CHOICE FROM LISTS IN GAME THEORY AND MECHANISM DESIGN

A Master's Thesis

by
SERKAN KARADEMİR

Department of
Economics
İhsan Doğramacı Bilkent University
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Serkan KARADEMİR

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By Serkan Karademir

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Nuh Aygün Dalkıran
Advisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Kemal Yıldız
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Serkan Küçükşenel
Examining Committee Member

Approval of the Graduate School of Economics and Social Sciences

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Refet S. Gürkaynak
Director

ABSTRACT

CHOICE FROM LISTS IN GAME THEORY AND MECHANISM DESIGN

Karademir, Serkan

M.A., Department of Economics

Supervisor: Asst. Prof. Nuh Aygün Dalkıran

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Game theory models choice via preferences or treats choice as primitive. This thesis studies strategic environments in which players choose from lists and integrates this perspective into game theory and mechanism design. It develops a list-based framework with a corresponding equilibrium notion and presents existence and uniqueness conditions. In mechanism design, the implementation problem within this framework is visited and necessary and sufficient conditions are derived. Libertarian Paternalist mechanisms, which preserve each player's full menu while steering choices via ordering, are analyzed, and the conditions for implementation are characterized. Finally, efficiency is analyzed through two different approaches: efficiency of mechanisms, a novel approach, and efficiency of social choice rules, the standard in the literature.

Keywords: Choice from lists, existence and uniqueness, implementation, Libertarian Paternalism, efficiency.

ÖZET

OYUN KURAMI VE MEKANİZMA TASARIMINDA LISTELERDEN SEÇİM

Karademir, Serkan

Yüksek Lisans, İktisat Bölümü

Tez Danışmanı: Dr. Öğr. Nuh Aygün Dalkıran

Eylül 2025

Oyun kuramı seçimleri tercihler üzerinden ya da seçimi ilksel bir unsur olarak ele alarak modeller. Bu tez, oyuncuların listelerden seçim yaptığı stratejik ortamları ele alır ve bu yaklaşımı oyun kuramı ve mekanizma tasarımına uygular. Liste temelli bir stratejik çerçeve ile buna karşılık gelen denge kavramı tanımlanır; genel varlık ve teklik koşulları verilir. Mekanizma tasarımında, bu çerçevede uygulanabilirlik problemi incelenir ve gereklilik-yeterlilik koşulları türetilir. Her oyuncunun tam menüsünü koruyup sıralamlar aracılığıyla seçimleri yönlendiren Liberteryen Paternalist mekanizmalar analiz edilir ve hangi koşullarda bir kuralı uyguladikları karakterize edilir. Son olarak, verimlilik iki farklı yaklaşımla incelenmektedir: yeni bir yaklaşım olarak mekanizmaların verimliliği ve literatürde standart olan sosyal seçim kurallarının verimliliği.

Anahtar Kelimeler: Listelerden seçim, varlık ve teklik, uygulanabilirlik, Libertaryen Paternalizm, verimlilik.

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TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Mainstream game theory predominantly focuses on *existential games*, which are mental models representing the objective reality of strategic interactions among economic agents. These models are standard mathematical expressions, where a theorist, for convenience, labels the players[1] and their corresponding strategies with indices, commonly using numbers, as such labels serve only to simplify the analysis and make it more comprehensible. They are constructed by theorists to describe the games both to themselves and to their audience, with the main objective of studying a predetermined equilibrium notion. However, this approach abstracts away from how players perceive or label their strategies and, more importantly, how they arrive at their optimal strategies.

While existential games provide a rigorous mathematical framework, discrepancies between models and experimental results remain. For instance, the concept of focal points, introduced by Schelling (1960), is well known in strategic interactions as outcomes to which players naturally gravitate toward due to their salience, symmetry, or shared cultural or contextual understanding, even without explicit communication. In coordination games with multiple equilibria, mainstream models struggle to explain why players converge on a particular focal point (outcome), as they overlook the process by which players arrive at their optimal strategies. To formalize focal points, Sugden (1995) highlights

[1]Throughout the thesis, "player" is used synonymously with "economic agent" in contexts involving strategic interaction.

the necessity of analyzing games from the players' perspectives, emphasizing that this requires understanding how players mentally describe and label the strategies they recognize. In line with this approach, this thesis argues that the order in which players evaluate their strategies during best-response analysis is a critical yet often neglected dimension. Although this sequential processing of available strategies remains invisible within the framework of existential games, it is apparent to the players themselves, provided they engage in a step-by-step evaluation of alternatives they can achieve given the strategies of others. Such processing may arise endogenously, as players prioritize strategies based on factors like social acceptability, or exogenously, from the structure of the game itself, such as the predetermined arrangement of candidates on a ballot in a voting game. Accordingly, I confine attention to games whose institutional structure exogenously imposes an ex ante ordering on the strategy sets of the players.

The step-by-step evaluation of strategies can be motivated by some recent developments in choice theory. Choice theory tends to represent choice problems as sets, assuming that decision-makers (DMs) perceive their choice problems as unordered collections of distinct alternatives. However, such representations overlook real-life cases where (i) the order of alternatives matters or (ii) the frequency with which a DM is exposed to the same alternative is relevant. In line with the seminal work of Salant and Rubinstein (2006), this thesis models choice problems as lists. Such models accurately capture real-life choice problems, where alternatives mostly come in structured formats, such as menus in restaurants or results from a web search, ordered by factors like popularity, price, or other relevant attributes. Even when an explicit order is absent, DMs are cognitively limited and unable to process alternatives in a choice problem in parallel. This creates an implicit list structure, where order and frequency shape decision-making.

Although models of choice from lists are well acknowledged in the choice theory literature, the game theory and mechanism design literature has largely overlooked this perspective. Regardless of the game's structure, players are assumed to engage in some form of best-response analysis based on context-independent utility or preference maximization. In a typical best-response analysis, a player fixes a strategy profile of other players and evaluates the set of outcomes obtainable through their possible strategies. This set is commonly referred to as the player's opportunity set, given the strategies of others. From this perspective, one can view every best-response analysis as a choice problem with respect to some opportunity set, and thus every game as essentially a collection of choice problems.

This thesis aims to project the concept of choice from lists onto game theory and mechanism design, where players view their opportunity sets as lists. In this framework, the order of alternatives achievable by a player is dictated by the sequence in which they evaluate their strategies (or messages, in the context of mechanism design) during best-response analysis, which contrasts with the typical implicit assumption that best-response analysis is independent of the sequence in which a player evaluates their possible strategies. The following examples illustrate how the order of alternatives in a best-response analysis can influence decision-making in game theory and mechanism design.

(a)(Satisficing Firms)(Simon (1955)) In a Cournot duopoly, Firm 1 adopts a satisficing approach in the following way: Given the quantity choice of Firm 2, Firm 1 seeks to maximize its own profit, π_1, until it achieves a minimum profit level of π^*. Once π^* is reached, Firm 1 stops further analysis and produces the first quantity required to meet π^*. This behavior can be motivated by managerial objectives, such as meeting minimum shareholder expectations while mitigating risk or cost of the analysis, or by strategic considerations, such as avoiding overly aggressive competition. In the best-response analysis, Firm 1 start from zero production and incrementally evaluates its output levels, selecting the first quantity that satisfies or exceeds its profit threshold. [2]

[2] I further analyze this example in Section 2.2.

(b)(Focal Points in Coordination Games)(Schelling (1960)) Consider a coordination game in which a student and their thesis advisor agree to meet next week without specifying a venue, without any possibility of communication, and with no shared history of meeting locations. The venues are the advisor's office and the student's office, between which the players are ex ante indifferent. Given cognitive limits, players select the first strategy that yields a Nash equilibrium. Social norms render the advisor's office salient; in a best-response analysis, players therefore consider meeting there first. The advisor's office thus emerges as the focal point. Yet standard game theory does not capture this because strategies are not labeled.

(c)(Incumbency and Recency Effect in Voting) Consider a voting game in which n voters choose among A, B, and abstention ($\emptyset$). The ballot lists A first as the incumbent and B second. Abstention is evaluated only after both candidates have been considered. This ordering induces a list structure over actions and a sequential processing in best-response evaluation. Voters with strict preferences choose their most preferred candidate. Voters indifferent between A and B intend to abstain, yet two forces may redirect them. The incumbency effect pulls indifferent voters toward A, with incumbency functioning as a default or focal status, increasing name recognition and perceived competence, and lowering justification costs. The recency effect pulls them toward B, since B is more salient once both options have been inspected. These opposing effects create a tension that shapes the final vote distribution, with the outcome determined by the relative strengths of these biases.

This thesis projects the choice-from-lists framework onto game theory and mechanism design. The rest of the thesis is accordingly organized in two parts. For game theory, in Chapter 2 the game-theoretic framework is developed formally, the concept of List-Nash equilibrium is introduced, and several examples are presented. In Chapter 3 existence and uniqueness results are established. For mechanism design, in Chapter 4 ordered mechanisms are defined and the classical implementation problem is revisited within the list-based framework, with necessary and sufficient conditions characterized. In Chapter 5 a specific class of mechanisms, referred to as Libertarian Paternalist mechanisms, is analyzed and the necessary and sufficient conditions for their implementability are derived. Chapter 6 presents results on efficiency from two perspectives: mechanisms and social choice rules. Lastly, Chapter 7 provides a brief summary, situates the work within the literature, and outlines directions for future research.

CHAPTER 2

CHOICE FROM LISTS IN GAME THEORY

In this chapter I present the model's preliminaries and introduce the equilibrium concept in a framework that defines players' choice functions on lists. I also provide a simple extension of the Cournot model to illustrate the framework.

2.1 Preliminaries

Let N be a finite, nonempty set of players, or the society, with $|N| \geq 2$, and X be a finite, nonempty set of outcomes. An element of X is denoted by x, and each $i \in N$ represents a particular player in the society. For convenience, define $\mathcal{X} = 2^X \setminus \{\emptyset\}$ as the collection of all nonempty subsets of X, and $\mathcal{N} = 2^N \setminus \{\emptyset\}$ as the collection of all nonempty subsets of N, whose elements are called coalitions. Let Θ be a finite, nonempty set of relevant states of the world regarding players' choices, and let θ denote a particular state of the world. Throughout the thesis, the true state of the world is assumed to be common knowledge among the players but unknown to the mechanism designer.[3]

The set of natural numbers is denoted with $\mathbb{N} = \{1, 2, \ldots\}$, as usual. For $k \in \mathbb{N}$, write $[k] := \{1, 2, \ldots, k\}$. A list over X is an ordered pair (k, f) with $k \in \mathbb{N}$ and $f \colon [k] \to X$; identifying (k, f) with the k-tuple $(x_1, \ldots, x_k)$ where $x_j = f(j)$ for every $j \in [k]$ gives the usual representation of a finite, nonempty sequence of

[3]Allowing agents to hold private information about the true state would lead to an incomplete-information setting, which lies beyond the present scope.

outcomes. For each $k \in \mathbb{N}$, define $X^k := \{(k, f) \mid f : [k] \to X\}$. The set of all possible lists is then

$$\mathcal{L} := \bigcup_{k \in \mathbb{N}} X^k.$$

Henceforth, any pair $(k, f) \in \mathcal{L}$ is written simply as $L = (x_1, x_2, \ldots, x_k)$. Its support is $A(L) := \{x_j \mid j \in [k]\} \subseteq X$, which is the set of distinct outcomes appearing in L. Each list $L \in \mathcal{L}$ constitutes a choice problem. A player's choice behavior is taken as primitive and represented through choice functions that may vary across states of the world. [4]

Definition 1. A choice function from lists for player i at state $\theta \in \Theta$ is any function $C_i^\theta : \mathcal{L} \to X$ such that $C_i^\theta(L) \in A(L)$ for all $L \in \mathcal{L}$.

When the context is unambiguous, the qualifier "from lists" is omitted and the term choice functions is used. Throughout the thesis, unless otherwise stated, the choices are assumed to be nonempty and single-valued. An immediate observation is as follows.

Observation 1. While $\mathcal{X}$ is finite, the domain $\mathcal{L}$ is countably infinite, so each player may face countably many distinct choice problems.

Proof. This is a trivial but important observation. Assume that X is finite, then $\mathcal{X}$ is obviously finite. Now, put $m := |X| \geq 1$. For every $k \in \mathbb{N}$, observe that $|X^k| = m^k < \infty$. The family $\{X^k\}_{k \in \mathbb{N}}$ is indexed by $\mathbb{N}$, hence countable, and each of its members is finite. Therefore $|\mathcal{L}| = \left|\bigcup_{k \in \mathbb{N}} X^k\right| \leq \aleph_0$, where $\aleph_0 := |\mathbb{N}|$. To obtain the reverse inequality fix an element $x_0 \in X$ and, for every $k \in \mathbb{N}$, define $f_k \colon [k] \to X$ by $f_k(j) = x_0$ for all $j \in [k]$. Set $L_k := (k, f_k) \in \mathcal{L}$. If $k \neq \ell$

[4] In many rational-choice frameworks, the state space is simply taken to be the set of all linear orders (that is, all complete, transitive, antisymmetric relations) on X. Such a definition offers operational or intuitive insight into what a "state" actually represents. The projection of this approach onto the list-based framework yields the ambient assignment space

$$\widetilde{\Theta} = \{\vartheta : N \times \mathcal{L} \to X \mid \forall i \in N, \forall L \in \mathcal{L}, \vartheta(i, L) \in A(L)\},$$

and then one can study a *relevant* finite subset $\Theta \subsetneq \widetilde{\Theta}$ for modelling purposes. Each $\theta \in \Theta$ thus fully specifies the counterfactual choices $\{\theta(i, L)\}_{i \in N, L \in \mathcal{L}}$.

then $L_k \neq L_\ell$, so the map $\varphi\colon \mathbb{N} \to \mathcal{L}$ where $\varphi(k) = L_k$ is injective and yields $\aleph_0 \leq |\mathcal{L}|$. Combining the two inequalities gives $|\mathcal{L}| = \aleph_0$. Thus, $\mathcal{L}$ is countably infinite.[5] $\square$

The standard approach instead represents a choice problem by a menu $B \in \mathcal{X}$, of which there can be only finitely many. By allowing both the presentation order and the multiplicity of alternatives to matter, list-based framework delivers a markedly richer domain of choice problems and thereby accommodates more intricate patterns of behavior.[6] In my framework, the players can be considered "behavioral," that is, I assume no additional internal consistency properties beyond the minimal requirements of non-emptiness and single-valuedness.

Given arbitrary sets A and B and a function $f\colon A \to B$ I write $f\colon A \xrightarrow{\sim} B$ to indicate that f is bijective. Let I be a nonempty index set and $f_i\colon A_i \to B_i$ a function for all $i \in I$. The Cartesian product of the family $(A_i)_{i\in I}$ is represented as $\bigtimes_{i\in I} A_i = \big\{(a_i)_{i\in I} \mid a_i \in A_i \text{ for all } i \in I\big\}$. The product function $\prod_{i\in I} f_i\colon \bigtimes_{i\in I} A_i \to \bigtimes_{i\in I} B_i$ is defined by $\big(\prod_{i\in I} f_i\big)(a) = (f_i(a_i))_{i\in I}$ at each $a = (a_i)_{i\in I} \in \bigtimes_{i\in I} A_i$. Let $k \in \mathbb{N}$ and for each $i \in [k]$ let A_i be an arbitrary (nonempty) set. Every element $v \in \bigtimes_{i\in[k]} A_i$ can be written as the k-tuple $v = (v_i)_{i\in[k]}$, where $v_i \in A_i$ is the i-th coordinate of v. For any fixed $j \in [k]$ set $v_{-j} = (v_i)_{i\in[k]\setminus\{j\}} = (v_1, \ldots, v_{j-1}, v_{j+1}, \ldots, v_k) \in \bigtimes_{i\in[k]\setminus\{j\}} A_i$ the $(k-1)$-tuple obtained by deleting the j-th coordinate of v.

Let $\mathscr{C}_i^\theta$ denote the set of all possible choice functions for player $i \in N$ at state $\theta \in \Theta$. Consequently, $\mathscr{C}^\theta = \bigtimes_{i\in N} \mathscr{C}_i^\theta$ is the set of all choice profiles for state θ. I then define $\mathscr{C} = \bigtimes_{\theta\in\Theta} \mathscr{C}^\theta$, that is, $\mathscr{C}$ is the product of all such profiles across every state in Θ.

[5]One may extend the size of X, and assume that it is countably infinite. Then, the same argument applies with m replaced by $\aleph_0$. Each X^k then has cardinality $\aleph_0$, so $|\mathcal{L}| \leq \aleph_0$. The injection φ constructed above is still available, giving $\aleph_0 \leq |\mathcal{L}|$. Hence $|\mathcal{L}| = \aleph_0$ in this case as well.

[6]For an axiomatization of choice from lists functions, see Salant and Rubinstein (2006); for its generalization to choice from frames, see Salant and Rubinstein (2008).

Definition 2. An ordered game form is a three-tuple $G = \langle N, (S_i, \pi_i)_{i\in N}, g\rangle$, where: (i) N is a finite nonempty set of players, (ii) for each $i \in N, S_i$ is a finite, nonempty set of strategies available to player i, (iii) for each $i \in N, \pi_i : [|S_i|] \xrightarrow{\sim} S_i$ is a strategy-order function for player i, (iv) $g : \bigtimes_{i\in N} S_i \to X$ is an outcome function.

The strategy-order function, hereinafter simply the order function, is state invariant and endows player i with a cognitively salient index where the primitive object is the unordered finite set S_i, and π_i merely attaches the tags $1, \ldots, |S_i|$ to her strategies. In the sense of Sugden (1995), (S_i, π_i) is a set of strategies that i has to choose from and is identified with labels, which are simply an enumeration in this framework. Because π_i is bijective, the integer label k carries no strategic substance, and any behavioral regularity attributed to order effects must therefore arise from the exogenous order function, not from intrinsic asymmetries in the game form.

For brevity, let $S = \bigtimes_{i\in N} S_i$ and $\pi = \prod_{i\in N} \pi_i$. Given a choice profile $C^\theta \in \mathscr{C}^\theta$, $G\big(C^\theta\big)$ denotes an (behavioral) ordered strategic game. For any profile $s = (s_j)_{j\in N} \in S$, set $s_{-i} = (s_j)_{j\in N\setminus\{i\}}$. The opportunity list of player i when the others play s_{-i} is the list of all outcomes obtainable by i via a unilateral deviation, namely $L_i(s_{-i}) = (g(\pi_i(k), s_{-i}))_{k=1}^{|S_i|}$. The support of an opportunity list is called the opportunity set, which is $O_i(s_{-i}) = A(L_i(s_{-i}))$, or equivalently, it can be written as $O_i(s_{-i}) = \{g(s_i, s_{-i}) : s_i \in S_i\}$.

Definition 3. Let $\theta \in \Theta$ and $C^\theta \in \mathscr{C}^\theta$. A strategy profile $s^* \in S$ is a (behavioral) List-Nash equilibrium of strategic game $G(C^\theta)$ at θ if, for all $i \in N$, $g\left(s_i^*, s_{-i}^*\right) = C_i^\theta\left(L_i(s_{-i}^*)\right)$.

The set of all List-Nash equilibria of an ordered strategic game $G(C^\theta)$ is denoted by LNE($G(C^\theta)$) $:= \big\{s \in S : \forall i \in N, g(s_i, s_{-i}) = C_i^\theta\left(L_i\left(s_{-i}\right)\right)\big\}$.

2.2 Example on Cournot Competition

To further understand the concepts of ordered games and Nash equilibria, consider a Cournot competition with two firms $N = \{1, 2\}$. Let $\epsilon > 0$ be a fixed grid size and $K \in \mathbb{N}$ be a capacity index. For each $i \in N$ let $S_i = \{k\epsilon : k \in \{0, 1, \ldots, K\}\}$ denote the set of strategies available to firm $i \in N$, i.e., the quantity levels that firm i is allowed to produce. The upper bound $K\epsilon$ represents a production capacity due to time or land scarcity, and the grid size ϵ captures technological indivisibilities such as lot sizes generated by machines operating at constant ratios. Taking ϵ sufficiently small and K sufficiently large provides a fine discretization while preserving these frictions.

Each firm $i \in N$ has a satisficing profit threshold $\tau_i > 0$. Given $s_{-i} \in S_{-i}$, firm i arranges its feasible strategies in the exogenous order given by π_i and, in that order, evaluates the profit each strategy would deliver against s_{-i}. It stops at the first strategy whose profit reaches or exceeds τ_i and plays that strategy. If no strategy reaches the threshold, it identifies the strategies that yield the highest profit and plays the one that appears first in its order.

Formally, let $P : S \to \mathbb{R}$ be an inverse demand function, $\kappa_i : S_i \to \mathbb{R}$ be the total cost function of firm $i \in N$. Profit functions of the firms are given as $u_i(s) = P(s_1, s_2)s_i - \kappa_i(s_i)$, for all $i \in N$. Define $g : S \to \mathbb{R}^2$ by $g(s) = (u_1(s), u_2(s))$. Let $X := g(S) \subsetneq \mathbb{R}^2$ and regard lists and choice functions in this section as taken over this finite outcome set X. Let $L = (x_1, \ldots, x_k) \in \mathcal{L}$ with entries $x_m = (x_m^1, x_m^2) \in X$. Define $T_i(L) = \{m \in [k] : x_m^i \geq \tau_i\}$ and $\arg\max_i(L) = \left\{m \in [k] : x_m^i = \max_{\ell \in [k]} x_\ell^i\right\}$. Set $\alpha_i(L) = \min T_i(L)$ if $T_i(L) \neq \emptyset$, and $\alpha_i(L) = \min \arg\max_i(L)$ otherwise; then $C_i(L) = x_{\alpha_i(L)} \in X$.

Example 1. Assume $\pi_i(k) = (k-1)\epsilon$ for $k \in [K+1]$, $P(s_1, s_2) = a - b(s_1 + s_2)$ with $a > 0$, $b > 0$, and $\kappa_i(s_i) = \bar{\kappa}_i s_i$ with $\bar{\kappa}_i \in [0, a)$.

Let $A_i = a - \bar{\kappa}_i > 0$. For $s_{-i} \in S_{-i}$ define $u_i(s_i, s_{-i}) = (A_i - bs_{-i} - bs_i)\, s_i$. Then

$\partial u_i/\partial s_i = A_i - bs_{-i} - 2bs_i$ and $\partial^2 u_i/\partial s_i^2 = -2b < 0$, so $u_i(., s_{-i})$ is strictly concave on its domain. The unique continuous maximizer is $s_i^{\mathrm{c}}(s_{-i}) = \frac{A_i - bs_{-i}}{2b}$, with value $u_i\left(s_i^{\mathrm{c}}(s_{-i}), s_{-i}\right) = \frac{(A_i - bs_{-i})^2}{4b}$. Imposing the capacity interval $[0, K\epsilon]$ gives the constrained continuous maximizer $t_i(s_{-i}) = \min\left\{\max\left\{s_i^{\mathrm{c}}(s_{-i}), 0\right\}, K\epsilon\right\}$.

For $z \in \mathbb{R}$ set $\lfloor z \rfloor_\epsilon = \epsilon \lfloor z/\epsilon \rfloor$ and $\lceil z \rceil_\epsilon = \epsilon \lceil z/\epsilon \rceil$. Let $\xi^{\downarrow}_{S_i}(t)$ denote the nearest grid point to t with ties resolved toward the smaller grid point, that is,

$$\xi^{\downarrow}_{S_i}(t) = \begin{cases} \lfloor t \rfloor_\epsilon & \text{if } t \leq \dfrac{\lfloor t \rfloor_\epsilon + \lceil t \rceil_\epsilon}{2}, \\ \lceil t \rceil_\epsilon & \text{if } t > \dfrac{\lfloor t \rfloor_\epsilon + \lceil t \rceil_\epsilon}{2}. \end{cases}$$

Strict concavity of $u_i(\cdot, s_{-i})$ implies that the profit maximizers on S_i should lie in $\{\lfloor t_i(s_{-i}) \rfloor_\epsilon,\ \lceil t_i(s_{-i}) \rceil_\epsilon\}$. If $t_i(s_{-i})$ is not exactly at a midpoint then the discrete maximizer is unique and equals the closer neighbor, if it is exactly at a midpoint then both neighbors maximize and the ascending order $\pi_i(k) = (k-1)\epsilon$ selects the smaller neighbor. Hence the profit maximizing discrete choice when the satisficing constraint is inactive equals $\hat{s}_i(s_{-i}) = \xi^{\downarrow}_{S_i}\big(t_i(s_{-i})\big)$. Introduce $\tau_i > 0$ and define the quadratic discriminant $\Delta_i(s_{-i}) = (A_i - bs_{-i})^2 - 4b\tau_i$. Solving $u_i(s_i, s_{-i}) = \tau_i$ for s_i yields the roots

$$r_{i,\pm}(s_{-i}) = \frac{A_i - bs_{-i} \pm \sqrt{\Delta_i(s_{-i})}}{2b}$$

whenever $\Delta_i(s_{-i}) \geq 0$. The roots satisfy $r_{i,-}(s_{-i})\, r_{i,+}(s_{-i}) = \frac{\tau_i}{b} > 0$ and $r_{i,-}(s_{-i}) + r_{i,+}(s_{-i}) = \frac{A_i - bs_{-i}}{b}$, hence they have the same sign and they are positive if and only if $A_i - bs_{-i} > 0$. Therefore

$$\{s_i \in \mathbb{R} : u_i(s_i, s_{-i}) \geq \tau_i\} = [r_{i,-}(s_{-i}), r_{i,+}(s_{-i})] \iff \Delta_i(s_{-i}) \geq 0.$$

For convenience, write $J_i(s_{-i}) = [\, r_{i,-}(s_{-i}),\ r_{i,+}(s_{-i}) \,] \cap S_i$. There exists a threshold reaching grid point if and only if $\Delta_i(s_{-i}) \geq 0$ and $J_i(s_{-i}) \neq \emptyset$. Equivalently $\max_{s_i \in S_i} u_i(s_i, s_{-i}) \geq \tau_i$ if and only if $J_i(s_{-i})$ is nonempty. In the latter case the first grid point that reaches the threshold in the ascending order is $\lceil r_{i,-}(s_{-i}) \rceil_\epsilon$, which necessarily satisfies $\lceil r_{i,-}(s_{-i}) \rceil_\epsilon \leq K\epsilon$. Now, define the following selector function.

$$R_i(s_{-i}) = \begin{cases} \min J_i(s_{-i}) & \text{if } J_i(s_{-i}) \neq \emptyset \\ \xi^{\downarrow}_{S_i}(t_i(s_{-i})) & \text{otherwise} \end{cases}$$

The opportunity list $L_i(s_{-i})$ scans S_i in increasing s_i. If the threshold is feasible on the grid, the rule picks the first grid point in $[r_{i,-}, r_{i,+}] \cap [0, K\epsilon]$, which is $\lceil r_{i,-}(s_{-i}) \rceil_\epsilon$. If the threshold is infeasible, the rule falls back to the earliest discrete maximizer, which is $\xi^{\downarrow}_{S_i}(t_i(s_{-i}))$ by the tie breaking built into $\xi^{\downarrow}_{S_i}$. Therefore $C_i(L_i(s_{-i})) = g(R_i(s_{-i}), s_{-i})$ on the entire strategy domain. A profile $s^* = (s_1^*, s_2^*) \in S_1 \times S_2$ is a List-Nash equilibrium if and only if $g(s_i^*, s_{-i}^*) = g(R_i(s_{-i}^*), s_{-i}^*)$ for $i \in N$. Fix i and first assume $s_{-i}^* > 0$. Then $u_{-i}(s_i, s_{-i}^*) = (A_{-i} - bs_{-i}^*) s_{-i}^* - bs_i s_{-i}^*$ is strictly decreasing in s_i. Therefore, $u_{-i}(s_i^*, s_{-i}^*) = u_{-i}(R_i(s_{-i}^*), s_{-i}^*)$ implies $s_i^* = R_i(s_{-i}^*)$. Next assume $s_{-i}^* = 0$. Then $u_{-i}(s_i, 0) = 0$ for all s_i, so the second coordinate gives no restriction. The first coordinate requires $(A_i - bs_i^*) s_i^* = (A_i - bR_i(0)) R_i(0)$. Rearranging gives $b(s_i^* - R_i(0))(A_i/b - (s_i^* + R_i(0))) = 0$. Hence $s_i^* \in \left\{R_i(0), \frac{A_i}{b} - R_i(0)\right\}$. Feasibility requires $s_i^* \in S_i$, so $\frac{A_i}{b} - R_i(0)$ is admissible only when it lies in this set. Define the correspondence

$$\Phi_i(s_{-i}) = \begin{cases} \{R_i(s_{-i})\} & \text{if } s_{-i} > 0, \\ \{R_i(0)\} \cup \left(\left\{\frac{A_i}{b} - R_i(0)\right\} \cap S_i\right) & \text{if } s_{-i} = 0. \end{cases}$$

These cases are mutually exclusive and exhaustive, and the argument above shows that s^* is a List-Nash equilibrium if and only if $s_i^* \in \Phi_i(s_{-i}^*)$ for $i \in N$.

Example 2. Assume $\pi_1(k) = (k-1)\epsilon$ and $\pi_2(k) = (K+1-k)\epsilon$ for $k \in [K+1]$, $P(s_1, s_2) = a - b(s_1 + s_2)$ with $a > 0$, $b > 0$, and $\kappa_i(s_i) = \bar{\kappa}_i s_i$ with $\bar{\kappa}_i \in [0, a)$.

Define the order-dependent nearest grid projections

$$\xi^{\downarrow}_{S_i}(t) = \begin{cases} \lfloor t \rfloor_\epsilon & \text{if } t \le \dfrac{\lfloor t \rfloor_\epsilon + \lceil t \rceil_\epsilon}{2}, \\ \lceil t \rceil_\epsilon & \text{if } t > \dfrac{\lfloor t \rfloor_\epsilon + \lceil t \rceil_\epsilon}{2}, \end{cases} \qquad \xi^{\uparrow}_{S_i}(t) = \begin{cases} \lfloor t \rfloor_\epsilon & \text{if } t < \dfrac{\lfloor t \rfloor_\epsilon + \lceil t \rceil_\epsilon}{2}, \\ \lceil t \rceil_\epsilon & \text{if } t \ge \dfrac{\lfloor t \rfloor_\epsilon + \lceil t \rceil_\epsilon}{2}. \end{cases}$$

The operator $\xi^{\downarrow}$ breaks midpoint ties toward the smaller grid point, while $\xi^{\uparrow}$ breaks midpoint ties toward the larger grid point. With π_1 ascending, firm 1 uses $\xi^{\downarrow}$. With π_2 descending, firm 2 uses $\xi^{\uparrow}$.

Given $\tau_i > 0$ set $\Delta_i(s_{-i}) = (A_i - bs_{-i})^2 - 4b\tau_i$ and, when $\Delta_i(s_{-i}) \ge 0$,

$$r_{i,\pm}(s_{-i}) = \frac{A_i - bs_{-i} \pm \sqrt{\Delta_i(s_{-i})}}{2b}.$$

The threshold-feasible intersection with the capacity grid is nonempty if and only if $\Delta_i(s_{-i}) \ge 0$ and $\max\{0, r_{i,-}(s_{-i})\} \le \min\{K\epsilon, r_{i,+}(s_{-i})\}$. Define $J_i(s_{-i}) = [\, r_{i,-}(s_{-i}),\, r_{i,+}(s_{-i}) \,] \cap S_i$. Now, write

$$R_1(s_{-1}) = \begin{cases} \min J_1(s_{-1}) & \text{if } J_1(s_{-1}) \ne \emptyset, \\ \xi^{\downarrow}_{S_1}\big(t_1(s_{-1})\big) & \text{otherwise,} \end{cases}$$

and

$$R_2(s_{-2}) = \begin{cases} \max J_2(s_{-2}) & \text{if } J_2(s_{-2}) \ne \emptyset, \\ \xi^{\uparrow}_{S_2}\big(t_2(s_{-2})\big) & \text{otherwise.} \end{cases}$$

The remainder is identical to the solution of Example 1, mutatis mutandis: player 2's selector uses $\max J_2$ in place of $\min J_2$ and $\xi^{\uparrow}_{S_2}$ in place of $\xi^{\downarrow}_{S_2}$. Hence for $i = 1, 2$ and all $s_{-i} \in S_{-i}$, $C_i(L_i(s_{-i})) = g(R_i(s_{-i}), s_{-i})$, and the same monotonicity and feasibility arguments yield $s_i^* \in \Phi_i(s_{-i}^*)$.

It is possible to interpret the firms' behavior in Example 1 and Example 2. Read k as the number of workers a firm hires and ϵ as the fixed amount of output produced by each worker. In the symmetric model both firms behave like private firms: conditional on meeting the profit threshold, each chooses the smallest workforce k among the quantities that satisfy τ_i. In contrast, in the asymmetric model firm 2 behaves like a public firm: conditional on meeting τ_2, it chooses the largest workforce k among the quantities that satisfy the threshold.[7]

Corollary 1. Consider the environments in Example 1 and Example 2. Suppose there exists $s^* = (s_1^*, s_2^*) \in S$ with $s_1^* > 0$ and $s_2^* > 0$ such that in the symmetric game $s_1^* = \min J_1(s_2^*)$ and $s_2^* = \min J_2(s_1^*)$, and $J_2(s_1^*)$ contains at least two distinct grid points. Then s^* is a List-Nash equilibrium in the symmetric game and is not a List-Nash equilibrium in the asymmetric game. In particular the two equilibrium sets differ whenever at some symmetric equilibrium $J_2(s_1^*)$ is not a singleton. The analogous statement holds with the roles of players 1 and 2 interchanged.

Proof. In the symmetric game the opportunity lists scan S_1 and S_2 in increasing order, so $C_i(L_i(s_{-i}^*)) = g(\min J_i(s_{-i}^*), s_{-i}^*)$ and the strict monotonicity argument gives $s_i^* = \min J_i(s_{-i}^*)$. In the asymmetric game player 2's induced choice at s_1^* equals $g(\max J_2(s_1^*), s_1^*)$, while $s_2^* = \min J_2(s_1^*) \neq \max J_2(s_1^*)$ because $J_2(s_1^*)$ is not a singleton. So $g(s_2^*, s_1^*) \neq C_2(L_2(s_1^*))$. Hence s^* fails the equilibrium condition there. The symmetric case for player 1 and the asymmetric case for player 2 swap roles verbatim. $\square$

To further illustrate the List-Nash equilibrium and the effects of presentation order, I run a simulation over a finite parameter grid. For each parameter, I fix a finite set of predetermined values, summarized in Table 2.2. For each parameter vector p, I compute three equilibrium correspondences on $S_1(p) \times S_2(p)$: the standard discretized Nash equilibria (Standard NE) solving $s_i \in \arg\max_{t \in S_i(p)} u_i(t, s_{-i})$ for $i \in \{1, 2\}$; the symmetric List Nash equilibria (Symmetric List NE) generated by $\pi_i(k) = (k-1)\epsilon$ for $i = 1, 2$ as in Example 1, with choice function C_i at threshold τ_i; and the asymmetric List Nash equilibria (Asymmetric List NE) generated by $\pi_1(k) = (k-1)\epsilon$ and $\pi_2(k) = (K+1-k)\epsilon$ as in Example 2.

[7] I thank Kemal Yıldız for suggesting this interpretation.

Symbol	Meaning	Set	Cardinality
E	Grid size ϵ	$\{0.02, 0.04, \ldots, 0.20\}$	$\|E\| = 10$
K	Capacity index	$\{5, 10, 15\}$	$\|K\| = 3$
T	Profit thresholds (τ_1, τ_2)	$\{0.05, 0.10, \ldots, 1.50\}$	$\|T\| = 30$
κ	Unit costs $(\bar{\kappa}_1, \bar{\kappa}_2)$	$\{0.1, 0.3, 0.5, 0.7, 0.9\}$	$\|\kappa\| = 5$
A	Intercept a of inverse demand	$\{2, 3\}$	$\|A\| = 2$
B	Slope b of inverse demand	$\{0.5, 1.0, 1.5\}$	$\|B\| = 3$
		Total configurations	$10 \cdot 3 \cdot 30 \cdot 30 \cdot 5 \cdot 5 \cdot 2 \cdot 3 = 4{,}050{,}000$

Notes: The capacity range satisfies $K\epsilon \in [0.10, 3.00]$. Thresholds are small to moderate, so threshold feasibility occurs often but not always. For all $a \in A$ and $\bar{\kappa}_i \in \kappa$, $A_i = a - \bar{\kappa}_i > 0$. The set B ranges from relatively flat to relatively steep inverse demand. The sets T and κ are applied independently to the two firms, which yields the factors $|T|^2$ and $|\kappa|^2$ in the total count.

The simulation results are in Table 2.2. On this grid the standard discretized Cournot game always has at least one equilibrium. A List equilibrium does not exists only when the thresholds τ_i cannot be met given the capacities. The symmetric and asymmetric List sets match in 56.59% of cases and differ in 43.41%. Hence the presentation order embedded in π alters equilibrium predictions at a substantial rate once behavior follows list-based satisficing.

	Standard NE	Symmetric List NE	Asymmetric List NE
No equilibria	0	121,802	92,069
	(0.00%)	(3.01%)	(2.27%)
Unique equilibrium	3,019,500	3,510,322	3,399,609
	(74.59%)	(86.67%)	(83.95%)
Multiple equilibria	1,030,500	417,876	558,322
	(25.41%)	(10.32%)	(13.78%)
Average count	1.31	1.07	1.14
Maximum count	3	7	5
Symmetric vs Asymmetric List NE			
Coincide rate:	56.59% (2,292,048 / 4,050,000)		

Percentages are relative to the total number of configurations. Numbers in parentheses are shares.

Note that the multiplicity in the standard Cournot game comes from discretizing strategies. The continuous best response is $s_i^{c}(s_{-i}) = (A_i - bs_{-i})/(2b)$, which is unique because $u_i(s_i, s_{-i}) = (A_i - bs_{-i} - bs_i)s_i$ is strictly concave and $\partial^2 u_i/\partial s_i^2 = -2b < 0$. On the grid $S_i = \{0, \epsilon, \ldots, K\epsilon\}$ the discrete maximizers are the grid neighbors of $t_i(s_{-i}) = \min\{\max\{s_i^c(s_{-i}), 0\}, K\epsilon\}$. If $t_i(s_{-i})$ falls strictly between grid points, the closer neighbor is unique. If it falls at a midpoint, both neighbors tie, so the discrete best response at s_{-i} is $\{\lfloor t_i(s_{-i}) \rfloor_\epsilon,\ \lceil t_i(s_{-i}) \rceil_\epsilon\}$. Each player's best response is then a nonincreasing step correspondence that can intersect the opponent's in more than one point. Hence midpoint ties for both players produce adjacent best replies and can generate multiple pure strategy equilibria.

CHAPTER 3

EXISTENCE AND UNIQUENESS OF LIST-NASH EQUILIBRIUM

List-Nash equilibrium extends Behavioral Nash equilibrium[8] by embedding it in a framework in which each player follows an ordered best-response procedure over an exogenously ordered strategy set. For completeness, I first state an existence and uniqueness theorem. However, prior to this, some auxiliary definitions are in order.

3.1 Existence of List-Nash Equilibrium

Definition 4. Let $G(C^\theta)$ be an ordered strategic game, and fix a coalition $J \in \mathcal{N}$. The J-coalitional order-domination relation $\trianglerighteq_J$ on $S_J = \bigtimes_{j \in J} S_j$ is the binary relation $\trianglerighteq_J \subseteq S_J \times S_J$ defined by: for all $s, \tilde{s} \in S_J$,

$$s \trianglerighteq_J \tilde{s} \iff \forall j \in J, \pi_j^{-1}(s_j) \leq \pi_j^{-1}(\tilde{s}_j).$$

For convenience, when $J = N$ simply write $\trianglerighteq$ instead of $\trianglerighteq_N$. Similarly, when $J = \{i\}$ for some $i \in N$, abuse the notation and write $\trianglerighteq_i$. Observe that, for any $J \in \mathcal{N}$ with $|J| = 1$, the pair $(S_i, \trianglerighteq_i)$ is a linearly ordered set, whereas if $|J| \geq 2$ the pair $(S_J, \trianglerighteq_J)$ is only partially ordered.

Let $G(C^\theta)$ be an ordered strategic game, $i \in N$. The best-response correspondence of player i is a one-to-many mapping $B_i : S_{-i} \to 2^{S_i} \setminus \{\emptyset\}$ defined by

[8] To read details of Behavioral Nash equilibrium and its implementation, see de Clippel (2014).

$B_i(s_{-i}) = \{\, s_i \in S_i \mid g(s_i, s_{-i}) = C_i^\theta(L_i(s_{-i}))\}$, at each $s_{-i} \in S_{-i}$. Observe that, for any s_{-i}, the choice function returns $C_i^\theta\left(L_i\left(s_{-i}\right)\right) \in O_i\left(s_{-i}\right)$. Hence there exists $s_i \in S_i$ with $g\left(s_i, s_{-i}\right) = C_i^\theta\left(L_i\left(s_{-i}\right)\right)$, so $B_i\left(s_{-i}\right) \neq \emptyset$.

Definition 5. Let $G(C^\theta)$ be an ordered strategic game. Let $\beta_i : S_{-i} \to S_i$ be given by $\beta_i(s_{-i}) := \min_{\trianglerighteq_i} B_i(s_{-i})$. The game $G(C^\theta)$ satisfies Monotone Opportunity condition if for all $i \in N$,

$$\forall s_{-i}, \tilde{s}_{-i} \in S_{-i} \quad \left[s_{-i} \trianglerighteq_{N\setminus\{i\}} \tilde{s}_{-i}\right] \Rightarrow \left[\beta_i\left(s_{-i}\right) \trianglerighteq_i \beta_i\left(\tilde{s}_{-i}\right)\right]$$

Note that as every $(S_i, \trianglerighteq_i)$ is a finite linearly ordered set, β_i is well-defined. The condition states that increasing the opponents' coordinates cannot decrease a player's upward-moving opportunities.

Theorem 1. *Every ordered strategic game $G(C^\theta)$ that satisfies the Monotone Opportunity condition has a List-Nash equilibrium.*

The proof of Theorem 1 invokes a well-known result from lattice theory, recalled below.

Theorem 2 (Knaster–Tarski Fixed Point Theorem)**.** *Let $(S, \leq)$ be a complete lattice and $f : S \to S$ isotone with respect to $\leq$. The set of fixed points of f is itself a complete lattice under $\leq$.*

For a detailed proof of the Knaster–Tarski theorem and necessary definitions and tools, see Knaster (1928) and Tarski (1955). As a complete lattice cannot be empty, Theorem 2 implies f has at least one fixed point.

Proof of Theorem 1. Fix an ordered strategic game $G(C^\theta)$ and assume the Monotone Opportunity condition holds. First, note that $(S, \trianglerighteq)$ is a complete lattice. To see this, let $\emptyset \neq A \subseteq S$. For each $i \in N$ define $u_i := \max_{\trianglerighteq_i}\{a_i : \ a \in A\}$, and $\ell_i := \min_{\trianglerighteq_i}\{a_i : \ a \in A\}$. Set $\sup_{\trianglerighteq} A := (u_i)_{i\in N}$ and $\inf_{\trianglerighteq} A := (\ell_i)_{i\in N}$. For any $a \in A$ and any $i \in N$ one has $a_i \trianglerighteq_i u_i$ and $\ell_i \trianglerighteq_i a_i$, hence $a \trianglerighteq \sup_{\trianglerighteq} A$ and $\inf_{\trianglerighteq} A \trianglerighteq a$.

If $v \in S$ is an upper bound of A, then $a_i \trianglerighteq_i v_i$ for all $a \in A$ and all i, which implies $u_i \trianglerighteq_i v_i$ for all i by maximality of u_i. Therefore $\sup_{\trianglerighteq} A \trianglerighteq v$. The dual argument shows that $\inf_{\trianglerighteq} A$ is the greatest lower bound of A. For the empty set define $\bot := (\min_{\trianglerighteq_i} S_i)_{i \in N} = (\pi_i(1))_{i \in N}$, and $\top := (\max_{\trianglerighteq_i} S_i)_{i \in N} = (\pi_i(|S_i|))_{i \in N}$, and set $\sup_{\trianglerighteq} \emptyset := \bot$ and $\inf_{\trianglerighteq} \emptyset := \top$. Hence $(S, \trianglerighteq)$ is a complete lattice.

Recall that $\beta : S \to S$ is given by $\beta(s) := (\beta_i(s_{-i}))_{i \in N}$. Now, I claim that β is isotone with respect to $\trianglerighteq$. Let $s, \tilde{s} \in S$ satisfy $s \trianglerighteq \tilde{s}$. Then for each $i \in N$ one has $s_{-i} \trianglerighteq_{N \setminus \{i\}} \tilde{s}_{-i}$. By the Monotone Opportunity condition it follows that $\beta_i(s_{-i}) \trianglerighteq_i \beta_i(\tilde{s}_{-i})$ for every i. Therefore $\beta(s) \trianglerighteq \beta(\tilde{s})$, which proves isotonicity.

By Theorem 2 the set of fixed points of β is nonempty. Pick $s^* \in S$ with $\beta(s^*) = s^*$. Then for each $i \in N$ one has $s_i^* = \beta_i(s_{-i}^*) \in B_i(s_{-i}^*)$, hence $g(s_i^*, s_{-i}^*) = C_i^\theta(L_i(s_{-i}^*))$. Therefore s^* is a List-Nash equilibrium of $G(C^\theta)$. This completes the proof. $\square$

Remark 1. *The Monotone Opportunity condition is a sufficient but not a necessary condition for the existence of List-Nash equilibria. Example 3 demonstrates this later. For details, see Remark 2.*

Corollary 2. Let $G(C^\theta)$ be an ordered strategic game that satisfies the Monotone Opportunity condition. Then:

1. $\mathrm{Fix}(\beta) = \{s \in S : \beta(s) = s\} \subseteq \mathrm{LNE}(G(C^\theta))$. Moreover, if for every $i \in N$ and every $s_{-i} \in S_{-i}$ the set $B_i(s_{-i})$ is a singleton, then $\mathrm{Fix}(\beta) = \mathrm{LNE}(G(C^\theta))$.

2. $\mathrm{Fix}(\beta)$ is a complete lattice under $\trianglerighteq$ whose least and greatest elements are obtained by the limits $s^{\min} = \lim_{t \to \infty} \beta^t(\underline{s})$ and $s^{\max} = \lim_{t \to \infty} \beta^t(\overline{s})$, where $\underline{s} := (\min_{\trianglerighteq_i} S_i)_{i \in N}$ and $\overline{s} := (\max_{\trianglerighteq_i} S_i)_{i \in N}$. In a finite S these limits are reached in at most $|S| - 1$ steps.

Proof. By Theorem 2, the fixed-point set $\mathrm{Fix}(\beta)$ is a nonempty complete lattice under $\trianglerighteq$. If $s \in \mathrm{Fix}(\beta)$ then for each $i \in N$ there is $s_i = \beta_i(s_{-i}) \in B_i(s_{-i})$, hence $g(s_i, s_{-i}) = C_i^\theta(L_i(s_{-i}))$. Thus $s \in$ LNE$(G(C^\theta))$, which proves $\mathrm{Fix}(\beta) \subseteq$ LNE$(G(C^\theta))$.

Conversely, let $s \in$ LNE$(G(C^\theta))$. Then $s_i \in B_i(s_{-i})$ for every i. If, in addition, each $B_i(s_{-i})$ is a singleton, it follows that $s_i = \beta_i(s_{-i})$ for all i, whence $\beta(s) = s$ and $s \in \mathrm{Fix}(\beta)$. This yields $\mathrm{Fix}(\beta) =$ LNE$(G(C^\theta))$ under the stated hypothesis.

For the iterative characterization, define $\beta^0(s) := s$ and $\beta^{t+1}(s) := \beta(\beta^t(s))$ for $t \in \mathbb{N}$. Isotonicity of β gives $\beta^t(\underline{s}) \trianglerighteq \beta^{t+1}(\underline{s})$ and $\beta^{t+1}(\overline{s}) \trianglerighteq \beta^t(\overline{s})$ for all t. Since S is finite, the nondecreasing sequence $(\beta^t(\underline{s}))_{t\geq 0}$ and the nonincreasing sequence $(\beta^t(\overline{s}))_{t\geq 0}$ stabilize in at most $|S| - 1$ steps; call the stabilized values $s^{\min}$ and $s^{\max}$ respectively. Both satisfy $\beta(s^{\min}) = s^{\min}$ and $\beta(s^{\max}) = s^{\max}$, hence lie in $\mathrm{Fix}(\beta)$. For any $s \in \mathrm{Fix}(\beta)$, it follows that $\underline{s} \trianglerighteq s \trianglerighteq \overline{s}$, so by isotonicity $\beta^t(\underline{s}) \trianglerighteq s \trianglerighteq \beta^t(\overline{s})$ for all t. Passing to the limits along the two chains yields $s^{\min} \trianglerighteq s \trianglerighteq s^{\max}$. Therefore $s^{\min}$ and $s^{\max}$ are the least and greatest elements of $\mathrm{Fix}(\beta)$. $\square$

The order functions $\{\pi_i\}_{i\in N}$ and the update rule β are common knowledge. Because the extreme profiles $\underline{s}$ and $\overline{s}$ use universally first or last labels, they are publicly salient. If players expect the sequence $\beta^t(\underline{s})$ to unfold, Corollary 2 implies that all higher-order beliefs converge on $s^{\min}$. Likewise, expectation of the sequence from $\overline{s}$ concentrates beliefs on $s^{\max}$. Hence $s^{\min}$ and $s^{\max}$ act as Schelling (1960) focal points: prominence of extreme labels and common knowledge of monotone dynamics suffice to single them out, independent of payoffs or communication.

3.2 Uniqueness of List-Nash Equilibrium

The Monotone Opportunity condition yields existence but does not preclude multiplicity, since the fixed-point set of β is a complete lattice and its extremal elements $s^{\min}$ and $s^{\max}$ may differ. Accordingly, a stronger hypothesis is needed to ensure uniqueness.

Let I be a nonempty index set and write $A = \bigtimes_{i\in I} A_i$. The Hamming set-difference is the function $d_H : A \times A \to 2^I$ defined by $d_H(a, a') = \{\, i \in I : a_i \neq a'_i \,\}$. Thus, for strategy profiles $s, s' \in S$ one can write $d_H(s, s') \subseteq N$. For convenience, write $h(s, \tilde{s}) = |d_H(s, \tilde{s})|$ for each $s, \tilde{s} \in S$.

Definition 6. Let $G(C^\theta)$ be an ordered strategic game. The game $G(C^\theta)$ satisfies the Hamming Contraction condition if there exists $m \in \mathbb{N}$ such that for all $s, \tilde{s} \in S$ with $s \trianglerighteq \tilde{s}$ and $s \neq \tilde{s}$, then $h\left(\beta^m(s), \beta^m(\tilde{s})\right) < h(s, \tilde{s})$.

Although the Hamming Contraction condition is strong, it is defined on β that chooses the $\trianglerighteq_i$-minimum of $B_i\left(s_{-i}\right)$ and only on comparable profiles. Under the Monotone Opportunity condition one has $s \trianglerighteq \tilde{s}$ implies $\beta(s) \trianglerighteq \beta(\tilde{s})$. In many instances a single update already yields $h(\beta(s), \beta(\tilde{s})) < h(s, \tilde{s})$. In finite games, even when one-step reduction fails for some comparable pairs, there exists $m \in \mathbb{N}$ such that $h\left(\beta^m(s), \beta^m(\tilde{s})\right) < h(s, \tilde{s})$ for all comparable $s \neq \tilde{s}$. The assumption targets uniqueness along chains while avoiding stronger requirements on all selections or incomparable profiles.

Lemma 1. *Assume Hamming Contraction condition. Then for any $s, \tilde{s} \in S$ with $s \trianglerighteq \tilde{s}$ one has $\beta^m(s) = \beta^m(\tilde{s})$. Consequently $\beta^k(s) = \beta^k(\tilde{s})$ for every $k \geq m$.*

Proof. Fix $s, \tilde{s} \in S$ with $s \trianglerighteq \tilde{s}$ and set $q := h(s, \tilde{s})$. If $q = 0$ there is nothing to prove. If $q \geq 1$, choose a chain $s = v^0 \trianglerighteq v^1 \trianglerighteq \cdots \trianglerighteq v^q = \tilde{s}$ with $h\left(v^{j-1}, v^j\right) = 1$ for all $j \in [q]$. For each adjacent pair $v^{j-1} \trianglerighteq v^j$, contraction gives $h\left(\beta^m\left(v^{j-1}\right), \beta^m\left(v^j\right)\right) < 1$, hence $\beta^m\left(v^{j-1}\right) = \beta^m\left(v^j\right)$. Transitivity yields $\beta^m(s) = \beta^m(\tilde{s})$. If $k \geq m$ write $k = m + r$ with $r \in \mathbb{N} \cup \{0\}$. Then $\beta^k(s) = \beta^r\left(\beta^m(s)\right) = \beta^r\left(\beta^m(\tilde{s})\right) = \beta^k(\tilde{s})$. □

Proposition 1. *The Monotone Opportunity condition does not imply the Hamming Contraction condition, and vice versa.*

Proof. Let $N = \{1, 2\}$. For each construction prescribe a reply rule $\widehat{\beta} = (\widehat{\beta}_i)_{i\in N}$ with $\widehat{\beta}_i : S_{-i} \to S_i$. On every opportunity list $L_i(s_{-i}) = (g(\pi_i(k), s_{-i}))_{r=1}^{|S_i|}$ define $C_i^\theta(L_i(s_{-i})) = g(\widehat{\beta}_i(s_{-i}), s_{-i})$. Then $B_i(s_{-i}) = \{\widehat{\beta}_i(s_{-i})\}$ and $\beta = \widehat{\beta}$.

First, take $S_1 = \{a, b\}$ with $\pi_1(1) = a$ and $\pi_1(2) = b$; and $S_2 = \{c, d\}$ with $\pi_2(1) = c$ and $\pi_2(2) = d$. Define $\widehat{\beta}_1(c) = a$, $\widehat{\beta}_1(d) = b$, $\widehat{\beta}_2(a) = c$, $\widehat{\beta}_2(b) = d$. If $s_{-1} \trianglerighteq_2 \tilde{s}_{-1}$ then $\widehat{\beta}_1(s_{-1}) \trianglerighteq_1 \widehat{\beta}_1(\tilde{s}_{-1})$, and if $s_{-2} \trianglerighteq_1 \tilde{s}_{-2}$ then $\widehat{\beta}_2(s_{-2}) \trianglerighteq_2 \widehat{\beta}_2(\tilde{s}_{-2})$. Hence the Monotone Opportunity condition holds. On the other hand, however, $\widehat{\beta}(a, c) = (a, c)$, $\widehat{\beta}(b, d) = (b, d)$, $\widehat{\beta}(a, d) = (b, c)$, $\widehat{\beta}(b, c) = (a, d)$. For $s = (a, c)$ and $\tilde{s} = (b, d)$ one has $s \trianglerighteq \tilde{s}$ and $h(s, \tilde{s}) = 2$, while $h(\widehat{\beta}^m(s), \widehat{\beta}^m(\tilde{s})) = 2$ for all $m \in \mathbb{N}$. Therefore the Hamming Contraction condition fails.

Second, take $S_1 = S_2 = \{0, 1\}$ with $\pi_1(1) = 0$, $\pi_1(2) = 1$, $\pi_2(1) = 0$, and $\pi_2(2) = 1$. Define $\widehat{\beta}_1(s_{-1}) = 1 - s_{-1}$ and $\widehat{\beta}_2(s_{-2}) = 1$. Then $\widehat{\beta}(s_1, s_2) = (1 - s_2, 1)$ and $\widehat{\beta}^2(s) = (0, 1)$ for all $s \in S$. Hence $h(\widehat{\beta}^2(s), \widehat{\beta}^2(\tilde{s})) = 0 < h(s, \tilde{s})$ whenever $s \trianglerighteq \tilde{s}$ and $s \neq \tilde{s}$, so Hamming contraction holds with $m = 2$. Monotone Opportunity fails because $0 \trianglerighteq_2 1$ yet $\widehat{\beta}_1(0) = 1 \not\trianglerighteq_1 \widehat{\beta}_1(1) = 0$. $\square$

Theorem 3. *Let $G(C^\theta)$ be an ordered strategic game. Assume the Monotone Opportunity condition and the Hamming Contraction condition hold for some $m \in \mathbb{N}$. Then $\mathrm{Fix}(\beta) = \{s^*\}$.*

Proof. By Theorem 2, $\mathrm{Fix}(\beta)$ is a nonempty complete lattice with least and greatest elements $s^{\min}$ and $s^{\max}$. If $s^{\min} \neq s^{\max}$ then, as both are fixed points, $\beta^m(s^{\min}) = s^{\min}$ and $\beta^m(s^{\max}) = s^{\max}$. Hence $h(\beta^m(s^{\min}), \beta^m(s^{\max})) = h(s^{\min}, s^{\max})$, which contradicts the Hamming Contraction condition. Therefore $s^{\min} = s^{\max}$. $\square$

Note that $\mathrm{Fix}(\beta)$ need not coincide with the set of List-Nash equilibria. In general $\mathrm{Fix}(\beta) \subseteq \mathrm{LNE}\left(G\left(C^\theta\right)\right)$, and the inclusion can be strict unless each $B_i\left(s_{-i}\right)$ is a singleton, in which case equality holds (see Corollary 2). Even a simple 2×2 construction can show that an equilibrium profile can fail to be a fixed point of β, so $\mathrm{Fix}(\beta)$ is not, by itself, an equilibrium selection without the unique chosen response property.

Definition 7. For $i \in N$ and $s_{-i} \in S_{-i}$ set $B_i(s_{-i}) = \{\, s_i \in S_i : g(s_i, s_{-i}) =$

$C_i^\theta(L_i(s_{-i}))\}$. The unique chosen response condition holds if $|B_i(s_{-i})| = 1$ for all i and all s_{-i}.

Theorem 4. *Let $G(C^\theta)$ be an ordered strategic game. Assume the Monotone Opportunity condition and the Hamming Contraction condition for some $m \in \mathbb{N}$. If, in addition, the unique chosen response condition holds, then $G(C^\theta)$ has a unique List-Nash equilibrium, equal to the unique element of* $\mathrm{Fix}(\beta)$.

Proof. By Theorem 3, $\mathrm{Fix}(\beta) = \{s^*\}$. If s is a List-Nash equilibrium, then $s_i \in B_i(s_{-i})$ for every i, hence $s_i = \min B_i(s_{-i}) = \beta_i(s_{-i})$ for every i, so $\beta(s) = s$ and $s = s^*$. Conversely, if $\beta(s) = s$, then $s_i \in B_i(s_{-i})$ for all i, so s is a List-Nash equilibrium. □

I acknowledge that the Monotone Opportunity condition and the Hamming Contraction condition, together with the unique chosen response assumption when invoked, are deliberately demanding. Nevertheless, in general strategic environments, equilibrium uniqueness is a demanding property that rarely obtains without substantive structure on primitives. Broad uniqueness results are therefore uncommon; when they do appear, they rest on hypotheses of comparable strength. The role of these assumptions is methodological rather than descriptive, as they delineate a tractable class of games for which equilibrium is uniquely determined.

CHAPTER 4

CHOICE FROM LISTS IN MECHANISM DESIGN

The preceding analysis treats the order functions as fixed primitives of an ordered game form, providing only a naïve account of how players perceive their strategies. In many applications, however, the way alternatives or messages are labeled or sequenced can itself be chosen by the designer. I therefore revisit the classic implementation problem within this richer framework and analyze what happens if a mechanism designer can endogenously determine the order functions, together with the usual outcome function and message spaces.

4.1 The Implementation Problem

The ordered tuple $\mathcal{E} = \langle N, X, \Theta, \mathscr{C} \rangle$ is called the environment in which the mechanism designer operates, and it is assumed to be common knowledge among the players. A social choice rule is a correspondence $F : \Theta \to \mathcal{X}$ with $\emptyset \neq F(\theta) \subseteq X$ for every $\theta \in \Theta$, and I denote the set of all social choice rules by $\mathcal{F} = \{ F \mid F : \Theta \to \mathcal{X} \}$. An ordered mechanism, hereinafter simply referred to as a mechanism, μ is an ordered tuple $\langle (M_i, \pi_i)_{i \in N}, g \rangle$ where M_i is the finite nonempty set of messages available to player i, π_i is the message-order function, which specifies the order in which agent i receives the messages, and $g : M \to X$ is the outcome function where $M = \bigtimes_{i \in N} M_i$.

Let $\mathcal{M}$ be the set of all possible mechanisms for the environment $\mathcal{E}$. For any $\theta \in \Theta$ and $C^\theta \in \mathscr{C}^\theta$, a mechanism $\mu \in \mathcal{M}$ and the profile C^θ induce an ordered strategic game $G^\mu(C^\theta) = \langle N,\ (M_i, \pi_i)_{i\in N},\ g\rangle$. For $m = (m_i)_{i\in N} \in M$ and $i \in N$ define the opportunity list of player i against m_{-i} by $L_i^\mu(m_{-i}) = \big(g(\pi_i(k), m_{-i})\big)_{k=1}^{|M_i|} \in \mathcal{L}$. A profile $m^* \in M$ is a List-Nash equilibrium of $G^\mu(C^\theta)$ if and only if for every $i \in N$, $g(m_i^*, m_{-i}^*) = C_i^\theta\big(L_i^\mu(m_{-i}^*)\big)$. Let $\text{LNE}\big(\mu,\theta\big) \subseteq M$ denote the set of List-Nash equilibrium message profiles, and define the associated set of equilibrium outcomes by $\text{LNE}_g(\mu,\theta) = \{\, g(m) :\ m \in \text{LNE}\big(\mu,\theta\big)\,\}\ \subseteq X$.

Definition 8. Let $\mathcal{E}$ be an environment, and $F \in \mathcal{F}$ be a social choice rule. A mechanism $\mu \in \mathcal{M}$ (fully) implements the social choice rule F in List-Nash equilibrium if for all $\theta \in \Theta$, $F(\theta) = \text{LNE}_g(\mu,\theta)$. If there exists such a mechanism, then the social choice rule F is said to be implementable in List-Nash equilibrium.

The main aim of the mechanism designer is to control the rules of the game, which is formalized as an ordered game form. In other words, the mechanism designer aims to ensure that, at every state of the world, alternatives chosen by the social choice rule coincide with the equilibrium outcomes of the game for the given state. A novelty here is that the designer also determines the order in which players receive messages, creating a *nudge* that helps implement the desired social choice rule.

Throughout this chapter I restrict attention to full implementation and derive necessary and sufficient conditions for it under the List-Nash equilibrium concept. Before stating the general results, I compare implementability under List-Nash and under the conventional Nash equilibrium: in the former, players' choice functions are defined on lists; in the latter, on sets. Because the choice domains differ, the formulations are not strictly comparable, so I allow a mild abuse of terminology for the sake of exposition. The examples in the next section make the comparison precise.

Example 3. Let $N = \{1,2\}, X = \{x,y,z\}$ and $\Theta = \{\theta_1, \theta_2\}$. The social choice rule F is given as $F(\theta_1) = \{y\}$ and $F(\theta_2) = \{z\}$. For each $i \in N$ and $\theta \in \Theta$ write $\succ_i^\theta$ for a strict preference[9] on X arranged in a column with the most preferred element on top. The boxed alternative in the second row marks the agent's personal aspiration level.

θ_1			θ_2	
$\succ_1^{\theta_1}$	$\succ_2^{\theta_1}$		$\succ_1^{\theta_2}$	$\succ_2^{\theta_2}$
x	y		y	y
$\boxed{y}$	$\boxed{z}$		$\boxed{z}$	$\boxed{z}$
z	x		x	x

Let v_i^θ denote the aspiration level of player i at state θ, i.e., the boxed alternative. Given a list $L = (x_1, \ldots, x_k) \in \mathcal{L}$ define $V_i^\theta(L) := \left\{m \in [k] \mid x_m \succsim_i^\theta v_i^\theta\right\}$. The choice function $C_i^\theta : \mathcal{L} \to X$ is defined by

$$C_i^\theta(L) = \begin{cases} x_{j^*}, & \text{if } V_i^\theta(L) \neq \emptyset \text{ and } j^* = \min V_i^\theta(L), \\ x_j, & \text{if } V_i^\theta(L) = \emptyset \text{ and } j \in \arg\max_{m \in [k]} x_m \text{ under } \succ_i^\theta . \end{cases}$$

at each $L = (x_1, x_2, \ldots, x_k) \in \mathcal{L}$. The players exhibit satisficing behavior in the following way: Given a list of alternatives, they aim to choose the first alternative that meets or exceeds their satisfaction level, starting from the beginning of the list. Once they find such an alternative, they consider themselves sufficiently satisfied and stop evaluating further alternatives. If no such alternative is found, they choose the most preferred alternative among those they have evaluated.[10]

[9]For each i, θ, let $\succsim_i^\theta$ be a reflexive, transitive and complete relation on X, with asymmetric and symmetric parts denoted by $\succ_i^\theta$ and $\sim_i^\theta$, respectively.

[10]Note that this aspiration level does not imply that the player is indifferent between their aspiration level and the alternatives they prefer to their aspiration level. Moreover, note that this choice behavior is a generalization of the choice behaviors represented in Example 1 and Example 2.

This satisficing behavior aligns with $\succsim$-maximization when the choice problem is presented to the players as a set. In this case, presenting a set implies that players can perceive all the alternatives simultaneously, meaning no "search" is involved, and they will simply choose the $\succ$-most preferred alternative within the set. I now show that F is not Nash implementable but List-Nash implementable. However, before conducting the proof, recall some well-known results from mechanism design literature. Let $F : \Theta \to \mathcal{X}$ be a single-valued social choice rule, that is $|F(\theta)| = 1$ for every $\theta \in \Theta$. Now, F is (Maskin-)monotonic if, for every $\theta, \theta' \in \Theta$ and every $x \in X$,

$$[x \in F(\theta) \wedge \forall i \in N, \forall y \in X, x \succsim_i^{\theta} y \implies x \succsim_i^{\theta'} y] \implies [x \in F(\theta')].$$

Maskin (1999) proves that monotonicity is a necessary condition for Nash implementation.

Now, F is not Nash implementable because F is not monotonic. To see this, observe that $y \in F(\theta_1)$ and for all $i \in N$, for all $a \in X$, $y \succsim_i^{\theta_1} a$ implies $y \succsim_i^{\theta_2} a$. However, $y \notin F(\theta_2)$. On the other hand, F is implementable in a List-Nash equilibrium by the following mechanism. Let $M_i = \{\overline{m}, \underline{m}\}$ and $\pi_i(1) = \overline{m}$, and $\pi_i(2) = \underline{m}$ for all $i \in N$. Now, consider the outcome function g, represented by the matrix below.

		Player 2	
		$\overline{m}$	$\underline{m}$
Player 1	$\overline{m}$	z	x
	$\underline{m}$	y	z

Computing the outcome $g(m)$, the two opportunity lists $L_1(m_{-1})$ and $L_2(m_{-2})$, and the values of the choice rule C_i^{θ} for every message profile $m \in M$ gives the table below.

$m=(m_1,m_2)$	$g(m)$	$C_1^{\theta_1}\big(L_1(m_{-1})\big)$	$C_2^{\theta_1}\big(L_2(m_{-2})\big)$	$C_1^{\theta_2}\big(L_1(m_{-1})\big)$	$C_2^{\theta_2}\big(L_2(m_{-2})\big)$
$(\overline{m},\overline{m})$	z	y	z	z	z
$(\overline{m},\underline{m})$	x	x	z	z	z
$(\underline{m},\overline{m})$	y	y	y	z	y
$(\underline{m},\underline{m})$	z	x	y	z	y

Inspection shows that the equilibrium equalities $g(m)=C_i^{\theta}\left(L_i\left(m_{-i}\right)\right)$ for $i=1,2$ hold only at $m=(\underline{m},\overline{m})$ when $\theta=\theta_1$ and only at $m=(\overline{m},\overline{m})$ when $\theta=\theta_2$. Therefore the mechanism yields precisely the outcomes prescribed by F and admits no additional List-Nash equilibria, i.e., it fully implements F.

Remark 2. *Consider Example 3 with the mechanism described above and fix $\theta=\theta_1$. Note that, $\pi_2^{-1}(\overline{m})\le\pi_2^{-1}(\underline{m})$ implies $\overline{m}\trianglerighteq_2\underline{m}$. On the other hand, $\beta_1(\overline{m})=\underline{m}$ and $\beta_1(\underline{m})=\overline{m}$, hence $\beta_1(\overline{m})\ntrianglerighteq_1\beta_1(\underline{m})$. Therefore the Monotone Opportunity condition is not satisfied. Nevertheless a List-Nash equilibrium exists. Thus the Monotone Opportunity condition is indeed sufficient for existence of List-Nash equilibrium, but not necessary.*

Example 4. Let $N=\{1,2\}, X=\{x,y\}$, and $\Theta=\{\theta_1,\theta_2\}$. For each $i\in N$ and $\theta\in\Theta$ let $\succ_i^{\theta}$ be the strict preference on X displayed below. Players follow the same satisficing rule introduced in Example 3. Define the social choice rule F by $F\left(\theta_1\right)=\{x\}$ and $F\left(\theta_2\right)=\{y\}$.

θ_1	
$\succ_1^{\theta_1}$	$\succ_2^{\theta_1}$
x	y
$\boxed{y}$	$\boxed{x}$

θ_2	
$\succ_1^{\theta_2}$	$\succ_2^{\theta_2}$
y	x
$\boxed{x}$	$\boxed{y}$

Now, observe that F is Nash implementable (as it is dictatorial), but it is not List-Nash implementable. To see this, suppose for a contradiction that there exists a mechanism $\mu=\langle(M_i,\pi_i)_{i\in N},g\rangle$ that List-Nash implements F. Then, $F(\theta_1)=$ $\mathrm{LNE}_g(\mu,\theta_1)=\{x\}$, i.e., there exists some $(\overline{m_1},\underline{m_2})\in M$ such that $g(\overline{m_1},\underline{m_2})\in$

$\text{LNE}_g(\mu, \theta_1)$ such that $g(\overline{m_1}, \underline{m_2}) = C_1^{\theta_1}\left(L_1(\underline{m_2})\right) = C_2^{\theta_1}\left(L_2(\overline{m_1})\right) = x$. Then, for all $m_2' \in M_2$ such that $g(\overline{m_1}, m_2') = y$, it follows $\pi_2^{-1}(\underline{m_2}) < \pi_2^{-1}(m_2')$. Similarly, $F(\theta_2) = \text{LNE}_g(\mu, \theta_2) = \{y\}$, i.e., there exists some $g(\underline{m_1}, \overline{m_2}) \in \text{LNE}_g(\mu, \theta_2)$ such that $g(\underline{m_1}, \overline{m_2}) = C_1^{\theta_2}\left(L_1(\overline{m_2})\right) = C_2^{\theta_2}\left(L_2(\underline{m_1})\right) = y$. Then, for all $m_1' \in M_1$ such that $g(m_1', \overline{m_2}) = x$, it follows $\pi_1^{-1}(\underline{m_1}) < \pi_1^{-1}(m_1')$. Now, without loss of generality, suppose $\pi_1^{-1}(\overline{m_1}) < \pi_1^{-1}(\underline{m_1})$. Then, $g(\overline{m_1}, \overline{m_2}) = y$. If $\pi_2^{-1}(\overline{m_2}) < \pi_2^{-1}(\underline{m_2})$, then there is a contradiction since $C_2^{\theta_1}\left(L_2(\overline{m_1})\right) = y$. On the other hand, if $\pi_2^{-1}(\underline{m_2}) < \pi_2^{-1}(\overline{m_2})$ then, there is a contradiction as $g(\overline{m_1}, \underline{m_2}) \in \text{LNE}_g(\mu, \theta_2)$ with $g(\overline{m_1}, \underline{m_2}) = x$. Thus, F is not List-Nash implementable.

I summarize the main points of these examples, highlighted in the remark below.

Remark 3. *There are social choice rules that are List-Nash implementable but not Nash implementable (see Example 3). Conversely, there are social choice rules that are Nash implementable but not List-Nash implementable. Moreover, a dictatorial social choice rule may not be List-Nash implementable (see Example 4).*

4.3 Necessary Conditions of List-Nash Implementation

In the standard rational domain, any social choice rule (function) that is implementable in Nash equilibrium is Maskin monotonic (Maskin, 1999). In the behavioral framework of de Clippel (2014), where choices need not be rational, implementation implies the existence of a collection of opportunity sets consistent with the target social choice rule. I seek an analogous necessary condition for List-Nash implementation in the list-based framework and derive it through a sequence of observations.

The first observation aligns with de Clippel (2014), that is, given $\theta \in \Theta$ and $x \in F(\theta)$, there exist lists for each player from which each player chooses x at θ. If the same lists also induce the choice of x at some $\theta' \in \Theta \setminus \{\theta\}$, then $x \in F(\theta')$. Moreover, given a mechanism, for each player the length of every opportunity list is fixed and independent of the others' message profile. Combining these two observations, I impose the following condition.

Definition 9. Let $\kappa \in \mathbb{N}^{|N|}$ be a vector of natural numbers indexed by players. Let $\Lambda = \left\{ \lambda_i^{(x,\theta)} : [\kappa_i] \to X \mid i \in N, x \in F(\theta), \theta \in \Theta \right\}$ be a family of functions. Then Λ is said to be κ-consistent with the social choice rule F if

(i) for every $i \in N, \theta \in \Theta$ and $x \in F(\theta)$, then $C_i^{\theta}((\lambda_i^{(x,\theta)}(r))_{r=1}^{\kappa_i}) = x$,

(ii) for every $\theta, \theta' \in \Theta$ and $x \in F(\theta)$, if $C_i^{\theta'}((\lambda_i^{(x,\theta)}(r))_{r=1}^{\kappa_i}) = x$ for all $i \in N$, then $x \in F(\theta')$.

If $\kappa_i = \kappa_j$ for all $i, j \in N$, then Λ is said to be κ-consistent with the social choice rule F.

The second observation is that κ-consistency does not by itself yield a well-defined mechanism. Prescribed lists can conflict at profiles lying on intersecting slices of the outcome table. I therefore impose a compatibility condition that allows the entire family of state-contingent lists to be embedded in a single mechanism while keeping the outcome single-valued on the union of the prescribed slices. The condition enforces invariance along opponent-fixed slices and agreement at their cross intersections, thereby eliminating contradictory prescriptions.

Definition 10. Let $\kappa \in \mathbb{N}^{|N|}$ be a vector of natural numbers indexed by players. Let $\Lambda = \left\{ \lambda_i^{(x,\theta)} : [\kappa_i] \to X \mid i \in N, x \in F(\theta), \theta \in \Theta \right\}$ be a family of functions. Then Λ is said to be compatible with F if

(i) for every $\theta \in \Theta$ and $x \in F(\theta)$ there exists a vector $c^{(x,\theta)} \in \bigtimes_{i \in N} [\kappa_i]$ satisfying $\lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = x$ for all $i \in N$, and $c^{(x,\theta)} = c^{(y,\theta')}$ implies $x = y$,

(ii) if $d_H(c^{(x,\theta)}, c^{(y,\theta')}) = \{i\}$ then $\lambda_i^{(x,\theta)} = \lambda_i^{(y,\theta')}$,

(iii) if $d_H(c^{(x,\theta)}, c^{(y,\theta')}) = \{i, j\}$ with $i \neq j$ then $\lambda_i^{(x,\theta)}(c_i^{(y,\theta')}) = \lambda_j^{(y,\theta')}(c_j^{(x,\theta)})$.

Theorem 5. *If F is List-Nash implementable, then there exist $\kappa \in \mathbb{N}^{|N|}$ and a corresponding family Λ that is κ-consistent and compatible with F.*

Proof. Assume F is List-Nash implementable. Then there exists a mechanism $\mu = \langle (M_i, \pi_i)_{i\in N}, g\rangle \in \mathcal{M}$ such that for every $\theta \in \Theta$, $\text{LNE}_g(\mu,\theta) = F(\theta)$. For each $i \in N$ set $\kappa_i := |M_i| \in \mathbb{N}$. Fix $\theta \in \Theta$ and $x \in F(\theta)$. Since $x \in \text{LNE}_g(\mu,\theta)$, there exists $m^{(x,\theta)} = (m_i^{(x,\theta)})_{i\in N} \in \text{LNE}(\mu,\theta)$ with $g(m^{(x,\theta)}) = x$. Define $c_i^{(x,\theta)} := \pi_i^{-1}(m_i^{(x,\theta)}) \in [\kappa_i]$, and $\lambda_i^{(x,\theta)} : [\kappa_i] \to X$ where $\lambda_i^{(x,\theta)}(r) := g(\pi_i(r), m_{-i}^{(x,\theta)})$. Let $\kappa = (\kappa_i)_{i\in N}$ and $\Lambda = \{\lambda_i^{(x,\theta)} : i \in N, x \in F(\theta), \theta \in \Theta\}$.

I need to show that Λ is κ-consistent with F. For any $i \in N$, $C_i^\theta((\lambda_i^{(x,\theta)}(r))_{r=1}^{\kappa_i}) = C_i^\theta(L_i(m_{-i}^{(x,\theta)})) = g(m_i^{(x,\theta)}, m_{-i}^{(x,\theta)}) = g(m^{(x,\theta)}) = x$, so condition (i) of κ-consistency holds. For $\theta' \in \Theta$, if $C_i^{\theta'}((\lambda_i^{(x,\theta)}(r))_{r=1}^{\kappa_i}) = x$ for all i, then for all i, $g(m_i^{(x,\theta)}, m_{-i}^{(x,\theta)}) = C_i^{\theta'}(L_i(m_{-i}^{(x,\theta)}))$, hence $m^{(x,\theta)} \in \text{LNE}(\mu,\theta')$. Therefore $x = g(m^{(x,\theta)}) \in \text{LNE}_g(\mu,\theta') = F(\theta')$. Thus condition (ii) of κ-consistency holds.

Now, I need to show that Λ is compatible with F. For any (x,θ) and any $i \in N$, $\lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = g(\pi_i(\pi_i^{-1}(m_i^{(x,\theta)})), m_{-i}^{(x,\theta)}) = g(m^{(x,\theta)}) = x$, so item (i) holds. Moreover, if $c^{(x,\theta)} = c^{(y,\theta')}$, then for every $i \in N$, $m_i^{(x,\theta)} = \pi_i\big(c_i^{(x,\theta)}\big) = \pi_i\big(c_i^{(y,\theta')}\big) = m_i^{(y,\theta')}$. Hence $m^{(x,\theta)} = m^{(y,\theta')}$ and, since g is single-valued, $x = g\big(m^{(x,\theta)}\big) = g\big(m^{(y,\theta')}\big) = y$. If $d_H(c^{(x,\theta)}, c^{(y,\theta')}) = \{i\}$, then $m_{-i}^{(x,\theta)} = m_{-i}^{(y,\theta')}$ and for all $r \in [\kappa_i]$, it follows that $\lambda_i^{(x,\theta)}(r) = g(\pi_i(r), m_{-i}^{(x,\theta)}) = g(\pi_i(r), m_{-i}^{(y,\theta')}) = \lambda_i^{(y,\theta')}(r)$, hence item (ii) holds. If $d_H(c^{(x,\theta)}, c^{(y,\theta')}) = \{i,j\}$ with $i \neq j$, then for all other coordinates $k \in N \setminus \{i,j\}$ one has $c_k^{(x,\theta)} = c_k^{(y,\theta')}$. Now, let $\widetilde{m} \in M$ be defined by

$$\widetilde{m}_k = \begin{cases} \pi_i(c_i^{(y,\theta')}), & \text{if } k = i, \\ \pi_j(c_j^{(x,\theta)}), & \text{if } k = j, \\ \pi_k(c_k^{(x,\theta)}) = \pi_k(c_k^{(y,\theta')}), & \text{if } k \in N \setminus \{i,j\}. \end{cases}$$

Then $\widetilde{m}_{-i} = m_{-i}^{(x,\theta)}$, and $\widetilde{m}_{-j} = m_{-j}^{(y,\theta')}$. Then $\lambda_i^{(x,\theta)}(c_i^{(y,\theta')}) = g(\pi_i(c_i^{(y,\theta')}), m_{-i}^{(x,\theta)}) = g(\widetilde{m}) = g(\pi_j(c_j^{(x,\theta)}), m_{-j}^{(y,\theta')}) = \lambda_j^{(y,\theta')}(c_j^{(x,\theta)})$, so item (iii) holds. Therefore Λ is κ-consistent and compatible with F, as required. $\square$

In general, a family Λ that is κ-consistent and compatible identifies only one-dimensional slices of the outcome table, so it specifies a mechanism on a proper subset of the message grid and leaves the remaining cells undefined. Filling those cells ad hoc can reshape players' opportunity lists, change best responses, and sever the link between the prescribed outcomes and the target social choice rule, thereby spoiling full implementation. To solve this issue, I first formalize an incomplete mechanism, which records the order functions together with a partial outcome map and treats all remaining profiles as undefined. I then seek mild but effective sufficient conditions under which this partial specification admits a coherent completion to a total outcome function that preserves the induced equilibrium behavior and achieves full implementation.

Definition 11. An incomplete mechanism is a tuple $\mu^* = \langle (M_i, \pi_i)_{i\in N}, g^*\rangle$ where M_i is the set of messages available to player i, π_i is the message-order function of player i, and $g^* : A \to X$ where $\emptyset \neq A \subsetneq M$.

Accordingly, a mechanism $\mu = \langle (M_i, \pi_i)_{i\in N}, g\rangle$ is called complete when g is defined on the entire message space. Consistent with the earlier notation, let $\mathcal{M}$ denote the set of all complete mechanisms and $\mathcal{M}^*$ the set of all incomplete mechanisms.

Definition 12. Let $\bar{\mu}^* = \left\langle \bar{M}, \bar{\pi}, \bar{g}^*\right\rangle$ and $\tilde{\mu}^* = \langle \tilde{M}, \tilde{\pi}, \tilde{g}^*\rangle$ be two incomplete mechanisms with $\bar{g}^* : \bar{A} \to X$ and $\tilde{g}^* : \tilde{A} \to X$. Now, $\bar{\mu}^*$ and $\tilde{\mu}^*$ are said to be compatible if: (i) $\bar{M} = \tilde{M}$, (ii) $\bar{\pi} = \tilde{\pi}$, (iii) $\bar{g}^*(m) = \tilde{g}^*(m)$ for all $m \in \bar{A} \cap \tilde{A}$.

Note that condition (i) is imposed purely for technical convenience, rather than any genuine epistemic restriction. Indeed, if it is possible to find an isomorphic relation between distinct message spaces of two incomplete mechanisms, that preserves the condition (ii), then relax (i). That relaxation is legitimate because I treat messages as purely formal symbols: only the ordering induced by π and the associated outcome function matter, not the specific names. Hence any order-preserving relabeling of the message spaces is without loss.

Definition 13. Let $\{\langle M, \pi, g_j^*\rangle\}_{j\in I}$ be a collection of compatible mechanisms where $\emptyset \neq I$ is an index set, and $g_j^* : A_j \to X$ for all $j \in I$. Now, a (possibly complete) mechanism $\langle \bar{M}, \bar{\pi}, \bar{g}^*\rangle$ is said to be induced by $\{\langle M, \pi, g_j^*\rangle\}_{j\in I}$ if $\bar{M} = M, \bar{\pi} = \pi$ and $\bar{g}^* : \bigcup_{j\in I} A_j \to X$ such that $\bar{g}^*(m) = g_j^*(m)$ for all $m \in A_j$ and $j \in I$.

Proposition 2. *Let $\kappa \in \mathbb{N}^{|N|}$ and Λ be a corresponding family of functions that is compatible with F. Then Λ induces an (possibly complete) incomplete mechanism.*

Proof. Fix $\kappa \in \mathbb{N}^{|N|}$ and a family Λ that is compatible with F. For each $i \in N$ choose a finite set M_i with $|M_i| = \kappa_i$ and a bijection $\pi_i : [\kappa_i] \xrightarrow{\sim} M_i$. For every (x, θ) with $x \in F(\theta)$ let $c^{(x,\theta)} \in \bigtimes_{i\in N}[\kappa_i]$ be given by compatibility item (i), and set $m^{(x,\theta)} = (\pi_i(c_i^{(x,\theta)}))_{i\in N} \in M$.

For each $i \in N$ define $A_i^{(x,\theta)} = \{(\pi_i(r), m_{-i}^{(x,\theta)}) : r \in [\kappa_i]\} \subseteq M$ and $A^{(x,\theta)} = \bigcup_{i\in N} A_i^{(x,\theta)}$. Define $g^{*(x,\theta)} : A^{(x,\theta)} \to X$ by $g^{*(x,\theta)}(\pi_i(r), m_{-i}^{(x,\theta)}) = \lambda_i^{(x,\theta)}(r)$. First, $g^{*(x,\theta)}$ is well-defined on $A^{(x,\theta)}$. If $m \in A_i^{(x,\theta)} \cap A_j^{(x,\theta)}$ with $i = j$ there is no issue. If $i \neq j$ then necessarily $m = (\pi_i(c_i^{(x,\theta)}), m_{-i}^{(x,\theta)}) = (\pi_j(c_j^{(x,\theta)}), m_{-j}^{(x,\theta)}) = m^{(x,\theta)}$. The compatibility (i) implies $\lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = \lambda_j^{(x,\theta)}(c_j^{(x,\theta)}) = x$, hence both prescriptions agree. Second, prescriptions agree across distinct pairs (x, θ) and (y, θ'). If $m \in A^{(x,\theta)} \cap A^{(y,\theta')}$, there exist $i, j \in N$ and $r \in [\kappa_i]$, $r' \in [\kappa_j]$ with $m = (\pi_i(r), m_{-i}^{(x,\theta)}) = (\pi_j(r'), m_{-j}^{(y,\theta')})$. If $i = j$ then $m_{-i}^{(x,\theta)} = m_{-i}^{(y,\theta')}$, so $d_H(c^{(x,\theta)}, c^{(y,\theta')}) = \{i\}$ and compatibility (ii) yields $\lambda_i^{(x,\theta)} = \lambda_i^{(y,\theta')}$, hence $g^{*(x,\theta)}(m) = g^{*(y,\theta')}(m)$. If $i \neq j$ then necessarily $r = c_i^{(y,\theta')}$, $r' = c_j^{(x,\theta)}$, and $c_k^{(x,\theta)} = c_k^{(y,\theta')}$ for all $k \notin \{i, j\}$. Compatibility (iii) gives $\lambda_i^{(x,\theta)}(c_i^{(y,\theta')}) = \lambda_j^{(y,\theta')}(c_j^{(x,\theta)})$, hence again $g^{*(x,\theta)}(m) = g^{*(y,\theta')}(m)$.

Therefore the family $\{\langle (M_i)_{i\in N}, (\pi_i)_{i\in N}, g^{*(x,\theta)}\rangle\}_{x\in F(\theta), \theta\in\Theta}$ is pairwise compatible. Define $g^* : \bigcup_{x,\theta} A^{(x,\theta)} \to X$ by $g^*(m) = g^{*(x,\theta)}(m)$ whenever $m \in A^{(x,\theta)}$. The previous paragraph shows this is well-defined. Set $\mu^* = \langle (M_i)_{i\in N}, (\pi_i)_{i\in N}, g^*\rangle$.

Then μ^* is an induced incomplete mechanism. If $\bigcup_{x,\theta} A^{(x,\theta)} = M$, it is complete.

$\square$

A social choice rule F may not be fully implementable, since κ-consistency and compatibility only yields an incomplete mechanism. The assumption below allows us to construct a complete mechanism.

Definition 14. Let $\kappa \in \mathbb{N}^{|N|}$ be a vector of natural numbers indexed by players. Let $\Lambda = \left\{ \lambda_i^{(x,\theta)} : [\kappa_i] \to X \mid i \in N, x \in F(\theta), \theta \in \Theta \right\}$ be a family of functions. Then Λ is said to be strongly compatible with F if

(i) Λ is compatible with F,

(ii) if for every $\theta \in \Theta$ and every $c \in \bigtimes_{i \in N} [\kappa_i]$ with $c \neq c^{(x,\theta)}$ for all $x \in F(\theta)$, there exist $i \neq j$ in N and $x_i, x_j \in F(\theta)$ with $x_i \neq x_j$ such that $c_{-i} = c_{-i}^{(x_i,\theta)}$ and $c_{-j} = c_{-j}^{(x_j,\theta)}$.

Theorem 6. *Assume there exist $\kappa \in \mathbb{N}^{|N|}$ and a family Λ that is κ-consistent and strongly compatible with F. Then F is List-Nash implementable.*

Proof. Fix $\kappa \in \mathbb{N}^{|N|}$ and a family $\Lambda = \{\lambda_i^{(x,\theta)} : [\kappa_i] \to X \mid i \in N,\ x \in F(\theta),\ \theta \in \Theta\}$ that is κ-consistent and strongly compatible with F. For each $i \in N$ choose a finite message set M_i with $|M_i| = \kappa_i$ and a bijection $\pi_i : [\kappa_i] \xrightarrow{\sim} M_i$. For every (x, θ) with $x \in F(\theta)$, let $c^{(x,\theta)} \in \bigtimes_{i \in N} [\kappa_i]$ be given by compatibility (i), and set $m^{(x,\theta)} = (\pi_i(c_i^{(x,\theta)}))_{i \in N} \in M := \bigtimes_{i \in N} M_i$.

For each such (x, θ) and every $i \in N$, define the i-slice $A_i^{(x,\theta)} = \{ (\pi_i(r),\ m_{-i}^{(x,\theta)}) : r \in [\kappa_i] \} \subseteq M$, and let $A^{(x,\theta)} = \bigcup_{i \in N} A_i^{(x,\theta)}$. Define a partial outcome map $g^{*(x,\theta)} : A^{(x,\theta)} \to X$ by $g^{*(x,\theta)}(\pi_i(r),\ m_{-i}^{(x,\theta)}) = \lambda_i^{(x,\theta)}(r)$. This is well-defined on $A^{(x,\theta)}$: if a profile m lies in $A_i^{(x,\theta)} \cap A_j^{(x,\theta)}$ with $i \neq j$, then necessarily $m = (\pi_i(c_i^{(x,\theta)}), m_{-i}^{(x,\theta)}) = (\pi_j(c_j^{(x,\theta)}), m_{-j}^{(x,\theta)}) = m^{(x,\theta)}$, and compatibility (i) gives $\lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = \lambda_j^{(x,\theta)}(c_j^{(x,\theta)}) = x$, so both prescriptions agree.

Prescriptions also agree across distinct pairs (x,θ) and (y,θ'). Indeed, if $m \in A^{(x,\theta)} \cap A^{(y,\theta')}$, then $m = (\pi_i(r), m_{-i}^{(x,\theta)}) = (\pi_j(r'), m_{-j}^{(y,\theta')})$ for some $i,j \in N$ and $r \in [\kappa_i], r' \in [\kappa_j]$. If $i = j$, then $m_{-i}^{(x,\theta)} = m_{-i}^{(y,\theta')}$, whence $d_H(c^{(x,\theta)}, c^{(y,\theta')}) = \{i\}$ and compatibility (ii) yields $\lambda_i^{(x,\theta)} = \lambda_i^{(y,\theta')}$ as functions on $[\kappa_i]$, so $g^{*(x,\theta)}(m) = g^{*(y,\theta')}(m)$. If $i \neq j$, then necessarily $r = c_i^{(y,\theta')}$, $r' = c_j^{(x,\theta)}$, and $c_k^{(x,\theta)} = c_k^{(y,\theta')}$ for all $k \notin \{i,j\}$; compatibility (iii) gives $\lambda_i^{(x,\theta)}(c_i^{(y,\theta')}) = \lambda_j^{(y,\theta')}(c_j^{(x,\theta)})$, hence again $g^{*(x,\theta)}(m) = g^{*(y,\theta')}(m)$. It follows that the union prescription $g^* : \bigcup_{x,\theta} A^{(x,\theta)} \to X$, defined by $g^*(m) = g^{*(x,\theta)}(m)$ whenever $m \in A^{(x,\theta)}$, is well-defined.

I now show that $\bigcup_{x,\theta} A^{(x,\theta)} = M$. Fix $\theta \in \Theta$ and an arbitrary $m = (\pi_i(c_i))_{i\in N} \in M$, writing $c = (c_i)_{i\in N} \in \bigtimes_{i\in N}[\kappa_i]$. If there exists $x \in F(\theta)$ with $c = c^{(x,\theta)}$, then $m \in A_i^{(x,\theta)}$ for every i. Otherwise, by strong compatibility there exist indices $i \neq j$ and $x_i, x_j \in F(\theta)$ with $x_i \neq x_j$ such that $c_{-i} = c_{-i}^{(x_i,\theta)}$ and $c_{-j} = c_{-j}^{(x_j,\theta)}$, hence $m \in A_i^{(x_i,\theta)} \cap A_j^{(x_j,\theta)}$. Since θ was arbitrary, this proves that the union of the slices is all of M. Define $g : M \to X$ by $g = g^*$. Set the mechanism $\mu = \langle (M_i, \pi_i)_{i\in N}, g\rangle$; this mechanism is complete.

Fix $\theta \in \Theta$. First, $F(\theta) \subseteq \text{LNE}_g(\mu,\theta)$. Let $x \in F(\theta)$ and consider the central profile $m^{(x,\theta)}$. For each $i \in N$, along the i-slice through $m^{(x,\theta)}$ the outcome function g coincides with $g^{*(x,\theta)}$, hence the opportunity list against $m_{-i}^{(x,\theta)}$ is $L_i(m_{-i}^{(x,\theta)}) = (g(\pi_i(r), m_{-i}^{(x,\theta)}))_{r=1}^{\kappa_i} = (\lambda_i^{(x,\theta)}(r))_{r=1}^{\kappa_i}$. By κ-consistency (i), $C_i^\theta(L_i(m_{-i}^{(x,\theta)})) = x$ for all i. At the central point, compatibility (i) gives $g(m^{(x,\theta)}) = \lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = x$ for every i. Thus the equalities $g(m^{(x,\theta)}) = C_i^\theta(L_i(m_{-i}^{(x,\theta)}))$ hold for all i, so $m^{(x,\theta)} \in \text{LNE}(\mu,\theta)$ and $x \in \text{LNE}_g(\mu,\theta)$.

Second, $\text{LNE}_g(\mu,\theta) \subseteq F(\theta)$. Let $m = (\pi_i(c_i))_{i\in N} \in \text{LNE}(\mu,\theta)$ and write $c = (c_i)_{i\in N}$. If $c = c^{(x,\theta)}$ for some $x \in F(\theta)$, then, as above, $g(m) = x \in F(\theta)$. Otherwise, strong compatibility indices $i \neq j$ and $x_i, x_j \in F(\theta)$ with $x_i \neq x_j$ and $c_{-i} = c_{-i}^{(x_i,\theta)}$, $c_{-j} = c_{-j}^{(x_j,\theta)}$. Hence $L_i(m_{-i}) = (\lambda_i^{(x_i,\theta)}(r))_{r=1}^{\kappa_i}$ and $L_j(m_{-j}) = (\lambda_j^{(x_j,\theta)}(r))_{r=1}^{\kappa_j}$ because g coincides with the corresponding prescriptions on the relevant slices. By κ-consistency (i), it follows that $C_i^\theta(L_i(m_{-i})) = \lambda_i^{(x_i,\theta)}(c_i^{(x_i,\theta)}) = x_i$ and

$C_j^{\theta}(L_j(m_{-j})) = \lambda_j^{(x_j,\theta)}(c_j^{(x_j,\theta)}) = x_j$. Since $x_i \neq x_j$, these outputs differ, contradicting the equilibrium equalities $g(m) = C_i^{\theta}(L_i(m_{-i}))$ for all i. Therefore every equilibrium profile at θ must be central, i.e., $m = m^{(x,\theta)}$ for some $x \in F(\theta)$, and at such a profile $g(m) = x \in F(\theta)$.

Combining the two inclusions, $\text{LNE}_g(\mu, \theta) = F(\theta)$ for every $\theta \in \Theta$. Hence F is fully implementable in List-Nash equilibrium. $\square$

4.5 Why the Canonical Mechanism Does Not Work in the List-Based Framework?

A classic approach in implementation theory is to implement a social choice rule via a canonical mechanism (i.e., a Maskin-Moore-Repullo type mechanism or a modulo mechanism) or one of its variants. These mechanisms provide a "cookbook": a systematic recipe for specifying an outcome function that fills the outcome table so that, under standard conditions (e.g., Maskin monotonicity together with no-veto power, or nearby relaxations), the prespecified equilibrium concept coincides with the designer's target rule F. For instance, de Clippel (2014, Prop. 2.b) shows that in behavioral environments with more than three players, if a social choice function respects unanimity and there exists a collection of opportunity sets that is strongly consistent with it, then the function is Nash implementable.[11] The proof proceeds by using the canonical mechanism, in which the message space takes the product form $\mathcal{M}_i = \mathcal{X} \times \Theta \times \mathbb{Z}_+$, and, leveraging the premise, constructs a well-defined outcome map that implements the target social choice function.

However, lists do not behave as nicely as sets. Consequently, one must specify each player's opportunity list exactly, including its length and the precise ordering of its elements, since even a slight permutation or the insertion of a single alternative may change a player's choice. I deliberately impose no additional

[11]See de Clippel (2014) for details.

consistency axioms on individual choice that would neutralize such sensitivity. This shift from sets to lists entails two further design requirements. First, a compatibility requirement, that is, when the state-contingent lists are embedded into the outcome table, all intersections must agree so that the induced outcome function is well-defined. In set-based canonical constructions, one can often pin an outcome to a cell by appealing to a property of a set (or to a canonical representative of that set); here, because choices depend on order, one cannot simply pick an arbitrary representative. Intersecting slices must literally yield the same outcome at their common cell. Second, a completion requirement that is, if the prescribed lists do not cover the entire message grid, the remaining cells cannot be filled arbitrarily, since ad hoc completions may generate spurious equilibria with outcomes outside F.

Ensuring both well-definedness and a safe completion of the partial outcome map motivates strengthening the compatibility requirement to strong compatibility. A further virtue of the construction is its uniformity in the number of agents: it applies for any N, avoiding the usual split between $|N| \geq 3$ (where canonical mechanism operate) and two-player environments. Moreover, I impose no structural restrictions on F such as no-veto power or unanimity. Within the list-based framework, and under the sufficiency conditions stated above, the designer can fully implement any target social choice rule.

CHAPTER 5

ON LIBERTARIAN PATERNALISM

Within the benchmark implementation framework, the designer cannot dictate agents' actions directly and so must construct a game form whose equilibrium correspondence coincides with the prescribed allocation rule. This approach embodies an implicit paternalism, assuming that any welfare enhancement (as if) legitimizes arbitrary degree of mechanism complexity. The common approach is therefore "anything is permissible, as long as it implements," which may yield mechanisms that while possible in theory, collapse under practical scrutiny. Pathological features include restricting a player's menu to a strict subset of the grand set of alternatives, soliciting messages from only a privileged subset of players, or inflating the message space to exponential proportions. A quintessential example is the canonical mechanism described in Maskin (1999) or Moore and Repullo (1990), whose message space cardinality is on the order of $|X| \times |\Theta| \times |\mathbb{N}| \times |N|$, is often criticized for being intractable in practical applications. Motivated by these observations, I introduce the Libertarian Paternalist mechanisms following Thaler and Sunstein (2009), in which each player is presented with a permutation of the grand set of alternatives at each opportunity list.

Definition 15. Let $\mu = \langle (M_i, \pi_i)_{i \in N}, g \rangle \in \mathcal{M}$ be a mechanism where $g : M \to X$. Then μ is said to be a Libertarian Paternalist mechanism (LP mechanism) if $|M_i| = |X|$ and $O_i^{\mu}(m_{-i}) = X$ for all $i \in N$ and $m_{-i} \in M_{-i}$.

Under a Libertarian Paternalist mechanism, agents retain a formal influence over outcomes and a freedom of choice, thereby preserving a sense of liberty. This influence is symbolic, and the freedom is, of course, nothing but a mere illusion. The designer knows the choices of the players and provides such liberty only on the understanding that any player who is able to deviate, does not wish to do so. In environments with rational players or even with behavioral players having set-based choice behavior, Libertarian Paternalist mechanisms may prove to be ineffective. Because each player receives the grand set of alternative at every best-response stage, they default to their ex ante optimum. Consequently, implementation succeeds only if the social choice rule's outcome coincides with each player's best-choice alternative, a requirement that is both overly restrictive and, paradoxically, too permissive. By contrast, list-based framework endows the designer with genuine flexibility by acknowledging players' choices may depend on the order in which alternatives are presented. Exploiting order-dependent best-response correspondences, the designer can provide paternalistic guidance with players' libertarian autonomy preserved.

In this chapter I examine the implementation problem under the constraint of Libertarian Paternalist mechanisms, and provide a complete characterization of List-Nash implementation in environments with an arbitrary number of players.

5.1 Necessary Conditions of List-Nash Implementation via Libertarian Paternalist Mechanisms

In the Libertarian Paternalist mechanisms every opportunity list has cardinality $|X|$ and enumerates X. Hence, for each player i and each fixed opponents' message m_{-i}, one has $L_i\left(m_{-i}\right)$ a permutation of X. This is the substantive departure from the earlier observations, where the lengths κ_i were arbitrary and surjectivity onto X was not imposed. Consequently the column maps

$\lambda_i^{(x,\theta)} : [|X|] \xrightarrow{\sim} X$ must be bijections, which motivates the adjective latin. [12]

Definition 16. Let $\tilde{\Lambda} = \left\{ \lambda_i^{(x,\theta)} : [|X|] \xrightarrow{\sim} X \mid i \in N, x \in F(\theta), \theta \in \Theta \right\}$ be a family of bijections. Then $\tilde{\Lambda}$ is said to be a latin consistent with the social choice rule F if

(i) for every $i \in N, \theta \in \Theta$ and $x \in F(\theta)$, then $C_i^{\theta}((\lambda_i^{(x,\theta)}(r))_{r=1}^{|X|}) = x$,

(ii) for every $\theta, \theta' \in \Theta$ and $x \in F(\theta)$, if $C_i^{\theta'}((\lambda_i^{(x,\theta)}(r))_{r=1}^{|X|}) = x$ for all $i \in N$, then $x \in F(\theta')$.

Definition 17. Let $\tilde{\Lambda} = \left\{ \lambda_i^{(x,\theta)} : [|X|] \xrightarrow{\sim} X \mid i \in N, x \in F(\theta), \theta \in \Theta \right\}$ be a family of bijections. Then $\tilde{\Lambda}$ is said to be latin compatible with F if

(i) For every $\theta \in \Theta$ and $x \in F(\theta)$ there exists a vector $c^{(x,\theta)} \in [|X|]^{|N|}$ satisfying $\lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = x$ for all $i \in N$, and $c^{(x,\theta)} = c^{(y,\theta')}$ implies $x = y$,

(ii) If $d_H(c^{(x,\theta)}, c^{(y,\theta')}) = \{i\}$ then $\lambda_i^{(x,\theta)} = \lambda_i^{(y,\theta')}$,

(iii) If $d_H(c^{(x,\theta)}, c^{(y,\theta')}) = \{i, j\}$ with $i \neq j$ then $\lambda_i^{(x,\theta)}(c_i^{(y,\theta')}) = \lambda_j^{(y,\theta')}(c_j^{(x,\theta)})$.

Latin consistency strengthens κ-consistency by enforcing the permutation requirement and latin compatibility strengthens the earlier compatibility by imposing agreement constraints at all one– and two–coordinate intersections of centered columns, now within the class of permutations.

Theorem 7. *If F is List–Nash implementable via a Libertarian Paternalist mechanism then there exists a family $\tilde{\Lambda}$ that is latin consistent and latin compatible with F.*

Proof. Let $\mu = \langle (M_i, \pi_i)_{i \in N}, g \rangle$ be a Libertarian Paternalist mechanism that implements F in List–Nash equilibrium. Thus $|M_i| = |X|$ for every i and $O_i^{\mu}(m_{-i}) = X$ for every i and every $m_{-i} \in M_{-i}$. Fix $\theta \in \Theta$ and $x \in F(\theta)$. Choose a List–Nash

[12] The term 'Latin' comes from combinatorics, since Libertarian Mechanisms are closely related to Latin squares.

profile $m^{(x,\theta)} = (m_i^{(x,\theta)})_{i\in N} \in M$ with $g(m^{(x,\theta)}) = x$. Define $c_i^{(x,\theta)} := \pi_i^{-1}(m_i^{(x,\theta)}) \in [|X|]$ and, for each $r \in [|X|]$, set $\lambda_i^{(x,\theta)}(r) := g\big(\pi_i(r),\, m_{-i}^{(x,\theta)}\big)$.

For each i, the opportunity list against $m_{-i}^{(x,\theta)}$ equals $\big(\lambda_i^{(x,\theta)}(1), \ldots, \lambda_i^{(x,\theta)}(|X|)\big)$. Since $O_i^{\mu}(m_{-i}^{(x,\theta)}) = X$ and the list has length $|X|$, every element of X appears at least once, and the pigeonhole principle forces each element of X to appear exactly once. Hence $\lambda_i^{(x,\theta)} : [|X|] \xrightarrow{\sim} X$. The remainder of the proof is similar to that of Theorem 5, but I include it for completeness.

First, I show that latin consistency holds. At $m^{(x,\theta)}$, the List-Nash equalities give $g(m_i^{(x,\theta)}, m_{-i}^{(x,\theta)}) = C_i^{\theta}\big(L_i(m_{-i}^{(x,\theta)})\big)$ for every i. The left side equals x and $L_i(m_{-i}^{(x,\theta)}) = (\lambda_i^{(x,\theta)}(r))_{r=1}^{|X|}$, so $C_i^{\theta}\big((\lambda_i^{(x,\theta)}(r))_{r=1}^{|X|}\big) = x$ for all i. If $\theta' \in \Theta$ satisfies $C_i^{\theta'}\big((\lambda_i^{(x,\theta)}(r))_{r=1}^{|X|}\big) = x$ for every i, then $g(m_i^{(x,\theta)}, m_{-i}^{(x,\theta)}) = C_i^{\theta'}(L_i(m_{-i}^{(x,\theta)}))$ for every i, hence $m^{(x,\theta)} \in \mathrm{LNE}\big(\mu, \theta'\big)$ and therefore $x = g(m^{(x,\theta)}) \in \mathrm{LNE}_g(\mu, \theta') = F(\theta')$.

Now, I show that latin compatibility holds. First, for each (x, θ) and each i, $\lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = g\big(\pi_i(c_i^{(x,\theta)}), m_{-i}^{(x,\theta)}\big) = g(m^{(x,\theta)}) = x$. If $c^{(x,\theta)} = c^{(y,\theta')}$, then $m^{(x,\theta)} = m^{(y,\theta')}$, so $x = g(m^{(x,\theta)}) = g(m^{(y,\theta')}) = y$. If $d_H\big(c^{(x,\theta)}, c^{(y,\theta')}\big) = \{i\}$, then $m_{-i}^{(x,\theta)} = m_{-i}^{(y,\theta')}$ and therefore for every $r \in [|X|]$ one has $\lambda_i^{(x,\theta)}(r) = g\big(\pi_i(r), m_{-i}^{(x,\theta)}\big) = g\big(\pi_i(r), m_{-i}^{(y,\theta')}\big) = \lambda_i^{(y,\theta')}(r)$. If $d_H\big(c^{(x,\theta)}, c^{(y,\theta')}\big) = \{i, j\}$ with $i \neq j$, then the profile $\big(\pi_i(c_i^{(y,\theta')}),\, m_{-i}^{(x,\theta)}\big)$ coincides with $\big(\pi_j(c_j^{(x,\theta)}),\, m_{-j}^{(y,\theta')}\big)$ because all other coordinates match. Single valuedness of g yields $\lambda_i^{(x,\theta)}\big(c_i^{(y,\theta')}\big) = g\big(\pi_i(c_i^{(y,\theta')}), m_{-i}^{(x,\theta)}\big) = g\big(\pi_j(c_j^{(x,\theta)}), m_{-j}^{(y,\theta')}\big) = \lambda_j^{(y,\theta')}\big(c_j^{(x,\theta)}\big)$.

Define $\tilde{\Lambda} = \{\lambda_i^{(x,\theta)} : [|X|] \xrightarrow{\sim} X \mid i \in N,\ x \in F(\theta),\ \theta \in \Theta\}$. The preceding paragraphs show that $\tilde{\Lambda}$ is latin consistent and latin compatible with F. $\square$

5.2 Sufficient Conditions of List-Nash Implementation via Libertarian Paternalist Mechanisms

The prescriptions encoded by $\tilde{\Lambda}$ determine only a partial outcome map restricted to the centered columns. A completion to a total Libertarian Paternalist mechanism is not guaranteed, hence the LP extension hypothesis, which posits a completion that preserves the column structure of opportunity lists everywhere. Even with such a completion, profiles outside the centered columns can generate off-path equilibria that are artifacts of the completion rather than of Λ. Unanimity is imposed precisely to rule these out, since in any LP mechanism an equilibrium entails common selection of a single outcome from bijective columns at the realized state, and unanimity forces any such consensus to lie in $F(\theta)$.

Definition 18. Let N and X be finite nonempty sets. An incomplete mechanism $\mu^* = \langle (M_i)_{i\in N}, (\pi_i)_{i\in N}, g^* \rangle \in \mathcal{M}^*$ with partial outcome map $g^* : A \to X$ and nonempty domain $A \subseteq M$ is a Libertarian Paternalist incomplete mechanism if and only if the following hold.

(i) For every $i \in N, |M_i| = |X|$ and $\pi_i : [|X|] \xrightarrow{\sim} M_i$.

(ii) There exists a nonempty set of centers $C \subseteq \bigtimes_{i\in N}[|X|]$ such that a profile $m \in M$ lies in A if and only if there exist $c \in C$ and $i \in N$ with $m_{-i} = (\pi_j(c_j))_{j\in N\setminus\{i\}}$. Equivalently, $A = \bigcup_{c\in C}\bigcup_{i\in N}\{(\pi_i(r), (\pi_j(c_j))_{j\neq i}) : r \in [|X|]\}$.

(iii) For every $c \in C$ and $i \in N$, the full column through c is in A and $r \mapsto g^*\left(\pi_i(r), (\pi_j(c_j))_{j\neq i}\right)$ is a bijection onto X; for any $i \in N$ and $m_{-i} \in M_{-i}$, the map $r \mapsto g^*(\pi_i(r), m_{-i})$ is injective on $\{r \in [|X|] : (\pi_i(r), m_{-i}) \in A\}$.

Proposition 3. *Let $\tilde{\Lambda} = \{\lambda_i^{(x,\theta)} : [|X|] \xrightarrow{\sim} X \mid i \in N,\ x \in F(\theta),\ \theta \in \Theta\}$ be latin compatible with F. Then $\tilde{\Lambda}$ induces a (possibly complete) Libertarian Paternalist incomplete mechanism.*

Proof. Fix a family $\tilde{\Lambda} = \{\lambda_i^{(x,\theta)} : [|X|] \xrightarrow{\sim} X \mid i \in N,\ x \in F(\theta),\ \theta \in \Theta\}$ that is latin compatible with F. For each $i \in N$ choose a finite message set M_i with $|M_i| = |X|$ and a bijection $\pi_i : [|X|] \xrightarrow{\sim} M_i$. For every pair (x, θ) with $x \in F(\theta)$, let $c^{(x,\theta)} = (c_i^{(x,\theta)})_{i \in N} \in \bigtimes_{i \in N}[|X|]$ be the central index vector given by latin compatibility item (i), so that $\lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = x$ for all i, and if $c^{(x,\theta)} = c^{(y,\theta')}$ then $x = y$.

For each such (x, θ) and each $i \in N$, define the slice $A_i^{(x,\theta)} = \{(\pi_i(r), (\pi_j(c_j^{(x,\theta)}))_{j \neq i}) : r \in [|X|]\} \subseteq M$ and let $A^{(x,\theta)} = \bigcup_{i \in N} A_i^{(x,\theta)}$. Define a partial outcome map $g^{*(x,\theta)} : A^{(x,\theta)} \to X$ by the rule $g^{*(x,\theta)}(\pi_i(r), (\pi_j(c_j^{(x,\theta)}))_{j \neq i}) = \lambda_i^{(x,\theta)}(r)$. I first show that $g^{*(x,\theta)}$ is well defined on $A^{(x,\theta)}$. If a profile m belongs to both $A_i^{(x,\theta)}$ and $A_j^{(x,\theta)}$ with $i = j$, there is nothing to prove. If $i \neq j$, then necessarily $m = (\pi_i(c_i^{(x,\theta)}), (\pi_k(c_k^{(x,\theta)}))_{k \neq i}) = (\pi_j(c_j^{(x,\theta)}), (\pi_k(c_k^{(x,\theta)}))_{k \neq j})$, which is the center profile determined by $c^{(x,\theta)}$. Latin compatibility item (i) gives $\lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = \lambda_j^{(x,\theta)}(c_j^{(x,\theta)}) = x$, so both prescriptions coincide at m.

Now, set $A = \bigcup_{x \in F(\theta),\ \theta \in \Theta} A^{(x,\theta)}$. If a profile m belongs to both $A^{(x,\theta)}$ and $A^{(y,\theta')}$, then there exist $i, j \in N$ and $r \in [|X|]$, $r' \in [|X|]$ such that $m = (\pi_i(r), (\pi_k(c_k^{(x,\theta)}))_{k \neq i}) = (\pi_j(r'), (\pi_k(c_k^{(y,\theta')}))_{k \neq j})$. If $i = j$, then $(\pi_k(c_k^{(x,\theta)}))_{k \neq i} = (\pi_k(c_k^{(y,\theta')}))_{k \neq i}$, hence $c^{(x,\theta)}$ and $c^{(y,\theta')}$ differ in at most the i-th coordinate. Latin compatibility item (ii) yields $\lambda_i^{(x,\theta)} = \lambda_i^{(y,\theta')}$, so $g^{*(x,\theta)}(m) = g^{*(y,\theta')}(m)$. If $i \neq j$, then equality of m forces $r = c_i^{(y,\theta')}$, $r' = c_j^{(x,\theta)}$, and $c_k^{(x,\theta)} = c_k^{(y,\theta')}$ for all $k \notin \{i, j\}$. Latin compatibility item (iii) gives $\lambda_i^{(x,\theta)}(c_i^{(y,\theta')}) = \lambda_j^{(y,\theta')}(c_j^{(x,\theta)})$, hence again $g^{*(x,\theta)}(m) = g^{*(y,\theta')}(m)$. Therefore there exists a well defined map $g^* : A \to X$ given by $g^*(m) = g^{*(x,\theta)}(m)$ whenever $m \in A^{(x,\theta)}$.

I claim that $\mu^* = \langle (M_i)_{i \in N}, (\pi_i)_{i \in N}, g^* \rangle$ is a Libertarian Paternalist incomplete mechanism. Property (i) in the definition holds by construction since $|M_i| = |X|$ and π_i is bijective. For property (ii), define the set of centers $C = \{c^{(x,\theta)} : x \in F(\theta),\ \theta \in \Theta\} \subseteq \bigtimes_{i \in N}[|X|]$. Then $A = \bigcup_{c \in C} \bigcup_{i \in N} \{(\pi_i(r), (\pi_j(c_j))_{j \neq i}) : r \in [|X|]\}$ by construction, so a profile lies in A if and only if it lies on a full column through some center vector c. For property (iii), fix $c \in C$ and $i \in N$. The entire column $\{(\pi_i(r), (\pi_j(c_j))_{j\neq i}) : r \in [|X|]\}$ is a subset of A by definition. Along this column the map $r \mapsto g^*(\pi_i(r), (\pi_j(c_j))_{j\neq i})$ equals $r \mapsto \lambda_i^{(x,\theta)}(r)$ for the unique pair (x, θ) with $c^{(x,\theta)} = c$. Since $\lambda_i^{(x,\theta)}$ is a bijection $[|X|] \xrightarrow{\sim} X$, this map is a bijection onto X. Finally, for arbitrary $i \in N$ and $m_{-i} \in M_{-i}$, the set $\{r \in [|X|] : (\pi_i(r), m_{-i}) \in A\}$ is either empty or all of $[|X|]$. In the latter case $m_{-i} = (\pi_j(c_j))_{j\neq i}$ for some $c \in C$ and the map $r \mapsto g^*(\pi_i(r), m_{-i})$ is the bijection $r \mapsto \lambda_i^{(x,\theta)}(r)$ for the corresponding (x, θ). In either case the injectivity requirement in property (iii) holds.

Thus $\tilde{\Lambda}$ induces the Libertarian Paternalist incomplete mechanism μ^*. If the union of the columns covers M, the mechanism is complete; otherwise it is incomplete. □

Definition 19. Let $\mu^* = \langle (M_i)_{i\in N}, (\pi_i)_{i\in N}, g^*\rangle \in \mathcal{M}^*$ be a Libertarian Paternalist incomplete mechanism with domain $A \subseteq M$. A complete mechanism $\mu = \langle (M_i)_{i\in N}, (\pi_i)_{i\in N}, g\rangle \in \mathcal{M}$ is an LP–extension of μ^* if and only if:

(i) μ is Libertarian Paternalist mechanism,

(ii) g extends g^*, that is $g(m) = g^*(m)$ for every $m \in A$.

μ^* is said to be extendable to a Libertarian Paternalist mechanism when such μ exists.

Definition 20. Let $\tilde{\Lambda}$ be latin compatible with F, and let $c^{(x,\theta)} \in [|X|]^{|N|}$ be the central indices from latin compatibility. Now, $\tilde{\Lambda}$ is said to be extendable to a Libertarian Paternalist mechanism if there exist finite message sets M_i with $|M_i| = |X|$, bijections $\pi_i : [|X|] \xrightarrow{\sim} M_i$, and a total map $g : M \to X$ such that:

(i) For every $i \in N$ and every $m_{-i} \in M_{-i}$, the map $r \mapsto g(\pi_i(r), m_{-i})$ is a bijection $[|X|] \xrightarrow{\sim} X$. Equivalently $O_i^{\mu}(m_{-i}) = X$ for all i and all m_{-i}.

(ii) For every $\theta \in \Theta$, $x \in F(\theta)$, $i \in N$, and $r \in [|X|]$, $g(\pi_i(r),\, (\pi_j(c_j^{(x,\theta)}))_{j\neq i}) = \lambda_i^{(x,\theta)}(r)$.

In this case the complete mechanism μ is a Libertarian Paternalist mechanism that extends the centered column prescriptions of $\tilde{\Lambda}$.

The next definition introduces unanimity for the list-based framework. At any state, if each player can be given a complete list of all alternatives, possibly in different orders, from which all select the same alternative, then the rule must include that alternative at that state, aligning classical unanimity with revealed choice from lists and excluding equilibria that contradict unanimity.

Definition 21. A social choice rule F satisfies unanimity if for every $\theta \in \Theta$ and $x \in X$, whenever there exist bijections $\sigma_i : [|X|] \xrightarrow{\sim} X$ for all $i \in N$ and $c \in [|X|]^{|N|}$ such that $\forall i \in N : C_i^\theta((\sigma_i(r))_{r=1}^{|X|}) = x$ and $\sigma_i(c_i) = x$, then $x \in F(\theta)$.

Theorem 8. *If there exists a family $\tilde{\Lambda}$ that is latin consistent and latin compatible with F which can be extended to a Libertarian Paternalist mechanism, and F satisfies unanimity, then F is List-Nash implementable via a Libertarian Paternalist mechanism.*

Proof. Fix $\theta \in \Theta$. Define $\mu = \langle (M_i, \pi_i)_{i\in N}, g\rangle$. Since $|M_i| = |X|$ for all i and $r \mapsto g(\pi_i(r), m_{-i})$ is a bijection $[|X|] \xrightarrow{\sim} X$ for all m_{-i}, the mechanism is Libertarian Paternalist and, for every i and m_{-i}, the opportunity set $O_i^\mu(m_{-i})$ equals X.

Let $x \in F(\theta)$ and set $c^{(x,\theta)} = (c_i^{(x,\theta)})_{i\in N} \in [|X|]^{|N|}$ as in latin compatibility. Define $m^{(x,\theta)} = (\pi_i(c_i^{(x,\theta)}))_{i\in N} \in M$. For each $i \in N$ one has $L_i(m_{-i}^{(x,\theta)}) = (g(\pi_i(r), m_{-i}^{(x,\theta)}))_{r=1}^{|X|} = (\lambda_i^{(x,\theta)}(r))_{r=1}^{|X|}$ by the extension property. Latin consistency yields $C_i^\theta(L_i(m_{-i}^{(x,\theta)})) = x$ for all i. At the center, $g(m^{(x,\theta)}) = \lambda_i^{(x,\theta)}(c_i^{(x,\theta)}) = x$ for every i by latin compatibility. Hence for all i one has $g(m^{(x,\theta)}) = C_i^\theta(L_i(m_{-i}^{(x,\theta)}))$. Therefore $m^{(x,\theta)} \in \text{LNE}(G^\mu(C^\theta))$ and $x = g(m^{(x,\theta)}) \in \text{LNE}_g(\mu, \theta)$. This proves $F(\theta) \subseteq \text{LNE}_g(\mu, \theta)$.

Let $m = (m_i)_{i\in N} \in \text{LNE}(G^\mu(C^\theta))$ and set $x := g(m)$. For each $i \in N$ define $\sigma_i : [|X|] \to X$ by $\sigma_i(r) := g(\pi_i(r), m_{-i})$. By the extension property each σ_i is a bijection $[|X|] \xrightarrow{\sim} X$. Since σ_i is bijective, there exists a unique $c_i \in [|X|]$ such that $\sigma_i(c_i) = x$. Note that $\pi_i(c_i) = m_i$ because $x = g(m) = g(\pi_i(c_i), m_{-i})$ and the column map is injective. The List-Nash equalities give, for every $i \in N$, that $C_i^\theta(L_i(m_{-i})) = g(m) = x$. Using $L_i(m_{-i}) = (\sigma_i(r))_{r=1}^{|X|}$ for each i, it follows that $C_i^\theta((\sigma_i(r))_{r=1}^{|X|}) = x$ and $\sigma_i(c_i) = x$ for all i. By unanimity, $x \in F(\theta)$. Therefore $g(m) \in F(\theta)$ for every $m \in$ LNE($G^\mu(C^\theta)$), that is LNE$_g(\mu, \theta) \subseteq F(\theta)$.

Combining the inclusions yields LNE$_g(\mu, \theta) = F(\theta)$. Since $\theta \in \Theta$ was arbitrary, LNE$_g(\mu, \theta) = F(\theta)$ holds for all $\theta \in \Theta$. □

CHAPTER 6

EFFICIENCY

Implementation theory conceptualizes the mechanism designer as a (benevolent) principal who designs a game form that implements a prespecified social choice rule. The literature has concentrated almost exclusively on exploring which social choice rules admit implementation under a predetermined equilibrium notion, and has assessed efficiency by imposing normative criteria directly on social choice rules over specified domains and analyzed whether such rules are implementable. On the other hand, however, the equally salient question of how to structure the mechanism itself has remained largely unexplored. That is, the ontological question "Can a mechanism implement this social choice rule?" has overshadowed the procedural question "How should a mechanism implement a social choice rule?" Chapter 5 engaged both of these questions within our list-based framework and, in particular, examined whether a Libertarian Paternalist mechanism, a mechanism that provides a sense of liberty to each player, can implement the target rule.

In this chapter I pass from feasibility to selection. Once the necessary and sufficient conditions developed in the preceding chapter are satisfied, the implementability correspondence associates to each target social choice rule a nonempty set of implementing mechanisms. The designer's problem is no longer whether implementation is attainable but which member of this set ought to be used in practice. I address this selection problem by advancing an axiomatic program that defines efficiency criteria directly on mechanisms. The contribution is simple in structure yet novel in focus, offering an alternative perspective on efficiency within mechanism design.

For the sake of completeness, I conclude by revisiting efficiency at the level of social choice rules and by situating the efficiency notions used in the behavioral implementation literature within this framework.

6.1 Efficiency of Mechanisms

Let $F \in \mathcal{F}$ be a social choice rule, and define $\mathcal{M}_F := \{\mu \in \mathcal{M} \mid \text{for every } \theta \in \Theta,\ \text{LNE}_g(\mu, \theta) = F(\theta)\}$. For $\mu \in \mathcal{M}_F$, write M_i^μ for player i's message set in μ. The next two definitions formalize the standard notion of mechanism efficiency adopted in this chapter.

Definition 22. For $i \in N$, $\theta \in \Theta$, and $\mu^1, \mu^2 \in \mathcal{M}_F$, say that μ^1 provides at least as much liberty to player i as μ^2 at θ, and write $\mu^1 \succeq^L_{i,\theta} \mu^2$ if for every $m^2 \in$ LNE(μ^2, θ) there exists $m^1 \in$ LNE(μ^1, θ) such that $O_i^{\mu^2}((m^2)_{-i}) \subseteq O_i^{\mu^1}((m^1)_{-i})$. Write $\mu^1 \succ^L_{i,\theta} \mu^2$ if the inclusion is strict for some such m^1, m^2. Define $\succeq^L_i$ and $\succ^L_i$ by requiring the relation for every $\theta \in \Theta$.

This simple reasoning behind this measure is that efficiency of implementation should not come at the expense of opportunities, so a mechanism that never narrows and sometimes expands a player's options is normatively preferable.

Definition 23. For $i \in N$ and $\mu^1, \mu^2 \in \mathcal{M}_F$, say that μ^1 is no more complex for player i than μ^2, and write $\mu^1 \succeq^C_i \mu^2$ if $|M_i^{\mu^1}| \leq |M_i^{\mu^2}|$; write $\mu^1 \succ^C_i \mu^2$ if $|M_i^{\mu^1}| < |M_i^{\mu^2}|$.

Complexity measure, on the other hand, serves as a parsimonious proxy for communication and cognitive burden, favoring mechanisms that achieve the same rule with fewer actions available to the player. The following proposition is simple and intuitive. LP mechanisms grants each player the entire outcome set as attainable at equilibrium, so any other mechanism can only match, never expand, these opportunities, and a smaller message space can only contract them.

Proposition 4. *Let $F \in \mathcal{F}$. If $\mathcal{M}_F$ contains a Libertarian Paternalist mechanism μ^{LP}, then for every player i, every state θ, and every $\mu \in \mathcal{M}_F$ one has $\mu^{LP} \succeq^L_{i,\theta} \mu$. Moreover, if $|M_i^\mu| < |X|$, then for every state θ one has $\mu^{LP} \succ^L_{i,\theta} \mu$; in particular, there is no $\mu \in \mathcal{M}_F$ with $\mu \succ^C_i \mu^{LP}$ and $\mu \succeq^L_i \mu^{LP}$.*

Proof. Pick θ and i. Since $\mu^{LP} \in \mathcal{M}_F$, there exists $m^{LP} \in \mathrm{LNE}(\mu^{LP}, \theta)$ and $O_i^{\mu^{LP}}((m^{LP})_{-i}) = X$. For any $\mu \in \mathcal{M}_F$ and any $m \in \mathrm{LNE}(\mu, \theta)$, one has $O_i^\mu(m_{-i}) \subseteq X$, so $\mu^{LP} \succeq^L_{i,\theta} \mu$. If $|M_i^\mu| < |X|$, then $|O_i^\mu(m_{-i})| \leq |M_i^\mu| < |X|$, hence the inclusion is strict. □

Definition 24. For $\mu^1, \mu^2 \in \mathcal{M}_F$, write $\mu^1 \succeq^L \mu^2$ if $\mu^1 \succeq^L_{i,\theta} \mu^2$ holds for every $i \in N$ and every $\theta \in \Theta$; write $\mu^1 \succ^L \mu^2$ if, in addition, strictness holds for some i, θ. Define $\succeq^C$ and $\succ^C$ analogously via $\succeq^C_i$ and $\succ^C_i$. A mechanism $\mu^\star$ is liberty–complexity efficient if there is no $\tilde{\mu} \in \mathcal{M}_F$ such that $\tilde{\mu} \succeq^L \mu^\star$ and $\tilde{\mu} \succeq^C \mu^\star$ with at least one strict inequality.

Definition 25. For each player i, fix a nonempty $\mathcal{L}_i^{\mathrm{adm}} \subseteq \mathcal{L}$. A mechanism μ is admissible if $L_i^\mu(m_{-i}) \in \mathcal{L}_i^{\mathrm{adm}}$ for every player i and every m_{-i}.

If $\mathcal{L}_i^{\mathrm{adm}}$ contains singletons and choices are inside the displayed support, then any $x \in X$ is witnessed by (x), so $\ell_i(x, \theta) = 1$. To avoid trivial bounds one can restrict $\mathcal{L}_i^{\mathrm{adm}}$ (e.g., to permutations of X, or lists of length at least 2).

Lemma 2. *Fix F. For each player i, alternative x, and state θ, let $\ell_i(x, \theta) := \min\{\, |L| : L \in \mathcal{L}_i^{\mathrm{adm}}$ and $C_i^\theta(L) = x \,\}$. Then every $\mu \in \mathcal{M}_F$ satisfies $|M_i^\mu| \geq \max_{\theta \in \Theta} \max_{x \in F(\theta)} \ell_i(x, \theta)$ for every i.*

Proof. Take $\mu \in \mathcal{M}_F$, i, θ, and $x \in F(\theta)$. Let $m \in \mathrm{LNE}(\mu, \theta)$ with $g(m) = x$. Admissibility gives $L_i^\mu(m_{-i}) \in \mathcal{L}_i^{\mathrm{adm}}$ and $C_i^\theta(L_i^\mu(m_{-i})) = x$. Feasibility of the interface gives $|L_i^\mu(m_{-i})| \leq |M_i^\mu|$, hence the claim. □

The length lower bound asserts that the designer must give a player at least as many messages as the largest, across states and targeted outcomes, minimal length of an admissible list that makes that outcome the player's choice, since the interface cannot display a longer list than the player has messages and equilibrium choices must be justified by some admissible list.

Definition 26. For each player $i \in N$ define the requirement index $\kappa_i^{\text{req}}(F) := \max_{\theta\in\Theta} \max_{x\in F(\theta)} \ell_i(x,\theta) \in \mathbb{N} \cup \{\infty\}$.

The requirement index can be interpreted as the worst case minimal length for a player across all states and targeted outcomes, and it gives a necessary lower bound on that player's message set in any mechanism that implements the rule.

Corollary 3. If F is List–Nash implementable, then $\kappa_i^{\text{req}}(F)$ is finite for every player i, and every $\mu \in \mathcal{M}_F$ satisfies $|M_i^\mu| \geq \kappa_i^{\text{req}}(F)$.

Proof. If F is implementable, there exists $\mu \in \mathcal{M}_F$ with finite $|M_i^\mu|$ for all i. Apply Lemma 2. $\square$

The following theorem further emphasizes the efficiency of LP mechanisms. If an LP mechanism exists, it is liberty maximal; if its message sets are minimal, it is also liberty–complexity efficient, so no alternative can preserve liberty while reducing complexity.

Theorem 9. *Let F be a social choice rule and suppose $\mu^{LP} \in \mathcal{M}_F$ is Libertarian Paternalist. Then μ^{LP} is L-maximal, that is, $\mu^{LP} \succeq^L \mu$ for every $\mu \in \mathcal{M}_F$. If, in addition, $|M_i^{\mu^{LP}}| = \kappa_i^{\text{req}}(F)$ for every player i, then μ^{LP} is liberty–complexity efficient.*

Proof. L-maximality is Proposition 4. For the efficiency part, take $\tilde{\mu} \in \mathcal{M}_F$ with $\tilde{\mu} \succeq^L \mu^{LP}$ and $\tilde{\mu} \succeq^C \mu^{LP}$. Corollary 3 and $|M_i^{\mu^{LP}}| = \kappa_i^{\text{req}}(F)$ force $|M_i^{\tilde{\mu}}| = |M_i^{\mu^{LP}}|$ for all i. By L-maximality, $\mu^{LP} \succeq^L \tilde{\mu}$; hence no strict improvement in either dimension. $\square$

The analysis delves more deeply into the efficiency of mechanisms; to this end, several additional efficiency notions are introduced.

Definition 27. For $\mu \in \mathcal{M}_F$, $\theta \in \Theta$, and $m \in \text{LNE}(\mu,\theta)$, let $v_i(\mu,\theta,m) := |O_i^\mu(m_{-i})|$ and $\Delta(v) := \max_{i\in N} v_i - \min_{i\in N} v_i$. Mechanism μ has balanced liberty if $\Delta(v(\mu,\theta,m)) = 0$ for every θ and every such m. For $\mu^1,\mu^2 \in \mathcal{M}_F$, write $\mu^1 \succeq^{BL} \mu^2$ if for every θ there exist equilibria $m^1 \in \text{LNE}(\mu^1,\theta)$ and $m^2 \in \text{LNE}(\mu^2,\theta)$ with $\Delta(v(\mu^1,\theta,m^1)) \leq \Delta(v(\mu^2,\theta,m^2))$; write $\mu^1 \succ^{BL} \mu^2$ if the inequality is strict for some θ.

One can view balanced liberty as a measure that ties efficiency to equalized opportunity across participants, with lower dispersion in attainable-set sizes marking a fairer and more symmetric allocation of freedom.

Definition 28. Mechanism μ is neutral if for every permutation $\sigma : X \to X$ there exist bijections $\tau_i : M_i^\mu \to M_i^\mu$ (one for each $i \in N$) such that for every $m \in M$, $g((\tau_i(m_i))_{i\in N}) = \sigma(g(m))$ and $A(L_i^\mu(\tau_{-i}(m_{-i}))) = \sigma(A(L_i^\mu(m_{-i})))$.

Definition 29. Mechanism μ is anonymous if for every permutation $\gamma : N \to N$ there exist bijections $\phi_i : M_i^\mu \to M_{\gamma(i)}^\mu$ such that relabeling players by γ and messages by ϕ preserves the induced outcome and opportunity sets up to that relabeling.

Neutrality and anonymity are standard in the literature. Neutrality makes efficiency a judgment of procedure rather than labels, since any outcome relabeling leaves the evaluation unchanged. On the other hand, anonymity makes efficiency independent of identity, since permuting players leaves both behavior and assessment unchanged. The following definition formalizes coalitional liberty, requiring group level robustness by ruling out mechanisms that preserve individual opportunity sets while shrinking a coalition's collective opportunity.

Definition 30. For a coalition $S \subseteq N$ and $\mu^1,\mu^2 \in \mathcal{M}_F$, write $\mu^1 \succeq_S^L \mu^2$ if for every state θ and every $m^2 \in \text{LNE}(\mu^2,\theta)$ there exists $m^1 \in \text{LNE}(\mu^1,\theta)$ such that

$O_i^{\mu^2}((m^2)_{-i}) \subseteq O_i^{\mu^1}((m^1)_{-i})$ for every $i \in S$. Write $\mu^1 \succ_S^L \mu^2$ if some inclusion is strict for some $i \in S$ at some θ.

Corollary 4. Under the assumptions of Proposition 4, $\mu^{LP} \succeq_S^L \mu$ for every coalition $S \subseteq N$ and every $\mu \in \mathcal{M}_F$. If, for some $i \in S$, state θ, and equilibrium $m \in \text{LNE}(\mu, \theta)$, one has $|O_i^\mu(m_{-i})| < |X|$, then $\mu^{LP} \succ_S^L \mu$.

Proof. For any θ, $O_i^{\mu^{LP}}(\cdot) = X$ for all i, so $O_i^\mu(\cdot) \subseteq X$ for all $i \in S$. If some $|O_i^\mu(m_{-i})| < |X|$, the inclusion is strict. □

Corollary 5. Under the assumptions of Proposition 4, $\mu^{LP} \succeq^{BL} \mu$ for every $\mu \in \mathcal{M}_F$. Moreover, if for some θ and $m \in \text{LNE}(\mu, \theta)$ either $\Delta(v(\mu, \theta, m)) > 0$ or $\min_{i \in N} |O_i^\mu(m_{-i})| < |X|$, then $\mu^{LP} \succ^{BL} \mu$.

Proof. At any θ, an equilibrium m^{LP} of μ^{LP} yields all $|O_i^{\mu^{LP}}((m^{LP})_{-i})| = |X|$, so dispersion is 0. Any μ has $|O_i^\mu(m_{-i})| \leq |X|$. Hence weakly larger dispersion; strict whenever one inequality is strict. □

Proposition 5. *If $\mathcal{M}_F$ contains a Libertarian Paternalist mechanism μ^{LP}, then there exists $\tilde{\mu} \in \mathcal{M}_F$ obtained by one-to-one renaming of each player's messages (no outcome relabeling) such that $|\tilde{M}_i| = |X|$ and $O_i^{\tilde{\mu}}(m_{-i}) = X$ for every player i and every m_{-i}. Furthermore:*

(i) If for every permutation σ of X there exist bijections $\{\tau_i^\sigma : \tilde{M}_i \to \tilde{M}_i\}_{i \in N}$ with $\tilde{g}((\tau_i^\sigma(m_i))_{i \in N}) = \sigma(\tilde{g}(m))$ and $A(\tilde{L}_i(\tau_{-i}^\sigma(m_{-i}))) = \sigma(A(\tilde{L}_i(m_{-i})))$, then $\tilde{\mu}$ is neutral.

(ii) If for every permutation γ of players there exist bijections $\{\phi_i^\gamma : \tilde{M}_i \to \tilde{M}_{\gamma(i)}\}_{i \in N}$ with $\tilde{g}((\phi_i^\gamma(m_i))_{i \in N}) = \tilde{g}(m)$ and $O_{\gamma(i)}^{\tilde{\mu}}(\phi_{-i}^\gamma(m_{-i})) = O_i^{\tilde{\mu}}(m_{-i})$ for every i, then $\tilde{\mu}$ is anonymous.

Proof. For each i and fixed m_{-i}, the map $m_i \mapsto g^{\mu^{LP}}(m_i, m_{-i})$ is a bijection from $M_i^{\mu^{LP}}$ to X. Fix a reference $\bar{m}$ and use these bijections to rename messages to X. Now, define $\tilde{g}$ accordingly. Then $|\tilde{M}_i| = |X|$ and $O_i^{\tilde{\mu}}(m_{-i}) = X$ for all i and all m_{-i}. The neutrality/anonymity parts follow directly from the stated renamings. $\square$

Definition 31. A mechanism μ has off-path opportunity invariance if, for every player i and for all opponents' message profiles m_{-i} and m'_{-i}, one has $O_i^{\mu}(m_{-i}) = O_i^{\mu}(m'_{-i})$.

Theorem 10. *Assume F is List–Nash implementable with $F(\theta) \neq \emptyset$ for every state, and $\mathcal{M}_F$ contains a Libertarian Paternalist mechanism. Suppose $\kappa_i^{\mathrm{req}}(F) = |X|$ for every player i. If $\mu \in \mathcal{M}_F$ is L-maximal, message-minimal, and neutral/anonymous, then on the equilibrium path at every state there exist one-to-one renamings of players' messages such that each player's message directly selects the outcome: for each i and each $m_i \in X$, $g^{\mu}(m_i, m_{-i}) = m_i$ and $O_i^{\mu}(m_{-i}) = X$. If, in addition, μ has off-path opportunity invariance, then $O_i^{\mu}(m_{-i}) = X$ for all opponents' messages and every i. Hence μ is Libertarian Paternalist.*

Proof. Fix a state θ and take $m \in \mathrm{LNE}(\mu, \theta)$. By Proposition 4 and L-maximality, each player i can attain X against m_{-i}, and message-minimality forces $|M_i^{\mu}| = |X|$. Hence the map $m_i \mapsto g^{\mu}(m_i, m_{-i})$ is a bijection $M_i^{\mu} \to X$. Rename messages along this bijection so that, on this path, $g^{\mu}(m_i, m_{-i}) = m_i$ and $O_i^{\mu}(m_{-i}) = X$ for every i. Since the state was arbitrary and neutrality/anonymity allow consistent renamings across states and players, this holds on the equilibrium path at every state. If, in addition, off-path opportunity invariance holds, then equality of opportunity sets across all opponents' messages implies $O_i^{\mu}(\cdot) = X$ for every i, so $|M_i^{\mu}| = |X|$ and $O_i^{\mu}(\cdot) = X$, i.e., the mechanism is Libertarian Paternalist. $\square$

Proposition 5 places LP mechanisms in a canonical representation. By a one-to-one relabeling of messages, each player's message set can be identified with X and yields the full opportunity set against every opponents' profile. Under the displayed permutations, neutrality and anonymity then follow as closure under relabelings of outcomes and players.

Theorem 10 gives a converse style characterization. Under implementability, liberty maximality, minimal message sizes, and neutrality or anonymity, any mechanism must, on the equilibrium path at every state, behave as direct selection with full opportunities. Moreover, with off path opportunity invariance this extends to all profiles, hence the mechanism is LP.

6.2 Efficiency of Social Choice Rules

Consider a rational environment, that is, an environment in which players have complete and transitive preference orderings over the set of alternatives. Then, the Pareto social choice rule on Θ, denoted by $F_R^{\text{eff}} : \Theta \to \mathcal{X}$, is defined pointwise by

$$F_R^{\text{eff}}(\theta) := \left\{ x \in X \mid \nexists\, y \in X \text{ such that } \forall i \in N,\ y \succsim_i^\theta x \text{ and } \exists j \in N :\ y \succ_j^\theta x \right\},$$

for every $\theta \in \Theta$. This correspondence regards the social outcome as efficient whenever no alternative can make all players weakly better off, with a strict gain for at least one player, according to the state-contingent preferences $(\succsim_i^\theta)_{i \in N}$. In behavioral environments, that is, players have choice correspondences that do not necessarily obey any consistency assumption, we consider the efficiency rule introduced by de Clippel (2014). When each C_i^θ is a set-based choice function $C_i^\theta : \mathcal{X} \to X$, define $F_B^{\text{eff}} : \Theta \to \mathcal{X}$ by

$$F_B^{\text{eff}}(\theta) := \left\{ x \in X \mid \exists\, (S_i)_{i \in N} \in \mathcal{X}^{|N|} \text{ with } x = C_i^\theta\,(S_i)\, \forall i \in N \text{ and } \bigcup_{i \in N} S_i = X \right\},$$

for every $\theta \in \Theta$. Here efficiency is defined by player-specific menus whose union covers the set of all alternatives. We now adapt behavioral efficiency to our list-based framework.

Definition 32. For $\theta \in \Theta$, the list efficiency correspondence $F_L^{\text{eff}} : \Theta \to \mathcal{X}$ is defined by

$$F_L^{\text{eff}}(\theta) := \left\{ x \in X \mid \exists\, (L_i)_{i\in N} \in \mathcal{L}^{|N|} \text{ with } \bigcap_{i\in N}\{C_i^\theta(L_i)\} = \{x\} \text{ and } \bigcup_{i\in N} A(L_i) = X \right\},$$

at each $\theta \in \Theta$.

Thus $x \in F_L^{\text{eff}}(\theta)$ if and only if there exist player-specific lists such that each player i chooses x from L_i at θ, and the union of supports $\bigcup_{i\in N} A(L_i)$ equals X.

Given a length vector $\kappa \in \mathbb{N}^{|N|}$, a κ constrained variant is obtained by restricting to lists with $|L_i| = \kappa_i$ for all $i \in N$. These constraints capture institutional or technological frictions that limit how many entries a designer can place on each player's list, while leaving the ordering within the list endogenous. We now link our efficiency notion to the behavioral efficiency notion introduced by de Clippel (2014).

Proposition 6. *Assume order and multiplicity invariance of choices, that is, for every $i \in N$ and $\theta \in \Theta$, if $A(L) = A(L')$ then $C_i^\theta(L) = C_i^\theta(L')$ for all $L, L' \in \mathcal{L}$. Define the induced set choice $\widehat{C}_i^\theta : \mathcal{X} \to X$ by $\widehat{C}_i^\theta(S) := C_i^\theta(L)$ for any $L \in \mathcal{L}$ with $A(L) = S$. Then for every $\theta \in \Theta$, $F_L^{\text{eff}}(\theta) = F_B^{\text{eff}}(\theta)$.*

Proof. Suppose $x \in F_L^{\text{eff}}(\theta)$. Choose $(L_i)_{i\in N}$ with $\bigcap_{i\in N}\{C_i^\theta(L_i)\} = \{x\}$ and $\bigcup_{i\in N} A(L_i) = X$. Set $S_i := A(L_i)$. Then $x = C_i^\theta(L_i) = \widehat{C}_i^\theta(S_i)$ for all i, and $\bigcup_{i\in N} S_i = X$, so $x \in F_B^{\text{eff}}(\theta)$. Conversely, suppose $x \in F_B^{\text{eff}}(\theta)$. Then there exist $(S_i)_{i\in N}$ with $x = \widehat{C}_i^\theta(S_i)$ for all i and $\bigcup_{i\in N} S_i = X$. For each i, choose any $L_i \in \mathcal{L}$ with $A(L_i) = S_i$. Order and multiplicity invariance gives $C_i^\theta(L_i) = \widehat{C}_i^\theta(S_i) = x$ for all i, and $\bigcup_{i\in N} A(L_i) = \bigcup_{i\in N} S_i = X$. Hence $x \in F_L^{\text{eff}}(\theta)$. □

In particular, for every $\theta \in \Theta$ one has $F_L^{\text{eff}}(\theta) \subseteq X$, hence $F_L^{\text{eff}}(\theta) \in \mathcal{X} \cup \{\emptyset\}$. Without order or multiplicity invariance the behavioral and list notions need not coincide; depending on C^θ, one can have $F_L^{\text{eff}}(\theta) \subsetneq F_B^{\text{eff}}(\theta)$ or $F_B^{\text{eff}}(\theta) \subsetneq F_L^{\text{eff}}(\theta)$.

Observation 2. F_L^{eff} is List-Nash implementable if and only if there exist $\kappa \in \mathbb{N}^{|N|}$ and a family Λ that is κ-consistent and strongly compatible with F.

Proof. The equivalence follows by applying Theorems 5 and 6 to the specific rule $F = F_L^{\mathrm{eff}}$. $\square$

CHAPTER 7

CONCLUSION

This thesis studies games and mechanisms where players choose from lists rather than sets, represent behavior by state-contingent list choice functions, and define List-Nash equilibrium as the fixed point of order-dependent best replies. Existence follows from the Monotone Opportunity condition via Knaster-Tarski, uniqueness follows from a Hamming Contraction together with uniquely chosen responses. For implementation, the key primitives are columns that encode opportunity lists: κ-consistency and compatibility are necessary, and adding strong compatibility makes it sufficient. In Libertarian Paternalist mechanisms every column is a permutation of X; latin consistency and compatibility, plus unanimity, deliver implementation while keeping a sense of freedom of choice. Lastly, by taking a novel approach, I assess efficiency at the level of mechanisms and show that, when efficiency means maximizing individual freedom while keeping message space simple, order-based Libertarian Paternalist mechanisms deliver the optimal tradeoff and are essentially unique under mild symmetry, with supporting lower bounds and fairness comparisons.

7.1 Literature Review

The choice-from-lists literature begins with Salant and Rubinstein (2006), who develop a model in which decision-makers choose from lists rather than unordered sets. Salant and Rubinstein (2008) later extend this approach by introducing a general framework for choice with frames, where the same feasible set can yield different choices under different presentations, including orderings, labels, and contextual decorations. Subsequent literature studies choice from lists and framing effects, both theoretically and empirically. Nevertheless, to the best of my knowledge, this is the first work to explicitly connect choice-from-lists models in choice theory with game theory and mechanism design. My contribution is to embed list-based choice directly into strategic interaction by equipping each player's strategy set with an exogenous order and interpreting best responses as choice from opportunity lists, thereby providing the missing link between presentation-sensitive individual choice and equilibrium analysis.

For finite normal-form games, equilibrium existence is due to Nash (1950, 1951), who allow mixed strategies and establish existence via fixed-point arguments. Nash (1950) invokes Brouwer's fixed-point theorem; Nash (1951) applies Kakutani's fixed-point theorem to the best-response correspondence on the product of mixed-strategy simplices. In the existence result, I take a different route, employing lattice-theoretic tools and the Knaster–Tarski fixed-point theorem. In my framework the order structure enables lattice methods for existence and uniqueness. If moving opponents "up" in their orders cannot move a player's minimal chosen response "down" in her order, the resulting best-response operator is isotone on the product order over labels. The Knaster–Tarski fixed-point theorem then guarantees existence of a fixed point, hence a List-Nash equilibrium (Knaster, 1928; Tarski, 1955). This Monotone Opportunity condition is the analogue, in the label order, of strategic complementarities in supermodular games (Vives, 1990; Milgrom and Roberts, 1990; Topkis, 1998). Beyond existence, I obtain uniqueness via a Hamming contraction of iterated best responses and identify constructive extremal equilibria $s^{\min}$ and $s^{\max}$ that serve as focal selections in ordered strategy spaces.

Implementation theory, a central branch of mechanism design (Dasgupta et al., 1979; Maskin, 1999), seeks conditions under which a mechanism's equilibria coincide with a target social choice rule. Hurwicz (1986) investigates implementation when standard rationality assumptions are relaxed. Moore and Repullo (1990) provide necessary and sufficient conditions for Nash implementation in environments with three or more players, and Dutta and Sen (1991) provide such conditions in two-player environments. In behavioral domains, de Clippel (2014) introduce the notion of consistency and show that a consistent collection of opportunity sets is necessary for Nash implementation; with at least three agents, strong consistency together with unanimity is sufficient. Independently, Korpela (2012) analyze Nash implementability without imposing restrictions on individual choice behavior. My implementation analysis aligns with the behavioral implementation literature, which has been studied extensively, including work on incomplete information, ex post and interim equilibria, and strong Nash equilibria (Barlo and Dalkıran, 2023a,b, 2024; Hayashi et al., 2023).

7.2 Discussion and Future Work

I confined attention to games whose institutional structure exogenously imposes an ex ante ordering on each player's strategy set. While some games indeed come with such an exogenous order (e.g., voting), others admit endogenous order structures. A first natural extension, therefore, is to endogenize the ordering. Players could choose, alongside their strategies or messages, an order function that orders their own available strategies as a function of the state, beliefs, or observed play. This can also be analyzed in an incomplete-information setting, in which the designer does not observe how players will order the messages they are given.

Another direction replaces orders with frames. Following Salant and Rubinstein (2008), who extend choice-from-lists models to frames, one can interpret an order over strategies as a particular kind of frame. Under frames, players need not impose full linear orders on their strategies; instead, they attach labels that shape cognition.

A further observation is that by ordering strategies I have ordered alternatives within each best response, but not the best responses themselves. One can also place a list over deviations—for instance, a list of opponents to react to—capturing the idea that not only what one inspects inside a best response matters, but also which best response one inspects first.

Finally, the choice-from-lists primitive can be swapped out for other behavioral maps without changing the architecture. Any choice-from-lists model from the choice literature (e.g., revealed-attention models, sequential or iterative selection rules, or limited-attention correspondences) can be analyzed within this framework. I leave these extensions for future work.

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