. The list of the companies included in the study for portfolio construction purposes is as follows:

Table 3. 1 *The list of sample companies*

AKBNK.E	Akbank T.A.Ş.
ARCLK.E	Arçelik A.Ş.
EKGYO.E	Emlak Konut Gayrimenkul Yatırım Ortaklığı A.Ş.
EREGL.E	Ereğli Demir ve Çelik Fabrikaları T.A.Ş.
GARAN.E	T. Garanti Bankası A.Ş.
HALKB.E	Türkiye Halk Bankası A.Ş.
ISCTR.E	T. İş Bankası A.Ş.
KCHOL.E	Koç Holding A. Ş.
KRDMD.E	Kardemir Karabük Demir Çelik Sanayi ve Ticaret A. Ş.
PETKM.E	Petkim Petrokimya Holding A.Ş.
PGSUS.E	Pegasus Hava Tasimacılığı A. Ş.
SAHOL.E	Hacı Ömer Sabancı Holding A.Ş
SISE.E	Türkiye Şişe ve Cam Fabrikaları A.Ş
TCELL.E	Turkcell İletişim Hizmetleri A.Ş
THYAO.E	Türk Hava Yolları A.O.,
TOASO.E	Tofas Türk Otomobil Fabrikası A.Ş.
TTKOM.E	Türk Telekomünikasyon A.Ş.
TUPRS.E	Türkiye Petrol Rafinerileri A.Ş.
VAKBN.E	Türkiye Vakıflar Bankası T.A.O.
YKBNK.E	Yapi ve Kredi Bankası A.Ş.

3.2 Portfolio Optimization

Optimal portfolios were constructed with R code (the R codes are presented in the appendix). We applied monthly rebalancing during portfolio optimization. Rebalancing is a process of buying and selling of securities to bring the portfolio for setting the weights of each asset back to its original state according to an investor's tolerance of risk and investment

strategy has changed. The investor can use rebalancing to readjust the weighting of each asset or security class in the portfolio in order to fulfill a newly devised asset allocation.

3.2.1 Minimum variance optimization

Minimum variance portfolio also known as minimum risk portfolio is a risk-based approach in portfolio construction. It is different from Markowitz portfolio selection in that instead of using both return and risk, minimum variance portfolio is constructed using only measure of risk. If an investor desires investing in the portfolio with the least risk, he/she does not think about the expected return, however the only thing that he/she wants is the lowest possible risk. The minimum variance portfolio is related with modern portfolio theory and efficient frontier. It is the only portfolio lying on the efficient frontier, with a minimum level of standard deviation. In this situation, the variance is also minimal. That is the reason why this portfolio is also called the minimum variance portfolio.

The minimum variance portfolio can be calculated by minimizing the variance to the necessary constraint. This is called as budget constraint, which is the amount of capital the investor has to invest. The reason why investors want to optimize the minimum variance portfolio is that it is very hard to estimate the future or expected return, but it is easy to measure the risk.

The optimization problem of mean-variance portfolio can be written as:

$$\min_w \quad w^T \hat{\Sigma} w - w^T \hat{\mu} \quad (3.2)$$

$$\text{s.t } w^T e = 1 \quad (3.3)$$

The purpose of the first equation is to minimize the difference of portfolio return variance $w^T \hat{\Sigma} w$ and its mean $w^T \hat{\mu}$. The second equation assures that the portfolio weights resume to one.

The minimum-variance portfolio problem solution is as follows:

$$w_{min} = \frac{\hat{\Sigma}^{-1} e}{e^T \hat{\Sigma}^{-1} e} \quad (3.4)$$

The covariance matrix $\hat{\Sigma}$ can be decomposed into correlation and volatility matrices.

$$\hat{\Sigma} = \text{diag}(\hat{\sigma})\hat{\Omega}\text{diag}(\hat{\sigma})$$

Where, $\text{diag}(\hat{\sigma})$ is the diagonal matrix with stock volatilities on the diagonal, and $\hat{\Omega}$ is the matrix of correlation. For obtaining the optimal weights of portfolio on the covariance matrix sample, we need to estimate two quantities: correlations $\hat{\Omega}$ and volatilities ($\hat{\sigma}$).

3.2.2 Mean-variance optimization

Mean-variance model of portfolio selection was published in 1952 by Harry Markowitz. This model is one part of Modern Portfolio Theory (MPT) and still inspires empirical and theoretical research today. Markowitz suggests that portfolio selection process can be divided into two stages: the first stage starts with observations and beliefs about future performance of assets and in the second stage above mentioned beliefs are used for constructing the portfolio (Markowitz, 1952). The most important aim of the portfolio theory is optimally allocating investment budget among different assets. Mean-variance optimization (MVO) is the quantitative tool, which allows the investor to make allocation according to trade-off among risk and return. As we have already mentioned, Markowitz divides portfolio selection process into 2 stages, mean variance approach is about second stage, portfolio selection.

Every investor's aim is to maximize the capitalized or discounted return of future or expected return. Although the future is not known exactly, it is "anticipated" or "expected" return which Markowitz discounts. The hypothesis indicates that the investor puts all her or his funds in the asset with the highest discount value. If two or more assets have the same value, then any of these combinations of these is as good as any other. Let us assume there are N securities, and consider that r_{it} is an anticipated return, at time t per dollar invested in asset i, d_{it} be a rate at which the return on i^{th} asset at time t will be discounted back to the present. If we assume that X_i is the relative amount invested in asset i. Markowitz excludes

short sales, so $X_i \geq 0$ for all i . Then the discounted expected return of a portfolio is calculated with the following formula: (Markowitz, 1952).

$$R = \sum_{t=1}^{\infty} \sum_{i=1}^N d_{it} r_{it} X_i = \sum_{i=1}^N X_i (\sum_{t=1}^{\infty} d_{it} r_{it}) \quad (3.5)$$

$$R_i = \sum_{t=1}^{\infty} d_{it} r_{it} \quad (3.6)$$

this is the discounted return of the i^{th} asset, thus

$$R = \sum X_i R_i \quad (3.7)$$

Where,

R_i is independent of X_i since $X_i \geq 0$ for all i and $\sum X_i = 1$

R is a weighted average of R_i , with X_i as non-negative weights

In order to maximize R , Markowitz consider $X_i = 1$ for i with maximum R_i

$R = \sum X_i R_i$ defines the flow of returns from the portfolio as an entire.

If the investor's aim is to maximize the expected return, then he or she should place all the funds in the assets with the maximum expected return. Markowitz's model does not derive n-asset case, but his model represents for 3 or 4 asset cases. The portfolios return (R) is a sum of the random variables, investor can choose the weights of assets. The expected return of the portfolio is calculated as follows:

$$E = \sum_{i=1}^N X_i \mu_i \quad (3.8)$$

The variance of portfolio is calculated as follows:

$$V = \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} X_i X_j \quad (3.9)$$

Investors have to choose different combinations of E and V according to their choice of portfolio $X_1, \dots, X_N$. Assume that the set of all accessible (E, V) combination is as on figure below.

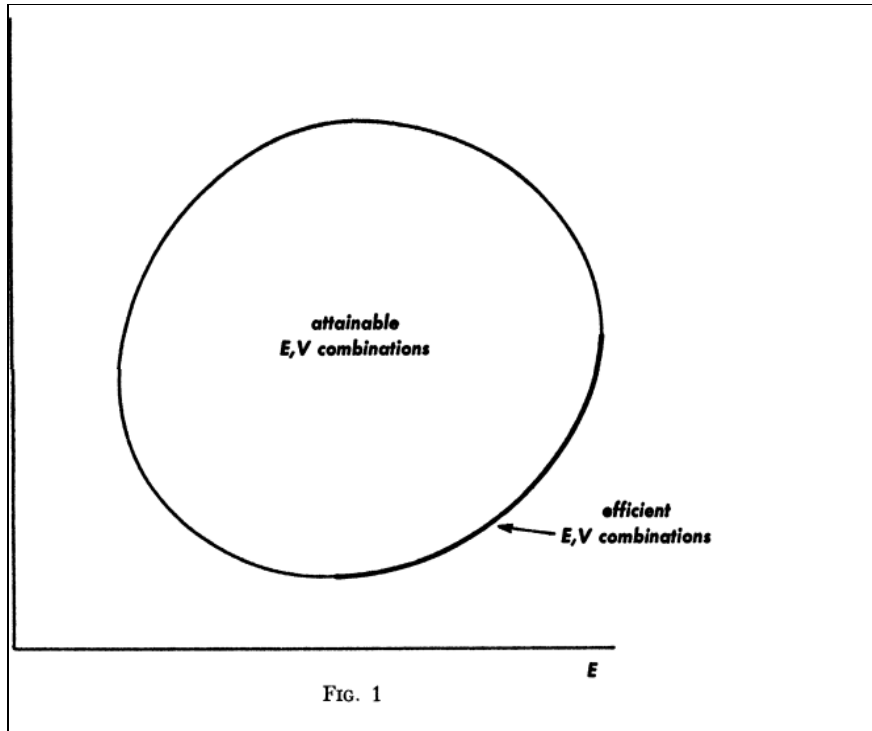


Figure 3. 1 E-V combination (Source: Adopted from H. Markowitz (1952))

According to E-V rule the investors should choose one of those portfolios that give increase to the E-V combinations shown as efficient in the figure above. With minimum V for a given E and maximum E for a given V. In order to calculate the efficient surface, two conditions are needed. First, the investors must act according to E-V maxim and second, we had to be able to arrive at reasonable μ_i and σ_{ij} .

In the three securities case, Markowitz's model reduces to the following:

$$E = \sum_{i=1}^3 X_i \mu_i \quad (3.10)$$

$$V = \sum_{i=1}^3 \sum_{j=1}^3 X_i X_j \sigma_{ij} \quad (3.11)$$

$$\sum_{i=1}^3 X_i = 1 \quad (3.12)$$

$$X_i \geq 0 \text{ for } i = 1, 2, 3.$$

3.2.3 Optimization with option implied information

We built a new estimator of covariance matrix using forward-looking information from a cross-section of option prices. The minimum variance portfolio is based on the covariance metrics where historical information is replaced by option implied information. For this, we employed:

- Option-implied volatility
- Option-implied correlation

3.2.3.1 Implied volatility (*“Black-Scholes option pricing model”*)

Expectations regarding the volatility or any other moments of returns are usually estimated according to historical data. However, historical data may not be accurate for providing the best volatility forecast of expected risks, as it contains past information. In contrast, option trading data contains forward-looking information. Thus, implied volatility derived from option trading data may have significant information for expected risk, which historical data lacks (Francis J.C & Kim.D, 2013). Implied volatility ignores historical information, alternatively, determines the option σ based on the actual prices of options. While historical volatility is a backward-looking estimator, implied volatility is a forward-looking estimator.

Implied volatility is not directly measurable therefore it is usually derived from the “Black-Scholes Option Pricing Model”, which was developed by three economists: Myron Scholes, Fischer Black and Robert Merton and is possibly the most well-known option pricing model in the world. The above, mentioned economists introduced this model in their paper in 1973, called “The Pricing of Options and Corporate Liabilities”. The economists were awarded Nobel Prizes in economics in 1997 for their work of a new method in determining the value of derivatives. The model makes certain assumptions (Black & Scholes, 1973).

- The option is European and will be expired only on expiration date
- There is no dividend during the lifetime of option;
- Markets are efficient (for example, market movements cannot be anticipated);

- There are no any transaction costs during buying the options
- The volatility and risk-free rate are known and constant
- The returns on the underlying are normally distributed

Black-Scholes formula is also appropriate for corporate bonds, corporate liabilities and warrants. Generally, the formula can also be used to obtain the discount which should be applied to the corporate bond for default possibility.

For call option,

$$C = SN(d_1) - N(d_2)X_e^{-rT} \quad (3.13)$$

and for put option,

$$P = X_e^{-rT}N(-d_2) - SN(-d_1) \quad (3.14)$$

Where,

C is price of the call option

P is price of the put option

S is current stock price

t is time until option exercise

K is option strike price

r is risk free interest rate

N is cumulative normal standard distribution

e is exponential term.

Calculation of implied volatility requires using the following above mentioned five other model inputs:

1. The underlying price of the stock
2. The market price of the option
3. The expiration time
4. The strike price of an option
5. The risk-free interest rate

When we know option prices, the first step is using this information to back out implied volatilities for predicting volatility (Benninga, 2008). By replacing the model price C in the above, mentioned equation with the market price of the call option, we can figure out this equation regarding the volatility parameter σ by repeating the method, as the rest of variables, S , X , r , and T can be detected uniquely. The value which is obtained by backed out of σ is called implied volatility. It is implied by the Black-Scholes option pricing formula and values of other parameters (Benninga, 2008).

Implied volatility has several advantages and disadvantages. Regarding advantages: first, it might shift forward-looking information of underlying asset. Second, since the underlying asset's trading price is available, implied volatility can be calculated. Hence, implied volatility can be calculated any time the market is operates.

Regarding disadvantages: first, the estimation of implied volatility depends on the choice of option pricing model, so if the option price is calculated incorrectly, then implied volatility will provide deceiving assessment of expected risk. If market is efficient and option pricing model is specified correctly, implied volatility obtained by options with the same expiration date and with the same underlying asset should be the same, even if strike price is different. However, according to empirical studies, implied volatility calculated by Black-Scholes option pricing model displays a suspicious pattern across strike price. Second, implied volatility computed from deep-out-of-the-money and deep-in-the-money options has, a tendency to be higher rather than calculated from at-the-money options on the same underlying asset. Third, if there is no closed-form solution for the option, computing of implied volatility would be difficult, though several model free methods can be suggested. Fourth, there are measurement problems in implied volatility. These problems generally come from nonsynchronous trading. To specify more, timing of option closing prices and underlying asset may be different. For example, for rarely traded options the last trade of the option might appear before to the last trade of the day. Transaction cost is another reason of measurement error and fifth one, if there is no actively traded option on the underlying asset, estimation of implied volatility is impossible (Francis J.C & Kim.D, 2013).

According to preliminary studies, historical volatility provides better forecasting for future volatility rather than implied volatility. However, recent studies demonstrate that implied volatility is much more informationally efficient for forecasting future volatility

rather than historical volatility. Yet, the comparison results can be sensitive to the options, option pricing model, sample period, market data.

In empirical finance, usually, historical information was used to build the future expectations. Option prices contain valuable information about the future moments of asset prices (Black & Scholes, 1973). In order to improve the quality of moment estimations, a number of researchers use forward-looking information instead of historical data. Strategies based on fully-implied estimators are a good choice as they significantly outperform the benchmark strategies in most cases and are never beaten by any other strategies (Kempf & Korn, 2012).

For our research, we need to calculate historical and implied volatilities. For this purpose, researches use different kind of models. For calculating historical volatility, we decided to use the rolling-window methodology with 3 years window length. For calculating implied volatility, some researchers follow Black-Scholes model and some of them follow Bakshi, Kapadia and Madan's model.

3.2.3.2 Implied correlations

After predicting the volatility using options, we consider another option-implied information, implied correlation, as a portfolio optimization under the unbiased measure.

Implied correlations are not calculated directly from the options' price. (Buss & Vilkov, 2012) create their model for computing implied correlations. They list the technical conditions and empirical formal observations, which must be satisfied for the implied correlation matrix. Then, they defined a simple parametric equation for calculating implied correlations, which is constant with the following empirical and technical observations.

Especially, for determination of the implied correlations, $\rho_{i,j,t}^Q$, they only determine constraints which equates the observed implied variance of a market index $(\sigma_{M,t}^Q)^2$ with the calculated implied variance of the portfolio for all market index elements $i=1, \dots, N$: (Buss & Vilkov, 2012)

$$(\sigma_{M,t}^Q)^2 = \sum_{i=1}^N \sum_{j=i}^N w_i w_j \sigma_{i,t}^Q \sigma_{j,t}^Q \rho_{ij,t}^Q \quad (3.15)$$

Where,

$\sigma_{i,t}^Q$ defines the implied volatility of i stock in the index, and w_i is the index weight.

Implied correlation should satisfy the following technical conditions: first, correlation matrix should be positive and second, all correlations $\rho_{i,j,t}^Q$ should not exceed one. Furthermore, the implied correlation should be constant with two following empirical observations: First, the implied correlation $\rho_{i,j,t}^Q$ should not be higher than the correlation under the true measure $\rho_{i,j,t}^P$, and second the correlation risk premium should be higher in magnitude for stocks pairs. This provides larger diversification profits (for example, negatively or low correlated stocks), and hereafter are showing to a larger risk of losing diversification in bad times categorized by strong correlations. According to Mueller, Stathopoulos, & Vedolin (2012) the second observation is supported by a negative correlation between the correlation risk premium and correlation under the objective measure.

Taking into consideration the above mentioned empirical and technical conditions (Buss & Vilkov, 2012) developed the following parametric form for calculation of implied correlation $\rho_{i,j,t}^Q$:

$$\rho_{i,j,t}^Q = \rho_{i,j,t}^P - \alpha_t (1 - \rho_{i,j,t}^P) \quad (3.16)$$

Where,

$\rho_{i,j,t}^P$ is expected correlation under the objective measure

α_t represents the parameter to be identified, it relates to the implied and realized correlations of the index and is constant in the interval (-1, 1). If α is supposed to be negative the implied correlation is going to be higher rather than the realized correlation, and if α is positive the opposite result will be obtained. (Eklund & Estaifo, 2018).

Replacing the implied correlations (3.15) into constraints (3.16), α can be calculated as following.

$$\alpha_t = -\frac{(\sigma_{M,t}^Q)^2 - \sum_{i=1}^N \sum_{j=i}^N w_i w_j \sigma_{i,t}^Q \sigma_{j,t}^Q \rho_{ij,t}^Q}{\sum_{i=1}^N \sum_{j=i}^N w_i w_j \sigma_{i,t}^Q \sigma_{j,t}^Q (1 - \rho_{ij,t}^Q)} \quad (3.17)$$

After that, we should use equation (2) in order to distinguish the full implied correlation matrix Γ_t^Q , with $\rho_{i,j,t}^Q$ element.

If $-1 < \alpha_t \leq 0$, then the model is satisfied for technical conditions mentioned above, and also constant with empirical observations (Buss & Vilkov, 2012).

3.3 Performance Measurement

Most investors assess their portfolios' performance based only on returns, just few of them consider risk. There are several kind of performance measurement tools, which are able to evaluate portfolio performance. Sharpe and Treynor ratios combine risk and return performance in a single value, though each of them is slightly different. These performance measures are based on security market line and capital market line. (Francis J.C & Kim.D, 2013).

For the evaluation of portfolio performance, we used the following three criteria: annualized return annualized portfolio return, annualized portfolio volatility (standard deviation), portfolio Sharpe ratio.

3.3.1 Portfolio return

The actual return on a portfolio of securities over some specific period of time is the weighted average of the individual stocks in the portfolio, it can be computed using the following formula (Fabozzi & Markowitz, 2011).

$$R_p = w_1 R_1 + w_2 R_2 + \cdots w_G R_G \quad (3.18)$$

Where,

R_p is rate of return of the portfolio over the period of time

R_g is the rate of return of asset g over the period of time

w_g is the weight of security g in the portfolio

G is the number securities in the portfolio

Shortly the above formula can be written as following:

$$R_p = \sum_{g=1}^G w_g R_g \quad (3.19)$$

3.3.2 Portfolio volatility (Standard deviation)

Volatility or standard deviation is the statistical measurement of annual rate of return of investment. Higher standard deviation indicates greater risk and more volatility. Correlation is the measure of degree to which assets move in relation to each other, in our case between stocks.

In order to calculate the portfolio standard deviation, we need to use correlation and co-variance Using correlation we can calculate portfolio standard deviation, with the following formula:

$$\sigma_p = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2} \quad (3.20)$$

Where,

σ_p is portfolio standard deviation

w is asset weight

σ is asset volatility

$\rho_{1,2}$ is correlation between assets 1 and 2

Co-variance measures the joint variability of two random variables and in finance co-variance matrix is often used for portfolio optimization and risk management processes. We can calculate portfolio standard deviation using covariance. We can collapse the above, mentioned portfolio standard deviation formula into a much simpler from using a covariance matrix, with the following formula:

$$\sigma_{portfolio} = \sqrt{w_T \cdot \Sigma \cdot w} \quad (3.21)$$

Where,

$\sigma_{portfolio}$ is portfolio volatility

Σ is a co-variance matrix of returns

W is portfolio weights (w_T is transposed portfolio weights)

$\cdot$ is the dot-multiplication operator

3.3.3 Sharpe ratio

Sharpe ratio was first developed by (Tobin, 1965) as a linear risk-return modeling technique and later extended by (Sharpe, 1966). The Sharpe ratio is the average return earned in excess of the risk-free investment rate (such as U.S. government bond) per unit of total risk or volatility.

According to Modern Portfolio Theory, adding assets to the portfolio, which are weakly correlated with each other (correlation coefficient of less than one) may decrease the risk of the portfolio without losing the return. Such kind of diversifications are made for increasing of portfolio's Sharpe ratio. The greater the Sharpe ratio the more attractive the return with respect to risk. Sharpe ratio can be computed using the formula below:

$$S_p = \frac{\text{Excess return (or risk premium)}}{\text{Total risk}} = \frac{\overline{r_P} - r_f}{\sigma_P} \quad (3.22)$$

Where, $\overline{R_p}$ is average return on portfolio, r_f is average return of risk-free interest rate and σ_p is standard deviation of portfolio.

The average holding period return (HPR) above the risk-free interest rate is known as the risk premium or excess return. It measures the additional return, that is earned by investing in risky assets. Portfolio's risk premium is divided by the standard deviation of returns, σ , which is the measure of portfolio's total risk (Francis J.C & Kim.D, 2013).

Using Sharpe's ratio, the managers can check if a portfolio's excess mean return is sufficient in order to compensate for higher risk taken by investing in risky asset portfolio rather than the market portfolio. Using Sharpe ratio, we can rank and compare performance of investment portfolios with different risk classes, which have different average return and risk.

3.3.4 T-test

Eventually, to test our above, mentioned hypothesis, in other words, to compare the performance of portfolios constructed with historical information and forward-looking information, we conduct t-test. T-test is a statistically significant test used testing hypothesis to compare two group means and the probability differences of the samples (Fraser, 2016).

4 FINDINGS

This chapter covers the results and interpretations of results of the thesis. The first section shows the relationship between implied and historical volatilities and stock total returns. The second section presents analysis based on implied and historical information (implied and historical correlation matrixes). The third section presents evaluation of stock performance, while the fourth and fifth sections show the summary of portfolio weights and portfolio performance.

4.1 Historical and Implied Volatilities

As indicated in the charts below, for every stock, the historical volatility is flatter than implied volatility, which means that implied volatility fluctuates more and shows a similar pattern adjusted returns of the stocks.

The stock volatility impacts the value of call and put options positively. It means that the higher the volatility, the higher is the chance to make a profit for the buyer, thus the higher the value of option. This is the reason why options are more valuable during volatile times.

The full purpose of understanding the concept of option implied volatility is to get some important insights about how to trade options. Implied volatility measures the implied risk, which traders assign to the price of option. Generally, implied volatility is an indicator of how much risk traders are attaching to the stock. Higher implied volatility means that the market is becoming riskier and vice versa. Traders usually look at implied volatility to measure the market direction. Implied volatility is purely about expected volatility. Increasing implied volatility means that the volatility expectations are going up.

Historical volatility is calculated using the 3 years rolling -window method. As reported in the charts, historical volatility line seems flat for most of the stocks. For equity markets volatility is a risk measure. However, both call and put options benefit from greater volatility.

According to implied volatilities, we can say that the riskiest stocks are AKBNK, YKBNK and PETKM. On the other hand, KRDM, PGSUS and THYAO have almost the same historical and implied volatilities. Both lines are close to each other. These stocks are

fairly stable (data with respect to implied and historical volatilities as well as stock returns are presented in appendix).

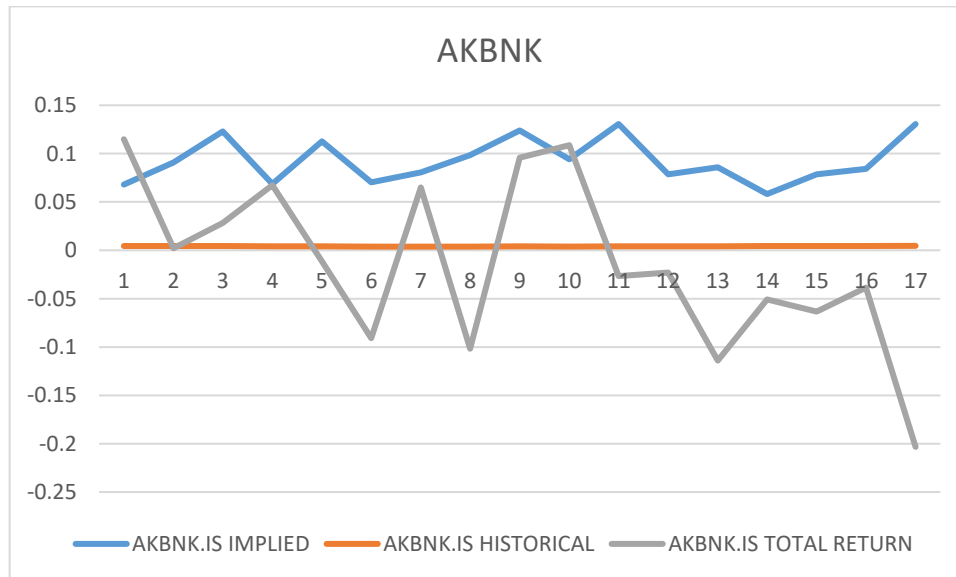


Figure 4. 1 The relationship between implied and historical volatilities and stock total return (AKBNK).

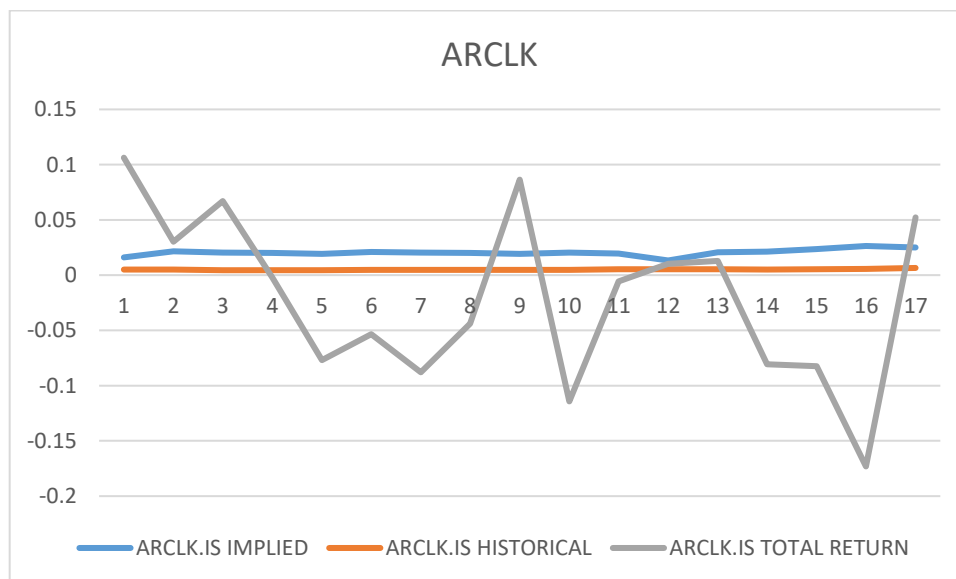


Figure 4. 2 The relationship between implied and historical volatilities and stock total return (ARCLK).

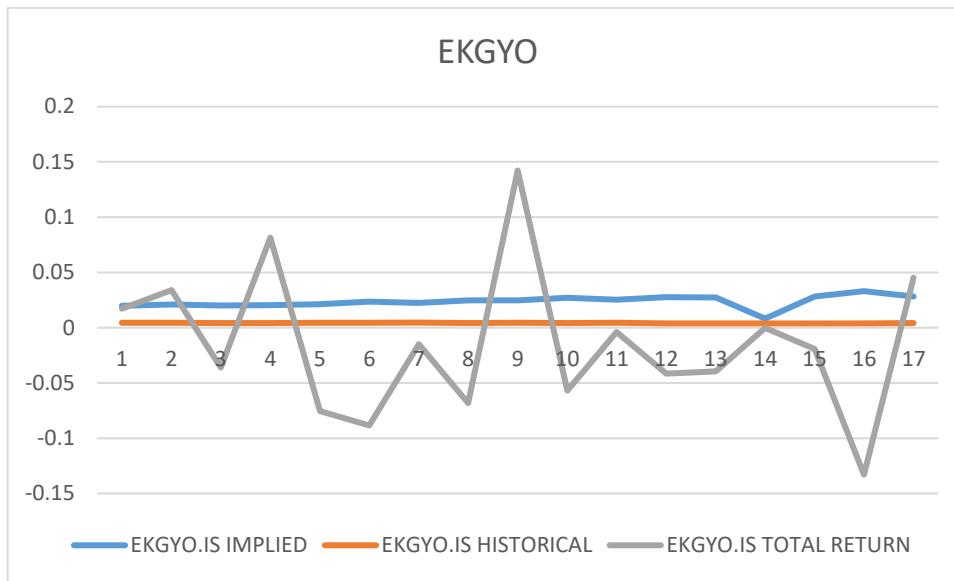


Figure 4. 3 The relationship between implied and historical volatilities and stock total return (EKGYO).

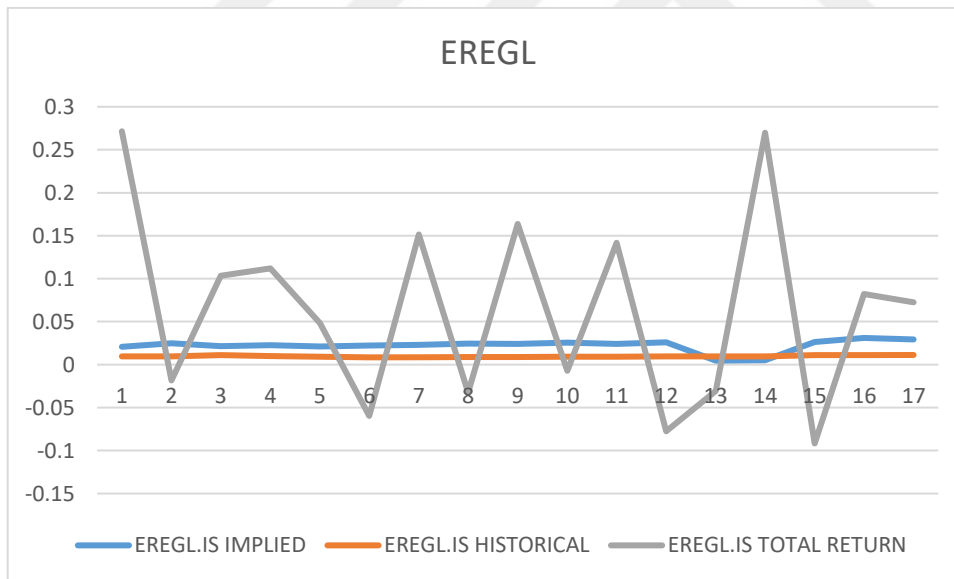


Figure 4. 4 The relationship between implied and historical volatilities and stock total return (EREGL).

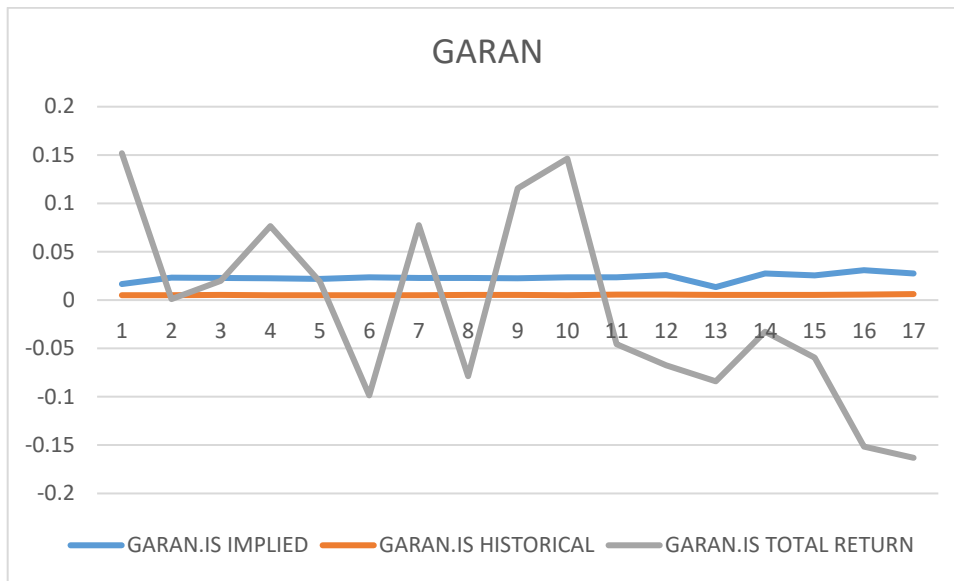


Figure 4. 5 The relationship between implied and historical volatilities and stock total return (GARAN).

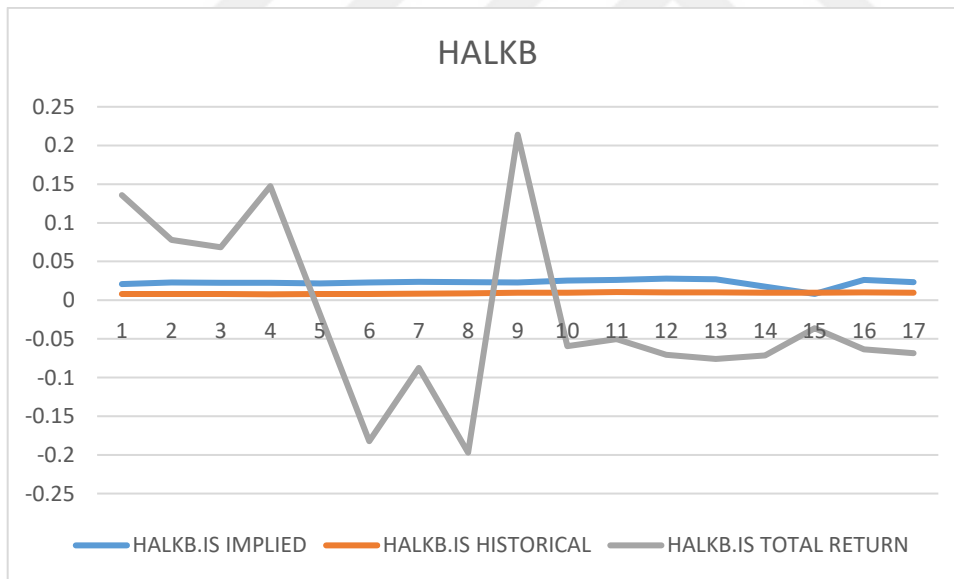


Figure 4. 6 The relationship between implied and historical volatilities and stock total return (HALKB).

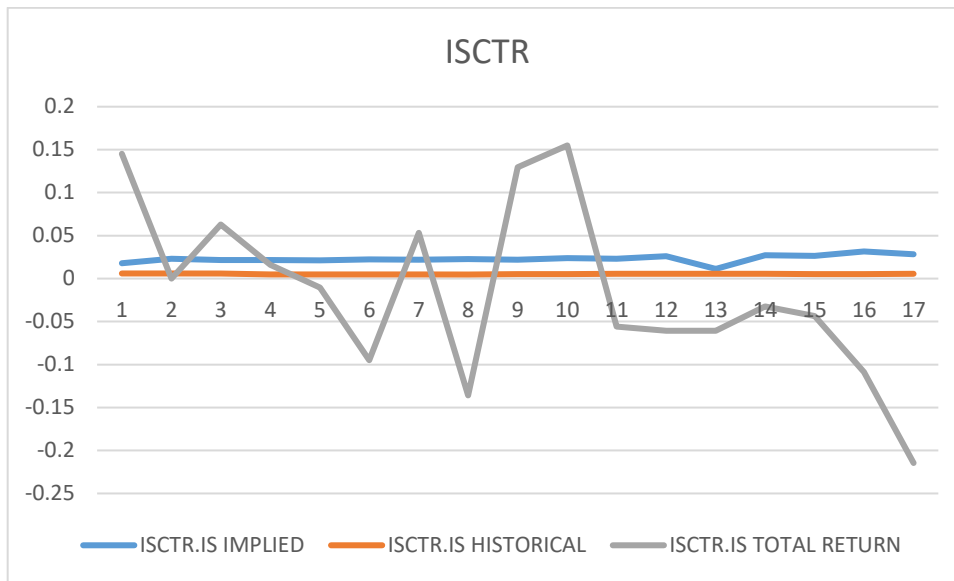


Figure 4. 7 The relationship between implied and historical volatilities and stock total return (ISCTR).

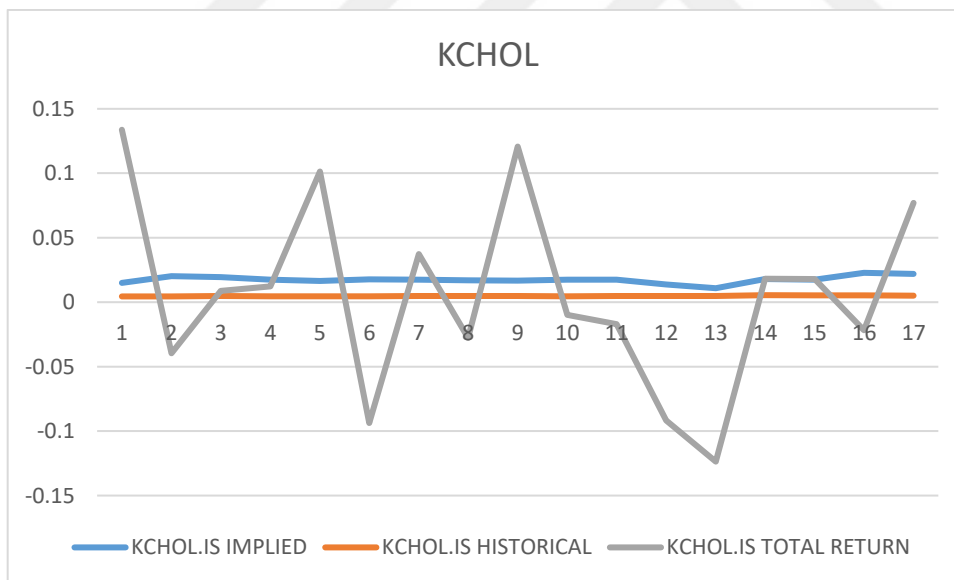


Figure 4. 8 The relationship between implied and historical volatilities and stock total return (KCHOL).

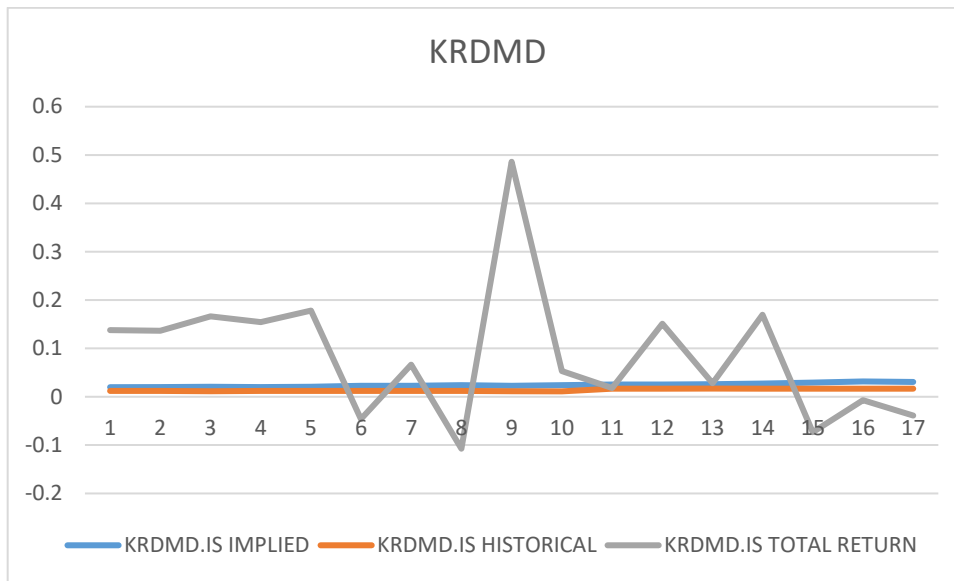


Figure 4. 9 The relationship between implied and historical volatilities and stock total return (KRDMD).

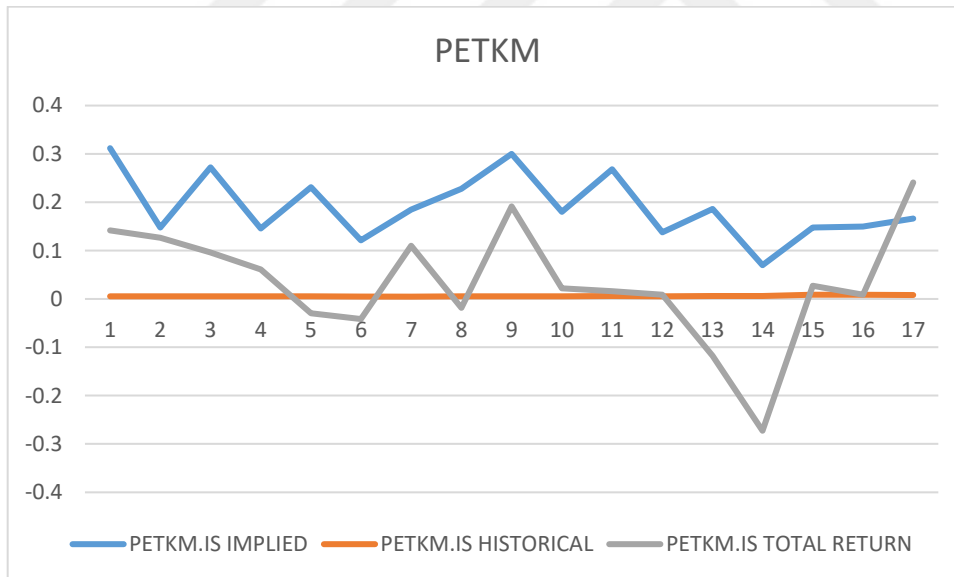


Figure 4. 10 The relationship between implied and historical volatilities and stock total return (PETKM).

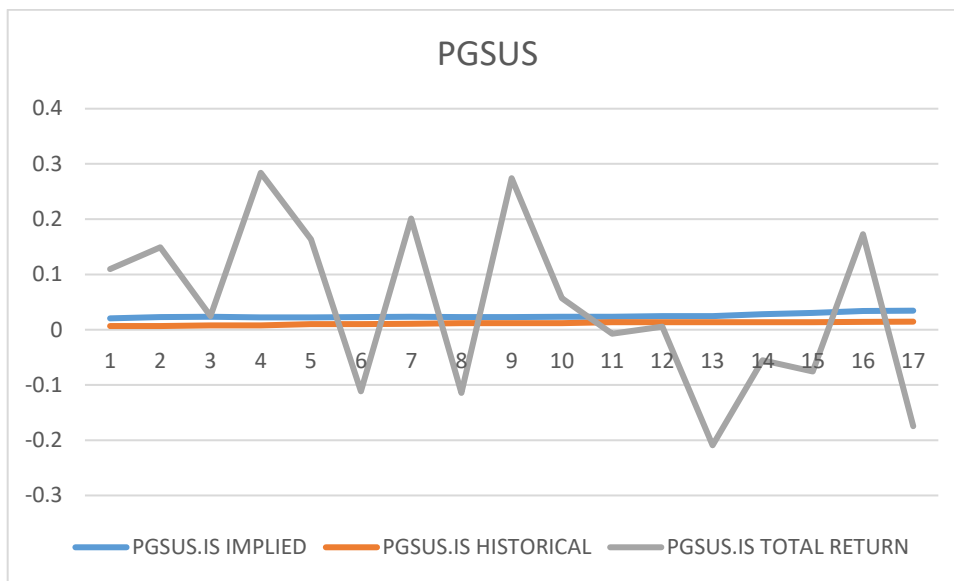


Figure 4. 11 The relationship between implied and historical volatilities and stock total return (PGSUS).

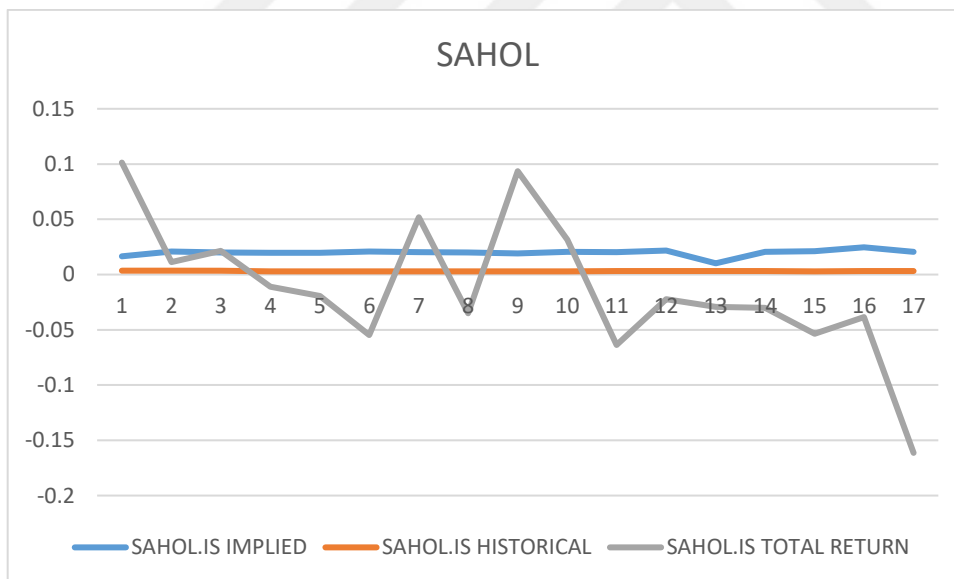


Figure 4. 12 The relationship between implied and historical volatilities and stock total return (SAHOL).

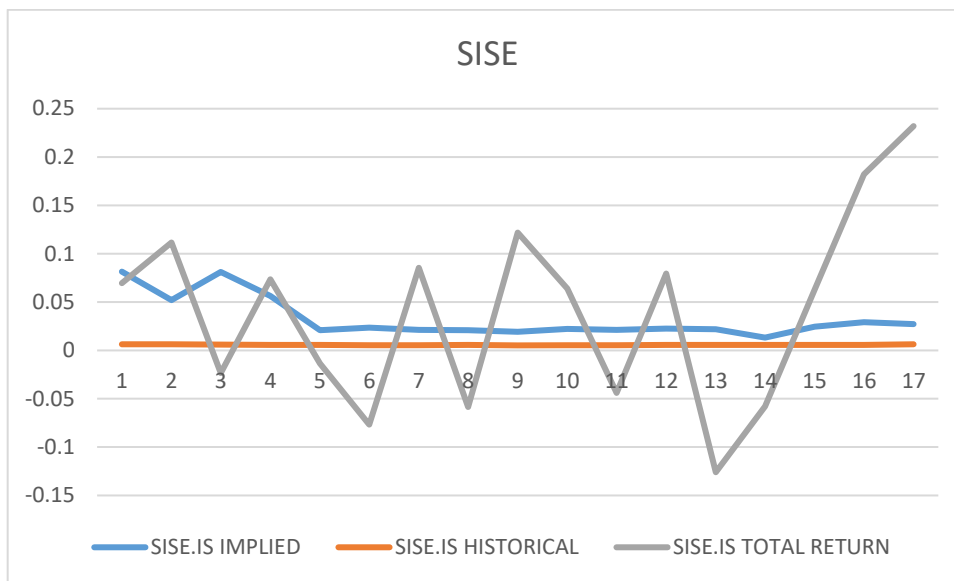


Figure 4. 13 The relationship between implied and historical volatilities and stock total return (SISE).

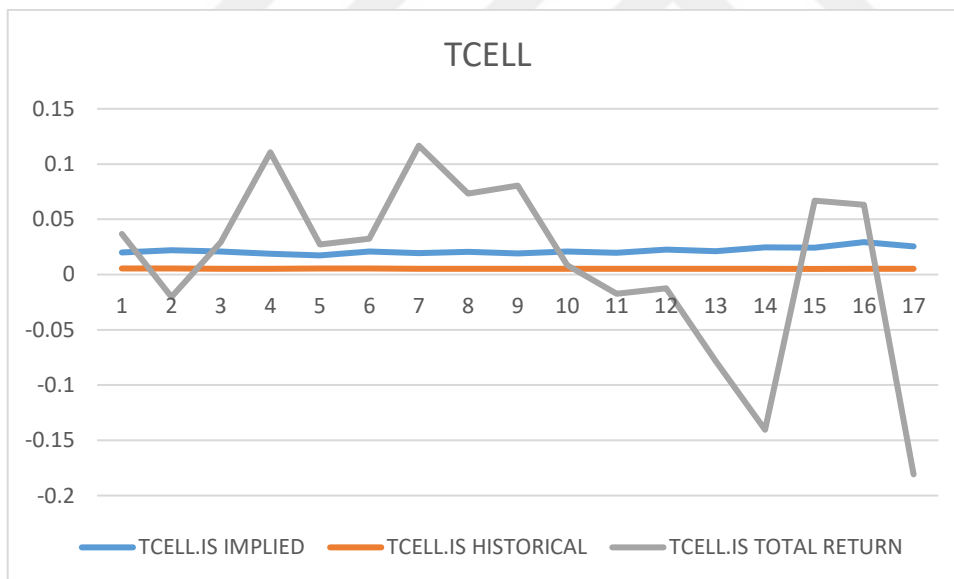


Figure 4. 14 The relationship between implied and historical volatilities and stock total return (TCELL).

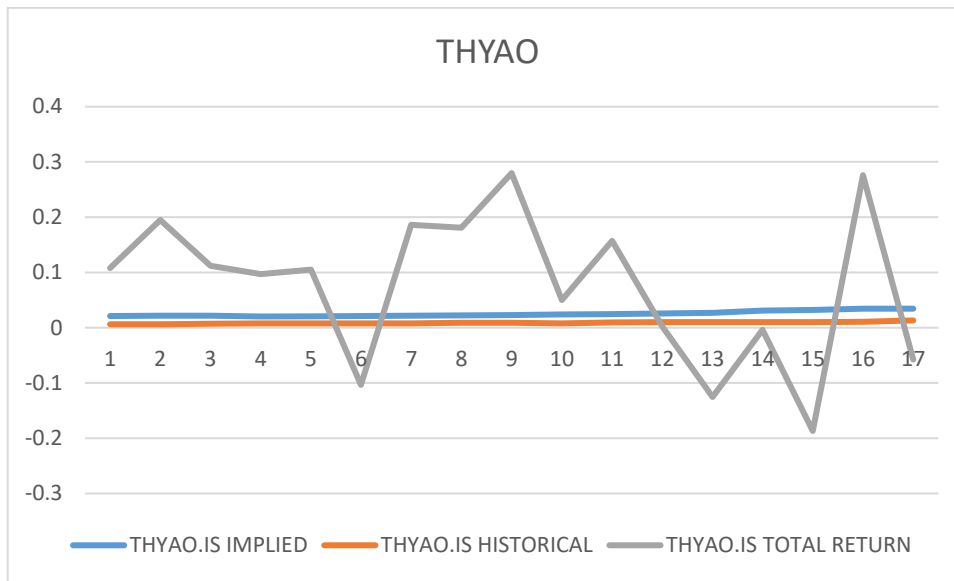


Figure 4. 15 The relationship between implied and historical volatilities stock total return (THYAO).

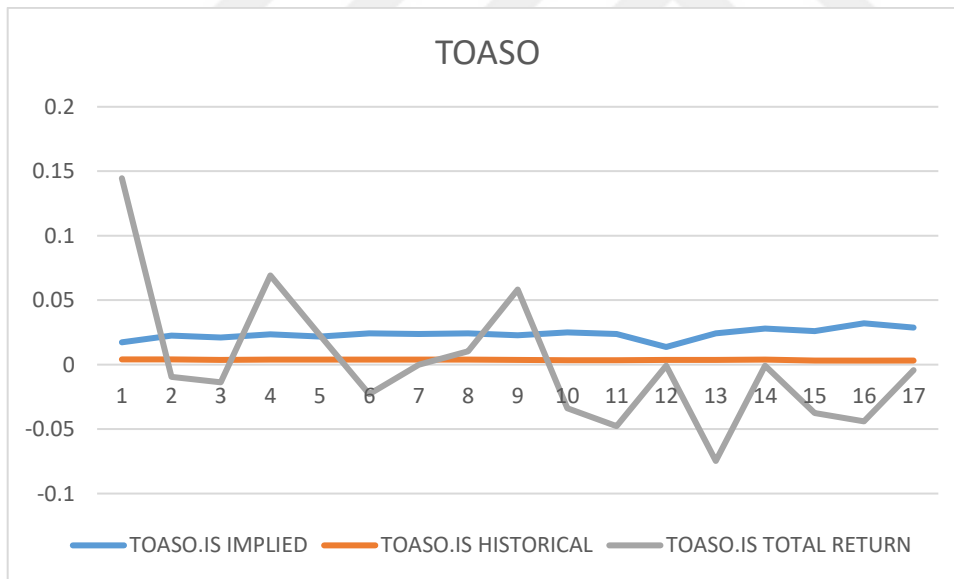


Figure 4. 16 The relationship between implied and historical volatilities and stock total return (TOASO).

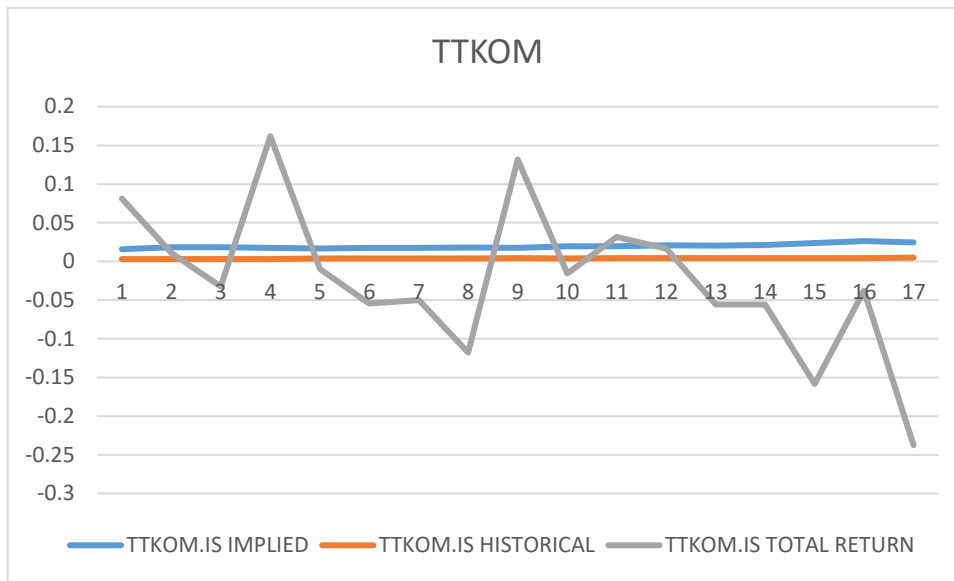


Figure 4. 17 The relationship between implied and historical volatilities and stock total return (TTKOM).

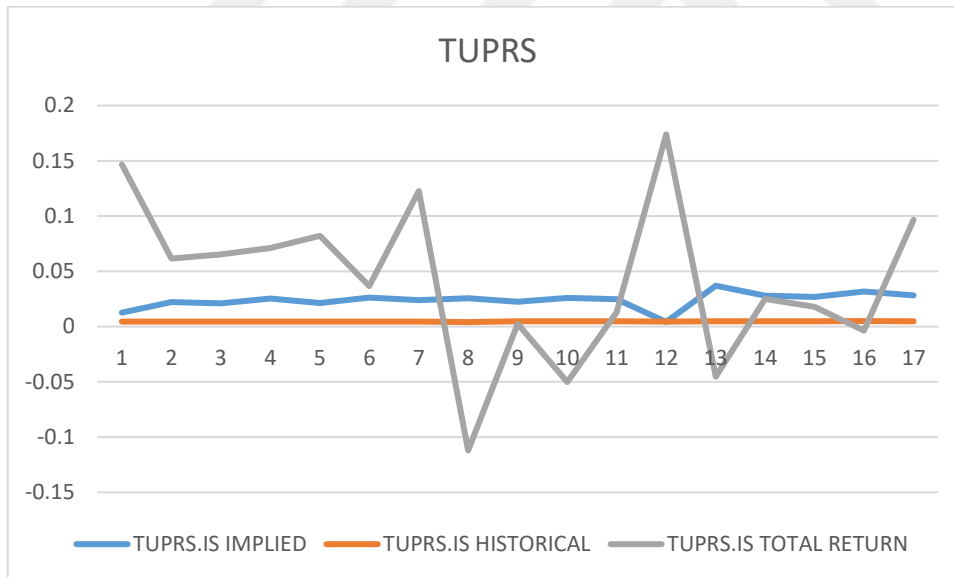


Figure 4. 18 The relationship between implied and historical volatilities and stock total return (TUPRS).

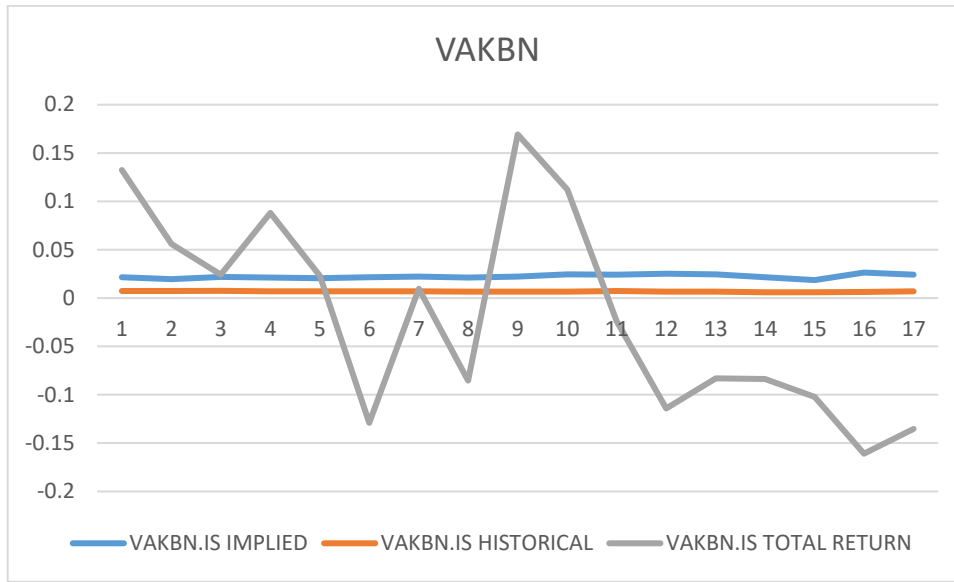


Figure 4. 19 The relationship between implied and historical volatilities and stock total return (VAKBN).

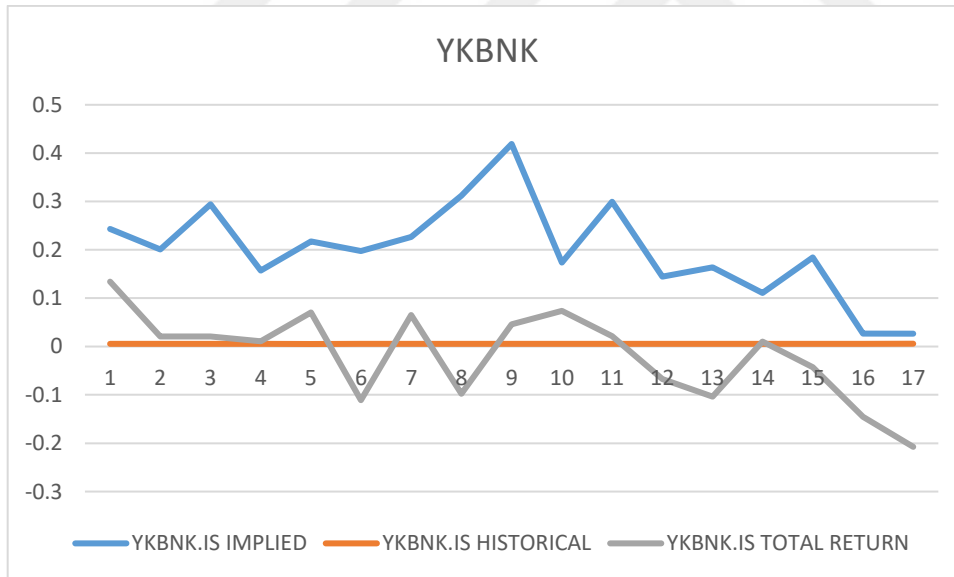


Figure 4. 20 The relationship between implied and historical volatilities and stock total return (YKBNK).

4.2 Historical Correlations

Correlation is a statistical measure of relationship of two variables. In our research, we have 20 stocks and we consider each stock's correlations with other stocks. The relationship of per stock with all other stocks are as the following:

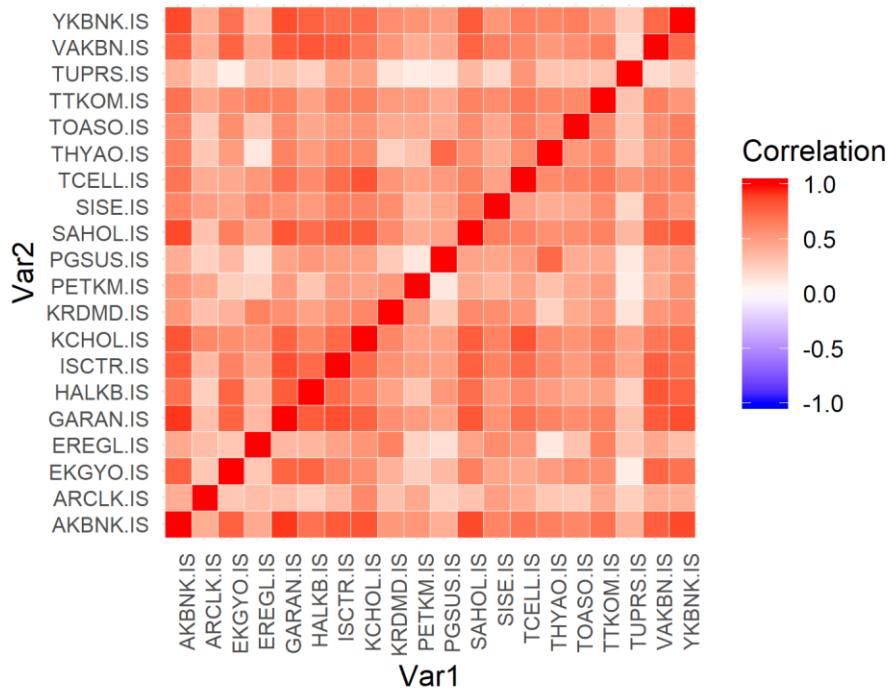


Figure 4. 21 The implied correlation matrix, between stock adjusted returns (Heat map).

- AKBNK.IS has the lowest positive correlation with TUPRS.IS and PGSUS.IS, (0.409294 and 0.418239), and has the highest positive correlation with YKBNK.IS and GARAN.IS (0.865758 and 0.920365).
- ARCLK.IS has the lowest positive correlation with PGSUS.IS and HALKB.IS, (0.249732 and 0.0.2572), and has the highest positive correlation with SISE.IS and KCHOL.IS (0.499434 and 0.605018).
- EKGYO.IS has the lowest positive correlation with TUPRS.IS and PETKM.IS (0.094682 and 0.261217), and has the highest correlation with VAKBN.IS and AKBNK.IS (0.765747 and 0.778438).
- EREGL.IS has the lowest positive correlation THYAO.IS and PGSUS.IS (0.122076 and 0.166356) and the highest positive correlation with KRDMD.IS and TTKOM.IS (0.636876 and 0.638668).

- GARAN.IS has the lowest correlation with TUPRS.IS and ARCLK.IS (0.326065 and 0.332568) and the highest positive correlation with YKBNK.IS and AKBNK.IS (0.850539 and 0.920365).
- HALKB.IS has the lowest correlation with TUPRS.IS and ARCLK.IS (0.24569 and 0.2572), and has the highest correlation with GARAN.IS and VAKBN.IS (0.797995 and 0.81843).
- ISCTR.IS has the lowest positive correlation with ARCLK.IS and TUPRS.IS (0.363373 and 0.46631), and has the highest correlation with AKBNK.IS and GARAN.IS (0.80061 and 0.844211).
- KCHOL.IS has lowest positive correlation with PETKM.IS and TUPRS.IS (0.482676 and 0.485753) and has the highest correlation with AKBNK.IS and TCELL.IS (0.828906 and 0.830927).
- KRDMD.IS has the lowest correlation with TUPRS.IS and THYAO.IS (0.152279 and 0.245183), and has the highest correlation with KCHOL.IS and EREGL.IS (0.612896 and 0.705496).
- PETKM.IS has the lowest correlation with TUPRS.IS and PGSUS.IS (0.101229 and 0.1254), and the highest correlation with AKBNK.IS and TUPRS.IS (0.54018 and 0.550839).
- PGSUS.IS has the lowest correlation with TUPRS.IS and PETKM.IS (0.12205 and 0.1254) and the highest correlation with HALKB.IS and THYAO.IS (0.531596 and 0.746089).
- SAHOL.IS has the lowest correlation with ARCLK.IS and TUPRS.IS (0.317981 and 0.372048), and has the highest correlation with GARAN.IS and AKBNK.IS (0.817031 and 0.865264).
- SISE.IS has the lowest correlation with TUPRS.IS and PETKM.IS (0.209242 and 0.377284), and has the highest correlation with VAKBN.IS and SAHOL.IS (0.641646 and 0.654696).
- TCELL.IS has the lowest correlation with ARCLK.IS and EKGYO.IS (0.431604 and 0.450934) and has the highest correlation with ISCTR.IS and KCHOL.IS (0.731675 and 0.830927).

- THYAO.IS has the lowest correlation with EREGL.IS and KRDMD.IS (0.122076 and 0.245183), and the highest correlation with AKBNK.IS and PGSUS.IS (0.647053 and 0.746089).
- TOASO.IS has the lowest correlation with ARCLK.IS and TUPRS.IS (0.27407 and 0.314875) and has the highest correlation with TCELL.IS and YKBNK.IS (0.628133 and 0.65352).
- TTKOM.IS has the lowest correlation with TUPRS.IS and PGSUS.IS (0.316638 and 0.439006) and has the highest correlation with TCELL.IS and AKBNK.IS (0.675538 and 0.710175).
- TUPRS.IS has the lowest positive correlation with EKGYO.IS and PETKM.IS (0.094682 and 0.101229) and has the highest correlation with KCHOL.IS and TCELL.IS (0.485753 and 0.545106).
- VAKBN.IS has the lowest correlation with TUPRS.IS and ARCLK.IS (0.193777 and 0.419738) and the highest correlation with GARAN.IS and HALKB.IS (0.800962 and 0.81843).
- YKBNK.IS has the lowest correlation with TUPRS.IS and EREGL.IS (0.260332 and 0.340138) and the highest correlation with GARAN.IS and AKBNK.IS (0.850539 and 0.865758).

4.3 Implied Correlation

As we have already mentioned implied correlation was calculated by Buss&Vilkov method, (the data of results is in the appendix). The implied correlation of per stock with all other stocks are as the following:

- AKBNK.IS has the lowest negative correlation with TUPRS.IS and PGSUS.IS (-0.18141 and -0.16352), the lowest positive correlation is with KRDMD.IS and PETKM.IS (0.069473 and 0.08036), and the highest correlation is with YKBNK.IS and GARAN.IS (0.731516 and 0.84073).

- ARCLK.IS has the lowest negative correlation with PGSUS.IS and HALKB.IS (-0.500536 and -0.485600), and the highest positive correlation with KCHOL.IS 0.210036.
- EKGYO.IS has the lowest negative correlation with TUPRS.IS and PETKM.IS (-0.810635 and -0.477567) and the lowest positive correlation with THYAO.IS and KCHOL.IS (0.027165 and 0.152399), and has the highest correlation with VAKBN.IS and AKBNK.IS (0.531495 and 0.556876).
- EREGL.IS has the lowest negative correlation with THYAO.IS and PGSUS.IS (-0.755885 and -0.667288), while TCELL.IS and KCHOL.IS have the lowest positive correlation of (0.062933 and 0.091282), and KRDM.D.IS and TTKOM.IS have the highest positive correlation of (0.273752 and 0.277336).
- GARAN.IS has the lowest negative correlation with TUPRS.IS and ARCLK.IS (-0.347871 and -0.334863), the lowest positive correlation with PETKM.IS and SISE.IS (0.028579 and 0.118852) and the highest positive correlation with YKBNK.IS and AKBNK.IS (0.806464 and 0.819711).
- HALKB.IS has the lowest negative correlation with TUPRS.IS and ARCLK.IS (-0.508620 and -0.485601), while the lowest positive correlation is with THYAO.IS and SISE.IS (0.022718 and 0.038520), and the highest positive correlation is with GARAN.IS and VAKBN.IS (0.595989 and 0.636860).
- ISCTR.IS has the lowest negative correlation with ARCLK.IS and TUPRS.IS (-0.273254 and -0.067380), has the lowest positive correlation with PETKM.IS and TOASO.IS (0.007787 and 0.041978), and has the highest positive correlation with AKBNK.IS and GARAN.IS (0.601220 and 0.688421).
- KCHOL.IS has lowest negative correlation with PETKM.IS and TUPRS.IS (-0.034645 and -0.028495), has lowest positive correlation with EREGL.IS and TOASO.IS (0.091282 and 0.109032) and has the highest positive correlation with AKBNK.IS and TCELL.IS (0.0657812 and 0.661854).
- KRDM.D.IS has the lowest negative correlation with TUPRS.IS and THYAO.IS (-0.695443 and -0.509633), has the lowest positive correlation with TTKOM.IS and

PETKM.IS (0.047593 and 0.052338), and has the highest positive correlation with KCHOL.IS and EREGL.IS (0.225792 and 0.273752).

- PETKM.IS has the lowest negative correlation with TUPRS.IS and PGSUS.IS (-0.797542 and -0.749200), has the lowest positive correlation with ISCTR.IS and TTKOM.IS (0.007787 and 0.026722), and has the highest positive correlation with AKBNK.IS and YKBNK.IS (0.080360 and 0.101677).
- PGSUS.IS has the lowest negative correlation with TUPRS.IS and PETKM.IS (-0.7559 and -0.7492), has the lowest positive correlation with YKBNK.IS and TCELL.IS (0.026829 and 0.048405), and has the highest positive correlation with HALKB.IS and THYAO.IS (0.063193 and 0.492178).
- SAHOL.IS has the lowest negative correlation with ARCLK.IS and TUPRS.IS (-0.36404 and -0.2559), has the lowest positive correlation with THYAO.IS and TOASO.IS (0.133603 and 0.169289), and has the highest positive correlation with GARAN.IS and AKBNK.IS (0.634063 and 0.730528).
- SISE.IS has the lowest negative correlation with TUPRS.IS and PETKM.IS (-0.58152 and -0.24543), has the lowest positive correlation with HALKB.IS and YKBNK.IS (0.03852 and 0.083282), and has the highest positive correlation with VAKBN.IS and SAHOL.IS (0.283293 and 0.309392).
- TCELL.IS has the lowest negative correlation with ARCLK.IS and EKGYO.IS (-0.13679 and -0.09813), has the lowest positive correlation with PGSUS.IS and EREGL.IS (0.048405 and 0.062933), and has the highest positive correlation with ISCTR.IS and KCHOL.IS (0.46335 and 0.661854).
- THYAO.IS has the lowest negative correlation with EREGL.IS and KRDMD.IS (-0.75585 and -0.50963), has the lowest positive correlation with HALKB.IS and EKGYO.IS (0.022718 and 0.027165), and has the highest positive correlation with AKBNK.IS and PGSUS.IS (0.294107 and 0.492178).
- TOASO.IS has the lowest negative correlation with ARCLK.IS and TUPRS.IS (-0.45186 and -0.37025), has the lowest positive correlation with ISCTR.IS and THYAO.IS (0.041978 and 0.062764), and has the highest positive correlation with TCELL.IS and YKBNK.IS (0.256267 and 0.307041).

- TTKOM.IS has the lowest negative correlation with TUPRS.IS and PGSUS.IS (-0.36672 and -0.12199), has the lowest positive correlation with PETKM.IS and KRDMD.IS (0.026722 and 0.047593), and has the highest positive correlation with TCELL.IS and AKBNK.IS (0.351076 and 0.42035).
- TUPRS.IS has the lowest negative correlation with EKGYO.IS and PETKM.IS (-0.81064 and -0.79754) and has the highest positive correlation with TCELL.IS 0.090211.
- VAKBN.IS has the lowest negative correlation with TUPRS.IS and ARCLK.IS (-0.61245 and -0.16052), has the lowest positive correlation with THYAO.IS and KRDMD.IS (0.053247 and 0.074821), and has the highest positive correlation with GARAN.IS and HALKB.IS (0.601924 and 0.63686).
- YKBNK.IS has the lowest negative correlation with TUPRS.IS and EREGL.IS (-0.47934 and -0.31972), has the lowest positive correlation with PGSUS.IS and SISE.IS (0.026838 and 0.0832282), and the highest correlation with GARAN.IS and AKBNK.IS (0.701078 and 0.731516).

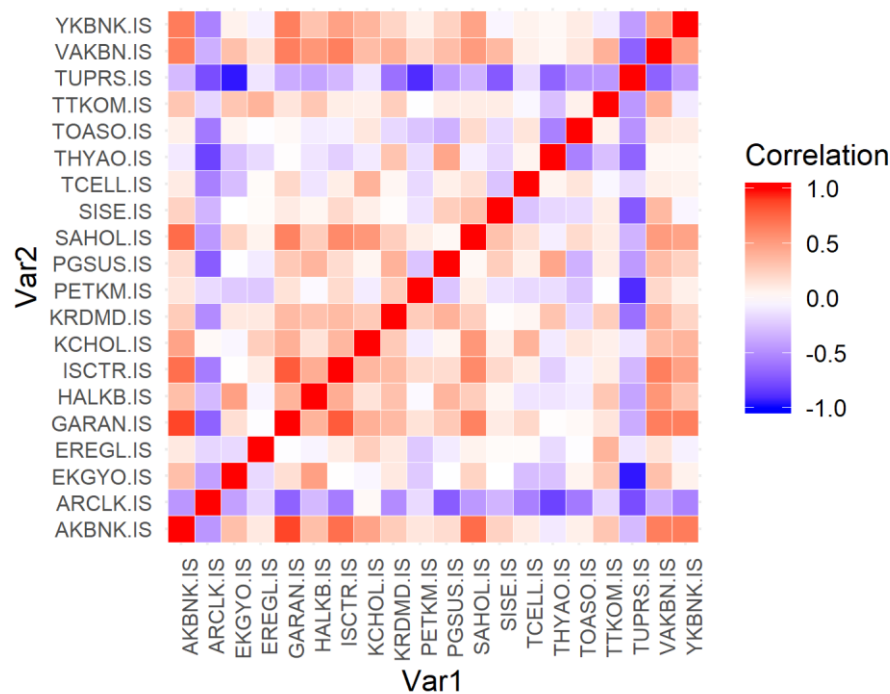


Figure 4-1 The implied correlation matrix, between stock adjusted returns (Heat map).

4.4 Stock Performance

We present information about annualized return, annualized standard deviation and annualized Sharpe ratio of each individual stocks. The three companies with the lowest annualized return are TTKOM with -0.276, ARCLK with -0.251, and HALKB with -0.238, and the three companies with the highest annualized return are KRDMD with 1.039, THYAO with 0.898 and EREGL with 0.776.

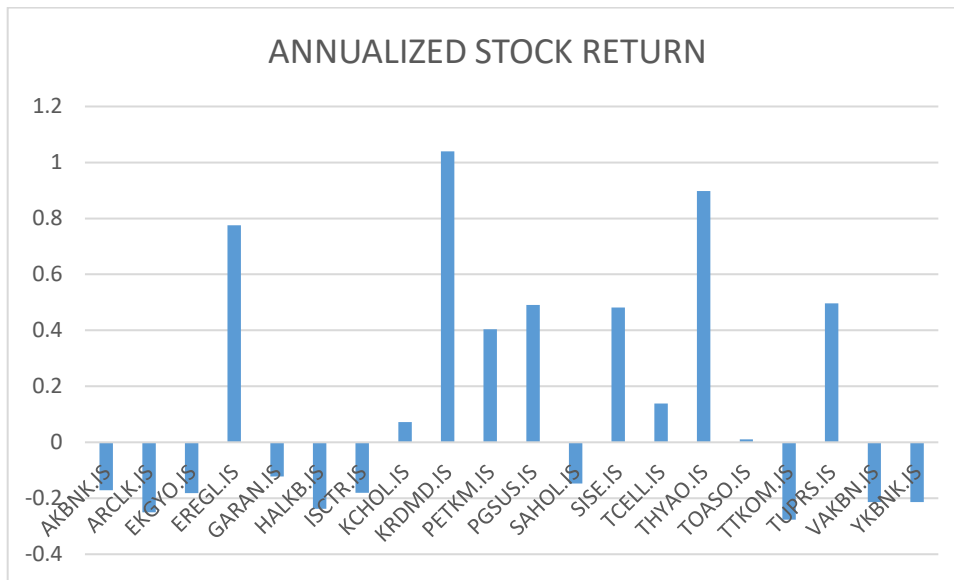


Figure 4. 22 Annualized stock return

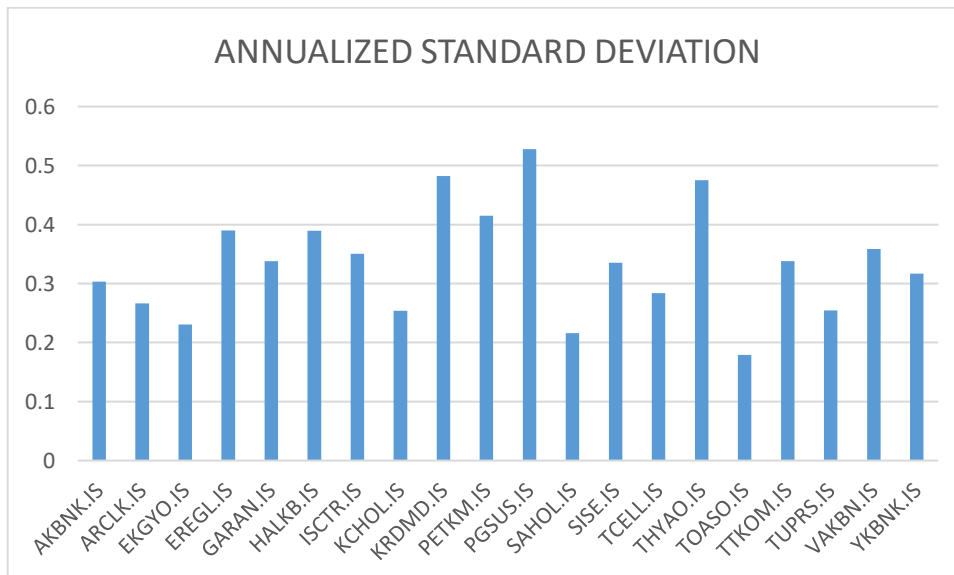


Figure 4. 23 Annualized standard deviation

The next performance measure is annualized standard deviation. Standard deviation is used for measuring stock volatility. The more an individual stock's returns fluctuates from its mean return, the more volatile the stock. The higher the volatility, the riskier the stock, and vice versa.

In our sample, stocks with the highest annualized standard deviation are the following three companies: PGSUS (Pegasus), KRDM (Kardemir) and THYAO (Turkish Airlines), which are respectively 0.528, 0.482 and 0.475.

The stocks with the lowest annualized standard deviation are TOASO (Tofas), SAHOL (Sabanci Holding) and EKGYO (Emlak Konut) which are respectively 0.179, 0.216 and 0.230.

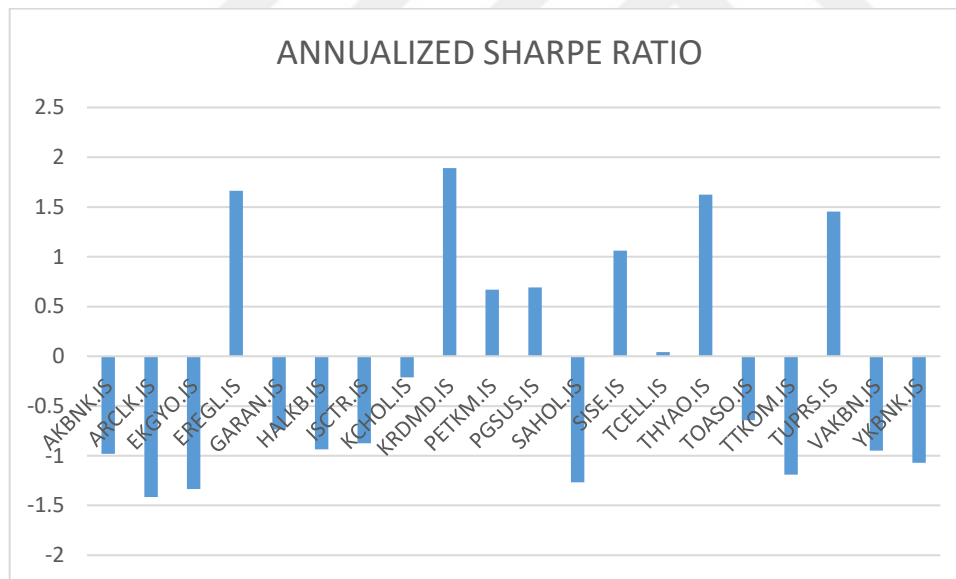


Figure 4. 24 Annualized Sharpe ratio

The last measure of stock performance is the Sharpe ratio. Simply, the Sharpe ratio is a return per unit of risk, which is characterized by variance. The higher the Sharpe ratio, the better is the combination of return and risk.

In our sample, the following 3 stocks have the highest Sharpe ratios: KRDM (Kardemir), EREGL ((Eregli) and THYAO (Turkish Airlines) which are respectively 1.892,

1.664 and 1.623. The stocks with the lowest Sharpe ratio are: ARCLK (Arcelik), EKGYO (Emlak Konut) and SAHOL (Sabanci Holding), which are respectively -1.416, -1.336 and -1.266.

4.5 Optimized Portfolio Weights

The technique of portfolio analysis does not indicate the amount of money to be invested in each security. Preferably, the proportion of each security the optimum portfolio should presume. The proportions or weights is symbolized as w_i s. Thus, w_i is the of portfolio's total value which should be invested in i security.

Assuming the all funds in the portfolio are accounted for, the following constraint is placed for all portfolios:

$$\sum_{i=1}^n w_i = 1$$

In other words, the total sum of investment in portfolio is 100%.

The graphs below show weights of portfolios developed by different methods. We can see that weights obtained by using implied volatility show much greater variation of weights throughout the time. Especially, the minimum variance implied weights changes throughout the time, which indicates that it is dynamically adjusting the weights based on how the market perceives future volatility. The flat graphs of portfolios based on historical information reflect the slight changes in the mean returns and the sample covariance matrix from month to month, thus the sample weights are the nearly the same. When option-implied information is used in portfolio optimization, implied volatility changes more from one month to another and as a result the weights of optimal portfolios fluctuate in time.

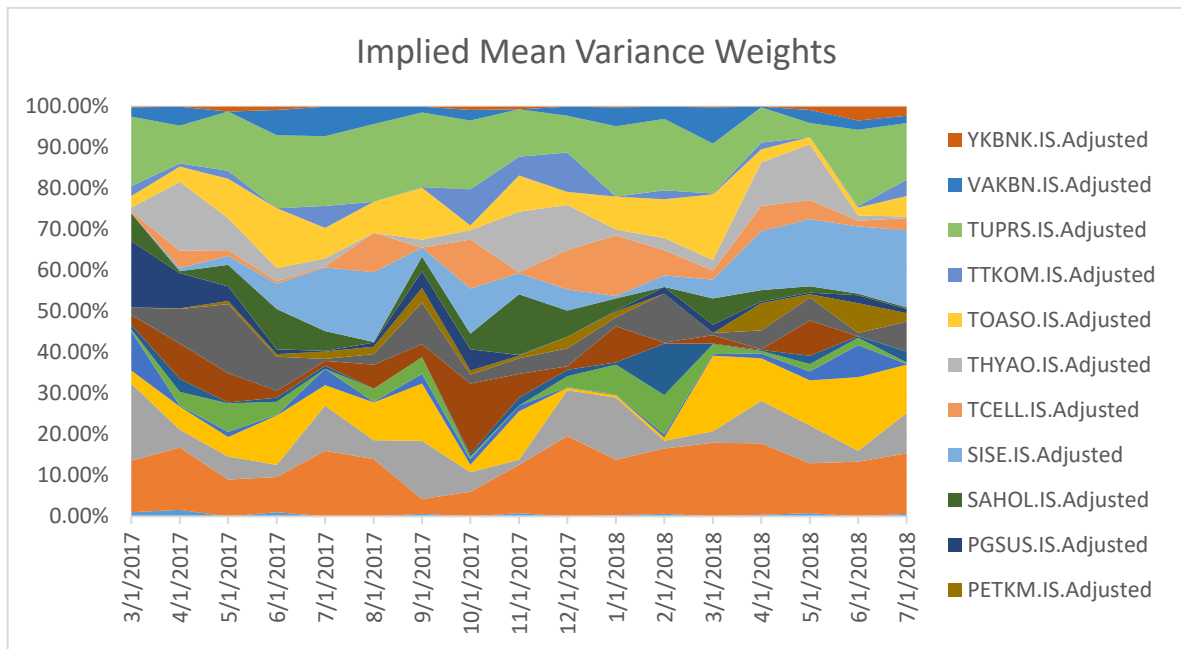


Figure 4. 25 Implied mean variance weights allocation

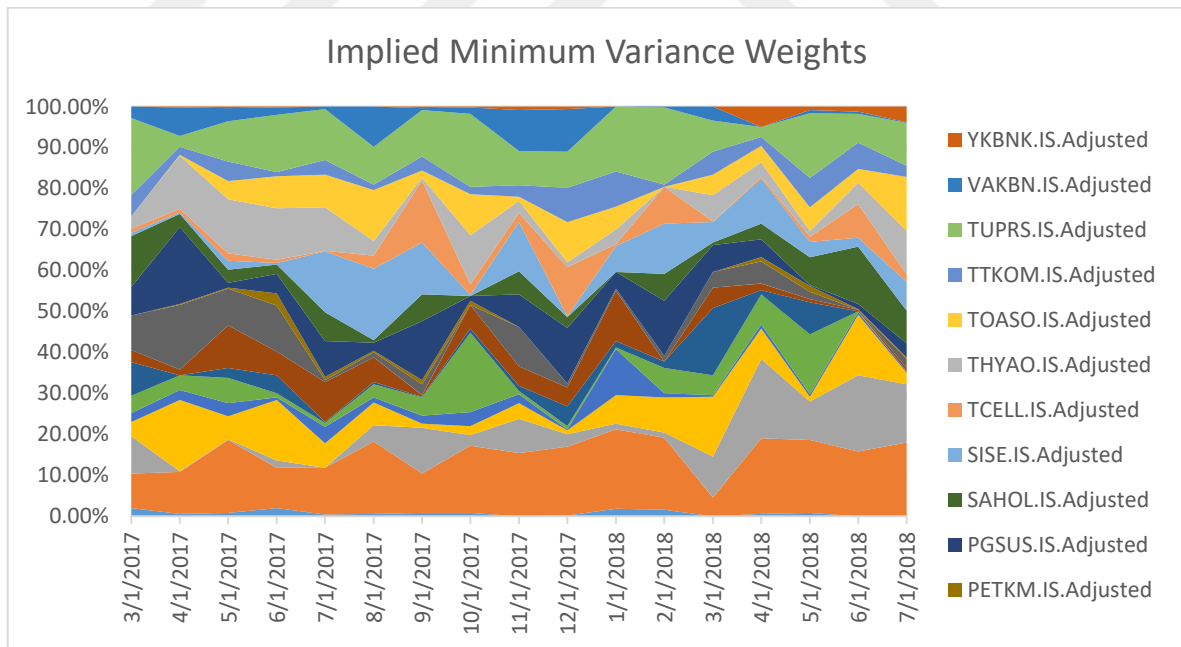


Figure 4. 26 Implied minimum variance weights allocation

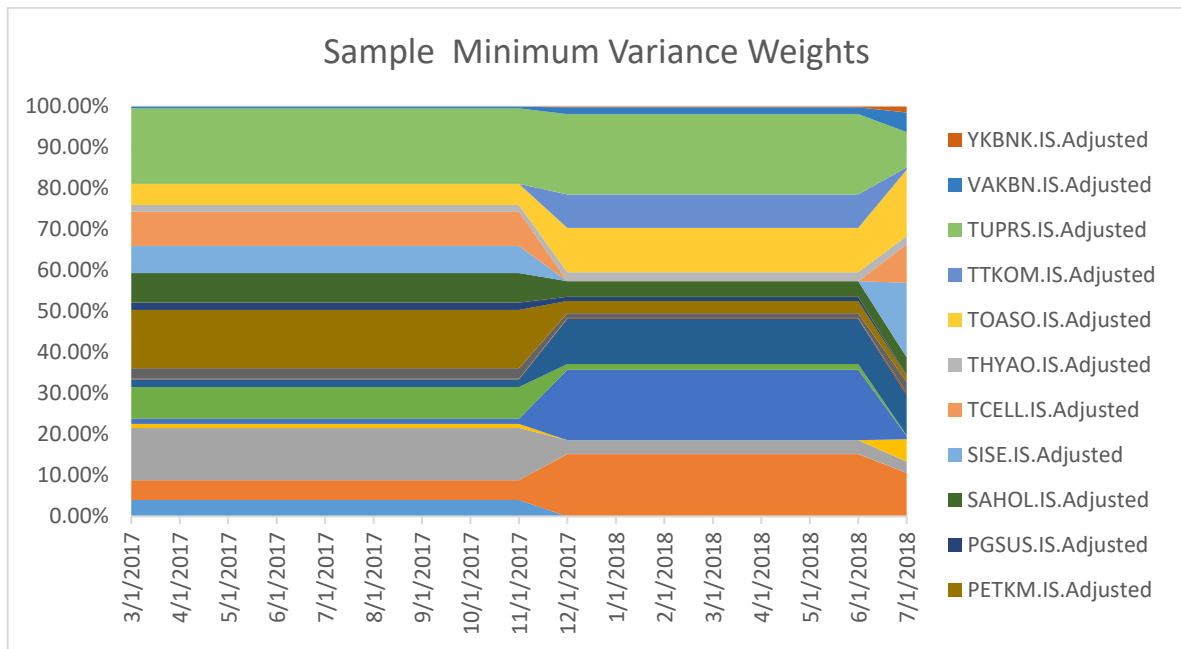


Figure 4. 27 Sample (historical) minimum variance weights allocation

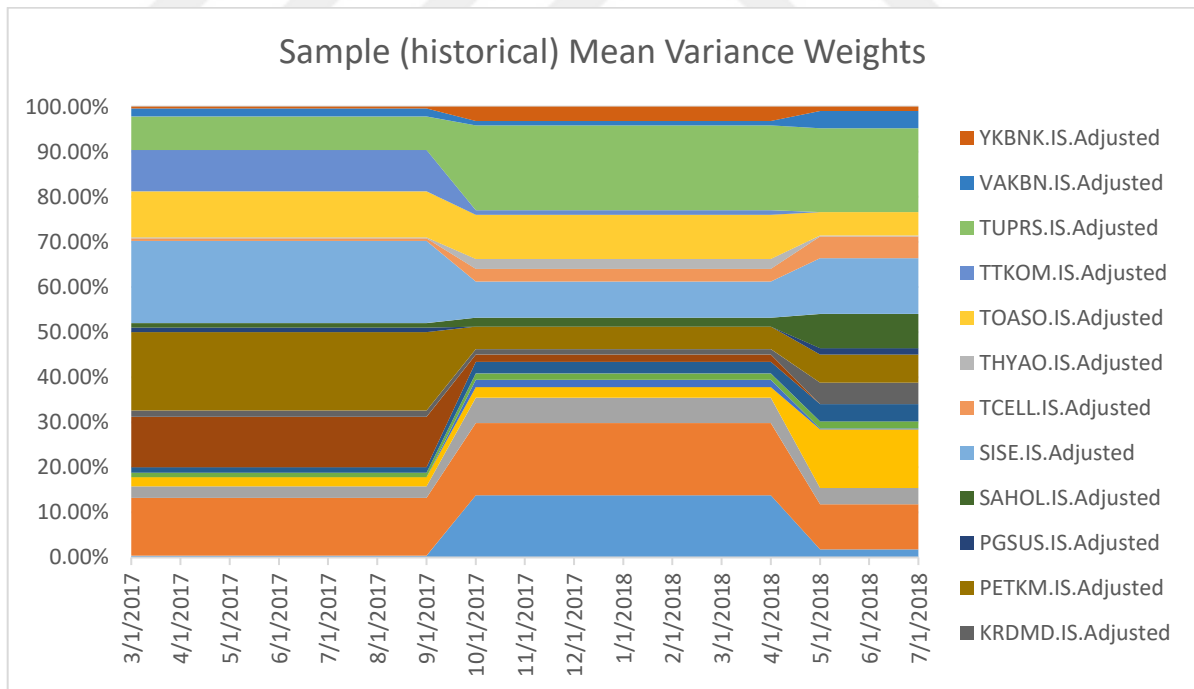


Figure 4. 28 Sample (historical) mean variance weights allocation

4.6 Portfolio Performances

This chart shows the cumulative returns of 5 different portfolios:

- Benchmark portfolio,
- Minimum Variance Sample (historical) portfolio,
- Minimum Variance Implied portfolio,
- Mean Variance Sample (historical) portfolio,
- Mean Variance Implied portfolio.

We can see that Minimum Variance Implied and Mean Variance Implied portfolios provide the highest level of cumulative return, especially Minimum Variance Implied, which has the highest return among the five portfolios during the whole sample period. The increase in the cumulative returns is noticeable beginning from November 2017, from about 0.42% to 0.60%. From April 2017 to November 2017, the benchmark portfolio had the least variance. From November 2017 to the end of sample period, the minimum variance sample (historical) portfolio had the least variance.

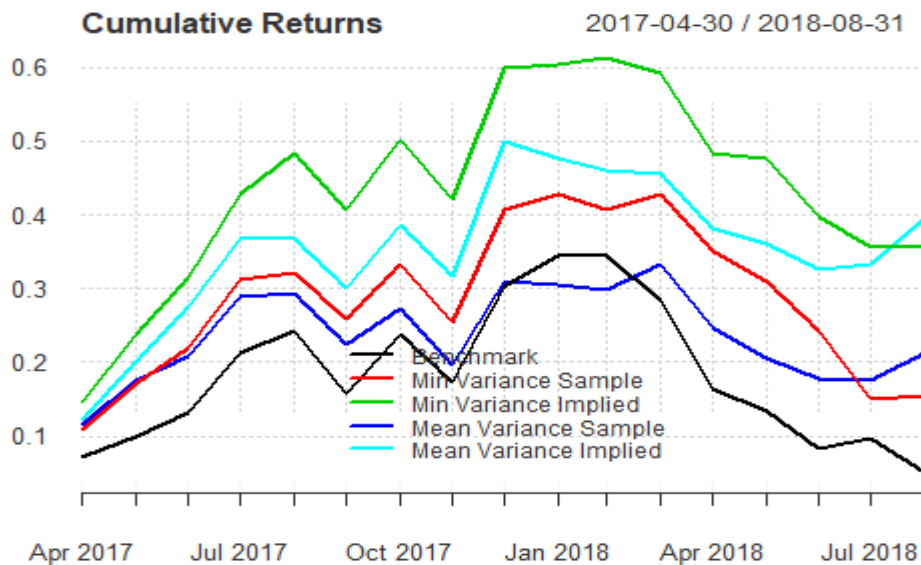


Figure 4. 29 Cumulative returns of 5 different portfolios (benchmark, minimum variance sample (historic), minimum variance implied, mean variance sample (historic) and mean variance implied)

In the Table 4.1 below, we make the comparison of performance of different portfolios. We can see that the Minimum Variance Implied portfolio has the highest annualized standard deviation (0.2272) and Mean Variance Implied portfolio has the highest annualized return and annualized Sharpe ratio (0.2657 and 0.5717).

Table 4. 1 *Performance of portfolios (benchmark, minimum variance sample (historical), minimum variance implied, mean variance sample (historical) and mean variance implied)*

	Bench mark	Minimum Variance Sample	Minimum Variance Implied	Mean Variance Sample	Mean Variance Implied
Annualized Return	0.0347	0.1068	0.2411	0.1452	0.2657
Annualized Std Dev	0.1994	0.2082	0.2272	0.1807	0.2051
Annualized Sharpe (Risk-free ratef=12.64%)	-0.4444	-0.1175	0.4192	0.0545	0.5717

Portfolios constructed using option implied information have higher annualized returns and annualized Sharpe ratios compared to portfolios constructed using historical information. In fact, Mean Variance Implied portfolio has the highest annualized return and annualized Sharpe ratio. Mean Variance Implied outperforms the Mean Variance Sample (historical) and benchmark portfolio. Implied Minimum Variance portfolio outperforms Minimum Variance Sample (historical) portfolio and benchmark portfolio.

The Table 4.2 below shows comparison of two portfolios: Implied mean variance and sample (historical) mean variance. This comparison illustrates our thesis hypothesis, compares two type of portfolios, portfolio based on the historical information and portfolio based on the implied information, namely mean variance sample (historical) and mean variance implied. The t-test results show that the mean returns of these two portfolios are different (statistically significant at 5% level) and our H_0 hypothesis (optimal portfolios based on option implied information do not perform better than optimal portfolios based

on historical information) can be rejected, which means that the portfolio based on implied information performs better rather than the portfolio based on historical information.

Table 4. 2 *T-test of portfolio performance (Mean Variance Implied versus Mean Variance Sample)*

	<i>Mean Variance Implied</i>	<i>Mean Variance Sample</i>
Mean	0.021419367	0.012621574
Variance	0.003505145	0.002722029
Observations	17	17
Pearson Correlation	0.959318809	
Hypothesized Mean Difference	0	
df	16	
t Stat	2.09166207	
P(T<=t) one-tail	0.026386877	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.052773753	
t Critical two-tail	2.119905299	

In the Table 4.3 below, we compare two portfolios: Minimum Variance Implied and Minimum Variance Sample (historical) portfolio. The t-test results show that the mean returns of these two portfolios are different (statistically significant at 5% level) and our H_0 hypothesis (optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information), can be rejected, which means that the portfolio based on implied information performs better than the portfolio based on historical information.

Table 4.3 *T-test of portfolio performance (Minimum Variance Implied versus Minimum Variance Sample)*

	<i>Min Variance Implied</i>	<i>Min Variance Sample</i>
Mean	0.020118846	0.010153347
Variance	0.004300208	0.003613047
Observations	17	17
Pearson Correlation	0.953678957	
Hypothesized Mean Difference	0	
df	16	
t Stat	2.067251785	
P(T<=t) one-tail	0.02764188	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.055283761	
t Critical two-tail	2.119905299	

In the Table 4.4, we compare the portfolios based on Mean Variance Implied and Benchmark portfolios. The t-test results show that the mean returns of these two portfolios are different (statistically significant at 5% level) and our H_0 hypothesis (optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information) can be rejected, which means that the Mean Variance Implied portfolio based on implied information performs better than the Benchmark portfolio.

Table 4.4 *T-test of portfolio performance (Mean Variance Implied versus Benchmark)*

	<i>Mean Variance Implied</i>	<i>Benchmark</i>
Mean	0.016820691	0.004398389
Variance	0.003302536	0.003314774
Observations	17	17
Pearson Correlation	0.927401208	
Hypothesized Mean Difference	0	
df	16	
t Stat	2.336772122	
P(T<=t) one-tail	0.016390914	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.032781829	
t Critical two-tail	2.119905299	

In the Table 4.5, we compare two portfolios: Minimum Variance Implied and Benchmark portfolios. The t-test results show that the mean returns of these two portfolios

are not different and our H_0 hypothesis (Optimal portfolios based on option implied information do not perform better than optimal portfolios based on historical information) cannot be rejected, which means that the Minimum Variance Implied portfolio based on implied information does not performs better than the Benchmark portfolio.

Table 4.5 *T-test of portfolio performance (Minimum Variance Implied versus Benchmark)*

	<i>Min Variance Implied</i>	<i>Benchmark</i>
Mean	0.007487883	0.004398389
Variance	0.002928062	0.003314774
Observations	17	17
Pearson Correlation	0.901092936	
Hypothesized Mean Difference	0	
df	16	
t Stat	0.508207171	
P(T<=t) one-tail	0.309121411	
t Critical one-tail	1.745883676	
P(T<=t) two-tail	0.618242822	
t Critical two-tail	2.119905299	

4.7 Comparison of the Results with Other Similar Research Results

As we have already mentioned in literature review, there is a limited number of similar research papers. Some frameworks used in our study are similar to the studies of other researchers. The following studies and results are similar to our studies:

One study by Kempt, Korn and Sassning (2012), developed the first estimator of the covariance matrix which is totally forward-looking, using information just from options. They tested the quality of the new estimator by examining the out-of-sample performance of a global minimum variance (GMVP) strategy and compared its performance with benchmark strategies based on historical information. According to their results, the implied estimator outperforms the historical estimator.

The aim of DeMiguel, Plyakha, Uppal and Vilkov (2013) in their study is to examine if option-implied information can be used in order to improve the selection of mean-variance portfolios with a large number of assets. They measure the performance of portfolio using

Sharpe ratio, volatility and turnover. Their empirical evidence shows that option-implied volatility helps to decrease volatility of the portfolio. Using the option-implied volatility, skewness and risk premium in order to adjust returns leads to a significant improvement in Sharpe ratio.

Vial (2013) in his master thesis analyzes the option-implied information in the portfolio optimization framework. They analyze different portfolio allocation strategies and study if option implied portfolios outperform historical portfolios. Their results show that options add forecasting power to the portfolio optimization problem. Although option-implied portfolios outperform the historical ones, differences are mostly insignificant. In specific situations, option-implied information is a rational alternative to the historical moment estimators.

Our findings are in line with the findings of Kempt, Korn (2012) and Sassning DeMiguel, Plyakha, Uppal and Vilkov (2013) and Vial (2013). As the findings imply, forward-looking information derived from option prices are superior indicators of future stock returns and it can be utilized to make better investment decisions.

5 CONCLUSION

Portfolio optimization is one of the most important topics in the field of investment management. Various portfolio optimization methodologies and techniques have been developed by researchers and practitioners to examine and determine optimal portfolios. According to many studies, option-implied volatility is a strong forecaster of future volatility in the equity market. According to other studies, option implied information is a strong forecaster of future volatility, that was also improved by our study and results. Thus, option implied information is used in several studies for portfolio optimization purposes.

The aim of our thesis was to construct different type of portfolios using different methods (based on historical and implied information); compare their performances and most importantly determine whether portfolios based on implied information outperform the portfolios constructed based on the historical information in the Turkish Stock Market. For this aim, we decided to compare 5 portfolios: the benchmark portfolio (BIST30 index), minimum variance sample (historical) portfolio, which was built using historical information, minimum variance implied portfolio, which was built using implied information, mean variance sample (historical) based on historical information, and mean variance implied portfolio based on implied information.

We used option prices of 20 stocks, which have been trading in the Futures and Options Market of Borsa İstanbul. The sample period is from March 2017 to July 2018. We developed portfolio optimization models using option prices as well as mean variance and minimum variance portfolios using historical stock price data.

We calculated implied volatility of the sample stocks from option prices using Black-Scholes option pricing model. We employed Buss and Vilkov's model for the calculation of implied correlations between stocks. Option implied mean variance and option implied minimum variance portfolios are based on the covariance metrics developed after historical information is replaced by option-implied information. After obtaining the optimal weights of portfolios, we made monthly rebalancing and we measured the performances of portfolios using annualized return, annualized standard deviation and annualized Sharpe ratio.

According to our results, portfolios based on implied information outperform benchmark and sample (historical) portfolios. Portfolios constructed using option implied

information have higher annualized returns and annualized Sharpe ratios compared to portfolios constructed using historical information. Most importantly, the minimum variance implied portfolio appears to show the best performance among other portfolios with respect to annualized return, standard deviation and Sharpe ratio.

Our study is limited with the sample period and the assumptions of the models employed in the methodology. Our study could be extended with a longer sample size and historical volatilities can be computed with shorter rolling window methods.

Our findings may enable local and foreign investors, future investors, researchers and policy makers to utilize forward –looking information implied in option prices in order to improve the out-of-sample performance of their portfolios. Our findings confirm that portfolios based on implied-information overperform portfolios based on historical information.

Our recommendation to future researchers who will work on the same topic could be to model the relations between implied volatility and stock returns and use implied volatility to forecast future stock returns.

To our knowledge this is one of the first studies in Turkey, which utilizes, stock option data for portfolio optimization purposes.

REFERENCES

- Anagnostopoulos, K. P., & Mamanis, G. (2011). Multiobjective evolutionary algorithms for complex portfolio optimization problems. *Computational Management Science*, 8(3), 259–279. <https://doi.org/10.1007/s10287-009-0113-8>
- Bahaludin, H., Abdullah, M. H., & Tolos, S. M. (2017). Asset allocation using option-implied moments. *Journal of Physics: Conference Series*, 890(1). <https://doi.org/10.1088/1742-6596/890/1/012158>
- Benninga, S. (2008). Financial Modeling. *Massachusetts Institute of Technology*. Retrieved from Massachusetts Institute of Technology
- Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, 81(3), 637-654.
- Boasson, V., Boasson, E., & Zhou, Z. (2011). Portfolio optimization in a mean-semivariance framework. *Investment Management and Financial Innovations*, 8(3), 58–68.
- Buss, A., Schoenleber, L., & Vilkov, G. (2017). Option-Implied Correlations, Factor Models, and Market Risk. *SSRN Electronic Journal*. <https://doi.org/10.2139/ssrn.2906484>
- Buss, A., & Vilkov, G. (2012). Measuring equity risk with option-implied correlations. *Review of Financial Studies*, 25(10), 3113–3140. <https://doi.org/10.1093/rfs/hhs087>
- Canina, L. & Figlewski, S. (1993). The Informational Content of Implied Volatility. *The Review of Financial Studies*, Vol. 6, No. 3 (1993), Pp. 659-681 Published by: Oxford University Press., 6(3), 659–681.
- Ceria, S. & Sivaramakrishnan, K.K. (2013). *Portfolio Optimization*. (March), 1–222. <https://doi.org/10.1201/b17178>
- CFA Institute. (2017). *CFA PROGRAM CURRICULUM 2017 LEVEL I*. Wiley.

- Chang, B. Y., Christoffersen, P., Jacobs, K., & Vainberg, G. (2012). Option-implied measures of equity risk. *Review of Finance*, 16(2), 385–428.
<https://doi.org/10.1093/rof/rfq029>
- Davari-Ardakani, H., Aminnayeri, M., & Seifi, A. (2016). Multistage portfolio optimization with stocks and options. *International Transactions in Operational Research*, 23(3), 593-622.
- Dobbins, R., Witt, S. F., & Fielding, J., Hart, E., Sim, K., Kamimura, K., Meredieu, C., Guyon, D., ... Gardiner, B. A. (1994). Portfolio theory and investment management. *Blackwell Business*, 9(2), 1–8. <https://doi.org/10.1007/978-3-319-19809-5>
- Driessen, J., Maenhout, P. J., & Vilkov, G. (2013). Option-Implied Correlations and the Price of Correlation Risk. *SSRN Electronic Journal*, (November).
<https://doi.org/10.2139/ssrn.2359380>
- Eisen D. (2000). *Using Options to Buy Stocks Build Wealth with Little Risk and No Capital*.
- Eklund, S., & Estaifo, R. (2018). *Modeling implied correlation matrices using option prices*.
- Fabozzi, F. J., & Markowitz, H. M. (2011). *Equity Valuation and Portfolio Management* (Vol. 199). John Wiley & Sons.
- Fama, E. F., & Roll, R. (1968). Some properties of symmetric stable distributions. *Journal of the American Statistical Association*, 63(323), 817-836.
- Francis J.C & Kim.D. (2013). *Modern Portfolio Theory*. John Wiley & Sons, Inc.
- Frank et al. (2011). *Decision Rules to Manage Sequence Risk in Retirement*. (November).
- Fraser, C. (2016). *Business Statistics for Competitive Advantage with Excel 2016*.
- Friedentag, H. C. (2009). *Option Income The Stock Generator*. John Wiley & Sons, Inc.
- Galsband, V. (2012). Downside risk of international stock returns. *Journal of Banking and*

Finance, 36(8), 2379–2388. <https://doi.org/10.1016/j.jbankfin.2012.04.019>

Grootveld, H., & Hallerbach, W. (1999). *Variance vs downside risk : Is there really that much difference ?* 114, 304–319.

Hull, J. C. (2003). *Options, Futures, Other Derivatives*. Pearson Education International, Inc., Upper Saddle River, New Jersey, 07458.

Jansen, D. W., & De Vries, C. G. (1991). On the frequency of large stock returns: Putting booms and busts into perspective. *The review of economics and statistics*, 18-24.

Kempf, A., & Korn, O. (2012). Portfolio Optimization Using Forward-Looking Portfolio Optimization Using Forward-Looking. *Finance*, (January), 1–24.
<https://doi.org/10.1093/rof/rfu006>

Konno, H., Waki, H., & Yuuki, A. (2002). Portfolio optimization under lower partial risk measures. *Asia-Pacific Financial Markets*, 9(2), 127–140.
<https://doi.org/10.1023/A:1022238119491>

Kostakis, A., Panigirtzoglou, N., & Skiadopoulos, G. (2010). Asset Allocation with Option-Implied Distributions : A Forward-Looking Approach. *Working Paper*.

Markowitz, H. (1991). Foundations of Portfolio Theory. *The Journal of Finance*, XLVI(2), 469–477. <https://doi.org/10.2139/ssrn.2791621>

Markowitz, H. (1952). Portfolio Selection. *The Journal Of Finance*, Vol. 7, No. 1. (Mar., 1952), Pp. 77-91. *Stable*, 7(1), 77–91.

Markowitz, H. M. (1927-). (1959). *Portfolio selection : efficient diversification of investments*. Retrieved from
<http://ezproxy.si.unav.es:2048/login?url=http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,url&db=edsnuk&AN=edsnuk.vtls001518435&lang=es&site=eds-live&scope=site>

Mueller, P., Stathopoulos, A., & Vedolin, A. (2012). International Correlation Risk. *In*

2012 Meeting Papers (No. 818). Society for Economic Dynamics.

Natenberg, S. (2014). *Option Volatility and Pricing*. Mc Graw Hill Education ISBN 9780071818780.

Plyakha, Y., Uppal, R., & Vilkov, G. (2012). Why Does an Equal-Weighted Portfolio Outperform Value- and Price-Weighted Portfolios? *SSRN Electronic Journal*, (March). <https://doi.org/10.2139/ssrn.1787045>

Rehman, Z., & Vilkov, G. (2011). Risk-Neutral Skewness: Return Predictability and Its Sources. *SSRN Electronic Journal*, (November 2008). <https://doi.org/10.2139/ssrn.1301648>

Roy, A. . D. . (1952). Safety First and the Holding of Assets Author. *Society, The Econometric*, 20(3), 431–449. <https://doi.org/10.1177/002795018109800105>

Sharpe, F. william. (1966). Mutual Fund Performance Author. *The Journal of Business* , Vol . 39 , No . 1 , Part 2 : Supplement on Security Prices Published by : The University of Chicago Press Stable URL : <https://www.jstor.org/stable/2351741> REFEREN, 39(1), 119–138.

Tobin.J. (1965). Money and Economic Growth Author. *The Econometric Society Stable URL : https://www.jstor.org/stable/1910352 The Econometric Society Is Collaborating with JSTOR to Digitize , Preserve and Extend Access to Econometrica*, 33(4), 671–684.

Tobin, J. (1958). Liquidity Preference as Behavior Towards Risk. *The Review of Economic Studies*, 25(2), 65. <https://doi.org/10.2307/2296205>

Vial, C. (2013). *Forward-Looking Information in Portfolio Selection*. 41(0), 84.

Vilkov, G., & Xiao, Y. (2013). Option-Implied Information and Predictability of Extreme Returns. *SSRN Electronic Journal*. <https://doi.org/10.2139/ssrn.2209654>

Ward, R. W. (2004). *OPTIONS AND OPTIONS TRADING*.

Yan, W., Miao, R., & Li, S. (2007). Multi-period semi-variance portfolio selection: Model and numerical solution. *Applied Mathematics and Computation*, 194(1), 128–134.

<https://doi.org/10.1016/j.amc.2007.04.036>

<https://www.onefpa.org/journal/Pages/Incorporating%20Time%20into%20the%20Efficient%20Frontier.aspx>

<https://www.investopedia.com/terms/c/capm.asp>

<https://www.borsaistanbul.com/en/data/data/viop-derivatives-market>

<https://www.msci.com/emerging-markets>

<https://www.borsaistanbul.com/docs/default-source/kurumsal-yonetim/borsa-2018-annual-report.pdf?sfvrsn=6>

<https://www.thebalance.com/msci-index-what-is-it-and-what-does-it-measure-3305948>

<https://www.msci.com/documents/1296102/15035999/USLetter-MIS-EM-May2019-cbren.pdf/fb580e1e-d54c-4c68-1314-977bbff69bd7?t=1559125400402>

<https://www.msci.com/documents/1296102/15035999/USLetter-MIS-EM-May2019-cbr-en.pdf/fb580e1e-d54c-4c68-1314-977bbff69bd7?t=155912540040>

<https://www.msci.com/documents/1296102/15035999/USLetter-MIS-ACWI-Apr2019-cbr-en.pdf/9de006ef-9cf0-8bd7-62ec-e67430e9155f?t=1559105406131>

<https://www.msci.com/documents/10199/ae0d3e1e-ef7f-47ed-a2a3-970532651d23>

<https://www.borsaistanbul.com/en/products-and-markets/products/options/single-stock-options>

<https://www.borsaistanbul.com/en/products-andmarkets/products/options/equity-index-options/bist30-index-options-contract-specification>

<https://www.forbes.com/pictures/eglg45gdjd/why-invest-in-emerging-markets-2/#46485a2572e0>

APPENDIX

Historical Covariance

	AKBNK	ARCLK	EKGYO	EREGL	GARA	HALKB	ISCTR.I	KCHOL	KRDM	PETK	PGSUS	SAHOL		TCELL	THYA	TOAS	TTKO	TUPRS	VAKB	YKBNK
	.IS	.IS	.IS	.IS	N.IS	.IS	S	.IS	D.IS	M.IS	.IS	.IS	SISE.IS	IS	O.IS	O.IS	M.IS	.IS	N.IS	.IS
AKBNK	0.0043	0.0019	0.0034	0.0029	0.0043	0.0041	0.0040	0.0036	0.0038	0.0025	0.0022	0.0033	0.0032	0.0033	0.0033	0.0025	0.0026	0.0018	0.0044	0.0043
.IS	43	89	68	09	04	76	16	57	38	76	5	79	22	52	63	87	11	11	58	44
ARCLK	0.0019	0.0049	0.0013	0.0024	0.0016	0.0016	0.0019	0.0028	0.0025	0.0023	0.0014	0.0013	0.0027	0.0022	0.0016	0.0012	0.0018	0.0012	0.0025	0.0021
IS	89	97	86	01	68	18	55	63	47	15	41	32	83	4	32	35	04	24	58	96
EKGYO	0.0034	0.0013	0.0045	0.0019	0.0036	0.0046	0.0032	0.0026	0.0029	0.0012	0.0020	0.0026	0.0024	0.0022	0.0027	0.0024	0.0021	0.0004	0.0044	0.0036
.IS	68	86	71	55	72	02	34	08	66	78	22	01	81	39	38	92	99	3	63	42
EREGL	0.0029	0.0024	0.0019	0.0097	0.0025	0.0033	0.0035	0.0036	0.0068	0.0016	0.0013	0.0027	0.0045	0.0038	0.0009	0.0020	0.0035	0.0021	0.0038	0.0025
IS	09	01	55	74	97	24	7	11	58	84	43	57	68	58	52	06	22	11	59	61
GARAN	0.0043	0.0016	0.0036	0.0025	0.0050	0.0050	0.0045	0.0036	0.0044	0.0026	0.0027	0.0034	0.0031	0.0037	0.0035	0.0026	0.0025	0.0015		0.0045
.IS	04	68	72	97	36	38	6	78	43	41	26	36	3	17	38	51	34	54	0.0049	96
HALKB	0.0041	0.0016	0.0046	0.0033	0.0050	0.0079	0.0049	0.0036	0.0046	0.0019	0.0038	0.0038	0.0036	0.0038	0.0035	0.0025	0.0024	0.0014	0.0062	0.0053
IS	76	18	02	24	38	16	85	82	63	63	62	24	42	89	88	81	03	68	77	01
ISCTR.I	0.0040	0.0019	0.0032	0.0035	0.0045	0.0049	0.0057	0.0037	0.0046	0.0027	0.0030	0.0035	0.0038	0.0040	0.0036	0.0025	0.0026	0.0023	0.0051	0.0041
S	16	55	34	7	6	85	93	92	66	75	87	25	1	9	21	28	82	83	47	94
KCHOL	0.0036	0.0028	0.0026	0.0036	0.0036	0.0036	0.0037	0.0044	0.0044	0.0023	0.0026	0.0031	0.0033	0.0040	0.0031	0.0023	0.0024	0.0021	0.0039	0.0037
IS	57	63	08	11	78	82	92	81	69	38	91	37	37	85	64	67	04	84	46	39
KRDM	0.0038	0.0025	0.0029	0.0068	0.0044	0.0046	0.0046	0.0044	0.0118	0.0041	0.0024	0.0038	0.0049	0.0043	0.0021	0.0031	0.0031	0.0011	0.0050	0.0048
D.IS	38	47	66	58	43	63	66	69	65	47	31	42	54	7	06	1	83	14	46	18
PETKM	0.0025	0.0023	0.0012	0.0016	0.0026	0.0019	0.0027	0.0023	0.0041	0.0052	0.0007	0.0018	0.0021	0.0025	0.0018	0.0020	0.0020	0.0004	0.0026	0.0030
.IS	76	15	78	84	41	63	75	38	47	36	41	79	52	51	47	42	72	92	7	35
PGSUS	0.0022	0.0014	0.0020	0.0013	0.0027	0.0038	0.0030	0.0026	0.0024	0.0007	0.0066	0.0022	0.0029	0.0031	0.0048	0.0022	0.0019	0.0006	0.0031	0.0031
IS	5	41	22	43	26	62	87	91	31	41	66	79	73	43	04	31	99	69	94	92
SAHOL	0.0033	0.0013	0.0026	0.0027	0.0034	0.0038	0.0035	0.0031	0.0038	0.0018	0.0022	0.0035	0.0030	0.0027	0.0026	0.0022	0.0020	0.0014	0.0039	0.0036
IS	79	32	01	57	36	24	25	37	42	79	79	11	58	39	49	09	94	8	04	37
	0.0032	0.0027	0.0024	0.0045	0.0031	0.0036	0.0038	0.0033	0.0049	0.0021	0.0029	0.0030	0.0062	0.0028	0.0026	0.0022	0.0025	0.0011	0.0043	0.0032
SISE.IS	22	83	81	68	3	42	1	37	54	52	73	58	15	23	89	82	93	08	6	51
TCELL.I	0.0033	0.0022	0.0022	0.0038	0.0037	0.0038	0.0040	0.0040	0.0043	0.0025	0.0031	0.0027	0.0028	0.0053	0.0034	0.0029	0.0027	0.0026		0.0036
S	52	4	39	58	17	89	9	85	7	51	43	39	23	92	35	41	67	88	0.0039	2
THYAO	0.0033	0.0016	0.0027	0.0009	0.0035	0.0035	0.0036	0.0031	0.0021	0.0018	0.0048	0.0026	0.0026	0.0034	0.0062	0.0026	0.0027	0.0016	0.0035	0.0037
.IS	63	32	38	52	38	88	21	64	06	47	04	49	89	35	2	72	05	52	8	58

TOASO	0.0025	0.0012	0.0024	0.0020	0.0026	0.0025	0.0025	0.0023	0.0031	0.0020	0.0022	0.0022	0.0022	0.0029	0.0026	0.0040	0.0021	0.0013	0.0031	0.0031
.IS	87	35	92	06	51	81	28	67	1	42	31	09	82	41	72	65	37	48	46	73
TTKOM	0.0026	0.0018	0.0021	0.0035	0.0025	0.0024	0.0026	0.0024	0.0031	0.0020	0.0019	0.0020	0.0025	0.0027	0.0027	0.0021	0.0031	0.0011	0.0031	0.0023
.IS	11	04	99	22	34	03	82	04	83	72	99	94	93	67	05	37	12	86	37	02
TUPRS.	0.0018	0.0012	0.0004	0.0021	0.0015	0.0014	0.0023	0.0021	0.0011	0.0004	0.0006	0.0014	0.0011	0.0026	0.0016	0.0013	0.0011	0.0045	0.0011	0.0013
IS	11	24	3	11	54	68	83	84	14	92	69	8	08	88	52	48	86	09	22	31
VAKBN	0.0044	0.0025	0.0044	0.0038		0.0062	0.0051	0.0039	0.0050	0.0026	0.0031	0.0039	0.0043		0.0035	0.0031	0.0031	0.0011	0.0074	0.0049
.IS	58	58	63	59	0.0049	77	47	46	46	7	94	04	6	0.0039	8	46	37	22	3	25
YKBNK.	0.0043	0.0021	0.0036	0.0025	0.0045	0.0053	0.0041	0.0037	0.0048	0.0030	0.0031	0.0036	0.0032	0.0036	0.0037	0.0031	0.0023	0.0013	0.0049	0.0057
IS	44	96	42	61	96	01	94	39	18	35	92	37	51	2	58	73	02	31	25	98

Historical Correlation

	AKBNK	ARCLK	EKGYO	EREGL	GARA	HALKB	ISCTR.I	KCHOL	KRDM	PETK	PGSUS	SAHOL		TCELL	THYA	TOAS	TTKO	TUPRS	VAKB	YKBNK
	.IS	.IS	.IS	.IS	N.IS	.IS	S	.IS	D.IS	M.IS	.IS	.IS	SISE.IS	IS	O.IS	O.IS	M.IS	.IS	N.IS	.IS
AKBNK		0.4268	0.7784	0.4464	0.9203	0.7122	0.8006	0.8289	0.5347	0.5401	0.4182	0.8652	0.6201	0.6926	0.6470	0.6157	0.7101	0.4092	0.7847	0.8657
.IS	1	72	38	87	65	9	1	06	36	8	39	64	78	15	53	3	75	94	05	58
ARCLK.	0.4268		0.2900	0.3435	0.3325		0.3633	0.6050	0.3308	0.4526	0.2497	0.3179	0.4994	0.4316	0.2926	0.2740	0.4574	0.2578	0.4197	0.4078
IS	72	1	96	96	68	0.2572	73	18	08	22	32	81	34	04	87	7	51	03	38	87
EKGYO	0.7784	0.2900		0.2925	0.7652	0.7650	0.6284		0.4027	0.2612	0.3663	0.6492	0.4655	0.4509	0.5135	0.5782	0.5830	0.0946	0.7657	0.7074
.IS	38	96	1	5	64	89	23	0.5762	25	17	06	62	3	34	82	05	34	82	47	84
EREGL.	0.4464	0.3435	0.2925		0.3701		0.4743	0.5456	0.6368	0.2354	0.1663		0.5861	0.5314	0.1220	0.3182	0.6386	0.3179	0.4528	0.3401
IS	87	96	5	1	75	0.3779	81	41	76	29	56	0.4707	73	67	76	02	68	15	2	38
GARAN	0.9203	0.3325	0.7652	0.3701		0.7979	0.8442	0.7742	0.5748	0.5142	0.4704	0.8170	0.5594	0.7132	0.6320	0.5857	0.6399	0.3260	0.8009	0.8505
.IS	65	68	64	75	1	95	11	32	17	89	75	31	26	81	8	99	95	65	62	39
HALKB.	0.7122		0.7650		0.7979		0.7360	0.6181	0.4812	0.3049	0.5315	0.7254	0.5192	0.5952	0.5113	0.4550	0.4842	0.2456	0.8184	0.7825
IS	9	0.2572	89	0.3779	95	1	72	55	19	13	96	03	6	42	59	12	07	9	3	15
ISCTR.I	0.8006	0.3633	0.6284	0.4743	0.8442	0.7360		0.7441	0.5627	0.5038	0.4967	0.7816	0.6350	0.7316	0.6031	0.5209	0.6316	0.4663	0.7845	0.7236
S	1	73	23	81	11	72	1	52	86	94	19	27	31	75	69	89	86	1	27	47
KCHOL.	0.8289	0.6050		0.5456	0.7742	0.6181	0.7441		0.6128	0.4826	0.4923	0.7908	0.6322	0.8309	0.5992	0.5545	0.6438	0.4857	0.6837	0.7335
IS	06	18	0.5762	41	32	55	52	1	96	76	13	78	43	27	17	16	85	53	53	58
KRDM	0.5347	0.3308	0.4027	0.6368	0.5748	0.4812	0.5627	0.6128		0.5261	0.2733	0.5953	0.5769	0.5463	0.2451	0.4478	0.5237	0.1522	0.5374	0.5808
D.IS	36	08	25	76	17	19	86	96	1	69	4	23	05	06	83	23	97	79	1	55
PETKM	0.5401	0.4526	0.2612	0.2354	0.5142	0.3049	0.5038	0.4826	0.5261			0.4382	0.3772	0.4801	0.3237	0.4425	0.5133	0.1012	0.4280	0.5508
.IS	8	22	17	29	89	13	94	76	69	1	0.1254	26	84	76	14	37	61	29	26	39

PGSUS.	0.4182	0.2497	0.3663	0.1663	0.4704	0.5315	0.4967	0.4923	0.2733			0.4710	0.4618	0.5242	0.7460	0.4285	0.4390	0.1220	0.4538	0.5134
IS	39	32	06	56	75	96	19	13	4	0.1254	1	46	97	02	89	58	06	5	7	19
SAHOL.	0.8652	0.3179	0.6492		0.8170	0.7254	0.7816	0.7908	0.5953	0.4382	0.4710		0.6546	0.6293	0.5668	0.5846	0.6335	0.3720	0.7642	0.8060
IS	64	81	62	0.4707	31	03	27	78	23	26	46	1	96	65	02	45	66	48	31	89
	0.6201	0.4994	0.4655	0.5861	0.5594	0.5192	0.6350	0.6322	0.5769	0.3772	0.4618	0.6546		0.4875	0.4325	0.4540	0.5896	0.2092	0.6416	0.5416
SISE.IS	78	34	3	73	26	6	31	43	05	84	97	96	1	66	07	58	51	42	46	41
TCELL.I	0.6926	0.4316	0.4509	0.5314	0.7132	0.5952	0.7316	0.8309	0.5463	0.4801	0.5242	0.6293	0.4875		0.5930	0.6281	0.6755	0.5451	0.6161	0.6473
S	15	04	34	67	81	42	75	27	06	76	02	65	66	1	83	33	38	06	19	39
THYAO	0.6470	0.2926	0.5135	0.1220	0.6320	0.5113	0.6031	0.5992	0.2451	0.3237	0.7460	0.5668	0.4325	0.5930		0.5313	0.6147	0.3118	0.5266	0.6258
.IS	53	87	82	76	8	59	69	17	83	14	89	02	07	83	1	82	76	75	23	33
TOASO	0.6157	0.2740	0.5782	0.3182	0.5857	0.4550	0.5209	0.5545	0.4478	0.4425	0.4285	0.5846	0.4540	0.6281	0.5313		0.6008	0.3148	0.5724	0.6535
.IS	3	7	05	02	99	12	89	16	23	37	58	45	58	33	82	1	3	75	34	2
TTKOM	0.7101	0.4574	0.5830	0.6386	0.6399	0.4842	0.6316	0.6438	0.5237	0.5133	0.4390	0.6335	0.5896	0.6755	0.6147	0.6008		0.3166	0.6524	0.5419
.IS	75	51	34	68	95	07	86	85	97	61	06	66	51	38	76	3	1	38	77	46
TUPRS.	0.4092	0.2578	0.0946	0.3179	0.3260	0.2456	0.4663	0.4857	0.1522	0.1012	0.1220	0.3720	0.2092	0.5451	0.3118	0.3148	0.3166		0.1937	0.2603
IS	94	03	82	15	65	9	1	53	79	29	5	48	42	06	75	75	38	1	77	32
VAKBN	0.7847	0.4197	0.7657	0.4528	0.8009	0.8184	0.7845	0.6837	0.5374	0.4280	0.4538	0.7642	0.6416	0.6161	0.5266	0.5724	0.6524	0.1937		0.7503
.IS	05	38	47	2	62	3	27	53	1	26	7	31	46	19	23	34	77	77	1	67
YKBNK.	0.8657	0.4078	0.7074	0.3401	0.8505	0.7825	0.7236	0.7335	0.5808	0.5508	0.5134	0.8060	0.5416	0.6473	0.6258	0.6535	0.5419	0.2603	0.7503	
IS	58	87	84	38	39	15	47	58	55	39	19	89	41	39	33	2	46	32	67	1

Implied Covariance

	AKBNK	ARCLK	EKGYO	EREGL	GARA	HALKB	ISCTR.I	KCHOL	KRDM	PETK	PGSUS	SAHOL		TCELL	THYA	TOAS	TTKO	TUPRS	VAKB	YKBNK
	.IS	.IS	.IS	.IS	N.IS	.IS	S	.IS	D.IS	M.IS	.IS	.IS	SISE.IS	IS	O.IS	O.IS	M.IS	.IS	N.IS	.IS
AKBNK.I	0.0678	0.0140	0.0285	0.0167	0.0307	0.0268	0.0279	0.0264	0.0196	0.0785	0.0156	0.0288	0.0461	0.0255	0.0243	0.0211	0.0232	0.0120	0.0300	0.1111
S	56	89	46	25	67	87	04	88	09	85	23	29	18	74	46	11	32	16	6	69
	0.0140	0.0160	0.0051	0.0062	0.0054	0.0047	0.0061	0.0094	0.0059	0.0320	0.0045	0.0051	0.0180	0.0077	0.0053	0.0045	0.0072	0.0036	0.0078	0.0254
ARCLK.IS	89	55	74	61	08	22	6	04	01	29	37	53	65	52	57	71	79	81	21	76
EKGYO.I	0.0285	0.0051	0.0198	0.0059	0.0138	0.0156	0.0118	0.0099	0.0079	0.0205	0.0073	0.0116	0.0187	0.0089	0.0104	0.0107	0.0103	0.0015	0.0158	0.0490
S	46	74	17	22	25	07	36	5	81	37	94	9	08	98	43	14	07	02	52	94
	0.0167	0.0062	0.0059	0.0206	0.0068	0.0078	0.0091	0.0096	0.0128	0.0189	0.0034	0.0086	0.0240	0.0108	0.0025	0.0060	0.0115	0.0051	0.0095	0.0241
EREGL.IS	25	61	22	79	31	75	27	25	93	07	3	58	63	33	36	23	33	52	76	11
GARAN.I	0.0307	0.0054	0.0138	0.0068	0.0164	0.0148	0.0144	0.0121	0.0103	0.0368	0.0086	0.0134	0.0204	0.0129	0.0117	0.0098	0.0103	0.0047	0.0151	0.0538
S	67	08	25	31	69	39	95	89	84	59	58	11	94	75	16	95	14	16	16	05
HALKB.I	0.0268	0.0047	0.0156	0.0078	0.0148	0.0209	0.0142	0.0109	0.0098	0.0246	0.0110	0.0134	0.0214	0.0122	0.0107	0.0086	0.0088	0.0040	0.0174	0.0558
S	87	22	07	75	39	98	71	88	16	76	46	45	8	26	03	78	11	12	4	95
	0.0279	0.0061	0.0118	0.0091	0.0144	0.0142	0.0179	0.0122		0.0376	0.0095	0.0133	0.0242	0.0138	0.0116	0.0091	0.0106	0.0070	0.0154	0.0477
ISCTR.IS	04	6	36	27	95	71	01	14	0.0106	52	3	76	55	76	57	75	14	31	36	27
KCHOL.I	0.0264	0.0094	0.0099	0.0096	0.0121	0.0109	0.0122	0.0150	0.0105	0.0330	0.0086	0.0124	0.0221	0.0144	0.0106	0.0089	0.0099	0.0067	0.0123	0.0443
S	88	04	5	25	89	88	14	49	84	68	6	09	41	49	17	54	19	16	35	59
KRDM.D	0.0196	0.0059	0.0079	0.0128	0.0103	0.0098		0.0105	0.0198	0.0413	0.0055	0.0107	0.0231	0.0109	0.0049	0.0082	0.0092	0.0024	0.0111	0.0403
IS	09	01	81	93	84	16	0.0106	84	17	67	18	19	84	01	85	98	6	16	25	07
PETKM.I	0.0785	0.0320	0.0205	0.0189	0.0368	0.0246	0.0376	0.0330	0.0413	0.3118	0.0100	0.0313	0.0601	0.0380	0.0261	0.0325	0.0360	0.0063	0.0351	0.1516
S	85	29	37	07	59	76	52	68	67	98	43	04	5	12	13	3	04	71	53	44
PGSUS.I	0.0156	0.0045	0.0073	0.0034	0.0086	0.0110	0.0095	0.0086	0.0055	0.0100	0.0205	0.0086	0.0189	0.0106	0.0154	0.0080	0.0079	0.0019	0.0095	0.0362
S	23	37	94	3	58	46	3	6	18	43	63	4	08	55	53	89	06	72	71	92
SAHOL.I	0.0288	0.0051	0.0116	0.0086	0.0134	0.0134	0.0133	0.0124	0.0107	0.0313	0.0086	0.0163	0.0239	0.0114	0.0104	0.0098	0.0101	0.0053	0.0143	0.0508
S	29	53	9	58	11	45	76	09	19	04	4	6	05	11	71	43	77	63	75	24
	0.0461	0.0180	0.0187	0.0240	0.0204	0.0214	0.0242	0.0221	0.0231	0.0601	0.0189	0.0239	0.0814	0.0197	0.0178	0.0170	0.0211	0.0067	0.0269	0.0762
SISE.IS	18	65	08	63	94	8	55	41	84	5	08	05	94	29	34	61	39	32	36	2
	0.0255	0.0077	0.0089	0.0108	0.0129	0.0122	0.0138	0.0144	0.0109	0.0380	0.0106	0.0114	0.0197	0.0200	0.0121	0.0117	0.0120	0.0087	0.0128	0.0452
TCELL.IS	74	52	98	33	75	26	76	49	01	12	55	11	29	92	43	19	25	08	43	32
THYAO.I	0.0243	0.0053	0.0104	0.0025	0.0117	0.0107	0.0116	0.0106	0.0049	0.0261	0.0154	0.0104	0.0178	0.0121	0.0208	0.0101	0.0111	0.0050	0.0111	0.0445
S	46	57	43	36	16	03	57	17	85	13	53	71	34	43	63	02	51	77	86	59
TOASO.I	0.0211	0.0045	0.0107	0.0060	0.0098	0.0086	0.0091	0.0089	0.0082	0.0325	0.0080	0.0098	0.0170	0.0117	0.0101	0.0173	0.0099	0.0046	0.0110	0.0424
S	11	71	14	23	95	78	75	54	98	3	89	43	61	19	02	24	31	71	8	02

TTKOM.I	0.0232	0.0072	0.0103	0.0115	0.0103	0.0088	0.0106	0.0099	0.0092	0.0360	0.0079	0.0101	0.0211	0.0120	0.0111	0.0099	0.0157	0.0044	0.0120	0.0335
S	32	79	07	33	14	11	14	19	6	04	06	77	39	25	51	31	7	81	49	48
TUPRS.I	0.0120	0.0036	0.0015	0.0051	0.0047	0.0040	0.0070	0.0067	0.0024	0.0063	0.0019	0.0053	0.0067	0.0087	0.0050	0.0046	0.0044	0.0127	0.0032	0.0144
S	16	81	02	52	16	12	31	16	16	71	72	63	32	08	77	71	81	01	12	63
VAKBN.I	0.0300	0.0078	0.0158	0.0095	0.0151	0.0174	0.0154	0.0123	0.0111	0.0351	0.0095	0.0143	0.0269	0.0128	0.0111	0.0110	0.0120	0.0032	0.0216	0.0543
S	6	21	52	76	16	4	36	35	25	53	71	75	36	43	86	8	49	12	25	94
YKBNK.I	0.1111	0.0254	0.0490	0.0241	0.0538	0.0558	0.0477	0.0443	0.0403	0.1516	0.0362	0.0508	0.0762	0.0452	0.0445	0.0424	0.0335	0.0144	0.0543	0.2429
S	69	76	94	11	05	95	27	59	07	44	92	24	2	32	59	02	48	63	94	89

Implied Correlation

	AKBNK .IS	ARCLK .IS	EKGYO .IS	EREGL .IS	GARA N.IS	HALKB .IS	ISCTR.I S	KCHOL .IS	KRDM D.IS	PETK M.IS	PGSUS .IS	SAHOL .IS	SISE.IS	TCELL IS	THYA O.IS	TOAS O.IS	TTKO M.IS	TUPRS .IS	VAKB N.IS	YKBNK .IS
		-		-							-							-		
AKBNK .IS	1	0.1462 6	0.5568 76	0.1070 3	0.8407 3	0.4245 81	0.6012 2	0.6578 12	0.0694 73	0.0803 6	0.1635 2	0.7305 28	0.2403 56	0.3852 3	0.2941 07	0.2314 6	0.4203 5	0.1814 1	0.5694 11	0.7315 16
	-		-	-	-		-		-	-	-	-	-	-	-	-		-	-	-
ARCLK. IS	0.1462 6	1	0.4198 1	0.3128 1	0.3348 6	- 0.4856	0.2732 5	0.2100 36	0.3383 8	0.0947 6	0.5005 4	0.3640 4	0.0011 3	0.1367 9	0.4146 3	0.4518 6	- 0.0851	0.4843 9	0.1605 2	0.1842 3
	-		-						-	-	-							-		
EKGYO .IS	0.5568 76	0.4198 1	1	- 0.4149	0.5305 28	0.5301 79	0.2568 46	0.1523 99	0.1945 5	0.4775 7	0.2673 9	0.2985 23	0.0689 4	0.0981 3	0.0271 65	0.1564 1	0.1660 68	0.8106 4	0.5314 95	0.4149 68
	-				-				-	-	-				-			-	-	-
EREGL. IS	0.1070 3	0.3128 1	- 0.4149		0.2596 5	- 0.2442	0.0512 4	0.0912 82	0.2737 52	0.5291 4	0.6672 9	- 0.0586	0.1723 46	0.0629 33	0.7558 5	- 0.3636	0.2773 36	0.3641 7	0.0943 6	0.3197 2
GARAN .IS	0.8407 3	0.3348 6	0.5305 28	0.2596 5		0.5959 89	0.6884 21	0.5484 63	0.1496 34	0.0285 79	0.0590 5	0.6340 63	0.1188 52	0.4265 62	0.2641 61	0.1715 98	0.2799 9	0.3478 7	0.6019 24	0.7010 78
									-	-						-	-	-		
HALKB. IS	0.4245 81	- 0.4856	0.5301 79	- 0.2442	0.5959 89		0.4721 43	0.2363 09	0.0375 6	0.3901 7	0.0631 93	0.4508 06	0.0385 2	0.1904 85	0.0227 18	0.0899 8	0.0315 9	0.5086 2	0.6368 6	0.5650 29
ISCTR.I S	0.6012 2	0.2732 5	0.2568 46	0.0512 4	0.6884 21	0.4721 43		0.4883 04	0.1255 72	0.0077 87	0.0065 6	0.5632 54	0.2700 61	0.4633 5	0.2063 38	0.0419 78	0.2633 72	0.0673 8	0.5690 54	0.4472 94

KCHOL. IS	0.6578	0.2100	0.1523	0.0912	0.5484	0.2363	0.4883		0.2257	0.0346	0.0153	0.5817	0.2644	0.6618	0.1984	0.1090	0.2877	0.0284	0.3675	0.4671
	12	36	99	82	63	09	04	1	92	5	7	56	85	54	33	32	71	9	06	16
KRDM D.IS	0.0694	0.3383	0.1945	0.2737	0.1496	0.0375	0.1255	0.2257		0.0523	0.4533	0.1906	0.1538	0.0926	0.5096	0.1043	0.0475	0.6954	0.0748	0.1617
	73	8	5	52	34	6	72	92	1	38	2	46	11	12	3	5	93	4	21	09
PETKM .IS	0.0803	0.0947	0.4775	0.5291	0.0285	0.3901	0.0077	0.0346	0.0523		-	0.1235	0.2454	0.0396	0.3525	0.1149	0.0267	0.7975	0.1439	0.1016
	6	6	7	4	79	7	87	5	38	1	0.7492	5	3	5	7	3	22	4	5	77
PGSUS. IS	0.1635	0.5005	0.2673	0.6672	0.0590	0.0631	0.0065	0.0153	0.4533	-		0.0579	0.0762	0.0484	0.4921	0.1428	0.1219	-	0.0922	0.0268
	2	4	9	9	5	93	6	7	2	0.7492	1	1	1	05	78	8	9	0.7559	6	38
SAHOL. IS	0.7305	0.3640	0.2985	-	0.6340	0.4508	0.5632	0.5817	0.1906	0.1235	0.0579		0.3093	0.2587	0.1336	0.1692	0.2671	-	0.5284	0.6121
	28	4	23	0.0586	63	06	54	56	46	5	1	1	92	31	03	89	31	0.2559	63	77
SISE.IS	0.2403	0.0011	0.0689	0.1723	0.1188	0.0385	0.2700	0.2644	0.1538	0.2454	0.0762	0.3093		0.0248	0.1349	0.0918	0.1793	0.5815	0.2832	0.0832
	56	3	4	46	52	2	61	85	11	3	1	92	1	7	9	8	02	2	93	82
TCELL.I S	0.3852	0.1367	0.0981	0.0629	0.4265	0.1904	0.4633	0.6618	0.0926	0.0396	0.0484	0.2587	0.0248		0.1861	0.2562	0.3510	0.0902	0.2322	0.2946
	3	9	3	33	62	85	5	54	12	5	05	31	7	1	66	67	76	11	39	78
THYAO .IS	0.2941	0.4146	0.0271	0.7558	0.2641	0.0227	0.2063	0.1984	0.5096	0.3525	0.4921	0.1336	0.1349	0.1861		0.0627	0.2295	0.3762	0.0532	0.2516
	07	3	65	5	61	18	38	33	3	7	78	03	9	66	1	64	52	5	47	66
TOASO .IS	0.2314	0.4518	0.1564	-	0.1715	0.0899	0.0419	0.1090	0.1043	0.1149	0.1428	0.1692	0.0918	0.2562	0.0627		0.2016	0.3702	0.1448	0.3070
	6	6	1	0.3636	98	8	78	32	5	3	8	89	8	67	64	1	6	5	67	41
TTKOM .IS	0.4203	-	0.1660	0.2773	0.2799	0.0315	0.2633	0.2877	0.0475	0.0267	0.1219	0.2671	0.1793	0.3510	0.2295	0.2016		0.3667	0.3049	0.0838
	5	0.0851	68	36	9	9	72	71	93	22	9	31	02	76	52	6	1	2	53	91
TUPRS. IS	0.1814	0.4843	0.8106	0.3641	0.3478	0.5086	0.0673	0.0284	0.6954	0.7975	-	-	0.5815	0.0902	0.3762	0.3702	0.3667		0.6124	0.4793
	1	9	4	7	7	2	8	9	4	4	0.7559	0.2559	2	11	5	5	2	1	5	4
VAKBN .IS	0.5694	0.1605	0.5314	0.0943	0.6019	0.6368	0.5690	0.3675	0.0748	0.1439	0.0922	0.5284	0.2832	0.2322	0.0532	0.1448	0.3049	0.6124		0.5007
	11	2	95	6	24	6	54	06	21	5	6	63	93	39	47	67	53	5	1	34
YKBNK. IS	0.7315	0.1842	0.4149	0.3197	0.7010	0.5650	0.4472	0.4671	0.1617	0.1016	0.0268	0.6121	0.0832	0.2946	0.2516	0.3070	0.0838	0.4793	0.5007	
	16	3	68	2	78	29	94	16	09	77	38	77	82	78	66	41	91	4	34	1

Sample Volatility

	AKBNK .IS	ARCLK. IS	EKGYO .IS	EREGL. IS	GARAN .IS	HALKB .IS	ISCTR.I S	KCHOL .IS	KRDMD .IS	PETKM .IS	PGSUS .IS	SAHOL .IS	SISE.IS	TCELL.I S	THYAO .IS	TOASO .IS	TTKOM .IS	TUPRS. IS	VAKBN .IS	YKBNK .IS
1	0.0043	0.0049	0.0045	0.0097	0.0050	0.0079	0.0057	0.0044	0.0118	0.0052	0.0066	0.0035	0.0062	0.0053	0.0062	0.0040	0.0031	0.0045	0.0074	0.0057
	43	97	71	74	36	16	93	81	65	36	66	11	15	92	2	65	12	09	3	98
2	0.0043	0.0049	0.0045	0.0097	0.0050	0.0079	0.0057	0.0044	0.0118	0.0052	0.0066	0.0035	0.0062	0.0053	0.0062	0.0040	0.0031	0.0045	0.0074	0.0057
	43	97	71	74	36	16	93	81	65	36	66	11	15	92	2	65	12	09	3	98
3	0.0043	0.0044	0.0042	0.0110	0.0053	0.0081	0.0058	0.0047	0.0117	0.0051	0.0076	0.0034	0.0059	0.0052	0.0073	0.0037	0.0030	0.0045	0.0074	0.0056
	76	38	64	62	04	02	95	07	19	69	12	31	46	78	83	8	86	38	95	77
4	0.0041	0.0044	0.0042	0.0100	0.0050	0.0075	0.0050	0.0045	0.0121	0.0052	0.0076	0.0029	0.0056	0.0051	0.0076	0.0038	0.0031	0.0045	0.0068	0.0053
	14	86	5	74	54	63	02	36	39	52	38	03	96	89	24	2	01	19	54	43
5	0.0041	0.0044	0.0043	0.0093	0.0051	0.0081	0.0049	0.0045	0.0119	0.0051	0.0099	0.0028	0.0057	0.0054	0.0075	0.0038	0.0037	0.0044	0.0069	0.0052
	01	43	87	61	26	24	18	33	76	65	27	74	55	69	63	5	38	56	86	52
6	0.0039	0.0047	0.0045	0.0086	0.0050	0.0081	0.0049	0.0046	0.0119	0.0049	0.0103	0.0028	0.0054	0.0054	0.0077	0.0038	0.0037	0.0045	0.0069	0.0052
	25	01	49	52	41	33	07	13	24	84	27	91	81	19	87	41	09	04	88	87
7	0.0039	0.0046	0.0046	0.0085	0.0050	0.0083	0.0048	0.0047	0.0122	0.0050	0.0105	0.0029	0.0054	0.0052	0.0079	0.0039	0.0037	0.0045	0.0071	0.0054
	79	91	54	7	99	46	4	07	32	93	1	73	09	36	45	2	74	17	64	23
8	0.0038	0.0048	0.0043	0.0089	0.0052	0.0087	0.0047	0.0048	0.0120	0.0051	0.0116	0.0029	0.0056	0.0052	0.0087	0.0037	0.0038	0.0040	0.0067	0.0055
	93	64	06	31	9	59	52	06	53	61	45	87	12	17	89	9	18	69	62	88
9	0.0041	0.0046	0.0044	0.0089	0.0053	0.0095	0.0051	0.0047	0.0115	0.0052	0.0119	0.0029	0.0052	0.0051	0.0092	0.0037	0.0041	0.0046	0.0066	0.0056
	05	73	1	63	61	99	77	48	01	52	16	51	04	82	01	07	67	91	29	54
10	0.0038	0.0047	0.0040	0.0091	0.0051	0.0097	0.0050	0.0046	0.0112	0.0053	0.0117	0.0029	0.0053	0.0051	0.0077	0.0035	0.0039	0.0048	0.0067	0.0055
	6	9	94	14	13	27	49	68	01	42	66	87	35	84	02	06	54	25	23	63
11	0.0041	0.0052	0.0046	0.0092	0.0057	0.0104	0.0056	0.0048	0.0169	0.0058	0.0135	0.0030	0.0055	0.0051	0.0094	0.0035	0.0043	0.0047	0.0074	0.0054
	85	14	05	95	47	97	86	53	56	36	35	77	14	53	39	3	02	77	09	38
12	0.0042	0.0052	0.0039	0.0094	0.0056	0.0100	0.0057	0.0048	0.0169	0.0057	0.0133	0.0032	0.0056	0.0051	0.0098	0.0036	0.0043	0.0046	0.0067	0.0054
	08	04	27	94	26	13	54	77	02	06	13	08	09	59	88	81	21	48	55	07
13	0.0040	0.0053	0.0040	0.0095	0.0051	0.0102	0.0054	0.0047	0.0169	0.0058	0.0134	0.0031	0.0056	0.0050	0.0100	0.0037	0.0044	0.0047	0.0067	0.0053
	64	03	2	57	95	32	37	4	61	07	6	56	82	92	67	72	41	44	01	21
14	0.0043	0.0051	0.0039	0.0096	0.0053	0.0096	0.0054	0.0054	0.0165	0.0058	0.0138	0.0030	0.0055	0.0052	0.0102	0.0038	0.0043	0.0048	0.0060	0.0053
	66	4	09	48	75	97	1	47	54	78	41	88	59	41	68	46	28	61	75	42
15	0.0044	0.0053	0.0038	0.0109	0.0054	0.0095	0.0053	0.0052	0.0169	0.0084	0.0137	0.0030	0.0056	0.0050	0.0102	0.0032	0.0043	0.0048	0.0060	0.0052
	62	73	45	21	14	93	19	13	52	99	78	25	84	86	84	25	01	5	21	87
16	0.0045	0.0055	0.0039	0.0110	0.0055	0.0098	0.0053	0.0052	0.0167	0.0086	0.0141		0.0057	0.0051	0.0105	0.0031	0.0042	0.0049	0.0061	0.0054
	86	25	25	08	59	66	33	27	15	77	6	0.0031	67	99	76	24	18	88	78	28
17	0.0046	0.0065	0.0041	0.0112	0.0062	0.0095	0.0056	0.0050	0.0166	0.0083	0.0145	0.0031	0.0063	0.0051	0.0132	0.0032	0.0047	0.0048	0.0069	0.0058
	26	02	42	28	11	75	38	82	59	6	48	42	64	39	22	15	98	83	72	6

Implied Volatility

	AKBNK .IS	ARCLK. IS	EKGYO .IS	EREGL. IS	GARAN .IS	HALKB .IS	ISCTR.I S	KCHOL .IS	KRDMD .IS	PETKM .IS	PGSUS .IS	SAHOL .IS	SISE.IS	TCELL.I S	THYAO .IS	TOASO .IS	TTKOM .IS	TUPRS. IS	VAKBN .IS	YKBNK .IS
1	0.0678 56	0.0160 55	0.0198 17	0.0206 79	0.0164 69	0.0209 98	0.0179 01	0.0150 49	0.0198 17	0.3118 98	0.0205 63	0.0163 6	0.0814 94	0.0200 92	0.0208 63	0.0173 24	0.0157 7	0.0127 01	0.0216 25	0.2429 89
2	0.0909 0.1231	36 0.0205	38 0.0201	22 0.0216	03 0.0227	54 0.0225	92 0.0216	45 0.0193	83 0.0204	99 0.2720	76 0.0232	56 0.0200	54 0.0811	65 0.0208	28 0.0218	29 0.0214	38 0.0224	15 0.0181	03 0.0223	27 0.0195
3	15 0.0688	28 0.0199	74 0.0203	78 0.0224	98 0.0225	05 0.0224	6 0.0217	53 0.0173	09 0.0203	52 0.1460	55 0.0225	83 0.0197	4 0.0564	79 0.0188	39 0.0203	95 0.0234	44 0.0174	85 0.0252	93 0.0210	35 0.1572
4	41 0.1127	81 0.0192	85 0.0212	94 0.0211	76 0.0217	26 0.0215	45 0.0212	97 0.0164	53 0.0204	01 0.0221	36 0.0196	83 0.0208	92 0.0173	14 0.0208	08 0.0214	1 0.0216	79 0.0167	37 0.0214	85 0.0205	35 0.2172
5	06 0.0702	77 0.0209	39 0.0236	62 0.0222	61 0.0234	95 0.0228	9 0.0222	91 0.0177	21 0.0225	0.2314 0.1210	25 0.0230	35 0.0207	56 0.0234	13 0.0207	2 0.0214	86 0.0241	47 0.0176	09 0.0261	25 0.0215	82 0.1976
6	25 0.0807	33 0.0202	14 0.0223	37 0.0228	3 0.0227	45 0.0237	63 0.0220	82 0.0173	3 0.0224	02 0.1850	96 0.0235	26 0.0202	6 0.0213	81 0.0194	02 0.0218	59 0.0237	9 0.0174	67 0.0238	82 0.0220	25 0.2264
7	17 0.0984	96 0.0202	9 0.0247	89 0.0243	25 0.0230	63 0.0234	26 0.0228	76 0.0170	53 0.0238	54 0.2279	56 0.0227	18 0.0201	01 0.0207	51 0.0204	17 0.0221	96 0.0242	59 0.0179	39 0.0256	97 0.0210	27 0.3118
8	89 0.1239	53 0.0193	5 0.0247	93 0.0239	38 0.0224	92 0.0227	82 0.0220	44 0.0166	86 0.0224	13 0.3000	04 0.0226	18 0.0191	59 0.0192	13 0.0190	95 0.0227	23 0.0227	06 0.0175	87 0.0225	92 0.0221	79 0.4189
9	46 0.0940	52 0.0202	68 0.0269	85 0.0256	27 0.0236	66 0.0253	22 0.0238	41 0.0174	63 0.0239	09 0.1804	91 0.0234	76 0.0205	05 0.0221	21 0.0207	64 0.0242	96 0.0248	52 0.0195	22 0.0259	68 0.0244	67 0.1733
10	64 0.1306	78 0.0196	91 0.0251	04 0.0240	73 0.0235	58 0.0260	51 0.0229	46 0.0173	22 0.0253	14 0.2680	89 0.0235	62 0.0201	62 0.0213	11 0.0195	15 0.0247	94 0.0236	76 0.0197	8 0.0248	31 0.0241	69 0.2995
11	25 0.0786	56 0.0132	91 0.0275	27 0.0259	68 0.0257	77 0.0278	88 0.0260	43 0.0138	76 0.0250	95 0.1378	44 0.0246	58 0.0216	26 0.0225	99 0.0225	1 0.0260	46 0.0136	55 0.0208	6 0.0042	56 0.0253	06 0.1442
12	96 0.0859	73 0.0207	67 0.0274	12 0.0046	7 0.0134	68 0.0269	02 0.0112	29 0.0108	64 0.0259	58 0.1860	97 0.0243	44 0.0101	57 0.0218	98 0.0212	15 0.0267	96 0.0241	18 0.0204	55 0.0369	44 0.0246	78 0.1636
13	63 0.0582	6 0.0212	85 0.0082	71 0.0052	21 0.0273	14 0.0175	51 0.0270	49 0.0178	53 0.0271	14 0.0698	37 0.0278	72 0.0204	6 0.0132	61 0.0245	58 0.0309	7 0.0279	12 0.0213	63 0.0279	67 0.0214	78 0.1110
14	13 0.0787	07 0.0236	5 0.0282	95 0.0262	87 0.0254	85 0.0080	82 0.0265	4 0.0174	87 0.0291	16 0.1474	63 0.0306	62 0.0212	46 0.0245	07 0.0243	73 0.0319	85 0.0259	73 0.0236	42 0.0267	14 0.0186	5 0.1841
15	75 0.0842	62 0.0263	95 0.0330	98 0.0310	94 0.0308	91 0.0261	98 0.0315	36 0.0227	21 0.0316	8 0.1497	33 0.0339	29 0.0246	14 0.0292	46 0.0293	03 0.0344	71 0.0320	65 0.0262	72 0.0318	11 0.0263	86 0.0269
16	27 0.1305	85 0.0251	84 0.0281	53 0.0294	97 0.0276	76 0.0230	95 0.0281	88 0.0219	88 0.0305	5 0.1662	09 0.0344	49 0.0206	51 0.0271	35 0.0255	48 0.0343	61 0.0288	83 0.0244	22 0.0283	99 0.0243	74 0.0265
17	46	27	68	88	35	97	73	51	79	48	6	51	81	92	56	48	92	49	01	77

Minimum Variance Sample Weights

	AKBNK .IS	ARCLK. IS	EKGYO .IS	EREGL. IS	GARAN .IS	HALKB. IS	ISCTR. IS	KCHOL. IS	KRDMD .IS	PETKM .IS	PGSUS. IS	SAHOL. IS	SISE. IS	TCELL. IS	THYAO .IS	TOASO .IS	TTKOM .IS	TUPRS. IS	VAKBN .IS	YKBNK. IS
1	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
2	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
3	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
4	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
5	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
6	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
7	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
8	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
9	0.04	0.048	0.128	0.01	0.012	0.078	0.018	0.002	0.026	0.142	0.018	0.072	0.066	0.084	0.016	0.052	0	0.184	0.004	0
10	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
11	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
12	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
13	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
14	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
15	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
16	0	0.152	0.034	0	0.172	0.014	0.11	0.002	0.012	0.03	0.01	0.038	0	0	0.022	0.108	0.082	0.196	0.016	0.002
17	0	0.106	0.028	0.054	0.008	0.002	0.098	0.006	0.026	0.014	0.002	0.044	0.182	0.094	0.02	0.162	0.006	0.086	0.048	0.014
18	0	0.106	0.028	0.054	0.008	0.002	0.098	0.006	0.026	0.014	0.002	0.044	0.182	0.094	0.02	0.162	0.006	0.086	0.048	0.014

Mean Variance Sample Weights

	AKBNK.IS	ARCLK.IS	EKGYO.IS	EREGL.IS	GARAN.IS	HALKB.IS	ISCTR.IS	KCHOL.IS	KRDMD.IS	PETKM.IS	PGSUS.IS	SAHOL.IS	SISE.IS	TCELL.IS	THYAO.IS	TOASO.IS	TTKOM.IS	T
1	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
2	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
3	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
4	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
5	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
6	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
7	0.004	0.128	0.026	0.02	0	0.01	0.012	0.112	0.014	0.174	0.01	0.01	0.182	0.006	0.002	0.102	0.092	
8	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
9	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
10	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
11	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
12	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
13	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
14	0.138	0.16	0.056	0.024	0.016	0.014	0.026	0.016	0.012	0.05	0	0.02	0.08	0.028	0.022	0.098	0.01	
15	0.018	0.1	0.036	0.13	0.002	0.016	0.038	0	0.048	0.062	0.014	0.076	0.124	0.048	0.002	0.052	0	
16	0.018	0.1	0.036	0.13	0.002	0.016	0.038	0	0.048	0.062	0.014	0.076	0.124	0.048	0.002	0.052	0	
17	0.018	0.1	0.036	0.13	0.002	0.016	0.038	0	0.048	0.062	0.014	0.076	0.124	0.048	0.002	0.052	0	
18	0.018	0.1	0.036	0.13	0.002	0.016	0.038	0	0.048	0.062	0.014	0.076	0.124	0.048	0.002	0.052	0	

Implied Minimum Variance Weights

	AKBN K.IS	ARCLK .IS	EKGYO .IS	EREGL. .IS	GARAN .IS	HALKB .IS	ISCTR. .IS	KCHOL .IS	KRDMD .IS	PETKM .IS	PGSUS .IS	SAHOL .IS	SISE.I S	TCELL. .IS	THYAO .IS	TOASO .IS	TTKOM .IS	TUPRS. .IS	VAKBN .IS	YKBN K.IS
1	0.02	0.084	0.092	0.034	0.022	0.042	0.082	0.03	0.084	0	0.07	0.124	0.008	0.01	0.03	0	0.052	0.188	0.028	0
2	0.006	0.102	0.002	0.174	0.024	0.036	0	0.014	0.158	0.002	0.188	0.032	0.004	0.008	0.13	0.002	0.02	0.026	0.07	0.002
3	0.008	0.178	0.002	0.056	0.032	0.062	0.024	0.104	0.09	0.002	0.012	0.032	0.022	0.018	0.132	0.044	0.048	0.098	0.032	0.004
4	0.02	0.098	0.018	0.148	0.006	0.01	0.044	0.058	0.112	0.03	0.048	0.022	0.004	0.008	0.126	0.078	0.01	0.14	0.018	0.002
5	0.004	0.114	0	0.06	0.04	0.01	0.002	0.098	0.004	0.008	0.088	0.07	0.148	0.002	0.106	0.08	0.036	0.124	0.006	0
6	0.006	0.176	0.04	0.056	0.012	0.032	0.006	0.06	0.012	0.004	0.02	0.006	0.174	0.032	0.036	0.124	0.014	0.092	0.098	0
7	0.008	0.096	0.112	0.01	0.02	0.044	0.002	0.004	0.026	0.01	0.144	0.066	0.126	0.152	0.006	0.018	0.034	0.114	0.004	0.004
8	0.008	0.164	0.026	0.022	0.034	0.194	0.01	0.058	0	0.01	0.012	0	0.002	0.026	0.12	0.1	0.018	0.178	0.016	0.002
9	0	0.154	0.084	0.038	0.022	0.008	0.012	0.048	0.096	0	0.08	0.056	0.12	0.022	0.03	0.01	0.028	0.084	0.1	0.008
10	0.002	0.168	0.03	0.01	0.002	0.008	0.048	0.046	0.01	0	0.136	0.026	0.002	0.12	0.012	0.098	0.084	0.088	0.104	0.006
11	0.018	0.194	0.014	0.07	0.112	0.004	0.016	0.122	0.006	0	0.04	0	0.064	0.004	0.036	0.056	0.086	0.158	0	0
12	0.016	0.176	0.012	0.086	0.01	0.062	0.016	0	0.014	0	0.134	0.066	0.122	0.09	0	0	0.006	0.188	0.002	0
13	0	0.046	0.1	0.146	0.004	0.048	0.164	0.05	0.038	0	0.066	0.006	0.05	0.002	0.064	0.05	0.056	0.076	0.034	0
14	0.006	0.184	0.194	0.074	0.01	0.074	0.01	0.016	0.054	0.01	0.044	0.038	0.11	0.004	0.036	0.04	0.022	0.024	0	0.05
15	0.008	0.178	0.094	0.012	0.006	0.146	0.078	0.008	0.018	0.014	0.002	0.068	0.038	0.012	0.014	0.058	0.072	0.158	0.008	0.008
16	0	0.158	0.186	0.148	0.006	0.002	0	0.004	0	0.002	0.012	0.14	0.022	0.082	0.052	0.034	0.064	0.07	0.006	0.012
17	0	0.18	0.142	0.028	0.004	0	0	0.008	0.022	0.004	0.034	0.08	0.07	0.018	0.106	0.132	0.028	0.104	0.002	0.038

Implied Mean Variance Weights

	AKBNK .IS	ARCLK .IS	EKGYO .IS	EREGL. .IS	GARAN .IS	HALKB .IS	ISCTR. .IS	KCHOL .IS	KRDMD .IS	PETKM .IS	PGSUS .IS	SAHOL .IS	SISE.I S	TCELL. .IS	THYAO .IS	TOASO .IS	TTKOM .IS	TUPRS .IS	VAKBN .IS	YKBN K.IS
1	0.01	0.126	0.188	0.032	0.096	0.004	0.012	0.024	0.018	0	0.162	0.068	0	0.004	0.01	0.028	0.024	0.17	0.022	0.002
2	0.016	0.152	0.044	0.056	0.002	0.034	0.032	0.086	0.084	0.002	0.086	0.004	0.008	0.042	0.168	0.038	0.008	0.092	0.046	0
3	0	0.09	0.056	0.048	0.012	0.07	0.002	0.072	0.168	0.008	0.036	0.052	0.022	0.014	0.078	0.096	0.02	0.144	0	0.012

4	0.01	0.086	0.03	0.12	0.002	0.032	0.01	0.016	0.084	0.006	0.012	0.098	0.062	0.006	0.032	0.146	0	0.178	0.062	0.008
5	0	0.16	0.11	0.05	0.04	0	0.008	0.012	0.004	0.018	0.002	0.048	0.156	0.002	0.02	0.074	0.054	0.17	0.072	0
6	0	0.14	0.046	0.092	0.002	0.032	0	0.058	0.026	0.018	0.01	0.002	0.17	0.096	0	0.076	0	0.19	0.042	0
7	0.006	0.036	0.142	0.14	0.024	0.04	0	0.032	0.102	0.036	0.04	0.036	0.022	0	0.02	0.126	0.002	0.182	0.014	0
8	0	0.06	0.048	0.018	0.014	0.006	0.006	0.172	0.022	0.01	0.052	0.038	0.11	0.12	0.022	0.012	0.088	0.168	0.026	0.008
9	0.008	0.118	0.012	0.118	0.014	0	0.02	0.058	0.038	0.006	0.002	0.148	0.052	0.002	0.148	0.088	0.046	0.116	0	0.006
10	0	0.196	0.112	0.006	0.002	0.026	0.016	0.008	0.044	0.028	0	0.064	0.052	0.096	0.11	0.032	0.096	0.09	0.022	0
11	0.002	0.136	0.152	0.006	0.002	0.072	0.006	0.088	0.022	0.014	0.002	0.03	0.006	0.148	0.014	0.08	0	0.172	0.046	0.002
12	0.006	0.16	0.018	0.008	0.008	0.096	0.126	0.002	0.12	0	0.016	0	0.028	0.062	0.03	0.094	0.022	0.174	0.03	0
13	0	0.18	0.028	0.184	0.004	0.024	0.002	0.02	0.006	0	0.02	0.064	0.046	0.022	0.026	0.16	0.002	0.122	0.088	0.002
14	0.004	0.174	0.104	0.104	0.012	0.006	0.002	0.002	0.046	0.066	0.004	0.028	0.144	0.062	0.106	0.032	0.016	0.086	0.002	0
15	0.008	0.122	0.092	0.11	0.022	0.018	0.02	0.086	0.056	0.008	0.004	0.016	0.164	0.046	0.136	0.018	0	0.034	0.032	0.008
16	0	0.134	0.026	0.18	0.078	0.018	0.004	0	0.008	0.072	0.02	0.004	0.164	0.014	0.012	0.02	0.004	0.186	0.022	0.034
17	0.006	0.148	0.098	0.118	0.002	0.004	0.026	0	0.074	0.02	0.01	0.004	0.188	0.03	0.004	0.05	0.04	0.138	0.018	0.022

Stock Returns

	AKBNK .IS	ARCLK. .IS	EKGYO .IS	EREGL. .IS	GARAN .IS	HALKB .IS	ISCTR.I S	KCHOL .IS	KRDMD .IS	PETKM .IS	PGSUS .IS	SAHOL .IS	SISE.IS	TCELL.I S	THYAO .IS	TOASO .IS	TTKOM .IS	TUPRS. IS	VAKBN .IS	YKBNK .IS
1	0.114889	0.106287	0.017241	0.271493	0.151871	0.135969	0.145241	0.133755	0.137931	0.142032	0.110028	0.101336	0.069544	0.036759	0.107861	0.144583	0.081356	0.146638	0.132463	0.134211
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0.002103	0.03038	0.033898	0.01843	0.001043	0.078098	1.23E-12	0.03952	0.136364	0.126273	0.14931	0.011342	0.111459	0.02015	0.194719	0.00947	0.010972	0.061453	0.056013	0.020882

3	0.0283 32	0.0671 58	0.0360 7	0.1032 86	0.0197 92	0.0682 85	0.0627 67	0.0087 28	0.1666 67	0.0958 41	0.0245 63	0.0214 95	0.0233 5	0.0291 26	0.1118 78	0.0136 5	0.0325 6	0.0652 63	0.0241 95	0.0204 55
4	0.0673 47	- 0.0023	0.0816 33	0.1120 57	0.0766 09	0.1475 29	0.0161 07	0.0123 61	0.1542 86	0.0610 56	0.2839 64	0.0109 8	0.0737 01	0.1105 35	0.0968 94	0.0692 04	0.1618 59	0.0711 46	0.0880 99	0.0111 36
5	0.0114 7	0.0769 2	0.0754 7	0.0484 69	0.0189 75	0.0178 9	0.0105 7	0.1013 43	0.1782 18	0.0295 5	0.1634 85	0.0194 3	0.0133 3	0.0272 16	0.1053 23	0.0226 54	0.0096 6	0.0821 03	0.0227 27	0.0704 85
6	0.0909 1	0.0533 3	0.0884 4	0.0596 1	- 0.0987	0.1821 9	0.0947 9	0.0105 8	0.1782 2	0.0295 7	0.1634 7	0.0194 2	0.0133 8	0.0272 48	0.1053 8	0.0226 8	0.0096 2	0.0821 58	0.0227 7	0.0704 1
7	0.0648 94	0.0880 3	0.0149 3	0.1513 58	0.0774 79	0.0874 6	0.0530 97	0.0373 09	0.0660 79	0.1103 68	0.2014 45	0.0518 96	0.0853 66	0.1166 27	0.1862 86	2.62E- 11	0.0500 7	0.1225 33	0.0095 69	0.0648 15
8	- 0.1019	0.0439 2	0.0681 8	0.0325 8	0.0786 2	0.1971 1	0.1358 5	0.0277 1	0.1074 4	0.0180 7	0.1142 3	- 0.0351	0.0584 3	0.0733 94	0.1811 18	0.0103 63	0.1178 3	0.1120 9	0.0853 1	0.0978 3
9	0.0956 62	0.0863 2	0.1422 76	0.1637 63	0.1155 05	0.2139 64	0.1296 6	0.1206 79	0.4861 11	0.1917 18	0.2745 1	0.0934 12	0.1217 18	0.0806 15	0.2797 72	0.0583 33	0.1318 1	0.0024 75	0.1692 57	0.0457 83
10	0.1086 29	0.1143 1	0.0569 4	0.0069 9	0.1464 55	0.0593 7	0.1549 5	0.0097 4	0.0529 6	0.0218 79	0.0568 05	0.0314 75	0.0638 3	0.0083 98	0.0503 51	0.0339 2	0.0155 3	0.0502 1	0.1122 6	0.0737 33
11	0.0265 6	0.0057 7	0.0037 7	0.1417 09	0.0455 7	- 0.0503	- 0.0559	0.0169 4	0.0177 51	0.0163 73	0.0072 8	0.0636 4	-0.044 0.0173	- 6	0.1571 6	0.0476 5	0.0315 46	0.0129 98	- 0.0239	0.0214 59
12	-0.023	0.0103 72	0.0416 7	0.0774 6	0.0673 5	0.0706 1	0.0605 3	0.0917 2	0.1511 63	0.0086 74	0.0056 4	0.0223 5	0.0794 98	0.0123 9	0.0020 98	0.0007 8	0.0168 2	0.1739 31	0.1142 9	0.0672 3
13	0.1141 4	0.0128 85	0.0395 3	0.0305 3	0.0841 1	0.0759 8	0.0607 2	0.1236 6	0.0277 78	0.1179 4	- 0.2092	0.0292 6	0.1259 7	0.0785 5	0.1255 9	- 0.0747	0.0556 4	- 0.0452	0.0829 5	- 0.1036
14	0.0508 3	0.0807 5	0.0002 8	0.2696 88	0.0325 7	0.0713 4	0.0324 7	0.0182 22	0.1695 33	- 0.2726	0.0553 2	0.0301 5	0.0577 5	- 0.1404	0.0035 9	- 0.0008	0.0557 3	0.0251 21	0.0837 5	0.0100 5
15	0.0635 1	0.0824 3	0.0193 2	0.0917 2	0.0594 8	0.0364 6	0.0436 2	0.0178 95	0.0735 3	0.0276 01	0.0758 3	0.0535 9	0.0628 14	0.0667 56	0.1867 9	- 0.0376	0.1585 2	0.0179 08	0.1023 8	0.0426 6

	-	-		-	-	-	-	-		-	-	-	-	-	-	-	-	-	-	
16	0.03856	0.17311	-0.133	0.082353	0.15155	0.06351	0.10877	-0.0218	-0.0068	0.009091	0.17303	0.03851	0.182033	0.06327	0.276219	0.04406	0.03808	-0.0037	-0.1609	0.14516
	-	-		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
17	0.20332	0.052339	0.045455	0.072464	0.16315	0.06848	0.21457	0.076923	0.03881	0.240991	0.17452	0.16137	0.232	0.18083	0.05787	0.00435	-0.2375	0.096654	0.13537	0.20755

Portfolio Returns

	Benchmark	Min Variance Sample	Min Variance Implied	Mean Variance Sample	Mean Variance Implied
1	0.070191242	0.107393572	0.145086254	0.115881769	0.122721231
2	0.026406287	0.056521286	0.08146239	0.052488688	0.06906799
3	0.029742096	0.041507411	0.061214952	0.028828373	0.061824585
4	0.071019142	0.07663783	0.087008918	0.066020471	0.073914554
5	0.025455685	0.006966634	0.039114166	0.005090235	0.000109779
6	-0.068761233	-0.047393694	-0.052516352	-0.055017004	-0.050528035
7	0.070840387	0.058860439	0.068539675	0.03973587	0.065957056
8	-0.052765488	-0.059218694	-0.055007928	-0.060430459	-0.049888599
9	0.110924887	0.123446213	0.127098307	0.095150802	0.138715839
10	0.030736938	0.013403174	0.002854448	-0.001898175	-0.014846726
11	0.000501091	-0.014410655	0.004689785	-0.006807013	-0.010660567
12	-0.043842573	0.015496351	-0.012867585	0.026542719	-0.003506796
13	-0.094201326	-0.054039354	-0.068134952	-0.064311053	-0.050716558
14	-0.025839785	-0.031394355	-0.005246908	-0.032808342	-0.014668342
15	-0.044841628	-0.050921863	-0.053061319	-0.0236396	-0.026623009
16	0.011400978	-0.074736066	-0.028869442	-0.002191415	0.005769515
17	-0.042194082	0.004488678	0.000655979	0.03193089	0.047487321

R CODES USED IN THE STUDY

```
if(!require(dplyr)) install.packages("dplyr"); library(dplyr)
if(!require(readr)) install.packages("readr"); library(readr)
if(!require(quantmod)) install.packages("quantmod"); library(quantmod)
if(!require(PortfolioAnalytics)) install.packages("PortfolioAnalytics"); library(PortfolioAnalytics)
if(!require(ROI)) install.packages("ROI"); library(ROI)
if(!require(ROI.plugin.glpk)) install.packages("ROI.plugin.glpk"); library(ROI.plugin.glpk)
if(!require(ROI.plugin.quadprog)) install.packages("ROI.plugin.quadprog"); library(ROI.plugin.quadprog)
if(!require(ROI.plugin.symphony)) install.packages("ROI.plugin.symphony"); library(ROI.plugin.symphony)
if(!require(RQuantLib)) install.packages("RQuantLib"); library(RQuantLib)
if(!require(bsts)) install.packages("bsts"); library(bsts)
if(!require(MASS)) install.packages("MASS"); library(MASS)
if(!require(lubridate)) install.packages("lubridate"); library(lubridate)
if(!require(derivmkt)) install.packages("derivmkt"); library(derivmkt)
if(!require(ggplot2)) install.packages("ggplot2"); library(ggplot2)
if(!require(reshape2)) install.packages("reshape2"); library(reshape2)
if(!require(plm)) install.packages("plm"); library(plm)
if(!require(broom)) install.packages("broom"); library(broom)

stocks <- read_csv("Stocks.csv")
index <- read_csv("Index.csv")
# stocks[, 2] <- stocks[, 2]/1000
options <- read_csv("Month-End Options.csv")
dates <- read_csv("Dates.csv")
dates <- as.matrix(dates)
divYield <- read_csv("Dividend Yield.csv")
getOption <- function(stock) {
  singleStock <- filteredStocks %>% dplyr::select(stock)
  singleOption <- filteredOptions %>% filter(UNDERLYING == stock)
  callNumber <- which(abs(singleOption$`strike price` - as.numeric(singleStock)) == min(abs(singleOption$`strike price` - as.numeric(singleStock))))
  callNumber <- callNumber[1]
  finalOption <- singleOption[callNumber, ]
  return(finalOption)
}
uniqueStocks <- colnames(unique(stocks)[-1])
finalOptions <- list()
```

```

for (i in 1:nrow(dates)) {
  tradeDate <- dates[i, 1]
  expiryDate <- dates[i, 2]
  filteredStocks <- stocks %>% filter(Date == tradeDate)
  filteredOptions <- options %>% filter(`TRADE DATE` == tradeDate &
    `End of Month Date` == expiryDate &
    `C/P` == "C")

  tempOptions <- c()
  for (j in 1:length(uniqueStocks)) {
    tempOptions <- rbind(tempOptions, getOption(uniqueStocks[j]))
  }
  finalOptions[[i]] <- tempOptions
}
spotPrices <- c()
for (i in 1:nrow(dates)) {
  tradeDate <- dates[i, 1]
  filteredStocks <- stocks %>% filter(Date == tradeDate)
  spotPrices <- rbind(spotPrices, filteredStocks)
}
for (i in 1:nrow(dates)) {
  finalOptions[[i]] <- cbind(finalOptions[[i]], t(spotPrices[i, -1]))
  finalOptions[[i]] <- cbind(finalOptions[[i]], t(divYield[i, -1]))
  finalOptions[[i]] <- cbind(finalOptions[[i]],
    (as.numeric(finalOptions[[i]]$`End of Month Date`-finalOptions[[i]]$`TRADE DATE`)/365))
  colnames(finalOptions[[i]])[27:29] <- c("Spot", "DivYield", "Maturity")
}
finalIndexOption <- list()
for (i in 1:nrow(dates)) {
  tradeDate <- dates[i, 1]
  expiryDate <- dates[i, 2]
  filteredStocks <- index %>% filter(Date == tradeDate)
  filteredOptions <- options %>% filter(`TRADE DATE` == tradeDate &
    `End of Month Date` == expiryDate &
    `C/P` == "C")
  tempOptions <- getOption("D_XU030D")
  finalIndexOption[[i]] <- tempOptions
}

```

```

spotPrices <- c()
for (i in 1:nrow(dates)) {
  tradeDate <- dates[i, 1]
  filteredStocks <- index %>% filter(Date == tradeDate)
  spotPrices <- rbind(spotPrices, filteredStocks)
}
for (i in 1:nrow(dates)) {
  finalIndexOption[[i]] <- cbind(finalIndexOption[[i]], t(spotPrices[i, -1]))
  finalIndexOption[[i]] <- cbind(finalIndexOption[[i]], 0)
  finalIndexOption[[i]] <- cbind(finalIndexOption[[i]],
    (as.numeric(finalIndexOption[[i]]$`End of Month Date`-finalIndexOption[[i]]$`TRADE
DATE`)/365))
  colnames(finalIndexOption[[i]][27:29] <- c("Spot", "DivYield", "Maturity")
}

finalOptionsOutput <- c()
finalOptionsIndexOutput <- c()
for (i in 1:nrow(dates)) {
  finalOptionsOutput <- rbind(finalOptionsOutput, finalOptions[[i]])
  finalOptionsIndexOutput <- rbind(finalOptionsIndexOutput, finalIndexOption[[i]])
}

write.csv(finalOptionsOutput, "Stock Options Used.csv")
write.csv(finalOptionsIndexOutput, "Index Options Used.csv")
start <- as.Date("2014-03-01")
end <- as.Date("2018-08-31")
returns <- read.csv("Total Returns.csv")
returnDates <- as.Date(returns[, 1])
returns <- returns[, -1]
returns <- xts(returns, order.by = returnDates)
index(returns) <- LastDayInMonth(as.Date(as.yearmon(index(returns), "%mm/%Y")))
returns <- returns/100
write.csv(returns, "Returns.csv")
rf <- 0.126411475409836/12
sampleReturns <- last(returns, 17)
stockMeans <- sapply(sampleReturns, mean) * 12
stockStdDev <- sapply(sampleReturns, sd) * 12^0.5
stockSharpe <- (stockMeans - rf*12)/(stockStdDev)

```

```

write.csv(rbind(stockMeans, stockStdDev, stockSharpe), "Stock Performance.csv")
weights <- read_csv("Weights.csv")[, -1]
historicWeightsOptim <- function(x, startDate, sampleOption, portOption, endDate, rf, i) {
  sampleData <- window(x, start = startDate, end = endDate)
  sampleVol <- apply(sampleData, 2, var)
  sampleStdDev <- apply(sampleData, 2, sd)*(12)^0.5
  # colnames(sampleOption)[7:9] <- c("Spot", "divYield", "maturity")
  # colnames(portOption)[7:9] <- c("Spot", "divYield", "maturity")
  sampleImpVol <- c()
  for (j in 1:nrow(sampleOption)) {
    sampleImpVol <- c(sampleImpVol,
                      bscallimpvol(price=sampleOption$`SETTLEMENT PRICE`[j],
                                   s=sampleOption$Spot[j],
                                   k=sampleOption$`strike price`[j],
                                   d=sampleOption$DivYield[j],
                                   r=rf*12,
                                   tt=sampleOption$Maturity[j]))
  }
  portImpVol <- bscallimpvol(price=portOption$`SETTLEMENT PRICE`,
                             s=portOption$Spot,
                             k=portOption$`strike price`,
                             d=portOption$DivYield,
                             r=rf*12,
                             tt=portOption$Maturity)

  sampleImpVol <- sampleImpVol/12
  sampleImpSD <- sampleImpVol ^ 0.5
  sampleCor <- cor(sampleData)
  portWeights <- weights[i, ]
  samplePortVar <- as.matrix(portWeights) %*% diag(sampleImpSD) %*% sampleCor %*% diag(sampleImpSD)
  %*% t(as.matrix(portWeights))
  L <- matrix(-1/20, 20, 20)
  diag(L) <- 1
  alphaWnum <- portImpVol - samplePortVar
  alphaWden <- as.matrix(portWeights) %*% diag(sampleImpSD) %*% (L - sampleCor) %*% diag(sampleImpSD)
  %*% t(as.matrix(portWeights))
  alphaW <- c(alphaWnum/alphaWden)

```

```

alphaW <- max(alphaW, -1)
impCor <- sampleCor + alphaW * (matrix(1, 20, 20) - sampleCor)
impCov <- diag(sampleImpSD) %*% sampleCor %*% diag(sampleImpSD)
port <- portfolio.spec(assets = colnames((sampleData)))
port <- add.constraint(portfolio = port,
                      type = "full_investment")
port <- add.constraint(portfolio = port,
                      type = "long_only")
port <- add.constraint(portfolio = port,
                      type = "box",
                      min = 0.0,
                      max = 0.2)

minVar.port <- add.objective(portfolio = port,
                            type = "risk",
                            name = "StdDev")
meanVar.port <- add.objective(portfolio = minVar.port,
                             type = "return",
                             name = "mean")

num_assets = ncol(x)
momentargs = list()
momentargs$mu = colMeans(x)
momentargs$sigma = impCov
momentargs$m3 = matrix(0, nrow = num_assets, ncol = num_assets ^ 2)
momentargs$m4 = matrix(0, nrow = num_assets, ncol = num_assets ^ 3)
minVarSample.opt <- optimize.portfolio(R = sampleData,
                                     portfolio = minVar.port,
                                     optimize_method = "random",
                                     momentargs = momentargs,
                                     trace = TRUE)

meanVarSample.opt <- optimize.portfolio(R = sampleData,
                                     portfolio = meanVar.port,
                                     optimize_method = "random",
                                     momentargs = momentargs,
                                     trace = TRUE)

return(list(minVarWeights = extractWeights(minVarSample.opt),
           meanVarWeights = extractWeights(meanVarSample.opt),

```

```

        covar = impCov,
        var = sampleImpVol,
        impCor = impCor,
        portImpVol = portImpVol,
        samplePortVar = samplePortVar,
        alphaW = alphaW,
        impVol = sampleImpVol
    ))
}
minVarWeights <- c()
meanVarWeights <- c()
coVars <- list()
variance <- c()
impCor <- list()
impVol <- list()
for (i in 1:nrow(dates)) {
    startDate <- as.Date(dates[i, 1])
    startDate <- startDate %m-% months(36)
    endDate <- as.Date(dates[i, 1])
    sampleOptimisation <- historicWeightsOptim(x = returns,
                                                startDate = startDate,
                                                endDate = endDate,
                                                rf = rf,
                                                i = i,
                                                sampleOption = finalOptions[[i]],
                                                portOption = finalIndexOption[[i]])
    minVarWeights <- rbind(minVarWeights,
                           sampleOptimisation$minVarWeights)
    meanVarWeights <- rbind(meanVarWeights,
                             sampleOptimisation$meanVarWeights)
    coVars[[i]] <- sampleOptimisation$covar
    variance <- rbind(variance, sampleOptimisation$var)
    write.csv(coVars[[i]], paste0("Implied Covariance ", i, ".csv"))
    impCor[[i]] <- sampleOptimisation$impCor
    impVol[[i]] <- sampleOptimisation$impVol
    write.csv(impCor[[i]], paste0("Implied Correlation ", i, ".csv"))
}

```

[illegible]

```

        training_period = 36,
        rolling_window = 36)
meanVarSample.opt <- optimize.portfolio.rebalancing(R = returns,
        portfolio = meanVar.port,
        optimize_method = "random",
        trace = TRUE,
        rebalance_on = "months",
        training_period = 36,
        rolling_window = 36)
test <- optimize.portfolio(R = returns,
        portfolio = meanVar.port,
        optimize_method = "random",
        trace = TRUE)
chart.Weights(minVarSample.opt, main = "Min Var Sample")
chart.Weights(meanVarSample.opt, main = "Mean Var Sample")

retMinVarSample <- Return.portfolio(R = returns,
        weights = extractWeights(minVarSample.opt))
retMeanVarSample <- Return.portfolio(R = returns,
        weights = extractWeights(meanVarSample.opt))
retMinVarImplied <- Return.portfolio(R = returns,
        weights = minVarWeights)
retMeanVarImplied <- Return.portfolio(R = returns,
        weights = meanVarWeights)
portImpliedRet <- cbind(retMinVarImplied, retMeanVarImplied)
portRetAll <- cbind(benchmarkReturns,
        retMinVarSample,
        retMinVarImplied,
        retMeanVarSample,
        retMeanVarImplied)
colnames(portRetAll) <- c("Benchmark",
        "Min Variance Sample",
        "Min Variance Implied",
        "Mean Variance Sample",
        "Mean Variance Implied")
write.csv(table.AnnualizedReturns(portRetAll, Rf = rf), "Port Performance.csv")
write.csv(portRetAll, "portRetAll.csv")

```

```

chart.CumReturns(portRetAll, main = "Cumulative Returns", legend.loc = "bottom")
chart.Drawdown(portRetAll, main = "Drawdown", legend.loc = "bottom")
minVarSampleWeights <- extractWeights(minVarSample.opt)
meanVarSampleWeights <- extractWeights(meanVarSample.opt)
write.csv(minVarSampleWeights, "Min Var Sample Weights.csv")
write.csv(meanVarSampleWeights, "Mean Var Sample Weights.csv")
covarCalc <- function(x, startDate, endDate, impliedCor, i) {
  sampleData <- window(x, start = startDate, end = endDate)
  sampleCov <- cov(sampleData)
  sampleCor <- cor(sampleData)
  melted_cormat <- melt(sampleCor)
  melted_imp_cormat <- melt(impliedCor)
  # colnames(melted_cormat) <- c("Stock1", "Stock2")
  heatmap <- ggplot(data = melted_cormat, aes(Var1, Var2, fill = value))+
    geom_tile(color = "white")+
    scale_fill_gradient2(low = "blue", high = "red", mid = "white",
                        midpoint = 0, limit = c(-1,1), space = "Lab",
                        name="Correlation") +
    theme_minimal()+
    theme(axis.text.x = element_text(angle = 90, vjust = 1,
                                      size = 8, hjust = 1))+
    coord_fixed() +
    theme(axis.text.y = element_text(size = 8))
  heatmap2 <- ggplot(data = melted_imp_cormat, aes(Var1, Var2, fill = value))+
    geom_tile(color = "white")+
    scale_fill_gradient2(low = "blue", high = "red", mid = "white",
                        midpoint = 0, limit = c(-1,1), space = "Lab",
                        name="Correlation") +
    theme_minimal()+
    theme(axis.text.x = element_text(angle = 90, vjust = 1,
                                      size = 8, hjust = 1))+
    coord_fixed() +
    theme(axis.text.y = element_text(size = 8))
  return(list(sampleCov = sampleCov,
              sampleVar = diag(sampleCov),
              sampleCor = sampleCor,
              heatmap = heatmap,

```

```

        heatmap2 = heatmap2))
    }
    sampleCov <- list()
    sampleVar <- c()
    sampleCor <- list()
    heatmap <- list()
    impHeatmap <- list()
    for (i in 1:length(dates)) {
        startDate <- dates[i]
        startDate <- startDate %m-% months(36)
        endDate <- dates[i]
        calc <- covarCalc(returns, startDate, endDate, impCor, i)
        sampleCov[[i]] <- calc$sampleCov
        sampleVar <- rbind(sampleVar, calc$sampleVar)
        sampleCor[[i]] <- calc$sampleCor
        heatmap[[i]] <- calc$heatmap
        impHeatmap[[i]] <- calc$heatmap2
        ggsave(paste0("Heatmap ", i, ".png"), heatmap[[i]])
        ggsave(paste0("Implied Heatmap ", i, ".png"), impHeatmap[[i]])
        write.csv(sampleCov[[i]], paste0("Sample Covariance ", i, ".csv"))
        write.csv(sampleCor[[i]], paste0("Correlation ", i, ".csv"))
    }
    colnames(variance) <- colnames(sampleVar)
    write.csv(variance, "Implied Volatility.csv")
    write.csv(sampleVar, "Sample Volatility.csv")

```

CURRICULUM VITAE

PERSONAL INFORMATION

Name Surname -Thea Anguridze

✉ teaanguridze@gmail.com

Sex Female | Date of birth 07 April 1990 | Nationality Georgian

EDUCATION AND TRAINING

2013 - 2019

PhD in Finance

Anadolu University

Eskisehir, Turkey

15.09.2017- 19.01.2018

Exchange student

Nanjing University of Aeronautics and Astronautics

Nanjing, China

01.10.2011 – 15.07.2013

Master in business-management (Finance)

International Black Sea University

Tbilisi, Georgia

03.10.2007 – 26.08.2011

Bachelor in business administration (Major Banking and Finance, Minor English language and literature)

Ilia State University

Tbilisi, Georgia

25.09.2013 – 20.06.2014

Turkish language course (TÖMER)

Eskisehir Osmangazi University

Eskisehir, Turkey

September 2012 – January 2013

ACCA qualification in financial Accounting
F3 (Financial Accounting)

Tbilisi, Georgia

Languages:

Georgian, English, Turkish, Russian