REACTIVE POWER MARKET DESIGN FOR PV-RICH DISTRIBUTION NETWORKS

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DECLARATION OF ORIGINALITY

I hereby declare that I am the sole author of this thesis and that this is the true copy of my thesis, including the final revisions, approved by my thesis committee. All data and information have been obtained, produced, and presented in accordance with the rules of research ethics and principles of academic honesty. As required by these rules, to the best of my knowledge I have acknowledged ideas, thoughts, and any copyrighted material in accordance with the standard referencing rules. I certify that any part of this thesis has not been submitted for a degree or diploma in another educational institution.

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ABSTRACT

The increasing integration of distributed photovoltaic (PV) generation sources into medium-voltage distribution networks offers significant opportunities for sustainable energy systems while introducing new technical and economic challenges. As PV generation meets local consumption, the active power drawn from the transmission network decreases, whereas reactive power demand persists; this situation increases the Q/P ratio at the High-Voltage/Medium-Voltage (HV/MV) connection point and imposes financial penalties on Distribution System Operators (DSO). Modern PV inverters can function as flexible reactive power sources owing to their dynamic P–Q capability curves. This thesis proposes a local market model for procuring reactive power as an ancillary service from PV inverters at the distribution level. The proposed model is structured in which the DSO serves as both the system operator and the market operator. The market-clearing problem is formulated as an optimal power flow problem with a 24-hour time horizon and hourly settlement periods. The model's original contributions include a sustainability threshold, a $\pm 10\%$ tolerance-band mechanism, and an Expected Payment Function comprising components for capacity, utilization, and opportunity cost. This conceptual framework reduces the penalty risk for DSOs, creates a new revenue stream for PV owners, and establishes the theoretical foundation for market-based reactive power management in networks with high PV penetration.
Keywords: Reactive Power Market, Photovoltaic Inverters, Distribution System Operator, Expected Payment Function, Ancillary Services

ÖZET

Dağıtık fotovoltaik (PV) üretim kaynaklarının orta gerilim dağıtım şebekelerine artan entegrasyonu, sürdürülebilir enerji sistemleri açısından önemli fırsatlar sunmakla birlikte yeni teknik ve ekonomik zorluklar ortaya çıkarmaktadır. PV üretimi yerel tüketimi karşıladıkça iletim şebekesinden çekilen aktif güç azalırken reaktif güç talebi devam etmekte; bu durum Yüksek Gerilim/Orta Gerilim (YG/OG) bağlantı noktasındaki Q/P oranını artırmakta ve Dağıtım Sistemi Operatörlerine (DSO) mali cezalar uygulanmasına neden olmaktadır. Modern PV inverterleri dinamik P–Q kapasite eğrileri sayesinde esnek reaktif güç kaynakları olarak işlev görebilmektedir. Bu tez, dağıtım seviyesinde PV inverterlerinden reaktif gücün yan hizmet olarak tedarik edilmesine yönelik yerel bir piyasa modeli önermektedir.

Önerilen model, Sabit Reaktif Güç Modu üzerine kurgulanmış olup DSO'nun hem sistem operatörü hem de piyasa operatörü rolünü üstlendiği bir yapıya sahiptir. Piyasa takas problemi, 24 saatlik zaman ufku ve saatlik uzlaştırma periyodu ile optimal güç akışı problemi olarak formüle edilmiştir. Modelin özgün katkıları arasında sürdürülebilirlik eşiği, $\pm\%10$ tolerans bandı mekanizması ve kapasite, kullanım ile fırsat maliyeti bileşenlerinden oluşan Beklenen Ödeme Fonksiyonu yer almaktadır. Bu kavramsal çerçeve, DSO'ların ceza riskini azaltırken PV sahiplerine yeni bir gelir akışı yaratmakta ve yüksek PV penetrasyonlu şebekelerde piyasa tabanlı reaktif güç yönetiminin teorik temelini oluşturmaktadır.

Anahtar Kelimeler: Reaktif Güç Piyasası, Fotovoltaik İnverterler, Dağıtım Sistemi Operatörü, Beklenen Ödeme Fonksiyonu, Yan Hizmetler

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LIST OF ACRONYMS AND ABBREVIATIONS

AC	Alternating Current
APV	Active Power-Voltage Control Mode
AVR	Automatic Voltage Regulator
BS EN	British Standard European Norm
CAPEX	Capital Expenditure
CHP	Combined Heat and Power
DAM	Day-Ahead Market
DC	Direct Current
DER	Distributed Energy Resource
DG	Distributed Generation
DLMP	Distribution Locational Marginal Price
DSO	Distribution System Operator
ENTSO-E	European Network of Transmission System Operators for Electricity
EPDK	Energy Market Regulatory Authority (Türkiye)
EPF	Expected Payment Function
FACTS	Flexible AC Transmission Systems
FFR	Fast Frequency Response

FRT Fault Ride-Through

GA Genetic Algorithm

GW Gigawatt

GWO Grey Wolf Optimizer

HV High Voltage

HVDC High Voltage Direct Current

IBR Inverter-Based Resource

IEA International Energy Agency

IEC International Electrotechnical Commission

IEEE Institute of Electrical and Electronics Engineers

IET Institution of Engineering and Technology

IGBT Insulated Gate Bipolar Transistor

IRENA International Renewable Energy Agency

ISO Independent System Operator

LCOE Levelized Cost of Energy

LMP Locational Marginal Price

LV Low Voltage

MIMO Multiple-Input Multiple-Output

MOSFET Metal-Oxide-Semiconductor Field-Effect Transistor

MPP	Maximum Power Point
MPPT	Maximum Power Point Tracking
MSC	Mechanically Switched Capacitor
MSR	Mechanically Switched Reactor
MV	Medium Voltage
MVA	Mega Volt-Ampere
MVAR	Mega Volt-Ampere Reactive
MW	Megawatt
NGC	National Grid Company
NPV	Net Present Value
NREL	National Renewable Energy Laboratory
OLTC	On-Load Tap Changer
OPF	Optimal Power Flow
OPEX	Operating Expenditure
ORPD	Optimal Reactive Power Dispatch
P	Active Power
PERC	Passivated Emitter and Rear Cell
PF	Power Factor
PJM	Pennsylvania-New Jersey-Maryland Interconnection

PQ	Active-Reactive Power Control Mode
PSO	Particle Swarm Optimization
PV	Photovoltaic
PVPS	Photovoltaic Power Systems Programme
PWM	Pulse Width Modulation
Q	Reactive Power
RES	Renewable Energy Source
RMS	Root Mean Square
RP	Reactive Power
RTM	Real-Time Market
S	Apparent Power
SOA	Safe Operating Area
STATCOM	Static Synchronous Compensator
SVC	Static VAR Compensator
TSO	Transmission System Operator
TW	Terawatt
TWh	Terawatt-hour
VAR	Volt-Ampere Reactive
VCA	Voltage Control Area

VQ Voltage-Reactive Power Control Mode

VSC Voltage Source Converter

VV Volt-VAR Control Mode

VW Volt-Watt Control Mode

kV Kilovolt

kW Kilowatt

1. INTRODUCTION

Access to energy is one of the fundamental factors that directly affect the welfare of societies and their economic development [1, 2]. The reliable and continuous provision of electrical energy is regarded as the essential infrastructure of modern economies [3]. When the energy sector is examined within the framework of industrial growth, urbanization, and technological advancements, it is observed that investments are largely directed towards meeting the increasing electricity demand.

Global energy demand exhibits a rapid upward trend driven by population growth, economic expansion, urbanization, digitalization, and electrification. As illustrated in Figure 1.1, global electricity consumption reached 31,153 TWh in 2024, representing a record increase of 4.3% compared to 2023 [4]. Future projections indicate that this growth will continue; demand is expected to grow at an annual average rate of 2.5% to 3.2% in the coming years, depending on the direction of adopted energy policies [5].

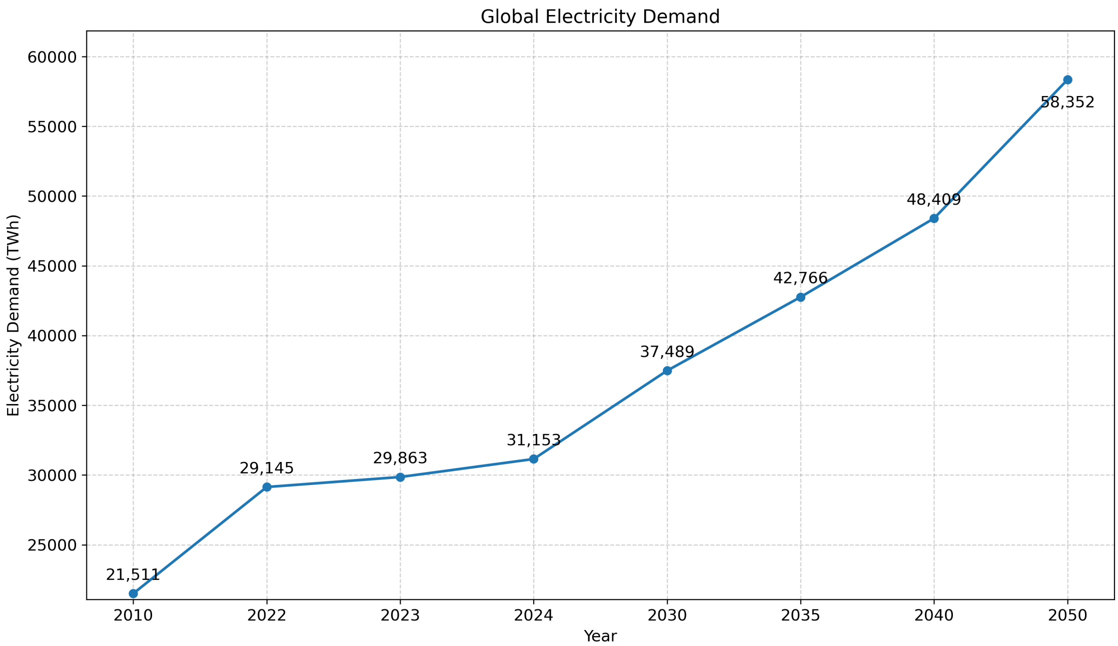

Figure 1.1: *Global electricity demand: historical data and projections (2010–2050, TWh)*

However, meeting this increasing global demand primarily through fossil fuels creates serious structural challenges in terms of energy supply security and environmental sustainability. While fossil fuels, including coal, oil, and natural gas, continue to play a dominant role in global energy production, their economic and ecological limitations are becoming increasingly apparent. Consequently, the energy transition is now considered a necessity rather than a choice; countries are developing and implementing new energy policies to support this transformation in line with their social and economic development goals [5].

At the center of this energy transition are renewable energy sources, particularly solar energy. As presented in Figure 1.2, global Photovoltaic (PV) capacity reached 1.6 TW by the end of 2023, with roughly 400 GW of new installations compared to 2022 [6]. Data from 2024 indicate that global PV capacity has exceeded 2.2 TW, with an estimated 600 GW of additional capacity recorded in a single year [7]. This capacity corresponds to approximately 2,700 TWh of annual electricity generation, meeting roughly 9% of global electricity demand.

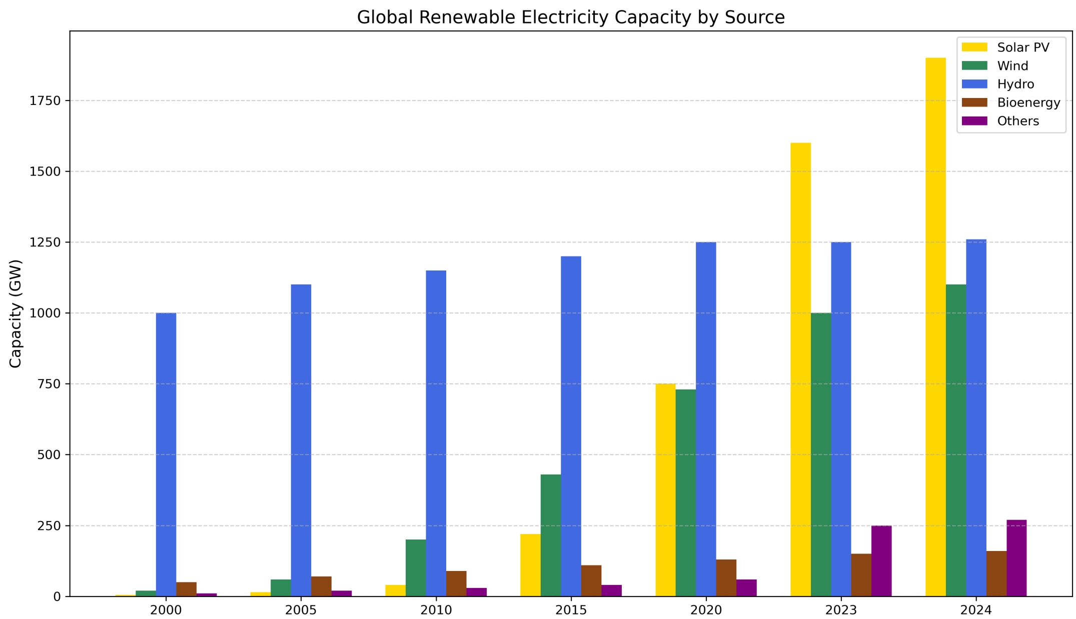

Figure 1.2: *Global renewable electricity capacity by source (2000–2024, GW)*

This rapid growth trend in solar energy at the global scale is similarly reflected in Türkiye. As shown in Figure 1.3, the share of distribution-level PV plants in total installed capacity reached approximately 15% by 2024, rising to roughly 18 GW. The electricity generated by these plants constitutes approximately 6% of the country's total production [8]. This rapid increase fundamentally challenges traditional grid operation approaches, creating new technical requirements and operational difficulties, particularly in voltage stability, reactive power management, and bidirectional power flow [9, 10].

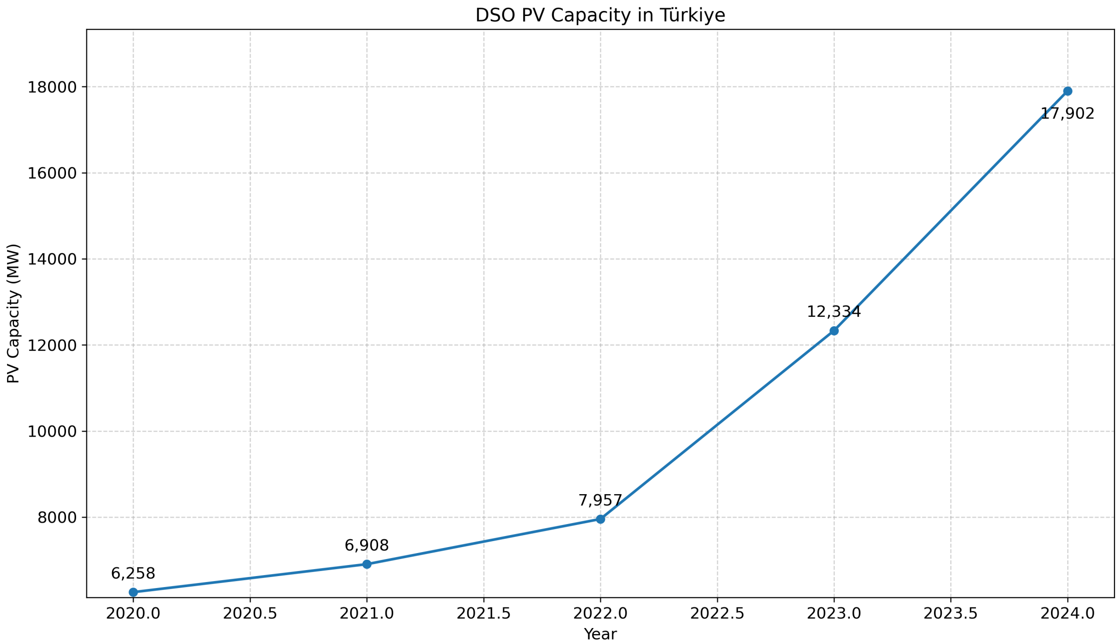

Figure 1.3: *Distribution-connected PV capacity growth in Türkiye (2020–2024, MW)*

1.1 Problem Statement

Traditional distribution networks were designed as passive structures where electricity flows unidirectionally from centralized generation facilities to consumers. However, high PV penetration is transforming this fundamental paradigm, converting networks into dynamic systems with bidirectional power flow [11].

The most prominent regulatory consequence of this transformation is the exceedance of specified limits in the Q/P ratio measured at High Voltage (HV)/Medium Voltage (MV) transformer substations. Under the current regulatory framework, PV plants at the distribution level typically operate at unity power factor ($\cos\varphi = 1$) and generate only active power. Since PV generation meets a portion of local consumption, the net active power drawn from the transmission system decreases. In contrast, the network's reactive power demand remains constant or may even increase due to the capacitive effect of underground cables. A reduction in the active energy value in the denominator of ratio calculations increases the Q/P ratio and causes regulatory limits to be exceeded.

In many countries, this issue is addressed through reactive energy ratio limits applied monthly at HV/MV transformer substations. Financial penalties are imposed on distribution companies when these limits are exceeded. However, current regulatory frameworks generally contain a significant gap: generation facilities with installed capacity below a specified threshold are exempt from reactive power control obligations, and this exemption covers the vast majority of PV plants connected to the distribution network.

This regulatory structure creates a contradictory situation. Distribution companies face the risk of Q/P penalties arising from PV penetration beyond their control. PV plants, despite having the capacity to provide reactive power support, have no obligation to utilize this capability; neither incentives nor requirements exist. This situation compels distribution companies to invest in traditional compensation equipment such as capacitors or reactors.

1.2 Motivation

The problem defined in Section 1.1 also presents a significant opportunity. Modern PV inverters have evolved beyond passive devices that merely perform Direct Current (DC)/Alternating Current (AC) conversion; they can now provide grid-support functions, such as reactive power control. These inverters can generate reactive power as long as their instantaneous active power output remains below the inverter's rated apparent power [12, 13]. This represents a significant reactive power potential, particularly during low-irradiance periods (morning, evening, and cloudy days).

In networks where distribution companies are directing investments towards compensation equipment, the unutilized reactive power capacity of PV inverters already exists. A mechanism that matches the needs and resources of these two parties can create value for both stakeholders.

A market-based approach offers a suitable framework for matching Distribution System Operator (DSO) needs with available inverter VAR capability. The market mechanism can transform reactive power support into an economic opportunity. Thus, while DSOs obtain the required reactive power support from existing PV plants, PV owners can generate additional revenue from their existing infrastructure.

Two developments enhance the relevance of this approach. First, PV penetration in distribution networks is increasing rapidly globally, and this trend is expected to continue. Second, international standards (IEEE 1547-2018, BS EN 50549) have defined the grid support functions of inverters, and regulatory frameworks have begun to mature.

Although comprehensive studies on reactive power markets exist in the literature, most focus on the transmission level [14]. At the distribution level, Q/P ratio-oriented reactive power market design in networks with high PV penetration is a relatively new research area. This thesis aims to address this gap in the field.

1.3 Scope and Work

This thesis aims to develop a market-based solution framework to address the reactive power management challenges caused by high PV penetration in distribution networks.

The scope of the study is to develop a local reactive power market model for PV plants connected to medium-voltage distribution networks. Unlike existing studies, the proposed market model treats maintaining the Q/P ratio within regulatory limits at the HV/MV interface as the primary market motivation, rather than bus-based voltage regulation.

In the proposed model, the DSO is positioned as a single buyer (monopsonist) and market operator. The sellers are PV plant owners providing inverter-based reactive power at the MV level. The market operates with an hourly settlement period and is cleared over a day-ahead planning horizon.

The remainder of this thesis is organized as follows. Chapter 2 examines the relevant literature and identifies the research gap. Chapters 3 and 4 provide the theoretical background on power systems and inverter-based distributed generation. Chapter 5 evaluates existing approaches to reactive power markets. Chapter 6 presents the proposed market model. Chapter 7 discusses policy recommendations and concludes the thesis with directions for future research.

2. BACKGROUND AND RELATED WORK

This chapter provides an overview of the main research areas that form the foundation of this thesis—reactive power management in distribution networks and emerging market mechanisms for procuring reactive power. The review is structured around four main themes: (i) reactive power management in power systems, (ii) ancillary service markets and reactive power procurement mechanisms, (iii) the reactive power potential of distributed energy resources, and (iv) the research gap in the literature. This framework establishes the theoretical foundation for the technical and economic concepts presented in the subsequent chapters of the thesis.

2.1 Reactive Power Management in Power Systems

Reactive power management is an area that has been extensively addressed in the power systems literature. Comparative technical assessments of reactive power compensation at the distribution level cover a wide range of technologies, from conventional capacitor applications to Flexible AC Transmission Systems (FACTS) [15]. These studies highlight performance differences among these technologies under varying operating conditions.

Reactive power planning at the microgrid scale is an area where metaheuristic optimization methods (PSO, GA, GWO) are widely applied due to its nonlinear and highly constrained structure [16]. Comparative analyses show that hybrid approaches combining individual algorithms achieve faster and more consistent convergence.

Optimal Reactive Power Dispatch (ORPD) can be formulated as a non-convex constrained optimization problem [17]. The fundamental difficulty in solving this problem is that the objectives of minimizing network losses and improving the voltage profile often move in opposite directions.

Three fundamental architectural approaches are defined for reactive power control in distribution networks: centralized, decentralized, and distributed [18]. Among these approaches, distributed coordinated control strategies demonstrate better performance than both traditional droop-based and consensus-based algorithms, particularly when supported by flexible leader assignment mechanisms [19]. This advantage becomes evident in networks with high penetration of Distributed Energy Resources (DERs).

A robust MIMO design has been proposed for coordinated voltage control of multiple PV systems in low-voltage distribution networks [20]. The study demonstrated that the proposed approach exhibits more stable performance than conventional methods in scenarios in which inter-controller interactions can cause oscillations.

2.2 Ancillary Service Markets: Global Overview

There is extensive literature on ancillary service markets and reactive power procurement mechanisms. Comparative analysis of ancillary service markets on a global scale reveals that procurement mechanisms can be classified into four main categories: mandatory provision obligations, bilateral contract arrangements, tender-based competitive procurement, and spot market mechanisms [21]. Market structures in Australia, PJM, various regions of Europe, and New Zealand adopt different combinations of these categories.

Reactive power ancillary service markets can be categorized into two groups based on pricing approach: value-based structures that measure the contribution of reactive power to the grid, and cost-based structures that rely on production costs [22]. In particular, two-tier market frameworks involving TSO-DSO coordination require detailed design in terms of participant diversity, bid formats, and settlement protocols.

The feasibility of competitive reactive power procurement from DERs at the distribution level has been tested under different regulatory frameworks. For example, the Power Potential initiative in Great Britain, as the most comprehensive pilot application in this field, has produced valuable results in terms of both technical performance and commercial sustainability [23].

The reactive power market design literature addresses pricing mechanisms in four main categories: system-wide uniform pricing, geographic zone-based zonal pricing, bus-based nodal pricing, and bid-based market-clearing pricing [14]. Although academic interest has shifted toward distribution-level markets since 2011, the rate at which these theoretical studies have translated into operational implementation has remained relatively limited.

Approaches such as Optimal Power Flow (OPF)-based marginal cost pricing and reactive power procurement market design within a stochastic programming framework have been developed. Studies at the distribution level have shown that DER owners can obtain more than 10% of their revenues from reactive power services [24, 25, 26].

2.3 Reactive Power Potential of Distributed Energy Resources

The reactive power provision capacity of PV inverters is limited by three fundamental physical constraints: the current-carrying capacity of semiconductor switching elements, voltage regulation requirements at the point of grid connection, and DC-link capacitor voltage stability [27]. Adaptation of these constraints to dynamically changing operating conditions requires advanced droop control strategies.

IEEE 1547-2018 and BS EN 50549-2:2019 define the reactive power requirements and control modes for DERs. These standards mandate that smart inverters support various control modes such as constant power factor, volt-Var, and constant reactive power modes [12, 13].

Regarding the economic participation of DERs in grid services, it has been demonstrated that distribution systems providing reactive power services to the transmission network offer significant economic and technical advantages [28]. Similarly, the contribution of PV inverters to voltage regulation in distribution networks has been experimentally validated [29, 30].

2.4 Research Gap

The literature review identifies research gaps in reactive power management and market mechanisms. Despite increased interest in distribution-level markets in the post-2011 literature, this interest has not yet translated into concrete practical applications [14]. Existing reactive power markets predominantly operate at the transmission level, and characteristics specific to distribution networks (high R/X ratio, radial topology, and unbalanced three-phase operation) are not accounted for. Apart from the Power Potential initiative in Great Britain, there is no systematic market implementation for competitive reactive power procurement from DERs at the distribution level.

Studies in the literature emphasize that TSO-DSO coordination is essential for stable operation of reactive power markets and optimal reactive power resource scheduling [22]. However, most existing publications consider only two voltage levels or two horizontal connection points between distribution systems [14]. The potential of distribution systems to provide reactive power services to the transmission network is not adequately addressed by current market structures and regulatory frameworks, and clear methodologies for cost and benefit sharing at the TSO-DSO interface remain lacking.

A critical deficiency of existing reactive power market designs also emerges in participant behavior modeling [14]. The vast majority of studies in the literature assume that market actors bid based on marginal cost; however, they disregard the reality that participants will exhibit strategic behavior aiming for profit maximization under actual market conditions. This simplifying assumption raises questions about the resilience of proposed market mechanisms against strategic manipulation and market power abuse.

Additional gaps have been identified in emerging electricity markets. In most emerging markets, small-scale generation facilities connected to the distribution network are exempt from reactive power obligations. Regulatory frameworks exclude this service from market mechanisms. Furthermore, penalty mechanisms target DSOs while offering no incentives to generation facilities and consumers. The reactive power capacity of PV plants remains unutilized due to the absence of appropriate regulatory frameworks and incentive mechanisms, leading to significant resource waste.

In the following chapter, the technical infrastructure necessary to address these research gaps—the fundamentals of power systems and the physical principles of reactive power control—is examined in detail.

3. FUNDAMENTALS OF POWER SYSTEMS

This chapter addresses the fundamental concepts of power systems and the physical infrastructure of reactive power management. First, the electricity value chain and network structures are examined, followed by a detailed discussion of power concepts in AC systems. Finally, the traditional methods and limitations of reactive power control in distribution networks are discussed.

Modern economies are structurally dependent on an uninterrupted and high-quality electricity supply. Traditional power systems are built on a centralized architecture in which energy generated at large-scale power plants flows unidirectionally toward end users via high-voltage transmission lines. This structure has been operated effectively on the assumption of passive distribution networks for many years.

However, the rapid proliferation of distributed energy resources, particularly PV systems, at the distribution level is transforming this traditional grid operation paradigm. Increasing PV penetration renders power flow bidirectional and creates new operational challenges in voltage regulation and reactive power balance. These developments indicate that distribution networks are no longer passive networks but are evolving into dynamic systems that require active management.

In this context, a thorough understanding of reactive power flow and voltage behavior, particularly at the distribution level, is critical. Unlike transmission systems, distribution networks possess distinctive physical characteristics such as a high resistance/reactance (R/X) ratio, radial topology, and local voltage sensitivity. These characteristics create a strong interaction between active and reactive power flows, limiting the direct application of conventional transmission-level approaches to distribution networks. Therefore, reactive power management strategies and market-based mechanisms to be developed at the distribution level must be designed taking these physical realities into account.

The purpose of this chapter is to establish the physical and mathematical foundation for the Q/P ratio-based reactive power market design developed in the subsequent chapters. First, the electricity supply chain and the roles of generation, transmission, and distribution are introduced; then, the physical meanings and mathematical relationships of active, reactive, and apparent power in AC systems are explained. Finally, the chapter examines key characteristics of distribution networks, the impact of the R/X ratio on voltage sensitivity, and how the integration of distributed generation transforms these relationships.

This fundamental conceptual framework demonstrates that the reactive power capacity of PV inverters provides significant value at the distribution level and that this capacity must be effectively utilized through market-based mechanisms.

3.1 Electricity Value Chain

Electricity is produced through a complex and integrated value chain that delivers it from primary sources to end consumers. This chain is not merely a technical flow; it also encompasses economic, environmental, and social dimensions. While ensuring uninterrupted energy flow from generation to consumption, this value chain plays a critical role in maintaining supply-demand balance, sustaining market functionality, and ensuring energy security.

The process by which electrical energy reaches consumers from its source is structured as an integrated value chain consisting of four sequential stages. The generation stage, where primary energy sources are converted into electricity, is followed by transmission, where energy is transported to main load centers through high-voltage lines. Subsequently, distribution networks deliver this energy to end users, and the process is completed at the retail stage. This layered structure gained strategic significance with the liberalization reforms that gained momentum from the 1990s onwards [31]. These reforms moved the electricity sector away from a vertically integrated public monopoly structure by unbundling natural-monopoly network activities (transmission and distribution) from segments that could be opened to competition (generation and retail). Thus, electricity transformed from a regulated public service into a tradable commodity. The general structure of this value chain is schematically illustrated in Figure 3.1.

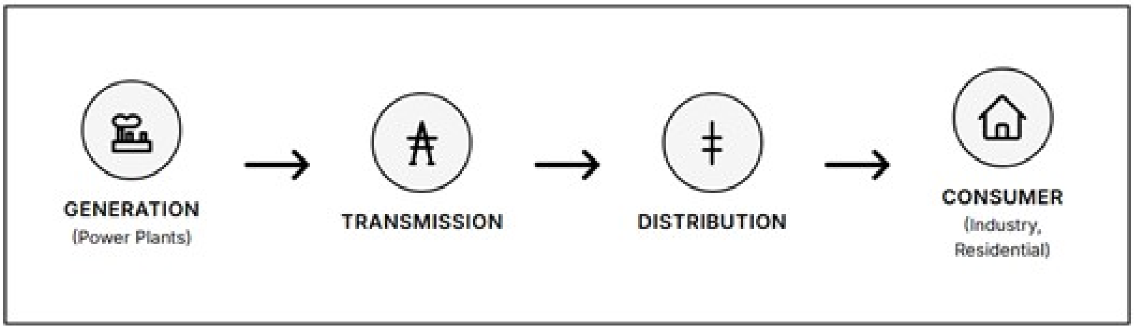

Figure 3.1: *Main Stages of the Electricity Value Chain*

From a physical perspective, this chain consists of an extensive power network originating at generation stations. The produced electricity is conveyed to urban centers and major industrial loads via high-voltage transmission lines. It is then delivered to homes and businesses via medium- and low-voltage distribution networks [32]. The fundamental purpose of this structure is to generate electricity reliably, stably, and economically, and to deliver it to end users in a usable form at a specified voltage and frequency.

The distinctive characteristic of electrical systems is the requirement to balance generation and consumption simultaneously. Despite advances in battery and other storage technologies, economical storage at the grid scale remains limited. This physical constraint compels system operators to continuously monitor supply-demand balance on the order of seconds and to maintain frequency and voltage parameters within specified bands.

Furthermore, the rapidly increasing share of renewable energy sources, particularly wind and solar, is making this operational balance more complex. Variable and unpredictable generation profiles necessitate the development of new tools beyond conventional balancing mechanisms.

In summary, this complex journey of electrical energy brings various technical and operational challenges. These challenges encompass resource diversity in generation, distance and capacity limitations in transmission, voltage and power quality requirements in distribution, and the need for instantaneous demand fulfillment. Consequently, each stage of the electricity value chain plays a critical role in delivering energy from its source to end users in a safe, continuous, and high-quality manner. However, increasing electrification, the variability of renewable sources, and digitalization processes today necessitate treating this chain not merely as a structure divided into stages, but as an integrated system.

3.1.1 Generation

The generation stage, which constitutes the first and most fundamental link of the electricity value chain, encompasses the process of converting primary energy sources into electrical energy, including fossil fuels such as coal, natural gas, and oil, as well as renewable sources such as nuclear energy, hydroelectric, wind, solar, biomass, and geothermal. Historically, electricity generation has been carried out through large-scale centralized power plants, which have formed the backbone of energy supply security for many years. However, today, due to changes in energy policies, technological advances, and environmental concerns, distributed generation is gaining increasing importance, with flexible, localized, and renewable-based solutions coming to the forefront in networks [33].

The physical basis of conventional electricity generation is electromagnetic induction. In synchronous generators, the rotor, driven by a primary energy source (thermal, hydraulic, or wind), rotates within the stator windings to produce AC. This electromechanical conversion constitutes the fundamental energy source of today's grids [33]. The electrical energy obtained at these plants is typically generated at voltage levels of 11–25 kV at the plant output. To enable efficient transmission over long distances through the transmission network, this voltage is stepped up to 132 kV, 220 kV, 400 kV, or higher [33]. Thus, the process starting from generation is delivered to consumers safely and efficiently through transmission and distribution networks.

This centralized and synchronized grid structure, which has formed the foundation of grid stability for decades, is now undergoing a fundamental transformation. The rapid increase of distributed and variable renewable energy sources, particularly PV systems, located near consumption points, is fundamentally changing the electricity generation paradigm. Unlike conventional synchronous generators, these new-generation sources connected to the grid via power-electronics-based inverters enable bidirectional power flow; however, because they lack rotating masses, they do not provide natural inertia to the grid [34]. High PV penetration reduces the system's total inertia, making the grid more sensitive and vulnerable to frequency variations [35].

However, contrary to conventional views, inverter-based generation units can provide a much faster and more precise reactive power response than synchronous generators when equipped with appropriate control strategies. Although inverters have limited capacity for short-circuit current, their technical flexibility in reactive power modulation makes them valuable assets for grid stability. Nevertheless, due to the lack of current market regulations and incentive mechanisms, this technical potential generally remains unutilized, and inverters are primarily operated at unity power factor, focusing only on active power generation.

Ultimately, the electricity generation structure is evolving from a traditional configuration consisting of centralized, dispatchable, and high-inertia plants to a hybrid generation architecture that also includes distributed, variable, inverter-based, and low-inertia sources. This transformation necessitates new ancillary service mechanisms and market designs for reactive power management and grid stability, particularly at the distribution level.

3.1.2 Transmission

The transmission system constitutes the backbone of the electrical energy value chain. This high-voltage network transports electricity generated at large-scale power plants to main consumption centers safely and efficiently. The system also provides the connection between geographically dispersed generation sources and load centers. By enabling energy trade between different regions, it plays a central role in maintaining the supply-demand balance. The increasingly complex and interconnected structure of modern power systems is driven mainly by the robustness and operational flexibility of transmission networks [36].

High-voltage transmission is the fundamental engineering solution that ensures the economic feasibility of long-distance energy transportation. Thermal losses in transmission lines are directly proportional to the square of the current according to Joule's law ($P_{\text{loss}} = I^2R$). Considering the power equation ($P = V \cdot I$), increasing voltage at a constant power level reduces current inversely proportionally, resulting in a quadratic reduction in losses [32]. This physical necessity is the fundamental rationale for stepping up the energy at 11–25 kV level at the plant output to 154 kV, 380 kV, or even 765 kV through step-up transformers before feeding it into the network.

However, the energy transition of the 21st century poses new challenges for traditional transmission networks. Large-scale wind and solar plants are located in regions with high energy potential, and these regions are often far from consumption centers. This situation causes bottlenecks in existing transmission lines and increases the need for new transmission corridors. In this context, two key technologies stand out. The first is High Voltage Direct Current (HVDC) transmission technologies. HVDC offers lower losses compared to alternating current in long-distance and submarine energy transmission, while also enabling connection between asynchronous grids [37]. Today, the development of

VSC-HVDC (Voltage Source Converter) technology has increased the feasibility of multi-terminal HVDC networks and provided flexible solutions for renewable energy integration [38]. The second is FACTS. These power-electronics-based technologies (e.g., Static VAR Compensator (SVC), Static Synchronous Compensator (STATCOM)) increase the power-transfer capacity of existing lines and improve grid stability by controlling reactive power flow instantaneously [39].

In short, modern transmission systems are no longer passive networks that merely transport energy from one point to another. Advanced control mechanisms, power electronics applications, and communication infrastructure support these systems. Thus, transmission networks have transformed into dynamic structures that actively manage the safe and economic flow of energy.

3.1.3 Distribution

Distribution networks, which constitute the final link of the electrical energy supply chain, ensure that energy received from the transmission system is delivered to end users reliably and with high quality. Energy transmitted at high voltage levels is gradually stepped down through step-down transformer substations. Distribution networks operate in a different environment from transmission systems due to their numerous connection points and local load characteristics. This physical structure makes voltage regulation and reactive power management at the distribution level a distinct problem from those in transmission systems. In particular, short line lengths and local load density render voltage behavior more sensitive and localized.

In this network structure operated by the DSO, energy is stepped down to low-voltage levels through distribution transformers and delivered to end users. The low-voltage levels commonly used in European and Turkish applications are 230 V (single-phase) and 400 V (three-phase). Energy is transmitted through feeder lines that originate from medium voltage busbars and supply a specific geographic area; consumer connections are provided through distribution transformers positioned along these feeders. A simplified single-line diagram illustrating the transition from transmission to distribution levels, along with a typical medium-voltage distribution structure, is presented in Figure 3.2.

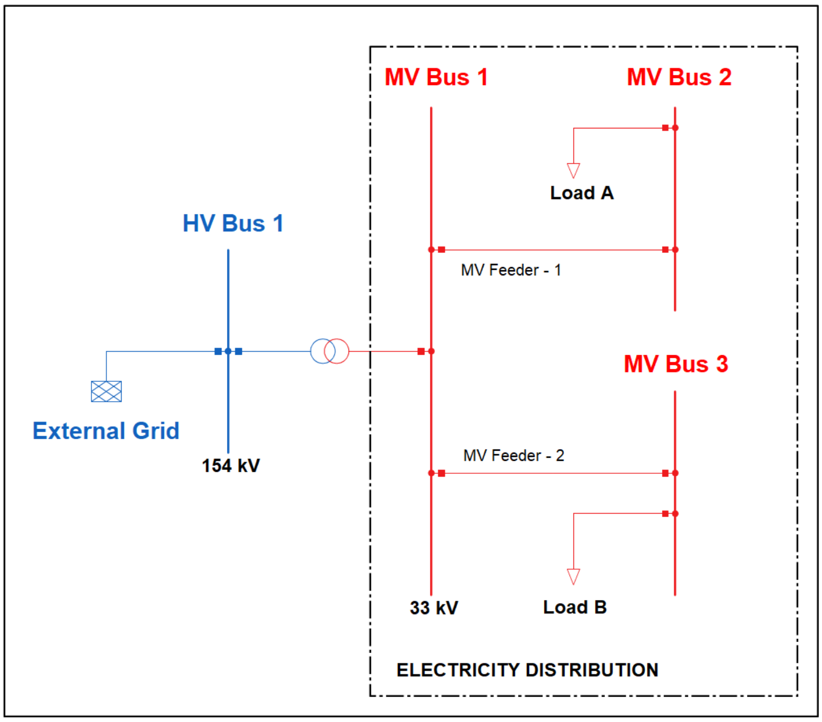

Figure 3.2: *Sample single-line diagram showing electricity distribution at the MV level*

The fundamental distinction between distribution and transmission networks manifests itself in the line impedance profile. At the distribution level, particularly in networks where underground cables are prevalent, the resistance component (R) increases significantly compared to the reactance component (X), resulting in high resistance/reactance (R/X) ratios [40]. This causes voltage variations to be highly sensitive not only to reactive power flow but also to active power flow. Consequently, voltage regulation in distribution networks cannot be solved using the conventional decoupled power-flow approaches employed in transmission systems.

In distribution networks with high R/X ratios, reactive power plays a determining role not only in voltage control but also in line losses and equipment capacity utilization. Uncontrolled increases in reactive power flow raise line currents, thereby increasing I^2R losses and limiting the apparent power capacity of transformers and cables. Therefore, the location, capacity, and timing of reactive power sources at the distribution level are critical to system performance. This physical reality explains why power-electronics-based flexible units, such as PV inverters, create value in distribution networks.

Within this framework, distribution networks can no longer be viewed merely as networks where energy is passively transmitted. These systems are dynamic structures that require active management of voltage, losses, and power quality objectives. The structural and physical characteristics at the distribution level elevate reactive power management beyond a mere technical necessity. With appropriate mechanisms, this service has also become economically viable. The Q/P ratio-based reactive power market approach addressed in this thesis is built upon these distinctive characteristics of distribution networks.

3.2 Power Concepts in AC Systems

Power analysis in AC systems forms the foundation for evaluating system efficiency, stability, and power quality in the generation, transmission, and consumption stages of electrical energy. Electrical power is divided into three fundamental components according to their functions in the system and their mathematical properties: active power (P), reactive power (Q), and apparent power (S). Each of these components directly affects load behavior, energy utilization efficiency of the system, and capacity utilization of network equipment such as transformers and lines [41, 42].

In addition to their physical meanings, the mathematical relationships among these power components are critical in engineering design. In this context, power factor is a metric that directly determines energy utilization efficiency in the system, and low power factor increases system losses, causing overloading of transmission elements and voltage imbalances [43, 44].

In AC circuits, voltage and current waveforms in steady state are expressed as sinusoidal functions of time. In a general case, voltage $v(t)$ and current $i(t)$ can be defined as follows:

$$v(t) = V_m \cos(\omega t + \theta_v) \tag{3.1}$$

$$i(t) = I_m \cos(\omega t + \theta_i) \tag{3.2}$$

where V_m and I_m represent the peak (maximum) values of voltage and current respectively, ω represents the angular frequency ($2\pi f$), and θ_v and θ_i represent the phase angles of voltage and current respectively.

Power at any instant in a circuit, namely instantaneous power $p(t)$, equals the product of instantaneous voltage and current:

$$p(t) = v(t) \cdot i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \tag{3.3}$$

When this expression is rearranged using trigonometric product-to-sum formulas, a deeper understanding of the power structure emerges [32]:

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) \tag{3.4}$$

This equation clearly demonstrates that instantaneous power consists of two fundamental components:

Active Power Component: The first term of the equation, $\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$, is a time-independent constant value. This component represents net energy transfer in the circuit and is called average power or active power (P). Its unit is Watt (W). This power is the power that performs actual work by converting to heat across resistors, performing mechanical work in motors, or producing light in lamps.

Reactive Power Component: The second term, $\frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$, is a time-varying component oscillating at twice the grid frequency (2ω). This oscillation indicates that energy periodically flows back and forth between the source and load; no net work is performed. This energy exchange is necessary to establish and maintain the magnetic fields of inductors and the electric fields of capacitors. The amplitude of this oscillating power forms the basis of the reactive power (Q) concept.

The correct interpretation of these power components is critical to understanding the technical limits and economic consequences of reactive power management at the distribution level.

3.2.1 Active Power

Active power, which constitutes the horizontal component of the power triangle, represents the portion of electrical energy that is actually consumed or converted. From the perspective of PV systems, this value corresponds to the amount of power that the inverter delivers to the grid, and that is subject to sale in the energy market. Its unit is Watt (W), and it is expressed in MW at the grid scale.

Active power is expressed as the product of effective (RMS) voltage and current values and the power factor [45]:

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\varphi) \tag{3.5}$$

3.2.2 Reactive Power

Reactive power is the component of AC power that does not perform net work or energy transfer in an AC system; however, it is periodically exchanged between the source and load to establish and maintain the electric and magnetic fields required by inductive and capacitive elements. Inductive loads, such as motors, transformers, and ballasts, must draw reactive current from the grid to establish their magnetic fields, given their operating principles [32]. Although this power occupies the apparent power capacity of grid equipment (cables, transformers) by continuously oscillating within the system, it does not result in net energy consumption, unlike active power [46].

The presence of reactive power directly affects system efficiency and stability. Uncontrolled or excessive reactive power flow increases the total current in lines, causing overloading of transformers and generators, increasing I^2R losses, and also causing serious voltage drops by interacting with line impedance [47].

Despite not performing direct work, the mathematical formulation of reactive power, which is essential for the functioning of AC systems, is as follows:

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\varphi) \tag{3.6}$$

Its unit is the Volt-Ampere Reactive (VAr).

The sign and direction of reactive power varies according to the type of load:

Inductive Circuit (Lagging Power Factor): Current lags voltage by $90°$ ($\varphi = +90°$). In this case, $\sin(90°) = 1$ and reactive power takes a positive value. Inductive loads "consume" or draw reactive power from the grid. Figure 3.3 shows the current-voltage phase difference in an inductive load.

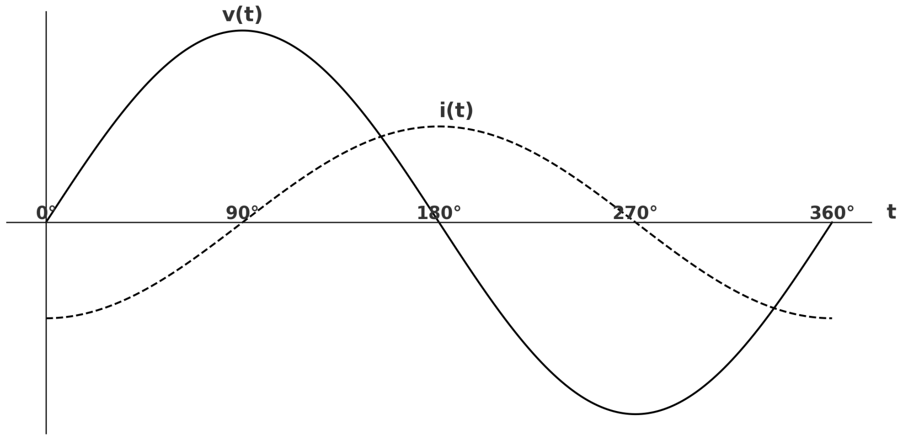

Figure 3.3: *Current-voltage relationship in an inductive load*

Capacitive Circuit (Leading Power Factor): Current leads voltage by $90°$ ($\varphi = -90°$). In this case, $\sin(-90°) = -1$ and reactive power takes a negative value. Capacitive loads generate or supply reactive power to the grid. The graph representing this situation is shown in Figure 3.4.

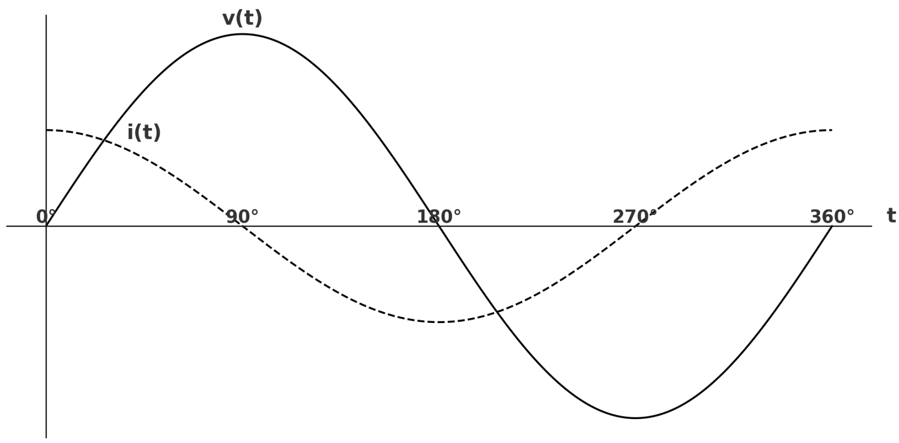

Figure 3.4: *Current-voltage relationship in capacitive load*

In both cases (purely inductive or purely capacitive), since the phase difference is $90°$, the power factor is zero ($\cos(\pm 90°) = 0$) and no average active power consumption occurs in the circuit. For this reason, “power factor correction” is performed by adding capacitor banks to systems with heavy inductive loads to control reactive power [42].

Although this energy oscillation does not perform "net work," it creates very real costs and constraints on transmission and distribution infrastructure:

Capacity Occupation: The oscillating energy passes through conductors and transformers as current (I). This situation fills the apparent power capacity (S) of equipment, thereby limiting the capacity available for carrying active power (P) that performs useful work.

Active Losses: Reactive current increases I^2R losses that convert to heat as it passes through the resistance of conductors. In other words, "non-working" reactive power causes "real" energy loss in the system.

Consequently, reactive power constitutes a physical necessity and a systemic burden. This physical reality demonstrates why reactive power must be carefully managed by grid operators (DSO/TSO). At the same time, it establishes the fundamental legitimacy for compensating the sources that provide this management (e.g., PV inverters) through market mechanisms in return for the "capacity allocation" service they provide.

3.2.3 Apparent Power

Apparent power (S) represents the total power drawn by an AC circuit from the grid and represents the actual load of the system. Apparent power is the "apparent" power of the system, obtained by direct multiplication of effective (RMS) voltage and current values [33].

$$S = V_{\text{rms}} I_{\text{rms}} \tag{3.7}$$

The unit of apparent power is the Volt-Ampere (VA), expressed in kilovolt-ampere (kVA) or megavolt-ampere (MVA), depending on the scale of the power system. Apparent power alone does not provide information about the energy conversion efficiency of the system; rather, it is a comprehensive quantity that reflects the sum of work-performing

Active Power (P) and non-work-performing Reactive Power (Q). From an engineering perspective, the most critical function of apparent power is its use as the fundamental quantity for sizing electrical infrastructure elements (transformers, cables, generators, and switchgear). The basic physical reasons for this are:

Thermal Limits: The conductors and windings of equipment must withstand the heating (I^2R) caused by the total current (I_{rms}) passing through them. Whether the current originates from active or reactive components does not change the heating effect; the conductor is exposed to both conditions.

Insulation Limits: The insulation levels of equipment are designed according to the system voltage (V_{rms}) they are exposed to.

Therefore, equipment in power systems is rated not by active power (Watt) but by apparent power (VA), which is the resultant of current and voltage [32]. This situation reveals a critical consequence in terms of system efficiency: A system with low power factor draws higher apparent power (S) and consequently higher current from the grid to transfer the same amount of active power (P). This additional capacity utilization necessitates larger-sized and more costly infrastructure investments.

3.2.4 Power Triangle

The fundamental relationship among active power (P), reactive power (Q), and apparent power (S) is geometrically represented by a right-angled triangle called the "power triangle." This triangle, shown in Figure 3.5, concretizes the vectorial relationship of energy components in AC power systems.

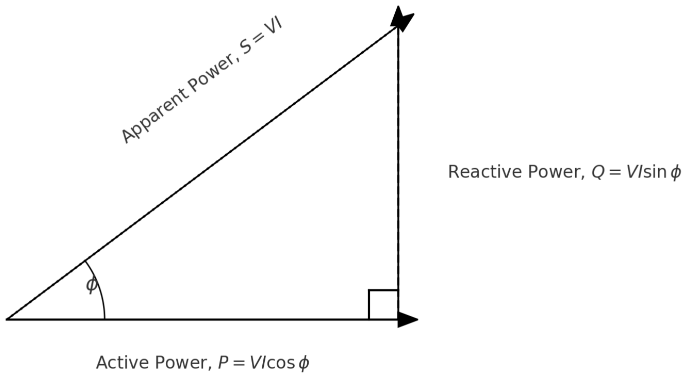

Figure 3.5: *Power triangle showing the relationship between P, Q, and S*

Due to this geometric relationship, the Pythagorean theorem applies among these three power components:

$$S^2 = P^2 + Q^2 \tag{3.8}$$

The power triangle can be located in different regions of the coordinate plane depending on the load characteristics. This situation is shaped by the vectorial direction change of reactive power relative to active power depending on whether reactive power is positive or negative (Figure 3.6).

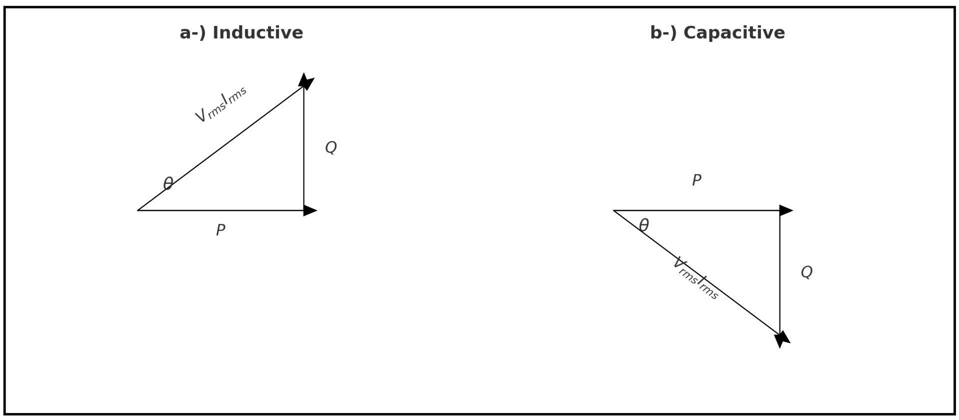

Figure 3.6: *Power triangle representation for inductive and capacitive loads*

Inductive Load (Lagging PF): Inductive loads such as motors consume reactive power ($Q > 0$) to generate magnetic fields. Therefore, the reactive power vector is directed upward on the positive imaginary axis. This situation is shown in Figure 3.6(a), where reactive power extends upward as a perpendicular component relative to active power in the power triangle.

Capacitive Load (Leading Power Factor / Leading PF): Capacitive loads such as capacitor banks or PV inverters operating in appropriate mode supply the reactive power needed by the system ($Q < 0$). In this case, since reactive power is negative, the reactive power vector in the power triangle is directed downward (toward the negative imaginary axis). This vectorial behaviour of reactive power is presented in Figure 3.6(b). This characteristic is used as a fundamental tool in power factor correction and voltage regulation strategies.

3.2.5 Power Factor

Power Factor (PF) is a fundamental performance metric indicating how efficiently energy is used in an electrical system. It expresses how much of the total apparent power (S) drawn by the system from the grid is converted into active power (P) that performs actual work. Mathematically, it is defined as the ratio of active power to apparent power:

$$PF = \frac{P}{S} \tag{3.9}$$

Starting from the power triangle geometry, it can be seen that this ratio is also equal to the cosine of the phase difference angle φ between voltage and current phasors:

$$PF = \cos(\varphi) \tag{3.10}$$

Although power factor can theoretically range from -1 to 1, in practice it takes a value between 0 and 1 at consumption points. In an ideal system, the power factor is 1 ($\cos(\varphi) = 1$). This means that all power drawn is active power, and the reactive power component is zero (Unity Power Factor). As the power factor deviates from 1, system efficiency decreases, and the reactive power component increases.

A low power factor leads to serious technical and economic problems for both consumers and energy providers (DSO) [42, 48]:

Increased System Losses: For a given active power (P) demand, according to the formula $I = P/(V \cdot PF)$, as power factor decreases, the current drawn from the grid increases. Since energy losses are proportional to the square of current ($P_{\text{loss}} = I^2R$), low PF increases grid losses exponentially.

Inefficient Use of Equipment Capacity: Transformers, cables, and generators are sized according to apparent power (kVA). A low PF causes a large portion of the capacity of this equipment to be consumed ("occupied") for carrying non-working reactive power.

Voltage Drop: Increased current causes greater voltage drop ($V_{\text{drop}} = I \cdot Z$) across the impedance of transmission and distribution lines, degrading power quality.

Economic Penalties: Energy market regulations impose reactive power penalties on distribution companies that exceed specified limits (20% inductive, 15% capacitive in Türkiye) due to the additional burden and losses imposed on the grid.

Under high PV penetration at the distribution level, the power factor problem originates not only from consumer behaviour but also from changes in active power injection.

3.3 Distribution Network Characteristics

The physical structure and topology of distribution networks directly determine the nature of voltage regulation, power flow, and reactive power management problems. Unlike the meshed structure commonly used in transmission systems, distribution networks are typically built on radial or limited-loop topologies. This topological difference causes power flow at the distribution level to become more local, directional, and operation-condition-sensitive.

3.3.1 Radial Systems

The most commonly used structure in distribution networks is radial systems. In this topology, each load point is connected to the source via a single electrical path, and power flow is largely unidirectional [42]. While the radial structure offers simplicity and cost advantages for network planning and protection coordination, it also has structural characteristics that limit operational flexibility. A typical radial distribution network topology and feeder-based power-flow structure are schematically shown in Figure 3.7.

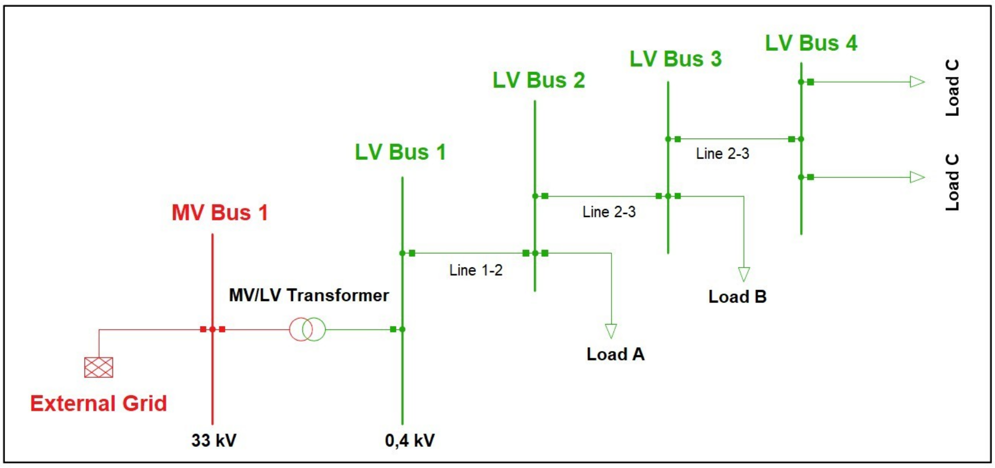

Figure 3.7: *Radial distribution system – single line diagram*

The sequential progression of power flow along the feeder in radial systems renders voltage levels highly sensitive to line impedance and load distribution. As seen in Figure 3.7, increasing line impedance along the feeder increases the risk of voltage drop at end points or voltage rise in the case of distributed generation. Since distributed sources such as PV generation facilities connected to distribution networks perform active power injection unlike traditional load profiles, phenomena such as voltage rise and reverse power flow can emerge more prominently in radial structures.

Under this topological structure, voltage regulation cannot be addressed with a centralized and holistic control approach. Instead, feeder-based and local interventions are required. The unidirectional feeding structure clearly demonstrates why voltage control must be based on local reactive power sources. Therefore, reactive power management in radial distribution networks is not merely a technical balancing tool. It is also a strategic operational element that enables local voltage control. Distributed and fast-response reactive power sources, such as PV inverters, provide significant value to the system in this structure.

3.3.2 Loop Systems

In ring (loop) distribution systems, load points have multiple supply paths. A limited meshed structure exists even under normal operating conditions. This structure offers greater operational flexibility and supply security than radial systems. Providing alternative supply capability during fault conditions is a significant advantage. However, the operation and protection of loop systems require more complex control and coordination mechanisms.

In ring systems, power flow can occur through different paths depending not only on load demand but also on switch configurations and the distribution of network impedances. A typical loop distribution network topology is shown in Figure 3.8.

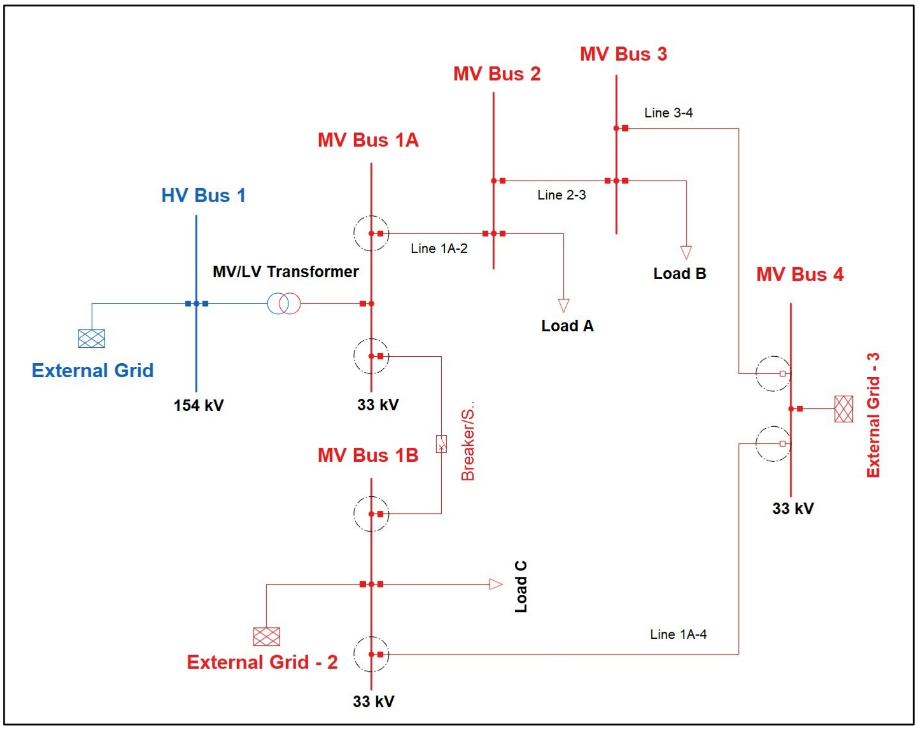

Figure 3.8: *Ring distribution system – single line diagram*

As shown in Figure 3.8, switching states and the network's impedance distribution directly affect the direction and magnitude of active and reactive power flows.

Although ring systems offer a topologically more flexible structure, the multiple supply paths presented in Figure 3.8 do not render the reactive power–voltage relationship a secondary issue due to the high R/X ratios at the distribution level. On the contrary, reactive power–voltage interaction plays a dominant role in these systems as well, and the voltage control problem emerges as an operational requirement that must be solved at the local level.

3.3.3 Voltage Level Classification

Distribution networks are classified not only by their topological structures but also by their operational voltage levels. This classification is based on international standards and establishes a fundamental framework for the system's technical design, equipment selection (insulation and breaker capacity), safety procedures, and cost analysis. Distribution systems are classified into two main categories: Low Voltage (LV) and MV.

3.3.3.1 LV Networks

LV networks are defined by the International Electrotechnical Commission (IEC) in the IEC 60038 standard as systems generally with nominal values up to 1 kV (≤ 1 kV) [49]. These networks are the final link in the distribution chain, starting after distribution transformers that step down energy from medium voltage to low voltage and delivering it directly to end users.

The standard phase-to-neutral voltage commonly used in Türkiye and Europe is 230 V and the phase-to-phase voltage is 400 V (previous standard 220/380 V). These three-phase, four-wire (3 phases + 1 neutral) systems can supply both single-phase residential loads and three-phase commercial and light-industrial loads [50]. However, LV networks operate at high currents due to limited voltage levels; this condition makes line losses and voltage drops more pronounced.

3.3.3.2 MV Networks

MV networks are the backbone of the electrical distribution system, receiving energy from primary step-down substations (HV/MV), distributing it over wide geographical areas, and feeding LV distribution transformers. According to IEC standards, this category generally covers voltage levels between 1 kV and 36 kV, although some definitions extend the range up to 52 kV [42, 49]. The most commonly used MV levels in Türkiye and worldwide include 10 kV, 15.8 kV, 33 kV, and 34.5 kV.

Since MV networks can carry the same power with lower currents compared to LV, I^2R losses and voltage drop problems are much less significant. This enables energy to be transported efficiently through feeders spanning several kilometers. These systems feed not only numerous distribution transformers supplying residential areas but also large consumers such as large commercial buildings, hospitals, shopping centers, and medium-scale industrial facilities that receive energy directly from MV [33].

In this context, MV networks play a critical role not only in energy transmission but also in voltage regulation, reactive power management, and the integration of distributed generation. The predominant connection of PV systems at the MV level reinforces this situation. These networks have become a strategic focal point for reactive power control and flexibility services.

3.3.4 R/X Ratio and Voltage Sensitivity

The difference in the R/X ratio is the fundamental reason underlying the distinct voltage–reactive power relationships observed in transmission and distribution networks. Table 3.1 quantitatively illustrates this distinction using typical R/X ratios and impedance angles.

Table 3.1: *Transmission and Distribution Networks: A Fundamental Distinction*

Parameter	**Transmission (HV)**	**Distribution (MV/LV)**
Typical R/X Ratio	0.1 – 0.3	0.5 – 4.0
Dominant Component	Reactance ($X >> R$)	Resistance ($R \approx X$ or $R > X$)
Impedance Angle	70° – 85°	25° – 60°

The complex impedance of a power line is expressed as follows:

$$Z = R + jX = |Z|\angle\theta_Z \tag{3.11}$$

where R represents resistance, X represents reactance (predominantly inductive), and $\theta_Z = \arctan(X/R)$ represents the impedance angle.

In transmission networks, high X/R ratio (low R/X) means that reactance dominates the line impedance. This characteristic leads to the well-known approach in literature: active power flow is primarily dependent on phase angle differences, while reactive power flow is primarily dependent on voltage magnitude differences [47].

However, in distribution networks, resistance becomes comparable to reactance or even exceeds it. This situation creates a dependency between active and reactive power flows, meaning:

$$\Delta V \approx \frac{R \cdot P + X \cdot Q}{V} \tag{3.12}$$

When the R/X ratio approaches or exceeds unity, both active and reactive power injections significantly affect the voltage profile. This coupling has critical implications for voltage regulation strategies in distribution systems with high DER penetration.

Consequently, the high R/X ratios observed in distribution networks necessitate a reassessment of fundamental assumptions regarding reactive power control. Control strategies based solely on reactive power injection may prove insufficient for voltage regulation under certain operating conditions. In situations where active power flows are significant, active power curtailment may need to be considered in addition to reactive power support to achieve effective voltage control. The decoupled P–δ and Q–V approaches commonly used for transmission systems are largely invalid due to the high R/X ratio structure of distribution networks. The pronounced effect of the resistance component on voltage drop causes local reactive power sources to gain higher system value at the distribution level [51].

3.3.5 Voltage-Reactive Power Relationship in Distribution Networks

A mathematical understanding of the relationship between voltage and reactive power is essential for designing effective voltage-regulation strategies and reactive-power markets at the distribution level. This is demonstrated by first addressing the voltage-reactive power relationship in distribution networks with traditional unidirectional power flow, and then showing how Distributed Generation (DG) integration transforms this relationship.

3.3.5.1 Conventional Distribution Feeder Model without DG

Traditional distribution networks are typically designed as passive radial systems, with active and reactive power flowing unidirectionally from the high-voltage grid bus to low-voltage load points [52]. For the example single-line distribution feeder shown in Figure 3.9, the relationship between the sending end (V_1) and the receiving end (V_2) can be expressed mathematically.

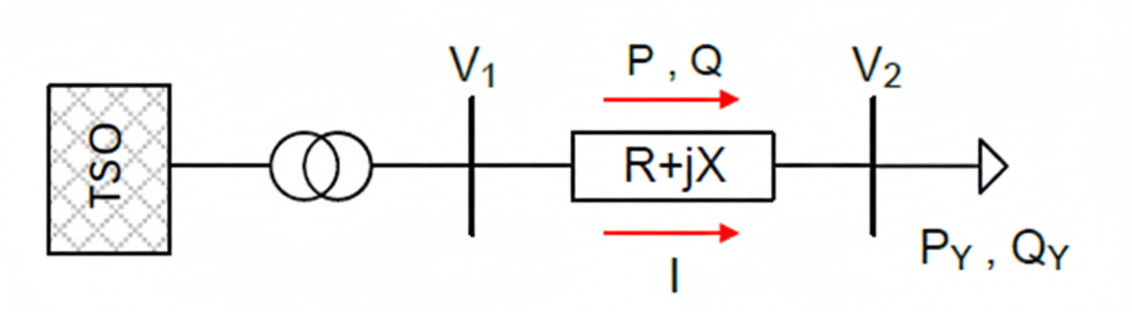

Figure 3.9: *Sample single-line distribution feeder example*

According to Figure 3.9, V_1 represents the voltage at the line head, V_2 represents the voltage at the line end, P_Y represents the active power of the load, Q_Y represents the reactive power of the load, P and Q represent the active and reactive power supplied by the grid, and I represents the current.

$$\hat{V}_1 = \hat{V}_2 + \hat{I}(R + jX) \tag{3.13}$$

where $\hat{I}$ is the phasor representation of current flowing along the line. The complex power supplied by the grid is expressed as:

$$S = P + jQ = \hat{V}_1 \hat{I}^* \tag{3.14}$$

From this equation, the current can be derived as:

$$\hat{I} = \frac{P - jQ}{\hat{V}_1^*} \tag{3.15}$$

According to these equations, the voltage at the line head is obtained as:

$$\hat{V}_1 = \hat{V}_2 + \frac{RP + XQ}{\hat{V}_1} + j\frac{XP - RQ}{\hat{V}_1} \tag{3.16}$$

Accordingly, the voltage change between the line head and line end can be written as:

$$\Delta\hat{V} = \hat{V}_1 - \hat{V}_2 = \frac{RP + XQ}{\hat{V}_1} + j\frac{XP - RQ}{\hat{V}_1} \tag{3.17}$$

In this expression:

- Real part $\frac{RP+XQ}{V}$: Determines the voltage magnitude change
- Imaginary part $\frac{XP-RQ}{V}$: Determines the voltage angle difference

Since voltage angle differences are generally small in distribution networks, the following approximate expression is commonly used for voltage drop:

$$\Delta V \approx \frac{RP + XQ}{V} \tag{3.18}$$

This equation demonstrates that the voltage magnitude at any bus is affected by both active and reactive power flows through the line impedance.

3.3.5.2 Distribution Feeder Model with DG

With the integration of distributed generation facilities into the grid, the unidirectional structure of power flow is fundamentally changing [53]. As a result of DG integration, unidirectional power flow in the network is being replaced by bidirectional power flow. The voltage at the connection point of DG units may exceed the voltage at the source bus. A sample single-line distribution feeder with a connected distributed generation unit is shown in Figure 3.10.

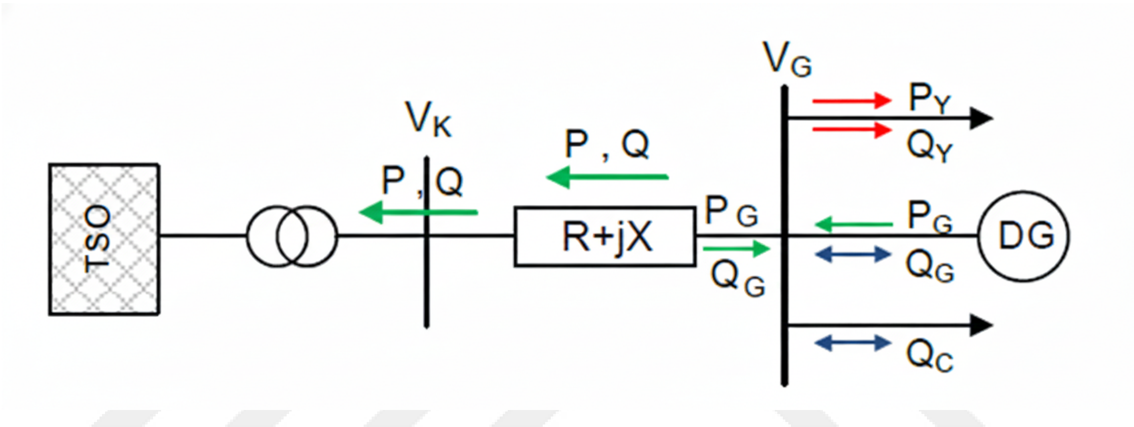

Figure 3.10: *Sample network with distributed generation*

For a distribution feeder containing DG, the relationship between the voltage at the generator bus (V_G) and the voltage at the source bus (V_K) is expressed as:

$$V_G \approx V_K + \frac{RP + XQ}{V_G} \tag{3.19}$$

According to the representation in Figure 3.10:

- P_G: Active power generated by the generator
- Q_G: Reactive power supplied to or drawn from the system by the generator
- P_Y, Q_Y: Active and reactive powers of loads connected to the bus
- Q_C: Capacity of existing compensation units

Net power flows are defined as follows:

$$P = P_G - P_Y \tag{3.20}$$

$$Q = (\pm Q_C - Q_Y \pm Q_G) \tag{3.21}$$

In this case, the voltage change is obtained as:

$$\Delta V = V_G - V_K \approx R(P_G - P_Y) + X(\pm Q_C - Q_Y \pm Q_G) \tag{3.22}$$

This equation clearly demonstrates that DG units can directly affect the network voltage through reactive power control.

3.3.5.3 Voltage Sensitivity to Reactive Power Injection

The sensitivity of bus voltage to reactive power injection can be expressed in terms of Jacobian matrix elements in power flow analysis. In this context, the sensitivity of voltage magnitude at bus i to reactive power injected at the same bus is defined as follows:

$$\frac{\partial V_i}{\partial Q_j} = J^{-1}_{VQ,ij} \tag{3.23}$$

These sensitivity coefficients in distribution networks differ significantly from those in transmission systems. Due to the high R/X ratio of distribution lines, the effect of reactive power injections on voltage is generally greater. However, this effect is not uniformly distributed throughout the system; it is more pronounced at buses closest to the reactive power injection. Particularly as feeder endpoints are approached, voltage variations exhibit a more sensitive structure due to the cumulative effect of line impedance.

These characteristics cause the Q–V relationship in distribution networks to possess a strong, local, and topology-sensitive character. Consequently, the reactive power control problem becomes a distributed problem that must be addressed on a feeder and node basis, rather than with a centralized approach as in transmission systems. The fundamental differences in the Q–V relationship between transmission and distribution networks are summarized in Table 3.2.

Table 3.2: *Comparison of Q-V Relationship Characteristics in Transmission and Distribution Networks*

Characteristic	**Transmission**	**Distribution**
Q–V Coupling	Strong and dominant	Strong but mixed with P
P–V Coupling	Weak (negligible)	Significant (high R/X)
Sensitivity Propagation	Wide area	Local
Control Structure	Centralized	Distributed
Reactive Power Transportability	Relatively high	Limited

3.3.5.4 Implications for DER-Based Voltage Control

The local character of the voltage–reactive power relationship in distribution networks has important implications for voltage control strategies based on distributed energy resources such as PV inverters. The effect of reactive power injection on voltage is strongly dependent not only on the magnitude of reactive power provided but also on the location where the injection occurs and the network topology.

In line with the relationship expressed in Equation 3.22, PV systems located closer to feeder endpoints can achieve greater voltage regulation than systems near the source bus. This situation indicates that reactive power services carry location-sensitive system value at the distribution level. Therefore, local effects must be accounted for in the pricing of these services.

However, uncoordinated reactive power injection by multiple PV inverters can lead to voltage oscillations and control instabilities [54]. Therefore, for effective voltage control in distribution networks, distributed control strategies or centralized optimization-based coordination approaches must be considered together.

Under high PV generation conditions, when active power injection exceeds load demand ($P_G > P_Y$), Equation (3.22) produces a positive ΔV and the voltage rise problem emerges. This voltage rise can be effectively limited through reactive power absorption ($Q_G < 0$). In the literature, it has been demonstrated that coordinated reactive power control approaches that consider the capacity limits of generators and inverters can significantly reduce voltage rise [55].

Within this framework, the reactive power capacity of distributed generation units emerges as a determining factor for the effectiveness of voltage control in distribution networks. While Equation 3.22 reveals the direct effect of the reactive power that PV inverters can provide on voltage behaviour, the dynamic P–Q capability curves of inverters limit the maximum amount of reactive power that can be provided at any operating moment. This topic is critical to determining the technical and economic limits of reactive power services.

3.4 Traditional Reactive Power Control Methods

The stable operation of power systems depends on maintaining not only active power balance (frequency control) but also reactive power balance (voltage control). The fundamental parameter determining the voltage profile in transmission and distribution networks is reactive power flow. Due to the predominantly inductive character of network impedance ($X >> R$), there is a strong sensitivity relationship between changes in voltage (dV) and changes in reactive power (dQ). This relationship reveals that voltage control is essentially a reactive power management problem [56].

In traditional network architecture, this management is performed in a "hierarchical" structure through centralized generation plants and passive or active compensation devices placed at critical nodes of the network. These methods are generally solutions that require high initial investment cost and are based on centralized planning [57].

Within this scope, the traditional reactive power control mechanisms commonly used in power systems and the technical and operational limitations of these methods are addressed in detail in the following subsections.

3.4.1 Synchronous Generators

Synchronous generators are the primary sources of reactive power in traditional power systems. A synchronous machine can provide reactive power to the grid (over-excitation – capacitive effect) or draw reactive power from the grid (under-excitation – inductive effect) by changing the excitation current. Equipped with Automatic Voltage Regulators (AVR), these machines provide continuous and dynamic voltage control to keep terminal voltage within specified limits [47].

However, the reactive power provision capabilities of synchronous generators are directly related to active power generation and are limited by the P–Q capability curve. This capability curve shown in Figure 3.11 is determined by three fundamental constraints that define the operating limits of the generator [58]. Under over-excitation conditions, the field current limit, representing thermal limits in the rotor windings, requires reduction of active power output during high reactive power generation ($Q > 0$). The armature current limit reflects the thermal capacity of stator windings. This limit imposes the condition $P^2 + Q^2 \leq S^2_{\text{rated}}$. Under low excitation conditions, the under-excitation limit becomes effective. This limit aims to prevent excessive heating and stability problems in stator end regions during reactive power absorption.

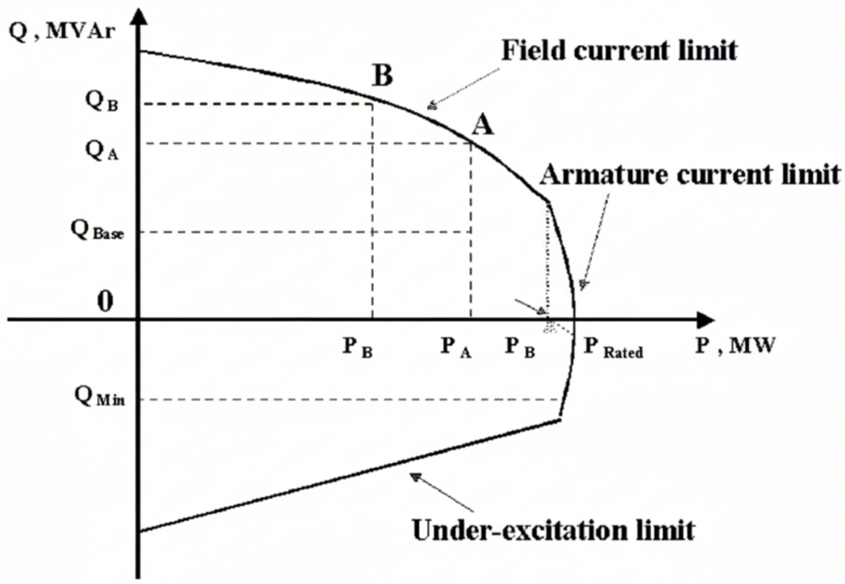

Figure 3.11: *Generator P–Q Capability Curve (Adapted from Bhattacharya & Zhong, 2002)*

This capability curve forms the basis for Expected Payment Function (EPF) calculations in reactive power markets. The Q_{Base} point represents the reactive power required for the generator's own needs, the Q_A point represents the maximum reactive power that can be generated without active power restriction, and the Q_B point represents the maximum reactive power achievable in exchange for active power reduction [59]. This structure clearly demonstrates that reactive power is not merely a technical service but also carries an economic opportunity cost element that may require forgoing active power.

3.4.2 Shunt Capacitors and Reactors

Mechanically switched shunt capacitors and reactors are the most commonly used passive compensation elements in distribution networks. Capacitors inject reactive power ($+Q$) into the grid to compensate for voltage drop caused by inductive loads and to correct power factor. Reactors absorb reactive power ($-Q$) to dampen overvoltages (Ferranti Effect) that occur during light loading conditions (nighttime hours).

These devices have long been preferred in distribution networks for their low investment costs and simple structures. However, the control capabilities of shunt capacitors and reactors may prove insufficient in meeting the dynamic operating requirements of modern distribution networks. The step-wise rather than continuous nature of their reactive power output limits precise and constant voltage regulation. Additionally, the switching of these elements through mechanical switching equipment causes relatively long response times. This situation makes it difficult to respond quickly to sudden load or generation changes.

In addition, high-frequency transient voltage oscillations (switching transients) can occur in the network during switching operations of shunt compensation elements. This situation can adversely affect power quality. These limitations do not change the fact that shunt capacitors and reactors are effective reactive power sources in traditional distribution networks. However, they cannot provide an adequate solution on their own in modern systems, where high penetration of distributed generation and rapid voltage changes are present.

3.4.3 On-Load Tap Changing Transformers (OLTC)

The fundamental voltage control mechanism in distribution networks is the On-Load Tap Changers (OLTC) integrated into power transformers. OLTC maintains the voltage on the secondary side within the desired band (e.g., 1.0 ± 0.05 p.u.) by adjusting the transformer's turn ratio. Similarly, Step Voltage Regulators are used on long distribution lines to compensate for voltage drop along the line [48].

Although OLTCs are effective in adjusting voltage level, they do not directly control reactive power flows and only shift voltage system-wide. Therefore, they offer a limited solution to problems arising from reactive power–voltage interaction. Additionally, due to their mechanical-switching structure, rapid voltage fluctuations in networks with high

PV penetration cause OLTCs to switch frequently (hunting effect), thereby shortening equipment life and increasing maintenance costs [60].

3.4.4 Flexible AC Transmission Systems (FACTS)

FACTS devices encompass power-electronics-based reactive-power control devices, such as SVC and STATCOM. These systems can respond on the order of milliseconds compared to mechanical compensation elements using semiconductor switches such as thyristors or IGBTs; thus providing highly effective solutions for improving voltage regulation and system stability [39].

However, the high installation and operating costs (CAPEX and OPEX) of FACTS devices limit their widespread adoption in distribution networks. Therefore, FACTS solutions are most commonly used in transmission systems and in special industrial applications with high and rapid reactive power demands, such as arc furnaces. At the distribution level, the use of these devices is generally not considered economically feasible.

3.4.5 Limitations of Traditional Methods in Distribution Networks

Traditional reactive power control methods have been effectively used for decades in power systems based on centralized generation. However, the rapid penetration of distributed energy resources, particularly PV systems, in distribution networks is making the structural limitations of these methods increasingly apparent. Traditional compensation devices have been designed for relatively slow-changing, predictable load profiles and struggle to adapt to the dynamic operating conditions of modern distribution networks.

Foremost among these limitations is temporal mismatch. PV generation can fluctuate by 50–80% on the order of seconds due to meteorological effects such as cloud passages [61, 62]. In contrast, typical response times of mechanically switched capacitor banks are 60–180 seconds, and response times of on-load tap changers are in the range of 20–60 seconds [63]. This pronounced timescale difference clearly demonstrates that traditional compensation equipment cannot respond in time to rapid PV fluctuations and significantly limits the effectiveness of voltage regulation in distribution networks with high PV penetration.

Another fundamental limitation is spatial mismatch. Traditional reactive power sources are generally located at transformer substations or centralized generation points; in contrast, voltage problems in modern distribution networks mostly occur at DER connection points, particularly at feeder ends and distributed throughout the network. Considering the local character of reactive power, centralized compensation approaches cannot effectively and efficiently solve these localized voltage problems. This situation demonstrates that distributed and location-sensitive reactive power sources offer a more effective solution in modern distribution networks [16].

High DER penetration also introduces another structural problem for traditional methods: bidirectional power flow. Distribution networks have historically been designed under the assumption of unidirectional power flow, yet reverse power flows occur during peak PV generation and low-load conditions. Under these operating conditions, the voltage rise problem becomes dominant over the voltage drop; furthermore, the overvoltage issue can be exacerbated when shunt capacitors are in service. This situation demonstrates that traditional compensation approaches have become incompatible with the fundamental operating assumptions of modern distribution networks.

In addition to these technical limitations, economic inefficiency also constrains the applicability of traditional approaches. Installing separate reactive power compensation equipment at every potential voltage problem point is not economically sustainable for distribution networks. In contrast, modern PV inverters already possess significant reactive power capacity that is unused mainly when operated at a unity power factor. As summarized in Table 3.3, PV inverters offer lower capital cost, much faster response time, distributed positioning advantage, and high control flexibility compared to traditional devices.

Table 3.3: *General Comparison of Traditional Compensation Systems*

Criterion	Traditional Devices	PV Inverters
Capital Cost	High (additional investment)	Low (existing capacity)
Response Speed	Slow–Medium	Very Fast
Spatial Distribution	Centralized	Distributed
Control Flexibility	Limited	High
Maintenance Cost	High (mechanical)	Low (electronic)
Scalability	Difficult	Easy

Within this framework, the temporal, spatial, and economic limitations of traditional reactive power control methods clearly reveal the need for a new approach in modern distribution networks. Effective utilization of the reactive power capacity of existing inverter-based sources can reduce capital-intensive infrastructure investments. At the same time, it offers flexible solutions that can adapt to local and dynamic system needs. However, for this potential to be realized, market-based coordination mechanisms that appropriately incentivize distributed reactive power services, namely reactive power markets, must be designed.

Given these technical limitations, the following chapter examines the potential for reactive power control offered by distributed generation sources, particularly PV inverters.

4. DISTRIBUTED GENERATION AND INVERTER TECHNOLOGIES

This chapter examines distributed generation technologies, particularly the reactive power capacity of PV inverters. First, the concept and classification of distributed generation are presented, followed by an explanation of the fundamentals of PV systems. Finally, inverter control modes and the concept of active power curtailment are addressed, establishing the technical foundations of the proposed market model.

In the traditional power system approach, electrical energy was generated at large-scale centralized power plants and delivered to end users through a unidirectional flow via transmission/distribution networks. In this structure, the distribution network was considered a passive endpoint where energy flow was only "consumed." However, over the past two decades, environmental concerns, decarbonization targets, and technological advances have placed the distribution of generation sources to points close to load centers, namely the concept of DG, at the center of the energy sector.

Distributed Generation is generally defined as "generation units that are not centrally planned, are usually connected to the distribution network, and have a capacity smaller than 50 MW–100 MW" [64]. The integration of these sources into the grid is transforming distribution systems from a passive structure to Active Distribution Networks, where power flow is bidirectional and local generators actively contribute to grid operation.

The increasing penetration of DG units, particularly inverter-based sources, creates technical challenges for grid operators such as protection coordination and voltage rise. However, the power-electronics interfaces of these devices exhibit strong reactive-power control capability.

Unlike traditional synchronous generators, inverter-based DG systems do not require a rotating mass or magnetic field windings to generate reactive power. Instead, they can digitally control the phase angle between current and voltage on the order of milliseconds using power electronics switching techniques. With the recognition of this potential, the IEEE 1547-2018 standard has mandated that DG units connected to the distribution network possess specific reactive power capabilities (e.g., constant power factor, Volt-Var control) to support grid voltage and stability [12].

In this chapter, the types of distributed generation sources, the fundamental operating principles of PV systems, and, in particular, the advanced reactive power control capabilities offered by inverter technologies are addressed. This technical infrastructure provides the physical and operational foundation for the reactive power market model proposed in this thesis.

4.1 Distributed Generation: Definition and Classification

DG refers to the production of electrical energy using relatively small-scale units located near consumption points or directly integrated into the distribution network, rather than centralized large-scale power plants. The IEEE 1547-2018 standard defines a distributed energy resource as “a power source capable of generating or storing electrical energy whose primary purpose is to provide energy to the electric power system in an area,” emphasizing that these units are active system components operating in interaction with the grid [12].

DG systems encompass a wide range of technologies including PV panels, wind turbines, biomass systems, micro turbines, and fuel cells. The main driving forces behind the proliferation of these systems today are increasing global energy demand, environmental sustainability targets aimed at reducing carbon emissions, and the need to enhance energy supply security [64, 65]. Indeed, the declining costs of renewable energy sources such as PV and wind are accelerating this transformation. Unlike the traditional “Unidirectional” (Generation $\rightarrow$ Transmission $\rightarrow$ Distribution) flow, DG units inject energy directly into the distribution network or at the prosumer point. This structural transformation and the position of different DG technologies in the grid hierarchy are illustrated in Figure 4.1.

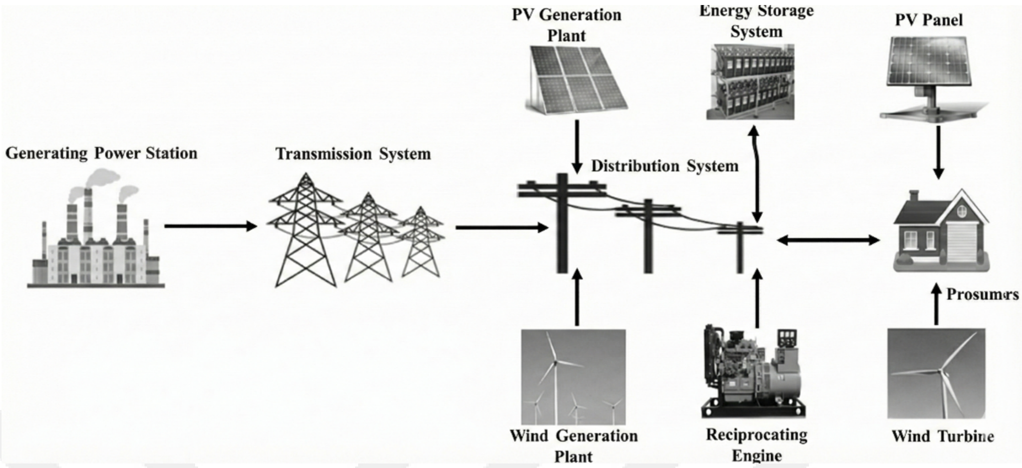

Figure 4.1: *Distributed generation sources*

The effects and control capabilities of distributed generation sources on the grid depend on the interface technology through which the source is connected, rather than on the primary energy source. In the power systems literature, DG units are classified into two main categories based on their grid-connection interfaces: Rotating-Machine-Based Sources and Inverter-Based Resources (IBR).

4.1.1 Rotating Machine-Based Resources

The first category of DG technologies comprises rotating-machine-based sources that use synchronous or asynchronous generators for electromechanical energy conversion. This category encompasses small-scale hydroelectric plants, biomass-fired power stations, geothermal facilities, combined heat and power (CHP) systems, gas turbine generators, reciprocating engine generators, and certain wind turbine technologies (Type 1 and Type 2 configurations).

The reactive power control capacity of rotating machines is limited by the response time and thermal limits of excitation systems. Synchronous generators can provide reactive power support through field current adjustment. However, response time is typically on the order of seconds, and the available reactive power range is limited by the generator capability curve, which is determined by armature and field current heating limits. These characteristics limit the voltage-support capacity of rotating-machine-based DG sources and reduce their flexibility in modern distribution networks with rapidly changing operating conditions.

4.1.2 Inverter-Based Resources

The second main category of distributed generation sources comprises Inverter-Based Resources (IBR) that connect to the grid via power-electronics-based converters. This category includes PV systems, modern variable-speed wind turbines (Type 3 doubly-fed induction generators and Type 4 full-converter configurations), battery energy storage systems, and fuel cells. The defining characteristic of IBR is its connection to the grid via voltage-sourced or current-sourced converters, rather than via directly connected rotating machines.

Unlike rotating machines, IBR provides zero or very low natural inertia to the power system since they lack rotating masses directly connected to the grid. This characteristic has raised concerns regarding frequency stability in systems with high IBR penetration [26]. However, this apparent disadvantage can be offset by the superior controllability and rapid response capabilities of power-electronics interfaces.

The fundamental advantage of IBR lies in their ability to independently and rapidly control active and reactive power output. Modern inverters can perform reactive power adjustments on millisecond time scales by digitally processing current and voltage waveforms; this offers faster control capability compared to traditional synchronous generators [66]. When this rapid-response capacity is combined with the ability to operate across a wide range of power factors, it makes IBR highly valuable for dynamic voltage support and reactive power services.

4.2 Fundamentals of Photovoltaic Systems

From the perspective of the proposed market mechanism, the most critical characteristic of PV systems is their stochastic, non-deterministic generation. The output power of a PV panel is not constant but exhibits a nonlinear character ($I - V$ Characteristic) depending on instantaneous solar irradiance and cell temperature. While the generated current is directly proportional to solar irradiance, increasing ambient temperature reduces the efficiency of crystalline silicon cells with a negative temperature coefficient (approximately $-0.4\%/°$C), thereby lowering the open-circuit voltage. In addition, shading effects, particularly under partial cloud cover, can cause mismatch losses across panel arrays, thereby affecting generation unpredictably [67]. For example, a 50% reduction in irradiance causes approximately 50% decrease in maximum power output. Rapid fluctuations in production can occur due to cloud passage, with 50–80% power variations possible within seconds to minutes. These environmental variables constitute the primary source of uncertainty in determining the capacity available in the reactive power market.

Despite these environmental variables, Maximum Power Point Tracking (MPPT) algorithms are used in inverters to enable the system to operate at maximum efficiency. MPPT optimizes the operating voltage of the system by continuously scanning for the point that yields the highest power on the P-V curve of the panel [68]. In the traditional operating approach in the literature, MPPT is treated merely as a tool for energy maximization. However, in the model proposed in this thesis, the MPPT point is redefined as a dynamic reference limit that determines the inverter's instantaneous maximum active power capacity. If the grid operator demands reactive power for voltage control and the inverter's apparent power capacity ($S_{\max}$) is limited, the operating point must be deviated from this MPPT reference and active power curtailment must be performed. This situation provides the technical basis for the "opportunity cost" component of the proposed market model.

Although PV systems can operate independently of the grid (off-grid), depending on their application areas, the primary focus of modern power systems and electricity markets is on grid-connected (On-Grid) systems. The high penetration of these systems into the distribution network introduces technical challenges, including bidirectional power flow, voltage rise at feeder ends, and reduced system inertia [69, 70]. These challenges necessitate the transformation of PV systems from passive sources that merely "inject energy" into manageable assets that actively support grid stability.

4.3 Inverter Technologies

In PV systems, the inverter is not merely an intermediate component that converts DC to grid-compatible AC, but rather a central power electronics unit that determines the system's performance, reliability, and interaction with the grid [71]. The efficiency and control capabilities of the inverter directly affect the specific energy yield (kWh/kWp) obtained from the PV system and consequently the Levelized Cost of Energy (LCOE) of the investment [72].

PV inverters are designed with different topologies according to their connection architectures to PV arrays, with a fundamental distinction between central- and string-inverter structures. Central inverters offer lower unit-cost advantages in large-scale solar power plants because their power capacities range from hundreds of kilowatts to megawatt levels [73]. However, having a single point of failure and being unable to manage string-based mismatch losses effectively constitute significant disadvantages [74]. String inverters, on the other hand, provide higher energy yield and operational flexibility, particularly in residential and commercial applications, through multiple MPPT structures that allow independent control of each PV string [75]. These two fundamental architectures are shown in Figure 4.2.

As the share of renewable energy sources in the grid has increased, the role of inverters has also transformed. These devices are no longer passive power converters but

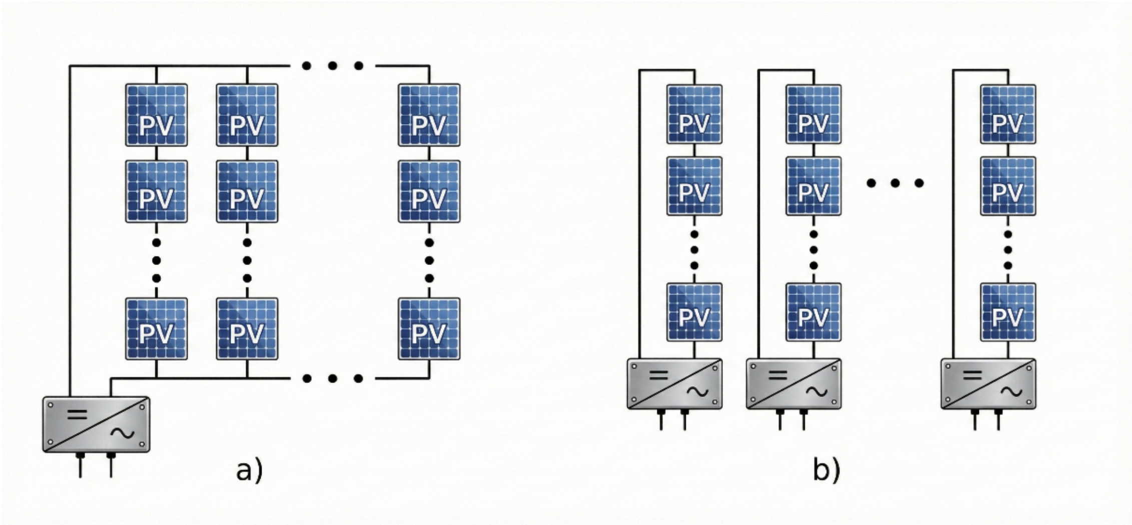

Figure 4.2: *PV inverter system topologies: a) Central inverter configuration, b) String inverter configuration*

smart grid assets that actively contribute to grid stability. These next-generation devices, referred to as "Smart Inverters," are equipped with advanced control algorithms that can communicate with grid operators and provide a range of Ancillary Grid Services. These functions are mandated by updated grid codes worldwide, including IEEE 1547-2018 and similar regulations [76, 12].

The main grid support functions offered by smart inverters are as follows: Volt–Var control provides reactive power injection or absorption in response to voltage variations at the point of connection. Frequency–Watt and Fast Frequency Response (FFR) mechanisms perform active power adjustment in response to frequency deviations. Fault Ride-Through (FRT) capabilities enable remaining connected to the grid during short-term voltage sags or swells [12, 66]. In addition, ramp-rate control helps limit the adverse effects of sudden changes in PV generation on the grid [76].

These advanced control functions transform inverter-based PV systems from merely energy-generating sources into strategic assets capable of providing fast, flexible, and location-sensitive reactive power services in distribution networks. These characteristics constitute the fundamental technical infrastructure that enables inverter participation in reactive power markets and makes it meaningful.

4.4 Reactive Power Capability of PV Inverters

The increasing integration of PV systems in distribution networks is changing the voltage and reactive power control approach traditionally provided by synchronous generators. Modern power-electronics-based inverters have evolved from merely DC/AC conversion interfaces to active grid assets that can dynamically generate or consume reactive power to regulate grid voltage and enhance stability [66, 77].

However, the reactive power control capability of a PV inverter is not unlimited. A dynamic Safe Operating Area (SOA) constrains this capability, determined by the inverter's hardware limits (current, voltage), environmental conditions (solar irradiance, temperature), and the selected control strategy (Active/Reactive Power Priority). Therefore, the inverter's instantaneous reactive power capability varies with both electrical and operating conditions.

Figure 4.3 shows the safe operation area of a PV inverter. As seen in the figure, while the maximum active power that PV panels can generate ($P_{\text{max -array}}$) varies depending on solar irradiance and temperature, the maximum active power that the inverter can deliver to the grid is limited by the inverter capacity ($P_{\text{max -inverter}}$) [78]. Inverter capacity is generally selected to exceed the maximum power that the PV module can generate. This design approach ensures that the inverter remains within the safe operation area under different irradiance and temperature conditions [78].

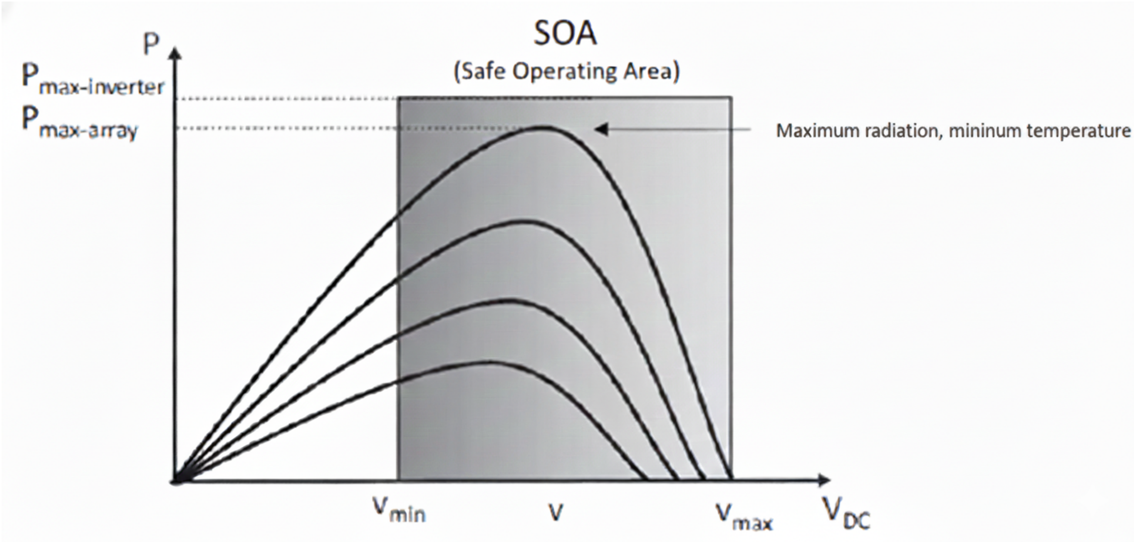

Figure 4.3: *Safe Operation Area of a PV Generator (SOA: Safe Operation Area; Pmax-inverter: Maximum Inverter Power; Pmax-array: Maximum PV Array Power) (Adapted from Cabrera-Tobar et al., 2019)*

Within the SOA, two fundamental limits determine the inverter's reactive power capacity: current and voltage limits. These limits define the mathematical relationship between the active and reactive power components of the inverter and determine the shape of the instantaneous P–Q capability curve. The analytical expression of these limits and the dynamic P–Q capabilities of the inverter are addressed in detail in the following subsection.

4.4.1 Dynamic P–Q Capability Curve

In the literature, the power capacity of a PV inverter is generally modeled as a static semicircle expressed by the equation $S_{\text{nom}}^2 = P^2 + Q^2$. However, under real-field conditions, the inverter's reactive power provision capability is not constant. As a result of the interaction between the inverter's hardware limits and environmental conditions (solar irradiance, temperature), active and reactive power limits are defined by a dynamic P–Q curve that varies with time.

Figure 4.4 shows the active and reactive power output curve of a PV inverter [79].

The P–Q capability curve presented in the figure is formed by the combined effect of three fundamental physical mechanisms.

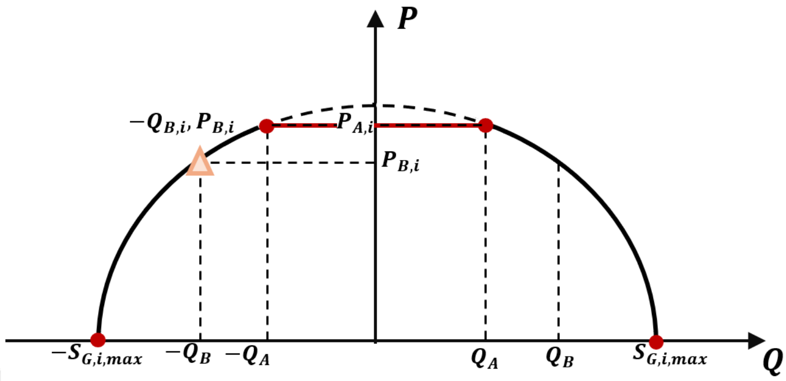

Figure 4.4: *Active and Reactive Power Output Curve of a PV Inverter*

First, the outer limit of the inverter's reactive power capacity is determined by the current limit. This limit, shown as a semicircle in Figure 4.4, corresponds to the thermal withstand capability of power electronics switching elements (IGBT/MOSFET). Since the maximum current ($I_{\max}$) that the inverter can deliver to the grid is constant, this also limits the maximum apparent power. The inverter cannot exceed this limit under any condition, and this constraint is mathematically expressed as:

$$P^2 + Q^2 \leq (V_g \cdot I_i)^2 \tag{4.1}$$

where V_g is the grid voltage and I_i is the inverter current [80].

Second, the inverter's instantaneous active power output is constrained by the photovoltaic DC power limit. The $P_{A,i}$ and $P_{B,i}$ values shown as vertical lines in Figure 4.4 represent the DC power obtainable from the PV array under different irradiance and temperature conditions. Under high solar irradiance, the active power reaches $P_{A,i}$; under low irradiance, it approaches $P_{B,i}$. This limit continuously changes depending on environmental conditions, independent of inverter hardware.

As a result of these two constraints, the inverter's dynamic reactive power capacity becomes directly dependent on instantaneous active power generation. As active power generation decreases, the available capacity remaining within the P–Q circle increases, and the inverter can provide more reactive power. This relationship allows the lower and upper limits of reactive power to be defined as a function of active power generation as follows:

$$\underline{Q}_t^2 = f(P_{\text{DER},t}) = -\overline{S_i^2} + (\mu\, P_{\text{DER},t})^2 \tag{4.2}$$

$$\overline{Q}_t^2 = f(P_{\text{DER},t}) = \overline{S_i^2} - (\mu\, P_{\text{DER},t})^2 \tag{4.3}$$

where $\overline{S_i}$ represents the apparent power capacity of the inverter and μ represents the scaling factor [79].

As shown in Figure 4.4, $P_{A,i}$ and $P_{B,i}$ represent active power levels under different irradiance conditions, while Q_A and Q_B represent the reactive power limits corresponding to these conditions. This situation demonstrates that the reactive power capacity of the inverter is not static but a stochastic and time-varying quantity.

In the market model proposed within the scope of this thesis, the instantaneous available reactive power capacity of the inverter (Q_{avail}) is defined as the "idle capacity" remaining within the apparent power capacity after active power generation:

$$Q_{\text{avail}} = \sqrt{S_{\text{max}}^2 - P_{\text{ac}}^2} \tag{4.4}$$

This expression indicates the maximum amount of reactive power that the inverter can provide without forgoing active power generation. If the grid operator demands reactive power above this value, the inverter is required to implement active power curtailment. This technical requirement forms the basis of the opportunity cost concept defined in the proposed market model.

4.4.2 Reactive Power Control Modes

Modern grid standards (IEEE 1547-2018, EN 50549-2) have defined various operating modes for inverters to adjust their reactive power output autonomously or remotely according to grid conditions. In the market model proposed by this thesis, which of these modes the inverters will operate in is determined by the DSO according to the current needs of the grid (local voltage issues or general reactive power balance).

4.4.2.1 Voltage–Reactive Power Control

This is the most effective and commonly used method for voltage regulation in distribution networks. In this mode, the inverter continuously monitors the grid voltage at the point of connection and dynamically adjusts the reactive power output according to a defined characteristic curve (droop curve). The Volt-Var Characteristic Curve based on the IEEE 1547-2018 standard is shown in Figure 4.5[12].

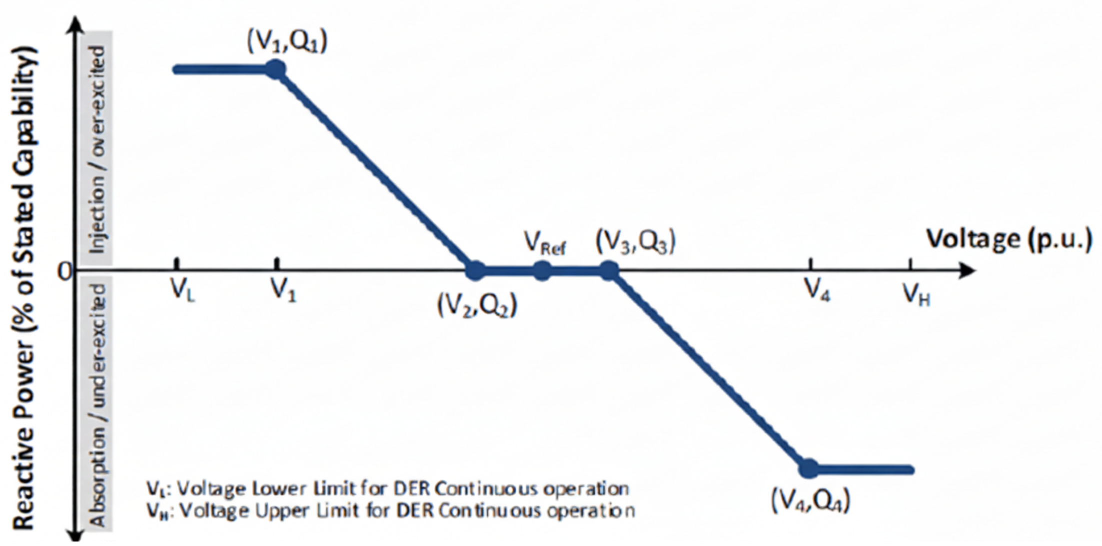

Figure 4.5: *A Volt-Var characteristic curve based on IEEE 1547-2018 standard*

This characteristic curve begins with a dead-band region defined around the nominal voltage; the inverter does not generate reactive power within this range. When the voltage drops below the lower limit, the inverter injects capacitive reactive power to raise the voltage, while when the upper limit is exceeded, it absorbs inductive reactive power to dampen the voltage rise. The slope of the curve determines the inverter's sensitivity to voltage changes and the aggressiveness of intervention. The most important advantage of Volt–Var mode is its ability to operate autonomously. It can respond to local voltage problems on the order of milliseconds without requiring a centralized communication infrastructure.

4.4.2.2 Constant Power Factor Mode

In this mode, the inverter adjusts the reactive power output proportionally to the current active power generation. The objective is to maintain a constant power factor (e.g., 0.95) at the plant's grid connection point. This mode is a "passive" control. For example, when active power decreases on a cloudy day, the inverter's reactive power output also decreases automatically. However, the grid may need more voltage support at that moment. Therefore, it is not as effective as Volt-Var mode in distribution feeders experiencing dynamic voltage issues.

4.4.2.3 Constant Reactive Power Mode

In this mode, the inverter generates a constant reactive power setpoint (Q_{ref}) sent by the grid operator (DSO) or plant controller, independent of grid voltage or active power output. The reactive power market model proposed in this thesis is structured based on the Constant Reactive Power Mode (Q_{set}).

4.4.3 Control Priorities and Active Power Curtailment

Under boundary conditions where a PV inverter's apparent power capacity (S_{nom}) reaches full utilization and the grid operator simultaneously demands additional reactive power ($P_{\text{mppt}}^2 + Q_{\text{req}}^2 > S_{\text{nom}}^2$), which power component the control system prioritizes is a critical economic decision that determines the plant's revenue stream, beyond being a technical preference. This decision mechanism is referred to in the literature as "Control Priority."

The market model proposed in this thesis presents the transition from the current "Active Power Priority" structure to a "Reactive Power Priority" structure that is responsive to grid needs.

4.4.3.1 Active Power Priority Control (Active Power Priority / P-Priority)

Active power priority control is the most common and default operating mode for PV plants. In this approach, the inverter's primary objective is to maximize revenue from energy sales by maximizing active power generation (P) under current environmental conditions. Reactive power support is provided only within the residual capacity remaining within the capability curve, in a manner that does not affect active power generation.

Therefore, while the inverter can meet the required Q_{set} during hours of low solar irradiance, reactive power support remains limited when active power reaches its maximum, and the inverter approaches its capacity limit. This approach prevents revenue loss from energy generation. However, it restricts the reactive power service that can be offered to the grid. In markets where reactive power support is not mandatory or economically incentivized, it is a feasible method for plants to provide limited contribution without experiencing active power loss.

In this mode, the inverter allocates the entire nominal capacity (S_{nom}) primarily to active power generation; reactive power is provided only through the headroom remaining after active power. This operating behavior is illustrated in Figure 4.6, which shows situations in which the inverter continues to operate at the maximum power point (P_{MPP}) and reactive power is limited to the region remaining within the P–Q capability curve. The maximum amount of reactive power that can be provided is expressed as:

$$Q_{\max} = \sqrt{S_{\text{nom}}^2 - P_{\text{mppt}}^2} \tag{4.5}$$

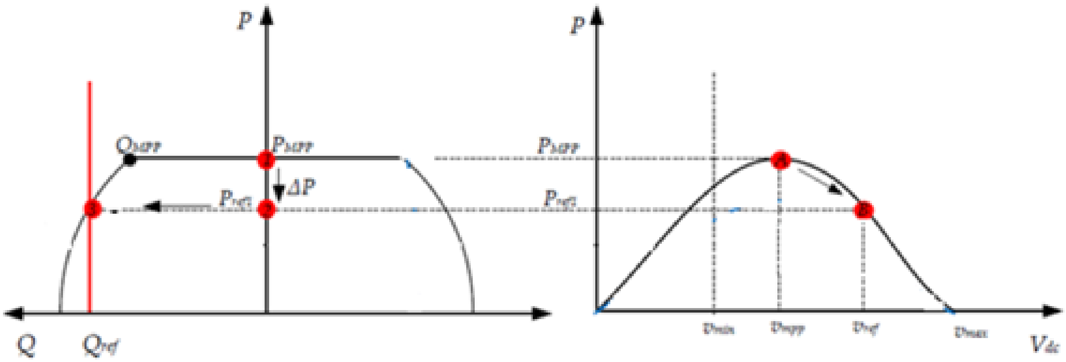

Figure 4.6: *Reactive power control of a PV inverter operating at PMPP Point*

4.4.3.2 Reactive Power Priority Control (Reactive Power Priority / Q-Priority)

In reactive power priority control, the inverter's primary task is to meet the grid operator-determined reactive power reference (Q_{ref}) under all operating conditions. The operation of this control philosophy is schematically shown in Figure 4.6. When additional reactive power is required beyond its capacity, an inverter operating at the maximum power point must shift its operating point to meet the demand.

This process occurs by shifting the inverter's DC operating voltage to the V_{ref} value corresponding to the maximum power point. The inevitable consequence of this is a reduction (ΔP) in active power generation. In other words, the inverter foregoes active power generation until it provides the demanded (Q_{ref}) value by prioritizing grid stability.

Reactive power priority control may be technically mandatory in situations where serious voltage problems occur or grid security is critical. However, this approach directly reduces the energy production and consequently the financial revenues of the PV plant. Therefore, the economic compensation of revenue loss caused by active power curtailment constitutes the fundamental motivation of the proposed reactive power market model.

4.4.4 *Opportunity Cost Concept*

The implementation of reactive power priority mode leads to a mandatory change in the production profile of the PV plant. The difference between the maximum power the plant could potentially generate (P_{mppt}) and the actual production curtailed for grid stability ($P_{\text{curtailed}}$) represents the energy that cannot be delivered to the system:

$$\Delta P_{\text{loss}} = P_{\text{mppt}} - P_{\text{curtailed}} \tag{4.6}$$

This energy loss is a direct revenue loss for the plant owner. In the literature and in this thesis, this loss is defined as the alternative gain foregone to provide reactive power service, namely “Opportunity Cost.”

The fundamental objective of the proposed market mechanism is to compensate for this opportunity cost incurred by PV plants providing the reactive power support needed by the distribution network in a transparent, measurable, and fair manner. This approach aims to incentivize the provision of reactive power services through voluntary participation while ensuring economic balance between active power generation and grid support services.

Following the presentation of the technical capabilities of PV inverters and the concept of opportunity cost, the following chapter examines the approaches in the literature regarding how these services can be priced through market mechanisms.

5. REACTIVE POWER MARKET DESIGN APPROACHES

Reactive power markets exhibit different characteristics compared to traditional energy markets, and this situation requires specialized market designs. This chapter examines the fundamental principles of reactive power market design, cost components, and settlement methods; subsequently addressing the different market models proposed in the literature in a systematic manner.

5.1 Principles and Economics of Reactive Market Design

Reactive power markets exhibit significantly different characteristics from active power markets. Unlike active power, reactive power cannot be transmitted over long distances, which creates a distinct local characteristic [81]. This locality results in reactive power having different values depending on the location of its injection into the grid. As with active power, reactive power flows must also be continuously balanced; however, the management of this constraint is more difficult due to its local nature.

An important characteristic of reactive power markets is that the local grid operator (DSO or TSO) is generally the sole buyer of reactive power as an ancillary service. This situation inherently gives reactive power markets a monopsonistic structure [14]. As another consequence of locality, only a limited number of providers can be considered for some reactive power demands in the system, and this situation can create market power for these providers.

Unlike active power, reactive power generation is relatively inexpensive because it does not require fuel [14]. When sufficient competition is ensured, market prices can approach zero. However, in most cases, reactive power setpoints cannot be freely determined. This is because reactive capacities are closely linked to the active power feed of the units. This interaction can be defined by P–Q curves. Since active power curtailment may be required to provide reactive power, market analysis becomes complex.

This situation leads to opportunity costs and affects power flows in the system.

Therefore, it is not possible for the grid operator to simply accept the cheapest bids; the grid model and current system conditions must be taken into account [58]. Additionally, the limited number of supply providers and tight coupling with the physical energy system can cause dramatic fluctuations and sudden spikes in reactive power prices. All these characteristics—locality, monopsony, few providers, low generation costs, connections with active power, and fluctuating prices—complicate market design for reactive power.

5.1.1 Cost Components (Loss Cost, Switching Cost, Opportunity Cost)

The market price of reactive power depends not only on technical requirements but also on the cost components incurred during the provision of this service. Costs related to reactive power supply play a fundamental role in determining the market bids of supply providers. According to the classification proposed by Lamont and Fu, costs related to reactive power supply are addressed under three main components: loss cost, switching (degradation) cost, and lost opportunity cost [82].

Cost of Loss arises from the occurrence of additional active power losses in the system as a result of reactive power provision or absorption. Reactive power support causes additional losses both in the field windings of synchronous generators and in power electronics elements of inverter-based sources. The reactive power provided or absorbed by a generator increases the real power loss in field windings, and this loss emerges as a nonlinear function of reactive power [24]. Although this cost component is generally smaller than the system's total losses, it should not be neglected in precise market designs.

Switching Cost emerges particularly in reactive power sources containing mechanical switching elements. Capacitor banks and some inverter configurations use switches with limited mechanical life. Each switching operation consumes some of the equipment's lifespan. In the long term, this situation leads to maintenance or replacement costs. If the market mechanism does not take this degradation effect into account, the willingness of supply providers to offer reactive power services may decrease.

Lost Opportunity Cost is one of the most critical cost components of reactive power markets. In both synchronous generators and inverter-based sources, reactive power capacity is constrained by apparent power limits and in most cases is directly related to active power generation. To provide more reactive power, active power output may need to be reduced. This situation causes a decrease in revenue that could be obtained from active power sales and consequently the emergence of opportunity cost. This cost component is of determining importance particularly for sources where active and reactive power components share the same hardware capacity, such as PV inverters, since in these sources the P–Q balance is directly defined by the inverter capability curve.

5.1.2 Expected Payment Function (EPF)

Generators and inverter-based sources providing reactive power services incur different types of costs depending on operating conditions. While some of these costs can be directly measured, the disaggregation and quantification of some components, particularly opportunity cost, is quite difficult. In liberalized market structures, a financial mechanism that is compatible with technical realities and economically rational is needed to cover these costs. The EPF provides a mathematical framework in which these costs incurred by reactive power service providers are expressed in terms of expected payment [24].

The EPF approach represents a transition from traditional cost-based compensation mechanisms to a more market-oriented structure. While reactive power costs were mostly modeled with fixed and exogenously defined cost functions in previous studies, in a free market environment, potential reactive power providers are treated as actors who autonomously determine their bids—and consequently their expected payments. Nevertheless, the EPF structure is not an entirely abstract pricing mechanism but is based on the physical and economic costs underlying reactive power generation. In the literature, EPF is generally modeled as a quadratic function [24].

A typical EPF consists of three fundamental components. **Capacity (availability) payment** is a fixed component that incentivizes a source to keep a certain reactive power capacity ready for the market. **Utilization payment** is the payment component that varies depending on the amount of reactive power actually provided and corresponds to the use of the service. **Opportunity cost payment** aims to compensate for the revenue loss incurred by the provider when active power generation must be curtailed to provide reactive power. The joint consideration of these three components enables the creation of a fair and transparent payment mechanism. This mechanism encompasses both the technical constraints and economic consequences of reactive power service.

5.1.3 Settlement Methods (Uniform, Nodal, Pay-as-Bid)

The settlement price in reactive power markets can be determined using marginal pricing or pay-as-bid mechanisms. From the perspective of game-theoretic justifications and economic efficiency, marginal pricing methods are generally preferred, as in active energy markets [83]. However, due to the distinct local character of reactive power, the application of marginal pricing requires different spatial arrangements depending on the level of detail of the grid model.

In the **uniform pricing** approach, a single clearing price is determined in the market without considering network constraints and locational differences [84]. All accepted bids are settled at the highest accepted bid. While ease of implementation and low computational requirements are the main advantages of this method, its inability to reflect the location-sensitive value of reactive power constitutes a significant disadvantage.

In cases where the network model is explicitly considered, the **nodal pricing** approach can be used. In this method, a separate price is determined for each node in the network, and the local value of reactive power is directly reflected in price signals [85]. At the distribution level, this approach is addressed within the framework of Distribution Locational Marginal Price (DLMP), providing holistic pricing that includes components such as congestion, voltage constraints, and losses. While nodal pricing is the method that best ensures economic efficiency, it poses challenges in implementation due to high data requirements and computational complexity.

Between these two extreme approaches, **zonal pricing** is considered a practical intermediate solution [86]. The network is divided into voltage or control zones, and a single price is determined for each zone. In this way, locational differences are partially reflected while computational complexity is also limited. The creation of localized reactive power markets by defining voltage control areas in distribution networks is possible within this framework.

As an alternative to marginal pricing, in the **pay-as-bid settlement** method, each supply provider receives payment at the bid price they submitted [87]. This approach can reduce the risk of market power and sudden price spikes by preventing the formation of a single marginal price. However, it requires careful design due to its potential to induce strategic bidding behavior among participants and reduce economic efficiency.

In summary, the selection of settlement method in reactive power markets depends on factors such as the level of detail of the network model, computational capacity, market power concerns, and local voltage control needs. The local and physically constrained nature of reactive power makes the direct application of uniform settlement approaches commonly used in active energy markets unsuitable in most cases.

5.2 Reactive Power Market Models

Due to advances in technology, regulatory changes, and the evolving dynamics of global electricity markets, different reactive power market designs have been proposed in the literature. In this section, the main market models proposed for determining and managing the cost of reactive power are systematically examined. The approaches in the literature can be grouped into four main categories based on their operational scope and the problems they address.

5.2.1 Cost-Based Purchase Agreement Model

In the cost-based purchase agreement model, compensation for newly connected components is provided through a cost-based incentive mechanism. This compensation is based on the amount of reactive power capacity that these components offer to the system controller to provide the required services [88, 89].

Using this model, sources are incentivized to be allocated to increase available reactive power capacity at nodes that can significantly affect the voltage of neighboring buses [90]. However, the total amount of incentive provided may be limited; in this case, there may not be sufficient reactive power capacity for the system controller to manage expected operational scenarios [91].

Among the main advantages of this model are ease of implementation and transparency. Since the cost structure is predetermined, uncertainty for participants is reduced. However, due to the lack of market signals, economic efficiency may remain limited and sufficient incentive may not be provided for optimal positioning of reactive power sources.

5.2.2 *Capacity-Based Auction Model*

In the capacity-based auction model, the DSO and pre-selected supply providers enter into long-term contracts, typically through annual auctions, to create a stable ancillary services market for reactive power [92]. These contracts can be extended to cover multiple years to provide stronger investment signals to market participants and encourage sustainable infrastructure development.

The annual VAR capacity auction proposed by Frias et al. treats reactive power generation and consumption capacities as separate market products and is optimized to maximize system security [19]. The annual time horizon provides a sufficient signal for new reactive power capacity investments or capacity upgrades of existing sources.

However, capacity-based markets may be inadequate in reflecting rapid changes in short-term operating conditions and offer limited flexibility for real-time voltage problems.

5.2.3 *Localized Market Model*

The localized market model is a design approach that directly takes into account the local characteristic of reactive power. In this model, the power system is divided into voltage control areas and a separate competitive market is created for each area [86]. The concept of electrical distance is used to define different voltage control areas within the power system.

This structure proposed by Zhong et al. reflects the location-dependent value of reactive power by creating separate uniform prices for each voltage control area [23]. The differential payment mechanism aims to fairly compensate service providers according to their positions in the system.

While the localized market approach is advantageous in terms of regionally limiting the effects of price manipulation and reducing the payment burden, it requires the correct definition of control areas and detailed analysis of the network model.

5.2.4 OPF-Based and Two-Level Market Models

OPF-based market models are commonly used to optimize reactive power supply [93]. In these approaches, reactive power providers submit their cost functions to the system operator, and the operator obtains an AC-OPF solution that minimizes total cost under security and operating constraints.

Dandachi et al. jointly optimized reactive power utilization costs and switching operations using security-constrained OPF [93]. However, these early studies assume integrated systems where all reactive power sources are centrally controlled.

Two-level market structures have been proposed to reduce high volatility in marginal prices [22]. In this framework, the long-term reactive capacity market provides investment and availability signals, while the short-term reactive energy market manages real-time operation. The two-stage structure developed by El-Samahy et al. combines long-term contracts and short-term fixed-price dispatch mechanisms [94].

In contrast, it has also been emphasized in the literature that long-term contracts may limit competition and reduce market flexibility [95]. These studies reveal that reactive power market design should be adapted to the operating level and system structure rather than adopted as a one-size-fits-all solution.

After evaluating the strengths and weaknesses of these approaches in the literature, the following chapter proposes an original reactive power market model that accounts for the specific characteristics of distribution networks.

6. PROPOSED REACTIVE POWER MARKET MODEL

Reactive power markets exhibit different physical and economic characteristics compared to active energy markets, thus necessitating their own unique market design approaches. While a significant portion of studies in the literature address reactive power directly within the framework of the voltage regulation problem, this thesis adopts a different perspective. The proposed model addresses voltage regulation indirectly, taking as the market driving force the power factor requirements applied at the HV/MV grid connection point and the associated penalty mechanisms.

The framework presented in this chapter first explains how the proposed market model will operate in practice, explicitly defines the roles of the DSO and PV inverter owners, and presents the market processes step by step from procurement to settlement. Considering the local nature of reactive power and the limited number of supply providers at the distribution level, a pricing mechanism that is compatible with local market constraints while encouraging competitive behavior has been designed. To ensure sustainable participation, a fair payment structure is proposed that explicitly includes opportunity costs arising from active power curtailment and enables continuous participation of supply providers without incurring financial losses.

The proposed market model is structured based on the Constant Reactive Power Mode (Q_{set}). This choice provides the following fundamental advantages:

1. The Q_{set} value determined as a result of market clearing can be directly applied as an inverter control reference.

2. Unlike $Q(V)$ or active power-dependent $Q(P)$ control modes, the deterministic nature of reactive power output significantly simplifies settlement and payment calculations.

3. It provides a control structure that is naturally compatible with the operational

requirements of the DSO, which holds central planning and coordination responsibility.

The market design developed within this framework enables reactive power to be treated as a sustainable and market-based ancillary service by integrating the technical capabilities of inverter-based reactive power sources at the distribution level with economic incentives.

6.1 Conceptual Framework

The conceptual framework of the proposed reactive power market is structured around one buyer, multiple supply providers, and a market operation layer, as illustrated in Figure 6.1. Within this framework, the DSO can assume the role of both system operator (demand side) and market operator depending on the organization of the market; however, institutional separation of these two functions is also possible. The supply side consists of small and medium-scale PV inverters connected to the distribution network.

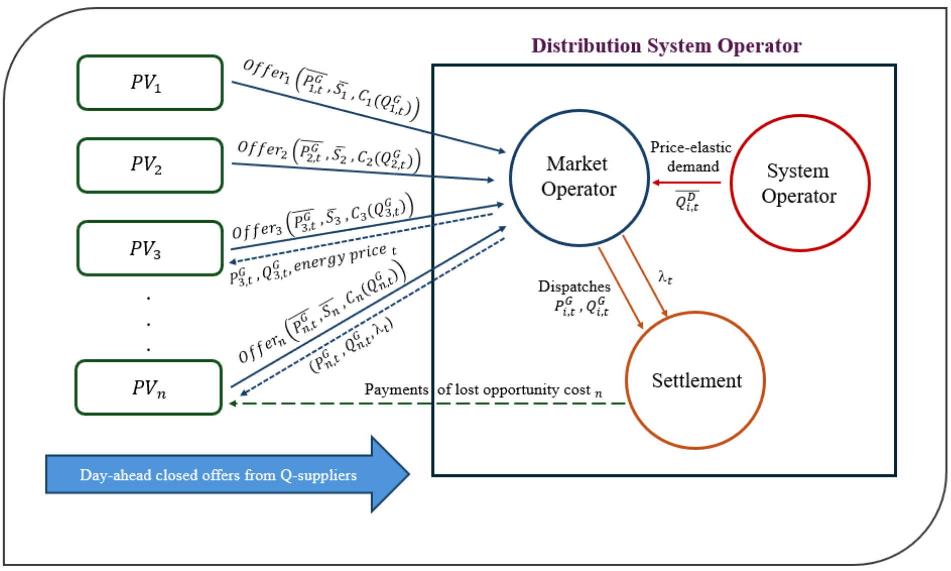

Figure 6.1: *Information exchange among the market parties*

The proposed market has an hourly settlement period and is cleared day-ahead for the subsequent 24-hour time horizon. This structure offers a practical balance between the predictability level of PV generation and the operating requirements in the distribution network.

PV plants submit their active power generation forecasts, inverter apparent power capacities, and cost (price) functions related to reactive power generation to the DSO for each time period. The DSO, in turn, defines price-elastic reactive power demand in accordance with network operating conditions and power factor limits determined by the Transmission System Operator (TSO), and executes the market clearing.

Figure 6.1 illustrates the information and bid flow among market participants, while Figure 6.2 schematically presents the time-sequential operational steps of the market.

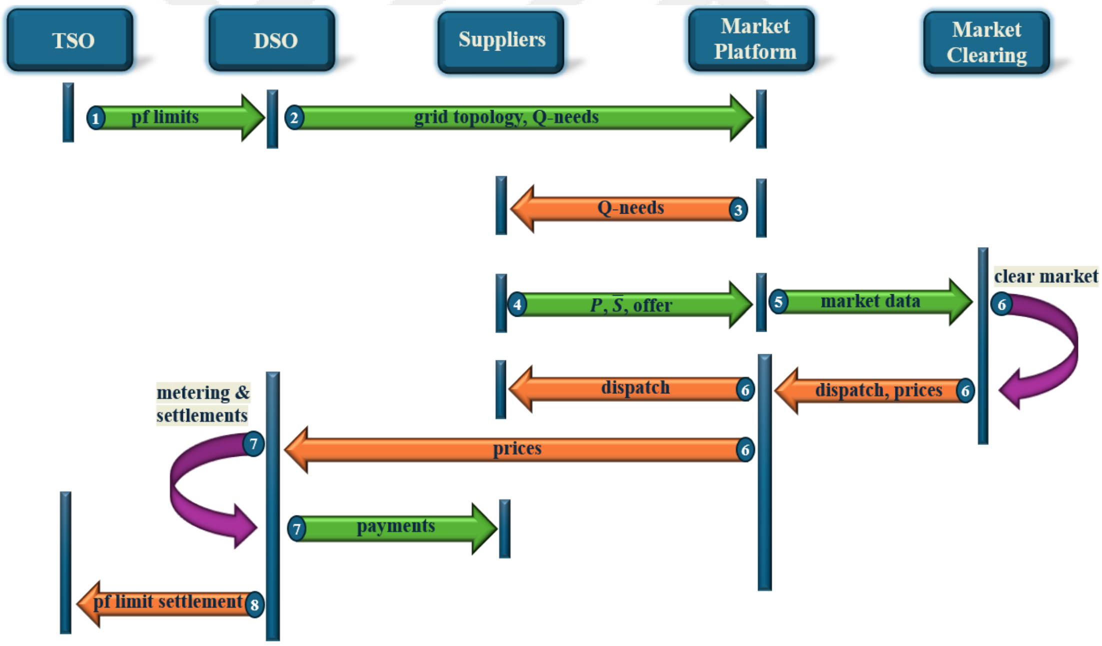

Figure 6.2: *Operational steps of the market*

1. The TSO notifies the DSO of the power factor (PF) limits applicable for the HV/MV connection point.

2. The DSO determines the price-elastic reactive power demand ($Q_{D_i,t}$) for each time period in accordance with network topology, forecasted nodal active power demands ($P_{D_i,t}$), and operating constraints, and transmits it to the market platform.

3. The market platform announces the hourly reactive power requirements defined by the DSO to supply providers.

4. Supply providers (PV inverters) submit their sealed bids. These bids include: active power generation forecast ($P_{G_i,t}$), inverter apparent power capacity (S_i), and cost (bid) function for reactive power generation ($C_i(Q_{G_i,t})$).

5. All collected market data are transferred to the optimization module that executes the market clearing.

6. Supply-demand matching is performed and nodal reactive power clearing prices are determined along with active and reactive power dispatches for each supply provider.

7. The DSO verifies actual active and reactive power generation through real-time measurements and calculates settlement payments.

8. In the monthly settlement phase, whether the total Q/P ratio at the HV/MV connection point remains within defined inductive and capacitive limits is verified jointly with the TSO, and penalty or balancing payments are applied if applicable.

This conceptual structure enables reactive power to be operated as a measurable, priceable, and sustainable ancillary service by integrating the technical reactive power capabilities of PV inverters with local distribution-level needs and market-based incentive mechanisms.

6.2 Mathematical Formulation

This section presents the mathematical formulation of the proposed reactive power market model. The model is a day-ahead, deterministic, AC-OPF-based market-clearing problem that aims to economically efficiently procure reactive power from PV inverters connected to the distribution network under the technical operating constraints of the DSO. The temporal structure is defined with an hourly solution horizon for the subsequent 24-hour period, similar to active power day-ahead markets.

6.2.1 Objective Function

The objective function of the market-clearing problem aims to minimize total net social cost from the DSO perspective. This net cost is defined as the difference between the costs incurred corresponding to the reactive power provided by reactive power supply providers and the benefit obtained by the DSO through preventing power factor violations and ensuring network operating security:

$$\min \sum_{t} \left\{ \sum_{i \in \Omega_G} C_i(Q_{G_i,t}) - B\left(\sum_{i \in \Omega_G} Q_{G_i,t} \right) \right\} \tag{6.1}$$

where $C_i(Q_{G_i,t})$ represents the cost or bid function related to reactive power generation of the i-th supply provider (PV inverter), and $B(\cdot)$ represents the reactive power benefit function obtained by the DSO through maintaining power factor limits at the HV/MV connection point and preventing associated penalties.

This formulation addresses reactive power procurement not merely as a cost minimization problem but also within the framework of economic benefit–cost balance.

6.2.2 Nodal Power Balance Constraints

The following nodal balance equations ensure active and reactive power balance at each system node:

$$P_{G_i,t} = P_{D_i,t} + \sum_j P_{ij,t} - \sum_j P_{ji,t} + \sum_j LP_{ji,t} \quad \forall i \in N,\ t \in T \tag{6.2}$$

$$Q_{G_i,t} = Q_{D_i,t} + \sum_j Q_{ij,t} - \sum_j Q_{ji,t} + \sum_j LQ_{ji,t} \quad \forall i \in N,\ t \in T \tag{6.3}$$

These constraints ensure active- and reactive-power balance by matching net injection and withdrawal at each node.

6.2.3 Network Model and Voltage Constraints

The voltage–power relationship in the distribution network is modeled using DistFlow equations to represent high R/X ratios accurately:

$$(V_{j,t})^2 = (V_{i,t})^2 - 2(r_{ij}P_{ij,t} + x_{ij}Q_{ij,t}) + (r_{ij}^2 + x_{ij}^2) \cdot \frac{(P_{ij,t})^2 + (Q_{ij,t})^2}{(V_{i,t})^2} \tag{6.4}$$

The voltage at node i is calculated as a function of power flow in the line and line impedance. This expression is derived from DistFlow equations, which provide a nonlinear yet accurate representation of voltage drops by considering both resistive and reactive components in distribution lines.

Active and reactive power losses on the line are defined respectively as:

$$LP_{ij,t} = r_{ij} \cdot \frac{(P_{ij,t})^2 + (Q_{ij,t})^2}{(V_{i,t})^2} \tag{6.5}$$

$$LQ_{ij,t} = x_{ij} \cdot \frac{(P_{ij,t})^2 + (Q_{ij,t})^2}{(V_{i,t})^2} \tag{6.6}$$

To ensure network operating security, node voltages are maintained within the following limits:

$$\underline{V}_i \leq V_{i,t} \leq \overline{V}_i \quad \forall i \in N,\ t \in T \tag{6.7}$$

6.2.4 Line Flow Limits

The thermal limits of lines are modeled with apparent power constraints:

$$(P_{ij,t})^2 + (Q_{ij,t})^2 \leq (\overline{S}_{ij})^2 \quad \forall(i,j) \in L,\ t \in T \tag{6.8}$$

This constraint guarantees safe operation by preventing overloading of distribution lines.

6.2.5 Inverter Capability and Sustainability Constraints

The active and reactive power generation of PV inverters is limited by the inverter apparent power capacity:

$$(P_{G_i,t})^2 + (Q_{G_i,t})^2 \leq (\overline{S}_{G_i})^2 \quad \forall i \in \Omega_G,\ t \in T \tag{6.9}$$

In addition, an original sustainability constraint is defined within the scope of this thesis:

$$\mu \cdot \overline{P}_{G_i,t} \leq P_{G_i,t} \leq \overline{P}_{G_i,t} \quad \forall i \in \Omega_G,\ t \in T \tag{6.10}$$

where the parameter $\mu \in (0,1)$ is defined as the sustainability threshold. This constraint aims to prevent excessive or strategic curtailment of active power generation for the purpose of providing reactive power. Thus, PV generators are prevented from completely disabling their active power generation solely to earn revenue from the reactive power market, ensuring that energy production remains at the center of system operation.

6.3 Model Characteristics

The proposed market-clearing model possesses the following characteristics:

- AC DistFlow-based, nonlinear, and non-convex structure,

- Day-ahead, hourly time resolution,
- P–Q capability constraints specific to PV inverters,
- Power factor-based market driving force,
- EPF-compatible cost structure incorporating opportunity costs.

This mathematical framework ensures effective and sustainable procurement of reactive power by integrating the technical capabilities of inverter-based reactive power sources at the distribution level with market-based incentive mechanisms.

6.4 Market Operation Rules

In the proposed market, supply providers submit sealed bids to the market platform. These bids consist of three fundamental components:

1. Active power generation forecast for the relevant settlement period,
2. Nominal apparent power capacity of the inverter,
3. Reactive power price function defined based on the amount of reactive power to be provided.

The active power generation forecast reflects the expected PV generation of the supply provider for the relevant settlement period. The apparent power capacity represents the nominal capacity of the inverter and determines the balance between active and reactive power generation. The price function defines the fee demanded by the supply provider based on the amount of reactive power.

6.4.1 Pricing Mechanism

The market settlement price for reactive power can be determined using marginal pricing or pay-as-bid methods. As commonly accepted in energy markets, the marginal pricing approach has been adopted in this study due to game-theoretic justifications and transparency advantages. Depending on whether network constraints are incorporated into the model, marginal pricing can be applied in three different spatial structures:

1. Uniform pricing,
2. Zonal pricing,
3. Nodal pricing.

When network constraints are neglected, a single settlement price is formed for all market participants. However, when real network topology and physical constraints are considered, a different reactive power value emerges for each node, and this situation necessitates nodal pricing. Alternatively, zonal pricing can also be applied by dividing the network into voltage control areas.

In this study, nodal marginal pricing has been preferred as it most accurately reflects the local character of reactive power. Supply providers are compensated on a nodal basis through DLMP. The DLMP structure offers a fair and physically consistent pricing mechanism for radial medium-voltage distribution networks by explicitly incorporating congestion, voltage, and loss components.

6.5 Payment Mechanism

The proposed reactive power market framework is structured around the EPF, which systematically encompasses different cost components associated with reactive power provision. The EPF is designed to reflect the technical and economic costs incurred by supply providers while offering reactive power services, and consists of three fundamental components: capacity payment, utilization payment, and opportunity cost payment.

$$\text{EPF} = \text{Capacity Payment} + \text{Utilization Payment} + \text{Opportunity Cost Payment} \quad (6.11)$$

Figure 6.3 schematically illustrates how the EPF is shaped depending on the amount of reactive power support. As can be observed, the EPF possesses a symmetric structure and is applied with the same principles for both capacitive and inductive reactive power support.

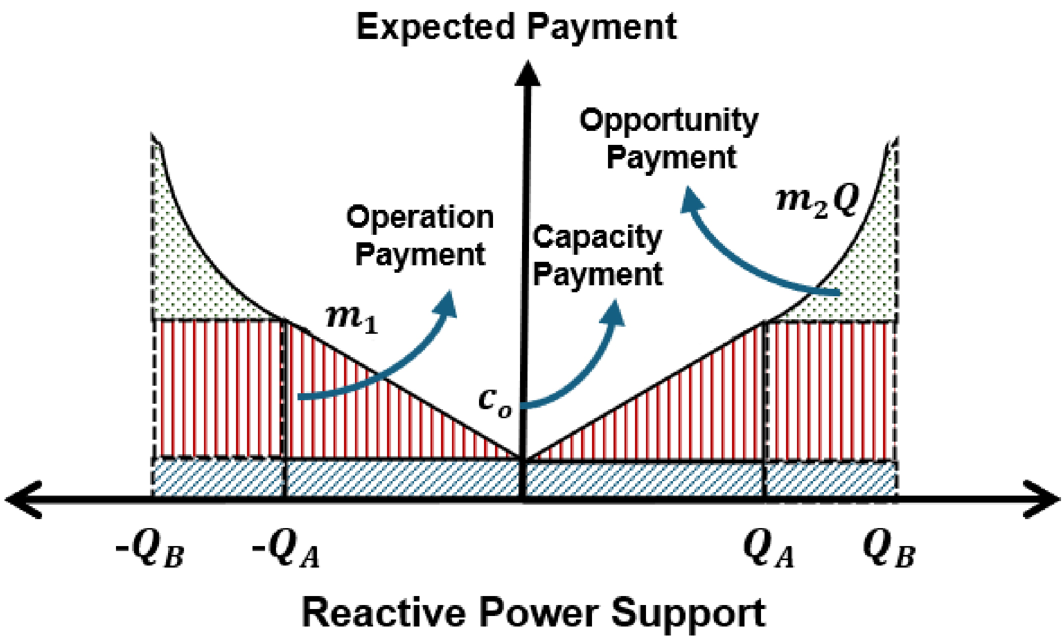

Figure 6.3: *Structure of the Expected Payment Function of the Q-suppliers*

6.5.1 Capacity Payment

Capacity payment (c_0) is a fixed payment defined to incentivize the participation of supply providers in the market. This payment is based on the supply provider being technically capable of providing reactive power and committing to the defined maximum reactive power capacity Q_B. The capacity payment is valid throughout the entire $[-Q_B, +Q_A]$ range regardless of actual reactive power dispatch.

The fundamental purpose of intentionally keeping this payment low is solely to guarantee market access and prevent participants who do not dispatch from obtaining unfair gains. Thus, the market is structured to reward supply providers who deliver actual operational contribution.

6.5.2 Utilization Payment

In the region where reactive power output is $|Q| \leq Q_A$, inverters can provide reactive power without forgoing active power generation. The costs incurred in this region predominantly arise from increased system losses and additional operating loads on power electronics components.

Therefore, in the $|Q| \leq Q_A$ range, the EPF includes a linear utilization payment in addition to the capacity payment. The utilization payment is expressed as the product of the m_1 coefficient, which represents the marginal reactive power price, and the amount of reactive power provided. This structure offers a linear payment mechanism that incentivizes small and medium-level reactive power support.

6.5.3 Opportunity Cost Payment

When reactive power output exceeds the Q_A value ($Q_A < |Q| \leq Q_B$), reduction of active power generation becomes mandatory due to the apparent power capacity limitations of the inverter. In this case, the PV plant foregoes active power that it could sell in the energy market and incurs direct revenue loss. This loss is defined as opportunity cost in the literature and within the scope of this thesis.

In this region, the EPF includes an opportunity cost payment that aims to compensate for the revenue loss arising from active power curtailment, in addition to capacity and utilization payments. In Figure 6.3, this situation is represented by a second linear region

(m_2) with increasing slope after the Q_A point. Selecting this slope such that $m_2 > m_1$ ensures that reactive power support requiring active power forgoing is compensated at a higher rate.

6.5.4 Sustainability Threshold and EPF Relationship

The sustainability threshold (μ) introduced in this study is defined as an external technical constraint that prevents excessive and systematic curtailment of active power generation for the purpose of providing reactive power. This threshold guarantees that supply providers cannot reduce their active power generation below a certain proportion (μ) of the forecasted value while providing reactive power support.

In this context, the EPF is applied to be valid within the feasible operating region defined by the sustainability threshold. The Q_B point shown in Figure 6.3 reflects not only the apparent power capacity of the inverter but also the maximum active power curtailment permitted by the sustainability threshold. In other words, the EPF is not an unlimited mechanism that incentivizes the complete sacrifice of active power generation; rather, it is an economic incentive tool constrained to preserve the environmental and systemic sustainability of renewable generation.

6.5.5 Tolerance Band Mechanism

The reference according to which opportunity cost is paid is of critical importance for the sustainability of the market. In this study, a mechanism based on a $\pm 10\%$ tolerance band is proposed to address the drawbacks of approaches based solely on forecast or solely on actual production. As long as the actual active power generation of the supply provider remains within $\pm 10\%$ of the forecasted value, the opportunity payment is calculated based on actual production. If this band is exceeded, the payment is constrained by the limit around the forecast value.

This mechanism both prevents market manipulation and supports the long-term functioning of the market by incentivizing forecast accuracy.

6.5.6 Payment Mechanism Summary

In conclusion, the proposed payment mechanism addresses:

1. The local and technical nature of reactive power,
2. The dynamic P–Q capability limits of PV inverters, and
3. Active power curtailment constrained by the sustainability threshold

within a single holistic framework. This definition of the EPF forms the foundation of a reactive power market structure that is fair and predictable for supply providers, and economically and environmentally sustainable for the DSO.

7. CONCLUSIONS

The rapid expansion of renewable energy capacity globally is forcing traditional power grids to undergo a structural transformation. In particular, the integration of PV systems into distribution networks has accelerated the transition from passive grid structures to active distribution networks. As increasing PV generation meets local consumption, the active power drawn from the HV/MV connection point decreases. However, the reactive power demand of network loads continues, causing the Q/P ratio at the HV/MV point to increase and the power factor to deteriorate. In many countries, including Türkiye, exceeding regulatory power factor limits results in significant financial penalties for DSOs.

Distribution networks, unlike transmission systems, have higher R/X ratios. This characteristic increases the impact of reactive power flow on the voltage profile while limiting the effectiveness of traditional compensation methods. Modern PV inverters are not limited to active power generation alone; they can function as flexible reactive power sources capable of responding within milliseconds owing to their dynamic P–Q capability curves. International standards recognize this capacity. However, when active power must be curtailed to provide reactive power support, the PV owner's lost energy sales revenue constitutes an opportunity cost. In the existing literature, there is no market design at the distribution level that explicitly incorporates this opportunity cost and is specific to PV inverters.

The fundamental cost components in reactive power market design consist of loss cost, switching cost, and opportunity cost. The EPF integrates these components within a single framework. Settlement methods include uniform, nodal, or pay-as-bid mechanisms. A market structure where the DSO is positioned as the sole buyer (monopsonist) offers the opportunity to match the network's reactive power requirements with the idle capacity of PV inverters. In such a market, DLMP can be applied to reflect the high loss rates and voltage constraints of distribution networks. The payment structure can include capacity payment, utilization payment, and opportunity-cost compensation components.

The proposed market model is structured based on the Constant Reactive Power Mode (Q_{set}). The DSO assumes the role of both system operator and market operator, while PV inverters are positioned as supply providers. In the market operation, the TSO first communicates the applicable power factor limits for the HV/MV connection point to the DSO. PV plants then submit their bids by conveying active power generation forecasts, inverter capacities, and reactive power cost functions. The DSO executes the market clearing accordingly. The market-clearing problem is formulated as an OPF with a 24-hour time horizon, a day-ahead planning horizon, and hourly settlement periods. The model includes voltage limits, line flow limits, and P–Q capability constraints specific to PV inverters.

The sustainability threshold (μ) is one of the original elements of the model. It is defined to prevent excessive and systematic curtailment of active power generation for providing reactive power services. This threshold guarantees that supply providers do not reduce their active power generation below a certain proportion of the forecasted value. Thus, renewable energy generation remains prioritized, and environmental sustainability is preserved. The reference for opportunity-cost payments is also critical to market sustainability. To address the disadvantages of approaches based solely on forecast or actual production values, a $\pm 10\%$ tolerance band mechanism is proposed. As long as actual active power generation remains within this band, the opportunity cost payment is calculated based on actual production. When the band is exceeded, the payment is constrained by the forecast value limit. This structure prevents market manipulation and incentivizes forecast accuracy.

EPF addresses the local and technical nature of reactive power within a single holistic framework. It incorporates the dynamic P–Q capability limits of PV inverters and active power curtailment constrained by the sustainability threshold. The EPF consists of three components: a capacity payment that guarantees market access but is intentionally kept low, a utilization payment that increases linearly in the region where active power curtailment is not required, and an opportunity cost payment compensated at a higher slope where curtailment becomes mandatory. This payment structure rewards supply providers who deliver actual operational contribution while preventing non-dispatching participants from obtaining unfair gains.

Although the proposed model offers a technically feasible and economically consistent structure, an independent local reactive power market mechanism does not yet exist in many countries. The EPF definition forms the foundation of a market structure that is fair and predictable for supply providers and economically sustainable for the DSO. However, realizing this structure requires evolution of the existing regulatory framework. In current regulations, financial penalties imposed on DSOs are generally based on inductive and capacitive power factor limits at network connection points. These limits are defined as fixed values in most countries and do not account for locational differences.

The first policy recommendation is a spatial aggregation approach involving the assignment of different reactive power limits by the TSO on a transformer basis. The limits for each MV/HV transformer substation can be differentiated by considering local network characteristics and reactive power sensitivities. Meanwhile, the overall Q/P ratio at the DSO level is maintained to ensure system-wide power quality and operational security. This arrangement will enable more supply providers to participate in the market by accurately reflecting the local value of reactive power. Increased participation will enhance price flexibility and contribute to competitive outcomes in local markets. Through spatial differentiation, economic signals will become stronger at nodes where reactive power is genuinely needed.

The second policy recommendation envisions seasonal adjustment of reactive power limits. PV generation and load profiles exhibit distinct seasonal variations throughout the year. Reactive power requirements display different characteristics in summer and winter months. Current fixed limits do not reflect this variability; they prove unnecessarily restrictive in some periods while remaining inadequate in others. Seasonally adjusted limits will increase market effectiveness by accurately reflecting varying reactive power demand. This approach will reduce the risk of penalties for DSOs and create a more predictable market environment for supply providers.

The proposed spatial aggregation and seasonal adjustment approaches can be implemented gradually in a manner compatible with existing regulations. These approaches enable the evolution of reactive power markets through existing penalty and limit mechanisms without requiring radical regulatory changes. A practical and low-risk transition pathway is thus offered for regulatory authorities and DSOs toward local reactive power markets. These policy recommendations contribute to a regulatory framework that accounts for the local and time-dependent nature of reactive power, promotes competition, and aligns the technical potential of PV inverters with economic signals. Such regulations will enable reactive power at the distribution level to be treated not merely as a technical necessity but also as a manageable market service.

Despite this potential, some significant limitations must be acknowledged. The proposed market model has not been tested through numerical simulations or case studies using real network data. The model is presented as a conceptual and theoretical framework; validation of its practical applicability is left to future studies. Future research should first validate the proposed model through simulations on IEEE test systems (e.g., IEEE 33-bus or IEEE 69-bus distribution systems) or real distribution network data. These simulations should evaluate model performance under different PV penetration scenarios, load profiles, and network topologies.

In conclusion, this thesis establishes a conceptual foundation demonstrating that reactive power at the distribution level can be treated not only as a technical necessity but also as a manageable and economic ancillary service. The Q/P ratio deterioration caused by high PV penetration at the HV/MV connection point can be addressed more effectively through market-based mechanisms rather than traditional compensation methods. The proposed framework reduces penalty risk forDSOs while creating a new revenue stream for PV owners, generating value for both parties. This conceptual model provides the theoretical infrastructure for market-based reactive power management in networks with high PV penetration and constitutes a starting point for future empirical studies.

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APPENDIX — NOMENCLATURE

This appendix provides the nomenclature used in the mathematical formulation of the proposed reactive power market model presented in Section 6.

Sets

Symbol	Description
N	Set of network nodes
L	Set of distribution lines
Ω_G	Set of PV generators (supply providers)
T	Set of time periods (hours)

Variables

Symbol	Description	Unit
$P_{G_i,t}$	Active power generation at node i, time t	MW
$Q_{G_i,t}$	Reactive power generation at node i, time t	MVAr
$P_{ij,t}$	Active power flow on line connecting nodes i and j, time t	MW
$Q_{ij,t}$	Reactive power flow on line connecting nodes i and j, time t	MVAr
$V_{i,t}$	Voltage magnitude at node i, time t	p.u.
$LP_{ij,t}$	Active power loss on line i–j, time t	MW
$LQ_{ij,t}$	Reactive power loss on line i–j, time t	MVAr

Parameters

Symbol	Description	Unit
$P_{D_i,t}$	Active power demand at node i, time t	MW
$Q_{D_i,t}$	Reactive power demand at node i, time t	MVAr
r_{ij}	Resistance of line connecting nodes i and j	Ω
x_{ij}	Reactance of line connecting nodes i and j	Ω
$\overline{V}_i$	Maximum voltage magnitude at node i	p.u.
$\underline{V}_i$	Minimum voltage magnitude at node i	p.u.
$\overline{S}_{ij}$	Maximum apparent power flow limit on line i–j	MVA
S_{G_i}	Nominal apparent power capacity of inverter at node i	MVA
$\overline{P}_{G_i,t}$	Forecasted active power generation at node i, time t	MW
μ	Sustainability threshold	–

Functions

Symbol	Description	Unit
$C_i(Q_{G_i,t})$	Cost/bid function for reactive power generation of supply provider i	\$/MVAr
$B(\cdot)$	Benefit function obtained by DSO through power factor compliance	\$

Indices

Symbol	Description
i, j	Node indices
t	Time period index (hour)
(i, j)	Line index connecting nodes i and j

Acronyms Used in Formulation

Acronym	Description
OPF	Optimal Power Flow
EPF	Expected Payment Function
DSO	Distribution System Operator
TSO	Transmission System Operator
HV/MV	High Voltage / Medium Voltage
p.u.	Per unit