

Advanced concepts of piezoelectric patch-based energy harvesting and locally resonant bandgap formation in thin plates

by

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**Advanced concepts of piezoelectric patch-based energy harvesting and
locally resonant bandgap formation in thin plates**

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Graduate School of Sciences and Engineering

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To my beloved Niloofar

ABSTRACT

Vibrational energy harvesting using piezoelectric transducers has received significant attention over the past decade for enabling self-powered embedded devices in health monitoring applications, and thus reducing the maintenance costs of discarded batteries. In parallel, researchers have extensively investigated the piezoelectric shunt damping technique for efficiently attenuating the vibrations of flexible structures. More recently, in conjunction with shunt damping, locally resonant piezoelectric structures have been employed in the concept of metamaterial-based electromechanical structures to create bandgaps. This work is centered on the electromechanical modeling of piezoelectric patches integrated to thin plate-like structures, which are widely available in aerospace, space, and automotive industries, for the applications of vibration-based energy harvesting and locally resonant bandgap formation.

Vibration-based energy harvesters in the literature are typically made of cantilevered beams with piezoceramic layers which generate power under the base excitations. As alternative to these beam-based harvesters, piezoelectric patches can be integrated to plate structures in order to directly exploit the vibration modes of the host system without adding significant mass and volumetric occupancy of the cantilevers, thus enabling a compact two-dimensional energy harvesting system. Meanwhile, in order to charge the storage components such as a battery or a supercapacitor, a stable DC electrical signal is required. However, majority of the studies on beam and plate-like harvesters have used a simple resistance as an external electrical load. Generally, existing studies on the electrical part have oversimplified the mechanical models, while the works on the mechanical domain have overlooked the practical electrical circuits. Considering these needs, the first part of this thesis aims to address

those problems by developing unified analytical and numerical frameworks to fully bridge the electrical and mechanical domains, and hence formulating a complete electromechanically coupled system. In chapter 2, a multi-modal equivalent circuit model (ECM) for a single piezo-patch energy harvester on a thin plate is developed using the analogy between the second-order circuitry equation and the governing electromechanical equation. The equivalent circuit parameters are obtained from the analytical modal analysis solution of a rectangular plate with a piezo-patch, and account for multiple vibration modes of the host structure as well as the two-way electromechanical coupling. In chapter 3, an equivalent impedance electroelastic model is developed by integrating the equivalent impedance load of the practical harvesting circuit into the electroelastic equations of the plate with multiple piezo-patch energy harvesters (MPEHs). Moreover, a generalized ECM for the MPEHs is established which is compatible with different interconnections of the piezo-patches (series, parallel, or mixed series-parallel configurations). Finally, a respective rectification technique as a solution for avoiding charge cancellations, due to opposite current signs of piezo-patches at higher vibration modes, is proposed.

In the second part of the thesis, a type of two-dimensional electromechanical metastructures composed of locally resonant piezo-patches integrated to a thin plate is introduced and investigated. Analogous to metamaterial-based mechanical structures where an array of purely mechanical resonators is integrated to the structure, many piezoelectric elements with resonant shunt circuits can be integrated to a thin plate in order to generate bandgaps in the host structure, enabling a compact and tunable shunt damping mechanism. In the literature of metamaterials, most studies assume an infinite structure made of repeated assembly of unit-cells and use Bloch theorem to model the bandgaps of structure. However, for practical implementation of locally resonant metamaterials where finite structures with distributed array of resonators are employed, a high-fidelity model is required to bridge the modal interaction of the host structure with the electromechanical behavior of the resonant piezoelectric elements.

To this end, chapter 4 presents an electroelastic modeling of a thin plate with array of piezo-patches connected to resonant shunt circuits for investigating the bandgap formation. Displacement FRF surface plots are presented for a set of piezoelectric materials such as ceramics and single crystals, where the bandgaps are observed at frequency bandwidth with a very low vibration amplitude. It is shown that bandgaps are formed at different target frequencies and that their size is widened by the increase in the thickness of the piezo-patches.

ÖZETÇE

Piezoelektrik transdüserleri kullanarak enerji hasatlama, son on yılda, özellikle, yapı titreşimlerinin izlenmesi uygulamalarında kullanılan, kendi kendine çalışan gömülü aygıtlara enerji sağlamak amacıyla kullanılmakta ve böylece kullanılan pillerin bakım maliyetlerinin önemli bir miktarda düşmesini sağlamaktadır. Buna paralel olarak, araştırmacılar, esnek yapıların titreşimlerini etkin bir şekilde azaltmak için piezoelektrik şant sönümleme tekniğini kapsamlı bir şekilde kullanmaktadırlar. Daha yakın zamanda, şant sönümleme ile birlikte, lokal rezonansa sahip piezoelektrik yapılar, farklı rezonans bantlarında sönümleme oluşturmak için meta materyal tabanlı elektromekanik yapılar kapsamında kullanılmaktadır. Yukarıda verilen uygulamalara destek sağlamak amacıyla, bu çalışma, havacılık, uzay ve otomotiv endüstrilerinde yaygın olarak kullanılan ince levha benzeri yapılara, farklı şekillerde entegre edilen piezoelektrik yamaların, elektromekanik modellemesine odaklanmaktadır. Bu çalışmalar titreşim esaslı enerji hasatlama ve lokal rezonans frekans bantlarında sönümleme uygulamalarına odaklanmıştır.

Literatürdeki titreşim esaslı enerji hasatlama çalışmaları çoğunlukla, taban titreşiminden güç üreten piezo-seramik plakalardan oluşmuş dirsekli kirişler üzerinedir. Bu kiriş tabanlı hasatlayıcılara alternatif olarak, piezoelektrik yamalar plaka yapılara entegre edilerek, yapıların hacimsel miktarını önemli miktarda artırmadan, ana sistemin titreşim modlarını doğrudan kullanabilir ve böylece kompakt iki boyutlu enerji hasat sistemi olarak kullanılabilirler . Bu arada, pil veya kondansatör gibi depolama sistemlerini şarj etmek için kararlı bir DC elektrik sinyali gerekir. Bununla birlikte, kiriş ve levha benzeri hasatlayıcılar ile ilgili yapılan çalışmaların çoğu, basit bir direnç harici bir elektrik yükü olarak kullanılmaktadır. Genel olarak, elektrik alanındaki

mevcut çalışmalar mekanik modelleri aşırı basitleştirirken, mekanik alan üzerindeki çalışmalar pratik elektrik devrelerini tam olarak modelleyememektedir. Bu tezin ilk kısmı, bu ihtiyaçları göz önüne alarak, elektriksel ve mekanik alanları tam olarak entegre etmek üzere, analitik ve sayısal modeller geliştirerek bu sorunları çözmeyi amaçlamaktadır. Tezin 2. bölümünde, ince levhadaki tek bir piezo-yama enerji hasatlayıcısı için çok-modlu eşdeğer devre modeli (ECM) devre denklemi ve elektromekanik denklem arasındaki benzetim kullanılarak geliştirilmiştir. Eşdeğer devre parametreleri, piezo-yama ile dikdörtgen plakanın analitik modal analiz çözümünden elde edilir ve iki yönlü elektromekanik bağlantının yanı sıra, ana yapının çoklu titreşim modlarını da hesaba katmaktadır. Bölüm 3'te, pratik hasat devresinin eşdeğer empedans yükünü, plakanın elektroelastik denklemlerine, çoklu piezo yama enerjisi hasatlayıcıyla (MPEH'ler) entegre ederek eşdeğer empedans elektroelastik modeli geliştirilmiştir. Ayrıca, piezo-yamaların farklı bağlantıları (seri, paralel veya karışık seri-paralel konfigürasyonlar) ile uyumlu MPEH'ler için genelleştirilmiş bir ECM oluşturulmuştur. Son olarak, daha yüksek titreşim modlarında piezo-yama zıt akım işaretlerine bağlı olarak, yük iptallerinden kaçınmak için çözüm olarak ilgili bir düzeltme tekniği önerilmektedir.

Tezin ikinci bölümünde, yerel plakalara entegre edilmiş lokal rezonanslı piezo-yamalar içeren iki boyutlu elektromekanik meta-materyaller için çalışmalar ve ilgili sonuçlar verilmiştir. Meta-materyal tabanlı mekanik yapılara benzer şekilde, rezonant şant devrelerine sahip birçok piezoelektrik eleman, ana yapıda, farklı rezonans bant frekanslarında sönümlenme aralıkları oluşturmak için ince bir plakaya entegre edilebilir. Meta-material literatüründe, çoğu çalışma, birim hücrelerin tekrar tekrar bir araya getirilerek, sonsuz bir yapı varsayımını kullanarak, yapının frekans bant aralıklarını modellemek için Bloch teoremini kullanmaktadır. Bununla birlikte, çoklu rezonatörler dizisiyle sonlu yapıların kullanıldığı lokal olarak rezonant metamateryallerin pratik olarak uygulanması için, rezonant piezoelektrik elemanların elektromekanik davranışı ile ana yapıya ait modal etkileşimi anlayabilmek için yüksek kalitede modellere ihtiyaç duyulmaktadır. Bu amaçla, bölüm 4, frekans bant genişliği oluşumunu araştırmak

için rezonant şant devrelerine bağı sıralı piezo-yamaların ince bir plakaya entegre edildiklerinde, elektroelastik modellemesini detaylı bir şekilde anlatmaktadır. FRF yüzey grafikleri seramik ve tek kristaller gibi bir dizi piezoelektrik malzeme için incelenmiş; ayrıca hedeflenen frekans bant aralıklarında çok düşük bir titreşim miktarı gözlemlenmiştir. Son olarak da, bant aralıklarının farklı hedef frekanslarda oluştuğu ve boyutlarının piezo-yama kalınlığındaki artışla genişlediği de gösterilmiştir.



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TABLE OF CONTENTS

List of Tables	xiv
List of Figures	xv
Nomenclature	xxii
Chapter 1: Introduction	1
1.1 Vibration Energy Harvesting Using Piezoelectricity	1
1.2 Review of Existing Analytical and Equivalent Circuit Models for Piezo- electric Energy Harvesters with Nonlinear Circuits	4
1.3 Piezoelectric Shunt damping and Locally Resonant Bandgaps in Metamaterial- Based Structures	6
1.4 Dissertation Outline	8
Chapter 2: Equivalent Circuit Modeling of a Piezo-patch on a Thin plate	10
2.1 Introducion	10
2.2 Mathematical Modeling	11
2.3 Equivalent Circuit Model	13
2.4 Model Validation	15
2.4.1 AC input - AC output problem	15

2.4.2	AC input - DC output problem	17
2.5	Conclusion	19
Chapter 3: Equivalent Impedance Electroelastic Modeling of MPEHs with Nonlinear Circuits		22
3.1	Introduction	22
3.2	Distributed Parameter Model	24
3.2.1	Electromechanical equations for series configuration	24
3.2.2	Electromechanical equations for parallel configuration	27
3.3	Equivalent Impedance Electroelastic Model	28
3.3.1	Equivalent impedance load in series configuration	28
3.3.2	Equivalent impedance load in parallel configuration	33
3.4	Equivalent Circuit Modeling of MPEHs	36
3.5	Model Validation Results and Discussions	38
3.5.1	Experimental setup	38
3.5.2	AC input - AC output case	40
3.5.3	AC input - DC output case	41
3.6	Comments on the Electrical Connection Configurations	43
3.7	Conclusion	49
Chapter 4: Investigation of Bandgap Formation in Two-dimensional Electromechanical Metastructures		59
4.1	Introduction	59

4.2	Electroelastic modeling of a thin plate with array of resonant piezo-patches	61
4.3	Experimental Validation of the Electromechanical FRFs	69
4.4	Mass and stiffness contribution of the piezo-patches	71
4.5	Electromechanical Coupling	72
4.6	Bandgap Formation via Locally Resonant Piezo-patches	75
4.7	Conclusion	77
Chapter 5:	Conclusion, Contributions and Future Work	83
5.1	Vibrational Energy Harvesting Using Piezoelectric Patches on Thin plates	83
5.2	Locally Resonant Bandgaps in Two-dimensional Electromechanical Metas-structures	85
	Bibliography	88

LIST OF TABLES

2.1	Analogy between electrical and mechanical domains of piezo-patch harvester integrated to a thin plate.	15
2.2	Geometric and material properties	16
3.1	Analogy between electrical and mechanical domains OF MPEHs . .	38
3.2	Geometric, material and electroelastic properties	40
3.3	Equivalent capacitance for different connection types.	45
4.1	Geometric, material and electroelastic properties	70
4.2	Typical electroelastic properties and piezoelectric material coupling coefficient for two piezoelectric ceramics and three single crystal materials [Cao et al., 2004, Erturk and Inman, 2011]	78

LIST OF FIGURES

1.1	Examples of piezoelectric energy harvesters: (a) Cantilevered beam harvester with a tip mass [Erturk and Inman, 2011] (b) Airfoil-based piezoelectric energy harvester [Carlos De Marqui and Erturk, 2013] (c) Piezo-patch energy harvester on a plate [Aridogan et al., 2014a] (d) Corrugated piezoelectric energy harvester [Harne, 2012]	3
2.1	Schematics of a piezoelectric energy harvesting using a patch structurally integrated to a thin plate under transverse vibrations.	11
2.2	Multi-modal equivalent circuit representation of a piezo-patch harvester shunted to a resistive load (a total of $m \times n$ vibration modes).	14
2.3	Voltage FRFs of the piezo-patch harvester under point force excitation, the arrow indicates the increase in the resistive load value from short-circuit to open-circuit condition.	16
2.4	Comparison of analytical multi-mode and SPICE single-mode voltage FRFs for the first four vibration modes ($R_l = 100\Omega$).	17
2.5	Piezo-patch energy harvester on a thin plate with AC-DC conversion circuit, consisting of a full-wave rectifier, a smoothing capacitor, and a resistive load.	18
2.6	Comparison of DC voltage outputs for the first four vibration modes: solid line shows the reference analytical model and the SPICE single-mode simulations are shown by blue circles.	20

2.7	Comparison of average peak power outputs for the first four vibration modes: solid line shows the reference analytical model and the SPICE single-mode simulations are shown by blue circles.	21
3.1	Schematics of MPEHs attached to a thin plate and connected to an AC-DC converter with two connection configurations: (a) Series (b) Parallel. MPEHs: multiple piezo-patch energy harvesters.	25
3.2	Electrical circuit representation of MPEHs combined in series and connected to a standard AC-DC converter. Each piezo-patch energy harvester is indicated as a parallel branch of a proportional current generator and a piezoelectric internal capacitance.	26
3.3	Electrical circuit representation of MPEHs in parallel configuration connected to a standard AC-DC converter. Again, each piezo-patch harvester is equivalently shown by a proportional current generator and a piezo capacitance connected in parallel.	27
3.4	The top diagram shows array of piezo-patch harvesters connected in series to a standard AC-DC circuit, where each harvester is represented by an equivalent voltage source connected to a piezo capacitance in series. In the bottom diagram, the overall capacitance plus the AC-DC circuit is replaced by an equivalent impedance Z_{eq}^{ser} connected to the overall voltage source.	31
3.5	The top diagram shows array of piezo-patch harvesters connected in parallel to a standard AC-DC circuit, where each harvester is represented by an equivalent current source connected to a piezo capacitance in parallel. In the bottom diagram, the overall capacitance plus the AC-DC circuit is replaced by an equivalent impedance Z_{eq}^{par} connected to the overall current source.	34

3.6	This diagram illustrates the procedure for developing the multi-mode equivalent circuit model of the MPEHs. Arrows indicate that the k-th piezo-patch harvester branch in series or parallel configuration shown in (a), is represented by a set of mn secondary-order circuits connected in parallel to a piezo capacitance (each sub-branch represents a single vibration mode of the host system), as shown in (b). The equivalent circuit model of the whole system can then be obtained by connecting a total of branches in series or parallel configuration.	37
3.7	Experimental setup: (a) PZT-1, (b) PZT-2, (c) PZT-3, (d) aluminum plate, (e) clamping frame, (f) electrical circuit (g) modal shaker with a force transducer, (h) data acquisition unit, (i) signal generator and (j) laser vibrometer.	39
3.8	AC input - AC output case: Comparison of experiment, analytical model and SPICE simulations for voltage FRF of MPEHs in (a) series and (b) parallel configurations at open-circuit condition.	41
3.9	Variations of the first mode velocity and DC voltage outputs of the harvester for the range of resistive loads subjected to 1 N point force excitation: (a) Series configuration (b) Parallel configuration. Experiment data are shown in the left column and analytical results in the right column. Arrows indicate the direction of increase in resistive load value.	43
3.10	Comparison of peak power outputs for the AC input AC output and AC input DC output cases versus load resistance. Analytical model and SPICE simulation: (a) first mode (b) second mode (c) third mode, experiment data: (d) first mode (e) second mode (f) third mode. . . .	44

3.11	Different configurations for connecting three piezo-patches: SSS, SSP, PPS, and PPP. The labels S and P stand for the series and parallel connections, respectively.	45
3.12	Comparison of voltage FRFs for different connection types when $R_l = 100k\Omega$. The voltage FRF amplitude for the SSS configuration is larger than the others.	46
3.13	Voltage and power FRF surface plots for different connection types: Vibration mode 1.	48
3.14	Voltage and power FRF surface plots for different connection types: Vibration mode 2.	50
3.15	Voltage and power FRF surface plots for different connection types: Vibration mode 3.	51
3.16	Voltage and power FRF surface plots for different connection types: Vibration mode 4.	52
3.17	Voltage and power FRF surface plots for different connection types: Vibration mode 5.	53
3.18	Voltage and power FRF surface plots for different connection types: Vibration mode 6.	54
3.19	Voltage FRFs for each piezo-patch harvesters at open-circuit condition. Analytical and SPICE simulation results accurately match with the experimental data.	55
3.20	Imaginary part of the Voltage FRFs for each piezo-patch harvesters at open-circuit condition. The amplitudes are normalized, and positive/negative sign indicates the positive/negative strain sign under the covered patch area	56

3.21	Different AC-DC converter circuit configurations: (a) Single rectifier (b) Multiple or respective rectifiers	57
3.22	Comparison of DC Voltage response nearby the 6th vibration mode for single and multiple rectification cases.	58
4.1	Schematic of a two-dimensional electromechanical metastructure. Close- up view shows the cross section of the thin plate with (a) single patch and (b) patch pair connected to kth shunt admittance. In case (b) patches are symmetrically placed on two surface sides and connected in series with the poling directions pointed out by white arrows. . . .	62
4.2	Experimental setup: (a)-(c) three piezo-patches (d) Aluminum plate (e) fully clamped frame (f) modal shaker (g) signal generator (h) signal analyzer (i) laser Doppler vibrometer (LDV).	70
4.3	Electromechanical FRFs at open-circuit condition: (a) displacement FRF and (b) voltage FRF. Analytical results (solid line) are in good agreement with experimental data (dashed line).	71
4.4	Distribution of piezo-patches on the host structure.	73
4.5	Contours of the first four normalized natural frequencies of the elec- tromechanical system at short-circuit condition for the several number of PZT-5A patches with different thickness ratios: (a) unsymmetrical and (b) symmetrical configurations. First four natural frequencies of the plain plate are 53.7, 104.7, 114.2, and 161.5 Hz.	74

4.6	Comparison of displacement FRFs for two electrical connection cases: (a),(c) All piezo-patches in parallel (b),(d) piezo-patches are independent. Here, the number of PZT-5A patches and their normalized thickness are selected as $P = 9$ and , and the patches are placed symmetrically on top and bottom of the plate.	75
4.7	The system-level electromechanical coupling map for the first four vibration modes of the plate with P number of piezo-patches and different patch-to-plate thickness ratios: (a) unsymmetrical and (b) symmetrical configurations. The piezoelectric materials electromechanical coupling coefficient is for PZT-5A ceramic.	76
4.8	Displacement frequency response functions and resonant frequencies versus patch-to-plate thickness ratio $\hat{h}$ for (a)-(b) PZT-5A (d)-(d) PZT-5H (e)-(f) PMN-PT (33%) piezoelectric materials in unsymmetrical (shown in the left column) and symmetrical (the right column) configurations. The thin plate with array of piezo-patches ($P = 25$) is excited by a point force at position $(3a/4, b/4)$ and displacement response is calculated at $(a/4, b/4)$; resonant frequencies are indicated by gray circles, target frequency is $\omega_t = 100\text{Hz}$ and $R_k = 100\Omega$	79
4.9	Transmittance plot for the PMN-PT (33%) material with $\hat{h} = 0.45$ in symmetrical configuration. The bandgap starts from the target frequency of $\omega_t = 100\text{Hz}$ and ends at a lower frequency value. Force and displacement locations are same as shown in Figure4.6(a).	80

4.10	Displacement frequency response functions and resonant frequencies versus patch-to-plate thickness ratio $\hat{h}$ for (a) $\omega_t = 54\text{Hz}$ (b) $\omega_t = 100\text{Hz}$ (c) $\omega_t = 160\text{Hz}$. Piezoelectric material is selected as PMN-PT (33%) and patches ($P = 25$) are in symmetrical configuration. The thin plate with array of piezo-patches is excited by a point force at position $(3a/4, b/4)$ and displacement response is calculated at $(a/4, b/4)$; resonant frequencies are indicated by gray circles, and $R_k = 100\Omega$. .	81
4.11	Transmittance plot for the PMN-PT (33%) material with $\hat{h} = 0.5$ at different target frequencies (shown in the legends)in symmetrical configuration.	82

NOMENCLATURE

1D: One dimensional

2D: Two dimensional

CCCC: All four edges clamped

ECM: Equivalent Circuit Modeling

FEM: Finite-element method

FRFs: Frequency response functions

MPEHs: Multiple piezo-patch energy harvesters

MDOFs: Multiple degrees of freedoms

PZT: Lead zirconate titanate

SDOF: Single degree of freedom

SSHI: Synchronized switch energy harvesting on inductor

SPICE: Simulation program with integrated circuit emphasis

Chapter 1

INTRODUCTION

1.1 Vibration Energy Harvesting Using Piezoelectricity

Vibration-based piezoelectric energy harvesting has received significant attention over the last decades to power small electronic components and thereby eliminating the need for battery replacement. There are various vibration-to-electricity conversion mechanisms such as electrostatic [Torres and Rincón-Mora, 2009, Basset et al., 2009, Le and Halvorsen, 2012], electromagnetic [Beeby et al., 2007, Yang et al., 2009, Zorlu et al., 2011], magnetostrictive [Wang and Yuan, 2008, Adly et al., 2010], and piezoelectric [Roundy and Wright, 2004, Sodano et al., 2004, Kim et al., 2009, Erturk and Inman, 2011] transductions as well as electroactive [Ren et al., 2007, Tiwari et al., 2008] and electrostrictive [Liu et al., 2005, Lallart et al., 2012] polymers. Particularly, piezoelectric transducers have drawn the most attention due to their high power density and ease of application at different geometric scales [Anton and Sodano, 2007, Cook-Chennault et al., 2008, Erturk and Inman, 2011].

A large body of research in piezoelectric energy harvesting have focused on cantilever beam configurations with piezoelectric layers/patches bonded to them. Piezoelectric cantilevers are typically mounted on a vibratory surface to exploit the bending vibration modes of the flexible structures under the excitation frequencies matching their resonance frequency. Due to their effective harvesting performance at resonance frequency, such cantilever designs are also referred to as resonant energy harvesters. Analytical and experimental studies [Anton and Sodano, 2007, DuToit and Wardle,

2007, Erturk and Inman, 2009, Elvin and Elvin, 2009b, Erturk, 2012] as well as finite-element modeling [Yang and Tang, 2009, Elvin and Elvin, 2009a] of piezoelectric beam harvesters are widely established in the literature.

As an alternative to piezoelectric beam-like configurations such as cantilevers, piezoelectric patches can be easily integrated to thin plates to extract vibrational energy of their host structure and convert it into electrical energy. This implementation is convenient especially for thin plate structures employed in many engineering applications such as aerospace, automotive, and marine to enable compact and broadband energy harvesting. Integrating piezo-patch energy harvesters to such host plates reduces the extra mass loading and volumetric occupancy, the major problem with the base-excited cantilever design. Very few studies have focused on energy harvesting from two-dimensional structures. De Marqui et al. [Junior et al., 2009] developed an electromechanical finite element model (FEM) for a piezoelectric energy harvester embedded in a cantilever plate, and later on extended this model to the airflow excitation problem by adding electroaeroelastic coupling [De Marqui Jr et al., 2010, De Marqui et al., 2011] for energy harvesting from aeroelastic flutter. Rupp et al. [Rupp et al., 2009] performed FEM-based topology optimization in cantilever plates and shells for maximizing the power generation. Erturk [Erturk, 2011] developed an analytical formulation for piezoelectric patch-based energy harvesting from surface strain variations of large host structures such as a bridge through the one-way electromechanical coupling. A two-dimensional corrugated harvester design was introduced by Harne et al. [Harne, 2012], who later analyzed the electroelastic dynamics of a vibrating panel of a public bus with corrugated piezoelectric springs attached to it [Harne, 2013]. More recently, Aridogan et al. [Aridogan et al., 2014a, Aridogan et al., 2014b] developed analytical distributed-parameter models and presented experimental validations for single and multiple piezoelectric patch-based energy harvesters structurally integrated to fully clamped plates under harmonic point force excitations. Later, they [Aridogan et al., 2016] presented analytical and numerical solutions to energy harvesting using multiple piezo-patches from stationary random vibrations of the plate and verified

the predicted mean power and velocity outputs of the electromechanical system with experiments.

Such thin plate structures frequently employed in many engineering applications, when coupled with piezoelectric patches, enable a compact and portable energy harvesting system to power the low-power wireless sensors/nodes in nearby. For this to happen, power conditioning circuits which are inherently nonlinear are required to convert the AC signal to a stable DC signal. The modeling of such patch-based energy harvesting systems with nonlinear circuits forms the first part of this work.

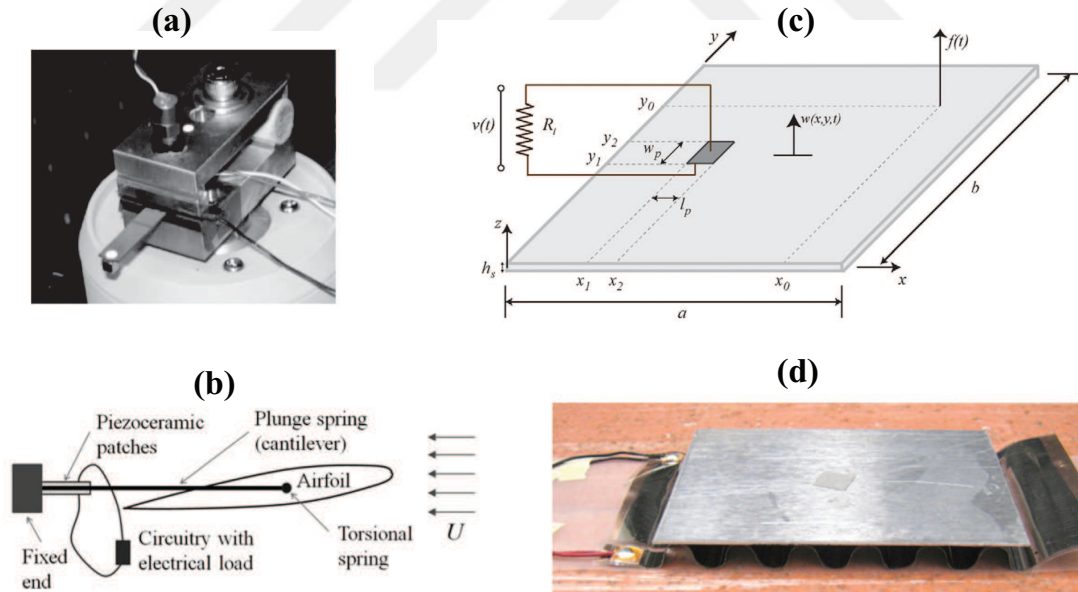


Figure 1.1: Examples of piezoelectric energy harvesters: (a) Cantilevered beam harvester with a tip mass [Erturk and Inman, 2011] (b) Airfoil-based piezoelectric energy harvester [Carlos De Marqui and Erturk, 2013] (c) Piezo-patch energy harvester on a plate [Aridogan et al., 2014a] (d) Corrugated piezoelectric energy harvester [Harne, 2012]

1.2 Review of Existing Analytical and Equivalent Circuit Models for Piezoelectric Energy Harvesters with Nonlinear Circuits

From the electrical domain point of view, most studies have used a simple resistive load for modeling the AC power output of both cantilever beam and plate-like harvesters. It is true that the harvested power is in the form of alternating current due to the kind of vibrations which is harmonic and stochastic in nature. But, for practical implementation of the energy harvesters, more complex interface circuits are required to provide a stable DC signal for charging a storage device (e.g. supercapacitor or battery). A standard AC-DC converter circuit includes a rectifier bridge which consists of diodes and smoothing capacitors. In order to match the impedance of the harvester and the storage device for maximizing the power transfer, usually a DC-DC converter is employed right after the rectifier circuit for signal regulation [Ottman et al., 2002]. Later, more sophisticated circuits such as synchronized switch energy harvesting on inductor (SSHI) [Guyomar et al., 2005, Badel et al., 2006] were proposed to increase the power output in weakly coupled energy harvesters. This circuit interface adds a branch of an inductor with a switch connected in parallel to the piezoelectric element before the rectifier circuit.

In the literature of piezoelectric energy harvesting, studies focusing on the electrical part have used simplified mechanical models (e.g. mass-spring-damper representation of the harvester) [Shu and Lien, 2006, Shu et al., 2007], whereas the ones focusing on the complexities of the mechanical domain have overlooked the practical electrical circuits [Erturk and Inman, 2009, Aridogan et al., 2014a]. Efforts in numerical modeling of the coupled dynamics of both nonlinear electrical circuits and flexible mechanical structures started by introducing the equivalent circuit model (ECM) for piezoelectric energy harvesters. In this modeling approach, each vibration mode of the host structure (e.g. a beam) is represented by a set of resistor-capacitor-inductor elements in series/parallel and the electromechanical coupling is modeled via an ideal transformer. Afterwards, by building the equivalent circuit components of the energy

harvester in an electronic circuit simulator such as SPICE, the dynamics of the harvesting system with nonlinear circuit components (e.g. rectifier and/or SSHI circuits) can be predicted. Yang and Tang [Yang et al., 2009] and Elvin and Elvin [Elvin and Elvin, 2009b] developed equivalent circuit models including multiple-degrees-of-freedom (MDOFs) of a cantilever beam and backward electromechanical coupling. Elvin and Elvin [Elvin and Elvin, 2009b] obtained the equivalent circuit parameters from Rayleigh-Ritz method, whereas Yang and Tang [Yang et al., 2009] used the analogy between the second-order circuit equation of an electrical resonator and electromechanical coupled equation of a piezoelectric beam referred to as the distributed parameter model in [Erturk and Inman, 2008a]. In parallel, Elvin and Elvin [Elvin and Elvin, 2009a] developed a coupled FEM-SPICE model although it required extra computation time for updating the model at each iteration in both FEM and SPICE software. Yang and Tang [Yang et al., 2009] also extracted equivalent circuit parameters from FEM simulations for simple and complex mechanical geometries. Note that these studies only focused on the one-dimensional cantilever configurations for energy harvesting from base excitation.

On the other hand, several researchers proposed analytical models for predicting the average DC power output of single-degree-of-freedom (SDOF) harvesting systems with nonlinear circuits. Using lumped parameter modeling of the electromechanical system, Shu and Lien [Shu and Lien, 2006] derived analytic expressions for the AC input - DC output power under steady-state condition. Their derivation improved the early models proposed by Ottman et al. [Ottman et al., 2002], who neglected backward coupling of the electrical load on the system and Guyomar et al. [Guyomar et al., 2005] who assumed in-phase relation between the velocity output and force input. Later, Shu et al. [Shu et al., 2007] and Lien et al. [Lien et al., 2010] extended their lumped parameter electromechanical modeling to the parallel-SSHI and series-SSHI circuit topologies, respectively and validated their analytical model by numerical simulations in SPICE. The same group proposed the ideal of equivalent impedance for investigating the electrical response of the array of SDOF harvesters

connected in parallel [Lien and Shu, 2012] or series [Lin et al., 2013]. This method is realized by defining an equivalent impedance load as a function of the nonlinear circuit dynamics, excitation frequency and the piezoelectric capacitance, and using the principle of charge and energy balance to derive a closed-form expression for electrical response of the array of harvesters. In the same vein, Liang and Liao [Liang and Liao, 2012] proposed the equivalent impedance idea, but this time with considering the fundamental Fourier mode of the harvesting circuit.

Although the aforementioned analytical models are useful for predicting the electrical response of the electromechanical system in a close neighborhood of a single vibration mode, they neglect the effect of higher modes and the distributed-parameters in continuous systems (e.g., beams and plates). Such issues with oversimplifications in SDOF modeling are summarized in [Erturk and Inman, 2008b].

1.3 Piezoelectric Shunt damping and Locally Resonant Bandgaps in Metamaterial-Based Structures

In conjunction with the vibration energy harvesting using direct piezoelectricity effect, piezoelectric shunt damping techniques have been widely studied for attenuating the structural vibrations of flexible structures. The aim of this research is to replace the conventional bulky mass-spring-damper isolating systems with piezoelectric transducers that add no significant mass and volumetric occupancy to the host structure. Piezoelectric patches/layers can be directly integrated to the vibrating host structures and thereby convert the mechanical energy into the electrical energy which can be dissipated in various electronic circuit components. In 1979, Forward [Forward, 1979] utilized the shunt damping for an optical structure using one piezoelectric patch as a sensor and another one as an actuator driven by a negative feedback. Twelve years later, Hagood and von Flotow [Hagood and von Flotow, 1991] showed that a piezoelectric patch shunted to an inductive circuit forms an electrical resonance, acting as equivalent to a pure mechanical vibration damping system. Several research groups have used network of multi-modal damping resonant shunt circuits [Hollkamp,

1994, Wu, 1998, Wu, 1999, Behrens et al., 2001] for broadband vibration control of flexible structures, or implemented negative capacitance circuits [Behrens et al., 2003, Collet et al., 2012, Beck et al., 2013]. Others have proposed using array of structurally integrated piezoelectric patches connected to electrical networks with the partial differential equations analogous to those of the host beam [Lossouarn et al., 2015] and plate [Alessandroni et al., 2005, Lossouarn et al., 2016] structures. In addition, nonlinear switch-based circuits have been considered as effective semi-passive methods for shunt damping [Clark, 2000, Lefeuvre et al., 2006, Lallart et al., 2008], although they add up extra complexities to the system.

Recently, piezoelectric shunt damping techniques jointly with locally resonant metamaterials have been used for bandgap formation, a frequency range in which vibration wave does not propagate through the elastic medium. Inspired by locally resonant metamaterial-based mechanical structures [Liu et al., 2000, Sugino et al., 2016, Sugino et al., 2017b], electromechanical metamaterials are realized by integrating many piezoelectric elements to the host structure, enabling the creation of bandgap at wavelengths much larger than the lattice constant [Casadei et al., 2009, Airoidi and Ruzzene, 2011, Casadei et al., 2012, Chen et al., 2013]. Analogous to an array of purely mechanical resonators, an array of resonant shunt circuits can be employed in electromechanical structures to display low-frequency bandgaps for vibration/noise attenuation. For predicting the band structure of metamaterials, most studies assume an infinite structure made of repeated assembly of unit-cells and use finite-element methods defined in Bloch theorem. However, for practical implementation of locally resonant metamaterials where finite structures with distributed array of resonators (and not necessary periodic) are employed, this type of modeling cannot bridge the information between the bandgaps formation and modal analysis of such metamaterials. For this reason, Sugino et. al [Sugino et al., 2017b] recently developed a general framework for bandgap estimation in locally resonant 1D and 2D mechanical metastructures (i.e. metamaterial-based finite structures), using modal analysis approach. Later, they presented a distributed-parameter model and modal analysis of

a locally resonant 1D electromechanical metastructure, where a finite-size bimorph piezoelectric beam with many segmented electrodes connected to inductive circuits is used for bandgap formation [Sugino et al., 2017a]. With assuming infinite number of electrodes on piezoelectric layers, they showed that admittance of resonant shunt circuitry introduces a frequency-dependent stiffness term to the structure, as opposed to the frequency-dependent mass terms in purely mechanical metastructures. Closed-form expression for bandgap edge frequencies along with numerical validations for a finite number of segmented electrodes were also presented.

1.4 Dissertation Outline

After reviewing the literature of vibration energy harvesting using piezoelectric transduction and the existing efforts in modeling of the practical harvesting circuits, it is noted that the major studies focusing on electrical parts have overlooked the multi-modal nature of the flexible mechanical structures and the ones focusing on the mechanical part have employed simplified electrical models such as a resistance. Likewise, electroelastic modeling of bandgaps formation in two-dimensional plates using array of resonant piezoelectric patches is not present. The goal of this dissertation is to address those problems by taking into account the distributed parameters of the mechanical part, dynamics of the practical harvesting circuits in piezoelectric energy harvesting research, and dynamics of the resonant shunt circuits in concept of metamaterials. Therefore, the outlined chapters are as follows: Chapter 2 introduces the multi-modal equivalent circuit modeling of a single piezo-patch integrated to a thin plate and then extends it to multiple piezoelectric patches. Chapter 3 deals with the derivation of the equivalent impedance electroelastic model of the multiple piezo-patch energy harvesters on a thin plate with AC-DC conversion and presents the numerical and experimental validations. Chapter 4 investigates the piezoelectric shunt damping and bandgap formation in thin plates using array of locally resonant piezo-patches and their potential use in applications of vibration/noise attenuation. Chapter 5 concludes this dissertation by summarizing the novelties and giving fu-

ture directions in the fields of piezoelectric energy harvesting and electromechanical metastructures.



Chapter 2

EQUIVALENT CIRCUIT MODELING OF A PIEZO-PATCH ON A THIN PLATE

2.1 *Introduction*

Piezo-patch energy harvesters attached to thin plates can be readily integrated to plate-like structures in automotive, marine, and aerospace applications, in order to directly exploit structural vibrations of the host system. This implementation eliminates the mass loading and volumetric occupancy of cantilever attachments. For predicting the DC electrical response, simulation of piezoelectric energy harvesters with nonlinear circuit elements (e.g., AC-DC converter and/or SSHI circuits) requires implementing equivalent circuit models in circuit simulation software (e.g. SPICE) for single/multiple modes of vibration.

In the following, a multi-mode equivalent circuit model of a piezo-patch energy harvester integrated to a thin plate is developed and coupled with a standard AC-DC conversion circuit. First, a mathematical description for a distributed-parameter analytical model of a thin plate with single piezo-patch connected to a resistive load is presented for the AC input - AC output problem. The equivalent circuit model ECM with the system parameters obtained from the analytical model is developed by accounting for two-way electromechanical coupling. Specifically, for AC input - DC output problem, an analytical single-mode model for the plate is coupled with the standard rectifier model of Shu and Lien Shu2006. The ECM of the electromechanical system is built in a circuit simulator SPICE and simulations are validated against the analytical models.

2.2 Mathematical Modeling

A brief description of the analytical formulation of a thin plate with a piezoelectric patch harvester (figure 2.1) is presented based on the Kirchhoff-Love plate theory [Aridogan et al., 2014a]. This theory assumes a mid-plane representation of the three-dimensional plate in two-dimensional form which is practical for plates with very thin thickness as compared to the other two dimensions. The boundary conditions of the host plate are given by all four edges clamped (CCCC) as shown in figure 2.1.

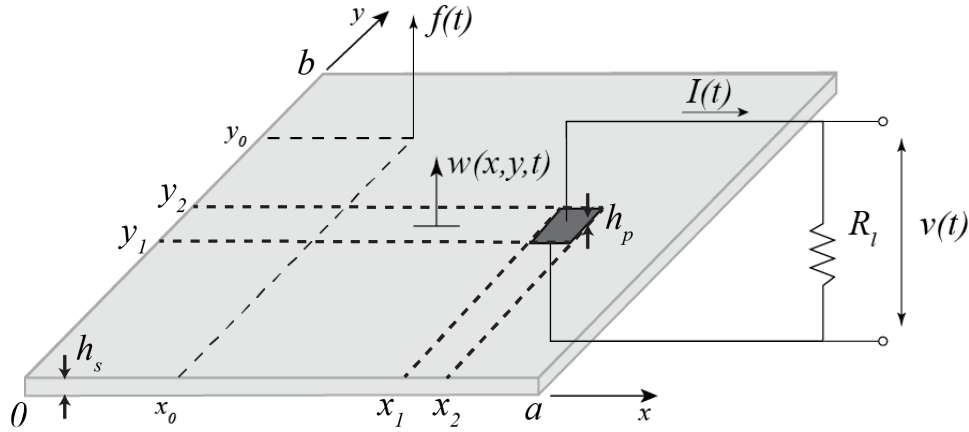


Figure 2.1: Schematics of a piezoelectric energy harvesting using a patch structurally integrated to a thin plate under transverse vibrations.

The host plate is excited by a transverse point force $f(t)$ at the position coordinates (x_0, y_0) . The plate is assumed to be thin where the effects of transverse shear deformation are neglected based on the Kirchhoff plate theory. A piezo-patch with a length of l_p and width of w_p is placed on a rectangular region with two corners at (x_1, y_1) and (x_2, y_2) . The length and width of the host plate are a and b , respectively. Thicknesses of the host plate and the piezo-patch in the z -direction are indicated as h_s and h_p , respectively. A resistive load R_l is employed as an external electrical load connected to the two electrodes which are conductive electrode layers with negligible thickness covering the top and bottom surfaces of the piezo-patch. Moreover, it is assumed that the piezoelectric volume is much smaller than that of the host structure,

and the patch is coupled to the host only electromechanically (with insignificant mass and stiffness contribution) and it exhibits piezoelectrically induced moments at the electrode boundaries.

The electromechanically coupled equation governing the dynamics of transverse vibration of the host plate with piezo-patch can be written as [Aridogan et al., 2014a]

$$\begin{aligned}
 D \left(\frac{\partial^4 w(x, y, t)}{\partial x^4} + 2 \frac{\partial^4 w(x, y, t)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x, y, t)}{\partial y^4} \right) + c \frac{\partial w(x, y, t)}{\partial t} + \rho_s h_s \frac{\partial^2 w(x, y, t)}{\partial t^2} \\
 - \theta v(t) \left\{ \left[\frac{d\delta(x - x_1)}{dx} - \frac{d\delta(x - x_2)}{dx} \right] [H(y - y_1) - H(y - y_2)] \right. \\
 \left. + \left[\frac{d\delta(x - x_1)}{dy} - \frac{d\delta(y - y_2)}{dy} \right] [H(x - x_1) - H(x - x_2)] \right\} \\
 = f(t) \delta(x - x_0) \delta(y - y_0)
 \end{aligned} \tag{2.1}$$

where $w(x, y, t)$ is transverse deflection of the plate in z-direction. The bending stiffness of the plate is $D = Y_s h_s^3 / (12 - 12\nu_s^2)$, where Y_s and ν_s are the Young's modulus and Poisson's ratio. The mass density of the plate is ρ_s and c is the coefficient of viscous damping. The Dirac delta functions are $\delta(x)$ and $\delta(y)$ along the x and y directions; $H(x)$ and $H(y)$ are the Heaviside functions indicating the position of the patch edges on the plate surface; $v(t)$ is the electrode voltage across the external resistive load R_l . The electromechanical term is defined as $\theta = e_{31} \bar{h}_{h_{pc}}$, the multiplication of effective piezoelectric constant $\bar{e}_{31}$ and reference distance h_{pc} (i.e. the distance from the neutral surface level of the plate to mid-plane of the piezo-patch). The governing equation of the coupled circuit dynamics can be expressed as

$$C_p \frac{dv(t)}{dt} + \frac{v(t)}{R_l} + \theta \left\{ \int_{y_1}^{y_2} \int_{x_1}^{x_2} \left[\frac{\partial^3 w(x, y, t)}{\partial x^2 \partial t} + \frac{\partial^3 w(x, y, t)}{\partial y^2 \partial t} \right] dx dy \right\} = 0 \tag{2.2}$$

where C_p is the capacitance of the piezo-patch and defined as $C_p = \bar{\epsilon}_{33}^S w_p l_p / h_p$. Equations (2.1) and (2.2) are the distributed-parameter electroelastic model of piezo-patch harvester in physical coordinates. By conducting the modal analysis procedure for the two-dimensional structure, electromechanically coupled ordinary differential equations of the plate in modal coordinates are given by [Aridogan et al., 2014a]

$$\frac{d^2 \eta_{mn}(t)}{dt^2} + 2\zeta_{mn} \omega_{mn} \frac{d\eta_{mn}(t)}{dt} + \omega_{mn}^2 \eta_{mn} - \tilde{\theta}_{mn} v(t) = f_{mn}(t) \tag{2.3}$$

$$C_p \frac{dv(t)}{dt} + \frac{v(t)}{R_l} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{\theta}_{mn} \frac{d\eta_{mn}(t)}{dt} = 0 \quad (2.4)$$

Here, ω_{mn} is the undamped natural frequency for the mn th vibration mode and ζ_{mn} is the modal damping ratio. The modal forcing in equation (2.3) is

$$f_{mn}(t) = \int_0^b \int_0^a f(t) \delta(x - x_0) \delta(y - y_0) \phi_{mn}(x, y) dx dy = f(t) \phi_{mn}(x_0, y_0) \quad (2.5)$$

where the electromechanical coupling term $\tilde{\theta}_{mn}$ is given by

$$\tilde{\theta}_{mn} = \theta \left[\int_{y_1}^{y_2} \frac{\partial \phi_{mn}(x, y)}{\partial x} \Big|_{x_1}^{x_2} dy + \int_{x_1}^{x_2} \frac{\partial \phi_{mn}(x, y)}{\partial y} \Big|_{y_1}^{y_2} dx \right] \quad (2.6)$$

where ϕ_{mn} is the mass-normalized mode shape for the mn th vibration mode [Aridogan et al., 2014a].

2.3 Equivalent Circuit Model

Although the analytical distributed-parameter model of the plate (described in previous section) is capable of predicting the AC power output of the system accurately, its practical use is limited with linear circuit components such as a simple resistive load. When a resistive load is only considered for the AC input - AC output problem, the distributed parameter model is very efficient and accurate as it accounts for effect of higher vibration modes. However, in the presence of nonlinear circuit components (e.g. diodes and switches), another electronically compatible approach is required to predict the system response. In this section, an equivalent circuit modeling and analysis of a piezo-patch harvester integrated to a thin plate while connected to a full-wave rectifier is presented. This model will be based on the equivalent circuit modeling technique which is based on the superposition of infinite number of lumped-parameter models for each vibration mode. Figure 2.2 shows the equivalent circuit model of the piezoelectric energy harvester integrated to a thin plate for the case of simple resistive loading. Each vibrational mode of the harvester is represented as a second-order circuit connected to a piezo-patch capacitance C_p and a resistive load R_l through an ideal transformer. By applying Kirchhoffs voltage law to the

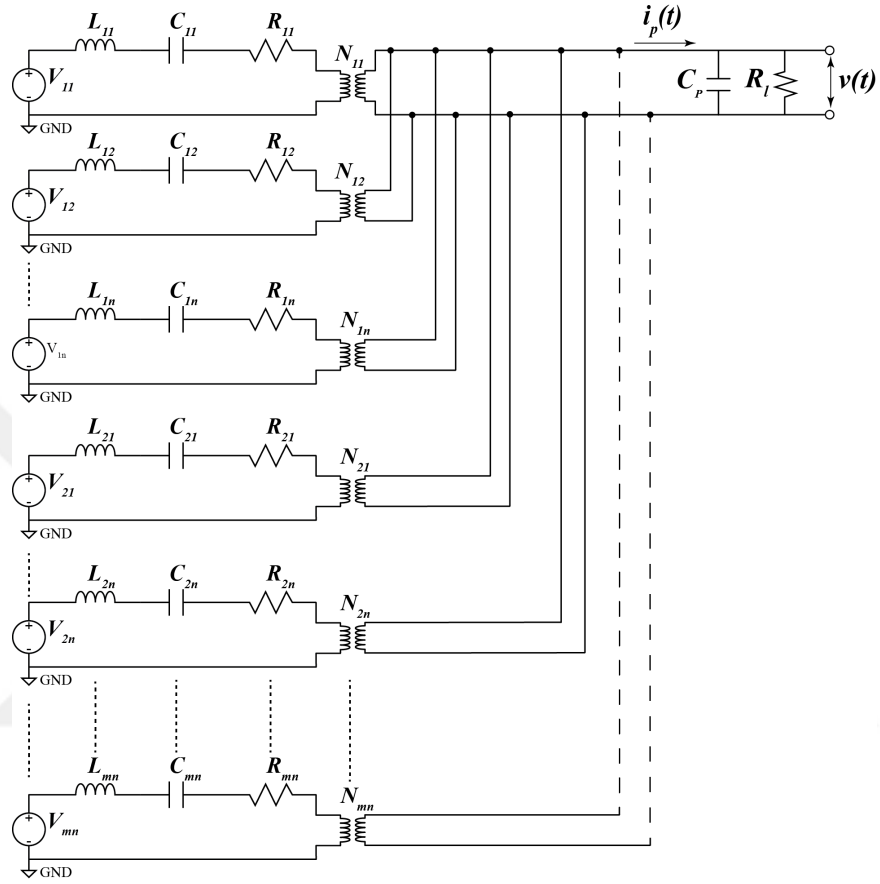


Figure 2.2: Multi-modal equivalent circuit representation of a piezo-patch harvester shunted to a resistive load (a total of $m \times n$ vibration modes).

multi-vibration mode circuit and by analogy with equation (2.3), system parameters of the equivalent circuit model can be obtained. Table 2.1 gives a summary of the analogy between the analytical distributed-parameter expression and the equivalent circuit model. Note that, the $i_p(t)$ term is the dependent current source which is the sum of currents in the parallel branches with piezoelectric capacitance and resistive load. Therefore, by applying Kirchhoffs current law and using analogy with equation (2.4), the current flowing to piezoelectric capacitance and the current source can be expressed as:

$$i_c(t) = C_p \frac{dv(t)}{dt} \quad (2.7)$$

$$i_p(t) = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{\theta}_{mn} \frac{d\eta_{mn}(t)}{dt} \quad (2.8)$$

Table 2.1: Analogy between electrical and mechanical domains of piezo-patch harvester integrated to a thin plate.

Equivalent circuit parameters	Mechanical counterpart terms
Voltage source: $V_{mn}(t)$	$f_{mn}(t)$
Inductance: L_{mn}	1
Resistance: R_{mn}	$2\zeta_{mn}\omega_{mn}$
Capacitance: C_{mn}	$(\omega_{mn})^{-1}$
Transformer ratio: N_{mn}	$-\tilde{\theta}_{mn}$
Electrical charge: $q_{mn}(t)$	$\eta_{mn}(t)$

2.4 Model Validation

In this section, equivalent circuit model of a fully clamped plate with a perfectly bonded piezo-patch is constructed in SPICE software and simulation results are presented for the standard AC-AC and AC-DC problems. Equivalent circuit model results for AC configuration are validated against the distributed parameter model developed by Aridogan et al. [Aridogan et al., 2014a]. For AC-DC case, a single-mode analytical model of the plate is obtained using the lumped-parameter model of Shu and Lien [Shu and Lien, 2006] and compared with numerical SPICE simulations. The geometric and material properties of the aluminum plate and piezo-patch are given in Table 2.2.

2.4.1 AC input - AC output problem

In the standard AC input - AC output problem, the energy harvesting circuit is represented by a resistor attached to the piezo-patch as shown in figure 2.1. Parameters of the multi-mode equivalent circuit model (ECM) are identified from the analytical model considering the first 9 vibration modes of the plate. Steady-state voltage fre-

Table 2.2: Geometric and material properties

Property	Aluminum	Piezoceramic
Length (mm)	580	72.4
Width (mm)	540	72.4
Thickness (mm)	1.96	0.267
Youngs modulus (GPa)	70	69
Mass density (kg/m^3)	2700	7800
Piezoelectric constant (C/m^2)		-190
Permittivity constant (nF/m)		9.57
Damping ratio	0.01	0.01

quency response functions (FRFs) of the piezo-patch harvester under the effect of a harmonic point force are shown in figure 2.3 for a range of resistive loads from 100Ω to $1M\Omega$. It can be seen that the voltage output results of the analytical model match perfectly with equivalent circuit model simulated in SPICE for different resistive load cases when parameters are identified from the analytical model (using table 2.1).

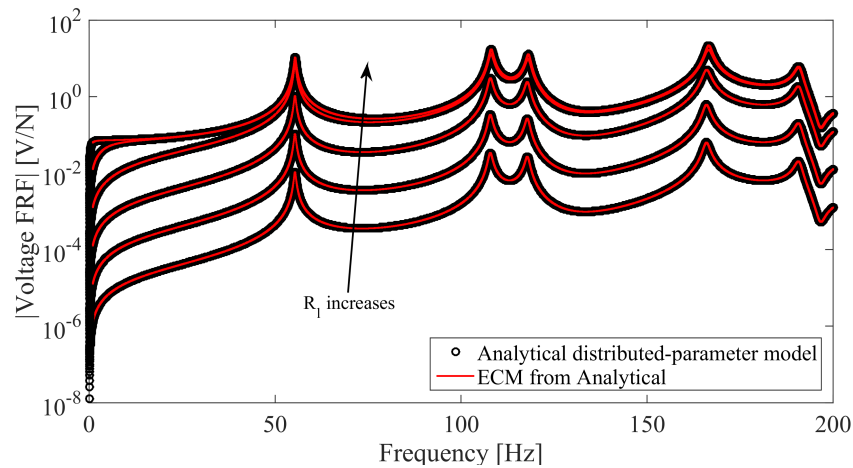


Figure 2.3: Voltage FRFs of the piezo-patch harvester under point force excitation, the arrow indicates the increase in the resistive load value from short-circuit to open-circuit condition.

In order to confirm the validity of single-mode expressions in the vicinity of the resonant frequencies, one can use single branch of the ECM and compared it with the multi-mode solution. Figure 2.4 demonstrates that single-mode results of the equivalent circuit model are in good agreement with the analytical multi-mode results in the neighborhood of the resonance peaks. Since the neighboring modes are not included in the analysis, the single-mode solution becomes unreliable and inaccurate as the excitation frequency deviates from the resonance condition. Having validated the results analytical (modal analysis) with ECM, the AC-DC conversion problem is explored by adding the standard rectification circuit.

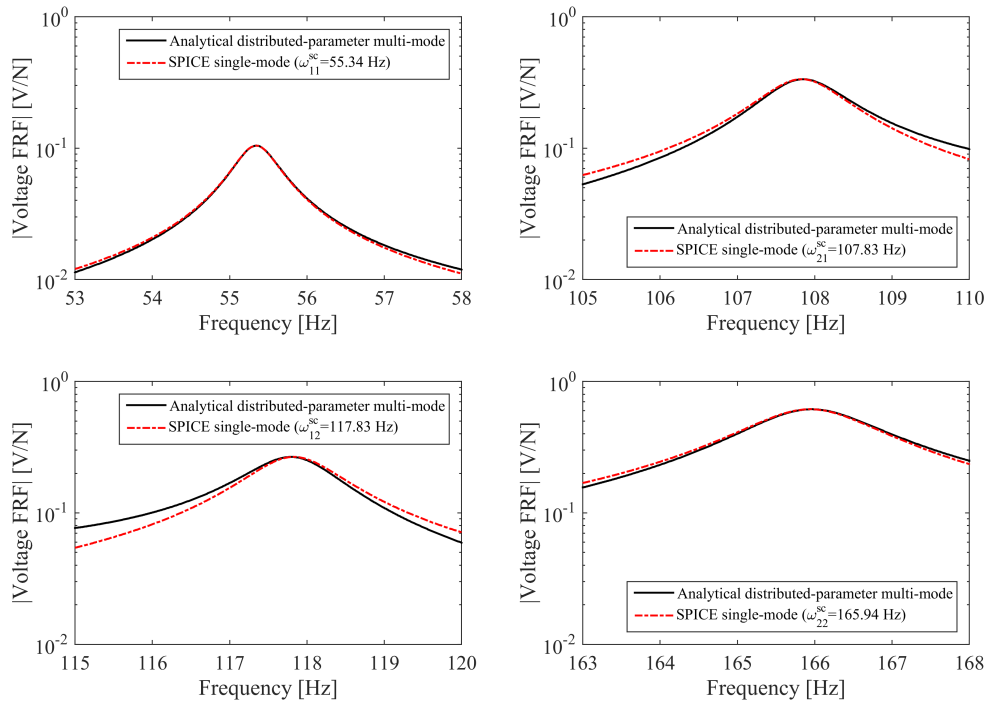


Figure 2.4: Comparison of analytical multi-mode and SPICE single-mode voltage FRFs for the first four vibration modes ($R_l = 100\Omega$).

2.4.2 AC input - DC output problem

Figure 2.5 shows the standard AC-DC piezoelectric energy harvesting circuit, which consists of four diodes in a full-bridge configuration, a smoothing capacitance, and a

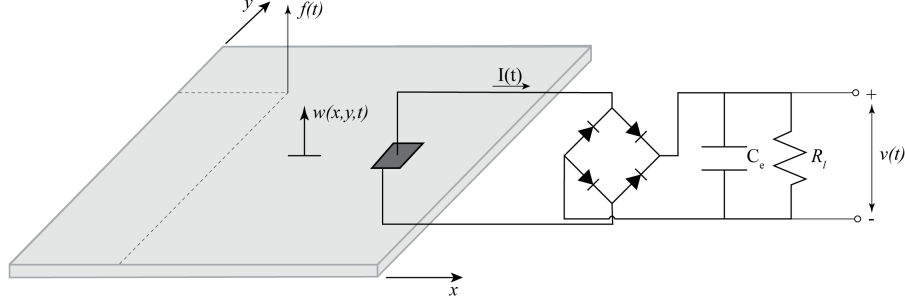


Figure 2.5: Piezo-patch energy harvester on a thin plate with AC-DC conversion circuit, consisting of a full-wave rectifier, a smoothing capacitor, and a resistive load.

resistive load. The lumped-parameter AC-DC model developed by Shu and Lien [Shu and Lien, 2006] is the reference used for this study, however, since it deals with the cantilever problem. Therefore, the SDOF representation of a plate with a piezo-patch is adopted to the lumped parameters model of Shu and Lien (equations (2.9) and (2.10)) and used for analytical simulations. Having validated the single-mode AC input - AC output case (in the resonance neighborhood) as well as the multi-mode cases, the rectified voltage and power output of the equivalent circuit model of the harvester are shown to be validated in the standard AC input - DC output problem. To this end, using the lumped-parameters for each vibrational mode, AC-DC harvesting circuit is simulated in SPICE software and validated with the modified analytical model originally proposed by Shu and Lien [Shu and Lien, 2006]. By implementing this model, the steady-state voltage V_c for the piezo-patch harvester can be written as:

$$V_c = \frac{L_{mn}\omega R_l}{R_l C_p \omega + \pi/2} u_0 \quad (2.9)$$

where the displacement amplitude u_0 is derived in terms of the force amplitude and the system parameters as

$$u_0 = \frac{F_0 \phi_{mn}(x_0, y_0)}{\left[\left(R_{mn}\omega + \frac{2\omega N_{mn}^2 R_l}{(C_p \omega R_l + \pi/2)^2} \right)^2 + \left(1/C_{mn} - \omega^2 L_{mn} + \frac{\omega N_{mn}^2 R_l}{C_p \omega R_l + \pi/2} \right)^2 \right]^{0.5}} \quad (2.10)$$

Here, the fundamental assumption is that the voltage loss in the diode is negligible as compared to the piezoelectric voltage output, therefore the diodes act like a piece-wise switch. In order to generate a stable DC voltage V_c with negligible ripple, smoothing capacitor C_e must be large enough so that a constant DC voltage output can be generated. Ideal transformer and diodes are used in the equivalent circuit model simulations in SPICE to compare numerical results with Shu and Liens analytical model. Analysis of the harvesting circuit is performed for each vibration mode in SPICE software. By conducting time-domain analysis for the harvester circuit, steady-state voltage values are extracted for various values of the resistive loads. Figures 2.6 and 2.7 present the SPICE simulation results for the resonant DC voltage and peak power outputs together with the analytical results for a wide range of resistive loads. The results are in excellent agreement when the assumptions in the SPICE model agree with the analytical approach. Clearly the equivalent circuit model can handle more circuit complexities, such as non-ideal diode behavior and other losses, as well as more complex switching (e.g. SSHI interfaces) and DC-DC regulation circuits, when used in conjunction with SPICE.

2.5 Conclusion

Piezoelectric patch-based energy harvesters attached to thin plates can be integrated to plate-like structures in engineering applications such as automotive, marine, and aerospace, directly exploiting structural vibrations of the host system. With this implementation, mass loading and volumetric occupancy of base excited cantilever attachments can be reduced. In this work, an equivalent circuit model of a piezo-patch-base harvester attached on a thin plate was developed and validated for the standard AC-AC and AC-DC circuit configurations. The equivalent circuit parameters considering multiple vibration modes of the harvester were identified from the analytical distributed-parameter model. Then, using these system parameters, numerical simulations were carried out in SPICE. The simulation results of voltage frequency response functions in the AC-AC configuration were presented for a wide

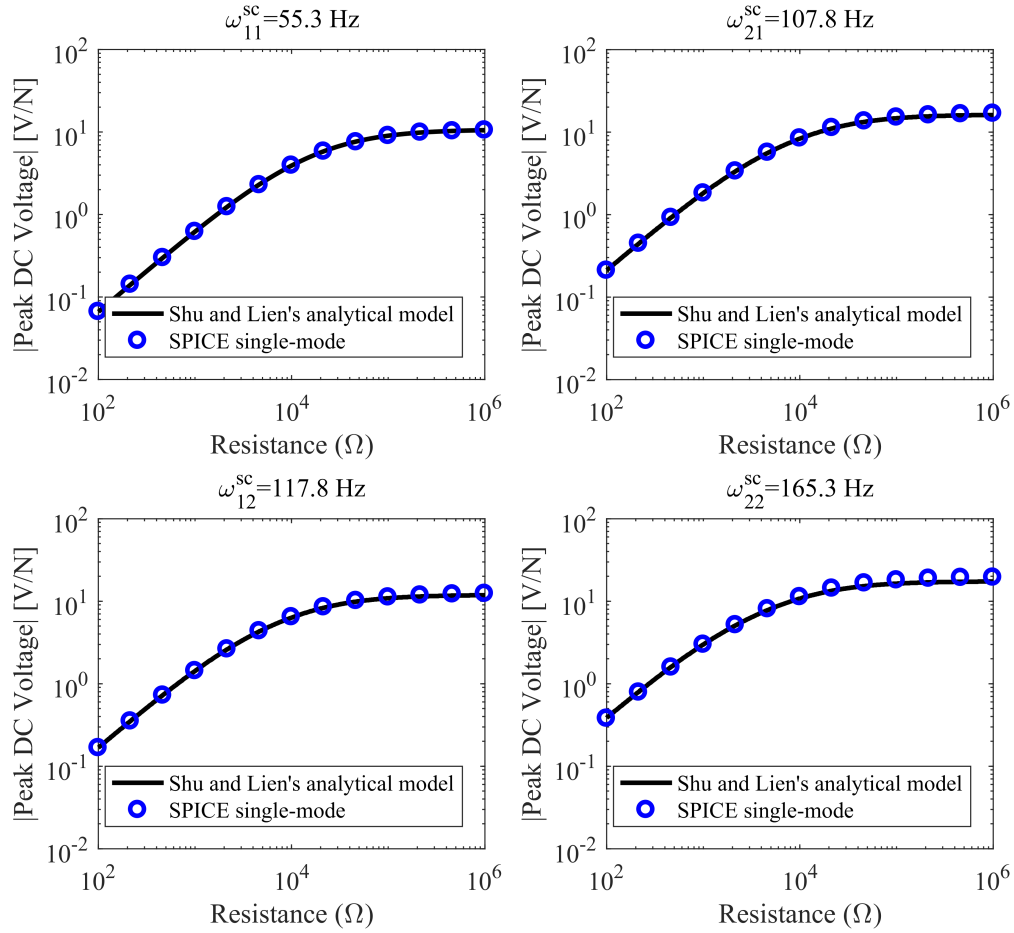


Figure 2.6: Comparison of DC voltage outputs for the first four vibration modes: solid line shows the reference analytical model and the SPICE single-mode simulations are shown by blue circles.

range of resistive load cases and verified against the analytical model. It was shown that equivalent circuit model accurately predicts the coupled electromechanical behavior of the harvester. Equivalent circuit single-mode representation was also verified with the analytical multi-mode solution to confirm the correct approximation of the SDOF model near the vibration modes. Then, the single-mode equivalent circuit model was connected to a full-wave rectifier, a smoothing capacitor, and a resistive load as a standard AC-DC conversion circuit. Steady-state electrical responses were investigated for the first four modes of the harvester. Resonant DC voltage and power

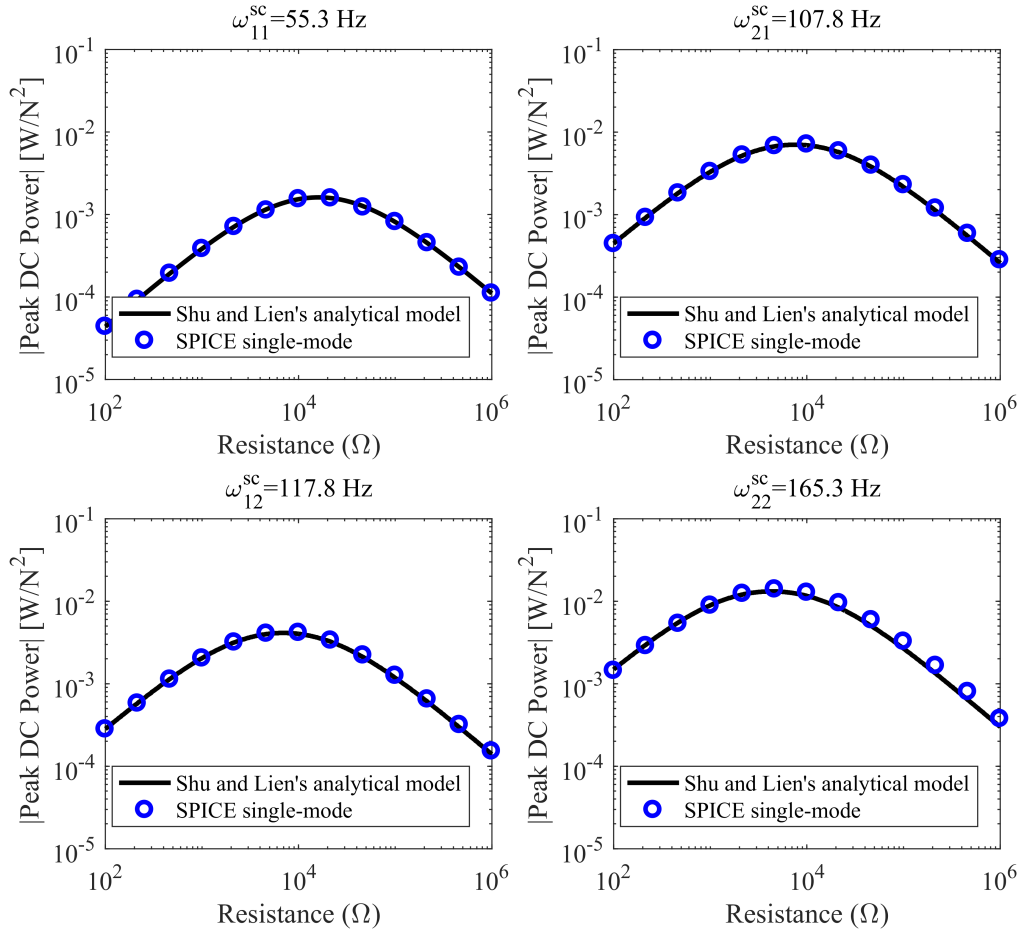


Figure 2.7: Comparison of average peak power outputs for the first four vibration modes: solid line shows the reference analytical model and the SPICE single-mode simulations are shown by blue circles.

outputs across different resistive loads were computed and validated with the existing analytical single-degree-of-freedom model. The multi-mode equivalent circuit model developed in this study enables exploring the harvesting performance of the piezo-electric patch-based harvesters attached to thin plates with any linear and nonlinear electrical components, including non-ideal circuit components as well as DC-DC regulation circuits when used along with software such as SPICE for system-level design and optimization.

Chapter 3

EQUIVALENT IMPEDANCE ELECTROELASTIC MODELING OF MPEHS WITH NONLINEAR CIRCUITS

3.1 Introduction

It has been well established over the past decades that piezoelectric energy harvesting is a promising solution for reducing the maintenance costs due to the battery replacement in embedded health monitoring and wireless sensing systems. As an alternative to beam-based harvester configurations, piezoelectric patches can be readily integrated to thin plate-like structures, to extract the vibrational energy of their host structure and convert it into electrical energy. The vast availability of thin plates-based structures in aerospace, automotive and marine applications makes this implementation convenient to employ and provide compact and broadband energy harvesting. In addition, piezo-patch energy harvesters attached to a thin plate have the merits of removing extra mass loading and volumetric occupancy as well as exhibiting a relatively large number of vibrational modes in a specific frequency range over the conventional cantilever designs. Besides the extensive studies on the beam harvesters in the literature [Anton and Sodano, 2007, Erturk and Inman, 2009, Erturk and Inman, 2011], few works have focused on energy harvesting from two-dimensional plate-like structures [Kim et al., 2005, Rupp et al., 2009, Junior et al., 2009, Arioglan et al., 2014a]. Furthermore, in the majority of modeling of both cantilever beam and plate-like harvesters, a resistance load is used to predict the AC power output. However, in practice, to transfer the electrically harvested power to the practical end devices (e.g. a battery or a storage capacitor), a standard rectifier circuit followed by a DC-DC converter is required to provide a stable DC voltage signal.

Therefore, predicting the dynamic electromechanical response of the harvester(s) connected to practical circuits require high-fidelity analytical and/or numerical models. Liang and Liao [Liang and Liao, 2012], and Lien and Shu [Lien and Shu, 2012] proposed equivalent load impedance idea to obtain the steady-state electrical response of the harvester system connected to nonlinear circuits. The first group obtained the system impedance using the fundamental Fourier mode of the harvesting circuit, whereas, the second group used the principles of charge and energy balance. Although these studies emphasized on the effect of different nonlinear circuits on dynamics of the mechanical part, they modeled the host structure (i.e., cantilever beam) as a single-degree-of-freedom (SDOF) system that was similarly adopted in [Ottman et al., 2002, Wickenheiser et al., 2010, Wu et al., 2015]. This type of modeling neglects the distributed-parameters in continuous systems (e.g., beams and plates) along with other deficiencies that are reported in [Erturk and Inman, 2008b].

Despite the efforts on the lumped-parameter modeling of harvesters with nonlinear circuit interfaces, studies involving analytical closed-form solutions of two-dimensional structures (e.g., thin plates) with single/multiple piezo-patch harvesters connected to AC-DC circuits are not reported in the literature. In the previous chapter, an equivalent circuit model for a single piezo-patch on a thin plate with AC-DC conversion was introduced as a numerical approach to predict the electrical response of the harvester in presence of nonlinear circuits. This chapter presents an equivalent impedance electroelastic modeling of multiple piezo-patch energy harvesters (MPEHs) integrated into a thin plate while connected in series/parallel to a standard AC-DC converter circuit. The proposed analytical approach integrates the equivalent load impedance of the harvester circuit into the distributed-parameter solution of the plate, thus, provides closed-form expressions for both the vibration and electrical responses of MPEHs with a practical nonlinear circuit. In addition, a generalized equivalent circuit modeling of a plate with MPEHs is developed to be compatible with any connection types (i.e., series, parallel, and mixed series-parallel) of the patch harvesters, which fully incorporates the interconnection of piezo-patches, multiple vibration modes of the plates,

and two-way electromechanical coupling effect. Using the proposed analytical and ECM models, electrical power outputs of MPEHs with the series and parallel configurations for a wide range of resistive loads in AC input - AC output and AC input - DC output cases are calculated. Experimental validations are carried out and shown to be in good agreement with the developed models.

3.2 Distributed Parameter Model

In this section, a brief description of the distributed-parameter model of a thin plate with MPEHs is presented based on Kirchhoff-Love plate theory. Figure 3.1 shows a fully clamped thin plate with structurally integrated piezoelectric patches in series and parallel configurations connected to a standard AC-DC converter circuit. The host plate with the length and width of a and b is excited by a transverse point force $f(t)$ applied to the position coordinates (x_0, y_0) . The plate thickness h_s is assumed to be much smaller than other two dimensions, hence the effects of transverse shear deformation are neglected based on the Kirchhoffs assumption. Note that here and hereafter, the subscripts s and p stand for the host structure and piezoelectric patch, respectively. The total number of piezo-patch harvesters is denoted by P and the thickness of k th piezo-patch is $h_{p,k}$. Because of the small volumetric ratio of the piezo-patches to the plate substrate, mass and stiffness contributions of the piezoelectric material is neglected, and only piezoelectrically induced moments are modeled in the electromechanical equations. The following subsections express the brief electromechanical equations of MPEHs in series and parallel configurations while a simple resistance is used as an external electrical load.

3.2.1 Electromechanical equations for series configuration

The coupled circuit equation for each piezo-patch harvester (shown in figure 3.2) can be written as

$$C_{p,k} \frac{dv_k^{ser}(t)}{dt} + i_c^{ser}(t) = i_k^{ser}(t) \quad (3.1)$$

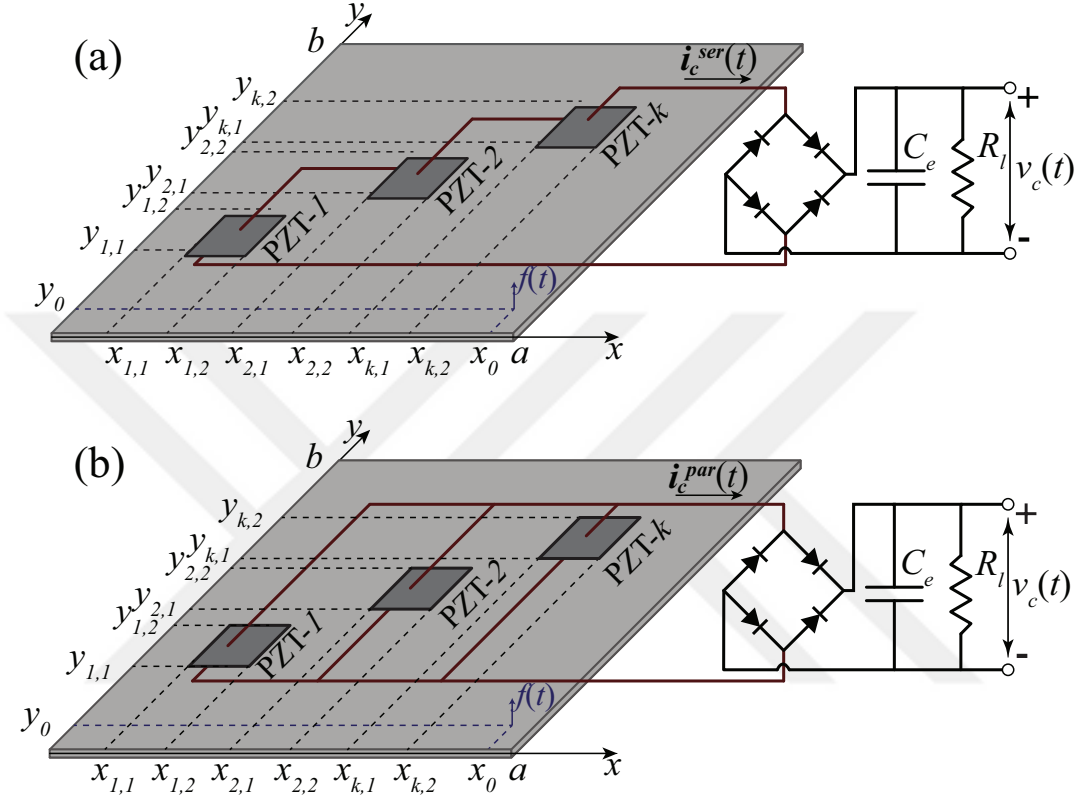


Figure 3.1: Schematics of MPEHs attached to a thin plate and connected to an AC-DC converter with two connection configurations: (a) Series (b) Parallel. MPEHs: multiple piezo-patch energy harvesters.

where $k = 1, 2, \dots, P$, $v_k^{ser}(t)$ and $i_k^{ser}(t)$ are the voltage across the electrodes and current source of the k th piezo-patch harvester, and the current flowing through the harvesting circuit is denoted by $i_c^{ser}(t)$. The internal piezoelectric capacitance is given by [Aridogan et al., 2014b]

$$C_{p,k} = (\bar{\epsilon}_{33}^s)_k \frac{l_{p,k} w_{p,k}}{h_{p,k}} \quad (3.2)$$

where the piezo-patch dimension terms $l_{p,k}$, $w_{p,k}$, and $h_{p,k}$ are the length, width, and thickness of the k th patch harvester, respectively; $(\bar{\epsilon}_{33}^s)_k$ is the permittivity term for the k th piezo-patch at constant strain. The governing electromechanical equations in

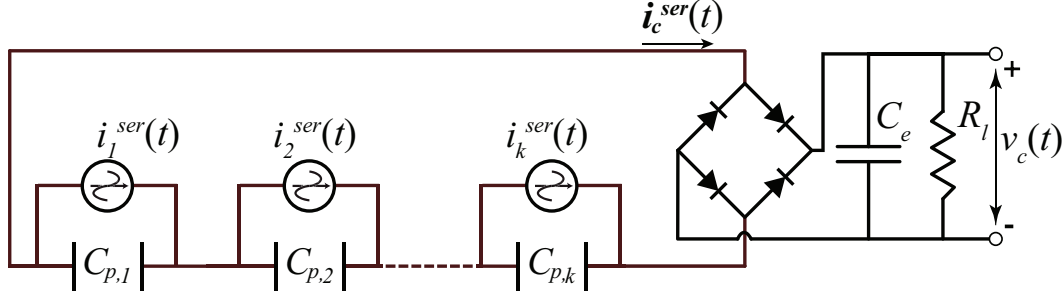


Figure 3.2: Electrical circuit representation of MPEHs combined in series and connected to a standard AC-DC converter. Each piezo-patch energy harvester is indicated as a parallel branch of a proportional current generator and a piezoelectric internal capacitance.

modal coordinates for point force excitation can be given by [Aridogan et al., 2014b]

$$\frac{d^2 \eta_{mn}^{ser}(t)}{dt^2} + 2\zeta_{mn}\omega_{mn} \frac{d\eta_{mn}^{ser}(t)}{dt} + \omega_{mn}^2 \eta_{mn}^{ser}(t) - \sum_{k=1}^P \tilde{\theta}_{mn,k} v_k^{ser}(t) = f_{mn}(t) \quad (3.3)$$

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{\theta}_{mn,k} \frac{d\eta_{mn}^{ser}(t)}{dt} + C_{p,k} \frac{dv_k^{ser}(t)}{dt} + i_c^{ser}(t) = 0 \quad (3.4)$$

where ω_{mn} and ζ_{mn} are the corresponding undamped natural frequency and modal damping ratio for the mn th vibration mode. From the electrical equations (3.1) and (3.4), the velocity dependent current source of the k th harvester for the series configuration can be obtained as

$$i_k^{ser}(t) = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{\theta}_{mn,k} \frac{d\eta_{mn}^{ser}(t)}{dt} \quad (3.5)$$

where $\eta_{mn}^{ser}(t)$ is the modal coordinate of the plate for the mn th vibration mode and the modal electromechanical coupling term $\tilde{\theta}_{mn,k}$ can be obtained as

$$\tilde{\theta}_{mn,k} = \theta_k \int_{y_{k,1}}^{y_{k,2}} \int_{x_{k,1}}^{x_{k,2}} \left[\frac{\partial^2 \phi_{mn}(x, y)}{\partial x^2} + \frac{\partial^2 \phi_{mn}(x, y)}{\partial y^2} \right] dx dy \quad (3.6)$$

where $\phi_{mn}(x, y)$ is the mass-normalized mode-shape of the plate at (x, y) location. The electromechanical term θ_k is defined as $\theta_k = h_{pc,k} \times \bar{e}_{31,k}$ with $\bar{e}_{31,k}$ being the effective piezoelectric constant, and $h_{pc,k}$ being the reference distance (i.e. vertical distance

from the mid-plane of the patch from the neutral surface of the plate). Moreover, the modal forcing input in (3.3) is given by [Aridogan et al., 2014b]

$$f_{mn}(t) = f(t)\phi_{mn}(x_0, y_0) \quad (3.7)$$

3.2.2 Electromechanical equations for parallel configuration

The electrical circuit representation of the parallel configuration of MPEHs is shown in figure 3.3. The coupled circuit equations can be written as

$$\frac{dv^{par}(t)}{dt} \sum_{k=1}^P C_{p,k} + i_c^{par}(t) = i^{par}(t) \quad (3.8)$$

where $i^{par}(t)$ is the summation of all current sources, that is $i^{par}(t) = \sum_{k=1}^P i_k^{par}(t)$, and $C_{p,k}$ is the equivalent piezoelectric capacitance defined in (3.2). The voltage across the all of piezo-patches electrodes is equal together which means $v^{par}(t) = v_k^{par}(t)$ for $k = 1, \dots, P$, and the current flowing through the interface circuit is denoted by $i_c^{par}(t)$. Likewise the series configuration, the electromechanically equations in modal

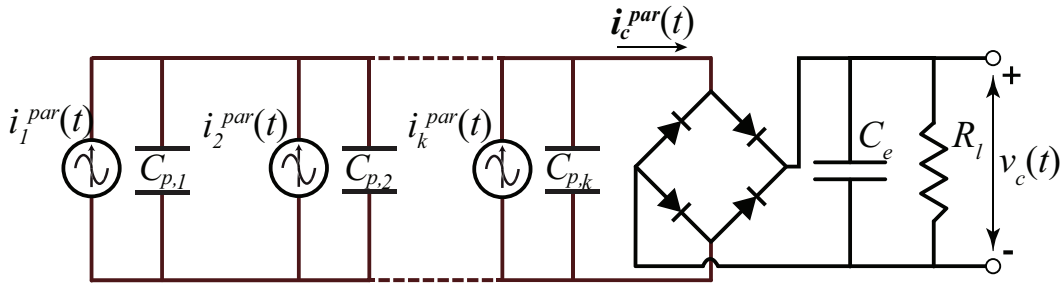


Figure 3.3: Electrical circuit representation of MPEHs in parallel configuration connected to a standard AC-DC converter. Again, each piezo-patch harvester is equivalently shown by a proportional current generator and a piezo capacitance connected in parallel.

coordinates can be written for the parallel configuration of the MPEHs as [Aridogan

et al., 2014b]

$$\frac{d^2 \eta_{mn}^{par}(t)}{dt^2} + 2\zeta_{mn}\omega_{mn} \frac{d\eta_{mn}^{par}(t)}{dt} + \omega_{mn}^2 \eta_{mn}^{par}(t) - v^{par}(t) \sum_{k=1}^P \tilde{\theta}_{mn,k} = f_{mn}(t) \quad (3.9)$$

$$\sum_{k=1}^P \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{\theta}_{mn,k} \frac{d\eta_{mn}^{par}(t)}{dt} + \frac{dv^{par}(t)}{dt} \sum_{k=1}^P C_{p,k} + i_c^{par}(t) = 0 \quad (3.10)$$

Consequently, from (3.8) and (3.10) the velocity dependent current source for the k th patch harvester can be expressed as

$$i_k^{par} = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{\theta}_{mn,k} \frac{d\eta_{mn}^{par}(t)}{dt} \quad (3.11)$$

where $\eta_{mn}^{par}(t)$ is the modal coordinate for MPEHs in parallel configuration at the m nth vibration mode.

3.3 Equivalent Impedance Electroelastic Model

Solving the electromechanical equations given in the previous section under steady-state conditions for the series/parallel configurations of MPEHs connected to a single resistance would be straightforward [Aridogan et al., 2014b]. However, for the practical harvesting circuits which consist of nonlinear elements such as diodes, another approach is required to derive the exact solution. For this reason, the following subsections propose the idea of using equivalent load impedance to develop closed-form expressions for the mechanical and electrical responses of MPEHs.

3.3.1 Equivalent impedance load in series configuration

For the standard AC-DC converter circuit shown in figure 3.2, the current flowing through the ideal rectifier circuit i_c^{ser} can be defined as the piecewise equation

$$i_c^{ser}(t) = \begin{cases} C_e \dot{v}_c^{ser}(t) + \frac{v_c^{ser}(t)}{R_l} & \text{if } v^{ser} = v_c^{ser} \\ -C_e \dot{v}_c^{ser}(t) - \frac{v_c^{ser}(t)}{R_l} & \text{if } v^{ser} = -v_c^{ser} \\ 0 & \text{if } |v^{ser}| < v_c^{ser} \end{cases} \quad (3.12)$$

where $v^{ser}(t)$ is the overall piezoelectric voltage defined as $v^{ser}(t) = \sum_{k=1}^P v_k^{ser}(t)$, and $v_c^{ser}(t)$ is the rectified DC voltage across the resistive load for the series configuration.

If the smoothing capacitor is wisely chosen so that the time constant $R_l C_e$ is much larger than the period of the excitation, the DC voltage output will take a constant V_c^{ser} magnitude, and the ripple in the rectified voltage would be negligible for large values of smoothing capacitor [Shu and Lien, 2006]. The current balance equation in (3.4) can be rearranged and written in the following form

$$\frac{dv_k^{ser}(t)}{dt} + \frac{i_c^{ser}(t)}{C_{p,k}} = \frac{-\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{\theta}_{mn,k} \frac{d\eta_{mn}^{ser}(t)}{dt}}{C_{p,k}} \quad (3.13)$$

Let the equivalent voltage source for the k th piezo-patch harvester be defined as

$$v_k(t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \hat{v}_{mn,k}(t); \text{ where } \hat{v}_{mn,k}(t) = -\frac{\tilde{\theta}_{mn,k} \eta_{mn}^{ser}(t)}{C_{p,k}} \quad (3.14)$$

and taking the summation of both sides in equation (3.13) over k from 1 to P while using (3.14) gives

$$\frac{dv^{ser}(t)}{dt} + \frac{i_c^{ser}(t)}{C_p^{eq}} = \frac{d\hat{v}(t)}{dt} \quad (3.15)$$

where $\hat{v}(t) = \sum_{k=1}^P \hat{v}_k(t)$ is referred to the overall equivalent voltage source and $C_p^{eq} = \left(\sum_{k=1}^P (C_{p,k})^{-1}\right)^{-1}$. Now, based on the linear system assumption, if the transverse point force applied on the surface of the plate is of the form $f(t) = F_0 e^{j\omega t}$, the steady-state responses of modal coordinate of the plate, the k th piezoelectric capacitance voltage, and the k th equivalent voltage source can be written as

$$\eta_{mn}^{ser} = H_{mn}^{ser} e^{j\omega t} \quad (3.16)$$

$$v_k^{ser}(t) = V_k^{ser} e^{j\omega t} \quad (3.17)$$

$$\hat{v}_k(t) = -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\tilde{\theta}_{mn,k} H_{mn}^{ser} e^{j\omega t}}{C_{p,k}} = \hat{V}_k e^{j\omega t} \quad (3.18)$$

where H_{mn}^{ser} , V_k^{ser} , and $\hat{V}_k$ are the complex valued amplitudes. Further, the steady-state form of the overall equivalent voltage source is $\hat{v}(t) = \hat{V} e^{j\omega t}$ with $\hat{V} = \sum_{k=1}^P \hat{V}_k$.

Having substituted the steady-state forms of the η_{mn}^{ser} and $v_k^{ser}(t)$ into (3.3) and (3.4), the complex model response H_{mn}^{ser} can be obtained by extracting the V_k^{ser} from

(3.4) and substituting into (3.3)

$$H_{mn}^{ser} = \frac{F_0 \phi_{mn}(x_0, y_0) + \sum_{k=1}^P \tilde{\theta}_{mn,k} V_k^{ser}}{\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega} \quad (3.19)$$

Integrating the equation (3.15) over the half-cycle period of the excitation (e.g., time interval in which $\hat{v}(t)$ goes from minimum $-\hat{V}$ to maximum $\hat{V}$), and using the piecewise equation in (3.15) yields the DC voltage output

$$V_c^{ser} = \frac{C_p^{eq}\omega R_l}{\pi/2 + C_p^{eq}\omega R_l} \hat{V} \quad (3.20)$$

The next step is to find the complex equivalent voltages $\hat{V}_k$ and the complex voltages V_k^{ser} for $k = 1, 2, \dots, P$, so that the DC voltage output can be obtained readily from equation (3.20), and the complex modal amplitude H_{mn}^{ser} can be obtained from (3.19). Figure 3.4 shows the electrical circuit representation of equation (3.15) for the MPEHs in series configuration. This electrical representation unlike the one shown in figure 3.2, can facilitate the analytical modeling. In particular, the harvesting circuit (i.e., AC-DC converter) plus the overall capacitance of piezo-patches is represented as an equivalent load impedance Z_{eq}^{ser} . Suppose that the Z_{eq}^{ser} is known for a range of excitation frequencies, then the current flowing through it is

$$I_c^{ser} = \frac{1}{Z_{eq}^{ser}} \hat{V} = \frac{1}{Z_{eq}^{ser}} \sum_{k=1}^P \hat{V}_k \quad (3.21)$$

where I_c^{ser} is the complex amplitude of the current $i_c^{ser}(t)$. Here, to determine the equivalent impedance load, we use the model proposed by Lin et al. [Lin et al., 2013], who obtained the equivalent load impedance for the array of SDOF harvesters connected in series to the AC-DC converter circuit and showed that the equivalent impedance is only the function of excitation frequency, equivalent piezoelectric capacitance, and resistive load. Therefore, it is independent of the number of vibrational modes and can be safely adopted for our case. The complex impedance model is defined as $Z_{eq}^{ser} = (Z_R^{ser} - jZ_I^{ser})^{-1}$, where the real and imaginary parts are given by [Lin et al., 2013]

$$Z_R^{ser} = \frac{2(C_p^{eq})^2\omega^2 R_l}{\pi/2 + C_p^{eq}\omega R_l}; Z_I^{ser} = \frac{-\pi C_p^{eq}\omega}{\pi + 2C_p^{eq}\omega R_l} \quad (3.22)$$

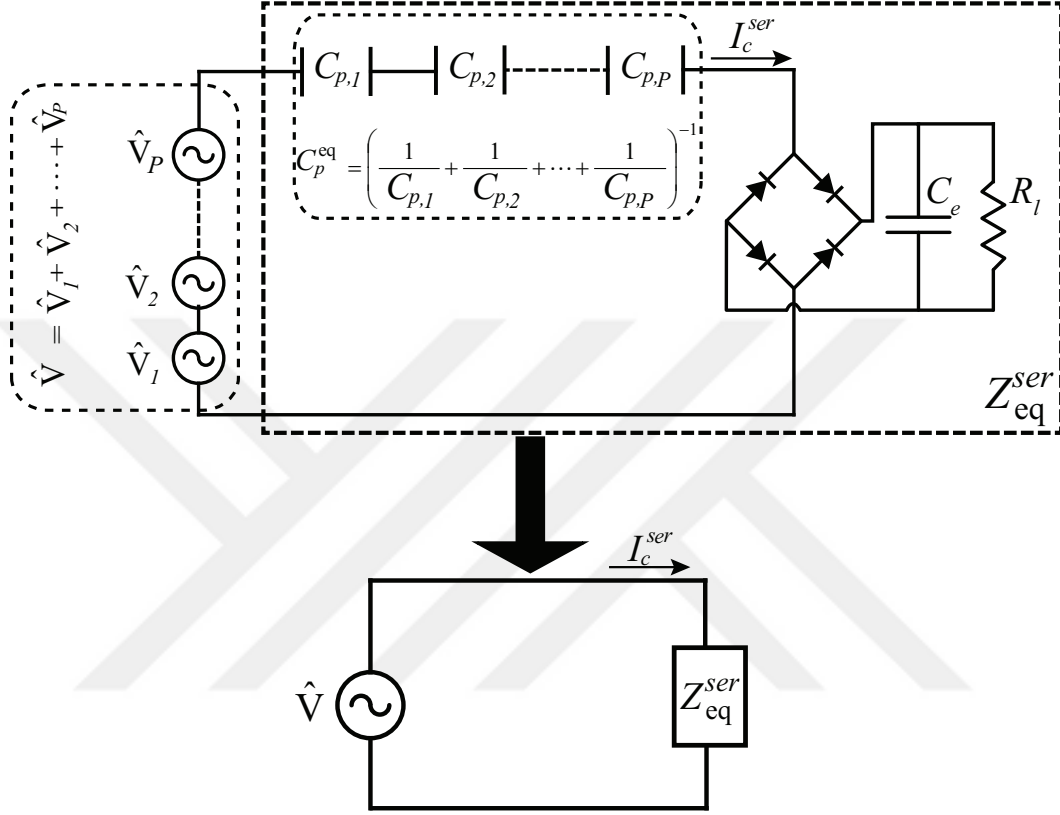


Figure 3.4: The top diagram shows array of piezo-patch harvesters connected in series to a standard AC-DC circuit, where each harvester is represented by an equivalent voltage source connected to a piezo capacitance in series. In the bottom diagram, the overall capacitance plus the AC-DC circuit is replaced by an equivalent impedance $Z_{\text{eq}}^{\text{ser}}$ connected to the overall voltage source.

Now, taking a time derivative of (3.3), replacing $dv_k^{\text{ser}}(t)/dt$ with that of (3.13), and substituting the steady-state forms from (3.16), (3.18), and (3.21), the modal mechanical equation reduces to

$$\begin{aligned}
 & (\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega) \times H_{mn}^{\text{ser}} + \sum_{k=1}^P \left[\frac{1}{j\omega Z_{\text{eq}}^{\text{ser}}} \sum_{l=1}^P \left(\frac{\tilde{\theta}_{mn,l}}{C_{p,l}} \right) - \tilde{\theta}_{mn,k} \right] \hat{V}_k \\
 & = F_0 \phi_{mn}(x_0, y_0)
 \end{aligned} \tag{3.23}$$

Then, multiplying both sides in (24) by $\tilde{\theta}_{mn,k}/C_{p,k} \times (\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega)^{-1}$, and taking double summation over m by n number of vibrational modes, the resulting

equation can be rewritten in a matrix equation of the form

$$(\mathbf{Q} - \mathbf{I}) \hat{\mathbf{V}} = \hat{\mathbf{F}} \quad (3.24)$$

where $\hat{\mathbf{V}}$ is a $P \times 1$ unknown vector which is expressed as $\hat{\mathbf{V}} = [\hat{V}_1, \hat{V}_2, \dots, \hat{V}_P]^T$, $\mathbf{I}$ is the identity matrix with the dimensions of $P \times P$; $\mathbf{Q}$ is a square matrix with the dimensions of $P \times P$ and $\hat{\mathbf{F}}$ is a column vector with the dimensions of $P \times P$, given by

$$Q_{ij} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{\tilde{\theta}_{mn,i}}{C_{p,i}} \times \frac{1/j\omega Z_{eq}^{ser} \sum_{k=1}^P \frac{\tilde{\theta}_{mn,k}}{C_{p,k}} - \tilde{\theta}_{mn,j}}{(\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega)} \right] \quad (3.25)$$

$$\hat{f}_i = F_0 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{\tilde{\theta}_{mn,i}}{C_{p,i}} \times \frac{\phi_{mn}(x_0, y_0)}{(\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega)} \right] \quad (3.26)$$

Having solved for each $\hat{V}_k$ in (3.24), the steady-state magnitude of DC voltage across the resistive load can be obtained from (3.20). Meanwhile, the complex equivalent voltage $\hat{V}_k$ can be substituted into (3.15), considering (3.17) and (3.18), to obtain the complex voltage V_k^{ser} as

$$V_k^{ser} = \hat{V}_k - \frac{1}{j\omega C_{p,k} Z_{eq}^{ser}} \sum_{k=1}^P \hat{V}_k \quad (3.27)$$

The resulting complex voltage V_k^{ser} can be substituted into (3.19) to obtain the complex amplitudes H_{mn}^{ser} . Next, the transverse physical displacement of the plate at point (x, y) , and time t can be obtained based on the modal expansion theorem, by substituting η_{mn}^{ser} into the following

$$\begin{aligned} w^{ser}(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \phi_{mn}(x, y) \eta_{mn}^{ser}(t) \\ &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\phi_{mn}(x, y)}{(\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega)} \right. \\ &\quad \left[F_0 \phi_{mn}(x_0, y_0) + \sum_{k=1}^P \tilde{\theta}_{mn,k} \hat{V}_k - \frac{1}{j\omega Z_{eq}^{ser}} \sum_{l=1}^P \frac{\tilde{\theta}_{mn,l}}{C_{p,l}} \sum_{k=1}^P \hat{V}_k \right] \Big\} e^{j\omega t} \end{aligned} \quad (3.28)$$

Having formulated the steady-state vibration response, the electromechanical FRF can be extracted for the transverse displacement-to-force input ratio using $\beta^{ser}(\omega, x, y) =$

$w^{ser}(x, y, t)/F_0 e^{j\omega t}$. The velocity FRF defined as velocity-to-force input ratio, can be directly derived from the displacement FRF using $\beta^{ser, velocity}(\omega, x, y) = j\omega \beta^{ser}(\omega, x, y)$. It should be noted that all the complex $\hat{V}_k$ inherit the amplitude F_0 from (3.23), which eventually cancels out with the F_0 in the denominator of displacement FRF.

3.3.2 Equivalent impedance load in parallel configuration

Likewise the series configuration of MPEHs, the current flowing into the AC-DC converter circuit, i_c^{par} shown in (3.3), is related to the rectified voltage $v_c^{par}(t)$ by the piecewise equation given in (3.12).

For the harmonic excitations at frequency ω , the applied force input is of the form $f(t) = F_0 e^{j\omega t}$. The steady-state mechanical modal response $\eta_{mn}^{par}(t)$, and the steady-state voltage response of piezo-patch harvesters $v^{par}(t)$ based on the linear system assumption can be expressed as

$$\eta_{mn}^{par}(t) = H_{mn}^{par} e^{j\omega t} \quad (3.29)$$

$$v^{par}(t) = V^{par} e^{j\omega t} \quad (3.30)$$

where H_{mn}^{par} and V^{par} are both complex valued. Next, by inserting the steady-state forms into (3.8) and (3.9) the complex modal response H_{mn}^{par} can be obtained as

$$H_{mn}^{par} = \frac{F_0 \phi_{mn}(x_0, y_0) + V^{par} \sum_{k=1}^P \tilde{\theta}_{mn,k}}{(\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega)} \quad (3.31)$$

The velocity dependent current source in (3.11) can be expressed with the steady-state form of $i_k^{par}(t) = I_k^{par} e^{j\omega t}$ where the complex current amplitude is

$$I_k^{par} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} I_{mn,k}^{par} = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} j\omega \tilde{\theta}_{mn,k} H_{mn}^{par} \quad (3.32)$$

and the overall velocity proportional current source can be written as the summation of all the dependent current source terms

$$i^{par}(t) = I^{par} e^{j\omega t} = \sum_{k=1}^P I_k^{par} e^{j\omega t} \quad (3.33)$$

The average DC voltage output of the harvester, under the steady-state operation, can be obtained by integrating the current balance equation in (3.8) over the half-cycle

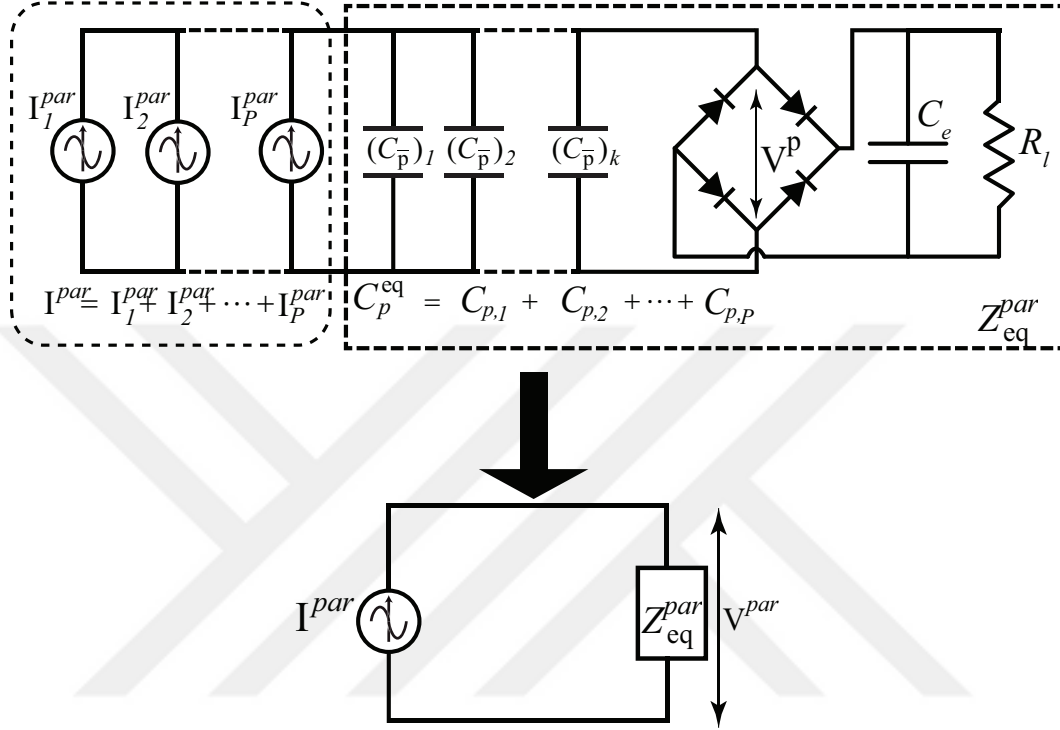


Figure 3.5: The top diagram shows array of piezo-patch harvesters connected in parallel to a standard AC-DC circuit, where each harvester is represented by an equivalent current source connected to a piezo capacitance in parallel. In the bottom diagram, the overall capacitance plus the AC-DC circuit is replaced by an equivalent impedance Z_{eq}^{par} connected to the overall current source.

period of the excitation

$$V_c^{par} = \frac{R_l}{\pi/2 + R_l \omega C_p^{eq} I^{par}} \quad (3.34)$$

Here, C_p^{eq} is the overall equivalent piezoelectric capacitance for the piezo-patches in parallel, which is obtained as $C_p^{eq} = \sum_{k=1}^P C_{p,k}$. Now for determining the DC voltage in (3.35), all the I_k^{par} for $k = 1, 2, \dots, P$ should be calculated. To do so, let Z_{eq}^{par} be the equivalent impedance for the parallel configuration which consists of the AC-DC circuit and the overall equivalent piezo capacitance as shown in figure 3.5. Lien and Shu [Lien and Shu, 2012] proposed the equivalent impedance model for the array of SDOF harvesters connected in parallel to an AC-DC converter which is adopted here as well. The reason for validity of this model was the independency to the mechanical

modal behavior. The complex equivalent impedance is defined as $Z_{eq}^{par} = Z_R^{par} + jZ_I^{par}$ where the real and imaginary parts are given by

$$Z_R^{par} = \frac{2R_l}{(\pi/2 + C_p^{eq} R_l \omega)^2}, Z_I^{par} = -\frac{R_l}{\pi/2 + C_p^{eq} R_l \omega} \quad (3.35)$$

The complex magnitudes of equivalent current source and voltage across Z_{eq}^{par} in figure 3.5 are related by

$$V^{par} = Z_{eq}^{par} I^{par} = Z_{eq}^{par} \sum_{k=1}^{par} I_k^{par} \quad (3.36)$$

Substituting the steady-state forms of $v^{par}(t)$ and $\eta_{mn}^{par}(t)$ into (3.9), the mechanical equation for the parallel configuration of MPEHs reduces to

$$\begin{aligned} (\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega) H_{mn}^{par} - Z_{eq}^{par} \sum_{l=1}^P I_l^{par} \sum_{k=1}^P \tilde{\theta}_{mn,k} \\ = F_0 \phi_{mn}(x_0, y_0) \end{aligned} \quad (3.37)$$

Then, dividing both sides of the above equation by $\sum_{k=1}^P \tilde{\theta}_{mn,k}$, and using (3.32), the remaining can be rearranged to be in the form of matrix equation as

$$\mathbf{Z}^* \mathbf{I}^* = \mathbf{V}^* \quad (3.38)$$

where $\mathbf{I}^*$ is the unknown column vector with the dimensions of mn including the equivalent current amplitudes corresponding to the vibration modes as $\mathbf{I}^* = [I_{11}^p, I_{12}^p, \dots, I_{mn}^p]^T$. In addition, the known column vector $\mathbf{V}^*$ with the dimensions of $mn \times 1$ defined as $\mathbf{V}^* = [V_{11}^p, V_{12}^p, \dots, V_{mn}^p]^T$ and the square matrix $\mathbf{Z}^*$ with the dimensions of $mn \times mn$, given by

$$V_{ij}^* = \frac{-F_0 \phi_{ij}(x_0, y_0)}{\sum_{k=1}^P \tilde{\theta}_{ij,k}}, \quad \text{for } i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (3.39)$$

$$\mathbf{Z}^* = \text{diag}([Z_{11}, Z_{21}, \dots, Z_{mn}]) + Z_{eq}^{par} \mathbf{J} \quad (3.40)$$

where $\mathbf{J}$ is an all-ones matrix with the dimensions of $m \times n$ times $m \times n$, Z_{rs} for $r = 1, \dots, m$ and $s = 1, 2, \dots, n$ are

$$Z_{rs} = \frac{2\zeta_{rs}\omega_{rs}}{\left(\sum_{k=1}^P \tilde{\theta}_{rs,k}\right)^2} + j \frac{\omega^2 - \omega_{rs}^2}{\omega \left(\sum_{k=1}^P \tilde{\theta}_{rs,k}\right)^2} \quad (3.41)$$

Now, by solving the linear equations in (3.38) for the unknown vector $\mathbf{I}^*$, the steady-state magnitude of DC voltage across the resistive load can be readily calculated from (3.34).

Additionally, the complex H_{mn}^{par} can be extracted from (3.37) to determine the mechanical modal coordinate $\eta_{mn}^{par}(t)$ which can be substituted in the following equation to obtain the physical transverse displacement response of the plate at point (x, y) , and time t

$$\begin{aligned} w^{par}(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \phi_{mn}(x, y) \eta_{mn}^{par}(t) \\ &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\phi_{mn}(x, y)}{(\omega_{mn}^2 - \omega^2 + j2\zeta_{mn}\omega_{mn}\omega)} \right. \\ &\quad \left. \left[F_0 \phi_{mn}(x_0, y_0) + Z_{eq}^{par} \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} I_{rs}^{par} \sum_{k=1}^P \tilde{\theta}_{mn,k} \right] \right\} e^{j\omega t} \end{aligned} \quad (3.42)$$

The transverse displacement-to-force input FRF can be expressed as $\beta^{par}(\omega, x, y) = w^{par}(x, y, t)/(F_0 e^{j\omega t})$, and the velocity FRF can be obtained as $\beta^{par, velocity}(\omega, x, y) = j\beta^{par}(\omega, x, y)$. Again, notice that all the complex I_{mn}^{par} s include the force amplitude F_0 from Equation (3.38) resulting in the cancellation of the force amplitude term in the displacement FRF.

3.4 Equivalent Circuit Modeling of MPEHs

Equivalent electrical representation of the mechanical part of piezoelectric energy harvesters, in a linear regime, is well established in the literature. Researchers have presented an equivalent circuit model for a piezoelectric layer on a beam [Elvin and Elvin, 2009b, Yang and Tang, 2009, Tang et al., 2015], a piezo-patch on the plate [Aghakhani et al., 2015, Bayik et al., 2016], and array of individual cantilever harvesters in series [Lin et al., 2013] and parallel [Lien and Shu, 2012] configurations. In all these models, each piezoelectric energy harvester is bonded to a single host structure. However, in our case, multiple piezo-patch harvesters share a common host structure to exploit the multiple vibration modes, thus, a certain ECM is required to consider the interconnection of piezo-patches as well as their electromechanical coupling effect

on the plate. Figure 3.6 demonstrates the equivalent circuit modeling for the k th

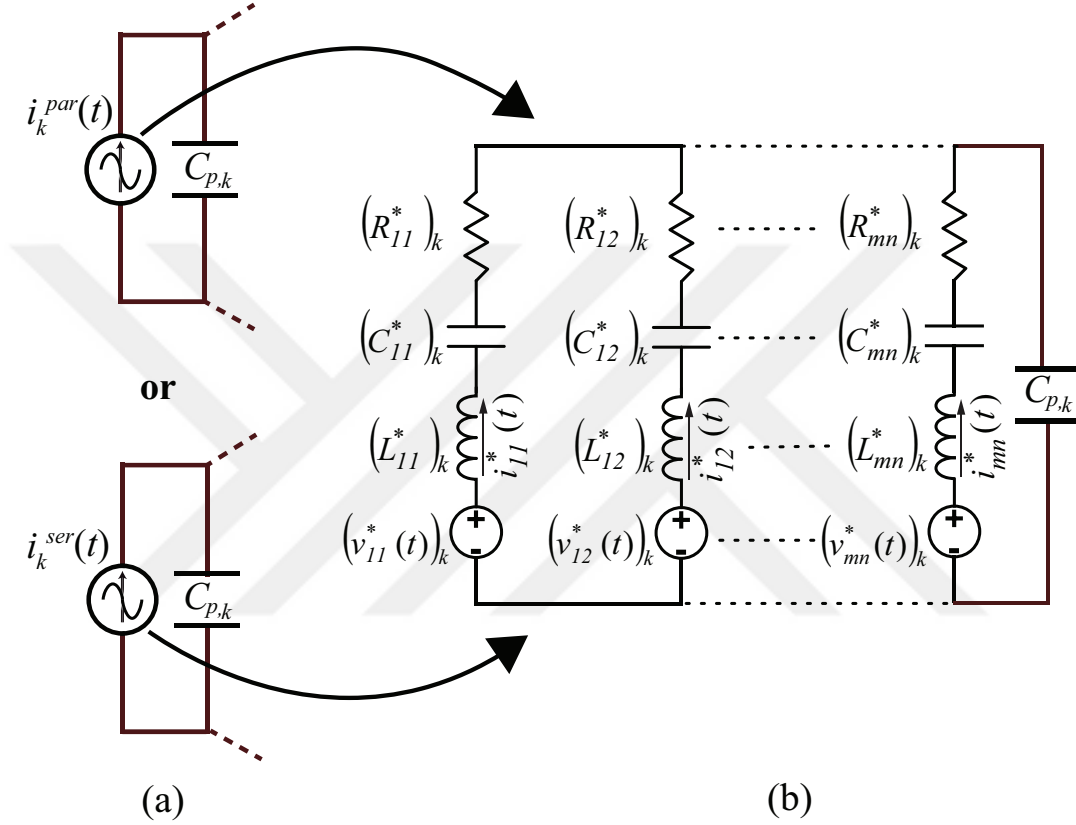


Figure 3.6: This diagram illustrates the procedure for developing the multi-mode equivalent circuit model of the MPEHs. Arrows indicate that the k -th piezo-patch harvester branch in series or parallel configuration shown in (a), is represented by a set of mn secondary-order circuits connected in parallel to a piezo capacitance (each sub-branch represents a single vibration mode of the host system), as shown in (b). The equivalent circuit model of the whole system can then be obtained by connecting a total of branches in series or parallel configuration.

piezo-patch harvester, where the dependent current source is represented by a set of secondary-order circuits in parallel. Having repeated this procedure for $k = 1, 2, \dots, P$, in parallel or series configurations (see figures 3.2 and 3.3), the full electromechanical behavior of the multiple piezo-patch harvesters while connected to a practical interface circuit can be estimated via an electronic circuit simulator (e.g. SPICE). With analogy between the analytical distributed-parameter expressions given in (3.3)

Table 3.1: Analogy between electrical and mechanical domains OF MPEHs

Equivalent circuit parameters	Mechanical counterpart terms
Electrical current: $i_{mn,k}^*(t)$	$-\tilde{\theta}_{mn,k}\eta_{mn}^{par}(t)$ or $-\tilde{\theta}_{mn,k}\eta_{mn}^{ser}(t)$
Voltage source: $v_{mn,k}^*(t)$	$-f_{mn}(t) \times \left(\sum_{k=q}^P \tilde{\theta}_{mn,k}\right)^{-1}$
Inductance: $L_{mn,k}^*$	$\left(\tilde{\theta}_{mn,k} \sum_{l=1}^P \tilde{\theta}_{mn,l}\right)^{-1}$
Resistance: $R_{mn,k}^*$	$2\zeta_{mn}\omega_{mn} \times \left(\tilde{\theta}_{mn,k} \sum_{l=1}^P \tilde{\theta}_{mn,l}\right)^{-1}$
Capacitance: $C_{mn,k}^*$	$\left(\tilde{\theta}_{mn,k} \sum_{l=1}^P \tilde{\theta}_{mn,l}\right) \times (\omega_{mn}^2)$

and (3.9), and the equivalent circuit model in figure 3.6, system parameters are obtained and given in table table 3.1. Its worth pointing out the dissimilarities in the system parameters for single versus multiple piezo-patch harvesters integrated into a thin plate. In the single patch harvester case shown in the previous chapter, for each vibration mode, there exists just a single coupling term $\tilde{\theta}_{mn}$ in the elements of mechanical impedance with $R_{mn}^* = 2\zeta_{mn}\omega_{mn}/\tilde{\theta}_{mn}^2$, $L_{mn}^* = 1/\tilde{\theta}_{mn}^2$, $C_{mn}^* = \tilde{\theta}_{mn}^2/\omega_{mn}^2$ and the equivalent voltage source as $v_{mn}^* = -f_{mn}(t)/\tilde{\theta}_{mn}$. However, in the case of multiple patch harvesters, these equivalent terms of the single harvester cannot be simply mapped into the k-th piezo-patch harvester and regarded as an independent parameter, which is why the system parameters are different for the MPEHs.

3.5 Model Validation Results and Discussions

3.5.1 Experimental setup

Figure 3.7 presents the experimental setup used for analyzing the harvesting performance of multiple piezo-patch configurations and validating the analytical and ECM models. The transversely isotropic piezoceramic patches (T105-A4E-602 manufactured by Piezo Systems, Inc.) are installed as piezo-patch energy harvesters on the aluminum plate. The dynamic point force loading is measured by a force transducer (PCB 208C02) attached between the shakers stinger and the plate surface at position (0.085,0.085) m away from the bottom right corner of the plate. The velocity outputs

are measured via a laser Doppler vibrometer (Polytec PDV 100). Two general experiment cases are examined in this study, namely AC input - AC output case and AC input - DC output case. In the first case, the aluminum plate is excited by a linear sine sweep signal over the bandwidth of 20-260 Hz through a modal shaker. The voltage output-to-force input FRFs are directly measured from the two terminals of the resistive load. For the second experiment case, the sweep rate is selected as slow enough to ensure considerable charge time for the smoothing capacitor, and as fast enough to avoid lengthy experiment data. The rectifier circuit consists of four commercially available Schottky diodes (BAT85), which are known for their fast switching performance and the low voltage drop (typically around 0.2 V [Tiwari et al., 2013]), and a smoothing capacitor followed by a resistive load. For both case studies, the geometric, material, and electroelastic properties are given in Table 2.

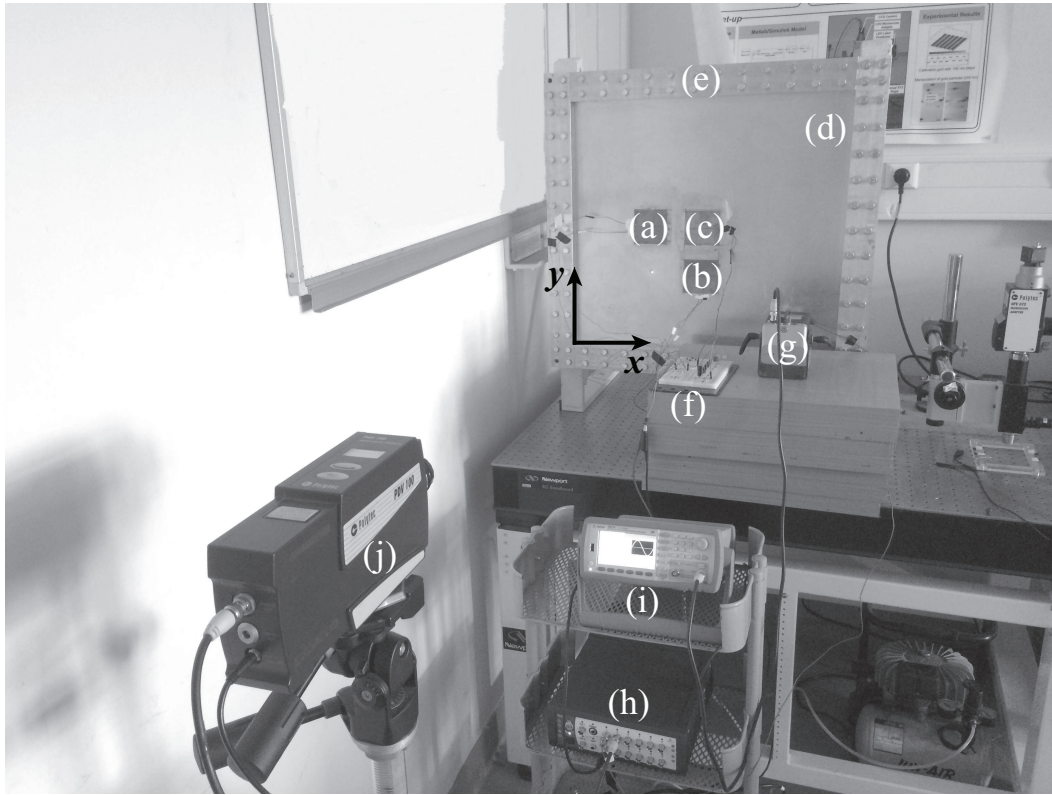


Figure 3.7: Experimental setup: (a) PZT-1, (b) PZT-2, (c) PZT-3, (d) aluminum plate, (e) clamping frame, (f) electrical circuit (g) modal shaker with a force transducer, (h) data acquisition unit, (i) signal generator and (j) laser vibrometer.

Table 3.2: Geometric, material and electroelastic properties

Property	Aluminum	Piezoceramic
Length (mm)	580	72.4
Width (mm)	540	72.4
Thickness (mm)	1.9	0.267
Young's modulus (GPa)	65.1	66
Mass density (kg/m_3)	2575	7800
Piezoelectric constant (pC/V)	-	-190
Permittivity constant (nF/m)	-	10.38

3.5.2 AC input - AC output case

Figure 3.8 presents the steady-state voltage FRFs of the MPEHs with series and parallel configurations under a harmonic point force excitation. The electrical load is close to the open-circuit condition ($R_l = 1M\Omega$). The AC-AC numerical results obtained from SPICE simulations, considering 5×5 vibration modes of the plate, accurately match with the existing analytical model presented by Aridogan et al. [Aridogan et al., 2014b] and experimental data. The modal damping ratios can be extracted from the experimental velocity or voltage FRFs with an arbitrary resistive load used in the harvesting circuit [Erturk and Inman, 2011]. It can be seen that the overall open-circuit voltage output magnitudes of series configuration (figure 3.8a) are larger than the ones in the parallel configuration (figure 3.8b), which is in line with the series connection assumption. Also in the fourth, fifth, sixth and seventh vibration modes, peak voltages are significantly smaller than the other resonance frequencies, due to the charge cancellation of piezo-patches which is directly dependent on the strain distribution under the patch covered area.

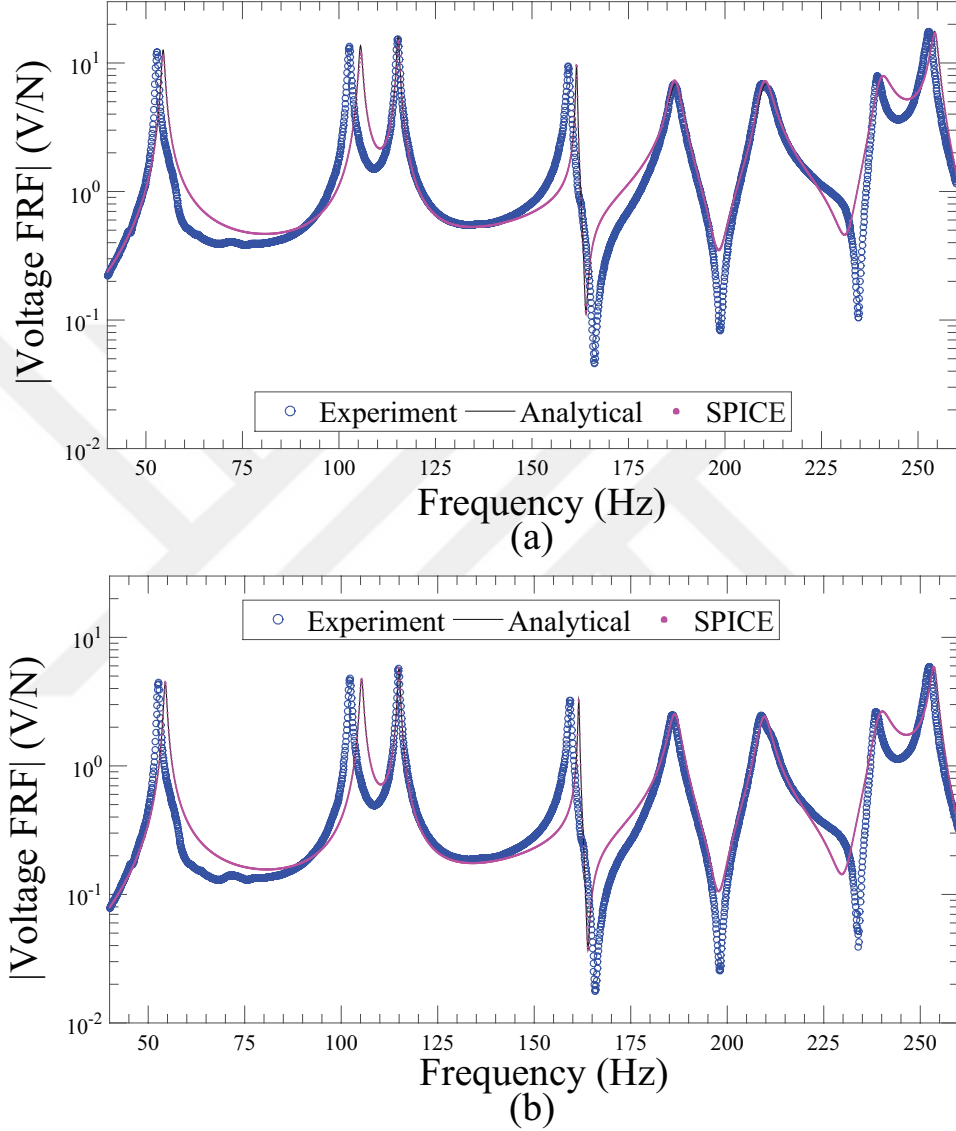


Figure 3.8: AC input - AC output case: Comparison of experiment, analytical model and SPICE simulations for voltage FRF of MPEHs in (a) series and (b) parallel configurations at open-circuit condition.

3.5.3 AC input - DC output case

The analytical and experimental velocity and DC voltage outputs of the patch harvesters at the excitation frequencies around the first vibration mode for eight values of resistive loads in series and parallel configurations are shown in figure 3.9, respectively. The harmonic force excitation magnitude for the first vibration mode is around

1 N. Although the analytical model developed in this work ignores the real behavior of the diodes, it reasonably follows the trend of experimental velocity and voltage responses.

After validating the electromechanical responses of MPEHs connected to a standard rectifier circuit for a range of resistive loads, the power outputs of the multiple harvesters are calculated for both AC-AC and AC-DC cases. Fig. 10 shows the analytical, numerical and experimental results of average power output versus load resistance for the first three vibration mode, where the peak amplitudes are extracted at the open-circuit resonance frequencies. The load resistance values used in the experiments are 100Ω , 470Ω , $1k\Omega$, $5.56k\Omega$, $15k\Omega$, $47k\Omega$, $100k\Omega$, $475k\Omega$, and $1M\Omega$. Here, the magnitudes of sinusoidal external force for the first three vibration modes, in the open-circuit conditions, are measured by the force transducer as 0.72 N, 0.53 N and 0.32 N, when the sine-sweep signals with a constant peak voltage amplitude are transmitted to the shaker. The graphs in the upper row exhibit the analytical predictions and Spice simulations for the peak AC and DC voltage outputs of MPEHs for the first three vibration modes, whereas the graphs in the bottom row show the experimental data. The overall average power levels for each vibration mode in AC-DC case are lower than the corresponding AC-AC, which can be referred mainly to the electrical loss. The optimum resistive load value, where the average power is maximum, is larger for the series configuration than that for the parallel configuration in both AC-AC and AC-DC cases. Additionally, the optimal load resistance of both the series and parallel configuration in AC-DC is slightly shifted to a larger value, when compared to the AC-AC case. The SPICE simulations always underestimate the analytical model predictions, by taking into account voltage drop of the rectifier circuit, and tend to be closer to the experiment results. Overall, the analytical and SPICE predictions agree well with the experimental data for a wide range of resistive loads.

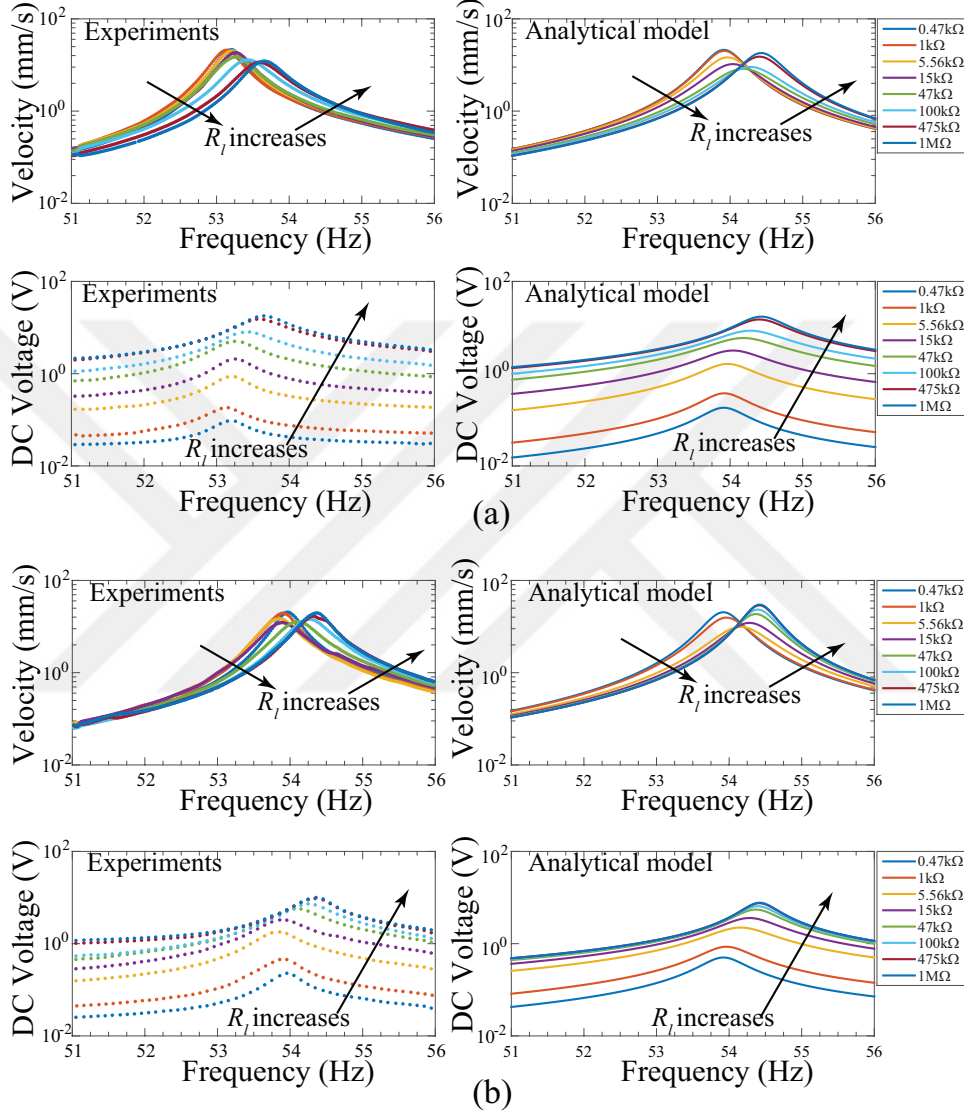


Figure 3.9: Variations of the first mode velocity and DC voltage outputs of the harvester for the range of resistive loads subjected to 1 N point force excitation: (a) Series configuration (b) Parallel configuration. Experiment data are shown in the left column and analytical results in the right column. Arrows indicate the direction of increase in resistive load value.

3.6 Comments on the Electrical Connection Configurations

In the previous chapter, the analytical and SPICE results for both AC and DC electrical outputs of MPEHs connected in series/parallel were presented together with

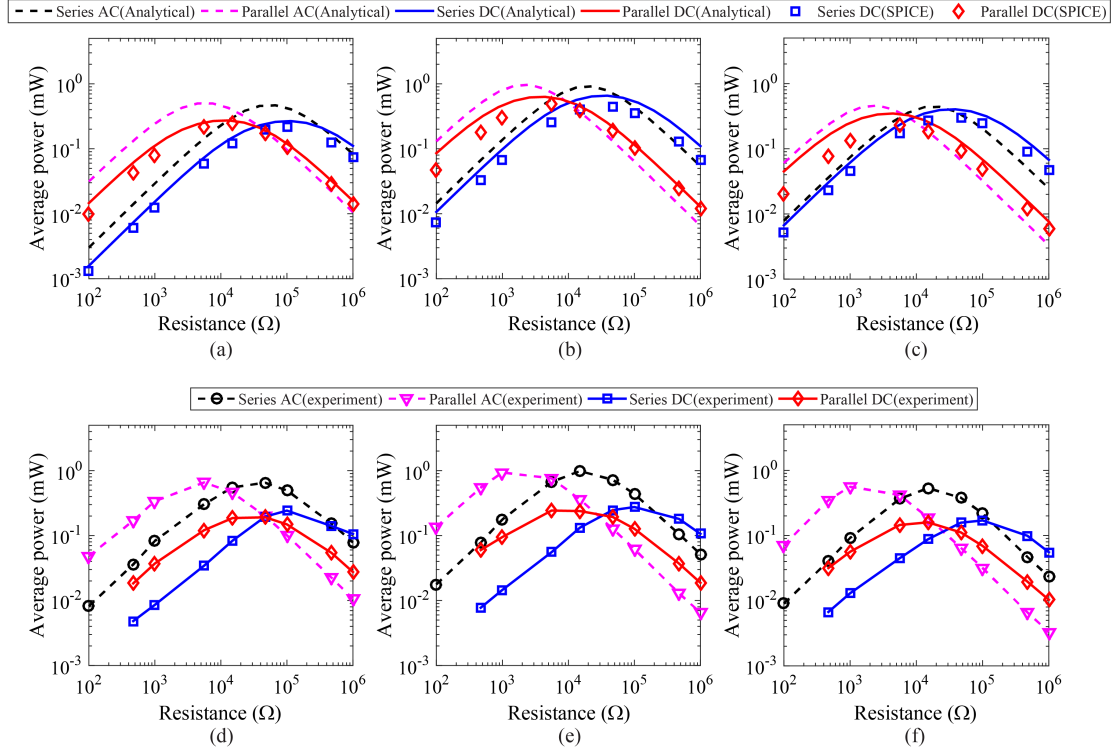


Figure 3.10: Comparison of peak power outputs for the AC input AC output and AC input DC output cases versus load resistance. Analytical model and SPICE simulation: (a) first mode (b) second mode (c) third mode, experiment data: (d) first mode (e) second mode (f) third mode.

the experimental validations. When the number of piezo-patch harvesters increases, other connection types for the electrodes can be realized such as mixed series-parallel connection. This section details the differences in electrical output behavior for a set of mixed electrical connections. Figure 3.11 illustrates four possible connection types for three piezo-patches. The ordering of the patches in connection is not considered since the piezoceramic patches used in this study have the same internal capacitance. Labels are assigned for the different connection types are PPP, PPS, SSP, and SSS, where P and S stand for parallel and series connections, respectively. Under harmonic point force excitations, the voltage outputs for these four connections are shown in figure 3.12. It can be seen that the voltage level across an arbitrary resistive load ($R_l = 100k\Omega$) is the largest for the SSS connection type which has the lowest

equivalent piezoelectric capacitance value. The equivalent capacitance of the three piezo-patches in different connection configurations are given in table 3.3. The internal capacitance of each piezo-patch is measured by the multimeter as approximately $C_p = 257 \text{ nF}$.

Table 3.3: Equivalent capacitance for different connection types.

Connection Types	SSS	PPS	SSP	PPP
Equivalent Capacitance	$\frac{C_p}{3}$	$\frac{2C_p}{3}$	$\frac{3C_p}{2}$	$3C_p$
Values	85.6 nF	171.3 nF	385.5 nF	777 nF

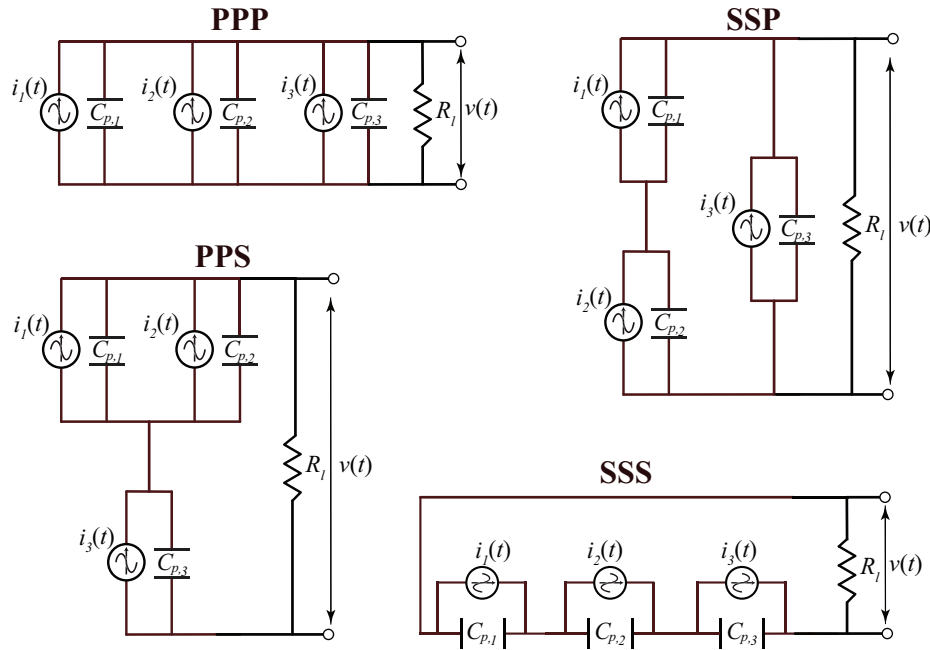


Figure 3.11: Different configurations for connecting three piezo-patches: SSS, SSP, PPS, and PPP. The labels S and P stand for the series and parallel connections, respectively.

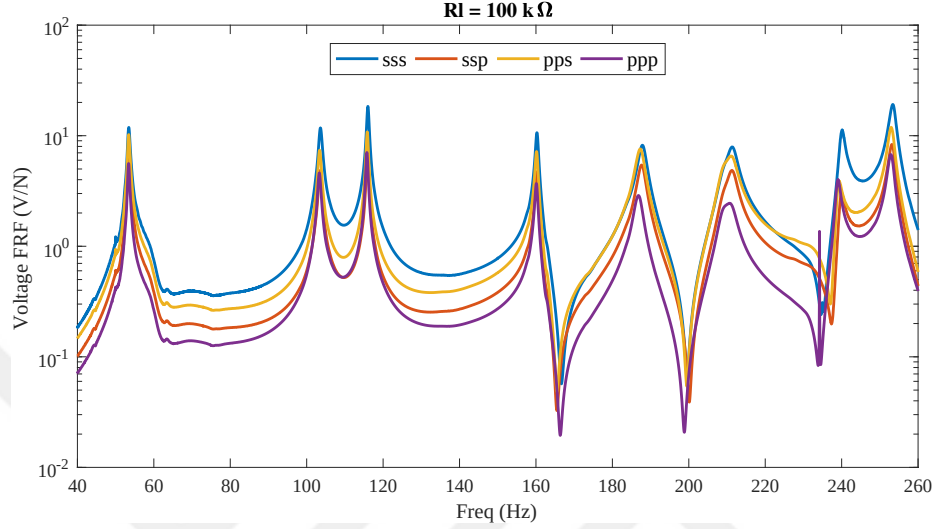


Figure 3.12: Comparison of voltage FRFs for different connection types when $R_l = 100k\Omega$. The voltage FRF amplitude for the SSS configuration is larger than the others.

Having shown the difference in the voltage FRFs of SSS, SSP, PPS, and PPP configurations, FRF experiments are conducted for a range of resistive loads (13 values from 100Ω to $1M\Omega$ and for all four possible connection types). The corresponding voltage and power FRF surfaces are presented in figures 3.13 - 3.18 for the first 6 vibration modes. The maximum power output for the optimum resistive load is marked with a blue circle. Notice that the optimum resistance has the smallest value in the PPP connection and the largest in the SSS connection. This is expected because the optimum resistance has a reverse relation with the piezoelectric capacitance, which can be obtained from the steady-state power FRF expression as [Erturk and Inman, 2011]:

$$R_l^{opt} = \frac{1}{\omega C_p} \quad (3.43)$$

Besides, the maximum power output is almost equal for all the connection types, indicating that one configuration does not have an advantage over the other one in generating electrical power. However, depending on the impedance of the sensor device, the optimum connection configuration could be chosen in order to match the

impedances for maximizing the energy flow.

On the other hand, it is important to observe the electrical response of each piezo-patch, in order to understand their phase difference which is a key factor in charge cancellation phenomena. Figure 3.19 shows the SPICE and analytical simulations for the individual voltage FRFs of the three piezo-patches with the experimental data. During the experiments, the voltage signal across one piezo-patch was separately recorded while the other two piezo-patches were short circuited. The results indicate that each patch harvester has different voltage behavior, for instance, the resonant voltage peaks at the second and third vibration modes are not pronounced for the PZT-1, while clearly seen for PZT-2 and PZT-3. This is because of the fact that the center of the PZT-1 is on the nodal line at the second and third vibration bending modes, in which the positive and negative charges are eventually cancelled out and no current is generated in the harvester.

Meanwhile, for comparing the voltage phases the piezo-patch harvesters, the imaginary part of the voltage FRFs are shown in figure 3.20. Using the imaginary data of the voltage response, one can decide how to combine the electrodes. For example, consider the 6th vibration mode where the PZT-1 and PZT-3 have negative imaginary peaks while PZT-2 has a positive imaginary peak value. For connecting the piezoelectric electrodes in a right combination, the terminals of the PZT-2 should be reversed when connecting the patches together (in series, parallel, or mixed series-parallel) so that all the patches have the same phase, hence no charge cancellation occurs.

Another approach to remedy the problem with the charge cancelation can be the use of respective rectifiers instead of single rectifier as shown in figure 3.21. Using one AC-DC converter circuit for each patch harvester, and connecting them together in series/parallel to a resistive load can eliminates the summation of negative and positive voltages since they would be all positive signals with small ripples. To give an example, again consider the 6th vibration mode of the plate where two piezo-patches having the same voltage sign and the other piezo-patch having the opposite

voltage sign. The DC voltage responses across the resistive load for the single and respective rectification cases are shown in figure 3.22. There is a drastic increase in the electrical output when multiple AC-DC converters are used as compared to single converter. Therefore, such respective rectification method can be safely implemented for the network of patch harvesters, although it adds up extra circuit components which have inherent electrical loss.

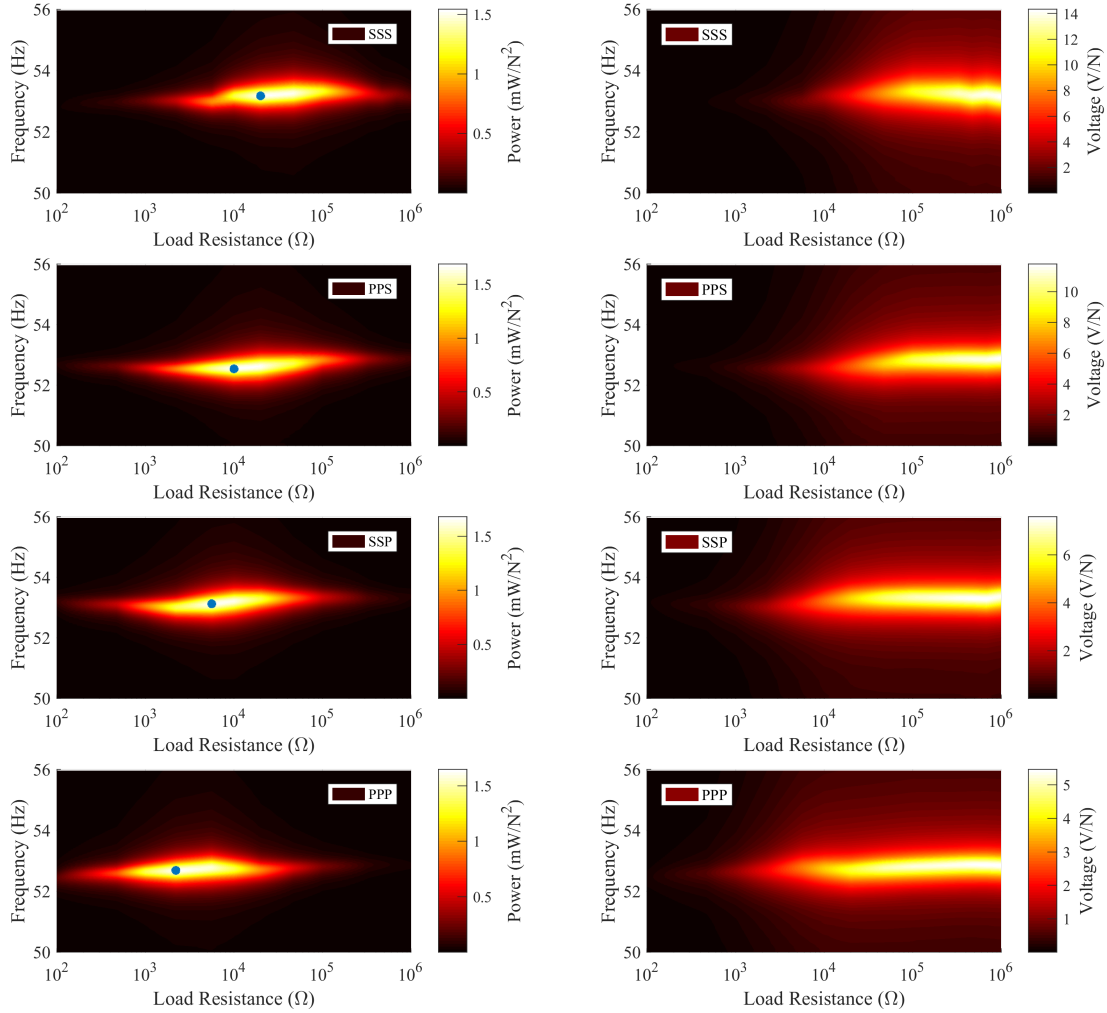


Figure 3.13: Voltage and power FRF surface plots for different connection types: Vibration mode 1.

3.7 Conclusion

In this chapter, an analytical and a numerical approach was proposed for modeling MPEHs attached to thin plates with practical circuit interface. The analytical model integrates the equivalent impedance of the harvesting circuit into the distributed-parameter electroelastic model of the plate with multiple patch harvesters. Analytical predictions for the steady-state velocity response and DC voltage output of the MPEHs in series and parallel configurations were presented for a wide range of resistive loads around the first resonance frequency and validated by the experimental results. The ECM of the electromechanical system was developed for any connection types of the harvesters and validated for the AC-AC case by the existing analytical solution and experiments. Moreover, peak power outputs of the harvester with standard AC-AC and AC-DC configurations are predicted for the first three vibration modes as a function of varying load resistance. Furthermore, in the last section, different connection configurations were discussed for MPEHs. It was shown that respective rectifiers can drastically increase the DC power output of the patch harvesters as multiple vibration modes of the host plate can be exploited without any charge cancellations. In sum, the equivalent impedance electroelastic model and the ECM developed in this work, which can be extended to other distributed-parameter systems with integrated single/multiple piezo-patch harvesters, allows evaluating the performance of such systems for design improvement and power optimization.

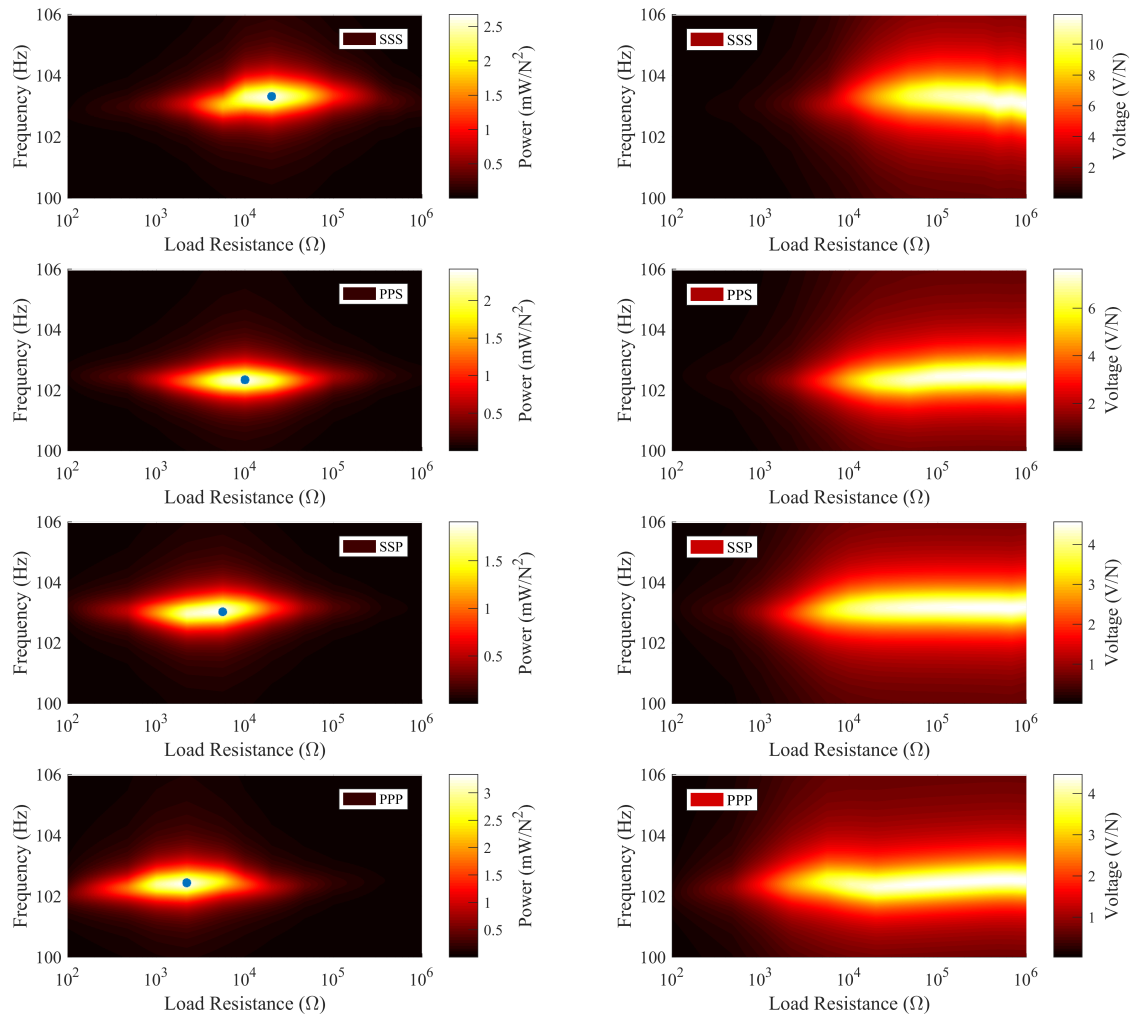


Figure 3.14: Voltage and power FRF surface plots for different connection types: Vibration mode 2.

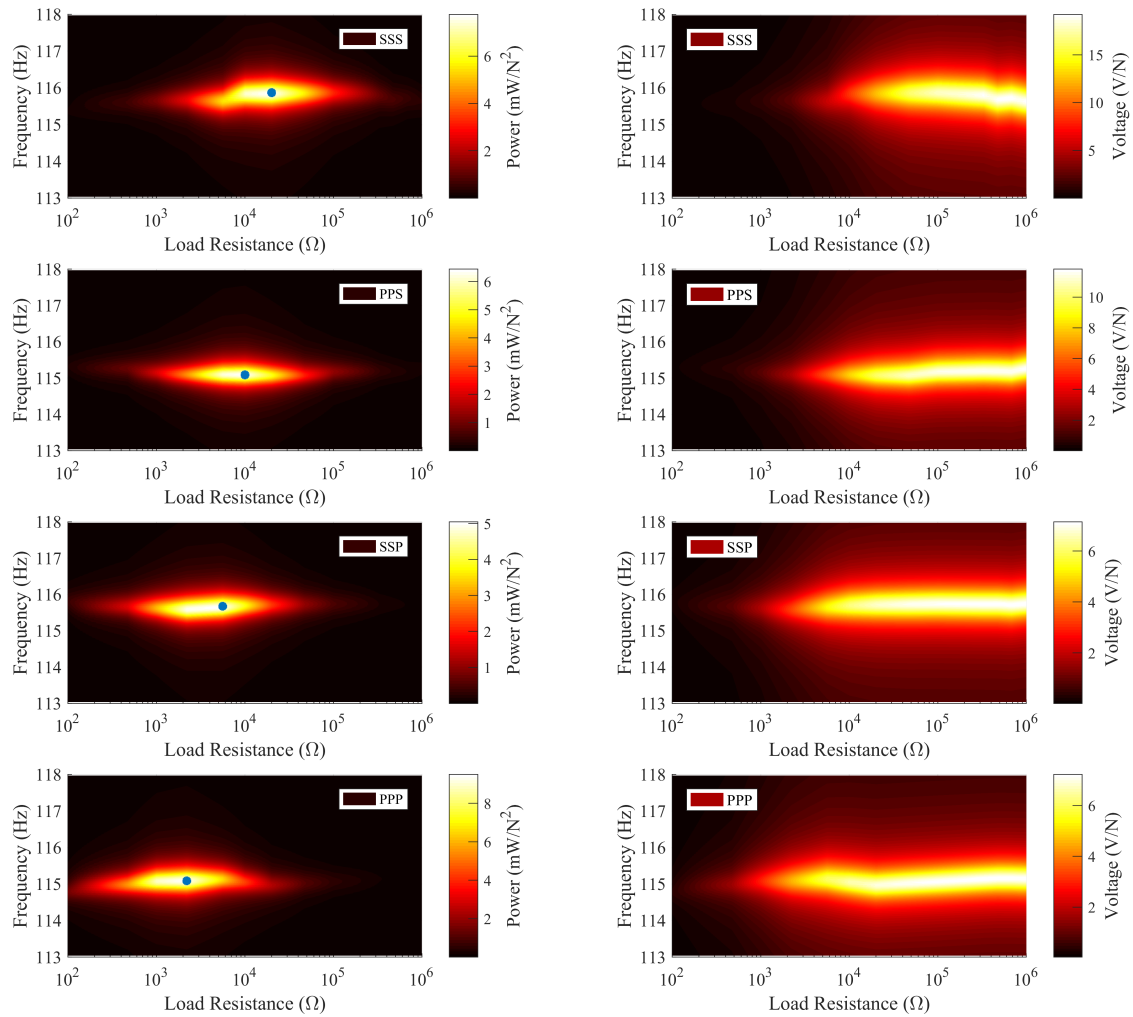


Figure 3.15: Voltage and power FRF surface plots for different connection types: Vibration mode 3.

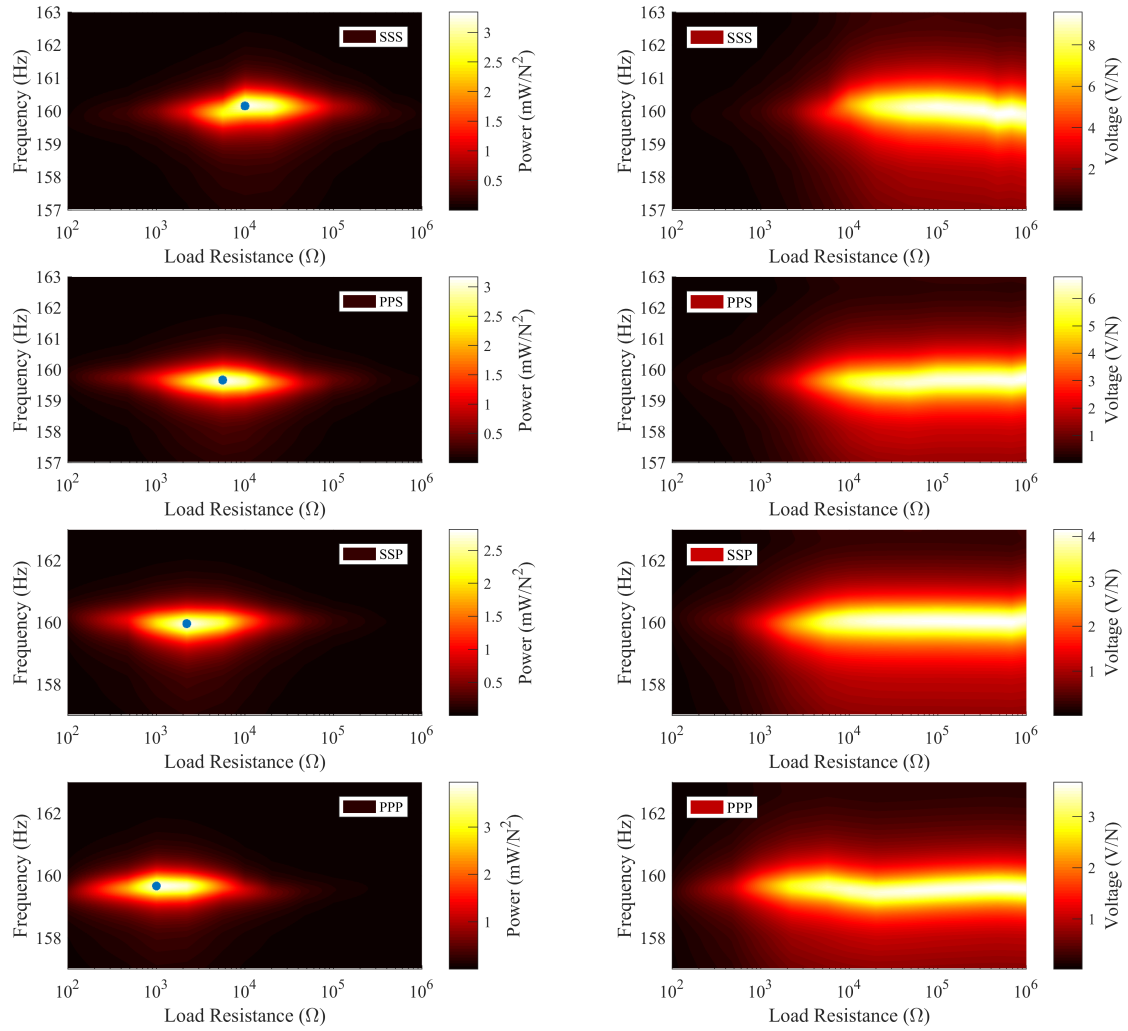


Figure 3.16: Voltage and power FRF surface plots for different connection types: Vibration mode 4.

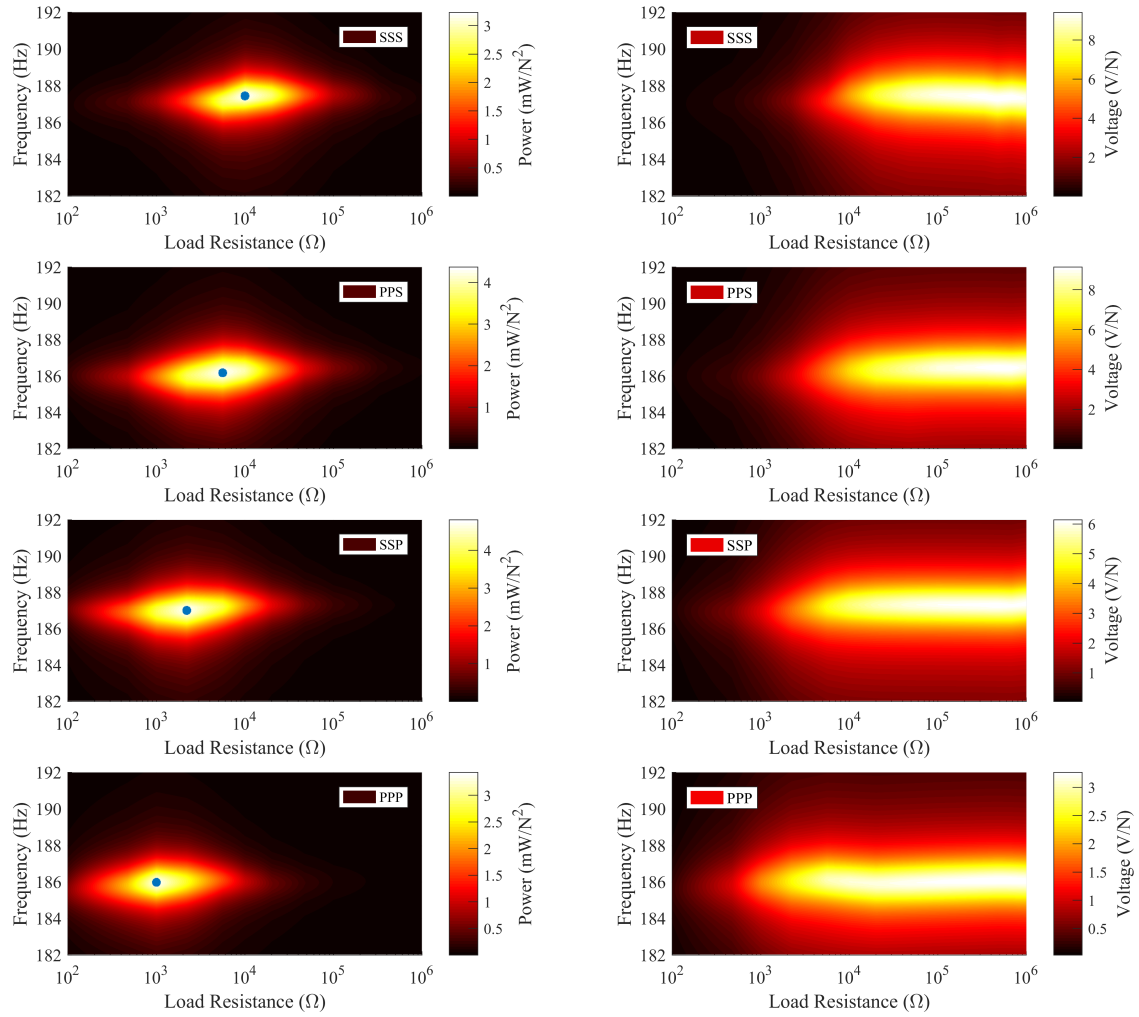


Figure 3.17: Voltage and power FRF surface plots for different connection types: Vibration mode 5.

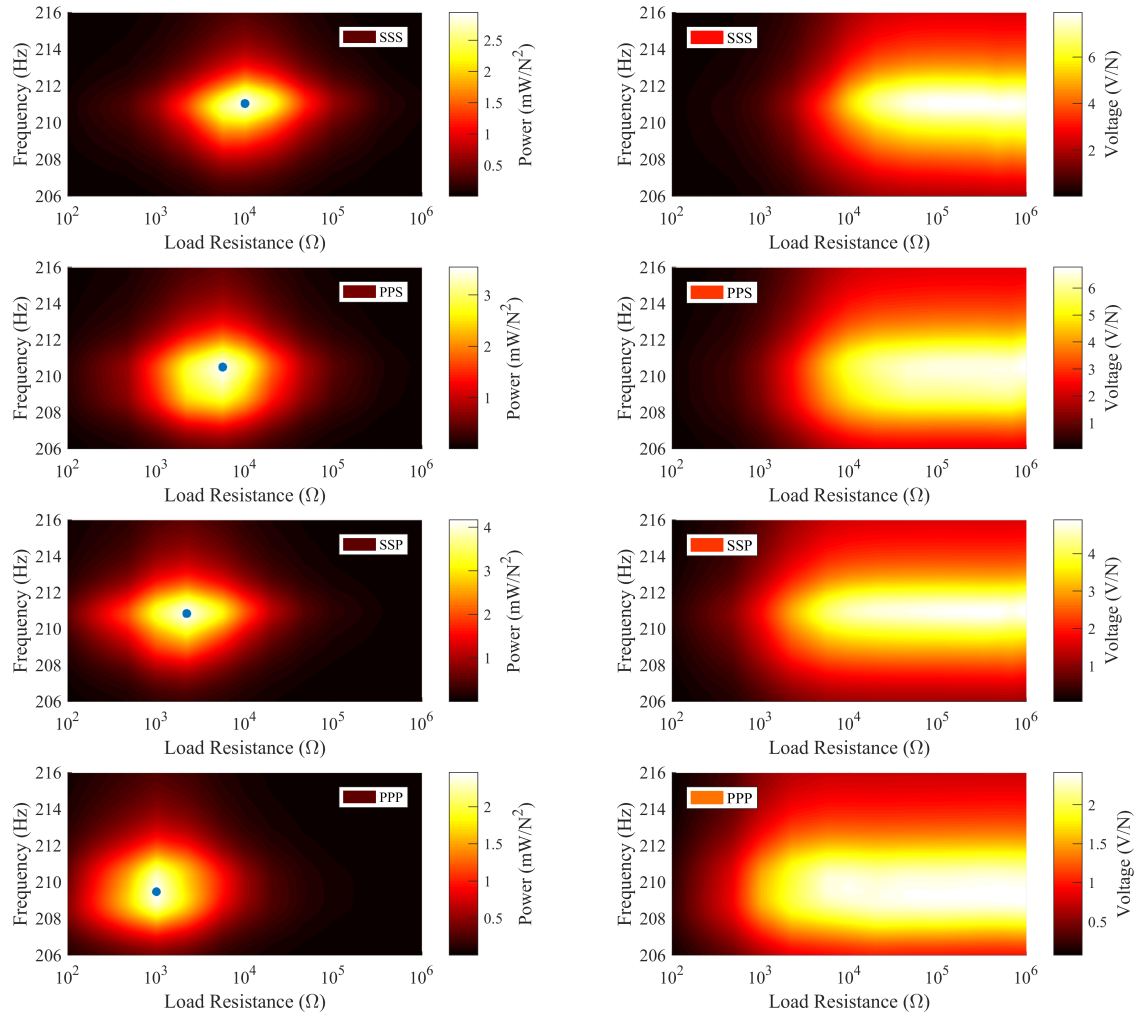


Figure 3.18: Voltage and power FRF surface plots for different connection types: Vibration mode 6.

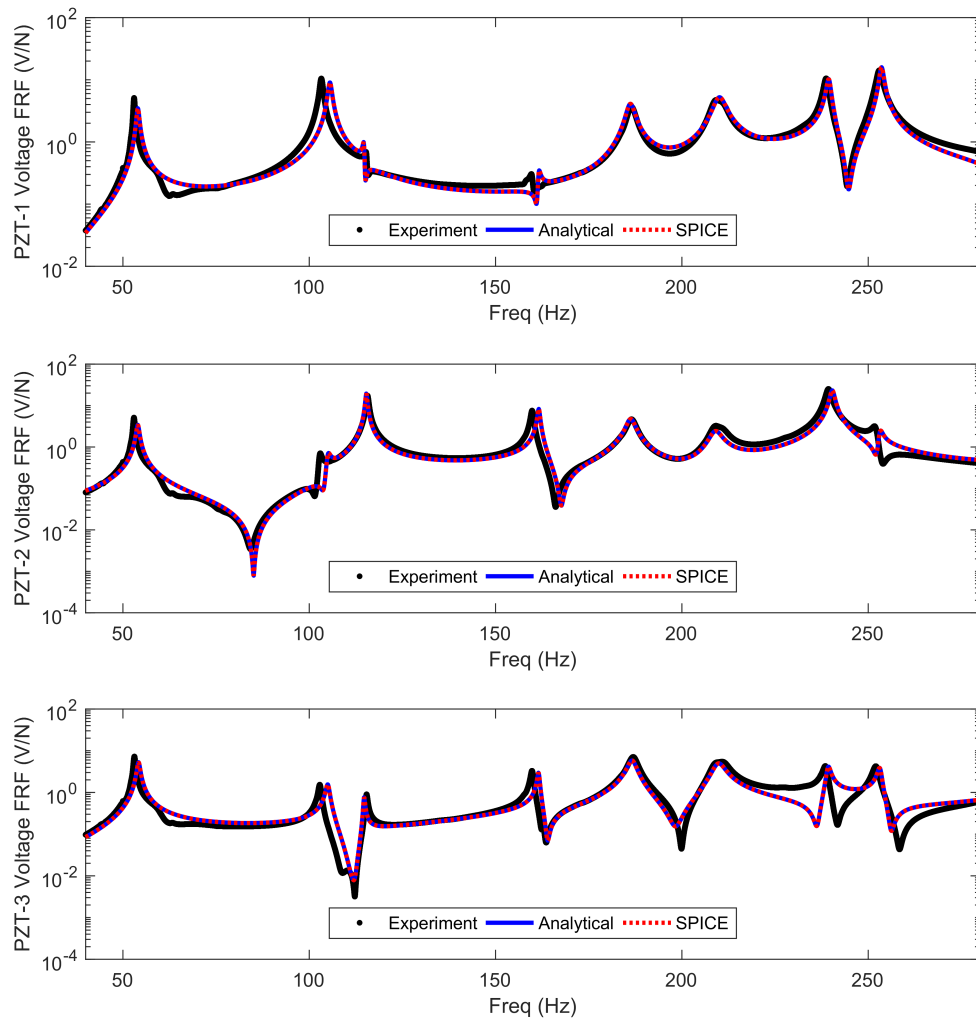


Figure 3.19: Voltage FRFs for each piezo-patch harvesters at open-circuit condition. Analytical and SPICE simulation results accurately match with the experimental data.

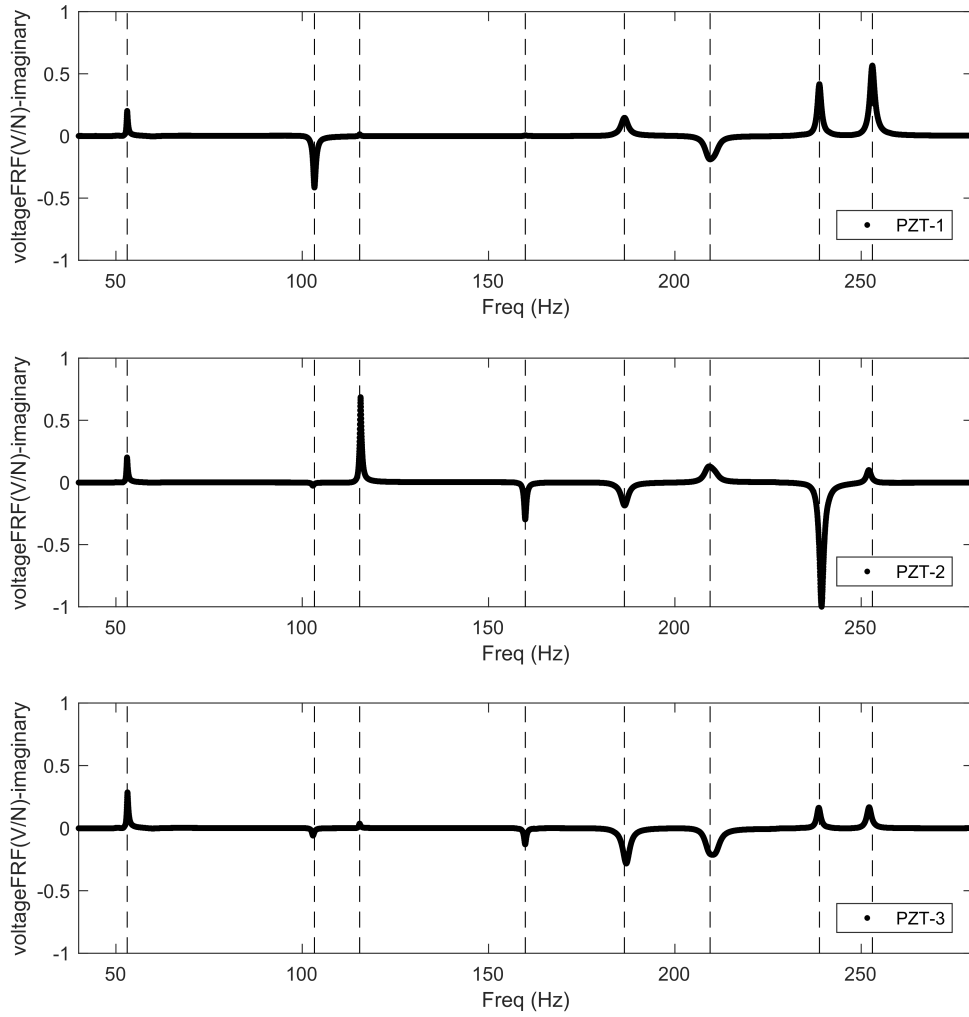


Figure 3.20: Imaginary part of the Voltage FRFs for each piezo-patch harvesters at open-circuit condition. The amplitudes are normalized, and positive/negative sign indicates the positive/negative strain sign under the covered patch area

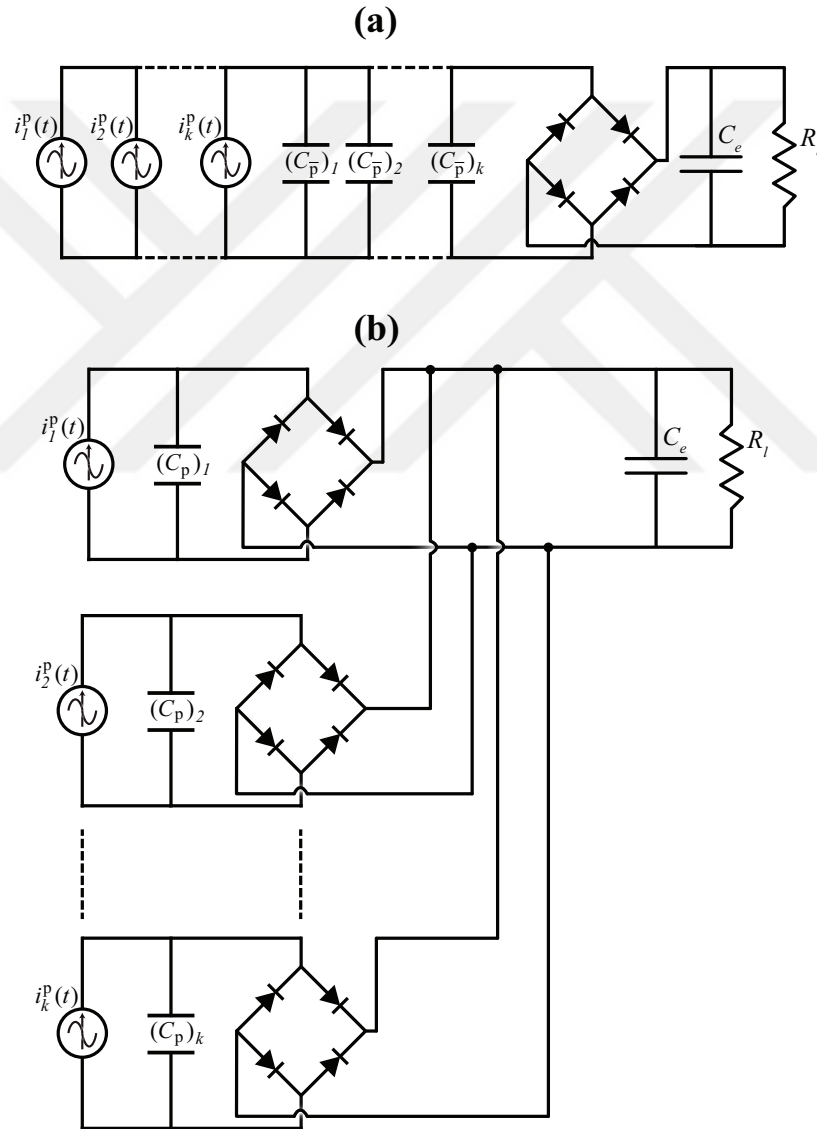


Figure 3.21: Different AC-DC converter circuit configurations: (a) Single rectifier (b) Multiple or respective rectifiers

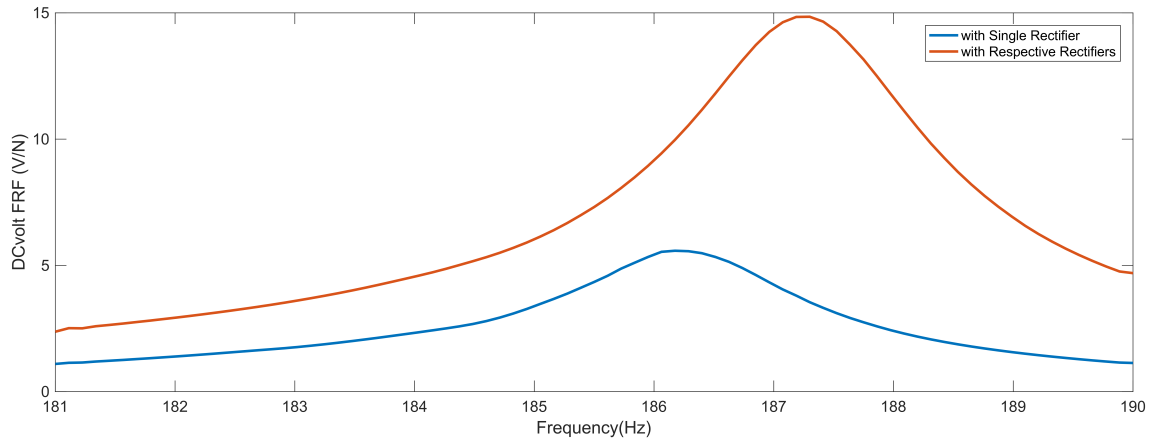


Figure 3.22: Comparison of DC Voltage response nearby the 6th vibration mode for single and multiple rectification cases.

Chapter 4

INVESTIGATION OF BANDGAP FORMATION IN TWO-DIMENSIONAL ELECTROMECHANICAL METASTRUCTURES

4.1 *Introduction*

Piezoelectric shunt damping techniques have been widely studied over the past decades as an alternative to the conventional bulky mass-spring-damper systems for attenuating the structural vibrations of flexible structures. Piezoelectric transducers can be directly integrated to vibrating host structures, without adding significant mass and volumetric occupancy, and convert the mechanical energy into the electrically energy which can be dissipated in various electrical circuit components. More recently, piezoelectric shunt damping techniques have been used jointly with locally resonant metamaterials for bandgap formation, a frequency range in which vibration wave does not propagate through the elastic medium. The metamaterial-based electromechanical structures are realized by having many piezoelectric elements integrated to the host structure, enabling the creation of bandgap at wavelengths much larger than the lattice constant [Casadei et al., 2009, Airoidi and Ruzzene, 2011, Casadei et al., 2012, Chen et al., 2013, Sugino et al., 2017a]. In the literature of metamaterials field, most studies assume an infinite structure made of repeated assembly of unit-cells and use finite-element methods defined in Bloch theorem for estimating the bandgaps formation. However, in practice, finite structures with distributed array of resonators (and not necessary periodic) are employed, where this type of modeling cannot bridge the information between the bandgaps formation and modal analysis of such meta-

materials. Lately, Sugino et.al. [Sugino et al., 2017a] demonstrated modal analysis based modeling of a locally resonant 1D electromechanical metastructure, where a finite-size bimorph piezoelectric beam with many segmented electrodes shunted to inductive circuits is used for creating bandgaps.

This chapter investigates the bandgaps formation in a 2D electromechanical structure made from a finite-sized thin plate with an array of resonant piezo-patches. First, modal analysis of the electromechanical system is formulated using Rayleigh-Ritz method for two configurations of piezo-patches arrangement: (a) unsymmetrical configuration, where patches are located on just one surface of the plate; (b) symmetrical configuration, where patch pairs are symmetrically located on both top and bottom surface of the plate. Next, electromechanical frequency response functions (FRFs) of the structure are obtained in presence of the resonant shunt circuits. The displacement and voltage FRFs are validated by the experimental data for three number of piezo-patches in open-circuit condition. Effect of piezoelectric materials mass and stiffness contribution on the natural frequencies of the host system is demonstrated for the first four vibration modes, when the number of piezo-patches and their thicknesses are increased. System-level electromechanical coupling coefficients are shown to be dependent on the number of piezo-patches as well as their thickness. The interconnection of piezo-patches and how it can directly affect the shunt damping are discussed. Next, formation of locally resonant bandgaps is investigated in the electromechanical system by tuning resonant frequencies of each shunt circuit to the target frequency of interest. Since the system electromechanical coupling depends on the geometry and properties of the piezo-patches, the bandgaps are shown for different piezoelectric materials such as ceramics and single crystals. Finally, creation of the locally resonant bandgaps at different target frequencies are presented.

4.2 Electroelastic modeling of a thin plate with array of resonant piezo-patches

Consider a thin plate structure with an array of piezoelectric patches integrated to it as shown in figure 4.1. The piezo-patches can be placed either symmetrically on both sides (figure 4.1a) or only on one surface-side (figure 4.1b) of the host plate. In the symmetrical arrangement, the patch pairs can be connected in a series or parallel configuration, where the substrate can be the conductive layer between the top and bottom patches. For this study, the piezo-patch pairs are poled in the opposite directions under the transverse vibrations, meaning that the electrodes are connected in series. The electrode layers covering the both surfaces of the piezo-patches and the bonding layers are assumed to have negligible thickness. The thin plate with an array of structurally integrated piezo-patches is modeled based on the Kirchhoff plate theory by assuming that the effects of transverse shear deformation is neglected. Under the point force excitation, the partial differential equation for the undamped vibrations of the composite plate can be written as

$$-\frac{\partial^2 M_1}{\partial x^2} - 2\frac{\partial^2 M_6}{\partial x \partial y} - \frac{\partial^2 M_2}{\partial y^2} + m(x, y) \frac{\partial^2 w(x, y, t)}{\partial t^2} = f(t) \delta(x - x_0) \delta(y - y_0) \quad (4.1)$$

where M_1 and M_2 are the internal bending moments along the x, y axes respectively, and M_6 in the xy -plane. The mass per unit area is indicated as $m(x, y)$, and $w(x, y, t)$ is the transverse vibrations of the plate at position (x, y) and time t . The transverse point force $f(t)$ is applied at (x_0, y_0) coordinates which is specified in the equation by the Dirac delta functions of $\delta(x)$ and $\delta(y)$. The aluminum substrate is assumed to be isotropic, therefore, the stress-strain relations of the plate structure reduces to

$$\begin{bmatrix} T_1^s \\ T_2^s \\ T_6^s \end{bmatrix} = \frac{Y_s}{1 - \nu_s^2} \begin{bmatrix} 1 & \nu_s & 0 \\ \nu_s & 1 & 0 \\ 0 & 0 & (1 - \nu_s)/2 \end{bmatrix} \begin{bmatrix} S_1^s \\ S_2^s \\ S_6^s \end{bmatrix} \quad (4.2)$$

where T_1^s and T_2^s are the normal stress components along the x and y axes respectively, T_6^s is the strain component in the xy -plane, Y_s is the Young's modulus

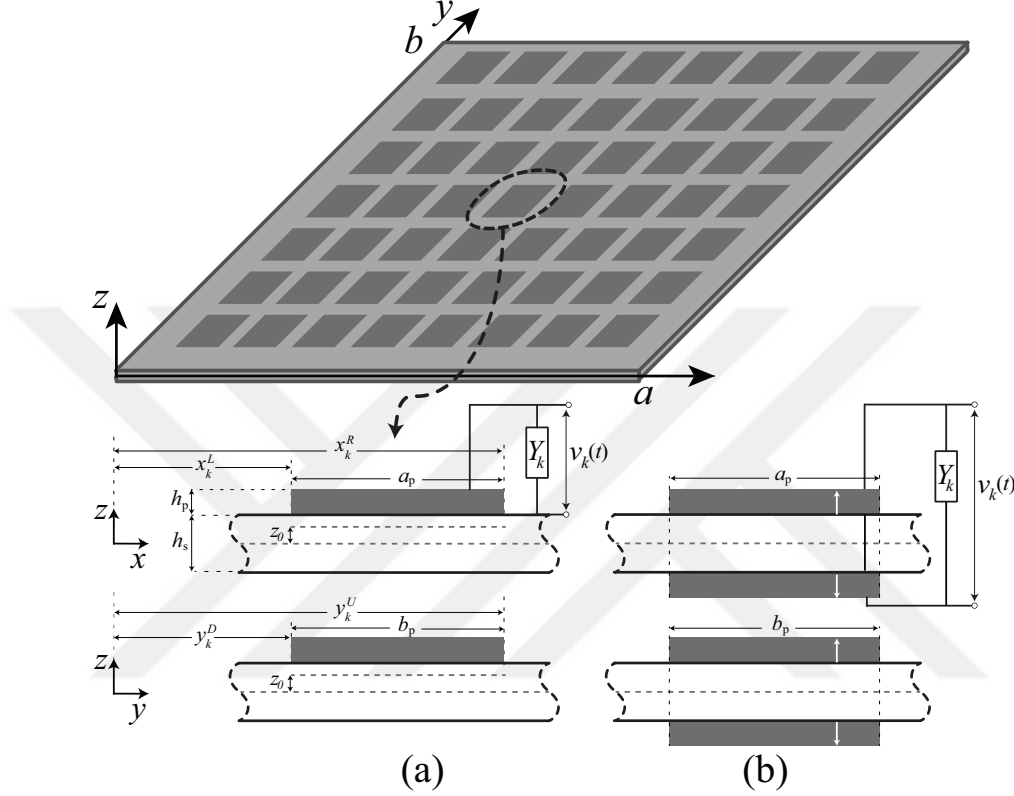


Figure 4.1: Schematic of a two-dimensional electromechanical metastructure. Close-up view shows the cross section of the thin plate with (a) single patch and (b) patch pair connected to k th shunt admittance. In case (b) patches are symmetrically placed on two surface sides and connected in series with the poling directions pointed out by white arrows.

and ν_s is the Poisson's ratio, S_1^s and S_2^s are the normal strain components, S_6^s is the strain component in the xy -plane. Considering the thin two-dimensional piezoelectric patch, the reduced form of the linear constitutive equations is given by

$$\begin{bmatrix} T_1^p \\ T_2^p \\ T_6^p \\ D_3 \end{bmatrix} = \begin{bmatrix} \bar{c}_{11}^E & \bar{c}_{12}^E & 0 & -\bar{e}_{31} \\ \bar{c}_{12}^E & \bar{c}_{11}^E & 0 & -\bar{e}_{31} \\ 0 & 0 & \bar{c}_{66}^E & 0 \\ \bar{e}_{31} & \bar{e}_{31} & 0 & -\bar{\epsilon}_{33}^S \end{bmatrix} \begin{bmatrix} S_1^p \\ S_2^p \\ S_6^p \\ E_3 \end{bmatrix} \quad (4.3)$$

where T_1^p and T_2^p are the normal stress, S_1^p and S_2^p are the axial strain components along the x and y axes, whereas, T_6^p and S_6^p are the shear stress and strain com-

ponents in the xy -plane; D_3 is the electric displacement, and E_3 is the electric field in the z direction. Further, $\bar{c}_{11}^E$, $\bar{c}_{12}^E$, and $\bar{c}_{66}^E$ are the elastic stiffness components at constant electric field, $\bar{c}_{31}$ is the effective piezoelectric stiffness constant, and $\bar{\epsilon}_{33}$ is the permittivity component at constant strain. Note that, herein and hereafter, the superscripts or subscripts "s" and "p" stand for the substrate and piezoelectric patch, respectively. The in-plane strain components at a certain level $\bar{z}$ under the Kirchhoffs assumptions can be written as:

$$\begin{aligned} S_1(x, y, z, t) &= -\bar{z} \frac{\partial^2 w(x, y, yt)}{\partial x^2}, \quad S_2(x, y, z, t) = -\bar{z} \frac{\partial^2 w(x, y, yt)}{\partial y^2}, \\ S_6(x, y, z, t) &= -2\bar{z} \frac{\partial^2 w(x, y, yt)}{\partial x \partial y} \end{aligned} \quad (4.4)$$

where $\bar{z}$ for the areas that the neutral surface is not coincident with the xy -plane (see figure 4.1) is expressed as $\bar{z} = z - z_0$, and for the other (x, y) locations as $\bar{z} = z$. For the unsymmetrical configuration (shown in 4.1a), the internal bending moment terms in (4.1) are the first moments of the stress field over the cross-section:

$$\begin{aligned} M_q &= \left(1 - \sum_{k=1}^P \bar{H}_k(x, y)\right) \int_{-h_s/2}^{h_s/2} T_q^s z \, dz + \sum_{k=1}^P \bar{H}_k(x, y) \int_{-h_s/2}^{h_s/2} T_q^s (z - z_0) \, dz \\ &\quad + \sum_{k=1}^P \bar{H}_k(x, y) \int_{h_s/2}^{h_s/2+h_p} T_{q,k}^p (z - z_0) \, dz, \text{ for } q = 1, 2, 6 \end{aligned} \quad (4.5)$$

and for the symmetrical configuration (shown in figure 4.1b)

$$\begin{aligned} M_q &= \int_{-h_s/2}^{h_s/2} T_q^s z \, dz + \sum_{k=1}^P \bar{H}_k(x, y) \left(\int_{-h_s/2}^{h_s/2} T_{q,k}^p z \, dz + \int_{h_s/2}^{h_s/2+h_p} T_{q,k}^p z \, dz \right) \\ &\quad \text{for } q = 1, 2, 6 \end{aligned} \quad (4.6)$$

Here, $\bar{H}_k(x, y)$ is the cross product of the two heavy-side functions in x and y directions, indicating the edge coordinates of the k th piezo-patch:

$$\begin{aligned} \bar{H}_k(x, y) &= [H(x - x_k^L) - H(x - x_k^R)] \times [H(y - y_k^D) - H(y - y_k^U)] \\ &\quad \text{for } k = 1, 2, \dots, P \end{aligned} \quad (4.7)$$

Having substituted the internal moments in equations (4.5), (4.6) into (4.1), the governing partial differential equation of the plate with array of piezo-patches, and

current balance equation in each piezo-patch are

$$D_{11} \left(\frac{\partial^4 w(x, y, t)}{\partial x^4} + \frac{\partial^4 w(x, y, t)}{\partial y^4} \right) + 2D_{12} \frac{\partial^4 w(x, y, t)}{\partial x^2 \partial y^2} + m \frac{\partial^2 w(x, y, t)}{\partial t^2} - \vartheta \sum_{k=1}^P v_k(t) \left(\frac{\partial^2 \bar{H}_k(x, y)}{\partial x^2} + \frac{\partial^2 \bar{H}_k(x, y)}{\partial y^2} \right) = f(t) \delta(x - x_0) \delta(y - y_0) \quad (4.8)$$

$$C_{p,k} \frac{dv_k}{dt} + Y_k v_k(t) + \vartheta \int_{y_k^D}^{y_k^U} \int_{x_k^L}^{x_k^R} \left(\frac{\partial^3 w(x, y, t)}{\partial y^2 \partial t} + \frac{\partial^3 w(x, y, t)}{\partial x^2 \partial t} \right) dx dy = 0 \quad \text{for } k = 1, 2, \dots, P \quad (4.9)$$

where ϑ is the electromechanical coupling term in physical coordinates; $C_{p,k}$, Y_k , and $v_k(t)$ are the piezoelectric capacitance, the load admittance, and the voltage across the k th electrodes of single or paired patch respectively, D_{11} and D_{12} are the flexural rigidity terms, $m(x, y)$ is the mass per unity, for the unsymmetrical patch configuration given by

$$\begin{aligned} D_{11} &= \frac{Y_s}{(1 - \nu_s^2)} \left[I_s + h_s z_0^2 \sum_{k=1}^P \bar{H}_k(x, y) \right] + \bar{c}_{11}^E I_p \sum_{k=1}^P \bar{H}_k(x, y) \\ D_{12} &= \frac{Y_s}{(1 - \nu_s^2)} \left[I_s + h_s z_0^2 \sum_{k=1}^P \bar{H}_k(x, y) \right] + (\bar{c}_{12}^E + 2\bar{c}_{66}^E) I_p \sum_{k=1}^P \bar{H}_k(x, y) \\ I_s &= \frac{h_s^3}{12}, I_p = \frac{1}{3} \left[\left(h_p + \frac{h_s}{2} - z_0 \right)^3 - \left(\frac{h_s}{2} - z_0 \right)^3 \right] \\ m &= \rho_s h_s + \rho_p h_p \sum_{k=1}^P \bar{H}_k(x, y) \\ \vartheta &= \frac{\bar{e}_3 1}{2h_p} \left[\left(h_p + \frac{h_s}{2} - z_0 \right)^2 - \left(\frac{h_s}{2} - z_0 \right)^2 \right] \\ C_{p,k} &= \bar{\epsilon}_{33}^S \frac{a_p b_p}{h_p} \end{aligned} \quad (4.10)$$

and for the symmetrical configuration

$$\begin{aligned}
 D_{11} &= \frac{Y_s I_s}{(1 - \nu_s^2)} + 2\bar{c}_{11}^E I_p \sum_{k=1}^P \bar{H}_k(x, y) \\
 D_{12} &= \frac{Y_s I_s}{(1 - \nu_s^2)} + 2(\bar{c}_{12}^E + 2\bar{c}_{66}^E) I_p \sum_{k=1}^P \bar{H}_k(x, y) \\
 I_s &= \frac{h_s^3}{12}, I_p = \frac{1}{3} \left[\left(h_p + \frac{h_s}{2} \right)^3 - \left(\frac{h_s}{2} \right)^3 \right] \\
 m &= \rho_s h_s + 2\rho_p h_p \sum_{k=1}^P \bar{H}_k(x, y) \\
 \vartheta &= \frac{\bar{e}_3 1}{h_p} \left[\left(h_p + \frac{h_s}{2} \right)^2 - \left(\frac{h_s}{2} \right)^2 \right] \\
 C_{p,k} &= \bar{\varepsilon}_{33}^S \frac{a_p b_p}{2h_p}
 \end{aligned} \tag{4.11}$$

Based on the assumed modes expansion, the transverse displacement of the beam can be expressed as

$$w(x, y, t) = \sum_{m=1}^M \sum_{n=1}^N \varphi_{mn}(x, y) \mu_{mn}(t) \tag{4.12}$$

where $\mu_{mn}(t)$ are the modal coordinates; $\varphi_{mn}(x, y)$ are the assumed mode shapes of the plate with array of piezo-patches. Because of the geometric discontinuities introduced by the patches, using the exact solution would be cumbersome. Therefore, the natural frequencies are obtained using the characteristic orthogonal polynomials as the trial functions in the Rayleigh-Ritz method which is an approximate solution to the differential eigenvalue problem [Bhat, 1985, Meirovitch and Parker, 2001]. By normalizing the spatial coordinates as $\eta = x/a$ and $\xi = y/b$, the orthonormal polynomials in can be generated through Gram-Schmidt process in the intervals of

$0 \leq \eta \leq 1$ and $0 \leq \xi \leq 1$ as [Bhat, 1985]

$$\begin{aligned}
 X_1(\eta) &= (\eta - B_1)X_0(\eta), \\
 X_m(\eta) &= (\eta - B_m)X_{m-1}(\eta)C_mX_{m-2}(\eta), \\
 B_m &= \frac{\int_0^1 \eta X_{m-1}^2(\eta) d\eta}{\int_0^1 X_{m-1}^2(\eta) d\eta}, \\
 C_m &= \frac{\int_0^1 \eta X_{m-1}(\eta)X_{m-2}(\eta) d\eta}{\int_0^1 X_{m-2}^2(\eta) d\eta}
 \end{aligned} \tag{4.13}$$

The first polynomial component $X_0(\eta)$ is a normalized trial function which satisfies the boundary conditions, a fully cantilevered plate (CCCC) in this case, given by [Bhat, 1985]

$$X_0(\eta) = \frac{(\eta^2 - 2\eta^3 + \eta^4)}{\sqrt{\left(\int_0^1 (\eta^2 - 2\eta^3 + \eta^4)^2 d\eta\right)}} \tag{4.14}$$

The same procedure can be followed for constructing the $Y_n(\xi)$ polynomials with $n = 1, 2, \dots, N$. Next, writing the Rayleigh quotient in the form of a ratio of the potential energy to the kinetic energy of the electromechanical system (see [Bhat, 1985, Yoon et al., 2016] for the details of the energy expressions), and minimizing it with respect to the coefficients A_{ij} , yields the following algebraic eigenvalue equation

$$[K_{mn,ij} - \lambda_{mn}M_{mn,ij}] A_{mn} = 0 \tag{4.15}$$

where for the unsymmetrical configuration, the mass and stiffness matrix of the elec-

tromechanical system are

$$\begin{aligned}
 K_{mn,ij} = & E_{mi}^{(2,2)} F_{nj}^{(0,0)} + \left(\frac{h_s z_0^2}{I_s} + \frac{\bar{c}_{11}^E I_p}{Y_s I_s / (1 - \nu_s^2)} \right) \sum_{k=1}^P \left(\hat{E}_{mi}^{(2,2)} \right)_k \left(\hat{F}_{nj}^{(0,0)} \right)_k \\
 & + \alpha^4 \left[E_{mi}^{(0,0)} F_{nj}^{(2,2)} + \left(\frac{h_s z_0^2}{I_s} + \frac{\bar{c}_{11}^E I_p}{Y_s I_s / (1 - \nu_s^2)} \right) \sum_{k=1}^P \left(\hat{E}_{mi}^{(0,0)} \right)_k \left(\hat{F}_{nj}^{(2,2)} \right)_k \right] \\
 & + \alpha^2 \left[E_{mi}^{(0,2)} F_{nj}^{(2,0)} + E_{mi}^{(2,0)} F_{nj}^{(0,2)} + \left(\frac{\nu_s h_s z_0^2}{I_s} + \frac{\bar{c}_{12}^E I_p}{Y_s I_s / (1 - \nu_s^2)} \right) \right. \\
 & \left. \sum_{k=1}^P \left(\left(\hat{E}_{mi}^{(0,2)} \right)_k \left(\hat{F}_{nj}^{(2,0)} \right)_k + \left(\hat{E}_{mi}^{(2,0)} \right)_k \left(\hat{F}_{nj}^{(0,2)} \right)_k \right) \right] \\
 & + 4\alpha^2 \left[E_{mi}^{(1,1)} F_{nj}^{(1,1)} + \left(\frac{(1 - \nu_s) h_s z_0^2}{2I_s} + \frac{\bar{c}_{66}^E I_p}{Y_s I_s / (1 - \nu_s^2)} \right) \sum_{k=1}^P \left(\hat{E}_{mi}^{(1,1)} \right)_k \left(\hat{F}_{nj}^{(1,1)} \right)_k \right] \\
 M_{mni,j} = & E_{mi}^{(0,0)} F_{nj}^{(0,0)} + \frac{\rho_p h_p}{\rho_s h_s} \sum_{k=1}^P \left(\hat{E}_{mi}^{(0,0)} \right)_k \left(\hat{F}_{nj}^{(0,0)} \right)_k
 \end{aligned} \tag{4.16}$$

where

$$\begin{aligned}
 E_{mi}^{(r,s)} &= \int_0^1 \frac{d^r X_m}{d\eta^r} \frac{d^s X_i}{d\eta^s} d\eta; \left(\hat{E}_{mi}^{(r,s)} \right)_k = \int_0^1 H(\eta - x_k^L/a) - H(\eta - x_k^R/a) \frac{d^r X_m}{d\eta^r} \frac{d^s X_i}{d\eta^s} d\eta, \\
 F_{nj}^{(r,s)} &= \int_0^1 \frac{d^r Y_n}{d\xi^r} \frac{d^s Y_j}{d\xi^s} d\xi; \left(\hat{F}_{nj}^{(r,s)} \right)_k = \int_0^1 H(\xi - y_k^D/b) - H(\xi - y_k^U/b) \frac{d^r Y_n}{d\xi^r} \frac{d^s Y_j}{d\xi^s} d\xi, \\
 \alpha &= a/b; m, i = 1, \dots, M; n, j = 1, \dots, N; r, s = 0, 1, 2; \\
 \lambda &= \rho_s h_s \alpha^4 \omega^2 (1 - \nu_s^2) / Y_s I_s
 \end{aligned} \tag{4.17}$$

One can obtain the mass and stiffness matrix for the symmetrical configuration too by multiplying the piezoelectric related components with two and removing all the terms associated with the neutral surface term z_0 . Further, the assumed mode shapes, referred to as Ritz eigenfunction [Meirovitch and Parker, 2001] are

$$\varphi_{mn}(x, y) = \sum_{i=1}^M \sum_{j=1}^N A_{mni,j} X_i(x) Y_j(y) \tag{4.18}$$

Analogous to the orthogonality conditions of the eigenfunctions in distributed parameter systems, substituting equation (4.12) into (4.8), multiplying by some plate mode shape, and integrating over the surface area of the plate [Aridogan et al., 2014a, Ari-

dogan et al., 2014b], the governing equations in modal coordinates can be obtained as

$$\ddot{\eta}_{mn}(t) + 2\zeta_{mn}\omega_{mn}\dot{\eta}_{mn}(t) + \omega_{mn}^2\eta_{mn}(t) - \sum_{k=1}^P \theta_{mn,k}v_k(t) = f_{mn}(t) \quad (4.19)$$

$$C_{p,k}\dot{v}(t) + Y_kv_k(t) + \sum_{m=1}^M \sum_{n=1}^N \theta_{mn,k}\dot{\eta}_{mn}(t) = 0 \quad \text{for } k = 1, 2, \dots, P \quad (4.20)$$

where $\theta_{mn,k}$ is the electromechanical coupling term for the k th piezo-patch, and $f_{mn}(t)$ is the modal forcing input, written as

$$\theta_{mn,k} = \vartheta \int_{y_k^D}^{y_K^U} \int_{x_k^L}^{x_k^R} \left(\frac{\partial^2 \varphi_{mn}(x, y)}{\partial x^2} + \frac{\partial^2 \varphi_{mn}(x, y)}{\partial y^2} \right) dx dy \quad (4.21)$$

$$f_{mn}(t) = f(t)\varphi_{mn}(x_0, y_0) \quad (4.22)$$

Applying the Laplace transformation on equations (4.19) and (4.20), the modal coordinates can be written in terms of the voltages as

$$H_{mn}(s) = \frac{F_{mn}(s) + \sum_{k=1}^P \theta_{mn,k}V_k(s)}{s^2 + 2s\zeta_{mn}\omega_{mn} + \omega_{mn}^2} \quad (4.23)$$

where the voltages can be obtained using the matrix inversion as

$$\mathbf{V} = [\text{diag}[sC_{p,1} + Y_1, \dots, sC_{p,P} + Y_P] + \mathbf{A}]^{-1} \mathbf{F} \quad (4.24)$$

here, $\mathbf{V} = [V_1(s), V_1(s), \dots, V_P(s)]^T$ includes the voltage across the electrodes of a single(paired) piezo-patch; $\mathbf{A}$ is a square matrix of size $P \times P$, and $\mathbf{F}$ is column vector of size $P \times 1$, given by

$$A_{ij} = s \sum_{m=1}^M \sum_{n=1}^N \frac{\theta_{mn,i}\theta_{mn,j}}{s^2 + 2s\zeta_{mn}\omega_{mn} + \omega_{mn}^2}$$

$$F_i = -sF_0 \sum_{m=1}^M \sum_{n=1}^N \frac{\theta_{mn,i}\varphi_{mn}(x_0, y_0)}{s^2 + 2s\zeta_{mn}\omega_{mn} + \omega_{mn}^2}, \quad i, j = 1, \dots, P \quad (4.25)$$

where F_0 is the input force amplitude. Note that, if the piezo-patches are all connected in parallel to an admittance $Y(s)$, the voltages $V_k(s)$ for $k = 1, \dots, P$ would be equal to a single voltage $V(s)$. Therefore, by taking a summation of Laplace transform

of equations (3.19 and 3.20) over $k = 1, \dots, P$, the unknown $V_k(s)$ can be obtained without matrix conversion as

$$V(s) = \frac{-sF_0 \sum_{k=1}^P \sum_{m=1}^M \sum_{n=1}^N \frac{\theta_{mn,k} \varphi_{mn}(x_0, y_0)}{s^2 + 2s\zeta_{mn}\omega_{mn} + \omega_{mn}^2}}{sP \sum_{k=1}^P C_{p,k} + Y(s) + s \sum_{k=1}^P \sum_{m=1}^M \sum_{n=1}^N \frac{\theta_{mn,k} \sum_{l=1}^P \theta_{mn,l}}{s^2 + 2s\zeta_{mn}\omega_{mn} + \omega_{mn}^2}} \quad (4.26)$$

4.3 Experimental Validation of the Electromechanical FRFs

In order to validate the displacement and voltage FRFs under the point force excitation, an experimental setup is employed as shown in figure 4.2. The setup is comprised of three piezoceramic patches (Piezo Systems, Inc. Model PSI-5A4E) attached to one surface of a thin aluminum plate, and excited by a modal shaker (The modal Shop, Inc. Model K2007E01) over a frequency bandwidth of 10-300 Hz. The point force input is measured by a force transducer (PCB group, Inc. Model 208C02) which is mounted to the stinger and attached to the plate surface. The transverse velocity measurements are made by a laser Doppler vibrometer (Polytec, Inc. PDV-100). A data acquisition unit (01dB-Metravib, Inc. NetdB) is used to collect and analyze the velocity, force and voltage signals. The mechanical and geometrical properties of the experimental setup are given in table 4.1.

Figure 4.3 presents the experimental validations for the displacement and voltage FRFs of the thin plate with three surface-bonded piezo-patches. The modal damping ratios are obtained using the system identification technique [Erturk and Inman, 2011]. The electromechanical FRFs are obtained at open-circuit condition, that is a very large value of resistive load (close to $1M\Omega$) in the analytical model. Based on the Rayleigh-Ritz modeling approach, number of modes are chosen as 15×15 to ensure the convergence of the system eigenvalues [Meirovitch and Parker, 2001]. Analytical FRFs are in good agreement with the experimental ones, however, the differences at some vibration modes are due to the uncertainty in material properties and imperfect clamped boundary conditions of the plate.

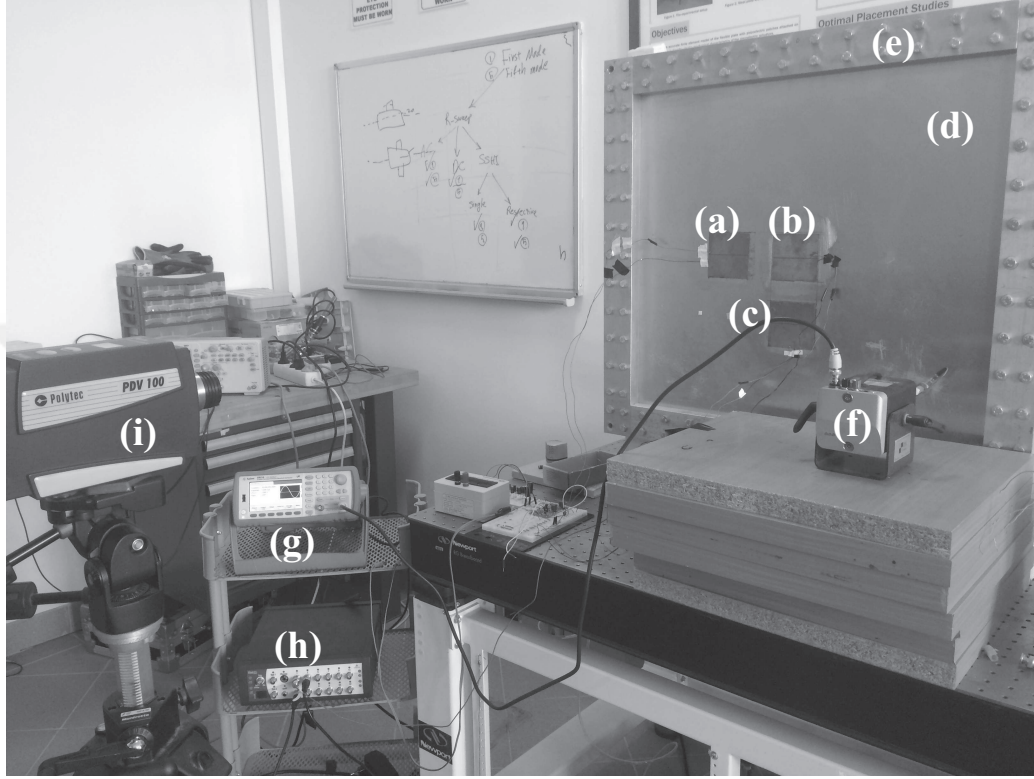


Figure 4.2: Experimental setup: (a)-(c) three piezo-patches (d) Aluminum plate (e) fully clamped frame (f) modal shaker (g) signal generator (h) signal analyzer (i) laser Doppler vibrometer (LDV).

Table 4.1: Geometric, material and electroelastic properties

Property	Aluminum plate	Piezoceramic (PSI-5A4E)
Length (mm)	580	72.4
Width (mm)	540	72.4
Thickness (mm)	1.9	0.267
Youngs modulus (GPa)	69	66
Mass density (kg/m_3)	2700	7800
Piezoelectric constant (pm/V)	-	-190
Permittivity constant (nF/m)	-	10.38

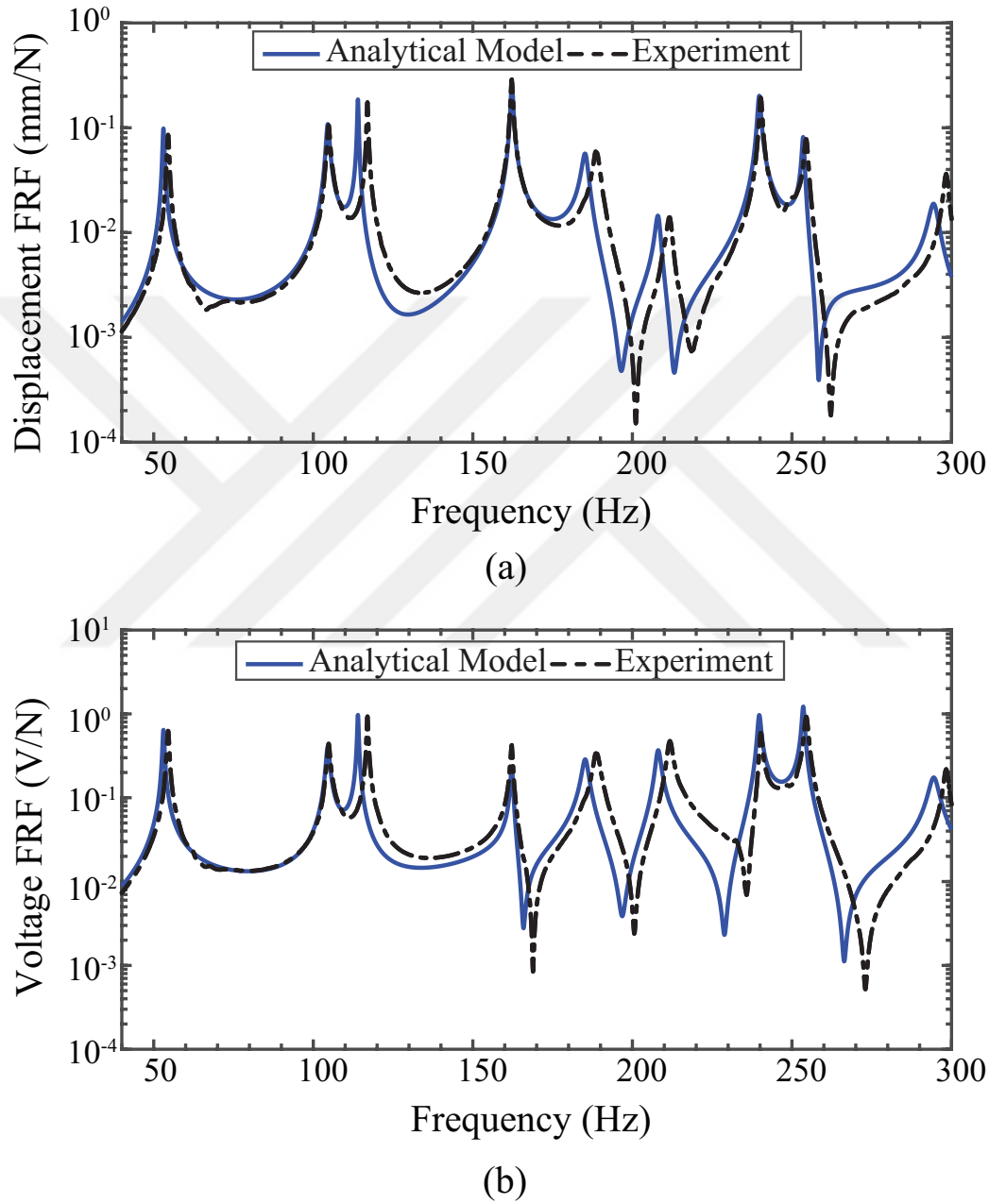


Figure 4.3: Electromechanical FRFs at open-circuit condition: (a) displacement FRF and (b) voltage FRF. Analytical results (solid line) are in good agreement with experimental data (dashed line).

4.4 Mass and stiffness contribution of the piezo-patches

In order to investigate how array of piezo-patches affect the dynamics of the host plate, modal analysis simulations are performed for the several number of patches

composed of different thicknesses. Figure 3.3 demonstrates the even distribution of the patches from the middle to the edges of the plate surface by keeping the symmetry at the plate center. For the representative plate size ($580 \times 540 \text{ mm}^2$) in this study, maximum number of 36 piezo-patches (each $72.4 \times 72.4 \text{ mm}^2$) can cover the plate surface without touching the clamped edges. Note that for the symmetrical configuration, same number of the patches are attached to the bottom surface. Solving the modal analysis equations in (4.15), the first four short-circuit natural frequencies of the electromechanical system are calculated for each number of patches with different thicknesses for both unsymmetrical and symmetrical configurations, and normalized with the corresponding vibration modes of the plain plate as shown in figure 3.4. The dimensionless parameters are

$$\hat{h} = \frac{h_p}{h_s}, \quad \hat{\omega}_i = \frac{\omega_i}{\omega_{s,i}}, \quad \text{for } i = 1, 2, \dots \quad (4.27)$$

where $\omega_{s,i}$ terms are the natural frequencies of the host plate without any piezoelectric patch attachment, and ω_i are the natural frequencies of the electromechanical system. Note that for the regions with the dimensionless vibration modes below 1, addition of the piezo-patches increases the effective modal mass components, which causes the natural frequency of the system to drop to the lower values. This is especially seen for all the number of patches with different thicknesses at the first three vibration modes of the unsymmetrical configuration (figure 4.5a) and the first mode of the symmetrical configuration (figure 4.5b). However, in some regions of the other vibration modes, $\hat{\omega}_i$ are larger than 1 which indicates the domination of the effective modal stiffness components over the modal mass ones.

4.5 Electromechanical Coupling

By increasing the number of patches, choosing their terminal combinations and electrical boundary conditions can directly affect the device electromechanical coupling, indicating how strongly the piezoelectric material is coupled to the host plate. A general definition of the system-level electromechanical coupling for i th mode is given based on the difference between the short-circuit and open-circuit natural frequen-

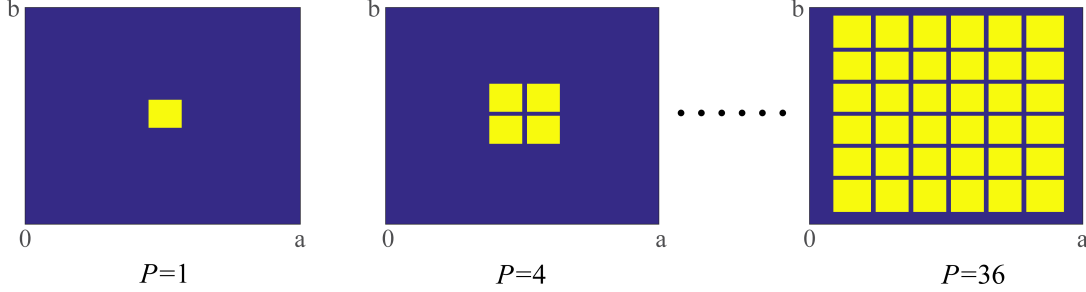


Figure 4.4: Distribution of piezo-patches on the host structure.

cies [Lesieutre and Davis, 1997]

$$k_i^2 = \frac{f_{oc,i}^2 - f_{sc,i}^2}{f_{oc,i}^2}, \quad \text{for } i = 1, 2, \dots \quad (4.28)$$

Focusing on equation (4.21), it is evident that the electromechanical coupling term is directly proportional to the integral of the strain over the area that is covered by the piezo-patch. Thus, opposite electrical charges would be generated in the piezo-patches at higher vibration modes due to the opposite strain signs. Considering this effect, if the patches are first connected in parallel, series, or even mixed series-parallel configurations, there would be a drastic charge cancellation and hence a very low electromechanical coupling. For example, consider two cases: (a) the piezo-patches are all connected in parallel and shunted to a resistive load; (b) the piezo-patches are independently shunted to a resistive load. The effect of electromechanical coupling at each vibration mode for these two cases can be seen on the corresponding short-circuit and open-circuit displacement FRFs of an arbitrary location on the plate (at center of the bottom left quarter) under point force excitation (at center of the bottom right quarter), as shown in figure 4.6. There is almost no electrically induced stiffness effect at the second, third, fourth, fifth, sixth, seventh vibration modes for case (a). However, in case (b), no electrical cancellations occur between the piezo-patches since they are independently shunted. Therefore, for the shunt damping applications, its best to employ the piezo-patches independently for exploiting all vibration modes of the host system. Moreover, they can be utilized as locally resonant metastructures

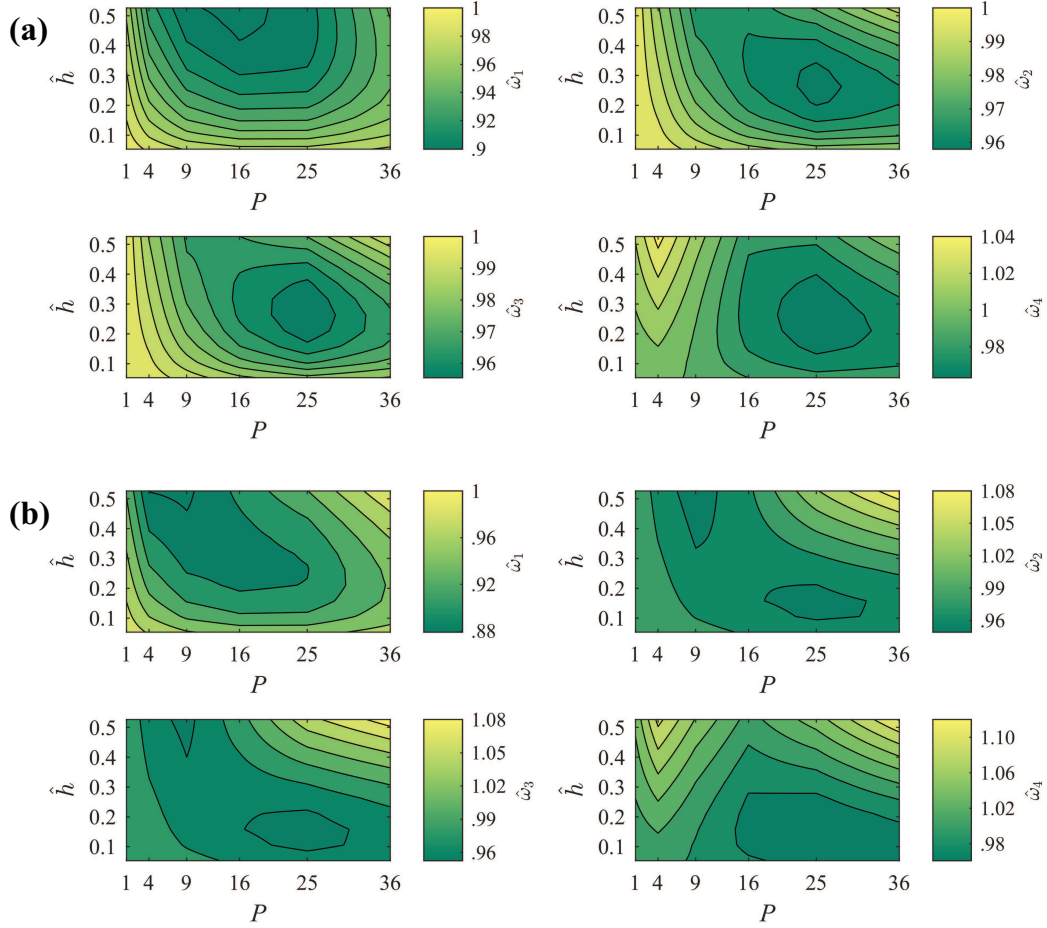


Figure 4.5: Contours of the first four normalized natural frequencies of the electromechanical system at short-circuit condition for the several number of PZT-5A patches with different thickness ratios: (a) unsymmetrical and (b) symmetrical configurations. First four natural frequencies of the plain plate are 53.7, 104.7, 114.2, and 161.5 Hz.

which will be discussed in the next section. Having shown the advantages of locally shunted piezo-patches, the system electromechanical coupling coefficient are obtained for a different number of PZT-5A patches and a range of thickness ratios. Figure 4.7 shows the electromechanical coupling map for the first four vibration modes, which are normalized by the electromechanical coupling coefficient of the piezoelectric material [Lesieutre and Davis, 1997, IEEE, 1988]

$$k_{31}^2 = \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T} \quad (4.29)$$

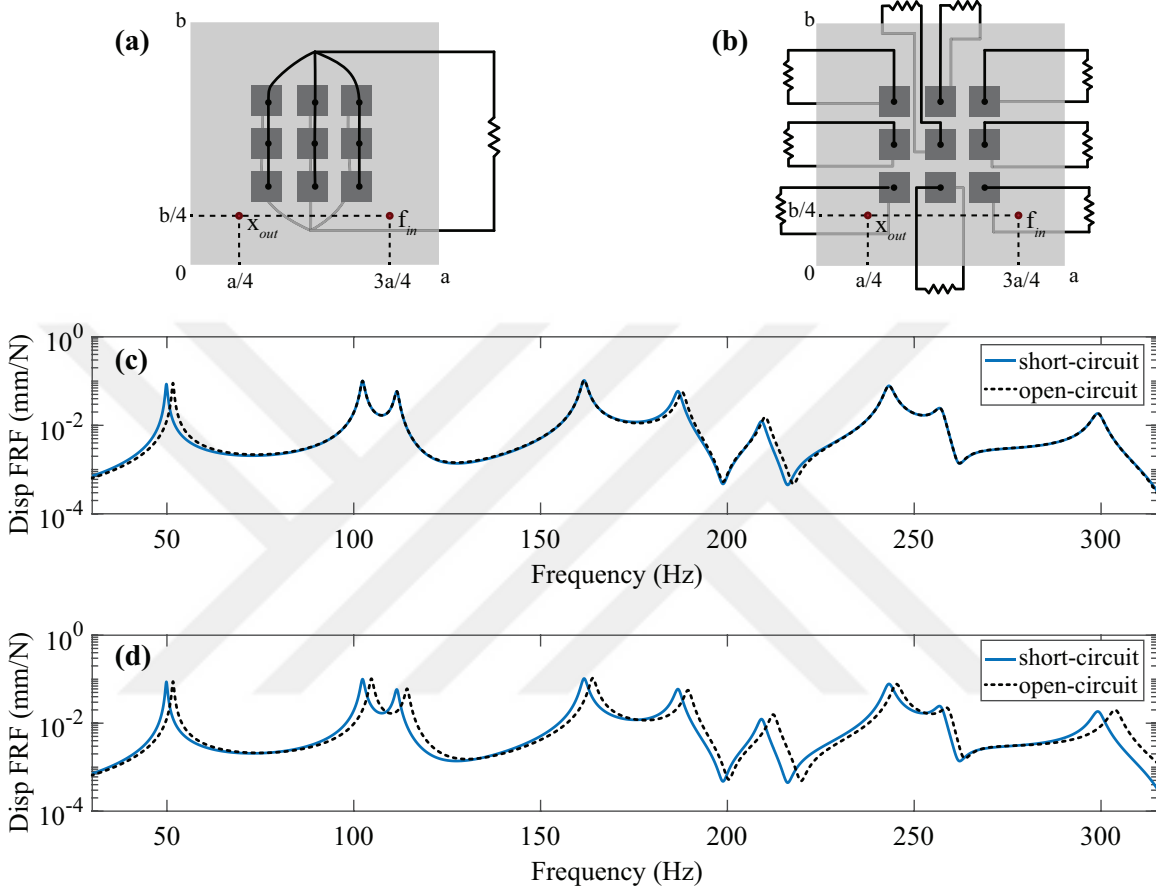


Figure 4.6: Comparison of displacement FRFs for two electrical connection cases: (a),(c) All piezo-patches in parallel (b),(d) piezo-patches are independent. Here, the number of PZT-5A patches and their normalized thickness are selected as $P = 9$ and , and the patches are placed symmetrically on top and bottom of the plate.

4.6 Bandgap Formation via Locally Resonant Piezo-patches

The closed-form expressions presented in section 4.2 can be used to obtain the forced response of the electromechanical system when each piezo-patch is connected to a shunt circuit. To create a resonant bandgap at a target frequency of interest, let each shunt circuit admittance be

$$Y_k(s) = (L_k s + R_k)^{-1} \quad (4.30)$$

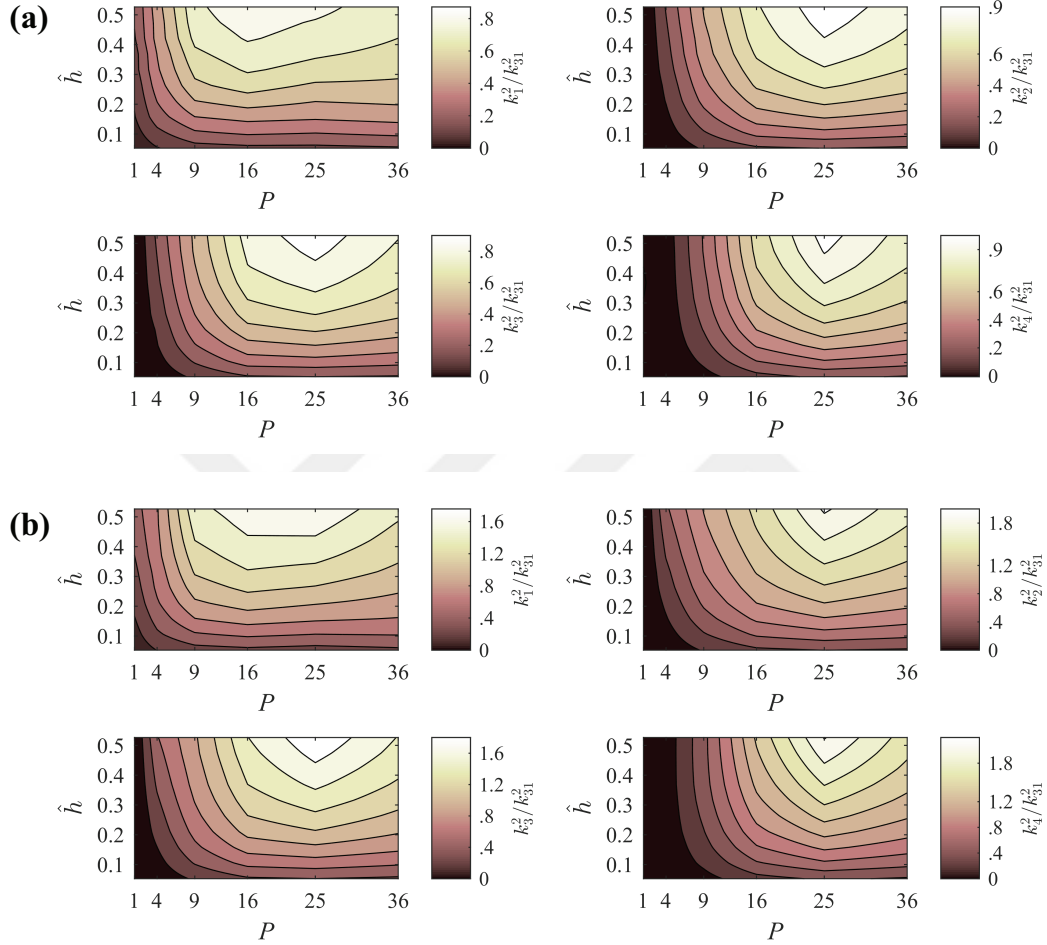


Figure 4.7: The system-level electromechanical coupling map for the first four vibration modes of the plate with P number of piezo-patches and different patch-to-plate thickness ratios: (a) unsymmetrical and (b) symmetrical configurations. The piezo-electric materials electromechanical coupling coefficient is for PZT-5A ceramic.

where L_k and R_k are the inductance and resistance applied to the k th shunt circuits. The inductance value can be selected according to the target frequency ω_t as

$$L_k = (C_{p,k}\omega_t^2)^{-1} \quad (4.31)$$

To measure the response of the structure, frequency response functions (FRFs) defined as ratio of displacement output at an arbitrary position to a force input excitation are calculated. For the representative plate dimensions ($580 \times 540 \times 1.9 \text{ mm}^3$) and 25 pair of

piezo-patches bonded symmetrically to top and bottom surface of the host structure, FRF surface plots are shown in figure 4.8 for a set of different piezoelectric materials given in table 4.2. For one of the system parameters in figure 4.8(f) with $\hat{h} = 0.45$, the transmittance plot is shown in figure 4.9. The bandgap starts at $\omega_t = 100\text{Hz}$ and ends at lower than frequency value where the vibration is reduced significantly. Figure 4.10 also presents the surface plots for different target frequencies with the piezoelectric crystal PMN-PT (33% PT). Figure 4.11 displays the transmittance plots for the different target frequencies with the piezoelectric crystal PMN-PT (33% PT) and $\hat{h} = 0.5$.

In these simulations, the bandgap is displayed by a sudden drop in the magnitude of the FRFs in the frequency bandwidth below the target frequency. Note that for a given number of piezo-patches ($P = 25$ in figures 4.8 and 4.10), as the thickness of the piezo-patches increases, wider bandgap is generated, indicating the direct relation with the system-level electromechanical coupling (see figure 4.7 for the maximum values). Additionally, the resonance frequencies (indicated by gray circles) seem to veer in a direction away from the target frequency by increasing the piezo-to-plate thickness ratio, allowing a wideband vibration/noise reduction.

4.7 Conclusion

Two-dimensional electromechanical metastructures made from a thin plate with array of piezoelectric elements connected to resonant shunt circuits can display bandgaps for vibration reduction in aerospace, marine and automotive applications. Similar to the purely mechanical resonators, locally resonant piezoelectric patches can produce bandgaps in the elastic substrate at wavelength much larger than the lattice size, enabling a compact and tunable mechanism for shunt damping applications. In order to efficiently design such finite-sized metamaterial-based structures in real-life applications, its necessary to bridge the modal interaction of the host structure with the electromechanical coupling behavior of the resonant piezo-patches. Therefore, in this study, we developed an electroelastic modeling of a thin plate with array of

Table 4.2: Typical electroelastic properties and piezoelectric material coupling coefficient for two piezoelectric ceramics and three single crystal materials [Cao et al., 2004, Erturk and Inman, 2011]

Piezoelectric Material	ρ_p ($kg.m^{-3}$)	$\bar{c}_{11}^E$ (GPa)	$\bar{c}_{12}^E$ (GPa)	$\bar{c}_{66}^E$ (GPa)	$\bar{e}_{31}$ ($C.m^{-2}$)	ϵ_{33}^S ($nF.m^{-1}$)	k_{31}^2
PZT-5A	7750	69.4	24.3	22.6	-16.0	9.6	0.118
PZT-5H	7500	66.1	19.2	23.5	-23.4	17.3	0.151
PMN-PT (30% PT)	8040	22.1	8.1	65.8	-27.8	17.8	0.236
PMN-PT (33% PT)	8060	14.9	2.4	65.8	-23.0	11.5	0.353

piezo-patches shunted to resonating circuits for investigating the bandgap formation. Using Rayleigh-Ritz method, mass and stiffness of each piezo-patch was included in the modal analysis. The effect of increase in the number and thickness of patches on the natural frequency of the plate and system-level electromechanical coupling were presented for the first four vibration modes. It was shown that the interconnection of the patches plays an important role in electromechanical coupling and thus in locally resonant bandgaps. The displacement frequency response functions under the point force excitation were calculated for a range of piezo-to-plate thickness ratios and for several piezoelectric materials such as ceramics and single crystals. The bandgaps size, indicated by the frequency bandwidth with a very low vibration amplitude, was shown to be wider for the single crystals (e.g., PMN-PT, 33%PT) rather than piezo-ceramic ones (e.g., PZT-5A). Finally, it was shown that by increasing the ratio of the piezo-patch to the plate thickness, wider bandgaps were created and also the nearby modes were diverged in the opposite direction of the targeted frequency.

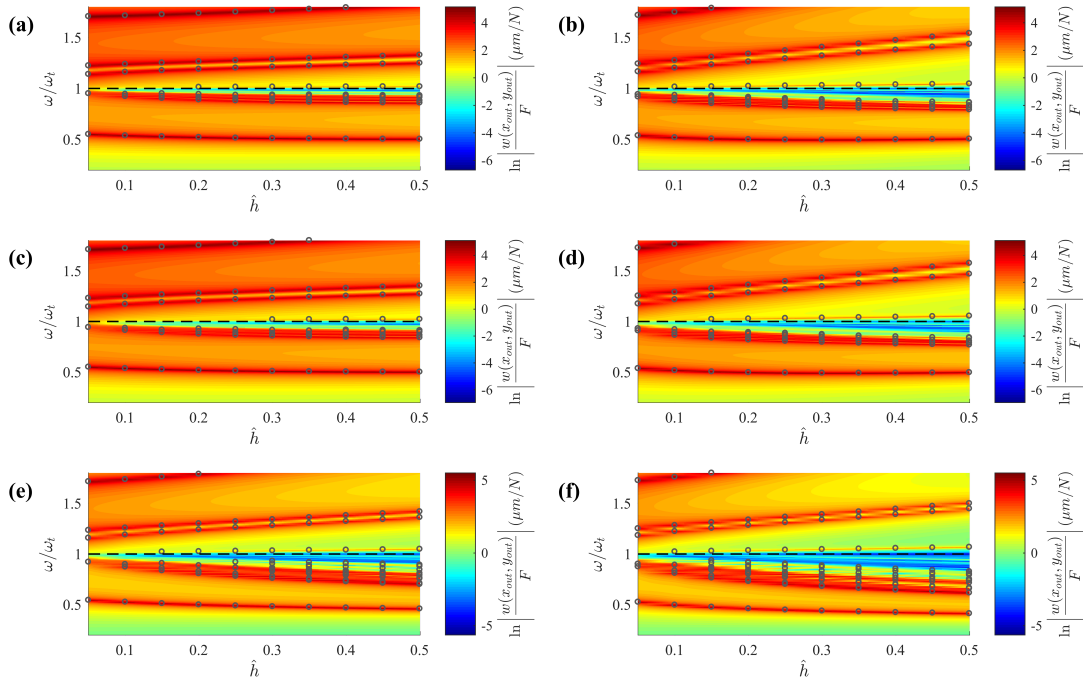


Figure 4.8: Displacement frequency response functions and resonant frequencies versus patch-to-plate thickness ratio $\hat{h}$ for (a)-(b) PZT-5A (d)-(d) PZT-5H (e)-(f) PMN-PT (33%) piezoelectric materials in unsymmetrical (shown in the left column) and symmetrical (the right column) configurations. The thin plate with array of piezopatches ($P = 25$) is excited by a point force at position $(3a/4, b/4)$ and displacement response is calculated at $(a/4, b/4)$; resonant frequencies are indicated by gray circles, target frequency is $\omega_t = 100\text{Hz}$ and $R_k = 100\Omega$.

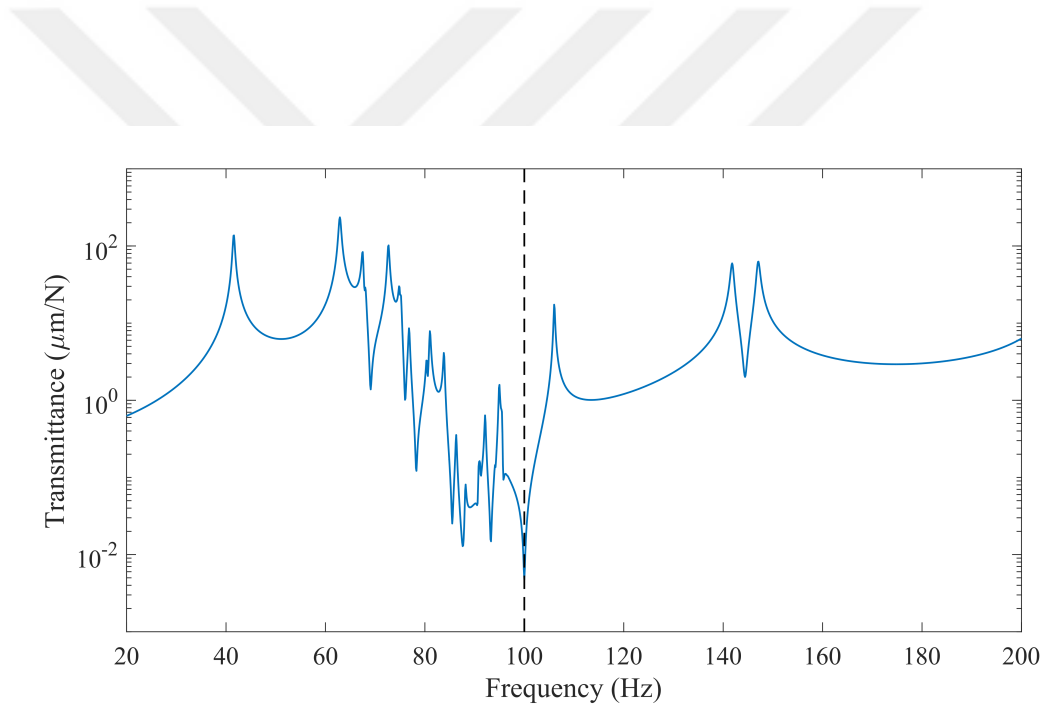


Figure 4.9: Transmittance plot for the PMN-PT (33%) material with $\hat{h} = 0.45$ in symmetrical configuration. The bandgap starts from the target frequency of $\omega_t = 100\text{Hz}$ and ends at a lower frequency value. Force and displacement locations are same as shown in Figure 4.6(a).

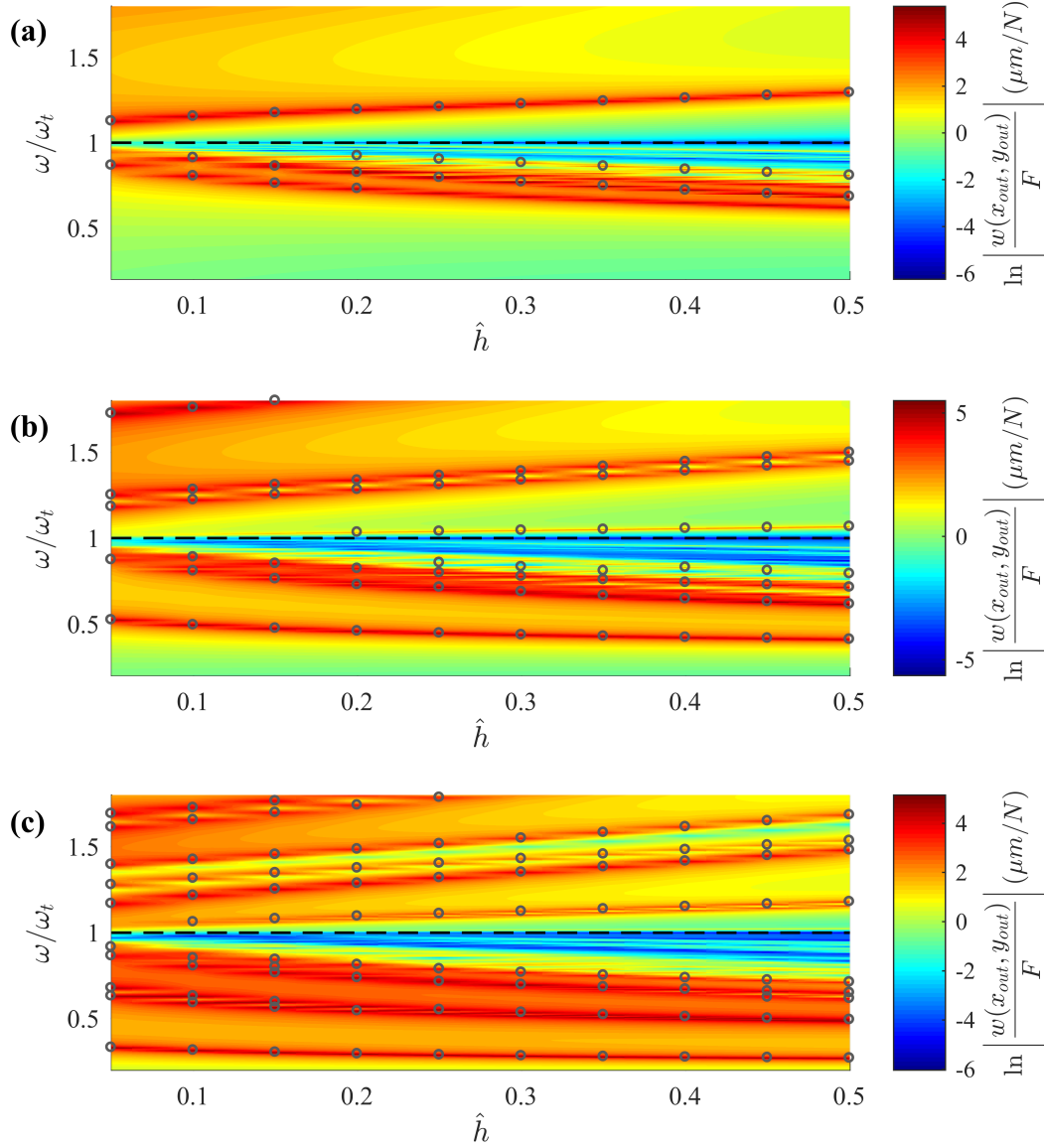


Figure 4.10: Displacement frequency response functions and resonant frequencies versus patch-to-plate thickness ratio $\hat{h}$ for (a) $\omega_t = 54\text{Hz}$ (b) $\omega_t = 100\text{Hz}$ (c) $\omega_t = 160\text{Hz}$. Piezoelectric material is selected as PMN-PT (33%) and patches ($P = 25$) are in symmetrical configuration. The thin plate with array of piezo-patches is excited by a point force at position $(3a/4, b/4)$ and displacement response is calculated at $(a/4, b/4)$; resonant frequencies are indicated by gray circles, and $R_k = 100\Omega$.

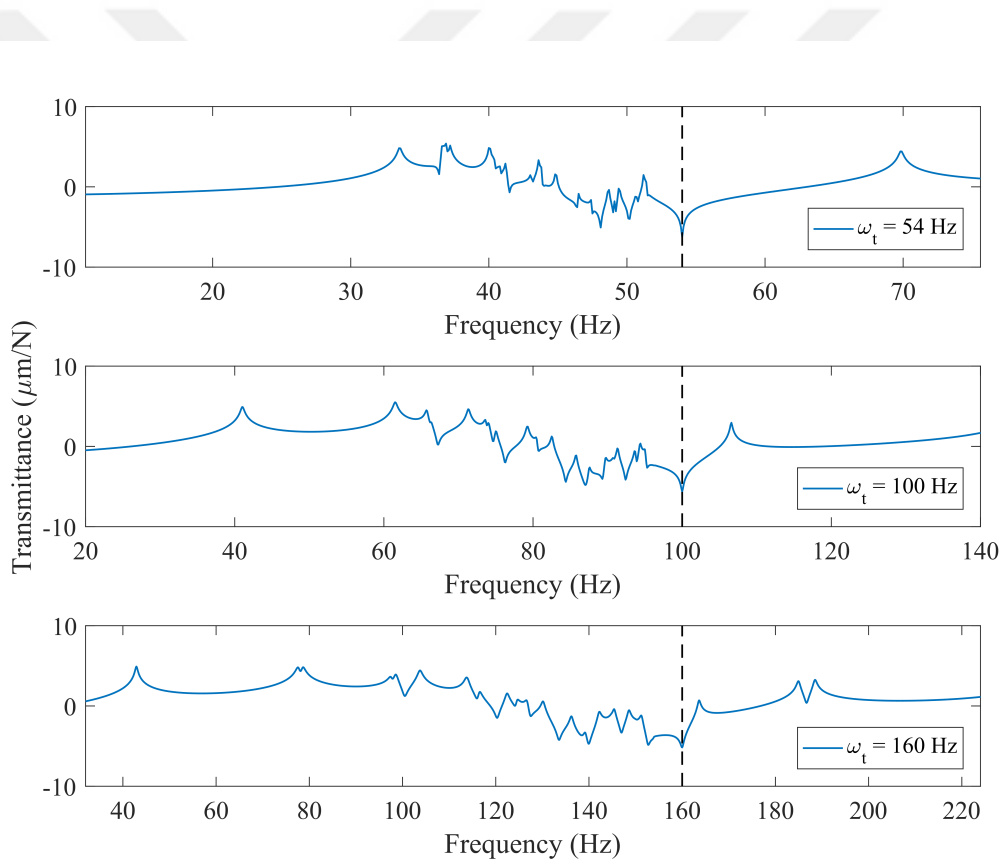


Figure 4.11: Transmittance plot for the PMN-PT (33%) material with $\hat{h} = 0.5$ at different target frequencies (shown in the legends) in symmetrical configuration.

Chapter 5

CONCLUSION, CONTRIBUTIONS AND FUTURE WORK

5.1 Vibrational Energy Harvesting Using Piezoelectric Patches on Thin plates

Piezoelectric patches attached to thin plates can be integrated to plate-like structures in engineering applications such as automotive, marine, and aerospace, to directly harvest the structural vibrations of the host system and power the nearby sensors. With this implementation, mass loading and volumetric occupancy of base excited cantilever attachments can be reduced, enabling a compact two-dimensional energy harvesting system. Literature of piezoelectric patch-based energy harvesting have largely focused on the mechanical and electrical aspects in separate domains. Existing studies on the electrical part have oversimplified the mechanical models (by using SDOF representation of the harvesters), while the modeling efforts for the mechanical domain have overlooked the practical electrical circuits (such as switch-based interfaces). Two chapters of this thesis were dedicated to address those problems by developing unified analytical and numerical frameworks to fully bridge the electrical and mechanical domain, and hence formulating a complete electromechanically coupled system. The following bulleted list summarizes the major contributions of each chapter:

- Development of a multi-modal equivalent circuit model (ECM) for a piezo-patch energy harvester integrated to a thin plate using the analogy between the second-order circuitry equation and the governing electromechanical equation. The

equivalent circuit parameters were identified from the analytical modal analysis solution, by taking into account the two-way electromechanical coupling and multiple vibration modes of the host structure (Chapter 2);

- Generating the whole electromechanical system in an electronic circuit simulator SPICE, conducting numerical simulations for two standard AC input (harmonic point force excitation) - AC output (alternating voltage across the resistive load) and AC input - DC output (rectified voltage across the resistive load) cases, and validation of the electrical output results against the existing analytical models (Chapter 2);
- Establishment of a generalized ECM for the multiple piezo-patch energy harvesters (MPEHs) by accounting for the compatibility with different electrical interconnection types (series, parallel, or mixed series-parallel configurations), and validation by the experiments and existing analytical models for the AC-AC case (Chapter 3);
- Analytical modeling of multiple piezo-patch energy harvesters (MPEHs) attached to a thin plate in series/parallel configurations. A unified analytical framework was achieved by integrating the equivalent impedance load of the harvesting circuit into the electroelastic equations of the plate with MPEHs (Chapter 3);
- Obtaining the closed-form expressions for vibration response and electrical outputs of MPEHs and presenting experimental validations for the case of MPEHs connected in series and/or parallel to a standard AC-DC converter circuit. Prediction of AC and DC peak power outputs of MPEHs by analytical and SPICE simulations and showing good agreement with the experimental measurements (chapter 3);
- Exploring different connection configurations of MPEHs (SSS, PPP, SSP, PPS)

by showing that maximum peak power generation is same in all and only the optimum resistive load varies depending on the equivalent capacitance of interconnected piezo-patches (chapter 3);

- Unveiling the reasons behind the charge cancellation in interconnected MPEHs by inspecting individual voltage FRFs and correlating the positive and negative strain signs under the piezo-patches with the sign of imaginary voltage response. Introducing the respective rectification technique (an AC-DC converter for each piezo-patch) for avoiding the charge cancellations in interconnected MPEHs and thus increasing the power output (chapter 3).

For the future work, the equivalent impedance electroelastic model and the ECM developed in this thesis can be extended to other flexible structures (such as beam-based or shell-like structures) with integrated single/multiple piezo-patch energy harvesters connected to nonlinear circuits. Moreover, nonlinear switch-based interfaces such as SSHI circuits can be used to further increase the power outputs of MPEHs on thin plates which are known as weakly-coupled electromechanical systems. Meanwhile, for identifying the equivalent impedance load of more sophisticated harvesting circuits where there is no analytical expression, empirical models can be obtained using experimental measurements or numerical approximations in SPICE and then fed into the developed analytical model.

5.2 Locally Resonant Bandgaps in Two-dimensional Electromechanical Metastructures

Array of piezoelectric patches connected to resonant shunt circuits can display locally resonant bandgaps in thin plate structures for vibration reduction in aerospace, marine and automotive applications. Analogous to the purely mechanical resonators, locally resonant piezoelectric patches can produce bandgaps in the elastic substrate at wavelength much larger than the lattice size, enabling a compact and tunable shunt damping mechanism. For designing such finite-sized metamaterial-based electrome-

chanical structures or electromechanical metastructures in realistic applications, it is necessary to bridge the modal interaction of the host structure with the electromechanical coupling behavior of the resonant piezo-patches. To this end, chapter 4 presented an electroelastic modeling of a thin plate with array of piezo-patches shunted to resonating circuits for investigating the bandgap formation. The major contribution of this chapter is summarized as follows:

- Development of an analytical framework for the array of resonant piezoelectric patches integrated to thin plates in unsymmetrical and symmetrical configurations. The governing equation of motion were derived based on the Kirchhoff-Love plate theory and the modal analysis was performed using the Rayleigh-Ritz method;
- Development of an analytical framework for the array of resonant piezoelectric patches integrated to thin plates in unsymmetrical and symmetrical configurations. The governing equation of motion were derived based on the Kirchhoff-Love plate theory and the modal analysis was performed using the Rayleigh-Ritz method;
- Obtaining closed form expression for vibration response of the host plate with the array piezo-patches connected to resonant shunt circuits using matrix inversion;
- Illustration of the effect of the number and thickness of the piezo-patches on the natural frequencies of the plate and also on the system-level electromechanical coupling;
- Presenting the displacement FRF surface plots for a set of piezoelectric materials such as ceramics and single crystals and observing the bandgap formation (indicated by the frequency bandwidth with a very low vibration amplitude)

at different target frequencies (which can be selected by tuning the resonant frequency of the shunt circuits)

- Demonstrating the widening of the bandgaps and veering of the resonant frequencies in the opposite directions by the increase in the thickness of the piezopatches, and showing that piezoelectric single crystals (e.g., PMN-PT, 33%PT) produce wider bandgaps than piezoceramics (e.g., PZT-5A).

Electromechanical metastructures can be a good alternative to the purely mechanical metastructures for they are easily tunable and add no significant mass and volumetric occupancies. As a future direction in this field, piezoelectric patches in different arrangements such as circle-shape or other irregular form can be explored for creating wider bandgaps at regions of interest. Further, negative capacitance shunt circuits can be used to effectively increase the width of the bandgaps, although they introduce other complexities such as requiring an external power supply and decreasing the electromechanical coupling. Another interesting approach will be the concept of hybrid metamaterial energy harvesting systems, where the array of locally resonant piezoelectric patches can produce large bandgaps for vibration attenuation and generate high power outputs for the storage components at the same time. The developed electroelastic model in chapter 4 can be implemented to design the efficient and practical shunt plus harvesting circuits for such hybrid two-dimensional structures.

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