DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

PERFORMANCE COMPARISON OF NONPARAMETRIC CUSUM AND EWMA CONTROL CHARTS UNDER MEAN SHIFTS

by

Fatma KAYMAKAMTORUNLARI DENİZ

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İZMİR

PERFORMANCE COMPARISON OF NONPARAMETRIC CUSUM AND EWMA CONTROL CHARTS UNDER MEAN SHIFTS

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled **"PERFORMANCE COMPARISON OF NONPARAMETRIC CUSUM AND EWMA CONTROL CHARTS UNDER MEAN SHIFTS"** completed by **FATMA KAYMAKAMTORUNLARI DENİZ** under supervision of **ASSOC. PROF. DR. ALİ RIZA FİRUZAN** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Ali Rıza FİRUZAN

Supervisor

Prof. Dr. Dursun AYDIN

(Jury Member)

Doç. Dr. Süleyman ALPAYKUT

(Jury Member)

Prof.Dr. Kadriye ERTEKİN

Director

Graduate School of Natural and Applied Sciences

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PERFORMANCE COMPARISON OF NONPARAMETRIC CUSUM AND EWMA CONTROL CHARTS UNDER MEAN SHIFTS

ABSTRACT

Generally, Shewhart control charts, which require normality hypothesis, are used in monitoring process mean. However, Shewhart control charts may not show adequate performance when the distribution of the process in question does not suit the normal distribution and when there are small shifts in the process mean. For this reason, in cases in which the process distribution is not known, it is more beneficial to use nonparametric control charts that do not require any hypothesis about the distribution.

In addition, if there are shifts less than 1.5 sigma, which can be defined as small in the process, preferring the CUSUM (cumulative sum) and EWMA (exponentially weighted moving average) charts, developed as alternatives to Shewhart control charts, would yield more accurate results.

In this study, the nonparametric CUSUM control chart and the nonparametric EWMA control chart, designed with the change-point model and the Mann-Whitney Statistic, were introduced and the simulation study was conducted using the R statistical programming language. In this simulation study, data from four different distributions were generated and the average run length (ARL) values for both control charts were calculated. As a result of the calculated ARL values, both control charts were compared in terms of performance and it was observed that the CUSUM chart performed better than the EWMA chart for all distributions applied under the determined conditions.

Keywords: Statistical process control, CUSUM control chart, EWMA control chart, Average run length

ORTALAMADA KAYMALAR OLDUĞUNDA PARAMETRİK OLMAYAN CUSUM VE EWMA KONTROL KARTLARININ PERFORMANSLARININ KARŞILAŞTIRILMASI

ÖZ

Genellikle, süreç ortalamasının izlenmesinde normallik varsayımı gerektiren Shewhart kontrol kartları kullanılmaktadır. Ancak, Shewhart kontrol kartları ilgilenilen sürecin dağılımı, normal dağılıma uygun olmadığı ve süreç ortalamasında küçük kaymalar söz konusu olduğunda yeterli performansı gösteremeyebilir. Bu nedenle, süreç dağılımının bilinmediği durumlarda, dağılım hakkında herhangi bir varsayım gerektirmeyen parametrik olmayan kontrol kartlarının kullanılması daha yararlı olur.

Ayrıca, süreçte küçük olarak tanımlayabileceğimiz 1,5 sigmanın altında sapmalar mevcut ise shewhart kontrol kartlarına alternatif olarak geliştirilen CUSUM (kümülatif toplam) ve EWMA (üstel ağırlıklandırılmış hareketli ortalama) kontrol kartlarını tercih etmek daha hassas sonuçlar sağlar.

Bu çalışmada, değişim noktası ve Mann-Whitney istatistiği kullanılarak tasarlanan parametrik olmayan CUSUM kontrol kartı ile parametrik olmayan EWMA kontrol kartı tanıtılmış ve R istatistiksel programlama dili kullanılarak simülasyon çalışması yapılmıştır. Bu simülasyon çalışması yapılırken dört farklı dağılımlardan veriler üretilmiş ve her iki kontrol kartına ait ortalama işletim uzunluğu (ARL) değerleri hesaplanmıştır. Hesaplanan ARL değerleri sonucunda her iki kontrol kartı performans açısından karşılaştırılmış ve belirlenen koşullar altında, uygulanan tüm dağılımlar için CUSUM kartının, EWMA kartından daha iyi bir performans sergilediği görülmüştür.

Anahtar Kelimeler: İstatistiksel süreç kontrolü, CUSUM kontrol kartı, EWMA kontrol kartı, Ortalama işletim uzunluğu

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CHAPTER ONE
INTRODUCTION

The increasing competition and the complex industry environment today renders the statistical quality control studies for process improvement rather significant. The control charts, among the statistical quality control tools, are frequently used to monitor the process variability.

In order for the quality control to be performed accurately and reliably, the variability in the services/products due to general and special reasons should be determined correctly and an effort should be spent to minimize the variability arising from special reasons. In this respect, control charts, among other statistical quality control tools, are used frequently in quality studies to eliminate the problems in the process by monitoring the process variability.

Shewhart (1931) combined quality with statistics for the first time and introduced the Shewhart control charts. Although these control charts are quite practical in monitoring the process, it is a significant disadvantage that they are not adequate in small mean shifts and they only work under normality hypothesis. Therefore, the CUSUM control chart and the EWMA control chart, that can identify the minute shifts in the process mean, were developed as alternatives to Shewhart control chart.

When the process distribution is not known, the nonparametric control charts, used independently from the distribution, are preferred. There are many studies in the literature on nonparametric control charts. Hawkins & Qiu (2003) proposed the change-point formulation as a suitable methodology to detect the process variability when the process parameters are not known.

Chakraborti & Wiel (2008), proposed a nonparametric control chart using the Mann-Whitney statistic, and showed that the performance of this control chart is superior to the performance of the classical Shewhart $\bar{x}$ control chart. Zhou, Zou, Zhang & Wang (2009) designed and proposed the nonparametric EWMA control chart using a test statistic they developed based on the change-point model and Mann-Whitney statistic.

Using the moving average sign statistics Riaz developed a sensitive nonparametric EWMA control chart in 2015. In-control performance of the chart was measured with regard to different measures including ARL, SDRL, EQL, RARL, and PCI. Riaz's NPSSEWMA chart showed better performance than the existing NPSEWMA, NPASEWMA and NPSCUSUM charts (Riaz, 2015).

Wang, Zhang & Xiong (2016), proposed a nonparametric CUSUM control chart based on Mann-Whitney statistic and examined the performance of this chart using the ARL approach. Shirke & Barale (2018) proposed the nonparametric CUSUM control chart for process variability and examined the performance of this chart using the average run length (ARL) approach.

My study comprises of six chapters. In the first chapter, general information about the study is given. In the second chapter, definitions about quality and quality control are given. The concept of quality is defined, and the eight dimensions of quality defined by Darvin (1984) are explained. In addition, the concept of quality control is mentioned, and the benefits of quality control are included.

In the third chapter, the topics of statistical process control are defined and the quality control charts, one of the most important tools, are elaborated. Specifically, CUSUM and EWMA control charts, which are effective in small shifts in the process mean, are emphasized. This chapter also describes the average run length (ARL) concept, which allows us to examine the performance of control charts.

The fourth chapter presents the design of the CUSUM and EWMA control charts, which are developed using the test statistic based on change-point model and Mann-Whitney statistic.

In the fifth chapter, a simulation study is carried out using four different distributions including normal, chi-square, student t and lognormal. R statistical programming language is used for simulation study and performances of these two control charts are compared over ARL values. In the sixth and last chapter, the data obtained at the end of the simulation study are examined and the performance of the control charts is interpreted.

CHAPTER TWO
QUALITY CONCEPT

2.1 Definitions of Quality

Despite its multifarious definitions in the literature, it is possible to define quality as the conformance to design purpose and suitability to use, in general. While the American Society for Quality Control (ASQC) defines quality as all of the characteristics reflecting the competencies of a product or service to meet the requirements, Joseph M. Juran (1988) defines it as the suitability for use.

The concept of quality includes subjective values due to its structure varying from person to person, as the concept of quality requires the demands and expectations of the person that would use the service or product be fulfilled. On the other hand, the quality of a service or product that can be measured and determined via quality standards is the objective quality.

In order to question the quality of any product or service, the functional and qualitative properties of the product or service should be known (Montgomery, 2009).

- Functional Properties: All of the properties a product or service should have in order to fulfil the desired purpose (properties that can be expressed numerically such as the dimensions of the rims of a car, the number of questions in a survey, the flexibility of a handbag, etc.)

- Qualitative Properties: All of the properties required to produce the product or service better or at least at the same standard at all times (the conformance of a rim to a certain size, the conformance of the questions in a survey to a standard, the suitability degree of footwear leather, etc.)

2.1.1 Dimensions of Quality

Eight dimensions of quality defined by D. Garvin in 1984 (Ergüven, 2016) are:

- Performance: Performance refers to the functional properties of a product. The primary features that a product must have, such as the battery life of a mobile phone, the fuel consumption of a car, the processor speed of a computer and the customer's requirements. Since the performance characteristics of a product are generally measurable values, it is possible to compare similar products in terms of their performance.

- Features: These are the second-order criteria that enable the product or service affect the customer and complete the basic function. For instance, the quiet operation of a vacuum cleaner, the exterior design of a mobile phone, etc.

- Reliability: Reliability is a criterion whether the product or service will fulfil all the functions adequately to meet the expectations of the customers within the lifetime of the product. For example, the number of times a new dishwasher fails in the warranty period is a sign of reliability.

- Conformance: Conformance is the criterion of how well both the design and the functional properties of the product comply with the predetermined standards. It provides the user with information on the technical dimension of product quality.

- Durability: Durability is a feature related to the lifetime of the product. We can define the technological durability as the period of use until a deformation occurs in a product.

- Serviceability: In case of any questions or problems regarding the product, it is possible to provide concepts such as courtesy, speed, competence and easy accessibility for the user.

- Aesthetic: Aesthetic is one of the subjective quality characteristics. It contains different features such as appearance, sound, smell, taste of a product or the feelings it creates in the users. It is the feature that makes the product attractive to the user even if it has no effect on the performance of the product.

- Perceived Quality: When purchasing a product, users may not always know all the features of the product and may therefore have to make comparisons. In such cases the reputation of the manufacturer is directly related to the demand for that product. For example, for a user who wants to buy a mobile phone, the phone produced by a famous telephone company will be more attractive than the other less well-known company with closer design and features.

2.1.2 Importance of Quality

Producing quality product or services is of vital importance for businesses. The businesses need continuity and sustainability to survive. Therefore, they should set off from the focus point of quality, the customer demand and expectations. The product or service produced should meet the exact demands and expectations of the customers, even surpass these. This situation is due to the competitive market; because, it is possible to access various businesses that offer the same product or service.

The presence of a competitive environment, in fact, is one of the significant factors in the increase in the quality of a product or service arriving to the customer. While quality is not enough alone for gaining customer loyalty, the increase in the quality level is of great importance. What is important is to increase the level of quality to ensure customer satisfaction. Constituting a continuous and increasing level of quality would yield considerable returns for the businesses; because, when developed correctly, quality prevents errors, does not require corrective actions, increases net production, removes delays and prevents overstocks.

Providing quality products and services to customers reduce the costs for the businesses, as well as it prevents waste of time and ensures businesses' existence in commercial markets by increasing customer confidence. To form quality properly, the "Total Quality Management" should be implemented.

2.2 Quality Control

Quality control is the sum of the techniques and actions used to control the conformance of a product or service to predetermined standards. Quality control is a method depending on the continuous monitoring of the quality using statistical methods, and the increase of product quality by taking precautions when necessary.

Quality control is of great significance with regard to determining whether the job is done correctly and to the optimum use of the tools and equipment required for the production, the early detection of defective products or production, if any, and thus preventing the economic loss, providing the customer with the products of the desired characteristics, determining the economic life of the products, ensuring trust in the domestic and foreign markets, and increasing competition.

Therefore, quality control should be performed accurately and reliably. To conduct an accurate and reliable quality control, the defects in the products due to the systematic and random errors should be detected accurately and the systematic errors should be minimized. Statistical quality control is carried out in two ways. The first one is the quality control in the manufacturing process of the product, and it is generally called the "Process Control". The latter is the quality control carried out upon the receipt of the product (Demir, 2008).

2.2.1 Benefits of Quality Control

Before quality control, the inspection operation, which is a function of quality, was being conducted to control the conformance of an end product to the targeted specifications. However, this method is a retrospective method since it controls the conformance of the products, for which the production process is completed. Therefore, after this process the product would be valid or waste. This poses a great risk for the businesses. Because this method causes the businesses to lose time and money.

Quality control, on the other hand, aims at the filtering of products that do not conform to the specifications by ensuring the control of the product in the production process and at taking prudential precautions by preventing the production of these products. In addition, it increases productivity by decreasing the number of waste products and thus the complaints from the customers. By this means, the risk of the businesses to lose time, money and reputation could be minimized.

Quality control is rather a significant tool for both the businesses supplying the product or the service and the customers. Another purpose of quality control practices is to ensure the production of a product or service in the most economical manner possible and the supply of that product or service to the customers with the lowest price possible, without compromise on quality. And by this means, the purpose is to increase the profitability of the businesses by gaining the highest benefits with the best costs, and to increase customer satisfaction at the same time.

CHAPTER THREE
STATISTICAL PROCESS CONTROL

3.1 Definition of Statistical Process Control

A process comprises of certain elements such as operation, material, environmental conditions and operators. These elements may cause variability in the process. The variability that may occur in the process affects the quality of the product/service to be offered. Therefore, the variability in a process should be constantly monitored, analysed and the process should be taken in-control statistically. Using statistical methods to take the process in-control and to maintain control is called statistical process control.

The aims of the statistical process control are to minimize the production costs, to render productions that would meet the specifications, and to ensure its sustainability.

The solution of the problems and continuous improvement depend on conducting statistical interpretations using realistic data. Results obtained incidentally, and the decisions taken depending on these results are not in conformance with the total quality management philosophy. Control of the output properties could be ensured by taking all the process variability and the input in-control.

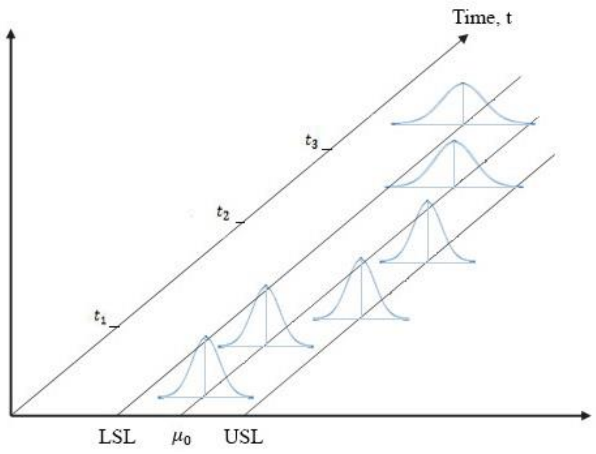
The losses due to delays are prevented by identifying the problems at their origin using statistical process control. In addition, it increases productivity via timely completion by ensuring rapid reactions for solutions of problems identified in the processes. Correspondingly, continuous improvement in the processes and thus the user satisfaction can be provided. Therefore, statistical process control is of great importance for the examination of the process.

The main reason for the problems in the processes is variability. The variability in the processes has two types as chance and assignable causes (Montgomery, 2005).

- Chance causes: These variabilities occur randomly in a process. They are generally caused by environmental effects, e.g. ambient moisture and dust, the ambient temperature and voltage fluctuations, etc. Although it is possible to take precautions for this type of variability since they are foreseeable, it is not possible to eliminate them definitely or it is very expensive to eliminate.

- Assignable causes: There is a certain reason in the process that causes the variability. This reason causes the process to get out-of-control and it is not possible to eliminate variability without eliminating this reason. This kind of variabilities are the ones that are possible to eliminate despite they are not foreseeable, e.g. changing the adjustment of the machinery, inexperienced operators, change in material, etc.

These chance causes and assignable causes are showed in Figure 3.1. Since any value, exceeding the lower and upper specification limit or any mean-shift is present in the process until t_1 time we may assert that the process is in-control. The variability to occur in this process could only arise from chance causes. At t_1 time, variability occurs due to an assignable cause and it causes the operation mean to shift from μ_0 value to μ_1 value. At t_2 time another variability due to another assignable cause occurs. This variability shifts the process standard deviation from σ_0 to σ_1 despite it does not affect the process mean. The variability due to another assignable cause at t_3 time causes the shift of both the process mean and the process standard deviations. As a result, it is clearly seen in Figure 3.1 that the process gets out-of-control att_1, t_2 and t_3 times due to assignable causes (Montgomery, 2009).



Process quality characteristic, x

Figure 3.1 Chance causes and assignable causes of variation (Montgomery, 2009)

While determining the ideal performance levels of the quality characteristics, a room for flexibility to reach to this level should be left. These values between these limits, called the specification limits, are accepted as close to the required level and it is thought that there is not any problem in the process performance. The highest value a quality characteristic can obtain is called the upper specification limit and the lowest value is called the lower specification limit (Adams, 1994). While the specification limits indicate the state a process to have, and the process limits indicate how a process actually behaves.

Statistical process control consists of seven tools: such as pareto diagrams, cause and effect diagrams, check sheets, process flow diagrams, scatter diagrams, histograms, and control charts.

3.2 Control Charts

Control charts are among the most important process monitoring tools used in statistical process control. Control charts exhibit the variations in time of the measurement values obtained from the samples taken in certain periods of the production. By means of these charts, the production process is monitored, any variation is noticed, the reasons for the variability are questioned and it is eliminated. In addition, control charts can be defined as statistical tools used for ensuring and maintaining the targeted level of quality.

Using the control charts, the process ability could be investigated by estimating the parameters of the relevant process. Even though it is not possible to eliminate the process variability definitely most of the time, which is the main purpose of statistical process control, it is possible to minimise the variability using these charts (Montgomery, 2009).

The control chart, as seen in Figure 3.2, comprises of three elements, i.e. the upper control limit (UCL), the lower control limit (LCL) and the central line (CL). The central line shows the mean value of the quality characteristic (target value).

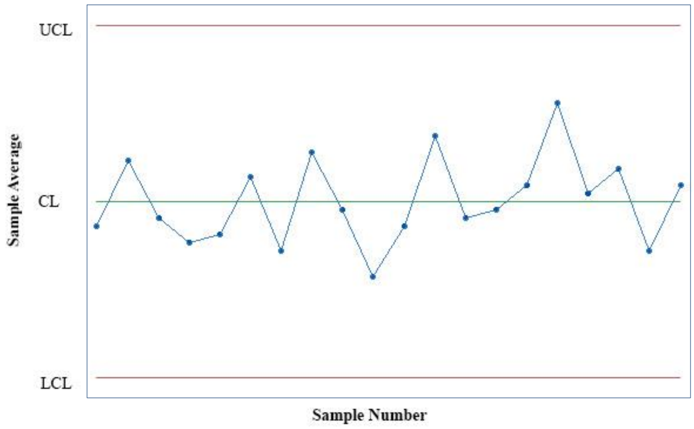


Figure 3.2 A typical control chart

When a control chart is created, first the temporary control limits are defined. Later the process sufficiency is checked, and it is confirmed whether the process is in-control or not. In the event of the process being in-control and being sufficient, the control limits defined temporarily become permanent and the control chart can be used. Otherwise, the control chart should be recreated.

Shewhart control charts are significant tools in taking a process in-control in an economic and reliable manner using statistical methods. As it is known, there are numerous control charts. However, each control chart is used to investigate one single characteristic. For each quality characteristic, a separate control chart should be created.

On the other hand, the products created by a process are evaluated with regard to the measurability and assess ability of a quality control to be investigated. Control charts can be examined in two groups as variable control charts and attribute control charts. If measurable characteristics are in question, the quantitative control charts are used; and if assessable characteristics are in question, the control charts for attributes are used.

3.2.1 Control Charts for Variables

Control charts for variables are used when the characteristics of the products obtained from the process to be controlled are measurable. Measurable quality characteristics such as length, depth, width, temperature and diameter or volume are called variables. When a measurable quality characteristic is at hand, both the mean and the standard deviation of these variables should be controlled.

It is possible to classify the control charts into two groups according to the design structure of the charts, as memoryless control charts and memory control charts. Shewhart control charts are in the memoryless category since they do not consider previous information. CUSUM control charts and EWMA control charts are the most commonly preferred memory control charts in terms of using previous information (Abbas, Riaz & Does, 2014).

3.2.1.1 Shewhart Control Charts

In these charts, it is hypothesised that the points on the chart, which is based on the central limit theorem, exhibit a normal distribution. According to the central limit theorem, regardless of the distribution of the relevant population, the mean distributions of the samples taken from this population converges to normal distribution. In addition, increasing convergence of the means to normal distribution is obtained by increasing the sample size.

Shewhart established the confidence interval of the process mean using the z-test or the t-test as a decision criterion in process control. Accordingly, when a point falls out of the confidence interval the out-of-control signal is formed. Dudding and Jennet, in their 1944 study, asserted the concept of warning limits (Taylor, 2014). In this study, the structure of the Shewhart control charts used today has been established by the combination of a secondary confidence interval with Shewhart's confidence interval by Dudding and Jennet in their aforementioned studies.

99.73% of the observation conforming to normal distribution should be between the control limits and 95.45% of them should be between the warning limits. If the 100 samples are taken in to account, all of the 100 points should be between the control limits. Approximately 5 points are expected to fall between the control limits and the warning limits (Kartal, 1999). These distribution percentages are presented in Figure 3.3.

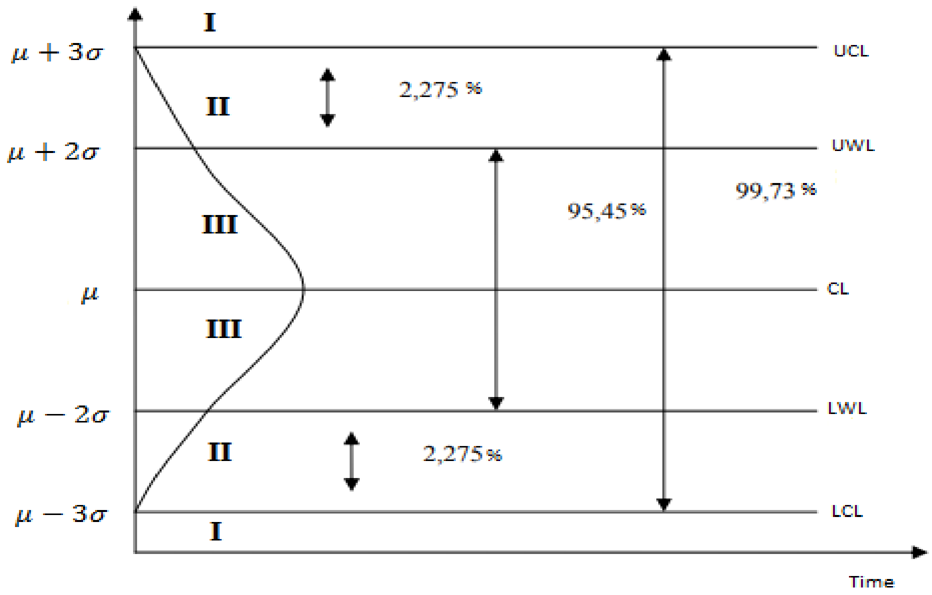


Figure 3.3 Elements and regions of Shewhart control chart

Process variability, in addition to process mean, should be monitored since generally explicit information is required about a process. Both the mean and the variation of a process can be monitored using Shewhart control charts.

Shewhart control charts have two types as $\bar{x} - s$ and $\bar{x} - R$. Process mean is monitored using the $\bar{x}$ control chart, while s and R control charts are used to monitor process variability. While s control chart is a chart established based on the standard deviation values, the R control chart is formed depending on the range values. In cases where the sample size is smaller than 10 the $\bar{x} - R$ control chart is used; and when the sample size is equal to or greater than 10, the $\bar{x} - s$ control chart is used. Generally, the $\bar{x} - R$ control chart is used more frequently due to its easily constituable structure (Montgomery, 2009).

3.2.2 Control Charts for Attributes

Control charts for attributes are used when the products, obtained from the process to be taken under control statistically, are inconvenient for measurement. In these cases, since the controlled quality characteristics do not have any measurement unit, they are classified as binary-categories such as faulty-faultless, appropriate-inappropriate, good-bad, approved-disapproved, regular-irregular, etc. The attribute variables include diameter conformance, surface roughness, and opaqueness level of a product color, etc. (Işığıçok, 2004).

In-control charts for attributes, two basic concepts are used, defect and defective. Defect is a characteristic that does not meet the specifications of the product obtained; defective is a part that has more defects than the permitted number of defects. Control charts for attributes can be classified into four groups as below:

- Fraction Defective (p) Control Chart
- The Number of Defectives (np) Control Chart
- The Number of Defects (c) Control Chart
- The Number of Defects per Unit (u) Control Chart

3.2.3 Other Control Charts

In this chapter, the CUSUM and the EWMA control charts, which were developed as alternative to, and which perform better than the Shewhart control charts in determining the small shifts in the process mean, are explained.

3.2.3.1 CUSUM Control Charts

CUSUM control charts were first asserted by Page (1954) and they were improved by many researchers such as Ewan & Kemp (1960), Johnson & Leone (1962), Lucas (1976), Hawkins & Olwell (1998) (Peerajit, Areepong & Sukparungsee, 2016).

CUSUM control chart considers all the information in the sample data by marking the cumulative sum (C_i) of the algebraic deviations from the target value on the chart. By means of this structure, it can identify small shifts in the process mean faster than the Shewhart control charts. Duncan (1974), Lucas (1976) and Hawkins (1981) state that CUSUM control charts are more useful than the Shewhart control charts when the size of the shift is below 1.5σ (Koshti, 2011).

In industry for the detection of small shifts in process location and dispersion, are frequently used the CUSUM control chart (Abujiya, Riaz & Lee, 2015).

Let $\bar{x}_j = j$ be the arithmetic mean of the sample and μ_0 be the target for the process mean, C_i (i. The cumulative sum value of the sample) is calculated as below (Montgomery, 2009):

$$C_i = \sum_{j=1}^{i}(\bar{x}_j - \mu_0) \tag{3.1}$$

The $\bar{x}$ mean values, which are obtained from process, are distributed around the target value μ_0, the number of the sample values greater than μ would be approximately same as the number of the sample values smaller than μ, and the CUSUM chart would display a horizontal trend. This indicates that the process under control statistically (Banks, 1989).

Since $\bar{x}_j - \mu_0$ differences would increase positively if the mean of the process were gradually increasing, the CUSUM chart would exhibit a positive trend. Similarly, the chart would exhibit a negative trend if the mean of the process were decreasing gradually. The change of the trend in the CUSUM chart is caused by the change in the data. The appearance of a positive or a negative trend in the CUSUM chart indicates that the process shows a trend of going out-of-control or the mean of the process has changed (Murdoch, 1979).

To use C_i value in CUSUM charts, the CUSUM equation is transformed into the form in Equation 3.2 (Osanaiye & Talabi, 1989):

$$C_i = (x_i - \mu_0) + C_{i-1} \tag{3.2}$$

The intended purposes of the cumulative sum techniques can be listed as below (Woodward & Goldsmith, 1964):

- Providing information for the correction of the process by determining small shifts in the mean of the process,
- determining from which sample the shift in the mean of the process has started,
- estimating the current mean of the process reliably,
- making short-term estimations about the future mean of the process.

While determining the changes in the mean of the process by monitoring the changes in the trend of the chart is the most important advantage of the CUSUM control chart, it is a great disadvantage that it fails taking the right decision about the process when the process outcome values exhibit periodical fluctuations (Kartal, 1999). In cases where samples of n = 1 volume are used, the CUSUM control chart is more effective in identifying the changes in the mean of the process, when compared to the Shewhart control charts (Mitra, 1993).

In order to detect whether the process in-control using the CUSUM control charts the V mask procedure proposed by Barnard (1959) is used (Oktay & Özçomak, 2001).

Figure 3.4 presents the V mask. To form the V mask, θ angle and the d length should be known. A line of length d is drawn from the point containing the last C_i value, parallel to the horizontal axis. The P point of the line is the point where the arms of the V mask unite. The arms of the V mask express the control limits of the CUSUM control chart (Svolba & Bauer, 1999).

If all the cumulative sum values, marked in the CUSUM chart, remain between the arms of the V mask, it can be argued that the process is in-control. If any of the cumulative sum values falls outside the arms of the V mask, it is understood that the process has got out-of-control (Mitra, 1993).

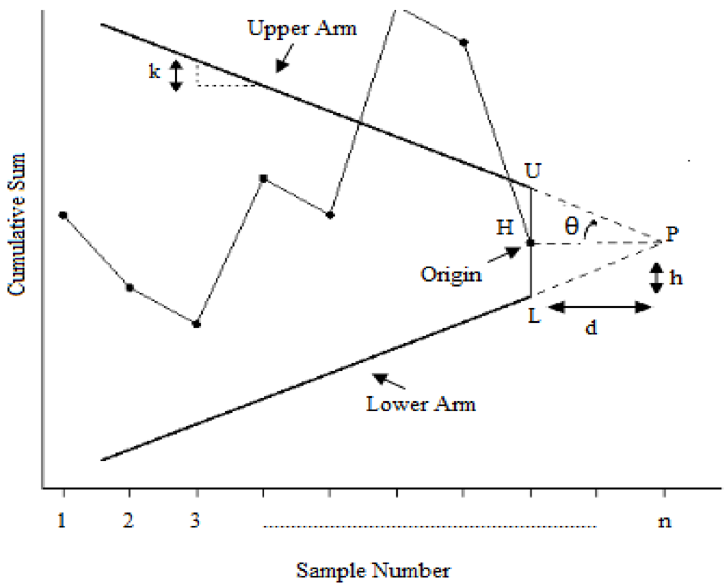


Figure 3.4 V-Mask

- $\sigma_{\bar{x}}$: Standard error for sample mean ($\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$)
- H: Proceeding decision interval, OU or OL length
- h: The value that gives the decision interval in case the sample statistic is multiplied by itself H=h$\sigma_{\bar{x}}$)
- K: inclination of the V mask arms
- k: the value of the slope of the V mask branches in the multiplied result of the sample statistic (K=k$\sigma_{\bar{x}}$))
- d: value of OP length
- θ: Angle between center line and arm (Demir, 2008).

While the cumulative sum values are presented in the vertical axis, the sample numbers are presented in the horizontal axis. When the process generates products at the target values, the number of the positive shifts obtained from the cumulative sum would be the same as the number of the negative shifts, and the chart would generate a horizontal trend. If an upward trend appears in the CUSUM chart, this indicates there is a positive shift. Similarly, a downward trend is observed when there is a negative shift (Lucas, 1976).

3.2.3.2 EWMA Control Charts

The exponentially weighted moving average (EWMA) control chart was introduced by Roberts (1959). EWMA control chart, like the CUSUM control chart, was developed as an alternative to Shewhart control charts and it is effective in small shifts. Its performance is similar to the CUSUM control chart's performance (Patel & Divecha, 2011). However, it is the most important advantage of the EWMA control chart over the CUSUM control chart that it can be formed more easily.

EWMA control charts are used in time series, in addition to the process control. In some cases, they are used in the estimation of the next observation. EWMA control chart can be considered as the weighted mean of all previous observations and the present observation. Therefore, it does not require the normality hypothesis to be supported. It is ideal for cases, in which the sample volume is one (Baray, 2008).

EWMA chart detects shifts through changes accumulated under exponential smoothing (Patel & Divecha, 2011). EWMA is a weighted moving averages method. However, in this method, the last observation gets the greatest weight and the weights of the previous observations decrease geometrically. By this means, it is ensured that the most recent data is considered more.

Let x_i obsrevations be a random variable from a normal distribution with a mean μ , the EWMA statistics is calculated as presented in 3.3:

$$z_i = \lambda x_i + (1-\lambda) z_{i-1} \tag{3.3}$$

where i is the sample number ($i = 1,2..n$), λ is known as the smoothing constant and is chosen such that $0 < \lambda \leq 1$. Since EWMA control charts belong to the memory control charts, determining the λ parameter high would decrease the effect of the previous observations (Fleischer, Lanza & Schlipf, 2008). When $\lambda = 1$, the EWMA value would depend on the last observation only, and the previous observation would not have any effect.

Roberts (1959), Crowder (1987) and Lucas-Saccucci (1987) exhibit that greater λ values yielded optimal results in determining the greater shifts, and the smaller λ values in determining the smaller shifts in the mean-shift (Özçil, 2015).

The quantity z_{i-1} represents the past information. When $i = 1$ initial value is assumed equal to the target mean or the average of the preliminary samples. The in-control mean and the variance of the EWMA statistic are,

$$\mathrm{E}(z_i) = \mu_0, \qquad \mathrm{V}(z_i) = \sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]\right) \tag{3.4}$$

where μ_0 represents the target mean of X and $\sigma_{\bar{X}}$ represents the standard deviation of $\overline{\mathrm{X}}$, $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}$ (Abbas, Riaz & Does, 2014). The center line and control limits for the EWMA control chart are as follows (Montgomery, 2005).

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \tag{3.5}$$

$$CL = \mu_0 \tag{3.6}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \tag{3.7}$$

The factor L indicates the width of the control limits. As the term $\left[1-(1-\lambda)^{2i}\right]$ increases, EWMA control limits will approach steady-state values given by

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{3.8}$$

$$CL = \mu_0 \tag{3.9}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}\,. \tag{3.10}$$

3.2.4 Average Run Length (ARL)

Average Run Length (*ARL*) is the expected number of times a production process will be sampled, before the control chart in use signals a shift of a given size in the process level (Govindaraju, 2005). When the probability of the point representing a quality, characteristic going out-of-control limits is expressed as p, the probability for the first point going out of the control limits are as presented in 3.11.

$$P(run\ length = 1) = p \tag{3.11}$$

The probability for the second point to go out-of-control limits, when the first point is in-control limits is given in equation 3.12. While k-1 points are in-control limits, the probability of getting out-of-control limits at the kth point can be calculated using the equation 3.13 (Burrill & Ledoter, 1999).

$$P(run\ length = 2) = p(1-p) \tag{3.12}$$

$$\vdots$$

$$P(run\ length = k) = p(1-p)^{k-1}, \quad k = 1,2,\ldots \tag{3.13}$$

The mean of this geometric distribution is called the average run length and is calculated as below:

$$ARL = \frac{1}{p} \tag{3.14}$$

The ARL values for any Shewhart chart, in the in-control and out-of-control states, can be calculated using the Type I and Type II errors as follows.

- In-Control ARL: It is the average number of points in a control chart plotted before a false alarm. α (Type I error) is the probability of a point to go out-of-control when the process is in-control, the ARL_0 can be calculated from (Pongpullponsak & Jayathavaj, 2014b);

$$ARL_0 = \frac{1}{\alpha}. \tag{3.15}$$

The ARL is expected to be large when the process is under control.

- Out-of-Control ARL: It is the average number of points after the process goes out-of-control, before the control chart detects it. When the chart does not signal the out-of-control when the process is actually out-of-control, the β (Type II error) error occurs. $1-\beta$ (the power of the test) is the probability of a point to go out-of-control when the process is already out-of-control, and the ARL_1 can be calculated from

$$ARL_1 = \frac{1}{1-\beta}. \tag{3.16}$$

In cases, where the process is out-of-control the ARL is expected to be smaller. Because, when the process goes out-of-control, the quickness of the control chart to detect the situation is acknowledged as an indicator of the performances of control charts (Baray, 2008).

For a better understanding of the term ARL, an example with a sample of size n=5 is presented on a Shewhart $\bar{x}$ chart. If the mean and standard deviation of the process have the standard values μ and σ respectively, the control limits pertaining to these are, (assuming $\mu = 0, \sigma = 1$)

$$LCL = \mu - 3\frac{\sigma}{\sqrt{n}}$$

$$UCL = \mu + 3\frac{\sigma}{\sqrt{n}}.$$

Let us first assume that the process is static (in-control) with regard to the standard values of the process parameters. ($\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{5}}$) and if the process outcomes are normal, then the random variable $\bar{x}$ is normal too. In this case,
$P(\text{one point plots out of control}) = \alpha$

$$\alpha = P[\bar{x} < LCL\ or\ \bar{x} > UCL]$$

$$= 1 - P[LCL < \bar{x} < UCL]$$

$$= 1 - P\left[\frac{LCL-\mu}{\frac{\sigma}{\sqrt{n}}} < \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} < \frac{UCL-\mu}{\frac{\sigma}{\sqrt{n}}}\right]$$

$$= 1 - P\left[\frac{\mu-3\frac{\sigma}{\sqrt{n}}-\mu}{\frac{\sigma}{\sqrt{n}}} < \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} < \frac{\mu+3\frac{\sigma}{\sqrt{n}}-\mu}{\frac{\sigma}{\sqrt{n}}}\right]$$

$$\alpha = 1 - P[-3 < Z < 3] = 0.0027.$$

$P[-3 < Z < 3]$, this probability was obtained using the standard normal distribution table.

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0027} = 370$$

If we interpret this, even though all process outcomes are in-control, (i.e. the process is static, and the parameters are at their standard values), the $\bar{x}$ chart gives an average of one signal in each 370 outcomes.

Assume that the mean of the process shifter 1 standard deviation above its standard value $(\mu + \sigma)$ (when the process standard deviation is at its standard value). In this case,

$$\beta = P[LCL < \bar{x} < UCL]$$

$$= P\left[\frac{LCL-(\mu+\sigma)}{\frac{\sigma}{\sqrt{n}}} < \frac{\bar{x}-(\mu+\sigma)}{\frac{\sigma}{\sqrt{n}}} < \frac{UCL-(\mu+\sigma)}{\frac{\sigma}{\sqrt{n}}}\right]$$

$$= P[-5.24 < Z < 0.76] = 0.7764$$

$$ARL_1 = \frac{1}{(1-\beta)} = \frac{1}{(1-0.7764)} = 4.5.$$

According to this result, when the mean of the process is above almost 1 process standard deviation, we may argue that a 4.5 sample with an average n=5 sample size is required to detect such anomaly.

CHAPTER FOUR
NONPARAMETRIC CONTROL CHARTS

In this chapter, the nonparametric change-point formulation is explained using the Mann-Whitney statistic. An essential recognized nonparametric approach to the case, with an unknown underlying distribution of the process, is the rank based method, with Mann-Whitney statistic being a favoured and robust one.

In the further chapter, the designs of the nonparametric CUSUM and the nonparametric EWMA control charts are elaborated, which are developed based on the test statistic, change-point model and Mann-Whitney statistic.

4.1 The Change-Point Model and Mann-Whitney Statistic

The conventional change-point model can be described as noted below. Let $\{X_i,\ i = 1,2 \ldots n\}$ be of independent random variables, and

$$X_i \sim \begin{cases} F(x;\ \mu_1, \sigma_1^2), & i = 1,2, \ldots \ldots \ldots \ldots . \ldots \tau, \\ F(x;\ \mu_2, \sigma_2^2), & i = \tau + 1, \tau + 2, \ldots, n \end{cases} \tag{4.1}$$

where F denote a continuous distribution function containing unknown types, μ_1, μ_2 denote the process mean, and σ_1^2, σ_2^2 the process variance. If $\mu_1 \neq \mu_2$ or $\sigma_1^2 \neq \sigma_2^2$, then τ is said to be a change-point else the process is said to be in-control. In this study, only shifts in the process mean were examined and let $\sigma_1^2 = \sigma_2^2$ (Hawkins & Qiu, 2003).

To determine the change-point occurring in the process, nonparametric Mann-Whitney two-sample test can be used. For any $1 \leq t < n$, the Mann-Whitney statistics is defined as

$$MW_{t,n} = \sum_{i=1}^{t} \sum_{j=t+1}^{n} I(x_j < x_i) \tag{4.2}$$

where

$$I(x_j < x_i) = \begin{cases} 1, & x_j < x_i, \\ 0, & x_j \geq x_i. \end{cases} \tag{4.3}$$

The expected value and variance of the $MW_{t,n}$ test statistic in the case of in-control is as follows (Zhou et al., 2009).

$$E_0(MW_{t,n}) = \frac{t(n-t)}{2},\ Var_0(MW_{t,n}) = \frac{t(n-t)(n+1)}{12}. \tag{4.4}$$

When there are dependent observations in the data, the variance of $MW_{t,n}$ is usually multiplied by the correction factor,

$$1 - \sum_{i=1}^{r} g_i(g_i^2 - 1)n^{-1}(n^2 - 1)^{-1}. \tag{4.5}$$

where in the n observations r is the distinct number of values and the i^{th} value occurs with a frequency $g_i (\sum_{i=1}^{r} g_i = n)$. In this condition, the in-control variance of $MW_{t,n}$ is

$$Var_0(MW_{t,n}) = \frac{t(n-t)(n+1)}{12}\left(1 - \sum_{i=1}^{r} g_i(g_i^2 - 1)n^{-1}(n^2 - 1)^{-1}\right). \tag{4.6}$$

The standardized Mann-Whitney statistic $SMW_{t,n}$ is denoted by

$$SMW_{t,n} = \frac{MW_{t,n} - E_0(MW_{t,n})}{\sqrt{Var_0(MW_{t,n})}}. \tag{4.7}$$

The distribution of $SMW_{t,n}$ would be symmetric about zero if the process is in-control. Furthermore, large values of $SMW_{t,n}$ show a negative shift in the process, while small values of $SMW_{t,n}$ show a positive shift in the process (Mann & Whitney, 1947). A test statistic for the hypothesis $H_0: \mu_1 = \mu_2$ is proposed by Pettitt (1979) as follows (Zhou et al., 2009).

$$T_n = max_{1 \le t \le n-1} \left| SMW_{t,n} \right|. \tag{4.8}$$

If T_n test statistic is greater than h_n critical value, it can be said that there is a shift in the process mean (i.e., hypothesis H_0 is rejected). If the T_n test statistic is less than or equal to the h_n critical value, there is not enough evidence to say that there is a shift in the process mean. The limiting distribution of T_n proposed by Pettitt (1979) can be used to find the appropriate h_n critical values (Wang et al., 2016).

The CUSUM and EWMA control charts are frequently suggested when a minute shift in the mean is detected at the process's change-point. Assuming that the change-point occurred specifies a mean shift, then the assumption of the Mann-Whitney statistic becomes larger at the change-point, and the sample mean around the change-point might occur as a small shift. In this case, it is difficult to detect the mean shift rapidly via the statistic T_n, since it only relies on the finite individual observations regardless of all historical sample information. For this reason, nonparametric CUSUM and EWMA control charts were constructed according to the standardized Mann-Whitney statistic $SMW_{t,n}$.

Zhou et al. (2009) modified the maximal standardized Mann-Whitney statistic T_n to approximate it to the case, which contains all m IC historical individual and the n future observations, and they recommended the SMW chart, including a test statistic as following

$$T_{m,n} = max_{m-m_0 \le t < m+n} \left| SMW_{t,(m+n)} \right|. \tag{4.9}$$

4.2 Nonparametric CUSUM Control Chart Based on Mann-Whitney Statistic

The test statistic, which is proposed for nonparametric CUSUM control chart by Wang at al. (2017),

$$S_j^+(m,n) = max\{0, S_{j-1}^+(m,n) + SMW_{j,(m+n)} - k\} \quad (4.10)$$

$$S_j^-(m,n) = max\{0, S_{j-1}^-(m,n) + SMW_{j,(m+n)} + k\} \quad (4.11)$$

where $j = m - m_0, m - m_0 + 1, \ldots, m - m_0 + n - 1,$ $0 \leq m_0 \leq m,$ $S_{m-m_0-1}^+(m,n) = S_{m-m_0-1}^-(m,n) = 0,$ k is the reference value and constant k is 0.5 in this study. Set

$$S_{max}^+(m,n) = max_{m-m_0 \leq j \leq m+n-1} S_j^+(m,n), \quad (4.12)$$

$$S_{min}^-(m,n) = min_{m-m_0 \leq j \leq m+n-1} S_j^-(m,n). \quad (4.13)$$

Control rule is as follows,

- After the n[th] future sample is obtained, calculate $S_{max}^+(m,n)$ or $S_{min}^-(m,n)$.
- Let $h_{m,n}$ be the decision value, which is selected to acquire the given IC ARL. If $S_{max}^+(m,n) \leq h_{m,n}$ or $S_{min}^-(m,n) \geq -h_{m,n}$, then finalize as the shift cannot be proven and continue to obtain the (n+1) [st] future sample.
- If $S_{max}^+(m,n) > h_{m,n}$ or $S_{min}^-(m,n) < -h_{m,n}$, then an out-of-control signal is triggered (Wang at al., 2016).

The decision value $h_{m,n}$ can be calculated using the following equations, for the given false alarm probability (type one error) α.

$$Pr\left(S_{max}^{+}(m,n) > h_{m,n}(\alpha) \middle| S_{max}^{+}(m,i) \leq h_{m,i}(\alpha), 1 \leq i < n\right) = \alpha, n > 1, \quad (4.14)$$

$$Pr\left(S_{max}^{+}(m,1) > h_{m,1}(\alpha)\right) = \alpha.$$

Or

$$Pr\left(S_{min}^{-}(m,n) < -h_{m,n}(\alpha) \middle| S_{min}^{-}(m,i) \leq h_{m,i}(\alpha), 1 \leq i < n\right) = \alpha, n > 1, \quad (4.15)$$

$$Pr\left(S_{min}^{-}(m,1) < -h_{m,1}(\alpha)\right) = \alpha.$$

Due to the complexity of this conditional probability, it does not seem possible to solve it analytically. For this reason, these values are calculated by Wang et al. (2017) using one million sequences, coming from the standard normal distribution to estimate, of length 500. It is assumed that the m (historical sample) size is greater than 10.

Table 4.1 indicates the control limit of CUSUM control chart for α values of 0.01, 0.005, 0.0027, and 0.002, that correspond to in-control ARL_s of 100, 200, 370, 500, for m=10 and 50, m_0=4, and n values within the range 1-490. As shown in Table 4.1, $h_{m,n}$ initially increases, but then stabilizes and optimal decision values are obtained using this estimation approach. Furthermore, using the last entries in the same column, it is possible to estimate the missing values in Table 4.1.

Table 4.1 The decision values $h_{m,n}(\alpha)$ of CUSUM control chart (Wang et al., 2016)

ARL_0								
	m=10				**m=50**			
n	**100**	**200**	**370**	**500**	**100**	**200**	**370**	**500**
1	1.120	1.185	1.184	1.114	1.182	1.192	1.235	1.254
3	1.263	1.329	1.216	1.412	1.346	1.395	1.459	1.502
5	1.462	1.586	1.668	1.693	1.554	1.660	1.748	1.827
7	1.611	1.752	1.865	1.914	1.696	1.847	1.974	2.038
9	1.689	1.845	1.972	2.021	1.774	1.945	2.082	2.150
11	1.744	1.916	2.050	2.119	1.833	2.023	2.160	2.238
13	1.798	1.988	2.129	2.167	1.887	2.082	2.238	2.316
15	1.836	2.027	2.178	2.246	1.996	2.121	2.297	2.375
17	1.873	2.076	2.227	2.295	1.950	2.160	2.336	2.484
19	1.897	2.090	2.261	2.334	1.970	2.180	2.360	2.453
22	1.924	2.129	2.295	2.393	1.989	2.198	2.394	2.472
26	1.951	2.170	2.349	2.432	1.999	2.238	2.434	2.522
30	1.985	2.191	2.283	2.481	2.038	2.192	2.453	2.551
35	2.065	2.237	2.422	2.510	2.043	2.297	2.483	2.581
40	2.026	2.268	2.452	2.549	2.063	2.367	2.502	2.600
50	2.049	2.292	2.496	2.588	2.078	2.327	2.522	2.620
60	2.071	2.317	2.530	2.628	2.097	2.346	2.552	2.649
70	2.083	2.333	2.546	2.638	2.107	2.356	2.571	2.669
80	2.091	2.341	2.559	2.657	2.112	2.366	2.591	2.688
90	2.095	2.356	2.569	2.667	2.117	2.386	2.600	2.698
115	2.106	2.372	2.589	2.686	2.122	2.386	2.605	2.708
140	2.118	2.380	2.600	2.690	2.127	2.390	2.610	2.718
165	2.152	2.388	2.608	2.706	2.129	2.395	2.655	2.727
190	2.136	2.405	2.628	2.726	2.142	2.415	2.630	2.742
240	2.134	2.413	2.633	2.735	2.147	2.420	2.635	2.747
290		2.421	2.643	2.745		2.425	2.620	2.752
390			2.658	2.760			2.659	2.767
490				2.795				2.800

The choice of decision values $h_{m,n}(\alpha)$ for different m are considered for the empirical distribution of the CUSUM chart run length (RL). It is seen in Table 4.1 that there is a very small difference of control limits for m = 10 and m = 50. Because, when m_1, m_2 are large enough for CUSUM chart, the distribution of $S_j^+(m_1,n)(S_j^-(m_1,n))$ is very similar to the distribution of $S_j^-(m_2,n)(S_j^+(m_2,n))$. Therefore, the control limits of the CUSUM chart for different m should be close.

4.3 Nonparametric EWMA Control Chart Based on Mann-Whitney Statistic

To better detect the change-point in the process, an EWMA chart is established based on $SMW_{t,(m+n)}$ by Zhou et al. using charting statistic

$$Y_j(m,n) = \lambda \, . SMW_{j,(m+n)} + (1-\lambda). Y_{j-1}(m,n), \tag{4.16}$$

where $j = m - m_0, m - m_0 + 1, \ldots, m + n - 1$, $Y_{m-m_0-1}(m,n) = 0$ and λ ($0 < \lambda \leq 1$) is a smoothing constant. In this study we assume $\lambda = 0.2$. Let

$$Y_{max}(m,n) = max_{m-m_0 \leq j < m+n} \left| Y_j(m,n) \right|. \tag{4.17}$$

Control rule is as follows,

- Following the nth future sample, calculate $Y_{max}(m,n)$.

- Let $h_{m,n}$ be the decision value, which is select to acquire the given IC ARL. If $Y_{max}(m,n) \leq h_{m,n}$then finalize that there is no evidence of a shift and continue to obtain the (n+1)st future sample.

- If $Y_{max}(m,n) > h_{m,n}$, then an out-of-control signal is triggered (Zhou at al., 2009).

After monitoring the $(m+n)$th sample, the difference between the EWMA chart and the CUSUM chart can be obtained as follows: while the EWMA chart is used for calculating the maximum value of exponentially moving average of $SMW_{t,(m+n)}$, the CUSUM chart, for each t, $m - m_0 \leq t < m + n$, calculates the maximum value of the cumulative sum of $SMW_{t,(m+n)} - k$ (the "sum" takes value of zero when the "sum" is less than zero) or the minimum value of the cumulative sum of $SMW_{t,(m+n)} + k$ (the "sum" takes value of zero when the "sum" is larger than zero, then) (Wang et al. 2016).

The smoothing constant λ is taken to be 0.2 for the EWMA chart. Generally, a smoothing constant relatively larger is required to perform the detection of larger shifts quicker and a smoothing constant relatively smaller is required to perform the detection of smaller shifts quicker (Li, 2017).

The proper value of m_0 can be select at the range of $[4,10]$ for $m \geq 10$ (Zhou et al. 2009) and we use $m_0 = 4$ for the CUSUM and EWMA charts.

The decision value $h_{m,n}$ can be calculated using the following equations, for the given false alarm probability (type one error) α.

$$Pr\big(Y_{max}(m,n) > h_{m,n}(\alpha) \big| Y_{max}(m,i) \leq h_{m,i}(\alpha), 1 \leq i < n\big) = \alpha, n > 1, \quad (4.18)$$

$$Pr\left(Y_{max}(m,1) > h_{m,1}(\alpha)\right) = \alpha.$$

This conditional probability does not seem to be possible to solve analytically due to its computational complexity. Therefore, these values are calculated by Zhou et al. (2009) using one million sequences coming from the standard normal distribution to estimate, with a length of 500. The historical sample size is assumed greater than 10.

The control limit of EWMA control chart for α values of 0.01, 0.005, 0.0027, and 0.002, corresponding to in-control ARL_s of 100, 200, 370, 500, for m=10 and 50, $m_0 = 4$, and n values with the range 1-490 are shown in Table 4.2. The $h_{m,n}$ initially increases before stabilizing and optimal decision values are obtained using this estimation approach. Also, last entries in the same column are used to approximate the missing values.

Table 4.2 The decision values $h_{m,n}(\alpha)$ of EWMA control chart (Zhou et al., 2009)

	ARL_0							
	m=10				**m=50**			
N	**100**	**200**	**370**	**500**	**100**	**200**	**300**	**500**
1	1.250	1.305	1.340	1.354	1.310	1.376	1.425	1.444
2	1.355	1.430	1.481	1.501	1.420	1.508	1.576	1.605
3	1.453	1.539	1.604	1.633	1.518	1.625	1.708	1.752
4	1.535	1.641	1.721	1.750	1.605	1.732	1.820	1.869
5	1.602	1.719	1.818	1.853	1.684	1.820	1.918	1.967
6	1.660	1.789	1.896	1.936	1.742	1.889	2.006	2.055
7	1.711	1.852	1.965	2.014	1.796	1.947	2.074	2.133
8	1.754	1.906	2.019	2.072	1.840	2.006	2.133	2.191
9	1.789	1.945	2.072	2.121	1.874	2.045	2.182	2.250
10	1.820	1.992	2.116	2.170	1.903	2.084	2.221	2.289
11	1.844	2.016	2.150	2.219	1.933	2.123	2.260	2.338
12	1.875	2.047	2.180	2.248	1.957	2.152	2.299	2.377
13	1.891	2.078	2.219	2.277	1.977	2.172	2.328	2.406
14	1.914	2.102	2.248	2.316	1.996	2.191	2.357	2.436
15	1.926	2.117	2.268	2.336	2.006	2.211	2.387	2.465
16	1.941	2.133	2.287	2.355	2.021	2.230	2.406	2.484
17	1.953	2.156	2.307	2.375	2.030	2.240	2.416	2.504
18	1.965	2.168	2.326	2.404	2.045	2.250	2.426	2.523
19	1.977	2.180	2.341	2.414	2.050	2.260	2.440	2.533
20	1.988	2.195	2.355	2.434	2.060	2.270	2.455	2.543
22	2.004	2.219	2.385	2.473	2.069	2.289	2.484	2.562
24	2.016	2.234	2.414	2.492	2.079	2.309	2.494	2.582
26	2.031	2.250	2.429	2.512	2.089	2.318	2.504	2.602
28	2.039	2.266	2.448	2.531	2.099	2.328	2.514	2.611
30	2.055	2.281	2.463	2.551	2.108	2.338	2.523	2.621
35	2.070	2.297	2.482	2.570	2.113	2.357	2.543	2.641
40	2.086	2.328	2.512	2.609	2.123	2.367	2.562	2.660
50	2.109	2.352	2.556	2.648	2.138	2.387	2.582	2.680
60	2.121	2.369	2.580	2.678	2.147	2.396	2.602	2.699
70	2.133	2.383	2.596	2.688	2.157	2.406	2.621	2.719
80	2.141	2.391	2.609	2.707	2.162	2.416	2.641	2.738
90	2.145	2.406	2.619	2.717	2.167	2.426	2.650	2.748
115	2.156	2.422	2.639	2.736	2.172	2.436	2.655	2.758
140	2.168	2.430	2.650	2.746	2.177	2.440	2.660	2.768
165	2.172	2.438	2.658	2.756	2.179	2.445	2.665	2.777
190	2.176	2.445	2.668	2.766	2.182	2.455	2.670	2.782
240	2.184	2.453	2.673	2.775	2.187	2.460	2.675	2.787
290		2.461	2.683	2.785		2.465	2.680	2.792
390			2.688	2.790			2.689	2.797
490				2.795				2.800

The choice of decision values $h_{m,n}(\alpha)$ for different m is a significant issue to be taken into consideration. Table 4.2 indicates that there is a very small difference between the control limits for m = 10 and m = 50. Because, when m_1, m_2 are large enough for the EWMA chart, the distribution of $Y_j(m_1, n)$ is very similar to the distribution of $Y_j(m_2, n)$. Therefore, the control limits of the EWMA chart for different m should be close (Zhou et al. 2009).

Table 4.1 and Table 4.2 allow us to choose the control limits for m = 10 and m = 50. These control limits are also expected to be applicable for other values of m. Table 4.1 and Table 4.2 give the run length distribution of CUSUM and EWMA charts for $\alpha = 0.005$, $m = 25$ using the control limits of $h_{10,t}$ and these values are very close to the geometric distribution.

We also obtained the same behaviour the run length distribution for $m = 100, 300, 500$ using the decision values $h_{50,t}(0.005)$ in the simulations. When there is not a strict requirement of in-control behaviour of run-length, using $h_{50,t}(\alpha)$ as the limits for the cases $m > 50$ seems reasonable (Wang et al. 2017).

In practice, quality control required two tasks to be completed. The first one is to determine whether the process is in-control. The latter one is to indicate the shift's position, if there is any. Determining the position of the process change would facilitate and accelerate the detection of the special cause by engineers. Comparable to Zhou et al. (2009), the two-sided CUSUM chart can be supported by obtaining an estimator of the change-point, based on the Mann-Whitney statistic.

By means of the CUSUM control chart it can be inferred that an out-of-control signal is present at $m + n$ observation; in other words, it can be argued that an upward or downward shift has appeared after the τth future sample, $m \leq \tau < m + n$. It can be asserted that a nonparametric estimator, as proposed by Zhou et al. (2009), of the change-point τ of a step shift as

$$\hat{\tau} = arg_{m \leq \tau < m+n} max \left| SMW_{t,(m+n)} \right|, \tag{4.19}$$

taking the properties of the two-test statistic $S_j^+(m_1, n), S_j^-(m_1, n)$ into consideration. Although there are other estimators, obtained by parametric likelihood methods, which are more effective than this one, yet this estimator is an accurate and useful estimator of the position of the change-point (Zhou et al. 2009).

The equality$MW_{t,n} = W_{t,n} - \frac{t(t+1)}{2}$, where $W_{t,n} = \sum_{i=1}^{t} R_i$ and R_i denotes the rank of the ith observation x_i in the total n observations, indicates the equivalence between the Wilcoxon rank-sum statistic and $W_{t,n}$ and the Mann-Whitney statistic$W_{t,n}$. This equivalence can be used to minimize the computational complexity of the Mann-Whitney statistic (Wang et al. 2016).

CHAPTER FIVE
APPLICATION

In this chapter, two simulation examples will be presented with the CUSUM control chart and the EWMA control chart to show the detection of change-point in a process, in which the in-control observation times are fixed and which contains several future observations.

First, a small data set with 4 degree of freedom student t distribution was created and the performance of the CUSUM and the EWMA control charts, based on the Mann-Whitney statistics, were compared. Later, data were generated from different distribution and the performance of the CUSUM and the EWMA control charts were compared with a 10000-repetitition simulation study, which used the generated data.

This simulation study compares the CUSUM and EWMA control charts for $N(0{,}1)$, $\chi^2(4)$, $t(4)$ and lognormal$(0{,}1)$ data with regard to ARL and it contains $m = 10$, $\alpha = 0.005$ and different values of τ. In this tables, δ is the standard deviation coefficient to measure the size of the shift while τ is the change-point, with values 10, 50, 100 and 250 selected for the illustration to be representative (under, k = 0.5 is the reference value and the smoothing constant λ = 0.2).

In the following chapter, two simulation examples will be presented with the CUSUM control chart and the EWMA control chart to show how change-point in a process is detected, in which the in-control observation times are fixed and which contains several future observations.

5.1 An Illustrative Example

To compare CUSUM and EWMA control charts, a small data set was obtained from 4 degrees of freedom t distribution. The first 15 values of this data refer to historical observations and assume that there is an increasing shift in the mean with a standard deviation of 0.75, after the 5th future observation, shown in table 5.1 and table 5.3.

Table 5.1 shows a simulation conducted for the CUSUM charts. The control limits of the CUSUM chart $h_{10,n}(0.005)$ for $m = 10$ are used which yield nearly in-control $ARL \approx 200$. Table 5.1 gives the $S^{+}_{max}(15, n)$ and $S^{-}_{min}(15, n)$ values obtained by using the $h_{10,n}(0.005)$ (for $n = 1,2,3 \ldots 7$) control limits.

Since it is known that the first 15 observation values are under control, the checking starts from the 16th observation value.

Since it yielded $S^{-}_{min}(15,7) = -6.878 < -1.752$ in the 22nd observation value, we can argue that the out-of-control signal is received at this point. According to this result, we may assert that the CUSUM control chart caught the shift at the 20th point in the 22nd point, by giving the out-of-control signal two steps after.

Table 5.1 The CUSUM statistics and control limits in case of a shift after the 20th sample value in the process mean

N	x_n	$S_{max}^{+}(15, n)$	$S_{min}^{-}(15, n)$	$h_{10,n}(0.005)$
1	0.193			
2	0.562			
3	0.727			
4	0.22			
5	0.324			
6	2.176			
7	-0.858			
8	-1.32			
9	-0.781			
10	0.539			
11	-1.314			
12	-0.331			
13	0.672			
14	0.159			
15	0.838			
16	-0.555	0	-0.903	1.185
17	-0.237	0	-1.231	1.276
18	-1.006	0.175	-0.343	1.329
19	-0.833	0.478	-0.176	1.461
20	-0.704	1.096	-0.04	1.589
21	1.454	0	-1.49	1.661
22	1.95	0	-6.878	1.752

In addition, the maximum of $|SMW_{t,(m+n)}|$ for $n = 1,2, \dots ,12$, as shown table 5.2, is the $|SMW_{20,22}| = 2.284$. This value indicates that the estimator of the location of the shift (the change-point) is $\hat{\tau} = 20$, accurately.

Table 5.2 $\boldsymbol{SMW_{t,(m+n)}}$ values for CUSUM chart

n	$\boldsymbol{SMW_{t,(m+n)}}$
1	-0.236
2	0.343
3	0.909
4	0.851
5	0.901
6	1.474
7	0.811
8	0.068
9	-0.434
10	-0.198
11	-0.821
12	-1.253
13	-0.902
14	-1.092
15	-0.599
16	-0.663
17	-0.509
18	-1.022
19	-1.579
20	-2.284
21	-1.655

Table 5.3 presents the simulation study conducted for the EWMA control charts. The control limits of the EWMA chart $h_{10,n}(0.005)$ for $m = 10$ are used, which yield nearly in-control ARL≈200. Table 5.3 gives the values obtained by using the $h_{10,n}(0.005)$ (for n $=$ 1,2,3 ... 12) control limits.

Since it is known that the first 15 observation values are under control, the checking starts from the 16th observation value. Since it yielded the value $Y_{max}(m,n) = 2.244 > 2.047$ in the 27nd observation value, we may argue that the out-of-control signal is received at this point.

According to this result, we may assert that the EWMA control chart caught the shift at the 20th point in the 27th point, by giving the out-of-control signal seven steps after. In this context, as the CUSUM control chart signals after two out-of-control observations, it is possible to argue that the CUSUM chart is quicker than the EWMA chart to detect the shift in the process.

Table 5.3 The EWMA statistic and control limits in case of a shift after the 20th sample value in the process mean

N	x_n	$Y_{max}(m,n)$	$h_{10,n}(0.005)$
1	0.193		
2	0.562		
3	0.727		
4	0.22		
5	0.324		
6	2.176		
7	-0.858		
8	-1.32		
9	-0.781		
10	0.539		
11	-1.314		
12	-0.331		
13	0.672		
14	0.159		
15	0.838		
16	-0.555	0.442	1.305
17	-0.237	0.489	1.430
18	-1.006	0.332	1.539
19	-0.833	0.269	1.641
20	-0.704	0.425	1.719
21	1.454	0.597	1.789
22	1.95	1.247	1.852
23	-0.795	0.79	1.906
24	0.078	1.139	1.945
25	1.563	1.547	1.992
26	2.109	2.004	2.016
27	1.279	2.244	2.047

Also, the maximum of $|SMW_{t,(m+n)}|$ for $n = 1,2,\dots,12$, as shown table 5.4, is the $|SMW_{20,27}| = 2.988$. This value indicates that the estimator of the location of the shift (the change-point) is $\hat{\tau} = 20$, accurately.

Table 5.4 $SMW_{t,(m+n)}$ values for EWMA chart

n	$SMW_{t,(m+n)}$
1	-0.385
2	0.000
3	0.386
4	0.273
5	0.250
6	0.758
7	0.111
8	-0.584
9	-1.080
10	-0.954
11	-1.530
12	-1.952
13	-1.747
14	-1.941
15	-1.610
16	-1.727
17	-1.707
18	-2.160
19	-2.602
20	-2.988
21	-2.566
22	-1.997
23	-2.594
24	-2.469
25	-1.944
26	-1.027

When the results in Table 5.1 and Table 5.3 are considered, the CUSUM and EWMA control charts obtained by using the data set containing a process with a 0.75 shift in the mean and which has a $t(4)$ distribution. When these values are examined, it is seen that the CUSUM chart signalled at the 22nd point for a change-point in the 20th point, while the EWMA chart signalled at the 27th point.

It can be argued that the chart signalling quicker has shown a better performance. Therefore, the CUSUM chart can be argued to perform better than the EWMA chart, since it signalled quicker. Wang et al. (2016) said that the cause of the signal delay of the EWMA chart may be the inertial properties of the EWMA chart.

Haq (2017) proposed a new nonparametric EWMA control chart for monitoring process variability. In the study, extensive Monte Carlo simulations are used to compute the run length profiles of the proposed EWMA chart. The EWMA chart's ability to detect small to moderate shifts in the parameters of the processes render this chart to be preferred frequently. The comparison of the proposed EWMA chart with a recent existing EWMA chart, which is known to have a better performance of other control charts, was conducted to perform a better comparison between the two.

It is found that the performance of the proposed EWMA control chart is essentially and consistently better than the existing powerful EWMA chart. This study also contains a demonstration of the working and implementation of the two aforementioned charts.

An economic design of exponentially weighted moving average control chart was proposed by Hariba & Tukaram (2016), using sign statistic to control the location parameter of the process. Different shifts were used to evaluate the economic performance of this chart. The results yielded that the sample size to detect the shift and the loss cost from the process decreased as the shift in the process location increased. The increasing shift also increases the power of the chart.

With this design, the chart performs better, economically and statistically, for large shifts in the process. In any process with known or unknown process outcome distribution, this economic procedure can be used. In order to check the effect on the statistical and economic performance of the design, a design sensitivity analysis was also carried out, due to the changes in different time and cost parameters.

Li (2017) proposed a new nonparametric adaptive CUSUM chart for detecting arbitrary distributional changes. Any tuning parameter is not needed for the control chart, and it performs efficient computations. The self-starting feature of the proposed control chart renders the chart to be applicable when there is not any sufficiently large reference data. With its built-in post-signal diagnostic function, the proposed control chart can determine the type of distributional changes emerging after an alarm. On the other hand, it is proven that the performance of the proposed control chart is sufficient for a broad spectrum of settings. The said performance compares approvingly with the nonparametric control charts in the literature.

5.2 Simulation Study and Results

In this chapter, we design a simulation study with R statistical programming language for analysing the performances of CUSUM control chart and EWMA control chart. In order to compare performances of the control charts, we simulate data sets from some distributions, namely, normal, chi-square, student t and lognormal. The chi-square, student t, lognormal distributions are chosen because of can represent a wide variety of shapes such as symmetric, skewed, heavy-tailed distributions.

Using the data sets obtained from these distribution, a 10000-repetition simulation study was conducted using the parameters m = 10, $\alpha = 0.005$, change-point $\tau = 10, 50, 100,250$ and the standard deviation coefficient to measure the size of the shift $\delta = 0.0(0.25)3$. The values obtained at the end of these simulations are presented in Tables 5.5, 5.6, 5.7 and 5.8. Here, k = 0.5 is the reference value and the smoothing constant $\lambda = 0.2$.

Table 5.5 (10,000 simulations) presents the comparison between CUSUM and EWMA control charts for $N(0,1)$, data with regard to ARL and it contains m = 10, $\alpha = 0.005$ and different values of τ. In this table, δ is the coefficient of standard deviation for measuring the shift size while τ is the change-point, with values 10, 50, 100 and 250 selected for the illustration to be representative.

It is assumed that the underlying in-control distribution is the standard normal distribution without any loss of generality and the values obtained as a result of the simulation study performed under these conditions are presented in Table 5.5.

We can interpret Table 5.5 as follows:

- Both the CUSUM and the EWMA control charts become rather shift-sensitive as the future observed in-control data are added and these observations update the previous information.

- The performances for both charts in detecting a relatively large mean-shift look alike; however, the sensitivity of the EWMA control chart is sometimes minutely higher than the CUSUM chart. The sensitivity of both charts decreases when the mean shifts in question are relatively large.

- Both charts are comparably sensitive to small shifts depending on the rank information of the data. With smaller means shifts, CUSUM chart has a better sensitivity than the EWMA chart. For example, the ARL of the CUSUM chart is 99.1, equalling approximately to 95.6% of EWMA's, when the mean shift size is about $\delta = 0.75$ standard deviation, and when $\tau = 10$.

Table 5.5 The ARL comparison between CUSUM chart and EWMA chart for $\boldsymbol{N(0,1)}$ data and $\boldsymbol{m = 10}$, $\boldsymbol{\alpha = 0.005}$

	τ = 10		**τ = 50**		**τ = 100**		**τ = 250**	
δ	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**
0.00	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
0.25	174.3	178.9	122.4	128.5	97.4	100.2	74.1	78.3
0.50	139.6	145.1	41.5	45.3	32.1	38.0	25.4	27.9
0.75	99.1	103.7	11.6	15.4	11.3	13.3	11.2	13.1
1.00	51.4	55.7	8.5	11.5	8.6	8.9	9.1	9.8
1.25	25.6	31.2	6.8	9.91	7.2	8.1	7.6	7.4
1.50	13.7	16.7	6.5	7.3	6.7	6.9	6.0	6.1
1.75	7.9	10.3	5.1	5.7	5.3	5.4	5.4	5.1
2.00	6.0	7.4	4.7	5.5	4.6	4.9	4.8	4.8
2.25	5.6	5.8	4.5	5.3	4.5	4.6	4.7	4.5
2.50	4.7	4.5	4.4	4.2	4.4	4.4	4.6	4.3
2.75	4.4	4.3	4.2	4.1	4.3	4.1	4.4	4.1
3.00	4.0	4.0	4.1	4.0	4.0	4.0	4.2	4.0

As it is known, the CUSUM and EWMA control charts for small shifts in the means of the processes, respond to different types of distributions in the underlying process. Normally, any knowledge of the underlying distribution is not present in start-up or short-run situations, and in these situations, a nonparametric or distribution-free scheme is convenient to use, such as the CUSUM chart and EWMA chart used in this study.

The $\chi^2(4)$ distribution, $t(4)$ distribution and the $lognormal\ (0,1)$ distribution are chosen since they could exhibit a spectrum of shapes including symmetric, skewed and heavy-tailed distributions. The process mean shifts of δ = 0.0(0.25)3 times standard deviation are considered. Tables 5.6, 5.7 and 5.8 summarize the performance of both charts simulated with the values m = 10, α = 0.005 and τ = 10, 50, 100, 250.

The data presented in these tables could be interpreted as follows:

- The CUSUM and the EWMA control charts exhibit similar performance results for all types of distribution. Both charts are shift-sensitive at the moderate and small level. However, the CUSUM control chart detects smaller mean shifts somewhat quicker than the EWMA control chart. Therefore, the CUSUM chart is slightly more advantageous.

- Similar performances of the EWMA chart were found under the aforementioned distributions and the normal case in detecting large shifts; but the non-symmetry of the underlying process distribution influenced very slightly the efficacy of the CUSUM chart (as the control limits are generated from the case with a symmetric distribution). Thus, the CUSUM chart does not exhibit a performance similar to the normal cases in detecting large mean shifts. The CUSUM chart performs slightly slower in detecting the large mean-shift than the EWMA chart in cases of normality.

Table 5.6 The ARL comparison between CUSUM chart and EWMA chart for $\chi^2(4)$ data and $m = 10$, $\alpha = 0.005$

	τ = 10		**τ = 50**		**τ = 100**		**τ = 250**	
δ	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**
0.00	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
0.25	193.3	198.6	129.4	134.0	92.9	98.9	61.2	68.5
0.50	164.5	171.4	35.1	39.8	25.4	26.1	18.2	21.3
0.75	117.3	128.1	14.6	16.1	12.6	12.9	9.4	12.2
1.00	78.9	83.4	8.8	9.7	8.3	8.8	6.9	7.5
1.25	44.2	52.3	6.6	8.0	6.2	6.7	6.4	6.7
1.50	23.4	26.7	5.7	6.3	5.4	5.6	5.8	6.0
1.75	17.7	17.2	5.0	5.2	5.1	5.3	5.3	5.1
2.00	12.9	12.1	4.8	5.1	4.8	4.9	4.8	4.9
2.25	7.1	6.9	4.5	4.7	4.6	4.6	4.5	4.7
2.50	6.3	6.0	4.4	4.5	4.4	4.3	4.3	4.2
2.75	5.9	5.8	4.3	4.2	4.4	4.3	4.2	4.1
3.00	5.8	5.5	4.3	4.1	4.3	4.2	4.1	4.0

Table 5.7 The ARL comparison between CUSUM chart and EWMA chart for $\boldsymbol{t(4)}$ data and $\boldsymbol{m = 10}$, $\boldsymbol{\alpha = 0.005}$

	τ = 10		**τ = 50**		**τ = 100**		**τ = 250**	
δ	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**
0.00	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
0.25	163.4	166.9	107.9	111.6	77.3	82.4	53.6	59.1
0.50	129.3	134.1	28.4	33.2	19.7	21.4	16.8	19.4
0.75	73.2	82.8	11.2	13.4	9.3	12.0	10.4	11.2
1.00	34.4	41.2	8.4	8.9	7.3	8.2	7.5	8.0
1.25	18.8	20.5	6.8	7.1	5.9	6.6	6.7	6.9
1.50	12.2	14.6	5.9	6.2	5.2	5.7	5.8	6.0
1.75	7.5	8.7	5.0	5.1	4.5	4.9	5.3	5.4
2.00	6.5	6.9	4.5	4.8	4.3	4.7	4.7	4.9
2.25	5.3	5.6	4.3	4.4	4.2	4.4	4.4	4.6
2.50	4.6	4.8	4.1	4.2	4.1	4.2	4.2	4.3
2.75	4.2	4.5	4.0	4.1	4.1	4.2	4.0	4.1
3.00	4.0	4.2	3.9	4.1	4.0	4.1	3.9	4.0

Table 5.8 The ARL comparison between CUSUM chart and EWMA chart for $\boldsymbol{lognormal(0,1)}$ data and $\boldsymbol{m=10}$, $\boldsymbol{\alpha=0.005}$

	τ=10		**τ=50**		**τ=100**		**τ=250**	
δ	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**	**CUSUM**	**EWMA**
0.00	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
0.25	177.9	186.3	137.9	141.6	102.3	108.1	69.1	73.0
0.50	158.2	160.1	41.0	46.3	22.7	28.7	19.2	23.6
0.75	120.0	126.2	18.6	21.2	12.8	15.1	12.0	12.8
1.00	95.2	99.4	11.4	12.7	10.1	10.9	8.4	9.8
1.25	70.1	73.8	8.0	8.9	8.4	8.7	7.9	8.4
1.50	47.7	60.1	6.6	7.5	6.8	7.0	6.7	7.1
1.75	35.8	40.2	6.3	6.6	6.4	6.7	6.0	6.2
2.00	30.1	34.4	5.8	6.0	6.1	6.3	5.6	5.9
2.25	25.3	29.1	5.1	5.3	5.6	5.6	5.2	5.5
2.50	19.4	22.8	5.0	5.3	5.4	5.5	5.1	5.3
2.75	15.6	17.4	4.8	5.1	5.3	5.3	5.0	5.1
3.00	14.9	15.7	4.7	5.0	5.2	5.1	4.9	5.0

CHAPTER SIX
CONCLUSION

In the nonparametric literature, the Wilcoxon and Mann-Whitney tests are powerful tests for testing location differences, while the Ansari-Bradley test is a robust test for scale differences. Each test can be regarded as a rank-sum test. The Wilcoxon-Mann-Whitney test ranks the data from the smallest to the largest, while the Ansari-Bradley test ranks the data from the center to outward (Li,2017).

In this study, the performances of the nonparametric CUSUM and EWMA control charts, which are created to detect the shifts in the process mean and based on the Mann-Whitney statistic, were compared using the ARL values.

For this comparison, data were generated from different distribution such as $N(0,1)$, $\chi^2(4)$, $t(4)$, $\text{lognormal}(0,1)$ and ARL values were calculated via the simulation studies. With this simulation study, it was found that both control charts performed well for small and moderate shifts, for all types of distribution. It was also found that both control charts performed better as the size of the shift and the change-point value increased.

In addition to these results, the study found that the ARL value of the CUSUM control chart was found slightly lower the ARL value of the EWMA control chart. This means that when k = 0.5 is the reference value and the smoothing constant λ = 0.2, the CUSUM control chart performed slightly quicker than the EWMA control chart in catching small sized shifts in the process mean. This is because the EWMA chart is not very effective in the shifts in the mean of the process.

When the studies in the literature are reviewed, it is seen that this EWMA chart is scale increase-sensitive only and is not that robust in detecting other types of distributional changes when compared to its competitor when used for location shifts, considering Ross and Adams' simulation studies (2012).

The detection power of the resulting control chart would be dependent on the different choices of the λ. A smaller λ would render the EWMA chart to be more robust to smaller changes and the chart with a larger λ would be more robust to detect larger changes (Luccas & Saccucci, 1990). Therefore, according to Wang (2017), it is almost possible to consider the EWMA chart as a CUSUM chart, when the smoothing constant $\lambda < 0.01$.

Although both control charts are advantageous as they are both easily designable and they can be used independently from the distribution, their biggest disadvantage is that they are insufficient when a greater shift occurs in the process mean.

Consequently, it is seen in the study that the performances of the CUSUM and the EWMA control charts are very close and both charts have acceptable performances in the event of small shifts in the process mean. However, our study confirms other studies in the literature that the CUSUM control chart performs slightly better in small shifts in the process mean when compared to the EWMA chart. Accordingly, it is concluded that using the CUSUM control chart to detect shifts in the location parameter would yield more accurate results than the EWMA chart would.

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