On the Prompt Fission Neutron Energy Distributions

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Rifat SOYLU

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Prof. Dr. A. Necmeddin YAZICI

Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Ramazan KOÇ

Head of Department

This is to certify that we have read this thesis and that in our consensus opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. Mehmet KOÇAK

Supervisor

Examining Committee Members:	Signature
Associate Prof. Dr. Mustafa YILMAZ	……………….
Associate Prof. Dr. M. Fatih HASOĞLU	……………….
Assist. Prof. Dr. Mehmet KOÇAK	……………….

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Rifat SOYLU

ABSTRACT

ON THE PROMPT FISSION NEUTRONS

ENERGY DISTRIBUTION

SOYLU, Rifat
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Supervisor: Assist. Prof. Dr. Mehmet KOÇAK
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Prompt fission neutron energy spectrum (PFNS) from spontaneous fission of ^{240}Pu (sf) and ^{242}Pu (sf) is calculated and compared quantitatively with the experimentally measured data. Calculations were done with three different methods; the first method is based on neutron's average center of mass kinetic energy, for the second fission fragment excitation energy distribution was used, and for the third method standardization of ^{252}Cf (sf) was preferred. In each method aim is to calculate nuclear temperature parameters of light and heavy fission fragments. The last method has also been used in this thesis to calculate several crucial parameters of some well-known fissile nuclei, such as neutron average center of mass energy, excitation energy and multiplicity. We have shown that all calculations well describes the observed data.

Key words: Fission, prompt neutron, energy spectrum, excitation energy, multiplicity, center of mass system, laboratory system.

ÖZ

ANİ FİZYON NÖTRONLARININ ENERJİ

DAĞILIMLARI ÜZERİNE

SOYLU, Rifat

Yüksek Lisans Tezi, Fizik Mühendisliği

Tez Yöneticisi: Dr. Öğr. Üyesi Mehmet KOÇAK

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43 sayfa

^{240}Pu (sf) ve ^{242}Pu (sf) izotoplarının kendiliğinden fizyonunda ortaya çıkan ani nötronların enerji spektrumu hesaplanarak deneysel olarak ölçülen verilerle kantitatif olarak karşılaştırılmıştır. Hesaplamalar üç farklı metodla yapılmaktadır; ilk metod nötronun kütle merkezi sisteminde ortalama kinetik enerjisine dayanmakta, ikincisi için fizyon ürün çekirdeklerinin uyarılma enerji dağılımları kullanılmakta ve üçüncü metot için ^{252}Cf (sf) standardizasyonu tercih edilmektedir. Bu metotların her birinde amaç ağır ve hafif ürün çekirdeklerin sıcaklık parametrelerini bulmaktır. Sonuncu metot bu tezde aynı zamanda bazı iyi bilinen fizyon yapan atom çekirdeklerinin kütle merkezi sisteminde nötron enerjisi, uyartılma enerjisi ve ortalama ani fizyon nötronu gibi çok önemli parametrelerini hesaplamada da kullanılmıştır. Bu tezde yapılan bütün hesaplamaların deneysel verileri iyi tasvir ettiğini gösterdik.

Anahtar kelimeler: Fizyon, ani nötronlar, enerji spectrumu, uyarılma enerjisi, açığa çıkan nötron sayısı, kütle merkezi sistemi, laboratuvar sistemi.

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LIST OF ABBREVIATIONS

A	Mass number of compound nucleus
$A_{L,H}$	Light and heavy fragments mass numbers
TKE	Total kinetic energy of fission fragment
PFNS	Prompt fission neutron spectrum
sf	Spontaneous fission
a	Level density parameter
R_T	Ratio of maximum temperature
E_n	Neutron incident energy
B_n	Neutron banding energy
E^*	Total excitation energy of fission fragment
$E^*_{L,H}$	Light and heavy total excitation energy of fission fragments
$\bar{\epsilon}$	Average center of mass energy of fission fragment
$\bar{\epsilon}_{L,H}$	Light and heavy average center of mass energy of fission fragments
$\bar{E_f}$	Average fission fragment kinetic energy per nucleon
$\bar{E}_f^{L,H}$	Light and heavy average fission fragments kinetic energy per nucleons
$\bar{E}$	Average neutron lab energy

T_m	Maximum temperature of fission fragment
$T_{mL,H}$	Light and heavy maximum temperature of fission fragments
T_M	Thermodynamic temperature of Maxwell spectrum
$T_{ML,H}$	Light and heavy thermodynamic temperatures of Maxwell spectrum
T_W	Thermodynamic temperature of Watt spectrum
$T_{WL,H}$	Light and heavy thermodynamic temperatures of Watt spectrum
T_{LC}	Thermodynamic temperature of Le Couteur spectrum
$T_{LCL,H}$	Light and heavy thermodynamic temperatures of Le Couteur spectrum
T_{Wf}	Thermodynamic temperature of Weisskopf spectrum
$T_{WfL,H}$	Light and heavy thermodynamic temperatures of Weisskopf spectrum

CHAPTER I

INTRODUCTION

Nuclear fission is a nuclear reaction in which a heavy atomic nucleus splits into smaller parts. It may happen spontaneously or can be induced in a variety of ways such as (thermal or fast) neutron induced fission, heavy-ion induced fission, photofission, and delayed fission. The possibility of such a process was totally unexpected and its discovery came as a shock to the scientific community. Nevertheless, once the existence of fission was established experimentally, it took a remarkably short time to put this spectacular process to practical use. Although nuclear fission has been studied intensely for over seventy years, it is still not well understood in detail and it remains an active field at the forefront of modern physics.

Once the neutron discovered by Chadwick in 1932, it was quickly turned into a useful tool in nuclear physics, the main advantage being that its electric neutrality enables it to readily enter the nucleus. Particularly, Enrico Fermi and his colleagues initiated a series of experiments in which they irradiated natural uranium with neutrons in 1934. They realized that such irradiations might lead to the production of new elements with a new activity so called beta decay; subsequently Fermi proposed the theory of the beta (β) radioactivity. Hahn and Strassman published an article to confirm Fermi's theory and then they discovered nuclear fission in 1938. According to this theory when uranium, plutonium or any other actinide nucleus is bombarded by a neutron it is divided into two fragments with a few neutrons, photons usually in the forms of gamma rays and subsequently beta and alpha particles. The process is accompanied by the release of large amount of energy in the form of heat, roughly 200 MeV.

Neutrons which emitted directly from fission are called prompt neutrons. They are emitted within very short time of about 10^{-14} seconds. It is known that prompt neutrons trigger the nuclear chain reaction, in the neutron induced fission, to produce energy in nuclear power plants.

Prompt neutrons from spontaneous fissions of fissile nuclei have become subject of many theoretical and experimental investigations due to their applications in nuclear reactors, such as in criticality and nuclear reactor shielding calculations. But up to day the form of pure energy distributions of neutrons in laboratory system preserve some peculiarities which needs to further investigations of the problem, and the aim is to clarify the subject matter. Over one-third of the energy produced in most nuclear power plants comes from plutonium. Isotopes of plutonium with mass numbers 240 through 242 are made along with ^{239}Pu in nuclear reactors. ^{240}Pu and ^{242}Pu exhibit a high rate of spontaneous fission the neutron flux of any sample containing it. This makes reactor-grade plutonium entirely unsuitable for use in a bomb. These two even mass numbered isotope of plutonium in addition to be used in commercial and military purposes they are also important to produce nuclear energy in nuclear power plants.

Mass and energy characteristics of fission fragments emitted in the spontaneous fission of $^{240,242}Pu$ are discussed in terms of the random neck rupture model as well as in terms of the scission point model [1]. The prompt neutron spectra for $^{240}Pu(sf)$ and $^{242}Pu(sf)$ were calculated with the modified Madland-Nix model [2] with consideration to the multimodal nature of the fission process. Los Alamos model with multiple fission changes upgraded with the dependence of the average fission fragment kinetic energy [3] was used for spontaneous fission of ^{240}Pu. Integral neutron spectra [4] in the ^{240}Pu and ^{242}Pu spontaneous fission were measured using the time-of-flight method in the energy range *0.1-15 MeV* relative to the standard neutron spectrum in ^{252}Cf spontaneous fission. The modified Los Alamos model which was used an analytical expression of the inverse reaction cross section that taking scission neutron contribution into account [5] was used to predict experimental prompt neutron fission spectrum (PNFS) data of $^{240}Pu(sf)$ and $^{242}Pu(sf)$. The point by point model which is the based on the prompt neutron multiplicity $P(\upsilon)$ that depends on the multi parametric matrix $\upsilon(A, E^*)$ and on the fission fragment distribution [6] is applied to calculate PNFS of $^{240}Pu(sf)$ and

$^{242}Pu(sf)$. A method of total excitation energy *(E^*)* partition between fully accelerated fission fragment based exclusively on the systematic behavior of experimental multiplicity, *υ(A)*, method [7] is applied on some spontaneous and neutron induced fissions but not on $Pu(sf)$. The event-by-event model FREYA [8] is extended to study spontaneous fission spectrum of ^{240}Pu. The even-odd effects in the prompt emission of $^{240}Pu(sf)$ and $^{242}Pu(sf)$ and other even isotopes of Pu are studied by Tudora and Hambsch [9].

As given in the literature survey above there are so many valuable studies on spontaneous fission of the even isotopes of Plutonium. But the studies on the prompt neutron energy spectrum of these isotopes are just a few. The existing studies in this field not always well represent the experimentally measured data.

In this thesis work we aimed to describe experimental observed data with the calculated spectra. Therefore, in the next chapter prompt neutron energy spectra of $^{240}Pu(sf)$ and $^{242}Pu(sf)$ are calculated with three handy methods; (i) neutron's average kinetic energy model, (ii) non-equitemperature model and (iii) total excitation energy partition model. Calculations are done by using three well-known theoretical functions: Maxwellian, Watt and Le Couteur spectrums. Results of the calculations are presented and discussed in Chapter three. In that chapter while we compare the performance of the calculation methods in description of the experimental data we also test the ability of the three theoretical spectrums to carry out this task. We finally noted that these smart and flexible models works for all fissile nuclei whether spontaneous or neutron induced fission and especially the third method is very successful in calculating some critical characteristic properties of fissile nuclei in nuclear fission.

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CHAPTER II

NEUTRON ENERGY SPECTRA

2.1 Introduction

Studies on prompt fission neutron spectrum have become subject of many theoretical and experimental investigations [10-13]. The center of mass energy spectrum of neutrons depends on many effects such as multiple neutron emissions from fission fragments, initial excitation energy distribution of fission fragments, fission fragment charge and mass distribution, competition between neutron and γ-emissions from fragments with relatively lower excitation energies and so on [14-19,26]. Weisskopf evaporation theory [19,20] take all of these effects into consideration and gives the following expression for the neutron energy distribution in the range $\epsilon, \epsilon + d\epsilon$

$$\Phi(\epsilon)d\epsilon = const.\,\epsilon\sigma_c(\epsilon)W(\epsilon_{max} - \epsilon)d\epsilon \qquad (2.1)$$

where $\sigma_c(\epsilon)$ is the compound nucleus formation cross section at the energy ϵ, W is the energy level density of residual nucleus, ϵ_{max} is the maximum possible energy of emitted neutron. Relating the nuclear level density with the thermodynamic temperature [19,20] of residual nucleus, $T(\epsilon_{max})$, Eq.(2.1) is transformed to the form of

$$\Phi(\epsilon)d\epsilon = const.\,\epsilon\sigma_c(\epsilon)e^{-\epsilon/T}d\epsilon \qquad (2.2)$$

If we take $\sigma_c(\epsilon)$ as constant then one comes to form of Weisskopf's neutron energy spectrum,

$$\Phi_{W_f}(\epsilon)d\epsilon = const.\,\epsilon e^{-\epsilon/T}d\epsilon \qquad (2.3)$$

where W_f refers to the Weisskopf spectrum. When $\sigma_c(\epsilon) \sim 1/\sqrt{\epsilon}$, ($1/v$ law), then the Maxwell form of energy spectrum is obtained,

$$\Phi_M(\epsilon)d\epsilon = const.\sqrt{\epsilon}e^{-\epsilon/T}d\epsilon \qquad (2.4)$$

For the so called Le Couteur spectrum [21,22], the power of ϵ is $5/11$. This spectrum was calculated in the framework of cascade neutron evaporation from a highly excited nucleus Le Couteur form of neutron energy spectrum is in this case expressed in the form

$$\frac{1}{\sigma_c(\epsilon)}\Phi(\epsilon)d\epsilon = const.\,\epsilon^{\ell-1}e^{-\epsilon/T}d\epsilon \tag{2.5}$$

where $\ell \approx 16/11$, $T \approx (11/12)\,T_m$ within the Fermi-Gas model and T_m is the temperature corresponding to first neutron emission. The expression in Eq. (2.5) can be written in the usual form of

$$\Phi(\epsilon) = const.\,\epsilon^{5/11}e^{-\epsilon/T} \tag{2.6}$$

2.2 Center of mass and laboratory spectra

The energy spectra forms of Weisskopf, Maxwell and Le Couteur when normalized to unity in the energy range of emitted neutrons, $(0,\infty)$, are

$$\Phi_{W_f}(\epsilon) = \frac{\epsilon}{T_{Wf}{}^2}e^{-\epsilon/T_{Wf}} \tag{2.7}$$

$$\Phi_M(\epsilon) = \frac{2\sqrt{\epsilon}}{\sqrt{\pi T_M{}^3}}e^{-\epsilon/T_M} \tag{2.8}$$

$$\Phi_{LC}(\epsilon) = \frac{\epsilon^{5/11}e^{-\epsilon/T_{LC}}}{\Gamma(16/11)T_{LC}{}^{16/11}} \tag{2.9}$$

where T_{Wf}, T_M *a*nd T_{LC} are the corresponding temperature parameters, respectively and $\Gamma(x) = \int_0^\infty t^{x-1}\,e^{-t}dt$. Average energies of these spectra are, correspondingly

$$\bar{\epsilon}_{Wf} = 2T_{Wf}, \quad \bar{\epsilon}_M = \frac{3}{2}T_M, \quad \bar{\epsilon}_{LC} = \frac{16}{11}T_{LC} \tag{2.10}$$

Laboratory spectrum, for isotropic distribution of neutrons in the center of mass system of fission fragment having constant speed is

$$N(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2}\frac{\Phi(\epsilon)d\epsilon}{4\sqrt{\epsilon E_f}} \tag{2.11}$$

where E_f is the kinetic energy of fission fragment per nucleon. Laboratory spectrums corresponding to the center of mass system given by Eqs. (2.7-2.9) have respectively forms of;

$$N_F(E) = \frac{1}{4\sqrt{E_f T}}\left[\left(-x_1 e^{-x_1{}^2} + \mathrm{erf}(\mathrm{x}_1)\right) - \left|\left(-x_2 e^{-x_2{}^2} - \mathrm{erf}(x_2)\right)\right|\right] \qquad (2.12)$$

where $x_1 = \sqrt{\frac{E}{T}} + \sqrt{\frac{E_f}{T}}$, $x_2 = \sqrt{\frac{E}{T}} - \sqrt{\frac{E_f}{T}}$ and $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$

$$N_W(E) = \frac{1}{\sqrt{\pi T E_f}} e^{-E_f/T}\, e^{-E/T} \sinh\left(\frac{2}{T}\sqrt{E E_f}\right) \qquad (2.13)$$

and

$$N_{LC}(E) = \frac{\Gamma(^{21}/_{22})}{\Gamma(^{16}/_{11})}\frac{1}{4\sqrt{E_f T}}\left[\Gamma\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, \frac{21}{22}\right) - \Gamma\left(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, \frac{21}{22}\right)\right] \qquad (2.14)$$

where $\Gamma(x, a)$ is the incomplete gamma function defined by

$$\Gamma(x, a) = \frac{1}{\Gamma(a)}\int_0^x e^{-t}\, t^{a-1}dt \qquad (2.15)$$

Here N_F is the Feather spectrum [23] while N_W is the Watt spectrum [24] and N_{LC} is the Le Couteur spectrum which is transformed to the laboratory system. The detailed calculation for the derivations of Eqs. (2.12-2.14) is given in Appendix A, B and C. Note that, throughout this work we use LCL abbreviation for the Le Couteur laboratory spectrum. It is also noted that, Terrell in his work [14] has shown that single Feather spectrum cannot represent experimental data.

CHAPTER III

METHODS AND CALCULATIONS

We used three different methods to calculate the temperature parameter for the representation of experimental prompt neutron energy spectra of $^{240}Pu(sf)$ and $^{242}Pu(sf)$.

For a quantitative comparison of the spectrums with experimental data of Gerasimenko [4] we used chi-square (X^2) statistical equation given by

$$X^2 = \sum_{i=1}^{n=92} \left(\frac{N'_{exp,i} - N'_{th,i}}{\sigma_i}\right)^2 \tag{3.1}$$

$$X_v^2 = X^2/(n-2). \tag{3.2}$$

Here $N'_{exp,i}$ and $N'_{th,i}$ are the experimental and calculated neutron energy spectrums, respectively and σ_i is the experimental error. The number "2" in Eq. (3.2) refers to number of the free parameter which are average fission fragment kinetic energy per nucleon $\bar{E_f}$ and temperature parameter T.

3.1 Method 1: Average Kinetic Energy Model

This method simply requires knowing average neutron center of mass kinetic energy ($\bar{\epsilon}$) to calculate thermodynamic (or nuclear) temperature parameter T. It is basically obtained from the relation

$$\bar{E} = \bar{E_f} + \bar{\epsilon} \tag{3.3}$$

where $\bar{E_f}$ is the average fission fragment kinetic energy per nucleon and $\bar{E}$ is the average neutron laboratory energy. Average neutron center of mass kinetic energy of the neutron for many fissile nuclei has not been measured or calculated directly, especially for light and heavy fragments. In that case we can use Eq. (3.3) and inherent ratio between two fragments as in Eq. (3.8).

Average fission fragment kinetic energies per nucleon for light and heavy fragments are defined respectively by

$$\bar{E_f}^{L} = \frac{A_H}{A_L}\frac{<TKE>}{A}, \qquad \bar{E_f}^{H} = \frac{A_L}{A_H}\frac{<TKE>}{A} \tag{3.4}$$

where $< TKE >$ is the total kinetic energy of the compound nucleus and A is the compound nucleus mass number which is equal to the summation of mass numbers of light and heavy fragments; $A = A_L + A_H$. The average fission fragment kinetic energy per nucleon is described in terms of the light and heavy fragment kinetic energies per nucleon as

$$\bar{E_f} = \frac{1}{2}(\bar{E_f}^{L} + \bar{E_f}^{H}). \tag{3.5}$$

Similarly, the average center of mass neutron kinetic energy $\bar{\epsilon}$ can be expressed as the mean value of the center of mass neutron kinetic energy of the light ($\bar{\epsilon}_L$) and heavy ($\bar{\epsilon}_H$) fission fragments

$$\bar{\epsilon} = \frac{1}{2}(\bar{\epsilon}_L + \bar{\epsilon}_H). \tag{3.6}$$

Relation between mean center of mass neutron energy and temperature parameter of the Maxwellian and Le Couteur spectrums are given by

$$\bar{\epsilon}_M = \frac{3}{2}\, T_M\,, \qquad \bar{\epsilon}_{\mathrm{LC}} = \frac{16}{11}\, T_{LC}\,. \tag{3.7}$$

Because of the direct relation between center of mass neutron energy and temperature parameter, the thermodynamic ratio between T parameters of light and heavy fragments described by Ohsawa [27], $R_T = \frac{T_L}{T_H}$, can also be used for center of mass energy of light and heavy fragments

$$R_T = \frac{\bar{\epsilon}_L}{\bar{\epsilon}_H}. \tag{3.8}$$

3.1.1 Calculation of $^{240}Pu(sf)$ with Average Kinetic Energy Model

We began our calculation with spontaneous fission of $^{240}Pu(sf)$. The average neutron laboratory energy for this Pu isotope was measured by Gerasimenko et al. [4] as $2.044\ MeV$. Demattè et al. [1] measured its total fission fragment kinetic energy as $178.5\ MeV$ and most probable mass numbers of its light and heavy fragments as 101 and 139, respectively. By knowing these data we can easily calculate the average fission fragment kinetic energy per nucleon for two fission fragments by using Eq. (3.4): $\bar{E_f}^L = 1.024\ MeV$ and $\bar{E_f}^H = 0.540\ MeV$. Then, from Eq. (3.5) we find total average kinetic energy per nucleon as $0.782\ MeV$. From then, Eq. (3.3) helps us to predict the average center of mass energy of the prompt neutron as 1.262 MeV. Eq. (3.6) and Eq. (3.8) are useful to calculate the center of mass energy light and heavy fragments. In the related literature the value of R_T can varies from 1.1 to 1.8 but it is most probable between 1.1 and 1.4. When we predict $\bar{\epsilon}_L$ and $\bar{\epsilon}_H$ we can calculate temperature parameters of Watt and Le Couteur spectrums by using Eq. (3.7). Temperature parameter of Maxwellian spectrum, which is $T_M = 1.371\ MeV$, is calculated by Gerasimenko [4] in fitting his experimental data to this spectrum. The parameters ϵ and T corresponding to suitable R_T values are presented in Table 3.1. The calculated chi-square values for each spectrum are listed in the Table 3.2. Remind that the temperature parameters are same for center of mass and laboratory systems. It is clear in Table 3.2 that we get the best description of the observed data for three theoretical spectrums when $R_T = 1.3$. The relation between theoretical and experimental data is showed in Figure 3.1 and Figure 3.2. Two figures are identical but in order to see the details clearly in lower and higher energies we used logarithmic scales in that figures.

Table 3.1 Center of mass energy and temperature parameters of light and heavy fragments in comparison with different R_T ratios for the $^{240}Pu(sf)$.

Parameters (MeV)	$R_T = 1.1$	$R_T = 1.2$	$R_T = 1.3$	$R_T = 1.4$
$\bar{\epsilon}_L$	1.322	1.377	1.427	1.472
$\bar{\epsilon}_H$	1.202	1.147	1.097	1.052
T_{LCL}	0.909	0.947	0.981	1.012
T_{LCH}	0.826	0.789	0.754	0.723
T_{WL}	0.881	0.918	0.951	0.982
T_{WH}	0.801	0.765	0.732	0.701

Table 3.2 Chi-squares values of the theoretical spectrums for different R_T values calculated for $^{240}Pu(sf)$.

Chi-Square	$R_T = 1.1$	$R_T = 1.2$	$R_T = 1.3$	$R_T = 1.4$
$X_v^2(LC)$	1.625	1.134	0.925	1.023
$X_v^2(W)$	2.012	1.355	0.978	0.909
$X_v^2(M)$	1.672	1.672	1.672	1.672

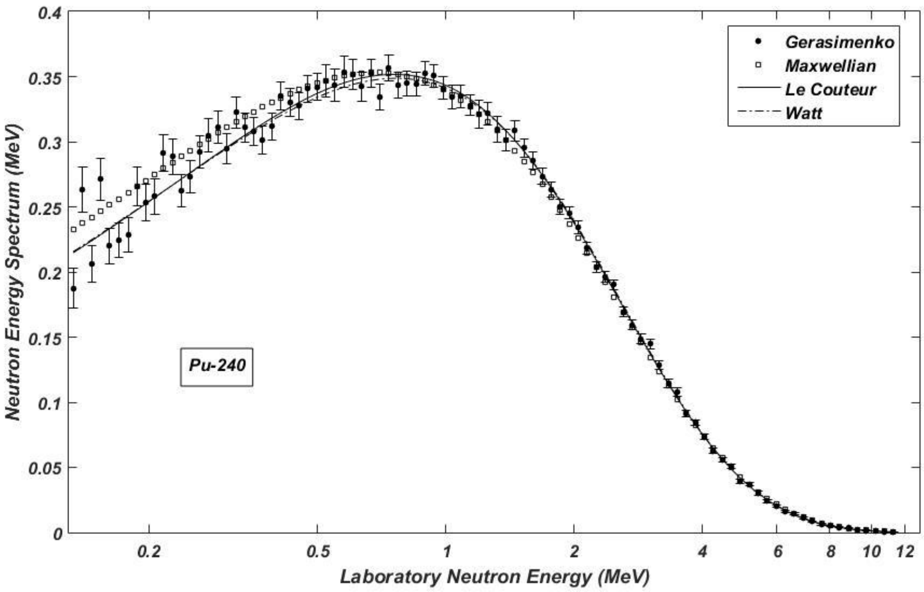

Figure 3.1 Comparison of the theoretical spectra with experimental data of Gerasimenko [4] for $R_T = 1.3$ for the spontaneous fission of $^{240}Pu(sf)$.

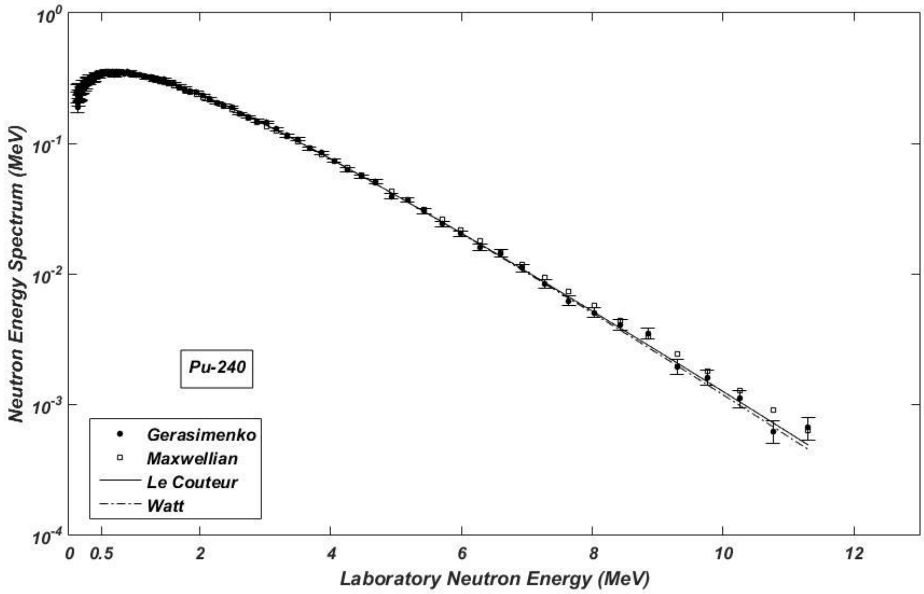

Figure 3.2 Comparison of the theoretical spectra with experimental data of Gerasimenko [4] for $R_T = 1.3$ for the spontaneous fission of $^{240}Pu(sf)$.

3.1.2 Calculation of $^{242}Pu(sf)$ with Average Kinetic Energy Model

Now we repeat the calculations in previous subsection for the second even isotope of Plutonium, namely, $^{242}Pu(sf)$. Neutron's average laboratory energy is measured by Gerasimenko et al. [4] as $2.011\ MeV$. Demattè et al. [1] measured the total fission fragment kinetic energy as $180.49\ MeV$ and average light and heavy fission fragment's mass numbers as 104 and 138, respectively. Using the Eq. (3.4) we calculated light and heavy average fission fragment kinetic energies per nucleon as $\bar{E}_f^{\ L} = 0.990$ MeV and $\bar{E}_f^{\ H} = 0.562$ MeV. Average center of mass neutron energy is thus found to be $1.235\ MeV$. For the most probable R_T values varies from 1.2 to 1.5 center of mass energies and temperature parameters of light and heavy fragments are listed in Table 3.3. For the Maxwellian spectrum $T_M = 1.343\ MeV$ is constant and obtained from fitting $^{242}Pu(sf)$ experimental data to this spectrum [4]. Chi-square results for each R_T are given in Table 3.4. In this table it is clearly seen that Le Couteur, Maxwellian and Watt spectrums better represent the experimental data when $R_T = 1.5$. Comparison of the theoretical and experimental data (for this ratio) is shown in Figure 3.3 and Figure 3.4.

Table 3.3 Center of mass energy and temperature parameters of light and heavy fragments in comparison with different R_T ratios for the $^{242}Pu(sf)$.

Parameters (MeV)	$\boldsymbol{R_T = 1.2}$	$\boldsymbol{R_T = 1.3}$	$\boldsymbol{R_T = 1.4}$	$\boldsymbol{R_T = 1.5}$
$\bar{\epsilon}_L$	1.347	1.396	1.441	1.482
$\bar{\epsilon}_H$	1.123	1.074	1.029	0.988
T_{LCL}	0.926	0.960	0.991	1.019
T_{LCH}	0.772	0.738	0.708	0.679
T_{WL}	0.898	0.931	0.961	0.988
T_{WH}	0.749	0.716	0.686	0.659

Table 3.4 Chi-squares values of the theoretical spectrums for different R_T values calculated for $^{242}Pu(sf)$.

Chi-Square	$R_T = 1.2$	$R_T = 1.3$	$R_T = 1.4$	$R_T = 1.5$
$X_v^2(LC)$	4.236	2.870	2.018	1.725
$X_v^2(W)$	5.195	3.516	2.348	1.740
$X_v^2(M)$	1.649	1.649	1.649	1.649

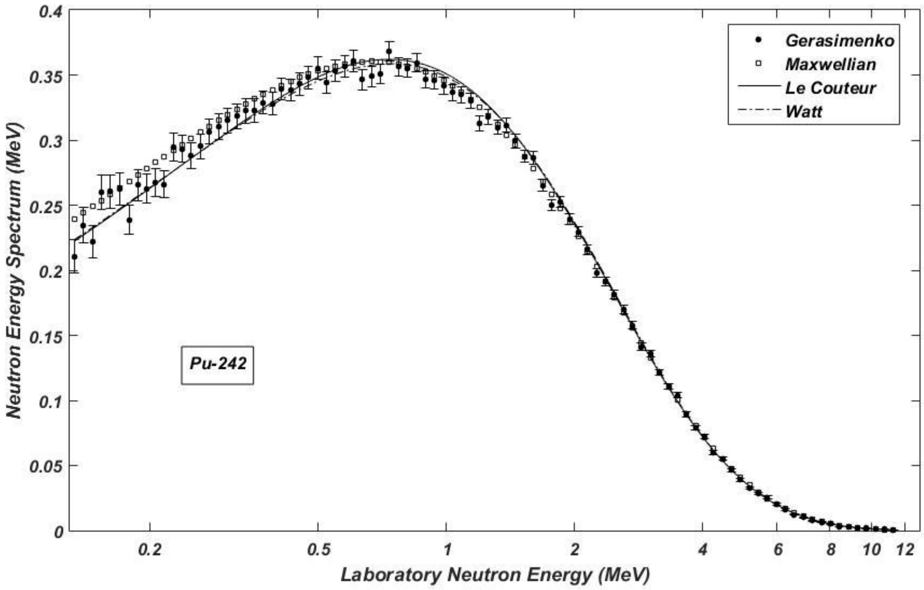

Figure 3.3 Comparison of the theoretical spectra with experimental data of Gerasimenko [4] for $R_T = 1.5$ for the spontaneous fission of $^{242}Pu(sf)$.

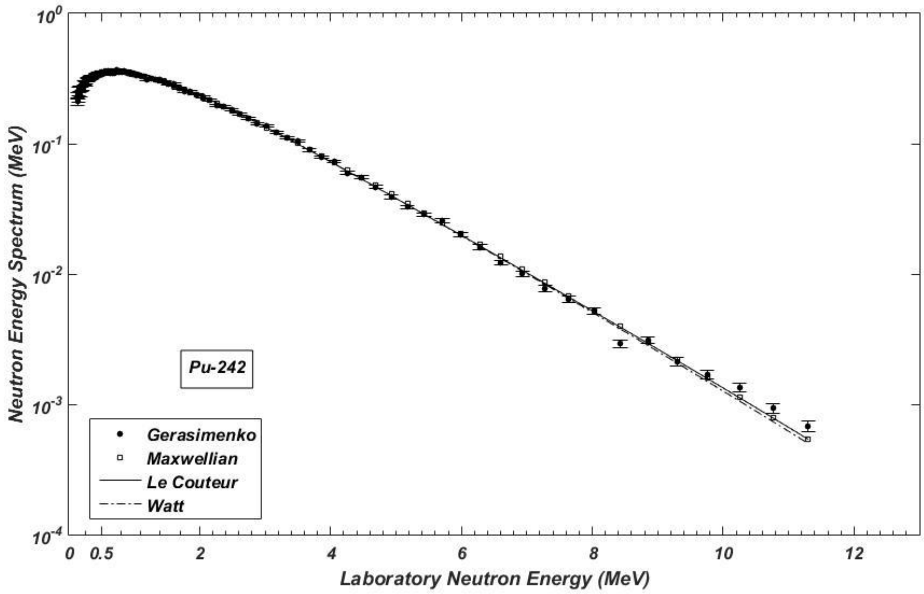

Figure 3.4 Comparison of the theoretical spectra with experimental data of Gerasimenko [4] for $R_T = 1.5$ for the spontaneous fission of $^{242}Pu(sf)$.

3.2 Method 2: Non-Equitemperature Model

This model is based on the non-equality of the nuclear temperatures of the light and heavy fission fragments. It simply uses the idea that each fragment has a maximum temperature T_m when they reach the maximum kinetic energies. It was inspired from the works of Madland and Nix [26].

This model is applicable when the total excitation energy E^* and/or the light fragment excitation energy E_L^* and heavy fragment excitation energy E_H^* of a fissile nuclei is known. Total excitation energy of the fissile nuclei is given by

$$E^* = a\,T_m^2\,, \qquad E^* = E_L^* + E_H^* \tag{3.9}$$

where a is known as level density parameter and described in terms of a mass number and level density constant C (MeV) as $a = {}^{A}/_{C}$. For Plutonium isotopes constant C is suggested by Madland-Nix [26] as $C = 10\ MeV$. We also observed as a result of our calculations on both isotopes of Plutonium that $C = 10\ MeV$ gives the best approximation for the neutron energy spectrum.

Using the ratio $R_T = {}^{T_{mL}}/_{T_{mH}}$ of the maximum temperatures of light and heavy fragments Ohsawa [27] derived the following useful relation for the light and heavy fragments,

$$T_{mL} = \left[\frac{A\,R_T^2}{A_L\,R_T^2 + A_H}\right]^{1/2} T_m\,; \qquad T_{mH} = \left[\frac{A}{A_L\,R_T^2 + A_H}\right]^{1/2} T_m. \quad (3.10)$$

There is a practical relation between T and T_m: $T = \frac{4}{3(\lambda+1)} T_m$, where λ is 1 Maxwellian spectrum and $5/11$ for Le Couteur spectrum. Because T for Maxwellian in center of mass system is same for Watt in laboratory system, we have

$$T_{LC}{}^{L,H} = \frac{11}{12}\,T_m{}^{L,H}\,; \qquad T_W{}^{L,H} = \frac{8}{9}\,T_m{}^{L,H} \quad (3.11)$$

This method is applied to ${}^{240}Pu(sf)$ and ${}^{242}Pu(sf)$ in the following sections.

3.2.1 Calculation of $^{240}\boldsymbol{Pu(sf)}$ with Non-Equitemperature Model

Excitation energy of ${}^{240}Pu(sf)$ is simply calculated by using the equation relating to spontaneous fission in the form of Madland-Nix [26]; $< E^* > = < E_r > - < TKE >$. Average energy release in fission of ${}^{240}Pu(sf)$ is calculated by Tudora et al. [6] as $E_r = 199.63\ MeV$ and average total kinetic energy $TKE = 178.5\ MeV$ is measured by Demattè et al. [1]. Thus, we have $E^* = 21.13\ MeV$ and the maximum temperature $T_m = 0.938\ MeV$.

Eq. (3.10) and Eq. (3.11) provide to calculate T_{mL}, T_{mH}, T_L, T_H. And all these temperature parameters corresponding to different R_T ratios are listed in Table 3.5. The quantitative comparison of calculated spectrums with R_T is done in Table 3.6. Figures 3.5 and 3.6 are drawn for $R_T = 1.3$ which provides the best description of the experimental data.

Table 3.5 Temperature parameters calculated for different R_T values in spontaneous fission of $^{240}Pu(sf)$ using non-equitemperature model.

Parameters (MeV)	$R_T = 1.1$	$R_T = 1.2$	$R_T = 1.3$	$R_T = 1.4$
T_{mL}	0.970	1.014	1.053	1.087
T_{mH}	0.882	0.845	0.810	0.777
T_{LCL}	0.889	0.930	0.965	0.997
T_{LCH}	0.808	0.775	0.743	0.712
T_{WL}	0.862	0.902	0.936	0.966
T_{WH}	0.784	0.751	0.720	0.690

Table 3.6 Chi-squares calculations for different R_T values in spontaneous fission of $^{240}Pu(sf)$ using non-equitemperature model.

Chi-Square	$R_T = 1.1$	$R_T = 1.2$	$R_T = 1.3$	$R_T = 1.4$
$X_v^2(LC)$	1.807	1.275	1.058	1.169
$X_v^2(W)$	2.161	1.473	1.092	1.039
$X_v^2(M)$	1.672	1.672	1.672	1.672

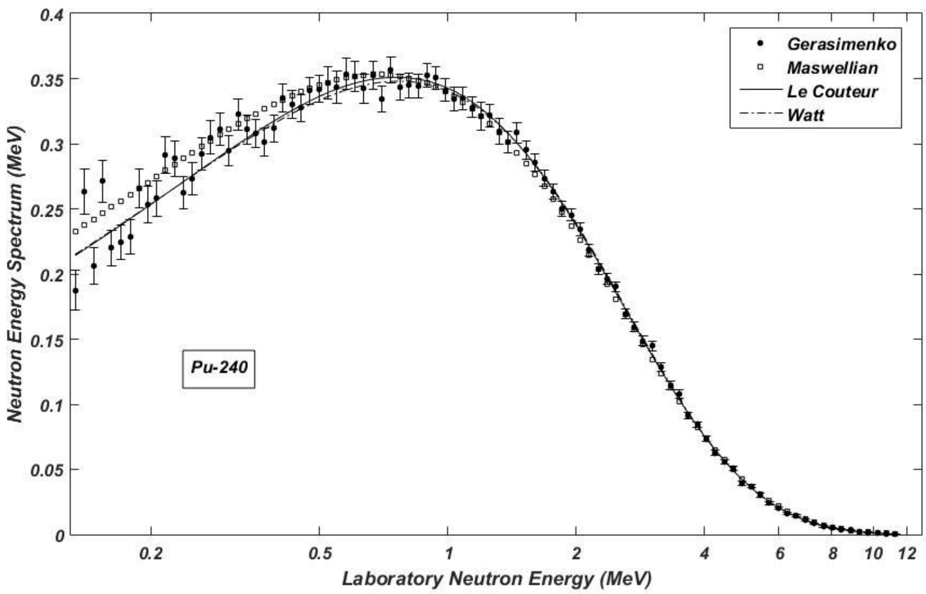

Figure 3.5 Comparison of theoretical spectrums with experimental data of Gerasimenko [4] for the spontaneous fission of $^{240}Pu(sf$ for $R_T = 1.3$.

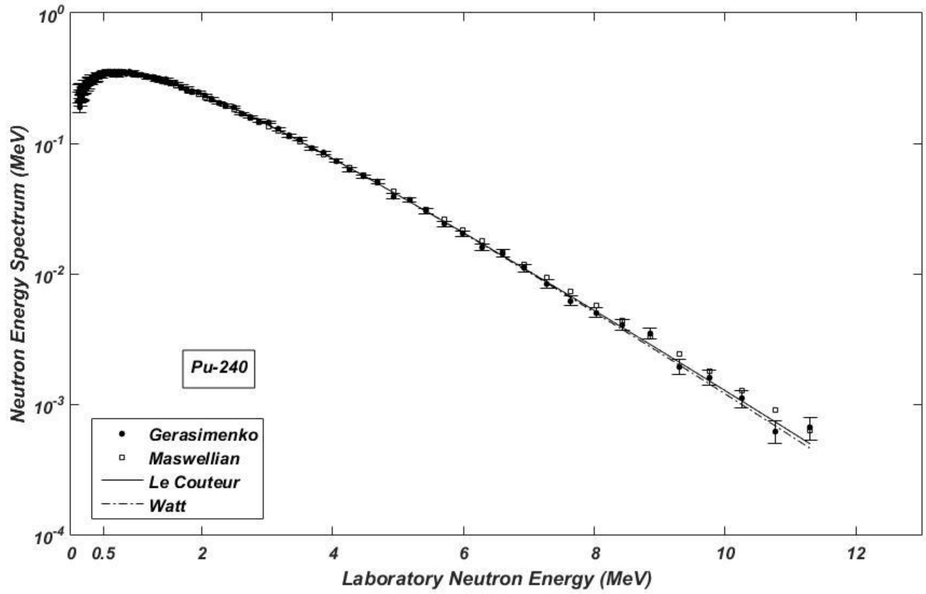

Figure 3.6 Comparison of theoretical spectrums with experimental data of Gerasimenko [4] for the spontaneous fission of $^{240}Pu(sf$ for $R_T = 1.3$.

3.2.2 Calculation of $^{242}Pu(sf)$ with Non-Equitemperature Model

In the case of $^{242}Pu(sf)$, average released energy is $E_r = 200.85\ MeV$ [6] and the total average fission fragment kinetic energy is $TKE = 180.49\ MeV$ [1]. Using this equation; $< E^* > = < E_r > - < TKE >$ we find the excitation energy as $E^* = 20.36\ MeV$. Maximum temperature $T_m = 0.917\ MeV$ is obtained from Eq. (3.9). After calculating the parameters T_{mL} and T_{mH} for specific R_T values with Eq. (3.10) we can get T_L and T_H for theoretical spectrums. Calculation result for this isotope is given in Table 3.7 and Table 3.8. The best chi-square result is taken when $R_T = 1.5$. Thus, in Figure 3.7 and Figure 3.8 calculated spectrums represent the observed data for $R_T = 1.5$ in spontaneous fission of $^{242}Pu(sf)$.

Table 3.7 Temperature parameters calculated for different R_T values in spontaneous fission of $^{242}Pu(sf)$ using non-equitemperature model.

Parameters (MeV)	$R_T = 1.2$	$R_T = 1.3$	$R_T = 1.4$	$R_T = 1.5$
T_{mL}	1.009	1.047	1.080	1.110
T_{mH}	0.841	0.806	0.772	0.740
T_{LCL}	0.925	0.960	0.990	1.017
T_{LCH}	0.771	0.738	0.707	0.678
T_{WL}	0.897	0.931	0.960	0.986
T_{WH}	0.748	0.716	0.686	0.658

Table 3.8 Chi-squares calculations for different R_T values in spontaneous fission of $^{242}Pu(sf)$ using non-equitemperature model.

Chi-Square	$R_T = 1.2$	$R_T = 1.3$	$R_T = 1.4$	$R_T = 1.5$
$X_v^2(LC)$	4.252	2.869	2.021	1.743
$X_v^2(W)$	5.209	3.516	2.350	1.757
$X_v^2(M)$	1.649	1.649	1.649	1.649

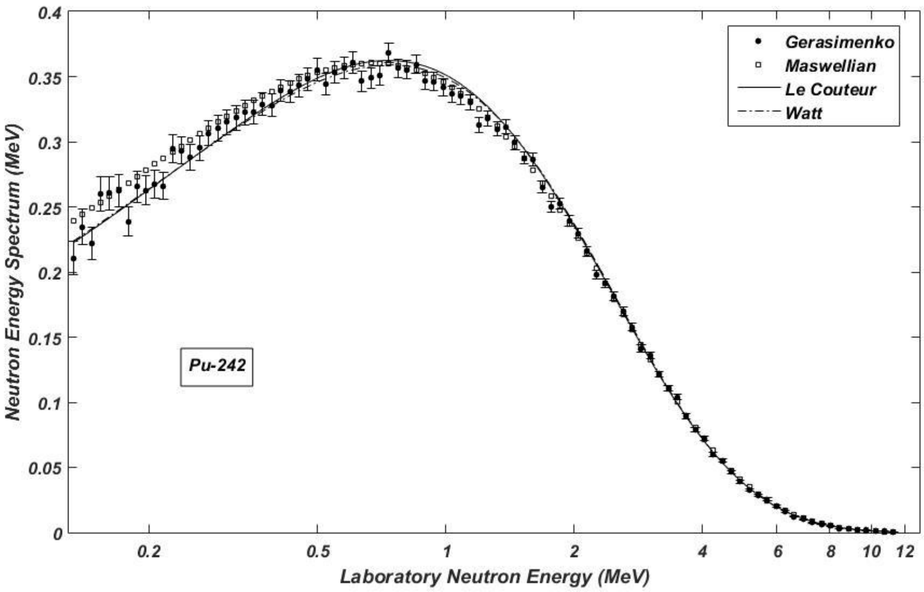

Figure 3.7 Comparison of theoretical spectrums with experimental data of Gerasimenko [4] for the spontaneous fission of $^{242}Pu(sf$ for $R_T = 1.5$.

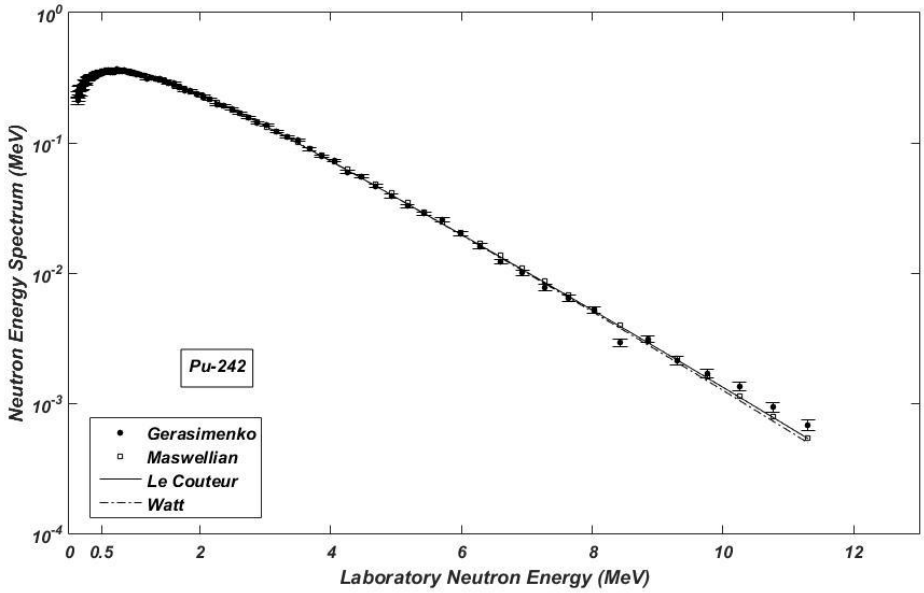

Figure 3.8 Comparison of theoretical spectrums with experimental data of Gerasimenko [4] for the spontaneous fission of $^{242}Pu(sf$ for $R_T = 1.5$.

3.3 Method 3: Total Excitation Energy Partition Model

There is no doubt that $^{252}Cf(sf)$ is the most studied spontaneously fissioning nucleus. It's almost every property is measured or calculated by several researchers [28-38]. It is chosen as a shape standard for all fissioning systems. That's why we also used this nucleus as a standard to calculate parameters of Pu isotopes as Kornilov et. al. [25] did in their valuable study.

Let's remember again Eq. (3.10) which is

$$E^* = \frac{A}{C}\, T_m^2 \,.$$

For Californium (indicated with subscript Cf) and an unknown nuclei (indicated with subscript x) this equation is rewritten as

$$E^{*Cf} = \frac{A^{Cf}}{C^{Cf}} \left(T_m^{Cf}\right)^2 \;; \qquad E^{*x} = \frac{A^x}{C^x}\,(T_m^x)^2. \qquad (3.12)$$

Parameter T of unknown nuclei is written in terms of the parameter T of Cf for light (L) and heavy (H) fragment as

$${\mathrm{T}_{L,H}}^{\mathrm{X}} = {\mathrm{T}_{L,H}}^{\mathrm{Cf}} \sqrt{\frac{E_X^{*L,H}\mathrm{C}^{\mathrm{X}} A_{L,H}^{Cf}}{\mathrm{E}_{\mathrm{Cf}}^{*L,H}\mathrm{C}^{\mathrm{Cf}} A_{L,H}^{X}}}\,. \qquad (3.13)$$

This equation is a fulcrum for the third method. Eq. (3.7) also allows us to relate parameter T and center of mass energy ϵ of Cf and unknown nuclei x as $\frac{T^X}{T^{Cf}} = \frac{\epsilon^X}{\epsilon^{Cf}}$

Then, we can redefine Eq. (3.12) in a new form

$${\epsilon_{L,H}}^{\mathrm{X}} = {\epsilon_{L,H}}^{\mathrm{Cf}} \sqrt{\frac{E_X^{*L,H}\mathrm{C}^{\mathrm{X}} A_{L,H}^{Cf}}{\mathrm{E}_{\mathrm{Cf}}^{*L,H}\mathrm{C}^{\mathrm{Cf}} A_{L,H}^{X}}}\,. \qquad (3.14)$$

Excitation energy of fission fragments for most of the fissile nuclei, Pu isotopes are also included in it, is not known. This is probably because of the hardness in measuring them. Thanks to the relation between excitation energy and neutron multiplicity (v) [7] we can overcome this problem,

$$E_L^* = \frac{v_L}{v_p} E^* \;; \qquad E_H^* = \frac{v_H}{v_p} E^* \qquad (3.15)$$

where $\nu_{\mathrm{p}} = \nu_L + \nu_H$ is the total multiplicity. From the Eq. (15) it is evident that excitation energy and neutron multiplicity are directly proportional and the ratio of the two terms in this equation yields

$$\frac{E_L^*}{E_H^*} = \frac{\nu_L}{\nu_H} \,. \tag{3.16}$$

Substituting the relation $E_{L,H}^* = \frac{A_{L,H}}{C}\left({T_m}^{L,H}\right)^2$ into the Eq. (3.15) together with $E^* = \frac{A}{C}\, T_m^2$ we get a useful relation between T_m^L and T_m^H with T_m ;

$$T_m^L = T_m\sqrt{\frac{\nu_L}{\nu_p}\frac{A}{A_L}} \;;\quad T_m^H = T_m\sqrt{\frac{\nu_H}{\nu_p}\frac{A}{A_H}} \,. \tag{3.17}$$

It is reminded that the temperature ratio was $R_T = \frac{{T_m}^L}{{T_m}^H}$, Eq. (3.17) gives

$$R_T = \sqrt{\frac{\nu_L A_H}{\nu_H A_L}}. \tag{3.18}$$

This final equation is a key to find the required parameters, such that for a suitable R_T we get the ratio ν_L/ν_H, and this ratio allows us to predict excitation energies of light and heavy fragments by using Eq. (15). Finally we obtain temperature parameters of light and heavy fragments with Eq. (3.13). Center of mass energies of two fragments also can now be calculated by Eq. (3.14). This method is applied to $^{240}Pu(sf)$ and $^{242}Pu(sf)$ in the following sections.

3.3.1 Calculation of $^{240}\boldsymbol{Pu(sf)}$ and $^{242}\boldsymbol{Pu(sf)}$ with Total Excitation Energy Partition Model

From the literature reviews we found the required fission properties of $^{252}Cf(sf)$. Total excitation energy 32.9 MeV was measured by Kornilov et al. [25]. Center of mass energy of light and heavy fragments are 1.42 MeV and 1.3 MeV, respectively [28]. Temperature parameters of the light and heavy fragments calculated by Koçak et al. [28,29] for theoretical spectrums Maxwellian, and Le Couteur are $T_{ML} = 0.947\ MeV$, $T_{MH} = 0.87\ MeV$, $T_{LCL} = 0.976\ MeV$ and $T_{LCH} = 0.890\ MeV$. Mass number of light fragment is 108 and of heavy fragment is 144. Vorobyev et al. [39] was measured the total multiplicity $\nu_{\mathrm{pair}} = 3.76$, light fragment's multiplicity $\nu_L = 2.051$ and heavy fragment's multiplicity as $\nu_H = 1.698$. Total multiplicity of

$^{240}Pu(sf)$ is [40] as $\nu_{\text{pair}} = 2.156$ and that's of $^{242}Pu(sf)$ is [40] $\nu_{\text{pair}} = 2.145$. In the first two models we noticed that temperature ratio for $^{240}Pu(sf)$ is $R_T = 1.3$ and for $^{242}Pu(sf)$ is $R_T = 1.5$. Thus, we used these values in this model, too.

Table 3.9 Calculated parameters of $^{240}Pu(sf)$ and $^{242}Pu(sf)$ isotopes using the excitation energy partition model.

Parameters (MeV)	$^{240}Pu(sf)$ $R_T = 1.3$	$^{242}Pu(sf)$ $R_T = 1.5$
$\bar{\epsilon}$	1.250	1.210
$\bar{\epsilon}_L$	1.322	1.366
$\bar{\epsilon}_H$	1.177	1.054
T_{mL}	0.992	1.025
T_{mH}	0.883	0.791
ν_L/ν_H**(Unitless)**	1.228	1.696
ν_L**(Unitless)**	1.188	1.349
ν_H**(Unitless)**	0.968	0.796
E_L^*	11.646	12.807
E_H^*	9.484	7.553
T_{LCL}	0.909	0.939
T_{LCH}	0.809	0.725
T_{WL}	0.882	0.911
T_{WH}	0.791	0.708

Calculated parameters for two isotopes of Pu are listed in Table (3.9) and their chi-square results by using three spectrums are given in the next table, Table (3.10). Comparison of calculated and experimentally observed prompt neutron spectrums for two plutonium isotopes are given in Figures (3.9), (3.10), (3.11) and (3.12).

Table 3.10 Chi-squares for $^{240}Pu(sf)$ and $^{242}Pu(sf)$ isotopes using the excitation energy partition model.

Chi-Square	$^{240}\boldsymbol{Pu(sf)}\ \boldsymbol{R_T = 1.3}$	$^{242}\boldsymbol{Pu(sf)}\ \boldsymbol{R_T = 1.5}$
$\boldsymbol{X_v^2(LC)}$	1.519	3.530
$\boldsymbol{X_v^2(W)}$	1.900	4.135
$\boldsymbol{X_v^2(M)}$	1.672	1.649

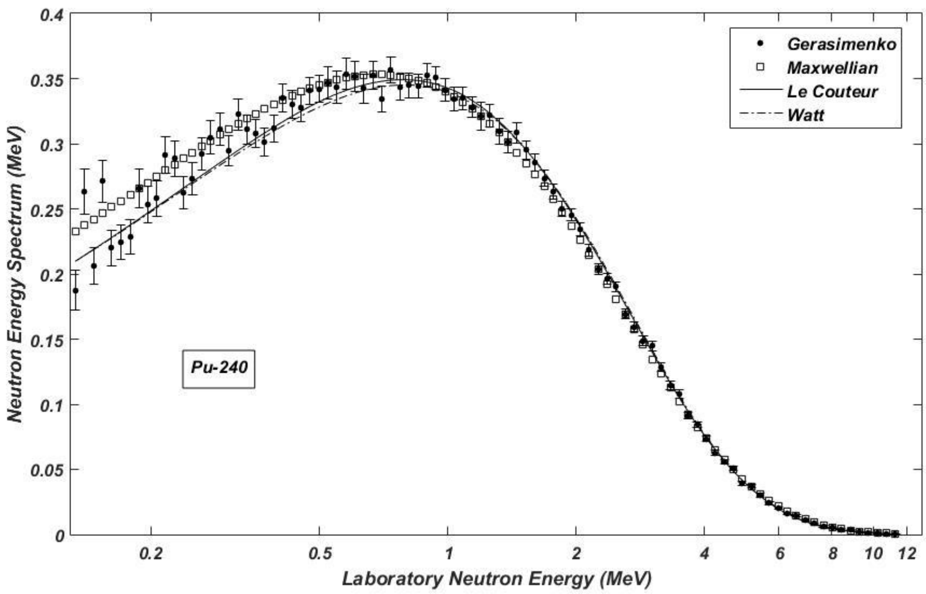

Figure 3.9 Comparison of the theoretical spectrums with experimental data of $^{240}Pu(sf)$ using the excitation energy partition model.

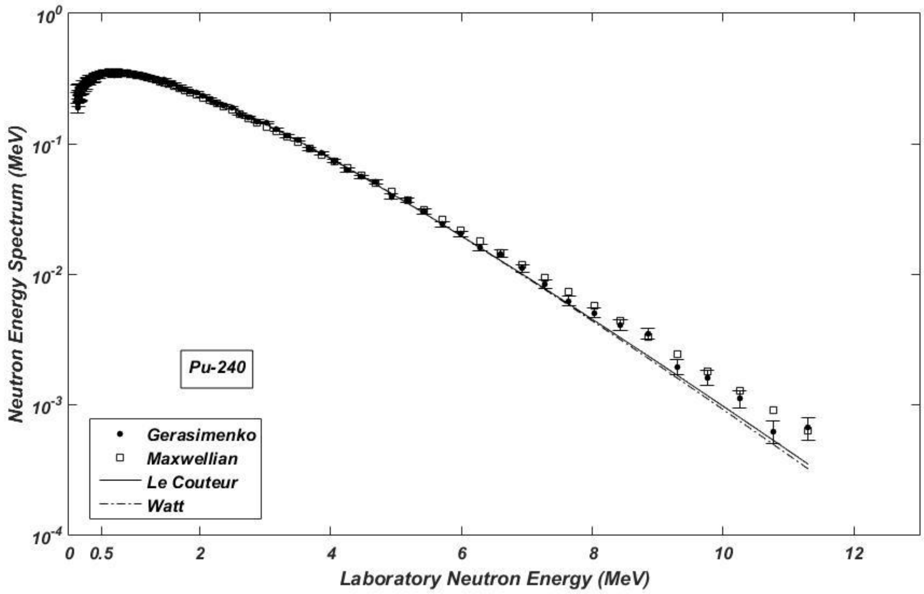

Figure 3.10 Comparison of the theoretical spectrums with experimental data of $^{240}Pu(sf)$ using the excitation energy partition model.

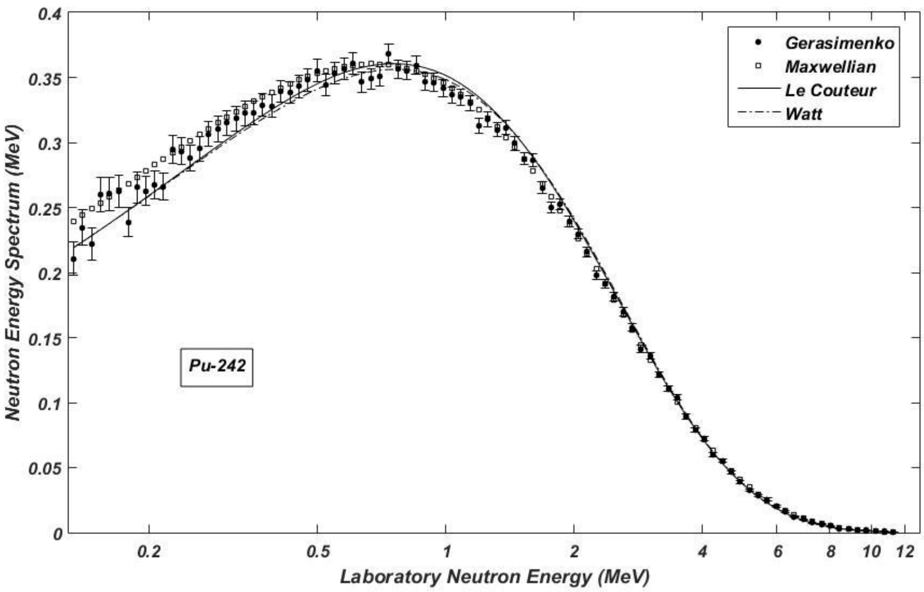

Figure 3.11 Comparison of the theoretical spectrums with experimental data of $^{242}Pu(sf)$ using the excitation energy partition model.

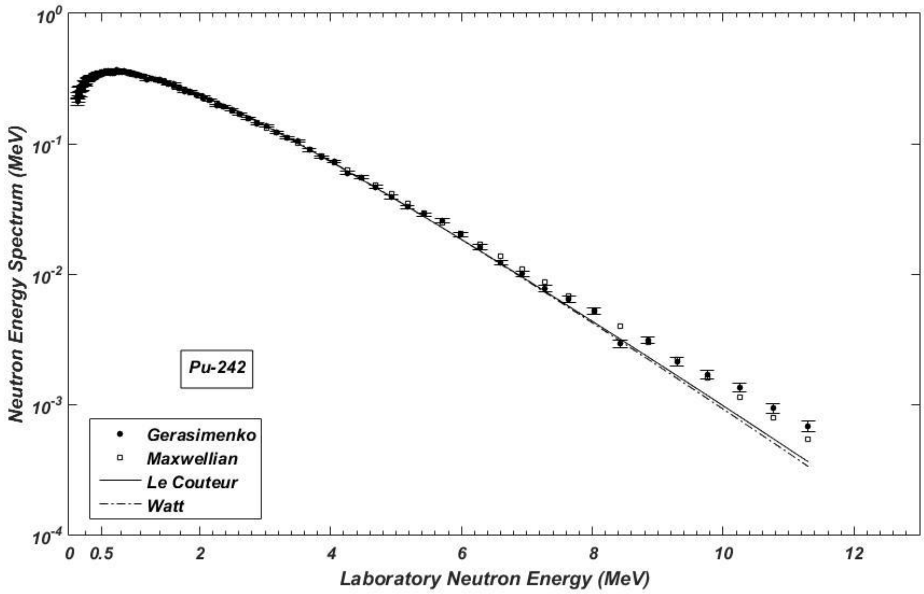

Figure 3.12 Comparison of the theoretical spectrums with experimental data of $^{242}Pu(sf)$ using the excitation energy partition model.

We applied the third model to calculate the center of mass energy, multiplicity and excitation energy of the light and heavy fragments of some neutron induced fission making actinides i.e. $^{235}U(n,f)$, $^{238}U(n,f)$ and $^{239}Pu(n,f)$. All calculated parameters are compared with the results (if exist) in the literature given in Table 3.11. In calculating these parameters some measured (or calculated) quantities corresponding to these actinides were used as given below.

^{235}U

Total kinetic energy $TKE = 171.95\ MeV$ [41],

Average released energy $E_r = 188.946\ MeV$ [41],

Incident neutron energy $E_n = 0.53\ MeV$ [26],

Binding energy $B_n = 6.546\ MeV$ [26],

$A_H = 139,\ A_L = 97$ [41].

$\upsilon_p = 2.43$ [42,43].

^{238}U

Total kinetic energy $TKE = 170.263\ MeV$ [44],

Average released energy $E_r = 185.766\ MeV$ [44],

Incident neutron energy $E_n = 4.8\ MeV$ [44],

Binding energy $B_n = 6.154\ MeV$ [44],

$A_H = 141,\ A_L = 98$ [26].

$\upsilon_p = 2.86$ [45].

^{239}Pu

Total kinetic energy $TKE = 177.60\ MeV$ [46],

Average released energy $E_r = 196.987\ MeV$ [46],

Incident neutron energy $E_n = 0.50\ MeV$ [46],

Binding energy $B_n = 6.534\ MeV$ [46],

$A_H = 140,\ A_L = 100$ [46].

$\upsilon_p = 2.8776$ [3].

In Table 3.11 the calculated center of mass and excitation energies of light and heavy fragments could not be compared with the literature's data because they have neither been measured nor calculated.

Table 3.11 Comparison of the calculated parameters of $^{235}U(n,f)$, $^{238}U(n,f)$ and $^{239}Pu(n,f)$ using the total excitation energy partition model with the literature.

Parameters	$^{235}\boldsymbol{U(n_{th},f)}$		$^{238}\boldsymbol{U(n,f)}$		$^{239}\boldsymbol{Pu(n_{th},f)}$	
(MeV)	This study	Literature	This study	Literature	This study	Literature
$\bar{\boldsymbol{\epsilon}}$	1.411	1.358[26]	1.473		1.467	1.514[26]
$\bar{\boldsymbol{\epsilon}}_{\boldsymbol{L}}$	1.555		1.576		1.541	
$\bar{\boldsymbol{\epsilon}}_{\boldsymbol{H}}$	1.268		1.371		1.394	
$\boldsymbol{\nu_p}$**(Unitless)**	2.43	2.43[42,43]	2.86	2.799[26]	2.8776	2.90[49]
$\boldsymbol{\nu_L}$**(Unitless)**	1.42	1.42[42,43]	1.577		1.552	1.56[49]
$\boldsymbol{\nu_H}$**(Unitless)**	1.01	1.01[42,43]	1.283		1.326	1.34[49]
$\boldsymbol{E^*}$	24.072	21.51[47]	26.457	21.702[26]	26.421	25.137[50]
$\boldsymbol{E_L^*}$	14.072	13.7[47]	14.590		14.247	
$\boldsymbol{E_H^*}$	10.00	8.81[47]	11.867		12.174	
C	11	11[48]	11	11[48]	11	11[48]

CHAPTER IV

RESULTS AND DISCCUSSIONS

Since the discovery of fission, there have been several efforts to describe the observed fission neutron spectrum either spontaneously or neutron induced with deterministic approximations. Among them are Los Alamos model [26], multimodal fission model [32], the point-by-point model [51].

All calculations in thesis work aims to represent the experimentally observed prompt neutron energy spectrum. We did calculations in three ways. In each way the aim is to calculate nuclear temperature parameter of the fission fragments.

In the first method we used the center of mass kinetic energy of the prompt neutrons emitted from the light and heavy fragments. It is preferred when you know the measured value of this parameter. But, unfortunately, we generally don't have for most of the fissioning nuclei,$^{240}Pu(sf)$ and $^{242}Pu(sf)$ are also included in. Owing to this reason we used a commonly accepted approach based on the ratio between the parameters of two fission fragments. The ratio, R_T , generally larger than one and smaller than two. The ratio approach was used also in other two calculation methods. We did calculations for a number of R_T values. We observed that in each method $R_T = 1.3$ for $^{240}Pu(sf)$ and $R_T = 1.5$ for $^{242}Pu(sf)$.

The first method is the easiest one because it requires less parameters in calculation and it is the best among the chosen three models for the isotope $^{240}Pu(sf)$. It is also better for $^{242}Pu(sf)$ when we use the Watt spectrum for calculations. Chi-square is less than one with Watt and Le Couteur spectrums for $^{240}Pu(sf)$ and around 1.7 for $^{242}Pu(sf)$ as given in Tables 3.2 and 3.4. It means that this model well represent the observed spectrum comparing with others especially in case of $^{240}Pu(sf)$.

The second method in our calculation was called “Non-Equitemperature Model”. This name is chosen because it argues that the maximum temperature parameters (T_m^L and T_m^H) of two fission fragments should be different from each other. Chi-squares as given in Tables 3.6 and 3.8 are approximately one with Watt and Le Couteur spectrums for $^{240}Pu(sf)$. In case of the isotope $^{242}Pu(sf)$ this method is better comparing with the first one when we use Le Couteur spectrum. The calculation results are not bad at all especially comparing with other deterministic methods in the literature.

The last method which is called Total Excitation Energy Partition Model basically depends on the spontaneous fission of $^{252}Cf(sf)$ and its fission quantities. The procedure to obtain the temperature parameters of the fission fragments is longer and needs more parameters comparing with the first two models. Although the chi-square results of Watt and Le Couteur spectrums for two Pu isotopes given in Table 3.10 are larger than that of first two models they are less than two for $^{240}Pu(sf)$. For the isotope $^{242}Pu(sf)$ chi-square is around four with Watt and Le Couteur spectrums but it is less than two with Maxwellian spectrum. Thus, we should choose Maxwellian spectrum in calculating the fission spectrum of $^{242}Pu(sf)$. There is a superiority of this model over the other two is that we can find some very important fission quantities of the light and heavy fragments of any fissioning nuclei. Among these quantities is center of mass energy, excitation energy and prompt neutron multiplicity. It is very important in fission studies to know such quantities. For the application except for two even isotopes we calculated several parameters of the neutron induced fission of $^{235}U(n,f)$, $^{238}U(n,f)$, and $^{239}Pu(n,f)$ nuclei and list in the Table 3.11. In that table we compared also our calculated results with the measured (or calculated) ones given in the literature if they exist. Chi-square comparisons of three models for both isotopes are given in Table 4.1 and Table 4.2 as follows.

Table 4.1 Chi-square comparisons for $^{240}Pu(sf)$ with theoretical spectrums.

Chi-Square	First Model	Second Model	Third Model
$X_v^2(LC)$	0.925	1.058	1.519
$X_v^2(W)$	0.978	1.092	1.900
$X_v^2(M)$	1.672	1.672	1.672

Table 4.2 Chi-square comparisons for $^{242}Pu(sf)$ with theoretical spectrums.

Chi-Square	First Model	Second Model	Third Model
$X_v^2(LC)$	1.725	1.743	3.530
$X_v^2(W)$	1.740	1.757	4.135
$X_v^2(M)$	1.649	1.649	1.649

CHAPTER V

CONCLUSION

In this thesis work we tried to do a simple representation of prompt neutron spectrum in the spontaneous fission of $^{240}Pu(sf)$ and $^{242}Pu(sf)$ isotopes. To accomplish this goal we applied Maxwellian, Watt and Le Couteur in laboratory spectrums to three different calculation methods. Calculations are compared with the most recent experimental data measured by Gerasimenkov et.al.

The common parameter R_T was determined carefully as 1.3 for $^{240}Pu(sf)$ and 1.5 for $^{242}Pu(sf)$. This parameter is very useful to relate corresponding parameters of light and heavy fragments. For the comparison of theoretical and experimental data there are two essential parameters; thermodynamic (or nuclear) temperature parameter and fission fragment kinetic energy per nucleon. The latter can be calculated easily from the experimentally measured total kinetic energy in fission. For the former, parameter T, the mentioned calculation methods were used.

For the quantitative comparisons we used chi-square (χ^2) statistical method within the matlab programming code. As chi-square results corresponding to each Pu isotopes and each method are listed in tables Le Couteur in Lab spectrum is very successful, in general, in calculating the neutron energy spectrum of two even isotopes of plutonium in comparison to other two spectrums. While the first method, average kinetic energy model, works well for $^{240}Pu(sf)$ the second method, non-equitemperature model, is better for $^{242}Pu(sf)$. Although the third method, excitation energy partition model, not better than the other two models in representing the observed data, it is very useful to calculate the center of mass energy, neutron multiplicity and excitation energy parameters of the fission fragments. The aforementioned parameters of the individual fragments of most fissile nuclei are unfortunately not known either of measured or calculated. It is quite successful in calculating these parameters as seen in Table 3.11 which is done for neutron induced fission of $^{235}U(n,f)$, $^{238}U(n,f)$ and $^{239}Pu(n,f)$.

We finally note that the chi-square results for $^{240}Pu(sf)$ are smaller than that of $^{242}Pu(sf)$. This is probably due to the fact that the probability to make fission is higher for the lighter isotope.

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APPENDIX A

Transformation from Maxwellian to Watt Spectrum:

The center of mass Maxwellian spectrum can be expressed as in Eq. (2.8),

$$\Phi_M(\epsilon) = \frac{2\sqrt{\epsilon}}{\sqrt{\pi T_M{}^3}} e^{-\epsilon/T_M} \qquad (A.1)$$

Using the transformation function given in Eq. (2.11) for Maxwellian spectrum

$$N(E) = \int \frac{\Phi(\varepsilon)d\varepsilon}{4\sqrt{E_f\varepsilon}} = \frac{1}{4\sqrt{E_f\varepsilon}} \int \frac{2}{\sqrt{\pi}T^{3/2}} \frac{\sqrt{\varepsilon}e^{-\varepsilon/T}d\varepsilon}{\sqrt{\varepsilon}} d\varepsilon = \frac{1}{2\sqrt{\pi}\sqrt{E_f}T^{3/2}} \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} e^{-\varepsilon/T} d\varepsilon \qquad (A.2)$$

$$= \frac{1}{2\sqrt{\pi}\sqrt{E_f}T^{3/2}} (-T)(e^{-\varepsilon/T}) \Big| {}_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2}) = \frac{1}{2\sqrt{\pi E_f T}} (e^{-(\sqrt{E}+\sqrt{E_f})^2/T} - e^{-(\sqrt{E}-\sqrt{E_f})^2/T})$$

$$= \frac{(-1)}{2\sqrt{\pi E_f T}} (e^{-\left(E/T + E_f/T + 2\sqrt{E}\sqrt{E_f/T}\right)} - e^{-\left(E/T + E_f/T - 2\sqrt{E}\sqrt{E_f/T}\right)}$$

$$= \frac{(-1)}{\sqrt{\pi E_f T}} e^{-E/T} e^{-E_f/T} (\frac{e^{2\sqrt{E}\sqrt{E_f/T}} - e^{-2\sqrt{E}\sqrt{E_f/T}}}{2})$$

Then, we get,

$$N(E) = \frac{e^{-E_f/T}}{(\pi E_f T)^{1/2}} e^{-E/T} \sinh[{}^{2(EE_F)^{1/2}}/_{T}] \qquad (A.3)$$

This equation (distribution) is called as the Watt spectrum.

APPENDIX B

Transformation from Weisskopf's Spectrum to Feather Spectrum:

The center of mass Weisskopf spectrum can be expressed as in Eq. (2.7),

$$\Phi_{W_f}(\epsilon) = \frac{\epsilon}{T_{Wf}{}^2} e^{-\epsilon/T_{Wf}} \tag{B.1}$$

Using the transformation function given in Eq. (2.11

$$N(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \frac{\phi(\varepsilon)d\varepsilon}{4\sqrt{E_f\varepsilon}} = \frac{1}{4\sqrt{E_f}}\int \frac{(\varepsilon/_{T^2})e^{-\varepsilon/_T}}{\sqrt{\varepsilon}}d\varepsilon = \frac{1}{4\sqrt{E_f}}\int \frac{\sqrt{\varepsilon}}{T^2}e^{-\varepsilon/_T}\,d\varepsilon \tag{B.2}$$

$$N(E) = \frac{1}{4\sqrt{E_f}}\left[\frac{-\sqrt{\varepsilon}}{Te^{\varepsilon/_T}} - \sqrt{\pi\varepsilon}erfc(\sqrt{\varepsilon/_T})\frac{1}{2T\sqrt{\varepsilon/_T}}\right]_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \tag{B.3}$$

$$N(E) = \frac{1}{4\sqrt{E_f}}\left[-\frac{\sqrt{E}+\sqrt{E_f}}{(\sqrt{E}+\sqrt{E_f})^2/_T} - \sqrt{\pi}(\sqrt{E}+\sqrt{E_f})erfc(\frac{\sqrt{E}+\sqrt{E_f}}{\sqrt{T}})\frac{1}{2\sqrt{T}(\sqrt{E}+\sqrt{E_f})}\right] -$$

$$\left[-\frac{\sqrt{E}-\sqrt{E_f}}{(\sqrt{E}-\sqrt{E_f})^2/_T} - \sqrt{\pi}(\sqrt{E}-\sqrt{E_f})erfc(\frac{\sqrt{E}-\sqrt{E_f}}{\sqrt{T}})\frac{1}{2\sqrt{T}(\sqrt{E}-\sqrt{E_f})}\right] \tag{B.4}$$

where

$$erfc(x) = 1 - \mathrm{erf}(x) = \frac{2}{\pi}\int_0^\infty \exp(-t^2)\,dt$$

and

$$erf(x) = 1 - \mathrm{erf}(x) = \frac{2}{\pi}\int_0^x \exp(-t^2)\,dt$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}(\frac{1}{2}\int_{-x}^{x}\exp(-t^2)dt) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x}\exp(-t^2)d\mathrm{as}$$

$$t = \frac{u}{\sqrt{2}} \Rightarrow dt = \frac{du}{\sqrt{2}}\ ;\ t^2 = \frac{u^2}{2}$$

Then,

$$\mathrm{erf}(x) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x} \exp(-\frac{u^2}{2})\frac{du}{\sqrt{2}} = \frac{1}{\sqrt{2\pi}}\int_{-x}^{x} exp\left(-\frac{u^2}{2}\right) du$$

$$erfc(x) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-x}^{x} exp\left(-\frac{u^2}{2}\right) du$$

Therfore,

$$\mathrm{N(E)}=\frac{-1}{4\sqrt{E_f}}\frac{(\sqrt{E}+\sqrt{E_f})}{T} exp\left[-\left.(\sqrt{E}+\sqrt{E_f})^2\right/T\right] - \frac{1}{2}\sqrt{\frac{\pi}{T}}\frac{1}{4\sqrt{E_f}}(1 -$$

$$\frac{1}{\sqrt{2\pi}}\int_{-x_1}^{x_1} exp\left(-\frac{u^2}{2}\right) du \ +$$

$$\frac{1}{4\sqrt{E_f}}\frac{(\sqrt{E}-\sqrt{E_f})}{T} exp\left[-\left.(\sqrt{E}-\sqrt{E_f})^2\right/T\right] - \frac{1}{2}\sqrt{\frac{\pi}{T}}\frac{1}{4\sqrt{E_f}}(1 -$$

$$\frac{1}{\sqrt{2\pi}}\int_{-x_2}^{x_2} exp\left(-\frac{u^2}{2}\right) du \qquad (B.5)$$

where $x_1 = \frac{(\sqrt{E}+\sqrt{E_f})}{T}$; $x_2 = \frac{(\sqrt{E}-\sqrt{E_f})}{T}$, which leads to

$$\mathrm{N(E)} = -\frac{1}{4\sqrt{E_f}}\frac{(\sqrt{E}+\sqrt{E_f})}{T}\exp(-{x_1}^2) - \frac{1}{8}\sqrt{\frac{\pi}{TE_f}} + \frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\mathrm{erf}(x_1) +$$

$$\frac{1}{4\sqrt{E_f}}\frac{(\sqrt{E}-\sqrt{E_f})}{T}\exp(-{x_2}^2) + \frac{1}{8}\sqrt{\frac{\pi}{TE_f}} - \frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\mathrm{erf}(x_2) \qquad (B.6)$$

in which,

$$\exp(-{x_1}^2) = \exp\left[-\left(\sqrt{\frac{E}{T}} + \sqrt{\frac{E_f}{T}}\right)^2\right] = \exp\left[-\frac{1}{2}\left(\sqrt{\frac{2E}{T}} + \sqrt{\frac{2E_f}{T}}\right)^2\right]$$

and

$$\exp(-{x_2}^2) = \exp\left[-\left(\sqrt{\frac{E}{T}} + \sqrt{\frac{E_f}{T}}\right)^2\right] = \exp\left[-\frac{1}{2}\left(\sqrt{\frac{2E}{T}} + \sqrt{\frac{2E_f}{T}}\right)^2\right]$$

Finally, we have

$$N(E) = -\frac{\sqrt{2}}{8\sqrt{TE_f}}\frac{(\sqrt{E}+\sqrt{E_f})}{T}\exp\left(-{x_1'}^2\right) + \frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\operatorname{erf}(x_1) +$$

$$\frac{\sqrt{2}}{8\sqrt{TE_f}}\frac{(\sqrt{E}-\sqrt{E_f})}{T}\exp\left(-{x_2'}^2\right) - \frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\operatorname{erf}(x_2)$$

where $x_1' = \sqrt{\frac{2E}{T}} + \sqrt{\frac{2E_f}{T}}$ and $x_2' = \sqrt{\frac{2E}{T}} - \sqrt{\frac{2E_f}{T}}$

and,

$$\frac{1}{4\sqrt{E_f}}\frac{(\sqrt{E}\pm\sqrt{E_f})}{T} = \frac{1}{4\sqrt{E_f}\sqrt{T}}\frac{\sqrt{2}}{2}\left(\sqrt{\frac{2E}{T}} \pm \sqrt{\frac{2E_f}{T}}\right) = \frac{\sqrt{2}}{8\sqrt{E_f T}}\left(\sqrt{\frac{2E}{T}} \pm \sqrt{\frac{2E_f}{T}}\right)$$

which also equals to;

$$= \frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\frac{2}{\sqrt{2\pi}}\left(\sqrt{\frac{2E}{T}} \pm \sqrt{\frac{2E_f}{T}}\right).$$

Hence,

$$N(E) = \left(\frac{\pi^{\frac{1}{2}}}{8E^{1/2}T^{1/2}}\right)\begin{bmatrix} -(2\pi)^{-\frac{1}{2}}\,2\left(\sqrt{\frac{2E}{T}} + \sqrt{\frac{2E_f}{T}}\right)\exp\left(\frac{-{x_1}^2}{2}\right) + \operatorname{erf}(x_1) + \\ (2\pi)^{-\frac{1}{2}}\,2\left(\sqrt{\frac{2E}{T}} - \sqrt{\frac{2E_f}{T}}\right)\exp\left(\frac{-{x_2}^2}{2}\right) - \operatorname{erf}(x_2) \end{bmatrix}$$

$$N(E) = \left(\frac{\pi^{\frac{1}{2}}}{8E^{1/2}T^{1/2}}\right)\left\{F\left[\left(\frac{2E}{T}\right)^{1/2} + \left(\frac{2E_f}{T}\right)^{1/2}\right] - F\left[\left|\left(\frac{2E}{T}\right)^{1/2} - \left(\frac{2E_f}{T}\right)^{1/2}\right|\right]\right\} \qquad (B.7)$$

in which,

$$\mathrm{F}(x) = 2x(2\pi)^{1/2}\exp\left(-x^2/2\right) + (2\pi)^{1/2}\int_{-x}^{x}\exp\left(-t^2/2\right)dt,$$

Eqn. (B.7) can also be written as

$$N_F(E) = \frac{1}{4\sqrt{E_f T}}\left[\left(-x_1 e^{-{x_1}^2} + \operatorname{erf}(x_1)\right) - \left|\left(-x_2 e^{-{x_2}^2} - \operatorname{erf}(x_2)\right)\right|\right]$$

which is known as Feather spectrum.

APPENDIX C

Transformation of the Le-Couter Spectrum to the LAB system:

The Le Couter Spectrum in Center of Mass System is;

$$\Phi(\varepsilon) = C\varepsilon^{5/11} e^{-\varepsilon/T} \qquad (C.1)$$

where $C = \frac{1}{\Gamma\left(\frac{16}{11}\right)\left(\frac{11}{12}T_m\right)^{16/11}} = (\Gamma(\frac{16}{11})T^{16/11})^{-1}$.

The relation between center of mass and LAB system is,

$$N(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \frac{\phi(\varepsilon)d\varepsilon}{4\sqrt{E_f\varepsilon}} \qquad (C.2)$$

so,

$$N(E) = \int \frac{C\varepsilon^{5/11}e^{-\varepsilon/T}}{4\sqrt{E_f\varepsilon}} d\varepsilon = \frac{C}{4\sqrt{E_f}} \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \varepsilon^{-1/22} e^{-\varepsilon/T} d\varepsilon \qquad (C.3)$$

and

$$\mathrm{I} = \int \varepsilon^{-1/22} e^{-\varepsilon/T} d\varepsilon = -\left(T\left(\frac{x}{T}\right)^{1/22} igamma\left(21/22, x/T\right)\right)/x^{1/22} . \qquad (C.4)$$

As

$$F(a) = \int_0^\infty e^{-t} t^{a-1} dt$$

and

$\Gamma(x, a) = \frac{1}{\Gamma(a)}\int_0^x e^{-t}t^{a-1}dt$ or $gamma\ inc(x, a)$ is incomplete gamma function in (C.4), remembering the relation

$$igamma(a, x) = gamma(a)[1 - gammainc(x, a)], \qquad (C.5)$$

we can rearrange Eqn. (C.4) as

$$\mathrm{I} = \int_{x^-}^{x^+} \varepsilon^{-1/22} e^{-\varepsilon/T} d\varepsilon = -T^{21/22}\, igamma(21/22\, \varepsilon/T) \Big|_{x^-}^{x^+} \qquad (C.6)$$

$$\mathrm{I} = -T^{21/22}\left[igamma\left(21/22, \frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}\right) - igamma(21/22, \frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T})\right] \qquad (C.7)$$

where $\quad x^+ = \left(\sqrt{E} + \sqrt{E_f}\right)^2 \quad$ and $\quad x^- = \left(\sqrt{E} - \sqrt{E_f}\right)^2$

Substituting Equation (C.5) into (C.7) we have

$$\mathrm{I} = -T^{21/22}\begin{bmatrix} gamma(21/22)\left\{1 - gammainc(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, 21/22\right\} + \\ -gamma(21/22)\left\{1 - gammainc(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, 21/22\right\} \end{bmatrix}$$

$$\mathrm{I} = -T^{21/22}\left[-gammainc\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, 21/22\right) + gammainc\left(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, 21/22\right)\right]$$

or basically,

$$\mathrm{I} = T^{21/22}\left[gammainc\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, 21/22\right) - gammainc(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, 21/22\right]$$

Thus,

$$\mathrm{N(E)}=\left(\frac{1}{4\sqrt{E_f T}}\right)\left(\frac{gamma(21/22)}{gamma(16/11)}\right)\left[gammainc\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, \frac{21}{22}\right) - gammainc(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, \frac{21}{22})\right]. \qquad (C.8)$$

This equation can also be written as

$$N_{LC}(E) = \frac{\Gamma(21/22)}{\Gamma(16/11)}\frac{1}{4\sqrt{E_f T}}\left[\Gamma\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, \frac{21}{22}\right) - \Gamma\left(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, \frac{21}{22}\right)\right]. \qquad (C.9)$$

The equation in (C.9) is called the Le Couteur spectrum in Lab system or basically the LCL spectrum.