CASH REPLENISHMENT STRATEGIES AND VEHICLE ROUTING PROBLEM FOR AUTOMATED TELLER MACHINES

(OTOMATİK VEZNE MAKİNELERİ İÇİN ARAÇ ROTALAMA PROBLEMİ VE NAKİT İKMAL STRATEJİLERİ)

by

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Thesis

Submitted in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE

in

LOGISTICS AND FINANCIAL MANAGEMENT

in the

GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

of

GALATASARAY UNIVERSITY

July 2023

ACKNOWLEDGEMENTS

I would like to express how thankful I am to work with my supervisor Prof. Dr. Müjde EROL GENEVOIS. Thanks so much for her guidance for the last two years. Without her support and motivation, I wouldn't go so far in this work.

I would want to thank Dr. Michele Cedolin for his guidance with the best advice and knowledge. Thanks for the help and support whenever I needed.

I would like to thank Erkut Çokal for his endless support and confidence in me.

July 2023
Deniz Orhan

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LIST OF SYMBOLS

ATM : Automatic Teller Machines
ACO : Ant Colony Optimization Algorithm
ANN : Artificial Neural Network
ARIMA : Autoregressive Integrated Moving Average
ARP : Arc Routing Problems
CIT : Cash in Transit
DNN : Deep Neural Network
DSS : Decision Support System
ETS : Exponential Smoothing Model
GA : Genetic Algorithm
GIS : Geographic Information Systems
GRNN : General Regression Neural Network
HGA : Hybrid Genetic Algorithm
IRP : Inventory Routing Problem
LSTM : Long Short Term Memory
LNS : Large Neighbourhood Search
PSTN : Public Switched Telephone Network
PSO : Particle Swarm Optimization
RCTVR : Risk-Constrained Cash-in-Transit Vehicle Routing Problem
NNM : Nearest Neighbor Method
MDVRPTW : Multi-Depot Vehicle Routing Problem with a Time Window
MLFF : Multi-Layer Feed Forward Neural Network
SARIMA : Seasonal Autoregressive Integrated Moving Average
SMAPE : Symmetric Mean Absolute Percentage
SNNs : Simulated Neural Networks
SVR : Support Vector Regression
TTRP : Truck and Trailer Routing Problem
TSP : Travelling Salesman Problem
MASA : Multiple ARIMA Subsequences Aggregate Time Series Model
VRP : Vehicle Routing Problem
VRPPD : Vehicle Routing Problem with Pickups and Deliveries
VRPTW : Vehicle Routing Problem with Time Windows
VRPSPD : VRP with Simultaneous Delivery and Pick-up
WNN : Wavelet Neural Network

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ABSTRACT

Logistics has become one of the most significant parts of the process in many business areas. For an efficient logistics system, each stage of the operation needs to be designed carefully. Logistics approaches are also applied in the financial sector such as ATM cash management. Automatic Teller Machines (ATMs) provide efficient service for a financial institution to its customers through a self-service, time-independent, and accessibility mechanism. For daily money transactions, ATM is the fastest, safest, and most practical banking tool. Many challenges have risen in the network design of the cash and these problems can be solved by using the optimized solution. This solution aims to meet customer demand at the ATM and at the same time, minimize loss for banks. The NN5 Competition dataset includes of daily time series; of 111 randomly selected different cash machines at different locations. The data is integrated into 13 ATMs and the system is run for 56 days. ATMs were randomly selected in the Beşiktaş area of İstanbul. In addition, because of security concerns about money-fixed assets, operations are only made in the daytime. This paper states the solution by a combination of cash replenishment of ATMs and vehicle routing problems. The replenishment policy starts with forecasting cash withdrawals by applying various methods. These are statical methodologies which are ARIMA and SARIMA, and machine learning methods which are Prophet, and DNN. Firstly, the ARIMA method is applied to the data because it is a better understanding of data or predicting upcoming series points, ARIMA gave effective results for this purpose. However, in this case, seasonality has a huge impact on data since on some days people are willing to withdraw cash more than on ordinary days such as salary days. SARIMA is generated to overcome seasonality since it takes into account seasonality as a parameter. Therefore, it would great opportunity to analyze the two statical methods for the same data set to identify their differences. Machine learning methods become popular in academic studies last years. Because, in today's world, machine learning methods show better results by using present conditions parameters in

their model to handle challenging problems. For this purpose, ANN (Artificial Neural Network) method is first analyzed and then SVR (Support Vector Regression), and LSTM (Long Short Term Memory) are analyzed. Models of those methods impair the prediction ability of the models when the data is increasingly complicated or changes dramatically. Due to the fact that, these are not suitable for the data set since data is noisy. Research continues with DNN (Deep Neural Network) which uses advanced math modeling to analyze input in complicated ways. Since data is noisy in this case, DNN can show more effective results than other machine learning methods. Prophet may be classified as a new method among others since it was released in 2017 as a time series prediction model. Prophet is a Facebook-developed open-source library based on a combination of trends, seasonality, and holidays. In the Prophet, unlike ARIMA models, the measurements do not need to be evenly spaced, and missing data are not interpolated. In most of the studies, ARIMA compares with the Prophet. Prophet makes a difference in seasonality and the holiday's approach between other methods. Prophet also takes into account the holidays as a parameter. This is an important feature for holidays that change dates every year such as Ramadan and Thanksgiving. The VRP Spreadsheet Solver is an open-source tool which is implemented in this study as a real-world application. Real time traffic data is used in the solver. Also, visualization of the routes is made at Bings Map. The main inputs of VRP Spreadsheet Solver are pickup and delivery amounts, fixed cost, fuel cost per km, and cost of visiting or not visiting. The linear programming of the general aggregate planning problem is applied to calculate delivery amounts. Capacity and distance limits are determined as infinite in the model. VRP Spreadsheet Solver tool is introduced to find optimal routes of trucks when the cost is minimized by using forecasted data. By creating a decision support system, several scenarios are applied that visit ATMs. All ATMs have no stock on the first day and they are filled with forecasted data at the beginning of the day. Scenarios in order, if the stock level become below safety stock level and the solver decides to visit or not the rest of the ATMs, visiting all ATMs every two days and visiting all ATMs every three days. In the first scenario, the service level is determined as 90% since the service industry generally determines its service level as 90%. After the calculating average withdrawal rate of a certain day, 10% of the average value is determined as the stock level for each ATM. Whenever the amount of cash in the ATM is placed below this stock level, the vehicle must visit the ATM for filing. In the second and third scenarios, this stock level didn't determine because each ATM visited regardless of stock level. As a second strategy, ATMs are clustered according to the nearest neighbor algorithm. 3 group is determined in this manner. The ratio is found by dividing the total demand amount of the group by the demand amount of each ATM. The delivery amount is determined by multiplying this ratio with the ATMs in the group by the total demand for that day. The same calculations were also made for the other two scenarios, for the second scenario, the sum of the forecast amount of two days, and for the third scenario, the sum of the forecast amount of three days was used in the calculation. After analyzing all scenarios, the strategy with the lowest cost was chosen. This paper presents models and solutions inspired by automated teller machine (ATM) replenishment and vehicle routing problems for armored trucks.

ÖZET

Lojistik, birçok iş alanında sürecin en önemli parçalarından biri haline geldi. Verimli bir lojistik sistem için operasyonun her aşamasının özenle tasarlanması gerekir. ATM nakit yönetimi gibi finans sektöründe de lojistik yaklaşımlar uygulanmaktadır. Otomatik Vezne Makineleri (ATM'ler), kolay kullanım, zamandan bağımsız ve erişilebilirlik mekanizmasıyla bir finans kuruluşunun müşterilerine verimli hizmet vermesini sağlar. Günlük para işlemleri için en hızlı, en güvenli ve en pratik bankacılık aracı ATM'dir. Nakit ikmali planlamadında ortaya çıkan birçok zorluk, optimize edilmiş çözüm kullanılarak çözülebilir. Bu çözüm, ATM'deki müşterin talebini karşılamayı ve aynı zamanda bankaların zararını en aza indirmeyi amaçlamaktadır. NN5 Yarışması veri seti günlük zaman serilerini içerir ve farklı konumlarda 111 rastgele seçilmiş farklı bankamatikler bu veri setinin içinde yer alır. Veriler 13 ATM olacak şekilde birleştirilmiş ve sistem 56 gün boyunca çalıştırılmıştır. ATM’ler İstanbul'un Beşiktaş bölgesinde rastgele seçilmiştir. Ayrıca, para ile ilgili güvenlik endişeleri nedeniyle, operasyonlar sadece gündüz yapılacak şekilde planlanmıştır. Bu tez, ATM'lerin nakit ikmali ve araç rotalama problemlerinin bir kombinasyonunu oluşturarak bir çözüm sunmamaktadır. İkmal politikası, çeşitli yöntemler uygulayarak nakit çekimlerini tahmin etmekle başlar. Bunlar, ARIMA ve SARIMA olan statik metodolojiler ve Prophet ve DNN olan makine öğrenme yöntemleridir. Verilerin daha iyi anlaşılması veya gelecek seri noktalarını tahmin etmesi nedeniyle öncelikle verilere ARIMA yöntemi uygulanmış, ARIMA bu amaçla etkili sonuçlar vermiştir. Ancak bu durumda mevsimselliğin veriler üzerinde büyük bir etkisi vardır çünkü maaş günü gibi bazı günlerde insanlar para çekmeye daha meyillidir. SARIMA, mevsimselliği bir parametre olarak dikkate aldığı için mevsimselliğin üstesinden gelmek için kullanılmıştır. Bu nedenle, aynı veri seti için bu iki statik yöntemi analiz etmek, farklılıklarını belirlemeyi sağlamıştır. Makine öğrenimi yöntemleri son yıllarda akademik çalışmalarda popüler hale gelmiştir. Çünkü günümüz dünyasında makine öğrenmesi yöntemleri zorlu problemlerin üstesinden gelmek için

modellerinde mevcut durum parametrelerini kullanarak daha iyi sonuçlar vermektedir. Bu amaçla önce YSA (Yapay Sinir Ağı) yöntemi incelenmiş, ardından SVR (Support Vector Regression) ve LSTM (Long Short Term Memory) analiz edilmiştir. Bu yöntemlerin modelleri, veriler giderek karmaşıklaştığında veya önemli ölçüde değiştiğinde modellerin tahmin yeteneğini bozar. Çünkü veri seti düzgün olmadığından bu yönetemler uygun değildir. Girdiyi karmaşık şekillerde analiz etmek için gelişmiş matematik modelleme kullanan DNN (Derin Sinir Ağı) ile çalışmaya devam edildiğinde daha etkili bir sonuç ortaya çıkacağı öngörülmüştür. Bu durumda veriler düzensiz olduğu için DNN, diğer makine öğrenimi yöntemlerinden daha etkili sonuçlar gösterebilir. Prophet, 2017'de zaman serisi tahmin modeli olarak piyasaya çıktığından beri diğerleri arasında yeni bir yöntem olarak sınıflandırılabilir. Prophet, trend, mevsimsellik ve tatillerin birleşimine dayalı, Facebook tarafından geliştirilmiş açık kaynaklı bir yöntemdir. Prophet, ARIMA modellerinden farklı olarak, ölçümlerin eşit aralıklı olması gerekmez ve eksik veriler enterpolasyona tabi tutulmaz. Araştırmaların çoğunda Prophet, mevsimselliği ve tatil günlerine yaklaşımı açısından diğer yöntemler arasında fark yaratır. Prophet metodunda bayramları da bir parametre olarak dikkate alır. Bu, Ramazan ve Şükran Günü gibi her yıl tarihleri değişen bayramlar için önemli bir özelliktir. VRP Spreadsheet Solver, bu çalışmada uygulanan açık kaynaklı bir araçtır. Çözücüde gerçek zamanlı trafik verileri kullanılır. Ayrıca rotaların görselleştirmesi Bings Map'te yapılmaktadır. VRP Spreadsheet Solver'ın ana girdileri, teslim alma ve teslim tutarları, sabit maliyet, km başına yakıt maliyeti ve ziyaret edip etmeme maliyetidir. Teslimat miktarlarını hesaplamak için karma tamsayılı doğrusal programlama uygulanır. Modelde kapasite ve mesafe limitleri sonsuz olarak belirlenmiştir. VRP Spreadsheet Solver aracı, tahmini veriler kullanılarak maliyet en aza indirildiğinde en uygun kamyon rotalarını bulmak için uygulandı. Bir karar destek sistemi oluşturularak, ATM'leri ziyaret eden çeşitli senaryolar uygulanır. Tüm ATM'lerde ilk gün stok bulunmamakta ve gün başında tahmin verileri ile doldurulmaktadır. Senoryalar sırasıyla; stok seviyesinin güvenli stok seviyesinin altına düşmesi ve çözücünün geri kalan ATM'leri ziyaret edip etmemeye karar vermesi, tüm ATM'leri iki günde bir ve tüm ATM'leri üç günde bir ziyaret etmesiolarak belirlenmiştir. Birinci senaryoda hizmet sektöründe genellikle hizmet düzeyi %90 olarak belirlediği için bu çalışmada da hizmet düzeyi %90 olarak belirlenmiştir. Belirli bir gündeki para çekme oranı ortalaması hesaplandıktan sonra her ATM için ortalama değerin %10'u stok seviyesi olarak belirlenir. ATM'deki nakit miktarı bu stok seviyesinin altına düştüğünde, araç nakit ikmali için ATM'ye uğramalıdır. İkinci ve üçüncü senaryoda, stok seviyesi ne olursa olsun her ATM ziyaret ediliği için stok seviye belirlenmedi. İkinci bir strateji olarak, ATM'ler en yakın komşu algoritmasına göre kümelenir. 3 grup ATM bu şekilde belirlenir. Grubun toplam talep miktarının her bir ATM'nin talep miktarına bölünmesiyle ATM'nin gruba ağırlık oranı bulunur. Bu oran gruptaki ATM'lerin toplam talebi ile çarpılmasıyla, teslimat tutarı belirlenir. Diğer iki senaryo için de aynı hesaplamalar yapılmış, ikinci senaryo için iki günlük tahmin tutarının toplamı, üçüncü senaryo için üç günlük tahmin tutarı toplamı hesaplamada kullanılmıştır. Tüm senaryoların analizi sonucunda en düşük maliyetli strateji seçilmiştir. Bu makale, zırhlı kamyonlar için otomatik vezne makinesi (ATM) nakit ikmali ve araç rotalama problemlerinden ilham alan modeller ve çözümler sunmaktadır.

1. INTRODUCTION

In logistics activities, meeting customer expectations for a wide range of high-quality goods is a significant obstacle. To provide high-quality service, various areas need to be arranged effectively. In my research, the problem is divided into two different but attached areas. Firstly, determine the cash replenishment policy and then design the vehicle routing problem for trucks that carries valuable items. These two problems can't examine separately since the constraints of models link to each other. For instance, the maximum amount of money based on security issues that are carried by trucks may affect the demand planning of cash that is loaded onto a truck. Therefore, a significant challenge of this problem is combining these two components in harmony.

As crucial as the accuracy of the cash prediction is, the forecast must be made in such a way that it can meet consumer demands while also not loading too much money that is not withdrawn from the ATM for a long time. There needs to be an optimized relationship between holding cost and the service level customer to construct effective stocking management; a huge amount of inventory entails significant financial expenses, yet an adequate inventory level is crucial to meet consumer cash demand (Baker et al., 2013). As a result, a suitable estimated cash management strategy should be devised. The first solution may be refilling the ATM with a low amount of money but this solution has some negative sides such as service cost. This causes dissatisfaction among customers which is the most undesirable situation. To solve this problem, loading the ATM with a lot of cash can cause idle cash. Idle cash means inactive money in the machine for many more days. This idle cash may classify as a loss for banks due to not being traded at the exchange. Moreover, this unprofitable strategy of banks is the second most undesirable situation. As a result, the goal is to prevent the replenishment of more money than is required. The most efficient way to optimize ATM cash management is to maintain these constraints in balance.

The research into the VRP has resulted in main improvements in the fields of exact algorithms and heuristics. In recent years, the development of extremely unique mathematical programming decomposition algorithms and optimized metaheuristics for the VRP. The classic vehicle routing problem seeks to find the best route such as the shortest or fastest depending on the user's desire for vehicles to service several people, every vehicle starts and ends at the same point where is a depot, and each customer gets a service only once by one vehicle, and constraints of the model need to be satisfied. As opposed to Arc Routing Problems (ARP), where the requests are separated throughout the arcs, or street segments of the underlying road network, in this type of problem, which is referred to as a Vehicle Routing Problem (VRP), the transportation requests to be serviced are typically gathering in specific points of a road mapping (Toth & Vigo, 2014). VRP is one of the most selected methods of routing and delivery issues in the service industry today. With the fast increase in the social economy, the quantity of cash in circulation has played an essential role in commerce and trade, and it is expected to continue to do so in the near future. In many nations, the amount of cash in circulation has increased dramatically during the last ten years (Xu et al., 2019). A real-life application of the vehicle routing problem is the challenge of cash transit. In reality, logistics businesses transport banknotes and coins from the depot to customers to deal with the transfer of cash and valuables in this business. The cash transportation industry is now a huge industry, with a global cost of handling cash exceeding $300 billion per year (Talarico, 2016). Increasing the number of service sites without increasing the number of service vehicles, on the other hand, creates delivery and pickup delays. This issue leads to an increase in overtime costs and a decrease in client satisfaction. As a result, each bank makes an effort to improve its transportation service in order to improve client expectations (Boonsam et al., 2011). Generally, the transportation of money which is an important part of cash management, is managed by third parties. Banks made an agreement with firms and they carry the cash for different banks. Loading money to each bank's ATM becomes their business. Cash is transferred between cash centers and ATMs by companies known as "cash in transit (CIT)". For each ATM visit, banks pay the CIT a certain amount, and this money is used to pay for logistic costs, a significant portion of operating expenses.

The problem with ATM cash management derives from the trade-off between these expenses: an ATM cash management system should reduce total idle cash and logistic costs while guaranteeing that ATMs do not run out of cash and that consumers receive quality service, i.e., be able to put the right amount of money that should be placed in ATMs to the customer makes happy when they found the money they want.

Table 1.1: Conceptual model

INPUT	PROCESS	OUTPUT
-Realized withdrawal amount of cash of 15 days for 13 ATMs - Forecasted withdrawal rate -Locations of 13 ATMs and 1 depot -Cost of operation -Vehicles for used	-Statical and machine learning methodologies are applied for data and the most accurate method is selected according to MAPE analysis -VRP spreadsheet solver is used to minimize cost with parameters of pickup, delivery, and cost amounts -Various scenarios are applied	-Best routes are determined for vehicles by minimizing cost when meeting customer demand -Visualisation of routes is obtained -Duration of each route is found -Scenarios are compared according to cost amounts and the scenario with the lowest cost is chosen

However, there is not enough concentration on the cash transportation problem. In the literature, few studies focus on this problem. Firstly, it is hard to find data for this problem because of the security policy of banks. Many researchers focus on the development of models to get better results than previous ones. Studies that include real-time data and optimized models are rare and valuable for this approach. The objective of this research is to improve cash distribution and cash replenishment while using the existing resources. In this paper, the objective function of the model is minimizing cash management costs. To improve the routing of trucks for certain areas and to determine the optimized value of cash. Instead of e-banking is developing, withdrawing cash is never an irreplaceable transaction. People always need cash in their pockets. Therefore, this topic never become old fashioned. The main reason for this study is to generate the routing of cash distribution by using ATM distance. In the literature, a combination of routing and replenishment can be rarely seen. But this daily life problem needs more attention for the rising quality of life. Because the literature lacks studies, this area needs to be improved more.

2. DETAILED INFORMATION

An Automated Teller Machine, or ATM, is a customized computer that allows bank account users to handle their money more easily. People want to make to print a record of account activity, check their account balances, withdraw or deposit cash, transfer money between accounts, and even purchase stamps. The Bankograph, designed by an American called Luther George Simjian in 1960, was a machine that allowed consumers to deposit cash and cheques. In June 1967, the first ATM was installed on a street in Enfield, London, at a Barclays bank branch. John Shepherd-Barron, a British inventor, is credited with inventing it. Customers could only withdraw a maximum of GBP10 at a time from the machine. With the development of technology, transactions that can be made by ATMs have improved. Furthermore, ATMs are classified separately based on their innovation. ATMs are divided into two types. The first type is a basic one that allows customers can only withdraw cash and get updated account balances. The second type has more extended features. Deposits are accepted, line-of-credit payments and transfers are facilitated, and account information is accessed on more advanced devices. Types of ATMs can have placed according to the needs of certain regions. For example, in small towns, most people don’t use the advanced features of ATMs therefore basic ATMs for withdrawing or load money may be adequate for kind of locations. Need of customers can change but one thing is certain, the need for cash will never change. In the near future, the prediction about ATM development is that number of ATMs will rise. Rather than bank tellers, ATMs become full-service transaction points. The issue of ATM cash distribution, which will be always popular, should be further developed for these reasons. The purpose of this article is to optimize replenishment and routing while reducing operational costs.

3. LITERATURE REVIEW

Service points problems can be seen in many various sectors such as finance, entertainment, manufacturing, etc. ATM is the main service point for the banking industry. ATMs are daily service providers for many people. Therefore, an efficient system is important to rise customer loyalty. In this chapter, articles that are interested in service point problems are classified based on their problem types and purpose. For problem types, 3 main problems are selected. These are Assignment Problems, Inventory problems, and Forecasting problems. Moreover, Cost Minimization and Time Minimization headlines were selected according to the article's objective. Afterward, for the vehicle routing part of the study, literature is examined for the grouping of ATMs based on customer withdrawal behavior.

Firstly, I search the keywords which are Vehicle Routing Problem, Automated Teller Machines, Cash Replenishment, Cash in the database of EBSCO Discovery Service EDS, and ABI/INFORM Global. Web of Science, Scopus, and Science Direct websites are used for research. In addition, I eliminated some research that was not suitable for the scope of the study.

Routing is the process of deciding on a path for traffic to take either inside, between, or between many networks. A variety of networks, including digital networks like the Internet and circuit-switched networks like the public switched telephone network (PSTN), engage in routing. Although routing has been extensively studied in the literature, little of it has been done in the service sector, particularly in the banking sector.

3.1. Service Point Problems Based on Applied Methodologies

3.1.1 Assignment Problem

Assignment Problem is the methodology of linking with the duties and persons that may provide the most beneficial result. Assignment problems can combine with vehicle routing for creating optimal cash distribution. Boonsam claimed that goal of the assignment of branches to distribution centers is to minimize distance and not exceed the capacity (Boonsam et al., 2011). Basic assignment problems can ensure efficient grouping for branches.

3.1.2 Inventory Problem

In some cases, inventory and assignment problems cannot consider separately. Therefore, the model may be developed by combining this two problem. Larrain state that storing management and delivering issue originates in the financial service business as an application of technique and to assess its performance. This problem has many aspects of a rich inventory-routing problem (IRP) that mixed vehicle routing problem and management of stock, across a multi-period time horizon. A decision maker should decide the time of visiting each ATM, the amount of loading cash every time, and how to assign ATMs to distinct vehicle routes in this IRP. A significant point of aim is to reduce overall inventory management and distribution costs while providing excellent service to the demand (Larrain et al., 2017).

3.1.3 Forecasting Problem

The difficulty of forecasting the right quantity of cash for each day's customer needs which means that the minimal amount of cash is always accessible until the next replenishment is problematic. There will be no client unhappiness because of not loaded

ATM. A machine learning technique to predict ATM replenishment amounts utilizing a data-driven approach for estimating the correct money for all ATMs or a set of ATMs can be implemented for this problem (Asad et al., 2020). Machine learning technique is the most common method to solve forecasting problems in recent years. Another machine learning code for problem represented to solve forecasting of daily cash withdraws from ATMs. By using Artificial Neural Networks, daily cash withdrawals for certain ATMs are claimed as predictable. In addition, the study provides a cost function for determining the most efficient money and replenishment days. Cash withdrawals adopt an estimated seasonal pattern depending on date and time parameters. The suggested model reduces the idle balance (Serengil & Ozpinar, 2019).

The pioneering studies in the vehicle routing literature are by Dantzig et al. (1959) first defined the vehicle routing problem in 1959 and presented a solution approach based on linear programming. Their study included optimum routing for gasoline trucks that start from the terminal to service stations by using the shortest routes. One of the significant constraints was meeting the demand for stations. After this research, Clarke and Wright invented the heuristic solution approach known as the savings method. Fletcher and Clarke presented a paper at the Operational Research Society's conference in England in 1963. For the illustrative case that they used, their heuristic was better than Dantzig and Ramser’s. Clarke and Wright developed Dantzig and Ramser's solution to add the decrease in distance achieved by connecting two customers to one way instead of servicing them on different routes. In their technique, Dantzig and Ramser focused primarily on the distance between a couple of consumers, tying them into a comparable path (Dantzig & Ramser, 1959). The new method used the previous method as a basis and improved the model by creating a connection between the customer’s routes. This feature provided a shorter route that is more desirable because of the cost. VRP methodology can be implemented in many business areas such as repairable products for VRP with simultaneous pickup and delivery.

3.2 Service Point Problems Based on Purpose

3.2.1 Cost Minimizing

In addition, time window repair and pick-up can have integrated with vehicle routing problems. Rabbani et al. (2018) research considers the multi-depot vehicle routing problem with a time window considering two repair and pickup vehicles (MDVRPTW). In that paper, the objective function is not only minimizing total traveling cost but also time window violations. Genetic algorithm (GA) and Hybrid Genetic Algorithm (HGA) are applied to this problem. A comparison of their results points out HGA has better performance than GA. Routing problems started to be applied later to the service sector. Today's service sector faces many challenges, such as providing quality services and trying to keep service costs low. Researchers and professionals are frequently seeking to use optimization and information technology due to economic problems. In the service sector, it is common to be needed to solve real problems. One of the important research which used real data from the bank was generated by Van Anholt et al. (2016) who introduced, designed, and solved a complex multiperiod inventory-routing problem involving pickups and deliveries inspired by ATM replenishment in the Netherlands. The authors used a clustering technique to separate the problem into many more manageable subproblems, which they generalized by correcting several variables. Valid inequalities are then used to enhance the resulting subproblems. An efficient branch-and-cut algorithm is then produced after that. By offering cash supply chain parties based on increasing cash management operations and cutting expenses, this research foresees this growth. Bolduc et al. (2010) introduce the new model, which specifies which clients will be serviced by a certain carrier as well as the delivery routes for those provided by the private fleet, in order to reduce total transportation and inventory expenses. They describe two new neighbor reduction algorithms as well as a new tabu search algorithm. Outcomes showed an improvement in overall solution quality.

3.2.2 Time Minimizing

Anbuudayasankar et al. (2009) state that VRPTW (Vehicle Routing Problem with Time Windows) solutions must be more practical and efficient than optimal, according to businesses. To deliver such real-time resources, such as processing a service request over the internet, the solution for a VRPTW must be obtained in seconds or minutes and be as feasible and implementable as possible. The goal of the study is to minimize the total route by using a minimum number of vehicles. To solve VRPSPD (VRP with Simultaneous Delivery and Pick-up) with uncertainty demands, simultaneous pick-ups and deliveries, a metaheuristic algorithm and SA (Simulated Annealing) are implemented. SA is a metaheuristic for forecasting global optimization in a search field. Results showed that SA has better results than the Min method according to solution quality and computation time. Moreover, time spread constraints can be combined with time window constraints. According to Michallet et al. (2014), the second option is uncertain arrival times and periodic vehicle routing with no-wait time windows are very hard to handle. As minimizing route costs and dispersing arrival times are two goals that are in opposition to one another, the overall cost of the routes is minimized with the restriction that the hours of any two routes to the same client must differ by a specific time constant. Furthermore, it is possible to combine VRP with GIS (Geographic Information System) for transportation operations. Koç et al. (2018) indicate that the joint multiple depots, pickup and delivery, multi-trip, and homogeneous fixed vehicle fleet are all addressed in the rich VRP. Initially, a mathematical formulation was represented. Then a GIS-based solution strategy was applied, which uses a Tabu search algorithm to evaluate and show all data, as well as model solutions, in a geographic format. They used a bank dataset and a GIS-based solution methodology to solve the problem. Finally, by examining the trade-offs between various limitations, present significant management and policy insights. Also, the tabu search heuristic can be suitable for vehicle routing problems. Furthermore, a hybrid genetic algorithm may be used to solve vehicle routing problems. Jeon et al. (2007) use a hybrid evolutionary algorithm to focus on heterogeneous vehicles, duplicate trips, and various depots in order to discover the best feasible answer to the vehicle routing problem. A hybrid genetic algorithm (HGA), a mathematical programming model with a new numerical formula that shows the distribution amount, and sub-tour elimination that incorporates the improvement of generation for an initial solution, were offered in the study. Particle swarm optimization (PSO) which is combined with VRP, is a computational method for solving problems by iteratively trying to improve a potential solution against a criteria of quality. The PSO method is beneficial to solve VRPSPD thanks to a combination of outcomes. The concept of vehicle orientation covers the route to a specific area. The cheapest insertion heuristic and 2-opt approach, which are used during route creation, rise the solution quality. An algorithm reduces the number of vehicles available to service consumers. PSO's methodology for generating several solutions and keeping the best one identified during the iteration phase (Ai & Kachitvichyanukul, 2009). It is possible to generate a model in terms of environmental concerns. Recycling ATMs are expensive, thus their deployment should be carefully planned. Batı et al., (2017) imply that optimized ATM networks according to cash-related costs. Integer linear program is used optimization of problem that jointly determines when to visit an ATM, how much money to load to which ATM, and the route for cash distribution to ATMs. They also decide which ATMs should be replaced with recycling ATMs in the network. Then, based on cash cost and the choice to recycle ATMs, offer a polynomial-time heuristic approach and compare it to the optimal formulation. Developments in this VRP cannot be ignored this is due to the fact that the service sector is in constant change. I briefly summarized the development of studies on the Vehicle Routing Problem.

3.3 ATM as a Service Point

ATM’s classified as a service point because they provide money to the customer. When “provide” and “customer” words come together, they create the service environment. Customers can withdraw money, check their balance, deposit money for payments, transfer between accounts, and also transfer funds. To do these actions, they need a service provider point which is an ATM. ATM management is not the business of the banks. Generally, third parties operate these movements for several banks. ATM as a service delegated the management of the ATM channel to a third party, allowing financial institutions to focus on their important activities. As a result, ATM as a service lowers costs simplifies maintenance and system updates, and gives consumers the experience they want. Best delivery or service shipping is important to ensure the efficiency of service by minimizing costs (Bozkaya et al., 2012).

3.3.1 ATM Problems

When customers need to cash, they do not want to face the issue in the ATM. To prevent the problems, proactive behaviors need to be taken. Regular control and maintenance can avoid unsatisfied customers who need money. Some failures are the most common ones failing to dispense cash or the requested amount of cash. This problem occurs when ATMs do not have sufficient cash which is an inventory problem or software issue in the head of the ATM. Moreover, some machines can not read the cards correctly which is an easy problem to solve by changing with a new reader. One of the challenging tasks may be viruses can conquer the ATM software and this problem may spread to other ones. Proactive actions need to be taken such as setup virus protection programs or generating strength software. These all problems can be experienced by customers but solutions to these are simple. A complicated and most significant problem is cash and inventory of ATMs which is the main scope of this study.

3.4 Methodologies for ATM Clustering

Various approaches can be implemented for the grouping of ATMs. ATM clustering is a useful methodology for forecasting demand and creating routing. Most used methods are determined according to a literature survey. The first one is grouping ATMs based on the cash withdrawal approach of customers and the second one is grouping of nearest ATMs together.

3.4.1 Cash Demand of ATM

Similar cash withdrawal behaviors can be observed in some places. For instance, creating 3 clusters of ATMs that are placed in core business areas, touristic areas, and residential places. The proposed method aims to group the ATMs according to cash withdrawal attitude (Ekinci et al., 2021). Forecasting of cash become easier by this method and accurate results are obtained. In addition, grouping by more specific patterns can be implanted for this problem. Vankatesh et al. (2014) claim that clusters the ATMs showed resemble withdrawal rates for the same weekdays. Seasonality is taken an account and the time series model is improved due to the paydays or holidays. Moreover, it is possible to group the ATM based on the withdrawal rate of each day. Days of the week are separated into 3 groups based on the behavior of the cash withdrawal. The normal distribution is observed for each cluster and the standard deviation is calculated for each group (Pala, 2014). This research focuses to think from various perspectives. Other studies' goal is to group the ATM but this study aims to cluster the days afterward and assign the ATMs to each cluster. There is a lack of atm grouping in terms of cash demand in the literature. Therefore, this area needs to improve more to achieve accurate results.

3.4.2 Location of ATM

A cluster replenishment strategy might save ATMs in similar geographic areas enormous cash in terms of operational expenses. Consider combining adjacent ATMs and averaging their daily withdrawals, then averaging the data over a seven-day period. Results claimed that the cluster mediocre forecast has shown more successful performance than the individual forecast (Ekinci et al., 2015). Different location patterns can be used for the problem. Depot is the most important place for vehicle routing and putting this to the core of the problem can be reasonable. Similar to nearest neighbor heuristics, the heuristic provides every client to the most closed warehouse and after that construct the routes for a group of client of any depot (Bae & Moon, 2016). Customers symbolize ATM and depot is the cash depot for our model. Moreover, various methods can be suitable to determine the truck routes in terms of their location. According to the inventory policy, there is a low level of cash in such an ATM that should be fulfilled immediately. The shortest route by time window VRP algorithm is served for requesting ATMs (Scholtz et al., 2012). To sum up, a high service level with low cost is stated by this methodology. Furthermore, different algorithms can be used and results would be compared to identify the better one. Firstly, the nearest neighbor method is applied to each ATM that is a service place. Secondly, the ant colony optimization algorithm can be beneficial to solve the problem. To find the shortest and most reliable path, ants follow each other's track on the road. If a group of ants chose a certain path rest of the colony follow it and in the future they tend to use the same path as well (Zelenka et al., 2012). As a result, the ACO method provides a better solution rather than NNA.

In addition, instead of these two methodologies, clustering the ATMs with the lowest robbery risk is one of the significant strategies however it is rare to find these studies in the literature. Routes can be detected and become defenseless against rubbery attacks. To avoid this risk, the model is updated daily to ensure the most recent solution for cash transportation. The likelihood of selecting selected links in prior successive times decreases with each step (Ghannadpour & Zandiyeh, 2020). By putting the robbery risk at the center of the problem, routes can be generated by combining particular perspectives. To conclude, the most accurate grouping strategies for our study are the demand for cash at ATMs and the place of ATMs. In the literature, there is a lack of knowledge about which one is better. Therefore, we will implement two methodologies into the model separately and present the results of these to compare each other.

4. METHODOLOGIES

4.1 Forecasting Methods

Businesses can profit from forecasting due to the fact that it allows them to make logical decisions and establish strategies. Process and cash based issues are determined in accordance with the current market structure as well as forecast for later. To find out the patterns, previous data is examined which are then utilized to forecast future cycles and changes. To find efficient forecasts, many methods can be applied to data. Instead of traditional ones computer-based methodologies especially machine learning algorithms become common in the last years. In this section, several methods are mentioned respectively and the best one for our data is clarified lastly.

4.1.1 Prophet Method

Prophet is a Facebook-developed open-source library based on decomposable (trend+seasonality+holidays) models. It is one of the most useful machine-learning methods in the literature. It allows users to construct accurate time series forecasts using basic understandable criteria, and it also supports the inclusion of unique seasons and holidays. Prophet is a time series prediction model released in 2017. Taylor & Letham claim that a basic, modular regression model was used that frequently works well with default settings and allows analysts to pick and choose which components are important to their forecasting challenge and make changes as needed. The Fourier series is used to create an effective model of seasonal effects (Taylor & Letham, 2018). Prophet has various aspects from other machine learning methodologies. Seasonality and the holiday approach of the Prophet are the core differences when we compare them with forecasting strategies. Dates of holidays may be changed every year. For instance, the beginning of Ramadan starts approximately 11 days earlier each year. To overcome this kind of challenge, Prophet proposes a parameter that includes days surrounding the holiday, and these surroundings also classify as a holiday in the model. This helpful creature is significant for seasonal data. On the other hand, in the literature, the prophet method is not used for the ATM withdrawal forecasting approach. Sales forecasting, price forecasting, traffic prediction, and time series forecasting are some areas that Prophet is obtained. Trend term, seasonal period term, and holidays are the three essential aspects of the model. Unlike other forecasting approaches, the Prophet framework is straightforward to understand. Aside from excellent prediction abilities, Prophet can also efficiently cope with data having periodic and cyclic changes, as well as significant anomalous values (Wang et al., 2021). Because of the many features, ATM data have seasonal changes and also include noise data. When used to estimate ATM withdrawal, the Prophet method may transcend the constraints of standard forecasting models and provide optimal forecasting results. In the literature, Prophet is not used to estimate the cash demand of ATMs before. It is generally used for production planning or sales forecasting. But it can be also suitable for our problem.

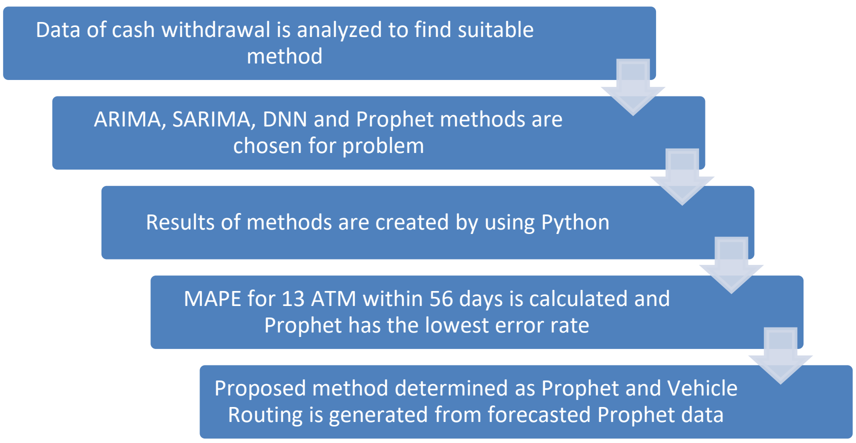

Figure 4.1.1.1: Steps of Prophet Forecasting

Forecasting is a curial accomplishment for business in terms of production planning schedules, workload prediction, or inspecting weak points. Obtaining an efficient forecast is not an easy task, it includes many challenges such as the method should be used easily due to nonexpert workers, offering flexibility for time series, and eliminating the inefficient forecast to find the optimal one. The model included a modular regression model with interpretable parameters that analysts with domain knowledge of the time series may naturally alter.

4.1.2 Arima and Sarima Methods

A statistical analysis technique which is known as ARIMA (autoregressive integrated moving average) uses time series data to improve data analysis or predict future trends. A kind of regression analysis called an autoregressive integrated moving average model assesses how stable one dependent variable is in comparison to other fluctuating variables. By focusing on the variations between values in a series rather than the correct numbers, the model seeks to predict future trends in financial markets or securities. An ARIMA model has three constants in terms of; *p* for a number of autoregressive terms, *d* for the order of differencing, and *q* for the number of moving-average terms. Generally, ARIMA duration would be expressed as ARIMA (*p*, *d*, *q*). To determine this forecast properly by ARIMA, several steps are required according to Nahmias. Box-Jenkins models are also known as ARIMA models. There are four core steps to implement the Box-Jenkins forecasting method. The first one is data transformation and this method begins with stationary time series. Even though the mean of the series is essentially consistent, the variance may not be, necessitating data transformation. The second one is model identification that is which ARIMA model is suitable for certain problems. To find patterns that match those of known processes, it is significantly more beneficial to look at sample autocorrelations and partial autocorrelations. The third one is parameter estimation which is determined by either least squares fitting methods, the method of maximum likelihood, or computer programs. The fourth step is forecasting which can be done by Box-Jenkins models which ensure one step ahead and multiple step ahead forecasts. The last step is evaluation. Forecast errors are a significant tool for this. When the residuals show patterns, it indicates that the forecasting model may be developed (Nahmias & Olsen, 2015). To forecast the cash in the ATM, ARIMA is the most common method for this problem. Khanarsa, P. et al. claim that after producing and fitting a certain number of subsequence patterns with ARIMA models, the best model is found using the symmetric mean absolute percentage error (SMAPE). Results claimed that the multiple

ARIMA subsequences aggregate time series model (MASA) has shown lower SMAPE than the SARIMA model and ETS exponential smoothing model (Khanarsa & Sinapiromsaran, 2017).

Moreover, to overcome seasonality in the data SARIMA can be a useful method. SARIMA may be represented as a seasonal ARIMA. SARIMA similarly not also used past data like ARIMA but also takes into account the seasonality of the pattern as a parameter. SARIMA become more suitable for forecasting complex data spaces containing trends. SARIMA duration would be expressed $(P, D, Q)_m$. m is the seasonality term which is the number of observations per year. SARIMA become the most suitable method where seasonality is known. The ARIMA model and the vector time series model's forecasting accuracy are examined and contrasted with two basic models used as a benchmark. When compared to the vector time series model, the seasonal ARIMA model had a greater predicting accuracy (Wagner, 2010). Comparing the techniques to forecast the cash demand is the common method to find the best solution. Moreover, SARIMA models for ATM data can be a new technique. Because switch ARIMA models match the data properly, especially ex-post projections, this type of model should perform well in forecasting ATM withdrawals. SARIMA models applied to certain subseries may be valuable tools in characterizing the structure of daily withdrawals regardless of ATM location, according to the empirical findings (Gurgul & Suder, 2013). ARIMA is also implemented for this problem but it gets worse results than SARIMA.

4.1.3 Neural Network Method

Instead of statical methodologies such as ARIMA and SARIMA, machine learning also has a huge share in the literature to forecast the optimal solutions. Traditional time series prediction algorithms such as Box Jenkins and ARIMA presume a linear connection between inputs and outcomes. The benefit of neural networks is that they can approximate nonlinear functions (Oancea & Ciucu, 2014). Especially neural networks have become the most popular trends in machine learning. Artificial neural networks

(ANNs) and simulated neural networks (SNNs) belong to the machine learning method and they are significant components of deep learning. The human brain made an inspiration by those because their name and structure come from this, and they have very similar communication types to real neurons. Artificial neural networks (ANNs) are forecasting techniques based on simple brain mathematical models. Complex nonlinear connections between the response variable and its predictors are possible with them. A neural network may be conceived of as a layer-based network of "neurons." The bottom layer is made up of predictors (or inputs), and the top layer is made up of forecasts (or outputs). There may also be "hidden neurons" in intermediary levels. This diagram depicts a multilayer feed-forward network, input comes to the layer from the previous layer output. A weighted linear combination is applicable to make a combination of the inputs to each node. Before being output, the result is changed by a nonlinear function (Hyndman & Athanasopoulos, 2018).

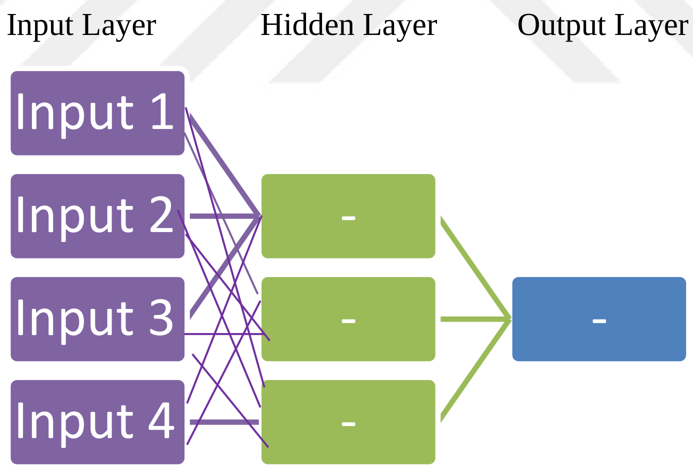

Figure 4.1.3.1: Multilayer Feed-Forward Network

Neural networks may be used for cash demand forecasting. Several types of neural networks can be implemented for this problem. Venkatesh et al. (2014) state that four neural networks are used which are general regression neural network (GRNN), multi-layer feed forward neural network (MLFF), group method of data handling (GMDH), and wavelet neural network (WNN) built to predict an ATM center’s cash demand and GRNN has shown the best result according to the SMAPE. Various neural networks may be suitable for data and results are analyzed based on the error method. Different neural networks are applied. The unknown future values of a collection of 100-time series of daily ATM cash withdrawals were modeled and forecasted using a multi-layer feed forward neural network model. The outputs of the forecasting model depict the complicated dynamics of the time series data that have been properly described (Atsalaki et al., 2011). But in this study, seasonality is not into account. One of the best ways to find efficient solutions is to compare the different methods and classify their results. ANN and SPR methods are used to predict daily cash demand for ATMs. The simulation studies and experimental tests showed that both proposed methods of forecasting are tolerable. Results showed that ANN offers better results than SPR (Simutis et al., 2008).

4.1.4. Comparison of Methodologies

Analysis of several methodologies which come from different perspectives provides an efficient solution. With this strategy, advantages and detrimental sides can detect in terms of improving the model. The determination of the method depends on the data and goal of the study. The structure of data and desired outcomes are the main features when finding the best method. According to this approach, the most beneficial method may be applied. In this section, a comparison of various methods is mentioned.

Firstly, machine learning methods may be compared with each other. Nonlinear prediction methods, such as artificial neural networks (ANNs) and support vector regression (SVR), have been devised and widely employed in artificial intelligence domains to overcome the flaws of linear models. Moreover, the constraints of SVR and LSTM (Long Short Term Memory) models impair the prediction ability of the models when the data is increasingly complicated or changes dramatically. Therefore, these methods are not suitable for noisy data.

In the Prophet, unlike ARIMA models, evenly spaced doesn’t require for the measurements, and missing data are not interpolated. For example, eliminating outliers

Taylor & Letham, 2018). In most of the studies, ARIMA compare with the Prophet and Prophet method offer better results than ARIMA mostly.

4.2 Routing Methods

4.2.1 Vehicle Routing Problem

It is possible to apply various methodologies for cash management of an automated teller machine. Firstly, using fuzzy constraints for truck routing problems can be reasonable due to the uncertainty present in the data. In addition, the Vehicle Routing Problem is not only a method that is implemented but also a Truck and Trailer Routing Problem that can be applied to many problems in transport and logistics Torres et al. (2013) claim that the difference between TTRP and VRP is the usage of trailers, which is often overlooked in the VRP. TTRP is a VRP version in which a subset of vehicles is permitted to pull trailers for the advantage of increased capacity. When the problem's elements are unclear, imprecise, and inexact, a fuzzy parametric technique is given as a generic strategy to solve it. Furthermore, VRP is the most common method which is used for logistics operations. Scholtz et al. (2012) claim that the integration of four operations research methods which is the vehicle routing problem (VRP), discrete-event, knapsack problem, and continuous review policy can be implemented for Automated Teller Machines Replenishment. These four methods are combined under the title of decision support system (DSS). Study outcome proof that implementation of the VRP to an ATM network results in a cheap service, and increased quality. The findings and related decision support indicate that many operation research approaches can be combined into a decision support system for ATMs. That research claims that different kinds of methodologies can be integrated for problems and effective solutions can obtain for real data. In the modern world, it becomes the protection of properties which are cash, coin, social media accounts, etc. Many properties have similar points that is money. Hackers or thieves want to receive money from you and search for the easiest way. Bank trucks that road from depot to ATM can be an easy target for robbery. That’s why, the routes of trucks need to design not only to include the shortest route for minimizing cost but also risk factor due to safety issues. Yan et al. (2012) create a model for secure cash delivery routes. A model that formulates potential movements of cash transportation vehicles among all demand sites in time and space dimensions is built using the time-space network technique. This model includes a novel idea of time and space similarity for routing and scheduling, which is anticipated to assist security carriers in creating more adaptable routing and scheduling methods. Cash transportation operations can classify as high-risk because of robberies etc. To handle this risk factor, routes may be redesigned due to the protection of valuables. Bozkaya et al. (2017) consider a bi-objective function that is to keep the total transportation cost and security risk of transferring assets along the planned routes as low as possible. For risk constraint, two indicators are determined using the same or very similar routes and passing neighborhoods with low socio-economic status in the routes. Also, the capacity of vehicles is constructed in terms of required by insurance requirements, a monetary value carried. To overcome this, the composite risk measure is used to choose different routes from origin to destination over a period of days. Moreover, it is possible to construct a model according to risk constraints. Talarico et al. (2015) introduced the risk-constrained Cash-in-Transit Vehicle Routing Problem (RCTVR). Three metaheuristic approaches are developed to solve RCTVR. First, four distinct methods are created to arrive at a preliminary solution. Heuristic of Clarke and Wright TSP nearest neighbor with greedy randomized selection, TSP Lin-Kerninghan with splitting, and nearest neighbor with greedy randomized selection method. Metaheuristics' output has the potential to quickly produce solutions of high quality. VRP may be created for a vehicle routing problem with time windows (VRPTW). By using not heterogeneous vehicles that have a certain capacity a similar depot location for the beginning and ending location of vehicles, and a set of customers who have demanded and imposed time windows, a VRPTW attempts to find the most cost-effective way to deliver products to these customers within their specified time windows (Bozkaya et al., 2012). The routes must be designed so that each node is only visited once by a certain vehicle within a specified time interval, that all routes begin and terminate at the depot, and that the combined demands of all locations on the route do not exceed the vehicle's capacity. VRPTW heuristics are generally examined on two factors: solution quality according to objective function value and speed. Gendreau et al. (2005) claim that good heuristics must also have a basic implementation, adaptability, and robustness. After the literature review of cash transportation for ATMs, I narrow down the scope of the research. I have listed the studies using different methods in this section. Various methodologies may be applied such as fuzzy constraint, DSS, risk constraint, and time windows constraint. Similar objective functions are shaped in terms of these methods. In this paper, I decided to choose a basic vehicle routing problem for modeling. However, my ultimate aim is to develop the VRP model by adding one of the methods that I listed.

4.2.2 Features of Vehicle Routing Problem

Based on the fundamental vehicle routing problem (VRP), the vehicle routing problem has a lot of several improved versions in both academic study and real-world applications, such as:

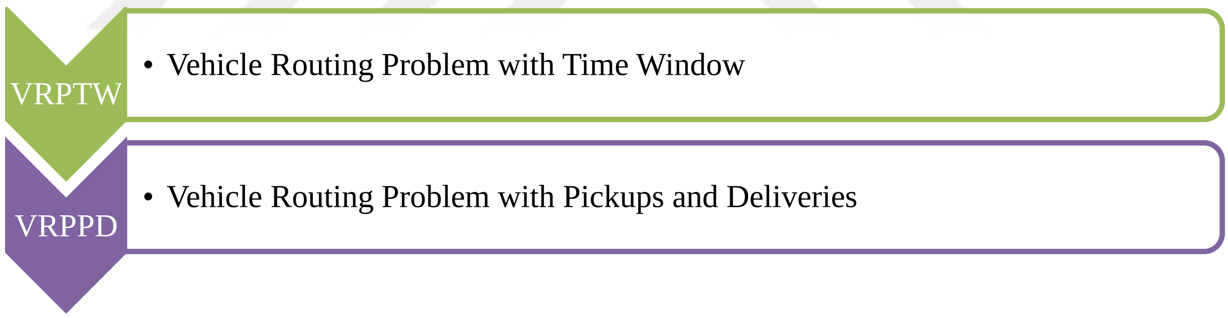

Figure 4.2.2.1: Types of Vehicle Routing Problems

There are several VRP variants, but Vehicle Routing Problem with Time Window and Vehicle Routing Problem with Pickups and Deliveries are the most suitable problems for my study based on my expected outcome and constraints. A heterogeneous vehicle fleet at different points must meet with the customer demands in the VRP with Pickup and Delivery (VRPPD). A pickup point, a matching delivery point, and the requirement to be transferred between these places characterize each request. In most of the models, the objective functions aim to minimize costs (Desaulniers et al., 2002). Vehicle Routing

Problem with Time Windows (VRPTW) is the time slot for providing a service to customers. These VRPs are available to be implemented in data. In the later stage of the problem, these two problems will be combined to find an optimal solution.

4.2.3 VRP Spreadsheet Solver

The VRP Spreadsheet Solver is introduced in this study as a solution to the issue that the realistic effect of VRP has unique requirements to which the software can be changed by the user. This software overcomes these issues through the familiarity of its user interface, usability, flexibility, and accessibility. Due to their dynamic nature, the journey distance and duration data must be continuously collected from a GIS, which either creates a recurrent acquisition expense or necessitates the need for in-house specialized expertise. Selective client visits, taking and delivering at the same time, time windows, fleet composition, distance restrictions, and the last stop of the vehicles are all aspects of the VRP Spreadsheet Solver. Spreadsheet for VRP Solver uses a rising step of information and retains the information about the components of a VRP on distinct worksheets. The VRP Solver Console's main worksheet provides the problem's parameters. The following spreadsheets are created in the order indicated by their indices: 1. Locations, 2. Distances, 3. Vehicles, 4. Solution, and 5. Visualization (Erdoğan, 2017). Details of the main worksheet can be seen in Figure 4.1. I decided to use Bing Maps Key for real-time traffic data and visualization of routes. In this way, the model becomes more realistic in this manner. 1 depot and 13 ATMs are determined for study. Duration is calculated by the real-time traffic data of Bing Maps. Average speed and travel type are determined by predicted information. Every truck should come back to the depot at the sun goes down and routes need to be finished in the specific timeslot. CPU time limit is stated as 60 as recommended by the program.

Sequence	Parameter	Value	Remarks
0.Interface	Language	English	Please refer to the manual for modifying the interface.
	Optional - Bing Maps Key	Aq9Jly5EKQQdPh33NXrJ5VQd1jvTaPy	You can get a free trial key at https://www.bingmapsportal.com/
1.Locations	Number of depots	1	[1, 20]
	Number of customers	13	[5, 200]
2.Distances	Distance computation method	Bing Maps driving distances (km)	Recommendation: Use 'postcode, country' format for addresses
	Duration computation method	Bing Maps driving durations	
	Bing Maps travel mode	Truck	
	Bing Maps route type	Fastest - Real Time Traffic	Recommendation: Use 'Fastest'
	Bing Maps route detail level	2	[0, 10]
	Average vehicle speed	70	
3.Vehicles	Number of vehicle types	1	
4.Solution	Do the vehicles return to their depot(s)?	Yes - only once at the end	
	Time window type	Hard	
	Backhauls?	No	If activated, delivery locations must be visited before pickup locations
5.Optional - Visualization	Visualization background	Bing Maps	
	Location labels	Location IDs	
6.Solver	Warm start?	Yes	
	Show progress on the status bar?	No	
	CPU time limit (seconds)	60	Recommendation: At least 60 seconds

VRP Solver Console | 1.Locations | 2.Distances | 3.Vehicles | 4.Solution | 5.Visualization

Figure 4.2.3.1: VRP solver console

After then, locations of ATMs are added to the system. A desirable time slot for visiting is stated if the vehicle can't visit in that time penalty cost will be added to the cost calculation. Visiting options are determined according to various strategies. Service time is the time of filling and withdrawing the cash to the ATM. By using the most suitable forecasting method data which is Prophet pickup and delivery amounts are determined. Profit is calculated by the remaining cash of the ATM and the bank interest rate on a certain day.

Location ID	Name	Address	Latitude (y)	Longitude (x)	Time window start	Time window end	Must be visited?	Service time	Pickup amount	Delivery amount	Profit
0	Depot	Yapı Kredi D Blok Plaza Bi	41,0807910	29,0129130	08:00	18:00	Starting location	0:00	0	0	0
1	ATM1	İBB Beşiktaş Ortaköy Mh.	41,0485280	29,0266820	08:00	18:00	Don't visit	0:15	- 712,47	15.826,00	-0,05
2	ATM2	İBB Beşiktaş Barbaros Blv	41,0434700	29,0070900	08:00	18:00	Don't visit	0:15	- 575,33	15.032,00	-0,02
3	ATM3	İBB Beşiktaş Kuruçeşme M	41,0601040	29,0364920	08:00	18:00	Don't visit	0:15	281,44	17.962,00	-0,69
4	ATM4	4.Levent Şb 2ATM Kodu:	41,0866070	29,0086190	08:00	18:00	Don't visit	0:15	- 4.875,10	17.102,00	-0,6
5	ATM5	Superonline BeşiktaşATM	41,0503810	29,0070070	08:00	18:00	Don't visit	0:15	- 432,98	7.770,00	-0,25
6	ATM6	İBB Beşiktaş CdATM Kodu	41,0417520	29,0052680	08:00	18:00	Don't visit	0:15	228,98	20.884,00	-1,31
7	ATM7	Gayrettepe Türk Telekom	41,0651500	29,0086630	08:00	18:00	Don't visit	0:15	- 718,07	13.477,00	-0,02
8	ATM8	İBB Bebek Cevdet Paşa Cı	41,0793560	29,0479520	08:00	18:00	Don't visit	0:15	- 15.635,27	14.665,00	-0,28
9	ATM9	Barbaros Plaza AtakuleAT	41,0578720	29,0095890	08:00	18:00	Don't visit	0:15	- 797,57	15.720,00	-0,48
10	ATM10	İBB Beşiktaş İskelesiATM	41,0452180	29,0073090	08:00	18:00	Don't visit	0:15	- 1.628,34	12.824,00	-0,5
11	ATM11	Koza İş Merkezi C BlokAT	41,0566670	29,0119440	08:00	18:00	Must be visited	0:15	2.100,91	18.491,00	0,67
12	ATM12	Akmerkez AVMATM Kodu	41,0768990	29,0268020	08:00	18:00	Don't visit	0:15	1.887,11	22.791,00	-3,95
13	ATM13	Etiler Alkent SitesiATM Kc	41,0849140	29,0327780	08:00	18:00	Don't visit	0:15	- 689,80	12.457,00	-0,84

Figure 4.2.3.2: Location sheet

Distance and duration time between ATMs are calculated by Bing Maps in the next sheet.

In the vehicles sheet, we only have 1 type of vehicle therefore we added their features. Capacity and distance limits are determined as infinite to allow the system to make an optimal decision about visiting. Fixed cost per trip includes vehicle rent cost and driver wage. Cost per unit distance is the fuel cost of the vehicle per km. Driving time is the same as the previous sheet to construct the model sensible. Every vehicle needs to start from the depot and should return to the depot end of every route. The number of vehicles is classified as 3 since my outcomes showed that 3 is the most optimal number for this model from Python results for VRP previously.

Starting depot	Vehicle type	Capacity	Fixed cost per trip	Cost per unit distance	Duration multiplier	Distance limit	Work start time	Driving time limit	Working time limit	Return depot	Number of vehicles
Depot	T1	1000000	119,00	18,46	1,00	1000000,00	08:00	9:00	10:00	Depot	3

Figure 4.2.3.3: Vehicle sheet

In the solution sheet, the system gives the results according to my parameters and my preference about don't visit, must be visit or may be visit. Net profit is calculated by the total profit values from the location sheet, fixed cost per trip, and cost per km multiplied by the distance traveled and penalty cost of the time window. In my study, this objective function is changed based on my model since minimizing the cost needed to find. Therefore, minus values of costs are added to the system and the objective function becomes maximize minus cost which means minimizing cost. After this crucial alteration, several scenarios are applied to the system to find the most optimal one.

Total net profit:	**-246,79**							
Vehicle:	**V1**	**Stops:**	**2**	**Net profit:**	**-246,79**			
Stop count	**Location Name**	**Distance travelled**	**Driving time**	**Arrival time**	**-246,7896533**	**Working time**	**Profit collected**	**Load**
0	Depot	0,00	0:00		08:00	0:00	-	18.491,00
1	ATM11	3,45	0:20	08:20	08:35	0:35	0,67	2.100,91
2	Depot	6,96	0:44	08:59		0:59	0,67	-
3								
4								

Figure 4.2.3.4: Results sheet

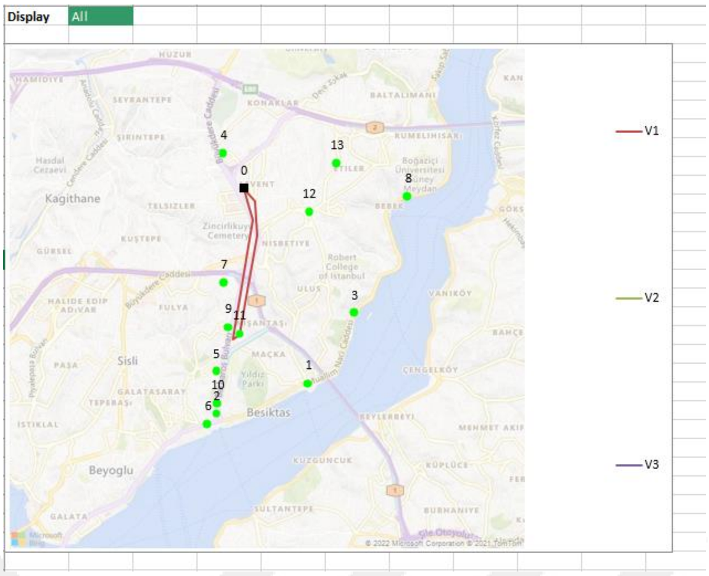

Figure 4.2.3.5: Visualization sheet

4.2.3.1 Large Neighborhood Search Algorithm

Large Neighborhood search is a mathematical optimization strategy that continually optimizes the latest inference into a new solution in an attempt to identify effectively ideal answers to an optimization problem. A solution's neighborhood is a collection of related solutions acquired by making small adjustments to the first implication. The main idea of this algorithm is to gather the similar solution to find the optimal solution. The neighborhood is enourmoıus and exponentially scaled for a very large-scale neighborhood search. A large neighborhood search algorithm is an iterative process that, using a specified neighborhood definition approach, destroys a portion of the existing answer at each iteration and reoptimizes it to find the best possible solution. A subgroup of consumers who are not linked to their present routes constitutes a neighborhood. In light of the fact that some of the routes in the present solution are fixed, the re-optimization issue that results corresponds to the original VRPTW (Desaulniers et al., 2007). One of the features of the algorithm is it aims to reduce the number of vehicles and minimize the total travel distance by adding or removing the operators (Konstantakopoulos et al., 2020). In addition, there are several versions of large neighborhood search algorithms in the literature but Adaptive Large Neighborhood Search (ALNS) is the most applied the large neighborhood search method in academic studies. A heuristic technique which is Adaptive Large Neighborhood Search (ALNS) has been successfully used to address a variety of challenging path planning issues.

In contrast to other neighborhood search algorithms, ALNS exhibits the traits of rapid convergence and avoids slipping into local optimum, allowing it to finish the reconstruction and optimization of the solution rapidly. A team of demolition and repair operators is employed in the large neighborhood search algorithm (LNS) to search the neighborhood. The ideal solution is of poorer quality in the smaller neighborhood (Huang et al., 2021).

5. MATHEMATICAL FORMULATION

5.1 Prophet Method

Taylor et al. (2018) are introduced Prophet's mathematical formulation. Prophet employs a decomposable time series model that consists of three primary components: trend, seasonality, and holidays. The following equation combines them.

$$y(t) = g(t) + s(t) + h(t) + \varepsilon \qquad (5.\ 1)$$

g(t) is a trend item, s(t) is a seasonal change, h(t) is the holiday factor, ε is a error term. Trend parameters discuss in two ways. The first one is nonlinear growth which reaches a saturation point.

$$g(t) = \frac{C(t)}{1+\exp\left(-(k+a(t)^T\delta\left(t-(m+a(t)^T\gamma)\right)\right)} \qquad (5.\ 2)$$

C(t) is the time-varying capacity, k is the base rate, m is offset, $k + a(t)^T\delta$ is a growth rate of time varying, to connect the endpoints of segments $m + a(t)^T\gamma$ is adjusted as a offset parameter and δ is the growth rate alteration.

For linear growth, a piece-wise constant rate of growth ensures an efficient model most of the time.

$$g(t) = (k + a(t)^T\delta)t + (m + a(t)^T\gamma) \tag{5. 3}$$

Where k is the growth rate, δ is rate adjustments, m is offset , and γ is to make the function continuous.

Seasonality models must be expressed as periodic functions of t in order to t and anticipate seasonal impacts. In order to imitate seasonality, Fourier terms are used in regression models by using sine and cosine terms.

$$s(t) = \sum_{n=1}^{N} \left(a_n \cos(\frac{2\pi nt}{P}) + b_n \sin(\frac{2\pi nt}{P})\right) \tag{5. 4}$$

P is the regular period that is expected, 365 for annual data and 7 for weekly data. N= 10 and N= 3 for yearly and weekly seasonality work well for most cases.

Some holidays are certain dates in the year but some of them can be varying every year such as religious holidays or Thanksgiving day. Therefore, the model needs to fit these changes. Resuming that the impacts of vacations are independent. For every holiday parameter κ_i is added, which is the corresponding change in the prediction, and an indicator function showing whether time t is during holiday i.

$$h(t) = Z(t)\kappa \tag{5. 5}$$

5.2 VRP Spreadsheet Solver

An LNS (Large Neighbourhood Search) algorithm is implemented within VRP Spreadsheet Solver. The mathematical formulation of VRP Spreadsheet Solver is adopted from Erdoğan [13] as below; V_D is the vertex set and determined to include the depot(s), V_C to represent the clients, and $V = V_D \cup V_C$. Moreover, $V_M \subseteq V_C$ is the client group for delivering. Let G = (V, A) is the network for solving the VRP. The profit of delivering a cash to customer is claimed as $i \in V_C$ as p_i, the amount of money to take for the client as q_i, the distribution cost as $\hat{q}i$, and the servicing time for a client is s_i. Furthermore, the time interval is accomplished for the customer as [a_i, b_i]. An important point is time interval is assigned for each depot vertex.

The vehicle cluster is represented as K and assigned for each vehicle as $k \in K$ the origin depot of the vehicle as $o^k \in V_D$, the time of start to ride the vehicle as τ^k, the fixed cost of the truck as f^k the capacity is Q^k, the limit of distance as D^k, the driving time limit as $\hat{\hat{D}}^k$, the limit of working time as W^k, and the truck return to the depot as r^k. Connected with each arc (i, j) ∈ A, there is a distance d_{ij} and driving time $\hat{d}_{ij}$. Moreover, for each vehicle $k \in K$, travel cost c_{ij}^k on arc (i, j). Parameters according to process is stated. Ω to be equal to 1 if the truck should came back to certain depot and 0 otherwise. Also, β to be 1 if there is a backhaul constraint, and 0 otherwise. Furthermore, Θ to be equal to 1 if truck doesn't serve service in time window at the cost of a penalty Π per unit of time, and 0 otherwise.

Decision variables are stated too. Let x_{ij}^k be equal to 1 if vehicle k traverses arc (i, j) and 0 otherwise. Also, y_i^k be equal to 1 if vehicle k come and serving vertex i, and 0 otherwise. The number of taking amount and delivering amount processed by vehicle k on arc (i, j) is determined as w_{ij}^k and z_{ij}^k, respectively. t_i^k as the time at that truck k comes at vertex i, and v_i as the amount of break of the time window of vertex i. The mathematical formulation for the VRP is then:

$$\text{Max } \sum_{i\epsilon V_C}\sum_{k\epsilon K} p_i y_i^k - \sum_{(i,j)\varepsilon A}\sum_{k\varepsilon K} c_{ij}^k x_{ij}^k - \sum_{j\varepsilon V_C}\sum_{k\varepsilon K} f^k x_{o^k.j}^k - \prod \sum_{i\epsilon V} v_i \qquad (5.\ 6)$$

Subject to

$$\sum_{k\epsilon K} y_i^k = 1\ \forall i \in V_M, \qquad (5.\ 7)$$

$$\sum_{k \epsilon K} y_i^k \leq 1 \quad \forall i \, \epsilon V_C \setminus V_M, \tag{5. 8}$$

$$\sum_{j \epsilon V \setminus \{i\}} x_{ij}^k \leq \sum_{j \epsilon V \setminus \{i\}} x_{ji}^k \quad \forall j \in V_c, k \epsilon K, \tag{5. 9}$$

$$\sum_{p \epsilon S, q \epsilon V \setminus S} x_{pq}^k \geq y_i^k \; \forall i \in V_c, k \epsilon K, S \subset V : o^k \, \epsilon \, S, i \, \epsilon \, V \setminus S, \tag{5. 10}$$

$$\sum_{p \epsilon S, q \, \epsilon \, V \setminus S} x_{pq}^k \geq \Omega y_i^k \; \forall i \in V_c, k \epsilon K, S \subset V : i \, \epsilon \, S, r^k \, \epsilon \, V \setminus S, \tag{5. 11}$$

$$\sum_{j \in V_C} x_{o^k, j}^k \leq 1 \;\; \forall k \in K, \tag{5. 12}$$

$$\sum_{k \in K} x_{ij}^k \leq 1 - \beta \quad \forall (i, j) \in A : \; q_i > 0 \; and \; \widehat{q_j} > 0, \tag{5. 13}$$

$$\sum_{j \in V \setminus \{i\}} w_{ij}^k - \sum_{j \in V \setminus \{i\}} w_{ji}^k = \; q_i y_i^k \; \forall i \; \in V_C, k \in K, \tag{5. 14}$$

$$\sum_{i \in V_C} w_{i, r^k}^k = \; \sum_{j \in V_C} q_j y_j^k \quad \forall k \in K, \tag{5. 15}$$

$$\sum_{j \in V\{i\}} z_{ji}^k - \sum_{j \in V\{i\}} z_{ij}^k = \; \hat{q}_i y_i^k \; \forall i \; \in \; V_C, k \in K, \tag{5. 16}$$

$$\sum_{i \in V_C} z_{o^k, j}^k = \; \sum_{i \in V_C} \hat{q}_i y_i^k \;\; \forall k \in K, \tag{5. 17}$$

$$t_i^k + \left(\hat{d}_{ij} + s_i\right) x_{ij}^k - W^k \left(1 - x_{ij}^k\right) \leq t_j^k \;\; \forall (i, j) \in A : j \in V_C, k \in K, \tag{5. 18}$$

$$a_i \leq t_i^k \leq b_i - s_i + v_i \quad \forall i \in V_C, k \in K, \tag{5. 19}$$

$$v_i \leq M . \Theta \;\; \forall i \in V_C, \tag{5. 20}$$

$$t_{o^k}^k = \tau^k \quad \forall k \in K, \tag{5. 21}$$

$$t_i^k + \left(s_i + \hat{d}_{ij}\right) x_{i, r^k}^k \leq b_{r^k} + v_{r^k} + M \; (1 - \Omega) \;\; \forall (i, j) \in A : i \in V_C, k \in K, \tag{5. 22}$$

$$w_{ij}^k + z_{ij}^k \le Q^k x_{ij}^k \quad \forall (i,j) \in A, k \in K, \tag{5. 23}$$

$$\sum_{(i,j)\in A} d_{ij}\, x_{ij}^k \le D^k \quad \forall (i,j) \in A, k \in K, \tag{5. 24}$$

$$\sum_{(i,j)\in A} \hat{d}_{ij}\, x_{ij}^k \le \widehat{D}^k \quad \forall (i,j) \in A, k \in K, \tag{5. 25}$$

$$\sum_{i\in V_C} s_i y_i^k + \sum_{(i,j)\in A} \hat{d}_{ij}\, x_{ij}^k \le W^k \quad \forall (i,j) \in A, k \in K, \tag{5. 26}$$

$$x_{ij}^k \in \{0,1\} \quad \forall (i,j) \in A, k \in K, \tag{5. 27}$$

$$y_i^k \in \{0, 1\} \quad \forall i \in V_C, k \in K, \tag{5. 28}$$

$$v_i \ge 0 \quad \forall i \in V_C, \tag{5. 29}$$

$$w_{ij}^k \ge 0 \quad \forall (i,j) \in A, k \in K, \tag{5. 30}$$

$$z_{ij}^k \ge 0 \quad \forall (i,j) \in A, k \in K. \tag{5. 31}$$

The objective function (5.6) maximizes the overall profit obtained less the fixed cost of employing trucks, the fine for exceeding time frames, and the cost of vehicle travel. However, since my goal is cost minimization, profit values are expressed as negative costs, and the objective function is changed to max cost minus min cost. The term "constraints" refers to the rules that the vehicles impose for the customers' visits. Constraint (5.7) makes sure that each client is only visited once, whereas constraint (5.8) compels a visit to the clients who should be visited. The VRP versions are supported by constraint set (5.9), which demands an inflow if there is an outflow in that truck doesn't need to come back to the depot. Constraints (5.10) establish a balance between the depot of vehicle k and the clients it serves, while constraints (5.11) force the vehicle to return to its depot when necessary. Each vehicle may only be utilized once, according to constraint (5.12), whereas constraint (5.13) enforces the backhaul limitation. The limitations that determine the customer needs are then presented. Constraints (5.14) and (5.15) offer flow conservation for the pickup commodity. Constraints (5.16) and (5.17) also offer flow conservation for the delivery commodity. The foundation for the time windows is provided by constraints (5.18), which are based on the Miller-Tucker-Zemlin sub-tour elimination constraints (Miller et al., 1960). The time window limits for every client, and the amount of not obeying this are stated in constraints (5.19) and (5.20). The last set of constraints claims the limitations for trucks. Constraints (5.21) and (5.22) are the beginning time of the driving for truck k and it don’t need to be come back unless it is required. Constraint (5.23) prohibits the violation of vehicle capacities. Constraints (5.24) – (5.26) explain the distance, driving time, and working time requirements for each vehicle. Finally, constraints (5.27) – (5.31) are integrality and nonnegativity constraints.

6. CASE STUDY

The NN5 Competition dataset composed of daily time series originated from the observation of daily withdrawals at 111 randomly selected different cash machines at different locations. We integrated the data into 13 ATMs for 56 days to solve it more easily.

We selected the ATMs randomly in the Beşiktaş area of İstanbul. These ATMs are owned by one of the biggest banks in Turkey. Also, the location of 1 depot is determined according to the bank headquarters building. Moreover, security concerns about money-fixed assets, and processes only operating in the daytime.

For analyzing the results effectively and finding the optimal solution, several scenarios are applied to the data by using VRP Spreadsheet Solver.

In the first scenario, all ATMs have no stock on the first day and they are filled with forecasted data at the beginning of the day. In the following days, the solver decides to visit the ATM or not according to their stock level. At the end of 15 days, the cost value is obtained for comparison.

After careful examination of order quantity methods, we found out that many companies in the service industry aim for a 90% service level in their process. Thus, we decided to keep 10% of the forecasted demand amount in the ATMs every day. After calculating the average amount of withdrawals from all ATMs for every day, a 10% value is found. To calculate the delivery amounts, pickup amounts, and cost of these operations, linear programming of the general aggregate planning problem is applied. The mathematical formulation of the problem is taken from Nahmias et al. (2015), some of the variables and parameters are modified based on our model.

Cost parameters and given information are; D_t withdrawal rate in period t, P forecast of demand level in period t. Variable is I_t Inventory level in period t,

$$I_t = I_{t-1} + P_t - D_t \;\; for\; 1 \leq t \leq T \tag{6. 1}$$

Constraints (6.1) ensure the inventory balance of each ATM.

In the second scenario, all ATMs are visited on the first day with 2-day forecast data loaded but on the second day, none of the ATMs are visited. It is cost saving option since the trip costs of a vehicle are expensive. In the third scenario, all ATMs are visited every 3 days. The forecasted value for the three days is filled into the ATMs.

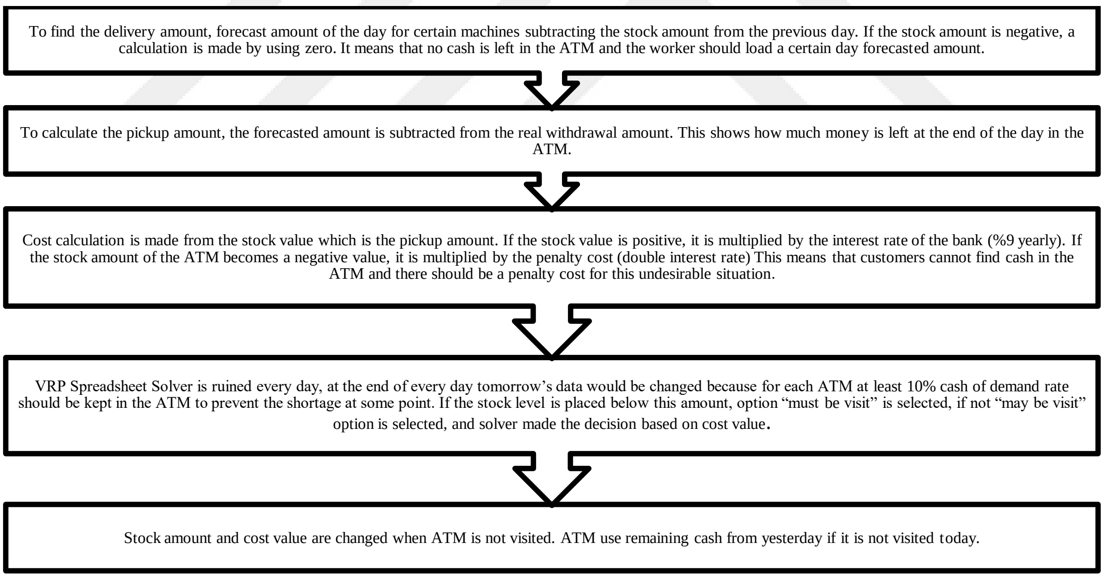

Figure 6.1: Steps of calculating delivery amounts, pickup amounts, and cost

After these scenarios, another strategy is applied for data which is to group the ATMs according to the nearest neighbor methodology. To assign the ATMs to the vehicles, the average amount of forecasted data is calculated for each ATM, after then a total of these 13 ATM averages is divided into three since 3 vehicles is used.

In addition, each ATMs contribution to the final total is calculated as a percentage. To find the replenishment amount, the contribution percentage of the ATM is multiplied by the total demand of ATMs in the group. Moreover, the same scenarios are also applied to grouping ATMs.

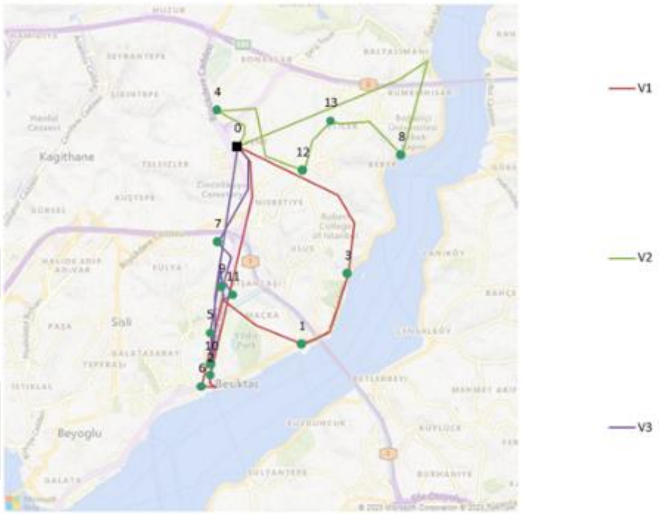

Figure 6.2: Groups of ATMs

7. FINDINGS

Statical methods, machine learning methods, and hybrid methods can be implemented for cash replenishment problems of ATMs. Our data include the withdrawal amount of cash for 13 ATMs for 56 days. After examining the literature, the SARIMA method is chosen from statical methods because of the seasonality parameter which is crucial for our data. Many neural networks are placed in the machine learning group. The most common one is ANN which is not the proper one for data due to the fact that our data include 56 days and ANN needs a longer dataset to conclude an efficient solution. DNN revealed better results among neural network methods. Furthermore, the newest machine learning method of this study which is Prophet is also implemented. Our comparison includes ARIMA-SARIMA-DNN-Prophet with the realized amount of cash.

We decided to run the model in terms of forecast the last 8 days by using the first 48 days' data. In addition, the first 48 days are recalculated based on the standard deviation of the forecasted 8 days. Thanks to that, the whole data become compatible data sets among each other. In the graph, below, a comparison of methods is illustrated with the original amount of cash for one ATM (103) which is randomly selected.

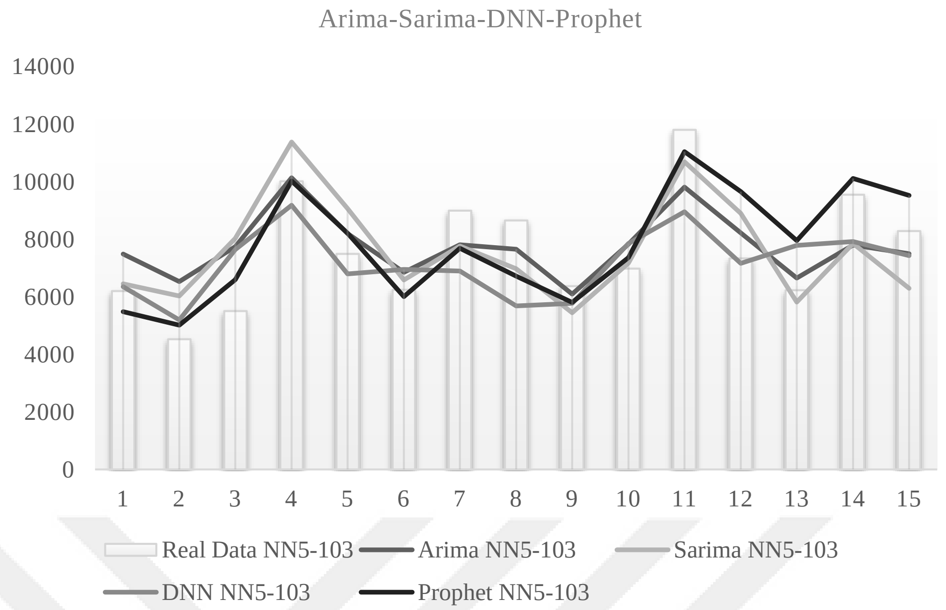

Figure 7.1: Methods comparison for ATM 103

In addition, the mean absolute percentage error (MAPE) is calculated for all methods. MAPE of ARIMA, SARIMA, DNN, and Prophet is 19,60 %, 19,72 %, 19,05 %, and 16,09 % respectively. Prophet is the best methodology by the smallest error rate in this group for ATM 103.

After that, MAPE is calculated for the whole data to identify the data in the aggregate. The results of the methods are shown below.

Table 7.1: MAPE results

MAPE													
	Average11	Average111	NN5-101	NN5-102	NN5-103	NN5-104	NN5-105	NN5-106	NN5-107	NN5-108	NN5-109	NN5-110	NN5-111
ARIMA	10,94	12,36	16,83	60,09	19,60	26,47	23,73	16,72	435,83	26,32	15,34	32,47	19,15
SARIMA	10,38	13,01	17,59	47,28	19,72	21,71	26,77	18,92	486,34	32,66	12,53	34,38	18,96
DNN	12,25	14,66	14,77	37,27	19,05	23,52	26,66	18,10	563,20	26,62	13,31	32,36	21,51
PROPHET	9,91	9,62	13,85	26,85	17,89	17,36	23,59	14,68	498,51	21,40	10,88	38,53	17,77

When we examine the averages of MAPE percentages for each method, the results are almost similar to the ATM results. 23,33%, 22,82%, 21,67%, and 18,52% for ARIMA, SARIMA, DNN, and Prophet respectively. ARIMA and Prophet showed the best results.

We decided with Prophet for the second part of our problem which is vehicle routing of trucks to 13 ATMs.

Vehicle routing problems are used. Starting point of vehicles' is the head office of the bank. The vehicle number is determined as 3 because when we select 4, one vehicle visit only one ATM and this becomes costlier than 3. Therefore, 3 truck is optimal for our model. All ATMs are visited by the fastest real time traffic approach. 3 different scenario is applied to find pickup and delivery amounts after then these scenarios are compared to each other based on cost value to determine the efficient strategy.

The first scenario starts with zero stock of ATMs on the first day and replenishment amounts are loaded based on Prophet forecast for the first day. At the end of the first day, the remaining stock in the ATM is multiplied by the daily bank interest rate if there is money left in the ATM unless the remaining stock in the ATM is multiplied by the twofold daily bank interest rate to calculate customer dissatisfaction about not finding cash in the ATM. Data was pound so exchanged the cash rate to TL. The first day all ATMs are visited. Which ATMs to visit in the coming days were determined according to the amount of safety stock. The safety stock level is determined at 44.893 for all ATMs and if the amount of money falls below the specified safety stock value, that ATM must be visited the next day. If it is above the safety stock, may be visit option is selected in Excel and the system decides whether to visit or not based on cost calculation. Some ATMs are not visited and their cost of not visiting is also added to the cost function. May be visited option can be applied on all days but some ATMs may go days without money being loaded. Because the routing of the vehicle is stopped when total cost value exceded the fixed cost and cost per unit value in may be visit option. At the end of every day, next day data of delivery, pickup, and cost values are recalculated according to the previous day. It was decided according to this logic whether the ATM would be visited in the following days and how much cash to be loaded.

EOQ calculation can be applied to find the optimal loading quantity. But in the EOQ formula annual holding cost per unit need to be known. In our case, this cost value may vary depending on which strategy will be applied. Therefore, this quantity calculation is no longer suitable for the problem. After careful consideration, we found out that many companies in the service industry aim for a 90% service level in their process. Thus, we decided to keep 10% of the forecasted demand amount in the ATMs every day. After calculating the average amount of withdrawals from all ATMs for every day, a 10% value is found.

The second scenario is visiting all the ATMs every 2 days. On the first day, all ATMs are visited but on the second day no ATMs are visited, and customers withdraw money left in stock from the previous day. In this scenario, first day ATMs are loaded with a total of today’s and next day forecast data value. Some ATMs are not visited on certain days and their cost of not visiting is also added to the cost function.

The third scenario is very close to the second one. Visit all the ATMs every 3 days. On the first day of routing. The estimated amount of cash for three days was filled into ATMs. These scenarios are run for 56 days in the system. Comparison of results includes the total of each scenario costs for 56 days.

Visit almost every day has the smallest amount of value for 56 days between other scenarios. This means visiting all ATMs every almost every day is the cheapest solution for this data and parameters.

After these scenarios, another strategy is applied for data which is to group the ATMs according to the nearest neighborhood methodology. To assign the ATMs to the vehicles, the average amount of withdrawal cash is calculated for each ATM, after then a total of these 13 ATM averages is divided into three. Because 3 vehicles will be used to distribute. 87.016 is found to decisive number for vehicles. Assigned the ATMs of the vehicle starting with most close ATMs when vehicle capacity exceeds the decisive number, no more ATMs are added to the group. After then, each ATMs contribution to the final total is calculated as a percentage.

The next step is finding the sum of the real withdrawal rate of each group for every day. Moreover, the same scenarios are also applied to grouping ATMs. Firstly, the sum of 1 day withdrawal amount is founded, after the sum of 2 days amount is found and lastly sum of 3 days quantity is calculated. To find the replenishment amount, the contribution percentage of ATM which is calculated previous step is multiplied by the sum amount.

In the next stages of calculations are the same as the previous strategy of individuals. A comparison of three scenarios of grouping ATMs is made by analyzing cost values for 56 days. In the end, visiting almost every day individually has the smallest cost rate but visiting almost every day gruping strategy follows this with small difference.

In the end, two strategies are analyzed. The first one is analyzing the data for individual ATMs, the second one is analyzing the data for a group of ATMs. Results showed that it is more profitable to deal with ATMs individually. The decision support system finds out that the optimized model becomes examine the ATM individually and visit all ATMs almost every day.

8. CONCLUSION

The starting point of this study was constructing a model to mix two different aspect of ATM process. Firstly, stages of this process are determined and 2 of them were chosen among steps. These are cash replenishment for ATMs and routing for trucks. Secondly, after careful consideration of foresting methods, 4 methods selected to be tested on the dataset. 2 statical method are applied to data which are ARIMA and SARIMA. However, machine learning methods also implemented, and they showed better results than statical methodologies. The reason behind that is data was noisy and machine learning methods are better handling for those kind of data sets due to effective parameters of model. DNN and Prophet was 2 machine learning method that used in study. All methods are analyzed by metric system. Thirdly, vehicle routing method is applied to the values of the forecasted method that gave the best results which is Prophet. For this part of the study, delivery amounts, pickup amounts and cost amounts entered to the VRP Spreadsheet Solver with parameters. Fourthly, several scenarios are created to find out the optimized one according to cost function in the solver. Lastly, all results are evaluated, and best strategy is selected for this data set and parameters. The decision support system is constructed for ATM cash replenishment and vehicle routing policy. To create this system, VRP Spreadsheet Solution, an open-source spreadsheet solver for the VRP and its several variants, is used in this study. In the ATM case study, it is seen that a solver is effective for this problem. On many benchmark situations from the literature, I demonstrated the importance of the problem and why it should be done optimally. As a future work time window specifications may change for each ATM. In that time, maybe more optimal results can occur. Also, determining a certain time window for each ATM would be another case study in future studies.

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APPENDICES

Appendix A. Python code of Prophet

```
import pandas as pd
from pandas import read_csv
from pandas import to_datetime
from pandas import DataFrame
from prophet import Prophet
from matplotlib import pyplot
from sklearn.metrics import mean_absolute_error
import pickle

# load data as .csv
path = 'C:\\Users\\Unknown\\Downloads \\data.csv' #data's path
df = read_csv(path, sep=';')

#drop empty line
df.drop([48],inplace=True)
df.reset_index(drop=True,inplace=True) #reseting indexes after line dropping

#generate dates from given interval
days = pd.date_range(start='01-JAN-2008', end='25-FEB-2008')

#atm begin-end columns
for i in range(2,13):

    #take 1 atm datas
```

```
df_train = df.iloc[:,[i]]

#concat dates to data
df_train.insert(0, 'ds', days)

#training dataframe
df_new_train = df_train.iloc[:48]

# prepare expected column names
df_new_train.columns = ['ds', 'y']
df_new_train['ds']= to_datetime(df_new_train['ds'])

# define the model
model = Prophet(daily_seasonality = True)

# fit the model
model.fit(df_new_train)

# define the period for which we want a prediction
future = model.make_future_dataframe(periods=8,freq='D')

#model saving
filename=f'E:\\facing\\NN5-{99 + i}.model'
pickle.dump(model, open(filename, 'wb'))

# use the model to make a forecast
forecast = model.predict(future)

filename=f'E:\\facing\\NN5-{99 + i}.png'
# plot forecast
model.plot(forecast)
pyplot.savefig(filename)
```

```
pyplot.show()
forecast['org'] = df_train.iloc[:,-1]
forecast[['ds', 'org', 'yhat', 'yhat_lower', 'yhat_upper']].to_csv(rf'E:\\facing\\NN5-{99 +
i}.xlsx', index=False)#Data Saving
#print(forecast[['ds', 'org', 'yhat', 'yhat_lower', 'yhat_upper']])

# calculate MAE between expected and predicted values for december
y_true = df_train.iloc[:,-1].values[48:]
y_pred = forecast['yhat'].values[48:]
mae = mean_absolute_error(y_true, y_pred)
print('MAE: %.3f' % mae)

# plot expected vs actual
filename=f'E:\\facing\\NN5-{99 + i}_PREDICTION.png'
pyplot.plot(y_true, label='Actual')
pyplot.plot(y_pred, label='Predicted')
pyplot.text(0,y_true.max(),'MAE: %.3f' % mae, fontsize=10,
    bbox = dict(facecolor = 'red', alpha = 0.5))
pyplot.legend()
#Data Plot Saving
pyplot.savefig(filename)
pyplot.show()
```

Appendix B. Individual ATM Results of VRP Spreadsheet Solver

Cost Tables (TL Version)

Days/Frequency of visits	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Vısıted ATMs	-793	-466	-983	-758	-945	-697	-735	-744
Not visited ATMs		-1.909	-349	-2.041	-641	-334	-663	-346
Sum	-773	-2.375	-1.332	-2.800	-1.587	-1.031	-1.398	-1.090
Visit every 2 days	-1.901	-957	-3.085	-1.091	-1.727	-1.038	-2.103	-1.239
Visit every 3 days	-3.480	-2.409	-1.204	-3.771	-1.660	-1.134	-3.258	-2.064

Days/Frequency of visits	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Vısıted ATMs	-651	-628	-629	-927	-660	-711	-731	-802
Not visited ATMs	-542	-571	-1.693		-325		-399	-764
Sum	-1.193	-1.199	-2.322	-927	-985	-711	-1.130	-1.565
Visit every 2 days	-2.596	-1.158	-3.105	-1.265	-2.238	-1.202	-2.246	-1.122
Visit every 3 days	-1.242	-5.184	-3.088	-1.233	-3.352	-2.108	-1.138	-4.627

Days/Frequency of visits	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Vısıted ATMs	-1.570	-1.274	-565	-847	-1.157	-766	-889	-793
Not visited ATMs			-2.525		-562	-929		-490
Sum	-1.570	-1.274	-3.090	-847	-1.719	-1.695	-889	-1.283
Visit every 2 days	-2.637	-1.171	-1.921	-1.261	-2.091	-1.599	-2.480	-1.445
Visit every 3 days	-2.555	-1.214	-3.075	-2.004	-1.704	-3.608	-2.321	-1.530

Days/Frequency of visits	25	26	27	28	29	30	31	32
Vısıted ATMs	-693	-962	-610	-773	-691	-764	-610	-744
Not visited ATMs	-2.238	-436	-742		-303	-655	-1.570	-972
Sum	-2.931	-1.398	-1.351	-773	-994	-1.419	-2.180	-1.716
Visit every 2 days	-3.107	-1.519	-2.371	-1.330	-2.614	-1.638	-3.891	-1.795
Visit every 3 days	-4.001	-1.913	-1.499	-3.532	-2.543	-1.558	-5.796	-3.666

Days/Frequency of visits	33	34	35	36	37	38	39	40
Vısıted ATMs	-1.278	-633	-613	-631	-801	-911	-972	-1.097
Not visited ATMs	-209	-844	-821	-494	-488	-358	-794	-1.014
Sum	-1.487	-1.477	-1.433	-1.125	-1.289	-1.269	-1.766	-2.111
Visit every 2 days	-3.613	-2.789	-4.000	-2.895	-4.407	-2.686	-4.467	-2.245
Visit every 3 days	-2.555	-5.037	-3.915	-2.809	-6.411	-4.690	-2.495	-4.283

Days/Frequency of visits	41	42	43	44	45	46	47	48
Vısıted ATMs	-972	-966	-713	-637	-714	-580	-784	-558
Not visited ATMs			-1.141	-339		-1.983	-499	-898
Sum	-972	-966	-1.854	-976	-714	-2.564	-1.284	-1.456
Visit every 2 days	-3.186	-1.909	-2.916	-1.754	-3.996	-1.892	-2.849	-1.981
Visit every 3 days	-3.106	-1.868	-4.431	-3.270	-1.839	-4.778	-2.782	-1.914

Days/Frequency of visits	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**Sum**
Vısıted ATMs	-684	-637	-705	-662	-770	-956	-693	-695	-44.234
Not visited ATMs	-417	-1.009	-394	-1.268	-1.450		-434	-486	-37.338
Sum	-1.101	-1.646	-1.098	-1.931	-2.221	-956	-1.126	-1.181	-81.552
Visit every 2 days	-3.260	-2.192	-3.921	-2.538	-4.369	-2272,76	-3.531	-2.478	-133.088
Visit every 3 days	-4.363	-3.295	-2.270	-6.542	-4.302	-2.216	-3.467	-2.414	-168.522

Appendix C. Group of ATM Results of VRP Spreadsheet Solver

Cost Tables (TL Version)

Days/Frequency of visits	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Vısıted ATMs	-788	-548	-747	-1.041	-1.099	-779	-689	-801
Not visited ATMs	-	-1.341	-654	-1.214	-499	-587	-827	-474
Sum	-788	-1.889	-1.400	-2.255	-1.597	-1.365	-1.517	-1.275
Visit every 2 days	-1.901	-985	-2.750	-1.407	-1.373	-1.111	-1.740	-1.008
Visit every 3 days	-3.480	-2.409	-1.180	-3.275	-1.297	-1.553	-2.908	-1.714

Days/Frequency of visits	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Vısıted ATMs	-756	-635	-914	-1.149	-942	-612	-747	-852
Not visited ATMs	-456	-808	-966	-564	-466	-1.072	-396	-630
Sum	-1.212	-1.443	-1.881	-1.713	-1.408	-1.684	-1.143	-1.482
Visit every 2 days	-2.113	-884	-2.505	-1.521	-1.715	-1.019	-1.719	-946
Visit every 3 days	-1.057	-4.711	-2.615	-1.591	-2.851	-1.606	-1.111	-4.126

Days/Frequency of visits	17	18	19	20	21	22	23	24
Vısıted ATMs	-1.289	-992	-740	-1.118	-1.110	-876	-922	-692
Not visited ATMs	-1.064	-690	-2.477	-487	-888	-796	-	-865
Sum	-2.353	-1.682	-3.217	-1.605	-1.998	-1.672	-922	-1.557
Visit every 2 days	-2.154	-1.258	-1.407	-1.393	-1.783	-1.452	-2.019	-1.063
Visit every 3 days	-2.055	-1.415	-2.602	-1.804	-1.835	-3.137	-1.850	-1.331

Days/Frequency of visits	25	26	27	28	29	30	31	32
Vısıted ATMs	-810	-1.119	-814	-728	-647	-802	-564	-1.020
Not visited ATMs	-1.307	-571	-817	-779	-778	-781	-1.069	-1.356
Sum	-2.117	-1.691	-1.631	-1.507	-1.426	-1.583	-1.633	-2.376
Visit every 2 days	-2.535	-1.328	-1.742	-993	-1.874	-1.046	-2.815	-1.217
Visit every 3 days	-3.507	-1.426	-1.303	-2.968	-1.979	-1.204	-4.794	-2.664

Days/Frequency of visits	33	34	35	36	37	38	39	40
Vısıted ATMs	-1.210	-539	-698	-644	-893	-1.006	-1.095	-1.165
Not visited ATMs	-345	-552	-865	-605	-	-792	-786	-1.198
Sum	-1.555	-1.091	-1.563	-1.249	-893	-1.798	-1.881	-2.363
Visit every 2 days	-2.397	-1.680	-1.768	-1.055	-2.069	-1.363	-2.338	-1.746
Visit every 3 days	-1.867	-3.039	-1.917	-1.247	-4.159	-2.437	-1.968	-2.407

Days/Frequency of visits	41	42	43	44	45	46	47	48
Vısıted ATMs	-1.130	-1.018	-808	-788	-672	-762	-896	-764
Not visited ATMs	-630	-575	-835	-	-885	-1.783	-499	-721
Sum	-1.761	-1.594	-1.643	-788	-1.557	-2.545	-1.394	-1.485
Visit every 2 days	-1.651	-1.758	-1.563	-1.095	-2.798	-1.018	-1.514	-905
Visit every 3 days	-1.461	-2.034	-3.146	-1.985	-1.030	-3.497	-1.500	-1.062

Days/Frequency of visits	49	50	51	52	53	54	55	56	Sum
Vısıted ATMs	-566	-703	-704	-746	-815	-1.153	-689	-638	-47.443
Not visited ATMs	-941	-538	-1.077	-1.222	-783	-	-719	-708	-42.740
Sum	-1.508	-1.241	-1.781	-1.968	-1.598	-1.153	-1.408	-1.346	-90.184
Visit every 2 days	-1.835	-910	-2.286	-903	-2.388	-1554,51	-1.829	-941	-90.141
Visit every 3 days	-3.006	-1.938	-1.123	-4.829	-2.589	-1.642	-1.829	-941	-126.010

BIOGRAPHICAL SKETCH

In 2014, Deniz Orhan graduated from 75. Yıl Anatolian High School. She pursued a bachelor’s degree in industrial engineering at TED University whit exclusively in English. She has been achieving a deep study through her master’s thesis about a more optimal way for cash forecasting and routing by analyzing new methods and several strategies. Her research interest and focus are in areas of interest including cash replenishment, inventory control, and decision support system.