

ESSAYS IN REVERSE LOGISTICS FOR RETAILERS

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ESSAYS IN REVERSE LOGISTICS FOR RETAILERS

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To my beloved Hakan, Kaan, and Elif Su...

ABSTRACT

As the retail industry expands and evolves, particularly with the growth of e-commerce, retailers are increasingly faced with higher levels of product returns. With this increase, it is no longer feasible to dispatch returned products to outlets or landfills; hence, retailers must re-evaluate returns in their inventories to maximize profits while minimizing the environmental impact of unsold merchandise. In the fast fashion retail sector, handling product returns has become a significant challenge due to rapidly changing consumer preferences and high product return rates. In Chapter 2, we aim to study a retailer's optimal inventory control policy under product returns to maximize expected profit. We model a period's returns to be stochastically dependent on the previous period's sales quantity. Using dynamic programming formulation, we solve for the optimal periodic review inventory policy and provide structural results on the optimal policy of the final period. Through numerical studies, we show that incorporating detailed sales-dependent returns could increase a retailer's expected profit by 23%. Ignoring this dependency in determining the optimal inventory policy results in increased order frequency, higher levels of backorders, and more leftovers, but above all could lead to a significant overestimation of the resulting profit.

In managing inventories, retailers must consider product return flows arriving from various sales periods and are obligated to track detailed sales and return data meticulously. In Chapter 3 we present an optimal inventory control policy for a retailer facing stochastic product returns over multiple periods to maximize expected profit during a single selling season consisting of finite periods. The problem is formulated using dynamic programming, and due to its computational complexity, we propose an Approximate Dynamic Programming value iteration algorithm using basis functions.

Our proposed algorithm reduces the solution time drastically without a significant sacrifice from optimality and generates solutions for instances that are not solvable exactly otherwise. We assess the significance of utilizing detailed return information and show that our proposed model enhances retailers' profits compared to a model aggregating return information. We also show the increase in profit improvement under conditions of mark-down pricing or order capacity constraints.

Retailers often employ various sales channels, concurrently managing both brick-and-mortar and online stores. Implementing an omnichannel approach aims to harmonize the customer experience across all platforms; hence, customers encounter uniform pricing, service quality, and return conditions whether they purchase in-store or online. In Chapter 4, we present the optimal solution for the finite horizon, lost sales model, considering the complex structure of the omnichannel system and the distinctive dynamics of the fast fashion retail sector. In our model, the retailer has a warehouse for online sales and a brick-and-mortar store for offline sales. Customers can purchase items online and return them to the warehouse or the brick-and-mortar store. In the case of offline sales, returns are restricted to the brick-and-mortar store, resulting in two distinct return flows with different return rates for the brick-and-mortar store. We aim to determine the optimal pre-season order quantity for the warehouse, considering the sales of both channels, along with the transshipment order quantity from the warehouse to the brick-and-mortar store during the season. We formulate the problem using dynamic programming and present a sensitivity analysis that provides valuable business insights.

This dissertation introduces an inventory control model where stochastic returns depend on previous sales. The model is enhanced by integrating returns from multiple periods and multiple channels with various extensions. Through an extensive computational study, we propose several managerial insights regarding optimal inventory policies.

ÖZETÇE

Perakende firmaları, e-ticaret satışlarının artmasıyla birlikte ürün iadelerinin artan düzeyleri ile başa çıkmak zorunda kalmışlardır. Bu artışla birlikte iade ürünleri artık outlet mağazaları veya atık sahalarına göndermek uygun olmamaktadır; bu nedenle perakendeciler, karlarını maksimize etmek ve çevresel etkileri minimize etmek için ürün iadelerini tekrar değerlendirmelidir. Bölüm 2’de, ürün iadelerinin önceki dönem satışlarına stokastik olarak bağlı olduğu orijinal ve zengin detayları olan bir stok yönetimi modeli çalışmaktayız. Optimal periyodik envanter yönetimi modelini dinamik programlama ile çözmekte, optimal kar politikasının bazı yapısal özelliklerini ve optimal envanter politikasının son periyodunu göstermekteyiz. Ürün iadelerini dikkate alan ve ürün iadelerini dikkate almadan net talep yöntemini kullanan iki firmanın karlılık karşılaştırmalarını, simülasyon kullanarak yaptığımız ön çalışmada, net talep kullanan modele göre karlılığın %23 arttığını gözlemledik. Bu artışın özellikle sabit satın alma maliyeti, ön sipariş maliyeti ve iade oranına daha hassas olduğunu gözlemledik. Ürün iadelerini dikkate almayıp gerçek talep yerine net talep kullanıldığında iade oranları arttıkça ön siparişlerin ve buna bağlı olarak da sipariş sıklığının arttığını gözlemledik. Ürün iadelerini dikkate almamak, optimal stok politikasını belirlemede artan sipariş sıklığına, daha yüksek seviyede ön siparişlere ve daha fazla atığa neden olabilir, ancak en önemlisi, elde edilen kârın önemli bir şekilde olduğundan fazla tahmin edilmesine yol açabilir.

Hızlı moda perakende sektöründe, hızla değişen tüketici tercihleri ve yüksek ürün iade oranları nedeniyle ürün iadeleri ile başa çıkmak önemli bir zorluk haline gelmiştir. Perakendeciler, envanterlerini yönetirken ürün iadelerini ve çevresel etkilerini dikkate

almalıdır. Bölüm 3'te, bir perakendecinin satış sezonu boyunca beklenen kârı maksimize etmek amacı ile çok sayıda dönemden gelen stokastik ürün iadelerinin dahil edildiği optimal bir envanter kontrol politikası sunuyoruz. Detaylı iade bilgilerinin kullanımının değerini ölçümlüyor ve önerdiğimiz modelin özellikle dönem içinde fiyat indirimi uygulandığında veya sipariş kapasitesi kısıtlamaları olduğunda perakendecilerin kârını nasıl pozitif anlamda etkilediğini gösteriyoruz. Fakat bu sefer birden fazla periyoddan gelen satış ve iade bilgisinin hesaplama karmaşıklığı nedeniyle, dinamik programlamaya alternatif olarak Yaklaşık Dinamik Programlama değer iterasyonu algoritması da önermekteyiz. Önerdiğimiz algoritma, optimal çözüm sonucundan önemli bir fedakarlık yapmadan çözüm süresini büyük ölçüde azaltmakta ve başka türlü kesin olarak çözülemeyen durumlar için bir alternatif üretmektedir.

Perakendeciler genellikle çeşitli satış kanallarını kullanır, hem fiziksel hem de çevrimiçi (online) mağazaları aynı anda yönetirler. Bu kanallar genellikle bağımsız olarak işlev görmüş olsa da, çok kanallı (omnichannel) sistem yaklaşımıyla tüm platformlarda müşteri deneyimini birleştirmeyi amaçlarlar. Bu, müşterinin mağazada veya çevrimiçi olarak satın alma yapmasına bağlı olarak, aynı fiyat, hizmet kalitesi ve iade koşullarıyla karşılaşması anlamına gelir. Hedef, müşterilere alışveriş deneyimleri boyunca tek bir firmayla etkileşimde bulunduklarından emin olarak, onlara sorunsuz bir servis sağlamaktır. Bölüm 4'te, çok kanallı sistemin karmaşık yapısını ve hızlı moda perakende sektörünün karakteristik dinamiklerini dikkate alarak, sürekli dönem, kayıp satış modeli için optimal bir çözüm sunuyoruz. Modelimizde, perakendecinin, çevrimiçi satışlar için bir deposu ve fiziki satışlar için de bir mağazası bulunmaktadır. Müşteriler, çevrimiçi satın alabilir ve ürünleri depoya veya mağazaya iade edebilirler; offline satışlar için iadeler, mağazayla sınırlıdır ve mağaza, farklı iade oranlarına sahip iki farklı iade akışıyla karşılaşır. Sezon boyunca her iki kanal için de optimal sipariş miktarını ve her dönemde deponun mağazaya sevk edilecek envanter miktarını belirlemeyi amaçlamaktayız.

Bu tez, stokastik iadelerin önceki satışlara bağlı olduğu bir envanter kontrol modelini sunmaktadır. Model, birden çok dönemden ve kanaldan gelen iadelerin entegrasyonu ile geliştirilmiş olup, çeşitli uzantıları da çalışılmıştır. Kapsamlı bir hesaplama çalışması kullanarak, optimal envanter politikaları ile ilgili bir dizi yönetimsel içgörü de önermekteyiz.



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CHAPTER I

INTRODUCTION

The retail industry, valued at nearly 24 trillion in 2019 [1] is enlarging and evolving with digitalization. Consumers are becoming more demanding and aware of quality or fashion, and their purchasing behavior is changing more quickly. To meet these evolving consumer expectations, retailers are expanding their product offerings, introducing greater model and size variability tailored to different seasons and cultural preferences. This strategic approach addresses the dynamic nature of consumer preferences in an industry shaped by digital advancements and shifting market trends.

Most retail chains sell through both offline and online stores, providing customers with a consistent experience across both channels. Customers experience smooth and easy online operations, and this experience increases sales, but the lack of "Power of Touch" experience through online stores also increases the number of returns [2]. The National Retail Federation states that \$260 billion in merchandise was returned in 2016 in the US, a 66% increase from five years ago. E-commerce returns are 20% of the sales, whereas this ratio is 10% at a brick-and-mortar store. Holiday e-commerce returns increase to 30% of the sales and even 50% of the sales with expensive products [3].

When product returns are not reintegrated into inventory during the selling season, they may stay in warehouses for extended periods, incurring potential holding costs. The issue is compounded when returns become obsolete due to falling out of fashion or missing the season, leading to landfill disposal, thereby conflicting with companies' sustainability policies [4]. Despite these negative effects, retailers often accept returns compelled by legal obligations or competitive reasons. The "No Questions Asked

Return Policy” is emerging as a novel marketing tool to increase consumer satisfaction and loyalty [2].

Accurately estimating product returns’ quantity and arrival times in inventory management is important. The timing of returns holds significance, with the endowment effect gaining importance as return periods extend. The endowment effect signifies that as customers retain products for longer durations, they develop a stronger attachment, reducing their inclination to return the purchased items (see, for example, [5]). Companies are aware of this effect and increase the returns periods accordingly. However, retailers generally prefer returns within shorter time frames to facilitate selling at full price during the season rather than resorting to discounted sales at the season’s end. Hence, if companies can meticulously track return data with the arrival times, they can benefit by efficiently re-selling these returned products within the same season.

The omnichannel concept has emerged in the rapidly advancing retail sector, particularly emphasizing its seamless integration with technology. Companies in the retail sector often leverage multiple channels, combining brick-and-mortar and online stores simultaneously. These stores are often considered to operate independently, but implementing an omnichannel approach seeks to unify the customer experience across all channels. This means that regardless of whether a customer makes a purchase in-store or online, they should encounter consistent pricing, service quality, and return conditions. KPMG survey shows that 96% of items purchased in-store are also returned in-store, while one-third of items purchased online were also returned in-store [6]. Leading fast fashion retailers such as Zara, Mango, and GAP have embraced easy return policies, allowing customers to return items to any department store, regardless of the original purchase location, including online. A simple return policy enhances customer loyalty and may increase the company’s revenue; however, retailers should manage the return uncertainty to minimize the related costs in the

omnichannel setting [7].

This dissertation presents an inventory control model in which stochastic returns depend on the most recent previous sales. The model is then refined by incorporating returns from multiple periods and channels. While the findings are relevant to various retail companies, the assumptions are particularly well-suited for fast fashion retailers specializing in affordable, stylish clothing. The fast fashion industry, characterized by short product life cycles, dynamic assortments, and high demand and supply uncertainties, is the primary context for this research ([8], [9]). Fashion products, known for their rapid turnover, also experience high return rates. Remarkably, 80% of these returns can be resold as new, with 50% requiring only new packaging for resale [10]. Retailers, therefore, can either resell these returned items or return them to the manufacturer for a refund of the wholesale price [11].

In the second chapter, we show a model where the returns depend on the previous sales, and we focus on the benefit of keeping detailed sales and return quantities. Our model has a finite horizon and includes details such as fixed ordering cost, both versions of unmet demand as a backorder and a lost sales setting, and a detailed terminal cost period. Thus, it is a relevant real-life setting for the practitioners in the sector. Using dynamic programming, we derive an optimal periodic review inventory policy for our problem and demonstrate that an (s, S) type inventory policy may no longer be optimal. We also show the structural properties of the optimal profit and inventory policy for the last period. We benchmark our model with a model that considers net demand, meaning it ignores the dependency of the return on the previous sales and aggregates the season's returns as a negative demand flow. We also show the profit improvement of using our model compared to a net demand model in the full factorial experiment.

In the third chapter, we propose a model where the returns depend on previous sales of multiple periods and focus on the benefit of keeping detailed sales and return

quantities and their respective arrival times. The model permits customers to return not only in the period after purchase but for various subsequent periods, mirroring a more realistic scenario. Employing dynamic programming, the study formulates an optimal periodic review inventory policy. However, due to the increased number of state variables associated with sales from multiple periods, the curse of dimensionality is introduced. The study adopts approximate dynamic programming (ADP) as a solution method to address this challenge. The results reveal a profit gap averaging 1% between optimal and ADP solutions, highlighting the effectiveness of ADP in significantly reducing solution time. Furthermore, the model is benchmarked against the net demand model, showing profit improvements in a one-way sensitivity analysis. The study also investigates the impact of using exact sales and return information on overall outcomes, particularly in scenarios involving order capacity and markdown pricing.

In the fourth chapter, we present the optimal solution for the lost sales model, considering the complex structure of the omnichannel system and the distinctive dynamics of the fast fashion retail sector. We introduce a retailer with a warehouse managing online sales and a brick-and-mortar store for offline sales. Customers can purchase online and return items to the warehouse or the brick-and-mortar store, while returns for offline sales are limited to the brick-and-mortar store. In this context, the brick-and-mortar store experiences two distinct return flows with different return rates. The retailer must decide on the optimal order quantity for online and offline sales for the season and determine the inventory quantity to be transferred from the warehouse to the brick-and-mortar store in each period.

The organization of the dissertation is as follows. We discuss the related literature, provide further details for each problem, and present the analysis in Chapter 2 for the inventory control model in which stochastic returns depend on the most recent previous sales, in Chapter 3 for inventory control model in which stochastic returns

depend on multiple previous sales, and finally in Chapter 4 for inventory control model in which stochastic returns depend on most recent previous sales from multiple channels. Conclusions and possible future work are summarized in Chapter 6. All the proofs are presented in Chapter 5.

1.1 Literature Review

Product returns have been studied in the literature along several lines of research such as inventory management with return flow (see, for example, [10]), the effects of return cost (see, for example, [12]), and pricing strategy under product returns with money-back guarantee policy (see, for example, [13]). We focus on the inventory management implications of product returns, which have been studied in literature since the late seventies. We review papers that use stochastic models and refer the readers to papers with deterministic models to [14] as one of the earlier works and [15] for various extensions. We will explore two streams of papers. The papers in the first stream consider the return of used products that cannot be resold unless processed, i.e., remanufactured. All returns may not be directly added to inventory upon receipt: The remanufactured return amount is a critical decision variable, especially if the new product price is sensitive to sales quantity (see, for example, [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27]). Papers in the second stream study inventory control problems under commercial returns. Before being sold as new items, the returns are added back to inventory either without further processing (e.g., merchandise, containers, blood) or sometimes after minimal processing but lead time or cost involved in such operations is generally ignored. (see, for example, [28], [29], [30], [31], [32], [33]). The models used in the first stream are similar to those in the second one when all returns are remanufactured under negligible lead time. Moreover, the modeling of dependence between returns and product sales/demand is common to both streams.

Our contribution to the literature presents a model wherein stochastic return flows depend on the sales from the previous period. In the second chapter, our model incorporates a fixed ordering cost within a finite horizon and a detailed terminal period cost structure. Expanding on these settings, we introduce another model in the third chapter, featuring stochastic returns arriving from multiple periods with distinct return rates. This extended model includes order capacity and mark-down pricing constraints. Moreover, we propose an ADP algorithm in the third chapter, significantly reducing calculation costs with an average optimality gap of only 1%. Lastly, in the fourth chapter, we put forth a comprehensive omnichannel inventory control model that accommodates various return flows from different sales channels. This intricate setting allows a store to receive return flows from both its own sales and those of another store, enriching the complexity of the model.

In each chapter (Chapter 2, 3, and 4) we delve into the literature more comprehensively, focusing on specific aspects such as multiperiod returns, multiperiod channels, capacity constraints, or ADP.

CHAPTER II

SINGLE ITEM PERIODIC REVIEW INVENTORY CONTROL WITH SALES DEPENDENT STOCHASTIC RETURN FLOWS

2.1 *Introduction*

The retail industry is gigantic and was worth nearly \$24 trillion in 2019 [1]. Large retail chains mostly run the industry since they have the cost advantage of buying huge amounts of inventory. Walmart, the largest retail chain in the world, has been top-ranked among all industries in the Fortune Global 500 list for years. As the retail industry expands and evolves with digitalization, consumers with dynamic purchasing behaviors also push for higher levels of personalization and quality. To overcome this consumer hunger, retailers increase the number of products they offer. For example, Walmart offers more than 75 million stock-keeping units for sale on its online channel [34].

Most retail chains sell through both offline and online stores, with an increasing focus on the latter, allowing customers to experience a smooth service from both channels. The Chinese company, Alibaba Group, significantly increased its online retail sales with 580 million active monthly users while European and US countries face brick-and-mortar store closures [35]. During the COVID-19 pandemic, the retail industry has even struggled with jumping online sales due to “stay at home” obligations. [36] states that Walmart’s online sales increased by 74%, lifting overall sales by nearly 9% in two months. Adobe’s Digital Economy Index for July 2020 reveals that US online sales increased 55% from last year, despite a slowdown in the growth due to reopened states [37]. According to the report, online shopping was preferred

for cheaper prices before the pandemic but now it seems that this differentiation is no longer valid and in fact, online sales are substituting offline sales.

As retail e-commerce grows rapidly, product returns are also growing. Either for legislative or competitive reasons, many companies accept returns without any reason declared under the “No Questions Asked Return Policy” [2], and this customer experience increases sales. The National Retail Federation states that \$260 billion in merchandise was returned in 2016 in the US, which is a 66% increase from five years ago. The lack of power of touch at online stores further increases the number of returns. [3] presents that a brick-and-mortar store faces returns as 10% of the sales, whereas this ratio is 20% with e-commerce returns. Holiday e-commerce returns increase to 30% of the sales and even to 50% of the sales for expensive products. Increasing return rates are also attributable to strategic customers who are abusing the return policies of retail companies and ordering more than they need to resolve fit uncertainty and then simply returning the unfit fashion items [2]. [38] reports that for catalog retailers return rates on fashion items are generally around 35–40% and could increase to 75% of the sales for some products.

Product returns are often accepted for longer periods when a defect is under the product warranty. This return process is often protected by law. Companies also may accept returns of no-defect products for a shorter declared period even if it is not always obligatory, which is the focus of this dissertation. Retailers can increase consumer satisfaction and loyalty and collect data regarding consumer behavior or product performance by allowing returns of no-defect products ([39], [40]). Retailers also have the advantage of using the funds obtained from the returned products’ revenue until the product is eventually returned and the customer is refunded, which could take a substantial amount of time [41]. However, handling product returns could be costly especially when the returns are not resold. If the returns are not added back to the inventory during the selling season, they could stay in the warehouse,

generating holding costs until they are resold or salvaged, or could go to a landfill generating obsolescence or environmental costs [4]. Hence, retailers are in a position to carefully manage their inventories to address these challenges to reap the benefits of product returns.

Inventory management under product returns requires detailed sales and returns data. Trivially, the number of product returns depends on the previous sales: as sales increase, returns in the following period are also likely to increase. When the return rates are low, the net demand approach (i.e., recording only the amount of demand less the returns) is convenient [21]. However, relevant data to measure the dependency between sales and returns is needed for high-return products. [42] shows the importance of tracking sales data and exchanging electronic sales and related information. [40] argues that offering a mobile application that tracks user purchases and returns could translate into higher profits via increased sales and returns. However, not all companies keep separate sales and return data records for practical or economic reasons; instead, they keep track of net demand.

This dissertation will present an inventory control model in which stochastic returns depend on previous sales. Retailers could leverage our model to quantify the benefits of keeping detailed sales and returns data and how to leverage this data in managing product inventories. The results of this dissertation interest many retail companies, but the assumptions of this dissertation best suit fast fashion retailers that offer stylish clothes at affordable prices. This industry is characterized by short life-cycle products, dynamic assortments, high demand, and supply uncertainties, and highly responsive supply chains ([8], [9]). In fast fashion retail, the seasons and the related assortments change rapidly such as 4-5 different seasonal assortments in a year. According to [42], 35% of the U.S. apparel market contains fashion products with only a 10-week product life. As discussed above, fashion products are also known to have high return rates. 80% of these returns can be resold as new, of which 50% need

just new packaging [10]. Hence, retailers can either resell them or send the returns back to the manufacturer for a refund of the wholesale price [11].

In the literature, many papers on inventory management consider returns of used products that are repurposed via remanufacturing or recycling (see, for example, [21], [25]). Another stream of literature focuses on commercial returns: These items can be sold as new products after minimal processing (see, for example, [32] and [33]). We contribute to the latter stream of literature by incorporating the dependency of returns on previous sales. Specifically, we assume that product returns are stochastically dependent on the previous period's sales quantity. In our model, we maximize a retailer's expected total revenue less the costs of fixed order, procurement, backorder, holding, and salvage incurred during a finite horizon.

Our main contributions to the existing literature can be summarized as follows: (1) The form of stochastic dependency of returns on the previous sales modeled in this dissertation is original and rich in detail and thus operationally more relevant for the practitioners. (2) Using dynamic programming, we derive an optimal periodic review inventory policy for our problem and demonstrate that an (s, S) type inventory policy may no longer be optimal. (3) We show some structural properties of the optimal profit and the optimal inventory policy in the last period. (4) We compare the profits of a *return-smart* retailer who manages its inventory by keeping separate records of sales and returns and following our results and a *return-naive* retailer who does not track sales explicitly and simply uses the net demand approach (i.e., considers only the product demand less the product returns). Using Monte Carlo simulation, we find that the return-smart retailer could enjoy, on average 23% more profit than the return-naive retailer. We find that this profit improvement percentage is most sensitive to the fixed order cost, backorder cost, and return rates. The return-smart retailer considers the possible incoming product returns and orders less frequently (thus saving from the fixed order cost) than the return-naive retailer. The latter

option leads to lower order-up-to levels which increase backorders as the return rates increase. (5) Finally, we provide an extension of our model with backorders to the case with lost sales.

The second chapter of this dissertation is organized further as follows. In Section 2.2, we review the related literature. In Section 2.3, we describe the details of the model and our assumptions. Analytical results of the formulated problem are discussed in Section 2.4. We exhibit the results of a numerical study along with a sensitivity analysis in the next section. We present an extension with the lost sales case in Section 2.6. Finally, we summarize the results and conclusions and give an outlook for future research in Section 2.7.

2.2 Literature Review

As we have mentioned in Chapter 1, we consider two streams of papers. The papers in the first stream consider the return of used products that cannot be resold unless processed, i.e., remanufactured. In the second stream, papers study commercial returns that are directly added back to inventory. The modeling of dependence between returns and product sales/demand is common to both streams which is at the core of this dissertation.

The first stream includes papers focusing on hybrid manufacturing, remanufacturing, and disposal processes. Demand and return distributions are mostly assumed to be independent across periods, and a net demand approach is followed. Only a handful of these papers assume that demand and returns are correlated albeit within the same period. The papers in this stream could be further grouped based on the type of inventory review.

The first group of papers studies continuous review policies. [43] aims to determine the manufacturing quantity and [44] studies an extension where excess returns are sent to a central warehouse. In his model, a remanufacturing cycle is initiated when

the inventory of returned items exceeds a certain value. [16] and [17] study settings where a customer needs a brand new product upon return of an old product that reached its end of life. [18] extends further by adding positive lead time and fixed order cost under both single and two-echelon systems. [19] revises this work by allowing returns to be added back to inventory upon receipt and show the optimality of the conventional (s, Q) inventory policy. A multi-echelon version of the problem studied in this group of papers is studied in [23].

The papers in the second group consider periodic review policies. [16] finds the optimal solution to an n -period repairable return inventory problem. [17] incorporates positive lead times to [16]’s model. [20] suggests effective heuristics to solve the models presented in the preceding two papers. [21] studies the average cost optimality of an (s, S) manufacturing policy with return flow using net demand in an infinite horizon, assuming dependence of stochastic demand and returns within the same period. [22] extends the first two papers in this group to a serial multi-echelon inventory system when remanufactured items flow into the most upstream stage. [24] extends this study with correlated demand and returns within the same period. [25] considers a setting where the return amount depends on the useful life of the products sold and the probability of returns. [26] incorporates positive lead times to [16] but assumes that unmet demand is outsourced from an external source instead of a backorder model. More recently, [27] studies a two-period model that assumes stochastic dependency of the returns and sales within the same period.

Papers in the second stream study inventory control problems under commercial returns. These returns are added back to inventory either without further processing (e.g., merchandise, containers, blood) or after minimal processing with a negligible lead time or cost. Few papers in this stream specifically focus on fashion retailers: [45] investigates optimal order quantities under a two-period newsvendor model, assuming a fixed fraction of sales returns to be sold immediately to the consumers. In a single

selling season, during which a fixed fraction of sales returns, [10] studies optimal pre-season procurement quantity and evaluates the value of using returns to fulfill seasonal demand.

Next, we discuss a subset of the papers in the second stream that do not necessarily focus on fashion retailers but those in which demand and returns are assumed to be dependent across periods as in this chapter. [28] studies a periodic review inventory system with recycling in which returns are deterministic and modeled as a fixed fraction of demand. The paper assumes lost sales in a finite horizon with no fixed order cost or lead time. [29] studies a similar model with stochastic returns. Using simulation, the authors compare different return forecasting methods based on the granularity of the return and demand data (e.g., only aggregate, past period(s) demand, past period(s) return) and find the reorder point given the return forecast. [30] employs dynamic programming to solve the similar model of the preceding work under finite horizon. [31] studies a continuous review (s, S) inventory system for rental products in which stochastic returns depend on demand. In a finite period model, [32] studies a periodic review inventory system where product returns depend on demand under positive processing lead time. [33] uses [31]’s model and compares the demand-dependent return model with an independent one using a queuing system formulation and investigates the optimality of base-stock policies.

Table 1 provides a taxonomy of the most relevant papers to this work. First, we group papers based on the type of returns studied: used product returns (U) and commercial returns (C). In the second column, we group the papers based on the studied inventory control method, e.g., periodic review (P) and continuous review (C). The next column specifies the return dependency on demand under five clusters: demand-dependent within the same period (DWP), demand-dependent across periods (DAP), sales-dependent within the same period (SWP), and sales-dependent across

periods (SAP). In the fourth column, we indicate the employed solution methodology: dynamic programming (DP), Markov Chain modeling (MC), simulation (S), or heuristics (H). We also label the unmet demand assumption as backorder (BO), lost sales (LS), or an instantaneous outsourced supply (NA) in column five. The next one lists whether the model is created under a finite (F) or an infinite horizon (IF). Columns seven and eight denote papers with fixed ordering costs or positive processing lead times, respectively, with an asterisk. The last two columns specify the distributions used to model demand and return in the main model and the computational study in parenthesis.

Table 1: Detailed comparison of closely related papers

	Return Type	Review Type	Return Dependency	Solution Method	Unmet Demand	Horizon	Fixed Cost	Lead Time	Demand Distribution	Return Distribution
Simpson (1978)	U	P	DWP	DP	BO	F			Generic	Generic
Kelle and Silver (1989)	C	P	DAP	S	NA	F		*	Normal	Binomial
Inderfurth (1997)	U	P	DWP	DP	BO	F		*	Generic	Generic
Yuan and Cheung (1998)	C	C	DAP	MC/H	BO	IF	*		Poisson	Exponential
Buchanan and Abad (1998)	C	P	DAP	DP	BO	F			Generic (Exp.)	Generic (Uni.)
Kiesmüller and van der Laan (2001)	C	P	DAP	MC	BO	F		*	Poisson	Poisson
Fleischmann and Kuik (2003)	U	P	DWP	MC	BO	IF	*	*	Generic (Poi.)	Generic (Poi.)
De croix (2006)	U	P	DWP	DP	BO	F		*	Generic	Generic
Mitra (2013)	C	P	DWP	S	BO	IF	*		Normal	Normal
Benedito and Corominas (2013)	U	P	DAP	MC/H	NA	IF			Generic	Generic (Bin.)
Zerhouni et al. (2013)	C	C	DAP	MC/H	LS	IF			Poisson	Exponential
Calmon and Graves (2017)	U	P	DWP	S/H	NA	F		*	Generic	Generic
Fu et al. (2019)	U	P	SWP	DP	LS	F			Generic (Uni.)	Generic (Uni.)
Our Model	C	P	SAP	DP	BO/LS	F	*		Generic (Poi.)	Binomial

Our most significant contribution to the existing literature is to present and study a model with stochastic return flows that depend on the actual sales in the previous period. This is a critical extension, as the retailer’s ordering decisions now indirectly affect the return quantity in the following periods. We also incorporate fixed ordering cost in a finite horizon setting, with a detailed terminal period cost structure. All these modeling choices lead to a rich model setting, which translates into practically relevant conclusions about managing inventories under return flows.

2.3 *Model Formulation*

We consider a fast-fashion retailer's finite horizon periodic review inventory control problem where the product returns depend on the previous period's sales quantity. We assume that the retailer has a single store and focuses on a single item that can be returned within one period upon purchase without any restrictions. The returned product is assumed to be in almost perfect condition and is directly put on the shelf for resale as a new product upon arrival to the store after minor processing which is assumed to have negligible cost and takes negligible time.

The retailer reviews its net inventory at the beginning of each period t , I_t and gives a replenishment order, O_t , if necessary. For example, [46] discusses that Zara, one of the biggest fast-fashion retailers, replenishes product inventories on a fixed schedule (i.e., periodic-review inventory policy). We assume that the order is received after a negligible replenishment lead time. On the manufacturing side, the production and distribution to warehouses must be completed before the selling season. Since assortments are "floor-ready" to deliver, with the proper labels, price tags, and hangers, there is no production lead time and delivery lead time is negligible [42]. Next, product returns, R_t , are accepted, and demand (D_t) and resulting sales (SQ_t) are realized. In this dissertation, we assume that any unsatisfied demand is backordered without a fulfillment time limit during the season. With the increasing use of e-commerce channels along with retail stores, unsatisfied orders are more conveniently backordered as these orders could be taken online and then delivered to the customer ([47], [48]). At the end of the period, holding or backorder costs are incurred. We assume that the selling season for this product ends after T periods. After the end of the selling season, either the final shortage is satisfied via final procurement or the remaining inventory is salvaged. A list of notations is provided in Table 2.

We assume that the demand across periods, D_t , is independent, identically distributed, and a discrete random variable with probability mass function $f(x)$ and

Table 2: List of notations used in this chapter

Parameters	
c	Unit procurement cost.
K	Fixed ordering cost.
h	Holding cost per unit per period.
b	Backorder cost per unit per period.
r	Return credit per period.
s	Salvage value per unit.
p	Retail price per unit.
α	Return probability
T	Number of periods in the selling season.
Variables	
I_t	Net inventory at the beginning of period t .
O_t	Replenishment order in period t .
D_t	Demand realized during period t .
SQ_t	Sales realized during period t .
R_t	Returns received during period t .
S_t	Inventory level just after the replenishment order but before observing demand and returns in period t .

mean μ . Customers pay a stationary unit price of p to purchase the product. Each sold product in period t is assumed to be returned in period $t + 1$ with probability α . The return period for a backordered item starts with the fulfillment of this order. We assume that return probability is independent of the sales period, but our model can be extended to incorporate nonidentical return probabilities across sales periods. Given the previous period sales SQ_{t-1} , one could deduce that; R_t , follows a binomial distribution with parameters α and SQ_{t-1} . Immediately upon receiving a return, the retailer must pay the return credit, r , back to the customer. Extending our model to incorporate nonzero unit return processing cost is trivial by simply increasing the value of r accordingly.

The retailer begins the initial period with zero inventory. Then the total cost of ordering in any period has two parts: unit procurement cost, c , and fixed ordering cost, K . Thus, the total cost of ordering O_t units is denoted by $\bar{c}$ and can be stated

as follows:

$$\bar{c}(O_t) = \begin{cases} K + cO_t & \text{if } O_t > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Any excess inventory at the end of a period incurs a unit holding cost, h . Similarly, any shortage at the end of a period incurs unit backorder cost, b .

We formulate the retailer's multi-period inventory control problem using dynamic programming. The state variable at each period t has two components: the net inventory at the beginning of this period (I_t) and the sales quantity in the previous period (SQ_{t-1}). We aim to find the optimal net inventory after replenishment in each period, $S_t := I_t + O_t$, to maximize the expected profit. Using the principle of optimality, one could write the backward recursive formulation for the expected total profit-to-go function $P_t(I_t, SQ_{t-1})$ as follows:

$$P_t(I_t, SQ_{t-1}) = \max_{S_t \geq I_t} \{ p\mathbb{E}_{D_t, R_t}[SQ_t] - \bar{c}(S_t - I_t) - r\mathbb{E}[R_t] - J(S_t, SQ_{t-1}) + \mathbb{E}_{D_t, R_t}[P_{t+1}(I_{t+1}, SQ_t)] \} \quad (2)$$

where the first term is the expected revenue from sales, the next two terms are the ordering costs and return credits, and the fourth term is the current period's total expected backorder and holding costs, which could be written as:

$$J(S_t, SQ_{t-1}) = h\mathbb{E}_{D_t, R_t}[(S_t + R_t - D_t)^+] + b\mathbb{E}_{D_t, R_t}[(D_t - S_t - R_t)^+]. \quad (3)$$

In this chapter, we use the notations $x^+ := \max\{0, x\}$ and $x^- := \max\{0, -x\}$. The net inventory cannot be salvaged or destroyed during the selling season. Hence, the only constraint on the admissible inventory policies is that $S_t \geq I_t$: the net inventory after replenishment cannot be less than the beginning net inventory. Both parts of the state variable at period $t + 1$, I_{t+1} , and SQ_t , depend on demand and return of period t . Hence, the expectation before the next period's profit-to-go function in equation (2) is over D_t and R_t . Finally, we assume that there is no discounting as the selling season is comparatively short for many retail products.

The transition of the state variable can be explained in two parts. Given the sequence of events, the net inventory at the beginning of period $t + 1$, I_{t+1} , can be simply written as $I_{t+1} = I_t + O_t + R_t - D_t = S_t + R_t - D_t$. Next, the sales quantity at time t can be written as:

$$\begin{aligned} SQ_t &= \min\{I_t^+ + O_t + R_t, D_t + I_t^-\} = \min\{I_t^+ + S_t - I_t + R_t, D_t + I_t^-\} \\ &= \min\{S_t + R_t, D_t\} + I_t^-, \end{aligned} \quad (4)$$

where the first equality follows from the definition of O_t and the second equality follows from the fact that $x = x^+ - x^-$ for any $x \in \mathbb{R}$. Equation (4) formulates that the sales quantity of period t is the minimum of the inventory on hand plus the replenishment order plus the returns received and the current period's demand plus the backordered demand quantity from the previous period (i.e., the negative part of net inventory).

At the end of the selling season, which is denoted by period $T + 1$, any excess inventory is salvaged at a unit profit of s , and any shortage is purchased and delivered to the customers. During end-of-season operations, inventories of all product types are reviewed together, final orders are placed for backordered items, and unsold merchandise is sent back to the warehouse. Thus, a fixed ordering cost would be incurred independent of this particular product. We ignore this constant term in our objective function (i.e., fixed cost for this last order is excluded). This assumption is consistent with the vast literature on inventory management (see, for example, [49], [50], [51], [52], [53]). We account for any returns of the satisfied backorder demand after the end of the season by including the return credit and the salvage profit due to these returns. Thus, the terminal expected profit could be written as follows:

$$\begin{aligned} P_{T+1}(I_{T+1}, SQ_T) &= p(I_{T+1})^- + s\mathbb{E}_{R_{T+1}}[(I_{T+1} + R_{T+1})^+] \\ &\quad - c\mathbb{E}_{R_{T+1}}[(I_{T+1} + R_{T+1})^-] - r\mathbb{E}[R_{T+1}] + \alpha(s - r)(I_{T+1})^- \end{aligned} \quad (5)$$

In this equation, each term accounts for the following: profit from the backordered items, salvage from the leftover inventory, final procurement cost to fulfill backordered

demand, and the return credit. The last term accounts for the products sold and then returned at the end of the selling season.

2.4 *Structure of the Optimal Profit and Inventory Policy*

This section presents some analytical results based on the formulated problem. First, we provide an alternative cost-centric formulation, showing that these two formulations are equivalent. Next, we provide some monotonicity properties of the profit-to-go function. Finally, the structure of the optimal inventory policy is analyzed utilizing the alternative formulation.

We choose a profit-centric approach in this dissertation. Alternatively, one could follow a more traditional cost-centric formulation and study the optimal inventory policy of the retailer that minimizes total expected cost. Specifically, one could use a dynamic programming formulation using cost-to-go and terminal cost functions as stated below:

$$C_t(I_t, SQ_{t-1}) = \min_{S_t \geq I_t} \{ \bar{c}(S_t - I_t) + r\mathbb{E}[R_t] + J(S_t, SQ_{t-1}) \\ + \mathbb{E}_{D_t, R_t}[C_{t+1}(I_{t+1}, SQ_t)] \}, \quad (6)$$

$$C_{T+1}(I_{T+1}, SQ_T) = c\mathbb{E}_{R_{T+1}}[(I_{T+1} + R_{T+1})^-] - s\mathbb{E}_{R_{T+1}}[(I_{T+1} + R_{T+1})^+] \\ + r\mathbb{E}[R_{T+1}] + \alpha(r - s)(I_{T+1})^- \quad (7)$$

Next, we show that these two formulations are basically equivalent.

Given any state variable pairs I_t and SQ_{t-1} the profit-to-go and cost-to-go functions of the two formulations stated in equations (2, 5) and (6, 7) have the following relationship:

$$P_t(I_t, SQ_{t-1}) = p(\mu(T - t + 1) + I_t^-) - C_t(I_t, SQ_{t-1}), \quad \forall 1 \leq t \leq T. \quad (8)$$

All the proofs are relegated to the appendix. Proposition 1 implies that for each period t , the profit-to-go function is equal to total revenue from the expected demand during the remainder of the selling season and any backordered demand from previous

periods less the cost-to-go function for all values of the net inventory and previous period sales quantity. Hence, the optimal ordering decisions are exactly the same under both formulations for any state variable pairs I_t and SQ_{t-1} . In particular, one can easily deduce that $P_1(0,0) = p\mu T - C_1(0,0)$. This implies that, due to our assumption of backordered unmet demand, one could easily find the optimal expected profit by subtracting the optimal expected total cost from the revenue of expected total demand during the selling season.

One can show that the optimal profit-to-go function is monotone in the return probability.

Assume that $r \geq c \geq s$ and $r \geq b$. Given any state variables I_t and SQ_{t-1} , the profit-to-go function is decreasing in the return probability.

As expected, the profit decreases as customers are more likely to return products. Proposition 2 requires some parametric assumptions, which eliminates trivial cases: The return credit is more than the product cost, which is larger than the salvage value. Similarly, the per unit and period backorder cost is lower than the return credit. Following the same logic in the proof, one can trivially show that the profit also decreases in all the cost parameters (c, K, h, b) and increases in the salvage value.

We now present analytical results on the structural properties of the optimal inventory policy. In the rest of this section, we assume that there is no fixed ordering cost, and there are only two periods: one selling period and one terminal period.

The next proposition provides structural results on the optimal inventory policy for the last period: Let $K = 0$ and $c \geq s$. Then the following statements hold:

- i) $C_T(I_T, SQ_{T-1})$ is submodular on $(I_T, -SQ_{T-1})$.
- ii) Let $S_T^*(I_T, SQ_{T-1})$ be the set of optimal order-up-to levels. Then $S_T^*(.)$ is non-decreasing in I_T and nonincreasing in SQ_{T-1} .

Using Proposition 1 and Proposition 2, it is trivial to show that the following

corollary is true:

Let $K = 0$ and $c \geq s$. Then the following statements hold:

- i) $P_T(I_T, SQ_{T-1})$ is supermodular on $(I_T, -SQ_{T-1})$.
- ii) Let $S_T^*(I_T, SQ_{T-1})$ be the set of optimal order-up-to levels in the profit-centric formulation. Then $S_T^*(.)$ is nondecreasing in I_T and nonincreasing in SQ_{T-1} .

Corollary 1 shows that the optimal net inventory after replenishment is non-decreasing in the net beginning inventory and non-increasing in the sales quantity of the previous period. The former result is expected given the zero fixed ordering cost assumption and indestructibility of net inventory. The latter result also holds since a higher level of sales quantity in the previous period implies a higher number of possible returns in the current period. This structural result is quite substantial: One could utilize this result to find the optimal inventory decisions more quickly by narrowing down the search space using the optimal decisions for adjacent values of the state variables.

The structural results hold under two assumptions. Nonzero fixed ordering cost could lead the cost-to-go function to be non-submodular. The assumption of $c \geq s$ is satisfied for cases worth studying: Otherwise, it is optimal to procure a very large quantity in the last period and salvage them at the end of the selling season for profit. Unfortunately, we could not extend the results to other periods in the selling season, which requires convexity of the cost-to-go function which may not hold for many cases. Hence we resort to numerical experiments to analyse the structure of the optimal policy for interim periods under nonzero fixed ordering cost.

2.5 Computational Results

In this section, we present the results of an extensive numerical study to better understand and quantify the results of our proposed model. First, we study the

structure of the optimal inventory control policy using the model proposed in Section 2.3. Then, we discuss the value of incorporating detailed return flow into the retailer’s inventory control problem and how the problem parameters affect this value.

We refer to the retailer that uses our proposed model in managing her inventory as the “*return-smart*” retailer who keeps detailed records of sales and returns and considers item returns as dependent on the *previous period sales*. As a benchmark, we also consider an alternative retailer who does not track previous period sales explicitly, *ignores the dependency of the return flow on sales* and instead simply uses a net demand approach (i.e., considers only the product demand less the returns). For this “*return-naive*” retailer, it is known that the optimal inventory control policy is a traditional (s, S) policy where an order is given to increase net inventory to S only if the net inventory is less than or equal to s (see, for example, [49]).

In this computational study, we assume that the demand for each period follows Poisson distribution with mean λ_D . The return-smart retailer considers that each sold item could be returned in the next period with probability α . On the other hand, the return-naive retailer simply uses the net demand in managing the inventory. In our case, one can show that the net demand follows Poisson distribution with mean $(1 - \alpha)\lambda_D$.

For the rest of this chapter, we consider a particular parameter set, called the base case scenario, to study the differences between the inventory control policies and profits of return-smart and return-naive retailers. The parameters are set following [23] which also considers an inventory system with product returns. Since retail price is not used in [23], we set the value for this parameter as twice the procurement cost, implying 50% unit profit margin which is common for retailers. The mean of the Poisson distribution is taken as $\lambda_D = 4$ and $\alpha = 0.50$, hence the return-naive retailer uses the net demand with mean $(1 - \alpha)\lambda_D = 2$. Since the support of the Poisson distribution is unbounded, we truncate the demand from a level such that

the cumulative probability exceeds 0.9972. We assume that each period lasts a month and that there are 4 periods in a season ($T = 4$). Inventory holding cost rate is taken as 5% per unit per month, $b = \$20$ per unit per month, $K = \$25$ per order, $c = \$40$ per unit, and $p = \$80$ per unit. We also assume that there is no salvage value at the end of the period ($s = 0$) and the customer is given full credit for returns and any related processing cost is ignored ($r = p$). In section 5.3, we provide an analysis on the sensitivity of our results on this base case scenario.

2.5.1 Analysis of the Optimal Inventory Control Policy

We calculate the optimal inventory decisions for the return-smart retailer using our proposed model and also for the return-naive retailer following [49]. The calculation for the return-smart retailer follows a backwards solution approach: We solve for the optimal inventory decision of the last period, keep the optimal values in a matrix, then solve for the previous period. Since the demand follows Poisson distribution we evaluate only the integer valued decisions. This procedure continues in a recursive fashion. For each period and each state, we calculate the value of the objective function for each possible inventory level after ordering (between the lower bound of current net inventory and the upper bound of remaining periods times the maximum demand per period) and record the one that yields the best value (i.e., exhaustive search).

For the base case scenario, the optimal inventory decisions in period 2 for both types of retailers are presented in Figure 1 given that the previous period sales are equal to zero. Let's first compare the form of the optimal policies. As expected, the return-naive retailer follows an (s, S) policy where she places an order only if the net inventory is below or equal to 1 (i.e., re-order point $s = 1$) and increases the inventory to 4 (i.e., order-up-to level $S = 4$). On the other hand, the return-smart retailer does not follow an (s, S) type inventory policy. She places an order

only if the inventory level is below or equal to 4, however, the order-up-to level is not constant for all values of the net inventory. When the net inventory is below zero (i.e., there is positive backorder from previous periods), the order-up-to level decreases as the number of backorders increases. This could be seen as counter-intuitive at first sight as the retailer could be expected to order more especially when the backordered demand is high. However, the return-smart retailer knows that each fulfilled backordered demand is going to turn into sales in the current period which then could be returned with probability α in the next period. Thus she reduces the order size taking into account the higher number of possible returns from these fulfilled backordered demand. Using the same logic, the return-smart retailer keeps the order-up-to level constant when there is no backordered demand as each product in the inventory has to be first sold and then returned, a lower probability event, in order to be used in fulfilling demand in future periods.

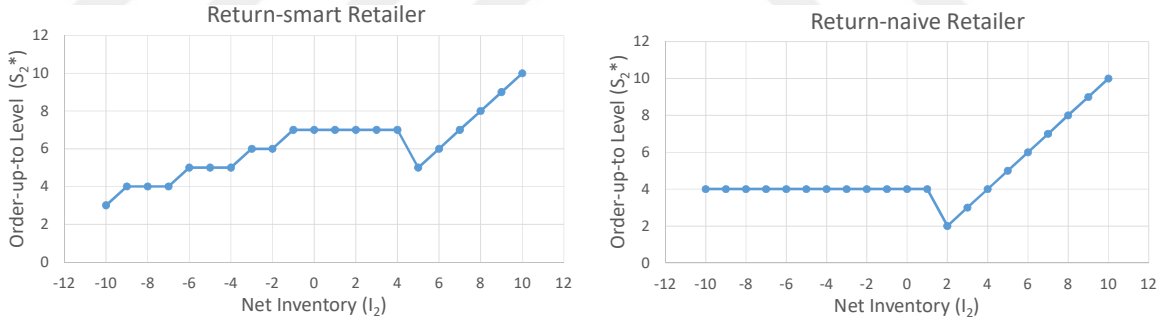


Figure 1: The optimal order-up-to levels of both retailers for the base case scenario at period 2 ($SQ_1 = 0$)

Comparing the inventory levels below which an inventory order is given, we observe that the return-smart retailer places an order whenever the net inventory is at or below 4 which is much smaller for the corresponding threshold with the return-naive retailer (i.e., 1). This is because the return-smart retailer expects a higher demand rate of 4 assuming no returns will arrive in period 2 as $SQ_1 = 0$. However, the return-naive retailer ignores the sales-dependency of the returns and expects a lower net demand

rate of $0.5 * 4 = 2$. This phenomenon also results in the return-naive retailer ordering less (4) than the return-smart retailer (7) when she orders and there is no backorder. Thus we expect the return-naive retailer to have higher backorder costs and higher fixed order cost due to increased order frequency to compensate for the increased backorders. We also find similar observations for the order level in the first period: The return-smart retailer orders 8 units whereas the return-naive retailer orders only 6 units. Later on, whenever the inventory decreases below the re-order point, the return-naive retailer always orders less compared to the return-smart retailer.

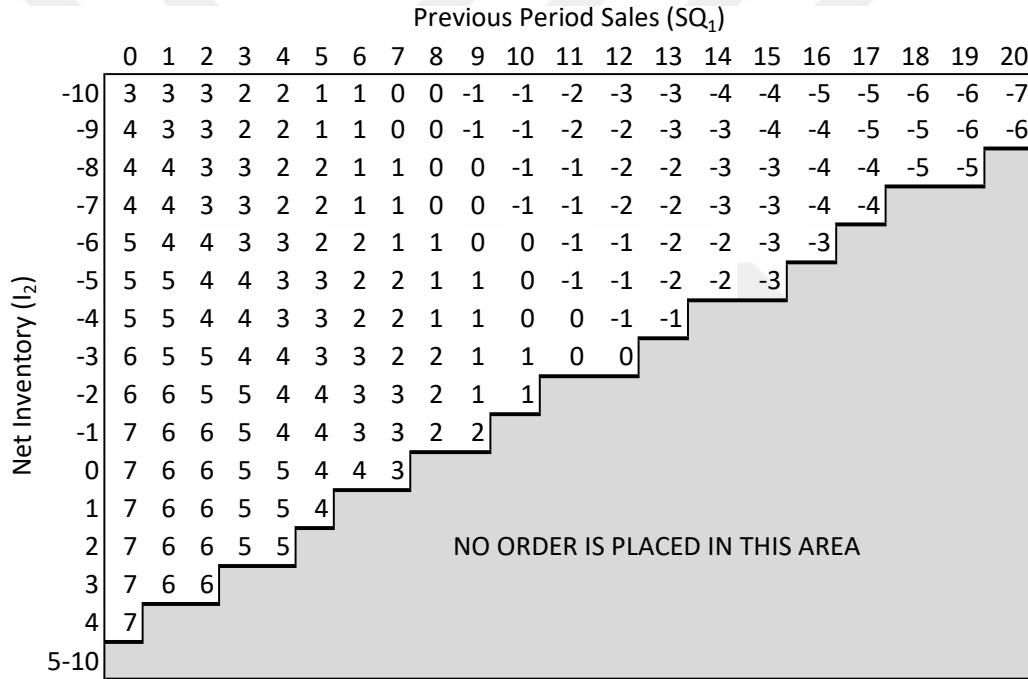


Figure 2: The optimal order-up-to levels of the return-smart retailer for the base case scenario at period 2 as the previous period sales quantity varies

Given the effect of net inventory level on inventory decisions, next we investigate the effect of previous period sales. By definition, the return-naive retailer does not use this information. For the return-smart retailer, Figure 2 shows the optimal inventory levels after ordering in period 2 as a function of the net inventory level before ordering and the previous period sales. Notice that the values in the first column of Figure 2

are also presented in the left part of Figure 1 with $SQ_1 = 0$. Figure 2 presents optimal order-up-to levels for all values of SQ_1 . In the grey shaded area (bottom right) no replenishment order is given. In the white area, the return-smart retailer places a positive replenishment order. For a fixed net inventory level, the order quantity decreases monotonically in the previous period sales. The amount of returns in the current period increases stochastically in the previous sales quantity; thus the retailer orders less expecting a larger number of returns.

2.5.2 Value of Using Sales-Dependent Returns

In this section, we quantify the value of considering sales-dependent returns in making inventory decisions. Specifically, we calculate the improvement in the profit of the return-smart retailer versus the return-naïve one. Given the optimal inventory decisions of both retailers as discussed in the previous subsection, we evaluate the expected profit of both retailers using the same Monte-Carlo simulation. The Monte-Carlo simulation generates a random demand value for each period in the horizon and using each retailer's inventory policy, evaluates the resulting average total profit across simulation runs. Clearly, the expected profit of the return-smart retailer is asymptotically the same if one uses simulation instead of the dynamic programming model of Section 3. Our dynamic programming formulation could be used for the return-naïve retailer as well, however we specifically employ simulation to be able to calculate the components of the total profit and better understand the underlying factors for differences in the profits. To minimize any variation due to the generated random numbers, we use identical random demand values while running simulations. Finally, we compare the evaluated total profit of the return-smart retailer for the whole season to that of the return-naïve one, assuming zero initial inventory. In this subsection, we use the base case scenario parameters and conduct a one-way sensitivity analysis in the next subsection.

In our Monte Carlo simulation, we determine to use 100,000 samples. After 10 replications, we find that the standard deviation of the return-smart retailer is \$0.37($CV = 0.0029$), supporting our choice of the number of samples. Even though both retailers face the same demand realization (i.e., same random numbers), they order according to their separate inventory control policies. Thus they may have different sales and return quantities in each period and eventually different total profits from each other.

For the base case scenario, Table 3 provides the total expected profits of both retailers as well as their cost breakdowns. It has four sections: The first section shows a breakdown of the total cost during the selling season into procurement, ordering, holding, and backorder costs. The next section presents the costs incurred after the selling season ends that include the final purchasing cost of remaining backorders, the return credits for these fulfilled backorders, the salvage value of on-hand inventory, if any; after the remaining backorders are fulfilled and the sales revenue of the fulfilled backorders. The third section displays the net revenue after the returns are credited from the total revenue during the selling season. Finally, in the last section, we deduct both total season and end-of-season costs from the net revenue to find the total profit of each retailer.

Table 3: Total profit breakdowns for both retailer types using base case scenario

Retailers	Procurement Cost B1	Ordering Cost B2	Holding Cost B3	Backorder Cost B4	Total Season Cost B=B1+B2+B3+B4	End-of-Season Cost C	Season Revenue A1	Return Credit A2	Season Net Revenue A=A1-A2	Total Profit A-B-C
Return-naive	398.69	50.39	12.77	50.69	512.54	113.14	1,180.48	452.22	728.26	102.59
Return-smart	380.23	36.57	17.13	46.88	480.80	97.96	1,161.86	456.58	705.28	126.52

Table 3 shows that the return-smart retailer has a significant 23% increase in profit compared to the return-naive one. The latter retailer has higher costs in all categories except the holding cost. The underlying reason for this observation stems

from the optimal inventory decisions studied in the previous subsection: The return-naive retailer never considers the incoming returns and orders more frequently in smaller quantities. The total order quantity during the season is also slightly (5%) higher. However, the timing of these orders is not perfect: The return-naive retailer faces more backorders leading to slightly a lower product fill rate (87%) compared to the return-smart retailer (89%) as expected. The return-smart retailer orders fewer units at the right time since she considers the possible returns and treats them as a secondary supply source. Moreover, the return-naive retailer has to satisfy a higher number of backorders at the end of the season and then has to pay for any return credits associated with these sales without being able to resell.

On a final note, we observe that the return-naive retailer, which uses the net demand approach (i.e., considers only the product demand less the returns), evaluates her optimal profit as \$218.95 using the associated dynamic programming formulation. However, simulating the optimal inventory policy derived from this approach under stochastic sales-dependent returns results in a much lower expected profit of \$102.45 which is 53.21% lower than the evaluated optimal profit. However, this discrepancy does not exist for the return-smart retailer since she considers returns in evaluating the optimal profit. This level of overestimation would be problematic for many reasons including budget planning. Thus another advantage of our model is the correct evaluation of the actual profit resulting from a chosen inventory control strategy.

2.5.3 Sensitivity Analysis

For the base case scenario, we find that the value of incorporating sales-dependent returns in finding the optimal inventory policy is 23%. In this section, we first present a one-way sensitivity analysis on this value with respect to problem parameters. For each parameter, we modify the number used in the base case scenario and report the change in the percentage difference in the retailers' profits. Then, we show the results

of a full factorial experiment.

Figure 3 provides the results of sensitivity analysis with respect to the fixed order cost. As expected both retailers' profits decrease as the fixed cost increases. However, the decrease in the return-naive retailer's profit is steeper as this retailer orders more frequently as compared to the return-smart one. The value of incorporating sales-dependent returns increases to about 70% when the fixed cost increases to 50\$.

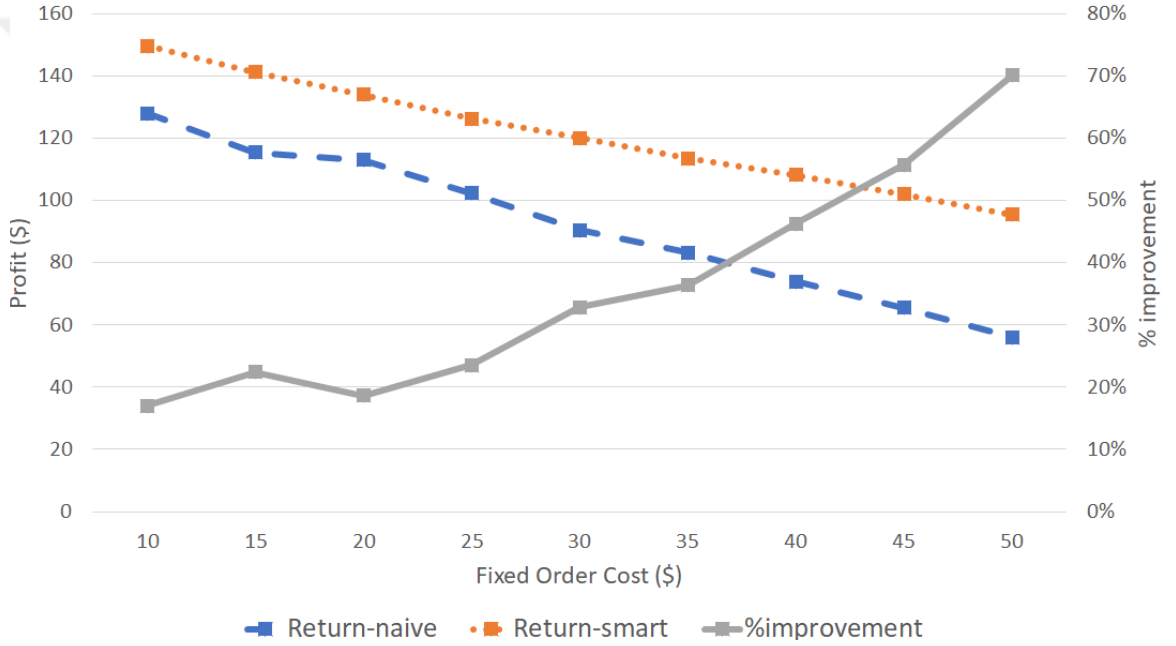


Figure 3: % profit improvement of incorporating sales-dependent returns as the fixed order cost varies

The effect of the backorder cost on the value of incorporating sales-dependent returns is shown in Figure 4. There are two main observations: (1) When the backorder cost is as low as the holding cost (i.e., $h = b = \$2$), the return-naive retailer places no order in the first period to save from the fixed order cost. As a result, she does not sell any products in the first period and loses the opportunity of incoming returns in the second period. On the other hand, the return-smart retailer places a positive initial

order and uses the returns in the following periods as a second source of supply resulting in a 36% profit gap between the two retailers' profits. (2) When the backorder cost is quite large, the return-naïve retailer orders more frequently than necessary to save from backorder cost. Thus she incurs high fixed order cost and the percentage profit improvement of incorporating sales-dependent returns could be as high as 90%. For moderate values of the backorder cost, the percentage profit difference is lower.



Figure 4: % profit improvement of incorporating sales-dependent returns as the backorder cost varies

The significant effect of the return probability is evident from Figure 5 which shows the percentage change in the retailer's profit as the return probability (α) varies. The value of incorporating sales-dependent returns increases monotonically with α : With higher values of the return probability, the return-naïve retailer chooses a lower initial order quantity expecting a lower net demand. However, the actual demand is

much higher in the first period which leads to high backorder costs, low sales, and low returns in each period which ends with a loss at the end of the season.

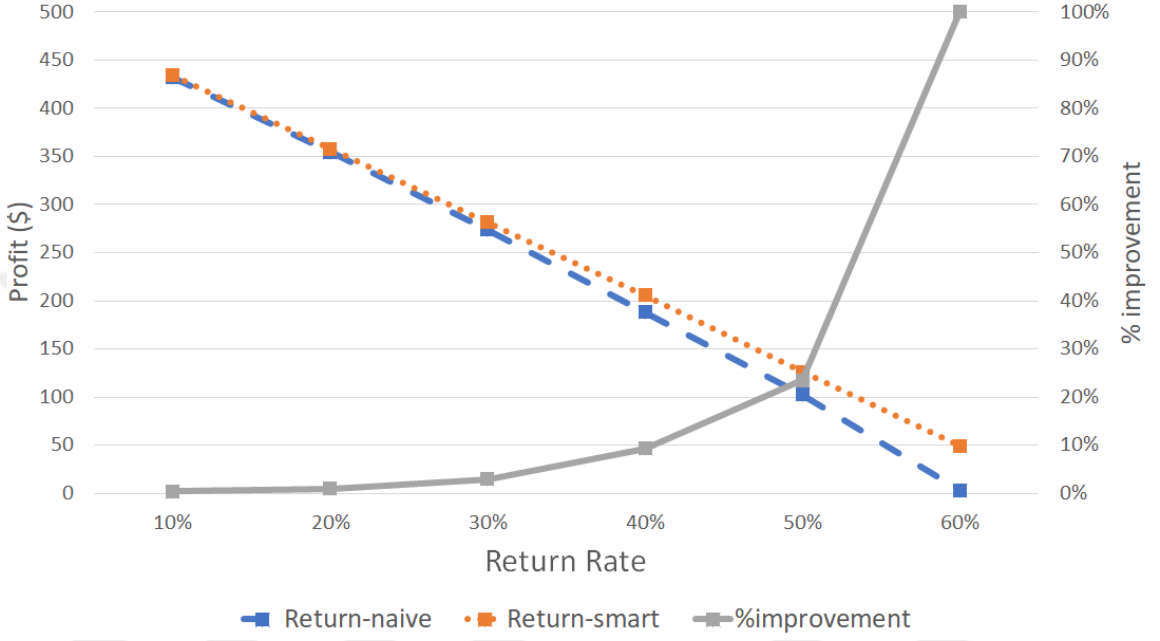


Figure 5: % profit improvement of incorporating sales-dependent returns as the return probability varies

We conduct an additional one-way sensitivity analysis for the remaining cost parameters and the results are presented in Table 4. Three statistics (minimum, median, and maximum) for % improvement are presented for each parameter as the value of that parameter changes in the given interval while keeping the others at base values. As the procurement cost increases, the percentage improvement in the retailer's profit reaches 52%. As the profit margin decreases with the higher procurement cost, the return-naive retailer has difficulty in controlling the comparatively higher fixed order costs. Since the total holding cost is the lowest cost category (see, Table 3), the percentage improvement in the retailer's profit varies only slightly as the holding cost rate increases from 0.5% to 4%. Finally, we find that the percentage improvement decreases with the salvage value. However, the return-smart retailer's profit is still

4% higher than the return-naive retailer even when the salvage value is the same as the procurement cost.

Table 4: % profit improvement in the retailer’s profit as procurement cost, holding cost, and salvage value parameters vary

Parameter	Instance Set	Min	Med	Max
Procurement unit cost	[10,15,...,45]	3%	9%	52%
Holding cost	[1%,2%,...,6%]	23%	27%	29%
Salvage value	[0,10,...,40]	4%	15%	23%

The resulting service levels could be a separate concern for the retailer. Table 5 shows the percentage difference in the cycle service level (type I) and the product fill rate (type II) as the problem parameters vary. The percentage differences in both service level measures are most sensitive to the backorder cost as expected from our earlier results. Moreover, the return-naive retailer has better service levels (i.e., the percentage difference is negative) when $\alpha \leq 0.2$ at the expense of higher costs, but both the service level and the profit are smaller for larger values of the return probability.

Table 5: The percentage difference in the service levels as the problem parameters vary

Parameter	Instance	Type I			Type II		
		Min	Med	Max	Min	Med	Max
Procurement unit cost	[10,15,...,45]	-3%	-2%	9%	2%	3%	3%
Holding cost	[1%,2%,...,6%]	-4%	-2%	3%	2%	3%	4%
Backorder cost	[8,14,...,50]	-7%	-3%	8%	1%	3%	8%
Fixed order cost	[10,15,...,50]	-8%	1%	2%	1%	4%	4%
Return probability	[10%,20%,...,60%]	-6%	-3%	5%	0%	2%	5%
Salvage value	[0,10,...,40]	-5%	-2%	8%	2%	3%	3%

To assess the robustness of our one-way sensitivity results, we design a full factorial experiment with 69.984 instances varying all parameters. However, some of the

parameter value combinations result in very small or even negative profits for the retailers. Thus, we removed 4488 instances which results in an operating margin of less than 10%. Three statistics for each parameter are reported in Table 6, which support our conclusions from the one-way sensitivity results: Return rate has the highest impact on the percentage improvement in retailer's profit, while salvage value and the holding cost rate have very little effect. The average improvement monotonically increases with the fixed ordering cost and the procurement cost. When the procurement cost is very large ($c = 45$), however, the improvement slightly drops since the profit margin in these instances is the smallest. Finally, the average improvement is highest when the backorder cost is very small or high.

Table 6: The statistics for the % profit improvement in the retailer's profit in the full factorial experiment

Backorder Cost	2	8	14	20	26	32	38	44	50
10th Percentile	0.2%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%
Average	8.8%	5.3%	5.5%	5.9%	6.2%	6.4%	6.5%	6.6%	6.7%
90th Percentile	24.7%	14.9%	15.6%	16.5%	17.7%	18.3%	18.6%	18.7%	19.4%
Fixed Order Cost	10	15	20	25	30	35	40	45	50
10th Percentile	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%
Average	4.7%	5.0%	5.4%	5.9%	6.5%	6.9%	7.5%	7.9%	8.3%
90th Percentile	12.9%	13.9%	15.2%	16.4%	18.4%	19.8%	21.3%	22.9%	24.0%
Procurement Unit Cost	10	15	20	25	30	35	40	45	
10th Percentile	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	
Average	2.5%	2.8%	4.3%	5.2%	7.7%	7.7%	8.4%	7.3%	
90th Percentile	7.6%	7.9%	12.4%	14.3%	21.6%	22.0%	25.8%	21.3%	
Return Rate	0.10	0.20	0.30	0.40	0.50	0.60			
10th Percentile	0.0%	0.2%	0.5%	1.4%	3.2%	6.8%			
Average	0.2%	0.8%	2.3%	5.7%	11.9%	22.8%			
90th Percentile	0.4%	1.4%	4.4%	11.5%	25.9%	47.9%			
Holding Cost Rate	0.01	0.02	0.03	0.04	0.05	0.06			
10th Percentile	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%			
Average	6.2%	6.3%	6.3%	6.5%	6.6%	6.7%			
90th Percentile	17.6%	17.7%	18.0%	18.6%	18.8%	18.7%			
Salvage Value	0	10	20	30	40				
10th Percentile	0.1%	0.1%	0.1%	0.1%	0.1%				
Average	7.0%	6.2%	6.2%	6.1%	6.0%				
90th Percentile	19.4%	17.4%	17.7%	17.8%	18.1%				

2.6 *Extension: Lost Sales Case*

Our main formulation introduced in Section 3 assumes that unmet demand is back-ordered in line with most of the related literature in this field. For online sales, this assumption is even more plausible (see, for example, [54], [55], and [56]). However, an alternative assumption would be to assume that any unmet demand is lost (see references in Table 1). Hence in this section, we revisit our research question under the lost sales assumption.

Under the lost sales assumption, the transition of the state variable comprising of the net inventory at the start of period $t + 1$ and the previous sales quantity at period $t + 1$ could be written as follows:

$$I_{t+1} = (S_t + R_t - D_t)^+ \quad (9)$$

$$SQ_t = \min\{S_t + R_t, D_t\} \quad (10)$$

The profit-to-go functions of the dynamic programming formulation also require some modifications. First, expected unmet demand should be multiplied by the lost sales cost. For our base case scenario, we assume that the lost sales cost is zero to focus on the effect of losing sales revenue. Moreover, the terminal profit function is also modified such that it includes only the salvage value of the ending inventory and the credits issued for the returns of the products sold in the last period.

Under the base case scenario, our numerical results show that the return-smart retailer now follows an (s, S) inventory policy as well. The (s, S) values of the return-smart retailer depend on the previous sales quantity since she considers the return flow as a second source of supply and decreases its orders as the previous period sales increase. During period 2, she follows a $(3, 6)$ inventory policy when $SQ_1 = 0$ but switches to using a $(1, 4)$ inventory policy when $SQ_1 \in \{3, 4\}$. Similar to our findings with the main model, we find that the return-naive retailer orders fewer quantities compared to the return-smart retailer. For example, she uses a $(2, 5)$ inventory policy

in the second period. Comparing with the main model, the return-naive retailer has higher (s, S) values under lost sales setting, pointing towards a possible reason for lower profit.

Under the lost sales assumption, the return-smart retailer's profit is 56% higher using the base case scenario as shown in Table 7. Compared to the 23% profit improvement under the backordered demand setting, we conclude that the value of incorporating sales-dependent returns is higher if unmet demand is lost. Notice that the return-naive retailer has a higher revenue, but the increase in costs more than offsets increase in the revenue. On average the return-naive retailer orders twice, whereas the return-smart retailer only orders in the initial period. In addition, the return-naive retailer also orders more units (10) than the return-smart retailer (8) on average. Since the salvage cost is zero, excess inventory at the end of the selling season is worthless.

Table 7: Total profit breakdowns for both retailer types using the base case scenario and the lost sales assumption

Retailers	Procurement Cost B1	Ordering Cost B2	Holding Cost B3	Total Season Cost B=B1+B2+B3	End-of-Season Cost C	Season Revenue A1	Return Credit A2	Season Net Revenue A=A1-A2	Total Profit A-B-C
Return-naive	407.24	55.48	16.73	479.45	130.70	1,154.88	446.61	708.27	98.12
Return-smart	321.44	25.30	16.00	362.74	87.20	1,031.80	428.73	603.08	153.14

Table 8 shows the results of the one-way sensitivity analysis on the retailer's percentage improvement in profit as the problem parameters vary from the base case scenario. The percentage improvement increases with the procurement unit cost, the holding cost rate, the fixed ordering cost, the return probability and decreases with the salvage value. Compared with the results for the backordered demand setting in Section 2.5.3, there is a significant increase in the variation of the percentage improvement.

Table 8: % improvement in the retailer’s profit as the problem parameters vary assuming unmet demand is lost

Parameter	Instance	Min	Med	Max
Procurement unit cost	[10,15,...,45]	4%	14%	128%
Holding cost	[1%,2%,...,6%]	39%	48%	60%
Fixed order cost	[10,15,...,50]	23%	44%	71%
Return probability	[10%,20%,...,60%]	0%	12%	126%
Salvage value	[0,10,...,40]	3%	17%	56%

Finally, we investigate the effect of a positive lost sales cost on the percentage profit improvement and present the results in Figure 6. As the lost sales cost increases moderately from zero, the return-smart retailer slightly increases her order frequency and orders in some periods in addition to the initial one. Thus the profit gap between the retailers decreases. When the lost sales cost is very high, however, the return-naïve retailer increases order frequency, even more, to not incur high lost sales cost. The return-smart retailer does not react drastically as she acknowledges possible returns in the subsequent periods. Thus the percentage improvement increases again to more than 70% when the lost sales cost is \$40.

2.7 Conclusions

We investigate how a return-smart retailer could utilize the product returns to maximize her total profit over a finite selling season. She could use the returns as a second supply source and thus limit her order frequency and backorders. A comparison with the profit of a return-naïve retailer, which ignores the dependency of returns on previous sales quantifies the value of incorporating detailed return information. Using a base case scenario, we find that the return-smart retailer could obtain a 23% higher profit. Our sensitivity analysis shows that this value could be much higher especially when the fixed ordering cost is high, the backorder cost is very small or high, the

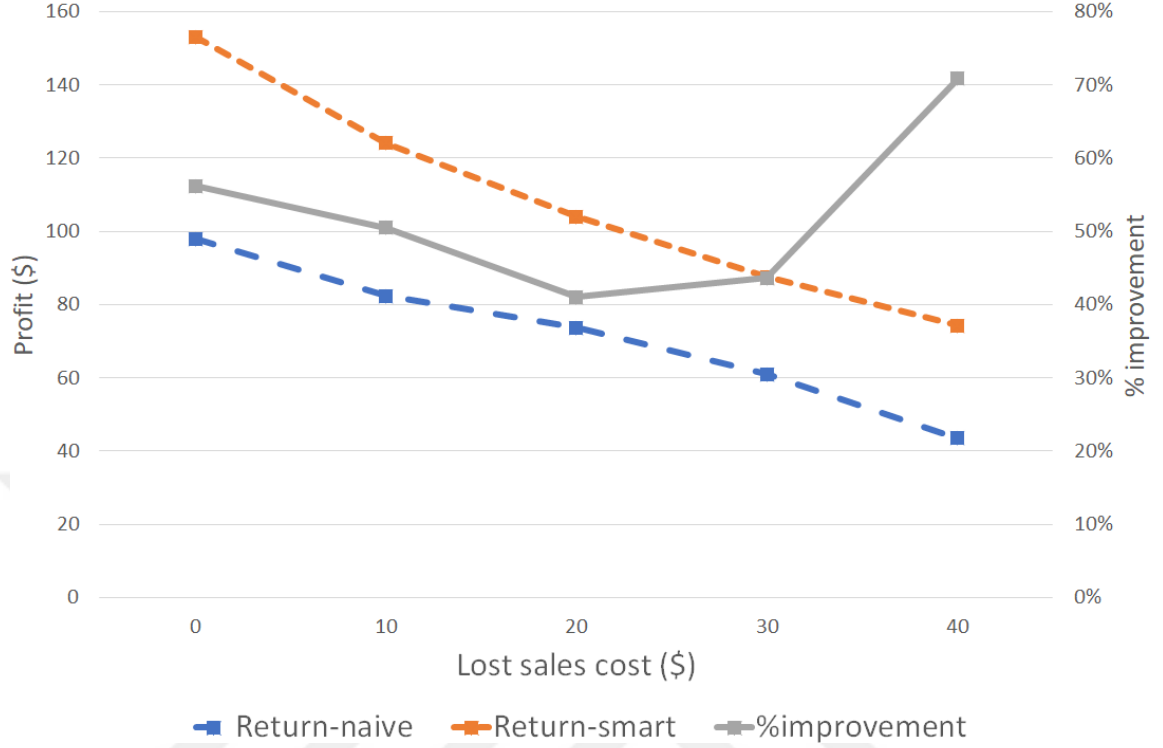


Figure 6: % profit improvement of incorporating sales-dependent returns as the lost sales cost varies

return probability is high, or the procurement unit cost is high.

Customers increasingly opt to return products, and with the higher share of e-commerce channel sales, the product return rates are rising above 30 – 50% in some categories. Due to the COVID-19 pandemic, offline sales are substituted by online sales even more, thus subject to higher return rates. Retailers, specifically those in the fast fashion industry, which operates under the main assumptions used in our dissertation, could benefit from the results shared for two related issues.

First, return-smart retailers should reconsider their inventory replenishment decisions by incorporating return flows. For example, slightly increasing the initial order sizes, compared to a return-naive setting, could be a smart strategy over the rest of the selling season, especially if the return rate and fixed ordering cost are high. Inflated initial order sizes could also be used to identify products with high return

rates early in the season and react accordingly. Second, retailers could quantify the value of using detailed return information. Our proposal requires firms to keep separate meticulous records of both product sales and returns, which could be costly from IT and labor points of view. Moreover, product return process flows should be redesigned to collect this additional information. Our results highlight when and if these additional costs are worthwhile for the retailer.



CHAPTER III

SINGLE ITEM PERIODIC REVIEW INVENTORY CONTROL WITH SALES DEPENDENT STOCHASTIC RETURN FLOWS FROM MULTIPLE PERIODS

3.1 *Introduction*

The retail industry changes parallel to the world's economic conditions, like a global slowdown, war, or a pandemic. In 2020, during the COVID-19 outbreak, shopping in physical stores stopped temporarily due to pandemic restrictions, prompting the retail industry to adapt to these new conditions. Online sales increased enormously due to these restrictions, and people experienced the comfort of online purchasing from their homes. The pandemic has finished, but the preference for online purchasing has stayed, and it is still a growing business. National Retail Federation in the US reported that in 2022, \$761 billion worth of clothing was predicted to be returned, representing 20.8% of all online purchases [57]. According to a recent survey ([6]), up to 91% of returns result from the retailer's "no-cost return offer" to attract new customers. One major reason for high levels of return due to online sales is the lack of touch. Especially in apparel retail, size, color, and flexibility can be different than expected when bought online. Moreover, when considering the timing of customer returns for products purchased online, the decision to return may vary between offline and online sales. Returns may not arrive at the same pace even if a sale is realized on the same day.

A company's return window sets the maximum number of days customers can return a product after purchase. [58] explores strategies for optimizing this return

window to maximize the company’s overall profit. They also present analytical findings based on data from Zalando, Europe’s largest online fashion retailer. Zalando strategically optimized both its return window and internal processing of returns, re-introducing over two-thirds of returns for resale within three or four days. As a result, most returned items re-enter the supply chain twice a month. According to [59], shorter return periods are typically applied to seasonal products, such as fashion items. This is because items returned early have more opportunities for resale, whereas products returned later tend to have a lower salvage value.

In inventory management, estimating the arrival times of the product returns is as important as estimating their quantity. As the return periods increase, the endowment effect increases, and customers tend to attach to the product they purchase and decrease their willingness to return the product (see, for example, [5]). Companies are aware of this endowment effect and increase the returns periods accordingly. Customers tend to return the products by the end of return periods (see, for example, [41] and [60]). Still, retailers prefer to receive a return in a shorter period to sell at full price in the season rather than sell at the end of the season with a discount. According to McKinsey’s 2021 survey ([61]), retailers can receive in-store returns 12-16 days earlier than online sales. Hence, if companies can track the return data with a time detail, they can benefit from these return products as they can re-sell them within the season.

We introduce a model designed to monitor returns across multiple periods, facilitating optimal ordering decisions for retailers. Our model aims to maximize profit by considering revenue net of procurement, fixed ordering, holding and backorder costs, subtracting return credits for items sold in multi-periods, and factoring in the salvage value of unsold items in the terminal period. We establish a customer return policy that allows product returns from various purchase periods. The model meticulously tracks the return potential, incorporating different return probabilities.

Through comparative analysis with a benchmark model using net demand, which calculates returns on an aggregate basis rather than a per-period basis, we demonstrate the substantial increase in profit achievable with our model. Furthermore, we present a sensitivity analysis, and our findings show significant profit improvement with our model, particularly in fast fashion retail industries where decreasing sales prices over periods are common. Moreover, we show a profit improvement with our model where there are order capacity constraints.

To overcome the curse of dimensionality while solving the model with numerous state variables, we present an Approximate Dynamic Programming (ADP) value iteration algorithm using basis functions, effectively reducing the computational cost. Our proposed algorithm drastically reduces the solution time without a significant sacrifice from optimality and generates solutions for larger instances in terms of dimensionality.

Our primary contribution in this chapter is the presentation and analysis of a model featuring stochastic return flows arising from sales across different periods within a finite horizon. This model is enriched with a fixed ordering cost and is further extended to include a capacity constraint and mark-down pricing. Additionally, we introduce an ADP algorithm that yields near-optimal results with significantly reduced computational costs compared to obtaining optimal results.

The remainder of this chapter is organized as follows. Section 3.2 provides a literature review. Section 3.3 introduces the model. Section 3.4 shows the solution methods including dynamic programming and heuristics. Section 3.5 shows the computational results, and finally, we discuss the managerial insights and future research opportunities in Section 3.6.

3.2 Literature Review

This dissertation examines inventory management for reverse supply chains with product returns. Product returns have been extensively studied in the existing literature. Most papers have concentrated on re-manufacturing the returns in which the quantity of the re-manufactured returns is a decision variable. Many of them concentrated on the return policy leniency or salvaging returns, the return window policies, or the sales price of a product in a closed-loop system. Few of the papers have studied inventory management for reverse supply chains, and again, few concentrated on retail returns where all returns may be added back to inventory. We will first examine the papers that study inventory management with stochastic returns and then examine the models that receive product returns in multi-periods along with reviewing the models with order capacity constraints. Later, we will look at alternative heuristic solution methods and finally review our contribution to the literature.

3.2.1 Inventory Management with Stochastic Returns

We can divide the returns into two main groups as the ones that need to be remanufactured before re-selling (see, for example, [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27]), and the commercial ones that need negligible or none processing cost/time before re-selling (see, for example, [28], [29], [30], [31], [32], [33]). In all these papers, product returns are dependent on demand either within the same period or across the periods, and in few of them, returns are sales-dependent, which is a more realistic assumption (see, for example, [27], [62], [63]).

Papers discussing commercial returns are more closely aligned with our research. [30] study the (s, S) policy in a closed loop system using the net demand model in a finite horizon. [31] study a continuous review (s, S) inventory system where stochastic returns depend on demand. [33] use a demand-dependent return model and study the optimality of base-stock policies. [27] study a model that assumes stochastic

dependency of the returns and sales within the same period. [62] shows a system where the product returns are sales-dependent across the periods. They present a model with stochastic return flows that depend on the actual sales in the previous period. Their model also involves fixed ordering cost in a finite-horizon setting, with a detailed terminal period cost structure. They also provide a detailed taxonomy for the above-mentioned papers.

3.2.2 Multi-period Returns

Most of the literature that uses multi-period return data is concerned with the leniency policies or the company's managerial decisions regarding the return window [58] show how to set a company's return window to maximize its profit. [59] show that as the return period increases, consumer valuation becomes better. [64] study the number of products sold and the selling price of the product as a function of the return products in single and multi-period problems. There are various studies regarding the distribution network that study multi-period returns. (see, for example, [65], [66]).

Some studies use classical newsvendor problems with an extension of consumer returns. This group of papers focuses on return policy leniency and salvaging returns. Few of these papers use a single selling season with two periods representing regular sales and clearance (see, for example, [67], [68], [69]). [67] show that reducing the product recovery cost of returned commodity products is valuable in improving financial performance. [68] analyze the optimal ordering decisions considering online shoppers' purchasing behavior. They consider a lenient return policy and recommend that retailers apply a low-transaction-cost return process, like offering pre-labeled return boxes for high-margin products. [69] show that when a retailer offers resale service, she can increase the price, reduce the size of consumer returns, and increase the order quantity.

Other than the leniency policies or the company’s managerial decisions regarding the return window, we concentrate on the impact of inventory decisions on the return window decision. In literature, product returns that arrive in multiple periods are mostly evaluated in the industries where the returns are re-manufactured. This kind of product has either unlimited or very long return periods. [70] evaluate the computer returns where they are re-manufactured and sold as second-quality products. They investigate the optimal quantity of re-manufactured returns. Their model forecasts the timing of collecting the available product returns based on the sales estimates. [71] study a similar model for the returns of used phones. They show that the returns arrive in a uniform distribution with multi-periods. Similar models are studied in the literature (see, for example, [72]). [63] study a previous sales-dependent stochastic returns model where the return window allows customers to return in multi-periods in an infinite horizon. They also study and compare the computational performance of different heuristic models to solve the system. [73] study optimizing product collection in the milk industry where the product returns are continual. Although this sector, where there are continual compulsory returns, is very different from the retail sector, they show the benefits of collecting detailed return data through periods. [74] work to establish an optimal network for collecting apparel products sold online during the season.

Furthermore, we explore the impact of product order capacity within a closed-loop system. This inquiry is prompted by the prevalent challenges in the retail industry, where constraints such as budget availability, diverse production and supply schedules for various products, and limited shared storage space are common. [75] examine how storage and order capacity limitations can alter the optimal replenishment policy in a multi-product system. Their findings indicate that the optimal replenishment priority and quantity depend on the products’ inventory levels and capacities. Moreover, capacity restrictions can potentially reverse the stocking priorities of products. [76]

investigate a closed-loop supply chain’s dynamic behavior with capacity restrictions in manufacturing and re-manufacturing. Their study reveals that manufacturing capacity solely influences the forward flow of materials, whereas re-manufacturing capacity has a broader impact on the dynamics of the entire system.

3.2.3 Approximate dynamic programming in inventory management

ADP is an extension of dynamic programming and Bellman’s equation that is preferred to be used when there is a ‘curse of dimensionality’ in the model. In ADP, look-up tables or value functions are used in an algorithm to overcome recursiveness. ADP has recently been studied in the literature; we can refer to a structured coverage of ADP for [77] and [78]. Various inventory management models use an approximate dynamic programming method (see, for example, [79], [80] [81]).

[79] show ADP application to fleet management problems in the context of dynamic resource allocation. [82] use a dynamic programming model to find optimal base-stock levels. [83] use the ADP approach to solve the problem of determining inventory levels in a distributed aircraft spare parts warehouse network. [80] show a workable model for managing an inventory bank holding perishable blood platelets with a short shelf life. They study ADP for three different supply policies and then compare them to a myopic policy that does not consider the inventory changes during the day. [81] use a similar dynamic programming method for inventory decisions involving multi-production and multi-products. They compare the ADP algorithm and a myopic deterministic policy. The ADP algorithm archives near optimal costs in their study, whereas the myopic deterministic policy stays weak.

3.2.4 Contribution to the Literature

There are two main contributions to the literature in this chapter. First, we demonstrate a model that tracks the returns from multiple periods and accordingly gives the optimal ordering decisions. With our model, the retailer maximizes its profit as

revenue minus procurement cost, fixed ordering cost, holding and backorder cost minus return credits for the returns that are sold in multi-periods, and finally, a salvage value of unsold items in the terminal period. Our model keeps track of the return potential. So, the model includes the conditional return probability of the products sold but not yet returned. We show the increase in profit when using our model when benchmarked against a model that uses net demand, meaning calculating the returns on an aggregate basis rather than a detailed calculation for each period. We also show this model's sensitivity analysis with various cost parameters. It is also very interesting to observe that when our model is used instead of a net demand model, the improvement in profit increases when there is order capacity or markdown pricing. The second contribution is to apply an ADP algorithm to our model and to decrease the computational cost without sacrificing the optimal solution.

3.3 Model Formulation

We model an apparel retail store where customers can return purchased items within a given return window. Specifically, a customer may return the purchased item in k periods, $k = 1, 2, \dots, L$ where L is the length of the return window. Moreover, there are given fractions specifying the percentage of an item returned in k (where $k \leq L$) periods.

The retailer uses a periodic review inventory control policy, and any unsatisfied demand is backordered. The retailer, along with the inventory position of the product, keeps track of the previously sold items with return potential and gives inventory replenishment decisions accordingly over a finite planning horizon. For instance, the environment described is typical in the fast fashion apparel industry, where returns over a window are common and a different product concept or season is introduced every 2-3 month.

In what follows, we describe our model and introduce the notation used. A summary of our notation is provided in Table 9. We assume that demand in period t , D_t , is a non-negative integer-valued random variable with mean μ and demands in successive periods are independent and identical distributed (iid) random variables. Once a product is sold in period t , a customer may return this product in period $t + k$ with a stationary probability α_k , $k = 1, 2, \dots, L$ where L is the length of the return window. For completeness, let α_{L+1} be the probability that a sold item is never returned back to the store.

Table 9: List of notations used in this chapter

Parameters	
c	Unit procurement cost.
K	Fixed ordering cost.
h	Holding cost per unit per period.
b	Backorder cost per unit per period.
r_t	Return credit per period.
s	Salvage value per unit.
p_t	Retail price per unit per period.
C_t	Order capacity per period.
α_k	The probability that an item sold at t is returned in period $t + k$, $k = 1, 2, \dots, L$.
T	Number of periods in the selling season.
Variables	
I_t	Net inventory at the beginning of period t .
O_t	Replenishment order in period t .
D_t	Demand realized during period t .
SQ_t	Sales realized during period t .
$\mathbf{RP}_t$	Vector of return potential information available at time t .
S_t	Inventory level just after the replenishment order but before observing demand and returns in period t .

Let $RP_{t-k,t}$ be the return potential in period t of the sales that occurred in period $t - k$, $k = 1, 2, \dots, L$: the amount sold in period $t - k$, but not yet returned as of the beginning of period t . For $k = 1$, $RP_{t-1,t}$ equals the amount sold in the previous period $t - 1$. We denoted this quantity as SQ_{t-1} . Let $\mathbf{RP}_t$ be the vector of return potential information available at time t :

$$\mathbf{RP}_t = (RP_{t-1,t}, RP_{t-2,t}, \dots, RP_{t-L,t}).$$

Accordingly, we define $R_{t-k,t}$ as the random variable denoting the number of returns at time t from the sales realized at the $t - k$.

Lemma 1 *Given the return potential information $\mathbf{RP}_t$ at the beginning of period t , the distribution of $R_{t-k,t}$ is given by a Binomial distribution with parameters $RP_{t-k,t}$ (the number of trials) and $\alpha_k / \sum_{j=k}^{L+1} \alpha_j$ (the success probability) for $k = 1, 2, \dots, L$.*

By decomposition of Bernoulli trials: any item sold in period $t - k$ but not yet returned by the beginning of period t will either return in t , or will return in periods $t + 1, t + 2, \dots, t - k + L$, or will never return with respective probabilities $\alpha_k, \alpha_{k+1}, \alpha_{k+2}, \dots, \alpha_L, \alpha_{L+1}$. As these are the only possible events, they redefine the sample space, and therefore, the probability of return in period t is given by

$$\frac{\alpha_k}{\alpha_k + \sum_{j=k+1}^L \alpha_j + \alpha_{L+1}}.$$

Let I_t be the net inventory at the beginning of period t before the ordering decision. The retailer, after observing $\mathbf{RP}_t$ places an order O_t . The inventory after ordering becomes $S_t = I_t + O_t$. The lead time is assumed to be zero. Following the order given, we observe any returns from the sales in the previous periods: $R_{t-k,t}$ for $k = 1, 2, \dots, L$. Afterwards, the demand in period t , D_t is observed. As any unsatisfied demand is backordered, the net inventory at the end of the period becomes:

$$I_{t+1} = S_t + \sum_{k=1}^L R_{t-k,t} - D_t. \quad (11)$$

The amount of sales in period t is given by:

$$SQ_t = \min\{S_t + \sum_{k=1}^L R_{t-k,t}, D_t\} + I_t^-, \quad (12)$$

where $I_t^- = \max\{0, -I_t\}$.

Given the return potential information $\mathbf{RP}_t$ in period t , the return potential in

period $t + 1$, $\mathbf{RP}_{t+1}$ is obtained as

$$\begin{aligned}
RP_{t-L+1,t+1} &= RP_{t-L+1,t} - R_{t-L+1,t} \\
RP_{t-L+2,t+1} &= RP_{t-L+2,t} - R_{t-L+2,t} \\
&\vdots = \vdots \\
RP_{t-1,t+1} &= RP_{t-1,t} - R_{t-1,t} \\
RP_{t,t+1} &= SQ_t.
\end{aligned}$$

We aim to find the optimal inventory policy for the retailer to maximize the expected total profit. Hence, we can use the following recursive dynamic programming formulation.

$$\begin{aligned}
P_t(I_t, \mathbf{RP}_t) &= \max_{I_t \leq S_t \leq C_t + I_t} \{p_t \mathbb{E}_{D_t, \mathbf{RP}_t}[SQ_t] \\
&\quad - \bar{c}(S_t - I_t) \\
&\quad - \sum_{k=1}^L r_{t-k} \mathbb{E}_{D_t, \mathbf{RP}_t}[R_{t-k,t}] \\
&\quad - J(S_t, \mathbf{RP}_t) \\
&\quad + \mathbb{E}_{D_t, \mathbf{RP}_t}[P_{t+1}(I_{t+1}, \mathbf{RP}_{t+1})]\}.
\end{aligned} \tag{13}$$

In Equation 13, the first term is the period's revenue, and the second and third terms are the ordering cost and return credits. P_{t+1} is the profit-to-go function of the next period. We do not consider a discount factor since the selling season is assumed to be very short. C_t is the order capacity for each period. The J term is the current period's total expected backorder and inventory holding costs, which are given as:

$$\begin{aligned}
J(S_t, \mathbf{RP}_t) &= h \mathbb{E}_{D_t, \mathbf{RP}_t}[(S_t + \sum_{k=1}^L R_{t-k,t} - D_t)^+] \\
&\quad + b \mathbb{E}_{D_t, \mathbf{RP}_t}[(D_t - S_t - \sum_{k=1}^L R_{t-k,t})^+].
\end{aligned}$$

At time $T+1$, the season ends, and no demand is realized. The equation accounts for the profit from the backordered items, salvage from the leftover inventory, final procurement cost to fulfill the backordered demand, and the return credit. The returns that arrive at the season's end are credited and salvaged. Thus, the ending period cost function could be written as follows:

$$\begin{aligned}
P_{T+1}(I_{T+1}, \mathbf{RP}_{T+1}) &= p(I_{T+1})^- \\
&- c\mathbb{E}_{\mathbf{RP}_T}[(I_{T+1} + \sum_{k=T-L+1}^T R_{k,T+1})^-] \\
&+ s\mathbb{E}_{\mathbf{RP}_T}[(I_{T+1} + \sum_{k=T-L+1}^T R_{k,T+1})^+] \\
&- \sum_{k=T-L+1}^{T+1} r_k \mathbb{E}[R_{k,T+1}] \\
&- (\sum_{k=1}^L \alpha_k)(r_T - s)(I_{T+1})^-. \tag{14}
\end{aligned}$$

3.4 Solution Methods

This section presents three solution methods for our problem: dynamic programming, approximate dynamic programming, and a distribution-free myopic approach. Our model can be solved using dynamic programming with Bellman's equation for an optimal solution for small instances. However, as the state space enlarges, computing the exact solution for our model in a reasonable amount of time is not possible. The fundamental premise of ADP is the extension of dynamic programming and Bellman's equation to address problems involving large state or action spaces. We propose an ADP value iteration algorithm using basis functions. We also search for a myopic approach that makes decisions based on the current state without considering the cumulative impact on the overall objective function.

3.4.1 Dynamic Programming

The model outlined in Section 3.3 incorporates $L + 1$ dimensional state variables: inventory at the beginning of each period (1) and the return potential during the return window (L). We calculate expectation over $L + 1$ random variables, one for the period's demand distribution and L expectations for returns from each of the previous L sales period affecting the processing time.

The model's dimensions of the state space are large since we use a backorder setting meaning that the state space is also affected by the number of periods since the retailer can fulfill the unmet demand in any period. Also, the maximum demand per period increases the state space affecting inventory at the beginning of each period (I_t), inventory level after the replenishment order of each period (S_t), demand of each period (D_t), previous sale quantity of L different periods (SQ_t), return potential of L different periods ($\mathbf{RP}_t$). As the state space gets larger, memory usage increases.

As the return window expands, the model's dimensions grow, leading to the curse of dimensionality which is the main problem of most dynamic optimization models. Even with the minimal number of periods or mean demand, determining the optimal solution becomes hard to calculate.

3.4.2 Approximate Dynamic Programming

ADP is an alternative method for solving recursive stochastic optimization models with computational costs. We design an ADP algorithm as shown in [78] which attempts to approximate the value function by fixed basis function and its linear weights. An alternative to basis functions would be to use look-up tables, but our experiments show that the multidimensional information variables still use high memory to keep look-up table representations for the value functions; hence, we decided

to use a basis function as shown at the last part of Equation (15):

$$\begin{aligned}\hat{P}_t(I_t, \mathbf{RP}_t) = & \max_{I_t \leq S_t \leq C_t + I_t} \{p_t \mathbb{E}[SQ_t] - \bar{c}(S_t - I_t) - \sum_{k=1}^L r_{t-k} \mathbb{E}[R_{t-k,t}] - J(S_t, \mathbf{RP}_t) \\ & + \sum_f \theta_{f,t+1}^{n-1} \phi_{f,t}((I_t, \mathbf{RP}_t)^n, S_t^n)\}\end{aligned}\quad (15)$$

Value functions are approximated by utilizing the quantities from the state variables using basis functions with features. Although specified as linear models, they may also consist of nonlinear behaviors. There are two problems in estimating the basis function: the form of this basis function, $\phi_{f,t}(I_t, \mathbf{RP}_t)$ and the value of the linear weights, $\theta_{f,t}$. For the first problem, we conducted a regression analysis to find the structure of the optimal profit function. As we plot the data, we observe that the system is nonlinear, but we can observe an increasing and then decreasing function with respect to inventory. The form of the function increases linearly and then nonlinearly up to a breakpoint, then decreases nonlinearly and then linearly after this breakpoint. Thus, we decided to form the basis function with previous sales, linear and nonlinear versions of the inventory up to a breakpoint and inventory after the breakpoint. We approximate the value function by expressing it in quantities derived from state variables. These quantities are represented using basis functions denoted as $\phi_f(\cdot)$, where $f \in F$ signifies a feature. Hence, we formed the basis function for the multi-period return flow model as:

$$\phi(I_t, \mathbf{RP}_t) = [1, \mathbf{RP}_t, (I_t - bp_t)^+, ((I_t - bp_t)^+)^2, (I_t - bp_t)^-, ((I_t - bp_t)^-)^2] \quad (16)$$

Breakpoint, bp_t , is formed for the multi-period return flow as follows:

$$bp_t = \lambda(T - t) - \sum_{k=1}^L \frac{\alpha_k}{\sum_{j=k}^{L+1} \alpha_j} RP_{t-k,t} \quad (17)$$

Using the basis function, our approximate value function for multi-period return flow with θ coefficients is formed as:

$$\begin{aligned}\tilde{P}_t(I_t, \mathbf{RP}_t) = & \theta_{0,t} + \sum_{k=1}^L \theta_{k,t} R P_{t-k,t} + \theta_{L+1,t} (I_t - bp_t)^+ + \theta_{L+2,t} ((I_t - bp_t)^+)^2 \\ & + \theta_{L+3,t} (I_t - bp_t)^- + \theta_{L+4,t} ((I_t - bp_t)^-)^2\end{aligned}\tag{18}$$

The principle of ADP is to replace the true value function $P_t(I_t, \mathbf{RP}_t)$ with a statistical approximation, $\tilde{P}_t(I_t, \mathbf{RP}_t)$. The function $\phi_f(\cdot)$ captures specific qualities from the state variable considered explanatory. We use a recursive least squares method to update parameter vector θ_t across replications. We start with an initial estimate θ_0^1 , after which θ is updated following each observation as shown in Algorithm 1. We iterate a fixed number of times and then use the θ vector to make the value function.

Algorithm 1 ADP Basis Function Algorithm

Input : Cost Parameters

Output: Approximate Value Function $\theta_{f,t}^N$, for all t and f

Initialization:

$\theta_t^0 \leftarrow [0, 0, \dots, 0]$ for all f and t

$[I_1^0, \mathbf{RP}_1^0] \leftarrow [0, \dots, 0]$

Set $n \leftarrow 1$

for each time step t **do**

Set $t \leftarrow 1$

while $t \leq T$ **do**

Solve:

$$\hat{P}_t^n(I_t, \mathbf{RP}_t) \leftarrow \max_{I_t \leq S_t \leq C_t + I_t} \{p_t \mathbb{E}[SQ_t] - \bar{c}(S_t - I_t) - \sum_{k=1}^L r_{t-k} \mathbb{E}[R_{t-k,t}] - J(S_t, \mathbf{RP}_t) + \sum_f \theta_{f,t+1}^{n-1} \phi_{f,t}(I_t^n, \mathbf{RP}_t^n)\}$$

Let S_t^n be the value of S_t that solves $\hat{P}_t^n(I_t, \mathbf{RP}_t)$

Update the value function recursively to obtain $\theta_{f,t}^n \leftarrow \theta_{f,t}^{n-1} - H_t^n \phi_t^n(I_t, \mathbf{RP}_t) \hat{\epsilon}_t^n$

where,

$$H_t^n \leftarrow \frac{1}{\gamma_t^n} B_t^{n-1}$$

$$\hat{\epsilon}_t^n \leftarrow \tilde{P}_t(I_t, \mathbf{RP}_t)^{n-1} - \hat{P}_t^n$$

$$B_t^n \leftarrow B_t^{n-1} - \frac{1}{\gamma_t^n} (B_t^{n-1} \phi_t^n(I_t, \mathbf{RP}_t) (\phi_t^n(I_t, \mathbf{RP}_t))^\top B_t^{n-1})$$

$$\gamma_t^n \leftarrow 1 + (\phi_{f,t}^n(I_t, \mathbf{RP}_t))^\top B_t^{n-1} \phi_{f,t}^n(I_t, \mathbf{RP}_t)$$

Choose a sample $W^{t+1} = W(w^{t+1})$ and determine the next stage.

end

if $t \leq T$ **then**

| Increment t

else

| Increment n if $n \leq N$

end

end

return $\bar{\theta}_{f,t}^N$, for all f and t

3.4.3 Myopic Approach

The myopic model neglects future implications and concentrates solely on the total reward within the immediate decision-making period. In our model, even if we focus solely on calculating the period cost, we still have to compute expectations for $L + 1$ instances, leading to a relatively high calculation cost. Hence, we study a myopic approach where only the mean and variance of the demand and return flow are known in a case with a fixed ordering cost. We consider the model in a single period; hence, we drop the unnecessary subscript t and define the state variables as:

$$SQ = \min(I + O + R, D) + I^+ = D - (D - S - R)^+$$

$$J = h(S + R - D)^+ + b(D - S - R)^+ = (h + b)(D - S - R)^+ + h(S + R - D)$$

The profit function is written as:

$$\begin{aligned} P(I, \mathbf{RP}) = \max\{ & -hS - (p + h + b)\mathbb{E}[(D - S - R)^+] \\ & - (h + r)\mathbb{E}[R] + (h + p)\mathbb{E}[D] + pI^- - \bar{c}(S - I) \} \end{aligned} \quad (19)$$

Following [84], we can pursue a distribution-free approach to:

$$P(I, \mathbf{RP}) = \max_{I \leq S} \{ (h + c)S + (p + h + b)\mathbb{E}[(D - S - R)^+] - cI + K \}. \quad (20)$$

We define:

$$d = \frac{h + c}{c}, m = \frac{p + b - c}{c}$$

The ordering rule is: order up to S^* units if $I \leq s^*$ and do not order otherwise, where:

$$\begin{aligned} S^* = & \lambda - SQ_{-1}u_1 - SQ_{-2}u_2 \\ & + \frac{1}{2} \sqrt{\sigma_D^2 + SQ_{-1}u_1(1 - u_1) + SQ_{-2}u_2(1 - u_2)} \left(\sqrt{\frac{m}{d}} - \sqrt{\frac{d}{m}} \right). \end{aligned} \quad (21)$$

where,

$$u_1 = \alpha_1, u_2 = \frac{\alpha_2}{1 - \alpha_1}.$$

and

$$s^* = \lambda - SQ_{-1}u_1 - SQ_{-2}u_2 + \frac{(m-d)\hat{A} - (m+d)\sqrt{(\hat{A}^2 - md\sigma^2)}}{2md}. \quad (22)$$

where,

$$\hat{A} = \sigma\sqrt{md} + \frac{K}{c} + \frac{(m-d)\hat{A} - (m+d)\sqrt{(\hat{A}^2 - md\sigma^2)}}{2md}.$$

and,

$$\sigma = \sqrt{\sigma_D^2 + SQ_{-1}u_1(1-u_1) + SQ_{-2}u_2(1-u_2)}.$$

3.5 Computational Results

In this section, we present the results of the numerical study. We show the value of multiple return flows in the retailer's inventory control problem and the effect of problem parameters on the profit. We refer to the retailer that uses our proposed model in managing her inventory as the “*data-rich*” retailer, as she uses rich data of the exact return rates, sales, and return quantity for each period during the customer return window. Additionally, as a benchmark we refer to another retailer that uses the net demand model in managing her inventory as the “*data-poor*” retailer as she aggregates the return rate information and doesn't track the previous sale and return data.

We clarify the specifics of our computational study setup in Section 3.5.1. In Section 3.5.2, we present the results and corresponding managerial insights addressing key inquiries for the following questions:

- What is the computational performance of the solution methods? (3.5.2.1)
- How does the performance of a data-rich retailer compare to that of a data-poor retailer under different parameter settings? (3.5.2.2)
- How does maintaining exact sales and return information with precise return rates in each period impact the retailer's expected profit? (3.5.2.3)

- What is the effect on expected profit for tracking detailed sales returns data when there is order capacity (3.5.2.4) or markdown pricing (3.5.2.5)?

3.5.1 Experimental Setup

In this computational study, we examine a specific base case scenario to compare the ordering policies and the expected profits of the data-rich and data-poor retailers. We assume the season has 5 periods, $T = 5$, with a customer return window of 2 periods, $L = 2$. We assume the mean of the demand is fixed at 6, however to understand different variance around this mean, we assume three different demand distributions: (1) Poisson distribution with parameter 6 ($x_p \sim \text{Pois}(6)$), (2) Uniform distribution with parameters between 0 and 12 ($x_1 \sim U(0, 12)$), and (3) Uniform distribution with parameters between 4 and 8 ($x_2 \sim U(4, 8)$). In our computations, we truncate demand values exceeding a probability of 0.9912 for the Poisson distribution case.

Returns arrive with $\alpha_1 = 0.10$ and $\alpha_2 = 0.40$, respectively, in the two periods allowed at the consumer's return policy. Consequently, the data-poor retailer assumes a demand mean of 3 and no return flow. We set the rest of the cost parameters as $h = 2\$$ per unit per period, $b = 20\$$ per unit per period, and $K = 25\$$ per order. We set unit procurement cost and retail price parameters as $c = 40\$$ per unit and $p_t = 100\$$ per unit, respectively. We also assume that there is no salvage value at the end of the period ($s = 0$), the customer is given full credit for returns, and any related processing cost is ignored ($r_t = p_t$).

Utilizing the inventory decisions generated by both models, we employ Monte Carlo simulation to compute the expected average profit and average values associated with the breakdown of costs across 100,000 simulation replications.

We also present a one-way sensitivity analysis on the value of incorporating multi-period sales-dependent returns with respect to problem parameters. We change the number used in the base case scenario for each parameter and show the percentage

difference in the retailers' profit. We define this difference as profit gain percentage, meaning the profit gain from using the exact sales and return information, and calculate the profit gain percentage as the difference between the expected profit of the data-rich retailer and the data-poor retailer as a ratio of the expected profit of the data-poor retailer.

For the sensitivity analysis, we changed the procurement cost $[10, 15, \dots, 45]$, holding cost $[0.40, 0.80, \dots, 2.40]$, backorder cost $[2, 8, \dots, 50]$, fixed ordering cost $[10, 15, \dots, 50]$, salvage value $[0, 10, \dots, 40]$, and finally return rates $[0.10, 0.20, \dots, 0.60]$ for a total of 53 cases. For the return rates' sensitivity analysis, we study 21 different cases, but we represent the return rates as the total of two periods here. To be more specific, for example for the total of 60% return rate, our cases were as: $(\alpha_1 = 0.10, \alpha_2 = 0.50)$, $(\alpha_1 = 0.30, \alpha_2 = 0.30)$, $(\alpha_1 = 0.50, \alpha_2 = 0.10)$, etc.

3.5.2 Results

In this section, we present the computational findings and associated managerial insights. Initially, we highlight the optimality gap between the exact and ADP solutions. Then, we present the computational results of the data-rich retailer for various scenarios, comparing them with the data-poor retailer's results. Consequently, we conduct a one-way sensitivity analysis regarding the parameters of interest. Later, we show the effect of tracking detailed returns data in scenarios involving order capacity and markdown pricing.

3.5.2.1 Computational performance of solution methods

The time required to solve the given problems varies significantly among the proposed solutions. As expected, the myopic approach can be solved in less than a second but yields the least favorable solution, with a substantial error of 20% for the base case cost parameter values. In contrast, the remaining approaches demonstrate comparable results. Specifically, the dynamic programming solution takes 514.32 minutes (8.57

hours) to compute the multi-period optimal solution. Meanwhile, the ADP basis function algorithm enables the computation of a near-optimal multi-period return flow solution with a modest 0.38% error in 31.9 minutes. The benchmarked net demand approach solution takes 6.33 minutes to solve but exhibits an error of 9.41%.

We show the sensitivity analysis regarding the optimality gap of ADP in Table 10. This one-way sensitivity analysis shows that the profit gap between the optimal and ADP solutions for the data-rich retailer averages 1.05%. The maximum disparity is 7.87%, occurring in an extreme case with a 60% total return rate.

Encouraged by these positive results, we opt for the ADP algorithm over dynamic programming for our computational studies and exclude the myopic approach throughout the rest of the chapter. We will present the remaining computational findings derived from the ADP algorithm.

Table 10: One-way sensitivity analysis of the profit gap between the optimal and ADP solutions for the data-rich retailer

Statistics	% Optimality Gap Proc. unit cost	% Optimality Gap Holding cost	% Optimality Gap Backorder cost	% Optimality Gap Fixed ord. cost	% Optimality Gap Salvage value	% Optimality Gap Return rate	% Optimality Gap
Min	0.38	0.37	0.25	0.38	0.33	0.00	0.00
10 th Percentile	0.50	0.38	0.27	0.40	0.35	0.02	0.10
Average	1.01	0.74	1.01	1.10	0.76	1.06	1.05
90 th Percentile	1.66	1.08	1.94	1.74	1.40	2.78	1.96
Max	1.73	1.18	2.24	2.51	1.93	7.87	7.87

3.5.2.2 Performance of the data-rich retailer and the data-poor retailer

For the base case scenario, in Table 11, the first two columns show which demand distribution and solution approach of a retailer is used while calculating the profit values. The percentage profit gain of the data-rich retailer varies between 7 – 10%. As the demand variance increases, we observe a higher profit gain, highlighting the data-rich retailer’s advantage in achieving higher profits than the data-poor retailer, especially in scenarios with high demand variance. We can observe the reason for this higher profit gain in Table 13 that the data-rich retailer orders in larger quantities

and decreases the backorder cost when the variance is high compared to the data-poor retailer.

Table 11: Cost breakdowns and percentage profit gain of data-rich retailer compared to the data-poor retailer

Distributions	Retailer	Unit Cost	Fixed Order Cost	Holding Cost	Backorder Cost	Terminal Cost	Return Credit	Profit	%Profit Gain
Poisson $x_p \sim \text{Pois}(6)$	Data-Rich	762.00	36.90	45.95	71.51	599.77	934.84	541.00	8.99
	Data-Poor	826.86	64.54	26.37	65.18	573.15	939.80	496.38	
Uniform 1 $x_1 \sim U(0, 12)$	Data-Rich	822.99	32.79	64.11	78.75	613.68	930.35	456.53	9.66
	Data-Poor	866.10	66.74	32.03	93.00	604.23	919.00	416.32	
Uniform 2 $x_2 \sim U(4, 8)$	Data-Rich	746.49	55.98	27.60	61.93	579.71	951.29	577.07	6.79
	Data-Poor	812.55	68.85	22.12	46.79	555.86	952.22	540.37	

The gap between the data-rich and data-poor retailers averages 6.71% with a maximum discrepancy of 31.65% as shown in Table 12.

Table 12: One-way sensitivity analysis - Statistics of the percentage profit gain of data-rich retailer compared to the data-poor retailer

Statistics	% Profit Gain Proc. unit cost	% Profit Gain Holding cost	% Profit Gain Backorder cost	% Profit Gain Fixed ord. cost	% Profit Gain Salvage value	% Profit Gain Return rate	% Profit Gain
Min	1.61	7.62	4.12	4.80	0.77	(0.33)	(0.33)
10 th Percentile	1.65	7.80	4.78	6.27	1.11	0.20	0.79
Average	4.93	8.26	8.47	8.82	4.41	6.36	6.71
90 th Percentile	9.55	8.83	11.39	11.25	7.96	17.41	11.33
Max	10.86	8.99	11.41	11.71	8.99	31.65	31.65

Figure 7 illustrates the sensitivity analysis results related to fixed order and back-order costs. Both retailers experience a decline in profits with an increase in fixed costs. The data-poor retailer must place more frequent orders than the data-rich retailer. Incorporating sales-dependent multi-period returns becomes more advantageous, exhibiting an increase of 11.71% when the fixed cost increases to \$50.

As the backorder cost increases, the data-poor retailer opts for more frequent orders to mitigate backorder costs, consequently increasing average order quantities as

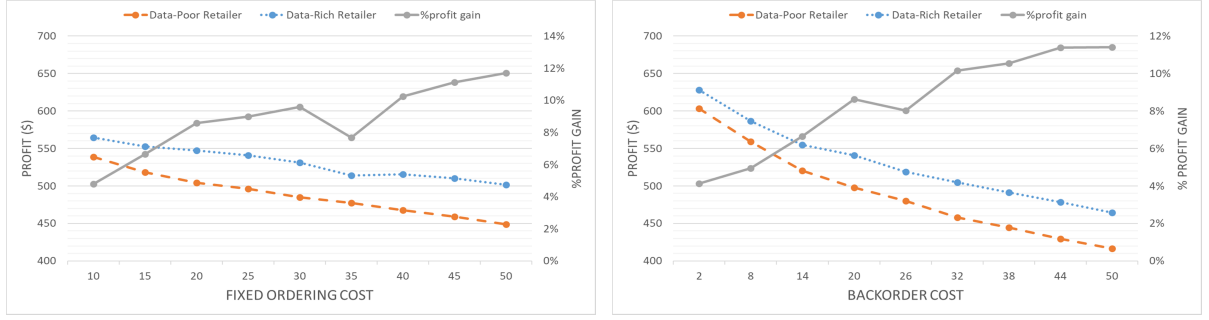


Figure 7: Sensitivity Analysis - Fixed Order Cost and Backorder Cost

seen in Table 13. The percentage profit gain by integrating sales-dependent multi-period returns can be as substantial as 11.41%. With an escalation in procurement costs, the percentage gain in the retailer's profit reaches 10.86%. Notably, total holding costs are not so sensitive to increases, with the percentage gain in profit remaining 8.26% on average.

Table 13: Average target stock levels (S_t^*) and average order quantities (O_t) across simulation replications.

Distributions	Retailer	S_1^*	S_2^*	S_3^*	S_4^*	S_5^*	O_1	O_2	O_3	O_4	O_5
Poisson $x_p \sim \text{Pois}(6)$	Data-Rich	17.00	11.02	6.10	4.24	1.62	17.00	-	0.47	1.14	0.42
	Data-Poor	10.00	6.10	6.30	5.17	2.87	10.00	2.10	5.59	1.99	0.99
Uniform 1 $x_1 \sim U(0, 12)$	Data-Rich	19.00	12.99	7.91	5.81	3.09	19.00	-	0.31	0.90	0.36
	Data-Poor	10.00	7.07	5.87	5.44	3.77	10.00	3.08	4.22	2.74	1.61
Uniform 2 $x_2 \sim U(4, 8)$	Data-Rich	13.00	7.00	6.12	3.92	1.16	13.00	-	4.52	0.82	0.32
	Data-Poor	8.00	7.39	6.37	4.85	2.57	8.00	5.39	4.38	1.57	0.99

The sensitivity to salvage value reveals a diminishing percentage gain. Specifically, when the salvage value is zero, the retailer's profit gain is 8.99%. However, this value diminishes to 0.77% when the salvage value aligns with the procurement cost.

We analyze the effect on profit of receiving the returns in the earlier periods in Table 14. We show some cases regarding the arrival time of the returns between the earliest and latest periods. Receiving returns earlier in the season has an increasing

Table 14: Effect of return window and product sojourn time on the expected profit of data-rich retailer

1 period return lag α_1	2 period return lag α_2	Profit(\$) Poisson ($x_p \sim \text{Pois}(6)$)	Profit(\$) Uniform 1 ($x_1 \sim U(0, 12)$)	Profit(\$) Uniform 2 ($x_2 \sim U(4, 8)$)
0.00	0.50	511.15	425.72	557.47
0.10	0.40	541.00	456.53	577.07
0.25	0.25	575.62	492.20	601.92
0.40	0.10	608.18	535.53	643.83
0.50	0.00	630.92	566.63	667.64

profit impact. Even if the total return rate of the return window is 50%, the difference between having most of the returns in the first period (earlier) results in a 119.77 surplus in the profit for the base case scenario. [60] suggests that consumers tend to return by the end of the return period, whereas retailers prefer to receive a return in a shorter period to sell full-price in the season rather than at the end with a discount. Retailers, particularly in industries like fast fashion, aim for shorter return periods to capitalize on re-selling opportunities within the season (see, for example, [59]).

In Figure 8, we show retailers' percentage profit gain of sales-dependent multi-period returns. The graphs show the total return rate of the return window. The first graph illustrates scenarios where the return rate in the first period exceeds that of the second period, while the second graph represents the reverse. Both percentage profit gains (11% and 25%) show an increasing trend as the total return probability rises. With higher return probabilities, the data-poor retailer anticipates lower demand and places smaller orders than the data-rich retailer. This ordering preference results in higher backorder costs, lower sales, reduced returns due to lower sales, and, ultimately, a loss at the end of the season. Retailers may consider investing in customer incentives to influence a shorter product sojourn time during returns, which can yield greater profit benefits.

3.5.2.3 Effect of keeping exact sales and return information

Shorter return windows are particularly beneficial for seasonal products, such as fashion items, as early returns present more opportunities for resale [59]. Consumers tend

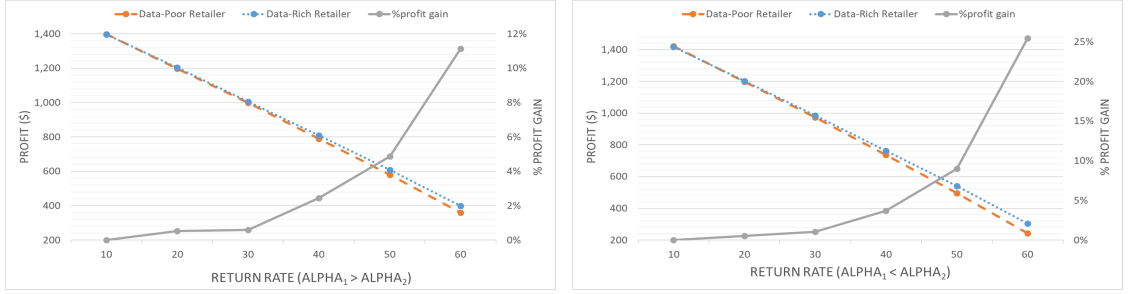


Figure 8: Sensitivity Analysis - Return Rates

to initiate returns toward the end of the period, but retailers prefer to receive returns within a shorter time frame to facilitate full-price sales during the season, as opposed to discounted sales at the season's end.

In Table 15, we present the variations in profit corresponding to changes in return data information within an environment characterized by the arrival of returns in two distinct periods for 3 different cases with different return rates as: $(\alpha_1 = 0.10, \alpha_2 = 0.40)$, $(\alpha_1 = 0.40, \alpha_2 = 0.10)$, and $(\alpha_1 = 0.25, \alpha_2 = 0.25)$. For each case, the first row of the table illustrates the profit when the retailer uses exact sales and return information regarding both periods. This row is the data-rich retailer's case as the ordering decisions and the simulation use the same return rate information. Subsequently, for each case, the second row depicts the profit when possessing the sales quantity of the most recent sales and the associated return probability solely; earlier sales and return information is disregarded. For each case, the third row maintains the most recent sales quantity but aggregates the return probabilities of both periods. Finally, last row shows the data-poor retailer's case. Those rows show the profits associated with the mentioned different return rates but are simulated with the exact (real-life) return rate information.

We highlight the advantages of maintaining comprehensive sales and return data; however, it is crucial to acknowledge that retaining such detailed information incurs

substantial costs for companies. Opting to disregard earlier data and focusing solely on the most recent information may be a practical preference, particularly when it is established that most returns originate from recent sales as shown in Case 2 where profit loss is only 1.48% and 0.45% when the return rates are aggregated for the return window.

Table 15: Effect of keeping exact sales and return information

Cases	Return Rates $\alpha_1 - \alpha_2$	Profit $x_p \sim \text{Pois}(6)$	%Profit Loss	Profit $x_p \sim \text{Pois}(9)$	%Profit Loss
Case 1	0.10 – 0.40	541.00		876.02	
	0.10 – 0.00	415.27	30.28	665.73	31.59
	0.50 – 0.00	518.72	4.29	852.22	2.79
	Data-poor Retailer	496.38	8.99	818.55	7.02
Case 2	0.40 – 0.10	610.11		973.62	
	0.40 – 0.00	601.20	1.48	960.54	1.36
	0.50 – 0.00	607.36	0.45	973.27	0.04
	Data-poor Retailer	579.65	5.25	945.58	2.96
Case 3	0.25 – 0.25	575.31		922.51	
	0.25 – 0.00	518.45	10.97	825.95	11.69
	0.50 – 0.00	566.56	1.54	915.13	0.81
	Data-poor Retailer	539.23	6.69	882.82	4.50

3.5.2.4 Effect of order capacity

Order capacities define the maximum quantity of products a retailer can order or replenish in a given period. These capacities are pivotal in balancing supply and demand, preventing overstocking or stockouts. In inventory management with product returns, order capacities become even more crucial. Retailers must account for returned items, considering their condition and potential for resale. In this section, we investigate the effect of a less restricted order capacity's effect on the retailers' profit.

Figure 9 illustrates the change in profit with increasing order capacity. When the order capacity is very small, even lower than the mean demand of the period, both retailers have to make the same decision, even if the data-rich retailer wants to order more. Then, they have similar expected profits; however, as the order capacity

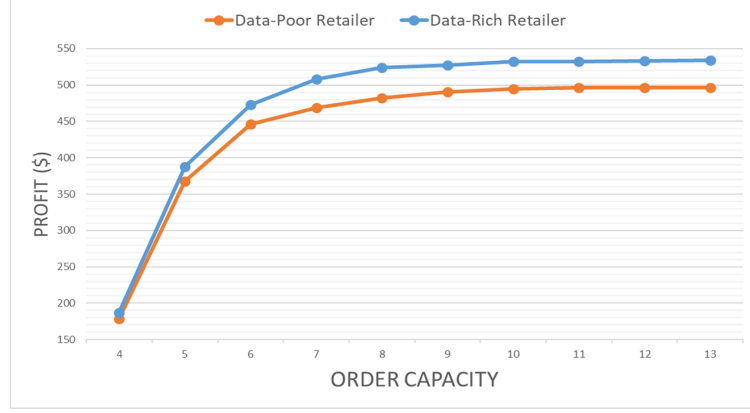


Figure 9: Profit changes of both retailers as the order capacity increases

increases, they can make different decisions since the feasible region expands. Hence, we see that the difference between the profits increases as the order capacity increases.

Table 16: Percentage profit loss when exact sales and return information is not used, with order capacity constraint

Cases	Return Rates $\alpha_1 - \alpha_2$	%Profit Loss Low Capacity:4	%Profit Loss Medium Capacity:6	%Profit Loss High Capacity:8	%Profit Loss Uncapacitated
Case 1	0.10 – 0.40				
	0.10 – 0.00	14.25	21.09	25.38	30.28
	0.50 – 0.00	1.64	2.12	2.41	4.29
	Data-Poor Retailer	4.69	5.99	8.67	8.99
Case 2	0.40 – 0.10				
	0.40 – 0.00	0.85	1.56	1.48	1.48
	0.50 – 0.00	0.38	0.09	0.07	0.45
	Data-Poor Retailer	2.24	3.23	3.83	5.25
Case 3	0.25 – 0.25				
	0.25 – 0.00	5.56	8.98	10.24	10.97
	0.50 – 0.00	0.28	0.99	1.22	1.54
	Data-Poor Retailer	1.92	4.39	5.95	6.69

Next, we study the order capacity effect on the percentage profit gain when exact sales and return information is used as shown in Table 16. Due to constrained ordering decisions, the percentage profit gain diminishes with lower order capacities. In scenarios where customers choose to return at the end rather than the beginning of the return window, the profit gain for not retaining exact sales and return data escalates significantly, rising from 14.25% to 30.28%, as the order capacity increases.

While maintaining detailed sales and return data can be expensive for retailers,

keeping track of the most recent data may be more practical and cost-effective. The percentage profit gain decreases when customers return purchased products earlier in the return window as in Case 2. We already observed this effect when there is no order capacity, but interestingly, we observe an increase in the percentage profit gain when the order capacity is equal to the mean demand of the period.

3.5.2.5 Effect of mark-down pricing

Our model incorporates the realistic assumption that retail sales prices may not remain constant and could decrease over time, a common occurrence in sectors such as apparel retail. Table 17 presents four distinct cases of decreasing sales prices. In each scenario, the profit for the data-rich retailer is consistently higher than that of the data-poor retailer. This significant difference in profit is attributed to the missed opportunity to resell returned items at a higher price. We note a significant percentage gain in profit when the data-poor retailer is compared to the data-rich retailer, from 18% to 45%, as the retailers apply discounts in the earlier periods.

The data-poor retailer places orders in smaller quantities, resulting in backorders and reduced sales quantity. This approach enables her to fulfill demand in the later periods of the season, but these later periods yield lower profits than earlier. Consequently, the time shift in ordering decisions leads to diminished profits at the end of the season.

In such cases where the customers return products by the end of the return window, we notice a higher profit gain. This is primarily because markdown pricing makes the chance to resell the returned items less profitable. The data-poor retailer tends to place orders in smaller quantities, reducing the overall returns. However, those returns that arrive in smaller quantities have a lower profit when re-sold, resulting in less profit.

Table 17: Effect of markdown pricing with various return rates

Cases	%Profit Gain ($\alpha_1 : 0.10 - \alpha_2 : 0.40$)	%Profit Gain ($\alpha_1 : 0.40 - \alpha_2 : 0.10$)	%Profit Gain ($\alpha_1 : 0.25 - \alpha_2 : 0.25$)
Retail Price = {100, 100, 80, 80, 60}	18.13	6.61	8.09
Retail Price = {100, 80, 80, 80, 60}	18.42	6.99	9.25
Retail Price = {100, 80, 80, 60, 60}	30.58	8.47	15.43
Retail Price = {100, 100, 60, 60, 60}	44.60	14.85	13.95

3.6 Conclusions

We investigate how a data-rich retailer could maximize profit over a finite selling season by strategically utilizing product returns from different sales periods. A comparison with the profit of a data-poor retailer, ignoring the dependency of returns on previous sales, quantifies the value of incorporating detailed return information. The data-rich retailer could obtain a 9% higher profit in the base case scenario in such a setting. Our sensitivity analysis indicates that this value could be even higher, particularly when facing high fixed ordering costs, a high return probability, or a markdown pricing.

We model the periods in detail, allowing for product returns from sales of multiple previous periods with different return rates. Additionally, we explore non-stationary product prices and incorporate order capacity constraints into our model to deepen our research. With all these extensions, we present an ADP algorithm to solve the model. Using ADP, we drastically decreased the solution time (514.32 minutes to 31.9 minutes for the base case scenario) without sacrificing the optimality (the average optimality gap for ADP is 1% resulting from 53 different cases).

We present a one-way sensitivity analysis for different parameter settings. As the backorder or fixed ordering costs increase, we observe a higher percentage profit gain for the data-rich retailer. The effect of return rates is profound on the profit. We observe that as most returns arrive in the earlier periods, the retailer's profit increases due to the re-selling opportunity. With higher return probabilities, the data-poor retailer expects lower mean demand and places smaller orders, resulting in

higher backorder costs, lower sales, and fewer returns.

We show that keeping exact data regarding returns in terms of return rate and arrival times increases the retailer's profit. This effect is higher when there is ordering capacity especially when fewer returns arrive in the earlier periods. A similar trend is observed in the presence of mark-down pricing, reducing the profitability of reselling returned items. In situations where returns are unavoidable, retailers should implement strategic measures, such as promoting early returns through offers like free shipping or expedited refunds, to mitigate the impact on profit.

CHAPTER IV

SINGLE ITEM PERIODIC REVIEW INVENTORY CONTROL WITH SALES DEPENDENT STOCHASTIC RETURN FLOWS FROM MULTIPLE CHANNELS

4.1 *Introduction*

In the context of fast fashion, omnichannel retail represents a dynamic and interconnected approach to meet consumers' evolving demands. Fast fashion brands seamlessly integrate various sales channels, such as brick-and-mortar stores, e-commerce platforms, mobile applications, and social media, to create a cohesive shopping experience. By leveraging technology and data analytics, fast fashion brands can tailor marketing efforts, promotions, and product assortments to individual customer preferences, fostering loyalty in an increasingly competitive market.

A company with an omnichannel approach gains an advantage/preference in customer satisfaction. The customer is confident that regardless of which store of the company they shop from, they will have the same price or discount. The ability to return products to any store, not just where the purchase was made, is also a crucial criterion for customer satisfaction. Particularly during holidays/New Year/special occasion gift shopping, individuals in different cities prefer brands with an omnichannel presence due to the ease of the return process. Consequently, retailers must strategically navigate the complexities of returns processing to meet and exceed customer expectations, optimize inventory management, and minimize the environmental impact of returns.

A straightforward return policy significantly enhances customer loyalty. 66% of

consumers review a retailer's return policy before making a purchase, and a noteworthy 92% expresses a willingness to make repeat orders if they have a positive return experience. Major fast fashion apparel retailers like Zara, Mango, and GAP have adopted user-friendly return policies. These policies often allow consumers to return items to any department store, including online purchases, irrespective of the original purchase location. While a simple return policy can potentially boost a company's revenue, retailers must address the complexity of return uncertainty strategically to minimize associated costs, particularly within the omnichannel framework ([7]).

According to a study by KPMG, 96% of products purchased from stores are returned to stores. In addition, one-third of products purchased from online stores are returned to a brick-and-mortar store. A simple return process enhances customer satisfaction and loyalty. Companies with a simple return process can increase their revenues. Still, they should also consider the uncertainties and costs arising from increased returns due to omnichannel while planning their inventory management.

In Chapter 4, we provide the optimal solution for the lost sales model, taking into account the complex structure of the system and the unique dynamics of the fast fashion retail sector.

4.2 Literature Review

With the rapid integration of technology into contemporary life, one of the sectors affected is the retail industry. Particularly during the COVID-19 pandemic, customers have increased their online shopping habits. In parallel with this trend in the sector, companies have shown a tendency to open online stores in addition to their existing brick-and-mortar stores. The alignment of the shopping experience between online and offline stores in the eyes of the customer is a relatively new development. Therefore, very few studies in the literature address the concept of omnichannel.

Buy Online Return Physical (BORP) option is commonly used in retail. [85] work

on a model that considers purchasing from an online store and returning to a physical (brick-and-mortar) store in their study, which they examine as a game theory. They suggested that offering the same option might not always be profitable for companies if their competitors choose this policy. Before implementing this option, they propose examining the customer characteristics of retailers, evaluating the effectiveness and adequacy of the return process within the channel, and investigating the similarities between their channels and those of their competitors.

In their paper, [86] demonstrate that, compared to a model with only an online store, supporting the online store with a brick-and-mortar store is more profitable for the company. [87] design a model for a retail company with both online and brick-and-mortar stores using the newsvendor model, where returns also may arrive at the store. However, in their model, return items are not treated as new but are resold at a lower price. Their study shows that customers receiving more services without additional charges increases customer satisfaction and makes the company more profitable. [88] show that for last-mile e-commerce distribution, shifting the collection from individual customers to an aggregate form, like collecting from collection and delivery points, is more cost-beneficial for the companies (For example, shifting 20% of the deliveries to delivery points leads to 5.5% of total cost savings).

In an omnichannel context, retailers make rationing and ordering decisions. In a study by [63], a model is explored in which an omnichannel retailer is responsible for these decisions, with the assumption that returns can only originate from online sales. They propose a Deep Reinforcement Learning (DRL) algorithm to address large-scale instances. Their findings indicate that an increase in online returns reduces profit and service levels in the offline channel, particularly in shorter return windows.

We present an omnichannel model that considers the system’s intricate structure and captures the distinct dynamics of the fast fashion retail sector. This model incorporates a warehouse for online sales and a brick-and-mortar store for offline

sales. Our approach focuses on optimizing the initial inventory to be purchased for the season across both channels and determining the order quantity to be transferred from the warehouse to the brick-and-mortar store in each period. Our contribution to the literature is our presentation of a detailed real-life model that includes three different return flows within the system. Customers can return online-purchased products to both the warehouse and the brick-and-mortar store, with the latter experiencing two distinct return flows from both offline and online sales. Our model contributes to the literature by presenting the solution of the optimal season order and the optimal transshipment quantity for the brick-and-mortar store.

4.3 Model Formulation

We model an omnichannel retailer wherein customers can purchase the same item online or offline. The retailer employs a periodic review inventory control policy and any unsatisfied demand results in lost sales. The retailer tracks the inventory position of products in the warehouse where online sales occur and in the brick-and-mortar store where the offline sales occur. Additionally, the retailer keeps records of previously sold items from both online and offline sales and gives the replenishment decision of the offline store accordingly over a finite horizon. The retailer does not replenish warehouse inventory since seasonal orders are placed once at the beginning of the season.

Throughout the selling season, the sequence of events unfolds: the warehouse initiates the season by placing a comprehensive order with the supplier, incurring unit costs. Periodically, during the season, the offline store assesses its inventory needs and, if necessary, issues a replenishment order to the warehouse, incurring a fixed shipment cost and a transshipment cost per unit. At the start of each period, the offline store accepts orders. The warehouse processes returns for online sales from the preceding period, incurring return credit. Simultaneously, the offline store

accepts returns from online and offline sales in the prior period, with corresponding return credits. As demand and sales are realized, unsatisfied demand from both the warehouse and offline store results in lost sales, incurring lost sales costs. At the end of each period, the warehouse and offline store incur holding costs for the on-hand inventory. A list of notations is provided in Table 18.

Table 18: List of notations used in Chapter 3

Parameters	
c	Unit procurement cost.
K	Fixed shipment cost from warehouse to the brick-and-mortar store.
f	Transshipment unit cost from warehouse to the brick-and-mortar store.
h_b	Holding cost of the brick-and-mortar store per unit per period.
h_w	Holding cost of the online store per unit per period.
l_w	Lost sales cost of online store per unit per period.
l_b	Lost sales cost of the brick-and-mortar store per unit per period.
r	Return credit of a product sold in the brick-and-mortar store or online.
s_w	Salvage value of the product not sold at the warehouse.
s_b	Salvage value of the product not sold at the brick-and-mortar store.
p	Retail price of a product.
α_w	Return probability of a product sold online and returned to the warehouse.
α_{wb}	Return probability of a product sold online and returned to the brick-and-mortar store.
α_b	Return probability of a product sold offline and returned to the brick-and-mortar store.
T	Number of periods in the selling season.
Variables	
$I_{w,t}$	Net inventory of warehouse store at the beginning of period t .
$I_{b,t}$	Net inventory of the brick-and-mortar store at the beginning of period t .
O_t	Replenishment order of the brick-and-mortar store in period t .
$D_{w,t}$	Demand realized during period t at the online store.
$D_{b,t}$	Demand realized during period t at the brick-and-mortar store.
$SQ_{w,t}$	Sales realized at the online store during period t .
$SQ_{b,t}$	Sales realized at the brick-and-mortar store during period t .
$R_{w,t}$	Returns that are sold online received at the warehouse during period t .
$R_{wb,t}$	Returns that are sold online received at the brick-and-mortar store during period t .
$R_{b,t}$	Returns that are sold offline received at the brick-and-mortar store during period t .
S_t	Inventory level at the brick-and-mortar store just after replenishment order but before demand and returns.

We assume that the demand for both channels $D_{w,t}$ and $D_{b,t}$ are independent across periods. However, the demand across periods is identically distributed for each channel. Customers pay a stationary unit price of p to purchase the product. Each sold product in period t is assumed to be returned in period $t + 1$ with probability α_w for the products that are sold online and returned to the warehouse, α_{wb} for products

sold online and returned to the brick-and-mortar store, and α_b for products sold and returned in the brick-and-mortar store. Given the previous period online sales $SQ_{w,t-1}$, one could deduce that $R_{w,t}$ and $R_{wb,t}$, follows a multinomial distribution with parameters α_w, α_{wb} and $SQ_{w,t-1}$. Given the previous period offline sales $SQ_{b,t-1}$, one could deduce that $R_{b,t}$ follows a binomial distribution with parameters α_b and $SQ_{b,t-1}$. Immediately upon receiving a return, the retailer must pay the return credit, r , back to the customer. Extending our model to incorporate nonzero unit return processing cost is trivial by simply increasing the value of r accordingly.

The retailer begins the initial period with inventory at the warehouse for both channels. This order is purchased just once at the beginning of the season. Then, the warehouse sends orders to the brick-and-mortar store when needed. The total cost of ordering in any period has two parts: unit transshipment cost, f , and fixed ordering cost, K . Thus, the total cost of ordering O_t units from the warehouse to the brick-and-mortar store is denoted by $\bar{c}$ and can be stated as follows:

$$\bar{c}(O_t) = \begin{cases} K + f O_t & \text{if } O_t > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (23)$$

Any excess inventory at the end of a period incurs a unit holding cost, h_w and h_b for the warehouse and the brick-and-mortar store respectively. Similarly, any shortage at the end of a period incurs unit lost sales cost, l_w and l_b .

We formulate the retailer's inventory control problem using dynamic programming. The state variable at each period t has four components: the net inventory of warehouse and brick-and-mortar store at the beginning of this period ($I_{w,t}$ and $I_{b,t}$) and the online and offline sales quantities in the previous period ($SQ_{w,t-1}$ and $SQ_{b,t-1}$). To maximize the expected profit for the season, we aim to find the optimal net inventory after replenishment in each period, $S_t := I_{b,t} + O_t$. Using the principle of optimality, one could write the backward recursive formulation for the expected

total profit-to-go function $P_t(I_{w,t}, I_{b,t}, SQ_{w,t-1}, SQ_{b,t-1})$ as follows:

$$\begin{aligned}
P_t(I_{w,t}, I_{b,t}, SQ_{w,t-1}, SQ_{b,t-1}) = & \max_{I_{b,t} \leq S_{b,t} \leq I_{w,t} + I_{b,t}} \\
& \{p\mathbb{E}_{D_{w,t}, D_{b,t}, R_{w,t}, R_{b,t}, R_{wb,t}}[SQ_{w,t} + SQ_{b,t}] \\
& - \bar{c}(S_{b,t} - I_{b,t}) \\
& - r\mathbb{E}[R_{w,t}] - r\mathbb{E}[R_{b,t}] - r\mathbb{E}[R_{wb,t}] \\
& - h_w\mathbb{E}_{D_{w,t}, R_{w,t}}[(I_{w,t} - O_{b,t} + R_{w,t} - D_{w,t})^+] \\
& - h_b\mathbb{E}_{D_{b,t}, R_{b,t}, R_{wb,t}}[(I_{b,t} + O_{b,t} + R_{b,t} + R_{wb,t} - D_{b,t})^+] \\
& - l_w\mathbb{E}_{D_{w,t}, R_{w,t}}[(D_{w,t} - I_{w,t} + O_{b,t} - R_{w,t})^+] \\
& - l_b\mathbb{E}_{D_{b,t}, R_{b,t}, R_{wb,t}}[(D_{b,t} - I_{b,t} - O_{b,t} - R_{b,t} - R_{wb,t})^+] \\
& + \mathbb{E}_{D_{w,t}, D_{b,t}, R_{w,t}, R_{b,t}, R_{wb,t}}[P_{t+1}(I_{w,t+1}, I_{b,t+1}, SQ_{w,t}, SQ_{b,t})]\}
\end{aligned} \tag{24}$$

where the first term is the expected revenue from sales, the next two terms are the ordering costs and return credits, and the fourth to seventh terms are the current period's total expected lost sales and holding costs for the periods. Equation 24 gives the optimal decisions for the transshipment quantities. We then decide how much to order at the warehouse before the season starts. That problem can be formulated as follows:

$$P_0 = \max_{0 \leq I_{w,1}} \{P_1(I_{w,1}, 0, 0, 0) - cI_{w,1}\}. \tag{25}$$

In Equation 25, P_0 is the net season profit; hence, it is equal to the regular season profit ($P_1(I_{w,1}, 0, 0, 0)$) minus the pre-season ordering cost.

In this chapter, we use the notations $x^+ := \max\{0, x\}$ and $x^- := \max\{0, -x\}$. The net inventory cannot be salvaged or destroyed during the selling season. Hence, there are two constraints on the admissible inventory policies as $I_{b,t} \leq S_{b,t}$ and $S_{b,t} \leq I_{w,t} + I_{b,t}$: the net inventory after replenishment of the brick-and-mortar store cannot be less than the beginning net inventory of it, and the order quantity of the brick-and-mortar store can not be more than the inventory of the warehouse for the same period. Both parts of the state variable at period $t + 1$, $I_{w,t+1}$, $I_{b,t+1}$, $SQ_{w,t}$ and

$SQ_{b,t}$ depend on demand and returns of period t . Hence, the expectation before the next period's profit-to-go function in Equation 20 is over $D_{w,t}$, $D_{b,t}$, $R_{w,t}$, $R_{b,t}$ and $R_{wb,t}$. Finally, we assume there is no discounting as the selling season for many retail products is comparatively short.

The transition of the state variable can be explained in two parts. Given the sequence of events, the net inventory at the beginning of period $t + 1$ of both offline and online stores, $I_{w,t+1}$ and $I_{b,t+1}$, can be written as:

$$\begin{aligned} I_{w,t+1} &= (I_{w,t} - S_{b,t} + I_{b,t} + R_{w,t} - D_{w,t})^+ \\ I_{b,t+1} &= (S_{b,t} + R_{b,t} + R_{wb,t} - D_{b,t})^+ \end{aligned} \quad (26)$$

Next, the sales quantities of both channels at time t can be written as:

$$\begin{aligned} SQ_{w,t} &= \min\{I_{w,t} - S_{b,t} + I_{b,t} + R_{w,t}, D_{w,t}\} \\ SQ_{b,t} &= \min\{(S_{b,t} + R_{b,t} + R_{wb,t}), D_{b,t}\} \end{aligned} \quad (27)$$

Equation 27 formulates that the sales quantity of period t is the minimum of the inventory on hand, the replenishment order, the returns received, and the current period's demand quantity from the previous period.

At the end of the selling season, denoted by period $T + 1$, any excess inventory is salvaged at a unit profit of s_w at the warehouse and s_b at the brick-and-mortar store. We also account for the return credit. Thus, the terminal expected profit could be written as follows:

$$\begin{aligned} P_{T+1}(I_{w,T+1}, I_{b,T+1}, SQ_{w,T}, SQ_{b,T}) &= \\ r\mathbb{E}[R_{w,T+1}] - r\mathbb{E}[R_{b,T+1}] - r\mathbb{E}[R_{wb,T+1}] & \\ -s_w\mathbb{E}_{R_{w,T+1}}[I_{w,T+1} + R_{w,T+1}] & \\ -s_b\mathbb{E}_{R_{b,T+1}, R_{wb,T+1}}[I_{b,T+1} + R_{b,T+1} + R_{wb,T+1}] & \end{aligned} \quad (28)$$

In this equation, each term accounts for the following: the return credit and the salvage from the leftover inventory for both channels.

4.4 *Computational Results*

This section thoroughly examines inventory policy parameters derived from the model defined in Section 4.3 using dynamic programming. Subsequently, we present the details of our computational study setup and show the results of the system, including a one-way sensitivity analysis.

4.4.1 **Experimental Setup**

In this computational study, we use a base case scenario parameter settings. The retailer initiates an order for the season to stock the warehouse, where online sales are managed, incurring a unit purchasing cost of $c_t = 40$. On the manufacturing side, production and warehouse distribution must be completed before the selling season. Due to assortments being "floor-ready" with appropriate labels, price tags, and hangers, there is no production lead time, and the delivery lead time is deemed negligible [42]. Subsequently, the retailer assesses the net inventory of the brick-and-mortar store at the outset of each period t , denoted as $I_{b,t}$, and issues a replenishment order, O_t , if necessary. This order is fulfilled from the warehouse inventory, $I_{w,t}$, incurring a fixed ordering cost of $K = 25$ and a transshipment cost per unit, $f = 4$. The order is assumed to be received with a negligible replenishment lead time. The return rates for online sales, set at $\alpha_w = 0.20$ for returns to the warehouse, $\alpha_{wb} = 0.10$ for returns to the brick-and-mortar store, and $\alpha_b = 0.20$ for returns from offline sales to the brick-and-mortar store, are also considered in the base case scenario.

The Poisson distribution's mean values are $\lambda_{w,t} = 3$ for offline sales and $\lambda_{b,t} = 2$ for online sales. The model assumes a season of four periods ($T = 4$). Any unsatisfied demand is treated as lost, incurring costs of $l_w = 20$ for online and $l_b = 10$ for offline channels. The inventory holding cost is $h_w = 2$ per unit per period for the warehouse and $h_b = 20$ per unit per period for the brick-and-mortar store. In the omnichannel setting, the retail sales price is consistent across both channels at $p_t = 100$ per unit.

It is assumed that the salvage value at the end of the period is $s_w = 20$ for the warehouse and $s_b = 10$ for the brick-and-mortar store. Customer returns are credited in full, and associated processing costs are disregarded ($r = p$).

4.4.2 Results

In this section, we present the computational findings and the corresponding business insights. We present the retailer’s computational outcomes across various scenarios and subsequently conduct a sensitivity analysis for these cases. In one-way sensitivity analysis, all cost or model parameters are held constant at the base case scenario, while only one group of parameters is systematically altered at a time. The results are then observed accordingly.

Table 19 shows the profit and ordering decisions corresponding to varying periods in a season. It demonstrates that, with all other parameters held constant, the average profit per period increases as the number of periods in a season rises when utilizing our omnichannel model with return flow.

Table 19: Profit and ordering decisions for various number of periods.

# of Periods	Total Season Profit	Average Profit per Period	Initial Season Order	Total Shipment to Brick-and-mortar Store
2	196.73	98.37	9.00	3.02
3	334.71	111.57	12.00	3.56
4	461.77	115.44	16.00	4.71
5	587.42	117.48	19.00	5.54
6	708.01	118.00	22.00	6.37

We demonstrate in Table 20 that when the mean demand for online sales exceeds that of the brick-and-mortar store, the profit is higher than in the reverse scenario. Online sales consistently yield higher profitability for a retailer as they exclude shipment costs associated with offline sales.

When we compare the mean demands of the online and offline channels, the impact of return rates is also crucial and must be considered alongside the mean demand.

Table 20: Profit and ordering decisions for various means of demand.

λ_w	λ_b	Total Season Profit	Initial Season Order	Total Shipment to Brick-and-mortar Store
3	2	461.77	16.00	4.71
2	4	564.99	19.00	11.28
4	2	602.96	19.00	4.34
3	3	580.53	19.00	7.78
4	4	844.79	25.00	10.50

Table 21 presents various combinations of return rates when the mean demand for online sales exceeds that of offline sales. The highest observed profit (464.73\$) occurs when $\alpha_w = 0.3$, $\alpha_{wb} = 0$, and $\alpha_b = 0.2$. This result can be attributed to the highest mean demand associated with online sales. When most returns are directed back to the warehouse, the opportunity for re-selling is maximized. This argument is supported by the lowest profit, 431.05\$, observed when the highest return rate is to the brick-and-mortar store, especially when it does not have a similarly high mean demand. This situation arises in the case where $\alpha_w = 0.2$, $\alpha_{wb} = 0.1$, and $\alpha_b = 0.3$. In this scenario, most returns are directed to the brick-and-mortar store, where the mean demand is insufficient for reselling these returns. Additionally, as the salvage value of the brick-and-mortar store is lower than that of the warehouse, the resulting profit is the lowest.

Table 21: Profit and ordering decisions for various return rates.

α_w	α_{wb}	α_b	Total Season Profit	Initial Season Order	Total Shipment to Brick-and-mortar Store
0.2	0.1	0.2	463.24	16.00	4.71
0.1	0.2	0.2	460.37	16.00	4.02
0.0	0.3	0.2	447.83	16.00	3.58
0.3	0.0	0.2	464.73	16.00	5.55
0.2	0.1	0.3	431.05	15.00	3.92

Table 22 illustrates that as the fixed ordering cost and transshipment cost resulting from orders sent from the warehouse to the brick-and-mortar store increase, the profit decreases accordingly, as expected. An interesting observation is that when the fixed ordering cost is exceptionally high, the warehouse refrains from sending any orders to the brick-and-mortar store, effectively reducing the initial season order and

channeling all sales to the online store. However, the brick-and-mortar store still receives returns from online sales, which are resold. Of course, the retailer's profit in this scenario is significantly lower compared to other cases.

Table 22: Fixed ordering and transshipment costs for orders from the warehouse to the Brick-and-mortar store.

		Total Season Profit	Initial Season Order	Total Shipment to Brick-and-mortar Store
Fixed Ordering Cost	0	518.43	16.00	5.03
	25	463.20	16.00	4.70
	50	426.01	16.00	4.60
	100	370.05	15.00	4.05
	200	321.08	11.00	0.00
Transshipment Cost	0	481.44	16.00	4.81
	4	461.77	16.00	4.71
	10	436.84	15.00	4.11
	20	398.02	15.00	3.82
	30	362.35	14.00	3.28

Finally, Table 23 presents different scenarios for the holding cost, lost sales cost, and salvage value. As anticipated, profits decrease with rising holding or lost sales costs and increase with a higher salvage value.

Table 23: Sensitivity analysis for holding cost, lost sales cost, and salvage value.

	w	b	Total Season Profit	Initial Season Order	Total Shipment to Brick-and-mortar Store
Holding Cost	2	20	462.57	16.00	4.69
	2	2	566.56	18.00	6.59
	4	4	520.43	16.00	5.70
	8	9	412.49	15.00	5.48
	16	18	265.46	12.00	4.26
Lost Sales Cost	20	10	462.37	16.00	4.70
	10	20	450.86	16.00	5.03
	10	10	470.95	15.00	4.43
	0	20	457.81	15.00	4.85
	20	0	486.86	15.00	4.07
Salvage Value	20	10	461.77	16.00	4.71
	20	15	467.40	16.00	4.76
	15	20	463.80	15.00	4.59
	15	15	458.80	15.00	4.50
	0	0	418.97	14.00	4.18

4.5 *Conclusion*

In the final chapter of this dissertation, we introduced an omnichannel model in which a retailer operates through two distinct sales channels: online and offline. In this model, the retailer places a pre-season order and subsequently makes decisions regarding inventory transshipment to the brick-and-mortar store in each period. The complexity of this model arises from the omnichannel structure, presenting two optimal decisions: one for the quantity to be transshipped and another for the initial season order.

In this system, customers can return online sales to both the warehouse and the brick-and-mortar store. However, offline sales can only be returned to the brick-and-mortar store. Consequently, the brick-and-mortar store experiences two distinct return flows; notably, the return quantity may exceed its sales quantity. It is essential to highlight that there are three return flows in the system, requiring the retailer to decide on the initial season order and transshipment quantity to the brick-and-mortar store accordingly.

We conducted a one-way sensitivity analysis under various parameter settings. Initially, we demonstrate that as the number of periods in the season increases, the average profit per period increases when our model is used. Furthermore, we underscore the significance of return rates alongside the mean demand values for both channels. When excess returns occur in a channel with a low mean demand value, profits decline, given the lower chance of reselling returns in that particular channel. Additionally, if the channel's salvage value is lower, the negative impact on profit can be higher. Moreover, increased fixed ordering costs or transshipment costs lead to decreased transshipment quantity, resulting in lower profits. Lastly, profits decrease with rising holding and lost sales costs, while they increase with a higher salvage value.

CHAPTER V

PROOF OF PROPOSITIONS IN CHAPTER 1

5.1 *Proof of Proposition 1 in Chapter 1*

[Proof] of Proposition 1: We will complete the proof in two steps. First, we will show that the claim of the proposition holds for $t = T$. Next, we will show that the claim holds for $t - 1$, assuming it holds for t . These two steps ensure that the claim holds $\forall 1 \leq t \leq T$.

Step 1: Using the transition of the state variables I_{T+1} and SQ_T , we can rewrite the profit-to-go function for period T multiplied by -1 as follows:

$$\begin{aligned}
 -P_T(I_T, SQ_{T-1}) &= \min_{S_T \geq I_T} \{ \bar{c}(S_T - I_T) + r\mathbb{E}[R_T] + J(S_T, SQ_{T-1}) \\
 &\quad - \mathbb{E}_{D_T, R_T}[P_{T+1}(I_{T+1}, SQ_T)] \\
 &\quad - p\mathbb{E}_{D_T, R_T}[SQ_T] \} \\
 &= \min_{S_T \geq I_T} \{ \bar{c}(S_T - I_T) + r \alpha SQ_{T-1} \\
 &\quad + h\mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] \\
 &\quad + b\mathbb{E}_{D_T, R_T}[(D_T - S_T - R_T)^+] \\
 &\quad - \mathbb{E}_{D_T, R_T}[P_{T+1}(S_T + R_T - D_T, \min\{S_T + R_T, D_T\} + I_T^-)] \\
 &\quad - p\mathbb{E}_{D_T, R_T}[\min\{S_T + R_T, D_T\} + I_T^-] \}
 \end{aligned} \tag{29}$$

Replacing the terminal profit function with equation (5) and using the expectation

of the binomial distribution, one can further expand $P_T(I_T, SQ_{T-1})$ as:

$$\begin{aligned}
-P_T(I_T, SQ_{T-1}) = & \min_{S_T \geq I_T} \{ \bar{c}(S_T - I_T) + r \alpha SQ_{T-1} + h \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] \\
& + b \mathbb{E}_{D_T, R_T}[(D_T - S_T - R_T)^+] \\
& + c \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^-] \\
& - s \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
& + r \alpha \mathbb{E}_{D_T, R_T}[\min\{S_T + R_T, D_T\} + I_T^-] \\
& - \alpha(s - r) \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^-] \\
& - p \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^- + \min\{S_T + R_T, D_T\} + I_T^-] \}
\end{aligned} \tag{30}$$

Following similar steps, one could also expand the cost-to-go function for period T as follows:

$$\begin{aligned}
C_T(I_T, SQ_{T-1}) = & \min_{S_T \geq I_T} \{ \bar{c}(S_T - I_T) + r \alpha SQ_{T-1} \\
& + h \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] \\
& + b \mathbb{E}_{D_T, R_T}[(D_T - S_T - R_T)^+] \\
& + c \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^-] \\
& - s \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
& + r \alpha \mathbb{E}_{D_T, R_T}[\min\{S_T + R_T, D_T\} + I_T^-] \\
& + \alpha(r - s)(S_T + R_T - D_T)^- \}
\end{aligned} \tag{31}$$

Let's define

$$\begin{aligned}
f_T(S_T, I_T, SQ_{T-1}) := & \bar{c}(S_T - I_T) + r \alpha SQ_{T-1} \\
& + h \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] \\
& + b \mathbb{E}_{D_T, R_T}[(D_T - S_T - R_T)^+] \\
& + c \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^-] \\
& - s \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
& + r \alpha \mathbb{E}_{D_T, R_T}[\min\{S_T + R_T, D_T\} + I_T^-] \\
& + \alpha(r - s)(S_T + R_T - D_T)^-.
\end{aligned} \tag{32}$$

Notice that

$$\begin{aligned}
(S_T + R_T - D_T)^- + \min\{S_T + R_T, D_T\} &= -\min\{S_T + R_T - D_T, 0\} \\
&+ \min\{S_T + R_T, D_T\} = D_T.
\end{aligned} \tag{33}$$

Hence equation (30) could be rewritten as:

$$\begin{aligned}
-P_T(I_T, SQ_{T-1}) &= \min_{S_T \geq I_T} \{f_T(S_T, I_T, SQ_{T-1}) - p \mathbb{E}_{D_T, R_T}[D_T + I_T^-]\} \\
&= \min_{S_T \geq I_T} \{f_T(S_T, I_T, SQ_{T-1})\} - p(\mu + I_T^-) \\
&= C_T(I_T, SQ_{T-1}) - p(\mu + I_T^-),
\end{aligned} \tag{34}$$

where the first equality follows from the fact that the second term inside the minimization function does not depend on the decision variable S_T and the second equality uses the expanded form for the cost-to-go function in equation (31). Thus completing the proof that the claim of the proposition holds for $t = T$.

Step 2: Next we are going to show that the claim holds for period $t - 1$ assuming it holds for period t . In other words assume that the following is true for all I_t and SQ_{t-1} :

$$P_t(I_t, SQ_{t-1}) = p(\mu(T - t + 1) + I_t^-) - C_t(I_t, SQ_{t-1}).$$

Using this assumption, one could write the profit-to-go function for period $t - 1$

as:

$$\begin{aligned}
-P_{t-1}(I_{t-1}, SQ_{t-2}) = & \min_{S_{t-1} \geq I_{t-1}} \{ \bar{c}(S_{t-1} - I_{t-1}) + r \alpha SQ_{t-2} \\
& + h \mathbb{E}_{D_{t-1}, R_{t-1}} [(S_{t-1} + R_{t-1} - D_{t-1})^+] \\
& + b \mathbb{E}_{D_{t-1}, R_{t-1}} [(D_{t-1} - S_{t-1} - R_{t-1})^+] \\
& + \mathbb{E}_{D_{t-1}, R_{t-1}} [C_t(I_t, SQ_{t-1})] \\
& - p \mathbb{E}_{D_{t-1}, R_{t-1}} [\mu(T - t + 1) + I_t^-] \\
& - p \mathbb{E}_{D_{t-1}, R_{t-1}} [\min\{S_{t-1} + R_{t-1}, D_{t-1}\} + I_{t-1}^-] \}
\end{aligned} \tag{35}$$

The cost-to-go function for period $t - 1$ is:

$$C_{t-1}(I_{t-1}, SQ_{t-2}) = \min_{S_{t-1} \geq I_{t-1}} \{ f_{t-1}(S_{t-1}, I_{t-1}, SQ_{t-2}) \}, \tag{36}$$

where

$$\begin{aligned}
f_{t-1}(S_{t-1}, I_{t-1}, SQ_{t-2}) := & \bar{c}(S_{t-1} - I_{t-1}) + r \alpha SQ_{t-2} \\
& + h \mathbb{E}_{D_{t-1}, R_{t-1}} [(S_{t-1} + R_{t-1} - D_{t-1})^+] \\
& + b \mathbb{E}_{D_{t-1}, R_{t-1}} [(D_{t-1} - S_{t-1} - R_{t-1})^+] \\
& + \mathbb{E}_{D_{t-1}, R_{t-1}} [C_t(I_t, SQ_{t-1})].
\end{aligned} \tag{37}$$

Now, using the last two equalities and recalling that $I_t = S_{t-1} + R_{t-1} - D_{t-1}$, equation (35) could be rewritten as:

$$\begin{aligned}
-P_{t-1}(I_{t-1}, SQ_{t-2}) = & \min_{S_{t-1} \geq I_{t-1}} \{ f_{t-1}(S_{t-1}, I_{t-1}, SQ_{t-2}) \\
& - p(\mu(T - t + 1) + \mathbb{E}_{D_{t-1}, R_{t-1}} [(S_{t-1} + R_{t-1} - D_{t-1})^-] \\
& + \min\{S_{t-1} + R_{t-1}, D_{t-1}\} + I_{t-1}^-) \} \\
= & \min_{S_{t-1} \geq I_{t-1}} \{ f_{t-1}(S_{t-1}, I_{t-1}, SQ_{t-2}) - p(\mu(T - t + 1) \\
& + \mathbb{E}_{D_{t-1}, R_{t-1}} [D_{t-1} + I_{t-1}^-]) \} \\
= & \min_{S_{t-1} \geq I_{t-1}} \{ f_{t-1}(S_{t-1}, I_{t-1}, SQ_{t-2}) \} - p(\mu(T - t + 2) + I_{t-1}^-) \\
= & C_{t-1}(I_{t-1}, SQ_{t-2}) - p(\mu(T - t + 2) + I_{t-1}^-),
\end{aligned}$$

where the third to last equality follows using an equivalent version of equation (33) for period t , and the second to last equality follows as the second part of the minimization function does not depend on the decision variable S_{t-1} , and the last equality is by equation (36). Thus we complete the proof that the claim of the proposition holds for period $t - 1$, assuming it holds for period t .

5.2 Proof of Proposition 2 in Chapter 1

[Proof] of Proposition 2: The proof is by induction. Let $t = T + 1$. Following Proposition 1 and the fact that $x^- = x^+ - x$, one can write that

$$\begin{aligned} P_{T+1}(I_{T+1}, SQ_T) = & pI_{T+1}^- - (c - s)\mathbb{E}_{R_{T+1}}(I_{T+1} + R_{T+1})^+ + c(I_{T+1} + \alpha SQ_T) \\ & - r\alpha SQ_T - \alpha(r - s)I_{T+1}^- \end{aligned} \quad (38)$$

Taking the derivative with respect to α yields:

$$\begin{aligned} \frac{\partial P_{T+1}(I_{T+1}, SQ_T)}{\partial \alpha} = & -(c - s)\frac{\partial \mathbb{E}_{R_{T+1}}(I_{T+1} + R_{T+1})^+}{\partial \alpha} \\ & - (r - c)SQ_T - (r - s)I_{T+1}^- \end{aligned}$$

Consider two random variables $R_{T+1}^1 \sim \text{Bin}(\alpha_1, SQ_T)$ and $R_{T+1}^2 \sim \text{Bin}(\alpha_2, SQ_T)$, where $\alpha_1 < \alpha_2$. In this case, R_{T+1}^2 is first-order stochastically dominant over R_{T+1}^1 , which implies that

$$\mathbb{E}_{R_{T+1}^1}(I_{T+1} + R_{T+1}^1)^+ \leq \mathbb{E}_{R_{T+1}^2}(I_{T+1} + R_{T+1}^2)^+$$

Since $r \geq c \geq s$, we deduce that the terminal profit function is decreasing in α . For the second part of the proof, assume that $P_{t+1}(I_{t+1}, SQ_t)$ is decreasing in α for all I_{t+1} and SQ_t . Now, using Proposition 1, equations (3) and (6) and utilizing the envelope function and dominated convergence theorems, one can show that:

$$\begin{aligned} \frac{\partial P_t(I_t, SQ_{t-1})}{\partial \alpha} = & -\frac{\partial C_t(I_t, SQ_{t-1})}{\partial \alpha} \\ = & -(r - b)SQ_{t-1} - (h + b)\frac{\partial \mathbb{E}_{D_t, R_t}(S_t^* + R_t - D_t)^+}{\partial \alpha} \\ & - \mathbb{E}_{D_t, R_t}\left[\frac{\partial C_{t+1}(I_{t+1}, SQ_t)}{\partial \alpha}\right] \end{aligned}$$

The first term is negative as $r \geq b$. Using similar arguments as in part one, it is easy to see that the second term is also negative. Finally, due to the induction assumption, the last term is also negative. Thus we deduce that the profit-to-go function is decreasing in α . Next, we present Lemma 1, along with its proof, which is used later in the proof of Proposition 3. Let $\mathcal{C}_t$ denote the set of feasible states and actions:

$$\mathcal{C}_t := \{(I_t, -SQ_{t-1}, S_t) \mid I_t \in \mathbb{Z}, SQ_{t-1} \in \mathbb{Z}^+, S_t \geq I_t\}.$$

Then $\mathcal{C}_t$ is a lattice.

[Proof] of Lemma 1: Let x_t^1, x_t^2 be elements of $\mathcal{C}_t$.

Consider

$$x_t^1 \wedge x_t^2 := (\min(I_t^1, I_t^2), \min(-SQ_{t-1}^1, -SQ_{t-1}^2), \min(S_t^1, S_t^2)).$$

Clearly $\min(I_t^1, I_t^2) \in \mathbb{Z}$ and $\min(-SQ_{t-1}^1, -SQ_{t-1}^2) \in \mathbb{Z}^+$.

Moreover, we know that $S_t^1 \geq I_t^1$ and $S_t^2 \geq I_t^2$.

Thus $\min(S_t^1, S_t^2) \geq \min(I_t^1, I_t^2)$, which implies that $x_t^1 \wedge x_t^2 \in \mathcal{C}_t$.

Next, consider

$$x_t^1 \vee x_t^2 := (\max(I_t^1, I_t^2), \max(-SQ_{t-1}^1, -SQ_{t-1}^2), \max(S_t^1, S_t^2)).$$

Now, $\max(I_t^1, I_t^2) \in \mathbb{Z}$ and $\max(-SQ_{t-1}^1, -SQ_{t-1}^2) \in \mathbb{Z}^+$.

Also, one can conclude that $\max(S_t^1, S_t^2) \geq \max(I_t^1, I_t^2)$, implying $x_t^1 \vee x_t^2 \in \mathcal{C}_t$. Since both $x_t^1 \wedge x_t^2$ and $x_t^1 \vee x_t^2$ are elements of $\mathcal{C}_t$, we can conclude that $\mathcal{C}_t$ is a lattice. Now,

we are ready to prove Proposition 3: [Proof] of Proposition 3:

We first rewrite the cost-to-go function using equations (3), (4), (6), and (7):

$$\begin{aligned}
C_T(I_T, SQ_{T-1}) &= \min_{S_T \geq I_T} \{ \bar{c}(S_T - I_T) + r \alpha SQ_{T-1} \\
&\quad + h \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] \\
&\quad + b \mathbb{E}_{D_T, R_T}[(D_T - S_T - R_T)^+] \\
&\quad + c \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^-] \\
&\quad - s \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
&\quad + r \alpha \mathbb{E}_{D_T, R_T}[\min\{S_T + R_T, D_T\} + I_T^-] \\
&\quad - \alpha(s - r) \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^-] \} \\
&= \min_{S_T \geq I_T} \{ \bar{c}(S_T - I_T) + r \alpha SQ_{T-1} \\
&\quad + (h + b) \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] \\
&\quad + b(\mu - S_T - \alpha SQ_{T-1}) \\
&\quad + (c - s) \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})^+] \\
&\quad - c \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T - D_T + R_{T+1})] \\
&\quad + r \alpha \mathbb{E}_{D_T, R_T}[\min\{S_T + R_T, D_T\} + I_T^-] \\
&\quad - \alpha(s - r) \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^-] \}
\end{aligned}$$

where the last equality follows from using the equation $(-a)^+ = a^- = a^+ - a$ for any $a \in \mathbb{R}$. Now, we derive a set of useful equalities.

(i)

$$\begin{aligned}
\mathbb{E}_{D_T, R_T}[\min\{S_T + R_T, D_T\} + I_T^-] &= S_T + \alpha SQ_{T-1} \\
&\quad - \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] + I_T^-
\end{aligned}$$

(ii)

$$\begin{aligned}
\mathbb{E}[R_{T+1}] &= \mathbb{E}_{D_T, R_T}[\mathbb{E}[R_{T+1}|D_T, R_T]] = \mathbb{E}_{D_T, R_T}[\alpha(\min\{S_T + R_T, D_T\} + I_T^-)] \\
&= \alpha \mathbb{E}_{D_T, R_T}[S_T + R_T - (S_T + R_T - D_T)^+ + I_T^-] \\
&= \alpha S_T + \alpha^2 S Q_{T-1} - \alpha \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] \\
&\quad + \alpha I_T^-
\end{aligned}$$

(iii)

$$\mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^-] = \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] - S_T - \alpha S Q_{T-1} + \mu$$

Using (i)-(iii) and rearranging terms gives us:

$$\begin{aligned}
C_T(I_T, S Q_{T-1}) &= \min_{S_T \geq I_T} \{ \mu(b + c - \alpha(s - r)) + I_T^- \alpha(r - c) \\
&\quad + \bar{c}(S_T - I_T) - S_T(b + (1 + \alpha)c - \alpha s) \\
&\quad + \alpha S Q_{T-1}(r - b - c(1 + \alpha) + \alpha s) \\
&\quad + (h + b + \alpha(c - s)) \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+] \\
&\quad + (c - s) \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T + R_{T+1} - D_T)^+] \} \\
&:= \min_{S_T \geq I_T} \{ \tilde{C}_T(I_T, S Q_{T-1}, S_T) \}
\end{aligned}$$

Now, we would like to show that $\tilde{C}_T(I_T, S Q_{T-1}, S_T)$ is submodular on $\mathcal{C}_T$. In order to show that, let $x_T^1 := (I_T^1, -S Q_{T-1}^1, S_T^1)$ and $x_T^2 := (I_T^2, -S Q_{T-1}^2, S_T^2)$ be elements of $\mathcal{C}_T$. We need to prove that:

$$\tilde{C}_T(x_T^1) + \tilde{C}_T(x_T^2) \geq \tilde{C}_T(x_T^1 \wedge x_T^2) + \tilde{C}_T(x_T^1 \vee x_T^2). \quad (39)$$

Define $W_T(x_T) := \mathbb{E}_{D_T, R_T}[(S_T + R_T - D_T)^+]$ and $Y_T(x_T) := \mathbb{E}_{D_T, R_T, R_{T+1}}[(S_T + R_T + R_{T+1} - D_T)^+]$ where $x_T \in \mathcal{C}_T$ and $R_T \sim \text{Bin}(S Q_{T-1}, \alpha)$. Using the definition of $\tilde{C}_T(\cdot)$ and the fact that $y + z = \min(y, z) + \max(y, z) \forall y, z \in \mathbb{R}$, rearranging the terms of inequality (.11) gives us:

$$\begin{aligned}
& \bar{c}(S_T^1 - I_T^1) + \bar{c}(S_T^2 - I_T^2) + (h + b + \alpha c - \alpha s)(W_T(x_T^1) + W_T(x_T^2)) \\
& + (c - s)(Y_T(x_T^1) + Y_T(x_T^2)) \geq \bar{c}(\min(S_T^1, S_T^2) - \min(I_T^1, I_T^2)) \\
& \bar{c}(\max(S_T^1, S_T^2) - \max(I_T^1, I_T^2)) + (h + b + \alpha(c - s))(W_T(x_T^1 \wedge x_T^2) + W_T(x_T^1 \vee x_T^2)) \\
& + (c - s)(Y_T(x_T^1 \wedge x_T^2) + Y_T(x_T^1 \vee x_T^2)).
\end{aligned} \tag{40}$$

Part 1: Without loss of generality, assume that $S_T^1 \leq S_T^2$. If $-SQ_{T-1}^1 \leq -SQ_{T-1}^2$, then $W_T(x_T^1 \wedge x_T^2) = W_T(x_T^1)$ and $W_T(x_T^1 \vee x_T^2) = W_T(x_T^2)$. Thus $W_T(x_T^1) + W_T(x_T^2) \geq W_T(x_T^1 \wedge x_T^2) + W_T(x_T^1 \vee x_T^2)$.

Otherwise $W_T(x_T^1 \wedge x_T^2) = \mathbb{E}_{D_T, R_T^1}[(S_T^1 + R_T^2 - D_T)^+]$ and $W_T(x_T^1 \vee x_T^2) = \mathbb{E}_{D_T, R_T^2}[(S_T^2 + R_T^1 - D_T)^+]$ where $R_T^i \sim \text{Bin}(SQ_{T-1}^i, \alpha)$. Using [89], since $SQ_{T-1}^1 \leq SQ_{T-1}^2$, we conclude that R_T^2 dominates R_T^1 under first-order stochastic dominance (FSD).

Consider $u_T(x) := \mathbb{E}_{D_T}[(S_T^2 + x - D_T)^+ - (S_T^1 + x - D_T)^+]$. Since $S_T^1 \leq S_T^2$, $u_T(x)$ is a non-decreasing function. Thus, by [90], we can deduce that

$$\begin{aligned}
\mathbb{E}_{D_T, R_T^2}[(S_T^2 + R_T^2 - D_T)^1 - (S_T^1 + R_T^2 - D_T)^+] & \geq \mathbb{E}_{D_T, R_T^1}[(S_T^2 + R_T^1 - D_T)^1 \\
& - (S_T^1 + R_T^1 - D_T)^+]
\end{aligned}$$

which implies that $W_T(x_T^2) + W_T(x_T^1) \geq W_T(x_T^1 \wedge x_T^2) + W_T(x_T^1 \vee x_T^2)$.

Part 2: Following similar steps of part 1, we now show that

$$Y_T(x_T^2) + Y_T(x_T^1) \geq Y_T(x_T^1 \wedge x_T^2) + Y_T(x_T^1 \vee x_T^2).$$

Again, without loss of generality, assume that $S_T^1 \leq S_T^2$.

The case with $-SQ_{T-1}^1 \leq -SQ_{T-1}^2$ is obvious. For $SQ_{T-1}^1 \leq SQ_{T-1}^2$, let $R_{T+1}^i(x) \sim \text{Bin}(\{S_T^i + x, D_T\} + I_T^-, \alpha)$ for any $x \in \mathbb{Z}^+$. Define

$$\begin{aligned}
V_T(x) &:= \mathbb{E}_{D_T, R_{T+1}^2}[(S_T^2 + x + R_{T+1}^2(x) - D_T)^+ | x] \\
& - \mathbb{E}_{D_T, R_{T+1}^1}[(S_T^1 + x + R_{T+1}^1(x) - D_T)^+ | x]
\end{aligned}$$

Since $S_T^1 \leq S_T^2$, $R_{T+1}^2(x)$ dominates $R_{T+1}^1(x)$ under FSD for any $x \in \mathbb{Z}^+$. Thus it is trivial to see that $V_T(x)$ is a non-decreasing function. Also recalling that R_T^2 dominates R_T^1 under FSD leads to $\mathbb{E}_{R_T^2}[V_T(R_T^2)] \geq \mathbb{E}_{R_T^1}[V_T(R_T^1)]$ which further implies that:

$$\begin{aligned} & \mathbb{E}_{R_T^2}[\mathbb{E}_{D_T, R_{T+1}^2}[(S_T^2 + R_T^2 + R_{T+1}^2(R_T^2) - D_T)^+ | R_T^2]] \\ & - \mathbb{E}_{R_T^2}[\mathbb{E}_{D_T, R_{T+1}^1}[(S_T^1 + R_T^2 + R_{T+1}^1(R_T^2) - D_T)^+ | R_T^2]] \\ & \geq \mathbb{E}_{R_T^1}[\mathbb{E}_{D_T, R_{T+1}^2}[(S_T^2 + R_T^1 + R_{T+1}^2(R_T^1) - D_T)^+ | R_T^1]] \\ & - \mathbb{E}_{R_T^1}[\mathbb{E}_{D_T, R_{T+1}^1}[(S_T^1 + R_T^1 + R_{T+1}^1(R_T^1) - D_T)^+ | R_T^1]] \end{aligned}$$

thus showing that $Y_T(x_T^2) + Y_T(x_T^1) \geq Y_T(x_T^1 \wedge x_T^2) + Y_T(x_T^1 \vee x_T^2)$.

Using the results obtained in parts 1 and 2, and recalling the assumptions that $K = 0$ and $c \geq s$, we can conclude that inequality (.12) holds completing the proof that $\tilde{C}_T(I_T, SQ_{T-1}, S_T)$ is submodular on $\mathcal{C}_T$.

Following [91], we can thus deduce that the set of optimal net inventory after replenishment decisions, $S_T^*(I_T, SQ_{T-1})$, is non-decreasing in I_T and non-increasing in SQ_{T-1} by definition of $\mathcal{C}_T$ in Lemma 1. Moreover, preservation of submodularity under minimization (see, [92]) implies that $C_T(I_T, SQ_{T-1})$ is also submodular on $(I_T, -SQ_{T-1})$.

CHAPTER VI

CONCLUSIONS

We investigate the impact of maintaining detailed sales and return information on retailers' profits throughout a finite selling season. Our model incorporates returns that are dependent on the previous sales quantity. Retailers can effectively reduce order frequency and mitigate backorders or lost sales by utilizing returns as a secondary supply source. In the second chapter, we introduce a simple inventory control model where stochastic returns depend on the most recent previous sales. Subsequently, in the third chapter, we expand this model to incorporate returns arriving from multiple periods, enhancing it with extensions such as order capacity constraints and markdown pricing. Finally, in the fourth chapter, we further extend the model to include returns dependent on previous sales from multiple channels with varying return rates, thereby adopting an omnichannel approach tailored for fast fashion retail.

The second chapter demonstrates that retailers can leverage product returns to maximize their total profit over a finite selling season. Retailers can effectively reduce order frequency and backorders by utilizing returns as a secondary supply source. A comparison with a retailer's profit ignoring the dependency of returns on previous sales quantifies the value of incorporating detailed return information. In a base case scenario, we find that the profit penalty of disregarding this dependency could be as high as 23%. Furthermore, this value may escalate significantly, particularly when the fixed ordering cost, backorder cost, or the return probability is high.

In the third chapter, we release the return policy for the customers and allow them to return the purchased products in various periods. This relaxation results in an expansion of the tracked periods for sales and return information. However, the increase

in state variables introduced the curse of dimensionality in dynamic programming, diverting the introduction of the ADP algorithm to address this challenge. Our findings demonstrate that the ADP algorithm significantly reduced solution time, decreasing it from 514.32 minutes to 31.9 minutes in the base case scenario while maintaining an average optimality gap of just 1%. Through a sensitivity analysis, we established that return rates substantially impact profit. Specifically, receiving returns in earlier periods enhances overall profitability for retailers. This effect becomes even more pronounced in situations involving order capacity or mark-down pricing.

In the fourth chapter, we introduce an omnichannel structure. Our model involves two distinct sales channels for the retailer: a warehouse for online sales and a brick-and-mortar store for offline sales. Our system incorporates three return flows, as customers can return products purchased online to the warehouse or the brick-and-mortar store. These two return flows from online sales and the return flow from offline sales back to the brick-and-mortar store constitute the three return flows in our system. Using our model the retailer gives two optimal decisions, The first one is the initial pre-season order to fulfill the demand of both channels during the season as it can not be repeated.

The second decision involves determining the transshipment quantity from the warehouse to the brick-and-mortar store for each period during the season. Given fixed ordering and transshipment costs per unit ordered for this quantity, the model enables the retailer to optimize transshipment considering two different factors: the cost of lost sales and the salvage value of the channels. We highlight the significance of return rates in conjunction with the mean demand values for both channels. In this context, the brick-and-mortar store may experience returns exceeding its sales due to multiple return flows from both online and offline channels. When excess returns are directed to a channel with a lower demand value, the likelihood of reselling

those returned items diminishes, negatively impacting overall profit. Moreover, increased fixed ordering costs or transshipment costs lead to decreased transshipment quantity, resulting in lower profits. Lastly, profits decrease with rising holding and lost sales costs, while they increase with a higher salvage value. Furthermore, higher fixed ordering costs or transshipment costs are associated with reduced transshipment quantity, resulting in lower profits.

Retailers, specifically those in the fast fashion industry, which operates under the main assumptions used in this dissertation, could benefit from the results shared for the related issues. First, retailers should reconsider their inventory replenishment decisions by incorporating return flows. For example, slightly increasing the initial order sizes, compared to a net demand setting, could be a smart strategy over the rest of the selling season, especially if the return rate and fixed ordering cost are high. Inflated initial order sizes could also be used to identify products with high return rates early in the season and react accordingly. Second, retailers could quantify the value of using detailed return information. Our proposal requires firms to keep separate meticulous records of both product sales and returns, which could be costly from IT and labor points of view. Moreover, product return process flows should be redesigned to collect this additional information. Our results highlight when and if these additional costs are worthwhile for the retailer.

Our results complement the existing literature in the field of inventory management under commercial returns in multiple directions. Most of the papers in this stream assume that returns are demand-dependent, which makes incoming returns independent of inventory management decisions made in previous periods. The stochastic dependency of returns on the previous sales as assumed in [30] and [25] translates into a dependency of returns and demand, as any unmet demand can be satisfied from an external supply source in these papers. In a two-period model for a perishable product, [27] assumes that returns depend only on the current period's sales.

In our previous sales-dependent return model, return quantity was indirectly affected by the retailer's actions. This fundamental difference in our formulation brings forth new insights. For example, our results suggest that raising the initial order amount a bit could save fixed ordering costs in the following periods due to returns from probabilistically higher sales amounts in the initial period. The finite horizon model coupled with a detailed terminal period cost structure, as compared to zero terminal cost assumption, is another unique feature of this research. Under these distinct and realistic elements of our models, we quantify the value of incorporating returns in inventory management as in [29], [30], and [25], which is increasing in the return rate ([32], [33]).

The results of this dissertation could alleviate some of the environmental concerns prominent in the retail sector. After the selling season ends, leftover inventory mostly lacks salvage value due to rapid trend changes or seasonal reasons. Hence, they could be a costly burden for the firm even after markdown sales and could eventually end up in a landfill. According to [4], five billion pounds of returned goods end up in the US landfills each year, and this landfill waste from returns alone contributes 15 million metric tons of carbon dioxide to the atmosphere. Our model in the first chapter results in less leftover inventory (62% in the backorder setting and 87% in the lost sales setting) compared to a model that ignores this dependency. Thus, our models could indirectly lower the environmental impact of a retailer by limiting landfill waste. Faced with such a retailer that treats returns as a second source of supply, the upstream firms are expected to lower their respective orders accordingly which, in turn, could decrease the leftover inventory at the rest of the supply chain as well. As the retailer could absorb some of the demand variability using returns, the bullwhip effect would be reduced leading to positive impacts on the environmental performance of supply chains [93]. Sharing detailed return information with the upstream parties could further reduce the bullwhip effect.

Future research focusing on detailed operational models of product returns has the potential to enhance our comprehension of the challenges faced by retailers. Our work can be extended along various avenues. Customers often opt for substitute products, particularly when uncertain about size or color during the online shopping experience, inadvertently contributing to an increase in returns. Enhancing inventory replenishment decisions can be achieved by refining and improving strategies based on substitute product information.



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