Pricing Ambiguity and Ambiguity Aversion in the Cross Section of Stocks

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This is to certify that I have examined this copy of a doctoral dissertation by

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and have found that it is complete and satisfactory in all respects,

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To all my heroes, for the inspiration they ignite.

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ABSTRACT

Pricing Ambiguity and Ambiguity Aversion in the Cross-Section of Stocks

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Based on a theoretical model that relies on the theory of smooth ambiguity aversion, this thesis empirically investigates whether ambiguity and aversion towards it is priced in the cross-section of stocks and tests whether the model has explanatory power for resolving some of the cross sectional stock anomalies. The literature investigating whether ambiguity is priced in stock returns typically studies the research question at the aggregate market level. The main finding in the thesis is that ambiguity is priced in the cross section of stocks. Abnormal excess returns of stocks (i.e. CAPM alphas) in the cross section are shown to increase with stocks' ambiguity exposure which is defined as the difference between the ambiguity beta of a stock and its CAPM beta, where ambiguity beta is the stock's exposure to market portfolio ambiguity. In order to estimate stocks' ambiguity betas, macroeconomic uncertainty index of Jurado, Ludvigson and Ng (2015) is employed as a proxy for market portfolio ambiguity. The thesis shows when stocks are ranked into quintile portfolios according to their ambiguity exposure and ambiguity exposure is controlled for in the CAPM regression as a separate factor, alpha of the high-minus-low quintile as well as the alphas of the individual quintiles disappear. The thesis also shows that ambiguity and ambiguity aversion fully explain the beta anomaly, while providing partial explanations to operating profitability and momentum anomalies.

ÖZETÇE

Muğlaklık ve Muğlaklıktan Kaçınmanın
Hisse Senetleri Kesidinde Fiyatlanması

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Bu tez pürüzsüz muğlaklıktan kaçınma teorisine dayanan teorik bir modeli kullanarak muğlaklık ve muğlaklıktan kaçınmanın kesitsel olarak hisse senetlerinde fiyatlanıp fiyatlanmadığını ve kullanılan modelin kesitsel hisse senedi anormalliklerini açıklama gücünü araştırmıştır. Literatürde muğlaklığın hisse senetlerinde fiyatlanıp fiyatlanmadığını araştıran çalışmalar genellikle araştırma sorusuna toplam piyasa düzeyinde bakmıştır. Tezdeki ana bulgu muğlaklığın kesitsel olarak hisse senetlerinde fiyatlandığıdır. Hisse senetlerinin kesitte risksiz getiri oranı üzerindeki anormal getirilerinin (yani sermaye varlık fiyatlandırma modelindeki (SVFM) alfalar) hisselerin muğlaklığa olan duyarlılığıyla arttığı gösterilmiştir. Muğlaklık duyarlılığı şirketin muğlaklık betası ve SVFM betasının farkı olarak tanımlanmaktadır; muğlaklık betası ise hissenin piyasa portföyü muğlaklığına olan duyarlılığıdır. Hisselerin muğlaklık betalarını tahmin edebilmek için, Jurado, Ludvigson ve Ng'nin (2015) oluşturduğu makroekonomik belirsizlik endeksi piyasa portföyü muğlaklığını temsil için kullanılmıştır. Tez, hisseler muğlaklık duyarlılıklarına göre sıralanıp beş ayrı portföye bölündüğünde ve muğlaklık duyarlılığı bir faktör olarak SVFM regresyonunda kullanıldığında, hem en yüksek duyarlılıklı portföyün alınıp en düşük duyarlılıklı portföyün satıldığı fark portföyünün SVFM alfasının, hem de beş portföyün ayrı ayrı SVFM alfalarının yokolduğunu göstermiştir. Tez ayrıca muğlaklık ve muğlaklıktan kaçınmanın beta anomalisini tümüyle, operasyonel karlılık ve momentum anomalilerini de kısmen açıkladığını göstermektedir.

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Chapter 1

Introduction

Markets respond to myriad of newsflow everyday at every instant of the trading hours; macro, financial, political, sector or firm-specific newsflow affect stock prices in a seemingly random fashion and stock prices reflect equivalent of overall expectations of market players at any point in time. Proper gauging of aggregate macro/financial uncertainty based on real data, therefore, is an important task in order to quantify uncertainty, which in turn, may help to uncover why stock returns deviate in a statistically significant manner from what is anticipated by existing theories.

The main aim of this thesis is to empirically investigate the role of macroeconomic uncertainty in the cross sectional returns of individual stocks based on a theoretical model and test whether the model has explanatory power for resolving some of the cross sectional stock anomalies. As probably the most repeated word throughout the text in this thesis, it is crucial to elaborate the term uncertainty right away, as it will refer to a specific context in modern decision theory rather than an umbrella term to describe randomness. The terms uncertainty and ambiguity are used interchangeably in the thesis.

It was Frank Knight who first distinguished uncertainty from risk in his book published in 1921. Knight expressed that known risks have a clearly defined probability distribution, whereas uncertainty stems from an unknown probability distribution or a distribution with unknown parameters. A good analogy might be the words of Donald Rumsfeld (who served as the Secretary of Defense in the US) about Iraq: “there are known knowns, there are known unknowns, and there are unknown unknowns." Known unknowns might be thought of as risk, while unknown unknowns might be considered as uncertainty. Such uncertainty in decision theory is thus referred to as Knightian uncertainty or ambiguity. An important distinction between ambiguity and risk is that ambiguity is uncertainty about probabilities, while risk is uncertainty about outcomes.

When there is uncertainty about a data generating process or about a probability distribution and/or its parameters, decision makers' subjective beliefs come into play and they subjectively assume data originates from a set of possible models. Interestingly, Ellsberg (1961) showed that when there is ambiguity, decision maker's choice is inconsistent with expected utility theory such that decision maker strictly favors the bet with known probability and utility outcome, over the ambiguous ones with same or even greater utility. The notion that resolved the paradox was ambiguity aversion which may be defined as decision makers' dislike of ambiguous bets and preference for bets with quantifiable risks.

Existence of ambiguity in stock pricing context is intuitive. At any point in time, portfolio managers are faced with a whole host of uncertainties and should make investment decisions by subjectively relying on a set of models that seem reasonable. Yet, a sizable portion of the financial literature, disregarding ambiguity, assumes investors behave as if they know the distributions of returns, which is hard to justify.

Theoretical model employed in the thesis relies on the theory of smooth ambiguity aversion of Klibanoff et al. (2005). The key distinguishing factor of the theory is the clear separation of ambiguity and ambiguity aversion of the decision maker. Another advantage is that the well-developed framework for dealing with risk attitudes can be adapted to accommodate attitudes towards ambiguity.

Heterogeneity in ambiguity of assets is also intuitive. For instance, a firm's stock return compared to its bond return would be structurally more exposed to uncertainty. In addition, bond returns are subject to only downside uncertainty, while stock returns are subject to uncertainty in either direction. Another example would be firms in diverse industries have varying degrees of exposure to aggregate uncertainty shocks.

Incorporating not only heterogeneity in ambiguity of assets, but also heterogeneity in ambiguity aversion of investors to the asset pricing picture is critical, since Hara and Honda (2018) show that the mutual fund theorem of Tobin (1958) continues to hold if agents are homogeneously ambiguity averse. Yet, mutual fund theorem (i.e. to maximize return for any risk level, just hold the market portfolio) is inconsistent with the asset allocation puzzle of Canner et al. (1997), which in contrast to the mutual fund theorem refers to the finding that the common advice in the market being conservative investors should hold a higher ratio of bonds to stocks in their portfolios than aggressive investors.

The theoretical framework of Mukerji, Ozsoylev, Tallon (2022) employed in the thesis incorporates both ambiguity and ambiguity aversion, and establishes a single-factor CAPM-like asset pricing formula with heterogeneously ambiguous assets and heterogeneously ambiguity averse agents. In the framework, ambiguity about the return distribution is taken to be the uncertainty about the mean parameter. As in the CAPM, the single pricing factor is the excess return of the market portfolio, however the factor loading now accounts for the existence of ambiguity as well as risk. The overall adjustment to the CAPM due to ambiguity and ambiguity aversion is encapsulated in the CAPM alpha, α, which has a positive association with the difference of ambiguity beta and CAPM beta, i.e. $\beta^{Amb} - \beta^{CAPM}$, which is referred to as ambiguity exposure. In other words, expected return of an asset is composed of two uncertainty premia: (CAPM-predicted) risk premium, and ambiguity premium, i.e. α. Note that CAPM beta β^{CAPM} is the sensitivity of a stock's excess return to the market portfolio excess return, whereas ambiguity beta β^{Amb} is the sensitivity of a stock's mean excess return to the market portfolio's mean excess return. That is, ambiguity beta is stock's exposure to systematic ambiguity.

Ambiguity exposure of a stock involves two parameters, namely CAPM beta and ambiguity beta. Estimation of CAPM beta is quite standard in the literature, while for estimating ambiguity beta the thesis relies on the macroeconomic uncertainty index (MEU) of Jurado et al. (2015). MEU seems to be a proper proxy for systematic ambiguity, as MEU as an aggregate entity reflecting episodes of varying sentiment/uncertainty for the economy as a whole for period(s) ahead should be a good measure for systematic ambiguity. As a result, stock returns are regressed against the JLN index and the regression coefficient is β_i^{MEU}, ambiguity beta of stock i.

In order to empirically test the theory, the first check is whether the CAPM alpha, α_i, has a positive association with $\beta_i^{MEU} - \beta_i^{CAPM}$. At the end of each month $t-1$, stocks are ranked into quintile portfolios according to their $\beta_{i,t-1}^{MEU} - \beta_{i,t-1}^{CAPM}$, and the portfolio excess returns for the post-ranking month t are linked across months to form one time-series of post-ranking excess returns for each quintile portfolio. For post-ranking excess returns of quintile portfolios, both equal-weighted and value-weighted returns are computed.

By using quintile portfolios post-ranking excess returns, CAPM and conditional CAPM alphas are obtained. Monotonic increase in alphas from low to high quintile together with economically and statistically significant positive alpha for high-minus-low quintile are shown. For robustness, the procedure is repeated in the presence of Fama-French three and five factor models.

Sensitivity of ambiguity exposure to numerous factors that are reported in the literature to affect cross-sectional stock returns are calculated using Fama Macbeth regressions. Sensitivities make economic sense.

As another robustness check, power of ambiguity exposure in explaining stocks' abnormal excess returns are tested for persistence when a subset or the whole set of other factors which have explanatory power for stocks' returns are controlled for in the same regression. Explanatory power of ambiguity exposure for stocks' abnormal excess returns are persistent with and without controlling for industry effect.

As an event study for periods of heightened uncertainty in the markets, days around the Federal Open Market Committee (FOMC) meetings between 1982 and 2015 are investigated. The main finding is that the CAPM alpha difference between the highest and the lowest ambiguity exposure quintiles significantly widens just days before the meeting, as ambiguity aversion among market players becomes more pronounced. As might be expected, ambiguity aversion subsides immediately after the meeting.

For cross-sectional tests, finally, ambiguity exposure was established as a separate explanatory factor for cross-sectional excess stock returns. When the factor for ambiguity exposure is controlled for in the CAPM regression, alpha of the high-minus-low quintile as well as the alphas of the individual quintiles are shown to disappear.

Overall, the novel contribution of this thesis is empirical evidence of ambiguity and aversion towards it in the pricing of stocks in the cross-section, and what this implies for asset pricing. The findings in the thesis are in accordance with the other research in the literature that point to the strong relation between ambiguity and excess equity returns at the aggregate market level. The literature investigating whether ambiguity is priced in stock returns typically studies the research question at the aggregate market level. The research in the thesis is one of the few in the literature that explores the pricing of ambiguity in the cross-section of stock returns, and its distinguishing strength stems from its foundation in a (decision) theory-based model. The compatibility of the fundamental assumptions of the model with the real world fortifies that strength.

This thesis also conducts empirical research on the explanatory power of ambiguity and aversion towards it according to the employed theory on cross-sectional asset pricing anomalies. The empirical results in this thesis show that ambiguity and ambiguity aversion fully explain the beta anomaly, while providing partial explanations to operating profitability and momentum anomalies. These findings constitute an additional novel contribution to the literature.

The thesis proceeds as follows. In Chapter 2, as part of the literature review, smooth ambiguity aversion model of Klibanoff et al. (2005) as the foundation for the theoretical model employed in the thesis is discussed with emphasis on its comparative strength, alongside the separate literature on empirical evidence of ambiguity aversion and its asset pricing implications. Novel contributions of the thesis vis-a-vis some other studies in the literature are also mentioned. In Chapter 3, theoretical model employed for empirical tests is presented in detail. Chapter 4 looks into the data with specifics on source, definitions, data construction, and runs through the empirical testing methodology. Chapter 5 lays out the empirical results of the thesis on pricing of ambiguity in the cross-section of stocks and underlines the thesis' novel contributions to the literature. Chapter 6 reports the empirical results obtained in the thesis on the power of ambiguity to explain major cross-sectional asset pricing anomalies. Chapter 7 concludes.

Chapter 2

Literature Review

It was Frank Knight who first distinguished uncertainty from risk in his book published in 1921. He asserted that in most economic problems where decision makers face uncertainty, measuring probabilities is difficult. Knight also pointed out that known risks have a clearly defined probability distribution, whereas uncertainty stems from an unknown probability distribution or a distribution with unknown parameters. Such uncertainty is thus referred to as Knightian uncertainty or ambiguity. Hence, ambiguity is uncertainty about probabilities, while risk is about uncertainty about outcomes. Tangibly, Brenner and Izhakian (2018) proposed degree of ambiguity measured by the expected volatility of probabilities, analogous to risk in financial literature measured by volatility of returns (notice that, asset return is an outcome in the finance context).

Ambiguity is often referred to describe uncertainty about a data generating process in modern decision theory. When there is uncertainty about the probability distribution and/or its parameters or the data generating process in general, decision maker's subjective beliefs come into play. The decision maker presumes the data comes from an unknown member of a set of possible models. Ellsberg (1961) showed that when there is ambiguity, decision maker's choice is inconsistent with expected utility theory. In the Ellsberg paradox, decision maker strictly favors the bet in which (s)he knows the probability and utility outcome, over the ambiguous bets even though the ambiguous ones have the same or even greater utility. One explanation that resolves the paradox is the notion of ambiguity aversion, a simple definition of which is the dislike of decision makers for the ambiguous bets and favoring of bets with quantifiable risks.

Why is ambiguity important in asset pricing context in finance? The financial literature, to a large extent, assumes investors behave as if they know the distributions of returns, disregarding ambiguity. However, this is hard to justify. It is well established that first moments (i.e. means) of return distributions are extremely difficult to estimate (Merton (1980), Cochrane (1997), Blanchard (1993), and Anderson et al. (2003)), although arguably, estimation errors for second moments may be virtually eliminated by finer sampling. Moreover, recent contributions (such as Maenhout (2004), Epstein and Schneider (2008), Illeditsch (2011), Condie et al. (2020), Mele and Sangiorgi (2015), Ju and Miao (2012), Hansen and Sargent (2010), and Collard et al. (2018)) have shown that ambiguity aversion can eloquently help to explain empirical regularities involving price of aggregate uncertainty. Many of the new theories at the frontline of consumption-based asset pricing, which provide strong explanations for aggregate puzzles such as equity premium, excess volatility, and their dynamics, do not have similar explanatory power for cross-sectional puzzles (Lettau and Ludvigson (2009)). In that respect, ambiguity has strong potential, and has motivated the research in this thesis.

In an ambiguous data generating process as in the case of ever changing prices and returns of financial assets, the decision maker will have to choose from a set of distributions of expected utility values induced by the set of models believed to be possible. Task of a portfolio manager lays a concrete example. Almost at any point in time, the portfolio manager, with access to all publicly available data, is faced with a range of return distributions that seem plausible. They as decision makers are inclined to choose a portfolio position whose value is more robust to (or equivalently, less affected by) parameter uncertainty. Indeed, Knight and Ellsberg had also intuitively argued that concern about parameter uncertainty induces a decision maker to follow rules that is more robust amongst the set of all subjectively possible return distributions. Investigation of set of rules robust to model uncertainty or misspecification has been central to the literature and some pioneering contributions came forth by Schmeidler (1989), Gilboa and Schmeidler (1989), followed by the theory of smooth ambiguity aversion by Klibanoff et al. (2005) (hereafter referred to as KMM) and subsequent work of Hansen and Sargent (2008) involving robust control theory. Amongst the decision making models, this thesis adopts smooth ambiguity aversion of KMM (2005) as a theoretical foundation.

Amongst the popular decision making models that incorporate ambiguity and explain Ellsberg paradox at the same time, Gilboa and Schmeidler's (GS) maxmin expected utility (MMEU) model is important to mention for contrasting purposes. In the MMEU, the decision maker has a (subjective) set of priors (probabilities) which he thinks are relevant over the (finite) states of the world. He computes the minimal expected utility for each act (note that an act is a function from the set of states of the world to the set of outcomes), where the prior ranges on the set of priors. He then chooses the act with the maximum minimal expected utility amongst all the minimal expected utilities of acts. An important attribute of the GS model is that ambiguity and ambiguity attitude of the decision maker is inseparable. The model implicitly assumes that the decision maker exhibits infinite ambiguity aversion, by taking the minimal expected utility of each act.

As mentioned previously, this thesis relies on the theory of KMM (2005) which as a model differs from many others in the literature from several angles. The key distinguishing factor of the model is that it achieves a clear separation of ambiguity, identified as a characteristic of the decision maker's beliefs, and ambiguity attitude, a characteristic of the decision maker's tastes. In fact, KMM (2005) show in their paper that when ambiguity aversion of the decision maker is taken to infinity, the smooth ambiguity model essentially exhibits a maxmin expected utility (of GS). Another significant advantage of the model is that the well-developed framework for dealing with risk attitudes can be accommodated to ambiguity attitudes.

Smooth ambiguity aversion model, expressed as $V_S(f) = E_\mu \, \Phi(E_{p \in P}(u(f)))$ (where f is an act defined on the set of states of the world S and u is a von Neumann-Morgenstern utility function), actually introduces a second-order probability measure over the set of priors in P. In other words, there is a second-order probability of probabilities, representing the decision maker's subjective uncertainty about the different probabilities deemed possible, which is in this sense, a second-order belief. Ambiguity attitude, on the other hand, is characterized by the shape of Φ, which for ambiguity averse decision makers is an increasing concave utility function. Overall, the separation of ambiguity and ambiguity attitude in the smooth ambiguity model is thus achieved by the introduction of second-order beliefs and a second utility function to represent ambiguity attitudes. Going back to the portfolio manager example, a smooth ambiguity-averse investor will dislike mean-preserving spreads in his choices of portfolios.

Note that, there is a separate literature on empirical evidence on ambiguity aversion and asset pricing implications. Brenner and Izhakian (2018) indicate that ambiguity in the equity market is priced. They have the following findings: i) when ambiguity is introduced alongside risk, the latter has a positive and significant effect on the equity premium, ii) for a high expected probability of favorable returns, the ambiguity premium is positive, implying ambiguity-averse behaviour, iii) for a high expected probability of unfavorable returns, the ambiguity premium is negative, implying ambiguity-loving behaviour, iv) aversion to ambiguity increases with the expected probability of favorable returns, and love for ambiguity increases with the expected probability of unfavorable returns. Another important finding in the paper is that when ambiguity is included in the pricing model, the effect of risk is positive and significant, while its effect is insignificant when ambiguity is not accounted for. Overall, the paper is about aggregate equity premium.

Drechsler (2013) uses variance premium as a proxy for ambiguity, and shows that fluctuations in model uncertainty and thus the variance premium may help predict stock returns. The author uses VIX (the option implied volatility of S&P500) for risk neutral expectation of variance, and calculates the realized variance from S&P500 futures. The paper shows that model uncertainty (ambiguity) concerns help to capture the large premium in the prices of equity index options. As above, Drechsler's (2013) empirical investigation between ambiguity and stock returns is at aggregate market level, yet confirms the strong relation between ambiguity and excess premium in equities.

There has also been some research on the cross-sectional implications of macroeconomic uncertainty. Bali et al. (2017) use the macroeconomic uncertainty (MEU) index to sort portfolios according to MEU beta, which is the slope coefficient of the MEU index in an OLS regression where excess returns of the stock are regressed against the said index as well as many other well-known risk factors. They find that portfolio abnormal returns are decreasing in MEU beta, which is intriguing. If the macroeconomic uncertainty (measured by the MEU index) is interpreted as macroeconomic ambiguity, then the finding seems to suggest that assets which are more exposed to systematic ambiguity should carry a lower abnormal return.

Empirical findings in this thesis using our theory-based methodology suggest otherwise. Empirical tests in the thesis include: i) investigation of the relation between ambiguity and excess stock returns in the cross-section, ii) robustness checks of the explanatory power of ambiguity for excess stock returns when other well-known factors are accounted for, iii) checking the relation between ambiguity with other well-established factors which have an impact on stock returns, iv) testing of the dynamic relation between ambiguity and excess stock returns on the time-varying macroeconomic uncertainty level as well as the FOMC announcements when the ambiguity is typically high, v) and lastly, inquiring whether ambiguity as a factor to explain excess stock returns might resolve part of the cross-sectional asset pricing anomalies.

The novel contribution of this thesis is empirical evidence of ambiguity and aversion towards it in the pricing of stocks in the cross-section, and what this implies for asset pricing. Our findings are in accordance with the research that point to the strong relation between ambiguity and excess equity returns at the aggregate market level. The empirical methodology of the thesis is devised according to the findings of an asset pricing theory built on the smooth ambiguity aversion model of KMM (2005). The asset pricing theory accomodates both heterogeneity in ambiguity and heterogeneity in ambiguity aversion; therefore, applicable to real world.

Comparing the empirical results in this thesis with those of Bali et.al (2017) as another research about the impact of (macroeconomic) uncertainty on the pricing in the cross-section of stocks, the findings are in conflict. Their results show that stocks with a higher exposure to macroeconomic uncertainty should carry a lower abnormal return. Although the index (which is MEU, as detailed in Chapter 4) proxying macroeconomic uncertainty is identical for their research and this thesis, they estimate the exposure to macroeconomic uncertainty by MEU beta alone, which is the coefficient of the MEU index in the OLS regression of excess stock return against the index (in addition to many other well known risk factors). This thesis differs in the methodology, which is based on the theory. Proposition 1 in the Chapter 3 implies that exposure to systematic ambiguity (which we proxy by macroeconomic uncertainty) is a linear function of $\beta_i^{MEU} - \beta_i^{CAPM}$, not β_i^{MEU} alone. Chapter 5 does verify that abnormal returns (alphas in CAPM and conditional CAPM) are indeed increasing in $\beta_i^{MEU} - \beta_i^{CAPM}$.

This thesis also conducts empirical research on the explanatory power of ambiguity and aversion towards it according to the employed theory on cross-sectional asset pricing anomalies. An extensive list of cross-sectional anomalies had been listed in Hou et al. (2015), and the paper had indicated that the comprehensive examination of nearly 80 anomalies revealed half of the anomalies were insignificant in the broad cross section. Their evidence suggests that many claims in the anomalies literature seem exaggerated due to equal-weighting and the due impact of small caps.

Accordingly, the focus in the thesis for anomalies has been on the prominent and persisting ones; namely, the beta, size, book-to-market, investment (asset growth), operating profitability, and momentum anomalies. The empirical results in this thesis show that ambiguity and ambiguity aversion fully explain the beta anomaly, while providing only partial explanations to operating profitability and momentum anomalies (methodology, empirical results and discussion are detailed in Chapter 6). These findings constitute an additional novel contribution to the literature.

Empirical results showing resolution of the beta anomaly is fully consistent with the theory employed in the thesis. Recall that, security market line being too flat compared to what is predicted by the CAPM is called the beta anomaly. The anomaly points to the fact that low beta stocks carry a positive ambiguity premium, while high beta stocks carry a negative ambiguity premium. This is consistent with the theory employed in this thesis. Lower cross-sectional variation in ambiguity beta with respect to CAPM beta results in ambiguity beta being higher than low CAPM betas and being lower than high CAPM betas.

Chapter 3

Theory

3.1 Model

Theoretical framework employed in this thesis is based on Mukerji, Ozsoylev and Tallon (2022).[1] This section starts by describing the domain of choice, agents, their beliefs and preferences. The model involves two uncertain assets $i = 1, 2$ and a risk-free asset f. The price of the risk-free asset, p_f, is normalized to 1. The price of uncertain asset i is denoted by p_i. The set of agents is finite, $\{1, \cdots, n, \cdots, N\}$. Agent n's holdings of the assets are denoted by $q_{i,n}$; conveniently, $q_n = (q_{1,n}, q_{2,n})$ for agent n's holdings in two assets. Then, $a_{i,n}$, the monetary amount invested in asset i by agent n, is $a_{i,n} \equiv p_i q_{i,n}$, and $a_n = (a_{1,n}, a_{2,n})$ to denote the agent n's monetary investment (or equivalently, monetary holdings) in uncertain assets. Since $p_f = 1$ due to normalization, $a_{f,n}$, agent n's monetary holding of the risk-free asset, is $a_{f,n} = q_{f,n}$. Endowment of asset i is $e_{i,n}$ for agent n, and e_i denotes the aggregate endowment of the asset. All agents have zero endowment of the risk-free asset, and therefore there is zero aggregate supply of the risk-free asset so that $e_f = 0$. Risk-free and uncertain returns are both exogenous. The gross (monetary) returns of the risk-free asset and uncertain assets are R_f and R_i, $i = 1, 2$, respectively and let $R \equiv (R_1, R_2)$. If agent n invests $a_{j,n}$ in asset j, where $j = f, 1, 2$, then his payoff is $R_j a_{j,n}$.

[1] Sections 3.1 and 3.2 are based on and borrow from Mukerji, Ozsoylev and Tallon (2022). Sections 3.3 and 3.4 are based on a paper which I am a co-author of, Mukerji, Ozsoylev, Savran and Tallon (2022).

The uncertain returns are ambiguous, which is to say that agents are uncertain about the probability distribution governing each return: they believe that the returns data are generated by an unknown member of a set of possible models. Formally, we have a random vector $M \equiv (M_1, M_2)$, the model, the realization of which fixes the vector of conditional distribution of returns $R|M \equiv (R_1|M_1, R_2|M_2)$. The uncertainty about returns conditional on a model M (i.e., $R|M$) is referred to as the first-order uncertainty, while uncertainty about the model M itself is referred to as the second-order uncertainty. The setup of Hara and Honda (2018) to describe the agents' common beliefs about the uncertainty governing returns is adapted.

In that setup, both first- and second-order uncertainties are Gaussian. Mukerji, Ozsoylev, Tallon (2022) further impose the following assumptions.

Assumption 1. *The mean return of asset i conditional on model M is $M_i, i = 1, 2$. That is,*

$$E\left[R \mid M\right] = M.$$

Assumption 2. *Models and asset returns are jointly normally distributed with*

$$cov\left(R, M\right) = var\left(M\right) \equiv \Sigma_M.$$

That is,

$$\begin{pmatrix} M \\ R \end{pmatrix} \sim N\left(\begin{pmatrix} E[M] \\ E[R] \end{pmatrix}, \begin{pmatrix} \Sigma_M & \Sigma_M \\ \Sigma_M & \Sigma_R \end{pmatrix}\right)$$

where

$$\Sigma_M = \begin{pmatrix} \left(\sigma_1^M\right)^2 & \sigma_{12}^M \\ \sigma_{12}^M & \left(\sigma_2^M\right)^2 \end{pmatrix} \; and \;\; \Sigma_R = \begin{pmatrix} {\sigma_1}^2 & \sigma_{12} \\ \sigma_{12} & {\sigma_2}^2 \end{pmatrix}.$$

Assumption 2 is actually all about the joint normality of R and M and the restriction that $cov\left(R, M\right) = var\left(M\right)$ does not result in loss of generality as pointed out by Hara and Honda (2018). The matrix Σ_R is the variance-covariance matrix of the unconditional distribution of returns R. Thus, $(\sigma_i^M)^2$ and σ_i^M are referred to as the *model variance of asset i* and *model standard deviation of asset i*, respectively, and Σ_M as the model variance-covariance matrix. The Projection Theorem, together with Assumptions 1 and 2, yields

$$M = E\left[R \mid M\right] = E\left[R\right] + \left[cov(R, M)\right]\left[var\left(M\right)\right]^{-1}\left(M - E[M]\right) = E[R] + M - E[M],$$

which in turn implies that $E[R] = E[M] \equiv \mu \equiv (\mu_1, \mu_2)$. It is assumed that $\mu_i > R_f$, $i = 1, 2$, so that risk- and ambiguity-averse agents do not rule out investment into uncertain assets. Also, following Assumption 2 and the Projection Theorem, it holds that

$$\Sigma \equiv var\left(R \mid M\right) = var\left(R\right) - \left[cov(R, M)\right]\left[var\left(M\right)\right]^{-1}\left[cov(R, M)\right] = \Sigma_R - \Sigma_M. \quad (3.1)$$

Hence, the conditional asset return is distributed as:

$$R \mid M \sim N\left(M, \Sigma_R - \Sigma_M\right).$$

In particular, the variance of returns conditioned on the realization of M is independent of the realized value and the uncertainty about the model only affects the (conditional) mean of the return, not the (conditional) variance. Therefore, model uncertainty reduces to *parameter uncertainty* about the mean.

How the agents incorporate uncertainty in the evaluation of portfolios is based on the framework of the smooth ambiguity aversion model of Klibanoff et al. (2005). According to that model, if the agent knows the realization of M and thus is faced with no parameter uncertainty, the evaluation is the usual expected utility evaluation using the random variable $R|M$. Uncertainty about M makes the expected utility evaluation (based on $R|M$) uncertain; the agent is ambiguity averse if he dislikes mean preserving spreads in this uncertainty. More specifically, consider a portfolio $(a_{f,n}, a_{1,n}, a_{2,n})$ which yields a final contingent wealth equal to $W(a_{f,n}, a_{1,n}, a_{2,n}) =$

$a_{f,n}R_f + a_{1,n}R_1 + a_{2,n}R_2$. Agent n evaluates such a portfolio according to:[2]

$$E_M \left[\phi_n \left(E_{R|M}[u_n \left(W(a_{f,n}, a_{1,n}, a_{2,n})\right)]\right)\right], \tag{3.2}$$

where u_n, a utility function, incorporates the agent's attitude to risk, and ϕ_n, an increasing concave function, reflects the agent's ambiguity aversion. Thus, the ambiguity and the ambiguity aversion are represented distinctly through the random variable M and the function ϕ_n, respectively. This parametric separation is important and useful, as we can independently change ambiguity and ambiguity attitude. For example, it is possible to hold an agent's beliefs (i.e. perceived ambiguity) fixed, while varying their ambiguity attitude, say from aversion to neutrality (i.e., replacing a concave ϕ_n with an affine one reduces the preference to expected utility while retaining the same beliefs). As in Hara and Honda (2018), $u_n(x) = -exp(-\theta_n x)$ and $\phi_n(y) = -(-y)^{\gamma_n/\theta_n}$. Note, if $\gamma_n = \theta_n$, then agent n is ambiguity neutral and a CARA (expected) utility maximizer. If $\gamma_n > \theta_n$, the agent is ambiguity averse. Denote by

$$\eta_n \equiv -\, y\, \frac{\phi_n''(y)}{\phi_n'(y)} = \frac{\gamma_n}{\theta_n} - 1 = \frac{\gamma_n - \theta_n}{\theta_n},$$

the coefficient of (relative) ambiguity aversion of the agent. Another assumption employed is that $\eta_n \geq 0$ for all n, which rules out ambiguity seeking attitude.

Given Assumptions 1 and 2, and the specifications of u_n and ϕ_n as above, the agents' maximization problem given in (3.2) is equivalent to choosing a portfolio $(a_{f,n}, a_{1,n}, a_{2,n})$ that maximizes the following, as shown by Lemma 1 of Hara and Honda (2018):

$$V_n(a_{f,n}, a_{1,n}, a_{2,n}) \;\equiv\; \underbrace{a_{f,n}R_f + \mu^\top a_n}_{\text{mean return}} - \frac{\theta_n}{2}\, \underbrace{a_n^\top \Sigma_R a_n}_{\text{risk}} - \frac{\gamma_n - \theta_n}{2}\, \underbrace{a_n^\top \Sigma_M a_n}_{\text{ambiguity}}, \tag{3.3}$$

where $a_n = (a_{1,n}, a_{2,n})$. This formulation therefore generalizes Markowitz' (1952) standard mean-variance model: $\frac{\theta_n}{2}\, a_n^\top \Sigma_R a_n$ is the standard risk adjustment to the eval-

[2] E_X denotes an expectation operator which integrates over the realization of the random variable X.

uation while $\frac{\gamma_n - \theta_n}{2}\ a_n^\top \Sigma_M a_n$ introduces an ambiguity adjustment. Maccheroni et al. (2013) arrive at the formulation (3.3), which they refer to as the *robust mean-variance model*, as a second-order (Arrow-Pratt) approximation of the certainty equivalent of a smooth ambiguity evaluation where u_n, ϕ_n and beliefs are arbitrarily specified. This is analogous to the fact that the standard mean-variance formulation may be seen as a quadratic approximation of the certainty equivalent of an expected utility evaluation where utility and beliefs are specified arbitrarily.

Note that the terms in the formulation in (3.3) may be arranged as follows:

$$V_n(a_{f,n}, a_{1,n}, a_{2,n}) \ = \ a_{f,n}R_f + \mu^\top a_n - \frac{\theta_n}{2}\ a_n^\top \left(\Sigma_R + \eta_n \Sigma_M\right) a_n. \tag{3.4}$$

As a result, ambiguity averse agent's evaluation of the portfolio can be read as if it were the evaluation of a standard (as opposed to robust) mean-variance utility agent with absolute risk aversion parameter θ_n and an as if belief that uncertain assets' return distribution is given by $N(\mu, \Sigma_R + \eta_n \Sigma_M)$. Thus, the population of robust mean-variance agents with identical beliefs but heterogeneous ambiguity aversion may be equivalently seen as a population of standard mean-variance agents with heterogeneous as if beliefs which differ in the variance but not in the mean of returns. The disagreement in as if beliefs is in the term $\eta_n \Sigma_M$, and hence the differences are magnified by Σ_M. Given that the ambiguity is about the mean returns, one might have expected that more ambiguity averse decision makers would behave as if they believe the mean returns were lower, but they instead behave as if the return variances are higher. This as if beliefs interpretation is helpful in gaining intuition for some of the results in the subsequent analysis.

Remark 1. *As if beliefs, characterized by $\eta_n \Sigma_M$, are connected to objective and estimable properties of return distributions encapsulated in Σ_M. Ambiguity aversion, which forges the link in this connection, is also measurable (see, e.g., Dimmock et al. (2016)). Hence, these as if beliefs can be disciplined by objective returns data, yielding testable predictions. Indeed, an important pricing implication of ambiguity, namely, which assets carry a positive CAPM alpha (i.e, ambiguity premium) versus a* *negative one (i.e., ambiguity discount), can be determined solely by Σ_M without any information on agents' possibly heterogeneous ambiguity aversion.*

3.2 Portfolio Choice

This section is based on Mukerji, Ozsoylev and Tallon (2022) and discusses how the composition of the optimal portfolio is determined by ambiguity aversion given exogenous asset prices. Consider agent n with initial wealth $W_n \equiv p_1 e_{1,n} + p_2 e_{2,n}$. The maximization problem he faces is

$$\begin{aligned} \max_{a_{f,n},\ a_{1,n},\ a_{2,n}} \quad & a_{f,n} R_f + \mu^\top a_n - \frac{\theta_n}{2} a_n^\top \Sigma_R a_n - \frac{\gamma_n - \theta_n}{2} a_n^\top \Sigma_M a_n \\ \text{s. to} \quad & a_{f,n} + a_{1,n} + a_{2,n} \leq W_n. \end{aligned} \tag{3.5}$$

Solving for the optimal monetary holdings of uncertain assets, $a_n = (a_{1,n}, a_{2,n})$, yields:

$$a_n \;=\; \frac{1}{\theta_n} (\Sigma_R + \eta_n \Sigma_M)^{-1} (\mu - R_f 1), \tag{3.6}$$

or more explicitly,

$$a_{i,n} = \frac{(\mu_i - R_f) A_{j,n} - (\mu_j - R_f) B_{12,n}}{A_{1,n} A_{2,n} + (B_{12,n})^2}, \quad i = 1, 2, \tag{3.7}$$

where

$$\begin{aligned} A_{i,n} &= \theta_n \sigma_i^2 + (\gamma_n - \theta_n)(\sigma_i^M)^2 \;=\; \theta_n \left[\sigma_i^2 + \eta_n (\sigma_i^M)^2 \right], \\ B_{12,n} &= \theta_n \sigma_{12} + (\gamma_n - \theta_n) \sigma_{12}^M \;=\; \theta_n \left[\sigma_{12} + \eta_n \sigma_{12}^M \right]. \end{aligned}$$

Hence,

$$\frac{a_{1,n}}{a_{2,n}} = \frac{(\mu_1 - R_f)\left[\sigma_2^2 + \eta_n (\sigma_2^M)^2\right] - (\mu_2 - R_f)\left[\sigma_{12} + \eta_n \sigma_{12}^M\right]}{(\mu_2 - R_f)\left[\sigma_1^2 + \eta_n (\sigma_1^M)^2\right] - (\mu_1 - R_f)\left[\sigma_{12} + \eta_n \sigma_{12}^M\right]}. \tag{3.8}$$

As a result, the ratio of monetary investments in uncertain assets is independent of the agent's risk aversion parameter θ_n. If everyone is ambiguity neutral (i.e., $\eta_n = 0$ for all n), the above formula implies the classical mutual fund theorem (Tobin

(1958)), which states that ambiguity neutral agents, regardless of their risk aversion, invest in the same proportion across uncertain assets, and therefore hold the same portfolio of uncertain assets. Yet, the ratio in (3.8) does depend on the agent's ambiguity aversion parameter η_n. The mutual fund theorem continues to hold if agents' ambiguity attitude is homogenous (e.g. they are homogeneously ambiguity averse). However, generically, two agents n and n' with different ambiguity aversion parameters will have different ratios of monetary investments in uncertain assets. Noting $\frac{a_{1,n}}{a_{2,n}} = \frac{p_1 q_{1,n}}{p_2 q_{2,n}}$, this also means that they will hold uncertain assets in different proportions, i.e., $\frac{q_{1,n}}{q_{2,n}} \neq \frac{q_{1,n'}}{q_{2,n'}}$. The following remark summarizes these observations:

Remark 2. *If agents are homogeneous in ambiguity aversion, i.e., $\eta_n = \eta_{n'}$ for all n, n', then the mutual fund theorem holds, that is, for optimal portfolio choices it holds that $\frac{a_{1,n}}{a_{2,n}} = \frac{a_{1,n'}}{a_{2,n'}}$ and $\frac{q_{1,n}}{q_{2,n}} = \frac{q_{1,n'}}{q_{2,n'}}$ for all n, n'. If, on the other hand, agents are heterogeneous in ambiguity aversion, then the mutual fund theorem generically fails.*

To see the intuition, the formula in (3.4) and the ensuing discussion is guiding. When agents are homogeneously ambiguity averse (i.e., identical η_n's for all agents), they act as if they have standard mean-variance preferences with the same beliefs, and hence the mutual fund theorem follows. On the other hand, agents with heterogeneous ambiguity aversion act as if they have different beliefs about return variances, which leads them to hold different portfolios as they disagree on how to optimally diversify. The latter point is apparent in (3.6), as optimal monetary holdings are a function of $\eta_n \Sigma_M$, the distinguishing aspect of the as if belief. That the mutual fund theorem holds with homogeneous ambiguity aversion and fails with heterogeneous ambiguity aversion was already noted in Hara and Honda (2018).

3.3 Equilibrium Asset Pricing

This section is based on Mukerji, Ozsoylev, Savran and Tallon (2022) and derives the equilibrium of a static multi-agent economy and properties of equilibrium asset prices as a function of asset ambiguity and agent ambiguity aversion. In particular, it establishes a single-factor CAPM-like asset pricing formula with heterogeneously ambiguous assets and heterogeneously ambiguity averse agents. As in the CAPM, the single pricing factor is the excess return of the market portfolio, however the factor loading now accounts for the existence of ambiguity as well as risk. In the formula to be shown, the standard CAPM beta is adjusted depending on the extent to which the ambiguity of the asset return is correlated with the ambiguity of the market portfolio return. The extent of the adjustment determines the ambiguity premium.

A CAPM-like pricing formula with heterogeneously ambiguity averse agents is surprising since the results in the previous section show that with such heterogeneity the classical mutual fund theorem fails, and therefore agents hold portfolios distinct from the market portfolio. The single-factor pricing formula will emerge by constructing what is effectively a representative agent for pricing purposes.

Heterogeneity in ambiguity attitudes is only natural and to be expected; arguably, a substantial part of the investor population is of the expected utility type and they co-exist with ambiguity averse investors. Since the argument in Ruffino (2014) completely rides on the fact that the mutual fund theorem holds under homogeneity of ambiguity attitudes, it leaves no reason for one to think that the result is valid for such mixed investor populations. This puts into question the empirical relevance and testability of the theory under the homogeneity assumption. Apart from adding crucial empirical relevance, extending the theory to heterogeneous ambiguity attitudes adds to our understanding of the conceptual significance of ambiguity aversion in the determination of asset prices by showing that, even in an economy with a substantial presence of expected utility agents who have the "correct" beliefs, ambiguity averse agents still influence prices in a marked way.

An *equilibrium* of the (static) economy is given by asset holdings $\{(q^*_{f,n}, q^*_{1,n}, q^*_{2,n})\}_{n=1}^N$ and asset prices (p^*_1, p^*_2) such that monetary holdings $\{(q^*_{f,n}, p^*_1 q^*_{1,n}, p^*_2 q^*_{2,n})\}_{n=1}^N$ are a solution to the agents' maximization problem (3.5), and market clears, i.e.,

$$\sum_n q^*_{i,n} = \sum_n e_{i,n} = e_i, \ i = 1, 2, \text{ and } \sum_n q^*_{f,n} = 0.$$

To state the asset pricing result, some further notation is necessary. The return of a generic portfolio of uncertain assets held by agent n is denoted by $R(a_n)$ where,

$$R(a_n) \equiv \frac{a_{1,n}R_1 + a_{2,n}R_2}{a_{1,n} + a_{2,n}} = \frac{p_1 q_{1,n} R_1 + p_2 q_{2,n} R_2}{p_1 q_{1,n} + p_2 q_{2,n}}.$$

Also let

$$\begin{aligned}
cov(R(a_n), R_i) &\equiv \frac{a_{1,n}\sigma_{1i} + a_{2,n}\sigma_{2i}}{a_{1,n} + a_{2,n}}, \\
var(R(a_n)) &\equiv \frac{(a_{1,n})^2(\sigma_1)^2 + 2a_{1,n}a_{2,n}\sigma_{12} + (a_{2,n})^2(\sigma_2)^2}{(a_{1,n} + a_{2,n})^2}, \\
cov^M(R(a_n), R_i) &\equiv \frac{a_{1,n}\sigma^M_{1i} + a_{2,n}\sigma^M_{2i}}{a_{1,n} + a_{2,n}}, \\
var^M(R(a_n)) &\equiv \frac{(a_{1,n})^2(\sigma^M_1)^2 + 2a_{1,n}a_{2,n}\sigma^M_{12} + (a_{2,n})^2(\sigma^M_2)^2}{(a_{1,n} + a_{2,n})^2}.
\end{aligned}$$

As is standard, agent n is said to *hold the market portfolio* if his holdings of uncertain assets is proportional to the aggregate endowment of uncertain assets, i.e., $(q_{1,n}, q_{2,n}) = (k_n e_1, k_n e_2)$ for some real scalar k_n. This implies that agent n holds the market portfolio if and only if $\frac{q_{1,n}}{q_{2,n}} = \frac{e_1}{e_2}$. We let R_{market} denote *the market portfolio return*, that is,

$$R_{market} \equiv \frac{p^*_1 e_1 R_1 + p^*_2 e_2 R_2}{p^*_1 e_1 + p^*_2 e_2}. \tag{3.9}$$

Note that the market portfolio return, R_{market}, is determined in equilibrium as it depends on equilibrium prices (p^*_1, p^*_2), even though asset returns R_i are exogenous. Let us define asset i's *beta*, β_i, in the standard way, and *ambiguity beta*, β^{Amb}_i, analogously:

$$\beta_i \equiv \frac{cov(R_{market}, R_i)}{var(R_{market})}, \quad \beta^{Amb}_i \equiv \frac{cov^M(R_{market}, R_i)}{var^M(R_{market})}.$$

The anticipated result is as follows:

Proposition 1. *Let* $\underline{\eta} \equiv \min_n \{\eta_n\}$ *and* $\bar{\eta} \equiv \max_n \{\eta_n\}$. *Then, there exists a unique*

$\eta_{market} \in \left[\underline{\eta}, \bar{\eta}\right]$ *such that the following equality holds:*

$$E[R_i] - R_f = \frac{cov(R_{market}, R_i) + \eta_{market}\ cov^M(R_{market}, R_i)}{var(R_{market}) + \eta_{market}\ var^M(R_{market})} \left(E[R_{market}] - R_f\right) \tag{3.10}$$

$$= \alpha_i + \beta_i \left(E[R_{market}] - R_f\right), \tag{3.11}$$

where

$$\alpha_i \equiv \frac{\eta_{market}\ var^M(R_{market})}{var(R_{market}) + \eta_{market}\ var^M(R_{market})} \left(\beta_i^{Amb} - \beta_i\right) \left(E[R_{market}] - R_f\right). \tag{3.12}$$

Furthermore, $\eta_{market} > 0$ *if and only if* $\bar{\eta} > 0$.

The proof of Proposition 1 is relegated to the Appendix. Also, Lemma A.1 in the Appendix shows that, given an agent's optimal portfolio choice, expected excess asset returns can be explained by a single-factor pricing formula, where the single factor is the excess return of the agent's optimal portfolio (of the uncertain assets). With homogeneous ambiguity aversion, all agents' optimal portfolios coincide with the market portfolio since the mutual fund theorem holds. Therefore, setting η_{market} equal to the agents' common ambiguity aversion coefficient, the CAPM-like formulation in Proposition 1 obtains (as Ruffino (2014) had shown). The novel contribution here is showing that the same formula continues to hold under heterogeneous ambiguity aversion, a case where the mutual fund theorem fails. That follows from the fact that they effectively establish a representative agent for pricing purposes. If one was to regress the excess returns of asset i on the excess returns of the market portfolio over time, he would get $\beta_i = \frac{cov(R_{market}, R_i)}{var(R_{market})}$ as the OLS slope coefficient. Note, this OLS coefficient is the CAPM factor loading and therefore the sole determinant for the cross-sectional expected returns in CAPM. Absent ambiguity aversion, the asset pricing prediction of the theory is identical to that of CAPM. However, even with a single ambiguity averse agent in the economy, there is an additional term in the excess return predicted by the theory of Mukerji et.al (2020), the ambiguity premium α_i, driven by asset i's exposure to systematic ambiguity as can be seen from (3.10).

Thus, by that theory, the cross-sectional expected returns are not only determined by the standard CAPM predictor, β_i, based on the OLS coefficient, but also by i's ambiguity exposure.

The excess return of an asset is the compensation for *systematic* (non-diversifiable) uncertainty in that asset. Like in the standard CAPM, the single pricing factor is the expected excess return on the market portfolio. The difference ambiguity and ambiguity aversion bring is the adjustment to the factor loading, as can be seen from (3.10). The overall adjustment to the standard CAPM formula is encapsulated in α_i, which we refer to as asset i's *ambiguity premium.* Asset i's ambiguity premium is zero if (i) all agents are ambiguity neutral so that $\eta_{market} = 0$, or (ii) all uncertain assets carry no ambiguity so that $var^M(R_{market}) = 0$, or (iii) $\beta_i^{Amb} = \beta_i$. Given some ambiguity and ambiguity aversion, $\alpha_i > 0$ if and only if $\beta_i^{Amb} > \beta_i$, that is, if the asset's exposure to systematic ambiguity (or, exposure to systematic second-order uncertainty) is greater than its exposure to systematic unconditional uncertainty. Note, however, even if an asset's returns are ambiguous, the asset will carry an ambiguity discount if its systematic ambiguity exposure is relatively low.

There is a literature that looks into asset pricing implications of heterogeneity in subjective beliefs about the variance-covariance matrix of asset returns. Chiarella et al. (2010) and Gerber and Hens (2017) discuss how such heterogeneity leads to deviations from the mutual fund theorem and the standard CAPM (i.e., CAPM with homogeneous beliefs). In particular, from Proposition 3.1 of Gerber and Hens (2017), if the heterogeneity is only about the covariances of returns then one can derive a CAPM alpha (i.e., return in excess of that predicted by the standard CAPM) for each asset i given by $\left(\bar{\beta}_i - \beta_i\right)(E[R_{market}] - R_f)$, where β_i is the OLS slope coefficient described earlier and $\bar{\beta}_i$ is the "average belief" about the market beta of asset i. This average belief reflects the weighted average of individual agents' subjective beliefs and the weights depend on agents' risk aversion and initial wealth.

However, the nature and substance of the prediction for CAPM alpha in the theory employed by the thesis and those in the theory of subjective beliefs just described are quite distinct. First, the relative ease of learning variances makes it implausible that subjective beliefs can explain the sustained CAPM alphas observed in data. Secondly, our theory can predict which assets will command a positive or negative CAPM alpha based purely on publicly observed returns data. In our theory, an asset i will command a premium (discount) if and only if $\beta_i^{Amb} - \beta_i > 0$ (respectively < 0); in the other theory, premium (discount) corresponds to $\bar{\beta}_i - \beta_i > 0$ (respectively < 0). Note, β_i^{Amb} is purely a function of Σ_M, whose elements may be estimated on publicly available data on returns. Hence, in our theory whether an asset has a premium or a discount can be determined, in principle, using only publicly observed data. On the other hand, estimation of $\bar{\beta}_i$ requires information on individual agents' subjective beliefs about covariances of returns as well as their risk preferences and wealth levels. There is no such publicly available information for a representative sample of investor population. Furthermore, the ambiguity-based theory generates the insight that the class of assets that command premia are those with relatively high exposure to systematic ambiguity. The insight may be falsified by data of course, but this is an insight which is distinct from what may emerge from the subjective beliefs theory. The empirical implications of this insight are examined in the following section.

Notice from (3.12) that, if $var^M(R_{market})$ were to go up relative to $var(R_{market})$ (without β_i^{Amb} and β_i changing), then the direct effect of this is that $|\alpha_i|$ increases for *all* assets. The bigger the ratio $\frac{var^M(R_{market})}{var(R_{market})}$, the bigger the share of ambiguity in the unconditional uncertainty about aggregate market returns. This suggests the departure from (standard) CAPM across all assets is more pronounced at times when the proportion of systematic ambiguity relative to the systematic unconditional uncertainty is greater.

Therefore, the two main take-aways from Proposition 1 are, one, that the cross-sectional heterogeneity in exposure to systematic ambiguity is a determinant for the variation in cross-sectional expected returns, and second, that the failure of CAPM is more pronounced in times of higher systematic ambiguity. These have empirical implications, as discussed next.

3.4 Discussion of Cross-Sectional Empirical Implications

In order to check if ambiguity has explanatory power for the systematic departures from CAPM one needs to test, first and foremost, if the CAPM alpha, α_i, has a positive association to $\beta_i^{Amb} - \beta_i$. A short cut approach to this, that does not require the estimation of Σ_M, is by proxying cross-sectional variation in exposure to systematic ambiguity with that in exposure to macroeconomic uncertainty (MEU) index developed by Jurado et al. (2015). In particular, stock returns can be regressed against this index and stocks with higher slope coefficients can be identified as those with higher exposure to macroeconomic uncertainty and therefore likely with higher exposure to systematic ambiguity. Hence, this slope coefficient can be used as a proxy for β_i^{Amb} for ranking (ordinal) purposes. The parameter β_i is the standard CAPM factor loading and can be estimated in the usual way, as the slope coefficient of an OLS regression where excess returns of asset i are regressed against the excess returns of the market portfolio over time. This testing approach is inspired by some intriguing findings in the recent empirical literature which associate exposure to macroeconomic uncertainty with a return discount rather than a premium. In particular, Bali et al. (2017) use the MEU index to sort portfolios according to β_i^{MEU}, the slope coefficient of the MEU index in an OLS regression in which excess portfolio returns are regressed against the said index as well as numerous well-known risk factors. They find that portfolio abnormal returns are decreasing in β_i^{MEU} and conclude that exposure to macroeconomic uncertainty generates a return discount. If the macroeconomic uncertainty were interpreted as macroeconomic ambiguity, then this finding seems to suggest that assets which are more exposed to systematic ambiguity should carry a lower abnormal return. However, Proposition 1 suggests that the correct ranking of abnormal returns due to exposure to systematic ambiguity obtains from the difference $\beta_i^{\mathrm{MEU}} - \beta_i$.

The empirical literature also established that even though there is a positive relation between an asset's CAPM beta and its average return, this relation (i.e., the security market line) is too flat in the data (Black et al. (1972) and Treynor and Black (1973)). That is, compared to what is predicted by the CAPM, the returns on the low beta assets are too high and the returns on the high beta assets are too low. Our theory is consistent with this observation, the so-called *beta anomaly*: a flatter security market line is obtained when low beta assets carry a positive ambiguity premium and high beta assets carry a negative ambiguity premium. This would happen if the cross-sectional variation in ambiguity beta were lower than the cross-sectional variation in beta so that ambiguity beta would be higher than beta for low beta assets and lower for high beta assets. In our framework, ambiguity beta is driven by exposure to aggregate shocks that affect models governing the return generation processes (more specifically, means of returns) whereas beta captures both such exposure and also exposure to aggregate shocks that affect return realizations given the models. Consider a recession: it implies a lower *mean return* for most firms due to its economy-wide and persistent effect, however *realized returns* depend on firm characteristics such as brand loyalty, disposable income of the client base and price elasticity of the product or service offered. Therefore, arguably, there is less cross-sectional variation in ambiguity betas compared to betas.

Next, consider the *size* and *value premia* documented in the literature critiquing CAPM (e.g., see Fama and French (1992) and (1993)). These premia refer to the observation that, controlling for CAPM betas, firms with small market capitalizations and high book-to-market equity ratios generate, on average, higher stock returns. There is no theoretical consensus on why these premia exist. It has been suggested that size premium may be partly an outcome of the fact that small firms are more dependent on outside financing, leaving them more exposed to business cycles and broader financial conditions (see, e.g., Perez-Quiros and Timmermann (2000)). When it comes to value premium, many associate it with financial distress risk since distressed firms tend to have high book-to-market equity ratios (see, e.g., Fama and French (1995) and Vassalou and Xing (2004)). These informal, uncertainty based explanations can be linked to our theory: arguably, ambiguity premium (hence, CAPM alpha) is larger for firms which are more dependent on outside financing and those which are financially distressed as such firms would have relatively more exposure to systematic ambiguity (i.e., they would likely have higher $\beta_i^{Amb} - \beta_i$). Why? Since stock returns are operating returns net of outside financing costs, exposure to uncertain financing costs driven by broad financial conditions makes them less predictable. Therefore, if the financing conditions worsen due to business or financial cycles, firms more reliant on outside financing, such as small and distressed ones, generate stock returns that are more ambiguous from the perspective of investors. The ambiguity is also systematic: dependence on outside financing implies higher exposure to economy-wide conditions, and therefore all small and distressed firms are commonly and strongly affected by cycles.

One may test whether ambiguity has explanatory power for the beta, size or value anomalies using the following empirical strategy. Start by constructing quantile portfolios sorted according to $\beta_i^{Amb} - \beta_i$. Then, within each such quantile, further sort stocks into portfolios ranked according to their CAPM betas, book- to-market ratios or market capitalizations. Sorting according to $\beta_i^{Amb} - \beta_i$ allows us to control for cross-sectional differences in exposure to systematic ambiguity. If, for instance, the difference between the abnormal returns of high and low book-to-market quantile stocks is significantly lower when exposure to systematic ambiguity is controlled for (i.e., within each $\beta_i^{Amb} - \beta_i$ quantile) compared to the case when this exposure is not controlled for, then one would have supporting evidence for ambiguity (at least, partially) explaining value premium.

Overall, the theoretical framework employed in this thesis predicts departures from CAPM are, on average, more pronounced at times when the perception of systematic ambiguity is high. Such perceptions, i.e., perceptions associated with macroeconomic ambiguity, are arguably higher than usual at turning points of business cycles, especially downturns. On the other hand, such perceptions can be exceptionally lower following events associated with revelation of news that significantly resolve incumbent macroeconomic ambiguity. The latter suggestion thus offers a possible explanation for the findings in Savor and Wilson (2014) that the standard CAPM predictions perform relatively better on days when news about inflation, unemployment, or Federal Open

Markets Committee interest rate decisions are announced than on non-announcement days.

Chapter 4

Data & Methodology

This chapter describes the data used in this thesis, which includes stock level variables (such as beta, size, book-to-market, operational profitability, investments, momentum), macro economic uncertainty index, FOMC meeting dates, analysts related data, institutional ownership, financial constructs (such as VXO beta, Whited-Wu index, etc.) and outlines the methodology, the empirical strategy and the rationale for the empirical results obtained.

4.1 Data

4.1.1 Stock Level Variables

The stock sample employed in the thesis includes all common stocks traded on the NYSE, AMEX and NASDAQ exchanges and it runs from August 1960 to May 2015. Sample selection period is made in conjunction with the period of macroeconomic uncertainty index (more in detail below) which is used to estimate ambiguity betas of individual stocks.

Stocks' market data (price, return, shares outstanding, etc.) are obtained from the CRSP database, while stocks' accounting data are from the CRSP/Compustat merged database. The thesis relies on Kenneth French's website for the definitions to construct Fama French factors of individual stocks.

Specifically, Size (or market equity) is price times shares outstanding (divided by 1,000,000) at month-end. Book-to-market is computed at the end of every June and held constant from June to May of next year. Book equity used in June of year t is the book equity for the last fiscal year end in $t-1$, whereas the market equity is price times shares outstanding at the end of December of $t-1$. Book equity is computed by adding deferred taxes and investment tax credit to, and by subtracting preferred stock from stockholder' equity. Operating profitability and investments are also calculated at the end of each June and are held constant from June to May of next year. Operating Profitability for June of year t is annual revenues minus cost of goods sold, SGA expenses, and interest expense divided by book equity for the last fiscal year end in $t-1$. Investment for June of year t is the change in total assets from the fiscal year ending in $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets. Momentum is the cumulative return of a stock, expressed as a percentage (0.5 is a 0.5% return; hence the cumulative return is multiplied by 100), during the 11-month period covering months $t-11$ through $t-1$.

Although monthly stock data is predominantly utilized, daily is also used for parts of the analysis in the thesis.

4.1.2 Macroeconomic Uncertainty Index

As a proxy of macroeconomic uncertainty, the macroeconomic uncertainty index (MEU) constructed by Jurado et al. (JLN) (2015) is employed. The index, obtained from Sydney Ludvigson's website, is a monthly time series and starts from July 1960, and for the stock sample in the thesis, the period runs from July 1960 to April 2015.

JLN (2015) regards uncertainty as the conditional volatility of the purely unforecastable component of the economic activity. The MEU index shows aggregate macro uncertainty level by incorporating a broad range of macroeconomic indicators such as real output and income, employment and hours, real retail, manufacturing and trade sales, consumer spending, housing starts, inventories and inventory sales ratios, orders and unfulfilled orders, compensation and labor costs, capacity utilization measures,

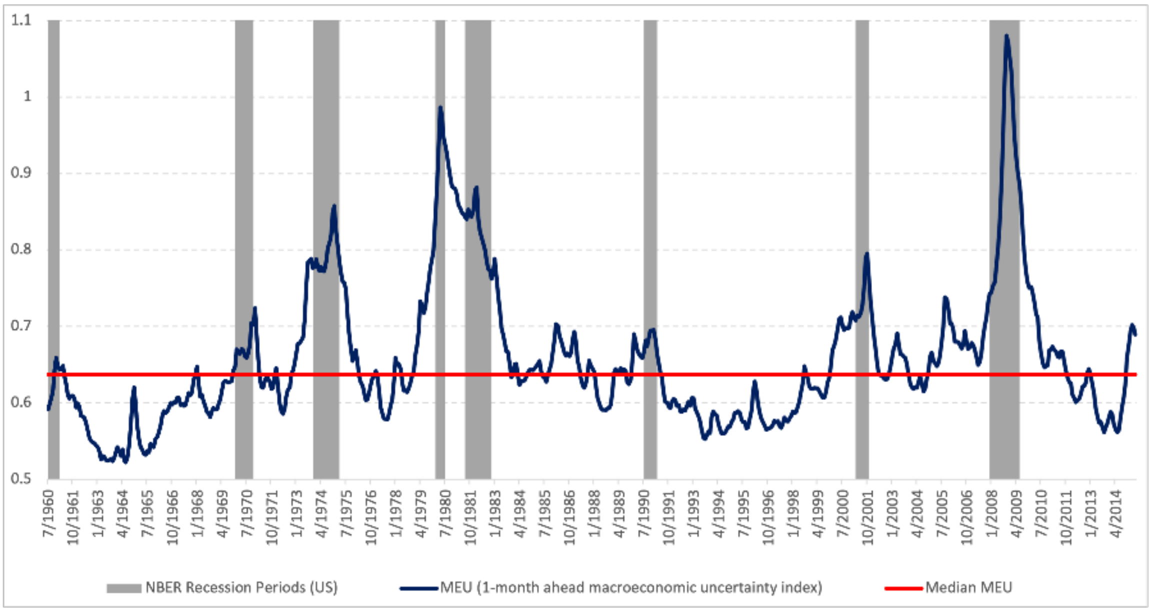

Figure 4.1: One-month ahead macroeconomic uncertainty index (MEU)

price indexes, bond and stock market indexes, and foreign exchange measures.

The MEU indicates h-period ahead uncertainty, and there are three separate indexes available with h being equal to 1-month, 3-months, and 12-months. 1-month ahead time series was utilized in the thesis.

4.1.3 Other Data

Equity analysts data, including number of analysts covering a stock and stock level analyst forecast dispersion, are obtained from Institutional Brokers' Estimate System (I/B/E/S) dataset and covers the period from January 1976 to May 2015.

Institutional Ownership is from Thomson-Reuters 13F Ownership data and is available from March 1980 onwards. For the stock sample in the thesis, data period runs from March 1980 to May 2015.

CBOE S&P100 Volatility Index (also called the 'VXO'), used for estimating VXO beta for individual stocks, is taken from FRED website, maintained by the Research Department at the Federal Reserve Bank of St. Louis. VXO starts from January 1986 and for the stock sample in the thesis runs until May 2015. As detailed in Ang et al.

(2006), VXO beta, which is estimated using VXO as a proxy for market volatility, denotes the sensitivity of a stock to innovations in aggregate volatility. Estimation is performed by regressing excess stock returns on the excess market returns and the changes in implied volatility using daily data in month t:

$$R_{i,d} - R_{f,d} = \alpha_{i,daily} + \beta_{i,daily}^{MKT} \left(R_{MKT,d} - R_{f,d}\right) + \beta_{i,daily}^{VXO} \Delta VIX_d + \epsilon_{i,d}. \tag{4.1}$$

$\beta_{i,daily}^{VXO}$ estimated by the above regression is the implied market volatility beta of stock i in month t (VXO beta for a particular month is calculated only for stocks with at least 15 days of return data in that month). Stocks' VXO betas are calculated for the period between January 1986 and May 2015.

In order to estimate the level of financial constraints/distress companies experience, Whited-Wu (2006) index is utilized. Whited-Wu's (WW) stock sample used for estimation covers the period from January 1975 to April 2001. The index, constructed using GMM estimation of an investment Euler equation, is given by the following for stock i at month t:

$$\begin{aligned} WW_{i,t} &= -0.091CF_{i,t} - 0.062DIVPOS_{i,t} + 0.021TLTD_{i,t} - 0.044LNTA_{i,t} \\ &\quad +0.102ISG_{i,t} + 0.035SG_{i,t}. \end{aligned} \tag{4.2}$$

In the above equation, CF is the ratio of cash flow to total assets; DIVPOS is a dummy with value equal to 1 if the firm pays cash dividends; TLTD is the ratio of the long-term debt to total assets; LNTA is the natural log of total assets; ISG is firm's 3-digit industry sales growth and SG is firm's sales growth. Stock level accounting data are obtained from CRSP/Compustat merged database. Regulated and financial firms are excluded from the sample, while all firms with missing firm-year observations, zero or negative total assets or sales and short term debt exceeding total debt are also dropped. A firm is included only if it has at least eight consecutive quarters of complete data and if it never has more than two quarters of negative sales growth in the last eight quarters. This last criterion is emphasized by WW, as they like to differentiate firms with external finance constraints from the ones in serious financial distress. The stock sample in the thesis with their WW-index computed according to above caveats runs from July 1978 to April 2001.

Illiquidity measure of Amihud (2002) for stock i in month t requires usage of daily data. The measure is the monthly average of the ratio of the daily absolute stock return to the daily trading volume in million dollars, that is:

$$ILLIQ_{i,t} = \frac{\sum_{i=1}^{N} \frac{(|R_{i,d}|)}{VOLD_{i,d}}}{N}, \tag{4.3}$$

where $R_{i,d}$ and $VOLD_{i,d}$ are the daily return and the trading volume in million dollars of stock i on day d, and N is the number of trading days of stock i in month t with return and volume data. For a particular month, only stocks with at least 15 trading days are included in the sample. Stock sample for the thesis with illiquidity measures computed is from July 1965 to May 2015.

FOMC meeting dates are obtained from Cieslak et al. (2019). For the purposes of this thesis, FOMC meeting dates between September 1982 and June 2015 are considered for the analysis.

4.2 Methodology

Backbone of the empirical tests in the thesis is the estimation of the ambiguity exposure for each stock in our sample. As detailed in the Theory chapter, ambiguity exposure of a stock involves two parameters, namely CAPM beta and ambiguity beta. Estimation of CAPM beta is quite standard in the literature, while for estimating ambiguity beta the thesis relies on the MEU index of JLN (2015). MEU seems to be a proper proxy for systematic ambiguity, as MEU as an aggregate entity reflecting episodes of varying sentiment/uncertainty for the economy as a whole for period(s) ahead should be a good measure for systematic ambiguity.

For each stock i and month $t-1$ in our sample, two separate rolling OLS regressions are run; one for CAPM beta by regressing excess stock returns on excess market

SUMMARY STATS

	Mean	Median	Min	Max	Stdev
MEU (Jul60-Apr15)	0.657	0.637	0.522	1.080	0.092
Excess Mkt. Ret.(Jul60-Apr15)	0.005	0.087	-0.232	0.161	0.044
β^{MEU} (Jul65-May15)*	0.067	0.051	-0.743	0.677	0.187
β^{CAPM} (Jul65-May15)*	0.702	0.698	0.0004	1.413	0.343
β^{MEU} - β^{CAPM} (Jul65-May15)*	-0.636	-0.653	-1.733	0.501	0.414

CROSS CORRELATIONS

	MEU	Exc.Mkt.Ret.
MEU (Jul60-Apr15)	1	-0.098
Excess Mkt.Return (Jul60-Apr15)	-0.098	1

	β^{MEU}	β^{CAPM}	β^{MEU} - β^{CAPM}
β^{MEU} (Jul65-May15)*	1	-0.144	0.571
β^{CAPM} (Jul65-May15)*	-0.144	1	-0.894
β^{MEU} - β^{CAPM} (Jul65-May15)*	0.571	-0.894	1

* Time series formed by cross-sectional means

Table 4.1: Summary statistics and cross-correlations

The table reports summary statistics and cross-correlations for the variables used in the estimation of systematic ambiguity exposure, namely macroeconomic uncertainty index (MEU), excess market portfolio return, MEU beta (β^{MEU}), CAPM beta (β^{CAPM}), as well as the systematic ambiguity exposure estimate itself, $\beta^{MEU} - \beta^{CAPM}$. The reported statistics and cross-correlations are the time-series means of the statistics and cross-correlations computed for the cross-section of stocks. The sample periods for each variable is reported in the table.

returns, and a second one by regressing excess stock returns on the MEU index of JLN:

$$R_{i,\tau} - R_{f,\tau} = \alpha_{i,t-1} + \beta_{i,t-1}^{CAPM}(R_{MKT,\tau} - R_{f,\tau}) + \epsilon_{i,\tau}, \tag{4.4}$$

$$R_{i,\tau} - R_{f,\tau} = a_{i,t-1} + \beta_{i,t-1}^{MEU} MEU_{\tau-1} + \eta_{i,\tau}, \tag{4.5}$$

where $\{\tau : \tau = t-60, \cdots, t-1\}$.Note that in equation (4.5), the excess stock return at time τ is regressed on the MEU value at $\tau - 1$, as the MEU index indicating one month ahead uncertainty is utilized.

Since the MEU index used for the thesis starts at July 1960 and ends at April 2015, MEU_{t-1} starts at August 1960 and runs until May 2015. Duly, excess stock and market returns are obtained from the CRSP from August 1960 to May 2015. The estimation procedure at each month-end uses only data available at that time. Eligible stocks at month-end $t-1$ are defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with at least 24 monthly return observations in the 60-month period $[t-60, t-1]$.

Using the slope coefficients in equations (4.4) and (4.5), we obtain the month-end systematic ambiguity exposures of all eligible stocks measured by $\beta_{i,t-1}^{MEU} - \beta_{i,t-1}^{CAPM}$ for month $t-1$. We are thus ready to sort stocks each month in our sample according to their ambiguity exposure. Sorting each month uses only data available at that time. The summary statistics and cross-correlations for the variables related to the estimation of systematic ambiguity exposure are reported in Table 4.1.

Note that, in order to empirically check the theory, one needs to test, first and foremost, whether the CAPM alpha α_i (which we refer to as stock i's ambiguity premium) has a positive association with $\beta_i^{MEU} - \beta_i^{CAPM}$. Sorting will serve ranking (ordinal) purposes such that stocks which are more sensitive to (macroeconomic) ambiguity will be checked whether they have a higher ambiguity premium. Consequently, at the end of each month t-1, stocks are ranked into quintile portfolios according to their $\beta_{i,t-1}^{MEU} - \beta_{i,t-1}^{CAPM}$, and the portfolio excess returns for the post-ranking month t are linked across months to form one time-series of post-ranking excess returns for each quintile portfolio. For post-ranking excess returns of quintile portfolios, both equal-weighted and value-weighted returns are computed. Yet, mostly the value-weighted dataset is used for the tests, to better reflect aggregate market behaviour.

Although monthly stock data is used for bulk of the empirical tests, daily stock data is also utilized to observe the relation between abnormal stock returns (i.e. α_i) and ambiguity exposure (i.e. $\beta_i^{MEU} - \beta_i^{CAPM}$) just days before and after major economic events such as FOMC meetings.

4.2.1 Empirical Strategy: What to Test, How to Test and Why

Regressing quintile portfolios' post-ranking excess returns (both equal weighted and value-weighted) on excess market returns and reporting the intercepts as estimated CAPM alphas is the first step. To justify higher the ambiguity exposure, higher the alpha in excess stock returns, one needs to see a monotonic increase in alphas from low ambiguity exposure quintile to high ambiguity exposure quintile. It will be important to see an economically and statistically significant positive alpha for high-minus-low quintile portfolio. In addition to alphas, average excess returns of quintiles (please refer to results displayed in Table 5.1 in Chapter 5) need to be reported.

The above procedure is repeated to estimate conditional CAPM alphas, as well as Fama-French 3 factor (FF3) and Fama-French 5 factor (FF5) alphas. The aim is to investigate whether the positive correlation between abnormal stock returns and ambiguity exposure is robust to factors other than excess market premium, such as those in conditional CAPM framework (i.e. lagged dividend yield, lagged term premium, lagged default premium and lagged risk-free rate interacting with excess market premium for time-varying beta), or size and Book-to-Market in FF3, and Asset Growth (Investment) and Operating Profitability as the additional two factors in FF5 (please refer to results displayed in Table 5.1 in Chapter 5).

In addition to testing the relation between alpha and ambiguity exposure at all times in the stock sample, check the relation separately at times of high MEU and low MEU in the CAPM and conditional CAPM frameworks. This will shed light on the strength of the relation during times of low and high uncertainty (please refer to results displayed in Table 5.2 in Chapter 5).

In the zoo of factors that are reported in the literature to affect cross-sectional stock returns, it is critical to measure how stocks' ambiguity exposure correlate with those factors. The factors specified are size, book-to-market, operating profitability, investment (asset growth), momentum, illiquidity, VXO beta, financial constraints/distress, institutional ownership, analyst dispersion, and number of analyst estimates. I report the time-series averages of the standardized slope coefficients from the Fama-Macbeth regressions of the ambiguity exposure on each factor separately, and all in one regression. Sensitivity of ambiguity exposure, if any, to the factor in question should make economic sense (please refer to results displayed in Table 5.4 in Chapter 5).

I check whether the power of ambiguity exposure in explaining stocks' abnormal excess returns (i.e. alpha) persists when a subset or the whole set of other factors which have explanatory power are controlled for in the same regression. Multiple regressions each having ambiguity exposure as one of regressors together with differing sets of explanatory variables are run without and with controlling for industry effect by assigning each stock to one of ten industries based on its four-digit SIC code (refer to results displayed in Table 5.5 in Chapter 5). This is again another robustness check for ambiguity exposure in explaining abnormal excess returns in the cross-section.

There may be periods during the year when macro or financial uncertainty rises. Typical of those are days of important macro/financial data releases, earnings announcement periods, days around FOMC meetings, episodes of looming recessions, etc. It is worthwhile to look closer to those periods when ambiguity aversion becomes an overriding theme in the markets and the link between abnormal returns and ambiguity exposure may be fortified.

As such, I investigate days around the FOMC meetings with the expectation to see the alpha difference between the highest and the lowest ambiguity exposure quintiles to markedly widen just days before the meeting, as ambiguity aversion among market players becomes more pronounced. It would not be surprising to see ambiguity aversion subside immediately after the meeting (please refer to results displayed in Table 5.3 in Chapter 5). Obviously, daily return data is used for the test. Quintile portfolios are formed according to ambiguity exposure of stocks in the preceding month and portfolio compositions are kept fixed throughout the days of the current month. Value weights are according to market capitalizations as of the end of the preceding month.

After the above tests, I use a proxy for ambiguity exposure as a separate explanatory factor for cross-sectional excess stock returns, in addition to CAPM beta. For this, I employ the Fama-Macbeth coefficients, which form a time series, from the regressions of CAPM alpha on ambiguity exposure. One should see alpha as an abnormal excess return disappear, when the factor for ambiguity exposure is controlled for in the CAPM regression (please refer to results displayed in Table 5.6 in Chapter 5).

As the final step, I investigate whether the factor for ambiguity exposure can resolve major cross-sectional asset anomalies such as beta, size, book-to-market, investment, operating profitability, and momentum. For the particular anomaly, say beta, I rank stocks into quintile portfolios according to their beta at the end of each month $t-1$ and the value-weighted portfolio excess returns for the post-ranking month t are linked across months to form one time-series of post-ranking excess returns for each quintile portfolio. Then I perform a two-stage process:

(i) I run CAPM and conditional CAPM for the returns of each quintile portfolio to report alphas for each as well as the CAPM beta,

(ii) I regress CAPM alpha time series for each quintile portfolio, obtained by deducting CAPM beta times the excess market risk premium from the excess returns of the quintile portfolio, on the factor for ambiguity exposure. I report the coefficient of the factor and the intercept (which I call *ambiguity alpha*).

One would expect to see the *ambiguity alpha* to be economically and statistically insignificant (please refer to results displayed in Table 5.6 in Chapter 5).

Chapter 5

Empirical Results I

This chapter is based on a paper which I am a co-author of, Mukerji, Ozsoylev, Savran and Tallon (2022), and it explores how exposure to systematic ambiguity is priced in the cross-section of stocks.

5.1 Ambiguity and Alphas

We first test the hypothesis that CAPM alphas, α, are increasing in exposure to systematic ambiguity measured by $\beta^{MEU} - \beta^{CAPM}$. To that end, we run the following CAPM regression for quintile portfolios sorted according to systematic ambiguity exposure

$$R_{p,t} - R_{f,t} = \alpha_p + \beta_p(R_{MKT,t} - R_{f,t}) + \epsilon_{p,t} \tag{5.1}$$

for the whole sample period. As can be seen from Table 5.1, CAPM alphas of quintile portfolios, α_p, $p = Low, 2, 3, 4, High$, are monotonically increasing in $\beta_p^{MEU} - \beta_p^{CAPM}$, in the most part. The only exception to this monotonic relationship is observed between value-weighted portfolios 4 and $High$, where the CAPM alpha difference becomes economically and statistically insignificant. However, *the ambiguity spread*, i.e., the difference between CAPM alphas of the Low and $High$ quintile value-weighted portfolios is strongly significant. In the case of value-weighted portfolios, the ambiguity spread is 40 basis points per month (with a Newey-West t-statistic of 2.35) and a similar estimate is obtained for the equal-weighted case.

In the same table, we examine the conditional CAPM alphas, where the CAPM beta coefficient is time-varying and dependent on business cycle predictors such as dividend yield ($DivYield$), term premium ($TermPrem$), default premium ($DefPrem$) and risk-free rate (R_f). In particular, we run the following conditional CAPM regression for the quintile portfolios for the whole sample period:

$$\begin{aligned} R_{p,t} - R_{f,t} &= \alpha_p^c + \beta_{0,p}(R_{MKT,t} - R_{f,t}) \\ &\quad + \beta_{1,p} R_{f,t-1} * (R_{MKT,t} - R_{f,t}) \\ &\quad + \beta_{2,p} DivYield_{t-1} * (R_{MKT,t} - R_{f,t}) \\ &\quad + \beta_{3,p} DefPrem_{t-1}(R_{MKT,t} - R_{f,t}). \\ &\quad + \beta_{4,p} TermPrem_{t-1} * (R_{MKT,t} - R_{f,t}) + \epsilon_{p,t}^c. \end{aligned} \tag{5.2}$$

The results are stronger both in terms of monotonicity of alphas and the size of the ambiguity spread (i.e., High-minus-Low alpha) in the case of conditional CAPM.

We estimate the Fama-French 3-factor and 5-factor alphas for quintile portfolios by running the following regression for the whole sample period:

$$R_{p,t} - R_{f,t} = \alpha_p^{FF} + \beta_p^{FF} X_t + +\epsilon_{p,t}^{FF}, \tag{5.3}$$

where X_t is the collection of market excess return, size factor (SMB) and book-to-market factor (HML) in month t in the case of the 3-factor model, with operating profitability (OP) and investment (INV) factors added to the collection in the case of the 5-factor model. Notice from the table that exposure to systematic ambiguity seems to be distinct from Fama-French factors size and book-to-market as an explanatory factor for abnormal returns. However, the results in the table suggest that there may be a relationship between ambiguity and the new Fama-French factors investment and operating profitability.

Portfolios sorted on β^{MEU} - β^{CAPM}	Panel A: Equal weighted					Panel B: Value weighted				
	Excess return	CAPM alpha	Conditional CAPM alpha	FF3 alpha	FF5 alpha	Excess return	CAPM alpha	Conditional CAPM alpha	FF3 alpha	FF5 alpha
Low	0.84% (2.28)	0.11% (0.58)	0.12% (0.55)	-0.17% (-1.18)	0.07% (0.33)	0.45% (1.53)	-0.23% (-2.07)	-0.23% (-2.09)	-0.24% (-2.37)	-0.13% (-0.95)
2	0.84% (2.98)	0.26% (1.85)	0.22% (1.58)	-0.01% (-0.14)	-0.02% (-0.15)	0.45% (2.03)	-0.09% (-1.27)	-0.13% (-1.94)	-0.13% (-1.89)	-0.21% (-2.91)
3	0.85% (3.51)	0.36% (2.91)	0.32% (2.65)	0.10% (1.58)	0.05% (0.72)	0.59% (3.19)	0.14% (2.76)	0.11% (2.53)	0.10% (2.14)	-0.01% (-0.17)
4	0.83% (3.75)	0.41% (3.43)	0.40% (3.42)	0.19% (2.92)	0.16% (2.59)	0.60% (3.40)	0.19% (2.66)	0.19% (2.88)	0.14% (2.15)	0.09% (1.00)
High	0.91% (3.44)	0.48% (2.79)	0.55% (3.10)	0.29% (2.67)	0.41% (3.40)	0.59% (2.93)	0.18% (1.66)	0.25% (2.08)	0.17% (1.54)	0.22% (1.43)
High - Low	0.07% (0.38)	0.36% (2.31)	0.43% (2.54)	0.46% (2.61)	0.34% (1.30)	0.14% (0.68)	0.40% (2.35)	0.47% (2.60)	0.41% (2.26)	0.35% (1.30)

Table 5.1: Alphas of portfolios sorted on systematic ambiguity exposure

At the end of each month $t-1$, eligible stocks are sorted into quintile portfolios according to their month-end historical ambiguity exposure measured by $\beta_{t-1}^{\mathrm{MEU},i} - \beta_{t-1}^{\mathrm{CAPM},i}$. The parameters $\beta_{t-1}^{\mathrm{CAPM},i}$ and $\beta_{t-1}^{\mathrm{MEU},i}$ are the slope coefficients from equations (4.4) and (4.5), respectively. The estimation and sorting procedure at each month-end uses only data available at that time. Eligible stocks at month-end $t-1$ are defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with at least 24 monthly return observations in the 60-month period $[t-60, t-1]$. The portfolio returns for the post-ranking month t are linked across months to form one time-series of post-ranking returns for each quintile. The columns present the post-ranking excess returns and alphas for the equal-weighted and value-weighted quintile portfolios, in percent per month. The alphas are estimated as intercepts from the regressions of excess portfolio post-ranking returns on excess market returns, on conditional CAPM factors (i.e. market return, product of market return and lagged risk-free rate, product of market return and lagged dividend yield, product of market return and lagged default premium, product of market return and lagged term premium), on the Fama-French 3-factor returns, and on the Fama-French 5-factor returns. The last row presents excess return and alpha differences between quintile 5 (High) and quintile 1 (Low). The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation of excess returns and alphas is August 1965–June 2015.

The ranking parameter for exposure to systematic ambiguity, $\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}$, depends on the MEU beta and the CAPM beta and one may wonder whether the difference is determined solely by the latter, i.e., the CAPM beta. Table 5.4 shows that when $\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}$ is regressed against $\beta_{i,t}^{MEU}$ and $\beta_{i,t}^{CAPM}$ separately, both betas individually have similar explanatory powers over the ranking parameter $\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}$. In particular, the standardized coefficient for $\beta_{i,t}^{MEU}$ is 0.58 with a Newey-West t-statistic of 23.96 and that of $\beta_{i,t}^{CAPM}$ is -0.73 with a Newey-West t-statistic of -36.51. That is, the variation in our ranking parameter is affected by both MEU beta and CAPM beta, and in comparable measure.

5.2 Dynamics of Ambiguity Pricing

Having demonstrated the strong relation between systematic ambiguity exposure and CAPM alphas, we proceed by examining how the said relation is dynamically affected by the macroeconomic uncertainty (MEU) level. As discussed in Section 3.4, our theoretical framework predicts that departures from CAPM are, on average, more pronounced at times when the perception of systematic ambiguity is high. Therefore we expect to see higher CAPM alphas associated with systematic ambiguity exposure during times of high macroeconomic uncertainty. To test this, we first divide the months into those with MEU levels lower than the time-series median MEU (Low MEU) and those with MEU's higher than the time-series median MEU (High MEU). Then we run the following monthly CAPM regressions for the quintile portfolios sorted according to systematic ambiguity exposure for the whole sample period:

$$
\begin{aligned}
R_{p,t} - R_{f,t} &= \alpha_{p,L} + \kappa_p H_t + \beta_p (R_{MKT,t} - R_{f,t}) + \epsilon_{p,t}, && (5.4)\\
R_{p,t} - R_{f,t} &= \alpha_{p,H} + \lambda_p L_t + \bar{\beta}_p (R_{MKT,t} - R_{f,t}) + \eta_{p,t}, && (5.5)
\end{aligned}
$$

where H_t is a dummy variable that is equal to 1 if month t is a High MEU month and L_t is a dummy variable that is equal to 1 if month t is a Low MEU month. $\alpha_{p,L}$ and $\alpha_{p,H}$ are quintile p's CAPM alphas for Low MEU and High MEU periods, respectively.

By making similar adjustments to the conditional CAPM regression equations with dummy variables, we also obtain conditional CAPM alphas for High MEU and Low MEU periods separately. Table 5.2 reports the regression results. As expected, the High MEU CAPM alphas are higher than Low MEU CAPM alphas for almost all quintile portfolios sorted on systematic ambiguity exposure. The same is true for the High-minus-Low CAPM alpha and the High-minus-Low conditional CAPM alpha.

As a further test of the hypothesis that CAPM alphas associated with systematic ambiguity exposure are higher during times of higher uncertainty, we also examine the time-variation in CAPM alphas around the FOMC meeting dates. To that end, we run the following daily CAPM regressions for the quintile portfolios for the whole sample period:

$$\begin{aligned} R_{p,t} - R_{f,t} &= \alpha_{p,W} + \gamma_p NW_t + \beta_{0,p}(R_{MKT,t} - R_{f,t}) \\ &\quad + \beta_{1,p}(R_{MKT,t-1} - R_{f,t-1}) \\ &\quad + \beta_{2,p}\frac{(R_{MKT,t-2} - R_{f,t-2}) + (R_{MKT,t-3} - R_{f,t-3}) + (R_{MKT,t-4} - R_{f,t-4})}{3} \\ &\quad + \epsilon_{p,t}, \end{aligned} \tag{5.6}$$

where W denotes the time window of interest and NW_t is a dummy variable that is equal to 1 if day d does not belong to the time window W. In particular, we examine the 3-day period before the FOMC meeting, the day of the FOMC meeting and the 3-day period after the FOMC meeting as time windows of interest. $\alpha_{p,W}$ is quintile p's CAPM alpha for the time window W. As daily returns are used for the regression, we include both current and lagged market excess returns in the regression equation following Dimson (1979). Table 5.3 reports the regression results. Once again, as expected, the CAPM alphas are statistically significant and highest in magnitude during the 3-day period before the FOMC meeting for almost all quintile portfolios and the alphas vanish, in both economic and statistical significance, in the 3-day period after the FOMC meeting. The same is true for the dynamics of High-minus-Low CAPM alpha and the pattern is more pronounced during times of high macroeconomic uncertainty (High MEU).

	VW portfolios sorted on β^{MEU} - β^{CAPM}	Conditional on MEU quintiles within above specified window: Low 50% MEU	High 50% MEU
CAPM alpha	*Low*	-0.22% (-1.49)	-0.24% (-1.51)
	2	-0.14% (-1.86)	-0.04% (-0.35)
	3	0.07% (1.07)	0.21% (2.93)
	4	0.13% (1.84)	0.25% (2.20)
	High	0.10% (0.67)	0.26% (1.94)
	High - Low	0.32% (1.40)	0.49% (2.04)
Cond. CAPM alpha	*High - Low*	0.38% (1.67)	0.58% (2.27)

Table 5.2: Alphas of portfolios sorted on systematic ambiguity exposure in relation to levels of macroeconomic uncertainty (MEU)

The table displays the CAPM and conditional CAPM alphas of quintile portfolios conditional on macroeconomic uncertainty (MEU) levels lower than and higher than the time-series median MEU. To run CAPM and conditional CAPM regressions of value-weighted quintile portfolios conditional on two distinct MEU levels, regression equations involve dummies for each MEU level as controls, in addition to CAPM and conditional CAPM factors. High-Low rows present alpha differences between quintile 5 (High) and quintile 1 (Low). Only High-Low row is presented for conditional CAPM alphas for conciseness. All alphas are in percent per month. The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation of alphas is August 1965–May 2015.

Ai and Bansal (2018) show that stock returns realized around pre-scheduled macroeconomic announcements, such as the employment report and the FOMC statements, account for 55% of the equity premium during the 1961-2014 period, and virtually 100% of it during the later period of 1997-2014, where more announcement data are available. They also theoretically establish that such an announcement premium identifies a significant deviation from expected utility and constitutes a market based evidence for a large class of non-expected models that features aversion to "Knightian uncertainty". The results reported in Table 5.3 are a novel contribution to the literature as they establish a similar dynamic pattern and announcement premium for the cross-section of stocks and they also help strengthen the relationship between the macroeconomic announcement premia and ambiguity (uncertainty) aversion by sorting the test portfolios according to systematic ambiguity exposure.

5.3 Ambiguity and Stock Characteristics

In this section, we examine the relation between a stock's systematic ambiguity exposure and its average characteristics based on the Fama and MacBeth (1973) cross-sectional regressions. Monthly cross-sectional regressions are run according to the following specification:

$$\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM} = a_{1,t} C_{i,t} + \epsilon_{i,t}, \tag{5.7}$$

where $\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}$ is the systematic ambiguity exposure of stock i in month t and $C_{i,t}$ is either one of or a collection of variables specific to stock i observable in month t (CAPM beta, MEU beta, VXO beta, size, book-to-market, operating profitability, investment, momentum, illiquidity, financial constraints/distress, number of analysts providing an earnings estimate, analyst dispersion and institutional ownership). Table 5.4 reports the time-series averages of the standardized slope coefficients from the above regressions.[1]

[1]Note that we drop the intercept in the regression equation (5.7), as the OLS estimate of the intercept is always zero if all the variables are standardized in the regression.

VW* portfolios sorted on β^{MEU} - β^{CAPM}	All Days	3-Day window preceding FOMC	FOMC day	3-Day window after FOMC	Other Days
Low	-0.010% (-1.68)	-0.080% (-4.96)	0.010% (0.31)	-0.004% (-0.25)	-0.002% (-0.35)
2	-0.000 (-0.04)	-0.009% (-0.92)	0.014% (0.78)	0.007% (0.65)	-0.000 (-0.12)
3	0.012% (3.46)	0.027% (2.81)	0.021% (1.32)	0.010% (0.99)	0.009% (2.51)
4	0.012% (3.01)	0.026% (2.47)	0.026% (1.40)	0.004% (0.30)	0.010% (2.36)
High	0.013% (1.99)	0.033% (1.64)	0.057% (1.94)	0.020% (1.05)	0.009% (1.12)
High - Low	0.023% (2.16)	0.113% (3.79)	0.047% (0.98)	0.024% (0.78)	0.011% (0.89)
High - Low (High MEU)	0.023% (1.36)	0.148% (3.16)	0.039% (0.49)	-0.015% (-0.28)	0.011% (0.57)
High - Low (Low MEU)	0.024% (1.86)	0.079% (2.21)	0.051% (0.98)	0.062% (1.92)	0.012% (0.80)

* by end-of-month MCaps

Table 5.3: Alphas of portfolios sorted on systematic ambiguity exposure in relation to FOMC meeting announcements

The table displays the CAPM alphas of quintile portfolios for time windows on and around the FOMC meeting announcement days. To run CAPM regressions of value-weighted quintile portfolios conditional on the said time windows, regression equations involve dummies for each time window as controls, in addition to CAPM and conditional CAPM factors. High-Low rows present alpha differences between quintile 5 (High) and quintile 1 (Low). High-Low alpha differences are also computed for the same time windows conditional on the MEU level being lower than or higher than the median MEU. All alphas are in percent per day. The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation of alphas is September 1982–May 2015.

Arguably, cyclical and financially constrained/distressed firms have higher exposure to systematic ambiguity. As discussed in Section 3.4, small firms are more dependent on outside financing thus more exposed to business cycles (Perez-Quiros and Timmermann (2000)) and financially distressed firms tend to have high book-to-market equity ratios (Fama and French (1995) and Vassalou and Xing (2004)). This is consistent with the average slope coefficient on size being significantly negative and average slope coefficients on book-to-market and financial constraints/distress being significantly positive in the table.

Lack of transparency would also likely increase a stock's exposure to systematic ambiguity. Higher institutional ownership is associated with greater management disclosure and analyst following, resulting in higher transparency and information production (Boone and White (2015)). Similarly, Lang et al. (2012) find strong positive association between liquidity (as measured by lower bid-ask spreads and fewer zero-return days) and transparency (as measured by less evidence of earnings management, better accounting standards, higher quality auditors, more analyst following, and more accurate analyst forecasts). The table reports the average slope coefficients on number of analyst estimates and institutional ownership to be significantly negative whereas the slope coefficient on illiquidity to be significantly positive. These findings are as expected in light of the discussion above.

Overall, the table indicates that stocks with high systematic ambiguity exposure are small, illiquid, financially constrained or distressed, followed by a small number of analysts and that they have high book-to-market equity ratios, low investment characteristics (i.e, low asset growth), less disagreement among analysts following them and low institutional ownership. The last column in the table shows that when all variables are included simultaneously, the relations between systematic ambiguity exposure and most of the stock characteristics become weaker. The characteristics that remain significantly associated with systematic ambiguity exposure are size and institutional ownership after controlling for all other variables.

$\beta^{MEU} - \beta^{CAPM}$ regressed on	β^{CAPM}	β^{MEU}	VXO_beta	Size	B/M	OP	INV.	MOM.	LIQ.	FIN.DIST.	AN.EST.	AN.DISP.	INST.OWN.	ALL
β^{CAPM}	-0.729													-
	(-36.51)													
β^{MEU}		0.576												-
		(23.96)												
VXO_beta			-0.010											-0.003
			(-3.56)											(-0.33)
Size				-0.067										-0.120
				(-5.83)										(-3.66)
Book-to-Market					0.068									0.035
					(5.40)									(1.32)
Operating profitability						0.002								0.037
						(0.61)								(2.62)
Investment							-0.035							-0.011
							(-3.70)							(-0.68)
Momentum								0.042						-0.005
								(1.67)						(-0.09)
Illiquidity									0.015					0.008
									(3.45)					(0.55)
Financial constraint/distress										0.094				-0.055
										(6.74)				(-1.60)
Number of Analyst Estimates											-0.058			0.010
											(-6.74)			(0.56)
Analyst dispersion												-0.028		-0.020
												(-6.43)		(-1.60)
Institutional ownership													-0.123	-0.067
													(-15.19)	(-5.74)
Data time span	Jul65-May15	Jul65-May15	Jan86-May15	Jul65-May15	Jul65-May15	Jul65-May15	Jul65-May15	Jul65-May15	Jul65-May15	Jul78-Apr01	Jan76-May15	Jan76-May15	Mar80-May15	Jan86-Apr01
Total Obs. (firm-month)	2,939,508	2,939,508	2,107,299	2,934,594	2,172,402	2,172,349	2,441,222	2,925,551	2,371,497	97,898	1,451,565	1,200,162	2,235,164	45,069
Obs. (average firms/month)	4,907	4,907	5,970	4,899	3,627	3,627	4,075	4,884	3,959	357	3,069	2,537	5,284	245
Adjusted R^2	56.73%	38.35%	0.18%	1.68%	1.93%	0.11%	0.97%	6.35%	0.23%	1.80%	0.85%	0.21%	1.92%	16.46%

Table 5.4: Systematic ambiguity exposure and stock characteristics

Assembled using the procedure detailed in Table 5.1, the panel data of eligible stocks, with their month-end systematic ambiguity exposure measured by $\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}$ for month t, is employed to measure the sensitivity of each stock's ambiguity exposure to various stock-level characteristics; namely, CAPM beta (β^{CAPM}), MEU beta (β^{MEU}), VXO beta, Size, Book-to-Market, Operating Profitability, Investment, Momentum, Illiquidity, Financial Constraints/Distress, Number of Analyst Estimates, Analyst Dispersion and Institutional Ownership. The table reports the time-series averages of the standardized slope coefficients from the Fama-Macbeth regressions of the ambiguity exposure on each characteristic (equation (5.7)). The rightmost column displays the average slope coefficients from the Fama Macbeth regression of the ambiguity exposure on the entire set of characteristics. The Newey-West adjusted t-statistics are given in parentheses. The sample period is typically July 1965–May 2015, although it differs for some characteristics due to limitations of the data.

The characteristics that are significantly positively associated with systematic ambiguity exposure are known to have high average excess returns and CAPM alphas. In particular, small, high book-to market (Fama and French (1992, 1993)) and illiquid (Amihud (2002)) stocks with low investment characteristics (Fama and French (2015) and Hou at al. (2015)) and low analyst dispersion (Diether et al. (2002)) generate high one-month ahead excess returns and alphas. This suggests that the alpha associated with high systematic ambiguity exposure may be due to the interaction between the latter and the aforementioned stock characteristics. In the next section, we test whether the positive relation between the systematic ambiguity exposure and the CAPM alpha still holds once we control for these characteristics in Fama-MacBeth cross-sectional regressions.

5.4 Stock Level Cross-Sectional Regression Results

In our tests examining the significance of systematic ambiguity exposure as a determinant of CAPM alphas, the analysis has been at the portfolio level so far. However, the portfolio-level analysis does not utilize all available information in the cross-section due to aggregation and it is difficult to control for a multitude of stock characteristics simultaneously at the portfolio level. Consequently, we now examine the cross-sectional relation between systematic ambiguity exposure and CAPM alphas at the stock level using the Fama and MacBeth (1973) regressions.

In our analysis, we first estimate stock i's month $t+1$ CAPM alpha as

$$CAPM\alpha_{i,t+1} \;=\; (R_{i,t+1} - R_{f,t+1}) - \beta_{i,t}^{CAPM}(R_{MKT,t+1} - R_{f,t+1}), \qquad (5.8)$$

where $\beta_{i,t}^{CAPM}$ is the CAPM beta of stock i in month t. We then run a monthly cross-sectional regression according to the following specification:

$$CAPM\alpha_{i,t+1} \;=\; \theta_{0,t} + \theta_{1,t}(\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}) + \theta_{2,t}C_{i,t} + \epsilon_{i,t+1}, \qquad (5.9)$$

where $\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}$ is the systematic ambiguity exposure of stock i in month t

and $C_{i,t}$ is a collection of variables specific to stock i observable in month t (VXO beta, size, book-to-market, operating profitability, investment, momentum, illiquidity, financial constraints/distress, number of analysts providing an earnings estimate, analyst dispersion and institutional ownership). We also determine the significance of systematic ambiguity exposure in the explanation of excess returns by running monthly cross-sectional regressions according to the following specification:

$$R_{i,t+1} - R_{f,t+1} \;=\; \mu_{0,t} + \mu_{1,t}(\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}) + \eta_{i,t+1}, . \qquad (5.10)$$

Table 5.5 reports the time-series averages of the slope coefficients from the above regressions. The average slopes provide standard Fama-MacBeth tests for determining which explanatory variables on average have non-zero premiums.

The univariate regression result reported in the second column of Table 5.5 indicates a positive and statistically significant relation between systematic ambiguity exposure and CAPM alphas. The average slope from the monthly regressions of $CAPM\alpha_{i,t+1}$ on $\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}$ alone is 0.0061 with a Newey-West t-statistic of 6.27. As can be seen from the third to sixth columns of the table, the average slope coefficients on systematic ambiguity exposure are positive, highly significant, and have similar magnitudes regardless of the set of stock-level characteristics (or factors) controlled for.

In the last six columns of the table, we control for the industry effect. In each month, each stock is assigned to one of the 10 industries based on the four-digit SIC code and we replicate our earlier stock-level cross-sectional regressions now controlling for the industry effect. We observe from the table that the industry effect has almost no influence on the economic significance of the cross-sectional relation between systematic ambiguity exposure and CAPM alphas. In all regression specifications except for the one where all stock-level characteristics are controlled for, the average slope coefficients on systematic ambiguity exposure remain positive and highly significant.

Fama-Macbeth regressions	WITHOUT CONTROLLING FOR INDUSTRY EFFECT						CONTROLLING FOR INDUSTRY EFFECT					
	Rexcess regressed on	CAPM alpha regressed on					Rexcess regressed on	CAPM alpha regressed on				
	(β^{MEU} - β^{CAPM})	(β^{MEU} - β^{CAPM})	(FF5)	(FF5+MOM)	(ALL excl. WW-index)	(ALL)	(β^{MEU} - β^{CAPM})	(β^{MEU} - β^{CAPM})	(FF5)	(FF5+MOM)	(ALL excl. WW-index)	(ALL)
β^{MEU} - β^{CAPM}	0.0004	0.0061	0.0059	0.0058	0.0066	0.0056	0.0005	0.0060	0.0058	0.0057	0.0062	0.0048
	(0.44)	(6.27)	(5.43)	(5.01)	(5.55)	(2.08)	(0.66)	(6.09)	(5.43)	(5.00)	(5.44)	(1.75)
VXO_beta					-0.0033	-0.0067					-0.0031	-0.0081
					(-1.90)	(-1.04)					(-1.93)	(-1.08)
Size			-0.0014	-0.0015	-0.0004	-0.0013			-0.0014	-0.0014	-0.0003	-0.0017
			(-3.15)	(-3.35)	(-0.83)	(-0.76)			(-3.37)	(-3.53)	(-0.69)	(-0.86)
Book-to-Market			0.0017	0.0018	0.0012	-0.0005			0.0020	0.0021	0.0014	0.0001
			(3.27)	(3.50)	(1.93)	(-0.17)			(4.59)	(5.02)	(3.09)	(0.03)
Operating profitability			0.0005	0.0004	0.0011	-0.0027			0.0003	0.0003	0.0011	-0.0066
			(0.97)	(1.06)	(2.62)	(-0.47)			(0.83)	(0.97)	(2.72)	(-1.02)
Investment			-0.0039	-0.0041	-0.0033	-0.0096			-0.0041	-0.0042	-0.0032	-0.0099
			(-6.19)	(-6.82)	(-5.80)	(-2.23)			(-6.64)	(-7.21)	(-6.07)	(-2.20)
Momentum				0.00004	0.00004	0.0001				0.00004	0.00003	0.0001
				(2.25)	(1.25)	(2.82)				(1.93)	(0.99)	(2.70)
Liquidity					0.0003	0.0019					0.0001	0.0024
					(0.35)	(0.72)					(0.12)	(0.67)
Leverage					-0.0001	0.0030					-0.00004	0.0020
					(-1.24)	(1.22)					(-1.05)	(0.73)
Financial constraint/distress (WW-index)						-0.0251						-0.02
						(-0.74)						(-0.49)
Institutional ownership					0.0005	-0.0010					0.0006	-0.0007
					(0.24)	(-0.14)					(0.32)	(-0.10)
Analyst dispersion					-0.0010	-0.0142					-0.0010	-0.0244
					(-2.96)	(-1.60)					(-3.20)	(-2.58)
Intercept	0.0089	0.0056	0.0131	0.0118	0.0057	0.0068	0.0089	0.0059	0.0124	0.0111	0.0034	0.0136
	(3.85)	(2.49)	(3.36)	(3.16)	(1.11)	(0.72)	(3.67)	(2.48)	(3.33)	(3.08)	(0.69)	(1.31)
Data time span	Jul65-May15	Jul65-May15	Jul65-May15	Jul65-May15	Jan86-May15	Jan86-Apr01	Jul65-May15	Jul65-May15	Jul65-May15	Jul65-May15	Jan86-May15	Jan86-Apr01
Total Obs. (firm-month)	2,939,508	2,939,508	2,105,706	2,100,389	787,645	45,104	2,722,084	2,722,084	2,097,168	2,092,955	787,188	45,081
Obs. (average firms/month)	4907	4907	3515	3506	2231	245	4544	4544	3501	3494	2230	245
Adjusted R^2	1.10%	1.58%	3.56%	4.38%	5.31%	7.26%	3.05%	3.55%	5.18%	5.87%	7.98%	9.26%

Table 5.5: Fama-Macbeth cross-sectional regressions

This table reports the time series averages of slope coefficients obtained from cross-sectional Fama Macbeth regressions of one-month-ahead CAPM alphas on select subsets of stock characteristics used in Table 5.4, with and without controlling for industry effect (equation (5.9)). For both with and without industry effect panels, the first columns display the time series averages of slope coefficients from the Fama-Macbeth regressions of one-month-ahead excess returns on ambiguity exposure (equation (5.10)), unlike other columns where the dependent variable of the Fama Macbeth regressions is one-month-ahead CAPM alpha. The Newey-West adjusted t-statistics are given in parentheses. The sample period is July 1965–May 2015, except for the columns where almost all characteristics are used as controls.

5.5 A (Pseudo) Factor for Pricing Ambiguity

Our analysis so far demonstrates that systematic ambiguity exposure is a significant determinant of CAPM alphas. As we know from Proposition 1 in Chapter 3, our theoretical framework predicts that equity premium (i.e., expected market portfolio excess return) is the only factor for pricing systematic risk as well as systematic ambiguity. However, a pseudo factor, which captures the portion of CAPM alpha associated with systematic ambiguity exposure, can be generated. To do so, we once again estimate stock i's month $t+1$ CAPM alpha as

$$CAPM\alpha_{i,t+1} \;=\; (R_{i,t+1} - R_{f,t+1}) - \beta_{i,t}^{CAPM}(R_{MKT,t+1} - R_{f,t+1}),$$

where $\beta_{i,t}^{CAPM}$ is the CAPM beta of stock i in month t. We then run the following monthly cross-sectional regression:

$$CAPM\alpha_{i,t+1} \;=\; \delta_{0,t+1} + \delta_{1,t+1}(\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}) + \epsilon_{i,t+1}, \tag{5.11}$$

where $\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}$ is the systematic ambiguity exposure of stock i in month t. The estimated slope coefficient, $\hat{\delta}_{1,t}$, is the premium associated with systematic ambiguity exposure in month t and we use its time series as our monthly pseudo factor capturing the *price of ambiguity* (or, to be more precise, adjustment to the CAPM formula for exposure to systematic ambiguity).

We can use the pseudo factor for ambiguity, namely $\hat{\delta}_{1,t}$, as a separate explanatory variable for cross-sectional variation in individual stock returns. Specifically, we propose a 2-stage regression process to price a test portfolio p. First, run the following rolling time-series regression for portfolio p and months $\tau = t-60, \cdots, t-1$

$$R_{p,\tau} - R_{f,\tau} = \alpha_{p,t-1} + \beta_{p,t-1}^{CAPM}(R_{MKT,\tau} - R_{f,\tau}) + \epsilon_{p,\tau} \tag{5.12}$$

in order to estimate CAPM beta of portfolio p in month $t-1$. Then use the following equation to estimate the CAPM alpha for portfolio p in month t:

$$CAPM\alpha_{p,t} = (R_{p,t} - R_{f,t}) - \beta_{p,t-1}^{CAPM}(R_{MKT,t} - R_{f,t}). \tag{5.13}$$

Finally, in the second regression stage, run the following monthly time-series regression:

$$CAPM\alpha_{p,t} = \alpha_p^{2S} + \lambda_p^{2S}\,\hat{\delta}_{1,t} + \eta_{i,t}, \tag{5.14}$$

where $\hat{\delta}_{1,t}$'s are the estimated slope coefficients of monthly cross-sectional regressions (5.11).

The quintile portfolios sorted according to systematic ambiguity exposure and used in Sections 3.1 and 3.2 constitute a natural testing ground for our proposed 2-stage pricing model. To the extent that the proposed model works well in pricing a test portfolio p, the residual α_p^{2S} from the 2-stage regression process should be close to 0. Table 5.6 reports the estimated CAPM alphas and betas as well as the estimated residual α_p^{2S} and slope coefficient λ_p^{2S} from the 2-stage pricing model for the said quintile portfolios. The residual from the 2-stage pricing model is statistically insignificant for virtually all quintile portfolios. Also, when the High-minus-Low CAPM alpha is compared to the High-minus-Low residual for the 2-stage model, we notice a sharp drop in both statistical significance and economic significance: the High-minus-Low CAPM alpha is statistically significant and 40 basis points per month whereas the High-minus-Low residual for the 2-stage model is 1 basis point per month and statistically insignificant.

VW portfolios sorted on β^{MEU} - β^{CAPM}	**Two-stage Process:** i) CAPM regression ii) Regress CAPM alpha on the ambiguity exposure (i.e. β^{MEU} - β^{CAPM}) factor CAPM alpha	Alpha (2^{nd} factor) (Intercept in the 2nd regr.)	Beta	Lambda (Coef. of amb. exp. factor)
Low	-0.225% (-2.07)	0.041% (0.38)	1.364 (33.66)	-0.434 (-5.48)
2	-0.090% (-1.27)	0.001% (0.02)	1.096 (39.34)	-0.148 (-2.69)
3	0.138% (2.76)	0.097% (1.90)	0.922 (39.91)	0.067 (1.24)
4	0.190% (2.66)	0.080% (1.44)	0.828 (31.73)	0.180 (2.85)
High	0.177% (1.66)	0.032% (0.33)	0.832 (20.95)	0.238 (2.54)
High - Low	0.403% (2.35)	-0.009% (-0.06)	-0.533 (-7.46)	0.672 (4.32)

Table 5.6: A pseudo factor for pricing ambiguity

This table reports CAPM alphas together with intercepts and coefficients if CAPM alphas of quintiles are regressed on a proposed factor representing ambiguity exposure. Monthly cross-sectional regressions of CAPM alphas on ambiguity exposure (measured by $\beta_(i,t)^{MEU} - \beta_(i,t)^{CAPM}$) are run for all eligible stocks (NYSE, AMEX and NASDAQ) at time t; slope coefficients $\delta_{1,t}$ form a time series to be used as a factor to represent ambiguity (equation (5.11)). Stocks are sorted into quintile portfolios according to their month-end systematic ambiguity exposure and we run a CAPM regression for each quintile portfolio (equation (5.12)). By using (equation (5.13)), we obtain CAPM alpha time series for each quintile. We then regress CAPM alphas on the proposed ambiguity factor (i.e. $\delta_{1,t}$) and report the intercept and the coefficient for each quintile (equation (5.14)). The Newey-West adjusted t-statistics are given in parentheses. The sample period is July 1965–May 2015.

Chapter 6

Empirical Results II

6.1 Ambiguity Pricing and Anomalies

This chapter reports the empirical results of the research on the power of ambiguity to explain major cross-sectional asset pricing anomalies. The research looks into beta, size, book-to-market, operating profitability, investment and momentum from major anomalies, and investigates whether ambiguity when used as a factor to explain abnormal excess returns at the cross-section resolves those anomalies.

6.2 Bottom Line and Discussion

The empirical results show that ambiguity when used as a factor to explain abnormal excess returns at the cross-section fully explains beta anomaly, while providing a partial reduction in abnormal excess returns observed in operating profitability and momentum anomalies. The results have not produced a favorable outcome in size, book-to-market and investment anomalies; that is, ambiguity did not drive down alphas neither economically nor statistically when used as a separate factor in the case of those anomalies.

In the case of beta anomaly, high minus low quintile CAPM alpha which stood at a negative 45 bps per month was slashed to negative 13 bps with no statistical significance, when CAPM alpha is regressed on the ambiguity factor (see Table 6.1).

As implied by the theory in this thesis, low beta stocks have a higher ambiguity exposure and ambiguity premium, unlike high beta stocks' ambiguity discount due to lower ambiguity exposure. Empirical results validate the theory's expectation.

For operating profitability (OP) and momentum anomalies (see Tables 6.5 and 6.6, respectively), use of ambiguity factor as a control variable for CAPM alphas of high minus low quintiles visibly reduces economic significance of alphas, yet, statistical significance persists for the case of momentum, and weakens albeit marginally falling above 5% significance level in the OP case.

To better understand the nature of anomalies and the corresponding factors and make sense of the findings, the following exercise is devised: the entire stock sample between 1965-2015 is split into three categories according to size (i.e. market capitalization) at the end of each month; micro/small caps is the lowest 30%, mid caps are in 30%-70% of the distribution, and big caps occupy the highest 30%.

Under each stock size category, stocks are divided into quintiles at the end of each month according to the specific factor and the portfolio returns for the post-ranking month are linked across months to form one time-series of post-ranking (value-weighted) excess returns for the high-minus-low quintile (also called the hedge portfolio). Five different dummies to denote different macro uncertainty levels (based on the MEU series) are created for the months of the same time series. Finally, CAPM is run for each high-minus-low quintile portfolio of the specific factor for each of the five uncertainty levels (i.e. by using four dummies as controls for the remaining uncertainty levels) to identify at which uncertainty level(s) CAPM alpha as abnormal excess return emerges. Results are shown in the following figures.

Ambiguity exposure generates alphas in all stock size categories and in the significant portion of the uncertainty domain for the hedge portfolio. In an environment where uncertainty is very low, ambiguity aversion is also low and investors are less sensitive to ambiguity exposure of the stocks they choose for their portfolios. When uncertainty peaks (like in the onset of a recessionary period), abnormal excess return to be gained by longing ambiguity sensitive and shorting ambiguity defensive stocks also fades, potentially due to earlier positioning (i.e. before uncertainty reaches the

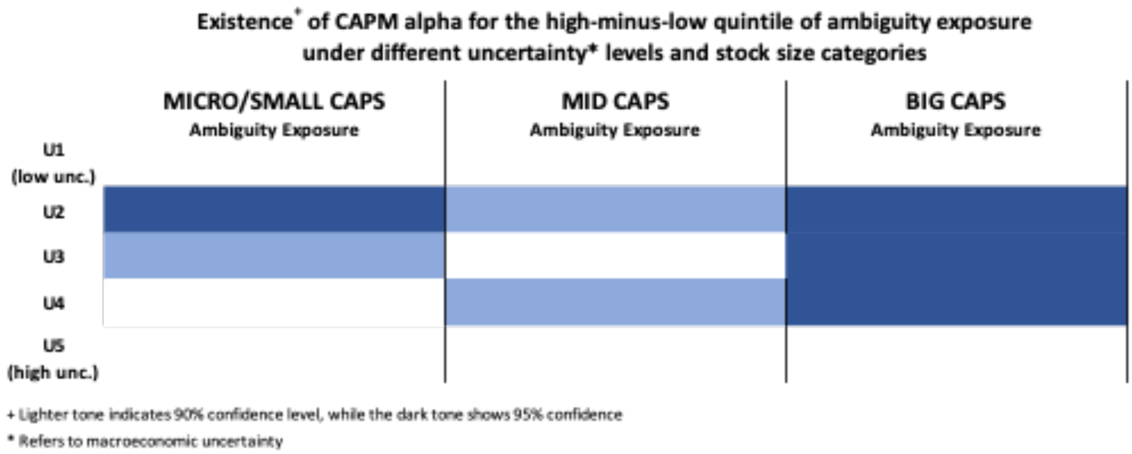

Figure 6.1: High-minus-low abnormal returns of quintiles sorted according to ambiguity exposure in relation to macroeconomic uncertainty levels and market capitalizations

highest level).

Beta anomaly seems to persist in mid caps and big caps only; it is non-existent in micro and small caps. It also exists in moderate to upper uncertainty levels.

Size anomaly is dominant in micro and small caps only, and looks marginal in mid and big caps (in line with the result in Fama and French (2008)). The anomaly is pervasive across all uncertainty levels, however. Dominance of the anomaly in micro caps and its pervasiveness in all uncertainty levels suggest financial distress and paint a picture of low-rated stocks facing either deteriorating or improving credit conditions, which may either be macro related or idiosyncratic (i.e. firm performance dependent) or both.

Book-to-market (value) anomaly is non-existent in big-caps and exists in all uncertainty levels except for periods of very high uncertainty. As documented in Avromov et al. (2010), one potential explanation for the anomaly is the market upgrade of low-rated firms which survive financial distress and experience high subsequent returns.

Operating profitability anomaly is prevalent in low uncertainty levels, and dominant in mid and big caps. Dominance in mid and big caps hints firm maturity for an established record of profitability. It is an accrual anomaly (past performance determining future stock returns). Sticky expectations theory of Bouchaud et al. (2018)

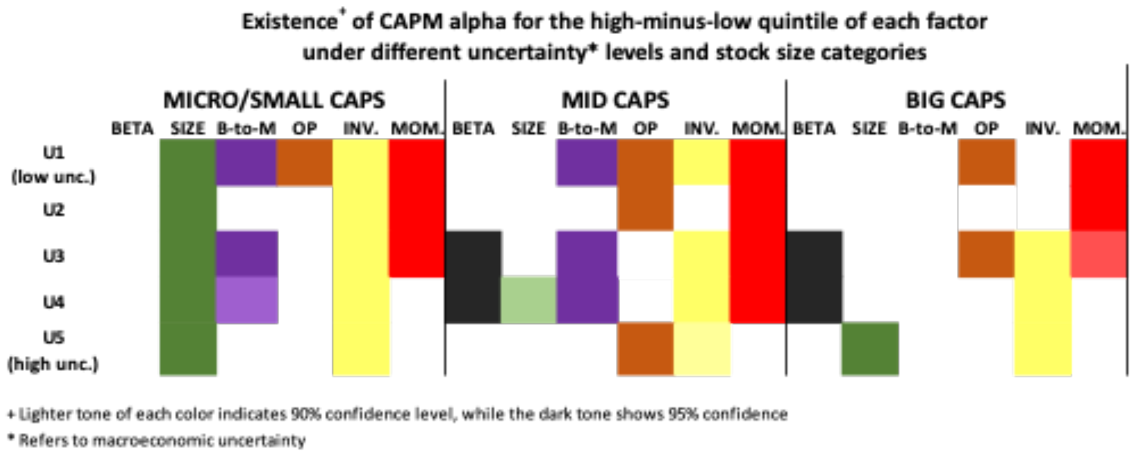

Figure 6.2: High-minus-low abnormal returns of quintiles sorted according to Fama-French 5-factor and momentum categories in relation to macroeconomic uncertainty levels and market capitalizations

is one potential explanation. Prevalence in low uncertainty levels is not surprising, due to ample profitability in expansionary periods.

Investments (asset growth) and momentum anomalies prove to be the most dominant anomalies. Both exist in all stock size categories. Investments are prevalent in almost all uncertainty levels, while momentum is apparent in mid to lower uncertainties.

Note that value-weighted portfolio excess returns are calculated for the quintiles of each factor. This, in contrast to equal-weighting where microcaps dominate the outcome, has an advantage but a pitfall as well. The downside is microcaps and small caps are practically out of the picture. This is particularly important for anomalies such as size and book-to-market where all the action is either in microcaps or small caps.

Overall, in addition to full explanatory power of ambiguity for the beta anomaly, the declines in the CAPM alphas of OP and Momentum when ambiguity factor comes into play is encouraging, yet the case of investment anomaly is curious. For a better verdict, more work shall be necessary.

6.3 Background

A brief description of the anomalies covered in this research would be in order, before delving into procedural details of the research:

- *Beta anomaly* is the empirical security market line being flatter than the one predicted by the CAPM (Black et al. (1972) and Treynor and Black (1973)). Empirically, high beta stocks, typically, have a negative alpha, while low beta stocks, typically, have a positive alpha. Negative and positive CAPM alphas denote ambiguity discount and ambiguity premium respectively, according to the theory this thesis is based on.

- *Size anomaly* is stock returns of small firms being higher than stock returns of larger firms (Fama and French (1992)).

- *Book-to-market anomaly* is stock returns of high book-to-market firms being greater than stock returns of low book-to-market firms (Fama and French (1993)).

- *Operating profitability anomaly* is the positive relation between accounting rates of return on operating profitability and future stock returns. Put differently, the anomaly is the power of past (operating) profitability in forecasting future stock returns (Fama and French (2006), Novy-Marx (2013)).

- *Investment anomaly* refers to the negative cross-sectional relation between corporate investments and future stock returns. That is, there is a persistent negative predictability in future stock returns of firms following large investments (as measured by the percentage change in total assets) by those firms (Titman, Wei, Xie 2004)). Operating profitability and investment anomalies are categorized under the broad name of accruals anomaly.

- *Momentum anomaly*, a prominent one in the asset pricing literature, is the continuation of the outperformance of stocks that have performed well in the medium-term past, which is typically three to twelve months (Jegadeesh and

Titman (1993)). By longing outperformers (winners) and shorting underperformers (losers) in the recent past, a momentum strategy makes profits. Persistence of the anomaly is clearly at odds with both game theoretic positioning and efficient market hypothesis.

6.4 Procedure

Calculation of beta, size, book-to-market, operating profitability, investment and momentum are as detailed in Chapter 4. Note that all of the above listed anomalies are, in fact, about factors that were shown in the literature to affect cross-sectional stock returns. Hence, for each of those factors, stocks are sorted into classes, and it is checked whether ambiguity as a distinct factor has explanatory power for abnormal excess return (i.e. CAPM alpha) of each class for the particular anomaly. The employed methodology is common to all anomalies.

Specifically, eligible stocks are sorted into quintile portfolios at the end of each month $t-1$ according to the factor in question (i.e. beta, size, book-to-market, operating profitability, investment, momentum) and the value-weighted portfolio returns for the post-ranking month t are linked across months to form one time-series of post-ranking excess returns for each quintile of the particular factor. CAPM is then run for each quintile portfolio of the specific factor:

$$R_{FactorQ_n,t} - R_{f,t} \;=\; \alpha_{FactorQ_n} + \beta_{FactorQ_n}(R_{MKT,t} - R_{f,t}) + \epsilon_{FactorQ_n,t}. \tag{6.1}$$

Using the above equation, CAPM alpha time series for each quintile of the specific factor is obtained by the following:

$$\alpha_{FactorQ_n,t} \;=\; (R_{FactorQ_n,t} - R_{f,t}) - \beta_{FactorQ_n}(R_{MKT,t} - R_{f,t}). \tag{6.2}$$

As a result, there are five CAPM alpha time series (i.e. one for each quintile) for each factor. To check whether ambiguity, as a separate factor, has explanatory power for the abnormal excess returns of the particular anomaly, one needs to regress CAPM alpha time series of each quintile on the ambiguity factor. Regression constant, named as ambiguity alpha in Chapter 4, for the high minus low quintile of the particular anomaly is of particular interest. Weakening or diminishing economic/statistical significance of high minus low ambiguity alpha when compared with that of high minus low CAPM alpha for a particular anomaly would signify ambiguity's explanatory power for that particular anomaly.

Hence, the ambiguity factor presented in Chapter 5 will be of use. As might be recalled, ambiguity factor is obtained by running the below monthly cross-sectional (Fama Macbeth) regression for all eligible stocks (NYSE+AMEX+NASDAQ) for all months in the stock sample, where slope coefficients, $\delta_{1,t}$, form a time series to be used as the factor representing ambiguity:

$$CAPM\alpha_{i,t+1} = \delta_{0,t} + \delta_{1,t}(\beta_{i,t}^{MEU} - \beta_{i,t}^{CAPM}) + \epsilon_{i,t+1}. \quad (6.3)$$

Note that CAPM alpha for stock i for month $t+1$ is given by

$$CAPM\alpha_{i,t+1} = (R_{i,t+1} - R_{f,t+1}) - \beta_{i,t}^{CAPM}(R_{MKT,t+1} - R_{f,t+1}). \quad (6.4)$$

And, finally using (6.2) and (6.3), ambiguity alpha for quintiles of the factor in question is the constant term in the following regression:

$$\alpha_{FactorQ_n,t} = \alpha_{Ambiguity,FactorQ_n} + \lambda_{FactorQ_n}\delta_{1,t} + \epsilon_{FactorQ_n,t}. \quad (6.5)$$

Results of the procedure explained above are shown in the following tables for beta, size, book-to-market, operating profitability, investment and momentum factors, respectively.

	Two-stage Process: i) CAPM regression ii) Regress CAPM alpha on the ambiguity exposure (i.e. β^{MEU} - β^{CAPM}) factor				
Beta Portfolios	CAPM alpha	Conditional CAPM alpha	Beta	Lambda (Coef. of amb. exp. factor)	Alpha (2^{nd} factor) (Intercept in the 2nd regr.)
Beta1 *(Low)*	0.170% (1.76)	0.225% (2.38)	0.728 (21.05)	0.194 (3.71)	0.051% (0.58)
Beta2	0.196% (2.62)	0.166% (2.55)	0.796 (35.55)	0.097 (2.13)	0.136% (2.03)
Beta3	0.062% (0.97)	0.021% (0.43)	0.928 (31.30)	0.029 (0.41)	0.044% (0.76)
Beta4	-0.106% (-1.61)	-0.142% (-2.20)	1.119 (41.38)	-0.124 (-2.47)	-0.030% (-0.48)
Beta5 *(High)*	-0.282% (-2.23)	-0.251% (-2.02)	1.422 (33.64)	-0.334 (-4.77)	-0.077% (-0.67)
Beta5minus1 *(High - Low)*	-0.452% (-2.46)	-0.476% (-2.71)	0.694 (10.48)	-0.528 (-5.82)	-0.129% (-0.80)

Table 6.1: Ambiguity pricing and beta anomaly

The table reports CAPM alphas, conditional CAPM alphas, CAPM betas of beta quintiles, together with intercepts and coefficients when CAPM alphas of quintiles are regressed on a proposed factor representing ambiguity exposure. At the end of each month $t-1$, eligible stocks are sorted into quintile portfolios according to their month-end historical CAPM beta measured by $\beta_{i,t-1}^{CAPM}$. The estimation and sorting procedure at each month-end uses only data available at that time. Eligible stocks at month-end $t-1$ are defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with at least 24 monthly return observations in the 60-month period $[t-60, t-1]$. Value-weighted portfolio returns for the post-ranking month t are linked across months to form one time-series of post-ranking value-weighted returns for each beta quintile. The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation is August 1965–June 2015.

	Two-stage Process: i) CAPM regression ii) Regress CAPM alpha on the ambiguity exposure (i.e. $\beta^{MEU} - \beta^{CAPM}$) factor				
Size Portfolios	CAPM alpha	Conditional CAPM alpha	Beta	Lambda (Coef. of amb. exp. factor)	Alpha (2^{nd} factor) (Intercept in the 2nd regr.)
Size1 (Low)	0.545% (2.31)	0.567% (2.34)	1.017 (19.43)	-0.612 (-4.58)	0.920% (3.46)
Size2	0.157% (0.89)	0.153% (0.85)	1.074 (23.35)	-0.437 (-5.08)	0.425% (2.17)
Size3	0.128% (0.93)	0.122% (0.87)	1.101 (28.64)	-0.326 (-4.86)	0.328% (2.19)
Size4	0.176% (1.72)	0.154% (1.50)	1.102 (37.15)	-0.181 (-4.02)	0.287% (2.72)
Size5 (High)	0.004% (0.19)	-0.002% (-0.10)	0.964 (144.00)	0.025 (1.61)	-0.011% (-0.52)
Size5m1 (High - Low)	-0.541% (-2.19)	-0.569% (-2.25)	-0.053 (-0.96)	0.637 (4.39)	-0.931% (-3.36)

Table 6.2: Ambiguity pricing and size anomaly

The table reports CAPM alphas, conditional CAPM alphas, CAPM betas of size quintiles, together with intercepts and coefficients when CAPM alphas of quintiles are regressed on a proposed factor representing ambiguity exposure. At the end of each month $t-1$, eligible stocks are sorted into quintile portfolios according to their month-end market capitalization. Value-weighted portfolio returns for the post-ranking month t are linked across months to form one time-series of post-ranking value-weighted excess returns for each size quintile. The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation is August 1965–June 2015.

	Two-stage Process: i) CAPM regression ii) Regress CAPM alpha on the ambiguity exposure (i.e. β^{MEU} - β^{CAPM}) factor				
Book-to-Market Portfolios	CAPM alpha	Conditional CAPM alpha	Beta	Lambda (Coef. of amb. exp. factor)	Alpha (2^{nd} factor) (Intercept in the 2nd regr.)
BM1 (Low)	-0.088% (-1.13)	-0.092% (-1.23)	1.035 (57.43)	0.022 (0.73)	-0.102% (-1.28)
BM2	0.008% (0.15)	-0.012% (-0.23)	0.988 (50.01)	0.012 (0.35)	0.001% (0.02)
BM3	0.069% (0.89)	0.039% (0.58)	0.915 (32.05)	0.037 (0.78)	0.046% (0.63)
BM4	0.197% (1.90)	0.119% (1.43)	0.898 (22.57)	0.012 (0.14)	0.190% (2.10)
BM5 (High)	0.327% (3.12)	0.260% (2.71)	0.954 (26.22)	-0.111 (-1.87)	0.394% (3.87)
BM5m1 (High - Low)	0.415% (2.52)	0.353% (2.28)	-0.081 (-1.67)	-0.132 (-1.68)	0.496% (3.06)

Table 6.3: Ambiguity pricing and book-to-market anomaly

The table reports CAPM alphas, conditional CAPM alphas, CAPM betas of book-to-market quintiles, together with intercepts and coefficients when CAPM alphas of quintiles are regressed on a proposed factor representing ambiguity exposure. At the end of each month $t-1$, eligible stocks are sorted into quintile portfolios according to their book-to-market (book equities get updated every June; book equity used in June of year $t-1$ is the book equity for the last fiscal year end in $t-2$, while the market equity is as of the end of December of year $t-2$). Value-weighted portfolio returns for the post-ranking month t are linked across months to form one time-series of post-ranking value-weighted excess returns for each book-to-market quintile. The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation is August 1965–June 2015.

	Two-stage Process: i) CAPM regression ii) Regress CAPM alpha on the ambiguity exposure (i.e. β^{MEU} - β^{CAPM}) factor				
Investments Portfolios	CAPM alpha	Conditional CAPM alpha	Beta	Lambda (Coef. of amb. exp. factor)	Alpha (2nd factor) (Intercept in the 2nd regr.)
Inv1 (Low)	0.253% (2.60)	0.212% (2.30)	1.039 (35.16)	-0.081 (-1.38)	0.302% (3.03)
Inv2	0.186% (2.88)	0.137% (2.42)	0.885 (39.84)	0.039 (0.88)	0.163% (2.86)
Inv3	0.098% (2.07)	0.062% (1.70)	0.904 (46.91)	0.047 (1.21)	0.069% (1.83)
Inv4	-0.006% (-0.13)	-0.015% (-0.36)	1.011 (72.55)	0.008 (0.35)	-0.010% (-0.23)
Inv5 (High)	-0.233% (-2.99)	-0.185% (-2.48)	1.214 (52.46)	-0.077 (-1.66)	-0.186% (-2.53)
Inv5m1 (High - Low)	-0.486% (-3.56)	-0.397% (-3.16)	0.175 (3.86)	0.004 (0.04)	-0.488% (-3.67)

Table 6.4: Ambiguity pricing and investment anomaly

The table reports CAPM alphas, conditional CAPM alphas, CAPM betas of investment quintiles, together with intercepts and coefficients when CAPM alphas of quintiles are regressed on a proposed factor representing ambiguity exposure. At the end of each month $t-1$, eligible stocks are sorted into quintile portfolios according to their change in total assets, or investment in short (change in total assets get updated every June. Change in total assets is total assets in fiscal year ending in year $t-2$ minus total assets in fiscal year ending in year $t-3$, divided by total assets in fiscal year ending in year $t-3$). Value-weighted portfolio returns for the post-ranking month t are linked across months to form one time-series of post-ranking value-weighted excess returns for each investment quintile. The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation is August 1965–June 2015.

			Two-stage Process: i) CAPM regression ii) Regress CAPM alpha on the ambiguity exposure (i.e. β^{MEU} - β^{CAPM}) factor		
OP Portfolios	CAPM alpha	Conditional CAPM alpha	Beta	Lambda (Coef. of amb. exp. factor)	Alpha (2nd factor) (Intercept in the 2nd regr.)
OP1 *(Low)*	-0.455% (-2.47)	-0.383% (-2.34)	1.330 (21.41)	-0.191 (-1.28)	-0.338% (-1.89)
OP2	-0.200% (-2.99)	-0.201% (-3.07)	1.063 (37.77)	-0.113 (-3.01)	-0.131% (-2.01)
OP3	0.044% (0.88)	0.023% (0.49)	0.925 (60.50)	-0.010 (-0.47)	0.051% (1.03)
OP4	-0.010% (-0.20)	-0.025% (-0.56)	0.987 (70.97)	0.047 (1.85)	-0.039% (-0.73)
OP5 *(High)*	0.093% (1.85)	0.077% (1.60)	0.968 (61.09)	0.036 (1.05)	0.070% (1.33)
OP5m1 *(High - Low)*	0.548% (2.55)	0.459% (2.43)	-0.362 (-4.86)	0.228 (1.28)	0.408% (1.94)

Table 6.5: Ambiguity pricing and operating profitability anomaly

The table reports CAPM alphas, conditional CAPM alphas, CAPM betas of operating profitability quintiles, together with intercepts and coefficients when CAPM alphas of quintiles are regressed on a proposed factor representing ambiguity exposure. At the end of each month $t-1$, eligible stocks are sorted into quintile portfolios according to their operating profitability (operating profitability is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for the last fiscal year end in $t-2$). Value-weighted portfolio returns for the post-ranking month t are linked across months to form one time-series of post-ranking value-weighted excess returns for each operating profitability quintile. The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation is August 1965–June 2015.

	Two-stage Process: i) CAPM regression ii) Regress CAPM alpha on the ambiguity exposure (i.e. β^{MEU} - β^{CAPM}) factor				
Momentum Portfolios	CAPM alpha	Conditional CAPM alpha	Beta	Lambda (Coef. of amb. exp. factor)	Alpha (2nd factor) (Intercept in the 2nd regr.)
M1 *(Low)*	-0.828% (-4.65)	-0.867% (-4.93)	1.323 (18.22)	-0.537 (-3.94)	-0.499% (-2.37)
M2	-0.191% (-1.88)	-0.232% (-2.31)	1.039 (21.30)	-0.234 (-3.32)	-0.048% (-0.45)
M3	-0.069% (-1.15)	-0.090% (-1.51)	0.922 (46.03)	-0.055 (-1.34)	-0.035% (-0.61)
M4	0.171% (2.99)	0.147% (2.48)	0.922 (40.92)	0.036 (0.79)	0.149% (2.45)
M5 *(High)*	0.398% (3.49)	0.446% (3.73)	1.084 (24.85)	0.151 (1.87)	0.305% (2.62)
M5m1 *(High - Low)*	1.226% (4.99)	1.313% (5.33)	-0.239 (-2.21)	0.688 (3.41)	0.804% (2.82)

Table 6.6: Ambiguity pricing and momentum anomaly

The table reports CAPM alphas, conditional CAPM alphas, CAPM betas of momentum quintiles, together with intercepts and coefficients when CAPM alphas of quintiles are regressed on a proposed factor representing ambiguity exposure. At the end of each month $t-1$, eligible stocks are sorted into quintile portfolios according to their month-end momentum as defined in Section 4.1.1. We require: i) a minimum of nine available monthly returns in the 11-month period $[t-12, t-2]$, ii) price data for the month $t-12$, iii) return data for the month $t-2$. Value-weighted portfolio returns for the post-ranking month t are linked across months to form one time-series of post-ranking value-weighted returns for each momentum quintile. The Newey-West adjusted t-statistics are given in parentheses. The sample period for estimation is August 1965–June 2015.

Chapter 7

Conclusion

A sizable portion of the financial literature, disregarding ambiguity, assumes investors behave as if they know the distributions of returns, which is hard to justify. Risk in equity markets implies future returns are realized with known probabilities. Ambiguity, on the other hand, alludes to circumstances where probabilities about those future realizations are neither known nor assigned uniquely. This thesis, based on a theoretical model built on the smooth ambiguity aversion model (Klibanoff, Marinacci, Mukerji (2005)) in decision theory, empirically shows ambiguity and aversion towards it is priced in the cross-section of stocks in the equity market. A separate event study, investigating pricing of ambiguity and its aversion in periods of heightened systematic uncertainty such as days around the FOMC meetings, points to a very significant CAPM alpha in the high minus low quintile and almost perfect monotonicity from low to high quintile during the 3-day window preceding the meeting, and thus further strengthens the case. These are novel contributions to the literature.

The two fundamental assumptions of the theoretical model the thesis relies on are heterogeneity in ambiguity of assets and heterogeneity in ambiguity aversion of agents, both of which make the model tenable. The model asserts that expected return of a stock is composed of two uncertainty premia: (CAPM-predicted) risk premium, and ambiguity premium. Abnormal return of stock i (i.e. economically and statistically significant alpha) is captured in ambiguity premium, which has a positive association with ambiguity exposure, i.e. $\beta_i^{MEU} - \beta_i^{CAPM}$, the first term of which is stock i's ambiguity beta. Ambiguity beta represents a stock's exposure to systematic ambiguity. For estimating ambiguity betas of stocks, the thesis utilizes the macroeconomic uncertainty index (MEU) of Jurado, Ludvigson, Ng (2015). MEU seems to be a proper proxy for systematic ambiguity, as MEU as an aggregate entity reflecting episodes of varying sentiment/uncertainty for the economy as a whole for period(s) ahead should be a good measure for systematic ambiguity.

Univariate portfolio-level analyses show that quintile portfolios obtained by sorting stocks every month according to their ambiguity exposure yield a monotonic increase in CAPM and conditional CAPM alphas from low to high quintile together with economically and statistically significant positive alphas for high-minus-low quintile. Annualized risk-adjusted CAPM and conditional CAPM alphas for the high minus low quintile stand at 5% and 6% respectively. Bivariate portfolio-level analyses controlling for Fama French factors indicate the monotonicity in alphas from low to high quintile, while significance of positive alphas in the CAPM and conditional CAPM of the high minus low quintile persists.

Sensitivity of ambiguity exposure to numerous factors, which have been shown to affect cross-sectional stock returns, such as size, book-to-market, operating profitability, investments (asset growth), momentum, VXO beta, (il)liquidity, financial constraints/distress, number of analyst estimates, analyst dispersion, institutional ownership are calculated using Fama Macbeth regressions. Sensitivities make economic sense.

Power of ambiguity exposure in explaining stocks' abnormal excess returns are tested for persistence when a subset or the whole set of other factors which have explanatory power for stocks' returns are controlled for in the same regression. Explanatory power of ambiguity exposure for stocks' abnormal excess returns are persistent with and without controlling for industry effect.

As a final step for testing pricing of ambiguity in cross-sectional stock returns, ambiguity exposure was established as a separate explanatory factor. When the factor for ambiguity exposure is controlled for in the CAPM regression, alpha of the high-minus-low quintile (as well as the alphas of the individual quintiles) is shown to disappear.

As a separate event study for heightened uncertainty in the markets, days around FOMC meetings between 1982 and 2015 are investigated. 3-day window preceding the FOMC, the FOMC day, and the 3-day window after the FOMC are in focus. The main finding is that alpha difference between the highest and the lowest ambiguity exposure quintiles significantly widen just days before the meeting, as ambiguity aversion among market players becomes more pronounced. As might be expected, ambiguity aversion subsides immediately after the meeting. There is almost perfect monotonicity from low to high quintile in the 3-day window preceding the FOMC, and more importantly, annualized risk-adjusted CAPM alpha of the high minus low quintile stands at 28% (with a t-stat of 3.79).

The thesis also explores the explanatory power of ambiguity and aversion towards it on cross-sectional asset pricing anomalies. The empirical results in this thesis show that ambiguity and ambiguity aversion fully explain the beta anomaly, while providing partial explanations to operating profitability and momentum anomalies. These findings constitute an additional novel contribution to the literature.

Measurement and estimation of systematic uncertainty is a difficult task. Better proxies for systematic uncertainty may be expected to improve the estimation of the ambiguity beta and enhance the explanatory power of ambiguity as a factor in stock returns. Emergence of new proxies would provide a natural exercise along the lines of this thesis for future research.

Momentum, investments and operating profitability are dominant anomalies across all stock size categories. Another area of future research would be the investigation of why ambiguity and aversion towards it bring only a partial explanation for the two, while failing to explain the investment anomaly.

Extreme periods of high and low uncertainty such as onset of recessionary periods or rosy boom times in the economy also demand further research in terms of asset pricing in the ambiguity context.

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Appendix

This appendix provides the proofs of Proposition 1 and an additional related result, Lemma 1. The arabic numerals in parantheses, (x), refer to equations in the main text. The equations introduced in the Appendix are numbered in the form (A.x).

Lemma 1. *For any given $n = 1, \ldots, N$, let a_n be agent n's optimal portfolio so that a_n solves the maximization problem (3.5). Then the following holds:*

$$E[R_i] - R_f = \frac{cov(R(a_n), R_i) + \eta_n cov^M(R(a_n), R_i)}{var(R(a_n)) + \eta_n var^M(R(a_n))}(E[R(a_n)] - R_f), \quad i = 1, 2. \tag{A.1}$$

Proof of Lemma 1. Recall that the optimal portfolio is given by (3.7):

$$a_{i,n} = \frac{(\mu_i - R_f)A_{j,n} - (\mu_j - R_f)B_{12,n}}{A_{1,n}A_{2,n} + B_{12,n}^2}, \quad i, j = 1, 2, i \neq j$$

where $A_{i,n} = \theta_n \left[\sigma_i^2 + \eta_n(\sigma_i^M)^2\right]$ and $B_{12} = \theta_n \left[\sigma_{12} + \eta_n \sigma_{12}^M\right]$. Hence, we have the following:

$$\mu_i - R_f = (a_{i,n}A_{i,n} + a_{j,n}B_{12,n}) \frac{A_{1,n}A_{2,n} + B_{12,n}^2}{A_{1,n}A_{2,n} - B_{12,n}^2} \tag{A.2}$$

This further implies

$$\begin{aligned} E[R(a_n)] - R_f &\equiv \frac{a_{1,n}(\mu_1 - R_f) + a_{2,n}(\mu_2 - R_f)}{a_{1,n} + a_{2,n}} \\ &= \frac{(a_{1,n})^2 A_{1,n} + 2a_{1,n}a_{2,n}B_{12,n} + (a_{2,n})^2 A_{2,n}}{a_{1,n} + a_{2,n}} \times \frac{A_{1,n}A_{2,n} + B_{12,n}^2}{A_{1,n}A_{2,n} - B_{12,n}^2}. \end{aligned}$$

Together with (A.2), this implies that:

$$\frac{\mu_i - R_f}{a_{i,n}A_{i,n} + a_{j,n}B_{12,n}} = \frac{E[R(a_n)] - R_f}{\frac{(a_{1,n})^2 A_{1,n} + 2a_{1,n}a_{2,n}B_{12,n} + (a_{2,n})^2 A_{2,n}}{a_{1,n}+a_{2,n}}}. \tag{A.3}$$

Next, observe the following:

$$\begin{aligned} a_{i,n}A_{i,n} + a_{j,n}A_{j,n} &= \theta_n(a_{1,n} + a_{2,n})\left(cov(R(a_n), R_i) + \eta_n cov^M(R(a_n), R_i)\right), \\ (a_{1,n})^2 A_{1,n} + 2a_{1,n}a_{2,n}B_{12,n} + (a_{2,n})^2 A_{2,n} &= \theta_n(a_{1,n} + a_{2,n})^2\left(var(R(a_n)) + \eta_n var^M(R(a_n))\right). \end{aligned}$$

Therefore, (A.3) can be re-written as

$$E[R_i] - R_f = \frac{cov(R(a_n), R_i) + \eta_n cov^M(R(a_n), R_i)}{var(R(a_n)) + \eta_n var^M(R(a_n))}(E[R(a_n)] - R_f),$$

which is the desired result. □

Proof of Proposition 1. Proving there exists $\eta_{market} \in [\underline{\eta}, \bar{\eta}]$ such that an agent with ambiguity aversion parameter η_{market} would optimally hold the market portfolio is sufficient to establish existence of $\eta_{market} \in [\underline{\eta}, \bar{\eta}]$ such that (3.10)-(3.12) hold. This is so because the antecedent implies via Lemma 1 that (3.10) holds, which can then be re-written as (3.11)-(3.12). From (3.7), we know that agent n's optimal monetary holding of asset i is $a_{i,n} = \frac{K_i(\eta_n)}{\theta_n}$ where

$$K_i(\eta_n) \equiv \frac{(\mu_i - R_f)(\sigma_j^2 + \eta_n(\sigma_j^M)^2) - (\mu_j - R_f)(\sigma_{12} + \eta_n\sigma_{12}^M)}{(\sigma_1^2 + \eta_n(\sigma_1^M)^2)(\sigma_2^2 + \eta_n(\sigma_2^M)^2) + (\sigma_{12} + \eta_n\sigma_{12}^M)^2}, \quad j \neq i.$$

Therefore agent n's optimal portfolio return is given by

$$\frac{a_{1,n}R_1 + a_{2,n}R_2}{a_{1,n} + a_{2,n}} = \frac{K_1(\eta_n)R_1 + K_2(\eta_n)R_2}{K_1(\eta_n) + K_2(\eta_n)}.$$

Following the equation above and (3.9), the optimal portfolio return of an agent with ambiguity aversion parameter η_{market} is equal to the market portfolio return if and only if

$$\frac{K_1(\eta_{market})R_1 + K_2(\eta_{market})R_2}{K_1(\eta_{market}) + K_2(\eta_{market})} = \frac{p_1^* e_1 R_1 + p_2^* e_2 R_2}{p_1^* e_1 + p_2^* e_2}. \tag{A.4}$$

Moreover, if the above equality holds for all realizations of exogenous asset returns R_1 and R_2, then the agent with ambiguity aversion η_{market} optimally holds the market portfolio. Now note that market clearing implies

$$\sum_n \frac{K_i(\eta_n)}{\theta_n} = p_i^* e_i \quad i = 1, 2. \tag{A.5}$$

Hence, following (A.4)-(A.5), an agent with ambiguity aversion parameter η_{market} optimally holds the market portfolio if and only if[1]

$$\frac{K_1(\eta_{market})}{K_1(\eta_{market}) + K_2(\eta_{market})} = \frac{\sum_n \frac{K_1(\eta_n)}{\theta_n}}{\sum_n \frac{K_1(\eta_n)}{\theta_n} + \sum_n \frac{K_2(\eta_n)}{\theta_n}}. \tag{A.6}$$

Re-writing the above equation, we obtain

$$\begin{aligned} & \frac{(\mu_1 - R_f)\sigma_2^2 - (\mu_2 - R_f)\sigma_{12} + \eta_{market}((\mu_1 - R_f)(\sigma_2^M)^2 - (\mu_2 - R_f)\sigma_{12}^M)}{\sum_{i=1,2,\ j\neq i}[(\mu_i - R_f)\sigma_j^2 - (\mu_j - R_f)\sigma_{12} + \eta_{market}((\mu_i - R_f)(\sigma_j^M)^2 - (\mu_j - R_f)\sigma_{12}^M)]} \\ = & \frac{\sum_n \frac{1}{\theta_n} \frac{(\mu_1 - R_f)\sigma_2^2 - (\mu_2 - R_f)\sigma_{12} + \eta_n((\mu_1 - R_f)(\sigma_2^M)^2 - (\mu_2 - R_f)\sigma_{12}^M)}{(\sigma_1^2 + \eta_n(\sigma_1^M)^2)(\sigma_2^2 + \eta_n(\sigma_2^M)^2) + (\sigma_{12} + \eta_n\sigma_{12}^M)^2}}{\sum_{i=1,2,\ j\neq i}\sum_n \frac{1}{\theta_n} \frac{(\mu_i - R_f)\sigma_j^2 - (\mu_j - R_f)\sigma_{12} + \eta_n((\mu_i - R_f)(\sigma_j^M)^2 - (\mu_j - R_f)\sigma_{12}^M)}{(\sigma_1^2 + \eta_n(\sigma_1^M)^2)(\sigma_2^2 + \eta_n(\sigma_2^M)^2) + (\sigma_{12} + \eta_n\sigma_{12}^M)^2}}. \end{aligned}$$

This uniquely yields

$$\eta_{market} = \left(\sum_{k=1}^N \Pi_k\right)^{-1} \sum_{n=1}^N \Pi_n \eta_n, \tag{A.7}$$

where, for $n = 1, \cdots, N$,

$$\Pi_n = \left(\frac{1}{\theta_n}\right)\left(\frac{1}{(\sigma_1^2 + \eta_n(\sigma_1^M)^2)\,(\sigma_2^2 + \eta_n(\sigma_2^M)^2) + (\sigma_{12} + \eta_n\sigma_{12}^M)^2}\right). \tag{A.8}$$

[1]Note that, if (A.6) holds, then it necessarily holds that

$$\frac{K_2(\eta_{market})}{K_1(\eta_{market}) + K_2(\eta_{market})} = \frac{\sum_n \frac{K_2(\eta_n)}{\theta_n}}{\sum_n \frac{K_1(\eta_n)}{\theta_n} + \sum_n \frac{K_2(\eta_n)}{\theta_n}}.$$

Since $\Pi_n > 0$ for all n, it follows that $\min_n\{\eta_n\} = \underline{\eta} < \eta_{market} < \bar{\eta} = \max_n\{\eta_n\}$ provided that $\underline{\eta} \neq \bar{\eta}$. Hence, we have the desired result. $\square$