

THREE ESSAYS ON MONETARY POLICY MODELING: APPLICATIONS  
OF INFLATION TARGETING

A Ph.D. Dissertation

by

EBRU YÜKSEL

Department of Economics  
Bilkent University  
Ankara

February 2008



*To My Family*

THREE ESSAYS ON MONETARY POLICY MODELING: APPLICATIONS  
OF INFLATION TARGETING

The Institute of Economics and Social Sciences  
of  
Bilkent University

by

EBRU YÜKSEL

In Partial Fulfillment of the Requirements for the Degree of  
DOCTOR OF PHILOSOPHY

in

THE DEPARTMENT OF  
ECONOMICS  
BILKENT UNIVERSITY  
ANKARA

February 2008

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

-----  
Assoc. Prof. Kıvılcım Metin-Özcan  
Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

-----  
Assoc. Prof. Syed F. Mahmud  
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

-----  
Assist. Prof. Nedim Alemdar  
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

-----  
Assoc. Prof. Levent Akdeniz  
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy in Economics.

-----  
Prof. Serdar Sayan  
Examining Committee Member

Approval of the Institute of Economics and Social Sciences

-----  
Prof. Erdal Erel  
Director

## **ABSTRACT**

### **THREE ESSAYS ON MONETARY POLICY MODELING: APPLICATIONS OF INFLATION TARGETING**

Yüksel, Ebru

Ph.D., Department of Economics

Supervisor: Assoc. Prof. KIVILCIM Metin-Özcan

February 2008

This dissertation is made up of three essays on modeling monetary policy in a New Keynesian framework. The first essay presents an overview of the evolution of New Keynesian view. Since most of the studies in monetary policy literature employ New Keynesian models due to their power in accounting for price rigidities, microeconomic foundations and various monetary policy rules; such a survey improves our understanding of the type of theoretical and empirical research that has so far been conducted to analyze monetary policy within a New Keynesian framework. This first essay also gives a detailed derivation of structural relationships developed from microfoundations.

The second essay examines the behavior of Taylor-type monetary policy rule by introducing interest rate pass-through in a New Keynesian setting with backward looking components. A simulation is performed to analyze the behavior of policy instrument and pass-through relationship under inflation targeting. The main contribution of this essay is the introduction of interest rate pass-through into a New Keynesian structural model for the first time. Besides, as differently from the previous literature, the structural model

allows for time-varying parameters (TVP) not only for the parameters of the monetary policy rule but also for the coefficients of the interest rate pass-through and other dynamics of the system. This is a salient feature of the analysis here, since previous studies in this field typically allow for variation over time of parameters of the monetary policy rule alone. However, having TVP specification for all parameters of the model provides the flexibility of examining the impact of policy changes over the monetary policy rule, interest rate pass-through and other dynamics of the system. The last important aspect of the second essay is the use of Extended Kalman Filter (EKF) as the estimation technique. That EKF is not widely employed for estimating non-linear systems in this field makes this study significant in demonstrating the strength of EKF in predicting TVP models. The results of the simulations carried out within this essay revealed that long-term interest rate and interest rate pass-through specification are essential ingredients to be included in monetary policy analysis.

The last essay investigates whether inflation targeting programs have altered the pattern of inflation and its variability for five developed countries and four emerging economies implementing inflation targeting programs. A generalized autoregressive conditional heteroscedasticity (GARCH) specification is used to model inflation variability, which accounts for public perception of the future levels of inflation variability – conditional variance. We found that implementation of inflation targeting program has changed the public perception towards inflation only in Australia, Chile, Sweden and the UK, indicating limited empirical support for the lower inflation and its variability for the inflation targeting regimes.

Keywords: EKF, GARCH, Inflation Targeting, Inflation Variability, Interest Rate Pass-Through, Microfoundations, Monetary Policy Analysis, and New Keynesian Framework

## ÖZET

### PARA POLİTİKASI MODELLEMESİ ÜZERİNE ÜÇ MAKALE: ENFLASYON HEDEFLEMESİ UYGULAMALARI

Yüksel, Ebru

Doktora, İktisat Bölümü

Tez Yöneticisi: Doç. Dr. Kıvılcım Metin-Özcan

Şubat 2008

Bu çalışma, para politikası modellemesi üzerine New Keynesian görüş çerçevesinde şekillendirilmiş üç makaleden oluşmaktadır. İlk makale New Keynesian görüşün gelişmesine dair genel bir incelemedir. Para politikası literatüründeki birçok çalışma fiyat katılığını, mikroekonomik yapıtaşlarını ve değişik para politikası kurallarını açıklamadaki gücünden dolayı New Keynesian modelleri kullandıkları için, böyle bir inceleme bugüne kadar New Keynesian görüş çerçevesinde para politikalarını incelemek amacıyla yapılan teorik ve ampirik çalışmaları daha iyi anlamamıza yardımcı olacaktır. İlk makale ayrıca mikroekonomik yapıtaşlarından geliştirilen ve ekonomideki yapısal bağıntıları açıklayan denklemlerin nasıl elde edildiğini ayrıntılı bir biçimde açıklamaktadır.

İkinci makale New Keynesian görüş çerçevesinde oluşturulan Taylor para politikası kuralını ve faiz oranı aktarım mekanizmasını kullanan bir modeli incelemektedir. Para politikası kuralının ve faiz oranı aktarım mekanizmasının davranışlarını enflasyon hedeflemesi altında inceleyen bir simülasyon çalışması gerçekleştirilmiştir. Bu çalışmanın literatüre esas katkısı faiz oranı aktarım mekanizmasının ilk defa bir New Keynesian modelde

kullanılmasıdır. Bunun yanı sıra literatürden farklı olarak, sadece para politikası kuralındaki katsayıların değil diğer tüm yapısal denklemlerdeki ve faiz oranı aktarım mekanizmasındaki katsayıların zamana bağlı olarak değişmesine izin verilmiştir. Bu yapı makalenin en belirgin özelliklerinden biridir çünkü bu alanda daha önce yapılmış çalışmalar sadece para politikası kuralındaki katsayıların değişmesine izin vermektedir. Ancak modeldeki bütün katsayıların zamana bağlı olarak değişmesi politika değişikliklerinin ekonomi üzerindeki etkilerini incelemek için esneklik sağlamaktadır. Bu makalenin sonuncu özelliği ise tahmin etme metodu olarak Genişletilmiş Kalman Filtresinin (EKF) kullanılmasıdır. Bu alanda, lineer olmayan modellerin tahmininde EKF yaygın bir şekilde kullanılmadığı için bu çalışma EKF'nin zamana bağlı katsayıların olduğu modelleri tahmin etme gücünü göstermesi bakımından önemlidir. Bu makale çerçevesinde yapılan simülasyonlar, para politikası analizinde uzun dönemli faiz oranlarının ve faiz oranı aktarım mekanizmasının önemli elemanlar olduğunu ortaya çıkarmıştır.

Son makale enflasyon hedeflemesi programlarının enflasyon ve enflasyon oynaklığı üzerindeki etkilerini incelemektedir. Bunun için enflasyon hedeflemesi programı uygulayan beş gelişmiş, dört gelişmekte olan ülke seçilmiştir. Enflasyon oynaklığını modellemek için tanım olarak kamunun enflasyon algılamasını da içeren GARCH metodu kullanılmıştır. Bu çalışmanın sonucunda sadece Avustralya, Şili, İsveç ve İngiltere'deki enflasyon hedeflemesi programlarının halkın enflasyon algısını değiştirdiği ve bu yöndeki ampirik desteğin sınırlı olduğu ortaya çıkmıştır.

Anahtar Kelimeler: Enflasyon Hedeflemesi, Enflasyon Oynaklığı, Faiz Oranı Aktarım Mekanizması, GARCH, Genişletilmiş Kalman Filtresi (EKF), Mikroekonomik Yapıtaşları, New Keynesian Çerçeve ve Para Politikası Analizi

## ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my supervisor, Prof. Kivılcım Metin-Özcan for her excellent guidance, encouragement and support she provided throughout the development and improvement of this study. This dissertation would not have been completed without her invaluable understanding and patience. She was/will be more than an advisor to me.

I am very grateful to Prof. Hakan Berument who significantly contributed to this dissertation and my graduate experience. I am indebted to him for supporting me with his guidance and valuable advices.

I would like to thank Prof. Nedim Alemdar for his valuable contributions to this dissertation. I also would like to thank the other members of my committee who are Prof. Levent Akdeniz, Prof. Syed F. Mahmud and Prof. Serdar Sayan for their noteworthy comments and advices.

I am very grateful to Dr. Vuslat Us for her contribution and support during my studies. I would like to thank Carl E. Walsh for his help in completing derivations.

Many thanks also go to my colleagues working at Cankaya University, in the Department of Industrial Engineering and at Hacettepe University, in the Department of Industrial Engineering for their friendship, support and advices. I also would like to thank Meltem Sagturk for her friendship and continuous support. Lastly, I would like to thank Haluk Cakir for his kind help.

I am absolutely grateful to my family, my father Alaittin Yüksel, my mother Yüksel Yüksel and my brother Baki Yüksel, for their endless love, continuous support, encouragement, patience and understanding they have provided to me in my entire life. Thank you for being always beside me.

## **TABLE OF CONTENTS**

|                                                                                                        |             |
|--------------------------------------------------------------------------------------------------------|-------------|
| <b>ABSTRACT .....</b>                                                                                  | <b>iii</b>  |
| <b>ÖZET .....</b>                                                                                      | <b>v</b>    |
| <b>ACKNOWLEDGEMENTS .....</b>                                                                          | <b>vii</b>  |
| <b>TABLE OF CONTENTS .....</b>                                                                         | <b>viii</b> |
| <b>LIST OF TABLES .....</b>                                                                            | <b>xi</b>   |
| <b>CHAPTER 1 INTRODUCTION .....</b>                                                                    | <b>1</b>    |
| <b>CHAPTER 2 RE-WORKING ON NEW KEYNESIAN FRAMEWORK<br/>AND RE-EXAMINATION OF MICROFOUNDATIONS.....</b> | <b>7</b>    |
| 2.1. INTRODUCTION .....                                                                                | 7           |
| 2.2. HISTORICAL BACKGROUND .....                                                                       | 8           |
| 2.3. EMERGENCE OF NEW KEYNESIAN FRAMEWORK .....                                                        | 11          |
| 2.4. TIME INCONSISTENCY .....                                                                          | 17          |
| 2.5. ANALYSIS OF POLICY RULES .....                                                                    | 21          |
| 2.6. INVESTIGATION ON THE EFFECTIVENESS OF POLICY<br>RULES .....                                       | 25          |

|                                                              |            |
|--------------------------------------------------------------|------------|
| 2.7. HISTORICAL ANALYSIS OF POLICY RULES.....                | 28         |
| 2.8. MICROFOUNDATIONS OF NEW KEYNESIAN MODEL .....           | 31         |
| 2.8.1. Households.....                                       | 33         |
| 2.8.2. Firms .....                                           | 38         |
| 2.8.3. Equilibrium .....                                     | 41         |
| 2.8.4. Monetary Policy Rule .....                            | 53         |
| 2.9. BRIDGING GAP FOR MONETARY POLICY ANALYSIS .....         | 55         |
| <br><b>CHAPTER 3 MONETARY POLICY ANALYSIS WITH TVP</b>       |            |
| <b>INTEREST RATE PASS-THROUGH.....</b>                       | <b>57</b>  |
| 3.1. INTRODUCTION .....                                      | 57         |
| 3.2. STATE SPACE MODELS.....                                 | 71         |
| 3.2.1. Linear State Space Models and Kalman Filter.....      | 72         |
| 3.2.2. Non-Linear State Space Models and EKF .....           | 74         |
| 3.3. THE MODEL AND STATE SPACE REPRESENTATION.....           | 79         |
| 3.3.1. The Baseline Model.....                               | 79         |
| 3.3.2. Alternative Model.....                                | 86         |
| 3.4. DATA GENERATION AND SIMULATION.....                     | 90         |
| 3.5. SIMULATION RESULTS AND FINDINGS.....                    | 93         |
| 3.5.1. Coefficients of Monetary Policy Rule .....            | 94         |
| 3.5.2. Coefficients of Interest Rate Pass-Through .....      | 97         |
| 3.6. CONCLUSION.....                                         | 99         |
| <br><b>CHAPTER 4 EFFECTS OF ADOPTING INFLATION TARGETING</b> |            |
| <b>REGIMES ON INFLATION VARIABILITY .....</b>                | <b>102</b> |
| 4.1. INTRODUCTION .....                                      | 102        |

|                                   |            |
|-----------------------------------|------------|
| 4.2. METHODOLOGY .....            | 107        |
| 4.3. EMPIRICAL EVIDENCE .....     | 112        |
| 4.4. CONCLUSION.....              | 121        |
| <b>CHAPTER 5 CONCLUSION .....</b> | <b>124</b> |
| <b>BIBLIOGRAPHY .....</b>         | <b>129</b> |
| <b>APPENDICES .....</b>           | <b>145</b> |
| APPENDIX A .....                  | 146        |
| APPENDIX B.....                   | 147        |
| APPENDIX C.....                   | 155        |

## LIST OF TABLES

|                                                          |     |
|----------------------------------------------------------|-----|
| Table 1: Summary of Literature on TVP Policy Rules ..... | 66  |
| Table 2: ARCH-LM Test for the Inflation Series .....     | 113 |
| Table 3: Estimation Results of the Model 1 .....         | 114 |
| Table 4: Estimation Results of the Model 2 .....         | 116 |
| Table 5: Estimation Results of the Model 3 .....         | 118 |

## **CHAPTER 1**

### **INTRODUCTION**

For the last 50 years, there has been a great tendency in working on monetary policy implemented by a monetary authority and its influence over the economy. This tendency is motivated by the experience/observation that monetary policy implemented in a country has significant impact on real activity of an economy in short run.

Academicians and policymakers interested in monetary economics want to know/understand the relationship/causality among macroeconomic variables such as output, interest rates, inflation, employment, money stock and exchange rates to see what type of behavioral relations construct a stable economy in a country. Simply, rise of monetary policy is the consequence of a desire to smooth business cycle fluctuations in an economy. Therefore, the connection between real aggregate variables and nominal variables is the study area of monetary economics. Examining long-run and short-run relations

between nominal and real variables gives insight to the dynamics of an economy, monetary policies implemented and welfare of the public, which is a great concern for the politicians. Investigation of monetary policy rules could provide us with the advantage of seeing benefits, limitations, implementation easiness'/difficulties of the policies, propagation mechanisms of the economy, explanatory power of the policies for the dynamics of economy. Thus, it is important to study theoretical and empirical aspects of monetary economics and its evolution.

In the first essay of this dissertation, a brief summary about the development of New Keynesian framework will be given. The reasoning of this review is grounded on the fact that currently, most of the studies in monetary economics rely on New Keynesian view due to its power in explaining/modeling price rigidities, microeconomic foundations, rational expectations and various monetary policy rules. Such a literature study provides us to see what type of theoretical and empirical research has been performed related to monetary policy analysis under a New Keynesian composition.

In addition to the literature study, the first essay will give details of how to reach reduced form equations developed from microfoundations. In general, the articles in this field do not include these derivations and begin their analysis with the reduced form equations. However, in order to understand inherent assumptions of the New Keynesian world and direct/form subsequent research,

the microfoundations of the economy, assumptions and dynamics that shape this world and the way how linear relationships are derived from such a microfounded model should be known clearly. Since most of the studies skip this part, it is not so easy to see properties of these derivations. Therefore, the last part of the first essay will be allocated to mathematics of formation of a New Keynesian model. We hope that such an analytical examination could offer a baseline for many researchers in their studies.

The literature review given in the first essay reveals that recently, inflation targeting has been favored by most of the studies for stabilizing prices in the economy. Therefore, the other two essays of this dissertation are shaped around the implementation of inflation targeting.

The second essay is going to investigate the behavior of Taylor-type monetary policy rule with interest rate pass-through in a New Keynesian setting with backward looking components. A simulation study will be performed to analyze the behavior of policy instrument and pass-through relationship under inflation targeting. This essay has three distinctive features, which are not common in the literature:

The main contribution of this essay is the introduction of interest rate pass-through to a New Keynesian setting with/in addition to a monetary policy rule. As far as we know, appearance of interest rate pass-through in a simulated

structural model is the first time in the literature. The reason of such an extension is the following: Aggregate demand is affected by long-term interest rate, regarding the consumption and investment behavior of agents, rather than short-term policy rate. Thus, it is suggested that monetary transmission mechanism should have a component that includes long-term interest rates. Accordingly, interest rate pass-through, which explains the relationship between policy rate and long-term market rates, is added to the structural model.

Another significant point of the second essay is that, this article has time-varying parameter (TVP) property so that, all parameters of the model are time-dependent. Such a characterization gives us the opportunity of examining the influence of policy changes over the monetary policy rule, interest rate pass-through and other dynamics of the system. Here, it is necessary to state that we allow changes not only in the parameters of the monetary policy rule but also in the coefficients of the interest rate pass-through and other dynamics of the system. This is a distinctive property as generally; the studies accomplished in this area consider time-varying property only for monetary policy rule. A model with TVP feature provides us with the opportunity of examining the influence of policy changes over the economy and through which channels these changes are disseminated.

Furthermore, the last important aspect of the second essay is the estimation technique employed, which is Extended Kalman Filter (EKF). The model has a non-linear characteristic since we employ time-varying parameters. Therefore, it is necessary to use an estimation algorithm, which is appropriate for non-linear systems. Although the usual Kalman Filter is powerful for linear systems, it loses its strength when the non-linear systems are under consideration. Thus, we prefer using EKF, which is produced for non-linear models. Use of EKF in the field of TVP monetary policy analysis is not so broad hence; our study will be a leading one demonstrating the strength of EKF in predicting TVP models.

Finally, the third essay of this dissertation will be an empirical study about the effect of inflation targeting policy on inflation and its variability. In this study, we are going to investigate whether inflation targeting programs have altered the perception of public towards inflation and its variability in 5 developed and 4 developing countries implementing inflation targeting programs. In the literature, there are similar studies investigating the effect of inflation targeting programs on various macroeconomic variables however, none of them have attempted to measure public perception about inflation, which was our main contribution. We will use autoregressive conditional heteroscedasticity (ARCH)/ generalized autoregressive conditional heteroscedasticity (GARCH) type of conditional variability to measure inflation variability, which was not made before in the literature. Our work has two folds: one is whether

implementation of these programs, in other words commitment to such a monetary policy rule, really changed the public assessment of inflation and its variability. In this respect, our study makes contributions to the rules vs. discretion debate. That is, it provides a basis for judging the effect of inflation targeting programs on convincing people's view about the intentions of the monetary policy authority. The second point is the comparison of this behavior between developed and developing countries. We are going to explore whether opinion of people about inflation variability display differences with respect to the economic state of the country. As to the consequences of the third essay shortly, it was found that inflation targeting programs decreased inflation variability in one developed and one emerging country significantly, and in some of the other countries insignificantly. The conclusion of the third essay revealed that implementation of inflation targeting program has really changed the public perception towards inflation in some countries, that is, expectation of people about inflation level and its variability has decreased during the implementation of the program and after. This result was observed both in developed and developing countries indicating that the economic state of the country does not create big differences about the effectiveness of inflation targeting programs in reducing inflation variability.

## **CHAPTER 2**

### **RE-WORKING ON NEW KEYNESIAN FRAMEWORK AND RE-EXAMINATION OF MICROFOUNDATIONS**

#### ***2.1. Introduction***

In this chapter, a brief summary of the evolution of New Keynesian framework will be given. The developments that are highly influential in monetary policy analysis and New Keynesian modeling will be mentioned. Since so many works are employing structural models based on New Keynesian view for conducting monetary policy analysis, this literature study presents a summary of theoretical and empirical research on monetary policy analysis under a New Keynesian arrangement.

After the literature review, a mathematical model with microfoundations will be introduced that explains the dynamics, building blocks and assumptions of

New Keynesian world. Starting from this microfounded model, linear structural relationships, which are Phillip's curve and aggregate demand curve (in other words, investment-saving, shortly IS, curve) will be derived in detail.

## ***2.2. Historical Background***

New Keynesian view takes its roots from the macroeconomic framework of Keynes (1936) after the period of Great Depression, which was the consequence of a policy of economic liberalism, proposing that private sector and markets can act optimally without any government interference. During Great Depression, aggregate output was at a highly low value with decreasing employment and capital utilization. From this experience, it was realized that a liberal market economy was unsuccessful in managing supply and demand. Therefore, government interference was necessary to control and direct them. In Keynesian view, behavior of individuals determines the aggregate demand and hence macroeconomic trends. Consumption and investment behavior of consumers shape the aggregate demand, which is the driving force of the economy so that, short run variations in real activity- output and employment- are figured as the consequence of variations in aggregate demand. At this point, government intervention has the role of implementing a macroeconomic stabilization policy to control aggregate demand and smooth business cycle fluctuations. After World War II, Keynesian view was widely accepted and

investment-saving (IS)/liquidity of money (LM) framework with Phillip's curve was utilized to study the dynamics of the economy and control economic activity.

In 1960s, high inflation as well as inflation-output trade off became a central policy item for governments. Phillip's curve, a model on the relationship between inflation and unemployment, was started to be the focus of many works. Furthermore, policymakers perceived Phillip's curve as a powerful tool showing the importance and strong implications of monetary policy. A policy that favors increasing demand for goods and services results in high inflation and reduced unemployment in the short-run. However, quantity theory of money by Friedman (1970, 1971) stated that the trade off between employment and inflation disappear in the long-run, that is; when the agents adjust to high inflation rate, unemployment rises again. To keep unemployment at low levels, continuously increasing inflation is needed. Additionally, Friedman (1968) discussed the failure of the Phillips' curve from the microeconomic side and concluded that the link between unemployment and inflation may not work since unemployment is affected from not only money growth but also labor supply and demand, which are not considered in the Phillips' curve.

In 1970s, oil shocks and other productivity related problems showed that IS/LM model with Phillip's curve framework was not sufficient to explain stagflation, rising inflation together with increasing unemployment. This was

due to the vision that Keynesian model was focused on the aggregate demand side of the economy and supply side was having secondary importance but, the problems arising in 1970s were related to supply side of the economy. Besides, Keynesian model was also open to Lucas' critique due to Lucas (1976) claiming that Phillips' curve failed to explain period of stagflation since it was derived from empirical forecasting models not from a theoretical model with microfoundations.

Lucas' critique maintains that the relationships among macroeconomic variables alter when the macroeconomic policy changes so, using reduced form equations are not enough to make economic analysis. Macroeconomic models based on microfoundations are more reliable in assessing the influence of policy changes on economy assuming that macroeconomic policies do not change the behavior of micro building blocks of the model like preferences of the individuals, technology and market structure. Thus, empirically obtained relationships are subject to policy in effect, that is, the association between inflation and unemployment in a low inflation environment is different from the mentioned relationship in a high inflation environment since policy regimes implemented in these periods are different and empirical correlations are affected from the policy changes. Since IS/LM framework was composed of functional equations relating macroeconomics variables such as output, inflation, unemployment, consumption to each other, this view had the lack of microeconomic foundations. Furthermore, behavior of individuals and firms

were not considered in the IS/LM models so, it was not possible to derive tough conclusions about the impact of policy on economic activity. In order to assess the impact of a policy on economy, a microfounded model should be built in which, the micro blocks such as preferences of individuals, technology, and budget constraint are not affected from the policy change.

Due to the shortcomings of IS/LM framework and Phillip's curve, revisions and some fundamental adjustments were carried out to overcome these limitations. As a consequence of some major modifications, New Keynesian framework has emerged, which will be discussed in the next section.

### ***2.3. Emergence of New Keynesian Framework***

In the late 1970s, New Keynesian framework was started to be pronounced for modeling economy with an increasing interest in monetary policy modeling. New Keynesian framework combines IS/LM and Phillip's curve models with microfounded building blocks, that is, household and firm's behavior are modeled in micro level and they are aggregated to derive functional forms/relationships among macroeconomic variables. In this respect, it can be said that New Keynesian view was a response to Keynesian view due to Lucas' critique. Furthermore, there were new extensions that make New Keynesian models operational in policy analysis.

One extension is the introduction of rational expectations initiated by Muth (1961) and later developed by Lucas (1972). Rational expectations hypothesis is about the way of forecasting future events, which influence the current actions of the agents. The hypothesis states that agents use all available information and make predictions with perfect foresight. In this way, the rational expectations outcomes can be regarded as equilibrium results and if there is a deviation from the equilibrium, it is not a systematic mistake but random error. In other words, individuals form their expectations optimally so that predictions of economic theory are in conformance with the predictions of agents.

The other expansion is about the nominal rigidities. New Keynesian framework suggests that prices and wages cannot be adjusted quickly in the short run, which is considered as a market imperfection leading to inefficiency in the economy. Then government or central bank, which implements a monetary policy, can produce more efficient results. This view promotes/increases interest into design and implementation of optimal monetary policies. In this picture, it will not be wrong to state that macroeconomic analysis and macroeconomic policies mostly overlapped with the monetary policy analysis.

Many economists contributed to this area in different directions. Although their concentration is disseminated in separate branches, they share a common thought that in the long-run, impact of money on prices decreases and

influence on output is little. However, in the short-run, effect of money on real activity is significant. Initially, most of the studies explored the relationship between money and output, empirically. Friedman and Schwartz (1963) presented empirically that money stock growth produces changes in output in the short-run. Besides the examination of the association between money stock and output, some articles investigated the factors that are used to forecast output. For instance, Sims (1980) showed that in addition to money stock, short-term nominal interest rate can be more effective in forecasting output since short-term nominal interest rate is more informative about the monetary policy actions. Friedman and Kuttner (1992) examined the relationship between output and different money stock definitions with different interest rates. Similarly, Bernanke and Blinder (1992) showed that federal funds rate is more efficient in explaining movements in real variables compared to money stock and bond rates.

After the adoption of rational expectations, many economists used this approach in their researches. For instance, Sargent and Wallace (1975) investigated the effectiveness and equilibrium properties of different monetary policy rules under the assumption that public's expectations are formed rationally. They found that money supply rule have some influence on prices but not on output. Another work that used rational expectations was Fischer's, (1977) arguing that monetary policy can affect short-run real output in a sticky-wage framework. This study is influential for the construction of New

Keynesian models with sticky prices. Mankiw (1988, 1990) summarized advances in macroeconomics including New Keynesian approach with an emphasis on rational expectations.

Following Lucas' critique due to Lucas (1976), microfounded models with rational expectations hypothesis, so called dynamic stochastic general equilibrium models, were started to be constructed in 1980s and later. The first study that shapes a general equilibrium model relying on microeconomic foundations belongs to Kydland and Prescott (1982). Using such a new model, they contributed to business cycle literature by showing that besides being the source of long-run growth, technology shocks can also be an important source of short-run output fluctuations in a perfectly competitive environment without market frictions. Later, McCallum and Nelson (1999a) attempted to combine IS/LM framework with microeconomic foundations. It was illustrated that with a little modification to IS equation, addition of expected future income, the IS/LM equations can reasonably be used to express aggregate demand side of the economy, which is drawn from the solution of the optimization problem of the agents. Moreover, this representation can be merged with various aggregate supply patterns.

While Kydland and Prescott (1982) introduced the usage of microfoundations for macroeconomic models, which is one of the essentials for New Keynesian framework, Taylor (1980) and Gordon (1982) studied on nominal rigidity

feature of the economy. They demonstrated that under rational expectations, wages and prices adjust gradually to aggregate demand shocks, that is, wages and prices are sticky and they do not adjust in the short-run but they can fully adjust in the long-run. Rotemberg (1982) verified empirically that prices are sticky in the US. The nominal rigidity takes its roots from the costly price adjustment behavior of firms. The concept of price adjustment cost was used by Schmitt-Grohe and Uribe (2002) to introduce price rigidity into their microfounded model.

Imperfect market structure, in which wages and prices can not adjust immediately, was one of the arguments used to explain economic fluctuations. The other item, which is used to explore the impact of money on output, was the price setting behavior of firms. Monopolistic competition is more powerful compared to perfect competition in explaining the price setting behavior of firms and output fluctuations in response to changes in demand. Hence, interest was on the behavior of monopolistically competitive firms and differentiated products. Some works on this issue are Mankiw (1985), Blanchard and Kiyotaki (1987) and Rotemberg (1987).

The building blocks of New Keynesian models that are used to analyze monetary policies to stabilize the fluctuations in an economy are outlined above. These are IS/LM framework, Phillips' curve, rational expectations, microeconomic foundations of the structural model, nominal rigidity (sticky

prices) and monopolistic competition behavior of firms. In addition to individuals and private firms, it is certain that an authority, whether central bank or government, is included in the models to implement monetary policy rule. Many authors used in the past and still are using, these elements in their works to study the influence of monetary policy over the economy. One of the main concerns is to find an optimal monetary policy rule to stabilize inflation and output growth. This issue also includes the determination of the policy instrument that will be used to implement the monetary policy. There is a vast literature on this subject and one part examines the optimal monetary policies on theoretical grounds with microfounded models (dynamic general equilibrium models); the remaining part, on the other hand, associates the conclusions of the former with real data using different econometric specifications and simulation tools.

In 1980s and later, many studies used the dynamic general equilibrium models with a New Keynesian perspective to examine dynamics of the economy. Ball et al. (1988) explored the advances that New Keynesian approach brings into macroeconomic analysis. They showed that sticky prices and nominal aggregate demand shocks are the driving force of the fluctuations in real output in the short-run, which is a support for the theory of New Keynesians about output fluctuations. Another argument was about the relationship between slope of the Phillips' curve and average inflation, in other words inflation-output trade off, stating that in a low average inflation environment, nominal

aggregate demand shocks have sizeable impact on real output while in a high average inflation environment, price level is the one, which is affected highly from the nominal shocks.

Elaboration of dynamics in a microfounded New Keynesian model attracted the attention into the issue of monetary policy design. Many economists directed their research to monetary policy rules, transmission mechanisms, policy instruments and properties of these policies both in closed and open economies. These studies also gave rise to an important issue about monetary policy implementation: Should monetary policy authority stick to a predefined policy rule or can discretionary policy create more preferable results? In fact, exploration of monetary policy rules was carried out with time inconsistency problem and rule vs. discretion debate, simultaneously. Here, it will be practical to mention about time inconsistency concept, which is explained in the next section.

#### ***2.4. Time Inconsistency***

As it is known from the equilibrium models, stability of the economy (or equilibrium) depends on current and future behavior of the variables. In this sense, equilibrium also relies on the monetary policy rule implemented by the central bank; since expectations of the agents are formed assuming that central

bank will obey the policy rule in the future. According to this behavior, a policy rule can be found for central bank to optimize its objective function. However, there is a lot of debate on what guarantees that central bank will obey the policy rule specified before? Sometimes, it can be preferred to deviate from the policy rule, although agents acted and formed expectations presuming that central bank will implement the policy rule declared before. If discretion is possible, that is, deviation from the rule is likely, then agents will be aware of this scenario and act considering the possibility of this deviation, resultantly expectations of them will not be based on the policy rule previously announced. If there are not certain rules ensuring that central bank will obey the policy rule stated previously, and/or there is the opportunity of deviating from the rule, then central bank may find it optimal to use incentives, which are not consistent with the policy rule. Such policies are called time inconsistent policies, that is, if an action proposed at time  $t$  for time  $t+k$ , is not optimal to implement when time  $t+k$  arrives, then these policies are time inconsistent. On the other hand, if an action proposed at time  $t$  for time  $t+k$ , is still optimal to implement when time  $t+k$  arrives, then these are called time consistent policies. It can be said that time consistent policies are still optimal to implement even new information arrives and new events happen.

To find and conduct an optimal monetary policy, time inconsistency subject has great importance to discuss. It was stated that effectiveness of a monetary policy relies on both current actions and future actions or expectations of the

agents about future policy implementations. Then, in order to understand how policy rules operate and affect economy, we need to understand how agents react to policy actions so that we can have idea about formation of the expectations. To do this, policymakers should follow a well-defined policy rule and the scenarios for potential deviations from the rule can be investigated. Besides, examination of time inconsistency may provide guidance for understanding the decision making problems and designing policymaking bodies such as central banks.

In addition to introduction of microfoundations to macroeconomic models, other great contribution of Kydland and Prescott is about time consistency of economic policies. Kydland and Prescott (1977) showed that a government with rational expectations and forward-looking behavior can find it optimal to implement discretionary policy when it takes into account expectations of private sector for policymaking. However, it was proved that welfare loss is greater under discretionary policy compared to the case when government announces and sticks to a predefined rule.

Barro and Gordon (1983a, 1983b) supported the view of Kydland and Prescott (1977) so that commitment to a predefined rule is superior to discretionary policy with respect to macroeconomic results. Barro and Gordon (1983b) stressed the importance and credibility of monetary institution, as well. Another study that underlined the credibility of government is Backus and

Driffill (1985). They argued that without full credibility of government, output loss is too high for keeping inflation at low levels. Rules vs. discretion debate were also discussed by Taylor (1996). The author pointed out that under the policy goal of price stability, obeying a policy rule, instead of discretion, is preferable in responding different shocks concerning different aspects such as accountability of the performance of the monetary authority, time inconsistency problem, stipulation of future events of monetary policy, reducing the uncertainty about the future monetary policy actions, formation of events for policymakers to achieve policy goals. However, it was argued that some discretion might still be necessary, to lesser extent, while working with a policy rule.

Another study, which contributes to discretion vs. rule debate, belongs to Dwyer (1993). In this study, it was claimed that although they are powerful, policy rules do not remove discretion totally, that is, there is still room for discretion. Then, design of monetary authority and monetary policy draws the attention. As for the policy rules to be implemented, time consistency of monetary policy, use of feedback rules, response of the monetary authority to the current state of the economy, effect of feedback policy on the future behavior of the economy become the main concerns of the discussion.

## ***2.5. Analysis of Policy Rules***

Having seen the superiority of commitment to policy rules instead of discretion in producing better economic results, many researchers examined different monetary policy rules. Henderson and McKibbin (1993) compared alternative monetary policy rules in a two-country world by the help of scenarios composed of shocks to money demand/goods demand/productivity, interest rate tool with full/partial adjustment, different policy targets like money supply/nominal income/output plus inflation, existence/absence of nominal wage persistence. It was stated that all these aspects, type of shocks, adjustment of policy tool, policy target, nominal wage persistence are important for the design of an effective monetary policy.

A famous monetary policy rule, called Taylor rule, belonged John B. Taylor. Taylor (1993) suggested this practical policy rule so that policy interest rate should be responsive to changes in inflation and real output. This econometrically supported result has been highly influential in monetary policy literature and Taylor rule was employed in numerous models. Although many economists discussed pros of this operational rule, it was also criticized in some works. For instance, Orphanides (1998) pointed out the importance of the timing of information, which is necessary for implementation of policy rule. The problems with regard to real-time data, uncertainty inherent in the data, misleading conclusions obtained due to utilization of ex-post revised data in

monetary policy analysis were illustrated via Taylor rule. Having in mind the concerns related to real-time data, policymakers should use variables with minimum uncertainty so that the monetary policy can be implemented with high efficiency. Obviously, monetary policy is designed to stabilize inflation and output in an economy and monetary policy rule should be respondent to these variables. Due to this sensitivity, it is clear that any mismeasurement in output and inflation can have considerable negative impact on the implementation of policy rules and their consequences.

Starting from the argument of Orphanides (2001), Leitemo and Lønning (2006) points out that having precise data on output gap can improve the efficiency of monetary policy rule considerably. Since Taylor rule needs current output gap data, which cannot be observed currently but is available after some time has passed, estimation of output gap becomes compulsory. However, due to the complications about the definition of natural output level, it is difficult to measure output gap in real-time. This leads to uncertainty in it, which can bring out problems in implementation of policy rules and deviations from the desired policy. In order to overcome this problem, Leitemo and Lønning (2006) proposed the use of proxies, simple and expectation-based proxies, developed from the relationship between inflation and output gap, in place of output gap in the Taylor rule as benchmark rule. The results of the study recommended that proxy-based policy rules considerably decreases the uncertainty of the model with respect to those based on current estimate of output gap. Levin et

al. (1998) proposed that instead of level of the short-term interest rate in the Taylor rule, using first difference of the policy rate could generate better results in achieving low inflation-output volatility. Also, it was stated that this type of rules were more robust with respect to producing the similar results, at least for the models used in the study.

Design of monetary policy rule was emphasized also by McCallum (1997) by highlighting the essence of commitment to a policy rule even in the existence of discretionary pressures. Despite the value of optimal monetary policy rule, which is a rule specific to a particular model, robust rules, which can suit to various models, were raised. Growth rate targets for inflation and output were suggested as central bank's policy target while both short-term interest rate and monetary base were discussed for being policy instrument. Finally, monetary and fiscal policy relationship and collaboration of monetary and fiscal authorities were mentioned.

Ireland (1997) evaluated various monetary policy rules using a dynamic general equilibrium model calibrated for the US. It was revealed that aggregate output fluctuations were mostly due to technology shocks. Besides, decreasing average inflation rate and achieving price stability would provide gains regarding to welfare.

Issues in the design of monetary policy rules, employing the suitable policy targets and instruments were also discussed by Ball (1997). The macroeconomic framework was based on three linear equations, without dealing with the microfoundations, the IS curve, Phillips' curve and monetary policy rule. Three types of monetary policy rules were examined with respect to their efficiencies in reducing total variances of output and inflation; Taylor rule, inflation targeting and nominal income targeting. It was stated that the Taylor rules are efficient but efficiency depends on the parameters used; moreover, an efficient Taylor rule can be treated as inflation targeting policy. Tightness of inflation targeting regime depends on the preferences about inflation and output volatility, that is, if low inflation volatility is desired, then a strict policy is needed. Nominal income targeting is found to be inefficient in minimizing the output and inflation variance. Similar and prior to Orphanides (1998), Ball (1997) criticized the Taylor rule with respect to measurement of parameters like potential output level and sensitivity of them to policy variables.

During and after 1990s, desirability of price stability led to rise of inflation targeting. In conducting monetary policy, adoption of price level path or target inflation rate were highly popularized. Some countries, Canada, Finland, New Zealand, Switzerland, and the UK adopted this regime and many studies elaborated on the consequences of implementation and future policy actions. Inflation targeting and price level targeting as policy rule/framework,

implementation circumstances of these regimes, forward looking behavior in building inflation expectations, issues related to monetary authority, policy instruments and policy targets, trade-off regarding to inflation-output volatility were discussed both theoretically and empirically by many researchers such as Smith (1994), Cecchetti (1995), Green (1996), Svensson (1996), Bernanke and Mishkin (1997), Rudebusch and Svensson (1998), Svensson (1995), Mishkin and Posen (1997), Clarida et al. (1997).

Issues in the design of monetary policy rules were followed by concerns about the efficiency of these rules in providing price and output stability. Therefore, some studies started to evaluate monetary policy rules on the basis of establishing a stable economy. The next section mentions some of the studies examining the effectiveness of monetary policy rules and transmission mechanisms.

## ***2.6. Investigation on the Effectiveness of Policy Rules***

An assessment on monetary transmission mechanism, which gives important insights for the design of a policy rule, was made by Taylor (1995). This study highlighted common crucial characteristics of monetary transmission models, which are extracted empirically from the relationships existing among the macroeconomic variables. A transmission mechanism that considers exchange

rate, short and long-term interest rates was particularly discussed. A flexible exchange rate system with Taylor-type interest rate rule, due to Taylor (1993), was favored according to empirical studies.

Being another extension, Goodfriend and King (1997) explained the components of the new research field of macroeconomics, new neoclassical synthesis, which is the combination of neoclassical principles in microeconomic analysis and Keynesian approach in determination of aggregate output. In this framework, nature of monetary transmission mechanisms, role of monetary policies and interaction of inflation with real activity were illustrated and it was established that in a rational expectations setting, inflation targeting is the optimal monetary policy. The implementation attributes of inflation targeting were also stated, such as, response to price shocks, output-inflation variability trade-off, use of interest rate rules.

Rotemberg and Woodford (1998) contributed to this literature by investigating effectiveness of different monetary policy rules over the welfare of the public using an optimization-based model. As for calibration of the theoretical model, they used vector autoregressive (VAR) specification for modeling actual time series data. Two monetary policies, Taylor rule and constrained-optimal policy, were evaluated with respect to a utility based loss function for monetary authority. Instead of imposing an ad hoc loss function for government, as done by previous works, a welfare loss function is derived from the households'

lifetime utility function. The conclusions were similar to previous studies in that, Taylor-type monetary policy rule decreases volatility of inflation and inflation stabilization is the optimal policy. However, such a policy goes along with high output volatility and high interest rate volatility, which requires high average inflation leading to a trade-off for the optimal policy. On the other hand, constrained-optimal policy achieves a better trade-off between inflation rate variability and interest rate volatility by allowing inflationary shocks, which rise average inflation, in fact. The latter policy lowers both average inflation rates, thus interest rate volatility, and inflation variability. Nonetheless, since constrained-optimal policy permits to supply shocks, contrary to historical policy, output variability increases under this strategy.

A similar work was accomplished by Clarida et al. (1999). In a New Keynesian framework, advances in monetary policy design and implementation were examined using a simple theoretical model.

Schmitt-Grohe and Uribe (2002) investigated both fiscal and monetary policy (instead of just monetary policy) under sticky prices and imperfect competition (instead of flexible prices and perfect competition) and the results were compared both with the flexible price-perfect competition model and flexible price-imperfect competition model. The results depicted that although Friedman rule (zero nominal interest rate) was found to be optimal for the flexible price-perfect competition model, it was not the case anymore for sticky

price-imperfect competition model. Also, volatility of inflation highly decreases in sticky price-imperfect competition model compared to other models since, it was found optimal for governments to design policies that support stable prices in order to reduce welfare loss due to price stickiness. Apart from these, implementation of Taylor-type policy rules in this structure were not favored.

As the experience in the design and implementation of monetary policy rules increased, attention was directed, by some authors, to a research field about the assessment of policy rules in time, in other words ex-post monetary policy analysis. The subsequent section briefly introduces the advances in historical monetary policy analysis.

### ***2.7. Historical Analysis of Policy Rules***

While some studies investigated efficient monetary policy rules, which are optimal for the models used, some other works by making historical monetary policy analysis demonstrated that monetary policy rules alter as dynamics of the economy changes. For instance, Judd and Rudebusch (1998) performed a historical analysis associating economic events with Fed's actions. The study concluded that Taylor rule framework was a suitable tool for the formation of an effective monetary policy rule, to some extent.

Another study elaborating on the applicability of the Taylor rules was Kozicki (1999). Although the Taylor rule was inadequate due to problems of real-time data reliability and problem of robustness to changing specifications of the variables used, it was pointed in the article that Taylor rule framework is simple, easy to understand and practical to use as a starting point for monetary policy analysis and implementation.

Taylor (1999a) examined the monetary policy history of the US based on the monetary policy rules used. The influences of different monetary policy rules on the behavior of economy were evaluated and these rules were linked to political events in time. It is concluded that the short term interest rates should respond to inflation and output at high degrees. Changes in the monetary policy directly affect the economic stability and economic outcomes.

Similarly, Clarida et al. (2000) analyzed the monetary policies maintained in the US. They concluded that one of the factors that should be considered in formulating monetary policy is the view of policymaker about the state and dynamics of the economy. Parallel to Taylor (1999a) and Clarida et al. (2000), McCallum (2000) examined the suitability of different policy instruments and targets for the US, the UK and Japan using historical ex-post data for monetary policy analysis. Motivated by McCallum and Nelson (1999b, 1999c) and Taylor (1993), in McCallum (2000), efficiency of different policy rules (interest rate rule, monetary base rule and their variants) and target variables

(inflation target, nominal income growth hybrid target) were evaluated for each country separately using ex-post revised data despite the conceptual problems inherent in definitions and way of gathering these data. The results showed that monetary base rules can give better policy recommendations to some extent compared to interest rate instrument based on ex-post data. Furthermore, efficiency of a rule was found to be more dependent on the right instrument selection rather than target choice, as long as output gap measure is not used.

Analysis and conduct of monetary policy literature constitutes the implications of new policy rules, interaction with the fiscal policy, changes in the monetary transmission mechanism and monetary policy rules, targeting regimes, and injection of exchange rate dynamics in open economies as well as various extensions regarding to euro area. However, it is clear that all these works based on the macroeconomic relationships derived from theoretical models with microfoundations. In this respect, it is important to understand the dynamics of a simple theoretical model and derivation of macroeconomic relationships from these models. The following sections explain the derivation of the Phillip's curve and the IS curve from a theoretical microfounded model and imposing the monetary policy rule.

## ***2.8. Microfoundations of New Keynesian Model***

The works achieved in the field of monetary policy analysis generally start directly with baseline structural equations, which are Phillip's curve and IS curve, with the explanation of monetary policy rule. After modeling economy with these equations, they study on the policy implications of the monetary policy rules under consideration and other aspects of the economy. These works mostly skip the derivation of structural equations from a microfounded model and they employ already-developed Phillip's curve and IS curve. However, as it was mentioned in the previous sections, it is also of importance to know the way of acquiring structural relations that explain the economy. On this ground, this section constitutes the re-derivation of structural equations in much more detail. Such a thorough study at this level of technicality cannot be seen in any article.

The theoretical model and the notation that will be used are mostly adapted from Walsh (2003). As it is inferred from the previous sections, during the 1970s and 1980s, monetary policy analysis were performed mostly using standard IS/LM framework or through a quantity theory of money with random disturbances. Although these models were powerful in explaining relationships among the macroeconomic variables and the dynamics of the economy, the weak side of them was their theoretical foundations. Later, some work was carried out to form a theoretical base for these models linking them to

optimizing agent behavior and so, contemporary dynamic general equilibrium models are shaped.

Our baseline macroeconomic framework is described with a dynamic general equilibrium model containing money. We will simply consider closed economy case. Frictions in the economy are provided by nominal price rigidities, which make the model more realistic (and exclude perfectly flexible price setting behavior). This rigidity is provided using Calvo-type sticky price setting. Firms are monopolistically competitive and they produce differentiated goods, implying that goods markets are also monopolistically competitive. Infinitely lived households are the owner of the firms, that is, we have producer/consumer agents. The central bank uses short-term nominal interest rate as the monetary policy instrument. Therefore, money supply is determined endogenously to achieve determined level of nominal interest rate.

Households buy consumption goods, supply labor and hold bonds (via a financial agent) and money. Firms hire labor, produce differentiated goods and sell them in monopolistically competitive goods markets (Simple monopolistic competition model is given in Dixit and Stiglitz, 1977). Each period some firms can adjust their prices whereas remaining firms cannot. Firms adjusting their prices are selected randomly and fraction of them is  $1-\omega$  so,  $\omega$  fraction of all firms cannot be able to adjust their prices. Then, it can be said that for a firm, the probability of not adjusting the price of a product between two

periods,  $t$  and  $t+s$ , is given by  $\omega^s$ . This price stickiness is introduced by Calvo (1983). Here the parameter  $\omega$  refers to the intensity of price rigidity so that high  $\omega$  means a few of all firms can adjust their prices and degree of price stickiness is high. Households and firms display optimizing behavior meaning that households maximize expected present worth of their utility and firms maximize expected present worth of their profit however, central bank does not behave optimally in controlling nominal interest rates.

### 2.8.1. Households

Using the notation of Walsh (2003), utility function of the household is described as a function of consumption of differentiated goods  $C_t$ , real money balances  $\frac{M_t}{P_t}$ , and time allocated to employment  $N_t$ . Objective function of the household is to maximize present worth of expected future utility, which is given by

$$E \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_t}{P_t} \right)^{1-b} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right] \quad (1)$$

In this formulation  $C_t$  is a composite variable consisting of differentiated goods produced by monopolistically competitive firms. If we locate all firms in

an interval of  $(0, 1)$ , and accept that good  $c_j$  is produced by firm  $j$  then composite consumption  $C_t$  becomes

$$C_t = \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \quad (2)$$

where  $\theta$  is the price elasticity of demand. Households make their decisions in two steps; first they minimize cost of purchasing consumption goods for an implicit level of  $C_t$ . Second, after finding the cost of any level of  $C_t$ , they choose optimally  $C_t$ ,  $M_t$  and  $N_t$ .

The first step includes the following optimization problem:

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj \quad (3)$$

subject to

$$\left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t \quad (4)$$

where  $p_{jt}$  represents price of good  $j$  at time  $t$ . Taking  $\psi_t$  as the Lagrange multiplier of the constraint, the following equations make up the solution of the above optimization problem:

$$L = \int_0^1 p_{jt} c_{jt} dj - \psi_t \left\{ \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - C_t \right\} \quad (5)$$

with the first order condition

$$\frac{\partial L}{\partial c_{jt}} = p_{jt} - \psi_t \frac{\theta}{\theta - 1} \left[ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{1}{\theta-1}} \frac{\theta-1}{\theta} c_{jt}^{\frac{-1}{\theta}} = 0$$

Solving this first order condition for  $c_{jt}$  yields

$$c_{jt} = \left[ \frac{p_{jt}}{\psi_t} \right]^{-\theta} C_t \quad (6)$$

Substituting equation (6) in the definition of  $C_t$ , given by equation (2), gives

the following relationship:

$$C_t = \psi_t \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} C_t \quad (7)$$

Solving equation (7) for the Lagrange multiplier  $\psi_t$  gives

$$\psi_t = \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \equiv P_t \quad (8)$$

meaning that the Lagrange multiplier is the price index for consumption goods.

Then, the relation for consumption good  $c_{jt}$  becomes

$$c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t \quad (9)$$

As  $\theta$  gets larger, consumption goods become closer substitutes and the market approaches perfect competition behavior. This concludes the first step of the decision making process of the households.

Given the definition of aggregated price index  $P_t$ , the second stage develops as follows:

$$\max_{C_t, M_t, N_t, B_t} E \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_t}{P_t} \right)^{1-b} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right] \quad (10)$$

subject to the budget constraint

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1}) \left( \frac{B_{t-1}}{P_t} \right) + \Pi_t \quad (11)$$

where  $B_t$  is the nominal holdings of one-period bonds,  $W_t$  is the nominal wage,  $\Pi_t$  real profit transferred from firms and  $i_t$  is the nominal interest rate faced by households that bonds pay. Letting  $\lambda_t$  be the Lagrange multiplier, solution of this optimization problem can be outlined as follows:

$$\begin{aligned} L = E \sum_{t=0}^{\infty} \beta^t & \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_t}{P_t} \right)^{1-b} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right] \\ & + \lambda_t \left[ \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + (1+i_{t-1}) \left( \frac{B_{t-1}}{P_t} \right) + \Pi_t - C_t - \frac{M_t}{P_t} - \frac{B_t}{P_t} \right] \end{aligned} \quad (12)$$

with the first order conditions

$$\frac{\partial L}{\partial C_t} = \beta^t C_t^{-\sigma} - \lambda_t = 0 \quad (13)$$

$$\frac{\partial L}{\partial M_t} = \beta^t \frac{\gamma}{1-b} (1-b) \left( \frac{M_t}{P_t} \right)^{-b} \frac{1}{P_t} - \lambda_t \frac{1}{P_t} + \lambda_{t+1} \frac{1}{P_{t+1}} = 0 \quad (14)$$

$$\frac{\partial L}{\partial N_t} = -\beta^t \chi N_t^{-\eta} + \lambda_t \frac{W_t}{P_t} = 0 \quad (15)$$

$$\frac{\partial L}{\partial B_t} = -\lambda_t \frac{1}{P_t} + \lambda_{t+1} (1+i_t) \frac{1}{P_{t+1}} = 0 \quad (16)$$

Combination of equations (13) and (16) gives the following consumption-price relationship:

$$C_t^{-\sigma} = \beta(1+i_t)E\left(\frac{P_t}{P_{t+1}}\right)C_{t+1}^{-\sigma} \quad (17)$$

Combination of equations (13) and (15) yields the real wage-consumption equation as follows:

$$\frac{\chi N_t^{-\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \quad (18)$$

Equations (13), (14) and (16) shape the connection between real money balances and consumption, which is given below:

$$\gamma \left( \frac{M_t}{P_t} \right)^{-b} \frac{1}{C_t^{-\sigma}} = \frac{i_t}{1+i_t} \quad (19)$$

Equation (17) states that intertemporal allocation of consumption goods is determined by their prices, that is, expected inflation is an important variable for consumption decision. Equation (18) implies that for any time period  $t$ , trade-off between consumption and employment depends on the real wage. Equation (19) indicates that intratemporal substitution between real money balances and consumption is dependent on the nominal interest rate, opportunity cost of holding money.

### 2.8.2. Firms

Firm's problem is simply to maximize profit, which is the difference between money earned from the sale of the products and money spent for labor input and production cost. The capital component of production is disregarded for simplicity hence; the only input for production is labor. The production function of a firm takes the following form:

$$c_{jt} = Z_t N_{jt}^a \quad (20)$$

where  $c_{jt}$  is the quantity produced from product  $j$  in period  $t$ ,  $Z_t$  is the aggregate productivity parameter in period  $t$ ,  $N_{jt}$  is the labor input used to produce product  $j$  in period  $t$  and  $a$  is the parameter used to determine increasing/constant/decreasing returns to scale property of the production function. If  $a > 1$ , then the production function has increasing returns to scale property; if  $a = 1$ , it has constant returns to scale and if  $a < 1$ , decreasing returns to scale shapes the behavior of production function. It is assumed that the production function has constant returns to scale property, that is  $a = 1$ , and the expected value of the productivity is one, that is,  $E(Z_t) = 1$ .

Firm's profit maximization problem is constrained by three restrictions, which are production function given by equation (20), demand function that the firm faces for its products given by equation (9) and price stickiness mentioned before. Similar to the household's decision making, firm's problem can be

analyzed in two stages, too: The first phase includes cost minimization and is formulated as follows:

$$\min_{N_{jt}} \left( \frac{W_t}{P_t} \right) N_{jt} \quad (21)$$

subject to the production technology introduced by equation (20). Calling  $\varphi_t$  as the Lagrange multiplier, cost minimization problem is transformed to

$$L = \left( \frac{W_t}{P_t} \right) N_{jt} - \varphi_t (c_{jt} - Z_t N_{jt}) \quad (22)$$

The first order condition suggests

$$\frac{\partial L}{\partial N_{jt}} = \frac{W_t}{P_t} - \varphi_t Z_t = 0$$

which, in turn, means that

$$\varphi_t = \frac{W_t/P_t}{Z_t}. \quad (23)$$

Equation (23) indicates that the lagrange multiplier represents the firm's real marginal cost.

The second stage, profit maximization problem, can be called pricing decision since the firm picks the price level of the goods that maximizes profit, which is restricted by demand curves of the products and price stickiness. The formulation of the problem is as follows:

$$\max_{p_{jt}} E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right) c_{j,t+i} - \varphi_{t+i} c_{j,t+i} \right] \quad (24)$$

where  $\beta^i (C_{t+i}/C_t)^{-\sigma}$  constitutes the discount factor of future profit,  $\omega^i$  is the probability of unchanging price of good  $j$  from period  $t$  to period  $t+i$ . From the household's consumption goods purchasing cost minimization problem, we can substitute equation (9) in place of  $c_{j,t+i}$ . Now, firm's pricing decision, equation (24), becomes

$$\max_{p_{jt}} E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i} \quad (25)$$

for which the first order condition is

$$\frac{\partial}{\partial p_{jt}} = E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma} \left[ (1-\theta) \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} P_{t+i}^{-1} + \varphi_{t+i} \theta \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta-1} P_{t+i}^{-1} \right] C_{t+i} = 0$$

Rearrangement of the above first order condition results in

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma} (\theta-1) \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} P_{t+i}^{-1} C_{t+i} =$$

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left( \frac{C_{t+i}}{C_t} \right)^{-\sigma} \varphi_{t+i} \theta \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta-1} P_{t+i}^{-1} C_{t+i}$$

Carrying the terms without subscript  $i$  to the outside of expectation operator produces

$$C_t^{\sigma} (\theta-1) p_{jt}^{-\theta} E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} P_{t+i}^{\theta-1} = C_t^{\sigma} \theta p_{jt}^{-\theta-1} E_t \sum_{i=0}^{\infty} \omega^i \beta^i \varphi_{t+i} C_{t+i}^{1-\sigma} P_{t+i}^{\theta}$$

Canceling  $C_t^{\sigma}$  and  $p_{jt}^{-\theta}$ , and dividing both sides of the equality with  $P_t^{\theta}$  leads to

$$\frac{\theta-1}{P_t} E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} = \theta p_{jt}^{-1} E_t \sum_{i=0}^{\infty} \omega^i \beta^i \phi_{t+i} C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}$$

Rearranging the above condition yields the following optimal pricing decision rule for  $p_{jt}$ :

$$\frac{p_{jt}}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \phi_{t+i} C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}} \quad (26)$$

Since all price adjusting firms face the same pricing problem, as expressed by equation (26), the optimal price set by these firms will be the same. If we convey the optimal price as  $p_t^*$ , then the equation (26) becomes

$$\frac{p_t^*}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \phi_{t+i} C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}} \quad (27)$$

### 2.8.3. Equilibrium

We start with flexible price equilibrium, that is all firms are able adjust the prices of their products every period hence, there is no price rigidity. This equilibrium requires that  $\omega$ , fraction of firms that do not adjust prices in a period, equals to zero,  $\omega = 0$ . In order to find the flexible price equilibrium, we

need to take limit of equation (27) when  $\omega$  goes to zero and use L'hospital rule. The following steps are used to get this equilibrium condition:

$$\begin{aligned}
\frac{p_t^*}{P_t} &= \left( \frac{\theta}{\theta-1} \right) \lim_{\omega \rightarrow 0} \left\{ \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \varphi_{t+i} C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}} \right\} \\
\frac{p_t^*}{P_t} &= \left( \frac{\theta}{\theta-1} \right) \lim_{\omega \rightarrow 0} \left\{ \frac{E_t \sum_{i=1}^{\infty} i \omega^{i-1} \beta^i \varphi_{t+i} C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=1}^{\infty} i \omega^{i-1} \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}} \right\} \\
\frac{p_t^*}{P_t} &= \left( \frac{\theta}{\theta-1} \right) \lim_{\omega \rightarrow 0} \left\{ \frac{\overbrace{\omega^0}^1 \beta E_t \varphi_{t+1} C_{t+1}^{1-\sigma} \left( \frac{P_{t+1}}{P_t} \right)^{\theta} + E_t \sum_{i=2}^{\infty} i \omega^{i-1} \beta^i \varphi_{t+i} C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{\underbrace{\omega^0}_1 \beta E_t C_{t+1}^{1-\sigma} \left( \frac{P_{t+1}}{P_t} \right)^{\theta-1} + E_t \sum_{i=2}^{\infty} i \omega^{i-1} \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}} \right\} \\
\frac{p_t^*}{P_t} &= \left( \frac{\theta}{\theta-1} \right) \lim_{\omega \rightarrow 0} \left\{ \frac{\beta E_t \varphi_{t+1} C_{t+1}^{1-\sigma} \left( \frac{P_{t+1}}{P_t} \right)^{\theta} + \overbrace{E_t \sum_{i=2}^{\infty} i \omega^{i-1} \beta^i \varphi_{t+i} C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}^0}{\beta E_t C_{t+1}^{1-\sigma} \left( \frac{P_{t+1}}{P_t} \right)^{\theta-1} + \underbrace{E_t \sum_{i=2}^{\infty} i \omega^{i-1} \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}_0} \right\} \\
\frac{p_t^*}{P_t} &= \left( \frac{\theta}{\theta-1} \right) \left\{ \frac{\beta E_t \varphi_{t+1} C_{t+1}^{1-\sigma} \left( \frac{P_{t+1}}{P_t} \right)^{\theta}}{\beta E_t C_{t+1}^{1-\sigma} \left( \frac{P_{t+1}}{P_t} \right)^{\theta-1}} \right\}
\end{aligned}$$

$$\frac{p_t^*}{P_t} = \left( \frac{\theta}{\theta - 1} \right) E_t \varphi_{t+1} \left( \frac{P_{t+1}}{P_t} \right)$$

When we impose the equilibrium conditions,  $\varphi_{t+1} = \varphi_t$  and  $P_{t+1} = P_t$ , the above expression becomes

$$\frac{p_t^*}{P_t} = \frac{\theta}{\theta - 1} \varphi_t \quad (28)$$

Equation (28) establishes the pricing rule of all firms in flexible price equilibrium. It is clear that when all firms are able to adjust prices, since they face the same constraints, they will set the same price so that  $p_t^* = P_t$ . Then real marginal cost of firms,  $\varphi_t$ , will be  $(\theta - 1)/\theta$ .

Equation (23) states that firm's real marginal cost also equals to  $(W_t/P_t)/Z_t$ , resulting in that

$$\varphi_t = \frac{\theta - 1}{\theta} = \frac{W_t/P_t}{Z_t} \quad (29)$$

This equality produces real wage as a function of productivity, which can be given by

$$\frac{W_t}{P_t} = \frac{\theta - 1}{\theta} Z_t \quad (30)$$

If we combine equation (30) with equation (18), we have the following flexible price equilibrium condition

$$\frac{W_t}{P_t} = \frac{\theta - 1}{\theta} Z_t = \frac{\chi N_t^\eta}{C_t^{-\sigma}} \quad (31)$$

When we approximate equation (31) around the steady state, it will have the following functional form (see, Appendix A, for the explanations about approximation procedure):

$$Z_t = \frac{\theta}{\theta-1} \chi \frac{N_t^\eta}{C_t^{-\sigma}} \Rightarrow Z^{ss} (1 + \hat{z}_t) = \frac{\theta}{\theta-1} \chi \frac{(N^{ss})^\eta (1 + \hat{n}_t^f)^\eta}{(C^{ss})^{-\sigma} (1 + \hat{c}_t^f)^\sigma}$$

Since at steady state  $Z^{ss} = (\theta/\theta-1) \chi \left( (N^{ss})^\eta / (C^{ss})^{-\sigma} \right)$ , canceling the terms from both sides and using the approximation rules generate

$$\begin{aligned} (1 + \hat{z}_t) &= \frac{(1 + \hat{n}_t^f)^\eta}{(1 + \hat{c}_t^f)^\sigma} = \frac{(1 + \eta \hat{n}_t^f)}{(1 + (-\sigma) \hat{c}_t^f)} = (1 + \eta \hat{n}_t^f) (1 + (-\sigma) \hat{c}_t^f)^{-1} \\ &= (1 + \eta \hat{n}_t^f) (1 + \sigma \hat{c}_t^f) = (1 + \eta \hat{n}_t^f + \sigma \hat{c}_t^f) \end{aligned}$$

$$\hat{z}_t = \eta \hat{n}_t^f + \sigma \hat{c}_t^f \quad (32)$$

The production function given by equation (20) is approximated around steady state as follows:

$$C^{ss} (1 + \hat{c}_t^f) = Z^{ss} (1 + \hat{z}_t) N^{ss} (1 + \hat{n}_t^f) = Z^{ss} N^{ss} (1 + \hat{z}_t + \hat{n}_t^f)$$

Since at steady state  $C^{ss} = Z^{ss} N^{ss}$ , canceling the related terms produces

$$\hat{c}_t^f = \hat{n}_t^f + \hat{z}_t \quad (33)$$

Using the fact that at equilibrium  $\hat{y}_t^f = \hat{c}_t^f$ , equation (33) turns into

$$\hat{y}_t^f = \hat{n}_t^f + \hat{z}_t \quad (34)$$

When we combine equations (32) and (34) with the equilibrium condition,  $\hat{y}_t^f = \hat{c}_t^f$ , we end up with the approximation to output around steady state (in a flexible price framework) as follows:

$$\hat{y}_t^f = \left( \frac{1+\eta}{\sigma+\eta} \right) \hat{z}_t \quad (35)$$

After finding output in a flexible price setting, we continue with the sticky price analysis, that is  $\omega \neq 0$ . As we know, if prices are sticky all firms cannot adjust their prices every period. Some randomly selected firms, whose fraction is  $1-\omega$ , can adjust their prices, remaining firms, with the fraction  $\omega$ , continue to use previous period's prices. Then, the average price in a period will have two components, as expressed below:

$$P_t^{1-\theta} = (1-\omega) \left( p_t^* \right)^{1-\theta} + \omega P_{t-1}^{1-\theta} \quad (36)$$

Now, we can use equations (27) and (36) to derive a representation for the deviations of inflation rate around its steady state. For this derivation, the initial assumption we made is that steady state level of inflation rate is zero. If we let  $Q_t = p_t^* / P_t$ , ratio of newly selected price to the average price in a period, it can be inferred that the steady state value of  $Q_t$  is one,  $Q_t = 1$ . Here, one thing should be mentioned: Although we are working with sticky prices, approximating the expressions around zero inflation steady state requires  $Q_t = 1$  and this equality holds also when all firms can adjust their prices every

period, i.e. flexible price equilibrium. If we divide both sides of equation (36)

by  $P_t^{1-\theta}$ , we will obtain

$$1 = (1 - \omega)Q_t^{1-\theta} + \omega \left( \frac{P_{t-1}}{P_t} \right)^{1-\theta} \quad (37)$$

The following steps show how to approximate equation (37) around a steady state with zero inflation rate:

$$1 = (1 - \omega) \left( \underbrace{Q^{ss}}_1 \right)^{1-\theta} (1 + \hat{q}_t)^{1-\theta} + \omega \left( \frac{P^{ss}(1 + \hat{p}_{t-1})}{P^{ss}(1 + \hat{p}_t)} \right)^{1-\theta}$$

$$1 = (1 - \omega)(1 + \hat{q}_t)^{1-\theta} + \omega \left( (1 + \hat{p}_{t-1})(1 + \hat{p}_t)^{-1} \right)^{1-\theta}$$

$$1 = (1 - \omega)(1 + (1 - \theta)\hat{q}_t) + \omega(1 + \hat{p}_{t-1} - \hat{p}_t)^{1-\theta}$$

At this point, it is clear that we can substitute the inflation rate equation,

$\pi_t = \hat{p}_t - \hat{p}_{t-1}$ . Then, we can continue to approximation as follows:

$$1 = (1 - \omega)(1 + (1 - \theta)\hat{q}_t) + \omega(1 + \hat{p}_{t-1} - \hat{p}_t)^{1-\theta}$$

$$1 = (1 - \omega)(1 + (1 - \theta)\hat{q}_t) + \omega(1 - \pi_t)^{1-\theta}$$

$$1 = (1 - \omega)(1 + (1 - \theta)\hat{q}_t) + \omega(1 - (1 - \theta)\pi_t)$$

$$1 = 1 + (1 - \theta)\hat{q}_t - \omega(1 + (1 - \theta)\hat{q}_t) + \omega(1 - (1 - \theta)\pi_t)$$

$$1 = 1 + (1 - \theta)\hat{q}_t - \omega(1 + (1 - \theta)\hat{q}_t - 1 + (1 - \theta)\pi_t)$$

$$0 = (1 - \theta)\hat{q}_t - \omega(1 - \theta)(\hat{q}_t + \pi_t) = \hat{q}_t - \omega(\hat{q}_t + \pi_t) = (1 - \omega)\hat{q}_t - \omega\pi_t$$

When we arrange the terms in the above equality, we can reach to the following equilibrium condition, which links inflation to relative price of products:

$$\hat{q}_t = \left( \frac{\omega}{1-\omega} \right) \pi_t \quad (38)$$

Using the definition of the term  $Q_t$ , equation (27) becomes

$$\left[ E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} \right] Q_t = \frac{\theta}{\theta-1} \left[ E_t \sum_{i=0}^{\infty} \omega^i \beta^i \varphi_{t+i} C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} \right] \quad (39)$$

Equation (39) can be approximated around zero inflation steady state as follows (for this approximation, it should be remembered that for small numbers  $\hat{a}_t$  and  $\hat{b}_t$ ,  $\hat{a}_t \hat{b}_t \approx 0$ ):

$$\underbrace{Q^{ss}}_1 (1 + \hat{q}_t) \left[ E_t \sum_{i=0}^{\infty} \omega^i \beta^i (C^{ss})^{1-\sigma} (1 + \hat{c}_{t+i})^{1-\sigma} \left( \frac{P^{ss} (1 + \hat{p}_{t+i})}{P^{ss} (1 + \hat{p}_t)} \right)^{\theta-1} \right] =$$

$$\frac{\theta}{\theta-1} \left[ E_t \sum_{i=0}^{\infty} \omega^i \beta^i \varphi^{ss} (1 + \hat{\varphi}_{t+i}) (C^{ss})^{1-\sigma} (1 + \hat{c}_{t+i})^{1-\sigma} \left( \frac{P^{ss} (1 + \hat{p}_{t+i})}{P^{ss} (1 + \hat{p}_t)} \right)^{\theta} \right]$$

$$(C^{ss})^{1-\sigma} (1 + \hat{q}_t) \sum_{i=0}^{\infty} \omega^i \beta^i (1 + (1-\sigma) E_t \hat{c}_{t+i}) (1 + (\theta-1) (E_t \hat{p}_{t+i} - \hat{p}_t)) =$$

$$(C^{ss})^{1-\sigma} \varphi^{ss} \frac{\theta}{\theta-1} \sum_{i=0}^{\infty} \omega^i \beta^i (1 + (1-\sigma) E_t \hat{c}_{t+i}) (1 + \theta (E_t \hat{p}_{t+i} - \hat{p}_t)) (1 + E_t \hat{\varphi}_{t+i})$$

$$\begin{aligned}
& \left(C^{ss}\right)^{1-\sigma} (1+\hat{q}_t) \sum_{i=0}^{\infty} \omega^i \beta^i \left(1 + (1-\sigma)E_t \hat{c}_{t+i} + (\theta-1)(E_t \hat{p}_{t+i} - \hat{p}_t)\right) = \\
& \left(C^{ss}\right)^{1-\sigma} \varphi^{ss} \frac{\theta}{\theta-1} \sum_{i=0}^{\infty} \omega^i \beta^i \left(1 + E_t \hat{\phi}_{t+i} + (1-\sigma)E_t \hat{c}_{t+i} + \theta(E_t \hat{p}_{t+i} - \hat{p}_t)\right) \\
& \left(C^{ss}\right)^{1-\sigma} (1+\hat{q}_t) \left[ \underbrace{\sum_{i=0}^{\infty} \omega^i \beta^i}_{1/(1-\omega\beta)} + \sum_{i=0}^{\infty} \omega^i \beta^i \left((1-\sigma)E_t \hat{c}_{t+i} + (\theta-1)(E_t \hat{p}_{t+i} - \hat{p}_t)\right) \right] = \\
& \left(C^{ss}\right)^{1-\sigma} \varphi^{ss} \frac{\theta}{\theta-1} \left[ \underbrace{\sum_{i=0}^{\infty} \omega^i \beta^i}_{1/(1-\omega\beta)} + \sum_{i=0}^{\infty} \omega^i \beta^i \left(E_t \hat{\phi}_{t+i} + (1-\sigma)E_t \hat{c}_{t+i} + \theta(E_t \hat{p}_{t+i} - \hat{p}_t)\right) \right] \\
& \left(\frac{\left(C^{ss}\right)^{1-\sigma}}{1-\omega\beta}\right) (1+\hat{q}_t) + \left(C^{ss}\right)^{1-\sigma} \sum_{i=0}^{\infty} \omega^i \beta^i \left[(1-\sigma)E_t \hat{c}_{t+i} + (\theta-1)(E_t \hat{p}_{t+i} - \hat{p}_t)\right] = \\
& \left(\frac{\theta}{\theta-1}\right) \varphi^{ss} \left(C^{ss}\right)^{1-\sigma} \left\{ \left(\frac{1}{1-\omega\beta}\right) + \sum_{i=0}^{\infty} \omega^i \beta^i \left[E_t \hat{\phi}_{t+i} + (1-\sigma)E_t \hat{c}_{t+i} + \theta(E_t \hat{p}_{t+i} - \hat{p}_t)\right] \right\} \\
& \tag{40}
\end{aligned}$$

Combining the flexible price equilibrium condition, equation (28), and

definition of  $Q_t$  imposes that  $Q_t = \frac{\theta}{\theta-1} \varphi_t = 1$ , at zero inflation steady state.

Using this equality and canceling the terms that are seen on both sides of equation (40) reduces it to

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t = \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\phi}_{t+i} + E_t \hat{p}_{t+i} - \hat{p}_t) \quad (41)$$

Carrying  $\hat{p}_t$  to the left hand side of the equation (41) results in

$$\hat{q}_t + \hat{p}_t = (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \hat{\phi}_{t+i} + E_t \hat{p}_{t+i}) \quad (42)$$

At this point, it is necessary to remember equilibrium condition

$Q_t = \frac{\theta}{\theta-1} \varphi_t = 1$ , again. Approximating this equality around the steady state

levels of the variables yields  $\hat{q}_t = \hat{\phi}_t$  (This approximation can be done as

follows:  $Q^{ss}(1+\hat{q}_t) = \frac{\theta}{\theta-1} \varphi^{ss}(1+\hat{\phi}_t)$ . At steady state  $Q^{ss} = \frac{\theta}{\theta-1} \varphi^{ss}$  then,

$(1+\hat{q}_t) = (1+\hat{\phi}_t) \Rightarrow \hat{q}_t = \hat{\phi}_t$ ). Using this steady state expression, we are able to

write equation (42) for  $i = 0, 1$  as follows:

$$\begin{aligned} \hat{q}_t + \hat{p}_t &= (1-\omega\beta) \left[ (\hat{\phi}_t + \hat{p}_t) + \omega\beta (E_t \hat{q}_{t+1} + E_t \hat{p}_{t+1}) \right] \\ \hat{q}_t &= (1-\omega\beta) \hat{\phi}_t + \omega\beta (E_t \hat{q}_{t+1} + E_t \hat{p}_{t+1} - \hat{p}_t) \\ \hat{q}_t &= (1-\omega\beta) \hat{\phi}_t + \omega\beta (E_t \hat{q}_{t+1} + E_t \pi_{t+1}) \end{aligned} \quad (43)$$

Equation (38) is used to remove  $\hat{q}_t$  from the above expression. Then, we get

$$\begin{aligned} \left(\frac{\omega}{1-\omega}\right) \pi_t &= (1-\omega\beta) \hat{\phi}_t + \omega\beta \left[ \left(\frac{\omega}{1-\omega}\right) E_t \pi_{t+1} + E_t \pi_{t+1} \right] \\ \left(\frac{\omega}{1-\omega}\right) \pi_t &= (1-\omega\beta) \hat{\phi}_t + \omega\beta \left(\frac{1}{1-\omega}\right) E_t \pi_{t+1} \end{aligned}$$

$$\pi_t = (1 - \omega\beta) \left( \frac{1 - \omega}{\omega} \right) \hat{\phi}_t + \beta E_t \pi_{t+1} \quad (44)$$

If we let  $\tilde{\kappa}$  be  $(1 - \omega\beta)((1 - \omega)/\omega)$ , then we can obtain forward looking New Keynesian Phillips curve as follows:

$$\pi_t = \tilde{\kappa} \hat{\phi}_t + \beta E_t \pi_{t+1} \quad (45)$$

Here it is worth mentioning that this Phillips curve is a function of real marginal cost instead of an output gap (difference between actual output and potential output) measure. However, we can express the Phillips curve as a function of output gap. For this, equation (23) is approximated around zero inflation steady state, initially.

$$\varphi^{ss}(1 + \hat{\phi}_t) = \frac{W^{ss}(1 + \hat{w}_t)/P^{ss}(1 + \hat{p}_t)}{Z^{ss}(1 + \hat{z}_t)}$$

$$\varphi^{ss}(1 + \hat{\phi}_t) = \frac{W^{ss}/P^{ss}}{Z^{ss}} (1 + \hat{w}_t)(1 + \hat{p}_t)^{-1}(1 + \hat{z}_t)^{-1}$$

$$\varphi^{ss}(1 + \hat{\phi}_t) = \frac{W^{ss}/P^{ss}}{Z^{ss}} (1 + \hat{w}_t - \hat{p}_t - \hat{z}_t)$$

Since at steady state  $\varphi^{ss} = \frac{W^{ss}/P^{ss}}{Z^{ss}}$ , we can put

$$\hat{\phi}_t = (\hat{w}_t - \hat{p}_t) - \hat{z}_t \quad (46)$$

Next, equation (31) should be approximated around zero inflation steady state as follows:

$$\frac{W^{ss}(1+\hat{w}_t)}{P^{ss}(1+\hat{p}_t)} = \chi \frac{(N^{ss})^\eta (1+\hat{n}_t)^\eta}{(C^{ss})^{-\sigma} (1+\hat{c}_t)^{-\sigma}}$$

$$\frac{W^{ss}}{P^{ss}}(1+\hat{w}_t)(1+\hat{p}_t)^{-1} = \chi \frac{(N^{ss})^\eta}{(C^{ss})^{-\sigma}}(1+\hat{n}_t)^\eta(1+\hat{c}_t)^\sigma$$

$$\frac{W^{ss}}{P^{ss}}(1+\hat{w}_t-\hat{p}_t) = \chi \frac{(N^{ss})^\eta}{(C^{ss})^{-\sigma}}(1+\eta\hat{n}_t+\sigma\hat{c}_t)$$

The steady state expression  $W^{ss}/P^{ss} = \chi \left( (N^{ss})^\eta / (C^{ss})^{-\sigma} \right)$  reduces the above

equality to the following:

$$\hat{w}_t - \hat{p}_t = \eta\hat{n}_t + \sigma\hat{c}_t \quad (47)$$

In addition to equation (47), if we substitute the other equilibrium conditions that are equation (34) and  $\hat{y}_t = \hat{c}_t$ , into equation (46) we can reach to the following equality:

$$\begin{aligned} \hat{\phi}_t &= (\hat{w}_t - \hat{p}_t) - \hat{z}_t = (\eta\hat{n}_t + \sigma\hat{c}_t) - \hat{z}_t = (\eta(\hat{y}_t - \hat{z}_t) + \sigma\hat{y}_t) - \hat{z}_t \\ \hat{\phi}_t &= (\sigma + \eta) \left[ \hat{y}_t - \left( \frac{1+\eta}{\sigma+\eta} \right) \hat{z}_t \right] \end{aligned} \quad (48)$$

Using equation (35), the real marginal cost of the firm can be written as

$$\hat{\phi}_t = (\sigma + \eta) (\hat{y}_t - \hat{y}_t^f). \quad (49)$$

Accepting the term  $\hat{y}_t^f$  as the potential output (the flexible price equilibrium output), if we let  $x_t$  be the output gap, which can be expressed as the

difference between actual output and potential output, then the Phillips curve, equation (45), turns out to be a function of output gap as follows:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \quad (50)$$

where  $\kappa = (\sigma + \eta)(1 - \omega)((1 - \omega\beta)/\omega)$ .

After deriving Phillips curve that relates current inflation rate to current output gap and expected future inflation rate, we should also set up the dynamics of output determination, that is the IS equation.

The IS curve is derived completely from the consumption-price relationship of the household's problem, which is equation (17). Here it is convenient to state equation (17) again with small modifications, which are the substitution of inflation rate for price terms and introduction of gross real interest rate.

$$C_t^{-\sigma} = \beta(1 + i_t)E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma} = \beta(1 + i_t)E_t \left( \frac{1}{1 + \pi_{t+1}} \right) C_{t+1}^{-\sigma} = \beta E_t \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) C_{t+1}^{-\sigma}$$

$$C_t^{-\sigma} = \beta E_t (R_t) C_{t+1}^{-\sigma} \quad (51)$$

where  $1 + \pi_{t+1} = P_{t+1}/P_t$  and gross real interest rate,  $R_t = E_t((1 + i_t)/(1 + \pi_{t+1}))$ .

Steady state form of equation (51) implies that

$$(C^{ss})^{-\sigma} = \beta R^{ss} (C^{ss})^{-\sigma} \Rightarrow \beta R^{ss} = 1 \quad (52)$$

Using the steady state condition,  $\hat{y}_t = \hat{c}_t$ , and equation (52), we can end up with the IS curve by approximating the equation (51) around zero inflation steady state, as follows:

$$\begin{aligned}
\left(C^{ss}\right)^{-\sigma} (1 + \hat{c}_t)^{-\sigma} &= \underbrace{\beta R^{ss}}_1 E_t (1 + \hat{r}_t) \left(C^{ss}\right)^{-\sigma} (1 + \hat{c}_{t+1})^{-\sigma} \\
1 - \sigma \hat{c}_t &= E_t (1 + \hat{r}_t - \sigma \hat{c}_{t+1}) = 1 + E_t \hat{r}_t - E_t \sigma \hat{c}_{t+1} \\
-\sigma \hat{y}_t &= -\sigma E_t \hat{y}_{t+1} + E_t (\hat{i}_t - \pi_{t+1}) \\
\hat{y}_t &= E_t \hat{y}_{t+1} - \left(\frac{1}{\sigma}\right) (\hat{i}_t - E_t \pi_{t+1}) \tag{53}
\end{aligned}$$

If we substitute the output gap formulation,  $x_t = \hat{y}_t - \hat{y}_t^f$ , to equation (53), the IS curve can be expressed as a function of output gap, too:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right) (\hat{i}_t - E_t \pi_{t+1}) + u_t \tag{54}$$

where  $u_t = E_t \hat{y}_{t+1}^f - \hat{y}_t^f$  is an exogenous productivity shock defined employing equation (35).

It can be concluded that structure of an economy can be represented by two equations, expectations-augmented Phillips curve and forward looking IS curve given by equations (50) and (54), respectively.

#### 2.8.4. Monetary Policy Rule

It is well established that in an economy, monetary policy that relates nominal and real macroeconomic variables, influences real activity in short time periods

(see for example, Bernanke and Blinder, 1992; Gali, 1992; Bernanke and Mihov, 1997; Christiano et al., 1996; Christiano et al., 1998; Bernanke et al., 1997). Thus, monetary policy decisions are used to establish a stable economy in terms of prices and output growth. In this respect, it is necessary to transmit monetary policy actions to macroeconomic variables such as inflation, output, employment, money stock, so that interrelations among these variables can produce the grounds of a steady economy. For this purpose, different transmission mechanisms are used to distribute changes in the policy variables to other real and nominal variables. Some of these transmission channels are interest rate, exchange rate, asset prices. In our model, we used interest rate as the monetary policy tool that disseminates actions of monetary authority to the financial system and real activity.

In order to specify monetary transmission mechanism, it is necessary to determine the behavior of monetary policy implemented by central bank. The general tendency in the literature is to use Taylor (1993) type monetary policy rule, in which short-term interest rate responds to both inflation and output gap. Then the Taylor-type monetary policy rule is expressed below:

$$\hat{i}_t = \delta_\pi \pi_t + \delta_x x_t + v_t \quad (55)$$

where  $v_t$  stands for the shocks to short-term interest rate. To guarantee the existence of a unique equilibrium, it is required that the nominal interest rate should respond greater than one to changes in inflation rate, that is  $\delta_\pi > 1$ .

Otherwise, if policy rate, nominal short-term interest rate, reacts to changes in inflation less than one-for-one, then an increase in the inflation results in a decrease in the real interest rate. A fall in the real interest rate leads to an increase in aggregate demand, which raises the inflation and so forth.

As a result, we can represent a closed economy via three equations derived from a theoretical model. These are Phillip's curve, equation (50), IS curve, equation (54) and monetary policy rule, equation (55). However, although there are expectation-based variables in these equations, various versions of them were/are adopted in different studies including lagged variables and several explanatory variables.

## ***2.9. Bridging Gap for Monetary Policy Analysis***

Previous sections of this essay outlined the conventional method of obtaining structural linear relationships that explain the dynamics of a New Keynesian economy. After having these equations, the general tendency in the literature is to perform a simulation and/or empirical study for conducting monetary policy analysis and extracting policy suggestions using aforementioned structural equations and monetary policy rule (see for instance, Ball, 1997; Rudebusch and Svensson, 1998; Clarida et al., 1999; Schmitt-Grohe and Uribe, 2002; Leitemo and Lønning, 2006). Therefore, the same methodology will be our

road map for the next chapter, so that, we will describe a New Keynesian world with structural relationships and monetary policy rule obtained in this chapter and then, conduct a research on monetary policy analysis using that framework.

## **CHAPTER 3**

### **MONETARY POLICY ANALYSIS WITH TVP INTEREST RATE PASS-THROUGH**

#### ***3.1. Introduction***

Recently, New Keynesian models with microfoundations have gained a lot of attention. These models raise interest in reducing inflation volatility and accordingly inflation targeting in order to establish macroeconomic stability. Today, aims of central banking are to achieve price stability primarily and, to a lesser extent, output stability. In establishing price stability, design of optimal monetary policies and theoretically supported monetary policy rules are essential according to New Keynesian models. In this context, short-term interest rates, used as policy instruments, are the key variables as suggested by Taylor (1993) and Woodford (2003). Importance of interest rate pattern for building efficient monetary policy rules, especially for inflation targeting

countries, was further stressed by Taylor (1999a). Hence, it becomes essential for central banks to determine accurate interest rate rules to reach price stability in an economy. For this purpose, a lot of effort has been spent to establish precise policy rules (see, for example, Taylor, 1993; Clarida et al., 1999 and papers in Taylor, 1999b).

Practically, monetary policy rules are based on the attitude of policymakers towards the structure of the economy and importance attached by policymakers to the opposing objectives of the policy. From this perspective, parameters of the interest rate rules implemented by central banks are subject to change because of changing nature of the policy objectives and behavior of policymakers. For instance, central banks do not give the identical responses to same economic circumstances due to variations in the weights assigned to policy objectives. These variations result from changes in the preferences/priorities of policymakers about the implementation of policy rules.

Besides, central banks utilize broader range of information set while building policy decisions instead of relying only on a policy rule equation. Thus, for example, if the policy rule is a Taylor-type interest rate rule, then the same levels of output gap and inflation may not produce the same level of interest rate in different periods since the information set used by central banks will be

different in these periods. This implies changing behavior of the parameters employed in the interest rate rule.

Variations in the monetary policy transmission mechanism, that is the structure of the economy, can be another reason for unstable nature of the coefficients of a policy rule. In this point of view, interest rate rules should be treated as they are dynamic instead of being static. Intuitively, it should be noted that, due to shifts in the coefficients of policy rules, studies that use interest rate rules with stable parameters may be misleading or inefficient in formulating policy advices. This statement is also in line with the Lucas (1976) critique (Lucas' critique maintains that the relationships among macroeconomic variables alter when the macroeconomic policy changes), so that in order to conduct empirical policy analysis, changing parameter models, contrary to fixed parameter models, are appropriate for accounting policy shifts.

A lot of research is devoted to monetary policy changes, their importance and time-varying parameter (TVP) specification of monetary policy rules. Since numerous works studied implications of various versions of Taylor rule for different countries following Taylor (1993), this line of research raised the utilization of Taylor-type interest rate rules in analyzing policy shifts. Cogley and Sargent (2001) showed empirically that after World War II, inflation-unemployment dynamics of the US changed significantly and accordingly they pointed out the change in conductance of monetary policy using Bayesian

vector autoregression (VAR) with time-varying parameters in a nonlinear stochastic model framework. They demonstrated that policy actions alter significantly with respect to the status of the economy using time-dependent parameter models, implying that coefficients in the policy rule are changing with time.

Judd and Rudebusch (1998) illustrated how the Fed's reaction function has changed over time using subsample analysis. They divided monetary policy history of the US into three periods, which are identified with corresponding Fed chairman, and econometrically estimated Taylor-type policy rule for each. The study resulted in that Taylor-type reaction function estimated for each subperiod significantly differed, implying that monetary policy regime displayed considerable variations in time. This study was also an interesting example in revealing the dependency of monetary policy on the attitude of policymakers towards the structure of the economy.

Similarly, Clarida et al. (2000) presented that the US monetary policy has changed significantly using a subsample analysis. They considered two periods for analysis and estimated forward-looking Taylor rule for each of them using generalized method of moments (GMM). Estimation results were in line with the work of Judd and Rudebusch (1998) in that monetary policy regime displayed important differences across the periods considered. Parallel to these studies, Orphanides (2004) provided evidence that there have been changes in

the interest rate rule of the US using the estimations of a forward-looking Taylor rule for two subperiods.

Apart from the above mentioned works, which are empirical studies about the monetary policy shifts in the US, recently extensive amount of effort has been spent on characterization of Taylor-type rules with TVPs. In this line of research, Boivin (2006) estimated forward-looking Taylor rule with time-varying coefficients following driftless random walks for the US. TVPs of the model were estimated through maximum likelihood estimation (MLE) technique and likelihood function was constructed using Kalman filter. It was revealed that there are gradual adjustments in the coefficients of the policy rule in the US, which cannot be captured adequately by sub-sample analysis (A similar conclusion was stated by Jalil, 2004 using a TVP backward-looking Taylor rule with ex-post and real-time data for the US.). McCulloch (2007) also estimated TVP forward-looking Taylor rule for the US. However, this study differs from the previous one on methodological grounds so that it used adaptive least squares (Adaptive LS) algorithm to model time-varying structural VAR framework employed for structural relations and policy rule (see also McCulloch, 2005 for Adaptive LS algorithm).

Following Boivin (2006), Kim and Nelson (2006) estimated a forward-looking monetary policy rule with TVP using ex post data. However, besides TVP property, they also considered the uncertainty included in the forecasts of

future inflation and output gap in their estimations. Two-step MLE procedure was applied using Kalman filter for estimating the model. Their empirical result was the division of monetary policy history of the US into three periods instead of two. That is, there are three periods that can be captured in which monetary policy of the Fed has changed. Partouche (2007) estimated a forward-looking TVP monetary policy rule for the US, as well. However, this paper adopted a new technique for estimation procedure, which combined GMM framework with smoothing splines. Such a technique is not restrictive on econometric terms in the sense that it imposes no constraints on the form of heteroscedasticity of the shock terms and the correlations between the regressors and disturbances.

Orphanides and Williams (2005) adopted time variation in parameters of the model in a different way, which allows agents to update their expectations about the structure of the economy and monetary policy. Changes in the view of policymakers and resultant changes in the monetary policy conducted in the US were analyzed empirically.

Canova and Gambetti (2004) examined changes in the structure of the US economy in terms of inflation and output processes and the changes in the monetary policy using time-varying coefficients structural VAR framework. It was found that changes in the coefficients of structural equations and changes in the variance of structural shocks have explanatory power in the analysis of

monetary policy of the US. Parallel to this study, Sims and Zha (2006) documented inferences about monetary policy changes in the US by allowing time variation both in the coefficients of the Taylor rule and variances of disturbances within a structural VAR framework. Among various regime specifications, the model allowing time-dependence only in the variances of disturbances and the model allowing time variation only in the parameters of the monetary policy rule were found to be the best-fit models for the US data.

Time-varying coefficient specification of the Taylor rule was used by Plantier and Scrimgeour (2002) to reveal that neutral real interest rate of New Zealand follows a downward trend in recent years. Ordinary least squares (LS) method and Kalman filter were used to estimate different specifications of TVP Taylor rule. Similarly, Trehan and Wu (2007) predicted a Taylor rule for the backward-looking US economy focusing on time-varying equilibrium real interest rate. In order to estimate the state of the economy in real time, Kalman filter was used. It was interpreted that taking into account the time-variation in the equilibrium real interest rate makes substantial difference in the assessment of monetary policy.

Another study analyzing the time-varying property of equilibrium interest rates belongs to Horvath (2006). This paper estimated various forms of fixed-parameter and TVP Taylor-type policy rules for the Czech Republic, using GMM and Kalman filter in order to evaluate changes in equilibrium interest

rate. It was found that equilibrium interest rate has decreased steadily over time. Time-varying property of natural rate of interest in the euro area was examined by Mesonnier and Renne (2007). A Taylor-type monetary policy rule with time-varying natural rate of interest was estimated for the euro area using Kalman filter.

Mandler (2007) employed Taylor rule with TVPs and unobserved components model for the output gap together to predict uncertainty in the future values of the Federal Funds Rate. For the estimation procedure, MLE technique via Kalman filter was used. It was found that this uncertainty can be divided into three components, which are uncertainty due to time-dependent coefficients of the Taylor rule, uncertainty about the future economic events and residual uncertainty.

Trecroci and Vassalli (2006) estimated forward-looking TVP Taylor rule for the UK, Germany, France and Italy, in addition to the US, using Kalman filter (The same analysis was performed for the same countries previously by Wesche, 2003 using a different estimation procedure, which is Markov-switching model with independent switching processes for the TVPs of the Taylor rule and variances of disturbances.). It was demonstrated that the countries analyzed have different interest rate rules and TVP Taylor rules are preferred, compared to fixed parameter rules, in capturing the variations in the policy rates for the US, the UK, Germany, France and Italy. Furthermore, the

coefficients of the policy rules are changing over time in a gradual fashion as it was found by Boivin (2006). For Germany, Kuzin (2006) estimated a backward-looking Taylor rule with time-dependent coefficients using Markov switching models and Kalman filter. The conclusion was similar to the work of Trecroci and Vassalli (2006) in the sense that TVP Taylor rule performed well in capturing the policy shifts and time-varying reactions to inflationary developments in Germany. A similar research was achieved by Elkhoury (2006), which examined the TVP monetary policy rule for an open economy, Switzerland. The study used Kalman filter to embed policy shifts and structural changes into the model and found that uncertainty associated with the policy rule was mostly due to time-varying characterization of the parameters and, to a lesser extent, monetary shocks. The summary of the literature on TVP policy rules is presented in Table 1.

In the literature, the common practice is to specify TVP characteristic only for monetary policy rule while parameters of the structural relationships are taken fixed. However, this paper takes the parameters of both monetary policy rule and structural equations as variable. Since such an exercise is not common in the literature, the current study can be regarded as a leading attempt in this field. That is, being different from the TVP Taylor rule papers, not only coefficients of the Taylor rule but also coefficients of structural equations are time dependent.

**Table 1:** Summary of Literature on TVP Policy Rules

|                  |                                |                                |
|------------------|--------------------------------|--------------------------------|
| Kalman Filter    | Trehan and Wu (2007)           |                                |
|                  | Trecroci and Vassalli (2006)   |                                |
|                  | Elkhoury (2006)                |                                |
|                  | MLE                            | Boivin (2006)                  |
|                  |                                | Jalil (2004)                   |
|                  |                                | Kim and Nelson (2006)          |
|                  |                                | Mandler (2007)                 |
| VAR              | Cogley and Sargent (2001)      |                                |
|                  | Canova and Gambetti (2004)     |                                |
|                  | Mesonnier and Renne (2007)     |                                |
| LS               | Plantier and Scrimgeour (2002) |                                |
| GMM              | Horvath (2006)                 |                                |
| Markov Switching | Kuzin (2006)                   |                                |
| Subsample        | LS                             | Judd and Rudebusch (1998)      |
|                  |                                | Orphanides (2004)              |
|                  | GMM                            | Clarida et.al (2000)           |
| VAR              | Sims and Zha (2006)            |                                |
|                  | LS                             | Orphanides and Williams (2005) |
| MLE              | Markov Switching               | Wesche (2003)                  |
| GMM              | Partouche (2007)               |                                |
| Adaptive LS      | Mcculloch (2007)               |                                |

Accordingly, the aim of this paper is to examine changes in inflation targeting monetary policy in a New Keynesian framework composed of a Taylor rule with time-varying coefficients, IS curve with time-varying coefficients and Phillip's curve with time-varying coefficients. Having TVP specification in structural relationships allows us to capture changes in the structure of the economy, which carries crucial information for policymakers in monetary policymaking.

In the structural models of monetary policy analysis, monetary transmission mechanism is of importance, too. Since policy decisions impact/manipulate the economy through these transmission channels, dynamics of propagation mechanisms should be included in policy analysis. In the literature, the structural models assume that central bank employs short-term interest rates as the policy instrument, that is; policy decisions are transmitted to the economy via short-term policy interest rate. Such a construction implicitly assumes that households/firms realize policy rate without any media and, macroeconomic variables such as inflation, aggregate output are affected by policy rate, directly. Nevertheless, monetary transmission mechanism does not work in this way because households/investors in the economy do not realize policy rates directly. On the contrary, short-term policy interest rates are indirectly recognized by agents. There are financial institutions that buy bonds from the central bank using deposits collected from the households and, issue credits to households. Since these deposits and credits, in other words investments,

spread on long horizons, investors require high interest rates due to less liquidity. It is obvious that financial agents (e.g. banks) collect deposits from and provide loans to households with different maturities according to risk and time horizon preference of the households. However, we can say that generally maturity of these deposits and loans are long-term thus, simply it can be assumed that the market interest rate is a longer-term interest rate compared to short-term policy rate. Hence, the interest rate faced by the households is not the short-term policy interest rate but it is the long-term market interest rate. Changes in the policy rate lead to changes in long-term rate, which results in changes in investment, consumption and finally aggregate output. Thus, the monetary transmission mechanism for policy decisions is from policy rate to long-term rate and then output and inflation.

Up to now, studies related to monetary policy analysis impose that policy decisions are disseminated through the economy via short-term policy interest rate. Therefore, it is presumed that aggregate demand is influenced by short-term policy interest rate. However, it is widely accepted that aggregate demand is dependent on long-term interest rates rather than short-term interest rates (see, for instance, Taylor, 1995; Rotemberg and Woodford, 1999; Woodford, 1999). Transmission mechanism uses long-term interest rate, as explained above. Therefore, since central bank uses only short-term interest rate as the monetary policy tool, there should be a transition equation between market interest rate faced by the households and policy rate, with the intention that by

changing the latter, central bank can impact the economy according to this connection. Although interest rate pass-through and term structure of interest rate literature has elaborated on this issue, no attempt has been made to consider interest rate pass-through specification in a structural model of monetary policy analysis, as to our knowledge. Therefore, the main contribution of the present paper is the inclusion of interest rate pass-through specification with Taylor-type monetary policy rule in a New Keynesian model. Particularly, a TVP interest rate pass-through specification is added to structural model in a simple way to account for monetary transmission mechanism in the economy. It will not be wrong to claim that such a composition can provide the chance of examining transition from short-term to long-term interest rates under different monetary policies.

In order to estimate such a model, extended Kalman filter (EKF) is used, which is a new application in the field of TVP Taylor rule analysis. The studies listed above revealed that monetary policy changes are sensitive to the way of how time variation in the coefficients of the interest rate rule is modeled and the estimation techniques used. While initial studies were using subsample analysis to indicate changes in monetary policies, later different techniques were started to be used like Markov-switching models, GMM, Kalman filter. Especially Kalman filter is widely utilized as a recursive estimation technique due to its appropriateness in estimating past, present and future states of a model even exact form of the model is not known. Although it is an influential technique in

addressing the estimation of linear transformations, as stated by Harvey (1990), it fails to be reliable for non-linear state space forms. While modeling simultaneously monetary policy rule and structural equations with TVPs, the system takes a non-linear form and usage of EKF becomes compulsory as the appropriate estimation tool. As to the knowledge of us, despite being powerful, use of EKF is not so common in the monetary policy analysis with TVPs. Hence, our study can be a good preliminary example demonstrating the strength of EKF in estimating monetary policy models.

The organization of the paper is as follows: In Section 3.2, we briefly explain linear and non-linear state space models with applications of the Kalman filter and the EKF, respectively. In Section 3.3, we will describe our baseline model with Phillip's curve, IS curve, Taylor-type monetary policy rule and interest rate pass-through with other structural relationships of the economy. Then an alternative model without interest rate pass-through specification will be introduced. Moreover, the state space representations of these two models (baseline model and alternative model) are given in this section. We simulate our baseline model (the economy with interest rate pass-through) and alternative model (the economy without interest rate pass-through) in Section 3.4 by generating artificial data. This section explains the data generation process initially, and then application of EKF to the simulated models is given. Section 3.5 discusses the results and findings of estimations about the time-

dependent parameters of the monetary policy rule and interest rate pass-through. Lastly, Section 3.6 concludes the study.

### ***3.2. State Space Models***

State space models, in other words dynamic models, are used to associate observations of a system to unobserved states or parameters of the system. State space representation is powerful especially for multi-input, multi-output and/or time-dependent systems. In general, a state space model is composed of two sets of equations, the system equations and the observation equations. The system (dynamic) equations represent the evolution of state variables and the observation (output) equations model the observed state variables. Given the observations of a system, estimation and prediction of state variables are of primary concern for state space models. State space models can be both linear and non-linear. The next two sections will explain the specifications and estimation procedures of linear and non-linear state space models, sequentially (Explanations about the Kalman filter and EKF are standard and adopted from Welch and Bishop, 2006; Pasricha, 2006 and Ribeiro, 2004).

### 3.2.1. Linear State Space Models and Kalman Filter

For the estimation and prediction of linear state space models, Kalman filter, which was introduced by Kalman (1960) as a recursive linear filter, is widely used. Kalman filter supplies predictions of state variables for a linear state space model using system and observation equations. Recent economics literature includes various applications of Kalman filter, such as Bahmani and Brown (2004), Ljungqvist and Sargent (2000), Cheung (1993).

Consider a system with linear time-varying dynamics that has the following state space representation:

$$x_{k+1} = A_k x_k + B_k u_k + G w_k \quad k \geq 0 \quad (56)$$

$$y_k = C_k x_k + v_k \quad (57)$$

where  $x_k \in R^n$  is the state vector with n elements,  $y_k \in R^r$  is the observation vector with r elements,  $u_k \in R^m$  is the exogenous vector with m elements,  $w_k \in R^n$  and  $v_k \in R^r$  are Gaussian noises with n and r elements, respectively.

$A_k$  is the  $nxn$  transition matrix,  $C_k$  is the  $rxn$  observation matrix. The sequence  $\{u_k\}$  is deterministic while the sequences  $\{w_k\}$  and  $\{v_k\}$  are white, zero mean disturbances with

$$E(w_k) = E(v_k) = 0 \quad (58)$$

and covariance matrix

$$E \begin{bmatrix} \begin{pmatrix} w_k \\ v_k \end{pmatrix} \begin{pmatrix} w_k^T & v_k^T \end{pmatrix} \end{bmatrix} = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix}. \quad (59)$$

The initial state of the system,  $x_0$ , is defined as a Gaussian random vector with mean

$$E(x_0) = \bar{x}_0 \quad (60)$$

and covariance matrix

$$E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T] = P_0. \quad (61)$$

It is assumed that initial state of the system and shocks hitting the system are independent so that

$$E[x_0 w_k^T] = E[x_0 v_k^T] = 0. \quad (62)$$

Kalman filter simply predicts the unobserved components of the system, which is the estimated state vector,  $\hat{x}_k$ , given the observation vector  $Y_k = \{y_0, y_1, \dots, y_k\}$ . Given the following initial conditions

$$\hat{x}_{0|-1} = \bar{x}_0 \text{ and } P_{0|-1} = P_0 \quad (63)$$

estimation of the state vector for the next period is completed by

$$\hat{x}_{k+1|k} = A_k \hat{x}_{k|k} + B_k u_k \quad (64)$$

$$P_{k+1|k} = A_k P_{k|k} A_k^T + G_k Q_k G_k^T. \quad (65)$$

Finally, the filtering procedure is characterized by the following conditions:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} [y_{k+1} - C_{k+1} \hat{x}_{k+1|k}] \quad (66)$$

$$K_{k+1} = P_{k+1|k} C_{k+1}^T [C_{k+1} P_{k+1|k} C_{k+1}^T + R_{k+1}]^{-1} \quad (67)$$

$$P_{k+1|k+1} = [I - K_{k+1}C_{k+1}]P_{k+1|k} . \quad (68)$$

Here, equation (66) is the updated estimate of state vector, equation (67) is the Kalman gain and equation (68) is the updated estimate of error covariance matrix for the state vector.

### 3.2.2. Non-Linear State Space Models and EKF

Non-linear state space models are obtained by dropping linearity assumptions in equations (56) or (57). Since Kalman filter is restricted with linear models, such a non-linear system requires a different filtering procedure called EKF. EKF linearizes the current mean and covariance using the partial derivatives of the process and measurement functions to compute estimates even within the non-linear relationships.

Consider the following non-linear state space model without exogenous variables:

$$x_{k+1} = f_k(x_k) + w_k \quad (69)$$

$$y_k = h_k(x_k) + v_k \quad (70)$$

where  $x_k \in R^n$  is the state vector,  $y_k \in R^r$  is the observation vector,  $w_k \in R^n$  and  $v_k \in R^r$  are disturbances to the system.  $f_k(x_k): R^n \rightarrow R^n$  and

$h_k(x_k) : R^n \rightarrow R^r$  are the vector-valued functions representing transition and observation processes, respectively. The sequences  $\{w_k\}$  and  $\{v_k\}$  are independent, white Gaussian noises with mean zero

$$E(w_k) = E(v_k) = 0 \quad (71)$$

and covariance matrix

$$E \begin{bmatrix} w_k \\ v_k \end{bmatrix} \begin{bmatrix} w_k^T & v_k^T \end{bmatrix} = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix}. \quad (72)$$

The initial state of the system,  $x_0$ , is considered as a Gaussian random vector with the following mean and covariance matrix

$$x_0 \sim N(\bar{x}_0, P_0). \quad (73)$$

Given the observation matrix of the system,  $Y_1^k = \{y_1, y_2, \dots, y_k\}$ , EKF predicts state of the system. Iterations of the EKF can be explained in five steps:

1. Take the last filtered state estimate  $\hat{x}_{k|k}$
2. Linearize the system equation,  $x_{k+1} = f(x_k) + w_k$ , around  $\hat{x}_{k|k}$
3. Employ the prediction algorithm of the Kalman filter to get  $\hat{x}_{k+1|k}$  and  $P_{k+1|k}$
4. Linearize the observation equation,  $y_k = h(x_k) + v_k$ , around  $\hat{x}_{k+1|k}$
5. Apply the filtering algorithm of the Kalman filter to get  $\hat{x}_{k+1|k+1}$  and  $P_{k+1|k+1}$ .

For linearization procedure, EKF uses Jacobian matrices of  $f(\cdot)$  and  $h(\cdot)$  specified by

$$F_k = \frac{\partial f_k}{\partial x_k}(\hat{x}_{k|k}) \quad (74)$$

$$H_{k+1} = \frac{\partial h_{k+1}}{\partial x_{k+1}}(\hat{x}_{k+1|k}). \quad (75)$$

Then the prediction equations for the EKF can be summarized as

$$\hat{x}_{k+1|k} = f_k(\hat{x}_{k|k}) \quad (76)$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k. \quad (77)$$

The filtering cycle, mentioned in Chui and Chen (1991), is outlined as

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} [y_{k+1} - h_{k+1}(\hat{x}_{k+1|k})] \quad (78)$$

$$K_{k+1} = P_{k+1|k} H_{k+1}^T [H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1}]^{-1} \quad (79)$$

$$P_{k+1|k+1} = [I - K_{k+1} H_{k+1}] P_{k+1|k}. \quad (80)$$

It should be noted that EKF is not an optimal filter; instead it relies on a set of approximations. If the linearizations are not good approximations of the linear model, then EKF may diverge.

In the above dynamic system, vector-valued functions are assumed to depend on some unknown parameter vector  $\theta$  and all matrices can be interpreted in terms of  $\theta$ . In a TVP approach, the aim is to identify time-dependent pattern of  $\theta$  hence in such a case, it is treated as another state variable that should be

estimated via EKF. This treatment was used in studies like Ozbek and Efe (2001, 2004), Ozbek et al. (2003), Bacchetta and Gerlach (1997) as a standard application. For this algorithm, the following dynamic (transition), observation and parameter equations constitute the basics of the system:

$$x_{k+1} = A_k(\theta_k)x_k + G_k(\theta_k)w_k \quad (81)$$

$$y_k = C_k(\theta_k)x_k + v_k \quad (82)$$

$$\theta_{k+1} = \theta_k + \zeta_k \quad (83)$$

where it is allowed that  $\theta$  follows a random walk process;  $\zeta_k$  is a white noise disturbance with zero mean and  $Var(\zeta_k) = S_k = S > 0$  for all  $k$ . Here, it is obvious that if  $S = 0$ , then  $\theta$  will not be time-dependent any longer and EKF cannot be implemented. Exogenous input  $u_k$  can also be considered but here it is ignored for simplicity. Based on the above equations, the non-linear state space model can be written as

$$\begin{bmatrix} x_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} A_k(\theta_k)x_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} G_k(\theta_k)w_k \\ \zeta_k \end{bmatrix} \quad (84)$$

$$y_k = \begin{bmatrix} C_k(\theta_k) & 0 \end{bmatrix} \begin{bmatrix} x_k \\ \theta_k \end{bmatrix} + v_k \quad (85)$$

Now, EKF algorithm can be applied to the above model in order to estimate unobserved state variables, which also include parameter vector  $\theta$  as one of its components. The initial state of the system is defined as

$$\begin{bmatrix} E(x_0) \\ E(\theta_0) \end{bmatrix} = \begin{bmatrix} \bar{x}_0 \\ \bar{\theta}_0 \end{bmatrix} \quad (86)$$

with the covariance matrix

$$P_0 = \begin{bmatrix} \text{cov}(x_0) & 0 \\ 0 & S_0 \end{bmatrix}. \quad (87)$$

The prediction cycle yields the following equations for  $k=1,2,\dots$

$$\begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{\theta}_{k+1|k} \end{bmatrix} = \begin{bmatrix} A_k(\hat{\theta}_{k|k})\hat{x}_{k|k} \\ \hat{\theta}_{k|k} \end{bmatrix} \quad (88)$$

$$\begin{aligned} P_{k+1|k} = & \begin{bmatrix} A_k(\hat{\theta}_{k|k}) & \frac{\partial}{\partial \theta}(A_k(\hat{\theta}_{k|k}))\hat{x}_{k|k} \\ 0 & I \end{bmatrix} P_{k|k} \begin{bmatrix} A_k(\hat{\theta}_{k|k}) & \frac{\partial}{\partial \theta}(A_k(\hat{\theta}_{k|k}))\hat{x}_{k|k} \\ 0 & I \end{bmatrix}^T \\ & + \begin{bmatrix} G_k(\hat{\theta}_{k|k})Q_kG_k^T(\hat{\theta}_{k|k}) & 0 \\ 0 & S_k \end{bmatrix} \end{aligned} \quad (89)$$

The update or filtering algorithm is characterized by

$$\begin{bmatrix} \hat{x}_{k+1|k+1} \\ \hat{\theta}_{k+1|k+1} \end{bmatrix} = \begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{\theta}_{k+1|k} \end{bmatrix} + K_{k+1} \left[ y_{k+1} - \begin{bmatrix} C_{k+1}(\hat{\theta}_{k|k})\hat{x}_{k+1|k} \end{bmatrix} \right] \quad (90)$$

$$K_{k+1} = P_{k+1|k} \begin{bmatrix} C_{k+1}(\hat{\theta}_k) & 0 \end{bmatrix}^T \left[ \begin{bmatrix} C_{k+1}(\hat{\theta}_k) & 0 \end{bmatrix} P_{k+1|k} \begin{bmatrix} C_{k+1}(\hat{\theta}_k) & 0 \end{bmatrix}^T + R_{k+1} \right]^{-1} \quad (91)$$

$$P_{k+1} = [I - K_{k+1} \begin{bmatrix} C_{k+1}(\hat{\theta}_k) & 0 \end{bmatrix}] P_{k+1|k} \quad (92)$$

### 3.3. The Model and State Space Representation

In this part of the study, we will explain the building blocks of our baseline model and the alternative model with their state space formulations.

#### 3.3.1. The Baseline Model

The baseline model is based on New Keynesian framework so that it consists of basically three equations, which are Phillip's curve, IS curve and monetary policy rule. In order to model inflation process, backward-looking Phillip's curve is used in the following form (Backward-looking Phillips curve and IS curve were adopted in various studies such as Rudebusch and Svensson, 1999; Rudebusch and Svensson, 2002; Ball, 1997; Ball, 1999; Ball, 2000; Peersman and Smets, 1999; Goodhart and Hofmann, 2000):

$$\pi_{k+1} = \alpha_{1,k}\pi_k + \alpha_{2,k}\pi_{k-1} + \alpha_{3,k}ygap_k + \alpha_{4,k}z_k + \varepsilon_k^\pi \quad (93)$$

where  $\pi_k$  stands for inflation rate for period  $k$ ,  $ygap_k$  is the output gap for period  $k$ ,  $z_k$  and  $\varepsilon_k^\pi$  are the exogenous input and inflation disturbance for period  $k$ , respectively. In this relationship obviously,  $\alpha_{1,k}$  and  $\alpha_{2,k}$  are expected to be positive.  $\alpha_{3,k}$  is a coefficient that represents the impact of output gap on inflation hence, it is also supposed to be positive.  $z_k$  can be

considered as the change in nominal exchange rate to take into account cost-push inflation. In such a case  $\alpha_{4,k}$  is thought to be greater than zero showing the degree of pass through of exchange rate to inflation.

As it is mentioned previously, a number of studies about monetary transmission mechanism accept that aggregate demand is affected by long-term interest rate rather than policy rate determined by central bank (see, for example, Taylor, 1995; Rotemberg and Woodford, 1999; Woodford, 1999; Basci et al., 2007; Coricelli et al., 2006). Since consumption and investment decisions of the agents are affected primarily by long-term interest rate, it can be said that aggregate output is largely dependent on the long-term interest rate. However, the common practice in the literature asserts that aggregate demand is directly affected by short-term policy interest rate so, the IS curve consists of short-term policy rate instead of long-term interest rate. Nonetheless, contrary to literature, we follow the claim that aggregate output depends on mostly long-term interest rate and accordingly, we replace short-term policy rate with long-term interest rate, in the IS curve. As far as we know, inclusion of long-term interest rate in the IS curve was not made before in a New Keynesian framework hence, our study can be considered as a leading attempt to consider a monetary transmission mechanism via long-term interest rate in a structural model.

The IS curve that characterizes the aggregate demand has the following components:

$$ygap_{k+1} = \beta_{1,k} ygap_k + \beta_{2,k} (i_k^l - \pi_k) + \beta_{3,k} z_k + \varepsilon_k^{ygap} \quad (94)$$

where  $\varepsilon_k^{ygap}$  represents the shock hitting the output gap and  $i_k^l$  denotes the long-term market interest rate.  $\beta_{1,k}$  represents the persistence of output gap and is considered to be greater than zero while  $\beta_{2,k}$  stands for the influence of real interest rate on output gap, which is negative.  $\beta_{3,k}$  indicating the effect of nominal exchange rate on output gap is considered to be positive.

After the construction of structural relationships, Taylor-type monetary policy rule is added, which implies that short-term policy interest rate is a function of output gap and inflation. Similar to previously cited studies, interest rate smoothing is maintained and a Taylor-type interest rate rule, which is modeled below, is reached:

$$i_{k+1}^p = r_k^* + \pi_k + \gamma_{1,k} (\pi_k - \pi_k^*) + \gamma_{2,k} ygap_k + \gamma_{3,k} i_k^p + \varepsilon_k^{i^p} \quad (95)$$

where  $i_k^p$  is the short-term policy rate set by central bank for period  $k$ ,  $r_k^*$  and  $\pi_k^*$  are the equilibrium short-term real interest rate and inflation target for period  $k$  and  $\varepsilon_k^{i^p}$  is the disturbance term for the policy rate. As it is clear in the related literature that all coefficients of the policy rule are expected to be greater than zero. However, since there is debate on the level of  $\gamma_{1,k}$ , we do not

make a presumption that whether it is greater or less than one. Apart from this discussion, there are time subscripts for inflation target and equilibrium real interest rate which means that they can change over time. Such a relationship also allows us to incorporate changes in targeted inflation and equilibrium real interest rate, which provides a flexibility to examine time-dependent properties of these variables as noted by Woodford (2003).

Up to now, the components of standard models of monetary policy analysis have been given. Our contribution to this model is the inclusion of interest rate pass through as a different monetary transmission mechanism. While monetary policy rule determines the short-term policy interest rate, aggregate output is a function of long-term interest rate. The transition from policy rate to aggregate output and inflation is realized by means of long-term interest rate. That is, changes in the policy interest rate lead to changes in the long-term market interest rate, which is faced by households and firms in the economy. After then, long-term interest rate affects aggregate output. Our input takes place at this part and in order to complete the above model, we incorporate interest rate pass through relationship, which is the first in the literature, as to our knowledge. The interest rate pass through relationship establishes the link between short-term policy rate and long-term interest rate. Similar to Kwapil and Scharler (2006), Crespo-Cuaresma et al. (2004), Sørensen and Werner (2006), long-term interest rate is modeled as a function of its lag and policy rate, which is given below:

$$i_{k+1}^l = \lambda_k + \lambda_{1,k} i_k^l + \lambda_{2,k} i_k^p + \varepsilon_k^l \quad (96)$$

where  $\varepsilon_k^l$  stands for error term for the long-term interest rate. The interest rate pass through literature states that long-term interest rate is positively dependent on its lag and policy rate hence; all the coefficients in this equation are expected to be positive.

In addition to the core equations, given above, there are some other relationships that explain the dynamics and time-dependent property of the model. One of these equations is related to output. It is assumed that output follows the Beveridge-Nelson (1981) decomposition given as

$$y_k = y_k^* + gap_k \quad (97)$$

where  $y_k$  and  $y_k^*$  represent the output and potential output, respectively.

Potential output and equilibrium rate of real interest are assumed to follow a local linear trend model:

$$y_{k+1}^* = y_k^* + \mu_k + \varepsilon_k^{y^*} \quad (98)$$

$$r_{k+1}^* = r_k^* + \eta_k + \varepsilon_k^{r^*} \quad (99)$$

where the two stochastic trends  $\mu_k$  and  $\eta_k$ , as well as trend component of interest rate pass through,  $\lambda_k$ , are assumed to follow autoregressive, AR(1), process:

$$\mu_{k+1} = \rho_{\mu,k} \mu_k + \varepsilon_k^\mu \quad (100)$$

$$\eta_{k+1} = \rho_{\eta,k} \eta_k + \varepsilon_k^\eta \quad (101)$$

$$\lambda_{k+1} = \rho_{\lambda,k} \lambda_k + \varepsilon_k^\lambda \quad (102)$$

Clearly,  $\varepsilon_k^{y^*}$ ,  $\varepsilon_k^{r^*}$ ,  $\varepsilon_k^\mu$ ,  $\varepsilon_k^\eta$ ,  $\varepsilon_k^\lambda$  are error terms.

After introducing the functional relationships among the macroeconomic variables of the economy, it is needed to write state space representation of the model. The state space formulation is necessary for the application of EKF to estimate TVPs of the model because such a representation forms a basis for the execution of the filtering and updating algorithm of the EKF. The state space representation of the model can be given as:

$$x_{k+1} = Ax_k + Bu_k + w_k \quad (103)$$

$$y_k = Cx_k + v_k \quad (104)$$

where  $x_k$  is the state vector,  $y_k$  is the observation vector,  $u_k$  is the vector of exogenous variables. Besides,  $A$  denotes the transition matrix,  $B$  is the matrix of exogenous variables and  $C$  stands for the observation matrix. Furthermore,  $w_k$  and  $v_k$  are the vectors of normally distributed i.i.d shocks, which are assumed to be uncorrelated and have covariance matrices  $R_w$  and  $R_v$ , respectively.

Accordingly, the state space model, that is dynamic equations and observation equations, is written as follows:

$$\begin{bmatrix} \pi_{k+1} \\ \pi_k \\ ygap_{k+1}^* \\ y_{k+1} \\ i_{k+1}^l \\ i_{k+1}^p \\ r_{k+1}^* \\ \mu_{k+1} \\ \eta_{k+1} \\ \lambda_{k+1} \end{bmatrix} = \begin{bmatrix} \alpha_{1,k} & \alpha_{2,k} & \alpha_{3,k} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\beta_{2,k} & 0 & \beta_{1,k} & 0 & \beta_{2,k} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_{1,k} & \lambda_{2,k} & 0 & 0 & 0 & 1 \\ 1+\gamma_{1,k} & 0 & \gamma_{2,k} & 0 & 0 & \gamma_{3,k} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\mu,k} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\eta,k} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\lambda,k} \end{bmatrix} \begin{bmatrix} \pi_k \\ \pi_{k-1} \\ ygap_k^* \\ y_k \\ i_k^l \\ i_k^p \\ r_k^* \\ \mu_k \\ \eta_k \\ \lambda_k \end{bmatrix} \\
 + \begin{bmatrix} \alpha_{4,k} & 0 \\ 0 & 0 \\ \beta_{3,k} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\gamma_{1,k} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_k \\ \pi_k^* \end{bmatrix} + \begin{bmatrix} \varepsilon_k^\pi \\ 0 \\ \varepsilon_k^{ygap} \\ \varepsilon_k^{y^*} \\ \varepsilon_k^{i^l} \\ \varepsilon_k^{i^p} \\ \varepsilon_k^{r^*} \\ \varepsilon_k^r \\ \varepsilon_k^\mu \\ \varepsilon_k^\eta \\ \varepsilon_k^\lambda \end{bmatrix}$$

$$\begin{bmatrix} \pi_k \\ y_k \\ i_k^l \\ i_k^p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_k \\ \pi_{k-1} \\ ygap_k^* \\ y_k \\ i_k^l \\ i_k^p \\ r_k^* \\ \mu_k \\ \eta_k \\ \lambda_k \end{bmatrix}$$

The above state space formulation is used to express our baseline model in the form of equations (84) and (85) with the parameter vector of  $\theta_k$ , which was given in equation (83), specified as:

$$\theta_k = [\alpha_{1,k} \ \alpha_{2,k} \ \alpha_{3,k} \ \alpha_{4,k} \ \beta_{1,k} \ \beta_{2,k} \ \beta_{3,k} \ \lambda_{1,k} \ \lambda_{2,k} \ \gamma_{1,k} \ \gamma_{2,k} \ \gamma_{3,k} \ \rho_{\mu,k} \ \rho_{\eta,k} \ \rho_{\lambda,k}]^T$$

### 3.3.2. Alternative Model

The alternative model is based on New Keynesian framework, too. The main difference between our baseline model and the alternative model is the inclusion of interest rate pass-through relationship. That is, the alternative model does not consist of interest rate pass-through and related components. In this way, we can assess the influence of interest rate pass-through over the economy by comparing the baseline model and the alternative model. In other words, there will be a chance for evaluating the efficiency of interest rate pass-

through in transmitting policy decisions to macroeconomic variables. In this respect, the alternative model is made up of Phillip's curve, IS curve and monetary policy rule. Such a structure is widely used in monetary policy analysis. Here, one important point should be mentioned regarding the IS curve: The alternative model assumes that in the IS curve, aggregate demand is directly affected by short-term policy interest rate as it is common in the literature. That is to say, policy decisions are disseminated through the economy via short-term policy interest rate rather than long-term market interest rate. This is the main difference from the baseline model so that the alternative model does not need an interest rate pass-through equation for monetary transmission purposes. The remaining relationships are similar to those of the baseline model. Resultantly, the following set of equations present the structure of the alternative model with accompanying relationships:

$$\pi_{k+1} = \alpha_{1,k}\pi_k + \alpha_{2,k}\pi_{k-1} + \alpha_{3,k}ygap_k + \alpha_{4,k}z_k + \varepsilon_k^\pi \quad (105)$$

$$ygap_{k+1} = \beta_{1,k}ygap_k + \beta_{2,k}(i_k^p - \pi_k) + \beta_{3,k}z_k + \varepsilon_k^{ygap} \quad (106)$$

$$i_{k+1}^p = r_k^* + \pi_k + \gamma_{1,k}(\pi_k - \pi_k^*) + \gamma_{2,k}ygap_k + \gamma_{3,k}i_k^p + \varepsilon_k^{i^p} \quad (107)$$

$$y_k = y_k^* + ygap_k \quad (108)$$

$$y_{k+1}^* = y_k^* + \mu_k + \varepsilon_k^y \quad (109)$$

$$r_{k+1}^* = r_k^* + \eta_k + \varepsilon_k^r \quad (110)$$

$$\mu_{k+1} = \rho_{\mu,k}\mu_k + \varepsilon_k^\mu \quad (111)$$

$$\eta_{k+1} = \rho_{\eta,k} \eta_k + \varepsilon_k^\eta \quad (112)$$

where equation (105) is the Phillip's curve formulating inflation, equation (106) is the IS curve modeling the aggregate demand, equation (107) is the monetary policy rule. Beveridge-Nelson (1981) decomposition of output, potential output and equilibrium real interest rate are given in equations (108), (109) and (110), respectively. Accordingly, equations (111) and (112) model the stochastic trends of potential output and equilibrium real interest rate.

Similar to the baseline model, the state space formulation of the alternative model should be given now for estimation purposes. The state space representation of the alternative model, that is dynamic equations and observation equations, can be written as:

$$\begin{bmatrix} \pi_{k+1} \\ \pi_k \\ ygap_{k+1}^* \\ y_{k+1} \\ i_{k+1}^p \\ r_{k+1}^* \\ \mu_{k+1} \\ \eta_{k+1} \end{bmatrix} = \begin{bmatrix} \alpha_{1,k} & \alpha_{2,k} & \alpha_{3,k} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\beta_{2,k} & 0 & \beta_{1,k} & 0 & \beta_{2,k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1+\gamma_{1,k} & 0 & \gamma_{2,k} & 0 & \gamma_{3,k} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\mu,k} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\eta,k} \end{bmatrix} \begin{bmatrix} \pi_k \\ \pi_{k-1} \\ ygap_k^* \\ y_k \\ i_k^p \\ r_k^* \\ \mu_k \\ \eta_k \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha_{4,k} & 0 \\ 0 & 0 \\ \beta_{3,k} & 0 \\ 0 & 0 \\ 0 & -\gamma_{1,k} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_k^* \\ \pi_k^* \end{bmatrix} + \begin{bmatrix} \varepsilon_k^\pi \\ 0 \\ \varepsilon_k^{ygap} \\ \varepsilon_k^{y^*} \\ \varepsilon_k^{i^p} \\ \varepsilon_k^{r^*} \\ \varepsilon_k^\mu \\ \varepsilon_k^\eta \end{bmatrix}$$

$$\begin{bmatrix} \pi_k \\ y_k \\ i_k^p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_k \\ \pi_{k-1} \\ ygap_k^* \\ y_k \\ i_k^p \\ r_k^* \\ \mu_k \\ \eta_k \end{bmatrix}$$

The state space formulation given above presents the alternative model in the form of equations (84) and (85). Accordingly the parameter vector of the alternative model will be the following:

$$\theta_k = [\alpha_{1,k} \ \alpha_{2,k} \ \alpha_{3,k} \ \alpha_{4,k} \ \beta_{1,k} \ \beta_{2,k} \ \beta_{3,k} \ \gamma_{1,k} \ \gamma_{2,k} \ \gamma_{3,k} \ \rho_{\mu,k} \ \rho_{\eta,k}]^T$$

Now, the baseline model and the alternative model will be estimated separately. Estimation of state variables and unknown parameters requires application of prediction and filtering cycles of EKF explained in equations (86) through (92). The next section will explain data generation process and application of EKF, sequentially.

### 3.4. Data Generation and Simulation

So far, we have explained dynamics and relationships of our structural models with their state space representations. Now, we will produce artificial data for these economies.

Initially, we will simulate the baseline model and obtain the time series of observed variables. For data generation process, the model described in equations (93) through (102) are used with the following, arbitrarily selected, initial values of state variables and parameters:

$$\begin{aligned} & \left[ \pi_0 \quad \pi_{-1} \quad ygap_0 \quad y_0^* \quad i_0^l \quad i_0^p \quad r_0^* \quad \mu_0 \quad \eta_0 \quad \lambda_0 \right]^T = [3 \quad 2 \quad 1 \quad 4 \quad 5 \quad 5 \quad 5.5 \quad 1 \quad 1 \quad 1]^T \\ & \left[ \alpha_{1,k} \quad \alpha_{2,k} \quad \alpha_{3,k} \quad \alpha_{4,k} \quad \beta_{1,k} \quad \beta_{2,k} \quad \beta_{3,k} \quad \lambda_{1,k} \quad \lambda_{2,k} \quad \gamma_{1,k} \quad \gamma_{2,k} \quad \gamma_{3,k} \quad \rho_{\mu,k} \quad \rho_{\eta,k} \quad \rho_{\lambda,k} \right]^T = \\ & [ .456 \quad .474 \quad .186 \quad .143 \quad .800 \quad -.158 \quad .007 \quad .422 \quad .479 \quad .600 \quad .100 \quad .351 \quad .422 \quad .001 \quad .200 ]^T \end{aligned}$$

Disturbance terms are produced from normal distribution with mean zero and variance one so that  $w_k \sim N(0,1)$  and  $v_k \sim N(0,1)$ .

After data generating process, application of EKF to the artificial data takes place. This operation requires determination of initial values both for the state variables and parameters again. For the initiation of EKF, the same starting values, given above, are selected for state variables and parameters of the system with following additional parameters:

$$Q_k = \begin{bmatrix} 189.69 & 0 & \dots & \dots & 0 \\ 0 & 188.69 & & & \\ & & 46.32 & & \ddots \\ \vdots & & & 754.98 & \vdots \\ & & & & 115.35 \\ \vdots & & & & & 171.16 \\ & & & & & & 285.24 \\ \vdots & & & & & & & 1.29 \\ & & & & & & & & .90 & 0 \\ 0 & & \dots & & \dots & & & & 0 & 1.11 \end{bmatrix}$$

$$R_k = \begin{bmatrix} 191 & 0 & 0 & 0 \\ 0 & 105 & 0 & 0 \\ 0 & 0 & 116 & 0 \\ 0 & 0 & 0 & 172 \end{bmatrix} \quad P_0 = I_{10 \times 10} * 0.1 \quad \text{and} \quad S_0 = I_{15 \times 15} * 0.001$$

Upon specifying the required starting values, EKF is applied to generated data using the algorithm given by equations (88) through (92).

Likewise, we will simulate the alternative model. For this, we will generate artificial data using equations (105) through (112). The following set gives the starting values of the state variables, coefficients and other parameters of the alternative model.

$$\begin{bmatrix} \pi_0 & \pi_{-1} & ygap_0 & y_0^* & i_0^p & r_0^* & \mu_0 & \eta_0 \end{bmatrix}^T = [3 \ .2 \ .1 \ .4 \ .5 \ .55 \ .1 \ .1]^T$$

$$\begin{bmatrix} \alpha_{1,0} & \alpha_{2,0} & \alpha_{3,0} & \alpha_{4,0} & \beta_{1,0} & \beta_{2,0} & \beta_{3,0} & \gamma_{1,0} & \gamma_{2,0} & \gamma_{3,0} & \rho_{\mu,0} & \rho_{\eta,0} \end{bmatrix}^T =$$

$$[.456 \ .474 \ .186 \ .143 \ .800 \ -.158 \ .007 \ .600 \ .100 \ .351 \ .422 \ .001]^T$$

$$Q_k = \begin{bmatrix} 73.9 & 0 & & \dots & & 0 \\ 0 & 73.8 & & & & \\ & & 25.3 & & \ddots & \vdots \\ \vdots & & & 661.5 & & \\ & & & & 99.8 & \vdots \\ \vdots & & \ddots & & & 112.1 \\ & & & & & 1.3 & 0 \\ 0 & & \dots & & & 0 & 0.9 \end{bmatrix}$$

$$R_k = \begin{bmatrix} 75.7 & 0 & 0 \\ 0 & 626.6 & 0 \\ 0 & 0 & 100.4 \end{bmatrix} \quad P_0 = I_{8 \times 8} * 0.1 \quad \text{and} \quad S_0 = I_{12 \times 12} * 0.001$$

Lastly, the error terms are distributed normally, that is,  $w_k \sim N(0,1)$  and  $v_k \sim N(0,1)$ .

The next section elaborates on the results and findings of simulations. It should be noted that some data points from the beginning and end of the series are dropped for avoiding start up and termination errors, respectively.

### ***3.5. Simulation Results and Findings***

The results of the data generation process and application of EKF to this generated data have been given in Appendix B. The initial five figures (Figures B1 through B5) compare artificial series produced by means of data generation process (real value) and estimated series through EKF (estimation) for inflation, output, output gap policy interest rate and long-term interest rate of the baseline model. The next four figures (Figures B6 through B9) illustrate the simulated series (real value) and estimated series (estimation) for the inflation, output, output gap and policy interest rate of the alternative model. As a starting point it should be mentioned that when these nine figures (Figures B1 through B9) are examined, it is seen that the EKF algorithm mimics the behavior of generated data quite well, which means that EKF can be regarded as a powerful tool for estimating TVP models.

When we compare the corresponding figures of the baseline model and the alternative model, we can see that the baseline model is superior to the alternative model in following points: Inflation series of the alternative model is more volatile compared to inflation series of the baseline model indicating that interest rate pass-through specification has some role in reducing the volatility of the inflation. In other words, monetary policy rule alone is not very efficient in stabilizing inflation. This supports our view such that interest rate

pass-through is an influential monetary transmission mechanism that should be considered in monetary policy analysis.

Also, if we compare output series of both models, we can easily see that output increases in the baseline model while it falls in the alternative model. This clearly shows that using monetary policy rule alone shrinks the economy. Nonetheless, output series of the baseline model follows an increasing pattern indicating growth in economic activity. This again validate our assumption so that aggregate demand is dependent more on long-term interest rate so, interest rate pass-through is significant in transmitting changes in short-term policy interest rate to long-term market interest rate and resultantly to aggregate demand. Consequently, we can claim that the baseline model with interest rate pass-through is preferred over the alternative model without interest rate pass-through according to inflation and output processes. Having the superiority of the baseline model in mind, the next subsections elaborates on time varying parameters of the monetary policy rule and interest rate pass-through of the baseline model.

### **3.5.1. Coefficients of Monetary Policy Rule**

Figures B10, B11 and B12 display the patterns of time-varying coefficients of monetary policy rule  $(\gamma_1, \gamma_2, \gamma_3)$  of the baseline model. These graphs

demonstrate that, the parameters of monetary policy rule are greater than zero as expected. According to Figure B10, the coefficient of inflation on policy interest rate follows a decreasing pattern but it is greater than zero as expected. The weight of inflation on policy interest rate,  $\gamma_1$ , is falling from 0.62 to 0.56. Loosely speaking, inflation targeting policy is relaxed a little bit.

Figure B11 shows the effect of output gap on policy interest rate. The weight of output gap in policy rule,  $\gamma_2$ , is first increasing from 0.11 up to 0.19 and then decreasing back to 0.10. This implies that output gap stability has some emphasis initially but after the period 700 this policy changes in the opposite direction. Finally, Figure B12 illustrates that the smoothing parameter of policy interest rate,  $\gamma_3$ , is greater than 0.31 and displays an increasing trend with large cycles.

When we analyze Figure B10 with Figure B3 contemporaneously, output gap follows a relatively stable path until period 700 indicating that as  $\gamma_2$  rises, output gap stability can be achieved. However, the weight of inflation in policy rule,  $\gamma_1$ , follows a path with subsequent ups and downs around a decreasing trend. Examination of Figure B10,  $\gamma_1$ , with Figure B1, inflation, simultaneously shows that as  $\gamma_1$  decreases, monetary policy goes away from inflation targeting, which in turn results in high inflation. Such a configuration

may help to explain the growth in output so that, inflation results in rising output as being a driving force.

On the other hand, after the period 700, although  $\gamma_1$  is still decreasing, inflation begins to go down. When we link this observation with the fall in  $\gamma_2$ , in Figure B11, it can be argued that inflation targeting policy (in other words, decrease in inflation) is dependent more on the weight of output gap in policy rule,  $\gamma_2$ , so that fall in  $\gamma_2$  goes with the decrease in inflation. As a result, it can be concluded that if the aim is to decrease inflation, then the gap between  $\gamma_1$  and  $\gamma_2$  should be high and policy interest rate should be more dependent on inflation.

Policy interest rate of the baseline model, Figure B4, does follow slightly an upward trend, but volatility is too high. This volatility can be attributed to the relatively unstable pattern of output gap transferred via  $\gamma_2$ . Examination of inflation, output gap and policy interest rate series of the baseline model, Figures B1, B3 and B4, altogether points out that large oscillations in inflation and output gap are reflected in policy rate indicating that policy interest rate is highly sensitive to output gap and inflation. The amplitude of oscillations in policy interest rate is greater compared to those in inflation and output gap, which means that high value of  $\gamma_1$  and growth in  $\gamma_2$  amplifies the scale of

fluctuations in the inflation and output gap, correspondingly, while transferring them to policy rate.

High volatility in policy rate indicates also that interest rate smoothing is not so powerful. Nonetheless, after period 700, the smoothing parameter,  $\gamma_3$ , in Figure B12, begins to rise following a path opposite of  $\gamma_2$ . If the decrease in  $\gamma_2$  is considered as a sign of inflation targeting policy, or fall in inflation, then increasing  $\gamma_3$  can be regarded as a signal that backward-looking component of policy rate is remarkable during inflation targeting.

### **3.5.2. Coefficients of Interest Rate Pass-Through**

Time varying paths of the coefficients of the interest rate pass-through ( $\lambda_1, \lambda_2$ ) of the baseline model are demonstrated in Figure B13 and B14. Figure B13 illustrates that long-term interest rate smoothing parameter,  $\lambda_1$ , is between 0.34 and 0.48. Figure B14 demonstrates that the parameter displaying the impact of policy rate on long-term interest rate,  $\lambda_2$ , is between 0.45 and 0.54. The interest rate pass-through relationship asserts that long-term interest rate positively depends on both its own lag and policy interest rate so, the positive

values for  $\lambda_1$  and  $\lambda_2$  confirms the expectations about the signs of the parameters of the pass-through relationship.

If the pass-through component of long-term interest rate,  $\lambda_2$ , in Figure B14, is examined, it can be seen that long-term interest rate is highly affected by policy interest rate implying that pass-through from policy rate to long-term interest rate is increasing. Hence, it can be claimed that monetary transmission mechanism is relying also on long-term interest rate so that pass-through is high. This supports our argument that policy decisions are disseminated to the economy via long-term interest rate and monetary policy analysis should take into account interest rate pass-through.

Since volatility of policy rate is transmitted to long-term interest rate via this parameter, instability in long-term interest rate can be explained with the rise in  $\lambda_2$ . Moreover, downward trend in long-term interest rate smoothing parameter,  $\lambda_1$ , can be another explanation for the volatility of long-term interest rate. Nevertheless, after period 700,  $\lambda_1$  starts to increase suggesting that inflation targeting policy increases the weight of smoothing parameter in the pass-through equation similar to smoothing parameter in the monetary policy rule,  $\gamma_3$ . On the other hand, long-term interest rate is a little less volatile than policy interest rate, suggesting that smoothing parameter for long-term interest rate,  $\lambda_1$ , is more effective than the corresponding parameter for policy interest rate,

$\gamma_3$ . Higher average level of  $\lambda_1$ , compared to  $\gamma_3$ , indicates that backward looking behavior of long-term interest rate is stronger than that of policy interest rate.

Finally, when both of the policy interest rates, in Figure B4, and long-term interest rates, in Figure B5, are analyzed, it is observed that although their volatilities are high, there are no clear upward/downward trends in their pattern. However, a weak upward trend in both of the interest rates is recognized. High volatility can be attributed partially to disturbance terms and autoregressive components of the interest rates.

### **3.6. Conclusion**

In this study, we modeled a New Keynesian economy introducing interest rate pass-through, for the first time, as monetary transmission mechanism. The model consists of Phillip's curve, IS curve, monetary policy rule, interest rate pass-through relationship and other dynamics of the economy. Three points in this framework are noteworthy:

Firstly, introduction of interest rate pass-through relationship to a New Keynesian model in this kind of setting is the first, as far as we know. We imposed a monetary transmission mechanism by claiming that contrary to

previous literature, output gap is dependent on long-term interest rate rather than short-term policy interest rate. Comparison of the baseline model, which includes interest rate pass-through, with the alternative model, which does not have interest rate pass-through, revealed that the baseline model is superior over the alternative one in reducing the volatility of inflation and in achieving output growth. Therefore, it can be claimed that interest rate pass-through is an important ingredient in monetary policy analysis so that attention should be paid for including long-term interest rate in structural models. However, there is still room for further research on this issue. Our model is backward-looking so inclusion of forward-looking components can be the topic of a future study. Moreover, different forms of interest rate pass-through and policy rule can be inserted and these models can be tested with real-time data applications.

Secondly, we have used time-varying parameter specification for this system. Although this feature is not new, we have used this specification not only for monetary policy rule, as it is common in the related literature, but also for all parameters of the system. Hence, we have the flexibility of examining time-dependent pattern of all system dynamics. Even if such a dynamic model may decrease the estimation power of the technique used, the estimation results show that the model and estimation tool work quite well.

Lastly, due to the specification of the relationships among variables and parameters, the model has a non-linear characteristic and basket of suitable

estimation techniques became restricted. Although some tools are powerful for linear systems, they become improper for nonlinear systems therefore, it is of importance to employ an appropriate estimation technique, which is EKF. Similar to the first point, use of EKF in such a setting is new, to the knowledge of us. The estimation results showed that EKF is highly powerful in capturing the unobserved state variables and time-dependent parameters of the non-linear system.

## **CHAPTER 4**

### **EFFECTS OF ADOPTING INFLATION TARGETING REGIMES ON INFLATION VARIABILITY**

#### ***4.1. Introduction***

In the 1990s, the monetary policy concerns of many countries changed in the direction of low and stable inflation with the purpose of economic growth, low inflation uncertainty and growth sustainability. Therefore, many developed and developing countries started to adopt inflation targeting programs to improve their economic performance. Although inflation targeting programs have been mostly successful in achieving the targeted inflation levels, there are various debates on the performance of these programs.

One of these discussions is about the effect of inflation targeting programs on inflation variability. It is valuable to analyze the behavior of inflation uncertainty because it affects macroeconomic variables such as output,

investment, interest rate. For example, Friedman (1977), Froyen and Waud (1987) and Holland (1988) argue the presence of the adverse effect of inflation uncertainty on output; Hafer (1986) and Holland (1986) elaborate on the negative effect of inflation uncertainty on employment; Berument (1999) shows that inflation variability raises UK three-month treasury-bill rates.

There is contradictory evidence about the performance of inflation targeting programs in achieving lower inflation volatility. Some studies claim that the adoption of inflation targeting programs does have an impact on reducing the inflation variability, while some argue that this effect is not present.

Regarding inflation uncertainty, some argue that inflation variability and inflation expectations decrease after the adoption of inflation targeting programs. Dittmar et al. (1999) and Gavin (2003) elaborated on the performance of these programs in reducing the volatility of inflation. The former study gives evidence from G-10 countries about the behavior of inflation and its variance, and then explains analytically how inflation targeting programs can be successful in decreasing inflation variability. Gavin (2003) further supports this proposition by reviewing the experience of other inflation targeting countries.

Some studies assert that, in the inflation targeting countries, a decrease in the inflation expectations and inflation variability should be attributed to the

effectiveness of inflation decreasing programs not to the adoption of inflation targeting regimes. Cecchetti and Ehrman (1999) claimed that inflation targeting programs are not the sole factor in lowering inflation volatility and inflation expectations. Cecchetti and Ehrman (1999) discussed both theoretically and empirically on the link between inflation variability and output volatility under inflation targeting programs. In this study, data is used from 9 inflation targeting countries (Australia, Canada, Chile, Finland, Israel, New Zealand, Spain, Sweden, the UK) and 14 non-inflation targeting countries (Austria, Belgium, Denmark, France, Germany, Ireland, Italy, Japan, Korea, Mexico, Netherlands, Portugal, Switzerland, the US). They argued that the monetary policymakers in these countries preferred to take actions that favor a reduction in the volatility of inflation compared to a drop in output volatility. Thus, it is not the inflation targeting programs but the general tendency of the policymakers to reduce inflation that can be considered the main reason for the drop in the level of inflation in the countries analyzed (see, for example, Cecchetti, 1997; Groenfeld, 1998; Bernanke et al., 1999; Arestis et al., 2002 and Arestis and Mouratidis, 2004 for further support of this issue).

Two of the other studies elaborating on the effectiveness of inflation targeting programs on inflation uncertainty are Johnson (2002) and Johnson (2003). In these works, he used inflation surveys of experts to measure the expected inflation and variability of inflation and showed that an inflation targeting regime decreases the level of expected inflation; however, the variability of

inflation forecast errors does not decrease significantly in the targeting countries (Australia, Canada, New Zealand, Sweden, the UK) compared to not targeting countries (France, Germany, Italy, Japan, Netherlands, the US).

To quantify inflation variability, a variety of measures are employed by different studies, but none of these studies measures the perception of the public on inflation risk. In existing studies in the literature, only the observed variability was measured, only short-run dynamics of the variability were assessed or the measurement of perception was biased. Cecchetti and Ehrman (1999) and Dittmar et al. (1999) used the deviation of realized inflation from the targeted value as a measure of inflation variability. This type of specification measures observed changes in inflation. In contrast, Johnson (2002, 2003) employed standard deviation of inflation forecasts collected through a survey and conducted among professional forecasters. Although this method is a good measure of variability of expected inflation, as Bomberger (1996) argued, survey based studies have the problem of biased or unreliable data. The people, who take part in the survey may be biased, may not give an objective forecast or may not be able to use all the available information. As a result, the standard deviation of these forecasts would not be a reliable measure of the perception toward inflation risk, which is an important measure of the credibility of the program. Another measure of inflation variability, the bivariate stochastic volatility framework, is employed by Arestis et al. (2002) and Arestis and Mouratidis (2004). This type of specification is advantageous

for modeling inflation-output variability and is designed to evaluate the short-run dynamics of inflation-output variability trade-off but fails to capture the public's perception of the inflation risk.

Unlike the above models, this study uses ARCH/GARCH type of conditional inflation variability specification as a measure of inflation variability. This specification measures the perception of the public on inflation variability rather than the variance itself. Inflation uncertainty series are examined to determine whether inflation targeting programs have a significant effect on the evolution of inflation variability. In this way, we will be able to see whether inflation targeting programs can really convince the public in the variability of inflation has been reduced. This is the contribution of this study to the existing literature, in which public perception has not been considered before.

To perform this analysis, five developed countries and five emerging economies, which implement inflation targeting programs were selected. The analysis of both developed and developing country data will serve as a basis for the comparison of these two groups with respect to the efficiency of the inflation targeting programs in reducing the inflation variability if there is any discrepancy. The selection was based on the country classification made by World Bank and International Monetary Fund (IMF). World Bank classified the countries with respect to their 2006 gross national income (GNI) per capita as low income countries, whose 2006 GNI per capita was less than \$905; lower

middle income countries, whose 2006 GNI per capita was between \$906 and \$3595; upper middle income countries, whose between \$3596 and \$11115; and high income countries, whose 2006 GNI per capita was greater than \$11116 (web site: <http://go.worldbank.org/K2CKM78CC0>). IMF World Economic Outlook (2007) also grouped countries mainly as advanced countries and emerging and developing countries. According to these classifications, Australia, Canada, New Zealand, Sweden and the UK were selected as developed countries and Brazil, Chile, Colombia, Mexico and South Africa were selected as emerging markets. However, for Mexico, the results of the estimation changed according to the sample used. Due to the instability of the Mexican estimates, Mexico was eliminated from the study.

The outline of the study is as follows: The next section will explain the methodology used in this work. Then empirical evidence will be given. The last section is the conclusion.

#### ***4.2. Methodology***

Parallel to earlier studies like Engle (1982), Grier and Perry (1998, 2000), we selected the autoregressive (AR) process to model inflation. Berument (1999) also included real wage as an explanatory variable in the inflation specification for the UK, but it was found to be statistically insignificant. Hence, we do not

use other variables such as real wages in the inflation specification in order to avoid over-parameterization. It might have been possible to include additional variables for the nine countries that we considered. However, the models would likely not include the same variables for the inflation specification for the nine countries. Thus, we did not pursue this avenue further for the parallelism. Consequently, in order to model inflation, we used an  $n^{th}$  order autoregressive process, AR(n):

$$\pi_t = \beta_0 + \sum_{i=1}^n \beta_i \pi_{t-i} + \varepsilon_t \quad (113)$$

where  $\pi_t$  is the inflation level at time  $t$ ,  $n$  is the number of lags,  $\varepsilon_t$  is the residual term at time  $t$ . Here, inflation has an autoregressive expression at the lag length of  $n$  to account for the effect of autocorrelated residuals and we assume that  $\varepsilon_t$  has a zero mean and time varying variance of  $h_t^2$ . To model the time varying variance, Engle (1982) used autoregressive conditional heteroscedastic (ARCH) model, which is a conditional variance of the inflation equation

$$h_t^2 = \alpha_0 + \sum_{j=1}^q \alpha_j \varepsilon_{t-j}^2. \quad (114)$$

Later, Bollerslev (1986) included past values of  $h_t^2$  in addition to the lagged values of the squared residuals to capture the conditional variance – generalized autoregressive conditional heteroscedasticity model: GARCH (p,q).

$$h_t^2 = \alpha_o + \sum_{i=1}^p \alpha_{1i} h_{t-i}^2 + \sum_{j=1}^q \alpha_{2j} \varepsilon_{t-j}^2 \quad (115)$$

In this specification, two points need attention as sufficient conditions; firstly due to the non-negativity property of variances, the constant and the coefficients should be positive ( $\alpha_o, \alpha_{1i}, \alpha_{2j} > 0$ , for all  $i, j$ ). Secondly, conditional variance should not explode; hence, to provide non-explosiveness, the sum of the coefficients except the constant should be less than 1:

$$\sum_{i=1}^p \alpha_{1i} + \sum_{j=1}^q \alpha_{2j} < 1.$$

In the literature, studies exist that also include some exogenous variables in the GARCH specification; for instance, Karolyi (1995) included variance of stock market yields of foreign countries to model the variance of stock market yields of the home country. As another example, Berument and Kiymaz (2001) and Kiymaz and Berument (2003) included daily dummies to stock market returns and its conditional variance to account for the day of the week effect in stock market conditional variance. With this in mind, in this study, we included some exogenous variables in the mean inflation and GARCH equations to capture the impact of inflation targeting programs on inflation volatility.

To differentiate the periods before and after the inflation targeting program adoption, an exogenous dummy variable is used to represent the time of

implementation. The dummy variable  $d_t$  takes the value of one for the periods that inflation targeting is adopted, zero otherwise. In addition to this dummy, eleven monthly dummy variables are included to account for the seasonality in inflation. Thus, the new inflation equation, formed by employing the dummy variables (mean equation), is as follows:

$$\begin{aligned} \pi_t = & \beta_0 + \beta_0^d d_t + \beta_{Jan}^d d_{Jan,t} + \beta_{Feb}^d d_{Feb,t} + \beta_{Mar}^d d_{Mar,t} + \beta_{Apr}^d d_{Apr,t} + \\ & \beta_{May}^d d_{May,t} + \beta_{Jun}^d d_{Jun,t} + \beta_{Jul}^d d_{Jul,t} + \beta_{Aug}^d d_{Aug,t} + \beta_{Sep}^d d_{Sep,t} + \\ & \beta_{Oct}^d d_{Oct,t} + \beta_{Nov}^d d_{Nov,t} + \beta_{Dec}^d d_{Dec,t} + \sum_{i=1}^n \beta_i \pi_{t-i} + \varepsilon_t \end{aligned} \quad (116)$$

$d_{Jan,t}$  to  $d_{Dec,t}$  are the monthly seasonal dummy variables equal to one for the respective months and zero for the others.

In order to estimate the inflation equation shown by equation (116), three different GARCH (1,1) specifications are employed separately. Here, it is noteworthy to clarify why GARCH (1,1) models were selected in the current analysis. We could also employ different GARCH orders or ARCH type of classifications. There are two reasons for not doing so. First, GARCH (1,1) is the most commonly used specification for the conditional variances in general. Second, a set of robustness statistics that we performed on GARCH (1,1) specification provides evidence on the adequacy of the specification. We could adopt more complicated models, which would require additional parameters to estimate. In this paper we were able to find limited evidence on the decrease in

conditional variance of inflation. It would be more difficult to observe the decrease in conditional variance with more parameters to be estimated. The first one is the basic GARCH (1,1) model, in which no differentiation is made for inflation targeting periods, given below:

$$h_t^2 = \alpha_0 + \alpha_1 h_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 \quad (117)$$

The second model has an additive dummy variable for the inflation targeting era inserted in the GARCH (1,1) specification, in order to see if there is an exogenous decrease in the conditional variance of inflation:

$$h_t^2 = \alpha_0 + \alpha_0^d d_t + \alpha_1 h_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 \quad (118)$$

In the third conditional variance expression, besides the additive dummy variable (used for inflation targeting period) two multiplicative dummy variables were included. The first multiplicative dummy is employed with the lagged conditional variance, the second one is used with the lag of squared residual term. In this way, we are able to observe the behavior of persistence and impulse variability of inflation, respectively, after the inflation targeting is adopted. This last conditional variance equation is given as follows:

$$h_t^2 = \alpha_0 + \alpha_0^d d_t + \alpha_1 h_{t-1}^2 + \alpha_1^d d_t h_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 + \alpha_2^d d_t \varepsilon_{t-1}^2 \quad (119)$$

### ***4.3. Empirical Evidence***

Before modeling inflation with GARCH specification, we will check whether the inflation series has an ARCH effect. Hence, the autoregressive conditional heteroscedasticity – lagrangian multiplier (ARCH-LM) test is performed for the inflation targeting countries concerned. First, the inflation series of each country is regressed on a constant, eleven monthly dummy variables and the inflation lags, where Final Prediction Error (FPE) Criteria is applied to determine the optimum number of lags for which the autocorrelation existing in the residuals of the inflation equation is eliminated (Jansen and Cosimona, 1988 argues that ARCH-LM tests of autocorrelated residuals wrongly suggest the presence of an ARCH effect, even when there is no ARCH effect). After that, the residuals of this regression are squared and regressed on constant and lagged squared residuals. Here, the number of lags for residuals, which are the regressors, are chosen arbitrarily as 3, 6, 12, 18 and 24. Table 2 reports the  $p$ -values of these regressions for each inflation targeting country.

The null hypothesis of ‘there is no heteroscedasticity’ is rejected for all the countries for the 3 and 6 lags at 1% level. We could not reject the null only for New Zealand at 18 and 24 lags and at the 5% level for the remaining lags. Thus, we assume that the ARCH effect exists for all the countries that we consider.

**Table 2:** ARCH-LM Test for the Inflation Series<sup>1</sup>

| Country      | ARCH-LM (3) | ARCH-LM (6) | ARCH-LM (12) | ARCH-LM (18) | ARCH-LM (24) |
|--------------|-------------|-------------|--------------|--------------|--------------|
| Australia    | 0.000**     | 0.000**     | 0.000**      | 0.000**      | 0.000**      |
| Brazil       | 0.000**     | 0.000**     | 0.000**      | 0.000**      | 0.000**      |
| Canada       | 0.000**     | 0.000**     | 0.000**      | 0.000**      | 0.000**      |
| Chile        | 0.000**     | 0.000**     | 0.000**      | 0.000**      | 0.000**      |
| Colombia     | 0.000**     | 0.000**     | 0.000**      | 0.000**      | 0.000**      |
| New Zealand  | 0.000**     | 0.001**     | 0.034*       | 0.224        | 0.552        |
| South Africa | 0.000**     | 0.000**     | 0.000**      | 0.000**      | 0.000**      |
| Sweden       | 0.000**     | 0.000**     | 0.000**      | 0.000**      | 0.000**      |
| UK           | 0.000**     | 0.000**     | 0.000**      | 0.000**      | 0.000**      |

In the next step, the variability of the inflation residuals is modeled with a GARCH specification. Table 3 reports the estimation results of the first model, specified with equation (116) for the mean inflation and equation (117) for the conditional variance. In this table, the coefficient of the dummy variable for the inflation targeting period,  $\beta_0^d$ , is negative for all the countries except Brazil, indicating that empirical evidence for Australia, Canada, Chile, Colombia, New Zealand, South Africa, Sweden and the UK is consistent with the expectation that implementation of inflation targeting program decreases the conditional inflation, but it is only significant for Sweden and Chile. Note that one cannot interpret the estimated coefficient for  $\beta_0^d$  as an indication of lower

---

<sup>1</sup> \*\* and \* indicate 1% and 5% levels of significance, respectively.

inflation. Lower steady state inflation requires that  $\beta_0^d / \left(1 - \sum_{i=1}^n \beta_i\right)$  be negative. Negative  $\beta_0^d$  suggests a lower conditional inflation at a given level of past inflation, which will be called lower conditional expectation from now on.

**Table 3:** Estimation Results of the Model 1<sup>2, 3</sup>

| Country      | $\beta_0^d$ | $\alpha_0$   | $\alpha_1$ | $\alpha_2$ | Likelihood |
|--------------|-------------|--------------|------------|------------|------------|
| Australia    | -0.116      | 3894.642     | 0.029      | 0.008      | -1018.458  |
|              | (0.161)     | (447595.088) | (1.801)    | (0.981)    |            |
| Brazil       | 0.031       | 0.061*       | 0.058      | 0.674*     | 36.319     |
|              | (0.050)     | (0.029)      | (0.229)    | (0.337)    |            |
| Canada       | -0.023      | 0.005        | 0.875**    | 0.096**    | 165.362    |
|              | (0.028)     | (0.003)      | (0.041)    | (0.035)    |            |
| Chile        | -0.288**    | 0.013        | 0.771**    | 0.225**    | -106.702   |
|              | (0.069)     | (0.007)      | (0.051)    | (0.070)    |            |
| Colombia     | -0.199      | 0.000        | 0.921**    | 0.076**    | -305.753   |
|              | (0.102)     | (0.001)      | (0.020)    | (0.024)    |            |
| New Zealand  | -0.110      | 0.524**      | 0.025      | 0.144      | -272.071   |
|              | (0.073)     | (0.183)      | (0.272)    | (0.084)    |            |
| South Africa | -0.010      | 0.002        | 0.948**    | 0.049*     | -176.572   |
|              | (0.067)     | (0.003)      | (0.022)    | (0.024)    |            |
| Sweden       | -0.091*     | 0.015        | 0.880**    | 0.099      | 26.203     |
|              | (0.040)     | (0.010)      | (0.049)    | (0.054)    |            |
| UK           | -0.030      | 0.001        | 0.948**    | 0.047**    | 39.465     |
|              | (0.033)     | (0.001)      | (0.015)    | (0.016)    |            |

<sup>2</sup> \*\* and \* indicate 1% and 5% levels of significance, respectively.

<sup>3</sup> Values in parenthesis show standard deviations of the coefficients.

For the coefficients of conditional variance,  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$ , the estimation results satisfy the sufficient condition explained in the methodology part for all the countries. That is to say, all of the coefficients are positive (non-negativity) and the sum of  $\alpha_1$  and  $\alpha_2$  is less than 1 (non-explosiveness). The table also reports the log likelihood of the estimations in the last column. We did not report the coefficients of the lagged inflation variables in order to save space. We also calculated Ljung and Box autocorrelation and ARCH-LM tests for the standardized residuals  $(\varepsilon_t/h_t)$  for various lags for the current and other specifications that are used in this paper. These test statistics are reported in the Appendix C. Even if the ARCH-LM test fails to reject the ARCH effect for Chile and Colombia, after allowing conditional variance to change with adoption of inflation targeting regimes, this problem is mostly eliminated. Overall, we could reject the presence of autocorrelation and heteroscedasticity for our specifications. This further supports the validity of our specifications.

After estimating inflation volatility using the general GARCH (1,1) specification, the second model is estimated, in which the conditional variance equation has an additive dummy variable for the periods of inflation targeting, as well. Table 4 reports the estimations of the second model as specified by equations (116) and (118). The estimated coefficient for  $\beta_0^d$  is negative again for all the countries except Australia and Brazil. For the remaining countries, although the effect of the program is as expected (negative),  $\beta_0^d$  is significant

just for Chile and Sweden. When we look at the effect of the dummy for the inflation targeting period on the conditional variability of inflation,  $\alpha_0^d$ , it is negative for all except South Africa, illustrating that conditional inflation variability decreases after the implementation of an inflation targeting program, but it is significant only for Chile.

**Table 4:** Estimation Results of the Model 2<sup>4, 5</sup>

| Country      | $\beta_0^d$ | $\alpha_0^d$ | $\alpha_0$   |
|--------------|-------------|--------------|--------------|
| Australia    | 0.075       | -4.273       | 15023.032    |
|              | (0.174)     | (2818.614)   | (687822.037) |
| Brazil       | 0.023       | -0.032       | 0.083        |
|              | (0.054)     | (0.039)      | (0.044)      |
| Canada       | -0.021      | -0.002       | 0.008        |
|              | (0.027)     | (0.003)      | (0.004)      |
| Chile        | -0.219**    | -0.112*      | 0.133*       |
|              | (0.078)     | (0.053)      | (0.058)      |
| Colombia     | -0.119      | -0.001       | 0.000        |
|              | (0.100)     | (0.001)      | (0.001)      |
| New Zealand  | -0.140      | -0.343       | 0.674        |
|              | (0.073)     | (0.219)      | (0.400)      |
| South Africa | -0.110      | 0.000        | 0.003        |
|              | (0.074)     | (0.005)      | (0.003)      |
| Sweden       | -0.106*     | -0.103       | 0.152        |
|              | (0.043)     | (0.066)      | (0.094)      |
| UK           | -0.032      | -0.003       | 0.005        |
|              | (0.034)     | (0.003)      | (0.004)      |

<sup>4</sup> \*\* and \* indicate 1% and 5% levels of significance, respectively.

<sup>5</sup> Values in parenthesis show standard deviations of the coefficients.

**Table 4** (cont'd)

| Country      | $\alpha_1$ | $\alpha_2$ | Likelihood |
|--------------|------------|------------|------------|
| Australia    | 0.014      | 0.080      | -896.937   |
|              | (1.058)    | (3.677)    |            |
| Brazil       | 0.070      | 0.613*     | 36.590     |
|              | (0.220)    | (0.312)    |            |
| Canada       | 0.854**    | 0.100**    | 165.852    |
|              | (0.051)    | (0.038)    |            |
| Chile        | 0.601**    | 0.320**    | -123.772   |
|              | (0.075)    | (0.097)    |            |
| Colombia     | 0.974**    | 0.020**    | -393.581   |
|              | (0.007)    | (0.007)    |            |
| New Zealand  | 0.022      | 0.069      | -265.269   |
|              | (0.532)    | (0.066)    |            |
| South Africa | 0.935**    | 0.057*     | -157.611   |
|              | (0.028)    | (0.028)    |            |
| Sweden       | 0.477*     | 0.212      | 37.384     |
|              | (0.229)    | (0.153)    |            |
| UK           | 0.920**    | 0.063**    | 40.062     |
|              | (0.028)    | (0.024)    |            |

Table 4 also reports the values of the coefficients of conditional variance equation,  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$ , and the log likelihood of the estimation. As explained in Section 4.2, the non-negativity and non-explosiveness conditions are satisfied, which is required for a good estimation.

Equations (116) and (119) form the specifications of our third model, in which conditional variance of inflation includes multiplicative dummies in addition to the additive dummy for the inflation targeting period. Table 5 reports the

estimation results of the third model. The impact of the inflation targeting dummy variable in the mean inflation,  $\beta_0^d$ , is negative for all countries but only significant for Australia, Chile and Sweden. Therefore, it can be said that inflation targeting programs reach their goals to some extent; that is, the conditional expectation of inflation falls during the inflation targeting period.

**Table 5:** Estimation Results of the Model 3<sup>6, 7</sup>

| Country      | $\beta_0^d$ | $\alpha_0^d$ | $\alpha_1^d$ | $\alpha_2^d$ |
|--------------|-------------|--------------|--------------|--------------|
| Australia    | -1.236**    | -79.808      | 0.051        | 0.278        |
|              | (0.446)     | (64146.658)  | (2.123)      | (26.502)     |
| Brazil       | -0.019      | -0.168       | 0.634        | -0.022       |
|              | (0.048)     | (0.148)      | (0.669)      | (0.487)      |
| Canada       | -0.033      | 0.081**      | -1.066**     | 0.213        |
|              | (0.028)     | (0.023)      | (0.160)      | (0.176)      |
| Chile        | -0.268**    | -0.318**     | 0.791**      | -0.788**     |
|              | (0.079)     | (0.095)      | (0.114)      | (0.262)      |
| Colombia     | -0.174      | 0.062        | -1.163       | -0.019       |
|              | (0.100)     | (0.061)      | (1.077)      | (0.149)      |
| New Zealand  | -0.143      | -0.297       | -0.147       | 0.041        |
|              | (0.073)     | (0.766)      | (1.452)      | (0.143)      |
| South Africa | -0.103      | 0.028        | -0.283       | 0.181        |
|              | (0.075)     | (0.064)      | (0.431)      | (0.293)      |
| Sweden       | -0.109*     | -0.071       | -0.405       | -0.001       |
|              | (0.043)     | (0.102)      | (0.295)      | (0.324)      |
| UK           | -0.034      | -0.334**     | 0.254        | 0.322        |
|              | (0.036)     | (0.129)      | (0.468)      | (0.307)      |

<sup>6</sup> \*\* and \* indicate 1% and 5% levels of significance, respectively.

<sup>7</sup> Values in parenthesis show standard deviations of the coefficients.

**Table 5** (cont'd)

| Country      | $\alpha_0$    | $\alpha_1$ | $\alpha_2$ | Likelihood |
|--------------|---------------|------------|------------|------------|
| Australia    | 28108.384     | 0.045      | 0.283      | -957.532   |
|              | (2668281.173) | (1.818)    | (26.965)   |            |
| Brazil       | 0.218         | -0.504     | 0.421      | 38.717     |
|              | (0.144)       | (0.553)    | (0.353)    |            |
| Canada       | 0.002         | 0.919**    | 0.071*     | 167.805    |
|              | (0.002)       | (0.033)    | (0.031)    |            |
| Chile        | 0.321**       | 0.139      | 0.821**    | -118.174   |
|              | (0.095)       | (0.104)    | (0.264)    |            |
| Colombia     | 0.002         | 0.958**    | 0.034*     | -390.401   |
|              | (0.002)       | (0.014)    | (0.014)    |            |
| New Zealand  | 0.670         | 0.043      | 0.049      | -265.223   |
|              | (0.642)       | (0.871)    | (0.075)    |            |
| South Africa | 0.003         | 0.939**    | 0.054*     | -156.489   |
|              | (0.003)       | (0.027)    | (0.027)    |            |
| Sweden       | 0.191         | 0.342      | 0.257      | 38.884     |
|              | (0.111)       | (0.278)    | (0.195)    |            |
| UK           | 0.372**       | -0.175     | 0.120      | 33.972     |
|              | (0.130)       | (0.266)    | (0.077)    |            |

When the effect of the additive dummy variable on the conditional variance of inflation,  $\alpha_0^d$ , is analyzed, it is negative for all except Canada, Colombia and South Africa. Importantly for the UK and Chile, the negative effect of the inflation targeting program on the variability of inflation is significant; however, for Canada, the positive effect of inflation targeting program is significant. Hence, the conclusion can be drawn that inflation targeting programs do not have a uniform effect on the variability of inflation.

Not only the level of variance but the persistence of variance might be affected by inflation targeting. If the coefficient of  $\varepsilon_{t-1}^2$  decreases, this suggests that impulse variability decreases. If the coefficient of  $h_{t-1}^2$  decreases, then what we call the memory of shock also decreases. Accordingly, for Canada, Colombia, New Zealand, South Africa and Sweden, the effect of the multiplicative dummy with the lagged value of conditional variance,  $\alpha_1^d$ , is negative, which indicates that the persistence of inflation variability decreases during the inflation targeting period but it is significant only for Canada. However, for the case of Chile, the persistence of inflation volatility during the inflation targeting period is positive and significant.

The other multiplicative dummy used with the lag of squared residual,  $\alpha_2^d$ , has a negative influence on the inflation variability of Brazil, Chile, Colombia and Sweden but is significant only for Chile. This negative coefficient suggests that the impulse variability of inflation decreases during the inflation targeting period in these countries. When the values of the remaining coefficients ( $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$ ) are analyzed, although the non-explosiveness requirement is satisfied for all the countries, the non-negativity requirement is not fulfilled for Brazil and the UK. We did not explore this further since *i.* the added interactive terms were not statistically significant and *ii.* when they were dropped, the conventional restrictions were satisfied (see Table 4). The last column displays the log likelihood of the estimation.

Lower and statistically significant evidence is observed for Chile and the UK for the conditional evidence. This could not be generalized for other inflation targeting countries. Therefore, we claim that lower variability is not observed for all the inflation targeting countries. This is similar to the results of Johnson (2002, 2003) and others.

#### ***4.4. Conclusion***

This paper examines whether inflation targeting programs altered the pattern of inflation and its variability for the five developed countries and the four emerging economies that have implemented inflation targeting programs. All three types of models under consideration revealed that in general inflation targeting programs decrease conditional inflation expectations in both developed and developing countries. That is, sticking to a monetary policy rule changes perceptions of public towards inflation however, this result is not statistically significant for all countries. We find statistically significant lower conditional inflation expectations only for Australia, Chile and Sweden under various specifications. This observation points out that economic state of the country does not make any difference regarding the efficiency of inflation targeting programs in reducing inflationary expectations.

Estimation results of the second model showed that level of conditional variance decreases for both developed and developing countries, indicating that inflation targeting programs are efficient in reducing conditional variability of inflation. In this respect, we can claim that in both developed and developing countries, monetary authorities have some credibility with respect to reducing inflation variability however; this result is statistically significant only for Chile, which is a developing country. If we presume that monetary authorities in developed countries are more credible than those in developing countries then our results does not support this line of thought.

Finally, when we analyze the findings of the third model, we can not reach a general conclusion either for developed countries or for emerging markets with respect to level of conditional variance, persistence of inflation variability and impulse variability of inflation. According to estimation results, it seems that inflation targeting programs reduce inflation variability in some of the developed and developing countries while these programs increase or do not affect inflation variability in remaining developed and developing countries. Moreover, statistical significance can be observed for very few countries. In general, the conditional variance decreased statistically significantly only for Chile, a developing country, and the UK, a developed country. Therefore, the empirical support for the lower inflation and variability for the inflation targeting regimes is limited. Also, we could not reach a differentiation between

developed countries and developing countries with regard to the efficiency of inflation targeting programs in reducing inflation variability.

## **CHAPTER 5**

### **CONCLUSION**

Within years, monetary policies implemented by central banks, their influence over the economy and monetary transmission mechanism have been the research topic of several academicians and policymakers. Under the general tendency of carrying out monetary policy analysis and examining its impact over the economy, different directions/questions/motivations were adopted for various investigations.

In the first essay, a brief literature review was given about the evolution of monetary policy analysis and its future directions in a New Keynesian framework. Recently, most of the studies in monetary policy analysis have relied on the building blocks of the New Keynesian world while constructing their models and performing investigations about monetary policies. Hence, it is necessary to know the developments in monetary policy analysis and to understand the basics of New Keynesian models. Starting from the IS/LM

framework, today monetary policy field has various analysis subjects including changes in the monetary policy rules and monetary transmission mechanisms, implementation of inflation targeting programs, open economy and exchange rate features, euro area extensions. However, these works have some common properties related to developments in New Keynesian models, which are structural relationships derived from microeconomic foundations, rational expectations, monopolistic competition behavior of firms, nominal rigidities, different forms of Phillips' curve, aggregate demand relationship, a monetary policy rule and policy instrument.

Although many studies do not include the derivation of linear reduced form equations from microeconomic foundations and they employ directly the linear relationships that simulates the economy, it is of importance to know/understand how these equations are derived from microfoundations, since micro structure of the economy, behavior of households and firms, nature of connection among different variables and similar aspects are necessary to conduct future research. Therefore, a stylized closed economy was analyzed and reduced form equations, which are Phillips' curve and aggregate demand, were derived from a structural microfounded model.

The second essay analyzed monetary policy introducing interest rate pass-through relationship to a New Keynesian world with backward looking behavior. This study has three important features to mention: The first one is

the inclusion of interest rate pass-through relationship between short-term policy interest rate and long-term market interest rate as a new monetary transmission mechanism, for the first time in the literature, along with a Taylor-type monetary policy rule. The second is the use of time-varying parameter specification not only for coefficients of monetary policy rule, as it is common in the literature, but also for the coefficients of structural relationships in the model, which is not observed before in the literature. The third issue is the application of EKF, which is a new estimation algorithm. The estimation results showed us that long-term interest rate is an important component to be considered in structural models of monetary policy analysis since its influence on aggregate demand is increasing in time and pass-through from policy rate to long-term interest rate has an increasing weight. Apart from this result, it is also worth mentioning that performance of EKF in estimating models with TVPs is quite well. The model adopted in this study is backward-looking so including forward-looking components can be a valuable extension for future study. Furthermore, real-time data applications with different interest rate pass-through and policy rule specifications can be worth exploring. Such models can be modified for open economies so that exchange rate can be incorporated as an additional dynamic to the system.

The third essay investigated the impact of inflation targeting policy on inflation and its variability, empirically. Recently, many countries adopted inflation targeting policies in order to achieve low and stable inflation with maintaining

economic growth. Even though these programs succeed in achieving the targeted inflation levels, there is no common argument about the effect of them on inflation variability. Such an examination is worth to conduct since inflation uncertainty affects macroeconomic variables such as output, investment, interest rate. There are two main contributions of this essay to the inflation variability literature: The first input is the measurement of public perception towards inflation, which has not been made before. The second main contribution is the investigation of whether commitment to such a monetary policy rule really changed the public assessment of inflation and its variability. For this, unlike the existing models, this study used ARCH/GARCH type of conditional inflation variability specification as a measure of inflation variability. Another critical issue to be mentioned is the comparison of performance of inflation targeting programs between developed and developing countries. This comparison has the advantage of assessing whether economic state of the country is an important factor in evaluating performance of the inflation targeting programs in reducing inflation variability. The results of the study pointed out that inflation targeting programs convince public in reducing inflation and its variability both in developed and developing countries, although there are some exceptions and some of the results are not statistically significant. This result revealed that economic state of the economy does not have much influence on the effectiveness of inflation targeting programs in decreasing inflation variability. However, this conclusion can not be generalized to all inflation targeting countries since the conclusion

was not valid for some countries examined. For the countries in which inflation targeting programs are not successful in reducing inflation variability, concerns about the credibility of the monetary policy authority can be the reason of failure.

## BIBLIOGRAPHY

- Arestis, P. and K. Mouratidis. 2004, "Is There a Trade-off Between Inflation Variability and Output-Gap Variability in the EMU Countries?," *Scottish Journal of Political Economy* 51(5): 691-706.
- Arestis, P., G. M. Caporale and A. Cipollini. 2002. "Does Inflation Targeting Affect the Trade-off Between Output Gap and Inflation Variability?," *The Manchester School* 70(4): 528-545.
- Bacchetta, P. and S. Gerlach. 1997. "Consumption and Credit Constraints: International Evidence," *Journal of Monetary Economics* 40(2): 207-238.
- Backus, David and John Driffill. 1985. "Rational Expectations and Policy Credibility Following a Change in Regime," *Review of Economic Studies* 52(2): 211-221.
- Bahmani, Osokee and Ford Brown. 2004. "Kalman Filter Approach to Estimate the Demand for International Reserves," *Applied Economics* 36(15): 1655-1668.
- Ball, Laurence. 1997. "Efficient Rules for Monetary Policy," *NBER Working Papers* 5952.
- Ball, Laurence. 1999. "Policy Rules for Open Economies," in John B. Taylor, ed., *Monetary Policy Rules*. NBER Conference Report Series, University of Chicago Press, 127-144.

- Ball, Laurence. 2000. "Policy Rules and External Shocks," *NBER Working Papers* 7910.
- Ball, L., N. G. Mankiw and D. Romer. 1988. "The New Keynesian Economics and the Output-Inflation Trade-off," *Brookings Papers on Economic Activity* 1: 1-65.
- Banerjee, A, R. L. Lumsdaine and J. H. Stock. 1992. "Recursive and Sequential Tests of the Unit Root and Trend-Break Hypotheses: Theory and International Evidence," *Journal of Business & Economic Statistics* 10(3): 271-287.
- Barro, Robert J. and David B. Gordon. 1983a. "Rules, Discretion and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics* 12(1): 101-121.
- Barro, Robert J. and David B. Gordon. 1983b. "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy* 91(4): 589-610.
- Basci, E., O. Ozel and C. Sarikaya. 2007. "The Monetary Transmission Mechanism in Turkey: New Developments," *Central Bank of the Republic of Turkey, Research and Monetary Policy Department Working Paper* 07/04.
- Bernanke, Ben S. and Alan S. Blinder. 1992. "The Federal Funds Rate and the Channels of Monetary Transmission," *American Economic Review* 82(4): 901-921.
- Bernanke, Ben S. and Frederic S. Mishkin. 1997. "Inflation Targeting: A New Framework for Monetary Policy?," *Journal of Economic Perspectives* 11(2): 97-116.
- Bernanke, Ben S. and Ilian Mihov. 1997. "What Does the Bundesbank Target?," *European Economic Review* 41(6): 1025-1053.

- Bernanke, B. S., M. Gertler and M. Watson. 1997. "Systematic Monetary Policy and the Effects of Oil Price Shocks," *Brookings Papers on Economic Activity* 1: 91-157.
- Bernanke, B. S., T. Laubach, F. S. Mishkin and A. Posen. 1999. *Inflation Targeting: Lessons From The International Experience*. Princeton, NJ: Princeton University Press.
- Berument, H. 1999. "The Impact of Inflation Uncertainty on the Interest Rates in the UK," *Scottish Journal of Political Economy* 46(2): 207-218.
- Berument, H. and N. Dincer. 2005. "Inflation and Inflation Uncertainty in the G-7 Countries," *Physica A* 348: 371-379.
- Berument, H., Z. Kilinc, U. Ozlale. 2005. "The Missing Link Between Inflation Uncertainty and Interest Rates," *Scottish Journal of Political Economy* 52(2): 222-241.
- Berument, H. and H. Kiymaz. 2001. "The Day of the Week Effect on Stock Market Volatility," *Journal of Economics and Finance* 25(2): 181-193.
- Beveridge, S. and C. R. Nelson. 1981. "A New Approach to the Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle," *Journal of Monetary Economics* 7(2): 151-174.
- Blanchard, Olivier Jean and Nobuhiro Kiyotaki. 1987. "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review* 77(4): 647-666.
- Blanchard, O. J. and S. Fischer. 1989. *Lectures on Macroeconomics*. Cambridge, MA.: MIT Press.
- Boivin, Jean. 2006. "Has U.S. Monetary Policy Changed? Evidence from Drifting Coefficients and Real-Time Data," *Journal of Money, Credit and Banking* 38(5): 1149-1173.

- Bollerslev, T., R. Chou, and K. Kroner. 1991. "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics* 52: 5-59.
- Bomberger T. 1996. "Disagreement as a Measure of Uncertainty," *Journal of Money, Credit and Banking* 28(3): 381-392.
- Calvo, Guillermo A. 1983. "Staggered Prices in Utility-Maximizing Framework," *Journal of Monetary Economics* 12(3): 383-398.
- Canova, Fabio and Luca Gambetti. 2004. "Structural Changes in the US Economy: Bad Luck or Bad Policy?," *C.E.P.R. Discussion Papers* 5457.
- Cecchetti, Stephen G. 1995. "Inflation Indicators and Inflation Policy," *NBER/Macroeconomics Annual* 10(1): 189-219.
- Cecchetti S. G. 1997. "Central Bank Policy Rules: Conceptual Issues and Practical Considerations," *NBER Working Papers* 6306.
- Cecchetti, S. G. and M. Ehrmann. 1999. "Does Inflation Targeting Increase Output Volatility? An International Comparison of Policymakers' Preferences and Outcomes," *NBER Working Papers* 7426.
- Cheung, Yin-Wong. 1993. "Exchange Rate Risk Premiums," *Journal of International Money and Finance* 12: 182-194.
- Christiano, L. J., M. Eichenbaum and C. Evans. 1996. "The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds," *Review of Economics and Statistics* 78(1): 16-34.
- Christiano, L. J., M. Eichenbaum and C. Evans. 1998. "Monetary Policy Shocks: What Have We Learned and To What End?," *NBER Working Papers* 6400.

- Chui, C. K. and G. Chen. 1991. *Kalman Filtering with Real-Time Applications*. NY: Springer Verlag.
- Clarida, R., J. Gali and M. Gertler. 1997. "Monetary Policy Rules in Practice: Some International Evidence," *NBER Working Papers* 6254.
- Clarida, R., J. Gali and M. Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature* 37(4): 1661-1707.
- Clarida, R., J. Gali and M. Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115(1): 147-180.
- Cogley, Timothy and Thomas J. Sargent. 2001. "Evolving Post-World War II U.S. Inflation Dynamics," *NBER/Macroeconomics Annual* 16(1): 331-373.
- Coricelli, F., B. Egert and R. MacDonald. 2006. "Monetary Transmission Mechanism in Central & Eastern Europe: Gliding on a Wind of Change," *The William Davidson Institute at the University of Michigan Working Paper* 850.
- Crespo-Cuaresma, J., B. Egert and T. Reininger. 2004. "Interest Rate Pass-Through in New EU Member States: The Case of the Czech Republic, Hungary and Poland," *The William Davidson Institute at the University of Michigan Working Paper* 671.
- Dittmar, R., W. T. Gavin and F. E. Kydland. 1999. "Price-Level Uncertainty and Inflation Targeting," *Federal Reserve Bank of St. Louis Review* 81(4): 23-33.
- Dixit, Avinash K. and Joseph E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity," *American Economic Review* 67(3): 297-308.

- Dwyer Jr., Gerald P. 1993. "Rules and Discretion in Monetary Policy," *Review Federal Reserve Bank of St. Louis*, 75(3): 3-13.
- Elkhoury, Marwan. 2006. "A Time-Varying Parameter Model of A Monetary Policy Rule for Switzerland. The Case of the Lucas and Friedman Hypothesis," *HEI Working Papers* 01-2006.
- Engle, R. 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica* 50: 987-1007.
- Evans, M. 1991. "Discovering the Links Between Inflation Rates and Inflation Uncertainty," *Journal of Money, Credit and Banking* 23(2): 169-184.
- Fischer, Stanley. 1977. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," *Journal of Political Economy* 85(1): 191-205.
- Friedman, Milton. 1968. "The Role of Monetary Policy," *American Economic Review* 58(1): 1-17.
- Friedman, Milton. 1970. "A Theoretical Framework for Monetary Analysis," *Journal of Political Economy* 78(2): 193-238.
- Friedman, Milton. 1971. "A Monetary Theory of Nominal Income," *Journal of Political Economy* 79(2): 323-337.
- Friedman, Milton. 1977. "Nobel Lecture: Inflation and Unemployment," *Journal of Political Economy* 85: 451-472.
- Friedman, M. and A. Schwartz. 1963. *A Monetary History of the United States, 1867-1960* Princeton: Princeton University Press.
- Friedman, M. and Kenneth N. Kuttner. 1992. "Money, Income, Prices and Interest Rates," *American Economic Review* 82(3): 472-492.

- Froyen, R. and R. Waud. 1987. "An Examination of Aggregate Price Uncertainty in Four Countries and Some Implications for Real Output," *International Economic Review* 28(2): 353–373.
- Gali, Jordi. 1992. "How Well Does the IS/LM Model Fit Post-War U.S. Data?," *Quarterly Journal of Economics* 92: 709-738.
- Gavin, W.T. 2003. "Inflation Targeting: Why It Works and How To Make It Work Better," *Federal Reserve Bank of St. Louis Working Paper* 2003-027B.
- Goodfriend, Marvin and Robert G. King. 1997. "The New Neoclassical Synthesis and the Role of Monetary Policy," *NBER/Macroeconomics Annual* 12(1): 231-283.
- Goodhart, C. and B. Hofmann. 2000. "Financial Variables and the Conduct of Monetary Policy," *Sveriges Riksbank Working Paper* 112.
- Gordon, Robert J. 1982. "Price Inertia and Policy Ineffectiveness in the United States, 1890-1980," *Journal of Political Economy* 90(6): 1087-1117.
- Green, John H. 1996. "Inflation Targeting: Theory and Policy Implications," *International Monetary Fund Staff Papers* 43(4): 779-795.
- Grier, K. B. and M. J. Perry. 1998. "On Inflation and Inflation Uncertainty in the G- 7 Countries," *Journal of International Money and Finance* 17(4): 671-689.
- Grier, K. B. and M. J. Perry. 2000. "The Effects of Real and Nominal Uncertainty on Inflation and Output Growth: Some GARCH-M Evidence," *Journal of Applied Econometrics* 15: 45-58.
- Groenvelde, J. M. 1998. *Inflation Patterns and Monetary Policy: Lessons For The European Central Bank*. Cheltenham: Edward Elgar Press.

- Hafet, R. W. 1986. "Inflation Uncertainty and a Test of the Friedman Hypothesis," *Journal of Macroeconomics* 8(3): 365–372.
- Harvey, A. C. 1990. *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge: Cambridge University Press.
- Henderson, Dale W. and Warwick J. McKibbin. 1993. "A Comparison of Some Basic Monetary Policy Regimes for Open Economies: Implications of Different Degrees of Instrument Adjustment and Wage Persistence," *Carnegie-Rochester Conference Series on Public Policy* 39: 221-317.
- Holland, S. 1986. "Wage Indexation and the Effect of Inflation Uncertainty on Employment: An Empirical Analysis," *American Economic Review* 76(1): 235–243.
- Holland, S. 1988. "Indexation and the Effect of Inflation Uncertainty on Real GNP," *Journal of Business* 61(4): 473–484.
- Horvath, Roman. 2006. "Real-Time Time-Varying Equilibrium Interest Rates: Evidence on the Czech Republic," *MPRA Paper* 845.
- IMF. 2007. *World Economic Outlook*. Washington D.C.: International Monetary Fund, Publication Services.
- Ireland, Peter N. 1997. "A Small, Structural, Quarterly Model for Monetary Policy Evaluation," *Carnegie-Rochester Conference Series on Public Policy* 47: 83-108.
- Jalil, Munir A. 2004. "Monetary Policy in Retrospective: A Taylor Rule Inspired Exercise," Manuscript. University of California, San Diego.
- Jansen, D. W. and T. F. Cosimona. 1988. "Estimates of the Variance of US Inflation Based Upon the ARCH Model: Comment (in notes, comments, replies)," *Journal of Money, Credit and Banking* 20(3): 409–421.

- Johnson, D. R. 2002 "The Effect of Inflation Targeting on The Behaviour of Expected Inflation: Evidence From An 11 Country Panel," *Journal of Monetary Economics* 49(8): 1521-1538.
- Johnson, D. R. 2003. "The Effect of Inflation Targets on the Level of Expected Inflation in Five Countries," *Review of Economics and Statistics* 85(4): 1076-1081.
- Judd, John P. and Glenn D. Rudebusch. 1998. "Taylor's Rule and the Fed: 1970-1997," *Federal Reserve Bank of San Francisco Economic Review* 3: 3-16.
- Kalman, R. E. 1960. "A New Approach to Linear Filtering and Prediction Problems," *Journal of Basic Engineering* 35-45.
- Karolyi, A. 1995. "A Multivariate GARCH Model of International Transmission of Stock Returns and Volatility: The Case of the United States and Canada," *Journal of Business and Statistics* 13: 11-25.
- Keynes, J. M. 1936. *The General Theory of Employment, Interest and Money*. London: McMillan.
- Kim, Chang-Jin and Charles R. Nelson. 2006. "Estimation of a Forward-Looking Monetary Policy Rule: A Time-Varying Parameter Model Using Ex-Post Data," *Journal of Monetary Economics* 53(8): 1949-1966.
- Kiyamaz, H. and H. Berument. 2003. "The Day of the Week Effect on Stock Market Volatility and Volume: International Evidence," *Review of Financial Economics* 12: 363-380.
- Kontonikas, A. 2004. "Inflation and Inflation Uncertainty in the United Kingdom: Evidence from GARCH Modelling," *Economic Modelling* 21(3): 525-543.
- Kozicki, Sharon. 1999. "How Useful Are Taylor Rules for Monetary Policy?," *Economic Review* 84(2): 5-33.

- Kuzin, Vladimir. 2006. "The Inflation Aversion of the Bundesbank: A State Space Approach," *Journal of Economic Dynamics and Control* 30(9-10): 1671-1686.
- Kwapil, Claudia and Johann Scharler. 2006. "Interest Rate Pass-Through, Monetary Policy Rules and Macroeconomic Stability," *Austrian Central Bank Working Papers* 118.
- Kydland, Finn E. and Edward C. Prescott. 1977. "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy* 85(3): 473-491.
- Kydland, Finn E. and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations," *Econometrica* 50(6): 1345-1370.
- Leitemo, Kai and Ingunn Lønning. 2006. "Simple Monetary Policymaking without the Output Gap," *Journal of Money, Credit and Banking* 38(6): 1619-1640.
- Levin, A. T. and J. M. Piger. 2003. "Is Inflation Persistence Intrinsic in Industrial Economies?," *The Federal Reserve Bank of St. Louis Working Paper Series* 2002-023E.
- Levin, A., V. Wieland and J. C. Williams. 1998. "Robustness of Simple Monetary Policy Rules under Model Uncertainty," *NBER Working Papers* 6570.
- Ljungqvist, L. and T. Sargent. 2000. *Recursive Macroeconomic Theory*. Cambridge and London: MIT Press.
- Lucas, R. 1972. "Expectations and the Neutrality of Money," *Journal of Economic Theory* 4: 103-124.
- Lucas, R. 1976. "Econometric Policy Evaluation: A Critique," *Journal of Monetary Economics* supplement: 19-46.

- Mandler, Martin. 2007. "The Taylor Rule and Interest Rate Uncertainty in the U.S. 1955-2006," *MPRA Paper* 2340.
- Mankiw, N. G. 1985. "Small Menu Costs and Large Business Cycles: A Macroeconomic Model," *Quarterly Journal of Economics* 100(2): 529-538.
- Mankiw, N. G. 1988. "Recent Developments in Macroeconomics: A Very Quick Refresher Course," *Journal of Money, Credit and Banking* 20(3): 436-449.
- Mankiw, N. G. 1990. "A Quick Refresher Course in Macroeconomics," *Journal of Economic Literature* 28(4): 1645-1660.
- McCallum, Bennett T. 1997. "Issues in the Design of Monetary Policy Rules," *NBER Working Papers* 6016.
- McCallum, Bennett T. 2000. "Alternative Monetary Policy Rules: A Comparison with Historical Settings for the United States, the United Kingdom, and Japan," *Economic Quarterly* Federal Reserve Bank of Richmond 86(1): 49-79.
- McCallum, Bennett T. and Edward Nelson. 1999a. "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis," *Journal of Money, Credit and Banking* 31(3): 296-316.
- McCallum, Bennett T. and Edward Nelson. 1999b. "Nominal Income Targeting in an Open-Economy Optimizing Model," *Journal of Monetary Economics* 43(3): 553-578.
- McCallum, Bennett T. and Edward Nelson. 1999c. "Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model," in John B. Taylor, ed., *Monetary Policy Rules*. NBER Conference Report Series, University of Chicago Press, 15-45.
- McCulloch, J. Huston. 2005. "The Kalman Foundations of Adaptive Least Squares," *Ohio State University Working Paper*.

- McCulloch, J. Huston. 2007. "Adaptive Least Squares Estimation of the Time-Varying Taylor Rule," *Ohio State University Working Paper*.
- Mesonnier, Jean-Stephane and Jean-Paul Renne. 2007. "Does Uncertainty Make a Time-Varying Natural Rate of Interest Irrelevant For the Conduct of Monetary Policy?," *Banque de France Working Paper* 175.
- Mishkin, Frederic S. and Adam S. Posen. 1997. "Inflation Targeting: Lessons From Four Countries," *Economic Policy Review* 3(3): 9-110.
- Muth, John F. 1961. "Rational Expectations and the Theory of Price Movements," *Econometrica* 29(3): 315-335.
- Orphanides, Athanasios. 1998. "Monetary Policy Rules Based on Real-Time Data," *Finance and Economics Discussion Series* Board of Governors of the Federal Reserve System, 1998-03.
- Orphanides, Athanasios. 2001. "Monetary Policy Rules Based on Real-Time Data," *American Economic Review*, 91(4): 964-985.
- Orphanides, Athanasios. 2004. "Monetary Policy Rules, Macroeconomic Stability, and Inflation: A View from the Trenches," *Journal of Money, Credit and Banking* 36(2): 151-175.
- Orphanides, Athanasios and John C. Williams. 2005. "The Decline of Activist Stabilization Policy: Natural Rate Misperceptions, Learning, and Expectations," *Journal of Economic Dynamics and Control* 29(11): 1927-1950.
- Ozbek, L. and M. Efe. 2001. "Online Estimation of State and Parameters in Compartmental Models Using Extended Kalman Filter." in W. Sulis and I. Trofimova, eds., *NATO Science Series, Nonlinear Dynamics in Life and Social Sciences*. IOS Press, 320: 262-271.
- Ozbek, L. and M. Efe. 2004. "An Adaptive Extended Kalman Filter with Application to Compartment Models," *Communication in Statistics-Simulation and Computation* 33(1): 145-158.

- Ozbek, L., U. Ozlale and M. Efe. 2003. "Employing Extended Kalman Filter in a Simple Macroeconomic Model," *Central Bank Review* 3(1): 53-65.
- Partouche, Harry. 2007. "Time-Varying Coefficients in a GMM Framework: Estimation of a Forward Looking Taylor Rule for the Federal Reserve," *Banque de France Working Paper* 177.
- Pasricha, Gurnain Kaur. 2006. "Kalman Filter and Its Economic Applications." Unpublished. Santa Cruz, CA: University of California, Santa Cruz.
- Peersman, G. and F. Smets. 1999. "The Taylor Rule: A Useful Monetary Policy Benchmark for the Euro Area?," *International Finance* 1: 85-116.
- Plantier, L. Christopher and Dean Scrimgeour. 2002. "Estimating a Taylor Rule for New Zealand with a Time-Varying Neutral Real Rate," *Reserve Bank of New Zealand Discussion Paper Series* DP2002/06.
- Ribeiro, Maria Isabel. 2004. "Kalman and Extended Kalman Filters: Concept, Derivation and Properties," Unpublished. Lisboa: Institute for Systems and Robotics, Lisboa.
- Rotemberg, Julio J. 1982. "Sticky Prices in the United States," *Journal of Political Economy* 90(6): 1187-1211.
- Rotemberg, Julio J. 1987. "The New Keynesian Microfoundations," *NBER/Macroeconomics Annual* 69-104.
- Rotemberg, Julio J. and Michael Woodford. 1998. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version," *NBER Technical Working Paper* 233.
- Rotemberg Julio J. and Michael Woodford. 1999. "Interest-Rate Rules in an Estimated Sticky-Price Model," in John B. Taylor, ed., *Monetary Policy Rules*. NBER Conference Report Series, University of Chicago Press, 57-126.

- Rudebusch, Glenn D. and Lars E. O. Svensson. 1998. "Policy Rules for Inflation Targeting," *NBER Working Papers* 6512.
- Rudebusch, G. D. and L. E. O. Svensson. 1999. "Policy Rules for Inflation Targeting," in John B. Taylor, ed., *Monetary Policy Rules*. NBER Conference Report Series, University of Chicago Press, 203-253.
- Rudebusch, Glenn D. and Lars E. O. Svensson. 2002. "Eurosystem Monetary Targeting: Lessons from U.S. Data," *European Economic Review* 46(3): 417-442.
- Sargent, Thomas J. and Neil Wallace. 1975. "'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy* 83(2): 241-244.
- Schmitt-Grohe, Stephanie and Martin Uribe. 2002. "Optimal Fiscal and Monetary Policy Under Sticky Prices," *NBER Working Papers* 9220.
- Sims, Christopher A. 1980. "Comparison of Interwar and Postwar Business Cycles: Monetarism Reconsidered," *American Economic Review* 70(2): 250-257.
- Sims, Christopher A. and Tao Zha. 2006. "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review* 96 (1): 54-81.
- Smith, Bruce D. 1994. "Efficiency and Determinacy of Equilibrium Under Inflation Targeting," *Economic Theory* 4(3): 327-344.
- Sørensen, Christoffer K. and Thomas Werner. 2006. "Bank Interest Rate Pass-Through in the Euro Area: A Cross Country Comparison," *European Central Bank Working Paper Series* 580.
- Stock, J. H. and M. W. Watson. 2003. "Forecasting Output and Inflation: The Role of Asset Prices," *Journal of Economic Literature* 41(3): 788-829.

- Svensson, Lars E. 1995. "The Swedish Experience of an Inflation Target," *C.E.P.R. Discussion Papers* 1103.
- Svensson, Lars E. 1996. "Price Level Targeting vs. Inflation Targeting: A Free Lunch?," *NBER Working Papers* 5719.
- Taylor, John B. 1980. "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy* 88(1): 1-23.
- Taylor, John B. 1993. "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39: 195-214.
- Taylor, John B. 1995. "The Monetary Transmission Mechanism: An Empirical Framework," *Journal of Economic Perspectives*, 9(4): 11-26.
- Taylor, John B. 1996. "How Should Monetary Policy Respond to Shocks While Maintaining Long-Run Price Stability?--Conceptual Issues," *Achieving Price Stability* A symposium sponsored by the Federal Reserve Bank of Kansas City 181-195.
- Taylor, John B. 1999a. "A Historical Analysis of Monetary Policy Rules," in John B. Taylor, ed., *Monetary Policy Rules*. NBER Conference Report Series, University of Chicago Press, 319-341.
- Taylor, John B. (ed.) 1999b. *Monetary Policy Rules*. NBER Conference Report Series Chicago and London: University of Chicago Press.
- Trecroci, Carmine and Matilde Vassalli. 2006. "Monetary Policy Regime Shifts: New Evidence from Time-Varying Interest Rate Rules," *University of Brescia, Department of Economics Discussion Paper* 0602.
- Trehan, Bharat and Tao Wu. 2007. "Time-Varying Equilibrium Real Rates and Monetary Policy Analysis," *Journal of Economic Dynamics and Control* 31(5): 1584-1609.

- Uhlig, Harald. 1999. "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily," in *Computational Methods for the Study of Dynamic Economies*. Oxford University Press, 30-61.
- Walsh, Carl E. 2003. *Monetary Theory and Policy* (2nd ed.). Cambridge and London: MIT Press.
- Welch, Greg and Gary Bishop. 2006. "An Introduction to Kalman Filter." Unpublished. Chapel Hill, NC: Department of Computer Science University of North Carolina, Chapel Hill.
- Wesche, Katrin. 2003. "Monetary Policy in Europe: Evidence from Time-Varying Taylor Rules," *Bonn Econ Discussion Papers* bgse21\_2003.
- Woodford, Michael. 1999. "Optimal Monetary Policy Inertia," *NBER Working Papers* 7261.
- Woodford, Michael. 2003. *Interest and Prices*. Princeton and Oxford: Princeton University Press.

## **APPENDICES**

## APPENDIX A

For a variable  $A_t$ ,  $\hat{a}_t$  denotes percentage deviation of the variable around steady state and superscript  $f$  represents flexible price equilibrium. Let  $A^{ss}$  be the steady state value of the variable  $A_t$  and  $k \geq 1$ , then we can write the following approximations, which are used to obtain approximations of first order conditions around a steady state (Uhlig, 1999):

$$A_t = A^{ss} e^{\hat{a}_t} = A^{ss} (1 + \hat{a}_t)$$

$$A_t^k = (A^{ss})^k (e^{\hat{a}_t})^k = (A^{ss})^k (1 + \hat{a}_t)^k = (A^{ss})^k (e^{k\hat{a}_t}) = (A^{ss})^k (1 + k\hat{a}_t)$$

When we introduce a second variable  $B_t = B^{ss} e^{\hat{b}_t} = B^{ss} (1 + \hat{b}_t)$  then, for  $l \geq 1$ ,

we can write

$$\begin{aligned} A_t^k B_t^l &= (A^{ss})^k (e^{\hat{a}_t})^k (B^{ss})^l (e^{\hat{b}_t})^l = (A^{ss})^k (1 + \hat{a}_t)^k (B^{ss})^l (1 + \hat{b}_t)^l \\ &= (A^{ss})^k (e^{k\hat{a}_t}) (B^{ss})^l (e^{l\hat{b}_t}) = (A^{ss})^k (1 + k\hat{a}_t) (B^{ss})^l (1 + l\hat{b}_t) \\ &= (A^{ss})^k (B^{ss})^l (e^{k\hat{a}_t + l\hat{b}_t}) = (A^{ss})^k (B^{ss})^l (1 + k\hat{a}_t + l\hat{b}_t) \end{aligned}$$

## APPENDIX B

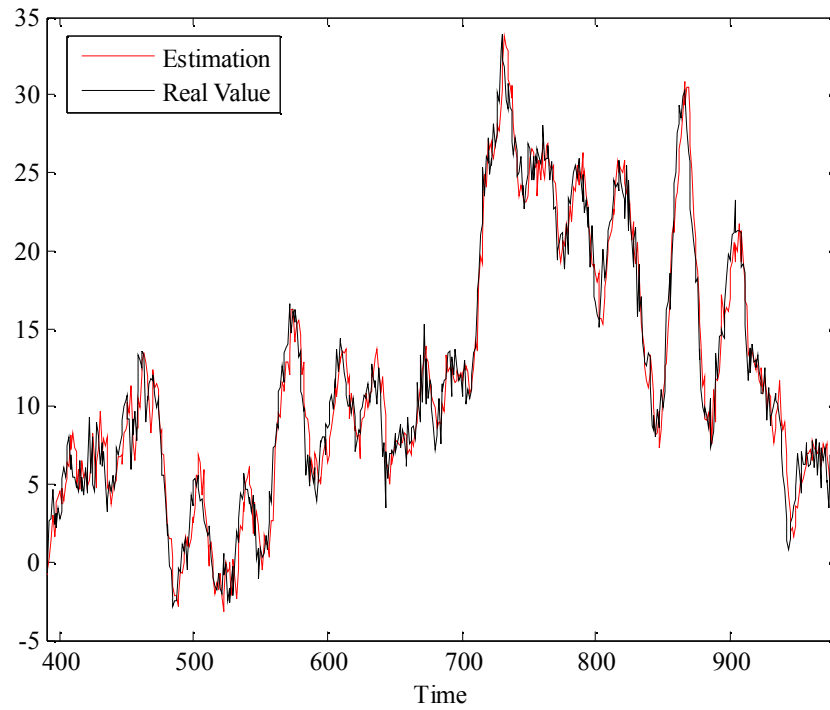


**Figure B1:** Inflation Series of the Baseline Model

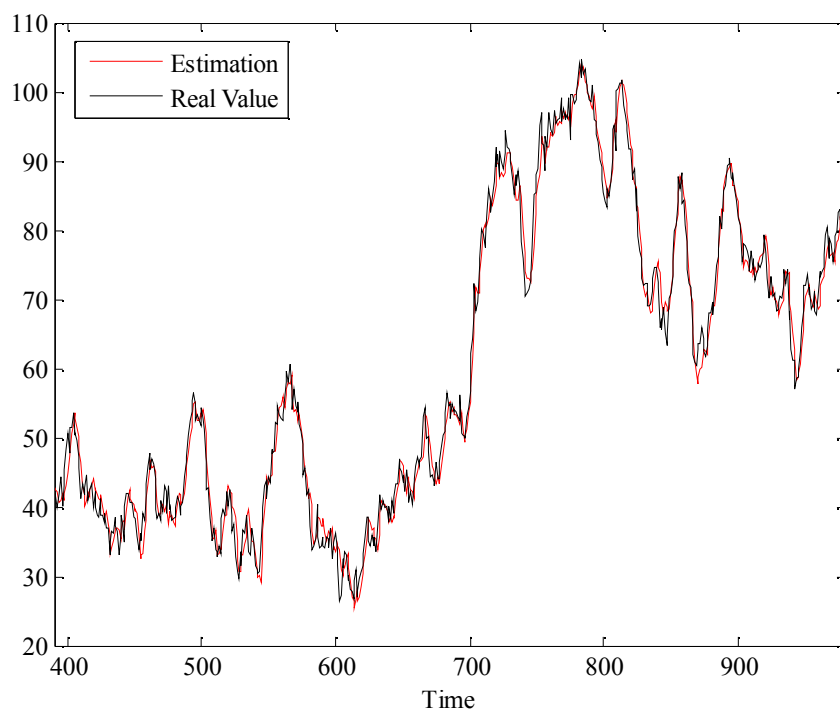


**Figure B2:** Output Series of the Baseline Model

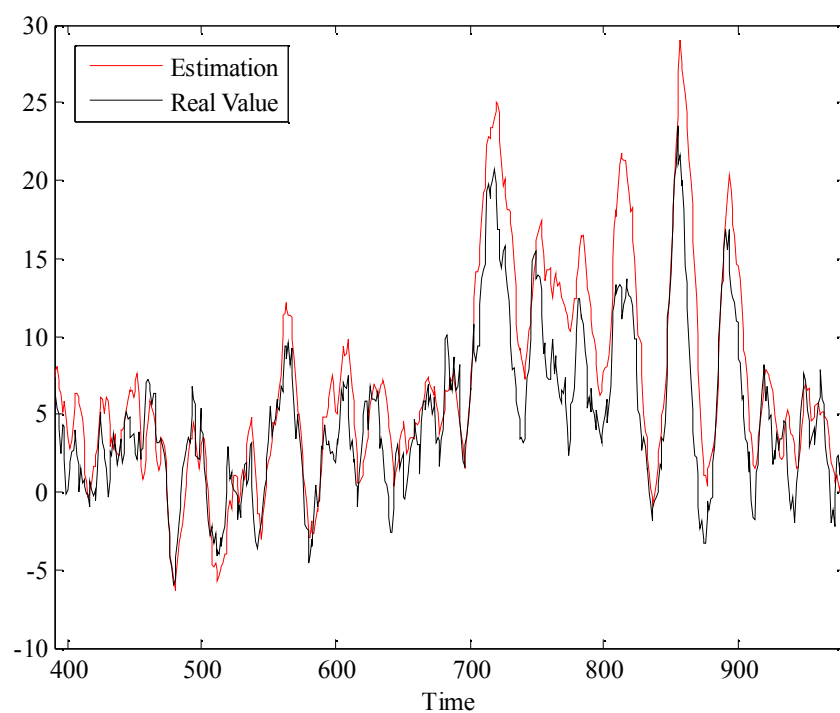


**Figure B3:** Output Gap Series of the Baseline Model

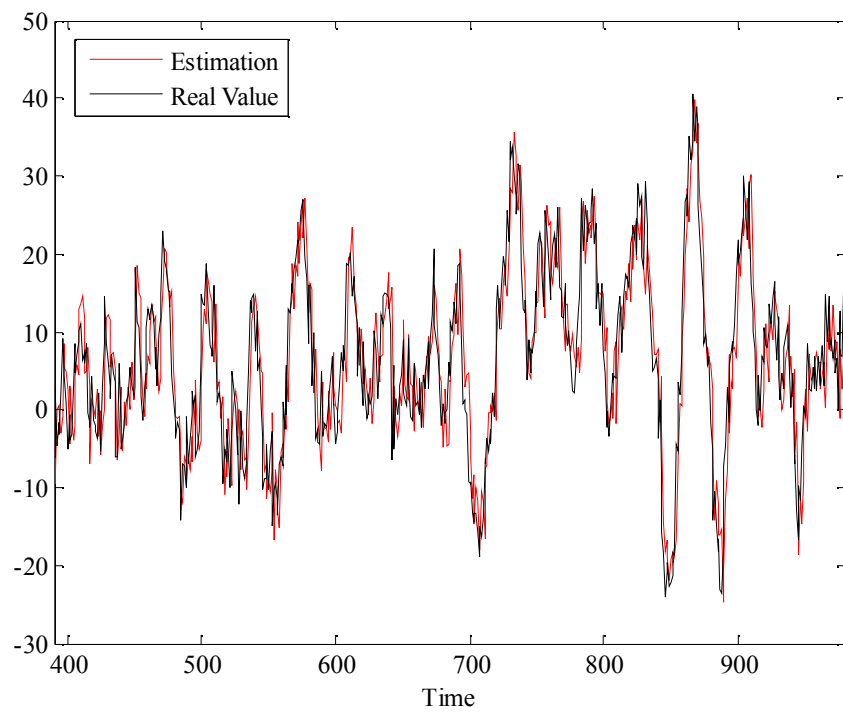


**Figure B4:** Policy Interest Rate Series of the Baseline Model

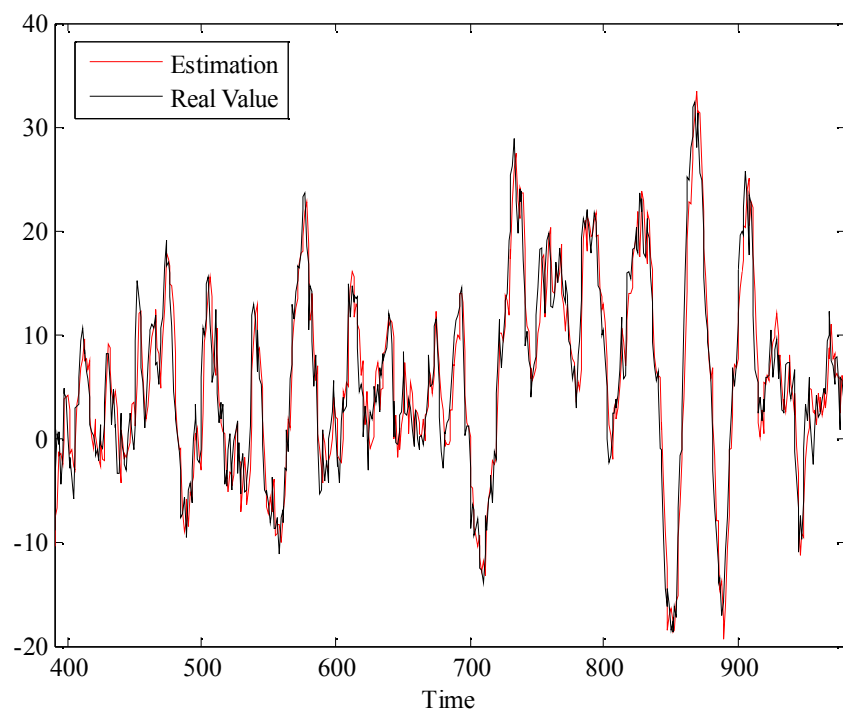


**Figure B5:** Long-term Interest Rate Series of the Baseline Model

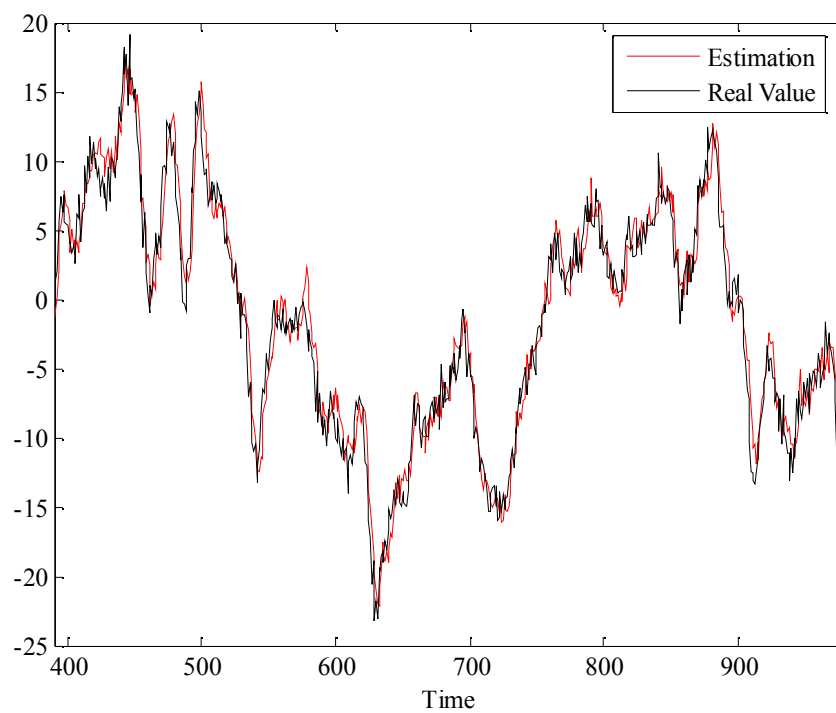


**Figure B6:** Inflation Series of the Alternative Model

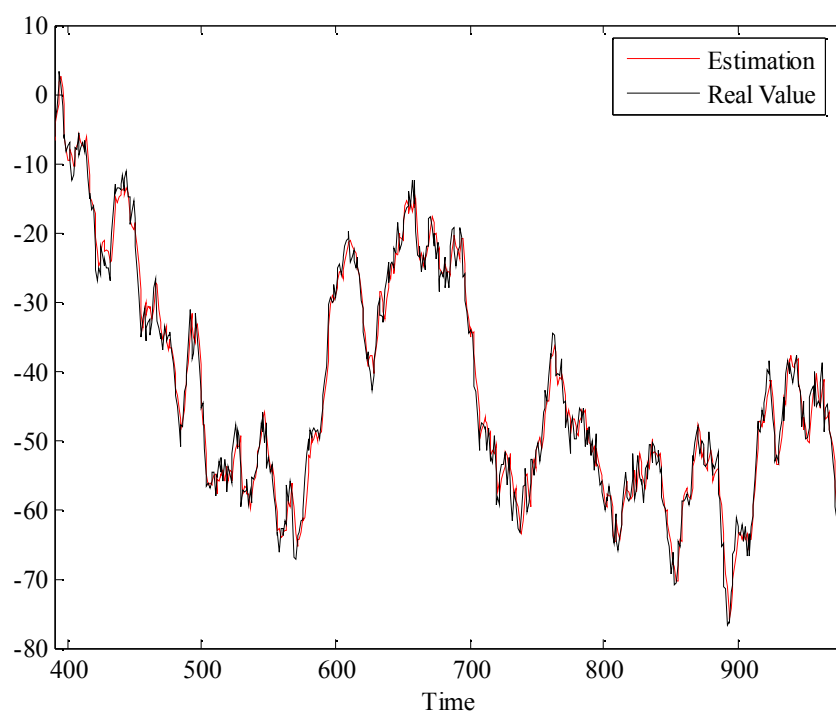


**Figure B7:** Output Series of the Alternative Model

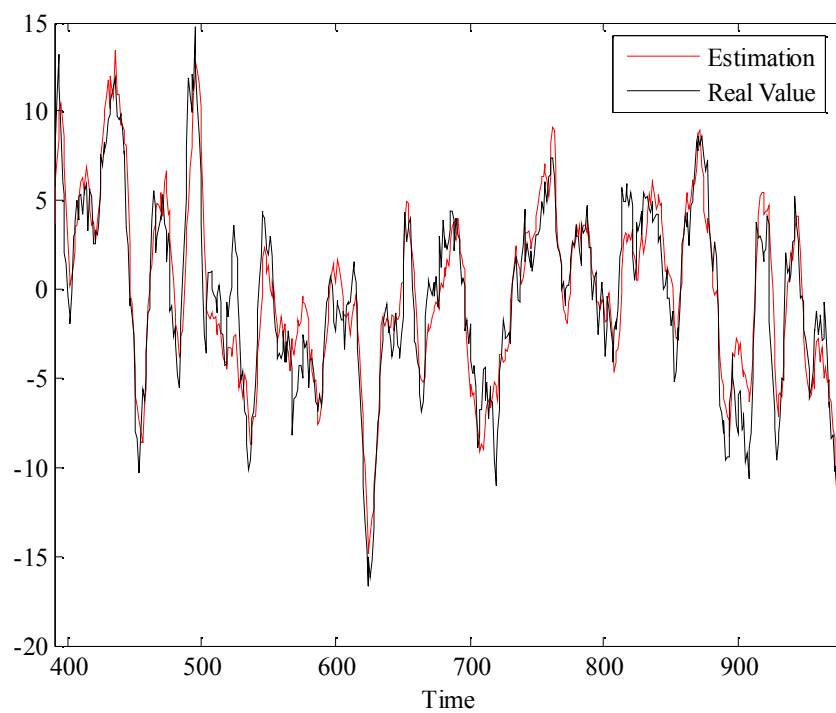


**Figure B8:** Output Gap Series of the Alternative Model

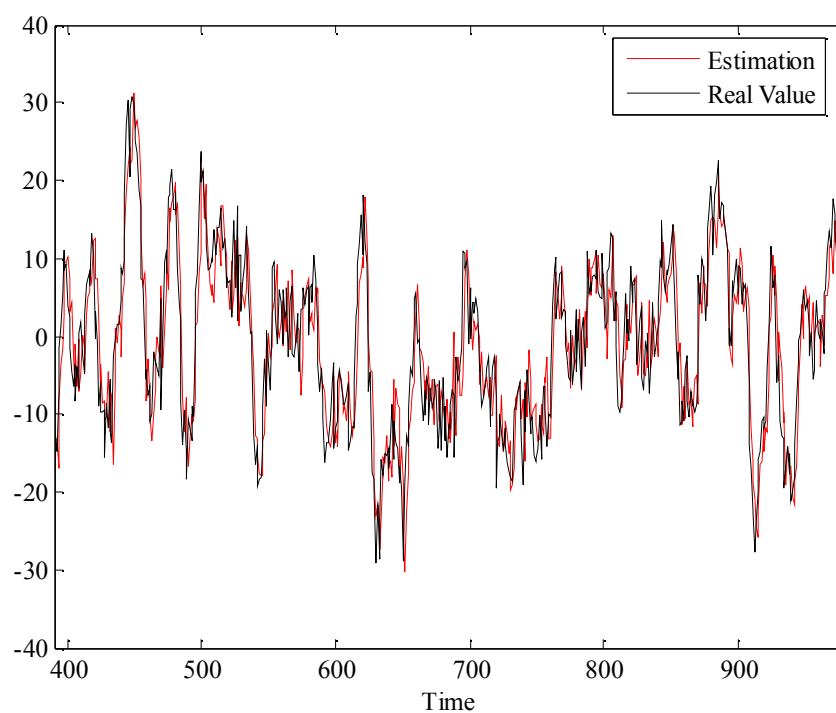


**Figure B9:** Policy Interest Rate Series of the Alternative Model

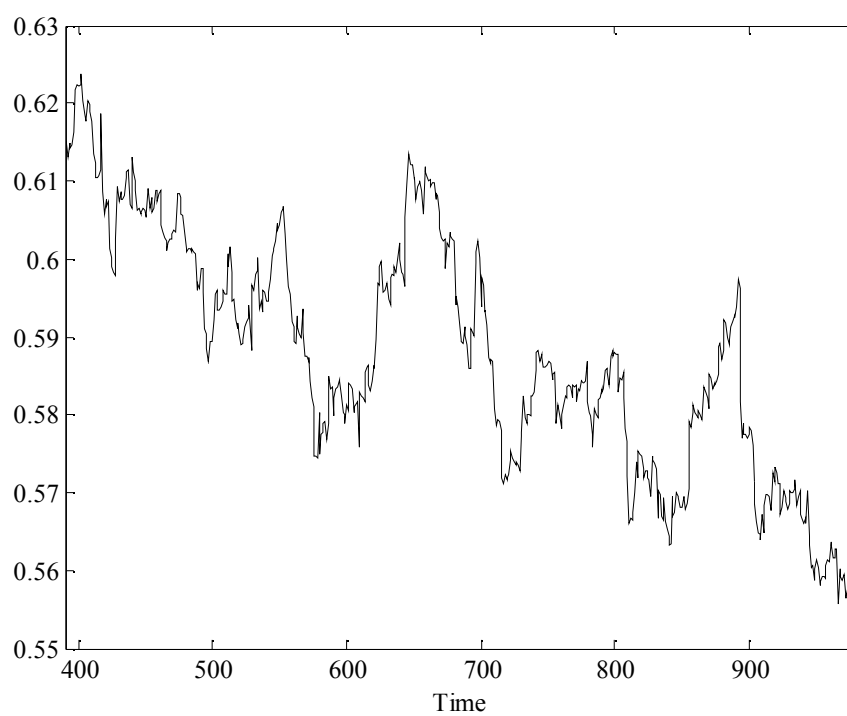


**Figure B10:** Monetary Policy Rule Parameter  $\gamma_1$  of the Baseline Model

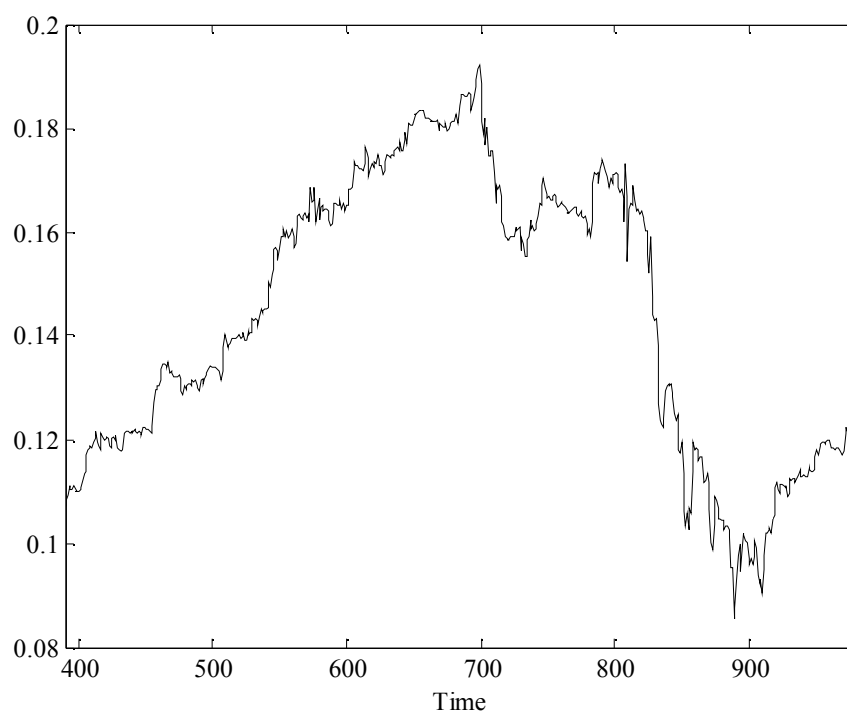


**Figure B11:** Monetary Policy Rule Parameter  $\gamma_2$  of the Baseline Model

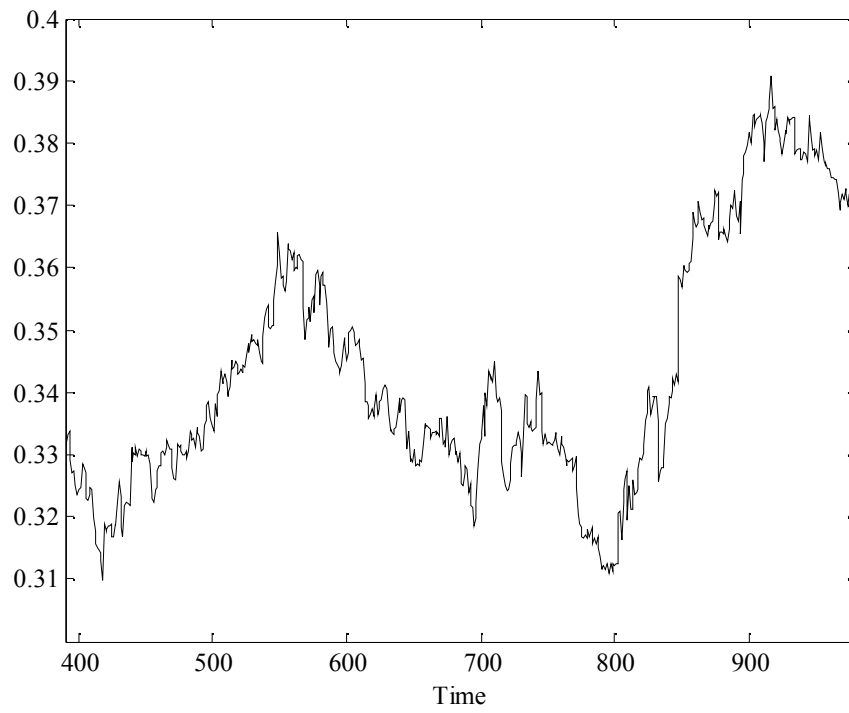


**Figure B12:** Monetary Policy Rule Parameter  $\gamma_3$  of the Baseline Model

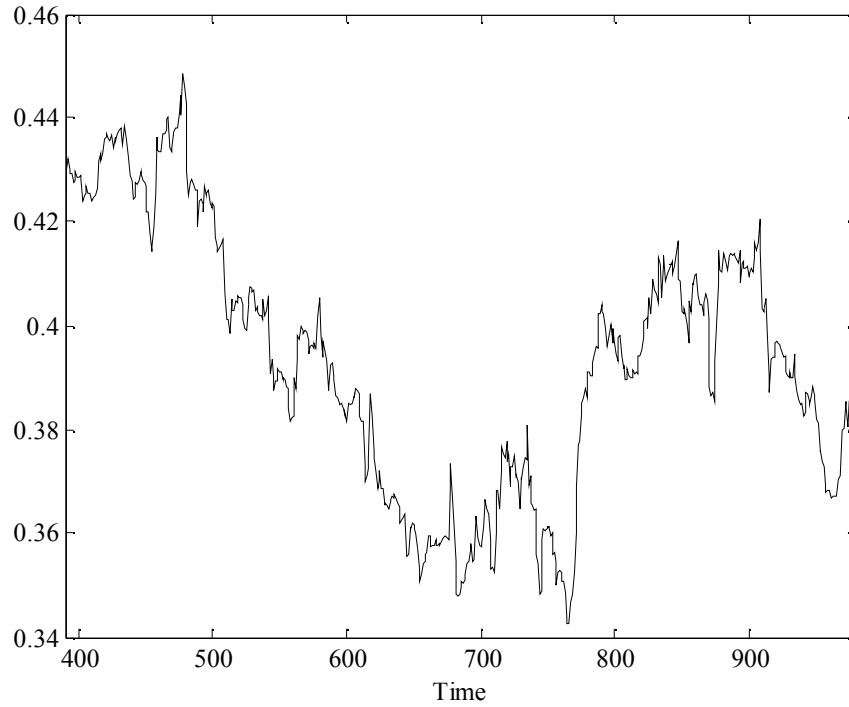


**Figure B13:** Interest Rate Pass-Through Parameter  $\lambda_1$  of the Baseline Model

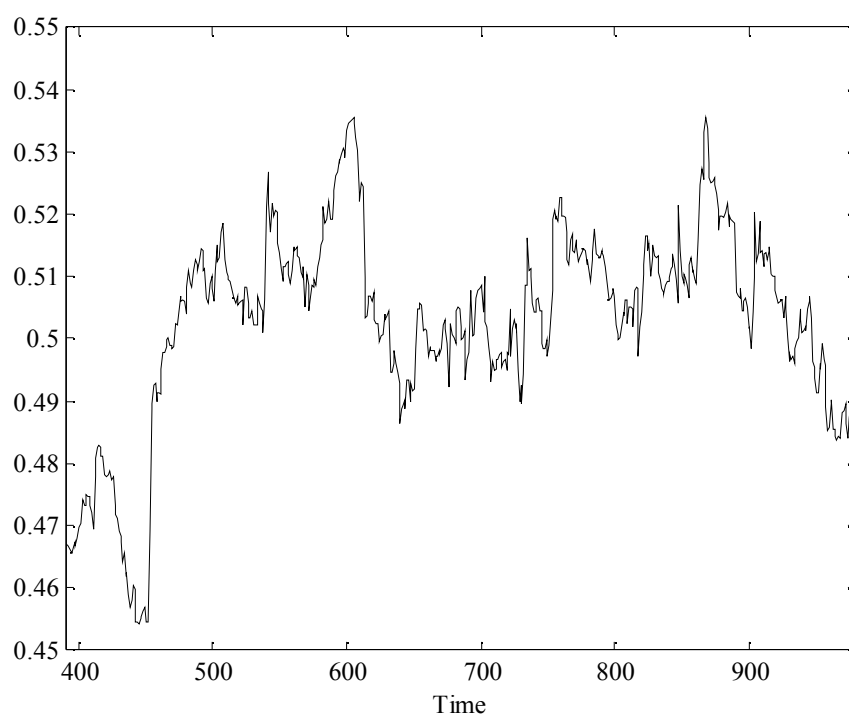


**Figure B14:** Interest Rate Pass-Through Parameter  $\lambda_2$  of the Baseline Model

## APPENDIX C

**Table C1:** Specification Tests of the Model 1 (*p*-values)

|              | Panel A: Autocorrelation Tests |             |             |             |
|--------------|--------------------------------|-------------|-------------|-------------|
| Country      | AR(6)                          | AR(12)      | AR(24)      | AR(36)      |
| Australia    | 1.000                          | 0.313       | 0.853       | 0.776       |
| Brazil       | 0.733                          | 0.466       | 0.856       | 0.295       |
| Canada       | 0.952                          | 0.989       | 0.748       | 0.764       |
| Chile        | 0.136                          | 0.420       | 0.037*      | 0.174       |
| Colombia     | 0.960                          | 0.967       | 0.825       | 0.556       |
| New Zealand  | 0.974                          | 0.990       | 0.994       | 0.846       |
| South Africa | 0.973                          | 0.926       | 0.185       | 0.214       |
| Sweden       | 0.780                          | 0.785       | 0.681       | 0.361       |
| UK           | 0.728                          | 0.835       | 0.476       | 0.372       |
|              | Panel B: ARCH-LM Tests         |             |             |             |
| Country      | ARCH-LM(6)                     | ARCH-LM(12) | ARCH-LM(24) | ARCH-LM(36) |
| Australia    | 0.986                          | 0.821       | 0.999       | 1.000       |
| Brazil       | 0.961                          | 0.946       | 0.968       | 0.988       |
| Canada       | 0.935                          | 0.962       | 0.997       | 1.000       |
| Chile        | 0.000**                        | 0.000**     | 0.999       | 1.000       |
| Colombia     | 0.906                          | 0.000**     | 0.000**     | 0.000**     |
| New Zealand  | 1.000                          | 1.000       | 1.000       | 1.000       |
| South Africa | 0.689                          | 0.907       | 0.732       | 0.851       |
| Sweden       | 0.925                          | 0.822       | 0.977       | 0.922       |
| UK           | 0.928                          | 0.976       | 0.999       | 1.000       |

\*\* and \* indicate 1% and 5% levels of significance, respectively.

**Table C2:** Specification Tests of the Model 2 (*p*-values)

|              | Panel A: Autocorrelation Tests |             |             |             |
|--------------|--------------------------------|-------------|-------------|-------------|
| Country      | AR(6)                          | AR(12)      | AR(24)      | AR(36)      |
| Australia    | 1.000                          | 0.929       | 0.980       | 0.983       |
| Brazil       | 0.725                          | 0.490       | 0.851       | 0.294       |
| Canada       | 0.965                          | 0.989       | 0.772       | 0.746       |
| Chile        | 0.794                          | 0.835       | 0.058       | 0.201       |
| Colombia     | 0.682                          | 0.649       | 0.277       | 0.130       |
| New Zealand  | 0.996                          | 1.000       | 0.999       | 0.975       |
| South Africa | 0.956                          | 0.949       | 0.215       | 0.217       |
| Sweden       | 0.820                          | 0.865       | 0.361       | 0.236       |
| UK           | 0.744                          | 0.797       | 0.532       | 0.434       |
|              | Panel B: ARCH-LM Tests         |             |             |             |
| Country      | ARCH-LM(6)                     | ARCH-LM(12) | ARCH-LM(24) | ARCH-LM(36) |
| Australia    | 0.970                          | 0.818       | 0.999       | 1.000       |
| Brazil       | 0.966                          | 0.915       | 0.985       | 0.997       |
| Canada       | 0.925                          | 0.928       | 0.998       | 1.000       |
| Chile        | 0.668                          | 0.842       | 0.303       | 0.632       |
| Colombia     | 0.961                          | 0.000**     | 0.000**     | 0.000**     |
| New Zealand  | 1.000                          | 1.000       | 1.000       | 1.000       |
| South Africa | 0.808                          | 0.947       | 0.918       | 0.989       |
| Sweden       | 0.940                          | 0.751       | 0.794       | 0.663       |
| UK           | 0.796                          | 0.930       | 0.998       | 1.000       |

\*\* and \* indicate 1% and 5% levels of significance, respectively.

**Table C3: Specification Tests of the Model 3 (*p*-values)**

|              | Panel A: Autocorrelation Tests |             |             |             |
|--------------|--------------------------------|-------------|-------------|-------------|
| Country      | AR(6)                          | AR(12)      | AR(24)      | AR(36)      |
| Australia    | 0.999                          | 0.999       | 0.944       | 0.979       |
| Brazil       | 0.704                          | 0.406       | 0.755       | 0.191       |
| Canada       | 0.905                          | 0.981       | 0.670       | 0.663       |
| Chile        | 0.978                          | 0.927       | 0.179       | 0.447       |
| Colombia     | 0.718                          | 0.728       | 0.406       | 0.262       |
| New Zealand  | 0.996                          | 1.000       | 0.999       | 0.974       |
| South Africa | 0.967                          | 0.932       | 0.204       | 0.194       |
| Sweden       | 0.857                          | 0.870       | 0.559       | 0.393       |
| UK           | 0.225                          | 0.226       | 0.091       | 0.026*      |
|              | Panel B: ARCH-LM Tests         |             |             |             |
| Country      | ARCH-LM(6)                     | ARCH-LM(12) | ARCH-LM(24) | ARCH-LM(36) |
| Australia    | 0.809                          | 0.746       | 0.999       | 1.000       |
| Brazil       | 0.990                          | 0.959       | 1.000       | 1.000       |
| Canada       | 0.907                          | 0.962       | 0.994       | 1.000       |
| Chile        | 0.692                          | 0.739       | 0.669       | 0.934       |
| Colombia     | 0.949                          | 0.000**     | 0.000**     | 0.000**     |
| New Zealand  | 1.000                          | 1.000       | 1.000       | 1.000       |
| South Africa | 0.760                          | 0.933       | 0.888       | 0.982       |
| Sweden       | 0.991                          | 0.967       | 0.995       | 0.867       |
| UK           | 0.175                          | 0.053       | 0.174       | 0.347       |

\*\* and \* indicate 1% and 5% levels of significance, respectively.