A MODEL OF DELEGATION IN BARGAINING

A Master's Thesis

by

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Ankara
June 2023

To my sister…

A MODEL OF DELEGATION IN BARGAINING

The Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

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In Partial Fulfillment of the Requirements for the Degree of
MASTER OF ARTS IN ECONOMICS

THE DEPARTMENT OF
ECONOMICS
İHSAN DOĞRAMACI BİLKENT UNIVERSITY
ANKARA

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

A MODEL OF DELEGATION IN BARGAINING

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July 2023

This thesis examines delegate selection in bargaining encounters where one side consists of a group. Employing a non-cooperative approach, we propose a three-stage bargaining game to analyze the trade-off that arises between the group's share of the surplus and the distribution within the group when a delegate holds a privileged position in the in-group bargaining stage. We show that there exist multiple equilibria and characterize them. We explore the trade-off by taking into account various delegate selection methods, individuals' time preferences, and in-group bargaining institutions. Our findings demonstrate that differences in these factors lead to substantial changes in equilibrium strategies. Specifically, under majority rule, granting a delegate the first proposer right in in-group bargaining results in an increase in the number of equilibria in which a weaker group member is elected, whereas a unanimity requirement leads to a stronger group member being preferred as the delegate in most of the equilibria. Furthermore, we illustrate that group members' discount factors are another important determinant of delegate selection.

Keywords: Delegation, Out-group Bargaining, In-group Bargaining,

ÖZET

PAZARLIKTA DELEGASYON MODELİ

Doğan, Gökçe
Yüksek Lisans, İktisat Bölümü
Tez Danışmanı: Doç. Dr. Emin Karagözoğlu

Temmuz 2023

Bu tez, en az bir tarafın grup olduğu pazarlık karşılaşmalarında delege seçiminin denge durumlarını analiz etmeyi amaçlamaktadır. İşbirliksiz bir yaklaşım ve üç aşamalı bir pazarlık oyunu aracılığı ile delegenin grup içi pazarlıkta ayrıcalığa sahip olduğu durumlarda grup tarafından alınan pay ile grup içi dağılım arasında bir tercih dengesi (trade-off) oluştuğu gösterilmektedir. Buna bağlı olarak, birden fazla denge noktasının var olduğunu gösteriyor ve bu denge noktalarını karakterize ediyoruz. Delege seçim yöntemleri, bireylerin zaman tercihleri ve grup içi pazarlık kurumu gibi çeşitli faktörleri dikkate alarak denge profillerindeki değişimi araştırıyoruz. Bulgularımız, bu faktörlerdeki farklılıkların denge stratejilerinde önemli değişikliklere yol açtığını göstermektedir. Özellikle, çoğulculuk kuralına göre, grup içi pazarlıkta bir delegeye ilk teklifi yapma yetkisinin verilmesi, daha zayıf grup üyesinin seçildiği denge noktalarının sayısında artışa neden olurken, oybirliği gereksinimi çoğu denge noktasında daha güçlü bir grup üyesinin delege olarak tercih edilmesine yol açar. Ayrıca, grup üyelerinin iskonto faktörlerinin arasındaki farkın da delege seçiminde önemli belirleyiciler

olduğunu göstermekteyiz.

Anahtar Kelimeler: Delegasyon, Grup Dışı Pazarlık, Grup İçi Pazarlık

ACKNOWLEDGMENTS

First of all, I would like to present my deepest gratitude to my thesis advisor, Emin Karagözoğlu, for his patience and unending support throughout this process. He was the best advisor that I could have wished for.

I would like to thank my examining committee members, Nuh Aygün Dalkıran and Kerim Keskin, for their valuable comments on the study, suggestions for further research, and time spared.

I must also acknowledge the insightful comments that my professors at Bilkent Economics, Tarık Kara, Refet Gürkaynak, and Alexandr Levkun, provided during my presentations. Moreover, I would like to thank Arda Gitmez for his valuable help.

I am indebted to Eray Yücel, Çağrı Sağlam, and Sang Seok Lee for their constant support during my educational journey.

A special thanks to my housemates and dearest friends Dolunay Arıcı and İrem Deşdemir for approaching me with patience and love at every step of this journey. I would like to thank Arlindo Skënderaj for his outstanding effort in supporting me and for believing in me even during moments of self-doubt.

Last but not least, I would like to thank my family, who supported and encouraged me to pursue my dreams all the way until now. Without their love, this journey would be impossible.

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CHAPTER 1

INTRODUCTION

The participation of a group in bargaining encounters is frequently seen in both practical and academic settings. One of the most standard examples of this is the labor union that sits at the bargaining table for wage bargaining. Workers decide to join labor unions as labor unions enable their members to form a stronger stance vis-à-vis the government and related companies by fostering collective action. In contrast to the aforementioned example, people sometimes do find themselves part of a group against their will, or groups may be given exogenously (Chae and Heidhues, 2004). Residents of a given city, for example, inadvertently establish a collective entity and are driven to collaborate in furthering the city's interests due to their shared location. Either way, while it is not very hard to come up with a decision when the group consists of a small number of similar people, it becomes increasingly challenging as the group size and heterogeneity of the members increase.

Addressing a group bargaining problem theoretically is another problem at hand. It is not easy to model the preferences and the decisions of a group that consists of infinitely many members. One of the approaches used in the bargaining literature to handle such situations is to treat the group as a single entity. However, this approach overlooks the importance of in-group composition (Chae and Moulin, 2010). Manzini and Mariotti (2005), on the other hand, suggests an approach in which group members bargain on the proposal to be made or answered, while Segendorff (1998) claims that consulting all group members at every step, can be very costly, particularly in terms of time, hence proposes delegation. Considering all the criticisms, the delegation of decision-making authority in bargaining encounters appears as one of the most attractive approaches. Since the elected delegate is a reflection of the members' preferences, the dynamics within the group are not totally ignored, and the decision-making process is facilitated. Contemplating this result, we are motivated to investigate the outcomes of delegation in bargaining encounters where at least one of the parties is a group.

While permitting one individual to take responsibility for decision-making on behalf of the group substantially simplifies the bargaining process, yet it introduces new queries that must be addressed. With rapid consideration, the first question that we can think of is which person will be selected as a delegate. The anticipated course of action entails the designation of an individual deemed best to reflect the group's objective, which may encompass diverse areas such as public good provision, political view, or allocation of a resource. It is frequently stated in the literature that the "best" delegate is the one with the strongest bargaining characteristics (e.g., more patient, less risk-averse) [1]. Arguably such a delegate could provide the greatest amount of share to the group, more public goods with the least contribution, or be the person closest to the group's political stance.

Within the scope of this study, we limit our attention to bargaining encounters for the division of a fixed amount of surplus and ask another relevant question: Is it truly the best strategy for the group to choose the "strongest" person as a delegate and get the biggest amount of surplus? What we have in mind while

[1]See Fershtman et al. (1991), Segendorff (1998), Cai (2000), Manzini and Mariotti (2005), Hamman et al. (2010), Harstad (2010), Cardona and Ponsatí (2015), Miller et al. (2018), Hagmann and Feiler (2020).

asking this particular question is in-group allocation. Obviously, if the surplus that the group receives were to be allocated evenly among all group members, then the immediate answer would be "yes," but why would anyone prefer to take on responsibility for decision-making on behalf of others if there is no additional benefit? In order to incentivize individuals for delegation, there must be a monetary or mental (i.e., prestige) privilege as a reciprocal arrangement. With this reasoning, we can update the question we asked as follows: If the designated delegate were to possess an advantageous position in in-group bargaining for the division of the obtained surplus, would the group members still prefer the "strongest" person as the delegate? The critical point here is that the strong person who can bring a larger surplus to the group is equally strong in sharing within the group. So, the strongest person wants a big share of the big pie, causing an uneven split of it. On the other hand, if the relatively weaker person becomes a delegate, she will be content with a smaller share of the group's pie, although she may not obtain a big pie for the group. Notice that there is no immediate answer to this updated question, and there is a trade-off to discover.

In this thesis, we try to investigate this trade-off. If a group were to allocate a fixed share of resources acquired through external bargaining among its members in a situation where the delegate possesses some privileges, would they still elect the strongest group member as a delegate? To the best of our knowledge, there are only a few studies on the delegate selection process, as also stated in Hagmann and Feiler (2020), and even fewer studies about the effect of this election on the distribution within the group.

To answer questions related to delegate selection in bargaining encounters, we propose a multi-stage bargaining game in which agents have different time preferences. First, the group chooses a delegate to bargain with an outsider over a fixed surplus. Then the obtained amount is shared within the group again through bargaining. In the first stage, group members' preferences are determined according to the payoffs they are expected to receive after in-group bargaining. Depending on these profiles, group members strategically vote to determine the delegate. Once a delegate is elected, the second stage of the game starts, and the delegate faces the outsider to bargain over a fixed surplus. Here, bargaining is carried out between two people according to alternating offer bargaining protocol (Rubinstein, 1982). Next, the delegate takes the amount she has earned to her own group, and the group proceeds to the final stage. At the final stage, group members bargain over the obtained amount according to multilateral bargaining protocol (Baron and Ferejohn, 1989).

One of the most important components of the game we study is that the selected delegate takes a privileged position in in-group bargaining. We model the "privilege" in two ways: (i) first proposal right and (ii) higher recognition probability. The main motivation for adding this privilege component is to create an incentive for group members to accept being a delegate. If there is no physical or status benefit from being a delegate, a person may not want to take on the responsibility of making decisions on behalf of the group. A similar approach can be observed within the existing body of literature. Cai (2000), for instance, presents a benefits scheme for the delegate, which is referred to as the "prestige" of being a delegate.

In this study, we show that there exist multiple equilibria in the delegate selection process a la majority voting rule of four different games, and characterize their properties. We then refine our results for each game using sequential pairwise voting. Combining these results, we show that electing weaker group members can be not only an equilibrium strategy but also a socially optimal one. By examining the preference profiles of individuals under different parameter values, we show that contrary to expectations, weaker group members prefer the strongest member's delegation when they also have enough strength to compete with her. We even find that given the parameter values, electing the strongest group member can be the worst possible outcome for society. Another important result that we find is that the way a proposal is voted in in-group bargaining affects delegate selection as well. Precisely, the majority rule multilateral bargaining with the first proposal right leads the game to result in the election of weaker members, whereas the unanimity rule version favors the strongest group member's delegation.

The rest of the thesis is organized as follows: Chapter 2 presents the literature review, Chapter 3 describes the model setup and presents the necessary assumptions; Chapter 4 provides the analytical results, the computational results, and the discussions; Chapter 5 concludes.

CHAPTER 2

LITERATURE REVIEW

Our study is related to the relatively small literature on delegation in group bargaining under the umbrella of non-cooperative game theory.

The delegation of decision-making authority in bargaining situations (although most of the time not in the group context) has been discussed in previous studies in the literature. The main factors that make delegation of decision-making authority preferable are: (i) the person who will make the decision may have more information on the subject or be an expert on the subject (Cai and Cont, 2004),(ii) the delegate can be used as a threat or commitment (Jones, 1989), (iii) the responsibility for making negative decisions may be shifted to someone else (Bartling and Fischbacher, 2012). We focus on the cases in which an exogenously given group has to bargain with an outsider [1] and consider the aspect of a delegation that accelerates and facilitates decision-making in bargaining situations [2]. While working on this aspect of delegation, we focus explicitly on the following question: What kind of individual would be selected as a delegate?

[1]With this approach, we are dealing with groups that resemble the "alliance" definition of Manzini and Mariotti (2005).

[2]We assume that the group has to elect a delegate in order to participate in the bargaining process. See, for example, Segendorff (1998) for the discussion on whether the group should delegate the decision-making or not.

Previous works almost unanimously suggest that the "strongest" person would be elected as a delegate. Cai (2000), Chae (2009), and Segendorff (1998) show that the delegation to the individual with stronger bargaining characteristics (e.g., more patient, less risk-averse or least costly) is the equilibrium strategy. Lammers (2010) shows that people prefer selfish delegates when interacting with outsiders. Hagmann and Feiler (2020) discuss that agents with more polarized beliefs are chosen to sit across the bargaining table. Cardona and Ponsatí (2015) show that more extremist members would be considered as the delegate. However, having the strongest person as the delegate is not always optimal for the principals or the constituency. For example, Cai (2000) shows that having “tough” agents on both sides of the bargaining causes costly delays, Hagmann and Feiler (2020) explains that having “strong” agents may turn the process into a Prisoners’ Dilemma and lead to socially undesirable outcomes. Similarly, Manzini and Mariotti (2005) indicates that although the existence of “stronger” members may be in benefit of alliance in terms of equilibrium outcome, their existence may force weaker agents to take actions that are suboptimal for them. Our model in which we can examine the effects of the delegate selection process on equilibria outcomes for the group is inspired by these latter results. The impact of delegate selection on the bargaining process is understudied in the literature.

The studies that are most similar to ours in terms of model setup are Segendorff (1998), Cai (2000), Manzini and Mariotti (2005), Chae (2009), and Sethi and Yoo (2022). Segendorff (1998) studies bargaining situations for the provision of the public good in which both sides of the bargaining are nations. In the first stage, a principal from the group assigns the delegate, and in the second stage, the delegate bargains over the provision of a public good. Here, the utility functions of the delegates are common knowledge, and a principal, by assigning a strong delegate, can pose a threat to the other nation. The study shows that when the delegates have too much power, the game turns into a

Prisoner's Dilemma, and the constituency ends up having less public good.

Cai (2000) studies bargaining situations in which one party is a group. The study presents a two-staged bargaining model with breakdown probability under the principal-agent framework. The characteristics of the agents are their private information, and the election takes place on whether to re-elect or change the representative after observing bargaining outcomes in the initial stage. The author shows that when the constituency does not know the exact amount, tough types may use the delay to signal their type, which may lead to inefficient equilibrium outcomes.

Manzini and Mariotti (2005) investigate a bargaining situation in which an alliance has to bargain with another individual according to alternating offer bargaining protocol as in Rubinstein (1982). The alliance has to come up with either a single proposal over the alternatives or a single response to the proposed policy. Members have a common interest but have different preference intensities. As the alliance tries to come up with a collective decision, they utilize (i) unanimity rule and (ii) majority rule. The study investigates how these different decision-making mechanisms affect the decision, and it shows that the equilibrium outcome for an alliance under the unanimity rule is better than the majority rule. Moreover, it shows that unanimity rules lead the alliance to behave as if the most aggressive member had been delegated to bargain on its behalf.

In Chae (2009), delegate selection and its impact on the equilibrium are examined in a 3-stage strategic bargaining game. There are two groups bargaining, and in the first stage, randomly assigned group members select their representatives. In the second stage, the two representatives bargain with each other in the fashion of the alternating offer protocol of Rubinstein (1982) with breakdown probability. In the third stage, the surplus is again bargained over within the group according to the Chae and Yang (1994) type n-person alternating-offer procedure with a fixed probability of breakdown. They show that each group chooses a representative who has the greatest marginal gain from increasing the total surplus of the group.

Sethi and Yoo (2022) also analyze non-cooperative bargaining between two groups over a fixed budget. In their model, a random member of one of the groups is chosen to make a proposal over the division of the surplus, both for her own group members and for the other group. Then another person is randomly chosen from the other group to decide on the allocation within her group. They refer *delegation* for the case in which a smaller group within the group has the proposal power. Through the delegation setup, the study shows that delegation of the proposal-making to a smaller group leads to a higher inequality in the in-group allocation.

One line of literature focuses on the impacts of voting decisions in in-group bargaining (see Manzini and Mariotti (2005), Bond and Eraslan (2007), Eraslan and Merlo (2017), and Miller et al. (2018)). In line with these studies, by comparing the expected payoffs obtained through unanimity rule and majority rule multilateral bargaining models, we show that not only the delegate's strength but also the method of in-group decision-making determines delegate selection.

CHAPTER 3

THE MODEL

We first describe the fundamentals that do not vary between the alternative models of the game. Consider a three-stage bargaining game with complete information. The set of all individuals in the game is denoted as $N = \{1, 2, ..., n\}$. From these, $n-1$ of the individuals form a group, and n^{th} player participates in the game as a single individual whom the group bargains over. For each individual $k \in N$, let $\delta_k \in (0, 1)$ be the time discount factor for the player k. The individuals in the game differ according to their δ_k. We restrict our analysis to the case where $n = 4$. Let the labels of the four individuals be A, B, C, and D, respectively, so the set of all individuals is $N = \{A, B, C, D\}$. Among these individuals, A, B, and C constitute a group denoted as G i.e. $G = \{A, B, C\}$. The time discount factor of each individual determines their strength level. In order to differentiate the strength of group members according to their bargaining characteristics, we assume that $\delta_A > \delta_B > \delta_C$. Hence, A, who has the highest time discount factor, is the strongest group member according to our assumption, and C is the softest.

3.1 Stage 1 Game: Delegate Selection

Before proceeding with the formal definitions and explanations, let us first emphasize that we use slightly less formal descriptions for this stage. The fundamental explanation for this action is that, while we are dealing with the utility function through the vast majority of the game, we are discussing at this stage preference relations as there is voting. We believe that this is the most convenient way to avoid the notation confusion that can be generated by combining these two different approaches and to deliver a more reader-friendly work.

In the first stage of the game, members of G determine the delegate that will bargain on their behalf in the second stage of the game. For this, each $i \in G$ ranks the delegation candidates according to their expected payoff under each case. To explain this with an example, let's consider A. Group member A compares her expected payoff at the end of stage 3 when she is the delegate to her expected payoff when B is the delegate and her expected payoff when C is the delegate. Since the utility function is linear in its return, she prefers the candidate that will provide the greatest expected payoff the most. Note that if the group members cannot agree on a delegate, given the voting rule, the group cannot proceed to the second stage, and they do not receive any share. Therefore, group members consider the zero payoff possibility as well, as they are forming their preferences and vote strategically given the preferences.

In the first stage of the game, only the members of a group $G = \{A, B, C\}$ interact to choose a delegate. The strategy of each member is to vote for a delegate from G, which includes voting for themselves. Let $\mathcal{G} = G \times G \times G$ be the set of all possible strategy profiles, i.e., voting strategy profiles. Each member ranks the individuals in G according to the expected payoff she gets at the end of stage 3, as explained above. Define the ranking function of each individual $i \in G$ by

$$\mathcal{T}_i : \{A, B, C\} \longrightarrow \{1, 2, 3\}.$$

We define $\mathcal{T}$ to be the set of all possible ranking profiles. That is

$$\mathcal{T} = \{(\mathcal{T}_A, \mathcal{T}_B, \mathcal{T}_C) \mid \mathcal{T}_i : \{A, B, C\} \longrightarrow \{1, 2, 3\}, \quad \forall i \in G\}$$

Given any ranking profile $r \in \mathcal{T}$, each player votes for a delegate. We denote by $\succsim_i^r$ the preferences of individual i over $\mathcal{G}$, the set of all voting strategy profiles. Thus, the set of all Nash Equilibria under the ranking profile r is

$$\mathcal{E}^r = \{a^* \in \mathcal{G} | (a_i^*, a_{-i}^*) \succsim_i^r (a_i, a_{-i}^*) \; \forall \; a_i \in G, \; \forall \; i \in G\}$$

Let Θ be a set of all possible outcomes i.e., $\Theta = \{A, B, C, \Phi\}$ where Φ denotes the outcome for the case in which delegate could not be determined according to the given voting rule. A voting rule in our setup is basically a mapping from the preference profiles to the set of outcomes. The results of the delegate selection are determined based on two different voting rules: majority rule and sequential pairwise voting rule with amendment agendas. From now on, let ϕ denote the elected delegate from the group G.

3.2 Stage 2 Game: Out-group Bargaining

In the second stage of the game, members of the group G have to reach an agreement over the division of a pie of size 1, according to alternating offer bargaining protocol (Rubinstein, 1982), with the individual D, which we will refer to as *outsider* from now on. This bilateral bargaining takes between the elected delegate ϕ of the group and the outsider. Bargaining lasts infinitely without a breakdown possibility, and the procedure is common knowledge between the players. In every odd period, the delegate makes a proposal for the division of the pie X, whereas, in every even period, the outsider makes the proposal. The responder in every period has two possible actions: Accept or Reject. If the responder accepts the division that is offered, the bargaining process ends and the sides share the pie according to the proposal. If the responder rejects the proposal, bargainers pass to the next period, and the responder becomes the proposer.

The share that is obtained by the delegate for the group is denoted as x_ϕ. Similarly, the outsider gets her own share from the bilateral bargaining problem, which is denoted as x_D. Let $X \equiv \{(x_\phi, x_D) \in \mathbb{R}^2_+ | \, x_\phi + x_D = 1\}$. The participants of the bargaining procedure discount the payoff based on their discount factors: Delay is costly. Hence shorter the bargaining procedure, the better for the bargainers. Rubinstein (1982) shows that there will be an immediate agreement at period 1 in which the first proposer offers her discounted continuation value to the responder.

3.3 Stage 3 Game: In-group Bargaining

In the last stage of the game, G has to reach an agreement over the division of the pie of size x_Φ according to the closed rule multilateral bargaining protocol (Baron and Ferejohn, 1989).

Let Z denote the set of feasible allocations, that is, $Z = \{x^{ij}_\phi \in \mathbb{R}^3_+ | \, \sum_{i=1}^n x^{ij}_\phi \leq x_\phi\}$ where x^{ij}_ϕ denotes the share for player i at the end of the stage 3 under ϕ's delegation[1]. The index j in the upper right corner denotes the owner of the proposal at the equilibrium. We assume that each player's utility function is linear in her own pie size only. Each player discounts the payoff in each period after the first one according to her discount factor. Hence, for group member i, her payoff at the period t is equal to $\delta_i^{t-1} x_i$ where x_i denotes i's payoff.

[1]In **higher recognition probability** variations, we do not have j on the upper right corner. The reason is that the first proposer is not certain in those variations. Hence we do not know at equilibrium whose proposal will be implemented.

The bargaining procedure can be described as follows: At the beginning of the initial period, a group member is selected as the proposer with probability p_i, where $\sum_{i \in G} p_i = 1$. The selected group member makes a proposal about the division of the obtained pie x_ϕ. Then this proposal is voted on according to a particular rule simultaneously among group members. Each group member has two possible actions given the proposal: Accept or Reject. If a necessary amount of people accept the proposal, it gets implemented, and the game ends. If rejected, the next period begins with again the determination of the proposer, and the rest is the same as the first period. Bargaining takes place for an infinite amount of periods without the possibility of breakdown until an allocation is accepted by the necessary amount of people.

As it is previously emphasized in the literature, we utilize the stationary subgame perfect equilibrium concept (SSPE), which is driven by the assumption of *stationary strategies*. Stationary strategies refer to the case in which all player takes the same action in each stage of the game. More specifically, if any group member i accepted (or rejected) a proposal x_i^j, then she will accept (or reject) the same offer in every other period. Similarly, if a group member j made the offer x_i^j to the member i at some stage, then she has to make the same offer at any other stage. In this game, as it is shown in Eraslan (2002), there exists an equilibrium, and it is unique.

The crucial thing that we do in this study is that show our results in different variations of the model in Baron and Ferejohn (1989). First, we will present the models in which the voting rule can be: (i) Majority Rule and (ii) Unanimity Rule [2]. In the majority rule BF Model, as will be discussed in detail in the next subsection, the proposer has to receive approval from the majority of the group members in order to implement the division. In the unanimity rule BF Model,

[2]We will refer to it as *BF Model* from now on.

a proposal should be unanimously approved to be implemented. The second source of variation comes from the important assumption: We assume that the delegate who bargains on behalf of the group has a privileged position as the pie is getting split within the group. This addition to the model is made in order to incentivize the elected group member for delegation duty. In agent-principal setups for delegation, agents (delegates) are offered either fixed-fee contracts or percentage-fee contracts in return for their service. Here we refrain from using "fee" for the delegation benefit. Instead, we propose more internalized incentive or privilege schemes for the delegate. First, we consider the case in which the delegate becomes the first proposer in in-group bargaining. In the second alternative, the delegate has a higher recognition probability in the BF Model.

The main reason we incentivize the delegate in the way that is described above is that we want to preserve the trade-off between the cake size and its distribution. This trade-off arises due to the existence of players with heterogeneous discount factors. With rapid consideration, we may realize that whether we fix the amount to be paid or the percentage of the share to be paid, this leads group members to elect the strongest person as the delegate who will bring the largest share of the pie to the group. However, we can also infer that this prevents us from observing the imbalances in the distribution of the pie.

3.3.1 Majority Rule BF Model

First Proposal Right

Recall that in the standard BF Model, the recognition probability p_i of the player i is equal to $\frac{1}{n}$. However, in this version of the model, we assume that the delegate is given the right to be the first proposer. More specifically, at date 0, the delegate who obtained the pie for the group is granted a recognition probability of $p_\phi = 1$.

If her proposal is not approved by the group, the delegate loses her advantage to be the proposer. Then the next period in which each $i \in G$ has a recognition probability of $p_i = \frac{1}{3}$ starts, and another proposer is recognized. The rest is the same as described previously in this chapter. Notice that the delegate can again be the proposer in the second period as well as other members of G.

In the majority voting rule BF Model, a proposer needs the approval of the majority of her group, including herself. Hence, for any group that has $n \geq 3$ players, a proposal has to be approved by $\frac{n+1}{2}$ players. In our model, since the delegate is the proposer at date 0, she needs $\frac{n-1}{2}$ more votes for her proposal to be accepted. Let v_i denote the continuation value for each $i \in G$. The *continuation value* v_i is the expected payoff of a member i that she might receive at a period given her stationary strategy. Hence, at date 0, the delegate offers their v_i to $\frac{n-1}{2}$ members of the group to leave them indifferent between continuing the game and accepting the proposal. All other specifications that are made previously in this chapter apply to this version as well.

Higher Recognition Probability

In this version of the BF Model, the recognition probabilities of the group members are heterogeneous. Specifically, in each period, the delegate has a recognition probability of $p_\phi = \frac{1}{2}$, and other group members have a recognition probability of $p_{-\phi} = \frac{1}{4}$ [3]. The intuition is that, in each period, the delegate is more likely to be the proposer compared to the other members of the group.

[3] Here one can consider any $p_\phi > \frac{1}{3}$ For simplicity, we took it as $\frac{1}{2}$.

3.3.2 Unanimity Rule BF Model

First Proposal Right

This variation is very similar to subsection 3.3.1 except this time, any proposal has to be voted according to Unanimity Rule. With that, a delegate, when she is appointed as the first proposer, has to receive the approval of all group members. This leads the proposer to offer their continuation values to each and every member of the group to implement her proposal. Similarly, if the proposal in the first period is not approved unanimously, then the next period starts, and a proposer i is recognized according to $p_i = \frac{1}{3}$.

Higher Recognition Probability

This model is the same as the one in subsection 3.3.1 except this time, any proposal has to be voted according to Unanimity Rule.

CHAPTER 4

MODEL RESULTS

4.1 Analytical Results

In this section, we analytically solve for the subgame perfect equilibrium of the game. In the first subsection, we present the results which are obtained through backward induction. In the second subsection, we present the equilibrium strategy for out-group bargaining, and in the final subsection, we present the result for in-group bargaining.

4.1.1 Stage 1: Delegate Selection

Before moving on to discussing first-stage results, let us elaborate on the SSPE payoffs, which are obtained using backward induction.

Majority Rule BF Model with First Proposal Right

At this stage, in order to determine their SSP equilibrium strategies, group members do backward induction. For this, group members plug the results that are shown in subsection 4.1.2 into their expected individual equilibrium payoffs in subsection 4.1.3. So, *Under A's delegation* the payoff allocations will be as follows:

$$\begin{aligned}
X_A^{AA} &= \frac{1-\delta_D}{1-\delta_D\delta_A} - \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_C(3-2\delta_B)}{9-3\delta_B-6\delta_C+\delta_C\delta_B} \\
X_A^{AB} &= 0 \\
X_A^{AC} &= \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_C(3-2\delta_B)}{9-3\delta_B-6\delta_C+\delta_C\delta_B}
\end{aligned} \tag{4.1}$$

Under *B's delegation*:

$$\begin{aligned}
X_B^{BA} &= 0 \\
X_B^{BB} &= \frac{1-\delta_D}{1-\delta_D\delta_B} - \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_C(3-2\delta_B)}{9-3\delta_B-6\delta_C+\delta_C\delta_B} \\
X_B^{BC} &= \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_C(3-2\delta_B)}{9-3\delta_B-6\delta_C+\delta_C\delta_B}
\end{aligned} \tag{4.2}$$

Under *C's delegation*:

$$\begin{aligned}
X_C^{CA} &= 0 \\
X_C^{CB} &= \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{3\delta_B(1-\delta_C)}{9-3\delta_B-6\delta_C+\delta_C\delta_B} \\
X_C^{CC} &= \frac{1-\delta_D}{1-\delta_D\delta_C} - \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{3\delta_B(1-\delta_C)}{9-3\delta_B-6\delta_C+\delta_C\delta_B}
\end{aligned} \tag{4.3}$$

Recall from chapter 3 that each group member forms her preference profile $\mathcal{T}_i$ based on these allocations. So that, A will compare X_A^{AA}, X_B^{BA} and X_C^{CA} which show her payoff under A's,B's and C's delegation respectively. B compares X_A^{AB}, X_B^{BB} and X_C^{CB} and similarly C compares X_A^{AC}, X_B^{BC} and X_C^{CC}. Later, based on these profiles, each group member i casts a vote [1].

Majority Rule BF Model with Higher Recognition Probability Right

Now let's observe the expected payoffs under the *higher recognition probability right* scenario. Here, different from the first proposal right case, group members' payoffs are not certain since the first proposer is not determined prior to the

[1]Later in this chapter, we will show these preference profiles in detail according to the different parameter values.

in-group bargaining. Hence they form their preferences based on the expected payoffs from each delegation. With this, the expected equilibrium payoffs are as follows:

Under A's delegation:

$$X_A^A = \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_A(1-\delta_C)(4-\delta_B)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \tag{4.4}$$
$$X_A^B = \delta_B V_B = \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{2\delta_B(1-\delta_C)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \tag{4.5}$$
$$X_A^C = \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_C(2-\delta_B)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \tag{4.6}$$

Under B's delegation:

$$X_B^A = \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_A(1-\delta_C)(4-\delta_B)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \tag{4.7}$$
$$X_B^B = \delta_B V_B = \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{8\delta_B(1-\delta_C)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \tag{4.8}$$
$$X_B^C = \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_C(4-3\delta_B)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \tag{4.9}$$

Under C's delegation:

$$X_C^A = \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{\delta_A(2-\delta_B)(1-\delta_C)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \tag{4.10}$$
$$X_B^C = \delta_B V_B = \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{2(1-\delta_C)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \tag{4.11}$$
$$X_C^C = \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{(4-3\delta_B)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \tag{4.12}$$

As aforementioned above, group members form their preference profiles based on these expected equilibrium payoffs. In order to decide about her vote for delegate selection, A compares X_A^A, X_B^A, X_C^C and receiving zero payoffs [2]. B compares X_A^B, X_B^B, X_C^B and zero payoff case. Similarly, C compares X_A^C, X_B^C, X_C^C and receiving nothing for failing to choose a delegate.

[2]Recall that zero payoffs occur in the case of failure of delegate selection

Unanimity Rule BF Model with First Proposal Right

In this version of the Baron Farejohn Model, note that the proposer(which coincides with the delegate) has to allocate the obtained surplus to each member of the group. Now let's look at the allocations based on the unanimity rule BF Model obtained with backward induction:

Now, If A is the delegate:

$$\begin{aligned}
X_A^{AA} &= X_A - \delta_B V_B - \delta_C V_C \\
&= \frac{1-\delta_D}{1-\delta_D\delta_A} - \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_B(1-\delta_A)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
&- \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_C(1-\delta_B)(1-\delta_A)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_A^{AB} &= \delta_B V_B = \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_B(1-\delta_A)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_A^{AC} &= \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_C(1-\delta_B)(1-\delta_A)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B}
\end{aligned} \tag{4.13}$$

If B is the delegate:

$$\begin{aligned}
X_B^{BA} &= \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_A(1-\delta_B)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_B^{BB} &= X_B - \delta_A V_A - \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_B} \\
&- \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_A(1-\delta_B)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
&- \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_C(1-\delta_B)(1-\delta_A)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_B^{BC} &= \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_C(1-\delta_B)(1-\delta_A)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B}
\end{aligned} \tag{4.14}$$

If C is the delegate:

$$
\begin{aligned}
X_C^{CA} &= \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{\delta_A(1-\delta_B)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_C^{CB} &= \delta_B V_B = \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{\delta_B(1-\delta_A)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_C^{CC} &= X_C - \delta_B V_B - \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_C} \\
&- \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{\delta_A(1-\delta_B)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
&- \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{\delta_B(1-\delta_C)(1-\delta_A)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B}
\end{aligned}
\tag{4.15}
$$

Unanimity Rule BF Model with Higher Recognition Probability Right

Now let's observe the expected payoffs under the *higher recognition probability right* scenario in the unanimity rule BF Model. Here, recall that - different from the first proposal right case - group members' payoffs are not certain since the first proposer is not determined prior to the in-group bargaining. Hence they form their preferences based on the expected payoffs from each delegation. With this, the expected equilibrium payoffs are as follows:

Under A's delegation:

$$
\begin{aligned}
X_A^A &= \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{2\delta_A(1-\delta_B)(1-\delta_C)}{4-2\delta_A-3\delta_B-3\delta_C+2\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_A^B &= \delta_B V_B = \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_B(1-\delta_A)(1-\delta_C)}{4-2\delta_A-3\delta_B-3\delta_C+2\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_A^C &= \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_A} \times \frac{\delta_C(1-\delta_A)(1-\delta_B)}{4-2\delta_A-3\delta_B-3\delta_C+2\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B}
\end{aligned}
\tag{4.16}
$$

Under B's delegation:

$$
\begin{aligned}
X_B^A &= \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_A(1-\delta_B)(1-\delta_C)}{4-3\delta_A-2\delta_B-3\delta_C+2\delta_A\delta_C+\delta_B\delta_C+\delta_A\delta_B} \\
X_B^B &= \delta_B V_B = \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{2\delta_B(1-\delta_A)(1-\delta_C)}{4-3\delta_A-2\delta_B-3\delta_C+2\delta_A\delta_C+\delta_B\delta_C+\delta_A\delta_B} \\
X_B^C &= \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_B} \times \frac{\delta_C(1-\delta_A)(1-\delta_B)}{4-3\delta_A-2\delta_B-3\delta_C+2\delta_A\delta_C+\delta_B\delta_C+\delta_A\delta_B}
\end{aligned}
\tag{4.17}
$$

Under C's delegation:

$$\begin{aligned}
X_C^A &= \delta_A V_A = \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{\delta_A(1-\delta_B)(1-\delta_C)}{4-3\delta_A-3\delta_B-2\delta_C+\delta_A\delta_C+\delta_B\delta_C+2\delta_A\delta_B} \\
X_C^B &= \delta_B V_B = \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{\delta_B(1-\delta_A)(1-\delta_C)}{4-3\delta_A-3\delta_B-2\delta_C+\delta_A\delta_C+\delta_B\delta_C+2\delta_A\delta_B} \\
X_C^C &= \delta_C V_C = \frac{1-\delta_D}{1-\delta_D\delta_C} \times \frac{2\delta_C(1-\delta_A)(1-\delta_B)}{4-3\delta_A-3\delta_B-2\delta_C+\delta_A\delta_C+\delta_B\delta_C+2\delta_A\delta_B}
\end{aligned} \tag{4.18}$$

In order to decide about her vote for delegate selection, A compares X_A^A, X_B^A, X_A^C and receiving zero payoff B compares X_A^B, X_B^B, X_C^B and zero payoff case. Similarly, C compares X_A^C, X_B^C, X_C^C and receiving nothing for failing to choose a delegate.

4.1.1.1 Majority Rule Voting

In this version of the first stage, group members decide on the delegate based on the majority rule voting. Recall that the majority rule implies that for a decision to be taken, at least $\frac{n+1}{2}$ votes are needed. The voting takes place simultaneously, and group members strategically make their decisions according to their (expected) payoffs from each delegation scenario. We discuss the majority rule voting results in the section 4.2.

4.1.1.2 Sequential Pairwise Voting

In this version of the first stage, group members decide on the delegate based on the sequential pairwise voting rule. The sequential pairwise voting presents more refined results compared to the majority voting rule, at least enabling us to eliminate the unlikely equilibrium outcomes emerging due to the Nash Equilibrium in some cases. We consider the amendment agendas Osborne (2023). In the amendment agenda, in an orderly fashion, two alternatives are compared. Players vote to choose surviving alternative. Next, surviving alternative is compared to a third alternative, and again, players vote to choose the better one, and so forth. Austen-Smith and Banks (2009) and Osborne (2023) model this binary agenda as a sequential game and find the sophisticated voting outcome (which corresponds to the subgame perfect equilibrium outcome) via backward induction. However, in our case, A is always indifferent between C's delegation and B's delegation, which makes it hard to find a sophisticated outcome. Still, we believe that sequential pairwise voting presents more refined results. We discuss the sequential pairwise voting rule results in the section 4.2 after observing preference profiles.

4.1.2 Stage 2: Out-group Bargaining

In this stage, a group member who has been elected as a delegate bargains a pie of size 1 with the outsider. The bargaining takes place in the fashion of Rubinstein (1982). The next proposition shows that there is a unique equilibrium payoff for the proposer and the responder in our context based on the result of Rubinstein (1982).

Proposition 1 *(Rubinstein (1982)) A subgame perfect equilibrium in the infinite horizon alternating offer bargaining game results in immediate acceptance. The unique subgame perfect equilibrium payoff for the delegate is:*

$$x_\phi = \frac{1 - \delta_D}{1 - \delta_D \delta_\phi} \tag{4.19}$$

Similarly, the unique subgame perfect equilibrium payoff for the outsider is:

$$x_D = \frac{\delta_D(1 - \delta_\phi)}{1 - \delta_D \delta_\phi} \tag{4.20}$$

Proof. Omitted. ■

For simplicity, we assume that the delegate is always the proposer in the out-group bargaining. [3]In regard, the payoff of each possible delegate are as follows:

$$X_A = \frac{1-\delta_D}{1-\delta_D\delta_A} \qquad X_B = \frac{1-\delta_D}{1-\delta_D\delta_B} \qquad X_C = \frac{1-\delta_D}{1-\delta_D\delta_C}$$

Recall that these are the amounts that a delegate will take to her group for in-group bargaining.

For each possible delegation scenario, the payoff of D would be as follows:

$$x_{AD} = \frac{\delta_D(1-\delta_A)}{1-\delta_D\delta_A} \qquad x_{BD} = \frac{\delta_D(1-\delta_B)}{1-\delta_D\delta_B} \qquad x_{CD} = \frac{\delta_D(1-\delta_C)}{1-\delta_D\delta_C}$$

where X_{iD} denotes the payoff for D under i's delegation.

Remark 1 Note that for all delegate candidates $\phi \in G$, her in-group expected SSP equilibrium payoff is a strictly increasing function of X_ϕ. This implies that when a delegate encounters D, her interests are completely aligned with the group's interests. This allows us to use Rubinstein (1982)'s SSPE results in our backward induction.

4.1.3 Stage 3: In-group Bargaining

In this stage, group members bargain over the division of the pie, which is obtained in the previous stage. Multilateral bargaining takes place in the fashion of Baron and Ferejohn (1989). The next proposition shows the SSPE equilibrium payoff vector as stated in Eraslan and Evdokimov (2019).

Proposition 2 *(Eraslan and Evdokimov, 2019) Let $p_i \in [0,1]$ and $\delta_i \in (0,1)$ denote player i's recognition probability and discount factor respectively, and assume $\sum_{i=1}^{n} p_i = 1$. Let $r_{ij} \in \{0,1\}$ denote the probability of being included in j'*

[3]According to our numeric calculations, this assumption does not have an impact on the equilibrium.

coalition for i. Then $v = (v_1, v_2, ..., vn)$ is an SSP equilibrium payoff vector if and only if it satisfies the following equation for all j:

$$v_j = p_j(1 - \sum_{i \neq j} r_{ji} v_i) + \sum_{i \neq j} p_i r_{ij} v_j \tag{4.21}$$

Proof. Omitted. ■

4.1.3.1 Majority Rule BF Model with First Proposal Right

At date 0 of the third stage, the delegate ϕ who obtained x_ϕ in the previous stage becomes the initial proposer in the in-group bargaining. Banks and Duggan (2000) show that agreement is reached immediately in any equilibrium under the majority rule BF Model[4]. We take this result as given. Let v_i denote the continuation value of any $i \in G$. Since $|G| = 3$, the delegate (proposer) need the approval of only *1 more group member.* for her proposal to be implemented.

Now suppose $A \in G$ is elected as the delegate of the group.

If A is the first proposer, A will propose the allocation $X_A = (X_A^{AA}, X_A^{AB}, X_A^{AC})$.

$$X_A^{AA} = X_A - \delta_C V_C \qquad X_A^{AB} = 0 \qquad X_A^{AC} = \delta_C V_C \tag{4.22}$$

As C is the least patient member of the group, she is cheaper than B to include in the winning coalition, so A will only offer C her continuation payoff to get her proposal accepted.

If B is the first proposer, B will propose the allocation $X_A = (X_A^{BA}, X_A^{BB}, X_A^{BC})$

$$X_A^{BA} = 0 \qquad X_A^{BB} = X_A - \delta_C V_C \qquad X_A^{BC} = \delta_C V_C \tag{4.23}$$

Similar to A, B would also only include C to her minimum winning coalition by

[4] Also see Kawamori (2004) for characterization of equilibrium values.

offering the continuation value of C.

If C is the first proposer, C will propose the allocation $X_A = (X_A^{CA}, X_A^{CB}, X_A^{CC})$.

$$X_A^{CA} = 0 \qquad X_A^{CB} = \delta_B V_B \qquad X_A^{CC} = X_A - \delta_B V_B \tag{4.24}$$

When C is the first proposer, C would include B to her winning coalition since $\delta_B < \delta_A$ which means that B is cheaper than A.

Now let's see the continuation values of the group members:

$$\begin{aligned} V_A &= \frac{1}{3}(X_A - \delta_C V_C) + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 0 \\ V_B &= \frac{1}{3} \cdot 0 + \frac{1}{3}(X_A - \delta_C V_C) + \frac{1}{3}\delta_B V_B \\ V_C &= \frac{1}{3}\delta_C V_C + \frac{1}{3}\delta_C V_C + \frac{1}{3}(X_A - \delta_B V_B) \end{aligned} \tag{4.25}$$

Let's solve these equations for V_A, V_B, and V_C we obtain:

$$\begin{aligned} V_A &= \frac{X_A(3-\delta_B)(1-\delta_C)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \\ V_B &= \frac{3X_A(1-\delta_C)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \\ V_C &= \frac{X_A(3-2\delta_B)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \end{aligned} \tag{4.26}$$

Then the allocations are as follows:

$$\begin{aligned} X_A^{AA} &= X_A - \delta_C V_C = X_A - \frac{X_A\delta_C(3-2\delta_B)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \\ X_A^{AB} &= 0 \\ X_A^{AC} &= \delta_C V_C = \frac{X_A\delta_C(3-2\delta_B)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B)} \end{aligned} \tag{4.27}$$

Now suppose $B \in G$ is elected as the delegate of the group.

If A is the first proposer, A will propose the allocation $X_B = (X_B^{AA}, X_B^{AB}, X_B^{AC})$.

$$X_B^{AA} = X_B - \delta_C V_C \qquad X_B^{AB} = 0 \qquad X_B^{AC} = \delta_C V_C \tag{4.28}$$

As C is the least patient member of the group, she is cheaper than B to include in the winning coalition, so A will only offer C her continuation payoff to get the proposal accepted.

If B is the first proposer, B will propose the allocation $X_B = (X_B^{BA}, X_B^{BB}, X_B^{BC})$.

$$X_B^{BA} = 0 \qquad X_B^{BB} = X_B - \delta_C V_C \qquad X_B^{BC} = \delta_C V_C \tag{4.29}$$

As C is the least patient member of the group, she is cheaper than A to include in the winning coalition, so B will only offer C her continuation payoff to get her proposal accepted.

If C is the first proposer, C will produce the allocation $X_B = (X_B^{CA}, X_B^{CB}, X_B^{CC})$.

$$X_B^{CA} = 0 \qquad X_B^{CB} = \delta_B V_B \qquad X_B^{CC} = X_B - \delta_B V_B \tag{4.30}$$

When C is the first proposer, C would include B to her winning coalition since $\delta_B < \delta_A$ which means that B is cheaper than A.

Now let's see the continuation values of the group members under B's delegation;

$$\begin{aligned} V_A &= \frac{1}{3}(X_B - \delta_C V_C) + \frac{1}{3}0 + \frac{1}{3}0 \\ V_B &= \frac{1}{3}0 + \frac{1}{3}(X_B - \delta_C V_C) + \frac{1}{3}\delta_B V_B \\ V_C &= \frac{1}{3}\delta_C V_C + \frac{1}{3}\delta_C V_C + \frac{1}{3}(X_B - \delta_B V_B) \end{aligned} \tag{4.31}$$

Let's solve these equations for V_A, V_B, and V_C;

$$\begin{aligned} V_A &= \frac{X_B(3-\delta_B)(1-\delta_C)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \\ V_B &= \frac{3X_B(1-\delta_C)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \\ V_C &= \frac{X_B(3-2\delta_B)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \end{aligned} \tag{4.32}$$

Then the allocations are as follows:

$$\begin{aligned} X_B^{BA} &= 0 \\ X_B^{BB} &= X_B - \delta_C V_C = X_B - \frac{X_B\delta_C(3-2\delta_B)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \\ X_B^{BC} &= \delta_C V_C = \frac{X_B\delta_C(3-2\delta_B)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \end{aligned} \tag{4.33}$$

Now suppose $C \in G$ is elected as the delegate of the group.

If A is the first proposer, A will propose the allocation $X_C = (X_C^{AA}, X_C^{AB}, X_C^{AC})$.

$$X_C^{AA} = X_C - \delta_C V_C \qquad X_C^{AB} = 0 \qquad X_C^{AC} = \delta_C V_C \tag{4.34}$$

If B is the first proposer, B will produce the allocation $X_C = (X_C^{BA}, X_C^{BB}, X_C^{BC})$.

$$X_C^{BA} = 0 \qquad X_C^{BB} = X_C - \delta_C V_C \qquad X_C^{BC} = \delta_C V_C \tag{4.35}$$

C will propose the allocation $X_C = (X_C^{CA}, X_C^{CB}, X_C^{CC})$.

$$X_C^{CA} = 0 \qquad X_C^{CB} = \delta_B V_B \qquad X_B^{CC} = X_C - \delta_B V_B \tag{4.36}$$

As B is the less patient member than A, she is cheaper to include in the winning coalition, so C will only offer B her continuation payoff to get the proposal accepted.

Now let's see the continuation values of the group members under C's delegation;

$$\begin{aligned} V_A &= \frac{1}{3}(X_C - \delta_C V_C) + \frac{1}{3}0 + \frac{1}{3}0 \\ V_B &= \frac{1}{3}0 + \frac{1}{3}(X_C - \delta_C V_C) + \frac{1}{3}\delta_B V_B \\ V_C &= \frac{1}{3}\delta_C V_C + \frac{1}{3}\delta_C V_C + \frac{1}{3}(X_C - \delta_B V_B) \end{aligned} \tag{4.37}$$

Let's solve these equations for V_A, V_B, and V_C;

$$\begin{aligned} V_A &= \frac{X_C(3-\delta_B)(1-\delta_C)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \\ V_B &= \frac{3X_C(1-\delta_C)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \\ V_C &= \frac{X_C(3-2\delta_B)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \end{aligned} \tag{4.38}$$

Then the allocations are as follows:

$$\begin{aligned} X_C^{CA} &= 0 \\ X_C^{CB} &= \delta_B V_B = \frac{3X_C\delta_B(1-\delta_C)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B)} \\ X_C^{CC} &= X_C - \delta_B V_B = X_C - \frac{3X_{2C}\delta_B(1-\delta_C)}{9 - 3\delta_B - 6\delta_C + \delta_C\delta_B} \end{aligned} \tag{4.39}$$

4.1.3.2 Majority Rule BF Model with Higher Recognition Probability

In this alternative, the delegate has a recognition probability of $p_\phi = \frac{1}{2}$, and other group members have a recognition probability of $p_{-\phi} = \frac{1}{4}$ throughout the third stage.

Note that the first proposer is not certain here. Hence different than **First Proposal Right alternative**, we cannot discuss the sure outcomes. Group members will consider their expected payoffs.

Now let's assume $A \in G$ is the delegate. Then the group's payoff from out-group bargaining is X_A.

If A is the first proposer, A will propose the allocation $X_A = (X_A^{AA}, X_A^{AB}, X_A^{AC})$.

$$X_A^{AA} = X_A - \delta_C V_C \qquad X_A^{AB} = 0 \qquad X_A^{AC} = \delta_C V_C \tag{4.40}$$

If B is the first proposer, B will propose the allocation $X_A = (X_A^{BA}, X_A^{BB}, X_A^{BC})$

$$X_A^{BA} = 0 \qquad X_A^{BB} = X_A - \delta_C V_C \qquad X_A^{BC} = \delta_C V_C \tag{4.41}$$

If C is the first proposer, C will propose the allocation $X_A = (X_A^{CA}, X_A^{CB}, X_A^{CC})$.

$$X_A^{CA} = 0 \qquad X_A^{CB} = \delta_B V_B \qquad X_A^{CC} = X_A - \delta_B V_B \tag{4.42}$$

Then the continuation values of the group members under A's delegation;

$$\begin{aligned} V_A &= \frac{1}{2}(X_A - \delta_C V_C) + \frac{1}{4}0 + \frac{1}{4}0 \\ V_B &= \frac{1}{4}(X_A - \delta_C V_C) + \frac{1}{4}\delta_B V_B + \frac{1}{2}0 \\ V_C &= \frac{1}{4}(X_A - \delta_B V_B) + \frac{1}{2}\delta_C V_C + \frac{1}{4}\delta_C V_C \end{aligned} \tag{4.43}$$

Let's solve these equations for V_A, V_B, and V_C;

$$\begin{aligned} V_A &= \frac{X_A(1-\delta_C)(4-\delta_B)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \\ V_B &= \frac{2X_A(1-\delta_C)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \\ V_C &= \frac{X_A(2-\delta_B)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \end{aligned} \tag{4.44}$$

Then the expected equilibrium payoffs are as follows:

$$\begin{aligned} X_A^A &= \delta_A V_A = \frac{X_A\delta_A(1-\delta_C)(4-\delta_B)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \\ X_A^B &= \delta_B V_B = \frac{2X_A\delta_B(1-\delta_C)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \\ X_A^C &= \delta_C V_C = \frac{X_A\delta_C(2-\delta_B)}{8-6\delta_C-2\delta_B+\delta_C\delta_B} \end{aligned} \tag{4.45}$$

Now let's assume $B \in G$ is the delegate. Then the group's payoff from out-group bargaining is X_B.

If A is the first proposer, A will propose the allocation $X_B = (X_B^{AA}, X_B^{AB}, X_B^{AC})$.

$$X_B^{AA} = X_B - \delta_C V_C \qquad X_B^{AB} = 0 \qquad X_B^{AC} = \delta_C V_C \tag{4.46}$$

If B is the first proposer, B will propose the allocation $X_B = (X_B^{BA}, X_B^{BB}, X_B^{BC})$.

$$X_B^{BA} = 0 \qquad X_B^{BB} = X_B - \delta_C V_C \qquad X_B^{BC} = \delta_C V_C \tag{4.47}$$

If C is the first proposer, C will produce the allocation $X_B = (X_B^{CA}, X_B^{CB}, X_B^{CC})$.

$$X_B^{BA} = 0 \qquad X_B^{BB} = \delta_B V_B \qquad X_B^{BC} = X_B - \delta_B V_B \tag{4.48}$$

Then the continuation values of the group members under B's delegation;

$$\begin{aligned} V_A &= \frac{1}{4}(X_B - \delta_C V_C) + \frac{1}{2}0 + \frac{1}{4}0 \\ V_B &= \frac{1}{2}(X_B - \delta_C V_C) + \frac{1}{4}\delta_B V_B + \frac{1}{4}0 \\ V_C &= \frac{1}{4}(X_B - \delta_B V_B) + \frac{1}{2}\delta_C V_C + \frac{1}{4}\delta_C V_C \end{aligned} \tag{4.49}$$

If we solve these equations for V_A, V_B, and V_C;

$$\begin{aligned} V_A &= \frac{X_B(1-\delta_C)(4-\delta_B)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \\ V_B &= \frac{8X_B(1-\delta_C)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \\ V_C &= \frac{X_B(4-3\delta_B)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \end{aligned} \tag{4.50}$$

Then expected equilibrium payoffs are as follows:

$$\begin{aligned} X_B^A &= \delta_A V_A = \frac{X_B\delta_A(1-\delta_C)(4-\delta_B)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \\ X_B^B &= \delta_B V_B = \frac{8X_B\delta_B(1-\delta_C)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \\ X_B^C &= \delta_C V_C = \frac{X_B\delta_C(4-3\delta_B)}{16-12\delta_C-4\delta_B+\delta_C\delta_B} \end{aligned} \tag{4.51}$$

Now let's assume $C \in G$ is the delegate. Then the group's payoff from out-group bargaining is X_C.

If A is the first proposer, A will propose the allocation $X_C = (X_C^{AA}, X_C^{AB}, X_C^{AC})$.

$$X_C^{AA} = X_C - \delta_C V_C \qquad X_C^{AB} = 0 \qquad X_C^{AC} = \delta_C V_C \tag{4.52}$$

If B is the first proposer, B will produce the allocation $X_C = (X_C^{BA}, X_C^{BB}, X_C^{BC})$.

$$X_C^{BA} = 0 \qquad X_C^{BB} = X_C - \delta_C V_C \qquad X_C^{BC} = \delta_C V_C \tag{4.53}$$

C will propose the allocation $X_C = (X_C^{CA}, X_C^{CB}, X_C^{CC})$.

$$X_C^{CA} = 0 \qquad X_C^{CB} = \delta_B V_B \qquad X_B^{CC} = X_C - \delta_B V_B \tag{4.54}$$

Then the continuation values of the group members under C's delegation;

$$\begin{aligned} V_A &= \frac{1}{4}(X_C - \delta_C V_C) + \frac{1}{2}0 + \frac{1}{4}0 \\ V_B &= \frac{1}{4}(X_C - \delta_C V_C) + \frac{1}{2}\delta_B V_B + \frac{1}{4}0 \\ V_C &= \frac{1}{2}(X_C - \delta_B V_B) + \frac{1}{4}\delta_C V_C + \frac{1}{4}\delta_C V_C \end{aligned} \tag{4.55}$$

If we solve these equations for V_A, V_B, and V_C;

$$\begin{aligned} V_A &= \frac{X_C(2-\delta_B)(1-\delta_C)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \\ V_B &= \frac{2X_C(1-\delta_C)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \\ V_C &= \frac{X_C(4-3\delta_B)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \end{aligned} \tag{4.56}$$

So the expected equilibrium payoffs are as follows:

$$\begin{aligned} X_C^A &= \delta_A V_A = \frac{X_C\delta_A(2-\delta_B)(1-\delta_C)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \\ X_C^B &= \delta_B V_B = \frac{2X_C\delta_B(1-\delta_C)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \\ X_C^C &= \delta_C V_C = \frac{X_C\delta_C(4-3\delta_B)}{8-4\delta_C-4\delta_B+\delta_B\delta_C} \end{aligned} \tag{4.57}$$

4.1.3.3 Unanimity Rule BF Model with First Proposal Right

At date 0 of the third stage, the delegate ϕ who obtained x_ϕ in the previous stage becomes the initial proposer in the in-group bargaining. Note that, in this alternative, the proposer needs to convince all of the group members to implement the proposal. Hence, the proposer must offer their continuation value to each $i \in G$.

Now assume $A \in G$ becomes the delegate, then the group will bargain over X_A in the third stage.

If A is the first proposer, A will propose the allocation $X_A = (X_A^{AA}, X_A^{AB}, X_A^{AC})$.

$$X_A^{AA} = X_A - \delta_B V_B - \delta_C V_C \qquad X_A^{AB} = \delta_B V_B \qquad X_A^{AC} = \delta_C V_C \tag{4.58}$$

If B is the first proposer, B will propose the allocation $X_A = (X_A^{BA}, X_A^{BB}, X_A^{BC})$.

$$X_A^{BA} = \delta_A V_A \qquad X_A^{BB} = X_A - \delta_A V_A - \delta_C V_C \qquad X_A^{BC} = \delta_C V_C \tag{4.59}$$

If C is the first proposer, C will propose the allocation $X_A = (X_A^{CA}, X_A^{CB}, X_A^{CC})$.

$$X_A^{CA} = \delta_A V_A \qquad X_A^{CB} = \delta_B V_B \qquad X_A^{CC} = X_A - \delta_A V_A - \delta_B V_B \tag{4.60}$$

Then the continuation values are be as follows:

$$\begin{aligned} V_A &= \frac{1}{3}(X_A - \delta_B V_B - \delta_C V_C) + \frac{2}{3}\delta_A V_A \\ V_B &= \frac{1}{3}(X_A - \delta_A V_A - \delta_C V_C) + \frac{2}{3}\delta_B V_B \\ V_C &= \frac{1}{3}(X_A - \delta_B V_B - \delta_A V_A) + \frac{2}{3}\delta_C V_C \end{aligned} \tag{4.61}$$

Let's solve these equations for V_A, V_B, and V_C:

$$\begin{aligned} V_A &= \frac{X_A(1-\delta_B)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ V_B &= \frac{X_A(1-\delta_A)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ V_C &= \frac{X_A(1-\delta_B)(1-\delta_A)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \end{aligned} \tag{4.62}$$

Since A is the delegate, she makes the first proposal. Therefore the equilibrium payoff allocations are:

$$\begin{aligned} X_A^{AA} &= X_A - \frac{X_A\delta_B(1-\delta_A)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ &- \frac{X_A\delta_C(1-\delta_B)(1-\delta_A)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ X_A^{AB} &= \delta_B V_B = \frac{X_A\delta_B(1-\delta_A)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ X_A^{AC} &= \delta_C V_C = \frac{X_A\delta_C(1-\delta_B)(1-\delta_A)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \end{aligned} \tag{4.63}$$

Now assume $B \in G$ becomes the delegate, then the group will bargain over X_B in the third stage.

If A is the first proposer, A will propose the allocation $X_B = (X_B^{AA}, X_B^{AB}, X_B^{AC})$.

$$X_B^{AA} = X_B - \delta_B V_B - \delta_C V_C \qquad X_B^{AB} = \delta_B V_B \qquad X_B^{AC} = \delta_C V_C \tag{4.64}$$

If B is the first proposer, B will propose the allocation $X_B = (X_B^{BA}, X_B^{BB}, X_B^{BC})$.

$$X_B^{BA} = \delta_A V_A \qquad X_B^{BB} = X_B - \delta_A V_A - \delta_C V_C \qquad X_B^{BC} = \delta_C V_C \tag{4.65}$$

If C is the first proposer, C will propose the following allocation $X_B = (X_B^{CA}, X_B^{CB}, X_{3B}^{CC})$.

$$X_B^{CA} = \delta_A V_A \qquad X_B^{CB} = \delta_B V_B \qquad X_B^{CC} = X_B - \delta_A V_A - \delta_B V_B \tag{4.66}$$

Then the continuation values are as follows:

$$\begin{aligned}
V_A &= \frac{1}{3}(X_B - \delta_B V_B - \delta_C V_C) + \frac{2}{3}\delta_A V_A \\
V_B &= \frac{1}{3}(X_B - \delta_A V_A - \delta_C V_C) + \frac{2}{3}\delta_B V_B \\
V_C &= \frac{1}{3}(X_B - \delta_B V_B - \delta_A V_A) + \frac{2}{3}\delta_C V_C
\end{aligned} \tag{4.67}$$

Let's solve these equations for V_A, V_B, and V_C:

$$\begin{aligned}
V_A &= \frac{X_B(1-\delta_B)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\
V_B &= \frac{X_B(1-\delta_A)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\
V_C &= \frac{X_B(1-\delta_B)(1-\delta_A)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B}
\end{aligned} \tag{4.68}$$

Since B is the delegate, she makes the first proposal. Therefore the equilibrium payoff allocations are:

$$\begin{aligned}
X_B^{BA} &= \delta_A V_A = \frac{X_B\delta_A(1-\delta_B)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\
X_B^{BB} &= X_B - \frac{X_B\delta_A(1-\delta_B)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\
&- \frac{X_B\delta_C(1-\delta_B)(1-\delta_A)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\
X_B^{BC} &= \delta_C V_C = \frac{X_B\delta_C(1-\delta_B)(1-\delta_A)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B}
\end{aligned} \tag{4.69}$$

Now assume $C \in G$ becomes the delegate, then the group will bargain over X_C in the third stage.

If A is the first proposer, A will propose the allocation $X_C = (X_C^{AA}, X_C^{AB}, X_C^{AC})$.

$$X_C^{AA} = X_C - \delta_B V_B - \delta_C V_C \qquad X_C^{AB} = \delta_B V_B \qquad X_C^{AC} = \delta_C V_C \tag{4.70}$$

If B is the first proposer, B will propose the allocation $X_C = (X_C^{BA}, X_C^{BB}, X_C^{BC})$.

$$X_C^{BA} = \delta_A V_A \qquad X_C^{BB} = X_C - \delta_A V_A - \delta_C V_C \qquad X_C^{BC} = \delta_C V_C \tag{4.71}$$

If C is the first proposer, C will propose the allocation $X_C = (X_C^{CA}, X_C^{CB}, X_C^{CC})$.

$$X_C^{CA} = \delta_A V_A \qquad X_C^{CB} = \delta_B V_B \qquad X_C^{CC} = X_C - \delta_A V_A - \delta_B V_B \tag{4.72}$$

Then the continuation values are as follows:

$$\begin{aligned} V_A &= \frac{1}{3}(X_C - \delta_B V_B - \delta_C V_C) + \frac{2}{3}\delta_A V_A \\ V_B &= \frac{1}{3}(X_C - \delta_A V_A - \delta_C V_C) + \frac{2}{3}\delta_B V_B \\ V_C &= \frac{1}{3}(X_C - \delta_B V_B - \delta_A V_A) + \frac{2}{3}\delta_C V_C \end{aligned} \tag{4.73}$$

Let's solve these equations for V_A, V_B, and V_C:

$$\begin{aligned} V_A &= \frac{X_C(1-\delta_B)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ V_B &= \frac{X_C(1-\delta_A)(1-\delta_C)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ V_C &= \frac{X_C(1-\delta_B)(1-\delta_A)}{3 - 2\delta_A - 2\delta_B - 2\delta_C + \delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \end{aligned} \tag{4.74}$$

Since C is the delegate, she makes the first proposal. Therefore the equilibrium payoff allocations are:

$$
\begin{aligned}
X_C^{CA} &= \delta_A V_A = \frac{X_C\delta_A(1-\delta_B)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_C^{CB} &= \delta_B V_B = \frac{X_C\delta_B(1-\delta_A)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} \\
X_C^{CC} &= X_C - \frac{X_C\delta_B(1-\delta_A)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B} - \\
&\frac{X_C\delta_A(1-\delta_B)(1-\delta_C)}{3-2\delta_A-2\delta_B-2\delta_C+\delta_B\delta_C+\delta_A\delta_C+\delta_A\delta_B}
\end{aligned} \tag{4.75}
$$

4.1.3.4 Unanimity Rule BF Model with Higher Recognition Probability

In this alternative, the delegate has a recognition probability of $p_\phi = \frac{1}{2}$, and other group members have a recognition probability of $p_{-\phi} = \frac{1}{4}$ throughout the third stage.

There are two things to remind at this point regarding this alternative: (i) Due to the unanimous decision requirement, the proposer needs to convince all of the group members to implement the proposal. Hence, the proposer must offer their continuation value to each $i \in G$. (ii) The first proposer is not certain here. Hence different than **First Proposal Right alternative**, we cannot discuss the sure outcomes. Group members will consider only their expected payoffs.

Now assume $A \in G$ becomes the delegate, then the group will bargain over X_A in the third stage.

If A is the first proposer, A will propose the allocation $X_A = (X_A^{AA}, X_A^{AB}, X_A^{AC})$.

$$
X_A^{AA} = X_A - \delta_B V_B - \delta_C V_C \qquad X_A^{AB} = \delta_B V_B \qquad X_A^{AC} = \delta_C V_C \tag{4.76}
$$

If B is the first proposer, B will propose the allocation $X_A = (X_A^{BA}, X_A^{BB}, X_A^{BC})$.

$$X_A^{BA} = \delta_A V_A \qquad X_A^{BB} = X_A - \delta_A V_A - \delta_C V_C \qquad X_A^{BC} = \delta_C V_C \tag{4.77}$$

If C is the first proposer, C will propose the allocation $X_{2A} = (X_A^{CA}, X_A^{CB}, X_A^{CC})$.

$$X_A^{CA} = \delta_A V_A \qquad X_A^{CB} = \delta_B V_B \qquad X_A^{CC} = X_A - \delta_A V_A - \delta_B V_B \tag{4.78}$$

Then the continuation values are as follows:

$$\begin{aligned} V_A &= \frac{1}{2}(X_A - \delta_B V_B - \delta_C V_C) + \frac{1}{4}\delta_A V_A + \frac{1}{4}\delta_A V_A \\ V_B &= \frac{1}{4}(X_A - \delta_A V_A - \delta_C V_C) + \frac{1}{4}\delta_B V_B + \frac{1}{2}\delta_B V_B \\ V_C &= \frac{1}{4}(X_A - \delta_B V_B - \delta_A V_A) + \frac{1}{2}\delta_C V_C + \frac{1}{4}\delta_C V_C \end{aligned} \tag{4.79}$$

Let's solve these equations for V_A, V_B, and V_C:

$$\begin{aligned} V_A &= \frac{2X_A(1-\delta_B)(1-\delta_C)}{4 - 2\delta_A - 3\delta_B - 3\delta_C + 2\delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ V_B &= \frac{X_A(1-\delta_A)(1-\delta_C)}{4 - 2\delta_A - 3\delta_B - 3\delta_C + 2\delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ V_C &= \frac{X_A(1-\delta_B)(1-\delta_A)}{4 - 2\delta_A - 3\delta_B - 3\delta_C + 2\delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \end{aligned} \tag{4.80}$$

Then the expected equilibrium payoffs are as follows:

$$\begin{aligned} X_A^A = \delta_A V_A &= \frac{2X_A\delta_A(1-\delta_B)(1-\delta_C)}{4 - 2\delta_A - 3\delta_B - 3\delta_C + 2\delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ X_A^B = \delta_B V_B &= \frac{X_A\delta_B(1-\delta_A)(1-\delta_C)}{4 - 2\delta_A - 3\delta_B - 3\delta_C + 2\delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \\ X_A^C = \delta_C V_C &= \frac{X_A\delta_C(1-\delta_A)(1-\delta_B)}{4 - 2\delta_A - 3\delta_B - 3\delta_C + 2\delta_B\delta_C + \delta_A\delta_C + \delta_A\delta_B} \end{aligned} \tag{4.81}$$

Now let's assume $B \in G$ is the delegate. Then the group's payoff from out-group bargaining is X_B.

If A is the first proposer, A will propose the allocation $X_B = (X_B^{AA}, X_B^{AB}, X_B^{AC})$.

$$X_B^{AA} = X_B - \delta_B V_B - \delta_C V_C \qquad X_B^{AB} = \delta_B V_B \qquad X_B^{AC} = \delta_C V_C \tag{4.82}$$

If B is the first proposer, B will propose the allocation $X_B = (X_B^{BA}, X_B^{BB}, X_B^{BC})$.

$$X_B^{BA} = \delta_A V_A \qquad X_B^{BB} = X_B - \delta_A V_A - \delta_C V_C \qquad X_B^{BC} = \delta_C V_C \tag{4.83}$$

If C is the first proposer, C will propose the allocation $X_B = (X_B^{CA}, X_B^{CB}, X_B^{CC})$.

$$X_B^{CA} = \delta_A V_A \qquad X_B^{CB} = \delta_B V_B \qquad X_B^{CC} = X_B - \delta_A V_A - \delta_B V_B \tag{4.84}$$

Then the continuation values are as follows:

$$\begin{aligned} V_A &= \frac{1}{4}(X_B - \delta_B V_B - \delta_C V_C) + \frac{3}{4}\delta_A V_A \\ V_B &= \frac{1}{2}(X_B - \delta_A V_A - \delta_C V_C) + \frac{1}{2}\delta_B V_B \\ V_C &= \frac{1}{4}(X_B - \delta_B V_B - \delta_A V_A) + \frac{3}{4}\delta_C V_C \end{aligned} \tag{4.85}$$

Let's solve these equations for V_A, V_B, and V_C:

$$\begin{aligned} V_A &= \frac{X_B(1-\delta_B)(1-\delta_C)}{4 - 3\delta_A - 2\delta_B - 3\delta_C + 2\delta_A\delta_C + \delta_B\delta_C + \delta_A\delta_B} \\ V_B &= \frac{2X_B(1-\delta_A)(1-\delta_C)}{4 - 3\delta_A - 2\delta_B - 3\delta_C + 2\delta_A\delta_C + \delta_B\delta_C + \delta_A\delta_B} \\ V_C &= \frac{X_B(1-\delta_B)(1-\delta_A)}{4 - 3\delta_A - 2\delta_B - 3\delta_C + 2\delta_A\delta_C + \delta_B\delta_C + \delta_A\delta_B} \end{aligned} \tag{4.86}$$

Then the expected equilibrium payoffs are as follows:

$$\begin{aligned} X_B^A = \delta_A V_A &= \frac{X_B\delta_A(1-\delta_B)(1-\delta_C)}{4 - 3\delta_A - 2\delta_B - 3\delta_C + 2\delta_A\delta_C + \delta_B\delta_C + \delta_A\delta_B} \\ X_B^B = \delta_B V_B &= \frac{2X_B\delta_B(1-\delta_A)(1-\delta_C)}{4 - 3\delta_A - 2\delta_B - 3\delta_C + 2\delta_A\delta_C + \delta_B\delta_C + \delta_A\delta_B} \\ X_B^C = \delta_C V_C &= \frac{X_B\delta_C(1-\delta_A)(1-\delta_B)}{4 - 3\delta_A - 2\delta_B - 3\delta_C + 2\delta_A\delta_C + \delta_B\delta_C + \delta_A\delta_B} \end{aligned} \tag{4.87}$$

Now let's assume $C \in G$ is the delegate. Then the group's payoff from out-group bargaining is X_C.

If A is the first proposer, A will propose the allocation $X_C = (X_C^{AA}, X_C^{AB}, X_C^{AC})$.

$$X_C^{AA} = X_C - \delta_B V_B - \delta_C V_C \qquad X_C^{AB} = \delta_B V_B \qquad X_C^{AC} = \delta_C V_C \tag{4.88}$$

If B is the first proposer, B will propose the allocation $X_C = (X_C^{BA}, X_C^{BB}, X_C^{BC})$.

$$X_C^{BA} = \delta_A V_A \qquad X_C^{BB} = X_C - \delta_A V_A - \delta_C V_C \qquad X_C^{BC} = \delta_C V_C \tag{4.89}$$

If C is the first proposer, C will propose the allocation $X_C = (X_C^{CA}, X_C^{CB}, X_C^{CC})$.

$$X_C^{CA} = \delta_A V_A \qquad X_C^{CB} = \delta_B V_B \qquad X_C^{CC} = X_C - \delta_A V_A - \delta_B V_B \tag{4.90}$$

Then the continuation values are as follows:

$$\begin{aligned} V_A &= \frac{1}{4}(X_C - \delta_B V_B - \delta_C V_C) + \frac{3}{4}\delta_A V_A \\ V_B &= \frac{1}{4}(X_C - \delta_A V_A - \delta_C V_C) + \frac{3}{4}\delta_B V_B \\ V_C &= \frac{1}{2}(X_C - \delta_B V_B - \delta_A V_A) + \frac{1}{2}\delta_C V_C \end{aligned} \tag{4.91}$$

Let's solve these equations for V_A, V_B, and V_C:

$$\begin{aligned} V_A &= \frac{X_C(1-\delta_B)(1-\delta_C)}{4 - 3\delta_A - 3\delta_B - 2\delta_C + \delta_A\delta_C + \delta_B\delta_C + 2\delta_A\delta_B} \\ V_B &= \frac{X_C(1-\delta_A)(1-\delta_C)}{4 - 3\delta_A - 3\delta_B - 2\delta_C + \delta_A\delta_C + \delta_B\delta_C + 2\delta_A\delta_B} \\ V_C &= \frac{2X_C(1-\delta_B)(1-\delta_A)}{4 - 3\delta_A - 3\delta_B - 2\delta_C + \delta_A\delta_C + \delta_B\delta_C + 2\delta_A\delta_B} \end{aligned} \tag{4.92}$$

Then the expected equilibrium payoffs are as follows:

$$\begin{aligned} X_C^A = \delta_A V_A &= \frac{X_C\delta_A(1-\delta_B)(1-\delta_C)}{4 - 3\delta_A - 3\delta_B - 2\delta_C + \delta_A\delta_C + \delta_B\delta_C + 2\delta_A\delta_B} \\ X_C^B = \delta_B V_B &= \frac{X_C\delta_B(1-\delta_A)(1-\delta_C)}{4 - 3\delta_A - 3\delta_B - 2\delta_C + \delta_A\delta_C + \delta_B\delta_C + 2\delta_A\delta_B} \\ X_C^C = \delta_C V_C &= \frac{2X_C\delta_C(1-\delta_A)(1-\delta_B)}{4 - 3\delta_A - 3\delta_B - 2\delta_C + \delta_A\delta_C + \delta_B\delta_C + 2\delta_A\delta_B} \end{aligned} \tag{4.93}$$

4.2 Computational Results

As seen earlier in this chapter, the analytical results obtained are difficult to interpret. For this reason, we continue with numerical analysis and characterize the delegation selection for various parameter values.

Accordingly, we change the time discount factors of A, B, C and D, in the $(0, 1)$ interval with steps size of 0.05, preserving the assumption we made at the beginning, $\delta_A > \delta_B > \delta_C$.

4.2.1 Majority Rule BF Model with First Proposal Right

First, let's observe all possible preference profiles of group members that are compatible with the payoff rankings:

Profile 1:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**C**
B $\sim$ C	C	A
	A	B

Profile 2:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**A**
B $\sim$ C	C	C
	A	B

Profile 3:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**A**
B $\sim$ C	C	B
	A	C

Among the profiles seen above, the most frequently encountered profile is 1. It is observed 18300 times in 18411 different parameter combinations, that is, in 99.4$\%$ of all cases. Profile 1 is followed by Profile 2 in terms of frequency, yet it is observed in only 99 cases which corresponds to 0.054$\%$. It is available where C is at least 0.35 and δ_A and δ_D are both 0.95. If $\delta_C \geq 0.65$, then this profile is also valid for the values of 0.9 of δ_A and δ_D. However, not for the case where $\delta_A = \delta_D = 0.9$. Specifically, Profile 2 can be visible where at most, one of δ_A and δ_D is 0.9. Profile 3, on the other hand, is only visible in 2 different parameter combinations. These are: (i) $\delta_A = \delta_D = 0.95$, $\delta_B = 0.9$ and $\delta_C = 0.65$. (ii) $\delta_A = \delta_D = 0.95$, $\delta_B = 0.9$ and $\delta_C = 0.7$. Consequently, for all other parameter values for δ_A, δ_B, δ_C and δ_D Profile 1 is observed.

Player A always prefers herself a delegate, as she is excluded from winning coalitions when anyone else is a delegate. *Player B*, on the other hand, is least likely to want *Player A* to be a delegate, as he is always outside the winning coalition when A is the delegate. This leads A and B to have preferences that are not affected by parameters. Unlike these two people, *Player C* is the one who has the room to make a proper comparison among alternatives since C can always get a positive payoff (is always included in the winning coalitions), as shown in the profiles above. Hence, in this variation, it is more important to examine the effects that cause the change in the payoffs of C, thus the change in the preference profiles.

Looking at $\mathcal{T}_C$s', the first thing we can observe is there exist two prevalent effects on C's payoffs: the *pie size effect* and the *first proposer effect*. The first proposer advantage is a well-known result in BF Model, as it is mentioned in Eraslan and Evdokimov (2019). Since the agreement is reached immediately in the first period, the first proposer i has a chance to obtain a payoff $x_i > \delta_i V_i$. Being the first to make the offer in the last stage makes C better off for most of the cases in the given parameter range. More specifically, her own delegation dominates A's delegation hence the bigger pie possibility.

On the other hand, in profile 2 and profile 3, we observe the opposite case. In those cases, the share that A would offer from her bigger pie dominates the first proposer advantage that C could get. The first condition for this to happen is that both A and D must be very "strong". Recall that in profiles 2 and 3, $\delta_A \geq 0.9$ and $\delta_D \geq 0.9$. The second and maybe more important condition is that C should not be very "weak". The lowest δ_C value for the case when $\delta_A = \delta_D = 0.95$ is 0.35, and 0.65 for the case when $\delta_A = 0.9$ and $\delta_D = 0.95$ [5]. This actually tells us that C prefers A's delegation when she has a chance against A, not when she

[5] or when $\delta_A = 0.95$ and $\delta_D = 0.9$

is too "weak". Although this is surprising at first, it is also a meaningful result because when C is too weak, even when A brings a significant amount of pie to the group, the pie is allocated even more unfairly. Hence, the increase in the group's pie size does not reflect C's individual payoff. She can get a meaningful amount only when she has some strength against A.

The difference that distinguishes profile 2 from profile 3 is C's second choice. She puts herself in second place in Profile 2 and prefers herself last in Profile 3. The fundamental reason for this change in C's preference is the difference between δ_C and δ_B. For example when $\delta_A = \delta_D = 0.95$, $\delta_B = 0.9$ and $\delta_C = 0.75$, we observe profile 2 whereas we observe profile 3 when $\delta_A = \delta_D = 0.95$, $\delta_B = 0.9$ and $\delta_C = 0.7$. This is because in the second case, since C is relatively weaker, after giving B her continuation value, the remaining amount falls below what B could offer her.

Equilibrium Voting Strategies for Majority Voting

Recall that $\mathcal{G}$ refers to the set of all possible voting strategy profiles for all $i \in G$ and $\mathcal{E}$ to the set of pure Nash Equilibria. Observe that $|\mathcal{G}| = 3^3 = 27$ and

$$\begin{aligned}\mathcal{G} = \{&(A,A,A),(A,A,B),(A,B,A),(A,A,C),(A,C,A),(A,B,C),(A,C,B),\\&(A,B,B),(A,C,C),(B,B,B),(B,B,C),(B,B,A),(B,A,B),(B,C,B),\\&(B,C,C),(B,A,A),(B,A,C),(B,C,A),(C,C,C),(C,C,A),(C,C,B),\\&(C,A,C),(C,B,C),(C,B,B),(C,A,A),(C,A,B),(C,B,A)\}\end{aligned}$$ [6]

If the preferences are as in **profile 1**, which constitutes the 99.4% of the cases, then the equilibrium voting strategy profiles are $\mathcal{E}^1 = \{(A,A,A),(B,B,B),(C,C,C),$

[6] Among those strategies above, the followings *can never be* an equilibrium strategy profile: (A,A,B),(A,A,C),(A,B,B),(A,B,C),(A,C,B),(B,A,B),(B,B,A),(B,A,C),(B,A,A),(B,C,A), (B,C,C),(C,A,C),(C,C,A),(C,C,B),(C,B,A),(C,A,B),(C,A,A) given the preference profiles of the group members. At least one $i \in G$ has a profitable deviation.

$(A, B, A), (A, C, C), (B, B, C), (C, B, C)\}$.[7] Recall that when the group members cannot choose a majority winner to be the delegate, they all get a payoff of 0. Hence, by strategically voting for their second best, they are better off, which leads to multiple equilibria. For example, instead of voting for herself, B can deviate to voting C given that C votes for herself and get $\delta_B V_B > 0$. Similarly, C can vote for A's delegation instead of her own and secure herself from not receiving a payoff. Among the group members, A has a different condition. Given that she would certainly not be included in any other winning coalitions, she is indifferent between C's delegation, B's delegation, and no delegation cases. This allows strategy profiles (B, B, C) and (C, B, C) to be equilibria at the same time.

If the preferences are as in **profile 2**, then the equilibrium voting strategy profiles are $\mathcal{E}^2 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, C, A), (B, B, C), (C, B, C)\}$. Electing B or C as a delegate can again be an equilibrium strategy for the group G, although least likely to happen. Given the parameter values that lead to this profile, A is more likely to be chosen as the delegate since the majority of the group members rank A's delegation as their top choice.

If the preferences are as in **profile 3**, then the equilibrium voting strategies are $\mathcal{E}^3 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, C, A), (B, C, B), (B, B, C), (C, B, B)\}$. Observe that in this profile, apart from (C, C, C), there is no other equilibrium profile induced by individuals' preferences in which C is chosen as a delegate. However, electing B is still a likely equilibrium strategy. This is not a surprising result, given the parameter values that lead to this profile. Recall that for **profile 3**, parameters should either be like $\delta_A = \delta_D = 0.95$, $\delta_B = 0.9$ and $\delta_C = 0.65$ or, $\delta_A = \delta_D = 0.95$, $\delta_B = 0.9$ and $\delta_C = 0.7$. As we discussed previously,

[7] Notice that the first three strategy profiles are equilibrium strategies because of the definition of majority voting rule and Nash Equilibrium. Later we switch to pairwise sequential voting to have more refined equilibrium results.

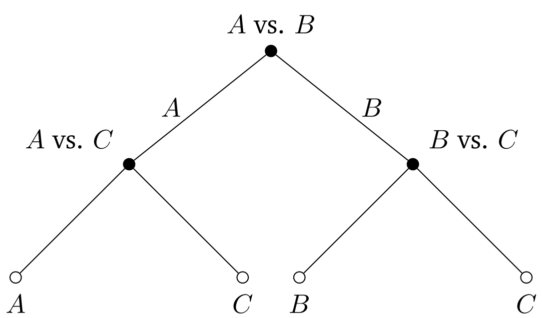
C, in these cases, has enough strength to reject small offers, while A and B have the potential to bring a relatively larger share to the group. Thus for C, a smaller share of the bigger pie dominates the strategy of getting the big part of the smaller pie.

As we can clearly see, electing C or B are also decisions that can be taken at equilibrium which tells us that electing the strongest member is not a unique strategy. Although the strongest group member might bring the biggest share to the group, it might be optimal as well to choose the less strong alternative and share the smaller pie more evenly.

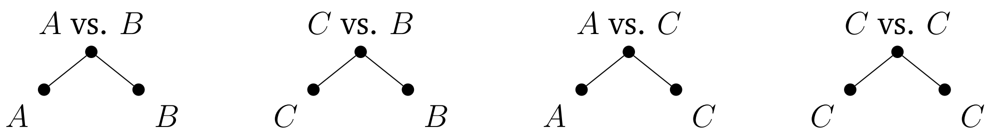
Equilibrium Voting Strategies for Sequential Pairwise Voting

We consider the sequential pairwise voting rule instead of the majority voting rule for delegate selection to refine our findings. Group members make sequential pairwise comparisons among alternatives, but they vote simultaneously. For example, group members first vote to eliminate A or B. Then a vote is taken on between surviving alternative and C. By using backward induction, we find the subgame perfect equilibria of this game. However, notice that for this particular game which includes majority rule multilateral bargaining protocol with the first proposal right, without further restriction, we still have multiple equilibria.

First, let's consider **Profile 1**. See that given the preference profile, group members may prefer casting a strategic vote which leads to the problem that the order of the pairs presented matters. Let G first vote over A and B, then over C and the surviving one. Then the voting tree is as follows:

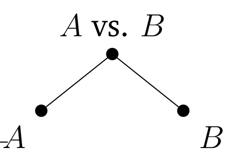


See that the equivalent of the last decision node on the left-hand side can be both A and C given the definition of Nash Equilibria since we do not restrict group members to undominated strategies [8]. On the right-hand side, again, both B and C can be the outcome. We end up with 4 possible cases when we move from the final node to the initial one.



As we see, given the preference profile, all alternatives can be the outcome of sequential pairwise voting. Even if we change the pairwise comparison order, the same result still exists. Notice that the very same condition exists in profile 2 as well. For this reason, we can say that sequential pairwise voting in profiles 1 and 2 is not sufficient to refine the results without further restriction.

Now let's observe Profile 3. In this profile, different from the others, we can have the restriction of weakly undominated strategies because the indifference in A's preferences does not affect the pairwise comparison results. Moreover, the result of the sequential pairwise voting procedure remains the same for every binary order of comparison. Hence, we only present the results for one specific order. Let G first vote over A and B, then over C and the surviving one. Since the voting tree resulting from this order is the same as in profiles 1 and 2, we do not draw it again. However, notice that the sophisticated equivalent of the voting tree becomes as seen below. Since A is the top-ranked alternative for both A and C, A is the winner of the voting.



[8]We do not impose this restriction because of the indifference in A's preferences.

Consequently, the results of the sequential pairwise voting approach are consistent with the results of majority voting for profiles 1 and 2 and are more refined for profile 3 compared to majority voting. Although we cannot say that this change we made in voting is very effective in this particular game, we will see that it is indeed effective in solving the multiple equilibria problem in other games.

4.2.2 Majority Rule BF Model with Higher Recognition Probability Right

First, let's observe all possible preference profiles of group members that are compatible with the payoff rankings:

Profile 1:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**A**
B	A	C
C	C	B

Profile 2:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**A**
B	A	B
C	C	C

Profile 3:

A	B	C
A	**A**	**A**
B	B	B
C	C	C

Profile 4:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**A**	**A**
B	B	C
C	C	B

Profile 5:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**C**
B	C	A
C	A	B

Profile 6:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**C**
B	A	A
C	C	B

Among the profiles seen above, the most frequently encountered profile is 6. It is observed 10887 times in 18411 different parameter combinations, that is, in 59% of all cases. Profile 6 is followed by Profile 1 in terms of frequency, yet it is observed in 3041 cases which corresponds to 16.5%. Profile 5 follows them with a frequency of 16%, while the least common profiles are 4, 2 and 3, respectively. The restrictions for profile 1 are as follows: $\delta_A \geq 0.55$ and $\delta_D \geq 0.45$. For group members to have preferences like profile 2, $\delta_A \geq 0.7$, $\delta_D \geq 0.75$, $\delta_B \geq 0.65$ and $\delta_C \leq 0.75$ must hold. For profile 3, the conditions applicable to profile 2 must be tightened: $\delta_C \leq 0.45$, $0.65 \leq \delta_B \leq 0.8$ and $\delta_A = \delta_D \geq 0.9$. Profile 4, on the other hand, necessitates conditions similar to those of profile 2, but more loosened versions as $\delta_C \leq 0.75$ and $\delta_A = \delta_D \geq 0.6$. The only condition that can be specified for profile 5 is $\delta_D \leq 0.4$. For all other parameter values of δ_A, δ_B , δ_C and δ_D Profile 6 is observed.

In all of the possible preference profiles, Player A strictly prefers her own delegation above others'.There are two interrelated reasons for this. The first is that when she's a delegate, in each round, she is more likely to receive a greater share of the bigger pie she provides to the group. This increases her expected payoff under her own delegation. But if we look at the other side of the coin, her expected payoff in others' delegation is decreasing because she is less likely to be part of the winning coalition.

Another important point to mention is that unlike the first proposal right variation, it now makes a difference for A whether B or C is the delegate. This is because in the former, player A is definitely excluded from the winning coalition regardless of who is chosen. Here, although the probability is relatively low, A might still get a payoff other than 0, even if one of the others is a delegate, because the first proposer is not pre-determined. A can still be the first proposer. In this case, B's delegation is strictly preferable to C's delegation, as she would want the bigger pie to be split.

The first thing to notice about B's preferences is A's position. Recall that, in the previous scenario where the delegate has the first proposal right, A was the least favorable alternative for B. However, in this current scenario, we observe that A becomes B's top alternative in some profiles. This is because even if A is the delegate and obtains the share of the group in out-group bargaining, B still can be the first proposer. Put more formally, in cases where B prefers A's delegation, the expected payoff she receives increases a lot, as A has the potential to obtain a significant of amount share for the group. As evidence of this situation, we can present the fact that $\frac{x_A}{x_B}$ >2 is strictly satisfied in all cases where B ranks A first.

For Player C, similar to what we observe in the previous case, the most preferred alternative can either be A or C, and A is always preferred better than B. In the latter case, the intuition is straightforward. Given the parameters, the continuation value of a player is constant, and it is multiplied by the pie size. Since $x_A > x_B$, C's expected payoff under A's delegation is always greater. For the former, the intuition gets rather complicated. Recall that in the first proposal right case, δ_C's value is a condition for C's decision. Here, C may prefer herself or A the best for any δ_C parameter. However, δ_A and δ_D matters. For C to prefer A's delegation best, δ_A must be at least 0.55 and $\delta_D \geq 0.45$. Hence, C cares more about how strong others are than how strong she is.

Equilibrium Voting Strategies for Majority Voting

Recall that $\mathcal{G}$ refers to the set of all possible voting strategy profiles for all $i \in G$ and $\mathcal{E}$ to the set of pure Nash Equilibria. Observe that $|\mathcal{G}| = 3^3 = 27$ and

$$\begin{aligned}\mathcal{G} = \{&(A,A,A),(A,A,B),(A,B,A),(A,A,C),(A,C,A),(A,B,C),(A,C,B),\\&(A,B,B),(A,C,C),(B,B,B),(B,B,C),(B,B,A),(B,A,B),(B,C,B),\\&(B,C,C),(B,A,A),(B,A,C),(B,C,A),(C,C,C),(C,C,A),(C,C,B),\\&(C,A,C),(C,B,C),(C,B,B),(C,A,A),(C,A,B),(C,B,A)\}\end{aligned}$$ [9]

If the preferences are as in **profile 1**, which constitutes the 59% of the cases, then the equilibrium voting strategy profiles are $\mathcal{E}^1 = \{(A,A,A),(B,B,B),(C,C,C),$ $(A,B,A),(A,C,A),(A,A,C),(B,B,C),(C,A,A)\}$. Electing B or C as a delegate can again be an equilibrium strategy for the group G, although least likely to happen. Given the parameter values that lead to this profile, A is more likely

[9] Among those strategies above, the followings *can never be* an equilibrium strategy profile: (A,B,B),(A,B,C),(A,C,B),(B,A,B),(B,B,A),(B,A,C),(B,C,A),(C,B,C),(B,C,C),(C,A,C), (C,C,A),(C,C,B),(C,B,A),(C,A,B), given the preference profiles of the group members. At least one $i \in G$ has a profitable deviation.

to be chosen as the delegate since the majority of the group members rank A's delegation as their top choice.

If the preferences are as in **profile 2**, then the equilibrium voting strategy profiles are $\mathcal{E}^2 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, C, A), (A, A, C), (B, C, B), (B, B, C), (C, B, B), (C, A, A)\}$. Electing C as a delegate can again be an equilibrium strategy for the group G, although least likely to happen. Given the parameter values that lead to this profile, A is more likely to be chosen as the delegate since the majority of the group members rank A's delegation as their top choice. However, B is also a strong candidate for being a majority winner, given that nobody ranks B as the worst.

If the preferences are as in **profile 3**, then the equilibrium voting strategies are $\mathcal{E}^3 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, C, A), (A, A, C), (A, A, B), (B, C, B), (B, A, A), (B, B, C), (C, B, B), (C, A, A)\}$. Also, for **profile 4**, the equilibrium voting strategy profiles are $\mathcal{E}^4 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, C, A), (A, A, C), (A, A, B), (B, A, A), (B, B, C), (C, A, A)\}$. Given the preferences of group members in these two profiles, we do not expect C to get elected as the delegate. In both of the profiles, A is more likely to be chosen as the delegate since the group members unanimously rank A's delegation as their top choice.

For **profile 5**, the equilibrium voting strategy profiles are $\mathcal{E}^5 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, C, C), (B, B, C)\}$. Notice that, in this profile, all candidates are equally likely to be selected because there is a Condorcet cycle among the preferences. If they vote sincerely for their top choice, they will end up with no majority winner or zero payoffs. Hence, by voting strategically voting, they can be better off. For example, instead of voting for herself, B can deviate to voting for C given that C votes for herself and get $\delta_B V_B > 0$. Similarly, C can vote for A's delegation instead of her own and secure herself from not receiving a payoff.

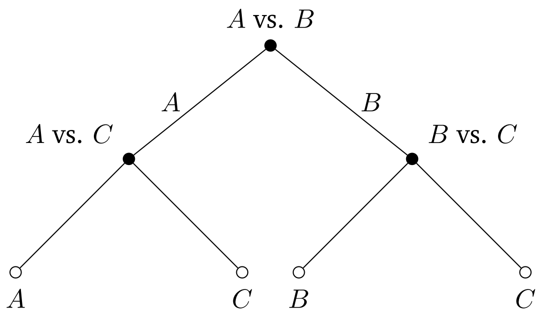
If the profile seen is **profile 6**, the equilibrium voting strategy profiles are $\mathcal{E}^6 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, A, C), (B, B, C)\}$. However, it would not be wrong to expect A or B to be elected because C is considered the worst option by two players. Players will vote strategically to sustain a delegate. Recall that profile 6 is the most prevalent one capturing 60% of the parameter combinations. Hence, we can safely say that for the majority of the cases, the election of a delegate different than "the strongest" player is feasible.

What we can infer from the equilibria voting strategies of this variation is that the indeterminacy of the first proposer leads group members to choose stronger delegates. When players have a positive probability of becoming the first proposer, pie size gains more importance. Players want to offer a division of the largest possible pie when they are the first to proposer. Therefore, the election of A or B appears as a better outcome.

Equilibrium Voting Strategies for Pairwise Sequential Voting

Now let's consider the sequential pairwise voting rule instead of the majority voting rule to refine our findings. In the game with *majority rule BF model with higher recognition probability right*, we observe that group members have strict preferences, and except for one profile, the preferences are single-peaked. Hence we will safely put the restriction of weakly undominated strategies for all profiles except 5. Also, notice that pairwise ordering of the alternatives does not change the winner. Let's assume G first votes over A and B, then over C and the surviving one[10].

[10]This assumption is for all of the profiles except profile 5. Since there is a Condorcet cycle, we examine that one differently.



First, let's consider **Profile 1**. The preferences are transitive on the triples. See that given the preference profile, the undominated strategy of each group member is to vote sincerely for the best of the two given alternatives. Hence, voting for C is eliminated on both sides of the game tree, and the voting decision boils down to A vs. B. Then the sophisticated voting outcome is A since $A \succ B$.

For **Profile 2**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is again made between A and B, which results in the election of A.

For **Profile 3**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is made between A and B, which results in the election of A.

For **Profile 4**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is made between A and B, which results in the election of A.

For **Profile 6**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is made between A and B, which results in the election of A.

In **Profile 5**, we observe the Condorcet cycle since $A \succ B$, $B \succ C$ but $C \succ A$ according to the preference orderings. The Condorcet cycle in preferences delivers a possibility that all available alternatives can be elected, regardless of which pairwise ordering we evaluate. Assume the first comparison is made between A and B. If A wins the first round, it encounters C and loses. C is elected as the delegate. However, both A and B would prefer B's delegation. In order not to elect C, A may vote for B in the first round. Since $B \succ C$ for A and B, they are better off. Yet, this makes C worse off. Instead of voting for herself during her encounter with A, she may vote for A and eliminate B's delegation.

In our game which starts with delegate selection vis. a vis. sequential pairwise voting and ends with a majority rule BF model with higher recognition probability right, we can safely expect A to be elected as a delegate. This deduction is also supported by the equilibrium results we obtained with the majority voting rule. The intuition of this result is that group members tend to maximize the group's share when the first proposer is uncertain in in-group bargaining. Thus, even if there is an allocation that a member does not prefer, allocating a bigger pie may compensate for it.

4.2.3 Unanimity Rule BF Model with First Proposal Right

First, let's observe all possible preference profiles of group members that are compatible with the payoff rankings:

Profile 1:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**A**	**C**
B	B	A
C	C	B

Profile 2:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**A**	**A**
B	B	B
C	C	C

Profile 3:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**A**	**A**
B	B	C
C	C	B

Profile 4:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**C**
B	A	A
C	C	B

Profile 5:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**A**
B	A	C
C	C	B

Among the profiles seen above, the most frequently encountered profile is 4. It is observed 18144 times in 18411 different parameter combinations, that is, in 98.5% of all cases. Profile 4 is followed by Profile 1 in terms of frequency, yet it is observed in only 166 cases which correspond to 0.90%. Profile 3 follows them with a frequency of 0.46%, while the least common profiles are 5 and 2, respectively. The restrictions for Profile 1 are as follows: $\delta_A \geq 0.9$, $\delta_D \geq 0.9$, $\delta_B \geq 0.35$ and $\delta_C \leq 0.6$. For profile 2 to be observed, $\delta_A = \delta_D = 0.95$, $\delta_B = 0.9$ and $\delta_C = 0.65$ or $\delta_C = 0.7$ must hold. For profile 3, we should have $\delta_A \geq 0.9$, $\delta_D \geq 0.9$, $\delta_B \geq 0.4$ and $\delta_C \geq 0.35$. Profile 5, which is only observed in 13 cases, necessitates $\delta_A \geq 0.9$, $\delta_D \geq 0.9$, $\delta_B \geq 0.75$ and $\delta_C \geq 0.55$ to hold. For the rest of the parameter combinations, profile 4 is observed.

In all of the possible preference profiles, Player A strictly prefers her own delegation above others'. The first and most fundamental reason for this is that when she becomes a delegate, the group will receive the largest possible share, and she will make the first proposal for the distribution of it. Hence, she will significantly be better off. As can be seen, A's secondary and tertiary preferences are also independent of parameter values because B and C are always weaker than her, so B, who is relatively stronger among them, can offer herself a larger share. This is due to the multiplication of the continuation value, which is constant in the given parameter values, by the larger amount of pie size.

The first thing to notice about B's preferences is A's position. Recall that, in the majority rule BF model with the first proposal right, A was the least favorable alternative for B. However, under unanimity rule voting, we observe that A becomes B's top alternative in some profiles and second favorite in the remaining ones. This is because even if A is the delegate and obtains the share of the group in out-group bargaining, she has to offer B her continuation value for the proposal to be implemented. Since we assume $\delta_A > \delta_C$, A becomes the second most profitable alternative regardless of the parameter values. The question to be asked at this point is what affects B's top preferences so that sometimes she ranks herself first and sometimes A? Although it does not have very sharp boundaries, the most fundamental factor is the difference between the bargaining characters of A, D, and B. More specifically, $\frac{\delta_A}{\delta_B}$ or $\frac{\delta_D}{\delta_B}$ ratio cannot be less than 1.055 when $A \succ_B B$ and cannot be more than 2.71. However, for the case where $B \succ_B A$, these ratios' minimum value becomes 0.055, and the maximum value reaches up to 9.5. This shows that when B is close to A and D in terms of bargaining power, she prefers the delegation of A realizing that A should offer herself a significant share. This even dominates the advantage of being the first proposer, which is, we believe, one of the crucial results.

The condition of having enough power against A and D also affects C's preferences. This can be seen much more clearly in C's preference orderings. The maximum value of $\frac{\delta_A}{\delta_C}$ when $A \succ_C C$ is 2.71 whereas the ratio reaches up to 19 when $C \succ_C A$. Similar change is observed in $\frac{\delta_D}{\delta_C}$ as well. Hence, we can conclude that when A is significantly stronger than C, C is better off being the first proposer, whereas when their strength level is similar, C enjoys A's offer more.

Equilibrium Voting Strategies for Majority Voting

Recall that $\mathcal{G}$ refers to the set of all possible voting strategy profiles for all $i \in G$

and $\mathcal{E}$ to the set of pure Nash Equilibria. Observe that $|\mathcal{G}| = 3^3 = 27$ and

$$\mathcal{G} = \{(A,A,A),(A,A,B),(A,B,A),(A,A,C),(A,C,A),(A,B,C),(A,C,B),$$
$$(A,B,B),(A,C,C),(B,B,B),(B,B,C),(B,B,A),(B,A,B),(B,C,B),$$
$$(B,C,C),(B,A,A),(B,A,C),(B,C,A),(C,C,C),(C,C,A),(C,C,B),$$
$$(C,A,C),(C,B,C),(C,B,B),(C,A,A),(C,A,B),(C,B,A)\}$$ [11]

If the preferences are as in **profile 1**, then the equilibrium voting strategy profiles are $\mathcal{E}^1 = \{(A,A,A),(B,B,B),(C,C,C),(A,B,A),(A,A,B),(A,A,C),$ $(B,A,A),(B,B,C)\}$. Notice that except for (C,C,C), there is no equilibrium strategy profile for group members in which C is elected as a delegate, given that two members rank C as the worst outcome. On the other hand, electing B as a delegate can be an equilibrium strategy for the group G as well as A.

If the preferences are as in **profile 2**, then the equilibrium voting strategy profiles are $\mathcal{E}^2 = \{(A,A,A),(B,B,B),(C,C,C),(A,B,A),(A,C,A),(A,A,B),(A,A,C),$ $(B,A,A),(B,B,C),(B,C,B),(C,A,A),(C,B,B)\}$. Except for (C,C,C), there is no equilibrium strategy profile for group members in which C is elected as a delegate. Given that all of them rank C as the worst outcome, this equilibrium can be expected to happen the least. Electing B as a delegate can again be an equilibrium strategy for the group G, although less likely to happen. Given it is the top-ranked alternative for all group members, A is more likely to be chosen as the delegate under this preference profile.

If the preferences are as in **profile 3**, then the equilibrium voting strategy profiles are $\mathcal{E}^3 = \{(A,A,A),(B,B,B),(C,C,C),(A,B,A),(A,A,B),(A,C,A),(A,A,C),$ $(B,A,A),(B,B,C),(C,A,A)\}$. Notice that except for (C,C,C), there is no equi-

[11]Among those strategy profiles above, the followings *can never be* an equilibrium strategy: (A,B,B),(A,C,C),(A,B,C),(A,C,B),(B,A,B),(B,B,A),(B,A,C),(B,C,A),(B,C,C),(C,A,C), (C,B,C),(C,C,A),(C,C,B),(C,B,A),(C,A,B), given the preference profiles of the group members. At least one $i \in G$ has a profitable deviation.

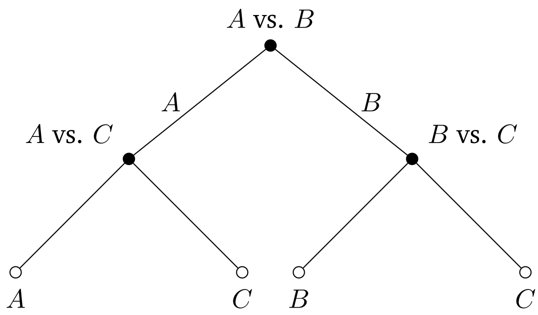
librium strategy profile for group members in which C is elected as a delegate, given that two members rank C as the worst outcome. Electing B as a delegate can be an equilibrium strategy for the group G as well as A although less likely.

If the preferences are as in **profile 4**, then the equilibrium voting strategies are $\mathcal{E}^4 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, A, C), (B, B, C)\}$. Notice that except for (C, C, C), there is no equilibrium strategy profile for group members in which C is elected as a delegate, given that two members rank C as the worst outcome. Electing B as a delegate can be an equilibrium strategy for the group G as well as A although less likely. Since A is never ranked as the worst alternative here, it is more likely for B and C to vote for A instead of their top choices.

If the preferences are as in **profile 5**, then the equilibrium voting strategy profiles are $\mathcal{E}^4 = \{(A, A, A), (B, B, B), (C, C, C), (A, B, A), (A, C, A), (A, A, C), (B, B, C),$ $(C, A, A)\}$. Notice that except for (C, C, C), there is no equilibrium strategy profile for group members in which C is elected as a delegate, given that two members rank C as the worst outcome. Electing B as a delegate can be an equilibrium strategy for the group G as well as A although less likely. Since A is never ranked as the worst alternative here, it is more likely for B and C to vote for A instead of their top choices.

Equilibrium Voting Strategies for Sequential Pairwise Voting

Now we consider the sequential pairwise voting rule instead of the majority voting rule for the initial stage of our game to refine our findings. We assume that no group member's strategy is weakly dominated in any subgame. Let G first vote over A and B, then over C and the surviving one. The voting tree is as follows:



First, let's consider **Profile 1**. The preferences are transitive on the triples. See that given the preference profile, the undominated strategy of each group member is to vote sincerely for the best of the two given alternatives. Hence, voting for C is eliminated, and the voting decision boils down to A vs. B. Then the sophisticated voting outcome is A since $A \succ B$.

For **Profile 2**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is again made between A and B, which results in the election of A.

For **Profile 3**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is made between A and B, which results in the election of A.

For **Profile 4**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is made between A and B, which results in the election of A.

For **Profile 5**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is made between A and B, which results in the election of A.

We can eliminate the multiple equilibria problem that arises under the majority voting with sequential pairwise voting. The results we found here are consistent with the equilibrium outcomes that occur in the majority voting yet are more refined. In light of these, we can say that group members are better off choosing the strongest person regardless of the parameter values.

4.2.4 Unanimity Rule BF Model with Higher Recognition Probability Right

First, let's observe all possible preference profiles of group members that are compatible with the payoff rankings:

Profile 1:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**C**
B	A	A
C	C	B

Profile 2:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**C**
B	A	B
C	C	A

Profile 3:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**C**
B	C	B
C	A	A

Profile 4:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**A**	**C**
B	B	A
C	C	B

Profile 5:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**A**	**A**
B	B	C
C	C	B

Profile 6:

$\mathcal{T}_A$	$\mathcal{T}_B$	$\mathcal{T}_C$
A	**B**	**C**
C	C	B
B	A	A

Among the profiles seen above, the most frequently encountered profile is 1. It is observed 7893 times in 18411 different parameter combinations, that is, in 42.87% of all cases. Profile 1 is followed by Profile 6 in terms of frequency, and it is observed in 4522 cases which corresponds to 24.56%. Profile 3 follows them with a frequency of 18.89%, while the least common profiles are 2, 5, and 4, respectively. The restrictions for Profile 2 are as follows: $\delta_A \geq 0.25$, $\delta_D \leq 0.85$, $\delta_B \geq 0.3$. For profile 3 to be observed, $\delta_A \geq 0.3$ and $0.1 \leq \delta_D \leq 0.75$ must hold. For profile 4, we should have $\delta_A \geq 0.7$, $\delta_D \geq 0.8$, $\delta_B \leq 0.7$ and $\delta_C \leq 0.55$. Profile 5, which is only observed in 293 cases, necessitates $\delta_A \geq 0.75$, $\delta_D \geq 0.8$, $\delta_B \leq 0.7$ and $\delta_C \leq 0.65$ to hold. For profile 6, $\delta_D \leq 0.6$ and for profile 1, $\delta_D \geq 0.3$ must hold.

In all of the possible preference profiles, Player A strictly prefers her own delegation above others'. The first and most fundamental reason for this is that when she becomes a delegate, the group will receive the largest possible share, and she will have a higher recognition probability of becoming the first proposer for the distribution of it. Hence, she will significantly be better off. So, this is independent of the parameter values.

A's second-best alternative varies, given the parameter values. This result seems surprising at first, given that C is always weaker than B. However, upon careful examination, the reason for this situation can be understood. For A, the probability of being the first proposer when B or C is delegated is $\frac{1}{4}$. With a probability of $\frac{3}{4}$, her continuation value will be presented to her. Obviously, the $\delta_A V_A$ from B is greater than the one from C because, as we showed earlier, the continuation value is multiplied by the group's share. What makes the difference in A's expected payoff is the amount left to A when A offers others their continuation values. In Profile 6, when A deducts the continuation values of others from the pie B obtained, the remaining pie is less than the remaining amount when we apply the same process to the pie brought by C. That is, although the amount brought by B is greater, A is worse off because the continuation values of the others increase even more. Hence, in Profile 6, A prefers the smaller pie brought by C if she herself is not going to be a delegate.

In all other models that we have, what we commonly observe [12] in B's preferences is, regardless of the parameter values, B ranks A higher than C given the assumption of $\delta_A > \delta_C$. However, here, in profiles 3 and 6, we observe an opposite preference ordering. The reason for such a change is the same one we mentioned for A: The bigger the pie size, the bigger it is the continuation values. Hence, B would need to offer much greater $\delta_A V_A$ and $\delta_C V_C$ to convince A and C if the pie that is allocated to the group is greater and settle for a less remaining amount. Therefore, in profiles 3 and 6, she prefers a smaller amount of group

[12]Except for the case of Majority Rule BF Model

pie if she herself is not the delegate.

Another important feature of B's preference orderings is that she ranks A first whenever A and D are significantly strong. Specifically, $\delta_A \geq 0.7$ and $\delta_D \geq 0.8$ must hold. Although this is a situation we have encountered before, there is a difference. In previous models, in cases where $A \succ_B B$, B is close to A and D in terms of bargaining power. In other words, she wants A to be a delegate when she can get enough offer from A. However, in the current model, she prefers A's delegation when she is noticeably weaker than A.

The first thing we will notice for C is that the number of orderings in which A is put in the first place has drastically decreased. For example, in the game where we have unanimity rule BF Model with the first proposal right model, C ranks A first in three out of five profiles. In this game, we observe this in only one profile. This shows us that it is no longer very profitable for C to have a greater group pie where the first proposer is not certain, and unanimity is required. This is also evident from the fact that C for the first time places B in a higher rank than A. C, as the weakest person in the group, does not have enough strength to divide a very large pie in a way that makes everyone happy. To be more specific, in the case where allocation is decided unanimously, the continuation values of others significantly increase in the pie size, which leaves C worse off.

Equilibrium Voting Strategies for Majority Voting

Recall that $\mathcal{G}$ refers to the set of all possible voting strategy profiles for all $i \in G$ and $\mathcal{E}$ to the set of pure Nash Equilibria of voting strategy profiles. Observe that

$|\mathcal{G}| = 3^3 = 27$ and

$$\mathcal{G} = \{(A,A,A),(A,A,B),(A,B,A),(A,A,C),(A,C,A),(A,B,C),(A,C,B),\\ (A,B,B),(A,C,C),(B,B,B),(B,B,C),(B,B,A),(B,A,B),(B,C,B),\\ (B,C,C),(B,A,A),(B,A,C),(B,C,A),(C,C,C),(C,C,A),(C,C,B),\\ (C,A,C),(C,B,C),(C,B,B),(C,A,A),(C,A,B),(C,B,A)\}^{13}$$

If the preferences are as in **profile 1**, then the equilibrium voting strategy profiles are $\mathcal{E}^1 = \{(A,A,A),(B,B,B),(C,C,C),(A,A,C),(A,B,A),(B,B,C)\}$. Notice that except for (C,C,C), there is no equilibrium strategy profile for group members in which C is elected as a delegate, given that two members rank C as the worst outcome. On the other hand, electing B as a delegate can be an equilibrium strategy for the group G as well as A.

If the preferences are as in **profile 2**, then the equilibrium voting strategy profiles are $\mathcal{E}^2 = \{(A,A,A),(B,B,B),(C,C,C),(A,A,C),(A,B,B),(B,B,C)\}$. Except for (C,C,C), there is no equilibrium strategy profile for group members in which C is elected as a delegate. Given that all of them rank C as the worst outcome, this equilibrium can be expected to happen the least. Electing A as a delegate can again be an equilibrium strategy for the group G, although less likely to happen. Given it is not ranked as the worst by anyone, B is more likely to be chosen as the delegate under this preference profile.

If the preferences are as in **profile 3**, then the equilibrium voting strategy profiles are $\mathcal{E}^3 = \{(A,A,A),(B,B,B),(C,C,C),(A,B,B),(A,C,C),(B,B,C)\}$. Notice that except for (A,A,A), there is no equilibrium strategy profile for group members in which A is elected as a delegate, given that two members rank A as the worst outcome. Electing C as a delegate can be an equilibrium strategy for

[13] Among those strategies above, the followings *can never be* an equilibrium strategy profile: (A,B,C),(A,C,B),(B,A,B),(B,B,A),(B,C,B),(B,A,C),(B,C,A),(B,C,C),(C,C,A),(C,C,B), (C,B,A),(C,A,B),(C,B,B) given the preference profiles of the group members. At least one $i \in G$ has a profitable deviation.

the group G as well as B although less likely.

If the preferences are as in **profile 4**, then the equilibrium voting strategy profiles are $\mathcal{E}^4 = \{(A,A,A),(B,B,B),(C,C,C),(A,A,B),(A,A,C),(A,B,A),(B,A,A),$ $(B,B,C)\}$. Notice that except for (C,C,C), there is no equilibrium strategy profile for group members in which C is elected as a delegate, given that two members rank C as the worst outcome. Electing B as a delegate can be an equilibrium strategy profile for the group G as well as A although less likely. Since A is never ranked as the worst alternative here, we can safely expect an equilibrium in which A is chosen to realize.

If the preferences are as in **profile 5**, then the equilibrium voting strategies profiles are $\mathcal{E}^4 = \{(A,A,A),(B,B,B),(C,C,C),(A,A,B),(A,B,A),(A,C,A),(A,A,C),$ $(B,B,A),(B,B,C),(C,A,A)\}$. Notice that except for (C,C,C), there is no equilibrium strategy profile for group members in which C is elected as a delegate, given that two members rank C as the worst outcome. Electing B as a delegate can be an equilibrium strategy for the group G as well as A although less likely. Since A is the best alternative $\forall i \in G$, we can expect A to be the delegate.

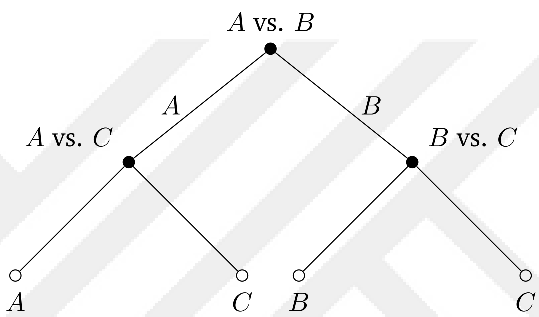
If the preferences are as in **profile 6**, then the equilibrium voting strategies are $\mathcal{E}^4 = \{(A,A,A),(B,B,B),(C,C,C),(A,C,C),(A,B,B),(C,B,C)\}$.
Notice that except for (A,A,A), there is no equilibrium strategy profile for group members in which A is elected as a delegate, given that two members rank A as the worst outcome. Electing B as a delegate can be an equilibrium strategy for the group G as well as C although less likely. Since C is never ranked as the worst alternative, C has the highest potential to be the delegate.

Notice that, in this model, the number of profiles in which A is likely to be chosen as a delegate dramatically decreases. In particular, we see the nearly opposite outcome of the majority rule counterpart of this model. There higher recognition probability right of the delegate leads the group members to consider

A's delegation more often. Whereas under the unanimity rule, members move away from choosing the strongest group member.

Equilibrium Voting Strategies for Sequential Pairwise Voting

Now we consider the sequential pairwise voting rule instead of the majority voting rule to refine our findings. We assume that no group member's strategy is weakly dominated in any subgame. Let G first vote over A and B, then over C and the surviving one. The voting tree is as follows:



First, let's consider **Profile 1**. The preferences are transitive on the triples. See that given the preference profile, the undominated strategy of each group member is to vote sincerely for the best of the two given alternatives. Hence, voting for C is eliminated, and the voting decision boils down to A vs. B. Then the sophisticated voting outcome is A since $A \succ B$.

For **Profile 2**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is again made between A and B, which results in the election of B.

For **Profile 3**, since $C \succ A$ and $B \succ C$, the sophisticated equivalent of the node on the left is C, and the other node has an equivalent of B. The final decision is made between C and B, which results in the election of B.

For **Profile 4**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is made between A and B, which results in the election of A.

For **Profile 5**, since $A \succ C$ and $B \succ C$, the sophisticated equivalent of the node on the left is A, and the other node has an equivalent of B. The final decision is made between A and B, which results in the election of A.

For **Profile 6**, since $C \succ A$ and $C \succ B$, the sophisticated equivalent of the node on the left is C, and the other node has an equivalent of C. Hence the sophisticated voting outcome is C.

As we can infer from the voting results, in this model of the game, the delegate choices of the people show significant differences according to the parameter values. This again presents evidence that electing the strongest group member as a delegate is not always the optimal action.

4.3 Comparative Statics

In this section, we will illustrate how the preference orderings of group members change as the discount factors change. Due to the intricacy inherent in our work, conducting comparative statistics for the equilibrium analytically becomes exceedingly challenging. Therefore, we provide visualizations for the changes occurring in specific parameter values instead. The determination of these parameter values is done in a manner that enables the detection of substantial variations in preference orderings.

4.3.1 Majority Rule BF Model with First Proposal Right

Recall that in the majority rule BF model with the first proposer right, A's and B's preference orderings are independent of the parameter values, whereas C has various preference orderings compatible with the payoffs. Specifically, for

C, the top-ranked alternative can be A or herself. Hence, here we examine the changes in $X_C^{CC} - X_A^{AC}$ to capture what leads C to have such preferences.

The first thing that the Figure 1 shows is as δ_A increases, the difference between the payoff that A can offer to C and the payoff C can obtain as the first proposer diminishes. In other words, as δ_A increases, we can claim that the size of the pie catches up with the first proposer advantage and then dominates.

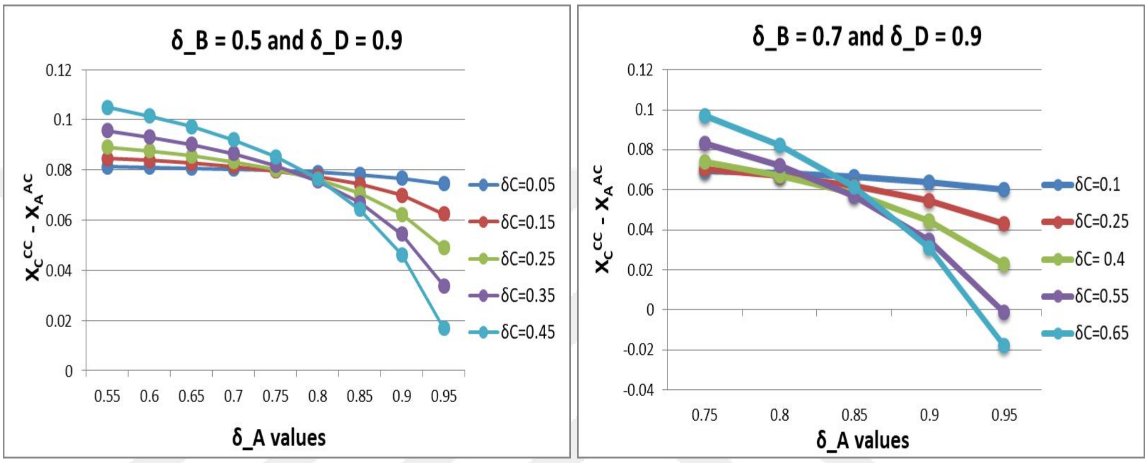


Figure 1: Effect of δ_A on $X_C^{CC} - X_A^{AC}$ for different values of δ_B

As we can see from the graph on the right-hand side, this difference even becomes negative as the value of δ_C increases. If we look at the curve where δ_C is 0.65, the difference is negative at the point where δ_A is 0.95, so A's delegation brings a higher payoff to C.

Another prominent detail is the slope of the curves given different δ_C values. We observe that, as δ_C increases, the difference between X_C^{CC} and X_A^{AC} shows more sharp changes. Figure 2 shows how this difference evolves with the changes in δ_D. A 0.05 increase in δ_D parameter (i) widens the range of $X_C^{CC} - X_A^{AC}$ for each δ_C level and (ii) and relatedly causes curves' x-intercept values to decrease.

Although not shown here, it is necessary to mention the effect of δ_B. Looking at the first figure, it can be thought that the differentiation is due to the increase in δ_B since we move from $\delta_B = 0.5$ to $\delta_B = 0.7$. But this is actually due to the fact that δ_C is allowed to take higher values. δ_B's effect on C's payoff is negligible and never changes the order of preference.

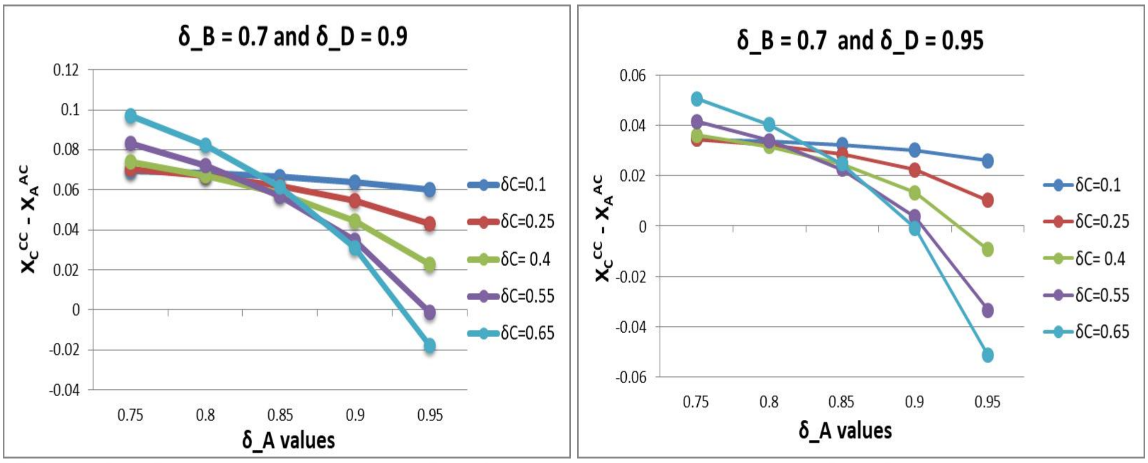


Figure 2: Effect of δ_A on $X_C^{CC} - X_A^{AC}$ for different values of δ_D

4.3.2 Majority Rule BF Model with Higher Recognition Probability Right

Recall that in the game with majority rule BF model with the higher recognition probability right, A's preference orderings are independent of the parameter values, whereas B and C have various preference orderings compatible with the payoffs. Specifically, for both B and C, the top-ranked alternative can be A or herself. Here, we examine the changes in $X_B^B - X_A^B$ to capture what leads B to have such preferences. We do not include the same visualization for C because whenever B ranks A first, C already does so.

The first thing that the Figure 3 shows is as δ_A increases, the difference between the expected payoff of B under A's delegation and her own diminishes. In other words, as δ_A increases, we can claim that the size of the pie catches up with the first proposer advantage and then dominates. As we can see from the graph on the right-hand side, this difference even becomes negative as the value of δ_B increases. If we look at the curve where δ_B is 0.6, the difference turns to be negative whenever $\delta_A \geq 0.85$, so A's delegation brings a higher payoff to B.

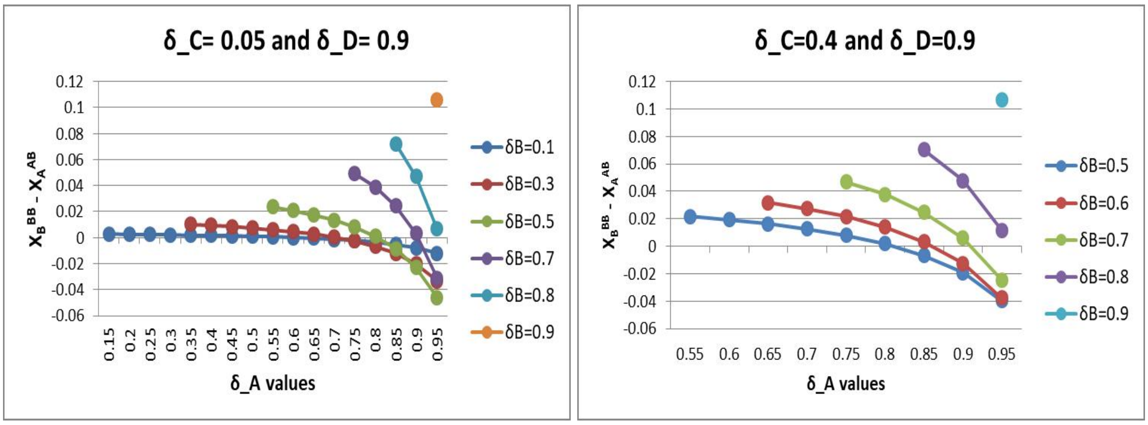


Figure 3: Effect of δ_A on $X_B^B - X_A^B$ for different values of δ_C

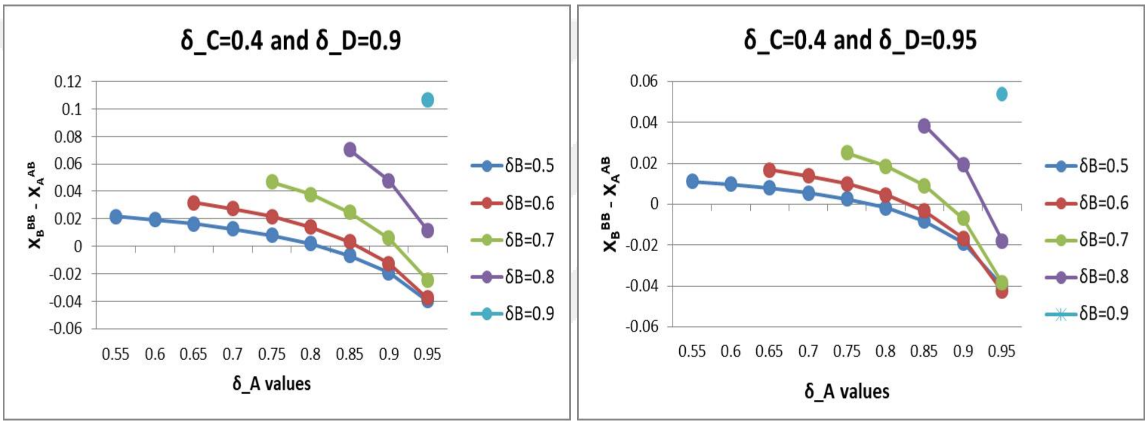


Figure 4: Effect of δ_A on $X_B^B - X_A^B$ for different values of δ_D

Figure 4, compares how the difference between X_B^B and X_A^B evolves with the changes in δ_D. A 0.05 increase in δ_D parameter (i) widens the range of $X_B^B - X_A^B$ for each δ_B level and (ii) and relatedly causes curves' x-intercept values to decrease. This leads to the conclusion that B gets more sensitive to the changes in δ_A.[14]

[14]The effect of δ_C is negligible.

4.3.3 Unanimity Rule BF Model with First Proposal Right

Recall that in the game with unanimity rule BF model with the first proposer right, A's preference orderings are independent of the parameter values, whereas B and C have various preference orderings compatible with the payoffs. Specifically, for both B and C, the top-ranked alternative can be A or herself. Here, we examine the changes in $X_B^{BB} - X_A^{AB}$ and $X_C^{CC} - X_A^{AC}$ to capture what leads B and C to have such preferences.

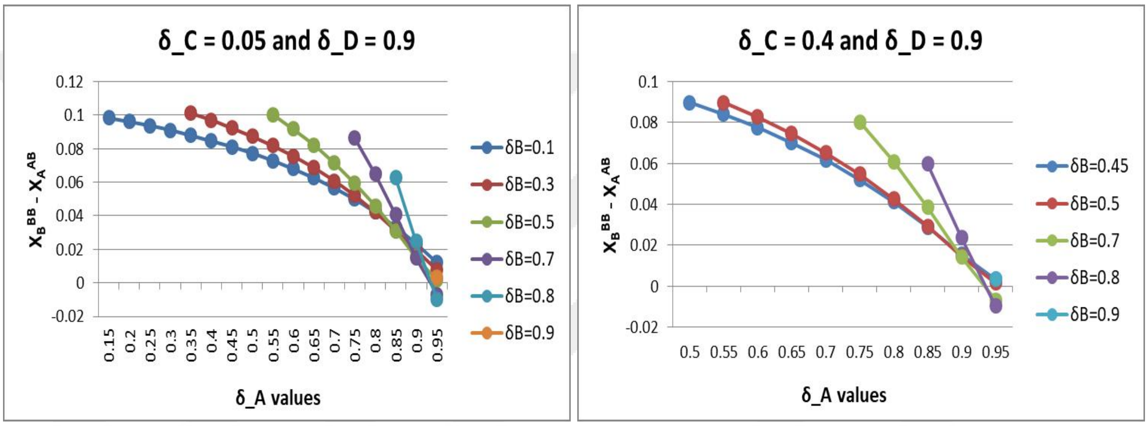


Figure 5: Effect of δ_A on $X_A^{AB} - X_B^{BB}$ for different values of δ_C

By looking at Figure 5, we observe that as δ_A increases, the difference between the expected payoff of B under A's delegation and her own diminishes. In other words, as δ_A increases, we can claim that the size of the pie catches up with the first proposer advantage and then dominates. However, also notice that $X_A^{AB} - X_B^{BB}$ obtains negative values only for higher values of B. For the given level of δ_C and δ_D parameters, we see this only when $\delta_B \geq 0.7$

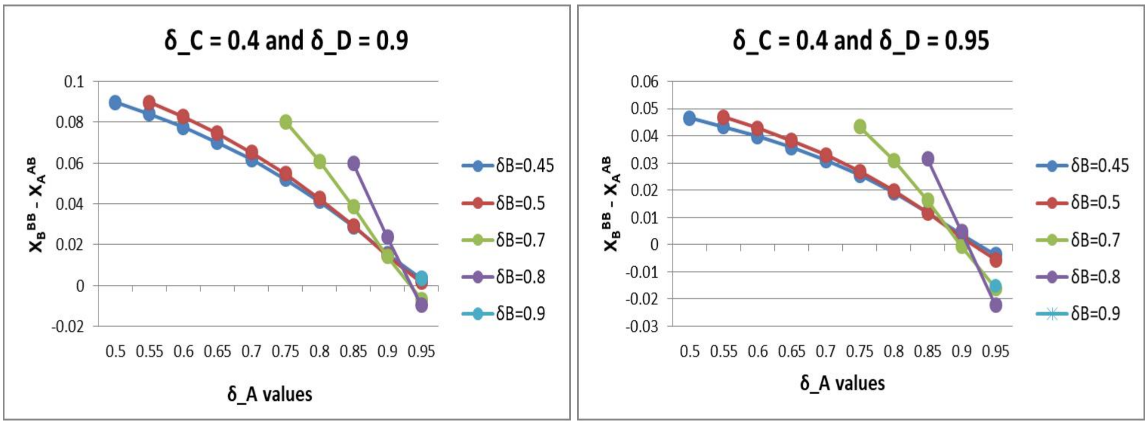


Figure 6: Effect of δ_A on $X_A^{AB} - X_B^{BB}$ for different values of δ_D

Observe from Figure 6 that the difference between X_B^{BB} and X_A^{AB} evolves with the changes in δ_D. A 0.05 increase in δ_D parameter (i) widens the range of $X_B^{BB} - X_A^{AB}$ for each δ_B level and (ii) and relatedly causes curves' x-intercept values to decrease. Since δ_D is one of the determinants of the group's pie size, an increase in δ_D causes the curves to shift downward. This leads to the conclusion that B's payoff under A's delegation increases as δ_D increases.

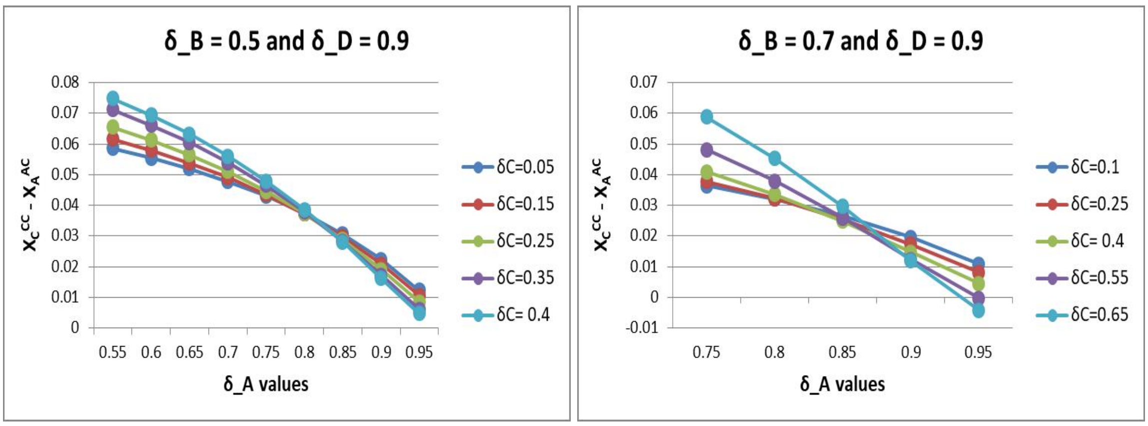


Figure 7: Effect of δ_A on $X_A^{AC} - X_C^{CC}$ for different values of δ_B

When we look at Figure 7, we see that the difference between the expected payoff of C under A's delegation and her own diminishes. In other words, as δ_A increases, we can claim that the size of the pie catches up with the first proposer

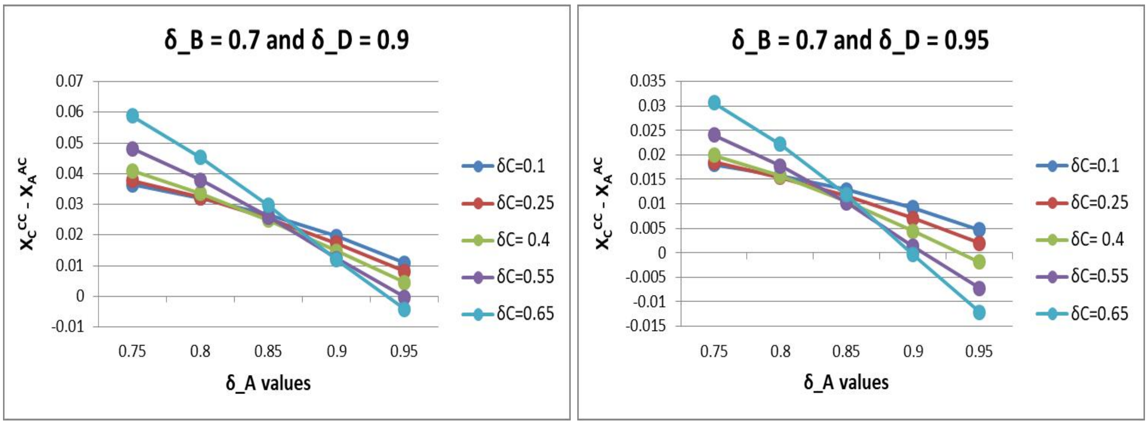


Figure 8: Effect of δ_A on $X_A^{AC} - X_C^{CC}$ for different values of δ_D

advantage and then dominates. However, also notice that $X_C^{CC} - X_A^{AC}$ obtains negative values only for higher values of C. For the given level of δ_B and δ_D parameters, we see this only when $\delta_B \geq 0.55$. Notice that this value is lower for C compared to B, which is an expected result. Another notable result is that the range of $X_C^{CC} - X_A^{AC}$ increases in each δ_C curve. That is, as δ_C increases, the slope also increases.

Figure 8, leads us to the same conclusion we draw in Figure 2, or Figure 4. the difference between X_C^{CC} and X_A^{AC} evolves with the changes in δ_D. A 0.05 increase in δ_D parameter (i) widens the range of $X_C^{CC} - X_A^{AC}$ for each δ_C level and (ii) and relatedly causes curves' x-intercept values to decrease. Since δ_D is one of the determinants of the group's pie size, an increase in δ_D causes the curves to shift downward. This leads to the conclusion that C's payoff under A's delegation increases as δ_D increases.

4.3.4 Unanimity Rule BF Model with Higher Recognition Probability Right

Unlike other games, in the game with unanimity rule multilateral bargaining model with higher recognition probability right, all group members' preference orderings are prone to change with the changes in the parameter values. Hence, we provide illustrations for each group member.

Let us first examine A's preferences. For A, regardless of the parameters, the top-ranked alternative is A. However, we observe that her second-best changes with respect to the parameters. Recall that in Profile 6, A ranks C better than B, which is not a result we obtain before. Given the frequency of Profile 6, we explore what drives $X_B^A - X_C^A$ to be negative or positive.

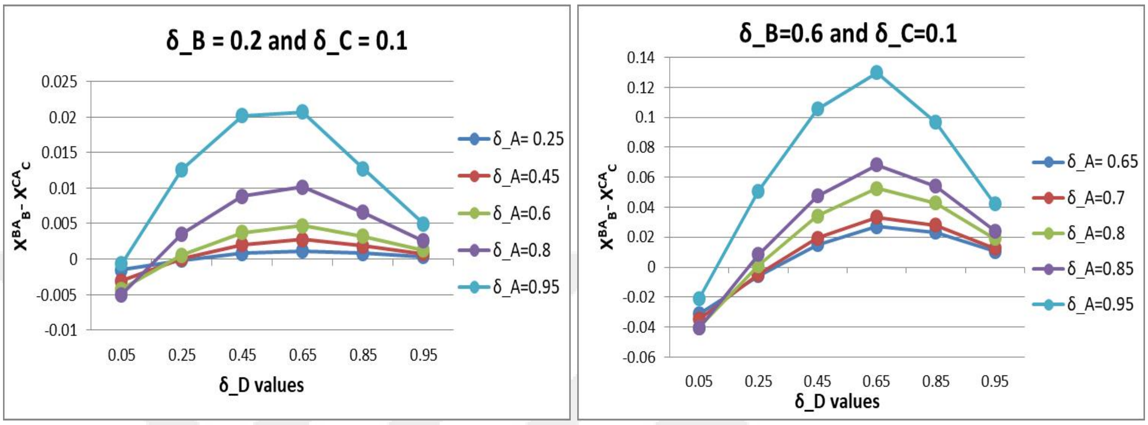


Figure 9: Effect of δ_D on $X_B^A - X_C^A$ for different values of δ_B

The first thing that we capture in Figure 9, as δ_D increases, the difference between X_B^A and X_C^A first increases and then decreases. This change is more noticeable for higher δ_A values. However, for all δ_A levels, the difference initially takes negative values, indicating that A prefers C better than B. So we can infer that, for lower values of δ_D, A prefers the weakest possible delegate if she herself would not be the delegate. When we switch to Figure 10, we observe that the shape of the curves remains the same, but their range shrinks as $\delta - C$ increases. More specifically, the lowest possible value of $X_B^A - X_C^A$ on the right-hand side is -0.04, and the maximum is 0.13, whereas, on the left-hand side, these are -0.02 and 0.06, respectively. Hence, an increase in the δ_C value brings $X_B^A - X_C^A$ closer to zero.

For B, we only investigate the changes in the top-ranked alternative. B also prefers C as her second-best alternative in some profiles, but we investigated this finding in-depth for A, so we do not investigate it independently for B. This

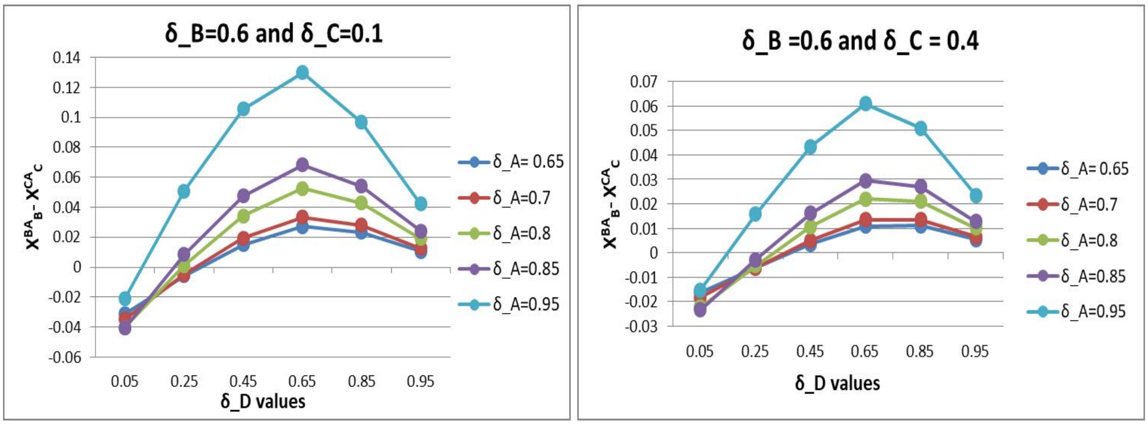


Figure 10: Effect of δ_D on $X_B^A - X_C^A$ for different values of δ_C

is due to the fact that whenever we see this shift in A's preferences, we also observe it in B's.

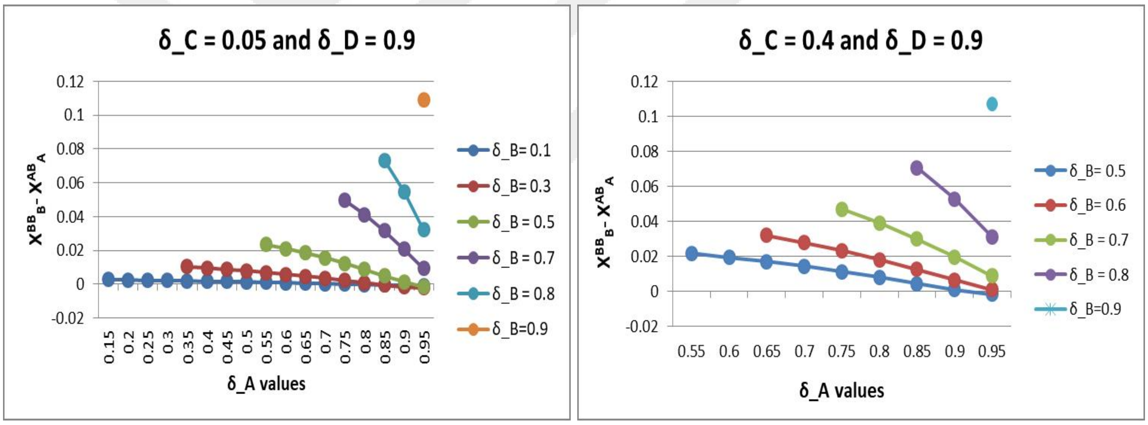


Figure 11: Effect of δ_A on $X_B^B - X_A^B$ for different values of δ_C

By looking at Figure 11, we observe that as δ_A increases, the difference between the expected payoff of B under A's delegation and her own diminishes. In other words, as δ_A increases, we can claim that the benefit of the pie size catches up with the first proposer advantage. However, also notice that $X_A^B - X_B^B$ obtains negative values only for lower values of B. For the given level of δ_C and δ_D parameters, we see this only when $\delta_B \leq 0.65$. For higher δ_B values, although this difference diminishes with the increase in δ_A, it never becomes negative, indicating that B benefits her own delegation more.

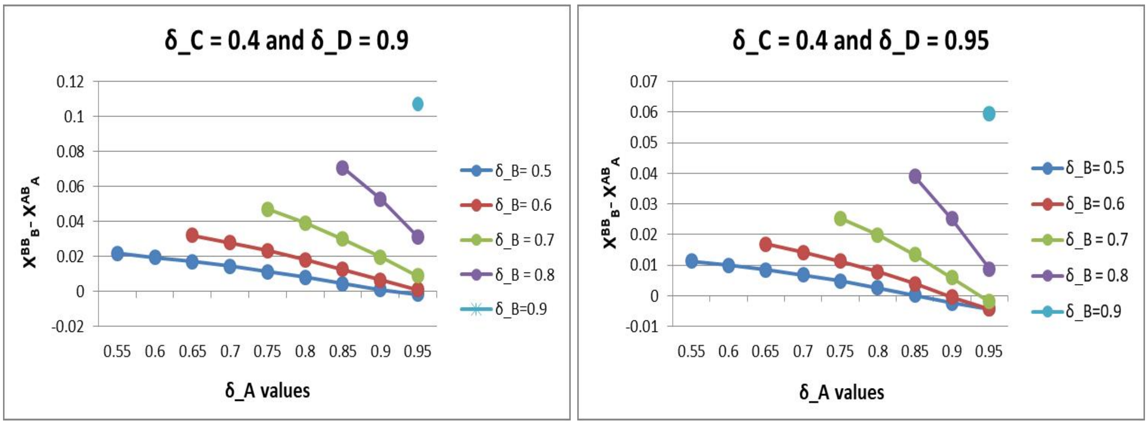


Figure 12: Effect of δ_A on $X_B^B - X_A^B$ for different values of δ_D

Figure 12, compares how the difference between X_B^B and X_A^B evolves with the changes in δ_D. A 0.05 increase in δ_D parameter (i) widens the range of $X_B^B - X_A^B$ for each δ_B level and (ii) and resultingly causes curves' x-intercept values to decrease. This leads to the conclusion that B gets more sensitive to the changes in δ_A. This is not a surprising result given that the change in δ_D is like an exogenous change in the model [15].

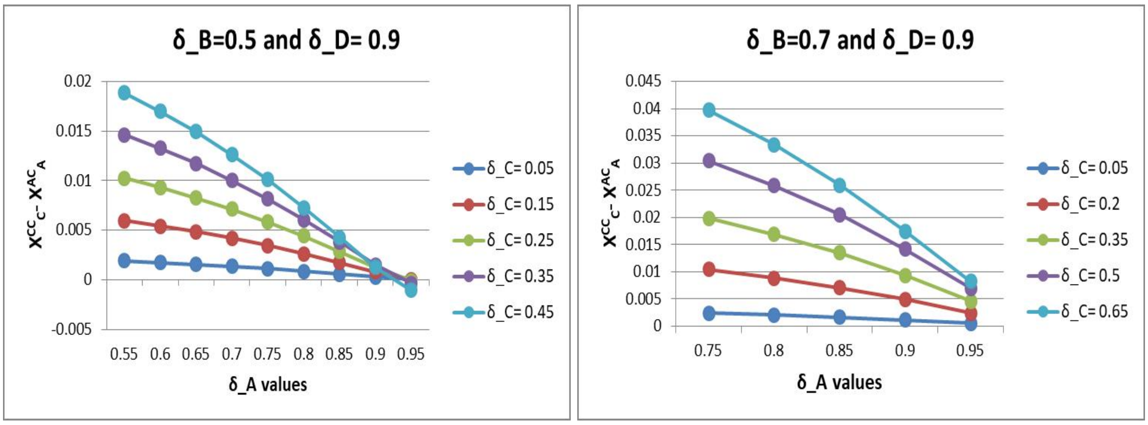


Figure 13: Effect of δ_A on $X_C^C - X_A^C$ for different values of δ_B

By looking at Figure 13, we observe that as δ_A increases, the difference between the expected payoff of C under A's delegation and her own diminishes. In other

[15] The effect of δ_C is negligible.

words, as δ_A increases, we can claim that the benefit of the pie size catches up with the first proposer advantage. Yet notice that this time δ_B also has an impact. When $\delta_B = 0.5$, at $\delta_A = 0.95$ level, we see that C prefers A's delegation when $\delta_C = 0.35$ or $\delta_C = 0.4$. However, when $\delta_B = 0.7$, we see that $X_C^C - X_A^C$ remains significantly above zero, indicating that when δ_B is higher C shifts to preferring her own delegation even when A is really strong. Another important detail to notice is that in this model, C prefers A's delegation only when she is too weak, opposite to other cases.

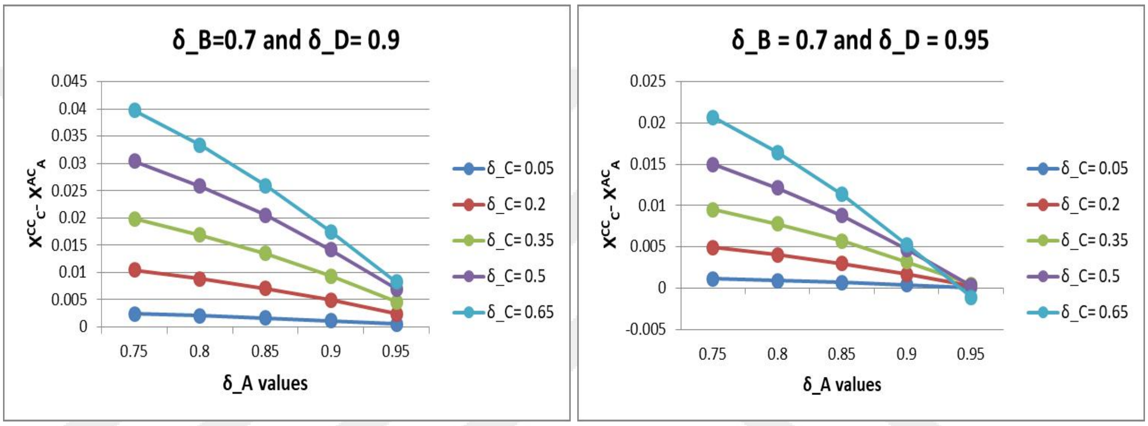


Figure 14: Effect of δ_A on $X_C^C - X_A^C$ for different values of δ_D

Figure 14 presents the results for different δ_D values. We see that when C is strong, she prefers A more. This is the exact opposite of the result we encountered in Figure 13. The increase in δ_D causes C to become relatively weaker at smaller values of δ_C and leads C to prefer herself, while for higher values of C, it leads C to prefer A. Observe that $X_C^C - X_A^C < 0$ when $\delta_C > 0.5$.

CHAPTER 5

DISCUSSION

The complexity of our findings makes it challenging for the reader to draw conclusions. We, therefore, summarize our findings in this discussion chapter.

First, let's consider the very first game we present in this thesis. The group members first elect a delegate through majority voting and sequential pairwise voting rules. The elected delegate ϕ encounters the outsider D on the bargaining table and obtains the group share. Next, this share is allocated among group members a la multilateral bargaining protocol in which the delegate has first proposal power. In this particular game, we show that *Player A* always prefers herself a delegate, as she is excluded from winning coalitions when anyone else is a delegate; *Player B*, is least likely to want *Player A* to be a delegate, as she is always outside the winning coalition when A is the delegate. *Player C*' preference orderings, on the other hand, are prone to change with the changes in the parameters. She may rank either A's delegation first or her own delegation. She prefers A over C when she has a chance against A, not when she is too "weak" because when C is too weak, even if A brings a significant amount of pie to the group, the pie is allocated even more unfairly. Given these preferences, in 2 out of 3 profiles, A is more likely to be elected given the majority rule voting rule. However, these profiles constitute only 0.06% of the parameter range. For the rest 99.4 %, we can expect the election of the weakest group member as verified by the sequential pairwise voting rule results of delegate selection.

In the second game, the only difference from the first one is the obtained share of the group is allocated among members a la multilateral bargaining protocol in which the delegate has a higher recognition probability. In this specific model, an important change occurs in B's preferences. We observe that A becomes B's top alternative in some profiles. In cases where B prefers A's delegation, the expected payoff she will receive increases a lot, as A has the potential to obtain a significant of amount share for the group. C, again, either ranks A as the first or herself, but in this game, she only cares about δ_A and δ_D values, so what causes a change in her expected payoff is others' strength. In this game, we observe that the tendency to elect the strongest group member increases. Both majority-rule voting and sequential pairwise voting equilibria verify this conclusion.

In the third game, keeping the first two stages of the game constant, we change the majority condition for acceptance of a proposal in multilateral bargaining to unanimity and offer the first proposer right to the delegate. Notice that, under the unanimity rule, all group members should be offered a positive payoff. We observe that for Player B, the delegation of A can be the best alternative, as well as her own delegation. In cases where B prefers A's delegation the most, we see that (i) both A and D are very strong and (ii) B is also strong enough to be offered a significant amount. For C, when A is significantly stronger than C, C is better off being the first proposer, whereas when their strength level is similar, C enjoys A's offer more.

In the final game, the only difference from the third one is the obtained share of the group is allocated among members a la unanimity rule bargaining protocol in which the delegate has a higher recognition probability. The first interesting result we see is that A ranks C as her second-best alternative in some profiles compared to B. This happens because the excess amount that B can provide to the group compared to C is not enough to overcome the increase in continuation values, so A ends up being worse off. We see the same result in B's orderings as well. We also observe that B ranks A as the best alternative whenever (i) A and D are very strong and (ii) B herself is weak. For C, it is no longer very profitable to have a greater group pie where the first proposer is not certain and unanimity is required. This is evident from the fact that C, for the first time, places B in a higher rank than A.

Consequently, as we move from the first proposer right to higher recognition probability in the games which have majority rule multilateral bargaining in their final stage, group members tend to rank A's delegation as the best alternative, whereas, in the unanimity rule cases, we observe the opposite. The determinacy of the first proposer affects members to preferences as well. When we keep the privilege of making the first proposal constant, as we move from majority voting to unanimity, people place A higher in their preference orderings, whereas in higher recognition probability scenarios, we observe the opposite. The indeterminacy of the first proposer and sharing the group's payoff with more members lead the group to prefer weaker delegates.

CHAPTER 6

CONCLUSION

In this thesis, we studied the delegation of decision-making authority in a multi-stage bargaining game in which at least one of the parties is a heterogeneous group in time discount factors. We first showed that there exist multiple equilibria when the delegate selection was to be made a la majority voting rule. By presenting the existence of multiple equilibria, we showed that delegating the decision-making authority to the strongest group member is not a unique equilibrium, contrary to what is claimed in the existing literature. We further showed that while the uncertainty of the first proposer in in-group bargaining leads to the request of the stronger delegate when a proposal is to be accepted by majority voting, it has the reverse impact when allocation necessitates unanimity. Later, by utilizing the sequential pairwise voting rule, we verified our findings and showed that although there are preference profiles in which the strongest group member is elected, electing weaker group members can also be a unique equilibrium outcome given the time discount factors of the players.

Although the literature on delegation in bargaining is well-developed, the number of studies focusing on delegation in group bargaining is surprisingly scarce. Existing studies in this line of the literature mostly concentrated on the effects of delegate selection on groups' aggregate payoff. Naturally, they draw the conclusion that the delegation of the strongest group member is the optimal outcome. However, how the share is divided is as important as the amount of share delivered to the group. To the best of our knowledge, this is the first study that draws attention to the consequences of electing a very powerful group member as the delegate. Due to offering a privileged position to the delegate in in-group bargaining, we can successfully capture the trade-off between greater group share and even in-group allocation.

Finally, we would like to express our ideas for future work. In this study, we focused on a complete information game, which is a strict assumption to make. Future research may elevate this assumption and explore the delegate selection decision under incomplete or asymmetric information. Incorporating beliefs into the current setup may provide more sophisticated conclusions. Another research agenda may include a social planner who values efficiency and equity. Instead of electing the delegate, the social planner may assign a group member as the delegate. Comparing the choice of the social planner with the delegate elected by the group can yield valuable insights.

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