PREDICTABLE DYNAMICS IN IMPLIED VOLATILITY SMIRK SLOPE: EVIDENCE FROM THE S&P 500 OPTIONS

A Master's Thesis

by
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Management
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July 2012

To My Mom,

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Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

by

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in

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ANKARA

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Business Administration.

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ABSTRACT

PREDICTABLE DYNAMICS IN IMPLIED VOLATILITY SMIRK SLOPE:
EVIDENCE FROM THE S&P 500 OPTIONS

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July, 2012

This study aims to investigate whether there are predictable patterns in the dynamics of implied volatility smirk slopes extracted from the intraday market prices of S&P 500 index options. I compare forecasts obtained from a short memory ARMA model and a long memory ARFIMA model within an out-of-sample context over various forecasting horizons. I find that implied volatility smirk slopes can be statistically forecasted and there is no statistically significant difference among competing models. Furthermore, I investigate whether these implied volatility smirk slopes have predictive power for future index returns. I find that slope measures have predictive ability up to 20 minutes.

Keywords: Implied Volatility Smirk, S&P 500, High-Frequency

ÖZET

ÖRTÜK OYNAKLIK SIRITMASININ EĞİMİNİN TAHMİN EDİLEBİLİR DİNAMİKLERİ: S&P 500 OPSİYONLARINDAN KANIT

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Temmuz, 2012

Bu çalışma, S&P 500 opsiyonlarının gün içi piyasa fiyatlarından elde edilen örtük oynaklık sırıtmasının eğimlerinde tahmin edilebilir dinamiklerin olup olmadığını araştırmayı amaçlamaktadır. Kısa hafızalı ARMA modeli ve uzun hafızalı ARFIMA modelinden elde edilen tahminleri örneklem dışı yaklaşımıyla çeşitli tahmin aralıklarında karşılaştırdım. Örtük oynaklık sırıtmasının eğiminin istatistiki olarak tahmin edilebileceğini buldum ve rakip modelller arasında istatistiki önemde hiç bir fark bulunmamaktadır. İlaveten, örtük oynaklık sırıtmasının eğimlerinin gelecekteki endeks getirilerini tahmin edebilme gücü olup olmadığını inceledim. Eğim ölçümlerinin 20 dakikaya kadar tahmin edebilme becerisi olduğunu buldum.

Anahtar Kelimeler: Örtük Oynaklık Sırıtması, S&P 500, Yüksek-Frekans

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CHAPTER 1

INTRODUCTION

A *derivative security* is a financial asset whose pay-off depends on the value of underlying asset such as stock, bond, index portfolio, currency, commodity etc. An *option* is a type of derivative security that gives the right, but not the obligation, to trade the underlying asset for a specified price at or before some maturity date. A call (put) option gives the holder right to buy (sell). The specified price in the contract is called the strike price or exercise price. Options can be classified as in-the-money, at-the-money or out-of-the-money, depending on whether their exercise prices are higher or lower than, or same with the spot price of underlying asset. *American options* can be exercised at any time up to the expiration date; however, *European options* can only be exercised on the expiration date.

The valuation of option contracts has a long history. This history actually begins with the Nobel prize winner seminal paper by Black-Scholes (1973). Their option pricing model depends on the no-arbitrage theory of finance. This model assumes that the price of the underlying asset follows geometric Brownian motion with constant volatility. However, empirical findings suggest that although it should be same, implied volatilities calculated from Black-Scholes model tend to differ across exercise prices and times to expiration. This anomaly is called in literature as implied volatility smile, smirk or skew depending on the shape of implied volatility function.

Subsequent option pricing models take form around this anomaly. These models extend the Black-Scholes model in order to reconcile the theory with market data. These models generalize the Black-Scholes model by focusing on finding the right distributional assumption and again depends on the no-arbitrage theory. Although, these models match the implied volatility smile anomaly in some cases, they remain insufficient to suggest satisfactory conclusions. Most of the cases these models converge to Black-Scholes model.

Under Black-Scholes no-arbitrage framework, competitive intermediaries can hedge perfectly their option inventories and so the demand and supply have no effect on option prices. However, recent model developed by Garleanu, Pedersen, and

Poteshman (2009) suggests that risk-averse options market makers cannot perfectly hedge their option inventories. This demand-based option pricing model suggests that the demand for an option can affect its price and so the implied volatility.

Investors with negative (positive) expectations demand put (call) options, and so the implied volatility of put options increase (decrease) relative to that of call options. The difference between these implied volatilities may reflect the investors' risk-aversion. This difference is named as implied volatility smile slope. Slope levels extracted from implied volatility function can contain information about the underlying market.

This thesis aims to investigate whether these slope levels can be predicted or not in high-frequency context. Unlike most of the recent studies that investigate the cross-sectional relationship between slope levels and stock returns in a daily basis, this study use unique high-frequency tick-by-tick S&P 500 (SPX) options data. This is the first study to examine the high-frequency time-series behavior implied volatility smirk slopes. Furthermore, it contributes to literature by examining the high-frequency relationship between slope levels and index returns.

I estimate the ARMA (*2, 1*) and ARFIMA (*2, d, 1*) models for in-sample period. Then I compare their out-of-sample forecasting abilities. I find that slope levels can be statistically predicted. Furthermore, there is no statistically significant differences in forecasting abilities of competing models. In addition to univariate analysis, I also conduct multivariate analysis through investigating the relationship between slope levels and index returns. I find that slope levels have predictive power of index returns up to 20 minutes.

The remainder of the paper is organized as follows: Chapter 2 provides information about the previous literature on option pricing and information flow between markets. Chapter 3 introduces the data used, how variables are calculated and screened. The second section of Chapter 3 gives the univariate and multivariate methodology used. Chapter 4 presents the results of univariate time series models and multivariate regression analyses. Chapter 5 concludes.

CHAPTER 2

LITERATURE REVIEW

2.1 – An Overview of Option Pricing Literature

There is extensive literature on how to value option contracts[1]. I review the most fundamental valuation models such as Black-Scholes, stochastic volatility, jump diffusions models and recent demand-based option pricing model. The no-arbitrage framework forms basis of the valuation of option contracts. Seminal paper by Black-Scholes (1973) shows how this framework can be used to value an option contract.

[1] Broadie and Detemple (2004) survey the literature on option pricing from its beginnings to the present. They provide an extensive review of valuation methods for European and American options.

Besides the assumptions of no taxes, unrestricted trading, transactions costs and portfolio constraints[2], the Black-Scholes option pricing model assumes that price of the underlying asset follows geometric Brownian motion with constant volatility. Consequently, whether they are out-of-the-money, in-the-money or at-the-money, all options on the same underlying asset should give us the same implied volatility. However, we empirically observe from the market that Black–Scholes implied volatilities tend to differ across exercise prices and times to expiration.

Although it should be flat, before October 1987 market crash[3], S&P 500 index option implied volatilities exhibit the so called smile pattern. In other words, out-of-the-money and in-the-money options have higher implied volatilities than at-the-money options. After the market crash, due to the significant change in investor attitudes toward downside risk, we observe a smirk pattern in implied volatilities. Namely, implied volatilities decrease monotonically across exercise prices.

Option prices are a monotonically increasing function of implied volatility. As long as the market price of an option does not violate the no-arbitrage condition, there

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[2] Some authors extend the Black-Scholes model by relaxing these assumptions. See Leland (1985), Hodeges and Neuberger (1989), Boyle and Vorst (1992), and Broadie, Cvitanic and Soner (1998).

[3] On October 19, 1987 the S&P 500 Index closed at 224.84, which represented a one-day log-return of -22.9% from the previous close price of 282.70.

exists a unique solution for implied volatility. Therefore, it is analogous to quote an option contract in terms of implied volatility. With this framework, smirk pattern of implied volatilities means that low-strike options are relatively more expensive than predicted by Black-Scholes model. Such evidence indicates that the implicit stock return distributions are negatively skewed with higher kurtosis than allowable in a Black-Scholes lognormal distribution.

Due to these anomalies, constant volatility and lognormal distribution of asset returns assumptions of Black-Scholes model has been criticized in the option pricing literature. Market option data suggest the need for an option pricing model where the distribution of log-returns of underlying asset have fatter tails and skewed compared to normal distribution. Then, various approaches, motivated by the abundant empirical evidence that the Black-Scholes model exhibits strong pricing biases across both moneyness and maturity, are developed in order to reconcile the theory with the data. Actually, these approaches generalize the Black–Scholes framework by focusing on finding the right distributional assumption[4].

[4] According to Bakshi, Cao and Chen (1997), every option pricing model has to make 3 assumptions; the distributional assumption, the interest rate process, and the market price of factor risks.

One approach is the *stochastic volatility* models that extend the Black–Scholes model by allowing the volatility of the return process to evolve randomly through time[5]. In order control for level of skewness and kurtosis, the correlation between volatility and underlying stock returns, and the volatility variation coefficient can help us respectively. Given the time series properties of asset volatility or option returns, when the volatility process is calibrated using parameter values, the stochastic volatility models are fairly close to Black-Scholes model with an updated volatility estimate (Bates 2003). Therefore, stochastic volatility models with plausible parameters cannot easily match observed volatility smiles and smirks. However, for instance when the asset price and volatility are negatively correlated, the stochastic volatility models of Heston (1993) and Hull and White (1987) can match the smile and smirk patterns.

The other approach is *jump-diffusion* models that augment the Black – Scholes returns distribution with a Poisson-drive jump process[6]. These models assert that the negative implicit skewness and high implicit kurtosis are the result of occasional, discontinuous jumps and crashes. When the mean jump is negative, the jump model of Bates (1996) can also match the smile and smirk patterns. However, when addressing

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[5] For work on stochastic volatility, see Hull and White (1987), Stein and Stein (1991), Heston (1993), Scott (1987), and Fouque et al. (2000).

[6] For work on jump-diffusions, see Merton (1976), Bates (1991), (1996), Amin (1993).

smiles and smirks, they can do fairly well for a single maturity, less well for multiple maturities (Bates 2003). According to Bates (2000), jump models converge towards Black-Scholes model at longer maturities due to the standard assumption of independent and identically distributed returns. In order to solve maturity related option pricing problems of standard jump models, Bates (2000) models jump intensities as stochastic.

Another approach is deterministic volatility function (DVF) approach. Derman and Kani (1994), Dupire (1994), and Rubinstein (1994) develop variations of this approach which assumes that the local volatility rate is flexible but deterministic function of the asset price and time. According to Dumas, Fleming and Whaley (1998), DVF models are the simplest since they pursue the arbitrage argument of the Black –Scholes model. They do not require additional assumptions about investor preferences for risk; they only require estimating the parameters that govern the volatility process.

These attempts to explain the behavior of implied volatility functions focus on relaxing the constant volatility or lognormal distribution of asset return assumptions of Black-Scholes model. These various parametric implications of Black-Scholes model depends on the no arbitrage theory that determines derivative prices independently of investor demand. Recent literature investigates option market participants' supply and demand for different option contracts (differ in exercise prices or expiration dates) in different option markets. This literature suggests that the demand for an option can affect its price.

Bollen and Whaley (2004) point out that the level of implied volatility will be higher or lower depending upon whether net public demand for particular option series is to buy or to sell. They investigate the relation between net buying pressure; which is the difference between the number of buyer motivated contracts and the number of seller motivated contracts, and the shape of implied volatility function[7]. According to Black-Scholes, option supply curves will be flat, so demand for options is not related to corresponding implied volatilities. However, due to the market frictions, supply curves are no longer flat, they are upward sloping. Therefore, excess demand will cause implied volatility to rise and excess supply will cause implied volatility to fall. For instance, out-of-the-money index put options are highly demanded for portfolio insurance by institutional investors[8]. Then we expect downward sloping implied

[7] There is considerable difference between trading volume and net buying pressure implied by Bollen and Whaley (2004). Volume and net buying pressure need not be highly correlated. They say that, although trading volume may be high on days with significant information flow, net buying pressure can be zero if as many public orders to buy as to sell.

[8] Bollen and Whaley (2004) summarize the trading activity in S&P 500 index options and 20 most actively traded options on individual stocks over the period of 1995 through 2000. For index options they show put options (55%) are more traded than call options (45%) compared to option trading activity of stock options. Furthermore, out-of-the-money put options (20.7%) on index are largest

volatility function from out-of-the-money put to at-the-money put option for S&P 500 Index.

In their empirical methodology, they regress the daily change in the average implied volatility of options in a particular moneyness category on contemporaneous measures of security return, security trading volume, and net buying pressure. In addition to these contemporaneous variables, the regression model also includes the lagged change in implied volatility as an explanatory variable. In this model, security return and trading volume are control variables for leverage and information flow effects respectively. With this model Bollen and Whaley test two alternative hypotheses, learning and limits to arbitrage hypothesis. Consistent with the trading activity results of index and individual stock options; for index options, net buying pressure on at-the-money put options has statistically significant power on explaining the change in the level of at-the-money call volatility. On the contrary, not surprisingly, for individual stock options, net buying pressure on at-the-money call options has greater impact on the level of call option implied volatility. Results of other moneyness category models have similar findings. For index put option trading drives the changes in call option implied volatility. However, in individual stocks, call

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trading activity. But, deep-out-of-the-money puts (13.9%) on index are also heavily traded as the at-the-money puts (15.7%). This evidence support the view on use of S&P 500 index puts as portfolio insurance by equity portfolio managers.

option trading drives the movements in the level and the slope of the call option implied volatility.

Garleanu, Pedersen, and Poteshman (2009) develop *demand based option pricing* model. Under Black-Scholes framework, competitive intermediaries can hedge perfectly their option inventories and so the demand pressure has no effect on option prices. Contrary, according to demand based model, risk-averse options market makers cannot perfectly hedge their option inventories because of the impossibility of trading continuously, stochastic volatility, jumps in the underlying asset and transaction costs, and thus demand for an option affects its price. They show that a marginal change in the demand pressure in an option contract increases its price by an amount proportional to the variance that is computed under certain probability measure depending on the demand, of the unhedgeable part of the option. Furthermore, demand pressure increases the price of other options by an amount proportional to the covariance of their unhedgeable parts. Therefore, they indicate that demand pressure increases the price of both particular option and the other options on the same underlying asset.

2.2 – Information Embedded in Option Prices and Predictability of Volatility Spreads

Easley, O'Hara, and Srinivas (1998) develop sequential trade model that features uninformed liquidity traders and informed investors. Unlike uninformed traders who trade in both the equity market and options market for exogenous reasons, informed investors have to decide whether to trade in the equity market, the options market or both. Due to the private information, prices are no longer full-efficient. In the pooling equilibrium of their model, investors with positive expectations can buy the stock, buy a call, or sell a put; in a similar manner, investors with negative signal can sell the stock, buy a put, or sell a call.

According to Easley et al. (1998), due to high leverage that options offer, or many informed traders in the stock market, or illiquidity of the particular stock; informed investors choose trade in options before they trade in the underlying stock. Therefore, changes in option prices can carry information that is predictive of future stock price movements. For instance, buying a call or selling a put increases call prices relative to put prices (call implied volatilities relative to put implied volatilities) and so carries positive information about future stock prices. In a like manner, buying a put or selling a call increases put prices relative to call prices and so carries negative information about future stock prices.

As expected informed traders with negative expectations prefer out-of-the-money puts as insurance for negative shocks when placing their trades. Therefore, the shape of implied volatility skew (the difference between out-of-the-money and at-the-money option implied volatilities) might reflect the risk of negative future news. Actually, according to Xing, Zhang, and Zhao (2010), the volatility skew contains at least 3 levels of information. These levels are “the likelihood of a negative price jump, the expected magnitude of the price jump, and the premium that compensates investors for both the risk of a jump and the risk that the jump could be large”.

Previous literature on information embedded in options market focuses on option trading volume. Pan and Poteshman (2006) investigate that whether option trading volume contains information or not about underlying stock returns. In order to examine the relationship between option trading volume and future stock returns, they form put-call ratios. They find that stocks with lowest put-call ratios (higher call option trading volume relative to put option) outperform stocks with highest put-call ratios (higher put option trading volume relative to call option) by more than 40 basis points on the next day and more than 1% in the next week.

Cao, Chen, and Griffin (2005), test the hypothesis that, in the presence of pending extreme informational events, the options market displaces the stock market as the primary place of informed trading and price discovery. They find substantial evidence of informed option trading prior to takeover announcements. They show that call volume imbalances are strongly correlated with next day's stock return prior to takeover announcements. Furthermore, their cross-sectional results indicate that the higher the takeover premiums, the higher the preannouncement call imbalance increases. They conclude that before the extreme announcement, the options market plays a more important role than the stock market when information asymmetry is severe.

With the motivation of sequential trade model, Cremers and Weinbaum (2010) show that predictability of volatility spread is reflection of informed trading; namely, informed investors first trade in options market. In this respect, they show that stocks with more asymmetric information more likely experience deviations. They use the term deviations from put-call parity not violation of put-call parity, because rather than pure arbitrage opportunities, they view the differences between put and call implied volatilities as proxies for price pressure. These differences may be the result of market imperfections and data related issues, or may due to the short sale constraints, or come from the trading activity of informed traders.

In order measure deviations from put-call parity, they take the difference between implied volatilities of call and put options that have same exercise price and time to maturity. Because equality of implied volatilities of pairs of put and call options implies put-call parity. Their main result is that these deviations contain economically and statistically significant information about future stock returns. They form a portfolio consists of long position in stocks with high volatility spread (relatively expensive calls) and short position in stocks with low volatility spread (relatively expensive puts). This portfolio has adjusted abnormal return of 50 basis points per week in January 1996 and December 2005 period. Furthermore, contrary to previous literature, they show that this result is not driven by short sale constraints. Thus they present strong evidence that option prices contain information that is not yet incorporated in stock prices.

Similar intuition with Cremers and Weinbaum (2010), but different approach, Xing, Zhang and Zhao (2010), show that the shape of the volatility smirk contains information about future equity returns. Stocks with steepest smirk meaning higher difference between out-of-the-money put implied volatility and at-the-money call implied volatility, underperform stocks with least volatility smirk by 10.9% per year on a risk-adjusted basis. They also test the persistency of predictability of volatility smirk, and find that predictability persists at least 6 months.

They investigate whether any relationship exists between volatility smirk and future earnings shock of individual stocks. They find that firms with steepest volatility smirks experience the worst earning shocks. Their results, therefore, indicate that the shape of volatility smirk comes from the firm fundamentals. Informed traders that are worried about possible future earnings shocks prefer out-of-the-money puts as insurance. Then, the price of these put options become expensive, and so their implied volatilities become higher compared to at-the-money call options. Consequently, we observe more pronounced smirk shapes in implied volatility functions.

Yan (2011) uses the slope of option implied volatility smile as a proxy for jump risk because options are forward-looking contracts and can provide ex ante measures of jump risk. In the presence of jump risk expected return is a function of the average jump size. According to jump processes literature, this implies there exists a negative relation between the slope of implied volatility smile and stock return. Contrary to previous literature, he does not find predictability power of difference between out-of-the-money put options' implied volatilities and at-the-money call option implied volatility. He uses local steepness that is calculated by taking the difference between implied volatility of put with a delta of -0.5 and implied volatility of call with a delta of 0.5, as a slope of implied volatility smile.

He forms five portfolios according to his local slope measure. Then, he shows that the average monthly portfolio returns decreases from 2.1% for quintile one (lowest slope) to 0.2% for quintile five (highest slope). The average monthly return of the long-short portfolio is 1.9% which is also economically and statistically significant.

As can be seen from the brief review of literature on implied volatility smirk slope levels, there is no study on high-frequency dynamics on these slope levels. This thesis aims to analyze the high-frequency predictability of implied volatility smirk slope levels. Furthermore, previous studies examine the relationship between slope levels and individual stock returns on cross-sectional basis. This study examines the time series relationship between slope levels and index returns in high-frequency context.

CHAPTER 3

DATA & METHODOLGY

3.1 Empirical Data

In this study my sample consists of tick-by-tick data of S&P 500 Index options (SPX) that covers the year 2006. It starts January 2006 and ends December 2006, comprising a total of 250 trading days. I use 3 month US T-Bill Secondary Market Rate as daily risk free rate in option implied volatility calculations.

Options data is obtained from the Berkeley Options Database that is derived from the Market Data Report (MDR file) of the Chicago Board Options Exchange (CBOE). This data provide entire information for every option quote. The information contains quote date, quote time (in seconds), expiration date, put – call code, exercise price, bid and ask prices and underlying instrument price. US T-Bill Secondary Market Rate and daily S&P 500 dividend yields are obtained from Datastream database.

One of the problems faced in high frequency data is the irregular time intervals between ticks. The raw financial time series data is not suitable to work with, because market ticks arrive at random time (stochastic). Time series are irregularly spaced and this is called *inhomogeneous* data problem in the literature. Regular time series econometrics tools cannot be applied to the inhomogeneous series (Daconogra et al., 2001). Because these tools mostly depend on backward operators and these operators will only work with regularly spaced time series data. For this reason, I use averages of prices for every 5 minute to transform our inhomogeneous time series to the homogeneous data[9].

Another problem is the nonsynchronous trading in the options market (CBOE) and the equity market (NYSE). Trading hours on the Chicago Board Options Exchange begin at 8:30 a.m. (CST) and end at 3:15 p.m. (CST); however, New York

[9] In high-frequency finance literature, this transformation is done in various ways. Some authors use the last tick available in the interval, some use linear interpolation.

Stock Exchange (NYSE) closes at 4:00 p.m. (EST) and this corresponds to 3.00 p.m. (CST). Therefore, I delete all option quotes after 3:00 p.m. (CST) in order not to have nonsynchronicity problem in my analysis. As a result, during each trading day, I have 78 5-minute intervals[10]. Data for each time interval consists of index level, bid and ask prices of call and put options, implied volatilities calculated from Black-Scholes model and slope measures. In the next section I will go in to the details of the estimation of the implied volatility and slope measures.

3.1.1 Implied Volatility Estimations

Implied volatility estimations are an important part of this research as I will be analyzing the predictable patterns of the difference in implied volatilities at various strike prices. Furthermore, I will be analyzing the relationship between these differences and the index level. Following the literature, first I filter the tick by tick options data based on maturity, no-arbitrage option boundaries and also obvious reporting errors and outliers.

[10] 2 of the 250 trading days operated until afternoon. Therefore, I have 45 5-minute intervals for these days.

I use options that have maturities between 15 and 45 trading days. According to Dumas et. al.(1998), the shorter term options have relatively small time premiums. As the options approaches to maturity, delivery can distort the prices. These options also have the highest liquidity. For this reason, due to the possible measurement errors and nonsynchronous option prices, implied volatility estimations are extremely sensitive. I also observe that options that have maturities shorter than 15 trading days are substantially unreliable when calculating option implied volatilities.

Another problem in calculating option implied volatilities is options with zero bid prices. Therefore, I use options that have bid price greater than zero[11]. Next, I also eliminate options that violate the no-arbitrage lower option boundaries in order to calculate implied volatilities.

For call options;

$$\text{Lower bound} \implies C \geq S_0 * e^{(-dy*t)} - K * e^{(-r*t)}$$

For put options;

$$\text{Lower bound} \implies P \geq K * e^{(-r*t)} - S_0 * e^{(-dy*t)}$$

[11] In a same manner, but a bit different approach, some authors use options with bid-ask midpoints higher than 0.125 or 0.25.

where C, P, K and S_0 are call, put, exercise and underlying index level respectively, *r* is the risk free rate, *dy* is the dividend yield and *t* is time to maturity.

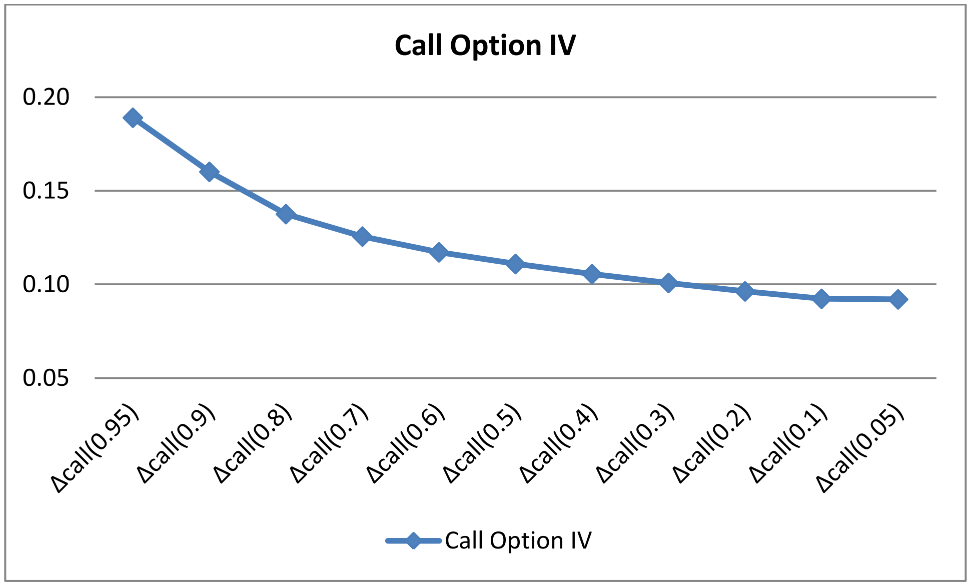
Put-Call parity violations are not filtered as they might contain information regarding the implied volatility calculations. Cremers and Weinbaum (2010), show that put-call parity implies the equality of implied volatilities of pairs of put and call options that have same exercise prices. They find that deviations of implied volatilities contain economically and statistically significant information about future stock returns.

3.1.2 Variable Construction

Implied volatilities of S&P 500 Index options are obtained from the 5 minute interval bid and ask option prices by using the Black-Scholes option pricing formula. Since S&P 500 Index options are European style, using the BS formula in implied volatility calculation does not pose a practical problem. I use option deltas as a measure of option moneyness. For instance, I take a call option with $\Delta_{call} = 0.5$ as at-the-money call option. Similarly, I take a put option with $\Delta_{put} = -0.5$ as at-the-money put option. Although these options are not exactly at-the-money, they are very close to being at-the-money (Yan, 2011). I denote fitted implied volatilities of put and call options with deltas equal to Δ_{put} and Δ_{call} as $\nu_{put}^{imp}(\Delta_{put})$ and $\nu_{call}^{imp}(\Delta_{call})$ respectively. Appendix A gives the descriptive statistics of implied volatilities across various moneyness levels.

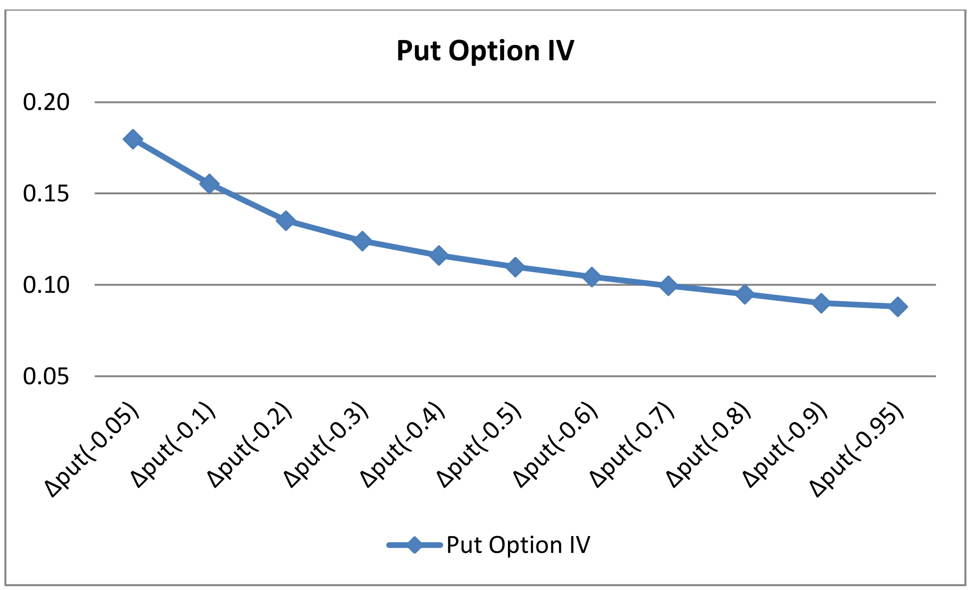
As it is discussed in the previous parts, I observe from the data that Black–Scholes implied volatilities tend to differ across moneyness. Although it should be flat theoretically, I observe a smirk pattern in implied volatilities as reported in previous studies. Namely, implied volatilities decrease monotonically with decreasing moneyness. For call options, option implied volatilities decrease from in-the-money to out-of-the-money. On the contrary, for put options, option implied volatilities decrease from out-of-the-money to in-the-money. Figure 3.1 and 3.2 show the averages of call option and put option implied volatilities across deltas respectively for the whole data period.

Figure 3.1: Call Option Implied Volatilities across Option Deltas



Note: This figure shows the averages of call option implied volatilities across call option deltas. In x-axis, from left to right, we move from deep-in-the-money call option (Δ_{call} = 0.95) to deep-out-of-the-money call option (Δ_{call} = 0.05).

Figure 3.2: Put Option Implied Volatilities across Option Deltas



Note: This figure shows the averages of put option implied volatilities across put option deltas. In x-axis, from left to right, we move from deep-out-of-the-money put option (Δ_{put} = -0.05) to deep-in-the-money put option (Δ_{put} = -0.95).

I define implied volatility smirk slope as the difference between put option implied volatility of specific moneyness and at-the-money call option implied volatility which is consistent with the recent literature. By using six different put options with respect to their deltas, I calculate six different slope levels as follows;

$$S_0 = \nu_{put}^{imp}(-0.5) - \nu_{call}^{imp}(0.5)$$

$$S_1 = \nu_{put}^{imp}(-0.4) - \nu_{call}^{imp}(0.5)$$

$$S_2 = \nu_{put}^{imp}(-0.3) - \nu_{call}^{imp}(0.5)$$

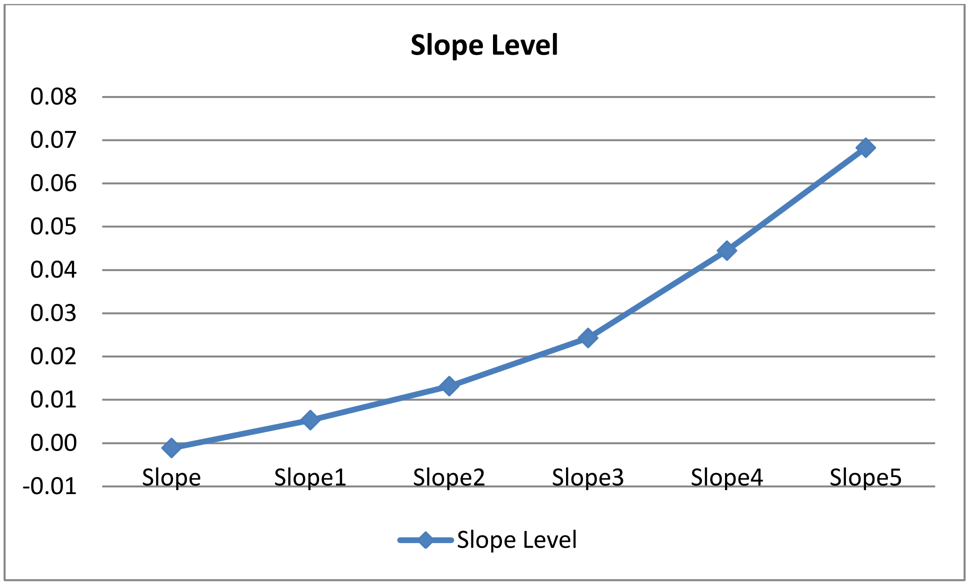
$$S_3 = \nu_{put}^{imp}(-0.2) - \nu_{call}^{imp}(0.5)$$

$$S_4 = \nu_{put}^{imp}(-0.1) - \nu_{call}^{imp}(0.5)$$

$$S_5 = \nu_{put}^{imp}(-0.05) - \nu_{call}^{imp}(0.5)$$

The slopes are calculated starting with the at-the-money put options and moving to deep-out-of-the-money put options from S_0 to S_5. As reported in earlier studies, I expect the slope to increase from S_0 to S_5. Because, put option implied volatility increases when we move from at-the-money put option to the out-of-the-money put option. Appendix A and B provides the descriptive statistics and time series plots for each slope levels respectively. Figure 3.3 depicts the averages of the slopes for the whole data period.

Figure 3.3: Implied Volatility Smirk Slope Levels



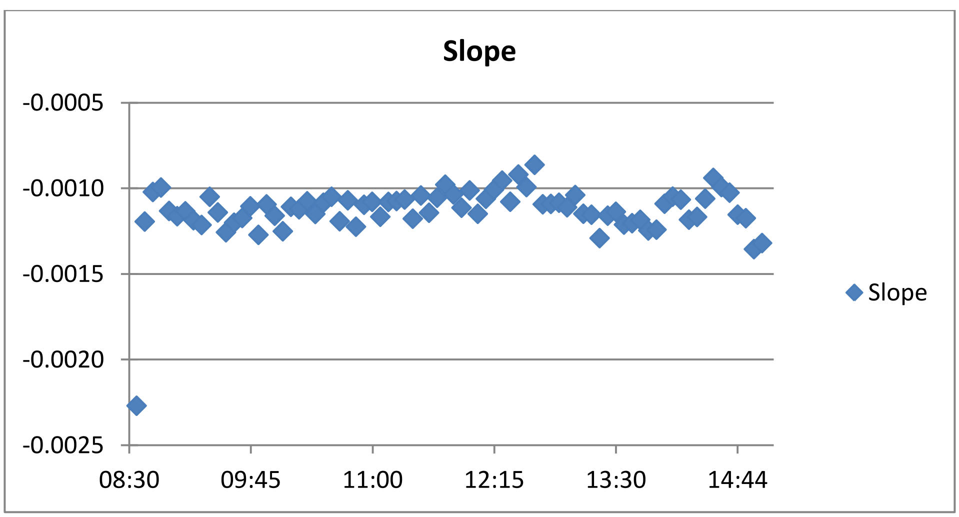
Note: This figure shows the pattern of slope levels across different slope definitions. As it can be observed from the figure, slope level increases when we move from at-the-money put option to deep-out-of-the-money put option.

3.1.3 Data Screening

In this section I perform diagnostic tests on the 5 minute intraday data. In high frequency analysis, it is important to examine the data for intraday patterns and to properly correct the data for further analysis. In addition to intraday patterns, stationarity conditions are also checked. As, non-stationarity can cause spurious regressions with erroneous estimates and results.

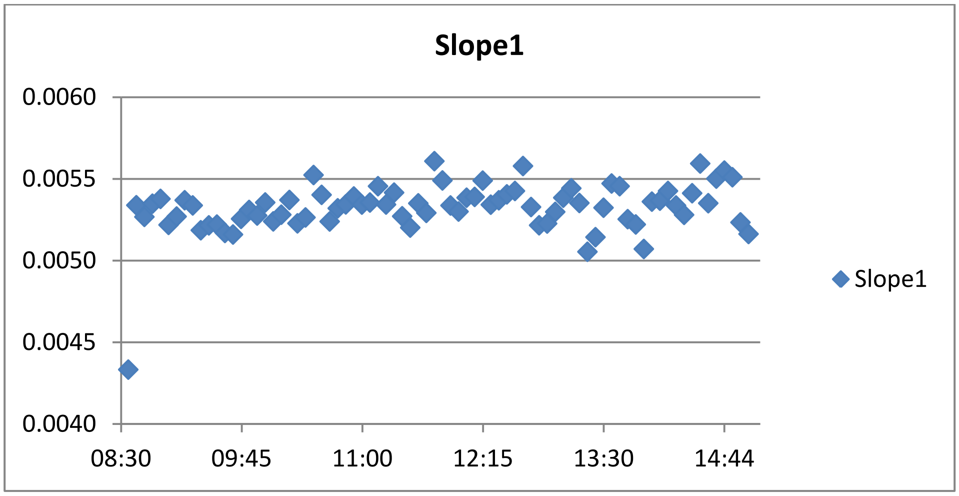
For intraday pattern diagnosis, I calculate the average of each variable for each 5 minute time interval in 250 trading day. Just for change in index level I use the absolute values when calculating average of time intervals in order not the change in prices cancel each other. Then I plot the averages across the time intervals whether any pattern exist or not. Figures 3.4a to 3.4f give the plots of average values of time intervals for slope measures.

Figure 3.4a: Intraday Pattern of S_0



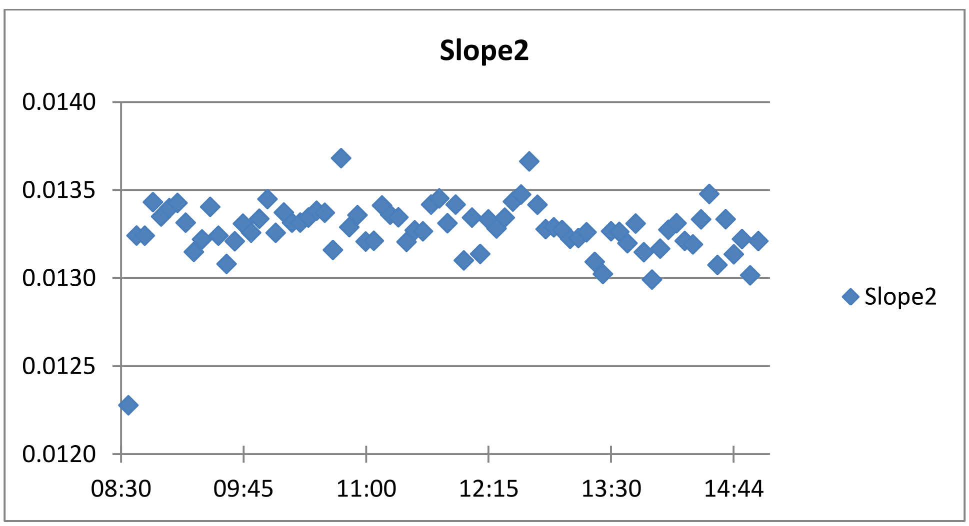
Note: This figure shows the intraday pattern of S_0. I take the average of whole period for each time interval.

Figure 3.4b: Intraday Pattern of S_1



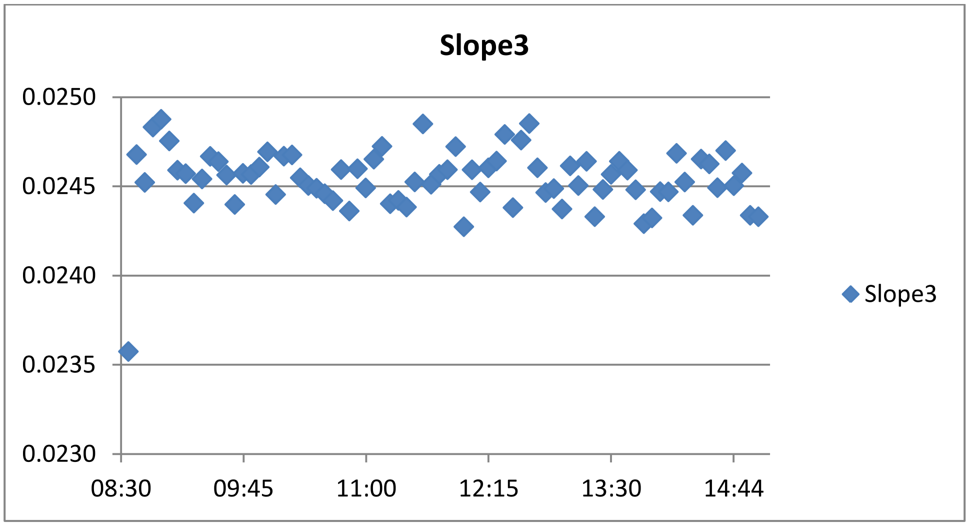
Note: This figure shows the intraday pattern of S_1. I take the average of whole period for each time interval.

Figure 3.4c: Intraday Pattern of S_2



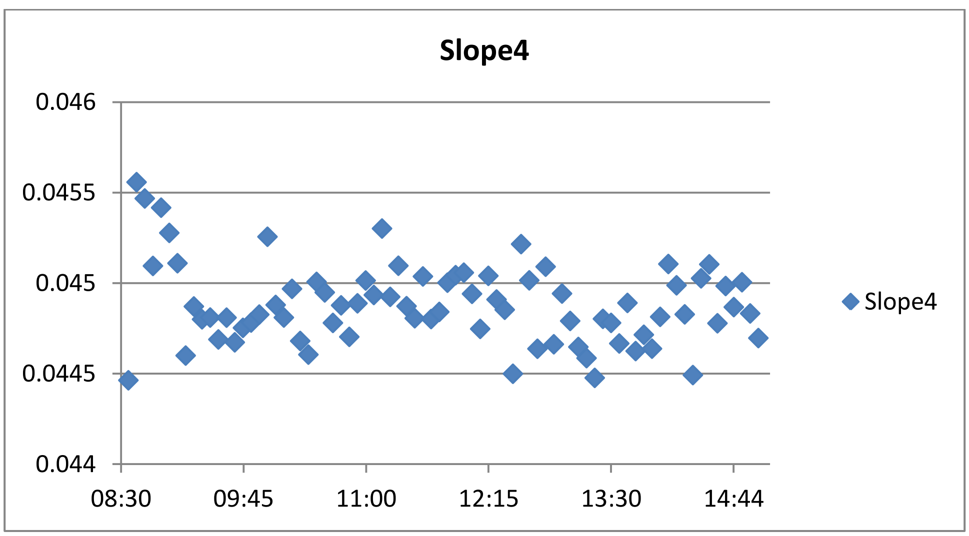
Note: This figure shows the intraday pattern of S_2. I take the average of whole period for each time interval.

Figure 3.4d: Intraday Pattern of S_3



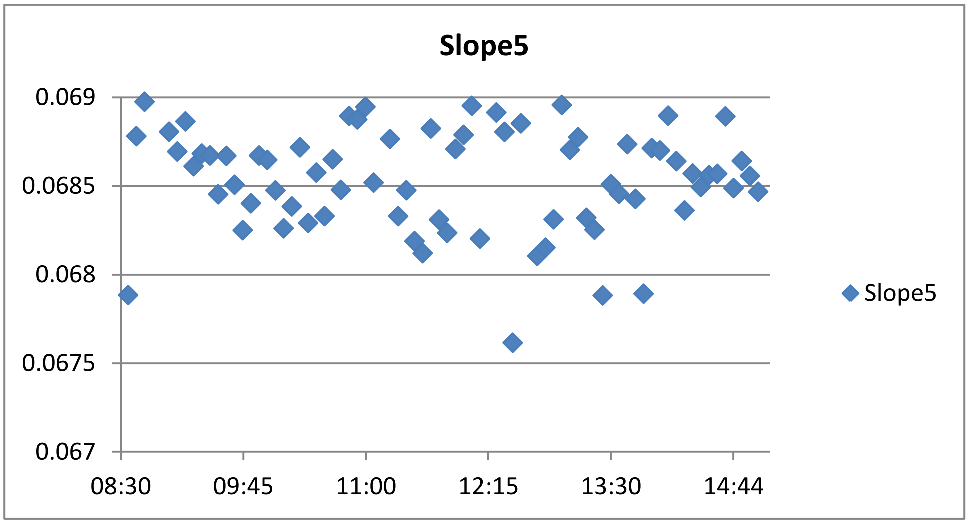
Note: This figure shows the intraday pattern of S_3. I take the average of whole period for each time interval.

Figure 3.4e: Intraday Pattern of S_4



Note: This figure shows the intraday pattern of S_4. I take the average of whole period for each time interval.

Figure 3.4f: Intraday Pattern of S_5



Note: This figure shows the intraday pattern of S_5. I take the average of whole period for each time interval.

As it can be seen from the figures, other than the opening time values, there seems no significant intraday pattern or outlier. Although the opening time value seems not problematic for S_4 and S_5, for the remaining slope measures it is about 6 standard deviations away from the mean of time intervals. Therefore, I throw away opening time from analysis in order to get reliable estimates.

For index returns, I observe from the data that there occur significant jumps from previous day's closing price to next day's opening price. Similar to slope measures, I throw away the opening time of index return. From Figure 4g, in addition to opening time problem, index returns exhibit the intraday u-shaped pattern. In order to get reliable estimates, I apply Flexible Fourier Form (FFF) transformation to intraday index returns. Following Andersen and Bollerslev (1997, 1998), the following decomposition of the intraday returns is considered;

$$r_{t,n} - E(r_{t,n}) = \frac{\sigma_t S_{t,n} Z_{t,n}}{\sqrt{N}}$$

where σ_t is the daily volatility factor, $S_{t,n}$ is the periodicity factor, $Z_{t,n}$ is the i.id. mean zero unit variance innovation term, N refers to the number of return intervals per day. By squaring and taking logs of both sides, the equation becomes;

$$2 \ln \frac{|R_{t,n} - \bar{R}|}{\widehat{\sigma_t}/\sqrt{N}} = f_{t,n} + \mu_{t,n}$$

The seasonal pattern is estimated by using ordinary least square estimation (OLS);

$$f_{t,n} = \mu_0 + \mu_1 \frac{n}{N_1} + \mu_2 \frac{n^2}{N_2} + \sum_{\mathrm{p}=1}^{\mathrm{P}} \left(\gamma_{\mathrm{p}} \cos \frac{2\mathrm{n}\pi\mathrm{p}}{\mathrm{N}} + \delta_{\mathrm{p}} \sin \frac{2\mathrm{n}\pi\mathrm{p}}{\mathrm{N}} \right) + \varepsilon_{\mathrm{t,n}}$$

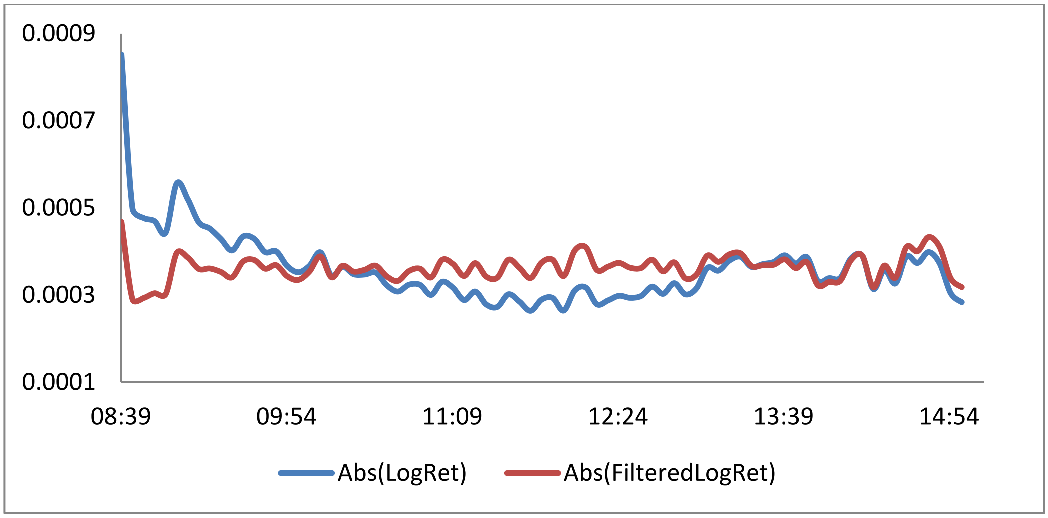
where $N_1 = (N + 1) / 2$ and $N_2 = (N + 1)(N + 2) / 6$ are normalizing constants. Based on Bayesian Information Criterion (BIC), the model sets $p = 2$. This specification allows the shape of the periodic pattern in the market to also depend on the overall level of the volatility. Also the combination of trigonometric functions and polynomial terms are likely to result in better approximation properties when estimating regularly recurring cycles. The intraday seasonal pattern is then determined by using fitted values of OLS;

$$\widetilde{s_{t,n}} = \frac{T exp(\frac{\widehat{f_{t,n}}}{2})}{\sum_{t=1}^{T/N} \sum_{n=1}^{N} exp(\frac{\widehat{f_{t,n}}}{2})}$$

Periodically filtered series is obtained by dividing the original series by the estimated seasonal pattern.

$$\widetilde{R_{t,n}} = \frac{R_{t,n}}{\widetilde{s_{t,n}}}$$

Figure 3.4g: Intraday Pattern of Index Return



Note: This figure shows the intraday pattern of absolute value of Index returns and the intraday pattern of deseasonalized of absolute value of Index returns. I take the average of whole period for each time interval.

As it can be seen from the figure 3.4g, the Flexible Fourier Form transformation makes the intraday pattern more smoother. The FFF representation provides an excellent overall characterization of the intraday periodicity. In the following sections of this study, I am going to use this transformed series.

Another problem that leads to unreliable estimates is the non-stationarity of variables. Thus, I perform some unit root test to each of the variable to test for stationarity. The first test that I perform is augmented Dickey-Fuller (ADF) test (Said and Dickey, 1984). The ADF test tests the null hypothesis that a time series is I(1) against the alternative that it is I(0), assuming that the dynamics in the data have an ARMA structure. Specification of the lag length is the most important practical issue for the implementation of ADF. When choosing lag length, I follow the data dependent lag length selection procedure of Ng and Perron (1995) that results in stables size of the test and minimal power loss. In addition ADF test, I also perform modified efficient Philips Perron (MPP) test (Ng and Perron, 2001). Compared to standard PP test, this test does not exhibit the severe size distortions for errors with large negative MA or AR roots, and they can have substantially higher power than the PP test. Table 3.1 gives the statistics of these tests for each variable.

Table 3.1: Unit Root Tests of Variables

	ΔS	**S_0**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**
ADF	-92,56**	-5,19**	-6,94**	-4,82**	-3,21*	-2,92*	-3,16*
MPP	-49,28**	-5,27**	-6,38**	-4,32**	-3,09**	-2,63**	-3,15**

Note: Table gives the t-values of the test statistics. ADF denotes the augmented Dickey-Fuller test; MPP denotes the modified efficient Philips-Perron test. (**) and (*) corresponds the 1% and 5% significance levels respectively.

From the unit root tests that I conduct, all of the variables seem stationary. Both ADF and MPP test give similar results. Almost all of the test statistics are significant at 1% significance level. Therefore, I strongly reject the hypothesis of there is unit root in time series for each of the variables that I investigate.

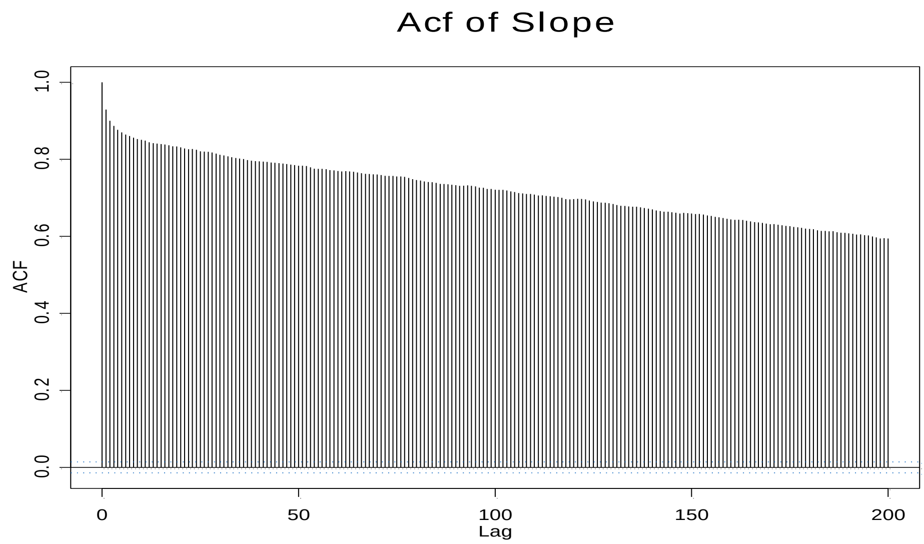
3.2 Methodology

3.2.1 Univariate ARIMA and ARFIMA models

I use a number of univariate time series models to examine whether the implied volatility smirks can be predicted. The reason for using competing models is the question of predictability is a joint hypothesis test of the forecasting model used.

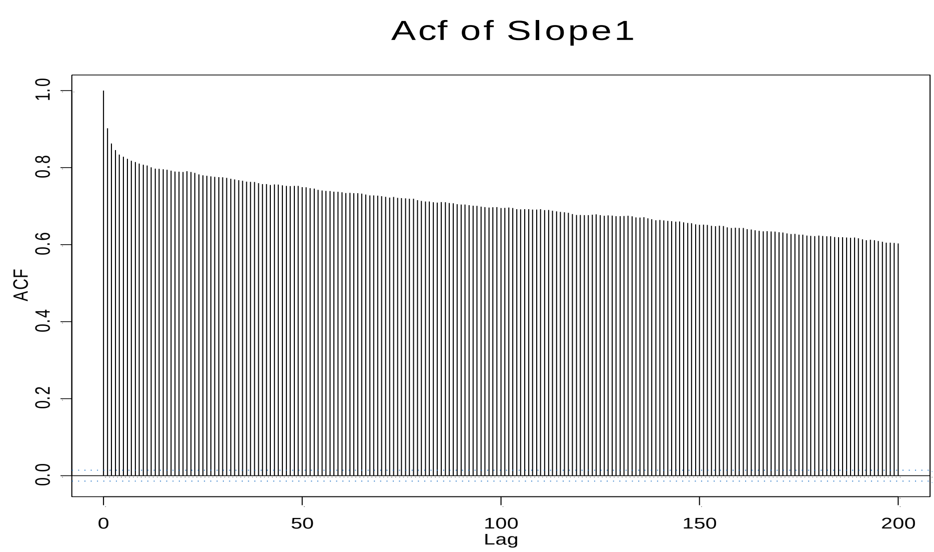
The autocorrelations of all slope measures are characterized by a slow decay. Figures 4.5a to 4.5f depict the ACF plots of slope variables. As it can be observed from the plots, all slope measures have significant persistency. The highly persistent autocorrelation behavior of the data suggests that its dynamics may be represented by a long memory process. In empirical settings, when a time series is highly persistent or appears to be non-stationary, researchers difference the time series once to achieve stationary. However, for some highly persistent economic and financial time series, it appears that an integer difference may be too much, which is indicated by the fact that the spectral density vanishes at zero frequency for the differenced time series. Therefore, instead of integer differencing, fractional differencing is suitable for long memory.

Figure 4.5a: ACF Plot of S_0



Note: This figure shows the ACF plot of S_0 up to 200 lags.

Figure 4.5b: ACF Plot of S_1

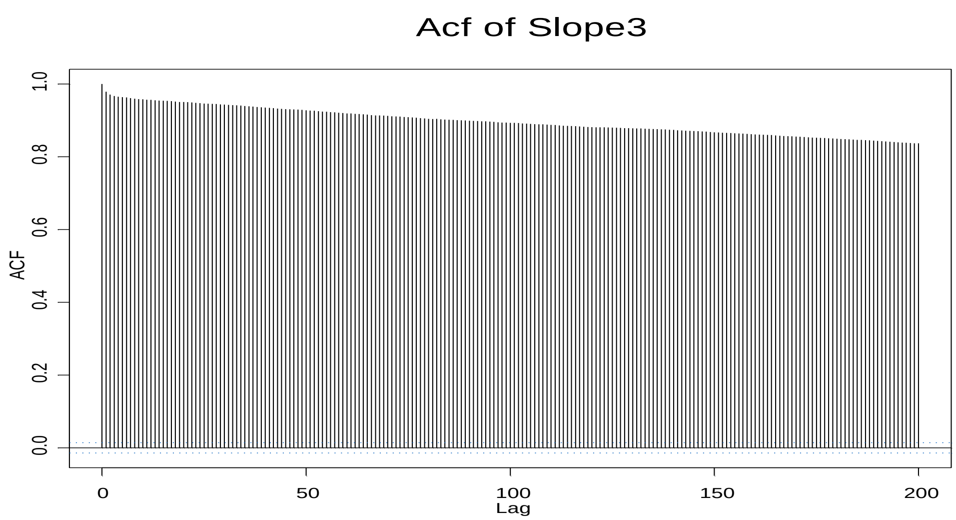


Note: This figure shows the ACF plot of S_1 up to 200 lags.

Figure 4.5c: ACF Plot of S_2

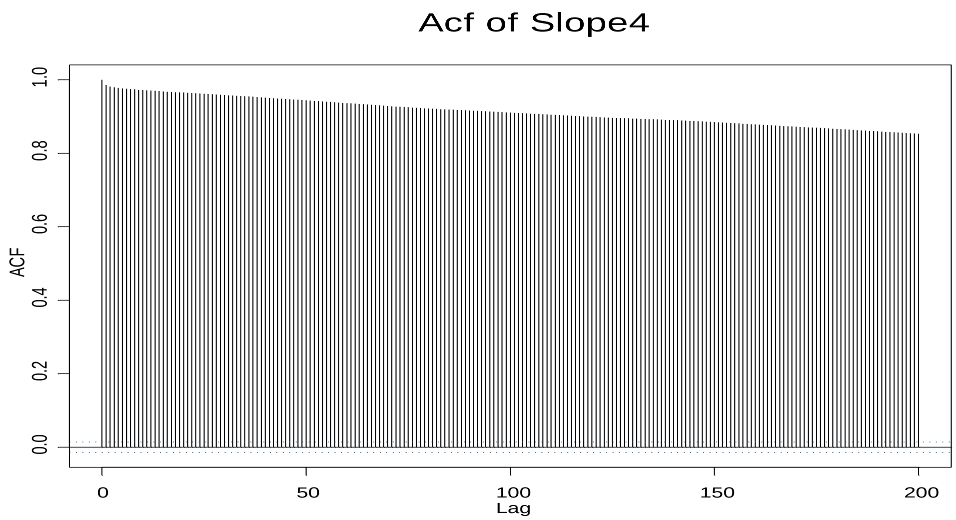
Note: This figure shows the ACF plot of S_2 up to 200 lags.

Figure 4.5d: ACF Plot of S_3



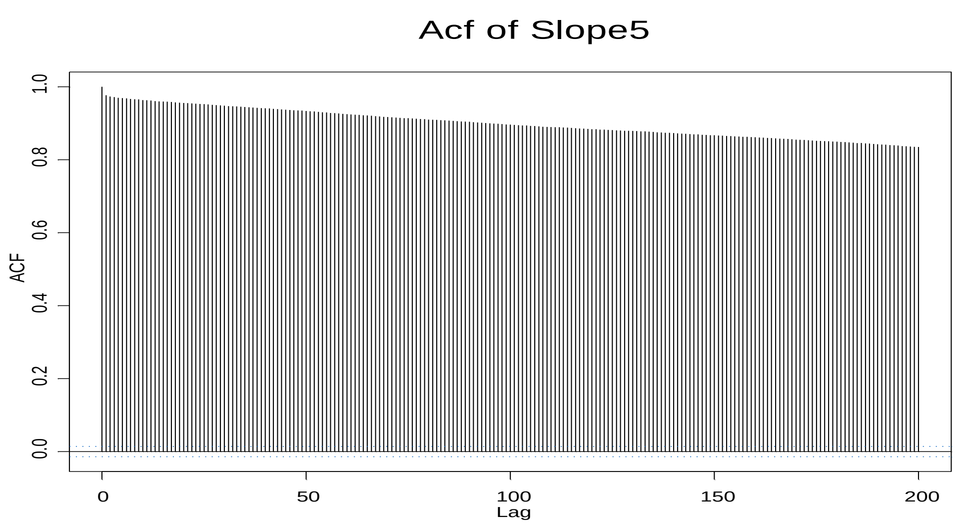
Note: This figure shows the ACF plot of S_3 up to 200 lags.

Figure 4.5e: ACF Plot of S_4



Note: This figure shows the ACF plot of S_4 up to 200 lags.

Figure 4.5f: ACF Plot of S_5



Note: This figure shows the ACF plot of S_5 up to 200 lags.

Granger and Joyeux (1980) and Hosking (1981) introduced a flexible class of long memory processes; called autoregressive fractionally integrated moving average models (ARFIMA)[12]. The ARFIMA (p, d, q) model for a process y_t is defined by

$$\phi(L)(1 - L)^d(y_t - \mu) = \theta(L)\varepsilon_t$$

where d denotes the non-integer order of fractional integration, $(1 - L)^d$ is the fractional difference operator, $\phi(L)$ and $\theta(L)$ denote the autoregressive component and moving average component respectively; and μ denotes the expected value of y_t processes and and ε_t is a Gaussian white noise process with zero mean and variance ξ^2. In the case where $|d| < 0.5$, the ARFIMA (p, d, q) process is invertible and second-order stationary. In particular, if $0 < d < 0.5$ ($-0.5 < d < 0$) the process is said to exhibit long-memory (antipersistent) in the sense that the sum of the autocorrelation functions diverges to infinity (a constant). The spectral density function of an ARFIMA(p, d, q) model is given by;

$$f_{arfima}(\omega) = \left(\frac{\xi^2}{2\pi}\right)\left|\theta\left(e^{-i\omega}\right)\right|^2\left|\emptyset\left(e^{-i\omega}\right)\right|^{-2}\left|1 - e^{-i\omega}\right|^{-2d}, \; 0 < \omega < 2\pi$$

[12] For an excellent survey of long memory processes, including applications to financial data, see Baillie (1996).

Gallant et al. (1999) show that the sum of a particular AR(1) models is an alternative to long memory that is also able to explain the highly persistent characteristics of volatility. With the same motivation, ARIMA (*p, d, q*) model is also employed to take into account the possible presence of short memory characteristics. The ARIMA (*p, d, q*) model for a process y_t is given by

$$\phi(L)(1 - L)^d(y_t - \mu) = \theta(L)\varepsilon_t$$

where d is an integer now that dictates the order of integration needed to produce a stationary and invertible process (in our case $d = 0$), L is the lag operator, $\phi(L) = 1 + \phi_1 L + \ldots + \phi_p L^p$ is the autoregressive polynomial, $\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q$ is the moving average polynomial, μ is the mean of y_t process and ε_t is a Gaussian white noise process with zero mean and variance ξ^2. The spectral density function of the general ARMA (*p, q*) model is given by;

$$f_{arma}(\omega) = \left(\frac{\xi^2}{2\pi}\right)|\theta(e^{-i\omega})|^2|\emptyset(e^{-i\omega})|^{-2}, \; 0 < \omega < 2\pi$$

3.2.2 Distributed Lag Models for Information Flows

I carry out a Granger causality test in order to observe lead-lag relationship between variables. The first step in the causality test procedure involves identification of the individual univariate time series using autoregressive integrated moving average (ARIMA) models to generate filtered series. In this test, each variable is regressed on a constant and *p* of its own lags as well as on *p* lags of other variable by the following vector autoregression VAR(*p*) model:

$$R_t = \phi_0 + \sum_{i=1}^{p} \Phi^{(i)}. R_{t-i} + \varepsilon_t$$

where R_t is the (2 x 1) vector of S&P 500 Index returns and the slope measure, ϕ_0 is the (2 x 1) vector of constants and $\Phi^{(i)}$ is the (2 x 2) matrix of autoregressive slope coefficients for lag i.

However this VAR model does not contain contemporaneous variables. We need these variables in order to test whether information flows immediately one market to another or not. The model that contains instantaneous relationships follows:

$$R_t = \phi_{-1} + \sum_{i=0}^{p} \phi^{(i)}.\ S_{t-i} + \xi_t$$

where R_t denotes the filtered time series of index returns, and S_t is the option volatility smile slope series of interest. Lags of slope measures are denoted by subscripts t - 1, t - 2,..., the coefficients with subscript -1 are constant terms, ξ_t is the disturbance term.

Given the filtering of our series, the constant term would be expected to pick up any remaining market frictions. The other terms pick up the interactions between the markets. If these interactions occur simultaneously, we would expect ϕ_0 to be significant, with none of the lag coefficients being significant. If, instead, market linkages take some time, then these lag effects would be identified by significance of the coefficients on the lagged slope series.

Formally, the null hypothesis that the markets are in a separating equilibrium and thus the option implied volatility slope series does not have predictive power for index returns. An extension of this hypothesis is to investigate the timing and direction of the predictive power of the joint series. This can be addressed by examining the behavior of the individual lag coefficients. In particular, the null hypothesis that the series are only contemporaneously related can be formally stated as follows:

Hypothesis: $\phi_i = 0$, individually for i = 1, 2, ……. , p

This hypothesis can be tested by simple t-tests of the significance of the individual coefficients.

CHAPTER 4

ANALYSIS

The aim of this study is to model the time series behavior of the implied volatility smirk slope levels and also to show the relationship between these slope levels and index returns. For these purposes two approaches, univariate analysis and time series regression analysis, are employed respectively.

The first part of this section presents the results of univariate analyses. In the next section the results of the regression analyses are discussed.

4.1 Univariate Analysis

4.1.1 In-Sample Evidence

In this section, I estimate the parameters of ARMA (*p, q*) and ARFIMA (*p, d, q*) models for the in-sample period. My in-sample period covers the period of January 3, 2006 to July 3, 2006. After the estimation of the models, I assess the out-of-sample performance of each model specification. The out-of-sample exercise is performed from July 4, 2006 to December 29, 2006.

When estimating the parameters of ARFIMA (*p, d, q*) models, I perform maximum likelihood estimation in the frequency domain by using the Whittle approximation of the Gaussian log-likelihood. The log-likelihood function in the frequency domain for a Gaussian process is as follows;

$$K = -\frac{1}{2}\sum_{j=1}^{T-1} ln(2\pi f(\omega_j)) + \hat{f}(\omega_j)/f(\omega_j)$$

where f is the theoretical spectral density of the process. The spectral density f at frequency $\omega_j = 2\pi j/T$ is given by the spectral density function of ARFIMA model, while $\hat{f}(\omega_j)$ is the value of the periodogram at frequency ω_j.

Similar to estimation of ARFIMA (*p*, *d*, *q*), I perform conditional maximum likelihood estimation when estimating the parameters of ARMA (*p*, *q*) model. The conditional likelihood treats the p initial values of the series as fixed and often sets the q initial values of the error terms to zero. The autoregressive and moving average orders are chosen according to Bayesian Information Criterion (BIC).

Table 4.1 presents the maximum likelihood estimates and their standard errors of the parameters of two competing models for 5-minute slope series. The values in white blocks are the standard errors of the parameters.

As table 4.1 suggests, for the ARFIMA (*2*, *d*, *1*) model, the fractional integration parameter is highly significant for all of the slope levels. It ranges from -0.26 to -0.11. The fractional integration parameter is negative and also its absolute value is lower than 0.5 for all slope measures. Therefore, ARFIMA (*2*, *d*, *1*) process is invertible and second-order stationary. Negative value of fractional integration parameter implies that, instead of long memory process, the slope series exhibit antipersistent process. It means that the sum of the autocorrelation functions diverges to a constant.

Autoregressive and moving-average parameters of the ARFIMA (*2, d, 1*) model is also highly significant for all of the slope measures. The sum of the two AR parameters is lower than 1 for each measure. The value of the moving-average parameter ranges from 0.72 to 0.84 and all of them are highly statistically significant.

In addition to ARFIMA (*2, d, 1*), table 4.1 gives ARMA (*2, 1*) maximum likelihood estimates and their corresponding standard errors. For all slope levels the estimates for the ARMA (*2, 1*) model can be interpreted as the sum of persistent and transient autoregressive components.

Table 4.1: Maximum likelihood estimation of ARMA and ARFIMA models for implied volatility smirk slope levels

	Slope		**Slope1**		**Slope2**		**Slope3**		**Slope4**		**Slope5**	
	Model 1 *(2, 0, 1)*	Model 2 *(2, d, 1)*	Model 1 *(2, 0, 1)*	Model 2 *(2, d, 1)*	Model 1 *(2, 0, 1)*	Model 2 *(2, d, 1)*	Model 1 *(2, 0, 1)*	Model 2 *(2, d, 1)*	Model 1 *(2, 0, 1)*	Model 2 *(2, d, 1)*	Model 1 *(2, 0, 1)*	Model 2 *(2, d, 1)*
d		-0.2419		-0.2585		-0.2588		-0.1366		-0.1165		-0.1081
		0.0221		0.0224		0.0194		0.0161		0.0140		0.0117
AR(1)	14.335	15.884	14.327	16.053	13.929	15.516	13.676	14.395	12.975	13.423	11.359	11.844
	0.0091	0.0222	0.0084	0.0222	0.0086	0.0205	0.0095	0.0187	0.0104	0.0176	0.0091	0.0166
AR(2)	-0.4349	-0.5889	-0.4331	-0.6057	-0.3930	-0.5518	-0.3676	-0.4398	-0.2975	-0.3426	-0.1359	-0.1849
	0.0090	0.0221	0.0083	0.0221	0.0086	0.0205	0.0095	0.0186	0.0104	0.0175	0.0091	0.0165
MA(1)	0.8830	0.8096	0.9093	0.8394	0.8953	0.8062	0.8545	0.7946	0.8051	0.7366	0.8347	0.7200
	0.0049	0.0045	0.0039	0.0042	0.0042	0.0046	0.0053	0.0048	0.0064	0.0058	0.0051	0.0065

Note: This table gives the estimates of the ARMA *(2,1)* and ARFIMA *(2, d, 1)* models for the 6 different 5-minute implied volatility smirk slope levels, from January 3, 2006 to July 3, 2006 (In-sample period). AR(1) and AR(2) are first and second order autoregressive parameters, MA(1) is the first order moving-average parameter, and "d " is the fractional integration. The estimates of the parameters are obtained by maximum likelihood estimation. The values in white blocks are the standard errors of the estimated parameters.

The estimates of the autoregressive and moving-average parameters of ARMA (*2*, *1*) model is almost similar to estimates of ARFIMA (*2*, *d*, *1*) model. The sum of the two autoregressive estimates is lower than 1 and the moving-average component ranges from 0.81 to 0.91. All of the estimates for each slope measure are highly statistically significant.

4.1.2 Out-of-sample forecasting performance

I assess the out-of-sample performance of ARMA (*2*, *1*) and ARFIMA (*2*, *d*, *1*) models that are estimated in the previous part. The out-of-sample exercise is performed from July 4, 2006 to December 29, 2006. I form the 1-day ahead and 3-day ahead point forecasts for each of slope measure.

I use two alternative metrics to assess the statistical significance of the out-of-sample point forecasts obtained. The first metric is the mean squared prediction error (MSE), calculated as the average squared deviations of the actual slope levels from the model based forecast, averaged over the number of observations. The second metric is the mean absolute prediction error (MAE), calculated as the average of the absolute differences between the actual slope levels and the model based forecast, averaged over the number of observations.

Table 4.2 and 4.3 gives the results of forecast metrics of 1-day ahead and 3-day ahead predictions respectively for each of the slope measures.

Table 4.2: 1-day ahead forecast performances of ARMA and ARFIMA models

	1 Day Ahead			
	MSE-ARMA	**MSE-ARFIMA**	**MAE-ARMA**	**MAE-ARFIMA**
Slope	0.000004	0.000007	0.0016	0.0021
Slope1	0.000005	0.000021	0.0018	0.0034
Slope2	0.000006	0.000014	0.0019	0.0029
Slope3	0.000009	0.000029	0.0023	0.0039
Slope4	0.000017	0.000052	0.0032	0.0050
Slope5	0.000041	0.000087	0.0049	0.0064

Note: This table gives the out of sample (from July 3, 2006 to December 29, 2006) one day forecast evaluation statistics of ARMA and ARFIMA models for each of the slope levels. MSE corresponds to mean squared error, and MAE corresponds to mean absolute error. Interval between forecasts equal to one day in order not to create overlapping data problem.

Table 4.3: 3-day ahead forecast performances of ARMA and ARFIMA models

	3 Day Ahead			
	MSE-ARMA	**MSE-ARFIMA**	**MAE-ARMA**	**MAE-ARFIMA**
Slope	0.000007	0.000007	0.0022	0.0023
Slope1	0.000006	0.000012	0.0018	0.0026
Slope2	0.000011	0.000014	0.0025	0.0028
Slope3	0.000025	0.000035	0.0039	0.0041
Slope4	0.000056	0.000092	0.0055	0.0071
Slope5	0.000071	0.000096	0.0061	0.0078

Note: This table gives the out of sample (from July 3, 2006 to December 29, 2006) three day forecast evaluation statistics of ARMA and ARFIMA models for each of the slope levels. MSE corresponds to mean squared error, and MAE corresponds to mean absolute error. Interval between forecasts equal to three day in order not to create overlapping data problem.

As Table 4.2 suggests, in 1-day forecasts, ARMA performs better than ARFIMA model both in terms of MSE and MAE for all of the slope measures. Similarly, in 3-day forecasts, ARMA model is still a bit better than the ARFIMA model, but the difference seems smaller compared to 1-day ahead forecasts. As a result, I can say that ARMA model has smaller prediction errors compared to ARFIMA model for all slope measures and forecast periods.

Although, there seems differences in the statistics of forecast metrics among competing models, these differences should have statistical meaning. For this reason, I use Diebold-Mariano (*DM*) (1995) statistics to determine if one model's forecast is more accurate than another's. The null hypothesis of this procedure is the two competing models have equal forecasting accuracy. Diebold and Mariano show that under the null hypothesis of equal predictive accuracy the *DM* statistic is asymptotically distributed *N*(0, 1). The Diebold-Mariano test statistic is as follows;

$$\mathrm{DM} = \frac{\bar{d}}{\widehat{\mathrm{lrv}}\,(\bar{d})^{1/2}}$$

where

$$\bar{d} = \frac{1}{N}\sum_{t=1}^{N} d_t$$

is the average loss differential, and $\widehat{\mathrm{lrv}}\,(\bar{d})$ is a consistent estimate of the long-run asymptotic variance of $\bar{d}$.

Table 4.4 gives the result of Diebold-Mariano test statistics of 1-day and 3-day evaluation periods for each of the slope measures. The results indicate that all of the statistics are inside the one standard deviation interval. Therefore, I cannot reject the null hypothesis of two competing models have equal predictive accuracy. There is no statistically significant differences in terms of predictive accuracy among ARMA (*2,1*) and ARFIMA (*2, d, 1*) models.

Table 4.4: DM Statistics of 1-day ahead and 3-day ahead forecasts

	1 Day Ahead		3 Day Ahead	
	DM-MSE	DM-MAE	DM-MSE	DM-MAE
Slope	0.3686	0.3649	0.0693	0.0881
Slope1	0.4200	0.5992	0.3713	0.5428
Slope2	0.3892	0.5374	0.2745	0.1401
Slope3	0.2444	0.4385	0.1679	0.0657
Slope4	0.3045	0.4778	0.3006	0.6550
Slope5	0.2110	0.2624	0.4901	0.8241

Note: This table gives the results of Diebold-Mariano (*DM*) statistics for 1-day and 3-day ahead evaluation periods. The *DM* statistic is asymptotically distributed as *N*(0, 1).

4.2 Regression Analysis

In the previous subsection, I investigate the time series dynamics of implied volatility smirk slope levels to understand whether they have predictable patterns or not. In this section, I test the hypothesis that the equity and derivatives markets are in separating equilibrium and so there is no lead-lag relation between them in a multivariate setting.

In this multivariate setting, I use distributed lag model that is explained in section 3.2.2 where the index return is the dependent variable, and the slope levels with lags are the independent variables. Table 4.5 gives the results of distributed lag model for each slope measure. According to F-statistics, all of the regressions seems statistically significant. Adjusted R^2 ranges from 2.05% to 0.2%. For S_0 measure where Yan (2011) defines as local steepness has predictive ability up to 4 lags which corresponds to 20 minutes. The other slope measures also have predictive ability ranges from 10 minutes to 20 minutes. Contemporaneous relationship is only highly significant for S_0 and S_1 slope measures.

To conclude, I can say that just the local steepness measure has contemporaneous relation with the index returns; the other measures have no such a relationship. However, their lags are highly statistically significant.

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Table 4.5: Results of distributed lag model for the relationship between slope levels and index returns

	Slope		**Slope1**		**Slope2**		**Slope3**		**Slope4**		**Slope5**	
	Value	p-value	Value	p-value	Value	p-value	Value	p-value	Value	p-value	Value	p-value
$Ø_{-1}$	0.0000	0.4399	0.0000	0.8964	0.0000	0.5286	0.0000	0.4632	0.0000	0.4801	0.0000	0.4287
$Ø_{0}$	0.0344**	0.0000	0.0170**	0.0000	0.0060	0.0574	-0.0020	0.5136	-0.0035	0.1147	-0.0022*	0.0356
$Ø_{1}$	-0.0660**	0.0000	-0.0358**	0.0000	-0.0269**	0.0000	-0.0232**	0.0000	-0.0108**	0.0000	-0.0046**	0.0001
$Ø_{2}$	0.0384**	0.0000	0.0272**	0.0000	0.0206**	0.0000	0.0221**	0.0000	0.0135**	0.0000	0.0056**	0.0000
$Ø_{3}$	-0.0131**	0.0003	-0.0115**	0.0002	-0.0060*	0.0272	-0.0045	0.1104	-0.0012	0.5489	0.0000	0.9814
$Ø_{4}$	0.0071*	0.0197	0.0034	0.1593	0.0058*	0.0127	0.0073**	0.0017	0.0018	0.2860	0.0010	0.2927
F-Stat	80.62		42.78		26.66		29.41		16.20		8.70	
Adj R^2	0.0205		0.0109		0.0067		0.0074		0.0040		0.0020	

Note: This table gives the results of distributed lag model that shows the relationship between implied volatility smirk slopes and the index returns. The data for these regressions cover the whole sample period. $Ø_{-1}$ is the constant term of the model, $Ø_{0}$ is the coefficient of the contemporaneous relationship, $Ø_{1}$ through to $Ø_{4}$ are the coefficients of lagged variables. (**) and (*) corresponds to 1% and 5% significance levels. All error terms are corrected by Newey-West (1987) Heteroskedasticity and Autocorrelation consistent covariance matrix.

CHAPTER 5

SUMMARY AND CONCLUSION

Option implied volatility smile is one of the most intriguing anomalies in the finance literature. Although all options on the same underlying asset should give us the same implied volatility, it is empirically observed from the market prices that Black-Scholes implied volatilities tend to differ across exercise prices and times to expiration.

This anomaly is the main reason for the criticism of fundamental assumptions of Black-Scholes option pricing model. With the motivation of this anomaly, numerous option pricing models such as stochastic volatility models, jump-diffusion models, are developed in order to match the implied volatility smile. Contrary to these models, recent literature suggests that demand can affect the option prices and so the implied volatilities.

Investors with positive expectations can buy the stock, buy a call, or sell a put; in a similar manner, investors with negative signal can sell the stock, buy a put, or sell a call. With the motivation of sequential trade model of Easley et al. (1998), due to the various features of derivatives markets, trades in option markets might reflect the investors' expectations before the underlying market. Recent studies show that implied volatility smirk can contain information about investors' expectations. The slope levels that are extracted from the implied volatility function can be used as a proxy for investor risk aversion or jump risk.

There are recent studies that show the cross-sectional relationship between implied volatility slopes and stock returns. But there is no study on intraday time-series dynamics of these slope levels. This thesis aims to analyze the high-frequency time-series behavior of implied volatility smirk slopes. Furthermore, the high-frequency relationship between these slope levels and index returns is also investigated.

My sample comprised of tick-by-tick data of S&P 500 options (SPX) that cover the period 2006 with a total of 250 trading days. This raw data is filtered according to maturity condition and no-arbitrage option boundaries. After filtering, the inhomogeneous tick-by-tick option series are converted 5-minute interval series.

Therefore, I have 78 5-minute intervals during each day. Data for each time interval consists of index level, bid and ask prices of call and put options, implied volatilities calculated from Black-Scholes model, option deltas. Then I use deltas as moneyness measure in order to calculate different levels of slope measures.

Consistent with the recent literature, I define implied volatility smirk slope as the difference between put option implied volatility of specific moneyness and at-the-money call option implied volatility. I calculate six different slope levels by using six different put options with respect to their deltas. Since the slope measures are the main variables that I investigate their time series behavior, it is important to screen the data in order to get reliable estimates. After screening, I remove the opening time due to the outlier behavior.

The main research question of this thesis is whether the implied volatility smirk slope levels can be predicted or not. For this purpose, I use two competing univariate time series models which are ARMA (*p, q*) and ARFIMA (*p, d, q*). The reason for ARFIMA (*p, d, q*) model is the highly persistent autocorrelation behavior of the slope levels. This dynamic may be represented by a long memory process. In addition to ARFIMA (*p, d, q*) model, ARMA (*p, q*) model is considered to take into account the possible presence of short memory characteristics. The autoregressive and moving average orders are determined by Bayesian Information Criteria (BIC).

I estimate the parameters of ARMA (*2*, *1*) and ARFIMA (*2*, *d*, *1*) for the in-sample period that is the first half of year 2006 by maximum likelihood method. Both models have similar coefficients for the autoregressive and moving average parameters. For all slope levels the fractional integration parameter of ARFIMA (*2, d*, *1*) model is highly statistically significant. Furthermore, the negative value of fractional integration parameter implies that all slope levels exhibits antipersistent process.

After the estimation of the models, I assess the out-of-sample performance of each model specification. The out-of-sample exercise is performed through the second half of year 2006. I form the 1-day ahead and 3-day ahead point forecasts for each of slope measure. I use Mean Absolute Error (MAE) and Mean Squared Error (MSE) metrics to assess the statistical significance of the out-of-sample point forecasts obtained. Both in 1-day and 3-day ahead forecasts, ARMA perform slightly better than the ARFIMA model for each of the slope measures. Since these differences should have a statistical meaning, I perform Diebold-Mariano (*DM*) (1995) statistics to determine if one model's forecast is more accurate than another's. The null hypothesis of this statistics is the two competing models have equal forecasting accuracy. I find that there is no statistically significant differences in terms of predictive accuracy among ARMA (*2,1*) and ARFIMA (*2, d, 1*) models. Therefore, I cannot reject the null hypothesis of two competing models have equal predictive accuracy.

My next research question in this thesis is whether the implied volatility smirk slope levels have high-frequency predictive power for subsequent index returns or not. For this purpose, I use distributed lag model to test the hypothesis of there is no relationship between the slope levels and index returns. In this model, index return is the dependent variable; and contemporaneous and lags of slope levels are the independent variables. Lag orders are determined by Bayesian Information Criteria (BIC) and order of 4 for all slope levels is seems appropriate for the analysis. I find that slope levels have predictive power for subsequent index returns up to 20 minutes.

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APPENDICES

APPENDIX A - DESCRIPTIVE STATISTICS

Table A-1: Descriptive statistics of call option implied volatilities

	Δcall(0.05)	Δcall(0.1)	Δcall(0.2)	Δcall(0.3)	Δcall(0.4)	Δcall(0.5)	Δcall(0.6)	Δcall(0.7)	Δcall(0.8)	Δcall(0.9)	Δcall(0.95)
Mean	0.0920	0.0923	0.0963	0.1008	0.1055	0.1109	0.1172	0.1256	0.1376	0.1601	0.1890
St. Dev.	0.0105	0.0110	0.0123	0.0138	0.0152	0.0168	0.0187	0.0214	0.0250	0.0325	0.0426
Skewness	0.5655	0.7743	0.9243	0.9831	0.9944	1.0822	1.0804	1.0482	1.0812	1.0091	0.6894
Kurtosis	0.7092	0.8971	0.9264	0.9134	0.9728	0.9292	0.8891	0.8251	0.8351	0.8520	0.2201

Note: This table shows the descriptive statistics of call option implied volatilities across different moneyness levels. The values in parenthesis are the call option deltas. From 0.05 to 0.95 call options move from out-of-the-money to in-the-money.

Table A-2: Descriptive statistics of put option implied volatilities

	Δput(-0.05)	Δput(-0.1)	Δput(-0.2)	Δput(-0.3)	Δput(-0.4)	Δput(-0.5)	Δput(-0.6)	Δput(-0.7)	Δput(-0.8)	Δput(-0.9)	Δput(-0.95)
Mean	0.1798	0.1554	0.1352	0.1240	0.1162	0.1098	0.1044	0.0996	0.0949	0.0900	0.0881
St. Dev.	0.0347	0.0292	0.0238	0.0207	0.0182	0.0164	0.0150	0.0137	0.0127	0.0126	0.0156
Skewness	1.2168	1.2117	1.2134	1.1752	1.1549	1.1280	1.0558	1.0233	0.8803	0.6339	0.6861
Kurtosis	1.1065	0.9995	0.9949	0.9785	0.9499	0.9355	0.8814	0.8364	0.6290	0.2107	0.3029

Note: This table shows the descriptive statistics of put option implied volatilities across different moneyness levels. The values in parenthesis are the put option deltas. From -0.05 to -0.95 put options move from out-of-the-money to in-the-money.

Table A-3: Descriptive statistics of slope levels

	Slope	Slope1	Slope2	Slope3	Slope4	Slope5
Mean	-0.0011	0.0053	0.0132	0.0243	0.0445	0.0682
St. Dev.	0.0035	0.0039	0.0054	0.0084	0.0140	0.0190
Skewness	-0.2144	-0.0294	0.6195	1.0098	1.1441	1.0658
Kurtosis	-0.1590	0.1334	0.6913	0.9163	1.2037	1.0930

Note: This table shows the descriptive statistics of slope levels.

APPENDIX B - TIME SERIES PLOTS

Figure B-1: Time series plot of S_0

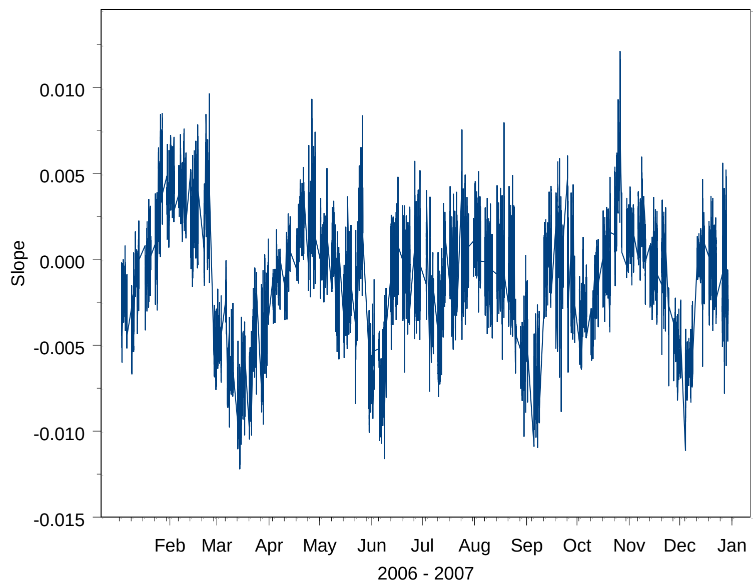

Figure B-2: Time series plot of S_1

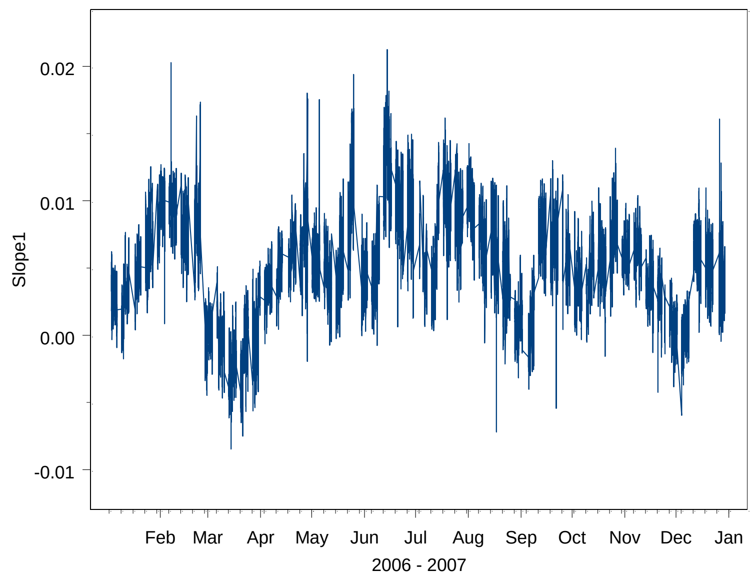

Figure B-3: Time series plot of S_2

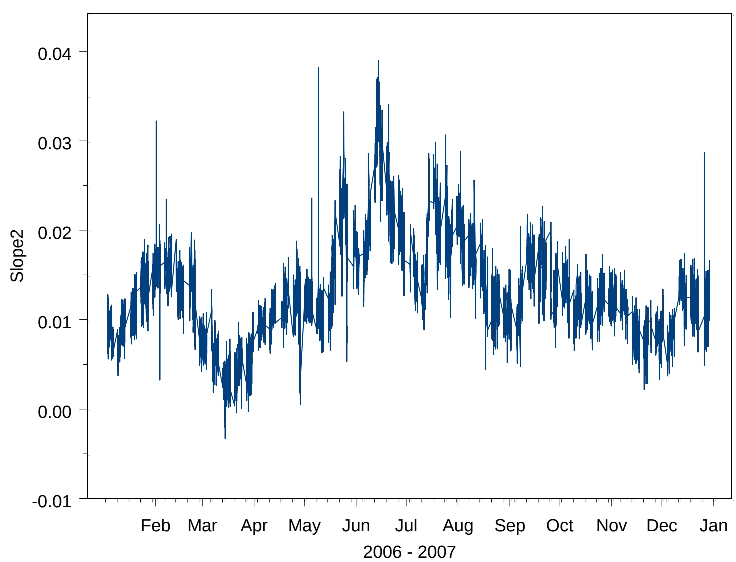

Figure B-4: Time series plot of S_3

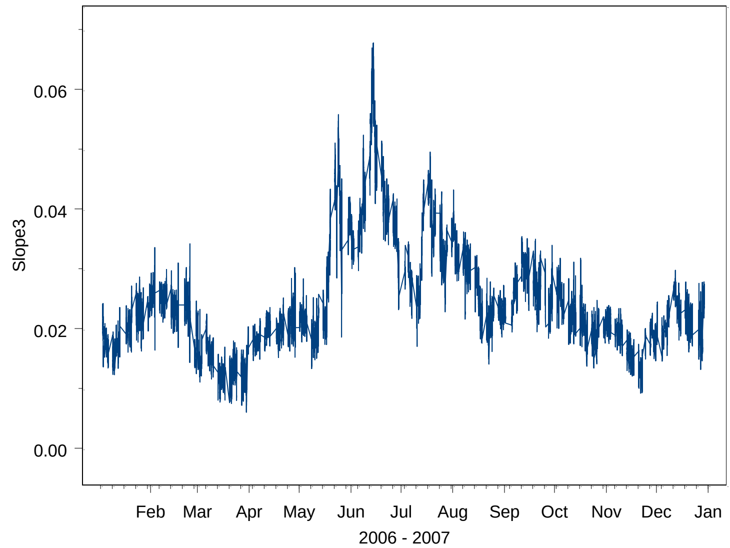

Figure B-5: Time series plot of S_4

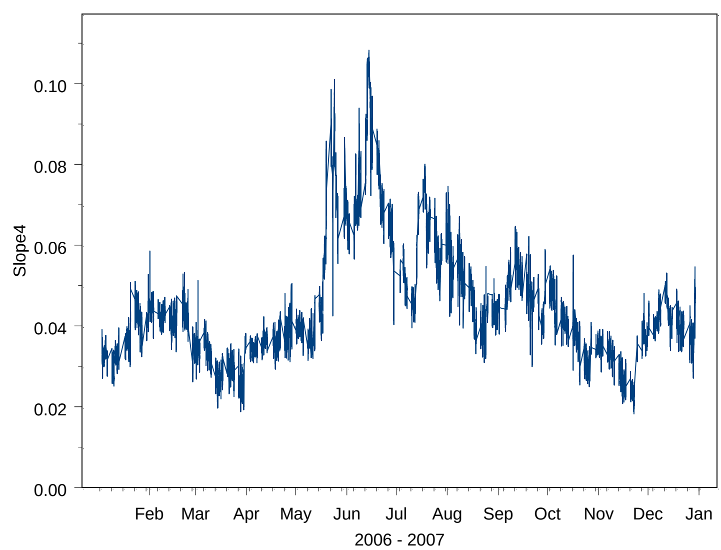

Figure B-6: Time series plot of S_5