ROBUST SCHEDULING FOR PARALLEL MACHINES WITH SEQUENCE DEPENDENT SETUP TIMES AND UNCERTAIN BREAKDOWNS

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To My Parents

ABSTRACT

In this study, machine scheduling activities in the plastic injection production facility of a company competing in electronic sector is investigated. The company designs the products and picks order from customers for these products. There is a frozen production period for the end-product production schedule and all materials to be used in the end-product are expected to be prepared before the production. Otherwise, tardiness of materials leads to penalty costs due to production disruptions and revisions in the frozen plan. There are parallel machines in the plastic injection production facility. Setup time is job and sequence dependent. Furthermore, there are random machine breakdowns in the plastic injection machines. A production schedule that considers sequence-dependent setups, uncertain machine breakdowns is developed with the objective of minimizing tardiness. The solution for the proposed mixed integer quadratic programming formulation provides a robust schedule, which takes possible effects of worst-case scenarios for machine breakdowns into account.

Keywords: parallel machine scheduling; sequence-dependent setup times; uncertain machine breakdown; robust scheduling

ÖZETÇE

Bu çalışmada, elektronik sektöründe çalışan bir şirketin plastik enjeksiyon üretimi yapan tesisinde makine çizelegeleme süreci incelenmiştir. Bu şirket, ürünlerini kendi tasarlar ve bu ürünler için müşterilerden sipariş toplar. Son ürün üretim çizelgesinde donmuş periyot vardır ve son üründe kullanlacak tüm malzemelerin üretim öncesinde hazır olması beklenmektedir. Aksi takdirde, malzemelerin gecikmesi, üretim duruşu veya donmuş plandaki revizyonlar nedeniyle ceza maliyetine sebep olmaktadır. Plastik enjeksiyon üretim tesisinde paralel makinalar vardır. Kurulum zamanı iş ve sıralama bağlıdır. Ayrıca, plastik enjeksiyon makinelerinde belirsiz makine arızaları vardır. Sıralamaya bağlı kurulumları ve belirsiz makine arızaları göz önüne alan bir üretim programı, gecikmeyi en aza indirmek amacıyla geliştirilmiştir. Önerilen karışık tamsayılı kuadratik programlama formülasyonu için çözüm, makine arızaları için en kötü durum senaryolarının olası etkilerini dikkate alan güçlü bir program sağlar.

Anahtar Kelimeler: paralel makine çizelgeleme; sıra bağımlı kurulum süresi; belirsiz makine arızaları; gürbüz çizelgeleme

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TABLE OF CONTENTS

DEDICATION . . . . **iii**

ABSTRACT . . . . **iv**

ÖZETÇE . . . . **v**

ACKNOWLEDGEMENTS . . . . **vi**

LIST OF TABLES . . . . **ix**

LIST OF FIGURES . . . . **x**

I INTRODUCTION **1**

II PROBLEM DEFINITION **3**

2.1 General View of Product Planning Process 3

2.2 Plastic Injection Material Planning and Scheduling 5

2.3 Plastic Injection Molds 6

2.4 Plastic Injection Machines 6

2.5 Setup Times 7

2.6 Unexpected Breakdown 8

2.7 Further Considerations 9

III LITERATURE REVIEW **12**

3.1 Machine Scheduling with Deterministic Parameters 13

3.2 Machine Scheduling with Stochastic Parameters 15

IV SOLUTION APPROACH **18**

4.1 Mathematical Models 18

4.1.1 Deterministic Model 18

4.1.2 Mathematical Model with Machine Breakdown 21

4.2 Robust Optimization 22

V RESULTS **24**

5.1 Instance Generation 24

5.2 Outputs of the Models 26

VI CONCLUSION **38**

VII FUTURE RESEARCH **39**

REFERENCES **40**

VITA **42**

LIST OF TABLES

1 A summary table for literature review 12

2 Solution approach of reviewed studies 13

3 Order-machine instances 24

4 Different levels of order-machine compatibility 25

5 Instances 26

6 Output of deterministic model with low compatibility 26

7 Output of deterministic model with medium compatibility 27

8 Output of deterministic model with high compatibility 27

9 Output of robust model with low compatibility 28

10 Output of robust model with medium compatibility 28

11 Output of robust model with high compatibility 29

12 Objective function values of robust model, by inputting sequence of deterministic model, with low compatibility 30

13 Objective function values of robust model, by inputting sequence of deterministic model, with medium compatibility 30

14 Objective function values of robust model, by inputting sequence of deterministic model, with high compatibility 31

15 Average OFV of model with breakdown, by inputting sequence of robust model, with low compatibility 32

16 Average OFV of model with breakdown, by inputting sequence of robust model, with medium compatibility 32

17 Average OFV of model with breakdown, by inputting sequence of robust model, with high compatibility 33

18 Average OFV of model with breakdown, by inputting sequence of deterministic model, with low compatibility 33

19 Average OFV of model with breakdown, by inputting sequence of deterministic model, with medium compatibility 34

20 Average OFV of model with breakdown, by inputting sequence of deterministic model, with high compatibility 34

LIST OF FIGURES

1 Flow Diagram for End-Product Planning Process 4

2 Hourly Production Number for Different Type of Materials 7

3 Demands of the product per months for last three years 9

4 A Diagram for Plastic Injection Machine Scheduling 10

5 Machine Schedule of Deterministic Model, without breakdown 35

6 Machine Schedule of Robust Model, with the worst case of breakdown values 35

7 Machine Schedule of Robust model, with given sequence of deterministic model, and with the worst case of breakdown values 36

8 Machine Schedule of Model with Breakdown, with given sequence of robust model, max OFV found by random generated breakdown values 36

9 Machine Schedule of Model with Breakdown, with given sequence of deterministic model, max OFV found by random generated breakdown values 36

CHAPTER I

INTRODUCTION

We study on the material planning process of a company in electronics sector. The company has their own product designs and receives order for them. There is high variety of products and this variety accompanies with high variety of material type for the products. Procurement for each material type is different. There are three different procurement methods: domestic procurement, foreign procurement and in-house production.

This study focuses on the plastic injection materials produced at in-house production facility. Reason for selecting this material type is complexity of the plastic injection production planning and scheduling. The complexity is mainly resulted by high variety of materials, low efficiency of production, capacity restrictions and unexpected capacity loss.

Different plastic injection material types are produced in the facility. Each plastic injection material can be produced with a certain plastic injection mold and an appropriate plastic injection machine. Setups are required to before producing with a plastic injection mold on a machine. Setup time is job and sequence dependent.

There is a frozen production period for the first seven days of end-product production schedule. Any change cannot be expected in the end-production schedule in the frozen production period. All materials to be used in the end-product are expected to be ready on the production time of it. Otherwise, tardiness of the materials results with penalty costs due to production disruptions and revisions in the frozen plan. Hence, producing plastic injection materials before due date is significant in order not to cause penalty cost for the tardiness.

In addition, there exists unexpected machine breakdown in the plastic injection production. This situation results in unexpected delay from the planned production time of the order. Thus, it may lead to unexpected tardiness.

In this study, we work on a production schedule of plastic injection material with the minimum total tardiness of the operations based on the end-product production schedule under unexpected machine breakdown.

After that, we introduce Problem Definition, and Literature Review for related problems. Then, we present Solution Approaches and Results for the proposed solution approaches. Finally, we summarize the results and share possible future works in Conclusion and Future Research.

CHAPTER II

PROBLEM DEFINITION

In this part, end-product production planning, and plastic injection production planning and scheduling process are illustrated.

2.1 General View of Product Planning Process

A flow chart for product planning process is shown in Figure 1. The customers demand orders on a specific time for designed product by the company. Then, demand planners define the due date of the order by considering the production capacity and material shortages. When the due dates of the orders are in one month period, the orders are included in the production plan. The production plan is prepared based on one-month rolling planning period. The orders are produced at least seven days later after included in the production plan due to the principles of frozen period and lead time of materials.

Meanwhile, there might be different conditions for orders such as incompatibilities or deficiencies in billing of materials (BOM) and technical problems. These problems are preventions to produce the orders and they lead to uncertainties in the production plan. Therefore, they are urgent to solve in order not to disrupt production plan.

Production of the orders is completed by the final assembly production lines where all components (raw materials, semi products etc.) are collected. The production schedule of the final assembly lines is formed based on the production plan by considering mainly the capacity of production lines and due date of the orders. The first seven day of the end-product production schedule is frozen. The schedule cannot be changed in the frozen period without penalty cost. If there is no alternative plan, production stop is inevitable. Therefore, any delay of components lead to penalty

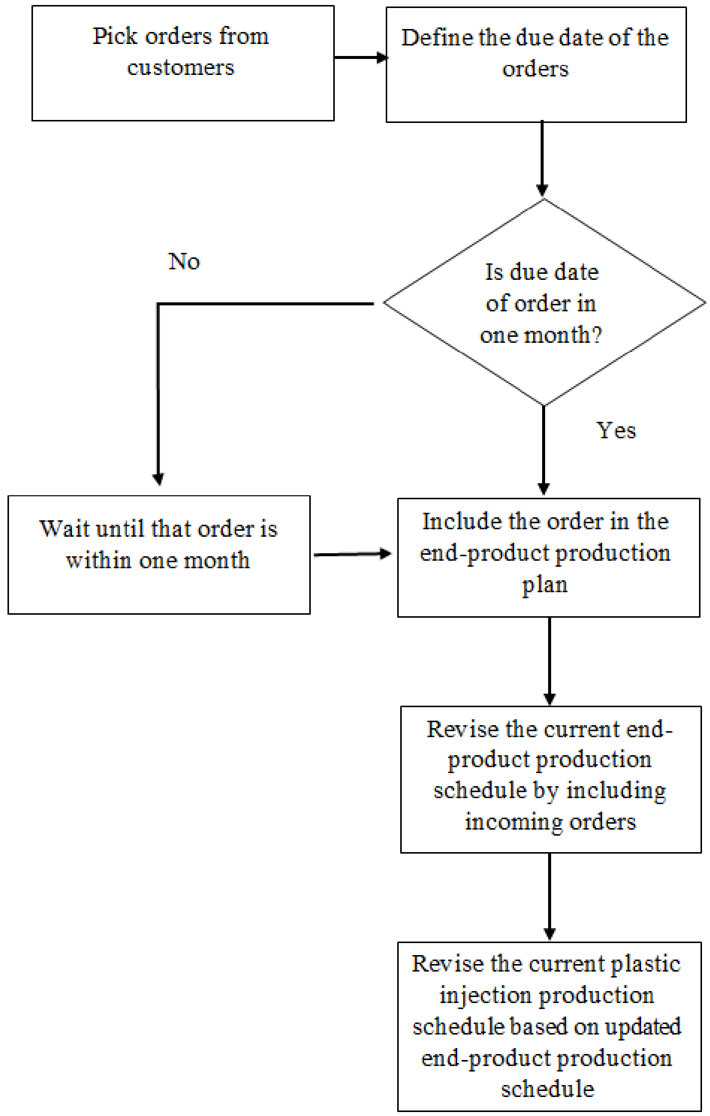

Figure 1: Flow Diagram for End-Product Planning Process

costs due to production stop and revisions in the frozen plan. Moreover, the next twenty day period is liquid period. Any changes can be done in the liquid period.

One month production schedule is studied for all components including plastic injection materials. Due to the frozen period, preparing all components before due date is important in order not to cause production disruptions or revisions in the frozen plan.

2.2 Plastic Injection Material Planning and Scheduling

After that point, plastic injection material planning process is introduced in detail. The plastic injection materials to be used in the end-products are definite based on product tree of them. The production schedule of end-product defines the required amount of plastic injection materials for each time period. Planning horizon of plastic injection production schedule is one month such as end-product production schedule. Frozen period is applied for first seven days of end-product production schedule that means any change in that schedule cannot be done in the first seven days without an additional penalty cost. Therefore, production starting time of each end-product in this period is accepted as strict due date for corresponding plastic injection materials. Any delay of the materials leads to production disruptions and penalty cost. The end-product production schedule is updated each day and the plastic injection production schedule is required to be updated depending on it.

Plastic injection materials are produced by an injection mold and machine. In the production process, the heated raw material is pressed by the machine into the mold to give the designed shape. After a while, the mold is pulled away and the plastic injection material is produced.

Plastic injection production plan is mainly composed by scheduling the mold operations to the machines. Thereafter, the main concept in the plastic injection machine scheduling is explained in detail.

2.3 Plastic Injection Molds

Each plastic injection material can be produced with a plastic injection mold. On the other hand, different plastic injection materials can be produced with the same plastic injection mold. These differences are resulted by versions on the mold specifying the design of plastic materials, or by material exchange on the plastic injection mold corresponding to the change of color or raw material. There are approximately 700 plastic injection materials and 360 plastic injection molds used in the plastic injection production plan.

Each plastic injection mold has certain cycle time and efficiency values. Therefore, molds have limited production capacity. Production capacity can be stated in terms of hourly production number. Hourly production number of each mold can be calculated as follows:

$$Hourly\,production\,number = (3600\text{ seconds}/1\text{hour})x\,Efficiency/\,Cycle\,Time(sec)$$

In case of capacity restrictions for a time period, having stock in the previous time period is preferred. Capacity of plastic injection of molds are relatively low when compared to capacity of other materials. In Figure 2, we show average hourly production number of four different materials types and Material Type A is for plastic injection materials. Due to low capacity, we encounter with scenarios to produce for stock frequently.

Moreover, there are technical features for molds defining compatibility with machines. A mold can be assigned to a machine, when the mold and machine are technically compatible.

2.4 Plastic Injection Machines

There are 85 plastic injection machines. They are parallel machines. Hourly production number of molds does not vary depending on the machines.

Machines have different characteristics. Compatibility of the molds with machines

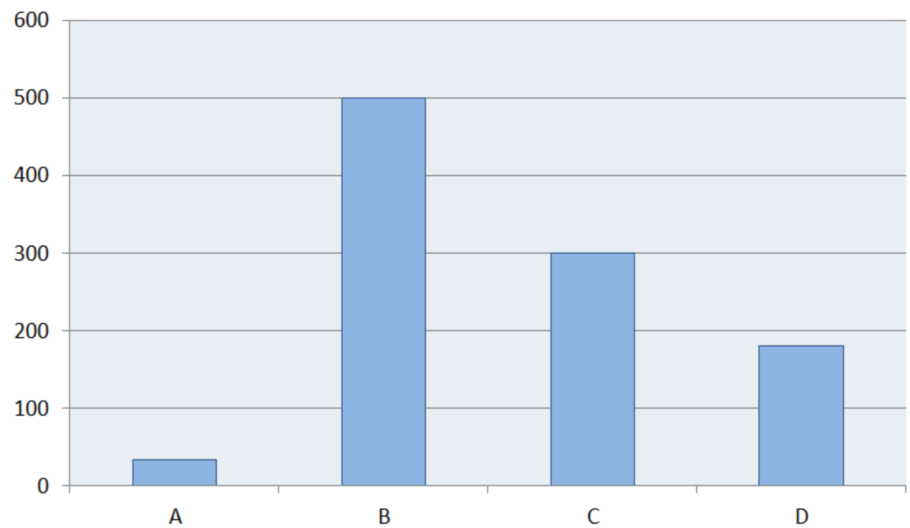

Figure 2: Hourly Production Number for Different Type of Materials

is determined by the characteristics of the machines and technical features of the molds.

Meanwhile, all of the machines are not located in the same location. There are two facilities; the one has 65 machines and the other has 20 machines. Transfer of the molds is not preferred. Fill rate and compatibility of the molds are periodically evaluated and assigned to the facilities to decrease the movement of the molds between facilities.

Due to characteristics and location of machines, eighty five machine can bu grouped into smaller machine groups. Likewise, molds can be divided in mold groups based on their technical features and locations. Assignment of molds to the machines can be performed based on these smaller groups.

2.5 Setup Times

Another consideration for the plastic injection machine scheduling is setup times. There are different kinds of setups: exchange of mold, version on the mold and material type. There is a given setup time table for each mold, version and material type.

1. Exchange of the mold means that assigning a new mold to the machine different from the mold on the machine.

2. Exchange of the version on the mold corresponds that a new version is assigned for the same mold on the machine. Depending on the molds, setup time of the version can be smaller or larger than the setup time of the mold exchange.

3. Material exchange includes the change of color or raw material. Exchange duration of material type depends on whether there is a special equipment on mold or not. If the mold has special equipment, the exchange duration is equal to exchange duration of the mold. Otherwise, the exchange duration is higher (approximately twenty four hours).

2.6 Unexpected Breakdown

The machine breakdown is encountered frequently in the plastic injection production. However, breakdown is an unexpected situation. It leads to unexpected capacity loss and delay in the production time of orders. If delayed production time is larger than the due date, then unexpected tardiness may occur.

In current scheduling process, inventory level is important to be able to tolerate possible tardiness due to unexpected breakdown. However, target inventory level is defined by material planner intuitively and it is not methodical. If inventory level for a material is not enough to overcome breakdown duration, there might be risk for failing to satisfy due date. On the other hand, a reverse situation might cause excessive inventory.

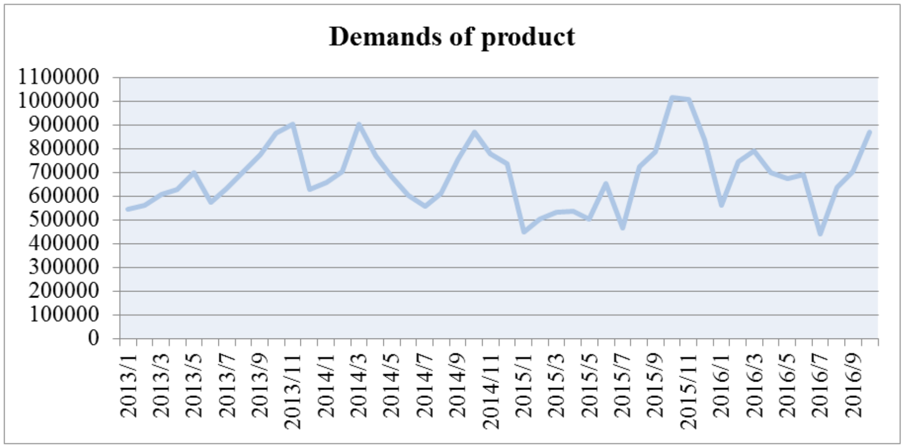

Figure 3: Demands of the product per months for last three years

2.7 Further Considerations

Customer demands for end-product show seasonality over the year as seen in the Figure 3. Monthly production number is determined by considering the monthly demand, option of keeping inventory for the next month or backlogging for the previous month, and capacity of final production lines. Thereafter, capacity of the plastic injection production facility is evaluated based on monthly production number for end-product. Number of working machines and fill rate of machines and molds are evaluated in order to satisfy demand on time. If there is a capacity exceed for molds and machines, keeping inventory for plastic injection material is evaluated instead of backlogging.

Until now, main concepts in the plastic injection machine scheduling are explained. They are also illustrated in Figure 4. The plastic injection material production planner conducts machine scheduling considering mainly: compatibility of molds and machines, due date of orders, setup times, possible risk for machine breakdown.

The main objective for the plastic injection machine scheduling is producing plastic injection materials on due date based on the end-product production schedule. Tardiness should be minimized.

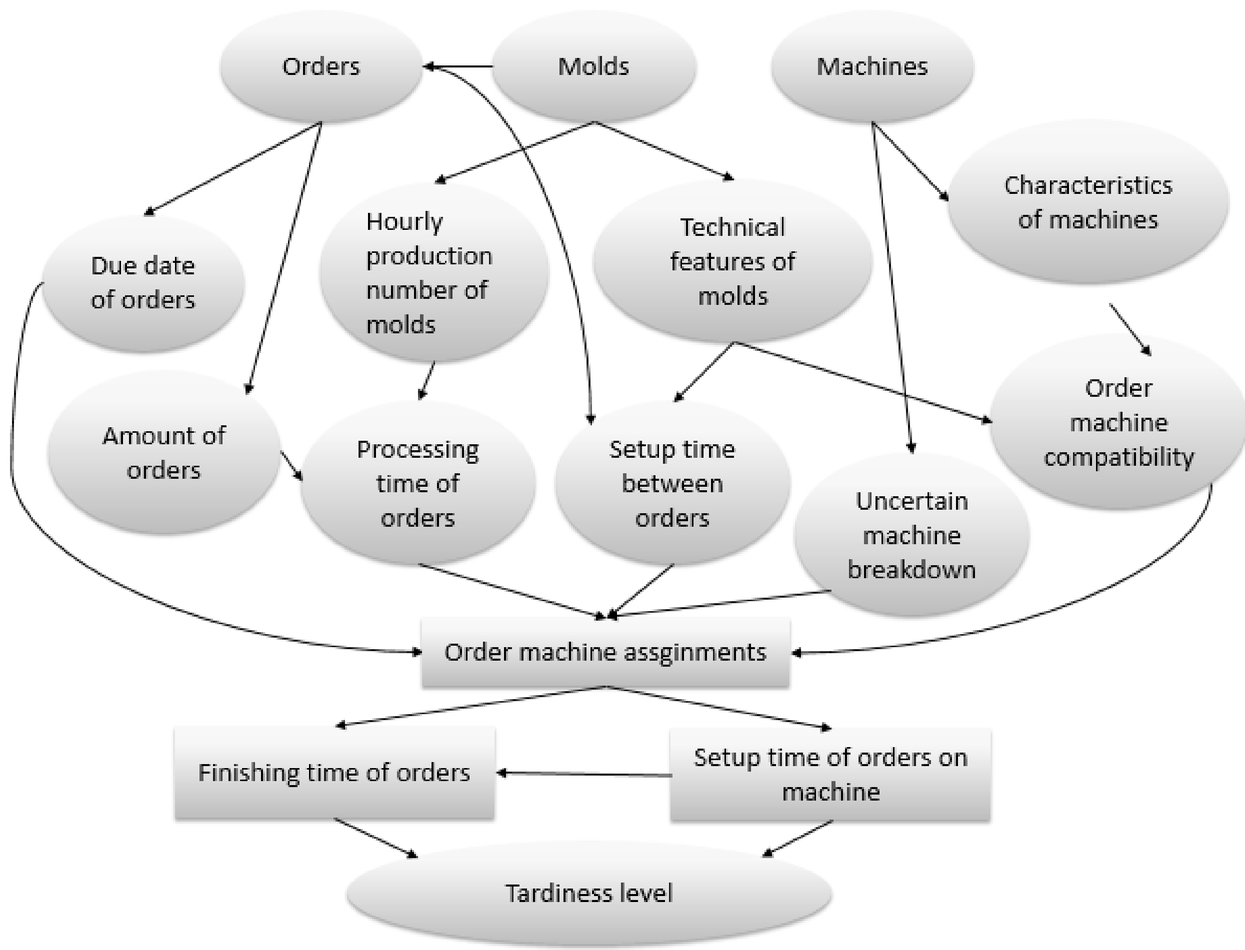

Figure 4: A Diagram for Plastic Injection Machine Scheduling

Meanwhile, machine breakdown affects directly the finishing time of planned orders and so tardiness. Therefore, managing uncertain machine breakdown requires a systematic scheduling method instead of an intuitive approach in order to improve machine scheduling process.

An effective scheduling process to reach main objectives and provide the other considerations introduced above is needed to be developed. This study focuses on preparing a plastic injection machine schedule minimizing tardiness and anticipating the possible delay in the production of orders due to uncertain machine breakdown.

CHAPTER III

LITERATURE REVIEW

In the literature, lots of scheduling problems are studied. Allahverdi et al. (2008) aims to provide a review for scheduling problems with setup times or costs. Allahverdi et al. (2008) states that scheduling problems are classified as batching or non-batching considerations, sequence-dependent or sequence-independent setup times in this study. It also categorizes scheduling problems based on shop environments which are single machine, parallel machines, flow shop, job shop and open shop. Based on these classifications, our problem has non-batch setup time, sequence-dependent setup time and parallel machines. In addition, our problem has an uncertain parameter due to machine breakdown.

Related studies are introduced in two parts by considering deterministic and stochastic parameters. Objectives and setup considerations are shown in Table 1. Table 1 also indicates whether stochastic parameters are used or not. In Table 2, solution methods of the studies are introduced.

Table 1: A summary table for literature review

Objective	Setup	Deterministic Parameters	Stochastic Parameters
Min ET and/or TT	NS	Federgruen and Mosheiov (1996),Mazdeh et al. (2010)	von Hoyningen-Huene and Kiesmüller (2015),O'Donovan et al. (1999),Li and Alfa (2005),Cai and Zhou (1999)
Min ET and/or TT	SIS		Mokhtari and Dadgar (2015)
Min ET and/or TT	SDS	Zhu and Heady (2000), Hiraishi et al. (2002), Balakrishnan et al. (1999)	
Min ET and/or TT and ST	SDS	Dastidar and Nagi (2005)	
Min ST	SIS	Aubry et al. (2008)	
Min MS	NS		Wang et al. (2015),von Hoyningen-Huene and Kiesmüller (2015),Hoyningen-Huene (2015),Jin-wei Gu (2014)
Min MS	SDS		Gholami et al. (2009)
Min MS	SIS		Allahverdi (1995)

The abbreviations used in Table 1 are as follows: Earliness time/costs (ET), Tardiness time/costs (TT), Setup time/costs (ST), Makespan/Total completion time (MS), No setup (NS), Sequence-dependent setup (SDS), Sequence-independent setup (SIS).

Table 2: Solution approach of reviewed studies

Article	Solution Method	Exact Solution or Not?
Federgruen and Mosheiov (1996)	Heuristics	Not Exact
Mazdeh et al. (2010)	Meta heuristics algorithm	Not Exact
von Hoyningen-Huene and Kiesmüller (2015)	Simulation study, Adjusted first fit decreasing algorithm	Not Exact
O'Donovan et al. (1999)	Predictable scheduling approach	Not Exact
SArticle9	Optimal scheduling policy	Exact
Cai and Zhou (1999)	Optimal scheduling policy	Exact
Mokhtari and Dadgar (2015)	Simulation-optimization framework	Not Exact
Zhu and Heady (2000)	Mixed integer programming	Exact
Hiraishi et al. (2002)	Transshipment problem	Exact
Balakrishnan et al. (1999)	Mixed integer programming	Exact
Balakrishnan et al. (1999)	Decomposition method	Exact
Dastidar and Nagi (2005)	Two-phase workcenter-based decomposition scheme	Not Exact
Aubry et al. (2008)	Transportation problem	Exact
Wang et al. (2015)	Multi-objective evolutionary algorithm	Not Exact
von Hoyningen-Huene and Kiesmüller (2015)	Approximations	Not Exact
Hoyningen-Huene (2015)	Three construction heuristics, Ant Colony System (ACS)	Not Exact
Jinwei Gu (2014)	the Longest Processing Time first (LPT) rule, the Shortest Processing Time first (SPT) rule	Exact
Gholami et al. (2009)	Immune algorithm	Not Exact
Allahverdi (1995)	Algorithm	Exact

3.1 Machine Scheduling with Deterministic Parameters

There are lots of studies on machine scheduling with deterministic parameters for single machine and multi machines. Since our problem is for multi machines, studies on scheduling problem for this type are reviewed in this part.

There is no setup consideration in (Federgruen and Mosheiov, 1996) and (Mazdeh et al., 2010). Heuristics for multi machines scheduling are studied in (Federgruen and Mosheiov, 1996) with the objective of minimizing total earliness and tardiness costs. Mazdeh et al. (2010) study on minimizing total tardiness and machine deteriorating costs for parallel uniform machines by solving with LP-metric method and meta heuristics algorithm.

In (Aubry et al., 2008), a mixed integer linear program and transportation problem are studied with parallel uniform machines, sequence independent setup times and load balanced machines by minimizing total setup costs.

Zhu and Heady (2000) work on non-uniform machines and sequence dependent setup times with objective of minimizing total earliness and tardiness costs by presenting a mixed integer programming. Proposed model gives optimal solution for the problems having about nine jobs and three machines. Hiraishi et al. (2002) model the scheduling problem as a transshipment problem minimizing the weighted number of early and tardy jobs for parallel machines. There are also sequence-dependent setup times. Balakrishnan et al. (1999) propose a mixed integer programming with uniform parallel machines, ready time of the jobs and sequence dependent setup times by the objective of minimizing the sum of earliness and tardiness. They also propose a decomposition method for solving larger problems. In (Dastidar and Nagi, 2005), a two-phase workcenter-based decomposition scheme is developed with non-uniform machines and sequence dependent setup times by the objective of minimizing total inventory holding costs, backlogging costs, and setup costs.

Solution methods in (Zhu and Heady, 2000), (Hiraishi et al., 2002), (Balakrishnan et al., 1999) and (Aubry et al., 2008) give exact solution. Our problem has similar considerations with (Zhu and Heady, 2000) and (Balakrishnan et al., 1999) by having sequence-dependent setup times, including tardiness related objective functions and looking for exact solutions. Apart from these, uncertain parameters are included in this study with robust approach.

3.2 Machine Scheduling with Stochastic Parameters

Number of studies in the literature for machine scheduling with stochastic parameters is relatively less than studies with deterministic parameters. These studies mostly focus on machine scheduling problem with stochastic machine breakdown and stochastic processing time. In our problem, processing time of orders are deterministic. Therefore, we review studies including stochastic machine breakdown.

At first, studies about single machine scheduling with stochastic machine breakdown are introduced.

O'Donovan et al. (1999) propose predictable scheduling approaches for single machine scheduling with stochastic machine breakdown by the objective of minimizing tardiness. Cai and Zhou (1999) study on single machine scheduling with stochastic processing times, deadlines and stochastic machine breakdown. They propose optimal scheduling policies minimizing the expected costs for earliness and tardiness of job completions about a common deadline. Li and Alfa (2005) study on single machine scheduling problem with stochastic processing times and stochastic machine breakdown. They study on three different objectives to be minimized: the weighted flow time, the weighted number of tardy jobs, and the weighted sum of job tardiness by proposing optimal scheduling policies. Jinwei Gu (2014) study on single machine scheduling problem with stochastic machine breakdown. It is assumed that the machine uptimes are independently and identically distributed and they are subject to a uniform distribution. They proved that the Longest Processing Time first (LPT) rule gives the minimum for expected makespan and the Shortest Processing Time first (SPT) rule gives minimum for expected value of total completion times. Wang et al. (2015) works on a single machine scheduling problem with the objective of minimizing operational costs including total completion time cost and resource consumption cost. A multi-objective evolutionary algorithm is developed in the study. von Hoyningen-Huene and Kiesmüller (2015) study on single machine scheduling problem with stochastic machine breakdown with the objective of minimizing the average cost including tardiness cost, and maintenance costs. They work on a simulation study and propose an adjusted First Fit Decreasing algorithm.

These studies with single machine scheduling have different objectives, mainly minimizing tardiness and/or earliness, and minimizing makespan. Li and Alfa (2005), Cai and Zhou (1999) and Jinwei Gu (2014) give exact solution, they show that proposed algorithm in their study give optimal results. However, other studies do not give exact solution for the problem studied.

Next, studies with multi machines are introduced. von Hoyningen-Huene and Kiesmüller (2015) study on an exact analytical expression for identical parallel machines with the objective of minimizing the expected makespan. Moreover, two different approximations with detailed numerical study are proposed due to high effort for reaching the exact solution. In (Hoyningen-Huene, 2015), three construction heuristics and an Ant Colony System algorithm are proposed for the scheduling problem with parallel identical machines and stochastic machine breakdown.

The studies mentioned until now for machine scheduling with stochastic machine breakdown do not include setup considerations. However, the next three studies regard setups.

Allahverdi (1995) studies on two machine flow shop scheduling problem with sequence independent setup times and stochastic machine breakdown. The objective is minimizing the makespan and it is shown that Yoshida and Hitomi's algorithm stochastically minimizes makespan under certain conditions on breakdowns. In (Gholami et al., 2009), a hybrid flow shop scheduling problem with sequence-dependent setup by the objective of minimizing expected makespan is studied. They propose an immune algorithm. Mokhtari and Dadgar (2015) study on flexible job shop system with stochastic machine breakdown and sequence-independent setup times by the objective of minimizing total number of tardy jobs. They present a mathematical model, and a simulation-optimization framework with simulated annealing (SA) optimizer and Monte Carlo (MC) simulator.

It is seen that studies for multi machines including stochastic machine breakdown and setup considerations is rare. Allahverdi (1995) refers to an algorithm in the literature with sequence-independent setup time and shows that the algorithm gives optimal results. However, other studies do not give exact solution to the problem.

This study focuses on machine scheduling process having parallel machines with sequence-dependent setups and uncertain machine breakdown by keeping the objective of minimizing tardiness. However, we seek for finding the exact solution and we work on robust optimization methods.

CHAPTER IV

SOLUTION APPROACH

We propose mathematical models to represent machine scheduling process in order to reach exact solution. At first, we present a model without breakdown. It is called as deterministic model in this study. Next, the model is revised to include machine breakdown. The second model has an uncertain term, breakdown. To deal with uncertainty, robust optimization method is preferred. We propose a new model, called as robust model.

4.1 Mathematical Models

In the model, there is no index for mold. Each order can be produced by a certain mold and it has characteristics of that mold. Therefore, molds are not necessary to include in the model. Required mold to produce order is known. Then, parameters including mold data can be revised. Processing time of order is calculated by amount of order and hourly production number of required mold. Compatibility of order with machines is also revised by regarding required mold. Also, we assume that there is no infeasible condition for molds while scheduling the orders on machines.

If there occurs breakdown on a machine, the order assigned to that machine cannot be produced anymore during breakdown. When breakdown is over, there is partial completion and only remaining amount of order is produced.

4.1.1 Deterministic Model

Parameters

I: Set of orders, $i \in I, i \in \{1..I_{max}\}$

J: Set of orders, $i \in J, j \in \{1..I_{max}\}$

M: Set of machines, $m \in M, m \in \{1..M_{max}\}$

P: Set of sequences, $p \in P, p \in \{1..P_{max}\}$

R_i: Processing time of order, $i \in I$

D_i: Due date of order, $i \in I$

$E_{i,m} :\in \{0,1\}$ 1 If order i can be assigned to machine $m, i \in I, m \in M$

$ST_{i,j}$: Setup time between order i and order $j, i \in I, j \in I$

Decision Variables

$X_{i,m,p} \in \{0,1\}$: 1 If order i is assigned to $p-th$ in sequence in machine m, $i \in I, m \in M, p \in P$

$Q_{i,j,m,p} \in \{0,1\}$: 1 If there is a setup between order i and order j in $p-th$ sequence in machine, $i \in I, j \in I, m \in M, p \in P$

$W_{i,j} \in \{0,1\}$: 1 If there a setup between order i and order j, $i \in I, j \in I$

T_i: Tardiness of order i, $i \in I$

F_i: Finishing time of order $i, i \in I$

Formulation

$$\min \quad z = \sum_i T_i \tag{1}$$

$$\text{subject to} \quad \sum_{m \in M, p \in P} X_{i,m,p} = 1 \qquad \forall i \in I \tag{2}$$

$$\sum_{i \in I} X_{i,m,p} \leq 1 \qquad \forall m \in M, \forall p \in P \tag{3}$$

$$\sum_{i \in I, p \in P} X_{i,m,p} \geq 1 \qquad \forall m \in M \tag{4}$$

$$X_{i,m,p} \leq E_{i,m} \qquad \forall i \in I, \forall m \in M, \forall p \in P \tag{5}$$

$$\sum_{i \in I} X_{i,m,p} \geq \sum_{j \in J} X_{j,m,p+1} \qquad \forall m \in M, \forall p \in P, \forall p \leq Pmax - 1 \tag{6}$$

$$Q_{i,j,m,p} + 1 \geq X_{i,m,p} + X_{j,m,p+1}$$
$$\forall i \in I, \forall j \in J, \forall m \in M, \forall p \in P, i \neq j, p \leq P_{max} - 1 \tag{7}$$

$$2Q_{i,j,m,p} \leq X_{i,m,p} + X_{j,m,p+1}$$
$$\forall i \in I, \forall j \in J, \forall m \in M, \forall p \in P, i \neq j, p \leq P_{max} - 1 \tag{8}$$

$$W_{i,j} = \sum_{m,p} Q_{i,j,m,p} \qquad \forall i, j \tag{9}$$

$$F_j \geq \sum_{i \neq j} (F_i W_{i,j} + W_{i,j} ST_{i,j}) + R_j \qquad \forall j \in I \tag{10}$$

$$T_i \geq 0 \qquad \forall i \in I \tag{11}$$

$$T_i \geq F_i - D_i \qquad \forall i \in I \tag{12}$$

$$F_i \geq 0 \qquad \forall i \in I \tag{13}$$

$$X_{i,m,p} \in \{0, 1\} \qquad \forall i \in I, \forall m \in M, \forall p \in P \tag{14}$$

$$Q_{i,j,m,p} \in \{0, 1\} \qquad \forall i \in I, \forall j \in J, \forall m \in M, \forall p \in P \tag{15}$$

$$W_{i,j} \in \{0, 1\} \qquad \forall i \in I, \forall j \in J \tag{16}$$

In this formulation, the objective is to minimize the total tardiness of orders. Constraint (2) ensures that each order must be assigned to any sequence of any machine. Constraint (3) states at most one order can be assigned to any sequence of any machine. Constraint (4) states that at least one order must be assigned to each machine.

Constraint (5) ensures that an order should be compatible with a machine to be able to assign that order to that machine. With Constraint (6), it is provided that an order can be assigned to a sequence of a machine, as well as there is an assigned order to the previous sequence of the machine. Constraint (7),(8) and (9) define whether there is setup between orders or not for consecutive sequence of the machines. Finishing time of the orders are calculated within Constraint (10). Constraint (11) and (12) define the tardiness of order. Constraint (13) defines the non-negativity of the variables. Constraint (14), (15) and (16) define the binary natures of the variables.

Meanwhile, there is nonlinearity in Constraint (10). To overcome with nonlinearity, we introduce a new decision variable for multiplication of two decision variables. Then, (10) is updated and four more constraints are added. New decision variable and constraints are illustrated below:

$Z_{i,j}$ Additional decision variable for nonlinear term, $i \in I, j \in J$

$$Z_{i,j} \leq MW_{i,j} \quad \forall i \in I, j \in J \tag{17}$$

$$Z_{i,j} \leq F_i \quad \forall i \in I, j \in J \tag{18}$$

$$Z_{i,j} \geq F_i - M(1 - W_{i,j}) \quad \forall i \in I, j \in J \tag{19}$$

$$Z_{i,j} \geq 0 \quad \forall i \in I, j \in J \tag{20}$$

Resulting model is Mixed Integer Linear Programming (MILP).

4.1.2 Mathematical Model with Machine Breakdown

To include the uncertain machine breakdown in the model, (10) is updated as follows:

$$RF_j \geq \sum_{i \neq j} (RF_i W_{i,j} + W_{i,j} ST_{i,j}) + R_j \quad \forall i \in I \tag{21}$$

where RF_i denotes the raw finishing time of order i excluding breakdowns.

A new parameter and constraint related to machine breakdown are added to the mathematical model. Note that constraints except (10) remain same.

$\hat{\epsilon}_{i,m}$ Duration of breakdown for order i, when it is assigned to the machine $m, i \in I, m \in M$

$$F_i \geq RF_i + \sum_{i' \in I} \sum_{p'=1}^{p} \hat{\epsilon}_{i',m} X_{i',m,p'} - M(1 - X_{i,m,p}) \quad \forall i \in I, m \in M, p \in P \tag{22}$$

where F_i are nonnegative variables.

F_i incorporates any breakdown that has been encountered before or during the time order i is processed. This is proportional to the processing time of order i and all orders before order i.

Constraint (22) includes an uncertain parameter, machine breakdown. To deal with uncertainty, we work on implementing the robust optimization methods for our problem.

4.2 Robust Optimization

Gorissen et al. (2015) introduce the methods and applications of robust optimization. In robust optimization, probability distributions of uncertain data are unknown. Uncertain data belong to so-called uncertainty set.

There are alternative uncertainty sets in the literature. We prefer to study with an ellipsoidal uncertainty set since it is less conservative. It is less pessimistic compared to the box uncertainty set. Also, model with an ellipsoidal uncertainty set gives a second order cone programming (SOCP) model which is relatively more tractable.

The uncertainty is modeled in the following way:

$$\hat{\epsilon}_{i,m} = R_i \xi_{i,m} \quad \forall i \in I, \ \forall m \in M \tag{23}$$

where $||\xi||_2 \leq \rho_m$. $\xi_{i,m}$ is the rate of breakdown for order i on machine m. ρ_m is the rate for breakdown on machine m. Both of these values change in [0,1].

Constraint (24) has uncertain $\xi_{i,m}$ values, and there infinitely many constraints.

$$F_i \geq RF_i + \sum_{i' \in I} \sum_{p'=1}^{p} R_i \xi_{i,m} X_{i',m,p'} - M(1 - X_{i,m,p}) \quad \forall i \in I, m \in M, p \in P, ||\xi||_2 \leq \rho_m \tag{24}$$

In order to reach relatively more tractable formulation, this constraint is reformulated by providing the worst case breakdown values. The resulting robust counterpart for an ellipsoidal uncertainty set is given as follows:

$$F_i \geq RF_i + \rho_m \sqrt{\sum_{p'=1}^{p} \sum_{i' \in I} (R_{i'} X_{i',m,p'})^2} - M(1 - X_{i,m,p}) \quad \forall i \in I, m \in M, p \in P \tag{25}$$

Constraints (2) - (21) except for (10) remain same. The resulting problem is a complex MIQP.

Meanwhile, constraint (25) is revised in order to reach a solvable model in the optimization engine. A new decision variable and four more constraints are added as follows:

$\gamma_{i,m,p}$ Additional variable2, $i \in I, m \in M, p \in P$

$$\gamma_{i,m,p} = F_i - RF_i + M(1 - X_{i,m,p}) \quad \forall i \in I, \forall m \in M, \forall p \in P \tag{26}$$

$$\gamma_{i,m,p}^2 \geq {\rho_m}^2 \sum_{p'=1}^{p} \sum_{i' \in I} (R_{i'} X_{i',m,p'})^2 \quad \forall i \in I, \forall m \in M, \forall p \in P \tag{27}$$

$$\gamma_{i,m,p} \geq 0 \quad \forall i \in I, \forall m \in M, \forall p \in P \tag{28}$$

The last version of this model is called as robust model in this study.

CHAPTER V

RESULTS

In this section, we explain how the instances are generated. Then, we present computational results.

5.1 Instance Generation

As mentioned earlier, we have machine groups formed by characteristics and locations of them. Also, orders are grouped based on technical features and locations of corresponding molds. Then, we can match related order and machine groups and work on scheduling them. Number of orders and number of machines in the instances which we study are indicated in Table 3.

Table 3: Order-machine instances

# of m/c's	# of orders
3	5
3	10
3	15
4	5
4	10
4	15
5	5
5	10
5	15
6	10
6	15

Meanwhile, this does not mean that all orders in a machine group are compatible with all machine in that group. Compatibility is defined by considering each pair of order and machine in the group.

$E_{i,m}$ is a binary parameter. It presents order-machine compatibility. It takes value of

1, if order i is compatible with machine m. Otherwise, it equals to 0.

In real life, orders are not compatible with all machines due to the characteristics of molds and machines. We work on different levels of order-machine compatibility as shown in Table 4.

Table 4: Different levels of order-machine compatibility

Data Type	Level of compatibility	Average rate of being compatible with a machine for an order
Data Type1	Low	%30
Data Type2	Medium	%50
Data Type3	High	%70

A typical plastic injection machine schedule include machine plan from one to four weeks. In this manner, processing time and due date of orders are generated. Time unit is day. Processing time orders randomly generated between [1,20]. Due date of orders are selected from {5,10,15,20}.

We focus on 11 order-machine matchings, and 3 levels of order machine compatibility in this study. Then, we have 33 different instances as shown in Table 5.We generate different processing time and due date for orders in each instance.

Meanwhile, we omit the instance with 6 machine, 5 orders due to nature of (4).

ρ_m is equal to the rate for breakdown duration on a machine. It gives worst case value. We calculate ρ_m as 0.1*the ratio of the total number of orders to the total number of machines in each instance in the study.

$ST_{i,j}$ is equal to setup times between orders. Setup times are selected from {0.1, 0.2, 0.3, 0.4, 0.5, 1}.

Proposed models, deterministic model and robust model, are solved by IBM ILOG CPLEX 12.7 which is an optimization engine. Computational results are taken by a computer having Intel Core i5-6200U CPU, @2.30 GHz and 8.00 GB RAM.

A material planner allocate three hours to prepare updated machine schedule every day. Therefore, we take solution time limit as 3 hours in CPLEX.

Table 5: Instances

# of m/c's	# of orders	Data Type1	Data Type2	Data Type3
3	5	Low	Medium	High
3	10	Low	Medium	High
3	15	Low	Medium	High
4	5	Low	Medium	High
4	10	Low	Medium	High
4	15	Low	Medium	High
5	5	Low	Medium	High
5	10	Low	Medium	High
5	15	Low	Medium	High
6	10	Low	Medium	High
6	15	Low	Medium	High

5.2 Outputs of the Models

At first, we presents outputs of deterministic model for each instance illustrated in Table 5.

Objective function values indicate tardiness level in terms of days. As seen in the Table 6 - 8, deterministic models are solved for these instances under 60 seconds even though solution time increases when order-machine compatibility rate increases.

Table 6: Output of deterministic model with low compatibility

# of m/c's	# of orders	OFV	OPT?	Opt. Gap	Solution Time
3	5	6.4	Optimal	-	1 seconds
3	10	5.4	Optimal	-	2 seconds
3	15	10.8	Optimal	-	3 seconds
4	5	3.1	Optimal	-	1 seconds
4	10	3.7	Optimal	-	2 seconds
4	15	10.2	Optimal	-	5 seconds
5	5	0	Optimal	-	1 seconds
5	10	4.7	Optimal	-	3 seconds
5	15	8.3	Optimal	-	3 seconds
6	10	6.1	Optimal	-	2 seconds
6	15	13.1	Optimal	-	5 seconds

Next, we present outputs of the robust model for same instances. The robust model gives a machine schedule for the worst case values of breakdown duration. Meanwhile, we

Table 7: Output of deterministic model with medium compatibility

# of m/c's	# of orders	OFV	OPT?	Opt. Gap	Solution Time
3	5	5	Optimal	-	1 seconds
3	10	10.9	Optimal	-	12 seconds
3	15	5.1	Optimal	-	12 seconds
4	5	0	Optimal	-	2 seconds
4	10	12.3	Optimal	-	9 seconds
4	15	13.7	Optimal	-	11 seconds
5	5	15	Optimal	-	2 seconds
5	10	19.8	Optimal	-	7 seconds
5	15	3.3	Optimal	-	25 seconds
6	10	3	Optimal	-	2 seconds
6	15	4	Optimal	-	15 seconds

Table 8: Output of deterministic model with high compatibility

# of m/c's	# of orders	OFV	OPT?	Opt. Gap	Solution Time
3	5	2.5	Optimal	-	1 seconds
3	10	9.8	Optimal	-	39 seconds
3	15	2.7	Optimal	-	42 seconds
4	5	0.3	Optimal	-	1 seconds
4	10	5.1	Optimal	-	13 seconds
4	15	3.1	Optimal	-	16 seconds
5	5	0	Optimal	-	1 seconds
5	10	5	Optimal	-	8 seconds
5	15	12.6	Optimal	-	8 seconds
6	10	10.3	Optimal	-	11 seconds
6	15	5.3	Optimal	-	25 seconds

use objective function values of deterministic model as lower bound to robust model for each instance. Because, we expect that finishing time of orders will increase when breakdown occurs. Hence, tardiness will be higher as well as it is different from 0. If tardiness is equal to 0, it may be higher or it may remain same depending on amount of increase in the finishing time of orders.

After integrating machine breakdown to model, objective function values change as seen in the Table 9 - 11. Solution time is higher compared to solution time of deterministic model.

We reach optimal solution except for the instance of 6 orders, 15 machines having medium and high compatibility.

Table 9: Output of robust model with low compatibility

# of m/c's	# of orders	OFV	OPT?	Opt. Gap	Solution Time
3	5	10.155	Optimal	-	1 seconds
3	10	16.037	Optimal	-	13 seconds
3	15	24.82	Optimal	-	1514 seconds
4	5	4.904	Optimal	-	1 seconds
4	10	11.075	Optimal	-	4 seconds
4	15	34.542	Optimal	-	284 seconds
5	5	1.5	Optimal	-	1 seconds
5	10	14.832	Optimal	-	12 seconds
5	15	13.887	Optimal	-	29 seconds
6	10	9.244	Optimal	-	2 seconds
6	15	28.65	Optimal	-	100 seconds

Table 10: Output of robust model with medium compatibility

# of m/c's	# of orders	OFV	OPT?	Opt. Gap	Solution Time
3	5	9	Optimal	-	1 seconds
3	10	23.332	Optimal	-	140 seconds
3	15	16.571	Optimal	-	2245 seconds
4	5	0.875	Optimal	-	1 seconds
4	10	23.048	Optimal	-	123 seconds
4	15	36.921	Optimal	-	1211 seconds
5	5	18.5	Optimal	-	1 seconds
5	10	36.474	Optimal	-	156 seconds
5	15	9.284	Optimal	-	399 seconds
6	10	11.068	Optimal	-	32 seconds
6	15	18.881	Not	44.73%	10800 seconds

Within robust model, we have a parameter for breakdown duration for each machine and we reach minimum tardiness values for machine schedule regarding breakdown. Within deterministic model, there are possible risks for unexpected tardiness in the machine schedule since breakdown duration is not included in the model.

Table 11: Output of robust model with high compatibility

# of m/c's	# of orders	OFV	OPT?	Opt. Gap	Solution Time
3	5	7.588	Optimal	-	2 seconds
3	10	27.054	Optimal	-	1507 seconds
3	15	11.154	Optimal	-	5222 seconds
4	5	5.013	Optimal	-	1 seconds
4	10	20.304	Optimal	-	3287 seconds
4	15	11.499	Optimal	-	6925 seconds
5	5	3.9	Optimal	-	1 seconds
5	10	12.686	Optimal	-	310 seconds
5	15	23.736	Optimal	-	6016 seconds
6	10	22.438	Optimal	-	764 seconds
6	15	25.568	Not	76.99%	10800 seconds

To evaluate the benefits of robust model on machine scheduling, we provide input the sequence of orders on the machines found by deterministic model to the robust model. We expect that new objective function values will be higher or will remain same compared to objective function values of robust model because robust model gives the best results by considering breakdown duration. New objective function values of robust model by inputting sequence found from deterministic model and difference of it from robust model are shown in Table 12 - 14.

New objective function values are higher as we expected, except for the instance with 6 orders and 15 machines with high machine compatibility. The reason is that we could not reach optimal solution for that instance as shown in Table 11.

By ignoring this exceptional case, we see that we cannot anticipate possible risks for tardiness if we implement machine scheduling without regarding breakdown. Also, we see that unanticipated tardiness increases when the ratio of the total number of orders to the total number of machines increases in general. When this ratio is higher, the robust model is more significant to prefer.

Until now, we have found objective function values and sequence of orders on machines for all instances regarding worst-case breakdown duration. However, we have indicated that uncertain breakdown values are in an ellipsoidal uncertainty set. Therefore, we expect that

Table 12: Objective function values of robust model, by inputting sequence of deterministic model, with low compatibility

# of m/c's	# of orders	OFV	Difference
3	5	10.155	0
3	10	16.037	0
3	15	27.707	2.887
4	5	4.904	0
4	10	11.175	0.1
4	15	37.531	2.989
5	5	2.5	1
5	10	15.107	0.275
5	15	17.399	3.512
6	10	9.244	0
6	15	29.041	0.391

Table 13: Objective function values of robust model, by inputting sequence of deterministic model, with medium compatibility

# of m/c's	# of orders	OFV	Difference
3	5	9.823	0.823
3	10	27.286	3.954
3	15	25.206	8.635
4	5	0.875	0
4	10	23.048	0
4	15	38.802	1.881
5	5	18.5	0
5	10	36.474	0
5	15	10.4	1.116
6	10	11.068	0
6	15	21.165	2.284

the worst case breakdown values for all order and machine assignments do not occur. Next, $\xi_{i,m}$ values are randomly generated based on ρ_m. $\xi_{i,m}$ values change between $[0, \rho_m]$ in ellipsoidal uncertainty set. We get 10 different samples of $\xi_{i,m}$ values for each instance. Based on these values, we calculate corresponding $\hat{\epsilon}_{i,m}$ for each sample with following equation:

$$\hat{\epsilon}_{i,m} = R_i \xi_{i,m} \quad \forall i \in I, \ \forall m \in M$$

Table 14: Objective function values of robust model, by inputting sequence of deterministic model, with high compatibility

# of m/c's	# of orders	OFV	Difference
3	5	7.588	0
3	10	29.933	2.879
3	15	18.111	6.957
4	5	5.013	0
4	10	22.613	2.309
4	15	15.261	3.762
5	5	3.9	0
5	10	13.031	0.345
5	15	31.745	8.009
6	10	22.438	0
6	15	24.315	-1.253

Then, we input $\hat{\epsilon}_{i,m}$ values to the model with breakdown introduced in Section 4.1.3. Also, we input the sequences found by robust model which we reach by the worst case breakdown values to this model. For each sample, we compute the objective function values of model. For each instance, we calculate the average objective function values of corresponding 10 samples. We expect that objective function values will be smaller because we do not take the worst case values of machine breakdown. Table 15 - 17 shows objective function values of robust model and average objective function values of model with breakdown by inputting sequences of robust model. We observe that tardiness values are smaller when we use breakdown values in the ellipsoidal uncertainty set as expected.

After that, we input sequences found by deterministic model to the model with breakdown in order to evaluate what we miss when breakdown duration is not considered. We compute the average objective function values of 10 samples for each instance.

Table 18 - 20 shows average objective function values of model with breakdown by inputting sequences of robust model and deterministic model and differences between them. Since breakdown duration vary depending on order and machine, sequences found by deterministic model give better solutions for some of the instances as expected. However, sequences found by robust model has better objective function values for most cases and in

Table 15: Average OFV of model with breakdown, by inputting sequence of robust model, with low compatibility

# of m/c's	# of orders	OFV-Robust Model	Avg OFV for the sequence of Robust Model
3	5	10.155	7.983
3	10	16.037	10.415
3	15	24.82	20.422
4	5	4.904	3.731
4	10	11.075	7.392
4	15	34.542	25.448
5	5	1.5	0.681
5	10	14.832	10.566
5	15	13.887	11.030
6	10	9.244	6.890
6	15	28.65	19.232

Table 16: Average OFV of model with breakdown, by inputting sequence of robust model, with medium compatibility

# of m/c's	# of orders	OFV-Robust Model	Avg OFV for the sequence of Robust Model
3	5	9	6.819
3	10	23.332	18.083
3	15	16.571	12.588
4	5	0.875	0.048
4	10	23.048	17.172
4	15	36.921	29.544
5	5	18.5	15.968
5	10	36.474	25.565
5	15	9.284	6.247
6	10	11.068	4.269
6	15	18.881	9.933

total.

Here is that we present Gantt Charts for the instance having 3 machines, 15 orders and medium compatibility.

Figure 5 shows the schedule found by deterministic model without regarding machine

Table 17: Average OFV of model with breakdown, by inputting sequence of robust model, with high compatibility

# of m/c's	# of orders	OFV-Robust Model	Avg OFV for the sequence of Robust Model
3	5	7.588	4.566
3	10	27.054	19.690
3	15	11.154	7.445
4	5	5.013	1.608
4	10	20.304	14.680
4	15	11.499	8.836
5	5	3.9	1.190
5	10	12.686	7.160
5	15	23.736	18.110
6	10	22.438	14.435
6	15	25.568	13.178

Table 18: Average OFV of model with breakdown, by inputting sequence of deterministic model, with low compatibility

# of m/c's	# of orders	Avg OFV for Robust Model	Avg OFV for Deterministic Model	Difference (Deterministic-Robust)
3	5	7.983	7.983	0
3	10	10.415	10.415	0
3	15	20.422	22.275	1.853
4	5	3.731	3.731	0
4	10	7.392	7.427	0.035
4	15	25.448	26.283	0.835
5	5	0.681	0.681	0
5	10	10.566	8.514	-2.052
5	15	11.030	12.622	1.592
6	10	6.890	6.890	0
6	15	19.232	18.422	-0.81

breakdowns. Figure 6 shows the schedule found by robust model regarding machine breakdowns. Figure 7 shows the schedule found by robust model which we input the sequence of deterministic model. Figure 6 and Figure 7 show schedules with the worst case of breakdown values. Next, we investigate tardiness values found by random generated breakdown

Table 19: Average OFV of model with breakdown, by inputting sequence of deterministic model, with medium compatibility

# of m/c's	# of orders	Avg OFV for Robust Model	Avg OFV for Deterministic Model	Difference (Deterministic-Robust)
3	5	6.819	6.708	-0.111
3	10	18.083	19.826	1.743
3	15	12.588	17.140	4.552
4	5	0.048	0.048	0
4	10	17.172	17.172	0
4	15	29.544	28.283	-1.261
5	5	15.968	16.099	0.131
5	10	25.565	25.565	0
5	15	6.247	6.266	0.019
6	10	4.269	4.336	0.067
6	15	9.933	8.011	-1.922

Table 20: Average OFV of model with breakdown, by inputting sequence of deterministic model, with high compatibility

# of m/c's	# of orders	Avg OFV for Robust Model	Avg OFV for Deterministic Model	Difference (Deterministic-Robust)
3	5	4.566	4.604	0.038
3	10	19.690	20.350	0.66
3	15	7.445	12.208	4.763
4	5	1.608	1.705	0.097
4	10	14.680	13.150	1.53
4	15	8.836	9.981	1.145
5	5	1.190	1.144	-0.046
5	10	7.160	7.163	.003
5	15	18.110	20.135	2.025
6	10	14.435	14.432	-0.003
6	15	13.178	9.780	-3.398

values in ellipsoidal uncertainty set. We select the maximum tardiness values for robust model and deterministic model. Figure 8 and Figure 9 shows the schedule for the selected tardiness values found sequence of the robust and deterministic model, respectively.

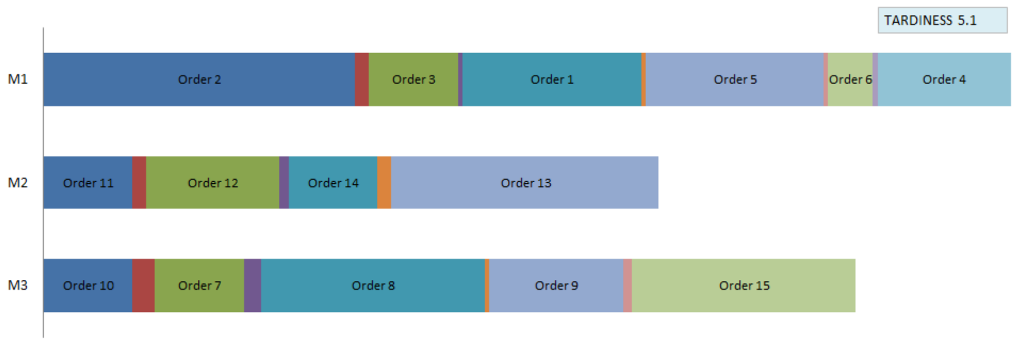

Figure 5: Machine Schedule of Deterministic Model, without breakdown

Figure 6: Machine Schedule of Robust Model, with the worst case of breakdown values

For these charts, the parts indicating the order number show the processing time of orders. In Figure 5, we see that there is one part between each order. They indicate setup times between orders. In Figure 6 - 9, we see that there are two parts between each order. First of them is breakdown duration of the previous order, other one is setup time between orders.

When we evaluate the comparative results with deterministic model, we observe that robust model is significant to recognize possible increases in the finishing time of orders and possible risks for tardiness due to uncertain machine breakdown. By realizing these risks earlier, required actions can be worked on instead of realizing them at last minutes.

When we investigate the penalty costs, we see that penalty cost due to production disruptions is higher than penalty cost due to revision in the frozen plan. If the tardy order is in frozen plan, production of end-product can be delayed with a penalty cost due to the revision in the frozen plan instead of causing a higher penalty cost with production

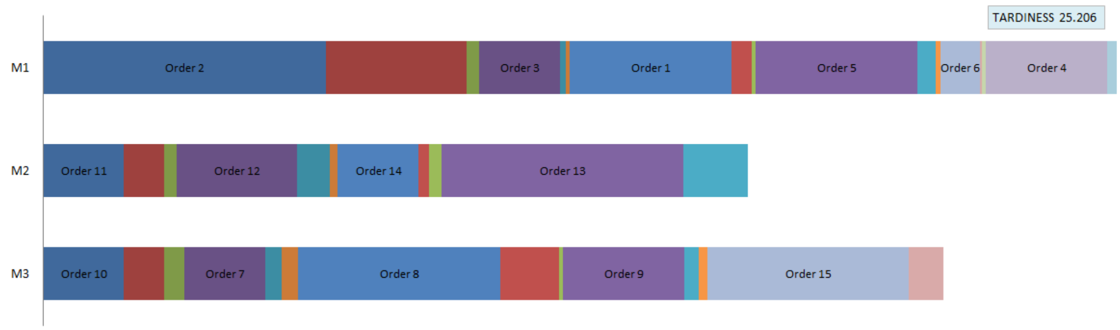

Figure 7: Machine Schedule of Robust model, with given sequence of deterministic model, and with the worst case of breakdown values

Figure 8: Machine Schedule of Model with Breakdown, with given sequence of robust model, max OFV found by random generated breakdown values

Figure 9: Machine Schedule of Model with Breakdown, with given sequence of deterministic model, max OFV found by random generated breakdown values

disruptions. If the tardy order is not in the frozen plan, the plan can be revised without any additional penalty cost.

On the other hand, when there is a expected tardiness for an order, working overtime for the machine which order is assigned or studies to increase capacity for the molds of the tardy order can be evaluated.

Meanwhile, we can keep more realistic inventory level by scheduling orders on machines based on the robust model. Within this model, breakdown values are considered when determining order machine assignments and orders are assigned earlier time units if possible. Thus, inventory level is defined by optimization model instead of an intuitive approach.

CHAPTER VI

CONCLUSION

In this study, we investigate plastic injection machine scheduling with sequence dependent setup time and unexpected machine breakdown. Orders for plastic injection materials have certain processing time defined by the capacity of corresponding mold and they have strict due date to satisfy. Thus, the objective is minimizing tardiness. We propose three different mathematical models within the study: deterministic model, model with breakdown and robust model. Deterministic model excluding machine breakdown does not take possible risks for breakdown into account. This may lead to unanticipated tardiness in the machine schedule. Therefore, we prepare a model including breakdown. Since this model has an uncertain parameter, we prefer to implement robust optimization which regarding worst-case analysis. When we evaluate outputs of the models, we observe that deterministic model fails to notice tardiness due to uncertain machine breakdown in worst-cases. Moreover, we examine the performance of models with sequence found by deterministic and robust model by integrating random generated breakdown values in ellipsoidal uncertainty set. Again, we observe that deterministic model fails to notice tardiness due to uncertain machine breakdown in most cases.

CHAPTER VII

FUTURE RESEARCH

We observe that proposed robust model does not give optimal solutions for the instance of 6 machines, 15 orders with medium and high compatibility in limited solution time (3 hours). As future research, robust models for more than 6 machines can be worked on in order to reach optimal solutions.

On the other hand, we observe that finding lower bound for objective function value increases the performance of robust model to reach optimal solution. To able to solve instances with higher number of machines and orders or to be able to reach solutions faster, lower bound for the objective functions can be improved.

Meanwhile, we work on the objective of minimizing tardiness. However, minimizing setup time can be other considerations in order to increase production efficiency as well as it does not increase the tardiness.

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