Essays on Monetary Unions

by

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To my love Merve

ABSTRACT

Essays on Monetary Unions
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This dissertation is composed of three chapters. The first chapter focuses on the derivation of an appropriate objective welfare function to compare monetary policies across different monetary regimes using the Linear Quadratic (LQ) Approach. Benigno and Benigno (2006) offers a method to derive quadratic welfare criteria for policy analysis using LQ Approach. The welfare criteria derived using this method ranks monetary policies accurately in a monetary regime. However, it does not necessarily provide accurate ranking of monetary policies across different regimes, namely, the flexible exchange rate regime, the cooperative flexible exchange rate regime, and the monetary union. Yet, choosing a monetary regime is as important as choosing the optimal monetary policy in a monetary regime. This chapter shows that Benigno and Benigno (2006)'s method can also compare policies across these regimes if it is assumed that the economy is in a steady state in the initial period. This assumption is equivalent to assuming that the monetary regime choice is announced one period in prior when the economy is in a steady state. Considering the fact that monetary regime changes are well communicated, this assumption remains fairly weak.

The second chapter studies the monetary regime choice between the monetary union and the flexible exchange rate regime in a large open economy framework. The classical approach argues that a monetary union should be established between countries with similar shocks so that the cost of the one-size-fits-all monetary policy is minimized. This chapter reveals that if there are inefficient shocks (i.e. shocks which create trade offs between output and inflation stabilization) and countries are large enough to affect each other, then the relative variance (mean preserving spread) of inefficient shocks should also be considered in the monetary regime choice. A country should choose the monetary union over the flexible exchange rate regime

only if the variance of foreign inefficient shocks is close to or larger than the variance of domestic inefficient shocks. This is on the condition that losing exchange rate flexibility has little or no cost. By joining a monetary union, a country "ties the hand of the other" countries joining the same union: it prevents welfare decreasing foreign monetary policy responses to foreign inefficient shocks. A monetary union is a Pareto Improvement only if the variances of member countries' inefficient shocks are close enough.

The third chapter continues to investigate the incentive to join a monetary union to "tie the hand of the others". It studies the effects of the price rigiditiy and the trade elasticity on this incentive and the monetary regime choice between the flexible exchange rate reigme and the monetary union. It finds that welfare gain from "tying the hand of the other" country decreases as prices become more flexible because more flexible prices compensate shocks and the exchange rate intertia better. The mational welfare cost of variation of the foreign output gap in the flexible exchange rate regime increases sharply with stickier prices. This increases the incentive to join the monetary union in order to "tie the hand of the other" country. Chapter 3 also concludes that under symmetric parameters and shocks, the need to "tie the hand of the other" country soars as the trade elasticity increases. However, if the trade elasticity is low, then welfare cost of losing the exchange rate flexibility may dominate welfare gain from "tying the hand of the other". Consequently, given a level of price rigidity, if the trade elasticity is low, then it is likely that a country prefers the flexible exchange rate regime over the monetary union. If the trade elasticity is high, then the monetary union dominates the flexible exchange rate regime.

ÖZETÇE

Parasal Birlikler Üstüne Denemeler
Cem Görgün
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Bu tez üç kısımdan oluşur. İlk kısımda farklı parasal rejimlerdeki para politikalarını Linear Quadratic (LQ) yöntem kullanarak doğru karşılaştıracak bir refah fonksiyonunun nasıl elde edileceği tartışılır. Benigno & Benigno (2006)'nun LQ yaklaşımını kullanarak bir kur rejiminde para politikalarını karşılaştırmak mümkündür. Ancak farklı rejimlerdeki para politikalarını karşılaştırmak mümkün değildir. Oysa kur rejimi seçmek en az bir rejimdeki para politikasına karar vermek kadar önemlidir. İlk kısım, eğer rejim değişikliği sırasında ekonominin istikrarlı dengesinde olduğu varsayılırsa, Benigno & Benigno (2006)'nun LQ metodunun farklı kur rejimlerindeki para politikalarını da doğru şekilde karşılaştırabildiğini gösterir. Kur rejimi ile ilgili değişikliklerin iletişiminin dikkatli ve uzun vadede yapıldığı göz önünde bulundurulursa, kur rejimi değişikliklerinde ekonominin istikrarlı dengesinde olduğu varsayımı zararsızdır.

İkinci kısım büyük ve açık ekonomilerde parasal birlik ile dalgalı kur rejimi arasındaki seçimi inceler. Klasik argüman ekonomik şoklar arasndaki korelasyonu pozitif ve yüksek olan ülkelerin parasal birlik oluşturmalarının sağlıklı olduğunu söyler. Ancak bu kısım, klasik argümanın yeterli olmadğını ortaya koyar. Maliyet şokları yaşayan ülkelerde, kur rejimi tercihi maliyet şoklarının görece varyansları da göz önünde bulundurularak yapılmalıdır. Bir ülkenin başka ülkelerle parasal birlik kurarak refahını arttırabilimesi için maliyet şokları parasal birlik kuracağı diğer ülkelere oranla daha küçük olmalıdır. Yani ülkenin maliyet şoklarının varyansı diğer ülkelerin maliyet şoklarının varyansına yakın ya da daha küçük olması gerekir. Bir parasal birliğin Pareto Optimum olabilmesi için ise parasal birliği kuracak ülkelerin maliyet şoklarının varyanslarının birbirlerine yakın olması gerekir.

Üçüncü kısım fiyat yapışkanlığının ve ticaret elastikliğinin ikinci kısımda anlatılan parasal birlik ile dalgalı kur rejimi arasındaki refah farkını nasıl etkilediklerini

inceler. Fiyat yapışkanlığı azaldıkça refah ekonomik şoklardan ve parasal birlikteki sabit kur rejiminin yarattığı fiyat katılıklarından daha az etkilenir. Bir ülke, dalgalı kur rejiminde fiyat yapışkanlığı arttıkça, yabancı ülkelerin çıktı açıklarındaki dalgalanmadan daha fazla etkilenir. Bu durum parasal birlik kurma insiyatifini arttırır. Dahası, simetrik parametreler ve şoklar altında, ticaret elastikliği arttıkça parasal birlik kurma insiyatifi artar. Ancak ticaret elastikliği düşük ise, parasal birlikteki sabit kur rejimin maliyeti yüksek olabilir. Bu sebeple, dalgalı kur rejiminde, refah parasal birliğe göre daha yüksek kalabilir. Dolaysyla, belli bir fiyat yapışkanlıında, ticaret esnekliği yüksek ise parasal birlik, ticaret esnekliği düşük ise dalgalı kur rejiminin avantajlı olma olasılığı yüksektir.

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NOMENCLATURE

CB	Central Bank
CP	Central Planner
CPI	Consumer's Price Index
DSGE	Dynamic Stochastic General Equilibrium
IRF	Impulse Response Function
LCP	Local Currency Pricing
LoOP	Law of One Price
LQ	Linear Quadratic
NK	New-Keynesian
PCP	Producer's Currency Pricing
PI	Pareto Improvement
PPI	Producer's Price Index
SOE	Small Open Economy

Chapter 1

COMPARING MONETARY REGIMES USING LINEAR QUADRATIC APPROACH IN A TWO-COUNTRY NEW-KEYNESIAN MODEL

1.1 Introduction

Chapter 1 investigates the derivation of an appropriate objective welfare function (i.e. welfare criteria) to compare monetary policies across different monetary regimes using LQ Approach. LQ Approach can be broadly summarized as evaluating welfare using quadratic welfare criterion (social welfare function approximated up to the second order) and linearized structural equations (approximated up to the first order). Benigno and Benigno (2006) offers a method to obtain appropriate objective welfare functions in a two-country economy. This method accurately ranks monetary policies and determines optimal monetary policy rules in a given monetary regime for a country or the economy. However, this method fails to compare policies of different regimes accurately without further assumption. Yet, choosing a monetary regime is as important as determining optimal policy in a given regime. Even though this is a general problem for this method, this chapter specifically focuses on comparing the flexible exchange rate regime, the cooperative flexible exchange rate regime, and the monetary union.[1]

[1] These regimes are well described in the NK literature. The flexible exchange rate regime refers to the "Nash Case" in Clarida et al. (2002) or the "non-cooperative regime" in Benigno and Benigno (2006). The cooperative flexible exchange rate regime is called the "cooperative equilibrium" in both Clarida et al. (2002) and Benigno and Benigno (2006). The monetary union is defined in Groll and Monacelli (2019). These regimes are described in details in later chapters.

This chapter offers a trivial assumption for Benigno and Benigno (2006)'s LQ Approach method so that approximated objective functions - welfare criteria derived using LQ Approach - accurately compare welfare across these regimes. It shows that Benigno and Benigno (2006)'s method accurately ranks monetary policies across the regimes if it is assumed that the economy is in a steady state in the initial period.

It is a standard assumption that an economy is in a steady state prior to the periods of analysis unless a study is particularly interested in a case in which the economy is away from its steady state (e.g. Yun, 2005). Furthermore, the assumption that this chapter proposes is equivalent to assuming that a change in the monetary regime is announced one period in prior when the economy is in a steady state. Considering the fact that in general monetary regime changes are well communicated and do not come by surprise, the assumption proposed in this chapter remains fairly weak.

Section 1.2 revises the literature and explains the methodology. Section 1.3 introduces a two-country economy model and the monetary regimes. Section 1.4 describes LQ Approach of Benigno and Benigno (2006). Section 1.5 presents the problem and provides the solution. Section 1.6 is the conclusion.

1.2 Literature Review and Methodology

Many studies try to determine optimal monetary/fiscal policies under different environments or regimes. Studies on optimal polices frequently rely on NK-DSGE models and these models are often non-linear (see Friedman and Woodford, 2010, Chapters 13, 14, 15). Researchers prefer to linearize these models because they are

(i) unsolvable analytically or time-consuming to solve, or

(ii) not simple enough to provide any intuition.

There are various ways to linearize an optimal policy problem (also known as the Ramsey Problem). A method is to determine optimality conditions of the non-linear Ramsey Problem, then linearizing optimality conditions and structural equations to solve for an optimal policy (e.g. Khan et al., 2003). However, most studies prefer LQ Approach: they linearize structural equations and derive a quadratic approximation to the objective function. Then, they solve the approximated Ramsey Problem to determine optimal policies.[2] There are various methods to apply LQ Approach. Rotemberg and Woodford (1998) presents a most common method to derive a quadratic welfare criterion in a closed economy model.[3] Gali and Monacelli (2005) extends this method to SOE models. However, this method requires that the original objective function is approximated around the efficient steady state (or around a steady state sufficiently close to the efficient steady state).[4] This limits number of cases which can be studied. Benigno and Woodford (2005) offers a similar method which produces a quadratic welfare criterion without necessarily approximating around the efficient steady state in a closed economy.

The limitation of RW method is particularly problematic and cannot be used to compare national welfare between the flexible exchange rate regime, the cooperative exchange rate regime, and the monetary union in two country (or multi-country) models because the efficient steady state depends on the monetary regime and the country. Following Benigno and Woodford (2005), Benigno and Benigno (2006) presents a method to derive quadratic welfare criteria in a two-country economy without necessarily relying on an efficient steady state. This should have been sufficient to compare monetary regimes in terms of national welfare. However, BB method imposes initial conditions on PPIs which restricts accurate ranking of policies across different regimes. This chapter shows that if it is assumed that the economy is in a steady state in the initial period in the monetary regimes compared,

[2]The difference between the two methods is small but pivotal. In the former method, the original optimal policy problem is solved. Then, conditions derived from the original problem are linearized. In the latter method, LQ Approach, the original problem is transformed into an approximated problem and then solved.

[3]I refer Rotemberg and Woodford (1998)'s method as RW method.

[4]The efficient steady state is the steady state which maximizes welfare of the relevant country/region. For details, see Woodford (2011); Galí (2015).

then monetary policies in these regimes can be compared using BB method.[5]

To this end, a two-country economy model based on Corsetti et al. (2010) is built. Then, the flexible exchange rate regime, the cooperative flexible exchange rate regime, and the monetary union are introduced. The reason why BB method fails to compare monetary regimes is explained. Later, it is shown that assuming the economy is in a steady state in the first period mitigates this problem.

1.3 The Model

This section describes a two-country economy model with nominal price rigidity, producer's currency pricing (PCP), and complete asset markets based on Corsetti et al. (2010). The model is similar to the models of Benigno and Benigno (2006) and Groll and Monacelli (2019). The former has the government spending and the latter has home bias and consumption preference shocks. Compared to Groll and Monacelli (2019) and using its notation, this model assumes $Z_{C,t} = Z^*_{C,t} = 1$, $\gamma = 1-n$, and $\gamma^* = n$. Compared to Benigno and Benigno (2006), it assumes $s_c = 1$ and $G_t = G^*_t = 0 \ \forall t$.

There are two countries, A and B, and infinitely many households aligned between 0 and 1. They have n and $1-n$ shares of population respectively. A household j in Country i produces a differentiated good and consumes a bundle of domestic and foreign goods. It maximizes the following utility function.

$$\tilde{U}^j \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[U(C_t^j) - V(y_t(j); a_t^i) \Big] \tag{1.1}$$

$V(y_t(j); a_t^i) \equiv (a_t^i)^{-\eta} \frac{y_t(j)^{1+\eta}}{1+\eta}$ is disutility from labor. a_t^i denotes stochastic and country-dependent productivity shocks, $y_t(j)$ is the differentiated goods produced by household j at time t with the inverse elasticity of goods production $\eta \geq 0$. $\beta \in (0;1)$ is the intertemporal discount factor. $U(C_t^j) \equiv \frac{(C_t^j)^{1-\rho}}{1-\rho}$ is utility gained from consuming goods and $\rho > 0$ is the inverse elasticity of intertemporal substitu-

[5] I refer to the flexible exchange rate regime, the cooperative flexible exchange rate regime, and the monetary union.

tion. C_t^j is a Dixit-Stiglitz aggregator for household j's consumption constituted of A's and B's goods bundles.

$$C_t^j \equiv \left[n^{1/\theta}(C_{A,t}^j)^{(\theta-1)/\theta} + (1-n)^{1/\theta}(C_{B,t}^j)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)} \tag{1.2}$$

$\theta > 0$ is the elasticity of substitution of domestic and foreign goods (also referred as the trade elasticity or the intratemporal elasticity of substitution). The model does not allow for home bias.[6] $C_{A,t}^j$ is the bundle of differentiated goods of Country A.

$$C_{A,t}^j \equiv \left[\left(\frac{1}{n}\right)^{1/\sigma} \int_0^n c^j(h)^{(\sigma-1)/\sigma} dh\right]^{\sigma/(\sigma-1)} \tag{1.3}$$

$C_{B,t}^j$ is the bundle of differentiated goods of Country B.

$$C_{B,t}^j \equiv \left[\left(\frac{1}{1-n}\right)^{1/\sigma} \int_n^1 c^j(f)^{(\sigma-1)/\sigma} df\right]^{\sigma/(\sigma-1)} \tag{1.4}$$

$\sigma > 0$ is the elasticity of substitution across goods produced within a country. Let $p_t^i(k)$ be the price of a differentiated good produced by a household k in terms of Country i's currency. Then, the Producer's Price Index (PPI) of Country A in terms of Country i's currency is

$$P_{A,t}^i \equiv \left[\left(\frac{1}{n}\right) \int_0^n p_t^i(h)^{1-\sigma} dh\right]^{1/(1-\sigma)} \tag{1.5}$$

Similarly, PPI of Country B in terms of Country i's currency is

$$P_{B,t}^i \equiv \left[\left(\frac{1}{1-n}\right) \int_n^1 p_t^i(f)^{1-\sigma} df\right]^{1/(1-\sigma)} \tag{1.6}$$

The Consumer's Price Index (CPI) in terms of Country i's currency is

$$P_t^i \equiv \left[n{P_{A,t}^i}^{1-\theta} + (1-n){P_{B,t}^i}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{1.7}$$

[6]For a similar model with home bias, I refer to Faia and Monacelli (2008).

S_t is the exchange rate at time t. The model assumes PCP meaning that sellers tag one price in their own currency for their goods in all markets. Then, the law of one price (LoOP) holds.[7]

$$p_t(k) = S_t p_t^*(k) \quad \forall t \tag{1.8}$$

(1.8) implies that $P_t = S_t P_t^* \ \forall t$. In the absence of home bias, this means that purchasing power parity (PPP) holds at any t. $T_t \equiv \frac{P_{B,t}}{P_{A,t}}$ is the terms of trade. Then, CPI can also be written as

$$\begin{aligned} P_t^i &= \left[(1-n) + n{T_t}^{1-\theta}\right]^{\frac{1}{1-\theta}} P_{A,t}^i \\ &= \left[(1-n){T_t}^{-(1-\theta)} + n\right]^{\frac{1}{1-\theta}} P_{B,t}^i \end{aligned} \tag{1.9}$$

The law of motion of the exchange rate is

$$\frac{T_t}{T_{t-1}} = \frac{S_t}{S_{t-1}} \frac{\Pi_{B,t}^*}{\Pi_{A,t}} \quad \forall t \tag{1.10}$$

If the exchange rate is fixed, i.e. $\Delta S_t = 0$, (1.10) becomes

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_{B,t}^*}{\Pi_{A,t}} \quad \forall t \tag{1.11}$$

Households trade one-period nominal state contingent assets denominated in A's currency. $v_{t,t+1} \equiv v(h^{t+1}|h^t)$ is the period-t risk adjusted price of an asset which gives 1 domestic currency if h^{t+1} is realized, 0 otherwise. Asset markets are complete both within and across countries. Households have identical initial (0) wealth. Then, consumption risk is perfectly insured between all agents implying

$$C_t^j = C_t \quad \forall j, t \tag{1.12}$$

A's one period nominal interest rate at time t is $(1+i_t) \equiv \left[\mathbb{E}_t v_{t,t+1}\right]^{-1}$ whereas B's one period nominal interest rate at time t is $(1+i_t^*) \equiv \left[\mathbb{E}_t v_{t,t+1} \frac{S_{t+1}}{S_t}\right]^{-1}$.

[7]For simplicity, throughout the paper, I suppress superscripts of A's currency and denote B's currency with $*$. (1.8) means $p_t^A(k) = S_t p_t^B(k)$

Household j is the monopolistic producer of one of the differentiated goods. Total demand for the differentiated good j produced at Country i at time t is

$$y_t(j) = \left(\frac{p_t^i(j)}{P_{i,t}^i}\right)^{-\sigma}\left(\frac{P_{i,t}^i}{P_t^i}\right)^{-\theta} C_t \tag{1.13}$$

Producers have price setting mechanism à la Calvo (1983): Each producer is allowed to change the price of its good at any period with probability $1-\alpha$. A producer sets the price of its differentiated product, $\tilde{p}_t(j)$, to maximize the expected discounted value of its net profits taking $P_{A,t}$, $P_{B,t}$, P_t (or $P_{A,t}^*$, $P_{B,t}^*$, P_t^*) as given. Pricing problem of a seller which can change its price at time t is

$$\max_{\tilde{p}_t(j)} \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \Big[\frac{U(C_{t+k})}{P_{t+k}}(1-\tau_{i,t+k})\tilde{p}_t(j)y_{t+k}(j) - V(y_{t+k}(j); a_{t+k}^i)\Big] \tag{1.14}$$

$\tau_{i,t}$ is an exogenous time-varying tax on sales of Country i. The optimal choice of $\tilde{p}_t(j)$ is

$$\tilde{p}_t(j) = \frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\alpha\beta)^k V_y(y_{t+k}(j); a_{t+k})y_{t+k}(j)}{\mathbb{E}_t \sum_{k=0}^{\infty}(\alpha\beta)^k \frac{1}{\mu_{i,t+k}}\frac{U_C(C_{t+k})}{P_{t+k}} y_{t+k}(h)} \tag{1.15}$$

$\mu_{i,t+k} \equiv \frac{\sigma}{(1-\tau_{i,t})(\sigma-1)}$ is the time-varying markup shock of Country i.[8] Given the Calvo price setting mechanism, the evolution of the domestic price index of Country i is

$$P_{i,t}{}^{1-\sigma} = \alpha P_{i,t-1}{}^{1-\sigma} + (1-\alpha)\tilde{p}_t(j)^{1-\sigma} \tag{1.16}$$

Finally, the economy is constituted of four types of shocks: productivity and markup shocks for A and B. $\xi_t \equiv [a_{A,t}\ a_{B,t}\ \mu_{A,t}\ \mu_{B,t}]'$ is the vector of all exogenous shocks at period t. A type of shock follows an AR(1) process with the persistence γ. Let x_t be a shock, then

$$x_t = \gamma x_{t-1} + \epsilon_{x,t}, \quad \epsilon_x \sim \text{i.i.d}(0, \sigma_x^2) \tag{1.17}$$

[8] Even though actual variation originates from the sales tax, the model is treated as if markups move stochastically, not the sales tax.

1.3.1 Monetary Regimes

This section describes the flexible exchange rate regime ($\mathcal{F}$), the cooperative flexible exchange rate regime ($\mathcal{C}$), and the monetary union ($\mathcal{M}$).

In $\mathcal{F}$, two national CBs (monetary authority) exist, each is attached to a country. A CB aims to maximize national welfare of the country it is attached to: $W_A \equiv \int_0^n \tilde{U}^j dj$ for Country A and $W_B \equiv \int_n^1 \tilde{U}^j dj$ for Country B. National welfare (sum of the expected discounted utilities of households) of A is

$$W_A = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[U(C_{t+k}) - \frac{1}{n} \int_0^n V(y_{t+k}(j); a_{t+k}^A) dj \right] \tag{1.18}$$

$$= \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[U(C_{t+k}) - V(Y_{A,t+k}; a_{t+k}^A) \mathit{\Delta}_t \right] \tag{1.19}$$

where $Y_{A,t} \equiv \left(\frac{P_{A,t}}{P_t}\right)^{-\theta} C_t$ and $\mathit{\Delta}_t \equiv \frac{1}{n} \int_0^n \left(\frac{p_t(j)}{P_{A,t}}\right)^{-\sigma(1+\eta)} dj$ is the price dispersion term of A. Similarly, welfare of B is

$$W_B = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[U(C_{t+k}) - V(Y_{B,t+k}; a_{t+k}^B) \mathit{\Delta}_t^* \right] \tag{1.20}$$

with $Y_{B,t} \equiv \left(\frac{P_{A,t}}{P_t}\right)^{-\theta} C_t$ and $\mathit{\Delta}_t^* \equiv \frac{1}{1-n} \int_n^1 \left(\frac{p_t(j)}{P_{B,t}}\right)^{-\sigma(1+\eta)} dj$ is the price dispersion term of B. To maximize welfare of its country, a CB chooses a strategy to set $\{\Pi_{i,t}\}_{t=0}^{\infty}$ at $t = 0$ and takes other CB's strategy as given. Policymakers commit to their strategies they declared at $t = 0$. The exchange rate floats.

In $\mathcal{C}$, the exchange rate is flexible. A CP chooses $\{Y_{A,t}, Y_{B,t}, \Pi_{A,t}, \Pi_{B,t}^*, T_t\}_{t=0}^{\infty}$ to maximize welfare of the world economy $W \equiv \int_0^1 U^j dj$. Welfare of the economy, i.e. $W = nW_A + (1-n)W_B$, can be written as

$$W = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[U(C_{t+k}) - nV(Y_{A,t+k}, a_{t+k}^i) \mathit{\Delta}_t - (1-n)V(Y_{B,t+k}, a_{t+k}^i) \mathit{\Delta}_t^* \right] \tag{1.21}$$

In $\mathcal{M}$, A and B adopt a common currency and transfer their monetary authorities powers to the CB of the monetary union. In this setting, the fixed exchange rate condition (1.11) immediately implies a single policy rate and a common currency. The new monetary authority (the CB of the monetary union) chooses a sequence $\{Y_{A,t}, Y_{B,t}, \Pi_{A,t}, \Pi^{*}_{B,t}, T_t\}_{t=0}^{\infty}$ to maximize welfare in the union.

In all three regimes, the CBs or the CP pursue monetary policies under commitment meaning that they can efficiently alter expectations.

1.4 Linear Quadratic Approach

This section explains BB method in LQ Approach. To avoid spurious welfare comparisons, welfare criteria must accurately rank policies

(i) in a monetary regime, and

(ii) across monetary regimes.

The latter condition requires that objective functions and structural equations are approximated around the same steady state in all monetary regimes. Benigno and Benigno (2006) shows that there exist a common zero-inflation deterministic steady state for $\mathcal{F}$ and $\mathcal{C}$. Appendix A.1 provides that $\mathcal{M}$ has the identical zero-inflation deterministic steady state. Given this steady state, Table 1.1 presents the linearized structural equations when $\bar{\mu} = 1$. To keep the model simple, it is assumed that $\bar{\mu} = 1$. However, the results can be generalized to $\bar{\mu} \neq 1$ cases. (1.30) is the law of motion of the exchange rate when the exchange rate is flexible. When the exchange rate is fixed, as supposed to be in $\mathcal{M}$, this equation is replaced with (1.31).

The former condition requires that welfare criteria are quadratic in order to evaluate monetary policies using linearized structural equations accurately. RW method to derive quadratic welfare criteria is presented by Rotemberg and Woodford (1998), Woodford (2011) and subsequent works. It uses a second order approximation to an objective function and evaluate it using linearized structural equation. To evaluate optimal policy problems accurately using RW method, the approximated objective

Table 1.1: Structural Equations

Aggregate Demand	$\hat{C}_t = \mathbb{E}_t\hat{C}_{t+1} - \rho^{-1}(i_t - \mathbb{E}_t\pi_{A,t+1} - (1-n)\mathbb{E}_t\Delta\hat{T}_{t+1})$	(1.22)
	$\hat{C}^*_t = \mathbb{E}_t\hat{C}^*_{t+1} - \rho^{-1}(i^*_t - \mathbb{E}_t\pi^*_{B,t+1} + n\mathbb{E}_t\Delta\hat{T}_{t+1})$	(1.23)
Market Clearing Condition	$\hat{Y}_{A,t} = \hat{C}_t + \theta(1-n)\hat{T}_t$	(1.24)
	$\hat{Y}_{B,t} = \hat{C}^*_t - \theta n\hat{T}_t$	(1.25)
Risk Sharing	$\hat{C}_t = \hat{C}^*_t$	(1.26)
Aggregate Supply	$\pi_{A,t} = \beta\mathbb{E}_t\pi_{A,t+1} + \kappa(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}) + (1-n)\kappa\psi(\hat{T}_t - \tilde{T}^w_t) + u_t$	(1.27)
	$\pi^*_{B,t} = \beta\mathbb{E}_t\pi^*_{B,t+1} + \kappa^*(\hat{Y}_{B,t} - \tilde{Y}^w_{B,t}) - n\kappa^*\psi(\hat{T}_t - \tilde{T}^w_t) + u^*_t$	(1.28)
Terms of Trade	$\hat{T}_t - \tilde{T}^w_t = \theta^{-1}\left[(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}) - (\hat{Y}_{B,t} - \tilde{Y}^w_{B,t})\right]$	(1.29)
Exchange Rate (flexible)	$\Delta\hat{S}_t = \Delta\hat{T}_t + \pi_{A,t} - \pi^*_{B,t}$	(1.30)
Exchange Rate (fixed)	$0 = \Delta\hat{T}_t + \pi_{A,t} - \pi^*_{B,t}$	(1.31)

$\kappa \equiv (\rho+\eta)\frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\sigma\eta)}$, $\kappa^* \equiv (\rho+\eta)\frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*(1+\sigma\eta)}$, $\psi \equiv \frac{1-\rho\theta}{\rho+\eta}$, $u_t \equiv \kappa\frac{\hat{\mu}_{A,t}}{\eta+\rho}$, $u^*_t \equiv \kappa^*\frac{\hat{\mu}_{B,t}}{\eta+\rho}$, $\tilde{Y}^w_{A,t} \equiv \tilde{C}_t + (1-n)\theta\tilde{T}^w_t$, $\tilde{Y}^w_{B,t} \equiv \tilde{C}_t + -n\theta\tilde{T}^w_t$, $\tilde{T}^w_t = \frac{\eta}{(1+\theta\eta)}[\hat{a}_{R,t}]$, $\tilde{C}_t \equiv \frac{\eta}{(\eta+\rho)}(\hat{a}_{W,t})$, $\hat{\mu}^W_t \equiv n\hat{\mu}_{A,t} + (1-n)\hat{\mu}_{B,t}$, $\hat{\mu}^R_t \equiv \hat{\mu}_{A,t} - \hat{\mu}_{B,t}$, $\hat{a}^W_t \equiv n\hat{a}_{A,t} + (1-n)\hat{a}_{B,t}$, $\hat{a}^R_t \equiv \hat{a}_{A,t} - \hat{a}_{B,t}$.

function - the welfare criterion - should not include linear terms (terms should be second or higher order). Rotemberg and Woodford (1998) provides that if the approximation is around the efficient steady state, then linear terms in the approximation vanish.[9] However, the efficient steady state depends on the monetary regime and the Ramsey Problem. For example, in the monetary union, the CB aims to maximize welfare of the union, i.e averaged welfare of the two countries combined. To derive appropriate welfare criterion with RW method, the efficient steady state for the union should be considered. However, the efficient steady state of the union is different from the efficient steady state of a single nation.[10] If the efficient steady state from the monetary union's perspective is used to derive welfare criterion of a country, then linear terms in the second order approximation to this country's objective function do not disappear. Therefore, the welfare criterion of this country does not accurately rank monetary policies of the monetary union. This means

[9] I refer to Woodford (2011, Chapter 6.1) for details of an efficient steady state.

[10] For further discussion of the problem, I refer to Corsetti and Pesenti (2001), Kim and Kim (2003), Woodford (2011) Chapter 6.

that national welfare criteria and welfare criterion of the union cannot be derived simultaneously using the same steady state with RW method. Alternatively, if two different steady states are used, then the latter condition is violated.

BB method solves this problem. It replaces linear terms in the second order approximation to an objective welfare function with an appropriate linear combination of the second approximation of structural equations. Structural equations of the second order are used only to replace the linear terms in the second order approximation to objective functions. Once the quadratic welfare criteria are obtained, the optimal monetary policy problem is evaluated with the linearized structural equations. Therefore, BB method obtains quadratic welfare criteria for any monetary regime or group of countries without necessarily relying on a special case of steady state. It provides that welfare of Country-A can be approximated quadratically

$$W_A = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[\lambda_{y_A}\big(\hat{Y}_{A,t} - \tilde{Y}^A_{A,t}\big)^2 + \lambda_{y_B}\big(\hat{Y}_{B,t} - \tilde{Y}^A_{B,t}\big)^2 + \lambda_q\big(\hat{T}_t - \tilde{T}^A_t\big)^2 + \lambda_{\pi_A}{\pi_{A,t}}^2 + \lambda_{\pi_B}{\pi^*_{B,t}}^2\Big] \tag{1.32}$$

conditional on $\pi_{A,0}$. (1.32) appropriately ranks monetary policies for Country-A. Similarly, B's welfare loss function is

$$W_B = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[\lambda^*_{y_A}\big(\hat{Y}_{A,t} - \tilde{Y}^B_{A,t}\big)^2 + \lambda^*_{y_B}\big(\hat{Y}_{B,t} - \tilde{Y}^B_{B,t}\big)^2 + \lambda^*_q\big(\hat{T}_t - \tilde{T}^B_t\big)^2 + \lambda^*_{\pi_A}{\pi_{A,t}}^2 + \lambda^*_{\pi_B}{\pi^*_{B,t}}^2\Big] \tag{1.33}$$

conditional on $\pi_{B,0}$. Finally, welfare loss function of the world economy can be written as

$$\begin{aligned} W = -\frac{\lambda^w_y}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[& n\big(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}\big)^2 + (1-n)\big(\hat{Y}_{B,t} - \tilde{Y}^w_{B,t}\big)^2 \\ & + n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}^w_t\big)^2 + \frac{\sigma n}{\kappa}{\pi_{A,t}}^2 + \frac{\sigma(1-n)}{\kappa^*}{\pi^*_{B,t}}^2\Big] \end{aligned} \tag{1.34}$$

conditional on $\pi_{A,0}$ and $\pi_{B,0}$. $\lambda^w_y \equiv (\eta + \rho)$. λ_{y_A}, λ_{y_B}, λ_{π_A}, λ_{π_B}, λ_q, $\lambda^*_{y_A}$, $\lambda^*_{y_B}$, $\lambda^*_{\pi_A}$, $\lambda^*_{\pi_B}$, λ^*_q, $\tilde{Y}^A_{A,t}$, $\tilde{Y}^A_{B,t}$, $\tilde{T}^A_t$, $\tilde{Y}^B_{A,t}$, $\tilde{Y}^B_{B,t}$, $\tilde{T}^B_t$ are defined in Appendix A.3.

The key notion is that welfare criteria - i.e. (1.32), (1.33), (1.34) - are derived conditional on initial PPIs: $\{\pi_{i,0}\}_{i\in\{A,B\}}$. Mathematically, this means that monetary policy problem in all monetary regimes should have additional constraints

$$\pi_{A,0} = \bar{\pi}_{A,0} \tag{1.35}$$

$$\pi_{B,0} = \bar{\pi}_{B,0} \tag{1.36}$$

$\{\bar{\pi}_{i,0}\}_{i\in\{A,B\}}$ is arbitrary but welfare criteria rank monetary policies accurately conditional on $\{\bar{\pi}_{i,0}\}_{i\in\{A,B\}}$. Intuitively, this indicates that (1.32) is capable of accurately ranking only monetary policies which satisfy (1.35). Similarly, (1.33) accurately ranks monetary policies which satisfy (1.36); (1.34) accurately ranks monetary policies which satisfy (1.35) and (1.36).

In $\mathcal{F}$, the CB of A maximizes national welfare (1.32) subject to (1.27), (1.28), (1.29), and (1.35). Similarly, the CB of B maximizes (1.33) subject to (1.27), (1.28), (1.29), and (1.36). In $\mathcal{M}$, the CB of the union maximizes welfare of the union (1.34) subject to (1.27), (1.28), (1.29), (1.31), (1.35), and (1.36). In $\mathcal{C}$, the CP maximizes (1.34) subject to (1.27), (1.28), (1.29), (1.35), and (1.36). Given the path of exogenous variables, all endogenous variables in a monetary regime are determined using first order conditions of relevant monetary problem(s) and structural equations.

1.5 Comparing Different Monetary Regimes

To be able to compare two monetary regimes, the initial conditions - (1.35) and (1.36) - must be identical across the regimes. Otherwise the comparison is not accurate. To circumvent this problem, this chapter proposes to assume that there are no shocks in the economy at $t = 0$ in $\mathcal{F}$, $\mathcal{C}$, or $\mathcal{M}$. Then, $\pi_{A,0} = \pi_{B,0} = \pi_A = \pi_B = 0$ hold for all three regimes. (1.35) and (1.36) are satisfied and welfare criteria - (1.32), (1.33), and (1.34) - work appropriately. Ad hoc conditions - (1.35) and (1.36) - are avoided and they can be ignored. This result is presented in Proposition 1.

Proposition 1. *Let* $\xi_t^{\mathcal{R}} \equiv [a_{A,t}^{\mathcal{R}}\ a_{B,t}^{\mathcal{R}}\ \mu_{A,t}^{\mathcal{R}}\ \mu_{B,t}^{\mathcal{R}}]'$ *be the vector of all exogenous shocks* *and* $\pi^{\mathcal{R}}_{i,t}$ *be PPI of Country-i in a monetary regime* $\mathcal{R} \in \{\mathcal{F}, \mathcal{C}, \mathcal{M}\}$. *If* $\xi^{\mathcal{R}}_0 = \xi = [a^{\mathcal{R}}_A \; a^{\mathcal{R}}_B \; \mu^{\mathcal{R}}_A \; \mu^{\mathcal{R}}_B]'$, *then* $\pi^{\mathcal{R}}_{i,0} = 0$ *for* $i \in \{A, B\}$ *and* $\mathcal{R} \in \{\mathcal{F}, \mathcal{C}, \mathcal{M}\}$

Proof. See Appendix A.7. □

Proposition 1 says that if there are no shocks in the first period, then PPIs remain 0. Proposition 1 can be extended that if shocks are absent in the first period, then the economy remains in the steady state in the first period.[11]

It is possible to assume

$$\pi^{\mathcal{R}}_{A,0} = \bar{\pi}_{A,0} \quad \text{for} \quad \mathcal{R} \in \{\mathcal{F}, \mathcal{C}, \mathcal{M}\} \tag{1.37}$$

$$\pi^{\mathcal{R}}_{B,0} = \bar{\pi}_{B,0} \quad \text{for} \quad \mathcal{R} \in \{\mathcal{F}, \mathcal{C}, \mathcal{M}\} \tag{1.38}$$

However, in this case, the optimal monetary policies and welfare depend on the ad hoc conditions (1.37) and (1.38). On the other hand, if shocks are absent in the first period, the economy remains in the steady state in any of these regimes. Since the steady state is identical across the regimes, the initial condition to be able to compare monetary regimes is automatically satisfied. Meanwhile, welfare effect of (1.37) and (1.38) is avoided.

It is easy to see that the initial period $t = 0$ can be changed into $t = -1$ and that the monetary regime is announced at the beginning of $t = -1$. Therefore, assuming that the economy is in the steady state in the first period is equivalent to assuming that the monetary regime is announced one period in prior and when the economy is in the steady state. In general, monetary regime changes are well communicated, therefore, this assumption remains as a fairly week one.

[11] Obviously, it is assumed that in the previous period ($t = -1$) the economy is in the steady state as well. If the economy is not in the steady state at $t = -1$, then the optimal policy does not necessarily say that the economy should continue to be in the steady state at $t = 0$. This is a common and warranted assumption unless a study specifically intends to investigate optimal policies when the economy significantly deviates from its steady state (e.g. Yun, 2005). An economy's history, i.e. its features at $t = -1$, affects t_0-optimal monetary policy. Assuming that it is in the steady state at $t = -1$ isolates the optimal policy analysis from the history of the economy.

1.6 Conclusion

This chapter aims to contribute to LQ Approach in the monetary regime choice problems. LQ Approach can be briefly described as a method to solve optimal policy problems using quadratic approximation to objective functions and linearized structural equations. Benigno and Benigno (2006) offers a method to apply LQ Approach to a broad range of optimal policy problems. However, this method cannot be applied to the monetary regime choice problems without further assumptions. In other words, LQ Approach presented by Benigno and Benigno (2006) accurately determine optimal monetary policy in a given monetary regime. However, it does not necessarily rank policies of different regimes accurately, namely the flexible exchange rate regime, the cooperative flexible exchange rate regime, and the monetary union.

This chapter proposes a weak assumption so that optimal policies in these regime can be compared with one another. It shows that if shocks are absent in the first period, then Benigno and Benigno (2006)'s LQ Approach accurately compares monetary policies across these regimes. This assumption is equivalent to assuming that the monetary regime choice is declared one period in prior when the economy is in the steady state. In general, monetary regime shifts are well communicated. Therefore, if the particular interest of a study is not the monetary regime choice in an economy deviated from its steady state, then this assumption is fairly weak.

Even though it is not shown in this study. This result can be extended to a broad range of problems. It is easy to see that this assumption can be applied to compare any two monetary regimes with identical steady states.

Chapter 2

TYING THE HAND OF THE OTHER: A DECISION TO JOIN A MONETARY UNION

2.1 Introduction

The classical approach (Friedman, 1953; Mundell, 1961) emphasizes that monetary unions are inherently costly because a single monetary policy tool cannot effectively respond to different shocks of members of a monetary union. On the other hand, monetary unions offer advantages in monetary policymaking, increase trade, decrease transaction costs etc. Dellas and Tavlas (2009) summarizes the costs and benefits of monetary unions and discusses the monetary unions literature extensively. In this sense, the conventional argument says that countries with similar shocks should establish a monetary union so that the welfare cost of the one-size-fits-all monetary policy is minimized while countries enjoy the benefits of a monetary union. Chari et al. (2019) calls this the Mundellion Criterion.

Even though it is subject to criticism, the Mundellian Criterion stands as the one to assess monetary unions. In a small open economy (SOE) framework, Clerc et al. (2011) concludes that countries with similar shocks should form monetary unions. In a large open economy (LOE) framework, Pappa (2004) finds that the Mundellian Criterion holds when shocks are efficient.[1] This study revisits the monetary regime

[1]Following Groll and Monacelli (2019), shocks which create a trade off between output and inflation stabilization are defined as inefficient shocks, i.e. they break so-called "the divine coincidence". They are known as cost push shocks. Goods markup shocks or wage markup shocks (Clarida et al., 2002) are examples of inefficient shocks. On the other hand, efficient shocks allow a monetary authority to stabilize both inflation and output simultaneously. These shocks are extensively discussed in Blanchard and Galí (2007). They are also called Mundellian shocks (Chari et al., 2019). Aggregate demand shocks can be given as an example of efficient shocks.

choice between the monetary union and the flexible exchange rate regime in a LOE framework with inefficient shocks.

The key contribution of this paper is to show that if there are inefficient shocks and countries are large enough to affect each other, then the relative variance (mean preserving spread) of inefficient shocks should be accounted for in the monetary regime choice: the Mundellion Criterion is not sufficient. A monetary union is welfare increasing compared to a flexible exchange rate regime only if the variance of foreign inefficient shocks is close to or larger than the variance of domestic inefficient shocks. This is on the condition that adopting a common currency (fixing the exchange rate between countries) has little or no cost.

Many studies argue that a country "ties its hand" (abandons its monetary autonomy) by joining a monetary union to exploit the benefits of the union (Giavazzi and Pagano, 1988; Soffritti and Zanetti, 2008; Clerc et al., 2011). Countries without a credible monetary policy suffer from welfare loss because of

(i) inflationary bias à la Barro and Gordon (1983), and

(ii) insufficiency in managing expectations (i.e. discretionary monetary policy).

They eliminate this welfare loss overnight by participating in a monetary union with a credible monetary authority. It is often argued that this was the main motivation of many countries that joined the European Monetary Union (EMU). This trade off between monetary autonomy and credibility is rather a national decision independent of other countries.

In this study, however, the main motivation of a country in joining a monetary union is to "tie the hand of the other". If countries are large enough, the monetary policy responses of a CB spillover to other countries and can damage their welfare, especially if the CB in question disregards these spillovers: Inefficient shocks create trade offs between inflation and output stabilization. The optimal trade off from a national CB's perspective in a flexible exchange rate regime is suboptimal for other countries. Therefore, the monetary policy responses of a foreign CB to inefficient shocks in its country decreases welfare of other countries in a flexible exchange rate regime. On the other hand, a monetary union provides a tool to enforce a country to consider welfare spillovers. Monetary policy implicitly imposes cooperation. The CB of the union, maximizing welfare of the union, works as a CP but is restricted with a single policy tool and the fixed exchange rate regime (i.e. a common currency). As opposed to national CBs in a flexible exchange rate regime, the CB of the union considers welfare spillovers.

From a national welfare perspective, this cooperation particularly improves responses to inefficient foreign shocks. In return, it worsens responses to domestic ones.[2] Therefore, a country suffering from relatively close or larger inefficient shocks in its neighbor prefers to establish a monetary union. It consents to transfer its monetary autonomy by establishing a monetary union because it expects larger welfare gain from other potential members losing their monetary autonomies. This relationship is reciprocal, therefore, a monetary union is a PI if inefficient shocks of countries have close variances.

Groll and Monacelli (2019) and Chari et al. (2019) find that the monetary regime choice strongly depends on the types of shocks (efficient or inefficient) as well as monetary policy credibility. Even though it substantially differs from these papers in terms of its objective and its framework, this study also emphasizes the distinction between efficient and inefficient shocks in the monetary regime choice. If inefficient shocks dominate the economy, the monetary regime choice should depend on the relative variance of inefficient shocks. Otherwise, the Mundellion Criterion stands as Pappa (2004) finds.

The remainder of this study has the following structure: Section 2.2 describes the methodology, Section 2.3 presents the welfare analysis, Section 2.4 is the conclusion.

2.2 Methodology

The results are presented using the two-country economy model described in the previous chapter. It is a dynamic general equilibrium model with nominal price sticki-

[2] From a CP's perspective (in terms of the welfare of the world economy), Clarida et al. (2002) shows that monetary policy cooperation weakly dominates monetary policy competition.

ness, PCP, and complete asset markets based on Corsetti et al. (2010).[3] Markup and productivity shocks are used to represent inefficient and efficient shocks respectively. The results can be generalized to any efficient or inefficient shock.

Two monetary regimes are compared: the monetary union ($\mathcal{M}$) and the flexible exchange rate regime ($\mathcal{F}$). In the former, the two countries adopt a common currency. Furthermore, they delegate their monetary policymaking capacity to a supranational authority which aims to maximize welfare of the world economy.[4] However, in the flexible exchange rate regime, policymakers play a strategic game. The CB of each country maximizes the welfare of their respective country and takes the other CB's actions as given. Monetary authorities pursue monetary policy with commitment (i.e. they are credible) independent of the monetary regime. To provide the intuition, this chapter also refers to the cooperative flexible exchange rate regime ($\mathcal{C}$).

It is possible to solve the model analytically by transforming it into the accurate LQ form. However, it requires time consuming and inconvenient calculations.[5] Therefore, this study relies on a numerical analysis after transforming the model in to the LQ form. The numerical analysis provides sufficient intuition. The model is calibrated based on Gali et al. (2007); Christiano et al. (2010); Groll and Monacelli (2019). First, a special case - the intertemporal elasticity is equal to the intratemporal elasticity and both are equal to unity - is studied. This case significantly simplifies the model and is crucial to provide the intuition of the "tying the hand

[3]The model used in this chapter and the previous one is a representative agent new keynesian (RANK) model. This study uses a RANK model instead of a heterogeneous agent new keynesian (HANK) model (see for example Debortoli and Galí, 2017; Kaplan et al., 2018; Bilbiie, 2018) because it specifically seeks (in)efficiencies brought by monetary policy in a monetary union and asymmetries across countries rather than (in)efficiencies originating from asymmetries within countries.

[4]In a two-country economy model, welfare of the monetary union corresponds to the welfare of the world economy.

[5]For various ways to solve a linearized model analytically, see Blanchard and Kahn (1980); Uhlig et al. (1995); Klein (2000); Sims (2002).

of the other" incentive. IRFs in $\mathcal{F}$, $\mathcal{C}$, and $\mathcal{M}$ are presented. Then, the results for the main calibration are provided. Finally, The model is simulated for 200 periods using random shocks. Ex-post national welfare loss difference between $\mathcal{M}$ and $\mathcal{F}$ as a function of the relative standard deviation is provided and the monetary regimes are compared accordingly.

2.3 Welfare Analysis

Throughout the study it is assumed that $\bar{\mu} = 1$ and the price stickiness is symmetric between countries, i.e. $\alpha = \alpha^*$. The discount factor β is 0.99. A's relative population, n, is 0.5. Average price contract durations are 3 periods: $\alpha = \alpha^* = 0.66$. The elasticity of substitution between differentiated goods, σ, is 9 implying a steady-state markup of prices over marginal costs of 12.5 percent. The value of η has been a focus of debate in macroeconomics. Following Gali et al. (2007) and Christiano et al. (2010), η is taken as 0.89. Nevertheless, robustness of results are checked using alternative values of η and results remain intact (not provided in the dissertation). Groll and Monacelli (2019) sets the intertemporal elasticity of substitution, ρ, to 1 and the trade elasticity to 2. ρ is set to 1. Two different values are adopted for the trade elasticity. In the special case, θ is set to 1 to obtain $\rho = \theta = 1$. In the general case, θ is switched to 2 as in Groll and Monacelli (2019). Table 2.1 summarizes the calibration. Unless stated otherwise, the specifications at Table 2.1 are used.

2.3.1 A Special Case

This section studies the special case of $\rho = \theta = 1$, i.e. the intertemporal elasticity of substitution is equal to the intratemporal elasticity of substitution and both are equal to unity. In this case, the terms of trade is insulated from the welfare criteria, i.e. $\tilde{\tilde{\lambda}}_q = \tilde{\tilde{\lambda}}_q^* = 0$.[6] The welfare loss function of A becomes

[6]Double tilde refers to the respective values evaluated at $\rho = \theta = 1$. In Appendix A.3, I present these values explicitly.

Table 2.1: Calibration

Parameter	Variable name	Value
β	Discount factor	0.99
ρ	Inverse elasticity of intertemporal substitution	1.00
η	Inverse elasticity of goods production	0.89
σ	Elasticity of substitution between goods produced in a country	9.00
θ	Elasticity of substitution between A and B goods	2.00
α	Calvo parameter for $i = \{A, B\}$	0.66
n	Relative size of domestic country	0.50
γ	Persistence of shocks $\gamma_{\mu_A} = \gamma_{\mu_B} = \gamma_{a_A} = \gamma_{a_B}$	0.70
σ_k^2	Variance of shocks' innovations $\sigma^2_{\hat{\mu}_A} = \sigma^2_{\hat{\mu}_B} = \sigma^2_{a_A} = \sigma^2_{a_B}$	0.01

$$\tilde{\tilde{W}}_A = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[\tilde{\tilde{\lambda}}_{y_A}\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^A_{A,t}\big)^2 + \tilde{\tilde{\lambda}}_{y_B}\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^A_{B,t}\big)^2 + \tilde{\tilde{\lambda}}_{\pi_A}\pi_{A,t}{}^2 + \tilde{\tilde{\lambda}}_{\pi_B}\pi^*_{B,t}{}^2\Big] \quad (2.1)$$

Similarly, the welfare loss function of B becomes

$$\tilde{\tilde{W}}_B = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[\tilde{\tilde{\lambda}}^*_{y_A}\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^B_{A,t}\big)^2 + \tilde{\tilde{\lambda}}^*_{y_B}\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^B_{B,t}\big)^2 + \tilde{\tilde{\lambda}}^*_{\pi_A}\pi_{A,t}{}^2 + \tilde{\tilde{\lambda}}^*_{\pi_B}\pi^*_{B,t}{}^2\Big] \quad (2.2)$$

A particular reason to begin with this special case is that it is possible to separate the objective function of a national CB from the welfare loss function of its country. The terms of trade is eliminated from the aggregate supply equations because $\tilde{\tilde{\psi}} = 0$. (1.27) boils down to

$$\pi_{A,t} = \beta\mathbb{E}_t\pi_{A,t+1} + \kappa\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^w_{A,t}\big) + u_t \quad (2.3)$$

(1.28) becomes

$$\pi^*_{B,t} = \beta\mathbb{E}_t\pi^*_{B,t+1} + \kappa^*\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^w_{B,t}\big) + u^*_t \quad (2.4)$$

This means that in $\mathcal{F}$, monetary authorities cannot change the other country's variables (foreign output and foreign PPI) through the terms of trade. A national monetary authority is able to set only its own output and PPI. Therefore, the objective function of A's CB is

$$\tilde{\tilde{L}}_A = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left[\tilde{\tilde{\lambda}}_{y_A}\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}_{A,t}^{A}\big)^2 + \tilde{\tilde{\lambda}}_{\pi_A}\pi_{A,t}{}^2\right] \tag{2.5}$$

The objective function of B's CB is

$$\tilde{\tilde{L}}_B = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left[\tilde{\tilde{\lambda}}^*_{y_B}\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}_{B,t}^{B}\big)^2 + \tilde{\tilde{\lambda}}^*_{\pi_B}\pi^*_{B,t}{}^2\right] \tag{2.6}$$

(2.5) is the part of (2.1) that domestic monetary policy can affect, i.e. domestic output and domestic PPI. Similarly, from B's perspective, (2.6) is the part of (2.2) that domestic monetary policy can affect. The rest of national welfare which depend on foreign output and foreign PPI is under the control of the foreign CB, therefore, it is outside of a national CB's objective function. Consequently, the monetary policy problem of A is to determine a path for $\{\hat{Y}_{A,t}, \pi_{A,t}\}_{t=0}^{\infty}$ to maximize (2.5) subject to (2.3). The optimality condition of this problem is

$$-\sigma\pi_{A,t} = \Delta\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}_{A,t}^{A}\big) \tag{2.7}$$

The CB of B determines $\{\hat{Y}_{B,t}, \pi^*_{B,t}\}_{t=0}^{\infty}$ to maximize (2.6) subject to (2.4). The optimality condition is

$$-\sigma\pi_{B,t} = \Delta\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}_{B,t}^{B}\big) \tag{2.8}$$

It is trivial that national welfare depends on monetary policy decisions of both countries.

The first best monetary policy for A, on the other hand, can easily be found by determining the path of $\{\hat{Y}_{A,t}, \hat{Y}_{B,t}, \pi_{A,t}, \pi^*_{B,t}\}_{t=0}^{\infty}$ which maximizes $\tilde{\tilde{W}}_A$. The optimal relationship between endogenous variables is

$$-\sigma\pi_{A,t} = \Delta\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^{A}_{A,t}\big) \tag{2.9}$$

$$-\sigma\pi_{B,t} = \Delta\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^{A}_{B,t}\big) \tag{2.10}$$

Comparing (2.7) and (2.8) to (2.9) and (2.10), the relationship between domestic output and domestic PPI in $\mathcal{F}$ matches with the first best choice of A because it is decided by the CB of A itself. The relationship between foreign output and foreign PPI, however, is not the first best for A because it is chosen by the CB of B to maximize its own objective $\tilde{\tilde{L}}_B$.

In order to provide the intuition of the "tying the hand of the other" effect, $\mathcal{C}$ should be studied. $\mathcal{M}$ differs from $\mathcal{F}$ in two terms: the exchange rate regime, and the nature of monetary policymaking. In $\mathcal{M}$, a common currency is adopted whereas in $\mathcal{F}$ exchange rate floats. Moreover, in $\mathcal{M}$, monetary policy competition is replaced by monetary policy cooperation in the sense that the CB maximizes welfare of the union instead of any national welfare and takes into account welfare spillovers. $\mathcal{C}$ is used to distinguish welfare effect of each change. In $\mathcal{C}$, the exchange rate is flexible and a CP chooses $\{Y_{A,t}, Y_{B,t}, \Pi_{A,t}, \Pi^*_{B,t}\}^{\infty}_{t=0}$ to maximize welfare of the world economy W. $\mathcal{C}$ corresponds to the "cooperative case" defined in Clarida et al. (2002). It differs from $\mathcal{M}$ only in terms of the exchange rate regime. In both cases, the monetary authorities have identical objectives, however, in $\mathcal{M}$, the CB of the union is restricted with the common currency. On the other hand, both in $\mathcal{C}$ and $\mathcal{F}$ exchange rate floats but $\mathcal{C}$ has policy cooperation unlike $\mathcal{F}$. Therefore, welfare difference between $\mathcal{F}$ and $\mathcal{C}$ provides the welfare effect of monetary policy cooperation over competition while welfare difference between $\mathcal{C}$ and $\mathcal{M}$ determine welfare cost or benefit of common currency. In $\mathcal{C}$, the CP maximizes welfare of the world economy $\tilde{\tilde{W}}$.

$$\tilde{\tilde{W}} = -\frac{\tilde{\tilde{\lambda}}^w_y}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[n\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^w_{A,t}\big)^2 + (1-n)\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^w_{B,t}\big)^2 + \frac{\sigma n}{\kappa}{\pi_{A,t}}^2 + \frac{\sigma(1-n)}{\kappa^*}{\pi^*_{B,t}}^2\Big] \tag{2.11}$$

$\tilde{\tilde{Y}}_{i,t}^{w}$ corresponds to the averaged output targets, i.e. $\tilde{\tilde{Y}}_{i,t}^{w} = n\tilde{\tilde{Y}}_{i,t}^{A} + (1-n)\tilde{\tilde{Y}}_{i,t}^{B}$. The CP chooses a path for $\{\hat{Y}_{A,t}, \hat{Y}_{B,t}, \pi_{A,t}, \pi_{B,t}^{*}\}_{t=0}^{\infty}$ which maximizes $\tilde{\tilde{W}}$ subject to (2.3) and (2.4). The optimality conditions are

$$-\sigma\pi_{A,t} = \Delta\big(\hat{Y}_{A,t} - \tilde{Y}_{A,t}^{w}\big) \tag{2.12}$$

$$-\sigma\pi_{B,t} = \Delta\big(\hat{Y}_{B,t} - \tilde{Y}_{B,t}^{w}\big) \tag{2.13}$$

In $\mathcal{C}$, A diverges from its first best relationship between domestic PPI and domestic output because the CP tries to stabilize A's output at a level which maximizes welfare of the world economy. Consequently, A loses welfare with this new cooperative equilibrium in terms of domestic parts, $\tilde{\tilde{L}}_{A}$. On the other hand, (2.13) improves national welfare of A compared to (2.8) because the CP tries to stabilize B's output at a level closer to A's first best. It follows that if welfare gains of switching from (2.13) to (2.8) outweigh welfare losses of moving away from (2.7) to (2.12), then A benefits from the new monetary regime $\mathcal{C}$.

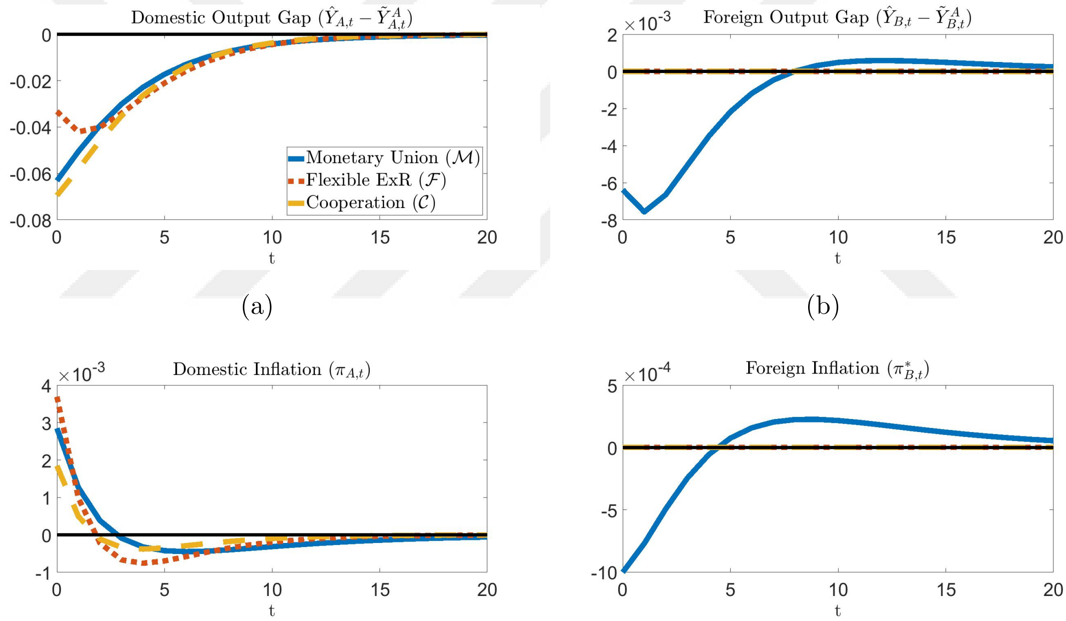
In $\mathcal{M}$, the monetary authority of the union chooses $\{\hat{Y}_{A,t}, \hat{Y}_{B,t}, \pi_{A,t}, \pi_{B,t}^{*}\}$ to maximize $\tilde{\tilde{W}}$ as in $\mathcal{C}$ but is additionally restricted by a common currency - expressed with the additional constraint (1.31) in this model. The optimality condition is

$$-\sigma\big(n\pi_{A,t} + (1-n)\pi_{B,t}\big) = n\Delta\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}_{A,t}^{w}\big) + (1-n)\Delta\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}_{B,t}^{w}\big) \tag{2.14}$$

A common currency restraints monetary policy in $\mathcal{M}$ compared to $\mathcal{C}$ and causes the one-size-fits-all cost in the following way. In this special case, a foreign markup shock does not require any change in domestic interest rate because domestic output and domestic PPI is not affected from a foreign markup shock. However, in $\mathcal{M}$, stabilization in domestic (foreign) output and domestic (foreign) PPI cannot be insulated from foreign (domestic) output and foreign (domestic) PPI. A's output and PPI are automatically affected from a monetary response to B's markup shock. For this reason, the CB of the union tempers its response to a markup shock in B because the optimal response to the shock violates the optimal relationship between

A's output and PPI. This tempered response would have been unnecessary if the exchange rate was flexible. Nevertheless, targets of the monetary authority in $\mathcal{M}$ are identical to targets of the CP in $\mathcal{C}$. For this reason, it is said that $\mathcal{M}$ is a constrained cooperation.

This result is illustrated below. Figure 2.1 depicts impulse responses of A's domestic and foreign output gaps, domestic and foreign PPIs to a positive markup shock in A under $\mathcal{F}$, $\mathcal{C}$, and $\mathcal{M}$ given the specification at Table 2.1 except the trade elasticity θ is set 1.[7] In $\mathcal{F}$, a positive shock to A's markup pushes domestic PPI



(a) (b) (c) (d)

Figure 2.1: Impulse responses to a positive markup shock in A ($\hat{\mu}_A$) given the parametrization at Table 2.1 except $\theta = 1$.

upwards. Foreign output and foreign PPI are not affected because the terms of trade is insulated from the aggregate supply relation of A and from the welfare objectives. Therefore, B's monetary authority remains unresponsive to a markup

[7] In the specification at Table 2.1, I have $\rho = 1$, therefore, I obtain $\rho = \theta = 1$ when I set $\theta = 1$. I also note that in the absence of productivity shocks $\tilde{T}^i_t = 0$ for $i \in \{A, B\}$ and $\tilde{Y}^i_{B,t} = 0$ for $i \in \{A, B\}$ if $\hat{\mu}_{B,t} = 0$.

shock in A. The CB of A responds to the markup shock optimally considering its output target, $\tilde{\tilde{Y}}^{A}_{A,t}$, and domestic PPI, $\pi_{A,t}$. In other words, it balances domestic output gap and domestic PPI at the optimal (2.7). In $\mathcal{C}$, a positive shock to A's markup pushes domestic PPI upwards. B's output and PPI remain unchanged again for the same reason explained in $\mathcal{F}$. This time, the CP maximizes welfare of the world economy, therefore, it stabilizes domestic output to minimize deviation from domestic output target of the world economy $\tilde{\tilde{Y}}^{w}_{A,t}$, rather than domestic output target of A $\tilde{\tilde{Y}}^{A}_{A,t}$. This indicates a higher deviation for A from its own domestic output target. Domestic PPI is lower in $\mathcal{C}$ compared to $\mathcal{F}$, however, the change and its effect on welfare is considerably smaller. Consequently, cooperation causes loss in welfare for A when a domestic markup shock hits the economy. In $\mathcal{M}$, a response to a shock (an interest rate increase or decrease) to stabilize A's output and inflation automatically incites an unwanted shift in foreign output and foreign PPI. Therefore, the CB of the union reacts less aggressively then it does in $\mathcal{C}$. This, in turn, tempers A's loss from inefficient response of domestic output gap. Still, the monetary policy response causes foreign output and foreign PPI to deviate from their targets. This is in fact an example of the one-size-fits-all cost of the monetary policy in a monetary union. This cost remains relatively small.

Figure 2.2 presents same graphs for a positive markup shock in B. Cooperation ($\mathcal{C}$) improves responses to A's foreign output gap because the CP aims to stabilize B's output at a level closer to the first best of A compared to $\mathcal{F}$. Foreign PPI also decreases compared to $\mathcal{F}$ but the change is relatively small. In $\mathcal{M}$, a monetary policy response to B's markup shock affects A's output gap and PPI as in the previous case. Therefore, A suffers from the one-size-fits-all cost. Nevertheless, the CB of the union tries to stabilize the economy at the same level as the CP in $\mathcal{C}$ does. Overall, in $\mathcal{M}$, A experiences a smaller deviation from its foreign output target compared to $\mathcal{F}$.

From a national welfare perspective, the monetary union improves responses to foreign inefficient shocks whereas it worsens responses to domestic inefficient shocks due to inherent cooperation. It follows that as domestic shocks outweigh foreign

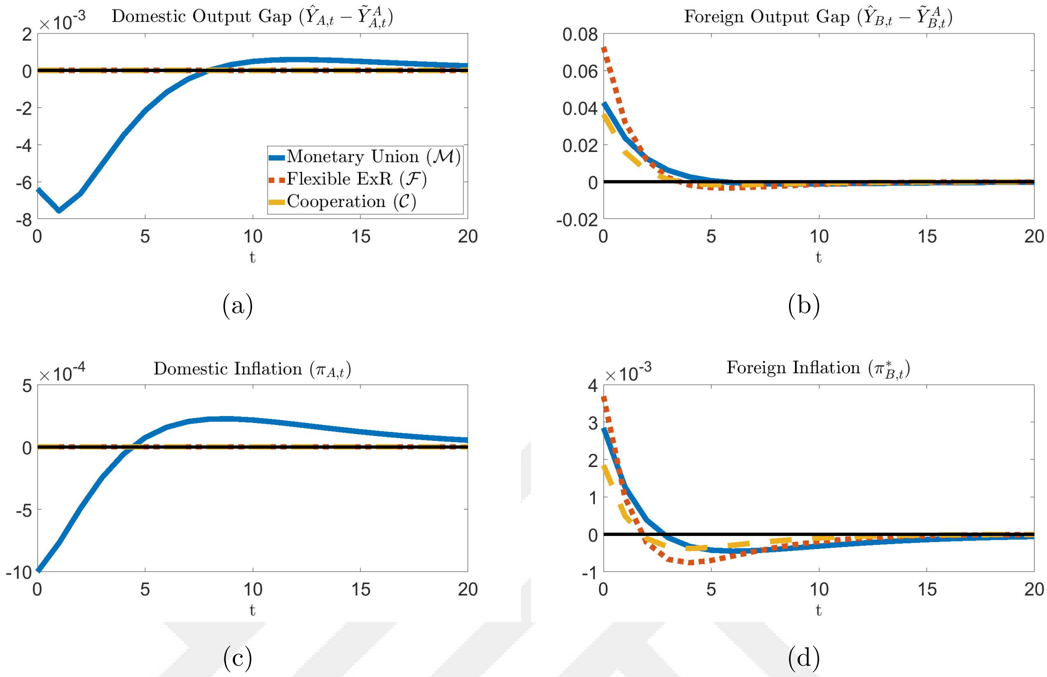
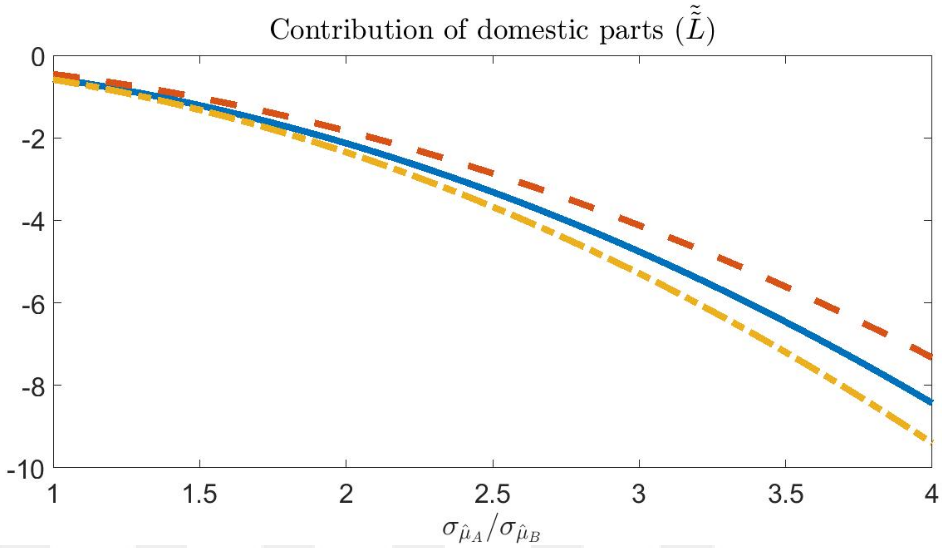


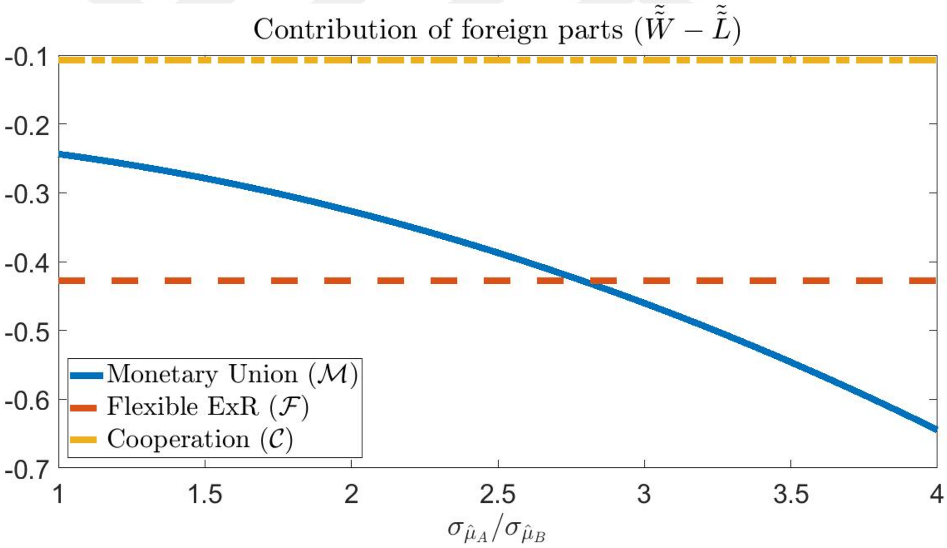
Figure 2.2: Impulse responses to a positive markup shock in B ($\hat{\mu}_B$) given the parametrization at Table 2.1 with $\theta = 1$.

shocks, $\mathcal{F}$ becomes preferable for a country because the domestic country prefers to dismiss the foreign country when it responds to its large domestic markup shocks. It is willing to bear the relatively small (because foreign inefficient shocks are relatively smaller than domestic shocks) cost of spillovers of independent foreign monetary policy which ignores domestic welfare. If foreign shocks are larger, then a country particularly suffers from independent foreign monetary policy. Then, this country prefers $\mathcal{M}$ over $\mathcal{F}$ because in $\mathcal{M}$ the CB considers welfare spillovers. In this way, by the nature of the monetary union, this country compels the other to cooperate in monetary policy. In other words, a country has an incentive to join a monetary union to "tie the hand of the other" in monetary policymaking.

Figure 2.3 shows contributions of domestic ($\tilde{\tilde{L}}_A$) and foreign ($\tilde{\tilde{W}}_A - \tilde{\tilde{L}}_A$) parts to national welfare as a function of the relative standard deviation for $\mathcal{F}$, $\mathcal{C}$, and $\mathcal{M}$. The standard deviation of B's markup shocks are kept constant but the standard deviation of A's markup shocks is changed. In all monetary regimes, the domestic



(a)



(b)

Figure 2.3: Contributions of the different parts to welfare loss function of A as a function of nominal price rigidity given the parametrization at Table 2.1 except $\theta = 1$.

part of national welfare decreases as the relative standard deviation increases because the standard deviation of A's markup shocks increase. Meanwhile, welfare loss difference between $\mathcal{M}$ and $\mathcal{F}$ also increases, making $\mathcal{M}$ more costly. On the other hand, the contribution of foreign parts to national welfare in $\mathcal{F}$ and $\mathcal{C}$ do not change based on the relative standard deviation of inefficient shocks because the standard deviation of B's markup shocks is constant and B's output and PPI are set independently from A's shocks and A's monetary policy. Nevertheless, $\mathcal{F}$ is welfare decreasing compared to $\mathcal{C}$ in terms of the foreign part. $\mathcal{M}$ improves A's national welfare loss through the foreign part up to a point. National welfare decreases as the relative standard deviation increases because domestic inefficient shocks become larger, welfare loss originating from the one-size-fits-all cost increases. After a while the cost exceeds gains from cooperation and $\mathcal{M}$ worsens A's welfare in terms of the foreign part as well.

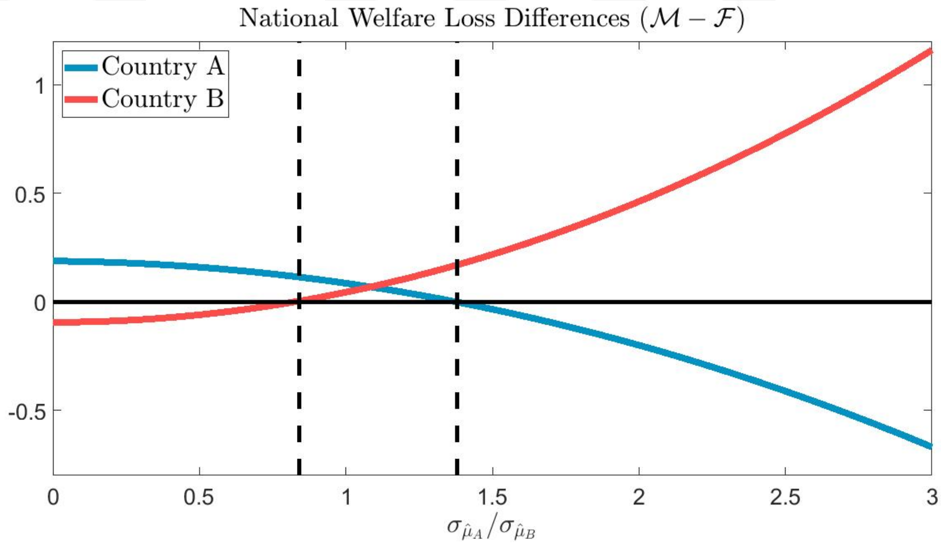


Figure 2.4: Welfare loss difference of A and B as a function of the relative standard deviation ($\sigma_{\hat{\mu}_A}/\sigma_{\hat{\mu}_B}$) given the parametrization at Table 2.1 except $\theta = 1$.

Since this mechanism applies to B as well, $\mathcal{M}$ becomes preferable for B as the standard deviation of A's markups increases and vice versa. If the standard deviations of A's and B's markup shocks are close, then the monetary union is a PI. Figure 2.4 presents national welfare loss differences of both countries between $\mathcal{M}$ and $\mathcal{F}$ as a function of the relative standard deviation. As in Figure 2.3, the standard deviation of B's markup shocks is fixed and the standard deviation of A's markup shocks changes. The horizontal black line is the zeroline. Above the zeroline, $\mathcal{M}$ dominates $\mathcal{F}$, below the zeroline $\mathcal{F}$ dominates $\mathcal{M}$. At the beginning, $\mathcal{M}$ is welfare increasing for A but decreasing for B because A's markup shocks are relatively smaller than B's markup shocks. As A's markup shocks get larger, $\mathcal{M}$ becomes welfare increasing for B as well. $\mathcal{M}$ is a PI between the dashed lines. However, when A's shocks become much larger than B's shocks, $\mathcal{M}$ becomes welfare decreasing for A. This leads to the conclusion that for a monetary union to be a PI, the variance of inefficient shocks in countries should be close.

2.3.2 The General Case

In the general case, in addition to aforementioned mechanisms, the beggar-thy-neighbor effect (Corsetti and Pesenti, 2001) exists across countries: a change in the terms of trade affects countries' outputs and PPIs through the aggregate supply equations in all monetary regimes. This means that domestic policy affects foreign output and foreign PPI. Therefore, a CB reacts to foreign shocks to adjust the terms of trade. Appendix C presents IRFs of A's domestic and foreign output gaps, domestic and foreign inflation, and the terms of trade gap to positive markup shocks (domestic and foreign) in $\mathcal{F}$, $\mathcal{C}$, and $\mathcal{M}$.[8] Nevertheless, the main mechanism remains intact. $\mathcal{C}$ improves responses to foreign shocks but worsens responses to domestic shocks slightly less. In $\mathcal{M}$, output gaps (except the domestic output gap when domestic markup shock hits) and PPIs are larger compared to $\mathcal{C}$, however, they are still better off relative to $\mathcal{F}$ in terms of domestic welfare.

Figure 2.5 shows welfare loss differences of A and B as a function of the relative standard deviation of markup shocks as in Figure 2.4 given the specification at Table 2.1. As in the previous case, $\mathcal{M}$ is welfare decreasing for a country which has considerably larger shocks. Therefore, $\mathcal{M}$ is a PI only if the variance of inefficient shocks are close.

2.3.3 Efficient vs Inefficient Shocks

Efficient shocks do not create trade off between output gap and inflation: The CP stabilizes A's and B's outputs at averaged targets and PPIs simultaneously.

[8]In the special case, the calibration in Table 2.1 is used except the trade elasticity θ is set to 1. In the general case, the calibration in Table 2.1 is used exactly.

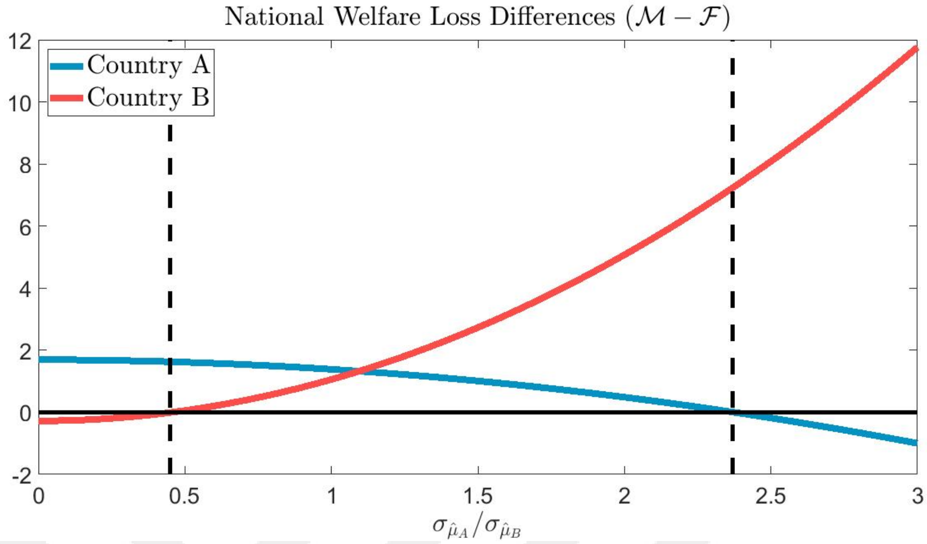


Figure 2.5: Welfare loss difference of A and B as a function of the relative standard deviation ($\sigma_{\hat{\mu}_A}/\sigma_{\hat{\mu}_B}$) given the parametrization at Table 2.1.

Inherent cooperation in $\mathcal{M}$ becomes desirable only to terminate the beggar-thy-neighbor effect in $\mathcal{F}$. In return, countries suffer from the one-size-fits-all cost of the monetary union. Consequently, as Pappa (2004) finds, the monetary regime choice depends on whichever dominates the other which in turn is determined by model parameters, e.g. the price rigidity and the trade elasticity. As shocks become similar, the cost of the one-size-fits-all monetary policy decreases. If $\rho = \theta = 1$, then the monetary union is welfare decreasing in national level (implying that it is welfare decreasing economy-wide) because (i) output targets of both countries, therefore, cooperative targets coincide, (ii) gains from shifting the terms of trade (the beggar-thy-neighbor effect) is 0. Then, $\mathcal{F}$ mimics $\mathcal{C}$'s allocations. In this case, $\mathcal{M}$ (being a constrained version of $\mathcal{C}$) cannot make national welfare strictly better off than $\mathcal{C}$. If efficient shocks are perfectly correlated, then welfare loss difference between $\mathcal{F}$ and $\mathcal{M}$ is 0 because in addition to (i) and (ii) the one-size-fits-all cost disappears.

This leads to the conclusion that if the variance of efficient shocks outweigh the variance of inefficient shocks in an economy, the classical criterion stands: Countries which experience similar efficient shocks should establish monetary unions. Otherwise, if inefficient shocks dominate the economy, countries should form a monetary union only if the variances of inefficient shocks between different countries are close.

2.4 Conclusion

This chapter explores the monetary regime choice between the monetary union and the flexible exchange rate regime. It reveals that if an economy has inefficient shocks creating trade offs in macroeconomic stabilization, and countries are large enough to have an impact on each other, then the relative variance (mean preserving spread) of inefficient shocks should be accounted for in the monetary regime choice.

In a flexible exchange rate regime, a country suffers significantly from the foreign monetary authority's policy responses to foreign inefficient shocks. A monetary union, however, inherently imposes cooperation in the sense that the CB of the union takes into account welfare spillovers across countries. Cooperation improves responses to foreign inefficient shocks whereas it worsens responses to domestic ones. Therefore, a country chooses to establish a monetary union with another only if the variance of foreign inefficient shocks are close to or larger than variance of domestic inefficient shocks. This is on the condition that adopting a common currency (the fixed exchange rate) has little or no cost.

It is commonly accepted that joining a monetary union entails a trade off between monetary autonomy and credibility. This trade off is rather independent of other members of the union. However, in the case being examined (LOE framework with inefficient shocks), a country uses the union as a tool to "tie the hand of the other country", i.e. to compel a foreign country to cooperate and consider monetary policy spillovers. This means that a country has an incentive to join a monetary union if other prospective members experience relatively large inefficient shocks, and they are large enough to impact the domestic country. Since this is a reciprocal relationship, a monetary union is a PI only if the inefficient shocks of its members have close variances (mean preserving spreads).

Chapter 3

TYING THE HAND OF THE OTHER: THE EFFECTS OF THE PRICE RIGIDITY AND THE TRADE ELASTICITY

3.1 Introduction

Chapter 2 presents the incentive to join a monetary union in order to "tie the hand of the other" countries. It discusses that in the flexible exchange rate regime a country's monetary responses to domestic cost-push shocks may harm another's welfare through spillovers. In brief, cost-push shocks create trade offs between output and inflation stabilization. The optimal trade off from a national CB's perspective is suboptimal for other countries.[1] In a monetary union, the CB which aims to maximize welfare of the union considers welfare of all members of the union. In a sense, it imposes an inherent cooperation between the union members in monetary policymaking. This cooperation worsens monetary responses to domestic cost-push shocks but improves monetary responses to foreign cost-push shocks from a nation's perspective. Consequently, a country prefers to join the monetary union if foreign countries' cost-push shocks outweigh or are close to the domestic cost-push shocks. In other words, the monetary regime choice between the flexible exchange rate regime and the monetary union should depend on the relative variance (mean preserving spread) of cost-push shocks. Since this effect is reciprocal, in an economy dominated by the cost-push shocks, a monetary union is a PI if cost-push shocks of prospective union members have close mean preserving spreads.

Chapter 2 shows this result using a two-country economy model based on Corsetti

[1]Chapter 2 refers to cost-push shocks as inefficient shocks. This difference does not have a particular reason.

et al. (2010). It relies on a numerical analysis. It calibrates the two-country model produces ex-post national welfare loss differences between the flexible exchange rate regime and the monetary union as a function of the relative standard deviation of cost-push shocks. However, it is silent about the effects of the important characteristics of an economy on the "tying the hand of the other" incentive.

This chapter investigates the effects of the price rigidity and the trade elasticity on the "tying the hand of the other" incentive.[2] It finds that this incentive depends strongly on the price rigidity and the trade elasticity. More flexible prices compensate monetary policy shocks and the exchange rate inertia more effectively. Therefore, the need to "tie the hand of the other" country diminishes. As the price rigidity increases, welfare loss due to a foreign country's monetary responses to the foreign country's own cost-push shocks soars. This is significantly reflected in welfare loss due to the variation in foreign output gap. The incentive of "tying the hand of the other" increases and the cost of the monetary union decreases or the monetary union dominates the flexible exchange rate regime.

The trade elasticity has two main effects on national welfare. As the trade elasticity increases, output becomes more sensitive to the term of trade. Welfare spillovers of a country's monetary responses increase. Therefore, the importance of the inherent cooperation in the monetary union increases with higher trade elasticity. Chapter 2 shows that the inherent cooperation can be welfare increasing or decreasing depending on the relative variances of cost-push shocks. Under symmetric parameters and shock structures,[3] as the trade elasticity increases, the incentive to "tie the hand of the other" country appreciates. On the other hand, the exchange rate inertia in the monetary union is compensated much easier if the trade elastic-

[2]As mentioned in Chapter 2, the trade elasticity is also referred as the intratemporal elasticity of substitution. In fact, it is the elasticity of substitution between Country-A's final good and Country-B's final good. It is equal to θ.

[3]"symmetric parameters and shock structures" means that the price rigidity in countries are identical, both countries have 1/2 weight in population, shock persistences and variances are identical. However, this does not mean that shocks in each country (e.g. markup shocks in A and B) are identical.

ity is high. Under symmetric parameters and shocks, the exchange rate inertia is welfare decreasing for both countries. Therefore, as the trade elasticity increases the national welfare cost of losing the exchange rate flexibility diminishes. If the trade elasticity is low, then it is possible that welfare loss from the exchange rate inertia dominates gains from "tying the hand of the other" country. In this case, the flexible exchange rate regime dominates the monetary union. If the trade elasticity is high, it is likely the monetary union dominates the flexible exchange rate regime.

Section 3.2 presents the methodology, Section 3.3 discusses the effect of the price rigidity and the trade elasticity on the monetary regime choice and the "tying the hand of the other" incentive, Section 3.4 concludes.

3.2 Methodology

Chapter 3 continues with the model described in Chapter 1. The model and LQ Approach is explained in details in Chapter 1 and Chapter 2. Chapter 2 simulates the model using random shocks and computes ex-post national welfare loss difference for 200 periods. It repeats this for many values of the relative standard deviation of cost-push shocks and obtain welfare loss difference between $\mathcal{F}$ and $\mathcal{M}$ as a function of the relative variances of cost-push shocks.[4] Chapter 3 follows a similar numerical analysis. It computes ex-post welfare loss differences using the same method. It uses the calibration provided in Table 2.1. For convenience, the calibration is presented again in Table 3.1.

In order to observe the effect of the price rigidity, it computes ex-post national welfare loss differences between $\mathcal{F}$ and $\mathcal{M}$ for 101 different values of $\alpha \in [0; 1]$. Then, it obtains welfare loss difference between $\mathcal{F}$ and $\mathcal{M}$ as a function of the price rigidity (α) between 0 and 1. To avoid possible changes due to random shocks, the same set of random shocks is used for every simulation of welfare loss difference with a different α. To check robustness, different sets of randoms shocks are tried (not provided in the dissertation). Results remain intact. This exercise is repeated for

[4] $\mathcal{M}$ and $\mathcal{F}$ stand for the monetary union and the flexible exchange rate regime respectively. They are described in details in Chapter 2.

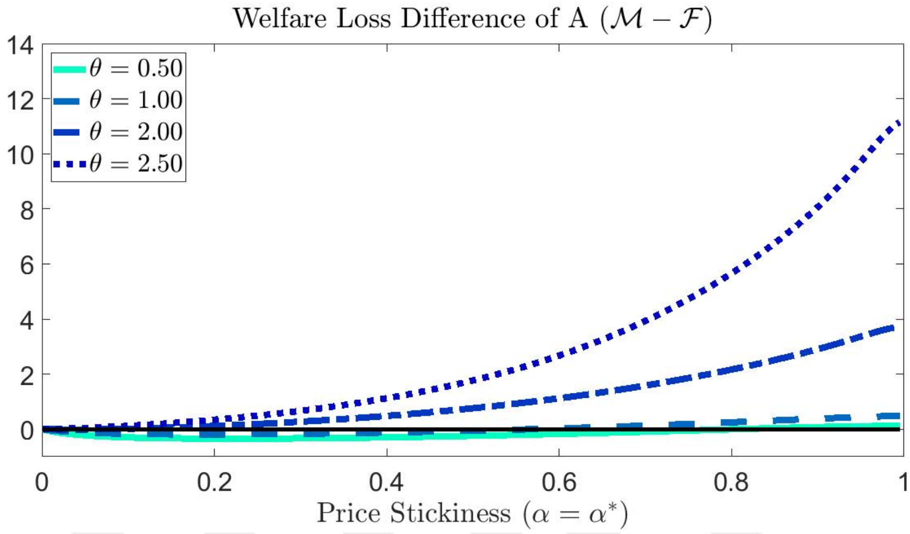
Table 3.1: Calibration

Parameter	Variable name	Value
β	Discount factor	0.99
ρ	Inverse elasticity of intertemporal substitution	1.00
η	Inverse elasticity of goods production	0.89
σ	Elasticity of substitution between goods produced in a country	9.00
θ	Elasticity of substitution between A and B goods	2.00
α	Calvo parameter for $i = \{A, B\}$	0.66
n	Relative size of domestic country	0.50
γ	Persistence of shocks $\gamma_{\mu_A} = \gamma_{\mu_B} = \gamma_{a_A} = \gamma_{a_B}$	0.70
σ_k^2	Variance of shocks' innovations $\sigma^2_{\hat{\mu}_A} = \sigma^2_{\hat{\mu}_B} = \sigma^2_{a_A} = \sigma^2_{a_B}$	0.01

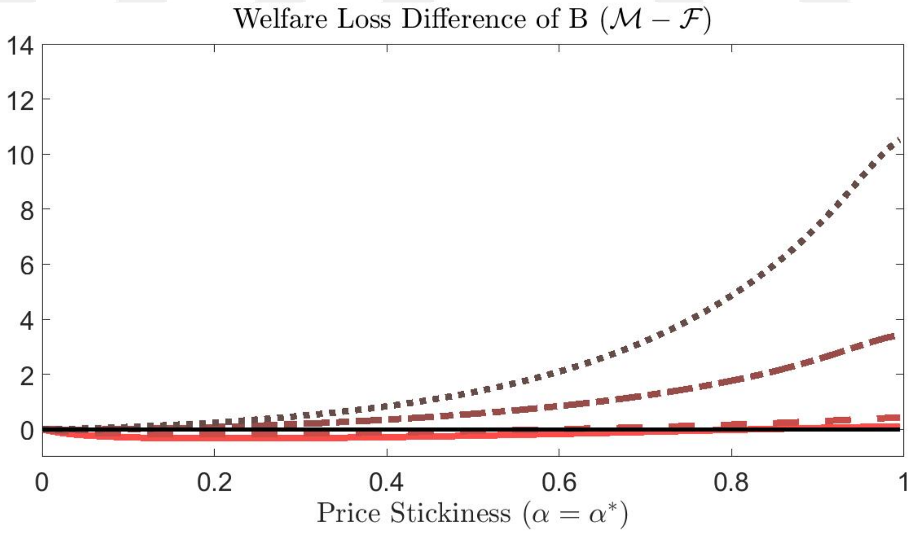
different levels of the trade elasticity $\theta \in \{0.5,\ 1.0,\ 2.0,\ 2.5\}$. As a result, national welfare loss differences as a function of the price rigidity (α) for different levels of the trade elasticity (θ) are obtained.

3.3 Welfare Analysis

Figure 3.1 presents welfare loss differences of A and B as a function of the nominal price rigidity given the parametrization at Table 3.1 for alternative levels of the trade elasticity (θ). It is easily seen that welfare loss differences of A and B are fairly similar but not identical. This is because ex-post welfare loss differences are used. In other words, welfare loss differences are computed for a set of random shocks. As a result, small differences between welfare loss functions of A and B are observed. Ex-ante welfare loss functions of A and B should be identical due to symmetric parameters and shock structures.



(a)



(b)

Figure 3.1: (a) Welfare loss difference of A between $\mathcal{M}$ and $\mathcal{F}$ as a function of the nominal price rigidity; (b) Welfare loss difference of B between $\mathcal{M}$ and $\mathcal{F}$ as a function of the nominal price rigidity.

3.3.1 The Price Rigidity

The dynamics between welfare change and the price rigidity can easily be grasped by analyzing different parts which contribute to national welfare. Figure 3.2 presents contributions of the parts of the national welfare loss function of A as a function of the price rigidity given the parametrization at Table 3.1.

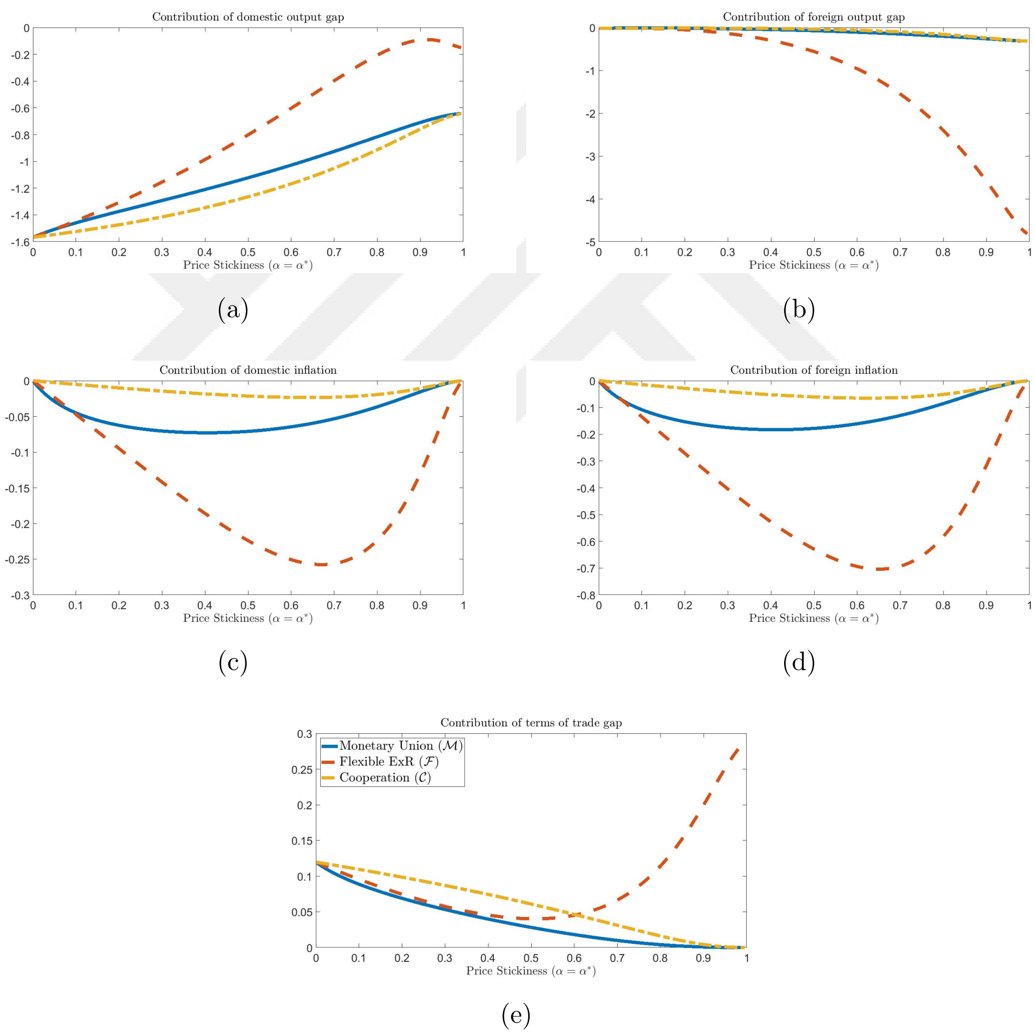


Figure 3.2: Contribution of (a) the domestic output gap, (b) the foreign output gap, (c) the domestic inflation, (d) the foreign inflation, (e) the terms of trade to the national welfare loss of A as a function of the nominal price rigidity given the parametrization at Table 3.1.

More flexible prices compensate any exchange rate inertia and monetary policy shock better independent of the underlying parameters. Output and the terms of trade gaps in $\mathcal{F}$, $\mathcal{C}$, and $\mathcal{M}$ converge to each other as $\alpha \to 0$. For the same reason, contributions of both domestic and foreign PPIs to national welfare loss decrease. This also indicates that the need for inherent cooperation imposed by $\mathcal{M}$ decreases. At the limiting case of fully flexible prices ($\alpha = \alpha^* = 0$), the monetary regime becomes irrelevant for any value of the trade elasticity. This can be observed in Figure 3.1.

Domestic and foreign PPIs have U-shape non-monotonic contribution (considered as a function of the price rigidity $\alpha = \alpha^*$) to national welfare. They do not have any contribution to welfare if prices are fully fixed just like if prices are fully flexible. This is because under full price rigidity ($\alpha = \alpha^* = 1$) inflation is always 0 irrespective of the monetary regime. In $\mathcal{M}$, the terms of trade converges to 0 as $\alpha \to 1$ for it depends only on domestic and foreign inflation under a common currency which can be observed from (1.31). In $\mathcal{F}$, the terms of trade depends also on the exchange rate affected by the monetary policies. Therefore, the monetary policies change output and the welfare loss difference does not necessarily converge to 0 when $\alpha \to 1$.

Groll and Monacelli (2019) shows that if prices are fully fixed, then welfare of the world economy is indifferent between $\mathcal{C}$ and $\mathcal{M}$. This study takes this result one step forward: A country is indifferent between $\mathcal{C}$ and $\mathcal{M}$. Since this is true for both countries, this validates Groll and Monacelli (2019)'s result. Nevertheless, Figure 3.2 shows that even if prices are fully fixed, then the monetary regime is relevant between $\mathcal{F}$ and $\mathcal{M}$.

A key feature of Figure 3.2b is that as prices become less flexible, welfare loss from variation in the foreign output gap in $\mathcal{F}$ soars. Therefore, the incentive to join the monetary union in order to "tie the hand of the other" country substantially increases.

3.3.2 The Trade Elasticity

From Figure 3.1, it is obvious that the monetary regime choice is strongly related with the trade elasticity (θ) as well. The trade elasticity increases (decreases) if θ appreciates (depreciates). The trade elasticity should be considered in two levels. Firstly, as the substitutability between international goods decreases, outputs become less sensitive to fluctuations in the terms of trade which can be observed from (1.29). Therefore, the importance of the inherent cooperation diminishes. At the limiting case of $\theta \to 0$, each country consumes a constant share of domestic and foreign goods. Then, welfare gains/losses from "tying the hand of the other" vanish. Depending on the shock structures, e.g. variance of shocks, the regime choice of a country may change. This is extensively studied in Chapter 2. Under symmetric parameters and shocks, as the trade elasticity increases the incentive to "tie the hand of the other", hence, the incentive to join the monetary union rises.

Secondly, as θ decreases, the inefficiency in the adjustment of the international relative prices rises which in turn, coupled with the price rigidity, increases the importance of the exchange rate flexibility. A country may prefer that the international relative prices adjust poorly depending on asymmetric parameters and shock structures. This is beyond the scope of this chapter. In this symmetric case, the inefficiency in the exchange rate adjustment deteriorates welfare of both countries.

In the symmetric case, a country faces a trade off in the monetary regime choice. In $\mathcal{M}$, it benefits from the inherent cooperation but loses the exchange rate flexibility. Whichever dominates the other strongly depends on the trade elasticity as can be seen from Figure 3.1. Given a level of price rigidity, if the trade elasticity is high, the incentive to "tie the hand of the other" country tends to outweigh the cost of the common currency. Then, $\mathcal{M}$ dominates $\mathcal{F}$. On the contrary, if the trade elasticity is low, then welfare gain of "tying the hand of the other" country remains small compared to the cost of losing the exchange rate flexibility. Consequently, $\mathcal{F}$ dominates $\mathcal{M}$.

3.4 Conclusion

This chapter continues to study the monetary regime choice problem and the incentive to join a monetary union to "tie the hand of the other" country. It investigates how the price rigidity and the trade elasticity affect this incentive. It uses the two-country model and LQ Approach described in Chapter 1. It relies on the numerical analysis method presented in Chapter 2.

It finds that as prices become more flexible, they compensate monetary policy shocks or any exchange rate inertia more effectively. Therefore, the incentive to "tie the hand of the other" country diminishes as well as any other incentive to choose a monetary regime. At the limiting case of full price flexibility, the regime choice becomes irrelevant. On the other hand, as the price rigidity increases, foreign monetary policy actions in the flexible exchange rate regime substantially harm domestic welfare. Welfare cost of variation in the foreign output gap soars. Therefore, under symmetric parameters and shock structures, the regime choice shifts on behalf of the monetary union above a level of price rigidity.

Furthermore, Chapter 3 concludes that the monetary regime choice significantly depend on the trade elasticity. As the trade elasticity increases, outputs become more sensitive to the variation in the terms of trade, therefore, to the foreign monetary policy actions. The incentive to impose the inherent cooperation through the monetary union increases from a country's perspective. In addition to this incentive, if the trade elasticity is high, then the exchange rate inertia in the monetary union is compensated more effectively. Under symmetric parameters, both countries benefit from "tying the hand of the other" and suffer from the fixed exchange rate - the common currency - in the monetary union. Given a level of the price rigidity, if the trade elasticity is low, then it is likely that a country prefers the flexible exchange rate regime over the monetary union. This is because the welfare cost of the common currency outweigh the benefit of "tying the hand of the other" countries. On the contrary, if the trade elasticity is high, then welfare gain of "tying the hand of the other" countries dominates the welfare cost of losing the exchange rate flexibility.

Then, the monetary union dominates the flexible exchange rate regime.

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Appendix A

APPENDIX FOR CHAPTER 1

A.1 Deterministic Steady State

This section shows that there exist a zero-inflation steady state for the deterministic problem of $\mathcal{F}$, $\mathcal{C}$ and $\mathcal{M}$. First, it is possible to define

$$F_t \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \frac{1}{\mu_{t+k}} C_{t+k}{}^{-\rho} Y_{A,t+k} \left(\frac{P_{A,t+k}}{P_{t+k}}\right) \left(\frac{P_{A,t+k}}{P_t}\right)^{\sigma-1} \tag{A.1}$$

$$F_t^* \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \frac{1}{\mu_{t+k}^*} C_{t+k}{}^{-\rho} Y_{B,t+k} \left(\frac{P_{B,t+k}^*}{P_{t+k}^*}\right) \left(\frac{P_{B,t+k}^*}{P_t^*}\right)^{\sigma-1} \tag{A.2}$$

Similarly,

$$K_t \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k a_{t+k}{}^{-\eta} Y_{A,t+k}{}^{1+\eta} \left(\frac{P_{A,t+k}}{P_{A,t}}\right)^{\sigma(1+\eta)} \tag{A.3}$$

$$K_t^* \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k a_{t+k}^*{}^{-\eta} Y_{B,t+k}{}^{1+\eta} \left(\frac{P_{B,t+k}^*}{P_{B,t}^*}\right)^{\sigma(1+\eta)} \tag{A.4}$$

Then, using the above equations, it can be written that

$$\frac{1-\alpha\Pi_{A,t}{}^{\sigma-1}}{1-\alpha} = \left(\frac{F_t}{K_t}\right)^{\frac{\sigma-1}{1+\sigma\eta}} \tag{A.5}$$

$$\frac{1-\alpha\Pi_{B,t}^*{}^{\sigma-1}}{1-\alpha} = \left(\frac{F_t^*}{K_t^*}\right)^{\frac{\sigma-1}{1+\sigma\eta}} \tag{A.6}$$

Also, the law of motion of the price dispersion is

$$\varDelta_t = \alpha \varDelta_{t-1} {\Pi_{A,t}}^{\sigma(1+\eta)} + (1-\alpha)\left(\frac{1-\alpha {\Pi_{A,t}}^{\sigma-1}}{1-\alpha}\right)^{\frac{-\sigma(1+\eta)}{1-\sigma}} \tag{A.7}$$

$$\varDelta_t^* = \alpha^* \varDelta_{t-1}^* {\Pi_{B,t}^*}^{\sigma(1+\eta)} + (1-\alpha^*)\left(\frac{1-\alpha {\Pi_{B,t}^*}^{\sigma-1}}{1-\alpha^*}\right)^{\frac{-\sigma(1+\eta)}{1-\sigma}} \tag{A.8}$$

If the government spending is closed i.e. $G_t = \forall t$ and $\bar{G} = 0$ (equivalent to $s_c = 1$), then Benigno and Benigno (2006)'s model boils down to the model described in Chapter 1. The flexible exchange rate regime ($\mathcal{F}$) and the cooperative exchange rate regime ($\mathcal{C}$) correspond to the "non-cooperative regime" and the "cooperative regime" in Benigno and Benigno (2006) respectively. Then, following Benigno and Benigno (2006), it is trivial that for $F_0 = \bar{F}_0$, $F_0^* = \bar{F}_0^*$, $K_0 = \bar{K}_0$, $K_0^* = \bar{K}_0^*$ such that $\frac{\bar{F}_0}{\bar{K}_0} = 1$ and $\frac{\bar{F}_0^*}{\bar{K}_0^*} = 1$, there exist a zero inflation steady state for $\mathcal{F}$ and $\mathcal{C}$. This steady state can be identified by

$$\bar{C} = (\bar{a}^{\eta} \bar{\mu}^{-1})^{\frac{1}{\eta+\rho}} \tag{A.9}$$

$$\bar{Y}_A = \bar{Y}_B = \bar{Y} = \bar{C} \tag{A.10}$$

$$\bar{F} = \bar{K} = \bar{\mu}^{-1} \bar{C}^{-\rho} \bar{Y} \tag{A.11}$$

and $\frac{P_{A,t}}{P_t} = \frac{P_{B,t}}{P_t} = \Pi_{A,t} = \Pi_{B,t} = 1$, $C_t = \bar{C}$, $Y_{A,t} = Y_{B,t} = \bar{Y}_A = \bar{Y}_B = \bar{Y}$, $F_t = K_t = F_t^* = K_t^* = \bar{F} = \bar{K} = \bar{F}^* = \bar{K}^*$.

Then, it must be shown that $\mathcal{M}$ has the same deterministic steady state. $\mathcal{M}$ differs from $\mathcal{C}$ only in terms of the exchange rate regime: it additionally has the fixed exchange rate condition (1.11). Therefore, in order to show that $\mathcal{M}$ has the same steady state, it is sufficient to prove that this steady state satisfies (1.11).[1] The fixed exchange rate condition can be written as

[1]This comes from the following result: Let $x^* = \arg\max f(x)$ subject to $g(x) = 0$ and also let $h(x^*) = 0$. Then, $x^* = \arg\max f(x)$ subject to $g(x) = 0$ and $h(x^*) = 0$.

$$\begin{aligned}
&\frac{T_t}{T_{t-1}} = \frac{S_t}{S_{t-1}}\frac{\Pi^*_{B,t}}{\Pi_{A,t}}\\
\iff &\frac{P_{B,t}}{P_{A,t}}\frac{P_{A,t-1}}{P_{B,t-1}} = \frac{S_t}{S_{t-1}}\frac{\Pi^*_{B,t}}{\Pi_{A,t}}\\
\iff &\frac{P_{B,t}}{P_t}\frac{P_t}{P_{A,t}}\frac{P_{t-1}}{P_{B,t-1}}\frac{P_{A,t-1}}{P_{t-1}} = \frac{S_t}{S_{t-1}}\frac{\Pi^*_{B,t}}{\Pi_{A,t}}
\end{aligned}$$

Given the steady state variables, the above equation becomes

$$\begin{aligned}
&\frac{S_t}{S_{t-1}} = 1 \quad \forall t\\
\iff &\Delta S_t = 0 \quad \forall t
\end{aligned}$$

The steady state defined above satisfies (1.11) meaning that $\mathcal{M}$ has the same zero-inflation steady state as $\mathcal{F}$ and $\mathcal{C}$, and it is the one defined above.

A.2 Structural Equations

This section derives the structural equations provided in Table 1.1. The aggregate supply relations (1.27) and (1.28), and the terms of trade - output relation (1.29) are derived in Benigno and Benigno (2006). This section additionally derives the risk sharing condition, the aggregate demand equations, the market clearing conditions and the law of motion of the exchange rate.

By the market completeness assumption and the consumer's problem, for any household pair $\{j, k\}$

$$\begin{aligned}
&\frac{C^j_{t+1}}{C^k_{t+1}} = \frac{C^j_t}{C^k_t} \quad \forall t\\
\iff &\frac{C^j_t}{C^k_t} = \frac{C^j_0}{C^k_0} \quad \forall t
\end{aligned}$$

Ex-ante identical household assumption leads to $C^j_t = C^k_t \implies \hat{C}_t = \hat{C}^*_t$. For the aggregate demand equation, one should refer again to the consumer's problem. By Woodford (2011, Chapter 2), it can be found that

$$\mathbb{E}_t \frac{{C_t}^{-\rho}}{{C_{t+1}}^{-\rho}} \frac{P_{t+1}}{P_t} = (1+i_t)^{-1} \tag{A.12}$$

Similarly, $(1+i_t^*)^{-1} = \mathbb{E}_t \frac{{C_t^*}^{-\rho}}{{C_{t+1}^*}^{-\rho}} \frac{P_{t+1}^*}{P_t^*}$. Using (1.9), the above equation can be written as

$$\mathbb{E}_t \frac{{C_t}^{-\rho}}{{C_{t+1}}^{-\rho}} \frac{\left[n+(1-n){T_{t+1}}^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\left[n+(1-n){T_t}^{1-\theta}\right]^{\frac{1}{1-\theta}}} \Pi_{A,t+1} = (1+i_t)^{-1} \tag{A.13}$$

The aggregate demand equations are obtained by log-linearizing the above equations. Log-linearizing the market clearing condition $Y_{A,t} = \left(\frac{P_{A,t}}{P_t}\right)^{-\theta} C_{A,t}$ gives

$$Y_{A,t} = \left(\frac{1}{\left[n+(1-n){T_{t+1}}^{1-\theta}\right]^{1/(1-\theta)}}\right)^{-\theta} C_{A,t} \tag{A.14}$$

$$\bar{Y}\hat{Y}_{A,t} = \bar{C}\hat{C}_t + (1-n)\theta\bar{C}\hat{T}_t$$

$$\Longleftrightarrow \bar{Y}\hat{Y}_{A,t} = \bar{C}\hat{C}_t + (1-n)\theta\bar{C}\hat{T}_t \tag{A.15}$$

$$\Longleftrightarrow \hat{Y}_{A,t} = \hat{C}_t + \theta(1-n)\hat{T}_t \tag{A.16}$$

For Country B, it is

$$\hat{Y}_{B,t} = \hat{C}_t - \theta n\hat{T}_t$$

The law of motion for the exchange rate is the following:

$$\frac{P_{B,t}}{P_{A,t}} \frac{P_{A,t-1}}{P_{B,t-1}} = \frac{S_t}{S_{t-1}} \frac{P_{B,t}^*}{P_{A,t}} \frac{P_{A,t-1}}{P_{B,t-1}^*}$$

$$\Longleftrightarrow \frac{T_t}{T_{t-1}} = \frac{S_t}{S_{t-1}} \frac{\Pi_{B,t}^*}{\Pi_{A,t}} \tag{A.17}$$

The above equation is the law of motion of the exchange rate for the flexible exchange rate regime $\mathcal{F}$. First order log-linearization of (A.17) is (1.30). If the exchange rate is fixed, then $S_t = S_{t-1}\ \forall t$. Therefore,

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_{B,t}^*}{\Pi_{A,t}} \tag{A.18}$$

Similarly, (1.31) is obtained by log-linearizing (A.18).

A.3 Welfare Criteria

Welfare criteria are identical to the ones in Benigno and Benigno (2006). Nevertheless, for convenience this section provides a sketch of derivations. The Technical Appendix of the relevant paper can be referred for further details.[2] Second order approximation to period t utility is

$$\begin{aligned} w_t = \bar{U}_C\bar{C}\Big[&\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 - \bar{\mu}^{-1}\hat{Y}_{A,t} \\ &- \frac{1}{2}\bar{\mu}^{-1}(1+\eta)\hat{Y}_{A,t}^2 + \bar{\mu}^{-1}\eta\hat{a}\hat{Y}_{A,t} \\ &- \frac{1}{2}\bar{\mu}^{-1}(\sigma^{-1}+\eta)\mathrm{var}_h\hat{y}_t(h)\Big] \\ &+ \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.19}$$

Using Woodford (2011, Chapter 6), it is found that

$$\sum_{t=0}^{\infty}\beta^t\mathrm{var}_h\hat{y}_t(h) = \sum_{t=0}^{\infty}\beta^t\mathrm{var}_h\hat{y}_t(h) = \frac{1}{k(1+\sigma\eta)}\sigma^2\sum_{t=0}^{\infty}\beta^t{\pi_{A,t}}^2 = \frac{\sigma^2}{k(1+\sigma\eta)}\sum_{t=0}^{\infty}\beta^t{\pi_{A,t}}^2 \tag{A.20}$$

The following equation is obtained by merging the two equations above and summing over t with discount factor β.

$$\begin{aligned} W_A = \bar{U}_C\bar{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[&\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 - \bar{\mu}^{-1}\hat{Y}_{A,t} \\ &- \frac{1}{2}\bar{\mu}^{-1}(1+\eta)\hat{Y}_{A,t}^2 + \bar{\mu}^{-1}\eta\hat{a}\hat{Y}_{A,t} \\ &- \frac{1}{2}\bar{\mu}^{-1}\sigma k^{-1}{\pi_{A,t}}^2\Big] \\ &+ \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.21}$$

Approximating to the structural equations up to second order and incorporating them into the second order approximation of the welfare function gives

[2] The technical appendix can be found at `http://personal.lse.ac.uk/BENIGNO/`

$$\begin{aligned} W_A = -\frac{1}{2}\bar{U}_C\bar{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[x_t'Q_x x_t + 2x_t'Q_\xi\xi_t + q_{\pi_A}{\pi_{A,t}}^2 + q_{\pi_B}{\pi_{B,t}^*}^2\big] \\ - \bar{U}_C\bar{C}\zeta_1\big[a_x'x_0 + a_\xi'\xi_0 + \frac{1}{2}x_0'A_\xi\xi_0 + \frac{1}{2}a_{\pi_A}{\pi_{A,0}}^2\big] \\ - \bar{U}_C\bar{C}\zeta_2\big[b_x'x_0 + b_\xi'\xi_0 + \frac{1}{2}b_0'B_\xi\xi_0 + \frac{1}{2}b_{\pi_B}{\pi_{B,0}}^2\big] \\ K_0 + \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.22}$$

where $K_0 \equiv \bar{U}_C\bar{C}\big[\zeta_1 V_0 + \zeta_2 V_0^*\big]$ and $x_t' \equiv [\hat{Y}_{A,t}\ \hat{C}_t\ \hat{p}_{A,t}\ \hat{Y}_{A,t}\ \hat{C}_t^*\ \hat{p}_{B,t}^*\ \hat{T}_t]$. Following the same process for the foreign country gives

$$\begin{aligned} W_B = -\frac{1}{2}\bar{U}_C\bar{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[x_t'Q_x^* x_t + 2x_t'Q_\xi^*\xi_t + q_{\pi_A}^*{\pi_{A,t}}^2 + q_{\pi_B}^*{\pi_{B,t}^*}^2\big] \\ - \bar{U}_C\bar{C}\zeta_1^*\big[a_x'x_0 + a_\xi'\xi_0 + \frac{1}{2}x_0'A_\xi\xi_0 + \frac{1}{2}a_{\pi_A}{\pi_{A,0}}^2\big] \\ - \bar{U}_C\bar{C}\zeta_2^*\big[b_x'x_0 + b_\xi'\xi_0 + \frac{1}{2}b_0'B_\xi\xi_0 + \frac{1}{2}b_{\pi_B}{\pi_{B,0}}^2\big] \\ K_0^* + \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.23}$$

where $K_0^* \equiv \bar{U}_C\bar{C}\big[\zeta_1^* V_0 + \zeta_2^* V_0^*\big]$. Then, following Benigno and Benigno (2006) again, the two equations below are obtained.

$$\begin{aligned} W_A = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[\lambda_{y_A}\big(\hat{Y}_{A,t} - \tilde{Y}_{A,t}^A\big)^2 + \lambda_{y_B}\big(\hat{Y}_{B,t} - \tilde{Y}_{B,t}^A\big)^2 \\ + \lambda_q\big(\hat{T}_t - \tilde{T}_t^A\big)^2 + \lambda_{\pi_A}{\pi_{A,t}}^2 + \lambda_{\pi_B}{\pi_{B,t}^*}^2\Big] \\ + K_0 + \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.24}$$

$$\begin{aligned} W_B = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[\lambda_{y_A}^*\big(\hat{Y}_{A,t} - \tilde{Y}_{A,t}^B\big)^2 + \lambda_{y_B}^*\big(\hat{Y}_{B,t} - \tilde{Y}_{B,t}^B\big)^2 \\ + \lambda_q^*\big(\hat{T}_t - \tilde{T}_t^B\big)^2 + \lambda_{\pi_A}^*{\pi_{A,t}}^2 + \lambda_{\pi_B}^*{\pi_{B,t}^*}^2\Big] \\ + K_0^* + \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.25}$$

Welfare criterion of the monetary authority of the union is

$$
\begin{aligned}
W = -\frac{\lambda_y^w}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[& n\big(\hat{Y}_{A,t}-\tilde{Y}^w_{A,t}\big)^2+(1-n)\big(\hat{Y}_{B,t}-\tilde{Y}^w_{B,t}\big)^2 \\
&+n(1-n)\theta\psi\big(\hat{T}_t-\tilde{T}^w_t\big)^2+\frac{\sigma n}{\kappa}{\pi_{A,t}}^2+\frac{\sigma(1-n)}{\kappa^*}{\pi^*_{B,t}}^2\Big] \\
& K_0^W+\text{t.i.p}+\mathcal{O}(||\xi||^3)
\end{aligned}
\tag{A.26}
$$

where $K_0^W \equiv nK_0+(1-n)K_0^*$. V_0 and V_0^* are defined in Benigno and Woodford (2005) and Benigno and Benigno (2006). They are constituted of $t=0$ inflation, and output at $t=0$ and in succeeding terms. This means that they depend on monetary policy. Let V_0 and V_0^* be given, then K_0, K_0^*, and K_0^W are also determined. It follows that first order approximations to V_0 and V_0^* are $\pi_{A,0}$ and $\pi^*_{A,0}$ respectively. Then, conditional on $\pi_{A,0}$ and $\pi^*_{B,0}$.

$$
\begin{aligned}
W_A = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[&\lambda_{y_A}\big(\hat{Y}_{A,t}-\tilde{Y}^A_{A,t}\big)^2+\lambda_{y_B}\big(\hat{Y}_{B,t}-\tilde{Y}^A_{B,t}\big)^2 \\
&+\lambda_q\big(\hat{T}_t-\tilde{T}^A_t\big)^2+\lambda_{\pi_A}{\pi_{A,t}}^2+\lambda_{\pi_B}{\pi^*_{B,t}}^2\Big] \\
&+\text{t.i.p}+\mathcal{O}(||\xi||^3)
\end{aligned}
\tag{A.27}
$$

$$
\begin{aligned}
W_B = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[&\lambda^*_{y_A}\big(\hat{Y}_{A,t}-\tilde{Y}^B_{A,t}\big)^2+\lambda^*_{y_B}\big(\hat{Y}_{B,t}-\tilde{Y}^B_{B,t}\big)^2 \\
&+\lambda^*_q\big(\hat{T}_t-\tilde{T}^B_t\big)^2+\lambda^*_{\pi_A}{\pi_{A,t}}^2+\lambda^*_{\pi_B}{\pi^*_{B,t}}^2\Big] \\
&+\text{t.i.p}+\mathcal{O}(||\xi||^3)
\end{aligned}
\tag{A.28}
$$

$$
\begin{aligned}
W = -\frac{\lambda_y^w}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[& n\big(\hat{Y}_{A,t}-\tilde{Y}^w_{A,t}\big)^2+(1-n)\big(\hat{Y}_{B,t}-\tilde{Y}^w_{B,t}\big)^2 \\
&+n(1-n)\theta\psi\big(\hat{T}_t-\tilde{T}^w_t\big)^2+\frac{\sigma n}{\kappa}{\pi_{A,t}}^2+\frac{\sigma(1-n)}{\kappa^*}{\pi^*_{B,t}}^2\Big] \\
&+\text{t.i.p}+\mathcal{O}(||\xi||^3)
\end{aligned}
\tag{A.29}
$$

$\bar{\pi}_{A,0}$ and $\bar{\pi}_{B,0}$ are arbitrary. Given $\pi_{A,0}$ and/or $\pi_{B,0}$ (depending on monetary regime), welfare criteria rank monetary policies accurately in a monetary regime $\mathcal{F}$, $\mathcal{C}$, or $\mathcal{M}$. Benigno and Woodford (2005) shows that this initial condition for PPIs are compatible with monetary policy from timeless perspective described in Woodford (1999). Benigno and Benigno (2006) shows that initial conditions for domestic inflation in two-country model is also compatible with monetary policy from timeless perspective. However, to be able to compare the two monetary regimes initial inflation conditions, $\bar{\pi}_{A,0}$ and $\bar{\pi}_{B,0}$, must be identical in both monetary regimes.

Weights of output, inflation, and the terms of trade targets are

$$\lambda_{y_A} \equiv \lambda_y^w\left[1-(1-n)\tfrac{\theta(1+\eta)}{1+\eta\theta}\right] \qquad \lambda^*_{y_A} \equiv n\left[\lambda_y^w\tfrac{\theta(1+\eta)}{1+\eta\theta}\right]$$

$$\lambda_{y_B} \equiv (1-n)\left[\lambda_y^w\tfrac{\theta(1+\eta)}{1+\eta\theta}\right] \qquad \lambda^*_{y_B} \equiv \lambda_y^w\left[1-n\tfrac{\theta(1+\eta)}{1+\eta\theta}\right]$$

$$\lambda_q \equiv (1-\theta\rho)\theta(1-n)\left[n\tfrac{\theta(1+\eta)}{1+\eta\theta}+(1-n)\tfrac{1-\theta}{1+\eta\theta}\right] \qquad \lambda^*_q \equiv (1-\theta\rho)\theta n\left[(1-n)\tfrac{\theta(1+\eta)}{1+\eta\theta}+n\tfrac{1-\theta}{1+\eta\theta}\right]$$

$$\lambda_{\pi_A} \equiv \tfrac{\sigma}{k}\left[n\tfrac{\theta(1+\eta)}{1+\eta\theta}+\tfrac{1-\theta}{1+\eta\theta}\right] \qquad \lambda^*_{\pi_A} \equiv \tfrac{\sigma}{k}n\tfrac{\theta(1+\eta)}{1+\eta\theta}$$

$$\lambda_{\pi_B} \equiv \tfrac{\sigma}{k}(1-n)\tfrac{\theta(1+\eta)}{1+\eta\theta} \qquad \lambda^*_{\pi_B} \equiv \tfrac{\sigma}{k}\left[(1-n)\tfrac{\theta(1+\eta)}{1+\eta\theta}+\tfrac{1-\theta}{1+\eta\theta}\right]$$

Output and the terms of trade targets are

$$\tilde{Y}^A_{A,t} \equiv \frac{1}{\lambda_{y_A}}\Big[\eta\frac{1-\theta+n\theta(1+\eta)-n(1-n)(1-\theta\rho)}{1+\eta\theta}\hat{a}_A+\frac{(1-n)n\eta(1-\rho\theta)}{1+\eta\theta}\hat{a}_{B,t}+(1-n)\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{A,t}\Big] \tag{A.30}$$

$$\tilde{Y}^A_{B,t} \equiv \frac{1}{\lambda_{y_B}}\Big[\frac{(1-n)n\eta(1-\rho\theta)}{1+\eta\theta}\hat{a}_{A,t}+(1-n)\eta\frac{\theta(1+\eta)-n(1-\theta\rho)}{1+\eta\theta}\hat{a}_{B,t}-(1-n)\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{B,t}\Big] \tag{A.31}$$

$$\tilde{T}^A_t \equiv \frac{1}{\lambda_q}\eta\theta\frac{1-\theta\rho}{1+\eta\theta}(\hat{a}_{A,t}-\hat{a}_{B,t}) \tag{A.32}$$

$$\tilde{Y}^B_{A,t} \equiv \frac{1}{\lambda^*_{y_A}}\Big[n\eta\frac{\theta(1+\eta)-(1-n)(1-\theta\rho)}{1+\eta\theta}\hat{a}_{A,t}+\frac{(1-n)n\eta(1-\rho\theta)}{1+\eta\theta}\hat{a}_B-n\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{A,t}\Big] \tag{A.33}$$

$$\tilde{Y}^B_{B,t} \equiv \frac{1}{\lambda^*_{y_B}}\Big[\frac{(1-n)n\eta(1-\rho\theta)}{1+\eta\theta}\hat{a}_{A,t}+\eta\frac{1-\theta+(1-n)\theta(1+\eta)-n(1-n)(1-\theta\rho)}{1+\eta\theta}\hat{a}_{B,t}+n\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{B,t}\Big] \tag{A.34}$$

$$\tilde{T}^B_t \equiv \frac{1}{\lambda^*_q}\eta\theta\frac{1-\theta\rho}{1+\eta\theta}(\hat{a}_{A,t}-\hat{a}_{B,t}) \tag{A.35}$$

Below, national targets as well as weights of outputs, PPIs, and the terms of trade in welfare criteria when $\rho=\theta=1$ are presented.

$$
\begin{aligned}
&\tilde{\tilde{\lambda}}_y^w &&= (1+\eta) \\
&\tilde{\tilde{\lambda}}_{y_A} &&= \tilde{\tilde{\lambda}}_{y_A}^* &&= (1+\eta)n \\
&\tilde{\tilde{\lambda}}_{y_A} &&= \tilde{\tilde{\lambda}}_{y_A}^* &&= (1+\eta)(1-n) \\
&\tilde{\tilde{\lambda}}_q &&= \tilde{\tilde{\lambda}}_q^* &&= 0 \\
&\tilde{\tilde{\lambda}}_{\pi_A} &&= \tilde{\tilde{\lambda}}_{\pi_A}^* &&= \tfrac{\sigma n}{k} \\
&\tilde{\tilde{\lambda}}_{\pi_B} &&= \tilde{\tilde{\lambda}}_{\pi_B}^* &&= \tfrac{\sigma(1-n)}{k}
\end{aligned}
$$

Output and the terms of trade targets when $\rho = \theta = 1$ are

$$\tilde{\tilde{Y}}_{A,t}^A \equiv \frac{1}{\tilde{\tilde{\lambda}}_{y_A}}\Big[\eta n\hat{a}_{A,t} + (1-n)\hat{\mu}_{A,t}\Big] \tag{A.36}$$

$$\tilde{\tilde{Y}}_{B,t}^A \equiv \frac{1}{\tilde{\tilde{\lambda}}_{y_B}}\Big[\eta(1-n)\hat{a}_{B,t} - (1-n)\hat{\mu}_{B,t}\Big] \tag{A.37}$$

$$\tilde{\tilde{T}}_t^A \equiv \frac{\eta}{(1-n)n(1+\eta)}(\hat{a}_{A,t} - \hat{a}_{B,t}) \tag{A.38}$$

$$\tilde{\tilde{Y}}_{A,t}^B \equiv \frac{1}{\tilde{\tilde{\lambda}}_{y_A}^*}\Big[n\eta\hat{a}_{A,t} - n\hat{\mu}_{A,t}\Big] \tag{A.39}$$

$$\tilde{\tilde{Y}}_{B,t}^B \equiv \frac{1}{\tilde{\tilde{\lambda}}_{y_B}^*}\Big[(1-n)\eta\hat{a}_{B,t} + n\hat{\mu}_{B,t}\Big] \tag{A.40}$$

$$\tilde{\tilde{T}}_t^B \equiv \frac{\eta}{n(1-n)(1+\eta)}(\hat{a}_{A,t} - \hat{a}_{B,t}) \tag{A.41}$$

A.4 Monetary Policy Problems in $\mathcal{F}$

In the flexible exchange rate regime, the optimal monetary policy problem for A's monetary authority is to maximize (2.1) subject to (1.27), (1.28), (1.29). Lagrangian of the problem can be written as

$$
\begin{aligned}
\mathcal{L} =& \mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\bigg\{-\frac{1}{2}\Big[\lambda_{y_A}(\hat{Y}_{A,t} - \tilde{Y}_{A,t}^A)^2 + \lambda_{y_B}(\hat{Y}_{B,t} - \tilde{Y}_{B,t}^A)^2 + \lambda_q(\hat{T}_t - \tilde{T}_t^A)^2 + \lambda_{\pi_A}\pi_{A,t}{}^2 + \lambda_{\pi_B}\pi_{B,t}^*{}^2\Big] \\
&+ \vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{A,t+1} + \kappa(\hat{Y}_{A,t} - \tilde{Y}_{A,t}^w) + (1-n)\kappa\psi(\hat{T}_t - \tilde{T}_t^w) + u_t - \pi_{A,t}\Big] \\
&+ \vartheta_{\pi,t}^*\Big[\beta\mathbb{E}_t\pi_{B,t+1}^* + \kappa^*(\hat{Y}_{B,t} - \tilde{Y}_{B,t}^w) - n\kappa^*\psi(\hat{T}_t - \tilde{T}_t^w) + u_t^* - \pi_{B,t}^*\Big] \\
&+ \vartheta_{\mathcal{T},t}\Big[\theta^{-1}[(\hat{Y}_{A,t} - \tilde{Y}_{A,t}^w) - (\hat{Y}_{B,t} - \tilde{Y}_{B,t}^w)] - (\hat{T}_t - \tilde{T}_t^w)\Big]\bigg\}
\end{aligned} \tag{A.42}
$$

$\vartheta_{\pi,t}$, $\vartheta_{\pi,t}^*$, $\vartheta_{\mathcal{T},t}$ are time dependent lagrange multipliers for (1.27), (1.28), (1.29) respectively. First order conditions with respect to $\hat{Y}_{A,t}$, $\hat{Y}_{B,t}$, $\pi_{A,t}$, and $\hat{T}_t$ are respectively

$$-\lambda_{y_A}\left(\hat{Y}_{A,t} - \tilde{Y}^{A}_{A,t}\right) + \vartheta_{\pi,t}\kappa + \vartheta_{\mathfrak{T},t}\theta^{-1} = 0 \tag{A.43}$$

$$-\lambda_{y_B}\left(\hat{Y}_{B,t} - \tilde{Y}^{A}_{B,t}\right) + \vartheta^{*}_{\pi,t}\kappa^{*} - \vartheta_{\mathfrak{T},t}\theta^{-1} = 0 \tag{A.44}$$

$$-\lambda_{\pi_A}\pi_{A,t} - \vartheta_{\pi,t} + \vartheta_{\pi,t-1} = 0 \tag{A.45}$$

$$-\lambda_{q}\left(\hat{T} - \tilde{T}^{A}_{t}\right) + \vartheta_{\pi,t}(1-n)\kappa\psi - \vartheta^{*}_{\pi,t}n\kappa^{*}\psi - \vartheta_{\mathfrak{T},t} = 0 \tag{A.46}$$

B's monetary problem is symmetric to A's problem. $\varpi_{\pi,t}$, $\varpi^{*}_{\pi,t}$, $\varpi_{\mathfrak{T},t}$ are lagrange multipliers for (1.27), (1.28), (1.29) respectively for B. First order conditions to B's problem are

$$-\lambda^{*}_{y_A}\left(\hat{Y}_{A,t} - \tilde{Y}^{B}_{A,t}\right) + \varpi_{\pi,t}\kappa + \varpi_{\mathfrak{T},t}\theta^{-1} = 0 \tag{A.47}$$

$$-\lambda^{*}_{y_B}\left(\hat{Y}_{B,t} - \tilde{Y}^{B}_{B,t}\right) + \varpi^{*}_{\pi,t}\kappa^{*} - \varpi_{\mathfrak{T},t}\theta^{-1} = 0 \tag{A.48}$$

$$-\lambda^{*}_{\pi_B}\pi^{*}_{B,t} - \varpi^{*}_{\pi,t} + \varpi^{*}_{\pi,t-1} = 0 \tag{A.49}$$

$$-\lambda^{*}_{q}\left(\hat{T} - \tilde{T}^{B}_{t}\right) + \varpi_{\pi,t}(1-n)\kappa\psi - \varpi^{*}_{\pi,t}n\kappa^{*}\psi - \varpi_{\mathfrak{T},t} = 0 \tag{A.50}$$

A.5 Monetary Policy Problem in $\mathcal{M}$

Monetary problem in the union is to maximize W subject to (1.27), (1.28), (1.29), (1.31). Lagrangian of the problem is

$$\begin{aligned}
\mathcal{L} =& \mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big\{ -\frac{\lambda_y^w}{2}\Big[n\big(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}\big)^2 + (1-n)\big(\hat{Y}_{B,t} - \tilde{Y}^w_{B,t}\big)^2 + n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}^w_t\big)^2 + \frac{\sigma n}{\kappa}{\pi_{A,t}}^2 + \frac{\sigma(1-n)}{\kappa^*}{\pi^*_{B,t}}^2\Big] \\
&+ \vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{A,t+1} + \kappa\big(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}\big) + (1-n)\kappa\psi\big(\hat{T}_t - \tilde{T}^w_t\big) + u_t - \pi_{A,t}\Big] \\
&+ \vartheta^*_{\pi,t}\Big[\beta\mathbb{E}_t\pi^*_{B,t+1} + \kappa^*\big(\hat{Y}_{B,t} - \tilde{Y}^w_{B,t}\big) - n\kappa^*\psi\big(\hat{T}_t - \tilde{T}^w_t\big) + u^*_t - \pi^*_{B,t}\Big] \\
&+ \vartheta_{\mathfrak{T},t}\Big[\theta^{-1}\big[\big(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}\big) - \big(\hat{Y}_{B,t} - \tilde{Y}^w_{B,t}\big)\big] - \big(\hat{T}_t - \tilde{T}^w_t\big)\Big] \\
&+ \vartheta_{\mathrm{Ex},t}\Big[\Delta\hat{T}_t + \pi_{A,t} - \pi^*_{B,t} = 0\Big]\Big\}
\end{aligned} \tag{A.51}$$

First order conditions are

$$-n\big(\hat{Y}_{A,t}-\tilde{Y}^{w}_{A,t}\big)+\vartheta_{\pi,t}\kappa+\vartheta_{\mathcal{T},t}\theta^{-1}=0 \tag{A.52}$$

$$-(1-n)\big(\hat{Y}_{B,t}-\tilde{Y}^{w}_{B,t}\big)+\vartheta_{\pi,t}\kappa^{*}-\vartheta_{\mathcal{T},t}\theta^{-1}=0 \tag{A.53}$$

$$-\frac{\sigma n}{\kappa}\pi_{A,t}-\vartheta_{\pi,t}+\vartheta_{\pi,t-1}+\vartheta_{\mathrm{Ex},t}=0 \tag{A.54}$$

$$-\frac{\sigma(1-n)}{\kappa^{*}}\pi^{*}_{B,t}-\vartheta_{\pi,t}+\vartheta_{\pi,t-1}-\vartheta_{\mathrm{Ex},t}=0 \tag{A.55}$$

$$-n(1-n)\theta\psi\big(\hat{T}_{t}-\tilde{T}^{w}_{t}\big)+(1-n)\kappa\psi\vartheta_{\pi,t}-n\kappa^{*}\psi\vartheta^{*}_{\pi,t}-\vartheta_{\mathcal{T},t}+\vartheta_{\mathrm{Ex},t}-\beta\vartheta_{\mathrm{Ex},t+1}=0 \tag{A.56}$$

Under the assumption that $\kappa=\kappa^{*}$, it is obtained that

$$-\sigma(n\pi_{A,t}+(1-n)\pi^{*}_{B,t})=n\Delta\big(\hat{Y}_{A,t}-\tilde{Y}^{w}_{A,t}\big)+(1-n)\Delta\big(\hat{Y}_{B,t}-\tilde{Y}^{w}_{B,t}\big) \tag{A.57}$$

A.6 Monetary Policy Problem in $\mathcal{C}$

Monetary problem in the cooperative flexible exchange rate regime is to maximize W subject to (1.27), (1.28), and (1.29). Lagrangian of the problem is

$$\begin{aligned}\mathcal{L}=&\mathbb{E}_0\sum_{t=1}^{\infty}\beta^{t}\Big\{-\frac{\lambda^{w}_{y}}{2}\Big[n(\hat{Y}_{A,t}-\tilde{Y}^{w}_{A,t})^{2}+(1-n)(\hat{Y}_{B,t}-\tilde{Y}^{w}_{B,t})^{2}+n(1-n)\theta\psi(\hat{T}_{t}-\tilde{T}^{w}_{t})^{2}+\frac{\sigma n}{\kappa}{\pi_{A,t}}^{2}+\frac{\sigma(1-n)}{\kappa^{*}}{\pi^{*}_{B,t}}^{2}\Big]\\&+\vartheta_{\pi,t}\Big[\beta\mathbb{E}_{t}\pi_{A,t+1}+\kappa(\hat{Y}_{A,t}-\tilde{Y}^{w}_{A,t})+(1-n)\kappa\psi(\hat{T}_{t}-\tilde{T}^{w}_{t})+u_{t}-\pi_{A,t}\Big]\\&+\vartheta^{*}_{\pi,t}\Big[\beta\mathbb{E}_{t}\pi^{*}_{B,t+1}+\kappa^{*}(\hat{Y}_{B,t}-\tilde{Y}^{w}_{B,t})-n\kappa^{*}\psi(\hat{T}_{t}-\tilde{T}^{w}_{t})+u^{*}_{t}-\pi^{*}_{B,t}\Big]\\&+\vartheta_{\mathcal{T},t}\Big[\theta^{-1}[(\hat{Y}_{A,t}-\tilde{Y}^{w}_{A,t})-(\hat{Y}_{B,t}-\tilde{Y}^{w}_{B,t})]-(\hat{T}_{t}-\tilde{T}^{w}_{t})\Big]\Big\}\end{aligned} \tag{A.58}$$

First order conditions are

$$-n\big(\hat{Y}_{A,t}-\tilde{Y}^{w}_{A,t}\big)+\vartheta_{\pi,t}\kappa+\vartheta_{\mathcal{T},t}\theta^{-1}=0 \tag{A.59}$$

$$-(1-n)\big(\hat{Y}_{B,t}-\tilde{Y}^{w}_{B,t}\big)+\vartheta_{\pi,t}\kappa^{*}-\vartheta_{\mathcal{T},t}\theta^{-1}=0 \tag{A.60}$$

$$-\frac{\sigma n}{\kappa}\pi_{A,t}-\vartheta_{\pi,t}+\vartheta_{\pi,t-1}=0 \tag{A.61}$$

$$-\frac{\sigma(1-n)}{\kappa^{*}}\pi^{*}_{B,t}-\vartheta_{\pi,t}+\vartheta_{\pi,t-1}=0 \tag{A.62}$$

$$-n(1-n)\theta\psi\big(\hat{T}_{t}-\tilde{T}^{w}_{t}\big)+(1-n)\kappa\psi\vartheta_{\pi,t}-n\kappa^{*}\psi\vartheta^{*}_{\pi,t}-\vartheta_{\mathcal{T},t}=0 \tag{A.63}$$

A.7 Proof of Proposition 1

This section provides the proof of Proposition 1. It shows that in $\mathcal{F}$, $\mathcal{C}$, and $\mathcal{M}$, if $\xi_0 = \xi = [a_A\ a_B\ \mu_A\ \mu_B]'$, then the optimal monetary policy satisfies $\pi_{A,0} = \pi_{B,0} = 0$. Notice that this is in fact a trivial proposition. If the economy is in the steady state at $t = -1$ and shocks are absent in $t = 0$, then the economy is in the steady state in $t = 0$ as well. It follows that in all three regimes, in the steady state, $\pi_A = \pi_B = 0$. Then, $\pi_{A,0} = \pi_{B,0}$.

Nevertheless, the section shows that for each regime $\xi_0 = \xi$ implies $\pi_{A,0} = \pi_{B,0} = \pi_A = \pi_B$.

It is noted that in all three regimes, if $\xi_0 = [a_A\ a_B\ \mu_A\ \mu_B]$, then

$$\tilde{Y}^w_{A,0} = \tilde{Y}^w_{B,0} = \tilde{Y}^A_{A,0} = \tilde{Y}^A_{B,0} = \tilde{Y}^B_{A,0} = \tilde{Y}^B_{B,0} = \tilde{T}^A_0 = \tilde{T}^B_0 = \tilde{T}^W_0 = 0 \tag{A.64}$$

First, $\mathcal{M}$ is inspected. Combining (1.27) and (1.28) gives

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \mathcal{Y}_t + u_{W,t} \tag{A.65}$$

$$\pi_{R,t} = \beta \mathbb{E}_t \pi_{R,t+1} + \kappa \mathcal{Y}_{R,t} + u_{R,t} \tag{A.66}$$

with $\pi_t = n\pi_{A,t} + (1-n)\pi_{B,t}$ and $\mathcal{Y}_t = n\hat{Y}_{A,t} + (1-n)\hat{Y}_{B,t}$, $\pi_{R,t} = \pi_{A,t} - \pi^*_{B,t}$ and $\mathcal{Y}_{R,t} = \hat{Y}_{A,t} - \hat{Y}_{B,t}$. Similarly, the optimality condition in $\mathcal{M}$ given by (A.57) can be written as

$$-\sigma\pi_t = \Delta\mathcal{Y}_t \tag{A.67}$$

Then, the economy wide average inflation and average output can be solved using (A.65) and (A.67). Combining (A.65) and (A.67) gives

$$\begin{aligned}
&\pi_t = \beta\mathbb{E}\pi_{t+1} - \sigma\kappa\pi_{-1,t} + u_{W,t} \\
\Leftrightarrow &\pi_{-1,t} - \pi_{-1,t-1} = \beta\mathbb{E}_t\pi_{-1,t+1} - \beta\pi_{-1,t} - \sigma\kappa\pi_{-1,t} + u_{W,t} \\
\Leftrightarrow &\mathbb{E}_t\pi_{-1,t+1} = \frac{1+\beta+\sigma\kappa}{\beta}\pi_{-1,t} - \frac{1}{\beta}\pi_{-1,t-1} - \frac{1}{\beta}u_{W,t}
\end{aligned} \tag{A.68}$$

where $\pi_{-1,t} = \sum_{k=0}^{t} \pi_t$. This is a second order stochastic difference equation and it can be shown that

$$\pi_{-1,0} = \pi_0 = \frac{\Phi}{1-\beta\gamma\Phi} u_{W,0} \tag{A.69}$$

where $\Phi \in (0,1)$ is the eigenvalue of (A.68). $u_{W,0} = 0$, therefore,

$$\pi_0 = 0 \tag{A.70}$$

$$\mathcal{Y}_0 = 0 \tag{A.71}$$

Using (A.66), (1.29), and (1.31), it is also possible to derive the following equation

$$\mathbb{E}_t \hat{T}_{t+1} = \frac{1+\beta+\kappa\theta+\kappa\psi)}{\beta} \hat{T}_t - \frac{1}{\beta} \hat{T}_{t-1} + \frac{1}{\beta} u_{R,t} \tag{A.72}$$

(A.72) is fairly similar to (A.68). It is trivial that for $u_{R,0} = u_0 - u_0^* = u - u^* = 0$

$$\hat{T}_0 = 0 \tag{A.73}$$

Combining (A.73) with (1.31) gives $\pi_{A,0} = \pi_{B,0}$. It follows that using (A.70), it can be obtained that

$$\pi_{A,0} = \pi_{B,0} = 0 \tag{A.74}$$

Then, it is concluded that if $\xi_0 = \xi$, then $\pi_{A,0} = \pi_{B,0} = 0$ in $\mathcal{M}$.

In $\mathcal{C}$, (A.65) and (A.66) hold because they are derived from (1.27) and (1.28) independent of the monetary regime. It follows that combining first order conditions of monetary policy problem in $\mathcal{C}$ gives

$$-\sigma\pi_{A,t} = \Delta\hat{Y}_{A,t} \tag{A.75}$$

$$-\sigma\pi_{B,t} = \Delta\hat{Y}_{B,t} \tag{A.76}$$

Averaging the optimality conditions above, (A.67) is obtained. Then, as in $\mathcal{M}$, (A.70) and (A.71) holds. This is an irrelevance result provided by Groll and Monacelli (2019, Proposition 3). It is also possible to combine (A.75) and (A.76) with (1.29) to get

$$\theta(\hat{T}_t - \hat{T}_{t-1}) = -\sigma\pi_{R,t} \tag{A.77}$$

Then, (A.66), (A.67), and (A.77) are used to get

$$\mathbb{E}_t\hat{T}_{t+1} = \frac{\sigma^{-1}\theta + \sigma^{-1}\theta\beta - \kappa\theta + \kappa\psi}{\sigma^{-1}\theta\beta}\hat{T}_t - \frac{1}{\beta}\hat{T}_{t-1} - \frac{1}{\sigma^{-1}\theta\beta}u_{R,t} \tag{A.78}$$

Then, as in (A.68) and (A.72), it can be shown that if $\xi_0 = \xi$ the above equation gives

$$\hat{T}_0 = 0 \tag{A.79}$$

(1.29), (A.71), and (A.79) implies that $\hat{Y}_{A,0} = \hat{Y}_{B,0} = 0$. (A.75) and (A.76) shows that $\pi_{A,0} = \pi_{B,0} = 0$. This concludes that if $\xi_0 = \xi$, then $\pi_{A,0} = \pi_{B,0} = 0$ in $\mathcal{C}$.

In $\mathcal{F}$, combining (A.43), (A.44), and (A.46) gives

$$-\lambda_q\hat{T}_t + (1-n)\psi\lambda_{y_A}\hat{Y}_{A,t} - n\psi\lambda_{y_B}\hat{Y}_{B,t} - (1+\psi\theta^{-1})\vartheta_{\mathcal{T},t} = 0 \tag{A.80}$$

Combining (A.43) and (A.45) gives

$$-\lambda_{\pi_A}\pi_{A,t} - \frac{\lambda_{y_A}}{\kappa}\Delta\hat{Y}_{A,t} + \frac{1}{\theta\kappa}(\vartheta_{\mathcal{T},t} - \vartheta_{\mathcal{T},t-1}) = 0 \tag{A.81}$$

Combining (A.80) and (A.81) to get rid of $\vartheta_{\mathcal{T},t}$ gives

$$-\lambda_{\pi_A}\pi_{A,t} - \Big(\frac{\lambda_{y_A}}{\kappa} + \frac{(1-n)\psi\lambda_{y_A}}{\theta\kappa(1+\kappa\theta^{-1})}\Big)\Delta\hat{Y}_{A,t} - \frac{n\psi\lambda_{y_B}}{\theta\kappa(1+\kappa\theta^{-1})}\hat{Y}_{B,t} - \frac{\lambda_q}{\theta\kappa(1+\kappa\theta^{-1})}\hat{T}_t = 0 \tag{A.82}$$

Following the same method for Country-B gives

$$-\lambda_q^* \hat{T}_t + (1-n)\psi\lambda_{y_A}^*(\hat{Y}_{A,t} - \tilde{Y}_{A,t}^B) - n\psi\lambda_{y_B}^*\hat{Y}_{B,t} - (1+\psi\theta^{-1})\varpi_{\mathcal{T},t} = 0 \quad \text{(A.83)}$$

and

$$-\lambda_{\pi_B}^* \pi_{B,t}^* - \frac{\lambda_{y_B}^*}{\kappa}\Delta\hat{Y}_{B,t} + \frac{1}{\theta\kappa}(\varpi_{\mathcal{T},t} - \varpi_{\mathcal{T},t-1}) = 0 \quad \text{(A.84)}$$

Again, combining (A.83) and (A.84) to get rid of $\varpi_{\mathcal{T},t}$ gives

$$-\lambda_{\pi_B}^* \pi_{B,t}^* + \frac{(1-n)\psi\lambda_{y_A}^*}{\theta\kappa}\Delta\hat{Y}_{A,t} - \Big(\frac{\lambda_{y_B}^*}{\kappa} + \frac{n\psi\lambda_{y_B}^*}{\theta\kappa}\Big)\Delta\hat{Y}_{B,t} - \frac{\lambda_q^*}{\theta\kappa}\hat{T}_t = 0 \quad \text{(A.85)}$$

(A.82), (A.85), (1.27), (1.28), (1.29) constructs a system of equations composed of a vector of endogenous variables $X_t \equiv [\hat{Y}_{A,t}\ \hat{Y}_{B,t}\ \hat{T}_t\ \pi_{A,t}\ \pi_{B,t}]'$ and exogenous variables $U_t = [u_t\ u_t^*]'$. If $\xi_0 = \xi$, then $U_0 = 0$ and following Blanchard and Kahn (1980) method, it is easy to see that $X_0 = 0$. Therefore, if $\xi_0 = \xi$, then $\pi_{A,0} = \pi_{B,0} = 0$ in $\mathcal{F}$.

$\square$

Appendix B

APPENDIX FOR CHAPTER 2

B.1 Monetary Policy Problems in $\mathcal{F}$ ($\rho = \theta = 1$ Case)

Under the assumption of $\rho = \theta = 1$, A's monetary policy is to maximize (2.1) subject to (2.4). Lagrangian of the problem is

$$\begin{aligned}\mathcal{L} =&\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Bigg\{ -\frac{1}{2}\Big[\lambda_{y_A}\big(\hat{Y}_{A,t} - \tilde{Y}^B_{A,t}\big)^2 + \lambda y_B\big(\hat{Y}_{B,t} - \tilde{Y}^B_{B,t}\big)^2 + \lambda_{\pi_A}{\pi_{A,t}}^2 + \lambda_{\pi_B}{\pi^*_{B,t}}^2\Big] \\ &+ \vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{A,t+1} + \kappa\big(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}\big) + u_t - \pi_{A,t}\Big]\Bigg\}\end{aligned} \tag{B.1}$$

First order conditions are

$$-\lambda_{y_A}\big(\hat{Y}_{A,t} - \tilde{Y}^B_{A,t}\big) + \vartheta_{\pi,t}\kappa = 0 \tag{B.2}$$

$$-\lambda_{\pi_A}\pi_{A,t} - \vartheta_{\pi,t} + \vartheta_{\pi,t-1} = 0 \tag{B.3}$$

Combining the first order conditions gives

$$\begin{aligned}-\lambda_{\pi_A}\pi_{A,t} &= \frac{\lambda_{y_A}}{\kappa}\Delta\big(\hat{Y}_{A,t} - \tilde{Y}^A_{A,t}\big) \\ \Leftrightarrow -\sigma\pi_{A,t} &= \Delta\big(\hat{Y}_{A,t} - \tilde{Y}^A_{A,t}\big)\end{aligned}$$

Following the same process for B gives

$$-\sigma\pi_{B,t} = \Delta\big(\hat{Y}_{B,t} - \tilde{Y}^B_{B,t}\big) \tag{B.4}$$

The two optimality condition along with (2.3) and (2.4) pin down all endogenous variables.

B.2 Monetary Policy Problem in $\mathcal{M}$ ($\rho = \theta = 1$ Case)

Monetary problem in the monetary union is to maximize W subject to (1.27), (1.28), (1.31). Lagrangian of the problem is

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Bigg\{ -\frac{\lambda_y^w}{2}\Big[n\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^w_{A,t}\big)^2 + (1-n)\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^w_{B,t}\big)^2 + \frac{\sigma n}{\kappa}{\pi_{A,t}}^2 + \frac{\sigma(1-n)}{\kappa^*}{\pi^*_{B,t}}^2 \Big] \\ &+ \vartheta_{\pi,t}\Big[\beta \mathbb{E}_t \pi_{A,t+1} + \kappa\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^w_{A,t}\big) + u_t - \pi_{A,t}\Big] \\ &+ \vartheta^*_{\pi,t}\Big[\beta \mathbb{E}_t \pi^*_{B,t+1} + \kappa^*\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^w_{B,t}\big) + u^*_t - \pi^*_{B,t}\Big] \\ &+ \vartheta_{\mathrm{Ex},t}\Big[\Delta \hat{T}_t + \pi_{A,t} - \pi^*_{B,t} = 0\Big]\Bigg\}\end{aligned} \tag{B.5}$$

First order conditions are

$$-n\big(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}\big) + \vartheta_{\pi,t}\kappa + \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.6}$$

$$-(1-n)\big(\hat{Y}_{B,t} - \tilde{Y}^w_{B,t}\big) + \vartheta_{\pi,t}\kappa^* - \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.7}$$

$$-\frac{\sigma n}{\kappa}\pi_{A,t} - \vartheta_{\pi,t} + \vartheta_{\pi,t-1} + \vartheta_{\mathrm{Ex},t} = 0 \tag{B.8}$$

$$-\frac{\sigma(1-n)}{\kappa^*}\pi^*_{B,t} - \vartheta_{\pi,t} + \vartheta_{\pi,t-1} - \vartheta_{\mathrm{Ex},t} = 0 \tag{B.9}$$

$$-\vartheta_{\mathcal{T},t} + \vartheta_{\mathrm{Ex},t} - \vartheta_{\mathrm{Ex},t+1} = 0 \tag{B.10}$$

Under the assumption that $\kappa = \kappa^*$,

$$-\sigma(n\pi_{A,t} + (1-n)\pi^*_{B,t}) = n\Delta\big(\hat{Y}_{A,t} - \tilde{Y}^w_{A,t}\big) + (1-n)\Delta\big(\hat{Y}_{B,t} - \tilde{Y}^w_{B,t}\big) \tag{B.11}$$

B.3 Monetary Policy Problem in $\mathcal{C}$ ($\rho = \theta = 1$ Case)

Monetary problem in the cooperative flexible exchange rate regime is to maximize W subject to (1.27), (1.28). Lagrangian of the problem is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Bigg\{ -\frac{\lambda_y^w}{2} \Big[n\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}_{A,t}^w\big)^2 + (1-n)\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}_{B,t}^w\big)^2 + \frac{\sigma n}{\kappa} {\pi_{A,t}}^2 + \frac{\sigma(1-n)}{\kappa^*} {\pi_{B,t}^*}^2 \Big]$$
$$+ \vartheta_{\pi,t} \Big[\beta \mathbb{E}_t \pi_{A,t+1} + \kappa\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}_{A,t}^w\big) + u_t - \pi_{A,t} \Big]$$
$$+ \vartheta_{\pi,t}^* \Big[\beta \mathbb{E}_t \pi_{B,t+1}^* + \kappa^*\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}_{B,t}^w\big) + u_t^* - \pi_{B,t}^* \Big] \Bigg\} \tag{B.12}$$

First order conditions are

$$-n\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}_{A,t}^w\big) + \vartheta_{\pi,t}\kappa = 0 \tag{B.13}$$
$$-(1-n)\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}_{B,t}^w\big) + \vartheta_{\pi,t}\kappa^* = 0 \tag{B.14}$$
$$-\frac{\sigma n}{\kappa}\pi_{A,t} - \vartheta_{\pi,t} + \beta\vartheta_{\pi,t-1} = 0 \tag{B.15}$$
$$-\frac{\sigma(1-n)}{\kappa^*}\pi_{B,t}^* - \vartheta_{\pi,t} + \beta\vartheta_{\pi,t-1} = 0 \tag{B.16}$$

Using first order condition,

$$-\sigma\pi_{A,t} = \Delta\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}_{A,t}^w\big) \tag{B.17}$$
$$-\sigma\pi_{B,t} = \Delta\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}_{B,t}^w\big) \tag{B.18}$$

B.4 First Best Solution for A in a Flexible Exchange Rate ($\rho = \theta = 1$ Case)

The first best monetary policy for A is the policy which maximizes W_A. Then, monetary policy problem is to maximize $\tilde{\tilde{W}}_A$ subject to (2.3) and (2.4). Lagrangian of the problem is

$$
\begin{aligned}
\mathcal{L} = \tilde{\tilde{W}}_A =& \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Bigg\{ -\frac{1}{2}\Big[\lambda_{y_A}\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^A_{A,t}\big)^2 + \lambda_{y_B}\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^A_{B,t}\big)^2 + \lambda_{\pi_A}{\pi_{A,t}}^2 + \lambda_{\pi_B}{\pi^*_{B,t}}^2\Big] \\
&+ \vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{A,t+1} + \kappa\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^w_{A,t}\big) + u_t - \pi_{A,t}\Big] \\
&+ \vartheta^*_{\pi,t}\Big[\beta\mathbb{E}_t\pi^*_{B,t+1} + \kappa^*\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^w_{B,t}\big) + u^*_t - \pi^*_{B,t}\Big]\Bigg\}
\end{aligned}
\tag{B.19}
$$

First order conditions of this problem are

$$ -\lambda_{y_A}\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^A_{A,t}\big) + \vartheta_{\pi,t}\kappa = 0 \tag{B.20} $$

$$ -\lambda_{\pi_A}\pi_{A,t} - \vartheta_{\pi,t} + \vartheta_{\pi,t-1} = 0 \tag{B.21} $$

$$ -\lambda_{y_B}\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^A_{B,t}\big) + \vartheta_{\pi,t}\kappa = 0 \tag{B.22} $$

$$ -\lambda_{\pi_B}\pi_{B,t} - \vartheta_{\pi,t} + \vartheta_{\pi,t-1} = 0 \tag{B.23} $$

First order conditions give

$$ -\sigma\pi_{A,t} = \Delta\big(\hat{Y}_{A,t} - \tilde{\tilde{Y}}^A_{A,t}\big) \tag{B.24} $$

$$ -\sigma\pi_{B,t} = \Delta\big(\hat{Y}_{B,t} - \tilde{\tilde{Y}}^A_{B,t}\big) \tag{B.25} $$

Appendix C

ADDITIONAL TABLES

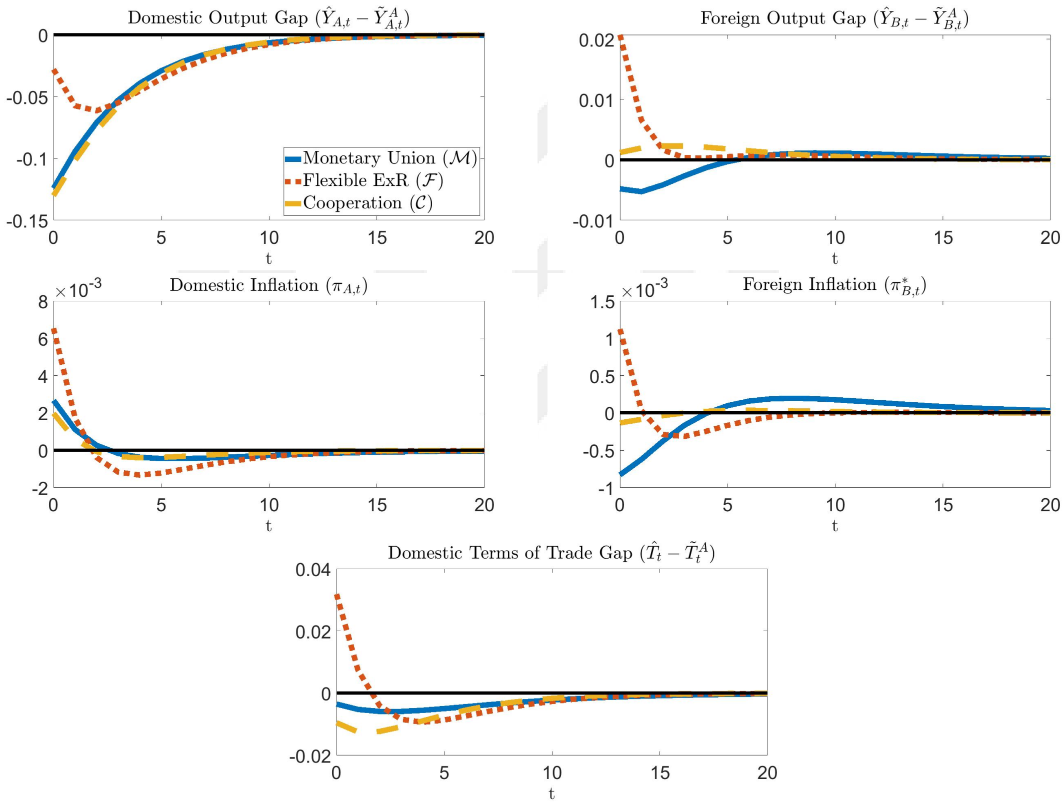

Figure C.1: Impulse response functions of a positive domestic markup shock ($\hat{\mu}_A$) given the parametrization at Table 2.1.

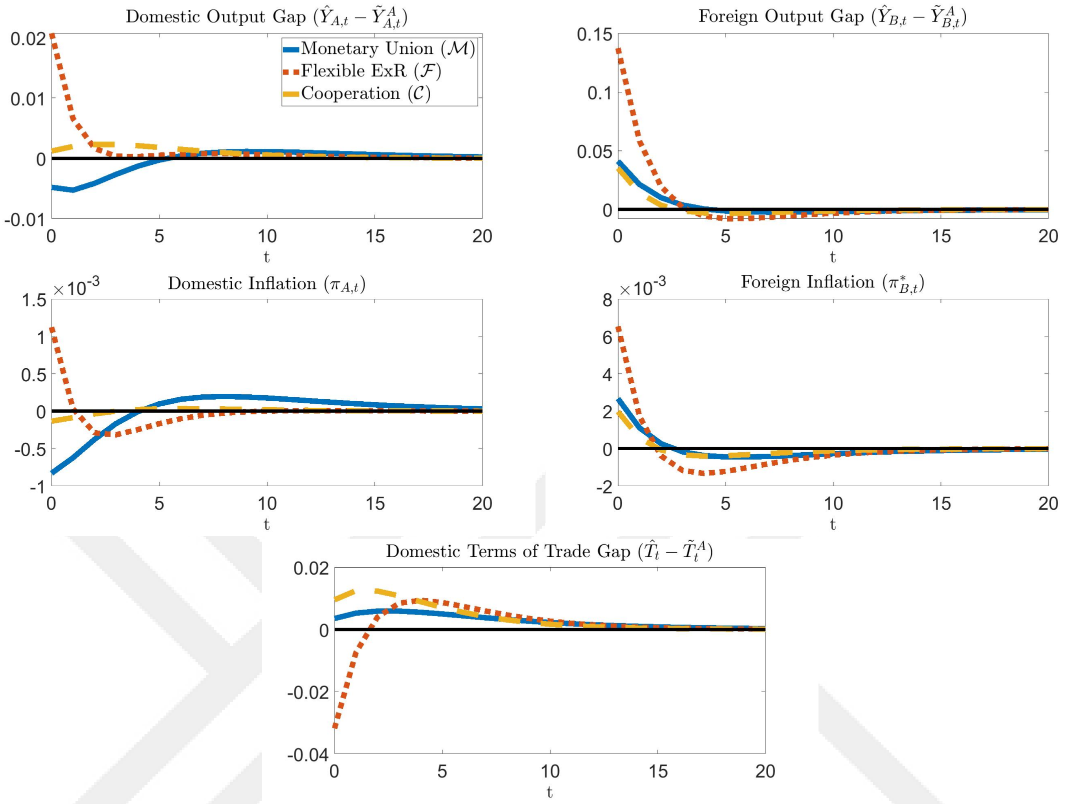

Figure C.2: Impulse response functions of a positive domestic markup shock ($\hat{\mu}_B$) given the parametrization at Table 2.1.