P-HUB MAXIMAL COVERING PROBLEM AND

EXTENSIONS FOR GRADUAL DECAY FUNCTIONS

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by

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July 2013

I certify that I have read this thesis and that in my opinion it is full adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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ABSTRACT

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M.S. in Industrial Engineering

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Hubs are special facilities that serve as switching, transshipment and sorting nodes in many to many distribution systems. The hub location problem deals with the selection of the locations of hub facilities and finding assignments of demand nodes to hubs simultaneously. The *p*-hub maximal covering problem, that is one of the variations of the hub location problems, aims to find locations of hubs so as to maximize the covered demand that are within the coverage distance with a predetermined number of hubs. In the literature of hub location, *p*-hub maximal covering problem is conducted in the framework of only binary coverage; origin-destination pairs are covered if the total path length is less than coverage distance and not covered at all if the path length exceeds the coverage distance. Throughout this thesis, we extend the definition of coverage and introduce *"partial coverage"* that changes with the distance, to the hub location literature. In this thesis, we study the *p*-hub maximal covering problem for single and multiple allocations and provide new formulations that are also valid for partial coverage. The problems are proved to be NP-Hard. We even show that assignment problem with a given set of hubs for the single allocation version of the problem is also NP-Hard. Computational results for all the proposed formulations with different data sets are presented and discussed.

Keywords: Hub location problem, *p*-hub maximal covering problem, partial coverage

ÖZET

P-ADÜ MAKSİMUM KAPSAMA PROBLEMLERİ VE KADEMELİ FONKSİYONLAR İÇİN GENİŞLETİLMESİ

Meltem Peker
Endüstri Mühendisliği Yüksek Lisans
Tez Yöneticisi: Doç Dr. Bahar Y. Kara
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Ana dağıtım üsleri (ADÜ) çoklu dağıtım sistemlerinde akışların toplandığı ve dağıtıldığı özelleşmiş merkezlerdir. ADÜ yer seçimi problemleri, ADÜ yer seçimlerinin yapılmasını ve talep noktalarının ADÜ'lere atanmasını içermektedir. ADÜ yer seçimi problemlerinin özel bir türü olan *p*-ADÜ maksimum kapsama problemi, belli bir sayıdaki ADÜ ile belli bir mesafe içindeki maksimum talebi karşılamak amacıyla ADÜ'leri yerleştirmeyi hedeflemektedir. Literatürde, ADÜ yer seçimi problemleri sadece *"ikili kapsama"* ile ele alınmıştır; başlangıç ve bitiş talep noktaları arasındaki toplam mesafe kapsama uzaklığından küçük ise bu talep noktaları arasındaki akış tamamen kapsanmış, kapsama uzaklığından büyük ise akış kapsanmamış olarak öngörülmüştür. Tezde, bu verilen tanım esnetilmiştir ve uzaklık artıkça azalan *"kısmi kapsama"*, *p*-ADÜ maksimum kapsama problemlerinde kullanılmıştır. İkili kapsama ve kısmi kapsama ile uygulanabilir tekli ve çoklu atama *p*-ADÜ maksimum kapsama problemleri için yeni matematiksel modeller geliştirilmiş ve *p*-ADÜ maksimum kapsama problemlerinin NP-Zor sınıfına ait olduğu ispatlanmıştır. Önerilen modeller, farklı veri setleri kullanılarak test edilmiş ve sonuçlar kıyaslanmıştır.

Anahtar Kelimeler: ADÜ yer seçimi problemi, *p*-ADÜ maksimum kapsama problemi, kısmi kapsama

To my precious family

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Chapter 1

Introduction

Hubs are special facilities which serve as switching, transshipment and sorting nodes in many large transportation and telecommunication networks. In *"many to many"* distribution systems instead of direct links for each origin-destination (*O-D*) pairs, flows are consolidated at specialized facilities (hubs) and the flows are distributed to their destination points through them. Due to the consolidation of flows at hubs, the number of links in the network is decreased and cost between any two hubs is discounted by exploiting economies of scale.

The hub location problem includes selection of the location of hub facilities and assignment of the demand nodes to these hubs in order to route the flow for each *O-D* pair. For the routing of flows, in the literature two types of assignment structures are defined. In *single allocation*, each demand node is assigned to only one hub; all the incoming and outgoing flows are served through that hub. In *multiple allocation*, there is not a restriction about the assignment of demand nodes to hub locations; the flows can be sent and received through more than one hub for any demand node. Since optimal assignments are affected by the optimal locations of hubs and the optimal locations of hubs are affected by optimal assignments of demand nodes, the decisions are needed to be considered simultaneously.

The hub location problem is first proposed by O'Kelly [1]. Later, Campbell [2] introduces the variants of hub location problem to the literature, and he divides the problem into four categories: *p*-hub median, the uncapacitated hub location, *p*-hub center and hub covering problems. Throughout this thesis, we study the *p*-hub maximal covering problem which is identified as a special case of the hub covering problem in the literature. Hub covering problem finds the minimum number of facilities that is needed to cover all the *O-D* pairs that are within a predefined coverage distance. However, i.e. due to the available amount of budget, covering all *O-D* pairs might not be possible. Thus, instead of covering all *O-D* pairs, maximizing the covered demand with a number of facilities can be aimed: The aim of the *p*-hub maximal covering problem is to maximize the demand covered within a specified critical distance or time with a fixed number of hubs. The detailed explanation about the problem and the variants of the hub location problem are given in Chapter 2.

The existing literature about the *p*-hub maximal covering problems is conducted in the framework of only binary coverage; any *O-D* pair is covered if the length of the path is within the maximum service distance (or time) and it is not covered at all if the length exceeds the maximum service distance. However, in real life there may be some situations where the coverage is not strict as given. Therefore, instead of binary (or constant) coverage, *"partial coverage"*, that changes with the distance, sometimes may yield more realistic solutions. To the best of our knowledge, there is no research about partial coverage in the hub location literature. Thus in this thesis, we introduce a new notion to the hub location literature for coverage and we provide new formulations for p-hub maximal covering problem which can also be applied with a coverage that can be defined via a gradual decay function.

The outline of this thesis is as follows. In Chapter 2, we give a detailed review of hub location problem. We also give a brief review of the covering problem from the general location literature. In Chapter 3, we provide the problem definition and model development for single allocation version of the problem. In Chapter 4, we explain the model development for multiple allocation and give the proposed mathematical formulation. In Chapter 5, we give the computational complexity analysis of the p-hub maximal covering problem. We present the computational results of the new formulation and comparison with the existing ones with both coverage types for single allocation in Chapter 6 and for multiple allocation in Chapter 7. The thesis concludes with some final remarks in Chapter 8.

Chapter 2

Literature Review

In this chapter, we review and classify the hub location problem. We also provide a short review of the covering problem from the general location literature.

Hub location problem involves n demand nodes that exchange flows. The set of origins, destinations and potential hub location sets from the n demand nodes are also identified. For each origin-destination (*O-D*) pair in the network; the characteristics such as flow, cost and time are assumed to be known. Due to the consolidation of demands at hubs, the cost for transferring demands between hubs is discounted by α, $0 \leq \alpha \leq 1$.Also in some variations of the hub location problems, the number of hubs *(p)* is predetermined. The aim of the problem is to find the hub locations and the allocation of demand nodes to hubs simultaneously.

Hub location problem is relatively a new research area in the location literature; the problem is first addressed by Goldman [3]. But, the increase of the research about the problem began with O'Kelly [4] that provides the first mathematical formulation of the hub location problem. The problem given by O'Kelly [4] is later referred as the *single allocation p-hub median problem*. Hub location problem receives an important interest and reputation with the paper of Campbell [2]. As well as improving the formulations of *p*-hub median and uncapacitated hub location problems, he introduces variations of the hub location problem with different objectives. He defines four types of the hub location problem and gives also their linear mathematical formulations: *p*-hub median, the uncapacitated hub location, *p*-hub center and hub covering problems.

The objective of the *p*-hub median problem is to minimize the total transportation cost while satisfying the assignments of the demand nodes to hubs with predetermined number of hubs (*p*). The single assignment version of the problem is formulated by O'Kelly [4] as a quadratic integer program. By studying a real data about airline passenger network, O'Kelly [4] also introduces a data set referred as *Civil Aeronautics Board (CAB)*. As the formulation is quadratic, it is hard to solve the problem optimally, so he provides a heuristic solution methodology to solve the problem with CAB data set. For the formulation of the problem, the network is considered as complete and no direct link between any two non-hub nodes is allowed. To discount the cost for transferring the flow between any two hubs α is introduced. In the formulation, the parameters c_{ij} represents the transportation cost and w_{ij} is the flow from node i to node j. The single assignment *p*-hub median problem given by O'Kelly [4] is as follows:

$$\min \sum_{i} \sum_{j} w_{ij} (\sum_{k} X_{ik} c_{ik} + \sum_{m} X_{jm} c_{jm} + \alpha \sum_{k} \sum_{m} X_{ik} X_{jm} c_{km}) \quad (2.1)$$

$$s.t \qquad X_{ik} \leq X_{kk} \ \ \forall\, i,k \qquad (2.2)$$

$$\sum_{k} X_{ik} = 1 \quad \forall\, i \qquad (2.3)$$

$$\sum_{k} X_{kk} = p \qquad (2.4)$$

$$X_{ik} \in \{0,1\} \ \ \forall\, i,k \qquad (2.5)$$

Let X_{ik} as 1 if node i is assigned to hub k, 0 otherwise. The objective function minimizes the total transportation cost; collections, distributions costs and cost of inter hub connections respectively. Constraint (2.2) satisfies that demand node i can be assigned to node j only if j is a hub. Constraint (2.3) satisfies the single assignment of each demand node and locating exactly p hub is guaranteed with constraint (2.4). Constraint set (2.5) is the domain constraints.

The first linear formulation of the problem is given by Campbell [2] with $O(n^4)$ variables and $O(n^4)$ constraints. But he did not provide any computational results for p-hub median problem. Later, Skorin Kapov [5] propose a new mathematical formulation for the single assignment version of p-hub median problem and conclude that their formulation yields stronger LP bounds than Campbell's formulation [2]. Moreover, they decrease the number constraints to $O(n^3)$ while keeping number of variables at $O(n^4)$. Later, Ernst and Krishnamoorthy [6] propose a new mathematical formulation for the single assignment version of the problem known as a network flow type formulation. They formulate the problem considering the flow on the links that are between the hubs and flow on the links for connecting demand nodes to hubs. They satisfy assignments of demand nodes by flow balance equations at hub nodes. Their formulation requires $O(n^3)$

variables and $O(n^2)$ constraints. In addition, they introduce *Australian Post (AP)* data set and test their formulation by using simulated annealing with this new data set. Ebery [7] achieves to formulate the problem with $O(n^2)$ variables and $O(n^2)$ constraints but the solution quality of the new formulation is not good as that of Ernst and Krishnamoorthy's [6].

The multiple assignment version of the *p*-hub median problem is first formulated by Campbell [8]. Similar to single allocation, Skorin Kapov [5] provide a new formulation which also yields stronger bounds for the LP relaxation of multiple assignment version of the problem. Later, Ernst and Krishnamoorthy [9] provide a better formulation by applying the network flow type formulation that is used in the single assignment version of the *p*-hub median problem. The new formulation is significantly better in terms of solution quality, number of variables and constraints; it requires $O(n^3)$ variables and $O(n^2)$ constraints. Garcia et al. [10] provide a new formulation with $O(n^2)$ variables for multiple assignment. The authors also state that based on this formulation they generate a branch and cut algorithm to solve the large instances that are not solved so far.

The uncapacitated hub location (UHL) problem differs from the *p*-hub median. In *p*-hub median problem there is no fixed cost for the opening hub locations or for establishing links for the assignments of demand nodes to hubs. Also, at UHL problem the number of hubs is not a parameter. The aim of the problem is similar to *p*-hub median; the objective function of UHL problem minimizes total transportation cost and the fixed cost associated with the located hubs. O'Kelly [11] is the first to formulate the UHL problem with quadratic objective function for the single assignment. Later Campbell [2] provides the linear formulations of UHL problem for single and multiple assignment versions. Also, in the same paper he reformulates the problem with adding capacity restriction for the hub nodes. Hamacher et al. [12] find a set of valid inequalities for uncapacitated facility location by using the polyhedral properties of the problem. By lifting the inequalities, they find a set of facet defining constraints for the multiple assignment version of UHL problem. Marin et al. [13] provide a new formulation without necessity of the satisfaction of the triangle inequality and with tighter constraints. They show that their formulation performs better than the other formulations that exist in the literature. For the comparison of them, they used both CAB and AP data sets.

The *p*-hub center and hub covering problems are first addressed by Campbell [2]. The aim of the *p*-hub center problem is to minimize the maximum distance (or time) between all *O-D* pairs while satisfying allocations of the demand nodes with *p* hubs. Kara and Tansel [14] give the proof of NP-Completeness of the single assignment *p*-hub center problem even if the locations of hubs are known. They also provide different linearization for the formulation given in Campbell [2] and they give a new formulation with $O(n^2)$ variables and $O(n^3)$ constraints. Ernst et al. [15] proves that multiple assignment of *p*-hub center problems are NP-Hard even if the discount factor (α) is zero. Also, they give a new formulation for single assignment version with $O(n^2)$ variables and $O(n^2)$ constraints based on the concept of "*radius of hub*". The new formulation requires less solution time and less number of nodes than Kara and Tansel's formulation [14]. They also provide a new mathematical formulation for the multiple assignment version with $O(n^3)$ variables and constraints.

Hub covering problems (analogously covering problems) are generally divided into two categories: set covering hub location problem and *p*-hub maximal covering problem. The aim of the set covering hub location problem is to minimize the number of hubs while satisfying service requirement for all *O-D* pairs; the distance between any *O-D* pair through located hubs should be less than predetermined coverage distance. Different service requirements for coverage of demand nodes are also defined by Campbell [2]. As well as providing quadratic formulations of the problem, he gives the linearization of the set covering hub location problem using different coverage perspectives. Kara and Tansel [16] provide different linearizations for the quadratic formulation of the problem given in Campbell [2]. In addition, they give a new formulation for the single assignment set covering hub location problem, and show that the new formulation performs better in terms of required solution times. They also prove the NP-Hardness of the problem. Later, Wagner [17] provides new formulations for set covering hub location problem. He improves the model given in Kara and Tansel [16] and achieves stronger LP relaxations by ruling out some assignments with preprocessing and aggregating some constraints. Ernst et al. [18] show that the formulation of Wagner [17] can also be tightened by lifting some of the constraints.

In the *p*-hub maximal covering problem, the aim is to maximize the demand covered that satisfy service requirement with a predetermined number of hubs. In this problem covering all *O-D* pairs is not a constraint for feasibility. The maximal covering hub location problem for both allocations is also posed by Campbell [2].

In the literature, hub covering problems have not received enough attention so far as also pointed in the review Alumur and Kara [19]. Moreover, there is not enough research in the maximal covering hub location problem (Karimi and Bashiri, [20]). Therefore, in this thesis we want to study maximal covering hub location problem with single and multiple allocations.

After Campbell [2], different formulations are also proposed for *p*-hub maximal covering problem by researchers. Hwang and Lee [21] develop a new model for single assignment version of the problem with $O(n^4)$ variables and constraints. They also provide two heuristics and the comparison of them with the solution of CPLEX is given. For multiple assignment version of *p*-hub maximal covering problem Weng et al. [22] give a new formulation with $O(n^2)$ variables and $O(n^2)$ constraints. By decreasing the order of the formulation, they state that it requires less CPU time for small scaled problems. The authors also show that multiple allocation *p*-hub maximal covering problem is NP-Hard. Thus, they generate two heuristics to solve the problem; genetic algorithm and tabu search. Qu and Weng [23] work on the solution technique for the problem. They use the formulation of Weng et al. [22] for their solution procedure and solve the problem by using the path relinking method. Zarandi et al. [24] discuss the necessity of covering each demand at least Q times. They model the *p*-hub maximal covering problem with having at least Q-coverage (or known as back-up coverage) and considering mandatory dispersion between hub locations.

In the literature, most of the researchers consider the service requirement as the total length of the path of *O-D* pairs through located hubs. Karimi and Bashiri [20] give the new formulations of hub set covering and *p*-hub maximal covering problems with different coverage types for single and multiple allocations (i.e. if the links on any path are smaller than the predetermined distance, than the *O-D* pair is considered as covered).

Apart from the discussed literature above, there are also some problems in the literature that are related to covering problem or hub covering problem. Tan and Kara [25] study the latest arrival hub covering problem with the computational results for the cargo delivery sector in Turkey. The aim is to minimize the number of hubs by considering departure times of the vehicles. Çetiner et al. [26] combine hub location and routing problem by considering time constraint in postal delivery service with a case study by using the data of Turkish postal delivery system. Mohammadi et al. [27] consider the hub covering problem with the crowdedness or congestion in the system; since hubs cannot serve all the trucks at the same time, the problem is modeled as a queuing system.

A recent review of hub location can be found in Kara and Taner [28]. The authors outline the research on hub location and provide a new taxonomy that serves for the classes of the hub location problem in the form of $\varepsilon/\varphi/\ \kappa/\lambda/\omega$. The fields correspond to objective criterion, allocation structure, capacity, inter-hub connectivity and other restrictions respectively. Objective criteria for *p*-hub median, uncapacitated hub location, *p*-hub center and hub covering problems are denoted as *pH-median*, *fix H-cost*, *pH-center* and *H-cover*. φ denotes the allocation structure as *single* or *multi*. The problems may be capacitated for *nodes* or *arcs*; uncapacitated version of the problems is denoted by U. The inter-hub connectivity can be *full*, or partial (i.e. *star*, *tree*). ω is for the other restrictions for the problems.

For the sake of completeness we also investigate covering problem from the general location literature. Similar to the hub covering location problem, covering problem is divided into two categories in the review of Schilling et al. [29]. Set covering problem is first defined by Hakimi [30] without introducing a mathematical formulation. The aim of the problem is to find the minimum number of vertices to cover all the nodes within a specified maximum distance. For the solution of the problem, he presents a method by using the Boolean function defined over the vertices for generating all coverings; hence enumeration of all feasible solutions is required. The first mathematical formulation of the set covering problem is posed by Toregas et al. [31].

For the second category of the problem, the maximal covering problem is first defined by Church and ReVelle [32]. They discuss that cost estimation for locating public facilities might be difficult, and so minimizing the cost of facilities (or number of facility) might yield unrealistic solutions. Therefore, the authors state that maximizing the population covered might be more realistic and they propose a formulation to maximize the population covered which can be served within a specified service distance (or time). About the extensions of the problems (set covering and maximal covering) more information can be found in a recent review of Farahani et al. [33].

In original maximal covering problems, a node is covered if the distance of a node from the facility is less than β and it is not covered if the distance is bigger than β. However in partial coverage beside the *"fully covered nodes"* (the nodes that their distances from any facility is less than β), there exists *"partially covered nodes"*. Different than binary coverage, in partial coverage generally two limits are defined: a lower limit corresponds for the distance for full coverage and an upper limit that beyond of this any node is not covered at all. For a distance between these limits, the nodes are covered partially depending on a function that changes gradually with the distance.

Church and Roberts [34] gives the first idea of partial coverage and they expand the notion of service coverage. They question that the demand nodes that are being β - ε away from the closest facility is fully covered but the demand nodes that are being $\beta + \varepsilon$ away from the closest facility is not covered at all. *T*hey develop a set of new models using piecewise linear step functions varying with the definition of the coverage; they state that at some cases coverage increases with the increment of the distance. Berman and Krass [35] formulate the same problem by defining k different zones for coverage and for each demand node, k different radii are defined. The function for the problem is defined as a nonincreasing step function; the nodes in the first zone are fully covered and beyond the k^{th} zone are not covered at all. Also for the solution of the problem, they develop greedy heuristic. Then this problem is generalized and applied for all the nonincreasing decay functions by Berman and Krass [36] and their previous paper is given as a special case of this general function. By defining a pair of radii specified for each demand node, they define *"fully covered"*, *"not covered"* and *"partially covered"* terms and they give the new formulation for all the nonincreasing decay functions. The same problem is also modeled in Karasakal and Karasakal [37] and they give a solution procedure based on Lagrangean relaxation. They also give the computational results; by using the randomly generated data and they compare the original maximal covering problem and the partial coverage version of the problem.

To the best of our knowledge, there is no study related with the partial coverage in the hub location problem which might be important in some real life applications of the hub maximal covering problem. Therefore, with this research we introduce the partial coverage to the hub location literature and provide new formulations that are applied to the also partial coverage.

Chapter 3

Single Allocation p-Hub Maximal Covering Problem

In this chapter we first provide the definition of the standard p-hub maximal covering problem. Then we define the partial coverage version which can be considered as an extension of the p-hub maximal covering problem. In Section 3.2 we provide the existing mathematical formulations for single allocation version of the problem. We also analyze the applicability of the existing formulations, which were proposed in the literature for binary coverage, to the partial coverage. In this section, we also propose our formulation for the problem. Our new formulation is applicable to both binary and partial coverage. We later show that the proposed formulation apart from being applicable to partial coverage performs significantly better in terms of CPU time. In Section 3.3, we propose valid inequalities for the proposed formulation.

For the rest of this thesis, we adopt the taxonomy explained in [28]. Since there is no short-hand notation for objective criterion ε for p-hub maximal covering problem, we used *max H-cover* for p-hub maximal covering problem and single allocation p-hub maximal covering problem is denoted as *max H-cover/single/U/full/{binary,partial} coverage*. *max H-cover* is the objective function of the problem, *single* refers to allocation type. *U* is for the uncapacitated version of the problem and *full* represents the full inter-hub connectivity case. Also we include *{binary,partial} coverage* to the taxonomy since the coverage type is an important factor for this research.

3.1 Problem Definition and Motivation

Standard hub covering problem (also known as hub set covering problem) aims to minimize the number of located hubs with allocation of nonhub nodes to hubs while satisfying service requirement. The service requirement is satisfied if distance (or time) of any path through located hubs is within a predetermined coverage distance. In the literature, generally three types of service requirements are used which are given first by Campbell [2]. d_{ij} represents the distance of each *O-D* pair and α is the economies of scale factor which is used to discount the distance between hubs where $0 \leq \alpha \leq 1$. β is the predetermined value for the maximum service distance. The types of coverage are:

- If total cost of the path length of any *i-j* (*O-D*) pair using hubs *k* and *m* is smaller than β, ($d_{ik} + \alpha d_{km} + d_{mj} \leq \beta$), the demand between the pair is covered.
- If the cost of the length of each link of the path *i-j* using hubs *k* and *m* is smaller than β, ($max(d_{ik}, \alpha d_{km}, d_{mj}) \leq \beta$), then the demand between the pair *i-j* is covered.
- If the cost of the length of links of the path *i-j* from nonhub nodes *i* and *j* to hubs *k* and *m* respectively is smaller than β, ($max(d_{ik}, d_{mj}) \leq \beta$), then the demand between the *i-j* pair is covered.

In the hub covering problem, covering all *O-D* pairs is required to satisfy the service requirement and for service requirement constraint, generally one of the given coverage definitions is used. Similarly, in the *p*-hub maximal covering problem, one of them is also used for service requirement constraint. But different than hub covering problem, covering all *O-D* pairs are not mandatory. The objective of the problem is to maximize the covered demands which are within the maximum service distance (β), while satisfying allocations of nonhub nodes with a fixed number of hubs. In the rest of this thesis, the first type of coverage given above is used for service requirement constraint. But other coverage types can also be used by only adapting the coverage definition that is used in any one of the models.

In the *p*-hub maximal covering problem, the coverage is considered as binary in the literature: any *O-D* pair is covered if the length of the path is within coverage distance (β), and it is not covered at all if the length exceeds β. However, such an assumption is not always reasonable and does not always yield rational solutions in real life applications; there may be some situations where the coverage is not strict as given. The first deficiency of binary coverage is the coverage may change drastically even with the small increment of β, i.e. if the length of a path is $\beta - \varepsilon$ it is considered as "*fully covered*" but if length is $\beta + \varepsilon$ it is considered as "*not covered*". Secondly, binary coverage does not distinguish the value of the coverage with the distance. Apart from these extreme points, in real life cases there also might be some zones that the coverage for each zone may not be the same. While the nearest zone's demand is considered as *fully covered*, the farthest zone's demand can only be covered partially. Therefore, instead of binary (or constant) coverage, "*partial coverage*", that changes with the distance, sometimes may yield more realistic solutions. For instance, in small package delivery sector (or cargo delivery sector), customers are generally time sensitive and they are looking for fast and on-time delivery services. Therefore, in today's competitive environment, companies try to decrease the time frame that packages are guaranteed to be delivered because customers generally have tendency to send their cargos with the company that has less time frame for service. But in order to decrease the overall time frame, some of the customers will not be served at all. In this situation, with binary coverage, only nearest zone's demand can be covered and there is no service for uncovered customers, therefore, the company loses all of the cargo of these customers. But with partial coverage case, the customers that are not in the nearest zone can now be served with a guarantee of longer time frame. Although, there is a service, due to the other factors (i.e. being less time sensitive or higher time frame) only some of these customers will choose to deliver their goods with that company. In the hub location literature, this issue has not been considered before. Thus, we introduce the partial coverage to the hub location problem for the *p*-hub maximal covering problem where the "*coverage*" can be defined with a gradual decay function.

3.2 Model Development for Single Allocation *p*-Hub Maximal Covering Problem

In this section we provide the existing models for single allocation and analyze their applicability to the partial coverage. We also provide the new mathematical formulation for single allocation which is also applicable to the partial coverage.

Let *N* be the node set, $H \subseteq N$ be the hub set and the graph be complete and undirected. In the *p*-hub maximal covering problem there is flow of demand (w_{ij}) between each *O-D* pair *i-j* such that $\forall\, i,j \in N$. Also, c_{ij}^{km} represents the "*cost*" of the total path length from origin node *i* to destination node *j* using hubs *k* and *m* respectively, such that $c_{ij}^{km} = \eta d_{ik} + \alpha d_{km} + \delta d_{jm}\ \forall\, i,j \in N, \forall\, k,m \in H$. For the inter hub connection, the cost of the distance between two hubs *k* and *m* is discounted by α. η is the transportation rate for collection from an origin to a hub and δ is the transportation rate for distribution from a hub to a destination, where generally $\alpha \leq \eta$ and $\alpha \leq \delta$. In the *p*-hub maximal covering problems, satisfying service requirement within a predetermined “time” bound might be more meaningful. In the rest of the thesis we use “*cost*”, “*distance*” and “*time*” interchangeably.

β_{ij} is the maximum allowable service distance (coverage distance) for each *O-D* pair *i-j* and *p* is the number of hubs to be located. For binary coverage, we define a new binary parameter a_{ij}^{km} to decide whether *O-D* pair (*i-j*) is covered by using hubs k and *m* respectively or not:

$$a_{ij}^{km} = \begin{cases} 1 & if\ c_{ij}^{km} \leq \beta_{ij} \\ 0 & otherwise \end{cases} \quad \forall\ i,j \in N \text{ and } \forall\ k,m \in H \qquad (3.1)$$

For the partial coverage case, only the definition of a_{ij}^{km} changes. Also, there is a new parameter for the upper bound, γ_{ij}, for the service level that can be provided partially.

Then, a_{ij}^{km} is as follows:

$$a_{ij}^{km} = \begin{cases} 1 & if\ c_{ij}^{km} \leq \beta_{ij} \\ f\left(c_{ij}^{km}\right) & if\ \ \beta_{ij} < c_{ij}^{km} \leq \gamma_{ij} \\ 0 & otherwise \end{cases} \quad \forall\ i,j \in N \text{ and } \forall\ k,m \in H \qquad (3.2)$$

where *f* is any nonincreasing decay function and the range of the function f is (0,1).

Note that, other types of coverage defined by Campbell [2] can also be used by only changing the definition of the parameter c_{ij}^{km}.

In the single allocation, any nonhub node should be assigned to exactly one hub to send and receive the flow. The first attempt to formulate the problem is given by Campbell [2] with the following linear mathematical formulation:

$Campbell$ [2] $$\max \sum_{i \in N} \sum_{j \in N} \sum_{k \in H} \sum_{m \in H} a_{ij}^{km} w_{ij} Y_{ijkm} \qquad (3.3)$$

$$s.t \qquad \sum_{k \in H} H_k = p \qquad (3.4)$$

$$\sum_{k \in H} \sum_{m \in H} Y_{ijkm} = 1 \qquad \forall\, i, j \in N \qquad (3.5)$$

$$X_{ik} \leq H_k \qquad \forall\, i \in N, k \in H \qquad (3.6)$$

$$\sum_{j \in N} \sum_{m \in H} (w_{ij} Y_{ijkm} + w_{ji} Y_{jimk}) = \sum_{j \in N} (w_{ij} + w_{ji}) X_{ik} \qquad \forall\, i \in N, k \in H \qquad (3.7)$$

$$H_k \in \{0,1\} \quad \forall\, k \in H \qquad (3.8)$$

$$0 \leq Y_{ijkm} \leq 1 \qquad \forall\, i, j \in N, \forall\, k, m \in H \qquad (3.9)$$

$$X_{ik} \in \{0,1\} \quad \forall\, i \in N, \forall\, k \in H \qquad (2.5)$$

The binary variable H_k takes 1 if a hub is located at node k and 0 otherwise. Similarly, X_{ik} takes 1 if node i is assigned to a hub located at node k and 0 otherwise. Y_{ijkm} is the fraction of coverage from origin node i to destination node j using hubs located at nodes k and m respectively. Objective function maximizes covered demand of *O-D* pairs. Constraint (3.4) guarantees that exactly p hubs are opened. Constraint (3.5) assures that the flow for every *O-D* pair is routed via some hub pair. Constraint (3.6) satisfies that node i can be assigned to node k if k is a hub. Constraint (3.7) guarantees the single allocation of each node using flow balance equality. Constraints (3.8), (3.9) and (2.5) are the domain constraints.

Since the model is formulated with route formulation (from origin node i to destination node j using hubs k and m respectively) it has $O(n^4)$ variables and $O(n^2)$ constraints.

The second formulation given by Hwang and Lee [21] has a similar modeling concept with that of Campbell [2]. They formulate the problem using the route formulation idea; they also keep track of the route for each *O-D* pair. The formulation of Hwang and Lee [21] is as follows:

$$\textit{Hwang and Lee } [21] \qquad \max \sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{m\in H} a_{ij}^{km} w_{ij} Y_{ijkm} \qquad (3.3)$$

$$s.t \qquad X_{ik} \leq X_{kk} \quad \forall\, i \in N, \forall\, k \in H \qquad (2.2)$$

$$\sum_{k\in H} X_{ik} = 1 \quad \forall\, i \in N \qquad (2.3)$$

$$\sum_{k\in H} X_{kk} = p \qquad (2.4)$$

$$2\, Y_{ijkm} \leq X_{ik} + X_{jm} \quad \forall\, i,j \in N, \forall\, k,m \in H \qquad (3.10)$$

$$Y_{ijkm} \in \{0,1\} \quad \forall\, i,j \in N, \forall\, k,m \in H \qquad (3.11)$$

$$X_{ik} \in \{0,1\} \quad \forall\, i \in N, \forall\, k \in H \qquad (2.5)$$

Instead of using two decision variables, Y_{ijkm} and H_k, they only define one binary decision variable addition to the X_{ik}: Y_{ijkm} takes 1 if from origin node *i* to destination node *j* the route uses hubs located at nodes *k* and *m* respectively, and 0 otherwise. So, it is the binary version of (3.9). The other decision variable X_{ik} is the same as that is of Campbell's formulation [2]. The aim of the constraints and the objective function are the same, the only difference is for guaranteeing the single assignments of nonhub nodes. Instead of constraint (3.7) in Campbell's [2] formulation, they use constraint (2.3) to satisfy the single allocations of nonhub nodes. The formulation has $O(n^4)$ variables and $O(n^4)$ constraints.

Although in both of the papers (Campbell, [2] and Hwang and Lee, [21]) only binary coverage is aimed, the formulations can easily be adapted to the partial coverage case.

In the papers, for the coverage of the routes (a_{ij}^{km}) only first definition (3.1) is given. However, instead of using (3.1), for the definition of a_{ij}^{km}, we can use (3.2) without changing anything else in the models. Thus, we can use both of the formulations for the *max H-cover/single/U/full/partial coverage*.

We propose a completely new formulation which has a different modeling perspective than the ones in the literature. We do not need to keep track of the route assignment for *O-D* pairs to hubs with the new formulation. Decision variables for assignments of demand nodes to hubs are adequate to calculate fraction of the coverage of each *O-D* pairs. Additionally, the coverage parameter (a_{ij}^{km}) takes place at the constraint level instead of at the objective function, therefore the number of terms at the objective function decreases to N^2 from N^4. The proposed formulation is as follows:

$$Peker\ and\ Kara\ (SA1) \qquad max \sum_{i\in N}\sum_{j\in N} w_{ij} Z_{ij} \qquad (3.12)$$

$$s.t \qquad Z_{ij} \le \sum_{k\in H} a_{ij}^{km} X_{ik} + \lambda_{ij}\left(1 - X_{jm}\right) \quad \forall\, i,j \in N, \forall\, m \in H \qquad (3.13)$$

$$X_{ik} \le X_{kk} \quad \forall\, i \in N, \forall\, k \in H \qquad (2.2)$$

$$\sum_{k\in H} X_{ik} = 1 \quad \forall\, i \in N \qquad (2.3)$$

$$\sum_{k\in H} X_{kk} = p \qquad (2.4)$$

$$X_{ik} \in \{0,1\} \quad \forall\, i \in N, \forall\, k \in H \qquad (2.5)$$

$$Z_{ij} \ge 0\ \forall\, i,j \in N \qquad (3.14)$$

The decision variable X_{ik} is same as in previous formulations; it takes 1 if node i is assigned to hub located at node k, and 0 otherwise. Z_{ij} is the fraction of coverage that is routed from origin node i to destination node j. Constraints (2.2)-(2.5) are the standard hub covering assignment constraints that are given in the previous formulations. Constraint (3.13) calculates the fraction of coverage of the *O-D* pair *i-j* with correct allocation of X_{ik} (origin node i to hub k) and X_{jm} (destination node j to hub m). Also to tighten the constraint we add $\lambda_{ij} = \max_{k,m\in H}\{a_{ij}^{km}\}$ to constraint (3.13). The aim of the objective function is to maximize the covered demands between every *O-D* pairs.

The new formulation is readily applicable to the partial coverage case. The proposed model has also less number of variables and constraints; $O(n^2)$ variables and $O(n^3)$ constraints.

The proposed formulation can also be improved for the symmetric distance matrices by changing constraint (3.13) to

$$Z_{ij} \leq \sum_{k\in H} a_{ij}^{km} X_{ik} + \lambda_{ij}(1 - X_{jm}) \quad \forall\, i,j \in N: i \leq j, \forall\, m \in H \qquad (3.15)$$

and that results with the reduction of more than half of the constraints from the constraint (3.13). Similarly, the objective function can also be improved for the symmetric flow data set and (3.12) can be changed with

$$max \sum_{i\in N} \sum_{j\in N: i\leq j} (w_{ij} + w_{ji}) Z_{ij} \qquad (3.16).$$

3.3 Strengthening the Proposed Formulations

In this section we drive several valid inequalities for *max H-cover/single/U/full/{binary, partial} coverage*.

Proposition 3.1: The following inequality is valid for single allocation *p*-hub maximal covering problem.

$$Z_{ij} \leq 1 \quad \forall\, i,j \in N \qquad (3.17)$$

Proof: In the formulation, Z_{ij} stands for the fraction of the coverage of flow between *O-D* pairs *i* and *j*. Therefore, the maximum value that it can take is one which corresponds to the full coverage of the *O-D* pair. However, the LP relaxation of the problem might take any value, which can be bigger than one. Therefore, to restrict Z_{ij} and thus to improve the solution quality we can add the constraint to the formulation.

Proposition 3.2: Inequality

$$Z_{ij} \geq \sum_{k \in H} a_{ij}^{km} X_{ik} + \left(X_{jm} - 1\right) \quad \forall\, i,j \in N, \forall\, m \in H \qquad (3.18)$$

is valid for the formulation.

Proof: Due to the constraint (2.3), ∃ a node *s* such that $X_{is} = 1$ and $X_{it} = 0\ \forall\, t \neq s$. Thus, $\sum_k a_{ij}^{km} X_{ik} = a_{ij}^{sm}$. Then, if destination node *j* is assigned to hub *m* ($X_{jm} = 1$), then (3.18) simplifies to $Z_{ij} \geq a_{ij}^{sm}$ and it is the correct coverage fraction via hubs *s* and *m*. If $X_{jm} = 0$, the inequality becomes $Z_{ij} \geq a_{ij}^{sm} - 1$ is found and it becomes a redundant constraint since $a_{ij}^{sm} \leq 1 \quad \forall i,j,m.$

Proposition 3.3: The following inequality is also valid.

$$Z_{ij} \leq \left(\sum_{m \in H} a_{ij}^{km} - \lambda_{ij}\right) X_{ik} + \lambda_{ij} \quad \forall\, i,j \in N, \forall\, k \in H \qquad (3.19)$$

Proof: If $X_{ik} = 0$, (3.19) becomes $Z_{ij} \leq \lambda_{ij}$ which is the valid inequality since λ_{ij} is the maximum coverage that can be for *O-D* pair. Due to the constraint (2.3), ∃ a node *s* such that $X_{is} = 1$, then the constraint simplifies to $Z_{ij} \leq \sum_m a_{ij}^{sm}$ and it is valid since $a_{ij}^{sm} \leq \sum_m a_{ij}^{sm}\ \forall\, i, j, m$.

Note that, even if (3.19) is stronger than (3.17), (3.17) can still strengthen the formulation by the same reason; the LP relaxation of (3.19) can yield fractional solutions and thus Z_{ij} might take any value bigger than one. The computational results for valid inequalities are given in Chapter 6. Based on the results, we decided to add (3.17) and (3.19) for computational results.

The improved formulation for symmetric distance matrices is named as *SA2* with the following constraints and objective function. We also reduce number of constraints of (3.17) and (3.19) by adding $i \leq j$ to the inequalities:

$$Peker\ and\ Kara\ (SA2) \qquad max \sum_{i \in N} \sum_{j \in N: i \leq j} (w_{ij} + w_{ji}) Z_{ij} \qquad (3.16)$$

$$s.t \qquad X_{ik} \leq X_{kk} \ \ \forall\, i \in N, \forall\, k \in H \qquad (2.2)$$

$$\sum_{k \in H} X_{ik} = 1 \quad \forall\, i \in N \qquad (2.3)$$

$$\sum_{k \in H} X_{kk} = p \qquad (2.4)$$

$$Z_{ij} \leq \sum_{k \in H} a_{ij}^{km} X_{ik} + \lambda_{ij}\left(1 - X_{jm}\right) \quad \forall i, j \in N: i \leq j, \forall\, m \in H \qquad (3.15)$$

$$Z_{ij} \leq \left(\sum_{m \in H} a_{ij}^{km} - \lambda_{ij} \right) X_{ik} + \lambda_{ij} \quad \forall\, i, j \in N: i \leq j, \forall\, k \in H \qquad (3.19)$$

$$Z_{ij} \leq 1 \quad \forall\, i, j \in N: i \leq j \qquad (3.17)$$

$$X_{ik} \in \{0,1\} \quad \forall\, i \in N, \forall\, k \in H \qquad (2.5)$$

$$Z_{ij} \geq 0\ \forall\, i, j \in N \qquad (3.14)$$

Chapter 4

Multiple Allocation *p*-Hub Maximal Covering Problem

In this section, we first provide the existing and the new formulations for *max H-cover/ multi/U/full/binary coverage*. We also discuss the applicability of the models to partial coverage. In Section 4.2 we propose several valid inequalities for the formulation.

4.1 Model Development for Multiple Allocation *p*-Hub Maximal Covering Problem

In the multiple allocation p-hub maximal covering problem, the allocation of nonhub nodes are not restricted and any nonhub node can be served by more than one hub. Campbell [2] poses the first linear formulation of the problem.

$Campbell$ [2]

$$\max \sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{m\in H} a_{ij}^{km} w_{ij} Y_{ijkm} \qquad (3.3)$$

$$s.t \qquad \sum_{k\in H} H_k = p \qquad (3.4)$$

$$\sum_{k\in H}\sum_{m\in H} Y_{ijkm} = 1 \qquad \forall\, i,j \in N \qquad (3.5)$$

$$Y_{ijkm} \le H_k \qquad \forall\, i,j \in N, \forall\, k,m \in H \qquad (4.1)$$

$$Y_{ijkm} \le H_m \qquad \forall\, i,j \in N, \forall\, k,m \in H \qquad (4.2)$$

$$H_k \in \{0,1\} \quad \forall\, k \in H \qquad (3.8)$$

$$0 \le Y_{ijkm} \le 1 \qquad \forall\, i,j \in N, \forall\, k,m \in H \qquad (3.9)$$

The notion of the given model is very similar to the Campbell's [2] formulation for the single assignment version, so objective function and constraints are similar. Since assignment to exactly one hub location is not necessary, the assignment variable X_{ik} is not used for the multiple version of the problem. Thus, for guaranteeing that only hubs are used for the route assignments of *O-D* pairs, instead of constraint (3.6), constraints (4.1) and (4.2) are added. The constraints satisfy that node *k* and *m* can be used in the route from origin node *i* to destination node *j* only if *k* and *m* are hubs. The formulation has $O(n^4)$ constraints and $O(n^4)$ variables since it is modeled with considering route for each *O-D* pair.

Similar to the Campbell's formulation [2] in the single allocation version, multiple allocation formulation can also be applicable to the partial coverage case by only changing definition of the coverage of the routes (a_{ij}^{km}). After changing only the definition we can use the model without changing anything else for the *max H-cover/ multi/U/full/partial coverage*.

Weng et al. [22] have a different formulation for the *max H-cover/multi/U/full/binary coverage*. They neither keep track the route for each *O-D* pair nor the assignments of the nonhub nodes to hubs. They only keep track the coverage for *O-D* pairs that can be covered with located hubs. The formulation is:

$$Weng\ et\ al.[22] \qquad \max \sum_{i \in N} \sum_{j \in N} w_{ij} U_{ij} \qquad (4.3)$$

$$s.t \qquad \sum_{k \in H} H_k = p \qquad (3.4)$$

$$U_{ij} \leq \sum_{k \in H} \sum_{m \in H} a_{ij}^{km} W_{km} \qquad \forall\, i, j \in N \qquad (4.4)$$

$$H_k + H_m \geq 2W_{km} \qquad \forall\, k, m \in H \qquad (4.5)$$

$$H_k \in \{0,1\} \quad \forall\, k \in H \qquad (3.8)$$

$$W_{km} \in \{0,1\} \ \forall\, k, m \in H \ \ and \ \ U_{ij} \in \{0,1\} \ \forall\, i, j \in N \qquad (4.6)$$

The decision variable U_{ij} takes 1 if the *O-D* pair *i-j* is covered by located hubs otherwise it is 0. Let H_k be 1 if a hub is located hub *k*, 0 otherwise. W_{km} is 1 if both nodes *k* and *m* are selected as hubs, 0 otherwise. Objective function maximizes the covered demand of *O-D* pairs by located hubs. Constraint (4.4) assures that *O-D* pair *i-j* can be covered if there exists two hubs (or same hub i.e. W_{kk}) that cover the path. Constraint (4.5) ensures if that W_{km} can be 1 only if H_k and H_m are 1. Constraint (4.6) is for the integrality of the decision variables. Since it does not include the path for each *O-D* pairs, the formulation has $O(n^2)$ constraints and $O(n^2)$ variables.

We remark that, in the formulation of Weng et al. [22], U_{ij} is defined as a binary variable and it is restricted to 0 and 1. However, by relaxing the variable and adding the constraint $U_{ij} \leq 1$ to the formulation, optimal solution can also be attained.

This formulation (Weng et al. [22]) cannot be applicable to the partial coverage since the model cannot calculate the correct coverage of *O-D* pair *i-j* for partial coverage case. U_{ij} is a binary variable so the formulation cannot take into account the partially covered nodes. Moreover, even if we change U_{ij} to a fractional decision variable, the formulation again may not calculate the correct coverage and there is also a possibility that it yields incorrect coverage of the *O-D* pairs. Thus for the multiple allocation version of the *p*-hub maximal covering problem, for the partial coverage case, we can only use the Campbell's formulation [2] from the literature.

We propose a new formulation that is different than both the formulations given above. Let X_{ijk} be 1 if the first hub of *O-D* pair *i-j* is k, and Y_{ijm} be 1 if the second hub of the *O-D* pair *i-j* is *m*. H_k takes 1 if node *k* is selected as a hub, otherwise it is zero. Z_{ij} is the fraction of the coverage that is routed from origin node i to destination node *j*. The proposed formulation is:

$$Peker\ and\ Kara\ \ (MA1) \qquad max \sum_{i \in N} \sum_{j \in N} w_{ij} Z_{ij} \tag{3.12}$$

$$s.t \qquad \sum_{k \in H} H_k = p \tag{3.4}$$

$$Z_{ij} \leq \sum_{k \in H} a_{ij}^{km} X_{ijk} + \lambda_{ij}\left(1 - Y_{ijm}\right) \qquad \forall\ i, j \in N, \forall\, m \in H \tag{4.7}$$

$$Z_{ij} \leq \sum_{m \in H} a_{ij}^{km} Y_{ijm} + \lambda_{ij}\left(1 - X_{ijk}\right) \qquad \forall\ i, j \in N, \forall\, k \in H \tag{4.8}$$

$$\sum_{k\in H} X_{ijk} = 1 \quad \forall\, i,j \in N \tag{4.9}$$

$$\sum_{m\in H} Y_{ijm} = 1 \quad \forall\, i,j \in N \tag{4.10}$$

$$X_{ijk} \le H_k \quad \forall\, i,j \in N, \forall\, k \in H \tag{4.11}$$

$$Y_{ijm} \le H_m \quad \forall\, i,j \in N, \forall\, m \in H \tag{4.12}$$

$$X_{ijk} \in \{0,1\} \quad \forall\, i,j \in N, \forall\, k \in H \tag{4.13}$$

$$Y_{ijm} \in \{0,1\} \quad \forall\, i,j \in N, \forall\, m \in H \tag{4.14}$$

$$H_k \in \{0,1\} \quad \forall\, k \in H \tag{3.8}$$

$$Z_{ij} \ge 0 \;\forall\, i,j \in N \tag{3.14}$$

Objective function maximizes the covered demand of all *i-j* pairs. Constraint (4.7) and (4.8) calculates the fraction of coverage of *O-D* pairs *i-j* using correct route allocations of X_{ijk} and Y_{ijm}. Constraint (4.9) guarantees that each path from origin node i to destination node *j* can be assigned to only one hub *k* as the first hub. Similarly, constraint (4.10) satisfies the same route can be assigned to only one hub *m* as the second hub. Constraint (4.11) and (4.12) satisfy that, the path *i-j* can be assigned to nodes *k* and *m* only if *k* and *m* are hubs, respectively. Constraints (4.13) and (4.14) are the domain constraints.

The new formulation can be used with partial coverage defined in (3.2) for asymmetric data sets. For symmetric data sets, we can add $i \le j$ to (3.12) and we can remove one set of binary variables (Y_{ijm}), that is for the route allocations and so almost half of the constraints are removed. Then instead of (4.7) and (4.8), the following can be used:

$$Z_{ij} \le \sum_{k\in H} a_{ij}^{km} X_{ijk} + \lambda_{ij}\left(1 - X_{jim}\right) \quad \forall\, i,j \in N, m \in H : i \le j \tag{4.15}$$

4.2 Strengthening the Proposed Formulations

Similar perspectives that are used for generating valid inequalities in Section 3.3 can be also applied to the *max H-cover/multi/U/full/{binary, partial} coverage*.

Inequality (3.17) is also valid for multiple allocation version of the problem.

Proposition 4.1: The following inequality is valid for the formulation MA1:

$$Z_{ij} \geq \sum_{k \in H} a_{ij}^{km} X_{ijk} + \left(Y_{ijm} - 1\right) \quad \forall\, i,j \in N, \forall\, m \in H \qquad (4.16)$$

Proof: Due to constraint (4.9), $\exists\ s$ such that $X_{ijs} = 1$ and $X_{ijt} = 0\ \forall\, t \neq s$. Thus, $\sum_k a_{ij}^{km} X_{ijk} = a_{ij}^{sm}$. Then if $Y_{ijm} = 1$, (4.16) simplifies to $Z_{ij} \geq a_{ij}^{sm}$ which is the coverage of *O-D* pairs via hubs s and m. If $Y_{ijm} = 0$, then $Z_{ij} \geq a_{ij}^{sm} - 1$ and it is a redundant constraint since $a_{ij}^{sm} \leq 1 \quad \forall\, i,j,m$ given that the range of function f is (0,1).

Proposition 4.2: The inequality

$$Z_{ij} \leq \left(\sum_{m \in H} a_{ij}^{km} - \lambda_{ij}\right) X_{ijk} + \lambda_{ij} \quad \forall\, i,j \in N, \forall\, k \in H \qquad (4.17)$$

is valid for *max H-cover/multi/U/full/{binary, partial} coverage*.

Proof: (4.17) is similar to the inequality (3.19) for *max H-cover/single/U/full/{binary, partial} coverage*. So the same proof also holds for the inequality (4.17) with X_{ijk} instead of X_{ik} in (3.19).

The computational results for those valid inequalities are given in Chapter 6. Based on the results, we decided to add (3.17) and (4.17) for computational results. For symmetric data sets, similar to the single allocation version of the problem, we can remove half of the constraints from (3.17) and (4.17) by adding $i \leq j$ to them. Then the formulation for symmetric data sets that is used for computational analysis is as follows:

$$Peker\ and\ Kara\ (MA2) \quad max \sum_{i \in N} \sum_{j \in N: i \leq j} (w_{ij} + w_{ji}) Z_{ij} \qquad (3.16)$$

$$s.t \qquad \sum_{k \in H} H_k = p \qquad (3.4)$$

$$\sum_{k \in H} X_{ijk} = 1 \quad \forall\, i, j \in N \qquad (4.9)$$

$$X_{ijk} \leq H_k \;\; \forall\, i, j \in N, \forall\, k \in H \qquad (4.11)$$

$$Z_{ij} \leq \sum_{k \in H} a_{ij}^{km} X_{ijk} + \lambda_{ij}(1 - X_{jim}) \quad \forall\, i, j \in N: i \leq j, \forall\, m \in H \qquad (4.15)$$

$$Z_{ij} \leq \left(\sum_{m \in H} a_{ij}^{km} - \lambda_{ij} \right) X_{ijk} + \lambda_{ij} \quad \forall\, i, j \in N: i \leq j, \forall\, k \in H \qquad (4.17)$$

$$Z_{ij} \leq 1 \quad \forall\, i, j \in N: i \leq j \qquad (3.17)$$

$$H_k \in \{0,1\} \;\; \forall\, k \in H \qquad (3.8)$$

$$Z_{ij} \geq 0\ \forall\, i, j \in N \qquad (3.14)$$

$$X_{ijk} \in \{0,1\} \;\; \forall\, i, j \in N, \forall\, k \in H \qquad (4.13)$$

Chapter 5

Complexity Analysis

In this chapter, we show that both *max H-cover/single/U/full/binary coverage* and *max H-cover/multi/U/full/binary coverage* are NP-Hard by reduction from the maximum coverage problem which is shown to be NP-Hard. Since binary coverage is a special case of partial coverage, the partial coverage variations of the problems are also NP-Hard. NP-Hardness of the multiple allocation version of the problem is known from Weng et al. [22], but it is not shown for single allocation in the related literature. We also demonstrate an alternative proof for the both allocation versions of p-hub maximal covering problem. As a special case of the p-hub maximal covering problem, NP-Hardness of allocation problem of single allocation is shown using transformation to hub center problem.

Proposition 5.1: *max H-cover/single/U/full/binary coverage* is NP-Complete even if *α=0, δ=0.*

Decision version of the *max H-cover/single/U/full/binary coverage*:

Instance: $G = (V, E), \eta, \alpha, \delta \geq 0$, $p \leq |V|$, β represents coverage distance, weight w_{ij} $\forall\, i, j \in V$, distance d_{ij} $\forall\, i, j \in V$ and $T \geq 0$.

Question: Is there a set of vertices $H \subseteq V$ and $|H| = p$ with an assignment vector u, where $u_i \in H$, such that $\sum_{i,j \in S} w_{ij} \geq T$ where S is a set of vertices with property $\eta d_{iu_i} + \alpha d_{u_i u_j} + \delta d_{u_j j} \leq \beta$ $\forall\, i, j \in S$ and $u_i, u_j \in H$?

Decision version of the maximum coverage problem (MCP):

Instance: $G' = (V', E')$, $p \leq |V'|$, β represents coverage distance, weight w_i' $\forall\, i \in V'$, distance d_{ij} $\forall\, i, j \in V'$ and $\mathrm{T}' \geq 0$.

Question: Is there a set of vertices $H' \subseteq V'$ and $|H'| = p$ such that $\sum_{\substack{i: \exists j \in H' \\ d_{ij} \leq \beta}} w_i \geq T'$?

Proof: *max H-cover/single/U/full/binary coverage* is in NP because finding the coverage of a given set *H* and assignment vector *u* can be done in polynomial time. To show NP-Completeness of the problem we can reduce the maximum coverage problem to *max H-cover/single/U/full/binary coverage* in polynomial time. For maximum coverage problem, consider an arbitrary instance of the graph *G'* such that *G'= (V', E')* where vertices *V'* denote the set of customers and potential sites for the facilities to be located. d_{ij} represents distances for edge set *E'* such that *i, j* $\in$ *V'*. Let w_i' be the demand of the customer *i* for each *i* $\in$ *V'* and if $d_{ij} \leq \beta$ *for* $i \in V'$, $j \in H'$ then w_i' is covered. At most *p* facilities can be located at the potential sites. The maximum coverage problem is NP-Hard [38]. Now consider an instance of *max H-cover/single/U/full/binary coverage* with the following data set: *G*=*G'* and *V* denotes the set of demand nodes and set of potential hub locations. The flow of demand for all $i, j \in V$ is $w_{ij}=w_i/|V-1|$ for $i \neq j$ and $w_{ii}=0$. $\alpha=\delta=0$ and hubs can be opened at most p of the locations. Then, the two problems are equivalent, because MCP has a solution with at most p facilities satisfying $\sum_{\substack{i:\exists j \in H' \\ d_{ij} \leq \beta}} w_i \geq T'$ if and only if *max H-cover/single/U/full/binary coverage* has a solution with at most p hubs, and with $\sum_{i,j \in S} w_{ij} \geq T$ where S is a set of vertices with property $\eta d_{iu_i} + \alpha d_{u_i u_j} + \delta d_{u_j j} \leq \beta \; \forall \, i,j \in S$ and $u_i, u_j \in H$. From the solution of MCP, an assignment vector u can be generated as follows: Let $k = \operatorname{argmin}_{l \in H'} d_{il}$ and $u_i = k$ $\forall \, i \in V'$ and it becomes a solution of *max H-cover/single/U/full/binary coverage*. Also, from the solution of the problem, a solution of MCP can be generated; if $d_{iu_i} \leq \beta$ then w_i is covered. Thus, *max H-cover/single/U/full/binary coverage* is NP-Complete. ■

Proposition 5.2: *max H-cover/multi/U/full/binary coverage* is NP-Complete even if $\alpha=0$, $\delta=0$.

Decision version of the problem:

Instance: $G = (V, E), \eta, \alpha, \delta \geq 0$, $p \leq |V|$, β represents coverage distance, weight w_{ij} $\forall \, i,j \in V$, distance d_{ij} $\forall \, i,j \in V$ and $T \geq 0$.

Question: Is there a set of vertices $H \subseteq V$ and $|H| = p$ with an assignment sets u_i where $u_i \in H \;\; \forall \, i \in V$ such that $\sum_{i,j \in S} w_{ij} \geq T$ where S is a set of vertices with property $\eta d_{iu_i} + \alpha d_{u_i u_j} + \delta d_{u_j j} \leq \beta \; \forall \, i,j \in S$ and $u_i, u_j \in H$?

Proof: The problem is in NP because finding the coverage of a given set H and assignment sets u_i $\forall \, i \in V$ can be done in polynomial time. To show NP-Completeness of the problem, we can use the same reduction used for the NP-Completeness of decision version of the *max H-cover/single/U/full/binary coverage* except generation of the assignments. Let $S_i = \{ l \in H' : d_{il} \leq \beta \}$. Then, $u_i = k$ $\forall \, k \in S_i$ and $\forall \, i \in V'$ and it becomes a solution of *max H-cover/multi/U/full/binary coverage*. Also, from the solution of the problem, a solution of MCP can be generated; if $d_{iu_i} \leq \beta$ holds at least one of the element of u_i, then w_i is covered. Thus, *max H-cover/multi/U/full/binary coverage* is NP-Complete. ■

Alternatively, we can prove the NP-Hardness of the problems by showing that a specific instance of the problems is equivalent to *p*-hub center problem. We will show for single allocation *p*-hub maximal covering problem. Multiple allocation version of it can be shown in similar way.

Alternative proof for Proposition 5.1:

Decision version of the *p*-hub center problem:

Instance: $G = (V, E), \eta, \alpha, \delta \geq 0$, $p \leq |V|$, β represents coverage distance, distance d_{ij} $\forall\, i, j \in V$.

Question: Does there exist a subset $H \subseteq V$ consisting $|H| \leq p$ with an assignment vector u where $u_i \in H \quad \forall\, i \in V$ such that $\eta d_{iu_i} + \alpha d_{u_i u_j} + \delta d_{u_j j} \leq \beta\ \forall\, i, j \in V$ and $u_i, u_j \in H$?

Proof: Let an instance of *max H-cover/single/U/full/binary coverage* such that $\eta, \alpha, \delta \leq 1$, distance d_{ij} weight w_{ij} $\forall\, i, j \in V$. For coverage distance, let $\beta = 3 \max_{(i,j) \in E} d_{ij}$ and $T = \sum_{(i,j) \in E} w_{ij}$. With these parameter settings, the condition $\sum_{i,j \in S} w_{ij} \geq T$ where S is a set of vertices with property $\eta d_{iu_i} + \alpha d_{u_i u_j} + \delta d_{u_j j} \leq \beta\ \forall\, i, j \in S$ and $u_i, u_j \in H$ of the decision version of *max H-cover/single/U/full/binary coverage* is directly satisfied. Thus, the decision version of *max H-cover/single/U/full/binary coverage* is equivalent to the decision version of *p*-hub center problem. Therefore, solving *p*-hub maximal covering with that instance will be as hard as solving *p*-hub center problem with that instance. Since single allocation hub center problem is in NP-Hard [14], *max H-cover/ single/U/full/binary coverage* is also in NP. ■

Similar conversion can be easily applied for the proof of NP-Hardness of multiple allocation version of the problem. Since multiple allocation version of the *p*-hub center problem is proved to be NP-Hard in [15], by using the same data set given above, *max H-cover/multi/U/full/binary coverage* reduces to multiple allocation hub center problem.

As a special case of the *p*-hub maximal covering problem, we also discuss the complexity of the allocation problems. The allocation problem is the problem of determining the assignments of nonhub nodes to hub(s) given that hub locations are fixed and known.

The allocation problem of multiple assignment *p*-hub maximal covering problem is polynomially solvable by solving $|N|^2$ shortest path for each origin-destination pair where $|N|$ is the cardinality of number of nonhub nodes.

However, allocation problem of *max H-cover/single/U/full/binary coverage* is in NP since a special instance of the problem is equivalent to allocation problem of *p*-hub center problem whose NP-Hardness are also proven by Ernst et.al [15].

Proposition 5.3: Allocation problem of *max H-cover/single/U/full/binary coverage* is NP-Complete.
Decision version of the allocation problem of *max H-cover/single/U/full/binary coverage*:

Instance: $G = (V, E), \eta, \alpha, \delta \geq 0$, given hub set $H \subseteq V$, β represents coverage distance, weight w_{ij} $\forall\, i,j \in V$, distance d_{ij} $\forall\, i,j \in V$ and $T \geq 0$.
Question: Is there an assignment vector u, where $u_i \in H$, such that $\sum_{i,j \in S} w_{ij} \geq T$ where S is set of vertices with property $\eta d_{iu_i} + \alpha d_{u_i u_j} + \delta d_{u_j j} \leq \beta$ $\forall\, i,j \in S$ and $u_i, u_j \in H$?

Decision version of the allocation problem of hub center problem:

Instance: $G = (V, E), \eta, \alpha, \delta \geq 0$, given hub set $H \subseteq V$, β represents coverage distance, distance d_{ij} $\forall\, i, j \in V$.

Question: Is there an assignment vector u, where $u_i \in H$ such that $\eta d_{iu_i} + \alpha d_{u_i u_j} + \delta d_{u_j j} \leq \beta \ \ \forall\, i, j \in V$ and $u_i, u_j \in H$?

Proof: Let an instance of *max H-cover/single/U/full/binary coverage* such that $\eta, \alpha, \delta \leq 1$, distance d_{ij} weight w_{ij} $\forall\, i, j \in V$. For coverage distance let $\beta = 3\max_{(i,j)\in E} d_{ij}$ and $T = \sum_{(i,j)\in E} w_{ij}$. Then the decision version of the allocation problem of *max H-cover/ single/U/full/binary coverage* can be modified with $\sum_{i,j\in S} w_{ij} \geq \sum_{(i,j)\in E} w_{ij}$ as done in the alternative proof of Proposition 5.1. Then the decision version of the allocation problem of *max H-cover/single/U/full/binary coverage* is equivalent to the decision version of the allocation problem of *p*-hub center problem. Thus, solving allocation problem of *p*-hub maximal covering with that instance would be as hard as solving the allocation problem of *p*-hub center problem with that instance. Since the allocation problem of hub center problem for single assignment is proven to be NP-Hard [15], allocation problem of *max H-cover/single/U/full/binary coverage* is also in NP. ■

Chapter 6

Computational Analysis for Single Allocation *p*-Hub Maximal Covering Problem

In this chapter, first data generation and then computational studies for *max H-cover/ single/U/full/{binary,partial} coverage* are explained. To test the behavior of mathematical models, two data sets are used: *Civil Aeronautics Board (CAB)* and *Turkish data set (TR)*. In both of the data sets, only distance and flow matrices between nodes are available; there is no data related with the partial coverage of the nodes, i.e. a nonincreasing function for coverage or maximum allowable service distance for the partial coverage (γ). So, in Section 6.1, generation of these parameters and the data sets are explained. In Section 6.2 computational results with valid inequalities are compared. In Section 6.3 computational results and discussions for the *max H-cover/single/U/full/ {binary, partial} coverage* are presented.

All the computational results are taken on a Linux environment with 4x AMD Opteron Interlagos 16C 6282SE 2.6G 16M 6400MT 96 GB RAM. Based on preliminary analyses, we decided to use CPLEX 12.4 for single allocation and Gurobi 5.0.2 for multiple allocation version of the p-hub maximal covering problem since their performances are better at these solvers.

6.1 Data Generation

O'Kelly [4] introduced a well-known data set (CAB) based on the airline passenger transportation between 25 U.S. cities. The locations and names (or numbers) of the cities are presented in Figure 6.1.

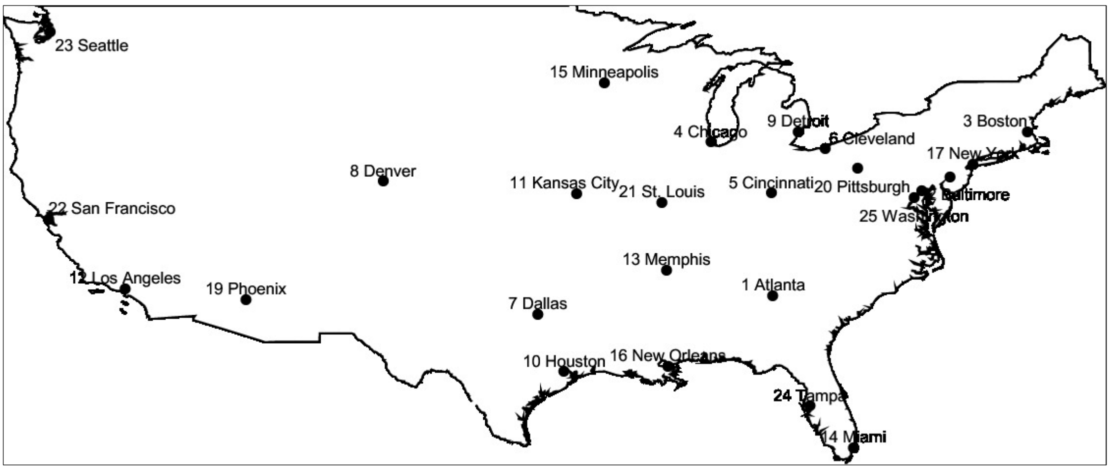


Figure 6.1: Locations of demand nodes and hubs for CAB data set

For CAB data set, all the demand nodes are considered as possible hub locations. Flow of demand and distance between the nodes are presented on the original data set. For obtaining the results we scaled the flow to 1000 and use the distance matrix as in the original data set. For the maximum service distance value, we generated parameters by using the results of the *p*-hub center problem [16]. The results of the problem are shown in Table 6.1.

Table 6.1: Solutions of *p*-hub center problem for CAB data set for α=0.2, 0.4, 0.6, 0.8 and p=2-5

α	p	β
0.2	2	2136
	3	1913
	4	1617
	5	1346
0.4	2	2401
	3	2099
	4	1881
	5	1597

α	p	β
0.6	2	2557
	3	2336
	4	2184
	5	2002
0.8	2	2713
	3	2552
	4	2457
	5	2307

Since the given values (*β*) are for covering all origin-destination (*O-D*) pairs, we reduce the maximum service distance for the *p*-hub maximal covering problem with 0.75 for the binary coverage parameter. While calculating c_{ij}^{km}, we take $\eta = \delta = 1$. Then parameter a_{ij}^{km} used for the binary coverage is:

$$a_{ij}^{km} = \begin{cases} 1 & if \ \ c_{ij}^{km} \leq \beta * 0.75 \\ 0 & otherwise \end{cases} \qquad (6.1)$$

Coverage distance values are changing with p and α, so obviously parameter a_{ij}^{km} changes for each combination of α and p.

For partial coverage, we use the same parameters for α, p and β that are used in binary coverage. We only need to adapt the definition of a_{ij}^{km} to the partial coverage. For the function given in 3.2, we use a stepwise function as shown below:

$$a_{ij}^{km} = \begin{cases} 1 & if\ c_{ij}^{km} \leq \beta * 0.75 \\ 0.75 & if\ \beta * 0.75 < \ c_{ij}^{km} \leq \beta * 0.80 \\ 0.5 & if\ \beta * 0.80 < \ c_{ij}^{km} \leq \beta * 0.85 \\ 0.25 & if\ \ \beta * 0.85 < c_{ij}^{km} \leq \beta * 0.90 \\ 0 & otherwise \end{cases} \qquad (6.2)$$

In order to test the performance of the formulations, we also use TR data set that is relatively larger than CAB data set. TR data set comprises of 81 nodes and 22 of them are selected as potential hub locations among the most populated and industrialized cities [25, 39]. All the nodes and 22 potential hub locations are shown in Figure 6.2. For testing the formulations number of hubs (p) is varied as 5, 10, 15 and 20.

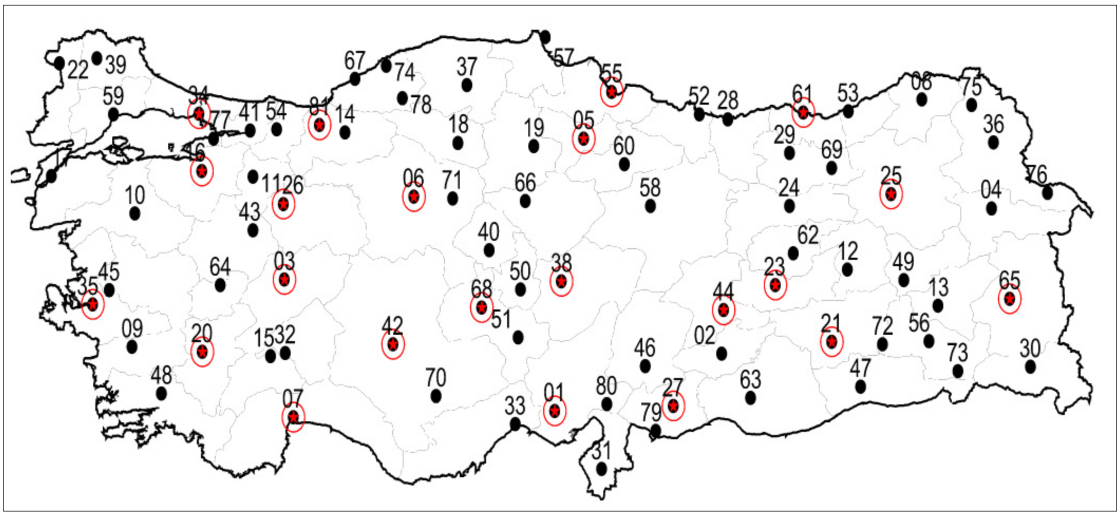


Figure 6.2: Locations of demand nodes and hubs for TR data set

For obtaining the results with TR data set, flow is scaled to 100,000. Besides the flow and distance data sets, there is another set of data which represents the travel time between the nodes. For that parameter, we assume that the travel time is directly proportional to the distance and so for the computational results, instead of distance, we use time for the covering parameter setting. The time and cost savings between two hubs are estimated as 10% and 20% by the cargo companies in Turkey [39]. Therefore, for the time reduction between hub facilities, we use both the reductions and α is taken as 0.8 and 0.9 for inter hub connections. For TR data set, transportation rates for collection from an origin to a hub (η) and for distribution from a hub to destination (δ) are set to 1.

For generating coverage parameter (*β)* for TR data set, first we use hub covering formulation with the worst parameter setting (α=0.9 and p=5) to find the distance for full coverage of the flows. Since hub covering result is about 24 hours (~ 24.1), we set the maximum allowable service time (γ) to 24 hours. For the binary coverage, we consider half-day service limit and binary coverage parameter setting is:

$$a_{ij}^{km} = \begin{cases} 1 & if \;\; t_{ij}^{km} \leq 12 \; hours \\ 0 & otherwise \end{cases} \qquad (6.3)$$

where $t_{ij}^{km} = t_{ik} + \alpha t_{km} + t_{jm}$ and t_{ij} is the time between two nodes i and j. For the partial coverage parameter setting, we again use a stepwise function:

$$a_{ij}^{km} = \begin{cases} 1 & if \; t_{ij}^{km} \leq 12\; h \\ 0.75 & if \;\; 12\; h < t_{ij}^{km} \leq 16\; h \\ 0.5 & if\, 16\; h < t_{ij}^{km} \leq 20\; h \\ 0.25 & if \; 20\; h < t_{ij}^{km} \leq 24\; h \\ 0 & otherwise \end{cases} \qquad (6.4)$$

6.2 Effects of Valid Inequalities

In chapters 3 and 4, valid inequalities for single and multiple allocation versions of the problem are proposed respectively. In this section, we present the results of the combinations of valid inequalities for both of the allocations and both of the coverage.

Recall that valid inequalities (3.17), (3.18) and (3.19) are proposed for single allocation. The results for preliminary analysis of both coverage types with TR data set are presented at tables 6.2 and 6.3. In each multi-row, the first line lists the best upper bound value at the root node and the second line lists the CPU time or the gap reported by the solver at the end of 1 hour time limit. At the tables, the columns 4 through 11 correspond to the all possible combinations of the valid inequalities.

The preliminary analyses show that the lowest best bounds at the root node are obtained when (3.19) is added to formulation (columns 7, 9, 10 and 11). Also, adding (3.19) decreases the solution time for binary coverage; the lowest CPU's are obtained at column 7. Adding (3.17) decreases the gaps for partial coverage. We also tested (3.18) with the combinations of the other valid inequalities but none of them did not improve the solution quality; adding (3.18) increases the solution time and the gap for almost all the instances at the preliminary test. Based on the tables 6.2 and 6.3, adding only (3.19) and adding (3.17) with (3.19) show similar performance. They yield the best improvements on the CPU time for binary coverage and on the solution quality for partial coverage. Although (3.19) provides slightly better results, due to the explanation at the proof of proposition 3.3, we used (3.17) and (3.19) together for the computational experiments of *max H-cover/single/U/full/{binary, partial} coverage.*

Table 6.2: Effect of the valid inequalities on the max H-cover/single/U/full/binary coverage

α	p		no val.ineq.	(3.17)	(3.18)	(3.19)	(3.17), (3.18)	(3.17), (3.19)	(3.18), (3.19)	(3.17), (3.18), (3.19)
0.8	5	best bd at root node	84931.0	84931.0	84931.0	82528	84931.0	82528	82528	82528
		CPU / gap(%)	255.77	264.79	516.19	171.21	533.42	171.72	619.11	593.82
	10	best bd at root node	88626.8	88626.8	88626.8	87608.9	88626.83	87608.9	87608.9	87608.9
		CPU / gap(%)	113.74	116.5	195.95	43.8	217.21	41.3	81.52	81.73
0.9	5	best bd at root node	79615.4	79615.4	79615.4	76857.4	79615.4	76857.4	76857.4	76857.4
		CPU / gap(%)	200.69	191.7	355.86	145.01	354.69	145.82	220.24	213.85
	10	best bd at root node	83287.7	83287.7	83287.7	82075.9	83287.7	82075.9	82075.9	82075.9
		CPU / gap(%)	92.02	87.31	159.34	16.77	168.53	16.46	33.95	34.86

Table 6.3: Effect of the valid inequalities on the max H-cover/single/U/full/partial coverage

α	p		no val.ineq.	(3.17)	(3.18)	(3.19)	(3.17), (3.18)	(3.17), (3.19)	(3.18), (3.19)	(3.17), (3.18), (3.19)
0.8	5	best bd at root node	168795.7	99206.0	168795.7	97114.6	99206.0	97114.6	97114.6	97114.6
		CPU / gap(%)	1h/79.17	1h/5.74	1h/80.61	1h/2.62	1h/5.53	1h/2.63	1h/2.48	1h/2.93
	10	best bd at root node	166966.4	99221.3	166966.4	97357.1	99221.3	97357.1	97357.1	97357.1
		CPU / gap(%)	1h/71.32	1h/2.49	1h/74.75	1h/0.44	1h/2.81	1h/0.46	1h/0.41	1h/0.41
0.9	5	best bd at root node	167160.5	99030.8	167160.5	95825.9	99030.8	95825.9	95825.9	95825.9
		CPU / gap(%)	1h/79.60	1h/6.11	1h/79.51	1h/2.10	1h/6.28	1h/2.16	1h/2.32	1h/2.58
	10	best bd at root node	165002.7	99006.1	165002.7	96054.5	99006.1	96054.5	96054.5	96054.5
		CPU / gap(%)	1h/71.13	1h/3.98	1h/77.01	1h/0.36	1h/4.27	1h/0.35	1h/0.42	1h/0.59

Recall that in Chapter 4, three valid inequalities (3.17), (4.16) and (4.17) are proposed for multiple allocation. To test the valid inequalities, we tried all the combinations of them with a preliminary test for both coverage types with TR data set. The results are presented at tables 6.4 and 6.5 for binary and partial coverage respectively. We again report best upper bounds at the root node and the solution time or gap at the end of 1 hour time limit.

Effect of the valid inequality (3.17) is easily observed from Table 6.5; for α=0.8, at the all columns that (3.17) is not used, at the all instances the best bound at the root node is always more than the value of full coverage. From the preliminary results, we also observe that adding (4.16) is generally resulted with the worse solutions compared to the other combinations of valid inequalities. (i.e. adding only (3.17) yields lower gaps than the adding (3.17) and (4.16) together.). Inequality (4.17) improves the solution quality and lower gaps are obtained with (4.17). Among the combinations that involve (3.17), the solutions with (3.17), (4.17) and (3.17), (4.16), (4.17) yield the lowest best bounds at the root nodes. Since the gaps of (3.17), (4.17) reported by the solver is slightly lower than the gaps of (3.17), (4.16), (4.17), we decided to use (3.17) and (4.17) for multiple allocation *p*-hub maximal covering problem.

Table 6.4: Effect of the valid inequalities on the max H-cover/multi/U/full/binary coverage

α	p		no val.ineq.	(3.17)	(4.16)	(4.17)	(3.17), (4.16)	(3.17), (4.17)	(4.16), (4.17)	(3.17), (4.16), (4.17)
0.8	5	best bd at root node	90173.9	90173.9	90173.9	90136.8	90173.9	90136.8	90136.8	90136.8
		CPU / gap(%)	1h/8.85	1h/8.85	1h/9.63	1h/3.2	1h/9.52	1h/3.18	1h/3.22	1h/3.13
	10	best bd at root node	90290.8	90290.8	90290.8	90271.3	90290.8	90271.3	90271.3	90271.3
		CPU / gap(%)	1h/1.61	1h/1.61	1h/1.64	1h/0.56	1h/1.7	1h/0.34	1h/0.43	1h/0.42
0.9	5	best bd at root node	85193.1	85193.1	85193.1	85174.9	85193.1	85174.9	85174.9	85174.9
		CPU / gap(%)	1h/11.0	1h/11.0	1h/8.91	1h/6.8	1h/9.04	1h/10.0	1h/6.18	1h/6.25
	10	best bd at root node	85256	85256	85256	85250.1	85256	85250.1	85250.1	85250.1
		CPU / gap(%)	2528.77	2496.88	2994.49	2126.34	3136.85	1952.55	2098.65	2040.75

Table 6.5: Effect of the valid inequalities on the max H-cover/multi/U/full/partial coverage

α	p		no val.ineq.	(3.17)	(4.16)	(4.17)	(3.17), (4.16)	(3.17), (4.17)	(4.16), (4.17)	(3.17), (4.16), (4.17)
0.8	5	best bnd at root node	171935.5	99849.4	171935.5	100663.5	99849.4	97649.1	100635.5	97643.7
		CPU / gap(%)	1h/66.8	1h/1.55	1h/68.4	1h/11.2	1h/2.6	1h/3.23	1h/2.85	1h/3.46
	10	best bnd at root node	172057.7	99878.7	172057.7	100663.5	99878.7	97678.4	100635.5	97672.9
		CPU / gap(%)	1h/57.8	1h/0.34	1h/56.2	1h/0.46	1h/0.37	1h/0.53	1h/1.61	1h/0.54
0.9	5	best bnd at root node	170568.7	99809.4	170568.7	99401.0	99809.4	96455.9	99370.3	96449.5
		CPU / gap(%)	1h/69.8	1h/2.12	1h/69.0	1h/1.49	1h/0.76	1h/2.42	1h/7.85	1h/1.66
	10	best bnd at root node	170683.3	99825.1	170683.3	99401.0	99825.1	96471.6	99370.3	96465.2
		CPU / gap(%)	1h/52.0	1h/0.18	1h/57.3	1h/0.23	1h/0.27	1h/0.24	1h/0.3	1h/0.39

6.3 Computational Analysis for Single Allocation *p*-Hub Maximal Covering Problem

In this section, computational experiments and discussions for *max H-cover/single/U/ full{binary, partial} coverage* are explained. In Chapter 3, three formulations are discussed for the single allocation *p*-hub maximal covering problem. In Section 6.3.1 we compare the results of them for the binary coverage case. For the computational results of the new formulation we use SA2 since both CAB and TR data sets have symmetric distance matrices.

As stated before, although applicability to partial coverage is not mentioned in the papers [2] and [21], the two given formulations are applicable to the partial coverage cases and the results of these models for partial coverage case are also presented at Section 6.3.2. Then, we also give discussion about the comparison of solutions for binary and partial coverage cases in Section 6.3.3. For all these sections, first we provide results with CAB data set, then with TR data set. Throughout the thesis we use a time limit of 1 hour for small data set (CAB) and 2 hours for large data set (TR) for the comparison purposes.

6.3.1 Computational Results of the Formulations with Binary Coverage

The preliminary analyses for *max H-cover/single/U/full/binary coverage* are shown at Table 6.6. We only report the results with α=0.6 and 0.8 due to the reason that formulations of Campbell [2] and Hwang and Lee [21] take too much CPU time for the problem and for also lower α values the formulations show similarities. Therefore, for comparison purposes, we set the time limit to one hour for the existing formulations.

For the tables in chapters 6 and 7, "*optimum value*" represents the optimum objective function value that is solved within time limit and "*CPU*" corresponds to the solution time of those instances. For the instances which are not solved to optimality within the time limit, we report "*best int*" that is the best integer solution, "*best upper bnd*" that is the best upper bound value reported by the solver at the end of the time limit and also "*opt. gap (%)*". For single allocation instances, the proposed formulation with CAB data set always finds the optimum solutions whereas the existing formulations in the literature usually have to be terminated due to the time limit. The gap provided by the solver for those instances were usually so high (i.e. 822.47% corresponding to the values of 837.02 for best integer and 7721.29 for best upper bound at the end of time limit) that we calculated those gaps based on the optimums found by our formulations, i.e. $\frac{optimum\ value - best\ int}{optimum\ value} x100$.

The results show that, the most up to date formulation in the literature (Hwang and Lee [21]) performs worse than even the basic formulation proposed by Campbell [2]. None of the 8 instances is solved to optimality within one hour. Also, at the end of the 1-hour time limit, none of the best integer values of the formulation [21] match with the optimum value. Moreover, the gaps and best upper bounds reported by the solver are too high. For example, at the instance with α=0.6, p=4, best integer value is 898.35, best bound is 6762.19 and the corresponding gap for these values is 652.73%. Note that in case of full coverage, for CAB data set objective value equals to 1000.

Although the basic formulation proposed by Campbell [2] performs better, it does not obtain the optimum value at 7 instances out of 8 in one hour. Only one of the 8 instances is solved optimally within one hour. The average optimality gap of these 7 non-optimum solutions at the end of one hour is 4.36%.

Table 6.6: Solutions of max H-cover/single/U/full/binary coverage for CAB data set

		Peker and Kara (SA2)		Campbell [2]		Hwang and Lee [21]		
α	p	optimum value	CPU (sec)	best int	CPU (sec)/ opt. gap (%)	best int	best upper bnd	CPU / opt. gap (%)
0.6	2	900.13	8.09	opt	2542.6 /0.00	792.42	9302.2	>1 h /11.97
	3	919.08	3.66	905.68	>1 h /1.46	893.06	7603.98	>1 h /2.83
	4	915.15	4.83	871.65	>1 h /4.75	898.35	6762.19	>1 h /1.84
	5	883.83	3.67	863.23	>1 h /2.33	869.01	5848.56	>1 h /1.68
0.8	2	877.91	1.01	804.63	>1 h /8.35	781.56	8790.8	>1 h /10.97
	3	873.47	2.11	812.95	>1 h /6.93	837.02	7721.29	>1 h /4.17
	4	873.02	4.52	825.59	>1 h /5.43	833.46	7212.84	>1 h /4.53
	5	862.32	2.04	851.21	>1 h /1.29	834.31	6530.93	>1 h /3.25

The complete results of the proposed formulation for binary coverage with SA2 are shown at Table 6.7. While both of the existing formulations fail to prove the optimality within one hour, the proposed formulation solves the problem in approximately at most 17 seconds.

Table 6.7: Solutions of max H-cover/single/U/full/binary coverage with SA2 for CAB data set

α	p	optimum value	CPU (sec)
0.2	2	926.62	16.74
	3	959.66	11.3
	4	956.69	10.8
	5	923.81	8.96
0.4	2	940.11	8.92
	3	953.96	8.54
	4	943.92	8.28
	5	891.95	5.65

α	p	optimum value	CPU (sec)
0.6	2	900.13	8.09
	3	919.08	3.66
	4	915.15	4.83
	5	883.83	3.67
0.8	2	877.91	1.01
	3	873.47	2.11
	4	873.02	4.52
	5	862.32	2.04

The solutions of the proposed model for TR data set are shown at Table 6.8. Since the TR network is a large data set and the existing formulations have $O(n^4)$ variables, we ran out of memory before obtaining any results with them and represented with *N/A*. On the other hand, with the proposed formulation, by decreasing the order of variables and constraints, we managed to solve a large data set within at most 3 minutes. Note that, for TR data set, in the case of full coverage objective value equals to 100,000 since the data is scaled to 100,000.

Table 6.8: Solutions of max H-cover/single/U/full/binary coverage for TR data set

		Peker and Kara (SA2)		Campbell [2]	Hwang and Lee [21]
α	p	optimum value	CPU (sec)		
0.8	5	78827.05	171.72	N/A	N/A
	10	86743.24	41.3	N/A	N/A
	15	89476.15	2.07	N/A	N/A
	20	90025.48	1.48	N/A	N/A
0.9	5	73781.13	145.82	N/A	N/A
	10	81712.81	16.46	N/A	N/A
	15	83538.9	3.77	N/A	N/A
	20	84380.01	0.66	N/A	N/A

6.3.2 Computational Results of the Formulations with Partial Coverage

For partial coverage case, we again only report the solutions of three formulations with α=0.6 and 0.8 for CAB data sets since the formulations in [2] and [21] take too much time for solving the problem. The results for *max H-cover/single/U/full/partial coverage* with a 1 hour time limit are given at Table 6.9.

The most up to date formulation again performs worst among the three formulations proposed for the partial coverage case. All the solutions' best bounds are again higher than 1000 (Table 6.9). But, at one instance out of 8, Hwang and Lee's formulation [21] finds the optimum solution as the best solution within 1 hour.

With Campbell's formulation [2] at one instance, the formulation proves the optimality within 1 hour. For other instances, best solutions at the end of 1 hour are not same with the optimum and on the average the optimality gap for these 7 instances is 3.38% which is less than the average optimality gap of the formulation [2] with binary coverage case.

Table 6.9: Solutions of max H-cover/single/U/full/partial coverage for CAB data set

		Peker and Kara (SA2)		Campbell [2]		Hwang and Lee [21]		
α	p	optimum value	CPU (sec)	best int	CPU (sec) / opt. gap (%)	best int	best bnd	CPU / opt. gap (%)
0.6	2	934.68	28.28	opt	3315.73 /0.0	opt	10946.4	> 1 h / 0.0
	3	940.19	41.59	926.53	> 1 h / 1.45	939.41	8902.86	> 1 h / 0.08
	4	946.24	42.79	882.03	> 1 h / 6.79	927.75	7862.19	> 1 h / 1.95
	5	930.00	140.41	914.66	> 1 h / 1.65	900.82	6840.15	> 1 h / 3.14
0.8	2	918.26	26.79	906.83	> 1 h / 1.24	845.23	10327	> 1 h / 7.95
	3	908.57	49.77	858.08	> 1 h / 5.56	891.93	8999.86	> 1 h / 1.83
	4	908.72	128.07	875.19	> 1 h / 3.69	888.79	8363.05	> 1 h / 2.19
	5	892.92	141.17	863.43	> 1 h / 3.30	870.13	7589.26	> 1 h / 2.55

From the preliminary analysis for CAB data set, it can be observed that, as expected, allowing partial coverage increases objective function values. Although generally the solution times increase with partial coverage, the proposed formulation finds the optimum solution within 3 minutes. The discussion between the binary and partial coverage will be explained in detail in the next subsection.

The solutions of the proposed and existing formulations for TR data set with a 2 hour time limit are shown at Table 6.10. At some instances the formulation does not prove the optimality and since the optimum solutions are not known for those instances, we report the "*gap (%)*" that is the gap reported by the solver. For those instances we present best integer values and for other instances, whose optimality is verified, we present the optimum values in column 3.

As stated, although allowing *"partially covered flows"* increases the solution times, the optimality of 4 instances out of 8 are proved within 2 hours. Also, we can discuss that

SA2 performs strictly better than the other formulations because while the gaps for the other instances of SA2 vary between 0.17% and 2.58%, other two formulations end within a few minutes without finding even an initial solution due to the memory limitation.

Table 6.10: Solutions of max H-cover/single/U/full/partial coverage for TR data set

α	p	Peker and Kara (SA2) optimum value / best int	Peker and Kara (SA2) CPU (sec) / gap (%)	Campbell [2]	Hwang and Lee [21]
0.8	5	94325.49	> 2 h / 2.58	N/A	N/A
	10	96677.16	> 2 h / 0.26	N/A	N/A
	15	97368.7	282.49	N/A	N/A
	20	97505.09	25.97	N/A	N/A
0.9	5	93307.93	> 2 h / 2.0	N/A	N/A
	10	95427.86	> 2 h / 0.17	N/A	N/A
	15	95884.38	1537.95	N/A	N/A
	20	96093.35	102.39	N/A	N/A

6.3.3 Comparison of the Solutions with Binary and Partial Coverage

In this subsection, we would like to analyze the effect of allowing partial coverage to solutions. Thus, the results of the proposed formulation with SA2 for binary and partial coverage are compared. For CAB data set, the objective values, solutions times and also hub locations for both coverage types are presented at Table 6.11.

At Table 6.11, columns 3, 4, 5 are the results of *max H-cover/single/U/full/binary coverage* and columns 6, 7 and 8 the results of the *max H-cover/single/U/full/partial coverage*. Firstly, we again emphasize that allowing partial coverage increases the solution times of the *p*-hub maximal covering problem and on the average, the increment in the CPU time is more than 50 seconds.

Table 6.11: Solutions of max H-cover/single/U/full/{binary, partial}coverage with SA2 for CAB data set

		Binary Coverage			Partial Coverage		
α	P	optimum value	CPU (sec)	hub locations	optimum value	CPU (sec)	hub locations
0.2	2	926.62	16.74	5,12	961.87	13.89	5,22
	3	959.66	11.3	2,12,13	977.57	15.59	2,13,22
	4	956.69	10.8	1,2,11,12	970.69	22.86	1,2,11,12
	5	923.81	8.96	4,7,12,18,24	946.76	68.74	4,7,12,18,24
0.4	2	940.11	8.92	5,12	965.35	16.4	5,12
	3	953.96	8.54	12,13,17	967.24	25.42	12,13,17
	4	943.92	8.28	11,12,14,18	964.92	28.55	11,12,14,18
	5	891.95	5.65	12,18,21,22,24	934.02	146.69	12,17,21,22,24
0.6	2	900.13	8.09	5,12	934.68	28.28	5,12
	3	919.08	3.66	5,11,12	940.19	41.59	12,13,17
	4	915.15	4.83	11,12,14,20	946.24	42.79	12,13,18,22
	5	883.83	3.67	4,7,12,17,24	930	140.41	11,12,18,22,24
0.8	2	877.91	1.01	8,21	918.26	26.79	8,21
	3	873.47	2.11	5,11,12	908.57	49.77	5,11,12
	4	873.02	4.52	5,11,19,22	908.72	128.07	8,12,17,21
	5	862.32	2.04	1,9,11,12,22	892.92	141.17	6,8,11,12,24

When we compare the results, as expected, the objective values of partial coverage case are higher than the objective values of binary coverage. On the average, 3.22% increment is achieved by allowing partially covered flows. One of the reasons of this increment is only due to allowing the partial coverage for the flow between the nodes. Even if at the cases where the optimal hub locations do not change, some additional pairs start to become partially covered and thus the objective value increase. For example, for the instance with α=0.2, p=4, with located hubs, the *O-D* pairs shown at Table 6.12 are partially covered pairs which were not covered at binary coverage (i.e. $Z_{ij}=Z_{ji}=0$ for the given *O-D* pairs). Recall that, Z_{ij} is the fraction of the coverage of *O-D* pairs.

Table 6.12: Partially covered *O-D* pairs for the instance with α =0.2, p=4

(i,j)	Z_{ij} (=Z_{ji})
(1,23)	0.5
(2,23)	0.25
(8,14)	0.75
(9,10)	0.75
(9,19)	0.75

(i,j)	Z_{ij} (=Z_{ji})
(9,22)	0.75
(10,14)	0.5
(10,19)	0.75
(10,22)	0.75

(i,j)	Z_{ij} (=Z_{ji})
(11,23)	0.75
(14,19)	0.5
(14,22)	0.5
(19,23)	0.5

Also, for the same instance, some of uncovered *O-D* pairs with binary coverage are also uncovered with partial coverage (Table 6.13). The remaining *O-D* pairs are fully covered with both coverage types.

Table 6.13: Uncovered *O-D* pairs for the instance with α =0.2, p=4

		Z_{ij} (=Z_{ji})
(i,j)	(3,23)	0
	(4,23)	0
	(5,23)	0
	(6,23)	0
	(7,23)	0
	(8,23)	0

		Z_{ij} (=Z_{ji})
(i,j)	(9,23)	0
	(10,23)	0
	(13,23)	0
	(14,23)	0
	(15,23)	0
	(16,23)	0

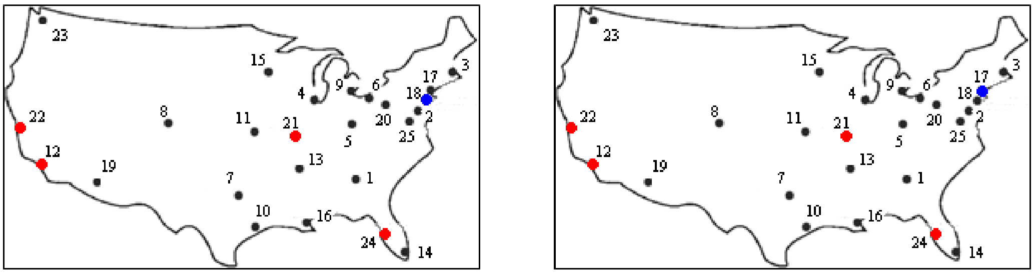
		Z_{ij} (=Z_{ji})
(i,j)	(17,23)	0
	(18,23)	0
	(20,23)	0
	(21,23)	0
	(24,23)	0
	(25,23)	0

From the Table 6.11, it can be observed that optimal hub locations do change at the 8 instances out of 16, therefore, the allocations of the nodes and the routes also change. So, changes in location of hubs are the other reason of the increment of objective values. At Table 6.14, the optimal hub locations for binary and partial coverage for these 8 instances are presented at columns 3 and 4 respectively. The effect of the hub location changes to the objective value for the 8 instances is also presented at the table. The objective values of partial coverage case with the hubs that are obtained from the binary coverage are calculated and presented at column 6. Although some of the % differences are small (i.e. 0.03%), on the average, the difference of the objective value with the "*wrong hubs*" is about 0.42%.

Table 6.14: Effect of using "wrong hubs" with partial coverage

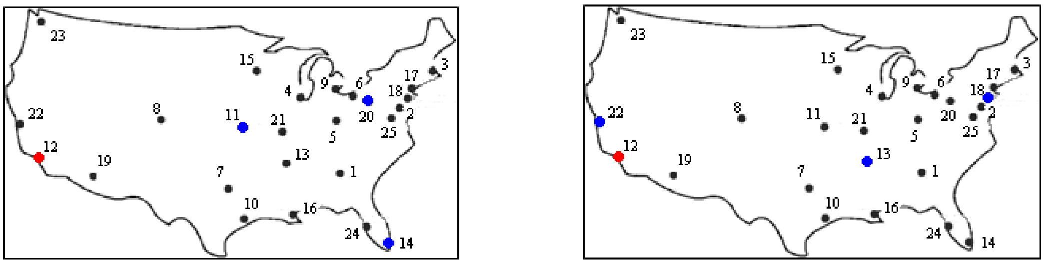
α	p	optimal hubs for partial coverage(*)	optimal hubs for binary coverage(**)	obj func value of partial coverage with (*)	obj func value of partial coverage with (**)	% difference between obj func values
0.2	2	5,22	5,12	961.87	961.59	0.03
	3	2,13,22	2,12,13	977.57	974.51	0.31
0.4	5	12,17,21,22,24	12,18,21,22,24	934.02	933.63	0.04
0.6	3	12,13,17	5,11,12	940.19	940.11	0.01
	4	12,13,18,22	11,12,14,20	946.24	940.7	0.59
	5	11,12,18,22,24	4,7,12,17,24	930	910.09	2.19
0.8	4	8,12,17,21	5,11,19,22	908.72	908.48	0.03
	5	6,8,11,12,24	1,9,11,12,22	892.92	890.98	0.22

At Appendix A, the location changes are shown with blue dots while red dots represent the hubs that are common in both coverage types. From the figures at Appendix A, it can be observed that at some instances the changes of locations are remarkable while at others they are not that significant such as in α=0.4, p=5; all the hubs at partial coverage are the same with binary coverage except node 17. When partial coverage is allowed, hub located at node 18 moves to node 17 (Figure 6.3). On the other hand, for the instance with α=0.6 and p=4, 3 of the hub locations change. At binary coverage node 14 is selected as hub, but at partial coverage the nearest hub to node 14 is node 13 and distance between them is almost half of the coverage distance. The other changes in the locations of hubs are presented at Figure 6.4.



binary coverage partial coverage

Figure 6.3: Locations of hubs for single allocation for the instance α=0.4, p=5 with both coverage types for CAB data set



binary coverage partial coverage

Figure 6.4: Locations of hubs for single allocation for the instance α=0.6, p=4 with both coverage types for CAB data set

Also, from Appendix A, it can be inferred that hub locations generally move towards to bigger cities such as New York (17) or San Francisco (22) when partial coverage is introduced. For the instance with α=0.6, p=3, this case can be observed easily. Instead of nodes 5 and 11, nodes 13 and 17 are selected as hubs. Also, this case can be seen for node 22. For some of the instances (i.e. α=0.6, p=5), even if node 12 (the nearest node to 22) is a hub for binary coverage, 22 is also selected as hub at the partial coverage case.

From the hub locations shown at Table 6.11, when α is small, peripheral cities are most likely chosen as hub locations. The difference between the places of hubs is evident especially for α=0.8. For instance, for p=2 a peripheral node (node 12 or 22) and a node located near to the center (node 5) are chosen for both coverage types for α=0.2, 0.4 and 0.6. However, for α=0.8, two nodes that are located near to the center (node 8 and 21) instead of peripheral nodes are chosen.

Next, we present a similar analysis with TR data set. The comparison of *max H-cover/single/U/full/binary coverage* and the *max H-cover/single/U/full/partial coverage* are also compared with TR data set. The results are shown at Table 6.15. Similar to CAB data set, CPU times increase when partial coverage is used and the increment is more observable for TR data set. For each combination of α and p the CPUs of partial coverage are higher than the CPUs of binary coverage. As expected, optimum values (or best integer solutions) increase with the partial coverage and the increments are higher than the CAB data set; on the average, the increment is 15.01%. Note that, there is a possibility of having higher increment in the objective value since at the 4 instances, the best solutions values of partial coverage are compared with the objective values of binary coverage.

Table 6.15: Solutions of max H-cover/single/U/full/{binary,partial}coverage with SA2 for TR data set

		Binary Coverage		Partial Coverage	
α	p	optimum value	CPU (sec)	optimum value / best int	CPU (sec) / gap (%)
0.8	5	78827.05	171.72	94325.49	> 2 h / 2.58
	10	86743.24	41.3	96677.16	> 2 h / 0.26
	15	89476.15	2.07	97368.7	282.49
	20	90025.48	1.48	97505.09	25.97
0.9	5	73781.13	145.82	93307.93	> 2 h /2.0
	10	81712.81	16.46	95427.86	> 2 h / 0.17
	15	83538.9	3.77	95884.38	1537.95
	20	84380.01	0.66	96093.35	102.39

We do not present any comparison of hub location changes since for the solved instances (p=15 and 20), the selected hubs are the same with both coverage cases and for p=5 and 10, since the solutions with partial coverage case may not optimum, the comparisons for that instances may be misleading.

Chapter 7

Computational Analysis for Multiple Allocation *p*-Hub Maximal Covering Problem

In this chapter, computational experiments of the formulations given in Chapter 4 for *max H-cover/multi/U/full/{binary, partial} coverage* are presented and discussed. Same data sets and parameters explained in Section 6.1 are used to test the performance of the proposed formulations.

As explained in Chapter 4, there are two different formulations in the literature for multiple allocation. Weng et al.'s formulation [22] works significantly better for *max H-cover/multi/U/full/binary coverage* since it has $O(n^2)$ decision variables and constraints. However, the formulation does not serve the aim of this research as it does not work with partial coverage. Therefore, in this chapter we did not perform any computational analysis using that formulation and we only compared the Campbell's formulation [2] and MA2 (since both data sets have symmetric distance matrices).

In Section 7.1, we compare the models for the binary coverage case followed by a discussion of the results for partial coverage case in Section 7.2. Also, we compare the results and give a discussion for the binary and partial coverage cases in Section 7.3. From the hubs that are selected for single and multiple allocations, some general differences are observed, so we conclude this section with a short comparison of single and multiple allocation versions of the problem.

7.1 Computational Results of the Formulations with Binary Coverage

The comparison of Campbell's formulation [2] and MA2 with CAB data set for binary coverage is shown at Table 7.1. We set time limit to 1 hour to compare the formulations. We provide the optimum values and CPU's for the instances whose optimum solutions are obtained within time limit. For other instances we present best integer values and the gaps reported by the solver at the end of time limit.

First observe that the proposed formulation performs better than the existing one [2]. Whereas the formulation [2] proves the optimality at 10 instances out of 16, at 15 of the instances, MA2 finds the optimum results within 1 hour. At one instance (α=0.2, p=5), both of them do not verify the optimality but still we can state that MA2 performs better for that instance because the gap of MA2 is less than the gap of Campbell's formulation [2] at the end of 1 hour. In terms of CPU time, MA2 also performs better. For the 10 instances that both of the formulations find optimums, the CPU times with MA2 are significantly less than the CPU times of Campbell's formulation [2].

Table 7.1: Solutions of max H-cover/multi/U/full/binary coverage for CAB data set

		Peker and Kara (MA2)		Campbell [2]	
α	p	optimum value/ best int	CPU (sec) / gap (%)	optimum value/ best int	CPU (sec) / gap (%)
0.2	2	930.98	94.84	868.73	> 1 h / 15.1
	3	965.8	237.34	965.8	> 1 h / 3.54
	4	957.11	951.16	957.11	> 1 h / 4.48
	5	926.99	>1 h / 3.34	926.99	> 1 h / 7.58
0.4	2	960.45	83.99	944.58	> 1 h / 5.86
	3	962.39	107.58	955.71	> 1 h /4.28
	4	950.07	2392.8	950.07	1961.37
	5	918.38	688.14	918.38	1346.49
0.6	2	939.63	73.54	939.63	1116.52
	3	931.67	100.86	931.67	1044.94
	4	936.33	96.59	936.33	724.6
	5	901.88	319.73	901.88	767.52
0.8	2	899.2	106.76	899.2	1732.31
	3	900.76	65.16	900.76	821.92
	4	896.09	159.75	896.09	612.8
	5	890.52	89.2	890.52	654.03

The results of binary coverage with TR data sets are presented at Table 7.2. We set time limit to 2 hours to obtain the results. Similar to the single allocation version of the problem, Campbell's (1994) formulation has $O(n^4)$ variables and constraints, so for TR data set, the formulation cannot even generate an initial solution due to the memory requirements. While Campbell's formulation cannot generate an initial solution, MA2 finds the optimum solutions at 6 instances within time limit for the other 2 instances that their optimality is not proved, the gaps are 2.54% and 9.82%.

Interestingly, for MA2, CPU times and gaps have similar structure for α and p with TR data set. For both α values, whereas the instances with p=5 are not solved within time limit, for other instances optimums are achieved. The solution times of these 6 instances are decreasing with the increment of p.

Table 7.2: Solutions of max H-cover/multi/U/full/binary coverage for TR data set

		Peker and Kara (MA2)		Campbell [2]
α	p	optimum value / best int	CPU (sec) / gap (%)	
0.8	5	84455.16	>2 h / 2.54%	N/A
	10	88625.7	4235.5	N/A
	15	89967.28	1445.93	N/A
	20	90325.57	35.81	N/A
0.9	5	76174.47	> 2 h / 9.82%	N/A
	10	84473.51	1952.55	N/A
	15	85007.21	516.11	N/A
	20	85248.3	102.1	N/A

7.2 Computational Results of the Formulations with Partial Coverage

The results of both formulations for partial coverage with CAB data set are shown at Table 7.3. We set time limit to 1 hour to compare the results. However, for partial coverage case with CAB data set, the proposed formulation does not perform strictly better than Campbell's formulation [2], since there are unsolved instances with both formulations. In terms of gap at the end of time limit and solution time for the solved instances, MA2 performs better.

Similar to the previous deductions, allowing partial coverage increases the solution time for both formulations, but for existing formulation, at two instances (α=0.6, p=2 and α=0.8, p=2) the CPU times decrease. Also observe here that, MA2 takes less CPU time at 4 instances out of 6 that are solved within the time limit with both formulations. Interestingly, MA2 requires more CPU time when *p* is large.

In terms of the solution quality, MA2 cannot prove the optimality of 7 instances out of 16 within the time limit. But the average gap of these instances is 1.46% at the end of 1 hour. Campbell's formulation [2] cannot prove the optimality for 5 instances and the average gap is 4.75%.

Table 7.3: Solutions of max H-cover/multi/U/full/partial coverage for CAB data set

		Peker and Kara (MA2)		Campbell [2]	
α	P	optimum value / best int	CPU (sec)/ gap (%)	optimum value / best int	CPU (sec) / gap (%)
0.2	2	970.13	508.64	933.64	> 1 h / 7.11
	3	982.62	1773.53	982.44	> 1 h / 1.79
	4	971.37	> 1 h / 0.44	926.74	> 1 h / 7.9
	5	949.09	> 1 h / 3.36	949.09	> 1 h / 4.6
0.4	2	977.05	380.71	977.05	> 1 h / 2.35
	3	972.31	2031.13	972.31	3461.27
	4	969.04	> 1 h / 0.61	969.04	2179.51
	5	945.3	> 1 h / 3.71	951.16	1819.72
0.6	2	962	538.74	962	1069.59
	3	957.45	3504.78	957.45	1358.43
	4	961.23	> 1 h / 0.6	961.23	1536.86
	5	946.1	> 1 h / 1.12	946.1	1145.15
0.8	2	941.65	571.9	941.65	1064.1
	3	936.57	790.83	936.57	969.32
	4	935.82	2408.08	935.82	985.04
	5	915.06	> 1 h / 0.38	915.06	878.69

The *max H-cover/multi/U/full/partial coverage* is also compared with TR data set (Table 7.4). The deductions made for binary coverage hold also for partial coverage case for TR data set. Since Campbell's formulation [2] has $O(n^4)$ constraints and variables, it ends without finding any solution due to the memory requirements.

For MA2, again allowing partial coverage increases CPU time and only at half of the instances, the optimums are achieved within 2 hour time limit. Also, from the results, we observe that, the gaps decrease with the increment of p for each α and finally for p=15 and 20, the gaps become zero. Similarly, for the solution times of the 4 instances that are solved (p=15 and 20), CPU's are decreasing for each α; CPU's of the instances with p=15 are smaller than the CPU's of the instances with p=20 for each α.

Table 7.4: Solutions of max H-cover/multi/U/full/partial coverage for TR data set

		Peker and Kara (MA2)		Campbell [2]
α	p	optimum value / best int	CPU (sec)/ gap (%)	
0.8	5	94552.57	> 2 h / 2.96	N/A
	10	97156.08	> 2 h / 0.26%	N/A
	15	97491.48	4953.65	N/A
	20	97581.05	80.72	N/A
0.9	5	95561.68	> 2 h / 0.55	N/A
	10	96118.04	> 2 h / 0.07	N/A
	15	96251.46	5206.41	N/A
	20	96311.73	76.73	N/A

7.3 Comparison of the Solutions with Binary and Partial Coverage

In this section, we analyze the effect of allowing partial coverage to the solution. For this purpose, we again present the objective function values (or best integer solutions) and CPU time results of binary and partial coverage and also locations of the hubs (Table 7.5). At three instances marked with (?), the optimum solutions are verified with neither new formulation nor Campbell's formulation [2]. Therefore, comparing hub locations for those instances may be misleading. (Note that, there are some instances whose optimality is not verified within time limit with MA2 but the optimal solutions of those instances are known from the results of the formulation [2] (Table 7.1 and Table 7.3))

As expected, introducing partial coverage to the formulation yields an increment in the optimum value. For obtaining the average, we ignore the instance with α=0.2, p=5 since the optimum solution is not verified for binary coverage case. The average improvement for the remaining 15 instances is more than 2.91% because there is a possibility of having higher value in the objective function value of the instance with α=0.2, p=4.

The increment in solution time can be realized easily from Table 7.5. This also leads to an increment in the number of unsolved instances for partial coverage. At 6 more instances, which are solved with binary coverage, the optimality is not proved within time limit.

Table 7.5: Solutions of max H-cover/multi/U/full/{binary, partial}coverage with SA2 for CAB data set

		Binary Coverage			Partial Coverage		
α	p	optimum value / best int	CPU (sec)/ gap (%)	hub locations	optimum value / best int	CPU (sec)/ gap (%)	hub locations
0.2	2	930.98	94.84	5,12	970.13	508.64	5,22
	3	965.8	237.34	2,12,13	982.62	1773.53	2,13,22
	4	957.11	951.16	1,11,12,25	971.37(?)	> 1 h /0.44	1,11,12,25
	5	926.99(?)	>1 h / 3.34	4,7,12,18,24	949.09(?)	> 1 h /3.36	4,7,12,18,24
0.4	2	960.45	83.99	5,12	977.05	380.71	5,12
	3	962.39	107.58	12,13,20	972.31	2031.13	12,13,17
	4	950.07	2392.8	11,12,14,18	969.04	> 1 h/ 0.61	12,13,18,23
	5	918.38	688.14	1,2,11,12,22	945.3	> 1 h /3.71	11,12,18,22,24
0.6	2	939.63	73.54	5,12	962	538.74	5,12
	3	931.67	100.86	5,8,12	957.45	3504.78	11,12,25
	4	936.33	96.59	12,13,18,22	961.23	> 1 h /0.6	12,13,18,22
	5	901.88	319.73	4,12,16,17,22	946.1	> 1 h /1.12	11,12,17,22,24
0.8	2	899.2	106.76	6,8	941.65	571.9	8,20
	3	900.76	65.16	2,11,12	936.57	790.83	11,12,20
	4	896.09	159.75	1,4,19,22	935.82	2408.08	12,13,21,22
	5	890.52	89.2	1,4,8,12,22	915.06	> 1 h /0.38	12,21,22,23,25

The changes in the locations of the hubs for both coverage types are shown at Appendix B. Since the optimality is not proved for the two instances with α=0.2, p=4 and α=0.2, p=5, we do not consider them and at 11 instances out of 14, the locations of hubs change when partial coverage is allowed. From the figures given in Appendix B, it can be observed that some changes in the hub locations are not significant. At 5 instances, only one of hubs with binary coverage moves to another node when partial coverage is introduced. Generally the new hub with partial coverage is the nearest node to the hub with binary coverage. For example, for the instance with α=0.2, p=3, all hubs are same except node 12 and instead of node 12, at partial coverage case 22 is selected as a hub location which is the nearest node to 12.

Similar conclusions as explained in single allocation version of the *p*-hub maximal covering problem also hold for multiple allocation version. When partial coverage is allowed, hubs are located at (or towards) more industrialized cities and peripheral cities are more likely to be selected as hubs. This deduction can be easily verified the instances with α=0.4, p=4 and α=0.8, p=5. At these instances node 23 is selected as hub which can be considered as the farthest node to every other node, but at none of the solutions with binary coverage, node 23 is selected as hub.

As a direct result of the changes in the locations of hubs, the assignments (and also the routes for each *O-D* pair) are affected and at the results, there exists both increment and decrement in the coverage of the pairs. One of the solution for the instance with α=0.4, p=4 is investigated and the coverage for each *O-D* pairs are presented at Appendix C. For instance, if we compare the coverage of node 23, at binary coverage only the three pairs (12, 23), (19, 23) and (22, 23) are covered. But at partial coverage every pair such that (i, 23) are fully covered for i=1…25 except node 14. Even node 14 is the farthest node to 23, the *O-D* pair is partially covered as $Z_{13,24}$ (=$Z_{24,13}$) =0.25.

The changes also decrease the coverage at some instances. For instance whereas *O-D* pairs (8, 14), (14, 19), (14, 22), (15, 19) and (15, 22) are covered at binary coverage case, they are not covered at all with located hubs at partial coverage.

The effects of changes in the locations of the hubs for the 11 instances are presented at Table 7.6. As discussed above, some changes are not significant; changing the location of only one of the hubs might not affect the objective function value significantly. For example, for the instance with α=0.8, p=3 objective function value of using "*wrong hubs*" effect only 0.03%. However, on the average for these 11 instances, the effect of using "*wrong hubs*" is 0.7%. Note that the results are obtained with MA2, therefore at one instance (α=0.4, p=5) the difference is negative, however, from Table 7.3, it can be observed that for that instance the optimum is 951.16 which is higher than the objective value of partial coverage with “wrong hubs”.

Table 7.6: Effect of using “wrong hubs” with partial coverage

α	p	Optimal hubs for partial coverage(*)	Optimal hubs for binary coverage(**)	obj func value of partial coverage with (*)	obj func value of partial coverage with (**)	% difference between obj func values
0.2	2	5,22	5,12	970.13	966.67	0.36
	3	2,13,22	2,12,13	982.62	977.58	0.52
0.4	3	12,13,17	12,13,20	972.31	971.84	0.05
	4	12,13,18,23	11,12,14,18	969.04	966.64	0.25
	5	11,12,18,22,24	1,2,11,12,22	945.3	948.45	-0.33
0.6	3	11,12,25	5,8,12	957.45	953.94	0.37
	5	11,12,17,22,24	4,12,16,17,22	946.1	942.02	0.43
0.8	2	8,20	6,8	941.65	941.04	0.06
	3	11,12,20	2,11,12	936.57	936.27	0.03
	4	12,13,21,22	1,4,19,22	935.82	891.66	4.95
	5	12,21,22,23,25	1,4,8,12,22	915.06	906.18	0.98

The results of *max H-cover/multi/U/full/partial coverage* are compared with binary and partial coverage cases for TR data set and they are presented at Table 7.7. Since at 2 and 4 instances the optimality of the solutions are not proved for binary and partial coverage cases respectively, for those instances we compare best solutions at the end of 2 hour. When the solutions are compared, on the average 12.92% increment is obtained. Note that, the average may be higher or lower than 12.92% due to the unsolved instance that its optimality is not verified within 2 hours.

When the gaps of the solutions at the end of 2 hours are compared, it can be observed that, the gaps are decreasing with the increment of p for each α and for both coverage types. The instances with p=5 for α=0.8 and 0.9 result with the highest gap at the end of 2 hours. But for binary coverage for p=10, 15 and 20, and for partial coverage for p=15 and 20, the gaps even decreases to zero.

In terms of CPU comparison, it is clear that, solution times increase when partial coverage is allowed. Due to the increment in the CPU time, at 2 of the instances that are solved at binary coverage, the optimums are not obtained with partial coverage case.

Table 7.7: Solutions of max H-cover/multi/U/full/{binary, partial}coverage with SA2 for TR data set

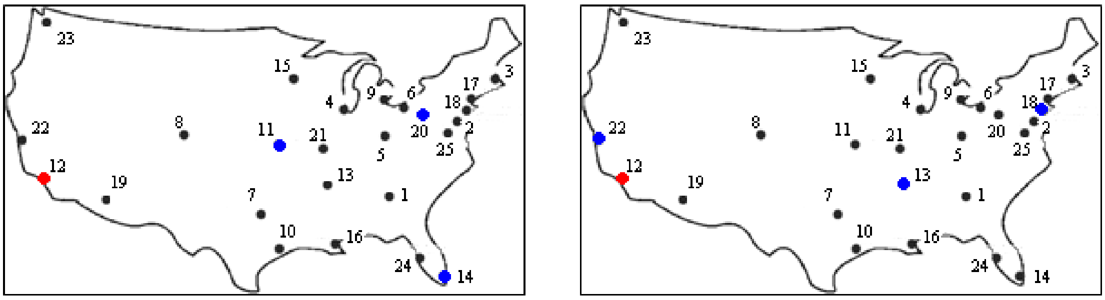
		Binary		Partial	
α	p	optimum value / best int	CPU (sec)/ gap (%)	optimum value / best int	CPU (sec)/ gap (%)
0.8	5	84455.16	>2 h / 2.54	94552.57	> 2 h / 2.96
	10	88625.7	4235.5	97156.08	> 2 h / 0.26
	15	89967.28	1445.93	97491.48	4953.65
	20	90325.57	35.81	97581.05	80.72
0.9	5	76174.47	> 2 h / 9.82	95561.68	> 2 h / 0.55
	10	84473.51	1952.55	96118.04	> 2 h / 0.07
	15	85007.21	516.11	96251.46	5206.41
	20	85248.3	102.1	96311.73	76.73

Similar to solutions of single allocation *p*-hub maximal covering problem with TR data set, allowing partial coverage do not change the locations of hubs for p=15 and 20. Since the other instances (p=5 and 10) may not be optimal, we do not compare the hub locations.

7.4 Comparison of the Solutions with Single and Multiple Allocations

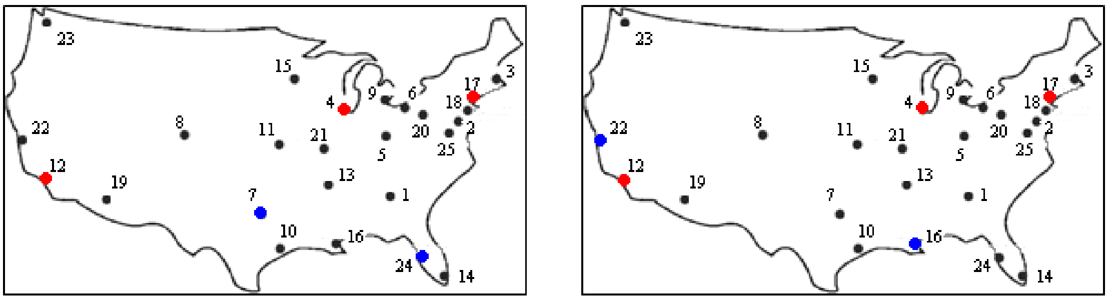
In this subsection, we analyze the difference in the solutions of single and multiple assignment versions of the *p*-hub maximal covering problem. For both coverage types, in multiple allocation, the locations of the hubs are more dispersed than the locations of the hubs in single allocation. From Figure 7.1, 7.2 and 7.3, the deduction can be observed easily. The location changes are shown with blue dots while red dots represent the hubs that are common in both assignment types.

For the figures given in Figure 7.1, at single allocation, nodes 11, 14 and 20 are selected as hubs, but at multiple allocation these 3 nodes are replaced with 13, 18 and 22 which are more distant to each other than 11, 14 and 20. Similarly, for the instance α=0.6, p=5, at single allocation whereas nodes 24 and 7 are hubs, at multiple allocation instead of these nodes, 16 and 22 are selected as hubs (Figure 7.2).



single allocation multiple allocation

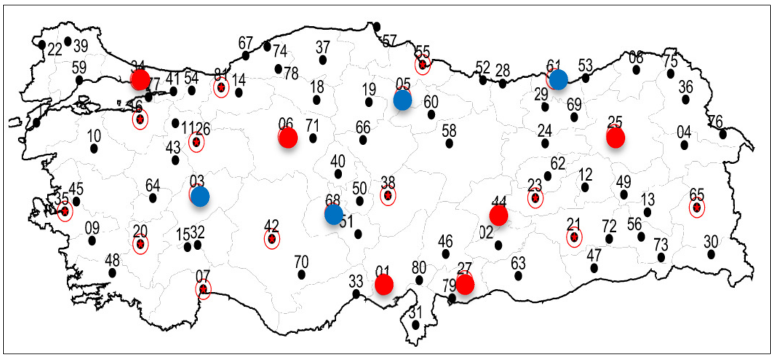
Figure 7.1: Locations of hubs for single and multiple allocations for the instance α=0.6, p=4 with binary coverage for CAB data set



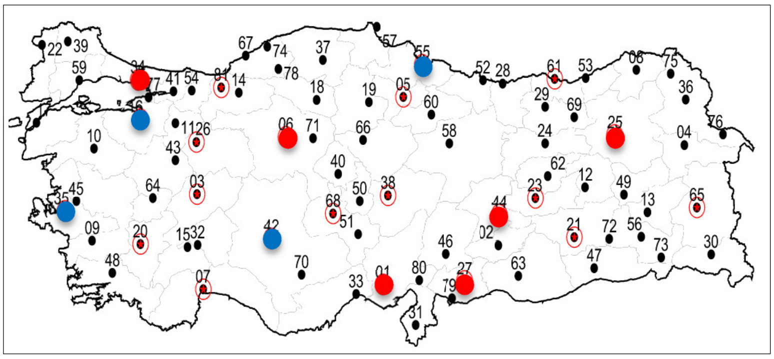
single allocation multiple allocation

Figure 7.2: Locations of hubs for single and multiple allocations for the instance α=0.6, p=5 with binary coverage for CAB data set

The analysis is also observable for the results with TR data set and one of them is presented at Figure 7.3. Four of the hub locations change and at multiple allocation, the changed locations are more distant than the hubs at single allocation.



single allocation



multiple allocation

Figure 7.3: Locations of hubs for single and multiple allocations for the instance α=0.9, p=10 with partial coverage for TR data set

Chapter 8

Conclusion and Future Research

In this thesis, we studied p-hub maximal covering problem which is a variation of the hub location problem. We observed that in the hub location literature, for service requirement of origin-destination (O-D) pairs only binary coverage is taken account; any O-D pair is considered as covered if the total path length is less than the predetermined coverage distance and it is not covered at all if the path length exceeds the coverage distance. However in some real life applications, a definition of coverage that changes with the distance might be more realistic. Thus, throughout this thesis, we extend the definition of coverage and introduce "*partial coverage*" to the hub location literature. We provide new formulations for single and multiple allocation versions of p-hub maximal covering problem which can also be applied with a coverage that can be defined via a gradual decay function.

In Chapter 2, we give a review of the studies on the hub location problem with a detailed explanation for p-hub maximal covering problem. To complete the discussion about the partial coverage, we provide a brief review of covering problems from the general location literature. In Chapter 3, first we explain the problem and the existing formulations for single allocation p-hub maximal covering problem. Then, we introduce an efficient formulation which performs significantly better than the competitive models of literature. In the next section, we explain the p-hub maximal covering problem for multiple allocation version. We provide the existing formulations and a discussion of applicability of the formulations to the partial coverage. We also propose a new formulation for multiple allocation with the valid inequalities for the model. In Chapter 5, we prove the NP-Hardness of the single allocation p-hub maximal covering problem by reducing the maximum coverage problem to it. We also provide an alternative proof for the NP-Hardness of multiple allocation version of the problem which is shown in Weng et.al. [22]. In addition, NP-Hardness of the allocation problem of the single assignment p-hub maximal covering problem is proved by using the transformation to allocation problem of single assignment p-hub center problem.

In chapters 6 and 7, we provide the computational results and comparisons for single and multiple allocations respectively. To test the formulations, we use CAB and TR data sets. We compare the formulations with the existing ones first for binary coverage, then for partial coverage with both data sets. We also compare and discuss the solutions of binary and partial coverage cases with the new formulations. We conclude with a short comparison of single and multiple allocation versions.

Based on the computational results, we can conclude that the new formulation for single allocation performs significantly better than the existing formulations for both coverage types. Even for a larger data set (TR data set) the optimum solutions are obtained within a few minutes for binary coverage but for partial coverage, there are some instances whose optimality is not proved within the time limit.

For multiple allocation, new formulation performs better than existing formulation for binary coverage, but for partial coverage, we cannot conclude that the new formulation is strictly better for the CAB data set. However, with the new formulation, we decrease the order of variables and constraints. Therefore, we manage to solve the problem for TR data set whereas the existing formulation [2] cannot generate any initial solution due to the memory limitations.

For future research, one may generate good valid inequalities to improve the solution quality of the formulations. Especially for multiple allocation, although optimum solutions are obtained at many instances, the best bounds are not good enough to prove the optimality. Therefore, adding some good valid inequalities may help to obtain the optimum solutions.

Another future research direction is to develop a heuristic algorithm for the p-hub maximal covering problem. The computational results show that, obtaining optimal solution with especially partial coverage takes longer solutions times. Therefore, developing a heuristic algorithm for the p-hub maximal covering problem by also considering partial coverage case helps to find good solutions in shorter times.

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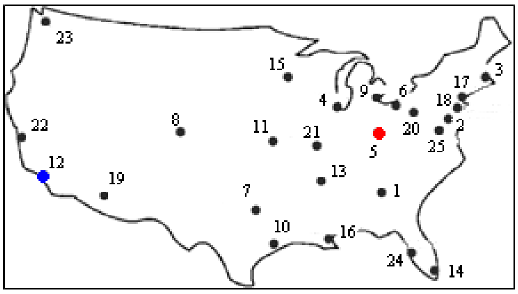
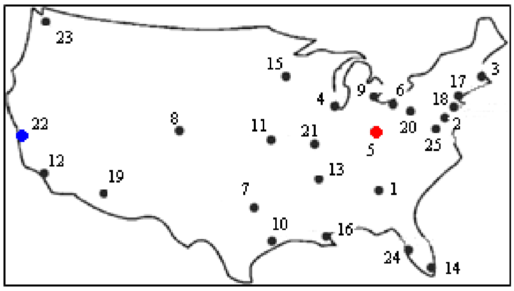
APPENDICES

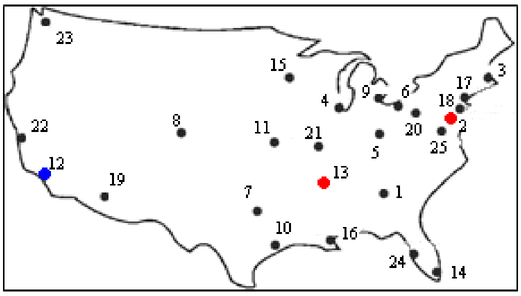
Appendix A: Changes in the Hub Locations of Single Allocation with Binary and Partial Coverage for CAB data set



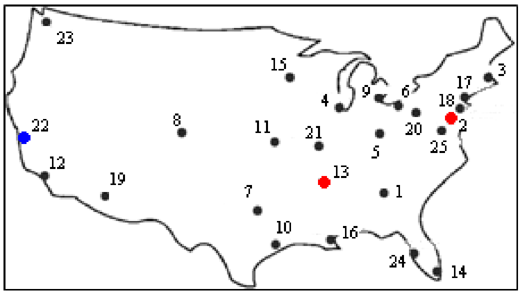
α=0.2, p=2 binary coverage



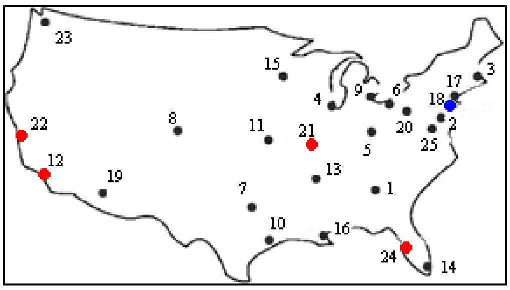
α=0.2, p=2 partial coverage



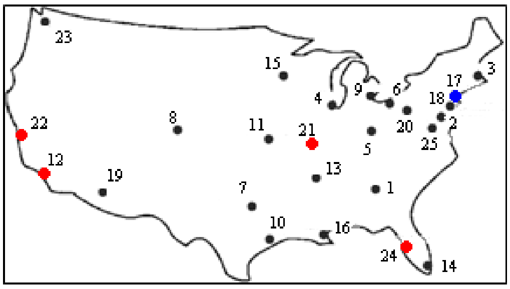
α=0.2, p=3 binary coverage



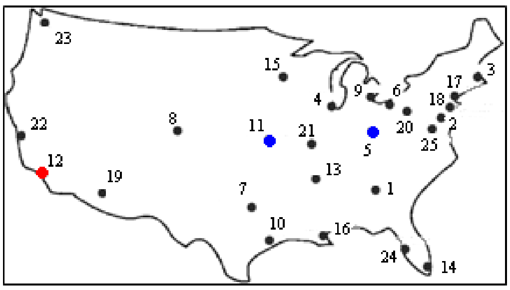
α=0.2, p=3 partial coverage



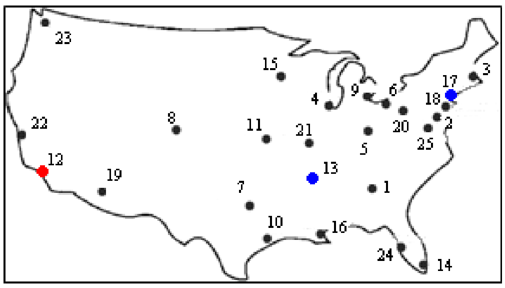
α=0.4, p=5 binary coverage



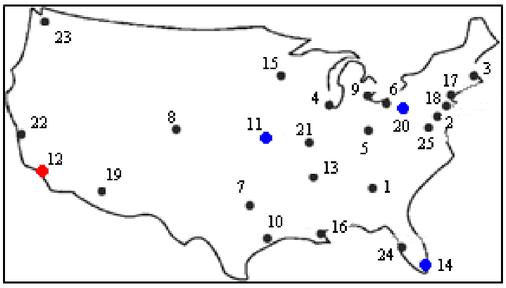
α=0.4, p=5 partial coverage

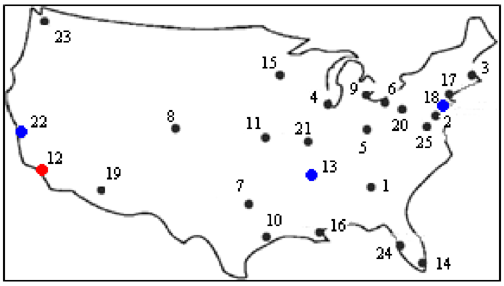


α=0.6, p=3 binary coverage



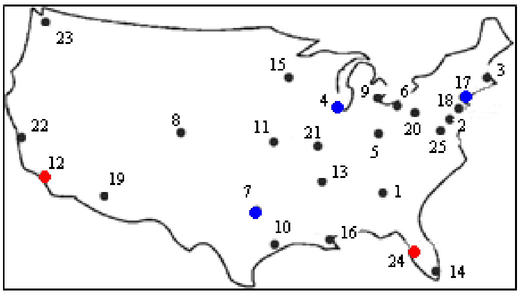
α=0.6, p=3 partial coverage

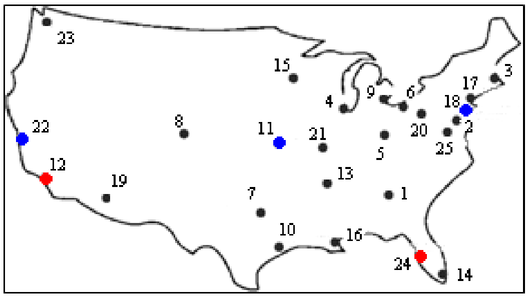




α=0.6, p=4 binary coverage

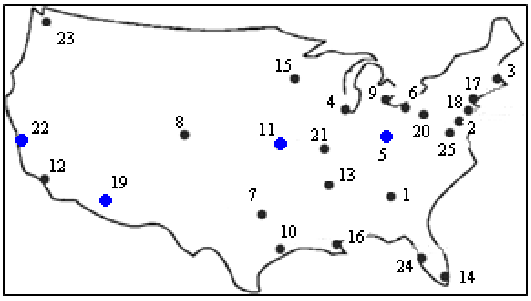
α=0.6, p=4 partial coverage

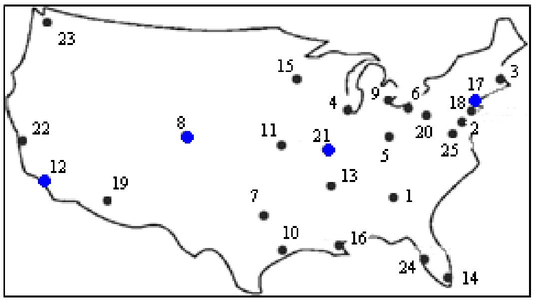




α=0.6, p=5 binary coverage

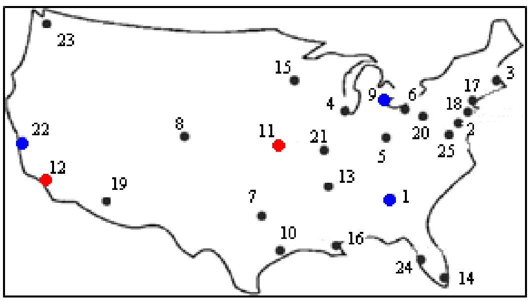
α=0.6, p=5 partial coverage

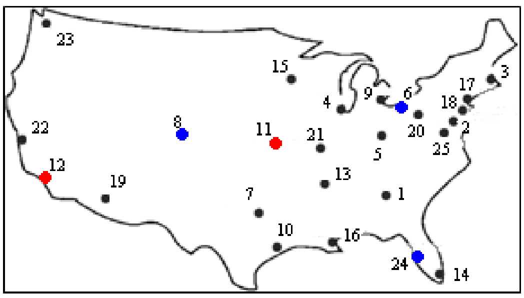




α=0.8, p=4 binary coverage

α=0.8, p=4 partial coverage

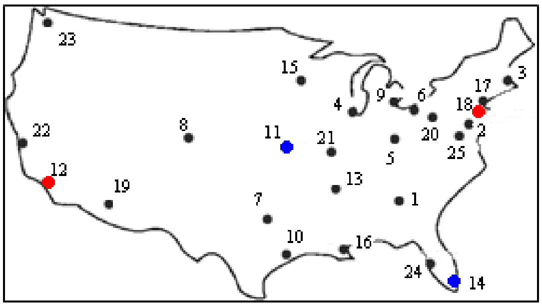




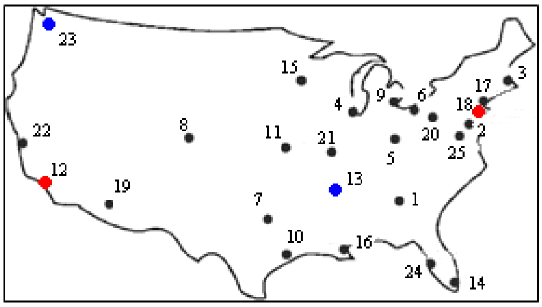
α=0.8, p=5 binary coverage

α=0.8, p=5 partial coverage

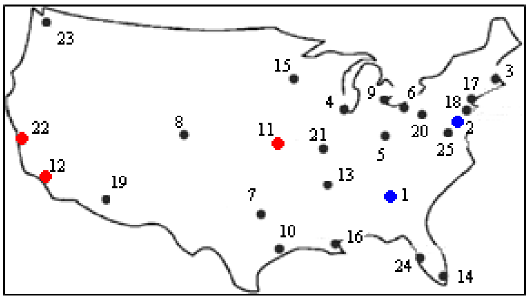
Appendix B: Changes in the Hub Locations of Multiple Allocation with Binary and Partial Coverage for CAB data set



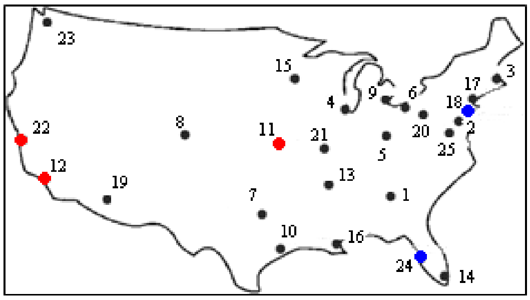
α=0.4, p=4 binary coverage



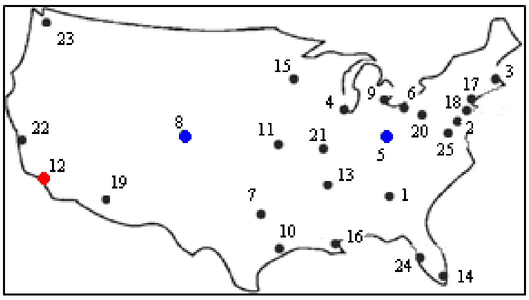
α=0.4, p=4 partial coverage



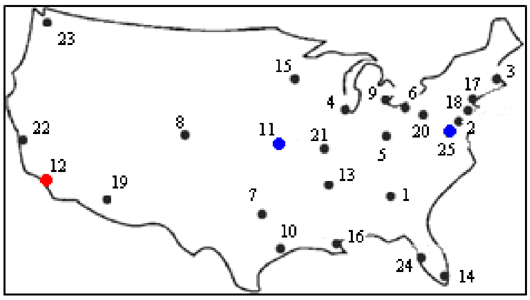
α=0.4, p=5 binary coverage



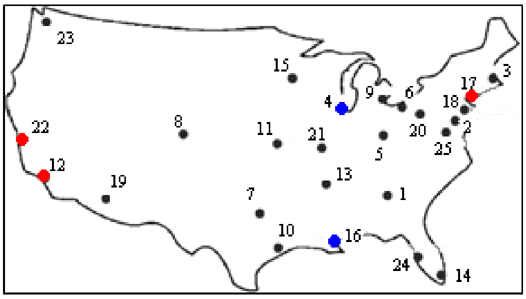
α=0.4, p=5 partial coverage



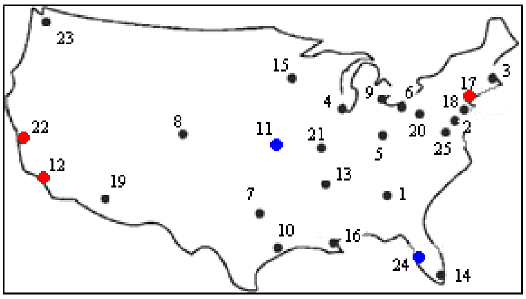
α=0.6, p=3 binary coverage



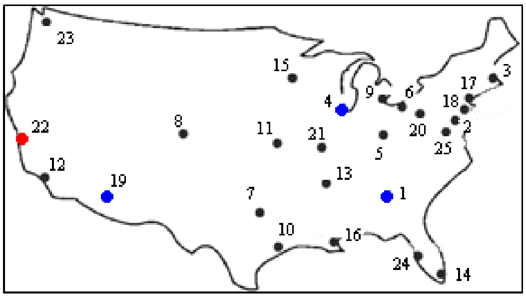
α=0.6, p=3 partial coverage



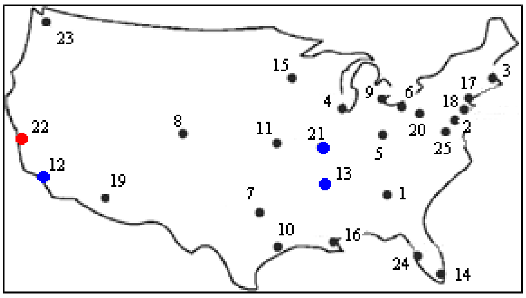
α=0.6, p=5 binary coverage



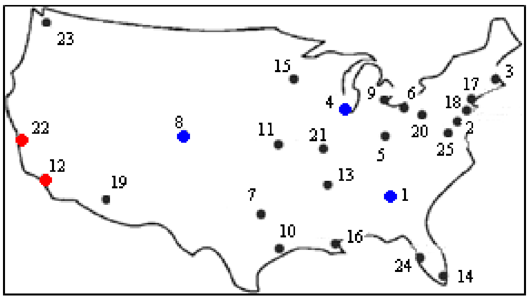
α=0.6, p=5 partial coverage



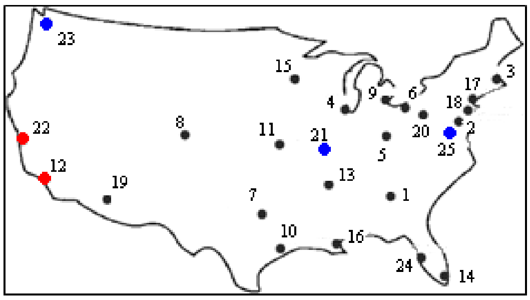
α=0.8, p=4 binary coverage



α=0.8, p=4 partial coverage



α=0.8, p=5 binary coverage



α=0.8, p=5 partial coverage

Appendix C: Coverage of *O-D* pairs for instance α=0.4, p=4 with both allocations for CAB data set

O-D pair	Binary Coverage	Partial Coverage
3 , 8	1	0.5
4 , 19	1	0.75
4 , 22	1	0.75
6 , 8	1	0.5
7 , 19	1	0.75
7 , 22	1	0.75
8 , 9	1	0.75
8 , 14	1	0
8 , 15	1	0.5
8 , 20	1	0.5
8 , 24	1	0.5
9 , 14	1	0.75
12 , 14	1	0.75
14 , 15	1	0.5
14 , 19	1	0
14 , 22	1	0
14 , 24	1	0.5
15 , 19	1	0
15 , 22	1	0
1 , 19	0	1
1 , 22	0	1

O-D pair	Binary Coverage	Partial Coverage
1 , 23	0	1
2 , 19	0	0.75
2 , 22	0	0.75
2 , 23	0	1
3 , 19	0	0.5
3 , 22	0	0.5
3 , 23	0	1
4 , 23	0	1
5 , 19	0	1
5 , 22	0	1
5 , 23	0	1
6 , 19	0	0.25
6 , 22	0	0.25
6 , 23	0	1
7 , 23	0	1
8 , 23	0	1
9 , 19	0	0.25
9 , 22	0	0.25
9 , 23	0	1
10 , 19	0	0.75
10 , 22	0	0.75

O-D pair	Binary Coverage	Partial Coverage
10 , 23	0	1
11 , 23	0	1
13 , 23	0	1
14 , 23	0	0.25
15 , 23	0	1
16 , 19	0	1
16 , 22	0	1
16 , 23	0	1
17 , 23	0	1
18 , 23	0	1
19 , 20	0	0.5
19 , 24	0	0.25
19 , 25	0	0.75
20 , 22	0	0.5
20 , 23	0	1
21 , 23	0	1
22 , 24	0	0.25
22 , 25	0	0.75
23 , 24	0	1
23 , 25	0	1