A NOVEL COMPREHENSIVE DECISION-MAKING APPROACH FOR PROJECT TYPE SELECTION IN THE SHIPBUILDING INDUSTRY

(ÖZGÜN BİR KAPSAMLI KARAR VERME YAKLAŞIMIYLA
GEMİ İNŞA SEKTÖRÜNDE PROJE TİPİ SEÇİMİ)

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LIST OF SYMBOLS

A	**:** Alternative
C	**:** Criteria
CI	**:** Choquet Integral
DM	**:** Decision Maker
k	**:** Consensus Ranking
MCDM	**:** Multi-Criteria Decision-Making
SCF	**:** Social Choice Function

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ABSTRACT

The ability of a nation to develop and produce is becoming more and more essential in today's world. The wealth, fortune, and safety of society are directly impacted by this aptitude. Nations must consider their particular peculiarities, such as geographic, environmental, scientific and technical factors, to continue to enhance this capability.

The shipbuilding industry deals with the creation of bigger (mostly oceangoing) vessels intended for the merchant fleet (cargo or passenger carriage), military resolutions or pleasure and private life. It also includes goods and services provided for the construction, modification, and upkeep of these vessels.

In recent years, it has become clear that one of the industries in which Türkiye excels in the area of meticulous design and production is shipbuilding, which increases our country's exports. In order to ensure greater sustainability and development in this area, it would be valuable to perform a study that will help specify the appropriate initiatives.

The main purpose of this study is to ascertain the most felicitous project type by taking into account the interaction between the criteria to appraise the project type selection in a multi-dimensional perspective with the details received from shipyards. In this study, after the criteria are identified, a well-known decision-making model will be used and implemented. High-tech production caliber shipyards can utilize the proposed strategy to decide on a project if adequate information has been gathered.

RÉSUMÉ

Dans le monde d'aujourd'hui, la capacité d'un pays à concevoir et à produire devient de plus en plus cruciale. Cette capacité a un impact direct sur la prospérité économique, la fortune et la sécurité de la société. Pour continuer à faire progresser cette capacité, les pays doivent tenir compte de leurs propres caractéristiques telles que géographiques, environnementales et scientifiques.

L'industrie de la construction navale s'occupe de la création de navires plus grands (principalement navigants) destinés à la flotte marchande (transport de marchandises ou de passagers), aux résolutions militaires ou à la plaisance et à la vie privée. Il comprend également les biens et services fournis pour la construction, la modification et l'entretien de ces navires.

Ces dernières années, on voit que la construction navale, qui améliore l'exportation de notre pays, est l'un des domaines où Türkiye réussit dans le domaine de la conception détaillée et de la production. Il serait intéressant de mener une étude qui permettra de préciser les bons projets afin d'assurer une meilleure pérennité et un meilleur développement de ce secteur.

L'objectif principal de cette étude est de déterminer le type de projet le plus approprié en tenant compte de l'interaction entre les critères d'évaluation de la sélection du type de projet dans une condition multidimensionnelle avec les données reçues des chantiers navals. Dans cette étude, une fois les critères identifiés, un modèle de prise de décision bien connu sera utilisé et mis en œuvre. La méthode suggérée peut être utilisée pour prendre une décision de projet pour les chantiers navals de haute technologie après la collecte de données suffisantes.

ÖZET

Günümüz dünyasında, bir ülkenin tasarlama ve üretme yeteneği giderek daha önemli hale geliyor. Bu yeteneğin toplumun ekonomik refahı, serveti ve güvenliği üzerinde doğrudan bir etkisi vardır. Bu yeteneği geliştirmeye devam edebilmek için ülkelerin coğrafi, çevresel ve bilimsel gibi belli başlı özelliklerini dikkate almaları gerekmektedir.

Gemi inşa endüstrisi, ticaret filosu (kargo veya yolcu taşımacılığı), askeri amaçlar veya zevk ve özel yaşam için tasarlanmış büyük gemilerin yaratılmasıyla ilgilenir. Ayrıca, bu gemilerin inşası, yenilenmesi ve bakımı için sağlanan mal ve hizmetleri de içerir.

Ülkemizin ihracatını artıran gemi inşa sektörü, geçtiğimiz senelerde Türkiye'nin detay tasarım ve üretim alanında başarılı olduğu alanlardan biridir. Bu sektörde daha iyi sürdürülebilirlik ve gelişme sağlamak için doğru projeleri belirlemeye yol gösterecek bir çalışma yapılması faydalı olacaktır.

Bu çalışmanın temel amacı, proje tipi seçimini çok boyutlu olarak değerlendirerek tersanelerden alınan veriler ile kriterler arasındaki etkileşimi dikkate alarak en uygun proje tipini belirlemektir. Bu çalışmada kriterler belirlendikten sonra literatürde kabul görmüş bir karar verme modeli kullanılacak ve uygulanacaktır. Önerilen yöntem, yeterli veri toplandıktan ve ilgili kriterler işlendikten sonra ileri teknoloji üretim kapasitesine sahip tersaneler için proje kararı vermede kullanılabilir.

1. INTRODUCTION

Since nature was unable to fully satisfy human requirements, early industrial activities were carried out in workshops based on handicraft. With the creation of the steam engine in 1764, high-volume typical items began to be produced in factories. From the beginning of time till the present, the idea of production has meant to make something or to alter the physical characteristics of the product.

These days, production-oriented corporations and organizations compete fiercely. Technical innovations and efficient production supervision are crucial for thriving in this cutthroat economy. Any industry's performance is influenced by a variety of factors. The choice of projects, both short-term and long-term, is one of the most central aspects in the success of any business driver (Kumar et al., 2009).

Strategies for project selection are crucial for businesses in project-based industries like the marine industry, especially in the primary phases (Mahdavi et al., 2019). Comparing the projects, each organized around diverse objectives, in order to discover which ones better satisfy the selection criteria, is a complicated task. Presenting a product that is technically great can provide competitive advantages, specifically for high-tech production organizations (Lawson et al., 2006). Not to be overlooked is the reality that a product is more than manufacturing. Design and R&D divisions are quite important.

As is common knowledge, Türkiye produces a wide range of yields. The shipbuilding industry is one of the most significant of these. Since the first shipyard was built during the Ottoman Empire in 1390, Türkiye has a wealth of practice when it comes to shipbuilding. Additionally, shipyards provide turn-key results for global marketplaces.

Depending on their intended role, ship designs can be divided into three categories: merchant ships, naval ships, and private yachts. Each class of ship has its unique design and manufacturing procedures. The ships should be built with the specified objective in mind. While every ship is dissimilar, they nevertheless have sure traits. The ships diverge in matters; speed, size, stability, maneuverability, safety and so forth. A ship requires a complicated series of operations that demand front-line technology and extensive knowledge as they are shaped lengthy and forceful period of time.

Ship building heavily relies on completing deadlines, staying within budget and fulfilling to technical specifications (Sæther & Karlsen, 2012). It is necessary for the shipyard to choose the right project to meet these requirements.

In this thesis, a project selection analysis was executed for advanced shipyards that can construct all of the aforementioned vessel types. The originality of this study is that it delivers evidence about the choice of ship projects in the shipbuilding dynamism and will serve shipyards decide in which path to empower with the support of MCDM which is Choquet Integral method. Also, with group decision making an appropriate ranking for the alternatives will be found with the intention to reliable. It will be a novel study since there is no similar research on this subject before.

2. LITERATURE REVIEW

Various decision support systems have been developed as a result of efforts to discover systematic approaches to solving project selection problems. In recent years, a number of initiatives have been established to ease firms make better project selection decisions.

2.1. Project Selection

Many companies must make informed selections when optimal investment allocation is challenging because of varied risk levels and resource requirements, thus project selection is critical. First reviewed study examined the use of decision support system to set regulated framework for project portfolio selection (Ghasemzadeh & Archer, 2000).

Construction companies rely on projects to stay afloat. Proper project selection is a multi-criteria decision that has a significant impact on the future of such companies. Next reviewed study provided a construction project selection model that mentions the effects of a company's existing projects. To determine a tender should be offered or not, the model uses a fuzzy multiplex decision making process (Ravanshadnia et al., 2010).

Increased energy demand is projected to be met by energy sources that are reliable and consistent. Solar energy systems are installed under the influence of fundamental technical, economic, and political variables. Next reviewed study remarked in his study that alternative solar energy plant projects are linguistically assessed using the major criteria based on the experts' knowledge (San Cristóbal, 2011).

Exogenous uncertainties that may affect project performance must be taken into account while selecting investment projects. This precariousness can be addressed by assessing how the projects respond in a variety of situations; however, assigning well-founded probability to such circumstances, or describing the decision makers' risk desires through a distinctively defined utility function, can be problematic. Next reviewed study cultivated a portfolio selection framework including apprehension imperfect information about scenario possibilities and utility functions; then, identified all non-dominated project portfolios based on this information, and provided decision support for project denial and acceptance (Liesiö & Salo, 2012).

Project-based businesses must define, evaluate, and direct their missions in line with their vision and objectives. A three-step hybrid method is advised (Tavana et al., 2015) to hand-pick an idyllic project combination. In order to achieve the greatest degree of synchronization between the final decision and the project grade, numerous administrative objectives have been taken into consideration.

From the time it is offered to the time it is finally selected, selecting a project portfolio is a difficult process including numerous elements and considerations. Given the importance of good selection, it is critical to construct logical arithmetical models to guide the association to its ultimate aim. To accomplish this, such models must accurately mirror both the organization's current position as well as its goals and desires. Because of the tight interconnection between projects and the dogmatic, financial, communal, and legal settings in which they are carried out, ambiguity and a lack of understanding about some figures is always an issue. A mathematical model is provided (Pérez et al., 2017) that extends the standard approach to these problems by integrating the inherent uncertainty.

There is a critical necessity for studies that factor sustainability considerations into decision-making. The quantifiable method of assessing sustainability in project investment is not widely used. As a result, next reviewed study has projected a cohesive innovative optimization practice in which the cost of sustainability is measured quantitatively and a reinvestment proposal is highly instigated. Using this method results in a win-win situation wherein environmental harm is prevented, finances are allocated effectively, and investment earnings are maximized (Kudratova et al., 2018).

A number of factors and attentions are taken into account throughout the knotty process of project selection. Additionally, since choosing the right initiatives is a challenging task, mathematical models must be created to direct the business toward its core objective. The priorities of the organization should be as fully disclosed by these models as is practicable. A multiuse method is declared (Baqeri et al., 2019) for selecting a project that maximizing proficiency and eminence while minimizing peril.

The foundation for organizational accomplishments is being laid by new projects as the global market has grown up more competitive in recent years. The foremost goal of this seeks, which preserved (Ma et al., 2020), is to ponder project selection from the viewpoint of sustainability in a decision-making situation which is erratic.

The choice of logical projects is essential for long-term growth. Operative sustainable upgrading depends on having the ability to choose the functioning bearable project. One of the most communal criteria used in the assessment of sustainable initiatives is project cost, along with modernization, hesitation, understanding, and knowledge handover. Next reviewed study attempted to use the fuzzy analytic hierarchy process, which handles blurred data to openly represent social belief, to rate project factors in terms of comparative meaning and impact on ongoing hustles (Alyamani & Long, 2020).

2.2. Selection in the Shipbuilding and Marine Industry

A new commercial ship costs a lot of money to build. In addition, a modern ship is an entirely complex facility that combines a variety of equipment, technology and disciplines. First reviewed study presented a method for picking a desirable ship as a reference ship for a new design from a set of candidates is described (Xie et al., 2008).

Ship management companies and shipyards have a close partnership that begins with the sketch of innovative ships and continues throughout the utilization of running ships. Episodic planning, which can be considered as the major logistics problem within the scope of the ship maintenance schedule, is a solemn process for ship owners. Therefore, the problem of selecting the most suitable shipyards based on various norms becomes even more important for ship supervision corporations. Next reviewed study examined a systematic assessment model of shipyard’s docking facilities to provide decision assistance to ship owners (Celik et al., 2009).

Recent advancements in nautical expertise have created a plethora of occasions for naval and marine engineers, particularly in terms of equipment preference. On the other hand, in order to control complete system veracity onboard ships, a key decision-making challenge has developed regarding the choice of equipment types. The fuzzy information axioms are used in (Cebi et al., 2012), to secure the best primary design layout for shipbuilding projects.

The grasp of the supply chain is lacking in the shipbuilding sector than others. Next reviewed study created a special supplier selection archetypal to help the shipbuilding manufacturing choose suppliers in a range of market circumstances (Zheng et al., 2013).

The process of owning and acquiring a ship involves numerous stages. Which ship is required must be determined, then a reliable shipyard ought to be chosen. This decision is conspicuous because it will touch on the ship's quality and completion date. Next reviewed study centered on providing shipyards with specifics about client viewpoints on the selection of action and help to recover their deficiencies (Kafali & Ozkok, 2015).

The shipbuilding industry places a premium on choosing qualified welders nowadays. Welder quality increases as a result of the intense rivalry among shipyards. Welding workmanship of higher value is an outcome of advanced welder quality. A shipyard has a competitive advantage over rivals if its workmanship is of a high caliber. The administration of the shipyard must therefore pay attention to the welder. Next reviewed study established the weights of the shipyard welder selection criteria (Ozkok, 2016).

When it comes to reducing waste and using less energy, shipping is one of the most environmentally friendly ways of transportation. Nevertheless, shipping still produces a sizable quantity of carbon emissions due to the vast scale of operations and the great number of boats used globally. In the next reviewed study, the choices made regarding the main engine energy efficacy and service cost in order to rally the sustainability of the shipbuilding supply chain and procurement activities (Xie et al., 2017).

Material selection in the process of engineering design is challenging and complicated because of the enormous variety of different materials that are readily available. Designers have to put into mind a variation of distinct qualitative and quantitative aspects when choosing the right material. Next reviewed study offered a hybrid line to decision-making for the accurate supplies in marine applications (Yadav et al., 2019).

The Arctic Northeast Passage may now be exploited as a cutoff between Asian and European harbors as a result of the opening of Arctic seas, which could result in significant economic gains. Several mentionable anchorages along the Russian Arctic coast, which deliver forward services for ships traveling the Arctic Northeast Passage, are difficult to build due to the rough environment and high prices. To pick fulcrum harbors among options to sustenance carriage through the Arctic Northeast Passage, a multi criteria decision model was built in the next reviewed study (Yao et al., 2020).

Most island countries have tremendous marine energy potential, which is contributing to the country's renewable energy portfolio. Given that deliberate energy preparation is susceptible to a couple of unpredictable aspects and multiplicate sustainability targets, Next reviewed study analyzed how managers may be supported in deciding which kinds of aquatic renewable energy plans should be preferred for advance (Akbari et al., 2021).

2.3. Literature Review Evaluation

As is evident, diverse studies have been done on the topic, and various MCDM techniques have been employed. The unifying goal of these studies is to identify the crucial project selection criteria and to make the best decision possible using the appropriate technique. Making the best decision will yield excellent results in terms of things like price, effectiveness, and quickness.

These studies show that it is important to set subject-based criteria. It is obvious that there is a lack of sight for the ship project type selection. Although the topics are kindly similar, the selection criteria should be determined with experts. For this reason, it was made sure that it would be beneficial to work on this subject in the study.

3. PRELIMINARIES

3.1. Project Types

The construction of a hefty ship is a lengthy, intricate, and demanding sequence of procedures that demand leading-edge technology, expertise and proficiency throughout. A new ship's design and creation must also have a specific objective. As was already explained, there are 3 types into which ships can be classed.

Merchant ships can essentially be understood of as ships carrying freight or passengers. Although the idea of using the sea to move goods has been around for more than 6000 years, the industrial revolution marked the beginning of the biggest strides in this area. In response to overwhelming demand, particularly in the last century, cargo ship numbers, varieties, speeds, and tonnage have all increased. General cargo, tanker, container, bulk carrier and ro-ro can be given as examples of cargo ships. On the other hand, over the past 30 years, passenger ships have undergone substantial alterations. Currently, cruise ships appear as modern ships carrying travelers and ferry boats carrying passengers and vehicles over short and medium distances. This kind of ship's owner can be a commercial and professional unity or a business person.

Naval ships are platforms that are specifically created for national defense in accordance with a country's tactical board plans and outfitted with weaponry appropriate for their function. Naval ships are equipped with cannons, machine guns, and guided projectiles to repel or eliminate enemy forces. Compared to the other two types of ships, the design process for naval ships is extra intricate and specialized. However, these are damage resistant, highly maneuverable and quick formations. Navy vessels have added features to survive opponent actions (Tupper, 2013). Aircraft carrier, corvette, destroyer, frigate, patrol boat and submarine can be given as example of naval crafts. This kind of ship's owner is solitary and obligatory the state of the country.

Private yachts, are specialized boats featuring a cabin, bathroom, basin, and galley that are used for charter and recreational activities. The idea of a yacht started to be viewed globally as a consumer good and an architectural creation in the early 1950s. Contrary to common opinion, socioeconomic strata that retain the economic power on land and carry it to the seas, rather than sailors, have shaped the historical identity of private yachts. As well, custom builds are typical pleasure vessels used for the entertaining and leisure purpose of its owner and his guests. Modern engineering solutions are provided in such projects by engineers and project managers. Motor yacht and sailing yacht can be said as samples of private yachts. This kind of ship's owner is frequently wealthy people and generally chartering companies.

3.2. Determination of Criteria

After inspecting the sources (Saracoglu, 2007; Solesvik, 2010; Papke-Shields, 2017; Favi, 2018; Sahin, 2020) correlated to the selection of the project and the shipbuilding industry, then combined the views of marine and defense industry specialists and academicians; superior and outstanding criteria in shaping the project to be nominated for high technology production competence shipyards are shown in the following table and then the criteria are defined. C1, C7, C9, C12 and C13 are industry specific and selected by experts.

Table 3.1: Important Criteria

C1- Modification	C6- Locality	C11- Quality Concern
C2- Environmental Degradation	C7- Overhaul Period	C12- Recognition Growth
C3- Facility Requirement	C8- Manpower Requirement	C13- Sister Ship
C4- Know-How Development	C9- Potential Customer	C14- Total Time
C5- Labor Challenge	C10- Profit Margin	C15- Waste of Material Used

C1- Modification (benefit): The changes' adeptness from the preparatory design stage to delivery period. Usually, modernization involves fluctuations and adjustments.

C2- Environmental Degradation (cost): This happens when the environment is harmed in some way, which results in a decrease in biological diversity and the atmosphere. This process proceeds naturally without human intervention or it may be accelerated.

C3- Facility Requirement (cost): Necessary capability and technology in the shipyard to be produced for ship developments that are grueling and intricate to building.

C4- Know-How Development (benefit): After a ship project is finished, it leaves behind a wealth of knowledge and expertise. Future projects may become more efficient overall as an upshot of these developments.

C5- Labor Challenge (cost): Work or handicraft skills help keep ship projects that are not mass produced or mechanized in production continuing. Production and workmanship grow more difficult as the design becomes more intricate.

C6- Locality (benefit): For a ship project, the ability to continue the shipbuilding with domestic services and field without going overseas for subcontracting.

C7- Overhaul Period (cost): A cluster of trials intended to maintain the functionality of an engineering structure for the duration of its estimated lifetime.

C8- Manpower Requirement (cost): Before submitting a proposal, it is needed to take into account the workforce that will be required for ship project. Managers, front-line personnel, and individuals with dedicated mastery may be involved in the description of manpower requirements for any project.

C9- Potential Customer (benefit): Someone who can become an owner or a client of products or services from an association. A shipyard can satiate the demands and desires of a potential customer.

C10- Profit Margin (benefit): It displays the proportion of sales that have generated profits. The ratio of a company's gross profit to its sales is known as the profit margin.

C11- Quality Concern (cost): The combination of a product's or service's features and qualities that determines how well it can satisfy explicit or implicit wants. Products with trailblazing engineering and radical technology are subject to high quality expectations. The importance attached to this expectation is known as quality concern.

C12- Recognition Growth (benefit): The intensification in prestige and popularity that happens when a major ship project is publicized to the globe under the influence of the media and if this project is noted and respected.

C13- Sister Ship (benefit): Each of two or more ships built to the identical design that share the matching fundamental dimensions, general arrangement, tank plan, and structural layout as the first ship in the series.

C14- Total Time (cost): It describes the total amount of time needed to complete a project from the outset to the customer's approval and delivery of the finished product.

C15- Waste of Material Used (cost): A product or substance that is no longer appropriate for intended use, or wasteful/unnecessary material that needs to be thrown away. Waste is defined as material that must be disposed of by the producer or holder.

In decision making with multiple criteria approach, certain number of alternatives with certain characteristics are graded by comparing values obtained from the criteria. The foundations or features that are momentous to the organization making the choice are known as decision criteria in a business setting. They should support in weighing the alternatives. Additively, the decision criteria should be quantifiable and relevant to the problem attempting to be solved.

3.3. Proposed Model

There is an exigency for a project selection decision process in which several criteria are examined from many viewpoints in light of all the existing situations, restrictions, and hazards in merchant, naval, and private shipbuilding.

MCDM approaches are now the most organized, reliable, and resilient ways to meet this demand. The sample structure of the MCDM hierarchy and the other notable related figures are shown below.

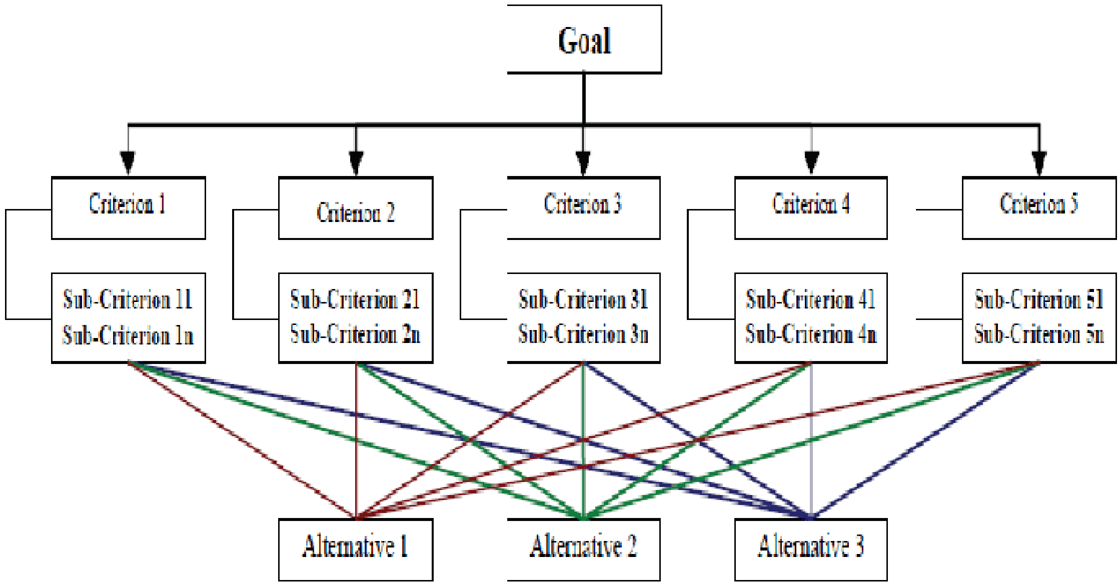

Figure 3.1: The sample structure of the MCDM hierarchy

MCDM comprises different elements and concepts based on the nature of the decision-making problem. The brief steps are shown below.

Figure 3.2: The brief steps of MCDM

The framework for decision-making is shown below.

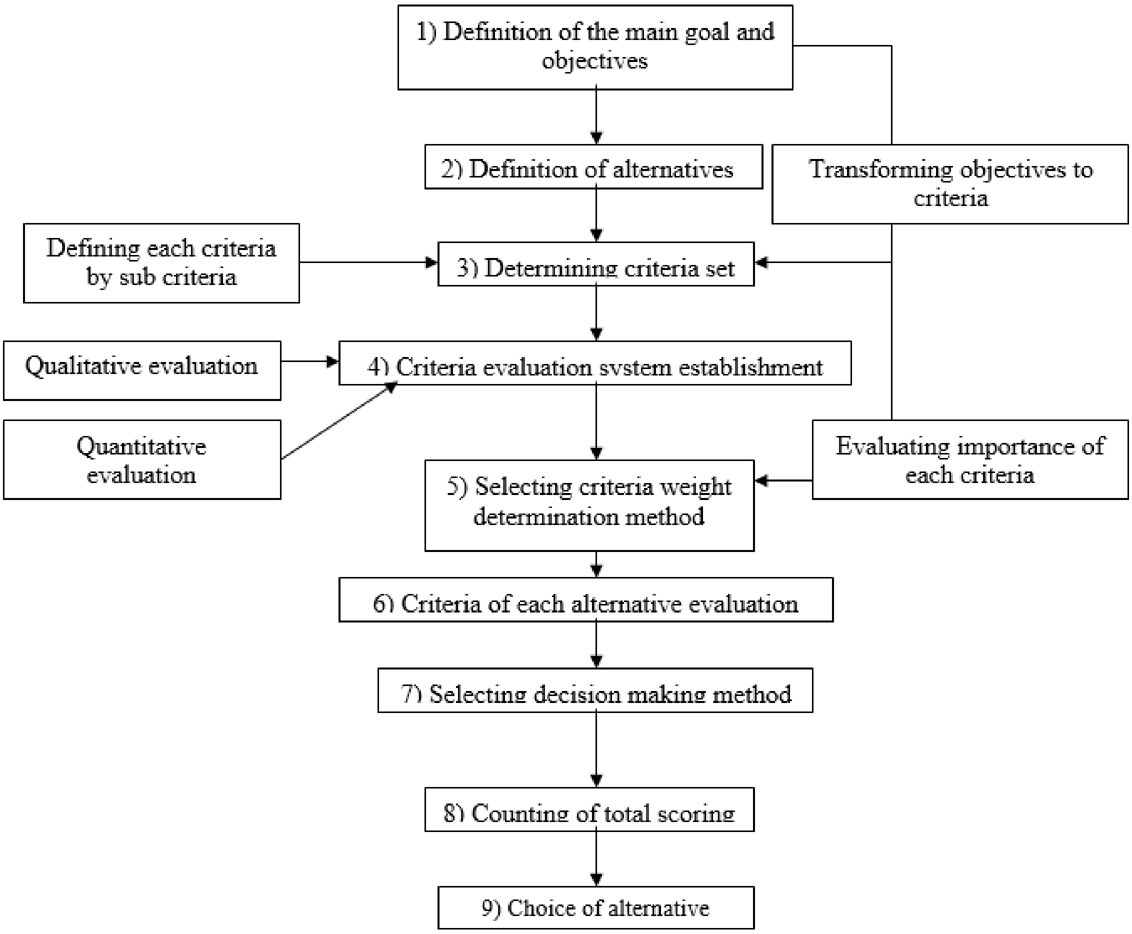

Figure 3.3: The framework for decision-making

3.4. Choquet Integral

The French mathematician Gustave Choquet developed the sub additive or super additive integral in 1953. Its original applications were in statistical mechanics and potential theory, but in the 1980s it made its way into decision theory, where it is used to quantify the expected value of an uncertain event. The Choquet Integral approach is an aggregation methodology defined in terms of a fuzzy measure that enables it to simulate the MCDM process by determining the relative weights of all potential groupings of criteria. The Shapley value and Interaction Index are two metrics that can be used to interpret the dependencies and interactions among the variables. Each criterion's total importance is estimated by the Shapley value, and the Interaction index calculates how well each criterion interacts with the others (Vu et al., 2014).

As long as the measure was considered to be 2-additive, practical situations just required the decision maker to provide importance and interaction indices, which are sufficient to indicate preferences over the alternatives. The decision-maker might not be able to provide exact values for these indices, though. Yet, this is not an issue because it is sensible to anticipate that the decision-maker will be able to provide ranges of values.

In order to convey intervals of preferences for multidimensional alternatives, it is therefore intended to offer a Choquet Integral. This will enable the creation of a straightforward yet precise model of preferences (Ceberio & Modave, 2006).

Let μ be a non-monotonic fuzzy measure on X and also, f a function on X with range $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ where $\alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_n$. The Choquet integral of a function $f: X \mapsto [0,1]$ in relation to μ is defined as:

$$(C) \int f d\mu = \sum_{i=1}^{n} (a_i - a_{i-1}).\, \mu(\{x | f(x) \ \geq \ a_i\}), where\ a_0 = 0 \tag{3.1}$$

3.4.1. Shapley value and Interaction index

Shapley value calculates each criterion's overall weight based on how much it grants to the final score for each group of criterions. Let μ be a fuzzy measure on X. The Shapley index for every $i \in X$ is defined as:

$$\Phi_i = \sum_{K \subset X \setminus \{i\}} \frac{(n - |K| - 1)!\, |K|!}{n!} [\mu(K \cup \{i\}) - \mu(K)] \tag{3.2}$$

Note that,

$$\sum_{i=1}^{n} \Phi_i = \mu(X)$$

Interaction index represent the behaviors of criteria in groups, and can be considered as a measuring of the interaction between criteria in the decision-making process. Let μ be a fuzzy measure on *X*. The interaction index among the elements *i* and *j* of *X* defined as:

$$I_{ij} = \sum_{K \subset N \setminus \{i,j\}} \frac{(n-|K|-2)!\,|K|!}{(n-1)!} \begin{bmatrix} \mu(K \cup \{i,j\}) - \mu(K \cup \{i\}) \\ -\mu(K \cup \{j\}) + \mu(K) \end{bmatrix} \tag{3.3}$$

Note that $I_{ij} \in [-1,1]$ together with if *Iij* is positive then interaction between *i* and *j* is said to be positive. In other words, if *Iij* is negative then interaction between *i* and *j* is said to negative ($\mathrm{I_{ij}} = I_{ij}$).

3.4.2. The 2-order model

The 2-order additivity will be the subject of the focus because it appears to be the most intriguing in utilitarian applications as it allows for the straightforward modeling of the interactions between criteria. Let t_i, $i = 1,\ldots,$n be the scores on the criteria. By using just, the interaction index; it is possible to express CI in the case of 2-additive measures as follows:

$$\begin{aligned} C_{\mu}(t_1,\ldots,t_n) = \sum_{I_{ij}>0} (t_i \wedge t_j) I_{ij} + \sum_{I_{ij}<0} (t_i \vee t_j) |I_{ij}| \\ + \sum_{i=1}^{n} t_i \left(\Phi_i - \frac{1}{2} \sum_{j \neq i} |I_{ij}| \right) \end{aligned} \tag{3.4}$$

With,

$$\Phi_i - \frac{1}{2} \sum_{j \neq i} |I_{ij}| \geq 0, \qquad \forall i = 1,\ldots,n$$

Note that the conjunctive, disjunctive, and additive parts of the CI for 2-additive measures correspond to the positive, negative interactions and the Shapley value, respectively (Mesiar, 2005). In the CI framework:

- Positive values of I_{ij} implies a conjunctive behavior between *i* and *j*. In a word, simultaneous satisfaction of both criteria *i* and *j* is significant for the global score.
- Negative values of I_{ij} implies a disjunctive behavior between *i* and *j*. In a word, satisfaction of either *i* or *j* is sufficient to have a significant effect on the global score.
- The Shapley value acts as a weight vector in a weighted arithmetic mean. This represents the linear part of CI.

3.4.3. 2-additive Choquet Integral

The weight of each elementary performance expression in respect to all other contributions to the overall performance will be determined and described in this section. To exemplify, the Shapley parameters vi and the interaction parameters Iij of any pair of selection criteria. The 2-additive CI expression given in (3.4) in the context of performance expression can be represented as follows (Berrah et al., 2008):

$$p_{Total} = \sum_{i=1}^{n} v_i p_i - \frac{1}{2} \sum_{\substack{\{i,j\} \\ i \neq j}} I_{ij} \left| p_i - p_j \right| \tag{3.5}$$

Where,

$$v_i - \frac{1}{2} \sum_{j=1}^{n} \left| I_{ij} \right| \geq 0 \quad \forall i \in [1, n] \; and \; j \neq i$$

The DMs are requested to include preferential details on the criterion and the pairs of criteria, including the degree of the preferences. With the Shapley and Interaction parameters acting as variables, a system of equations can be constructed with the support of this knowledge.

3.5. Social Choice Function

The study of collective decision-making processes and mechanisms is known as social choice theory. It is not a single theory, but a bunch of models and results concerning the accumulation of discrete inputs (such as votes, preferences, judgments, options) into collective outcomes (Fishburn, 1974).

3.5.1. Condorcet's Function

Condorcet function is to choose the candidate who knocks every other candidate under simple majority when such candidate exists. Whenever there is no simple majority along with the repeated majorities, Condorcet's function judges the worst a candidate does against others (Fishburn, 1977). As a result, the function is as follows, with a higher function value indicating a better candidate:

$$f_C(x) = \min_{y \in A \setminus \{x\}} \#(i: xP_i y) \tag{3.6}$$

3.5.2. Borda's Function

Borda function counts the number of individuals that preferred a candidate to each one of the other candidates (Black, 1958). As a result, the function is as follows, with a higher function value indicating a better candidate:

$$f_B(x) = \sum_{y \in A \setminus \{x\}} \#(i: xP_i y) \tag{3.7}$$

3.6. Agreement and Disagreement

There are functions used to find the agreement or disagreement measures for a probable ranking of the candidates. These functions defined on the set of all preference orders provided by various decision makers and which was to be proven to satisfy particular desirable properties as a good social choice method. Additionally, an alternative method or the SCF mentioned above may be used to determine the ranking.

3.6.1. Cook and Seiford's Function

As a way to gauge whether ranks are in agreement or disagreement, Cook and Seiford offer the distance function (Li et al., 2015). The entire absolute distance (disagreement) is then minimized by defining the consensus ranking.

In order to acquire the rankings of the alternatives for the group, they created a technique that minimized the difference between the final rankings of the alternatives and the ones provided by all decision-makers. Let r_{ij} characterize the rank given to the alternative j by the decision-maker i. In this case, the disagreement from the consensus ranking k will be:

$$d_j k = \sum_{i=1}^{n} |r_{ij} - k| \tag{3.8}$$

4. NUMERICAL APPLICATION

4.1. Criteria Grouping

Decision criteria are divided into 3 main criteria groups as Earning related (C1), Process related (C2), and Requirement and Regulation related (C3).

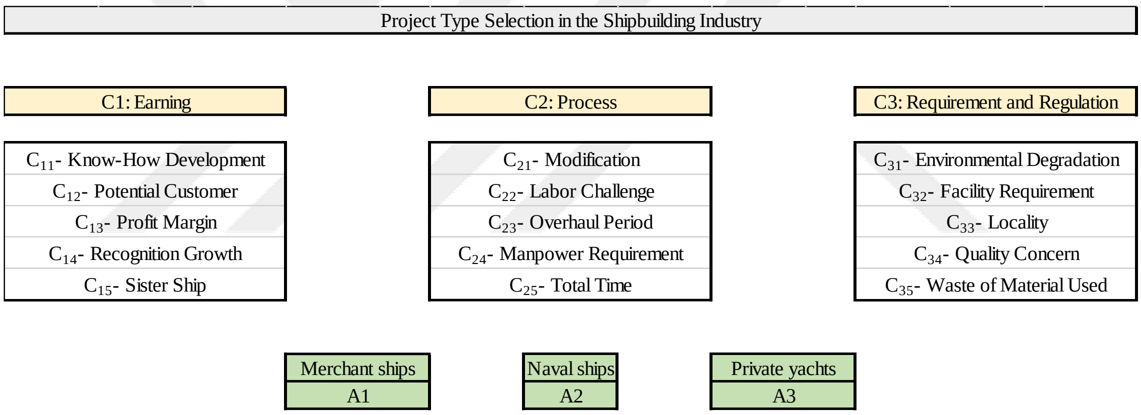

Figure 4.1: Criteria grouping

4.2. Relations between Criteria

Here, each DM can rank the components of options either directly or through pairwise comparisons in this situation. To obtain the ordinal judgment, each DM is firstly asked to express an opinion on the relative attractiveness of two components at a time. If the two aspects are not equally attractive, the DMs are next asked to make a qualitative assessment of the difference in attraction. The DMs are provided with the following six semantic categories of attractiveness difference to support streamline the process. Evaluated relations by the DMs are shared below:

- Very weak – VW

- Weak – W

- Moderate – M

- Strong – S

- Very strong – VS

- Extreme – E

For DM_1

$$C1\&C3 >^{M} C1\&C2 >^{M} C1 >^{W} C2\&C3 >^{M} C3 >^{W} C2 >^{S} 0$$

$$C_{12}\&C_{13} >^{M} C_{11}\&C_{13} >^{S} C_{11}\&C_{12} >^{S} C_{12}\&C_{14} >^{M} C_{12}\&C_{15} >^{VW} C_{13}\&C_{14} >^{W} C_{13}\&C_{15} >^{S} C_{11}\&C_{14} >^{W} C_{11}\&C_{15} >^{M} C_{14}\&C_{15} >^{VW} C_{13} >^{M} C_{12} >^{M} C_{11} >^{W} C_{14} >^{W} C_{15} >^{M} 0$$

$$C_{24}\&C_{25} >^{VS} C_{22}\&C_{25} >^{W} C_{22}\&C_{24} >^{W} C_{25} >^{M} C_{24} >^{S} C_{21}\&C_{25} >^{M} C_{21}\&C_{24} >^{W} C_{22}\&C_{23} >^{M} C_{21}\&C_{23} >^{S} C_{21}\&C_{22} >^{W} C_{22} >^{S} C_{23}\&C_{25} >^{W} C_{23}\&C_{24} >^{W} C_{23} >^{M} C_{21} >^{VS} 0$$

$$C_{33}\&C_{34} >^{S} C_{33} >^{M} C_{34} >^{W} C_{32}\&C_{34} >^{W} C_{32}\&C_{33} >^{M} C_{33}\&C_{35} >^{S} C_{31}\&C_{33} >^{M} C_{31}\&C_{34} >^{M} C_{31}\&C_{32} >^{W} C_{32}\&C_{35} >^{M} C_{32} >^{M} C_{34}\&C_{35} >^{S} C_{31}\&C_{35} >^{M} C_{35} >^{W} C_{31} >^{W} 0$$

For DM_2

$$C1\&C3 >^{S} C1\&C2 >^{S} C1 >^{W} C2\&C3 >^{W} C3 >^{M} C2 >^{M} 0$$

$$C_{12}\&C_{13} >^{M} C_{11}\&C_{12} >^{M} C_{11}\&C_{13} >^{M} C_{12} >^{W} C_{13}\&C_{14} >^{W} C_{13}\&C_{15} >^{S} C_{13} >^{W} C_{12}\&C_{14} >^{VW} C_{12}\&C_{15} >^{VW} C_{11}\&C_{14} >^{VW} C_{11}\&C_{15} >^{VS} C_{11} >^{W} C_{14}\&C_{15} >^{VS} C_{14} >^{VW} C_{15} >^{VW} 0$$

$$C_{24}\&C_{25} >^{M} C_{22}\&C_{24} >^{W} C_{23}\&C_{24} >^{S} C_{24} >^{W} C_{22}\&C_{25} >^{M} C_{23}\&C_{25} >^{VW} C_{21}\&C_{24} >^{W} C_{21}\&C_{25} >^{VS} C_{25} >^{M} C_{22}\&C_{23} >^{M} C_{21}\&C_{22} >^{S} C_{22} >^{VW} C_{21}\&C_{23} >^{S} C_{23} >^{W} C_{21} >^{VW} 0$$

$$C_{33}\&C_{34} >^{M} C_{34} >^{S} C_{34}\&C_{35} >^{M} C_{33}\&C_{35} >^{VW} C_{33} >^{S} C_{31}\&C_{35} >^{VW} C_{31}\&C_{33} >^{W} C_{32}\&C_{33} >^{VW} C_{32}\&C_{35} >^{M} C_{31}\&C_{34} >^{W} C_{35} >^{VW} C_{32}\&C_{34} >^{M} C_{31} >^{S} C_{31}\&C_{32} >^{M} C_{32} >^{M} 0$$

For DM_3

$$C1\&C2 >^{S} C1\&C3 >^{M} C2\&C3 >^{M} C1 >^{M} C2 >^{S} C3 >^{S} 0$$

$$C_{11}\&C_{13} >^{S} C_{12}\&C_{13} >^{M} C_{13} >^{W} C_{13}\&C_{15} >^{M} C_{13}\&C_{14} >^{VW} C_{11}\&C_{12} >^{M} C_{11}\&C_{15} >^{VW} C_{11}\&C_{14} >^{M} C_{11} >^{W} C_{12}\&C_{15} >^{VW} C_{12}\&C_{14} >^{M} C_{12} >^{VW} C_{14}\&C_{15} >^{VW} C_{15} >^{W} C_{14} >^{S} 0$$

$$C_{24}\&C_{25} >^{VW} C_{24} >^{W} C_{25} >^{VW} C_{23}\&C_{24} >^{M} C_{23}\&C_{25} >^{S} C_{21}\&C_{24} >^{VW} C_{22}\&C_{24} >^{W} C_{22}\&C_{25} >^{M} C_{21}\&C_{25} >^{VW} C_{22}\&C_{23} >^{M} C_{23} >^{W} C_{21}\&C_{23} >^{VW} C_{21}\&C_{22} >^{W} C_{22} >^{W} C_{21} >^{M} 0$$

$$C_{33}\&C_{34} >^{S} C_{33} >^{VS} C_{34} >^{S} C_{32}\&C_{34} >^{VW} C_{32}\&C_{33} >^{W} C_{33}\&C_{35} >^{S} C_{31}\&C_{33} >^{M} C_{31}\&C_{34} >^{W} C_{31}\&C_{32} >^{VW} C_{32}\&C_{35} >^{M} C_{34}\&C_{35} >^{S} C_{31}\&C_{35} >^{VW} C_{32} >^{W} C_{31} >^{VW} C_{35} >^{W} 0$$

The equation systems for each DM can be derived from the favored rankings and strength values provided by the DMs in the section. Equation systems for the Choquet Integral application are shared in Appendix B. Then, Matrix operations for the Choquet Integral application are shared in Appendix C. The weights of the criteria and the interactions between criteria are determined by the resolution of those matrix operations and are shared below:

Table 4.1: Criteria weights and interaction values from DM_1

	v_1	v_2	v_3	I_{12}	I_{13}	I_{23}
C	0.553	0.158	0.289	-0.053	0.000	-0.053

	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}
C1	0.205	0.268	0.248	0.148	0.131	0.107	0.114	0.047	0.047	0.114	0.094	0.087	0.047	0.047	0.040
C2	0.087	0.011	0.424	0.217	0.261	0.065	-0.196	0.261	0.261	-0.022	0.239	0.435	0.696	0.717	0.717
C3	0.261	0.076	0.054	0.228	0.380	-0.065	0.348	0.348	-0.022	0.457	0.348	0.022	0.696	0.304	0.630

Table 4.2: Criteria weights and interaction values from DM_2

	v_1	v_2	v_3	I_{12}	I_{13}	I_{23}
C	0.548	0.143	0.310	0.048	0.095	-0.048

	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}
C1	0.298	0.202	0.440	0.048	0.012	-0.071	0.048	0.095	0.095	-0.238	-0.286	-0.286	0.095	0.071	0.095
C2	0.038	0.182	0.144	0.341	0.295	0.045	0.045	-0.106	0.061	0.061	-0.030	0.045	0.015	0.076	-0.136
C3	0.314	0.333	0.078	0.255	0.020	0.137	0.294	0.569	0.020	0.196	0.490	-0.039	0.471	0.255	0.353

Table 4.3: Criteria weights and interaction values from DM_3

	v_1	v_2	v_3	I_{12}	I_{13}	I_{23}
C	0.448	0.345	0.207	0.069	0.069	0.069

	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}
C1	0.250	0.250	0.250	0.000	0.250	0.000	-0.500	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000
C2	0.313	0.260	0.146	0.094	0.188	0.021	0.104	0.271	0.354	0.042	0.333	0.333	0.271	0.292	0.562
C3	0.338	0.029	0.059	0.206	0.368	-0.176	0.559	0.500	-0.029	0.441	0.265	-0.176	0.765	0.412	0.647

4.3. The Decision Matrix

Information and data officially received from decision makers is shared below in order A1 (merchant ships), A2 (naval ships) and A3 (private yachts). Raw data is presented in Appendix A.

Table 4.4: Normalized values from the DM_1

C1	C2	C3	C4	C5
Modification	**Environmental Degradation**	**Facility Requirement**	**Know-How Development**	**Labor Challenge**
0.176	0.167	0.506	0.222	0.568
0.471	0.500	0.190	0.444	0.189
0.353	0.333	0.304	0.333	0.243

C6	C7	C8	C9	C10
Locality	**Overhaul Period**	**Manpower Requirement**	**Potential Customer**	**Profit Margin**
0.381	0.231	0.556	0.444	0.151
0.333	0.385	0.167	0.222	0.472
0.286	0.385	0.278	0.333	0.377

C11	C12	C13	C14	C15
Quality Concern	**Recognition Growth**	**Sister Ship**	**Total Time**	**Waste of Material Used**
0.486	0.227	0.533	0.516	0.517
0.243	0.409	0.267	0.161	0.207
0.270	0.364	0.200	0.323	0.276

Table 4.5: Normalized values from DM_2

C1	C2	C3	C4	C5
Modification	**Environmental Degradation**	**Facility Requirement**	**Know-How Development**	**Labor Challenge**
0.118	0.270	0.514	0.111	0.568
0.471	0.405	0.229	0.500	0.243
0.412	0.324	0.257	0.389	0.189

C6	C7	C8	C9	C10
Locality	**Overhaul Period**	**Manpower Requirement**	**Potential Customer**	**Profit Margin**
0.421	0.182	0.496	0.474	0.212
0.316	0.273	0.220	0.158	0.412
0.263	0.545	0.283	0.368	0.376

C11	C12	C13	C14	C15
Quality Concern	**Recognition Growth**	**Sister Ship**	**Total Time**	**Waste of Material Used**
0.526	0.182	0.444	0.375	0.462
0.263	0.364	0.333	0.250	0.231
0.211	0.455	0.222	0.375	0.308

Table 4.6: Normalized values from DM_3

C1	C2	C3	C4	C5
Modification	**Environmental Degradation**	**Facility Requirement**	**Know-How Development**	**Labor Challenge**
0.176	0.255	0.435	0.238	0.483
0.412	0.426	0.217	0.429	0.241
0.412	0.319	0.348	0.333	0.276

C6	C7	C8	C9	C10
Locality	**Overhaul Period**	**Manpower Requirement**	**Potential Customer**	**Profit Margin**
0.400	0.231	0.427	0.438	0.255
0.300	0.308	0.267	0.188	0.388
0.300	0.462	0.305	0.375	0.357

C11	C12	C13	C14	C15
Quality Concern	**Recognition Growth**	**Sister Ship**	**Total Time**	**Waste of Material Used**
0.459	0.286	0.471	0.435	0.481
0.287	0.381	0.294	0.217	0.231
0.255	0.333	0.235	0.348	0.288

4.4. Final Aggregation

Using the formula (3.4) or (3.5) with the values given above tables, calculations of the final performance scores for the project types with respect to each DM is shown:

Table 4.7: Final performance values from DM_1

Criteria			C1					C2					C3					P_{Ag}
Criteria Weights			0.553					0.158					0.289					
DM_1	A1	MS	0.234					0.035					0.233					0.213
	A2	NS	0.310					0.039					0.133					0.226
	A3	PY	0.307					0.231					0.256					0.283
Sub - Criteria			C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	
Sub - Criteria Weights			0.205	0.268	0.248	0.148	0.131	0.087	0.011	0.424	0.217	0.261	0.261	0.076	0.054	0.228	0.380	
DM_1	A1	MS	0.222	0.444	0.151	0.227	0.533	0.176	0.568	0.231	0.556	0.516	0.167	0.506	0.381	0.486	0.517	
	A2	NS	0.444	0.222	0.472	0.409	0.267	0.471	0.189	0.385	0.167	0.161	0.500	0.190	0.333	0.243	0.207	
	A3	PY	0.333	0.333	0.377	0.364	0.200	0.353	0.243	0.385	0.278	0.323	0.333	0.304	0.286	0.270	0.276	

Table 4.8: Final performance values from DM_2

Criteria			C1					C2					C3					P_{Ag}
Criteria Weights			0.548					0.143					0.310					
DM_2	A1	MS	0.282					0.398					0.266					0.294
	A2	NS	0.457					0.247					0.186					0.327
	A3	PY	0.384					0.318					0.175					0.302
Sub - Criteria			C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	
Sub - Criteria Weights			0.298	0.202	0.440	0.048	0.012	0.038	0.182	0.144	0.341	0.295	0.314	0.333	0.078	0.255	0.020	
DM_2	A1	MS	0.111	0.474	0.212	0.182	0.444	0.118	0.568	0.182	0.496	0.375	0.270	0.514	0.421	0.526	0.462	
	A2	NS	0.500	0.158	0.412	0.364	0.333	0.471	0.243	0.273	0.220	0.250	0.405	0.229	0.316	0.263	0.231	
	A3	PY	0.389	0.368	0.376	0.455	0.222	0.412	0.189	0.545	0.283	0.375	0.324	0.257	0.263	0.211	0.308	

Table 4.9: Final performance values from DM_3

Criteria			C1					C2					C3					P_{Ag}
Criteria Weights			0.448					0.345					0.207					
DM_3	A1	MS	0.346					0.169					0.269					0.257
	A2	NS	0.308					0.199					0.200					0.240
	A3	PY	0.296					0.261					0.228					0.265
Sub - Criteria			C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{31}	C_{32}	C_{33}	C_{34}	C_{35}	
Sub - Criteria Weights			0.250	0.250	0.250	0.000	0.250	0.313	0.260	0.146	0.094	0.188	0.338	0.029	0.059	0.206	0.368	
DM_3	A1	MS	0.238	0.438	0.255	0.286	0.471	0.176	0.483	0.231	0.427	0.435	0.255	0.435	0.400	0.459	0.481	
	A2	NS	0.429	0.188	0.388	0.381	0.294	0.412	0.241	0.308	0.267	0.217	0.426	0.217	0.300	0.287	0.231	
	A3	PY	0.333	0.375	0.357	0.333	0.235	0.412	0.276	0.462	0.305	0.348	0.319	0.348	0.300	0.255	0.288	

Hence the final rankings are as follows:

$$DM_1 \rightarrow A3 \succ A2 \succ A1$$

$$DM_2 \rightarrow A2 \succ A3 \succ A1$$

$$DM_3 \rightarrow A3 \succ A1 \succ A2$$

4.5. Application of SCF – Group Decision

$$\#(i : A1P_iA2) = 1 \quad \#(i : A2P_iA1) = 2 \quad \#(i : A3P_iA1) = 3$$
$$\#(i : A1P_iA3) = 0 \quad \#(i : A2P_iA3) = 1 \quad \#(i : A3P_iA2) = 2$$

Condorcet function values for the project types and their ranking according to Condorcet's function with using the formula (3.6) are as follows:

$$f_C\,(A1) = 0 \quad f_C\,(A2) = 1 \quad f_C\,(A3) = 2$$

$$A3 \succ A2 \succ A1$$

Borda function values for the project types and their ranking according to Borda's function with using the formula (3.7) are as follows:

$$f_B\,(A1) = 1 \quad f_B\,(A2) = 3 \quad f_B\,(A3) = 5$$

$$A3 \succ A2 \succ A1$$

4.6. Application of Agreement and Disagreement

Using the formula (3.8), d_jk values are as follows:

$$d_{A1}1 = 5 \quad d_{A1}2 = 2 \quad d_{A1}3 = 1 \quad d_{A2}1 = 3 \quad d_{A2}2 = 2 \quad d_{A2}3 = 3 \quad d_{A3}1 = 1 \quad d_{A3}2 = 2 \quad d_{A3}3 = 5$$

$j \setminus k$	1	2	3
A1	5	2	1
A2	3	2	3
A3	1	2	5

The Hungarian Algorithm can be used to solve and reveal the ranking: $A3 \succ A2 \succ A1$

5. CONCLUSION

Shipbuilding is regarded as one of the oldest, most open and highly competitive markets in the world. Shipbuilding projects are challenging, perplexed and long process that requires strong investment costs and labor intensive. In a number of countries, shipbuilding is a vital and strategic industry. Shipyards play a notable role in regional industrial infrastructure and national safety (naval ships).

Modern shipbuilders are developing tremendously sophisticated systems that are tested by one of the world's most demanding and perilous environments, from tankers to pleasure craft to ships constructed for defense and homeland security. Companies are looking to minimize both material costs and design & development costs as a result of market pressures to regulate output, decrease operating expenses, fulfill environmental concerns, maintain safeness, and improve human comfort.

Every day, decision-making by project managers and corporate executives is frequent. Many business decisions are knotty to comprehend and can have long-lasting effects on the bottom lines and welfare of organizations. Multiple criteria decision-making procedures consist of approaches and methods that aim to find the "best or finest" answer that satisfies all relevant but conflicting criteria.

The case was assessed using Choquet Integral in accordance with the nature of the relationships existing between the criteria, and the sub-criteria in order to handle the interactions. That way, all possible relations would be addressed without the need to try to create independent criteria or assume that the existing criteria are independent. These benefits make the CI a flexible tool for modeling relationships.

Social Choice Function was utilized to determine the project type's best fit ranking for this group of DMs once the findings for each DM have been composed. The addition of a group decision-making process clearly increased the level of impartiality.

When the results are examined, the first striking finding is that C1 has the highest weight for all decision makers **which is earning related**. Know-how development, potential customer and profit margin stands out; also, recognition growth and sister ship are in this group. Alternative 3, **which is private yachts**, provides superiority considering the final performances of the alternatives. In fact, the same rankings in the latest group decision-making practices shed light on a consistent study.

The aspect of personalization that a new build yacht brings is its greatest advantage. Nothing is off limits when creating a private yacht, including a hull design that is unique and a layout that is tailored to the needs of the owner. The wishes of the owners are taken into consideration when yacht building for those who want to protect their wellbeing while at sea, this might include luxurious modernity. Utilizing cutting edge eco-friendly technology, such as solar panels, wastewater recovery systems, and hybrid propulsion systems, a private yacht gives the choice of a smaller carbon footprint.

The long and the short of it, there is no project selection study for the shipbuilding industry in which studies on this subject are examined. For this reason, conducting a study in this field will make significant contributions to the literature and the sector.

Future effort after this study can be:
- applying the strategy across shipyards and ensuring that learn about, interact with, and are familiar with the approach.
- observing the consistency with the results obtained from the data that can be gained from the shipyards that are open to innovation and may invest.
- developing special software for the shipbuilding industry to shorten mathematical operations in a project development and decision.

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APPENDICES

Appendix A - The Scores of The Alternatives According to The Decision Makers

For DM_1:

Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate
Type	Benefit	Type	Cost	Type	Cost	Type	Benefit	Type	Cost
C1		C2		C3		C4		C5	
Modification		**Environmental Degradation**		**Facility Requirement**		**Know-How Development**		**Labor Challenge**	
3		6		3		4		3	
8		2		8		8		9	
6		3		5		6		7	

Unit	1-10 rate	Unit	month	Unit	1-10 rate	Unit	1-10 rate	Unit	percent
Type	Benefit	Type	Cost	Type	Cost	Type	Benefit	Type	Benefit
C6		C7		C8		C9		C10	
Locality		**Overhaul Period**		**Manpower Requirement**		**Potential Customer**		**Profit Margin**	
8		60		3		8		8	
7		36		10		4		25	
6		36		6		6		20	

Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	month	Unit	percent
Type	Cost	Type	Benefit	Type	Benefit	Type	Cost	Type	Cost
C11		C12		C13		C14		C15	
Quality Concern		**Recognition Growth**		**Sister Ship**		**Total Time**		**Waste of Material Used**	
5		5		8		15		8	
10		9		4		48		20	
9		8		3		24		15	

For DM_2:

Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate
Type	Benefit	Type	Cost	Type	Cost	Type	Benefit	Type	Cost
C1		C2		C3		C4		C5	
Modification		**Environmental Degradation**		**Facility Requirement**		**Know-How Development**		**Labor Challenge**	
2		6		4		2		3	
8		4		9		9		7	
7		5		8		7		9	

Unit	1-10 rate	Unit	month	Unit	1-10 rate	Unit	1-10 rate	Unit	percent
Type	Benefit	Type	Cost	Type	Cost	Type	Benefit	Type	Benefit
C6		C7		C8		C9		C10	
Locality		**Overhaul Period**		**Manpower Requirement**		**Potential Customer**		**Profit Margin**	
8		36		4		9		18	
6		24		9		3		35	
5		12		7		7		32	

Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	month	Unit	percent
Type	Cost	Type	Benefit	Type	Benefit	Type	Cost	Type	Cost
C11		C12		C13		C14		C15	
Quality Concern		**Recognition Growth**		**Sister Ship**		**Total Time**		**Waste of Material Used**	
4		2		8		24		20	
8		4		6		36		40	
10		5		4		24		30	

For DM_3:

Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate
Type	Benefit	Type	Cost	Type	Cost	Type	Benefit	Type	Cost
C1		C2		C3		C4		C5	
Modification		**Environmental Degradation**		**Facility Requirement**		**Know-How Development**		**Labor Challenge**	
3		5		4		5		4	
7		3		8		9		8	
7		4		5		7		7	

Unit	1-10 rate	Unit	month	Unit	1-10 rate	Unit	1-10 rate	Unit	percent
Type	Benefit	Type	Cost	Type	Cost	Type	Benefit	Type	Benefit
C6		C7		C8		C9		C10	
Locality		**Overhaul Period**		**Manpower Requirement**		**Potential Customer**		**Profit Margin**	
8		48		5		7		25	
6		36		8		3		38	
6		24		7		6		35	

Unit	1-10 rate	Unit	1-10 rate	Unit	1-10 rate	Unit	month	Unit	percent
Type	Cost	Type	Benefit	Type	Benefit	Type	Cost	Type	Cost
C11		C12		C13		C14		C15	
Quality Concern		**Recognition Growth**		**Sister Ship**		**Total Time**		**Waste of Material Used**	
5		6		8		20		12	
8		8		5		40		25	
9		7		4		25		20	

Appendix B - Equation Systems for the Choquet Integral Application

For DM_1:

Equation Systems Main Criteria (C1, C2, C3),

01. $P_{Ag}^{(1,0,1)} - P_{Ag}^{(1,1,0)} = 3a = v_3 - v_2 - ½ [I_{12} - I_{13}]$

02. $P_{Ag}^{(1,1,0)} - P_{Ag}^{(1,0,0)} = 3a = v_2 - ½ [I_{23} - I_{12}]$

03. $P_{Ag}^{(1,0,0)} - P_{Ag}^{(0,1,1)} = 2a = v_1 - v_2 - v_3$

04. $P_{Ag}^{(0,1,1)} - P_{Ag}^{(0,0,1)} = 3a = v_2 - ½ [I_{12} - I_{23}]$

05. $P_{Ag}^{(0,0,1)} - P_{Ag}^{(0,1,0)} = 2a = v_3 - v_2 - ½ [I_{13} - I_{12}]$

06. $P_{Ag}^{(0,1,0)} - P_{Ag}^{(0,0,0)} = 4a = v_2 - ½ [I_{12} + I_{23}]$

07. $P_{Ag}^{(1,1,1)} = 1 = v_1 + v_2 + v_3$

Equation Systems C1,

01. $P_{Ag}{}^{(0,1,1,0,0)} - P_{Ag}{}^{(1,0,1,0,0)} = 3a = v_2 - v_1 - ½\,[\,I_{13} + I_{24} + I_{25} - I_{14} - I_{15} - I_{23}\,]$

02. $P_{Ag}{}^{(1,0,1,0,0)} - P_{Ag}{}^{(1,1,0,0,0)} = 4a = v_3 - v_2 - ½\,[\,I_{12} + I_{34} + I_{35} - I_{13} - I_{24} - I_{25}\,]$

03. $P_{Ag}{}^{(1,1,0,0,0)} - P_{Ag}{}^{(0,1,0,1,0)} = 4a = v_1 - v_4 - ½\,[\,I_{13} + I_{15} + I_{24} - I_{12} - I_{34} - I_{45}\,]$

04. $P_{Ag}{}^{(0,1,0,1,0)} - P_{Ag}{}^{(0,1,0,0,1)} = 3a = v_4 - v_5 - ½\,[\,I_{14} + I_{25} + I_{34} - I_{15} - I_{24} - I_{35}\,]$

05. $P_{Ag}{}^{(0,1,0,0,1)} - P_{Ag}{}^{(0,0,1,1,0)} = 1a = v_2 + v_5 - v_3 - v_4 - ½\,[\,I_{12} + I_{15} - I_{13} - I_{14}\,]$

06. $P_{Ag}{}^{(0,0,1,1,0)} - P_{Ag}{}^{(0,0,1,0,1)} = 2a = v_4 - v_5 - ½\,[\,I_{14} + I_{24} + I_{35} - I_{15} - I_{25} - I_{34}\,]$

07. $P_{Ag}{}^{(0,0,1,0,1)} - P_{Ag}{}^{(1,0,0,1,0)} = 4a = v_3 + v_5 - v_1 - v_4 - ½\,[\,I_{23} + I_{25} - I_{12} - I_{24}\,]$

08. $P_{Ag}{}^{(1,0,0,1,0)} - P_{Ag}{}^{(1,0,0,0,1)} = 2a = v_4 - v_5 - ½\,[\,I_{15} + I_{24} + I_{34} - I_{14} - I_{25} - I_{35}\,]$

09. $P_{Ag}{}^{(1,0,0,0,1)} - P_{Ag}{}^{(0,0,0,1,1)} = 3a = v_1 - v_4 - ½\,[\,I_{12} + I_{13} + I_{45} - I_{15} - I_{24} - I_{34}\,]$

10. $P_{Ag}{}^{(0,0,0,1,1)} - P_{Ag}{}^{(0,0,1,0,0)} = 1a = v_4 + v_5 - v_3 - ½\,[\,I_{14} + I_{15} + I_{24} + I_{25} - I_{13} - I_{23}\,]$

11. $P_{Ag}{}^{(0,0,1,0,0)} - P_{Ag}{}^{(0,1,0,0,0)} = 3a = v_3 - v_2 - ½\,[\,I_{13} + I_{34} + I_{35} - I_{12} - I_{24} - I_{25}\,]$

12. $P_{Ag}{}^{(0,1,0,0,0)} - P_{Ag}{}^{(1,0,0,0,0)} = 3a = v_2 - v_1 - ½\,[\,I_{23} + I_{24} + I_{25} - I_{13} - I_{14} - I_{15}\,]$

13. $P_{Ag}{}^{(1,0,0,0,0)} - P_{Ag}{}^{(0,0,0,1,0)} = 2a = v_1 - v_4 - ½\,[\,I_{12} + I_{13} + I_{15} - I_{24} - I_{34} - I_{45}\,]$

14. $P_{Ag}{}^{(0,0,0,1,0)} - P_{Ag}{}^{(0,0,0,0,1)} = 2a = v_4 - v_5 - ½\,[\,I_{14} + I_{24} + I_{34} - I_{15} - I_{25} - I_{35}\,]$

15. $P_{Ag}{}^{(0,0,0,0,1)} - P_{Ag}{}^{(0,0,0,0,0)} = 3a = v_5 - ½\,[\,I_{15} + I_{25} + I_{35} + I_{45}\,]$

16. $P_{Ag}{}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

Equation Systems C2,

01. $P_{Ag}^{(0,0,0,1,1)} - P_{Ag}^{(0,1,0,0,1)} = 5a = v_4 - v_2 - ½ [I_{14} + I_{25} + I_{34} - I_{12} - I_{23} - I_{45}]$

02. $P_{Ag}^{(0,1,0,0,1)} - P_{Ag}^{(0,1,0,1,0)} = 2a = v_5 - v_4 - ½ [I_{23} + I_{24} + I_{35} - I_{13} - I_{14} - I_{34}]$

03. $P_{Ag}^{(0,1,0,1,0)} - P_{Ag}^{(0,0,0,0,1)} = 2a = v_2 + v_4 - v_5 - ½ [I_{12} + I_{14} + I_{23} + I_{34} - I_{15} - I_{35}]$

04. $P_{Ag}^{(0,0,0,0,1)} - P_{Ag}^{(0,0,0,1,0)} = 3a = v_5 - v_4 - ½ [I_{15} + I_{25} + I_{35} - I_{14} - I_{24} - I_{34}]$

05. $P_{Ag}^{(0,0,0,1,0)} - P_{Ag}^{(1,0,0,0,1)} = 4a = v_4 - v_1 - v_5 - ½ [I_{24} + I_{34} - I_{12} - I_{13} - I_{25} - I_{35}]$

06. $P_{Ag}^{(1,0,0,0,1)} - P_{Ag}^{(1,0,0,1,0)} = 3a = v_5 - v_4 - ½ [I_{14} + I_{25} + I_{35} - I_{15} - I_{24} - I_{34}]$

07. $P_{Ag}^{(1,0,0,1,0)} - P_{Ag}^{(0,1,1,0,0)} = 2a = v_1 + v_4 - v_2 - v_3 - ½ [I_{15} + I_{45} - I_{25} - I_{35}]$

08. $P_{Ag}^{(0,1,1,0,0)} - P_{Ag}^{(1,0,1,0,0)} = 3a = v_2 - v_1 - ½ [I_{13} + I_{24} + I_{25} - I_{14} - I_{15} - I_{23}]$

09. $P_{Ag}^{(1,0,1,0,0)} - P_{Ag}^{(1,1,0,0,0)} = 4a = v_3 - v_2 - ½ [I_{12} + I_{34} + I_{35} - I_{13} - I_{24} - I_{25}]$

10. $P_{Ag}^{(1,1,0,0,0)} - P_{Ag}^{(0,1,0,0,0)} = 2a = v_1 - ½ [I_{13} + I_{14} + I_{15} - I_{12}]$

11. $P_{Ag}^{(0,1,0,0,0)} - P_{Ag}^{(0,0,1,0,1)} = 4a = v_2 - v_3 - v_5 - ½ [I_{12} + I_{24} - I_{13} - I_{15} - I_{34} - I_{45}]$

12. $P_{Ag}^{(0,0,1,0,1)} - P_{Ag}^{(0,0,1,1,0)} = 2a = v_5 - v_4 - ½ [I_{15} + I_{25} + I_{34} - I_{14} - I_{24} - I_{35}]$

13. $P_{Ag}^{(0,0,1,1,0)} - P_{Ag}^{(0,0,1,0,0)} = 2a = v_4 - ½ [I_{14} + I_{24} + I_{45} - I_{34}]$

14. $P_{Ag}^{(0,0,1,0,0)} - P_{Ag}^{(1,0,0,0,0)} = 3a = v_3 - v_1 - ½ [I_{23} + I_{34} + I_{35} - I_{12} - I_{14} - I_{15}]$

15. $P_{Ag}^{(1,0,0,0,0)} - P_{Ag}^{(0,0,0,0,0)} = 5a = v_1 - ½ [I_{12} + I_{13} + I_{14} + I_{15}]$

16. $P_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

Equation Systems C3,

01. $P_{Ag}^{(0,0,1,1,0)} - P_{Ag}^{(0,0,1,0,0)} = 4a = v_4 - ½ [I_{14} + I_{24} + I_{45} - I_{34}]$

02. $P_{Ag}^{(0,0,1,0,0)} - P_{Ag}^{(0,0,0,1,0)} = 3a = v_3 - v_4 - ½ [I_{13} + I_{23} + I_{35} - I_{14} - I_{24} - I_{45}]$

03. $P_{Ag}^{(0,0,0,1,0)} - P_{Ag}^{(0,1,0,1,0)} = 2a = - v_2 - ½ [I_{24} - I_{12} - I_{23} - I_{25}]$

04. $P_{Ag}^{(0,1,0,1,0)} - P_{Ag}^{(0,1,1,0,0)} = 2a = v_4 - v_3 - ½ [I_{14} + I_{23} + I_{45} - I_{13} - I_{24} - I_{35}]$

05. $P_{Ag}^{(0,1,1,0,0)} - P_{Ag}^{(0,0,1,0,1)} = 3a = v_2 - v_5 - ½ [I_{12} + I_{24} + I_{35} - I_{15} - I_{23} - I_{45}]$

06. $P_{Ag}^{(0,0,1,0,1)} - P_{Ag}^{(1,0,1,0,0)} = 4a = v_5 - v_1 - ½ [I_{13} + I_{25} + I_{45} - I_{12} - I_{14} - I_{35}]$

07. $P_{Ag}^{(1,0,1,0,0)} - P_{Ag}^{(1,0,0,1,0)} = 3a = v_3 - v_4 - ½ [I_{14} + I_{23} + I_{35} - I_{13} - I_{24} - I_{45}]$

08. $P_{Ag}^{(1,0,0,1,0)} - P_{Ag}^{(1,1,0,0,0)} = 3a = v_4 - v_2 - ½ [I_{12} + I_{34} + I_{45} - I_{14} - I_{23} - I_{25}]$

09. $P_{Ag}^{(1,1,0,0,0)} - P_{Ag}^{(0,1,0,0,1)} = 2a = v_1 - v_5 - ½ [I_{13} + I_{14} + I_{25} - I_{12} - I_{35} - I_{45}]$

10. $P_{Ag}^{(0,1,0,0,1)} - P_{Ag}^{(0,1,0,0,0)} = 3a = v_5 - ½ [I_{15} + I_{35} + I_{45} - I_{25}]$

11. $P_{Ag}^{(0,1,0,0,0)} - P_{Ag}^{(0,0,0,1,1)} = 3a = v_2 - v_4 - v_5 - ½ [I_{12} + I_{23} - I_{14} - I_{15} - I_{34} - I_{35}]$

12. $P_{Ag}^{(0,0,0,1,1)} - P_{Ag}^{(1,0,0,0,1)} = 4a = v_4 - v_1 - ½ [I_{15} + I_{24} + I_{34} - I_{12} - I_{13} - I_{45}]$

13. $P_{Ag}^{(1,0,0,0,1)} - P_{Ag}^{(0,0,0,0,1)} = 3a = v_1 - ½ [I_{12} + I_{13} + I_{14} - I_{15}]$

14. $P_{Ag}^{(0,0,0,0,1)} - P_{Ag}^{(1,0,0,0,0)} = 2a = v_5 - v_1 - ½ [I_{25} + I_{35} + I_{45} - I_{12} - I_{13} - I_{14}]$

15. $P_{Ag}^{(1,0,0,0,0)} - P_{Ag}^{(0,0,0,0,0)} = 2a = v_1 - ½ [I_{12} + I_{13} + I_{14} + I_{15}]$

16. $P_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

For DM_2:

Equation Systems Main Criteria (C1, C2, C3),

01. $P_{Ag}^{(1,0,1)} - P_{Ag}^{(1,1,0)} = 4a = v_3 – v_2 – ½ [I_{12} – I_{13}]$

02. $P_{Ag}^{(1,1,0)} - P_{Ag}^{(1,0,0)} = 4a = v_2 – ½ [I_{23} – I_{12}]$

03. $P_{Ag}^{(1,0,0)} - P_{Ag}^{(0,1,1)} = 2a = 2a = v_1 – v_2 – v_3$

04. $P_{Ag}^{(0,1,1)} - P_{Ag}^{(0,0,1)} = 2a = v_2 – ½ [I_{12} – I_{23}]$

05. $P_{Ag}^{(0,0,1)} - P_{Ag}^{(0,1,0)} = 3a = v_3 – v_2 – ½ [I_{13} – I_{12}]$

06. $P_{Ag}^{(0,1,0)} - P_{Ag}^{(0,0,0)} = 3a = v_2 – ½ [I_{12} + I_{23}]$

07. $P_{Ag}^{(1,1,1)} = 1 = v_1 + v_2 + v_3$

Equation Systems C1,

01. $P_{Ag}^{(0,1,1,0,0)} - P_{Ag}^{(1,1,0,0,0)} = 3a = v_3 - v_1 - ½ [I_{12} + I_{34} + I_{35} - I_{14} - I_{15} - I_{23}]$

02. $P_{Ag}^{(1,1,0,0,0)} - P_{Ag}^{(1,0,1,0,0)} = 3a = v_2 - v_3 - ½ [I_{13} + I_{24} + I_{25} - I_{12} - I_{34} - I_{35}]$

03. $P_{Ag}^{(1,0,1,0,0)} - P_{Ag}^{(0,1,0,0,0)} = 3a = v_1 + v_3 - v_2 - ½ [I_{14} + I_{15} + I_{34} + I_{35} - I_{24} - I_{25}]$

04. $P_{Ag}^{(0,1,0,0,0)} - P_{Ag}^{(0,0,1,1,0)} = 2a = v_2 - v_3 - v_4 - ½ [I_{12} + I_{25} - I_{13} - I_{14} - I_{35} - I_{45}]$

05. $P_{Ag}^{(0,0,1,1,0)} - P_{Ag}^{(0,0,1,0,1)} = 2a = v_4 - v_5 - ½ [I_{14} + I_{24} + I_{35} - I_{15} - I_{25} - I_{34}]$

06. $P_{Ag}^{(0,0,1,0,1)} - P_{Ag}^{(0,0,1,0,0)} = 4a = v_5 - ½ [I_{15} + I_{25} + I_{45} - I_{35}]$

07. $P_{Ag}^{(0,0,1,0,0)} - P_{Ag}^{(0,1,0,1,0)} = 2a = v_3 - v_2 - v_4 - ½ [I_{13} + I_{35} - I_{12} - I_{14} - I_{25} - I_{45}]$

08. $P_{Ag}^{(0,1,0,1,0)} - P_{Ag}^{(0,1,0,0,1)} = 1a = v_4 - v_5 - ½ [I_{14} + I_{25} + I_{34} - I_{15} - I_{24} - I_{35}]$

09. $P_{Ag}^{(0,1,0,0,1)} - P_{Ag}^{(1,0,0,1,0)} = 1a = v_2 + v_5 - v_1 - v_4 - ½ [I_{23} + I_{35} - I_{13} - I_{34}]$

10. $P_{Ag}^{(1,0,0,1,0)} - P_{Ag}^{(1,0,0,0,1)} = 1a = v_4 - v_5 - ½ [I_{15} + I_{24} + I_{34} - I_{14} - I_{25} - I_{35}]$

11. $P_{Ag}^{(1,0,0,0,1)} - P_{Ag}^{(1,0,0,0,0)} = 5a = v_5 - ½ [I_{25} + I_{35} + I_{45} - I_{15}]$

12. $P_{Ag}^{(1,0,0,0,0)} - P_{Ag}^{(0,0,0,1,1)} = 2a = v_1 - v_4 - v_5 - ½ [I_{12} + I_{13} - I_{24} - I_{25} - I_{34} - I_{35}]$

13. $P_{Ag}^{(0,0,0,1,1)} - P_{Ag}^{(0,0,0,1,0)} = 5a = v_5 - ½ [I_{15} + I_{25} + I_{35} - I_{45}]$

14. $P_{Ag}^{(0,0,0,1,0)} - P_{Ag}^{(0,0,0,0,1)} = 1a = v_4 - v_5 - ½ [I_{14} + I_{24} + I_{34} - I_{15} - I_{25} - I_{35}]$

15. $P_{Ag}^{(0,0,0,0,1)} - P_{Ag}^{(0,0,0,0,0)} = 1a = v_5 - ½ [I_{15} + I_{25} + I_{35} + I_{45}]$

16. $P_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

Equation Systems C2,

01. $P_{Ag}^{(0,0,0,1,1)} - P_{Ag}^{(0,1,0,1,0)} = 3a = v_5 - v_2 - ½ [I_{15} + I_{24} + I_{35} - I_{12} - I_{23} - I_{45}]$

02. $P_{Ag}^{(0,1,0,1,0)} - P_{Ag}^{(0,0,1,1,0)} = 2a = v_2 - v_3 - ½ [I_{12} + I_{25} + I_{34} - I_{13} - I_{24} - I_{35}]$

03. $P_{Ag}^{(0,0,1,1,0)} - P_{Ag}^{(0,0,0,1,0)} = 4a = v_3 - ½ [I_{13} + I_{23} + I_{35} - I_{34}]$

04. $P_{Ag}^{(0,0,0,1,0)} - P_{Ag}^{(0,1,0,0,1)} = 2a = v_4 - v_2 - v_5 - ½ [I_{14} + I_{34} - I_{12} - I_{15} - I_{23} - I_{35}]$

05. $P_{Ag}^{(0,1,0,0,1)} - P_{Ag}^{(0,0,1,0,1)} = 3a = v_2 - v_3 - ½ [I_{12} + I_{24} + I_{35} - I_{13} - I_{25} - I_{34}]$

06. $P_{Ag}^{(0,0,1,0,1)} - P_{Ag}^{(1,0,0,1,0)} = 1a = v_3 + v_5 - v_1 - v_4 - ½ [I_{23} + I_{25} - I_{12} - I_{24}]$

07. $P_{Ag}^{(1,0,0,1,0)} - P_{Ag}^{(1,0,0,0,1)} = 2a = v_4 - v_5 - ½ [I_{15} + I_{24} + I_{34} - I_{14} - I_{25} - I_{35}]$

08. $P_{Ag}^{(1,0,0,0,1)} - P_{Ag}^{(0,0,0,0,1)} = 5a = v_1 - ½ [I_{12} + I_{13} + I_{14} - I_{15}]$

09. $P_{Ag}^{(0,0,0,0,1)} - P_{Ag}^{(0,1,1,0,0)} = 3a = v_5 - v_2 - v_3 - ½ [I_{15} + I_{45} - I_{12} - I_{13} - I_{24} - I_{34}]$

10. $P_{Ag}^{(0,1,1,0,0)} - P_{Ag}^{(1,1,0,0,0)} = 3a = v_3 - v_1 - ½ [I_{12} + I_{34} + I_{35} - I_{14} - I_{15} - I_{23}]$

11. $P_{Ag}^{(1,1,0,0,0)} - P_{Ag}^{(0,1,0,0,0)} = 4a = v_1 - ½ [I_{13} + I_{14} + I_{15} - I_{12}]$

12. $P_{Ag}^{(0,1,0,0,0)} - P_{Ag}^{(1,0,1,0,0)} = 1a = v_2 - v_1 - v_3 - ½ [I_{24} + I_{25} - I_{14} - I_{15} - I_{34} - I_{35}]$

13. $P_{Ag}^{(1,0,1,0,0)} - P_{Ag}^{(0,0,1,0,0)} = 4a = v_1 - ½ [I_{12} + I_{14} + I_{15} - I_{13}]$

14. $P_{Ag}^{(0,0,1,0,0)} - P_{Ag}^{(1,0,0,0,0)} = 2a = v_3 - v_1 - ½ [I_{23} + I_{34} + I_{35} - I_{12} - I_{14} - I_{15}]$

15. $P_{Ag}^{(1,0,0,0,0)} - P_{Ag}^{(0,0,0,0,0)} = 1a = v_1 - ½ [I_{12} + I_{13} + I_{14} + I_{15}]$

16. $P_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

Equation Systems C3,

01. $P_{Ag}{}^{(0,0,1,1,0)} - P_{Ag}{}^{(0,0,0,1,0)} = 3a = v_3 - ½ [I_{13} + I_{23} + I_{35} - I_{34}]$

02. $P_{Ag}{}^{(0,0,0,1,0)} - P_{Ag}{}^{(0,0,0,1,1)} = 4a = - v_5 - ½ [I_{45} - I_{15} - I_{25} - I_{35}]$

03. $P_{Ag}{}^{(0,0,0,1,1)} - P_{Ag}{}^{(0,0,1,0,1)} = 3a = v_4 - v_3 - ½ [I_{14} + I_{24} + I_{35} - I_{13} - I_{23} - I_{45}]$

04. $P_{Ag}{}^{(0,0,1,0,1)} - P_{Ag}{}^{(0,0,1,0,0)} = 1a = v_5 - ½ [I_{15} + I_{25} + I_{45} - I_{35}]$

05. $P_{Ag}{}^{(0,0,1,0,0)} - P_{Ag}{}^{(1,0,0,0,1)} = 4a = v_3 - v_1 - v_5 - ½ [I_{23} + I_{34} - I_{12} - I_{14} - I_{25} - I_{45}]$

06. $P_{Ag}{}^{(1,0,0,0,1)} - P_{Ag}{}^{(1,0,1,0,0)} = 1a = v_5 - v_3 - ½ [I_{13} + I_{25} + I_{45} - I_{15} - I_{23} - I_{34}]$

07. $P_{Ag}{}^{(1,0,1,0,0)} - P_{Ag}{}^{(0,1,1,0,0)} = 2a = v_1 - v_2 - ½ [I_{14} + I_{15} + I_{23} - I_{13} - I_{24} - I_{25}]$

08. $P_{Ag}{}^{(0,1,1,0,0)} - P_{Ag}{}^{(0,1,0,0,1)} = 1a = v_3 - v_5 - ½ [I_{13} + I_{25} + I_{34} - I_{15} - I_{23} - I_{45}]$

09. $P_{Ag}{}^{(0,1,0,0,1)} - P_{Ag}{}^{(1,0,0,1,0)} = 3a = v_2 + v_5 - v_1 - v_4 - ½ [I_{23} + I_{35} - I_{13} - I_{34}]$

10. $P_{Ag}{}^{(1,0,0,1,0)} - P_{Ag}{}^{(0,0,0,0,1)} = 2a = v_1 + v_4 - v_5 - ½ [I_{12} + I_{13} + I_{24} + I_{34} - I_{25} - I_{35}]$

11. $P_{Ag}{}^{(0,0,0,0,1)} - P_{Ag}{}^{(0,1,0,1,0)} = 1a = v_5 - v_2 - v_4 - ½ [I_{15} + I_{35} - I_{12} - I_{14} - I_{23} - I_{34}]$

12. $P_{Ag}{}^{(0,1,0,1,0)} - P_{Ag}{}^{(1,0,0,0,0)} = 3a = v_2 + v_4 - v_1 - ½ [I_{23} + I_{25} + I_{34} + I_{45} - I_{13} - I_{15}]$

13. $P_{Ag}{}^{(1,0,0,0,0)} - P_{Ag}{}^{(1,1,0,0,0)} = 4a = - v_2 - ½ [I_{12} - I_{23} - I_{24} - I_{25}]$

14. $P_{Ag}{}^{(1,1,0,0,0)} - P_{Ag}{}^{(0,1,0,0,0)} = 3a = v_1 - ½ [I_{13} + I_{14} + I_{15} - I_{12}]$

15. $P_{Ag}{}^{(0,1,0,0,0)} - P_{Ag}{}^{(0,0,0,0,0)} = 3a = v_2 - ½ [I_{12} + I_{23} + I_{24} + I_{25}]$

16. $P_{Ag}{}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

For DM_3:

Equation Systems Main Criteria (C1, C2, C3),

01. $P_{Ag}^{(1,1,0)} - P_{Ag}^{(1,0,1)} = 4a = v_2 – v_3 – ½ [I_{13} – I_{12}]$

02. $P_{Ag}^{(1,0,1)} - P_{Ag}^{(0,1,1)} = 3a = v_1 – v_2 – ½ [I_{23} – I_{13}]$

03. $P_{Ag}^{(0,1,1)} - P_{Ag}^{(1,0,0)} = 3a = v_2 + v_3 – v_1$

04. $P_{Ag}^{(1,0,0)} - P_{Ag}^{(0,1,0)} = 3a = v_1 – v_2 – ½ [I_{13} – I_{23}]$

05. $P_{Ag}^{(0,1,0)} - P_{Ag}^{(0,0,1)} = 4a = v_2 – v_3 – ½ [I_{12} – I_{13}]$

06. $P_{Ag}^{(0,0,1)} - P_{Ag}^{(0,0,0)} = 4a = v_3 – ½ [I_{13} + I_{23}]$

07. $P_{Ag}^{(1,1,1)} = 1 = v_1 + v_2 + v_3$

Equation Systems C1,

01. $P_{Ag}^{(1,0,1,0,0)} - P_{Ag}^{(0,1,1,0,0)} = 4a = v_1 - v_2 - ½ [I_{14} + I_{15} + I_{23} - I_{13} - I_{24} - I_{25}]$

02. $P_{Ag}^{(0,1,1,0,0)} - P_{Ag}^{(0,0,1,0,0)} = 3a = v_2 - ½ [I_{12} + I_{24} + I_{25} - I_{23}]$

03. $P_{Ag}^{(0,0,1,0,0)} - P_{Ag}^{(0,0,1,0,1)} = 2a = - v_5 - ½ [I_{35} - I_{15} - I_{25} - I_{45}]$

04. $P_{Ag}^{(0,0,1,0,1)} - P_{Ag}^{(0,0,1,1,0)} = 3a = v_5 - v_4 - ½ [I_{15} + I_{25} + I_{34} - I_{14} - I_{24} - I_{35}]$

05. $P_{Ag}^{(0,0,1,1,0)} - P_{Ag}^{(1,1,0,0,0)} = 1a = v_3 + v_4 - v_1 - v_2 - ½ [I_{35} + I_{45} - I_{15} - I_{25}]$

06. $P_{Ag}^{(1,1,0,0,0)} - P_{Ag}^{(1,0,0,0,1)} = 3a = v_2 - v_5 - ½ [I_{15} + I_{23} + I_{24} - I_{12} - I_{35} - I_{45}]$

07. $P_{Ag}^{(1,0,0,0,1)} - P_{Ag}^{(1,0,0,1,0)} = 1a = v_5 - v_4 - ½ [I_{14} + I_{25} + I_{35} - I_{15} - I_{24} - I_{34}]$

08. $P_{Ag}^{(1,0,0,1,0)} - P_{Ag}^{(1,0,0,0,0)} = 3a = v_4 - ½ [I_{24} + I_{34} + I_{45} - I_{14}]$

09. $P_{Ag}^{(1,0,0,0,0)} - P_{Ag}^{(0,1,0,0,1)} = 2a = v_1 - v_2 - v_5 - ½ [I_{13} + I_{14} - I_{23} - I_{24} - I_{35} - I_{45}]$

10. $P_{Ag}^{(0,1,0,0,1)} - P_{Ag}^{(0,1,0,1,0)} = 1a = v_5 - v_4 - ½ [I_{23} + I_{24} + I_{35} - I_{13} - I_{14} - I_{34}]$

11. $P_{Ag}^{(0,1,0,1,0)} - P_{Ag}^{(0,1,0,0,0)} = 3a = v_4 - ½ [I_{14} + I_{34} + I_{45} - I_{24}]$

12. $P_{Ag}^{(0,1,0,0,0)} - P_{Ag}^{(0,0,0,1,1)} = 1a = v_2 - v_4 - v_5 - ½ [I_{12} + I_{23} - I_{14} - I_{15} - I_{34} - I_{35}]$

13. $P_{Ag}^{(0,0,0,1,1)} - P_{Ag}^{(0,0,0,0,1)} = 1a = v_4 - ½ [I_{14} + I_{24} + I_{34} - I_{45}]$

14. $P_{Ag}^{(0,0,0,0,1)} - P_{Ag}^{(0,0,0,1,0)} = 2a = v_5 - v_4 - ½ [I_{15} + I_{25} + I_{35} - I_{14} - I_{24} - I_{34}]$

15. $P_{Ag}^{(0,0,0,1,0)} - P_{Ag}^{(0,0,0,0,0)} = 4a = v_4 - ½ [I_{14} + I_{24} + I_{34} + I_{45}]$

16. $P_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

Equation Systems C2,

01. $P_{Ag}^{(0,0,0,1,1)} - P_{Ag}^{(0,0,0,1,0)} = 1a = v_5 - ½ [I_{15} + I_{25} + I_{35} - I_{45}]$

02. $P_{Ag}^{(0,0,0,1,0)} - P_{Ag}^{(0,0,0,0,1)} = 2a = v_4 - v_5 - ½ [I_{14} + I_{24} + I_{34} - I_{15} - I_{25} - I_{35}]$

03. $P_{Ag}^{(0,0,0,0,1)} - P_{Ag}^{(0,0,1,1,0)} = 1a = v_5 - v_3 - v_4 - ½ [I_{15} + I_{25} - I_{13} - I_{14} - I_{23} - I_{24}]$

04. $P_{Ag}^{(0,0,1,1,0)} - P_{Ag}^{(0,0,1,0,1)} = 3a = v_4 - v_5 - ½ [I_{14} + I_{24} + I_{35} - I_{15} - I_{25} - I_{34}]$

05. $P_{Ag}^{(0,0,1,0,1)} - P_{Ag}^{(1,0,0,1,0)} = 4a = v_3 + v_5 - v_1 - v_4 - ½ [I_{23} + I_{25} - I_{12} - I_{24}]$

06. $P_{Ag}^{(1,0,0,1,0)} - P_{Ag}^{(0,1,0,1,0)} = 1a = v_1 - v_2 - ½ [I_{13} + I_{15} + I_{24} - I_{14} - I_{23} - I_{25}]$

07. $P_{Ag}^{(0,1,0,1,0)} - P_{Ag}^{(0,1,0,0,1)} = 2a = v_4 - v_5 - ½ [I_{14} + I_{25} + I_{34} - I_{15} - I_{24} - I_{35}]$

08. $P_{Ag}^{(0,1,0,0,1)} - P_{Ag}^{(1,0,0,0,1)} = 3a = v_2 - v_1 - ½ [I_{15} + I_{23} + I_{24} - I_{13} - I_{14} - I_{25}]$

09. $P_{Ag}^{(1,0,0,0,1)} - P_{Ag}^{(0,1,1,0,0)} = 1a = v_1 + v_5 - v_2 - v_3 - ½ [I_{14} + I_{45} - I_{24} - I_{34}]$

10. $P_{Ag}^{(0,1,1,0,0)} - P_{Ag}^{(0,0,1,0,0)} = 3a = v_2 - ½ [I_{12} + I_{24} + I_{25} - I_{23}]$

11. $P_{Ag}^{(0,0,1,0,0)} - P_{Ag}^{(1,0,1,0,0)} = 2a = - v_1 - ½ [I_{13} - I_{12} - I_{14} - I_{15}]$

12. $P_{Ag}^{(1,0,1,0,0)} - P_{Ag}^{(1,1,0,0,0)} = 1a = v_3 - v_2 - ½ [I_{12} + I_{34} + I_{35} - I_{13} - I_{24} - I_{25}]$

13. $P_{Ag}^{(1,1,0,0,0)} - P_{Ag}^{(0,1,0,0,0)} = 2a = v_1 - ½ [I_{13} + I_{14} + I_{15} - I_{12}]$

14. $P_{Ag}^{(0,1,0,0,0)} - P_{Ag}^{(1,0,0,0,0)} = 2a = v_2 - v_1 - ½ [I_{23} + I_{24} + I_{25} - I_{13} - I_{14} - I_{15}]$

15. $P_{Ag}^{(1,0,0,0,0)} - P_{Ag}^{(0,0,0,0,0)} = 3a = v_1 - ½ [I_{12} + I_{13} + I_{14} + I_{15}]$

16. $P_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

Equation Systems C3,

01. $P_{Ag}^{(0,0,1,1,0)} - P_{Ag}^{(0,0,1,0,0)} = 4a = v_4 – ½ [I_{14} + I_{24} + I_{45} – I_{34}]$

02. $P_{Ag}^{(0,0,1,0,0)} - P_{Ag}^{(0,0,0,1,0)} = 5a = v_3 – v_4 – ½ [I_{13} + I_{23} + I_{35} – I_{14} – I_{24} – I_{45}]$

03. $P_{Ag}^{(0,0,0,1,0)} - P_{Ag}^{(0,1,0,1,0)} = 4a = – v_2 – ½ [I_{24} – I_{12} – I_{23} – I_{25}]$

04. $P_{Ag}^{(0,1,0,1,0)} - P_{Ag}^{(0,1,1,0,0)} = 1a = v_4 – v_3 – ½ [I_{14} + I_{23} + I_{45} – I_{13} – I_{24} – I_{35}]$

05. $P_{Ag}^{(0,1,1,0,0)} - P_{Ag}^{(0,0,1,0,1)} = 2a = v_2 – v_5 – ½ [I_{12} + I_{24} + I_{35} – I_{15} – I_{23} – I_{45}]$

06. $P_{Ag}^{(0,0,1,0,1)} - P_{Ag}^{(1,0,1,0,0)} = 4a = v_5 – v_1 – ½ [I_{13} + I_{25} + I_{45} – I_{12} – I_{14} – I_{35}]$

07. $P_{Ag}^{(1,0,1,0,0)} - P_{Ag}^{(1,0,0,1,0)} = 3a = v_3 – v_4 – ½ [I_{14} + I_{23} + I_{35} – I_{13} – I_{24} – I_{45}]$

08. $P_{Ag}^{(1,0,0,1,0)} - P_{Ag}^{(1,1,0,0,0)} = 2a = v_4 – v_2 – ½ [I_{12} + I_{34} + I_{45} – I_{14} – I_{23} – I_{25}]$

09. $P_{Ag}^{(1,1,0,0,0)} - P_{Ag}^{(0,1,0,0,1)} = 1a = v_1 – v_5 – ½ [I_{13} + I_{14} + I_{25} – I_{12} – I_{35} – I_{45}]$

10. $P_{Ag}^{(0,1,0,0,1)} - P_{Ag}^{(0,0,0,1,1)} = 3a = v_2 – v_4 – ½ [I_{12} + I_{23} + I_{45} – I_{14} – I_{25} – I_{34}]$

11. $P_{Ag}^{(0,0,0,1,1)} - P_{Ag}^{(1,0,0,0,1)} = 4a = v_4 – v_1 – ½ [I_{15} + I_{24} + I_{34} – I_{12} – I_{13} – I_{45}]$

12. $P_{Ag}^{(1,0,0,0,1)} - P_{Ag}^{(0,1,0,0,0)} = 1a = v_1 + v_5 – v_2 – ½ [I_{13} + I_{14} + I_{35} + I_{45} – I_{23} – I_{24}]$

13. $P_{Ag}^{(0,1,0,0,0)} - P_{Ag}^{(1,0,0,0,0)} = 2a = v_2 – v_1 – ½ [I_{23} + I_{24} + I_{25} – I_{13} – I_{14} – I_{15}]$

14. $P_{Ag}^{(1,0,0,0,0)} - P_{Ag}^{(0,0,0,0,1)} = 1a = v_1 – v_5 – ½ [I_{12} + I_{13} + I_{14} – I_{25} – I_{35} – I_{45}]$

15. $P_{Ag}^{(0,0,0,0,1)} - P_{Ag}^{(0,0,0,0,0)} = 2a = v_5 – ½ [I_{15} + I_{25} + I_{35} + I_{45}]$

16. $P_{Ag}^{(1,1,1,1,1)} = 1 = v_1 + v_2 + v_3 + v_4 + v_5$

Appendix C - Matrix Operations for the Choquet Integral Application

For DM_1:

Matrix Operations Main Criteria (C1, C2, C3),

C	v_1	v_2	v_3	I_{12}	I_{13}	I_{23}	a	
1	0	-1	1	-0.5	0.5	0	-3	0
2	0	1	0	0.5	0	-0.5	-3	0
3	1	-1	-1	0	0	0	-2	0
4	0	1	0	-0.5	0	0.5	-3	0
5	0	-1	1	0.5	-0.5	0	-2	0
6	0	1	0	-0.5	0	-0.5	-4	0
7	1	1	1	0	0	0	0	1

Matrix Operations C1,

C1	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	-1	1	0	0	0	0	-0.5	0.5	0.5	0.5	-0.5	-0.5	0	0	0	-3	0
2	0	-1	1	0	0	-0.5	0.5	0	0	0	0.5	0.5	-0.5	-0.5	0	-4	0
3	1	0	0	-1	0	0.5	-0.5	0	-0.5	0	-0.5	0	0.5	0	0.5	-4	0
4	0	0	0	1	-1	0	0	-0.5	0.5	0	0.5	-0.5	-0.5	0.5	0	-3	0
5	0	1	-1	-1	1	-0.5	0.5	0.5	-0.5	0	0	0	0	0	0	-1	0
6	0	0	0	1	-1	0	0	-0.5	0.5	0	-0.5	0.5	0.5	-0.5	0	-2	0
7	-1	0	1	-1	1	0.5	0	0	0	-0.5	0.5	-0.5	0	0	0	-4	0
8	0	0	0	1	-1	0	0	0.5	-0.5	0	-0.5	0.5	-0.5	0.5	0	-2	0
9	1	0	0	-1	0	-0.5	-0.5	0	0.5	0	0.5	0	0.5	0	-0.5	-3	0
10	0	0	-1	1	1	0	0.5	-0.5	-0.5	0.5	-0.5	-0.5	0	0	0	-1	0
11	0	-1	1	0	0	0.5	-0.5	0	0	0	0.5	0.5	-0.5	-0.5	0	-3	0
12	-1	1	0	0	0	0	0.5	0.5	0.5	-0.5	-0.5	-0.5	0	0	0	-3	0
13	1	0	0	-1	0	-0.5	-0.5	0	-0.5	0	0.5	0	0.5	0	0.5	-2	0
14	0	0	0	1	-1	0	0	-0.5	0.5	0	-0.5	0.5	-0.5	0.5	0	-2	0
15	0	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	-0.5	-0.5	-3	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

Matrix Operations C2,

C2	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	0	-1	0	1	0	0.5	0	-0.5	0	0.5	0	-0.5	-0.5	0	0.5	-5	0
2	0	0	0	-1	1	0	0.5	0.5	0	-0.5	-0.5	0	0.5	-0.5	0	-2	0
3	0	1	0	1	-1	-0.5	0	-0.5	0.5	-0.5	0	0	-0.5	0.5	0	-2	0
4	0	0	0	-1	1	0	0	0.5	-0.5	0	0.5	-0.5	0.5	-0.5	0	-3	0
5	-1	0	0	1	-1	0.5	0.5	0	0	0	-0.5	0.5	-0.5	0.5	0	-4	0
6	0	0	0	-1	1	0	0	-0.5	0.5	0	0.5	-0.5	0.5	-0.5	0	-3	0
7	1	-1	-1	1	0	0	0	0	-0.5	0	0	0.5	0	0.5	-0.5	-2	0
8	-1	1	0	0	0	0	-0.5	0.5	0.5	0.5	-0.5	-0.5	0	0	0	-3	0
9	0	-1	1	0	0	-0.5	0.5	0	0	0	0.5	0.5	-0.5	-0.5	0	-4	0
10	1	0	0	0	0	0.5	-0.5	-0.5	-0.5	0	0	0	0	0	0	-2	0
11	0	1	-1	0	-1	-0.5	0.5	0	0.5	0	-0.5	0	0.5	0	0.5	-4	0
12	0	0	0	-1	1	0	0	0.5	-0.5	0	0.5	-0.5	-0.5	0.5	0	-2	0
13	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	0.5	0	-0.5	-2	0
14	-1	0	1	0	0	0.5	0	0.5	0.5	-0.5	0	0	-0.5	-0.5	0	-3	0
15	1	0	0	0	0	-0.5	-0.5	-0.5	-0.5	0	0	0	0	0	0	-5	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

Matrix Operations C3,

C3	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	0.5	0	-0.5	-4	0
2	0	0	1	-1	0	0	-0.5	0.5	0	-0.5	0.5	0	0	-0.5	0.5	-3	0
3	0	-1	0	0	0	0.5	0	0	0	0.5	-0.5	0.5	0	0	0	-2	0
4	0	0	-1	1	0	0	0.5	-0.5	0	-0.5	0.5	0	0	0.5	-0.5	-2	0
5	0	1	0	0	-1	-0.5	0	0	0.5	0.5	-0.5	0	0	-0.5	0.5	-3	0
6	-1	0	0	0	1	0.5	-0.5	0.5	0	0	0	-0.5	0	0.5	-0.5	-4	0
7	0	0	1	-1	0	0	0.5	-0.5	0	-0.5	0.5	0	0	-0.5	0.5	-3	0
8	0	-1	0	1	0	-0.5	0	0.5	0	0.5	0	0.5	-0.5	0	-0.5	-3	0
9	1	0	0	0	-1	0.5	-0.5	-0.5	0	0	0	-0.5	0	0.5	0.5	-2	0
10	0	0	0	0	1	0	0	0	-0.5	0	0	0.5	0	-0.5	-0.5	-3	0
11	0	1	0	-1	-1	-0.5	0	0.5	0.5	-0.5	0	0	0.5	0.5	0	-3	0
12	-1	0	0	1	0	0.5	0.5	0	-0.5	0	-0.5	0	-0.5	0	0.5	-4	0
13	1	0	0	0	0	-0.5	-0.5	-0.5	0.5	0	0	0	0	0	0	-3	0
14	-1	0	0	0	1	0.5	0.5	0.5	0	0	0	-0.5	0	-0.5	-0.5	-2	0
15	1	0	0	0	0	-0.5	-0.5	-0.5	-0.5	0	0	0	0	0	0	-2	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

For DM_2:

Matrix Operations Main Criteria (C1, C2, C3),

C	v_1	v_2	v_3	I_{12}	I_{13}	I_{23}	a	
1	0	-1	1	-0.5	0.5	0	-4	0
2	0	1	0	0.5	0	-0.5	-4	0
3	1	-1	-1	0	0	0	-2	0
4	0	1	0	-0.5	0	0.5	-2	0
5	0	-1	1	0.5	-0.5	0	-3	0
6	0	1	0	-0.5	0	-0.5	-3	0
7	1	1	1	0	0	0	0	1

Matrix Operations C1,

C1	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	-1	0	1	0	0	-0.5	0	0.5	0.5	0.5	0	0	-0.5	-0.5	0	-3	0
2	0	1	-1	0	0	0.5	-0.5	0	0	0	-0.5	-0.5	0.5	0.5	0	-3	0
3	1	-1	1	0	0	0	0	-0.5	-0.5	0	0.5	0.5	-0.5	-0.5	0	-3	0
4	0	1	-1	-1	0	-0.5	0.5	0.5	0	0	0	-0.5	0	0.5	0.5	-2	0
5	0	0	0	1	-1	0	0	-0.5	0.5	0	-0.5	0.5	0.5	-0.5	0	-2	0
6	0	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	0.5	-0.5	-4	0
7	0	-1	1	-1	0	0.5	-0.5	0.5	0	0	0	0.5	0	-0.5	0.5	-2	0
8	0	0	0	1	-1	0	0	-0.5	0.5	0	0.5	-0.5	-0.5	0.5	0	-1	0
9	-1	1	0	-1	1	0	0.5	0	0	-0.5	0	0	0.5	-0.5	0	-1	0
10	0	0	0	1	-1	0	0	0.5	-0.5	0	-0.5	0.5	-0.5	0.5	0	-1	0
11	0	0	0	0	1	0	0	0	0.5	0	0	-0.5	0	-0.5	-0.5	-5	0
12	1	0	0	-1	-1	-0.5	-0.5	0	0	0	0.5	0.5	0.5	0.5	0	-2	0
13	0	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	-0.5	0.5	-5	0
14	0	0	0	1	-1	0	0	-0.5	0.5	0	-0.5	0.5	-0.5	0.5	0	-1	0
15	0	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	-0.5	-0.5	-1	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

Matrix Operations C2,

C2	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	0	-1	0	0	1	0.5	0	0	-0.5	0.5	-0.5	0	0	-0.5	0.5	-3	0
2	0	1	-1	0	0	-0.5	0.5	0	0	0	0.5	-0.5	-0.5	0.5	0	-2	0
3	0	0	1	0	0	0	-0.5	0	0	-0.5	0	0	0.5	-0.5	0	-4	0
4	0	-1	0	1	-1	0.5	0	-0.5	0.5	0.5	0	0	-0.5	0.5	0	-2	0
5	0	1	-1	0	0	-0.5	0.5	0	0	0	-0.5	0.5	0.5	-0.5	0	-3	0
6	-1	0	1	-1	1	0.5	0	0	0	-0.5	0.5	-0.5	0	0	0	-1	0
7	0	0	0	1	-1	0	0	0.5	-0.5	0	-0.5	0.5	-0.5	0.5	0	-2	0
8	1	0	0	0	0	-0.5	-0.5	-0.5	0.5	0	0	0	0	0	0	-5	0
9	0	-1	-1	0	1	0.5	0.5	0	-0.5	0	0.5	0	0.5	0	-0.5	-3	0
10	-1	0	1	0	0	-0.5	0	0.5	0.5	0.5	0	0	-0.5	-0.5	0	-3	0
11	1	0	0	0	0	0.5	-0.5	-0.5	-0.5	0	0	0	0	0	0	-4	0
12	-1	1	-1	0	0	0	0	0.5	0.5	0	-0.5	-0.5	0.5	0.5	0	-1	0
13	1	0	0	0	0	-0.5	0.5	-0.5	-0.5	0	0	0	0	0	0	-4	0
14	-1	0	1	0	0	0.5	0	0.5	0.5	-0.5	0	0	-0.5	-0.5	0	-2	0
15	1	0	0	0	0	-0.5	-0.5	-0.5	-0.5	0	0	0	0	0	0	-1	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

Matrix Operations C3,

C3	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	0	0	1	0	0	0	-0.5	0	0	-0.5	0	0	0.5	-0.5	0	-3	0
2	0	0	0	0	-1	0	0	0	0.5	0	0	0.5	0	0.5	-0.5	-4	0
3	0	0	-1	1	0	0	0.5	-0.5	0	0.5	-0.5	0	0	-0.5	0.5	-3	0
4	0	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	0.5	-0.5	-1	0
5	-1	0	1	0	-1	0.5	0	0.5	0	-0.5	0	0.5	-0.5	0	0.5	-4	0
6	0	0	-1	0	1	0	-0.5	0	0.5	0.5	0	-0.5	0.5	0	-0.5	-1	0
7	1	-1	0	0	0	0	0.5	-0.5	-0.5	-0.5	0.5	0.5	0	0	0	-2	0
8	0	0	1	0	-1	0	-0.5	0	0.5	0.5	0	-0.5	-0.5	0	0.5	-1	0
9	-1	1	0	-1	1	0	0.5	0	0	-0.5	0	0	0.5	-0.5	0	-3	0
10	1	0	0	1	-1	-0.5	-0.5	0	0	0	-0.5	0.5	-0.5	0.5	0	-2	0
11	0	-1	0	-1	1	0.5	0	0.5	-0.5	0.5	0	0	0.5	-0.5	0	-1	0
12	-1	1	0	1	0	0	0.5	0	0.5	-0.5	0	-0.5	-0.5	0	-0.5	-3	0
13	0	-1	0	0	0	-0.5	0	0	0	0.5	0.5	0.5	0	0	0	-4	0
14	1	0	0	0	0	0.5	-0.5	-0.5	-0.5	0	0	0	0	0	0	-3	0
15	0	1	0	0	0	-0.5	0	0	0	-0.5	-0.5	-0.5	0	0	0	-3	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

For DM_3:

Matrix Operations Main Criteria (C1, C2, C3),

C	v_1	v_2	v_3	I_{12}	I_{13}	I_{23}	a	
1	0	1	-1	0.5	-0.5	0	-4	0
2	1	-1	0	0	0.5	-0.5	-3	0
3	-1	1	1	0	0	0	-3	0
4	1	-1	0	0	-0.5	0.5	-3	0
5	0	1	-1	-0.5	0.5	0	-4	0
6	0	0	1	0	-0.5	-0.5	-4	0
7	1	1	1	0	0	0	0	1

Matrix Operations C1,

C1	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	1	-1	0	0	0	0	0.5	-0.5	-0.5	-0.5	0.5	0.5	0	0	0	-4	0
2	0	1	0	0	0	-0.5	0	0	0	0.5	-0.5	-0.5	0	0	0	-3	0
3	0	0	0	0	-1	0	0	0	0.5	0	0	0.5	0	-0.5	0.5	-2	0
4	0	0	0	-1	1	0	0	0.5	-0.5	0	0.5	-0.5	-0.5	0.5	0	-3	0
5	-1	-1	1	1	0	0	0	0	0.5	0	0	0.5	0	-0.5	-0.5	-1	0
6	0	1	0	0	-1	0.5	0	0	-0.5	-0.5	-0.5	0	0	0.5	0.5	-3	0
7	0	0	0	-1	1	0	0	-0.5	0.5	0	0.5	-0.5	0.5	-0.5	0	-1	0
8	0	0	0	1	0	0	0	0.5	0	0	-0.5	0	-0.5	0	-0.5	-3	0
9	1	-1	0	0	-1	0	-0.5	-0.5	0	0.5	0.5	0	0	0.5	0.5	-2	0
10	0	0	0	-1	1	0	0.5	0.5	0	-0.5	-0.5	0	0.5	-0.5	0	-1	0
11	0	0	0	1	0	0	0	-0.5	0	0	0.5	0	-0.5	0	-0.5	-3	0
12	0	1	0	-1	-1	-0.5	0	0.5	0.5	-0.5	0	0	0.5	0.5	0	-1	0
13	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	-0.5	0	0.5	-1	0
14	0	0	0	-1	1	0	0	0.5	-0.5	0	0.5	-0.5	0.5	-0.5	0	-2	0
15	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	-0.5	0	-0.5	-4	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

Matrix Operations C2,

C2	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	0	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	-0.5	0.5	-1	0
2	0	0	0	1	-1	0	0	-0.5	0.5	0	-0.5	0.5	-0.5	0.5	0	-2	0
3	0	0	-1	-1	1	0	0.5	0.5	-0.5	0.5	0.5	-0.5	0	0	0	-1	0
4	0	0	0	1	-1	0	0	-0.5	0.5	0	-0.5	0.5	0.5	-0.5	0	-3	0
5	-1	0	1	-1	1	0.5	0	0	0	-0.5	0.5	-0.5	0	0	0	-4	0
6	1	-1	0	0	0	0	-0.5	0.5	-0.5	0.5	-0.5	0.5	0	0	0	-1	0
7	0	0	0	1	-1	0	0	-0.5	0.5	0	0.5	-0.5	-0.5	0.5	0	-2	0
8	-1	1	0	0	0	0	0.5	0.5	-0.5	-0.5	-0.5	0.5	0	0	0	-3	0
9	1	-1	-1	0	1	0	0	-0.5	0	0	0.5	0	0.5	0	-0.5	-1	0
10	0	1	0	0	0	-0.5	0	0	0	0.5	-0.5	-0.5	0	0	0	-3	0
11	-1	0	0	0	0	0.5	-0.5	0.5	0.5	0	0	0	0	0	0	-2	0
12	0	-1	1	0	0	-0.5	0.5	0	0	0	0.5	0.5	-0.5	-0.5	0	-1	0
13	1	0	0	0	0	0.5	-0.5	-0.5	-0.5	0	0	0	0	0	0	-2	0
14	-1	1	0	0	0	0	0.5	0.5	0.5	-0.5	-0.5	-0.5	0	0	0	-2	0
15	1	0	0	0	0	-0.5	-0.5	-0.5	-0.5	0	0	0	0	0	0	-3	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

Matrix Operations C3,

C3	v_1	v_2	v_3	v_4	v_5	I_{12}	I_{13}	I_{14}	I_{15}	I_{23}	I_{24}	I_{25}	I_{34}	I_{35}	I_{45}	a	
1	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	0.5	0	-0.5	-4	0
2	0	0	1	-1	0	0	-0.5	0.5	0	-0.5	0.5	0	0	-0.5	0.5	-5	0
3	0	-1	0	0	0	0.5	0	0	0	0.5	-0.5	0.5	0	0	0	-4	0
4	0	0	-1	1	0	0	0.5	-0.5	0	-0.5	0.5	0	0	0.5	-0.5	-1	0
5	0	1	0	0	-1	-0.5	0	0	0.5	0.5	-0.5	0	0	-0.5	0.5	-2	0
6	-1	0	0	0	1	0.5	-0.5	0.5	0	0	0	-0.5	0	0.5	-0.5	-4	0
7	0	0	1	-1	0	0	0.5	-0.5	0	-0.5	0.5	0	0	-0.5	0.5	-3	0
8	0	-1	0	1	0	-0.5	0	0.5	0	0.5	0	0.5	-0.5	0	-0.5	-2	0
9	1	0	0	0	-1	0.5	-0.5	-0.5	0	0	0	-0.5	0	0.5	0.5	-1	0
10	0	1	0	-1	0	-0.5	0	0.5	0	-0.5	0	0.5	0.5	0	-0.5	-3	0
11	-1	0	0	1	0	0.5	0.5	0	-0.5	0	-0.5	0	-0.5	0	0.5	-4	0
12	1	-1	0	0	1	0	-0.5	-0.5	0	0.5	0.5	0	0	-0.5	-0.5	-1	0
13	-1	1	0	0	0	0	0.5	0.5	0.5	-0.5	-0.5	-0.5	0	0	0	-2	0
14	1	0	0	0	-1	-0.5	-0.5	-0.5	0	0	0	0.5	0	0.5	0.5	-1	0
15	0	0	0	0	1	0	0	0	-0.5	0	0	-0.5	0	-0.5	-0.5	-2	0
16	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1

BIOGRAPHICAL SKETCH

The writer has finished his high school education in math / science. Then he completed his B.Sc. on Naval Architecture and Marine Engineering in Istanbul Technical University. His interests and focus are in the areas of multiple criteria decision making in the marine industry.